Computer Algebra
Independent Integration Tests

Summer 2024

8-Special-functions/354-8.5

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**136**]. This is test number [354].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (136)	0.00 (0)
Rubi	99.26 (135)	0.74 (1)
Maple	76.47 (104)	23.53 (32)
Reduce	41.18 (56)	58.82 (80)
Sympy	37.50 (51)	62.50 (85)
Fricas	25.00 (34)	75.00 (102)
Mupad	25.00 (34)	75.00 (102)
Giac	25.00 (34)	75.00 (102)
Maxima	25.00 (34)	75.00 (102)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

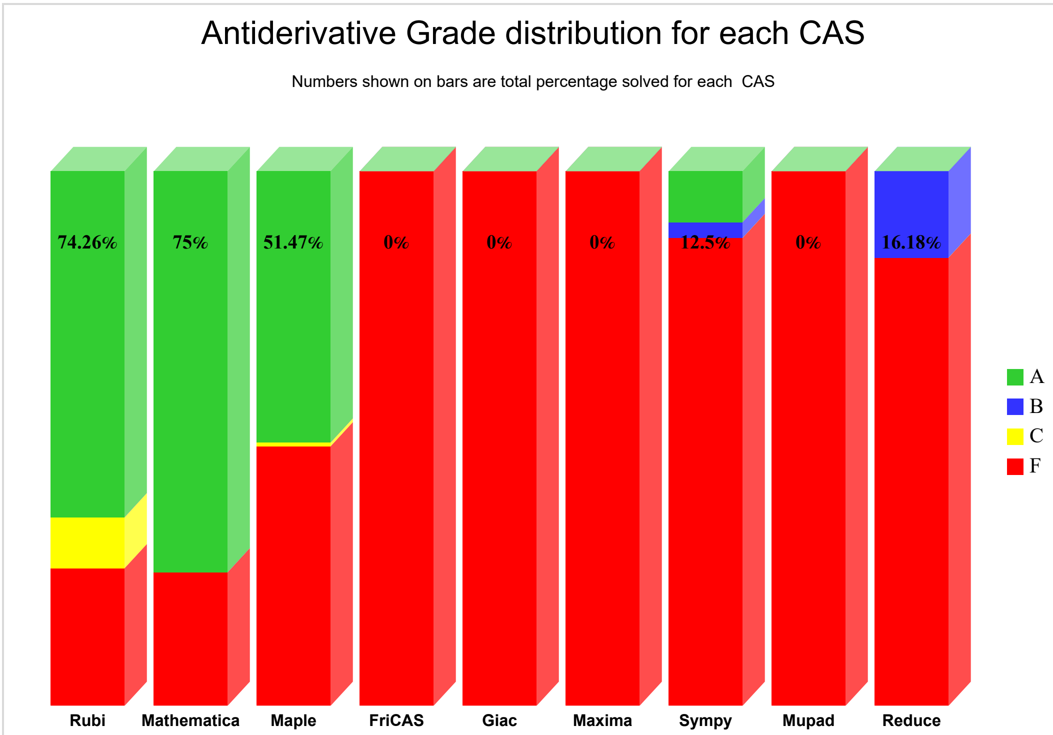
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

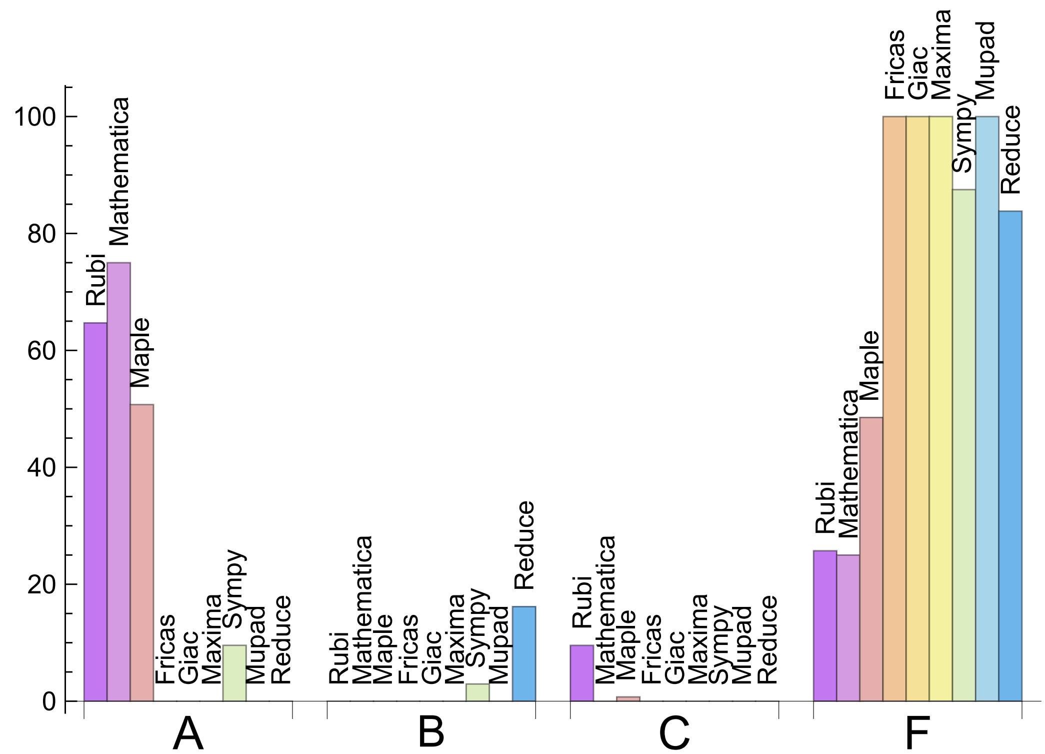
System	% A grade	% B grade	% C grade	% F grade
Mathematica	75.000	0.000	0.000	25.000
Rubi	64.706	0.000	9.559	25.735
Maple	50.735	0.000	0.735	48.529
Sympy	9.559	2.941	0.000	87.500
Fricas	0.000	0.000	0.000	100.000
Giac	0.000	0.000	0.000	100.000
Mupad	0.000	0.000	0.000	100.000
Maxima	0.000	0.000	0.000	100.000
Reduce	0.000	16.176	0.000	83.824

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	1	100.00	0.00	0.00
Maple	32	100.00	0.00	0.00
Reduce	80	100.00	0.00	0.00
Sympy	85	98.82	1.18	0.00
Fricas	102	100.00	0.00	0.00
Mupad	102	0.00	100.00	0.00
Giac	102	100.00	0.00	0.00
Maxima	102	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.08
Maxima	0.10
Giac	0.11
Reduce	0.21
Mathematica	0.53
Maple	0.61
Rubi	0.64
Sympy	1.14
Mupad	4.13

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Fricas	14.24	1.17	14.00	1.17
Mupad	14.24	1.17	14.00	1.17
Giac	14.24	1.17	14.00	1.17
Maxima	14.24	1.17	14.00	1.17
Reduce	29.93	1.12	16.00	1.17
Sympy	34.53	1.18	14.00	1.00
Maple	47.62	0.91	30.00	0.92
Mathematica	63.37	0.94	45.00	0.95
Rubi	92.61	1.12	56.00	1.05

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

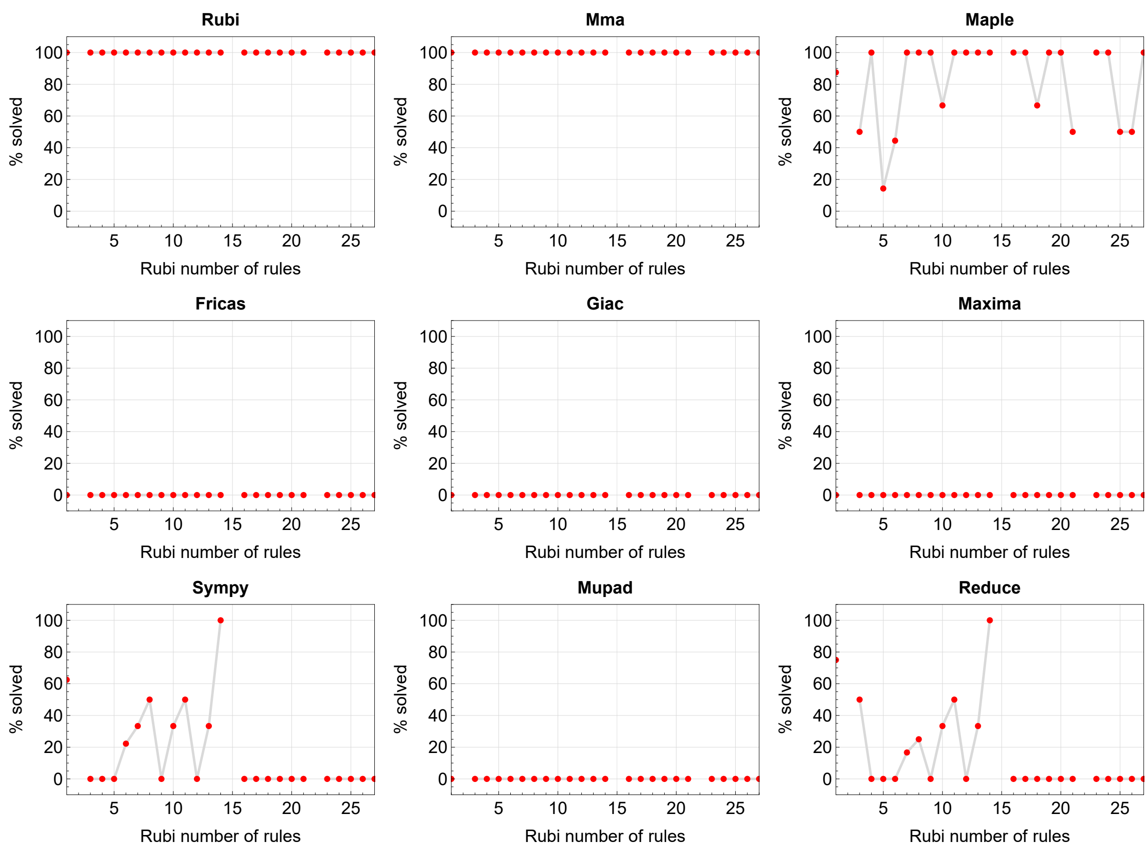


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

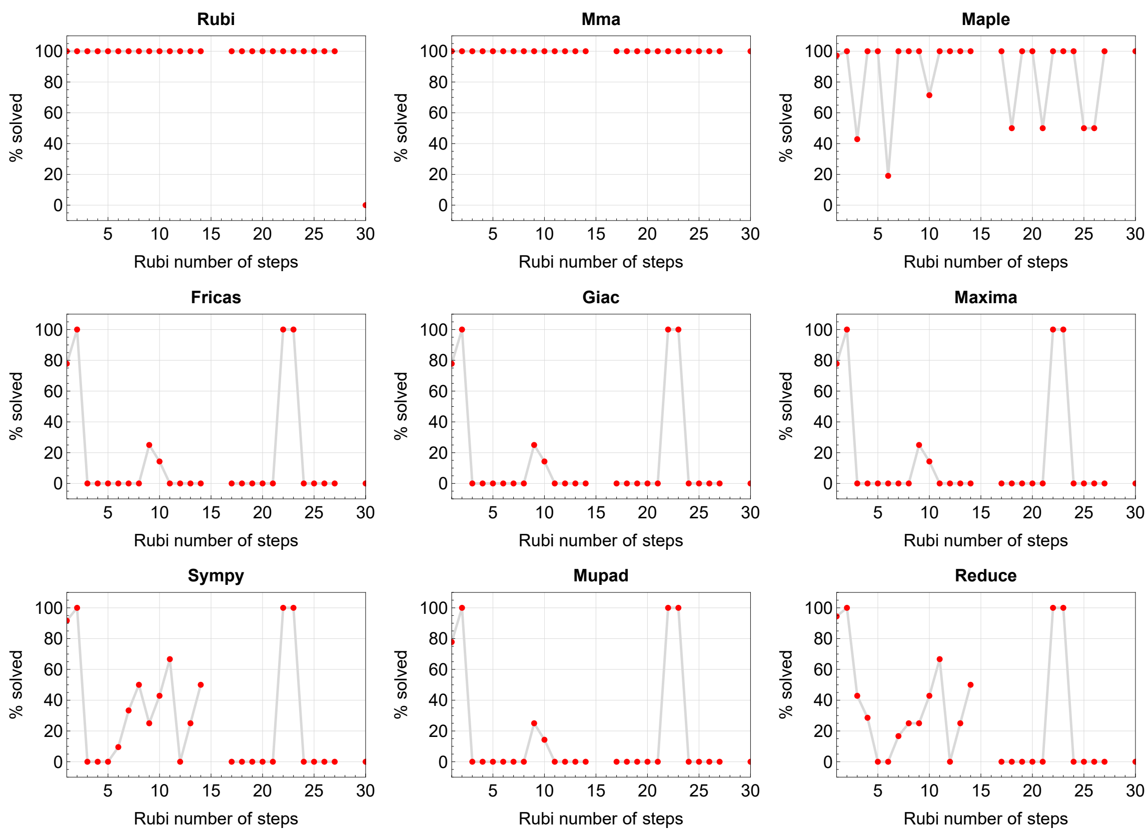


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

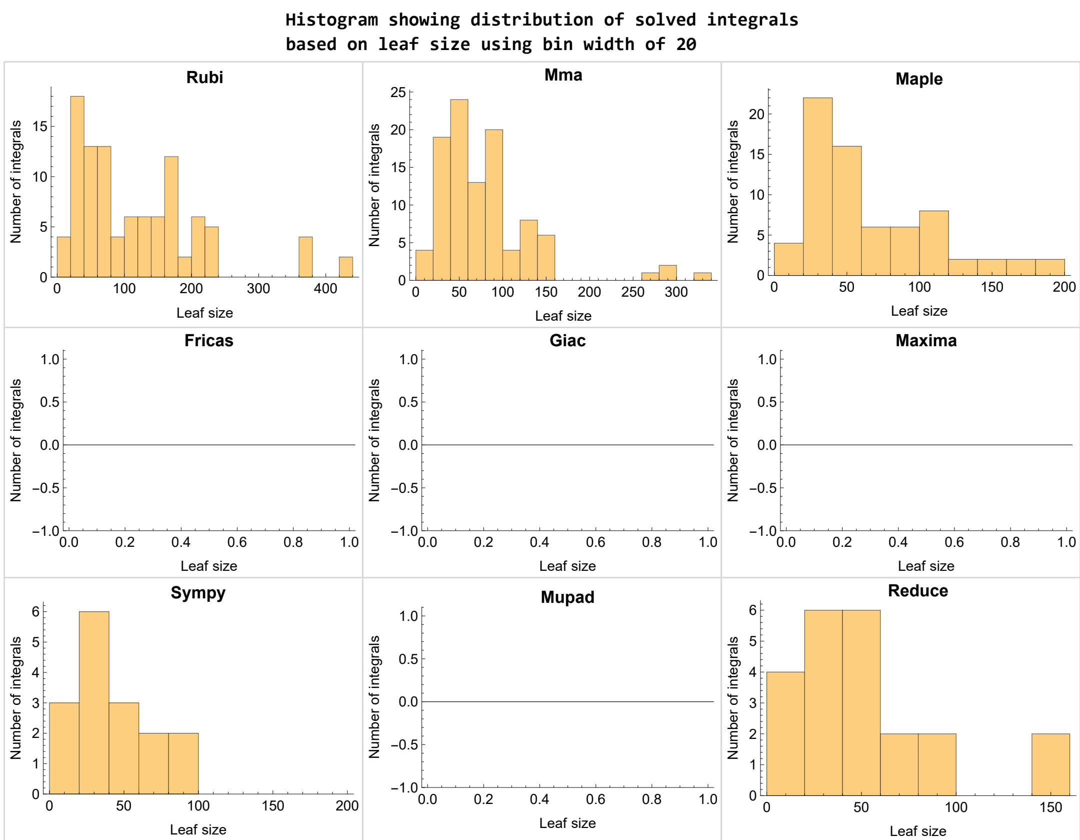


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

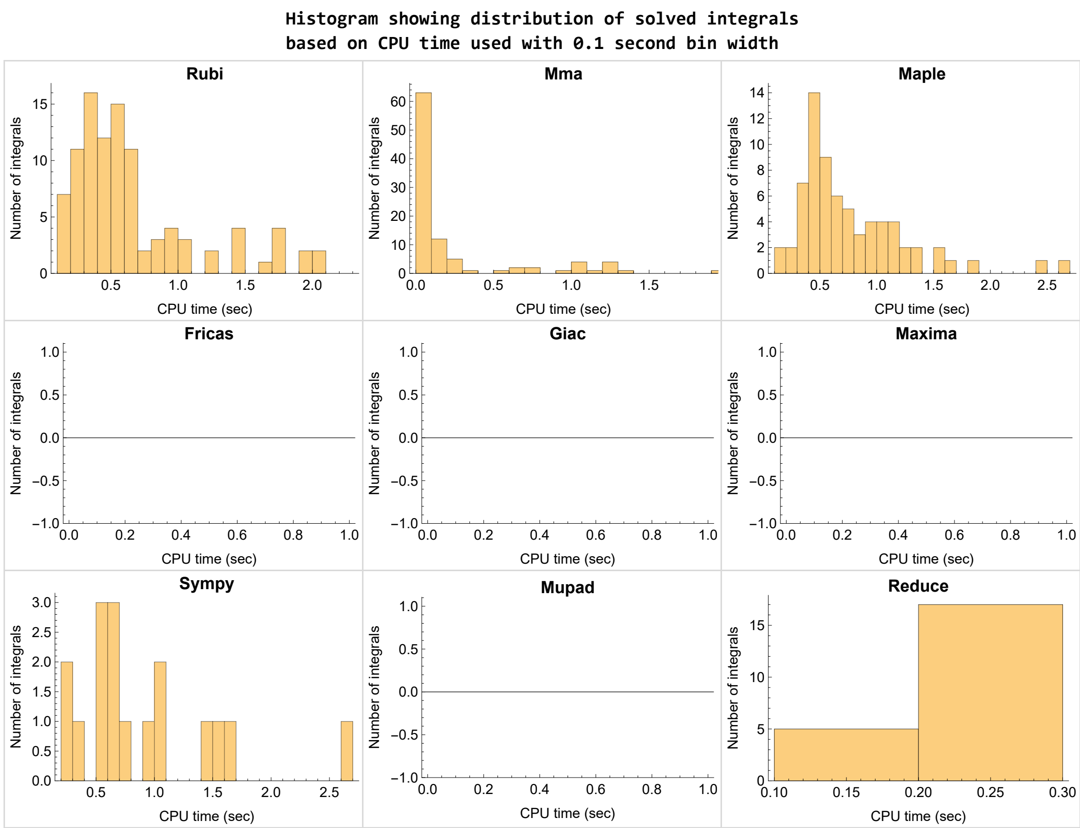


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

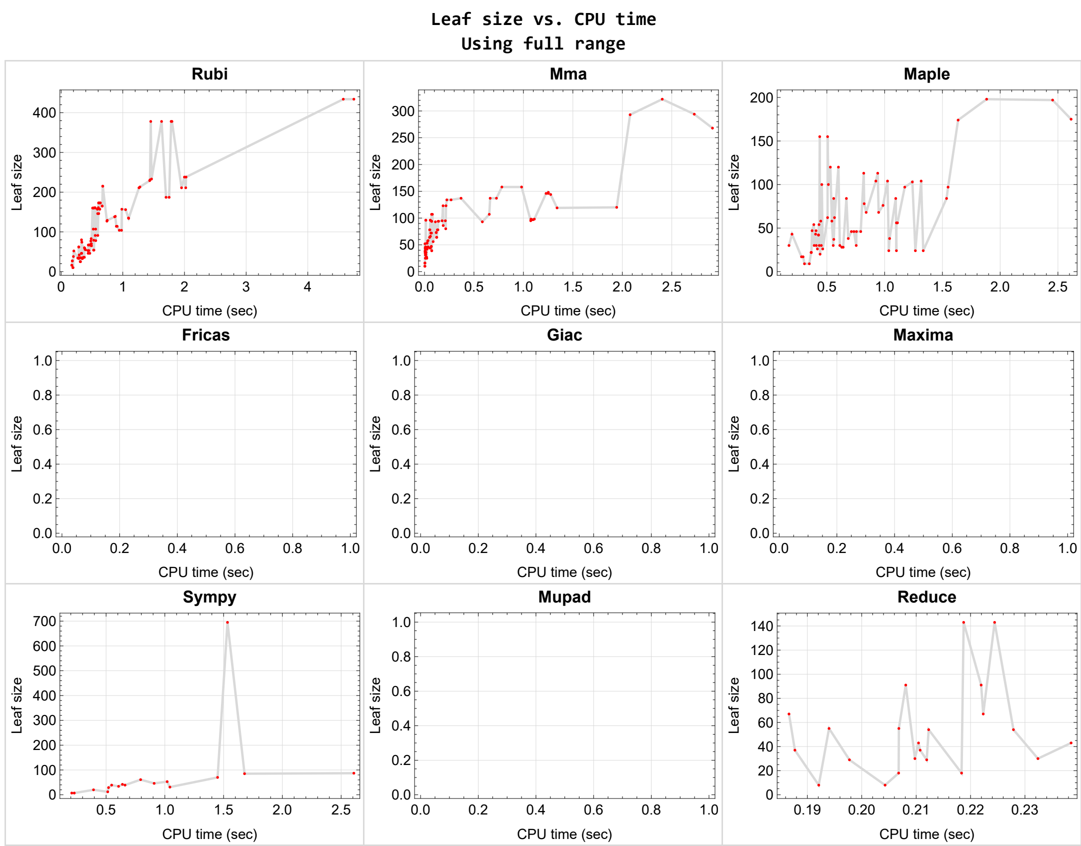


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{9, 14, 15, 16, 17, 22, 25, 29, 30, 31, 40, 46, 48, 58, 62, 65, 68, 77, 82, 83, 84, 85, 90, 93, 97, 98, 99, 108, 114, 116, 126, 130, 133, 136\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{35, 103\}$

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

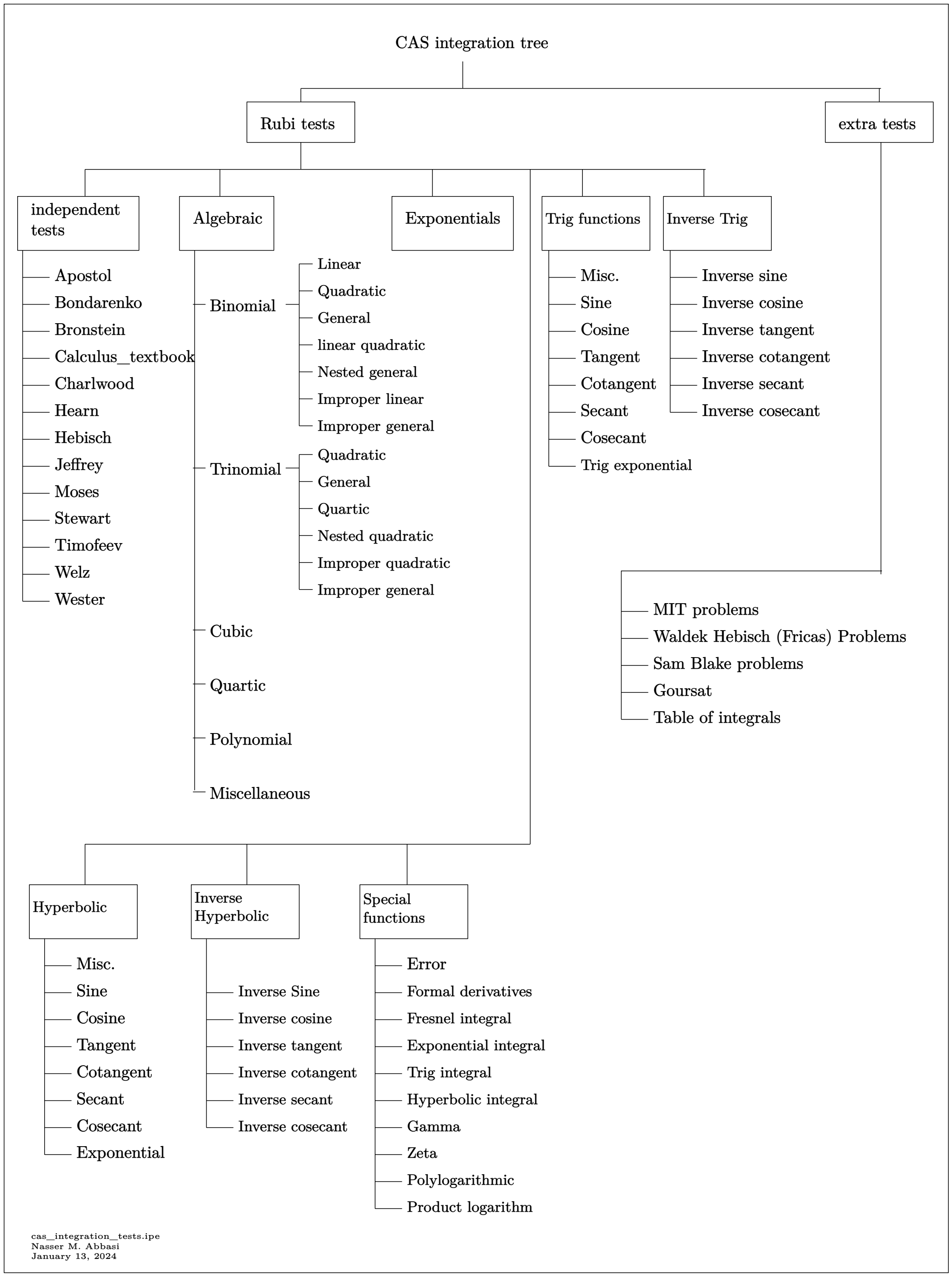
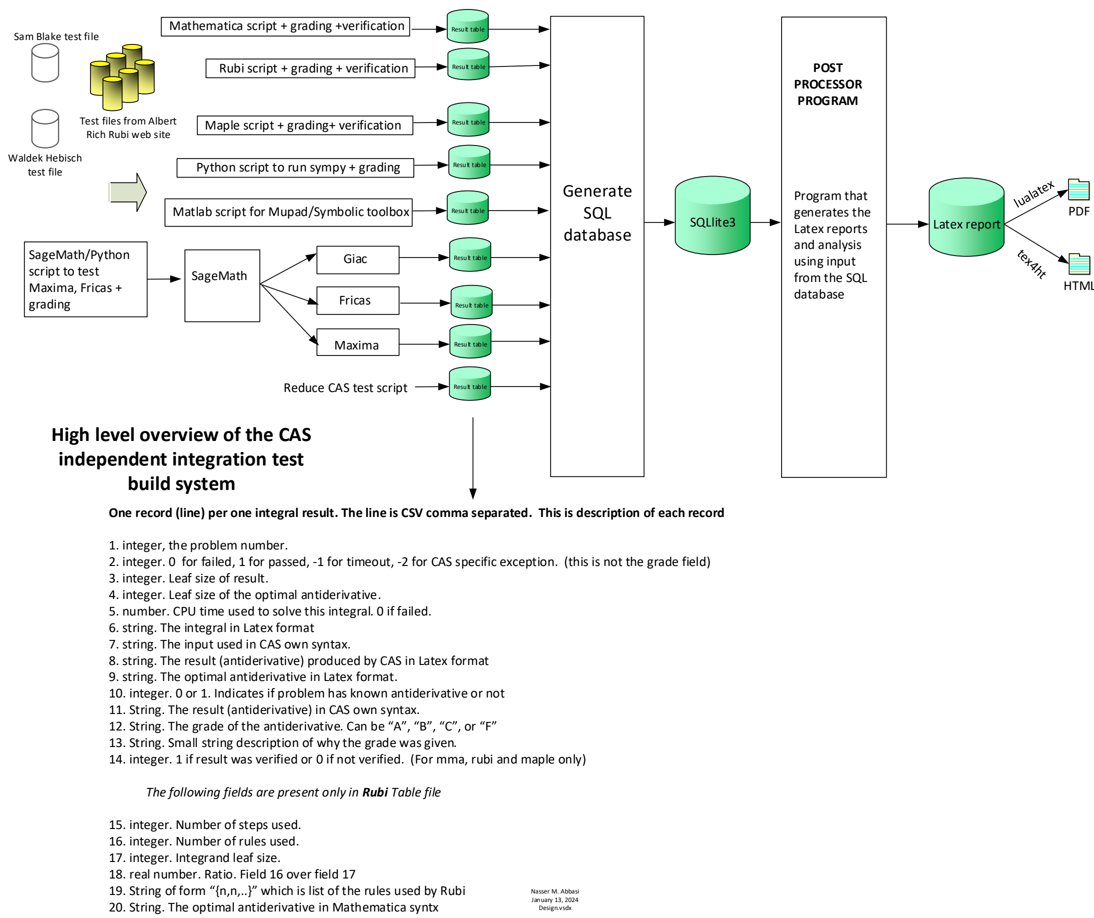


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 5, 6, 10, 11, 12, 13, 18, 19, 20, 21, 23, 24, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 41, 42, 43, 44, 45, 49, 50, 51, 53, 54, 55, 56, 57, 59, 60, 61, 63, 64, 66, 67, 69, 72, 73, 74, 75, 78, 79, 80, 81, 86, 87, 88, 89, 91, 92, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 131, 132, 134, 135 }

B grade { }

C grade { 1, 2, 3, 4, 7, 8, 39, 47, 70, 71, 76, 107, 115 }

F normal fail { 52 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 18, 19, 20, 21, 23, 24, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 63, 64, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 86, 87, 88, 89, 91, 92, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 131, 132, 134, 135 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 18, 19, 20, 21, 27, 28, 35, 41, 42, 43, 44, 45, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 70, 71, 72, 73, 75, 76, 78, 79, 80, 81, 86, 87, 88, 89, 95, 96, 103, 109, 110, 111, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129 }

B grade { }

C grade { 1 }

F normal fail { 23, 24, 26, 32, 33, 34, 36, 37, 38, 39, 47, 63, 64, 66, 67, 69, 74, 91, 92, 94, 100, 101, 102, 104, 105, 106, 107, 115, 131, 132, 134, 135 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 18, 19, 20, 21, 23, 24, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 63, 64, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 86, 87, 88, 89, 91, 92, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 131, 132, 134, 135 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 18, 19, 20, 21, 23, 24, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 63, 64, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 86, 87, 88, 89, 91, 92, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 131, 132, 134, 135 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 18, 19, 20, 21, 23, 24, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 63, 64, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 86, 87, 88, 89, 91, 92, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 131, 132, 134, 135 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 18, 19, 20, 21, 23, 24, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 63, 64, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 86, 87, 88, 89, 91, 92, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 131, 132, 134, 135 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 41, 70, 71, 72, 109 }

B grade { 69, 73, 75, 76 }

C grade { }

F normal fail { 10, 11, 12, 13, 18, 19, 20, 21, 23, 24, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 63, 64, 66, 67, 78, 79, 80, 81, 86, 87, 88, 89, 91, 92, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 131, 132, 134, 135 }

F(-1) timedout fail { 74 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 2, 3, 4, 5, 8, 18, 19, 20, 21, 35, 41, 70, 71, 72, 73, 76, 86, 87, 88, 89, 103, 109 }

C grade { }

F normal fail { 1, 6, 7, 10, 11, 12, 13, 23, 24, 26, 27, 28, 32, 33, 34, 36, 37, 38, 39, 42, 43, 44, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 63, 64, 66, 67, 69, 74, 75, 78, 79, 80, 81, 91, 92, 94, 95, 96, 100, 101, 102, 104, 105, 106, 107, 110, 111, 112, 113, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 131, 132, 134, 135 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	80	56	37	0	0	42	0	10	0
N.S.	1	1.05	0.74	0.49	0.00	0.00	0.55	0.00	0.13	0.00
time (sec)	N/A	0.330	0.088	0.560	0.000	0.000	0.639	0.000	0.228	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	83	53	54	0	0	61	0	55	0
N.S.	1	1.32	0.84	0.86	0.00	0.00	0.97	0.00	0.87	0.00
time (sec)	N/A	0.487	0.026	0.387	0.000	0.000	0.794	0.000	0.194	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	59	44	42	0	0	46	0	43	0
N.S.	1	1.20	0.90	0.86	0.00	0.00	0.94	0.00	0.88	0.00
time (sec)	N/A	0.382	0.019	0.427	0.000	0.000	0.909	0.000	0.238	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	42	35	30	0	0	29	0	29	0
N.S.	1	1.20	1.00	0.86	0.00	0.00	0.83	0.00	0.83	0.00
time (sec)	N/A	0.293	0.007	0.390	0.000	0.000	0.521	0.000	0.212	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	0	0	12	0	18	0
N.S.	1	1.00	1.00	1.06	0.00	0.00	0.75	0.00	1.12	0.00
time (sec)	N/A	0.175	0.004	0.294	0.000	0.000	0.514	0.000	0.207	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	20	0	0	20	0	10	0
N.S.	1	1.00	1.00	0.53	0.00	0.00	0.53	0.00	0.26	0.00
time (sec)	N/A	0.200	0.005	0.441	0.000	0.000	0.393	0.000	0.209	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	35	25	30	0	0	34	0	10	0
N.S.	1	1.40	1.00	1.20	0.00	0.00	1.36	0.00	0.40	0.00
time (sec)	N/A	0.318	0.009	0.555	0.000	0.000	0.606	0.000	0.223	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	55	46	47	0	0	39	0	37	0
N.S.	1	1.20	1.00	1.02	0.00	0.00	0.85	0.00	0.80	0.00
time (sec)	N/A	0.395	0.010	0.372	0.000	0.000	0.547	0.000	0.211	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.196	0.444	0.260	0.062	0.077	1.390	0.103	0.211	4.006

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	229	107	120	0	0	0	0	12	0
N.S.	1	1.54	0.72	0.81	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.436	0.080	0.600	0.000	0.000	0.000	0.000	0.218	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	156	78	84	0	0	0	0	12	0
N.S.	1	1.39	0.70	0.75	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.046	0.052	0.669	0.000	0.000	0.000	0.000	0.219	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	78	58	62	0	0	0	0	10	0
N.S.	1	1.05	0.78	0.84	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.562	0.036	0.566	0.000	0.000	0.000	0.000	0.200	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	36	31	30	0	0	0	0	8	0
N.S.	1	1.16	1.00	0.97	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.377	0.010	0.434	0.000	0.000	0.000	0.000	0.209	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20	1.20
time (sec)	N/A	0.188	0.113	0.131	0.064	0.072	1.128	0.103	0.257	4.082

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.195	0.123	0.215	0.093	0.066	1.070	0.108	0.220	4.216

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.191	0.146	0.191	0.069	0.063	1.074	0.109	0.194	4.168

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.480	6.061	0.306	0.056	0.081	0.626	0.107	0.225	4.151

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	165	94	155	0	0	0	0	143	0
N.S.	1	0.90	0.51	0.84	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.668	0.139	0.506	0.000	0.000	0.000	0.000	0.219	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	107	64	100	0	0	0	0	91	0
N.S.	1	0.91	0.54	0.85	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.560	0.124	0.513	0.000	0.000	0.000	0.000	0.208	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	67	47	58	0	0	0	0	54	0
N.S.	1	1.08	0.76	0.94	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	0.451	0.069	0.543	0.000	0.000	0.000	0.000	0.228	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	42	26	0	0	0	0	30	0
N.S.	1	1.00	1.56	0.96	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	0.182	0.025	0.464	0.000	0.000	0.000	0.000	0.232	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20	1.20
time (sec)	N/A	0.198	0.250	0.369	0.067	0.073	0.376	0.107	0.217	4.151

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	39	0	0	0	0	0	12	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.465	0.074	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	91	86	0	0	0	0	0	12	0
N.S.	1	0.82	0.77	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.595	0.189	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	12	14	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.00	1.17	1.17	1.17
time (sec)	N/A	0.221	4.502	0.263	0.063	0.072	1.323	0.106	0.221	4.166

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	333	434	158	0	0	0	0	0	14	0
N.S.	1	1.30	0.47	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	4.747	0.982	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	187	95	113	0	0	0	0	12	0
N.S.	1	1.21	0.62	0.73	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.701	0.211	0.940	0.000	0.000	0.000	0.000	0.244	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	53	41	43	0	0	0	0	10	0
N.S.	1	1.10	0.85	0.90	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.434	0.012	0.402	0.000	0.000	0.000	0.000	0.238	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	10	14	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	0.83	1.17	1.17	1.17
time (sec)	N/A	0.204	0.449	0.132	0.072	0.067	0.432	0.104	0.211	3.804

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	12	14	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.00	1.17	1.17	1.17
time (sec)	N/A	0.216	0.921	0.258	0.070	0.066	0.331	0.109	0.199	4.032

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	12	14	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.00	1.17	1.17	1.17
time (sec)	N/A	0.211	1.006	0.224	0.062	0.067	0.461	0.107	0.197	4.044

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	173	98	0	0	0	0	0	20	0
N.S.	1	1.35	0.77	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.637	1.111	0.000	0.000	0.000	0.000	0.000	0.241	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	173	98	0	0	0	0	0	18	0
N.S.	1	1.35	0.77	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.613	1.077	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	161	95	0	0	0	0	0	16	0
N.S.	1	1.35	0.80	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.604	1.074	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	62	96	56	0	0	0	0	67	0
N.S.	1	1.13	1.75	1.02	0.00	0.00	0.00	0.00	1.22	0.00
time (sec)	N/A	0.286	0.058	1.102	0.000	0.000	0.000	0.000	0.222	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	146	146	0	0	0	0	0	20	0
N.S.	1	1.20	1.20	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.610	1.229	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	157	148	0	0	0	0	0	20	0
N.S.	1	1.21	1.14	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.628	1.252	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	215	120	0	0	0	0	0	24	0
N.S.	1	1.29	0.72	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.676	1.943	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	134	96	0	0	0	0	0	14	0
N.S.	1	1.40	1.00	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.095	0.014	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	14	14	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.17	1.17	1.17	1.17
time (sec)	N/A	0.527	0.183	0.250	0.108	0.067	2.463	0.106	0.207	4.154

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	0	0	7	0	8	0
N.S.	1	1.00	1.00	0.90	0.00	0.00	0.70	0.00	0.80	0.00
time (sec)	N/A	0.193	0.004	0.348	0.000	0.000	0.229	0.000	0.204	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	0	0	0	0	11	0
N.S.	1	1.00	1.00	0.88	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.310	0.013	0.363	0.000	0.000	0.000	0.000	0.197	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	65	44	46	0	0	0	0	12	0
N.S.	1	1.07	0.72	0.75	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.492	0.061	0.712	0.000	0.000	0.000	0.000	0.216	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	138	64	68	0	0	0	0	14	0
N.S.	1	1.53	0.71	0.76	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.867	0.052	0.840	0.000	0.000	0.000	0.000	0.208	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	211	93	104	0	0	0	0	14	0
N.S.	1	1.69	0.74	0.83	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.262	0.110	0.924	0.000	0.000	0.000	0.000	0.216	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	14	14	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.17	1.17	1.17	1.17
time (sec)	N/A	1.026	0.304	0.245	0.108	0.084	3.145	0.107	0.222	4.100

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	54	44	0	0	0	0	0	14	0
N.S.	1	1.23	1.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.526	0.007	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	12	14	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.00	1.17	1.17	1.17
time (sec)	N/A	0.185	0.187	0.267	0.116	0.069	2.745	0.106	0.231	4.094

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	33	36	28	0	0	0	0	11	0
N.S.	1	0.97	1.06	0.82	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.285	0.015	0.629	0.000	0.000	0.000	0.000	0.240	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	69	46	46	0	0	0	0	12	0
N.S.	1	1.11	0.74	0.74	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.480	0.036	0.793	0.000	0.000	0.000	0.000	0.231	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	127	72	76	0	0	0	0	14	0
N.S.	1	1.30	0.73	0.78	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.742	0.064	0.986	0.000	0.000	0.000	0.000	0.214	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	0	94	104	0	0	0	0	14	0
N.S.	1	0.00	0.73	0.81	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.073	1.316	0.000	0.000	0.000	0.000	0.224	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	34	25	24	0	0	0	0	11	0
N.S.	1	1.17	0.86	0.83	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.270	0.024	1.266	0.000	0.000	0.000	0.000	0.210	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	34	25	24	0	0	0	0	11	0
N.S.	1	1.17	0.86	0.83	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.277	0.022	1.334	0.000	0.000	0.000	0.000	0.231	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	211	123	174	0	0	0	0	18	0
N.S.	1	1.21	0.71	1.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	2.021	0.217	1.636	0.000	0.000	0.000	0.000	0.199	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	104	73	84	0	0	0	0	16	0
N.S.	1	1.07	0.75	0.87	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.979	0.119	1.096	0.000	0.000	0.000	0.000	0.219	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	30	0	0	0	0	15	0
N.S.	1	1.00	0.97	0.91	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.340	0.013	0.612	0.000	0.000	0.000	0.000	0.232	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	39	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	2.44	1.12
time (sec)	N/A	0.279	0.412	0.316	0.187	0.077	1.223	0.108	0.200	4.075

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	238	134	197	0	0	0	0	18	0
N.S.	1	1.19	0.67	0.98	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	2.000	0.226	2.454	0.000	0.000	0.000	0.000	0.215	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	114	78	97	0	0	0	0	16	0
N.S.	1	1.05	0.72	0.89	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.895	0.136	1.549	0.000	0.000	0.000	0.000	0.213	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	38	0	0	0	0	15	0
N.S.	1	1.00	0.98	0.83	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.334	0.019	1.043	0.000	0.000	0.000	0.000	0.201	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12	1.12
time (sec)	N/A	0.270	0.514	0.329	0.195	0.068	1.495	0.109	0.196	4.103

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	378	294	0	0	0	0	0	16	0
N.S.	1	1.02	0.79	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.629	2.728	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	160	137	0	0	0	0	0	15	0
N.S.	1	1.05	0.90	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.527	0.664	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12	1.12
time (sec)	N/A	0.310	2.027	0.628	0.206	0.065	0.991	0.109	0.198	4.103

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	378	268	0	0	0	0	0	16	0
N.S.	1	1.02	0.72	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.784	2.911	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	160	137	0	0	0	0	0	15	0
N.S.	1	1.05	0.90	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.555	0.727	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12	1.12
time (sec)	N/A	0.266	1.804	0.647	0.199	0.071	1.181	0.106	0.238	4.126

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	74	66	0	0	0	695	0	10	0
N.S.	1	0.97	0.87	0.00	0.00	0.00	9.14	0.00	0.13	0.00
time (sec)	N/A	0.336	0.060	0.000	0.000	0.000	1.533	0.000	0.192	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	77	53	54	0	0	85	0	55	0
N.S.	1	1.22	0.84	0.86	0.00	0.00	1.35	0.00	0.87	0.00
time (sec)	N/A	0.482	0.026	0.432	0.000	0.000	1.678	0.000	0.207	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	60	44	42	0	0	70	0	43	0
N.S.	1	1.22	0.90	0.86	0.00	0.00	1.43	0.00	0.88	0.00
time (sec)	N/A	0.377	0.020	0.428	0.000	0.000	1.449	0.000	0.210	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	30	0	0	53	0	29	0
N.S.	1	1.00	1.00	0.86	0.00	0.00	1.51	0.00	0.83	0.00
time (sec)	N/A	0.284	0.007	0.408	0.000	0.000	1.020	0.000	0.198	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	0	0	31	0	18	0
N.S.	1	1.00	1.00	1.06	0.00	0.00	1.94	0.00	1.12	0.00
time (sec)	N/A	0.172	0.005	0.280	0.000	0.000	1.043	0.000	0.218	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	10	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.206	0.005	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	30	0	0	39	0	10	0
N.S.	1	1.00	1.00	1.20	0.00	0.00	1.56	0.00	0.40	0.00
time (sec)	N/A	0.310	0.009	0.452	0.000	0.000	0.661	0.000	0.219	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	56	46	47	0	0	87	0	37	0
N.S.	1	1.22	1.00	1.02	0.00	0.00	1.89	0.00	0.80	0.00
time (sec)	N/A	0.387	0.011	0.406	0.000	0.000	2.608	0.000	0.188	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.197	0.440	0.237	0.078	0.093	0.792	0.108	0.210	4.327

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	231	107	120	0	0	0	0	12	0
N.S.	1	1.41	0.65	0.73	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.442	0.071	0.530	0.000	0.000	0.000	0.000	0.191	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	157	78	84	0	0	0	0	12	0
N.S.	1	1.40	0.70	0.75	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.985	0.055	0.559	0.000	0.000	0.000	0.000	0.197	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	79	57	62	0	0	0	0	10	0
N.S.	1	1.07	0.77	0.84	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.529	0.032	0.506	0.000	0.000	0.000	0.000	0.214	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	36	31	30	0	0	0	0	8	0
N.S.	1	1.16	1.00	0.97	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.360	0.009	0.172	0.000	0.000	0.000	0.000	0.213	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20	1.20
time (sec)	N/A	0.182	0.109	0.128	0.065	0.070	0.562	0.108	0.219	4.380

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.190	0.125	0.186	0.082	0.072	0.435	0.106	0.202	4.364

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.192	0.139	0.184	0.063	0.068	0.510	0.109	0.215	4.501

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.463	6.215	0.278	0.061	0.085	0.656	0.108	0.196	4.548

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	165	94	155	0	0	0	0	143	0
N.S.	1	0.90	0.51	0.84	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.665	0.142	0.438	0.000	0.000	0.000	0.000	0.224	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	107	64	100	0	0	0	0	91	0
N.S.	1	0.91	0.54	0.85	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.521	0.121	0.456	0.000	0.000	0.000	0.000	0.222	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	67	47	58	0	0	0	0	54	0
N.S.	1	1.08	0.76	0.94	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	0.440	0.069	0.447	0.000	0.000	0.000	0.000	0.212	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	42	26	0	0	0	0	30	0
N.S.	1	1.00	1.56	0.96	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	0.181	0.025	0.418	0.000	0.000	0.000	0.000	0.210	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20	1.20
time (sec)	N/A	0.192	0.193	0.333	0.052	0.078	0.404	0.106	0.185	4.082

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	39	0	0	0	0	0	12	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.438	0.073	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	91	80	0	0	0	0	0	12	0
N.S.	1	0.82	0.72	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.553	0.215	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	12	14	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.00	1.17	1.17	1.17
time (sec)	N/A	0.218	2.370	0.283	0.074	0.085	1.244	0.111	0.236	3.931

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	332	434	158	0	0	0	0	0	14	0
N.S.	1	1.31	0.48	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	4.576	0.782	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	187	95	113	0	0	0	0	12	0
N.S.	1	1.21	0.62	0.73	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.752	0.178	0.819	0.000	0.000	0.000	0.000	0.194	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	53	41	43	0	0	0	0	10	0
N.S.	1	1.10	0.85	0.90	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.450	0.010	0.198	0.000	0.000	0.000	0.000	0.185	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	10	14	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	0.83	1.17	1.17	1.17
time (sec)	N/A	0.202	0.351	0.118	0.067	0.069	0.446	0.111	0.229	4.250

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	12	14	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.00	1.17	1.17	1.17
time (sec)	N/A	0.210	0.746	0.224	0.064	0.078	0.323	0.110	0.208	4.049

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	12	14	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.00	1.17	1.17	1.17
time (sec)	N/A	0.208	1.002	0.214	0.068	0.084	0.358	0.101	0.199	3.933

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	173	97	0	0	0	0	0	20	0
N.S.	1	1.35	0.76	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.629	1.099	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	173	97	0	0	0	0	0	18	0
N.S.	1	1.35	0.76	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.607	1.088	0.000	0.000	0.000	0.000	0.000	0.241	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	161	93	0	0	0	0	0	16	0
N.S.	1	1.35	0.78	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.548	0.585	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	62	96	56	0	0	0	0	67	0
N.S.	1	1.13	1.75	1.02	0.00	0.00	0.00	0.00	1.22	0.00
time (sec)	N/A	0.286	0.060	1.111	0.000	0.000	0.000	0.000	0.187	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	146	144	0	0	0	0	0	20	0
N.S.	1	1.20	1.18	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.592	1.276	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	157	146	0	0	0	0	0	20	0
N.S.	1	1.21	1.12	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.588	1.250	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	215	119	0	0	0	0	0	24	0
N.S.	1	1.29	0.71	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.674	1.340	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	135	96	0	0	0	0	0	14	0
N.S.	1	1.41	1.00	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.092	0.012	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	14	14	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.17	1.17	1.17	1.17
time (sec)	N/A	0.544	0.162	0.231	0.101	0.078	3.314	0.114	0.226	4.107

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	0	0	7	0	8	0
N.S.	1	1.00	1.00	0.90	0.00	0.00	0.70	0.00	0.80	0.00
time (sec)	N/A	0.190	0.004	0.306	0.000	0.000	0.208	0.000	0.192	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	0	0	0	0	11	0
N.S.	1	1.00	1.00	0.88	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.314	0.014	0.366	0.000	0.000	0.000	0.000	0.188	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	66	46	46	0	0	0	0	12	0
N.S.	1	1.08	0.75	0.75	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.479	0.035	0.734	0.000	0.000	0.000	0.000	0.208	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	139	64	68	0	0	0	0	14	0
N.S.	1	1.54	0.71	0.76	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.878	0.052	0.948	0.000	0.000	0.000	0.000	0.241	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	213	94	103	0	0	0	0	14	0
N.S.	1	1.50	0.66	0.73	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.278	0.071	1.240	0.000	0.000	0.000	0.000	0.196	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	14	14	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.17	1.17	1.17	1.17
time (sec)	N/A	1.013	0.337	0.234	0.130	0.077	3.895	0.121	0.195	3.882

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	54	44	0	0	0	0	0	14	0
N.S.	1	1.23	1.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.532	0.008	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	12	14	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.00	1.17	1.17	1.17
time (sec)	N/A	0.197	0.188	0.279	0.103	0.089	3.768	0.110	0.199	3.879

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	28	0	0	0	0	11	0
N.S.	1	1.00	1.06	0.82	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.281	0.014	0.641	0.000	0.000	0.000	0.000	0.192	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	70	44	46	0	0	0	0	12	0
N.S.	1	1.13	0.71	0.74	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.491	0.054	0.756	0.000	0.000	0.000	0.000	0.202	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	129	72	78	0	0	0	0	14	0
N.S.	1	1.18	0.66	0.72	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.748	0.071	0.823	0.000	0.000	0.000	0.000	0.223	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	233	94	104	0	0	0	0	14	0
N.S.	1	1.60	0.64	0.71	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.466	0.072	1.025	0.000	0.000	0.000	0.000	0.189	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	34	27	24	0	0	0	0	11	0
N.S.	1	1.17	0.93	0.83	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.278	0.021	1.036	0.000	0.000	0.000	0.000	0.189	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	34	27	24	0	0	0	0	11	0
N.S.	1	1.17	0.93	0.83	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.298	0.020	1.102	0.000	0.000	0.000	0.000	0.207	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	238	134	198	0	0	0	0	18	0
N.S.	1	1.18	0.67	0.99	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	2.026	0.269	1.883	0.000	0.000	0.000	0.000	0.206	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	114	78	97	0	0	0	0	16	0
N.S.	1	1.05	0.72	0.89	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.909	0.130	1.174	0.000	0.000	0.000	0.000	0.210	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	38	0	0	0	0	15	0
N.S.	1	1.00	0.98	0.83	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.328	0.021	0.685	0.000	0.000	0.000	0.000	0.203	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12	1.12
time (sec)	N/A	0.294	0.487	0.309	0.186	0.079	1.240	0.108	0.239	4.087

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	211	123	175	0	0	0	0	18	0
N.S.	1	1.21	0.71	1.01	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.953	0.188	2.614	0.000	0.000	0.000	0.000	0.211	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	104	73	84	0	0	0	0	16	0
N.S.	1	1.07	0.75	0.87	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.943	0.118	1.536	0.000	0.000	0.000	0.000	0.200	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	30	0	0	0	0	15	0
N.S.	1	1.00	0.97	0.91	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.347	0.013	0.754	0.000	0.000	0.000	0.000	0.203	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	39	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	2.44	1.12
time (sec)	N/A	0.247	0.370	0.289	0.155	0.089	1.262	0.108	0.222	4.007

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	378	322	0	0	0	0	0	16	0
N.S.	1	1.02	0.87	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.797	2.403	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	160	107	0	0	0	0	0	15	0
N.S.	1	1.05	0.70	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.510	0.655	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12	1.12
time (sec)	N/A	0.287	2.869	0.618	0.182	0.085	0.965	0.106	0.244	4.106

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	378	293	0	0	0	0	0	16	0
N.S.	1	1.02	0.79	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.453	2.079	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	160	137	0	0	0	0	0	15	0
N.S.	1	1.05	0.90	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.523	0.369	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12	1.12
time (sec)	N/A	0.255	2.675	0.641	0.179	0.094	0.924	0.108	0.183	4.388

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [10] had the largest ratio of [2.600000000000000009]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	C	6	6	1.05	8	0.750
2	C	13	13	1.32	8	1.625
3	C	11	11	1.20	8	1.375
4	C	7	7	1.20	6	1.167
5	A	1	1	1.00	4	0.250
6	A	1	1	1.00	8	0.125
7	C	7	7	1.40	8	0.875
8	C	11	11	1.20	8	1.375
9	N/A	1	0	1.00	10	0.000
10	A	27	26	1.54	10	2.600
11	A	21	21	1.39	10	2.100
12	A	14	13	1.05	8	1.625
13	A	8	8	1.16	6	1.333
14	N/A	1	0	1.00	10	0.000
15	N/A	1	0	1.00	10	0.000
16	N/A	1	0	1.00	10	0.000
17	N/A	2	0	1.00	10	0.000
18	A	3	3	0.90	10	0.300
19	A	3	3	0.91	10	0.300
20	A	3	3	1.08	8	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	1	1	1.00	6	0.167
22	N/A	1	0	1.00	10	0.000
23	A	3	3	1.00	10	0.300
24	A	3	3	0.82	10	0.300
25	N/A	1	0	1.00	12	0.000
26	A	26	26	1.30	12	2.167
27	A	18	18	1.21	10	1.800
28	A	7	7	1.10	8	0.875
29	N/A	1	0	1.00	12	0.000
30	N/A	1	0	1.00	12	0.000
31	N/A	1	0	1.00	12	0.000
32	A	6	5	1.35	17	0.294
33	A	6	5	1.35	15	0.333
34	A	6	5	1.35	13	0.385
35	A	4	3	1.13	17	0.176
36	A	6	5	1.20	17	0.294
37	A	6	5	1.21	17	0.294
38	A	6	5	1.29	19	0.263
39	C	21	21	1.40	12	1.750
40	N/A	10	0	1.00	12	0.000
41	A	1	1	1.00	12	0.083
42	A	7	7	1.00	9	0.778
43	A	13	12	1.07	10	1.200
44	A	20	20	1.53	12	1.667
45	A	26	25	1.69	12	2.083
46	N/A	23	0	1.00	12	0.000
47	C	10	10	1.23	12	0.833
48	N/A	1	0	1.00	12	0.000
49	A	6	6	0.97	9	0.667
50	A	13	13	1.11	10	1.300
51	A	19	18	1.30	12	1.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	F	0	0	N/A	0.000	N/A
53	A	4	4	1.17	9	0.444
54	A	4	4	1.17	9	0.444
55	A	12	12	1.21	16	0.750
56	A	10	10	1.07	14	0.714
57	A	6	6	1.00	13	0.462
58	N/A	1	0	1.00	16	0.000
59	A	11	11	1.19	16	0.688
60	A	9	9	1.05	14	0.643
61	A	5	5	1.00	13	0.385
62	N/A	1	0	1.00	16	0.000
63	A	6	6	1.02	14	0.429
64	A	3	3	1.05	13	0.231
65	N/A	1	0	1.00	16	0.000
66	A	6	6	1.02	14	0.429
67	A	3	3	1.05	13	0.231
68	N/A	1	0	1.00	16	0.000
69	A	6	6	0.97	8	0.750
70	C	14	14	1.22	8	1.750
71	C	10	10	1.22	8	1.250
72	A	8	8	1.00	6	1.333
73	A	1	1	1.00	4	0.250
74	A	1	1	1.00	8	0.125
75	A	8	8	1.00	8	1.000
76	C	10	10	1.22	8	1.250
77	N/A	1	0	1.00	10	0.000
78	A	25	24	1.41	10	2.400
79	A	20	20	1.40	10	2.000
80	A	13	12	1.07	8	1.500
81	A	8	8	1.16	6	1.333
82	N/A	1	0	1.00	10	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
83	N/A	1	0	1.00	10	0.000
84	N/A	1	0	1.00	10	0.000
85	N/A	2	0	1.00	10	0.000
86	A	3	3	0.90	10	0.300
87	A	3	3	0.91	10	0.300
88	A	3	3	1.08	8	0.375
89	A	1	1	1.00	6	0.167
90	N/A	1	0	1.00	10	0.000
91	A	3	3	1.00	10	0.300
92	A	3	3	0.82	10	0.300
93	N/A	1	0	1.00	12	0.000
94	A	25	25	1.31	12	2.083
95	A	17	17	1.21	10	1.700
96	A	7	7	1.10	8	0.875
97	N/A	1	0	1.00	12	0.000
98	N/A	1	0	1.00	12	0.000
99	N/A	1	0	1.00	12	0.000
100	A	6	5	1.35	17	0.294
101	A	6	5	1.35	15	0.333
102	A	6	5	1.35	13	0.385
103	A	4	3	1.13	17	0.176
104	A	6	5	1.20	17	0.294
105	A	6	5	1.21	17	0.294
106	A	6	5	1.29	19	0.263
107	C	18	18	1.41	12	1.500
108	N/A	9	0	1.00	12	0.000
109	A	1	1	1.00	12	0.083
110	A	7	7	1.00	9	0.778
111	A	12	11	1.08	10	1.100
112	A	19	19	1.54	12	1.583
113	A	24	23	1.50	12	1.917

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
114	N/A	22	0	1.00	12	0.000
115	C	10	10	1.23	12	0.833
116	N/A	1	0	1.00	12	0.000
117	A	5	5	1.00	9	0.556
118	A	12	12	1.13	10	1.200
119	A	17	16	1.18	12	1.333
120	A	27	27	1.60	12	2.250
121	A	4	4	1.17	9	0.444
122	A	4	4	1.17	9	0.444
123	A	10	10	1.18	16	0.625
124	A	9	9	1.05	14	0.643
125	A	4	4	1.00	13	0.308
126	N/A	1	0	1.00	16	0.000
127	A	12	12	1.21	16	0.750
128	A	9	9	1.07	14	0.643
129	A	6	6	1.00	13	0.462
130	N/A	1	0	1.00	16	0.000
131	A	6	6	1.02	14	0.429
132	A	3	3	1.05	13	0.231
133	N/A	1	0	1.00	16	0.000
134	A	6	6	1.02	14	0.429
135	A	3	3	1.05	13	0.231
136	N/A	1	0	1.00	16	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^m \mathbf{Shi}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 76

$$\int x^m \text{Shi}(bx)\,dx = -\frac{x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b(1+m)} - \frac{x^m(bx)^{-m}\Gamma(1+m,bx)}{2b(1+m)} + \frac{x^{1+m}\text{Shi}(bx)}{1+m}$$

output
```
-1/2*x^m*GAMMA(1+m,-b*x)/b/(1+m)/((-b*x)^m)-1/2*x^m*GAMMA(1+m,b*x)/b/(1+m)
/((b*x)^m)+x^(1+m)*Shi(b*x)/(1+m)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int x^m \text{Shi}(bx)\,dx = -\frac{x^m((-bx)^{-m}\Gamma(1+m,-bx) + (bx)^{-m}\Gamma(1+m,bx) - 2bx\text{Shi}(bx))}{2b(1+m)}$$

input
```
Integrate[x^m*SinhIntegral[b*x],x]
```

output
```
-1/2*(x^m*(Gamma[1 + m, -(b*x)]/(-(b*x))^m + Gamma[1 + m, b*x]/(b*x)^m - 2
*b*x*SinhIntegral[b*x]))/(b*(1 + m))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {7086, 27, 3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{Shi}(bx) \, dx$$

$$\Big\downarrow \text{ 7086}$$

$$\frac{x^{m+1}\text{Shi}(bx)}{m+1} - \frac{b\int \frac{x^m \sinh(bx)}{b}dx}{m+1}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^{m+1}\text{Shi}(bx)}{m+1} - \frac{\int x^m \sinh(bx)dx}{m+1}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x^{m+1}\text{Shi}(bx)}{m+1} - \frac{\int -ix^m \sin(ibx)dx}{m+1}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{x^{m+1}\text{Shi}(bx)}{m+1} + \frac{i\int x^m \sin(ibx)dx}{m+1}$$

$$\Big\downarrow \text{ 3789}$$

$$\frac{x^{m+1}\text{Shi}(bx)}{m+1} + \frac{i\left(\frac{1}{2}i\int e^{bx}x^m dx - \frac{1}{2}i\int e^{-bx}x^m dx\right)}{m+1}$$

$$\Big\downarrow \text{ 2612}$$

$$\frac{x^{m+1}\text{Shi}(bx)}{m+1} + \frac{i\left(\frac{ix^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b} + \frac{ix^m(bx)^{-m}\Gamma(m+1,bx)}{2b}\right)}{m+1}$$

input
```
Int[x^m*SinhIntegral[b*x],x]
```

output
```
(I*(((I/2)*x^m*Gamma[1 + m, -(b*x)])/(b*(-(b*x))^m) + ((I/2)*x^m*Gamma[1 +
 m, b*x])/(b*(b*x)^m)))/(1 + m) + (x^(1 + m)*SinhIntegral[b*x])/(1 + m)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 7086
```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(SinhIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Sinh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

method	result	size
meijerg	$\frac{b\,x^{2+m}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},1+\frac{m}{2}\right],\left[\frac{3}{2},\frac{3}{2},2+\frac{m}{2}\right],\frac{b^{2}x^{2}}{4}\right)}{2+m}$	37

input
```
int(x^m*Shi(b*x),x,method=_RETURNVERBOSE)
```

output
```
b/(2+m)*x^(2+m)*hypergeom([1/2,1+1/2*m],[3/2,3/2,2+1/2*m],1/4*b^2*x^2)
```

Fricas [F]

$$\int x^m \mathrm{Shi}(bx)\,dx = \int x^m \mathrm{Shi}(bx)\,dx$$

input
```
integrate(x^m*Shi(b*x),x, algorithm="fricas")
```

output
```
integral(x^m*sinh_integral(b*x), x)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int x^m \mathrm{Shi}(bx)\,dx = \frac{bx^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_3\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{3}{2},\frac{3}{2},\frac{m}{2}+2\end{matrix}\right|\frac{b^2x^2}{4}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}$$

input
```
integrate(x**m*Shi(b*x),x)
```

output
```
b*x**(m + 2)*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (3/2, 3/2, m/2 + 2), b**
2*x**2/4)/(2*gamma(m/2 + 2))
```

Maxima [F]

$$\int x^m \mathrm{Shi}(bx)\,dx = \int x^m \mathrm{Shi}(bx)\,dx$$

input
```
integrate(x^m*Shi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^m*Shi(b*x), x)
```

Giac [F]

$$\int x^m \mathrm{Shi}(bx)\,dx = \int x^m \mathrm{Shi}(bx)\,dx$$

input
```
integrate(x^m*Shi(b*x),x, algorithm="giac")
```

output
```
integrate(x^m*Shi(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \mathrm{Shi}(bx)\,dx = \int x^m\,\mathrm{sinhint}(b\,x)\,dx$$

input
```
int(x^m*sinhint(b*x),x)
```

output
```
int(x^m*sinhint(b*x), x)
```

Reduce [F]

$$\int x^m \mathrm{Shi}(bx)\,dx = \int x^m shi(bx)\,dx$$

input
```
int(x^m*Shi(b*x),x)
```

output
```
int(x**m*shi(b*x),x)
```

3.2 $\int x^3\mathbf{Shi}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 63

$$\int x^3\text{Shi}(bx)\,dx=-\frac{3x\cosh(bx)}{2b^3}-\frac{x^3\cosh(bx)}{4b}+\frac{3\sinh(bx)}{2b^4}+\frac{3x^2\sinh(bx)}{4b^2}+\frac{1}{4}x^4\text{Shi}(bx)$$

output
```
-3/2*x*cosh(b*x)/b^3-1/4*x^3*cosh(b*x)/b+3/2*sinh(b*x)/b^4+3/4*x^2*sinh(b*
x)/b^2+1/4*x^4*Shi(b*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^3\text{Shi}(bx)\,dx=-\frac{x(6+b^2x^2)\cosh(bx)}{4b^3}+\frac{3(2+b^2x^2)\sinh(bx)}{4b^4}+\frac{1}{4}x^4\text{Shi}(bx)$$

input
```
Integrate[x^3*SinhIntegral[b*x],x]
```

output
```
-1/4*(x*(6 + b^2*x^2)*Cosh[b*x])/b^3 + (3*(2 + b^2*x^2)*Sinh[b*x])/(4*b^4)
 + (x^4*SinhIntegral[b*x])/4
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.625$, Rules used = {7086, 27, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{Shi}(bx)\,dx$$

$$\Big\downarrow \text{7086}$$

$$\frac{1}{4}x^4\text{Shi}(bx)-\frac{1}{4}b\int \frac{x^3\sinh(bx)}{b}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}x^4\text{Shi}(bx)-\frac{1}{4}\int x^3\sinh(bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}x^4\text{Shi}(bx)-\frac{1}{4}\int -ix^3\sin(ibx)dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{4}x^4\text{Shi}(bx)+\frac{1}{4}i\int x^3\sin(ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{4}x^4\text{Shi}(bx)+\frac{1}{4}i\left(\frac{ix^3\cosh(bx)}{b}-\frac{3i\int x^2\cosh(bx)dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}x^4\text{Shi}(bx)+\frac{1}{4}i\left(\frac{ix^3\cosh(bx)}{b}-\frac{3i\int x^2\sin\left(ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{4}x^4\text{Shi}(bx)+\frac{1}{4}i\left(\frac{ix^3\cosh(bx)}{b}-\frac{3i\left(\frac{x^2\sinh(bx)}{b}-\frac{2i\int -ix\sinh(bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \; 26$$

$$\frac{1}{4}x^4\text{Shi}(bx)+\frac{1}{4}i\left(\frac{ix^3\cosh(bx)}{b}-\frac{3i\left(\frac{x^2\sinh(bx)}{b}-\frac{2\int x\sinh(bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}x^4\text{Shi}(bx)+\frac{1}{4}i\left(\frac{ix^3\cosh(bx)}{b}-\frac{3i\left(\frac{x^2\sinh(bx)}{b}-\frac{2\int -ix\sin(ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \; 26$$

$$\frac{1}{4}x^4\text{Shi}(bx)+\frac{1}{4}i\left(\frac{ix^3\cosh(bx)}{b}-\frac{3i\left(\frac{x^2\sinh(bx)}{b}+\frac{2i\int x\sin(ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \; 3777$$

$$\frac{1}{4}x^4\text{Shi}(bx)+\frac{1}{4}i\left(\frac{ix^3\cosh(bx)}{b}-\frac{3i\left(\frac{x^2\sinh(bx)}{b}+\frac{2i\left(\frac{ix\cosh(bx)}{b}-\frac{i\int\cosh(bx)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}x^4\text{Shi}(bx)+\frac{1}{4}i\left(\frac{ix^3\cosh(bx)}{b}-\frac{3i\left(\frac{x^2\sinh(bx)}{b}+\frac{2i\left(\frac{ix\cosh(bx)}{b}-\frac{i\int\sin\left(ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \; 3117$$

$$\frac{1}{4}x^4\text{Shi}(bx)+\frac{1}{4}i\left(\frac{ix^3\cosh(bx)}{b}-\frac{3i\left(\frac{x^2\sinh(bx)}{b}+\frac{2i\left(\frac{ix\cosh(bx)}{b}-\frac{i\sinh(bx)}{b^2}\right)}{b}\right)}{b}\right)$$

input
```
Int[x^3*SinhIntegral[b*x],x]
```

output
```
(I/4)*((I*x^3*Cosh[b*x])/b - ((3*I)*((x^2*Sinh[b*x])/b + ((2*I)*((I*x*Cosh
[b*x])/b - (I*Sinh[b*x])/b^2))/b))/b) + (x^4*SinhIntegral[b*x])/4
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7086
```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(SinhIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Sinh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

method	result	size
parts	$\frac{x^{4} \operatorname{Shi}(bx)}{4}-\frac{b^{3} x^{3} \cosh (bx)-3 b^{2} x^{2} \sinh (bx)+6 b x \cosh (bx)-6 \sinh (bx)}{4 b^{4}}$	54
derivativedivides	$\frac{\frac{b^{4} x^{4} \operatorname{Shi}(bx)}{4}-\frac{b^{3} x^{3} \cosh (bx)}{4}+\frac{3 b^{2} x^{2} \sinh (bx)}{4}-\frac{3 b x \cosh (bx)}{2}+\frac{3 \sinh (bx)}{2}}{b^{4}}$	56
default	$\frac{\frac{b^{4} x^{4} \operatorname{Shi}(bx)}{4}-\frac{b^{3} x^{3} \cosh (bx)}{4}+\frac{3 b^{2} x^{2} \sinh (bx)}{4}-\frac{3 b x \cosh (bx)}{2}+\frac{3 \sinh (bx)}{2}}{b^{4}}$	56
meijerg	$-\frac{4 i \sqrt{\pi}\left(-\frac{i x b\left(\frac{5 b^{2} x^{2}}{2}+15\right) \cosh (bx)}{40 \sqrt{\pi}}+\frac{i\left(\frac{15 b^{2} x^{2}}{2}+15\right) \sinh (bx)}{40 \sqrt{\pi}}+\frac{i x^{4} b^{4} \operatorname{Shi}(bx)}{16 \sqrt{\pi}}\right)}{b^{4}}$	69

input
```
int(x^3*Shi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*Shi(b*x)-1/4/b^4*(b^3*x^3*cosh(b*x)-3*b^2*x^2*sinh(b*x)+6*b*x*cosh
(b*x)-6*sinh(b*x))
```

Fricas [F]

$$\int x^{3}\operatorname{Shi}(bx)\,dx=\int x^{3}\operatorname{Shi}(bx)\,dx$$

input
```
integrate(x^3*Shi(b*x),x, algorithm="fricas")
```

output
```
integral(x^3*sinh_integral(b*x), x)
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int x^3 \mathrm{Shi}(bx)\,dx = \frac{x^4\,\mathrm{Shi}\,(bx)}{4} - \frac{x^3\cosh\,(bx)}{4b} + \frac{3x^2\sinh\,(bx)}{4b^2} - \frac{3x\cosh\,(bx)}{2b^3} + \frac{3\sinh\,(bx)}{2b^4}$$

input
```
integrate(x**3*Shi(b*x),x)
```

output
```
x**4*Shi(b*x)/4 - x**3*cosh(b*x)/(4*b) + 3*x**2*sinh(b*x)/(4*b**2) - 3*x*c
osh(b*x)/(2*b**3) + 3*sinh(b*x)/(2*b**4)
```

Maxima [F]

$$\int x^3 \mathrm{Shi}(bx)\,dx = \int x^3 \mathrm{Shi}(bx)\,dx$$

input
```
integrate(x^3*Shi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^3*Shi(b*x), x)
```

Giac [F]

$$\int x^3 \mathrm{Shi}(bx)\,dx = \int x^3 \mathrm{Shi}(bx)\,dx$$

input
```
integrate(x^3*Shi(b*x),x, algorithm="giac")
```

output
```
integrate(x^3*Shi(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \text{Shi}(bx)\,dx = \int x^3\,\text{sinhint}(b\,x)\;dx$$

input
```
int(x^3*sinhint(b*x),x)
```

output
```
int(x^3*sinhint(b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int x^3 \text{Shi}(bx)\,dx = \frac{-\cosh(bx)\,b^3x^3 - 6\cosh(bx)\,bx + shi(bx)\,b^4x^4 + 3\sinh(bx)\,b^2x^2 + 6\sinh(bx)}{4b^4}$$

input
```
int(x^3*Shi(b*x),x)
```

output
```
( - cosh(b*x)*b**3*x**3 - 6*cosh(b*x)*b*x + shi(b*x)*b**4*x**4 + 3*sinh(b*
x)*b**2*x**2 + 6*sinh(b*x))/(4*b**4)
```

3.3 $\int x^2 \text{Shi}(bx) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 49

$$\int x^2 \text{Shi}(bx) \, dx = -\frac{2\cosh(bx)}{3b^3} - \frac{x^2\cosh(bx)}{3b} + \frac{2x\sinh(bx)}{3b^2} + \frac{1}{3}x^3\text{Shi}(bx)$$

output
```
-2/3*cosh(b*x)/b^3-1/3*x^2*cosh(b*x)/b+2/3*x*sinh(b*x)/b^2+1/3*x^3*Shi(b*x
)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int x^2 \text{Shi}(bx) \, dx = -\frac{(2+b^2x^2)\cosh(bx)}{3b^3} + \frac{2x\sinh(bx)}{3b^2} + \frac{1}{3}x^3\text{Shi}(bx)$$

input
```
Integrate[x^2*SinhIntegral[b*x],x]
```

output
```
-1/3*((2 + b^2*x^2)*Cosh[b*x])/b^3 + (2*x*Sinh[b*x])/(3*b^2) + (x^3*SinhIn
tegral[b*x])/3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.375, Rules used = {7086, 27, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{Shi}(bx)\,dx$$

$$\Big\downarrow \text{ 7086}$$

$$\frac{1}{3}x^3\text{Shi}(bx)-\frac{1}{3}b\int \frac{x^2\sinh(bx)}{b}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}x^3\text{Shi}(bx)-\frac{1}{3}\int x^2\sinh(bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}x^3\text{Shi}(bx)-\frac{1}{3}\int -ix^2\sin(ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{3}x^3\text{Shi}(bx)+\frac{1}{3}i\int x^2\sin(ibx)dx$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{1}{3}x^3\text{Shi}(bx)+\frac{1}{3}i\left(\frac{ix^2\cosh(bx)}{b}-\frac{2i\int x\cosh(bx)dx}{b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}x^3\text{Shi}(bx)+\frac{1}{3}i\left(\frac{ix^2\cosh(bx)}{b}-\frac{2i\int x\sin\left(ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{1}{3}x^3\text{Shi}(bx)+\frac{1}{3}i\left(\frac{ix^2\cosh(bx)}{b}-\frac{2i\left(\frac{x\sinh(bx)}{b}-\frac{i\int -i\sinh(bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{3}x^3\text{Shi}(bx)+\frac{1}{3}i\left(\frac{ix^2\cosh(bx)}{b}-\frac{2i\left(\frac{x\sinh(bx)}{b}-\frac{\int\sinh(bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}x^3\text{Shi}(bx)+\frac{1}{3}i\left(\frac{ix^2\cosh(bx)}{b}-\frac{2i\left(\frac{x\sinh(bx)}{b}-\frac{\int-i\sin(ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{3}x^3\text{Shi}(bx)+\frac{1}{3}i\left(\frac{ix^2\cosh(bx)}{b}-\frac{2i\left(\frac{x\sinh(bx)}{b}+\frac{i\int\sin(ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{ 3118}$$

$$\frac{1}{3}x^3\text{Shi}(bx)+\frac{1}{3}i\left(\frac{ix^2\cosh(bx)}{b}-\frac{2i\left(\frac{x\sinh(bx)}{b}-\frac{\cosh(bx)}{b^2}\right)}{b}\right)$$

input
```
Int[x^2*SinhIntegral[b*x],x]
```

output
```
(I/3)*((I*x^2*Cosh[b*x])/b - ((2*I)*(-(Cosh[b*x]/b^2) + (x*Sinh[b*x])/b))/
b) + (x^3*SinhIntegral[b*x])/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7086
```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(SinhIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Sinh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
parts	$\frac{x^3 \operatorname{Shi}(bx)}{3} - \frac{b^2x^2\cosh(bx) - 2bx\sinh(bx) + 2\cosh(bx)}{3b^3}$	42
derivativedivides	$\frac{\frac{b^3x^3 \operatorname{Shi}(bx)}{3} - \frac{b^2x^2\cosh(bx)}{3} + \frac{2bx\sinh(bx)}{3} - \frac{2\cosh(bx)}{3}}{b^3}$	44
default	$\frac{\frac{b^3x^3 \operatorname{Shi}(bx)}{3} - \frac{b^2x^2\cosh(bx)}{3} + \frac{2bx\sinh(bx)}{3} - \frac{2\cosh(bx)}{3}}{b^3}$	44
meijerg	$\frac{2\sqrt{\pi}\left(\frac{1}{3\sqrt{\pi}} - \frac{\left(\frac{b^2x^2}{2}+1\right)\cosh(bx)}{3\sqrt{\pi}} + \frac{bx\sinh(bx)}{3\sqrt{\pi}} + \frac{b^3x^3 \operatorname{Shi}(bx)}{6\sqrt{\pi}}\right)}{b^3}$	60

input
```
int(x^2*Shi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*Shi(b*x)-1/3/b^3*(b^2*x^2*cosh(b*x)-2*b*x*sinh(b*x)+2*cosh(b*x))
```

Fricas [F]

$$\int x^2\text{Shi}(bx)\,dx = \int x^2\text{Shi}(bx)\,dx$$

input
```
integrate(x^2*Shi(b*x),x, algorithm="fricas")
```

output
```
integral(x^2*sinh_integral(b*x), x)
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int x^2\text{Shi}(bx)\,dx = \frac{x^3\,\text{Shi}\,(bx)}{3} - \frac{x^2\cosh\,(bx)}{3b} + \frac{2x\sinh\,(bx)}{3b^2} - \frac{2\cosh\,(bx)}{3b^3}$$

input
```
integrate(x**2*Shi(b*x),x)
```

output
```
x**3*Shi(b*x)/3 - x**2*cosh(b*x)/(3*b) + 2*x*sinh(b*x)/(3*b**2) - 2*cosh(b
*x)/(3*b**3)
```

Maxima [F]

$$\int x^2\text{Shi}(bx)\,dx = \int x^2\text{Shi}(bx)\,dx$$

input
```
integrate(x^2*Shi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^2*Shi(b*x), x)
```

Giac [F]

$$\int x^2 \text{Shi}(bx)\,dx = \int x^2 \text{Shi}(bx)\,dx$$

input
```
integrate(x^2*Shi(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*Shi(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \text{Shi}(bx)\,dx = \frac{x^3\,\text{sinhint}(b\,x)}{3} - \frac{\frac{2\cosh(b\,x)}{3} + \frac{b^2\,x^2\cosh(b\,x)}{3} - \frac{2\,b\,x\sinh(b\,x)}{3}}{b^3}$$

input
```
int(x^2*sinhint(b*x),x)
```

output
```
(x^3*sinhint(b*x))/3 - ((2*cosh(b*x))/3 + (b^2*x^2*cosh(b*x))/3 - (2*b*x*s
inh(b*x))/3)/b^3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int x^2 \text{Shi}(bx)\,dx = \frac{-\cosh(bx)\,b^2x^2 - 2\cosh(bx) + shi(bx)\,b^3x^3 + 2\sinh(bx)\,bx}{3b^3}$$

input
```
int(x^2*Shi(b*x),x)
```

output
```
( - cosh(b*x)*b**2*x**2 - 2*cosh(b*x) + shi(b*x)*b**3*x**3 + 2*sinh(b*x)*b
*x)/(3*b**3)
```

3.4 $\int x\mathbf{Shi}(bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 35

$$\int x\mathrm{Shi}(bx)\,dx = -\frac{x\cosh(bx)}{2b} + \frac{\sinh(bx)}{2b^2} + \frac{1}{2}x^2\mathrm{Shi}(bx)$$

output
```
-1/2*x*cosh(b*x)/b+1/2*sinh(b*x)/b^2+1/2*x^2*Shi(b*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int x\mathrm{Shi}(bx)\,dx = -\frac{x\cosh(bx)}{2b} + \frac{\sinh(bx)}{2b^2} + \frac{1}{2}x^2\mathrm{Shi}(bx)$$

input
```
Integrate[x*SinhIntegral[b*x],x]
```

output
```
-1/2*(x*Cosh[b*x])/b + Sinh[b*x]/(2*b^2) + (x^2*SinhIntegral[b*x])/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {7086, 27, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Shi}(bx) \, dx$$

$$\Big\downarrow \text{7086}$$

$$\frac{1}{2}x^2\text{Shi}(bx)-\frac{1}{2}b\int \frac{x\sinh(bx)}{b}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}x^2\text{Shi}(bx)-\frac{1}{2}\int x\sinh(bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2\text{Shi}(bx)-\frac{1}{2}\int -ix\sin(ibx)dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}x^2\text{Shi}(bx)+\frac{1}{2}i\int x\sin(ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2}x^2\text{Shi}(bx)+\frac{1}{2}i\left(\frac{ix\cosh(bx)}{b}-\frac{i\int \cosh(bx)dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2\text{Shi}(bx)+\frac{1}{2}i\left(\frac{ix\cosh(bx)}{b}-\frac{i\int \sin\left(ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{2}x^2\text{Shi}(bx)+\frac{1}{2}i\left(\frac{ix\cosh(bx)}{b}-\frac{i\sinh(bx)}{b^2}\right)$$

input
```
Int[x*SinhIntegral[b*x],x]
```

output
```
(I/2)*((I*x*Cosh[b*x])/b - (I*Sinh[b*x])/b^2) + (x^2*SinhIntegral[b*x])/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7086
```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(SinhIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Sinh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
parts	$\frac{x^2 \operatorname{Shi}(bx)}{2} - \frac{bx\cosh(bx)-\sinh(bx)}{2b^2}$	30
derivativedivides	$\frac{\frac{b^2x^2 \operatorname{Shi}(bx)}{2} - \frac{bx\cosh(bx)}{2} + \frac{\sinh(bx)}{2}}{b^2}$	32
default	$\frac{\frac{b^2x^2 \operatorname{Shi}(bx)}{2} - \frac{bx\cosh(bx)}{2} + \frac{\sinh(bx)}{2}}{b^2}$	32
meijerg	$\frac{i\sqrt{\pi}\left(\frac{ibx\cosh(bx)}{2\sqrt{\pi}} - \frac{i\sinh(bx)}{2\sqrt{\pi}} - \frac{ib^2x^2 \operatorname{Shi}(bx)}{2\sqrt{\pi}}\right)}{b^2}$	49

input
```
int(x*Shi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*Shi(b*x)-1/2/b^2*(b*x*cosh(b*x)-sinh(b*x))
```

Fricas [F]

$$\int x\text{Shi}(bx)\,dx = \int x\text{Shi}(bx)\ dx$$

input
```
integrate(x*Shi(b*x),x, algorithm="fricas")
```

output
```
integral(x*sinh_integral(b*x), x)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x\text{Shi}(bx)\,dx = \frac{x^2 \operatorname{Shi}(bx)}{2} - \frac{x\cosh(bx)}{2b} + \frac{\sinh(bx)}{2b^2}$$

input
```
integrate(x*Shi(b*x),x)
```

output
```
x**2*Shi(b*x)/2 - x*cosh(b*x)/(2*b) + sinh(b*x)/(2*b**2)
```

Maxima [F]

$$\int x\text{Shi}(bx)\,dx = \int x\text{Shi}(bx)\,dx$$

input
```
integrate(x*Shi(b*x),x, algorithm="maxima")
```

output
```
integrate(x*Shi(b*x), x)
```

Giac [F]

$$\int x\text{Shi}(bx)\,dx = \int x\text{Shi}(bx)\,dx$$

input
```
integrate(x*Shi(b*x),x, algorithm="giac")
```

output
```
integrate(x*Shi(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{Shi}(bx)\,dx = \frac{\frac{\sinh(b\,x)}{2} - \frac{b\,x\cosh(b\,x)}{2}}{b^2} + \frac{x^2\,\text{sinhint}(b\,x)}{2}$$

input
```
int(x*sinhint(b*x),x)
```

output
```
(sinh(b*x)/2 - (b*x*cosh(b*x))/2)/b^2 + (x^2*sinhint(b*x))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x\text{Shi}(bx)\,dx = \frac{-\cosh(bx)\,bx + shi(bx)\,b^2x^2 + \sinh(bx)}{2b^2}$$

input
```
int(x*Shi(b*x),x)
```

output
```
( - cosh(b*x)*b*x + shi(b*x)*b**2*x**2 + sinh(b*x))/(2*b**2)
```

3.5 $\int \mathbf{Shi}(bx)\, dx$

Optimal result

Integrand size = 4, antiderivative size = 16

$$\int \mathrm{Shi}(bx)\, dx = -\frac{\cosh(bx)}{b} + x\mathrm{Shi}(bx)$$

output
```
-cosh(b*x)/b+x*Shi(b*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \mathrm{Shi}(bx)\, dx = -\frac{\cosh(bx)}{b} + x\mathrm{Shi}(bx)$$

input
```
Integrate[SinhIntegral[b*x],x]
```

output
```
-(Cosh[b*x]/b) + x*SinhIntegral[b*x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7082}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Shi}(bx)\,dx$$

$$\Big\downarrow \text{7082}$$

$$x\text{Shi}(bx)-\frac{\cosh(bx)}{b}$$

input
```
Int[SinhIntegral[b*x],x]
```

output
```
-(Cosh[b*x]/b) + x*SinhIntegral[b*x]
```

Defintions of rubi rules used

rule 7082
```
Int[SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(SinhInte
gral[a + b*x]/b), x] - Simp[Cosh[a + b*x]/b, x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
parts	$-\frac{\cosh(bx)}{b}+x\ \text{Shi}\left(bx\right)$	17
derivativedivides	$\frac{\text{Shi}(bx)bx-\cosh(bx)}{b}$	19
default	$\frac{\text{Shi}(bx)bx-\cosh(bx)}{b}$	19
meijerg	$-\frac{\sqrt{\pi}\left(-\frac{2}{\sqrt{\pi}}+\frac{2\cosh(bx)}{\sqrt{\pi}}-\frac{2bx\ \text{Shi}(bx)}{\sqrt{\pi}}\right)}{2b}$	35

input
```
int(Shi(b*x),x,method=_RETURNVERBOSE)
```

output
```
-cosh(b*x)/b+x*Shi(b*x)
```

Fricas [F]

$$\int \mathrm{Shi}(bx)\,dx = \int \mathrm{Shi}(bx)\,dx$$

input
```
integrate(Shi(b*x),x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*x), x)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \mathrm{Shi}(bx)\,dx = x\,\mathrm{Shi}\,(bx) - \frac{\cosh{(bx)}}{b}$$

input
```
integrate(Shi(b*x),x)
```

output
```
x*Shi(b*x) - cosh(b*x)/b
```

Maxima [F]

$$\int \operatorname{Shi}(bx)\,dx = \int \operatorname{Shi}(bx)\,dx$$

input
```
integrate(Shi(b*x),x, algorithm="maxima")
```

output
```
integrate(Shi(b*x), x)
```

Giac [F]

$$\int \operatorname{Shi}(bx)\,dx = \int \operatorname{Shi}(bx)\,dx$$

input
```
integrate(Shi(b*x),x, algorithm="giac")
```

output
```
integrate(Shi(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{Shi}(bx)\,dx = x\,\mathrm{sinhint}(b\,x) - \frac{\cosh(b\,x)}{b}$$

input
```
int(sinhint(b*x),x)
```

output
```
x*sinhint(b*x) - cosh(b*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \mathrm{Shi}(bx)\,dx = \frac{-\cosh(bx) + shi(bx)\,bx}{b}$$

input
```
int(Shi(b*x),x)
```

output
```
( - cosh(b*x) + shi(b*x)*b*x)/b
```

3.6 $\int \frac{\text{Shi}(bx)}{x} dx$

Optimal result

Integrand size = 8, antiderivative size = 38

$$\int \frac{\text{Shi}(bx)}{x} dx = \frac{1}{2} bx\,{}_3F_3(1,1,1;2,2,2;-bx) + \frac{1}{2} bx\,{}_3F_3(1,1,1;2,2,2;bx)$$

output
```
1/2*b*x*hypergeom([1, 1, 1],[2, 2, 2],-b*x)+1/2*b*x*hypergeom([1, 1, 1],[2
, 2, 2],b*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\text{Shi}(bx)}{x} dx = \frac{1}{2} bx\,{}_3F_3(1,1,1;2,2,2;-bx) + \frac{1}{2} bx\,{}_3F_3(1,1,1;2,2,2;bx)$$

input
```
Integrate[SinhIntegral[b*x]/x,x]
```

output
```
(b*x*HypergeometricPFQ[{1, 1, 1}, {2, 2, 2}, -(b*x)])/2 + (b*x*Hypergeomet
ricPFQ[{1, 1, 1}, {2, 2, 2}, b*x])/2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7084}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(bx)}{x} \, dx$$

$$\Big\downarrow \text{7084}$$

$$\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;-bx)+\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;bx)$$

input
```
Int[SinhIntegral[b*x]/x,x]
```

output
```
(b*x*HypergeometricPFQ[{1, 1, 1}, {2, 2, 2}, -(b*x)])/2 + (b*x*Hypergeomet
ricPFQ[{1, 1, 1}, {2, 2, 2}, b*x])/2
```

Defintions of rubi rules used

rule 7084
```
Int[SinhIntegral[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[(1/2)*b*x*Hypergeometr
icPFQ[{1, 1, 1}, {2, 2, 2}, (-b)*x], x] + Simp[(1/2)*b*x*HypergeometricPFQ[
{1, 1, 1}, {2, 2, 2}, b*x], x] /; FreeQ[b, x]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

method	result	size
meijerg	$bx\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2}\right],\left[\frac{3}{2},\frac{3}{2},\frac{3}{2}\right],\frac{b^2x^2}{4}\right)$	20

input
```
int(Shi(b*x)/x,x,method=_RETURNVERBOSE)
```

output
```
b*x*hypergeom([1/2,1/2],[3/2,3/2,3/2],1/4*b^2*x^2)
```

Fricas [F]

$$\int \frac{\text{Shi}(bx)}{x}\,dx = \int \frac{\text{Shi}(bx)}{x}\,dx$$

input
```
integrate(Shi(b*x)/x,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*x)/x, x)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

$$\int \frac{\text{Shi}(bx)}{x}\,dx = bx\,{}_2F_3\left(\begin{matrix} \frac{1}{2}, \frac{1}{2} \\ \frac{3}{2}, \frac{3}{2}, \frac{3}{2} \end{matrix}\middle| \frac{b^2x^2}{4}\right)$$

input
```
integrate(Shi(b*x)/x,x)
```

output
```
b*x*hyper((1/2, 1/2), (3/2, 3/2, 3/2), b**2*x**2/4)
```

Maxima [F]

$$\int \frac{\text{Shi}(bx)}{x}\,dx = \int \frac{\text{Shi}(bx)}{x}\,dx$$

input
```
integrate(Shi(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x)/x, x)
```

Giac [F]

$$\int \frac{\mathrm{Shi}(bx)}{x}\,dx = \int \frac{\mathrm{Shi}(bx)}{x}\,dx$$

input
```
integrate(Shi(b*x)/x,x, algorithm="giac")
```

output
```
integrate(Shi(b*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\mathrm{Shi}(bx)}{x}\,dx = \int \frac{\mathrm{sinhint}(b\,x)}{x}\,dx$$

input
```
int(sinhint(b*x)/x,x)
```

output
```
int(sinhint(b*x)/x, x)
```

Reduce [F]

$$\int \frac{\mathrm{Shi}(bx)}{x}\,dx = \int \frac{shi(bx)}{x}dx$$

input
```
int(Shi(b*x)/x,x)
```

output
```
int(shi(b*x)/x,x)
```

3.7 $\int \frac{\text{Shi}(bx)}{x^2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \frac{\text{Shi}(bx)}{x^2}\,dx = b\text{Chi}(bx) - \frac{\sinh(bx)}{x} - \frac{\text{Shi}(bx)}{x}$$

output
```
b*Chi(b*x)-sinh(b*x)/x-Shi(b*x)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\text{Shi}(bx)}{x^2}\,dx = b\text{Chi}(bx) - \frac{\sinh(bx)}{x} - \frac{\text{Shi}(bx)}{x}$$

input
```
Integrate[SinhIntegral[b*x]/x^2,x]
```

output
```
b*CoshIntegral[b*x] - Sinh[b*x]/x - SinhIntegral[b*x]/x
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {7086, 27, 3042, 26, 3778, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(bx)}{x^2} \, dx$$

$$\Big\downarrow \text{ 7086}$$

$$b \int \frac{\sinh(bx)}{bx^2} dx - \frac{\text{Shi}(bx)}{x}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{\sinh(bx)}{x^2} dx - \frac{\text{Shi}(bx)}{x}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\text{Shi}(bx)}{x} + \int -\frac{i \sin(ibx)}{x^2} dx$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{\text{Shi}(bx)}{x} - i \int \frac{\sin(ibx)}{x^2} dx$$

$$\Big\downarrow \text{ 3778}$$

$$-\frac{\text{Shi}(bx)}{x} - i \left(ib \int \frac{\cosh(bx)}{x} dx - \frac{i \sinh(bx)}{x} \right)$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\text{Shi}(bx)}{x} - i \left(ib \int \frac{\sin\left(ibx + \frac{\pi}{2}\right)}{x} dx - \frac{i \sinh(bx)}{x} \right)$$

$$\Big\downarrow \text{ 3782}$$

$$-\frac{\text{Shi}(bx)}{x} - i \left(ib\text{Chi}(bx) - \frac{i \sinh(bx)}{x} \right)$$

input
```
Int[SinhIntegral[b*x]/x^2,x]
```

output
```
(-I)*(I*b*CoshIntegral[b*x] - (I*Sinh[b*x])/x) - SinhIntegral[b*x]/x
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 7086
```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(SinhIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Sinh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result
parts	$-\frac{\operatorname{Shi}(bx)}{x}+b\left(-\frac{\sinh(bx)}{bx}+\operatorname{Chi}(bx)\right)$
derivativedivides	$b\left(-\frac{\operatorname{Shi}(bx)}{bx}-\frac{\sinh(bx)}{bx}+\operatorname{Chi}(bx)\right)$
default	$b\left(-\frac{\operatorname{Shi}(bx)}{bx}-\frac{\sinh(bx)}{bx}+\operatorname{Chi}(bx)\right)$
meijerg	$\frac{\sqrt{\pi}\, b\left(\frac{8\gamma-16+8\ln(x)+8\ln(ib)}{\sqrt{\pi}}+\frac{16}{\sqrt{\pi}}-\frac{4\,\mathrm{e}^{bx}}{\sqrt{\pi}\, bx}+\frac{4\,\mathrm{e}^{-bx}}{\sqrt{\pi}\, bx}-\frac{4(-9bx+9)(-\gamma-\ln(-bx)-\operatorname{expIntegral}_1(-bx))}{9\sqrt{\pi}\, bx}+\frac{4(9bx+9)(-\gamma-\ln(bx)-\mathrm{e}}{9\sqrt{\pi}\, bx}}{8}$

input
```
int(Shi(b*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-Shi(b*x)/x+b*(-sinh(b*x)/b/x+Chi(b*x))
```

Fricas [F]

$$\int \frac{\operatorname{Shi}(bx)}{x^2}\, dx = \int \frac{\operatorname{Shi}(bx)}{x^2}\, dx$$

input
```
integrate(Shi(b*x)/x^2,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*x)/x^2, x)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{\operatorname{Shi}(bx)}{x^2}\, dx = \frac{b^3x^2\,{}_3F_4\left(\begin{matrix} 1,1,\frac{3}{2} \\ 2,2,\frac{5}{2},\frac{5}{2} \end{matrix}\middle| \frac{b^2x^2}{4}\right)}{36}+\frac{b\log(b^2x^2)}{2}$$

input
```
integrate(Shi(b*x)/x**2,x)
```

output
```
b**3*x**2*hyper((1, 1, 3/2), (2, 2, 5/2, 5/2), b**2*x**2/4)/36 + b*log(b**
2*x**2)/2
```

Maxima [F]

$$\int \frac{\mathrm{Shi}(bx)}{x^2}\,dx = \int \frac{\mathrm{Shi}(bx)}{x^2}\,dx$$

input
```
integrate(Shi(b*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x)/x^2, x)
```

Giac [F]

$$\int \frac{\mathrm{Shi}(bx)}{x^2}\,dx = \int \frac{\mathrm{Shi}(bx)}{x^2}\,dx$$

input
```
integrate(Shi(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(Shi(b*x)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\mathrm{Shi}(bx)}{x^2}\,dx = \int \frac{\mathrm{sinhint}(b\,x)}{x^2}\,dx$$

input
```
int(sinhint(b*x)/x^2,x)
```

output
```
int(sinhint(b*x)/x^2, x)
```

Reduce [F]

$$\int \frac{\mathrm{Shi}(bx)}{x^2}\,dx = \int \frac{shi(bx)}{x^2}dx$$

input
```
int(Shi(b*x)/x^2,x)
```

output
```
int(shi(b*x)/x**2,x)
```

3.8 $\int \frac{\text{Shi}(bx)}{x^3} dx$

Optimal result

Integrand size = 8, antiderivative size = 46

$$\int \frac{\text{Shi}(bx)}{x^3} dx = -\frac{b\cosh(bx)}{4x} - \frac{\sinh(bx)}{4x^2} + \frac{1}{4}b^2\text{Shi}(bx) - \frac{\text{Shi}(bx)}{2x^2}$$

output
```
-1/4*b*cosh(b*x)/x-1/4*sinh(b*x)/x^2+1/4*b^2*Shi(b*x)-1/2*Shi(b*x)/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\text{Shi}(bx)}{x^3} dx = -\frac{b\cosh(bx)}{4x} - \frac{\sinh(bx)}{4x^2} + \frac{1}{4}b^2\text{Shi}(bx) - \frac{\text{Shi}(bx)}{2x^2}$$

input
```
Integrate[SinhIntegral[b*x]/x^3,x]
```

output
```
-1/4*(b*Cosh[b*x])/x - Sinh[b*x]/(4*x^2) + (b^2*SinhIntegral[b*x])/4 - Sin
hIntegral[b*x]/(2*x^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.375, Rules used = {7086, 27, 3042, 26, 3778, 3042, 3778, 26, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(bx)}{x^3} \, dx$$

$$\Big\downarrow \text{7086}$$

$$\frac{1}{2} b \int \frac{\sinh(bx)}{bx^3} dx - \frac{\text{Shi}(bx)}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \int \frac{\sinh(bx)}{x^3} dx - \frac{\text{Shi}(bx)}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\text{Shi}(bx)}{2x^2} + \frac{1}{2} \int -\frac{i \sin(ibx)}{x^3} dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{\text{Shi}(bx)}{2x^2} - \frac{1}{2} i \int \frac{\sin(ibx)}{x^3} dx$$

$$\Big\downarrow \text{3778}$$

$$-\frac{\text{Shi}(bx)}{2x^2} - \frac{1}{2} i \left(\frac{1}{2} ib \int \frac{\cosh(bx)}{x^2} dx - \frac{i \sinh(bx)}{2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\text{Shi}(bx)}{2x^2} - \frac{1}{2} i \left(\frac{1}{2} ib \int \frac{\sin\left(ibx + \frac{\pi}{2}\right)}{x^2} dx - \frac{i \sinh(bx)}{2x^2}\right)$$

$$\Big\downarrow \text{3778}$$

$$-\frac{\text{Shi}(bx)}{2x^2} - \frac{1}{2} i \left(\frac{1}{2} ib \left(-\frac{\cosh(bx)}{x} + ib \int -\frac{i \sinh(bx)}{x} dx\right) - \frac{i \sinh(bx)}{2x^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-\frac{\text{Shi}(bx)}{2x^2}-\frac{1}{2}i\left(\frac{1}{2}ib\left(b\int\frac{\sinh(bx)}{x}dx-\frac{\cosh(bx)}{x}\right)-\frac{i\sinh(bx)}{2x^2}\right)$$
$$\Big\downarrow \; 3042$$
$$-\frac{\text{Shi}(bx)}{2x^2}-\frac{1}{2}i\left(\frac{1}{2}ib\left(-\frac{\cosh(bx)}{x}+b\int-\frac{i\sin(ibx)}{x}dx\right)-\frac{i\sinh(bx)}{2x^2}\right)$$
$$\Big\downarrow \; 26$$
$$-\frac{\text{Shi}(bx)}{2x^2}-\frac{1}{2}i\left(\frac{1}{2}ib\left(-\frac{\cosh(bx)}{x}-ib\int\frac{\sin(ibx)}{x}dx\right)-\frac{i\sinh(bx)}{2x^2}\right)$$
$$\Big\downarrow \; 3779$$
$$-\frac{\text{Shi}(bx)}{2x^2}-\frac{1}{2}i\left(\frac{1}{2}ib\left(b\text{Shi}(bx)-\frac{\cosh(bx)}{x}\right)-\frac{i\sinh(bx)}{2x^2}\right)$$

input
```
Int[SinhIntegral[b*x]/x^3,x]
```

output
```
-1/2*SinhIntegral[b*x]/x^2 - (I/2)*(((-1/2*I)*Sinh[b*x])/x^2 + (I/2)*b*(-(
Cosh[b*x]/x) + b*SinhIntegral[b*x]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 7086
```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(SinhIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Sinh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

method	result	size
parts	$-\frac{\mathrm{Shi}(bx)}{2x^{2}}+\frac{b^{2}\left(-\frac{\sinh(bx)}{2b^{2}x^{2}}-\frac{\cosh(bx)}{2bx}+\frac{\mathrm{Shi}(bx)}{2}\right)}{2}$	47
derivativedivides	$b^{2}\left(-\frac{\mathrm{Shi}(bx)}{2b^{2}x^{2}}-\frac{\sinh(bx)}{4b^{2}x^{2}}-\frac{\cosh(bx)}{4bx}+\frac{\mathrm{Shi}(bx)}{4}\right)$	48
default	$b^{2}\left(-\frac{\mathrm{Shi}(bx)}{2b^{2}x^{2}}-\frac{\sinh(bx)}{4b^{2}x^{2}}-\frac{\cosh(bx)}{4bx}+\frac{\mathrm{Shi}(bx)}{4}\right)$	48
meijerg	$\frac{i\sqrt{\pi}\,b^{2}\left(\frac{4i\cosh(bx)}{bx\sqrt{\pi}}+\frac{4i\sinh(bx)}{b^{2}x^{2}\sqrt{\pi}}+\frac{4i\left(-b^{2}x^{2}+2\right)\mathrm{Shi}(bx)}{b^{2}x^{2}\sqrt{\pi}}\right)}{16}$	69

input
```
int(Shi(b*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*Shi(b*x)/x^2+1/2*b^2*(-1/2/b^2/x^2*sinh(b*x)-1/2/b/x*cosh(b*x)+1/2*Sh
i(b*x))
```

