Computer Algebra Independent Integration Tests

Summer 2024

8-Special-functions/355-8.6

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**233**]. This is test number [355].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (233)	0.00 (0)
Mathematica	97.00 (226)	3.00 (7)
Maple	85.41 (199)	14.59 (34)
Fricas	79.40 (185)	20.60 (48)
Mupad	65.67 (153)	34.33 (80)
Sympy	54.94 (128)	45.06 (105)
Reduce	32.19 (75)	67.81 (158)
Maxima	31.76 (74)	68.24 (159)
Giac	21.89 (51)	78.11 (182)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

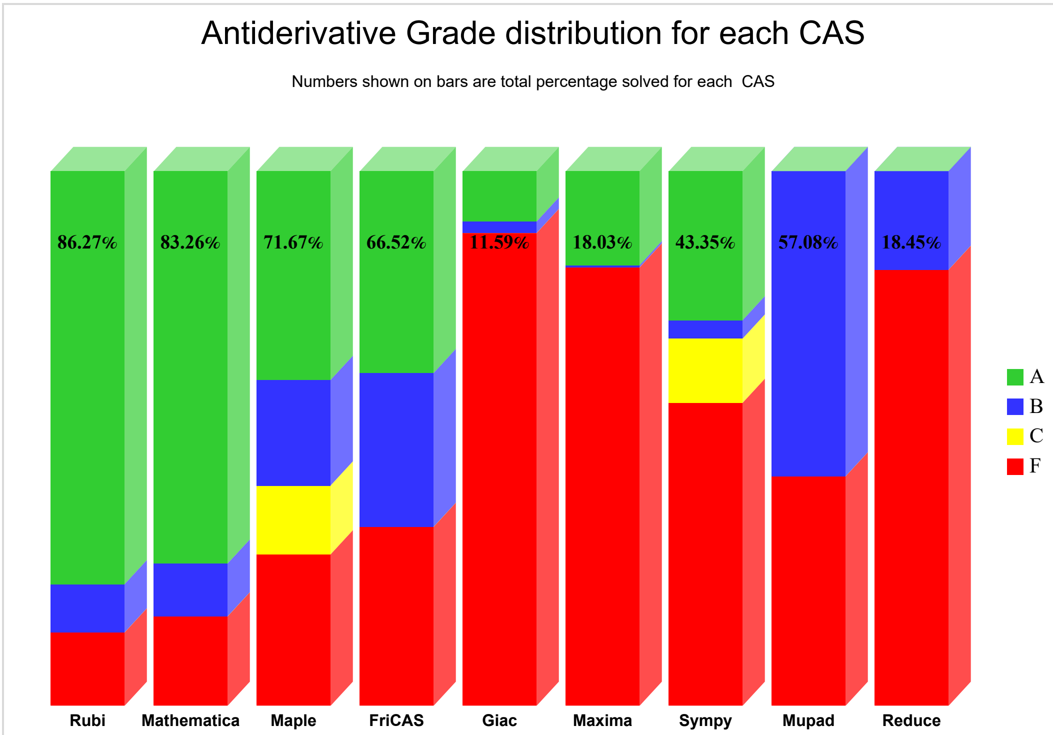
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

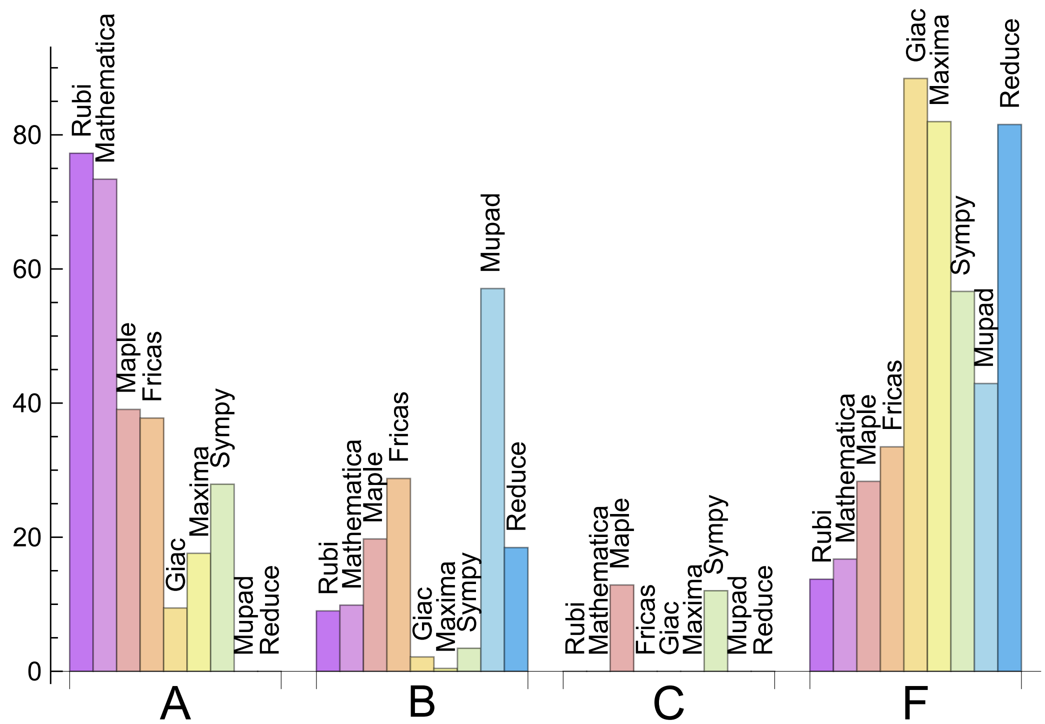
System	% A grade	% B grade	% C grade	% F grade
Rubi	77.253	9.013	0.000	13.734
Mathematica	73.391	9.871	0.000	16.738
Maple	39.056	19.742	12.876	28.326
Fricas	37.768	28.755	0.000	33.476
Sympy	27.897	3.433	12.017	56.652
Maxima	17.597	0.429	0.000	81.974
Giac	9.442	2.146	0.000	88.412
Mupad	0.000	57.082	0.000	42.918
Reduce	0.000	18.455	0.000	81.545

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	7	100.00	0.00	0.00
Maple	34	100.00	0.00	0.00
Fricas	48	62.50	0.00	37.50
Mupad	80	0.00	100.00	0.00
Sympy	105	63.81	34.29	1.90
Reduce	158	100.00	0.00	0.00
Maxima	159	100.00	0.00	0.00
Giac	182	92.86	0.00	7.14

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Giac	0.11
Fricas	0.14
Mathematica	0.20
Reduce	0.25
Rubi	0.37
Mupad	1.03
Maple	1.28
Sympy	3.82

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	35.31	1.35	17.00	1.00
Mathematica	70.31	1.23	29.50	1.00
Mupad	73.81	2.28	23.00	1.05
Rubi	80.39	1.22	29.00	1.00
Reduce	84.93	2.97	30.00	1.35
Giac	124.61	4.15	17.00	1.13
Maple	172.30	4.11	69.00	1.73
Sympy	202.42	4.68	54.00	1.58
Fricas	553.18	17.30	54.00	1.72

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

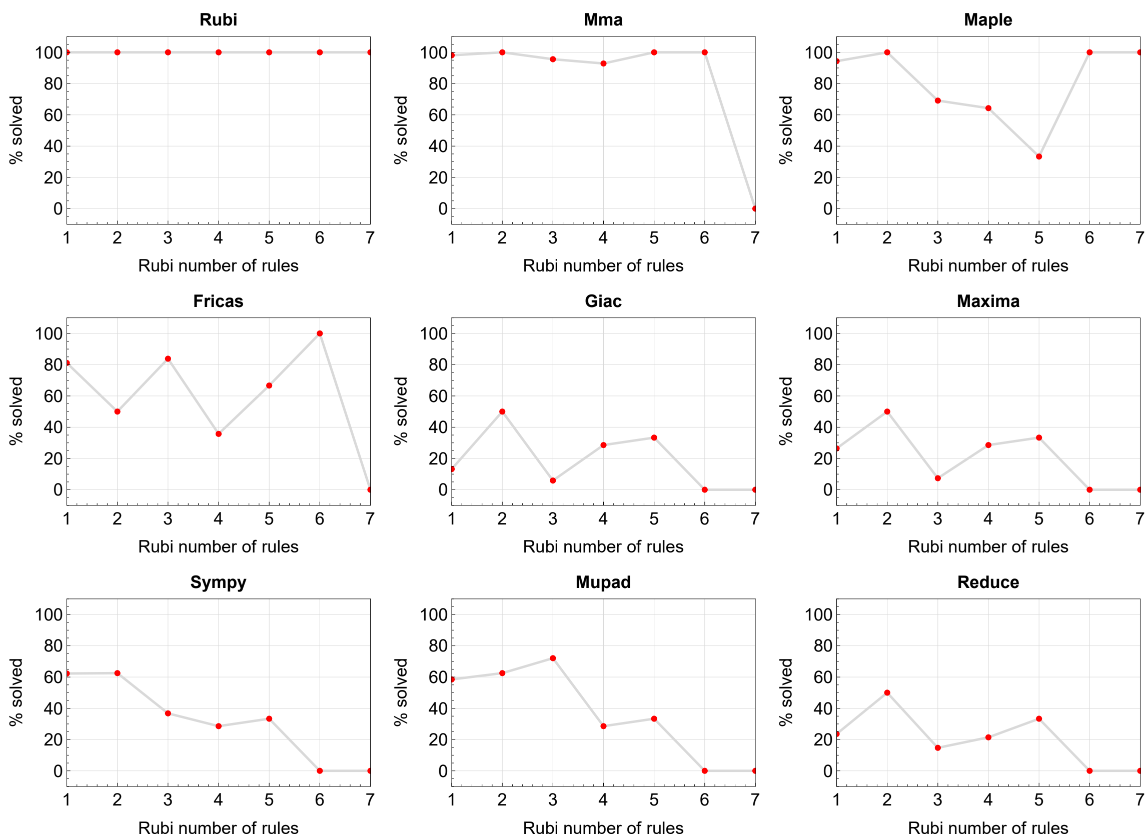


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

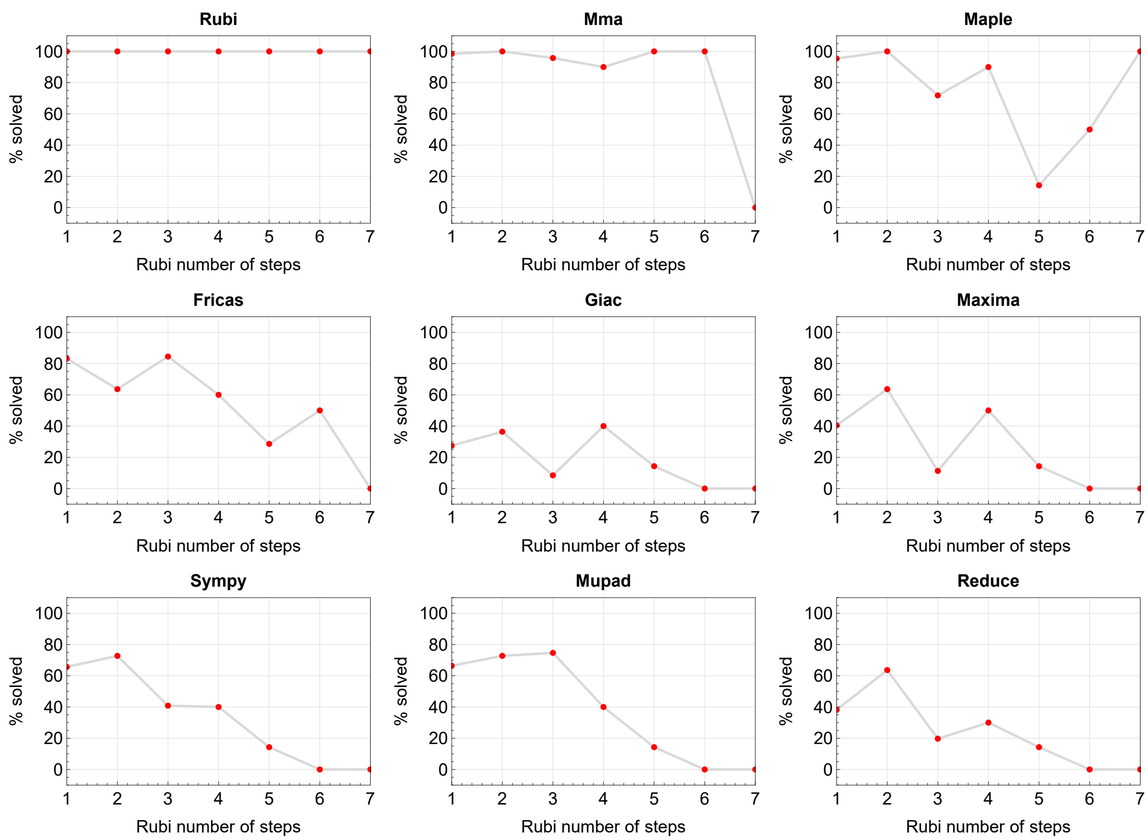


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

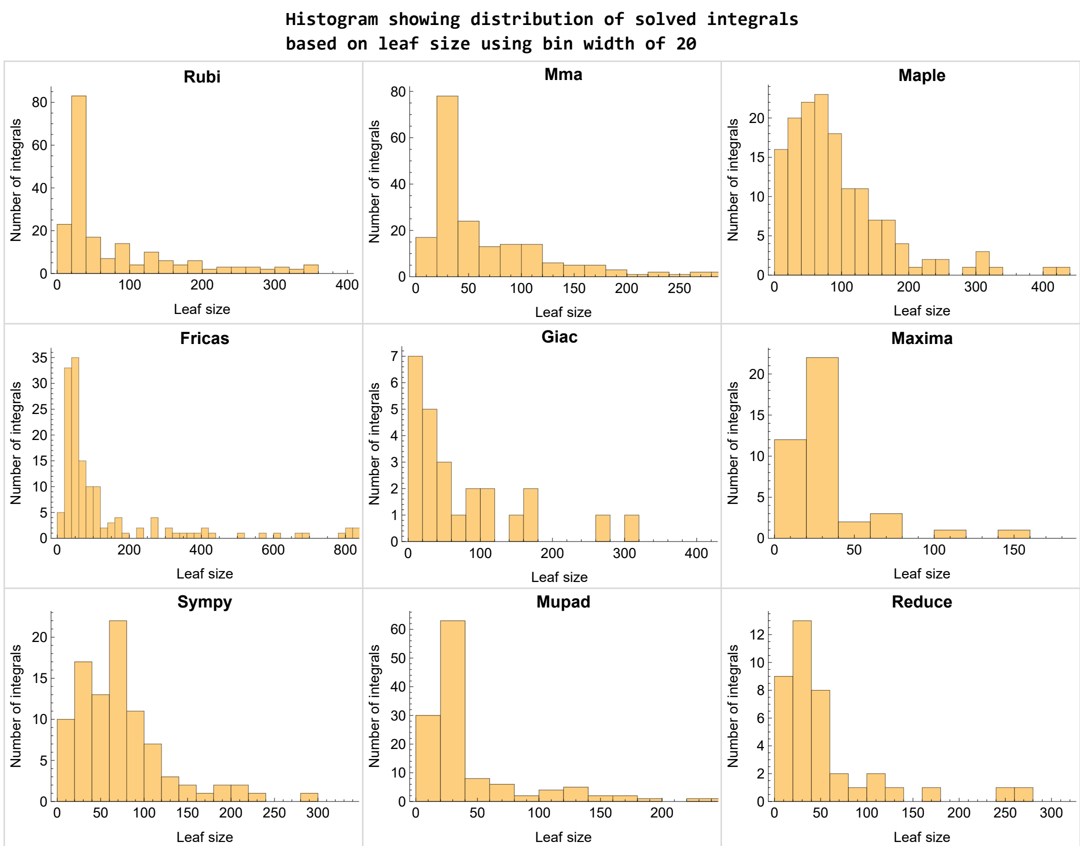


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

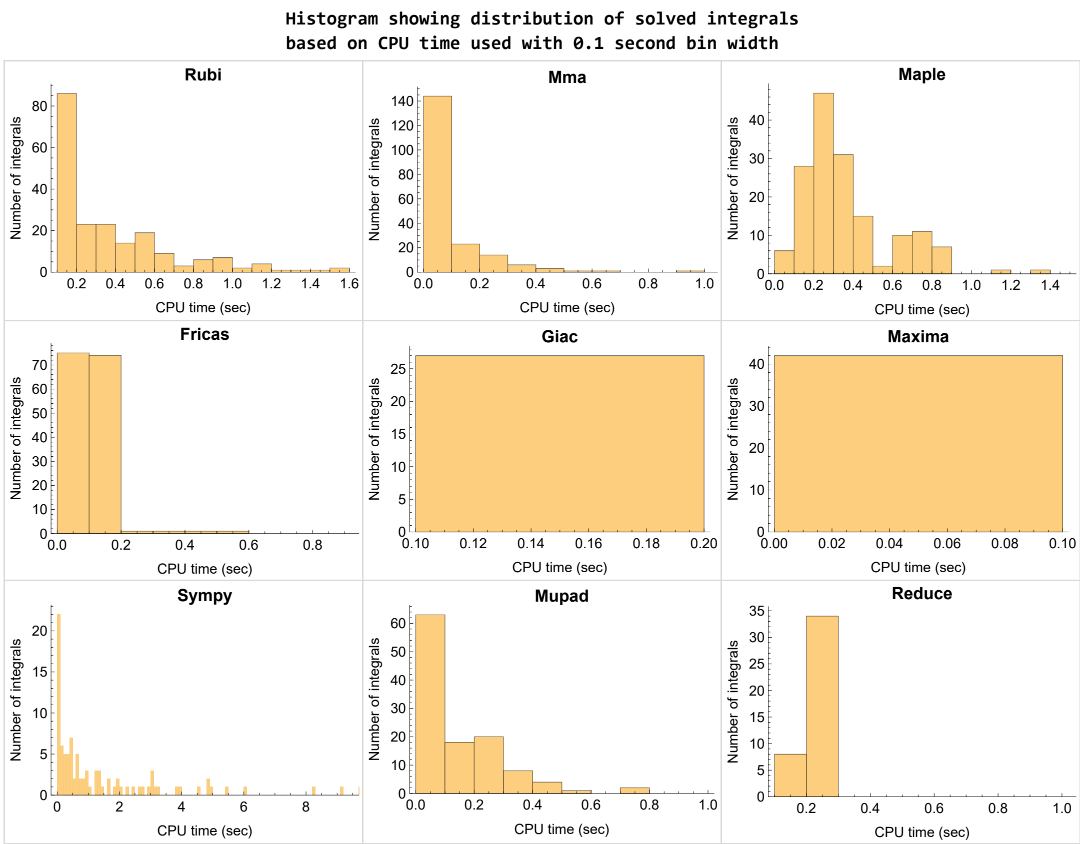


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

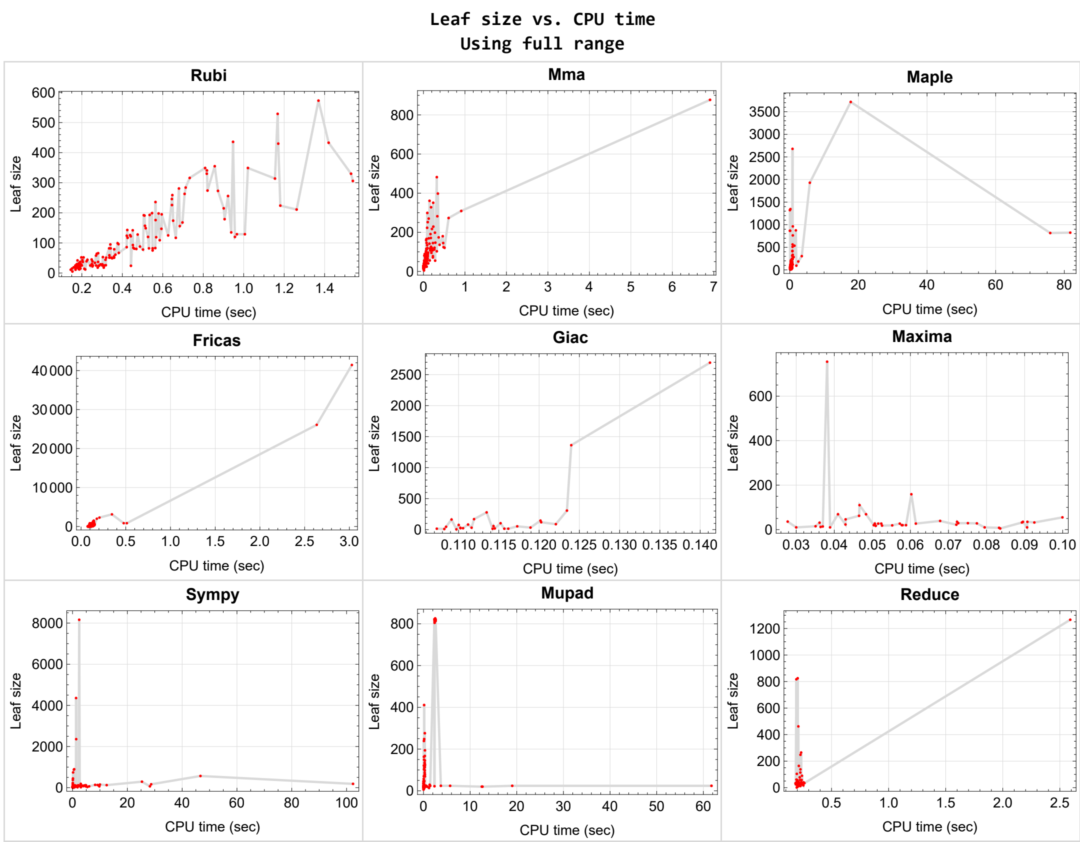


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{106, 141, 149, 157, 169, 170, 171, 172, 182, 183, 184, 185, 186, 187, 193, 194, 195, 207, 208, 209, 210, 211, 215, 216, 217, 222, 223, 224, 225, 226, 227, 228}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {106, 141, 149, 157, 169, 170, 171, 172, 182, 183, 184, 185}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple $\{25, 26, 27, 28, 29, 30, 31, 32, 128, 129, 130, 131\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

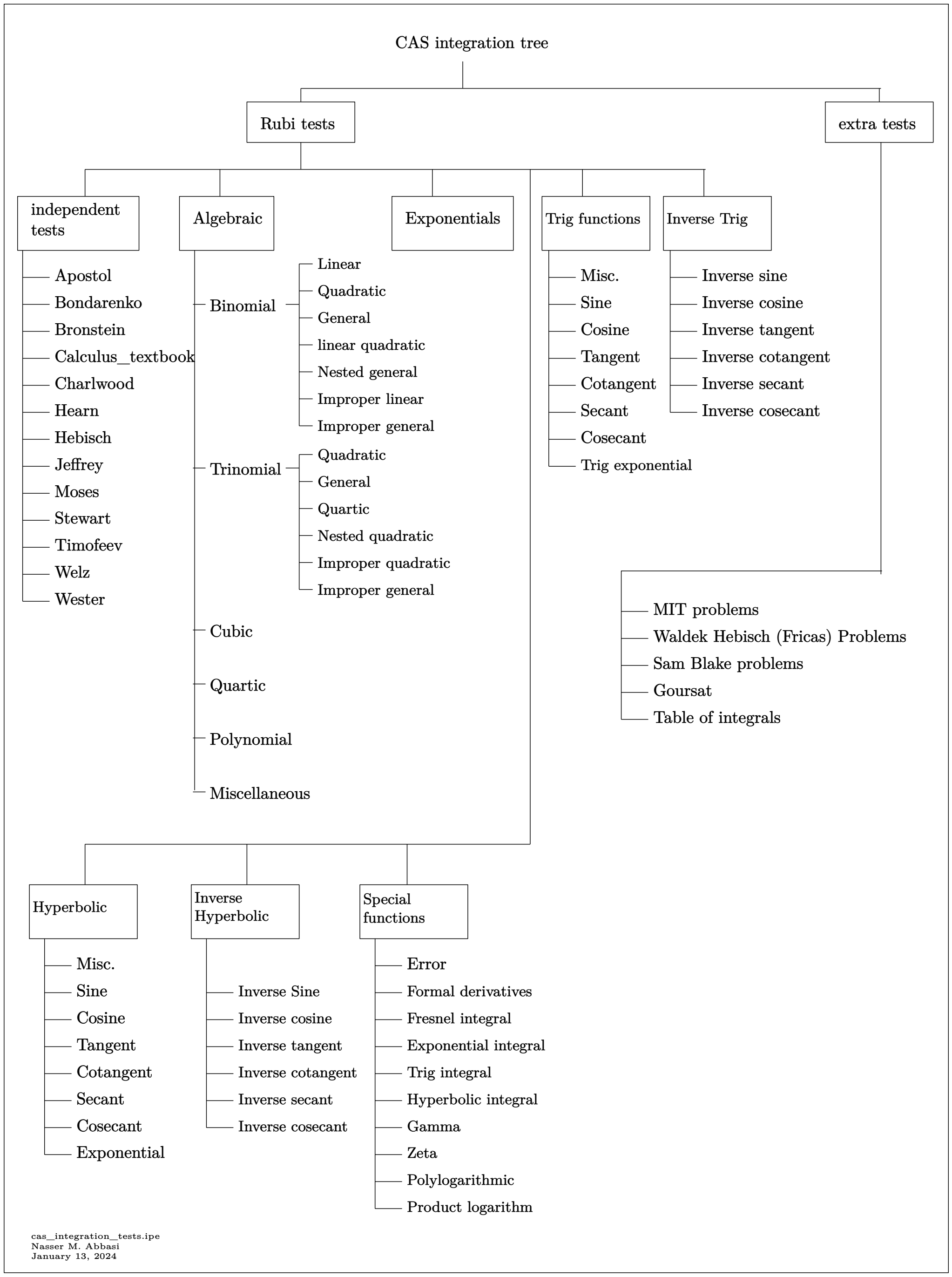
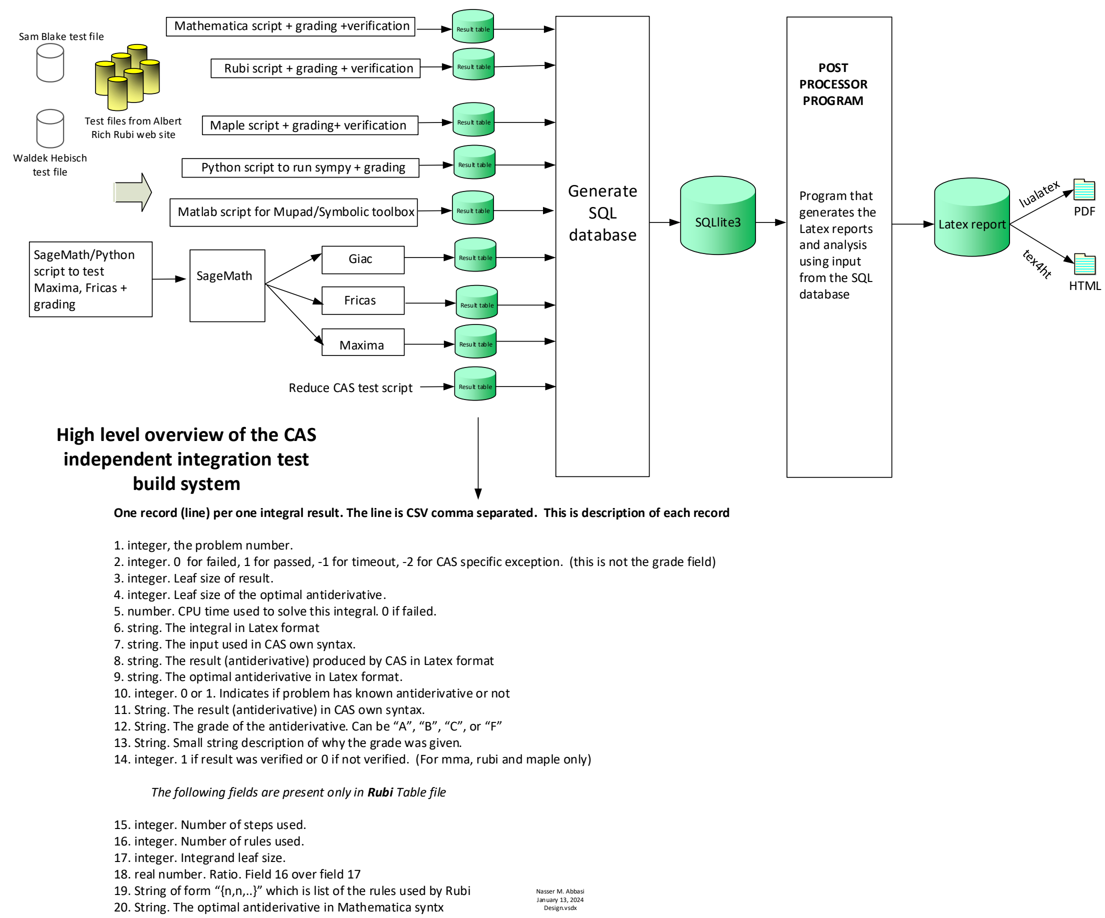


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 122, 123, 124, 125, 126, 131, 132, 133, 134, 135, 137, 138, 139, 140, 142, 143, 144, 145, 148, 150, 151, 152, 156, 165, 166, 167, 173, 174, 175, 176, 177, 178, 179, 180, 181, 188, 189, 190, 191, 192, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 212, 213, 214, 218, 219, 220, 221, 229, 230, 231, 232, 233 }

B grade { 119, 120, 121, 127, 128, 129, 130, 136, 146, 147, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 168 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 124, 125, 126, 127, 129, 130, 133, 134, 135, 138, 139, 140, 142, 143, 147, 150, 151, 158, 161, 162, 163, 164, 165, 166, 167, 168, 173, 174, 175, 176, 177, 178, 179, 180, 181, 188, 189, 190, 191, 192, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 212, 213, 214, 218, 219, 220, 221, 229, 232, 233 }

B grade { 5, 21, 28, 29, 38, 47, 55, 56, 63, 71, 119, 128, 131, 136, 137, 145, 146, 148, 153, 154, 155, 156, 159 }

C grade { }

F normal fail { 123, 132, 144, 152, 160, 230, 231 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 38, 47, 56, 61, 62, 63, 65, 66, 67, 69, 70, 71, 73, 74, 75, 90, 98, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 128, 129, 130, 131, 132, 161, 162, 163, 164, 165, 166, 167, 168, 203, 204, 212, 214, 218, 219, 220, 221, 229, 230, 231, 232, 233 }

B grade { 1, 8, 17, 25, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 68, 102, 124, 125, 126, 127, 133, 134, 135, 136, 188, 189, 190, 191, 192, 205, 206 }

C grade { 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 173, 174, 175, 176, 177, 178 }

F normal fail { 64, 72, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 179, 180, 181, 196, 197, 198, 199, 200, 201, 202, 213 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 9, 10, 11, 12, 13, 14, 15, 16, 20, 22, 23, 24, 30, 31, 32, 34, 35, 36, 37, 39, 40, 43, 44, 52, 61, 62, 63, 65, 66, 67, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 89, 97, 99, 110, 111, 112, 113, 114, 115, 116, 117, 118, 121, 122, 124, 125, 130, 133, 139, 140, 161, 162, 163, 164, 165, 166, 167, 168, 173, 174, 175, 176, 177, 178, 179, 180, 181, 191, 192, 199, 212, 213, 214 }

B grade { 17, 18, 19, 25, 26, 27, 28, 33, 41, 42, 45, 46, 48, 49, 50, 51, 53, 54, 55, 57, 58, 59, 60, 68, 86, 87, 88, 91, 92, 93, 94, 95, 96, 100, 101, 119, 120, 126, 127, 128, 129, 131, 134, 135, 136, 137, 138, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 188, 189, 190 }

C grade { }

F normal fail { 21, 29, 38, 47, 56, 64, 72, 90, 98, 123, 132, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 218, 219, 220, 221, 229, 230, 231, 232, 233 }

F(-1) timedout fail { }

F(-2) exception fail { 1, 2, 3, 4, 5, 6, 7, 8, 79, 102, 103, 104, 105, 106, 107, 108, 109, 182 }

Maxima

A grade { 4, 9, 10, 11, 12, 13, 14, 15, 16, 20, 28, 37, 46, 55, 61, 62, 63, 65, 66, 67, 78, 89, 97, 105, 110, 111, 112, 113, 114, 115, 116, 117, 118, 122, 131, 140, 148, 156, 181, 192, 199 }

B grade { 60 }

C grade { }

F normal fail { 1, 2, 3, 5, 6, 7, 8, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 64, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 138, 139, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 173, 174, 175, 176, 177, 178, 179, 180, 188, 189, 190, 191, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 212, 213, 214, 218, 219, 220, 221, 229, 230, 231, 232, 233 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 9, 10, 11, 12, 13, 14, 15, 16, 61, 62, 63, 69, 70, 71, 110, 111, 112, 113, 114, 115, 232, 233 }

B grade { 60, 68, 116, 117, 118 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 67, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 173, 174, 175, 176, 177, 178, 179, 180, 181, 188, 189, 190, 191, 192, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 212, 213, 214, 229, 230 }

F(-1) timedout fail { }

F(-2) exception fail { 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 231 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 63, 71, 79, 80, 81, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 119, 120, 121, 122, 128, 129, 130, 131, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 182, 183, 184, 185, 232, 233 }

C grade { }

F normal fail { }

F(-1) timedout fail { 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 115, 116, 117, 118, 123, 124, 125, 126, 127, 132, 133, 134, 135, 136, 179, 180, 181, 188, 189, 190, 191, 192, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 212, 213, 214, 218, 219, 220, 221, 229, 230, 231 }

F(-2) exception fail { }

Sympy

A grade { 2, 3, 4, 5, 6, 9, 10, 11, 12, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 38, 47, 56, 61, 62, 63, 65, 66, 67, 69, 70, 71, 73, 74, 75, 103, 104, 105, 111, 112, 113, 114, 119, 120, 121, 122, 129, 130, 131, 162, 163, 164, 165, 166, 167, 174, 175, 176, 177, 178 }

B grade { 7, 8, 76, 77, 78, 102, 110, 128 }

C grade { 13, 14, 15, 16, 34, 35, 36, 37, 39, 40, 41, 43, 44, 45, 46, 48, 49, 50, 52, 53, 54, 55, 57, 58, 59, 79, 80, 81 }

F normal fail { 64, 72, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 115, 116, 117, 118, 123, 124, 132, 133, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 191, 192, 203, 204, 205, 206, 212, 213, 214, 218, 219, 220, 221, 229, 230, 231, 232, 233 }

F(-1) timedout fail { 1, 33, 42, 51, 60, 68, 86, 94, 107, 108, 109, 125, 126, 127, 134, 135, 136, 161, 168, 173, 179, 182, 186, 187, 188, 189, 190, 194, 195, 196, 197, 198, 199, 200, 201, 202 }

F(-2) exception fail { 180, 181 }

Reduce

A grade { }

B grade { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 60, 61, 62, 63, 65, 110, 111, 112, 113, 114, 119, 120, 121, 122, 128, 129, 130, 131, 233 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 115, 116, 117, 118, 123, 124, 125, 126, 127, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 173, 174, 175, 176, 177, 178, 179, 180, 181, 188, 189, 190, 191, 192, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 212, 213, 214, 218, 219, 220, 221, 229, 230, 231, 232 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-2)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	1322	0	0	0	0	11	826
N.S.	1	1.00	1.00	52.88	0.00	0.00	0.00	0.00	0.44	33.04
time (sec)	N/A	0.176	0.003	0.026	0.000	0.000	0.000	0.000	0.283	2.509

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	46	0	0	54	0	11	44
N.S.	1	1.00	1.00	1.84	0.00	0.00	2.16	0.00	0.44	1.76
time (sec)	N/A	0.178	0.002	0.720	0.000	0.000	0.306	0.000	0.188	0.043

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	34	0	0	39	0	9	35
N.S.	1	1.00	1.00	1.36	0.00	0.00	1.56	0.00	0.36	1.40
time (sec)	N/A	0.173	0.002	0.429	0.000	0.000	0.228	0.000	0.201	0.017

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	10	0	19	0	7	17
N.S.	1	1.00	1.00	1.00	0.53	0.00	1.00	0.00	0.37	0.89
time (sec)	N/A	0.160	0.007	0.289	0.030	0.000	0.169	0.000	0.212	0.047

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	66	56	0	0	24	0	11	10
N.S.	1	1.00	2.06	1.75	0.00	0.00	0.75	0.00	0.34	0.31
time (sec)	N/A	0.186	0.091	0.223	0.000	0.000	0.417	0.000	0.203	0.096

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	27	0	0	20	0	11	26
N.S.	1	1.00	1.00	1.50	0.00	0.00	1.11	0.00	0.61	1.44
time (sec)	N/A	0.186	0.003	0.638	0.000	0.000	0.187	0.000	0.186	0.019

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	40	0	0	41	0	11	17
N.S.	1	1.00	1.00	1.60	0.00	0.00	1.64	0.00	0.44	0.68
time (sec)	N/A	0.184	0.003	0.647	0.000	0.000	0.246	0.000	0.216	0.018

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	54	0	0	54	0	11	17
N.S.	1	1.00	1.00	2.16	0.00	0.00	2.16	0.00	0.44	0.68
time (sec)	N/A	0.187	0.003	0.695	0.000	0.000	0.303	0.000	0.252	0.016

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	64	32	32	31	31	37	31	33	31
N.S.	1	1.23	0.62	0.62	0.60	0.60	0.71	0.60	0.63	0.60
time (sec)	N/A	0.274	0.018	0.149	0.036	0.101	0.052	0.112	0.208	0.043

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	44	24	24	23	23	29	23	25	23
N.S.	1	1.16	0.63	0.63	0.61	0.61	0.76	0.61	0.66	0.61
time (sec)	N/A	0.227	0.017	0.111	0.043	0.095	0.046	0.110	0.204	0.017

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	16	16	15	15	20	15	17	15
N.S.	1	1.00	0.67	0.67	0.62	0.62	0.83	0.62	0.71	0.62
time (sec)	N/A	0.176	0.014	0.116	0.035	0.094	0.041	0.107	0.201	0.039

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	10	10	10	10	12	10
N.S.	1	1.00	1.00	1.00	0.91	0.91	0.91	0.91	1.09	0.91
time (sec)	N/A	0.145	0.008	0.094	0.039	0.080	0.036	0.108	0.228	0.038

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	8	5	5	8	5	5	6
N.S.	1	1.00	1.00	1.60	1.00	1.00	1.60	1.00	1.00	1.20
time (sec)	N/A	0.153	0.012	0.216	0.084	0.078	0.378	0.110	0.200	0.013

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	8	19	20	19	27	17
N.S.	1	1.00	1.00	0.95	0.40	0.95	1.00	0.95	1.35	0.85
time (sec)	N/A	0.186	0.025	0.271	0.083	0.074	0.573	0.114	0.210	0.046

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	39	36	33	10	29	36	33	34	36
N.S.	1	0.98	0.90	0.82	0.25	0.72	0.90	0.82	0.85	0.90
time (sec)	N/A	0.223	0.020	0.267	0.079	0.091	0.851	0.119	0.263	0.042

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	58	43	46	10	37	49	46	43	45
N.S.	1	1.04	0.77	0.82	0.18	0.66	0.88	0.82	0.77	0.80
time (sec)	N/A	0.270	0.021	0.282	0.091	0.081	1.223	0.108	0.208	0.041

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	816	0	838	877	0	817	814
N.S.	1	1.00	1.00	32.64	0.00	33.52	35.08	0.00	32.68	32.56
time (sec)	N/A	0.177	0.003	75.899	0.000	0.148	0.497	0.000	0.192	2.606

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	0	54	44	0	33	31
N.S.	1	1.00	1.00	1.28	0.00	2.16	1.76	0.00	1.32	1.24
time (sec)	N/A	0.187	0.003	0.211	0.000	0.082	0.052	0.000	0.184	0.044

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	0	46	36	0	25	23
N.S.	1	1.00	1.00	0.96	0.00	1.84	1.44	0.00	1.00	0.92
time (sec)	N/A	0.179	0.002	0.152	0.000	0.105	0.059	0.000	0.246	0.059

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	30	16	20	33	20	0	17	15
N.S.	1	1.00	1.67	0.89	1.11	1.83	1.11	0.00	0.94	0.83
time (sec)	N/A	0.175	0.011	0.145	0.059	0.081	0.044	0.000	0.220	0.032

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	41	16	0	0	19	0	21	13
N.S.	1	1.00	2.93	1.14	0.00	0.00	1.36	0.00	1.50	0.93
time (sec)	N/A	0.212	0.014	0.214	0.000	0.000	1.827	0.000	0.210	0.039

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	11	0	20	8	0	12	10
N.S.	1	1.00	1.00	0.58	0.00	1.05	0.42	0.00	0.63	0.53
time (sec)	N/A	0.179	0.005	0.261	0.000	0.102	0.045	0.000	0.213	0.048

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	33	0	23	17	0	36	33
N.S.	1	1.00	1.00	1.32	0.00	0.92	0.68	0.00	1.44	1.32
time (sec)	N/A	0.184	0.003	0.288	0.000	0.094	0.399	0.000	0.227	0.066

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	46	0	37	19	0	42	46
N.S.	1	1.00	1.00	1.84	0.00	1.48	0.76	0.00	1.68	1.84
time (sec)	N/A	0.254	0.003	0.306	0.000	0.119	0.413	0.000	0.194	0.066

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	824	0	846	896	0	825	821
N.S.	1	1.00	1.00	32.96	0.00	33.84	35.84	0.00	33.00	32.84
time (sec)	N/A	0.270	0.003	81.734	0.000	0.098	0.607	0.000	0.203	2.606

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	40	0	62	63	0	41	39
N.S.	1	1.00	1.00	1.60	0.00	2.48	2.52	0.00	1.64	1.56
time (sec)	N/A	0.301	0.003	0.232	0.000	0.083	0.063	0.000	0.236	0.047

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	0	54	53	0	33	31
N.S.	1	1.00	1.00	1.28	0.00	2.16	2.12	0.00	1.32	1.24
time (sec)	N/A	0.286	0.002	0.195	0.000	0.085	0.052	0.000	0.186	0.073

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	38	24	20	41	37	0	25	23
N.S.	1	1.00	2.11	1.33	1.11	2.28	2.06	0.00	1.39	1.28
time (sec)	N/A	0.278	0.011	0.218	0.050	0.077	0.071	0.000	0.187	0.037

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	23	24	56	25	0	0	22	0	26	24
N.S.	1	1.04	2.43	1.09	0.00	0.00	0.96	0.00	1.13	1.04
time (sec)	N/A	0.443	0.018	0.231	0.000	0.000	3.045	0.000	0.213	0.054

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	0	29	12	0	17	15
N.S.	1	1.00	1.00	0.89	0.00	1.61	0.67	0.00	0.94	0.83
time (sec)	N/A	0.308	0.003	0.274	0.000	0.082	0.060	0.000	0.253	0.036

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	16	0	25	14	0	17	15
N.S.	1	1.00	1.00	0.62	0.00	0.96	0.54	0.00	0.65	0.58
time (sec)	N/A	0.318	0.007	0.346	0.000	0.073	0.063	0.000	0.201	0.053

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	46	0	23	32	0	44	46
N.S.	1	1.00	1.00	1.84	0.00	0.92	1.28	0.00	1.76	1.84
time (sec)	N/A	0.311	0.003	0.358	0.000	0.086	0.628	0.000	0.209	0.044

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	866	0	814	0	0	15	822
N.S.	1	1.00	1.00	34.64	0.00	32.56	0.00	0.00	0.60	32.88
time (sec)	N/A	0.200	0.003	0.068	0.000	0.111	0.000	0.000	0.241	2.341

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	84	0	38	73	0	15	46
N.S.	1	1.00	1.00	3.36	0.00	1.52	2.92	0.00	0.60	1.84
time (sec)	N/A	0.242	0.003	0.221	0.000	0.085	2.982	0.000	0.208	0.019

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	76	0	30	60	0	13	37
N.S.	1	1.00	1.00	3.04	0.00	1.20	2.40	0.00	0.52	1.48
time (sec)	N/A	0.189	0.002	0.223	0.000	0.077	1.678	0.000	0.183	0.017

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	68	0	25	46	0	11	22
N.S.	1	1.00	1.00	2.62	0.00	0.96	1.77	0.00	0.42	0.85
time (sec)	N/A	0.178	0.006	0.259	0.000	0.101	0.982	0.000	0.189	0.014

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	55	18	32	37	0	15	26
N.S.	1	1.00	0.94	3.06	1.00	1.78	2.06	0.00	0.83	1.44
time (sec)	N/A	0.171	0.005	0.252	0.053	0.085	0.855	0.000	0.255	0.019

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	103	77	0	0	42	0	15	14
N.S.	1	1.00	2.64	1.97	0.00	0.00	1.08	0.00	0.38	0.36
time (sec)	N/A	0.252	0.322	0.259	0.000	0.000	0.476	0.000	0.183	0.135

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	106	0	34	58	0	15	21
N.S.	1	1.00	1.00	5.89	0.00	1.89	3.22	0.00	0.83	1.17
time (sec)	N/A	0.186	0.003	0.280	0.000	0.085	0.906	0.000	0.183	0.018

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	141	0	39	78	0	15	21
N.S.	1	1.00	1.00	5.64	0.00	1.56	3.12	0.00	0.60	0.84
time (sec)	N/A	0.188	0.003	0.305	0.000	0.090	1.692	0.000	0.198	0.014

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	165	0	47	92	0	15	21
N.S.	1	1.00	1.00	6.60	0.00	1.88	3.68	0.00	0.60	0.84
time (sec)	N/A	0.183	0.003	0.311	0.000	0.081	3.099	0.000	0.221	0.016

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	866	0	806	0	0	15	814
N.S.	1	1.00	1.00	34.64	0.00	32.24	0.00	0.00	0.60	32.56
time (sec)	N/A	0.184	0.003	0.064	0.000	0.120	0.000	0.000	0.190	2.371

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	84	0	30	76	0	13	37
N.S.	1	1.00	1.00	3.36	0.00	1.20	3.04	0.00	0.52	1.48
time (sec)	N/A	0.184	0.003	0.224	0.000	0.087	3.238	0.000	0.199	0.016

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	76	0	25	63	0	11	22
N.S.	1	1.00	1.00	2.92	0.00	0.96	2.42	0.00	0.42	0.85
time (sec)	N/A	0.185	0.006	0.217	0.000	0.096	1.932	0.000	0.234	0.014

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	82	0	43	63	0	15	17
N.S.	1	1.00	1.00	3.28	0.00	1.72	2.52	0.00	0.60	0.68
time (sec)	N/A	0.179	0.002	0.253	0.000	0.103	1.338	0.000	0.207	0.018

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	32	98	20	37	60	0	15	21
N.S.	1	1.00	1.78	5.44	1.11	2.06	3.33	0.00	0.83	1.17
time (sec)	N/A	0.174	0.013	0.247	0.058	0.104	0.957	0.000	0.193	0.018

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	51	121	85	0	0	61	0	15	14
N.S.	1	0.93	2.20	1.55	0.00	0.00	1.11	0.00	0.27	0.25
time (sec)	N/A	0.359	0.507	0.243	0.000	0.000	0.648	0.000	0.186	1.184

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	135	0	38	80	0	15	21
N.S.	1	1.00	1.00	7.50	0.00	2.11	4.44	0.00	0.83	1.17
time (sec)	N/A	0.180	0.003	0.286	0.000	0.081	1.357	0.000	0.255	0.018

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	165	0	43	99	0	15	21
N.S.	1	1.00	1.00	6.60	0.00	1.72	3.96	0.00	0.60	0.84
time (sec)	N/A	0.182	0.003	0.300	0.000	0.091	2.628	0.000	0.190	0.013

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	189	0	51	112	0	15	21
N.S.	1	1.00	1.00	7.56	0.00	2.04	4.48	0.00	0.60	0.84
time (sec)	N/A	0.185	0.003	0.295	0.000	0.089	4.936	0.000	0.197	0.014

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	866	0	798	0	0	15	805
N.S.	1	1.00	1.00	34.64	0.00	31.92	0.00	0.00	0.60	32.20
time (sec)	N/A	0.175	0.003	0.073	0.000	0.102	0.000	0.000	0.229	2.387

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	84	0	25	76	0	11	22
N.S.	1	1.00	1.00	3.23	0.00	0.96	2.92	0.00	0.42	0.85
time (sec)	N/A	0.179	0.006	0.263	0.000	0.098	3.956	0.000	0.226	0.019

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	90	0	51	76	0	15	17
N.S.	1	1.00	1.00	3.60	0.00	2.04	3.04	0.00	0.60	0.68
time (sec)	N/A	0.178	0.003	0.271	0.000	0.086	2.744	0.000	0.196	0.016

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	131	0	43	76	0	15	21
N.S.	1	1.00	1.00	5.24	0.00	1.72	3.04	0.00	0.60	0.84
time (sec)	N/A	0.168	0.002	0.309	0.000	0.081	1.946	0.000	0.198	0.013

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	40	127	20	37	76	0	15	21
N.S.	1	1.00	2.22	7.06	1.11	2.06	4.22	0.00	0.83	1.17
time (sec)	N/A	0.166	0.018	0.271	0.058	0.086	1.461	0.000	0.225	0.054

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	67	145	94	0	0	71	0	15	14
N.S.	1	1.05	2.27	1.47	0.00	0.00	1.11	0.00	0.23	0.22
time (sec)	N/A	0.383	0.475	0.270	0.000	0.000	1.218	0.000	0.193	0.137

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	159	0	38	94	0	15	21
N.S.	1	1.00	1.00	8.83	0.00	2.11	5.22	0.00	0.83	1.17
time (sec)	N/A	0.180	0.003	0.305	0.000	0.079	2.248	0.000	0.184	0.019

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	189	0	43	114	0	15	21
N.S.	1	1.00	1.00	7.56	0.00	1.72	4.56	0.00	0.60	0.84
time (sec)	N/A	0.191	0.002	0.317	0.000	0.093	4.508	0.000	0.185	0.014

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	213	0	51	128	0	15	21
N.S.	1	1.00	1.00	8.52	0.00	2.04	5.12	0.00	0.60	0.84
time (sec)	N/A	0.187	0.003	0.339	0.000	0.100	9.173	0.000	0.243	0.016

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	1346	755	848	0	1362	1266	0
N.S.	1	1.00	1.00	46.41	26.03	29.24	0.00	46.97	43.66	0.00
time (sec)	N/A	0.185	0.003	0.187	0.038	0.512	0.000	0.124	2.591	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	72	69	64	110	88	90	0
N.S.	1	1.00	1.00	2.48	2.38	2.21	3.79	3.03	3.10	0.00
time (sec)	N/A	0.185	0.003	0.127	0.041	0.116	0.614	0.122	0.241	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	58	62	56	83	75	78	0
N.S.	1	1.00	1.00	2.00	2.14	1.93	2.86	2.59	2.69	0.00
time (sec)	N/A	0.180	0.003	0.121	0.047	0.116	0.379	0.110	0.221	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	50	43	46	42	46	53	61	47
N.S.	1	1.00	2.27	1.95	2.09	1.91	2.09	2.41	2.77	2.14
time (sec)	N/A	0.178	0.041	0.104	0.043	0.117	0.242	0.117	0.192	0.176