Fricas [F]

$$\int \frac{\text{Shi}(bx)}{x^3}\,dx = \int \frac{\text{Shi}(bx)}{x^3}\,dx$$

input
```
integrate(Shi(b*x)/x^3,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*x)/x^3, x)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{\text{Shi}(bx)}{x^3}\,dx = \frac{b^2\,\text{Shi}\,(bx)}{4} - \frac{b\cosh\,(bx)}{4x} - \frac{\sinh\,(bx)}{4x^2} - \frac{\text{Shi}\,(bx)}{2x^2}$$

input
```
integrate(Shi(b*x)/x**3,x)
```

output
```
b**2*Shi(b*x)/4 - b*cosh(b*x)/(4*x) - sinh(b*x)/(4*x**2) - Shi(b*x)/(2*x**
2)
```

Maxima [F]

$$\int \frac{\text{Shi}(bx)}{x^3}\,dx = \int \frac{\text{Shi}(bx)}{x^3}\,dx$$

input
```
integrate(Shi(b*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x)/x^3, x)
```

Giac [F]

$$\int \frac{\text{Shi}(bx)}{x^3}\,dx = \int \frac{\text{Shi}(bx)}{x^3}\,dx$$

input
```
integrate(Shi(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(Shi(b*x)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{Shi}(bx)}{x^3}\,dx = \frac{b^2\,\text{sinhint}(b\,x)}{4} - \frac{\frac{\text{sinhint}(b\,x)}{2} + \frac{\sinh(b\,x)}{4} + \frac{b\,x\cosh(b\,x)}{4}}{x^2}$$

input
```
int(sinhint(b*x)/x^3,x)
```

output
```
(b^2*sinhint(b*x))/4 - (sinhint(b*x)/2 + sinh(b*x)/4 + (b*x*cosh(b*x))/4)/
x^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{\text{Shi}(bx)}{x^3}\,dx = \frac{-\cosh(bx)\,bx + shi(bx)\,b^2x^2 - 2shi(bx) - \sinh(bx)}{4x^2}$$

input
```
int(Shi(b*x)/x^3,x)
```

output
```
( - cosh(b*x)*b*x + shi(b*x)*b**2*x**2 - 2*shi(b*x) - sinh(b*x))/(4*x**2)
```

3.9 $\int x^m \mathbf{Shi}(bx)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \text{Shi}(bx)^2 \, dx = \text{Int}\left(x^m \text{Shi}(bx)^2, x\right)$$

output
```
Defer(Int)(x^m*Shi(b*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Shi}(bx)^2 \, dx = \int x^m \text{Shi}(bx)^2 \, dx$$

input
```
Integrate[x^m*SinhIntegral[b*x]^2,x]
```

output
```
Integrate[x^m*SinhIntegral[b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{Shi}(bx)^2 \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int x^m \text{Shi}(bx)^2 dx$$

input
```
Int[x^m*SinhIntegral[b*x]^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{Shi}(bx)^2 \, dx$$

input
```
int(x^m*Shi(b*x)^2,x)
```

output
```
int(x^m*Shi(b*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \mathrm{Shi}(bx)^2 \, dx = \int x^m \mathrm{Shi}(bx)^2 \, dx$$

input
```
integrate(x^m*Shi(b*x)^2,x, algorithm="fricas")
```

output
```
integral(x^m*sinh_integral(b*x)^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \mathrm{Shi}(bx)^2 \, dx = \int x^m \operatorname{Shi}^2(bx) \, dx$$

input
```
integrate(x**m*Shi(b*x)**2,x)
```

output
```
Integral(x**m*Shi(b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \mathrm{Shi}(bx)^2 \, dx = \int x^m \mathrm{Shi}(bx)^2 \, dx$$

input
```
integrate(x^m*Shi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*Shi(b*x)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Shi}(bx)^2 \, dx = \int x^m \text{Shi}(bx)^2 \, dx$$

input
```
integrate(x^m*Shi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^m*Shi(b*x)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Shi}(bx)^2 \, dx = \int x^m \, \text{sinhint}(b\,x)^2 \, dx$$

input
```
int(x^m*sinhint(b*x)^2,x)
```

output
```
int(x^m*sinhint(b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Shi}(bx)^2 \, dx = \int x^m shi(bx)^2 \, dx$$

input
```
int(x^m*Shi(b*x)^2,x)
```

output
```
int(x**m*shi(b*x)**2,x)
```

3.10 $\int x^3\mathbf{Shi}(bx)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 149

$$\int x^3\text{Shi}(bx)^2\,dx=\frac{x^2}{2b^2}-\frac{3\text{Chi}(2bx)}{2b^4}+\frac{3\log(x)}{2b^4}-\frac{x\cosh(bx)\sinh(bx)}{b^3}+\frac{2\sinh^2(bx)}{b^4}+\frac{x^2\sinh^2(bx)}{4b^2}-\frac{3x\cosh(bx)\text{Shi}(bx)}{b^3}-\frac{x^3\cosh(bx)\text{Shi}(bx)}{2b}+\frac{3\sinh(bx)\text{Shi}(bx)}{b^4}+\frac{3x^2\sinh(bx)\text{Shi}(bx)}{2b^2}+\frac{1}{4}x^4\text{Shi}(bx)^2$$

output
```
1/2*x^2/b^2-3/2*Chi(2*b*x)/b^4+3/2*ln(x)/b^4-x*cosh(b*x)*sinh(b*x)/b^3+2*s
inh(b*x)^2/b^4+1/4*x^2*sinh(b*x)^2/b^2-3*x*cosh(b*x)*Shi(b*x)/b^3-1/2*x^3*
cosh(b*x)*Shi(b*x)/b+3*sinh(b*x)*Shi(b*x)/b^4+3/2*x^2*sinh(b*x)*Shi(b*x)/b
^2+1/4*x^4*Shi(b*x)^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.72

$$\int x^3\text{Shi}(bx)^2\,dx=\frac{3b^2x^2+8\cosh(2bx)+b^2x^2\cosh(2bx)-12\text{Chi}(2bx)+12\log(x)-4bx\sinh(2bx)-4(bx(6+b^2x^2)\cosh(}{8b^4}$$

input
```
Integrate[x^3*SinhIntegral[b*x]^2,x]
```

output
```
(3*b^2*x^2 + 8*Cosh[2*b*x] + b^2*x^2*Cosh[2*b*x] - 12*CoshIntegral[2*b*x]
+ 12*Log[x] - 4*b*x*Sinh[2*b*x] - 4*(b*x*(6 + b^2*x^2)*Cosh[b*x] - 3*(2 +
b^2*x^2)*Sinh[b*x])*SinhIntegral[b*x] + 2*b^4*x^4*SinhIntegral[b*x]^2)/(8*
b^4)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.54, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 2.600$, Rules used = {7090, 7096, 27, 5895, 3042, 25, 3791, 15, 7102, 27, 3042, 25, 3791, 15, 7096, 27, 3042, 26, 3044, 15, 7100, 27, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{Shi}(bx)^2\,dx$$

$$\Big\downarrow \text{7090}$$

$$\frac{1}{4}x^4\text{Shi}(bx)^2-\frac{1}{2}\int x^3\sinh(bx)\text{Shi}(bx)dx$$

$$\Big\downarrow \text{7096}$$

$$\frac{1}{2}\left(\frac{3\int x^2\cosh(bx)\text{Shi}(bx)dx}{b}+\int \frac{x^2\cosh(bx)\sinh(bx)}{b}dx-\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}\right)+\frac{1}{4}x^4\text{Shi}(bx)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{3\int x^2\cosh(bx)\text{Shi}(bx)dx}{b}+\frac{\int x^2\cosh(bx)\sinh(bx)dx}{b}-\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}\right)+\frac{1}{4}x^4\text{Shi}(bx)^2$$

$$\Big\downarrow \text{5895}$$

$$\frac{1}{2}\left(\frac{3\int x^2\cosh(bx)\text{Shi}(bx)dx}{b}+\frac{\frac{x^2\sinh^2(bx)}{2b}-\frac{\int x\sinh^2(bx)dx}{b}}{b}-\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}\right)+\frac{1}{4}x^4\text{Shi}(bx)^2$$

$\Big\downarrow$ 3042

$$\frac{1}{4}x^4\text{Shi}(bx)^2+\frac{1}{2}\left(\frac{3\int x^2\cosh(bx)\text{Shi}(bx)dx}{b}+\frac{\frac{x^2\sinh^2(bx)}{2b}-\frac{\int -x\sin(ibx)^2dx}{b}}{b}-\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{4}x^4\text{Shi}(bx)^2+\frac{1}{2}\left(\frac{3\int x^2\cosh(bx)\text{Shi}(bx)dx}{b}+\frac{\frac{x^2\sinh^2(bx)}{2b}+\frac{\int x\sin(ibx)^2dx}{b}}{b}-\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}\right)$$

$\Big\downarrow$ 3791

$$\frac{1}{2}\left(\frac{\frac{\frac{\int xdx}{2}+\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{3\int x^2\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}\right)+\frac{1}{4}x^4\text{Shi}(bx)^2$$

$\Big\downarrow$ 15

$$\frac{1}{2}\left(\frac{3\int x^2\cosh(bx)\text{Shi}(bx)dx}{b}+\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}-\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}\right)+\frac{1}{4}x^4\text{Shi}(bx)^2$$

$\Big\downarrow$ 7102

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}-\int\frac{x\sinh^2(bx)}{b}dx+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}-\frac{x}{}\right.$$
$$\frac{1}{4}x^4\text{Shi}(bx)^2$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}-\frac{\int x\sinh^2(bx)dx}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}-x^3\ldots\right.$$

$$\frac{1}{4}x^4\text{Shi}(bx)^2$$

$\downarrow$ 3042

$$\frac{1}{4}x^4\text{Shi}(bx)^2+$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}-\frac{\int -x\sin(ibx)^2dx}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}-x^3\ldots\right.$$

$\downarrow$ 25

$$\frac{1}{4}x^4\text{Shi}(bx)^2+$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{\int x\sin(ibx)^2dx}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}-\frac{x^3\text{S}\ldots}{}\right.$$

$\downarrow$ 3791

$$\frac{1}{2}\left(\frac{3\left(\frac{\frac{\int xdx}{2}+\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}}{b}-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+x\ldots}{b}\right.$$

$$\frac{1}{4}x^4\text{Shi}(bx)^2$$

$\downarrow$ 15

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+x^2\ldots}{b}\right.$$

$$\frac{1}{4}x^4\text{Shi}(bx)^2$$

$\downarrow$ 7096

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}-\int \frac{\cosh(bx)\sinh(bx)}{b}dx+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}\right.$$

$$\frac{1}{4}x^4\text{Shi}(bx)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\int \cosh(bx)\sinh(bx)dx}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}\right.$$

$$\frac{1}{4}x^4\text{Shi}(bx)^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}x^4\text{Shi}(bx)^2+$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\int -i\cos(ibx)\sin(ibx)dx}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}\right.$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{4}x^4\text{Shi}(bx)^2+$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}+\frac{i\int \cos(ibx)\sin(ibx)dx}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}\right.$$

$$\Big\downarrow \text{3044}$$

$$\frac{1}{4}x^4\text{Shi}(bx)^2+$$
$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(\frac{\int i\sinh(bx)d(i\sinh(bx))}{b^2}-\frac{\int\cosh(bx)\text{Shi}(bx)dx}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}\right.$$

$$\Big\downarrow \; 15$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\int\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(b}{4b^2}}{}\right.$$
$$\frac{1}{4}x^4\text{Shi}(bx)^2$$

$$\Big\downarrow \; 7100$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}-\int\frac{\sinh^2(bx)}{bx}dx}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}\right.$$
$$\frac{1}{4}x^4\text{Shi}(bx)^2$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}-\frac{\int\frac{\sinh^2(bx)}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}\right.$$

$$\frac{1}{4}x^4\text{Shi}(bx)^2$$

$\downarrow$ 3042

$$\frac{1}{4}x^4\text{Shi}(bx)^2+$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}-\frac{\int-\frac{\sin(ibx)^2}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right.}{b}\right.$$

$\downarrow$ 25

$$\frac{1}{4}x^4\text{Shi}(bx)^2+$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}+\frac{\int\frac{\sin(ibx)^2}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}\right.$$

$\downarrow$ 3793

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\frac{\int \left(\frac{1}{2x}-\frac{\cosh(2bx)}{2x}\right)dx}{b}+\frac{\text{Shi}(bx)\sinh(bx)}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sin}{b}\right.}{b}\right.$$

$$\frac{1}{4}x^4\text{Shi}(bx)^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\sinh^2(bx)}{2b^2}-\frac{\frac{\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}}{b}+\frac{\text{Shi}(bx)\sinh(bx)}{b}}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(}{b}\right.}{b}\right.$$

$$\frac{1}{4}x^4\text{Shi}(bx)^2$$

input
```
Int[x^3*SinhIntegral[b*x]^2,x]
```

output
```
(x^4*SinhIntegral[b*x]^2)/4 + (((x^2*Sinh[b*x]^2)/(2*b) + (x^2/4 - (x*Cosh
[b*x]*Sinh[b*x])/(2*b) + Sinh[b*x]^2/(4*b^2))/b)/b - (x^3*Cosh[b*x]*SinhIn
tegral[b*x])/b + (3*((x^2/4 - (x*Cosh[b*x]*Sinh[b*x])/(2*b) + Sinh[b*x]^2/
(4*b^2))/b + (x^2*Sinh[b*x]*SinhIntegral[b*x])/b - (2*(-1/2*Sinh[b*x]^2/b^
2 + (x*Cosh[b*x]*SinhIntegral[b*x])/b - ((-1/2*CoshIntegral[2*b*x] + Log[x
]/2)/b + (Sinh[b*x]*SinhIntegral[b*x])/b)/b))/b))/b)/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 7090
```
Int[(x_)^(m_.)*SinhIntegral[(b_.)*(x_)]^2, x_Symbol] :> Simp[x^(m + 1)*(Sin
hIntegral[b*x]^2/(m + 1)), x] - Simp[2/(m + 1)   Int[x^m*Sinh[b*x]*SinhInte
gral[b*x], x], x] /; FreeQ[b, x] && IGtQ[m, 0]
```

rule 7096
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7100
```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7102
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{b^4x^4\operatorname{Shi}(bx)^2}{4}-2\operatorname{Shi}(bx)\left(\frac{b^3x^3\cosh(bx)}{4}-\frac{3b^2x^2\sinh(bx)}{4}+\frac{3bx\cosh(bx)}{2}-\frac{3\sinh(bx)}{2}\right)+\frac{b^2x^2\cosh(bx)^2}{4}-bx\cosh(bx)\sinh(b}{b^4}$
default	$\frac{\frac{b^4x^4\operatorname{Shi}(bx)^2}{4}-2\operatorname{Shi}(bx)\left(\frac{b^3x^3\cosh(bx)}{4}-\frac{3b^2x^2\sinh(bx)}{4}+\frac{3bx\cosh(bx)}{2}-\frac{3\sinh(bx)}{2}\right)+\frac{b^2x^2\cosh(bx)^2}{4}-bx\cosh(bx)\sinh(b}{b^4}$

input
```
int(x^3*Shi(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/4*b^4*x^4*Shi(b*x)^2-2*Shi(b*x)*(1/4*b^3*x^3*cosh(b*x)-3/4*b^2*x^
2*sinh(b*x)+3/2*b*x*cosh(b*x)-3/2*sinh(b*x))+1/4*b^2*x^2*cosh(b*x)^2-b*x*c
osh(b*x)*sinh(b*x)+1/4*b^2*x^2+2*cosh(b*x)^2+3/2*ln(b*x)-3/2*Chi(2*b*x))
```

Fricas [F]

$$\int x^3\text{Shi}(bx)^2\,dx=\int x^3\text{Shi}(bx)^2\,dx$$

input
```
integrate(x^3*Shi(b*x)^2,x, algorithm="fricas")
```

output
```
integral(x^3*sinh_integral(b*x)^2, x)
```

Sympy [F]

$$\int x^3\text{Shi}(bx)^2\,dx=\int x^3\,\text{Shi}^2\,(bx)\,dx$$

input
```
integrate(x**3*Shi(b*x)**2,x)
```

output
```
Integral(x**3*Shi(b*x)**2, x)
```

Maxima [F]

$$\int x^3 \text{Shi}(bx)^2 \, dx = \int x^3 \text{Shi}(bx)^2 \, dx$$

input
```
integrate(x^3*Shi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*Shi(b*x)^2, x)
```

Giac [F]

$$\int x^3 \text{Shi}(bx)^2 \, dx = \int x^3 \text{Shi}(bx)^2 \, dx$$

input
```
integrate(x^3*Shi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^3*Shi(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \text{Shi}(bx)^2 \, dx = \int x^3 \, \text{sinhint}(b\,x)^2 \, dx$$

input
```
int(x^3*sinhint(b*x)^2,x)
```

output
```
int(x^3*sinhint(b*x)^2, x)
```

Reduce [F]

$$\int x^3 \text{Shi}(bx)^2 \, dx = \int shi(bx)^2 \, x^3 dx$$

input
```
int(x^3*Shi(b*x)^2,x)
```

output
```
int(shi(b*x)**2*x**3,x)
```

3.11 $\int x^2\mathbf{Shi}(bx)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 112

$$\begin{aligned}\int x^2\text{Shi}(bx)^2\,dx=&\ \frac{5x}{6b^2}-\frac{5\cosh(bx)\sinh(bx)}{6b^3}+\frac{x\sinh^2(bx)}{3b^2}\\&-\frac{4\cosh(bx)\text{Shi}(bx)}{3b^3}-\frac{2x^2\cosh(bx)\text{Shi}(bx)}{3b}\\&+\frac{4x\sinh(bx)\text{Shi}(bx)}{3b^2}+\frac{1}{3}x^3\text{Shi}(bx)^2+\frac{2\text{Shi}(2bx)}{3b^3}\end{aligned}$$

output
```
5/6*x/b^2-5/6*cosh(b*x)*sinh(b*x)/b^3+1/3*x*sinh(b*x)^2/b^2-4/3*cosh(b*x)*
Shi(b*x)/b^3-2/3*x^2*cosh(b*x)*Shi(b*x)/b+4/3*x*sinh(b*x)*Shi(b*x)/b^2+1/3
*x^3*Shi(b*x)^2+2/3*Shi(2*b*x)/b^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int x^2\text{Shi}(bx)^2\,dx=\frac{8bx+2bx\cosh(2bx)-5\sinh(2bx)-8\left(\left(2+b^2x^2\right)\cosh(bx)-2bx\sinh(bx)\right)\text{Shi}(bx)+4b^3x^3\text{Shi}(bx)^2+8}{12b^3}$$

input
```
Integrate[x^2*SinhIntegral[b*x]^2,x]
```

output
```
(8*b*x + 2*b*x*Cosh[2*b*x] - 5*Sinh[2*b*x] - 8*((2 + b^2*x^2)*Cosh[b*x] -
2*b*x*Sinh[b*x])*SinhIntegral[b*x] + 4*b^3*x^3*SinhIntegral[b*x]^2 + 8*Sin
hIntegral[2*b*x])/(12*b^3)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.39, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.100, Rules used = {7090, 7096, 27, 5895, 3042, 25, 3115, 24, 7102, 27, 3042, 25, 3115, 24, 7094, 27, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{Shi}(bx)^2\,dx$$

$$\Big\downarrow \text{7090}$$

$$\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2}{3}\int x^2\sinh(bx)\text{Shi}(bx)dx$$

$$\Big\downarrow \text{7096}$$

$$\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2}{3}\left(-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\int\frac{x\cosh(bx)\sinh(bx)}{b}dx+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2}{3}\left(-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\int x\cosh(bx)\sinh(bx)dx}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}\right)$$

$$\Big\downarrow \text{5895}$$

$$\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2}{3}\left(-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}-\frac{\int\sinh^2(bx)dx}{2b}}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2}{3}\left(-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}-\frac{\int -\sin(ibx)^2dx}{2b}}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}\right)$$

$\big\downarrow$ 25

$$\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2}{3}\left(-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\int \sin(ibx)^2dx}{2b}}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}\right)$$

$\big\downarrow$ 3115

$$\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2}{3}\left(-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\frac{\frac{\int 1dx}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}+\frac{x\sinh^2(bx)}{2b}}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}\right)$$

$\big\downarrow$ 24

$$\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2}{3}\left(-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)$$

$\big\downarrow$ 7102

$$\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\int\sinh(bx)\text{Shi}(bx)dx}{b}-\int\frac{\sinh^2(bx)}{b}dx+\frac{x\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)$$

$\big\downarrow$ 27

$$\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\int\sinh(bx)\text{Shi}(bx)dx}{b}-\frac{\int\sinh^2(bx)dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)$$

$\big\downarrow$ 3042

$$\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}-\frac{\int -\sin(ibx)^2dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right.$$

$\downarrow$ 25

$$\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}+\frac{\int \sin(ibx)^2dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right.$$

$\downarrow$ 3115

$$\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}+\frac{\frac{\int 1dx}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right.$$

$\downarrow$ 24

$$\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right.$$

$\downarrow$ 7094

$$\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\int\frac{\cosh(bx)\sinh(bx)}{bx}dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{x\sinh^2(bx)}{2b}\right.$$

$\downarrow$ 27

$$\frac{1}{3}x^3\text{Shi}(bx)^2 - \frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int\frac{\cosh(bx)\sinh(bx)}{x}dx}{b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(b}{2b}}{}\right.$$

$$\downarrow 5971$$

$$\frac{1}{3}x^3\text{Shi}(bx)^2 - \frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int\frac{\sinh(2bx)}{2x}dx}{b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+}{}\right.$$

$$\downarrow 27$$

$$\frac{1}{3}x^3\text{Shi}(bx)^2 - \frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int\frac{\sinh(2bx)}{x}dx}{2b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+}{}\right.$$

$$\downarrow 3042$$

$$\frac{1}{3}x^3\text{Shi}(bx)^2 - \frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int-\frac{i\sin(2ibx)}{x}dx}{2b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+}{}\right.$$

$$\downarrow 26$$

$$\frac{1}{3}x^3\text{Shi}(bx)^2 - \frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}+\frac{i\int\frac{\sin(2ibx)}{x}dx}{2b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+}{}\right.$$

$$\downarrow 3779$$

$$\frac{1}{3}x^3\text{Shi}(bx)^2 - \frac{2}{3}\left(\frac{x^2\text{Shi}(bx)\cosh(bx)}{b} - \frac{2\left(\frac{x\text{Shi}(bx)\sinh(bx)}{b} - \frac{\frac{\text{Shi}(bx)\cosh(bx)}{b} - \frac{\text{Shi}(2bx)}{2b}}{b} + \frac{\frac{x}{2} - \frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b} - \frac{\frac{x\sinh^2(bx)}{2b} + \frac{\frac{x}{2} - \frac{\sinh(}{}}{}}{b}\right.$$

input
```
Int[x^2*SinhIntegral[b*x]^2,x]
```

output
```
(x^3*SinhIntegral[b*x]^2)/3 - (2*(-(((x*Sinh[b*x]^2)/(2*b) + (x/2 - (Cosh[
b*x]*Sinh[b*x])/(2*b))/(2*b))/b) + (x^2*Cosh[b*x]*SinhIntegral[b*x])/b - (
2*((x/2 - (Cosh[b*x]*Sinh[b*x])/(2*b))/b + (x*Sinh[b*x]*SinhIntegral[b*x])
/b - ((Cosh[b*x]*SinhIntegral[b*x])/b - SinhIntegral[2*b*x]/(2*b))/b))/b))
/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7090
```
Int[(x_)^(m_.)*SinhIntegral[(b_.)*(x_)]^2, x_Symbol] :> Simp[x^(m + 1)*(Sin
hIntegral[b*x]^2/(m + 1)), x] - Simp[2/(m + 1)   Int[x^m*Sinh[b*x]*SinhInte
gral[b*x], x], x] /; FreeQ[b, x] && IGtQ[m, 0]
```

rule 7094
```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7096
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7102
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\frac{b^3x^3\,\mathrm{Shi}(bx)^2}{3}-2\,\mathrm{Shi}(bx)\left(\frac{b^2x^2\cosh(bx)}{3}-\frac{2bx\sinh(bx)}{3}+\frac{2\cosh(bx)}{3}\right)+\frac{bx\cosh(bx)^2}{3}-\frac{5\cosh(bx)\sinh(bx)}{6}+\frac{bx}{2}+\frac{2\,\mathrm{Shi}(2bx)}{3}}{b^3}$
default	$\frac{\frac{b^3x^3\,\mathrm{Shi}(bx)^2}{3}-2\,\mathrm{Shi}(bx)\left(\frac{b^2x^2\cosh(bx)}{3}-\frac{2bx\sinh(bx)}{3}+\frac{2\cosh(bx)}{3}\right)+\frac{bx\cosh(bx)^2}{3}-\frac{5\cosh(bx)\sinh(bx)}{6}+\frac{bx}{2}+\frac{2\,\mathrm{Shi}(2bx)}{3}}{b^3}$

input
```
int(x^2*Shi(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*b^3*x^3*Shi(b*x)^2-2*Shi(b*x)*(1/3*b^2*x^2*cosh(b*x)-2/3*b*x*si
nh(b*x)+2/3*cosh(b*x))+1/3*b*x*cosh(b*x)^2-5/6*cosh(b*x)*sinh(b*x)+1/2*b*x
+2/3*Shi(2*b*x))
```

Fricas [F]

$$\int x^2\mathrm{Shi}(bx)^2\,dx=\int x^2\mathrm{Shi}(bx)^2\;dx$$

input
```
integrate(x^2*Shi(b*x)^2,x, algorithm="fricas")
```

output
```
integral(x^2*sinh_integral(b*x)^2, x)
```

Sympy [F]

$$\int x^2 \mathrm{Shi}(bx)^2 \, dx = \int x^2 \,\mathrm{Shi}^2\,(bx) \, dx$$

input
```
integrate(x**2*Shi(b*x)**2,x)
```

output
```
Integral(x**2*Shi(b*x)**2, x)
```

Maxima [F]

$$\int x^2 \mathrm{Shi}(bx)^2 \, dx = \int x^2 \mathrm{Shi}(bx)^2 \, dx$$

input
```
integrate(x^2*Shi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^2*Shi(b*x)^2, x)
```

Giac [F]

$$\int x^2 \mathrm{Shi}(bx)^2 \, dx = \int x^2 \mathrm{Shi}(bx)^2 \, dx$$

input
```
integrate(x^2*Shi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2*Shi(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{Shi}(bx)^2\,dx=\int x^2\,\mathrm{sinhint}(b\,x)^2\,dx$$

input
```
int(x^2*sinhint(b*x)^2,x)
```

output
```
int(x^2*sinhint(b*x)^2, x)
```

Reduce [F]

$$\int x^2\text{Shi}(bx)^2\,dx=\int shi(bx)^2\,x^2dx$$

input
```
int(x^2*Shi(b*x)^2,x)
```

output
```
int(shi(b*x)**2*x**2,x)
```

3.12 $\int x\textbf{Shi}(bx)^2\,dx$

Optimal result

Integrand size = 8, antiderivative size = 74

$$\int x\text{Shi}(bx)^2\,dx = -\frac{\text{Chi}(2bx)}{2b^2}+\frac{\log(x)}{2b^2}+\frac{\sinh^2(bx)}{2b^2}-\frac{x\cosh(bx)\text{Shi}(bx)}{b}+\frac{\sinh(bx)\text{Shi}(bx)}{b^2}+\frac{1}{2}x^2\text{Shi}(bx)^2$$

output
```
-1/2*Chi(2*b*x)/b^2+1/2*ln(x)/b^2+1/2*sinh(b*x)^2/b^2-x*cosh(b*x)*Shi(b*x)
/b+sinh(b*x)*Shi(b*x)/b^2+1/2*x^2*Shi(b*x)^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int x\text{Shi}(bx)^2\,dx = \frac{\cosh(2bx)-2\text{Chi}(2bx)+2\log(x)+(-4bx\cosh(bx)+4\sinh(bx))\text{Shi}(bx)+2b^2x^2\text{Shi}(bx)^2}{4b^2}$$

input
```
Integrate[x*SinhIntegral[b*x]^2,x]
```

output
```
(Cosh[2*b*x] - 2*CoshIntegral[2*b*x] + 2*Log[x] + (-4*b*x*Cosh[b*x] + 4*Si
nh[b*x])*SinhIntegral[b*x] + 2*b^2*x^2*SinhIntegral[b*x]^2)/(4*b^2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.625$, Rules used = {7090, 7096, 27, 3042, 26, 3044, 15, 7100, 27, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Shi}(bx)^2\,dx$$

$$\Big\downarrow \text{7090}$$

$$\frac{1}{2}x^2\text{Shi}(bx)^2-\int x\sinh(bx)\text{Shi}(bx)dx$$

$$\Big\downarrow \text{7096}$$

$$\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}+\int \frac{\cosh(bx)\sinh(bx)}{b}dx+\frac{1}{2}x^2\text{Shi}(bx)^2-\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}+\frac{\int \cosh(bx)\sinh(bx)dx}{b}+\frac{1}{2}x^2\text{Shi}(bx)^2-\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}+\frac{\int -i\cos(ibx)\sin(ibx)dx}{b}+\frac{1}{2}x^2\text{Shi}(bx)^2-\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}-\frac{i\int \cos(ibx)\sin(ibx)dx}{b}+\frac{1}{2}x^2\text{Shi}(bx)^2-\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3044}$$

$$-\frac{\int i\sinh(bx)d(i\sinh(bx))}{b^2}+\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}+\frac{1}{2}x^2\text{Shi}(bx)^2-\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}+\frac{\sinh^2(bx)}{2b^2}+\frac{1}{2}x^2\text{Shi}(bx)^2-\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{7100}$$

$$\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}-\int\frac{\sinh^2(bx)}{bx}dx}{b}+\frac{\sinh^2(bx)}{2b^2}+\frac{1}{2}x^2\text{Shi}(bx)^2-\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}-\frac{\int\frac{\sinh^2(bx)}{x}dx}{b}}{b}+\frac{\sinh^2(bx)}{2b^2}+\frac{1}{2}x^2\text{Shi}(bx)^2-\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}-\frac{\int-\frac{\sin(ibx)^2}{x}dx}{b}}{b}+\frac{\sinh^2(bx)}{2b^2}+\frac{1}{2}x^2\text{Shi}(bx)^2-\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}+\frac{\int\frac{\sin(ibx)^2}{x}dx}{b}}{b}+\frac{\sinh^2(bx)}{2b^2}+\frac{1}{2}x^2\text{Shi}(bx)^2-\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\frac{\int\left(\frac{1}{2x}-\frac{\cosh(2bx)}{2x}\right)dx}{b}+\frac{\text{Shi}(bx)\sinh(bx)}{b}}{b}+\frac{\sinh^2(bx)}{2b^2}+\frac{1}{2}x^2\text{Shi}(bx)^2-\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sinh^2(bx)}{2b^2}+\frac{\frac{\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}}{b}+\frac{\text{Shi}(bx)\sinh(bx)}{b}}{b}+\frac{1}{2}x^2\text{Shi}(bx)^2-\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

input
```
Int[x*SinhIntegral[b*x]^2,x]
```

output
```
Sinh[b*x]^2/(2*b^2) - (x*Cosh[b*x]*SinhIntegral[b*x])/b + (x^2*SinhIntegra
l[b*x]^2)/2 + ((-1/2*CoshIntegral[2*b*x] + Log[x]/2)/b + (Sinh[b*x]*SinhIn
tegral[b*x])/b)/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7090
```
Int[(x_)^(m_.)*SinhIntegral[(b_.)*(x_)]^2, x_Symbol] :> Simp[x^(m + 1)*(Sin
hIntegral[b*x]^2/(m + 1)), x] - Simp[2/(m + 1)   Int[x^m*Sinh[b*x]*SinhInte
gral[b*x], x], x] /; FreeQ[b, x] && IGtQ[m, 0]
```

rule 7096
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7100
```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{b^2x^2\operatorname{Shi}(bx)^2}{2}-2\operatorname{Shi}(bx)\left(\frac{bx\cosh(bx)}{2}-\frac{\sinh(bx)}{2}\right)+\frac{\cosh(bx)^2}{2}+\frac{\ln(bx)}{2}-\frac{\operatorname{Chi}(2bx)}{2}}{b^2}$	62
default	$\frac{\frac{b^2x^2\operatorname{Shi}(bx)^2}{2}-2\operatorname{Shi}(bx)\left(\frac{bx\cosh(bx)}{2}-\frac{\sinh(bx)}{2}\right)+\frac{\cosh(bx)^2}{2}+\frac{\ln(bx)}{2}-\frac{\operatorname{Chi}(2bx)}{2}}{b^2}$	62

input
```
int(x*Shi(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*b^2*x^2*Shi(b*x)^2-2*Shi(b*x)*(1/2*b*x*cosh(b*x)-1/2*sinh(b*x))
+1/2*cosh(b*x)^2+1/2*ln(b*x)-1/2*Chi(2*b*x))
```

Fricas [F]

$$\int x\operatorname{Shi}(bx)^2\,dx=\int x\operatorname{Shi}(bx)^2\,dx$$

input
```
integrate(x*Shi(b*x)^2,x, algorithm="fricas")
```

output
```
integral(x*sinh_integral(b*x)^2, x)
```

Sympy [F]

$$\int x\text{Shi}(bx)^2\,dx = \int x\,\text{Shi}^2\,(bx)\,dx$$

input
```
integrate(x*Shi(b*x)**2,x)
```

output
```
Integral(x*Shi(b*x)**2, x)
```

Maxima [F]

$$\int x\text{Shi}(bx)^2\,dx = \int x\text{Shi}(bx)^2\,dx$$

input
```
integrate(x*Shi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x*Shi(b*x)^2, x)
```

Giac [F]

$$\int x\text{Shi}(bx)^2\,dx = \int x\text{Shi}(bx)^2\,dx$$

input
```
integrate(x*Shi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x*Shi(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{Shi}(bx)^2\,dx = \int x\,\text{sinhint}(b\,x)^2\,dx$$

input
```
int(x*sinhint(b*x)^2,x)
```

output
```
int(x*sinhint(b*x)^2, x)
```

Reduce [F]

$$\int x\text{Shi}(bx)^2\,dx = \int shi(bx)^2\,xdx$$

input
```
int(x*Shi(b*x)^2,x)
```

output
```
int(shi(b*x)**2*x,x)
```

3.13 $\int \text{Shi}(bx)^2\,dx$

Optimal result

Integrand size = 6, antiderivative size = 31

$$\int \text{Shi}(bx)^2\,dx = -\frac{2\cosh(bx)\text{Shi}(bx)}{b} + x\text{Shi}(bx)^2 + \frac{\text{Shi}(2bx)}{b}$$

output
```
-2*cosh(b*x)*Shi(b*x)/b+x*Shi(b*x)^2+Shi(2*b*x)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \text{Shi}(bx)^2\,dx = -\frac{2\cosh(bx)\text{Shi}(bx)}{b} + x\text{Shi}(bx)^2 + \frac{\text{Shi}(2bx)}{b}$$

input
```
Integrate[SinhIntegral[b*x]^2,x]
```

output
```
(-2*Cosh[b*x]*SinhIntegral[b*x])/b + x*SinhIntegral[b*x]^2 + SinhIntegral[
2*b*x]/b
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {7088, 7094, 27, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Shi}(bx)^2 \, dx$$

$$\Big\downarrow \text{7088}$$

$$x\text{Shi}(bx)^2 - 2\int \sinh(bx)\text{Shi}(bx)dx$$

$$\Big\downarrow \text{7094}$$

$$x\text{Shi}(bx)^2 - 2\left(\frac{\text{Shi}(bx)\cosh(bx)}{b} - \int \frac{\cosh(bx)\sinh(bx)}{bx}dx\right)$$

$$\Big\downarrow \text{27}$$

$$x\text{Shi}(bx)^2 - 2\left(\frac{\text{Shi}(bx)\cosh(bx)}{b} - \frac{\int \frac{\cosh(bx)\sinh(bx)}{x}dx}{b}\right)$$

$$\Big\downarrow \text{5971}$$

$$x\text{Shi}(bx)^2 - 2\left(\frac{\text{Shi}(bx)\cosh(bx)}{b} - \frac{\int \frac{\sinh(2bx)}{2x}dx}{b}\right)$$

$$\Big\downarrow \text{27}$$

$$x\text{Shi}(bx)^2 - 2\left(\frac{\text{Shi}(bx)\cosh(bx)}{b} - \frac{\int \frac{\sinh(2bx)}{x}dx}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$x\text{Shi}(bx)^2 - 2\left(\frac{\text{Shi}(bx)\cosh(bx)}{b} - \frac{\int -\frac{i\sin(2ibx)}{x}dx}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$x\text{Shi}(bx)^2 - 2\left(\frac{\text{Shi}(bx)\cosh(bx)}{b} + \frac{i\int \frac{\sin(2ibx)}{x}dx}{2b}\right)$$

$$\Big\downarrow \text{3779}$$

$$x\text{Shi}(bx)^2 - 2\left(\frac{\text{Shi}(bx)\cosh(bx)}{b} - \frac{\text{Shi}(2bx)}{2b}\right)$$

input
```
Int[SinhIntegral[b*x]^2,x]
```

output
```
x*SinhIntegral[b*x]^2 - 2*((Cosh[b*x]*SinhIntegral[b*x])/b - SinhIntegral[
2*b*x]/(2*b))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7088
```
Int[SinhIntegral[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(SinhIn
tegral[a + b*x]^2/b), x] - Simp[2   Int[Sinh[a + b*x]*SinhIntegral[a + b*x]
, x], x] /; FreeQ[{a, b}, x]
```

rule 7094
```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{\mathrm{Shi}(bx)^2 bx-2\cosh(bx)\,\mathrm{Shi}(bx)+\mathrm{Shi}(2bx)}{b}$	30
default	$\frac{\mathrm{Shi}(bx)^2 bx-2\cosh(bx)\,\mathrm{Shi}(bx)+\mathrm{Shi}(2bx)}{b}$	30

input
```
int(Shi(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(Shi(b*x)^2*b*x-2*cosh(b*x)*Shi(b*x)+Shi(2*b*x))
```

Fricas [F]

$$\int \mathrm{Shi}(bx)^2\,dx=\int \mathrm{Shi}(bx)^2\,dx$$

input
```
integrate(Shi(b*x)^2,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*x)^2, x)
```

Sympy [F]

$$\int \mathrm{Shi}(bx)^2\,dx = \int \mathrm{Shi}^2\left(bx\right)dx$$

input
```
integrate(Shi(b*x)**2,x)
```

output
```
Integral(Shi(b*x)**2, x)
```

Maxima [F]

$$\int \mathrm{Shi}(bx)^2\,dx = \int \mathrm{Shi}(bx)^2\,dx$$

input
```
integrate(Shi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x)^2, x)
```

Giac [F]

$$\int \mathrm{Shi}(bx)^2\,dx = \int \mathrm{Shi}(bx)^2\,dx$$

input
```
integrate(Shi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(Shi(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{Shi}(bx)^2\,dx = \int \operatorname{sinhint}(b\,x)^2\,dx$$

input
```
int(sinhint(b*x)^2,x)
```

output
```
int(sinhint(b*x)^2, x)
```

Reduce [F]

$$\int \operatorname{Shi}(bx)^2\,dx = \int shi(bx)^2\,dx$$

input
```
int(Shi(b*x)^2,x)
```

output
```
int(shi(b*x)**2,x)
```

3.14 $\int \frac{\mathbf{Shi}(bx)^2}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{Shi}(bx)^2}{x}\,dx = \text{Int}\left(\frac{\text{Shi}(bx)^2}{x},x\right)$$

output
```
Defer(Int)(Shi(b*x)^2/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Shi}(bx)^2}{x}\,dx = \int \frac{\text{Shi}(bx)^2}{x}\,dx$$

input
```
Integrate[SinhIntegral[b*x]^2/x,x]
```

output
```
Integrate[SinhIntegral[b*x]^2/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(bx)^2}{x} \, dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{\text{Shi}(bx)^2}{x} dx$$

input
```
Int[SinhIntegral[b*x]^2/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{Shi}\,(bx)^2}{x} dx$$

input
```
int(Shi(b*x)^2/x,x)
```

output
```
int(Shi(b*x)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Shi}(bx)^2}{x}\,dx = \int \frac{\text{Shi}(bx)^2}{x}\,dx$$

input
```
integrate(Shi(b*x)^2/x,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*x)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\text{Shi}(bx)^2}{x}\,dx = \int \frac{\text{Shi}^2\,(bx)}{x}\,dx$$

input
```
integrate(Shi(b*x)**2/x,x)
```

output
```
Integral(Shi(b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Shi}(bx)^2}{x}\,dx = \int \frac{\text{Shi}(bx)^2}{x}\,dx$$

input
```
integrate(Shi(b*x)^2/x,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x)^2/x, x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Shi}(bx)^2}{x} \, dx = \int \frac{\mathrm{Shi}(bx)^2}{x} \, dx$$

input
```
integrate(Shi(b*x)^2/x,x, algorithm="giac")
```

output
```
integrate(Shi(b*x)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Shi}(bx)^2}{x} \, dx = \int \frac{\mathrm{sinhint}(b\,x)^2}{x} \, dx$$

input
```
int(sinhint(b*x)^2/x,x)
```

output
```
int(sinhint(b*x)^2/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{Shi}(bx)^2}{x}\,dx = \int \frac{shi(bx)^2}{x}dx$$

input
```
int(Shi(b*x)^2/x,x)
```

output
```
int(shi(b*x)**2/x,x)
```

3.15 $\int \frac{\mathrm{Shi}(bx)^2}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\mathrm{Shi}(bx)^2}{x^2}\,dx = \mathrm{Int}\left(\frac{\mathrm{Shi}(bx)^2}{x^2},x\right)$$

output
```
Defer(Int)(Shi(b*x)^2/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Shi}(bx)^2}{x^2}\,dx = \int \frac{\mathrm{Shi}(bx)^2}{x^2}\,dx$$

input
```
Integrate[SinhIntegral[b*x]^2/x^2,x]
```

output
```
Integrate[SinhIntegral[b*x]^2/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(bx)^2}{x^2} \, dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{\text{Shi}(bx)^2}{x^2}dx$$

input
```
Int[SinhIntegral[b*x]^2/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{Shi}\left(bx\right)^2}{x^2}dx$$

input
```
int(Shi(b*x)^2/x^2,x)
```

output
```
int(Shi(b*x)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Shi}(bx)^2}{x^2}\,dx = \int \frac{\text{Shi}(bx)^2}{x^2}\,dx$$

input
```
integrate(Shi(b*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*x)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{Shi}(bx)^2}{x^2}\,dx = \int \frac{\text{Shi}^2\left(bx\right)}{x^2}\,dx$$

input
```
integrate(Shi(b*x)**2/x**2,x)
```

output
```
Integral(Shi(b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Shi}(bx)^2}{x^2}\,dx = \int \frac{\text{Shi}(bx)^2}{x^2}\,dx$$

input
```
integrate(Shi(b*x)^2/x^2,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x)^2/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Shi}(bx)^2}{x^2}\,dx = \int \frac{\mathrm{Shi}(bx)^2}{x^2}\,dx$$

input
```
integrate(Shi(b*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate(Shi(b*x)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Shi}(bx)^2}{x^2}\,dx = \int \frac{\mathrm{sinhint}(b\,x)^2}{x^2}\,dx$$

input
```
int(sinhint(b*x)^2/x^2,x)
```

output
```
int(sinhint(b*x)^2/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Shi}(bx)^2}{x^2}\,dx = \int \frac{shi(bx)^2}{x^2}dx$$

input
```
int(Shi(b*x)^2/x^2,x)
```

output
```
int(shi(b*x)**2/x**2,x)
```

3.16 $\int \frac{\text{Shi}(bx)^2}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{Shi}(bx)^2}{x^3} dx = \text{Int}\left(\frac{\text{Shi}(bx)^2}{x^3}, x\right)$$

output
```
Defer(Int)(Shi(b*x)^2/x^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Shi}(bx)^2}{x^3} dx = \int \frac{\text{Shi}(bx)^2}{x^3} dx$$

input
```
Integrate[SinhIntegral[b*x]^2/x^3,x]
```

output
```
Integrate[SinhIntegral[b*x]^2/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(bx)^2}{x^3} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\text{Shi}(bx)^2}{x^3}dx$$

input
```
Int[SinhIntegral[b*x]^2/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{Shi}\left(bx\right)^2}{x^3}dx$$

input
```
int(Shi(b*x)^2/x^3,x)
```

output
```
int(Shi(b*x)^2/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Shi}(bx)^2}{x^3} \, dx = \int \frac{\text{Shi}(bx)^2}{x^3} \, dx$$

input
```
integrate(Shi(b*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*x)^2/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{Shi}(bx)^2}{x^3} \, dx = \int \frac{\text{Shi}^2(bx)}{x^3} \, dx$$

input
```
integrate(Shi(b*x)**2/x**3,x)
```

output
```
Integral(Shi(b*x)**2/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Shi}(bx)^2}{x^3} \, dx = \int \frac{\text{Shi}(bx)^2}{x^3} \, dx$$

input
```
integrate(Shi(b*x)^2/x^3,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x)^2/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Shi}(bx)^2}{x^3} \, dx = \int \frac{\text{Shi}(bx)^2}{x^3} \, dx$$

input
```
integrate(Shi(b*x)^2/x^3,x, algorithm="giac")
```

output
```
integrate(Shi(b*x)^2/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Shi}(bx)^2}{x^3} \, dx = \int \frac{\text{sinhint}(b\,x)^2}{x^3} \, dx$$

input
```
int(sinhint(b*x)^2/x^3,x)
```

output
```
int(sinhint(b*x)^2/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{Shi}(bx)^2}{x^3}\,dx = \int \frac{shi(bx)^2}{x^3}dx$$

input
```
int(Shi(b*x)^2/x^3,x)
```

output
```
int(shi(b*x)**2/x**3,x)
```

3.17 $\int x^m \mathbf{Shi}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \text{Shi}(a+bx)\,dx = \frac{x^{1+m}\text{Shi}(a+bx)}{1+m} - \frac{b\text{Int}\left(\frac{x^{1+m}\sinh(a+bx)}{a+bx},x\right)}{1+m}$$

output
```
x^(1+m)*Shi(b*x+a)/(1+m)-b*Defer(Int)(x^(1+m)*sinh(b*x+a)/(b*x+a),x)/(1+m)
```

Mathematica [N/A]

Not integrable

Time = 6.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Shi}(a+bx)\,dx = \int x^m \text{Shi}(a+bx)\,dx$$

input
```
Integrate[x^m*SinhIntegral[a + b*x],x]
```

output
```
Integrate[x^m*SinhIntegral[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{Shi}(a+bx)\,dx$$

$$\Big\downarrow \text{ 7086}$$

$$\frac{x^{m+1}\text{Shi}(a+bx)}{m+1}-\frac{b\int \frac{x^{m+1}\sinh(a+bx)}{a+bx}dx}{m+1}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{x^{m+1}\text{Shi}(a+bx)}{m+1}-\frac{b\int \frac{x^{m+1}\sinh(a+bx)}{a+bx}dx}{m+1}$$

input
```
Int[x^m*SinhIntegral[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \,\text{Shi}\,(bx+a)\,dx$$

input
```
int(x^m*Shi(b*x+a),x)
```

output
```
int(x^m*Shi(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \mathrm{Shi}(a+bx)\,dx = \int x^m \mathrm{Shi}(bx+a)\,dx$$

input
```
integrate(x^m*Shi(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^m*sinh_integral(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \mathrm{Shi}(a+bx)\,dx = \int x^m \,\mathrm{Shi}\,(a+bx)\,dx$$

input
```
integrate(x**m*Shi(b*x+a),x)
```

output
```
Integral(x**m*Shi(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \mathrm{Shi}(a+bx)\,dx = \int x^m \mathrm{Shi}(bx+a)\,dx$$

input
```
integrate(x^m*Shi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^m*Shi(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \mathrm{Shi}(a+bx)\,dx = \int x^m \mathrm{Shi}(bx+a)\,dx$$

input
```
integrate(x^m*Shi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*Shi(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 4.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \mathrm{Shi}(a+bx)\,dx = \int x^m\,\mathrm{sinhint}(a+b\,x)\,dx$$

input
```
int(x^m*sinhint(a + b*x),x)
```

output
```
int(x^m*sinhint(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Shi}(a+bx)\,dx = \int x^m shi(bx+a)\,dx$$

input
```
int(x^m*Shi(b*x+a),x)
```

output
```
int(x**m*shi(a + b*x),x)
```

3.18 $\int x^3\mathbf{Shi}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 184

$$\int x^3\text{Shi}(a+bx)\,dx=\frac{a\cosh(a+bx)}{2b^4}+\frac{a^3\cosh(a+bx)}{4b^4}-\frac{3x\cosh(a+bx)}{2b^3}-\frac{a^2x\cosh(a+bx)}{4b^3}+\frac{ax^2\cosh(a+bx)}{4b^2}-\frac{x^3\cosh(a+bx)}{4b}+\frac{3\sinh(a+bx)}{2b^4}+\frac{a^2\sinh(a+bx)}{4b^4}-\frac{ax\sinh(a+bx)}{2b^3}+\frac{3x^2\sinh(a+bx)}{4b^2}-\frac{a^4\text{Shi}(a+bx)}{4b^4}+\frac{1}{4}x^4\text{Shi}(a+bx)$$

output
```
1/2*a*cosh(b*x+a)/b^4+1/4*a^3*cosh(b*x+a)/b^4-3/2*x*cosh(b*x+a)/b^3-1/4*a^
2*x*cosh(b*x+a)/b^3+1/4*a*x^2*cosh(b*x+a)/b^2-1/4*x^3*cosh(b*x+a)/b+3/2*si
nh(b*x+a)/b^4+1/4*a^2*sinh(b*x+a)/b^4-1/2*a*x*sinh(b*x+a)/b^3+3/4*x^2*sinh
(b*x+a)/b^2-1/4*a^4*Shi(b*x+a)/b^4+1/4*x^4*Shi(b*x+a)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.51

$$\int x^3\text{Shi}(a+bx)\,dx$$
$$=\frac{(2a+a^3-6bx-a^2bx+ab^2x^2-b^3x^3)\cosh(a+bx)+(6+a^2-2abx+3b^2x^2)\sinh(a+bx)+(-a^4+b}{4b^4}$$

input
```
Integrate[x^3*SinhIntegral[a + b*x],x]
```

output
```
((2*a + a^3 - 6*b*x - a^2*b*x + a*b^2*x^2 - b^3*x^3)*Cosh[a + b*x] + (6 +
a^2 - 2*a*b*x + 3*b^2*x^2)*Sinh[a + b*x] + (-a^4 + b^4*x^4)*SinhIntegral[a
 + b*x])/(4*b^4)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7086, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{Shi}(a+bx)\,dx$$
$$\Big\downarrow \text{7086}$$
$$\frac{1}{4}x^4\text{Shi}(a+bx)-\frac{1}{4}b\int \frac{x^4\sinh(a+bx)}{a+bx}dx$$
$$\Big\downarrow \text{7293}$$
$$\frac{1}{4}x^4\text{Shi}(a+bx)-$$
$$\frac{1}{4}b\int \left(\frac{\sinh(a+bx)a^4}{b^4(a+bx)}-\frac{\sinh(a+bx)a^3}{b^4}+\frac{x\sinh(a+bx)a^2}{b^3}-\frac{x^2\sinh(a+bx)a}{b^2}+\frac{x^3\sinh(a+bx)}{b}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4\text{Shi}(a+bx)-$$
$$\frac{1}{4}b\left(\frac{a^4\text{Shi}(a+bx)}{b^5}-\frac{a^3\cosh(a+bx)}{b^5}-\frac{a^2\sinh(a+bx)}{b^5}+\frac{a^2x\cosh(a+bx)}{b^4}-\frac{6\sinh(a+bx)}{b^5}-\frac{2a\cosh(a+bx)}{b^5}\right.$$

input
```
Int[x^3*SinhIntegral[a + b*x],x]
```

output
```
(x^4*SinhIntegral[a + b*x])/4 - (b*((-2*a*Cosh[a + b*x])/b^5 - (a^3*Cosh[a
 + b*x])/b^5 + (6*x*Cosh[a + b*x])/b^4 + (a^2*x*Cosh[a + b*x])/b^4 - (a*x^
2*Cosh[a + b*x])/b^3 + (x^3*Cosh[a + b*x])/b^2 - (6*Sinh[a + b*x])/b^5 - (
a^2*Sinh[a + b*x])/b^5 + (2*a*x*Sinh[a + b*x])/b^4 - (3*x^2*Sinh[a + b*x])
/b^3 + (a^4*SinhIntegral[a + b*x])/b^5))/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7086
```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(SinhIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Sinh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84

method	result
parts	$\frac{x^4 \operatorname{Shi}(bx+a)}{4} - \frac{a^4 \operatorname{Shi}(bx+a)-4a^3\cosh(bx+a)+6a^2((bx+a)\cosh(bx+a)-\sinh(bx+a))-4a\left((bx+a)^2\cosh(bx+a)-\right.}{}$
derivativedivides	$\frac{\frac{\operatorname{Shi}(bx+a)b^4x^4}{4}-\frac{a^4\operatorname{Shi}(bx+a)}{4}+a^3\cosh(bx+a)-\frac{3a^2((bx+a)\cosh(bx+a)-\sinh(bx+a))}{2}+a\left((bx+a)^2\cosh(bx+a)-2(bx+a)\,\mathrm{s}\right.}{b^4}$
default	$\frac{\frac{\operatorname{Shi}(bx+a)b^4x^4}{4}-\frac{a^4\operatorname{Shi}(bx+a)}{4}+a^3\cosh(bx+a)-\frac{3a^2((bx+a)\cosh(bx+a)-\sinh(bx+a))}{2}+a\left((bx+a)^2\cosh(bx+a)-2(bx+a)\,\mathrm{s}\right.}{b^4}$

input
```
int(x^3*Shi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*Shi(b*x+a)-1/4/b^4*(a^4*Shi(b*x+a)-4*a^3*cosh(b*x+a)+6*a^2*((b*x+a
)*cosh(b*x+a)-sinh(b*x+a))-4*a*((b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x+a
)+2*cosh(b*x+a))+(b*x+a)^3*cosh(b*x+a)-3*(b*x+a)^2*sinh(b*x+a)+6*(b*x+a)*c
osh(b*x+a)-6*sinh(b*x+a))
```

Fricas [F]

$$\int x^3 \operatorname{Shi}(a+bx)\,dx = \int x^3 \operatorname{Shi}(bx+a)\,dx$$

input
```
integrate(x^3*Shi(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^3*sinh_integral(b*x + a), x)
```

Sympy [F]

$$\int x^3\text{Shi}(a+bx)\,dx = \int x^3\,\text{Shi}\,(a+bx)\,dx$$

input
```
integrate(x**3*Shi(b*x+a),x)
```

output
```
Integral(x**3*Shi(a + b*x), x)
```

Maxima [F]

$$\int x^3\text{Shi}(a+bx)\,dx = \int x^3\text{Shi}(bx+a)\,dx$$

input
```
integrate(x^3*Shi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^3*Shi(b*x + a), x)
```

Giac [F]

$$\int x^3\text{Shi}(a+bx)\,dx = \int x^3\text{Shi}(bx+a)\,dx$$

input
```
integrate(x^3*Shi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*Shi(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \text{Shi}(a+bx)\,dx = \int x^3\,\text{sinhint}(a+b\,x)\,\,dx$$

input
```
int(x^3*sinhint(a + b*x),x)
```

output
```
int(x^3*sinhint(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.78

$$\int x^3 \text{Shi}(a+bx)\,dx = \frac{\cosh(bx+a)\,a^3-\cosh(bx+a)\,a^2bx+\cosh(bx+a)\,a\,b^2x^2+2\cosh(bx+a)\,a-\cosh(bx+a)\,b^3x^3-6$$

input
```
int(x^3*Shi(b*x+a),x)
```

output
```
(cosh(a + b*x)*a**3 - cosh(a + b*x)*a**2*b*x + cosh(a + b*x)*a*b**2*x**2 +
 2*cosh(a + b*x)*a - cosh(a + b*x)*b**3*x**3 - 6*cosh(a + b*x)*b*x - shi(a
 + b*x)*a**4 + shi(a + b*x)*b**4*x**4 + sinh(a + b*x)*a**2 - 2*sinh(a + b*
x)*a*b*x + 3*sinh(a + b*x)*b**2*x**2 + 6*sinh(a + b*x))/(4*b**4)
```

3.19 $\int x^2\mathbf{Shi}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 118

$$\int x^2\text{Shi}(a+bx)\,dx=-\frac{2\cosh(a+bx)}{3b^3}-\frac{a^2\cosh(a+bx)}{3b^3}+\frac{ax\cosh(a+bx)}{3b^2}-\frac{x^2\cosh(a+bx)}{3b}-\frac{a\sinh(a+bx)}{3b^3}+\frac{2x\sinh(a+bx)}{3b^2}+\frac{a^3\text{Shi}(a+bx)}{3b^3}+\frac{1}{3}x^3\text{Shi}(a+bx)$$

output
```
-2/3*cosh(b*x+a)/b^3-1/3*a^2*cosh(b*x+a)/b^3+1/3*a*x*cosh(b*x+a)/b^2-1/3*x
^2*cosh(b*x+a)/b-1/3*a*sinh(b*x+a)/b^3+2/3*x*sinh(b*x+a)/b^2+1/3*a^3*Shi(b
*x+a)/b^3+1/3*x^3*Shi(b*x+a)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.54

$$\int x^2\text{Shi}(a+bx)\,dx=-\frac{(2+a^2-abx+b^2x^2)\cosh(a+bx)+(a-2bx)\sinh(a+bx)-(a^3+b^3x^3)\,\text{Shi}(a+bx)}{3b^3}$$

input
```
Integrate[x^2*SinhIntegral[a + b*x],x]
```

output
```
-1/3*((2 + a^2 - a*b*x + b^2*x^2)*Cosh[a + b*x] + (a - 2*b*x)*Sinh[a + b*x
] - (a^3 + b^3*x^3)*SinhIntegral[a + b*x])/b^3
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7086, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{Shi}(a+bx)\,dx$$

$$\Big\downarrow \text{7086}$$

$$\frac{1}{3}x^3\text{Shi}(a+bx)-\frac{1}{3}b\int \frac{x^3\sinh(a+bx)}{a+bx}dx$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{3}x^3\text{Shi}(a+bx)-\frac{1}{3}b\int \left(-\frac{\sinh(a+bx)a^3}{b^3(a+bx)}+\frac{\sinh(a+bx)a^2}{b^3}-\frac{x\sinh(a+bx)a}{b^2}+\frac{x^2\sinh(a+bx)}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\text{Shi}(a+bx)-\frac{1}{3}b\left(-\frac{a^3\text{Shi}(a+bx)}{b^4}+\frac{a^2\cosh(a+bx)}{b^4}+\frac{a\sinh(a+bx)}{b^4}+\frac{2\cosh(a+bx)}{b^4}-\frac{2x\sinh(a+bx)}{b^3}-\frac{ax\cosh(a+bx)}{b^3}\right.$$

input
```
Int[x^2*SinhIntegral[a + b*x],x]
```

output
```
(x^3*SinhIntegral[a + b*x])/3 - (b*((2*Cosh[a + b*x])/b^4 + (a^2*Cosh[a +
b*x])/b^4 - (a*x*Cosh[a + b*x])/b^3 + (x^2*Cosh[a + b*x])/b^2 + (a*Sinh[a
+ b*x])/b^4 - (2*x*Sinh[a + b*x])/b^3 - (a^3*SinhIntegral[a + b*x])/b^4))/
3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7086
```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(SinhIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Sinh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

method	result
parts	$\frac{x^3 \operatorname{Shi}(bx+a)}{3} - \frac{-a^3 \operatorname{Shi}(bx+a)+3a^2 \cosh(bx+a)-3a((bx+a)\cosh(bx+a)-\sinh(bx+a))+(bx+a)^2 \cosh(bx+a)-2(b \cdots}{3b^3}$
derivativedivides	$\frac{\frac{\operatorname{Shi}(bx+a)b^3x^3}{3}+\frac{a^3 \operatorname{Shi}(bx+a)}{3}-a^2\cosh(bx+a)+a((bx+a)\cosh(bx+a)-\sinh(bx+a))-\frac{(bx+a)^2\cosh(bx+a)}{3}+\frac{2(bx+a)\sinh(\cdots}{3} \cdots}{b^3}$
default	$\frac{\frac{\operatorname{Shi}(bx+a)b^3x^3}{3}+\frac{a^3 \operatorname{Shi}(bx+a)}{3}-a^2\cosh(bx+a)+a((bx+a)\cosh(bx+a)-\sinh(bx+a))-\frac{(bx+a)^2\cosh(bx+a)}{3}+\frac{2(bx+a)\sinh(\cdots}{3} \cdots}{b^3}$

input
```
int(x^2*Shi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*Shi(b*x+a)-1/3/b^3*(-a^3*Shi(b*x+a)+3*a^2*cosh(b*x+a)-3*a*((b*x+a)
*cosh(b*x+a)-sinh(b*x+a))+(b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x+a)+2*co
sh(b*x+a))
```

Fricas [F]

$$\int x^2\text{Shi}(a+bx)\,dx = \int x^2\text{Shi}(bx+a)\;dx$$

input
```
integrate(x^2*Shi(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^2*sinh_integral(b*x + a), x)
```

Sympy [F]

$$\int x^2\text{Shi}(a+bx)\,dx = \int x^2\,\text{Shi}\,(a+bx)\,dx$$

input
```
integrate(x**2*Shi(b*x+a),x)
```

output
```
Integral(x**2*Shi(a + b*x), x)
```

Maxima [F]

$$\int x^2\text{Shi}(a+bx)\,dx = \int x^2\text{Shi}(bx+a)\;dx$$

input
```
integrate(x^2*Shi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^2*Shi(b*x + a), x)
```

Giac [F]

$$\int x^2\text{Shi}(a+bx)\,dx=\int x^2\text{Shi}(bx+a)\,dx$$

input
```
integrate(x^2*Shi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*Shi(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{Shi}(a+bx)\,dx=\int x^2\,\text{sinhint}(a+b\,x)\,dx$$

input
```
int(x^2*sinhint(a + b*x),x)
```

output
```
int(x^2*sinhint(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.77

$$\int x^2\text{Shi}(a+bx)\,dx$$
$$=\frac{-\cosh(bx+a)\,a^2+\cosh(bx+a)\,abx-\cosh(bx+a)\,b^2x^2-2\cosh(bx+a)+shi(bx+a)\,a^3+shi(bx}{3b^3}$$

input
```
int(x^2*Shi(b*x+a),x)
```

output
```
( - cosh(a + b*x)*a**2 + cosh(a + b*x)*a*b*x - cosh(a + b*x)*b**2*x**2 - 2
*cosh(a + b*x) + shi(a + b*x)*a**3 + shi(a + b*x)*b**3*x**3 - sinh(a + b*x
)*a + 2*sinh(a + b*x)*b*x)/(3*b**3)
```

3.20 $\int x\mathbf{Shi}(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 62

$$\int x\text{Shi}(a+bx)\,dx=\frac{(a-bx)\cosh(a+bx)}{2b^2}+\frac{\sinh(a+bx)}{2b^2}-\frac{a^2\text{Shi}(a+bx)}{2b^2}+\frac{1}{2}x^2\text{Shi}(a+bx)$$

output
```
1/2*(-b*x+a)*cosh(b*x+a)/b^2+1/2*sinh(b*x+a)/b^2-1/2*a^2*Shi(b*x+a)/b^2+1/
2*x^2*Shi(b*x+a)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int x\text{Shi}(a+bx)\,dx=\frac{(a-bx)\cosh(a+bx)+\sinh(a+bx)+(-a^2+b^2x^2)\,\text{Shi}(a+bx)}{2b^2}$$

input
```
Integrate[x*SinhIntegral[a + b*x],x]
```

output
```
((a - b*x)*Cosh[a + b*x] + Sinh[a + b*x] + (-a^2 + b^2*x^2)*SinhIntegral[a
 + b*x])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {7086, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Shi}(a+bx)\,dx$$

$$\Big\downarrow \ 7086$$

$$\frac{1}{2}x^2\text{Shi}(a+bx)-\frac{1}{2}b\int \frac{x^2\sinh(a+bx)}{a+bx}dx$$

$$\Big\downarrow \ 7293$$

$$\frac{1}{2}x^2\text{Shi}(a+bx)-\frac{1}{2}b\int \left(\frac{\sinh(a+bx)a^2}{b^2(a+bx)}-\frac{\sinh(a+bx)a}{b^2}+\frac{x\sinh(a+bx)}{b}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}x^2\text{Shi}(a+bx)-\frac{1}{2}b\left(\frac{a^2\text{Shi}(a+bx)}{b^3}-\frac{\sinh(a+bx)}{b^3}-\frac{a\cosh(a+bx)}{b^3}+\frac{x\cosh(a+bx)}{b^2}\right)$$

input
```
Int[x*SinhIntegral[a + b*x],x]
```

output
```
(x^2*SinhIntegral[a + b*x])/2 - (b*(-((a*Cosh[a + b*x])/b^3) + (x*Cosh[a +
 b*x])/b^2 - Sinh[a + b*x]/b^3 + (a^2*SinhIntegral[a + b*x])/b^3))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7086
```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(SinhIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Sinh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

method	result	size
parts	$\frac{x^2 \operatorname{Shi}(bx+a)}{2} - \frac{a^2 \operatorname{Shi}(bx+a) - 2a\cosh(bx+a) + (bx+a)\cosh(bx+a) - \sinh(bx+a)}{2b^2}$	58
derivativedivides	$\frac{\operatorname{Shi}(bx+a)\left(\frac{(bx+a)^2}{2} - (bx+a)a\right) + a\cosh(bx+a) - \frac{(bx+a)\cosh(bx+a)}{2} + \frac{\sinh(bx+a)}{2}}{b^2}$	60
default	$\frac{\operatorname{Shi}(bx+a)\left(\frac{(bx+a)^2}{2} - (bx+a)a\right) + a\cosh(bx+a) - \frac{(bx+a)\cosh(bx+a)}{2} + \frac{\sinh(bx+a)}{2}}{b^2}$	60

input
```
int(x*Shi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*Shi(b*x+a)-1/2/b^2*(a^2*Shi(b*x+a)-2*a*cosh(b*x+a)+(b*x+a)*cosh(b*
x+a)-sinh(b*x+a))
```

Fricas [F]

$$\int x\operatorname{Shi}(a+bx)\,dx = \int x\operatorname{Shi}(bx+a)\,dx$$

input
```
integrate(x*Shi(b*x+a),x, algorithm="fricas")
```

output
```
integral(x*sinh_integral(b*x + a), x)
```

Sympy [F]

$$\int x\mathrm{Shi}(a + bx)\, dx = \int x\,\mathrm{Shi}\,(a + bx)\, dx$$

input
```
integrate(x*Shi(b*x+a),x)
```

output
```
Integral(x*Shi(a + b*x), x)
```

Maxima [F]

$$\int x\mathrm{Shi}(a + bx)\, dx = \int x\mathrm{Shi}(bx + a)\, dx$$

input
```
integrate(x*Shi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*Shi(b*x + a), x)
```

Giac [F]

$$\int x\mathrm{Shi}(a + bx)\, dx = \int x\mathrm{Shi}(bx + a)\, dx$$

input
```
integrate(x*Shi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*Shi(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{Shi}(a+bx)\,dx$$
$$=\frac{\frac{e^{-a-b\,x}\,\left(a+e^{2\,a+2\,b\,x}+a\,e^{2\,a+2\,b\,x}-2\,a^2\,\mathrm{sinhint}(a+b\,x)\,e^{a+b\,x}-1\right)}{4}-\frac{b\,e^{-a-b\,x}\,\left(x+x\,e^{2\,a+2\,b\,x}\right)}{4}}{b^2}$$
$$+\frac{x^2\,\mathrm{sinhint}(a+b\,x)}{2}$$

input
```
int(x*sinhint(a + b*x),x)
```

output
```
((exp(- a - b*x)*(a + exp(2*a + 2*b*x) + a*exp(2*a + 2*b*x) - 2*a^2*sinhin
t(a + b*x)*exp(a + b*x) - 1))/4 - (b*exp(- a - b*x)*(x + x*exp(2*a + 2*b*x
)))/4)/b^2 + (x^2*sinhint(a + b*x))/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int x\text{Shi}(a+bx)\,dx$$
$$=\frac{\cosh(bx+a)\,a-\cosh(bx+a)\,bx-shi(bx+a)\,a^2+shi(bx+a)\,b^2x^2+\sinh(bx+a)}{2b^2}$$

input
```
int(x*Shi(b*x+a),x)
```

output
```
(cosh(a + b*x)*a - cosh(a + b*x)*b*x - shi(a + b*x)*a**2 + shi(a + b*x)*b*
*2*x**2 + sinh(a + b*x))/(2*b**2)
```

3.21 $\int \text{Shi}(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 27

$$\int \text{Shi}(a+bx)\,dx=-\frac{\cosh(a+bx)}{b}+\frac{(a+bx)\text{Shi}(a+bx)}{b}$$

output
```
-cosh(b*x+a)/b+(b*x+a)*Shi(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \text{Shi}(a+bx)\,dx=-\frac{\cosh(a)\cosh(bx)}{b}-\frac{\sinh(a)\sinh(bx)}{b}+\frac{a\text{Shi}(a+bx)}{b}+x\text{Shi}(a+bx)$$

input
```
Integrate[SinhIntegral[a + b*x],x]
```

output
```
-((Cosh[a]*Cosh[b*x])/b) - (Sinh[a]*Sinh[b*x])/b + (a*SinhIntegral[a + b*x
])/b + x*SinhIntegral[a + b*x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7082}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Shi}(a+bx)\,dx$$

$$\Big\downarrow \text{ 7082}$$

$$\frac{(a+bx)\text{Shi}(a+bx)}{b}-\frac{\cosh(a+bx)}{b}$$

input
```
Int[SinhIntegral[a + b*x],x]
```

output
```
-(Cosh[a + b*x]/b) + ((a + b*x)*SinhIntegral[a + b*x])/b
```

Defintions of rubi rules used

rule 7082
```
Int[SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(SinhInte
gral[a + b*x]/b), x] - Simp[Cosh[a + b*x]/b, x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\text{Shi}(bx+a)(bx+a)-\cosh(bx+a)}{b}$	26
default	$\frac{\text{Shi}(bx+a)(bx+a)-\cosh(bx+a)}{b}$	26
parts	$x\ \text{Shi}\left(bx+a\right)-\frac{\cosh(bx+a)-\text{Shi}(bx+a)a}{b}$	31

input
```
int(Shi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(Shi(b*x+a)*(b*x+a)-cosh(b*x+a))
```

Fricas [F]

$$\int \mathrm{Shi}(a+bx)\,dx = \int \mathrm{Shi}(bx+a)\,dx$$

input
```
integrate(Shi(b*x+a),x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*x + a), x)
```

Sympy [F]

$$\int \mathrm{Shi}(a+bx)\,dx = \int \mathrm{Shi}\,(a+bx)\,dx$$

input
```
integrate(Shi(b*x+a),x)
```

output
```
Integral(Shi(a + b*x), x)
```

Maxima [F]

$$\int \mathrm{Shi}(a+bx)\,dx = \int \mathrm{Shi}(bx+a)\,dx$$

input
```
integrate(Shi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(Shi(b*x + a), x)
```

Giac [F]

$$\int \operatorname{Shi}(a+bx)\,dx = \int \operatorname{Shi}(bx+a)\,dx$$

input
```
integrate(Shi(b*x+a),x, algorithm="giac")
```

output
```
integrate(Shi(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{Shi}(a+bx)\,dx = x\,\mathrm{sinhint}(a+b\,x) - \frac{e^{a+b\,x}}{2\,b} - \frac{e^{-a-b\,x}}{2\,b} + \frac{a\,\mathrm{sinhint}(a+b\,x)}{b}$$

input
```
int(sinhint(a + b*x),x)
```

output
```
x*sinhint(a + b*x) - exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) + (a*sinhin
t(a + b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \operatorname{Shi}(a+bx)\,dx = \frac{-\cosh(bx+a) + shi(bx+a)\,a + shi(bx+a)\,bx}{b}$$

input
```
int(Shi(b*x+a),x)
```

output
```
( - cosh(a + b*x) + shi(a + b*x)*a + shi(a + b*x)*b*x)/b
```

3.22 $\int \frac{\text{Shi}(a+bx)}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{Shi}(a+bx)}{x} \, dx = \text{Int}\left(\frac{\text{Shi}(a+bx)}{x}, x\right)$$

output
```
Defer(Int)(Shi(b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Shi}(a+bx)}{x} \, dx = \int \frac{\text{Shi}(a+bx)}{x} \, dx$$

input
```
Integrate[SinhIntegral[a + b*x]/x,x]
```

output
```
Integrate[SinhIntegral[a + b*x]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(a+bx)}{x} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\text{Shi}(a+bx)}{x}dx$$

input
```
Int[SinhIntegral[a + b*x]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{Shi}\left(bx+a\right)}{x}dx$$

input
```
int(Shi(b*x+a)/x,x)
```

output
```
int(Shi(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Shi}(a+bx)}{x}\,dx = \int \frac{\mathrm{Shi}(bx+a)}{x}\,dx$$

input
```
integrate(Shi(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\mathrm{Shi}(a+bx)}{x}\,dx = \int \frac{\mathrm{Shi}\,(a+bx)}{x}\,dx$$

input
```
integrate(Shi(b*x+a)/x,x)
```

output
```
Integral(Shi(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Shi}(a+bx)}{x}\,dx = \int \frac{\mathrm{Shi}(bx+a)}{x}\,dx$$

input
```
integrate(Shi(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Shi}(a+bx)}{x}\,dx=\int \frac{\mathrm{Shi}(bx+a)}{x}\,dx$$

input
```
integrate(Shi(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(Shi(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Shi}(a+bx)}{x}\,dx=\int \frac{\mathrm{sinhint}(a+b\,x)}{x}\,dx$$

input
```
int(sinhint(a + b*x)/x,x)
```

output
```
int(sinhint(a + b*x)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Shi}(a+bx)}{x}\,dx = \int \frac{shi(bx+a)}{x}dx$$

input
```
int(Shi(b*x+a)/x,x)
```

output
```
int(shi(a + b*x)/x,x)
```

3.23 $\int \frac{\text{Shi}(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{\text{Shi}(a+bx)}{x^2}\,dx = \frac{b\text{Chi}(bx)\sinh(a)}{a} + \frac{b\cosh(a)\text{Shi}(bx)}{a} - \frac{b\text{Shi}(a+bx)}{a} - \frac{\text{Shi}(a+bx)}{x}$$

output
```
b*Chi(b*x)*sinh(a)/a+b*cosh(a)*Shi(b*x)/a-b*Shi(b*x+a)/a-Shi(b*x+a)/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{\text{Shi}(a+bx)}{x^2}\,dx = \frac{bx\text{Chi}(bx)\sinh(a) + bx\cosh(a)\text{Shi}(bx) - (a+bx)\text{Shi}(a+bx)}{ax}$$

input
```
Integrate[SinhIntegral[a + b*x]/x^2,x]
```

output
```
(b*x*CoshIntegral[b*x]*Sinh[a] + b*x*Cosh[a]*SinhIntegral[b*x] - (a + b*x)
*SinhIntegral[a + b*x])/(a*x)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {7086, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{7086}$$

$$b\int \frac{\sinh(a+bx)}{x(a+bx)}dx-\frac{\text{Shi}(a+bx)}{x}$$

$$\Big\downarrow \text{7293}$$

$$b\int \left(\frac{\sinh(a+bx)}{ax}-\frac{b\sinh(a+bx)}{a(a+bx)}\right)dx-\frac{\text{Shi}(a+bx)}{x}$$

$$\Big\downarrow \text{2009}$$

$$b\left(\frac{\sinh(a)\text{Chi}(bx)}{a}-\frac{\text{Shi}(a+bx)}{a}+\frac{\cosh(a)\text{Shi}(bx)}{a}\right)-\frac{\text{Shi}(a+bx)}{x}$$

input
```
Int[SinhIntegral[a + b*x]/x^2,x]
```

output
```
-(SinhIntegral[a + b*x]/x) + b*((CoshIntegral[b*x]*Sinh[a])/a + (Cosh[a]*S
inhIntegral[b*x])/a - SinhIntegral[a + b*x]/a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7086
```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(SinhIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Sinh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\text{Shi}\left(bx+a\right)}{x^2}dx$$

input
```
int(Shi(b*x+a)/x^2,x)
```

output
```
int(Shi(b*x+a)/x^2,x)
```

Fricas [F]

$$\int \frac{\text{Shi}(a+bx)}{x^2}\,dx = \int \frac{\text{Shi}(bx+a)}{x^2}\,dx$$

input
```
integrate(Shi(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*x + a)/x^2, x)
```

Sympy [F]

$$\int \frac{\text{Shi}(a+bx)}{x^2}\,dx = \int \frac{\text{Shi}\left(a+bx\right)}{x^2}\,dx$$

input
```
integrate(Shi(b*x+a)/x**2,x)
```

output
```
Integral(Shi(a + b*x)/x**2, x)
```

Maxima [F]

$$\int \frac{\text{Shi}(a+bx)}{x^2}\,dx = \int \frac{\text{Shi}(bx+a)}{x^2}\,dx$$

input
```
integrate(Shi(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x + a)/x^2, x)
```

Giac [F]

$$\int \frac{\text{Shi}(a+bx)}{x^2}\,dx = \int \frac{\text{Shi}(bx+a)}{x^2}\,dx$$

input
```
integrate(Shi(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(Shi(b*x + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{Shi}(a+bx)}{x^2}\,dx = \int \frac{\text{sinhint}(a+b\,x)}{x^2}\,dx$$

input
```
int(sinhint(a + b*x)/x^2,x)
```

output
```
int(sinhint(a + b*x)/x^2, x)
```

Reduce [F]

$$\int \frac{\text{Shi}(a+bx)}{x^2}\,dx = \int \frac{shi(bx+a)}{x^2}dx$$

input
```
int(Shi(b*x+a)/x^2,x)
```

output
```
int(shi(a + b*x)/x**2,x)
```

3.24 $\int \frac{\mathrm{Shi}(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 111

$$\int \frac{\mathrm{Shi}(a+bx)}{x^3}\,dx = \frac{b^2\cosh(a)\mathrm{Chi}(bx)}{2a} - \frac{b^2\mathrm{Chi}(bx)\sinh(a)}{2a^2} - \frac{b\sinh(a+bx)}{2ax} - \frac{b^2\cosh(a)\mathrm{Shi}(bx)}{2a^2} + \frac{b^2\sinh(a)\mathrm{Shi}(bx)}{2a} + \frac{b^2\mathrm{Shi}(a+bx)}{2a^2} - \frac{\mathrm{Shi}(a+bx)}{2x^2}$$

output
```
1/2*b^2*cosh(a)*Chi(b*x)/a-1/2*b^2*Chi(b*x)*sinh(a)/a^2-1/2*b*sinh(b*x+a)/
a/x-1/2*b^2*cosh(a)*Shi(b*x)/a^2+1/2*b^2*sinh(a)*Shi(b*x)/a+1/2*b^2*Shi(b*
x+a)/a^2-1/2*Shi(b*x+a)/x^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{\mathrm{Shi}(a+bx)}{x^3}\,dx = \frac{b^2x^2\mathrm{Chi}(bx)(a\cosh(a)-\sinh(a)) - abx\sinh(a+bx) + b^2x^2(-\cosh(a)+a\sinh(a))\mathrm{Shi}(bx) - a^2\mathrm{Shi}(a+}{2a^2x^2}$$

input
```
Integrate[SinhIntegral[a + b*x]/x^3,x]
```

output
```
(b^2*x^2*CoshIntegral[b*x]*(a*Cosh[a] - Sinh[a]) - a*b*x*Sinh[a + b*x] + b
^2*x^2*(-Cosh[a] + a*Sinh[a])*SinhIntegral[b*x] - a^2*SinhIntegral[a + b*x
] + b^2*x^2*SinhIntegral[a + b*x])/(2*a^2*x^2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.82, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {7086, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(a+bx)}{x^3}dx$$

$$\Big\downarrow \text{7086}$$

$$\frac{1}{2}b\int \frac{\sinh(a+bx)}{x^2(a+bx)}dx-\frac{\text{Shi}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{2}b\int \left(\frac{\sinh(a+bx)b^2}{a^2(a+bx)}-\frac{\sinh(a+bx)b}{a^2x}+\frac{\sinh(a+bx)}{ax^2}\right)dx-\frac{\text{Shi}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}b\left(-\frac{b\sinh(a)\text{Chi}(bx)}{a^2}+\frac{b\text{Shi}(a+bx)}{a^2}-\frac{b\cosh(a)\text{Shi}(bx)}{a^2}+\frac{b\cosh(a)\text{Chi}(bx)}{a}+\frac{b\sinh(a)\text{Shi}(bx)}{a}-\frac{\sinh(a+b\right.$$
$$\frac{\text{Shi}(a+bx)}{2x^2}$$

input
```
Int[SinhIntegral[a + b*x]/x^3,x]
```

output
```
-1/2*SinhIntegral[a + b*x]/x^2 + (b*((b*Cosh[a]*CoshIntegral[b*x])/a - (b*
CoshIntegral[b*x]*Sinh[a])/a^2 - Sinh[a + b*x]/(a*x) - (b*Cosh[a]*SinhInte
gral[b*x])/a^2 + (b*Sinh[a]*SinhIntegral[b*x])/a + (b*SinhIntegral[a + b*x
])/a^2))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7086
```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(SinhIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Sinh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\mathrm{Shi}(bx + a)}{x^3} dx$$

input
```
int(Shi(b*x+a)/x^3,x)
```

output
```
int(Shi(b*x+a)/x^3,x)
```

Fricas [F]

$$\int \frac{\text{Shi}(a+bx)}{x^3}\,dx = \int \frac{\text{Shi}(bx+a)}{x^3}\,dx$$

input
```
integrate(Shi(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*x + a)/x^3, x)
```

Sympy [F]

$$\int \frac{\text{Shi}(a+bx)}{x^3}\,dx = \int \frac{\text{Shi}\,(a+bx)}{x^3}\,dx$$

input
```
integrate(Shi(b*x+a)/x**3,x)
```

output
```
Integral(Shi(a + b*x)/x**3, x)
```

Maxima [F]

$$\int \frac{\text{Shi}(a+bx)}{x^3}\,dx = \int \frac{\text{Shi}(bx+a)}{x^3}\,dx$$

input
```
integrate(Shi(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x + a)/x^3, x)
```

Giac [F]

$$\int \frac{\text{Shi}(a+bx)}{x^3}\,dx=\int \frac{\text{Shi}(bx+a)}{x^3}\,dx$$

input
```
integrate(Shi(b*x+a)/x^3,x, algorithm="giac")
```

output
```
integrate(Shi(b*x + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{Shi}(a+bx)}{x^3}\,dx=\int \frac{\text{sinhint}(a+b\,x)}{x^3}\,dx$$

input
```
int(sinhint(a + b*x)/x^3,x)
```

output
```
int(sinhint(a + b*x)/x^3, x)
```

Reduce [F]

$$\int \frac{\text{Shi}(a+bx)}{x^3}\,dx=\int \frac{shi(bx+a)}{x^3}dx$$

input
```
int(Shi(b*x+a)/x^3,x)
```

output
```
int(shi(a + b*x)/x**3,x)
```

3.25 $\int x^m \textbf{Shi}(a+bx)^2 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \text{Shi}(a+bx)^2 \, dx = \text{Int}\left(x^m \text{Shi}(a+bx)^2, x\right)$$

output
```
Defer(Int)(x^m*Shi(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 4.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \text{Shi}(a+bx)^2 \, dx = \int x^m \text{Shi}(a+bx)^2 \, dx$$

input
```
Integrate[x^m*SinhIntegral[a + b*x]^2,x]
```

output
```
Integrate[x^m*SinhIntegral[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{Shi}(a+bx)^2 \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int x^m \text{Shi}(a+bx)^2 dx$$

input
```
Int[x^m*SinhIntegral[a + b*x]^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{Shi}\left(bx+a\right)^2 dx$$

input
```
int(x^m*Shi(b*x+a)^2,x)
```

output
```
int(x^m*Shi(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \text{Shi}(a+bx)^2\,dx = \int x^m \text{Shi}(bx+a)^2\,dx$$

input
```
integrate(x^m*Shi(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m*sinh_integral(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \text{Shi}(a+bx)^2\,dx = \int x^m \,\text{Shi}^2\,(a+bx)\,dx$$

input
```
integrate(x**m*Shi(b*x+a)**2,x)
```

output
```
Integral(x**m*Shi(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \text{Shi}(a+bx)^2\,dx = \int x^m \text{Shi}(bx+a)^2\,dx$$

input
```
integrate(x^m*Shi(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*Shi(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \text{Shi}(a+bx)^2 \, dx = \int x^m \text{Shi}(bx+a)^2 \, dx$$

input
```
integrate(x^m*Shi(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*Shi(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \text{Shi}(a+bx)^2 \, dx = \int x^m \,\text{sinhint}(a+b\,x)^2 \, dx$$

input
```
int(x^m*sinhint(a + b*x)^2,x)
```

output
```
int(x^m*sinhint(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \text{Shi}(a+bx)^2 \, dx = \int x^m shi(bx+a)^2 \, dx$$

input
```
int(x^m*Shi(b*x+a)^2,x)
```

output
```
int(x**m*shi(a + b*x)**2,x)
```

3.26 $\int x^2\mathbf{Shi}(a+bx)^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 333

$$\begin{aligned}\int x^2\text{Shi}(a+bx)^2\,dx=&\ \frac{2x}{3b^2}-\frac{a\cosh(2a+2bx)}{6b^3}-\frac{(a-bx)\cosh(2a+2bx)}{6b^3}\\&+\frac{a\text{Chi}(2a+2bx)}{b^3}-\frac{a\log(a+bx)}{b^3}\\&-\frac{2\cosh(a+bx)\sinh(a+bx)}{3b^3}-\frac{\sinh(2a+2bx)}{12b^3}\\&-\frac{4\cosh(a+bx)\text{Shi}(a+bx)}{3b^3}-\frac{2a^2\cosh(a+bx)\text{Shi}(a+bx)}{3b^3}\\&+\frac{2ax\cosh(a+bx)\text{Shi}(a+bx)}{3b^2}-\frac{2x^2\cosh(a+bx)\text{Shi}(a+bx)}{3b}\\&-\frac{2a\sinh(a+bx)\text{Shi}(a+bx)}{3b^3}+\frac{4x\sinh(a+bx)\text{Shi}(a+bx)}{3b^2}\\&+\frac{a^2(a+bx)\text{Shi}(a+bx)^2}{3b^3}-\frac{ax(a+bx)\text{Shi}(a+bx)^2}{3b^2}\\&+\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}+\frac{2\text{Shi}(2a+2bx)}{3b^3}+\frac{a^2\text{Shi}(2a+2bx)}{b^3}\end{aligned}$$

output
```
2/3*x/b^2-1/6*a*cosh(2*b*x+2*a)/b^3-1/6*(-b*x+a)*cosh(2*b*x+2*a)/b^3+a*Chi
(2*b*x+2*a)/b^3-a*ln(b*x+a)/b^3-2/3*cosh(b*x+a)*sinh(b*x+a)/b^3-1/12*sinh(
2*b*x+2*a)/b^3-4/3*cosh(b*x+a)*Shi(b*x+a)/b^3-2/3*a^2*cosh(b*x+a)*Shi(b*x+
a)/b^3+2/3*a*x*cosh(b*x+a)*Shi(b*x+a)/b^2-2/3*x^2*cosh(b*x+a)*Shi(b*x+a)/b
-2/3*a*sinh(b*x+a)*Shi(b*x+a)/b^3+4/3*x*sinh(b*x+a)*Shi(b*x+a)/b^2+1/3*a^2
*(b*x+a)*Shi(b*x+a)^2/b^3-1/3*a*x*(b*x+a)*Shi(b*x+a)^2/b^2+1/3*x^2*(b*x+a)
*Shi(b*x+a)^2/b+2/3*Shi(2*b*x+2*a)/b^3+a^2*Shi(2*b*x+2*a)/b^3
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.47

$$\int x^2 \text{Shi}(a+bx)^2\,dx$$
$$= \frac{8a+8bx-4a\cosh(2(a+bx))+2bx\cosh(2(a+bx))+12a\text{Chi}(2(a+bx))-12a\log(a+bx)-5\sinh(2}{}$$

input
```
Integrate[x^2*SinhIntegral[a + b*x]^2,x]
```

output
```
(8*a + 8*b*x - 4*a*Cosh[2*(a + b*x)] + 2*b*x*Cosh[2*(a + b*x)] + 12*a*Cosh
Integral[2*(a + b*x)] - 12*a*Log[a + b*x] - 5*Sinh[2*(a + b*x)] - 8*((2 +
a^2 - a*b*x + b^2*x^2)*Cosh[a + b*x] + (a - 2*b*x)*Sinh[a + b*x])*SinhInte
gral[a + b*x] + 4*(a^3 + b^3*x^3)*SinhIntegral[a + b*x]^2 + 8*SinhIntegral
[2*(a + b*x)] + 12*a^2*SinhIntegral[2*(a + b*x)])/(12*b^3)
```

Rubi [A] (verified)

Time = 4.75 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.30, number of steps used = 26, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 2.167$, Rules used = {7092, 7092, 7088, 7094, 5971, 27, 3042, 26, 3779, 7096, 6151, 7100, 3042, 25, 3793, 2009, 7102, 7094, 5971, 27, 3042, 26, 3779, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{Shi}(a+bx)^2\,dx$$

$$\Big\downarrow \; 7092$$

$$-\frac{2}{3}\int x^2\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{2a\int x\text{Shi}(a+bx)^2dx}{3b}+\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$$\Big\downarrow \; 7092$$

$$-\frac{2}{3}\int x^2\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{2a\left(-\frac{a\int\text{Shi}(a+bx)^2dx}{2b}-\int x\sinh(a+bx)\text{Shi}(a+bx)dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}\right)}{3b}+\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$$\Big\downarrow \; 7088$$

$$-\frac{2}{3}\int x^2\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{2a\left(-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\int\sinh(a+bx)\text{Shi}(a+bx)dx\right)}{2b}-\int x\sinh(a+bx)\text{Shi}(a+bx)dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}\right)}{3b}+\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$$\Big\downarrow \; 7094$$

$$-\frac{2}{3}\int x^2\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{2a\left(-\int x\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\int\frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}dx\right)\right)}{2b}+x(a+bx)\right.}{3b}+\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$$\Big\downarrow \; 5971$$

$$-\frac{2}{3}\int x^2\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{2a\left(-\int x\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\int\frac{\sinh(2a+2bx)}{2(a+bx)}dx\right)\right)}{2b}+\frac{x(a+bx)\text{Shi}(a+}{2b}\right.}{3b}+\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$\Big\downarrow$ 27

$$-\frac{2}{3}\int x^2\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{2a\left(-\int x\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{1}{2}\int\frac{\sinh(2a+2bx)}{a+bx}dx\right)\right)}{2b}+\frac{x(a+bx)\text{Shi}(a\cdots}{2b}\right.}{3b}$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$\Big\downarrow$ 3042

$$-\frac{2}{3}\int x^2\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{2a\left(-\int x\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{1}{2}\int-\frac{i\sin(2ia+2ibx)}{a+bx}dx\right)\right)}{2b}+\frac{x(a+bx)\text{Sh}\cdots}{2b}\right.}{3b}$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$\Big\downarrow$ 26

$$-\frac{2}{3}\int x^2\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{2a\left(-\int x\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}+\frac{1}{2}i\int\frac{\sin(2ia+2ibx)}{a+bx}dx\right)\right)}{2b}+\frac{x(a+bx)\text{Shi}(\cdots}{2b}\right.}{3b}$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$\Big\downarrow$ 3779

$$-\frac{2}{3}\int x^2\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{2a\left(-\int x\sinh(a+bx)\text{Shi}(a+bx)dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}\right.}{3b}$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$\Big\downarrow$ 7096

$$-\frac{2}{3}\left(-\frac{2\int x\cosh(a+bx)\text{Shi}(a+bx)dx}{b}-\int\frac{x^2\cosh(a+bx)\sinh(a+bx)}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)-\frac{2a\left(\frac{\int\cosh(a+bx)\text{Shi}(a+bx)dx}{b}+\int\frac{x\cosh(a+bx)\sinh(a+bx)}{a+bx}dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{a\left(\frac{(a+bx)\text{Shi}(a\cdots}{b}\cdots\right)}{\cdots}\cdots\right)}{3b}$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$$\Big\downarrow \; 6151$$

$$-\frac{2}{3}\left(-\frac{2\int x\cosh(a+bx)\text{Shi}(a+bx)dx}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)-\frac{2a\left(\frac{\int\cosh(a+bx)\text{Shi}(a+bx)dx}{b}+\frac{1}{2}\int\frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}\cdots\right)}{\cdots}\cdots\right)}{3b}$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$$\Big\downarrow \; 7100$$

$$-\frac{2}{3}\left(-\frac{2\int x\cosh(a+bx)\text{Shi}(a+bx)dx}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)-\frac{2a\left(\frac{\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}-\int\frac{\sinh^2(a+bx)}{a+bx}dx}{b}+\frac{1}{2}\int\frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{a\left(\frac{(a+bx)\text{S}\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{3b}$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2}{3}\left(-\frac{2\int x\cosh(a+bx)\text{Shi}(a+bx)dx}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)-\frac{2a\left(\frac{\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}-\int-\frac{\sin(ia+ibx)^2}{a+bx}dx}{b}+\frac{1}{2}\int\frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{a\left(\frac{(a+b\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{3b}$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$$\Big\downarrow \; 25$$

$$-\frac{2}{3}\left(-\frac{2\int x\cosh(a+bx)\text{Shi}(a+bx)dx}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)-\frac{2a\left(\frac{\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\int\frac{\sin(ia+ibx)^2}{a+bx}dx}{b}+\frac{1}{2}\int\frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{a\left(\frac{(a+bx)\text{S}}{}\right.}{}\right.}{3b}$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$\big\downarrow$ 3793

$$-\frac{2}{3}\left(-\frac{2\int x\cosh(a+bx)\text{Shi}(a+bx)dx}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)-\frac{2a\left(\frac{\int\left(\frac{1}{2(a+bx)}-\frac{\cosh(2a+2bx)}{2(a+bx)}\right)dx+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}}{b}+\frac{1}{2}\int\frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right.}{3b}$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$\big\downarrow$ 2009

$$-\frac{2a\left(\frac{1}{2}\int\frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\int x\cosh(a+bx)\text{Shi}(a+bx)dx}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)+\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$\big\downarrow$ 7102

$$-\frac{2a\left(\frac{1}{2}\int\frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\left(-\frac{\int\sinh(a+bx)\text{Shi}(a+bx)dx}{b}-\int\frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Sh}}{}\right.$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$\big\downarrow$ 7094

$$-\frac{2a\left(\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\int\frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}dx}{b}-\int\frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a}{a+b}\right.$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$$\Big\downarrow \; 5971$$

$$-\frac{2a\left(\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\int\frac{\sinh(2a+2bx)}{2(a+bx)}dx}{b}-\int\frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+bx)}{a+bx}\right.$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$$\Big\downarrow \; 27$$

$$-\frac{2a\left(\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{1}{2}\int\frac{\sinh(2a+2bx)}{a+bx}dx}{b}-\int\frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+bx}{a+bx}\right.$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2a\left(\frac{1}{2}\int\frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{1}{2}\int-\frac{i\sin(2ia+2ibx)}{a+bx}dx}{b}-\int\frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+}{a+bx}\right.$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$$\downarrow \text{26}$$

$$-\frac{2a\left(\frac{1}{2}\int\frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}+\frac{1}{2}i\int\frac{\sin(2ia+2ibx)}{a+bx}dx}{b}-\int\frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+b}{a+bx}\right.$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$$\downarrow \text{3779}$$

$$-\frac{2a\left(\frac{1}{2}\int\frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\left(-\int\frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}\right)}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+bx))}{a+bx}d\right.$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$$\downarrow \text{7292}$$

$$-\frac{2a\left(\frac{1}{2}\int \frac{x\sinh(2a+2bx)}{a+bx}dx+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}-\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\left(-\int \frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2a+2bx)}{a+bx}dx\right.$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$\Big\downarrow$ 7293

$$-\frac{2}{3}\left(-\frac{1}{2}\int \left(\frac{\sinh(2a+2bx)a^2}{b^2(a+bx)}-\frac{\sinh(2a+2bx)a}{b^2}+\frac{x\sinh(2a+2bx)}{b}\right)dx-\frac{2\left(-\int \left(\frac{\sinh^2(a+bx)}{b}-\frac{a\sinh^2(a+bx)}{b(a+bx)}\right.\right.}{}\right.$$

$$\frac{2a\left(\frac{1}{2}\int \left(\frac{\sinh(2a+2bx)}{b}+\frac{a\sinh(2a+2bx)}{b(-a-bx)}\right)dx+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a\right.}{3b}$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

$\Big\downarrow$ 2009

$$-\frac{2}{3}\left(\frac{1}{2}\left(-\frac{a^2\text{Shi}(2a+2bx)}{b^3}+\frac{\sinh(2a+2bx)}{4b^3}+\frac{a\cosh(2a+2bx)}{2b^3}-\frac{x\cosh(2a+2bx)}{2b^2}\right)-\frac{2\left(\frac{a\text{Chi}(2a+2bx)}{2b^2}-\frac{a\log\right.}{}\right.$$

$$\frac{2a\left(\frac{1}{2}\left(\frac{\cosh(2a+2bx)}{2b^2}-\frac{a\text{Shi}(2a+2bx)}{b^2}\right)+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh}{b}\right.}{3b}$$

$$\frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b}$$

input
```
Int[x^2*SinhIntegral[a + b*x]^2,x]
```

output
```
(x^2*(a + b*x)*SinhIntegral[a + b*x]^2)/(3*b) - (2*a*(-((x*Cosh[a + b*x]*S
inhIntegral[a + b*x])/b) + (x*(a + b*x)*SinhIntegral[a + b*x]^2)/(2*b) + (
-1/2*CoshIntegral[2*a + 2*b*x]/b + Log[a + b*x]/(2*b) + (Sinh[a + b*x]*Sin
hIntegral[a + b*x])/b)/b + (Cosh[2*a + 2*b*x]/(2*b^2) - (a*SinhIntegral[2*
a + 2*b*x])/b^2)/2 - (a*(((a + b*x)*SinhIntegral[a + b*x]^2)/b - 2*((Cosh[
a + b*x]*SinhIntegral[a + b*x])/b - SinhIntegral[2*a + 2*b*x]/(2*b))))/(2*
b)))/(3*b) - (2*((x^2*Cosh[a + b*x]*SinhIntegral[a + b*x])/b + ((a*Cosh[2*
a + 2*b*x])/(2*b^3) - (x*Cosh[2*a + 2*b*x])/(2*b^2) + Sinh[2*a + 2*b*x]/(4
*b^3) - (a^2*SinhIntegral[2*a + 2*b*x])/b^3)/2 - (2*(x/(2*b) + (a*CoshInte
gral[2*a + 2*b*x])/(2*b^2) - (a*Log[a + b*x])/(2*b^2) - (Cosh[a + b*x]*Sin
h[a + b*x])/(2*b^2) + (x*Sinh[a + b*x]*SinhIntegral[a + b*x])/b - ((Cosh[a
 + b*x]*SinhIntegral[a + b*x])/b - SinhIntegral[2*a + 2*b*x]/(2*b))/b))/b)
)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6151
```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Simp[1/2^p   Int[u*Si
nh[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

rule 7088
```
Int[SinhIntegral[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(SinhIn
tegral[a + b*x]^2/b), x] - Simp[2   Int[Sinh[a + b*x]*SinhIntegral[a + b*x]
, x], x] /; FreeQ[{a, b}, x]
```

rule 7092
```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_) + (b_.)*(x_)]^2, x_Symbol]
 :> Simp[(a + b*x)*(c + d*x)^m*(SinhIntegral[a + b*x]^2/(b*(m + 1))), x] +
(-Simp[2/(m + 1)   Int[(c + d*x)^m*Sinh[a + b*x]*SinhIntegral[a + b*x], x],
 x] + Simp[(b*c - a*d)*(m/(b*(m + 1)))   Int[(c + d*x)^(m - 1)*SinhIntegral
[a + b*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

rule 7094
```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7096
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7100
```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7102
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int x^2 \operatorname{Shi}(bx+a)^2 \, dx$$

input
```
int(x^2*Shi(b*x+a)^2,x)
```

output
```
int(x^2*Shi(b*x+a)^2,x)
```

Fricas [F]

$$\int x^2 \text{Shi}(a+bx)^2 \, dx = \int x^2 \text{Shi}(bx+a)^2 \, dx$$

input
```
integrate(x^2*Shi(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^2*sinh_integral(b*x + a)^2, x)
```

Sympy [F]

$$\int x^2\mathrm{Shi}(a+bx)^2\,dx=\int x^2\,\mathrm{Shi}^2\,(a+bx)\,dx$$

input
```
integrate(x**2*Shi(b*x+a)**2,x)
```

output
```
Integral(x**2*Shi(a + b*x)**2, x)
```

Maxima [F]

$$\int x^2\mathrm{Shi}(a+bx)^2\,dx=\int x^2\mathrm{Shi}(bx+a)^2\,dx$$

input
```
integrate(x^2*Shi(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^2*Shi(b*x + a)^2, x)
```

Giac [F]

$$\int x^2\mathrm{Shi}(a+bx)^2\,dx=\int x^2\mathrm{Shi}(bx+a)^2\,dx$$

input
```
integrate(x^2*Shi(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*Shi(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \text{Shi}(a+bx)^2 \, dx = \int x^2 \, \text{sinhint}(a+b\,x)^2 \, dx$$

input
```
int(x^2*sinhint(a + b*x)^2,x)
```

output
```
int(x^2*sinhint(a + b*x)^2, x)
```

Reduce [F]

$$\int x^2 \text{Shi}(a+bx)^2 \, dx = \int shi(bx+a)^2 \, x^2 dx$$

input
```
int(x^2*Shi(b*x+a)^2,x)
```

output
```
int(shi(a + b*x)**2*x**2,x)
```

3.27 $\int x\textbf{Shi}(a+bx)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 154

$$\int x\text{Shi}(a+bx)^2\,dx = \frac{\cosh(2a+2bx)}{4b^2} - \frac{\text{Chi}(2a+2bx)}{2b^2} + \frac{\log(a+bx)}{2b^2} + \frac{a\cosh(a+bx)\text{Shi}(a+bx)}{b^2} - \frac{x\cosh(a+bx)\text{Shi}(a+bx)}{b} + \frac{\sinh(a+bx)\text{Shi}(a+bx)}{b^2} - \frac{a(a+bx)\text{Shi}(a+bx)^2}{2b^2} + \frac{x(a+bx)\text{Shi}(a+bx)^2}{2b} - \frac{a\text{Shi}(2a+2bx)}{b^2}$$

output
```
1/4*cosh(2*b*x+2*a)/b^2-1/2*Chi(2*b*x+2*a)/b^2+1/2*ln(b*x+a)/b^2+a*cosh(b*
x+a)*Shi(b*x+a)/b^2-x*cosh(b*x+a)*Shi(b*x+a)/b+sinh(b*x+a)*Shi(b*x+a)/b^2-
1/2*a*(b*x+a)*Shi(b*x+a)^2/b^2+1/2*x*(b*x+a)*Shi(b*x+a)^2/b-a*Shi(2*b*x+2*
a)/b^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.62

$$\int x\text{Shi}(a+bx)^2\,dx$$
$$=\frac{\cosh(2(a+bx))-2\text{Chi}(2(a+bx))+2\log(a+bx)+4((a-bx)\cosh(a+bx)+\sinh(a+bx))\text{Shi}(a+b}{4b^2}$$

input
```
Integrate[x*SinhIntegral[a + b*x]^2,x]
```

output
```
(Cosh[2*(a + b*x)] - 2*CoshIntegral[2*(a + b*x)] + 2*Log[a + b*x] + 4*((a
- b*x)*Cosh[a + b*x] + Sinh[a + b*x])*SinhIntegral[a + b*x] - 2*(a^2 - b^2
*x^2)*SinhIntegral[a + b*x]^2 - 4*a*SinhIntegral[2*(a + b*x)])/(4*b^2)
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.800, Rules used = {7092, 7088, 7094, 5971, 27, 3042, 26, 3779, 7096, 6151, 7100, 3042, 25, 3793, 2009, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Shi}(a+bx)^2\,dx$$

$$\Big\downarrow \text{7092}$$

$$-\frac{a\int \text{Shi}(a+bx)^2dx}{2b}-\int x\sinh(a+bx)\text{Shi}(a+bx)dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}$$

$$\Big\downarrow \text{7088}$$

$$-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\int \sinh(a+bx)\text{Shi}(a+bx)dx\right)}{2b}-\int x\sinh(a+bx)\text{Shi}(a+bx)dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}$$

$$\Big\downarrow \text{7094}$$

$$-\int x\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\int\frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}dx\right)\right)}{2b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}$$

$$\Big\downarrow \; 5971$$

$$-\int x\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\int\frac{\sinh(2a+2bx)}{2(a+bx)}dx\right)\right)}{2b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}$$

$$\Big\downarrow \; 27$$

$$-\int x\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{1}{2}\int\frac{\sinh(2a+2bx)}{a+bx}dx\right)\right)}{2b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}$$

$$\Big\downarrow \; 3042$$

$$-\int x\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{1}{2}\int-\frac{i\sin(2ia+2ibx)}{a+bx}dx\right)\right)}{2b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}$$

$$\Big\downarrow \; 26$$

$$-\int x\sinh(a+bx)\text{Shi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}+\frac{1}{2}i\int\frac{\sin(2ia+2ibx)}{a+bx}dx\right)\right)}{2b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}$$

$$\Big\downarrow \; 3779$$

$$-\int x\sinh(a+bx)\text{Shi}(a+bx)dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}$$

$$\Big\downarrow \; 7096$$

$$\frac{\int\cosh(a+bx)\text{Shi}(a+bx)dx}{b}+\int\frac{x\cosh(a+bx)\sinh(a+bx)}{a+bx}dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}$$

$$\Big\downarrow \text{6151}$$

$$\frac{\int \cosh(a+bx)\text{Shi}(a+bx)dx}{b}+\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}$$

$$\Big\downarrow \text{7100}$$

$$\frac{\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}-\int \frac{\sinh^2(a+bx)}{a+bx}dx}{b}+\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}-\int -\frac{\sin(ia+ibx)^2}{a+bx}dx}{b}+\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\int \frac{\sin(ia+ibx)^2}{a+bx}dx}{b}+\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{1}{2(a+bx)}-\frac{\cosh(2a+2bx)}{2(a+bx)}\right)dx+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}}{b}+\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}$$

$$\Big\downarrow \color{blue}{7292}$$

$$\frac{1}{2}\int \frac{x\sinh(2a+2bx)}{a+bx}dx+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}$$

$$\Big\downarrow \color{blue}{7293}$$

$$\frac{1}{2}\int \left(\frac{\sinh(2a+2bx)}{b}+\frac{a\sinh(2a+2bx)}{b(-a-bx)}\right)dx+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}$$

$$\Big\downarrow \color{blue}{2009}$$

$$\frac{1}{2}\left(\frac{\cosh(2a+2bx)}{2b^2}-\frac{a\text{Shi}(2a+2bx)}{b^2}\right)+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x(a+bx)\text{Shi}(a+bx)^2}{2b}-\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{a\left(\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}$$

input
```
Int[x*SinhIntegral[a + b*x]^2,x]
```

output
```
-((x*Cosh[a + b*x]*SinhIntegral[a + b*x])/b) + (x*(a + b*x)*SinhIntegral[a
 + b*x]^2)/(2*b) + (-1/2*CoshIntegral[2*a + 2*b*x]/b + Log[a + b*x]/(2*b)
+ (Sinh[a + b*x]*SinhIntegral[a + b*x])/b)/b + (Cosh[2*a + 2*b*x]/(2*b^2)
- (a*SinhIntegral[2*a + 2*b*x])/b^2)/2 - (a*(((a + b*x)*SinhIntegral[a + b
*x]^2)/b - 2*((Cosh[a + b*x]*SinhIntegral[a + b*x])/b - SinhIntegral[2*a +
 2*b*x]/(2*b))))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6151
```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Simp[1/2^p   Int[u*Si
nh[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

rule 7088
```
Int[SinhIntegral[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(SinhIn
tegral[a + b*x]^2/b), x] - Simp[2   Int[Sinh[a + b*x]*SinhIntegral[a + b*x]
, x], x] /; FreeQ[{a, b}, x]
```

rule 7092
```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_) + (b_.)*(x_)]^2, x_Symbol]
 :> Simp[(a + b*x)*(c + d*x)^m*(SinhIntegral[a + b*x]^2/(b*(m + 1))), x] +
(-Simp[2/(m + 1)   Int[(c + d*x)^m*Sinh[a + b*x]*SinhIntegral[a + b*x], x],
 x] + Simp[(b*c - a*d)*(m/(b*(m + 1)))   Int[(c + d*x)^(m - 1)*SinhIntegral
[a + b*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

rule 7094
```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7096
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7100
```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{\mathrm{Shi}(bx+a)^2\left(\frac{(bx+a)^2}{2}-(bx+a)a\right)-2\,\mathrm{Shi}(bx+a)\left(-a\cosh(bx+a)+\frac{(bx+a)\cosh(bx+a)}{2}-\frac{\sinh(bx+a)}{2}\right)-a\,\mathrm{Shi}(2bx+2a)+}{b^2}$
default	$\frac{\mathrm{Shi}(bx+a)^2\left(\frac{(bx+a)^2}{2}-(bx+a)a\right)-2\,\mathrm{Shi}(bx+a)\left(-a\cosh(bx+a)+\frac{(bx+a)\cosh(bx+a)}{2}-\frac{\sinh(bx+a)}{2}\right)-a\,\mathrm{Shi}(2bx+2a)+}{b^2}$

input
```
int(x*Shi(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(Shi(b*x+a)^2*(1/2*(b*x+a)^2-(b*x+a)*a)-2*Shi(b*x+a)*(-a*cosh(b*x+a)
+1/2*(b*x+a)*cosh(b*x+a)-1/2*sinh(b*x+a))-a*Shi(2*b*x+2*a)+1/2*cosh(b*x+a)
^2+1/2*ln(b*x+a)-1/2*Chi(2*b*x+2*a))
```

Fricas [F]

$$\int x\mathrm{Shi}(a+bx)^2\,dx=\int x\mathrm{Shi}(bx+a)^2\,dx$$

input
```
integrate(x*Shi(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x*sinh_integral(b*x + a)^2, x)
```

Sympy [F]

$$\int x\mathrm{Shi}(a+bx)^2\,dx=\int x\,\mathrm{Shi}^2\,(a+bx)\,dx$$

input
```
integrate(x*Shi(b*x+a)**2,x)
```

output
```
Integral(x*Shi(a + b*x)**2, x)
```

Maxima [F]

$$\int x\text{Shi}(a+bx)^2\,dx=\int x\text{Shi}(bx+a)^2\,dx$$

input
```
integrate(x*Shi(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x*Shi(b*x + a)^2, x)
```

Giac [F]

$$\int x\text{Shi}(a+bx)^2\,dx=\int x\text{Shi}(bx+a)^2\,dx$$

input
```
integrate(x*Shi(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x*Shi(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{Shi}(a+bx)^2\,dx=\int x\,\text{sinhint}(a+b\,x)^2\,dx$$

input
```
int(x*sinhint(a + b*x)^2,x)
```

output
```
int(x*sinhint(a + b*x)^2, x)
```

Reduce [F]

$$\int x\mathrm{Shi}(a+bx)^2\,dx=\int shi(bx+a)^2\,xdx$$

input
```
int(x*Shi(b*x+a)^2,x)
```

output
```
int(shi(a + b*x)**2*x,x)
```

3.28 $\int \mathbf{Shi}(a+bx)^2\,dx$

Optimal result

Integrand size = 8, antiderivative size = 48

$$\int \mathrm{Shi}(a+bx)^2\,dx=-\frac{2\cosh(a+bx)\mathrm{Shi}(a+bx)}{b}+\frac{(a+bx)\mathrm{Shi}(a+bx)^2}{b}+\frac{\mathrm{Shi}(2a+2bx)}{b}$$

output
```
-2*cosh(b*x+a)*Shi(b*x+a)/b+(b*x+a)*Shi(b*x+a)^2/b+Shi(2*b*x+2*a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \mathrm{Shi}(a+bx)^2\,dx =\frac{-2\cosh(a+bx)\mathrm{Shi}(a+bx)+(a+bx)\mathrm{Shi}(a+bx)^2+\mathrm{Shi}(2(a+bx))}{b}$$

input
```
Integrate[SinhIntegral[a + b*x]^2,x]
```

output
```
(-2*Cosh[a + b*x]*SinhIntegral[a + b*x] + (a + b*x)*SinhIntegral[a + b*x]^
2 + SinhIntegral[2*(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {7088, 7094, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Shi}(a+bx)^2 \, dx$$

$$\Big\downarrow \text{7088}$$

$$\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\int \sinh(a+bx)\text{Shi}(a+bx)dx$$

$$\Big\downarrow \text{7094}$$

$$\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}dx\right)$$

$$\Big\downarrow \text{5971}$$

$$\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\sinh(2a+2bx)}{2(a+bx)}dx\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{1}{2}\int \frac{\sinh(2a+2bx)}{a+bx}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{1}{2}\int -\frac{i\sin(2ia+2ibx)}{a+bx}dx\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}+\frac{1}{2}i\int \frac{\sin(2ia+2ibx)}{a+bx}dx\right)$$

$$\Big\downarrow \text{3779}$$

$$\frac{(a+bx)\text{Shi}(a+bx)^2}{b}-2\left(\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)$$

input
```
Int[SinhIntegral[a + b*x]^2,x]
```

output
```
((a + b*x)*SinhIntegral[a + b*x]^2)/b - 2*((Cosh[a + b*x]*SinhIntegral[a +
 b*x])/b - SinhIntegral[2*a + 2*b*x]/(2*b))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7088
```
Int[SinhIntegral[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(SinhIn
tegral[a + b*x]^2/b), x] - Simp[2   Int[Sinh[a + b*x]*SinhIntegral[a + b*x]
, x], x] /; FreeQ[{a, b}, x]
```

rule 7094
```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\mathrm{Shi}(bx+a)^2(bx+a)-2\cosh(bx+a)\,\mathrm{Shi}(bx+a)+\mathrm{Shi}(2bx+2a)}{b}$	43
default	$\frac{\mathrm{Shi}(bx+a)^2(bx+a)-2\cosh(bx+a)\,\mathrm{Shi}(bx+a)+\mathrm{Shi}(2bx+2a)}{b}$	43

input
```
int(Shi(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(Shi(b*x+a)^2*(b*x+a)-2*cosh(b*x+a)*Shi(b*x+a)+Shi(2*b*x+2*a))
```

Fricas [F]

$$\int \mathrm{Shi}(a+bx)^2\,dx = \int \mathrm{Shi}(bx+a)^2\,dx$$

input
```
integrate(Shi(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*x + a)^2, x)
```

Sympy [F]

$$\int \mathrm{Shi}(a+bx)^2\,dx = \int \mathrm{Shi}^2\left(a+bx\right)dx$$

input
```
integrate(Shi(b*x+a)**2,x)
```

output
```
Integral(Shi(a + b*x)**2, x)
```

Maxima [F]

$$\int \mathrm{Shi}(a+bx)^2\,dx = \int \mathrm{Shi}(bx+a)^2\,dx$$

input
```
integrate(Shi(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x + a)^2, x)
```

Giac [F]

$$\int \mathrm{Shi}(a+bx)^2\,dx = \int \mathrm{Shi}(bx+a)^2\,dx$$

input
```
integrate(Shi(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(Shi(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \mathrm{Shi}(a+bx)^2\,dx = \int \mathrm{sinhint}(a+b\,x)^2\,dx$$

input
```
int(sinhint(a + b*x)^2,x)
```

output
```
int(sinhint(a + b*x)^2, x)
```

Reduce [F]

$$\int \mathrm{Shi}(a+bx)^2\,dx = \int shi(bx+a)^2\,dx$$

input
```
int(Shi(b*x+a)^2,x)
```

output
```
int(shi(a + b*x)**2,x)
```

3.29 $\int \frac{\text{Shi}(a+bx)^2}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\text{Shi}(a+bx)^2}{x} \, dx = \text{Int}\left(\frac{\text{Shi}(a+bx)^2}{x}, x\right)$$

output
```
Defer(Int)(Shi(b*x+a)^2/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Shi}(a+bx)^2}{x} \, dx = \int \frac{\text{Shi}(a+bx)^2}{x} \, dx$$

input
```
Integrate[SinhIntegral[a + b*x]^2/x,x]
```

output
```
Integrate[SinhIntegral[a + b*x]^2/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(a+bx)^2}{x} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\text{Shi}(a+bx)^2}{x}dx$$

input
```
Int[SinhIntegral[a + b*x]^2/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\text{Shi}\left(bx+a\right)^2}{x}dx$$

input
```
int(Shi(b*x+a)^2/x,x)
```

output
```
int(Shi(b*x+a)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Shi}(a+bx)^2}{x}\,dx = \int \frac{\text{Shi}(bx+a)^2}{x}\,dx$$

input
```
integrate(Shi(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*x + a)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\text{Shi}(a+bx)^2}{x}\,dx = \int \frac{\text{Shi}^2\,(a+bx)}{x}\,dx$$

input
```
integrate(Shi(b*x+a)**2/x,x)
```

output
```
Integral(Shi(a + b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Shi}(a+bx)^2}{x}\,dx = \int \frac{\text{Shi}(bx+a)^2}{x}\,dx$$

input
```
integrate(Shi(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x + a)^2/x, x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Shi}(a+bx)^2}{x}\,dx = \int \frac{\text{Shi}(bx+a)^2}{x}\,dx$$

input
```
integrate(Shi(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(Shi(b*x + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 3.80 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Shi}(a+bx)^2}{x}\,dx = \int \frac{\text{sinhint}(a+b\,x)^2}{x}\,dx$$

input
```
int(sinhint(a + b*x)^2/x,x)
```

output
```
int(sinhint(a + b*x)^2/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Shi}(a+bx)^2}{x}\,dx = \int \frac{shi(bx+a)^2}{x}dx$$

input
```
int(Shi(b*x+a)^2/x,x)
```

output
```
int(shi(a + b*x)**2/x,x)
```

3.30 $\int \frac{\mathrm{Shi}(a+bx)^2}{x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\mathrm{Shi}(a+bx)^2}{x^2} \, dx = \mathrm{Int}\left(\frac{\mathrm{Shi}(a+bx)^2}{x^2}, x\right)$$

output
```
Defer(Int)(Shi(b*x+a)^2/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Shi}(a+bx)^2}{x^2} \, dx = \int \frac{\mathrm{Shi}(a+bx)^2}{x^2} \, dx$$

input
```
Integrate[SinhIntegral[a + b*x]^2/x^2,x]
```

output
```
Integrate[SinhIntegral[a + b*x]^2/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(a+bx)^2}{x^2} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\text{Shi}(a+bx)^2}{x^2}dx$$

input
```
Int[SinhIntegral[a + b*x]^2/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\text{Shi}\left(bx+a\right)^{2}}{x^{2}}dx$$

input
```
int(Shi(b*x+a)^2/x^2,x)
```

output
```
int(Shi(b*x+a)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Shi}(a+bx)^2}{x^2}\,dx = \int \frac{\text{Shi}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(Shi(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*x + a)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\text{Shi}(a+bx)^2}{x^2}\,dx = \int \frac{\text{Shi}^2\,(a+bx)}{x^2}\,dx$$

input
```
integrate(Shi(b*x+a)**2/x**2,x)
```

output
```
Integral(Shi(a + b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Shi}(a+bx)^2}{x^2}\,dx = \int \frac{\text{Shi}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(Shi(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x + a)^2/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Shi}(a+bx)^2}{x^2}\,dx = \int \frac{\text{Shi}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(Shi(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(Shi(b*x + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Shi}(a+bx)^2}{x^2}\,dx = \int \frac{\text{sinhint}(a+b\,x)^2}{x^2}\,dx$$

input
```
int(sinhint(a + b*x)^2/x^2,x)
```

output
```
int(sinhint(a + b*x)^2/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Shi}(a+bx)^2}{x^2}\,dx = \int \frac{shi(bx+a)^2}{x^2}dx$$

input
```
int(Shi(b*x+a)^2/x^2,x)
```

output
```
int(shi(a + b*x)**2/x**2,x)
```

3.31 $\int \frac{\text{Shi}(a+bx)^2}{x^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\text{Shi}(a+bx)^2}{x^3} dx = \text{Int}\left(\frac{\text{Shi}(a+bx)^2}{x^3}, x\right)$$

output
```
Defer(Int)(Shi(b*x+a)^2/x^3,x)
```

Mathematica [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Shi}(a+bx)^2}{x^3} dx = \int \frac{\text{Shi}(a+bx)^2}{x^3} dx$$

input
```
Integrate[SinhIntegral[a + b*x]^2/x^3,x]
```

output
```
Integrate[SinhIntegral[a + b*x]^2/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(a+bx)^2}{x^3}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\text{Shi}(a+bx)^2}{x^3}dx$$

input
```
Int[SinhIntegral[a + b*x]^2/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\text{Shi}\,(bx+a)^2}{x^3}dx$$

input
```
int(Shi(b*x+a)^2/x^3,x)
```

output
```
int(Shi(b*x+a)^2/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Shi}(a+bx)^2}{x^3}\,dx=\int \frac{\text{Shi}(bx+a)^2}{x^3}\,dx$$

input
```
integrate(Shi(b*x+a)^2/x^3,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*x + a)^2/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\text{Shi}(a+bx)^2}{x^3}\,dx=\int \frac{\text{Shi}^2\left(a+bx\right)}{x^3}\,dx$$

input
```
integrate(Shi(b*x+a)**2/x**3,x)
```

output
```
Integral(Shi(a + b*x)**2/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Shi}(a+bx)^2}{x^3}\,dx=\int \frac{\text{Shi}(bx+a)^2}{x^3}\,dx$$

input
```
integrate(Shi(b*x+a)^2/x^3,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x + a)^2/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Shi}(a+bx)^2}{x^3}\,dx = \int \frac{\mathrm{Shi}(bx+a)^2}{x^3}\,dx$$

input
```
integrate(Shi(b*x+a)^2/x^3,x, algorithm="giac")
```

output
```
integrate(Shi(b*x + a)^2/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Shi}(a+bx)^2}{x^3}\,dx = \int \frac{\mathrm{sinhint}(a+b\,x)^2}{x^3}\,dx$$

input
```
int(sinhint(a + b*x)^2/x^3,x)
```

output
```
int(sinhint(a + b*x)^2/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Shi}(a+bx)^2}{x^3}\,dx = \int \frac{shi(bx+a)^2}{x^3}dx$$

input
```
int(Shi(b*x+a)^2/x^3,x)
```

output
```
int(shi(a + b*x)**2/x**3,x)
```

3.32 $\int x^2 \textbf{Shi}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 128

$$\int x^2\text{Shi}(d(a+b\log(cx^n)))\,dx = \frac{1}{6}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\text{ExpIntegralEi}\left(\frac{(3-bdn)(a+b\log(cx^n))}{bn}\right) - \frac{1}{6}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\text{ExpIntegralEi}\left(\frac{(3+bdn)(a+b\log(cx^n))}{bn}\right) + \frac{1}{3}x^3\text{Shi}(d(a+b\log(cx^n)))$$

output
```
1/6*x^3*Ei((-b*d*n+3)*(a+b*ln(c*x^n))/b/n)/exp(3*a/b/n)/((c*x^n)^(3/n))-1/
6*x^3*Ei((b*d*n+3)*(a+b*ln(c*x^n))/b/n)/exp(3*a/b/n)/((c*x^n)^(3/n))+1/3*x
^3*Shi(d*(a+b*ln(c*x^n)))
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.77

$$\int x^2\text{Shi}(d(a+b\log(cx^n)))\,dx$$
$$=\frac{1}{6}x^3\left(e^{-\frac{3a}{bn}}(cx^n)^{-3/n}\left(\text{ExpIntegralEi}\left(-\frac{(-3+bdn)(a+b\log(cx^n))}{bn}\right)-\text{ExpIntegralEi}\left(\frac{(3+bdn)(a+\ldots}{bn}\right.\right.\right.$$
$$\left.+2\text{Shi}(d(a+b\log(cx^n)))\right)$$

input
```
Integrate[x^2*SinhIntegral[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^3*((ExpIntegralEi[-(((-3 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))] - ExpInte
gralEi[((3 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(E^((3*a)/(b*n))*(c*x^n)^(
3/n)) + 2*SinhIntegral[d*(a + b*Log[c*x^n])]))/6
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {7109, 27, 6065, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{Shi}(d(a+b\log(cx^n)))\,dx$$
$$\Big\downarrow \text{7109}$$
$$\frac{1}{3}x^3\text{Shi}(d(a+b\log(cx^n)))-\frac{1}{3}bdn\int\frac{x^2\sinh(d(a+b\log(cx^n)))}{d(a+b\log(cx^n))}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{3}x^3\text{Shi}(d(a+b\log(cx^n)))-\frac{1}{3}bn\int\frac{x^2\sinh(d(a+b\log(cx^n)))}{a+b\log(cx^n)}dx$$
$$\Big\downarrow \text{6065}$$

$$\frac{1}{3}x^3\text{Shi}(d(a+b\log(cx^n))) - \frac{1}{3}bn\left(\frac{1}{2}e^{ad}x^{-bdn}(cx^n)^{bd}\int\frac{x^{bdn+2}}{a+b\log(cx^n)}dx - \frac{1}{2}e^{-ad}x^{bdn}(cx^n)^{-bd}\int\frac{x^{2-bdn}}{a+b\log(cx^n)}dx\right)$$

$$\Big\downarrow \text{2747}$$

$$\frac{1}{3}x^3\text{Shi}(d(a+b\log(cx^n))) - \frac{1}{3}bn\left(\frac{x^3e^{ad}(cx^n)^{bd-\frac{bdn+3}{n}}\int\frac{(cx^n)^{\frac{bdn+3}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2n} - \frac{x^3e^{-ad}(cx^n)^{-3/n}\int\frac{(cx^n)^{\frac{3-bdn}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2n}\right)$$

$$\Big\downarrow \text{2609}$$

$$\frac{1}{3}x^3\text{Shi}(d(a+b\log(cx^n))) - \frac{1}{3}bn\left(\frac{x^3e^{ad-a\left(\frac{3}{bn}+d\right)}(cx^n)^{bd-\frac{bdn+3}{n}}\text{ExpIntegralEi}\left(\frac{(bdn+3)(a+b\log(cx^n))}{bn}\right)}{2bn} - \frac{x^3(cx^n)^{-3/n}e^{a\left(d-\frac{3}{bn}\right)-ad}\text{ExpIntegralE}}{2bn}\right.$$

input
```
Int[x^2*SinhIntegral[d*(a + b*Log[c*x^n])],x]
```

output
```
-1/3*(b*n*(-1/2*(E^(-(a*d) + a*(d - 3/(b*n)))*x^3*ExpIntegralEi[((3 - b*d*
n)*(a + b*Log[c*x^n]))/(b*n)])/(b*n*(c*x^n)^(3/n)) + (E^(a*d - a*(d + 3/(b
*n)))*x^3*(c*x^n)^(b*d - (3 + b*d*n)/n)*ExpIntegralEi[((3 + b*d*n)*(a + b*
Log[c*x^n]))/(b*n)])/(2*b*n))) + (x^3*SinhIntegral[d*(a + b*Log[c*x^n])])/
3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 6065
```
Int[(((e_.) + Log[(g_.)*(x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.)*
Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-E^((
-a)*d))*(i*x)^r*(1/((c*x^n)^(b*d)*(2*x^(r - b*d*n))))   Int[x^(r - b*d*n)*(
h*(e + f*Log[g*x^m]))^q, x], x] + Simp[E^(a*d)*(i*x)^r*((c*x^n)^(b*d)/(2*x^
(r + b*d*n)))   Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ
[{a, b, c, d, e, f, g, h, i, m, n, q, r}, x]
```

rule 7109
```
Int[((e_.)*(x_))^(m_.)*SinhIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(
d_.)], x_Symbol] :> Simp[(e*x)^(m + 1)*(SinhIntegral[d*(a + b*Log[c*x^n])]/
(e*(m + 1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*(Sinh[d*(a + b*Log[c*
x^n])]/(d*(a + b*Log[c*x^n]))), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[m, -1]
```

Maple [F]

$$\int x^2 \operatorname{Shi}\left(d(a+b\ln\left(c\,x^n\right))\right)dx$$

input
```
int(x^2*Shi(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x^2*Shi(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x^2 \text{Shi}(d(a+b\log(cx^n)))\,dx = \int x^2 \text{Shi}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(x^2*Shi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(x^2*sinh_integral(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x^2 \mathrm{Shi}(d(a + b\log{(cx^n)}))\ dx = \int x^2 \operatorname{Shi}{(ad + bd\log{(cx^n)})}\, dx$$

input
```
integrate(x**2*Shi(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x**2*Shi(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^2 \mathrm{Shi}(d(a + b\log{(cx^n)}))\ dx = \int x^2 \mathrm{Shi}((b\log{(cx^n)} + a)d)\ dx$$

input
```
integrate(x^2*Shi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x^2*Shi((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x^2 \mathrm{Shi}(d(a + b\log{(cx^n)}))\ dx = \int x^2 \mathrm{Shi}((b\log{(cx^n)} + a)d)\ dx$$

input
```
integrate(x^2*Shi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x^2*Shi((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{Shi}(d(a+b\log(cx^n)))\,dx=\int x^2\,\mathrm{sinhint}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)\,dx$$

input
```
int(x^2*sinhint(d*(a + b*log(c*x^n))),x)
```

output
```
int(x^2*sinhint(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x^2\text{Shi}(d(a+b\log(cx^n)))\,dx=\int shi(\log(x^nc)\,bd+ad)\,x^2dx$$

input
```
int(x^2*Shi(d*(a+b*log(c*x^n))),x)
```

output
```
int(shi(log(x**n*c)*b*d + a*d)*x**2,x)
```

3.33 $\int x\mathbf{Shi}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 128

$$\begin{aligned}
\int x\text{Shi}(d(a+b\log(cx^n)))\,dx &= \frac{1}{4}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\,\text{ExpIntegralEi}\left(\frac{(2-bdn)(a+b\log(cx^n))}{bn}\right) \\
&\quad -\frac{1}{4}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\,\text{ExpIntegralEi}\left(\frac{(2+bdn)(a+b\log(cx^n))}{bn}\right) \\
&\quad +\frac{1}{2}x^2\text{Shi}(d(a+b\log(cx^n)))
\end{aligned}$$

output

```
1/4*x^2*Ei((-b*d*n+2)*(a+b*ln(c*x^n))/b/n)/exp(2*a/b/n)/((c*x^n)^(2/n))-1/
4*x^2*Ei((b*d*n+2)*(a+b*ln(c*x^n))/b/n)/exp(2*a/b/n)/((c*x^n)^(2/n))+1/2*x
^2*Shi(d*(a+b*ln(c*x^n)))
```

Mathematica [A] (verified)

Time = 1.08 (sec), antiderivative size = 98, normalized size of antiderivative = 0.77

$$\int x\text{Shi}(d(a+b\log(cx^n)))\,dx$$
$$=\frac{1}{4}x^2\left(e^{-\frac{2a}{bn}}(cx^n)^{-2/n}\left(\text{ExpIntegralEi}\left(-\frac{(-2+bdn)(a+b\log(cx^n))}{bn}\right)-\text{ExpIntegralEi}\left(\frac{(2+bdn)(a+\ldots}{bn}\right.\right.\right.$$
$$+2\text{Shi}(d(a+b\log(cx^n)))\Big)$$

input
```
Integrate[x*SinhIntegral[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^2*((ExpIntegralEi[-(((-2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))] - ExpInte
gralEi[((2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(E^((2*a)/(b*n))*(c*x^n)^(
2/n)) + 2*SinhIntegral[d*(a + b*Log[c*x^n])]))/4
```

Rubi [A] (verified)

Time = 0.61 (sec), antiderivative size = 173, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7109, 27, 6065, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Shi}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{7109}$$

$$\frac{1}{2}x^2\text{Shi}(d(a+b\log(cx^n)))-\frac{1}{2}bdn\int\frac{x\sinh(d(a+b\log(cx^n)))}{d(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}x^2\text{Shi}(d(a+b\log(cx^n)))-\frac{1}{2}bn\int\frac{x\sinh(d(a+b\log(cx^n)))}{a+b\log(cx^n)}dx$$

$$\Big\downarrow \text{6065}$$

$$\frac{1}{2}x^2\text{Shi}(d(a+b\log(cx^n)))-\frac{1}{2}bn\left(\frac{1}{2}e^{ad}x^{-bdn}(cx^n)^{bd}\int\frac{x^{bdn+1}}{a+b\log(cx^n)}dx-\frac{1}{2}e^{-ad}x^{bdn}(cx^n)^{-bd}\int\frac{x^{1-bdn}}{a+b\log(cx^n)}dx\right)$$

$$\Big\downarrow \text{2747}$$

$$\frac{1}{2}x^2\text{Shi}(d(a+b\log(cx^n)))-\frac{1}{2}bn\left(\frac{x^2e^{ad}(cx^n)^{bd-\frac{bdn+2}{n}}\int\frac{(cx^n)^{\frac{bdn+2}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2n}-\frac{x^2e^{-ad}(cx^n)^{-2/n}\int\frac{(cx^n)^{\frac{2-bdn}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2n}\right)$$

$$\Big\downarrow \text{2609}$$

$$\frac{1}{2}x^2\text{Shi}(d(a+b\log(cx^n)))-\frac{1}{2}bn\left(\frac{x^2e^{ad-a\left(\frac{2}{bn}+d\right)}(cx^n)^{bd-\frac{bdn+2}{n}}\text{ExpIntegralEi}\left(\frac{(bdn+2)(a+b\log(cx^n))}{bn}\right)}{2bn}-\frac{x^2(cx^n)^{-2/n}e^{a\left(d-\frac{2}{bn}\right)-ad}\text{ExpIntegralE}}{2bn}\right.$$

input
```
Int[x*SinhIntegral[d*(a + b*Log[c*x^n])],x]
```

output
```
-1/2*(b*n*(-1/2*(E^(-(a*d) + a*(d - 2/(b*n)))*x^2*ExpIntegralEi[((2 - b*d*
n)*(a + b*Log[c*x^n]))/(b*n)])/(b*n*(c*x^n)^(2/n)) + (E^(a*d - a*(d + 2/(b
*n)))*x^2*(c*x^n)^(b*d - (2 + b*d*n)/n)*ExpIntegralEi[((2 + b*d*n)*(a + b*
Log[c*x^n]))/(b*n)])/(2*b*n))) + (x^2*SinhIntegral[d*(a + b*Log[c*x^n])])/
2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 6065
```
Int[(((e_.) + Log[(g_.)*(x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.)*
Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-E^((
-a)*d))*(i*x)^r*(1/((c*x^n)^(b*d)*(2*x^(r - b*d*n))))   Int[x^(r - b*d*n)*(
h*(e + f*Log[g*x^m]))^q, x], x] + Simp[E^(a*d)*(i*x)^r*((c*x^n)^(b*d)/(2*x^
(r + b*d*n)))   Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ
[{a, b, c, d, e, f, g, h, i, m, n, q, r}, x]
```

rule 7109
```
Int[((e_.)*(x_))^(m_.)*SinhIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(
d_.)], x_Symbol] :> Simp[(e*x)^(m + 1)*(SinhIntegral[d*(a + b*Log[c*x^n])]/
(e*(m + 1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*(Sinh[d*(a + b*Log[c*
x^n])]/(d*(a + b*Log[c*x^n]))), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[m, -1]
```

Maple [F]

$$\int x \,\mathrm{Shi}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

input
```
int(x*Shi(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x*Shi(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x\mathrm{Shi}(d(a+b\log\left(cx^{n}\right)))\,dx=\int x\mathrm{Shi}((b\log\left(cx^{n}\right)+a)d)\,dx$$

input
```
integrate(x*Shi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(x*sinh_integral(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x\text{Shi}(d(a+b\log(cx^n)))\,dx=\int x\,\text{Shi}\left(ad+bd\log\left(cx^n\right)\right)dx$$

input
```
integrate(x*Shi(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x*Shi(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x\text{Shi}(d(a+b\log(cx^n)))\,dx=\int x\text{Shi}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(x*Shi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x*Shi((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x\text{Shi}(d(a+b\log(cx^n)))\,dx=\int x\text{Shi}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(x*Shi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x*Shi((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{Shi}(d(a+b\log(cx^n)))\,dx=\int x\,\mathrm{sinhint}\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)\,dx$$

input
```
int(x*sinhint(d*(a + b*log(c*x^n))),x)
```

output
```
int(x*sinhint(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x\text{Shi}(d(a+b\log(cx^n)))\,dx=\int shi(\log(x^n c)\,bd+ad)\,xdx$$

input
```
int(x*Shi(d*(a+b*log(c*x^n))),x)
```

output
```
int(shi(log(x**n*c)*b*d + a*d)*x,x)
```

3.34 $\int \mathrm{Shi}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 119

$$\begin{aligned}\int \mathrm{Shi}(d(a+b\log(cx^n)))\,dx &= \frac{1}{2}e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\,\mathrm{ExpIntegralEi}\left(\frac{(1-bdn)(a+b\log(cx^n))}{bn}\right)\\ &\quad-\frac{1}{2}e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\,\mathrm{ExpIntegralEi}\left(\frac{(1+bdn)(a+b\log(cx^n))}{bn}\right)\\ &\quad+x\mathrm{Shi}(d(a+b\log(cx^n)))\end{aligned}$$

output
```
1/2*x*Ei((-b*d*n+1)*(a+b*ln(c*x^n))/b/n)/exp(a/b/n)/((c*x^n)^(1/n))-1/2*x*
Ei((b*d*n+1)*(a+b*ln(c*x^n))/b/n)/exp(a/b/n)/((c*x^n)^(1/n))+x*Shi(d*(a+b*
ln(c*x^n)))
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.80

$$\begin{aligned}\int \mathrm{Shi}(d(a+b\log(cx^n)))\,dx &= \frac{1}{2}e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\left(\mathrm{ExpIntegralEi}\left(-\frac{(-1+bdn)(a+b\log(cx^n))}{bn}\right)\right.\\ &\quad\left.-\mathrm{ExpIntegralEi}\left(\frac{(1+bdn)(a+b\log(cx^n))}{bn}\right)\right)+x\mathrm{Shi}(d(a+b\log(cx^n)))\end{aligned}$$

input
```
Integrate[SinhIntegral[d*(a + b*Log[c*x^n])],x]
```

output
```
(x*(ExpIntegralEi[-(((-1 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))] - ExpIntegra
lEi[((1 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)]))/(2*E^(a/(b*n))*(c*x^n)^n^(-1
)) + x*SinhIntegral[d*(a + b*Log[c*x^n])]
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {7106, 27, 6063, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Shi}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{7106}$$

$$x\text{Shi}(d(a+b\log(cx^n)))-bdn\int \frac{\sinh(d(a+b\log(cx^n)))}{d(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{27}$$

$$x\text{Shi}(d(a+b\log(cx^n)))-bn\int \frac{\sinh(d(a+b\log(cx^n)))}{a+b\log(cx^n)}dx$$

$$\Big\downarrow \text{6063}$$

$$x\text{Shi}(d(a+b\log(cx^n)))-bn\left(\frac{1}{2}e^{ad}x^{-bdn}(cx^n)^{bd}\int \frac{x^{bdn}}{a+b\log(cx^n)}dx-\frac{1}{2}e^{-ad}x^{bdn}(cx^n)^{-bd}\int \frac{x^{-bdn}}{a+b\log(cx^n)}dx\right)$$

$$\Big\downarrow \text{2747}$$

$$x\text{Shi}(d(a+b\log(cx^n)))-bn\left(\frac{xe^{ad}(cx^n)^{bd-\frac{bdn+1}{n}}\int \frac{(cx^n)^{\frac{bdn+1}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2n}-\frac{xe^{-ad}(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1-bdn}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2n}\right)$$

$$\Big\downarrow \text{2609}$$

$$x\text{Shi}(d(a+b\log(cx^n))) - bn\left(\frac{xe^{ad-a\left(\frac{1}{bn}+d\right)}(cx^n)^{bd-\frac{bdn+1}{n}}\text{ExpIntegralEi}\left(\frac{(bdn+1)(a+b\log(cx^n))}{bn}\right)}{2bn} - \frac{x(cx^n)^{-1/n}e^{a\left(d-\frac{1}{bn}\right)-ad}\text{ExpIntegralEi}\left(\right.}{2bn}\right.$$

input
```
Int[SinhIntegral[d*(a + b*Log[c*x^n])],x]
```

output
```
-(b*n*(-1/2*(E^(-(a*d) + a*(d - 1/(b*n)))*x*ExpIntegralEi[((1 - b*d*n)*(a
+ b*Log[c*x^n]))/(b*n)])/(b*n*(c*x^n)^n^(-1)) + (E^(a*d - a*(d + 1/(b*n)))
*x*(c*x^n)^(b*d - (1 + b*d*n)/n)*ExpIntegralEi[((1 + b*d*n)*(a + b*Log[c*x
^n]))/(b*n)])/(2*b*n))) + x*SinhIntegral[d*(a + b*Log[c*x^n])]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 6063
```
Int[(((e_.) + Log[(g_.)*(x_)^(m_.)]*(f_.))*(h_.))^(q_.)*Sinh[((a_.) + Log[(
c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-E^((-a)*d))*(1/((c*x^n)
^(b*d)*(2/x^(b*d*n))))   Int[(h*(e + f*Log[g*x^m]))^q/x^(b*d*n), x], x] + S
imp[E^(a*d)*((c*x^n)^(b*d)/(2*x^(b*d*n)))   Int[x^(b*d*n)*(h*(e + f*Log[g*x
^m]))^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, q}, x]
```

rule 7106
```
Int[SinhIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :>
 Simp[x*SinhIntegral[d*(a + b*Log[c*x^n])], x] - Simp[b*d*n   Int[Sinh[d*(a
 + b*Log[c*x^n])]/(d*(a + b*Log[c*x^n])), x], x] /; FreeQ[{a, b, c, d, n},
x]
```

Maple [F]

$$\int \operatorname{Shi}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

input
```
int(Shi(d*(a+b*ln(c*x^n))),x)
```

output
```
int(Shi(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int \operatorname{Shi}(d(a+b\log\left(cx^{n}\right)))\,dx=\int \operatorname{Shi}((b\log\left(cx^{n}\right)+a)d)\,dx$$

input
```
integrate(Shi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int \operatorname{Shi}(d(a+b\log\left(cx^{n}\right)))\,dx=\int \operatorname{Shi}\left(d(a+b\log\left(cx^{n}\right))\right)dx$$

input
```
integrate(Shi(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(Shi(d*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \mathrm{Shi}(d(a+b\log{(cx^n)}))\, dx = \int \mathrm{Shi}((b\log{(cx^n)}+a)d)\, dx$$

input
```
integrate(Shi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(Shi((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int \mathrm{Shi}(d(a+b\log{(cx^n)}))\, dx = \int \mathrm{Shi}((b\log{(cx^n)}+a)d)\, dx$$

input
```
integrate(Shi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(Shi((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \mathrm{Shi}(d(a+b\log{(cx^n)}))\, dx = \int \mathrm{sinhint}(d\,(a+b\,\ln{(c\,x^n)}))\, dx$$

input
```
int(sinhint(d*(a + b*log(c*x^n))),x)
```

output
```
int(sinhint(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int \mathrm{Shi}(d(a + b\log(cx^n)))\,dx = \int shi(\log(x^n c)\,bd + ad)\,dx$$

input
```
int(Shi(d*(a+b*log(c*x^n))),x)
```

output
```
int(shi(log(x**n*c)*b*d + a*d),x)
```

3.35 $\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x}\,dx = -\frac{\cosh(d(a+b\log(cx^n)))}{bdn} + \frac{(a+b\log(cx^n))\,\text{Shi}(d(a+b\log(cx^n)))}{bn}$$

output
```
-cosh(d*(a+b*ln(c*x^n)))/b/d/n+(a+b*ln(c*x^n))*Shi(d*(a+b*ln(c*x^n)))/b/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.75

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x}\,dx = -\frac{\cosh(ad)\cosh(bd\log(cx^n))}{bdn} - \frac{\sinh(ad)\sinh(bd\log(cx^n))}{bdn} + \frac{\log(cx^n)\,\text{Shi}(d(a+b\log(cx^n)))}{n} + \frac{a\text{Shi}(ad+bd\log(cx^n))}{bn}$$

input
```
Integrate[SinhIntegral[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
-((Cosh[a*d]*Cosh[b*d*Log[c*x^n]])/(b*d*n)) - (Sinh[a*d]*Sinh[b*d*Log[c*x^
n]])/(b*d*n) + (Log[c*x^n]*SinhIntegral[d*(a + b*Log[c*x^n])])/n + (a*Sinh
Integral[a*d + b*d*Log[c*x^n]])/(b*n)
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3039, 7281, 7082}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \text{Shi}(d(a+b\log(cx^n)))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int \text{Shi}(ad+b\log(cx^n)d)d(ad+b\log(cx^n)d)}{bdn}$$

$$\Big\downarrow \text{7082}$$

$$\frac{(ad+bd\log(cx^n))\text{Shi}(ad+b\log(cx^n)d)-\frac{x^{-n}(c^2x^{2n}+1)}{2c}}{bdn}$$

input
```
Int[SinhIntegral[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
(-1/2*(1 + c^2*x^(2*n))/(c*x^n) + (a*d + b*d*Log[c*x^n])*SinhIntegral[a*d
+ b*d*Log[c*x^n]])/(b*d*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 7082
```
Int[SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(SinhInte
gral[a + b*x]/b), x] - Simp[Cosh[a + b*x]/b, x] /; FreeQ[{a, b}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\operatorname{Shi}(ad+bd\ln(c\,x^{n}))(ad+bd\ln(c\,x^{n}))-\cosh(ad+bd\ln(c\,x^{n}))}{nbd}$
default	$\frac{\operatorname{Shi}(ad+bd\ln(c\,x^{n}))(ad+bd\ln(c\,x^{n}))-\cosh(ad+bd\ln(c\,x^{n}))}{nbd}$
parts	$\ln\left(x\right)\operatorname{Shi}\left(d(a+b\ln\left(c\,x^{n}\right))\right)-bn\left(-\frac{(\ln(c\,x^{n})-n\ln(x))\operatorname{Shi}(\ln(x)bdn+d(b(\ln(c\,x^{n})-n\ln(x))+a))}{b\,n^{2}}-\right.$

input
```
int(Shi(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b/d*(Shi(a*d+b*d*ln(c*x^n))*(a*d+b*d*ln(c*x^n))-cosh(a*d+b*d*ln(c*x^n)
))
```

Fricas [F]

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x}\,dx = \int \frac{\text{Shi}((b\log(cx^n)+a)d)}{x}\,dx$$

input
```
integrate(Shi(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*d*log(c*x^n) + a*d)/x, x)
```

Sympy [F]

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x}\,dx = \int \frac{\text{Shi}(ad+bd\log(cx^n))}{x}\,dx$$

input
```
integrate(Shi(d*(a+b*ln(c*x**n)))/x,x)
```

output
```
Integral(Shi(a*d + b*d*log(c*x**n))/x, x)
```

Maxima [F]

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x}\,dx = \int \frac{\text{Shi}((b\log(cx^n)+a)d)}{x}\,dx$$

input
```
integrate(Shi(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

output
```
integrate(Shi((b*log(c*x^n) + a)*d)/x, x)
```

Giac [F]

$$\int \frac{\text{Shi}(d(a + b\log(cx^n)))}{x} \, dx = \int \frac{\text{Shi}((b\log(cx^n) + a)d)}{x} \, dx$$

input
```
integrate(Shi(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

output
```
integrate(Shi((b*log(c*x^n) + a)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{Shi}(d(a + b\log(cx^n)))}{x} \, dx = \frac{\text{sinhint}(d\,(a + b\,\ln(c\,x^n)))\,\ln(c\,x^n)}{n} + \frac{a\,\text{sinhint}(d\,(a + b\,\ln(c\,x^n)))}{b\,n} - \frac{e^{a\,d}\,(c\,x^n)^{b\,d}}{2\,b\,d\,n} - \frac{e^{-a\,d}}{2\,b\,d\,n\,(c\,x^n)^{b\,d}}$$

input
```
int(sinhint(d*(a + b*log(c*x^n)))/x,x)
```

output
```
(sinhint(d*(a + b*log(c*x^n)))*log(c*x^n))/n + (a*sinhint(d*(a + b*log(c*x
^n))))/(b*n) - (exp(a*d)*(c*x^n)^(b*d))/(2*b*d*n) - exp(-a*d)/(2*b*d*n*(c*
x^n)^(b*d))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{\text{Shi}(d(a + b\log(cx^n)))}{x} \, dx = \frac{-\cosh(\log(x^n c)\,bd + ad) + \log(x^n c)\,shi(\log(x^n c)\,bd + ad)\,bd + shi(\log(x^n c)\,bd + ad)\,ad}{bdn}$$

input
```
int(Shi(d*(a+b*log(c*x^n)))/x,x)
```

output
```
( - cosh(log(x**n*c)*b*d + a*d) + log(x**n*c)*shi(log(x**n*c)*b*d + a*d)*b
*d + shi(log(x**n*c)*b*d + a*d)*a*d)/(b*d*n)
```

3.36 $\int \frac{\mathrm{Shi}(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 122

$$\int \frac{\mathrm{Shi}(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\,\mathrm{ExpIntegralEi}\left(-\frac{(1-bdn)(a+b\log(cx^n))}{bn}\right)}{2x} - \frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\,\mathrm{ExpIntegralEi}\left(-\frac{(1+bdn)(a+b\log(cx^n))}{bn}\right)}{2x} - \frac{\mathrm{Shi}(d(a+b\log(cx^n)))}{x}$$

output
```
1/2*exp(a/b/n)*(c*x^n)^(1/n)*Ei(-(-b*d*n+1)*(a+b*ln(c*x^n))/b/n)/x-1/2*exp
(a/b/n)*(c*x^n)^(1/n)*Ei(-(b*d*n+1)*(a+b*ln(c*x^n))/b/n)/x-Shi(d*(a+b*ln(c
*x^n)))/x
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.20

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x^2}\,dx$$
$$=\frac{1}{2}e^{-\frac{(-1+bdn)(a+b(-n\log(x)+\log(cx^n)))}{bn}}\left(\text{ExpIntegralEi}\left(\frac{(-1+bdn)(a+b\log(cx^n))}{bn}\right)\right.$$
$$\left.-\text{ExpIntegralEi}\left(-\frac{(1+bdn)(a+b\log(cx^n))}{bn}\right)\right)(\cosh(d(a+b(-n\log(x)+\log(cx^n))))$$
$$+\sinh(d(a+b(-n\log(x)+\log(cx^n)))))-\frac{\text{Shi}(d(a+b\log(cx^n)))}{x}$$

input
```
Integrate[SinhIntegral[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
((ExpIntegralEi[((-1 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)] - ExpIntegralEi[-
(((1 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))])*(Cosh[d*(a + b*(-(n*Log[x]) + L
og[c*x^n]))] + Sinh[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]))/(2*E^(((-1 + b
*d*n)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))) - SinhIntegral[d*(a + b*
Log[c*x^n])]/x
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {7109, 27, 6065, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x^2}\,dx$$
$$\Big\downarrow \text{7109}$$
$$bdn\int \frac{\sinh(d(a+b\log(cx^n)))}{dx^2(a+b\log(cx^n))}dx-\frac{\text{Shi}(d(a+b\log(cx^n)))}{x}$$
$$\Big\downarrow \text{27}$$

$$bn\int \frac{\sinh (d(a+b\log (cx^n)))}{x^2 (a+b\log (cx^n))}dx-\frac{\text{Shi}(d(a+b\log (cx^n)))}{x}$$

$\Big\downarrow$ 6065

$$bn\left(\frac{1}{2}e^{ad}x^{-bdn}(cx^n)^{bd}\int \frac{x^{bdn-2}}{a+b\log (cx^n)}dx-\frac{1}{2}e^{-ad}x^{bdn}(cx^n)^{-bd}\int \frac{x^{-bdn-2}}{a+b\log (cx^n)}dx\right)-\frac{\text{Shi}(d(a+b\log (cx^n)))}{x}$$

$\Big\downarrow$ 2747

$$bn\left(\frac{e^{ad}(cx^n)^{\frac{1}{n}}\int \frac{(cx^n)^{-\frac{1-bdn}{n}}}{a+b\log(cx^n)}d\log (cx^n)}{2nx}-\frac{e^{-ad}(cx^n)^{\frac{1}{n}}\int \frac{(cx^n)^{-\frac{bdn+1}{n}}}{a+b\log(cx^n)}d\log (cx^n)}{2nx}\right)-\frac{\text{Shi}(d(a+b\log (cx^n)))}{x}$$

$\Big\downarrow$ 2609

$$bn\left(\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{(1-bdn)(a+b\log(cx^n))}{bn}\right)}{2bnx}-\frac{(cx^n)^{\frac{1}{n}}e^{a\left(\frac{1}{bn}+d\right)-ad}\text{ExpIntegralEi}\left(-\frac{(bdn+1)(a+b\log(cx^n)}{bn}\right.}{2bnx}\right.$$
$$\frac{\text{Shi}(d(a+b\log (cx^n)))}{x}$$

input
```
Int[SinhIntegral[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
b*n*((E^(a/(b*n))*(c*x^n)^n^(-1)*ExpIntegralEi[-(((1 - b*d*n)*(a + b*Log[c
*x^n]))/(b*n))])/(2*b*n*x) - (E^(-(a*d) + a*(d + 1/(b*n)))*(c*x^n)^n^(-1)*
ExpIntegralEi[-(((1 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))])/(2*b*n*x)) - Sin
hIntegral[d*(a + b*Log[c*x^n])]/x
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 6065
```
Int[(((e_.) + Log[(g_.)*(x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.)*
Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-E^((
-a)*d))*(i*x)^r*(1/((c*x^n)^(b*d)*(2*x^(r - b*d*n))))   Int[x^(r - b*d*n)*(
h*(e + f*Log[g*x^m]))^q, x], x] + Simp[E^(a*d)*(i*x)^r*((c*x^n)^(b*d)/(2*x^
(r + b*d*n)))   Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ
[{a, b, c, d, e, f, g, h, i, m, n, q, r}, x]
```

rule 7109
```
Int[((e_.)*(x_))^(m_.)*SinhIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(
d_.)], x_Symbol] :> Simp[(e*x)^(m + 1)*(SinhIntegral[d*(a + b*Log[c*x^n])]/
(e*(m + 1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*(Sinh[d*(a + b*Log[c*
x^n])]/(d*(a + b*Log[c*x^n]))), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[m, -1]
```

Maple [F]

$$\int \frac{\mathrm{Shi}\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{2}}dx$$

input
```
int(Shi(d*(a+b*ln(c*x^n)))/x^2,x)
```

output
```
int(Shi(d*(a+b*ln(c*x^n)))/x^2,x)
```

Fricas [F]

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\text{Shi}((b\log(cx^n)+a)d)}{x^2}\,dx$$

input
```
integrate(Shi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*d*log(c*x^n) + a*d)/x^2, x)
```

Sympy [F]

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\text{Shi}(ad+bd\log(cx^n))}{x^2}\,dx$$

input
```
integrate(Shi(d*(a+b*ln(c*x**n)))/x**2,x)
```

output
```
Integral(Shi(a*d + b*d*log(c*x**n))/x**2, x)
```

Maxima [F]

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\text{Shi}((b\log(cx^n)+a)d)}{x^2}\,dx$$

input
```
integrate(Shi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

output
```
integrate(Shi((b*log(c*x^n) + a)*d)/x^2, x)
```

Giac [F]

$$\int \frac{\mathrm{Shi}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\mathrm{Shi}((b\log(cx^n)+a)d)}{x^2}\,dx$$

input
```
integrate(Shi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

output
```
integrate(Shi((b*log(c*x^n) + a)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\mathrm{Shi}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\mathrm{sinhint}(d\,(a+b\,\ln(c\,x^n)))}{x^2}\,dx$$

input
```
int(sinhint(d*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int(sinhint(d*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [F]

$$\int \frac{\mathrm{Shi}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{shi(\log(x^n c)\,bd+ad)}{x^2}\,dx$$

input
```
int(Shi(d*(a+b*log(c*x^n)))/x^2,x)
```

output
```
int(shi(log(x**n*c)*b*d + a*d)/x**2,x)
```

3.37 $\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 130

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x^3}\,dx = \frac{e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\text{ExpIntegralEi}\left(-\frac{(2-bdn)(a+b\log(cx^n))}{bn}\right)}{4x^2} - \frac{e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\text{ExpIntegralEi}\left(-\frac{(2+bdn)(a+b\log(cx^n))}{bn}\right)}{4x^2} - \frac{\text{Shi}(d(a+b\log(cx^n)))}{2x^2}$$

output
```
1/4*exp(2*a/b/n)*(c*x^n)^(2/n)*Ei(-(-b*d*n+2)*(a+b*ln(c*x^n))/b/n)/x^2-1/4
*exp(2*a/b/n)*(c*x^n)^(2/n)*Ei(-(b*d*n+2)*(a+b*ln(c*x^n))/b/n)/x^2-1/2*Shi
(d*(a+b*ln(c*x^n)))/x^2
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.14

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x^3}\,dx$$
$$= \frac{1}{4}e^{-\frac{(-2+bdn)(a+b(-n\log(x)+\log(cx^n)))}{bn}}\left(\text{ExpIntegralEi}\left(\frac{(-2+bdn)(a+b\log(cx^n))}{bn}\right) - \text{ExpIntegralEi}\left(-\frac{(2+bdn)(a+b\log(cx^n))}{bn}\right)\right)(\cosh(d(a+b(-n\log(x)+\log(cx^n)))) + \sinh(d(a+b(-n\log(x)+\log(cx^n))))) - \frac{\text{Shi}(d(a+b\log(cx^n)))}{2x^2}$$

input
```
Integrate[SinhIntegral[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
((ExpIntegralEi[((-2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)] - ExpIntegralEi[-
(((2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))])*(Cosh[d*(a + b*(-(n*Log[x]) + L
og[c*x^n]))] + Sinh[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]))/(4*E^(((-2 + b
*d*n)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))) - SinhIntegral[d*(a + b*
Log[c*x^n])]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {7109, 27, 6065, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x^3}\,dx$$
$$\Big\downarrow \text{7109}$$
$$\frac{1}{2}bdn\int \frac{\sinh(d(a+b\log(cx^n)))}{dx^3(a+b\log(cx^n))}dx - \frac{\text{Shi}(d(a+b\log(cx^n)))}{2x^2}$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{2}bn\int\frac{\sinh(d(a+b\log(cx^n)))}{x^3(a+b\log(cx^n))}dx-\frac{\text{Shi}(d(a+b\log(cx^n)))}{2x^2}$$

$\Big\downarrow$ 6065

$$\frac{1}{2}bn\left(\frac{1}{2}e^{ad}x^{-bdn}(cx^n)^{bd}\int\frac{x^{bdn-3}}{a+b\log(cx^n)}dx-\frac{1}{2}e^{-ad}x^{bdn}(cx^n)^{-bd}\int\frac{x^{-bdn-3}}{a+b\log(cx^n)}dx\right)-\frac{\text{Shi}(d(a+b\log(cx^n)))}{2x^2}$$

$\Big\downarrow$ 2747

$$\frac{1}{2}bn\left(\frac{e^{ad}(cx^n)^{2/n}\int\frac{(cx^n)^{-\frac{2-bdn}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2nx^2}-\frac{e^{-ad}(cx^n)^{2/n}\int\frac{(cx^n)^{-\frac{bdn+2}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2nx^2}\right)-\frac{\text{Shi}(d(a+b\log(cx^n)))}{2x^2}$$

$\Big\downarrow$ 2609

$$\frac{1}{2}bn\left(\frac{e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\text{ExpIntegralEi}\left(-\frac{(2-bdn)(a+b\log(cx^n))}{bn}\right)}{2bnx^2}-\frac{(cx^n)^{2/n}\,e^{a\left(\frac{2}{bn}+d\right)-ad}\,\text{ExpIntegralEi}\left(-\frac{(bdn+2)(a+b\log}{bn}\right.}{2bnx^2}\right.-\frac{\text{Shi}(d(a+b\log(cx^n)))}{2x^2}$$

input
```
Int[SinhIntegral[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
(b*n*((E^((2*a)/(b*n))*(c*x^n)^(2/n)*ExpIntegralEi[-(((2 - b*d*n)*(a + b*L
og[c*x^n]))/(b*n))])/(2*b*n*x^2) - (E^(-(a*d) + a*(d + 2/(b*n)))*(c*x^n)^(
2/n)*ExpIntegralEi[-(((2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))])/(2*b*n*x^2)
))/2 - SinhIntegral[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 6065
```
Int[(((e_.) + Log[(g_.)*(x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.)*
Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-E^((
-a)*d))*(i*x)^r*(1/((c*x^n)^(b*d)*(2*x^(r - b*d*n))))   Int[x^(r - b*d*n)*(
h*(e + f*Log[g*x^m]))^q, x], x] + Simp[E^(a*d)*(i*x)^r*((c*x^n)^(b*d)/(2*x^
(r + b*d*n)))   Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ
[{a, b, c, d, e, f, g, h, i, m, n, q, r}, x]
```

rule 7109
```
Int[((e_.)*(x_))^(m_.)*SinhIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(
d_.)], x_Symbol] :> Simp[(e*x)^(m + 1)*(SinhIntegral[d*(a + b*Log[c*x^n])]/
(e*(m + 1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*(Sinh[d*(a + b*Log[c*
x^n])]/(d*(a + b*Log[c*x^n]))), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[m, -1]
```

Maple [F]

$$\int \frac{\mathrm{Shi}\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{3}}dx$$

input
```
int(Shi(d*(a+b*ln(c*x^n)))/x^3,x)
```

output
```
int(Shi(d*(a+b*ln(c*x^n)))/x^3,x)
```

Fricas [F]

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\text{Shi}((b\log(cx^n)+a)d)}{x^3}\,dx$$

input
```
integrate(Shi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

output
```
integral(sinh_integral(b*d*log(c*x^n) + a*d)/x^3, x)
```

Sympy [F]

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\text{Shi}(ad+bd\log(cx^n))}{x^3}\,dx$$

input
```
integrate(Shi(d*(a+b*ln(c*x**n)))/x**3,x)
```

output
```
Integral(Shi(a*d + b*d*log(c*x**n))/x**3, x)
```

Maxima [F]

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\text{Shi}((b\log(cx^n)+a)d)}{x^3}\,dx$$

input
```
integrate(Shi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

output
```
integrate(Shi((b*log(c*x^n) + a)*d)/x^3, x)
```

Giac [F]

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\text{Shi}((b\log(cx^n)+a)d)}{x^3}\,dx$$

input
```
integrate(Shi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

output
```
integrate(Shi((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\text{sinhint}(d\,(a+b\,\ln(c\,x^n)))}{x^3}\,dx$$

input
```
int(sinhint(d*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int(sinhint(d*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [F]

$$\int \frac{\text{Shi}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{shi(\log(x^n c)\,bd+ad)}{x^3}dx$$

input
```
int(Shi(d*(a+b*log(c*x^n)))/x^3,x)
```

output
```
int(shi(log(x**n*c)*b*d + a*d)/x**3,x)
```

3.38 $\int (ex)^m \textbf{Shi}(d(a+b\log(cx^n))) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 167

$$\begin{aligned}\int (ex)^m \text{Shi}(d(a+b\log(cx^n))) \, dx &= \frac{e^{-\frac{a(1+m)}{bn}} x(ex)^m (cx^n)^{-\frac{1+m}{n}} \text{ExpIntegralEi}\left(\frac{(1+m-bdn)(a+b\log(cx^n))}{bn}\right)}{2(1+m)} \\ &\quad - \frac{e^{-\frac{a(1+m)}{bn}} x(ex)^m (cx^n)^{-\frac{1+m}{n}} \text{ExpIntegralEi}\left(\frac{(1+m+bdn)(a+b\log(cx^n))}{bn}\right)}{2(1+m)} \\ &\quad + \frac{(ex)^{1+m}\text{Shi}(d(a+b\log(cx^n)))}{e(1+m)}\end{aligned}$$

output
```
1/2*x*(e*x)^m*Ei((-b*d*n+m+1)*(a+b*ln(c*x^n))/b/n)/exp(a*(1+m)/b/n)/(1+m)/
((c*x^n)^((1+m)/n))-1/2*x*(e*x)^m*Ei((b*d*n+m+1)*(a+b*ln(c*x^n))/b/n)/exp(
a*(1+m)/b/n)/(1+m)/((c*x^n)^((1+m)/n))+(e*x)^(1+m)*Shi(d*(a+b*ln(c*x^n)))/
e/(1+m)
```

Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.72

$$\int (ex)^m \text{Shi}(d(a+b\log(cx^n)))\,dx = \frac{(ex)^m\left(e^{-\frac{(1+m)(a-bn\log(x)+b\log(cx^n))}{bn}}x^{-m}\left(\text{ExpIntegralEi}\left(\frac{(1+m-bdn)(a+b\log(cx^n))}{bn}\right)-\text{ExpIntegralEi}\left(\frac{(1+m+bdn}{}\right.\right.\right.}{2(1+m)}$$

input
```
Integrate[(e*x)^m*SinhIntegral[d*(a + b*Log[c*x^n])],x]
```

output
```
((e*x)^m*((ExpIntegralEi[((1 + m - b*d*n)*(a + b*Log[c*x^n]))/(b*n)] - Exp
IntegralEi[((1 + m + b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(E^(((1 + m)*(a -
b*n*Log[x] + b*Log[c*x^n]))/(b*n))*x^m) + 2*x*SinhIntegral[d*(a + b*Log[c*
x^n])]))/(2*(1 + m))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {7109, 27, 6065, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \text{Shi}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \; 7109$$

$$\frac{(ex)^{m+1}\text{Shi}(d(a+b\log(cx^n)))}{e(m+1)} - \frac{bdn\int \frac{(ex)^m\sinh(d(a+b\log(cx^n)))}{d(a+b\log(cx^n))}dx}{m+1}$$

$$\Big\downarrow \; 27$$

$$\frac{(ex)^{m+1}\text{Shi}(d(a+b\log(cx^n)))}{e(m+1)} - \frac{bn\int \frac{(ex)^m\sinh(d(a+b\log(cx^n)))}{a+b\log(cx^n)}dx}{m+1}$$

$$\Big\downarrow \; 6065$$

$$\frac{(ex)^{m+1}\text{Shi}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{bn\left(\frac{1}{2}e^{ad}(ex)^m(cx^n)^{bd}x^{-bdn-m}\int\frac{x^{m+bdn}}{a+b\log(cx^n)}dx-\frac{1}{2}e^{-ad}(ex)^m(cx^n)^{-bd}x^{bdn-m}\int\frac{x^{m-bdn}}{a+b\log(cx^n)}dx\right)}{m+1}$$

$$\Big\downarrow \; 2747$$

$$\frac{(ex)^{m+1}\text{Shi}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{bn\left(\frac{xe^{ad}(ex)^m(cx^n)^{bd-\frac{bdn+m+1}{n}}\int\frac{(cx^n)^{\frac{m+bdn+1}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2n}-\frac{xe^{-ad}(ex)^m(cx^n)^{-\frac{-bdn+m+1}{n}-bd}\int\frac{(cx^n)^{\frac{m-bdn+1}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2n}\right)}{m+1}$$

$$\Big\downarrow \; 2609$$

$$\frac{(ex)^{m+1}\text{Shi}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{bn\left(\frac{x(ex)^me^{ad-\frac{a(bdn+m+1)}{bn}}(cx^n)^{bd-\frac{bdn+m+1}{n}}\text{ExpIntegralEi}\left(\frac{(m+bdn+1)(a+b\log(cx^n))}{bn}\right)}{2bn}-\frac{x(ex)^me^{-\frac{a(-bdn+m+1)}{bn}-ad}(cx^n)^{-\frac{-bdn+m+1}{n}}}{2b}\right.}{m+1}$$

input
```
Int[(e*x)^m*SinhIntegral[d*(a + b*Log[c*x^n])],x]
```

output
```
-((b*n*(-1/2*(E^(-(a*d) - (a*(1 + m - b*d*n))/(b*n))*x*(e*x)^m*(c*x^n)^(-(
b*d) - (1 + m - b*d*n)/n)*ExpIntegralEi[((1 + m - b*d*n)*(a + b*Log[c*x^n]
))/(b*n)])/(b*n) + (E^(a*d - (a*(1 + m + b*d*n))/(b*n))*x*(e*x)^m*(c*x^n)^
(b*d - (1 + m + b*d*n)/n)*ExpIntegralEi[((1 + m + b*d*n)*(a + b*Log[c*x^n]
))/(b*n)])/(2*b*n)))/(1 + m)) + ((e*x)^(1 + m)*SinhIntegral[d*(a + b*Log[c
*x^n])])/(e*(1 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 6065
```
Int[(((e_.) + Log[(g_.)*(x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.)*
Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-E^((
-a)*d))*(i*x)^r*(1/((c*x^n)^(b*d)*(2*x^(r - b*d*n))))   Int[x^(r - b*d*n)*(
h*(e + f*Log[g*x^m]))^q, x], x] + Simp[E^(a*d)*(i*x)^r*((c*x^n)^(b*d)/(2*x^
(r + b*d*n)))   Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ
[{a, b, c, d, e, f, g, h, i, m, n, q, r}, x]
```

rule 7109
```
Int[((e_.)*(x_))^(m_.)*SinhIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(
d_.)], x_Symbol] :> Simp[(e*x)^(m + 1)*(SinhIntegral[d*(a + b*Log[c*x^n])]/
(e*(m + 1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*(Sinh[d*(a + b*Log[c*
x^n])]/(d*(a + b*Log[c*x^n]))), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[m, -1]
```

Maple [F]

$$\int (ex)^m \operatorname{Shi}(d(a + b\ln(c\,x^n)))\,dx$$

input
```
int((e*x)^m*Shi(d*(a+b*ln(c*x^n))),x)
```

output
```
int((e*x)^m*Shi(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int (ex)^m \text{Shi}(d(a + b\log(cx^n)))\, dx = \int (ex)^m \text{Shi}((b\log(cx^n) + a)d)\, dx$$

input
```
integrate((e*x)^m*Shi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral((e*x)^m*sinh_integral(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int (ex)^m \text{Shi}(d(a + b\log(cx^n)))\, dx = \int (ex)^m \text{Shi}(ad + bd\log(cx^n))\, dx$$

input
```
integrate((e*x)**m*Shi(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral((e*x)**m*Shi(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int (ex)^m \text{Shi}(d(a + b\log(cx^n)))\, dx = \int (ex)^m \text{Shi}((b\log(cx^n) + a)d)\, dx$$

input
```
integrate((e*x)^m*Shi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate((e*x)^m*Shi((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int (ex)^m \text{Shi}(d(a+b\log(cx^n)))\,dx = \int (ex)^m\,\text{Shi}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*Shi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate((e*x)^m*Shi((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \text{Shi}(d(a+b\log(cx^n)))\,dx = \int \text{sinhint}(d\,(a+b\,\ln(c\,x^n)))\,(e\,x)^m\,dx$$

input
```
int(sinhint(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

output
```
int(sinhint(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \text{Shi}(d(a+b\log(cx^n)))\,dx = e^m\left(\int x^m\,shi(\log(x^n c)\,bd+ad)\,dx\right)$$

input
```
int((e*x)^m*Shi(d*(a+b*log(c*x^n))),x)
```

output
```
e**m*int(x**m*shi(log(x**n*c)*b*d + a*d),x)
```

3.39 $\int \frac{\sinh(bx)\mathbf{Shi}(bx)}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 96

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x^3}\,dx = b^2\text{Chi}(2bx) - \frac{b\cosh(bx)\sinh(bx)}{2x} - \frac{\sinh^2(bx)}{4x^2} - \frac{b\sinh(2bx)}{4x} - \frac{b\cosh(bx)\text{Shi}(bx)}{2x} - \frac{\sinh(bx)\text{Shi}(bx)}{2x^2} + \frac{1}{4}b^2\text{Shi}(bx)^2$$

output
```
b^2*Chi(2*b*x)-1/2*b*cosh(b*x)*sinh(b*x)/x-1/4*sinh(b*x)^2/x^2-1/4*b*sinh(
2*b*x)/x-1/2*b*cosh(b*x)*Shi(b*x)/x-1/2*sinh(b*x)*Shi(b*x)/x^2+1/4*b^2*Shi
(b*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x^3}\,dx = b^2\text{Chi}(2bx) - \frac{b\cosh(bx)\sinh(bx)}{2x} - \frac{\sinh^2(bx)}{4x^2} - \frac{b\sinh(2bx)}{4x} - \frac{b\cosh(bx)\text{Shi}(bx)}{2x} - \frac{\sinh(bx)\text{Shi}(bx)}{2x^2} + \frac{1}{4}b^2\text{Shi}(bx)^2$$

input
```
Integrate[(Sinh[b*x]*SinhIntegral[b*x])/x^3,x]
```

output
```
b^2*CoshIntegral[2*b*x] - (b*Cosh[b*x]*Sinh[b*x])/(2*x) - Sinh[b*x]^2/(4*x
^2) - (b*Sinh[2*b*x])/(4*x) - (b*Cosh[b*x]*SinhIntegral[b*x])/(2*x) - (Sin
h[b*x]*SinhIntegral[b*x])/(2*x^2) + (b^2*SinhIntegral[b*x]^2)/4
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.40, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 1.750$, Rules used = {7098, 27, 3042, 25, 3795, 14, 25, 3042, 25, 3793, 2009, 7104, 27, 5971, 27, 3042, 26, 3778, 3042, 3782, 7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(bx)\sinh(bx)}{x^3}dx$$

$$\Big\downarrow \text{7098}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}dx+\frac{1}{2}b\int \frac{\sinh^2(bx)}{bx^3}dx-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}dx+\frac{1}{2}\int \frac{\sinh^2(bx)}{x^3}dx-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}dx+\frac{1}{2}\int -\frac{\sin(ibx)^2}{x^3}dx-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}dx-\frac{1}{2}\int \frac{\sin(ibx)^2}{x^3}dx-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{3795}$$

$$\frac{1}{2}\left(b^2\int \frac{1}{x}dx-2b^2\int -\frac{\sinh^2(bx)}{x}dx-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)+\frac{1}{2}b\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}dx-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \ 14$$

$$\frac{1}{2}\left(-2b^2\int -\frac{\sinh^2(bx)}{x}dx+b^2\log(x)-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)+\frac{1}{2}b\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}dx-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(2b^2\int \frac{\sinh^2(bx)}{x}dx+b^2\log(x)-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)+\frac{1}{2}b\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}dx-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\left(2b^2\int -\frac{\sin(ibx)^2}{x}dx+b^2\log(x)-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)+\frac{1}{2}b\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}dx-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(-2b^2\int \frac{\sin(ibx)^2}{x}dx+b^2\log(x)-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)+\frac{1}{2}b\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}dx-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \ 3793$$

$$\frac{1}{2}\left(-2b^2\int \left(\frac{1}{2x}-\frac{\cosh(2bx)}{2x}\right)dx+b^2\log(x)-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)+\frac{1}{2}b\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}dx-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}dx+\frac{1}{2}\left(-2b^2\left(\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}\right)+b^2\log(x)-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \ 7104$$

$$\frac{1}{2}b\left(b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx+b\int \frac{\cosh(bx)\sinh(bx)}{bx^2}dx-\frac{\text{Shi}(bx)\cosh(bx)}{x}\right)+\frac{1}{2}\left(-2b^2\left(\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}\right)+b^2\log(x)-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}b\left(b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx+\int \frac{\cosh(bx)\sinh(bx)}{x^2}dx-\frac{\text{Shi}(bx)\cosh(bx)}{x}\right)+\frac{1}{2}\left(-2b^2\left(\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}\right)+b^2\log(x)-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{1}{2}b\left(b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx+\int \frac{\sinh(2bx)}{2x^2}dx-\frac{\text{Shi}(bx)\cosh(bx)}{x}\right)+\frac{1}{2}\left(-2b^2\left(\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}\right)+b^2\log(x)-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}b\left(b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx+\frac{1}{2}\int \frac{\sinh(2bx)}{x^2}dx-\frac{\text{Shi}(bx)\cosh(bx)}{x}\right)+\frac{1}{2}\left(-2b^2\left(\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}\right)+b^2\log(x)-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}b\left(b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx+\frac{1}{2}\int -\frac{i\sin(2ibx)}{x^2}dx-\frac{\text{Shi}(bx)\cosh(bx)}{x}\right)+\frac{1}{2}\left(-2b^2\left(\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}\right)+b^2\log(x)-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{2}b\left(b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx-\frac{1}{2}i\int \frac{\sin(2ibx)}{x^2}dx-\frac{\text{Shi}(bx)\cosh(bx)}{x}\right)+\frac{1}{2}\left(-2b^2\left(\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}\right)+b^2\log(x)-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}b\left(b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx-\frac{1}{2}i\left(2ib\int \frac{\cosh(2bx)}{x}dx-\frac{i\sinh(2bx)}{x}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{x}\right)+\frac{1}{2}\left(-2b^2\left(\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}\right)+b^2\log(x)-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}b\left(b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx-\frac{1}{2}i\left(2ib\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx-\frac{i\sinh(2bx)}{x}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{x}\right)+\frac{1}{2}\left(-2b^2\left(\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}\right)+b^2\log(x)-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{3782}$$

$$\frac{1}{2}b\left(b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx-\frac{1}{2}i\left(2ib\text{Chi}(2bx)-\frac{i\sinh(2bx)}{x}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{x}\right)+\frac{1}{2}\left(-2b^2\left(\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}\right)+b^2\log(x)-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{7237}$$

$$\frac{1}{2}\left(-2b^2\left(\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}\right)+b^2\log(x)-\frac{\sinh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)+\frac{1}{2}b\left(-\frac{1}{2}i\left(2ib\text{Chi}(2bx)-\frac{i\sinh(2bx)}{x}\right)+\frac{1}{2}b\text{Shi}(bx)^2-\frac{\text{Shi}(bx)\cosh(bx)}{x}\right)-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}$$

input
```
Int[(Sinh[b*x]*SinhIntegral[b*x])/x^3,x]
```

output
```
(-2*b^2*(-1/2*CoshIntegral[2*b*x] + Log[x]/2) + b^2*Log[x] - (b*Cosh[b*x]*
Sinh[b*x])/x - Sinh[b*x]^2/(2*x^2))/2 - (Sinh[b*x]*SinhIntegral[b*x])/(2*x
^2) + (b*((-1/2*I)*((2*I)*b*CoshIntegral[2*b*x] - (I*Sinh[2*b*x])/x) - (Co
sh[b*x]*SinhIntegral[b*x])/x + (b*SinhIntegral[b*x]^2)/2))/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7098
```
Int[((e_.) + (f_.)*(x_))^(m_)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) +
 (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^(m + 1)*Sinh[a + b*x]*(SinhIntegr
al[c + d*x]/(f*(m + 1))), x] + (-Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)
*Cosh[a + b*x]*SinhIntegral[c + d*x], x], x] - Simp[d/(f*(m + 1))   Int[(e
+ f*x)^(m + 1)*Sinh[a + b*x]*(Sinh[c + d*x]/(c + d*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f}, x] && ILtQ[m, -1]
```

rule 7104
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^(m + 1)*Cosh[a + b*x]*(SinhInteg
ral[c + d*x]/(f*(m + 1))), x] + (-Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1
)*Sinh[a + b*x]*SinhIntegral[c + d*x], x], x] - Simp[d/(f*(m + 1))   Int[(e
 + f*x)^(m + 1)*Cosh[a + b*x]*(Sinh[c + d*x]/(c + d*x)), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && ILtQ[m, -1]
```

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{\sinh(bx)\operatorname{Shi}(bx)}{x^3}dx$$

input
```
int(sinh(b*x)*Shi(b*x)/x^3,x)
```

output
```
int(sinh(b*x)*Shi(b*x)/x^3,x)
```

Fricas [F]

$$\int \frac{\sinh(bx)\mathrm{Shi}(bx)}{x^3}\,dx = \int \frac{\mathrm{Shi}(bx)\sinh(bx)}{x^3}\,dx$$

input
```
integrate(sinh(b*x)*Shi(b*x)/x^3,x, algorithm="fricas")
```

output
```
integral(sinh(b*x)*sinh_integral(b*x)/x^3, x)
```

Sympy [F]

$$\int \frac{\sinh(bx)\mathrm{Shi}(bx)}{x^3}\,dx = \int \frac{\sinh(bx)\operatorname{Shi}(bx)}{x^3}\,dx$$

input
```
integrate(sinh(b*x)*Shi(b*x)/x**3,x)
```

output
```
Integral(sinh(b*x)*Shi(b*x)/x**3, x)
```

Maxima [F]

$$\int \frac{\sinh(bx)\mathrm{Shi}(bx)}{x^3}\,dx = \int \frac{\mathrm{Shi}(bx)\sinh(bx)}{x^3}\,dx$$

input
```
integrate(sinh(b*x)*Shi(b*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x)*sinh(b*x)/x^3, x)
```

Giac [F]

$$\int \frac{\sinh(bx)\mathrm{Shi}(bx)}{x^3}\,dx = \int \frac{\mathrm{Shi}(bx)\sinh(bx)}{x^3}\,dx$$

input
```
integrate(sinh(b*x)*Shi(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(Shi(b*x)*sinh(b*x)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(bx)\mathrm{Shi}(bx)}{x^3}\,dx = \int \frac{\mathrm{sinhint}(b\,x)\,\sinh(b\,x)}{x^3}\,dx$$

input
```
int((sinhint(b*x)*sinh(b*x))/x^3,x)
```

output
```
int((sinhint(b*x)*sinh(b*x))/x^3, x)
```

Reduce [F]

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x^3}\,dx = \int \frac{shi(bx)\sinh(bx)}{x^3}dx$$

input
```
int(sinh(b*x)*Shi(b*x)/x^3,x)
```

output
```
int((shi(b*x)*sinh(b*x))/x**3,x)
```

3.40 $\int \frac{\sinh(bx)\mathbf{Shi}(bx)}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sinh(bx)\mathrm{Shi}(bx)}{x^2}\,dx = -\frac{\sinh^2(bx)}{x} - \frac{\sinh(bx)\mathrm{Shi}(bx)}{x} + b\mathrm{Shi}(2bx) + b\mathrm{Int}\left(\frac{\cosh(bx)\mathrm{Shi}(bx)}{x},x\right)$$

output
```
-sinh(b*x)^2/x-sinh(b*x)*Shi(b*x)/x+b*Shi(2*b*x)+b*Defer(Int)(cosh(b*x)*Sh
i(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh(bx)\mathrm{Shi}(bx)}{x^2}\,dx = \int \frac{\sinh(bx)\mathrm{Shi}(bx)}{x^2}\,dx$$

input
```
Integrate[(Sinh[b*x]*SinhIntegral[b*x])/x^2,x]
```

output
```
Integrate[(Sinh[b*x]*SinhIntegral[b*x])/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(bx)\sinh(bx)}{x^2}\,dx$$

$$\Big\downarrow \text{ 7098}$$

$$b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx+b\int \frac{\sinh^2(bx)}{bx^2}dx-\frac{\text{Shi}(bx)\sinh(bx)}{x}$$

$$\Big\downarrow \text{ 27}$$

$$b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx+\int \frac{\sinh^2(bx)}{x^2}dx-\frac{\text{Shi}(bx)\sinh(bx)}{x}$$

$$\Big\downarrow \text{ 3042}$$

$$b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx+\int -\frac{\sin(ibx)^2}{x^2}dx-\frac{\text{Shi}(bx)\sinh(bx)}{x}$$

$$\Big\downarrow \text{ 25}$$

$$b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx-\int \frac{\sin(ibx)^2}{x^2}dx-\frac{\text{Shi}(bx)\sinh(bx)}{x}$$

$$\Big\downarrow \text{ 3794}$$

$$b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx-2ib\int \frac{i\sinh(2bx)}{2x}dx-\frac{\text{Shi}(bx)\sinh(bx)}{x}-\frac{\sinh^2(bx)}{x}$$

$$\Big\downarrow \text{ 27}$$

$$b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx+b\int \frac{\sinh(2bx)}{x}dx-\frac{\text{Shi}(bx)\sinh(bx)}{x}-\frac{\sinh^2(bx)}{x}$$

$$\Big\downarrow \text{ 3042}$$

$$b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx+b\int -\frac{i\sin(2ibx)}{x}dx-\frac{\text{Shi}(bx)\sinh(bx)}{x}-\frac{\sinh^2(bx)}{x}$$

$$\Big\downarrow \text{ 26}$$

$$b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx - ib\int \frac{\sin(2ibx)}{x}dx - \frac{\text{Shi}(bx)\sinh(bx)}{x} - \frac{\sinh^2(bx)}{x}$$

$$\Big\downarrow \text{ 3779}$$

$$b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx + b\text{Shi}(2bx) - \frac{\text{Shi}(bx)\sinh(bx)}{x} - \frac{\sinh^2(bx)}{x}$$

$$\Big\downarrow \text{ 7299}$$

$$b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx + b\text{Shi}(2bx) - \frac{\text{Shi}(bx)\sinh(bx)}{x} - \frac{\sinh^2(bx)}{x}$$

input
```
Int[(Sinh[b*x]*SinhIntegral[b*x])/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(bx)\,\text{Shi}(bx)}{x^2}dx$$

input
```
int(sinh(b*x)*Shi(b*x)/x^2,x)
```

output
```
int(sinh(b*x)*Shi(b*x)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}\,dx = \int \frac{\text{Shi}(bx)\sinh(bx)}{x^2}\,dx$$

input
```
integrate(sinh(b*x)*Shi(b*x)/x^2,x, algorithm="fricas")
```

output
```
integral(sinh(b*x)*sinh_integral(b*x)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.46 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh(bx)\mathrm{Shi}(bx)}{x^2}\,dx = \int \frac{\sinh(bx)\,\mathrm{Shi}(bx)}{x^2}\,dx$$

input
```
integrate(sinh(b*x)*Shi(b*x)/x**2,x)
```

output
```
Integral(sinh(b*x)*Shi(b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh(bx)\mathrm{Shi}(bx)}{x^2}\,dx = \int \frac{\mathrm{Shi}(bx)\sinh(bx)}{x^2}\,dx$$

input
```
integrate(sinh(b*x)*Shi(b*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x)*sinh(b*x)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}\,dx = \int \frac{\text{Shi}(bx)\sinh(bx)}{x^2}\,dx$$

input
```
integrate(sinh(b*x)*Shi(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(Shi(b*x)*sinh(b*x)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}\,dx = \int \frac{\text{sinhint}(b\,x)\ \sinh(b\,x)}{x^2}\,dx$$

input
```
int((sinhint(b*x)*sinh(b*x))/x^2,x)
```

output
```
int((sinhint(b*x)*sinh(b*x))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}\,dx = \int \frac{shi(bx)\sinh(bx)}{x^2}dx$$

input
```
int(sinh(b*x)*Shi(b*x)/x^2,x)
```

output
```
int((shi(b*x)*sinh(b*x))/x**2,x)
```

3.41 $\int \frac{\sinh(bx)\mathrm{Shi}(bx)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 10

$$\int \frac{\sinh(bx)\mathrm{Shi}(bx)}{x}\,dx = \frac{\mathrm{Shi}(bx)^2}{2}$$

output
```
1/2*Shi(b*x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(bx)\mathrm{Shi}(bx)}{x}\,dx = \frac{\mathrm{Shi}(bx)^2}{2}$$

input
```
Integrate[(Sinh[b*x]*SinhIntegral[b*x])/x,x]
```

output
```
SinhIntegral[b*x]^2/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(bx)\sinh(bx)}{x}\,dx$$

$$\Big\downarrow \text{ 7237}$$

$$\frac{\text{Shi}(bx)^2}{2}$$

input
```
Int[(Sinh[b*x]*SinhIntegral[b*x])/x,x]
```

output
```
SinhIntegral[b*x]^2/2
```

Defintions of rubi rules used

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\text{Shi}(bx)^2}{2}$	9
default	$\frac{\text{Shi}(bx)^2}{2}$	9

input
```
int(sinh(b*x)*Shi(b*x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*Shi(b*x)^2
```

Fricas [F]

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x}\,dx = \int \frac{\text{Shi}(bx)\sinh(bx)}{x}\,dx$$

input
```
integrate(sinh(b*x)*Shi(b*x)/x,x, algorithm="fricas")
```

output
```
integral(sinh(b*x)*sinh_integral(b*x)/x, x)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x}\,dx = \frac{\text{Shi}^2(bx)}{2}$$

input
```
integrate(sinh(b*x)*Shi(b*x)/x,x)
```

output
```
Shi(b*x)**2/2
```

Maxima [F]

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x}\,dx = \int \frac{\text{Shi}(bx)\sinh(bx)}{x}\,dx$$

input
```
integrate(sinh(b*x)*Shi(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x)*sinh(b*x)/x, x)
```

Giac [F]

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x}\,dx = \int \frac{\text{Shi}(bx)\sinh(bx)}{x}\,dx$$

input
```
integrate(sinh(b*x)*Shi(b*x)/x,x, algorithm="giac")
```

output
```
integrate(Shi(b*x)*sinh(b*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x}\,dx = \frac{\text{sinhint}(b\,x)^2}{2}$$

input
```
int((sinhint(b*x)*sinh(b*x))/x,x)
```

output
```
sinhint(b*x)^2/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x}\,dx = \frac{shi(bx)^2}{2}$$

input
```
int(sinh(b*x)*Shi(b*x)/x,x)
```

output
```
shi(b*x)**2/2
```

3.42 $\int \sinh(bx)\mathbf{Shi}(bx)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \sinh(bx)\mathrm{Shi}(bx)\,dx = \frac{\cosh(bx)\mathrm{Shi}(bx)}{b} - \frac{\mathrm{Shi}(2bx)}{2b}$$

output
```
cosh(b*x)*Shi(b*x)/b-1/2*Shi(2*b*x)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sinh(bx)\mathrm{Shi}(bx)\,dx = \frac{\cosh(bx)\mathrm{Shi}(bx)}{b} - \frac{\mathrm{Shi}(2bx)}{2b}$$

input
```
Integrate[Sinh[b*x]*SinhIntegral[b*x],x]
```

output
```
(Cosh[b*x]*SinhIntegral[b*x])/b - SinhIntegral[2*b*x]/(2*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {7094, 27, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Shi}(bx)\sinh(bx)\,dx$$

$$\Big\downarrow \text{7094}$$

$$\frac{\text{Shi}(bx)\cosh(bx)}{b}-\int \frac{\cosh(bx)\sinh(bx)}{bx}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int \frac{\cosh(bx)\sinh(bx)}{x}dx}{b}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int \frac{\sinh(2bx)}{2x}dx}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int \frac{\sinh(2bx)}{x}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int -\frac{i\sin(2ibx)}{x}dx}{2b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\text{Shi}(bx)\cosh(bx)}{b}+\frac{i\int \frac{\sin(2ibx)}{x}dx}{2b}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\text{Shi}(2bx)}{2b}$$

input
```
Int[Sinh[b*x]*SinhIntegral[b*x],x]
```

output
```
(Cosh[b*x]*SinhIntegral[b*x])/b - SinhIntegral[2*b*x]/(2*b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7094
```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\cosh(bx)\,\text{Shi}(bx)-\frac{\text{Shi}(2bx)}{2}}{b}$	22
default	$\frac{\cosh(bx)\,\text{Shi}(bx)-\frac{\text{Shi}(2bx)}{2}}{b}$	22

input
```
int(sinh(b*x)*Shi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b*(cosh(b*x)*Shi(b*x)-1/2*Shi(2*b*x))
```

Fricas [F]

$$\int \sinh(bx)\text{Shi}(bx)\,dx = \int \text{Shi}(bx)\sinh(bx)\,dx$$

input
```
integrate(sinh(b*x)*Shi(b*x),x, algorithm="fricas")
```

output
```
integral(sinh(b*x)*sinh_integral(b*x), x)
```

Sympy [F]

$$\int \sinh(bx)\text{Shi}(bx)\,dx = \int \sinh(bx)\,\text{Shi}(bx)\,dx$$

input
```
integrate(sinh(b*x)*Shi(b*x),x)
```

output
```
Integral(sinh(b*x)*Shi(b*x), x)
```

Maxima [F]

$$\int \sinh(bx)\text{Shi}(bx)\,dx = \int \text{Shi}(bx)\sinh(bx)\,dx$$

input
```
integrate(sinh(b*x)*Shi(b*x),x, algorithm="maxima")
```

output
```
integrate(Shi(b*x)*sinh(b*x), x)
```

Giac [F]

$$\int \sinh(bx)\text{Shi}(bx)\,dx = \int \text{Shi}(bx)\sinh(bx)\,dx$$

input
```
integrate(sinh(b*x)*Shi(b*x),x, algorithm="giac")
```

output
```
integrate(Shi(b*x)*sinh(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh(bx)\text{Shi}(bx)\,dx = \int \text{sinhint}(b\,x)\,\sinh(b\,x)\,dx$$

input
```
int(sinhint(b*x)*sinh(b*x),x)
```

output
```
int(sinhint(b*x)*sinh(b*x), x)
```

Reduce [F]

$$\int \sinh(bx)\text{Shi}(bx)\,dx = \int shi(bx)\sinh(bx)\,dx$$

input
```
int(sinh(b*x)*Shi(b*x),x)
```

output
```
int(shi(b*x)*sinh(b*x),x)
```

3.43 $\int x \sinh(bx)\mathbf{Shi}(bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 61

$$\int x\sinh(bx)\mathrm{Shi}(bx)\,dx=\frac{\mathrm{Chi}(2bx)}{2b^2}-\frac{\log(x)}{2b^2}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\cosh(bx)\mathrm{Shi}(bx)}{b}-\frac{\sinh(bx)\mathrm{Shi}(bx)}{b^2}$$

output
```
1/2*Chi(2*b*x)/b^2-1/2*ln(x)/b^2-1/2*sinh(b*x)^2/b^2+x*cosh(b*x)*Shi(b*x)/
b-sinh(b*x)*Shi(b*x)/b^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int x\sinh(bx)\mathrm{Shi}(bx)\,dx=-\frac{\cosh(2bx)-2\mathrm{Chi}(2bx)+2\log(x)+(-4bx\cosh(bx)+4\sinh(bx))\mathrm{Shi}(bx)}{4b^2}$$

input
```
Integrate[x*Sinh[b*x]*SinhIntegral[b*x],x]
```

output
```
-1/4*(Cosh[2*b*x] - 2*CoshIntegral[2*b*x] + 2*Log[x] + (-4*b*x*Cosh[b*x] +
 4*Sinh[b*x])*SinhIntegral[b*x])/b^2
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {7096, 27, 3042, 26, 3044, 15, 7100, 27, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Shi}(bx)\sinh(bx)\,dx$$

$$\Big\downarrow \text{7096}$$

$$-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}-\int \frac{\cosh(bx)\sinh(bx)}{b}dx+\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\int \cosh(bx)\sinh(bx)dx}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\int -i\cos(ibx)\sin(ibx)dx}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}+\frac{i\int \cos(ibx)\sin(ibx)dx}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int i\sinh(bx)d(i\sinh(bx))}{b^2}-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{7100}$$

$$-\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}-\int \frac{\sinh^2(bx)}{bx}dx}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}-\frac{\int \frac{\sinh^2(bx)}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}-\frac{\int -\frac{\sin(ibx)^2}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}+\frac{\int \frac{\sin(ibx)^2}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\frac{\int \left(\frac{1}{2x}-\frac{\cosh(2bx)}{2x}\right)dx}{b}+\frac{\text{Shi}(bx)\sinh(bx)}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sinh^2(bx)}{2b^2}-\frac{\frac{\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}}{b}+\frac{\text{Shi}(bx)\sinh(bx)}{b}}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

input
```
Int[x*Sinh[b*x]*SinhIntegral[b*x],x]
```

output
```
-1/2*Sinh[b*x]^2/b^2 + (x*Cosh[b*x]*SinhIntegral[b*x])/b - ((-1/2*CoshInte
gral[2*b*x] + Log[x]/2)/b + (Sinh[b*x]*SinhIntegral[b*x])/b)/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7096
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7100
```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\mathrm{Shi}(bx)(bx\cosh(bx)-\sinh(bx))-\frac{\cosh(bx)^2}{2}-\frac{\ln(bx)}{2}+\frac{\mathrm{Chi}(2bx)}{2}}{b^2}$	46
default	$\frac{\mathrm{Shi}(bx)(bx\cosh(bx)-\sinh(bx))-\frac{\cosh(bx)^2}{2}-\frac{\ln(bx)}{2}+\frac{\mathrm{Chi}(2bx)}{2}}{b^2}$	46

input
```
int(x*sinh(b*x)*Shi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(Shi(b*x)*(b*x*cosh(b*x)-sinh(b*x))-1/2*cosh(b*x)^2-1/2*ln(b*x)+1/2*
Chi(2*b*x))
```

Fricas [F]

$$\int x\sinh(bx)\mathrm{Shi}(bx)\,dx=\int x\mathrm{Shi}(bx)\sinh\left(bx\right)\,dx$$

input
```
integrate(x*sinh(b*x)*Shi(b*x),x, algorithm="fricas")
```

output
```
integral(x*sinh(b*x)*sinh_integral(b*x), x)
```

Sympy [F]

$$\int x\sinh(bx)\mathrm{Shi}(bx)\,dx=\int x\sinh\left(bx\right)\mathrm{Shi}\left(bx\right)dx$$

input
```
integrate(x*sinh(b*x)*Shi(b*x),x)
```

output
```
Integral(x*sinh(b*x)*Shi(b*x), x)
```

Maxima [F]

$$\int x\sinh(bx)\text{Shi}(bx)\,dx = \int x\text{Shi}(bx)\sinh(bx)\,dx$$

input
```
integrate(x*sinh(b*x)*Shi(b*x),x, algorithm="maxima")
```

output
```
integrate(x*Shi(b*x)*sinh(b*x), x)
```

Giac [F]

$$\int x\sinh(bx)\text{Shi}(bx)\,dx = \int x\text{Shi}(bx)\sinh(bx)\,dx$$

input
```
integrate(x*sinh(b*x)*Shi(b*x),x, algorithm="giac")
```

output
```
integrate(x*Shi(b*x)*sinh(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sinh(bx)\text{Shi}(bx)\,dx = \int x\,\text{sinhint}(b\,x)\,\sinh(b\,x)\,dx$$

input
```
int(x*sinhint(b*x)*sinh(b*x),x)
```

output
```
int(x*sinhint(b*x)*sinh(b*x), x)
```

Reduce [F]

$$\int x \sinh(bx)\mathrm{Shi}(bx)\, dx = \int shi(bx) \sinh{(bx)}\, x dx$$

input
```
int(x*sinh(b*x)*Shi(b*x),x)
```

output
```
int(shi(b*x)*sinh(b*x)*x,x)
```

3.44 $\int x^2 \sinh(bx)\mathbf{Shi}(bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int x^2 \sinh(bx)\text{Shi}(bx)\,dx = -\frac{5x}{4b^2}+\frac{5\cosh(bx)\sinh(bx)}{4b^3}-\frac{x\sinh^2(bx)}{2b^2}+\frac{2\cosh(bx)\text{Shi}(bx)}{b^3}+\frac{x^2\cosh(bx)\text{Shi}(bx)}{b}-\frac{2x\sinh(bx)\text{Shi}(bx)}{b^2}-\frac{\text{Shi}(2bx)}{b^3}$$

output
```
-5/4*x/b^2+5/4*cosh(b*x)*sinh(b*x)/b^3-1/2*x*sinh(b*x)^2/b^2+2*cosh(b*x)*S
hi(b*x)/b^3+x^2*cosh(b*x)*Shi(b*x)/b-2*x*sinh(b*x)*Shi(b*x)/b^2-Shi(2*b*x)
/b^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.71

$$\int x^2 \sinh(bx)\text{Shi}(bx)\,dx = \frac{-8bx-2bx\cosh(2bx)+5\sinh(2bx)+8\left((2+b^2x^2)\cosh(bx)-2bx\sinh(bx)\right)\text{Shi}(bx)-8\text{Shi}(2bx)}{8b^3}$$

input
```
Integrate[x^2*Sinh[b*x]*SinhIntegral[b*x],x]
```

output
```
(-8*b*x - 2*b*x*Cosh[2*b*x] + 5*Sinh[2*b*x] + 8*((2 + b^2*x^2)*Cosh[b*x] -
 2*b*x*Sinh[b*x])*SinhIntegral[b*x] - 8*SinhIntegral[2*b*x])/(8*b^3)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.53, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.667, Rules used = {7096, 27, 5895, 3042, 25, 3115, 24, 7102, 27, 3042, 25, 3115, 24, 7094, 27, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{Shi}(bx)\sinh(bx)\,dx$$

$$\Big\downarrow \text{7096}$$

$$-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\int \frac{x\cosh(bx)\sinh(bx)}{b}dx+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\int x\cosh(bx)\sinh(bx)dx}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{5895}$$

$$-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}-\frac{\int \sinh^2(bx)dx}{2b}}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}-\frac{\int -\sin(ibx)^2dx}{2b}}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\int \sin(ibx)^2dx}{2b}}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\frac{\frac{\int 1dx}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}+\frac{x\sinh^2(bx)}{2b}}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \text{7102}$$

$$-\frac{2\left(-\frac{\int\sinh(bx)\text{Shi}(bx)dx}{b}-\int\frac{\sinh^2(bx)}{b}dx+\frac{x\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\left(-\frac{\int\sinh(bx)\text{Shi}(bx)dx}{b}-\frac{\int\sinh^2(bx)dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(-\frac{\int\sinh(bx)\text{Shi}(bx)dx}{b}-\frac{\int-\sin(ibx)^2dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\left(-\frac{\int\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{\int\sin(ibx)^2dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{2\left(-\frac{\int\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{\frac{\int 1dx}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{2\left(-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \; 7094$$

$$-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\int \frac{\cosh(bx)\sinh(bx)}{bx}dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int \frac{\cosh(bx)\sinh(bx)}{x}dx}{b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \; 5971$$

$$-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int \frac{\sinh(2bx)}{2x}dx}{b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int \frac{\sinh(2bx)}{x}dx}{2b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int -\frac{i\sin(2ibx)}{x}dx}{2b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}+\frac{i\int\frac{\sin(2ibx)}{x}dx}{2b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \; 3779$$

$$\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{2\left(\frac{x\text{Shi}(bx)\sinh(bx)}{b}-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\text{Shi}(2bx)}{2b}}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

input
```
Int[x^2*Sinh[b*x]*SinhIntegral[b*x],x]
```

output
```
-(((x*Sinh[b*x]^2)/(2*b) + (x/2 - (Cosh[b*x]*Sinh[b*x])/(2*b))/(2*b))/b) +
 (x^2*Cosh[b*x]*SinhIntegral[b*x])/b - (2*((x/2 - (Cosh[b*x]*Sinh[b*x])/(2
*b))/b + (x*Sinh[b*x]*SinhIntegral[b*x])/b - ((Cosh[b*x]*SinhIntegral[b*x]
)/b - SinhIntegral[2*b*x]/(2*b))/b))/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7094
```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7096
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7102
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{\text{Shi}(bx)(b^2x^2\cosh(bx)-2bx\sinh(bx)+2\cosh(bx))-\frac{bx\cosh(bx)^2}{2}+\frac{5\cosh(bx)\sinh(bx)}{4}-\frac{3bx}{4}-\text{Shi}(2bx)}{b^3}$	68
default	$\frac{\text{Shi}(bx)(b^2x^2\cosh(bx)-2bx\sinh(bx)+2\cosh(bx))-\frac{bx\cosh(bx)^2}{2}+\frac{5\cosh(bx)\sinh(bx)}{4}-\frac{3bx}{4}-\text{Shi}(2bx)}{b^3}$	68

input
```
int(x^2*sinh(b*x)*Shi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(Shi(b*x)*(b^2*x^2*cosh(b*x)-2*b*x*sinh(b*x)+2*cosh(b*x))-1/2*b*x*co
sh(b*x)^2+5/4*cosh(b*x)*sinh(b*x)-3/4*b*x-Shi(2*b*x))
```

Fricas [F]

$$\int x^2\sinh(bx)\text{Shi}(bx)\,dx = \int x^2\text{Shi}(bx)\sinh(bx)\,dx$$

input
```
integrate(x^2*sinh(b*x)*Shi(b*x),x, algorithm="fricas")
```

output
```
integral(x^2*sinh(b*x)*sinh_integral(b*x), x)
```

Sympy [F]

$$\int x^2 \sinh(bx)\text{Shi}(bx)\,dx = \int x^2 \sinh\left(bx\right)\text{Shi}\left(bx\right)dx$$

input
```
integrate(x**2*sinh(b*x)*Shi(b*x),x)
```

output
```
Integral(x**2*sinh(b*x)*Shi(b*x), x)
```

Maxima [F]

$$\int x^2 \sinh(bx)\text{Shi}(bx)\,dx = \int x^2\text{Shi}(bx)\sinh\left(bx\right)\,dx$$

input
```
integrate(x^2*sinh(b*x)*Shi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^2*Shi(b*x)*sinh(b*x), x)
```

Giac [F]

$$\int x^2 \sinh(bx)\text{Shi}(bx)\,dx = \int x^2\text{Shi}(bx)\sinh\left(bx\right)\,dx$$

input
```
integrate(x^2*sinh(b*x)*Shi(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*Shi(b*x)*sinh(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh(bx)\text{Shi}(bx)\,dx = \int x^2\,\mathrm{sinhint}(b\,x)\,\sinh(b\,x)\,dx$$

input
```
int(x^2*sinhint(b*x)*sinh(b*x),x)
```

output
```
int(x^2*sinhint(b*x)*sinh(b*x), x)
```

Reduce [F]

$$\int x^2 \sinh(bx)\text{Shi}(bx)\,dx = \int shi(bx)\sinh(bx)\,x^2dx$$

input
```
int(x^2*sinh(b*x)*Shi(b*x),x)
```

output
```
int(shi(b*x)*sinh(b*x)*x**2,x)
```

3.45 $\int x^3 \sinh(bx)\mathbf{Shi}(bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 125

$$
\begin{aligned}
\int x^3 \sinh(bx)\text{Shi}(bx)\,dx = & -\frac{x^2}{b^2}+\frac{3\text{Chi}(2bx)}{b^4}-\frac{3\log(x)}{b^4} \\
& +\frac{2x\cosh(bx)\sinh(bx)}{b^3}-\frac{4\sinh^2(bx)}{b^4}-\frac{x^2\sinh^2(bx)}{2b^2} \\
& +\frac{6x\cosh(bx)\text{Shi}(bx)}{b^3}+\frac{x^3\cosh(bx)\text{Shi}(bx)}{b} \\
& -\frac{6\sinh(bx)\text{Shi}(bx)}{b^4}-\frac{3x^2\sinh(bx)\text{Shi}(bx)}{b^2}
\end{aligned}
$$

output
```
-x^2/b^2+3*Chi(2*b*x)/b^4-3*ln(x)/b^4+2*x*cosh(b*x)*sinh(b*x)/b^3-4*sinh(b
*x)^2/b^4-1/2*x^2*sinh(b*x)^2/b^2+6*x*cosh(b*x)*Shi(b*x)/b^3+x^3*cosh(b*x)
*Shi(b*x)/b-6*sinh(b*x)*Shi(b*x)/b^4-3*x^2*sinh(b*x)*Shi(b*x)/b^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.74

$$\int x^3 \sinh(bx)\mathrm{Shi}(bx)\,dx =$$
$$-\frac{3b^2x^2 + 8\cosh(2bx) + b^2x^2\cosh(2bx) - 12\mathrm{Chi}(2bx) + 12\log(x) - 4bx\sinh(2bx) - 4(bx(6+b^2x^2)\cos}{4b^4}$$

input
```
Integrate[x^3*Sinh[b*x]*SinhIntegral[b*x],x]
```

output
```
-1/4*(3*b^2*x^2 + 8*Cosh[2*b*x] + b^2*x^2*Cosh[2*b*x] - 12*CoshIntegral[2*
b*x] + 12*Log[x] - 4*b*x*Sinh[2*b*x] - 4*(b*x*(6 + b^2*x^2)*Cosh[b*x] - 3*
(2 + b^2*x^2)*Sinh[b*x])*SinhIntegral[b*x])/b^4
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.69, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 2.083$, Rules used = {7096, 27, 5895, 3042, 25, 3791, 15, 7102, 27, 3042, 25, 3791, 15, 7096, 27, 3042, 26, 3044, 15, 7100, 27, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\mathrm{Shi}(bx)\sinh(bx)\,dx$$

$$\Big\downarrow \; 7096$$

$$-\frac{3\int x^2\cosh(bx)\mathrm{Shi}(bx)dx}{b} - \int \frac{x^2\cosh(bx)\sinh(bx)}{b}dx + \frac{x^3\mathrm{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{3\int x^2\cosh(bx)\mathrm{Shi}(bx)dx}{b} - \frac{\int x^2\cosh(bx)\sinh(bx)dx}{b} + \frac{x^3\mathrm{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 5895$$

$$-\frac{3\int x^2\cosh(bx)\mathrm{Shi}(bx)dx}{b} - \frac{\frac{x^2\sinh^2(bx)}{2b} - \frac{\int x\sinh^2(bx)dx}{b}}{b} + \frac{x^3\mathrm{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int x^2\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\frac{x^2\sinh^2(bx)}{2b}-\frac{\int -x\sin(ibx)^2dx}{b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3\int x^2\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\frac{x^2\sinh^2(bx)}{2b}+\frac{\int x\sin(ibx)^2dx}{b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3791}$$

$$-\frac{\frac{\frac{\int xdx}{2}+\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}-\frac{3\int x^2\cosh(bx)\text{Shi}(bx)dx}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\int x^2\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{7102}$$

$$-\frac{3\left(-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}-\int\frac{x\sinh^2(bx)}{b}dx+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}-\frac{\int x\sinh^2(bx)dx}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}-\frac{\int -x\sin(ibx)^2dx}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3\left(-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{\int x\sin(ibx)^2dx}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3791}$$

$$-\frac{3\left(\frac{\frac{\int xdx}{2}+\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}}{b}-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\left(-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{7096}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int\cosh(bx)\text{Shi}(bx)dx}{b}-\int\frac{\cosh(bx)\sinh(bx)}{b}dx+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\int\cosh(bx)\sinh(bx)dx}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\int -i\cos(ibx)\sin(ibx)dx}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}+\frac{i\int \cos(ibx)\sin(ibx)dx}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{ 3044}$$

$$-\frac{3\left(-\frac{2\left(\frac{\int i\sinh(bx)d(i\sinh(bx))}{b^2}-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{ 7100}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}-\int\frac{\sinh^2(bx)}{bx}dx}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}-\frac{\int\frac{\sinh^2(bx)}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}-\frac{\int-\frac{\sin(ibx)^2}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}+\frac{\int\frac{\sin(ibx)^2}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \ 3793$$

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\int\left(\frac{1}{2x}-\frac{\cosh(2bx)}{2x}\right)dx}{b}+\frac{\text{Shi}(bx)\sinh(bx)}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(\cdots}{b}\right.}{b}$$

$$\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{3\left(-\frac{2\left(-\frac{\sinh^2(bx)}{2b^2}-\frac{\frac{\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}}{b}+\frac{\text{Shi}(bx)\sinh(bx)}{b}}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx\cdots}{b}\right.}{b}$$

$$\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Shi}(bx)\cosh(bx)}{b}$$

input
```
Int[x^3*Sinh[b*x]*SinhIntegral[b*x],x]
```

output
```
-((((x^2*Sinh[b*x]^2)/(2*b) + (x^2/4 - (x*Cosh[b*x]*Sinh[b*x])/(2*b) + Sinh
[b*x]^2/(4*b^2))/b)/b) + (x^3*Cosh[b*x]*SinhIntegral[b*x])/b - (3*((x^2/4
- (x*Cosh[b*x]*Sinh[b*x])/(2*b) + Sinh[b*x]^2/(4*b^2))/b + (x^2*Sinh[b*x]*
SinhIntegral[b*x])/b - (2*(-1/2*Sinh[b*x]^2/b^2 + (x*Cosh[b*x]*SinhIntegra
l[b*x])/b - ((-1/2*CoshIntegral[2*b*x] + Log[x]/2)/b + (Sinh[b*x]*SinhInte
gral[b*x])/b)/b))/b))/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 7096
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7100
```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7102
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\mathrm{Shi}(bx)\left(b^3x^3\cosh(bx)-3b^2x^2\sinh(bx)+6bx\cosh(bx)-6\sinh(bx)\right)-\frac{b^2x^2\cosh(bx)^2}{2}+2bx\cosh(bx)\sinh(bx)-\frac{b^2x^2}{2}-4\,\mathrm{co}}{b^4}$
default	$\frac{\mathrm{Shi}(bx)\left(b^3x^3\cosh(bx)-3b^2x^2\sinh(bx)+6bx\cosh(bx)-6\sinh(bx)\right)-\frac{b^2x^2\cosh(bx)^2}{2}+2bx\cosh(bx)\sinh(bx)-\frac{b^2x^2}{2}-4\,\mathrm{co}}{b^4}$

input
```
int(x^3*sinh(b*x)*Shi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(Shi(b*x)*(b^3*x^3*cosh(b*x)-3*b^2*x^2*sinh(b*x)+6*b*x*cosh(b*x)-6*s
inh(b*x))-1/2*b^2*x^2*cosh(b*x)^2+2*b*x*cosh(b*x)*sinh(b*x)-1/2*b^2*x^2-4*
cosh(b*x)^2-3*ln(b*x)+3*Chi(2*b*x))
```

Fricas [F]

$$\int x^3 \sinh(bx)\text{Shi}(bx)\,dx = \int x^3\text{Shi}(bx)\sinh(bx)\,dx$$

input
```
integrate(x^3*sinh(b*x)*Shi(b*x),x, algorithm="fricas")
```

output
```
integral(x^3*sinh(b*x)*sinh_integral(b*x), x)
```

Sympy [F]

$$\int x^3 \sinh(bx)\text{Shi}(bx)\,dx = \int x^3 \sinh(bx)\,\text{Shi}(bx)\,dx$$

input
```
integrate(x**3*sinh(b*x)*Shi(b*x),x)
```

output
```
Integral(x**3*sinh(b*x)*Shi(b*x), x)
```

Maxima [F]

$$\int x^3 \sinh(bx)\text{Shi}(bx)\,dx = \int x^3\text{Shi}(bx)\sinh(bx)\,dx$$

input
```
integrate(x^3*sinh(b*x)*Shi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^3*Shi(b*x)*sinh(b*x), x)
```