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	56	0	0	0	0	0	20	0
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.192	0.057	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	44	29	38	46	0	58	0
N.S.	1	1.00	1.00	2.00	1.32	1.73	2.09	0.00	2.64	0.00
time (sec)	N/A	0.191	0.003	0.132	0.075	0.107	0.274	0.000	0.238	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	59	32	49	78	0	71	0
N.S.	1	1.00	1.00	2.03	1.10	1.69	2.69	0.00	2.45	0.00
time (sec)	N/A	0.191	0.003	0.150	0.089	0.112	0.429	0.000	0.207	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	72	32	57	107	0	90	0
N.S.	1	1.00	1.00	2.48	1.10	1.97	3.69	0.00	3.10	0.00
time (sec)	N/A	0.195	0.003	0.131	0.093	0.124	0.687	0.000	0.207	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	2679	0	848	0	2693	0	0
N.S.	1	1.00	1.00	92.38	0.00	29.24	0.00	92.86	0.00	0.00
time (sec)	N/A	0.191	0.003	0.796	0.000	0.477	0.000	0.141	2.700	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	71	0	64	107	145	114	0
N.S.	1	1.00	1.00	2.45	0.00	2.21	3.69	5.00	3.93	0.00
time (sec)	N/A	0.186	0.002	0.325	0.000	0.122	1.027	0.120	0.204	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	59	0	56	85	119	102	0
N.S.	1	1.00	1.00	2.03	0.00	1.93	2.93	4.10	3.52	0.00
time (sec)	N/A	0.174	0.003	0.244	0.000	0.121	0.598	0.120	0.185	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	54	43	0	42	83	82	85	77
N.S.	1	1.00	2.45	1.95	0.00	1.91	3.77	3.73	3.86	3.50
time (sec)	N/A	0.176	0.071	0.117	0.000	0.110	0.758	0.111	0.231	0.272

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	78	0	0	0	0	0	29	0
N.S.	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.179	0.227	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	50	0	37	44	0	84	0
N.S.	1	1.00	1.00	2.27	0.00	1.68	2.00	0.00	3.82	0.00
time (sec)	N/A	0.252	0.002	0.218	0.000	0.110	0.276	0.000	0.234	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	70	0	49	66	0	79	0
N.S.	1	1.00	1.00	2.41	0.00	1.69	2.28	0.00	2.72	0.00
time (sec)	N/A	0.293	0.003	0.293	0.000	0.114	0.429	0.000	0.217	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	86	0	57	95	0	90	0
N.S.	1	1.00	1.00	2.97	0.00	1.97	3.28	0.00	3.10	0.00
time (sec)	N/A	0.301	0.003	0.368	0.000	0.107	0.751	0.000	0.206	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	228	0	100	185	0	135	0
N.S.	1	1.00	0.76	4.47	0.00	1.96	3.63	0.00	2.65	0.00
time (sec)	N/A	0.345	0.014	0.721	0.000	0.096	3.008	0.000	0.196	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	133	0	74	116	0	82	0
N.S.	1	1.00	0.76	2.61	0.00	1.45	2.27	0.00	1.61	0.00
time (sec)	N/A	0.358	0.013	0.444	0.000	0.099	2.058	0.000	0.188	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	39	19	21	54	0	43	0
N.S.	1	1.00	1.00	1.62	0.79	0.88	2.25	0.00	1.79	0.00
time (sec)	N/A	0.283	0.013	0.339	0.055	0.088	0.430	0.000	0.246	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	86	0	0	2358	0	15	12
N.S.	1	1.00	0.76	1.69	0.00	0.00	46.24	0.00	0.29	0.24
time (sec)	N/A	0.336	0.013	0.149	0.000	0.000	1.336	0.000	0.215	0.054

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	37	97	0	52	4357	0	21	19
N.S.	1	1.00	0.76	1.98	0.00	1.06	88.92	0.00	0.43	0.39
time (sec)	N/A	0.350	0.013	0.139	0.000	0.095	1.286	0.000	0.217	0.079

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	116	0	82	8162	0	21	19
N.S.	1	1.00	0.76	2.27	0.00	1.61	160.04	0.00	0.41	0.37
time (sec)	N/A	0.320	0.012	0.162	0.000	0.117	2.428	0.000	0.212	0.124

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	24	61	0	40	0	0	9	0
N.S.	1	1.00	0.86	2.18	0.00	1.43	0.00	0.00	0.32	0.00
time (sec)	N/A	0.197	0.028	0.165	0.000	0.093	0.000	0.000	0.194	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	38	81	0	62	0	0	11	0
N.S.	1	1.00	0.84	1.80	0.00	1.38	0.00	0.00	0.24	0.00
time (sec)	N/A	0.248	0.011	0.155	0.000	0.103	0.000	0.000	0.213	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	34	72	0	56	0	0	13	0
N.S.	1	1.00	0.79	1.67	0.00	1.30	0.00	0.00	0.30	0.00
time (sec)	N/A	0.203	0.028	0.135	0.000	0.095	0.000	0.000	0.209	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	40	91	0	83	0	0	15	0
N.S.	1	1.00	0.77	1.75	0.00	1.60	0.00	0.00	0.29	0.00
time (sec)	N/A	0.208	0.010	0.142	0.000	0.099	0.000	0.000	0.237	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	64	0	41441	0	0	11	0
N.S.	1	1.00	1.00	2.37	0.00	1534.85	0.00	0.00	0.41	0.00
time (sec)	N/A	0.191	0.003	0.109	0.000	3.028	0.000	0.000	0.212	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	64	0	85	0	0	11	0
N.S.	1	1.00	1.00	2.37	0.00	3.15	0.00	0.00	0.41	0.00
time (sec)	N/A	0.190	0.003	0.294	0.000	0.095	0.000	0.000	0.197	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	64	0	57	0	0	9	0
N.S.	1	1.00	1.00	2.37	0.00	2.11	0.00	0.00	0.33	0.00
time (sec)	N/A	0.180	0.002	0.244	0.000	0.100	0.000	0.000	0.210	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	106	22	35	0	0	7	0
N.S.	1	1.00	1.00	5.30	1.10	1.75	0.00	0.00	0.35	0.00
time (sec)	N/A	0.181	0.021	0.242	0.072	0.115	0.000	0.000	0.227	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	41	61	0	0	0	0	11	0
N.S.	1	1.00	1.32	1.97	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.191	0.084	0.143	0.000	0.000	0.000	0.000	0.199	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	148	0	41	0	0	11	0
N.S.	1	1.00	1.00	7.40	0.00	2.05	0.00	0.00	0.55	0.00
time (sec)	N/A	0.179	0.003	0.319	0.000	0.117	0.000	0.000	0.203	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	79	0	70	0	0	11	0
N.S.	1	1.00	1.00	2.93	0.00	2.59	0.00	0.00	0.41	0.00
time (sec)	N/A	0.184	0.003	0.398	0.000	0.111	0.000	0.000	0.256	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	79	0	101	0	0	11	0
N.S.	1	1.00	1.00	2.93	0.00	3.74	0.00	0.00	0.41	0.00
time (sec)	N/A	0.185	0.003	0.634	0.000	0.114	0.000	0.000	0.208	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	54	0	26081	0	0	11	0
N.S.	1	1.00	1.00	2.25	0.00	1086.71	0.00	0.00	0.46	0.00
time (sec)	N/A	0.188	0.003	0.092	0.000	2.635	0.000	0.000	0.189	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	54	0	62	0	0	11	0
N.S.	1	1.00	1.00	2.25	0.00	2.58	0.00	0.00	0.46	0.00
time (sec)	N/A	0.179	0.003	0.304	0.000	0.094	0.000	0.000	0.228	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	54	0	42	0	0	9	0
N.S.	1	1.00	1.00	2.25	0.00	1.75	0.00	0.00	0.38	0.00
time (sec)	N/A	0.176	0.002	0.280	0.000	0.092	0.000	0.000	0.222	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	97	17	27	0	0	7	0
N.S.	1	1.00	1.00	5.11	0.89	1.42	0.00	0.00	0.37	0.00
time (sec)	N/A	0.165	0.008	0.250	0.051	0.112	0.000	0.000	0.199	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	43	60	0	0	0	0	11	0
N.S.	1	1.00	1.39	1.94	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.183	0.069	0.134	0.000	0.000	0.000	0.000	0.200	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	133	0	37	0	0	11	0
N.S.	1	1.00	1.00	6.65	0.00	1.85	0.00	0.00	0.55	0.00
time (sec)	N/A	0.179	0.003	0.322	0.000	0.101	0.000	0.000	0.256	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	69	0	59	0	0	11	0
N.S.	1	1.00	1.00	3.14	0.00	2.68	0.00	0.00	0.50	0.00
time (sec)	N/A	0.180	0.003	0.424	0.000	0.093	0.000	0.000	0.202	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	69	0	82	0	0	11	0
N.S.	1	1.00	1.00	2.88	0.00	3.42	0.00	0.00	0.46	0.00
time (sec)	N/A	0.180	0.003	0.635	0.000	0.120	0.000	0.000	0.204	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	316	299	535	0	0	568	0	67	16
N.S.	1	1.63	1.54	2.76	0.00	0.00	2.93	0.00	0.35	0.08
time (sec)	N/A	0.733	0.093	1.369	0.000	0.000	46.675	0.000	0.230	0.212

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	199	166	282	0	0	291	0	46	276
N.S.	1	1.33	1.11	1.88	0.00	0.00	1.94	0.00	0.31	1.84
time (sec)	N/A	0.547	0.073	1.128	0.000	0.000	25.332	0.000	0.228	0.287

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	117	85	120	0	0	124	0	25	111
N.S.	1	1.10	0.80	1.13	0.00	0.00	1.17	0.00	0.24	1.05
time (sec)	N/A	0.426	0.066	0.874	0.000	0.000	12.459	0.000	0.203	0.178

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	40	30	12	0	34	0	9	70
N.S.	1	1.00	1.25	0.94	0.38	0.00	1.06	0.00	0.28	2.19
time (sec)	N/A	0.188	0.024	0.370	0.036	0.000	5.400	0.000	0.198	0.120

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	B
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	0	12	17	17	16
N.S.	1	1.00	1.13	1.00	1.13	0.00	0.80	1.13	1.13	1.07
time (sec)	N/A	0.198	0.201	0.023	0.069	0.000	21.278	0.109	0.255	0.080

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	83	78	93	0	0	0	0	28	16
N.S.	1	1.01	0.95	1.13	0.00	0.00	0.00	0.00	0.34	0.20
time (sec)	N/A	0.563	0.103	1.917	0.000	0.000	0.000	0.000	0.208	0.146

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	156	124	185	0	0	0	0	39	16
N.S.	1	1.14	0.91	1.35	0.00	0.00	0.00	0.00	0.28	0.12
time (sec)	N/A	0.684	0.484	2.481	0.000	0.000	0.000	0.000	0.203	0.166

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	273	180	304	0	0	0	0	50	16
N.S.	1	1.51	0.99	1.68	0.00	0.00	0.00	0.00	0.28	0.09
time (sec)	N/A	0.873	0.465	3.492	0.000	0.000	0.000	0.000	0.241	0.193

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	124	71	164	159	156	223	163	164	163
N.S.	1	1.11	0.63	1.46	1.42	1.39	1.99	1.46	1.46	1.46
time (sec)	N/A	0.422	0.226	0.507	0.060	0.093	0.092	0.109	0.214	0.095

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	95	56	104	110	101	146	103	104	103
N.S.	1	1.09	0.64	1.20	1.26	1.16	1.68	1.18	1.20	1.18
time (sec)	N/A	0.341	0.152	0.467	0.047	0.101	0.077	0.115	0.196	0.110

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	66	41	58	69	57	82	57	58	57
N.S.	1	1.06	0.66	0.94	1.11	0.92	1.32	0.92	0.94	0.92
time (sec)	N/A	0.281	0.115	0.464	0.048	0.082	0.074	0.114	0.224	0.050

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	24	24	36	23	34	23	26	23
N.S.	1	1.00	0.65	0.65	0.97	0.62	0.92	0.62	0.70	0.62
time (sec)	N/A	0.205	0.077	0.356	0.028	0.096	0.051	0.111	0.240	0.035

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	15	14	14	12	14	14	14
N.S.	1	1.00	1.00	1.00	0.93	0.93	0.80	0.93	0.93	0.93
time (sec)	N/A	0.151	0.002	0.108	0.037	0.090	0.034	0.114	0.206	0.060

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	29	40	28	31	0	29	26	0
N.S.	1	1.00	1.04	1.43	1.00	1.11	0.00	1.04	0.93	0.00
time (sec)	N/A	0.184	0.060	0.643	0.077	0.090	0.000	0.110	0.210	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	69	35	63	0	307	42	0
N.S.	1	1.00	1.02	1.30	0.66	1.19	0.00	5.79	0.79	0.00
time (sec)	N/A	0.304	0.087	0.681	0.072	0.088	0.000	0.123	0.225	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	86	72	106	35	107	0	169	58	0
N.S.	1	1.02	0.86	1.26	0.42	1.27	0.00	2.01	0.69	0.00
time (sec)	N/A	0.422	0.095	0.705	0.091	0.097	0.000	0.112	0.211	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	119	88	137	35	165	0	278	74	0
N.S.	1	1.07	0.79	1.23	0.32	1.49	0.00	2.50	0.67	0.00
time (sec)	N/A	0.567	0.104	0.697	0.090	0.094	0.000	0.113	0.203	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	314	223	240	0	279	374	0	248	248
N.S.	1	3.20	2.28	2.45	0.00	2.85	3.82	0.00	2.53	2.53
time (sec)	N/A	1.155	0.088	0.608	0.000	0.089	0.118	0.000	0.228	0.144

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	256	160	136	0	180	216	0	138	149
N.S.	1	2.67	1.67	1.42	0.00	1.88	2.25	0.00	1.44	1.55
time (sec)	N/A	0.923	0.064	0.602	0.000	0.087	0.085	0.000	0.228	0.123

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	198	99	54	0	100	92	0	57	70
N.S.	1	2.13	1.06	0.58	0.00	1.08	0.99	0.00	0.61	0.75
time (sec)	N/A	0.579	0.064	0.440	0.000	0.093	0.060	0.000	0.201	0.099

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	53	21	27	42	29	0	22	20
N.S.	1	1.00	1.83	0.72	0.93	1.45	1.00	0.00	0.76	0.69
time (sec)	N/A	0.191	0.018	0.168	0.057	0.089	0.048	0.000	0.231	0.037

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	0	107	0	0	0	0	99	0
N.S.	1	1.00	0.00	1.32	0.00	0.00	0.00	0.00	1.22	0.00
time (sec)	N/A	0.459	0.000	0.709	0.000	0.000	0.000	0.000	0.196	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	79	87	174	0	106	0	0	770	0
N.S.	1	1.08	1.19	2.38	0.00	1.45	0.00	0.00	10.55	0.00
time (sec)	N/A	0.360	0.056	0.722	0.000	0.096	0.000	0.000	0.212	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	126	94	314	0	179	0	0	0	0
N.S.	1	1.29	0.96	3.20	0.00	1.83	0.00	0.00	0.00	0.00
time (sec)	N/A	0.438	0.106	0.710	0.000	0.102	0.000	0.000	0.228	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	193	117	412	0	276	0	0	0	0
N.S.	1	1.97	1.19	4.20	0.00	2.82	0.00	0.00	0.00	0.00
time (sec)	N/A	0.535	0.162	0.764	0.000	0.124	0.000	0.000	0.207	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	259	191	510	0	384	0	0	0	0
N.S.	1	2.64	1.95	5.20	0.00	3.92	0.00	0.00	0.00	0.00
time (sec)	N/A	0.647	0.169	0.802	0.000	0.152	0.000	0.000	0.229	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	A	F	B	B	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	140	529	361	421	0	401	741	0	462	411
N.S.	1	3.78	2.58	3.01	0.00	2.86	5.29	0.00	3.30	2.94
time (sec)	N/A	1.168	0.146	0.773	0.000	0.096	0.199	0.000	0.209	0.134

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	137	436	252	252	0	265	447	0	265	239
N.S.	1	3.18	1.84	1.84	0.00	1.93	3.26	0.00	1.93	1.74
time (sec)	N/A	0.947	0.103	0.714	0.000	0.089	0.153	0.000	0.234	0.094

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	84	349	146	111	0	149	202	0	114	121
N.S.	1	4.15	1.74	1.32	0.00	1.77	2.40	0.00	1.36	1.44
time (sec)	N/A	0.809	0.075	0.552	0.000	0.087	0.101	0.000	0.227	0.087

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	81	39	27	64	65	0	40	43
N.S.	1	1.00	2.79	1.34	0.93	2.21	2.24	0.00	1.38	1.48
time (sec)	N/A	0.185	0.021	0.357	0.052	0.099	0.071	0.000	0.256	0.097

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	179	0	239	0	0	0	0	215	0
N.S.	1	1.10	0.00	1.48	0.00	0.00	0.00	0.00	1.33	0.00
time (sec)	N/A	0.906	0.000	0.839	0.000	0.000	0.000	0.000	0.214	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	122	88	335	0	164	0	0	1192	0
N.S.	1	1.28	0.93	3.53	0.00	1.73	0.00	0.00	12.55	0.00
time (sec)	N/A	0.442	0.103	0.852	0.000	0.109	0.000	0.000	0.190	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	157	126	563	0	274	0	0	0	0
N.S.	1	1.25	1.00	4.47	0.00	2.17	0.00	0.00	0.00	0.00
time (sec)	N/A	0.512	0.183	0.809	0.000	0.093	0.000	0.000	0.226	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	246	145	763	0	406	0	0	0	0
N.S.	1	1.73	1.02	5.37	0.00	2.86	0.00	0.00	0.00	0.00
time (sec)	N/A	0.645	0.229	0.878	0.000	0.105	0.000	0.000	0.229	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	341	328	963	0	572	0	0	0	0
N.S.	1	2.40	2.31	6.78	0.00	4.03	0.00	0.00	0.00	0.00
time (sec)	N/A	0.818	0.183	0.888	0.000	0.153	0.000	0.000	0.232	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	263	282	0	0	360	0	0	96	24
N.S.	1	1.89	2.03	0.00	0.00	2.59	0.00	0.00	0.69	0.17
time (sec)	N/A	0.708	0.334	0.000	0.000	0.102	0.000	0.000	0.220	0.188

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	176	171	0	0	223	0	0	68	24
N.S.	1	1.49	1.45	0.00	0.00	1.89	0.00	0.00	0.58	0.20
time (sec)	N/A	0.564	0.177	0.000	0.000	0.103	0.000	0.000	0.221	0.221

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	128	113	0	0	120	0	0	40	22
N.S.	1	1.47	1.30	0.00	0.00	1.38	0.00	0.00	0.46	0.25
time (sec)	N/A	0.476	0.082	0.000	0.000	0.089	0.000	0.000	0.198	0.402

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	48	0	27	52	0	0	17	21
N.S.	1	1.00	1.66	0.00	0.93	1.79	0.00	0.00	0.59	0.72
time (sec)	N/A	0.185	0.033	0.000	0.051	0.114	0.000	0.000	0.189	0.140

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	B
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	22	17	17	17	17	30	24
N.S.	1	1.00	1.13	1.47	1.13	1.13	1.13	1.13	2.00	1.60
time (sec)	N/A	0.195	0.338	0.278	0.098	0.112	0.693	0.107	0.258	0.267

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	135	121	0	0	349	0	0	54	24
N.S.	1	1.38	1.23	0.00	0.00	3.56	0.00	0.00	0.55	0.24
time (sec)	N/A	0.938	0.266	0.000	0.000	0.133	0.000	0.000	0.204	0.376

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	211	174	0	0	680	0	0	78	24
N.S.	1	1.38	1.14	0.00	0.00	4.44	0.00	0.00	0.51	0.16
time (sec)	N/A	1.262	0.367	0.000	0.000	0.115	0.000	0.000	0.207	0.465

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	B	F	F	F	B
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	197	330	0	0	0	1218	0	0	102	24
N.S.	1	1.68	0.00	0.00	0.00	6.18	0.00	0.00	0.52	0.12
time (sec)	N/A	1.531	0.000	0.000	0.000	0.144	0.000	0.000	0.210	0.581

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	274	398	0	0	506	0	0	140	24
N.S.	1	1.89	2.74	0.00	0.00	3.49	0.00	0.00	0.97	0.17
time (sec)	N/A	0.821	0.348	0.000	0.000	0.117	0.000	0.000	0.221	1.444

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	226	229	0	0	326	0	0	101	24
N.S.	1	2.02	2.04	0.00	0.00	2.91	0.00	0.00	0.90	0.21
time (sec)	N/A	0.645	0.238	0.000	0.000	0.100	0.000	0.000	0.197	1.390

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	195	142	0	0	178	0	0	62	22
N.S.	1	2.32	1.69	0.00	0.00	2.12	0.00	0.00	0.74	0.26
time (sec)	N/A	0.595	0.145	0.000	0.000	0.103	0.000	0.000	0.200	2.356

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	A	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	77	0	27	83	0	0	28	29
N.S.	1	1.00	2.66	0.00	0.93	2.86	0.00	0.00	0.97	1.00
time (sec)	N/A	0.192	0.040	0.000	0.061	0.091	0.000	0.000	0.251	1.322

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	B
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	22	17	17	19	17	54	24
N.S.	1	1.00	1.13	1.47	1.13	1.13	1.27	1.13	3.60	1.60
time (sec)	N/A	0.200	0.666	0.326	0.086	0.085	1.526	0.109	0.209	3.721

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	224	198	0	0	821	0	0	95	24
N.S.	1	1.87	1.65	0.00	0.00	6.84	0.00	0.00	0.79	0.20
time (sec)	N/A	1.181	0.277	0.000	0.000	0.113	0.000	0.000	0.197	5.709

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	306	309	0	0	1284	0	0	136	24
N.S.	1	1.71	1.73	0.00	0.00	7.17	0.00	0.00	0.76	0.13
time (sec)	N/A	1.540	0.910	0.000	0.000	0.133	0.000	0.000	0.228	19.027

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	B	F	F	F	B
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	433	0	0	0	1987	0	0	177	24
N.S.	1	2.00	0.00	0.00	0.00	9.16	0.00	0.00	0.82	0.11
time (sec)	N/A	1.420	0.000	0.000	0.000	0.176	0.000	0.000	0.182	61.743

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	355	482	0	0	662	0	0	184	24
N.S.	1	2.55	3.47	0.00	0.00	4.76	0.00	0.00	1.32	0.17
time (sec)	N/A	0.857	0.322	0.000	0.000	0.094	0.000	0.000	0.193	0.226

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	330	351	0	0	424	0	0	134	24
N.S.	1	3.03	3.22	0.00	0.00	3.89	0.00	0.00	1.23	0.22
time (sec)	N/A	0.818	0.235	0.000	0.000	0.107	0.000	0.000	0.227	0.205

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	284	270	0	0	235	0	0	84	22
N.S.	1	3.38	3.21	0.00	0.00	2.80	0.00	0.00	1.00	0.26
time (sec)	N/A	0.713	0.126	0.000	0.000	0.147	0.000	0.000	0.231	0.472

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	A	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	89	0	27	113	0	0	39	29
N.S.	1	1.00	3.07	0.00	0.93	3.90	0.00	0.00	1.34	1.00
time (sec)	N/A	0.186	0.031	0.000	0.052	0.086	0.000	0.000	0.198	0.180

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	B
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	22	17	17	19	17	78	24
N.S.	1	1.00	1.13	1.47	1.13	1.13	1.27	1.13	5.20	1.60
time (sec)	N/A	0.194	0.262	0.313	0.103	0.113	2.004	0.105	0.214	0.251

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	349	273	0	0	1517	0	0	136	24
N.S.	1	2.42	1.90	0.00	0.00	10.53	0.00	0.00	0.94	0.17
time (sec)	N/A	1.021	0.606	0.000	0.000	0.142	0.000	0.000	0.202	0.352

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	205	430	877	0	0	2285	0	0	194	24
N.S.	1	2.10	4.28	0.00	0.00	11.15	0.00	0.00	0.95	0.12
time (sec)	N/A	1.171	6.908	0.000	0.000	0.206	0.000	0.000	0.247	0.422

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	F	F	F	B	F	F	F	B
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	248	573	0	0	0	3116	0	0	252	24
N.S.	1	2.31	0.00	0.00	0.00	12.56	0.00	0.00	1.02	0.10
time (sec)	N/A	1.370	0.000	0.000	0.000	0.345	0.000	0.000	0.215	0.340

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	281	134	158	0	117	0	0	113	167
N.S.	1	3.51	1.68	1.98	0.00	1.46	0.00	0.00	1.41	2.09
time (sec)	N/A	0.680	0.088	0.322	0.000	0.108	0.000	0.000	0.224	0.283

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	236	117	134	0	104	165	0	92	132
N.S.	1	2.95	1.46	1.68	0.00	1.30	2.06	0.00	1.15	1.65
time (sec)	N/A	0.564	0.077	0.295	0.000	0.107	28.690	0.000	0.232	0.262

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	191	97	114	0	89	136	0	75	134
N.S.	1	2.39	1.21	1.42	0.00	1.11	1.70	0.00	0.94	1.68
time (sec)	N/A	0.510	0.064	0.290	0.000	0.112	8.245	0.000	0.198	0.229

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	142	76	76	0	70	99	0	61	87
N.S.	1	2.09	1.12	1.12	0.00	1.03	1.46	0.00	0.90	1.28
time (sec)	N/A	0.451	0.049	0.307	0.000	0.097	3.811	0.000	0.230	0.245

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	96	65	88	0	58	116	0	65	98
N.S.	1	1.39	0.94	1.28	0.00	0.84	1.68	0.00	0.94	1.42
time (sec)	N/A	0.381	0.052	0.351	0.000	0.107	3.156	0.000	0.210	0.267

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	99	69	120	0	61	151	0	75	145
N.S.	1	1.30	0.91	1.58	0.00	0.80	1.99	0.00	0.99	1.91
time (sec)	N/A	0.377	0.071	0.345	0.000	0.111	9.990	0.000	0.230	0.294

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	143	80	142	0	77	185	0	93	124
N.S.	1	1.79	1.00	1.78	0.00	0.96	2.31	0.00	1.16	1.55
time (sec)	N/A	0.425	0.102	0.319	0.000	0.100	102.344	0.000	0.226	0.319

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	192	96	166	0	91	0	0	114	194
N.S.	1	2.40	1.20	2.08	0.00	1.14	0.00	0.00	1.42	2.42
time (sec)	N/A	0.506	0.110	0.346	0.000	0.101	0.000	0.000	0.198	0.311

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	B
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	18	13	13	19	13	31	20
N.S.	1	1.00	1.15	1.38	1.00	1.00	1.46	1.00	2.38	1.54
time (sec)	N/A	0.184	4.495	0.181	0.079	0.101	2.012	0.110	0.210	0.779

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	B
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	18	13	13	19	13	30	20
N.S.	1	1.00	1.15	1.38	1.00	1.00	1.46	1.00	2.31	1.54
time (sec)	N/A	0.188	4.478	0.187	0.092	0.102	1.798	0.111	0.230	0.757

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	B
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	18	13	13	19	13	35	20
N.S.	1	1.00	1.15	1.38	1.00	1.00	1.46	1.00	2.69	1.54
time (sec)	N/A	0.183	2.774	0.174	0.088	0.135	2.267	0.106	0.197	12.643

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	B
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	18	13	13	19	13	38	20
N.S.	1	1.00	1.15	1.38	1.00	1.00	1.46	1.00	2.92	1.54
time (sec)	N/A	0.198	2.787	0.174	0.112	0.106	3.009	0.108	0.217	12.502

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	82	119	178	0	98	0	0	103	124
N.S.	1	1.02	1.49	2.22	0.00	1.22	0.00	0.00	1.29	1.55
time (sec)	N/A	0.335	0.203	0.463	0.000	0.107	0.000	0.000	0.233	0.275

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	82	95	115	0	83	65	0	86	114
N.S.	1	1.02	1.19	1.44	0.00	1.04	0.81	0.00	1.08	1.42
time (sec)	N/A	0.342	0.191	0.464	0.000	0.118	28.233	0.000	0.195	0.274

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	82	95	115	0	83	65	0	86	114
N.S.	1	1.02	1.19	1.44	0.00	1.04	0.81	0.00	1.08	1.42
time (sec)	N/A	0.550	0.065	0.386	0.000	0.134	9.775	0.000	0.202	0.253

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	82	80	142	0	65	65	0	73	79
N.S.	1	1.11	1.08	1.92	0.00	0.88	0.88	0.00	0.99	1.07
time (sec)	N/A	0.532	0.054	0.418	0.000	0.132	6.070	0.000	0.256	0.266

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	78	69	142	0	64	65	0	73	79
N.S.	1	1.15	1.01	2.09	0.00	0.94	0.96	0.00	1.07	1.16
time (sec)	N/A	0.502	0.070	0.408	0.000	0.109	4.820	0.000	0.207	0.276

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	75	54	186	0	51	68	0	63	69
N.S.	1	1.09	0.78	2.70	0.00	0.74	0.99	0.00	0.91	1.00
time (sec)	N/A	0.550	0.068	0.450	0.000	0.130	4.819	0.000	0.210	0.306

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	215	147	0	0	314	0	0	556	0
N.S.	1	0.97	0.67	0.00	0.00	1.42	0.00	0.00	2.52	0.00
time (sec)	N/A	0.901	0.287	0.000	0.000	0.133	0.000	0.000	0.237	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	149	117	0	0	154	0	0	204	0
N.S.	1	0.99	0.78	0.00	0.00	1.03	0.00	0.00	1.36	0.00
time (sec)	N/A	0.516	0.161	0.000	0.000	0.105	0.000	0.000	0.200	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	39	44	0	0	59	0
N.S.	1	1.00	1.00	0.00	0.75	0.85	0.00	0.00	1.13	0.00
time (sec)	N/A	0.198	0.030	0.000	0.068	0.088	0.000	0.000	0.181	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A	B
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	0	0	17	17	16
N.S.	1	1.00	1.13	1.00	1.13	0.00	0.00	1.13	1.13	1.07
time (sec)	N/A	0.191	0.629	0.024	0.075	0.000	0.000	0.109	0.191	0.128

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	B
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	22	17	17	19	17	24	24
N.S.	1	1.00	1.13	1.47	1.13	1.13	1.27	1.13	1.60	1.60
time (sec)	N/A	0.191	0.608	0.336	0.091	0.104	0.443	0.101	0.274	0.300

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	B
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	22	17	17	20	17	35	24
N.S.	1	1.00	1.13	1.47	1.13	1.13	1.33	1.13	2.33	1.60
time (sec)	N/A	0.189	0.752	0.296	0.082	0.106	0.818	0.116	0.192	0.342

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	B
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	22	17	17	20	17	46	24
N.S.	1	1.00	1.13	1.47	1.13	1.13	1.33	1.13	3.07	1.60
time (sec)	N/A	0.190	1.021	0.337	0.107	0.130	1.140	0.114	0.199	0.267

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	13	11	13	13	0	13	13	13
N.S.	1	1.00	1.18	1.00	1.18	1.18	0.00	1.18	1.18	1.18
time (sec)	N/A	0.179	0.487	0.036	0.091	0.100	0.000	0.105	0.256	0.101

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	0	17	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.00	1.13	1.13	1.13
time (sec)	N/A	0.188	0.734	0.158	0.094	0.159	0.000	0.111	0.229	0.097

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	174	217	3717	0	1080	0	0	88	0
N.S.	1	1.03	1.28	21.99	0.00	6.39	0.00	0.00	0.52	0.00
time (sec)	N/A	0.651	0.204	17.763	0.000	0.135	0.000	0.000	0.237	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	147	169	1929	0	618	0	0	67	0
N.S.	1	1.02	1.17	13.40	0.00	4.29	0.00	0.00	0.47	0.00
time (sec)	N/A	0.593	0.126	5.845	0.000	0.143	0.000	0.000	0.232	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	120	119	875	0	310	0	0	46	0
N.S.	1	1.04	1.03	7.61	0.00	2.70	0.00	0.00	0.40	0.00
time (sec)	N/A	0.527	0.097	1.774	0.000	0.102	0.000	0.000	0.231	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	93	65	315	0	127	0	0	25	0
N.S.	1	1.06	0.74	3.58	0.00	1.44	0.00	0.00	0.28	0.00
time (sec)	N/A	0.452	0.100	0.741	0.000	0.099	0.000	0.000	0.192	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	36	88	29	47	0	0	9	0
N.S.	1	1.00	1.16	2.84	0.94	1.52	0.00	0.00	0.29	0.00
time (sec)	N/A	0.189	0.023	0.206	0.073	0.095	0.000	0.000	0.186	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	12	17	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.80	1.13	1.13	1.13
time (sec)	N/A	0.199	0.272	0.157	0.088	0.099	51.539	0.110	0.227	0.141

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	28	0	17	28	17
N.S.	1	1.00	1.13	1.00	1.13	1.87	0.00	1.13	1.87	1.13
time (sec)	N/A	0.202	0.613	0.193	0.093	0.095	0.000	0.125	0.223	0.308

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	39	0	17	39	17
N.S.	1	1.00	1.13	1.00	1.13	2.60	0.00	1.13	2.60	1.13
time (sec)	N/A	0.193	0.617	0.202	0.101	0.102	0.000	0.112	0.199	0.368

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	129	112	0	0	0	0	0	21	0
N.S.	1	1.07	0.93	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	1.006	0.261	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	129	112	0	0	0	0	0	19	0
N.S.	1	1.07	0.93	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.966	0.244	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	120	107	0	0	0	0	0	17	0
N.S.	1	1.07	0.96	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.957	0.216	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	59	55	0	55	101	0	0	21	0
N.S.	1	0.97	0.90	0.00	0.90	1.66	0.00	0.00	0.34	0.00
time (sec)	N/A	0.336	0.282	0.000	0.100	0.110	0.000	0.000	0.230	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	117	107	0	0	0	0	0	21	0
N.S.	1	1.07	0.98	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.664	0.165	0.000	0.000	0.000	0.000	0.000	0.243	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	125	113	0	0	0	0	0	21	0
N.S.	1	1.07	0.97	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.627	0.175	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	168	136	0	0	0	0	0	25	0
N.S.	1	1.17	0.94	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.697	0.389	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	80	69	135	0	0	0	0	16	0
N.S.	1	1.08	0.93	1.82	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.469	0.041	0.404	0.000	0.000	0.000	0.000	0.207	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	55	49	99	0	0	0	0	16	0
N.S.	1	1.06	0.94	1.90	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.359	0.022	0.378	0.000	0.000	0.000	0.000	0.224	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	40	69	0	0	0	0	14	0
N.S.	1	1.00	1.33	2.30	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.250	0.023	0.303	0.000	0.000	0.000	0.000	0.194	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	24	0	0	0	0	8	0
N.S.	1	1.00	1.00	2.18	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.169	0.004	0.263	0.000	0.000	0.000	0.000	0.231	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.14	1.14
time (sec)	N/A	0.191	0.024	0.102	0.064	0.098	0.266	0.104	0.265	0.095

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	27	14	16	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.93	1.00	1.14	1.14	1.14
time (sec)	N/A	0.188	0.031	0.089	0.061	0.083	0.388	0.108	0.207	0.118

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	16	16
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	1.00	1.00
time (sec)	N/A	0.194	0.093	0.172	0.069	0.082	2.140	0.115	0.222	0.103

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	16	16
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	1.00	1.00
time (sec)	N/A	0.195	0.082	0.174	0.073	0.097	0.436	0.108	0.191	0.094

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	16	16
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	1.00	1.00
time (sec)	N/A	0.194	0.078	0.176	0.062	0.092	0.389	0.107	0.238	0.095

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	109	115	161	0	65	0	0	46	0
N.S.	1	1.17	1.24	1.73	0.00	0.70	0.00	0.00	0.49	0.00
time (sec)	N/A	0.585	0.069	0.434	0.000	0.099	0.000	0.000	0.228	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	84	104	0	0	42	0	0	25	0
N.S.	1	1.18	1.46	0.00	0.00	0.59	0.00	0.00	0.35	0.00
time (sec)	N/A	0.454	0.061	0.000	0.000	0.094	0.000	0.000	0.229	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	37	38	36	0	22	0	0	9	0
N.S.	1	1.23	1.27	1.20	0.00	0.73	0.00	0.00	0.30	0.00
time (sec)	N/A	0.280	0.009	0.248	0.000	0.106	0.000	0.000	0.237	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13	1.13
time (sec)	N/A	0.274	0.020	0.185	0.086	0.089	0.419	0.140	0.208	0.154

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	28	15	17	28	17
N.S.	1	1.00	1.13	1.00	1.13	1.87	1.00	1.13	1.87	1.13
time (sec)	N/A	0.268	0.060	0.077	0.082	0.089	2.104	0.165	0.224	0.152

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	0	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	0.00	1.13	1.13
time (sec)	N/A	0.191	0.045	0.170	0.059	0.114	0.425	0.000	0.214	0.149

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	88	77	112	0	0	0	0	67	0
N.S.	1	1.07	0.94	1.37	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.487	0.041	0.330	0.000	0.000	0.000	0.000	0.235	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	61	55	74	0	0	0	0	46	0
N.S.	1	1.05	0.95	1.28	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.366	0.022	0.342	0.000	0.000	0.000	0.000	0.214	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	46	36	0	0	0	0	25	0
N.S.	1	1.00	1.35	1.06	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.265	0.023	0.270	0.000	0.000	0.000	0.000	0.200	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	0	0	0	0	9	0
N.S.	1	1.00	1.00	1.08	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.185	0.004	0.237	0.000	0.000	0.000	0.000	0.217	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	12	0	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.80	0.00	1.13	1.13
time (sec)	N/A	0.198	0.023	0.083	0.070	0.080	0.221	0.000	0.261	0.194

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	28	14	0	28	17
N.S.	1	1.00	1.13	1.00	1.13	1.87	0.93	0.00	1.87	1.13
time (sec)	N/A	0.285	0.027	0.081	0.058	0.086	0.515	0.000	0.215	0.298

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	39	14	0	39	17
N.S.	1	1.00	1.13	1.00	1.13	2.60	0.93	0.00	2.60	1.13
time (sec)	N/A	0.393	0.030	0.083	0.069	0.079	0.563	0.000	0.226	0.308

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	15	17	17	15	0	38	17
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.88	0.00	2.24	1.00
time (sec)	N/A	0.410	0.089	0.197	0.090	0.088	2.111	0.000	0.272	0.182

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	15	17	17	15	0	16	17
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.88	0.00	0.94	1.00
time (sec)	N/A	0.293	0.068	0.169	0.087	0.095	0.413	0.000	0.195	0.159

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	15	17	17	15	0	18	17
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.88	0.00	1.06	1.00
time (sec)	N/A	0.200	0.066	0.179	0.067	0.080	0.348	0.000	0.207	0.171

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	15	17	35	15	0	30	17
N.S.	1	1.00	1.12	0.88	1.00	2.06	0.88	0.00	1.76	1.00
time (sec)	N/A	0.294	0.047	0.168	0.065	0.106	1.296	0.000	0.280	0.163

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	44	40	39	0	0	0	0	13	0
N.S.	1	1.13	1.03	1.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.317	0.009	0.251	0.000	0.000	0.000	0.000	0.196	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	0	13	0	0	0	0	30	0
N.S.	1	1.00	0.00	1.08	0.00	0.00	0.00	0.00	2.50	0.00
time (sec)	N/A	0.188	0.000	0.434	0.000	0.000	0.000	0.000	0.194	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	0	13	0	0	0	0	32	0
N.S.	1	1.00	0.00	1.08	0.00	0.00	0.00	0.00	2.67	0.00
time (sec)	N/A	0.217	0.000	0.448	0.000	0.000	0.000	0.000	0.246	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	0	0	0	15	17	15
N.S.	1	1.00	1.00	1.07	0.00	0.00	0.00	1.00	1.13	1.00
time (sec)	N/A	0.186	0.009	0.151	0.000	0.000	0.000	0.116	0.202	0.147

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	0	0	0	15	1	15
N.S.	1	1.00	1.00	1.07	0.00	0.00	0.00	1.00	0.07	1.00
time (sec)	N/A	0.193	0.004	0.161	0.000	0.000	0.000	0.116	0.195	0.018

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [214] had the largest ratio of [.5714289999999999965]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	9	0.111
2	A	1	1	1.00	9	0.111
3	A	1	1	1.00	7	0.143
4	A	1	1	1.00	5	0.200
5	A	1	1	1.00	9	0.111
6	A	1	1	1.00	9	0.111
7	A	1	1	1.00	9	0.111
8	A	1	1	1.00	9	0.111
9	A	4	4	1.23	10	0.400
10	A	3	3	1.16	10	0.300
11	A	2	2	1.00	8	0.250
12	A	1	1	1.00	6	0.167
13	A	1	1	1.00	10	0.100
14	A	2	2	1.00	10	0.200
15	A	3	3	0.98	10	0.300
16	A	4	4	1.04	10	0.400
17	A	1	1	1.00	9	0.111
18	A	1	1	1.00	9	0.111
19	A	1	1	1.00	7	0.143
20	A	1	1	1.00	5	0.200
21	A	2	2	1.00	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
22	A	1	1	1.00	9	0.111
23	A	1	1	1.00	9	0.111
24	A	1	1	1.00	9	0.111
25	A	1	1	1.00	9	0.111
26	A	1	1	1.00	9	0.111
27	A	1	1	1.00	7	0.143
28	A	1	1	1.00	5	0.200
29	A	3	3	1.04	9	0.333
30	A	1	1	1.00	9	0.111
31	A	1	1	1.00	9	0.111
32	A	1	1	1.00	9	0.111
33	A	1	1	1.00	9	0.111
34	A	1	1	1.00	9	0.111
35	A	1	1	1.00	9	0.111
36	A	1	1	1.00	7	0.143
37	A	1	1	1.00	5	0.200
38	A	2	2	1.00	9	0.222
39	A	1	1	1.00	9	0.111
40	A	1	1	1.00	9	0.111
41	A	1	1	1.00	9	0.111
42	A	1	1	1.00	9	0.111
43	A	1	1	1.00	9	0.111
44	A	1	1	1.00	9	0.111
45	A	1	1	1.00	7	0.143
46	A	1	1	1.00	5	0.200
47	A	3	3	0.93	9	0.333
48	A	1	1	1.00	9	0.111
49	A	1	1	1.00	9	0.111
50	A	1	1	1.00	9	0.111
51	A	1	1	1.00	9	0.111
52	A	1	1	1.00	9	0.111
53	A	1	1	1.00	9	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	1	1	1.00	7	0.143
55	A	1	1	1.00	5	0.200
56	A	4	4	1.05	9	0.444
57	A	1	1	1.00	9	0.111
58	A	1	1	1.00	9	0.111
59	A	1	1	1.00	9	0.111
60	A	1	1	1.00	11	0.091
61	A	1	1	1.00	11	0.091
62	A	1	1	1.00	9	0.111
63	A	1	1	1.00	7	0.143
64	A	1	1	1.00	11	0.091
65	A	1	1	1.00	11	0.091
66	A	1	1	1.00	11	0.091
67	A	1	1	1.00	11	0.091
68	A	1	1	1.00	11	0.091
69	A	1	1	1.00	11	0.091
70	A	1	1	1.00	9	0.111
71	A	1	1	1.00	7	0.143
72	A	1	1	1.00	11	0.091
73	A	1	1	1.00	11	0.091
74	A	1	1	1.00	11	0.091
75	A	1	1	1.00	11	0.091
76	A	1	1	1.00	11	0.091
77	A	1	1	1.00	11	0.091
78	A	1	1	1.00	12	0.083
79	A	1	1	1.00	11	0.091
80	A	1	1	1.00	11	0.091
81	A	1	1	1.00	11	0.091
82	A	1	1	1.00	7	0.143
83	A	1	1	1.00	9	0.111
84	A	1	1	1.00	9	0.111
85	A	1	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	1	1	1.00	9	0.111
87	A	1	1	1.00	9	0.111
88	A	1	1	1.00	7	0.143
89	A	1	1	1.00	5	0.200
90	A	1	1	1.00	9	0.111
91	A	1	1	1.00	9	0.111
92	A	1	1	1.00	9	0.111
93	A	1	1	1.00	9	0.111
94	A	1	1	1.00	9	0.111
95	A	1	1	1.00	9	0.111
96	A	1	1	1.00	7	0.143
97	A	1	1	1.00	5	0.200
98	A	1	1	1.00	9	0.111
99	A	1	1	1.00	9	0.111
100	A	1	1	1.00	9	0.111
101	A	1	1	1.00	9	0.111
102	A	3	3	1.63	15	0.200
103	A	3	3	1.33	15	0.200
104	A	3	3	1.10	13	0.231
105	A	1	1	1.00	7	0.143
106	N/A	1	0	1.00	15	0.000
107	A	3	3	1.01	15	0.200
108	A	3	3	1.14	15	0.200
109	A	3	3	1.51	15	0.200
110	A	5	5	1.11	18	0.278
111	A	4	4	1.09	18	0.222
112	A	3	3	1.06	18	0.167
113	A	2	2	1.00	16	0.125
114	A	1	1	1.00	10	0.100
115	A	1	1	1.00	18	0.056
116	A	2	2	1.00	18	0.111
117	A	3	3	1.02	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	4	4	1.07	18	0.222
119	B	3	3	3.20	15	0.200
120	B	3	3	2.67	15	0.200
121	B	3	3	2.13	13	0.231
122	A	1	1	1.00	7	0.143
123	A	4	4	1.00	15	0.267
124	A	3	3	1.08	15	0.200
125	A	3	3	1.29	15	0.200
126	A	3	3	1.97	15	0.200
127	B	3	3	2.64	15	0.200
128	B	3	3	3.78	15	0.200
129	B	3	3	3.18	15	0.200
130	B	3	3	4.15	13	0.231
131	A	1	1	1.00	7	0.143
132	A	7	7	1.10	15	0.467
133	A	3	3	1.28	15	0.200
134	A	3	3	1.25	15	0.200
135	A	3	3	1.73	15	0.200
136	B	3	3	2.40	15	0.200
137	A	3	3	1.89	15	0.200
138	A	3	3	1.49	15	0.200
139	A	3	3	1.47	13	0.231
140	A	1	1	1.00	7	0.143
141	N/A	1	0	1.00	15	0.000
142	A	3	3	1.38	15	0.200
143	A	3	3	1.38	15	0.200
144	A	3	3	1.68	15	0.200
145	A	3	3	1.89	15	0.200
146	B	3	3	2.02	15	0.200
147	B	3	3	2.32	13	0.231
148	A	1	1	1.00	7	0.143
149	N/A	1	0	1.00	15	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
150	A	3	3	1.87	15	0.200
151	A	3	3	1.71	15	0.200
152	A	3	3	2.00	15	0.200
153	B	3	3	2.55	15	0.200
154	B	3	3	3.03	15	0.200
155	B	3	3	3.38	13	0.231
156	A	1	1	1.00	7	0.143
157	N/A	1	0	1.00	15	0.000
158	B	3	3	2.42	15	0.200
159	B	3	3	2.10	15	0.200
160	B	3	3	2.31	15	0.200
161	B	3	3	3.51	13	0.231
162	B	3	3	2.95	13	0.231
163	B	3	3	2.39	13	0.231
164	B	3	3	2.09	13	0.231
165	A	3	3	1.39	13	0.231
166	A	3	3	1.30	13	0.231
167	A	3	3	1.79	13	0.231
168	B	3	3	2.40	13	0.231
169	N/A	1	0	1.00	13	0.000
170	N/A	1	0	1.00	13	0.000
171	N/A	1	0	1.00	13	0.000
172	N/A	1	0	1.00	13	0.000
173	A	3	3	1.02	13	0.231
174	A	3	3	1.02	13	0.231
175	A	3	3	1.02	13	0.231
176	A	3	3	1.11	13	0.231
177	A	3	3	1.15	13	0.231
178	A	3	3	1.09	13	0.231
179	A	3	3	0.97	15	0.200
180	A	3	3	0.99	15	0.200
181	A	1	1	1.00	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	N/A	1	0	1.00	15	0.000
183	N/A	1	0	1.00	15	0.000
184	N/A	1	0	1.00	15	0.000
185	N/A	1	0	1.00	15	0.000
186	N/A	1	0	1.00	11	0.000
187	N/A	1	0	1.00	15	0.000
188	A	3	3	1.03	15	0.200
189	A	3	3	1.02	15	0.200
190	A	3	3	1.04	15	0.200
191	A	3	3	1.06	13	0.231
192	A	1	1	1.00	7	0.143
193	N/A	1	0	1.00	15	0.000
194	N/A	1	0	1.00	15	0.000
195	N/A	1	0	1.00	15	0.000
196	A	5	4	1.07	18	0.222
197	A	5	4	1.07	16	0.250
198	A	6	5	1.07	14	0.357
199	A	4	3	0.97	18	0.167
200	A	5	4	1.07	18	0.222
201	A	5	4	1.07	18	0.222
202	A	5	4	1.17	20	0.200
203	A	4	4	1.08	14	0.286
204	A	3	3	1.06	14	0.214
205	A	2	2	1.00	12	0.167
206	A	1	1	1.00	6	0.167
207	N/A	1	0	1.00	14	0.000
208	N/A	1	0	1.00	14	0.000
209	N/A	1	0	1.00	16	0.000
210	N/A	1	0	1.00	16	0.000
211	N/A	1	0	1.00	16	0.000
212	A	6	6	1.17	15	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
213	A	5	5	1.18	13	0.385
214	A	4	4	1.23	7	0.571
215	N/A	3	0	1.00	15	0.000
216	N/A	3	0	1.00	15	0.000
217	N/A	1	0	1.00	15	0.000
218	A	4	4	1.07	15	0.267
219	A	3	3	1.05	15	0.200
220	A	2	2	1.00	13	0.154
221	A	1	1	1.00	7	0.143
222	N/A	1	0	1.00	15	0.000
223	N/A	2	0	1.00	15	0.000
224	N/A	3	0	1.00	15	0.000
225	N/A	3	0	1.00	17	0.000
226	N/A	2	0	1.00	17	0.000
227	N/A	1	0	1.00	17	0.000
228	N/A	2	0	1.00	17	0.000
229	A	3	3	1.13	11	0.273
230	A	1	1	1.00	25	0.040
231	A	1	1	1.00	27	0.037
232	A	1	1	1.00	16	0.062
233	A	1	1	1.00	17	0.059