Giac [F]

$$\int x^3 \sinh(bx)\text{Shi}(bx)\,dx = \int x^3\text{Shi}(bx)\sinh(bx)\,dx$$

input
```
integrate(x^3*sinh(b*x)*Shi(b*x),x, algorithm="giac")
```

output
```
integrate(x^3*Shi(b*x)*sinh(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \sinh(bx)\text{Shi}(bx)\,dx = \int x^3\,\text{sinhint}(b\,x)\,\sinh(b\,x)\,dx$$

input
```
int(x^3*sinhint(b*x)*sinh(b*x),x)
```

output
```
int(x^3*sinhint(b*x)*sinh(b*x), x)
```

Reduce [F]

$$\int x^3 \sinh(bx)\text{Shi}(bx)\,dx = \int shi(bx)\sinh(bx)\,x^3dx$$

input
```
int(x^3*sinh(b*x)*Shi(b*x),x)
```

output
```
int(shi(b*x)*sinh(b*x)*x**3,x)
```

3.46 $\int \frac{\cosh(bx)\mathbf{Shi}(bx)}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x^3}\,dx = -\frac{b\cosh(2bx)}{4x} - \frac{b\sinh^2(bx)}{2x} - \frac{\sinh(2bx)}{8x^2} - \frac{\cosh(bx)\text{Shi}(bx)}{2x^2} - \frac{b\sinh(bx)\text{Shi}(bx)}{2x} + b^2\text{Shi}(2bx) + \frac{1}{2}b^2\text{Int}\left(\frac{\cosh(bx)\text{Shi}(bx)}{x}, x\right)$$

output
```
-1/4*b*cosh(2*b*x)/x-1/2*b*sinh(b*x)^2/x-1/8*sinh(2*b*x)/x^2-1/2*cosh(b*x)
*Shi(b*x)/x^2-1/2*b*sinh(b*x)*Shi(b*x)/x+b^2*Shi(2*b*x)+1/2*b^2*Defer(Int)
(cosh(b*x)*Shi(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x^3}\,dx = \int \frac{\cosh(bx)\text{Shi}(bx)}{x^3}\,dx$$

input
```
Integrate[(Cosh[b*x]*SinhIntegral[b*x])/x^3,x]
```

output
```
Integrate[(Cosh[b*x]*SinhIntegral[b*x])/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(bx)\cosh(bx)}{x^3}dx$$

$$\Big\downarrow \text{ 7104}$$

$$\frac{1}{2}b\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}dx+\frac{1}{2}b\int \frac{\cosh(bx)\sinh(bx)}{bx^3}dx-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}b\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}dx+\frac{1}{2}\int \frac{\cosh(bx)\sinh(bx)}{x^3}dx-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{1}{2}b\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}dx+\frac{1}{2}\int \frac{\sinh(2bx)}{2x^3}dx-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}b\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}dx+\frac{1}{4}\int \frac{\sinh(2bx)}{x^3}dx-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}b\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}dx+\frac{1}{4}\int -\frac{i\sin(2ibx)}{x^3}dx-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{2}b\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}dx-\frac{1}{4}i\int \frac{\sin(2ibx)}{x^3}dx-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{2}b\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}dx-\frac{1}{4}i\left(ib\int \frac{\cosh(2bx)}{x^2}dx-\frac{i\sinh(2bx)}{2x^2}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}b\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}dx-\frac{1}{4}i\left(ib\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x^2}dx-\frac{i\sinh(2bx)}{2x^2}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{2}b\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}dx-\frac{1}{4}i\left(ib\left(-\frac{\cosh(2bx)}{x}+2ib\int -\frac{i\sinh(2bx)}{x}dx\right)-\frac{i\sinh(2bx)}{2x^2}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{2}b\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}dx-\frac{1}{4}i\left(ib\left(2b\int \frac{\sinh(2bx)}{x}dx-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}b\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}dx-\frac{1}{4}i\left(ib\left(-\frac{\cosh(2bx)}{x}+2b\int -\frac{i\sin(2ibx)}{x}dx\right)-\frac{i\sinh(2bx)}{2x^2}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{2}b\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}dx-\frac{1}{4}i\left(ib\left(-\frac{\cosh(2bx)}{x}-2ib\int \frac{\sin(2ibx)}{x}dx\right)-\frac{i\sinh(2bx)}{2x^2}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{ 3779}$$

$$\frac{1}{2}b\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}dx-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$$\Big\downarrow \text{ 7098}$$

$$\frac{1}{2}b\left(b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx+b\int \frac{\sinh^2(bx)}{bx^2}dx-\frac{\text{Shi}(bx)\sinh(bx)}{x}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}b\left(b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx+\int \frac{\sinh^2(bx)}{x^2}dx-\frac{\text{Shi}(bx)\sinh(bx)}{x}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$\Big\downarrow$ 3042

$$\frac{1}{2}b\left(b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx+\int -\frac{\sin(ibx)^2}{x^2}dx-\frac{\text{Shi}(bx)\sinh(bx)}{x}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{2}b\left(b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx-\int \frac{\sin(ibx)^2}{x^2}dx-\frac{\text{Shi}(bx)\sinh(bx)}{x}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$\Big\downarrow$ 3794

$$\frac{1}{2}b\left(b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx-2ib\int \frac{i\sinh(2bx)}{2x}dx-\frac{\text{Shi}(bx)\sinh(bx)}{x}-\frac{\sinh^2(bx)}{x}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{2}b\left(b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx+b\int \frac{\sinh(2bx)}{x}dx-\frac{\text{Shi}(bx)\sinh(bx)}{x}-\frac{\sinh^2(bx)}{x}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$\Big\downarrow$ 3042

$$\frac{1}{2}b\left(b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx+b\int -\frac{i\sin(2ibx)}{x}dx-\frac{\text{Shi}(bx)\sinh(bx)}{x}-\frac{\sinh^2(bx)}{x}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$\Big\downarrow$ 26

$$\frac{1}{2}b\left(b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx-ib\int \frac{\sin(2ibx)}{x}dx-\frac{\text{Shi}(bx)\sinh(bx)}{x}-\frac{\sinh^2(bx)}{x}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$\Big\downarrow$ 3779

$$\frac{1}{2}b\left(b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx + b\text{Shi}(2bx) - \frac{\text{Shi}(bx)\sinh(bx)}{x} - \frac{\sinh^2(bx)}{x}\right) - \frac{\text{Shi}(bx)\cosh(bx)}{2x^2} - \frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx) - \frac{\cosh(2bx)}{x}\right) - \frac{i\sinh(2bx)}{2x^2}\right)$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{1}{2}b\left(b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}dx + b\text{Shi}(2bx) - \frac{\text{Shi}(bx)\sinh(bx)}{x} - \frac{\sinh^2(bx)}{x}\right) - \frac{\text{Shi}(bx)\cosh(bx)}{2x^2} - \frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx) - \frac{\cosh(2bx)}{x}\right) - \frac{i\sinh(2bx)}{2x^2}\right)$$

input
```
Int[(Cosh[b*x]*SinhIntegral[b*x])/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(bx)\,\text{Shi}(bx)}{x^3}dx$$

input
```
int(cosh(b*x)*Shi(b*x)/x^3,x)
```

output
```
int(cosh(b*x)*Shi(b*x)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x^3}\,dx = \int \frac{\text{Shi}(bx)\cosh(bx)}{x^3}\,dx$$

input
```
integrate(cosh(b*x)*Shi(b*x)/x^3,x, algorithm="fricas")
```

output
```
integral(cosh(b*x)*sinh_integral(b*x)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\mathrm{Shi}(bx)}{x^3}\,dx = \int \frac{\cosh{(bx)}\,\mathrm{Shi}\,(bx)}{x^3}\,dx$$

input
```
integrate(cosh(b*x)*Shi(b*x)/x**3,x)
```

output
```
Integral(cosh(b*x)*Shi(b*x)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\mathrm{Shi}(bx)}{x^3}\,dx = \int \frac{\mathrm{Shi}(bx)\cosh{(bx)}}{x^3}\,dx$$

input
```
integrate(cosh(b*x)*Shi(b*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x)*cosh(b*x)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\mathrm{Shi}(bx)}{x^3}\,dx = \int \frac{\mathrm{Shi}(bx)\cosh{(bx)}}{x^3}\,dx$$

input
```
integrate(cosh(b*x)*Shi(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(Shi(b*x)*cosh(b*x)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\mathrm{Shi}(bx)}{x^3}\,dx = \int \frac{\mathrm{sinhint}(b\,x)\,\cosh(b\,x)}{x^3}\,dx$$

input
```
int((sinhint(b*x)*cosh(b*x))/x^3,x)
```

output
```
int((sinhint(b*x)*cosh(b*x))/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\mathrm{Shi}(bx)}{x^3}\,dx = \int \frac{\cosh{(bx)}\,shi(bx)}{x^3}dx$$

input
```
int(cosh(b*x)*Shi(b*x)/x^3,x)
```

output
```
int((cosh(b*x)*shi(b*x))/x**3,x)
```

3.47 $\int \frac{\cosh(bx)\mathbf{Shi}(bx)}{x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 44

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2} dx = b\text{Chi}(2bx) - \frac{\sinh(2bx)}{2x} - \frac{\cosh(bx)\text{Shi}(bx)}{x} + \frac{1}{2}b\text{Shi}(bx)^2$$

output
```
b*Chi(2*b*x)-1/2*sinh(2*b*x)/x-cosh(b*x)*Shi(b*x)/x+1/2*b*Shi(b*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2} dx = b\text{Chi}(2bx) - \frac{\sinh(2bx)}{2x} - \frac{\cosh(bx)\text{Shi}(bx)}{x} + \frac{1}{2}b\text{Shi}(bx)^2$$

input
```
Integrate[(Cosh[b*x]*SinhIntegral[b*x])/x^2,x]
```

output
```
b*CoshIntegral[2*b*x] - Sinh[2*b*x]/(2*x) - (Cosh[b*x]*SinhIntegral[b*x])/
x + (b*SinhIntegral[b*x]^2)/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {7104, 27, 5971, 27, 3042, 26, 3778, 3042, 3782, 7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(bx)\cosh(bx)}{x^2}dx$$

$$\Big\downarrow \text{7104}$$

$$b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx+b\int \frac{\cosh(bx)\sinh(bx)}{bx^2}dx-\frac{\text{Shi}(bx)\cosh(bx)}{x}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx+\int \frac{\cosh(bx)\sinh(bx)}{x^2}dx-\frac{\text{Shi}(bx)\cosh(bx)}{x}$$

$$\Big\downarrow \text{5971}$$

$$b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx+\int \frac{\sinh(2bx)}{2x^2}dx-\frac{\text{Shi}(bx)\cosh(bx)}{x}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx+\frac{1}{2}\int \frac{\sinh(2bx)}{x^2}dx-\frac{\text{Shi}(bx)\cosh(bx)}{x}$$

$$\Big\downarrow \text{3042}$$

$$b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx+\frac{1}{2}\int -\frac{i\sin(2ibx)}{x^2}dx-\frac{\text{Shi}(bx)\cosh(bx)}{x}$$

$$\Big\downarrow \text{26}$$

$$b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx-\frac{1}{2}i\int \frac{\sin(2ibx)}{x^2}dx-\frac{\text{Shi}(bx)\cosh(bx)}{x}$$

$$\Big\downarrow \text{3778}$$

$$b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx-\frac{1}{2}i\left(2ib\int \frac{\cosh(2bx)}{x}dx-\frac{i\sinh(2bx)}{x}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{x}$$

$$\Big\downarrow \text{3042}$$

$$b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx-\frac{1}{2}i\left(2ib\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx-\frac{i\sinh(2bx)}{x}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{x}$$

$$\Big\downarrow \text{3782}$$

$$b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}dx-\frac{1}{2}i\left(2ib\text{Chi}(2bx)-\frac{i\sinh(2bx)}{x}\right)-\frac{\text{Shi}(bx)\cosh(bx)}{x}$$

$$\Big\downarrow \text{7237}$$

$$-\frac{1}{2}i\left(2ib\text{Chi}(2bx)-\frac{i\sinh(2bx)}{x}\right)+\frac{1}{2}b\text{Shi}(bx)^2-\frac{\text{Shi}(bx)\cosh(bx)}{x}$$

input
```
Int[(Cosh[b*x]*SinhIntegral[b*x])/x^2,x]
```

output
```
(-1/2*I)*((2*I)*b*CoshIntegral[2*b*x] - (I*Sinh[2*b*x])/x) - (Cosh[b*x]*Si
nhIntegral[b*x])/x + (b*SinhIntegral[b*x]^2)/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7104
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^(m + 1)*Cosh[a + b*x]*(SinhInteg
ral[c + d*x]/(f*(m + 1))), x] + (-Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1
)*Sinh[a + b*x]*SinhIntegral[c + d*x], x], x] - Simp[d/(f*(m + 1))   Int[(e
 + f*x)^(m + 1)*Cosh[a + b*x]*(Sinh[c + d*x]/(c + d*x)), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && ILtQ[m, -1]
```

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{\cosh(bx)\,\mathrm{Shi}(bx)}{x^2}dx$$

input
```
int(cosh(b*x)*Shi(b*x)/x^2,x)
```

output
```
int(cosh(b*x)*Shi(b*x)/x^2,x)
```

Fricas [F]

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}\,dx = \int \frac{\text{Shi}(bx)\cosh{(bx)}}{x^2}\,dx$$

input
```
integrate(cosh(b*x)*Shi(b*x)/x^2,x, algorithm="fricas")
```

output
```
integral(cosh(b*x)*sinh_integral(b*x)/x^2, x)
```

Sympy [F]

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}\,dx = \int \frac{\cosh{(bx)}\,\text{Shi}{(bx)}}{x^2}\,dx$$

input
```
integrate(cosh(b*x)*Shi(b*x)/x**2,x)
```

output
```
Integral(cosh(b*x)*Shi(b*x)/x**2, x)
```

Maxima [F]

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}\,dx = \int \frac{\text{Shi}(bx)\cosh{(bx)}}{x^2}\,dx$$

input
```
integrate(cosh(b*x)*Shi(b*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x)*cosh(b*x)/x^2, x)
```

Giac [F]

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}\,dx = \int \frac{\text{Shi}(bx)\cosh(bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x)*Shi(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(Shi(b*x)*cosh(b*x)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}\,dx = \int \frac{\text{sinhint}(b\,x)\,\cosh(b\,x)}{x^2}\,dx$$

input
```
int((sinhint(b*x)*cosh(b*x))/x^2,x)
```

output
```
int((sinhint(b*x)*cosh(b*x))/x^2, x)
```

Reduce [F]

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}\,dx = \int \frac{\cosh(bx)\,shi(bx)}{x^2}\,dx$$

input
```
int(cosh(b*x)*Shi(b*x)/x^2,x)
```

output
```
int((cosh(b*x)*shi(b*x))/x**2,x)
```

3.48 $\int \frac{\cosh(bx)\textbf{Shi}(bx)}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x} dx = \text{Int}\left(\frac{\cosh(bx)\text{Shi}(bx)}{x}, x\right)$$

output
```
Defer(Int)(cosh(b*x)*Shi(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x} dx = \int \frac{\cosh(bx)\text{Shi}(bx)}{x} dx$$

input
```
Integrate[(Cosh[b*x]*SinhIntegral[b*x])/x,x]
```

output
```
Integrate[(Cosh[b*x]*SinhIntegral[b*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(bx)\cosh(bx)}{x}\,dx$$
$$\big\downarrow \text{ 7299}$$
$$\int \frac{\text{Shi}(bx)\cosh(bx)}{x}dx$$

input
```
Int[(Cosh[b*x]*SinhIntegral[b*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(bx\right)\text{Shi}\left(bx\right)}{x}dx$$

input
```
int(cosh(b*x)*Shi(b*x)/x,x)
```

output
```
int(cosh(b*x)*Shi(b*x)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x}\,dx = \int \frac{\text{Shi}(bx)\cosh{(bx)}}{x}\,dx$$

input
```
integrate(cosh(b*x)*Shi(b*x)/x,x, algorithm="fricas")
```

output
```
integral(cosh(b*x)*sinh_integral(b*x)/x, x)
```

Sympy [N/A]

Not integrable

Time = 2.75 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x}\,dx = \int \frac{\cosh{(bx)}\,\text{Shi}\,(bx)}{x}\,dx$$

input
```
integrate(cosh(b*x)*Shi(b*x)/x,x)
```

output
```
Integral(cosh(b*x)*Shi(b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x}\,dx = \int \frac{\text{Shi}(bx)\cosh{(bx)}}{x}\,dx$$

input
```
integrate(cosh(b*x)*Shi(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x)*cosh(b*x)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x}\,dx = \int \frac{\text{Shi}(bx)\cosh{(bx)}}{x}\,dx$$

input
```
integrate(cosh(b*x)*Shi(b*x)/x,x, algorithm="giac")
```

output
```
integrate(Shi(b*x)*cosh(b*x)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x}\,dx = \int \frac{\text{sinhint}(b\,x)\,\cosh(b\,x)}{x}\,dx$$

input
```
int((sinhint(b*x)*cosh(b*x))/x,x)
```

output
```
int((sinhint(b*x)*cosh(b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\text{Shi}(bx)}{x}\,dx = \int \frac{\cosh(bx)\,shi(bx)}{x}dx$$

input
```
int(cosh(b*x)*Shi(b*x)/x,x)
```

output
```
int((cosh(b*x)*shi(b*x))/x,x)
```

3.49 $\int \cosh(bx)\mathbf{Shi}(bx)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 34

$$\int \cosh(bx)\text{Shi}(bx)\,dx = -\frac{\text{Chi}(2bx)}{2b} + \frac{\log(x)}{2b} + \frac{\sinh(bx)\text{Shi}(bx)}{b}$$

output
```
-1/2*Chi(2*b*x)/b+1/2*ln(x)/b+sinh(b*x)*Shi(b*x)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \cosh(bx)\text{Shi}(bx)\,dx = -\frac{\text{Chi}(2bx)}{2b} + \frac{\log(bx)}{2b} + \frac{\sinh(bx)\text{Shi}(bx)}{b}$$

input
```
Integrate[Cosh[b*x]*SinhIntegral[b*x],x]
```

output
```
-1/2*CoshIntegral[2*b*x]/b + Log[b*x]/(2*b) + (Sinh[b*x]*SinhIntegral[b*x]
)/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {7100, 27, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Shi}(bx)\cosh(bx)\,dx$$

$$\Big\downarrow \text{ 7100}$$

$$\frac{\text{Shi}(bx)\sinh(bx)}{b}-\int \frac{\sinh^2(bx)}{bx}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\text{Shi}(bx)\sinh(bx)}{b}-\frac{\int \frac{\sinh^2(bx)}{x}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\text{Shi}(bx)\sinh(bx)}{b}-\frac{\int -\frac{\sin(ibx)^2}{x}dx}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\text{Shi}(bx)\sinh(bx)}{b}+\frac{\int \frac{\sin(ibx)^2}{x}dx}{b}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{\int \left(\frac{1}{2x}-\frac{\cosh(2bx)}{2x}\right)dx}{b}+\frac{\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}}{b}+\frac{\text{Shi}(bx)\sinh(bx)}{b}$$

input
```
Int[Cosh[b*x]*SinhIntegral[b*x],x]
```

output
```
(-1/2*CoshIntegral[2*b*x] + Log[x]/2)/b + (Sinh[b*x]*SinhIntegral[b*x])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7100
```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\sinh(bx)\,\mathrm{Shi}(bx)+\frac{\ln(bx)}{2}-\frac{\mathrm{Chi}(2bx)}{2}}{b}$	28
default	$\frac{\sinh(bx)\,\mathrm{Shi}(bx)+\frac{\ln(bx)}{2}-\frac{\mathrm{Chi}(2bx)}{2}}{b}$	28

input
```
int(cosh(b*x)*Shi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b*(sinh(b*x)*Shi(b*x)+1/2*ln(b*x)-1/2*Chi(2*b*x))
```

Fricas [F]

$$\int \cosh(bx)\text{Shi}(bx)\,dx = \int \text{Shi}(bx)\cosh\left(bx\right)\,dx$$

input
```
integrate(cosh(b*x)*Shi(b*x),x, algorithm="fricas")
```

output
```
integral(cosh(b*x)*sinh_integral(b*x), x)
```

Sympy [F]

$$\int \cosh(bx)\text{Shi}(bx)\,dx = \int \cosh\left(bx\right)\text{Shi}\left(bx\right)dx$$

input
```
integrate(cosh(b*x)*Shi(b*x),x)
```

output
```
Integral(cosh(b*x)*Shi(b*x), x)
```

Maxima [F]

$$\int \cosh(bx)\text{Shi}(bx)\,dx = \int \text{Shi}(bx)\cosh\left(bx\right)\,dx$$

input
```
integrate(cosh(b*x)*Shi(b*x),x, algorithm="maxima")
```

output
```
integrate(Shi(b*x)*cosh(b*x), x)
```

Giac [F]

$$\int \cosh(bx)\text{Shi}(bx)\,dx = \int \text{Shi}(bx)\cosh(bx)\,dx$$

input
```
integrate(cosh(b*x)*Shi(b*x),x, algorithm="giac")
```

output
```
integrate(Shi(b*x)*cosh(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(bx)\text{Shi}(bx)\,dx = \int \text{sinhint}(b\,x)\,\cosh(b\,x)\,dx$$

input
```
int(sinhint(b*x)*cosh(b*x),x)
```

output
```
int(sinhint(b*x)*cosh(b*x), x)
```

Reduce [F]

$$\int \cosh(bx)\text{Shi}(bx)\,dx = \int \cosh(bx)\,shi(bx)\,dx$$

input
```
int(cosh(b*x)*Shi(b*x),x)
```

output
```
int(cosh(b*x)*shi(b*x),x)
```

3.50 $\int x\cosh(bx)\mathbf{Shi}(bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int x\cosh(bx)\text{Shi}(bx)\,dx = \frac{x}{2b}-\frac{\cosh(bx)\sinh(bx)}{2b^2}-\frac{\cosh(bx)\text{Shi}(bx)}{b^2}+\frac{x\sinh(bx)\text{Shi}(bx)}{b}+\frac{\text{Shi}(2bx)}{2b^2}$$

output
```
1/2*x/b-1/2*cosh(b*x)*sinh(b*x)/b^2-cosh(b*x)*Shi(b*x)/b^2+x*sinh(b*x)*Shi
(b*x)/b+1/2*Shi(2*b*x)/b^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int x\cosh(bx)\text{Shi}(bx)\,dx = \frac{2bx-\sinh(2bx)+4(-\cosh(bx)+bx\sinh(bx))\text{Shi}(bx)+2\text{Shi}(2bx)}{4b^2}$$

input
```
Integrate[x*Cosh[b*x]*SinhIntegral[b*x],x]
```

output
```
(2*b*x - Sinh[2*b*x] + 4*(-Cosh[b*x] + b*x*Sinh[b*x])*SinhIntegral[b*x] +
2*SinhIntegral[2*b*x])/(4*b^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.300$, Rules used = {7102, 27, 3042, 25, 3115, 24, 7094, 27, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Shi}(bx)\cosh(bx)\,dx$$

$$\Big\downarrow \text{7102}$$

$$-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}-\int \frac{\sinh^2(bx)}{b}dx+\frac{x\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}-\frac{\int \sinh^2(bx)dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}-\frac{\int -\sin(ibx)^2dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}+\frac{\int \sin(ibx)^2dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}+\frac{\frac{\int 1dx}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}$$

$$\Big\downarrow \text{7094}$$

$$-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\int \frac{\cosh(bx)\sinh(bx)}{bx}dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int \frac{\cosh(bx)\sinh(bx)}{x}dx}{b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}$$

$$\Big\downarrow \; 5971$$

$$-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int \frac{\sinh(2bx)}{2x}dx}{b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int \frac{\sinh(2bx)}{x}dx}{2b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int -\frac{i\sin(2ibx)}{x}dx}{2b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}+\frac{i\int \frac{\sin(2ibx)}{x}dx}{2b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}$$

$$\Big\downarrow \; 3779$$

$$\frac{x\text{Shi}(bx)\sinh(bx)}{b}-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\text{Shi}(2bx)}{2b}}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}$$

input
```
Int[x*Cosh[b*x]*SinhIntegral[b*x],x]
```

output
```
(x/2 - (Cosh[b*x]*Sinh[b*x])/(2*b))/b + (x*Sinh[b*x]*SinhIntegral[b*x])/b
- ((Cosh[b*x]*SinhIntegral[b*x])/b - SinhIntegral[2*b*x]/(2*b))/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7094
```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7102
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{\mathrm{Shi}(bx)(bx\sinh(bx)-\cosh(bx))-\frac{\cosh(bx)\sinh(bx)}{2}+\frac{bx}{2}+\frac{\mathrm{Shi}(2bx)}{2}}{b^2}$	46
default	$\frac{\mathrm{Shi}(bx)(bx\sinh(bx)-\cosh(bx))-\frac{\cosh(bx)\sinh(bx)}{2}+\frac{bx}{2}+\frac{\mathrm{Shi}(2bx)}{2}}{b^2}$	46

input
```
int(x*cosh(b*x)*Shi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(Shi(b*x)*(b*x*sinh(b*x)-cosh(b*x))-1/2*cosh(b*x)*sinh(b*x)+1/2*b*x+
1/2*Shi(2*b*x))
```

Fricas [F]

$$\int x\cosh(bx)\mathrm{Shi}(bx)\,dx=\int x\mathrm{Shi}(bx)\cosh{(bx)}\,dx$$

input
```
integrate(x*cosh(b*x)*Shi(b*x),x, algorithm="fricas")
```

output
```
integral(x*cosh(b*x)*sinh_integral(b*x), x)
```

Sympy [F]

$$\int x \cosh(bx)\text{Shi}(bx)\, dx = \int x \cosh{(bx)} \,\text{Shi}\,(bx)\, dx$$

input
```
integrate(x*cosh(b*x)*Shi(b*x),x)
```

output
```
Integral(x*cosh(b*x)*Shi(b*x), x)
```

Maxima [F]

$$\int x \cosh(bx)\text{Shi}(bx)\, dx = \int x\text{Shi}(bx) \cosh{(bx)}\ dx$$

input
```
integrate(x*cosh(b*x)*Shi(b*x),x, algorithm="maxima")
```

output
```
integrate(x*Shi(b*x)*cosh(b*x), x)
```

Giac [F]

$$\int x \cosh(bx)\text{Shi}(bx)\, dx = \int x\text{Shi}(bx) \cosh{(bx)}\ dx$$

input
```
integrate(x*cosh(b*x)*Shi(b*x),x, algorithm="giac")
```

output
```
integrate(x*Shi(b*x)*cosh(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh(bx)\text{Shi}(bx)\,dx = \int x\,\text{sinhint}(b\,x)\,\cosh(b\,x)\,dx$$

input
```
int(x*sinhint(b*x)*cosh(b*x),x)
```

output
```
int(x*sinhint(b*x)*cosh(b*x), x)
```

Reduce [F]

$$\int x\cosh(bx)\text{Shi}(bx)\,dx = \int \cosh(bx)\,shi(bx)\,xdx$$

input
```
int(x*cosh(b*x)*Shi(b*x),x)
```

output
```
int(cosh(b*x)*shi(b*x)*x,x)
```

3.51 $\int x^2 \cosh(bx)\mathbf{Shi}(bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 98

$$\int x^2\cosh(bx)\text{Shi}(bx)\,dx = \frac{x^2}{4b}-\frac{\text{Chi}(2bx)}{b^3}+\frac{\log(x)}{b^3}-\frac{x\cosh(bx)\sinh(bx)}{2b^2}+\frac{5\sinh^2(bx)}{4b^3}-\frac{2x\cosh(bx)\text{Shi}(bx)}{b^2}+\frac{2\sinh(bx)\text{Shi}(bx)}{b^3}+\frac{x^2\sinh(bx)\text{Shi}(bx)}{b}$$

output
```
1/4*x^2/b-Chi(2*b*x)/b^3+ln(x)/b^3-1/2*x*cosh(b*x)*sinh(b*x)/b^2+5/4*sinh(
b*x)^2/b^3-2*x*cosh(b*x)*Shi(b*x)/b^2+2*sinh(b*x)*Shi(b*x)/b^3+x^2*sinh(b*
x)*Shi(b*x)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.73

$$\int x^2\cosh(bx)\text{Shi}(bx)\,dx = \frac{2b^2x^2+5\cosh(2bx)-8\text{Chi}(2bx)+8\log(x)-2bx\sinh(2bx)+8(-2bx\cosh(bx)+(2+b^2x^2)\sinh(bx))\,\text{S}}{8b^3}$$

input
```
Integrate[x^2*Cosh[b*x]*SinhIntegral[b*x],x]
```

output
```
(2*b^2*x^2 + 5*Cosh[2*b*x] - 8*CoshIntegral[2*b*x] + 8*Log[x] - 2*b*x*Sinh
[2*b*x] + 8*(-2*b*x*Cosh[b*x] + (2 + b^2*x^2)*Sinh[b*x])*SinhIntegral[b*x]
)/(8*b^3)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.30, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {7102, 27, 3042, 25, 3791, 15, 7096, 27, 3042, 26, 3044, 15, 7100, 27, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{Shi}(bx)\cosh(bx)\,dx$$

$$\Big\downarrow \text{7102}$$

$$-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}-\int \frac{x\sinh^2(bx)}{b}dx+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}-\frac{\int x\sinh^2(bx)dx}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}-\frac{\int -x\sin(ibx)^2dx}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{\int x\sin(ibx)^2dx}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{\frac{\int xdx}{2}+\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}}{b}-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2\int x\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 7096}$$

$$-\frac{2\left(-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}-\int \frac{\cosh(bx)\sinh(bx)}{b}dx+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2\left(-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\int \cosh(bx)\sinh(bx)dx}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\left(-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\int -i\cos(ibx)\sin(ibx)dx}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{2\left(-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}+\frac{i\int \cos(ibx)\sin(ibx)dx}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 3044}$$

$$-\frac{2\left(\frac{\int i\sinh(bx)d(i\sinh(bx))}{b^2}-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{2\left(-\frac{\int \cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 7100}$$

$$-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}-\int \frac{\sinh^2(bx)}{bx}dx}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}-\frac{\int \frac{\sinh^2(bx)}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}-\frac{\int -\frac{\sin(ibx)^2}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\sinh(bx)}{b}+\frac{\int \frac{\sin(ibx)^2}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 3793$$

$$-\frac{2\left(-\frac{\frac{\int\left(\frac{1}{2x}-\frac{\cosh(2bx)}{2x}\right)dx}{b}+\frac{\text{Shi}(bx)\sinh(bx)}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\left(-\frac{\sinh^2(bx)}{2b^2}-\frac{\frac{\frac{\log(x)}{2}-\frac{\text{Chi}(2bx)}{2}}{b}+\frac{\text{Shi}(bx)\sinh(bx)}{b}}{b}+\frac{x\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Shi}(bx)\sinh(bx)}{b}$$

input
```
Int[x^2*Cosh[b*x]*SinhIntegral[b*x],x]
```

output
```
(x^2/4 - (x*Cosh[b*x]*Sinh[b*x])/(2*b) + Sinh[b*x]^2/(4*b^2))/b + (x^2*Sin
h[b*x]*SinhIntegral[b*x])/b - (2*(-1/2*Sinh[b*x]^2/b^2 + (x*Cosh[b*x]*Sinh
Integral[b*x])/b - ((-1/2*CoshIntegral[2*b*x] + Log[x]/2)/b + (Sinh[b*x]*S
inhIntegral[b*x])/b)/b))/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7096
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7100
```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7102
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\mathrm{Shi}(bx)\left(b^2x^2\sinh(bx)-2bx\cosh(bx)+2\sinh(bx)\right)-\frac{bx\cosh(bx)\sinh(bx)}{2}+\frac{b^2x^2}{4}+\frac{5\cosh(bx)^2}{4}+\ln(bx)-\mathrm{Chi}(2bx)}{b^3}$	76
default	$\frac{\mathrm{Shi}(bx)\left(b^2x^2\sinh(bx)-2bx\cosh(bx)+2\sinh(bx)\right)-\frac{bx\cosh(bx)\sinh(bx)}{2}+\frac{b^2x^2}{4}+\frac{5\cosh(bx)^2}{4}+\ln(bx)-\mathrm{Chi}(2bx)}{b^3}$	76

input
```
int(x^2*cosh(b*x)*Shi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(Shi(b*x)*(b^2*x^2*sinh(b*x)-2*b*x*cosh(b*x)+2*sinh(b*x))-1/2*b*x*co
sh(b*x)*sinh(b*x)+1/4*b^2*x^2+5/4*cosh(b*x)^2+ln(b*x)-Chi(2*b*x))
```

Fricas [F]

$$\int x^2\cosh(bx)\mathrm{Shi}(bx)\,dx=\int x^2\mathrm{Shi}(bx)\cosh(bx)\,dx$$

input
```
integrate(x^2*cosh(b*x)*Shi(b*x),x, algorithm="fricas")
```

output
```
integral(x^2*cosh(b*x)*sinh_integral(b*x), x)
```

Sympy [F]

$$\int x^2\cosh(bx)\mathrm{Shi}(bx)\,dx=\int x^2\cosh(bx)\,\mathrm{Shi}(bx)\,dx$$

input
```
integrate(x**2*cosh(b*x)*Shi(b*x),x)
```

output
```
Integral(x**2*cosh(b*x)*Shi(b*x), x)
```

Maxima [F]

$$\int x^2 \cosh(bx)\text{Shi}(bx)\,dx = \int x^2\text{Shi}(bx)\cosh{(bx)}\,dx$$

input
```
integrate(x^2*cosh(b*x)*Shi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^2*Shi(b*x)*cosh(b*x), x)
```

Giac [F]

$$\int x^2 \cosh(bx)\text{Shi}(bx)\,dx = \int x^2\text{Shi}(bx)\cosh{(bx)}\,dx$$

input
```
integrate(x^2*cosh(b*x)*Shi(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*Shi(b*x)*cosh(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh(bx)\text{Shi}(bx)\,dx = \int x^2\,\text{sinhint}(b\,x)\,\cosh(b\,x)\,dx$$

input
```
int(x^2*sinhint(b*x)*cosh(b*x),x)
```

output
```
int(x^2*sinhint(b*x)*cosh(b*x), x)
```

Reduce [F]

$$\int x^2 \cosh(bx)\text{Shi}(bx)\,dx = \int \cosh(bx)\,shi(bx)\,x^2dx$$

input
```
int(x^2*cosh(b*x)*Shi(b*x),x)
```

output
```
int(cosh(b*x)*shi(b*x)*x**2,x)
```

3.52 $\int x^3 \cosh(bx)\textbf{Shi}(bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 128

$$\int x^3\cosh(bx)\text{Shi}(bx)\,dx=\frac{4x}{b^3}+\frac{x^3}{6b}-\frac{4\cosh(bx)\sinh(bx)}{b^4}-\frac{x^2\cosh(bx)\sinh(bx)}{2b^2}+\frac{2x\sinh^2(bx)}{b^3}-\frac{6\cosh(bx)\text{Shi}(bx)}{b^4}-\frac{3x^2\cosh(bx)\text{Shi}(bx)}{b^2}+\frac{6x\sinh(bx)\text{Shi}(bx)}{b^3}+\frac{x^3\sinh(bx)\text{Shi}(bx)}{b}+\frac{3\text{Shi}(2bx)}{b^4}$$

output
```
4*x/b^3+1/6*x^3/b-4*cosh(b*x)*sinh(b*x)/b^4-1/2*x^2*cosh(b*x)*sinh(b*x)/b^
2+2*x*sinh(b*x)^2/b^3-6*cosh(b*x)*Shi(b*x)/b^4-3*x^2*cosh(b*x)*Shi(b*x)/b^
2+6*x*sinh(b*x)*Shi(b*x)/b^3+x^3*sinh(b*x)*Shi(b*x)/b+3*Shi(2*b*x)/b^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.73

$$\int x^3\cosh(bx)\text{Shi}(bx)\,dx$$
$$=\frac{36bx+2b^3x^3+12bx\cosh(2bx)-24\sinh(2bx)-3b^2x^2\sinh(2bx)+12(-3(2+b^2x^2)\cosh(bx)+bx(6+}{12b^4}$$

input
```
Integrate[x^3*Cosh[b*x]*SinhIntegral[b*x],x]
```

output
```
(36*b*x + 2*b^3*x^3 + 12*b*x*Cosh[2*b*x] - 24*Sinh[2*b*x] - 3*b^2*x^2*Sinh
[2*b*x] + 12*(-3*(2 + b^2*x^2)*Cosh[b*x] + b*x*(6 + b^2*x^2)*Sinh[b*x])*Si
nhIntegral[b*x] + 36*SinhIntegral[2*b*x])/(12*b^4)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{Shi}(bx)\cosh(bx)\,dx$$

$$\Big\downarrow \ 7102$$

$$-\frac{3\int x^2\sinh(bx)\text{Shi}(bx)dx}{b}-\int\frac{x^2\sinh^2(bx)}{b}dx+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{3\int x^2\sinh(bx)\text{Shi}(bx)dx}{b}-\frac{\int x^2\sinh^2(bx)dx}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{3\int x^2\sinh(bx)\text{Shi}(bx)dx}{b}-\frac{\int -x^2\sin(ibx)^2dx}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \ 25$$

$$-\frac{3\int x^2\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{\int x^2\sin(ibx)^2dx}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \ 3792$$

$$\frac{\frac{\int -\sinh^2(bx)dx}{2b^2}+\frac{\int x^2dx}{2}+\frac{x\sinh^2(bx)}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}}{b}-\frac{3\int x^2\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \ 15$$

$$\frac{\frac{\int -\sinh^2(bx)dx}{2b^2}+\frac{x\sinh^2(bx)}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}-\frac{3\int x^2\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \ 25$$

$$\frac{-\frac{\int \sinh^2(bx)dx}{2b^2}+\frac{x\sinh^2(bx)}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}-\frac{3\int x^2\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\int -\sin(ibx)^2dx}{2b^2}+\frac{x\sinh^2(bx)}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}-\frac{3\int x^2\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \sin(ibx)^2dx}{2b^2}+\frac{x\sinh^2(bx)}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}-\frac{3\int x^2\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$\Big\downarrow$ 3115

$$\frac{\frac{\frac{\int 1dx}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}+\frac{x\sinh^2(bx)}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}-\frac{3\int x^2\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$\Big\downarrow$ 24

$$-\frac{3\int x^2\sinh(bx)\text{Shi}(bx)dx}{b}+\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$\Big\downarrow$ 7096

$$-\frac{3\left(-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\int\frac{x\cosh(bx)\sinh(bx)}{b}dx+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$\Big\downarrow$ 27

$$-\frac{3\left(-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\int x\cosh(bx)\sinh(bx)dx}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$\Big\downarrow$ 5895

$$-\frac{3\left(-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}-\frac{\int \sinh^2(bx)dx}{2b}}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}-\frac{\int -\sin(ibx)^2dx}{2b}}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3\left(-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\int \sin(ibx)^2dx}{2b}}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{3\left(-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}-\frac{\frac{\frac{\int 1dx}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}+\frac{x\sinh^2(bx)}{2b}}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{3\left(-\frac{2\int x\cosh(bx)\text{Shi}(bx)dx}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)}{b}+\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{7102}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}-\int \frac{\sinh^2(bx)}{b}dx+\frac{x\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)}{b}+\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}-\frac{\int \sinh^2(bx)dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)}{b}+\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}-\frac{\int -\sin(ibx)^2dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)}{b}+\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}+\frac{\int \sin(ibx)^2dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)}{b}+\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 3115}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}+\frac{\frac{\int 1dx}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)}{b}$$
$$\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\downarrow 24$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \sinh(bx)\text{Shi}(bx)dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)}{b}+\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\downarrow 7094$$

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\int\frac{\cosh(bx)\sinh(bx)}{bx}dx}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)}{b}+\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\downarrow 27$$

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int\frac{\cosh(bx)\sinh(bx)}{x}dx}{b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)}{b}+\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\downarrow 5971$$

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int\frac{\sinh(2bx)}{2x}dx}{b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)}{b}+\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$$\downarrow 27$$

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int\frac{\sinh(2bx)}{x}dx}{2b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\cdots}{b}$$

$$\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$\downarrow$ 3042

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\int-\frac{i\sin(2ibx)}{x}dx}{2b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cdots}{2\cdots}}{2b}}{b}\cdots}{b}$$

$$\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

$\downarrow$ 26

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Shi}(bx)\cosh(bx)}{b}+\frac{i\int\frac{\sin(2ibx)}{x}dx}{2b}}{b}+\frac{x\text{Shi}(bx)\sinh(bx)}{b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(bx)\cosh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cdots}{2b}}{2b}}{b}\cdots}{b}$$

$$\frac{\frac{x\sinh^2(bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Shi}(bx)\sinh(bx)}{b}$$

input

```
Int[x^3*Cosh[b*x]*SinhIntegral[b*x],x]
```

output

```
$Aborted
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\operatorname{Shi}(bx)\left(b^3x^3\sinh(bx)-3b^2x^2\cosh(bx)+6bx\sinh(bx)-6\cosh(bx)\right)-\frac{b^2x^2\cosh(bx)\sinh(bx)}{2}+\frac{b^3x^3}{6}+2bx\cosh(bx)^2-4\cos}{b^4}$
default	$\frac{\operatorname{Shi}(bx)\left(b^3x^3\sinh(bx)-3b^2x^2\cosh(bx)+6bx\sinh(bx)-6\cosh(bx)\right)-\frac{b^2x^2\cosh(bx)\sinh(bx)}{2}+\frac{b^3x^3}{6}+2bx\cosh(bx)^2-4\cos}{b^4}$

input
```
int(x^3*cosh(b*x)*Shi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(Shi(b*x)*(b^3*x^3*sinh(b*x)-3*b^2*x^2*cosh(b*x)+6*b*x*sinh(b*x)-6*c
osh(b*x))-1/2*b^2*x^2*cosh(b*x)*sinh(b*x)+1/6*b^3*x^3+2*b*x*cosh(b*x)^2-4*
cosh(b*x)*sinh(b*x)+2*b*x+3*Shi(2*b*x))
```

Fricas [F]

$$\int x^3\cosh(bx)\operatorname{Shi}(bx)\,dx=\int x^3\operatorname{Shi}(bx)\cosh(bx)\,dx$$

input
```
integrate(x^3*cosh(b*x)*Shi(b*x),x, algorithm="fricas")
```

output
```
integral(x^3*cosh(b*x)*sinh_integral(b*x), x)
```

Sympy [F]

$$\int x^3\cosh(bx)\operatorname{Shi}(bx)\,dx=\int x^3\cosh(bx)\operatorname{Shi}(bx)\,dx$$

input
```
integrate(x**3*cosh(b*x)*Shi(b*x),x)
```

output
```
Integral(x**3*cosh(b*x)*Shi(b*x), x)
```

Maxima [F]

$$\int x^3 \cosh(bx)\mathrm{Shi}(bx)\,dx = \int x^3\mathrm{Shi}(bx)\cosh(bx)\,dx$$

input
```
integrate(x^3*cosh(b*x)*Shi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^3*Shi(b*x)*cosh(b*x), x)
```

Giac [F]

$$\int x^3 \cosh(bx)\mathrm{Shi}(bx)\,dx = \int x^3\mathrm{Shi}(bx)\cosh(bx)\,dx$$

input
```
integrate(x^3*cosh(b*x)*Shi(b*x),x, algorithm="giac")
```

output
```
integrate(x^3*Shi(b*x)*cosh(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cosh(bx)\mathrm{Shi}(bx)\,dx = \int x^3\,\mathrm{sinhint}(b\,x)\,\cosh(b\,x)\,dx$$

input
```
int(x^3*sinhint(b*x)*cosh(b*x),x)
```

output
```
int(x^3*sinhint(b*x)*cosh(b*x), x)
```

Reduce [F]

$$\int x^3 \cosh(bx)\text{Shi}(bx)\,dx = \int \cosh(bx)\,shi(bx)\,x^3 dx$$

input
```
int(x^3*cosh(b*x)*Shi(b*x),x)
```

output
```
int(cosh(b*x)*shi(b*x)*x**3,x)
```

3.53 $\int \sinh(5x)\mathbf{Shi}(2x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 29

$$\int \sinh(5x)\text{Shi}(2x)\,dx = \frac{1}{5}\cosh(5x)\text{Shi}(2x) + \frac{\text{Shi}(3x)}{10} - \frac{\text{Shi}(7x)}{10}$$

output
```
1/5*cosh(5*x)*Shi(2*x)+1/10*Shi(3*x)-1/10*Shi(7*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \sinh(5x)\text{Shi}(2x)\,dx = \frac{1}{10}(2\cosh(5x)\text{Shi}(2x) + \text{Shi}(3x) - \text{Shi}(7x))$$

input
```
Integrate[Sinh[5*x]*SinhIntegral[2*x],x]
```

output
```
(2*Cosh[5*x]*SinhIntegral[2*x] + SinhIntegral[3*x] - SinhIntegral[7*x])/10
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {7094, 27, 5995, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Shi}(2x)\sinh(5x)\,dx$$

$$\Big\downarrow \ 7094$$

$$\frac{1}{5}\text{Shi}(2x)\cosh(5x)-\frac{2}{5}\int \frac{\cosh(5x)\sinh(2x)}{2x}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{5}\text{Shi}(2x)\cosh(5x)-\frac{1}{5}\int \frac{\cosh(5x)\sinh(2x)}{x}dx$$

$$\Big\downarrow \ 5995$$

$$\frac{1}{5}\text{Shi}(2x)\cosh(5x)-\frac{1}{5}\int \left(\frac{\sinh(7x)}{2x}-\frac{\sinh(3x)}{2x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{5}\left(\frac{\text{Shi}(3x)}{2}-\frac{\text{Shi}(7x)}{2}\right)+\frac{1}{5}\text{Shi}(2x)\cosh(5x)$$

input
```
Int[Sinh[5*x]*SinhIntegral[2*x],x]
```

output
```
(Cosh[5*x]*SinhIntegral[2*x])/5 + (SinhIntegral[3*x]/2 - SinhIntegral[7*x]
/2)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5995
```
Int[Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p
, 0] && IGtQ[q, 0]
```

rule 7094
```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\cosh(5x)\,\mathrm{Shi}(2x)}{5}+\frac{\mathrm{Shi}(3x)}{10}-\frac{\mathrm{Shi}(7x)}{10}$	24

input
```
int(sinh(5*x)*Shi(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/5*cosh(5*x)*Shi(2*x)+1/10*Shi(3*x)-1/10*Shi(7*x)
```

Fricas [F]

$$\int \sinh(5x)\text{Shi}(2x)\,dx = \int \text{Shi}(2\,x)\sinh\left(5\,x\right)\,dx$$

input
```
integrate(sinh(5*x)*Shi(2*x),x, algorithm="fricas")
```

output
```
integral(sinh(5*x)*sinh_integral(2*x), x)
```

Sympy [F]

$$\int \sinh(5x)\text{Shi}(2x)\,dx = \int \sinh\left(5x\right)\text{Shi}\left(2x\right)dx$$

input
```
integrate(sinh(5*x)*Shi(2*x),x)
```

output
```
Integral(sinh(5*x)*Shi(2*x), x)
```

Maxima [F]

$$\int \sinh(5x)\text{Shi}(2x)\,dx = \int \text{Shi}(2\,x)\sinh\left(5\,x\right)\,dx$$

input
```
integrate(sinh(5*x)*Shi(2*x),x, algorithm="maxima")
```

output
```
integrate(Shi(2*x)*sinh(5*x), x)
```

Giac [F]

$$\int \sinh(5x)\mathrm{Shi}(2x)\,dx = \int \mathrm{Shi}\left(2\,x\right)\sinh\left(5\,x\right)\,dx$$

input
```
integrate(sinh(5*x)*Shi(2*x),x, algorithm="giac")
```

output
```
integrate(Shi(2*x)*sinh(5*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh(5x)\mathrm{Shi}(2x)\,dx = \int \mathrm{sinhint}\left(2\,x\right)\,\sinh\left(5\,x\right)\,dx$$

input
```
int(sinhint(2*x)*sinh(5*x),x)
```

output
```
int(sinhint(2*x)*sinh(5*x), x)
```

Reduce [F]

$$\int \sinh(5x)\mathrm{Shi}(2x)\,dx = \int \mathit{shi}(2x)\sinh\left(5x\right)dx$$

input
```
int(sinh(5*x)*Shi(2*x),x)
```

output
```
int(shi(2*x)*sinh(5*x),x)
```

3.54 $\int \cosh(5x)\textbf{Shi}(2x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 29

$$\int \cosh(5x)\text{Shi}(2x)\,dx = \frac{\text{Chi}(3x)}{10} - \frac{\text{Chi}(7x)}{10} + \frac{1}{5}\sinh(5x)\text{Shi}(2x)$$

output
```
1/10*Chi(3*x)-1/10*Chi(7*x)+1/5*sinh(5*x)*Shi(2*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \cosh(5x)\text{Shi}(2x)\,dx = \frac{1}{10}(\text{Chi}(3x) - \text{Chi}(7x) + 2\sinh(5x)\text{Shi}(2x))$$

input
```
Integrate[Cosh[5*x]*SinhIntegral[2*x],x]
```

output
```
(CoshIntegral[3*x] - CoshIntegral[7*x] + 2*Sinh[5*x]*SinhIntegral[2*x])/10
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {7100, 27, 5993, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Shi}(2x)\cosh(5x)\,dx$$

$$\Big\downarrow \text{7100}$$

$$\frac{1}{5}\text{Shi}(2x)\sinh(5x)-\frac{2}{5}\int \frac{\sinh(2x)\sinh(5x)}{2x}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\text{Shi}(2x)\sinh(5x)-\frac{1}{5}\int \frac{\sinh(2x)\sinh(5x)}{x}dx$$

$$\Big\downarrow \text{5993}$$

$$\frac{1}{5}\text{Shi}(2x)\sinh(5x)-\frac{1}{5}\int \left(\frac{\cosh(7x)}{2x}-\frac{\cosh(3x)}{2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\left(\frac{\text{Chi}(3x)}{2}-\frac{\text{Chi}(7x)}{2}\right)+\frac{1}{5}\text{Shi}(2x)\sinh(5x)$$

input
```
Int[Cosh[5*x]*SinhIntegral[2*x],x]
```

output
```
(CoshIntegral[3*x]/2 - CoshIntegral[7*x]/2)/5 + (Sinh[5*x]*SinhIntegral[2*
x])/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5993
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(p_.)*Sinh[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Sinh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0
] && IGtQ[q, 0] && IntegerQ[m]
```

rule 7100
```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\mathrm{Chi}(3x)}{10}-\frac{\mathrm{Chi}(7x)}{10}+\frac{\sinh(5x)\,\mathrm{Shi}(2x)}{5}$	24

input
```
int(cosh(5*x)*Shi(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/10*Chi(3*x)-1/10*Chi(7*x)+1/5*sinh(5*x)*Shi(2*x)
```

Fricas [F]

$$\int \cosh(5x)\text{Shi}(2x)\,dx = \int \text{Shi}(2\,x)\cosh\left(5\,x\right)\,dx$$

input
```
integrate(cosh(5*x)*Shi(2*x),x, algorithm="fricas")
```

output
```
integral(cosh(5*x)*sinh_integral(2*x), x)
```

Sympy [F]

$$\int \cosh(5x)\text{Shi}(2x)\,dx = \int \cosh\left(5x\right)\text{Shi}\left(2x\right)dx$$

input
```
integrate(cosh(5*x)*Shi(2*x),x)
```

output
```
Integral(cosh(5*x)*Shi(2*x), x)
```

Maxima [F]

$$\int \cosh(5x)\text{Shi}(2x)\,dx = \int \text{Shi}(2\,x)\cosh\left(5\,x\right)\,dx$$

input
```
integrate(cosh(5*x)*Shi(2*x),x, algorithm="maxima")
```

output
```
integrate(Shi(2*x)*cosh(5*x), x)
```

Giac [F]

$$\int \cosh(5x)\text{Shi}(2x)\,dx = \int \text{Shi}(2\,x)\cosh\left(5\,x\right)\,dx$$

input
```
integrate(cosh(5*x)*Shi(2*x),x, algorithm="giac")
```

output
```
integrate(Shi(2*x)*cosh(5*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(5x)\text{Shi}(2x)\,dx = \int \text{sinhint}(2\,x)\,\cosh(5\,x)\,dx$$

input
```
int(sinhint(2*x)*cosh(5*x),x)
```

output
```
int(sinhint(2*x)*cosh(5*x), x)
```

Reduce [F]

$$\int \cosh(5x)\text{Shi}(2x)\,dx = \int \cosh\left(5x\right) \mathit{shi}(2x)\,dx$$

input
```
int(cosh(5*x)*Shi(2*x),x)
```

output
```
int(cosh(5*x)*shi(2*x),x)
```

3.55 $\int x^2 \sinh(a+bx)\textbf{Shi}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 174

$$\int x^2\sinh(a+bx)\text{Shi}(a+bx)\,dx = -\frac{x}{b^2}+\frac{(a-bx)\cosh(2a+2bx)}{4b^3}-\frac{a\text{Chi}(2a+2bx)}{b^3} + \frac{a\log(a+bx)}{b^3}+\frac{\cosh(a+bx)\sinh(a+bx)}{b^3} + \frac{\sinh(2a+2bx)}{8b^3}+\frac{2\cosh(a+bx)\text{Shi}(a+bx)}{b^3} + \frac{x^2\cosh(a+bx)\text{Shi}(a+bx)}{b} - \frac{2x\sinh(a+bx)\text{Shi}(a+bx)}{b^2} - \frac{\text{Shi}(2a+2bx)}{b^3}-\frac{a^2\text{Shi}(2a+2bx)}{2b^3}$$

output
```
-x/b^2+1/4*(-b*x+a)*cosh(2*b*x+2*a)/b^3-a*Chi(2*b*x+2*a)/b^3+a*ln(b*x+a)/b
^3+cosh(b*x+a)*sinh(b*x+a)/b^3+1/8*sinh(2*b*x+2*a)/b^3+2*cosh(b*x+a)*Shi(b
*x+a)/b^3+x^2*cosh(b*x+a)*Shi(b*x+a)/b-2*x*sinh(b*x+a)*Shi(b*x+a)/b^2-Shi(
2*b*x+2*a)/b^3-1/2*a^2*Shi(2*b*x+2*a)/b^3
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.71

$$\int x^2 \sinh(a+bx)\text{Shi}(a+bx)\,dx$$
$$= \frac{-8bx + 2a\cosh(2(a+bx)) - 2bx\cosh(2(a+bx)) - 8a\text{Chi}(2(a+bx)) + 8a\log(a+bx) + 5\sinh(2(a+b}{8}$$

input
```
Integrate[x^2*Sinh[a + b*x]*SinhIntegral[a + b*x],x]
```

output
```
(-8*b*x + 2*a*Cosh[2*(a + b*x)] - 2*b*x*Cosh[2*(a + b*x)] - 8*a*CoshIntegr
al[2*(a + b*x)] + 8*a*Log[a + b*x] + 5*Sinh[2*(a + b*x)] + 8*((2 + b^2*x^2
)*Cosh[a + b*x] - 2*b*x*Sinh[a + b*x])*SinhIntegral[a + b*x] - 8*SinhInteg
ral[2*(a + b*x)] - 4*a^2*SinhIntegral[2*(a + b*x)])/(8*b^3)
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {7096, 6151, 7102, 7094, 5971, 27, 3042, 26, 3779, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{Shi}(a+bx)\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{ 7096}$$

$$-\frac{2\int x\cosh(a+bx)\text{Shi}(a+bx)dx}{b} - \int \frac{x^2\cosh(a+bx)\sinh(a+bx)}{a+bx}dx + \frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{ 6151}$$

$$-\frac{2\int x\cosh(a+bx)\text{Shi}(a+bx)dx}{b} - \frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx + \frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{ 7102}$$

$$-\frac{2\left(-\frac{\int \sinh(a+bx)\text{Shi}(a+bx)dx}{b}-\int \frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \; 7094$$

$$-\frac{2\left(-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}dx}{b}-\int \frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \; 5971$$

$$-\frac{2\left(-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\sinh(2a+2bx)}{2(a+bx)}dx}{b}-\int \frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{1}{2}\int \frac{\sinh(2a+2bx)}{a+bx}dx}{b}-\int \frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\left(-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{1}{2}\int -\frac{i\sin(2ia+2ibx)}{a+bx}dx}{b}-\int \frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{2\left(-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}+\frac{1}{2}i\int \frac{\sin(2ia+2ibx)}{a+bx}dx}{b}-\int \frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \; 3779$$

$$-\frac{2\left(-\int \frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{7292}$$

$$-\frac{2\left(-\int \frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2a+2bx)}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{2}\int \left(\frac{\sinh(2a+2bx)a^2}{b^2(a+bx)}-\frac{\sinh(2a+2bx)a}{b^2}+\frac{x\sinh(2a+2bx)}{b}\right)dx-\frac{2\left(-\int \left(\frac{\sinh^2(a+bx)}{b}-\frac{a\sinh^2(a+bx)}{b(a+bx)}\right)dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2\text{Shi}(2a+2bx)}{b^3}+\frac{\sinh(2a+2bx)}{4b^3}+\frac{a\cosh(2a+2bx)}{2b^3}-\frac{x\cosh(2a+2bx)}{2b^2}\right)-\frac{2\left(\frac{a\text{Chi}(2a+2bx)}{2b^2}-\frac{a\log(a+bx)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b^2}+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}+\frac{x}{2b}\right)}{b}+\frac{x^2\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

input

```
Int[x^2*Sinh[a + b*x]*SinhIntegral[a + b*x],x]
```

output

```
(x^2*Cosh[a + b*x]*SinhIntegral[a + b*x])/b + ((a*Cosh[2*a + 2*b*x])/(2*b^
3) - (x*Cosh[2*a + 2*b*x])/(2*b^2) + Sinh[2*a + 2*b*x]/(4*b^3) - (a^2*Sinh
Integral[2*a + 2*b*x])/b^3)/2 - (2*(x/(2*b) + (a*CoshIntegral[2*a + 2*b*x]
)/(2*b^2) - (a*Log[a + b*x])/(2*b^2) - (Cosh[a + b*x]*Sinh[a + b*x])/(2*b^
2) + (x*Sinh[a + b*x]*SinhIntegral[a + b*x])/b - ((Cosh[a + b*x]*SinhInteg
ral[a + b*x])/b - SinhIntegral[2*a + 2*b*x]/(2*b))/b))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6151
```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Simp[1/2^p   Int[u*Si
nh[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

rule 7094
```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7096
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7102
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\operatorname{Shi}(bx+a)\left(a^2\cosh(bx+a)-2a((bx+a)\cosh(bx+a)-\sinh(bx+a))+(bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(bx+a)+2\cosh\right.}{}$
default	$\frac{\operatorname{Shi}(bx+a)\left(a^2\cosh(bx+a)-2a((bx+a)\cosh(bx+a)-\sinh(bx+a))+(bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(bx+a)+2\cosh\right.}{}$

input
```
int(x^2*sinh(b*x+a)*Shi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(Shi(b*x+a)*(a^2*cosh(b*x+a)-2*a*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))+(
b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x+a)+2*cosh(b*x+a))-1/2*a^2*Shi(2*b*
x+2*a)+a*cosh(b*x+a)^2-a*Chi(2*b*x+2*a)+a*ln(b*x+a)-1/2*(b*x+a)*cosh(b*x+a
)^2+5/4*sinh(b*x+a)*cosh(b*x+a)-3/4*b*x-3/4*a-Shi(2*b*x+2*a))
```

Fricas [F]

$$\int x^2\sinh(a+bx)\mathrm{Shi}(a+bx)\,dx=\int x^2\mathrm{Shi}(bx+a)\sinh(bx+a)\,dx$$

input
```
integrate(x^2*sinh(b*x+a)*Shi(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^2*sinh(b*x + a)*sinh_integral(b*x + a), x)
```

Sympy [F]

$$\int x^2\sinh(a+bx)\mathrm{Shi}(a+bx)\,dx=\int x^2\sinh(a+bx)\,\mathrm{Shi}(a+bx)\,dx$$

input
```
integrate(x**2*sinh(b*x+a)*Shi(b*x+a),x)
```

output
```
Integral(x**2*sinh(a + b*x)*Shi(a + b*x), x)
```

Maxima [F]

$$\int x^2\sinh(a+bx)\mathrm{Shi}(a+bx)\,dx=\int x^2\mathrm{Shi}(bx+a)\sinh(bx+a)\,dx$$

input
```
integrate(x^2*sinh(b*x+a)*Shi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^2*Shi(b*x + a)*sinh(b*x + a), x)
```

Giac [F]

$$\int x^2 \sinh(a+bx)\text{Shi}(a+bx)\,dx = \int x^2\text{Shi}(bx+a)\sinh(bx+a)\,dx$$

input
```
integrate(x^2*sinh(b*x+a)*Shi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*Shi(b*x + a)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh(a+bx)\text{Shi}(a+bx)\,dx = \int x^2\,\text{sinhint}(a+b\,x)\,\sinh(a+b\,x)\,dx$$

input
```
int(x^2*sinhint(a + b*x)*sinh(a + b*x),x)
```

output
```
int(x^2*sinhint(a + b*x)*sinh(a + b*x), x)
```

Reduce [F]

$$\int x^2 \sinh(a+bx)\text{Shi}(a+bx)\,dx = \int shi(bx+a)\sinh(bx+a)\,x^2dx$$

input
```
int(x^2*sinh(b*x+a)*Shi(b*x+a),x)
```

output
```
int(shi(a + b*x)*sinh(a + b*x)*x**2,x)
```

3.56 $\int x \sinh(a+bx)\textbf{Shi}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 97

$$\int x\sinh(a+bx)\text{Shi}(a+bx)\,dx = -\frac{\cosh(2a+2bx)}{4b^2}+\frac{\text{Chi}(2a+2bx)}{2b^2} - \frac{\log(a+bx)}{2b^2}+\frac{x\cosh(a+bx)\text{Shi}(a+bx)}{b} - \frac{\sinh(a+bx)\text{Shi}(a+bx)}{b^2}+\frac{a\text{Shi}(2a+2bx)}{2b^2}$$

output
```
-1/4*cosh(2*b*x+2*a)/b^2+1/2*Chi(2*b*x+2*a)/b^2-1/2*ln(b*x+a)/b^2+x*cosh(b
*x+a)*Shi(b*x+a)/b-sinh(b*x+a)*Shi(b*x+a)/b^2+1/2*a*Shi(2*b*x+2*a)/b^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int x\sinh(a+bx)\text{Shi}(a+bx)\,dx = \frac{-\cosh(2(a+bx))+2\text{Chi}(2(a+bx))-2\log(a+bx)+4(bx\cosh(a+bx)-\sinh(a+bx))\text{Shi}(a+bx)+}{4b^2}$$

input
```
Integrate[x*Sinh[a + b*x]*SinhIntegral[a + b*x],x]
```

output
```
(-Cosh[2*(a + b*x)] + 2*CoshIntegral[2*(a + b*x)] - 2*Log[a + b*x] + 4*(b*
x*Cosh[a + b*x] - Sinh[a + b*x])*SinhIntegral[a + b*x] + 2*a*SinhIntegral[
2*(a + b*x)])/(4*b^2)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {7096, 6151, 7100, 3042, 25, 3793, 2009, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Shi}(a+bx)\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{7096}$$

$$-\frac{\int \cosh(a+bx)\text{Shi}(a+bx)dx}{b}-\int \frac{x\cosh(a+bx)\sinh(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{6151}$$

$$-\frac{\int \cosh(a+bx)\text{Shi}(a+bx)dx}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{7100}$$

$$-\frac{\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}-\int \frac{\sinh^2(a+bx)}{a+bx}dx}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}-\int -\frac{\sin(ia+ibx)^2}{a+bx}dx}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\int \frac{\sin(ia+ibx)^2}{a+bx}dx}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{1}{2(a+bx)}-\frac{\cosh(2a+2bx)}{2(a+bx)}\right)dx+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{7292}$$

$$-\frac{1}{2}\int \frac{x\sinh(2a+2bx)}{a+bx}dx-\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{2}\int \left(\frac{\sinh(2a+2bx)}{b}+\frac{a\sinh(2a+2bx)}{b(-a-bx)}\right)dx-\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a\text{Shi}(2a+2bx)}{b^2}-\frac{\cosh(2a+2bx)}{2b^2}\right)-\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

input
```
Int[x*Sinh[a + b*x]*SinhIntegral[a + b*x],x]
```

output
```
(x*Cosh[a + b*x]*SinhIntegral[a + b*x])/b - (-1/2*CoshIntegral[2*a + 2*b*x
]/b + Log[a + b*x]/(2*b) + (Sinh[a + b*x]*SinhIntegral[a + b*x])/b)/b + (-
1/2*Cosh[2*a + 2*b*x]/b^2 + (a*SinhIntegral[2*a + 2*b*x])/b^2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6151
```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Simp[1/2^p   Int[u*Si
nh[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

rule 7096
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7100
```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\mathrm{Shi}(bx+a)((bx+a)\cosh(bx+a)-\sinh(bx+a)-a\cosh(bx+a))-\frac{\cosh(bx+a)^2}{2}-\frac{\ln(bx+a)}{2}+\frac{\mathrm{Chi}(2bx+2a)}{2}+\frac{a\,\mathrm{Shi}(2bx+2a)}{2}}{b^2}$
default	$\frac{\mathrm{Shi}(bx+a)((bx+a)\cosh(bx+a)-\sinh(bx+a)-a\cosh(bx+a))-\frac{\cosh(bx+a)^2}{2}-\frac{\ln(bx+a)}{2}+\frac{\mathrm{Chi}(2bx+2a)}{2}+\frac{a\,\mathrm{Shi}(2bx+2a)}{2}}{b^2}$

input
```
int(x*sinh(b*x+a)*Shi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(Shi(b*x+a)*((b*x+a)*cosh(b*x+a)-sinh(b*x+a)-a*cosh(b*x+a))-1/2*cosh
(b*x+a)^2-1/2*ln(b*x+a)+1/2*Chi(2*b*x+2*a)+1/2*a*Shi(2*b*x+2*a))
```

Fricas [F]

$$\int x\sinh(a+bx)\mathrm{Shi}(a+bx)\,dx = \int x\mathrm{Shi}(bx+a)\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x*sinh(b*x+a)*Shi(b*x+a),x, algorithm="fricas")
```

output
```
integral(x*sinh(b*x + a)*sinh_integral(b*x + a), x)
```

Sympy [F]

$$\int x\sinh(a+bx)\mathrm{Shi}(a+bx)\,dx = \int x\sinh\left(a+bx\right)\mathrm{Shi}\left(a+bx\right)dx$$

input
```
integrate(x*sinh(b*x+a)*Shi(b*x+a),x)
```

output
```
Integral(x*sinh(a + b*x)*Shi(a + b*x), x)
```

Maxima [F]

$$\int x \sinh(a+bx)\mathrm{Shi}(a+bx)\,dx = \int x\mathrm{Shi}(bx+a)\sinh(bx+a)\,dx$$

input
```
integrate(x*sinh(b*x+a)*Shi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*Shi(b*x + a)*sinh(b*x + a), x)
```

Giac [F]

$$\int x \sinh(a+bx)\mathrm{Shi}(a+bx)\,dx = \int x\mathrm{Shi}(bx+a)\sinh(bx+a)\,dx$$

input
```
integrate(x*sinh(b*x+a)*Shi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*Shi(b*x + a)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \sinh(a+bx)\mathrm{Shi}(a+bx)\,dx = \int x\,\mathrm{sinhint}(a+b\,x)\,\sinh(a+b\,x)\,dx$$

input
```
int(x*sinhint(a + b*x)*sinh(a + b*x),x)
```

output
```
int(x*sinhint(a + b*x)*sinh(a + b*x), x)
```

Reduce [F]

$$\int x \sinh(a+bx)\text{Shi}(a+bx)\,dx = \int shi(bx+a)\sinh(bx+a)\,xdx$$

input
```
int(x*sinh(b*x+a)*Shi(b*x+a),x)
```

output
```
int(shi(a + b*x)*sinh(a + b*x)*x,x)
```

3.57 $\int \sinh(a+bx)\textbf{Shi}(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \sinh(a+bx)\text{Shi}(a+bx)\,dx=\frac{\cosh(a+bx)\text{Shi}(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}$$

output
```
cosh(b*x+a)*Shi(b*x+a)/b-1/2*Shi(2*b*x+2*a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \sinh(a+bx)\text{Shi}(a+bx)\,dx=\frac{\cosh(a+bx)\text{Shi}(a+bx)}{b}-\frac{\text{Shi}(2(a+bx))}{2b}$$

input
```
Integrate[Sinh[a + b*x]*SinhIntegral[a + b*x],x]
```

output
```
(Cosh[a + b*x]*SinhIntegral[a + b*x])/b - SinhIntegral[2*(a + b*x)]/(2*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {7094, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Shi}(a+bx)\sinh(a+bx)\,dx$$

$$\Big\downarrow \ 7094$$

$$\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}dx$$

$$\Big\downarrow \ 5971$$

$$\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\sinh(2a+2bx)}{2(a+bx)}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{1}{2}\int \frac{\sinh(2a+2bx)}{a+bx}dx$$

$$\Big\downarrow \ 3042$$

$$\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{1}{2}\int -\frac{i\sin(2ia+2ibx)}{a+bx}dx$$

$$\Big\downarrow \ 26$$

$$\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}+\frac{1}{2}i\int \frac{\sin(2ia+2ibx)}{a+bx}dx$$

$$\Big\downarrow \ 3779$$

$$\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}$$

input
```
Int[Sinh[a + b*x]*SinhIntegral[a + b*x],x]
```

output
```
(Cosh[a + b*x]*SinhIntegral[a + b*x])/b - SinhIntegral[2*a + 2*b*x]/(2*b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7094
```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\cosh(bx+a)\,\mathrm{Shi}(bx+a)-\frac{\mathrm{Shi}(2bx+2a)}{2}}{b}$	30
default	$\frac{\cosh(bx+a)\,\mathrm{Shi}(bx+a)-\frac{\mathrm{Shi}(2bx+2a)}{2}}{b}$	30

input
```
int(sinh(b*x+a)*Shi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(cosh(b*x+a)*Shi(b*x+a)-1/2*Shi(2*b*x+2*a))
```

Fricas [F]

$$\int \sinh(a+bx)\text{Shi}(a+bx)\,dx = \int \text{Shi}(bx+a)\sinh(bx+a)\,dx$$

input
```
integrate(sinh(b*x+a)*Shi(b*x+a),x, algorithm="fricas")
```

output
```
integral(sinh(b*x + a)*sinh_integral(b*x + a), x)
```

Sympy [F]

$$\int \sinh(a+bx)\text{Shi}(a+bx)\,dx = \int \sinh(a+bx)\,\text{Shi}(a+bx)\,dx$$

input
```
integrate(sinh(b*x+a)*Shi(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*Shi(a + b*x), x)
```

Maxima [F]

$$\int \sinh(a+bx)\text{Shi}(a+bx)\,dx = \int \text{Shi}(bx+a)\sinh(bx+a)\,dx$$

input
```
integrate(sinh(b*x+a)*Shi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(Shi(b*x + a)*sinh(b*x + a), x)
```

Giac [F]

$$\int \sinh(a+bx)\text{Shi}(a+bx)\,dx = \int \text{Shi}(bx+a)\sinh(bx+a)\,dx$$

input
```
integrate(sinh(b*x+a)*Shi(b*x+a),x, algorithm="giac")
```

output
```
integrate(Shi(b*x + a)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh(a+bx)\text{Shi}(a+bx)\,dx = \int \text{sinhint}(a+b\,x)\,\sinh(a+b\,x)\,dx$$

input
```
int(sinhint(a + b*x)*sinh(a + b*x),x)
```

output
```
int(sinhint(a + b*x)*sinh(a + b*x), x)
```

Reduce [F]

$$\int \sinh(a+bx)\text{Shi}(a+bx)\,dx = \int \mathit{shi}(bx+a)\sinh(bx+a)\,dx$$

input
```
int(sinh(b*x+a)*Shi(b*x+a),x)
```

output
```
int(shi(a + b*x)*sinh(a + b*x),x)
```

3.58 $\int \frac{\sinh(a+bx)\text{Shi}(a+bx)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sinh(a+bx)\text{Shi}(a+bx)}{x}\,dx = \text{Int}\left(\frac{\sinh(a+bx)\text{Shi}(a+bx)}{x},x\right)$$

output
```
Defer(Int)(sinh(b*x+a)*Shi(b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)\text{Shi}(a+bx)}{x}\,dx = \int \frac{\sinh(a+bx)\text{Shi}(a+bx)}{x}\,dx$$

input
```
Integrate[(Sinh[a + b*x]*SinhIntegral[a + b*x])/x,x]
```

output
```
Integrate[(Sinh[a + b*x]*SinhIntegral[a + b*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(a+bx)\sinh(a+bx)}{x}dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{\text{Shi}(a+bx)\sinh(a+bx)}{x}dx$$

input
```
Int[(Sinh[a + b*x]*SinhIntegral[a + b*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(bx+a\right)\text{Shi}\left(bx+a\right)}{x}dx$$

input
```
int(sinh(b*x+a)*Shi(b*x+a)/x,x)
```

output
```
int(sinh(b*x+a)*Shi(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)\mathrm{Shi}(a+bx)}{x}\,dx = \int \frac{\mathrm{Shi}(bx+a)\sinh\left(bx+a\right)}{x}\,dx$$

input
```
integrate(sinh(b*x+a)*Shi(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(sinh(b*x + a)*sinh_integral(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sinh(a+bx)\mathrm{Shi}(a+bx)}{x}\,dx = \int \frac{\sinh\left(a+bx\right)\mathrm{Shi}\left(a+bx\right)}{x}\,dx$$

input
```
integrate(sinh(b*x+a)*Shi(b*x+a)/x,x)
```

output
```
Integral(sinh(a + b*x)*Shi(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)\mathrm{Shi}(a+bx)}{x}\,dx = \int \frac{\mathrm{Shi}(bx+a)\sinh\left(bx+a\right)}{x}\,dx$$

input
```
integrate(sinh(b*x+a)*Shi(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x + a)*sinh(b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)\text{Shi}(a+bx)}{x}\,dx = \int \frac{\text{Shi}(bx+a)\sinh(bx+a)}{x}\,dx$$

input
```
integrate(sinh(b*x+a)*Shi(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(Shi(b*x + a)*sinh(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)\text{Shi}(a+bx)}{x}\,dx = \int \frac{\text{sinhint}(a+b\,x)\,\sinh(a+b\,x)}{x}\,dx$$

input
```
int((sinhint(a + b*x)*sinh(a + b*x))/x,x)
```

output
```
int((sinhint(a + b*x)*sinh(a + b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{\sinh(a+bx)\text{Shi}(a+bx)}{x}\,dx = \left(\int \frac{shi(bx+a)\sinh(bx+a)}{b\,x^2+ax}dx\right)a+\frac{shi(bx+a)^2}{2}$$

input
```
int(sinh(b*x+a)*Shi(b*x+a)/x,x)
```

output
```
(2*int((shi(a + b*x)*sinh(a + b*x))/(a*x + b*x**2),x)*a + shi(a + b*x)**2)
/2
```

3.59 $\int x^2 \cosh(a+bx)\textbf{Shi}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 200

$$\int x^2 \cosh(a+bx)\text{Shi}(a+bx)\,dx = \frac{(a-bx)^2}{4b^3}+\frac{\cosh(2a+2bx)}{2b^3}-\frac{\text{Chi}(2a+2bx)}{b^3} -\frac{a^2\text{Chi}(2a+2bx)}{2b^3}+\frac{\log(a+bx)}{b^3}+\frac{a^2\log(a+bx)}{2b^3} +\frac{(a-bx)\cosh(a+bx)\sinh(a+bx)}{2b^3} +\frac{\sinh^2(a+bx)}{4b^3}-\frac{2x\cosh(a+bx)\text{Shi}(a+bx)}{b^2} +\frac{2\sinh(a+bx)\text{Shi}(a+bx)}{b^3} +\frac{x^2\sinh(a+bx)\text{Shi}(a+bx)}{b}-\frac{a\text{Shi}(2a+2bx)}{b^3}$$

output

```
1/4*(-b*x+a)^2/b^3+1/2*cosh(2*b*x+2*a)/b^3-Chi(2*b*x+2*a)/b^3-1/2*a^2*Chi(
2*b*x+2*a)/b^3+ln(b*x+a)/b^3+1/2*a^2*ln(b*x+a)/b^3+1/2*(-b*x+a)*cosh(b*x+a
)*sinh(b*x+a)/b^3+1/4*sinh(b*x+a)^2/b^3-2*x*cosh(b*x+a)*Shi(b*x+a)/b^2+2*s
inh(b*x+a)*Shi(b*x+a)/b^3+x^2*sinh(b*x+a)*Shi(b*x+a)/b-a*Shi(2*b*x+2*a)/b^
3
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.67

$$\int x^2\cosh(a+bx)\text{Shi}(a+bx)\,dx$$
$$= \frac{-4abx+2b^2x^2+5\cosh(2(a+bx))-4(2+a^2)\,\text{Chi}(2(a+bx))+8\log(a+bx)+4a^2\log(a+bx)+2a\,\text{s}}{}$$

input
```
Integrate[x^2*Cosh[a + b*x]*SinhIntegral[a + b*x],x]
```

output
```
(-4*a*b*x + 2*b^2*x^2 + 5*Cosh[2*(a + b*x)] - 4*(2 + a^2)*CoshIntegral[2*(
a + b*x)] + 8*Log[a + b*x] + 4*a^2*Log[a + b*x] + 2*a*Sinh[2*(a + b*x)] -
2*b*x*Sinh[2*(a + b*x)] + 8*(-2*b*x*Cosh[a + b*x] + (2 + b^2*x^2)*Sinh[a +
 b*x])*SinhIntegral[a + b*x] - 8*a*SinhIntegral[2*(a + b*x)])/(8*b^3)
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {7102, 7096, 6151, 7100, 3042, 25, 3793, 2009, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{Shi}(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{7102}$$

$$-\frac{2\int x\sinh(a+bx)\text{Shi}(a+bx)dx}{b}-\int \frac{x^2\sinh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{7096}$$

$$-\frac{2\left(-\frac{\int \cosh(a+bx)\text{Shi}(a+bx)dx}{b}-\int \frac{x\cosh(a+bx)\sinh(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\int \frac{x^2\sinh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{6151}$$

$$-\frac{2\left(-\frac{\int \cosh(a+bx)\text{Shi}(a+bx)dx}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\int \frac{x^2\sinh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$\Big\downarrow$ 7100

$$-\frac{2\left(-\frac{\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}-\int \frac{\sinh^2(a+bx)}{a+bx}dx}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\int \frac{x^2\sinh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$\Big\downarrow$ 3042

$$-\frac{2\left(-\frac{\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}-\int -\frac{\sin(ia+ibx)^2}{a+bx}dx}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\int \frac{x^2\sinh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$\Big\downarrow$ 25

$$-\frac{2\left(-\frac{\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\int \frac{\sin(ia+ibx)^2}{a+bx}dx}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\int \frac{x^2\sinh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$\Big\downarrow$ 3793

$$-\frac{2\left(-\frac{\int \left(\frac{1}{2(a+bx)}-\frac{\cosh(2a+2bx)}{2(a+bx)}\right)dx+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\int \frac{x^2\sinh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$\Big\downarrow$ 2009

$$-\frac{2\left(-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\int \frac{x^2\sinh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{7292}$$

$$-\frac{2\left(-\frac{1}{2}\int \frac{x\sinh(2a+2bx)}{a+bx}dx-\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\int \frac{x^2\sinh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(\frac{x\sinh^2(a+bx)}{b}+\frac{a^2\sinh^2(a+bx)}{b^2(a+bx)}-\frac{a\sinh^2(a+bx)}{b^2}\right)dx-\frac{2\left(-\frac{1}{2}\int \left(\frac{\sinh(2a+2bx)}{b}+\frac{a\sinh(2a+2bx)}{b(-a-bx)}\right)dx-\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)}{b}+\frac{x^2\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\text{Chi}(2a+2bx)}{2b^3}+\frac{a^2\log(a+bx)}{2b^3}+\frac{\sinh^2(a+bx)}{4b^3}+\frac{a\sinh(a+bx)\cosh(a+bx)}{2b^3}-\frac{2\left(\frac{1}{2}\left(\frac{a\text{Shi}(2a+2bx)}{b^2}-\frac{\cosh(2a+2bx)}{2b^2}\right)-\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}}{b}+\frac{x\text{Shi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{ax}{2b^2}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b^2}+\frac{x^2\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{x^2}{4b}$$

input
```
Int[x^2*Cosh[a + b*x]*SinhIntegral[a + b*x],x]
```

output
```
-1/2*(a*x)/b^2 + x^2/(4*b) - (a^2*CoshIntegral[2*a + 2*b*x])/(2*b^3) + (a^
2*Log[a + b*x])/(2*b^3) + (a*Cosh[a + b*x]*Sinh[a + b*x])/(2*b^3) - (x*Cos
h[a + b*x]*Sinh[a + b*x])/(2*b^2) + Sinh[a + b*x]^2/(4*b^3) + (x^2*Sinh[a
+ b*x]*SinhIntegral[a + b*x])/b - (2*((x*Cosh[a + b*x]*SinhIntegral[a + b*
x])/b - (-1/2*CoshIntegral[2*a + 2*b*x]/b + Log[a + b*x]/(2*b) + (Sinh[a +
 b*x]*SinhIntegral[a + b*x])/b)/b + (-1/2*Cosh[2*a + 2*b*x]/b^2 + (a*SinhI
ntegral[2*a + 2*b*x])/b^2)/2))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6151
```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Simp[1/2^p   Int[u*Si
nh[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

rule 7096
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7100
```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7102
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{\mathrm{Shi}(bx+a)\left(a^2\sinh(bx+a)-2a((bx+a)\sinh(bx+a)-\cosh(bx+a))+(bx+a)^2\sinh(bx+a)-2(bx+a)\cosh(bx+a)+2\sinh\right.}{}$
default	$\frac{\mathrm{Shi}(bx+a)\left(a^2\sinh(bx+a)-2a((bx+a)\sinh(bx+a)-\cosh(bx+a))+(bx+a)^2\sinh(bx+a)-2(bx+a)\cosh(bx+a)+2\sinh\right.}{}$

input
```
int(x^2*cosh(b*x+a)*Shi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(Shi(b*x+a)*(a^2*sinh(b*x+a)-2*a*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))+(
b*x+a)^2*sinh(b*x+a)-2*(b*x+a)*cosh(b*x+a)+2*sinh(b*x+a))+1/2*a^2*ln(b*x+a
)-1/2*a^2*Chi(2*b*x+2*a)+cosh(b*x+a)*sinh(b*x+a)*a-(b*x+a)*a-a*Shi(2*b*x+2
*a)-1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2+5/4*cosh(b*x+a)^2+ln
(b*x+a)-Chi(2*b*x+2*a))
```

Fricas [F]

$$\int x^2\cosh(a+bx)\mathrm{Shi}(a+bx)\,dx=\int x^2\mathrm{Shi}(bx+a)\cosh(bx+a)\,dx$$

input
```
integrate(x^2*cosh(b*x+a)*Shi(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^2*cosh(b*x + a)*sinh_integral(b*x + a), x)
```

Sympy [F]

$$\int x^2 \cosh(a + bx)\text{Shi}(a + bx)\, dx = \int x^2 \cosh(a + bx)\, \text{Shi}(a + bx)\, dx$$

input
```
integrate(x**2*cosh(b*x+a)*Shi(b*x+a),x)
```

output
```
Integral(x**2*cosh(a + b*x)*Shi(a + b*x), x)
```

Maxima [F]

$$\int x^2 \cosh(a + bx)\text{Shi}(a + bx)\, dx = \int x^2 \text{Shi}(bx + a) \cosh(bx + a)\, dx$$

input
```
integrate(x^2*cosh(b*x+a)*Shi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^2*Shi(b*x + a)*cosh(b*x + a), x)
```

Giac [F]

$$\int x^2 \cosh(a + bx)\text{Shi}(a + bx)\, dx = \int x^2 \text{Shi}(bx + a) \cosh(bx + a)\, dx$$

input
```
integrate(x^2*cosh(b*x+a)*Shi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*Shi(b*x + a)*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh(a+bx)\text{Shi}(a+bx)\,dx = \int x^2\,\text{sinhint}(a+b\,x)\,\cosh(a+b\,x)\,dx$$

input
```
int(x^2*sinhint(a + b*x)*cosh(a + b*x),x)
```

output
```
int(x^2*sinhint(a + b*x)*cosh(a + b*x), x)
```

Reduce [F]

$$\int x^2 \cosh(a+bx)\text{Shi}(a+bx)\,dx = \int \cosh(bx+a)\,shi(bx+a)\,x^2dx$$

input
```
int(x^2*cosh(b*x+a)*Shi(b*x+a),x)
```

output
```
int(cosh(a + b*x)*shi(a + b*x)*x**2,x)
```

3.60 $\int x \cosh(a+bx)\textbf{Shi}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 109

$$\begin{aligned}\int x\cosh(a+bx)\text{Shi}(a+bx)\,dx = {} & \frac{x}{2b}+\frac{a\text{Chi}(2a+2bx)}{2b^2}-\frac{a\log(a+bx)}{2b^2} \\ & -\frac{\cosh(a+bx)\sinh(a+bx)}{2b^2} \\ & -\frac{\cosh(a+bx)\text{Shi}(a+bx)}{b^2} \\ & +\frac{x\sinh(a+bx)\text{Shi}(a+bx)}{b}+\frac{\text{Shi}(2a+2bx)}{2b^2}\end{aligned}$$

output
```
1/2*x/b+1/2*a*Chi(2*b*x+2*a)/b^2-1/2*a*ln(b*x+a)/b^2-1/2*cosh(b*x+a)*sinh(
b*x+a)/b^2-cosh(b*x+a)*Shi(b*x+a)/b^2+x*sinh(b*x+a)*Shi(b*x+a)/b+1/2*Shi(2
*b*x+2*a)/b^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int x\cosh(a+bx)\text{Shi}(a+bx)\,dx$$
$$=\frac{2bx+2a\text{Chi}(2(a+bx))-2a\log(a+bx)-\sinh(2(a+bx))+4(-\cosh(a+bx)+bx\sinh(a+bx))\text{Shi}(a}{4b^2}$$

input
```
Integrate[x*Cosh[a + b*x]*SinhIntegral[a + b*x],x]
```

output
```
(2*b*x + 2*a*CoshIntegral[2*(a + b*x)] - 2*a*Log[a + b*x] - Sinh[2*(a + b*
x)] + 4*(-Cosh[a + b*x] + b*x*Sinh[a + b*x])*SinhIntegral[a + b*x] + 2*Sin
hIntegral[2*(a + b*x)])/(4*b^2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {7102, 7094, 5971, 27, 3042, 26, 3779, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Shi}(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{ 7102}$$

$$-\frac{\int \sinh(a+bx)\text{Shi}(a+bx)dx}{b}-\int \frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{ 7094}$$

$$-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}dx}{b}-\int \frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{ 5971}$$

$$-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\int\frac{\sinh(2a+2bx)}{2(a+bx)}dx}{b}-\int\frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{1}{2}\int\frac{\sinh(2a+2bx)}{a+bx}dx}{b}-\int\frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{1}{2}\int-\frac{i\sin(2ia+2ibx)}{a+bx}dx}{b}-\int\frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}+\frac{1}{2}i\int\frac{\sin(2ia+2ibx)}{a+bx}dx}{b}-\int\frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3779}$$

$$-\int\frac{x\sinh^2(a+bx)}{a+bx}dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}$$

$$\Big\downarrow \text{7293}$$

$$-\int\left(\frac{\sinh^2(a+bx)}{b}-\frac{a\sinh^2(a+bx)}{b(a+bx)}\right)dx+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\text{Chi}(2a+2bx)}{2b^2}-\frac{a\log(a+bx)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b^2}+\frac{x\text{Shi}(a+bx)\sinh(a+bx)}{b}-\frac{\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}+\frac{x}{2b}$$

input
```
Int[x*Cosh[a + b*x]*SinhIntegral[a + b*x],x]
```

output
```
x/(2*b) + (a*CoshIntegral[2*a + 2*b*x])/(2*b^2) - (a*Log[a + b*x])/(2*b^2)
 - (Cosh[a + b*x]*Sinh[a + b*x])/(2*b^2) + (x*Sinh[a + b*x]*SinhIntegral[a
 + b*x])/b - ((Cosh[a + b*x]*SinhIntegral[a + b*x])/b - SinhIntegral[2*a +
 2*b*x]/(2*b))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7094
```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7102
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\mathrm{Shi}(bx+a)((bx+a)\sinh(bx+a)-\cosh(bx+a)-a\sinh(bx+a))-\frac{\sinh(bx+a)\cosh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}+\frac{\mathrm{Shi}(2bx+2a)}{2}+a\left(-\frac{\ln(bx+}{2}\right.}{b^2}$
default	$\frac{\mathrm{Shi}(bx+a)((bx+a)\sinh(bx+a)-\cosh(bx+a)-a\sinh(bx+a))-\frac{\sinh(bx+a)\cosh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}+\frac{\mathrm{Shi}(2bx+2a)}{2}+a\left(-\frac{\ln(bx+}{2}\right.}{b^2}$

input
```
int(x*cosh(b*x+a)*Shi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(Shi(b*x+a)*((b*x+a)*sinh(b*x+a)-cosh(b*x+a)-a*sinh(b*x+a))-1/2*sinh
(b*x+a)*cosh(b*x+a)+1/2*b*x+1/2*a+1/2*Shi(2*b*x+2*a)+a*(-1/2*ln(b*x+a)+1/2
*Chi(2*b*x+2*a)))
```

Fricas [F]

$$\int x\cosh(a+bx)\mathrm{Shi}(a+bx)\,dx=\int x\mathrm{Shi}(bx+a)\cosh(bx+a)\,dx$$

input
```
integrate(x*cosh(b*x+a)*Shi(b*x+a),x, algorithm="fricas")
```

output
```
integral(x*cosh(b*x + a)*sinh_integral(b*x + a), x)
```

Sympy [F]

$$\int x\cosh(a+bx)\text{Shi}(a+bx)\,dx=\int x\cosh\left(a+bx\right)\text{Shi}\left(a+bx\right)dx$$

input
```
integrate(x*cosh(b*x+a)*Shi(b*x+a),x)
```

output
```
Integral(x*cosh(a + b*x)*Shi(a + b*x), x)
```

Maxima [F]

$$\int x\cosh(a+bx)\text{Shi}(a+bx)\,dx=\int x\text{Shi}(bx+a)\cosh\left(bx+a\right)\,dx$$

input
```
integrate(x*cosh(b*x+a)*Shi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*Shi(b*x + a)*cosh(b*x + a), x)
```

Giac [F]

$$\int x\cosh(a+bx)\text{Shi}(a+bx)\,dx=\int x\text{Shi}(bx+a)\cosh\left(bx+a\right)\,dx$$

input
```
integrate(x*cosh(b*x+a)*Shi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*Shi(b*x + a)*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\mathrm{Shi}(a+bx)\,dx = \int x\,\mathrm{sinhint}(a+b\,x)\,\cosh(a+b\,x)\,dx$$

input
```
int(x*sinhint(a + b*x)*cosh(a + b*x),x)
```

output
```
int(x*sinhint(a + b*x)*cosh(a + b*x), x)
```

Reduce [F]

$$\int x\cosh(a+bx)\mathrm{Shi}(a+bx)\,dx = \int \cosh(bx+a)\,shi(bx+a)\,xdx$$

input
```
int(x*cosh(b*x+a)*Shi(b*x+a),x)
```

output
```
int(cosh(a + b*x)*shi(a + b*x)*x,x)
```

3.61 $\int \cosh(a+bx)\mathbf{Shi}(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \cosh(a+bx)\text{Shi}(a+bx)\,dx=-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\log(a+bx)}{2b}+\frac{\sinh(a+bx)\text{Shi}(a+bx)}{b}$$

output
```
-1/2*Chi(2*b*x+2*a)/b+1/2*ln(b*x+a)/b+sinh(b*x+a)*Shi(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \cosh(a+bx)\text{Shi}(a+bx)\,dx=-\frac{\text{Chi}(2(a+bx))}{2b}+\frac{\log(a+bx)}{2b}+\frac{\sinh(a+bx)\text{Shi}(a+bx)}{b}$$

input
```
Integrate[Cosh[a + b*x]*SinhIntegral[a + b*x],x]
```

output
```
-1/2*CoshIntegral[2*(a + b*x)]/b + Log[a + b*x]/(2*b) + (Sinh[a + b*x]*Sin
hIntegral[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {7100, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Shi}(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{7100}$$

$$\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}-\int \frac{\sinh^2(a+bx)}{a+bx}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}-\int -\frac{\sin(ia+ibx)^2}{a+bx}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\int \frac{\sin(ia+ibx)^2}{a+bx}dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{1}{2(a+bx)}-\frac{\cosh(2a+2bx)}{2(a+bx)}\right)dx+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}$$

input
```
Int[Cosh[a + b*x]*SinhIntegral[a + b*x],x]
```

output
```
-1/2*CoshIntegral[2*a + 2*b*x]/b + Log[a + b*x]/(2*b) + (Sinh[a + b*x]*Sin
hIntegral[a + b*x])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7100
```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\sinh(bx+a)\,\mathrm{Shi}(bx+a)+\frac{\ln(bx+a)}{2}-\frac{\mathrm{Chi}(2bx+2a)}{2}}{b}$	38
default	$\frac{\sinh(bx+a)\,\mathrm{Shi}(bx+a)+\frac{\ln(bx+a)}{2}-\frac{\mathrm{Chi}(2bx+2a)}{2}}{b}$	38

input
```
int(cosh(b*x+a)*Shi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(sinh(b*x+a)*Shi(b*x+a)+1/2*ln(b*x+a)-1/2*Chi(2*b*x+2*a))
```

Fricas [F]

$$\int \cosh(a + bx)\text{Shi}(a + bx)\, dx = \int \text{Shi}(bx + a)\cosh(bx + a)\, dx$$

input
```
integrate(cosh(b*x+a)*Shi(b*x+a),x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*sinh_integral(b*x + a), x)
```

Sympy [F]

$$\int \cosh(a + bx)\text{Shi}(a + bx)\, dx = \int \cosh(a + bx)\,\text{Shi}(a + bx)\, dx$$

input
```
integrate(cosh(b*x+a)*Shi(b*x+a),x)
```

output
```
Integral(cosh(a + b*x)*Shi(a + b*x), x)
```

Maxima [F]

$$\int \cosh(a + bx)\text{Shi}(a + bx)\, dx = \int \text{Shi}(bx + a)\cosh(bx + a)\, dx$$

input
```
integrate(cosh(b*x+a)*Shi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(Shi(b*x + a)*cosh(b*x + a), x)
```

Giac [F]

$$\int \cosh(a+bx)\text{Shi}(a+bx)\,dx = \int \text{Shi}(bx+a)\cosh(bx+a)\,dx$$

input
```
integrate(cosh(b*x+a)*Shi(b*x+a),x, algorithm="giac")
```

output
```
integrate(Shi(b*x + a)*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\text{Shi}(a+bx)\,dx = \int \text{sinhint}(a+b\,x)\,\cosh(a+b\,x)\,dx$$

input
```
int(sinhint(a + b*x)*cosh(a + b*x),x)
```

output
```
int(sinhint(a + b*x)*cosh(a + b*x), x)
```

Reduce [F]

$$\int \cosh(a+bx)\text{Shi}(a+bx)\,dx = \int \cosh(bx+a)\,shi(bx+a)\,dx$$

input
```
int(cosh(b*x+a)*Shi(b*x+a),x)
```

output
```
int(cosh(a + b*x)*shi(a + b*x),x)
```

3.62 $\int \frac{\cosh(a+bx)\mathbf{Shi}(a+bx)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cosh(a+bx)\text{Shi}(a+bx)}{x}\,dx=\text{Int}\left(\frac{\cosh(a+bx)\text{Shi}(a+bx)}{x},x\right)$$

output
```
Defer(Int)(cosh(b*x+a)*Shi(b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Shi}(a+bx)}{x}\,dx=\int \frac{\cosh(a+bx)\text{Shi}(a+bx)}{x}\,dx$$

input
```
Integrate[(Cosh[a + b*x]*SinhIntegral[a + b*x])/x,x]
```

output
```
Integrate[(Cosh[a + b*x]*SinhIntegral[a + b*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Shi}(a+bx)\cosh(a+bx)}{x}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\text{Shi}(a+bx)\cosh(a+bx)}{x}dx$$

input
```
Int[(Cosh[a + b*x]*SinhIntegral[a + b*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(bx+a\right)\text{Shi}\left(bx+a\right)}{x}dx$$

input
```
int(cosh(b*x+a)*Shi(b*x+a)/x,x)
```

output
```
int(cosh(b*x+a)*Shi(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\mathrm{Shi}(a+bx)}{x}\,dx = \int \frac{\mathrm{Shi}(bx+a)\cosh(bx+a)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*Shi(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*sinh_integral(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\cosh(a+bx)\mathrm{Shi}(a+bx)}{x}\,dx = \int \frac{\cosh(a+bx)\,\mathrm{Shi}(a+bx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*Shi(b*x+a)/x,x)
```

output
```
Integral(cosh(a + b*x)*Shi(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\mathrm{Shi}(a+bx)}{x}\,dx = \int \frac{\mathrm{Shi}(bx+a)\cosh(bx+a)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*Shi(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(Shi(b*x + a)*cosh(b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Shi}(a+bx)}{x}\,dx = \int \frac{\text{Shi}(bx+a)\cosh\left(bx+a\right)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*Shi(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(Shi(b*x + a)*cosh(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Shi}(a+bx)}{x}\,dx = \int \frac{\text{sinhint}(a+b\,x)\,\cosh(a+b\,x)}{x}\,dx$$

input
```
int((sinhint(a + b*x)*cosh(a + b*x))/x,x)
```

output
```
int((sinhint(a + b*x)*cosh(a + b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Shi}(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)\,shi(bx+a)}{x}dx$$

input
```
int(cosh(b*x+a)*Shi(b*x+a)/x,x)
```

output
```
int((cosh(a + b*x)*shi(a + b*x))/x,x)
```

3.63 $\int x \sinh(a+bx)\mathbf{Shi}(c+dx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 371

$$\int x\sinh(a+bx)\text{Shi}(c+dx)\,dx = \frac{\cosh(a-c+(b-d)x)}{2b(b-d)} - \frac{\cosh(a+c+(b+d)x)}{2b(b+d)} - \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2b^2} + \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2b^2} - \frac{c\text{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2bd} + \frac{c\text{Chi}\left(\frac{c(b+d)}{d}+(b+d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2bd} - \frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2bd} - \frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2b^2} + \frac{x\cosh(a+bx)\text{Shi}(c+dx)}{b} - \frac{\sinh(a+bx)\text{Shi}(c+dx)}{b^2} + \frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2bd} + \frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2b^2}$$

output
```
1/2*cosh(a-c+(b-d)*x)/b/(b-d)-1/2*cosh(a+c+(b+d)*x)/b/(b+d)-1/2*cosh(a-b*c
/d)*Chi(c*(b-d)/d+(b-d)*x)/b^2+1/2*cosh(a-b*c/d)*Chi(c*(b+d)/d+(b+d)*x)/b^
2-1/2*c*Chi(c*(b-d)/d+(b-d)*x)*sinh(a-b*c/d)/b/d+1/2*c*Chi(c*(b+d)/d+(b+d)
*x)*sinh(a-b*c/d)/b/d-1/2*c*cosh(a-b*c/d)*Shi(c*(b-d)/d+(b-d)*x)/b/d-1/2*s
inh(a-b*c/d)*Shi(c*(b-d)/d+(b-d)*x)/b^2+x*cosh(b*x+a)*Shi(d*x+c)/b-sinh(b*
x+a)*Shi(d*x+c)/b^2+1/2*c*cosh(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b/d+1/2*sin
h(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b^2
```

Mathematica [A] (verified)