CHAPTER 3

Listing of integrals

3.1 $\int x^{100}\Gamma(0,ax)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int x^{100}\Gamma(0,ax)\,dx=\frac{1}{101}x^{101}\Gamma(0,ax)-\frac{\Gamma(101,ax)}{101a^{101}}$$

output
```
1/101*x^101*Ei(1,a*x)-1/101*GAMMA(101,a*x)/a^101
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^{100}\Gamma(0,ax)\,dx=\frac{1}{101}x^{101}\Gamma(0,ax)-\frac{\Gamma(101,ax)}{101a^{101}}$$

input
```
Integrate[x^100*Gamma[0, a*x],x]
```

output
```
(x^101*Gamma[0, a*x])/101 - Gamma[101, a*x]/(101*a^101)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{100}\Gamma(0,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{101}x^{101}\Gamma(0,ax)-\frac{\Gamma(101,ax)}{101a^{101}}$$

input
```
Int[x^100*Gamma[0, a*x],x]
```

output
```
(x^101*Gamma[0, a*x])/101 - Gamma[101, a*x]/(101*a^101)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1321 vs. 2(21) = 42.

Time = 0.03 (sec) , antiderivative size = 1322, normalized size of antiderivative = 52.88

Expression too large to display

input
```
int(x^100*Ei(1,x*a),x)
```

output
```
1/a^101*(-1055516033003632107099290624742660954185524161191648021896839298
559715737949168776242443226163456452025804390400000000000000000/101*x^29*
a^29*exp(-x*a)-90317312543607677060255118337640856431754876050390050366161
556141493867410651717110318010184819225395200000000000000000/101*x^35*a^35*
exp(-x*a)-331284225412682501619179520000/101*x^85*a^85*exp(-x*a)-113496347
63479915130099899190781300582640044743996215289213325790964685354292137378
95100243186512314006241280000000000000000000/101*x^31*a^31*exp(-x*a)-2365369
3694465530615609417728000000/101*x^83*a^83*exp(-x*a)-9332621544394415268169
92388562667004907159682643816214685929638952175999932299156089414639761565
18286253697920827223758251185210916864000000000000000000000000/101*exp(-x*
a)-16098703928453240136983769705676800000/101*x^81*a^81*exp(-x*a)-69028187
8632192000/101*x^91*a^91*exp(-x*a)-104319601456376996087654827692785664000
000/101*x^79*a^79*exp(-x*a)-2508814237322435473895975509378912678659857668
06639028794893211504149631696254769750883361624497848320000000000000000/10
1*x^36*a^36*exp(-x*a)-3060996495710533110587942811753716767138020067455779
263500833965823175640052589451103085355874023710874832732160000000000000000
000/101*x^28*a^28*exp(-x*a)-1498729784938957529766067890061756345678433710
45720003807587994398890718324290671405724972006126137054582663222880552700
66094080000000000000000000000/101*x^13*a^13*exp(-x*a)-19632565766406390410
9558167142400000/101*x^82*a^82*exp(-x*a)-231411335075716303160448476568...
```

Fricas [F(-2)]

Exception generated.

$$\int x^{100}\Gamma(0,ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^100*exp_integral_e(1,a*x),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated ((x
)^(((100)::EXPR INT)))*(exp_integral_e(((1)::EXPR INT),(a)*(x))):   There
are no library operations named exp_integral_e       Use HyperDoc Browse o
r issue
```

Sympy [F(-1)]

Timed out.

$$\int x^{100}\Gamma(0, ax)\,dx = \text{Timed out}$$

input
```
integrate(x**100*expint(1,a*x),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{100}\Gamma(0, ax)\,dx = \int x^{100} E_1(ax)\,dx$$

input
```
integrate(x^100*exp_integral_e(1,a*x),x, algorithm="maxima")
```

output
```
integrate(x^100*exp_integral_e(1, a*x), x)
```

Giac [F]

$$\int x^{100}\Gamma(0, ax)\,dx = \int x^{100} E_1(ax)\,dx$$

input
```
integrate(x^100*exp_integral_e(1,a*x),x, algorithm="giac")
```

output
```
integrate(x^100*exp_integral_e(1, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 826, normalized size of antiderivative = 33.04

$$\int x^{100}\Gamma(0,ax)\,dx = \text{Too large to display}$$

input
```
int(x^100*expint(a*x),x)
```

output
```
-(x^101*(exp(-a*x)*(1/(a*x) + 100/(a^2*x^2) + 9900/(a^3*x^3) + 970200/(a^4
*x^4) + 94109400/(a^5*x^5) + 9034502400/(a^6*x^6) + 858277728000/(a^7*x^7)
 + 80678106432000/(a^8*x^8) + 7503063898176000/(a^9*x^9) + 690281878632192
000/(a^10*x^10) + 62815650955529472000/(a^11*x^11) + 565340858599765248000
0/(a^12*x^12) + 503153364153791070720000/(a^13*x^13) + 442774960455336142
3360000/(a^14*x^14) + 3852142155961424437432320000/(a^15*x^15) + 331284225
412682501619179520000/(a^16*x^16) + 28159159160078012637630259200000/(a^17
*x^17) + 2365369369446553061560941772800000/(a^18*x^18) + 1963256576640639
0410955816714240000/(a^19*x^19) + 1609870392845324013698376970567680000/
(a^20*x^20) + 130399501820471245109568534615982080000/(a^21*x^21) + 10431
960145637699608765482769278566400000/(a^22*x^22) + 82412485150537826909242
7313877300674560000000/(a^23*x^23) + 642817384174195049892129048242945261568
8000000/(a^24*x^24) + 4949693858141301884169393671470678514073600000/(a^2
5*x^25) + 37617673321873894319687391903177156706959360000000/(a^26*x^26) +
282132549914054207397655439273828675302195200000000/(a^27*x^27) + 20877808
693640011347426502506263321972362444800000000/(a^28*x^28) + 15240800346357
20828362134682957222503982458470400000000/(a^29*x^29) + 109733762493771899
6420736971729200202867370098688000000000/(a^30*x^30) + 7791097137057804874
5872324992773214403583277006848000000000/(a^31*x^31) + 5453767995940463412211
06274949412500825082939047936000000000/(a^32*x^32) + 3763099917198919 7...
```

Reduce [F]

$$\int x^{100}\Gamma(0,ax)\,dx = \int ei(1,ax)\,x^{100}dx$$

input
```
int(x^100*Ei(1,a*x),x)
```

output
```
int(ei(1,a*x)*x**100,x)
```

3.2 $\int x^2\Gamma(0, ax)\, dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int x^2\Gamma(0, ax)\, dx = \frac{1}{3}x^3\Gamma(0, ax) - \frac{\Gamma(3, ax)}{3a^3}$$

output
```
1/3*x^3*Ei(1,a*x)-2/3*exp(-a*x)*(1+a*x+1/2*a^2*x^2)/a^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^2\Gamma(0, ax)\, dx = \frac{1}{3}x^3\Gamma(0, ax) - \frac{\Gamma(3, ax)}{3a^3}$$

input
```
Integrate[x^2*Gamma[0, a*x],x]
```

output
```
(x^3*Gamma[0, a*x])/3 - Gamma[3, a*x]/(3*a^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\Gamma(0,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{3}x^3\Gamma(0,ax)-\frac{\Gamma(3,ax)}{3a^3}$$

input
```
Int[x^2*Gamma[0, a*x],x]
```

output
```
(x^3*Gamma[0, a*x])/3 - Gamma[3, a*x]/(3*a^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

method	result	size
parts	$\frac{x^{3}\,\mathrm{expIntegral}_{1}(xa)}{3}-\frac{x^{2}a^{2}\mathrm{e}^{-xa}+2xa\,\mathrm{e}^{-xa}+2\,\mathrm{e}^{-xa}}{3a^{3}}$	46
derivativedivides	$\frac{\frac{x^{3}a^{3}\,\mathrm{expIntegral}_{1}(xa)}{3}-\frac{x^{2}a^{2}\mathrm{e}^{-xa}}{3}-\frac{2xa\,\mathrm{e}^{-xa}}{3}-\frac{2\,\mathrm{e}^{-xa}}{3}}{a^{3}}$	48
default	$\frac{\frac{x^{3}a^{3}\,\mathrm{expIntegral}_{1}(xa)}{3}-\frac{x^{2}a^{2}\mathrm{e}^{-xa}}{3}-\frac{2xa\,\mathrm{e}^{-xa}}{3}-\frac{2\,\mathrm{e}^{-xa}}{3}}{a^{3}}$	48
parallelrisch	$\frac{x^{3}a^{3}\,\mathrm{expIntegral}_{1}(xa)-x^{2}a^{2}\mathrm{e}^{-xa}-2xa\,\mathrm{e}^{-xa}-2\,\mathrm{e}^{-xa}}{3a^{3}}$	48
meijerg	$\frac{-\frac{\left(-\frac{1}{3}+\gamma+\ln(x)+\ln(a)\right)x^{3}a^{3}}{3}-\frac{x^{3}a^{3}}{9}+\frac{2}{3}-\frac{\left(4a^{2}x^{2}+8xa+8\right)\mathrm{e}^{-xa}}{12}-\frac{a^{3}x^{3}\left(-\gamma-\ln(xa)-\mathrm{expIntegral}_{1}(xa)\right)}{3}}{a^{3}}$	76

input
```
int(x^2*Ei(1,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*Ei(1,x*a)-1/3/a^3*(x^2*a^2*exp(-x*a)+2*x*a*exp(-x*a)+2*exp(-x*a))
```

Fricas [F(-2)]

Exception generated.

$$\int x^2\Gamma(0,ax)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^2*exp_integral_e(1,a*x),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated ((x
)^(((2)::EXPR INT))*(exp_integral_e(((1)::EXPR INT),(a)*(x))):   There ar
e no library operations named exp_integral_e       Use HyperDoc Browse or
issue
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int x^2\Gamma(0,ax)\,dx = \begin{cases} \frac{x^3\operatorname{E}_1(ax)}{3} - \frac{x^2e^{-ax}}{3a} - \frac{2xe^{-ax}}{3a^2} - \frac{2e^{-ax}}{3a^3} & \text{for } a\neq 0 \\ \frac{x^3\operatorname{E}_1(0)}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*expint(1,a*x),x)
```

output
```
Piecewise((x**3*expint(1, a*x)/3 - x**2*exp(-a*x)/(3*a) - 2*x*exp(-a*x)/(3
*a**2) - 2*exp(-a*x)/(3*a**3), Ne(a, 0)), (x**3*expint(1, 0)/3, True))
```

Maxima [F]

$$\int x^2\Gamma(0,ax)\,dx = \int x^2E_1(ax)\,dx$$

input
```
integrate(x^2*exp_integral_e(1,a*x),x, algorithm="maxima")
```

output
```
integrate(x^2*exp_integral_e(1, a*x), x)
```

Giac [F]

$$\int x^2\Gamma(0,ax)\,dx = \int x^2E_1(ax)\,dx$$

input
```
integrate(x^2*exp_integral_e(1,a*x),x, algorithm="giac")
```

output
```
integrate(x^2*exp_integral_e(1, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int x^2\Gamma(0,ax)\,dx = \frac{x^3\,\mathrm{expint}(a\,x)}{3} - \frac{x^3\,\mathrm{e}^{-a\,x}\left(\frac{1}{a\,x}+\frac{2}{a^2\,x^2}+\frac{2}{a^3\,x^3}\right)}{3}$$

input
```
int(x^2*expint(a*x),x)
```

output
```
(x^3*expint(a*x))/3 - (x^3*exp(-a*x)*(1/(a*x) + 2/(a^2*x^2) + 2/(a^3*x^3))
)/3
```

Reduce [F]

$$\int x^2\Gamma(0,ax)\,dx = \int ei(1,ax)\,x^2dx$$

input
```
int(x^2*Ei(1,a*x),x)
```

output
```
int(ei(1,a*x)*x**2,x)
```

3.3 $\int x\Gamma(0,ax)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 25

$$\int x\Gamma(0,ax)\,dx = \frac{1}{2}x^2\Gamma(0,ax) - \frac{\Gamma(2,ax)}{2a^2}$$

output
```
1/2*x^2*Ei(1,a*x)-1/2*exp(-a*x)*(a*x+1)/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\Gamma(0,ax)\,dx = \frac{1}{2}x^2\Gamma(0,ax) - \frac{\Gamma(2,ax)}{2a^2}$$

input
```
Integrate[x*Gamma[0, a*x],x]
```

output
```
(x^2*Gamma[0, a*x])/2 - Gamma[2, a*x]/(2*a^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\Gamma(0,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{2}x^2\Gamma(0,ax)-\frac{\Gamma(2,ax)}{2a^2}$$

input
```
Int[x*Gamma[0, a*x],x]
```

output
```
(x^2*Gamma[0, a*x])/2 - Gamma[2, a*x]/(2*a^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

method	result	size
parts	$\frac{x^{2} \operatorname{expIntegral}_{1}\left(x a\right)}{2}+\frac{-x a \,{\mathrm e}^{-x a}-{\mathrm e}^{-x a}}{2 a^{2}}$	34
derivativedivides	$\frac{\frac{x^{2} a^{2} \operatorname{expIntegral}_{1}\left(x a\right)}{2}-\frac{x a \,{\mathrm e}^{-x a}}{2}-\frac{{\mathrm e}^{-x a}}{2}}{a^{2}}$	35
default	$\frac{\frac{x^{2} a^{2} \operatorname{expIntegral}_{1}\left(x a\right)}{2}-\frac{x a \,{\mathrm e}^{-x a}}{2}-\frac{{\mathrm e}^{-x a}}{2}}{a^{2}}$	35
parallelrisch	$\frac{x^{2} a^{2} \operatorname{expIntegral}_{1}\left(x a\right)-x a \,{\mathrm e}^{-x a}-{\mathrm e}^{-x a}}{2 a^{2}}$	35
meijerg	$\frac{-\frac{\left(-\frac{1}{2}+\gamma+\ln\left(x\right)+\ln\left(a\right)\right) x^{2} a^{2}}{2}-\frac{a^{2} x^{2}}{4}+\frac{1}{2}-\frac{\left(3 x a+3\right) {\mathrm e}^{-x a}}{6}-\frac{x^{2} a^{2} \left(-\gamma-\ln\left(x a\right)-\operatorname{expIntegral}_{1}\left(x a\right)\right)}{2}}{a^{2}}$	68

input
```
int(x*Ei(1,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*Ei(1,x*a)+1/2/a^2*(-x*a*exp(-x*a)-exp(-x*a))
```

Fricas [F(-2)]

Exception generated.

$$\int x\Gamma(0,ax)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*exp_integral_e(1,a*x),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated (x)
*(exp_integral_e(((1)::EXPR INT),(a)*(x))):   There are no library operati
ons named exp_integral_e       Use HyperDoc Browse or issue
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int x\Gamma(0,ax)\,dx = \begin{cases} \frac{x^2 \operatorname{E}_1(ax)}{2} - \frac{xe^{-ax}}{2a} - \frac{e^{-ax}}{2a^2} & \text{for } a \neq 0 \\ \frac{x^2 \operatorname{E}_1(0)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*expint(1,a*x),x)
```

output
```
Piecewise((x**2*expint(1, a*x)/2 - x*exp(-a*x)/(2*a) - exp(-a*x)/(2*a**2),
 Ne(a, 0)), (x**2*expint(1, 0)/2, True))
```

Maxima [F]

$$\int x\Gamma(0,ax)\,dx = \int xE_1(ax)\,dx$$

input
```
integrate(x*exp_integral_e(1,a*x),x, algorithm="maxima")
```

output
```
integrate(x*exp_integral_e(1, a*x), x)
```

Giac [F]

$$\int x\Gamma(0,ax)\,dx = \int xE_1(ax)\,dx$$

input
```
integrate(x*exp_integral_e(1,a*x),x, algorithm="giac")
```

output
```
integrate(x*exp_integral_e(1, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int x\Gamma(0,ax)\,dx = \frac{x^2\,\mathrm{expint}(a\,x)}{2} - \frac{x^2\,\mathrm{e}^{-a\,x}\,\left(\frac{1}{a\,x}+\frac{1}{a^2\,x^2}\right)}{2}$$

input
```
int(x*expint(a*x),x)
```

output
```
(x^2*expint(a*x))/2 - (x^2*exp(-a*x)*(1/(a*x) + 1/(a^2*x^2)))/2
```

Reduce [F]

$$\int x\Gamma(0,ax)\,dx = \int ei(1,ax)\,xdx$$

input
```
int(x*Ei(1,a*x),x)
```

output
```
int(ei(1,a*x)*x,x)
```

3.4 $\int \Gamma(0, ax)\, dx$

Optimal result

Integrand size = 5, antiderivative size = 19

$$\int \Gamma(0, ax)\, dx = -\frac{e^{-ax}}{a} + x\Gamma(0, ax)$$

output
```
-1/a/exp(a*x)+x*Ei(1,a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \Gamma(0, ax)\, dx = -\frac{e^{-ax}}{a} + x\Gamma(0, ax)$$

input
```
Integrate[Gamma[0, a*x],x]
```

output
```
-(1/(a*E^(a*x))) + x*Gamma[0, a*x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(0,ax)\,dx$$

$$\Big\downarrow \text{7111}$$

$$x\Gamma(0,ax)-\frac{e^{-ax}}{a}$$

input
```
Int[Gamma[0, a*x],x]
```

output
```
-(1/(a*E^(a*x))) + x*Gamma[0, a*x]
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

method	result	size
parts	$x \operatorname{expIntegral}_1(xa)-\frac{{\mathrm e}^{-xa}}{a}$	19
derivativedivides	$\frac{xa \operatorname{expIntegral}_1(xa)-{\mathrm e}^{-xa}}{a}$	21
default	$\frac{xa \operatorname{expIntegral}_1(xa)-{\mathrm e}^{-xa}}{a}$	21
parallelrisch	$\frac{xa \operatorname{expIntegral}_1(xa)-{\mathrm e}^{-xa}}{a}$	21
meijerg	$\frac{-(\gamma-1+\ln(x)+\ln(a))xa-xa+1-{\mathrm e}^{-xa}-ax(-\gamma-\ln(xa)-\operatorname{expIntegral}_1(xa))}{a}$	50

input
```
int(Ei(1,x*a),x,method=_RETURNVERBOSE)
```

output
```
x*Ei(1,x*a)-exp(-x*a)/a
```

Fricas [F(-2)]

Exception generated.

$$\int \Gamma(0,ax)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(exp_integral_e(1,a*x),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated exp
_integral_e(((1)::EXPR INT),(a)*(x)):   There are no library operations na
med exp_integral_e       Use HyperDoc Browse or issue
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \Gamma(0,ax)\,dx = \begin{cases} x\,\mathrm{E}_1\left(ax\right) - \frac{e^{-ax}}{a} & \text{for}\ a \neq 0 \\ x\,\mathrm{E}_1\left(0\right) & \text{otherwise} \end{cases}$$

input
```
integrate(expint(1,a*x),x)
```

output
```
Piecewise((x*expint(1, a*x) - exp(-a*x)/a, Ne(a, 0)), (x*expint(1, 0), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int \Gamma(0,ax)\,dx = -\frac{E_2(ax)}{a}$$

input
```
integrate(exp_integral_e(1,a*x),x, algorithm="maxima")
```

output
```
-exp_integral_e(2, a*x)/a
```

Giac [F]

$$\int \Gamma(0,ax)\,dx = \int E_1(ax)\,dx$$

input
```
integrate(exp_integral_e(1,a*x),x, algorithm="giac")
```

output
```
integrate(exp_integral_e(1, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \Gamma(0,ax)\,dx = x\,\mathrm{expint}(a\,x) - \frac{\mathrm{e}^{-a\,x}}{a}$$

input
```
int(expint(a*x),x)
```

output
```
x*expint(a*x) - exp(-a*x)/a
```

Reduce [F]

$$\int \Gamma(0,ax)\,dx = \int ei(1,ax)\,dx$$

input
```
int(Ei(1,a*x),x)
```

output
```
int(ei(1,a*x),x)
```

3.5 $\int \frac{\Gamma(0,ax)}{x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 32

$$\int \frac{\Gamma(0,ax)}{x} \, dx = ax\, {}_3F_3(1,1,1;2,2,2;-ax) - \gamma \log(x) - \frac{1}{2}\log^2(ax)$$

output
```
a*x*hypergeom([1, 1, 1],[2, 2, 2],-a*x)-gamma*ln(x)-1/2*ln(a*x)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 66 vs. 2(32) = 64.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.06

$$\begin{aligned}\int \frac{\Gamma(0,ax)}{x} \, dx = {} & ax\, {}_3F_3(1,1,1;2,2,2;-ax) + \Gamma(0,ax)\log(ax) \\ & + \text{ExpIntegralEi}(-ax)(-\log(x) + \log(ax)) \\ & + \frac{1}{2}\log(x)(-2\Gamma(0,ax) + \log(x) - 2(\gamma + \log(ax)))\end{aligned}$$

input
```
Integrate[Gamma[0, a*x]/x,x]
```

output
```
a*x*HypergeometricPFQ[{1, 1, 1}, {2, 2, 2}, -(a*x)] + Gamma[0, a*x]*Log[a*
x] + ExpIntegralEi[-(a*x)]*(-Log[x] + Log[a*x]) + (Log[x]*(-2*Gamma[0, a*x
] + Log[x] - 2*(EulerGamma + Log[a*x])))/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7112}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(0,ax)}{x}dx$$

$$\Big\downarrow \; 7112$$

$$ax\,{}_3F_3(1,1,1;2,2,2;-ax) - \frac{1}{2}\log^2(ax) - \gamma\log(x)$$

input
```
Int[Gamma[0, a*x]/x,x]
```

output
```
a*x*HypergeometricPFQ[{1, 1, 1}, {2, 2, 2}, -(a*x)] - EulerGamma*Log[x] -
Log[a*x]^2/2
```

Defintions of rubi rules used

rule 7112
```
Int[Gamma[0, (b_.)*(x_)]/(x_), x_Symbol] :> Simp[b*x*HypergeometricPFQ[{1,
1, 1}, {2, 2, 2}, (-b)*x], x] + (-Simp[EulerGamma*Log[x], x] - Simp[(1/2)*L
og[b*x]^2, x]) /; FreeQ[b, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.75

method	result
meijerg	$-\frac{\pi^2}{12}-\frac{\ln(a)^2}{2}-\frac{\ln(x)^2}{2}-\frac{\gamma^2}{2}-\gamma\ln(x)-\ln(a)\gamma-\ln(x)\ln(a)+ax\,\mathrm{hypergeom}([1,1,1],[2,2,2]$

input
```
int(Ei(1,x*a)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/12*Pi^2-1/2*ln(a)^2-1/2*ln(x)^2-1/2*gamma^2-gamma*ln(x)-ln(a)*gamma-ln(
x)*ln(a)+a*x*hypergeom([1,1,1],[2,2,2],-x*a)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\Gamma(0,ax)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp_integral_e(1,a*x)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated ((x
)^(((-1)::EXPR INT))*(exp_integral_e(((1)::EXPR INT),(a)*(x))):   There a
re no library operations named exp_integral_e      Use HyperDoc Browse or
 issue
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\Gamma(0,ax)}{x}\,dx = ax\,{}_3F_3\left(\begin{matrix}1,1,1\\2,2,2\end{matrix}\middle|-ax\right)-\frac{\log(ax)^2}{2}-\gamma\log(ax)$$

input
```
integrate(expint(1,a*x)/x,x)
```

output
```
a*x*hyper((1, 1, 1), (2, 2, 2), -a*x) - log(a*x)**2/2 - EulerGamma*log(a*x
)
```

Maxima [F]

$$\int \frac{\Gamma(0,ax)}{x}\,dx = \int \frac{E_1(ax)}{x}\,dx$$

input
```
integrate(exp_integral_e(1,a*x)/x,x, algorithm="maxima")
```

output
```
integrate(exp_integral_e(1, a*x)/x, x)
```

Giac [F]

$$\int \frac{\Gamma(0,ax)}{x}\,dx = \int \frac{E_1(ax)}{x}\,dx$$

input
```
integrate(exp_integral_e(1,a*x)/x,x, algorithm="giac")
```

output
```
integrate(exp_integral_e(1, a*x)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.31

$$\int \frac{\Gamma(0,ax)}{x}\,dx = \int \frac{\mathrm{expint}(a\,x)}{x}\,dx$$

input
```
int(expint(a*x)/x,x)
```

output
```
int(expint(a*x)/x, x)
```

Reduce [F]

$$\int \frac{\Gamma(0, ax)}{x} \, dx = \int \frac{ei(1, ax)}{x} dx$$

input
```
int(Ei(1,a*x)/x,x)
```

output
```
int(ei(1,a*x)/x,x)
```

3.6 $\int \frac{\Gamma(0,ax)}{x^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int \frac{\Gamma(0,ax)}{x^2}\,dx = a\Gamma(-1,ax) - \frac{\Gamma(0,ax)}{x}$$

output
```
1/x*Ei(2,a*x)-Ei(1,a*x)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(0,ax)}{x^2}\,dx = a\Gamma(-1,ax) - \frac{\Gamma(0,ax)}{x}$$

input
```
Integrate[Gamma[0, a*x]/x^2,x]
```

output
```
a*Gamma[-1, a*x] - Gamma[0, a*x]/x
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(0,ax)}{x^2} \, dx$$

$$\Big\downarrow \text{ 7116}$$

$$a\Gamma(-1,ax) - \frac{\Gamma(0,ax)}{x}$$

input
```
Int[Gamma[0, a*x]/x^2,x]
```

output
```
a*Gamma[-1, a*x] - Gamma[0, a*x]/x
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

method	result
parallelrisch	$-\frac{xa\,\text{expIntegral}_1(xa)+\text{expIntegral}_1(xa)-e^{-xa}}{x}$
parts	$-\frac{\text{expIntegral}_1(xa)}{x}+a\left(\frac{e^{-xa}}{xa}-\text{expIntegral}_1(xa)\right)$
derivativedivides	$a\left(-\frac{\text{expIntegral}_1(xa)}{xa}+\frac{e^{-xa}}{xa}-\text{expIntegral}_1(xa)\right)$
default	$a\left(-\frac{\text{expIntegral}_1(xa)}{xa}+\frac{e^{-xa}}{xa}-\text{expIntegral}_1(xa)\right)$
meijerg	$a\left(\frac{1+\gamma+\ln(x)+\ln(a)}{xa}+\gamma-2+\ln(x)+\ln(a)-\frac{-8xa+4}{4xa}+\frac{e^{-xa}}{xa}+\frac{(4xa+4)(-\gamma-\ln(xa)-\text{expIntegr}}{4xa}\right.$

input
```
int(Ei(1,x*a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*(x*a*Ei(1,x*a)+Ei(1,x*a)-exp(-x*a))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\Gamma(0,ax)}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp_integral_e(1,a*x)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated ((x
)^(((-2)::EXPR INT)))*(exp_integral_e(((1)::EXPR INT),(a)*(x))):   There a
re no library operations named exp_integral_e       Use HyperDoc Browse or
 issue
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\Gamma(0, ax)}{x^2} \, dx = -a \operatorname{E}_1(ax) - \frac{\operatorname{E}_1(ax)}{x} + \frac{e^{-ax}}{x}$$

input
```
integrate(expint(1,a*x)/x**2,x)
```

output
```
-a*expint(1, a*x) - expint(1, a*x)/x + exp(-a*x)/x
```

Maxima [F]

$$\int \frac{\Gamma(0, ax)}{x^2} \, dx = \int \frac{E_1(ax)}{x^2} \, dx$$

input
```
integrate(exp_integral_e(1,a*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(exp_integral_e(1, a*x)/x^2, x)
```

Giac [F]

$$\int \frac{\Gamma(0, ax)}{x^2} \, dx = \int \frac{E_1(ax)}{x^2} \, dx$$

input
```
integrate(exp_integral_e(1,a*x)/x^2,x, algorithm="giac")
```

output
```
integrate(exp_integral_e(1, a*x)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{\Gamma(0,ax)}{x^2}\,dx = \frac{e^{-a\,x}}{x} - \frac{\operatorname{expint}(a\,x)}{x} - a\operatorname{expint}(a\,x)$$

input
```
int(expint(a*x)/x^2,x)
```

output
```
exp(-a*x)/x - expint(a*x)/x - a*expint(a*x)
```

Reduce [F]

$$\int \frac{\Gamma(0,ax)}{x^2}\,dx = \int \frac{ei(1,ax)}{x^2}dx$$

input
```
int(Ei(1,a*x)/x^2,x)
```

output
```
int(ei(1,a*x)/x**2,x)
```

3.7 $\int \frac{\Gamma(0,ax)}{x^3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{\Gamma(0,ax)}{x^3}\,dx = \frac{1}{2}a^2\Gamma(-2,ax) - \frac{\Gamma(0,ax)}{2x^2}$$

output
```
1/2/x^2*Ei(3,a*x)-1/2*Ei(1,a*x)/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(0,ax)}{x^3}\,dx = \frac{1}{2}a^2\Gamma(-2,ax) - \frac{\Gamma(0,ax)}{2x^2}$$

input
```
Integrate[Gamma[0, a*x]/x^3,x]
```

output
```
(a^2*Gamma[-2, a*x])/2 - Gamma[0, a*x]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(0,ax)}{x^3} \, dx$$

$$\Big\downarrow \; 7116$$

$$\frac{1}{2}a^2\Gamma(-2,ax) - \frac{\Gamma(0,ax)}{2x^2}$$

input
```
Int[Gamma[0, a*x]/x^3,x]
```

output
```
(a^2*Gamma[-2, a*x])/2 - Gamma[0, a*x]/(2*x^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

method	result
parallelrisch	$\frac{x^{2} a^{2} \operatorname{expIntegral}_{1}(x a)-x a \,\mathrm{e}^{-x a}-2 \operatorname{expIntegral}_{1}(x a)+\mathrm{e}^{-x a}}{4 x^{2}}$
parts	$-\frac{\operatorname{expIntegral}_{1}(x a)}{2 x^{2}}-\frac{a^{2}\left(-\frac{\mathrm{e}^{-x a}}{2 x^{2} a^{2}}+\frac{\mathrm{e}^{-x a}}{2 x a}-\frac{\operatorname{expIntegral}_{1}(x a)}{2}\right)}{2}$
derivativedivides	$a^{2}\left(-\frac{\operatorname{expIntegral}_{1}(x a)}{2 x^{2} a^{2}}+\frac{\mathrm{e}^{-x a}}{4 x^{2} a^{2}}-\frac{\mathrm{e}^{-x a}}{4 x a}+\frac{\operatorname{expIntegral}_{1}(x a)}{4}\right)$
default	$a^{2}\left(-\frac{\operatorname{expIntegral}_{1}(x a)}{2 x^{2} a^{2}}+\frac{\mathrm{e}^{-x a}}{4 x^{2} a^{2}}-\frac{\mathrm{e}^{-x a}}{4 x a}+\frac{\operatorname{expIntegral}_{1}(x a)}{4}\right)$
meijerg	$a^{2}\left(\frac{\frac{1}{2}+\gamma+\ln(x)+\ln(a)}{2 x^{2} a^{2}}-\frac{1}{a x}+\frac{1}{2}-\frac{\gamma}{4}-\frac{\ln(x)}{4}-\frac{\ln(a)}{4}-\frac{18 a^{2} x^{2}-36 x a+9}{36 x^{2} a^{2}}+\frac{(-9 x a+9)\mathrm{e}^{-x a}}{36 x^{2} a^{2}}+\frac{(-9 a^{2} x\right.$

input
```
int(Ei(1,x*a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/x^2*(x^2*a^2*Ei(1,x*a)-x*a*exp(-x*a)-2*Ei(1,x*a)+exp(-x*a))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\Gamma(0,ax)}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp_integral_e(1,a*x)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated ((x
)^(((-3)::EXPR INT)))*(exp_integral_e(((1)::EXPR INT),(a)*(x))):   There a
re no library operations named exp_integral_e       Use HyperDoc Browse or
 issue
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(19) = 38.

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{\Gamma(0,ax)}{x^3}\,dx = \frac{a^2 \operatorname{E}_1(ax)}{4} - \frac{ae^{-ax}}{4x} - \frac{\operatorname{E}_1(ax)}{2x^2} + \frac{e^{-ax}}{4x^2}$$

input
```
integrate(expint(1,a*x)/x**3,x)
```

output
```
a**2*expint(1, a*x)/4 - a*exp(-a*x)/(4*x) - expint(1, a*x)/(2*x**2) + exp(
-a*x)/(4*x**2)
```

Maxima [F]

$$\int \frac{\Gamma(0,ax)}{x^3}\,dx = \int \frac{E_1(ax)}{x^3}\,dx$$

input
```
integrate(exp_integral_e(1,a*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(exp_integral_e(1, a*x)/x^3, x)
```

Giac [F]

$$\int \frac{\Gamma(0,ax)}{x^3}\,dx = \int \frac{E_1(ax)}{x^3}\,dx$$

input
```
integrate(exp_integral_e(1,a*x)/x^3,x, algorithm="giac")
```

output
```
integrate(exp_integral_e(1, a*x)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{\Gamma(0,ax)}{x^3}\,dx = -\frac{\mathrm{expint}(a\,x) - \mathrm{expint}(3,a\,x)}{2\,x^2}$$

input
```
int(expint(a*x)/x^3,x)
```

output
```
-(expint(a*x) - expint(3, a*x))/(2*x^2)
```

Reduce [F]

$$\int \frac{\Gamma(0,ax)}{x^3}\,dx = \int \frac{ei(1,ax)}{x^3}dx$$

input
```
int(Ei(1,a*x)/x^3,x)
```

output
```
int(ei(1,a*x)/x**3,x)
```

3.8 $\int \frac{\Gamma(0,ax)}{x^4} dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{\Gamma(0,ax)}{x^4} dx = \frac{1}{3}a^3\Gamma(-3,ax) - \frac{\Gamma(0,ax)}{3x^3}$$

output
```
1/3/x^3*Ei(4,a*x)-1/3*Ei(1,a*x)/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(0,ax)}{x^4} dx = \frac{1}{3}a^3\Gamma(-3,ax) - \frac{\Gamma(0,ax)}{3x^3}$$

input
```
Integrate[Gamma[0, a*x]/x^4,x]
```

output
```
(a^3*Gamma[-3, a*x])/3 - Gamma[0, a*x]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(0,ax)}{x^4} \, dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{3}a^3\Gamma(-3,ax) - \frac{\Gamma(0,ax)}{3x^3}$$

input
```
Int[Gamma[0, a*x]/x^4,x]
```

output
```
(a^3*Gamma[-3, a*x])/3 - Gamma[0, a*x]/(3*x^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(21) = 42.

Time = 0.70 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

method	result
parallelrisch	$-\frac{x^3a^3\,\mathrm{expIntegral}_1(xa)-x^2a^2\mathrm{e}^{-xa}+xa\,\mathrm{e}^{-xa}+6\,\mathrm{expIntegral}_1(xa)-2\,\mathrm{e}^{-xa}}{18x^3}$
parts	$-\frac{\mathrm{expIntegral}_1(xa)}{3x^3}+\frac{a^3\left(\frac{\mathrm{e}^{-xa}}{3x^3a^3}-\frac{\mathrm{e}^{-xa}}{6x^2a^2}+\frac{\mathrm{e}^{-xa}}{6xa}-\frac{\mathrm{expIntegral}_1(xa)}{6}\right)}{3}$
derivativedivides	$a^3\left(-\frac{\mathrm{expIntegral}_1(xa)}{3x^3a^3}+\frac{\mathrm{e}^{-xa}}{9x^3a^3}-\frac{\mathrm{e}^{-xa}}{18x^2a^2}+\frac{\mathrm{e}^{-xa}}{18xa}-\frac{\mathrm{expIntegral}_1(xa)}{18}\right)$
default	$a^3\left(-\frac{\mathrm{expIntegral}_1(xa)}{3x^3a^3}+\frac{\mathrm{e}^{-xa}}{9x^3a^3}-\frac{\mathrm{e}^{-xa}}{18x^2a^2}+\frac{\mathrm{e}^{-xa}}{18xa}-\frac{\mathrm{expIntegral}_1(xa)}{18}\right)$
meijerg	$a^3\left(\frac{\frac{1}{3}+\gamma+\ln(x)+\ln(a)}{3x^3a^3}-\frac{1}{2x^2a^2}+\frac{1}{4ax}-\frac{13}{108}+\frac{\gamma}{18}+\frac{\ln(x)}{18}+\frac{\ln(a)}{18}-\frac{-104x^3a^3+216a^2x^2-432xa+96}{864x^3a^3}\right.$

input
```
int(Ei(1,x*a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/18/x^3*(x^3*a^3*Ei(1,x*a)-x^2*a^2*exp(-x*a)+x*a*exp(-x*a)+6*Ei(1,x*a)-2
*exp(-x*a))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\Gamma(0,ax)}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp_integral_e(1,a*x)/x^4,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated ((x
)^(((-4)::EXPR INT)))*(exp_integral_e(((1)::EXPR INT),(a)*(x))):   There a
re no library operations named exp_integral_e       Use HyperDoc Browse or
 issue
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(19) = 38$.

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \frac{\Gamma(0,ax)}{x^4}\,dx = -\frac{a^3\,\mathrm{E}_1(ax)}{18} + \frac{a^2e^{-ax}}{18x} - \frac{ae^{-ax}}{18x^2} - \frac{\mathrm{E}_1(ax)}{3x^3} + \frac{e^{-ax}}{9x^3}$$

input
```
integrate(expint(1,a*x)/x**4,x)
```

output
```
-a**3*expint(1, a*x)/18 + a**2*exp(-a*x)/(18*x) - a*exp(-a*x)/(18*x**2) -
expint(1, a*x)/(3*x**3) + exp(-a*x)/(9*x**3)
```

Maxima [F]

$$\int \frac{\Gamma(0,ax)}{x^4}\,dx = \int \frac{E_1(ax)}{x^4}\,dx$$

input
```
integrate(exp_integral_e(1,a*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(exp_integral_e(1, a*x)/x^4, x)
```

Giac [F]

$$\int \frac{\Gamma(0,ax)}{x^4}\,dx = \int \frac{E_1(ax)}{x^4}\,dx$$

input
```
integrate(exp_integral_e(1,a*x)/x^4,x, algorithm="giac")
```

output
```
integrate(exp_integral_e(1, a*x)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{\Gamma(0, ax)}{x^4} \, dx = -\frac{\mathrm{expint}(a\, x) - \mathrm{expint}(4, a\, x)}{3\, x^3}$$

input
```
int(expint(a*x)/x^4,x)
```

output
```
-(expint(a*x) - expint(4, a*x))/(3*x^3)
```

Reduce [F]

$$\int \frac{\Gamma(0, ax)}{x^4} \, dx = \int \frac{ei(1, ax)}{x^4} dx$$

input
```
int(Ei(1,a*x)/x^4,x)
```

output
```
int(ei(1,a*x)/x**4,x)
```

3.9 $\int e^{-ax}x^3\,dx$

Optimal result

Integrand size = 10, antiderivative size = 52

$$\int e^{-ax}x^3\,dx = -\frac{6e^{-ax}}{a^4} - \frac{6e^{-ax}x}{a^3} - \frac{3e^{-ax}x^2}{a^2} - \frac{e^{-ax}x^3}{a}$$

output
```
-6/a^4/exp(a*x)-6*x/a^3/exp(a*x)-3*x^2/a^2/exp(a*x)-x^3/a/exp(a*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int e^{-ax}x^3\,dx = -\frac{e^{-ax}(6 + 6ax + 3a^2x^2 + a^3x^3)}{a^4}$$

input
```
Integrate[x^3/E^(a*x),x]
```

output
```
-((6 + 6*a*x + 3*a^2*x^2 + a^3*x^3)/(a^4*E^(a*x)))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-ax} \, dx$$

$$\Big\downarrow \text{2607}$$

$$\frac{3 \int e^{-ax} x^2 dx}{a} - \frac{x^3 e^{-ax}}{a}$$

$$\Big\downarrow \text{2607}$$

$$\frac{3 \left(\frac{2 \int e^{-ax} x dx}{a} - \frac{x^2 e^{-ax}}{a}\right)}{a} - \frac{x^3 e^{-ax}}{a}$$

$$\Big\downarrow \text{2607}$$

$$\frac{3 \left(\frac{2 \left(\frac{\int e^{-ax} dx}{a} - \frac{x e^{-ax}}{a}\right)}{a} - \frac{x^2 e^{-ax}}{a}\right)}{a} - \frac{x^3 e^{-ax}}{a}$$

$$\Big\downarrow \text{2624}$$

$$\frac{3 \left(\frac{2 \left(-\frac{e^{-ax}}{a^2} - \frac{x e^{-ax}}{a}\right)}{a} - \frac{x^2 e^{-ax}}{a}\right)}{a} - \frac{x^3 e^{-ax}}{a}$$

input
```
Int[x^3/E^(a*x),x]
```

output
```
-(x^3/(a*E^(a*x))) + (3*(-(x^2/(a*E^(a*x))) + (2*(-(1/(a^2*E^(a*x))) - x/(
a*E^(a*x))))/a))/a
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{\left(x^{3} a^{3}+3 a^{2} x^{2}+6 x a+6\right) {\mathrm e}^{-x a}}{a^{4}}$	32
gosper	$-\frac{\left(x^{3} a^{3}+3 a^{2} x^{2}+6 x a+6\right) {\mathrm e}^{-x a}}{a^{4}}$	33
parallelrisch	$\frac{\left(-x^{3} a^{3}-3 a^{2} x^{2}-6 x a-6\right) {\mathrm e}^{-x a}}{a^{4}}$	33
orering	$-\frac{\left(x^{3} a^{3}+3 a^{2} x^{2}+6 x a+6\right) {\mathrm e}^{-x a}}{a^{4}}$	33
norman	$\left(-\frac{x^{3}}{a}-\frac{3 x^{2}}{a^{2}}-\frac{6 x}{a^{3}}-\frac{6}{a^{4}}\right) {\mathrm e}^{-x a}$	36
meijerg	$\frac{6-\frac{\left(4 x^{3} a^{3}+12 a^{2} x^{2}+24 x a+24\right) {\mathrm e}^{-x a}}{4}}{a^{4}}$	36
derivativedivides	$\frac{-x^{3} a^{3} {\mathrm e}^{-x a}-3 x^{2} a^{2} {\mathrm e}^{-x a}-6 x a \,{\mathrm e}^{-x a}-6 \,{\mathrm e}^{-x a}}{a^{4}}$	52
default	$\frac{-x^{3} a^{3} {\mathrm e}^{-x a}-3 x^{2} a^{2} {\mathrm e}^{-x a}-6 x a \,{\mathrm e}^{-x a}-6 \,{\mathrm e}^{-x a}}{a^{4}}$	52

input
```
int(x^3/exp(x*a),x,method=_RETURNVERBOSE)
```

output
```
-(a^3*x^3+3*a^2*x^2+6*a*x+6)/a^4*exp(-x*a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int e^{-ax}x^3\,dx = -\frac{(a^3x^3 + 3\,a^2x^2 + 6\,ax + 6)e^{(-ax)}}{a^4}$$

input
```
integrate(x^3/exp(a*x),x, algorithm="fricas")
```

output
```
-(a^3*x^3 + 3*a^2*x^2 + 6*a*x + 6)*e^(-a*x)/a^4
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.71

$$\int e^{-ax}x^3\,dx = \begin{cases} \frac{(-a^3x^3-3a^2x^2-6ax-6)e^{-ax}}{a^4} & \text{for } a^4 \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/exp(a*x),x)
```

output
```
Piecewise(((-a**3*x**3 - 3*a**2*x**2 - 6*a*x - 6)*exp(-a*x)/a**4, Ne(a**4,
 0)), (x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int e^{-ax}x^3\,dx = -\frac{(a^3x^3 + 3\,a^2x^2 + 6\,ax + 6)e^{(-ax)}}{a^4}$$

input
```
integrate(x^3/exp(a*x),x, algorithm="maxima")
```

output
```
-(a^3*x^3 + 3*a^2*x^2 + 6*a*x + 6)*e^(-a*x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int e^{-ax}x^3\,dx = -\frac{(a^3x^3 + 3\,a^2x^2 + 6\,ax + 6)e^{(-ax)}}{a^4}$$

input
```
integrate(x^3/exp(a*x),x, algorithm="giac")
```

output
```
-(a^3*x^3 + 3*a^2*x^2 + 6*a*x + 6)*e^(-a*x)/a^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int e^{-ax}x^3\,dx = -\frac{\mathrm{e}^{-a\,x}\,\left(a^3\,x^3 + 3\,a^2\,x^2 + 6\,a\,x + 6\right)}{a^4}$$

input
```
int(x^3*exp(-a*x),x)
```

output
```
-(exp(-a*x)*(6*a*x + 3*a^2*x^2 + a^3*x^3 + 6))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.63

$$\int e^{-ax}x^3\,dx = \frac{-a^3x^3 - 3a^2x^2 - 6ax - 6}{e^{ax}a^4}$$

input
```
int(x^3/exp(a*x),x)
```

output
```
( - a**3*x**3 - 3*a**2*x**2 - 6*a*x - 6)/(e**(a*x)*a**4)
```

3.10 $\int e^{-ax} x^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 38

$$\int e^{-ax} x^2 \, dx = -\frac{2e^{-ax}}{a^3} - \frac{2e^{-ax}x}{a^2} - \frac{e^{-ax}x^2}{a}$$

output
```
-2/a^3/exp(a*x)-2*x/a^2/exp(a*x)-x^2/a/exp(a*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int e^{-ax} x^2 \, dx = -\frac{e^{-ax}(2 + 2ax + a^2x^2)}{a^3}$$

input
```
Integrate[x^2/E^(a*x),x]
```

output
```
-((2 + 2*a*x + a^2*x^2)/(a^3*E^(a*x)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-ax} \, dx$$

$$\Big\downarrow \text{2607}$$

$$\frac{2 \int e^{-ax} x dx}{a} - \frac{x^2 e^{-ax}}{a}$$

$$\Big\downarrow \text{2607}$$

$$\frac{2\left(\frac{\int e^{-ax} dx}{a} - \frac{xe^{-ax}}{a}\right)}{a} - \frac{x^2 e^{-ax}}{a}$$

$$\Big\downarrow \text{2624}$$

$$\frac{2\left(-\frac{e^{-ax}}{a^2} - \frac{xe^{-ax}}{a}\right)}{a} - \frac{x^2 e^{-ax}}{a}$$

input
```
Int[x^2/E^(a*x),x]
```

output
```
-(x^2/(a*E^(a*x))) + (2*(-(1/(a^2*E^(a*x))) - x/(a*E^(a*x))))/a
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

method	result	size
risch	$-\frac{\left(a^{2} x^{2}+2 x a +2\right) {\mathrm e}^{-x a}}{a^{3}}$	24
gosper	$-\frac{\left(a^{2} x^{2}+2 x a +2\right) {\mathrm e}^{-x a}}{a^{3}}$	25
parallelrisch	$\frac{\left(-a^{2} x^{2}-2 x a -2\right) {\mathrm e}^{-x a}}{a^{3}}$	25
orering	$-\frac{\left(a^{2} x^{2}+2 x a +2\right) {\mathrm e}^{-x a}}{a^{3}}$	25
norman	$\left(-\frac{x^{2}}{a}-\frac{2 x}{a^{2}}-\frac{2}{a^{3}}\right) {\mathrm e}^{-x a}$	28
meijerg	$\frac{2-\frac{\left(3 a^{2} x^{2}+6 x a +6\right) {\mathrm e}^{-x a}}{3}}{a^{3}}$	28
derivativedivides	$\frac{-x^{2} a^{2} {\mathrm e}^{-x a}-2 x a \,{\mathrm e}^{-x a}-2 \,{\mathrm e}^{-x a}}{a^{3}}$	38
default	$\frac{-x^{2} a^{2} {\mathrm e}^{-x a}-2 x a \,{\mathrm e}^{-x a}-2 \,{\mathrm e}^{-x a}}{a^{3}}$	38

input
```
int(x^2/exp(x*a),x,method=_RETURNVERBOSE)
```

output
```
-(a^2*x^2+2*a*x+2)/a^3*exp(-x*a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int e^{-ax}x^2\,dx = -\frac{(a^2x^2 + 2\,ax + 2)e^{(-ax)}}{a^3}$$

input
```
integrate(x^2/exp(a*x),x, algorithm="fricas")
```

output
```
-(a^2*x^2 + 2*a*x + 2)*e^(-a*x)/a^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int e^{-ax}x^2\,dx = \begin{cases} \frac{(-a^2x^2-2ax-2)e^{-ax}}{a^3} & \text{for } a^3 \neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/exp(a*x),x)
```

output
```
Piecewise(((-a**2*x**2 - 2*a*x - 2)*exp(-a*x)/a**3, Ne(a**3, 0)), (x**3/3,
 True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int e^{-ax}x^2\,dx = -\frac{(a^2x^2 + 2\,ax + 2)e^{(-ax)}}{a^3}$$

input
```
integrate(x^2/exp(a*x),x, algorithm="maxima")
```

output
```
-(a^2*x^2 + 2*a*x + 2)*e^(-a*x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int e^{-ax}x^2\,dx = -\frac{(a^2x^2 + 2\,ax + 2)e^{(-ax)}}{a^3}$$

input
```
integrate(x^2/exp(a*x),x, algorithm="giac")
```

output
```
-(a^2*x^2 + 2*a*x + 2)*e^(-a*x)/a^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int e^{-ax}x^2\,dx = -\frac{\mathrm{e}^{-a\,x}\,(a^2\,x^2 + 2\,a\,x + 2)}{a^3}$$

input
```
int(x^2*exp(-a*x),x)
```

output
```
-(exp(-a*x)*(2*a*x + a^2*x^2 + 2))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int e^{-ax}x^2\,dx = \frac{-a^2x^2 - 2ax - 2}{e^{ax}a^3}$$

input
```
int(x^2/exp(a*x),x)
```

output
```
( - a**2*x**2 - 2*a*x - 2)/(e**(a*x)*a**3)
```

3.11 $\int e^{-ax} x \, dx$

Optimal result

Integrand size = 8, antiderivative size = 24

$$\int e^{-ax} x \, dx = -\frac{e^{-ax}}{a^2} - \frac{e^{-ax} x}{a}$$

output
```
-1/a^2/exp(a*x)-x/a/exp(a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int e^{-ax} x \, dx = -\frac{e^{-ax}(1 + ax)}{a^2}$$

input
```
Integrate[x/E^(a*x),x]
```

output
```
-((1 + a*x)/(a^2*E^(a*x)))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-ax}\,dx$$

$$\Big\downarrow \text{2607}$$

$$\frac{\int e^{-ax}dx}{a}-\frac{xe^{-ax}}{a}$$

$$\Big\downarrow \text{2624}$$

$$-\frac{e^{-ax}}{a^2}-\frac{xe^{-ax}}{a}$$

input
```
Int[x/E^(a*x),x]
```

output
```
-(1/(a^2*E^(a*x))) - x/(a*E^(a*x))
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{e^{-xa} (xa+1)}{a^{2}}$	16
gosper	$-\frac{e^{-xa} (xa+1)}{a^{2}}$	17
parallelrisch	$\frac{(-xa-1) e^{-xa}}{a^{2}}$	17
orering	$-\frac{e^{-xa} (xa+1)}{a^{2}}$	17
norman	$\left(-\frac{x}{a}-\frac{1}{a^{2}}\right) e^{-xa}$	20
meijerg	$\frac{1-\frac{(2xa+2) e^{-xa}}{2}}{a^{2}}$	20
derivativedivides	$\frac{-xa\,e^{-xa}-e^{-xa}}{a^{2}}$	24
default	$\frac{-xa\,e^{-xa}-e^{-xa}}{a^{2}}$	24