Time = 2.73 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.79

$$\int x\sinh(a+bx)\text{Shi}(c+dx)\,dx$$
$$=\frac{-\frac{(bc+d)e^{a-\frac{bc}{d}}\text{ExpIntegralEi}\left(\frac{(b-d)(c+dx)}{d}\right)}{d}+\frac{e^{-a-c-(b+d)x}\left(bd\left(b\left(-1+e^{2(a+bx)}\right)+d\left(1+e^{2(a+bx)}\right)\right)-(bc-d)\left(b^2-d^2\right)e^{\frac{(b+d)(c+dx)}{d}}\text{ExpInt}\right.}{(b-d)d(b+d)}}{}$$

input
```
Integrate[x*Sinh[a + b*x]*SinhIntegral[c + d*x],x]
```

output
```
(-(((b*c + d)*E^(a - (b*c)/d)*ExpIntegralEi[((b - d)*(c + d*x))/d])/d) + (
E^(-a - c - (b + d)*x)*(b*d*(b*(-1 + E^(2*(a + b*x))) + d*(1 + E^(2*(a + b
*x)))) - (b*c - d)*(b^2 - d^2)*E^(((b + d)*(c + d*x))/d)*ExpIntegralEi[-((
(b + d)*(c + d*x))/d)]))/((b - d)*d*(b + d)) - (b*d*E^c*(E^((-b + d)*x)/(-
b + d) + E^(2*a + (b + d)*x)/(b + d)) + (-(b*c) + d)*E^((b*c)/d)*ExpIntegr
alEi[-(((b - d)*(c + d*x))/d)] - (b*c + d)*E^(2*a - (b*c)/d)*ExpIntegralEi
[((b + d)*(c + d*x))/d])/(d*E^a) + 4*(b*x*Cosh[a + b*x] - Sinh[a + b*x])*S
inhIntegral[c + d*x])/(4*b^2)
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {7096, 7100, 5993, 2009, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sinh(a+bx)\text{Shi}(c+dx)\,dx$$

$$\Big\downarrow \text{7096}$$

$$-\frac{\int \cosh(a+bx)\text{Shi}(c+dx)dx}{b}-\frac{d\int \frac{x\cosh(a+bx)\sinh(c+dx)}{c+dx}dx}{b}+\frac{x\cosh(a+bx)\text{Shi}(c+dx)}{b}$$

$$\Big\downarrow \text{7100}$$

$$-\frac{\frac{\sinh(a+bx)\text{Shi}(c+dx)}{b}-\frac{d\int \frac{\sinh(a+bx)\sinh(c+dx)}{c+dx}dx}{b}}{b}-\frac{d\int \frac{x\cosh(a+bx)\sinh(c+dx)}{c+dx}dx}{b}+\frac{x\cosh(a+bx)\text{Shi}(c+dx)}{b}$$

$\downarrow$ 5993

$$-\frac{\frac{\sinh(a+bx)\text{Shi}(c+dx)}{b}-\frac{d\int \left(\frac{\cosh(a+c+(b+d)x)}{2(c+dx)}-\frac{\cosh(a-c+(b-d)x)}{2(c+dx)}\right)dx}{b}}{b}-\frac{d\int \frac{x\cosh(a+bx)\sinh(c+dx)}{c+dx}dx}{b}+\frac{x\cosh(a+bx)\text{Shi}(c+dx)}{b}$$

$\downarrow$ 2009

$$-\frac{d\int \frac{x\cosh(a+bx)\sinh(c+dx)}{c+dx}dx}{b}-\frac{\frac{\sinh(a+bx)\text{Shi}(c+dx)}{b}-\frac{d\left(-\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d}-\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\sinh}{}\right.}{b}}{b}$$
$$\frac{x\cosh(a+bx)\text{Shi}(c+dx)}{b}$$

$\downarrow$ 7293

$$-\frac{d\int \left(\frac{\cosh(a+bx)\sinh(c+dx)}{d}-\frac{c\cosh(a+bx)\sinh(c+dx)}{d(c+dx)}\right)dx}{b}-\frac{\frac{\sinh(a+bx)\text{Shi}(c+dx)}{b}-\frac{d\left(-\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d}-\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\sinh}{}\right.}{b}}{b}$$
$$\frac{x\cosh(a+bx)\text{Shi}(c+dx)}{b}$$

$\downarrow$ 2009

$$-\frac{d\left(\frac{c\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d^2}+\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\cosh\left(a-\frac{bc}{d}\right.}{}\right.}{b}$$
$$\frac{\frac{\sinh(a+bx)\text{Shi}(c+dx)}{b}-\frac{d\left(-\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d}-\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\sinh}{}\right.}{b}}{b}$$
$$\frac{x\cosh(a+bx)\text{Shi}(c+dx)}{b}$$

input
```
Int[x*Sinh[a + b*x]*SinhIntegral[c + d*x],x]
```

output
```
(x*Cosh[a + b*x]*SinhIntegral[c + d*x])/b - (d*(-1/2*Cosh[a - c + (b - d)*
x]/((b - d)*d) + Cosh[a + c + (b + d)*x]/(2*d*(b + d)) + (c*CoshIntegral[(
c*(b - d))/d + (b - d)*x]*Sinh[a - (b*c)/d])/(2*d^2) - (c*CoshIntegral[(c*
(b + d))/d + (b + d)*x]*Sinh[a - (b*c)/d])/(2*d^2) + (c*Cosh[a - (b*c)/d]*
SinhIntegral[(c*(b - d))/d + (b - d)*x])/(2*d^2) - (c*Cosh[a - (b*c)/d]*Si
nhIntegral[(c*(b + d))/d + (b + d)*x])/(2*d^2)))/b - ((Sinh[a + b*x]*SinhI
ntegral[c + d*x])/b - (d*(-1/2*(Cosh[a - (b*c)/d]*CoshIntegral[(c*(b - d))
/d + (b - d)*x])/d + (Cosh[a - (b*c)/d]*CoshIntegral[(c*(b + d))/d + (b +
d)*x])/(2*d) - (Sinh[a - (b*c)/d]*SinhIntegral[(c*(b - d))/d + (b - d)*x])
/(2*d) + (Sinh[a - (b*c)/d]*SinhIntegral[(c*(b + d))/d + (b + d)*x])/(2*d)
))/b)/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5993
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(p_.)*Sinh[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Sinh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0
] && IGtQ[q, 0] && IntegerQ[m]
```

rule 7096
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7100
```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int x \sinh(bx + a) \operatorname{Shi}(dx + c) \, dx$$

input
```
int(x*sinh(b*x+a)*Shi(d*x+c),x)
```

output
```
int(x*sinh(b*x+a)*Shi(d*x+c),x)
```

Fricas [F]

$$\int x \sinh(a + bx)\operatorname{Shi}(c + dx) \, dx = \int x\operatorname{Shi}(dx + c) \sinh(bx + a) \, dx$$

input
```
integrate(x*sinh(b*x+a)*Shi(d*x+c),x, algorithm="fricas")
```

output
```
integral(x*sinh(b*x + a)*sinh_integral(d*x + c), x)
```

Sympy [F]

$$\int x \sinh(a + bx)\operatorname{Shi}(c + dx) \, dx = \int x \sinh(a + bx) \operatorname{Shi}(c + dx) \, dx$$

input
```
integrate(x*sinh(b*x+a)*Shi(d*x+c),x)
```

output
```
Integral(x*sinh(a + b*x)*Shi(c + d*x), x)
```

Maxima [F]

$$\int x\sinh(a+bx)\text{Shi}(c+dx)\,dx = \int x\text{Shi}(dx+c)\sinh(bx+a)\,dx$$

input
```
integrate(x*sinh(b*x+a)*Shi(d*x+c),x, algorithm="maxima")
```

output
```
integrate(x*Shi(d*x + c)*sinh(b*x + a), x)
```

Giac [F]

$$\int x\sinh(a+bx)\text{Shi}(c+dx)\,dx = \int x\text{Shi}(dx+c)\sinh(bx+a)\,dx$$

input
```
integrate(x*sinh(b*x+a)*Shi(d*x+c),x, algorithm="giac")
```

output
```
integrate(x*Shi(d*x + c)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sinh(a+bx)\text{Shi}(c+dx)\,dx = \int x\,\text{sinhint}(c+d\,x)\,\sinh(a+b\,x)\,dx$$

input
```
int(x*sinhint(c + d*x)*sinh(a + b*x),x)
```

output
```
int(x*sinhint(c + d*x)*sinh(a + b*x), x)
```

Reduce [F]

$$\int x\sinh(a+bx)\text{Shi}(c+dx)\,dx=\int shi(dx+c)\sinh(bx+a)\,xdx$$

input
```
int(x*sinh(b*x+a)*Shi(d*x+c),x)
```

output
```
int(shi(c + d*x)*sinh(a + b*x)*x,x)
```

3.64 $\int \sinh(a+bx)\mathbf{Shi}(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 153

$$\int \sinh(a+bx)\text{Shi}(c+dx)\,dx = \frac{\text{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2b} - \frac{\text{Chi}\left(\frac{c(b+d)}{d}+(b+d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2b} + \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2b} + \frac{\cosh(a+bx)\text{Shi}(c+dx)}{b} - \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2b}$$

output
```
1/2*Chi(c*(b-d)/d+(b-d)*x)*sinh(a-b*c/d)/b-1/2*Chi(c*(b+d)/d+(b+d)*x)*sinh
(a-b*c/d)/b+1/2*cosh(a-b*c/d)*Shi(c*(b-d)/d+(b-d)*x)/b+cosh(b*x+a)*Shi(d*x
+c)/b-1/2*cosh(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\int \sinh(a+bx)\text{Shi}(c+dx)\,dx$$
$$= \frac{e^{-a-\frac{bc}{d}}\left(-e^{\frac{2bc}{d}}\text{ExpIntegralEi}\left(-\frac{(b-d)(c+dx)}{d}\right)+e^{2a}\text{ExpIntegralEi}\left(\frac{(b-d)(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\text{ExpIntegralEi}\left(-\left(\right.\right.\right.}{4b}$$

input
```
Integrate[Sinh[a + b*x]*SinhIntegral[c + d*x],x]
```

output
```
(E^(-a - (b*c)/d)*(-(E^((2*b*c)/d)*ExpIntegralEi[-(((b - d)*(c + d*x))/d)]
) + E^(2*a)*ExpIntegralEi[((b - d)*(c + d*x))/d] + E^((2*b*c)/d)*ExpIntegr
alEi[-(((b + d)*(c + d*x))/d)] - E^(2*a)*ExpIntegralEi[((b + d)*(c + d*x))
/d] + 4*E^(a + (b*c)/d)*Cosh[a + b*x]*SinhIntegral[c + d*x]))/(4*b)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7094, 5995, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\text{Shi}(c+dx)\,dx$$
$$\Big\downarrow \text{ 7094}$$
$$\frac{\cosh(a+bx)\text{Shi}(c+dx)}{b}-\frac{d\int \frac{\cosh(a+bx)\sinh(c+dx)}{c+dx}dx}{b}$$
$$\Big\downarrow \text{ 5995}$$
$$\frac{\cosh(a+bx)\text{Shi}(c+dx)}{b}-\frac{d\int \left(\frac{\sinh(a+c+(b+d)x)}{2(c+dx)}-\frac{\sinh(a-c+(b-d)x)}{2(c+dx)}\right)dx}{b}$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{\cosh(a+bx)\text{Shi}(c+dx)}{b} - \frac{d\left(-\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d}-\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\right.}{b}$$

input
```
Int[Sinh[a + b*x]*SinhIntegral[c + d*x],x]
```

output
```
(Cosh[a + b*x]*SinhIntegral[c + d*x])/b - (d*(-1/2*(CoshIntegral[(c*(b - d
))/d + (b - d)*x]*Sinh[a - (b*c)/d])/d + (CoshIntegral[(c*(b + d))/d + (b
+ d)*x]*Sinh[a - (b*c)/d])/(2*d) - (Cosh[a - (b*c)/d]*SinhIntegral[(c*(b -
 d))/d + (b - d)*x])/(2*d) + (Cosh[a - (b*c)/d]*SinhIntegral[(c*(b + d))/d
 + (b + d)*x])/(2*d)))/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5995
```
Int[Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p
, 0] && IGtQ[q, 0]
```

rule 7094
```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [F]

$$\int \sinh(bx+a)\,\mathrm{Shi}(dx+c)\,dx$$

input
```
int(sinh(b*x+a)*Shi(d*x+c),x)
```

output
```
int(sinh(b*x+a)*Shi(d*x+c),x)
```

Fricas [F]

$$\int \sinh(a+bx)\mathrm{Shi}(c+dx)\,dx = \int \mathrm{Shi}(dx+c)\sinh(bx+a)\,dx$$

input
```
integrate(sinh(b*x+a)*Shi(d*x+c),x, algorithm="fricas")
```

output
```
integral(sinh(b*x + a)*sinh_integral(d*x + c), x)
```

Sympy [F]

$$\int \sinh(a+bx)\mathrm{Shi}(c+dx)\,dx = \int \sinh(a+bx)\,\mathrm{Shi}(c+dx)\,dx$$

input
```
integrate(sinh(b*x+a)*Shi(d*x+c),x)
```

output
```
Integral(sinh(a + b*x)*Shi(c + d*x), x)
```

Maxima [F]

$$\int \sinh(a+bx)\text{Shi}(c+dx)\,dx = \int \text{Shi}(dx+c)\sinh(bx+a)\,dx$$

input
```
integrate(sinh(b*x+a)*Shi(d*x+c),x, algorithm="maxima")
```

output
```
integrate(Shi(d*x + c)*sinh(b*x + a), x)
```

Giac [F]

$$\int \sinh(a+bx)\text{Shi}(c+dx)\,dx = \int \text{Shi}(dx+c)\sinh(bx+a)\,dx$$

input
```
integrate(sinh(b*x+a)*Shi(d*x+c),x, algorithm="giac")
```

output
```
integrate(Shi(d*x + c)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh(a+bx)\text{Shi}(c+dx)\,dx = \int \text{sinhint}(c+d\,x)\,\sinh(a+b\,x)\,dx$$

input
```
int(sinhint(c + d*x)*sinh(a + b*x),x)
```

output
```
int(sinhint(c + d*x)*sinh(a + b*x), x)
```

Reduce [F]

$$\int \sinh(a+bx)\text{Shi}(c+dx)\,dx = \int shi(dx+c)\sinh(bx+a)\,dx$$

input
```
int(sinh(b*x+a)*Shi(d*x+c),x)
```

output
```
int(shi(c + d*x)*sinh(a + b*x),x)
```

3.65 $\int \frac{\sinh(a+bx)\text{Shi}(c+dx)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sinh(a+bx)\text{Shi}(c+dx)}{x}\,dx = \text{Int}\left(\frac{\sinh(a+bx)\text{Shi}(c+dx)}{x},x\right)$$

output
```
Defer(Int)(sinh(b*x+a)*Shi(d*x+c)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)\text{Shi}(c+dx)}{x}\,dx = \int \frac{\sinh(a+bx)\text{Shi}(c+dx)}{x}\,dx$$

input
```
Integrate[(Sinh[a + b*x]*SinhIntegral[c + d*x])/x,x]
```

output
```
Integrate[(Sinh[a + b*x]*SinhIntegral[c + d*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\text{Shi}(c+dx)}{x} \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\sinh(a+bx)\text{Shi}(c+dx)}{x}dx$$

input
```
Int[(Sinh[a + b*x]*SinhIntegral[c + d*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(bx+a\right)\text{Shi}\left(dx+c\right)}{x}dx$$

input
```
int(sinh(b*x+a)*Shi(d*x+c)/x,x)
```

output
```
int(sinh(b*x+a)*Shi(d*x+c)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)\text{Shi}(c+dx)}{x}\,dx = \int \frac{\text{Shi}(dx+c)\sinh(bx+a)}{x}\,dx$$

input
```
integrate(sinh(b*x+a)*Shi(d*x+c)/x,x, algorithm="fricas")
```

output
```
integral(sinh(b*x + a)*sinh_integral(d*x + c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sinh(a+bx)\text{Shi}(c+dx)}{x}\,dx = \int \frac{\sinh(a+bx)\,\text{Shi}(c+dx)}{x}\,dx$$

input
```
integrate(sinh(b*x+a)*Shi(d*x+c)/x,x)
```

output
```
Integral(sinh(a + b*x)*Shi(c + d*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)\text{Shi}(c+dx)}{x}\,dx = \int \frac{\text{Shi}(dx+c)\sinh(bx+a)}{x}\,dx$$

input
```
integrate(sinh(b*x+a)*Shi(d*x+c)/x,x, algorithm="maxima")
```

output
```
integrate(Shi(d*x + c)*sinh(b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)\text{Shi}(c+dx)}{x}\,dx = \int \frac{\text{Shi}(dx+c)\sinh(bx+a)}{x}\,dx$$

input
```
integrate(sinh(b*x+a)*Shi(d*x+c)/x,x, algorithm="giac")
```

output
```
integrate(Shi(d*x + c)*sinh(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)\text{Shi}(c+dx)}{x}\,dx = \int \frac{\text{sinhint}(c+d\,x)\,\sinh(a+b\,x)}{x}\,dx$$

input
```
int((sinhint(c + d*x)*sinh(a + b*x))/x,x)
```

output
```
int((sinhint(c + d*x)*sinh(a + b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)\text{Shi}(c+dx)}{x} \, dx = \int \frac{shi(dx+c)\sinh{(bx+a)}}{x} dx$$

input
```
int(sinh(b*x+a)*Shi(d*x+c)/x,x)
```

output
```
int((shi(c + d*x)*sinh(a + b*x))/x,x)
```

3.66 $\int x \cosh(a+bx)\mathbf{Shi}(c+dx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 371

$$\int x\cosh(a+bx)\text{Shi}(c+dx)\,dx = -\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2bd} + \frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2bd} - \frac{\text{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2b^2} + \frac{\text{Chi}\left(\frac{c(b+d)}{d}+(b+d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2b^2} + \frac{\sinh(a-c+(b-d)x)}{2b(b-d)} - \frac{\sinh(a+c+(b+d)x)}{2b(b+d)} - \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2b^2} - \frac{c\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2bd} - \frac{\cosh(a+bx)\text{Shi}(c+dx)}{b^2} + \frac{x\sinh(a+bx)\text{Shi}(c+dx)}{b} + \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2b^2} + \frac{c\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2bd}$$

output

```
-1/2*c*cosh(a-b*c/d)*Chi(c*(b-d)/d+(b-d)*x)/b/d+1/2*c*cosh(a-b*c/d)*Chi(c*
(b+d)/d+(b+d)*x)/b/d-1/2*Chi(c*(b-d)/d+(b-d)*x)*sinh(a-b*c/d)/b^2+1/2*Chi(
c*(b+d)/d+(b+d)*x)*sinh(a-b*c/d)/b^2+1/2*sinh(a-c+(b-d)*x)/b/(b-d)-1/2*sin
h(a+c+(b+d)*x)/b/(b+d)-1/2*cosh(a-b*c/d)*Shi(c*(b-d)/d+(b-d)*x)/b^2-1/2*c*
sinh(a-b*c/d)*Shi(c*(b-d)/d+(b-d)*x)/b/d-cosh(b*x+a)*Shi(d*x+c)/b^2+x*sinh
(b*x+a)*Shi(d*x+c)/b+1/2*cosh(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b^2+1/2*c*si
nh(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b/d
```

Mathematica [A] (verified)

Time = 2.91 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.72

$$\int x\cosh(a+bx)\text{Shi}(c+dx)\,dx$$
$$=\frac{e^{-a}\left(bde^{-c}\left(\frac{e^{-((b+d)x)}}{b+d}+\frac{e^{2a+bx-dx}}{b-d}\right)-(bc+d)e^{2a-\frac{bc}{d}}\text{ ExpIntegralEi}\left(\frac{(b-d)(c+dx)}{d}\right)+(bc-d)e^{\frac{bc}{d}}\text{ ExpIntegralEi}\left(-\frac{(b+d)(c+dx)}{d}\right)\right)}{d}+\frac{e^{-a}\left(b\right.}{}$$

input
```
Integrate[x*Cosh[a + b*x]*SinhIntegral[c + d*x],x]
```

output
```
(((b*d*(1/((b + d)*E^((b + d)*x)) + E^(2*a + b*x - d*x)/(b - d)))/E^c - (b
*c + d)*E^(2*a - (b*c)/d)*ExpIntegralEi[((b - d)*(c + d*x))/d] + (b*c - d)
*E^((b*c)/d)*ExpIntegralEi[-(((b + d)*(c + d*x))/d)])/(d*E^a) + (b*d*E^c*(
E^((-b + d)*x)/(-b + d) - E^(2*a + (b + d)*x)/(b + d)) + (-(b*c) + d)*E^((
b*c)/d)*ExpIntegralEi[-(((b - d)*(c + d*x))/d)] + (b*c + d)*E^(2*a - (b*c)
/d)*ExpIntegralEi[((b + d)*(c + d*x))/d])/(d*E^a) + 4*(-Cosh[a + b*x] + b*
x*Sinh[a + b*x])*SinhIntegral[c + d*x])/(4*b^2)
```

Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {7102, 6176, 2009, 7094, 5995, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh(a+bx)\text{Shi}(c+dx)\,dx$$
$$\Big\downarrow \text{ 7102}$$
$$-\frac{\int\sinh(a+bx)\text{Shi}(c+dx)dx}{b}-\frac{d\int\frac{x\sinh(a+bx)\sinh(c+dx)}{c+dx}dx}{b}+\frac{x\sinh(a+bx)\text{Shi}(c+dx)}{b}$$
$$\Big\downarrow \text{ 6176}$$

$$-\frac{\int \sinh(a+bx)\text{Shi}(c+dx)dx}{b}-\frac{d\int \left(\frac{x\cosh(a+c+(b+d)x)}{2(c+dx)}-\frac{x\cosh(a-c+(b-d)x)}{2(c+dx)}\right)dx}{b}+\frac{x\sinh(a+bx)\text{Shi}(c+dx)}{b}$$

$\Big\downarrow$ 2009

$$-\frac{\int \sinh(a+bx)\text{Shi}(c+dx)dx}{b}-\frac{d\left(\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d^2}+\frac{c\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\sinh\left(a-\frac{bc}{d}\right)\text{S}\right.}{b}+\frac{x\sinh(a+bx)\text{Shi}(c+dx)}{b}$$

$\Big\downarrow$ 7094

$$-\frac{\frac{\cosh(a+bx)\text{Shi}(c+dx)}{b}-\frac{d\int \frac{\cosh(a+bx)\sinh(c+dx)}{c+dx}dx}{b}}{b}-\frac{d\left(\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d^2}+\frac{c\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\sinh\left(a-\frac{bc}{d}\right)\text{S}\right.}{b}+\frac{x\sinh(a+bx)\text{Shi}(c+dx)}{b}$$

$\Big\downarrow$ 5995

$$-\frac{\frac{\cosh(a+bx)\text{Shi}(c+dx)}{b}-\frac{d\int \left(\frac{\sinh(a+c+(b+d)x)}{2(c+dx)}-\frac{\sinh(a-c+(b-d)x)}{2(c+dx)}\right)dx}{b}}{b}-\frac{d\left(\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d^2}+\frac{c\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\sinh\left(a-\frac{bc}{d}\right)\text{S}\right.}{b}+\frac{x\sinh(a+bx)\text{Shi}(c+dx)}{b}$$

$\Big\downarrow$ 2009

$$-\frac{d\left(\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d^2}+\frac{c\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\sinh\left(a-\frac{bc}{d}\right.}{}\right.}{b}-\frac{\frac{\cosh(a+bx)\text{Shi}(c+dx)}{b}-\frac{d\left(-\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d}-\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\cosh\right.}{}\right.}{b}}{b}+\frac{x\sinh(a+bx)\text{Shi}(c+dx)}{b}$$

input
```
Int[x*Cosh[a + b*x]*SinhIntegral[c + d*x],x]
```

output
```
(x*Sinh[a + b*x]*SinhIntegral[c + d*x])/b - (d*((c*Cosh[a - (b*c)/d]*CoshI
ntegral[(c*(b - d))/d + (b - d)*x])/(2*d^2) - (c*Cosh[a - (b*c)/d]*CoshInt
egral[(c*(b + d))/d + (b + d)*x])/(2*d^2) - Sinh[a - c + (b - d)*x]/(2*(b
- d)*d) + Sinh[a + c + (b + d)*x]/(2*d*(b + d)) + (c*Sinh[a - (b*c)/d]*Sin
hIntegral[(c*(b - d))/d + (b - d)*x])/(2*d^2) - (c*Sinh[a - (b*c)/d]*SinhI
ntegral[(c*(b + d))/d + (b + d)*x])/(2*d^2)))/b - ((Cosh[a + b*x]*SinhInte
gral[c + d*x])/b - (d*(-1/2*(CoshIntegral[(c*(b - d))/d + (b - d)*x]*Sinh[
a - (b*c)/d])/d + (CoshIntegral[(c*(b + d))/d + (b + d)*x]*Sinh[a - (b*c)/
d])/(2*d) - (Cosh[a - (b*c)/d]*SinhIntegral[(c*(b - d))/d + (b - d)*x])/(2
*d) + (Cosh[a - (b*c)/d]*SinhIntegral[(c*(b + d))/d + (b + d)*x])/(2*d)))/
b)/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5995
```
Int[Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p
, 0] && IGtQ[q, 0]
```

rule 6176
```
Int[(u_.)*Sinh[(a_.) + (b_.)*(x_)]^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.), x_
Symbol] :> Int[ExpandTrigReduce[u, Sinh[a + b*x]^m*Sinh[c + d*x]^n, x], x]
/; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 7094
```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7102
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(SinhIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Sinh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Sinh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int x\cosh\left(bx+a\right)\operatorname{Shi}\left(dx+c\right)dx$$

input
```
int(x*cosh(b*x+a)*Shi(d*x+c),x)
```

output
```
int(x*cosh(b*x+a)*Shi(d*x+c),x)
```

Fricas [F]

$$\int x\cosh(a+bx)\text{Shi}(c+dx)\,dx=\int x\text{Shi}(dx+c)\cosh\left(bx+a\right)\,dx$$

input
```
integrate(x*cosh(b*x+a)*Shi(d*x+c),x, algorithm="fricas")
```

output
```
integral(x*cosh(b*x + a)*sinh_integral(d*x + c), x)
```

Sympy [F]

$$\int x\cosh(a+bx)\text{Shi}(c+dx)\,dx=\int x\cosh\left(a+bx\right)\text{Shi}\left(c+dx\right)dx$$

input
```
integrate(x*cosh(b*x+a)*Shi(d*x+c),x)
```

output
```
Integral(x*cosh(a + b*x)*Shi(c + d*x), x)
```

Maxima [F]

$$\int x\cosh(a+bx)\text{Shi}(c+dx)\,dx = \int x\text{Shi}(dx+c)\cosh(bx+a)\,dx$$

input
```
integrate(x*cosh(b*x+a)*Shi(d*x+c),x, algorithm="maxima")
```

output
```
integrate(x*Shi(d*x + c)*cosh(b*x + a), x)
```

Giac [F]

$$\int x\cosh(a+bx)\text{Shi}(c+dx)\,dx = \int x\text{Shi}(dx+c)\cosh(bx+a)\,dx$$

input
```
integrate(x*cosh(b*x+a)*Shi(d*x+c),x, algorithm="giac")
```

output
```
integrate(x*Shi(d*x + c)*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\text{Shi}(c+dx)\,dx = \int x\,\text{sinhint}(c+d\,x)\,\cosh(a+b\,x)\,dx$$

input
```
int(x*sinhint(c + d*x)*cosh(a + b*x),x)
```

output
```
int(x*sinhint(c + d*x)*cosh(a + b*x), x)
```

Reduce [F]

$$\int x \cosh(a + bx)\text{Shi}(c + dx)\,dx = \int \cosh(bx + a)\,shi(dx + c)\,xdx$$

input
```
int(x*cosh(b*x+a)*Shi(d*x+c),x)
```

output
```
int(cosh(a + b*x)*shi(c + d*x)*x,x)
```

3.67 $\int \cosh(a+bx)\textbf{Shi}(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 153

$$\int \cosh(a+bx)\mathrm{Shi}(c+dx)\,dx = \frac{\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2b} - \frac{\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2b} + \frac{\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2b} + \frac{\sinh(a+bx)\mathrm{Shi}(c+dx)}{b} - \frac{\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2b}$$

output

```
1/2*cosh(a-b*c/d)*Chi(c*(b-d)/d+(b-d)*x)/b-1/2*cosh(a-b*c/d)*Chi(c*(b+d)/d
+(b+d)*x)/b+1/2*sinh(a-b*c/d)*Shi(c*(b-d)/d+(b-d)*x)/b+sinh(b*x+a)*Shi(d*x
+c)/b-1/2*sinh(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\int \cosh(a+bx)\text{Shi}(c+dx)\,dx$$
$$= \frac{e^{-a-\frac{bc}{d}}\left(e^{\frac{2bc}{d}}\text{ExpIntegralEi}\left(-\frac{(b-d)(c+dx)}{d}\right)+e^{2a}\text{ExpIntegralEi}\left(\frac{(b-d)(c+dx)}{d}\right)-e^{\frac{2bc}{d}}\text{ExpIntegralEi}\left(-\frac{(b+\cdots}{}\right.\right.}{4b}$$

input
```
Integrate[Cosh[a + b*x]*SinhIntegral[c + d*x],x]
```

output
```
(E^(-a - (b*c)/d)*(E^((2*b*c)/d)*ExpIntegralEi[-(((b - d)*(c + d*x))/d)] +
 E^(2*a)*ExpIntegralEi[((b - d)*(c + d*x))/d] - E^((2*b*c)/d)*ExpIntegralE
i[-(((b + d)*(c + d*x))/d)] - E^(2*a)*ExpIntegralEi[((b + d)*(c + d*x))/d]
 + 4*E^(a + (b*c)/d)*Sinh[a + b*x]*SinhIntegral[c + d*x]))/(4*b)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7100, 5993, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\text{Shi}(c+dx)\,dx$$
$$\Big\downarrow \text{7100}$$
$$\frac{\sinh(a+bx)\text{Shi}(c+dx)}{b}-\frac{d\int \frac{\sinh(a+bx)\sinh(c+dx)}{c+dx}dx}{b}$$
$$\Big\downarrow \text{5993}$$
$$\frac{\sinh(a+bx)\text{Shi}(c+dx)}{b}-\frac{d\int \left(\frac{\cosh(a+c+(b+d)x)}{2(c+dx)}-\frac{\cosh(a-c+(b-d)x)}{2(c+dx)}\right)dx}{b}$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sinh(a+bx)\text{Shi}(c+dx)}{b} - \frac{d\left(-\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d}-\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}}{}\right.}{b}$$

input
```
Int[Cosh[a + b*x]*SinhIntegral[c + d*x],x]
```

output
```
(Sinh[a + b*x]*SinhIntegral[c + d*x])/b - (d*(-1/2*(Cosh[a - (b*c)/d]*Cosh
Integral[(c*(b - d))/d + (b - d)*x])/d + (Cosh[a - (b*c)/d]*CoshIntegral[(
c*(b + d))/d + (b + d)*x])/(2*d) - (Sinh[a - (b*c)/d]*SinhIntegral[(c*(b -
 d))/d + (b - d)*x])/(2*d) + (Sinh[a - (b*c)/d]*SinhIntegral[(c*(b + d))/d
 + (b + d)*x])/(2*d)))/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5993
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(p_.)*Sinh[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Sinh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0
] && IGtQ[q, 0] && IntegerQ[m]
```

rule 7100
```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(SinhIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Sinh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [F]

$$\int \cosh (bx + a) \operatorname{Shi}(dx + c) \, dx$$

input
```
int(cosh(b*x+a)*Shi(d*x+c),x)
```

output
```
int(cosh(b*x+a)*Shi(d*x+c),x)
```

Fricas [F]

$$\int \cosh(a + bx)\operatorname{Shi}(c + dx) \, dx = \int \operatorname{Shi}(dx + c) \cosh (bx + a) \, dx$$

input
```
integrate(cosh(b*x+a)*Shi(d*x+c),x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*sinh_integral(d*x + c), x)
```

Sympy [F]

$$\int \cosh(a + bx)\operatorname{Shi}(c + dx) \, dx = \int \cosh (a + bx) \operatorname{Shi}(c + dx) \, dx$$

input
```
integrate(cosh(b*x+a)*Shi(d*x+c),x)
```

output
```
Integral(cosh(a + b*x)*Shi(c + d*x), x)
```

Maxima [F]

$$\int \cosh(a+bx)\mathrm{Shi}(c+dx)\,dx = \int \mathrm{Shi}(dx+c)\cosh\left(bx+a\right)\,dx$$

input
```
integrate(cosh(b*x+a)*Shi(d*x+c),x, algorithm="maxima")
```

output
```
integrate(Shi(d*x + c)*cosh(b*x + a), x)
```

Giac [F]

$$\int \cosh(a+bx)\mathrm{Shi}(c+dx)\,dx = \int \mathrm{Shi}(dx+c)\cosh\left(bx+a\right)\,dx$$

input
```
integrate(cosh(b*x+a)*Shi(d*x+c),x, algorithm="giac")
```

output
```
integrate(Shi(d*x + c)*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\mathrm{Shi}(c+dx)\,dx = \int \mathrm{sinhint}(c+d\,x)\,\cosh(a+b\,x)\,dx$$

input
```
int(sinhint(c + d*x)*cosh(a + b*x),x)
```

output
```
int(sinhint(c + d*x)*cosh(a + b*x), x)
```

Reduce [F]

$$\int \cosh(a+bx)\text{Shi}(c+dx)\,dx = \int \cosh(bx+a)\,shi(dx+c)\,dx$$

input
```
int(cosh(b*x+a)*Shi(d*x+c),x)
```

output
```
int(cosh(a + b*x)*shi(c + d*x),x)
```

3.68 $\int \frac{\cosh(a+bx)\mathbf{Shi}(c+dx)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cosh(a+bx)\text{Shi}(c+dx)}{x}\,dx = \text{Int}\left(\frac{\cosh(a+bx)\text{Shi}(c+dx)}{x},x\right)$$

output
```
Defer(Int)(cosh(b*x+a)*Shi(d*x+c)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Shi}(c+dx)}{x}\,dx = \int \frac{\cosh(a+bx)\text{Shi}(c+dx)}{x}\,dx$$

input
```
Integrate[(Cosh[a + b*x]*SinhIntegral[c + d*x])/x,x]
```

output
```
Integrate[(Cosh[a + b*x]*SinhIntegral[c + d*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)\text{Shi}(c+dx)}{x}dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\cosh(a+bx)\text{Shi}(c+dx)}{x}dx$$

input
```
Int[(Cosh[a + b*x]*SinhIntegral[c + d*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(bx+a)\,\text{Shi}(dx+c)}{x}dx$$

input
```
int(cosh(b*x+a)*Shi(d*x+c)/x,x)
```

output
```
int(cosh(b*x+a)*Shi(d*x+c)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Shi}(c+dx)}{x}\,dx = \int \frac{\text{Shi}(dx+c)\cosh(bx+a)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*Shi(d*x+c)/x,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*sinh_integral(d*x + c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\cosh(a+bx)\text{Shi}(c+dx)}{x}\,dx = \int \frac{\cosh(a+bx)\,\text{Shi}(c+dx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*Shi(d*x+c)/x,x)
```

output
```
Integral(cosh(a + b*x)*Shi(c + d*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Shi}(c+dx)}{x}\,dx = \int \frac{\text{Shi}(dx+c)\cosh(bx+a)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*Shi(d*x+c)/x,x, algorithm="maxima")
```

output
```
integrate(Shi(d*x + c)*cosh(b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Shi}(c+dx)}{x}\,dx = \int \frac{\text{Shi}(dx+c)\cosh\left(bx+a\right)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*Shi(d*x+c)/x,x, algorithm="giac")
```

output
```
integrate(Shi(d*x + c)*cosh(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Shi}(c+dx)}{x}\,dx = \int \frac{\text{sinhint}(c+d\,x)\,\cosh(a+b\,x)}{x}\,dx$$

input
```
int((sinhint(c + d*x)*cosh(a + b*x))/x,x)
```

output
```
int((sinhint(c + d*x)*cosh(a + b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Shi}(c+dx)}{x}\,dx=\int \frac{\cosh\left(bx+a\right)shi(dx+c)}{x}dx$$

input
```
int(cosh(b*x+a)*Shi(d*x+c)/x,x)
```

output
```
int((cosh(a + b*x)*shi(c + d*x))/x,x)
```

3.69 $\int x^m \mathbf{Chi}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 76

$$\int x^m \mathrm{Chi}(bx)\,dx = \frac{x^{1+m}\mathrm{Chi}(bx)}{1+m} - \frac{x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b(1+m)} + \frac{x^m(bx)^{-m}\Gamma(1+m,bx)}{2b(1+m)}$$

output
```
x^(1+m)*Chi(b*x)/(1+m)-1/2*x^m*GAMMA(1+m,-b*x)/b/(1+m)/((-b*x)^m)+1/2*x^m*
GAMMA(1+m,b*x)/b/(1+m)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int x^m \mathrm{Chi}(bx)\,dx = \frac{x^m\left(2x\mathrm{Chi}(bx) + \frac{(-b^2x^2)^{-m}(-(bx)^m\Gamma(1+m,-bx)+(-bx)^m\Gamma(1+m,bx))}{b}\right)}{2(1+m)}$$

input
```
Integrate[x^m*CoshIntegral[b*x],x]
```

output
```
(x^m*(2*x*CoshIntegral[b*x] + (-((b*x)^m*Gamma[1 + m, -(b*x)]) + (-(b*x))^
m*Gamma[1 + m, b*x])/(b*(-(b^2*x^2))^m)))/(2*(1 + m))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {7087, 27, 3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{Chi}(bx)\,dx$$

$$\Big\downarrow \text{ 7087}$$

$$\frac{x^{m+1}\text{Chi}(bx)}{m+1}-\frac{b\int \frac{x^m\cosh(bx)}{b}dx}{m+1}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^{m+1}\text{Chi}(bx)}{m+1}-\frac{\int x^m\cosh(bx)dx}{m+1}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x^{m+1}\text{Chi}(bx)}{m+1}-\frac{\int x^m\sin\left(ibx+\frac{\pi}{2}\right)dx}{m+1}$$

$$\Big\downarrow \text{ 3788}$$

$$\frac{x^{m+1}\text{Chi}(bx)}{m+1}-\frac{\frac{1}{2}i\int -ie^{bx}x^mdx-\frac{1}{2}i\int ie^{-bx}x^mdx}{m+1}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{x^{m+1}\text{Chi}(bx)}{m+1}-\frac{\frac{1}{2}\int e^{-bx}x^mdx+\frac{1}{2}\int e^{bx}x^mdx}{m+1}$$

$$\Big\downarrow \text{ 2612}$$

$$\frac{x^{m+1}\text{Chi}(bx)}{m+1}-\frac{\frac{x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b}-\frac{x^m(bx)^{-m}\Gamma(m+1,bx)}{2b}}{m+1}$$

input
```
Int[x^m*CoshIntegral[b*x],x]
```

output
```
(x^(1 + m)*CoshIntegral[b*x])/(1 + m) - ((x^m*Gamma[1 + m, -(b*x)])/(2*b*(
-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(2*b*(b*x)^m))/(1 + m)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 7087
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(CoshIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Cosh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Maple [F]

$$\int x^m \operatorname{Chi}(bx)\,dx$$

input
```
int(x^m*Chi(b*x),x)
```

output
```
int(x^m*Chi(b*x),x)
```

Fricas [F]

$$\int x^m \operatorname{Chi}(bx)\,dx = \int x^m \operatorname{Chi}(bx)\ dx$$

input
```
integrate(x^m*Chi(b*x),x, algorithm="fricas")
```

output
```
integral(x^m*cosh_integral(b*x), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. 2(60) = 120.

Time = 1.53 (sec) , antiderivative size = 695, normalized size of antiderivative = 9.14

$$\int x^m \operatorname{Chi}(bx)\,dx = \text{Too large to display}$$

input
```
integrate(x**m*Chi(b*x),x)
```

output
```
4*2**m*b*b**(-m - 1)*m*x*sqrt(exp(-2*m*log(2))*exp(m*log(b**2*x**2)))*log(
b**2*x**2)*gamma(m/2 + 5/2)/(8*m**2*gamma(m/2 + 5/2) + 16*m*gamma(m/2 + 5/
2) + 8*gamma(m/2 + 5/2)) + 8*2**m*EulerGamma*b*b**(-m - 1)*m*x*sqrt(exp(-2
*m*log(2))*exp(m*log(b**2*x**2)))*gamma(m/2 + 5/2)/(8*m**2*gamma(m/2 + 5/2
) + 16*m*gamma(m/2 + 5/2) + 8*gamma(m/2 + 5/2)) + 4*2**m*b*b**(-m - 1)*x*s
qrt(exp(-2*m*log(2))*exp(m*log(b**2*x**2)))*log(b**2*x**2)*gamma(m/2 + 5/2
)/(8*m**2*gamma(m/2 + 5/2) + 16*m*gamma(m/2 + 5/2) + 8*gamma(m/2 + 5/2)) -
 8*2**m*b*b**(-m - 1)*x*sqrt(exp(-2*m*log(2))*exp(m*log(b**2*x**2)))*gamma
(m/2 + 5/2)/(8*m**2*gamma(m/2 + 5/2) + 16*m*gamma(m/2 + 5/2) + 8*gamma(m/2
 + 5/2)) + 8*2**m*EulerGamma*b*b**(-m - 1)*x*sqrt(exp(-2*m*log(2))*exp(m*l
og(b**2*x**2)))*gamma(m/2 + 5/2)/(8*m**2*gamma(m/2 + 5/2) + 16*m*gamma(m/2
 + 5/2) + 8*gamma(m/2 + 5/2)) + b**(-m - 1)*b**(m + 3)*m**2*x**(m + 3)*gam
ma(m/2 + 3/2)*hyper((1, 1, m/2 + 3/2), (3/2, 2, 2, m/2 + 5/2), b**2*x**2/4
)/(8*m**2*gamma(m/2 + 5/2) + 16*m*gamma(m/2 + 5/2) + 8*gamma(m/2 + 5/2)) +
 2*b**(-m - 1)*b**(m + 3)*m*x**(m + 3)*gamma(m/2 + 3/2)*hyper((1, 1, m/2 +
 3/2), (3/2, 2, 2, m/2 + 5/2), b**2*x**2/4)/(8*m**2*gamma(m/2 + 5/2) + 16*
m*gamma(m/2 + 5/2) + 8*gamma(m/2 + 5/2)) + b**(-m - 1)*b**(m + 3)*x**(m +
3)*gamma(m/2 + 3/2)*hyper((1, 1, m/2 + 3/2), (3/2, 2, 2, m/2 + 5/2), b**2*
x**2/4)/(8*m**2*gamma(m/2 + 5/2) + 16*m*gamma(m/2 + 5/2) + 8*gamma(m/2 + 5
/2))
```

Maxima [F]

$$\int x^m \mathrm{Chi}(bx)\,dx = \int x^m \mathrm{Chi}(bx)\,dx$$

input
```
integrate(x^m*Chi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^m*Chi(b*x), x)
```

Giac [F]

$$\int x^m \mathrm{Chi}(bx)\,dx = \int x^m \mathrm{Chi}(bx)\,dx$$

input
```
integrate(x^m*Chi(b*x),x, algorithm="giac")
```

output
```
integrate(x^m*Chi(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \mathrm{Chi}(bx)\,dx = \int x^m \operatorname{coshint}(b\,x)\,dx$$

input
```
int(x^m*coshint(b*x),x)
```

output
```
int(x^m*coshint(b*x), x)
```

Reduce [F]

$$\int x^m \mathrm{Chi}(bx)\,dx = \int x^m \chi(bx)\,dx$$

input
```
int(x^m*Chi(b*x),x)
```

output
```
int(x**m*chi(b*x),x)
```

3.70 $\int x^3 \mathbf{Chi}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 63

$$\int x^3\mathrm{Chi}(bx)\,dx=\frac{3\cosh(bx)}{2b^4}+\frac{3x^2\cosh(bx)}{4b^2}+\frac{1}{4}x^4\mathrm{Chi}(bx)-\frac{3x\sinh(bx)}{2b^3}-\frac{x^3\sinh(bx)}{4b}$$

output
```
3/2*cosh(b*x)/b^4+3/4*x^2*cosh(b*x)/b^2+1/4*x^4*Chi(b*x)-3/2*x*sinh(b*x)/b
^3-1/4*x^3*sinh(b*x)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^3\mathrm{Chi}(bx)\,dx=\frac{3(2+b^2x^2)\cosh(bx)}{4b^4}+\frac{1}{4}x^4\mathrm{Chi}(bx)-\frac{x(6+b^2x^2)\sinh(bx)}{4b^3}$$

input
```
Integrate[x^3*CoshIntegral[b*x],x]
```

output
```
(3*(2 + b^2*x^2)*Cosh[b*x])/(4*b^4) + (x^4*CoshIntegral[b*x])/4 - (x*(6 +
b^2*x^2)*Sinh[b*x])/(4*b^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.750, Rules used = {7087, 27, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{Chi}(bx)\,dx$$

$$\Big\downarrow \text{7087}$$

$$\frac{1}{4}x^4\text{Chi}(bx)-\frac{1}{4}b\int \frac{x^3\cosh(bx)}{b}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}x^4\text{Chi}(bx)-\frac{1}{4}\int x^3\cosh(bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}x^4\text{Chi}(bx)-\frac{1}{4}\int x^3\sin\left(ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{4}x^4\text{Chi}(bx)+\frac{1}{4}\left(-\frac{x^3\sinh(bx)}{b}+\frac{3i\int -ix^2\sinh(bx)dx}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{4}\left(\frac{3\int x^2\sinh(bx)dx}{b}-\frac{x^3\sinh(bx)}{b}\right)+\frac{1}{4}x^4\text{Chi}(bx)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}x^4\text{Chi}(bx)+\frac{1}{4}\left(-\frac{x^3\sinh(bx)}{b}+\frac{3\int -ix^2\sin(ibx)dx}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{4}x^4\text{Chi}(bx)+\frac{1}{4}\left(-\frac{x^3\sinh(bx)}{b}-\frac{3i\int x^2\sin(ibx)dx}{b}\right)$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{4}x^4\text{Chi}(bx)+\frac{1}{4}\left(-\frac{x^3\sinh(bx)}{b}-\frac{3i\left(\frac{ix^2\cosh(bx)}{b}-\frac{2i\int x\cosh(bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}x^4\text{Chi}(bx)+\frac{1}{4}\left(-\frac{x^3\sinh(bx)}{b}-\frac{3i\left(\frac{ix^2\cosh(bx)}{b}-\frac{2i\int x\sin\left(ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{4}x^4\text{Chi}(bx)+\frac{1}{4}\left(-\frac{x^3\sinh(bx)}{b}-\frac{3i\left(\frac{ix^2\cosh(bx)}{b}-\frac{2i\left(\frac{x\sinh(bx)}{b}-\frac{i\int -i\sinh(bx)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{4}x^4\text{Chi}(bx)+\frac{1}{4}\left(-\frac{x^3\sinh(bx)}{b}-\frac{3i\left(\frac{ix^2\cosh(bx)}{b}-\frac{2i\left(\frac{x\sinh(bx)}{b}-\frac{\int \sinh(bx)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}x^4\text{Chi}(bx)+\frac{1}{4}\left(-\frac{x^3\sinh(bx)}{b}-\frac{3i\left(\frac{ix^2\cosh(bx)}{b}-\frac{2i\left(\frac{x\sinh(bx)}{b}-\frac{\int -i\sin(ibx)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{4}x^4\text{Chi}(bx)+\frac{1}{4}\left(-\frac{x^3\sinh(bx)}{b}-\frac{3i\left(\frac{ix^2\cosh(bx)}{b}-\frac{2i\left(\frac{x\sinh(bx)}{b}+\frac{i\int \sin(ibx)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3118}$$

$$\frac{1}{4}x^4\text{Chi}(bx)+\frac{1}{4}\left(-\frac{x^3\sinh(bx)}{b}-\frac{3i\left(\frac{ix^2\cosh(bx)}{b}-\frac{2i\left(\frac{x\sinh(bx)}{b}-\frac{\cosh(bx)}{b^2}\right)}{b}\right)}{b}\right)$$

input
```
Int[x^3*CoshIntegral[b*x],x]
```

output
```
(x^4*CoshIntegral[b*x])/4 + (-((x^3*Sinh[b*x])/b) - ((3*I)*((I*x^2*Cosh[b*
x])/b - ((2*I)*(-(Cosh[b*x]/b^2) + (x*Sinh[b*x])/b))/b))/b)/4
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7087
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(CoshIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Cosh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

method	result	size
parts	$\frac{x^4 \operatorname{Chi}(bx)}{4}-\frac{b^3x^3\sinh(bx)-3b^2x^2\cosh(bx)+6bx\sinh(bx)-6\cosh(bx)}{4b^4}$	54
derivativedivides	$\frac{\frac{b^4x^4\operatorname{Chi}(bx)}{4}-\frac{b^3x^3\sinh(bx)}{4}+\frac{3b^2x^2\cosh(bx)}{4}-\frac{3bx\sinh(bx)}{2}+\frac{3\cosh(bx)}{2}}{b^4}$	56
default	$\frac{\frac{b^4x^4\operatorname{Chi}(bx)}{4}-\frac{b^3x^3\sinh(bx)}{4}+\frac{3b^2x^2\cosh(bx)}{4}-\frac{3bx\sinh(bx)}{2}+\frac{3\cosh(bx)}{2}}{b^4}$	56

input
```
int(x^3*Chi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*Chi(b*x)-1/4/b^4*(b^3*x^3*sinh(b*x)-3*b^2*x^2*cosh(b*x)+6*b*x*sinh
(b*x)-6*cosh(b*x))
```

Fricas [F]

$$\int x^3 \operatorname{Chi}(bx)\,dx=\int x^3 \operatorname{Chi}(bx)\,dx$$

input
```
integrate(x^3*Chi(b*x),x, algorithm="fricas")
```

output
```
integral(x^3*cosh_integral(b*x), x)
```

Sympy [A] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.35

$$\int x^3 \operatorname{Chi}(bx)\,dx=-\frac{x^4\log(bx)}{4}+\frac{x^4\log(b^2x^2)}{8}+\frac{x^4\operatorname{Chi}(bx)}{4}-\frac{x^3\sinh(bx)}{4b}+\frac{3x^2\cosh(bx)}{4b^2}-\frac{3x\sinh(bx)}{2b^3}+\frac{3\cosh(bx)}{2b^4}$$

input
```
integrate(x**3*Chi(b*x),x)
```

output
```
-x**4*log(b*x)/4 + x**4*log(b**2*x**2)/8 + x**4*Chi(b*x)/4 - x**3*sinh(b*x
)/(4*b) + 3*x**2*cosh(b*x)/(4*b**2) - 3*x*sinh(b*x)/(2*b**3) + 3*cosh(b*x)
/(2*b**4)
```

Maxima [F]

$$\int x^3\text{Chi}(bx)\,dx = \int x^3\text{Chi}(bx)\,dx$$

input
```
integrate(x^3*Chi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^3*Chi(b*x), x)
```

Giac [F]

$$\int x^3\text{Chi}(bx)\,dx = \int x^3\text{Chi}(bx)\,dx$$

input
```
integrate(x^3*Chi(b*x),x, algorithm="giac")
```

output
```
integrate(x^3*Chi(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\text{Chi}(bx)\,dx = \int x^3\,\text{coshint}(b\,x)\,dx$$

input
```
int(x^3*coshint(b*x),x)
```

output
```
int(x^3*coshint(b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int x^3\mathrm{Chi}(bx)\,dx$$
$$= \frac{\chi(bx)\,b^4x^4 + 3\cosh(bx)\,b^2x^2 + 6\cosh(bx) - \sinh(bx)\,b^3x^3 - 6\sinh(bx)\,bx}{4b^4}$$

input
```
int(x^3*Chi(b*x),x)
```

output
```
(chi(b*x)*b**4*x**4 + 3*cosh(b*x)*b**2*x**2 + 6*cosh(b*x) - sinh(b*x)*b**3
*x**3 - 6*sinh(b*x)*b*x)/(4*b**4)
```

3.71 $\int x^2\text{Chi}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 49

$$\int x^2\text{Chi}(bx)\,dx = \frac{2x\cosh(bx)}{3b^2} + \frac{1}{3}x^3\text{Chi}(bx) - \frac{2\sinh(bx)}{3b^3} - \frac{x^2\sinh(bx)}{3b}$$

output
```
2/3*x*cosh(b*x)/b^2+1/3*x^3*Chi(b*x)-2/3*sinh(b*x)/b^3-1/3*x^2*sinh(b*x)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int x^2\text{Chi}(bx)\,dx = \frac{2x\cosh(bx)}{3b^2} + \frac{1}{3}x^3\text{Chi}(bx) - \frac{(2+b^2x^2)\sinh(bx)}{3b^3}$$

input
```
Integrate[x^2*CoshIntegral[b*x],x]
```

output
```
(2*x*Cosh[b*x])/(3*b^2) + (x^3*CoshIntegral[b*x])/3 - ((2 + b^2*x^2)*Sinh[
b*x])/(3*b^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {7087, 27, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{Chi}(bx)\,dx$$

$$\Big\downarrow \ 7087$$

$$\frac{1}{3}x^3\text{Chi}(bx)-\frac{1}{3}b\int \frac{x^2\cosh(bx)}{b}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}x^3\text{Chi}(bx)-\frac{1}{3}\int x^2\cosh(bx)dx$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}x^3\text{Chi}(bx)-\frac{1}{3}\int x^2\sin\left(ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \ 3777$$

$$\frac{1}{3}x^3\text{Chi}(bx)+\frac{1}{3}\left(-\frac{x^2\sinh(bx)}{b}+\frac{2i\int -ix\sinh(bx)dx}{b}\right)$$

$$\Big\downarrow \ 26$$

$$\frac{1}{3}\left(\frac{2\int x\sinh(bx)dx}{b}-\frac{x^2\sinh(bx)}{b}\right)+\frac{1}{3}x^3\text{Chi}(bx)$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}x^3\text{Chi}(bx)+\frac{1}{3}\left(-\frac{x^2\sinh(bx)}{b}+\frac{2\int -ix\sin(ibx)dx}{b}\right)$$

$$\Big\downarrow \ 26$$

$$\frac{1}{3}x^3\text{Chi}(bx)+\frac{1}{3}\left(-\frac{x^2\sinh(bx)}{b}-\frac{2i\int x\sin(ibx)dx}{b}\right)$$

$$\Big\downarrow \ 3777$$

$$\frac{1}{3}x^3\text{Chi}(bx)+\frac{1}{3}\left(-\frac{x^2\sinh(bx)}{b}-\frac{2i\left(\frac{ix\cosh(bx)}{b}-\frac{i\int\cosh(bx)dx}{b}\right)}{b}\right)$$

$\Big\downarrow$ 3042

$$\frac{1}{3}x^3\text{Chi}(bx)+\frac{1}{3}\left(-\frac{x^2\sinh(bx)}{b}-\frac{2i\left(\frac{ix\cosh(bx)}{b}-\frac{i\int\sin\left(ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)$$

$\Big\downarrow$ 3117

$$\frac{1}{3}x^3\text{Chi}(bx)+\frac{1}{3}\left(-\frac{x^2\sinh(bx)}{b}-\frac{2i\left(\frac{ix\cosh(bx)}{b}-\frac{i\sinh(bx)}{b^2}\right)}{b}\right)$$

input
```
Int[x^2*CoshIntegral[b*x],x]
```

output
```
(x^3*CoshIntegral[b*x])/3 + (-((x^2*Sinh[b*x])/b) - ((2*I)*((I*x*Cosh[b*x]
)/b - (I*Sinh[b*x])/b^2))/b)/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7087
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(CoshIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Cosh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
parts	$\frac{x^3 \operatorname{Chi}(bx)}{3} - \frac{b^2x^2 \sinh(bx) - 2bx\cosh(bx) + 2\sinh(bx)}{3b^3}$	42
derivativedivides	$\frac{\frac{b^3x^3 \operatorname{Chi}(bx)}{3} - \frac{b^2x^2 \sinh(bx)}{3} + \frac{2bx\cosh(bx)}{3} - \frac{2\sinh(bx)}{3}}{b^3}$	44
default	$\frac{\frac{b^3x^3 \operatorname{Chi}(bx)}{3} - \frac{b^2x^2 \sinh(bx)}{3} + \frac{2bx\cosh(bx)}{3} - \frac{2\sinh(bx)}{3}}{b^3}$	44

input
```
int(x^2*Chi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*Chi(b*x)-1/3/b^3*(b^2*x^2*sinh(b*x)-2*b*x*cosh(b*x)+2*sinh(b*x))
```

Fricas [F]

$$\int x^2 \operatorname{Chi}(bx)\,dx = \int x^2 \operatorname{Chi}(bx)\,dx$$

input
```
integrate(x^2*Chi(b*x),x, algorithm="fricas")
```

output
```
integral(x^2*cosh_integral(b*x), x)
```

Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int x^2\text{Chi}(bx)\,dx = -\frac{x^3\log(bx)}{3} + \frac{x^3\log(b^2x^2)}{6} + \frac{x^3\,\text{Chi}(bx)}{3} - \frac{x^2\sinh(bx)}{3b} + \frac{2x\cosh(bx)}{3b^2} - \frac{2\sinh(bx)}{3b^3}$$

input
```
integrate(x**2*Chi(b*x),x)
```

output
```
-x**3*log(b*x)/3 + x**3*log(b**2*x**2)/6 + x**3*Chi(b*x)/3 - x**2*sinh(b*x
)/(3*b) + 2*x*cosh(b*x)/(3*b**2) - 2*sinh(b*x)/(3*b**3)
```

Maxima [F]

$$\int x^2\text{Chi}(bx)\,dx = \int x^2\text{Chi}(bx)\,dx$$

input
```
integrate(x^2*Chi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^2*Chi(b*x), x)
```

Giac [F]

$$\int x^2\text{Chi}(bx)\,dx = \int x^2\text{Chi}(bx)\,dx$$

input
```
integrate(x^2*Chi(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*Chi(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \mathrm{Chi}(bx)\,dx = \frac{x^3\,\mathrm{coshint}(b\,x)}{3} - \frac{\frac{2\sinh(b\,x)}{3} + \frac{b^2\,x^2\sinh(b\,x)}{3} - \frac{2\,b\,x\cosh(b\,x)}{3}}{b^3}$$

input
```
int(x^2*coshint(b*x),x)
```

output
```
(x^3*coshint(b*x))/3 - ((2*sinh(b*x))/3 + (b^2*x^2*sinh(b*x))/3 - (2*b*x*c
osh(b*x))/3)/b^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int x^2 \mathrm{Chi}(bx)\,dx = \frac{\chi(bx)\,b^3x^3 + 2\cosh(bx)\,bx - \sinh(bx)\,b^2x^2 - 2\sinh(bx)}{3b^3}$$

input
```
int(x^2*Chi(b*x),x)
```

output
```
(chi(b*x)*b**3*x**3 + 2*cosh(b*x)*b*x - sinh(b*x)*b**2*x**2 - 2*sinh(b*x))
/(3*b**3)
```

3.72 $\int x\text{Chi}(bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 35

$$\int x\text{Chi}(bx)\,dx=\frac{\cosh(bx)}{2b^2}+\frac{1}{2}x^2\text{Chi}(bx)-\frac{x\sinh(bx)}{2b}$$

output
```
1/2*cosh(b*x)/b^2+1/2*x^2*Chi(b*x)-1/2*x*sinh(b*x)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int x\text{Chi}(bx)\,dx=\frac{\cosh(bx)}{2b^2}+\frac{1}{2}x^2\text{Chi}(bx)-\frac{x\sinh(bx)}{2b}$$

input
```
Integrate[x*CoshIntegral[b*x],x]
```

output
```
Cosh[b*x]/(2*b^2) + (x^2*CoshIntegral[b*x])/2 - (x*Sinh[b*x])/(2*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {7087, 27, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Chi}(bx)\,dx$$

$$\Big\downarrow \text{ 7087}$$

$$\frac{1}{2}x^2\text{Chi}(bx)-\frac{1}{2}b\int \frac{x\cosh(bx)}{b}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}x^2\text{Chi}(bx)-\frac{1}{2}\int x\cosh(bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}x^2\text{Chi}(bx)-\frac{1}{2}\int x\sin\left(ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{1}{2}x^2\text{Chi}(bx)+\frac{1}{2}\left(-\frac{x\sinh(bx)}{b}+\frac{i\int -i\sinh(bx)dx}{b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{2}\left(\frac{\int \sinh(bx)dx}{b}-\frac{x\sinh(bx)}{b}\right)+\frac{1}{2}x^2\text{Chi}(bx)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}x^2\text{Chi}(bx)+\frac{1}{2}\left(-\frac{x\sinh(bx)}{b}+\frac{\int -i\sin(ibx)dx}{b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{2}x^2\text{Chi}(bx)+\frac{1}{2}\left(-\frac{x\sinh(bx)}{b}-\frac{i\int \sin(ibx)dx}{b}\right)$$

$$\Big\downarrow \text{ 3118}$$

$$\frac{1}{2}\left(\frac{\cosh(bx)}{b^2}-\frac{x\sinh(bx)}{b}\right)+\frac{1}{2}x^2\text{Chi}(bx)$$

input
```
Int[x*CoshIntegral[b*x],x]
```

output
```
(x^2*CoshIntegral[b*x])/2 + (Cosh[b*x]/b^2 - (x*Sinh[b*x])/b)/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7087
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(CoshIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Cosh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
parts	$\frac{x^2\,\mathrm{Chi}(bx)}{2}-\frac{bx\sinh(bx)-\cosh(bx)}{2b^2}$	30
derivativedivides	$\frac{\frac{b^2x^2\,\mathrm{Chi}(bx)}{2}-\frac{bx\sinh(bx)}{2}+\frac{\cosh(bx)}{2}}{b^2}$	32
default	$\frac{\frac{b^2x^2\,\mathrm{Chi}(bx)}{2}-\frac{bx\sinh(bx)}{2}+\frac{\cosh(bx)}{2}}{b^2}$	32

input
```
int(x*Chi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*Chi(b*x)-1/2/b^2*(b*x*sinh(b*x)-cosh(b*x))
```

Fricas [F]

$$\int x\mathrm{Chi}(bx)\,dx=\int x\mathrm{Chi}(bx)\ dx$$

input
```
integrate(x*Chi(b*x),x, algorithm="fricas")
```

output
```
integral(x*cosh_integral(b*x), x)
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int x\mathrm{Chi}(bx)\,dx=-\frac{x^2\log(bx)}{2}+\frac{x^2\log(b^2x^2)}{4}+\frac{x^2\,\mathrm{Chi}(bx)}{2}-\frac{x\sinh(bx)}{2b}+\frac{\cosh(bx)}{2b^2}$$

input
```
integrate(x*Chi(b*x),x)
```

output
```
-x**2*log(b*x)/2 + x**2*log(b**2*x**2)/4 + x**2*Chi(b*x)/2 - x*sinh(b*x)/(
2*b) + cosh(b*x)/(2*b**2)
```

Maxima [F]

$$\int x\mathrm{Chi}(bx)\,dx = \int x\mathrm{Chi}(bx)\,dx$$

input
```
integrate(x*Chi(b*x),x, algorithm="maxima")
```

output
```
integrate(x*Chi(b*x), x)
```

Giac [F]

$$\int x\mathrm{Chi}(bx)\,dx = \int x\mathrm{Chi}(bx)\,dx$$

input
```
integrate(x*Chi(b*x),x, algorithm="giac")
```

output
```
integrate(x*Chi(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\mathrm{Chi}(bx)\,dx = \frac{\frac{\cosh(b\,x)}{2} - \frac{b\,x\,\sinh(b\,x)}{2}}{b^2} + \frac{x^2\,\mathrm{coshint}(b\,x)}{2}$$

input
```
int(x*coshint(b*x),x)
```

output
```
(cosh(b*x)/2 - (b*x*sinh(b*x))/2)/b^2 + (x^2*coshint(b*x))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x\text{Chi}(bx)\,dx = \frac{\chi(bx)\,b^2x^2 + \cosh(bx) - \sinh(bx)\,bx}{2b^2}$$

input
```
int(x*Chi(b*x),x)
```

output
```
(chi(b*x)*b**2*x**2 + cosh(b*x) - sinh(b*x)*b*x)/(2*b**2)
```

3.73 $\int \text{Chi}(bx)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 16

$$\int \text{Chi}(bx)\,dx = x\text{Chi}(bx) - \frac{\sinh(bx)}{b}$$

output
```
x*Chi(b*x)-sinh(b*x)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \text{Chi}(bx)\,dx = x\text{Chi}(bx) - \frac{\sinh(bx)}{b}$$

input
```
Integrate[CoshIntegral[b*x],x]
```

output
```
x*CoshIntegral[b*x] - Sinh[b*x]/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7083}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Chi}(bx)\,dx$$

$$\Big\downarrow \text{ 7083}$$

$$x\text{Chi}(bx)-\frac{\sinh(bx)}{b}$$

input
```
Int[CoshIntegral[b*x],x]
```

output
```
x*CoshIntegral[b*x] - Sinh[b*x]/b
```

Defintions of rubi rules used

rule 7083
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(CoshInte
gral[a + b*x]/b), x] - Simp[Sinh[a + b*x]/b, x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
parts	$x\ \text{Chi}\left(bx\right)-\frac{\sinh\left(bx\right)}{b}$	17
derivativedivides	$\frac{\text{Chi}\left(bx\right)bx-\sinh\left(bx\right)}{b}$	19
default	$\frac{\text{Chi}\left(bx\right)bx-\sinh\left(bx\right)}{b}$	19

input
```
int(Chi(b*x),x,method=_RETURNVERBOSE)
```

output
```
x*Chi(b*x)-sinh(b*x)/b
```

Fricas [F]

$$\int \mathrm{Chi}(bx)\,dx = \int \mathrm{Chi}(bx)\,dx$$

input
```
integrate(Chi(b*x),x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 1.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \mathrm{Chi}(bx)\,dx = -x\log(bx) + \frac{x\log(b^2x^2)}{2} + x\,\mathrm{Chi}(bx) - \frac{\sinh(bx)}{b}$$

input
```
integrate(Chi(b*x),x)
```

output
```
-x*log(b*x) + x*log(b**2*x**2)/2 + x*Chi(b*x) - sinh(b*x)/b
```

Maxima [F]

$$\int \mathrm{Chi}(bx)\,dx = \int \mathrm{Chi}(bx)\,dx$$

input
```
integrate(Chi(b*x),x, algorithm="maxima")
```

output
```
integrate(Chi(b*x), x)
```

Giac [F]

$$\int \mathrm{Chi}(bx)\,dx = \int \mathrm{Chi}(bx)\,dx$$

input
```
integrate(Chi(b*x),x, algorithm="giac")
```

output
```
integrate(Chi(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \mathrm{Chi}(bx)\,dx = x\,\mathrm{coshint}(b\,x) - \frac{\sinh(b\,x)}{b}$$

input
```
int(coshint(b*x),x)
```

output
```
x*coshint(b*x) - sinh(b*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \mathrm{Chi}(bx)\,dx = \frac{\chi(bx)\,bx - \sinh(bx)}{b}$$

input
```
int(Chi(b*x),x)
```

output
```
(chi(b*x)*b*x - sinh(b*x))/b
```

3.74 $\int \frac{\text{Chi}(bx)}{x} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 52

$$\int \frac{\text{Chi}(bx)}{x} \, dx = -\frac{1}{2} bx \, {}_3F_3(1,1,1;2,2,2;-bx) + \frac{1}{2} bx \, {}_3F_3(1,1,1;2,2,2;bx) + \gamma \log(x) + \frac{1}{2} \log^2(bx)$$

output
```
-1/2*b*x*hypergeom([1, 1, 1],[2, 2, 2],-b*x)+1/2*b*x*hypergeom([1, 1, 1],[
2, 2, 2],b*x)+gamma*ln(x)+1/2*ln(b*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\text{Chi}(bx)}{x} \, dx = -\frac{1}{2} bx \, {}_3F_3(1,1,1;2,2,2;-bx) + \frac{1}{2} bx \, {}_3F_3(1,1,1;2,2,2;bx) + \gamma \log(x) + \frac{1}{2} \log^2(bx)$$

input
```
Integrate[CoshIntegral[b*x]/x,x]
```

output
```
-1/2*(b*x*HypergeometricPFQ[{1, 1, 1}, {2, 2, 2}, -(b*x)]) + (b*x*Hypergeo
metricPFQ[{1, 1, 1}, {2, 2, 2}, b*x])/2 + EulerGamma*Log[x] + Log[b*x]^2/2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {7085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(bx)}{x} \, dx$$

$$\Big\downarrow \text{7085}$$

$$-\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;-bx)+\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;bx)+\frac{1}{2}\log^2(bx)+\gamma\log(x)$$

input
```
Int[CoshIntegral[b*x]/x,x]
```

output
```
-1/2*(b*x*HypergeometricPFQ[{1, 1, 1}, {2, 2, 2}, -(b*x)]) + (b*x*Hypergeo
metricPFQ[{1, 1, 1}, {2, 2, 2}, b*x])/2 + EulerGamma*Log[x] + Log[b*x]^2/2
```

Defintions of rubi rules used

rule 7085
```
Int[CoshIntegral[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[(-2^(-1))*b*x*Hypergeo
metricPFQ[{1, 1, 1}, {2, 2, 2}, (-b)*x], x] + (Simp[(1/2)*b*x*Hypergeometri
cPFQ[{1, 1, 1}, {2, 2, 2}, b*x], x] + Simp[EulerGamma*Log[x], x] + Simp[(1/
2)*Log[b*x]^2, x]) /; FreeQ[b, x]
```

Maple [F]

$$\int \frac{\text{Chi}\,(bx)}{x}dx$$

input
```
int(Chi(b*x)/x,x)
```

output
```
int(Chi(b*x)/x,x)
```

Fricas [F]

$$\int \frac{\text{Chi}(bx)}{x}\,dx = \int \frac{\text{Chi}(bx)}{x}\,dx$$

input
```
integrate(Chi(b*x)/x,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\text{Chi}(bx)}{x}\,dx = \text{Timed out}$$

input
```
integrate(Chi(b*x)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\mathrm{Chi}(bx)}{x}\,dx = \int \frac{\mathrm{Chi}(bx)}{x}\,dx$$

input
```
integrate(Chi(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x)/x, x)
```

Giac [F]

$$\int \frac{\mathrm{Chi}(bx)}{x}\,dx = \int \frac{\mathrm{Chi}(bx)}{x}\,dx$$

input
```
integrate(Chi(b*x)/x,x, algorithm="giac")
```

output
```
integrate(Chi(b*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\mathrm{Chi}(bx)}{x}\,dx = \int \frac{\mathrm{coshint}(b\,x)}{x}\,dx$$

input
```
int(coshint(b*x)/x,x)
```

output
```
int(coshint(b*x)/x, x)
```

Reduce [F]

$$\int \frac{\text{Chi}(bx)}{x}\,dx = \int \frac{\chi(bx)}{x}dx$$

input
```
int(Chi(b*x)/x,x)
```

output
```
int(chi(b*x)/x,x)
```

3.75 $\int \frac{\text{Chi}(bx)}{x^2} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \frac{\text{Chi}(bx)}{x^2} \, dx = -\frac{\cosh(bx)}{x} - \frac{\text{Chi}(bx)}{x} + b\text{Shi}(bx)$$

output
```
-cosh(b*x)/x-Chi(b*x)/x+b*Shi(b*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\text{Chi}(bx)}{x^2} \, dx = -\frac{\cosh(bx)}{x} - \frac{\text{Chi}(bx)}{x} + b\text{Shi}(bx)$$

input
```
Integrate[CoshIntegral[b*x]/x^2,x]
```

output
```
-(Cosh[b*x]/x) - CoshIntegral[b*x]/x + b*SinhIntegral[b*x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {7087, 27, 3042, 3778, 26, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(bx)}{x^2}dx$$

$$\Big\downarrow \text{7087}$$

$$b\int \frac{\cosh(bx)}{bx^2}dx-\frac{\text{Chi}(bx)}{x}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\cosh(bx)}{x^2}dx-\frac{\text{Chi}(bx)}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\text{Chi}(bx)}{x}+\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x^2}dx$$

$$\Big\downarrow \text{3778}$$

$$ib\int -\frac{i\sinh(bx)}{x}dx-\frac{\text{Chi}(bx)}{x}-\frac{\cosh(bx)}{x}$$

$$\Big\downarrow \text{26}$$

$$b\int \frac{\sinh(bx)}{x}dx-\frac{\text{Chi}(bx)}{x}-\frac{\cosh(bx)}{x}$$

$$\Big\downarrow \text{3042}$$

$$b\int -\frac{i\sin(ibx)}{x}dx-\frac{\text{Chi}(bx)}{x}-\frac{\cosh(bx)}{x}$$

$$\Big\downarrow \text{26}$$

$$-ib\int \frac{\sin(ibx)}{x}dx-\frac{\text{Chi}(bx)}{x}-\frac{\cosh(bx)}{x}$$

$$\Big\downarrow \text{3779}$$

$$-\frac{\text{Chi}(bx)}{x}+b\text{Shi}(bx)-\frac{\cosh(bx)}{x}$$

input
```
Int[CoshIntegral[b*x]/x^2,x]
```

output
```
-(Cosh[b*x]/x) - CoshIntegral[b*x]/x + b*SinhIntegral[b*x]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 7087
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(CoshIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Cosh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
parts	$-\frac{\mathrm{Chi}(bx)}{x}+b\left(-\frac{\cosh(bx)}{bx}+\mathrm{Shi}\left(bx\right)\right)$	30
derivativedivides	$b\left(-\frac{\mathrm{Chi}(bx)}{bx}-\frac{\cosh(bx)}{bx}+\mathrm{Shi}\left(bx\right)\right)$	32
default	$b\left(-\frac{\mathrm{Chi}(bx)}{bx}-\frac{\cosh(bx)}{bx}+\mathrm{Shi}\left(bx\right)\right)$	32

input
```
int(Chi(b*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-Chi(b*x)/x+b*(-1/b/x*cosh(b*x)+Shi(b*x))
```

Fricas [F]

$$\int \frac{\mathrm{Chi}(bx)}{x^2}\,dx=\int \frac{\mathrm{Chi}(bx)}{x^2}\,dx$$

input
```
integrate(Chi(b*x)/x^2,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x)/x^2, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

Time = 0.66 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{\mathrm{Chi}(bx)}{x^2}\,dx=\frac{b^2x_3F_4\left(\begin{matrix}\frac{1}{2},1,1\\ \frac{3}{2},\frac{3}{2},2,2\end{matrix}\middle|\frac{b^2x^2}{4}\right)}{4}-\frac{\log\left(b^2x^2\right)}{2x}-\frac{1}{x}-\frac{\gamma}{x}$$

input
```
integrate(Chi(b*x)/x**2,x)
```

output
```
b**2*x*hyper((1/2, 1, 1), (3/2, 3/2, 2, 2), b**2*x**2/4)/4 - log(b**2*x**2
)/(2*x) - 1/x - EulerGamma/x
```

Maxima [F]

$$\int \frac{\mathrm{Chi}(bx)}{x^2}\,dx = \int \frac{\mathrm{Chi}(bx)}{x^2}\,dx$$

input
```
integrate(Chi(b*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x)/x^2, x)
```

Giac [F]

$$\int \frac{\mathrm{Chi}(bx)}{x^2}\,dx = \int \frac{\mathrm{Chi}(bx)}{x^2}\,dx$$

input
```
integrate(Chi(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(Chi(b*x)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\mathrm{Chi}(bx)}{x^2}\,dx = \int \frac{\mathrm{coshint}(b\,x)}{x^2}\,dx$$

input
```
int(coshint(b*x)/x^2,x)
```

output
```
int(coshint(b*x)/x^2, x)
```

Reduce [F]

$$\int \frac{\mathrm{Chi}(bx)}{x^2}\,dx = \int \frac{\chi(bx)}{x^2}dx$$

input
```
int(Chi(b*x)/x^2,x)
```

output
```
int(chi(b*x)/x**2,x)
```

3.76 $\int \frac{\mathbf{Chi}(bx)}{x^3}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 46

$$\int \frac{\text{Chi}(bx)}{x^3}\,dx = -\frac{\cosh(bx)}{4x^2} + \frac{1}{4}b^2\text{Chi}(bx) - \frac{\text{Chi}(bx)}{2x^2} - \frac{b\sinh(bx)}{4x}$$

output
```
-1/4*cosh(b*x)/x^2+1/4*b^2*Chi(b*x)-1/2*Chi(b*x)/x^2-1/4*b*sinh(b*x)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\text{Chi}(bx)}{x^3}\,dx = -\frac{\cosh(bx)}{4x^2} + \frac{1}{4}b^2\text{Chi}(bx) - \frac{\text{Chi}(bx)}{2x^2} - \frac{b\sinh(bx)}{4x}$$

input
```
Integrate[CoshIntegral[b*x]/x^3,x]
```

output
```
-1/4*Cosh[b*x]/x^2 + (b^2*CoshIntegral[b*x])/4 - CoshIntegral[b*x]/(2*x^2)
 - (b*Sinh[b*x])/(4*x)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {7087, 27, 3042, 3778, 26, 3042, 26, 3778, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(bx)}{x^3} \, dx$$

$$\Big\downarrow \ 7087$$

$$\frac{1}{2} b \int \frac{\cosh(bx)}{bx^3} dx - \frac{\text{Chi}(bx)}{2x^2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2} \int \frac{\cosh(bx)}{x^3} dx - \frac{\text{Chi}(bx)}{2x^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\text{Chi}(bx)}{2x^2} + \frac{1}{2} \int \frac{\sin\left(ibx + \frac{\pi}{2}\right)}{x^3} dx$$

$$\Big\downarrow \ 3778$$

$$-\frac{\text{Chi}(bx)}{2x^2} + \frac{1}{2}\left(-\frac{\cosh(bx)}{2x^2} + \frac{1}{2} ib \int -\frac{i \sinh(bx)}{x^2} dx\right)$$

$$\Big\downarrow \ 26$$

$$\frac{1}{2}\left(\frac{1}{2} b \int \frac{\sinh(bx)}{x^2} dx - \frac{\cosh(bx)}{2x^2}\right) - \frac{\text{Chi}(bx)}{2x^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\text{Chi}(bx)}{2x^2} + \frac{1}{2}\left(-\frac{\cosh(bx)}{2x^2} + \frac{1}{2} b \int -\frac{i \sin(ibx)}{x^2} dx\right)$$

$$\Big\downarrow \ 26$$

$$-\frac{\text{Chi}(bx)}{2x^2} + \frac{1}{2}\left(-\frac{\cosh(bx)}{2x^2} - \frac{1}{2} ib \int \frac{\sin(ibx)}{x^2} dx\right)$$

$$\Big\downarrow \ 3778$$

$$-\frac{\text{Chi}(bx)}{2x^2}+\frac{1}{2}\left(-\frac{\cosh(bx)}{2x^2}-\frac{1}{2}ib\left(ib\int\frac{\cosh(bx)}{x}dx-\frac{i\sinh(bx)}{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\text{Chi}(bx)}{2x^2}+\frac{1}{2}\left(-\frac{\cosh(bx)}{2x^2}-\frac{1}{2}ib\left(ib\int\frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx-\frac{i\sinh(bx)}{x}\right)\right)$$

$$\Big\downarrow \text{3782}$$

$$-\frac{\text{Chi}(bx)}{2x^2}+\frac{1}{2}\left(-\frac{\cosh(bx)}{2x^2}-\frac{1}{2}ib\left(ib\text{Chi}(bx)-\frac{i\sinh(bx)}{x}\right)\right)$$

input
```
Int[CoshIntegral[b*x]/x^3,x]
```

output
```
-1/2*CoshIntegral[b*x]/x^2 + (-1/2*Cosh[b*x]/x^2 - (I/2)*b*(I*b*CoshIntegr
al[b*x] - (I*Sinh[b*x])/x))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 7087
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(CoshIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Cosh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

method	result	size
parts	$-\frac{\operatorname{Chi}(bx)}{2x^2}+\frac{b^2\left(-\frac{\cosh(bx)}{2b^2x^2}-\frac{\sinh(bx)}{2bx}+\frac{\operatorname{Chi}(bx)}{2}\right)}{2}$	47
derivativedivides	$b^2\left(-\frac{\operatorname{Chi}(bx)}{2b^2x^2}-\frac{\cosh(bx)}{4b^2x^2}-\frac{\sinh(bx)}{4bx}+\frac{\operatorname{Chi}(bx)}{4}\right)$	48
default	$b^2\left(-\frac{\operatorname{Chi}(bx)}{2b^2x^2}-\frac{\cosh(bx)}{4b^2x^2}-\frac{\sinh(bx)}{4bx}+\frac{\operatorname{Chi}(bx)}{4}\right)$	48

input
```
int(Chi(b*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*Chi(b*x)/x^2+1/2*b^2*(-1/2/b^2/x^2*cosh(b*x)-1/2*sinh(b*x)/b/x+1/2*Ch
i(b*x))
```

Fricas [F]

$$\int \frac{\operatorname{Chi}(bx)}{x^3}\,dx=\int \frac{\operatorname{Chi}(bx)}{x^3}\,dx$$

input
```
integrate(Chi(b*x)/x^3,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x)/x^3, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(39) = 78$.

Time = 2.61 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.89

$$\int \frac{\mathrm{Chi}(bx)}{x^3}\,dx = -\frac{b^2\log(bx)}{4} + \frac{b^2\log(b^2x^2)}{8} + \frac{b^2\,\mathrm{Chi}(bx)}{4} - \frac{b\sinh(bx)}{4x} + \frac{\log(bx)}{2x^2} - \frac{\log(b^2x^2)}{4x^2} - \frac{\cosh(bx)}{4x^2} - \frac{\mathrm{Chi}(bx)}{2x^2}$$

input
```
integrate(Chi(b*x)/x**3,x)
```

output
```
-b**2*log(b*x)/4 + b**2*log(b**2*x**2)/8 + b**2*Chi(b*x)/4 - b*sinh(b*x)/(
4*x) + log(b*x)/(2*x**2) - log(b**2*x**2)/(4*x**2) - cosh(b*x)/(4*x**2) -
Chi(b*x)/(2*x**2)
```

Maxima [F]

$$\int \frac{\mathrm{Chi}(bx)}{x^3}\,dx = \int \frac{\mathrm{Chi}(bx)}{x^3}\,dx$$

input
```
integrate(Chi(b*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x)/x^3, x)
```

Giac [F]

$$\int \frac{\mathrm{Chi}(bx)}{x^3}\,dx = \int \frac{\mathrm{Chi}(bx)}{x^3}\,dx$$

input
```
integrate(Chi(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(Chi(b*x)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\mathrm{Chi}(bx)}{x^3}\,dx = \frac{b^2\,\mathrm{coshint}(b\,x)}{4} - \frac{\frac{\mathrm{coshint}(b\,x)}{2} + \frac{\cosh(b\,x)}{4} + \frac{b\,x\,\sinh(b\,x)}{4}}{x^2}$$

input
```
int(coshint(b*x)/x^3,x)
```

output
```
(b^2*coshint(b*x))/4 - (coshint(b*x)/2 + cosh(b*x)/4 + (b*x*sinh(b*x))/4)/
x^2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{\mathrm{Chi}(bx)}{x^3}\,dx = \frac{\chi(bx)\,b^2x^2 - 2\chi(bx) - \cosh(bx) - \sinh(bx)\,bx}{4x^2}$$

input
```
int(Chi(b*x)/x^3,x)
```

output
```
(chi(b*x)*b**2*x**2 - 2*chi(b*x) - cosh(b*x) - sinh(b*x)*b*x)/(4*x**2)
```

3.77 $\int x^m \mathbf{Chi}(bx)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \mathrm{Chi}(bx)^2 \, dx = \mathrm{Int}\left(x^m \mathrm{Chi}(bx)^2, x\right)$$

output
```
Defer(Int)(x^m*Chi(b*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \mathrm{Chi}(bx)^2 \, dx = \int x^m \mathrm{Chi}(bx)^2 \, dx$$

input
```
Integrate[x^m*CoshIntegral[b*x]^2,x]
```

output
```
Integrate[x^m*CoshIntegral[b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{Chi}(bx)^2 \, dx$$

$$\Big\downarrow \; 7299$$

$$\int x^m \text{Chi}(bx)^2 dx$$

input
```
Int[x^m*CoshIntegral[b*x]^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{Chi}(bx)^2 \, dx$$

input
```
int(x^m*Chi(b*x)^2,x)
```

output
```
int(x^m*Chi(b*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Chi}(bx)^2 \, dx = \int x^m \text{Chi}(bx)^2 \, dx$$

input
```
integrate(x^m*Chi(b*x)^2,x, algorithm="fricas")
```

output
```
integral(x^m*cosh_integral(b*x)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \text{Chi}(bx)^2 \, dx = \int x^m \operatorname{Chi}^2(bx) \, dx$$

input
```
integrate(x**m*Chi(b*x)**2,x)
```

output
```
Integral(x**m*Chi(b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Chi}(bx)^2 \, dx = \int x^m \text{Chi}(bx)^2 \, dx$$

input
```
integrate(x^m*Chi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*Chi(b*x)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Chi}(bx)^2 \, dx = \int x^m \text{Chi}(bx)^2 \, dx$$

input
```
integrate(x^m*Chi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^m*Chi(b*x)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Chi}(bx)^2 \, dx = \int x^m \, \text{coshint}(b\, x)^2 \, dx$$

input
```
int(x^m*coshint(b*x)^2,x)
```

output
```
int(x^m*coshint(b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Chi}(bx)^2 \, dx = \int x^m \chi(bx)^2 \, dx$$

input
```
int(x^m*Chi(b*x)^2,x)
```

output
```
int(x**m*chi(b*x)**2,x)
```

3.78 $\int x^3\textbf{Chi}(bx)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 164

$$\begin{aligned}\int x^3\text{Chi}(bx)^2\,dx=&-\frac{x^2}{4b^2}+\frac{3\cosh^2(bx)}{8b^4}+\frac{3\cosh(bx)\text{Chi}(bx)}{b^4}\\&+\frac{3x^2\cosh(bx)\text{Chi}(bx)}{2b^2}+\frac{1}{4}x^4\text{Chi}(bx)^2-\frac{3\text{Chi}(2bx)}{2b^4}\\&-\frac{3\log(x)}{2b^4}-\frac{x\cosh(bx)\sinh(bx)}{b^3}-\frac{3x\text{Chi}(bx)\sinh(bx)}{b^3}\\&-\frac{x^3\text{Chi}(bx)\sinh(bx)}{2b}+\frac{13\sinh^2(bx)}{8b^4}+\frac{x^2\sinh^2(bx)}{4b^2}\end{aligned}$$

output
```
-1/4*x^2/b^2+3/8*cosh(b*x)^2/b^4+3*cosh(b*x)*Chi(b*x)/b^4+3/2*x^2*cosh(b*x
)*Chi(b*x)/b^2+1/4*x^4*Chi(b*x)^2-3/2*Chi(2*b*x)/b^4-3/2*ln(x)/b^4-x*cosh(
b*x)*sinh(b*x)/b^3-3*x*Chi(b*x)*sinh(b*x)/b^3-1/2*x^3*Chi(b*x)*sinh(b*x)/b
+13/8*sinh(b*x)^2/b^4+1/4*x^2*sinh(b*x)^2/b^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.65

$$\int x^3\text{Chi}(bx)^2\,dx$$
$$=\frac{-3b^2x^2+8\cosh(2bx)+b^2x^2\cosh(2bx)+2b^4x^4\text{Chi}(bx)^2-12\text{Chi}(2bx)-12\log(x)-4\text{Chi}(bx)\left(-3(2+\right.}{8b^4}$$

input
```
Integrate[x^3*CoshIntegral[b*x]^2,x]
```

output
```
(-3*b^2*x^2 + 8*Cosh[2*b*x] + b^2*x^2*Cosh[2*b*x] + 2*b^4*x^4*CoshIntegral
[b*x]^2 - 12*CoshIntegral[2*b*x] - 12*Log[x] - 4*CoshIntegral[b*x]*(-3*(2
+ b^2*x^2)*Cosh[b*x] + b*x*(6 + b^2*x^2)*Sinh[b*x]) - 4*b*x*Sinh[2*b*x])/(
8*b^4)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.41, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.400, Rules used = {7091, 7097, 27, 5895, 3042, 25, 3791, 15, 7103, 27, 3042, 3791, 15, 7097, 27, 3042, 26, 3044, 15, 7101, 27, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{Chi}(bx)^2\,dx$$
$$\Big\downarrow \text{7091}$$
$$\frac{1}{4}x^4\text{Chi}(bx)^2-\frac{1}{2}\int x^3\cosh(bx)\text{Chi}(bx)dx$$
$$\Big\downarrow \text{7097}$$
$$\frac{1}{2}\left(\frac{3\int x^2\text{Chi}(bx)\sinh(bx)dx}{b}+\int\frac{x^2\cosh(bx)\sinh(bx)}{b}dx-\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}\right)+\frac{1}{4}x^4\text{Chi}(bx)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{3\int x^2\text{Chi}(bx)\sinh(bx)dx}{b}+\frac{\int x^2\cosh(bx)\sinh(bx)dx}{b}-\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}\right)+\frac{1}{4}x^4\text{Chi}(bx)^2$$

$$\Big\downarrow \text{5895}$$

$$\frac{1}{2}\left(\frac{3\int x^2\text{Chi}(bx)\sinh(bx)dx}{b}+\frac{\frac{x^2\sinh^2(bx)}{2b}-\frac{\int x\sinh^2(bx)dx}{b}}{b}-\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}\right)+\frac{1}{4}x^4\text{Chi}(bx)^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2+\frac{1}{2}\left(\frac{3\int x^2\text{Chi}(bx)\sinh(bx)dx}{b}+\frac{\frac{x^2\sinh^2(bx)}{2b}-\frac{\int -x\sin(ibx)^2dx}{b}}{b}-\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2+\frac{1}{2}\left(\frac{3\int x^2\text{Chi}(bx)\sinh(bx)dx}{b}+\frac{\frac{x^2\sinh^2(bx)}{2b}+\frac{\int x\sin(ibx)^2dx}{b}}{b}-\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}\right)$$

$$\Big\downarrow \text{3791}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{\int xdx}{2}+\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{3\int x^2\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}\right)+\frac{1}{4}x^4\text{Chi}(bx)^2$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\left(\frac{3\int x^2\text{Chi}(bx)\sinh(bx)dx}{b}+\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}-\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}\right)+\frac{1}{4}x^4\text{Chi}(bx)^2$$

$$\Big\downarrow \text{7103}$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\int x\cosh(bx)\text{Chi}(bx)dx}{b}-\int\frac{x\cosh^2(bx)}{b}dx+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}-\right.$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\int x\cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\int x\cosh^2(bx)dx}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}-x\right.$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2+$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\int x\cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\int x\sin\left(ibx+\frac{\pi}{2}\right)^2dx}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}\right.$$

$$\Big\downarrow \; 3791$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{\frac{\int xdx}{2}-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}}{b}-\frac{2\int x\cosh(bx)\text{Chi}(bx)dx}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}}{b}\right.$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2$$

$$\Big\downarrow \; 15$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\int x\cosh(bx)\text{Chi}(bx)dx}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}}{b}+\right.$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2$$

$$\Big\downarrow \; 7097$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}-\int \frac{\cosh(bx)\sinh(bx)}{b}dx+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx}{b}\right.}{b}\right.$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\int \cosh(bx)\sinh(bx)dx}{b}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right.}{b}\right.$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2+$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\int -i\cos(ibx)\sin(ibx)dx}{b}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(b}{b}\right.}{b}\right.$$

$$\Big\downarrow \; 26$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2+$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}+\frac{i\int \cos(ibx)\sin(ibx)dx}{b}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right.}{b}\right.$$

$$\Big\downarrow \; 3044$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2+$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(\frac{\int i\sinh(bx)d(i\sinh(bx))}{b^2}-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right.}{b}\right.$$

$$\downarrow 15$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{\sin}{}\right.$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2$$

$$\downarrow 7101$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\int\frac{\cosh^2(bx)}{bx}dx}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(b}{b}\right.}{b}\right.$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2$$

$$\downarrow 27$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int\frac{\cosh^2(bx)}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(b}{b}\right.}{b}\right.$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2$$

$\downarrow$ 3042

$$\frac{1}{4}x^4\text{Chi}(bx)^2+$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int\frac{\sin\left(ibx+\frac{\pi}{2}\right)^2}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\text{co}}{b}\right.}{b}\right.$$

$\downarrow$ 3793

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int\left(\frac{\cosh(2bx)}{2x}+\frac{1}{2x}\right)dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)}{b}\cdots\right)}{b}\cdots\right.$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{3\left(-\frac{2\left(-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}}{b}}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\text{co}}{b}\cdots\right)}{b}\cdots\right.$$

$$\frac{1}{4}x^4\text{Chi}(bx)^2$$

input
```
Int[x^3*CoshIntegral[b*x]^2,x]
```

output
```
(x^4*CoshIntegral[b*x]^2)/4 + (-((x^3*CoshIntegral[b*x]*Sinh[b*x])/b) + (3
*((x^2*Cosh[b*x]*CoshIntegral[b*x])/b - (x^2/4 - Cosh[b*x]^2/(4*b^2) + (x*
Cosh[b*x]*Sinh[b*x])/(2*b))/b - (2*(-(((Cosh[b*x]*CoshIntegral[b*x])/b - (
CoshIntegral[2*b*x]/2 + Log[x]/2)/b)/b) + (x*CoshIntegral[b*x]*Sinh[b*x])/
b - Sinh[b*x]^2/(2*b^2)))/b))/b + ((x^2*Sinh[b*x]^2)/(2*b) + (x^2/4 - (x*C
osh[b*x]*Sinh[b*x])/(2*b) + Sinh[b*x]^2/(4*b^2))/b)/b)/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 7091
```
Int[CoshIntegral[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Cos
hIntegral[b*x]^2/(m + 1)), x] - Simp[2/(m + 1)   Int[x^m*Cosh[b*x]*CoshInte
gral[b*x], x], x] /; FreeQ[b, x] && IGtQ[m, 0]
```

rule 7097
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7101
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7103
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{\frac{b^4x^4\operatorname{Chi}(bx)^2}{4}-2\operatorname{Chi}(bx)\left(\frac{b^3x^3\sinh(bx)}{4}-\frac{3b^2x^2\cosh(bx)}{4}+\frac{3bx\sinh(bx)}{2}-\frac{3\cosh(bx)}{2}\right)+\frac{b^2x^2\cosh(bx)^2}{4}-bx\cosh(bx)\sinh(b}{b^4}$
default	$\frac{\frac{b^4x^4\operatorname{Chi}(bx)^2}{4}-2\operatorname{Chi}(bx)\left(\frac{b^3x^3\sinh(bx)}{4}-\frac{3b^2x^2\cosh(bx)}{4}+\frac{3bx\sinh(bx)}{2}-\frac{3\cosh(bx)}{2}\right)+\frac{b^2x^2\cosh(bx)^2}{4}-bx\cosh(bx)\sinh(b}{b^4}$

input
```
int(x^3*Chi(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/4*b^4*x^4*Chi(b*x)^2-2*Chi(b*x)*(1/4*b^3*x^3*sinh(b*x)-3/4*b^2*x^
2*cosh(b*x)+3/2*b*x*sinh(b*x)-3/2*cosh(b*x))+1/4*b^2*x^2*cosh(b*x)^2-b*x*c
osh(b*x)*sinh(b*x)-1/2*b^2*x^2+2*cosh(b*x)^2-3/2*ln(b*x)-3/2*Chi(2*b*x))
```

Fricas [F]

$$\int x^3\text{Chi}(bx)^2\,dx=\int x^3\text{Chi}(bx)^2\,dx$$

input
```
integrate(x^3*Chi(b*x)^2,x, algorithm="fricas")
```

output
```
integral(x^3*cosh_integral(b*x)^2, x)
```

Sympy [F]

$$\int x^3\text{Chi}(bx)^2\,dx=\int x^3\,\text{Chi}^2\,(bx)\,\,dx$$

input
```
integrate(x**3*Chi(b*x)**2,x)
```

output
```
Integral(x**3*Chi(b*x)**2, x)
```

Maxima [F]

$$\int x^3 \text{Chi}(bx)^2 \, dx = \int x^3 \text{Chi}(bx)^2 \, dx$$

input
```
integrate(x^3*Chi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*Chi(b*x)^2, x)
```

Giac [F]

$$\int x^3 \text{Chi}(bx)^2 \, dx = \int x^3 \text{Chi}(bx)^2 \, dx$$

input
```
integrate(x^3*Chi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^3*Chi(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \text{Chi}(bx)^2 \, dx = \int x^3 \operatorname{coshint}(b\,x)^2 \, dx$$

input
```
int(x^3*coshint(b*x)^2,x)
```

output
```
int(x^3*coshint(b*x)^2, x)
```

Reduce [F]

$$\int x^3 \mathrm{Chi}(bx)^2 \, dx = \int \chi(bx)^2 \, x^3 dx$$

input
```
int(x^3*Chi(b*x)^2,x)
```

output
```
int(chi(b*x)**2*x**3,x)
```

3.79 $\int x^2\mathbf{Chi}(bx)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 112

$$\int x^2\text{Chi}(bx)^2\,dx=-\frac{x}{2b^2}+\frac{4x\cosh(bx)\text{Chi}(bx)}{3b^2}+\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{5\cosh(bx)\sinh(bx)}{6b^3}-\frac{4\text{Chi}(bx)\sinh(bx)}{3b^3}-\frac{2x^2\text{Chi}(bx)\sinh(bx)}{3b}+\frac{x\sinh^2(bx)}{3b^2}+\frac{2\text{Shi}(2bx)}{3b^3}$$

output
```
-1/2*x/b^2+4/3*x*cosh(b*x)*Chi(b*x)/b^2+1/3*x^3*Chi(b*x)^2-5/6*cosh(b*x)*s
inh(b*x)/b^3-4/3*Chi(b*x)*sinh(b*x)/b^3-2/3*x^2*Chi(b*x)*sinh(b*x)/b+1/3*x
*sinh(b*x)^2/b^2+2/3*Shi(2*b*x)/b^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int x^2\text{Chi}(bx)^2\,dx=\frac{-8bx+2bx\cosh(2bx)+4b^3x^3\text{Chi}(bx)^2-8\text{Chi}(bx)\left(-2bx\cosh(bx)+\left(2+b^2x^2\right)\sinh(bx)\right)-5\sinh(2bx)}{12b^3}$$

input
```
Integrate[x^2*CoshIntegral[b*x]^2,x]
```

output
```
(-8*b*x + 2*b*x*Cosh[2*b*x] + 4*b^3*x^3*CoshIntegral[b*x]^2 - 8*CoshIntegr
al[b*x]*(-2*b*x*Cosh[b*x] + (2 + b^2*x^2)*Sinh[b*x]) - 5*Sinh[2*b*x] + 8*S
inhIntegral[2*b*x])/(12*b^3)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.40, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.000, Rules used = {7091, 7097, 27, 5895, 3042, 25, 3115, 24, 7103, 27, 3042, 3115, 24, 7095, 27, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{Chi}(bx)^2\,dx$$

$$\Big\downarrow \text{7091}$$

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\int x^2\cosh(bx)\text{Chi}(bx)dx$$

$$\Big\downarrow \text{7097}$$

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\int\frac{x\cosh(bx)\sinh(bx)}{b}dx+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\int x\cosh(bx)\sinh(bx)dx}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}\right)$$

$$\Big\downarrow \text{5895}$$

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}-\frac{\int\sinh^2(bx)dx}{2b}}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}-\frac{\int -\sin(ibx)^2dx}{2b}}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}\right)$$

$\big\downarrow$ 25

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\int \sin(ibx)^2dx}{2b}}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}\right)$$

$\big\downarrow$ 3115

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\frac{\frac{\int 1dx}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}+\frac{x\sinh^2(bx)}{2b}}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}\right)$$

$\big\downarrow$ 24

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)$$

$\big\downarrow$ 7103

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\int\cosh(bx)\text{Chi}(bx)dx}{b}-\int\frac{\cosh^2(bx)}{b}dx+\frac{x\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cos}{2b}}{2b}}{b}\right.$$

$\big\downarrow$ 27

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\int\cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\int\cosh^2(bx)dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh}{2b}}{2b}}{b}\right.$$

$\big\downarrow$ 3042

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\int\cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\int\sin\left(ibx+\frac{\pi}{2}\right)^2dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)}{2}}{2b}}{b}\right.$$

$\Big\downarrow$ 3115

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\int\cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\frac{\int 1dx}{2}+\frac{\sinh(bx)\cosh(bx)}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{x}{2}-}{b}\right.$$

$\Big\downarrow$ 24

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\int\cosh(bx)\text{Chi}(bx)dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(b}{2}}{}}{b}\right.$$

$\Big\downarrow$ 7095

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\int\frac{\cosh(bx)\sinh(bx)}{bx}dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(}{2b}}{}\right.$$

$\Big\downarrow$ 27

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\int\frac{\cosh(bx)\sinh(bx)}{\frac{x}{b}}dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(}{2b}}{}\right.$$

$\Big\downarrow$ 5971

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int\frac{\sinh(2bx)}{2x}dx}{b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+}{}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int\frac{\sinh(2bx)}{x}dx}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+}{}\right.$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int-\frac{i\sin(2ibx)}{x}dx}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}}{}\right.$$

$$\Big\downarrow \; 26$$

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}+\frac{i\int\frac{\sin(2ibx)}{x}dx}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+}{}\right.$$

$$\Big\downarrow \; 3779$$

$$\frac{1}{3}x^3\text{Chi}(bx)^2-\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\text{Shi}(2bx)}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{x}{2}-}{b}\right.$$

input
```
Int[x^2*CoshIntegral[b*x]^2,x]
```

output
```
(x^3*CoshIntegral[b*x]^2)/3 - (2*((x^2*CoshIntegral[b*x]*Sinh[b*x])/b - ((
x*Sinh[b*x]^2)/(2*b) + (x/2 - (Cosh[b*x]*Sinh[b*x])/(2*b))/(2*b))/b - (2*(
(x*Cosh[b*x]*CoshIntegral[b*x])/b - (x/2 + (Cosh[b*x]*Sinh[b*x])/(2*b))/b
- ((CoshIntegral[b*x]*Sinh[b*x])/b - SinhIntegral[2*b*x]/(2*b))/b))/b))/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7091
```
Int[CoshIntegral[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Cos
hIntegral[b*x]^2/(m + 1)), x] - Simp[2/(m + 1)   Int[x^m*Cosh[b*x]*CoshInte
gral[b*x], x], x] /; FreeQ[b, x] && IGtQ[m, 0]
```

rule 7095
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7097
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7103
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\frac{b^3x^3\,\mathrm{Chi}(bx)^2}{3}-2\,\mathrm{Chi}(bx)\left(\frac{b^2x^2\sinh(bx)}{3}-\frac{2bx\cosh(bx)}{3}+\frac{2\sinh(bx)}{3}\right)+\frac{bx\cosh(bx)^2}{3}-\frac{5\cosh(bx)\sinh(bx)}{6}-\frac{5bx}{6}+\frac{2\,\mathrm{Shi}(2bx)}{3}}{b^3}$
default	$\frac{\frac{b^3x^3\,\mathrm{Chi}(bx)^2}{3}-2\,\mathrm{Chi}(bx)\left(\frac{b^2x^2\sinh(bx)}{3}-\frac{2bx\cosh(bx)}{3}+\frac{2\sinh(bx)}{3}\right)+\frac{bx\cosh(bx)^2}{3}-\frac{5\cosh(bx)\sinh(bx)}{6}-\frac{5bx}{6}+\frac{2\,\mathrm{Shi}(2bx)}{3}}{b^3}$

input
```
int(x^2*Chi(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*b^3*x^3*Chi(b*x)^2-2*Chi(b*x)*(1/3*b^2*x^2*sinh(b*x)-2/3*b*x*co
sh(b*x)+2/3*sinh(b*x))+1/3*b*x*cosh(b*x)^2-5/6*cosh(b*x)*sinh(b*x)-5/6*b*x
+2/3*Shi(2*b*x))
```

Fricas [F]

$$\int x^2\mathrm{Chi}(bx)^2\,dx=\int x^2\mathrm{Chi}(bx)^2\,dx$$

input
```
integrate(x^2*Chi(b*x)^2,x, algorithm="fricas")
```

output
```
integral(x^2*cosh_integral(b*x)^2, x)
```

Sympy [F]

$$\int x^2\mathrm{Chi}(bx)^2\,dx=\int x^2\,\mathrm{Chi}^2\,(bx)\,dx$$

input
```
integrate(x**2*Chi(b*x)**2,x)
```

output
```
Integral(x**2*Chi(b*x)**2, x)
```

Maxima [F]

$$\int x^2 \text{Chi}(bx)^2 \, dx = \int x^2 \text{Chi}(bx)^2 \, dx$$

input
```
integrate(x^2*Chi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^2*Chi(b*x)^2, x)
```

Giac [F]

$$\int x^2 \text{Chi}(bx)^2 \, dx = \int x^2 \text{Chi}(bx)^2 \, dx$$

input
```
integrate(x^2*Chi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2*Chi(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \text{Chi}(bx)^2 \, dx = \int x^2 \, \text{coshint}(b \, x)^2 \, dx$$

input
```
int(x^2*coshint(b*x)^2,x)
```

output
```
int(x^2*coshint(b*x)^2, x)
```

Reduce [F]

$$\int x^2 \mathrm{Chi}(bx)^2\,dx = \int \chi(bx)^2\,x^2 dx$$

input
```
int(x^2*Chi(b*x)^2,x)
```

output
```
int(chi(b*x)**2*x**2,x)
```

3.80 $\int x\mathbf{Chi}(bx)^2\,dx$

Optimal result

Integrand size = 8, antiderivative size = 74

$$\int x\text{Chi}(bx)^2\,dx = \frac{\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{1}{2}x^2\text{Chi}(bx)^2 - \frac{\text{Chi}(2bx)}{2b^2} - \frac{\log(x)}{2b^2} - \frac{x\text{Chi}(bx)\sinh(bx)}{b} + \frac{\sinh^2(bx)}{2b^2}$$

output
```
cosh(b*x)*Chi(b*x)/b^2+1/2*x^2*Chi(b*x)^2-1/2*Chi(2*b*x)/b^2-1/2*ln(x)/b^2
-x*Chi(b*x)*sinh(b*x)/b+1/2*sinh(b*x)^2/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int x\text{Chi}(bx)^2\,dx = \frac{\cosh(2bx) + 2b^2x^2\text{Chi}(bx)^2 - 2\text{Chi}(2bx) - 2\log(x) + 4\text{Chi}(bx)(\cosh(bx) - bx\sinh(bx))}{4b^2}$$

input
```
Integrate[x*CoshIntegral[b*x]^2,x]
```

output
```
(Cosh[2*b*x] + 2*b^2*x^2*CoshIntegral[b*x]^2 - 2*CoshIntegral[2*b*x] - 2*L
og[x] + 4*CoshIntegral[b*x]*(Cosh[b*x] - b*x*Sinh[b*x]))/(4*b^2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {7091, 7097, 27, 3042, 26, 3044, 15, 7101, 27, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Chi}(bx)^2\,dx$$

$$\Big\downarrow \text{7091}$$

$$\frac{1}{2}x^2\text{Chi}(bx)^2 - \int x\cosh(bx)\text{Chi}(bx)dx$$

$$\Big\downarrow \text{7097}$$

$$\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b} + \int \frac{\cosh(bx)\sinh(bx)}{b}dx + \frac{1}{2}x^2\text{Chi}(bx)^2 - \frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b} + \frac{\int \cosh(bx)\sinh(bx)dx}{b} + \frac{1}{2}x^2\text{Chi}(bx)^2 - \frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b} + \frac{\int -i\cos(ibx)\sin(ibx)dx}{b} + \frac{1}{2}x^2\text{Chi}(bx)^2 - \frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b} - \frac{i\int \cos(ibx)\sin(ibx)dx}{b} + \frac{1}{2}x^2\text{Chi}(bx)^2 - \frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{3044}$$

$$-\frac{\int i\sinh(bx)d(i\sinh(bx))}{b^2} + \frac{\int \text{Chi}(bx)\sinh(bx)dx}{b} + \frac{1}{2}x^2\text{Chi}(bx)^2 - \frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}+\frac{\sinh^2(bx)}{2b^2}+\frac{1}{2}x^2\text{Chi}(bx)^2-\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{7101}$$

$$\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\int \frac{\cosh^2(bx)}{bx}dx}{b}+\frac{\sinh^2(bx)}{2b^2}+\frac{1}{2}x^2\text{Chi}(bx)^2-\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int \frac{\cosh^2(bx)}{x}dx}{b}}{b}+\frac{\sinh^2(bx)}{2b^2}+\frac{1}{2}x^2\text{Chi}(bx)^2-\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)^2}{x}dx}{b}}{b}+\frac{\sinh^2(bx)}{2b^2}+\frac{1}{2}x^2\text{Chi}(bx)^2-\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int \left(\frac{\cosh(2bx)}{2x}+\frac{1}{2x}\right)dx}{b}}{b}+\frac{\sinh^2(bx)}{2b^2}+\frac{1}{2}x^2\text{Chi}(bx)^2-\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sinh^2(bx)}{2b^2}+\frac{1}{2}x^2\text{Chi}(bx)^2-\frac{x\text{Chi}(bx)\sinh(bx)}{b}+\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}}{b}}{b}$$

input
```
Int[x*CoshIntegral[b*x]^2,x]
```

output
```
(x^2*CoshIntegral[b*x]^2)/2 + ((Cosh[b*x]*CoshIntegral[b*x])/b - (CoshInte
gral[2*b*x]/2 + Log[x]/2)/b)/b - (x*CoshIntegral[b*x]*Sinh[b*x])/b + Sinh[
b*x]^2/(2*b^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7091
```
Int[CoshIntegral[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Cos
hIntegral[b*x]^2/(m + 1)), x] - Simp[2/(m + 1)   Int[x^m*Cosh[b*x]*CoshInte
gral[b*x], x], x] /; FreeQ[b, x] && IGtQ[m, 0]
```

rule 7097
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7101
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{b^2x^2\operatorname{Chi}(bx)^2}{2}-2\operatorname{Chi}(bx)\left(\frac{bx\sinh(bx)}{2}-\frac{\cosh(bx)}{2}\right)+\frac{\cosh(bx)^2}{2}-\frac{\ln(bx)}{2}-\frac{\operatorname{Chi}(2bx)}{2}}{b^2}$	62
default	$\frac{\frac{b^2x^2\operatorname{Chi}(bx)^2}{2}-2\operatorname{Chi}(bx)\left(\frac{bx\sinh(bx)}{2}-\frac{\cosh(bx)}{2}\right)+\frac{\cosh(bx)^2}{2}-\frac{\ln(bx)}{2}-\frac{\operatorname{Chi}(2bx)}{2}}{b^2}$	62

input
```
int(x*Chi(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*b^2*x^2*Chi(b*x)^2-2*Chi(b*x)*(1/2*b*x*sinh(b*x)-1/2*cosh(b*x))
+1/2*cosh(b*x)^2-1/2*ln(b*x)-1/2*Chi(2*b*x))
```

Fricas [F]

$$\int x\operatorname{Chi}(bx)^2\,dx=\int x\operatorname{Chi}(bx)^2\,dx$$

input
```
integrate(x*Chi(b*x)^2,x, algorithm="fricas")
```

output
```
integral(x*cosh_integral(b*x)^2, x)
```

Sympy [F]

$$\int x\text{Chi}(bx)^2\,dx = \int x\,\text{Chi}^2\,(bx)\;dx$$

input
```
integrate(x*Chi(b*x)**2,x)
```

output
```
Integral(x*Chi(b*x)**2, x)
```

Maxima [F]

$$\int x\text{Chi}(bx)^2\,dx = \int x\text{Chi}(bx)^2\;dx$$

input
```
integrate(x*Chi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x*Chi(b*x)^2, x)
```