input
```
int(x/exp(x*a),x,method=_RETURNVERBOSE)
```

output
```
-exp(-x*a)*(a*x+1)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int e^{-ax}x\,dx = -\frac{(ax+1)e^{(-ax)}}{a^2}$$

input
```
integrate(x/exp(a*x),x, algorithm="fricas")
```

output
```
-(a*x + 1)*e^(-a*x)/a^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int e^{-ax}x\,dx = \begin{cases} \frac{(-ax-1)e^{-ax}}{a^2} & \text{for } a^2 \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x/exp(a*x),x)
```

output
```
Piecewise(((-a*x - 1)*exp(-a*x)/a**2, Ne(a**2, 0)), (x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int e^{-ax}x\,dx = -\frac{(ax+1)e^{(-ax)}}{a^2}$$

input
```
integrate(x/exp(a*x),x, algorithm="maxima")
```

output
```
-(a*x + 1)*e^(-a*x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int e^{-ax}x\,dx = -\frac{(ax+1)e^{(-ax)}}{a^2}$$

input
```
integrate(x/exp(a*x),x, algorithm="giac")
```

output
```
-(a*x + 1)*e^(-a*x)/a^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int e^{-ax}x\,dx = -\frac{\mathrm{e}^{-a\,x}\,(a\,x+1)}{a^2}$$

input
```
int(x*exp(-a*x),x)
```

output
```
-(exp(-a*x)*(a*x + 1))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int e^{-ax}x\,dx = \frac{-ax-1}{e^{ax}a^2}$$

input
```
int(x/exp(a*x),x)
```

output
```
( - (a*x + 1))/(e**(a*x)*a**2)
```

3.12 $\int e^{-ax}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int e^{-ax}\,dx=-\frac{e^{-ax}}{a}$$

output
```
-1/a/exp(a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{-ax}\,dx=-\frac{e^{-ax}}{a}$$

input
```
Integrate[E^(-(a*x)),x]
```

output
```
-(1/(a*E^(a*x)))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{2624\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-ax}\,dx$$

$$\Big\downarrow \; 2624$$

$$-\frac{e^{-ax}}{a}$$

input
```
Int[E^(-(a*x)),x]
```

output
```
-(1/(a*E^(a*x)))
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{e^{-xa}}{a}$	11
derivativedivides	$-\frac{e^{-xa}}{a}$	11
default	$-\frac{e^{-xa}}{a}$	11
norman	$-\frac{e^{-xa}}{a}$	11
risch	$-\frac{e^{-xa}}{a}$	11
parallelrisch	$-\frac{e^{-xa}}{a}$	11
orering	$-\frac{e^{-xa}}{a}$	11
meijerg	$\frac{1-e^{-xa}}{a}$	14

input
```
int(exp(-x*a),x,method=_RETURNVERBOSE)
```

output
```
-exp(-x*a)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^{-ax}\,dx = -\frac{e^{(-ax)}}{a}$$

input
```
integrate(exp(-a*x),x, algorithm="fricas")
```

output
```
-e^(-a*x)/a
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^{-ax}\,dx = \begin{cases} -\frac{e^{-ax}}{a} & \text{for } a \neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(exp(-a*x),x)
```

output
```
Piecewise((-exp(-a*x)/a, Ne(a, 0)), (x, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^{-ax}\,dx = -\frac{e^{(-ax)}}{a}$$

input
```
integrate(exp(-a*x),x, algorithm="maxima")
```

output
```
-e^(-a*x)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^{-ax}\,dx = -\frac{e^{(-ax)}}{a}$$

input
```
integrate(exp(-a*x),x, algorithm="giac")
```

output
```
-e^(-a*x)/a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^{-ax}\,dx = -\frac{\mathrm{e}^{-a\,x}}{a}$$

input
```
int(exp(-a*x),x)
```

output
```
-exp(-a*x)/a
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int e^{-ax}\,dx = -\frac{1}{e^{ax}a}$$

input
```
int(exp(-a*x),x)
```

output
```
( - 1)/(e**(a*x)*a)
```

3.13 $\int \frac{e^{-ax}}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 5

$$\int \frac{e^{-ax}}{x} dx = \text{ExpIntegralEi}(-ax)$$

output
```
Ei(-a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{e^{-ax}}{x} dx = \text{ExpIntegralEi}(-ax)$$

input
```
Integrate[1/(E^(a*x)*x),x]
```

output
```
ExpIntegralEi[-(a*x)]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-ax}}{x} \, dx$$

$$\Big\downarrow \text{2609}$$

$$\text{ExpIntegralEi}(-ax)$$

input
```
Int[1/(E^(a*x)*x),x]
```

output
```
ExpIntegralEi[-(a*x)]
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

method	result	size
derivativedivides	$-\text{expIntegral}_1(xa)$	8
default	$-\text{expIntegral}_1(xa)$	8
risch	$-\text{expIntegral}_1(xa)$	8
meijerg	$\ln(x) + \ln(a) - \ln(xa) - \text{expIntegral}_1(xa)$	19

input
```
int(1/exp(x*a)/x,x,method=_RETURNVERBOSE)
```

output
```
-Ei(1,x*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{e^{-ax}}{x} \, dx = \text{Ei}(-ax)$$

input
```
integrate(1/exp(a*x)/x,x, algorithm="fricas")
```

output
```
Ei(-a*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

$$\int \frac{e^{-ax}}{x} \, dx = \text{Ei}\left(axe^{i\pi}\right)$$

input
```
integrate(1/exp(a*x)/x,x)
```

output
```
Ei(a*x*exp_polar(I*pi))
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{e^{-ax}}{x}\,dx = \text{Ei}(-ax)$$

input
```
integrate(1/exp(a*x)/x,x, algorithm="maxima")
```

output
```
Ei(-a*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{e^{-ax}}{x}\,dx = \text{Ei}(-ax)$$

input
```
integrate(1/exp(a*x)/x,x, algorithm="giac")
```

output
```
Ei(-a*x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int \frac{e^{-ax}}{x}\,dx = -\text{expint}(a\,x)$$

input
```
int(exp(-a*x)/x,x)
```

output
```
-expint(a*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{e^{-ax}}{x} \, dx = ei(-ax)$$

input
```
int(1/exp(a*x)/x,x)
```

output
```
ei( - a*x)
```

3.14 $\int \frac{e^{-ax}}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 20

$$\int \frac{e^{-ax}}{x^2} dx = -\frac{e^{-ax}}{x} - a\,\text{ExpIntegralEi}(-ax)$$

output
```
-1/exp(a*x)/x-a*Ei(-a*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^{-ax}}{x^2} dx = -\frac{e^{-ax}}{x} - a\,\text{ExpIntegralEi}(-ax)$$

input
```
Integrate[1/(E^(a*x)*x^2),x]
```

output
```
-(1/(E^(a*x)*x)) - a*ExpIntegralEi[-(a*x)]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-ax}}{x^2} dx$$

$$\Big\downarrow \text{2608}$$

$$-a \int \frac{e^{-ax}}{x} dx - \frac{e^{-ax}}{x}$$

$$\Big\downarrow \text{2609}$$

$$-a \operatorname{ExpIntegralEi}(-ax) - \frac{e^{-ax}}{x}$$

input
```
Int[1/(E^(a*x)*x^2),x]
```

output
```
-(1/(E^(a*x)*x)) - a*ExpIntegralEi[-(a*x)]
```

Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{e^{-xa}}{x}+a\,\mathrm{expIntegral}_1(xa)$	19
derivativedivides	$a\left(-\frac{e^{-xa}}{xa}+\mathrm{expIntegral}_1(xa)\right)$	23
default	$a\left(-\frac{e^{-xa}}{xa}+\mathrm{expIntegral}_1(xa)\right)$	23
meijerg	$a\left(-\frac{1}{ax}+1-\ln(x)-\ln(a)+\frac{-2xa+2}{2xa}-\frac{e^{-xa}}{xa}+\ln(xa)+\mathrm{expIntegral}_1(xa)\right)$	57

input
```
int(1/exp(x*a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-exp(-x*a)/x+a*Ei(1,x*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{e^{-ax}}{x^2}\,dx=-\frac{ax\mathrm{Ei}(-ax)+e^{(-ax)}}{x}$$

input
```
integrate(1/exp(a*x)/x^2,x, algorithm="fricas")
```

output
```
-(a*x*Ei(-a*x) + e^(-a*x))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^{-ax}}{x^2}\,dx = -a\,\mathrm{Ei}\left(axe^{i\pi}\right) - \frac{e^{-ax}}{x}$$

input
```
integrate(1/exp(a*x)/x**2,x)
```

output
```
-a*Ei(a*x*exp_polar(I*pi)) - exp(-a*x)/x
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.40

$$\int \frac{e^{-ax}}{x^2}\,dx = -a\Gamma(-1, ax)$$

input
```
integrate(1/exp(a*x)/x^2,x, algorithm="maxima")
```

output
```
-a*gamma(-1, a*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{e^{-ax}}{x^2}\,dx = -\frac{ax\mathrm{Ei}(-ax) + e^{(-ax)}}{x}$$

input
```
integrate(1/exp(a*x)/x^2,x, algorithm="giac")
```

output
```
-(a*x*Ei(-a*x) + e^(-a*x))/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{e^{-ax}}{x^2} \, dx = a \operatorname{expint}(a\,x) - \frac{\mathrm{e}^{-a\,x}}{x}$$

input
```
int(exp(-a*x)/x^2,x)
```

output
```
a*expint(a*x) - exp(-a*x)/x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{e^{-ax}}{x^2} \, dx = \frac{-e^{ax}\,ei(-ax)\,ax - 1}{e^{ax}x}$$

input
```
int(1/exp(a*x)/x^2,x)
```

output
```
( - (e**(a*x)*ei( - a*x)*a*x + 1))/(e**(a*x)*x)
```

3.15 $\int \frac{e^{-ax}}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 40

$$\int \frac{e^{-ax}}{x^3} dx = -\frac{e^{-ax}}{2x^2} + \frac{ae^{-ax}}{2x} + \frac{1}{2}a^2 \operatorname{ExpIntegralEi}(-ax)$$

output
```
-1/2/exp(a*x)/x^2+1/2*a/exp(a*x)/x+1/2*a^2*Ei(-a*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{e^{-ax}}{x^3} dx = e^{-ax}\left(-\frac{1}{2x^2} + \frac{a}{2x}\right) + \frac{1}{2}a^2 \operatorname{ExpIntegralEi}(-ax)$$

input
```
Integrate[1/(E^(a*x)*x^3),x]
```

output
```
(-1/2*1/x^2 + a/(2*x))/E^(a*x) + (a^2*ExpIntegralEi[-(a*x)])/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2608, 2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-ax}}{x^3} \, dx$$

$$\Big\downarrow \text{2608}$$

$$-\frac{1}{2}a\int \frac{e^{-ax}}{x^2}dx - \frac{e^{-ax}}{2x^2}$$

$$\Big\downarrow \text{2608}$$

$$-\frac{1}{2}a\left(-a\int \frac{e^{-ax}}{x}dx - \frac{e^{-ax}}{x}\right) - \frac{e^{-ax}}{2x^2}$$

$$\Big\downarrow \text{2609}$$

$$-\frac{1}{2}a\left(-a\,\text{ExpIntegralEi}(-ax) - \frac{e^{-ax}}{x}\right) - \frac{e^{-ax}}{2x^2}$$

input
```
Int[1/(E^(a*x)*x^3),x]
```

output
```
-1/2*1/(E^(a*x)*x^2) - (a*(-(1/(E^(a*x)*x)) - a*ExpIntegralEi[-(a*x)]))/2
```

Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{\mathrm{e}^{-xa}}{2x^2}+\frac{a\,\mathrm{e}^{-xa}}{2x}-\frac{a^2\,\mathrm{expIntegral}_1(xa)}{2}$
derivativedivides	$a^2\left(-\frac{\mathrm{e}^{-xa}}{2x^2a^2}+\frac{\mathrm{e}^{-xa}}{2xa}-\frac{\mathrm{expIntegral}_1(xa)}{2}\right)$
default	$a^2\left(-\frac{\mathrm{e}^{-xa}}{2x^2a^2}+\frac{\mathrm{e}^{-xa}}{2xa}-\frac{\mathrm{expIntegral}_1(xa)}{2}\right)$
meijerg	$a^2\left(-\frac{1}{2x^2a^2}+\frac{1}{ax}-\frac{3}{4}+\frac{\ln(x)}{2}+\frac{\ln(a)}{2}+\frac{9a^2x^2-12xa+6}{12x^2a^2}-\frac{(-3xa+3)\mathrm{e}^{-xa}}{6x^2a^2}-\frac{\ln(xa)}{2}-\frac{\mathrm{expIntegral}_1(x}{2}\right.$

input
```
int(1/exp(x*a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(-x*a)/x^2+1/2*a*exp(-x*a)/x-1/2*a^2*Ei(1,x*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{e^{-ax}}{x^3}\,dx=\frac{a^2x^2\mathrm{Ei}(-ax)+(ax-1)e^{(-ax)}}{2\,x^2}$$

input
```
integrate(1/exp(a*x)/x^3,x, algorithm="fricas")
```

output
```
1/2*(a^2*x^2*Ei(-a*x) + (a*x - 1)*e^(-a*x))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{e^{-ax}}{x^3}\,dx = \frac{a^2\operatorname{Ei}\left(axe^{i\pi}\right)}{2} + \frac{ae^{-ax}}{2x} - \frac{e^{-ax}}{2x^2}$$

input
```
integrate(1/exp(a*x)/x**3,x)
```

output
```
a**2*Ei(a*x*exp_polar(I*pi))/2 + a*exp(-a*x)/(2*x) - exp(-a*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.25

$$\int \frac{e^{-ax}}{x^3}\,dx = -a^2\Gamma(-2, ax)$$

input
```
integrate(1/exp(a*x)/x^3,x, algorithm="maxima")
```

output
```
-a^2*gamma(-2, a*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{e^{-ax}}{x^3}\,dx = \frac{a^2x^2\operatorname{Ei}(-ax) + axe^{(-ax)} - e^{(-ax)}}{2\,x^2}$$

input
```
integrate(1/exp(a*x)/x^3,x, algorithm="giac")
```

output
```
1/2*(a^2*x^2*Ei(-a*x) + a*x*e^(-a*x) - e^(-a*x))/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{e^{-ax}}{x^3}\,dx = a^2\,\mathrm{e}^{-a\,x}\,\left(\frac{1}{2\,a\,x}-\frac{1}{2\,a^2\,x^2}\right)-\frac{a^2\,\mathrm{expint}(a\,x)}{2}$$

input
```
int(exp(-a*x)/x^3,x)
```

output
```
a^2*exp(-a*x)*(1/(2*a*x) - 1/(2*a^2*x^2)) - (a^2*expint(a*x))/2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{-ax}}{x^3}\,dx = \frac{e^{ax}\,ei(-ax)\,a^2x^2+ax-1}{2e^{ax}x^2}$$

input
```
int(1/exp(a*x)/x^3,x)
```

output
```
(e**(a*x)*ei( - a*x)*a**2*x**2 + a*x - 1)/(2*e**(a*x)*x**2)
```

3.16 $\int \frac{e^{-ax}}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 56

$$\int \frac{e^{-ax}}{x^4}\,dx = -\frac{e^{-ax}}{3x^3} + \frac{ae^{-ax}}{6x^2} - \frac{a^2e^{-ax}}{6x} - \frac{1}{6}a^3\,\text{ExpIntegralEi}(-ax)$$

output
```
-1/3/exp(a*x)/x^3+1/6*a/exp(a*x)/x^2-1/6*a^2/exp(a*x)/x-1/6*a^3*Ei(-a*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{e^{-ax}}{x^4}\,dx = -\frac{e^{-ax}(2 - ax + a^2x^2 + a^3e^{ax}x^3\,\text{ExpIntegralEi}(-ax))}{6x^3}$$

input
```
Integrate[1/(E^(a*x)*x^4),x]
```

output
```
-1/6*(2 - a*x + a^2*x^2 + a^3*E^(a*x)*x^3*ExpIntegralEi[-(a*x)])/(E^(a*x)*
x^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2608, 2608, 2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-ax}}{x^4}dx$$

$$\Big\downarrow \text{2608}$$

$$-\frac{1}{3}a\int \frac{e^{-ax}}{x^3}dx-\frac{e^{-ax}}{3x^3}$$

$$\Big\downarrow \text{2608}$$

$$-\frac{1}{3}a\left(-\frac{1}{2}a\int \frac{e^{-ax}}{x^2}dx-\frac{e^{-ax}}{2x^2}\right)-\frac{e^{-ax}}{3x^3}$$

$$\Big\downarrow \text{2608}$$

$$-\frac{1}{3}a\left(-\frac{1}{2}a\left(-a\int \frac{e^{-ax}}{x}dx-\frac{e^{-ax}}{x}\right)-\frac{e^{-ax}}{2x^2}\right)-\frac{e^{-ax}}{3x^3}$$

$$\Big\downarrow \text{2609}$$

$$-\frac{1}{3}a\left(-\frac{1}{2}a\left(-a\,\text{ExpIntegralEi}(-ax)-\frac{e^{-ax}}{x}\right)-\frac{e^{-ax}}{2x^2}\right)-\frac{e^{-ax}}{3x^3}$$

input
```
Int[1/(E^(a*x)*x^4),x]
```

output
```
-1/3*1/(E^(a*x)*x^3) - (a*(-1/2*1/(E^(a*x)*x^2) - (a*(-(1/(E^(a*x)*x)) - a
*ExpIntegralEi[-(a*x)]))/2))/3
```

Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{\mathrm{e}^{-xa}}{3x^3}+\frac{a\,\mathrm{e}^{-xa}}{6x^2}-\frac{a^2\mathrm{e}^{-xa}}{6x}+\frac{a^3\,\mathrm{expIntegral}_1(xa)}{6}$
derivativedivides	$a^3\left(-\frac{\mathrm{e}^{-xa}}{3x^3a^3}+\frac{\mathrm{e}^{-xa}}{6x^2a^2}-\frac{\mathrm{e}^{-xa}}{6xa}+\frac{\mathrm{expIntegral}_1(xa)}{6}\right)$
default	$a^3\left(-\frac{\mathrm{e}^{-xa}}{3x^3a^3}+\frac{\mathrm{e}^{-xa}}{6x^2a^2}-\frac{\mathrm{e}^{-xa}}{6xa}+\frac{\mathrm{expIntegral}_1(xa)}{6}\right)$
meijerg	$a^3\left(-\frac{1}{3x^3a^3}+\frac{1}{2x^2a^2}-\frac{1}{2ax}+\frac{11}{36}-\frac{\ln(x)}{6}-\frac{\ln(a)}{6}+\frac{-22x^3a^3+36a^2x^2-36xa+24}{72x^3a^3}-\frac{(4a^2x^2-4xa+8)\mathrm{e}^{-}}{24x^3a^3}\right.$

input
```
int(1/exp(x*a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*exp(-x*a)/x^3+1/6*a*exp(-x*a)/x^2-1/6*a^2*exp(-x*a)/x+1/6*a^3*Ei(1,x*
a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int \frac{e^{-ax}}{x^4}\,dx = -\frac{a^3x^3\text{Ei}(-ax) + (a^2x^2 - ax + 2)e^{(-ax)}}{6\,x^3}$$

input
```
integrate(1/exp(a*x)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(a^3*x^3*Ei(-a*x) + (a^2*x^2 - a*x + 2)*e^(-a*x))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{e^{-ax}}{x^4}\,dx = -\frac{a^3\,\text{Ei}\left(axe^{i\pi}\right)}{6} - \frac{a^2e^{-ax}}{6x} + \frac{ae^{-ax}}{6x^2} - \frac{e^{-ax}}{3x^3}$$

input
```
integrate(1/exp(a*x)/x**4,x)
```

output
```
-a**3*Ei(a*x*exp_polar(I*pi))/6 - a**2*exp(-a*x)/(6*x) + a*exp(-a*x)/(6*x*
*2) - exp(-a*x)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.18

$$\int \frac{e^{-ax}}{x^4}\,dx = -a^3\Gamma(-3, ax)$$

input
```
integrate(1/exp(a*x)/x^4,x, algorithm="maxima")
```

output
```
-a^3*gamma(-3, a*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{e^{-ax}}{x^4}\,dx = -\frac{a^3x^3\mathrm{Ei}(-ax) + a^2x^2e^{(-ax)} - axe^{(-ax)} + 2\,e^{(-ax)}}{6\,x^3}$$

input
```
integrate(1/exp(a*x)/x^4,x, algorithm="giac")
```

output
```
-1/6*(a^3*x^3*Ei(-a*x) + a^2*x^2*e^(-a*x) - a*x*e^(-a*x) + 2*e^(-a*x))/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{e^{-ax}}{x^4}\,dx = \frac{a^3\,\mathrm{expint}(a\,x)}{6} - a^3\,\mathrm{e}^{-a\,x}\left(\frac{1}{6\,a\,x} - \frac{1}{6\,a^2\,x^2} + \frac{1}{3\,a^3\,x^3}\right)$$

input
```
int(exp(-a*x)/x^4,x)
```

output
```
(a^3*expint(a*x))/6 - a^3*exp(-a*x)*(1/(6*a*x) - 1/(6*a^2*x^2) + 1/(3*a^3*
x^3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{e^{-ax}}{x^4}\,dx = \frac{-e^{ax}\,ei(-ax)\,a^3x^3 - a^2x^2 + ax - 2}{6e^{ax}x^3}$$

input
```
int(1/exp(a*x)/x^4,x)
```

output
```
( - e**(a*x)*ei( - a*x)*a**3*x**3 - a**2*x**2 + a*x - 2)/(6*e**(a*x)*x**3)
```

3.17 $\int x^{100}\Gamma(2,ax)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int x^{100}\Gamma(2,ax)\,dx=\frac{1}{101}x^{101}\Gamma(2,ax)-\frac{\Gamma(103,ax)}{101a^{101}}$$

output
```
1/101*x^101*exp(-a*x)*(a*x+1)-1/101*GAMMA(103,a*x)/a^101
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^{100}\Gamma(2,ax)\,dx=\frac{1}{101}x^{101}\Gamma(2,ax)-\frac{\Gamma(103,ax)}{101a^{101}}$$

input
```
Integrate[x^100*Gamma[2, a*x],x]
```

output
```
(x^101*Gamma[2, a*x])/101 - Gamma[103, a*x]/(101*a^101)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{7116\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{100}\Gamma(2,ax)\,dx$$

$$\Big\downarrow \; 7116$$

$$\frac{1}{101}x^{101}\Gamma(2,ax)-\frac{\Gamma(103,ax)}{101a^{101}}$$

input
```
Int[x^100*Gamma[2, a*x],x]
```

output
```
(x^101*Gamma[2, a*x])/101 - Gamma[103, a*x]/(101*a^101)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 815 vs. $2(26) = 52$.

Time = 75.90 (sec) , antiderivative size = 816, normalized size of antiderivative = 32.64

method	result	size
gosper	Expression too large to display	816
risch	Expression too large to display	816
orering	Expression too large to display	816
derivativedivides	Expression too large to display	1322
default	Expression too large to display	1322
parallelrisch	Expression too large to display	1322
meijerg	Expression too large to display	1632
parts	Expression too large to display	2635

input

```
int(x^100*exp(-x*a)*(a*x+1),x,method=_RETURNVERBOSE)
```

output

```
-(a^101*x^101+102*a^100*x^100+10200*a^99*x^99+1009800*a^98*x^98+98960400*a
^97*x^97+9599158800*a^96*x^96+921519244800*a^95*x^95+87544328256000*a^94*x
^94+8229166856064000*a^93*x^93+765312517613952000*a^92*x^92+70408751620483
584000*a^91*x^91+6407196397464006144000*a^90*x^90+576647675771760552960000
*a^89*x^89+51321643143686689213440000*a^88*x^88+451630459664442865078272000
00*a^87*x^87+392918499908065292618096640000*a^86*x^86+3379099099209361516
5156311040000*a^85*x^85+287223423432795728903828643840000*a^84*x^84+241267
67568354841227921606082560000*a^83*x^83+200252170817345182191749330485248
00000*a^82*x^82+164206780070223049397234450997903360000*a^81*x^81+1330074
918568806700117599053083017216000000*a^80*x^80+10640599348550453600940792424
664137728000000*a^79*x^79+8406073485354858344743226015484668805120000000*a
^78*x^78+655673731857678950889971629207804166799360000000*a^77*x^77+5048687
7353041279218527815449000920843550720000000*a^76*x^76+383700267883113722060
811397412406998410985472000000*a^75*x^75+287775200912335291545608548059305
24880823910400000000*a^74*x^74+212953648675128115743750325563885884118096
9369600000000*a^73*x^73+155456163532843524492937737661636695406210763980800
000000*a^72*x^72+11192843774364733763491517111637842069247175006617600000
00*a^71*x^71+7946919079798960972078977149262867869165494254698496000000000*
a^70*x^70+5562843355859272680455284004484007508415845978288947200000000*a
^69*x^69+383836191554289814951414596309396518080693372501937356800000000...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 838 vs. $2(21) = 42$.

Time = 0.15 (sec) , antiderivative size = 838, normalized size of antiderivative = 33.52

$$\int x^{100}\Gamma(2,ax)\,dx = \text{Too large to display}$$

input
```
integrate(x^100*gamma(2,a*x),x, algorithm="fricas")
```

output
```
1/101*(a^101*x^101*gamma(2, a*x) - (a^102*x^102 + 102*a^101*x^101 + 10302*
a^100*x^100 + 1030200*a^99*x^99 + 101989800*a^98*x^98 + 9995000400*a^97*x^
97 + 969515038800*a^96*x^96 + 93073443724800*a^95*x^95 + 8841977153856000*
a^94*x^94 + 831145852462464000*a^93*x^93 + 77296564279009152000*a^92*x^92
+ 7111283913668841984000*a^91*x^91 + 647126836143864620544000*a^90*x^90 +
58241415252947815848960000*a^89*x^89 + 5183485957512355610557440000*a^88*x
^88 + 456146764261087293729054720000*a^87*x^87 + 3968476849071459455442776
0640000*a^86*x^86 + 3412890090201455131680787415040000*a^85*x^85 + 2900956
576671236861928669302784000000*a^84*x^84 + 24368035244038389640200822143385
600000*a^83*x^83 + 20225469252551863401366682379010048000000*a^82*x^82 + 16
58488478709252798912067955078823936000000*a^81*x^81 + 134337566775449476711
87750436138473881600000*a^80*x^80 + 10747005342035958136950200348910779105
28000000*a^79*x^79 + 84901342202084069281906582756395154931712000000*a^78*
x^78 + 6622304691762557403988713454998822084673536000000*a^77*x^77 + 50991
7461265716920107130936034909300519862272000000*a^76*x^76 + 387537270561944
859281419511386531068395095326720000000*a^75*x^75 + 29065295292145864446106
46335398983012963214950400000000*a^74*x^74 + 21508318516187939690118782881
9524742959277906329600000000*a^73*x^73 + 157010725168171959737867115038253
0623602728716206080000000*a^72*x^72 + 11304772212108381101126432282754220
4899396467566837760000000*a^71*x^71 + 80263882705969505817997669207554...
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 877, normalized size of antiderivative = 35.08

$$\int x^{100}\Gamma(2,ax)\,dx = \text{Too large to display}$$

input
```
integrate(x**100*uppergamma(2,a*x),x)
```

output
```
Piecewise(((-a**101*x**101 - 102*a**100*x**100 - 10200*a**99*x**99 - 10098
00*a**98*x**98 - 98960400*a**97*x**97 - 9599158800*a**96*x**96 - 921519244
800*a**95*x**95 - 87544328256000*a**94*x**94 - 8229166856064000*a**93*x**9
3 - 765312517613952000*a**92*x**92 - 70408751620483584000*a**91*x**91 - 64
07196397464006144000*a**90*x**90 - 576647675771760552960000*a**89*x**89 -
51321643143686689213440000*a**88*x**88 - 4516304596644428650782720000*a**8
7*x**87 - 392918499908065292618096640000*a**86*x**86 - 3379099099209361516
5156311040000*a**85*x**85 - 2872234234327957289038286438400000*a**84*x**84
 - 241267675683548412279216060825600000*a**83*x**83 - 20025217081734518219
174933048524800000*a**82*x**82 - 1642067800702230493972344509979033600000*
a**81*x**81 - 133007491856880670011759905308301721600000*a**80*x**80 - 106
40599348550453600940792424664137728000000*a**79*x**79 - 840607348535485834
47432260154846688051200000*a**78*x**78 - 65567373185767895088997162920780
416679936000000*a**77*x**77 - 50486877353041279218527815449000920843550720
00000*a**76*x**76 - 383700267883113722060811397412406998410985472000000*a*
*75*x**75 - 287775200912335291545608548059305248808239104000000000*a**74*x*
*74 - 21295364867512811574375032556388588411809693696000000000*a**73*x**73
- 1554561635328435244929377376616366954062107639808000000000*a**72*x**72 -
111928437743647337634915171116378420692471750066176000000000*a**71*x**71 -
7946919079798960972078977149262867869165494254698496000000000*a**70*x**7...
```

Maxima [F]

$$\int x^{100}\Gamma(2,ax)\,dx = \int x^{100}\Gamma(2,ax)\;dx$$

input
```
integrate(x^100*gamma(2,a*x),x, algorithm="maxima")
```

output
```
integrate(x^100*gamma(2, a*x), x)
```

Giac [F]

$$\int x^{100}\Gamma(2,ax)\,dx=\int x^{100}\Gamma(2,ax)\;dx$$

input
```
integrate(x^100*gamma(2,a*x),x, algorithm="giac")
```

output
```
integrate(x^100*gamma(2, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 814, normalized size of antiderivative = 32.56

$$\int x^{100}\Gamma(2,ax)\,dx=\text{Too large to display}$$

input
```
int(x^100*exp(-a*x)*(a*x + 1),x)
```

output

```
-exp(-a*x)*((9519273975282303573533322363339203450053028762966925389796482
31731219519930945139211202932556796486519787718792437682334162089151352012
8000000000000000000000000*x)/a^100 + 9519273975282303573533322363339203450
05302876296692538979648231731219519930945139211202932556796486519787718792
4376823341620891513520128000000000000000000000000/a^101 + x^101 + (102*x^1
00)/a + (10200*x^99)/a^2 + (1009800*x^98)/a^3 + (98960400*x^97)/a^4 + (959
9158800*x^96)/a^5 + (921519244800*x^95)/a^6 + (87544328256000*x^94)/a^7 +
(8229166856064000*x^93)/a^8 + (765312517613952000*x^92)/a^9 + (70408751620
483584000*x^91)/a^10 + (6407196397464006144000*x^90)/a^11 + (5766476757717
60552960000*x^89)/a^12 + (51321643143686689213440000*x^88)/a^13 + (4516304
596644428650782720000*x^87)/a^14 + (392918499908065292618096640000*x^86)/a
^15 + (33790990992093615165156311040000*x^85)/a^16 + (28722342343279572890
38286438400000*x^84)/a^17 + (241267675683548412279216060825600000*x^83)/a^
18 + (20025217081734518219174933048524800000*x^82)/a^19 + (164206780070223
0493972344509979033600000*x^81)/a^20 + (1330074918568806700117599053083017
21600000*x^80)/a^21 + (10640599348550453600940792424664137728000000*x^79)/
a^22 + (840607348535485834474322601548466880512000000*x^78)/a^23 + (655673
731857678950889971629207804166799360000000*x^77)/a^24 + (504868773530412792
1852781544900092084355072000000*x^76)/a^25 + (3837002678831137220608113974
12406998410985472000000*x^75)/a^26 + (287775200912335291545608548059305...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 817, normalized size of antiderivative = 32.68

$$\int x^{100}\Gamma(2, ax)\, dx = \text{Too large to display}$$

input

```
int(x^100*exp(-a*x)*(a*x+1),x)
```

output

```
( - a**101*x**101 - 102*a**100*x**100 - 10200*a**99*x**99 - 1009800*a**98*
x**98 - 98960400*a**97*x**97 - 9599158800*a**96*x**96 - 921519244800*a**95
*x**95 - 87544328256000*a**94*x**94 - 8229166856064000*a**93*x**93 - 76531
2517613952000*a**92*x**92 - 70408751620483584000*a**91*x**91 - 64071963974
64006144000*a**90*x**90 - 576647675771760552960000*a**89*x**89 - 513216431
43686689213440000*a**88*x**88 - 4516304596644428650782720000*a**87*x**87 -
 392918499908065292618096640000*a**86*x**86 - 33790990992093615165156311040
0000*a**85*x**85 - 2872234234327957289038286438400000*a**84*x**84 - 241267
6756835484122792160608256000000*a**83*x**83 - 2002521708173451821917493304 8
524800000*a**82*x**82 - 16420678007022304939723445099790336000000*a**81*x**
81 - 133007491856880670011759905308301721600000*a**80*x**80 - 106405993485
5045360094079242466413772800000*a**79*x**79 - 840607348535485834474322601
54846688051200000*a**78*x**78 - 65567373185767895088997162920780416679936
000000*a**77*x**77 - 504868773530412792185278154490009208435507200000*a**
76*x**76 - 383700267883113722060811397412406998410985472000000*a**75*x**75
 - 28777520091233529154560854805930524880823910400000000*a**74*x**74 - 212
953648675128115743750325563885884118096936960000000*a**73*x**73 - 1554561
63532843524492937737661636695406210763980800000000*a**72*x**72 - 111928437
7436473376349151711163784206924717500661760000000*a**71*x**71 - 794691907
97989609720789771492628678691654942546984960000000*a**70*x**70 - 55628...
```

3.18 $\int x^2\Gamma(2,ax)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int x^2\Gamma(2,ax)\,dx = \frac{1}{3}x^3\Gamma(2,ax) - \frac{\Gamma(5,ax)}{3a^3}$$

output
```
1/3*x^3*exp(-a*x)*(a*x+1)-8*exp(-a*x)*(1+a*x+1/2*a^2*x^2+1/6*a^3*x^3+1/24*
a^4*x^4)/a^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^2\Gamma(2,ax)\,dx = \frac{1}{3}x^3\Gamma(2,ax) - \frac{\Gamma(5,ax)}{3a^3}$$

input
```
Integrate[x^2*Gamma[2, a*x],x]
```

output
```
(x^3*Gamma[2, a*x])/3 - Gamma[5, a*x]/(3*a^3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\Gamma(2,ax)\,dx$$

$$\Big\downarrow \; 7116$$

$$\frac{1}{3}x^3\Gamma(2,ax)-\frac{\Gamma(5,ax)}{3a^3}$$

input
```
Int[x^2*Gamma[2, a*x],x]
```

output
```
(x^3*Gamma[2, a*x])/3 - Gamma[5, a*x]/(3*a^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

method	result	size
gosper	$-\frac{(x^3a^3+4a^2x^2+8xa+8)\,\mathrm{e}^{-xa}}{a^3}$	32
risch	$-\frac{(x^3a^3+4a^2x^2+8xa+8)\,\mathrm{e}^{-xa}}{a^3}$	32
orering	$-\frac{(x^3a^3+4a^2x^2+8xa+8)\,\mathrm{e}^{-xa}}{a^3}$	32
norman	$-\frac{8\,\mathrm{e}^{-xa}}{a^3}-x^3\mathrm{e}^{-xa}-\frac{8x\,\mathrm{e}^{-xa}}{a^2}-\frac{4x^2\mathrm{e}^{-xa}}{a}$	46
derivativedivides	$-\frac{4x^2a^2\mathrm{e}^{-xa}+8xa\,\mathrm{e}^{-xa}+8\,\mathrm{e}^{-xa}+x^3a^3\mathrm{e}^{-xa}}{a^3}$	48
default	$-\frac{4x^2a^2\mathrm{e}^{-xa}+8xa\,\mathrm{e}^{-xa}+8\,\mathrm{e}^{-xa}+x^3a^3\mathrm{e}^{-xa}}{a^3}$	48
parallelrisch	$-\frac{4x^2a^2\mathrm{e}^{-xa}+8xa\,\mathrm{e}^{-xa}+8\,\mathrm{e}^{-xa}+x^3a^3\mathrm{e}^{-xa}}{a^3}$	48
meijerg	$\frac{6-\frac{(4x^3a^3+12a^2x^2+24xa+24)\,\mathrm{e}^{-xa}}{4}}{a^3}+\frac{2-\frac{(3a^2x^2+6xa+6)\,\mathrm{e}^{-xa}}{3}}{a^3}$	64
parts	$-x^3\mathrm{e}^{-xa}-\frac{x^2\mathrm{e}^{-xa}}{a}-\frac{\frac{3x^2a^2\mathrm{e}^{-xa}+6xa\,\mathrm{e}^{-xa}+6\,\mathrm{e}^{-xa}}{a}-\frac{2(-xa\,\mathrm{e}^{-xa}-\mathrm{e}^{-xa})}{a}}{a^2}$	87

input
```
int(x^2*exp(-x*a)*(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-(a^3*x^3+4*a^2*x^2+8*a*x+8)*exp(-x*a)/a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(21) = 42$.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int x^2\Gamma(2,ax)\,dx = \frac{a^3x^3\Gamma(2,ax) - (a^4x^4 + 4\,a^3x^3 + 12\,a^2x^2 + 24\,ax + 24)e^{(-ax)}}{3\,a^3}$$

input
```
integrate(x^2*gamma(2,a*x),x, algorithm="fricas")
```

output
```
1/3*(a^3*x^3*gamma(2, a*x) - (a^4*x^4 + 4*a^3*x^3 + 12*a^2*x^2 + 24*a*x +
24)*e^(-a*x))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int x^2\Gamma(2,ax)\,dx = \begin{cases} \frac{(-a^3x^3-4a^2x^2-8ax-8)e^{-ax}}{a^3} & \text{for } a^3 \neq 0 \\ \frac{ax^4}{4}+\frac{x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*uppergamma(2,a*x),x)
```

output
```
Piecewise((((-a**3*x**3 - 4*a**2*x**2 - 8*a*x - 8)*exp(-a*x)/a**3, Ne(a**3,
 0)), (a*x**4/4 + x**3/3, True))
```

Maxima [F]

$$\int x^2\Gamma(2,ax)\,dx = \int x^2\Gamma(2,ax)\,dx$$

input
```
integrate(x^2*gamma(2,a*x),x, algorithm="maxima")
```

output
```
integrate(x^2*gamma(2, a*x), x)
```

Giac [F]

$$\int x^2\Gamma(2,ax)\,dx = \int x^2\Gamma(2,ax)\,dx$$

input
```
integrate(x^2*gamma(2,a*x),x, algorithm="giac")
```

output
```
integrate(x^2*gamma(2, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int x^2\Gamma(2,ax)\,dx=-\frac{\mathrm{e}^{-a\,x}\,\left(a^3\,x^3+4\,a^2\,x^2+8\,a\,x+8\right)}{a^3}$$

input
```
int(x^2*exp(-a*x)*(a*x + 1),x)
```

output
```
-(exp(-a*x)*(8*a*x + 4*a^2*x^2 + a^3*x^3 + 8))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int x^2\Gamma(2,ax)\,dx=\frac{-a^3x^3-4a^2x^2-8ax-8}{e^{ax}a^3}$$

input
```
int(x^2*exp(-a*x)*(a*x+1),x)
```

output
```
( - a**3*x**3 - 4*a**2*x**2 - 8*a*x - 8)/(e**(a*x)*a**3)
```

3.19 $\int x\Gamma(2,ax)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 25

$$\int x\Gamma(2,ax)\,dx = \frac{1}{2}x^2\Gamma(2,ax) - \frac{\Gamma(4,ax)}{2a^2}$$

output
```
1/2*x^2*exp(-a*x)*(a*x+1)-3*exp(-a*x)*(1+a*x+1/2*a^2*x^2+1/6*a^3*x^3)/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\Gamma(2,ax)\,dx = \frac{1}{2}x^2\Gamma(2,ax) - \frac{\Gamma(4,ax)}{2a^2}$$

input
```
Integrate[x*Gamma[2, a*x],x]
```

output
```
(x^2*Gamma[2, a*x])/2 - Gamma[4, a*x]/(2*a^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\Gamma(2,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{2}x^2\Gamma(2,ax)-\frac{\Gamma(4,ax)}{2a^2}$$

input
```
Int[x*Gamma[2, a*x],x]
```

output
```
(x^2*Gamma[2, a*x])/2 - Gamma[4, a*x]/(2*a^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
gosper	$-\frac{\left(a^{2} x^{2}+3 x a+3\right) {\mathrm e}^{-x a}}{a^{2}}$	24
risch	$-\frac{\left(a^{2} x^{2}+3 x a+3\right) {\mathrm e}^{-x a}}{a^{2}}$	24
orering	$-\frac{\left(a^{2} x^{2}+3 x a+3\right) {\mathrm e}^{-x a}}{a^{2}}$	24
norman	$-\frac{3 \,{\mathrm e}^{-x a}}{a^{2}}-x^{2} {\mathrm e}^{-x a}-\frac{3 x \,{\mathrm e}^{-x a}}{a}$	33
derivativedivides	$\frac{-3 x a \,{\mathrm e}^{-x a}-3 \,{\mathrm e}^{-x a}-x^{2} a^{2} {\mathrm e}^{-x a}}{a^{2}}$	35
default	$\frac{-3 x a \,{\mathrm e}^{-x a}-3 \,{\mathrm e}^{-x a}-x^{2} a^{2} {\mathrm e}^{-x a}}{a^{2}}$	35
parallelrisch	$-\frac{x^{2} a^{2} {\mathrm e}^{-x a}+3 x a \,{\mathrm e}^{-x a}+3 \,{\mathrm e}^{-x a}}{a^{2}}$	35
parts	$-x^{2} {\mathrm e}^{-x a}-\frac{x \,{\mathrm e}^{-x a}}{a}-\frac{2 x a \,{\mathrm e}^{-x a}+3 \,{\mathrm e}^{-x a}}{a^{2}}$	45
meijerg	$\frac{2-\frac{\left(3 a^{2} x^{2}+6 x a+6\right) {\mathrm e}^{-x a}}{3}}{a^{2}}+\frac{1-\frac{\left(2 x a+2\right) {\mathrm e}^{-x a}}{2}}{a^{2}}$	48

input
```
int(x*exp(-x*a)*(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-(a^2*x^2+3*a*x+3)*exp(-x*a)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(21) = 42.

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int x\Gamma(2,ax)\,dx=\frac{a^{2}x^{2}\Gamma(2,ax)-(a^{3}x^{3}+3\,a^{2}x^{2}+6\,ax+6)e^{(-ax)}}{2\,a^{2}}$$

input
```
integrate(x*gamma(2,a*x),x, algorithm="fricas")
```

output
```
1/2*(a^2*x^2*gamma(2, a*x) - (a^3*x^3 + 3*a^2*x^2 + 6*a*x + 6)*e^(-a*x))/a
^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int x\Gamma(2,ax)\,dx=\begin{cases}\frac{(-a^2x^2-3ax-3)e^{-ax}}{a^2} & \text{for } a^2\neq 0\\ \frac{ax^3}{3}+\frac{x^2}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*uppergamma(2,a*x),x)
```

output
```
Piecewise(((-a**2*x**2 - 3*a*x - 3)*exp(-a*x)/a**2, Ne(a**2, 0)), (a*x**3/
3 + x**2/2, True))
```

Maxima [F]

$$\int x\Gamma(2,ax)\,dx=\int x\Gamma(2,ax)\,dx$$

input
```
integrate(x*gamma(2,a*x),x, algorithm="maxima")
```

output
```
integrate(x*gamma(2, a*x), x)
```

Giac [F]

$$\int x\Gamma(2,ax)\,dx=\int x\Gamma(2,ax)\,dx$$

input
```
integrate(x*gamma(2,a*x),x, algorithm="giac")
```

output
```
integrate(x*gamma(2, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x\Gamma(2,ax)\,dx = -\frac{e^{-a\,x}\,(a^2\,x^2+3\,a\,x+3)}{a^2}$$

input
```
int(x*exp(-a*x)*(a*x + 1),x)
```

output
```
-(exp(-a*x)*(3*a*x + a^2*x^2 + 3))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\Gamma(2,ax)\,dx = \frac{-a^2x^2-3ax-3}{e^{ax}a^2}$$

input
```
int(x*exp(-a*x)*(a*x+1),x)
```

output
```
( - a**2*x**2 - 3*a*x - 3)/(e**(a*x)*a**2)
```

3.20 $\int \Gamma(2, ax)\,dx$

Optimal result

Integrand size = 5, antiderivative size = 18

$$\int \Gamma(2, ax)\,dx = x\Gamma(2, ax) - \frac{\Gamma(3, ax)}{a}$$

output
```
x*exp(-a*x)*(a*x+1)-2*exp(-a*x)*(1+a*x+1/2*a^2*x^2)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \Gamma(2, ax)\,dx = e^{-ax}\left(-\frac{2}{a} - 2x - ax^2\right) + x\Gamma(2, ax)$$

input
```
Integrate[Gamma[2, a*x],x]
```

output
```
(-2/a - 2*x - a*x^2)/E^(a*x) + x*Gamma[2, a*x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(2,ax)\,dx$$

$$\Big\downarrow \text{7111}$$

$$x\Gamma(2,ax)-\frac{\Gamma(3,ax)}{a}$$

input
```
Int[Gamma[2, a*x],x]
```

output
```
x*Gamma[2, a*x] - Gamma[3, a*x]/a
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{(xa+2)\,\mathrm{e}^{-xa}}{a}$	16
risch	$-\frac{(xa+2)\,\mathrm{e}^{-xa}}{a}$	16
orering	$-\frac{(xa+2)\,\mathrm{e}^{-xa}}{a}$	16
norman	$-x\,\mathrm{e}^{-xa}-\frac{2\,\mathrm{e}^{-xa}}{a}$	20
parts	$-x\,\mathrm{e}^{-xa}-\frac{2\,\mathrm{e}^{-xa}}{a}$	20
derivativedivides	$-\frac{xa\,\mathrm{e}^{-xa}+2\,\mathrm{e}^{-xa}}{a}$	22
default	$-\frac{xa\,\mathrm{e}^{-xa}+2\,\mathrm{e}^{-xa}}{a}$	22
parallelrisch	$-\frac{xa\,\mathrm{e}^{-xa}+2\,\mathrm{e}^{-xa}}{a}$	22
meijerg	$\frac{1-\frac{(2xa+2)\,\mathrm{e}^{-xa}}{2}}{a}+\frac{1-\mathrm{e}^{-xa}}{a}$	34

input
```
int(exp(-x*a)*(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-(a*x+2)*exp(-x*a)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.83

$$\int \Gamma(2,ax)\,dx=\frac{ax\Gamma(2,ax)-(a^2x^2+2\,ax+2)e^{(-ax)}}{a}$$

input
```
integrate(gamma(2,a*x),x, algorithm="fricas")
```

output
```
(a*x*gamma(2, a*x) - (a^2*x^2 + 2*a*x + 2)*e^(-a*x))/a
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \Gamma(2,ax)\,dx = \begin{cases} \frac{(-ax-2)e^{-ax}}{a} & \text{for } a \neq 0 \\ \frac{ax^2}{2} + x & \text{otherwise} \end{cases}$$

input
```
integrate(uppergamma(2,a*x),x)
```

output
```
Piecewise(((-a*x - 2)*exp(-a*x)/a, Ne(a, 0)), (a*x**2/2 + x, True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \Gamma(2,ax)\,dx = \frac{ax\Gamma(2,ax) - \Gamma(3,ax)}{a}$$

input
```
integrate(gamma(2,a*x),x, algorithm="maxima")
```

output
```
(a*x*gamma(2, a*x) - gamma(3, a*x))/a
```

Giac [F]

$$\int \Gamma(2,ax)\,dx = \int \Gamma(2,ax)\,dx$$

input
```
integrate(gamma(2,a*x),x, algorithm="giac")
```

output
```
integrate(gamma(2, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \Gamma(2, ax)\, dx = -\frac{\mathrm{e}^{-a\,x}\,(a\,x+2)}{a}$$

input
```
int(exp(-a*x)*(a*x + 1),x)
```

output
```
-(exp(-a*x)*(a*x + 2))/a
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \Gamma(2, ax)\, dx = \frac{-ax-2}{e^{ax}a}$$

input
```
int(exp(-a*x)*(a*x+1),x)
```

output
```
( - a*x - 2)/(e**(a*x)*a)
```

3.21 $\int \frac{\Gamma(2,ax)}{x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{\Gamma(2,ax)}{x} \, dx = -e^{-ax} + \text{ExpIntegralEi}(-ax)$$

output
```
-exp(-a*x)+Ei(-a*x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 41 vs. 2(14) = 28.

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.93

$$\int \frac{\Gamma(2,ax)}{x} \, dx = -e^{-ax} + \text{ExpIntegralEi}(-ax) - e^{-ax}(1+ax)\log(ax) + \Gamma(2,ax)\log(ax)$$

input
```
Integrate[Gamma[2, a*x]/x,x]
```

output
```
-E^(-(a*x)) + ExpIntegralEi[-(a*x)] - ((1 + a*x)*Log[a*x])/E^(a*x) + Gamma
[2, a*x]*Log[a*x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {7113, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,ax)}{x}\,dx$$

$$\Big\downarrow \text{7113}$$

$$\int \frac{e^{-ax}}{x}dx - e^{-ax}$$

$$\Big\downarrow \text{2609}$$

$$\text{ExpIntegralEi}(-ax) - e^{-ax}$$

input
```
Int[Gamma[2, a*x]/x,x]
```

output
```
-E^(-(a*x)) + ExpIntegralEi[-(a*x)]
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 7113
```
Int[Gamma[n_, (b_.)*(x_)]/(x_), x_Symbol] :> -Simp[Gamma[n - 1, b*x], x] +
Simp[(n - 1)   Int[Gamma[n - 1, b*x]/x, x], x] /; FreeQ[b, x] && IGtQ[n, 1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$-\operatorname{expIntegral}_1(xa) - e^{-xa}$	16
default	$-\operatorname{expIntegral}_1(xa) - e^{-xa}$	16
risch	$-\operatorname{expIntegral}_1(xa) - e^{-xa}$	16
meijerg	$1 - e^{-xa} + \ln(x) + \ln(a) - \ln(xa) - \operatorname{expIntegral}_1(xa)$	27

input
```
int(exp(-x*a)*(a*x+1)/x,x,method=_RETURNVERBOSE)
```

output
```
-Ei(1,x*a)-exp(-x*a)
```

Fricas [F]

$$\int \frac{\Gamma(2,ax)}{x}\,dx = \int \frac{\Gamma(2,ax)}{x}\,dx$$

input
```
integrate(gamma(2,a*x)/x,x, algorithm="fricas")
```

output
```
integral(gamma(2, a*x)/x, x)
```

Sympy [A] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{\Gamma(2,ax)}{x}\,dx = a\left(\begin{cases} x & \text{for } a = 0 \\ -\frac{e^{-ax}}{a} & \text{otherwise} \end{cases}\right) + \operatorname{Ei}(-ax)$$

input
```
integrate(uppergamma(2,a*x)/x,x)
```

output
```
a*Piecewise((x, Eq(a, 0)), (-exp(-a*x)/a, True)) + Ei(-a*x)
```

Maxima [F]

$$\int \frac{\Gamma(2, ax)}{x}\,dx = \int \frac{\Gamma(2, ax)}{x}\,dx$$

input
```
integrate(gamma(2,a*x)/x,x, algorithm="maxima")
```

output
```
integrate(gamma(2, a*x)/x, x)
```

Giac [F]

$$\int \frac{\Gamma(2, ax)}{x}\,dx = \int \frac{\Gamma(2, ax)}{x}\,dx$$

input
```
integrate(gamma(2,a*x)/x,x, algorithm="giac")
```

output
```
integrate(gamma(2, a*x)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{\Gamma(2, ax)}{x}\,dx = \mathrm{ei}(-a\,x) - \mathrm{e}^{-a\,x}$$

input
```
int((exp(-a*x)*(a*x + 1))/x,x)
```

output
```
ei(-a*x) - exp(-a*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{\Gamma(2, ax)}{x} \, dx = \frac{e^{ax} ei(-ax) - 1}{e^{ax}}$$

input
```
int(exp(-a*x)*(a*x+1)/x,x)
```

output
```
(e**(a*x)*ei( - a*x) - 1)/e**(a*x)
```

3.22 $\int \frac{\Gamma(2,ax)}{x^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 19

$$\int \frac{\Gamma(2,ax)}{x^2} dx = ae^{-ax} - \frac{\Gamma(2,ax)}{x}$$

output
```
a/exp(a*x)-exp(-a*x)*(a*x+1)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(2,ax)}{x^2} dx = ae^{-ax} - \frac{\Gamma(2,ax)}{x}$$

input
```
Integrate[Gamma[2, a*x]/x^2,x]
```

output
```
a/E^(a*x) - Gamma[2, a*x]/x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,ax)}{x^2} \, dx$$

$$\Big\downarrow \ 7116$$

$$ae^{-ax} - \frac{\Gamma(2,ax)}{x}$$

input
```
Int[Gamma[2, a*x]/x^2,x]
```

output
```
a/E^(a*x) - Gamma[2, a*x]/x
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

method	result
gosper	$-\frac{e^{-xa}}{x}$
derivativedivides	$-\frac{e^{-xa}}{x}$
default	$-\frac{e^{-xa}}{x}$
norman	$-\frac{e^{-xa}}{x}$
risch	$-\frac{e^{-xa}}{x}$
parallelrisch	$-\frac{e^{-xa}}{x}$
orering	$-\frac{e^{-xa}}{x}$
meijerg	$a(\ln(x)+\ln(a)-\ln(xa)-\text{expIntegral}_1(xa))+a\Big(-\frac{1}{ax}+1-\ln(x)-\ln(a)+\frac{-2xa}{2xa}$

input
```
int(exp(-x*a)*(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-exp(-x*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\Gamma(2,ax)}{x^2}\,dx=\frac{axe^{(-ax)}-\Gamma(2,ax)}{x}$$

input
```
integrate(gamma(2,a*x)/x^2,x, algorithm="fricas")
```

output
```
(a*x*e^(-a*x) - gamma(2, a*x))/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{\Gamma(2,ax)}{x^2}\,dx = -\frac{e^{-ax}}{x}$$

input
```
integrate(uppergamma(2,a*x)/x**2,x)
```

output
```
-exp(-a*x)/x
```

Maxima [F]

$$\int \frac{\Gamma(2,ax)}{x^2}\,dx = \int \frac{\Gamma(2,ax)}{x^2}\,dx$$

input
```
integrate(gamma(2,a*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(gamma(2, a*x)/x^2, x)
```

Giac [F]

$$\int \frac{\Gamma(2,ax)}{x^2}\,dx = \int \frac{\Gamma(2,ax)}{x^2}\,dx$$

input
```
integrate(gamma(2,a*x)/x^2,x, algorithm="giac")
```

output
```
integrate(gamma(2, a*x)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int \frac{\Gamma(2, ax)}{x^2} \, dx = -\frac{e^{-a\,x}}{x}$$

input

```
int((exp(-a*x)*(a*x + 1))/x^2,x)
```

output

```
-exp(-a*x)/x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{\Gamma(2, ax)}{x^2} \, dx = -\frac{1}{e^{ax} x}$$

input

```
int(exp(-a*x)*(a*x+1)/x^2,x)
```

output

```
( - 1)/(e**(a*x)*x)
```

3.23 $\int \frac{\Gamma(2,ax)}{x^3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{\Gamma(2,ax)}{x^3}\,dx = \frac{1}{2}a^2\Gamma(0,ax) - \frac{\Gamma(2,ax)}{2x^2}$$

output
```
1/2*a^2*Ei(1,a*x)-1/2*exp(-a*x)*(a*x+1)/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(2,ax)}{x^3}\,dx = \frac{1}{2}a^2\Gamma(0,ax) - \frac{\Gamma(2,ax)}{2x^2}$$

input
```
Integrate[Gamma[2, a*x]/x^3,x]
```

output
```
(a^2*Gamma[0, a*x])/2 - Gamma[2, a*x]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,ax)}{x^3}\,dx$$

$$\Big\downarrow \; 7116$$

$$\frac{1}{2}a^2\Gamma(0,ax)-\frac{\Gamma(2,ax)}{2x^2}$$

input
```
Int[Gamma[2, a*x]/x^3,x]
```

output
```
(a^2*Gamma[0, a*x])/2 - Gamma[2, a*x]/(2*x^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{e^{-xa}}{2x^2}-\frac{a\,e^{-xa}}{2x}+\frac{a^2\,\mathrm{expIntegral}_1(xa)}{2}$
derivativedivides	$a^2\left(-\frac{e^{-xa}}{2x^2a^2}-\frac{e^{-xa}}{2xa}+\frac{\mathrm{expIntegral}_1(xa)}{2}\right)$
default	$a^2\left(-\frac{e^{-xa}}{2x^2a^2}-\frac{e^{-xa}}{2xa}+\frac{\mathrm{expIntegral}_1(xa)}{2}\right)$
meijerg	$a^2\left(-\frac{1}{ax}+1-\ln(x)-\ln(a)+\frac{-2xa+2}{2xa}-\frac{e^{-xa}}{xa}+\ln(xa)+\mathrm{expIntegral}_1(xa)\right)+a^2\left(-\right.$

input
```
int(exp(-x*a)*(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(-x*a)/x^2-1/2*a*exp(-x*a)/x+1/2*a^2*Ei(1,x*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\Gamma(2,ax)}{x^3}\,dx=-\frac{a^2x^2\mathrm{Ei}(-ax)+\Gamma(2,ax)}{2\,x^2}$$

input
```
integrate(gamma(2,a*x)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a^2*x^2*Ei(-a*x) + gamma(2, a*x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{\Gamma(2, ax)}{x^3} \, dx = -\frac{a \operatorname{E}_2(ax)}{x} - \frac{\operatorname{E}_3(ax)}{x^2}$$

input
```
integrate(uppergamma(2,a*x)/x**3,x)
```

output
```
-a*expint(2, a*x)/x - expint(3, a*x)/x**2
```

Maxima [F]

$$\int \frac{\Gamma(2, ax)}{x^3} \, dx = \int \frac{\Gamma(2, ax)}{x^3} \, dx$$

input
```
integrate(gamma(2,a*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(gamma(2, a*x)/x^3, x)
```

Giac [F]

$$\int \frac{\Gamma(2, ax)}{x^3} \, dx = \int \frac{\Gamma(2, ax)}{x^3} \, dx$$

input
```
integrate(gamma(2,a*x)/x^3,x, algorithm="giac")
```

output
```
integrate(gamma(2, a*x)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{\Gamma(2,ax)}{x^3}\,dx = -\frac{\frac{e^{-a\,x}}{2}+\frac{a\,x\,e^{-a\,x}}{2}}{x^2}-\frac{a^2\,\mathrm{ei}(-a\,x)}{2}$$

input
```
int((exp(-a*x)*(a*x + 1))/x^3,x)
```

output
```
- (exp(-a*x)/2 + (a*x*exp(-a*x))/2)/x^2 - (a^2*ei(-a*x))/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{\Gamma(2,ax)}{x^3}\,dx = \frac{-e^{ax}\,ei(-ax)\,a^2x^2-ax-1}{2e^{ax}x^2}$$

input
```
int(exp(-a*x)*(a*x+1)/x^3,x)
```

output
```
( - (e**(a*x)*ei( - a*x)*a**2*x**2 + a*x + 1))/(2*e**(a*x)*x**2)
```

3.24 $\int \frac{\Gamma(2,ax)}{x^4} dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{\Gamma(2,ax)}{x^4} dx = \frac{1}{3}a^3\Gamma(-1,ax) - \frac{\Gamma(2,ax)}{3x^3}$$

output
```
1/3*a^2/x*Ei(2,a*x)-1/3*exp(-a*x)*(a*x+1)/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(2,ax)}{x^4} dx = \frac{1}{3}a^3\Gamma(-1,ax) - \frac{\Gamma(2,ax)}{3x^3}$$

input
```
Integrate[Gamma[2, a*x]/x^4,x]
```

output
```
(a^3*Gamma[-1, a*x])/3 - Gamma[2, a*x]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,ax)}{x^4} \, dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{3}a^3\Gamma(-1,ax) - \frac{\Gamma(2,ax)}{3x^3}$$

input
```
Int[Gamma[2, a*x]/x^4,x]
```

output
```
(a^3*Gamma[-1, a*x])/3 - Gamma[2, a*x]/(3*x^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

method	result
risch	$-\frac{\mathrm{e}^{-xa}}{3x^{3}}-\frac{a\,\mathrm{e}^{-xa}}{3x^{2}}+\frac{a^{2}\mathrm{e}^{-xa}}{3x}-\frac{a^{3}\,\mathrm{expIntegral}_{1}(xa)}{3}$
derivativedivides	$-a^{3}\left(\frac{\mathrm{e}^{-xa}}{3x^{3}a^{3}}+\frac{\mathrm{e}^{-xa}}{3x^{2}a^{2}}-\frac{\mathrm{e}^{-xa}}{3xa}+\frac{\mathrm{expIntegral}_{1}(xa)}{3}\right)$
default	$-a^{3}\left(\frac{\mathrm{e}^{-xa}}{3x^{3}a^{3}}+\frac{\mathrm{e}^{-xa}}{3x^{2}a^{2}}-\frac{\mathrm{e}^{-xa}}{3xa}+\frac{\mathrm{expIntegral}_{1}(xa)}{3}\right)$
meijerg	$a^{3}\left(-\frac{1}{2x^{2}a^{2}}+\frac{1}{ax}-\frac{3}{4}+\frac{\ln(x)}{2}+\frac{\ln(a)}{2}+\frac{9a^{2}x^{2}-12xa+6}{12x^{2}a^{2}}-\frac{(-3xa+3)\mathrm{e}^{-xa}}{6x^{2}a^{2}}-\frac{\ln(xa)}{2}-\frac{\mathrm{expIntegral}_{1}(}{2}\right.$

input
```
int(exp(-x*a)*(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*exp(-x*a)/x^3-1/3*a*exp(-x*a)/x^2+1/3*a^2*exp(-x*a)/x-1/3*a^3*Ei(1,x*
a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{\Gamma(2,ax)}{x^4}\,dx = \frac{a^3x^3\mathrm{Ei}(-ax)+a^2x^2e^{(-ax)}-\Gamma(2,ax)}{3\,x^3}$$

input
```
integrate(gamma(2,a*x)/x^4,x, algorithm="fricas")
```

output
```
1/3*(a^3*x^3*Ei(-a*x) + a^2*x^2*e^(-a*x) - gamma(2, a*x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{\Gamma(2,ax)}{x^4}\,dx = -\frac{a\,\mathrm{E}_3\left(ax\right)}{x^2} - \frac{\mathrm{E}_4\left(ax\right)}{x^3}$$

input
```
integrate(uppergamma(2,a*x)/x**4,x)
```

output
```
-a*expint(3, a*x)/x**2 - expint(4, a*x)/x**3
```

Maxima [F]

$$\int \frac{\Gamma(2,ax)}{x^4}\,dx = \int \frac{\Gamma(2,ax)}{x^4}\,dx$$

input
```
integrate(gamma(2,a*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(gamma(2, a*x)/x^4, x)
```

Giac [F]

$$\int \frac{\Gamma(2,ax)}{x^4}\,dx = \int \frac{\Gamma(2,ax)}{x^4}\,dx$$

input
```
integrate(gamma(2,a*x)/x^4,x, algorithm="giac")
```

output
```
integrate(gamma(2, a*x)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{\Gamma(2,ax)}{x^4}\,dx = \frac{a^3\,\mathrm{ei}(-a\,x)}{3} - \frac{\frac{\mathrm{e}^{-a\,x}}{3} - \frac{a^2\,x^2\,\mathrm{e}^{-a\,x}}{3} + \frac{a\,x\,\mathrm{e}^{-a\,x}}{3}}{x^3}$$

input
```
int((exp(-a*x)*(a*x + 1))/x^4,x)
```

output
```
(a^3*ei(-a*x))/3 - (exp(-a*x)/3 - (a^2*x^2*exp(-a*x))/3 + (a*x*exp(-a*x))/
3)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \frac{\Gamma(2,ax)}{x^4}\,dx = \frac{e^{ax}ei(-ax)\,a^3x^3 + a^2x^2 - ax - 1}{3e^{ax}x^3}$$

input
```
int(exp(-a*x)*(a*x+1)/x^4,x)
```

output
```
(e**(a*x)*ei( - a*x)*a**3*x**3 + a**2*x**2 - a*x - 1)/(3*e**(a*x)*x**3)
```

3.25 $\int x^{100}\Gamma(3,ax)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int x^{100}\Gamma(3,ax)\,dx = \frac{1}{101}x^{101}\Gamma(3,ax) - \frac{\Gamma(104,ax)}{101a^{101}}$$

output
```
2/101*x^101*exp(-a*x)*(1+a*x+1/2*a^2*x^2)-1/101*GAMMA(104,a*x)/a^101
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^{100}\Gamma(3,ax)\,dx = \frac{1}{101}x^{101}\Gamma(3,ax) - \frac{\Gamma(104,ax)}{101a^{101}}$$

input
```
Integrate[x^100*Gamma[3, a*x],x]
```

output
```
(x^101*Gamma[3, a*x])/101 - Gamma[104, a*x]/(101*a^101)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{100}\Gamma(3,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{101}x^{101}\Gamma(3,ax)-\frac{\Gamma(104,ax)}{101a^{101}}$$

input
```
Int[x^100*Gamma[3, a*x],x]
```

output
```
(x^101*Gamma[3, a*x])/101 - Gamma[104, a*x]/(101*a^101)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 823 vs. 2(34) = 68.

Time = 81.73 (sec) , antiderivative size = 824, normalized size of antiderivative = 32.96

method	result	size
gosper	Expression too large to display	824
risch	Expression too large to display	824
orering	Expression too large to display	852
derivativedivides	Expression too large to display	1335
default	Expression too large to display	1335
parallelrisch	Expression too large to display	1335
meijerg	Expression too large to display	2461
parts	Expression too large to display	3968

input
```
int(2*x^100*exp(-x*a)*(1+x*a+1/2*a^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
-exp(-x*a)*(a^102*x^102+104*a^101*x^101+10506*a^100*x^100+1050600*a^99*x^9
9+104009400*a^98*x^98+10192921200*a^97*x^97+988713356400*a^96*x^96+9491648
2214400*a^95*x^95+9017065810368000*a^94*x^94+847604186174592000*a^93*x^93+
78827189314237056000*a^92*x^92+7252101416909809152000*a^91*x^91+6599412289
38792632832000*a^90*x^90+59394710604491336954880000*a^89*x^89+528612924379
9728988984320000*a^88*x^88+465179373454376151030620160000*a^87*x^87+404706
05490530725139663953920000*a^86*x^86+3480472072185642362011100037120000*a^
85*x^85+295840126135779600770943503155200000*a^84*x^84+2485057059540548646
4759254265036800000*a^83*x^83+2062597359418655376575018103998054400000*a^8
2*x^82+169132983472329740879151484527840460800000*a^81*x^81+13699771661258
709011211270246755077324800000*a^80*x^80+1095981732900696720896901619740406
185984000000*a^79*x^79+86582556899155040950855227959492088692736000000*a^
78*x^78+6753439438134093194166707780840382918033408000000*a^77*x^77+520014
836736325175950836499124709484688572416000000*a^76*x^76+395211275919607133
72263573933477920836331503616000000*a^75*x^75+2964084569397053502919768045
010844062724862771200000000*a^74*x^74+219342258135381959216062835330802460
64163984506880000000*a^73*x^73+1601198484388288302277258697914857962683970
8690022400000000*a^72*x^72+115286290875956757763962626249869773313245902 5
681612800000000*a^71*x^71+818532665219292980124134646374075390524045908233
94508800000000*a^70*x^70+57297286565350508608689425246185277336683213 57...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 846 vs. $2(21) = 42$.

Time = 0.10 (sec) , antiderivative size = 846, normalized size of antiderivative = 33.84

$$\int x^{100}\Gamma(3,ax)\,dx = \text{Too large to display}$$

input
```
integrate(x^100*gamma(3,a*x),x, algorithm="fricas")
```

output
```
1/101*(a^101*x^101*gamma(3, a*x) - (a^103*x^103 + 103*a^102*x^102 + 10506*
a^101*x^101 + 1061106*a^100*x^100 + 106110600*a^99*x^99 + 10504949400*a^98
*x^98 + 1029485041200*a^97*x^97 + 99860048996400*a^96*x^96 + 9586564703654
400*a^95*x^95 + 910723646847168000*a^94*x^94 + 85608022803633792000*a^93*x
^93 + 7961546120737942656000*a^92*x^92 + 732462243107890724352000*a^91*x^9
1 + 66654064122818055916032000*a^90*x^90 + 5998865771053625032442880000*a^
89*x^89 + 533899053623772627887416320000*a^88*x^88 + 469831167188919912540
92636160000*a^87*x^87 + 4087531154543603239106059345920000*a^86*x^86 + 351
527679290749878563121103749120000*a^85*x^85 + 29879852739713739677865293818
675200000*a^84*x^84 + 2509907630135954132940684680768716800000*a^83*x^83
+ 208322333301284193034076828503803494400000*a^82*x^82 + 17082431330705303
828794299937311886540800000*a^81*x^81 + 1383676937787129610132338294922262
809804800000*a^80*x^80 + 110694155022970368810587063593781024784384000000*
a^79*x^79 + 8744838246814659136036378023908700957966336000000*a^78*x^78 +
682097383251543412610837485864878674721374208000000*a^77*x^77 + 5252149851
0368842771034486411595657953545814016000000*a^76*x^76 + 399163388678803205
0598620967281270004469481865216000000*a^75*x^75 + 299372541509102403794896
572546095250335211139891200000000*a^74*x^74 + 2215356807167357788082234636
8411048524805624351948800000000*a^73*x^73 + 161721046923217118530003128489
4006542310810577692262400000000*a^72*x^72 + 116439153784716325341602252...
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 896, normalized size of antiderivative = 35.84

$$\int x^{100}\Gamma(3, ax)\, dx = \text{Too large to display}$$

input
```
integrate(x**100*uppergamma(3,a*x),x)
```

output
```
Piecewise(((-a**102*x**102 - 104*a**101*x**101 - 10506*a**100*x**100 - 105
0600*a**99*x**99 - 104009400*a**98*x**98 - 10192921200*a**97*x**97 - 98871
3356400*a**96*x**96 - 94916482214400*a**95*x**95 - 9017065810368000*a**94*
x**94 - 847604186174592000*a**93*x**93 - 78827189314237056000*a**92*x**92
- 7252101416909809152000*a**91*x**91 - 659941228938792632832000*a**90*x**9
0 - 59394710604491336954880000*a**89*x**89 - 5286129243799728988984320000*
a**88*x**88 - 465179373454376151030620160000*a**87*x**87 - 404706054905307
25139663953920000*a**86*x**86 - 3480472072185642362011100037120000*a**85*x
**85 - 295840126135779600770943503155200000*a**84*x**84 - 2485057059540548
6464759254265036800000*a**83*x**83 - 2062597359418655376575018103998054400
000*a**82*x**82 - 169132983472329740879151484527840460800000*a**81*x**81 -
 13699771661258709011211270246755077324800000*a**80*x**80 - 10959817329006
96720896901619740406185984000000*a**79*x**79 - 865825568991550409508552279
594920886927360000000*a**78*x**78 - 675343943813409319416670778084038291803
3408000000*a**77*x**77 - 520014836736325175950836499124709484688572416000
00*a**76*x**76 - 3952112759196071337226357393347792083633150361600000*a**
75*x**75 - 2964084569397053502919768045010844062724862771200000000*a**74*x
**74 - 219342258135381959216062835330802460641639845068800000000*a**73*x**
73 - 16011984843882883022772586979148579626839708690022400000000*a**72*x**
72 - 1152862908759567577639626262498697733132459025681612800000000*a**7...
```

Maxima [F]

$$\int x^{100}\Gamma(3, ax)\, dx = \int x^{100}\Gamma(3, ax)\, dx$$

input
```
integrate(x^100*gamma(3,a*x),x, algorithm="maxima")
```

output
```
integrate(x^100*gamma(3, a*x), x)
```

Giac [F]

$$\int x^{100}\Gamma(3,ax)\,dx = \int x^{100}\Gamma(3,ax)\,dx$$

input
```
integrate(x^100*gamma(3,a*x),x, algorithm="giac")
```

output
```
integrate(x^100*gamma(3, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 821, normalized size of antiderivative = 32.84

$$\int x^{100}\Gamma(3,ax)\,dx = \text{Too large to display}$$

input
```
int(2*x^100*exp(-a*x)*(a*x + (a^2*x^2)/2 + 1),x)
```

output
```
-exp(-a*x)*((9804852194540772680739322034239379553554619625855933151490376
78683156105528873493387539020533500381115381350356210812804186951825892573
1840000000000000000000000*x)/a^100 + a*x^102 + 9804852194540772680739322
03423937955355461962585593315149037678683156105528873493387539020533500381
1153813503562108128041869518258925731840000000000000000000000/a^101 + 10
4*x^101 + (10506*x^100)/a + (1050600*x^99)/a^2 + (104009400*x^98)/a^3 + (1
0192921200*x^97)/a^4 + (988713356400*x^96)/a^5 + (94916482214400*x^95)/a^6
 + (9017065810368000*x^94)/a^7 + (847604186174592000*x^93)/a^8 + (78827189
314237056000*x^92)/a^9 + (7252101416909809152000*x^91)/a^10 + (65994122893
8792632832000*x^90)/a^11 + (59394710604491336954880000*x^89)/a^12 + (52861
29243799728988984320000*x^88)/a^13 + (465179373454376151030620160000*x^87)
/a^14 + (40470605490530725139663953920000*x^86)/a^15 + (348047207218564236
2011100037120000*x^85)/a^16 + (295840126135779600770943503155200000*x^84)/
a^17 + (24850570595405486464759254265036800000*x^83)/a^18 + (2062597359418
655376575018103998054400000*x^82)/a^19 + (16913298347232974087915148452784
0460800000*x^81)/a^20 + (13699771661258709011211270246755077324800000*x^80
)/a^21 + (1095981732900696720896901619740406185984000000*x^79)/a^22 + (865
825568991550409508552279594920886927360000000*x^78)/a^23 + (675343943813409
3194166707780840382918033408000000*x^77)/a^24 + (5200148367363251759508364
99124709484688572416000000*x^76)/a^25 + (395211275919607133722635739334...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 825, normalized size of antiderivative = 33.00

$$\int x^{100}\Gamma(3,ax)\,dx = \text{Too large to display}$$

input
```
int(2*x^100*exp(-a*x)*(1+a*x+1/2*a^2*x^2),x)
```

output

```
( - a**102*x**102 - 104*a**101*x**101 - 10506*a**100*x**100 - 1050600*a**9
9*x**99 - 104009400*a**98*x**98 - 10192921200*a**97*x**97 - 988713356400*a
**96*x**96 - 94916482214400*a**95*x**95 - 9017065810368000*a**94*x**94 - 8
47604186174592000*a**93*x**93 - 78827189314237056000*a**92*x**92 - 7252101
416909809152000*a**91*x**91 - 659941228938792632832000*a**90*x**90 - 59394
710604491336954880000*a**89*x**89 - 5286129243799728988984320000*a**88*x**
88 - 465179373454376151030620160000*a**87*x**87 - 404706054905307251396639
53920000*a**86*x**86 - 3480472072185642362011100037120000*a**85*x**85 - 29
5840126135779600770943503155200000*a**84*x**84 - 2485057059540548646475925
4265036800000*a**83*x**83 - 2062597359418655376575018103998054400000*a**82
*x**82 - 169132983472329740879151484527840460800000*a**81*x**81 - 13699771
661258709011211270246755077324800000*a**80*x**80 - 10959817329006967208969
01619740406185984000000*a**79*x**79 - 865825568991550409508552279594920886
92736000000*a**78*x**78 - 675343943813409319416670778084038291803340800000
0*a**77*x**77 - 520014836736325175950836499124709484688572416000000*a**76*
x**76 - 39521127591960713372263573933477920836331503616000000*a**75*x**75
- 2964084569397053502919768045010844062724862771200000000*a**74*x**74 - 21
9342258135381959216062835330802460641639845068800000000*a**73*x**73 - 1601
1984843882883022772586979148579626839708690022400000000*a**72*x**72 - 1152
862908759567577639626262498697733132459025681612800000000*a**71*x**71 -...
```

3.26 $\int x^2\Gamma(3,ax)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int x^2\Gamma(3,ax)\,dx = \frac{1}{3}x^3\Gamma(3,ax) - \frac{\Gamma(6,ax)}{3a^3}$$

output
```
2/3*x^3*exp(-a*x)*(1+a*x+1/2*a^2*x^2)-40*exp(-a*x)*(1+a*x+1/2*a^2*x^2+1/6*
a^3*x^3+1/24*a^4*x^4+1/120*a^5*x^5)/a^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^2\Gamma(3,ax)\,dx = \frac{1}{3}x^3\Gamma(3,ax) - \frac{\Gamma(6,ax)}{3a^3}$$

input
```
Integrate[x^2*Gamma[3, a*x],x]
```

output
```
(x^3*Gamma[3, a*x])/3 - Gamma[6, a*x]/(3*a^3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\Gamma(3,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{3}x^3\Gamma(3,ax)-\frac{\Gamma(6,ax)}{3a^3}$$

input
```
Int[x^2*Gamma[3, a*x],x]
```

output
```
(x^3*Gamma[3, a*x])/3 - Gamma[6, a*x]/(3*a^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

method	result
gosper	$-\frac{e^{-xa}(a^4x^4+6x^3a^3+20a^2x^2+40xa+40)}{a^3}$
risch	$-\frac{e^{-xa}(a^4x^4+6x^3a^3+20a^2x^2+40xa+40)}{a^3}$
norman	$-\frac{40\,e^{-xa}}{a^3}-6x^3e^{-xa}-\frac{40x\,e^{-xa}}{a^2}-\frac{20x^2e^{-xa}}{a}-x^4a\,e^{-xa}$
derivativedivides	$-\frac{x^4a^4e^{-xa}+6x^3a^3e^{-xa}+20x^2a^2e^{-xa}+40xa\,e^{-xa}+40\,e^{-xa}}{a^3}$
default	$-\frac{x^4a^4e^{-xa}+6x^3a^3e^{-xa}+20x^2a^2e^{-xa}+40xa\,e^{-xa}+40\,e^{-xa}}{a^3}$
parallelrisch	$-\frac{x^4a^4e^{-xa}+6x^3a^3e^{-xa}+20x^2a^2e^{-xa}+40xa\,e^{-xa}+40\,e^{-xa}}{a^3}$
orering	$-\frac{2(a^4x^4+6x^3a^3+20a^2x^2+40xa+40)e^{-xa}(1+xa+\frac{1}{2}a^2x^2)}{a^3(a^2x^2+2xa+2)}$
meijerg	$\frac{4-\frac{2\left(3a^2x^2+6xa+6\right)e^{-xa}}{3}}{a^3}+\frac{12-\frac{\left(4x^3a^3+12a^2x^2+24xa+24\right)e^{-xa}}{2}}{a^3}+\frac{24-\frac{\left(5a^4x^4+20x^3a^3+60a^2x^2+120xa+120\right)e^{-xa}}{5}}{a^3}$
parts	$-x^4a\,e^{-xa}-2x^3e^{-xa}-\frac{2x^2e^{-xa}}{a}-\frac{-\frac{4\left(-xa\,e^{-xa}-e^{-xa}\right)}{a}+\frac{6x^2a^2e^{-xa}+12xa\,e^{-xa}+12\,e^{-xa}}{a}-\frac{4\left(-x^3a^3e^{-xa}\right.}{}}{a^2}$

input
```
int(2*x^2*exp(-x*a)*(1+x*a+1/2*a^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
-exp(-x*a)*(a^4*x^4+6*a^3*x^3+20*a^2*x^2+40*a*x+40)/a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.48

$$\int x^2\Gamma(3,ax)\,dx = \frac{a^3x^3\Gamma(3,ax)-(a^5x^5+5\,a^4x^4+20\,a^3x^3+60\,a^2x^2+120\,ax+120)e^{(-ax)}}{3\,a^3}$$

input
```
integrate(x^2*gamma(3,a*x),x, algorithm="fricas")
```

output
```
1/3*(a^3*x^3*gamma(3, a*x) - (a^5*x^5 + 5*a^4*x^4 + 20*a^3*x^3 + 60*a^2*x^
2 + 120*a*x + 120)*e^(-a*x))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int x^2\Gamma(3,ax)\,dx = \begin{cases} \frac{(-a^4x^4-6a^3x^3-20a^2x^2-40ax-40)e^{-ax}}{a^3} & \text{for } a^3 \neq 0 \\ \frac{a^2x^5}{5} + \frac{ax^4}{2} + \frac{2x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*uppergamma(3,a*x),x)
```

output
```
Piecewise(((-a**4*x**4 - 6*a**3*x**3 - 20*a**2*x**2 - 40*a*x - 40)*exp(-a*
x)/a**3, Ne(a**3, 0)), (a**2*x**5/5 + a*x**4/2 + 2*x**3/3, True))
```

Maxima [F]

$$\int x^2\Gamma(3,ax)\,dx = \int x^2\Gamma(3,ax)\,dx$$

input
```
integrate(x^2*gamma(3,a*x),x, algorithm="maxima")
```

output
```
integrate(x^2*gamma(3, a*x), x)
```

Giac [F]

$$\int x^2\Gamma(3,ax)\,dx=\int x^2\Gamma(3,ax)\,dx$$

input
```
integrate(x^2*gamma(3,a*x),x, algorithm="giac")
```

output
```
integrate(x^2*gamma(3, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int x^2\Gamma(3,ax)\,dx=-\frac{\mathrm{e}^{-a\,x}\,\left(a^4\,x^4+6\,a^3\,x^3+20\,a^2\,x^2+40\,a\,x+40\right)}{a^3}$$

input
```
int(2*x^2*exp(-a*x)*(a*x + (a^2*x^2)/2 + 1),x)
```

output
```
-(exp(-a*x)*(40*a*x + 20*a^2*x^2 + 6*a^3*x^3 + a^4*x^4 + 40))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int x^2\Gamma(3,ax)\,dx=\frac{-a^4x^4-6a^3x^3-20a^2x^2-40ax-40}{e^{ax}a^3}$$

input
```
int(2*x^2*exp(-a*x)*(1+a*x+1/2*a^2*x^2),x)
```

output
```
( - a**4*x**4 - 6*a**3*x**3 - 20*a**2*x**2 - 40*a*x - 40)/(e**(a*x)*a**3)
```

3.27 $\int x\Gamma(3, ax)\, dx$

Optimal result

Integrand size = 7, antiderivative size = 25

$$\int x\Gamma(3, ax)\, dx = \frac{1}{2}x^2\Gamma(3, ax) - \frac{\Gamma(5, ax)}{2a^2}$$

output
```
x^2*exp(-a*x)*(1+a*x+1/2*a^2*x^2)-12*exp(-a*x)*(1+a*x+1/2*a^2*x^2+1/6*a^3*
x^3+1/24*a^4*x^4)/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\Gamma(3, ax)\, dx = \frac{1}{2}x^2\Gamma(3, ax) - \frac{\Gamma(5, ax)}{2a^2}$$

input
```
Integrate[x*Gamma[3, a*x],x]
```

output
```
(x^2*Gamma[3, a*x])/2 - Gamma[5, a*x]/(2*a^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\Gamma(3,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{2}x^2\Gamma(3,ax)-\frac{\Gamma(5,ax)}{2a^2}$$

input
```
Int[x*Gamma[3, a*x],x]
```

output
```
(x^2*Gamma[3, a*x])/2 - Gamma[5, a*x]/(2*a^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

method	result	size
gosper	$-\frac{e^{-xa}\left(x^3a^3+5a^2x^2+12xa+12\right)}{a^2}$	32
risch	$-\frac{e^{-xa}\left(x^3a^3+5a^2x^2+12xa+12\right)}{a^2}$	32
norman	$-\frac{12\,e^{-xa}}{a^2}-5x^2e^{-xa}-\frac{12x\,e^{-xa}}{a}-x^3e^{-xa}a$	44
derivativedivides	$\frac{-x^3a^3e^{-xa}-5x^2a^2e^{-xa}-12xa\,e^{-xa}-12\,e^{-xa}}{a^2}$	48
default	$\frac{-x^3a^3e^{-xa}-5x^2a^2e^{-xa}-12xa\,e^{-xa}-12\,e^{-xa}}{a^2}$	48
parallelrisch	$-\frac{x^3a^3e^{-xa}+5x^2a^2e^{-xa}+12xa\,e^{-xa}+12\,e^{-xa}}{a^2}$	48
orering	$-\frac{2\left(x^3a^3+5a^2x^2+12xa+12\right)e^{-xa}\left(1+xa+\frac{1}{2}a^2x^2\right)}{a^2\left(a^2x^2+2xa+2\right)}$	60
parts	$-x^3e^{-xa}a-2x^2e^{-xa}-\frac{2x\,e^{-xa}}{a}-\frac{10xa\,e^{-xa}+12\,e^{-xa}+3x^2a^2e^{-xa}}{a^2}$	69
meijerg	$\frac{2-(2xa+2)e^{-xa}}{a^2}+\frac{4-\frac{2\left(3a^2x^2+6xa+6\right)e^{-xa}}{3}}{a^2}+\frac{6-\frac{\left(4x^3a^3+12a^2x^2+24xa+24\right)e^{-xa}}{4}}{a^2}$	85

input
```
int(2*x*exp(-x*a)*(1+x*a+1/2*a^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
-exp(-x*a)*(a^3*x^3+5*a^2*x^2+12*a*x+12)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int x\Gamma(3,ax)\,dx=\frac{a^2x^2\Gamma(3,ax)-(a^4x^4+4\,a^3x^3+12\,a^2x^2+24\,ax+24)e^{(-ax)}}{2\,a^2}$$

input
```
integrate(x*gamma(3,a*x),x, algorithm="fricas")
```

output
```
1/2*(a^2*x^2*gamma(3, a*x) - (a^4*x^4 + 4*a^3*x^3 + 12*a^2*x^2 + 24*a*x +
24)*e^(-a*x))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int x\Gamma(3,ax)\,dx = \begin{cases} \frac{(-a^3x^3-5a^2x^2-12ax-12)e^{-ax}}{a^2} & \text{for } a^2 \neq 0 \\ \frac{a^2x^4}{4}+\frac{2ax^3}{3}+x^2 & \text{otherwise} \end{cases}$$

input
```
integrate(x*uppergamma(3,a*x),x)
```

output
```
Piecewise(((-a**3*x**3 - 5*a**2*x**2 - 12*a*x - 12)*exp(-a*x)/a**2, Ne(a**
2, 0)), (a**2*x**4/4 + 2*a*x**3/3 + x**2, True))
```

Maxima [F]

$$\int x\Gamma(3,ax)\,dx = \int x\Gamma(3,ax)\,dx$$

input
```
integrate(x*gamma(3,a*x),x, algorithm="maxima")
```

output
```
integrate(x*gamma(3, a*x), x)
```

Giac [F]

$$\int x\Gamma(3,ax)\,dx = \int x\Gamma(3,ax)\,dx$$

input
```
integrate(x*gamma(3,a*x),x, algorithm="giac")
```

output
```
integrate(x*gamma(3, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int x\Gamma(3,ax)\,dx = -\frac{e^{-a\,x}\,\left(a^3\,x^3+5\,a^2\,x^2+12\,a\,x+12\right)}{a^2}$$

input
```
int(2*x*exp(-a*x)*(a*x + (a^2*x^2)/2 + 1),x)
```

output
```
-(exp(-a*x)*(12*a*x + 5*a^2*x^2 + a^3*x^3 + 12))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int x\Gamma(3,ax)\,dx = \frac{-a^3x^3-5a^2x^2-12ax-12}{e^{ax}a^2}$$

input
```
int(2*x*exp(-a*x)*(1+a*x+1/2*a^2*x^2),x)
```

output
```
( - a**3*x**3 - 5*a**2*x**2 - 12*a*x - 12)/(e**(a*x)*a**2)
```

3.28 $\int \Gamma(3, ax) \, dx$

Optimal result

Integrand size = 5, antiderivative size = 18

$$\int \Gamma(3, ax) \, dx = x\Gamma(3, ax) - \frac{\Gamma(4, ax)}{a}$$

output
```
2*x*exp(-a*x)*(1+a*x+1/2*a^2*x^2)-6*exp(-a*x)*(1+a*x+1/2*a^2*x^2+1/6*a^3*x
^3)/a
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(18) = 36.

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \Gamma(3, ax) \, dx = e^{-ax}\left(-\frac{6}{a} - 6x - 3ax^2 - a^2x^3\right) + x\Gamma(3, ax)$$

input
```
Integrate[Gamma[3, a*x],x]
```

output
```
(-6/a - 6*x - 3*a*x^2 - a^2*x^3)/E^(a*x) + x*Gamma[3, a*x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(3, ax) \, dx$$

$$\Big\downarrow \text{7111}$$

$$x\Gamma(3, ax) - \frac{\Gamma(4, ax)}{a}$$

input
```
Int[Gamma[3, a*x],x]
```

output
```
x*Gamma[3, a*x] - Gamma[4, a*x]/a
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

method	result	size
gosper	$-\frac{e^{-xa}(a^2x^2+4xa+6)}{a}$	24
risch	$-\frac{e^{-xa}(a^2x^2+4xa+6)}{a}$	24
norman	$-4x\,e^{-xa}-\frac{6\,e^{-xa}}{a}-x^2e^{-xa}a$	31
derivativedivides	$-\frac{x^2a^2e^{-xa}+4xa\,e^{-xa}+6\,e^{-xa}}{a}$	35
default	$-\frac{x^2a^2e^{-xa}+4xa\,e^{-xa}+6\,e^{-xa}}{a}$	35
parallelrisch	$-\frac{x^2a^2e^{-xa}+4xa\,e^{-xa}+6\,e^{-xa}}{a}$	35
orering	$-\frac{2(a^2x^2+4xa+6)e^{-xa}(1+xa+\frac{1}{2}a^2x^2)}{a(a^2x^2+2xa+2)}$	52
meijerg	$\frac{2-2\,e^{-xa}}{a}+\frac{2-(2xa+2)e^{-xa}}{a}+\frac{2-\frac{(3a^2x^2+6xa+6)e^{-xa}}{3}}{a}$	63
parts	$-x^2e^{-xa}a-2x\,e^{-xa}-\frac{2\,e^{-xa}}{a}-\frac{2(a\,e^{-xa}-a(-xa\,e^{-xa}-e^{-xa}))}{a^2}$	64

input
```
int(2*exp(-x*a)*(1+x*a+1/2*a^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
-exp(-x*a)*(a^2*x^2+4*a*x+6)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(18) = 36.

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \Gamma(3,ax)\,dx=\frac{ax\Gamma(3,ax)-(a^3x^3+3\,a^2x^2+6\,ax+6)e^{(-ax)}}{a}$$

input
```
integrate(gamma(3,a*x),x, algorithm="fricas")
```

output
```
(a*x*gamma(3, a*x) - (a^3*x^3 + 3*a^2*x^2 + 6*a*x + 6)*e^(-a*x))/a
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \Gamma(3, ax) \, dx = \begin{cases} \frac{(-a^2x^2-4ax-6)e^{-ax}}{a} & \text{for } a \neq 0 \\ \frac{a^2x^3}{3} + ax^2 + 2x & \text{otherwise} \end{cases}$$

input
```
integrate(uppergamma(3,a*x),x)
```

output
```
Piecewise(((-a**2*x**2 - 4*a*x - 6)*exp(-a*x)/a, Ne(a, 0)), (a**2*x**3/3 +
 a*x**2 + 2*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \Gamma(3, ax) \, dx = \frac{ax\Gamma(3, ax) - \Gamma(4, ax)}{a}$$

input
```
integrate(gamma(3,a*x),x, algorithm="maxima")
```

output
```
(a*x*gamma(3, a*x) - gamma(4, a*x))/a
```

Giac [F]

$$\int \Gamma(3, ax) \, dx = \int \Gamma(3, ax) \, dx$$

input
```
integrate(gamma(3,a*x),x, algorithm="giac")
```

output
```
integrate(gamma(3, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \Gamma(3,ax)\,dx = -\frac{\mathrm{e}^{-a\,x}\,\left(a^2\,x^2+4\,a\,x+6\right)}{a}$$

input
```
int(2*exp(-a*x)*(a*x + (a^2*x^2)/2 + 1),x)
```

output
```
-(exp(-a*x)*(4*a*x + a^2*x^2 + 6))/a
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \Gamma(3,ax)\,dx = \frac{-a^2x^2-4ax-6}{e^{ax}a}$$

input
```
int(2*exp(-a*x)*(1+a*x+1/2*a^2*x^2),x)
```

output
```
( - a**2*x**2 - 4*a*x - 6)/(e**(a*x)*a)
```

3.29 $\int \frac{\Gamma(3,ax)}{x}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int \frac{\Gamma(3,ax)}{x}\,dx = -2e^{-ax} + 2\,\text{ExpIntegralEi}(-ax) - \Gamma(2,ax)$$

output
```
-2/exp(a*x)+2*Ei(-a*x)-exp(-a*x)*(a*x+1)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 56 vs. 2(23) = 46.

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.43

$$\int \frac{\Gamma(3,ax)}{x}\,dx = e^{-ax}(-3-ax) + 2\,\text{ExpIntegralEi}(-ax) - e^{-ax}\left(2+2ax+a^2x^2\right)\log(ax) + \Gamma(3,ax)\log(ax)$$

input
```
Integrate[Gamma[3, a*x]/x,x]
```

output
```
(-3 - a*x)/E^(a*x) + 2*ExpIntegralEi[-(a*x)] - ((2 + 2*a*x + a^2*x^2)*Log[
a*x])/E^(a*x) + Gamma[3, a*x]*Log[a*x]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7113, 7113, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(3,ax)}{x} \, dx$$

$$\Big\downarrow \text{7113}$$

$$2\int \frac{\Gamma(2,ax)}{x}dx-\Gamma(2,ax)$$

$$\Big\downarrow \text{7113}$$

$$2\left(\int \frac{e^{-ax}}{x}dx-e^{-ax}\right)-\Gamma(2,ax)$$

$$\Big\downarrow \text{2609}$$

$$2\left(\text{ExpIntegralEi}(-ax)-e^{-ax}\right)-\Gamma(2,ax)$$

input
```
Int[Gamma[3, a*x]/x,x]
```

output
```
2*(-E^(-(a*x)) + ExpIntegralEi[-(a*x)]) - Gamma[2, a*x]
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 7113
```
Int[Gamma[n_, (b_.)*(x_)]/(x_), x_Symbol] :> -Simp[Gamma[n - 1, b*x], x] +
Simp[(n - 1)   Int[Gamma[n - 1, b*x]/x, x], x] /; FreeQ[b, x] && IGtQ[n, 1]
```

Maple [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-xa\,\mathrm{e}^{-xa}-3\,\mathrm{e}^{-xa}-2\,\mathrm{expIntegral}_1\left(xa\right)$	25
default	$-xa\,\mathrm{e}^{-xa}-3\,\mathrm{e}^{-xa}-2\,\mathrm{expIntegral}_1\left(xa\right)$	25
risch	$-xa\,\mathrm{e}^{-xa}-3\,\mathrm{e}^{-xa}-2\,\mathrm{expIntegral}_1\left(xa\right)$	25
meijerg	$2\ln\left(x\right)+2\ln\left(a\right)-2\ln\left(xa\right)-2\,\mathrm{expIntegral}_1\left(xa\right)+3-2\,\mathrm{e}^{-xa}-\frac{(2xa+2)\mathrm{e}^{-xa}}{2}$	44

input
```
int(2*exp(-x*a)*(1+x*a+1/2*a^2*x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
-x*a*exp(-x*a)-3*exp(-x*a)-2*Ei(1,x*a)
```

Fricas [F]

$$\int \frac{\Gamma(3,ax)}{x}\,dx=\int \frac{\Gamma(3,ax)}{x}\,dx$$

input
```
integrate(gamma(3,a*x)/x,x, algorithm="fricas")
```

output
```
integral(gamma(3, a*x)/x, x)
```

Sympy [A] (verification not implemented)

Time = 3.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\Gamma(3,ax)}{x}\,dx=-axe^{-ax}+2\,\mathrm{Ei}\left(-ax\right)-3e^{-ax}$$

input
```
integrate(uppergamma(3,a*x)/x,x)
```

output
```
-a*x*exp(-a*x) + 2*Ei(-a*x) - 3*exp(-a*x)
```

Maxima [F]

$$\int \frac{\Gamma(3, ax)}{x} \, dx = \int \frac{\Gamma(3, ax)}{x} \, dx$$

input
```
integrate(gamma(3,a*x)/x,x, algorithm="maxima")
```

output
```
integrate(gamma(3, a*x)/x, x)
```

Giac [F]

$$\int \frac{\Gamma(3, ax)}{x} \, dx = \int \frac{\Gamma(3, ax)}{x} \, dx$$

input
```
integrate(gamma(3,a*x)/x,x, algorithm="giac")
```

output
```
integrate(gamma(3, a*x)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{\Gamma(3, ax)}{x} \, dx = 2 \operatorname{ei}(-a\,x) - 3\,\mathrm{e}^{-a\,x} - a\,x\,\mathrm{e}^{-a\,x}$$

input
```
int((2*exp(-a*x)*(a*x + (a^2*x^2)/2 + 1))/x,x)
```

output
```
2*ei(-a*x) - 3*exp(-a*x) - a*x*exp(-a*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(3, ax)}{x} \, dx = \frac{2e^{ax} ei(-ax) - ax - 3}{e^{ax}}$$

input
```
int(2*exp(-a*x)*(1+a*x+1/2*a^2*x^2)/x,x)
```

output
```
(2*e**(a*x)*ei( - a*x) - a*x - 3)/e**(a*x)
```

3.30 $\int \frac{\Gamma(3,ax)}{x^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int \frac{\Gamma(3,ax)}{x^2} dx = a\Gamma(2,ax) - \frac{\Gamma(3,ax)}{x}$$

output
```
a*exp(-a*x)*(a*x+1)-2*exp(-a*x)*(1+a*x+1/2*a^2*x^2)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(3,ax)}{x^2} dx = a\Gamma(2,ax) - \frac{\Gamma(3,ax)}{x}$$

input
```
Integrate[Gamma[3, a*x]/x^2,x]
```

output
```
a*Gamma[2, a*x] - Gamma[3, a*x]/x
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(3,ax)}{x^2}\,dx$$

$$\Big\downarrow \; 7116$$

$$a\Gamma(2,ax) - \frac{\Gamma(3,ax)}{x}$$

input
```
Int[Gamma[3, a*x]/x^2,x]
```

output
```
a*Gamma[2, a*x] - Gamma[3, a*x]/x
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result
gosper	$-\frac{\mathrm{e}^{-xa}(xa+2)}{x}$
risch	$-\frac{\mathrm{e}^{-xa}(xa+2)}{x}$
norman	$\frac{-xa\,\mathrm{e}^{-xa}-2\,\mathrm{e}^{-xa}}{x}$
parallelrisch	$-\frac{xa\,\mathrm{e}^{-xa}+2\,\mathrm{e}^{-xa}}{x}$
derivativedivides	$-a\left(\frac{2\,\mathrm{e}^{-xa}}{xa}+\mathrm{e}^{-xa}\right)$
default	$-a\left(\frac{2\,\mathrm{e}^{-xa}}{xa}+\mathrm{e}^{-xa}\right)$
orering	$-\frac{2(xa+2)\mathrm{e}^{-xa}\left(1+xa+\frac{1}{2}a^2x^2\right)}{x(a^2x^2+2xa+2)}$
meijerg	$2a\left(-\frac{1}{ax}+1-\ln(x)-\ln(a)+\frac{-2xa+2}{2xa}-\frac{\mathrm{e}^{-xa}}{xa}+\ln(xa)+\mathrm{expIntegral}_1(xa)\right)+2a(\ln$

input
```
int(2*exp(-x*a)*(1+x*a+1/2*a^2*x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-exp(-x*a)*(a*x+2)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \frac{\Gamma(3,ax)}{x^2}\,dx = \frac{(a^2x^2+ax)e^{(-ax)}-\Gamma(3,ax)}{x}$$

input
```
integrate(gamma(3,a*x)/x^2,x, algorithm="fricas")
```

output
```
((a^2*x^2 + a*x)*e^(-a*x) - gamma(3, a*x))/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{\Gamma(3,ax)}{x^2}\,dx = \frac{(-ax-2)\,e^{-ax}}{x}$$

input
```
integrate(uppergamma(3,a*x)/x**2,x)
```

output
```
(-a*x - 2)*exp(-a*x)/x
```

Maxima [F]

$$\int \frac{\Gamma(3,ax)}{x^2}\,dx = \int \frac{\Gamma(3,ax)}{x^2}\,dx$$

input
```
integrate(gamma(3,a*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(gamma(3, a*x)/x^2, x)
```