Giac [F]

$$\int x\text{Chi}(bx)^2\,dx = \int x\text{Chi}(bx)^2\;dx$$

input
```
integrate(x*Chi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x*Chi(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{Chi}(bx)^2\,dx = \int x\,\text{coshint}(b\,x)^2\,dx$$

input
```
int(x*coshint(b*x)^2,x)
```

output
```
int(x*coshint(b*x)^2, x)
```

Reduce [F]

$$\int x\text{Chi}(bx)^2\,dx = \int \chi(bx)^2\,x dx$$

input
```
int(x*Chi(b*x)^2,x)
```

output
```
int(chi(b*x)**2*x,x)
```

3.81 $\int \text{Chi}(bx)^2\,dx$

Optimal result

Integrand size = 6, antiderivative size = 31

$$\int \text{Chi}(bx)^2\,dx = x\text{Chi}(bx)^2 - \frac{2\text{Chi}(bx)\sinh(bx)}{b} + \frac{\text{Shi}(2bx)}{b}$$

output
```
x*Chi(b*x)^2-2*Chi(b*x)*sinh(b*x)/b+Shi(2*b*x)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \text{Chi}(bx)^2\,dx = x\text{Chi}(bx)^2 - \frac{2\text{Chi}(bx)\sinh(bx)}{b} + \frac{\text{Shi}(2bx)}{b}$$

input
```
Integrate[CoshIntegral[b*x]^2,x]
```

output
```
x*CoshIntegral[b*x]^2 - (2*CoshIntegral[b*x]*Sinh[b*x])/b + SinhIntegral[2
*b*x]/b
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {7089, 7095, 27, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Chi}(bx)^2 \, dx$$

$$\Big\downarrow \text{7089}$$

$$x\text{Chi}(bx)^2 - 2\int \cosh(bx)\text{Chi}(bx)dx$$

$$\Big\downarrow \text{7095}$$

$$x\text{Chi}(bx)^2 - 2\left(\frac{\text{Chi}(bx)\sinh(bx)}{b} - \int \frac{\cosh(bx)\sinh(bx)}{bx}dx\right)$$

$$\Big\downarrow \text{27}$$

$$x\text{Chi}(bx)^2 - 2\left(\frac{\text{Chi}(bx)\sinh(bx)}{b} - \frac{\int \frac{\cosh(bx)\sinh(bx)}{x}dx}{b}\right)$$

$$\Big\downarrow \text{5971}$$

$$x\text{Chi}(bx)^2 - 2\left(\frac{\text{Chi}(bx)\sinh(bx)}{b} - \frac{\int \frac{\sinh(2bx)}{2x}dx}{b}\right)$$

$$\Big\downarrow \text{27}$$

$$x\text{Chi}(bx)^2 - 2\left(\frac{\text{Chi}(bx)\sinh(bx)}{b} - \frac{\int \frac{\sinh(2bx)}{x}dx}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$x\text{Chi}(bx)^2 - 2\left(\frac{\text{Chi}(bx)\sinh(bx)}{b} - \frac{\int -\frac{i\sin(2ibx)}{x}dx}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$x\text{Chi}(bx)^2 - 2\left(\frac{\text{Chi}(bx)\sinh(bx)}{b} + \frac{i\int \frac{\sin(2ibx)}{x}dx}{2b}\right)$$

$$\Big\downarrow 3779$$

$$x\text{Chi}(bx)^2 - 2\left(\frac{\text{Chi}(bx)\sinh(bx)}{b} - \frac{\text{Shi}(2bx)}{2b}\right)$$

input
```
Int[CoshIntegral[b*x]^2,x]
```

output
```
x*CoshIntegral[b*x]^2 - 2*((CoshIntegral[b*x]*Sinh[b*x])/b - SinhIntegral[
2*b*x]/(2*b))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7089
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(CoshIn
tegral[a + b*x]^2/b), x] - Simp[2   Int[Cosh[a + b*x]*CoshIntegral[a + b*x]
, x], x] /; FreeQ[{a, b}, x]
```

rule 7095
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{\mathrm{Chi}(bx)^2 bx-2\ \mathrm{Chi}(bx)\sinh(bx)+\mathrm{Shi}(2bx)}{b}$	30
default	$\frac{\mathrm{Chi}(bx)^2 bx-2\ \mathrm{Chi}(bx)\sinh(bx)+\mathrm{Shi}(2bx)}{b}$	30

input
```
int(Chi(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(Chi(b*x)^2*b*x-2*Chi(b*x)*sinh(b*x)+Shi(2*b*x))
```

Fricas [F]

$$\int \mathrm{Chi}(bx)^2\,dx=\int \mathrm{Chi}(bx)^2\ dx$$

input
```
integrate(Chi(b*x)^2,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x)^2, x)
```

Sympy [F]

$$\int \text{Chi}(bx)^2\,dx = \int \text{Chi}^2\left(bx\right)\,dx$$

input
```
integrate(Chi(b*x)**2,x)
```

output
```
Integral(Chi(b*x)**2, x)
```

Maxima [F]

$$\int \text{Chi}(bx)^2\,dx = \int \text{Chi}(bx)^2\,dx$$

input
```
integrate(Chi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x)^2, x)
```

Giac [F]

$$\int \text{Chi}(bx)^2\,dx = \int \text{Chi}(bx)^2\,dx$$

input
```
integrate(Chi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(Chi(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \mathrm{Chi}(bx)^2\,dx = \int \mathrm{coshint}(b\,x)^2\,dx$$

input
```
int(coshint(b*x)^2,x)
```

output
```
int(coshint(b*x)^2, x)
```

Reduce [F]

$$\int \mathrm{Chi}(bx)^2\,dx = \int \chi(bx)^2\,dx$$

input
```
int(Chi(b*x)^2,x)
```

output
```
int(chi(b*x)**2,x)
```

3.82 $\int \frac{\text{Chi}(bx)^2}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{Chi}(bx)^2}{x} dx = \text{Int}\left(\frac{\text{Chi}(bx)^2}{x}, x\right)$$

output
```
Defer(Int)(Chi(b*x)^2/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Chi}(bx)^2}{x} dx = \int \frac{\text{Chi}(bx)^2}{x} dx$$

input
```
Integrate[CoshIntegral[b*x]^2/x,x]
```

output
```
Integrate[CoshIntegral[b*x]^2/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(bx)^2}{x} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\text{Chi}(bx)^2}{x}dx$$

input
```
Int[CoshIntegral[b*x]^2/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{Chi}\left(bx\right)^2}{x}dx$$

input
```
int(Chi(b*x)^2/x,x)
```

output
```
int(Chi(b*x)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Chi}(bx)^2}{x}\,dx = \int \frac{\text{Chi}(bx)^2}{x}\,dx$$

input
```
integrate(Chi(b*x)^2/x,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\text{Chi}(bx)^2}{x}\,dx = \int \frac{\text{Chi}^2\,(bx)}{x}\,dx$$

input
```
integrate(Chi(b*x)**2/x,x)
```

output
```
Integral(Chi(b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Chi}(bx)^2}{x}\,dx = \int \frac{\text{Chi}(bx)^2}{x}\,dx$$

input
```
integrate(Chi(b*x)^2/x,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x)^2/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Chi}(bx)^2}{x}\,dx = \int \frac{\text{Chi}(bx)^2}{x}\,dx$$

input
```
integrate(Chi(b*x)^2/x,x, algorithm="giac")
```

output
```
integrate(Chi(b*x)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Chi}(bx)^2}{x}\,dx = \int \frac{\text{coshint}(b\,x)^2}{x}\,dx$$

input
```
int(coshint(b*x)^2/x,x)
```

output
```
int(coshint(b*x)^2/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Chi}(bx)^2}{x} \, dx = \int \frac{\chi(bx)^2}{x} dx$$

input
```
int(Chi(b*x)^2/x,x)
```

output
```
int(chi(b*x)**2/x,x)
```

3.83 $\int \frac{\text{Chi}(bx)^2}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{Chi}(bx)^2}{x^2}\,dx = \text{Int}\left(\frac{\text{Chi}(bx)^2}{x^2},x\right)$$

output
```
Defer(Int)(Chi(b*x)^2/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Chi}(bx)^2}{x^2}\,dx = \int \frac{\text{Chi}(bx)^2}{x^2}\,dx$$

input
```
Integrate[CoshIntegral[b*x]^2/x^2,x]
```

output
```
Integrate[CoshIntegral[b*x]^2/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(bx)^2}{x^2} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\text{Chi}(bx)^2}{x^2}dx$$

input
```
Int[CoshIntegral[b*x]^2/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{Chi}\left(bx\right)^{2}}{x^{2}}dx$$

input
```
int(Chi(b*x)^2/x^2,x)
```

output
```
int(Chi(b*x)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Chi}(bx)^2}{x^2}\,dx = \int \frac{\mathrm{Chi}(bx)^2}{x^2}\,dx$$

input
```
integrate(Chi(b*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{Chi}(bx)^2}{x^2}\,dx = \int \frac{\mathrm{Chi}^2(bx)}{x^2}\,dx$$

input
```
integrate(Chi(b*x)**2/x**2,x)
```

output
```
Integral(Chi(b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Chi}(bx)^2}{x^2}\,dx = \int \frac{\mathrm{Chi}(bx)^2}{x^2}\,dx$$

input
```
integrate(Chi(b*x)^2/x^2,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x)^2/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Chi}(bx)^2}{x^2}\,dx = \int \frac{\mathrm{Chi}(bx)^2}{x^2}\,dx$$

input
```
integrate(Chi(b*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate(Chi(b*x)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Chi}(bx)^2}{x^2}\,dx = \int \frac{\mathrm{coshint}(b\,x)^2}{x^2}\,dx$$

input
```
int(coshint(b*x)^2/x^2,x)
```

output
```
int(coshint(b*x)^2/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Chi}(bx)^2}{x^2}\,dx = \int \frac{\chi(bx)^2}{x^2}dx$$

input
```
int(Chi(b*x)^2/x^2,x)
```

output
```
int(chi(b*x)**2/x**2,x)
```

3.84 $\int \frac{\text{Chi}(bx)^2}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{Chi}(bx)^2}{x^3}\,dx = \text{Int}\left(\frac{\text{Chi}(bx)^2}{x^3},x\right)$$

output
```
Defer(Int)(Chi(b*x)^2/x^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Chi}(bx)^2}{x^3}\,dx = \int \frac{\text{Chi}(bx)^2}{x^3}\,dx$$

input
```
Integrate[CoshIntegral[b*x]^2/x^3,x]
```

output
```
Integrate[CoshIntegral[b*x]^2/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(bx)^2}{x^3} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\text{Chi}(bx)^2}{x^3}dx$$

input
```
Int[CoshIntegral[b*x]^2/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{Chi}\left(bx\right)^{2}}{x^{3}}dx$$

input
```
int(Chi(b*x)^2/x^3,x)
```

output
```
int(Chi(b*x)^2/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Chi}(bx)^2}{x^3}\,dx = \int \frac{\mathrm{Chi}(bx)^2}{x^3}\,dx$$

input
```
integrate(Chi(b*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x)^2/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{Chi}(bx)^2}{x^3}\,dx = \int \frac{\mathrm{Chi}^2\,(bx)}{x^3}\,dx$$

input
```
integrate(Chi(b*x)**2/x**3,x)
```

output
```
Integral(Chi(b*x)**2/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Chi}(bx)^2}{x^3}\,dx = \int \frac{\mathrm{Chi}(bx)^2}{x^3}\,dx$$

input
```
integrate(Chi(b*x)^2/x^3,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x)^2/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Chi}(bx)^2}{x^3}\,dx = \int \frac{\mathrm{Chi}(bx)^2}{x^3}\,dx$$

input
```
integrate(Chi(b*x)^2/x^3,x, algorithm="giac")
```

output
```
integrate(Chi(b*x)^2/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Chi}(bx)^2}{x^3}\,dx = \int \frac{\mathrm{coshint}(b\,x)^2}{x^3}\,dx$$

input
```
int(coshint(b*x)^2/x^3,x)
```

output
```
int(coshint(b*x)^2/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Chi}(bx)^2}{x^3}\,dx = \int \frac{\chi(bx)^2}{x^3}dx$$

input
```
int(Chi(b*x)^2/x^3,x)
```

output
```
int(chi(b*x)**2/x**3,x)
```

3.85 $\int x^m \mathbf{Chi}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \text{Chi}(a+bx)\,dx = \frac{x^{1+m}\text{Chi}(a+bx)}{1+m} - \frac{b\text{Int}\left(\frac{x^{1+m}\cosh(a+bx)}{a+bx},x\right)}{1+m}$$

output
```
x^(1+m)*Chi(b*x+a)/(1+m)-b*Defer(Int)(x^(1+m)*cosh(b*x+a)/(b*x+a),x)/(1+m)
```

Mathematica [N/A]

Not integrable

Time = 6.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Chi}(a+bx)\,dx = \int x^m \text{Chi}(a+bx)\,dx$$

input
```
Integrate[x^m*CoshIntegral[a + b*x],x]
```

output
```
Integrate[x^m*CoshIntegral[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{Chi}(a+bx) \, dx$$

$$\Big\downarrow \text{7087}$$

$$\frac{x^{m+1}\text{Chi}(a+bx)}{m+1}-\frac{b\int \frac{x^{m+1}\cosh(a+bx)}{a+bx}dx}{m+1}$$

$$\Big\downarrow \text{7299}$$

$$\frac{x^{m+1}\text{Chi}(a+bx)}{m+1}-\frac{b\int \frac{x^{m+1}\cosh(a+bx)}{a+bx}dx}{m+1}$$

input
```
Int[x^m*CoshIntegral[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \text{Chi}\,(bx+a)\,dx$$

input
```
int(x^m*Chi(b*x+a),x)
```

output
```
int(x^m*Chi(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Chi}(a+bx)\,dx = \int x^m \text{Chi}(bx+a)\,dx$$

input
```
integrate(x^m*Chi(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^m*cosh_integral(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \text{Chi}(a+bx)\,dx = \int x^m \,\text{Chi}\,(a+bx)\,dx$$

input
```
integrate(x**m*Chi(b*x+a),x)
```

output
```
Integral(x**m*Chi(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Chi}(a+bx)\,dx = \int x^m \text{Chi}(bx+a)\,dx$$

input
```
integrate(x^m*Chi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^m*Chi(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Chi}(a+bx)\,dx = \int x^m \text{Chi}(bx+a)\;dx$$

input
```
integrate(x^m*Chi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*Chi(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 4.55 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Chi}(a+bx)\,dx = \int x^m\,\text{coshint}(a+b\,x)\;dx$$

input
```
int(x^m*coshint(a + b*x),x)
```

output
```
int(x^m*coshint(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{Chi}(a+bx)\,dx = \int x^m \chi(bx+a)\,dx$$

input

```
int(x^m*Chi(b*x+a),x)
```

output

```
int(x**m*chi(a + b*x),x)
```

3.86 $\int x^3\textbf{Chi}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 184

$$\int x^3\text{Chi}(a+bx)\,dx=\frac{3\cosh(a+bx)}{2b^4}+\frac{a^2\cosh(a+bx)}{4b^4}-\frac{ax\cosh(a+bx)}{2b^3}+\frac{3x^2\cosh(a+bx)}{4b^2}-\frac{a^4\text{Chi}(a+bx)}{4b^4}+\frac{1}{4}x^4\text{Chi}(a+bx)+\frac{a\sinh(a+bx)}{2b^4}+\frac{a^3\sinh(a+bx)}{4b^4}-\frac{3x\sinh(a+bx)}{2b^3}-\frac{a^2x\sinh(a+bx)}{4b^3}+\frac{ax^2\sinh(a+bx)}{4b^2}-\frac{x^3\sinh(a+bx)}{4b}$$

output

```
3/2*cosh(b*x+a)/b^4+1/4*a^2*cosh(b*x+a)/b^4-1/2*a*x*cosh(b*x+a)/b^3+3/4*x^
2*cosh(b*x+a)/b^2-1/4*a^4*Chi(b*x+a)/b^4+1/4*x^4*Chi(b*x+a)+1/2*a*sinh(b*x
+a)/b^4+1/4*a^3*sinh(b*x+a)/b^4-3/2*x*sinh(b*x+a)/b^3-1/4*a^2*x*sinh(b*x+a
)/b^3+1/4*a*x^2*sinh(b*x+a)/b^2-1/4*x^3*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.51

$$\int x^3 \text{Chi}(a+bx)\,dx$$
$$= \frac{(6+a^2-2abx+3b^2x^2)\cosh(a+bx)+(-a^4+b^4x^4)\,\text{Chi}(a+bx)+(2a+a^3-6bx-a^2bx+ab^2x^2-b}{4b^4}$$

input
```
Integrate[x^3*CoshIntegral[a + b*x],x]
```

output
```
((6 + a^2 - 2*a*b*x + 3*b^2*x^2)*Cosh[a + b*x] + (-a^4 + b^4*x^4)*CoshInte
gral[a + b*x] + (2*a + a^3 - 6*b*x - a^2*b*x + a*b^2*x^2 - b^3*x^3)*Sinh[a
 + b*x])/(4*b^4)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {7087, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{Chi}(a+bx)\,dx$$

$$\Big\downarrow \text{ 7087}$$

$$\frac{1}{4}x^4\text{Chi}(a+bx)-\frac{1}{4}b\int \frac{x^4\cosh(a+bx)}{a+bx}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{1}{4}x^4\text{Chi}(a+bx)-$$
$$\frac{1}{4}b\int \left(\frac{\cosh(a+bx)a^4}{b^4(a+bx)}-\frac{\cosh(a+bx)a^3}{b^4}+\frac{x\cosh(a+bx)a^2}{b^3}-\frac{x^2\cosh(a+bx)a}{b^2}+\frac{x^3\cosh(a+bx)}{b}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}x^4\text{Chi}(a+bx)-\frac{1}{4}b\left(\frac{a^4\text{Chi}(a+bx)}{b^5}-\frac{a^3\sinh(a+bx)}{b^5}-\frac{a^2\cosh(a+bx)}{b^5}+\frac{a^2x\sinh(a+bx)}{b^4}-\frac{2a\sinh(a+bx)}{b^5}-\frac{6\cosh(a+bx)}{b^5}\right.$$

input
```
Int[x^3*CoshIntegral[a + b*x],x]
```

output
```
(x^4*CoshIntegral[a + b*x])/4 - (b*((-6*Cosh[a + b*x])/b^5 - (a^2*Cosh[a +
 b*x])/b^5 + (2*a*x*Cosh[a + b*x])/b^4 - (3*x^2*Cosh[a + b*x])/b^3 + (a^4*
CoshIntegral[a + b*x])/b^5 - (2*a*Sinh[a + b*x])/b^5 - (a^3*Sinh[a + b*x])
/b^5 + (6*x*Sinh[a + b*x])/b^4 + (a^2*x*Sinh[a + b*x])/b^4 - (a*x^2*Sinh[a
 + b*x])/b^3 + (x^3*Sinh[a + b*x])/b^2))/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7087
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(CoshIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Cosh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84

method	result
parts	$\frac{x^4 \operatorname{Chi}(bx+a)}{4} - \frac{a^4 \operatorname{Chi}(bx+a) - 4a^3 \sinh(bx+a) + 6a^2((bx+a)\sinh(bx+a) - \cosh(bx+a)) - 4a\left((bx+a)^2 \sinh(bx+a) - \right.}{}$
derivativedivides	$\frac{\frac{\operatorname{Chi}(bx+a) b^4 x^4}{4} - \frac{a^4 \operatorname{Chi}(bx+a)}{4} + a^3 \sinh(bx+a) - \frac{3a^2((bx+a)\sinh(bx+a) - \cosh(bx+a))}{2} + a\left((bx+a)^2 \sinh(bx+a) - 2(bx+a)\right.}{b^4}$
default	$\frac{\frac{\operatorname{Chi}(bx+a) b^4 x^4}{4} - \frac{a^4 \operatorname{Chi}(bx+a)}{4} + a^3 \sinh(bx+a) - \frac{3a^2((bx+a)\sinh(bx+a) - \cosh(bx+a))}{2} + a\left((bx+a)^2 \sinh(bx+a) - 2(bx+a)\right.}{b^4}$

input
```
int(x^3*Chi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*Chi(b*x+a)-1/4/b^4*(a^4*Chi(b*x+a)-4*a^3*sinh(b*x+a)+6*a^2*((b*x+a
)*sinh(b*x+a)-cosh(b*x+a))-4*a*((b*x+a)^2*sinh(b*x+a)-2*(b*x+a)*cosh(b*x+a
)+2*sinh(b*x+a))+(b*x+a)^3*sinh(b*x+a)-3*(b*x+a)^2*cosh(b*x+a)+6*(b*x+a)*s
inh(b*x+a)-6*cosh(b*x+a))
```

Fricas [F]

$$\int x^3 \operatorname{Chi}(a+bx)\,dx = \int x^3 \operatorname{Chi}(bx+a)\,dx$$

input
```
integrate(x^3*Chi(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^3*cosh_integral(b*x + a), x)
```

Sympy [F]

$$\int x^3\text{Chi}(a+bx)\,dx = \int x^3\,\text{Chi}\,(a+bx)\;dx$$

input
```
integrate(x**3*Chi(b*x+a),x)
```

output
```
Integral(x**3*Chi(a + b*x), x)
```

Maxima [F]

$$\int x^3\text{Chi}(a+bx)\,dx = \int x^3\text{Chi}(bx+a)\;dx$$

input
```
integrate(x^3*Chi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^3*Chi(b*x + a), x)
```

Giac [F]

$$\int x^3\text{Chi}(a+bx)\,dx = \int x^3\text{Chi}(bx+a)\;dx$$

input
```
integrate(x^3*Chi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*Chi(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\text{Chi}(a+bx)\,dx=\int x^3\,\text{coshint}(a+b\,x)\,dx$$

input
```
int(x^3*coshint(a + b*x),x)
```

output
```
int(x^3*coshint(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.78

$$\int x^3\text{Chi}(a+bx)\,dx$$
$$=\frac{-\chi(bx+a)\,a^4+\chi(bx+a)\,b^4x^4+\cosh(bx+a)\,a^2-2\cosh(bx+a)\,abx+3\cosh(bx+a)\,b^2x^2+6\cosh}{}$$

input
```
int(x^3*Chi(b*x+a),x)
```

output
```
( - chi(a + b*x)*a**4 + chi(a + b*x)*b**4*x**4 + cosh(a + b*x)*a**2 - 2*co
sh(a + b*x)*a*b*x + 3*cosh(a + b*x)*b**2*x**2 + 6*cosh(a + b*x) + sinh(a +
 b*x)*a**3 - sinh(a + b*x)*a**2*b*x + sinh(a + b*x)*a*b**2*x**2 + 2*sinh(a
 + b*x)*a - sinh(a + b*x)*b**3*x**3 - 6*sinh(a + b*x)*b*x)/(4*b**4)
```

3.87 $\int x^2 \mathbf{Chi}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 118

$$\int x^2 \text{Chi}(a+bx)\,dx = -\frac{a\cosh(a+bx)}{3b^3} + \frac{2x\cosh(a+bx)}{3b^2} + \frac{a^3\text{Chi}(a+bx)}{3b^3} + \frac{1}{3}x^3\text{Chi}(a+bx) - \frac{2\sinh(a+bx)}{3b^3} - \frac{a^2\sinh(a+bx)}{3b^3} + \frac{ax\sinh(a+bx)}{3b^2} - \frac{x^2\sinh(a+bx)}{3b}$$

output
```
-1/3*a*cosh(b*x+a)/b^3+2/3*x*cosh(b*x+a)/b^2+1/3*a^3*Chi(b*x+a)/b^3+1/3*x^
3*Chi(b*x+a)-2/3*sinh(b*x+a)/b^3-1/3*a^2*sinh(b*x+a)/b^3+1/3*a*x*sinh(b*x+
a)/b^2-1/3*x^2*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.54

$$\int x^2 \text{Chi}(a+bx)\,dx = -\frac{(a-2bx)\cosh(a+bx) - (a^3+b^3x^3)\,\text{Chi}(a+bx) + (2+a^2-abx+b^2x^2)\sinh(a+bx)}{3b^3}$$

input
```
Integrate[x^2*CoshIntegral[a + b*x],x]
```

output
```
-1/3*((a - 2*b*x)*Cosh[a + b*x] - (a^3 + b^3*x^3)*CoshIntegral[a + b*x] +
(2 + a^2 - a*b*x + b^2*x^2)*Sinh[a + b*x])/b^3
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {7087, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{Chi}(a+bx)\,dx$$

$$\Big\downarrow \text{7087}$$

$$\frac{1}{3}x^3\text{Chi}(a+bx)-\frac{1}{3}b\int \frac{x^3\cosh(a+bx)}{a+bx}dx$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{3}x^3\text{Chi}(a+bx)- \frac{1}{3}b\int \left(-\frac{\cosh(a+bx)a^3}{b^3(a+bx)}+\frac{\cosh(a+bx)a^2}{b^3}-\frac{x\cosh(a+bx)a}{b^2}+\frac{x^2\cosh(a+bx)}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\text{Chi}(a+bx)- \frac{1}{3}b\left(-\frac{a^3\text{Chi}(a+bx)}{b^4}+\frac{a^2\sinh(a+bx)}{b^4}+\frac{2\sinh(a+bx)}{b^4}+\frac{a\cosh(a+bx)}{b^4}-\frac{ax\sinh(a+bx)}{b^3}-\frac{2x\cosh(a+bx)}{b^3}\right.$$

input
```
Int[x^2*CoshIntegral[a + b*x],x]
```

output
```
(x^3*CoshIntegral[a + b*x])/3 - (b*((a*Cosh[a + b*x])/b^4 - (2*x*Cosh[a +
b*x])/b^3 - (a^3*CoshIntegral[a + b*x])/b^4 + (2*Sinh[a + b*x])/b^4 + (a^2
*Sinh[a + b*x])/b^4 - (a*x*Sinh[a + b*x])/b^3 + (x^2*Sinh[a + b*x])/b^2))/
3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7087
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(CoshIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Cosh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

method	result
parts	$\frac{x^3 \operatorname{Chi}(bx+a)}{3} - \frac{-a^3 \operatorname{Chi}(bx+a)+3a^2 \sinh(bx+a)-3a((bx+a)\sinh(bx+a)-\cosh(bx+a))+(bx+a)^2 \sinh(bx+a)-2(b}{3b^3}$
derivativedivides	$\frac{\frac{\operatorname{Chi}(bx+a) b^3 x^3}{3}+\frac{a^3 \operatorname{Chi}(bx+a)}{3}-a^2 \sinh(bx+a)+a((bx+a)\sinh(bx+a)-\cosh(bx+a))-\frac{(bx+a)^2 \sinh(bx+a)}{3}+\frac{2(bx+a)\cosh}{3}}{b^3}$
default	$\frac{\frac{\operatorname{Chi}(bx+a) b^3 x^3}{3}+\frac{a^3 \operatorname{Chi}(bx+a)}{3}-a^2 \sinh(bx+a)+a((bx+a)\sinh(bx+a)-\cosh(bx+a))-\frac{(bx+a)^2 \sinh(bx+a)}{3}+\frac{2(bx+a)\cosh}{3}}{b^3}$

input
```
int(x^2*Chi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*Chi(b*x+a)-1/3/b^3*(-a^3*Chi(b*x+a)+3*a^2*sinh(b*x+a)-3*a*((b*x+a)
*sinh(b*x+a)-cosh(b*x+a))+(b*x+a)^2*sinh(b*x+a)-2*(b*x+a)*cosh(b*x+a)+2*si
nh(b*x+a))
```

Fricas [F]

$$\int x^2\text{Chi}(a+bx)\,dx=\int x^2\text{Chi}(bx+a)\;dx$$

input
```
integrate(x^2*Chi(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^2*cosh_integral(b*x + a), x)
```

Sympy [F]

$$\int x^2\text{Chi}(a+bx)\,dx=\int x^2\,\text{Chi}\,(a+bx)\;dx$$

input
```
integrate(x**2*Chi(b*x+a),x)
```

output
```
Integral(x**2*Chi(a + b*x), x)
```

Maxima [F]

$$\int x^2\text{Chi}(a+bx)\,dx=\int x^2\text{Chi}(bx+a)\;dx$$

input
```
integrate(x^2*Chi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^2*Chi(b*x + a), x)
```

Giac [F]

$$\int x^2 \mathrm{Chi}(a+bx)\,dx = \int x^2 \mathrm{Chi}(bx+a)\,dx$$

input
```
integrate(x^2*Chi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*Chi(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \mathrm{Chi}(a+bx)\,dx = \int x^2\,\mathrm{coshint}(a+b\,x)\,dx$$

input
```
int(x^2*coshint(a + b*x),x)
```

output
```
int(x^2*coshint(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.77

$$\int x^2 \mathrm{Chi}(a+bx)\,dx$$
$$= \frac{\chi(bx+a)\,a^3 + \chi(bx+a)\,b^3x^3 - \cosh(bx+a)\,a + 2\cosh(bx+a)\,bx - \sinh(bx+a)\,a^2 + \sinh(bx+a)\,a}{3b^3}$$

input
```
int(x^2*Chi(b*x+a),x)
```

output
```
(chi(a + b*x)*a**3 + chi(a + b*x)*b**3*x**3 - cosh(a + b*x)*a + 2*cosh(a +
 b*x)*b*x - sinh(a + b*x)*a**2 + sinh(a + b*x)*a*b*x - sinh(a + b*x)*b**2*
x**2 - 2*sinh(a + b*x))/(3*b**3)
```

3.88 $\int x\textbf{Chi}(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 62

$$\int x\text{Chi}(a+bx)\,dx=\frac{\cosh(a+bx)}{2b^2}-\frac{a^2\text{Chi}(a+bx)}{2b^2}+\frac{1}{2}x^2\text{Chi}(a+bx)+\frac{(a-bx)\sinh(a+bx)}{2b^2}$$

output
```
1/2*cosh(b*x+a)/b^2-1/2*a^2*Chi(b*x+a)/b^2+1/2*x^2*Chi(b*x+a)+1/2*(-b*x+a)
*sinh(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int x\text{Chi}(a+bx)\,dx=\frac{\cosh(a+bx)+(-a^2+b^2x^2)\,\text{Chi}(a+bx)+(a-bx)\sinh(a+bx)}{2b^2}$$

input
```
Integrate[x*CoshIntegral[a + b*x],x]
```

output
```
(Cosh[a + b*x] + (-a^2 + b^2*x^2)*CoshIntegral[a + b*x] + (a - b*x)*Sinh[a
 + b*x])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {7087, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Chi}(a+bx)\,dx$$

$$\Big\downarrow \text{ 7087}$$

$$\frac{1}{2}x^2\text{Chi}(a+bx)-\frac{1}{2}b\int \frac{x^2\cosh(a+bx)}{a+bx}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{1}{2}x^2\text{Chi}(a+bx)-\frac{1}{2}b\int \left(\frac{\cosh(a+bx)a^2}{b^2(a+bx)}-\frac{\cosh(a+bx)a}{b^2}+\frac{x\cosh(a+bx)}{b}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}x^2\text{Chi}(a+bx)-\frac{1}{2}b\left(\frac{a^2\text{Chi}(a+bx)}{b^3}-\frac{a\sinh(a+bx)}{b^3}-\frac{\cosh(a+bx)}{b^3}+\frac{x\sinh(a+bx)}{b^2}\right)$$

input
```
Int[x*CoshIntegral[a + b*x],x]
```

output
```
(x^2*CoshIntegral[a + b*x])/2 - (b*(-(Cosh[a + b*x]/b^3) + (a^2*CoshIntegr
al[a + b*x])/b^3 - (a*Sinh[a + b*x])/b^3 + (x*Sinh[a + b*x])/b^2))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7087
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(CoshIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Cosh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

method	result	size
parts	$\frac{x^2 \operatorname{Chi}(bx+a)}{2} - \frac{a^2 \operatorname{Chi}(bx+a) - 2a\sinh(bx+a) + (bx+a)\sinh(bx+a) - \cosh(bx+a)}{2b^2}$	58
derivativedivides	$\frac{\operatorname{Chi}(bx+a)\left(\frac{(bx+a)^2}{2} - (bx+a)a\right) + a\sinh(bx+a) - \frac{(bx+a)\sinh(bx+a)}{2} + \frac{\cosh(bx+a)}{2}}{b^2}$	60
default	$\frac{\operatorname{Chi}(bx+a)\left(\frac{(bx+a)^2}{2} - (bx+a)a\right) + a\sinh(bx+a) - \frac{(bx+a)\sinh(bx+a)}{2} + \frac{\cosh(bx+a)}{2}}{b^2}$	60

input
```
int(x*Chi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*Chi(b*x+a)-1/2/b^2*(a^2*Chi(b*x+a)-2*a*sinh(b*x+a)+(b*x+a)*sinh(b*
x+a)-cosh(b*x+a))
```

Fricas [F]

$$\int x\operatorname{Chi}(a+bx)\,dx = \int x\operatorname{Chi}(bx+a)\,dx$$

input
```
integrate(x*Chi(b*x+a),x, algorithm="fricas")
```

output
```
integral(x*cosh_integral(b*x + a), x)
```

Sympy [F]

$$\int x\text{Chi}(a+bx)\,dx=\int x\,\text{Chi}\,(a+bx)\;dx$$

input
```
integrate(x*Chi(b*x+a),x)
```

output
```
Integral(x*Chi(a + b*x), x)
```

Maxima [F]

$$\int x\text{Chi}(a+bx)\,dx=\int x\text{Chi}(bx+a)\;dx$$

input
```
integrate(x*Chi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*Chi(b*x + a), x)
```

Giac [F]

$$\int x\text{Chi}(a+bx)\,dx=\int x\text{Chi}(bx+a)\;dx$$

input
```
integrate(x*Chi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*Chi(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{Chi}(a+bx)\,dx = \frac{x^2\,\text{coshint}(a+b\,x)}{2} + \frac{\frac{e^{-a-b\,x}\,(e^{2\,a+2\,b\,x}-a+a\,e^{2\,a+2\,b\,x}-2\,a^2\,\text{coshint}(a+b\,x)\,e^{a+b\,x}+1)}{4}+\frac{b\,e^{-a-b\,x}\,(x-x\,e^{2\,a+2\,b\,x})}{4}}{b^2}$$

input
```
int(x*coshint(a + b*x),x)
```

output
```
(x^2*coshint(a + b*x))/2 + ((exp(- a - b*x)*(exp(2*a + 2*b*x) - a + a*exp(
2*a + 2*b*x) - 2*a^2*coshint(a + b*x)*exp(a + b*x) + 1))/4 + (b*exp(- a -
b*x)*(x - x*exp(2*a + 2*b*x)))/4)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int x\text{Chi}(a+bx)\,dx = \frac{-\chi(bx+a)\,a^2+\chi(bx+a)\,b^2x^2+\cosh(bx+a)+\sinh(bx+a)\,a-\sinh(bx+a)\,bx}{2b^2}$$

input
```
int(x*Chi(b*x+a),x)
```

output
```
( - chi(a + b*x)*a**2 + chi(a + b*x)*b**2*x**2 + cosh(a + b*x) + sinh(a +
b*x)*a - sinh(a + b*x)*b*x)/(2*b**2)
```

3.89 $\int \text{Chi}(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 27

$$\int \text{Chi}(a+bx)\,dx = \frac{(a+bx)\text{Chi}(a+bx)}{b} - \frac{\sinh(a+bx)}{b}$$

output
```
(b*x+a)*Chi(b*x+a)/b-sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \text{Chi}(a+bx)\,dx = \frac{a\text{Chi}(a+bx)}{b} + x\text{Chi}(a+bx) - \frac{\cosh(bx)\sinh(a)}{b} - \frac{\cosh(a)\sinh(bx)}{b}$$

input
```
Integrate[CoshIntegral[a + b*x],x]
```

output
```
(a*CoshIntegral[a + b*x])/b + x*CoshIntegral[a + b*x] - (Cosh[b*x]*Sinh[a]
)/b - (Cosh[a]*Sinh[b*x])/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7083}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Chi}(a+bx)\,dx$$

$$\Big\downarrow \text{7083}$$

$$\frac{(a+bx)\text{Chi}(a+bx)}{b}-\frac{\sinh(a+bx)}{b}$$

input
```
Int[CoshIntegral[a + b*x],x]
```

output
```
((a + b*x)*CoshIntegral[a + b*x])/b - Sinh[a + b*x]/b
```

Defintions of rubi rules used

rule 7083
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(CoshInte
gral[a + b*x]/b), x] - Simp[Sinh[a + b*x]/b, x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\text{Chi}(bx+a)(bx+a)-\sinh(bx+a)}{b}$	26
default	$\frac{\text{Chi}(bx+a)(bx+a)-\sinh(bx+a)}{b}$	26
parts	$x\ \text{Chi}\left(bx+a\right)-\frac{\sinh(bx+a)-\text{Chi}(bx+a)a}{b}$	31

input
```
int(Chi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(Chi(b*x+a)*(b*x+a)-sinh(b*x+a))
```

Fricas [F]

$$\int \mathrm{Chi}(a+bx)\,dx = \int \mathrm{Chi}(bx+a)\ dx$$

input
```
integrate(Chi(b*x+a),x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x + a), x)
```

Sympy [F]

$$\int \mathrm{Chi}(a+bx)\,dx = \int \mathrm{Chi}\,(a+bx)\ dx$$

input
```
integrate(Chi(b*x+a),x)
```

output
```
Integral(Chi(a + b*x), x)
```

Maxima [F]

$$\int \mathrm{Chi}(a+bx)\,dx = \int \mathrm{Chi}(bx+a)\ dx$$

input
```
integrate(Chi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(Chi(b*x + a), x)
```

Giac [F]

$$\int \mathrm{Chi}(a+bx)\,dx = \int \mathrm{Chi}(bx+a)\,dx$$

input
```
integrate(Chi(b*x+a),x, algorithm="giac")
```

output
```
integrate(Chi(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \mathrm{Chi}(a+bx)\,dx = x\,\mathrm{coshint}(a+b\,x) - \frac{\mathrm{e}^{a+b\,x}}{2\,b} + \frac{\mathrm{e}^{-a-b\,x}}{2\,b} + \frac{a\,\mathrm{coshint}(a+b\,x)}{b}$$

input
```
int(coshint(a + b*x),x)
```

output
```
x*coshint(a + b*x) - exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) + (a*coshin
t(a + b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \mathrm{Chi}(a+bx)\,dx = \frac{\chi(bx+a)\,a + \chi(bx+a)\,bx - \sinh(bx+a)}{b}$$

input
```
int(Chi(b*x+a),x)
```

output
```
(chi(a + b*x)*a + chi(a + b*x)*b*x - sinh(a + b*x))/b
```

3.90 $\int \frac{\mathbf{Chi}(a+bx)}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{Chi}(a+bx)}{x}\,dx = \text{Int}\left(\frac{\text{Chi}(a+bx)}{x},x\right)$$

output
```
Defer(Int)(Chi(b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Chi}(a+bx)}{x}\,dx = \int \frac{\text{Chi}(a+bx)}{x}\,dx$$

input
```
Integrate[CoshIntegral[a + b*x]/x,x]
```

output
```
Integrate[CoshIntegral[a + b*x]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(a+bx)}{x} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\text{Chi}(a+bx)}{x} dx$$

input
```
Int[CoshIntegral[a + b*x]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{Chi}\left(bx+a\right)}{x} dx$$

input
```
int(Chi(b*x+a)/x,x)
```

output
```
int(Chi(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Chi}(a+bx)}{x}\,dx=\int \frac{\mathrm{Chi}(bx+a)}{x}\,dx$$

input
```
integrate(Chi(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\mathrm{Chi}(a+bx)}{x}\,dx=\int \frac{\mathrm{Chi}\,(a+bx)}{x}\,dx$$

input
```
integrate(Chi(b*x+a)/x,x)
```

output
```
Integral(Chi(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{Chi}(a+bx)}{x}\,dx=\int \frac{\mathrm{Chi}(bx+a)}{x}\,dx$$

input
```
integrate(Chi(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Chi}(a+bx)}{x}\,dx = \int \frac{\text{Chi}(bx+a)}{x}\,dx$$

input
```
integrate(Chi(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(Chi(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{Chi}(a+bx)}{x}\,dx = \int \frac{\text{coshint}(a+b\,x)}{x}\,dx$$

input
```
int(coshint(a + b*x)/x,x)
```

output
```
int(coshint(a + b*x)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{Chi}(a+bx)}{x}\,dx = \int \frac{\chi(bx+a)}{x}dx$$

input
```
int(Chi(b*x+a)/x,x)
```

output
```
int(chi(a + b*x)/x,x)
```

3.91 $\int \frac{\text{Chi}(a+bx)}{x^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{\text{Chi}(a+bx)}{x^2} \, dx = \frac{b\cosh(a)\text{Chi}(bx)}{a} - \frac{b\text{Chi}(a+bx)}{a} - \frac{\text{Chi}(a+bx)}{x} + \frac{b\sinh(a)\text{Shi}(bx)}{a}$$

output
```
b*cosh(a)*Chi(b*x)/a-b*Chi(b*x+a)/a-Chi(b*x+a)/x+b*sinh(a)*Shi(b*x)/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{\text{Chi}(a+bx)}{x^2} \, dx = \frac{bx\cosh(a)\text{Chi}(bx) - (a+bx)\text{Chi}(a+bx) + bx\sinh(a)\text{Shi}(bx)}{ax}$$

input
```
Integrate[CoshIntegral[a + b*x]/x^2,x]
```

output
```
(b*x*Cosh[a]*CoshIntegral[b*x] - (a + b*x)*CoshIntegral[a + b*x] + b*x*Sin
h[a]*SinhIntegral[b*x])/(a*x)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {7087, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(a+bx)}{x^2} \, dx$$

$$\Big\downarrow \text{ 7087}$$

$$b\int \frac{\cosh(a+bx)}{x(a+bx)}dx-\frac{\text{Chi}(a+bx)}{x}$$

$$\Big\downarrow \text{ 7293}$$

$$b\int \left(\frac{\cosh(a+bx)}{ax}-\frac{b\cosh(a+bx)}{a(a+bx)}\right)dx-\frac{\text{Chi}(a+bx)}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$b\left(-\frac{\text{Chi}(a+bx)}{a}+\frac{\cosh(a)\text{Chi}(bx)}{a}+\frac{\sinh(a)\text{Shi}(bx)}{a}\right)-\frac{\text{Chi}(a+bx)}{x}$$

input
```
Int[CoshIntegral[a + b*x]/x^2,x]
```

output
```
-(CoshIntegral[a + b*x]/x) + b*((Cosh[a]*CoshIntegral[b*x])/a - CoshIntegr
al[a + b*x]/a + (Sinh[a]*SinhIntegral[b*x])/a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7087
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(CoshIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Cosh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\text{Chi}\left(bx+a\right)}{x^2}dx$$

input
```
int(Chi(b*x+a)/x^2,x)
```

output
```
int(Chi(b*x+a)/x^2,x)
```

Fricas [F]

$$\int \frac{\text{Chi}(a+bx)}{x^2}\,dx = \int \frac{\text{Chi}(bx+a)}{x^2}\,dx$$

input
```
integrate(Chi(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x + a)/x^2, x)
```

Sympy [F]

$$\int \frac{\text{Chi}(a+bx)}{x^2}\,dx = \int \frac{\text{Chi}\left(a+bx\right)}{x^2}\,dx$$

input
```
integrate(Chi(b*x+a)/x**2,x)
```

output
```
Integral(Chi(a + b*x)/x**2, x)
```

Maxima [F]

$$\int \frac{\text{Chi}(a+bx)}{x^2}\,dx = \int \frac{\text{Chi}(bx+a)}{x^2}\,dx$$

input
```
integrate(Chi(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x + a)/x^2, x)
```

Giac [F]

$$\int \frac{\text{Chi}(a+bx)}{x^2}\,dx = \int \frac{\text{Chi}(bx+a)}{x^2}\,dx$$

input
```
integrate(Chi(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(Chi(b*x + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{Chi}(a+bx)}{x^2}\,dx = \int \frac{\text{coshint}(a+b\,x)}{x^2}\,dx$$

input
```
int(coshint(a + b*x)/x^2,x)
```

output
```
int(coshint(a + b*x)/x^2, x)
```

Reduce [F]

$$\int \frac{\mathrm{Chi}(a+bx)}{x^2}\,dx = \int \frac{\chi(bx+a)}{x^2}dx$$

input
```
int(Chi(b*x+a)/x^2,x)
```

output
```
int(chi(a + b*x)/x**2,x)
```

3.92 $\int \frac{\text{Chi}(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 111

$$\int \frac{\text{Chi}(a+bx)}{x^3}\,dx = -\frac{b\cosh(a+bx)}{2ax} - \frac{b^2\cosh(a)\text{Chi}(bx)}{2a^2} + \frac{b^2\text{Chi}(a+bx)}{2a^2} - \frac{\text{Chi}(a+bx)}{2x^2} + \frac{b^2\text{Chi}(bx)\sinh(a)}{2a} + \frac{b^2\cosh(a)\text{Shi}(bx)}{2a} - \frac{b^2\sinh(a)\text{Shi}(bx)}{2a^2}$$

output
```
-1/2*b*cosh(b*x+a)/a/x-1/2*b^2*cosh(a)*Chi(b*x)/a^2+1/2*b^2*Chi(b*x+a)/a^2
-1/2*Chi(b*x+a)/x^2+1/2*b^2*Chi(b*x)*sinh(a)/a+1/2*b^2*cosh(a)*Shi(b*x)/a-
1/2*b^2*sinh(a)*Shi(b*x)/a^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.72

$$\int \frac{\text{Chi}(a+bx)}{x^3}\,dx = \frac{(-a^2+b^2x^2)\,\text{Chi}(a+bx) + b^2x^2\text{Chi}(bx)(-\cosh(a)+a\sinh(a)) + bx(-a\cosh(a+bx)+bx(a\cosh(a) -}{2a^2x^2}$$

input
```
Integrate[CoshIntegral[a + b*x]/x^3,x]
```

output
```
((-a^2 + b^2*x^2)*CoshIntegral[a + b*x] + b^2*x^2*CoshIntegral[b*x]*(-Cosh
[a] + a*Sinh[a]) + b*x*(-(a*Cosh[a + b*x]) + b*x*(a*Cosh[a] - Sinh[a])*Sin
hIntegral[b*x]))/(2*a^2*x^2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.82, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {7087, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(a+bx)}{x^3}dx$$

$$\Big\downarrow \text{7087}$$

$$\frac{1}{2}b\int \frac{\cosh(a+bx)}{x^2(a+bx)}dx-\frac{\text{Chi}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{2}b\int \left(\frac{\cosh(a+bx)b^2}{a^2(a+bx)}-\frac{\cosh(a+bx)b}{a^2x}+\frac{\cosh(a+bx)}{ax^2}\right)dx-\frac{\text{Chi}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}b\left(\frac{b\text{Chi}(a+bx)}{a^2}-\frac{b\cosh(a)\text{Chi}(bx)}{a^2}-\frac{b\sinh(a)\text{Shi}(bx)}{a^2}+\frac{b\sinh(a)\text{Chi}(bx)}{a}+\frac{b\cosh(a)\text{Shi}(bx)}{a}-\frac{\cosh(a+bx)}{ax}\right)-\frac{\text{Chi}(a+bx)}{2x^2}$$

input
```
Int[CoshIntegral[a + b*x]/x^3,x]
```

output
```
-1/2*CoshIntegral[a + b*x]/x^2 + (b*(-(Cosh[a + b*x]/(a*x)) - (b*Cosh[a]*C
oshIntegral[b*x])/a^2 + (b*CoshIntegral[a + b*x])/a^2 + (b*CoshIntegral[b*
x]*Sinh[a])/a + (b*Cosh[a]*SinhIntegral[b*x])/a - (b*Sinh[a]*SinhIntegral[
b*x])/a^2))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7087
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^(m + 1)*(CoshIntegral[a + b*x]/(d*(m + 1))), x] - Simp[b/
(d*(m + 1))   Int[(c + d*x)^(m + 1)*(Cosh[a + b*x]/(a + b*x)), x], x] /; Fr
eeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\mathrm{Chi}\left(bx+a\right)}{x^3}dx$$

input
```
int(Chi(b*x+a)/x^3,x)
```

output
```
int(Chi(b*x+a)/x^3,x)
```

Fricas [F]

$$\int \frac{\mathrm{Chi}(a+bx)}{x^3}\,dx = \int \frac{\mathrm{Chi}(bx+a)}{x^3}\,dx$$

input
```
integrate(Chi(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x + a)/x^3, x)
```

Sympy [F]

$$\int \frac{\mathrm{Chi}(a+bx)}{x^3}\,dx = \int \frac{\mathrm{Chi}\,(a+bx)}{x^3}\,dx$$

input
```
integrate(Chi(b*x+a)/x**3,x)
```

output
```
Integral(Chi(a + b*x)/x**3, x)
```

Maxima [F]

$$\int \frac{\mathrm{Chi}(a+bx)}{x^3}\,dx = \int \frac{\mathrm{Chi}(bx+a)}{x^3}\,dx$$

input
```
integrate(Chi(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x + a)/x^3, x)
```

Giac [F]

$$\int \frac{\text{Chi}(a+bx)}{x^3} \, dx = \int \frac{\text{Chi}(bx+a)}{x^3} \, dx$$

input
```
integrate(Chi(b*x+a)/x^3,x, algorithm="giac")
```

output
```
integrate(Chi(b*x + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{Chi}(a+bx)}{x^3} \, dx = \int \frac{\text{coshint}(a+b\,x)}{x^3} \, dx$$

input
```
int(coshint(a + b*x)/x^3,x)
```

output
```
int(coshint(a + b*x)/x^3, x)
```

Reduce [F]

$$\int \frac{\text{Chi}(a+bx)}{x^3} \, dx = \int \frac{\chi(bx+a)}{x^3} dx$$

input
```
int(Chi(b*x+a)/x^3,x)
```

output
```
int(chi(a + b*x)/x**3,x)
```

3.93 $\int x^m \mathbf{Chi}(a+bx)^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \mathrm{Chi}(a+bx)^2\,dx = \mathrm{Int}\left(x^m \mathrm{Chi}(a+bx)^2, x\right)$$

output
```
Defer(Int)(x^m*Chi(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \mathrm{Chi}(a+bx)^2\,dx = \int x^m \mathrm{Chi}(a+bx)^2\,dx$$

input
```
Integrate[x^m*CoshIntegral[a + b*x]^2,x]
```

output
```
Integrate[x^m*CoshIntegral[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{Chi}(a+bx)^2 \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int x^m \text{Chi}(a+bx)^2 dx$$

input
```
Int[x^m*CoshIntegral[a + b*x]^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{Chi}\left(bx+a\right)^2 dx$$

input
```
int(x^m*Chi(b*x+a)^2,x)
```

output
```
int(x^m*Chi(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \mathrm{Chi}(a+bx)^2\,dx = \int x^m \mathrm{Chi}(bx+a)^2\,dx$$

input
```
integrate(x^m*Chi(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m*cosh_integral(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \mathrm{Chi}(a+bx)^2\,dx = \int x^m \,\mathrm{Chi}^2\,(a+bx)\,dx$$

input
```
integrate(x**m*Chi(b*x+a)**2,x)
```

output
```
Integral(x**m*Chi(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \mathrm{Chi}(a+bx)^2\,dx = \int x^m \mathrm{Chi}(bx+a)^2\,dx$$

input
```
integrate(x^m*Chi(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*Chi(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \text{Chi}(a+bx)^2 \, dx = \int x^m \text{Chi}(bx+a)^2 \, dx$$

input
```
integrate(x^m*Chi(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*Chi(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.93 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \text{Chi}(a+bx)^2 \, dx = \int x^m \operatorname{coshint}(a+b\,x)^2 \, dx$$

input
```
int(x^m*coshint(a + b*x)^2,x)
```

output
```
int(x^m*coshint(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \text{Chi}(a+bx)^2 \, dx = \int x^m \chi(bx+a)^2 \, dx$$

input
```
int(x^m*Chi(b*x+a)^2,x)
```

output
```
int(x**m*chi(a + b*x)**2,x)
```

3.94 $\int x^2\mathbf{Chi}(a+bx)^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 332

$$\begin{aligned}\int x^2\text{Chi}(a+bx)^2\,dx = &-\frac{2x}{3b^2}-\frac{a\cosh(2a+2bx)}{6b^3}-\frac{(a-bx)\cosh(2a+2bx)}{6b^3}\\&-\frac{2a\cosh(a+bx)\text{Chi}(a+bx)}{3b^3}+\frac{4x\cosh(a+bx)\text{Chi}(a+bx)}{3b^2}\\&+\frac{a^2(a+bx)\text{Chi}(a+bx)^2}{3b^3}-\frac{ax(a+bx)\text{Chi}(a+bx)^2}{3b^2}\\&+\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}+\frac{a\text{Chi}(2a+2bx)}{b^3}\\&+\frac{a\log(a+bx)}{b^3}-\frac{2\cosh(a+bx)\sinh(a+bx)}{3b^3}\\&-\frac{4\text{Chi}(a+bx)\sinh(a+bx)}{3b^3}-\frac{2a^2\text{Chi}(a+bx)\sinh(a+bx)}{3b^3}\\&+\frac{2ax\text{Chi}(a+bx)\sinh(a+bx)}{3b^2}-\frac{2x^2\text{Chi}(a+bx)\sinh(a+bx)}{3b}\\&-\frac{\sinh(2a+2bx)}{12b^3}+\frac{2\text{Shi}(2a+2bx)}{3b^3}+\frac{a^2\text{Shi}(2a+2bx)}{b^3}\end{aligned}$$

output
```
-2/3*x/b^2-1/6*a*cosh(2*b*x+2*a)/b^3-1/6*(-b*x+a)*cosh(2*b*x+2*a)/b^3-2/3*
a*cosh(b*x+a)*Chi(b*x+a)/b^3+4/3*x*cosh(b*x+a)*Chi(b*x+a)/b^2+1/3*a^2*(b*x
+a)*Chi(b*x+a)^2/b^3-1/3*a*x*(b*x+a)*Chi(b*x+a)^2/b^2+1/3*x^2*(b*x+a)*Chi(
b*x+a)^2/b+a*Chi(2*b*x+2*a)/b^3+a*ln(b*x+a)/b^3-2/3*cosh(b*x+a)*sinh(b*x+a
)/b^3-4/3*Chi(b*x+a)*sinh(b*x+a)/b^3-2/3*a^2*Chi(b*x+a)*sinh(b*x+a)/b^3+2/
3*a*x*Chi(b*x+a)*sinh(b*x+a)/b^2-2/3*x^2*Chi(b*x+a)*sinh(b*x+a)/b-1/12*sin
h(2*b*x+2*a)/b^3+2/3*Shi(2*b*x+2*a)/b^3+a^2*Shi(2*b*x+2*a)/b^3
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.48

$$\int x^2\text{Chi}(a+bx)^2\,dx$$
$$= \frac{-8a-8bx-4a\cosh(2(a+bx))+2bx\cosh(2(a+bx))+4(a^3+b^3x^3)\,\text{Chi}(a+bx)^2+12a\text{Chi}(2(a+bx)}{}$$

input
```
Integrate[x^2*CoshIntegral[a + b*x]^2,x]
```

output
```
(-8*a - 8*b*x - 4*a*Cosh[2*(a + b*x)] + 2*b*x*Cosh[2*(a + b*x)] + 4*(a^3 +
 b^3*x^3)*CoshIntegral[a + b*x]^2 + 12*a*CoshIntegral[2*(a + b*x)] + 12*a*
Log[a + b*x] - 8*CoshIntegral[a + b*x]*((a - 2*b*x)*Cosh[a + b*x] + (2 + a
^2 - a*b*x + b^2*x^2)*Sinh[a + b*x]) - 5*Sinh[2*(a + b*x)] + 8*SinhIntegra
l[2*(a + b*x)] + 12*a^2*SinhIntegral[2*(a + b*x)])/(12*b^3)
```

Rubi [A] (verified)

Time = 4.58 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.31, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 2.083$, Rules used = {7093, 7093, 7089, 7095, 5971, 27, 3042, 26, 3779, 7097, 6151, 7101, 3042, 3793, 2009, 7103, 7095, 5971, 27, 3042, 26, 3779, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{Chi}(a+bx)^2\,dx$$

$\downarrow$ 7093

$$-\frac{2}{3}\int x^2\cosh(a+bx)\text{Chi}(a+bx)dx-\frac{2a\int x\text{Chi}(a+bx)^2dx}{3b}+\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 7093

$$-\frac{2}{3}\int x^2\cosh(a+bx)\text{Chi}(a+bx)dx-\frac{2a\left(-\frac{a\int \text{Chi}(a+bx)^2dx}{2b}-\int x\cosh(a+bx)\text{Chi}(a+bx)dx+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}\right)}{3b}+\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 7089

$$-\frac{2}{3}\int x^2\cosh(a+bx)\text{Chi}(a+bx)dx-\frac{2a\left(-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\int \cosh(a+bx)\text{Chi}(a+bx)dx\right)}{2b}-\int x\cosh(a+bx)\text{Chi}(a+bx)dx+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}\right)}{3b}+\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 7095

$$-\frac{2}{3}\int x^2\cosh(a+bx)\text{Chi}(a+bx)dx-\frac{2a\left(-\int x\cosh(a+bx)\text{Chi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\int\frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}dx\right)\right)}{2b}+\frac{x(a+bx}{}\right.}{3b}+\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 5971

$$-\frac{2}{3}\int x^2\cosh(a+bx)\text{Chi}(a+bx)dx-\frac{2a\left(-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\int\frac{\sinh(2a+2bx)}{2(a+bx)}dx\right)\right)}{2b}-\int x\cosh(a+bx)\text{Chi}(a+bx)dx+\frac{x(a+bx)\text{Chi}(a-}{2b}\right.}{3b}+\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\Big\downarrow$ 27

$$\frac{2a\left(-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{1}{2}\int\frac{\sinh(2a+2bx)}{a+bx}dx\right)\right)}{2b}-\int x\cosh(a+bx)\text{Chi}(a+bx)dx+\frac{x(a+bx)\text{Chi}(}{2b}\right.}{3b}$$

$$-\frac{2}{3}\int x^2\cosh(a+bx)\text{Chi}(a+bx)dx-$$

$$\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\Big\downarrow$ 3042

$$\frac{2a\left(-\int x\cosh(a+bx)\text{Chi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{1}{2}\int-\frac{i\sin(2ia+2ibx)}{a+bx}dx\right)\right)}{2b}+\frac{x(a+bx)\text{C}}{2}\right.}{3b}$$

$$-\frac{2}{3}\int x^2\cosh(a+bx)\text{Chi}(a+bx)dx-$$

$$\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\Big\downarrow$ 26

$$\frac{2a\left(-\int x\cosh(a+bx)\text{Chi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}+\frac{1}{2}i\int\frac{\sin(2ia+2ibx)}{a+bx}dx\right)\right)}{2b}+\frac{x(a+bx)\text{Ch}}{2b}\right.}{3b}$$

$$-\frac{2}{3}\int x^2\cosh(a+bx)\text{Chi}(a+bx)dx-$$

$$\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\Big\downarrow$ 3779

$$-\frac{2a\left(-\int x\cosh(a+bx)\text{Chi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+}{2b}\right.}{3b}$$

$$\frac{2}{3}\int x^2\cosh(a+bx)\text{Chi}(a+bx)dx+\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\Big\downarrow$ 7097

$$-\frac{2a\left(\frac{\int \text{Chi}(a+bx)\sinh(a+bx)dx}{b}+\int \frac{x\cosh(a+bx)\sinh(a+bx)}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\int x\text{Chi}(a+bx)\sinh(a+bx)dx}{b}-\int \frac{x^2\cosh(a+bx)\sinh(a+bx)}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}\right)+\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 6151

$$-\frac{2a\left(\frac{\int \text{Chi}(a+bx)\sinh(a+bx)dx}{b}+\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+x(\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\int x\text{Chi}(a+bx)\sinh(a+bx)dx}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}\right)+\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 7101

$$-\frac{2a\left(\frac{\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\cosh^2(a+bx)}{a+bx}dx}{b}+\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2b}{2b}\right.\right.}{2b}\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\int x\text{Chi}(a+bx)\sinh(a+bx)dx}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}\right)+\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 3042

$$-\frac{2a\left(\frac{\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{a+bx}dx}{b}+\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a}{2}\right.\right.}{2b}\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\int x\text{Chi}(a+bx)\sinh(a+bx)dx}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}\right)+\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 3793

$$-\frac{2a\left(\frac{\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\int\left(\frac{\cosh(2a+2bx)}{2(a+bx)}+\frac{1}{2(a+bx)}\right)dx}{b}+\frac{1}{2}\int\frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}\right.\right.}{2b}\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\int x\text{Chi}(a+bx)\sinh(a+bx)dx}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}\right)+\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 2009

$$-\frac{2a\left(\frac{1}{2}\int\frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a-\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\int x\text{Chi}(a+bx)\sinh(a+bx)dx}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}\right)+\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 7103

$$-\frac{2a\left(\frac{1}{2}\int\frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a-\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\left(-\frac{\int\cosh(a+bx)\text{Chi}(a+bx)dx}{b}-\int\frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{C}}{}\right.$$

$$\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 7095

$$-\frac{2a\left(\frac{1}{2}\int\frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a-\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\int\frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}dx}{b}-\int\frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(}{a+}\right.$$

$$\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 5971

$$-\frac{2a\left(\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a-}{}\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\int \frac{\sinh(2a+2bx)}{2(a+bx)}dx}{b}-\int \frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx}{a+bx}\right.$$

$$\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 27

$$-\frac{2a\left(\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a-}{}\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{1}{2}\int \frac{\sinh(2a+2bx)}{a+bx}dx}{b}-\int \frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+b}{a+bx}\right.$$

$$\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 3042

$$-\frac{2a\left(\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a-}{}\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{1}{2}\int -\frac{i\sin(2ia+2ibx)}{a+bx}dx}{b}-\int \frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a-}{a+bx}\right.$$

$$\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 26

$$-\frac{2a\left(\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a-}{}\right.}{3b}$$

$$\frac{2}{3}\left(-\frac{2\left(-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}+\frac{1}{2}i\int\frac{\sin(2ia+2ibx)}{a+bx}dx}{b}-\int\frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+}{a+bx}\right.$$

$$\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 3779

$$-\frac{2}{3}\left(-\frac{2\left(-\int\frac{x\cosh^2(a+bx)}{a+bx}dx-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int\frac{x^2\sinh(2(a+bx)}{a+bx}\right.$$

$$\frac{2a\left(\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a+b}{}\right.}{3b}$$

$$\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 7292

$$-\frac{2}{3}\left(-\frac{2\left(-\int\frac{x\cosh^2(a+bx)}{a+bx}dx-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int\frac{x^2\sinh(2a+2bx)}{a+bx}\right.$$

$$\frac{2a\left(\frac{1}{2}\int \frac{x\sinh(2a+2bx)}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a+ba}{}\right.}{3b}$$

$$\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\downarrow$ 7293

$$-\frac{2}{3}\left(-\frac{1}{2}\int\left(\frac{\sinh(2a+2bx)a^2}{b^2(a+bx)}-\frac{\sinh(2a+2bx)a}{b^2}+\frac{x\sinh(2a+2bx)}{b}\right)dx-\frac{2\left(-\int\left(\frac{\cosh^2(a+bx)}{b}-\frac{a\cosh^2(a+bx)}{b(a+bx)}\right.\right.}{}\right.$$

$$\frac{2a\left(\frac{1}{2}\int\left(\frac{\sinh(2a+2bx)}{b}+\frac{a\sinh(2a+2bx)}{b(-a-bx)}\right)dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a}{2b}\right.}{3b}$$

$$\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

$\Big\downarrow$ 2009

$$-\frac{2}{3}\left(\frac{1}{2}\left(-\frac{a^2\text{Shi}(2a+2bx)}{b^3}+\frac{\sinh(2a+2bx)}{4b^3}+\frac{a\cosh(2a+2bx)}{2b^3}-\frac{x\cosh(2a+2bx)}{2b^2}\right)-\frac{2\left(\frac{a\text{Chi}(2a+2bx)}{2b^2}+\frac{a\,\text{lo}}{}\right.}{}\right.$$

$$\frac{2a\left(\frac{1}{2}\left(\frac{\cosh(2a+2bx)}{2b^2}-\frac{a\text{Shi}(2a+2bx)}{b^2}\right)-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}\right.}{3b}$$

$$\frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b}$$

input
```
Int[x^2*CoshIntegral[a + b*x]^2,x]
```

output
```
(x^2*(a + b*x)*CoshIntegral[a + b*x]^2)/(3*b) - (2*a*((x*(a + b*x)*CoshInt
egral[a + b*x]^2)/(2*b) + ((Cosh[a + b*x]*CoshIntegral[a + b*x])/b - CoshI
ntegral[2*a + 2*b*x]/(2*b) - Log[a + b*x]/(2*b))/b - (x*CoshIntegral[a + b
*x]*Sinh[a + b*x])/b + (Cosh[2*a + 2*b*x]/(2*b^2) - (a*SinhIntegral[2*a +
2*b*x])/b^2)/2 - (a*(((a + b*x)*CoshIntegral[a + b*x]^2)/b - 2*((CoshInteg
ral[a + b*x]*Sinh[a + b*x])/b - SinhIntegral[2*a + 2*b*x]/(2*b))))/(2*b)))
/(3*b) - (2*((x^2*CoshIntegral[a + b*x]*Sinh[a + b*x])/b + ((a*Cosh[2*a +
2*b*x])/(2*b^3) - (x*Cosh[2*a + 2*b*x])/(2*b^2) + Sinh[2*a + 2*b*x]/(4*b^3
) - (a^2*SinhIntegral[2*a + 2*b*x])/b^3)/2 - (2*(-1/2*x/b + (x*Cosh[a + b*
x]*CoshIntegral[a + b*x])/b + (a*CoshIntegral[2*a + 2*b*x])/(2*b^2) + (a*L
og[a + b*x])/(2*b^2) - (Cosh[a + b*x]*Sinh[a + b*x])/(2*b^2) - ((CoshInteg
ral[a + b*x]*Sinh[a + b*x])/b - SinhIntegral[2*a + 2*b*x]/(2*b))/b))/b))/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6151
```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Simp[1/2^p   Int[u*Si
nh[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

rule 7089
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(CoshIn
tegral[a + b*x]^2/b), x] - Simp[2   Int[Cosh[a + b*x]*CoshIntegral[a + b*x]
, x], x] /; FreeQ[{a, b}, x]
```

rule 7093
```
Int[CoshIntegral[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(a + b*x)*(c + d*x)^m*(CoshIntegral[a + b*x]^2/(b*(m + 1))), x] +
(-Simp[2/(m + 1)   Int[(c + d*x)^m*Cosh[a + b*x]*CoshIntegral[a + b*x], x],
 x] + Simp[(b*c - a*d)*(m/(b*(m + 1)))   Int[(c + d*x)^(m - 1)*CoshIntegral
[a + b*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

rule 7095
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7097
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7101
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7103
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int x^2 \operatorname{Chi}(bx+a)^2\,dx$$

input
```
int(x^2*Chi(b*x+a)^2,x)
```

output
```
int(x^2*Chi(b*x+a)^2,x)
```

Fricas [F]

$$\int x^2\operatorname{Chi}(a+bx)^2\,dx = \int x^2\operatorname{Chi}(bx+a)^2\,dx$$

input
```
integrate(x^2*Chi(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^2*cosh_integral(b*x + a)^2, x)
```

Sympy [F]

$$\int x^2\operatorname{Chi}(a+bx)^2\,dx = \int x^2 \operatorname{Chi}^2(a+bx)\,dx$$

input
```
integrate(x**2*Chi(b*x+a)**2,x)
```

output
```
Integral(x**2*Chi(a + b*x)**2, x)
```

Maxima [F]

$$\int x^2\text{Chi}(a+bx)^2\,dx=\int x^2\text{Chi}(bx+a)^2\,dx$$

input
```
integrate(x^2*Chi(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^2*Chi(b*x + a)^2, x)
```

Giac [F]

$$\int x^2\text{Chi}(a+bx)^2\,dx=\int x^2\text{Chi}(bx+a)^2\,dx$$

input
```
integrate(x^2*Chi(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*Chi(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{Chi}(a+bx)^2\,dx=\int x^2\,\text{coshint}(a+b\,x)^2\,dx$$

input
```
int(x^2*coshint(a + b*x)^2,x)
```

output
```
int(x^2*coshint(a + b*x)^2, x)
```

Reduce [F]

$$\int x^2\mathrm{Chi}(a+bx)^2\,dx=\int \chi(bx+a)^2\,x^2dx$$

input
```
int(x^2*Chi(b*x+a)^2,x)
```

output
```
int(chi(a + b*x)**2*x**2,x)
```

3.95 $\int x\text{Chi}(a+bx)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 154

$$\int x\text{Chi}(a+bx)^2\,dx=\frac{\cosh(2a+2bx)}{4b^2}+\frac{\cosh(a+bx)\text{Chi}(a+bx)}{b^2}-\frac{a(a+bx)\text{Chi}(a+bx)^2}{2b^2}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{\text{Chi}(2a+2bx)}{2b^2}-\frac{\log(a+bx)}{2b^2}+\frac{a\text{Chi}(a+bx)\sinh(a+bx)}{b^2}-\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{a\text{Shi}(2a+2bx)}{b^2}$$

output

```
1/4*cosh(2*b*x+2*a)/b^2+cosh(b*x+a)*Chi(b*x+a)/b^2-1/2*a*(b*x+a)*Chi(b*x+a
)^2/b^2+1/2*x*(b*x+a)*Chi(b*x+a)^2/b-1/2*Chi(2*b*x+2*a)/b^2-1/2*ln(b*x+a)/
b^2+a*Chi(b*x+a)*sinh(b*x+a)/b^2-x*Chi(b*x+a)*sinh(b*x+a)/b-a*Shi(2*b*x+2*
a)/b^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.62

$$\int x\text{Chi}(a+bx)^2\,dx$$
$$=\frac{\cosh(2(a+bx))-2(a^2-b^2x^2)\,\text{Chi}(a+bx)^2-2\text{Chi}(2(a+bx))-2\log(a+bx)+4\text{Chi}(a+bx)(\cosh(a}{4b^2}$$

input
```
Integrate[x*CoshIntegral[a + b*x]^2,x]
```

output
```
(Cosh[2*(a + b*x)] - 2*(a^2 - b^2*x^2)*CoshIntegral[a + b*x]^2 - 2*CoshInt
egral[2*(a + b*x)] - 2*Log[a + b*x] + 4*CoshIntegral[a + b*x]*(Cosh[a + b*
x] + (a - b*x)*Sinh[a + b*x]) - 4*a*SinhIntegral[2*(a + b*x)])/(4*b^2)
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.700, Rules used = {7093, 7089, 7095, 5971, 27, 3042, 26, 3779, 7097, 6151, 7101, 3042, 3793, 2009, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Chi}(a+bx)^2\,dx$$
$$\Big\downarrow \text{7093}$$
$$-\frac{a\int \text{Chi}(a+bx)^2dx}{2b}-\int x\cosh(a+bx)\text{Chi}(a+bx)dx+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}$$
$$\Big\downarrow \text{7089}$$
$$-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\int \cosh(a+bx)\text{Chi}(a+bx)dx\right)}{2b}-\int x\cosh(a+bx)\text{Chi}(a+bx)dx+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}$$
$$\Big\downarrow \text{7095}$$

$$-\int x\cosh(a+bx)\text{Chi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\int\frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}dx\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}$$

$\Big\downarrow$ 5971

$$-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\int\frac{\sinh(2a+2bx)}{2(a+bx)}dx\right)\right)}{2b}-\int x\cosh(a+bx)\text{Chi}(a+bx)dx+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}$$

$\Big\downarrow$ 27

$$-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{1}{2}\int\frac{\sinh(2a+2bx)}{a+bx}dx\right)\right)}{2b}-\int x\cosh(a+bx)\text{Chi}(a+bx)dx+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}$$

$\Big\downarrow$ 3042

$$-\int x\cosh(a+bx)\text{Chi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{1}{2}\int-\frac{i\sin(2ia+2ibx)}{a+bx}dx\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}$$

$\Big\downarrow$ 26

$$-\int x\cosh(a+bx)\text{Chi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}+\frac{1}{2}i\int\frac{\sin(2ia+2ibx)}{a+bx}dx\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}$$

$\Big\downarrow$ 3779

$$-\int x\cosh(a+bx)\text{Chi}(a+bx)dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}$$

$\Big\downarrow$ 7097

$$\frac{\int \text{Chi}(a+bx)\sinh(a+bx)dx}{b}+\int \frac{x\cosh(a+bx)\sinh(a+bx)}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{6151}$$

$$\frac{\int \text{Chi}(a+bx)\sinh(a+bx)dx}{b}+\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{7101}$$

$$\frac{\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\cosh^2(a+bx)}{a+bx}dx}{b}+\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{a+bx}dx}{b}+\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\int \left(\frac{\cosh(2a+2bx)}{2(a+bx)}+\frac{1}{2(a+bx)}\right)dx}{b}+\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{\log(a+bx)}{2b}}{b}$$

$\downarrow$ 7292

$$\frac{1}{2}\int \frac{x\sinh(2a+2bx)}{a+bx}dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{\log(a+bx)}{2b}}{b}$$

$\downarrow$ 7293

$$\frac{1}{2}\int \left(\frac{\sinh(2a+2bx)}{b}+\frac{a\sinh(2a+2bx)}{b(-a-bx)}\right)dx-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{\log(a+bx)}{2b}}{b}$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{\cosh(2a+2bx)}{2b^2}-\frac{a\text{Shi}(2a+2bx)}{b^2}\right)-\frac{a\left(\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)\right)}{2b}+\frac{x(a+bx)\text{Chi}(a+bx)^2}{2b}-\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}+\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{\log(a+bx)}{2b}}{b}$$

input
```
Int[x*CoshIntegral[a + b*x]^2,x]
```

output
```
(x*(a + b*x)*CoshIntegral[a + b*x]^2)/(2*b) + ((Cosh[a + b*x]*CoshIntegral
[a + b*x])/b - CoshIntegral[2*a + 2*b*x]/(2*b) - Log[a + b*x]/(2*b))/b - (
x*CoshIntegral[a + b*x]*Sinh[a + b*x])/b + (Cosh[2*a + 2*b*x]/(2*b^2) - (a
*SinhIntegral[2*a + 2*b*x])/b^2)/2 - (a*(((a + b*x)*CoshIntegral[a + b*x]^
2)/b - 2*((CoshIntegral[a + b*x]*Sinh[a + b*x])/b - SinhIntegral[2*a + 2*b
*x]/(2*b))))/(2*b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6151
```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Simp[1/2^p   Int[u*Si
nh[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

rule 7089
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(CoshIn
tegral[a + b*x]^2/b), x] - Simp[2   Int[Cosh[a + b*x]*CoshIntegral[a + b*x]
, x], x] /; FreeQ[{a, b}, x]
```

rule 7093
```
Int[CoshIntegral[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(a + b*x)*(c + d*x)^m*(CoshIntegral[a + b*x]^2/(b*(m + 1))), x] +
(-Simp[2/(m + 1)   Int[(c + d*x)^m*Cosh[a + b*x]*CoshIntegral[a + b*x], x],
 x] + Simp[(b*c - a*d)*(m/(b*(m + 1)))   Int[(c + d*x)^(m - 1)*CoshIntegral
[a + b*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

rule 7095
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7097
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7101
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{\operatorname{Chi}(bx+a)^2\left(\frac{(bx+a)^2}{2}-(bx+a)a\right)-2\operatorname{Chi}(bx+a)\left(-a\sinh(bx+a)+\frac{(bx+a)\sinh(bx+a)}{2}-\frac{\cosh(bx+a)}{2}\right)-a\operatorname{Shi}(2bx+2a)+}{b^2}$
default	$\frac{\operatorname{Chi}(bx+a)^2\left(\frac{(bx+a)^2}{2}-(bx+a)a\right)-2\operatorname{Chi}(bx+a)\left(-a\sinh(bx+a)+\frac{(bx+a)\sinh(bx+a)}{2}-\frac{\cosh(bx+a)}{2}\right)-a\operatorname{Shi}(2bx+2a)+}{b^2}$

input
```
int(x*Chi(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(Chi(b*x+a)^2*(1/2*(b*x+a)^2-(b*x+a)*a)-2*Chi(b*x+a)*(-a*sinh(b*x+a)
+1/2*(b*x+a)*sinh(b*x+a)-1/2*cosh(b*x+a))-a*Shi(2*b*x+2*a)+1/2*cosh(b*x+a)
^2-1/2*ln(b*x+a)-1/2*Chi(2*b*x+2*a))
```

Fricas [F]

$$\int x\operatorname{Chi}(a+bx)^2\,dx=\int x\operatorname{Chi}(bx+a)^2\,dx$$

input
```
integrate(x*Chi(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x*cosh_integral(b*x + a)^2, x)
```

Sympy [F]

$$\int x\operatorname{Chi}(a+bx)^2\,dx=\int x\operatorname{Chi}^2(a+bx)\,dx$$

input
```
integrate(x*Chi(b*x+a)**2,x)
```

output
```
Integral(x*Chi(a + b*x)**2, x)
```

Maxima [F]

$$\int x\text{Chi}(a+bx)^2\,dx = \int x\text{Chi}(bx+a)^2\,dx$$

input
```
integrate(x*Chi(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x*Chi(b*x + a)^2, x)
```

Giac [F]

$$\int x\text{Chi}(a+bx)^2\,dx = \int x\text{Chi}(bx+a)^2\,dx$$

input
```
integrate(x*Chi(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x*Chi(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{Chi}(a+bx)^2\,dx = \int x\,\text{coshint}(a+b\,x)^2\,dx$$

input
```
int(x*coshint(a + b*x)^2,x)
```

output
```
int(x*coshint(a + b*x)^2, x)
```

Reduce [F]

$$\int x\text{Chi}(a+bx)^2\,dx=\int \chi(bx+a)^2\,xdx$$

input
```
int(x*Chi(b*x+a)^2,x)
```

output
```
int(chi(a + b*x)**2*x,x)
```

3.96 $\int \mathrm{Chi}(a+bx)^2\,dx$

Optimal result

Integrand size = 8, antiderivative size = 48

$$\int \mathrm{Chi}(a+bx)^2\,dx=\frac{(a+bx)\mathrm{Chi}(a+bx)^2}{b}-\frac{2\mathrm{Chi}(a+bx)\sinh(a+bx)}{b}+\frac{\mathrm{Shi}(2a+2bx)}{b}$$

output
```
(b*x+a)*Chi(b*x+a)^2/b-2*Chi(b*x+a)*sinh(b*x+a)/b+Shi(2*b*x+2*a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \mathrm{Chi}(a+bx)^2\,dx=\frac{(a+bx)\mathrm{Chi}(a+bx)^2-2\mathrm{Chi}(a+bx)\sinh(a+bx)+\mathrm{Shi}(2(a+bx))}{b}$$

input
```
Integrate[CoshIntegral[a + b*x]^2,x]
```

output
```
((a + b*x)*CoshIntegral[a + b*x]^2 - 2*CoshIntegral[a + b*x]*Sinh[a + b*x]
 + SinhIntegral[2*(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {7089, 7095, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Chi}(a+bx)^2\,dx$$

$$\Big\downarrow \text{7089}$$

$$\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\int \cosh(a+bx)\text{Chi}(a+bx)dx$$

$$\Big\downarrow \text{7095}$$

$$\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\int \frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}dx\right)$$

$$\Big\downarrow \text{5971}$$

$$\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\int \frac{\sinh(2a+2bx)}{2(a+bx)}dx\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{1}{2}\int \frac{\sinh(2a+2bx)}{a+bx}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{1}{2}\int -\frac{i\sin(2ia+2ibx)}{a+bx}dx\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}+\frac{1}{2}i\int \frac{\sin(2ia+2ibx)}{a+bx}dx\right)$$

$$\Big\downarrow \text{3779}$$

$$\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-2\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\right)$$

input
```
Int[CoshIntegral[a + b*x]^2,x]
```

output
```
((a + b*x)*CoshIntegral[a + b*x]^2)/b - 2*((CoshIntegral[a + b*x]*Sinh[a +
 b*x])/b - SinhIntegral[2*a + 2*b*x]/(2*b))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7089
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(CoshIn
tegral[a + b*x]^2/b), x] - Simp[2   Int[Cosh[a + b*x]*CoshIntegral[a + b*x]
, x], x] /; FreeQ[{a, b}, x]
```

rule 7095
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\operatorname{Chi}(bx+a)^2(bx+a)-2\operatorname{Chi}(bx+a)\sinh(bx+a)+\operatorname{Shi}(2bx+2a)}{b}$	43
default	$\frac{\operatorname{Chi}(bx+a)^2(bx+a)-2\operatorname{Chi}(bx+a)\sinh(bx+a)+\operatorname{Shi}(2bx+2a)}{b}$	43

input
```
int(Chi(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(Chi(b*x+a)^2*(b*x+a)-2*Chi(b*x+a)*sinh(b*x+a)+Shi(2*b*x+2*a))
```

Fricas [F]

$$\int \operatorname{Chi}(a+bx)^2\,dx=\int \operatorname{Chi}(bx+a)^2\,dx$$

input
```
integrate(Chi(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x + a)^2, x)
```

Sympy [F]

$$\int \text{Chi}(a+bx)^2\,dx = \int \text{Chi}^2\,(a+bx)\,\,dx$$

input
```
integrate(Chi(b*x+a)**2,x)
```

output
```
Integral(Chi(a + b*x)**2, x)
```

Maxima [F]

$$\int \text{Chi}(a+bx)^2\,dx = \int \text{Chi}(bx+a)^2\,\,dx$$

input
```
integrate(Chi(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x + a)^2, x)
```

Giac [F]

$$\int \text{Chi}(a+bx)^2\,dx = \int \text{Chi}(bx+a)^2\,\,dx$$

input
```
integrate(Chi(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(Chi(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{Chi}(a+bx)^2\,dx=\int \operatorname{coshint}(a+b\,x)^2\,dx$$

input
```
int(coshint(a + b*x)^2,x)
```

output
```
int(coshint(a + b*x)^2, x)
```

Reduce [F]

$$\int \operatorname{Chi}(a+bx)^2\,dx=\int \chi(bx+a)^2\,dx$$

input
```
int(Chi(b*x+a)^2,x)
```

output
```
int(chi(a + b*x)**2,x)
```

3.97 $\int \frac{\mathbf{Chi}(a+bx)^2}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\text{Chi}(a+bx)^2}{x}\,dx = \text{Int}\left(\frac{\text{Chi}(a+bx)^2}{x},x\right)$$

output
```
Defer(Int)(Chi(b*x+a)^2/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Chi}(a+bx)^2}{x}\,dx = \int \frac{\text{Chi}(a+bx)^2}{x}\,dx$$

input
```
Integrate[CoshIntegral[a + b*x]^2/x,x]
```

output
```
Integrate[CoshIntegral[a + b*x]^2/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(a+bx)^2}{x} \, dx$$

$$\big\downarrow \; 7299$$

$$\int \frac{\text{Chi}(a+bx)^2}{x}dx$$

input
```
Int[CoshIntegral[a + b*x]^2/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\text{Chi}\left(bx+a\right)^{2}}{x}dx$$

input
```
int(Chi(b*x+a)^2/x,x)
```

output
```
int(Chi(b*x+a)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(a+bx)^2}{x}\,dx = \int \frac{\mathrm{Chi}(bx+a)^2}{x}\,dx$$

input
```
integrate(Chi(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x + a)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\mathrm{Chi}(a+bx)^2}{x}\,dx = \int \frac{\mathrm{Chi}^2\,(a+bx)}{x}\,dx$$

input
```
integrate(Chi(b*x+a)**2/x,x)
```

output
```
Integral(Chi(a + b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(a+bx)^2}{x}\,dx = \int \frac{\mathrm{Chi}(bx+a)^2}{x}\,dx$$

input
```
integrate(Chi(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x + a)^2/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(a+bx)^2}{x}\,dx = \int \frac{\mathrm{Chi}(bx+a)^2}{x}\,dx$$

input
```
integrate(Chi(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(Chi(b*x + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(a+bx)^2}{x}\,dx = \int \frac{\mathrm{coshint}(a+b\,x)^2}{x}\,dx$$

input
```
int(coshint(a + b*x)^2/x,x)
```

output
```
int(coshint(a + b*x)^2/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(a+bx)^2}{x}\,dx = \int \frac{\chi(bx+a)^2}{x}dx$$

input
```
int(Chi(b*x+a)^2/x,x)
```

output
```
int(chi(a + b*x)**2/x,x)
```

3.98 $\int \frac{\text{Chi}(a+bx)^2}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\text{Chi}(a+bx)^2}{x^2}\,dx = \text{Int}\left(\frac{\text{Chi}(a+bx)^2}{x^2},x\right)$$

output
```
Defer(Int)(Chi(b*x+a)^2/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Chi}(a+bx)^2}{x^2}\,dx = \int \frac{\text{Chi}(a+bx)^2}{x^2}\,dx$$

input
```
Integrate[CoshIntegral[a + b*x]^2/x^2,x]
```

output
```
Integrate[CoshIntegral[a + b*x]^2/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(a+bx)^2}{x^2} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\text{Chi}(a+bx)^2}{x^2}dx$$

input
```
Int[CoshIntegral[a + b*x]^2/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\text{Chi}\left(bx+a\right)^{2}}{x^{2}}dx$$

input
```
int(Chi(b*x+a)^2/x^2,x)
```

output
```
int(Chi(b*x+a)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(a+bx)^2}{x^2}\,dx = \int \frac{\mathrm{Chi}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(Chi(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x + a)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{Chi}(a+bx)^2}{x^2}\,dx = \int \frac{\mathrm{Chi}^2\,(a+bx)}{x^2}\,dx$$

input
```
integrate(Chi(b*x+a)**2/x**2,x)
```

output
```
Integral(Chi(a + b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(a+bx)^2}{x^2}\,dx = \int \frac{\mathrm{Chi}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(Chi(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x + a)^2/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(a+bx)^2}{x^2}\,dx = \int \frac{\mathrm{Chi}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(Chi(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(Chi(b*x + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(a+bx)^2}{x^2}\,dx = \int \frac{\mathrm{coshint}(a+b\,x)^2}{x^2}\,dx$$

input
```
int(coshint(a + b*x)^2/x^2,x)
```

output
```
int(coshint(a + b*x)^2/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Chi}(a+bx)^2}{x^2}\,dx = \int \frac{\chi(bx+a)^2}{x^2}dx$$

input
```
int(Chi(b*x+a)^2/x^2,x)
```

output
```
int(chi(a + b*x)**2/x**2,x)
```

3.99 $\int \frac{\text{Chi}(a+bx)^2}{x^3} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\text{Chi}(a+bx)^2}{x^3} \, dx = \text{Int}\left(\frac{\text{Chi}(a+bx)^2}{x^3}, x\right)$$

output
```
Defer(Int)(Chi(b*x+a)^2/x^3,x)
```

Mathematica [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Chi}(a+bx)^2}{x^3} \, dx = \int \frac{\text{Chi}(a+bx)^2}{x^3} \, dx$$

input
```
Integrate[CoshIntegral[a + b*x]^2/x^3,x]
```

output
```
Integrate[CoshIntegral[a + b*x]^2/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(a+bx)^2}{x^3} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\text{Chi}(a+bx)^2}{x^3}dx$$

input
```
Int[CoshIntegral[a + b*x]^2/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\text{Chi}\left(bx+a\right)^{2}}{x^{3}}dx$$

input
```
int(Chi(b*x+a)^2/x^3,x)
```

output
```
int(Chi(b*x+a)^2/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(a+bx)^2}{x^3}\,dx = \int \frac{\mathrm{Chi}(bx+a)^2}{x^3}\,dx$$

input
```
integrate(Chi(b*x+a)^2/x^3,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x + a)^2/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{Chi}(a+bx)^2}{x^3}\,dx = \int \frac{\mathrm{Chi}^2\,(a+bx)}{x^3}\,dx$$

input
```
integrate(Chi(b*x+a)**2/x**3,x)
```

output
```
Integral(Chi(a + b*x)**2/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(a+bx)^2}{x^3}\,dx = \int \frac{\mathrm{Chi}(bx+a)^2}{x^3}\,dx$$

input
```
integrate(Chi(b*x+a)^2/x^3,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x + a)^2/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Chi}(a+bx)^2}{x^3}\,dx = \int \frac{\text{Chi}(bx+a)^2}{x^3}\,dx$$

input
```
integrate(Chi(b*x+a)^2/x^3,x, algorithm="giac")
```

output
```
integrate(Chi(b*x + a)^2/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 3.93 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Chi}(a+bx)^2}{x^3}\,dx = \int \frac{\text{coshint}(a+b\,x)^2}{x^3}\,dx$$

input
```
int(coshint(a + b*x)^2/x^3,x)
```

output
```
int(coshint(a + b*x)^2/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(a+bx)^2}{x^3}\,dx = \int \frac{\chi(bx+a)^2}{x^3}dx$$

input

```
int(Chi(b*x+a)^2/x^3,x)
```

output

```
int(chi(a + b*x)**2/x**3,x)
```

3.100 $\int x^2\mathbf{Chi}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 128

$$\begin{aligned}\int x^2\text{Chi}(d(a+b\log(cx^n)))\,dx &= \frac{1}{3}x^3\text{Chi}(d(a+b\log(cx^n)))\\ &\quad -\frac{1}{6}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\,\text{ExpIntegralEi}\left(\frac{(3-bdn)(a+b\log(cx^n))}{bn}\right)\\ &\quad -\frac{1}{6}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\,\text{ExpIntegralEi}\left(\frac{(3+bdn)(a+b\log(cx^n))}{bn}\right)\end{aligned}$$

output
```
1/3*x^3*Chi(d*(a+b*ln(c*x^n)))-1/6*x^3*Ei((-b*d*n+3)*(a+b*ln(c*x^n))/b/n)/
exp(3*a/b/n)/((c*x^n)^(3/n))-1/6*x^3*Ei((b*d*n+3)*(a+b*ln(c*x^n))/b/n)/exp
(3*a/b/n)/((c*x^n)^(3/n))
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.76

$$\int x^2\text{Chi}(d(a+b\log(cx^n)))\,dx=\frac{1}{6}x^3\Bigg(2\text{Chi}(d(a+b\log(cx^n)))$$
$$-e^{-\frac{3a}{bn}}(cx^n)^{-3/n}\left(\text{ExpIntegralEi}\left(-\frac{(-3+bdn)(a+b\log(cx^n))}{bn}\right)+\text{ExpIntegralEi}\left(\frac{(3+bdn)(a+b\log(cx^n))}{bn}\right.\right.$$

input
```
Integrate[x^2*CoshIntegral[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^3*(2*CoshIntegral[d*(a + b*Log[c*x^n])] - (ExpIntegralEi[-(((-3 + b*d*n
)*(a + b*Log[c*x^n]))/(b*n))] + ExpIntegralEi[((3 + b*d*n)*(a + b*Log[c*x^
n]))/(b*n)])/(E^((3*a)/(b*n))*(c*x^n)^(3/n))))/6
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {7110, 27, 6066, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{Chi}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{7110}$$

$$\frac{1}{3}x^3\text{Chi}(d(a+b\log(cx^n)))-\frac{1}{3}bdn\int\frac{x^2\cosh(d(a+b\log(cx^n)))}{d(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\text{Chi}(d(a+b\log(cx^n)))-\frac{1}{3}bn\int\frac{x^2\cosh(d(a+b\log(cx^n)))}{a+b\log(cx^n)}dx$$

$$\Big\downarrow \text{6066}$$

$$\frac{1}{3}x^3\text{Chi}(d(a+b\log(cx^n)))-$$
$$\frac{1}{3}bn\left(\frac{1}{2}e^{-ad}x^{bdn}(cx^n)^{-bd}\int\frac{x^{2-bdn}}{a+b\log(cx^n)}dx+\frac{1}{2}e^{ad}x^{-bdn}(cx^n)^{bd}\int\frac{x^{bdn+2}}{a+b\log(cx^n)}dx\right)$$

$\big\downarrow$ 2747

$$\frac{1}{3}x^3\text{Chi}(d(a+b\log(cx^n)))-$$
$$\frac{1}{3}bn\left(\frac{x^3e^{-ad}(cx^n)^{-3/n}\int\frac{(cx^n)^{\frac{3-bdn}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2n}+\frac{x^3e^{ad}(cx^n)^{bd-\frac{bdn+3}{n}}\int\frac{(cx^n)^{\frac{bdn+3}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2n}\right)$$

$\big\downarrow$ 2609

$$\frac{1}{3}x^3\text{Chi}(d(a+b\log(cx^n)))-$$
$$\frac{1}{3}bn\left(\frac{x^3(cx^n)^{-3/n}e^{a\left(d-\frac{3}{bn}\right)-ad}\text{ExpIntegralEi}\left(\frac{(3-bdn)(a+b\log(cx^n))}{bn}\right)}{2bn}+\frac{x^3e^{ad-a\left(\frac{3}{bn}+d\right)}(cx^n)^{bd-\frac{bdn+3}{n}}\text{ExpIntegralE}}{2bn}\right.$$

input
```
Int[x^2*CoshIntegral[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^3*CoshIntegral[d*(a + b*Log[c*x^n])])/3 - (b*n*((E^(-(a*d) + a*(d - 3/(
b*n)))*x^3*ExpIntegralEi[((3 - b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(2*b*n*(
c*x^n)^(3/n)) + (E^(a*d - a*(d + 3/(b*n)))*x^3*(c*x^n)^(b*d - (3 + b*d*n)/
n)*ExpIntegralEi[((3 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(2*b*n)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 6066
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*(((e_.) + Log[(g_.)*(
x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.), x_Symbol] :> Simp[((i*x)
^r*(1/((c*x^n)^(b*d)*(2*x^(r - b*d*n)))))/E^(a*d)   Int[x^(r - b*d*n)*(h*(e
 + f*Log[g*x^m]))^q, x], x] + Simp[E^(a*d)*(i*x)^r*((c*x^n)^(b*d)/(2*x^(r +
 b*d*n)))   Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, m, n, q, r}, x]
```

rule 7110
```
Int[CoshIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^
(m_.), x_Symbol] :> Simp[(e*x)^(m + 1)*(CoshIntegral[d*(a + b*Log[c*x^n])]/
(e*(m + 1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*(Cosh[d*(a + b*Log[c*
x^n])]/(d*(a + b*Log[c*x^n]))), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[m, -1]
```

Maple [F]

$$\int x^2 \operatorname{Chi}\left(d(a+b\ln\left(c\,x^n\right))\right) dx$$

input
```
int(x^2*Chi(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x^2*Chi(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x^2 \mathrm{Chi}(d(a+b\log\left(cx^n\right)))\ dx = \int x^2 \mathrm{Chi}((b\log\left(cx^n\right)+a)d)\ dx$$

input
```
integrate(x^2*Chi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(x^2*cosh_integral(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x^2\mathrm{Chi}(d(a+b\log(cx^n)))\,dx=\int x^2\,\mathrm{Chi}\left(ad+bd\log(cx^n)\right)\,dx$$

input
```
integrate(x**2*Chi(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x**2*Chi(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^2\mathrm{Chi}(d(a+b\log(cx^n)))\,dx=\int x^2\mathrm{Chi}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(x^2*Chi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x^2*Chi((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x^2\mathrm{Chi}(d(a+b\log(cx^n)))\,dx=\int x^2\mathrm{Chi}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(x^2*Chi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x^2*Chi((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \mathrm{Chi}(d(a+b\log(cx^n)))\,dx = \int x^2\,\mathrm{coshint}(d\,(a+b\,\ln(c\,x^n)))\,dx$$

input
```
int(x^2*coshint(d*(a + b*log(c*x^n))),x)
```

output
```
int(x^2*coshint(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x^2 \mathrm{Chi}(d(a+b\log(cx^n)))\,dx = \int \chi(\log(x^n c)\,bd+ad)\,x^2dx$$

input
```
int(x^2*Chi(d*(a+b*log(c*x^n))),x)
```

output
```
int(chi(log(x**n*c)*b*d + a*d)*x**2,x)
```

3.101 $\int x\text{Chi}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 128

$$\begin{aligned}\int x\text{Chi}(d(a+b\log(cx^n)))\,dx &= \frac{1}{2}x^2\text{Chi}(d(a+b\log(cx^n)))\\ &\quad-\frac{1}{4}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\text{ExpIntegralEi}\left(\frac{(2-bdn)(a+b\log(cx^n))}{bn}\right)\\ &\quad-\frac{1}{4}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\text{ExpIntegralEi}\left(\frac{(2+bdn)(a+b\log(cx^n))}{bn}\right)\end{aligned}$$

output
```
1/2*x^2*Chi(d*(a+b*ln(c*x^n)))-1/4*x^2*Ei((-b*d*n+2)*(a+b*ln(c*x^n))/b/n)/
exp(2*a/b/n)/((c*x^n)^(2/n))-1/4*x^2*Ei((b*d*n+2)*(a+b*ln(c*x^n))/b/n)/exp
(2*a/b/n)/((c*x^n)^(2/n))
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.76

$$\int x\text{Chi}(d(a+b\log(cx^n)))\,dx = \frac{1}{4}x^2\Bigg(2\text{Chi}(d(a+b\log(cx^n))) -e^{-\frac{2a}{bn}}(cx^n)^{-2/n}\left(\text{ExpIntegralEi}\left(-\frac{(-2+bdn)(a+b\log(cx^n))}{bn}\right)+\text{ExpIntegralEi}\left(\frac{(2+bdn)(a+b\log(cx^n))}{bn}\right)\right)\Bigg)$$

input
```
Integrate[x*CoshIntegral[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^2*(2*CoshIntegral[d*(a + b*Log[c*x^n])] - (ExpIntegralEi[-(((-2 + b*d*n
)*(a + b*Log[c*x^n]))/(b*n))] + ExpIntegralEi[((2 + b*d*n)*(a + b*Log[c*x^
n]))/(b*n)])/(E^((2*a)/(b*n))*(c*x^n)^(2/n))))/4
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7110, 27, 6066, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Chi}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{7110}$$

$$\frac{1}{2}x^2\text{Chi}(d(a+b\log(cx^n))) - \frac{1}{2}bdn\int \frac{x\cosh(d(a+b\log(cx^n)))}{d(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}x^2\text{Chi}(d(a+b\log(cx^n))) - \frac{1}{2}bn\int \frac{x\cosh(d(a+b\log(cx^n)))}{a+b\log(cx^n)}dx$$

$$\Big\downarrow \text{6066}$$

$$\frac{1}{2}x^2\text{Chi}(d(a+b\log(cx^n)))-$$
$$\frac{1}{2}bn\left(\frac{1}{2}e^{-ad}x^{bdn}(cx^n)^{-bd}\int\frac{x^{1-bdn}}{a+b\log(cx^n)}dx+\frac{1}{2}e^{ad}x^{-bdn}(cx^n)^{bd}\int\frac{x^{bdn+1}}{a+b\log(cx^n)}dx\right)$$

$$\Big\downarrow \text{2747}$$

$$\frac{1}{2}x^2\text{Chi}(d(a+b\log(cx^n)))-$$
$$\frac{1}{2}bn\left(\frac{x^2e^{-ad}(cx^n)^{-2/n}\int\frac{(cx^n)^{\frac{2-bdn}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2n}+\frac{x^2e^{ad}(cx^n)^{bd-\frac{bdn+2}{n}}\int\frac{(cx^n)^{\frac{bdn+2}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2n}\right)$$

$$\Big\downarrow \text{2609}$$

$$\frac{1}{2}x^2\text{Chi}(d(a+b\log(cx^n)))-$$
$$\frac{1}{2}bn\left(\frac{x^2(cx^n)^{-2/n}e^{a\left(d-\frac{2}{bn}\right)-ad}\text{ExpIntegralEi}\left(\frac{(2-bdn)(a+b\log(cx^n))}{bn}\right)}{2bn}+\frac{x^2e^{ad-a\left(\frac{2}{bn}+d\right)}(cx^n)^{bd-\frac{bdn+2}{n}}\text{ExpIntegralE}}{2bn}\right.$$

input
```
Int[x*CoshIntegral[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^2*CoshIntegral[d*(a + b*Log[c*x^n])])/2 - (b*n*((E^(-(a*d) + a*(d - 2/(
b*n)))*x^2*ExpIntegralEi[((2 - b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(2*b*n*(
c*x^n)^(2/n)) + (E^(a*d - a*(d + 2/(b*n)))*x^2*(c*x^n)^(b*d - (2 + b*d*n)/
n)*ExpIntegralEi[((2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(2*b*n)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 6066
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*(((e_.) + Log[(g_.)*(
x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.), x_Symbol] :> Simp[((i*x)
^r*(1/((c*x^n)^(b*d)*(2*x^(r - b*d*n)))))/E^(a*d)   Int[x^(r - b*d*n)*(h*(e
 + f*Log[g*x^m]))^q, x], x] + Simp[E^(a*d)*(i*x)^r*((c*x^n)^(b*d)/(2*x^(r +
 b*d*n)))   Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, m, n, q, r}, x]
```

rule 7110
```
Int[CoshIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^
(m_.), x_Symbol] :> Simp[(e*x)^(m + 1)*(CoshIntegral[d*(a + b*Log[c*x^n])]/
(e*(m + 1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*(Cosh[d*(a + b*Log[c*
x^n])]/(d*(a + b*Log[c*x^n]))), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[m, -1]
```

Maple [F]

$$\int x\ \mathrm{Chi}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

input
```
int(x*Chi(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x*Chi(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x\mathrm{Chi}(d(a+b\log\left(cx^{n}\right)))\ dx=\int x\mathrm{Chi}((b\log\left(cx^{n}\right)+a)d)\ dx$$

input
```
integrate(x*Chi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(x*cosh_integral(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x\text{Chi}(d(a+b\log(cx^n)))\,dx=\int x\,\text{Chi}\left(ad+bd\log\left(cx^n\right)\right)\,dx$$

input
```
integrate(x*Chi(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x*Chi(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x\text{Chi}(d(a+b\log(cx^n)))\,dx=\int x\text{Chi}((b\log\left(cx^n\right)+a)d)\,dx$$

input
```
integrate(x*Chi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x*Chi((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x\text{Chi}(d(a+b\log(cx^n)))\,dx=\int x\text{Chi}((b\log\left(cx^n\right)+a)d)\,dx$$

input
```
integrate(x*Chi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x*Chi((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{Chi}(d(a+b\log(cx^n)))\,dx=\int x\,\mathrm{coshint}(d\,(a+b\,\ln(c\,x^n)))\,dx$$

input
```
int(x*coshint(d*(a + b*log(c*x^n))),x)
```

output
```
int(x*coshint(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x\text{Chi}(d(a+b\log(cx^n)))\,dx=\int \chi(\log(x^nc)\,bd+ad)\,xdx$$

input
```
int(x*Chi(d*(a+b*log(c*x^n))),x)
```

output
```
int(chi(log(x**n*c)*b*d + a*d)*x,x)
```

3.102 $\int \mathrm{Chi}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 119

$$\begin{aligned}\int \mathrm{Chi}(d(a+b\log(cx^n)))\,dx &= x\mathrm{Chi}(d(a+b\log(cx^n))) \\ &\quad -\frac{1}{2}e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\,\mathrm{ExpIntegralEi}\left(\frac{(1-bdn)(a+b\log(cx^n))}{bn}\right) \\ &\quad -\frac{1}{2}e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\,\mathrm{ExpIntegralEi}\left(\frac{(1+bdn)(a+b\log(cx^n))}{bn}\right)\end{aligned}$$

output
```
x*Chi(d*(a+b*ln(c*x^n)))-1/2*x*Ei((-b*d*n+1)*(a+b*ln(c*x^n))/b/n)/exp(a/b/
n)/((c*x^n)^(1/n))-1/2*x*Ei((b*d*n+1)*(a+b*ln(c*x^n))/b/n)/exp(a/b/n)/((c*
x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.78

$$\int \text{Chi}(d(a+b\log(cx^n)))\,dx$$
$$= x\text{Chi}(d(a+b\log(cx^n))) - \frac{1}{2}e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\left(\text{ExpIntegralEi}\left(-\frac{(-1+bdn)(a+b\log(cx^n))}{bn}\right) + \text{ExpIntegralEi}\left(\frac{(1+bdn)(a+b\log(cx^n))}{bn}\right)\right)$$

input
```
Integrate[CoshIntegral[d*(a + b*Log[c*x^n])],x]
```

output
```
x*CoshIntegral[d*(a + b*Log[c*x^n])] - (x*(ExpIntegralEi[-(((-1 + b*d*n)*(
a + b*Log[c*x^n]))/(b*n))] + ExpIntegralEi[((1 + b*d*n)*(a + b*Log[c*x^n])
)/(b*n)]))/(2*E^(a/(b*n))*(c*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {7107, 27, 6064, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Chi}(d(a+b\log(cx^n)))\,dx$$
$$\Big\downarrow \text{7107}$$
$$x\text{Chi}(d(a+b\log(cx^n))) - bdn\int \frac{\cosh(d(a+b\log(cx^n)))}{d(a+b\log(cx^n))}dx$$
$$\Big\downarrow \text{27}$$
$$x\text{Chi}(d(a+b\log(cx^n))) - bn\int \frac{\cosh(d(a+b\log(cx^n)))}{a+b\log(cx^n)}dx$$

$$\Big\downarrow \; 6064$$

$$x\text{Chi}(d(a+b\log(cx^n)))-bn\left(\frac{1}{2}e^{-ad}x^{bdn}(cx^n)^{-bd}\int\frac{x^{-bdn}}{a+b\log(cx^n)}dx+\frac{1}{2}e^{ad}x^{-bdn}(cx^n)^{bd}\int\frac{x^{bdn}}{a+b\log(cx^n)}dx\right)$$

$$\Big\downarrow \; 2747$$

$$x\text{Chi}(d(a+b\log(cx^n)))-bn\left(\frac{xe^{-ad}(cx^n)^{-1/n}\int\frac{(cx^n)^{\frac{1-bdn}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2n}+\frac{xe^{ad}(cx^n)^{bd-\frac{bdn+1}{n}}\int\frac{(cx^n)^{\frac{bdn+1}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2n}\right)$$

$$\Big\downarrow \; 2609$$

$$x\text{Chi}(d(a+b\log(cx^n)))-bn\left(\frac{x(cx^n)^{-1/n}e^{a\left(d-\frac{1}{bn}\right)-ad}\text{ExpIntegralEi}\left(\frac{(1-bdn)(a+b\log(cx^n))}{bn}\right)}{2bn}+\frac{xe^{ad-a\left(\frac{1}{bn}+d\right)}(cx^n)^{bd-\frac{bdn+1}{n}}\text{ExpIntegralEi}\left(\cdots\right.}{2bn}\right.$$

input
```
Int[CoshIntegral[d*(a + b*Log[c*x^n])],x]
```

output
```
x*CoshIntegral[d*(a + b*Log[c*x^n])] - b*n*((E^(-(a*d) + a*(d - 1/(b*n)))*
x*ExpIntegralEi[((1 - b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(2*b*n*(c*x^n)^n^
(-1)) + (E^(a*d - a*(d + 1/(b*n)))*x*(c*x^n)^(b*d - (1 + b*d*n)/n)*ExpInte
gralEi[((1 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(2*b*n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 6064
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*(((e_.) + Log[(g_.)*(
x_)^(m_.)]*(f_.))*(h_.))^(q_.), x_Symbol] :> Simp[1/((c*x^n)^(b*d)*(2/x^(b*
d*n)))/E^(a*d)   Int[(h*(e + f*Log[g*x^m]))^q/x^(b*d*n), x], x] + Simp[E^(a
*d)*((c*x^n)^(b*d)/(2*x^(b*d*n)))   Int[x^(b*d*n)*(h*(e + f*Log[g*x^m]))^q,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, q}, x]
```

rule 7107
```
Int[CoshIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :>
 Simp[x*CoshIntegral[d*(a + b*Log[c*x^n])], x] - Simp[b*d*n   Int[Cosh[d*(a
 + b*Log[c*x^n])]/(d*(a + b*Log[c*x^n])), x], x] /; FreeQ[{a, b, c, d, n},
x]
```

Maple [F]

$$\int \mathrm{Chi}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

input
```
int(Chi(d*(a+b*ln(c*x^n))),x)
```

output
```
int(Chi(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int \mathrm{Chi}(d(a+b\log\left(cx^{n}\right)))\;dx=\int \mathrm{Chi}((b\log\left(cx^{n}\right)+a)d)\;dx$$

input
```
integrate(Chi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int \operatorname{Chi}(d(a + b\log(cx^n)))\, dx = \int \operatorname{Chi}(d(a + b\log(cx^n)))\, dx$$

input
```
integrate(Chi(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(Chi(d*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \operatorname{Chi}(d(a + b\log(cx^n)))\, dx = \int \operatorname{Chi}((b\log(cx^n) + a)d)\, dx$$

input
```
integrate(Chi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(Chi((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int \operatorname{Chi}(d(a + b\log(cx^n)))\, dx = \int \operatorname{Chi}((b\log(cx^n) + a)d)\, dx$$

input
```
integrate(Chi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(Chi((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \mathrm{Chi}(d(a+b\log(cx^n)))\,dx = \int \mathrm{coshint}(d\,(a+b\,\ln(c\,x^n)))\,dx$$

input
```
int(coshint(d*(a + b*log(c*x^n))),x)
```

output
```
int(coshint(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int \mathrm{Chi}(d(a+b\log(cx^n)))\,dx = \int \chi(\log(x^n c)\,bd + ad)\,dx$$

input
```
int(Chi(d*(a+b*log(c*x^n))),x)
```

output
```
int(chi(log(x**n*c)*b*d + a*d),x)
```

3.103 $\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x}\,dx = \frac{\text{Chi}(d(a+b\log(cx^n)))(a+b\log(cx^n))}{bn} - \frac{\sinh(d(a+b\log(cx^n)))}{bdn}$$

output
```
Chi(d*(a+b*ln(c*x^n)))*(a+b*ln(c*x^n))/b/n-sinh(d*(a+b*ln(c*x^n)))/b/d/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.75

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x}\,dx = \frac{a\text{Chi}(ad+bd\log(cx^n))}{bn} + \frac{\text{Chi}(d(a+b\log(cx^n)))\log(cx^n)}{n} - \frac{\cosh(bd\log(cx^n))\sinh(ad)}{bdn} - \frac{\cosh(ad)\sinh(bd\log(cx^n))}{bdn}$$

input
```
Integrate[CoshIntegral[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
(a*CoshIntegral[a*d + b*d*Log[c*x^n]])/(b*n) + (CoshIntegral[d*(a + b*Log[
c*x^n])]*Log[c*x^n])/n - (Cosh[b*d*Log[c*x^n]]*Sinh[a*d])/(b*d*n) - (Cosh[
a*d]*Sinh[b*d*Log[c*x^n]])/(b*d*n)
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3039, 7281, 7083}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \text{Chi}(d(a+b\log(cx^n)))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int \text{Chi}(ad+b\log(cx^n)d)d(ad+b\log(cx^n)d)}{bdn}$$

$$\Big\downarrow \text{7083}$$

$$\frac{(ad+bd\log(cx^n))\text{Chi}(ad+b\log(cx^n)d)-\frac{x^{-n}(c^2x^{2n}-1)}{2c}}{bdn}$$

input
```
Int[CoshIntegral[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
(-1/2*(-1 + c^2*x^(2*n))/(c*x^n) + CoshIntegral[a*d + b*d*Log[c*x^n]]*(a*d
 + b*d*Log[c*x^n]))/(b*d*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 7083
```
Int[CoshIntegral[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(CoshInte
gral[a + b*x]/b), x] - Simp[Sinh[a + b*x]/b, x] /; FreeQ[{a, b}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\operatorname{Chi}(ad+bd\ln(c\,x^{n}))(ad+bd\ln(c\,x^{n}))-\sinh(ad+bd\ln(c\,x^{n}))}{nbd}$
default	$\frac{\operatorname{Chi}(ad+bd\ln(c\,x^{n}))(ad+bd\ln(c\,x^{n}))-\sinh(ad+bd\ln(c\,x^{n}))}{nbd}$
parts	$\ln(x)\operatorname{Chi}(d(a+b\ln(c\,x^{n})))-bn\Big(-\frac{(\ln(c\,x^{n})-n\ln(x))\operatorname{Chi}(\ln(x)bdn+d(b(\ln(c\,x^{n})-n\ln(x))+a))}{b\,n^{2}}-$

input
```
int(Chi(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b/d*(Chi(a*d+b*d*ln(c*x^n))*(a*d+b*d*ln(c*x^n))-sinh(a*d+b*d*ln(c*x^n)
))
```

Fricas [F]

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x}\,dx = \int \frac{\text{Chi}((b\log(cx^n)+a)d)}{x}\,dx$$

input
```
integrate(Chi(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*d*log(c*x^n) + a*d)/x, x)
```

Sympy [F]

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x}\,dx = \int \frac{\text{Chi}(ad+bd\log(cx^n))}{x}\,dx$$

input
```
integrate(Chi(d*(a+b*ln(c*x**n)))/x,x)
```

output
```
Integral(Chi(a*d + b*d*log(c*x**n))/x, x)
```

Maxima [F]

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x}\,dx = \int \frac{\text{Chi}((b\log(cx^n)+a)d)}{x}\,dx$$

input
```
integrate(Chi(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

output
```
integrate(Chi((b*log(c*x^n) + a)*d)/x, x)
```

Giac [F]

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x}\,dx = \int \frac{\text{Chi}((b\log(cx^n)+a)d)}{x}\,dx$$

input
```
integrate(Chi(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

output
```
integrate(Chi((b*log(c*x^n) + a)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x}\,dx = \frac{\ln(c\,x^n)\;\text{coshint}(d\,(a+b\,\ln(c\,x^n)))}{n} + \frac{a\,\text{coshint}(d\,(a+b\,\ln(c\,x^n)))}{b\,n} - \frac{e^{a\,d}\,(c\,x^n)^{b\,d}}{2\,b\,d\,n} + \frac{e^{-a\,d}}{2\,b\,d\,n\,(c\,x^n)^{b\,d}}$$

input
```
int(coshint(d*(a + b*log(c*x^n)))/x,x)
```

output
```
(log(c*x^n)*coshint(d*(a + b*log(c*x^n))))/n + (a*coshint(d*(a + b*log(c*x
^n))))/(b*n) - (exp(a*d)*(c*x^n)^(b*d))/(2*b*d*n) + exp(-a*d)/(2*b*d*n*(c*
x^n)^(b*d))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x}\,dx = \frac{\chi(\log(x^n c)\,bd+ad)\log(x^n c)\,bd + \chi(\log(x^n c)\,bd+ad)\,ad - \sinh(\log(x^n c)\,bd+ad)}{bdn}$$

input
```
int(Chi(d*(a+b*log(c*x^n)))/x,x)
```

output
```
(chi(log(x**n*c)*b*d + a*d)*log(x**n*c)*b*d + chi(log(x**n*c)*b*d + a*d)*a
*d - sinh(log(x**n*c)*b*d + a*d))/(b*d*n)
```

3.104 $\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 122

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x^2} \, dx = -\frac{\text{Chi}(d(a+b\log(cx^n)))}{x} + \frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}} \text{ExpIntegralEi}\left(-\frac{(1-bdn)(a+b\log(cx^n))}{bn}\right)}{2x} + \frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}} \text{ExpIntegralEi}\left(-\frac{(1+bdn)(a+b\log(cx^n))}{bn}\right)}{2x}$$

output
```
-Chi(d*(a+b*ln(c*x^n)))/x+1/2*exp(a/b/n)*(c*x^n)^(1/n)*Ei(-(-b*d*n+1)*(a+b
*ln(c*x^n))/b/n)/x+1/2*exp(a/b/n)*(c*x^n)^(1/n)*Ei(-(b*d*n+1)*(a+b*ln(c*x^
n))/b/n)/x
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.18

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{\text{Chi}(d(a+b\log(cx^n)))}{x} + \frac{1}{2}e^{-\frac{(-1+bdn)(a+b(-n\log(x)+\log(cx^n)))}{bn}}\left(\text{ExpIntegralEi}\left(\frac{(-1+bdn)(a+b\log(cx^n))}{bn}\right) + \text{ExpIntegralEi}\left(-\frac{(1+bdn)(a+b\log(cx^n))}{bn}\right)\right)(\cosh(d(a+b(-n\log(x)+\log(cx^n)))) + \sinh(d(a+b(-n\log(x)+\log(cx^n)))))$$

input
```
Integrate[CoshIntegral[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
-(CoshIntegral[d*(a + b*Log[c*x^n])]/x) + ((ExpIntegralEi[((-1 + b*d*n)*(a
 + b*Log[c*x^n]))/(b*n)] + ExpIntegralEi[-(((1 + b*d*n)*(a + b*Log[c*x^n])
)/(b*n))])*(Cosh[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))] + Sinh[d*(a + b*(-(
n*Log[x]) + Log[c*x^n]))]))/(2*E^(((-1 + b*d*n)*(a + b*(-(n*Log[x]) + Log[
c*x^n])))/(b*n)))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {7110, 27, 6066, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x^2}\,dx$$

$$\Big\downarrow \text{7110}$$

$$bdn\int \frac{\cosh(d(a+b\log(cx^n)))}{dx^2(a+b\log(cx^n))}dx - \frac{\text{Chi}(d(a+b\log(cx^n)))}{x}$$

$$\Big\downarrow \text{27}$$

$$bn\int \frac{\cosh (d(a+b\log (cx^n)))}{x^2 (a+b\log (cx^n))}dx-\frac{\text{Chi}(d(a+b\log (cx^n)))}{x}$$

$$\Big\downarrow \text{6066}$$

$$bn\left(\frac{1}{2}e^{-ad}x^{bdn}(cx^n)^{-bd}\int \frac{x^{-bdn-2}}{a+b\log (cx^n)}dx+\frac{1}{2}e^{ad}x^{-bdn}(cx^n)^{bd}\int \frac{x^{bdn-2}}{a+b\log (cx^n)}dx\right)-\frac{\text{Chi}(d(a+b\log (cx^n)))}{x}$$

$$\Big\downarrow \text{2747}$$

$$bn\left(\frac{e^{ad}(cx^n)^{\frac{1}{n}}\int \frac{(cx^n)^{-\frac{1-bdn}{n}}}{a+b\log(cx^n)}d\log (cx^n)}{2nx}+\frac{e^{-ad}(cx^n)^{\frac{1}{n}}\int \frac{(cx^n)^{-\frac{bdn+1}{n}}}{a+b\log(cx^n)}d\log (cx^n)}{2nx}\right)-\frac{\text{Chi}(d(a+b\log (cx^n)))}{x}$$

$$\Big\downarrow \text{2609}$$

$$bn\left(\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{(1-bdn)(a+b\log(cx^n))}{bn}\right)}{2bnx}+\frac{(cx^n)^{\frac{1}{n}}e^{a\left(\frac{1}{bn}+d\right)-ad}\text{ExpIntegralEi}\left(-\frac{(bdn+1)(a+b\log(cx^n)}{bn}\right.}{2bnx}\right.$$
$$\frac{\text{Chi}(d(a+b\log (cx^n)))}{x}$$

input
```
Int[CoshIntegral[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
-(CoshIntegral[d*(a + b*Log[c*x^n])]/x) + b*n*((E^(a/(b*n))*(c*x^n)^n^(-1)
*ExpIntegralEi[-(((1 - b*d*n)*(a + b*Log[c*x^n]))/(b*n))])/(2*b*n*x) + (E^
(-(a*d) + a*(d + 1/(b*n)))*(c*x^n)^n^(-1)*ExpIntegralEi[-(((1 + b*d*n)*(a
+ b*Log[c*x^n]))/(b*n))])/(2*b*n*x))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 6066
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*(((e_.) + Log[(g_.)*(
x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.), x_Symbol] :> Simp[((i*x)
^r*(1/((c*x^n)^(b*d)*(2*x^(r - b*d*n)))))/E^(a*d)   Int[x^(r - b*d*n)*(h*(e
 + f*Log[g*x^m]))^q, x], x] + Simp[E^(a*d)*(i*x)^r*((c*x^n)^(b*d)/(2*x^(r +
 b*d*n)))   Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, m, n, q, r}, x]
```

rule 7110
```
Int[CoshIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^
(m_.), x_Symbol] :> Simp[(e*x)^(m + 1)*(CoshIntegral[d*(a + b*Log[c*x^n])]/
(e*(m + 1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*(Cosh[d*(a + b*Log[c*
x^n])]/(d*(a + b*Log[c*x^n]))), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[m, -1]
```

Maple [F]

$$\int \frac{\mathrm{Chi}\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{2}}dx$$

input
```
int(Chi(d*(a+b*ln(c*x^n)))/x^2,x)
```

output
```
int(Chi(d*(a+b*ln(c*x^n)))/x^2,x)
```

Fricas [F]

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\text{Chi}((b\log(cx^n)+a)d)}{x^2}\,dx$$

input
```
integrate(Chi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*d*log(c*x^n) + a*d)/x^2, x)
```

Sympy [F]

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\text{Chi}(ad+bd\log(cx^n))}{x^2}\,dx$$

input
```
integrate(Chi(d*(a+b*ln(c*x**n)))/x**2,x)
```

output
```
Integral(Chi(a*d + b*d*log(c*x**n))/x**2, x)
```

Maxima [F]

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\text{Chi}((b\log(cx^n)+a)d)}{x^2}\,dx$$

input
```
integrate(Chi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

output
```
integrate(Chi((b*log(c*x^n) + a)*d)/x^2, x)
```

Giac [F]

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\text{Chi}((b\log(cx^n)+a)d)}{x^2}\,dx$$

input
```
integrate(Chi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

output
```
integrate(Chi((b*log(c*x^n) + a)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\text{coshint}(d\,(a+b\,\ln(c\,x^n)))}{x^2}\,dx$$

input
```
int(coshint(d*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int(coshint(d*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [F]

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\chi(\log(x^n c)\,bd+ad)}{x^2}dx$$

input
```
int(Chi(d*(a+b*log(c*x^n)))/x^2,x)
```

output
```
int(chi(log(x**n*c)*b*d + a*d)/x**2,x)
```

3.105 $\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 130

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{\text{Chi}(d(a+b\log(cx^n)))}{2x^2} + \frac{e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\text{ExpIntegralEi}\left(-\frac{(2-bdn)(a+b\log(cx^n))}{bn}\right)}{4x^2} + \frac{e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\text{ExpIntegralEi}\left(-\frac{(2+bdn)(a+b\log(cx^n))}{bn}\right)}{4x^2}$$

output
```
-1/2*Chi(d*(a+b*ln(c*x^n)))/x^2+1/4*exp(2*a/b/n)*(c*x^n)^(2/n)*Ei(-(-b*d*n
+2)*(a+b*ln(c*x^n))/b/n)/x^2+1/4*exp(2*a/b/n)*(c*x^n)^(2/n)*Ei(-(b*d*n+2)*
(a+b*ln(c*x^n))/b/n)/x^2
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.12

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{\text{Chi}(d(a+b\log(cx^n)))}{2x^2} + \frac{1}{4}e^{-\frac{(-2+bdn)(a+b(-n\log(x)+\log(cx^n)))}{bn}}\left(\text{ExpIntegralEi}\left(\frac{(-2+bdn)(a+b\log(cx^n))}{bn}\right) + \text{ExpIntegralEi}\left(-\frac{(2+bdn)(a+b\log(cx^n))}{bn}\right)\right)(\cosh(d(a+b(-n\log(x)+\log(cx^n)))) + \sinh(d(a+b(-n\log(x)+\log(cx^n)))))$$

input
```
Integrate[CoshIntegral[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
-1/2*CoshIntegral[d*(a + b*Log[c*x^n])]/x^2 + ((ExpIntegralEi[((-2 + b*d*n
)*(a + b*Log[c*x^n]))/(b*n)] + ExpIntegralEi[-(((2 + b*d*n)*(a + b*Log[c*x
^n]))/(b*n))])*(Cosh[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))] + Sinh[d*(a + b
*(-(n*Log[x]) + Log[c*x^n]))]))/(4*E^(((-2 + b*d*n)*(a + b*(-(n*Log[x]) +
Log[c*x^n])))/(b*n)))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {7110, 27, 6066, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x^3}\,dx$$

$$\Big\downarrow \text{7110}$$

$$\frac{1}{2}bdn\int \frac{\cosh(d(a+b\log(cx^n)))}{dx^3(a+b\log(cx^n))}dx - \frac{\text{Chi}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}bn\int \frac{\cosh(d(a+b\log(cx^n)))}{x^3(a+b\log(cx^n))}dx-\frac{\text{Chi}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{6066}$$

$$\frac{1}{2}bn\left(\frac{1}{2}e^{-ad}x^{bdn}(cx^n)^{-bd}\int \frac{x^{-bdn-3}}{a+b\log(cx^n)}dx+\frac{1}{2}e^{ad}x^{-bdn}(cx^n)^{bd}\int \frac{x^{bdn-3}}{a+b\log(cx^n)}dx\right)-\frac{\text{Chi}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{2747}$$

$$\frac{1}{2}bn\left(\frac{e^{ad}(cx^n)^{2/n}\int \frac{(cx^n)^{-\frac{2-bdn}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2nx^2}+\frac{e^{-ad}(cx^n)^{2/n}\int \frac{(cx^n)^{-\frac{bdn+2}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2nx^2}\right)-\frac{\text{Chi}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{2609}$$

$$\frac{1}{2}bn\left(\frac{e^{\frac{2a}{bn}}(cx^n)^{2/n}\text{ExpIntegralEi}\left(-\frac{(2-bdn)(a+b\log(cx^n))}{bn}\right)}{2bnx^2}+\frac{(cx^n)^{2/n}e^{a\left(\frac{2}{bn}+d\right)-ad}\text{ExpIntegralEi}\left(-\frac{(bdn+2)(a+b\log}{bn}\right.}{2bnx^2}\right.$$

$$\frac{\text{Chi}(d(a+b\log(cx^n)))}{2x^2}$$

input
```
Int[CoshIntegral[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
-1/2*CoshIntegral[d*(a + b*Log[c*x^n])]/x^2 + (b*n*((E^((2*a)/(b*n))*(c*x^
n)^(2/n)*ExpIntegralEi[-(((2 - b*d*n)*(a + b*Log[c*x^n]))/(b*n))])/(2*b*n*
x^2) + (E^(-(a*d) + a*(d + 2/(b*n)))*(c*x^n)^(2/n)*ExpIntegralEi[-(((2 + b
*d*n)*(a + b*Log[c*x^n]))/(b*n))])/(2*b*n*x^2)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 6066
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*(((e_.) + Log[(g_.)*(
x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.), x_Symbol] :> Simp[((i*x)
^r*(1/((c*x^n)^(b*d)*(2*x^(r - b*d*n)))))/E^(a*d)   Int[x^(r - b*d*n)*(h*(e
 + f*Log[g*x^m]))^q, x], x] + Simp[E^(a*d)*(i*x)^r*((c*x^n)^(b*d)/(2*x^(r +
 b*d*n)))   Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, m, n, q, r}, x]
```

rule 7110
```
Int[CoshIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^
(m_.), x_Symbol] :> Simp[(e*x)^(m + 1)*(CoshIntegral[d*(a + b*Log[c*x^n])]/
(e*(m + 1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*(Cosh[d*(a + b*Log[c*
x^n])]/(d*(a + b*Log[c*x^n]))), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[m, -1]
```

Maple [F]

$$\int \frac{\mathrm{Chi}\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{3}}dx$$

input
```
int(Chi(d*(a+b*ln(c*x^n)))/x^3,x)
```

output
```
int(Chi(d*(a+b*ln(c*x^n)))/x^3,x)
```

Fricas [F]

$$\int \frac{\mathrm{Chi}(d(a + b\log(cx^n)))}{x^3}\,dx = \int \frac{\mathrm{Chi}((b\log(cx^n) + a)d)}{x^3}\,dx$$

input
```
integrate(Chi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*d*log(c*x^n) + a*d)/x^3, x)
```

Sympy [F]

$$\int \frac{\mathrm{Chi}(d(a + b\log(cx^n)))}{x^3}\,dx = \int \frac{\mathrm{Chi}(ad + bd\log(cx^n))}{x^3}\,dx$$

input
```
integrate(Chi(d*(a+b*ln(c*x**n)))/x**3,x)
```

output
```
Integral(Chi(a*d + b*d*log(c*x**n))/x**3, x)
```

Maxima [F]

$$\int \frac{\mathrm{Chi}(d(a + b\log(cx^n)))}{x^3}\,dx = \int \frac{\mathrm{Chi}((b\log(cx^n) + a)d)}{x^3}\,dx$$

input
```
integrate(Chi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

output
```
integrate(Chi((b*log(c*x^n) + a)*d)/x^3, x)
```

Giac [F]

$$\int \frac{\mathrm{Chi}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\mathrm{Chi}((b\log(cx^n)+a)d)}{x^3}\,dx$$

input
```
integrate(Chi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

output
```
integrate(Chi((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\mathrm{Chi}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\mathrm{coshint}\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}{x^3}\,dx$$

input
```
int(coshint(d*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int(coshint(d*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [F]

$$\int \frac{\mathrm{Chi}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\chi(\log(x^n c)\,bd+ad)}{x^3}dx$$

input
```
int(Chi(d*(a+b*log(c*x^n)))/x^3,x)
```

output
```
int(chi(log(x**n*c)*b*d + a*d)/x**3,x)
```

3.106 $\int (ex)^m \mathbf{Chi}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 167

$$\int (ex)^m \text{Chi}(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}\text{Chi}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{e^{-\frac{a(1+m)}{bn}}x(ex)^m(cx^n)^{-\frac{1+m}{n}}\,\text{ExpIntegralEi}\left(\frac{(1+m-bdn)(a+b\log(cx^n))}{bn}\right)}{2(1+m)} - \frac{e^{-\frac{a(1+m)}{bn}}x(ex)^m(cx^n)^{-\frac{1+m}{n}}\,\text{ExpIntegralEi}\left(\frac{(1+m+bdn)(a+b\log(cx^n))}{bn}\right)}{2(1+m)}$$

output
```
(e*x)^(1+m)*Chi(d*(a+b*ln(c*x^n)))/e/(1+m)-1/2*x*(e*x)^m*Ei((-b*d*n+m+1)*(
a+b*ln(c*x^n))/b/n)/exp(a*(1+m)/b/n)/(1+m)/((c*x^n)^((1+m)/n))-1/2*x*(e*x)
^m*Ei((b*d*n+m+1)*(a+b*ln(c*x^n))/b/n)/exp(a*(1+m)/b/n)/(1+m)/((c*x^n)^((1
+m)/n))
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.71

$$\int (ex)^m \text{Chi}(d(a + b\log(cx^n)))\, dx$$
$$= \frac{(ex)^m \left(2x\text{Chi}(d(a + b\log(cx^n))) - e^{-\frac{(1+m)(a-bn\log(x)+b\log(cx^n))}{bn}} x^{-m} \left(\text{ExpIntegralEi}\left(\frac{(1+m-bdn)(a+b\log(cx^n))}{bn}\right)\right.\right.}{2(1+m)}$$

input
```
Integrate[(e*x)^m*CoshIntegral[d*(a + b*Log[c*x^n])],x]
```

output
```
((e*x)^m*(2*x*CoshIntegral[d*(a + b*Log[c*x^n])] - (ExpIntegralEi[((1 + m
- b*d*n)*(a + b*Log[c*x^n]))/(b*n)] + ExpIntegralEi[((1 + m + b*d*n)*(a +
b*Log[c*x^n]))/(b*n)])/(E^(((1 + m)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n)
)*x^m)))/(2*(1 + m))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {7110, 27, 6066, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \text{Chi}(d(a + b\log(cx^n)))\, dx$$
$$\Big\downarrow \text{ 7110}$$
$$\frac{(ex)^{m+1}\text{Chi}(d(a + b\log(cx^n)))}{e(m+1)} - \frac{bdn\int \frac{(ex)^m \cosh(d(a+b\log(cx^n)))}{d(a+b\log(cx^n))}dx}{m+1}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{(ex)^{m+1}\text{Chi}(d(a + b\log(cx^n)))}{e(m+1)} - \frac{bn\int \frac{(ex)^m \cosh(d(a+b\log(cx^n)))}{a+b\log(cx^n)}dx}{m+1}$$
$$\Big\downarrow \text{ 6066}$$

$$\frac{(ex)^{m+1}\text{Chi}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{bn\left(\frac{1}{2}e^{-ad}(ex)^m(cx^n)^{-bd}x^{bdn-m}\int\frac{x^{m-bdn}}{a+b\log(cx^n)}dx+\frac{1}{2}e^{ad}(ex)^m(cx^n)^{bd}x^{-bdn-m}\int\frac{x^{m+bdn}}{a+b\log(cx^n)}dx\right)}{m+1}$$

$\Big\downarrow$ 2747

$$\frac{(ex)^{m+1}\text{Chi}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{bn\left(\frac{xe^{-ad}(ex)^m(cx^n)^{-\frac{-bdn+m+1}{n}-bd}\int\frac{(cx^n)^{\frac{m-bdn+1}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2n}+\frac{xe^{ad}(ex)^m(cx^n)^{bd-\frac{bdn+m+1}{n}}\int\frac{(cx^n)^{\frac{m+bdn+1}{n}}}{a+b\log(cx^n)}d\log(cx^n)}{2n}\right)}{m+1}$$

$\Big\downarrow$ 2609

$$\frac{(ex)^{m+1}\text{Chi}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{bn\left(\frac{x(ex)^m e^{-\frac{a(-bdn+m+1)}{bn}-ad}(cx^n)^{-\frac{-bdn+m+1}{n}-bd}\text{ExpIntegralEi}\left(\frac{(m-bdn+1)(a+b\log(cx^n))}{bn}\right)}{2bn}+\frac{x(ex)^m e^{ad-\frac{a(bdn+m+1)}{bn}}(cx^n)^{bd-\frac{bdn+n}{n}}}{}\right.}{m+1}$$

input
```
Int[(e*x)^m*CoshIntegral[d*(a + b*Log[c*x^n])],x]
```

output
```
((e*x)^(1 + m)*CoshIntegral[d*(a + b*Log[c*x^n])])/(e*(1 + m)) - (b*n*((E^
(-(a*d) - (a*(1 + m - b*d*n))/(b*n))*x*(e*x)^m*(c*x^n)^(-(b*d) - (1 + m -
b*d*n)/n)*ExpIntegralEi[((1 + m - b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(2*b*
n) + (E^(a*d - (a*(1 + m + b*d*n))/(b*n))*x*(e*x)^m*(c*x^n)^(b*d - (1 + m
+ b*d*n)/n)*ExpIntegralEi[((1 + m + b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(2*
b*n)))/(1 + m)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 6066
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*(((e_.) + Log[(g_.)*(
x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.), x_Symbol] :> Simp[((i*x)
^r*(1/((c*x^n)^(b*d)*(2*x^(r - b*d*n)))))/E^(a*d)   Int[x^(r - b*d*n)*(h*(e
 + f*Log[g*x^m]))^q, x], x] + Simp[E^(a*d)*(i*x)^r*((c*x^n)^(b*d)/(2*x^(r +
 b*d*n)))   Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, m, n, q, r}, x]
```

rule 7110
```
Int[CoshIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^
(m_.), x_Symbol] :> Simp[(e*x)^(m + 1)*(CoshIntegral[d*(a + b*Log[c*x^n])]/
(e*(m + 1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*(Cosh[d*(a + b*Log[c*
x^n])]/(d*(a + b*Log[c*x^n]))), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[m, -1]
```

Maple [F]

$$\int (ex)^m \operatorname{Chi}(d(a+b\ln(c\,x^n)))\,dx$$

input
```
int((e*x)^m*Chi(d*(a+b*ln(c*x^n))),x)
```

output
```
int((e*x)^m*Chi(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int (ex)^m \text{Chi}(d(a + b\log(cx^n))) \, dx = \int (ex)^m \text{Chi}((b\log(cx^n) + a)d) \, dx$$

input
```
integrate((e*x)^m*Chi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral((e*x)^m*cosh_integral(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int (ex)^m \text{Chi}(d(a + b\log(cx^n))) \, dx = \int (ex)^m \text{Chi}(ad + bd\log(cx^n)) \, dx$$

input
```
integrate((e*x)**m*Chi(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral((e*x)**m*Chi(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int (ex)^m \text{Chi}(d(a + b\log(cx^n))) \, dx = \int (ex)^m \text{Chi}((b\log(cx^n) + a)d) \, dx$$

input
```
integrate((e*x)^m*Chi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate((e*x)^m*Chi((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int (ex)^m \text{Chi}(d(a+b\log(cx^n)))\,dx = \int (ex)^m \text{Chi}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*Chi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate((e*x)^m*Chi((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \text{Chi}(d(a+b\log(cx^n)))\,dx = \int \text{coshint}(d\,(a+b\,\ln(c\,x^n)))\,(e\,x)^m\,dx$$

input
```
int(coshint(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

output
```
int(coshint(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \text{Chi}(d(a+b\log(cx^n)))\,dx = e^m\left(\int x^m \chi(\log(x^n c)\,bd+ad)\,dx\right)$$

input
```
int((e*x)^m*Chi(d*(a+b*log(c*x^n))),x)
```

output
```
e**m*int(x**m*chi(log(x**n*c)*b*d + a*d),x)
```

3.107 $\int \frac{\cosh(bx)\mathbf{Chi}(bx)}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 96

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x^3}\,dx = -\frac{\cosh^2(bx)}{4x^2} - \frac{\cosh(bx)\text{Chi}(bx)}{2x^2} + \frac{1}{4}b^2\text{Chi}(bx)^2 + b^2\text{Chi}(2bx) - \frac{b\cosh(bx)\sinh(bx)}{2x} - \frac{b\text{Chi}(bx)\sinh(bx)}{2x} - \frac{b\sinh(2bx)}{4x}$$

output
```
-1/4*cosh(b*x)^2/x^2-1/2*cosh(b*x)*Chi(b*x)/x^2+1/4*b^2*Chi(b*x)^2+b^2*Chi
(2*b*x)-1/2*b*cosh(b*x)*sinh(b*x)/x-1/2*b*Chi(b*x)*sinh(b*x)/x-1/4*b*sinh(
2*b*x)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x^3}\,dx = -\frac{\cosh^2(bx)}{4x^2} - \frac{\cosh(bx)\text{Chi}(bx)}{2x^2} + \frac{1}{4}b^2\text{Chi}(bx)^2 + b^2\text{Chi}(2bx) - \frac{b\cosh(bx)\sinh(bx)}{2x} - \frac{b\text{Chi}(bx)\sinh(bx)}{2x} - \frac{b\sinh(2bx)}{4x}$$

input
```
Integrate[(Cosh[b*x]*CoshIntegral[b*x])/x^3,x]
```

output
```
-1/4*Cosh[b*x]^2/x^2 - (Cosh[b*x]*CoshIntegral[b*x])/(2*x^2) + (b^2*CoshIn
tegral[b*x]^2)/4 + b^2*CoshIntegral[2*b*x] - (b*Cosh[b*x]*Sinh[b*x])/(2*x)
 - (b*CoshIntegral[b*x]*Sinh[b*x])/(2*x) - (b*Sinh[2*b*x])/(4*x)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.41, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {7099, 27, 3042, 3795, 14, 3042, 3793, 2009, 7105, 27, 5971, 27, 3042, 26, 3778, 3042, 3782, 7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(bx)\cosh(bx)}{x^3}\,dx$$

$$\Big\downarrow \text{7099}$$

$$\frac{1}{2}b\int \frac{\text{Chi}(bx)\sinh(bx)}{x^2}dx+\frac{1}{2}b\int \frac{\cosh^2(bx)}{bx^3}dx-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}b\int \frac{\text{Chi}(bx)\sinh(bx)}{x^2}dx+\frac{1}{2}\int \frac{\cosh^2(bx)}{x^3}dx-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}b\int \frac{\text{Chi}(bx)\sinh(bx)}{x^2}dx+\frac{1}{2}\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)^2}{x^3}dx-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{3795}$$

$$\frac{1}{2}\left(b^2\left(-\int \frac{1}{x}dx\right)+2b^2\int \frac{\cosh^2(bx)}{x}dx-\frac{\cosh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)+\frac{1}{2}b\int \frac{\text{Chi}(bx)\sinh(bx)}{x^2}dx-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(2b^2\int\frac{\cosh^2(bx)}{x}dx+b^2(-\log(x))-\frac{\cosh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)+\frac{1}{2}b\int\frac{\text{Chi}(bx)\sinh(bx)}{x^2}dx-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(2b^2\int\frac{\sin\left(ibx+\frac{\pi}{2}\right)^2}{x}dx+b^2(-\log(x))-\frac{\cosh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)+\frac{1}{2}b\int\frac{\text{Chi}(bx)\sinh(bx)}{x^2}dx-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$\downarrow$ 3793

$$\frac{1}{2}\left(2b^2\int\left(\frac{\cosh(2bx)}{2x}+\frac{1}{2x}\right)dx+b^2(-\log(x))-\frac{\cosh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)+\frac{1}{2}b\int\frac{\text{Chi}(bx)\sinh(bx)}{x^2}dx-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$\downarrow$ 2009

$$\frac{1}{2}b\int\frac{\text{Chi}(bx)\sinh(bx)}{x^2}dx+\frac{1}{2}\left(2b^2\left(\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}\right)-b^2\log(x)-\frac{\cosh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$\downarrow$ 7105

$$\frac{1}{2}b\left(b\int\frac{\cosh(bx)\text{Chi}(bx)}{x}dx+b\int\frac{\cosh(bx)\sinh(bx)}{bx^2}dx-\frac{\text{Chi}(bx)\sinh(bx)}{x}\right)+\frac{1}{2}\left(2b^2\left(\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}\right)-b^2\log(x)-\frac{\cosh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$\downarrow$ 27

$$\frac{1}{2}b\left(b\int\frac{\cosh(bx)\text{Chi}(bx)}{x}dx+\int\frac{\cosh(bx)\sinh(bx)}{x^2}dx-\frac{\text{Chi}(bx)\sinh(bx)}{x}\right)+\frac{1}{2}\left(2b^2\left(\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}\right)-b^2\log(x)-\frac{\cosh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$\downarrow$ 5971

$$\frac{1}{2}b\left(b\int \frac{\cosh(bx)\text{Chi}(bx)}{x}dx+\int \frac{\sinh(2bx)}{2x^2}dx-\frac{\text{Chi}(bx)\sinh(bx)}{x}\right)+\frac{1}{2}\left(2b^2\left(\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}\right)-b^2\log(x)-\frac{\cosh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}b\left(b\int \frac{\cosh(bx)\text{Chi}(bx)}{x}dx+\frac{1}{2}\int \frac{\sinh(2bx)}{x^2}dx-\frac{\text{Chi}(bx)\sinh(bx)}{x}\right)+\frac{1}{2}\left(2b^2\left(\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}\right)-b^2\log(x)-\frac{\cosh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}b\left(b\int \frac{\cosh(bx)\text{Chi}(bx)}{x}dx+\frac{1}{2}\int -\frac{i\sin(2ibx)}{x^2}dx-\frac{\text{Chi}(bx)\sinh(bx)}{x}\right)+\frac{1}{2}\left(2b^2\left(\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}\right)-b^2\log(x)-\frac{\cosh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}b\left(b\int \frac{\cosh(bx)\text{Chi}(bx)}{x}dx-\frac{1}{2}i\int \frac{\sin(2ibx)}{x^2}dx-\frac{\text{Chi}(bx)\sinh(bx)}{x}\right)+\frac{1}{2}\left(2b^2\left(\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}\right)-b^2\log(x)-\frac{\cosh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}b\left(b\int \frac{\cosh(bx)\text{Chi}(bx)}{x}dx-\frac{1}{2}i\left(2ib\int \frac{\cosh(2bx)}{x}dx-\frac{i\sinh(2bx)}{x}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{x}\right)+\frac{1}{2}\left(2b^2\left(\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}\right)-b^2\log(x)-\frac{\cosh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}b\left(b\int\frac{\cosh(bx)\text{Chi}(bx)}{x}dx-\frac{1}{2}i\left(2ib\int\frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx-\frac{i\sinh(2bx)}{x}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{x}\right)+\frac{1}{2}\left(2b^2\left(\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}\right)-b^2\log(x)-\frac{\cosh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{3782}$$

$$\frac{1}{2}b\left(b\int\frac{\cosh(bx)\text{Chi}(bx)}{x}dx-\frac{\text{Chi}(bx)\sinh(bx)}{x}-\frac{1}{2}i\left(2ib\text{Chi}(2bx)-\frac{i\sinh(2bx)}{x}\right)\right)+\frac{1}{2}\left(2b^2\left(\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}\right)-b^2\log(x)-\frac{\cosh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}$$

$$\Big\downarrow \text{7237}$$

$$\frac{1}{2}\left(2b^2\left(\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}\right)-b^2\log(x)-\frac{\cosh^2(bx)}{2x^2}-\frac{b\sinh(bx)\cosh(bx)}{x}\right)-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}+\frac{1}{2}b\left(\frac{1}{2}b\text{Chi}(bx)^2-\frac{\text{Chi}(bx)\sinh(bx)}{x}-\frac{1}{2}i\left(2ib\text{Chi}(2bx)-\frac{i\sinh(2bx)}{x}\right)\right)$$

input
```
Int[(Cosh[b*x]*CoshIntegral[b*x])/x^3,x]
```

output
```
-1/2*(Cosh[b*x]*CoshIntegral[b*x])/x^2 + (-1/2*Cosh[b*x]^2/x^2 + 2*b^2*(Co
shIntegral[2*b*x]/2 + Log[x]/2) - b^2*Log[x] - (b*Cosh[b*x]*Sinh[b*x])/x)/
2 + (b*((b*CoshIntegral[b*x]^2)/2 - (CoshIntegral[b*x]*Sinh[b*x])/x - (I/2
)*((2*I)*b*CoshIntegral[2*b*x] - (I*Sinh[2*b*x])/x)))/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7099
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*Cosh[a + b*x]*(CoshInteg
ral[c + d*x]/(f*(m + 1))), x] + (-Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1
)*Sinh[a + b*x]*CoshIntegral[c + d*x], x], x] - Simp[d/(f*(m + 1))   Int[(e
 + f*x)^(m + 1)*Cosh[a + b*x]*(Cosh[c + d*x]/(c + d*x)), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && ILtQ[m, -1]
```

rule 7105
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_)*Sinh[(a_.) +
 (b_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^(m + 1)*Sinh[a + b*x]*(CoshIntegr
al[c + d*x]/(f*(m + 1))), x] + (-Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)
*Cosh[a + b*x]*CoshIntegral[c + d*x], x], x] - Simp[d/(f*(m + 1))   Int[(e
+ f*x)^(m + 1)*Sinh[a + b*x]*(Cosh[c + d*x]/(c + d*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f}, x] && ILtQ[m, -1]
```

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{\cosh(bx)\operatorname{Chi}(bx)}{x^3}dx$$

input
```
int(cosh(b*x)*Chi(b*x)/x^3,x)
```

output
```
int(cosh(b*x)*Chi(b*x)/x^3,x)
```

Fricas [F]

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x^3}\,dx = \int \frac{\text{Chi}(bx)\cosh(bx)}{x^3}\,dx$$

input
```
integrate(cosh(b*x)*Chi(b*x)/x^3,x, algorithm="fricas")
```

output
```
integral(cosh(b*x)*cosh_integral(b*x)/x^3, x)
```

Sympy [F]

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x^3}\,dx = \int \frac{\cosh(bx)\,\text{Chi}(bx)}{x^3}\,dx$$

input
```
integrate(cosh(b*x)*Chi(b*x)/x**3,x)
```

output
```
Integral(cosh(b*x)*Chi(b*x)/x**3, x)
```

Maxima [F]

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x^3}\,dx = \int \frac{\text{Chi}(bx)\cosh(bx)}{x^3}\,dx$$

input
```
integrate(cosh(b*x)*Chi(b*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x)*cosh(b*x)/x^3, x)
```

Giac [F]

$$\int \frac{\cosh(bx)\mathrm{Chi}(bx)}{x^3}\,dx = \int \frac{\mathrm{Chi}(bx)\cosh(bx)}{x^3}\,dx$$

input
```
integrate(cosh(b*x)*Chi(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(Chi(b*x)*cosh(b*x)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(bx)\mathrm{Chi}(bx)}{x^3}\,dx = \int \frac{\mathrm{coshint}(b\,x)\,\cosh(b\,x)}{x^3}\,dx$$

input
```
int((coshint(b*x)*cosh(b*x))/x^3,x)
```

output
```
int((coshint(b*x)*cosh(b*x))/x^3, x)
```

Reduce [F]

$$\int \frac{\cosh(bx)\mathrm{Chi}(bx)}{x^3}\,dx = \int \frac{\chi(bx)\cosh(bx)}{x^3}dx$$

input
```
int(cosh(b*x)*Chi(b*x)/x^3,x)
```

output
```
int((chi(b*x)*cosh(b*x))/x**3,x)
```

3.108 $\int \frac{\cosh(bx)\mathbf{Chi}(bx)}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}\,dx = -\frac{\cosh^2(bx)}{x} - \frac{\cosh(bx)\text{Chi}(bx)}{x} + b\text{Shi}(2bx) + b\text{Int}\left(\frac{\text{Chi}(bx)\sinh(bx)}{x},x\right)$$

output
```
-cosh(b*x)^2/x-cosh(b*x)*Chi(b*x)/x+b*Shi(2*b*x)+b*Defer(Int)(Chi(b*x)*sin
h(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}\,dx = \int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}\,dx$$

input
```
Integrate[(Cosh[b*x]*CoshIntegral[b*x])/x^2,x]
```

output
```
Integrate[(Cosh[b*x]*CoshIntegral[b*x])/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(bx)\cosh(bx)}{x^2}dx$$

$$\Big\downarrow \ 7099$$

$$b\int \frac{\text{Chi}(bx)\sinh(bx)}{x}dx+b\int \frac{\cosh^2(bx)}{bx^2}dx-\frac{\text{Chi}(bx)\cosh(bx)}{x}$$

$$\Big\downarrow \ 27$$

$$b\int \frac{\text{Chi}(bx)\sinh(bx)}{x}dx+\int \frac{\cosh^2(bx)}{x^2}dx-\frac{\text{Chi}(bx)\cosh(bx)}{x}$$

$$\Big\downarrow \ 3042$$

$$b\int \frac{\text{Chi}(bx)\sinh(bx)}{x}dx+\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)^2}{x^2}dx-\frac{\text{Chi}(bx)\cosh(bx)}{x}$$

$$\Big\downarrow \ 3794$$

$$b\int \frac{\text{Chi}(bx)\sinh(bx)}{x}dx+2ib\int -\frac{i\sinh(2bx)}{2x}dx-\frac{\text{Chi}(bx)\cosh(bx)}{x}-\frac{\cosh^2(bx)}{x}$$

$$\Big\downarrow \ 27$$

$$b\int \frac{\text{Chi}(bx)\sinh(bx)}{x}dx+b\int \frac{\sinh(2bx)}{x}dx-\frac{\text{Chi}(bx)\cosh(bx)}{x}-\frac{\cosh^2(bx)}{x}$$

$$\Big\downarrow \ 3042$$

$$b\int \frac{\text{Chi}(bx)\sinh(bx)}{x}dx+b\int -\frac{i\sin(2ibx)}{x}dx-\frac{\text{Chi}(bx)\cosh(bx)}{x}-\frac{\cosh^2(bx)}{x}$$

$$\Big\downarrow \ 26$$

$$b\int \frac{\text{Chi}(bx)\sinh(bx)}{x}dx-ib\int \frac{\sin(2ibx)}{x}dx-\frac{\text{Chi}(bx)\cosh(bx)}{x}-\frac{\cosh^2(bx)}{x}$$

$$\Big\downarrow \text{3779}$$

$$b\int \frac{\text{Chi}(bx)\sinh(bx)}{x}dx - \frac{\text{Chi}(bx)\cosh(bx)}{x} + b\text{Shi}(2bx) - \frac{\cosh^2(bx)}{x}$$

$$\Big\downarrow \text{7299}$$

$$b\int \frac{\text{Chi}(bx)\sinh(bx)}{x}dx - \frac{\text{Chi}(bx)\cosh(bx)}{x} + b\text{Shi}(2bx) - \frac{\cosh^2(bx)}{x}$$

input
```
Int[(Cosh[b*x]*CoshIntegral[b*x])/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(bx)\,\text{Chi}(bx)}{x^2}dx$$

input
```
int(cosh(b*x)*Chi(b*x)/x^2,x)
```

output
```
int(cosh(b*x)*Chi(b*x)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}\,dx = \int \frac{\text{Chi}(bx)\cosh(bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x)*Chi(b*x)/x^2,x, algorithm="fricas")
```

output
```
integral(cosh(b*x)*cosh_integral(b*x)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 3.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}\,dx = \int \frac{\cosh(bx)\,\text{Chi}\,(bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x)*Chi(b*x)/x**2,x)
```

output
```
Integral(cosh(b*x)*Chi(b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}\,dx = \int \frac{\text{Chi}(bx)\cosh(bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x)*Chi(b*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x)*cosh(b*x)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\mathrm{Chi}(bx)}{x^2}\,dx = \int \frac{\mathrm{Chi}(bx)\cosh{(bx)}}{x^2}\,dx$$

input
```
integrate(cosh(b*x)*Chi(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(Chi(b*x)*cosh(b*x)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\mathrm{Chi}(bx)}{x^2}\,dx = \int \frac{\mathrm{coshint}(b\,x)\,\cosh(b\,x)}{x^2}\,dx$$

input
```
int((coshint(b*x)*cosh(b*x))/x^2,x)
```

output
```
int((coshint(b*x)*cosh(b*x))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(bx)\mathrm{Chi}(bx)}{x^2}\,dx = \int \frac{\chi(bx)\cosh{(bx)}}{x^2}dx$$

input
```
int(cosh(b*x)*Chi(b*x)/x^2,x)
```

output
```
int((chi(b*x)*cosh(b*x))/x**2,x)
```

3.109 $\int \frac{\cosh(bx)\mathbf{Chi}(bx)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 10

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x}\,dx = \frac{\text{Chi}(bx)^2}{2}$$

output
```
1/2*Chi(b*x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x}\,dx = \frac{\text{Chi}(bx)^2}{2}$$

input
```
Integrate[(Cosh[b*x]*CoshIntegral[b*x])/x,x]
```

output
```
CoshIntegral[b*x]^2/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(bx)\cosh(bx)}{x}\,dx$$

$$\Big\downarrow \ 7237$$

$$\frac{\text{Chi}(bx)^2}{2}$$

input
```
Int[(Cosh[b*x]*CoshIntegral[b*x])/x,x]
```

output
```
CoshIntegral[b*x]^2/2
```

Defintions of rubi rules used

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\text{Chi}(bx)^2}{2}$	9
default	$\frac{\text{Chi}(bx)^2}{2}$	9

input
```
int(cosh(b*x)*Chi(b*x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*Chi(b*x)^2
```

Fricas [F]

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x}\,dx = \int \frac{\text{Chi}(bx)\cosh(bx)}{x}\,dx$$

input
```
integrate(cosh(b*x)*Chi(b*x)/x,x, algorithm="fricas")
```

output
```
integral(cosh(b*x)*cosh_integral(b*x)/x, x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x}\,dx = \frac{\text{Chi}^2(bx)}{2}$$

input
```
integrate(cosh(b*x)*Chi(b*x)/x,x)
```

output
```
Chi(b*x)**2/2
```

Maxima [F]

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x}\,dx = \int \frac{\text{Chi}(bx)\cosh(bx)}{x}\,dx$$

input
```
integrate(cosh(b*x)*Chi(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x)*cosh(b*x)/x, x)
```

Giac [F]

$$\int \frac{\cosh(bx)\mathrm{Chi}(bx)}{x}\,dx = \int \frac{\mathrm{Chi}(bx)\cosh(bx)}{x}\,dx$$

input
```
integrate(cosh(b*x)*Chi(b*x)/x,x, algorithm="giac")
```

output
```
integrate(Chi(b*x)*cosh(b*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(bx)\mathrm{Chi}(bx)}{x}\,dx = \frac{\mathrm{coshint}(b\,x)^2}{2}$$

input
```
int((coshint(b*x)*cosh(b*x))/x,x)
```

output
```
coshint(b*x)^2/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\cosh(bx)\mathrm{Chi}(bx)}{x}\,dx = \frac{\chi(bx)^2}{2}$$

input
```
int(cosh(b*x)*Chi(b*x)/x,x)
```

output
```
chi(b*x)**2/2
```

3.110 $\int \cosh(bx)\mathbf{Chi}(bx)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \cosh(bx)\text{Chi}(bx)\,dx = \frac{\text{Chi}(bx)\sinh(bx)}{b} - \frac{\text{Shi}(2bx)}{2b}$$

output
```
Chi(b*x)*sinh(b*x)/b-1/2*Shi(2*b*x)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cosh(bx)\text{Chi}(bx)\,dx = \frac{\text{Chi}(bx)\sinh(bx)}{b} - \frac{\text{Shi}(2bx)}{2b}$$

input
```
Integrate[Cosh[b*x]*CoshIntegral[b*x],x]
```

output
```
(CoshIntegral[b*x]*Sinh[b*x])/b - SinhIntegral[2*b*x]/(2*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {7095, 27, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Chi}(bx)\cosh(bx)\,dx$$

$$\Big\downarrow \text{7095}$$

$$\frac{\text{Chi}(bx)\sinh(bx)}{b}-\int \frac{\cosh(bx)\sinh(bx)}{bx}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int \frac{\cosh(bx)\sinh(bx)}{x}dx}{b}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int \frac{\sinh(2bx)}{2x}dx}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int \frac{\sinh(2bx)}{x}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int -\frac{i\sin(2ibx)}{x}dx}{2b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\text{Chi}(bx)\sinh(bx)}{b}+\frac{i\int \frac{\sin(2ibx)}{x}dx}{2b}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\text{Shi}(2bx)}{2b}$$

input
```
Int[Cosh[b*x]*CoshIntegral[b*x],x]
```

output
```
(CoshIntegral[b*x]*Sinh[b*x])/b - SinhIntegral[2*b*x]/(2*b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7095
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\mathrm{Chi}(bx)\sinh(bx)-\frac{\mathrm{Shi}(2bx)}{2}}{b}$	22
default	$\frac{\mathrm{Chi}(bx)\sinh(bx)-\frac{\mathrm{Shi}(2bx)}{2}}{b}$	22

input
```
int(cosh(b*x)*Chi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b*(Chi(b*x)*sinh(b*x)-1/2*Shi(2*b*x))
```

Fricas [F]

$$\int \cosh(bx)\mathrm{Chi}(bx)\,dx = \int \mathrm{Chi}(bx)\cosh(bx)\,dx$$

input
```
integrate(cosh(b*x)*Chi(b*x),x, algorithm="fricas")
```

output
```
integral(cosh(b*x)*cosh_integral(b*x), x)
```

Sympy [F]

$$\int \cosh(bx)\mathrm{Chi}(bx)\,dx = \int \cosh(bx)\,\mathrm{Chi}(bx)\,dx$$

input
```
integrate(cosh(b*x)*Chi(b*x),x)
```

output
```
Integral(cosh(b*x)*Chi(b*x), x)
```

Maxima [F]

$$\int \cosh(bx)\text{Chi}(bx)\,dx = \int \text{Chi}(bx)\cosh{(bx)}\,\,dx$$

input
```
integrate(cosh(b*x)*Chi(b*x),x, algorithm="maxima")
```

output
```
integrate(Chi(b*x)*cosh(b*x), x)
```

Giac [F]

$$\int \cosh(bx)\text{Chi}(bx)\,dx = \int \text{Chi}(bx)\cosh{(bx)}\,\,dx$$

input
```
integrate(cosh(b*x)*Chi(b*x),x, algorithm="giac")
```

output
```
integrate(Chi(b*x)*cosh(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(bx)\text{Chi}(bx)\,dx = \int \text{coshint}(b\,x)\,\cosh(b\,x)\,dx$$

input
```
int(coshint(b*x)*cosh(b*x),x)
```

output
```
int(coshint(b*x)*cosh(b*x), x)
```

Reduce [F]

$$\int \cosh(bx)\mathrm{Chi}(bx)\,dx = \int \chi(bx)\cosh{(bx)}\,dx$$

input
```
int(cosh(b*x)*Chi(b*x),x)
```

output
```
int(chi(b*x)*cosh(b*x),x)
```

3.111 $\int x\cosh(bx)\mathbf{Chi}(bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 61

$$\int x\cosh(bx)\mathrm{Chi}(bx)\,dx=-\frac{\cosh(bx)\mathrm{Chi}(bx)}{b^2}+\frac{\mathrm{Chi}(2bx)}{2b^2}+\frac{\log(x)}{2b^2}+\frac{x\mathrm{Chi}(bx)\sinh(bx)}{b}-\frac{\sinh^2(bx)}{2b^2}$$

output
```
-cosh(b*x)*Chi(b*x)/b^2+1/2*Chi(2*b*x)/b^2+1/2*ln(x)/b^2+x*Chi(b*x)*sinh(b
*x)/b-1/2*sinh(b*x)^2/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int x\cosh(bx)\mathrm{Chi}(bx)\,dx=\frac{-\cosh(2bx)+2\mathrm{Chi}(2bx)+2\log(x)+4\mathrm{Chi}(bx)(-\cosh(bx)+bx\sinh(bx))}{4b^2}$$

input
```
Integrate[x*Cosh[b*x]*CoshIntegral[b*x],x]
```

output
```
(-Cosh[2*b*x] + 2*CoshIntegral[2*b*x] + 2*Log[x] + 4*CoshIntegral[b*x]*(-C
osh[b*x] + b*x*Sinh[b*x]))/(4*b^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {7097, 27, 3042, 26, 3044, 15, 7101, 27, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Chi}(bx)\cosh(bx)\,dx$$

$$\Big\downarrow \text{ 7097}$$

$$-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}-\int \frac{\cosh(bx)\sinh(bx)}{b}dx+\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\int \cosh(bx)\sinh(bx)dx}{b}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\int -i\cos(ibx)\sin(ibx)dx}{b}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}+\frac{i\int \cos(ibx)\sin(ibx)dx}{b}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 3044}$$

$$\frac{\int i\sinh(bx)d(i\sinh(bx))}{b^2}-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{ 7101}$$

$$-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\int \frac{\cosh^2(bx)}{bx}dx}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int \frac{\cosh^2(bx)}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)^2}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 3793$$

$$-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int \left(\frac{\cosh(2bx)}{2x}+\frac{1}{2x}\right)dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}}{b}}{b}$$

input
```
Int[x*Cosh[b*x]*CoshIntegral[b*x],x]
```

output
```
-(((Cosh[b*x]*CoshIntegral[b*x])/b - (CoshIntegral[2*b*x]/2 + Log[x]/2)/b)
/b) + (x*CoshIntegral[b*x]*Sinh[b*x])/b - Sinh[b*x]^2/(2*b^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7097
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7101
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\mathrm{Chi}(bx)(bx\sinh(bx)-\cosh(bx))-\frac{\cosh(bx)^2}{2}+\frac{\ln(bx)}{2}+\frac{\mathrm{Chi}(2bx)}{2}}{b^2}$	46
default	$\frac{\mathrm{Chi}(bx)(bx\sinh(bx)-\cosh(bx))-\frac{\cosh(bx)^2}{2}+\frac{\ln(bx)}{2}+\frac{\mathrm{Chi}(2bx)}{2}}{b^2}$	46

input
```
int(x*cosh(b*x)*Chi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(Chi(b*x)*(b*x*sinh(b*x)-cosh(b*x))-1/2*cosh(b*x)^2+1/2*ln(b*x)+1/2*
Chi(2*b*x))
```

Fricas [F]

$$\int x\cosh(bx)\mathrm{Chi}(bx)\,dx=\int x\mathrm{Chi}(bx)\cosh(bx)\,dx$$

input
```
integrate(x*cosh(b*x)*Chi(b*x),x, algorithm="fricas")
```

output
```
integral(x*cosh(b*x)*cosh_integral(b*x), x)
```

Sympy [F]

$$\int x\cosh(bx)\mathrm{Chi}(bx)\,dx=\int x\cosh(bx)\,\mathrm{Chi}(bx)\,dx$$

input
```
integrate(x*cosh(b*x)*Chi(b*x),x)
```

output
```
Integral(x*cosh(b*x)*Chi(b*x), x)
```

Maxima [F]

$$\int x\cosh(bx)\text{Chi}(bx)\,dx=\int x\text{Chi}(bx)\cosh{(bx)}\,dx$$

input
```
integrate(x*cosh(b*x)*Chi(b*x),x, algorithm="maxima")
```

output
```
integrate(x*Chi(b*x)*cosh(b*x), x)
```

Giac [F]

$$\int x\cosh(bx)\text{Chi}(bx)\,dx=\int x\text{Chi}(bx)\cosh{(bx)}\,dx$$

input
```
integrate(x*cosh(b*x)*Chi(b*x),x, algorithm="giac")
```

output
```
integrate(x*Chi(b*x)*cosh(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh(bx)\text{Chi}(bx)\,dx=\int x\,\text{coshint}(b\,x)\,\cosh(b\,x)\,dx$$

input
```
int(x*coshint(b*x)*cosh(b*x),x)
```

output
```
int(x*coshint(b*x)*cosh(b*x), x)
```

Reduce [F]

$$\int x\cosh(bx)\mathrm{Chi}(bx)\,dx = \int \chi(bx)\cosh(bx)\,x dx$$

input
```
int(x*cosh(b*x)*Chi(b*x),x)
```

output
```
int(chi(b*x)*cosh(b*x)*x,x)
```

3.112 $\int x^2 \cosh(bx)\mathbf{Chi}(bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int x^2\cosh(bx)\text{Chi}(bx)\,dx=\frac{3x}{4b^2}-\frac{2x\cosh(bx)\text{Chi}(bx)}{b^2}+\frac{5\cosh(bx)\sinh(bx)}{4b^3}+\frac{2\text{Chi}(bx)\sinh(bx)}{b^3}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{x\sinh^2(bx)}{2b^2}-\frac{\text{Shi}(2bx)}{b^3}$$

output
```
3/4*x/b^2-2*x*cosh(b*x)*Chi(b*x)/b^2+5/4*cosh(b*x)*sinh(b*x)/b^3+2*Chi(b*x
)*sinh(b*x)/b^3+x^2*Chi(b*x)*sinh(b*x)/b-1/2*x*sinh(b*x)^2/b^2-Shi(2*b*x)/
b^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.71

$$\int x^2\cosh(bx)\text{Chi}(bx)\,dx=\frac{8bx-2bx\cosh(2bx)+8\text{Chi}(bx)\left(-2bx\cosh(bx)+(2+b^2x^2)\sinh(bx)\right)+5\sinh(2bx)-8\text{Shi}(2bx)}{8b^3}$$

input
```
Integrate[x^2*Cosh[b*x]*CoshIntegral[b*x],x]
```

output
```
(8*b*x - 2*b*x*Cosh[2*b*x] + 8*CoshIntegral[b*x]*(-2*b*x*Cosh[b*x] + (2 +
b^2*x^2)*Sinh[b*x]) + 5*Sinh[2*b*x] - 8*SinhIntegral[2*b*x])/(8*b^3)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.54, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.583$, Rules used = {7097, 27, 5895, 3042, 25, 3115, 24, 7103, 27, 3042, 3115, 24, 7095, 27, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{Chi}(bx)\cosh(bx)\,dx$$

$$\Big\downarrow \; 7097$$

$$-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\int \frac{x\cosh(bx)\sinh(bx)}{b}dx+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\int x\cosh(bx)\sinh(bx)dx}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 5895$$

$$-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}-\frac{\int \sinh^2(bx)dx}{2b}}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}-\frac{\int -\sin(ibx)^2dx}{2b}}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\int \sin(ibx)^2dx}{2b}}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 3115$$

$$-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\frac{\frac{\int 1dx}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}+\frac{x\sinh^2(bx)}{2b}}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \text{7103}$$

$$-\frac{2\left(-\frac{\int\cosh(bx)\text{Chi}(bx)dx}{b}-\int\frac{\cosh^2(bx)}{b}dx+\frac{x\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\left(-\frac{\int\cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\int\cosh^2(bx)dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(-\frac{\int\cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\int\sin\left(ibx+\frac{\pi}{2}\right)^2dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{2\left(-\frac{\int\cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\frac{\int 1dx}{2}+\frac{\sinh(bx)\cosh(bx)}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{2\left(-\frac{\int\cosh(bx)\text{Chi}(bx)dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \text{7095}$$

$$-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\int \frac{\cosh(bx)\sinh(bx)}{bx}dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int \frac{\cosh(bx)\sinh(bx)}{x}dx}{b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \; 5971$$

$$-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int \frac{\sinh(2bx)}{2x}dx}{b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int \frac{\sinh(2bx)}{x}dx}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int -\frac{i\sin(2ibx)}{x}dx}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}+\frac{i\int \frac{\sin(2ibx)}{x}dx}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

$$\Big\downarrow \text{3779}$$

$$-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\text{Shi}(2bx)}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}$$

input
```
Int[x^2*Cosh[b*x]*CoshIntegral[b*x],x]
```

output
```
(x^2*CoshIntegral[b*x]*Sinh[b*x])/b - ((x*Sinh[b*x]^2)/(2*b) + (x/2 - (Cos
h[b*x]*Sinh[b*x])/(2*b))/(2*b))/b - (2*((x*Cosh[b*x]*CoshIntegral[b*x])/b
- (x/2 + (Cosh[b*x]*Sinh[b*x])/(2*b))/b - ((CoshIntegral[b*x]*Sinh[b*x])/b
 - SinhIntegral[2*b*x]/(2*b))/b))/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7095
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7097
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7103
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{\text{Chi}(bx)\left(b^2x^2\sinh(bx)-2bx\cosh(bx)+2\sinh(bx)\right)-\frac{bx\cosh(bx)^2}{2}+\frac{5\cosh(bx)\sinh(bx)}{4}+\frac{5bx}{4}-\text{Shi}(2bx)}{b^3}$	68
default	$\frac{\text{Chi}(bx)\left(b^2x^2\sinh(bx)-2bx\cosh(bx)+2\sinh(bx)\right)-\frac{bx\cosh(bx)^2}{2}+\frac{5\cosh(bx)\sinh(bx)}{4}+\frac{5bx}{4}-\text{Shi}(2bx)}{b^3}$	68

input
```
int(x^2*cosh(b*x)*Chi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(Chi(b*x)*(b^2*x^2*sinh(b*x)-2*b*x*cosh(b*x)+2*sinh(b*x))-1/2*b*x*co
sh(b*x)^2+5/4*cosh(b*x)*sinh(b*x)+5/4*b*x-Shi(2*b*x))
```

Fricas [F]

$$\int x^2\cosh(bx)\text{Chi}(bx)\,dx=\int x^2\text{Chi}(bx)\cosh(bx)\,dx$$

input
```
integrate(x^2*cosh(b*x)*Chi(b*x),x, algorithm="fricas")
```

output
```
integral(x^2*cosh(b*x)*cosh_integral(b*x), x)
```

Sympy [F]

$$\int x^2\cosh(bx)\text{Chi}(bx)\,dx=\int x^2\cosh(bx)\,\text{Chi}(bx)\,dx$$

input
```
integrate(x**2*cosh(b*x)*Chi(b*x),x)
```

output
```
Integral(x**2*cosh(b*x)*Chi(b*x), x)
```

Maxima [F]

$$\int x^2 \cosh(bx)\text{Chi}(bx)\,dx = \int x^2\text{Chi}(bx)\cosh(bx)\,dx$$

input
```
integrate(x^2*cosh(b*x)*Chi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^2*Chi(b*x)*cosh(b*x), x)
```

Giac [F]

$$\int x^2 \cosh(bx)\text{Chi}(bx)\,dx = \int x^2\text{Chi}(bx)\cosh(bx)\,dx$$

input
```
integrate(x^2*cosh(b*x)*Chi(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*Chi(b*x)*cosh(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh(bx)\text{Chi}(bx)\,dx = \int x^2\,\text{coshint}(b\,x)\,\cosh(b\,x)\,dx$$

input
```
int(x^2*coshint(b*x)*cosh(b*x),x)
```

output
```
int(x^2*coshint(b*x)*cosh(b*x), x)
```

Reduce [F]

$$\int x^2 \cosh(bx)\text{Chi}(bx)\,dx = \int \chi(bx)\cosh(bx)\,x^2dx$$

input
```
int(x^2*cosh(b*x)*Chi(b*x),x)
```

output
```
int(chi(b*x)*cosh(b*x)*x**2,x)
```

3.113 $\int x^3 \cosh(bx)\mathbf{Chi}(bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 142

$$\int x^3\cosh(bx)\mathrm{Chi}(bx)\,dx=\frac{x^2}{2b^2}-\frac{3\cosh^2(bx)}{4b^4}-\frac{6\cosh(bx)\mathrm{Chi}(bx)}{b^4}-\frac{3x^2\cosh(bx)\mathrm{Chi}(bx)}{b^2}+\frac{3\mathrm{Chi}(2bx)}{b^4}+\frac{3\log(x)}{b^4}+\frac{2x\cosh(bx)\sinh(bx)}{b^3}+\frac{6x\mathrm{Chi}(bx)\sinh(bx)}{b^3}+\frac{x^3\mathrm{Chi}(bx)\sinh(bx)}{b}-\frac{13\sinh^2(bx)}{4b^4}-\frac{x^2\sinh^2(bx)}{2b^2}$$

output
```
1/2*x^2/b^2-3/4*cosh(b*x)^2/b^4-6*cosh(b*x)*Chi(b*x)/b^4-3*x^2*cosh(b*x)*C
hi(b*x)/b^2+3*Chi(2*b*x)/b^4+3*ln(x)/b^4+2*x*cosh(b*x)*sinh(b*x)/b^3+6*x*C
hi(b*x)*sinh(b*x)/b^3+x^3*Chi(b*x)*sinh(b*x)/b-13/4*sinh(b*x)^2/b^4-1/2*x^
2*sinh(b*x)^2/b^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.66

$$\int x^3 \cosh(bx)\text{Chi}(bx)\,dx = \frac{3b^2x^2 - 8\cosh(2bx) - b^2x^2\cosh(2bx) + 12\text{Chi}(2bx) + 12\log(x) + 4\text{Chi}(bx)\left(-3(2+b^2x^2)\cosh(bx) + bx\right.}{4b^4}$$

input
```
Integrate[x^3*Cosh[b*x]*CoshIntegral[b*x],x]
```

output
```
(3*b^2*x^2 - 8*Cosh[2*b*x] - b^2*x^2*Cosh[2*b*x] + 12*CoshIntegral[2*b*x]
+ 12*Log[x] + 4*CoshIntegral[b*x]*(-3*(2 + b^2*x^2)*Cosh[b*x] + b*x*(6 + b
^2*x^2)*Sinh[b*x]) + 4*b*x*Sinh[2*b*x])/(4*b^4)
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.50, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.917$, Rules used = {7097, 27, 5895, 3042, 25, 3791, 15, 7103, 27, 3042, 3791, 15, 7097, 27, 3042, 26, 3044, 15, 7101, 27, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{Chi}(bx)\cosh(bx)\,dx$$

$$\Big\downarrow \text{7097}$$

$$-\frac{3\int x^2\text{Chi}(bx)\sinh(bx)dx}{b} - \int \frac{x^2\cosh(bx)\sinh(bx)}{b}dx + \frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int x^2\text{Chi}(bx)\sinh(bx)dx}{b} - \frac{\int x^2\cosh(bx)\sinh(bx)dx}{b} + \frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{5895}$$

$$-\frac{3\int x^2\text{Chi}(bx)\sinh(bx)dx}{b} - \frac{\frac{x^2\sinh^2(bx)}{2b} - \frac{\int x\sinh^2(bx)dx}{b}}{b} + \frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\int x^2\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\frac{x^2\sinh^2(bx)}{2b}-\frac{\int -x\sin(ibx)^2dx}{b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{3\int x^2\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\frac{x^2\sinh^2(bx)}{2b}+\frac{\int x\sin(ibx)^2dx}{b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 3791$$

$$-\frac{\frac{\frac{\int xdx}{2}+\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}-\frac{3\int x^2\text{Chi}(bx)\sinh(bx)dx}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 15$$

$$-\frac{3\int x^2\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 7103$$

$$-\frac{3\left(-\frac{2\int x\cosh(bx)\text{Chi}(bx)dx}{b}-\int\frac{x\cosh^2(bx)}{b}dx+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{3\left(-\frac{2\int x\cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\int x\cosh^2(bx)dx}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\left(-\frac{2\int x\cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\int x\sin\left(ibx+\frac{\pi}{2}\right)^2dx}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \; 3791$$

$$-\frac{3\left(-\frac{\frac{\int xdx}{2}-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}}{b}-\frac{2\int x\cosh(bx)\text{Chi}(bx)dx}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\left(-\frac{2\int x\cosh(bx)\text{Chi}(bx)dx}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{7097}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int\text{Chi}(bx)\sinh(bx)dx}{b}-\int\frac{\cosh(bx)\sinh(bx)}{b}dx+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\int\cosh(bx)\sinh(bx)dx}{b}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\int -i\cos(ibx)\sin(ibx)dx}{b}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}-\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}+\frac{i\int \cos(ibx)\sin(ibx)dx}{b}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}$$

$$\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$\big\downarrow$ 3044

$$-\frac{3\left(-\frac{2\left(\frac{\int i\sinh(bx)d(i\sinh(bx))}{b^2}-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}$$

$$\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$\big\downarrow$ 15

$$-\frac{3\left(-\frac{2\left(-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}-$$

$$\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$\big\downarrow$ 7101

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\int\frac{\cosh^2(bx)}{bx}dx}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right.}{b}$$

$$\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$\big\downarrow$ 27

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int\frac{\cosh^2(bx)}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\cdots\right.}{b}$$

$$\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$\downarrow$ 3042

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int\frac{\sin\left(ibx+\frac{\pi}{2}\right)^2}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh\cdots}{b}\right.}{b}$$

$$\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$\downarrow$ 3793

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int\left(\frac{\cosh(2bx)}{2x}+\frac{1}{2x}\right)dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\,\text{c}\cdots}{b}\right.}{b}$$

$$\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

$\downarrow$ 2009

$$-\frac{3\left(-\frac{2\left(-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}}{b}}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}$$

$$\frac{\frac{\frac{\sinh^2(bx)}{4b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(bx)}{2b}}{b}+\frac{x^3\text{Chi}(bx)\sinh(bx)}{b}$$

input
```
Int[x^3*Cosh[b*x]*CoshIntegral[b*x],x]
```

output
```
(x^3*CoshIntegral[b*x]*Sinh[b*x])/b - (3*((x^2*Cosh[b*x]*CoshIntegral[b*x]
)/b - (x^2/4 - Cosh[b*x]^2/(4*b^2) + (x*Cosh[b*x]*Sinh[b*x])/(2*b))/b - (2
*(-(((Cosh[b*x]*CoshIntegral[b*x])/b - (CoshIntegral[2*b*x]/2 + Log[x]/2)/
b)/b) + (x*CoshIntegral[b*x]*Sinh[b*x])/b - Sinh[b*x]^2/(2*b^2)))/b))/b -
((x^2*Sinh[b*x]^2)/(2*b) + (x^2/4 - (x*Cosh[b*x]*Sinh[b*x])/(2*b) + Sinh[b
*x]^2/(4*b^2))/b)/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 7097
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7101
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7103
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{\mathrm{Chi}(bx)\left(b^{3}x^{3}\sinh(bx)-3b^{2}x^{2}\cosh(bx)+6bx\sinh(bx)-6\cosh(bx)\right)-\frac{b^{2}x^{2}\cosh(bx)^{2}}{2}+2bx\cosh(bx)\sinh(bx)+b^{2}x^{2}-4\,\mathrm{co}}{b^{4}}$
default	$\frac{\mathrm{Chi}(bx)\left(b^{3}x^{3}\sinh(bx)-3b^{2}x^{2}\cosh(bx)+6bx\sinh(bx)-6\cosh(bx)\right)-\frac{b^{2}x^{2}\cosh(bx)^{2}}{2}+2bx\cosh(bx)\sinh(bx)+b^{2}x^{2}-4\,\mathrm{co}}{b^{4}}$

input
```
int(x^3*cosh(b*x)*Chi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(Chi(b*x)*(b^3*x^3*sinh(b*x)-3*b^2*x^2*cosh(b*x)+6*b*x*sinh(b*x)-6*c
osh(b*x))-1/2*b^2*x^2*cosh(b*x)^2+2*b*x*cosh(b*x)*sinh(b*x)+b^2*x^2-4*cosh
(b*x)^2+3*ln(b*x)+3*Chi(2*b*x))
```

Fricas [F]

$$\int x^{3}\cosh(bx)\mathrm{Chi}(bx)\,dx=\int x^{3}\mathrm{Chi}(bx)\cosh\left(bx\right)\,dx$$

input
```
integrate(x^3*cosh(b*x)*Chi(b*x),x, algorithm="fricas")
```

output
```
integral(x^3*cosh(b*x)*cosh_integral(b*x), x)
```

Sympy [F]

$$\int x^3 \cosh(bx)\text{Chi}(bx)\, dx = \int x^3 \cosh{(bx)}\, \text{Chi}\,(bx)\; dx$$

input
```
integrate(x**3*cosh(b*x)*Chi(b*x),x)
```

output
```
Integral(x**3*cosh(b*x)*Chi(b*x), x)
```

Maxima [F]

$$\int x^3 \cosh(bx)\text{Chi}(bx)\, dx = \int x^3 \text{Chi}(bx) \cosh{(bx)}\; dx$$

input
```
integrate(x^3*cosh(b*x)*Chi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^3*Chi(b*x)*cosh(b*x), x)
```

Giac [F]

$$\int x^3 \cosh(bx)\text{Chi}(bx)\, dx = \int x^3 \text{Chi}(bx) \cosh{(bx)}\; dx$$

input
```
integrate(x^3*cosh(b*x)*Chi(b*x),x, algorithm="giac")
```

output
```
integrate(x^3*Chi(b*x)*cosh(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cosh(bx)\text{Chi}(bx)\,dx = \int x^3 \,\text{coshint}(b\,x)\,\cosh(b\,x)\,dx$$

input
```
int(x^3*coshint(b*x)*cosh(b*x),x)
```

output
```
int(x^3*coshint(b*x)*cosh(b*x), x)
```

Reduce [F]

$$\int x^3 \cosh(bx)\text{Chi}(bx)\,dx = \int \chi(bx)\cosh(bx)\,x^3 dx$$

input
```
int(x^3*cosh(b*x)*Chi(b*x),x)
```

output
```
int(chi(b*x)*cosh(b*x)*x**3,x)
```

3.114 $\int \frac{\text{Chi}(bx)\sinh(bx)}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x^3}\,dx = -\frac{b\cosh^2(bx)}{2x} - \frac{b\cosh(2bx)}{4x} - \frac{b\cosh(bx)\text{Chi}(bx)}{2x} - \frac{\text{Chi}(bx)\sinh(bx)}{2x^2} - \frac{\sinh(2bx)}{8x^2} + b^2\text{Shi}(2bx) + \frac{1}{2}b^2\text{Int}\left(\frac{\text{Chi}(bx)\sinh(bx)}{x},x\right)$$

output
```
-1/2*b*cosh(b*x)^2/x-1/4*b*cosh(2*b*x)/x-1/2*b*cosh(b*x)*Chi(b*x)/x-1/2*Ch
i(b*x)*sinh(b*x)/x^2-1/8*sinh(2*b*x)/x^2+b^2*Shi(2*b*x)+1/2*b^2*Defer(Int)
(Chi(b*x)*sinh(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x^3}\,dx = \int \frac{\text{Chi}(bx)\sinh(bx)}{x^3}\,dx$$

input
```
Integrate[(CoshIntegral[b*x]*Sinh[b*x])/x^3,x]
```

output
```
Integrate[(CoshIntegral[b*x]*Sinh[b*x])/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x^3}dx$$

$$\Big\downarrow \text{7105}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}dx+\frac{1}{2}b\int \frac{\cosh(bx)\sinh(bx)}{bx^3}dx-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}dx+\frac{1}{2}\int \frac{\cosh(bx)\sinh(bx)}{x^3}dx-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{5971}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}dx+\frac{1}{2}\int \frac{\sinh(2bx)}{2x^3}dx-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}dx+\frac{1}{4}\int \frac{\sinh(2bx)}{x^3}dx-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}dx+\frac{1}{4}\int -\frac{i\sin(2ibx)}{x^3}dx-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}dx-\frac{1}{4}i\int \frac{\sin(2ibx)}{x^3}dx-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}dx-\frac{1}{4}i\left(ib\int \frac{\cosh(2bx)}{x^2}dx-\frac{i\sinh(2bx)}{2x^2}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}dx-\frac{1}{4}i\left(ib\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x^2}dx-\frac{i\sinh(2bx)}{2x^2}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}dx-\frac{1}{4}i\left(ib\left(-\frac{\cosh(2bx)}{x}+2ib\int -\frac{i\sinh(2bx)}{x}dx\right)-\frac{i\sinh(2bx)}{2x^2}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}dx-\frac{1}{4}i\left(ib\left(2b\int \frac{\sinh(2bx)}{x}dx-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}dx-\frac{1}{4}i\left(ib\left(-\frac{\cosh(2bx)}{x}+2b\int -\frac{i\sin(2ibx)}{x}dx\right)-\frac{i\sinh(2bx)}{2x^2}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}dx-\frac{1}{4}i\left(ib\left(-\frac{\cosh(2bx)}{x}-2ib\int \frac{\sin(2ibx)}{x}dx\right)-\frac{i\sinh(2bx)}{2x^2}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}$$

$$\Big\downarrow \text{3779}$$

$$\frac{1}{2}b\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}dx-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$$\Big\downarrow \text{7099}$$

$$\frac{1}{2}b\left(b\int\frac{\text{Chi}(bx)\sinh(bx)}{x}dx+b\int\frac{\cosh^2(bx)}{bx^2}dx-\frac{\text{Chi}(bx)\cosh(bx)}{x}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{2}b\left(b\int\frac{\text{Chi}(bx)\sinh(bx)}{x}dx+\int\frac{\cosh^2(bx)}{x^2}dx-\frac{\text{Chi}(bx)\cosh(bx)}{x}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$\Big\downarrow$ 3042

$$\frac{1}{2}b\left(b\int\frac{\text{Chi}(bx)\sinh(bx)}{x}dx+\int\frac{\sin\left(ibx+\frac{\pi}{2}\right)^2}{x^2}dx-\frac{\text{Chi}(bx)\cosh(bx)}{x}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$\Big\downarrow$ 3794

$$\frac{1}{2}b\left(b\int\frac{\text{Chi}(bx)\sinh(bx)}{x}dx+2ib\int-\frac{i\sinh(2bx)}{2x}dx-\frac{\text{Chi}(bx)\cosh(bx)}{x}-\frac{\cosh^2(bx)}{x}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{2}b\left(b\int\frac{\text{Chi}(bx)\sinh(bx)}{x}dx+b\int\frac{\sinh(2bx)}{x}dx-\frac{\text{Chi}(bx)\cosh(bx)}{x}-\frac{\cosh^2(bx)}{x}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$\Big\downarrow$ 3042

$$\frac{1}{2}b\left(b\int\frac{\text{Chi}(bx)\sinh(bx)}{x}dx+b\int-\frac{i\sin(2ibx)}{x}dx-\frac{\text{Chi}(bx)\cosh(bx)}{x}-\frac{\cosh^2(bx)}{x}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$\Big\downarrow$ 26

$$\frac{1}{2}b\left(b\int\frac{\text{Chi}(bx)\sinh(bx)}{x}dx-ib\int\frac{\sin(2ibx)}{x}dx-\frac{\text{Chi}(bx)\cosh(bx)}{x}-\frac{\cosh^2(bx)}{x}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$\Big\downarrow$ 3779

$$\frac{1}{2}b\left(b\int \frac{\text{Chi}(bx)\sinh(bx)}{x}dx-\frac{\text{Chi}(bx)\cosh(bx)}{x}+b\text{Shi}(2bx)-\frac{\cosh^2(bx)}{x}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

$$\Big\downarrow \text{7299}$$

$$\frac{1}{2}b\left(b\int \frac{\text{Chi}(bx)\sinh(bx)}{x}dx-\frac{\text{Chi}(bx)\cosh(bx)}{x}+b\text{Shi}(2bx)-\frac{\cosh^2(bx)}{x}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}-\frac{1}{4}i\left(ib\left(2b\text{Shi}(2bx)-\frac{\cosh(2bx)}{x}\right)-\frac{i\sinh(2bx)}{2x^2}\right)$$

input
```
Int[(CoshIntegral[b*x]*Sinh[b*x])/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\text{Chi}\,(bx)\sinh\,(bx)}{x^3}dx$$

input
```
int(Chi(b*x)*sinh(b*x)/x^3,x)
```

output
```
int(Chi(b*x)*sinh(b*x)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x^3}\,dx=\int \frac{\text{Chi}(bx)\sinh\,(bx)}{x^3}\,dx$$

input
```
integrate(Chi(b*x)*sinh(b*x)/x^3,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x)*sinh(b*x)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 3.89 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(bx)\sinh(bx)}{x^3}\,dx = \int \frac{\sinh{(bx)}\,\mathrm{Chi}\,(bx)}{x^3}\,dx$$

input
```
integrate(Chi(b*x)*sinh(b*x)/x**3,x)
```

output
```
Integral(sinh(b*x)*Chi(b*x)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(bx)\sinh(bx)}{x^3}\,dx = \int \frac{\mathrm{Chi}(bx)\sinh{(bx)}}{x^3}\,dx$$

input
```
integrate(Chi(b*x)*sinh(b*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x)*sinh(b*x)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(bx)\sinh(bx)}{x^3}\,dx = \int \frac{\mathrm{Chi}(bx)\sinh(bx)}{x^3}\,dx$$

input
```
integrate(Chi(b*x)*sinh(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(Chi(b*x)*sinh(b*x)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(bx)\sinh(bx)}{x^3}\,dx = \int \frac{\mathrm{coshint}(b\,x)\,\sinh(b\,x)}{x^3}\,dx$$

input
```
int((coshint(b*x)*sinh(b*x))/x^3,x)
```

output
```
int((coshint(b*x)*sinh(b*x))/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(bx)\sinh(bx)}{x^3}\,dx = \int \frac{\chi(bx)\sinh(bx)}{x^3}dx$$

input
```
int(Chi(b*x)*sinh(b*x)/x^3,x)
```

output
```
int((chi(b*x)*sinh(b*x))/x**3,x)
```

3.115 $\int \frac{\text{Chi}(bx)\sinh(bx)}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 44

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x^2}\,dx = \frac{1}{2}b\text{Chi}(bx)^2 + b\text{Chi}(2bx) - \frac{\text{Chi}(bx)\sinh(bx)}{x} - \frac{\sinh(2bx)}{2x}$$

output
```
1/2*b*Chi(b*x)^2+b*Chi(2*b*x)-Chi(b*x)*sinh(b*x)/x-1/2*sinh(2*b*x)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x^2}\,dx = \frac{1}{2}b\text{Chi}(bx)^2 + b\text{Chi}(2bx) - \frac{\text{Chi}(bx)\sinh(bx)}{x} - \frac{\sinh(2bx)}{2x}$$

input
```
Integrate[(CoshIntegral[b*x]*Sinh[b*x])/x^2,x]
```

output
```
(b*CoshIntegral[b*x]^2)/2 + b*CoshIntegral[2*b*x] - (CoshIntegral[b*x]*Sin
h[b*x])/x - Sinh[2*b*x]/(2*x)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {7105, 27, 5971, 27, 3042, 26, 3778, 3042, 3782, 7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x^2}dx$$

$$\Big\downarrow \text{7105}$$

$$b\int \frac{\cosh(bx)\text{Chi}(bx)}{x}dx+b\int \frac{\cosh(bx)\sinh(bx)}{bx^2}dx-\frac{\text{Chi}(bx)\sinh(bx)}{x}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{\cosh(bx)\text{Chi}(bx)}{x}dx+\int \frac{\cosh(bx)\sinh(bx)}{x^2}dx-\frac{\text{Chi}(bx)\sinh(bx)}{x}$$

$$\Big\downarrow \text{5971}$$

$$b\int \frac{\cosh(bx)\text{Chi}(bx)}{x}dx+\int \frac{\sinh(2bx)}{2x^2}dx-\frac{\text{Chi}(bx)\sinh(bx)}{x}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{\cosh(bx)\text{Chi}(bx)}{x}dx+\frac{1}{2}\int \frac{\sinh(2bx)}{x^2}dx-\frac{\text{Chi}(bx)\sinh(bx)}{x}$$

$$\Big\downarrow \text{3042}$$

$$b\int \frac{\cosh(bx)\text{Chi}(bx)}{x}dx+\frac{1}{2}\int -\frac{i\sin(2ibx)}{x^2}dx-\frac{\text{Chi}(bx)\sinh(bx)}{x}$$

$$\Big\downarrow \text{26}$$

$$b\int \frac{\cosh(bx)\text{Chi}(bx)}{x}dx-\frac{1}{2}i\int \frac{\sin(2ibx)}{x^2}dx-\frac{\text{Chi}(bx)\sinh(bx)}{x}$$

$$\Big\downarrow \text{3778}$$

$$b\int \frac{\cosh(bx)\text{Chi}(bx)}{x}dx-\frac{1}{2}i\left(2ib\int \frac{\cosh(2bx)}{x}dx-\frac{i\sinh(2bx)}{x}\right)-\frac{\text{Chi}(bx)\sinh(bx)}{x}$$

$$\Big\downarrow \text{3042}$$

$$b\int \frac{\cosh(bx)\text{Chi}(bx)}{x}dx - \frac{1}{2}i\left(2ib\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx - \frac{i\sinh(2bx)}{x}\right) - \frac{\text{Chi}(bx)\sinh(bx)}{x}$$

$$\Big\downarrow \text{3782}$$

$$b\int \frac{\cosh(bx)\text{Chi}(bx)}{x}dx - \frac{\text{Chi}(bx)\sinh(bx)}{x} - \frac{1}{2}i\left(2ib\text{Chi}(2bx) - \frac{i\sinh(2bx)}{x}\right)$$

$$\Big\downarrow \text{7237}$$

$$\frac{1}{2}b\text{Chi}(bx)^2 - \frac{\text{Chi}(bx)\sinh(bx)}{x} - \frac{1}{2}i\left(2ib\text{Chi}(2bx) - \frac{i\sinh(2bx)}{x}\right)$$

input
```
Int[(CoshIntegral[b*x]*Sinh[b*x])/x^2,x]
```

output
```
(b*CoshIntegral[b*x]^2)/2 - (CoshIntegral[b*x]*Sinh[b*x])/x - (I/2)*((2*I)
*b*CoshIntegral[2*b*x] - (I*Sinh[2*b*x])/x)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7105
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_)*Sinh[(a_.) +
 (b_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^(m + 1)*Sinh[a + b*x]*(CoshIntegr
al[c + d*x]/(f*(m + 1))), x] + (-Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)
*Cosh[a + b*x]*CoshIntegral[c + d*x], x], x] - Simp[d/(f*(m + 1))   Int[(e
+ f*x)^(m + 1)*Sinh[a + b*x]*(Cosh[c + d*x]/(c + d*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f}, x] && ILtQ[m, -1]
```

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{\mathrm{Chi}\,(bx)\sinh\,(bx)}{x^2}dx$$

input
```
int(Chi(b*x)*sinh(b*x)/x^2,x)
```

output
```
int(Chi(b*x)*sinh(b*x)/x^2,x)
```

Fricas [F]

$$\int \frac{\text{Chi}(bx) \sinh(bx)}{x^2} \, dx = \int \frac{\text{Chi}(bx) \sinh(bx)}{x^2} \, dx$$

input
```
integrate(Chi(b*x)*sinh(b*x)/x^2,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x)*sinh(b*x)/x^2, x)
```

Sympy [F]

$$\int \frac{\text{Chi}(bx) \sinh(bx)}{x^2} \, dx = \int \frac{\sinh(bx) \text{Chi}(bx)}{x^2} \, dx$$

input
```
integrate(Chi(b*x)*sinh(b*x)/x**2,x)
```

output
```
Integral(sinh(b*x)*Chi(b*x)/x**2, x)
```

Maxima [F]

$$\int \frac{\text{Chi}(bx) \sinh(bx)}{x^2} \, dx = \int \frac{\text{Chi}(bx) \sinh(bx)}{x^2} \, dx$$

input
```
integrate(Chi(b*x)*sinh(b*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x)*sinh(b*x)/x^2, x)
```

Giac [F]

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x^2}\,dx = \int \frac{\text{Chi}(bx)\sinh{(bx)}}{x^2}\,dx$$

input
```
integrate(Chi(b*x)*sinh(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(Chi(b*x)*sinh(b*x)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x^2}\,dx = \int \frac{\text{coshint}(b\,x)\,\sinh(b\,x)}{x^2}\,dx$$

input
```
int((coshint(b*x)*sinh(b*x))/x^2,x)
```

output
```
int((coshint(b*x)*sinh(b*x))/x^2, x)
```

Reduce [F]

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x^2}\,dx = \int \frac{\chi(bx)\sinh{(bx)}}{x^2}dx$$

input
```
int(Chi(b*x)*sinh(b*x)/x^2,x)
```

output
```
int((chi(b*x)*sinh(b*x))/x**2,x)
```

3.116 $\int \frac{\text{Chi}(bx)\sinh(bx)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x}\,dx = \text{Int}\left(\frac{\text{Chi}(bx)\sinh(bx)}{x},x\right)$$

output
```
Defer(Int)(Chi(b*x)*sinh(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x}\,dx = \int \frac{\text{Chi}(bx)\sinh(bx)}{x}\,dx$$

input
```
Integrate[(CoshIntegral[b*x]*Sinh[b*x])/x,x]
```

output
```
Integrate[(CoshIntegral[b*x]*Sinh[b*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x}dx$$

input
```
Int[(CoshIntegral[b*x]*Sinh[b*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\text{Chi}\left(bx\right)\sinh\left(bx\right)}{x}dx$$

input
```
int(Chi(b*x)*sinh(b*x)/x,x)
```

output
```
int(Chi(b*x)*sinh(b*x)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x}\,dx = \int \frac{\text{Chi}(bx)\sinh{(bx)}}{x}\,dx$$

input
```
integrate(Chi(b*x)*sinh(b*x)/x,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x)*sinh(b*x)/x, x)
```

Sympy [N/A]

Not integrable

Time = 3.77 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x}\,dx = \int \frac{\sinh{(bx)}\,\text{Chi}\,(bx)}{x}\,dx$$

input
```
integrate(Chi(b*x)*sinh(b*x)/x,x)
```

output
```
Integral(sinh(b*x)*Chi(b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x}\,dx = \int \frac{\text{Chi}(bx)\sinh{(bx)}}{x}\,dx$$

input
```
integrate(Chi(b*x)*sinh(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x)*sinh(b*x)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x}\,dx = \int \frac{\text{Chi}(bx)\sinh{(bx)}}{x}\,dx$$

input
```
integrate(Chi(b*x)*sinh(b*x)/x,x, algorithm="giac")
```

output
```
integrate(Chi(b*x)*sinh(b*x)/x, x)
```

Mupad [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x}\,dx = \int \frac{\text{coshint}(b\,x)\,\sinh(b\,x)}{x}\,dx$$

input
```
int((coshint(b*x)*sinh(b*x))/x,x)
```

output
```
int((coshint(b*x)*sinh(b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\mathrm{Chi}(bx)\sinh(bx)}{x}\,dx = \int \frac{\chi(bx)\sinh{(bx)}}{x}dx$$

input
```
int(Chi(b*x)*sinh(b*x)/x,x)
```

output
```
int((chi(b*x)*sinh(b*x))/x,x)
```

3.117 $\int \mathrm{Chi}(bx)\sinh(bx)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 34

$$\int \mathrm{Chi}(bx)\sinh(bx)\,dx = \frac{\cosh(bx)\mathrm{Chi}(bx)}{b} - \frac{\mathrm{Chi}(2bx)}{2b} - \frac{\log(x)}{2b}$$

output
```
cosh(b*x)*Chi(b*x)/b-1/2*Chi(2*b*x)/b-1/2*ln(x)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \mathrm{Chi}(bx)\sinh(bx)\,dx = \frac{\cosh(bx)\mathrm{Chi}(bx)}{b} - \frac{\mathrm{Chi}(2bx)}{2b} - \frac{\log(bx)}{2b}$$

input
```
Integrate[CoshIntegral[b*x]*Sinh[b*x],x]
```

output
```
(Cosh[b*x]*CoshIntegral[b*x])/b - CoshIntegral[2*b*x]/(2*b) - Log[b*x]/(2*
b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {7101, 27, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Chi}(bx)\sinh(bx)\,dx$$

$$\Big\downarrow \text{ 7101}$$

$$\frac{\text{Chi}(bx)\cosh(bx)}{b}-\int \frac{\cosh^2(bx)}{bx}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int \frac{\cosh^2(bx)}{x}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)^2}{x}dx}{b}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int \left(\frac{\cosh(2bx)}{2x}+\frac{1}{2x}\right)dx}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}}{b}$$

input
```
Int[CoshIntegral[b*x]*Sinh[b*x],x]
```

output
```
(Cosh[b*x]*CoshIntegral[b*x])/b - (CoshIntegral[2*b*x]/2 + Log[x]/2)/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7101
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\cosh(bx)\operatorname{Chi}(bx)-\frac{\ln(bx)}{2}-\frac{\operatorname{Chi}(2bx)}{2}}{b}$	28
default	$\frac{\cosh(bx)\operatorname{Chi}(bx)-\frac{\ln(bx)}{2}-\frac{\operatorname{Chi}(2bx)}{2}}{b}$	28

input
```
int(Chi(b*x)*sinh(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b*(cosh(b*x)*Chi(b*x)-1/2*ln(b*x)-1/2*Chi(2*b*x))
```

Fricas [F]

$$\int \mathrm{Chi}(bx)\sinh(bx)\,dx = \int \mathrm{Chi}(bx)\sinh(bx)\;dx$$

input
```
integrate(Chi(b*x)*sinh(b*x),x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x)*sinh(b*x), x)
```

Sympy [F]

$$\int \mathrm{Chi}(bx)\sinh(bx)\,dx = \int \sinh(bx)\,\mathrm{Chi}(bx)\;dx$$

input
```
integrate(Chi(b*x)*sinh(b*x),x)
```

output
```
Integral(sinh(b*x)*Chi(b*x), x)
```

Maxima [F]

$$\int \mathrm{Chi}(bx)\sinh(bx)\,dx = \int \mathrm{Chi}(bx)\sinh(bx)\;dx$$

input
```
integrate(Chi(b*x)*sinh(b*x),x, algorithm="maxima")
```

output
```
integrate(Chi(b*x)*sinh(b*x), x)
```

Giac [F]

$$\int \operatorname{Chi}(bx)\sinh(bx)\,dx = \int \operatorname{Chi}(bx)\sinh(bx)\,dx$$

input
```
integrate(Chi(b*x)*sinh(b*x),x, algorithm="giac")
```

output
```
integrate(Chi(b*x)*sinh(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{Chi}(bx)\sinh(bx)\,dx = \int \operatorname{coshint}(b\,x)\,\sinh(b\,x)\,dx$$

input
```
int(coshint(b*x)*sinh(b*x),x)
```

output
```
int(coshint(b*x)*sinh(b*x), x)
```

Reduce [F]

$$\int \operatorname{Chi}(bx)\sinh(bx)\,dx = \int \chi(bx)\sinh(bx)\,dx$$

input
```
int(Chi(b*x)*sinh(b*x),x)
```

output
```
int(chi(b*x)*sinh(b*x),x)
```

3.118 $\int x\mathbf{Chi}(bx)\sinh(bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int x\text{Chi}(bx)\sinh(bx)\,dx = -\frac{x}{2b}+\frac{x\cosh(bx)\text{Chi}(bx)}{b}-\frac{\cosh(bx)\sinh(bx)}{2b^2} -\frac{\text{Chi}(bx)\sinh(bx)}{b^2}+\frac{\text{Shi}(2bx)}{2b^2}$$

output
```
-1/2*x/b+x*cosh(b*x)*Chi(b*x)/b-1/2*cosh(b*x)*sinh(b*x)/b^2-Chi(b*x)*sinh(
b*x)/b^2+1/2*Shi(2*b*x)/b^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.71

$$\int x\text{Chi}(bx)\sinh(bx)\,dx = -\frac{2bx+\text{Chi}(bx)(-4bx\cosh(bx)+4\sinh(bx))+\sinh(2bx)-2\text{Shi}(2bx)}{4b^2}$$

input
```
Integrate[x*CoshIntegral[b*x]*Sinh[b*x],x]
```

output
```
-1/4*(2*b*x + CoshIntegral[b*x]*(-4*b*x*Cosh[b*x] + 4*Sinh[b*x]) + Sinh[2*
b*x] - 2*SinhIntegral[2*b*x])/b^2
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {7103, 27, 3042, 3115, 24, 7095, 27, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Chi}(bx)\sinh(bx)\,dx$$

$$\Big\downarrow \text{7103}$$

$$-\frac{\int \cosh(bx)\text{Chi}(bx)dx}{b}-\int \frac{\cosh^2(bx)}{b}dx+\frac{x\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\int \cosh^2(bx)dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\int \sin\left(ibx+\frac{\pi}{2}\right)^2dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\int \cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\frac{\int 1dx}{2}+\frac{\sinh(bx)\cosh(bx)}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\int \cosh(bx)\text{Chi}(bx)dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}$$

$$\Big\downarrow \text{7095}$$

$$-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\int \frac{\cosh(bx)\sinh(bx)}{bx}dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int \frac{\cosh(bx)\sinh(bx)}{x}dx}{b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}$$

$\Big\downarrow$ 5971

$$-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int \frac{\sinh(2bx)}{2x}dx}{b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int \frac{\sinh(2bx)}{x}dx}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int -\frac{i\sin(2ibx)}{x}dx}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}$$

$\Big\downarrow$ 26

$$-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}+\frac{i\int \frac{\sin(2ibx)}{x}dx}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}$$

$\Big\downarrow$ 3779

$$-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\text{Shi}(2bx)}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}$$

input
```
Int[x*CoshIntegral[b*x]*Sinh[b*x],x]
```

output
```
(x*Cosh[b*x]*CoshIntegral[b*x])/b - (x/2 + (Cosh[b*x]*Sinh[b*x])/(2*b))/b
- ((CoshIntegral[b*x]*Sinh[b*x])/b - SinhIntegral[2*b*x]/(2*b))/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7095
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7103
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{\mathrm{Chi}(bx)(bx\cosh(bx)-\sinh(bx))-\frac{\cosh(bx)\sinh(bx)}{2}-\frac{bx}{2}+\frac{\mathrm{Shi}(2bx)}{2}}{b^2}$	46
default	$\frac{\mathrm{Chi}(bx)(bx\cosh(bx)-\sinh(bx))-\frac{\cosh(bx)\sinh(bx)}{2}-\frac{bx}{2}+\frac{\mathrm{Shi}(2bx)}{2}}{b^2}$	46

input
```
int(x*Chi(b*x)*sinh(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(Chi(b*x)*(b*x*cosh(b*x)-sinh(b*x))-1/2*cosh(b*x)*sinh(b*x)-1/2*b*x+
1/2*Shi(2*b*x))
```

Fricas [F]

$$\int x\mathrm{Chi}(bx)\sinh(bx)\,dx = \int x\mathrm{Chi}(bx)\sinh(bx)\;dx$$

input
```
integrate(x*Chi(b*x)*sinh(b*x),x, algorithm="fricas")
```

output
```
integral(x*cosh_integral(b*x)*sinh(b*x), x)
```

Sympy [F]

$$\int x\mathrm{Chi}(bx)\sinh(bx)\,dx = \int x\sinh(bx)\,\mathrm{Chi}(bx)\;dx$$

input
```
integrate(x*Chi(b*x)*sinh(b*x),x)
```

output
```
Integral(x*sinh(b*x)*Chi(b*x), x)
```

Maxima [F]

$$\int x\text{Chi}(bx)\sinh(bx)\,dx = \int x\text{Chi}(bx)\sinh{(bx)}\,dx$$

input
```
integrate(x*Chi(b*x)*sinh(b*x),x, algorithm="maxima")
```

output
```
integrate(x*Chi(b*x)*sinh(b*x), x)
```

Giac [F]

$$\int x\text{Chi}(bx)\sinh(bx)\,dx = \int x\text{Chi}(bx)\sinh{(bx)}\,dx$$

input
```
integrate(x*Chi(b*x)*sinh(b*x),x, algorithm="giac")
```

output
```
integrate(x*Chi(b*x)*sinh(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{Chi}(bx)\sinh(bx)\,dx = \int x\,\text{coshint}(b\,x)\,\sinh(b\,x)\,dx$$

input
```
int(x*coshint(b*x)*sinh(b*x),x)
```

output
```
int(x*coshint(b*x)*sinh(b*x), x)
```

Reduce [F]

$$\int x\mathrm{Chi}(bx)\sinh(bx)\,dx = \int \chi(bx)\sinh{(bx)}\,xdx$$

input
```
int(x*Chi(b*x)*sinh(b*x),x)
```

output
```
int(chi(b*x)*sinh(b*x)*x,x)
```

3.119 $\int x^2 \mathbf{Chi}(bx) \sinh(bx) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 109

$$\int x^2 \text{Chi}(bx) \sinh(bx) \, dx = -\frac{x^2}{4b} + \frac{\cosh^2(bx)}{4b^3} + \frac{2\cosh(bx)\text{Chi}(bx)}{b^3} + \frac{x^2\cosh(bx)\text{Chi}(bx)}{b} - \frac{\text{Chi}(2bx)}{b^3} - \frac{\log(x)}{b^3} - \frac{x\cosh(bx)\sinh(bx)}{2b^2} - \frac{2x\text{Chi}(bx)\sinh(bx)}{b^2} + \frac{\sinh^2(bx)}{b^3}$$

output
```
-1/4*x^2/b+1/4*cosh(b*x)^2/b^3+2*cosh(b*x)*Chi(b*x)/b^3+x^2*cosh(b*x)*Chi(
b*x)/b-Chi(2*b*x)/b^3-ln(x)/b^3-1/2*x*cosh(b*x)*sinh(b*x)/b^2-2*x*Chi(b*x)
*sinh(b*x)/b^2+sinh(b*x)^2/b^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.66

$$\int x^2 \text{Chi}(bx) \sinh(bx) \, dx = -\frac{2b^2x^2 - 5\cosh(2bx) + 8\text{Chi}(2bx) + 8\log(x) - 8\text{Chi}(bx)\left((2 + b^2x^2)\cosh(bx) - 2bx\sinh(bx)\right) + 2bx \sin}{8b^3}$$

input
```
Integrate[x^2*CoshIntegral[b*x]*Sinh[b*x],x]
```

output
```
-1/8*(2*b^2*x^2 - 5*Cosh[2*b*x] + 8*CoshIntegral[2*b*x] + 8*Log[x] - 8*Cos
hIntegral[b*x]*((2 + b^2*x^2)*Cosh[b*x] - 2*b*x*Sinh[b*x]) + 2*b*x*Sinh[2*
b*x])/b^3
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.18, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {7103, 27, 3042, 3791, 15, 7097, 27, 3042, 26, 3044, 15, 7101, 27, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{Chi}(bx)\sinh(bx)\,dx$$

$$\Big\downarrow \text{7103}$$

$$-\frac{2\int x\cosh(bx)\text{Chi}(bx)dx}{b}-\int \frac{x\cosh^2(bx)}{b}dx+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\int x\cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\int x\cosh^2(bx)dx}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int x\cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\int x\sin\left(ibx+\frac{\pi}{2}\right)^2dx}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3791}$$

$$-\frac{\frac{\int xdx}{2}-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}}{b}-\frac{2\int x\cosh(bx)\text{Chi}(bx)dx}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2\int x\cosh(bx)\text{Chi}(bx)dx}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{7097}$$

$$-\frac{2\left(-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}-\int \frac{\cosh(bx)\sinh(bx)}{b}dx+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\int \cosh(bx)\sinh(bx)dx}{b}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\left(-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\int -i\cos(ibx)\sin(ibx)dx}{b}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{2\left(-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}+\frac{i\int \cos(ibx)\sin(ibx)dx}{b}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 3044$$

$$-\frac{2\left(\frac{\int i\sinh(bx)d(i\sinh(bx))}{b^2}-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 15$$

$$-\frac{2\left(-\frac{\int \text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 7101$$

$$-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\int\frac{\cosh^2(bx)}{bx}dx}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int\frac{\cosh^2(bx)}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int\frac{\sin\left(ibx+\frac{\pi}{2}\right)^2}{x}dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 3793$$

$$-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\int\left(\frac{\cosh(2bx)}{2x}+\frac{1}{2x}\right)dx}{b}}{b}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\left(-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\text{Chi}(2bx)}{2}+\frac{\log(x)}{2}}{b}}{b}\right)}{b}-\frac{-\frac{\cosh^2(bx)}{4b^2}+\frac{x\sinh(bx)\cosh(bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\text{Chi}(bx)\cosh(bx)}{b}$$

input
```
Int[x^2*CoshIntegral[b*x]*Sinh[b*x],x]
```

output
```
(x^2*Cosh[b*x]*CoshIntegral[b*x])/b - (x^2/4 - Cosh[b*x]^2/(4*b^2) + (x*Co
sh[b*x]*Sinh[b*x])/(2*b))/b - (2*(-(((Cosh[b*x]*CoshIntegral[b*x])/b - (Co
shIntegral[2*b*x]/2 + Log[x]/2)/b)/b) + (x*CoshIntegral[b*x]*Sinh[b*x])/b
- Sinh[b*x]^2/(2*b^2)))/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7097
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7101
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7103
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{\mathrm{Chi}(bx)\left(b^{2}x^{2}\cosh(bx)-2bx\sinh(bx)+2\cosh(bx)\right)-\frac{bx\cosh(bx)\sinh(bx)}{2}-\frac{b^{2}x^{2}}{4}+\frac{5\cosh(bx)^{2}}{4}-\ln(bx)-\mathrm{Chi}(2bx)}{b^{3}}$	78
default	$\frac{\mathrm{Chi}(bx)\left(b^{2}x^{2}\cosh(bx)-2bx\sinh(bx)+2\cosh(bx)\right)-\frac{bx\cosh(bx)\sinh(bx)}{2}-\frac{b^{2}x^{2}}{4}+\frac{5\cosh(bx)^{2}}{4}-\ln(bx)-\mathrm{Chi}(2bx)}{b^{3}}$	78

input
```
int(x^2*Chi(b*x)*sinh(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(Chi(b*x)*(b^2*x^2*cosh(b*x)-2*b*x*sinh(b*x)+2*cosh(b*x))-1/2*b*x*co
sh(b*x)*sinh(b*x)-1/4*b^2*x^2+5/4*cosh(b*x)^2-ln(b*x)-Chi(2*b*x))
```