Giac [F]

$$\int \frac{\Gamma(3,ax)}{x^2}\,dx = \int \frac{\Gamma(3,ax)}{x^2}\,dx$$

input
```
integrate(gamma(3,a*x)/x^2,x, algorithm="giac")
```

output
```
integrate(gamma(3, a*x)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\Gamma(3,ax)}{x^2}\,dx = -\frac{\mathrm{e}^{-a\,x}\,(a\,x+2)}{x}$$

input
```
int((2*exp(-a*x)*(a*x + (a^2*x^2)/2 + 1))/x^2,x)
```

output
```
-(exp(-a*x)*(a*x + 2))/x
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\Gamma(3,ax)}{x^2}\,dx = \frac{-ax-2}{e^{ax}x}$$

input
```
int(2*exp(-a*x)*(1+a*x+1/2*a^2*x^2)/x^2,x)
```

output
```
( - a*x - 2)/(e**(a*x)*x)
```

3.31 $\int \frac{\Gamma(3,ax)}{x^3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 26

$$\int \frac{\Gamma(3,ax)}{x^3}\,dx = \frac{1}{2}a^2e^{-ax} - \frac{\Gamma(3,ax)}{2x^2}$$

output
```
1/2*a^2/exp(a*x)-exp(-a*x)*(1+a*x+1/2*a^2*x^2)/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(3,ax)}{x^3}\,dx = \frac{1}{2}a^2e^{-ax} - \frac{\Gamma(3,ax)}{2x^2}$$

input
```
Integrate[Gamma[3, a*x]/x^3,x]
```

output
```
a^2/(2*E^(a*x)) - Gamma[3, a*x]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(3,ax)}{x^3} \, dx$$

$$\Big\downarrow \; 7116$$

$$\frac{1}{2}a^2e^{-ax} - \frac{\Gamma(3,ax)}{2x^2}$$

input
```
Int[Gamma[3, a*x]/x^3,x]
```

output
```
a^2/(2*E^(a*x)) - Gamma[3, a*x]/(2*x^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

method	result
gosper	$-\frac{\mathrm{e}^{-xa}(xa+1)}{x^2}$
risch	$-\frac{\mathrm{e}^{-xa}(xa+1)}{x^2}$
norman	$\frac{-xa\,\mathrm{e}^{-xa}-\mathrm{e}^{-xa}}{x^2}$
parallelrisch	$\frac{-xa\,\mathrm{e}^{-xa}-\mathrm{e}^{-xa}}{x^2}$
derivativedivides	$a^2\left(-\frac{\mathrm{e}^{-xa}}{x^2a^2}-\frac{\mathrm{e}^{-xa}}{xa}\right)$
default	$a^2\left(-\frac{\mathrm{e}^{-xa}}{x^2a^2}-\frac{\mathrm{e}^{-xa}}{xa}\right)$
orering	$-\frac{2(xa+1)\mathrm{e}^{-xa}\left(1+xa+\frac{1}{2}a^2x^2\right)}{x^2(a^2x^2+2xa+2)}$
meijerg	$2a^2\left(-\frac{1}{2x^2a^2}+\frac{1}{ax}-\frac{3}{4}+\frac{\ln(x)}{2}+\frac{\ln(a)}{2}+\frac{9a^2x^2-12xa+6}{12x^2a^2}-\frac{(-3xa+3)\mathrm{e}^{-xa}}{6x^2a^2}-\frac{\ln(xa)}{2}-\frac{\mathrm{expIntegral}_1}{2}\right.$

input
```
int(2*exp(-x*a)*(1+x*a+1/2*a^2*x^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-exp(-x*a)*(a*x+1)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{\Gamma(3,ax)}{x^3}\,dx=\frac{a^2x^2e^{(-ax)}-\Gamma(3,ax)}{2\,x^2}$$

input
```
integrate(gamma(3,a*x)/x^3,x, algorithm="fricas")
```

output
```
1/2*(a^2*x^2*e^(-a*x) - gamma(3, a*x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int \frac{\Gamma(3,ax)}{x^3}\,dx = \frac{(-ax-1)\,e^{-ax}}{x^2}$$

input
```
integrate(uppergamma(3,a*x)/x**3,x)
```

output
```
(-a*x - 1)*exp(-a*x)/x**2
```

Maxima [F]

$$\int \frac{\Gamma(3,ax)}{x^3}\,dx = \int \frac{\Gamma(3,ax)}{x^3}\,dx$$

input
```
integrate(gamma(3,a*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(gamma(3, a*x)/x^3, x)
```

Giac [F]

$$\int \frac{\Gamma(3,ax)}{x^3}\,dx = \int \frac{\Gamma(3,ax)}{x^3}\,dx$$

input
```
integrate(gamma(3,a*x)/x^3,x, algorithm="giac")
```

output
```
integrate(gamma(3, a*x)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \frac{\Gamma(3, ax)}{x^3} \, dx = -\frac{\mathrm{e}^{-a\,x}\,(a\,x+1)}{x^2}$$

input
```
int((2*exp(-a*x)*(a*x + (a^2*x^2)/2 + 1))/x^3,x)
```

output
```
-(exp(-a*x)*(a*x + 1))/x^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{\Gamma(3, ax)}{x^3} \, dx = \frac{-ax-1}{e^{ax}x^2}$$

input
```
int(2*exp(-a*x)*(1+a*x+1/2*a^2*x^2)/x^3,x)
```

output
```
( - (a*x + 1))/(e**(a*x)*x**2)
```

3.32 $\int \frac{\Gamma(3,ax)}{x^4} dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{\Gamma(3,ax)}{x^4} dx = \frac{1}{3}a^3\Gamma(0,ax) - \frac{\Gamma(3,ax)}{3x^3}$$

output
```
1/3*a^3*Ei(1,a*x)-2/3*exp(-a*x)*(1+a*x+1/2*a^2*x^2)/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(3,ax)}{x^4} dx = \frac{1}{3}a^3\Gamma(0,ax) - \frac{\Gamma(3,ax)}{3x^3}$$

input
```
Integrate[Gamma[3, a*x]/x^4,x]
```

output
```
(a^3*Gamma[0, a*x])/3 - Gamma[3, a*x]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(3,ax)}{x^4}\,dx$$

$$\Big\downarrow \ 7116$$

$$\frac{1}{3}a^3\Gamma(0,ax)-\frac{\Gamma(3,ax)}{3x^3}$$

input
```
Int[Gamma[3, a*x]/x^4,x]
```

output
```
(a^3*Gamma[0, a*x])/3 - Gamma[3, a*x]/(3*x^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

method	result
risch	$-\frac{a^{2} {\mathrm e}^{-x a}}{3 x}+\frac{a^{3} \operatorname{expIntegral}_{1}\left(x a \right)}{3}-\frac{2 \,{\mathrm e}^{-x a}}{3 x^{3}}-\frac{2 a \,{\mathrm e}^{-x a}}{3 x^{2}}$
derivativedivides	$-a^{3} \left(\frac{{\mathrm e}^{-x a}}{3 x a}-\frac{\operatorname{expIntegral}_{1}\left(x a \right)}{3}+\frac{2 \,{\mathrm e}^{-x a}}{3 x^{3} a^{3}}+\frac{2 \,{\mathrm e}^{-x a}}{3 x^{2} a^{2}}\right)$
default	$-a^{3} \left(\frac{{\mathrm e}^{-x a}}{3 x a}-\frac{\operatorname{expIntegral}_{1}\left(x a \right)}{3}+\frac{2 \,{\mathrm e}^{-x a}}{3 x^{3} a^{3}}+\frac{2 \,{\mathrm e}^{-x a}}{3 x^{2} a^{2}}\right)$
meijerg	$2 a^{3} \left(-\frac{1}{3 x^{3} a^{3}}+\frac{1}{2 x^{2} a^{2}}-\frac{1}{2 a x}+\frac{11}{36}-\frac{\ln \left(x \right)}{6}-\frac{\ln \left(a \right)}{6}+\frac{-22 x^{3} a^{3}+36 a^{2} x^{2}-36 x a +24}{72 x^{3} a^{3}}-\frac{\left(4 a^{2} x^{2}-4 x a +8\right) e}{24 x^{3} a^{3}}\right.$

input
```
int(2*exp(-x*a)*(1+x*a+1/2*a^2*x^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2*exp(-x*a)/x+1/3*a^3*Ei(1,x*a)-2/3*exp(-x*a)/x^3-2/3*a*exp(-x*a)/x
^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\Gamma(3,ax)}{x^4}\,dx = -\frac{a^{3} x^{3} {\rm Ei}\left(-a x\right) + \Gamma\left(3, a x\right)}{3 \, x^{3}}$$

input
```
integrate(gamma(3,a*x)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(a^3*x^3*Ei(-a*x) + gamma(3, a*x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{\Gamma(3,ax)}{x^4}\,dx = -\frac{a^2\,\mathrm{E}_2\left(ax\right)}{x} - \frac{2a\,\mathrm{E}_3\left(ax\right)}{x^2} - \frac{2\,\mathrm{E}_4\left(ax\right)}{x^3}$$

input
```
integrate(uppergamma(3,a*x)/x**4,x)
```

output
```
-a**2*expint(2, a*x)/x - 2*a*expint(3, a*x)/x**2 - 2*expint(4, a*x)/x**3
```

Maxima [F]

$$\int \frac{\Gamma(3,ax)}{x^4}\,dx = \int \frac{\Gamma(3,ax)}{x^4}\,dx$$

input
```
integrate(gamma(3,a*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(gamma(3, a*x)/x^4, x)
```

Giac [F]

$$\int \frac{\Gamma(3,ax)}{x^4}\,dx = \int \frac{\Gamma(3,ax)}{x^4}\,dx$$

input
```
integrate(gamma(3,a*x)/x^4,x, algorithm="giac")
```

output
```
integrate(gamma(3, a*x)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{\Gamma(3,ax)}{x^4}\,dx = -\frac{a^3\,\mathrm{ei}(-a\,x)}{3} - \frac{\frac{2\,\mathrm{e}^{-a\,x}}{3} + \frac{a^2\,x^2\,\mathrm{e}^{-a\,x}}{3} + \frac{2\,a\,x\,\mathrm{e}^{-a\,x}}{3}}{x^3}$$

input
```
int((2*exp(-a*x)*(a*x + (a^2*x^2)/2 + 1))/x^4,x)
```

output
```
- (a^3*ei(-a*x))/3 - ((2*exp(-a*x))/3 + (a^2*x^2*exp(-a*x))/3 + (2*a*x*exp
(-a*x))/3)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{\Gamma(3,ax)}{x^4}\,dx = \frac{-e^{ax}\,ei(-ax)\,a^3x^3 - a^2x^2 - 2ax - 2}{3e^{ax}x^3}$$

input
```
int(2*exp(-a*x)*(1+a*x+1/2*a^2*x^2)/x^4,x)
```

output
```
( - e**(a*x)*ei( - a*x)*a**3*x**3 - a**2*x**2 - 2*a*x - 2)/(3*e**(a*x)*x**
3)
```

3.33 $\int x^{100}\Gamma(-1,ax)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int x^{100}\Gamma(-1,ax)\,dx=\frac{1}{101}x^{101}\Gamma(-1,ax)-\frac{\Gamma(100,ax)}{101a^{101}}$$

output
```
1/101*x^100/a*Ei(2,a*x)-1/101*GAMMA(100,a*x)/a^101
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^{100}\Gamma(-1,ax)\,dx=\frac{1}{101}x^{101}\Gamma(-1,ax)-\frac{\Gamma(100,ax)}{101a^{101}}$$

input
```
Integrate[x^100*Gamma[-1, a*x],x]
```

output
```
(x^101*Gamma[-1, a*x])/101 - Gamma[100, a*x]/(101*a^101)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{100}\Gamma(-1,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{101}x^{101}\Gamma(-1,ax)-\frac{\Gamma(100,ax)}{101a^{101}}$$

input
```
Int[x^100*Gamma[-1, a*x],x]
```

output
```
(x^101*Gamma[-1, a*x])/101 - Gamma[100, a*x]/(101*a^101)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. 2(24) = 48.

Time = 0.07 (sec) , antiderivative size = 866, normalized size of antiderivative = 34.64

Expression too large to display

input
```
int(x^99/a*Ei(2,x*a),x)
```

output
```
1/a^101*(1/101*(Psi(101)+gamma-1-Psi(102)+ln(x)+ln(a))*x^101*a^101+102/102
01*x^101*a^101+9332621544394415268169923885626670049071596826438162146859
2963895217599993229915608941463976156518286253697920827223758251185210916
864000000000000000000000000/101-1/20604*(-204*a^100*x^100+204*a^99*x^99+20196*
a^98*x^98+1979208*a^97*x^97+191983176*a^96*x^96+18430384896*a^95*x^95+1750
886565120*a^94*x^94+164583337121280*a^93*x^93+15306250352279040*a^92*x^92+
1408175032409671680*a^91*x^91+128143927949280122880*a^90*x^90+115329535154
35211059200*a^89*x^89+1026432862873733784268800*a^88*x^88+9032609193288857
3015654400*a^87*x^87+7858369998161305852361932800*a^86*x^86+67581981984187
2303303126220800*a^85*x^85+5744468468655914578076572876800*a^84*x^84+4825
35351367096824558432121651200*a^83*x^83+400504341634690364383498660970496
000*a^82*x^82+32841356014044609879446890199580672000*a^81*x^81+26601498371
37613400235198106166034432000*a^80*x^80+212811986971009072018815848493282
754560000*a^79*x^79+16812146970709716689486452030969337610240000*a^78*x^78+
1311347463715357901779943258415608333598720000*a^77*x^77+10097375470608255
84370556308980018416871014400000*a^76*x^76+7674005357662274441216227948248
1399682197094400000*a^75*x^75+575550401824670583091217096118610497616478208
000000*a^74*x^74+425907297350256231487500651127771768236193873920000000*a^73
*x^73+310912327065687048985875475323273390812421527961600000000*a^72*x^72+22
385687548729467526983034223275684138494350013235200000000*a^71*x^71+15893...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(21) = 42$.

Time = 0.11 (sec) , antiderivative size = 814, normalized size of antiderivative = 32.56

$$\int x^{100}\Gamma(-1, ax)\,dx = \text{Too large to display}$$

input
```
integrate(x^100*gamma(-1,a*x),x, algorithm="fricas")
```

output
```
1/101*(a^101*x^101*gamma(-1, a*x) - (a^99*x^99 + 99*a^98*x^98 + 9702*a^97*
x^97 + 941094*a^96*x^96 + 90345024*a^95*x^95 + 8582777280*a^94*x^94 + 8067
81064320*a^93*x^93 + 75030638981760*a^92*x^92 + 6902818786321920*a^91*x^91
 + 628156509555294720*a^90*x^90 + 56534085859976524800*a^89*x^89 + 5031533
641537910707200*a^88*x^88 + 442774960455336142233600*a^87*x^87 + 385214215
59614244374323200*a^86*x^86 + 3312842254126825016191795200*a^85*x^85 + 281
591591600780126376302592000*a^84*x^84 + 23653693694465530615609417728000*a
^83*x^83 + 1963256576640639041095581671424000*a^82*x^82 + 1609870392845324
0136983769705676800*a^81*x^81 + 1303995018204712451095685346159820800*a^
80*x^80 + 104319601456376996087654827692785664000*a^79*x^79 + 82412485150
53782690924731387730067456000*a^78*x^78 + 642817384174195049892129048242
9452615680000*a^77*x^77 + 494969385814130188416939367147067851407360000*a^7
6*x^76 + 37617673321873894319687391903177156706959360000*a^75*x^75 + 28213
25499140542073976554392738286753021952000000*a^74*x^74 + 20877808693640011
347426502506263321972362444800000*a^73*x^73 + 152408003463572082836213468
2957222503982458470400000*a^72*x^72 + 109733762493771899642073697172920020
28673700986880000*a^71*x^71 + 779109713705780487458723249927732144035832
77006848000000*a^70*x^70 + 545376799594046341221106274949412500825082939047
9360000000*a^69*x^69 + 376309991719891975442563329715094625569307227943075
840000000*a^68*x^68 + 2558907943695265433009430642062643453871289150...
```

Sympy [F(-1)]

Timed out.

$$\int x^{100}\Gamma(-1, ax)\, dx = \text{Timed out}$$

input
```
integrate(x**100*uppergamma(-1,a*x),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{100}\Gamma(-1,ax)\,dx = \int x^{100}\Gamma(-1,ax)\ dx$$

input
```
integrate(x^100*gamma(-1,a*x),x, algorithm="maxima")
```

output
```
integrate(x^100*gamma(-1, a*x), x)
```

Giac [F]

$$\int x^{100}\Gamma(-1,ax)\,dx = \int x^{100}\Gamma(-1,ax)\ dx$$

input
```
integrate(x^100*gamma(-1,a*x),x, algorithm="giac")
```

output
```
integrate(x^100*gamma(-1, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 822, normalized size of antiderivative = 32.88

$$\int x^{100}\Gamma(-1,ax)\,dx = \text{Too large to display}$$

input
```
int((x^99*expint(2, a*x))/a,x)
```

output
```
-(x^100*(exp(-a*x)*(1/(a*x) + 99/(a^2*x^2) + 9702/(a^3*x^3) + 941094/(a^4*
x^4) + 90345024/(a^5*x^5) + 8582777280/(a^6*x^6) + 806781064320/(a^7*x^7)
+ 75030638981760/(a^8*x^8) + 6902818786321920/(a^9*x^9) + 6281565095552947
20/(a^10*x^10) + 56534085859976524800/(a^11*x^11) + 5031533641537910707200
/(a^12*x^12) + 442774960455336142233600/(a^13*x^13) + 38521421559614244374
323200/(a^14*x^14) + 3312842254126825016191795200/(a^15*x^15) + 2815915916
007801263763025920000/(a^16*x^16) + 236536936944655306156094177280000/(a^17*
x^17) + 19632565766406390410955816714240000/(a^18*x^18) + 1609870392845324
0136983769705676800000/(a^19*x^19) + 130399501820471245109568534615982080000/(
a^20*x^20) + 10431960145637699608765482769278566400000/(a^21*x^21) + 824124
851505378269092473138773006745600000/(a^22*x^22) + 642817384174195049892129
048242945261568000000/(a^23*x^23) + 494969385814130188416939367147067851407
360000/(a^24*x^24) + 37617673321873894319687391903177156706959360000/(a^25*
x^25) + 2821325499140542073976554392738286753021952000000/(a^26*x^26) + 20
8778086936400113474265025062633219723624448000000/(a^27*x^27) + 1524080034
63572082836213468295722250398245847040000000/(a^28*x^28) + 109733762493771
89964207369717292002028673700986880000000/(a^29*x^29) + 779109713705780487
4587232499277321440358327700684800000000/(a^30*x^30) + 5453767995940463412211
0627494941250082508293904793600000000/(a^31*x^31) + 37630999171989197544256
3329715094625569307227943075840000000/(a^32*x^32) + 2558907943695265433 0...
```

Reduce [F]

$$\int x^{100}\Gamma(-1, ax)\,dx = \frac{\int ei(2, ax)\,x^{99}dx}{a}$$

input
```
int(x^99/a*Ei(2,a*x),x)
```

output
```
int(ei(2,a*x)*x**99,x)/a
```

3.34 $\int x^3\Gamma(-1,ax)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int x^3\Gamma(-1,ax)\,dx=\frac{1}{4}x^4\Gamma(-1,ax)-\frac{\Gamma(3,ax)}{4a^4}$$

output
```
1/4*x^3/a*Ei(2,a*x)-1/2*exp(-a*x)*(1+a*x+1/2*a^2*x^2)/a^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^3\Gamma(-1,ax)\,dx=\frac{1}{4}x^4\Gamma(-1,ax)-\frac{\Gamma(3,ax)}{4a^4}$$

input
```
Integrate[x^3*Gamma[-1, a*x],x]
```

output
```
(x^4*Gamma[-1, a*x])/4 - Gamma[3, a*x]/(4*a^4)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\Gamma(-1,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{4}x^4\Gamma(-1,ax)-\frac{\Gamma(3,ax)}{4a^4}$$

input
```
Int[x^3*Gamma[-1, a*x],x]
```

output
```
(x^4*Gamma[-1, a*x])/4 - Gamma[3, a*x]/(4*a^4)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(37) = 74.

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.36

method	result	size
meijerg	$\frac{\frac{\left(-\frac{5}{4}+\gamma+\ln(x)+\ln(a)\right)x^4a^4}{4}+\frac{5a^4x^4}{16}+\frac{1}{2}-\frac{\left(-5x^3a^3+5a^2x^2+10xa+10\right)\mathrm{e}^{-xa}}{20}+\frac{x^4a^4(-\gamma-\ln(xa)-\text{expIntegral}_1(xa))}{4}}{a^4}$	84

input
```
int(x^2/a*Ei(2,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/4*(-5/4+gamma+ln(x)+ln(a))*x^4*a^4+5/16*a^4*x^4+1/2-1/20*(-5*a^3*
x^3+5*a^2*x^2+10*a*x+10)*exp(-x*a)+1/4*x^4*a^4*(-gamma-ln(x*a)-Ei(1,x*a)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int x^3\Gamma(-1,ax)\,dx = \frac{a^4x^4\Gamma(-1,ax)-(a^2x^2+2\,ax+2)e^{(-ax)}}{4\,a^4}$$

input
```
integrate(x^3*gamma(-1,a*x),x, algorithm="fricas")
```

output
```
1/4*(a^4*x^4*gamma(-1, a*x) - (a^2*x^2 + 2*a*x + 2)*e^(-a*x))/a^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.98 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.92

$$\int x^3\Gamma(-1,ax)\,dx = \frac{\frac{ax^4\operatorname{Ei}\left(axe^{i\pi}\right)}{4}-\frac{i\pi ax^4}{4}+\frac{x^3e^{-ax}}{4}-\frac{x^2e^{-ax}}{4a}-\frac{xe^{-ax}}{2a^2}-\frac{e^{-ax}}{2a^3}}{a}$$

input
```
integrate(x**3*uppergamma(-1,a*x),x)
```

output
```
(a*x**4*Ei(a*x*exp_polar(I*pi))/4 - I*pi*a*x**4/4 + x**3*exp(-a*x)/4 - x**
2*exp(-a*x)/(4*a) - x*exp(-a*x)/(2*a**2) - exp(-a*x)/(2*a**3))/a
```

Maxima [F]

$$\int x^3\Gamma(-1,ax)\,dx = \int x^3\Gamma(-1,ax)\;dx$$

input
```
integrate(x^3*gamma(-1,a*x),x, algorithm="maxima")
```

output
```
integrate(x^3*gamma(-1, a*x), x)
```

Giac [F]

$$\int x^3\Gamma(-1,ax)\,dx = \int x^3\Gamma(-1,ax)\;dx$$

input
```
integrate(x^3*gamma(-1,a*x),x, algorithm="giac")
```

output
```
integrate(x^3*gamma(-1, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int x^3\Gamma(-1,ax)\,dx = -\frac{x^3\left(\mathrm{e}^{-a\,x}\left(\frac{1}{a\,x}+\frac{2}{a^2\,x^2}+\frac{2}{a^3\,x^3}\right)-\mathrm{expint}(2,a\,x)\right)}{4\,a}$$

input
```
int((x^2*expint(2, a*x))/a,x)
```

output
```
-(x^3*(exp(-a*x)*(1/(a*x) + 2/(a^2*x^2) + 2/(a^3*x^3)) - expint(2, a*x)))/
(4*a)
```

Reduce [F]

$$\int x^3\Gamma(-1,ax)\,dx = \frac{\int ei(2,ax)\,x^2dx}{a}$$

input
```
int(x^2/a*Ei(2,a*x),x)
```

output
```
int(ei(2,a*x)*x**2,x)/a
```

3.35 $\int x^2\Gamma(-1,ax)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int x^2\Gamma(-1,ax)\,dx=\frac{1}{3}x^3\Gamma(-1,ax)-\frac{\Gamma(2,ax)}{3a^3}$$

output
```
1/3*x^2/a*Ei(2,a*x)-1/3*exp(-a*x)*(a*x+1)/a^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^2\Gamma(-1,ax)\,dx=\frac{1}{3}x^3\Gamma(-1,ax)-\frac{\Gamma(2,ax)}{3a^3}$$

input
```
Integrate[x^2*Gamma[-1, a*x],x]
```

output
```
(x^3*Gamma[-1, a*x])/3 - Gamma[2, a*x]/(3*a^3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\Gamma(-1,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{3}x^3\Gamma(-1,ax)-\frac{\Gamma(2,ax)}{3a^3}$$

input
```
Int[x^2*Gamma[-1, a*x],x]
```

output
```
(x^3*Gamma[-1, a*x])/3 - Gamma[2, a*x]/(3*a^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(29) = 58.

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.04

method	result	size
meijerg	$\frac{\frac{\left(-\frac{4}{3}+\gamma+\ln(x)+\ln(a)\right)x^3a^3}{3}+\frac{4x^3a^3}{9}+\frac{1}{3}-\frac{\left(-8a^2x^2+8xa+8\right)e^{-xa}}{24}+\frac{a^3x^3\left(-\gamma-\ln(xa)-\text{expIntegral}_1(xa)\right)}{3}}{a^3}$	76

input
```
int(x/a*Ei(2,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/3*(-4/3+gamma+ln(x)+ln(a))*x^3*a^3+4/9*x^3*a^3+1/3-1/24*(-8*a^2*x
^2+8*a*x+8)*exp(-x*a)+1/3*a^3*x^3*(-gamma-ln(x*a)-Ei(1,x*a)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int x^2\Gamma(-1,ax)\,dx = \frac{a^3x^3\Gamma(-1,ax)-(ax+1)e^{(-ax)}}{3\,a^3}$$

input
```
integrate(x^2*gamma(-1,a*x),x, algorithm="fricas")
```

output
```
1/3*(a^3*x^3*gamma(-1, a*x) - (a*x + 1)*e^(-a*x))/a^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.68 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int x^2\Gamma(-1,ax)\,dx = \frac{\frac{ax^3\operatorname{Ei}\left(axe^{i\pi}\right)}{3}-\frac{i\pi ax^3}{3}+\frac{x^2e^{-ax}}{3}-\frac{xe^{-ax}}{3a}-\frac{e^{-ax}}{3a^2}}{a}$$

input
```
integrate(x**2*uppergamma(-1,a*x),x)
```

output
```
(a*x**3*Ei(a*x*exp_polar(I*pi))/3 - I*pi*a*x**3/3 + x**2*exp(-a*x)/3 - x*e
xp(-a*x)/(3*a) - exp(-a*x)/(3*a**2))/a
```

Maxima [F]

$$\int x^2\Gamma(-1,ax)\,dx = \int x^2\Gamma(-1,ax)\,dx$$

input
```
integrate(x^2*gamma(-1,a*x),x, algorithm="maxima")
```

output
```
integrate(x^2*gamma(-1, a*x), x)
```

Giac [F]

$$\int x^2\Gamma(-1,ax)\,dx = \int x^2\Gamma(-1,ax)\,dx$$

input
```
integrate(x^2*gamma(-1,a*x),x, algorithm="giac")
```

output
```
integrate(x^2*gamma(-1, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int x^2\Gamma(-1,ax)\,dx = -\frac{x^2\,\left(\mathrm{e}^{-a\,x}\,\left(\frac{1}{a\,x}+\frac{1}{a^2\,x^2}\right)-\mathrm{expint}(2,a\,x)\right)}{3\,a}$$

input
```
int((x*expint(2, a*x))/a,x)
```

output
```
-(x^2*(exp(-a*x)*(1/(a*x) + 1/(a^2*x^2)) - expint(2, a*x)))/(3*a)
```

Reduce [F]

$$\int x^2\Gamma(-1,ax)\,dx = \frac{\int ei(2,ax)\,xdx}{a}$$

input
```
int(x/a*Ei(2,a*x),x)
```

output
```
int(ei(2,a*x)*x,x)/a
```

3.36 $\int x\Gamma(-1, ax)\, dx$

Optimal result

Integrand size = 7, antiderivative size = 26

$$\int x\Gamma(-1, ax)\, dx = -\frac{e^{-ax}}{2a^2} + \frac{1}{2}x^2\Gamma(-1, ax)$$

output
```
-1/2/a^2/exp(a*x)+1/2*x/a*Ei(2,a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x\Gamma(-1, ax)\, dx = -\frac{e^{-ax}}{2a^2} + \frac{1}{2}x^2\Gamma(-1, ax)$$

input
```
Integrate[x*Gamma[-1, a*x],x]
```

output
```
-1/2*1/(a^2*E^(a*x)) + (x^2*Gamma[-1, a*x])/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\Gamma(-1, ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{2}x^2\Gamma(-1, ax) - \frac{e^{-ax}}{2a^2}$$

input
```
Int[x*Gamma[-1, a*x],x]
```

output
```
-1/2*1/(a^2*E^(a*x)) + (x^2*Gamma[-1, a*x])/2
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(23) = 46.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.62

method	result	size
meijerg	$\frac{\frac{\left(\gamma-\frac{3}{2}+\ln(x)+\ln(a)\right)x^2a^2}{2}+\frac{3a^2x^2}{4}+\frac{1}{2}-\frac{(-3xa+3)\mathrm{e}^{-xa}}{6}+\frac{x^2a^2(-\gamma-\ln(xa)-\mathrm{expIntegral}_1(xa))}{2}}{a^2}$	68

input
```
int(1/a*Ei(2,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/2*(gamma-3/2+ln(x)+ln(a))*x^2*a^2+3/4*a^2*x^2+1/2-1/6*(-3*a*x+3)*
exp(-x*a)+1/2*x^2*a^2*(-gamma-ln(x*a)-Ei(1,x*a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int x\Gamma(-1,ax)\,dx=\frac{a^2x^2\Gamma(-1,ax)-e^{(-ax)}}{2\,a^2}$$

input
```
integrate(x*gamma(-1,a*x),x, algorithm="fricas")
```

output
```
1/2*(a^2*x^2*gamma(-1, a*x) - e^(-a*x))/a^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int x\Gamma(-1,ax)\,dx=\frac{\frac{ax^2\operatorname{Ei}\left(axe^{i\pi}\right)}{2}-\frac{i\pi ax^2}{2}+\frac{xe^{-ax}}{2}-\frac{e^{-ax}}{2a}}{a}$$

input
```
integrate(x*uppergamma(-1,a*x),x)
```

output
```
(a*x**2*Ei(a*x*exp_polar(I*pi))/2 - I*pi*a*x**2/2 + x*exp(-a*x)/2 - exp(-a
*x)/(2*a))/a
```

Maxima [F]

$$\int x\Gamma(-1,ax)\,dx = \int x\Gamma(-1,ax)\,\,dx$$

input
```
integrate(x*gamma(-1,a*x),x, algorithm="maxima")
```

output
```
integrate(x*gamma(-1, a*x), x)
```

Giac [F]

$$\int x\Gamma(-1,ax)\,dx = \int x\Gamma(-1,ax)\,\,dx$$

input
```
integrate(x*gamma(-1,a*x),x, algorithm="giac")
```

output
```
integrate(x*gamma(-1, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x\Gamma(-1,ax)\,dx = \frac{x\,\mathrm{expint}(2,a\,x)}{2\,a} - \frac{\mathrm{e}^{-a\,x}}{2\,a^2}$$

input
```
int(expint(2, a*x)/a,x)
```

output
```
(x*expint(2, a*x))/(2*a) - exp(-a*x)/(2*a^2)
```

Reduce [F]

$$\int x\Gamma(-1, ax)\, dx = \frac{\int ei(2, ax)\, dx}{a}$$

input
```
int(1/a*Ei(2,a*x),x)
```

output
```
int(ei(2,a*x),x)/a
```

3.37 $\int \Gamma(-1, ax)\,dx$

Optimal result

Integrand size = 5, antiderivative size = 18

$$\int \Gamma(-1, ax)\,dx = x\Gamma(-1, ax) - \frac{\Gamma(0, ax)}{a}$$

output
```
1/a*Ei(2,a*x)-Ei(1,a*x)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \Gamma(-1, ax)\,dx = \frac{\text{ExpIntegralEi}(-ax)}{a} + x\Gamma(-1, ax)$$

input
```
Integrate[Gamma[-1, a*x],x]
```

output
```
ExpIntegralEi[-(a*x)]/a + x*Gamma[-1, a*x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(-1,ax)\,dx$$

$$\Big\downarrow \ 7111$$

$$x\Gamma(-1,ax)-\frac{\Gamma(0,ax)}{a}$$

input
```
Int[Gamma[-1, a*x],x]
```

output
```
x*Gamma[-1, a*x] - Gamma[0, a*x]/a
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(20) = 40.

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.06

method	result	size
meijerg	$\frac{\gamma+\ln(x)+\ln(a)+(\gamma-2+\ln(x)+\ln(a))xa+2xa+\mathrm{e}^{-xa}+\frac{(4xa+4)(-\gamma-\ln(xa)-\mathrm{expIntegral}_1(xa))}{4}}{a}$	55

input
```
int(1/a/x*Ei(2,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(gamma+ln(x)+ln(a)+(gamma-2+ln(x)+ln(a))*x*a+2*x*a+exp(-x*a)+1/4*(4*a*
x+4)*(-gamma-ln(x*a)-Ei(1,x*a)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \Gamma(-1, ax)\, dx = \frac{(a^2x^2 + ax)\Gamma(-1, ax) - e^{(-ax)}}{a^2x}$$

input
```
integrate(gamma(-1,a*x),x, algorithm="fricas")
```

output
```
((a^2*x^2 + a*x)*gamma(-1, a*x) - e^(-a*x))/(a^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \Gamma(-1, ax)\, dx = \frac{ax\,\mathrm{Ei}\left(axe^{i\pi}\right) - i\pi ax + \mathrm{Ei}\left(axe^{i\pi}\right) + e^{-ax}}{a}$$

input
```
integrate(uppergamma(-1,a*x),x)
```

output
```
(a*x*Ei(a*x*exp_polar(I*pi)) - I*pi*a*x + Ei(a*x*exp_polar(I*pi)) + exp(-a
*x))/a
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \Gamma(-1, ax) \, dx = \frac{ax\Gamma(-1, ax) + \text{Ei}(-ax)}{a}$$

input
```
integrate(gamma(-1,a*x),x, algorithm="maxima")
```

output
```
(a*x*gamma(-1, a*x) + Ei(-a*x))/a
```

Giac [F]

$$\int \Gamma(-1, ax) \, dx = \int \Gamma(-1, ax) \, dx$$

input
```
integrate(gamma(-1,a*x),x, algorithm="giac")
```

output
```
integrate(gamma(-1, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \Gamma(-1, ax) \, dx = \frac{e^{-a\,x}}{a} - \frac{\text{expint}(a\,x)}{a} - x\,\text{expint}(a\,x)$$

input
```
int(expint(2, a*x)/(a*x),x)
```

output
```
exp(-a*x)/a - expint(a*x)/a - x*expint(a*x)
```

Reduce [F]

$$\int \Gamma(-1, ax)\,dx = \frac{\int \frac{ei(2,ax)}{x}dx}{a}$$

input
```
int(1/a/x*Ei(2,a*x),x)
```

output
```
int(ei(2,a*x)/x,x)/a
```

3.38 $\int \frac{\Gamma(-1,ax)}{x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 39

$$\int \frac{\Gamma(-1,ax)}{x} \, dx = -\Gamma(-1,ax) - ax\, {}_3F_3(1,1,1;2,2,2;-ax) + \gamma \log(x) + \frac{1}{2} \log^2(ax)$$

output
```
-1/a/x*Ei(2,a*x)-a*x*hypergeom([1, 1, 1],[2, 2, 2],-a*x)+gamma*ln(x)+1/2*l
n(a*x)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 103 vs. 2(39) = 78.

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.64

$$\begin{aligned}\int \frac{\Gamma(-1,ax)}{x} \, dx = & -\frac{e^{-ax}}{ax} - ax\, {}_3F_3(1,1,1;2,2,2;-ax) + \gamma \log(x) + \Gamma(0,ax) \log(x) \\ & - \frac{\log^2(x)}{2} + \text{ExpIntegralEi}(-ax)(-1 + \log(x) - \log(ax)) \\ & - \frac{e^{-ax} \log(ax)}{ax} + \Gamma(-1,ax) \log(ax) + \log(x) \log(ax)\end{aligned}$$

input
```
Integrate[Gamma[-1, a*x]/x,x]
```

output
```
-(1/(a*E^(a*x)*x)) - a*x*HypergeometricPFQ[{1, 1, 1}, {2, 2, 2}, -(a*x)] +
 EulerGamma*Log[x] + Gamma[0, a*x]*Log[x] - Log[x]^2/2 + ExpIntegralEi[-(a
*x)]*(-1 + Log[x] - Log[a*x]) - Log[a*x]/(a*E^(a*x)*x) + Gamma[-1, a*x]*Lo
g[a*x] + Log[x]*Log[a*x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {7114, 7112}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-1,ax)}{x}\,dx$$

$$\Big\downarrow \ 7114$$

$$-\int \frac{\Gamma(0,ax)}{x}dx-\Gamma(-1,ax)$$

$$\Big\downarrow \ 7112$$

$$-ax\,{}_3F_3(1,1,1;2,2,2;-ax)+\frac{1}{2}\log^2(ax)-\Gamma(-1,ax)+\gamma\log(x)$$

input
```
Int[Gamma[-1, a*x]/x,x]
```

output
```
-Gamma[-1, a*x] - a*x*HypergeometricPFQ[{1, 1, 1}, {2, 2, 2}, -(a*x)] + Eu
lerGamma*Log[x] + Log[a*x]^2/2
```

Defintions of rubi rules used

rule 7112
```
Int[Gamma[0, (b_.)*(x_)]/(x_), x_Symbol] :> Simp[b*x*HypergeometricPFQ[{1,
1, 1}, {2, 2, 2}, (-b)*x], x] + (-Simp[EulerGamma*Log[x], x] - Simp[(1/2)*L
og[b*x]^2, x]) /; FreeQ[b, x]
```

rule 7114
```
Int[Gamma[n_, (b_.)*(x_)]/(x_), x_Symbol] :> Simp[Gamma[n, b*x]/n, x] + Sim
p[1/n   Int[Gamma[n + 1, b*x]/x, x], x] /; FreeQ[b, x] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.97

method	result
meijerg	$-\frac{1}{ax}+\frac{\pi^2}{12}+\frac{\ln(a)^2}{2}+\frac{\ln(x)^2}{2}+\frac{1}{2}+\frac{(1-\gamma)^2}{2}+\ln(x)\ln(a)-\ln(x)(1-\gamma)-\ln(a)(1-\gamma)-\frac{xa\,\mathrm{hyper}}{}$

input
```
int(1/a/x^2*Ei(2,x*a),x,method=_RETURNVERBOSE)
```

output
```
-1/a/x+1/12*Pi^2+1/2*ln(a)^2+1/2*ln(x)^2+1/2+1/2*(1-gamma)^2+ln(x)*ln(a)-l
n(x)*(1-gamma)-ln(a)*(1-gamma)-1/2*x*a*hypergeom([1,1,1],[2,2,3],-x*a)
```

Fricas [F]

$$\int \frac{\Gamma(-1,ax)}{x}\,dx=\int \frac{\Gamma(-1,ax)}{x}\,dx$$

input
```
integrate(gamma(-1,a*x)/x,x, algorithm="fricas")
```

output
```
integral(gamma(-1, a*x)/x, x)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{\Gamma(-1,ax)}{x}\,dx=\frac{-\frac{a^2x\,{}_3F_3\left(\begin{matrix}1,1,1\\2,2,3\end{matrix}\middle|-ax\right)}{2}+\frac{a\log{(ax)}^2}{2}-a\log{(ax)}+\gamma a\log{(ax)}-\frac{1}{x}}{a}$$

input
```
integrate(uppergamma(-1,a*x)/x,x)
```

output
```
(-a**2*x*hyper((1, 1, 1), (2, 2, 3), -a*x)/2 + a*log(a*x)**2/2 - a*log(a*x
) + EulerGamma*a*log(a*x) - 1/x)/a
```

Maxima [F]

$$\int \frac{\Gamma(-1,ax)}{x}\,dx=\int \frac{\Gamma(-1,ax)}{x}\,dx$$

input
```
integrate(gamma(-1,a*x)/x,x, algorithm="maxima")
```

output
```
integrate(gamma(-1, a*x)/x, x)
```

Giac [F]

$$\int \frac{\Gamma(-1,ax)}{x}\,dx=\int \frac{\Gamma(-1,ax)}{x}\,dx$$

input
```
integrate(gamma(-1,a*x)/x,x, algorithm="giac")
```

output
```
integrate(gamma(-1, a*x)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.36

$$\int \frac{\Gamma(-1, ax)}{x} \, dx = \int \frac{\mathrm{expint}(2, a\, x)}{a\, x^2} \, dx$$

input
```
int(expint(2, a*x)/(a*x^2),x)
```

output
```
int(expint(2, a*x)/(a*x^2), x)
```

Reduce [F]

$$\int \frac{\Gamma(-1, ax)}{x} \, dx = \frac{\int \frac{ei(2,ax)}{x^2} dx}{a}$$

input
```
int(1/a/x^2*Ei(2,a*x),x)
```

output
```
int(ei(2,a*x)/x**2,x)/a
```

3.39 $\int \frac{\Gamma(-1,ax)}{x^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int \frac{\Gamma(-1,ax)}{x^2}\,dx = a\Gamma(-2,ax) - \frac{\Gamma(-1,ax)}{x}$$

output
```
1/a/x^2*Ei(3,a*x)-1/a/x^2*Ei(2,a*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(-1,ax)}{x^2}\,dx = a\Gamma(-2,ax) - \frac{\Gamma(-1,ax)}{x}$$

input
```
Integrate[Gamma[-1, a*x]/x^2,x]
```

output
```
a*Gamma[-2, a*x] - Gamma[-1, a*x]/x
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-1,ax)}{x^2} \, dx$$
$$\Big\downarrow \text{7116}$$
$$a\Gamma(-2,ax) - \frac{\Gamma(-1,ax)}{x}$$

input
```
Int[Gamma[-1, a*x]/x^2,x]
```

output
```
a*Gamma[-2, a*x] - Gamma[-1, a*x]/x
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(26) = 52$.

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 5.89

method	result
meijerg	$a\left(-\frac{1}{2x^2a^2} - \frac{\gamma+\ln(x)+\ln(a)}{xa} - \frac{\gamma}{2} + \frac{5}{4} - \frac{\ln(x)}{2} - \frac{\ln(a)}{2} + \frac{-15a^2x^2+6}{12x^2a^2} - \frac{(6xa+6)\mathrm{e}^{-xa}}{12x^2a^2} - \frac{(6xa+12)(-\gamma-\ln(xa)-\mathrm{ex}}{12xa}\right.$

input
```
int(1/a/x^3*Ei(2,x*a),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/x^2/a^2-(gamma+ln(x)+ln(a))/x/a-1/2*gamma+5/4-1/2*ln(x)-1/2*ln(a)+
1/12/x^2/a^2*(-15*a^2*x^2+6)-1/12/x^2/a^2*(6*a*x+6)*exp(-x*a)-1/12/x/a*(6*
a*x+12)*(-gamma-ln(x*a)-Ei(1,x*a)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\Gamma(-1,ax)}{x^2}\,dx = -\frac{(a^2x^2 + 2\,ax)\Gamma(-1,ax) - e^{(-ax)}}{2\,ax^2}$$

input
```
integrate(gamma(-1,a*x)/x^2,x, algorithm="fricas")
```

output
```
-1/2*((a^2*x^2 + 2*a*x)*gamma(-1, a*x) - e^(-a*x))/(a*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.22

$$\int \frac{\Gamma(-1,ax)}{x^2}\,dx = \frac{-\frac{a^2\operatorname{Ei}\left(axe^{i\pi}\right)}{2} - \frac{a\operatorname{Ei}\left(axe^{i\pi}\right)}{x} + \frac{i\pi a}{x} - \frac{ae^{-ax}}{2x} - \frac{e^{-ax}}{2x^2}}{a}$$

input
```
integrate(uppergamma(-1,a*x)/x**2,x)
```

output
```
(-a**2*Ei(a*x*exp_polar(I*pi))/2 - a*Ei(a*x*exp_polar(I*pi))/x + I*pi*a/x
- a*exp(-a*x)/(2*x) - exp(-a*x)/(2*x**2))/a
```

Maxima [F]

$$\int \frac{\Gamma(-1,ax)}{x^2} dx = \int \frac{\Gamma(-1,ax)}{x^2} dx$$

input
```
integrate(gamma(-1,a*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(gamma(-1, a*x)/x^2, x)
```

Giac [F]

$$\int \frac{\Gamma(-1,ax)}{x^2} dx = \int \frac{\Gamma(-1,ax)}{x^2} dx$$

input
```
integrate(gamma(-1,a*x)/x^2,x, algorithm="giac")
```

output
```
integrate(gamma(-1, a*x)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{\Gamma(-1,ax)}{x^2} dx = -\frac{\mathrm{expint}(2, a\, x) - \mathrm{expint}(3, a\, x)}{a\, x^2}$$

input
```
int(expint(2, a*x)/(a*x^3),x)
```

output
```
-(expint(2, a*x) - expint(3, a*x))/(a*x^2)
```

Reduce [F]

$$\int \frac{\Gamma(-1, ax)}{x^2} dx = \frac{\int \frac{ei(2,ax)}{x^3} dx}{a}$$

input
```
int(1/a/x^3*Ei(2,a*x),x)
```

output
```
int(ei(2,a*x)/x**3,x)/a
```

3.40 $\int \frac{\Gamma(-1,ax)}{x^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{\Gamma(-1,ax)}{x^3} dx = \frac{1}{2}a^2\Gamma(-3,ax) - \frac{\Gamma(-1,ax)}{2x^2}$$

output
```
1/2/a/x^3*Ei(4,a*x)-1/2/a/x^3*Ei(2,a*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(-1,ax)}{x^3} dx = \frac{1}{2}a^2\Gamma(-3,ax) - \frac{\Gamma(-1,ax)}{2x^2}$$

input
```
Integrate[Gamma[-1, a*x]/x^3,x]
```

output
```
(a^2*Gamma[-3, a*x])/2 - Gamma[-1, a*x]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-1,ax)}{x^3} \, dx$$

$$\Big\downarrow \ 7116$$

$$\frac{1}{2}a^2\Gamma(-3,ax) - \frac{\Gamma(-1,ax)}{2x^2}$$

input
```
Int[Gamma[-1, a*x]/x^3,x]
```

output
```
(a^2*Gamma[-3, a*x])/2 - Gamma[-1, a*x]/(2*x^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(27) = 54.

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 5.64

method	result
meijerg	$a^2\Big(-\frac{1}{3x^3a^3} - \frac{-\frac{1}{2}+\gamma+\ln(x)+\ln(a)}{2x^2a^2} + \frac{1}{2ax} - \frac{7}{36} + \frac{\gamma}{12} + \frac{\ln(x)}{12} + \frac{\ln(a)}{12} + \frac{14x^3a^3-36a^2x^2-18xa+24}{72x^3a^3} - \frac{(-6a^2x^2+6\ldots}{72x\ldots}$

input
```
int(1/a/x^4*Ei(2,x*a),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/3/x^3/a^3-1/2*(-1/2+gamma+ln(x)+ln(a))/x^2/a^2+1/2/a/x-7/36+1/12*g
amma+1/12*ln(x)+1/12*ln(a)+1/72/x^3/a^3*(14*a^3*x^3-36*a^2*x^2-18*a*x+24)-
1/72/x^3/a^3*(-6*a^2*x^2+6*a*x+24)*exp(-x*a)-1/72/x^2/a^2*(-6*a^2*x^2+36)*
(-gamma-ln(x*a)-Ei(1,x*a)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{\Gamma(-1, ax)}{x^3} dx = -\frac{(ax-2)e^{(-ax)} - (a^3x^3 - 6ax)\Gamma(-1, ax)}{12ax^3}$$

input
```
integrate(gamma(-1,a*x)/x^3,x, algorithm="fricas")
```

output
```
-1/12*((a*x - 2)*e^(-a*x) - (a^3*x^3 - 6*a*x)*gamma(-1, a*x))/(a*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.12

$$\int \frac{\Gamma(-1, ax)}{x^3} dx = \frac{\frac{a^3 \operatorname{Ei}(axe^{i\pi})}{12} + \frac{a^2e^{-ax}}{12x} - \frac{a \operatorname{Ei}(axe^{i\pi})}{2x^2} + \frac{i\pi a}{2x^2} - \frac{ae^{-ax}}{12x^2} - \frac{e^{-ax}}{3x^3}}{a}$$

input
```
integrate(uppergamma(-1,a*x)/x**3,x)
```

output
```
(a**3*Ei(a*x*exp_polar(I*pi))/12 + a**2*exp(-a*x)/(12*x) - a*Ei(a*x*exp_po
lar(I*pi))/(2*x**2) + I*pi*a/(2*x**2) - a*exp(-a*x)/(12*x**2) - exp(-a*x)/
(3*x**3))/a
```

Maxima [F]

$$\int \frac{\Gamma(-1,ax)}{x^3}\,dx = \int \frac{\Gamma(-1,ax)}{x^3}\,dx$$

input
```
integrate(gamma(-1,a*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(gamma(-1, a*x)/x^3, x)
```

Giac [F]

$$\int \frac{\Gamma(-1,ax)}{x^3}\,dx = \int \frac{\Gamma(-1,ax)}{x^3}\,dx$$

input
```
integrate(gamma(-1,a*x)/x^3,x, algorithm="giac")
```

output
```
integrate(gamma(-1, a*x)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\Gamma(-1,ax)}{x^3}\,dx = -\frac{\mathrm{expint}(2,a\,x) - \mathrm{expint}(4,a\,x)}{2\,a\,x^3}$$

input
```
int(expint(2, a*x)/(a*x^4),x)
```

output
```
-(expint(2, a*x) - expint(4, a*x))/(2*a*x^3)
```

Reduce [F]

$$\int \frac{\Gamma(-1, ax)}{x^3}\, dx = \frac{\int \frac{ei(2,ax)}{x^4} dx}{a}$$

input
```
int(1/a/x^4*Ei(2,a*x),x)
```

output
```
int(ei(2,a*x)/x**4,x)/a
```

3.41 $\int \frac{\Gamma(-1,ax)}{x^4} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{\Gamma(-1,ax)}{x^4} \, dx = \frac{1}{3}a^3\Gamma(-4,ax) - \frac{\Gamma(-1,ax)}{3x^3}$$

output
```
1/3/a/x^4*Ei(5,a*x)-1/3/a/x^4*Ei(2,a*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(-1,ax)}{x^4} \, dx = \frac{1}{3}a^3\Gamma(-4,ax) - \frac{\Gamma(-1,ax)}{3x^3}$$

input
```
Integrate[Gamma[-1, a*x]/x^4,x]
```

output
```
(a^3*Gamma[-4, a*x])/3 - Gamma[-1, a*x]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-1,ax)}{x^4}\,dx$$

$$\Big\downarrow \ 7116$$

$$\frac{1}{3}a^3\Gamma(-4,ax)-\frac{\Gamma(-1,ax)}{3x^3}$$

input
```
Int[Gamma[-1, a*x]/x^4,x]
```

output
```
(a^3*Gamma[-4, a*x])/3 - Gamma[-1, a*x]/(3*x^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(27) = 54.

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 6.60

method	result
meijerg	$a^3\left(-\frac{1}{4a^4x^4}-\frac{-\frac{2}{3}+\gamma+\ln(x)+\ln(a)}{3x^3a^3}+\frac{1}{4x^2a^2}-\frac{1}{12ax}+\frac{29}{864}-\frac{\gamma}{72}-\frac{\ln(x)}{72}-\frac{\ln(a)}{72}+\frac{-145a^4x^4+360x^3a^3-1080a^2x}{4320x^4a^4}\right.$

input
```
int(1/a/x^5*Ei(2,x*a),x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/4/a^4/x^4-1/3*(-2/3+gamma+ln(x)+ln(a))/x^3/a^3+1/4/x^2/a^2-1/12/a/
x+29/864-1/72*gamma-1/72*ln(x)-1/72*ln(a)+1/4320/x^4/a^4*(-145*a^4*x^4+360
*a^3*x^3-1080*a^2*x^2-960*a*x+1080)-1/1440/x^4/a^4*(20*a^3*x^3-20*a^2*x^2+
40*a*x+360)*exp(-x*a)-1/1440/x^3/a^3*(20*a^3*x^3+480)*(-gamma-ln(x*a)-Ei(1
,x*a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(21) = 42$.