Fricas [F]

$$\int x^2\text{Chi}(bx)\sinh(bx)\,dx = \int x^2\text{Chi}(bx)\sinh(bx)\,dx$$

input
```
integrate(x^2*Chi(b*x)*sinh(b*x),x, algorithm="fricas")
```

output
```
integral(x^2*cosh_integral(b*x)*sinh(b*x), x)
```

Sympy [F]

$$\int x^2\text{Chi}(bx)\sinh(bx)\,dx = \int x^2\sinh(bx)\,\text{Chi}(bx)\,dx$$

input
```
integrate(x**2*Chi(b*x)*sinh(b*x),x)
```

output
```
Integral(x**2*sinh(b*x)*Chi(b*x), x)
```

Maxima [F]

$$\int x^2\text{Chi}(bx)\sinh(bx)\,dx = \int x^2\text{Chi}(bx)\sinh(bx)\,dx$$

input
```
integrate(x^2*Chi(b*x)*sinh(b*x),x, algorithm="maxima")
```

output
```
integrate(x^2*Chi(b*x)*sinh(b*x), x)
```

Giac [F]

$$\int x^2 \mathrm{Chi}(bx)\sinh(bx)\,dx = \int x^2 \mathrm{Chi}(bx)\sinh(bx)\,dx$$

input
```
integrate(x^2*Chi(b*x)*sinh(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*Chi(b*x)*sinh(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \mathrm{Chi}(bx)\sinh(bx)\,dx = \int x^2 \operatorname{coshint}(b\,x)\,\sinh(b\,x)\,dx$$

input
```
int(x^2*coshint(b*x)*sinh(b*x),x)
```

output
```
int(x^2*coshint(b*x)*sinh(b*x), x)
```

Reduce [F]

$$\int x^2 \mathrm{Chi}(bx)\sinh(bx)\,dx = \int \chi(bx)\sinh(bx)\,x^2dx$$

input
```
int(x^2*Chi(b*x)*sinh(b*x),x)
```

output
```
int(chi(b*x)*sinh(b*x)*x**2,x)
```

3.120 $\int x^3\textbf{Chi}(bx)\sinh(bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 146

$$\int x^3\text{Chi}(bx)\sinh(bx)\,dx = -\frac{5x}{2b^3}-\frac{x^3}{6b}+\frac{x\cosh^2(bx)}{2b^3}+\frac{6x\cosh(bx)\text{Chi}(bx)}{b^3}+\frac{x^3\cosh(bx)\text{Chi}(bx)}{b}-\frac{4\cosh(bx)\sinh(bx)}{b^4}-\frac{x^2\cosh(bx)\sinh(bx)}{2b^2}-\frac{6\text{Chi}(bx)\sinh(bx)}{b^4}-\frac{3x^2\text{Chi}(bx)\sinh(bx)}{b^2}+\frac{3x\sinh^2(bx)}{2b^3}+\frac{3\text{Shi}(2bx)}{b^4}$$

output

```
-5/2*x/b^3-1/6*x^3/b+1/2*x*cosh(b*x)^2/b^3+6*x*cosh(b*x)*Chi(b*x)/b^3+x^3*
cosh(b*x)*Chi(b*x)/b-4*cosh(b*x)*sinh(b*x)/b^4-1/2*x^2*cosh(b*x)*sinh(b*x)
/b^2-6*Chi(b*x)*sinh(b*x)/b^4-3*x^2*Chi(b*x)*sinh(b*x)/b^2+3/2*x*sinh(b*x)
^2/b^3+3*Shi(2*b*x)/b^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int x^3 \text{Chi}(bx) \sinh(bx)\,dx = \frac{-36bx - 2b^3x^3 + 12bx\cosh(2bx) + 12\text{Chi}(bx)\left(bx(6+b^2x^2)\cosh(bx) - 3(2+b^2x^2)\sinh(bx)\right) - 24\sinh(}{12b^4}$$

input
```
Integrate[x^3*CoshIntegral[b*x]*Sinh[b*x],x]
```

output
```
(-36*b*x - 2*b^3*x^3 + 12*b*x*Cosh[2*b*x] + 12*CoshIntegral[b*x]*(b*x*(6 +
 b^2*x^2)*Cosh[b*x] - 3*(2 + b^2*x^2)*Sinh[b*x]) - 24*Sinh[2*b*x] - 3*b^2*
x^2*Sinh[2*b*x] + 36*SinhIntegral[2*b*x])/(12*b^4)
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.60, number of steps used = 27, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 2.250$, Rules used = {7103, 27, 3042, 3792, 15, 3042, 3115, 24, 7097, 27, 5895, 3042, 25, 3115, 24, 7103, 27, 3042, 3115, 24, 7095, 27, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{Chi}(bx) \sinh(bx)\,dx$$

$$\Big\downarrow \text{7103}$$

$$-\frac{3\int x^2\cosh(bx)\text{Chi}(bx)dx}{b} - \int \frac{x^2\cosh^2(bx)}{b}dx + \frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int x^2\cosh(bx)\text{Chi}(bx)dx}{b} - \frac{\int x^2\cosh^2(bx)dx}{b} + \frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int x^2\cosh(bx)\text{Chi}(bx)dx}{b} - \frac{\int x^2\sin\left(ibx+\frac{\pi}{2}\right)^2dx}{b} + \frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3792}$$

$$-\frac{\frac{\int \cosh^2(bx)dx}{2b^2}+\frac{\int x^2dx}{2}-\frac{x\cosh^2(bx)}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}}{b}-\frac{3\int x^2\cosh(bx)\text{Chi}(bx)dx}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\frac{\int \cosh^2(bx)dx}{2b^2}-\frac{x\cosh^2(bx)}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}-\frac{3\int x^2\cosh(bx)\text{Chi}(bx)dx}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \sin\left(ibx+\frac{\pi}{2}\right)^2dx}{2b^2}-\frac{x\cosh^2(bx)}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}-\frac{3\int x^2\cosh(bx)\text{Chi}(bx)dx}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\frac{\frac{\int 1dx}{2}+\frac{\sinh(bx)\cosh(bx)}{2b}}{2b^2}-\frac{x\cosh^2(bx)}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}-\frac{3\int x^2\cosh(bx)\text{Chi}(bx)dx}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{3\int x^2\cosh(bx)\text{Chi}(bx)dx}{b}-\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{7097}$$

$$-\frac{3\left(-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\int \frac{x\cosh(bx)\sinh(bx)}{b}dx+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\int x\cosh(bx)\sinh(bx)dx}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 5895$$

$$-\frac{3\left(-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}-\frac{\int \sinh^2(bx)dx}{2b}}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\left(-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}-\frac{\int -\sin(ibx)^2dx}{2b}}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{3\left(-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\int \sin(ibx)^2dx}{2b}}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 3115$$

$$-\frac{3\left(-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}-\frac{\frac{\frac{\int 1dx}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}+\frac{x\sinh^2(bx)}{2b}}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}\right)}{b}-\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 24$$

$$-\frac{3\left(-\frac{2\int x\text{Chi}(bx)\sinh(bx)dx}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)}{b}-\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 7103$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \cosh(bx)\text{Chi}(bx)dx}{b}-\int \frac{\cosh^2(bx)}{b}dx+\frac{x\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)}{b}-\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\int \cosh^2(bx)dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)}{b}-\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\int \sin\left(ibx+\frac{\pi}{2}\right)^2dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)}{b}-\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 3115$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \cosh(bx)\text{Chi}(bx)dx}{b}-\frac{\frac{\int 1dx}{2}+\frac{\sinh(bx)\cosh(bx)}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)}{b}-\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \; 24$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \cosh(bx)\text{Chi}(bx)dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx)}{2b}}{2b}}{b}\right)}{b}-\cdots$$

$$\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{ 7095}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\int\frac{\cosh(bx)\sinh(bx)}{bx}dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{x}{2}-\cdots}{b}\cdots\right)}{b}\cdots$$

$$\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\int\frac{\cosh(bx)\sinh(bx)}{\frac{x}{b}}dx}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{x}{2}-\cdots}{b}\cdots\right)}{b}\cdots$$

$$\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{ 5971}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int\frac{\sinh(2bx)}{2x}dx}{b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cdots}{2b}}{2b}}{b}\cdots\right)}{b}\cdots$$

$$\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int\frac{\sinh(2bx)}{x}dx}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cdots}{2b}}{2b}}{b}\cdots}{b}$$

$$\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$\downarrow$ 3042

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\int-\frac{i\sin(2ibx)}{x}dx}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(b\cdots}{2\cdots}}{\cdots}}{b}\cdots}{b}$$

$$\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$\downarrow$ 26

$$-\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}+\frac{i\int\frac{\sin(2ibx)}{x}dx}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cdots}{2\cdots}}{2b}}{b}\cdots}{b}$$

$$\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}+\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

$\downarrow$ 3779

$$-\frac{-\frac{x\cosh^2(bx)}{2b^2}+\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(bx)\cosh(bx)}{2b}+\frac{x^3}{6}}{b}-$$

$$\frac{3\left(-\frac{2\left(-\frac{\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\text{Shi}(2bx)}{2b}}{b}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{\frac{\sinh(bx)\cosh(bx)}{2b}+\frac{x}{2}}{b}\right)}{b}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}-\frac{\frac{x\sinh^2(bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(bx)\cosh(bx\cdots}{2b}}{2b}}{b}\cdots}{b}$$

$$\frac{x^3\text{Chi}(bx)\cosh(bx)}{b}$$

input
```
Int[x^3*CoshIntegral[b*x]*Sinh[b*x],x]
```

output
```
(x^3*Cosh[b*x]*CoshIntegral[b*x])/b - (x^3/6 - (x*Cosh[b*x]^2)/(2*b^2) + (
x^2*Cosh[b*x]*Sinh[b*x])/(2*b) + (x/2 + (Cosh[b*x]*Sinh[b*x])/(2*b))/(2*b^
2))/b - (3*((x^2*CoshIntegral[b*x]*Sinh[b*x])/b - ((x*Sinh[b*x]^2)/(2*b) +
 (x/2 - (Cosh[b*x]*Sinh[b*x])/(2*b))/(2*b))/b - (2*((x*Cosh[b*x]*CoshInteg
ral[b*x])/b - (x/2 + (Cosh[b*x]*Sinh[b*x])/(2*b))/b - ((CoshIntegral[b*x]*
Sinh[b*x])/b - SinhIntegral[2*b*x]/(2*b))/b))/b))/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7095
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7097
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7103
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{\mathrm{Chi}(bx)\left(b^3x^3\cosh(bx)-3b^2x^2\sinh(bx)+6bx\cosh(bx)-6\sinh(bx)\right)-\frac{b^2x^2\cosh(bx)\sinh(bx)}{2}-\frac{b^3x^3}{6}+2bx\cosh(bx)^2-4\cos}{b^4}$
default	$\frac{\mathrm{Chi}(bx)\left(b^3x^3\cosh(bx)-3b^2x^2\sinh(bx)+6bx\cosh(bx)-6\sinh(bx)\right)-\frac{b^2x^2\cosh(bx)\sinh(bx)}{2}-\frac{b^3x^3}{6}+2bx\cosh(bx)^2-4\cos}{b^4}$

input
```
int(x^3*Chi(b*x)*sinh(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(Chi(b*x)*(b^3*x^3*cosh(b*x)-3*b^2*x^2*sinh(b*x)+6*b*x*cosh(b*x)-6*s
inh(b*x))-1/2*b^2*x^2*cosh(b*x)*sinh(b*x)-1/6*b^3*x^3+2*b*x*cosh(b*x)^2-4*
cosh(b*x)*sinh(b*x)-4*b*x+3*Shi(2*b*x))
```

Fricas [F]

$$\int x^3\mathrm{Chi}(bx)\sinh(bx)\,dx=\int x^3\mathrm{Chi}(bx)\sinh(bx)\,dx$$

input
```
integrate(x^3*Chi(b*x)*sinh(b*x),x, algorithm="fricas")
```

output
```
integral(x^3*cosh_integral(b*x)*sinh(b*x), x)
```

Sympy [F]

$$\int x^3 \mathrm{Chi}(bx) \sinh(bx)\, dx = \int x^3 \sinh(bx) \,\mathrm{Chi}(bx)\, dx$$

input
```
integrate(x**3*Chi(b*x)*sinh(b*x),x)
```

output
```
Integral(x**3*sinh(b*x)*Chi(b*x), x)
```

Maxima [F]

$$\int x^3 \mathrm{Chi}(bx) \sinh(bx)\, dx = \int x^3 \mathrm{Chi}(bx) \sinh(bx)\, dx$$

input
```
integrate(x^3*Chi(b*x)*sinh(b*x),x, algorithm="maxima")
```

output
```
integrate(x^3*Chi(b*x)*sinh(b*x), x)
```

Giac [F]

$$\int x^3 \mathrm{Chi}(bx) \sinh(bx)\, dx = \int x^3 \mathrm{Chi}(bx) \sinh(bx)\, dx$$

input
```
integrate(x^3*Chi(b*x)*sinh(b*x),x, algorithm="giac")
```

output
```
integrate(x^3*Chi(b*x)*sinh(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\text{Chi}(bx)\sinh(bx)\,dx=\int x^3\,\mathrm{coshint}(b\,x)\,\sinh(b\,x)\,dx$$

input
```
int(x^3*coshint(b*x)*sinh(b*x),x)
```

output
```
int(x^3*coshint(b*x)*sinh(b*x), x)
```

Reduce [F]

$$\int x^3\text{Chi}(bx)\sinh(bx)\,dx=\int \chi(bx)\sinh(bx)\,x^3dx$$

input
```
int(x^3*Chi(b*x)*sinh(b*x),x)
```

output
```
int(chi(b*x)*sinh(b*x)*x**3,x)
```

3.121 $\int \mathrm{Chi}(2x)\sinh(5x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 29

$$\int \mathrm{Chi}(2x)\sinh(5x)\,dx = \frac{1}{5}\cosh(5x)\mathrm{Chi}(2x) - \frac{\mathrm{Chi}(3x)}{10} - \frac{\mathrm{Chi}(7x)}{10}$$

output
```
1/5*cosh(5*x)*Chi(2*x)-1/10*Chi(3*x)-1/10*Chi(7*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \mathrm{Chi}(2x)\sinh(5x)\,dx = \frac{1}{10}(2\cosh(5x)\mathrm{Chi}(2x) - \mathrm{Chi}(3x) - \mathrm{Chi}(7x))$$

input
```
Integrate[CoshIntegral[2*x]*Sinh[5*x],x]
```

output
```
(2*Cosh[5*x]*CoshIntegral[2*x] - CoshIntegral[3*x] - CoshIntegral[7*x])/10
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {7101, 27, 5994, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Chi}(2x)\sinh(5x)\,dx$$

$$\Big\downarrow \ 7101$$

$$\frac{1}{5}\text{Chi}(2x)\cosh(5x)-\frac{2}{5}\int \frac{\cosh(2x)\cosh(5x)}{2x}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{5}\text{Chi}(2x)\cosh(5x)-\frac{1}{5}\int \frac{\cosh(2x)\cosh(5x)}{x}dx$$

$$\Big\downarrow \ 5994$$

$$\frac{1}{5}\text{Chi}(2x)\cosh(5x)-\frac{1}{5}\int \left(\frac{\cosh(3x)}{2x}+\frac{\cosh(7x)}{2x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{5}\left(-\frac{\text{Chi}(3x)}{2}-\frac{\text{Chi}(7x)}{2}\right)+\frac{1}{5}\text{Chi}(2x)\cosh(5x)$$

input
```
Int[CoshIntegral[2*x]*Sinh[5*x],x]
```

output
```
(Cosh[5*x]*CoshIntegral[2*x])/5 + (-1/2*CoshIntegral[3*x] - CoshIntegral[7
*x]/2)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5994
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) +
(f_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Cosh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0
] && IGtQ[q, 0] && IntegerQ[m]
```

rule 7101
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\cosh(5x)\,\mathrm{Chi}(2x)}{5}-\frac{\mathrm{Chi}(3x)}{10}-\frac{\mathrm{Chi}(7x)}{10}$	24

input
```
int(Chi(2*x)*sinh(5*x),x,method=_RETURNVERBOSE)
```

output
```
1/5*cosh(5*x)*Chi(2*x)-1/10*Chi(3*x)-1/10*Chi(7*x)
```

Fricas [F]

$$\int \mathrm{Chi}(2x)\sinh(5x)\,dx = \int \mathrm{Chi}(2\,x)\sinh\left(5\,x\right)\,dx$$

input
```
integrate(Chi(2*x)*sinh(5*x),x, algorithm="fricas")
```

output
```
integral(cosh_integral(2*x)*sinh(5*x), x)
```

Sympy [F]

$$\int \mathrm{Chi}(2x)\sinh(5x)\,dx = \int \sinh\left(5x\right)\mathrm{Chi}\left(2x\right)\,dx$$

input
```
integrate(Chi(2*x)*sinh(5*x),x)
```

output
```
Integral(sinh(5*x)*Chi(2*x), x)
```

Maxima [F]

$$\int \mathrm{Chi}(2x)\sinh(5x)\,dx = \int \mathrm{Chi}(2\,x)\sinh\left(5\,x\right)\,dx$$

input
```
integrate(Chi(2*x)*sinh(5*x),x, algorithm="maxima")
```

output
```
integrate(Chi(2*x)*sinh(5*x), x)
```

Giac [F]

$$\int \mathrm{Chi}(2x)\sinh(5x)\,dx = \int \mathrm{Chi}(2\,x)\sinh\left(5\,x\right)\,dx$$

input
```
integrate(Chi(2*x)*sinh(5*x),x, algorithm="giac")
```

output
```
integrate(Chi(2*x)*sinh(5*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \mathrm{Chi}(2x)\sinh(5x)\,dx = \int \mathrm{coshint}(2\,x)\,\sinh(5\,x)\,dx$$

input
```
int(coshint(2*x)*sinh(5*x),x)
```

output
```
int(coshint(2*x)*sinh(5*x), x)
```

Reduce [F]

$$\int \mathrm{Chi}(2x)\sinh(5x)\,dx = \int \chi(2x)\sinh\left(5x\right)dx$$

input
```
int(Chi(2*x)*sinh(5*x),x)
```

output
```
int(chi(2*x)*sinh(5*x),x)
```

3.122 $\int \cosh(5x)\textbf{Chi}(2x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 29

$$\int \cosh(5x)\text{Chi}(2x)\,dx = \frac{1}{5}\text{Chi}(2x)\sinh(5x) - \frac{\text{Shi}(3x)}{10} - \frac{\text{Shi}(7x)}{10}$$

output
```
1/5*Chi(2*x)*sinh(5*x)-1/10*Shi(3*x)-1/10*Shi(7*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \cosh(5x)\text{Chi}(2x)\,dx = \frac{1}{10}(2\text{Chi}(2x)\sinh(5x) - \text{Shi}(3x) - \text{Shi}(7x))$$

input
```
Integrate[Cosh[5*x]*CoshIntegral[2*x],x]
```

output
```
(2*CoshIntegral[2*x]*Sinh[5*x] - SinhIntegral[3*x] - SinhIntegral[7*x])/10
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {7095, 27, 5995, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Chi}(2x)\cosh(5x)\,dx$$

$$\Big\downarrow \text{7095}$$

$$\frac{1}{5}\text{Chi}(2x)\sinh(5x)-\frac{2}{5}\int \frac{\cosh(2x)\sinh(5x)}{2x}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\text{Chi}(2x)\sinh(5x)-\frac{1}{5}\int \frac{\cosh(2x)\sinh(5x)}{x}dx$$

$$\Big\downarrow \text{5995}$$

$$\frac{1}{5}\text{Chi}(2x)\sinh(5x)-\frac{1}{5}\int \left(\frac{\sinh(3x)}{2x}+\frac{\sinh(7x)}{2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\text{Chi}(2x)\sinh(5x)+\frac{1}{5}\left(-\frac{\text{Shi}(3x)}{2}-\frac{\text{Shi}(7x)}{2}\right)$$

input
```
Int[Cosh[5*x]*CoshIntegral[2*x],x]
```

output
```
(CoshIntegral[2*x]*Sinh[5*x])/5 + (-1/2*SinhIntegral[3*x] - SinhIntegral[7
*x]/2)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5995
```
Int[Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p
, 0] && IGtQ[q, 0]
```

rule 7095
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\mathrm{Chi}(2x)\sinh(5x)}{5}-\frac{\mathrm{Shi}(3x)}{10}-\frac{\mathrm{Shi}(7x)}{10}$	24

input
```
int(cosh(5*x)*Chi(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/5*Chi(2*x)*sinh(5*x)-1/10*Shi(3*x)-1/10*Shi(7*x)
```

Fricas [F]

$$\int \cosh(5x)\mathrm{Chi}(2x)\,dx = \int \mathrm{Chi}(2\,x)\cosh\left(5\,x\right)\,dx$$

input
```
integrate(cosh(5*x)*Chi(2*x),x, algorithm="fricas")
```

output
```
integral(cosh(5*x)*cosh_integral(2*x), x)
```

Sympy [F]

$$\int \cosh(5x)\mathrm{Chi}(2x)\,dx = \int \cosh\left(5x\right)\mathrm{Chi}\left(2x\right)\,dx$$

input
```
integrate(cosh(5*x)*Chi(2*x),x)
```

output
```
Integral(cosh(5*x)*Chi(2*x), x)
```

Maxima [F]

$$\int \cosh(5x)\mathrm{Chi}(2x)\,dx = \int \mathrm{Chi}(2\,x)\cosh\left(5\,x\right)\,dx$$

input
```
integrate(cosh(5*x)*Chi(2*x),x, algorithm="maxima")
```

output
```
integrate(Chi(2*x)*cosh(5*x), x)
```

Giac [F]

$$\int \cosh(5x)\text{Chi}(2x)\,dx = \int \text{Chi}\left(2\,x\right)\cosh\left(5\,x\right)\,dx$$

input
```
integrate(cosh(5*x)*Chi(2*x),x, algorithm="giac")
```

output
```
integrate(Chi(2*x)*cosh(5*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(5x)\text{Chi}(2x)\,dx = \int \text{coshint}(2\,x)\,\cosh(5\,x)\,dx$$

input
```
int(coshint(2*x)*cosh(5*x),x)
```

output
```
int(coshint(2*x)*cosh(5*x), x)
```

Reduce [F]

$$\int \cosh(5x)\text{Chi}(2x)\,dx = \int \chi(2x)\cosh\left(5x\right)dx$$

input
```
int(cosh(5*x)*Chi(2*x),x)
```

output
```
int(chi(2*x)*cosh(5*x),x)
```

3.123 $\int x^2\mathbf{Chi}(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 201

$$\int x^2\text{Chi}(a+bx)\sinh(a+bx)\,dx = -\frac{(a-bx)^2}{4b^3}+\frac{\cosh^2(a+bx)}{4b^3}+\frac{\cosh(2a+2bx)}{2b^3}+\frac{2\cosh(a+bx)\text{Chi}(a+bx)}{b^3}+\frac{x^2\cosh(a+bx)\text{Chi}(a+bx)}{b}-\frac{\text{Chi}(2a+2bx)}{b^3}-\frac{a^2\text{Chi}(2a+2bx)}{2b^3}-\frac{\log(a+bx)}{b^3}-\frac{a^2\log(a+bx)}{2b^3}+\frac{(a-bx)\cosh(a+bx)\sinh(a+bx)}{2b^3}-\frac{2x\text{Chi}(a+bx)\sinh(a+bx)}{b^2}-\frac{a\text{Shi}(2a+2bx)}{b^3}$$

output
```
-1/4*(-b*x+a)^2/b^3+1/4*cosh(b*x+a)^2/b^3+1/2*cosh(2*b*x+2*a)/b^3+2*cosh(b
*x+a)*Chi(b*x+a)/b^3+x^2*cosh(b*x+a)*Chi(b*x+a)/b-Chi(2*b*x+2*a)/b^3-1/2*a
^2*Chi(2*b*x+2*a)/b^3-ln(b*x+a)/b^3-1/2*a^2*ln(b*x+a)/b^3+1/2*(-b*x+a)*cos
h(b*x+a)*sinh(b*x+a)/b^3-2*x*Chi(b*x+a)*sinh(b*x+a)/b^2-a*Shi(2*b*x+2*a)/b
^3
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.67

$$\int x^2\text{Chi}(a+bx)\sinh(a+bx)\,dx =$$
$$-\frac{-4abx+2b^2x^2-5\cosh(2(a+bx))+4(2+a^2)\,\text{Chi}(2(a+bx))+8\log(a+bx)+4a^2\log(a+bx)-8}{}$$

input
```
Integrate[x^2*CoshIntegral[a + b*x]*Sinh[a + b*x],x]
```

output
```
-1/8*(-4*a*b*x + 2*b^2*x^2 - 5*Cosh[2*(a + b*x)] + 4*(2 + a^2)*CoshIntegra
l[2*(a + b*x)] + 8*Log[a + b*x] + 4*a^2*Log[a + b*x] - 8*CoshIntegral[a +
b*x]*((2 + b^2*x^2)*Cosh[a + b*x] - 2*b*x*Sinh[a + b*x]) - 2*a*Sinh[2*(a +
 b*x)] + 2*b*x*Sinh[2*(a + b*x)] + 8*a*SinhIntegral[2*(a + b*x)])/b^3
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {7103, 7097, 6151, 7101, 3042, 3793, 2009, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{Chi}(a+bx)\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{7103}$$

$$-\frac{2\int x\cosh(a+bx)\text{Chi}(a+bx)dx}{b}-\int \frac{x^2\cosh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{7097}$$

$$-\frac{2\left(-\frac{\int \text{Chi}(a+bx)\sinh(a+bx)dx}{b}-\int \frac{x\cosh(a+bx)\sinh(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\int \frac{x^2\cosh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{6151}$$

$$-\frac{2\left(-\frac{\int \text{Chi}(a+bx)\sinh(a+bx)dx}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\int \frac{x^2\cosh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{7101}$$

$$-\frac{2\left(-\frac{\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\cosh^2(a+bx)}{a+bx}dx}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\int \frac{x^2\cosh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(-\frac{\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{a+bx}dx}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\int \frac{x^2\cosh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{2\left(-\frac{\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\int \left(\frac{\cosh(2a+2bx)}{2(a+bx)}+\frac{1}{2(a+bx)}\right)dx}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}\right)}{b}-\int \frac{x^2\cosh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{\log(a+bx)}{2b}}{b}\right)}{b}-\int \frac{x^2\cosh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{7292}$$

$$-\frac{2\left(-\frac{1}{2}\int \frac{x\sinh(2a+2bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{\log(a+bx)}{2b}}{b}\right)}{b}-\int \frac{x^2\cosh^2(a+bx)}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(\frac{x\cosh^2(a+bx)}{b}+\frac{a^2\cosh^2(a+bx)}{b^2(a+bx)}-\frac{a\cosh^2(a+bx)}{b^2}\right)dx-\frac{2\left(-\frac{1}{2}\int \left(\frac{\sinh(2a+2bx)}{b}+\frac{a\sinh(2a+2bx)}{b(-a-bx)}\right)dx+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{\log(a+bx)}{2b}}{b}\right)}{b}+\frac{x^2\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\text{Chi}(2a+2bx)}{2b^3}-\frac{a^2\log(a+bx)}{2b^3}+\frac{\cosh^2(a+bx)}{4b^3}+\frac{a\sinh(a+bx)\cosh(a+bx)}{2b^3}-\frac{2\left(\frac{1}{2}\left(\frac{a\text{Shi}(2a+2bx)}{b^2}-\frac{\cosh(2a+2bx)}{2b^2}\right)+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{\log(a+bx)}{2b}}{b}\right)}{b}+\frac{ax}{2b^2}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b^2}+\frac{x^2\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{x^2}{4b}$$

input
```
Int[x^2*CoshIntegral[a + b*x]*Sinh[a + b*x],x]
```

output
```
(a*x)/(2*b^2) - x^2/(4*b) + Cosh[a + b*x]^2/(4*b^3) + (x^2*Cosh[a + b*x]*C
oshIntegral[a + b*x])/b - (a^2*CoshIntegral[2*a + 2*b*x])/(2*b^3) - (a^2*L
og[a + b*x])/(2*b^3) + (a*Cosh[a + b*x]*Sinh[a + b*x])/(2*b^3) - (x*Cosh[a
 + b*x]*Sinh[a + b*x])/(2*b^2) - (2*(-(((Cosh[a + b*x]*CoshIntegral[a + b*
x])/b - CoshIntegral[2*a + 2*b*x]/(2*b) - Log[a + b*x]/(2*b))/b) + (x*Cosh
Integral[a + b*x]*Sinh[a + b*x])/b + (-1/2*Cosh[2*a + 2*b*x]/b^2 + (a*Sinh
Integral[2*a + 2*b*x])/b^2)/2))/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6151
```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Simp[1/2^p   Int[u*Si
nh[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

rule 7097
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7101
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7103
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\mathrm{Chi}(bx+a)\left(a^2\cosh(bx+a)-2a((bx+a)\cosh(bx+a)-\sinh(bx+a))+(bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(bx+a)+2\cosh\right.}{}$
default	$\frac{\mathrm{Chi}(bx+a)\left(a^2\cosh(bx+a)-2a((bx+a)\cosh(bx+a)-\sinh(bx+a))+(bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(bx+a)+2\cosh\right.}{}$

input
```
int(x^2*Chi(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(Chi(b*x+a)*(a^2*cosh(b*x+a)-2*a*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))+(
b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x+a)+2*cosh(b*x+a))-1/2*a^2*ln(b*x+a
)-1/2*a^2*Chi(2*b*x+2*a)+cosh(b*x+a)*sinh(b*x+a)*a+(b*x+a)*a-a*Shi(2*b*x+2
*a)-1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2+5/4*cosh(b*x+a)^2-ln
(b*x+a)-Chi(2*b*x+2*a))
```

Fricas [F]

$$\int x^2\mathrm{Chi}(a+bx)\sinh(a+bx)\,dx=\int x^2\mathrm{Chi}(bx+a)\sinh(bx+a)\,dx$$

input
```
integrate(x^2*Chi(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^2*cosh_integral(b*x + a)*sinh(b*x + a), x)
```

Sympy [F]

$$\int x^2\text{Chi}(a+bx)\sinh(a+bx)\,dx=\int x^2\sinh{(a+bx)}\,\text{Chi}\,(a+bx)\;dx$$

input
```
integrate(x**2*Chi(b*x+a)*sinh(b*x+a),x)
```

output
```
Integral(x**2*sinh(a + b*x)*Chi(a + b*x), x)
```

Maxima [F]

$$\int x^2\text{Chi}(a+bx)\sinh(a+bx)\,dx=\int x^2\text{Chi}(bx+a)\sinh{(bx+a)}\;dx$$

input
```
integrate(x^2*Chi(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^2*Chi(b*x + a)*sinh(b*x + a), x)
```

Giac [F]

$$\int x^2\text{Chi}(a+bx)\sinh(a+bx)\,dx=\int x^2\text{Chi}(bx+a)\sinh{(bx+a)}\;dx$$

input
```
integrate(x^2*Chi(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*Chi(b*x + a)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{Chi}(a+bx)\sinh(a+bx)\,dx=\int x^2\,\mathrm{coshint}(a+b\,x)\,\sinh(a+b\,x)\,dx$$

input
```
int(x^2*coshint(a + b*x)*sinh(a + b*x),x)
```

output
```
int(x^2*coshint(a + b*x)*sinh(a + b*x), x)
```

Reduce [F]

$$\int x^2\text{Chi}(a+bx)\sinh(a+bx)\,dx=\int \chi(bx+a)\sinh\left(bx+a\right)x^2dx$$

input
```
int(x^2*Chi(b*x+a)*sinh(b*x+a),x)
```

output
```
int(chi(a + b*x)*sinh(a + b*x)*x**2,x)
```

3.124 $\int x\text{Chi}(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 109

$$\int x\text{Chi}(a+bx)\sinh(a+bx)\,dx=-\frac{x}{2b}+\frac{x\cosh(a+bx)\text{Chi}(a+bx)}{b}+\frac{a\text{Chi}(2a+2bx)}{2b^2}+\frac{a\log(a+bx)}{2b^2}-\frac{\cosh(a+bx)\sinh(a+bx)}{2b^2}-\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b^2}+\frac{\text{Shi}(2a+2bx)}{2b^2}$$

output
```
-1/2*x/b+x*cosh(b*x+a)*Chi(b*x+a)/b+1/2*a*Chi(2*b*x+2*a)/b^2+1/2*a*ln(b*x+
a)/b^2-1/2*cosh(b*x+a)*sinh(b*x+a)/b^2-Chi(b*x+a)*sinh(b*x+a)/b^2+1/2*Shi(
2*b*x+2*a)/b^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int x\text{Chi}(a+bx)\sinh(a+bx)\,dx$$
$$=\frac{-2bx+2a\text{Chi}(2(a+bx))+2a\log(a+bx)+4\text{Chi}(a+bx)(bx\cosh(a+bx)-\sinh(a+bx))-\sinh(2(a}{4b^2}$$

input
```
Integrate[x*CoshIntegral[a + b*x]*Sinh[a + b*x],x]
```

output
```
(-2*b*x + 2*a*CoshIntegral[2*(a + b*x)] + 2*a*Log[a + b*x] + 4*CoshIntegra
l[a + b*x]*(b*x*Cosh[a + b*x] - Sinh[a + b*x]) - Sinh[2*(a + b*x)] + 2*Sin
hIntegral[2*(a + b*x)])/(4*b^2)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {7103, 7095, 5971, 27, 3042, 26, 3779, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Chi}(a+bx)\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{7103}$$

$$-\frac{\int \cosh(a+bx)\text{Chi}(a+bx)dx}{b}-\int \frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{7095}$$

$$-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\int \frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}dx}{b}-\int \frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\int \frac{\sinh(2a+2bx)}{2(a+bx)}dx}{b}-\int \frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{1}{2}\int \frac{\sinh(2a+2bx)}{a+bx}dx}{b}-\int \frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{1}{2}\int -\frac{i\sin(2ia+2ibx)}{a+bx}dx}{b}-\int \frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}+\frac{1}{2}i\int\frac{\sin(2ia+2ibx)}{a+bx}dx}{b}-\int\frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{3779}$$

$$-\int\frac{x\cosh^2(a+bx)}{a+bx}dx-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{7293}$$

$$-\int\left(\frac{\cosh^2(a+bx)}{b}-\frac{a\cosh^2(a+bx)}{b(a+bx)}\right)dx-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\text{Chi}(2a+2bx)}{2b^2}+\frac{a\log(a+bx)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b^2}-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{x}{2b}$$

input
```
Int[x*CoshIntegral[a + b*x]*Sinh[a + b*x],x]
```

output
```
-1/2*x/b + (x*Cosh[a + b*x]*CoshIntegral[a + b*x])/b + (a*CoshIntegral[2*a
 + 2*b*x])/(2*b^2) + (a*Log[a + b*x])/(2*b^2) - (Cosh[a + b*x]*Sinh[a + b*
x])/(2*b^2) - ((CoshIntegral[a + b*x]*Sinh[a + b*x])/b - SinhIntegral[2*a
+ 2*b*x]/(2*b))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7095
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7103
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\mathrm{Chi}(bx+a)((bx+a)\cosh(bx+a)-\sinh(bx+a)-a\cosh(bx+a))-\frac{\sinh(bx+a)\cosh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}+\frac{\mathrm{Shi}(2bx+2a)}{2}+a\left(\frac{\ln(bx+a)}{2}\right.}{b^2}$
default	$\frac{\mathrm{Chi}(bx+a)((bx+a)\cosh(bx+a)-\sinh(bx+a)-a\cosh(bx+a))-\frac{\sinh(bx+a)\cosh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}+\frac{\mathrm{Shi}(2bx+2a)}{2}+a\left(\frac{\ln(bx+a)}{2}\right.}{b^2}$

input
```
int(x*Chi(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(Chi(b*x+a)*((b*x+a)*cosh(b*x+a)-sinh(b*x+a)-a*cosh(b*x+a))-1/2*sinh
(b*x+a)*cosh(b*x+a)-1/2*b*x-1/2*a+1/2*Shi(2*b*x+2*a)+a*(1/2*ln(b*x+a)+1/2*
Chi(2*b*x+2*a)))
```

Fricas [F]

$$\int x\mathrm{Chi}(a+bx)\sinh(a+bx)\,dx=\int x\mathrm{Chi}(bx+a)\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x*Chi(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
integral(x*cosh_integral(b*x + a)*sinh(b*x + a), x)
```

Sympy [F]

$$\int x\mathrm{Chi}(a+bx)\sinh(a+bx)\,dx=\int x\sinh\left(a+bx\right)\mathrm{Chi}\left(a+bx\right)\,dx$$

input
```
integrate(x*Chi(b*x+a)*sinh(b*x+a),x)
```

output
```
Integral(x*sinh(a + b*x)*Chi(a + b*x), x)
```

Maxima [F]

$$\int x\text{Chi}(a+bx)\sinh(a+bx)\,dx=\int x\text{Chi}(bx+a)\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x*Chi(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*Chi(b*x + a)*sinh(b*x + a), x)
```

Giac [F]

$$\int x\text{Chi}(a+bx)\sinh(a+bx)\,dx=\int x\text{Chi}(bx+a)\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x*Chi(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*Chi(b*x + a)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{Chi}(a+bx)\sinh(a+bx)\,dx=\int x\,\text{coshint}(a+b\,x)\,\sinh(a+b\,x)\,dx$$

input
```
int(x*coshint(a + b*x)*sinh(a + b*x),x)
```

output
```
int(x*coshint(a + b*x)*sinh(a + b*x), x)
```

Reduce [F]

$$\int x\text{Chi}(a+bx)\sinh(a+bx)\,dx=\int \chi(bx+a)\sinh(bx+a)\,xdx$$

input
```
int(x*Chi(b*x+a)*sinh(b*x+a),x)
```

output
```
int(chi(a + b*x)*sinh(a + b*x)*x,x)
```

3.125 $\int \mathbf{Chi}(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \text{Chi}(a+bx)\sinh(a+bx)\,dx=\frac{\cosh(a+bx)\text{Chi}(a+bx)}{b}-\frac{\text{Chi}(2a+2bx)}{2b}-\frac{\log(a+bx)}{2b}$$

output
```
cosh(b*x+a)*Chi(b*x+a)/b-1/2*Chi(2*b*x+2*a)/b-1/2*ln(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \text{Chi}(a+bx)\sinh(a+bx)\,dx=\frac{\cosh(a+bx)\text{Chi}(a+bx)}{b}-\frac{\text{Chi}(2(a+bx))}{2b}-\frac{\log(a+bx)}{2b}$$

input
```
Integrate[CoshIntegral[a + b*x]*Sinh[a + b*x],x]
```

output
```
(Cosh[a + b*x]*CoshIntegral[a + b*x])/b - CoshIntegral[2*(a + b*x)]/(2*b)
- Log[a + b*x]/(2*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {7101, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Chi}(a+bx)\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{7101}$$

$$\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\cosh^2(a+bx)}{a+bx}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{a+bx}dx$$

$$\Big\downarrow \text{3793}$$

$$\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\int \left(\frac{\cosh(2a+2bx)}{2(a+bx)}+\frac{1}{2(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{\log(a+bx)}{2b}$$

input

```
Int[CoshIntegral[a + b*x]*Sinh[a + b*x],x]
```

output

```
(Cosh[a + b*x]*CoshIntegral[a + b*x])/b - CoshIntegral[2*a + 2*b*x]/(2*b)
- Log[a + b*x]/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7101
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\cosh(bx+a)\operatorname{Chi}(bx+a)-\frac{\ln(bx+a)}{2}-\frac{\operatorname{Chi}(2bx+2a)}{2}}{b}$	38
default	$\frac{\cosh(bx+a)\operatorname{Chi}(bx+a)-\frac{\ln(bx+a)}{2}-\frac{\operatorname{Chi}(2bx+2a)}{2}}{b}$	38

input
```
int(Chi(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(cosh(b*x+a)*Chi(b*x+a)-1/2*ln(b*x+a)-1/2*Chi(2*b*x+2*a))
```

Fricas [F]

$$\int \mathrm{Chi}(a+bx)\sinh(a+bx)\,dx=\int \mathrm{Chi}(bx+a)\sinh\left(bx+a\right)\,dx$$

input
```
integrate(Chi(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x + a)*sinh(b*x + a), x)
```

Sympy [F]

$$\int \mathrm{Chi}(a+bx)\sinh(a+bx)\,dx=\int \sinh\left(a+bx\right)\mathrm{Chi}\left(a+bx\right)\,dx$$

input
```
integrate(Chi(b*x+a)*sinh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*Chi(a + b*x), x)
```

Maxima [F]

$$\int \mathrm{Chi}(a+bx)\sinh(a+bx)\,dx=\int \mathrm{Chi}(bx+a)\sinh\left(bx+a\right)\,dx$$

input
```
integrate(Chi(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(Chi(b*x + a)*sinh(b*x + a), x)
```

Giac [F]

$$\int \mathrm{Chi}(a+bx)\sinh(a+bx)\,dx=\int \mathrm{Chi}(bx+a)\sinh(bx+a)\,dx$$

input
```
integrate(Chi(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(Chi(b*x + a)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \mathrm{Chi}(a+bx)\sinh(a+bx)\,dx=\int \mathrm{coshint}(a+b\,x)\,\sinh(a+b\,x)\,dx$$

input
```
int(coshint(a + b*x)*sinh(a + b*x),x)
```

output
```
int(coshint(a + b*x)*sinh(a + b*x), x)
```

Reduce [F]

$$\int \mathrm{Chi}(a+bx)\sinh(a+bx)\,dx=\int \chi(bx+a)\sinh(bx+a)\,dx$$

input
```
int(Chi(b*x+a)*sinh(b*x+a),x)
```

output
```
int(chi(a + b*x)*sinh(a + b*x),x)
```

3.126 $\int \frac{\text{Chi}(a+bx)\sinh(a+bx)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{Chi}(a+bx)\sinh(a+bx)}{x}\,dx=\text{Int}\left(\frac{\text{Chi}(a+bx)\sinh(a+bx)}{x},x\right)$$

output
```
Defer(Int)(Chi(b*x+a)*sinh(b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{Chi}(a+bx)\sinh(a+bx)}{x}\,dx=\int \frac{\text{Chi}(a+bx)\sinh(a+bx)}{x}\,dx$$

input
```
Integrate[(CoshIntegral[a + b*x]*Sinh[a + b*x])/x,x]
```

output
```
Integrate[(CoshIntegral[a + b*x]*Sinh[a + b*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(a+bx)\sinh(a+bx)}{x} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\text{Chi}(a+bx)\sinh(a+bx)}{x}dx$$

input
```
Int[(CoshIntegral[a + b*x]*Sinh[a + b*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\text{Chi}\,(bx+a)\sinh\,(bx+a)}{x}dx$$

input
```
int(Chi(b*x+a)*sinh(b*x+a)/x,x)
```

output
```
int(Chi(b*x+a)*sinh(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{Chi}(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\text{Chi}(bx+a)\sinh{(bx+a)}}{x}\,dx$$

input
```
integrate(Chi(b*x+a)*sinh(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(cosh_integral(b*x + a)*sinh(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\text{Chi}(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\sinh{(a+bx)}\,\text{Chi}\,(a+bx)}{x}\,dx$$

input
```
integrate(Chi(b*x+a)*sinh(b*x+a)/x,x)
```

output
```
Integral(sinh(a + b*x)*Chi(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{Chi}(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\text{Chi}(bx+a)\sinh{(bx+a)}}{x}\,dx$$

input
```
integrate(Chi(b*x+a)*sinh(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x + a)*sinh(b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{Chi}(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\text{Chi}(bx+a)\sinh\left(bx+a\right)}{x}\,dx$$

input
```
integrate(Chi(b*x+a)*sinh(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(Chi(b*x + a)*sinh(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{Chi}(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\text{coshint}(a+b\,x)\,\sinh(a+b\,x)}{x}\,dx$$

input
```
int((coshint(a + b*x)*sinh(a + b*x))/x,x)
```

output
```
int((coshint(a + b*x)*sinh(a + b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{Chi}(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\chi(bx+a)\sinh(bx+a)}{x}dx$$

input
```
int(Chi(b*x+a)*sinh(b*x+a)/x,x)
```

output
```
int((chi(a + b*x)*sinh(a + b*x))/x,x)
```

3.127 $\int x^2 \cosh(a+bx)\mathbf{Chi}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 174

$$\int x^2\cosh(a+bx)\text{Chi}(a+bx)\,dx = \frac{x}{b^2}+\frac{(a-bx)\cosh(2a+2bx)}{4b^3} - \frac{2x\cosh(a+bx)\text{Chi}(a+bx)}{b^2}-\frac{a\text{Chi}(2a+2bx)}{b^3} - \frac{a\log(a+bx)}{b^3}+\frac{\cosh(a+bx)\sinh(a+bx)}{b^3} + \frac{2\text{Chi}(a+bx)\sinh(a+bx)}{b^3} + \frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}+\frac{\sinh(2a+2bx)}{8b^3} - \frac{\text{Shi}(2a+2bx)}{b^3}-\frac{a^2\text{Shi}(2a+2bx)}{2b^3}$$

output
```
x/b^2+1/4*(-b*x+a)*cosh(2*b*x+2*a)/b^3-2*x*cosh(b*x+a)*Chi(b*x+a)/b^2-a*Ch
i(2*b*x+2*a)/b^3-a*ln(b*x+a)/b^3+cosh(b*x+a)*sinh(b*x+a)/b^3+2*Chi(b*x+a)*
sinh(b*x+a)/b^3+x^2*Chi(b*x+a)*sinh(b*x+a)/b+1/8*sinh(2*b*x+2*a)/b^3-Shi(2
*b*x+2*a)/b^3-1/2*a^2*Shi(2*b*x+2*a)/b^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.71

$$\int x^2 \cosh(a+bx)\text{Chi}(a+bx)\,dx$$
$$= \frac{8bx + 2a\cosh(2(a+bx)) - 2bx\cosh(2(a+bx)) - 8a\text{Chi}(2(a+bx)) - 8a\log(a+bx) + 8\text{Chi}(a+bx)\,(-}{8}$$

input
```
Integrate[x^2*Cosh[a + b*x]*CoshIntegral[a + b*x],x]
```

output
```
(8*b*x + 2*a*Cosh[2*(a + b*x)] - 2*b*x*Cosh[2*(a + b*x)] - 8*a*CoshIntegra
l[2*(a + b*x)] - 8*a*Log[a + b*x] + 8*CoshIntegral[a + b*x]*(-2*b*x*Cosh[a
 + b*x] + (2 + b^2*x^2)*Sinh[a + b*x]) + 5*Sinh[2*(a + b*x)] - 8*SinhInteg
ral[2*(a + b*x)] - 4*a^2*SinhIntegral[2*(a + b*x)])/(8*b^3)
```

Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {7097, 6151, 7103, 7095, 5971, 27, 3042, 26, 3779, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{Chi}(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{7097}$$

$$-\frac{2\int x\text{Chi}(a+bx)\sinh(a+bx)dx}{b} - \int \frac{x^2\cosh(a+bx)\sinh(a+bx)}{a+bx}dx + \frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{6151}$$

$$-\frac{2\int x\text{Chi}(a+bx)\sinh(a+bx)dx}{b} - \frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx + \frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{7103}$$

$$-\frac{2\left(-\frac{\int \cosh(a+bx)\text{Chi}(a+bx)dx}{b}-\int \frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$\Big\downarrow$ 7095

$$-\frac{2\left(-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\int \frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}dx}{b}-\int \frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$\Big\downarrow$ 5971

$$-\frac{2\left(-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\int \frac{\sinh(2a+2bx)}{2(a+bx)}dx}{b}-\int \frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$\Big\downarrow$ 27

$$-\frac{2\left(-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{1}{2}\int \frac{\sinh(2a+2bx)}{a+bx}dx}{b}-\int \frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$\Big\downarrow$ 3042

$$-\frac{2\left(-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{1}{2}\int -\frac{i\sin(2ia+2ibx)}{a+bx}dx}{b}-\int \frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$\Big\downarrow$ 26

$$-\frac{2\left(-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}+\frac{1}{2}i\int \frac{\sin(2ia+2ibx)}{a+bx}dx}{b}-\int \frac{x\cosh^2(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$\Big\downarrow$ 3779

$$-\frac{2\left(-\int \frac{x\cosh^2(a+bx)}{a+bx}dx-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2(a+bx))}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{7292}$$

$$-\frac{2\left(-\int \frac{x\cosh^2(a+bx)}{a+bx}dx-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}-\frac{1}{2}\int \frac{x^2\sinh(2a+2bx)}{a+bx}dx+\frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{2}\int \left(\frac{\sinh(2a+2bx)a^2}{b^2(a+bx)}-\frac{\sinh(2a+2bx)a}{b^2}+\frac{x\sinh(2a+2bx)}{b}\right)dx-\frac{2\left(-\int \left(\frac{\cosh^2(a+bx)}{b}-\frac{a\cosh^2(a+bx)}{b(a+bx)}\right)dx-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}\right)}{b}+\frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2\text{Shi}(2a+2bx)}{b^3}+\frac{\sinh(2a+2bx)}{4b^3}+\frac{a\cosh(2a+2bx)}{2b^3}-\frac{x\cosh(2a+2bx)}{2b^2}\right)-\frac{2\left(\frac{a\text{Chi}(2a+2bx)}{2b^2}+\frac{a\log(a+bx)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b^2}-\frac{\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}}{b}+\frac{x\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{x}{2b}\right)}{b}+\frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

input

```
Int[x^2*Cosh[a + b*x]*CoshIntegral[a + b*x],x]
```

output

```
(x^2*CoshIntegral[a + b*x]*Sinh[a + b*x])/b + ((a*Cosh[2*a + 2*b*x])/(2*b^
3) - (x*Cosh[2*a + 2*b*x])/(2*b^2) + Sinh[2*a + 2*b*x]/(4*b^3) - (a^2*Sinh
Integral[2*a + 2*b*x])/b^3)/2 - (2*(-1/2*x/b + (x*Cosh[a + b*x]*CoshIntegr
al[a + b*x])/b + (a*CoshIntegral[2*a + 2*b*x])/(2*b^2) + (a*Log[a + b*x])/
(2*b^2) - (Cosh[a + b*x]*Sinh[a + b*x])/(2*b^2) - ((CoshIntegral[a + b*x]*
Sinh[a + b*x])/b - SinhIntegral[2*a + 2*b*x]/(2*b))/b))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6151
```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Simp[1/2^p   Int[u*Si
nh[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

rule 7095
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7097
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7103
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\mathrm{Chi}(bx+a)\left(a^2\sinh(bx+a)-2a((bx+a)\sinh(bx+a)-\cosh(bx+a))+(bx+a)^2\sinh(bx+a)-2(bx+a)\cosh(bx+a)+2\sinh\right.$
default	$\mathrm{Chi}(bx+a)\left(a^2\sinh(bx+a)-2a((bx+a)\sinh(bx+a)-\cosh(bx+a))+(bx+a)^2\sinh(bx+a)-2(bx+a)\cosh(bx+a)+2\sinh\right.$

input
```
int(x^2*cosh(b*x+a)*Chi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(Chi(b*x+a)*(a^2*sinh(b*x+a)-2*a*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))+(
b*x+a)^2*sinh(b*x+a)-2*(b*x+a)*cosh(b*x+a)+2*sinh(b*x+a))-1/2*a^2*Shi(2*b*
x+2*a)+a*cosh(b*x+a)^2-a*ln(b*x+a)-a*Chi(2*b*x+2*a)-1/2*(b*x+a)*cosh(b*x+a
)^2+5/4*sinh(b*x+a)*cosh(b*x+a)+5/4*b*x+5/4*a-Shi(2*b*x+2*a))
```

Fricas [F]

$$\int x^2 \cosh(a+bx)\text{Chi}(a+bx)\,dx = \int x^2\text{Chi}(bx+a)\cosh(bx+a)\,dx$$

input
```
integrate(x^2*cosh(b*x+a)*Chi(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^2*cosh(b*x + a)*cosh_integral(b*x + a), x)
```

Sympy [F]

$$\int x^2 \cosh(a+bx)\text{Chi}(a+bx)\,dx = \int x^2\cosh(a+bx)\,\text{Chi}(a+bx)\,dx$$

input
```
integrate(x**2*cosh(b*x+a)*Chi(b*x+a),x)
```

output
```
Integral(x**2*cosh(a + b*x)*Chi(a + b*x), x)
```

Maxima [F]

$$\int x^2 \cosh(a+bx)\text{Chi}(a+bx)\,dx = \int x^2\text{Chi}(bx+a)\cosh(bx+a)\,dx$$

input
```
integrate(x^2*cosh(b*x+a)*Chi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^2*Chi(b*x + a)*cosh(b*x + a), x)
```

Giac [F]

$$\int x^2 \cosh(a+bx)\text{Chi}(a+bx)\,dx = \int x^2\text{Chi}(bx+a)\cosh(bx+a)\,dx$$

input
```
integrate(x^2*cosh(b*x+a)*Chi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*Chi(b*x + a)*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh(a+bx)\text{Chi}(a+bx)\,dx = \int x^2\,\text{coshint}(a+b\,x)\,\cosh(a+b\,x)\,dx$$

input
```
int(x^2*coshint(a + b*x)*cosh(a + b*x),x)
```

output
```
int(x^2*coshint(a + b*x)*cosh(a + b*x), x)
```

Reduce [F]

$$\int x^2 \cosh(a+bx)\text{Chi}(a+bx)\,dx = \int \chi(bx+a)\cosh(bx+a)\,x^2dx$$

input
```
int(x^2*cosh(b*x+a)*Chi(b*x+a),x)
```

output
```
int(chi(a + b*x)*cosh(a + b*x)*x**2,x)
```

3.128 $\int x\cosh(a+bx)\mathbf{Chi}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 97

$$\int x\cosh(a+bx)\text{Chi}(a+bx)\,dx = -\frac{\cosh(2a+2bx)}{4b^2}-\frac{\cosh(a+bx)\text{Chi}(a+bx)}{b^2}+\frac{\text{Chi}(2a+2bx)}{2b^2}+\frac{\log(a+bx)}{2b^2}+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}+\frac{a\text{Shi}(2a+2bx)}{2b^2}$$

output
```
-1/4*cosh(2*b*x+2*a)/b^2-cosh(b*x+a)*Chi(b*x+a)/b^2+1/2*Chi(2*b*x+2*a)/b^2
+1/2*ln(b*x+a)/b^2+x*Chi(b*x+a)*sinh(b*x+a)/b+1/2*a*Shi(2*b*x+2*a)/b^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int x\cosh(a+bx)\text{Chi}(a+bx)\,dx = \frac{-\cosh(2(a+bx))+2\text{Chi}(2(a+bx))+2\log(a+bx)+4\text{Chi}(a+bx)(-\cosh(a+bx)+bx\sinh(a+bx)}{4b^2}$$

input
```
Integrate[x*Cosh[a + b*x]*CoshIntegral[a + b*x],x]
```

output
```
(-Cosh[2*(a + b*x)] + 2*CoshIntegral[2*(a + b*x)] + 2*Log[a + b*x] + 4*Cos
hIntegral[a + b*x]*(-Cosh[a + b*x] + b*x*Sinh[a + b*x]) + 2*a*SinhIntegral
[2*(a + b*x)])/(4*b^2)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {7097, 6151, 7101, 3042, 3793, 2009, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{Chi}(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{7097}$$

$$-\frac{\int \text{Chi}(a+bx)\sinh(a+bx)dx}{b}-\int \frac{x\cosh(a+bx)\sinh(a+bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{6151}$$

$$-\frac{\int \text{Chi}(a+bx)\sinh(a+bx)dx}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{7101}$$

$$-\frac{\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\cosh^2(a+bx)}{a+bx}dx}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{a+bx}dx}{b}-\frac{1}{2}\int \frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\int\left(\frac{\cosh(2a+2bx)}{2(a+bx)}+\frac{1}{2(a+bx)}\right)dx}{b}-\frac{1}{2}\int\frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\int\frac{x\sinh(2(a+bx))}{a+bx}dx+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{\log(a+bx)}{2b}}{b}$$

$$\Big\downarrow \text{7292}$$

$$-\frac{1}{2}\int\frac{x\sinh(2a+2bx)}{a+bx}dx+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{\log(a+bx)}{2b}}{b}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{2}\int\left(\frac{\sinh(2a+2bx)}{b}+\frac{a\sinh(2a+2bx)}{b(-a-bx)}\right)dx+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{\log(a+bx)}{2b}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a\text{Shi}(2a+2bx)}{b^2}-\frac{\cosh(2a+2bx)}{2b^2}\right)+\frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{\log(a+bx)}{2b}}{b}$$

input
```
Int[x*Cosh[a + b*x]*CoshIntegral[a + b*x],x]
```

output
```
-(((Cosh[a + b*x]*CoshIntegral[a + b*x])/b - CoshIntegral[2*a + 2*b*x]/(2*
b) - Log[a + b*x]/(2*b))/b) + (x*CoshIntegral[a + b*x]*Sinh[a + b*x])/b +
(-1/2*Cosh[2*a + 2*b*x]/b^2 + (a*SinhIntegral[2*a + 2*b*x])/b^2)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6151
```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Simp[1/2^p   Int[u*Si
nh[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

rule 7097
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7101
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\mathrm{Chi}(bx+a)((bx+a)\sinh(bx+a)-\cosh(bx+a)-a\sinh(bx+a))-\frac{\cosh(bx+a)^2}{2}+\frac{\ln(bx+a)}{2}+\frac{\mathrm{Chi}(2bx+2a)}{2}+\frac{a\,\mathrm{Shi}(2bx+2a)}{2}}{b^2}$
default	$\frac{\mathrm{Chi}(bx+a)((bx+a)\sinh(bx+a)-\cosh(bx+a)-a\sinh(bx+a))-\frac{\cosh(bx+a)^2}{2}+\frac{\ln(bx+a)}{2}+\frac{\mathrm{Chi}(2bx+2a)}{2}+\frac{a\,\mathrm{Shi}(2bx+2a)}{2}}{b^2}$

input
```
int(x*cosh(b*x+a)*Chi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(Chi(b*x+a)*((b*x+a)*sinh(b*x+a)-cosh(b*x+a)-a*sinh(b*x+a))-1/2*cosh
(b*x+a)^2+1/2*ln(b*x+a)+1/2*Chi(2*b*x+2*a)+1/2*a*Shi(2*b*x+2*a))
```

Fricas [F]

$$\int x\cosh(a+bx)\mathrm{Chi}(a+bx)\,dx=\int x\mathrm{Chi}(bx+a)\cosh\left(bx+a\right)\,dx$$

input
```
integrate(x*cosh(b*x+a)*Chi(b*x+a),x, algorithm="fricas")
```

output
```
integral(x*cosh(b*x + a)*cosh_integral(b*x + a), x)
```

Sympy [F]

$$\int x\cosh(a+bx)\mathrm{Chi}(a+bx)\,dx=\int x\cosh\left(a+bx\right)\mathrm{Chi}\left(a+bx\right)\,dx$$

input
```
integrate(x*cosh(b*x+a)*Chi(b*x+a),x)
```

output
```
Integral(x*cosh(a + b*x)*Chi(a + b*x), x)
```

Maxima [F]

$$\int x\cosh(a+bx)\text{Chi}(a+bx)\,dx = \int x\text{Chi}(bx+a)\cosh(bx+a)\,dx$$

input
```
integrate(x*cosh(b*x+a)*Chi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*Chi(b*x + a)*cosh(b*x + a), x)
```

Giac [F]

$$\int x\cosh(a+bx)\text{Chi}(a+bx)\,dx = \int x\text{Chi}(bx+a)\cosh(bx+a)\,dx$$

input
```
integrate(x*cosh(b*x+a)*Chi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*Chi(b*x + a)*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\text{Chi}(a+bx)\,dx = \int x\,\text{coshint}(a+b\,x)\,\cosh(a+b\,x)\,dx$$

input
```
int(x*coshint(a + b*x)*cosh(a + b*x),x)
```

output
```
int(x*coshint(a + b*x)*cosh(a + b*x), x)
```

Reduce [F]

$$\int x \cosh(a + bx)\text{Chi}(a + bx)\,dx = \int \chi(bx + a)\cosh(bx + a)\,xdx$$

input
```
int(x*cosh(b*x+a)*Chi(b*x+a),x)
```

output
```
int(chi(a + b*x)*cosh(a + b*x)*x,x)
```

3.129 $\int \cosh(a+bx)\mathbf{Chi}(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \cosh(a+bx)\text{Chi}(a+bx)\,dx=\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}$$

output
```
Chi(b*x+a)*sinh(b*x+a)/b-1/2*Shi(2*b*x+2*a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \cosh(a+bx)\text{Chi}(a+bx)\,dx=\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2(a+bx))}{2b}$$

input
```
Integrate[Cosh[a + b*x]*CoshIntegral[a + b*x],x]
```

output
```
(CoshIntegral[a + b*x]*Sinh[a + b*x])/b - SinhIntegral[2*(a + b*x)]/(2*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {7095, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{Chi}(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{7095}$$

$$\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\int \frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}dx$$

$$\Big\downarrow \text{5971}$$

$$\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\int \frac{\sinh(2a+2bx)}{2(a+bx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{1}{2}\int \frac{\sinh(2a+2bx)}{a+bx}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{1}{2}\int -\frac{i\sin(2ia+2ibx)}{a+bx}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}+\frac{1}{2}i\int \frac{\sin(2ia+2ibx)}{a+bx}dx$$

$$\Big\downarrow \text{3779}$$

$$\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}$$

input
```
Int[Cosh[a + b*x]*CoshIntegral[a + b*x],x]
```

output
```
(CoshIntegral[a + b*x]*Sinh[a + b*x])/b - SinhIntegral[2*a + 2*b*x]/(2*b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 7095
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\operatorname{Chi}(bx+a)\sinh(bx+a)-\frac{\operatorname{Shi}(2bx+2a)}{2}}{b}$	30
default	$\frac{\operatorname{Chi}(bx+a)\sinh(bx+a)-\frac{\operatorname{Shi}(2bx+2a)}{2}}{b}$	30

input
```
int(cosh(b*x+a)*Chi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(Chi(b*x+a)*sinh(b*x+a)-1/2*Shi(2*b*x+2*a))
```

Fricas [F]

$$\int \cosh(a+bx)\mathrm{Chi}(a+bx)\,dx = \int \mathrm{Chi}(bx+a)\cosh(bx+a)\,dx$$

input
```
integrate(cosh(b*x+a)*Chi(b*x+a),x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*cosh_integral(b*x + a), x)
```

Sympy [F]

$$\int \cosh(a+bx)\mathrm{Chi}(a+bx)\,dx = \int \cosh(a+bx)\,\mathrm{Chi}(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)*Chi(b*x+a),x)
```

output
```
Integral(cosh(a + b*x)*Chi(a + b*x), x)
```

Maxima [F]

$$\int \cosh(a+bx)\mathrm{Chi}(a+bx)\,dx = \int \mathrm{Chi}(bx+a)\cosh(bx+a)\,dx$$

input
```
integrate(cosh(b*x+a)*Chi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(Chi(b*x + a)*cosh(b*x + a), x)
```

Giac [F]

$$\int \cosh(a+bx)\text{Chi}(a+bx)\,dx = \int \text{Chi}(bx+a)\cosh(bx+a)\,dx$$

input
```
integrate(cosh(b*x+a)*Chi(b*x+a),x, algorithm="giac")
```

output
```
integrate(Chi(b*x + a)*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\text{Chi}(a+bx)\,dx = \int \text{coshint}(a+b\,x)\,\cosh(a+b\,x)\,dx$$

input
```
int(coshint(a + b*x)*cosh(a + b*x),x)
```

output
```
int(coshint(a + b*x)*cosh(a + b*x), x)
```

Reduce [F]

$$\int \cosh(a+bx)\text{Chi}(a+bx)\,dx = \int \chi(bx+a)\cosh(bx+a)\,dx$$

input
```
int(cosh(b*x+a)*Chi(b*x+a),x)
```

output
```
int(chi(a + b*x)*cosh(a + b*x),x)
```

3.130 $\int \frac{\cosh(a+bx)\text{Chi}(a+bx)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cosh(a+bx)\text{Chi}(a+bx)}{x}\,dx = \text{Int}\left(\frac{\cosh(a+bx)\text{Chi}(a+bx)}{x},x\right)$$

output
```
Defer(Int)(cosh(b*x+a)*Chi(b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Chi}(a+bx)}{x}\,dx = \int \frac{\cosh(a+bx)\text{Chi}(a+bx)}{x}\,dx$$

input
```
Integrate[(Cosh[a + b*x]*CoshIntegral[a + b*x])/x,x]
```

output
```
Integrate[(Cosh[a + b*x]*CoshIntegral[a + b*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{Chi}(a+bx)\cosh(a+bx)}{x} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\text{Chi}(a+bx)\cosh(a+bx)}{x} dx$$

input
```
Int[(Cosh[a + b*x]*CoshIntegral[a + b*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(bx+a)\,\text{Chi}(bx+a)}{x} dx$$

input
```
int(cosh(b*x+a)*Chi(b*x+a)/x,x)
```

output
```
int(cosh(b*x+a)*Chi(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Chi}(a+bx)}{x}\,dx = \int \frac{\text{Chi}(bx+a)\cosh(bx+a)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*Chi(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*cosh_integral(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\cosh(a+bx)\text{Chi}(a+bx)}{x}\,dx = \int \frac{\cosh(a+bx)\,\text{Chi}\,(a+bx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*Chi(b*x+a)/x,x)
```

output
```
Integral(cosh(a + b*x)*Chi(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Chi}(a+bx)}{x}\,dx = \int \frac{\text{Chi}(bx+a)\cosh(bx+a)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*Chi(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(Chi(b*x + a)*cosh(b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Chi}(a+bx)}{x}\,dx = \int \frac{\text{Chi}(bx+a)\cosh(bx+a)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*Chi(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(Chi(b*x + a)*cosh(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Chi}(a+bx)}{x}\,dx = \int \frac{\text{coshint}(a+b\,x)\,\cosh(a+b\,x)}{x}\,dx$$

input
```
int((coshint(a + b*x)*cosh(a + b*x))/x,x)
```

output
```
int((coshint(a + b*x)*cosh(a + b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{\cosh(a+bx)\text{Chi}(a+bx)}{x}\,dx = \frac{\chi(bx+a)^2}{2} + \left(\int \frac{\chi(bx+a)\cosh(bx+a)}{b\,x^2+ax}dx\right)a$$

input
```
int(cosh(b*x+a)*Chi(b*x+a)/x,x)
```

output
```
(chi(a + b*x)**2 + 2*int((chi(a + b*x)*cosh(a + b*x))/(a*x + b*x**2),x)*a)
/2
```

3.131 $\int x\textbf{Chi}(c+dx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 371

$$\int x\mathrm{Chi}(c+dx)\sinh(a+bx)\,dx = \frac{c\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2bd} + \frac{x\cosh(a+bx)\mathrm{Chi}(c+dx)}{b} + \frac{c\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2bd} + \frac{\mathrm{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2b^2} + \frac{\mathrm{Chi}\left(\frac{c(b+d)}{d}+(b+d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2b^2} - \frac{\mathrm{Chi}(c+dx)\sinh(a+bx)}{b^2} - \frac{\sinh(a-c+(b-d)x)}{2b(b-d)} - \frac{\sinh(a+c+(b+d)x)}{2b(b+d)} + \frac{\cosh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2b^2} + \frac{c\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2bd} + \frac{\cosh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2b^2} + \frac{c\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2bd}$$

output

```
1/2*c*cosh(a-b*c/d)*Chi(c*(b-d)/d+(b-d)*x)/b/d+x*cosh(b*x+a)*Chi(d*x+c)/b+
1/2*c*cosh(a-b*c/d)*Chi(c*(b+d)/d+(b+d)*x)/b/d+1/2*Chi(c*(b-d)/d+(b-d)*x)*
sinh(a-b*c/d)/b^2+1/2*Chi(c*(b+d)/d+(b+d)*x)*sinh(a-b*c/d)/b^2-Chi(d*x+c)*
sinh(b*x+a)/b^2-1/2*sinh(a-c+(b-d)*x)/b/(b-d)-1/2*sinh(a+c+(b+d)*x)/b/(b+d
)+1/2*cosh(a-b*c/d)*Shi(c*(b-d)/d+(b-d)*x)/b^2+1/2*c*sinh(a-b*c/d)*Shi(c*(
b-d)/d+(b-d)*x)/b/d+1/2*cosh(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b^2+1/2*c*sin
h(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b/d
```

Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.87

$$\int x\text{Chi}(c+dx)\sinh(a+bx)\,dx$$
$$=\frac{e^{-a-c-(b+d)x}\left(bd\left(d\left(-1+e^{2(c+dx)}\right)+b\left(1+e^{2(c+dx)}\right)\right)+(bc-d)\left(b^2-d^2\right)e^{\frac{(b+d)(c+dx)}{d}}\text{ExpIntegralEi}\left(-\frac{(b-}{}\right.\right.}{}$$

input
```
Integrate[x*CoshIntegral[c + d*x]*Sinh[a + b*x],x]
```

output
```
(E^(-a - c - (b + d)*x)*(b*d*(d*(-1 + E^(2*(c + d*x))) + b*(1 + E^(2*(c +
d*x)))) + (b*c - d)*(b^2 - d^2)*E^(((b + d)*(c + d*x))/d)*ExpIntegralEi[-(
((b - d)*(c + d*x))/d)] + (b*c - d)*(b^2 - d^2)*E^(((b + d)*(c + d*x))/d)*
ExpIntegralEi[-(((b + d)*(c + d*x))/d)]) + E^(a - (c*(b + d))/d)*(-(b*d*E^
((b*c)/d + b*x - d*x)*(b + d + b*E^(2*(c + d*x)) - d*E^(2*(c + d*x)))) + (
b*c + d)*(b^2 - d^2)*E^c*ExpIntegralEi[((b - d)*(c + d*x))/d] + (b*c + d)*
(b^2 - d^2)*E^c*ExpIntegralEi[((b + d)*(c + d*x))/d]) + 4*(b - d)*d*(b + d
)*CoshIntegral[c + d*x]*(b*x*Cosh[a + b*x] - Sinh[a + b*x]))/(4*b^2*(b - d
)*d*(b + d))
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {7103, 6177, 2009, 7095, 5995, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sinh(a+bx)\text{Chi}(c+dx)\,dx$$

$$\Big\downarrow \text{7103}$$

$$-\frac{\int \cosh(a+bx)\text{Chi}(c+dx)dx}{b}-\frac{d\int \frac{x\cosh(a+bx)\cosh(c+dx)}{c+dx}dx}{b}+\frac{x\cosh(a+bx)\text{Chi}(c+dx)}{b}$$

$$\Big\downarrow \text{6177}$$

$$-\frac{\int \cosh(a+bx)\text{Chi}(c+dx)dx}{b}-\frac{d\int\left(\frac{x\cosh(a-c+(b-d)x)}{2(c+dx)}+\frac{x\cosh(a+c+(b+d)x)}{2(c+dx)}\right)dx}{b}+\frac{x\cosh(a+bx)\text{Chi}(c+dx)}{b}$$

$\big\downarrow$ 2009

$$-\frac{\int \cosh(a+bx)\text{Chi}(c+dx)dx}{b}-\frac{d\left(-\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d^2}-\frac{c\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\sinh\left(a-\frac{bc}{d}\cdots\right.}{}\cdots\right)}{b}\cdots+\frac{x\cosh(a+bx)\text{Chi}(c+dx)}{b}$$

$\big\downarrow$ 7095

$$-\frac{\frac{\sinh(a+bx)\text{Chi}(c+dx)}{b}-\frac{d\int\frac{\cosh(c+dx)\sinh(a+bx)}{c+dx}dx}{b}}{b}-\frac{d\left(-\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d^2}-\frac{c\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\sinh\left(a-\frac{bc}{d}\cdots\right.}{}\cdots\right)}{b}\cdots+\frac{x\cosh(a+bx)\text{Chi}(c+dx)}{b}$$

$\big\downarrow$ 5995

$$-\frac{\frac{\sinh(a+bx)\text{Chi}(c+dx)}{b}-\frac{d\int\left(\frac{\sinh(a-c+(b-d)x)}{2(c+dx)}+\frac{\sinh(a+c+(b+d)x)}{2(c+dx)}\right)dx}{b}}{b}-\frac{d\left(-\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d^2}-\frac{c\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\sinh\left(a-\frac{bc}{d}\cdots\right.}{}\cdots\right)}{b}\cdots+\frac{x\cosh(a+bx)\text{Chi}(c+dx)}{b}$$

$\big\downarrow$ 2009

$$-\frac{d\left(-\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d^2}-\frac{c\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\sinh\left(a-\cdots\right.}{}\cdots\right)}{b}\cdots$$

$$\frac{\frac{\sinh(a+bx)\text{Chi}(c+dx)}{b}-\frac{d\left(\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\cosh\left(\cdots\right.}{}\cdots\right)}{b}\cdots}{b}\cdots$$

$$+\frac{x\cosh(a+bx)\text{Chi}(c+dx)}{b}$$

input
```
Int[x*CoshIntegral[c + d*x]*Sinh[a + b*x],x]
```

output
```
(x*Cosh[a + b*x]*CoshIntegral[c + d*x])/b - (d*(-1/2*(c*Cosh[a - (b*c)/d]*
CoshIntegral[(c*(b - d))/d + (b - d)*x])/d^2 - (c*Cosh[a - (b*c)/d]*CoshIn
tegral[(c*(b + d))/d + (b + d)*x])/(2*d^2) + Sinh[a - c + (b - d)*x]/(2*(b
 - d)*d) + Sinh[a + c + (b + d)*x]/(2*d*(b + d)) - (c*Sinh[a - (b*c)/d]*Si
nhIntegral[(c*(b - d))/d + (b - d)*x])/(2*d^2) - (c*Sinh[a - (b*c)/d]*Sinh
Integral[(c*(b + d))/d + (b + d)*x])/(2*d^2)))/b - ((CoshIntegral[c + d*x]
*Sinh[a + b*x])/b - (d*((CoshIntegral[(c*(b - d))/d + (b - d)*x]*Sinh[a -
(b*c)/d])/(2*d) + (CoshIntegral[(c*(b + d))/d + (b + d)*x]*Sinh[a - (b*c)/
d])/(2*d) + (Cosh[a - (b*c)/d]*SinhIntegral[(c*(b - d))/d + (b - d)*x])/(2
*d) + (Cosh[a - (b*c)/d]*SinhIntegral[(c*(b + d))/d + (b + d)*x])/(2*d)))/
b)/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5995
```
Int[Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p
, 0] && IGtQ[q, 0]
```

rule 6177
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(m_.)*Cosh[(c_.) + (d_.)*(x_)]^(n_.)*(u_.), x_
Symbol] :> Int[ExpandTrigReduce[u, Cosh[a + b*x]^m*Cosh[c + d*x]^n, x], x]
/; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 7095
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7103
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*Cosh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Cosh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int x \operatorname{Chi}(dx+c)\sinh(bx+a)\,dx$$

input
```
int(x*Chi(d*x+c)*sinh(b*x+a),x)
```

output
```
int(x*Chi(d*x+c)*sinh(b*x+a),x)
```

Fricas [F]

$$\int x\mathrm{Chi}(c+dx)\sinh(a+bx)\,dx = \int x\mathrm{Chi}(dx+c)\sinh(bx+a)\,dx$$

input
```
integrate(x*Chi(d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
integral(x*cosh_integral(d*x + c)*sinh(b*x + a), x)
```

Sympy [F]

$$\int x\mathrm{Chi}(c+dx)\sinh(a+bx)\,dx = \int x\sinh(a+bx)\operatorname{Chi}(c+dx)\,dx$$

input
```
integrate(x*Chi(d*x+c)*sinh(b*x+a),x)
```

output
```
Integral(x*sinh(a + b*x)*Chi(c + d*x), x)
```

Maxima [F]

$$\int x\mathrm{Chi}(c+dx)\sinh(a+bx)\,dx=\int x\mathrm{Chi}(dx+c)\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x*Chi(d*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*Chi(d*x + c)*sinh(b*x + a), x)
```

Giac [F]

$$\int x\mathrm{Chi}(c+dx)\sinh(a+bx)\,dx=\int x\mathrm{Chi}(dx+c)\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x*Chi(d*x+c)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*Chi(d*x + c)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\mathrm{Chi}(c+dx)\sinh(a+bx)\,dx=\int x\,\mathrm{coshint}(c+d\,x)\,\sinh(a+b\,x)\,dx$$

input
```
int(x*coshint(c + d*x)*sinh(a + b*x),x)
```

output
```
int(x*coshint(c + d*x)*sinh(a + b*x), x)
```

Reduce [F]

$$\int x\text{Chi}(c+dx)\sinh(a+bx)\,dx=\int \chi(dx+c)\sinh\left(bx+a\right)xdx$$

input
```
int(x*Chi(d*x+c)*sinh(b*x+a),x)
```

output
```
int(chi(c + d*x)*sinh(a + b*x)*x,x)
```

3.132 $\int \mathbf{Chi}(c+dx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 153

$$\int \mathrm{Chi}(c+dx)\sinh(a+bx)\,dx = -\frac{\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2b} + \frac{\cosh(a+bx)\mathrm{Chi}(c+dx)}{b} - \frac{\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2b} - \frac{\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2b} - \frac{\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2b}$$

output
```
-1/2*cosh(a-b*c/d)*Chi(c*(b-d)/d+(b-d)*x)/b+cosh(b*x+a)*Chi(d*x+c)/b-1/2*c
osh(a-b*c/d)*Chi(c*(b+d)/d+(b+d)*x)/b-1/2*sinh(a-b*c/d)*Shi(c*(b-d)/d+(b-d
)*x)/b-1/2*sinh(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.70

$$\int \text{Chi}(c+dx)\sinh(a+bx)\,dx = -\frac{-4\cosh(a+bx)\text{Chi}(c+dx)+e^{-a+\frac{bc}{d}}\left(\text{ExpIntegralEi}\left(-\frac{(b-d)(c+dx)}{d}\right)+\text{ExpIntegralEi}\left(-\frac{(b+d)(c+dx)}{d}\right)\right.}{4b}$$

input
```
Integrate[CoshIntegral[c + d*x]*Sinh[a + b*x],x]
```

output
```
-1/4*(-4*Cosh[a + b*x]*CoshIntegral[c + d*x] + E^(-a + (b*c)/d)*(ExpIntegr
alEi[-(((b - d)*(c + d*x))/d)] + ExpIntegralEi[-(((b + d)*(c + d*x))/d)])
+ E^(a - (b*c)/d)*(ExpIntegralEi[((b - d)*(c + d*x))/d] + ExpIntegralEi[((
b + d)*(c + d*x))/d]))/b
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7101, 5994, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\text{Chi}(c+dx)\,dx$$

$$\Big\downarrow \text{ 7101}$$

$$\frac{\cosh(a+bx)\text{Chi}(c+dx)}{b}-\frac{d\int \frac{\cosh(a+bx)\cosh(c+dx)}{c+dx}dx}{b}$$

$$\Big\downarrow \text{ 5994}$$

$$\frac{\cosh(a+bx)\text{Chi}(c+dx)}{b}-\frac{d\int \left(\frac{\cosh(a-c+(b-d)x)}{2(c+dx)}+\frac{\cosh(a+c+(b+d)x)}{2(c+dx)}\right)dx}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\cosh(a+bx)\text{Chi}(c+dx)}{b} - \frac{d\left(\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d} + \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d} + \frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d} + \frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x\right.}{2d}\right.}{b}$$

input
```
Int[CoshIntegral[c + d*x]*Sinh[a + b*x],x]
```

output
```
(Cosh[a + b*x]*CoshIntegral[c + d*x])/b - (d*((Cosh[a - (b*c)/d]*CoshInteg
ral[(c*(b - d))/d + (b - d)*x])/(2*d) + (Cosh[a - (b*c)/d]*CoshIntegral[(c
*(b + d))/d + (b + d)*x])/(2*d) + (Sinh[a - (b*c)/d]*SinhIntegral[(c*(b -
d))/d + (b - d)*x])/(2*d) + (Sinh[a - (b*c)/d]*SinhIntegral[(c*(b + d))/d
+ (b + d)*x])/(2*d)))/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5994
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) +
(f_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Cosh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0
] && IGtQ[q, 0] && IntegerQ[m]
```

rule 7101
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [F]

$$\int \mathrm{Chi}\left(dx+c\right)\sinh\left(bx+a\right)dx$$

input
```
int(Chi(d*x+c)*sinh(b*x+a),x)
```

output
```
int(Chi(d*x+c)*sinh(b*x+a),x)
```

Fricas [F]

$$\int \mathrm{Chi}(c+dx)\sinh(a+bx)\,dx=\int \mathrm{Chi}(dx+c)\sinh\left(bx+a\right)\,dx$$

input
```
integrate(Chi(d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
integral(cosh_integral(d*x + c)*sinh(b*x + a), x)
```

Sympy [F]

$$\int \mathrm{Chi}(c+dx)\sinh(a+bx)\,dx=\int \sinh\left(a+bx\right)\mathrm{Chi}\left(c+dx\right)\,dx$$

input
```
integrate(Chi(d*x+c)*sinh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*Chi(c + d*x), x)
```

Maxima [F]

$$\int \mathrm{Chi}(c+dx)\sinh(a+bx)\,dx=\int \mathrm{Chi}(dx+c)\sinh(bx+a)\,dx$$

input
```
integrate(Chi(d*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(Chi(d*x + c)*sinh(b*x + a), x)
```

Giac [F]

$$\int \mathrm{Chi}(c+dx)\sinh(a+bx)\,dx=\int \mathrm{Chi}(dx+c)\sinh(bx+a)\,dx$$

input
```
integrate(Chi(d*x+c)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(Chi(d*x + c)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \mathrm{Chi}(c+dx)\sinh(a+bx)\,dx=\int \mathrm{coshint}(c+d\,x)\,\sinh(a+b\,x)\,dx$$

input
```
int(coshint(c + d*x)*sinh(a + b*x),x)
```

output
```
int(coshint(c + d*x)*sinh(a + b*x), x)
```

Reduce [F]

$$\int \mathrm{Chi}(c+dx)\sinh(a+bx)\,dx=\int \chi(dx+c)\sinh\left(bx+a\right)dx$$

input
```
int(Chi(d*x+c)*sinh(b*x+a),x)
```

output
```
int(chi(c + d*x)*sinh(a + b*x),x)
```

3.133 $\int \frac{\text{Chi}(c+dx)\sinh(a+bx)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{Chi}(c+dx)\sinh(a+bx)}{x}\,dx = \text{Int}\left(\frac{\text{Chi}(c+dx)\sinh(a+bx)}{x},x\right)$$

output
```
Defer(Int)(Chi(d*x+c)*sinh(b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 2.87 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{Chi}(c+dx)\sinh(a+bx)}{x}\,dx = \int \frac{\text{Chi}(c+dx)\sinh(a+bx)}{x}\,dx$$

input
```
Integrate[(CoshIntegral[c + d*x]*Sinh[a + b*x])/x,x]
```

output
```
Integrate[(CoshIntegral[c + d*x]*Sinh[a + b*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\text{Chi}(c+dx)}{x} \, dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\sinh(a+bx)\text{Chi}(c+dx)}{x}dx$$

input
```
Int[(CoshIntegral[c + d*x]*Sinh[a + b*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\text{Chi}\,(dx+c)\sinh\,(bx+a)}{x}dx$$

input
```
int(Chi(d*x+c)*sinh(b*x+a)/x,x)
```

output
```
int(Chi(d*x+c)*sinh(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{Chi}(c+dx)\sinh(a+bx)}{x}\,dx = \int \frac{\text{Chi}(dx+c)\sinh(bx+a)}{x}\,dx$$

input
```
integrate(Chi(d*x+c)*sinh(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(cosh_integral(d*x + c)*sinh(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\text{Chi}(c+dx)\sinh(a+bx)}{x}\,dx = \int \frac{\sinh(a+bx)\,\text{Chi}(c+dx)}{x}\,dx$$

input
```
integrate(Chi(d*x+c)*sinh(b*x+a)/x,x)
```

output
```
Integral(sinh(a + b*x)*Chi(c + d*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{Chi}(c+dx)\sinh(a+bx)}{x}\,dx = \int \frac{\text{Chi}(dx+c)\sinh(bx+a)}{x}\,dx$$

input
```
integrate(Chi(d*x+c)*sinh(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(Chi(d*x + c)*sinh(b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\mathrm{Chi}(c+dx)\sinh(a+bx)}{x}\,dx = \int \frac{\mathrm{Chi}(dx+c)\sinh(bx+a)}{x}\,dx$$

input
```
integrate(Chi(d*x+c)*sinh(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(Chi(d*x + c)*sinh(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\mathrm{Chi}(c+dx)\sinh(a+bx)}{x}\,dx = \int \frac{\mathrm{coshint}(c+d\,x)\,\sinh(a+b\,x)}{x}\,dx$$

input
```
int((coshint(c + d*x)*sinh(a + b*x))/x,x)
```

output
```
int((coshint(c + d*x)*sinh(a + b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\mathrm{Chi}(c+dx)\sinh(a+bx)}{x}\,dx = \int \frac{\chi(dx+c)\sinh(bx+a)}{x}dx$$

input
```
int(Chi(d*x+c)*sinh(b*x+a)/x,x)
```

output
```
int((chi(c + d*x)*sinh(a + b*x))/x,x)
```

3.134 $\int x \cosh(a+bx)\mathbf{Chi}(c+dx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 371

$$\int x\cosh(a+bx)\text{Chi}(c+dx)\,dx = -\frac{\cosh(a-c+(b-d)x)}{2b(b-d)} - \frac{\cosh(a+c+(b+d)x)}{2b(b+d)} + \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2b^2} - \frac{\cosh(a+bx)\text{Chi}(c+dx)}{b^2} + \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2b^2} + \frac{c\text{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2bd} + \frac{c\text{Chi}\left(\frac{c(b+d)}{d}+(b+d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2bd} + \frac{x\text{Chi}(c+dx)\sinh(a+bx)}{b} + \frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2bd} + \frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2b^2} + \frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2bd} + \frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2b^2}$$

output
```
-1/2*cosh(a-c+(b-d)*x)/b/(b-d)-1/2*cosh(a+c+(b+d)*x)/b/(b+d)+1/2*cosh(a-b*
c/d)*Chi(c*(b-d)/d+(b-d)*x)/b^2-cosh(b*x+a)*Chi(d*x+c)/b^2+1/2*cosh(a-b*c/
d)*Chi(c*(b+d)/d+(b+d)*x)/b^2+1/2*c*Chi(c*(b-d)/d+(b-d)*x)*sinh(a-b*c/d)/b
/d+1/2*c*Chi(c*(b+d)/d+(b+d)*x)*sinh(a-b*c/d)/b/d+x*Chi(d*x+c)*sinh(b*x+a)
/b+1/2*c*cosh(a-b*c/d)*Shi(c*(b-d)/d+(b-d)*x)/b/d+1/2*sinh(a-b*c/d)*Shi(c*
(b-d)/d+(b-d)*x)/b^2+1/2*c*cosh(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b/d+1/2*si
nh(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b^2
```

Mathematica [A] (verified)

Time = 2.08 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.79

$$\int x\cosh(a+bx)\text{Chi}(c+dx)\,dx$$
$$= \frac{(bc+d)e^{a-\frac{bc}{d}}\text{ExpIntegralEi}\left(\frac{(b-d)(c+dx)}{d}\right)}{d} + \frac{e^{-a-c-(b+d)x}\left(-bd\left(d\left(-1+e^{2(a+bx)}\right)+b\left(1+e^{2(a+bx)}\right)\right)-(bc-d)\left(b^2-d^2\right)e^{\frac{(b+d)(c+dx)}{d}}\text{ExpInte}\right.}{(b-d)d(b+d)}$$

input
```
Integrate[x*Cosh[a + b*x]*CoshIntegral[c + d*x],x]
```

output
```
(((b*c + d)*E^(a - (b*c)/d)*ExpIntegralEi[((b - d)*(c + d*x))/d])/d + (E^(
-a - c - (b + d)*x)*(-(b*d*(d*(-1 + E^(2*(a + b*x))) + b*(1 + E^(2*(a + b*
x))))) - (b*c - d)*(b^2 - d^2)*E^(((b + d)*(c + d*x))/d)*ExpIntegralEi[-((
(b + d)*(c + d*x))/d)]))/((b - d)*d*(b + d)) + (b*d*E^c*(E^((-b + d)*x)/(-
b + d) - E^(2*a + (b + d)*x)/(b + d)) + (-(b*c) + d)*E^((b*c)/d)*ExpIntegr
alEi[-(((b - d)*(c + d*x))/d)] + (b*c + d)*E^(2*a - (b*c)/d)*ExpIntegralEi
[((b + d)*(c + d*x))/d])/(d*E^a) + 4*CoshIntegral[c + d*x]*(-Cosh[a + b*x]
 + b*x*Sinh[a + b*x]))/(4*b^2)
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {7097, 7101, 5994, 2009, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh(a+bx)\text{Chi}(c+dx)\,dx$$

$$\Big\downarrow \text{7097}$$

$$-\frac{\int \text{Chi}(c+dx)\sinh(a+bx)dx}{b} - \frac{d\int \frac{x\cosh(c+dx)\sinh(a+bx)}{c+dx}dx}{b} + \frac{x\sinh(a+bx)\text{Chi}(c+dx)}{b}$$

$$\Big\downarrow \text{7101}$$

$$-\frac{\frac{\cosh(a+bx)\text{Chi}(c+dx)}{b}-\frac{d\int \frac{\cosh(a+bx)\cosh(c+dx)}{c+dx}dx}{b}}{b}-\frac{d\int \frac{x\cosh(c+dx)\sinh(a+bx)}{c+dx}dx}{b}+\frac{x\sinh(a+bx)\text{Chi}(c+dx)}{b}$$

$$\Big\downarrow \; 5994$$

$$-\frac{\frac{\cosh(a+bx)\text{Chi}(c+dx)}{b}-\frac{d\int \left(\frac{\cosh(a-c+(b-d)x)}{2(c+dx)}+\frac{\cosh(a+c+(b+d)x)}{2(c+dx)}\right)dx}{b}}{b}-\frac{d\int \frac{x\cosh(c+dx)\sinh(a+bx)}{c+dx}dx}{b}+\frac{x\sinh(a+bx)\text{Chi}(c+dx)}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{d\int \frac{x\cosh(c+dx)\sinh(a+bx)}{c+dx}dx}{b}-\frac{\frac{\cosh(a+bx)\text{Chi}(c+dx)}{b}-\frac{d\left(\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\sinh\left(\cdots\right.}{\cdots}\cdots\right)}{b}}{b}+\frac{x\sinh(a+bx)\text{Chi}(c+dx)}{b}$$

$$\Big\downarrow \; 7293$$

$$-\frac{d\int \left(\frac{\cosh(c+dx)\sinh(a+bx)}{d}-\frac{c\cosh(c+dx)\sinh(a+bx)}{d(c+dx)}\right)dx}{b}-\frac{\frac{\cosh(a+bx)\text{Chi}(c+dx)}{b}-\frac{d\left(\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\sinh\left(\cdots\right.}{\cdots}\cdots\right)}{b}}{b}+\frac{x\sinh(a+bx)\text{Chi}(c+dx)}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{d\left(-\frac{c\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d^2}-\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d^2}-\frac{c\cosh\left(a-\cdots\right.}{\cdots}\cdots\right)}{b}-\frac{\frac{\cosh(a+bx)\text{Chi}(c+dx)}{b}-\frac{d\left(\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d}+\frac{\sinh\left(\cdots\right.}{\cdots}\cdots\right)}{b}}{b}+\frac{x\sinh(a+bx)\text{Chi}(c+dx)}{b}$$

input
```
Int[x*Cosh[a + b*x]*CoshIntegral[c + d*x],x]
```

output
```
(x*CoshIntegral[c + d*x]*Sinh[a + b*x])/b - (d*(Cosh[a - c + (b - d)*x]/(2
*(b - d)*d) + Cosh[a + c + (b + d)*x]/(2*d*(b + d)) - (c*CoshIntegral[(c*(
b - d))/d + (b - d)*x]*Sinh[a - (b*c)/d])/(2*d^2) - (c*CoshIntegral[(c*(b
+ d))/d + (b + d)*x]*Sinh[a - (b*c)/d])/(2*d^2) - (c*Cosh[a - (b*c)/d]*Sin
hIntegral[(c*(b - d))/d + (b - d)*x])/(2*d^2) - (c*Cosh[a - (b*c)/d]*SinhI
ntegral[(c*(b + d))/d + (b + d)*x])/(2*d^2)))/b - ((Cosh[a + b*x]*CoshInte
gral[c + d*x])/b - (d*((Cosh[a - (b*c)/d]*CoshIntegral[(c*(b - d))/d + (b
- d)*x])/(2*d) + (Cosh[a - (b*c)/d]*CoshIntegral[(c*(b + d))/d + (b + d)*x
])/(2*d) + (Sinh[a - (b*c)/d]*SinhIntegral[(c*(b - d))/d + (b - d)*x])/(2*
d) + (Sinh[a - (b*c)/d]*SinhIntegral[(c*(b + d))/d + (b + d)*x])/(2*d)))/b
)/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5994
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) +
(f_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Cosh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0
] && IGtQ[q, 0] && IntegerQ[m]
```

rule 7097
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^m*Sinh[a + b*x]*(CoshIntegral[c
+ d*x]/b), x] + (-Simp[d/b   Int[(e + f*x)^m*Sinh[a + b*x]*(Cosh[c + d*x]/(
c + d*x)), x], x] - Simp[f*(m/b)   Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*Cosh
Integral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

rule 7101
```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[Cosh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Cosh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int x\cosh\left(bx+a\right)\operatorname{Chi}\left(dx+c\right)dx$$

input
```
int(x*cosh(b*x+a)*Chi(d*x+c),x)
```

output
```
int(x*cosh(b*x+a)*Chi(d*x+c),x)
```

Fricas [F]

$$\int x\cosh(a+bx)\operatorname{Chi}(c+dx)\,dx=\int x\operatorname{Chi}(dx+c)\cosh\left(bx+a\right)\,dx$$

input
```
integrate(x*cosh(b*x+a)*Chi(d*x+c),x, algorithm="fricas")
```

output
```
integral(x*cosh(b*x + a)*cosh_integral(d*x + c), x)
```

Sympy [F]

$$\int x\cosh(a+bx)\operatorname{Chi}(c+dx)\,dx=\int x\cosh\left(a+bx\right)\operatorname{Chi}\left(c+dx\right)\,dx$$

input
```
integrate(x*cosh(b*x+a)*Chi(d*x+c),x)
```

output
```
Integral(x*cosh(a + b*x)*Chi(c + d*x), x)
```

Maxima [F]

$$\int x\cosh(a+bx)\text{Chi}(c+dx)\,dx=\int x\text{Chi}(dx+c)\cosh(bx+a)\,dx$$

input
```
integrate(x*cosh(b*x+a)*Chi(d*x+c),x, algorithm="maxima")
```

output
```
integrate(x*Chi(d*x + c)*cosh(b*x + a), x)
```

Giac [F]

$$\int x\cosh(a+bx)\text{Chi}(c+dx)\,dx=\int x\text{Chi}(dx+c)\cosh(bx+a)\,dx$$

input
```
integrate(x*cosh(b*x+a)*Chi(d*x+c),x, algorithm="giac")
```

output
```
integrate(x*Chi(d*x + c)*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\text{Chi}(c+dx)\,dx=\int x\,\text{coshint}(c+d\,x)\,\cosh(a+b\,x)\,dx$$

input
```
int(x*coshint(c + d*x)*cosh(a + b*x),x)
```

output
```
int(x*coshint(c + d*x)*cosh(a + b*x), x)
```

Reduce [F]

$$\int x\cosh(a+bx)\text{Chi}(c+dx)\,dx=\int\chi(dx+c)\cosh(bx+a)\,xdx$$

input

```
int(x*cosh(b*x+a)*Chi(d*x+c),x)
```

output

```
int(chi(c + d*x)*cosh(a + b*x)*x,x)
```

3.135 $\int \cosh(a+bx)\mathbf{Chi}(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 153

$$\int \cosh(a+bx)\mathrm{Chi}(c+dx)\,dx = -\frac{\mathrm{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2b} - \frac{\mathrm{Chi}\left(\frac{c(b+d)}{d}+(b+d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2b} + \frac{\mathrm{Chi}(c+dx)\sinh(a+bx)}{b} - \frac{\cosh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2b} - \frac{\cosh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2b}$$

output
```
-1/2*Chi(c*(b-d)/d+(b-d)*x)*sinh(a-b*c/d)/b-1/2*Chi(c*(b+d)/d+(b+d)*x)*sin
h(a-b*c/d)/b+Chi(d*x+c)*sinh(b*x+a)/b-1/2*cosh(a-b*c/d)*Shi(c*(b-d)/d+(b-d
)*x)/b-1/2*cosh(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\int \cosh(a+bx)\text{Chi}(c+dx)\,dx$$
$$= \frac{e^{-a-\frac{bc}{d}}\left(e^{\frac{2bc}{d}}\text{ExpIntegralEi}\left(-\frac{(b-d)(c+dx)}{d}\right)-e^{2a}\text{ExpIntegralEi}\left(\frac{(b-d)(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\text{ExpIntegralEi}\left(-\frac{(b+\cdots}{}\right.\right.}{4b}$$

input
```
Integrate[Cosh[a + b*x]*CoshIntegral[c + d*x],x]
```

output
```
(E^(-a - (b*c)/d)*(E^((2*b*c)/d)*ExpIntegralEi[-(((b - d)*(c + d*x))/d)] -
 E^(2*a)*ExpIntegralEi[((b - d)*(c + d*x))/d] + E^((2*b*c)/d)*ExpIntegralE
i[-(((b + d)*(c + d*x))/d)] - E^(2*a)*ExpIntegralEi[((b + d)*(c + d*x))/d]
 + 4*E^(a + (b*c)/d)*CoshIntegral[c + d*x]*Sinh[a + b*x]))/(4*b)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7095, 5995, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\text{Chi}(c+dx)\,dx$$
$$\Big\downarrow \ 7095$$
$$\frac{\sinh(a+bx)\text{Chi}(c+dx)}{b}-\frac{d\int \frac{\cosh(c+dx)\sinh(a+bx)}{c+dx}dx}{b}$$
$$\Big\downarrow \ 5995$$
$$\frac{\sinh(a+bx)\text{Chi}(c+dx)}{b}-\frac{d\int \left(\frac{\sinh(a-c+(b-d)x)}{2(c+dx)}+\frac{\sinh(a+c+(b+d)x)}{2(c+dx)}\right)dx}{b}$$
$$\Big\downarrow \ 2009$$

$$\frac{\sinh(a+bx)\text{Chi}(c+dx)}{b} - \frac{d\left(\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d} + \frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2d} + \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2d} + \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x\right.}{2d}\right.}{b}$$

input
```
Int[Cosh[a + b*x]*CoshIntegral[c + d*x],x]
```

output
```
(CoshIntegral[c + d*x]*Sinh[a + b*x])/b - (d*((CoshIntegral[(c*(b - d))/d
+ (b - d)*x]*Sinh[a - (b*c)/d])/(2*d) + (CoshIntegral[(c*(b + d))/d + (b +
 d)*x]*Sinh[a - (b*c)/d])/(2*d) + (Cosh[a - (b*c)/d]*SinhIntegral[(c*(b -
d))/d + (b - d)*x])/(2*d) + (Cosh[a - (b*c)/d]*SinhIntegral[(c*(b + d))/d
+ (b + d)*x])/(2*d)))/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5995
```
Int[Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p
, 0] && IGtQ[q, 0]
```

rule 7095
```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[Sinh[a + b*x]*(CoshIntegral[c + d*x]/b), x] - Simp[d/b   Int[Sinh[a +
 b*x]*(Cosh[c + d*x]/(c + d*x)), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [F]

$$\int \cosh(bx+a)\,\mathrm{Chi}(dx+c)\,dx$$

input
```
int(cosh(b*x+a)*Chi(d*x+c),x)
```

output
```
int(cosh(b*x+a)*Chi(d*x+c),x)
```

Fricas [F]

$$\int \cosh(a+bx)\mathrm{Chi}(c+dx)\,dx=\int \mathrm{Chi}(dx+c)\cosh(bx+a)\,dx$$

input
```
integrate(cosh(b*x+a)*Chi(d*x+c),x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*cosh_integral(d*x + c), x)
```

Sympy [F]

$$\int \cosh(a+bx)\mathrm{Chi}(c+dx)\,dx=\int \cosh(a+bx)\,\mathrm{Chi}(c+dx)\,dx$$

input
```
integrate(cosh(b*x+a)*Chi(d*x+c),x)
```

output
```
Integral(cosh(a + b*x)*Chi(c + d*x), x)
```

Maxima [F]

$$\int \cosh(a+bx)\text{Chi}(c+dx)\,dx = \int \text{Chi}(dx+c)\cosh(bx+a)\,dx$$

input
```
integrate(cosh(b*x+a)*Chi(d*x+c),x, algorithm="maxima")
```

output
```
integrate(Chi(d*x + c)*cosh(b*x + a), x)
```

Giac [F]

$$\int \cosh(a+bx)\text{Chi}(c+dx)\,dx = \int \text{Chi}(dx+c)\cosh(bx+a)\,dx$$

input
```
integrate(cosh(b*x+a)*Chi(d*x+c),x, algorithm="giac")
```

output
```
integrate(Chi(d*x + c)*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\text{Chi}(c+dx)\,dx = \int \text{coshint}(c+d\,x)\,\cosh(a+b\,x)\,dx$$

input
```
int(coshint(c + d*x)*cosh(a + b*x),x)
```

output
```
int(coshint(c + d*x)*cosh(a + b*x), x)
```

Reduce [F]

$$\int \cosh(a+bx)\mathrm{Chi}(c+dx)\,dx = \int \chi(dx+c)\cosh(bx+a)\,dx$$

input
```
int(cosh(b*x+a)*Chi(d*x+c),x)
```

output
```
int(chi(c + d*x)*cosh(a + b*x),x)
```

3.136 $\int \frac{\cosh(a+bx)\text{Chi}(c+dx)}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cosh(a+bx)\text{Chi}(c+dx)}{x} \, dx = \text{Int}\left(\frac{\cosh(a+bx)\text{Chi}(c+dx)}{x}, x\right)$$

output
```
Defer(Int)(cosh(b*x+a)*Chi(d*x+c)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Chi}(c+dx)}{x} \, dx = \int \frac{\cosh(a+bx)\text{Chi}(c+dx)}{x} \, dx$$

input
```
Integrate[(Cosh[a + b*x]*CoshIntegral[c + d*x])/x,x]
```

output
```
Integrate[(Cosh[a + b*x]*CoshIntegral[c + d*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)\text{Chi}(c+dx)}{x} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\cosh(a+bx)\text{Chi}(c+dx)}{x}dx$$

input
```
Int[(Cosh[a + b*x]*CoshIntegral[c + d*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(bx+a\right)\text{Chi}\left(dx+c\right)}{x}dx$$

input
```
int(cosh(b*x+a)*Chi(d*x+c)/x,x)
```

output
```
int(cosh(b*x+a)*Chi(d*x+c)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\mathrm{Chi}(c+dx)}{x}\,dx = \int \frac{\mathrm{Chi}(dx+c)\cosh(bx+a)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*Chi(d*x+c)/x,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*cosh_integral(d*x + c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\cosh(a+bx)\mathrm{Chi}(c+dx)}{x}\,dx = \int \frac{\cosh(a+bx)\,\mathrm{Chi}(c+dx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*Chi(d*x+c)/x,x)
```

output
```
Integral(cosh(a + b*x)*Chi(c + d*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\mathrm{Chi}(c+dx)}{x}\,dx = \int \frac{\mathrm{Chi}(dx+c)\cosh(bx+a)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*Chi(d*x+c)/x,x, algorithm="maxima")
```

output
```
integrate(Chi(d*x + c)*cosh(b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\mathrm{Chi}(c+dx)}{x}\,dx = \int \frac{\mathrm{Chi}(dx+c)\cosh(bx+a)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*Chi(d*x+c)/x,x, algorithm="giac")
```

output
```
integrate(Chi(d*x + c)*cosh(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\mathrm{Chi}(c+dx)}{x}\,dx = \int \frac{\mathrm{coshint}(c+d\,x)\,\cosh(a+b\,x)}{x}\,dx$$

input
```
int((coshint(c + d*x)*cosh(a + b*x))/x,x)
```

output
```
int((coshint(c + d*x)*cosh(a + b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\text{Chi}(c+dx)}{x}\,dx=\int \frac{\chi(dx+c)\cosh(bx+a)}{x}dx$$

input
```
int(cosh(b*x+a)*Chi(d*x+c)/x,x)
```

output
```
int((chi(c + d*x)*cosh(a + b*x))/x,x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational):  #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file