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \frac{\Gamma(-1,ax)}{x^4}\,dx = \frac{(a^2x^2 - 2\,ax + 6)e^{(-ax)} - (a^4x^4 + 24\,ax)\Gamma(-1,ax)}{72\,ax^4}$$

input
```
integrate(gamma(-1,a*x)/x^4,x, algorithm="fricas")
```

output
```
1/72*((a^2*x^2 - 2*a*x + 6)*e^(-a*x) - (a^4*x^4 + 24*a*x)*gamma(-1, a*x))/
(a*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.68

$$\int \frac{\Gamma(-1,ax)}{x^4}\,dx = \frac{-\frac{a^4\operatorname{Ei}\left(axe^{i\pi}\right)}{72} - \frac{a^3e^{-ax}}{72x} + \frac{a^2e^{-ax}}{72x^2} - \frac{a\operatorname{Ei}\left(axe^{i\pi}\right)}{3x^3} + \frac{i\pi a}{3x^3} - \frac{ae^{-ax}}{36x^3} - \frac{e^{-ax}}{4x^4}}{a}$$

input
```
integrate(uppergamma(-1,a*x)/x**4,x)
```

output
```
(-a**4*Ei(a*x*exp_polar(I*pi))/72 - a**3*exp(-a*x)/(72*x) + a**2*exp(-a*x)
/(72*x**2) - a*Ei(a*x*exp_polar(I*pi))/(3*x**3) + I*pi*a/(3*x**3) - a*exp(
-a*x)/(36*x**3) - exp(-a*x)/(4*x**4))/a
```

Maxima [F]

$$\int \frac{\Gamma(-1, ax)}{x^4} \, dx = \int \frac{\Gamma(-1, ax)}{x^4} \, dx$$

input
```
integrate(gamma(-1,a*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(gamma(-1, a*x)/x^4, x)
```

Giac [F]

$$\int \frac{\Gamma(-1, ax)}{x^4} \, dx = \int \frac{\Gamma(-1, ax)}{x^4} \, dx$$

input
```
integrate(gamma(-1,a*x)/x^4,x, algorithm="giac")
```

output
```
integrate(gamma(-1, a*x)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\Gamma(-1, ax)}{x^4} \, dx = -\frac{\operatorname{expint}(2, a\,x) - \operatorname{expint}(5, a\,x)}{3\,a\,x^4}$$

input
```
int(expint(2, a*x)/(a*x^5),x)
```

output
```
-(expint(2, a*x) - expint(5, a*x))/(3*a*x^4)
```

Reduce [F]

$$\int \frac{\Gamma(-1,ax)}{x^4}\,dx = \frac{\int \frac{ei(2,ax)}{x^5}dx}{a}$$

input
```
int(1/a/x^5*Ei(2,a*x),x)
```

output
```
int(ei(2,a*x)/x**5,x)/a
```

3.42 $\int x^{100}\Gamma(-2, ax)\, dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int x^{100}\Gamma(-2, ax)\, dx = \frac{1}{101}x^{101}\Gamma(-2, ax) - \frac{\Gamma(99, ax)}{101a^{101}}$$

output
```
1/101*x^99/a^2*Ei(3,a*x)-1/101*GAMMA(99,a*x)/a^101
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^{100}\Gamma(-2, ax)\, dx = \frac{1}{101}x^{101}\Gamma(-2, ax) - \frac{\Gamma(99, ax)}{101a^{101}}$$

input
```
Integrate[x^100*Gamma[-2, a*x],x]
```

output
```
(x^101*Gamma[-2, a*x])/101 - Gamma[99, a*x]/(101*a^101)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{100}\Gamma(-2,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{101}x^{101}\Gamma(-2,ax)-\frac{\Gamma(99,ax)}{101a^{101}}$$

input
```
Int[x^100*Gamma[-2, a*x],x]
```

output
```
(x^101*Gamma[-2, a*x])/101 - Gamma[99, a*x]/(101*a^101)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. 2(24) = 48.

Time = 0.06 (sec) , antiderivative size = 866, normalized size of antiderivative = 34.64

Expression too large to display

input
```
int(x^98/a^2*Ei(3,x*a),x)
```

output
```
1/a^101*(-1/202*(Psi(101)+gamma-3/2-Psi(102)+ln(x)+ln(a))*x^101*a^101-305/
40804*x^101*a^101+94268904488832477456261857430572424738096937640789516634
94238777294707070023223798882976159207729119823605850588608460429412647567
36000000000000000000000/101-1/61812*(306*a^100*x^100-306*a^99*x^99+612*a^
98*x^98+59976*a^97*x^97+5817672*a^96*x^96+558496512*a^95*x^95+53057168640*
a^94*x^94+4987373852160*a^93*x^93+463825768250880*a^92*x^92+42671970679080
960*a^91*x^91+3883149331796367360*a^90*x^90+349483439861673062400*a^89*x^8
9+31104026147688902553600*a^88*x^88+2737154300996623424716800*a^87*x^87+23
8132424186706237950361600*a^86*x^86+20479388480056736463731097600*a^85*x^8
5+1740748020804822599417143296000*a^84*x^84+146222833747605098351040036864
000*a^83*x^83+12136495201051223163136323059712000*a^82*x^82+99519260648620
0299377178490896384000*a^81*x^81+80610601125382224249551457762607104000*a^
80*x^80+6448848090030577939964116621008568320000*a^79*x^79+509458999112415
657257165213059676897280000*a^78*x^78+397378019307684212660588866186547979
87840000*a^77*x^77+30598107486691684374865342696364194450636800000*a^76*x^7
6+2325456168988568012489766044923678778248396800000*a^75*x^75+1744092126741
42600936732453369275908368629760000000*a^74*x^74+129062817378865524693182015
49326417219278602240000000*a^73*x^73+942158566865718330260228713100828457000
7337963520000000*a^72*x^72+678354168143317197787364673432596489045283333373
44000000*a^71*x^71+48163145938175521042902891813714350722215116695142400...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(21) = 42$.

Time = 0.12 (sec) , antiderivative size = 806, normalized size of antiderivative = 32.24

$$\int x^{100}\Gamma(-2,ax)\,dx = \text{Too large to display}$$

input
```
integrate(x^100*gamma(-2,a*x),x, algorithm="fricas")
```

output
```
1/101*(a^101*x^101*gamma(-2, a*x) - (a^98*x^98 + 98*a^97*x^97 + 9506*a^96*
x^96 + 912576*a^95*x^95 + 86694720*a^94*x^94 + 8149303680*a^93*x^93 + 7578
85242240*a^92*x^92 + 69725442286080*a^91*x^91 + 6345015248033280*a^90*x^90
 + 571051372322995200*a^89*x^89 + 50823572136746572800*a^88*x^88 + 4472474
348033698406400*a^87*x^87 + 389105268278931761356800*a^86*x^86 + 334630530
71988131476684800*a^85*x^85 + 2844359511118991175518208000*a^84*x^84 + 238
926198933995258743529472000*a^83*x^83 + 19830874511521606475712946176000*a
^82*x^82 + 1626131709944771731008461586432000*a^81*x^81 + 1317166685055265
10211685388500992000*a^80*x^80 + 10537333480442120816934831080079360000*a^
79*x^79 + 832449344954927544537851655326269440000*a^78*x^78 + 649310489064
84348473952429115449016320000*a^77*x^77 + 49996907657992948324943370418895
74256640000*a^76*x^76 + 3799764982007464072695696151836076435046400000*a^75
*x^75 + 284982373650559805452177211387705732628480000000*a^74*x^74 + 210886
95650141425603461113642690224214507520000000*a^73*x^73 + 153947478246032406
90526612959163863676590489600000000*a^72*x^72 + 1108421843371433329717916133
05979818471451525120000000*a^71*x^71 + 786979508793717664099720454472456711
147305828352000000*a^70*x^70 + 5508856561556023648698043181307196978031140
7984640000000*a^69*x^69 + 380111102747365631760164979510196591484148715094
0160000000*a^68*x^68 + 258475549868208629596912186066933682209221126263930
880000000*a^67*x^67 + 1731786184116997818299311646648455670801781545968...
```

Sympy [F(-1)]

Timed out.

$$\int x^{100}\Gamma(-2, ax)\, dx = \text{Timed out}$$

input
```
integrate(x**100*uppergamma(-2,a*x),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{100}\Gamma(-2,ax)\,dx = \int x^{100}\Gamma(-2,ax)\;dx$$

input
```
integrate(x^100*gamma(-2,a*x),x, algorithm="maxima")
```

output
```
integrate(x^100*gamma(-2, a*x), x)
```

Giac [F]

$$\int x^{100}\Gamma(-2,ax)\,dx = \int x^{100}\Gamma(-2,ax)\;dx$$

input
```
integrate(x^100*gamma(-2,a*x),x, algorithm="giac")
```

output
```
integrate(x^100*gamma(-2, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 814, normalized size of antiderivative = 32.56

$$\int x^{100}\Gamma(-2,ax)\,dx = \text{Too large to display}$$

input
```
int((x^98*expint(3, a*x))/a^2,x)
```

output
```
-(x^99*(exp(-a*x)*(1/(a*x) + 98/(a^2*x^2) + 9506/(a^3*x^3) + 912576/(a^4*x
^4) + 86694720/(a^5*x^5) + 8149303680/(a^6*x^6) + 757885242240/(a^7*x^7) +
 69725442286080/(a^8*x^8) + 6345015248033280/(a^9*x^9) + 57105137232299520
0/(a^10*x^10) + 50823572136746572800/(a^11*x^11) + 4472474348033698406400/
(a^12*x^12) + 389105268278931761356800/(a^13*x^13) + 33463053071988131476
684800/(a^14*x^14) + 2844359511118991175518208000/(a^15*x^15) + 23892619893
3995258743529472000/(a^16*x^16) + 19830874511521606475712946176000/(a^17*x
^17) + 1626131709944771731008461586432000/(a^18*x^18) + 131716668505526510
21168538850099200/(a^19*x^19) + 10537333480442120816934831080079360000/(a
^20*x^20) + 832449344954927544537851655326269440000/(a^21*x^21) + 64931048
906484348473952429115449016320000/(a^22*x^22) + 4999690765799294832494337041
88957425664000/(a^23*x^23) + 379976498200746407269569615183607643504640
000/(a^24*x^24) + 28498237365055980545217721138770573262848000000/(a^25*x^
25) + 21088695650141425603461113642690224214507520000000/(a^26*x^26) + 1539
474782460324069052661295916386367659048960000000/(a^27*x^27) + 110842184337
14333297179161330597981847145152512000000/(a^28*x^28) + 786979508793717664
09972045447245671114730582835200000/(a^29*x^29) + 550885656155602364869804
3181307196978031140798464000000/(a^30*x^30) + 380111102747365631760164979
51019659148414871509401600000/(a^31*x^31) + 25847554986820862959691218606
69336822092211262639308800000/(a^32*x^32) + 17317861841169978182993...
```

Reduce [F]

$$\int x^{100}\Gamma(-2,ax)\,dx=\frac{\int ei(3,ax)\,x^{98}dx}{a^2}$$

input
```
int(x^98/a^2*Ei(3,a*x),x)
```

output
```
int(ei(3,a*x)*x**98,x)/a**2
```

3.43 $\int x^3\Gamma(-2, ax)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int x^3\Gamma(-2,ax)\,dx = \frac{1}{4}x^4\Gamma(-2,ax) - \frac{\Gamma(2,ax)}{4a^4}$$

output
```
1/4*x^2/a^2*Ei(3,a*x)-1/4*exp(-a*x)*(a*x+1)/a^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^3\Gamma(-2,ax)\,dx = \frac{1}{4}x^4\Gamma(-2,ax) - \frac{\Gamma(2,ax)}{4a^4}$$

input
```
Integrate[x^3*Gamma[-2, a*x],x]
```

output
```
(x^4*Gamma[-2, a*x])/4 - Gamma[2, a*x]/(4*a^4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\Gamma(-2,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{4}x^4\Gamma(-2,ax)-\frac{\Gamma(2,ax)}{4a^4}$$

input
```
Int[x^3*Gamma[-2, a*x],x]
```

output
```
(x^4*Gamma[-2, a*x])/4 - Gamma[2, a*x]/(4*a^4)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(29) = 58.

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.36

method	result	size
meijerg	$\frac{-\frac{\left(-\frac{7}{4}+\gamma+\ln(x)+\ln(a)\right)x^4a^4}{8}-\frac{7a^4x^4}{32}+\frac{1}{4}-\frac{\left(15x^3a^3-15a^2x^2+30xa+30\right)\mathrm{e}^{-xa}}{120}-\frac{x^4a^4\left(-\gamma-\ln(xa)-\operatorname{expIntegral}_1(xa)\right)}{8}}{a^4}$	84

input
```
int(x/a^2*Ei(3,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(-1/8*(-7/4+gamma+ln(x)+ln(a))*x^4*a^4-7/32*a^4*x^4+1/4-1/120*(15*a^
3*x^3-15*a^2*x^2+30*a*x+30)*exp(-x*a)-1/8*x^4*a^4*(-gamma-ln(x*a)-Ei(1,x*a
)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int x^3\Gamma(-2,ax)\,dx = \frac{a^4x^4\Gamma(-2,ax)-(ax+1)e^{(-ax)}}{4\,a^4}$$

input
```
integrate(x^3*gamma(-2,a*x),x, algorithm="fricas")
```

output
```
1/4*(a^4*x^4*gamma(-2, a*x) - (a*x + 1)*e^(-a*x))/a^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.04

$$\int x^3\Gamma(-2,ax)\,dx = \frac{-\frac{a^2x^4\operatorname{Ei}\left(axe^{i\pi}\right)}{8}+\frac{i\pi a^2x^4}{8}-\frac{ax^3e^{-ax}}{8}+\frac{x^2e^{-ax}}{8}-\frac{xe^{-ax}}{4a}-\frac{e^{-ax}}{4a^2}}{a^2}$$

input
```
integrate(x**3*uppergamma(-2,a*x),x)
```

output
```
(-a**2*x**4*Ei(a*x*exp_polar(I*pi))/8 + I*pi*a**2*x**4/8 - a*x**3*exp(-a*x
)/8 + x**2*exp(-a*x)/8 - x*exp(-a*x)/(4*a) - exp(-a*x)/(4*a**2))/a**2
```

Maxima [F]

$$\int x^3\Gamma(-2, ax)\,dx = \int x^3\Gamma(-2, ax)\;dx$$

input
```
integrate(x^3*gamma(-2,a*x),x, algorithm="maxima")
```

output
```
integrate(x^3*gamma(-2, a*x), x)
```

Giac [F]

$$\int x^3\Gamma(-2, ax)\,dx = \int x^3\Gamma(-2, ax)\;dx$$

input
```
integrate(x^3*gamma(-2,a*x),x, algorithm="giac")
```

output
```
integrate(x^3*gamma(-2, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int x^3\Gamma(-2, ax)\,dx = -\frac{x^2\left(e^{-a\,x}\left(\frac{1}{a\,x}+\frac{1}{a^2\,x^2}\right)-\mathrm{expint}(3, a\,x)\right)}{4\,a^2}$$

input
```
int((x*expint(3, a*x))/a^2,x)
```

output
```
-(x^2*(exp(-a*x)*(1/(a*x) + 1/(a^2*x^2)) - expint(3, a*x)))/(4*a^2)
```

Reduce [F]

$$\int x^3\Gamma(-2,ax)\,dx = \frac{\int ei(3,ax)\,xdx}{a^2}$$

input
```
int(x/a^2*Ei(3,a*x),x)
```

output
```
int(ei(3,a*x)*x,x)/a**2
```

3.44 $\int x^2\Gamma(-2,ax)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 26

$$\int x^2\Gamma(-2,ax)\,dx = -\frac{e^{-ax}}{3a^3} + \frac{1}{3}x^3\Gamma(-2,ax)$$

output
```
-1/3/a^3/exp(a*x)+1/3*x/a^2*Ei(3,a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^2\Gamma(-2,ax)\,dx = -\frac{e^{-ax}}{3a^3} + \frac{1}{3}x^3\Gamma(-2,ax)$$

input
```
Integrate[x^2*Gamma[-2, a*x],x]
```

output
```
-1/3*1/(a^3*E^(a*x)) + (x^3*Gamma[-2, a*x])/3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\Gamma(-2,ax)\,dx$$

$$\Big\downarrow \ 7116$$

$$\frac{1}{3}x^3\Gamma(-2,ax)-\frac{e^{-ax}}{3a^3}$$

input
```
Int[x^2*Gamma[-2, a*x],x]
```

output
```
-1/3*1/(a^3*E^(a*x)) + (x^3*Gamma[-2, a*x])/3
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(23) = 46.

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.92

method	result	size
meijerg	$\frac{-\frac{\left(\gamma-\frac{11}{6}+\ln(x)+\ln(a)\right)x^{3}a^{3}}{6}-\frac{11x^{3}a^{3}}{36}+\frac{1}{3}-\frac{\left(4a^{2}x^{2}-4xa+8\right)\mathrm{e}^{-xa}}{24}-\frac{a^{3}x^{3}\left(-\gamma-\ln(xa)-\mathrm{expIntegral}_{1}(xa)\right)}{6}}{a^{3}}$	76

input
```
int(1/a^2*Ei(3,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/6*(gamma-11/6+ln(x)+ln(a))*x^3*a^3-11/36*x^3*a^3+1/3-1/24*(4*a^2
*x^2-4*a*x+8)*exp(-x*a)-1/6*a^3*x^3*(-gamma-ln(x*a)-Ei(1,x*a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int x^2\Gamma(-2,ax)\,dx = \frac{a^3x^3\Gamma(-2,ax) - e^{(-ax)}}{3\,a^3}$$

input
```
integrate(x^2*gamma(-2,a*x),x, algorithm="fricas")
```

output
```
1/3*(a^3*x^3*gamma(-2, a*x) - e^(-a*x))/a^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int x^2\Gamma(-2,ax)\,dx = \frac{-\frac{a^2x^3\operatorname{Ei}\left(axe^{i\pi}\right)}{6} + \frac{i\pi a^2x^3}{6} - \frac{ax^2e^{-ax}}{6} + \frac{xe^{-ax}}{6} - \frac{e^{-ax}}{3a}}{a^2}$$

input
```
integrate(x**2*uppergamma(-2,a*x),x)
```

output
```
(-a**2*x**3*Ei(a*x*exp_polar(I*pi))/6 + I*pi*a**2*x**3/6 - a*x**2*exp(-a*x
)/6 + x*exp(-a*x)/6 - exp(-a*x)/(3*a))/a**2
```

Maxima [F]

$$\int x^2\Gamma(-2,ax)\,dx = \int x^2\Gamma(-2,ax)\,dx$$

input
```
integrate(x^2*gamma(-2,a*x),x, algorithm="maxima")
```

output
```
integrate(x^2*gamma(-2, a*x), x)
```

Giac [F]

$$\int x^2\Gamma(-2,ax)\,dx = \int x^2\Gamma(-2,ax)\,dx$$

input
```
integrate(x^2*gamma(-2,a*x),x, algorithm="giac")
```

output
```
integrate(x^2*gamma(-2, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^2\Gamma(-2,ax)\,dx = \frac{x\,\mathrm{expint}(3,a\,x)}{3\,a^2} - \frac{e^{-a\,x}}{3\,a^3}$$

input
```
int(expint(3, a*x)/a^2,x)
```

output
```
(x*expint(3, a*x))/(3*a^2) - exp(-a*x)/(3*a^3)
```

Reduce [F]

$$\int x^2\Gamma(-2,ax)\,dx=\frac{\int ei(3,ax)\,dx}{a^2}$$

input
```
int(1/a^2*Ei(3,a*x),x)
```

output
```
int(ei(3,a*x),x)/a**2
```

3.45 $\int x\Gamma(-2,ax)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 25

$$\int x\Gamma(-2,ax)\,dx = \frac{1}{2}x^2\Gamma(-2,ax) - \frac{\Gamma(0,ax)}{2a^2}$$

output
```
1/2/a^2*Ei(3,a*x)-1/2*Ei(1,a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\Gamma(-2,ax)\,dx = \frac{1}{2}x^2\Gamma(-2,ax) - \frac{\Gamma(0,ax)}{2a^2}$$

input
```
Integrate[x*Gamma[-2, a*x],x]
```

output
```
(x^2*Gamma[-2, a*x])/2 - Gamma[0, a*x]/(2*a^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\Gamma(-2,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{2}x^2\Gamma(-2,ax)-\frac{\Gamma(0,ax)}{2a^2}$$

input
```
Int[x*Gamma[-2, a*x],x]
```

output
```
(x^2*Gamma[-2, a*x])/2 - Gamma[0, a*x]/(2*a^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(21) = 42$.

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.28

method	result	size
meijerg	$\frac{\frac{\gamma}{2}+\frac{\ln(x)}{2}+\frac{\ln(a)}{2}-\frac{(-2+\gamma+\ln(x)+\ln(a))x^2a^2}{4}-\frac{a^2x^2}{2}+\frac{(-9xa+9)e^{-xa}}{36}+\frac{\left(-9a^2x^2+18\right)(-\gamma-\ln(xa)-\text{expIntegral}_1(xa))}{36}}{a^2}$	82

input
```
int(1/x/a^2*Ei(3,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/2*gamma+1/2*ln(x)+1/2*ln(a)-1/4*(-2+gamma+ln(x)+ln(a))*x^2*a^2-1/
2*a^2*x^2+1/36*(-9*a*x+9)*exp(-x*a)+1/36*(-9*a^2*x^2+18)*(-gamma-ln(x*a)-E
i(1,x*a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int x\Gamma(-2,ax)\,dx = -\frac{(ax-1)e^{(-ax)} - (a^4x^4 - 2\,a^2x^2)\Gamma(-2,ax)}{2\,a^4x^2}$$

input
```
integrate(x*gamma(-2,a*x),x, algorithm="fricas")
```

output
```
-1/2*((a*x - 1)*e^(-a*x) - (a^4*x^4 - 2*a^2*x^2)*gamma(-2, a*x))/(a^4*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int x\Gamma(-2,ax)\,dx = \frac{-\frac{a^2x^2\operatorname{Ei}\left(axe^{i\pi}\right)}{4} + \frac{i\pi a^2x^2}{4} - \frac{axe^{-ax}}{4} + \frac{\operatorname{Ei}\left(axe^{i\pi}\right)}{2} + \frac{e^{-ax}}{4}}{a^2}$$

input
```
integrate(x*uppergamma(-2,a*x),x)
```

output
```
(-a**2*x**2*Ei(a*x*exp_polar(I*pi))/4 + I*pi*a**2*x**2/4 - a*x*exp(-a*x)/4
 + Ei(a*x*exp_polar(I*pi))/2 + exp(-a*x)/4)/a**2
```

Maxima [F]

$$\int x\Gamma(-2,ax)\,dx = \int x\Gamma(-2,ax)\,dx$$

input
```
integrate(x*gamma(-2,a*x),x, algorithm="maxima")
```

output
```
integrate(x*gamma(-2, a*x), x)
```

Giac [F]

$$\int x\Gamma(-2,ax)\,dx = \int x\Gamma(-2,ax)\,dx$$

input
```
integrate(x*gamma(-2,a*x),x, algorithm="giac")
```

output
```
integrate(x*gamma(-2, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int x\Gamma(-2,ax)\,dx = -\frac{\mathrm{expint}(a\,x) - \mathrm{expint}(3,a\,x)}{2\,a^2}$$

input
```
int(expint(3, a*x)/(a^2*x),x)
```

output
```
-(expint(a*x) - expint(3, a*x))/(2*a^2)
```

Reduce [F]

$$\int x\Gamma(-2, ax)\, dx = \frac{\int \frac{ei(3,ax)}{x}dx}{a^2}$$

input
```
int(1/x/a^2*Ei(3,a*x),x)
```

output
```
int(ei(3,a*x)/x,x)/a**2
```

3.46 $\int \Gamma(-2, ax)\, dx$

Optimal result

Integrand size = 5, antiderivative size = 18

$$\int \Gamma(-2, ax)\, dx = x\Gamma(-2, ax) - \frac{\Gamma(-1, ax)}{a}$$

output
```
1/x/a^2*Ei(3,a*x)-1/a^2/x*Ei(2,a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \Gamma(-2, ax)\, dx = -\frac{e^{-ax}}{a^2 x} - \frac{\text{ExpIntegralEi}(-ax)}{a} + x\Gamma(-2, ax)$$

input
```
Integrate[Gamma[-2, a*x],x]
```

output
```
-(1/(a^2*E^(a*x)*x)) - ExpIntegralEi[-(a*x)]/a + x*Gamma[-2, a*x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(-2, ax)\, dx$$

$$\Big\downarrow \text{ 7111}$$

$$x\Gamma(-2, ax) - \frac{\Gamma(-1, ax)}{a}$$

input
```
Int[Gamma[-2, a*x],x]
```

output
```
x*Gamma[-2, a*x] - Gamma[-1, a*x]/a
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 5.44

method	result	size
meijerg	$\frac{-\frac{1}{2ax}-\gamma-\ln(x)-\ln(a)-\frac{\left(\gamma-\frac{5}{2}+\ln(x)+\ln(a)\right)xa}{2}+\frac{-15a^2x^2+6}{12ax}-\frac{(6xa+6)\mathrm{e}^{-xa}}{12ax}-\frac{(6xa+12)(-\gamma-\ln(xa)-\mathrm{expIntegral}_1(xa))}{12}}{a}$	98

input
```
int(1/x^2/a^2*Ei(3,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/2/a/x-gamma-ln(x)-ln(a)-1/2*(gamma-5/2+ln(x)+ln(a))*x*a+1/12/a/x*(
-15*a^2*x^2+6)-1/12/a/x*(6*a*x+6)*exp(-x*a)-1/12*(6*a*x+12)*(-gamma-ln(x*a
)-Ei(1,x*a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \Gamma(-2, ax)\, dx = \frac{(a^3x^3 + 2\, a^2x^2)\Gamma(-2, ax) - e^{(-ax)}}{a^3x^2}$$

input
```
integrate(gamma(-2,a*x),x, algorithm="fricas")
```

output
```
((a^3*x^3 + 2*a^2*x^2)*gamma(-2, a*x) - e^(-a*x))/(a^3*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.33

$$\int \Gamma(-2, ax)\, dx = \frac{-\frac{a^2x\,\mathrm{Ei}\left(axe^{i\pi}\right)}{2} + \frac{i\pi a^2x}{2} - a\,\mathrm{Ei}\left(axe^{i\pi}\right) - \frac{ae^{-ax}}{2} - \frac{e^{-ax}}{2x}}{a^2}$$

input
```
integrate(uppergamma(-2,a*x),x)
```

output
```
(-a**2*x*Ei(a*x*exp_polar(I*pi))/2 + I*pi*a**2*x/2 - a*Ei(a*x*exp_polar(I*
pi)) - a*exp(-a*x)/2 - exp(-a*x)/(2*x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \Gamma(-2, ax)\, dx = \frac{ax\Gamma(-2, ax) - \Gamma(-1, ax)}{a}$$

input
```
integrate(gamma(-2,a*x),x, algorithm="maxima")
```

output
```
(a*x*gamma(-2, a*x) - gamma(-1, a*x))/a
```

Giac [F]

$$\int \Gamma(-2, ax)\, dx = \int \Gamma(-2, ax)\ dx$$

input
```
integrate(gamma(-2,a*x),x, algorithm="giac")
```

output
```
integrate(gamma(-2, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \Gamma(-2, ax)\, dx = -\frac{\mathrm{expint}(2, a\,x) - \mathrm{expint}(3, a\,x)}{a^2\,x}$$

input
```
int(expint(3, a*x)/(a^2*x^2),x)
```

output
```
-(expint(2, a*x) - expint(3, a*x))/(a^2*x)
```

Reduce [F]

$$\int \Gamma(-2, ax)\, dx = \frac{\int \frac{ei(3,ax)}{x^2} dx}{a^2}$$

input
```
int(1/a^2/x^2*Ei(3,a*x),x)
```

output
```
int(ei(3,a*x)/x**2,x)/a**2
```

3.47 $\int \frac{\Gamma(-2,ax)}{x} dx$

Optimal result

Integrand size = 9, antiderivative size = 55

$$\int \frac{\Gamma(-2,ax)}{x} dx = -\frac{1}{2}\Gamma(-2,ax) + \frac{\Gamma(-1,ax)}{2} + \frac{1}{2}ax\,{}_3F_3(1,1,1;2,2,2;-ax) - \frac{1}{2}\gamma\log(x) - \frac{1}{4}\log^2(ax)$$

output
```
-1/2/a^2/x^2*Ei(3,a*x)+1/2/a/x*Ei(2,a*x)+1/2*a*x*hypergeom([1, 1, 1],[2, 2
, 2],-a*x)-1/2*gamma*ln(x)-1/4*ln(a*x)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 121 vs. 2(55) = 110.

Time = 0.51 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.20

$$\int \frac{\Gamma(-2,ax)}{x} dx = \Gamma(-2,ax)\log(ax) + \frac{1}{4}\left(\frac{e^{-ax}(-1+3ax)}{a^2x^2} + 3\,\text{ExpIntegralEi}(-ax) + 2ax\,{}_3F_3(1,1,1;2,2,2;-ax) - 2\,\text{ExpIntegralEi}(-ax)\log(x) + \log^2(x) + \frac{2e^{-ax}(-1+ax)\log(ax)}{a^2x^2} + 2\,\text{ExpIntegralEi}(-ax)\log(ax) - 2\log(x)(\gamma+\Gamma(0,ax)+\log(ax))\right)$$

input
```
Integrate[Gamma[-2, a*x]/x,x]
```

output
```
Gamma[-2, a*x]*Log[a*x] + ((-1 + 3*a*x)/(a^2*E^(a*x)*x^2) + 3*ExpIntegralE
i[-(a*x)] + 2*a*x*HypergeometricPFQ[{1, 1, 1}, {2, 2, 2}, -(a*x)] - 2*ExpI
ntegralEi[-(a*x)]*Log[x] + Log[x]^2 + (2*(-1 + a*x)*Log[a*x])/(a^2*E^(a*x)
*x^2) + 2*ExpIntegralEi[-(a*x)]*Log[a*x] - 2*Log[x]*(EulerGamma + Gamma[0,
 a*x] + Log[a*x]))/4
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7114, 7114, 7112}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-2,ax)}{x}\,dx$$

$$\Big\downarrow \text{7114}$$

$$-\frac{1}{2}\int \frac{\Gamma(-1,ax)}{x}dx-\frac{1}{2}\Gamma(-2,ax)$$

$$\Big\downarrow \text{7114}$$

$$\frac{1}{2}\left(\int \frac{\Gamma(0,ax)}{x}dx+\Gamma(-1,ax)\right)-\frac{\Gamma(-2,ax)}{2}$$

$$\Big\downarrow \text{7112}$$

$$\frac{1}{2}\left(ax\,{}_3F_3(1,1,1;2,2,2;-ax)-\frac{1}{2}\log^2(ax)+\Gamma(-1,ax)-\gamma\log(x)\right)-\frac{\Gamma(-2,ax)}{2}$$

input
```
Int[Gamma[-2, a*x]/x,x]
```

output
```
-1/2*Gamma[-2, a*x] + (Gamma[-1, a*x] + a*x*HypergeometricPFQ[{1, 1, 1}, {
2, 2, 2}, -(a*x)] - EulerGamma*Log[x] - Log[a*x]^2/2)/2
```

Defintions of rubi rules used

rule 7112
```
Int[Gamma[0, (b_.)*(x_)]/(x_), x_Symbol] :> Simp[b*x*HypergeometricPFQ[{1,
1, 1}, {2, 2, 2}, (-b)*x], x] + (-Simp[EulerGamma*Log[x], x] - Simp[(1/2)*L
og[b*x]^2, x]) /; FreeQ[b, x]
```

rule 7114
```
Int[Gamma[n_, (b_.)*(x_)]/(x_), x_Symbol] :> Simp[Gamma[n, b*x]/n, x] + Sim
p[1/n   Int[Gamma[n + 1, b*x]/x, x], x] /; FreeQ[b, x] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

method	result
meijerg	$-\frac{1}{4x^2a^2}+\frac{1}{ax}-\frac{\pi^2}{24}-\frac{\ln(a)^2}{4}-\frac{\ln(x)^2}{4}-\frac{5}{16}-\frac{\left(\frac{3}{2}-\gamma\right)^2}{4}+\frac{\ln(a)\left(\frac{3}{2}-\gamma\right)}{2}+\frac{\ln(x)\left(\frac{3}{2}-\gamma\right)}{2}-\frac{\ln(x)\ln(a)}{2}+\frac{xa\,\text{hyperge}}{}$

input
```
int(1/x^3/a^2*Ei(3,x*a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^2/a^2+1/a/x-1/24*Pi^2-1/4*ln(a)^2-1/4*ln(x)^2-5/16-1/4*(3/2-gamma)^
2+1/2*ln(a)*(3/2-gamma)+1/2*ln(x)*(3/2-gamma)-1/2*ln(x)*ln(a)+1/6*x*a*hype
rgeom([1,1,1],[2,2,4],-x*a)
```

Fricas [F]

$$\int \frac{\Gamma(-2,ax)}{x}\,dx=\int \frac{\Gamma(-2,ax)}{x}\,dx$$

input
```
integrate(gamma(-2,a*x)/x,x, algorithm="fricas")
```

output
```
integral(gamma(-2, a*x)/x, x)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{\Gamma(-2,ax)}{x}\,dx = \frac{\frac{a^3x_3F_3\left(\begin{matrix}1,1,1\\2,2,4\end{matrix}\middle|-ax\right)}{6} - \frac{a^2\log{(ax)}^2}{4} - \frac{\gamma a^2\log{(ax)}}{2} + \frac{3a^2\log{(ax)}}{4} + \frac{a}{x} - \frac{1}{4x^2}}{a^2}$$

input
```
integrate(uppergamma(-2,a*x)/x,x)
```

output
```
(a**3*x**hyper((1, 1, 1), (2, 2, 4), -a*x)/6 - a**2*log(a*x)**2/4 - EulerGa
mma*a**2*log(a*x)/2 + 3*a**2*log(a*x)/4 + a/x - 1/(4*x**2))/a**2
```

Maxima [F]

$$\int \frac{\Gamma(-2,ax)}{x}\,dx = \int \frac{\Gamma(-2,ax)}{x}\,dx$$

input
```
integrate(gamma(-2,a*x)/x,x, algorithm="maxima")
```

output
```
integrate(gamma(-2, a*x)/x, x)
```

Giac [F]

$$\int \frac{\Gamma(-2,ax)}{x}\,dx = \int \frac{\Gamma(-2,ax)}{x}\,dx$$

input
```
integrate(gamma(-2,a*x)/x,x, algorithm="giac")
```

output
```
integrate(gamma(-2, a*x)/x, x)
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.25

$$\int \frac{\Gamma(-2,ax)}{x}\,dx = \int \frac{\mathrm{expint}(3,a\,x)}{a^2\,x^3}\,dx$$

input
```
int(expint(3, a*x)/(a^2*x^3),x)
```

output
```
int(expint(3, a*x)/(a^2*x^3), x)
```

Reduce [F]

$$\int \frac{\Gamma(-2,ax)}{x}\,dx = \frac{\int \frac{ei(3,ax)}{x^3}dx}{a^2}$$

input
```
int(1/a^2/x^3*Ei(3,a*x),x)
```

output
```
int(ei(3,a*x)/x**3,x)/a**2
```

3.48 $\int \frac{\Gamma(-2,ax)}{x^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int \frac{\Gamma(-2,ax)}{x^2} dx = a\Gamma(-3,ax) - \frac{\Gamma(-2,ax)}{x}$$

output
```
1/a^2/x^3*Ei(4,a*x)-1/a^2/x^3*Ei(3,a*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(-2,ax)}{x^2} dx = a\Gamma(-3,ax) - \frac{\Gamma(-2,ax)}{x}$$

input
```
Integrate[Gamma[-2, a*x]/x^2,x]
```

output
```
a*Gamma[-3, a*x] - Gamma[-2, a*x]/x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-2,ax)}{x^2} dx$$

$$\Big\downarrow \ 7116$$

$$a\Gamma(-3,ax) - \frac{\Gamma(-2,ax)}{x}$$

input
```
Int[Gamma[-2, a*x]/x^2,x]
```

output
```
a*Gamma[-3, a*x] - Gamma[-2, a*x]/x
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(26) = 52.

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 7.50

method	result
meijerg	$a\left(-\frac{1}{6x^3a^3} + \frac{1}{2x^2a^2} + \frac{-\frac{1}{2}+\gamma+\ln(x)+\ln(a)}{2xa} + \frac{\gamma}{6} - \frac{17}{36} + \frac{\ln(x)}{6} + \frac{\ln(a)}{6} + \frac{68x^3a^3+36a^2x^2-72xa+24}{144x^3a^3} - \frac{(-8a^2x^2-1}{48x}\right.$

input
```
int(1/x^4/a^2*Ei(3,x*a),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/6/x^3/a^3+1/2/x^2/a^2+1/2*(-1/2+gamma+ln(x)+ln(a))/x/a+1/6*gamma-17/
36+1/6*ln(x)+1/6*ln(a)+1/144/x^3/a^3*(68*a^3*x^3+36*a^2*x^2-72*a*x+24)-1/4
8/x^3/a^3*(-8*a^2*x^2-16*a*x+8)*exp(-x*a)+1/48/x/a*(8*a*x+24)*(-gamma-ln(x
*a)-Ei(1,x*a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{\Gamma(-2, ax)}{x^2}\, dx = -\frac{(a^3x^3 + 3\,a^2x^2)\Gamma(-2, ax) - e^{(-ax)}}{3\,a^2x^3}$$

input
```
integrate(gamma(-2,a*x)/x^2,x, algorithm="fricas")
```

output
```
-1/3*((a^3*x^3 + 3*a^2*x^2)*gamma(-2, a*x) - e^(-a*x))/(a^2*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 4.44

$$\int \frac{\Gamma(-2, ax)}{x^2}\, dx = \frac{\frac{a^3 \operatorname{Ei}\left(axe^{i\pi}\right)}{6} + \frac{a^2 \operatorname{Ei}\left(axe^{i\pi}\right)}{2x} - \frac{i\pi a^2}{2x} + \frac{a^2e^{-ax}}{6x} + \frac{ae^{-ax}}{3x^2} - \frac{e^{-ax}}{6x^3}}{a^2}$$

input
```
integrate(uppergamma(-2,a*x)/x**2,x)
```

output
```
(a**3*Ei(a*x*exp_polar(I*pi))/6 + a**2*Ei(a*x*exp_polar(I*pi))/(2*x) - I*p
i*a**2/(2*x) + a**2*exp(-a*x)/(6*x) + a*exp(-a*x)/(3*x**2) - exp(-a*x)/(6*
x**3))/a**2
```

Maxima [F]

$$\int \frac{\Gamma(-2, ax)}{x^2} \, dx = \int \frac{\Gamma(-2, ax)}{x^2} \, dx$$

input
```
integrate(gamma(-2,a*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(gamma(-2, a*x)/x^2, x)
```

Giac [F]

$$\int \frac{\Gamma(-2, ax)}{x^2} \, dx = \int \frac{\Gamma(-2, ax)}{x^2} \, dx$$

input
```
integrate(gamma(-2,a*x)/x^2,x, algorithm="giac")
```

output
```
integrate(gamma(-2, a*x)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{\Gamma(-2, ax)}{x^2} \, dx = -\frac{\mathrm{expint}(3, a\, x) - \mathrm{expint}(4, a\, x)}{a^2\, x^3}$$

input
```
int(expint(3, a*x)/(a^2*x^4),x)
```

output
```
-(expint(3, a*x) - expint(4, a*x))/(a^2*x^3)
```

Reduce [F]

$$\int \frac{\Gamma(-2, ax)}{x^2} \, dx = \frac{\int \frac{ei(3,ax)}{x^4} dx}{a^2}$$

input
```
int(1/a^2/x^4*Ei(3,a*x),x)
```

output
```
int(ei(3,a*x)/x**4,x)/a**2
```

3.49 $\int \frac{\Gamma(-2,ax)}{x^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{\Gamma(-2,ax)}{x^3} dx = \frac{1}{2}a^2\Gamma(-4,ax) - \frac{\Gamma(-2,ax)}{2x^2}$$

output
```
1/2/a^2/x^4*Ei(5,a*x)-1/2/a^2/x^4*Ei(3,a*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(-2,ax)}{x^3} dx = \frac{1}{2}a^2\Gamma(-4,ax) - \frac{\Gamma(-2,ax)}{2x^2}$$

input
```
Integrate[Gamma[-2, a*x]/x^3,x]
```

output
```
(a^2*Gamma[-4, a*x])/2 - Gamma[-2, a*x]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-2,ax)}{x^3}\,dx$$

$$\Big\downarrow \; 7116$$

$$\frac{1}{2}a^2\Gamma(-4,ax) - \frac{\Gamma(-2,ax)}{2x^2}$$

input
```
Int[Gamma[-2, a*x]/x^3,x]
```

output
```
(a^2*Gamma[-4, a*x])/2 - Gamma[-2, a*x]/(2*x^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(27) = 54.

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 6.60

method	result
meijerg	$a^2\Big(-\frac{1}{8a^4x^4}+\frac{1}{3x^3a^3}+\frac{\ln(x)+\ln(a)-1+\gamma}{4x^2a^2}-\frac{1}{6ax}+\frac{31}{576}-\frac{\gamma}{48}-\frac{\ln(x)}{48}-\frac{\ln(a)}{48}+\frac{-155a^4x^4+480x^3a^3+720a^2x^2-96}{2880x^4a^4}$

input
```
int(1/x^5/a^2*Ei(3,x*a),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/8/a^4/x^4+1/3/x^3/a^3+1/4*(ln(x)+ln(a)-1+gamma)/x^2/a^2-1/6/a/x+31
/576-1/48*gamma-1/48*ln(x)-1/48*ln(a)+1/2880/x^4/a^4*(-155*a^4*x^4+480*a^3
*x^3+720*a^2*x^2-960*a*x+360)-1/720/x^4/a^4*(15*a^3*x^3-15*a^2*x^2-150*a*x
+90)*exp(-x*a)+1/720/x^2/a^2*(-15*a^2*x^2+180)*(-gamma-ln(x*a)-Ei(1,x*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \frac{\Gamma(-2,ax)}{x^3}\,dx = -\frac{(ax-3)e^{(-ax)} - (a^4x^4 - 12\,a^2x^2)\Gamma(-2,ax)}{24\,a^2x^4}$$

input
```
integrate(gamma(-2,a*x)/x^3,x, algorithm="fricas")
```

output
```
-1/24*((a*x - 3)*e^(-a*x) - (a^4*x^4 - 12*a^2*x^2)*gamma(-2, a*x))/(a^2*x^
4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.63 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.96

$$\int \frac{\Gamma(-2,ax)}{x^3}\,dx = \frac{-\frac{a^4\operatorname{Ei}\left(axe^{i\pi}\right)}{48} - \frac{a^3e^{-ax}}{48x} + \frac{a^2\operatorname{Ei}\left(axe^{i\pi}\right)}{4x^2} - \frac{i\pi a^2}{4x^2} + \frac{a^2e^{-ax}}{48x^2} + \frac{5ae^{-ax}}{24x^3} - \frac{e^{-ax}}{8x^4}}{a^2}$$

input
```
integrate(uppergamma(-2,a*x)/x**3,x)
```

output
```
(-a**4*Ei(a*x*exp_polar(I*pi))/48 - a**3*exp(-a*x)/(48*x) + a**2*Ei(a*x*ex
p_polar(I*pi))/(4*x**2) - I*pi*a**2/(4*x**2) + a**2*exp(-a*x)/(48*x**2) +
5*a*exp(-a*x)/(24*x**3) - exp(-a*x)/(8*x**4))/a**2
```

Maxima [F]

$$\int \frac{\Gamma(-2, ax)}{x^3}\,dx = \int \frac{\Gamma(-2, ax)}{x^3}\,dx$$

input
```
integrate(gamma(-2,a*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(gamma(-2, a*x)/x^3, x)
```

Giac [F]

$$\int \frac{\Gamma(-2, ax)}{x^3}\,dx = \int \frac{\Gamma(-2, ax)}{x^3}\,dx$$

input
```
integrate(gamma(-2,a*x)/x^3,x, algorithm="giac")
```

output
```
integrate(gamma(-2, a*x)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\Gamma(-2, ax)}{x^3}\,dx = -\frac{\mathrm{expint}(3, a\,x) - \mathrm{expint}(5, a\,x)}{2\,a^2\,x^4}$$

input
```
int(expint(3, a*x)/(a^2*x^5),x)
```

output
```
-(expint(3, a*x) - expint(5, a*x))/(2*a^2*x^4)
```

Reduce [F]

$$\int \frac{\Gamma(-2, ax)}{x^3} \, dx = \frac{\int \frac{ei(3,ax)}{x^5} dx}{a^2}$$

input
```
int(1/a^2/x^5*Ei(3,a*x),x)
```

output
```
int(ei(3,a*x)/x**5,x)/a**2
```

3.50 $\int \frac{\Gamma(-2,ax)}{x^4} dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{\Gamma(-2,ax)}{x^4} dx = \frac{1}{3}a^3\Gamma(-5,ax) - \frac{\Gamma(-2,ax)}{3x^3}$$

output
```
1/3/a^2/x^5*Ei(6,a*x)-1/3/a^2/x^5*Ei(3,a*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(-2,ax)}{x^4} dx = \frac{1}{3}a^3\Gamma(-5,ax) - \frac{\Gamma(-2,ax)}{3x^3}$$

input
```
Integrate[Gamma[-2, a*x]/x^4,x]
```

output
```
(a^3*Gamma[-5, a*x])/3 - Gamma[-2, a*x]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-2,ax)}{x^4} \, dx$$

$$\Big\downarrow \; 7116$$

$$\frac{1}{3}a^3\Gamma(-5,ax) - \frac{\Gamma(-2,ax)}{3x^3}$$

input
```
Int[Gamma[-2, a*x]/x^4,x]
```

output
```
(a^3*Gamma[-5, a*x])/3 - Gamma[-2, a*x]/(3*x^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(27) = 54.

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 7.56

method	result
meijerg	$a^3\left(-\frac{1}{10x^5a^5}+\frac{1}{4a^4x^4}+\frac{-\frac{7}{6}+\gamma+\ln(x)+\ln(a)}{6x^3a^3}-\frac{1}{12x^2a^2}+\frac{1}{48ax}-\frac{157}{21600}+\frac{\gamma}{360}+\frac{\ln(x)}{360}+\frac{\ln(a)}{360}+\frac{314x^5a^5-900a\right.$

input
```
int(1/x^6/a^2*Ei(3,x*a),x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/10/x^5/a^5+1/4/a^4/x^4+1/6*(-7/6+gamma+ln(x)+ln(a))/x^3/a^3-1/12/x
^2/a^2+1/48/a/x-157/21600+1/360*gamma+1/360*ln(x)+1/360*ln(a)+1/43200/x^5/
a^5*(314*a^5*x^5-900*a^4*x^4+3600*a^3*x^3+8400*a^2*x^2-10800*a*x+4320)-1/2
880/x^5/a^5*(-8*a^4*x^4+8*a^3*x^3-16*a^2*x^2-432*a*x+288)*exp(-x*a)+1/2880
/x^3/a^3*(8*a^3*x^3+480)*(-gamma-ln(x*a)-Ei(1,x*a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(21) = 42$.

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{\Gamma(-2, ax)}{x^4}\, dx = \frac{(a^2x^2 - 3\,ax + 12)e^{(-ax)} - (a^5x^5 + 60\,a^2x^2)\Gamma(-2, ax)}{180\,a^2x^5}$$

input
```
integrate(gamma(-2,a*x)/x^4,x, algorithm="fricas")
```

output
```
1/180*((a^2*x^2 - 3*a*x + 12)*e^(-a*x) - (a^5*x^5 + 60*a^2*x^2)*gamma(-2,
a*x))/(a^2*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.94 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.48

$$\int \frac{\Gamma(-2, ax)}{x^4}\, dx = \frac{\frac{a^5\,\mathrm{Ei}\left(axe^{i\pi}\right)}{360} + \frac{a^4e^{-ax}}{360x} - \frac{a^3e^{-ax}}{360x^2} + \frac{a^2\,\mathrm{Ei}\left(axe^{i\pi}\right)}{6x^3} - \frac{i\pi a^2}{6x^3} + \frac{a^2e^{-ax}}{180x^3} + \frac{3ae^{-ax}}{20x^4} - \frac{e^{-ax}}{10x^5}}{a^2}$$

input
```
integrate(uppergamma(-2,a*x)/x**4,x)
```

output
```
(a**5*Ei(a*x*exp_polar(I*pi))/360 + a**4*exp(-a*x)/(360*x) - a**3*exp(-a*x
)/(360*x**2) + a**2*Ei(a*x*exp_polar(I*pi))/(6*x**3) - I*pi*a**2/(6*x**3)
+ a**2*exp(-a*x)/(180*x**3) + 3*a*exp(-a*x)/(20*x**4) - exp(-a*x)/(10*x**5
))/a**2
```

Maxima [F]

$$\int \frac{\Gamma(-2, ax)}{x^4} \, dx = \int \frac{\Gamma(-2, ax)}{x^4} \, dx$$

input
```
integrate(gamma(-2,a*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(gamma(-2, a*x)/x^4, x)
```

Giac [F]

$$\int \frac{\Gamma(-2, ax)}{x^4} \, dx = \int \frac{\Gamma(-2, ax)}{x^4} \, dx$$

input
```
integrate(gamma(-2,a*x)/x^4,x, algorithm="giac")
```

output
```
integrate(gamma(-2, a*x)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\Gamma(-2, ax)}{x^4} \, dx = -\frac{\text{expint}(3, a\,x) - \text{expint}(6, a\,x)}{3\,a^2\,x^5}$$

input
```
int(expint(3, a*x)/(a^2*x^6),x)
```

output
```
-(expint(3, a*x) - expint(6, a*x))/(3*a^2*x^5)
```

Reduce [F]

$$\int \frac{\Gamma(-2, ax)}{x^4} \, dx = \frac{\int \frac{ei(3,ax)}{x^6} dx}{a^2}$$

input
```
int(1/a^2/x^6*Ei(3,a*x),x)
```

output
```
int(ei(3,a*x)/x**6,x)/a**2
```

3.51 $\int x^{100}\Gamma(-3,ax)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int x^{100}\Gamma(-3,ax)\,dx = \frac{1}{101}x^{101}\Gamma(-3,ax) - \frac{\Gamma(98,ax)}{101a^{101}}$$

output
```
1/101*x^98/a^3*Ei(4,a*x)-1/101*GAMMA(98,a*x)/a^101
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^{100}\Gamma(-3,ax)\,dx = \frac{1}{101}x^{101}\Gamma(-3,ax) - \frac{\Gamma(98,ax)}{101a^{101}}$$

input
```
Integrate[x^100*Gamma[-3, a*x],x]
```

output
```
(x^101*Gamma[-3, a*x])/101 - Gamma[98, a*x]/(101*a^101)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{100}\Gamma(-3,ax)\,dx$$

$$\Big\downarrow \text{ 7116}$$

$$\frac{1}{101}x^{101}\Gamma(-3,ax)-\frac{\Gamma(98,ax)}{101a^{101}}$$

input
```
Int[x^100*Gamma[-3, a*x],x]
```

output
```
(x^101*Gamma[-3, a*x])/101 - Gamma[98, a*x]/(101*a^101)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. 2(24) = 48.

Time = 0.07 (sec) , antiderivative size = 866, normalized size of antiderivative = 34.64

Expression too large to display

input
```
int(x^97/a^3*Ei(4,x*a),x)
```

output

```
1/a^101*(1/606*(Psi(101)+gamma-11/6-Psi(102)+ln(x)+ln(a))*x^101*a^101+1117
/367236*x^101*a^101+961927596824821198533284259495636987123438139191729761
581044773193337456124818754988058791755890726512612841896796781676470678 32
3200000000000000000000/101-1/247248*(-408*a^100*x^100+408*a^99*x^99-816*
a^98*x^98+2448*a^97*x^97+237456*a^96*x^96+22795776*a^95*x^95+2165598720*a^
94*x^94+203566279680*a^93*x^93+18931664010240*a^92*x^92+1741713088942080*a
^91*x^91+158495891093729280*a^90*x^90+14264630198435635200*a^89*x^89+12695
5208766077153 2800*a^88*x^88+1117205837141478948864 00*a^87*x^87+97196907831
30866855116800*a^86*x^86+835893407349254549540044800*a^85*x^85+71050939624
686636710903808000*a^84*x^84+5968278928473677483715919872000*a^83*x^83+495
367151063315231148421349376000*a^82*x^82+406201063871918489541705506488320
00*a^81*x^81+3290228617362539765287814602555392000*a^80*x^80+2632182893890
03181223025168204431360000*a^79*x^79+207942448617312513166189882881500 7744
0000*a^78*x^78+16219510992150376026962810864757060403 20000*a^77*x^77+12489
02346395578954076136436586293651046400 00*a^76*x^76+949165783260640005097 86
369180558317479526400 00*a^75*x^75+711874337445480003823397768854187381096 4
48000000*a^74*x^74+5267870097096552028293143489520986620113715 2000000*a^73
*x^73+384554517088048298065399474735032023268301209600 0000*a^72*x^72+27687
925230339477460708762180922305675317687091200 0000*a^71*x^71+19658426913541
0289971032211484548370294755578347520 00000*a^70*x^70+137608988394787202...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(21) = 42$.

Time = 0.10 (sec) , antiderivative size = 798, normalized size of antiderivative = 31.92

$$\int x^{100}\Gamma(-3,ax)\,dx = \text{Too large to display}$$

input

```
integrate(x^100*gamma(-3,a*x),x, algorithm="fricas")
```

output
```
1/101*(a^101*x^101*gamma(-3, a*x) - (a^97*x^97 + 97*a^96*x^96 + 9312*a^95*
x^95 + 884640*a^94*x^94 + 83156160*a^93*x^93 + 7733522880*a^92*x^92 + 7114
84104960*a^91*x^91 + 647450535551360*a^90*x^90 + 5827054819622400*a^89*x^89
 + 518607878946393600*a^88*x^88 + 45637493347282636800*a^87*x^87 + 3970461
921213589401600*a^86*x^86 + 341459725224368688537600*a^85*x^85 + 290240766
4407133852569600*a^84*x^84 + 243802243810199243615846400*a^83*x^83 + 202
35586236246537220115251200*a^82*x^82 + 16593180713722160520494505984000*a
^81*x^81 + 1344047637811495002160054984704000*a^80*x^80 + 1075238110249196
0017280439877632000*a^79*x^79 + 84943810709686484136515475033292800*a^7
8*x^78 + 66256172353555457626482070525968384000*a^77*x^77 + 5101725271223
77023723911943049956556800*a^76*x^76 + 3877311206130065380301730767179669
831680000*a^75*x^75 + 29079834045975490352262980753847523737600000*a^74*
x^74 + 2151907719402186286067460575784716756582400000*a^73*x^73 + 1570892
63516359598882924622032284323230515200000*a^72*x^72 + 113104269731778911
1957057278632447127259709440000000*a^71*x^71 + 80304031509563026948951066782
903746035439370240000000*a^70*x^70 + 562128220566941188642657467480326222248
0755916800000000*a^69*x^69 + 38786847219118942016343365256142509335117215825
92000000000*a^68*x^68 + 2637505610900088057111348837417690634787970676162560
0000000*a^67*x^67 + 17671287593030589982646037210698527253079403530289152
0000000*a^66*x^66 + 11663049811400189388546384559061027987032406329990 8...
```

Sympy [F(-1)]

Timed out.

$$\int x^{100}\Gamma(-3, ax)\,dx = \text{Timed out}$$

input
```
integrate(x**100*uppergamma(-3,a*x),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{100}\Gamma(-3,ax)\,dx = \int x^{100}\Gamma(-3,ax)\,dx$$

input
```
integrate(x^100*gamma(-3,a*x),x, algorithm="maxima")
```

output
```
integrate(x^100*gamma(-3, a*x), x)
```

Giac [F]

$$\int x^{100}\Gamma(-3,ax)\,dx = \int x^{100}\Gamma(-3,ax)\,dx$$

input
```
integrate(x^100*gamma(-3,a*x),x, algorithm="giac")
```

output
```
integrate(x^100*gamma(-3, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 805, normalized size of antiderivative = 32.20

$$\int x^{100}\Gamma(-3,ax)\,dx = \text{Too large to display}$$

input
```
int((x^97*expint(4, a*x))/a^3,x)
```

output
```
(x^98*(expint(4, a*x) - exp(-a*x)*(1/(a*x) + 97/(a^2*x^2) + 9312/(a^3*x^3)
 + 884640/(a^4*x^4) + 83156160/(a^5*x^5) + 7733522880/(a^6*x^6) + 71148410
4960/(a^7*x^7) + 64745053551360/(a^8*x^8) + 5827054819622400/(a^9*x^9) + 5
18607878946393600/(a^10*x^10) + 45637493347282636800/(a^11*x^11) + 3970461
921213589401600/(a^12*x^12) + 341459725224368688537600/(a^13*x^13) + 29024
076644071338525696000/(a^14*x^14) + 2438022438101992436158464000/(a^15*x^1
5) + 202355862362465372201152512000/(a^16*x^16) + 165931807137221605204945
05984000/(a^17*x^17) + 1344047637811495002160054984704000/(a^18*x^18) + 10
7523811024919600172804398776320000/(a^19*x^19) + 849438107096864841365154 7
503329280000/(a^20*x^20) + 66256172353555457626482070525968384000 0/(a^21*x
^21) + 510172527122377023723911943049956556800000/(a^22*x^22) + 38773112061
300653803017307671796698316800000/(a^23*x^23) + 290798340459754903522629807
538475237376000000/(a^24*x^24) + 2151907719402186286067460575784716756582 4
000000/(a^25*x^25) + 15708926351635959888292462203228432323051520000 00/(a^
26*x^26) + 113104269731778911195705727863244712725970944000000/(a^27*x^27)
 + 803040315095630269489510667829037460354393702400000 0/(a^28*x^28) + 5621
282205669411886426574674803262222480755916800000 00/(a^29*x^29) + 387868472
19111894201634336525614250933511721582592000000 0/(a^30*x^30) + 263750561090
0088057111348837417690634787970676162560000000/(a^31*x^31) + 1767128759303
0589982646037210698527253079403530289152000000 0/(a^32*x^32) + 116630498...
```

Reduce [F]

$$\int x^{100}\Gamma(-3,ax)\,dx = \frac{\int ei(4,ax)\,x^{97}dx}{a^3}$$

input
```
int(x^97/a^3*Ei(4,a*x),x)
```

output
```
int(ei(4,a*x)*x**97,x)/a**3
```

3.52 $\int x^3\Gamma(-3, ax)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 26

$$\int x^3\Gamma(-3,ax)\,dx = -\frac{e^{-ax}}{4a^4} + \frac{1}{4}x^4\Gamma(-3,ax)$$

output
```
-1/4/a^4/exp(a*x)+1/4*x/a^3*Ei(4,a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^3\Gamma(-3,ax)\,dx = -\frac{e^{-ax}}{4a^4} + \frac{1}{4}x^4\Gamma(-3,ax)$$

input
```
Integrate[x^3*Gamma[-3, a*x],x]
```

output
```
-1/4*1/(a^4*E^(a*x)) + (x^4*Gamma[-3, a*x])/4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\Gamma(-3,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{4}x^4\Gamma(-3,ax)-\frac{e^{-ax}}{4a^4}$$

input
```
Int[x^3*Gamma[-3, a*x],x]
```

output
```
-1/4*1/(a^4*E^(a*x)) + (x^4*Gamma[-3, a*x])/4
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(23) = 46.

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.23

method	result	size
meijerg	$\frac{\frac{\left(\gamma-\frac{25}{12}+\ln(x)+\ln(a)\right)x^4a^4}{24}+\frac{25a^4x^4}{288}+\frac{1}{4}-\frac{\left(-5x^3a^3+5a^2x^2-10xa+30\right){\rm e}^{-xa}}{120}+\frac{x^4a^4\left(-\gamma-\ln(xa)-\text{expIntegral}_1(xa)\right)}{24}}{a^4}$	84

input
```
int(1/a^3*Ei(4,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/24*(gamma-25/12+ln(x)+ln(a))*x^4*a^4+25/288*a^4*x^4+1/4-1/120*(-5
*a^3*x^3+5*a^2*x^2-10*a*x+30)*exp(-x*a)+1/24*x^4*a^4*(-gamma-ln(x*a)-Ei(1,
x*a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int x^3\Gamma(-3,ax)\,dx = \frac{a^4x^4\Gamma(-3,ax) - e^{(-ax)}}{4\,a^4}$$

input
```
integrate(x^3*gamma(-3,a*x),x, algorithm="fricas")
```

output
```
1/4*(a^4*x^4*gamma(-3, a*x) - e^(-a*x))/a^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.96 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.92

$$\int x^3\Gamma(-3,ax)\,dx = \frac{\frac{a^3x^4\operatorname{Ei}\left(axe^{i\pi}\right)}{24} - \frac{i\pi a^3x^4}{24} + \frac{a^2x^3e^{-ax}}{24} - \frac{ax^2e^{-ax}}{24} + \frac{xe^{-ax}}{12} - \frac{e^{-ax}}{4a}}{a^3}$$

input
```
integrate(x**3*uppergamma(-3,a*x),x)
```

output
```
(a**3*x**4*Ei(a*x*exp_polar(I*pi))/24 - I*pi*a**3*x**4/24 + a**2*x**3*exp(
-a*x)/24 - a*x**2*exp(-a*x)/24 + x*exp(-a*x)/12 - exp(-a*x)/(4*a))/a**3
```

Maxima [F]

$$\int x^3\Gamma(-3,ax)\,dx = \int x^3\Gamma(-3,ax)\,dx$$

input
```
integrate(x^3*gamma(-3,a*x),x, algorithm="maxima")
```

output
```
integrate(x^3*gamma(-3, a*x), x)
```

Giac [F]

$$\int x^3\Gamma(-3,ax)\,dx = \int x^3\Gamma(-3,ax)\,dx$$

input
```
integrate(x^3*gamma(-3,a*x),x, algorithm="giac")
```

output
```
integrate(x^3*gamma(-3, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^3\Gamma(-3,ax)\,dx = \frac{x\,\mathrm{expint}(4,a\,x)}{4\,a^3} - \frac{\mathrm{e}^{-a\,x}}{4\,a^4}$$

input
```
int(expint(4, a*x)/a^3,x)
```

output
```
(x*expint(4, a*x))/(4*a^3) - exp(-a*x)/(4*a^4)
```

Reduce [F]

$$\int x^3\Gamma(-3,ax)\,dx = \frac{\int ei(4,ax)\,dx}{a^3}$$

input
```
int(1/a^3*Ei(4,a*x),x)
```

output
```
int(ei(4,a*x),x)/a**3
```

3.53 $\int x^2\Gamma(-3,ax)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int x^2\Gamma(-3,ax)\,dx=\frac{1}{3}x^3\Gamma(-3,ax)-\frac{\Gamma(0,ax)}{3a^3}$$

output
```
1/3/a^3*Ei(4,a*x)-1/3*Ei(1,a*x)/a^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^2\Gamma(-3,ax)\,dx=\frac{1}{3}x^3\Gamma(-3,ax)-\frac{\Gamma(0,ax)}{3a^3}$$

input
```
Integrate[x^2*Gamma[-3, a*x],x]
```

output
```
(x^3*Gamma[-3, a*x])/3 - Gamma[0, a*x]/(3*a^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\Gamma(-3,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{3}x^3\Gamma(-3,ax)-\frac{\Gamma(0,ax)}{3a^3}$$

input
```
Int[x^2*Gamma[-3, a*x],x]
```

output
```
(x^3*Gamma[-3, a*x])/3 - Gamma[0, a*x]/(3*a^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(21) = 42.

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.60

method	result	siz
meijerg	$\frac{\frac{\gamma}{3}+\frac{\ln(x)}{3}+\frac{\ln(a)}{3}+\frac{\left(-\frac{13}{6}+\gamma+\ln(x)+\ln(a)\right)x^3a^3}{18}+\frac{13x^3a^3}{108}+\frac{\left(16a^2x^2-16xa+32\right)e^{-xa}}{288}+\frac{\left(16x^3a^3+96\right)\left(-\gamma-\ln(xa)-\text{expIntegral}_1(xa)\right)}{288}}{a^3}$	90

input
```
int(1/x/a^3*Ei(4,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/3*gamma+1/3*ln(x)+1/3*ln(a)+1/18*(-13/6+gamma+ln(x)+ln(a))*x^3*a^
3+13/108*x^3*a^3+1/288*(16*a^2*x^2-16*a*x+32)*exp(-x*a)+1/288*(16*a^3*x^3+
96)*(-gamma-ln(x*a)-Ei(1,x*a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(21) = 42$.

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int x^2\Gamma(-3,ax)\,dx = -\frac{(a^2x^2-ax+2)e^{(-ax)}-(a^6x^6+6\,a^3x^3)\Gamma(-3,ax)}{3\,a^6x^3}$$

input
```
integrate(x^2*gamma(-3,a*x),x, algorithm="fricas")
```

output
```
-1/3*((a^2*x^2 - a*x + 2)*e^(-a*x) - (a^6*x^6 + 6*a^3*x^3)*gamma(-3, a*x))
/(a^6*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.74 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.04

$$\int x^2\Gamma(-3,ax)\,dx = \frac{\frac{a^3x^3\,\mathrm{Ei}\left(axe^{i\pi}\right)}{18}-\frac{i\pi a^3x^3}{18}+\frac{a^2x^2e^{-ax}}{18}-\frac{axe^{-ax}}{18}+\frac{\mathrm{Ei}\left(axe^{i\pi}\right)}{3}+\frac{e^{-ax}}{9}}{a^3}$$

input
```
integrate(x**2*uppergamma(-3,a*x),x)
```

output
```
(a**3*x**3*Ei(a*x*exp_polar(I*pi))/18 - I*pi*a**3*x**3/18 + a**2*x**2*exp(
-a*x)/18 - a*x*exp(-a*x)/18 + Ei(a*x*exp_polar(I*pi))/3 + exp(-a*x)/9)/a**
3
```

Maxima [F]

$$\int x^2\Gamma(-3,ax)\,dx=\int x^2\Gamma(-3,ax)\;dx$$

input
```
integrate(x^2*gamma(-3,a*x),x, algorithm="maxima")
```

output
```
integrate(x^2*gamma(-3, a*x), x)
```

Giac [F]

$$\int x^2\Gamma(-3,ax)\,dx=\int x^2\Gamma(-3,ax)\;dx$$

input
```
integrate(x^2*gamma(-3,a*x),x, algorithm="giac")
```

output
```
integrate(x^2*gamma(-3, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int x^2\Gamma(-3,ax)\,dx=-\frac{\mathrm{expint}(a\,x)-\mathrm{expint}(4,a\,x)}{3\,a^3}$$

input
```
int(expint(4, a*x)/(a^3*x),x)
```

output
```
-(expint(a*x) - expint(4, a*x))/(3*a^3)
```

Reduce [F]

$$\int x^2\Gamma(-3,ax)\,dx = \frac{\int \frac{ei(4,ax)}{x}dx}{a^3}$$

input
```
int(1/x/a^3*Ei(4,a*x),x)
```

output
```
int(ei(4,a*x)/x,x)/a**3
```

3.54 $\int x\Gamma(-3, ax)\, dx$

Optimal result

Integrand size = 7, antiderivative size = 25

$$\int x\Gamma(-3, ax)\, dx = \frac{1}{2}x^2\Gamma(-3, ax) - \frac{\Gamma(-1, ax)}{2a^2}$$

output
```
1/2/x/a^3*Ei(4,a*x)-1/2/a^3/x*Ei(2,a*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\Gamma(-3, ax)\, dx = \frac{1}{2}x^2\Gamma(-3, ax) - \frac{\Gamma(-1, ax)}{2a^2}$$

input
```
Integrate[x*Gamma[-3, a*x],x]
```

output
```
(x^2*Gamma[-3, a*x])/2 - Gamma[-1, a*x]/(2*a^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\Gamma(-3,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{2}x^2\Gamma(-3,ax)-\frac{\Gamma(-1,ax)}{2a^2}$$

input
```
Int[x*Gamma[-3, a*x],x]
```

output
```
(x^2*Gamma[-3, a*x])/2 - Gamma[-1, a*x]/(2*a^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(27) = 54.

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 5.24

method	result
meijerg	$\frac{-\frac{1}{3ax}+\frac{1}{4}-\frac{\gamma}{2}-\frac{\ln(x)}{2}-\frac{\ln(a)}{2}+\frac{xa}{2}+\frac{\left(-\frac{7}{3}+\gamma+\ln(x)+\ln(a)\right)x^2a^2}{12}+\frac{14x^3a^3-36a^2x^2-18xa+24}{72xa}-\frac{\left(-6a^2x^2+6xa+24\right)\mathrm{e}^{-xa}}{72xa}-\frac{\left(-6a^2x^2+36\right)(-\gamma}{}}{a^2}$

input
```
int(1/x^2/a^3*Ei(4,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/3/a/x+1/4-1/2*gamma-1/2*ln(x)-1/2*ln(a)+1/2*x*a+1/12*(-7/3+gamma
+ln(x)+ln(a))*x^2*a^2+1/72/x/a*(14*a^3*x^3-36*a^2*x^2-18*a*x+24)-1/72/x/a*
(-6*a^2*x^2+6*a*x+24)*exp(-x*a)-1/72*(-6*a^2*x^2+36)*(-gamma-ln(x*a)-Ei(1,
x*a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int x\Gamma(-3,ax)\,dx = -\frac{(ax-2)e^{(-ax)} - (a^5x^5 - 6\,a^3x^3)\Gamma(-3,ax)}{2\,a^5x^3}$$

input
```
integrate(x*gamma(-3,a*x),x, algorithm="fricas")
```

output
```
-1/2*((a*x - 2)*e^(-a*x) - (a^5*x^5 - 6*a^3*x^3)*gamma(-3, a*x))/(a^5*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.04

$$\int x\Gamma(-3,ax)\,dx = \frac{\frac{a^3x^2\operatorname{Ei}\left(axe^{i\pi}\right)}{12} - \frac{i\pi a^3x^2}{12} + \frac{a^2xe^{-ax}}{12} - \frac{a\operatorname{Ei}\left(axe^{i\pi}\right)}{2} - \frac{ae^{-ax}}{12} - \frac{e^{-ax}}{3x}}{a^3}$$

input
```
integrate(x*uppergamma(-3,a*x),x)
```

output
```
(a**3*x**2*Ei(a*x*exp_polar(I*pi))/12 - I*pi*a**3*x**2/12 + a**2*x*exp(-a*
x)/12 - a*Ei(a*x*exp_polar(I*pi))/2 - a*exp(-a*x)/12 - exp(-a*x)/(3*x))/a*
*3
```

Maxima [F]

$$\int x\Gamma(-3,ax)\,dx=\int x\Gamma(-3,ax)\;dx$$

input
```
integrate(x*gamma(-3,a*x),x, algorithm="maxima")
```

output
```
integrate(x*gamma(-3, a*x), x)
```

Giac [F]

$$\int x\Gamma(-3,ax)\,dx=\int x\Gamma(-3,ax)\;dx$$

input
```
integrate(x*gamma(-3,a*x),x, algorithm="giac")
```

output
```
integrate(x*gamma(-3, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int x\Gamma(-3,ax)\,dx=-\frac{\mathrm{expint}(2,a\,x)-\mathrm{expint}(4,a\,x)}{2\,a^3\,x}$$

input
```
int(expint(4, a*x)/(a^3*x^2),x)
```

output
```
-(expint(2, a*x) - expint(4, a*x))/(2*a^3*x)
```

Reduce [F]

$$\int x\Gamma(-3, ax)\, dx = \frac{\int \frac{ei(4,ax)}{x^2} dx}{a^3}$$

input
```
int(1/x^2/a^3*Ei(4,a*x),x)
```

output
```
int(ei(4,a*x)/x**2,x)/a**3
```

3.55 $\int \Gamma(-3, ax)\, dx$

Optimal result

Integrand size = 5, antiderivative size = 18

$$\int \Gamma(-3, ax)\, dx = x\Gamma(-3, ax) - \frac{\Gamma(-2, ax)}{a}$$

output
```
1/x^2/a^3*Ei(4,a*x)-1/a^3/x^2*Ei(3,a*x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. $2(18) = 36$.

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.22

$$\int \Gamma(-3, ax)\, dx = \frac{1}{2}\left(\frac{e^{-ax}(-1 + ax)}{a^3x^2} + \frac{\text{ExpIntegralEi}(-ax)}{a} + 2x\Gamma(-3, ax)\right)$$

input
```
Integrate[Gamma[-3, a*x],x]
```

output
```
((-1 + a*x)/(a^3*E^(a*x)*x^2) + ExpIntegralEi[-(a*x)]/a + 2*x*Gamma[-3, a*
x])/2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(-3,ax)\,dx$$

$$\Big\downarrow \text{7111}$$

$$x\Gamma(-3,ax)-\frac{\Gamma(-2,ax)}{a}$$

input
```
Int[Gamma[-3, a*x],x]
```

output
```
x*Gamma[-3, a*x] - Gamma[-2, a*x]/a
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 7.06

method	result
meijerg	$\frac{-\frac{1}{6x^2a^2}+\frac{1}{2ax}-\frac{1}{4}+\frac{\gamma}{2}+\frac{\ln(x)}{2}+\frac{\ln(a)}{2}+\frac{\left(\gamma-\frac{17}{6}+\ln(x)+\ln(a)\right)xa}{6}+\frac{68x^3a^3+36a^2x^2-72xa+24}{144a^2x^2}-\frac{\left(-8a^2x^2-16xa+8\right)e^{-xa}}{48a^2x^2}+\frac{(8xa+24)(-\gamma-\ln(}{}}{a}$

input
```
int(1/x^3/a^3*Ei(4,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/6/x^2/a^2+1/2/a/x-1/4+1/2*gamma+1/2*ln(x)+1/2*ln(a)+1/6*(gamma-17/
6+ln(x)+ln(a))*x*a+1/144/a^2/x^2*(68*a^3*x^3+36*a^2*x^2-72*a*x+24)-1/48/a^
2/x^2*(-8*a^2*x^2-16*a*x+8)*exp(-x*a)+1/48*(8*a*x+24)*(-gamma-ln(x*a)-Ei(1
,x*a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \Gamma(-3, ax)\, dx = \frac{(a^4x^4 + 3\, a^3x^3)\Gamma(-3, ax) - e^{(-ax)}}{a^4x^3}$$

input
```
integrate(gamma(-3,a*x),x, algorithm="fricas")
```

output
```
((a^4*x^4 + 3*a^3*x^3)*gamma(-3, a*x) - e^(-a*x))/(a^4*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 76, normalized size of antiderivative = 4.22

$$\int \Gamma(-3, ax)\, dx = \frac{\frac{a^3x \operatorname{Ei}\left(axe^{i\pi}\right)}{6} - \frac{i\pi a^3x}{6} + \frac{a^2 \operatorname{Ei}\left(axe^{i\pi}\right)}{2} + \frac{a^2e^{-ax}}{6} + \frac{ae^{-ax}}{3x} - \frac{e^{-ax}}{6x^2}}{a^3}$$

input
```
integrate(uppergamma(-3,a*x),x)
```

output
```
(a**3*x*Ei(a*x*exp_polar(I*pi))/6 - I*pi*a**3*x/6 + a**2*Ei(a*x*exp_polar(
I*pi))/2 + a**2*exp(-a*x)/6 + a*exp(-a*x)/(3*x) - exp(-a*x)/(6*x**2))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \Gamma(-3, ax)\,dx = \frac{ax\Gamma(-3, ax) - \Gamma(-2, ax)}{a}$$

input
```
integrate(gamma(-3,a*x),x, algorithm="maxima")
```

output
```
(a*x*gamma(-3, a*x) - gamma(-2, a*x))/a
```

Giac [F]

$$\int \Gamma(-3, ax)\,dx = \int \Gamma(-3, ax)\,dx$$

input
```
integrate(gamma(-3,a*x),x, algorithm="giac")
```

output
```
integrate(gamma(-3, a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \Gamma(-3, ax)\,dx = -\frac{\mathrm{expint}(3, a\,x) - \mathrm{expint}(4, a\,x)}{a^3\,x^2}$$

input
```
int(expint(4, a*x)/(a^3*x^3),x)
```

output
```
-(expint(3, a*x) - expint(4, a*x))/(a^3*x^2)
```

Reduce [F]

$$\int \Gamma(-3, ax)\, dx = \frac{\int \frac{ei(4,ax)}{x^3}dx}{a^3}$$

input
```
int(1/a^3/x^3*Ei(4,a*x),x)
```

output
```
int(ei(4,a*x)/x**3,x)/a**3
```

3.56 $\int \frac{\Gamma(-3,ax)}{x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 64

$$\int \frac{\Gamma(-3,ax)}{x} \, dx = -\frac{1}{3}\Gamma(-3,ax) + \frac{\Gamma(-2,ax)}{6} - \frac{\Gamma(-1,ax)}{6} - \frac{1}{6}ax \, {}_3F_3(1,1,1;2,2,2;-ax) + \frac{1}{6}\gamma \log(x) + \frac{1}{12}\log^2(ax)$$

output
```
-1/3/a^3/x^3*Ei(4,a*x)+1/6/a^2/x^2*Ei(3,a*x)-1/6/a/x*Ei(2,a*x)-1/6*a*x*hyp
ergeom([1, 1, 1],[2, 2, 2],-a*x)+1/6*gamma*ln(x)+1/12*ln(a*x)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 145 vs. 2(64) = 128.

Time = 0.48 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.27

$$\int \frac{\Gamma(-3,ax)}{x} \, dx = \Gamma(-3,ax)\log(ax) + \frac{1}{36}\left(\frac{e^{-ax}(-4+5ax-11a^2x^2)}{a^3x^3} - 11\,\text{ExpIntegralEi}(-ax) - 6ax \, {}_3F_3(1,1,1;2,2,2;-ax) + 6\,\text{ExpIntegralEi}(-ax)\log(x) - 3\log^2(x) - \frac{6e^{-ax}(2-ax+a^2x^2+a^3e^{ax}x^3\,\text{ExpIntegralEi}(-ax))\log(ax)}{a^3x^3} + 6\log(x)(\gamma+\Gamma(0,ax)+\log(ax))\right)$$

input
```
Integrate[Gamma[-3, a*x]/x,x]
```

output
```
Gamma[-3, a*x]*Log[a*x] + ((-4 + 5*a*x - 11*a^2*x^2)/(a^3*E^(a*x)*x^3) - 1
1*ExpIntegralEi[-(a*x)] - 6*a*x*HypergeometricPFQ[{1, 1, 1}, {2, 2, 2}, -(
a*x)] + 6*ExpIntegralEi[-(a*x)]*Log[x] - 3*Log[x]^2 - (6*(2 - a*x + a^2*x^
2 + a^3*E^(a*x)*x^3*ExpIntegralEi[-(a*x)])*Log[a*x])/(a^3*E^(a*x)*x^3) + 6
*Log[x]*(EulerGamma + Gamma[0, a*x] + Log[a*x]))/36
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {7114, 7114, 7114, 7112}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-3,ax)}{x}\,dx$$

$$\Big\downarrow \text{7114}$$

$$-\frac{1}{3}\int \frac{\Gamma(-2,ax)}{x}dx-\frac{1}{3}\Gamma(-3,ax)$$

$$\Big\downarrow \text{7114}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{\Gamma(-1,ax)}{x}dx+\frac{\Gamma(-2,ax)}{2}\right)-\frac{\Gamma(-3,ax)}{3}$$

$$\Big\downarrow \text{7114}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\int \frac{\Gamma(0,ax)}{x}dx-\Gamma(-1,ax)\right)+\frac{\Gamma(-2,ax)}{2}\right)-\frac{\Gamma(-3,ax)}{3}$$

$$\Big\downarrow \text{7112}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-ax\,{}_3F_3(1,1,1;2,2,2;-ax)+\frac{1}{2}\log^2(ax)-\Gamma(-1,ax)+\gamma\log(x)\right)+\frac{\Gamma(-2,ax)}{2}\right)-\frac{\Gamma(-3,ax)}{3}$$

input
```
Int[Gamma[-3, a*x]/x,x]
```

output
```
-1/3*Gamma[-3, a*x] + (Gamma[-2, a*x]/2 + (-Gamma[-1, a*x] - a*x*Hypergeom
etricPFQ[{1, 1, 1}, {2, 2, 2}, -(a*x)] + EulerGamma*Log[x] + Log[a*x]^2/2)
/2)/3
```

Defintions of rubi rules used

rule 7112
```
Int[Gamma[0, (b_.)*(x_)]/(x_), x_Symbol] :> Simp[b*x*HypergeometricPFQ[{1,
1, 1}, {2, 2, 2}, (-b)*x], x] + (-Simp[EulerGamma*Log[x], x] - Simp[(1/2)*L
og[b*x]^2, x]) /; FreeQ[b, x]
```

rule 7114
```
Int[Gamma[n_, (b_.)*(x_)]/(x_), x_Symbol] :> Simp[Gamma[n, b*x]/n, x] + Sim
p[1/n   Int[Gamma[n + 1, b*x]/x, x], x] /; FreeQ[b, x] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

method	result
meijerg	$-\frac{1}{9x^3a^3}+\frac{1}{4x^2a^2}-\frac{1}{2ax}+\frac{\pi^2}{72}+\frac{\ln(a)^2}{12}+\frac{\ln(x)^2}{12}+\frac{49}{432}+\frac{\left(\frac{11}{6}-\gamma\right)^2}{12}+\frac{\ln(x)\ln(a)}{6}-\frac{\ln(x)\left(\frac{11}{6}-\gamma\right)}{6}-\frac{\ln(a)\left(\frac{11}{6}-\gamma\right)}{6}$

input
```
int(1/x^4/a^3*Ei(4,x*a),x,method=_RETURNVERBOSE)
```

output
```
-1/9/x^3/a^3+1/4/x^2/a^2-1/2/a/x+1/72*Pi^2+1/12*ln(a)^2+1/12*ln(x)^2+49/43
2+1/12*(11/6-gamma)^2+1/6*ln(x)*ln(a)-1/6*ln(x)*(11/6-gamma)-1/6*ln(a)*(11
/6-gamma)-1/24*x*a*hypergeom([1,1,1],[2,2,5],-x*a)
```

Fricas [F]

$$\int \frac{\Gamma(-3, ax)}{x} \, dx = \int \frac{\Gamma(-3, ax)}{x} \, dx$$

input
```
integrate(gamma(-3,a*x)/x,x, algorithm="fricas")
```

output
```
integral(gamma(-3, a*x)/x, x)
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \frac{\Gamma(-3, ax)}{x} \, dx = \frac{-\frac{a^4 x_3 F_3\left(\begin{matrix} 1, 1, 1 \\ 2, 2, 5 \end{matrix}\middle| -ax\right)}{24} + \frac{a^3 \log(ax)^2}{12} - \frac{11a^3 \log(ax)}{36} + \frac{\gamma a^3 \log(ax)}{6} - \frac{a^2}{2x} + \frac{a}{4x^2} - \frac{1}{9x^3}}{a^3}$$

input
```
integrate(uppergamma(-3,a*x)/x,x)
```

output
```
(-a**4*x*hyper((1, 1, 1), (2, 2, 5), -a*x)/24 + a**3*log(a*x)**2/12 - 11*a
**3*log(a*x)/36 + EulerGamma*a**3*log(a*x)/6 - a**2/(2*x) + a/(4*x**2) - 1
/(9*x**3))/a**3
```

Maxima [F]

$$\int \frac{\Gamma(-3, ax)}{x} \, dx = \int \frac{\Gamma(-3, ax)}{x} \, dx$$

input
```
integrate(gamma(-3,a*x)/x,x, algorithm="maxima")
```

output
```
integrate(gamma(-3, a*x)/x, x)
```

Giac [F]

$$\int \frac{\Gamma(-3, ax)}{x} \, dx = \int \frac{\Gamma(-3, ax)}{x} \, dx$$

input
```
integrate(gamma(-3,a*x)/x,x, algorithm="giac")
```

output
```
integrate(gamma(-3, a*x)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.22

$$\int \frac{\Gamma(-3, ax)}{x} \, dx = \int \frac{\text{expint}(4, a\,x)}{a^3\,x^4} \, dx$$

input
```
int(expint(4, a*x)/(a^3*x^4),x)
```

output
```
int(expint(4, a*x)/(a^3*x^4), x)
```

Reduce [F]

$$\int \frac{\Gamma(-3, ax)}{x} \, dx = \frac{\int \frac{ei(4,ax)}{x^4} dx}{a^3}$$

input
```
int(1/a^3/x^4*Ei(4,a*x),x)
```

output
```
int(ei(4,a*x)/x**4,x)/a**3
```

3.57 $\int \frac{\Gamma(-3,ax)}{x^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int \frac{\Gamma(-3,ax)}{x^2}\,dx = a\Gamma(-4,ax) - \frac{\Gamma(-3,ax)}{x}$$

output
```
1/a^3/x^4*Ei(5,a*x)-1/a^3/x^4*Ei(4,a*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(-3,ax)}{x^2}\,dx = a\Gamma(-4,ax) - \frac{\Gamma(-3,ax)}{x}$$

input
```
Integrate[Gamma[-3, a*x]/x^2,x]
```

output
```
a*Gamma[-4, a*x] - Gamma[-3, a*x]/x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{7116\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-3, ax)}{x^2} \, dx$$
$$\Big\downarrow \; 7116$$
$$a\Gamma(-4, ax) - \frac{\Gamma(-3, ax)}{x}$$

input
```
Int[Gamma[-3, a*x]/x^2,x]
```

output
```
a*Gamma[-4, a*x] - Gamma[-3, a*x]/x
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(26) = 52$.

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 8.83

method	result
meijerg	$a\left(-\frac{1}{12a^4x^4}+\frac{1}{6x^3a^3}-\frac{1}{4x^2a^2}-\frac{-\frac{5}{6}+\gamma+\ln(x)+\ln(a)}{6xa}-\frac{\gamma}{24}+\frac{37}{288}-\frac{\ln(x)}{24}-\frac{\ln(a)}{24}+\frac{-185a^4x^4-200x^3a^3+360a^2x^2}{1440x^4a^4}\right.$

input
```
int(1/x^5/a^3*Ei(4,x*a),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/12/a^4/x^4+1/6/x^3/a^3-1/4/x^2/a^2-1/6*(-5/6+gamma+ln(x)+ln(a))/x/a-
1/24*gamma+37/288-1/24*ln(x)-1/24*ln(a)+1/1440/x^4/a^4*(-185*a^4*x^4-200*a
^3*x^3+360*a^2*x^2-240*a*x+120)-1/240/x^4/a^4*(10*a^3*x^3+30*a^2*x^2-20*a*
x+20)*exp(-x*a)-1/240/x/a*(10*a*x+40)*(-gamma-ln(x*a)-Ei(1,x*a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(18) = 36.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{\Gamma(-3,ax)}{x^2}\,dx = -\frac{(a^4x^4+4\,a^3x^3)\Gamma(-3,ax)-e^{(-ax)}}{4\,a^3x^4}$$

input
```
integrate(gamma(-3,a*x)/x^2,x, algorithm="fricas")
```

output
```
-1/4*((a^4*x^4 + 4*a^3*x^3)*gamma(-3, a*x) - e^(-a*x))/(a^3*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 5.22

$$\int \frac{\Gamma(-3,ax)}{x^2}\,dx = \frac{-\frac{a^4\,\mathrm{Ei}\,(axe^{i\pi})}{24}-\frac{a^3\,\mathrm{Ei}\,(axe^{i\pi})}{6x}+\frac{i\pi a^3}{6x}-\frac{a^3e^{-ax}}{24x}-\frac{a^2e^{-ax}}{8x^2}+\frac{ae^{-ax}}{12x^3}-\frac{e^{-ax}}{12x^4}}{a^3}$$

input
```
integrate(uppergamma(-3,a*x)/x**2,x)
```

output
```
(-a**4*Ei(a*x*exp_polar(I*pi))/24 - a**3*Ei(a*x*exp_polar(I*pi))/(6*x) + I
*pi*a**3/(6*x) - a**3*exp(-a*x)/(24*x) - a**2*exp(-a*x)/(8*x**2) + a*exp(-
a*x)/(12*x**3) - exp(-a*x)/(12*x**4))/a**3
```

Maxima [F]

$$\int \frac{\Gamma(-3, ax)}{x^2} \, dx = \int \frac{\Gamma(-3, ax)}{x^2} \, dx$$

input
```
integrate(gamma(-3,a*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(gamma(-3, a*x)/x^2, x)
```

Giac [F]

$$\int \frac{\Gamma(-3, ax)}{x^2} \, dx = \int \frac{\Gamma(-3, ax)}{x^2} \, dx$$

input
```
integrate(gamma(-3,a*x)/x^2,x, algorithm="giac")
```

output
```
integrate(gamma(-3, a*x)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{\Gamma(-3, ax)}{x^2} \, dx = -\frac{\mathrm{expint}(4, a\,x) - \mathrm{expint}(5, a\,x)}{a^3\,x^4}$$

input
```
int(expint(4, a*x)/(a^3*x^5),x)
```

output
```
-(expint(4, a*x) - expint(5, a*x))/(a^3*x^4)
```

Reduce [F]

$$\int \frac{\Gamma(-3, ax)}{x^2} \, dx = \frac{\int \frac{ei(4,ax)}{x^5} dx}{a^3}$$

input

```
int(1/a^3/x^5*Ei(4,a*x),x)
```

output

```
int(ei(4,a*x)/x**5,x)/a**3
```

3.58 $\int \frac{\Gamma(-3,ax)}{x^3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{\Gamma(-3,ax)}{x^3}\,dx = \frac{1}{2}a^2\Gamma(-5,ax) - \frac{\Gamma(-3,ax)}{2x^2}$$

output
```
1/2/a^3/x^5*Ei(6,a*x)-1/2/a^3/x^5*Ei(4,a*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(-3,ax)}{x^3}\,dx = \frac{1}{2}a^2\Gamma(-5,ax) - \frac{\Gamma(-3,ax)}{2x^2}$$

input
```
Integrate[Gamma[-3, a*x]/x^3,x]
```

output
```
(a^2*Gamma[-5, a*x])/2 - Gamma[-3, a*x]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-3,ax)}{x^3}\,dx$$

$$\Big\downarrow \ 7116$$

$$\frac{1}{2}a^2\Gamma(-5,ax)-\frac{\Gamma(-3,ax)}{2x^2}$$

input
```
Int[Gamma[-3, a*x]/x^3,x]
```

output
```
(a^2*Gamma[-5, a*x])/2 - Gamma[-3, a*x]/(2*x^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(27) = 54.

Time = 0.32 (sec) , antiderivative size = 189, normalized size of antiderivative = 7.56

method	result
meijerg	$a^2\left(-\frac{1}{15x^5a^5}+\frac{1}{8a^4x^4}-\frac{1}{6x^3a^3}-\frac{-\frac{4}{3}+\gamma+\ln(x)+\ln(a)}{12x^2a^2}+\frac{1}{24ax}-\frac{167}{14400}+\frac{\gamma}{240}+\frac{\ln(x)}{240}+\frac{\ln(a)}{240}+\frac{501x^5a^5-1800a}{}\right.$

input
```
int(1/x^6/a^3*Ei(4,x*a),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/15/x^5/a^5+1/8/a^4/x^4-1/6/x^3/a^3-1/12*(-4/3+gamma+ln(x)+ln(a))/x
^2/a^2+1/24/a/x-167/14400+1/240*gamma+1/240*ln(x)+1/240*ln(a)+1/43200/x^5/
a^5*(501*a^5*x^5-1800*a^4*x^4-4800*a^3*x^3+7200*a^2*x^2-5400*a*x+2880)-1/2
160/x^5/a^5*(-9*a^4*x^4+9*a^3*x^3+162*a^2*x^2-126*a*x+144)*exp(-x*a)-1/216
0/x^2/a^2*(-9*a^2*x^2+180)*(-gamma-ln(x*a)-Ei(1,x*a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \frac{\Gamma(-3,ax)}{x^3}\,dx = -\frac{(ax-4)e^{(-ax)} - (a^5x^5 - 20\,a^3x^3)\Gamma(-3,ax)}{40\,a^3x^5}$$

input
```
integrate(gamma(-3,a*x)/x^3,x, algorithm="fricas")
```

output
```
-1/40*((a*x - 4)*e^(-a*x) - (a^5*x^5 - 20*a^3*x^3)*gamma(-3, a*x))/(a^3*x^
5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.51 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.56

$$\int \frac{\Gamma(-3,ax)}{x^3}\,dx = \frac{\frac{a^5\,\mathrm{Ei}\left(axe^{i\pi}\right)}{240} + \frac{a^4e^{-ax}}{240x} - \frac{a^3\,\mathrm{Ei}\left(axe^{i\pi}\right)}{12x^2} + \frac{i\pi a^3}{12x^2} - \frac{a^3e^{-ax}}{240x^2} - \frac{3a^2e^{-ax}}{40x^3} + \frac{7ae^{-ax}}{120x^4} - \frac{e^{-ax}}{15x^5}}{a^3}$$

input
```
integrate(uppergamma(-3,a*x)/x**3,x)
```

output
```
(a**5*Ei(a*x*exp_polar(I*pi))/240 + a**4*exp(-a*x)/(240*x) - a**3*Ei(a*x*e
xp_polar(I*pi))/(12*x**2) + I*pi*a**3/(12*x**2) - a**3*exp(-a*x)/(240*x**2
) - 3*a**2*exp(-a*x)/(40*x**3) + 7*a*exp(-a*x)/(120*x**4) - exp(-a*x)/(15*
x**5))/a**3
```

Maxima [F]

$$\int \frac{\Gamma(-3, ax)}{x^3} \, dx = \int \frac{\Gamma(-3, ax)}{x^3} \, dx$$

input
```
integrate(gamma(-3,a*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(gamma(-3, a*x)/x^3, x)
```

Giac [F]

$$\int \frac{\Gamma(-3, ax)}{x^3} \, dx = \int \frac{\Gamma(-3, ax)}{x^3} \, dx$$

input
```
integrate(gamma(-3,a*x)/x^3,x, algorithm="giac")
```

output
```
integrate(gamma(-3, a*x)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\Gamma(-3, ax)}{x^3} \, dx = -\frac{\mathrm{expint}(4, a\, x) - \mathrm{expint}(6, a\, x)}{2\, a^3\, x^5}$$

input
```
int(expint(4, a*x)/(a^3*x^6),x)
```

output
```
-(expint(4, a*x) - expint(6, a*x))/(2*a^3*x^5)
```

Reduce [F]

$$\int \frac{\Gamma(-3,ax)}{x^3}\,dx = \frac{\int \frac{ei(4,ax)}{x^6}dx}{a^3}$$

input
```
int(1/a^3/x^6*Ei(4,a*x),x)
```

output
```
int(ei(4,a*x)/x**6,x)/a**3
```

3.59 $\int \frac{\Gamma(-3,ax)}{x^4} dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{\Gamma(-3,ax)}{x^4} dx = \frac{1}{3}a^3\Gamma(-6,ax) - \frac{\Gamma(-3,ax)}{3x^3}$$

output
```
1/3/a^3/x^6*Ei(7,a*x)-1/3/a^3/x^6*Ei(4,a*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(-3,ax)}{x^4} dx = \frac{1}{3}a^3\Gamma(-6,ax) - \frac{\Gamma(-3,ax)}{3x^3}$$

input
```
Integrate[Gamma[-3, a*x]/x^4,x]
```

output
```
(a^3*Gamma[-6, a*x])/3 - Gamma[-3, a*x]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-3,ax)}{x^4}\,dx$$

$$\Big\downarrow \; 7116$$

$$\frac{1}{3}a^3\Gamma(-6,ax)-\frac{\Gamma(-3,ax)}{3x^3}$$

input
```
Int[Gamma[-3, a*x]/x^4,x]
```

output
```
(a^3*Gamma[-6, a*x])/3 - Gamma[-3, a*x]/(3*x^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(27) = 54.

Time = 0.34 (sec) , antiderivative size = 213, normalized size of antiderivative = 8.52

method	result
meijerg	$a^3\Big(-\frac{1}{18a^6x^6}+\frac{1}{10x^5a^5}-\frac{1}{8a^4x^4}-\frac{-\frac{3}{2}+\gamma+\ln(x)+\ln(a)}{18x^3a^3}+\frac{1}{48x^2a^2}-\frac{1}{240ax}+\frac{167}{129600}-\frac{\gamma}{2160}-\frac{\ln(x)}{2160}-\frac{\ln(a)}{2160}+$

input
```
int(1/x^7/a^3*Ei(4,x*a),x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/18/a^6/x^6+1/10/x^5/a^5-1/8/a^4/x^4-1/18*(-3/2+gamma+ln(x)+ln(a))/
x^3/a^3+1/48/x^2/a^2-1/240/a/x+167/129600-1/2160*gamma-1/2160*ln(x)-1/2160
*ln(a)+1/907200/x^6/a^6*(-1169*a^6*x^6+3780*a^5*x^5-18900*a^4*x^4-75600*a^
3*x^3+113400*a^2*x^2-90720*a*x+50400)-1/60480/x^6/a^6*(28*a^5*x^5-28*a^4*x
^4+56*a^3*x^3+3192*a^2*x^2-2688*a*x+3360)*exp(-x*a)-1/60480/x^3/a^3*(28*a^
3*x^3+3360)*(-gamma-ln(x*a)-Ei(1,x*a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(21) = 42.

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{\Gamma(-3,ax)}{x^4}\,dx = \frac{(a^2x^2-4\,ax+20)e^{(-ax)}-(a^6x^6+120\,a^3x^3)\Gamma(-3,ax)}{360\,a^3x^6}$$

input
```
integrate(gamma(-3,a*x)/x^4,x, algorithm="fricas")
```

output
```
1/360*((a^2*x^2 - 4*a*x + 20)*e^(-a*x) - (a^6*x^6 + 120*a^3*x^3)*gamma(-3,
 a*x))/(a^3*x^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 5.12

$$\int \frac{\Gamma(-3,ax)}{x^4}\,dx$$
$$= \frac{-\frac{a^6\,\mathrm{Ei}\left(axe^{i\pi}\right)}{2160}-\frac{a^5e^{-ax}}{2160x}+\frac{a^4e^{-ax}}{2160x^2}-\frac{a^3\,\mathrm{Ei}\left(axe^{i\pi}\right)}{18x^3}+\frac{i\pi a^3}{18x^3}-\frac{a^3e^{-ax}}{1080x^3}-\frac{19a^2e^{-ax}}{360x^4}+\frac{2ae^{-ax}}{45x^5}-\frac{e^{-ax}}{18x^6}}{a^3}$$

input
```
integrate(uppergamma(-3,a*x)/x**4,x)
```

output
```
(-a**6*Ei(a*x*exp_polar(I*pi))/2160 - a**5*exp(-a*x)/(2160*x) + a**4*exp(-
a*x)/(2160*x**2) - a**3*Ei(a*x*exp_polar(I*pi))/(18*x**3) + I*pi*a**3/(18*
x**3) - a**3*exp(-a*x)/(1080*x**3) - 19*a**2*exp(-a*x)/(360*x**4) + 2*a*ex
p(-a*x)/(45*x**5) - exp(-a*x)/(18*x**6))/a**3
```

Maxima [F]

$$\int \frac{\Gamma(-3, ax)}{x^4}\, dx = \int \frac{\Gamma(-3, ax)}{x^4}\, dx$$

input
```
integrate(gamma(-3,a*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(gamma(-3, a*x)/x^4, x)
```

Giac [F]

$$\int \frac{\Gamma(-3, ax)}{x^4}\, dx = \int \frac{\Gamma(-3, ax)}{x^4}\, dx$$

input
```
integrate(gamma(-3,a*x)/x^4,x, algorithm="giac")
```

output
```
integrate(gamma(-3, a*x)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\Gamma(-3, ax)}{x^4}\, dx = -\frac{\mathrm{expint}(4, a\, x) - \mathrm{expint}(7, a\, x)}{3\, a^3\, x^6}$$

input
```
int(expint(4, a*x)/(a^3*x^7),x)
```

output
```
-(expint(4, a*x) - expint(7, a*x))/(3*a^3*x^6)
```

Reduce [F]

$$\int \frac{\Gamma(-3, ax)}{x^4} \, dx = \frac{\int \frac{ei(4,ax)}{x^7} dx}{a^3}$$

input
```
int(1/a^3/x^7*Ei(4,a*x),x)
```

output
```
int(ei(4,a*x)/x**7,x)/a**3
```

3.60 $\int x^{100}\Gamma\left(\frac{1}{2},ax\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int x^{100}\Gamma\left(\frac{1}{2},ax\right)\,dx=\frac{1}{101}x^{101}\Gamma\left(\frac{1}{2},ax\right)-\frac{\Gamma\left(\frac{203}{2},ax\right)}{101a^{101}}$$

output
```
1/101*x^101*Pi^(1/2)*erfc((a*x)^(1/2))-1/101*GAMMA(203/2,a*x)/a^101
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^{100}\Gamma\left(\frac{1}{2},ax\right)\,dx=\frac{1}{101}x^{101}\Gamma\left(\frac{1}{2},ax\right)-\frac{\Gamma\left(\frac{203}{2},ax\right)}{101a^{101}}$$

input
```
Integrate[x^100*Gamma[1/2, a*x],x]
```

output
```
(x^101*Gamma[1/2, a*x])/101 - Gamma[203/2, a*x]/(101*a^101)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{100}\Gamma\left(\frac{1}{2},ax\right)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{101}x^{101}\Gamma\left(\frac{1}{2},ax\right)-\frac{\Gamma\left(\frac{203}{2},ax\right)}{101a^{101}}$$

input
```
Int[x^100*Gamma[1/2, a*x],x]
```

output
```
(x^101*Gamma[1/2, a*x])/101 - Gamma[203/2, a*x]/(101*a^101)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1345 vs. 2(25) = 50.

Time = 0.19 (sec) , antiderivative size = 1346, normalized size of antiderivative = 46.41

method	result	size
parts	Expression too large to display	1346
derivativedivides	Expression too large to display	1354
default	Expression too large to display	1354

input
```
int(x^100*Pi^(1/2)*erfc((x*a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/101*x^101*Pi^(1/2)*erfc((x*a)^(1/2))+2/101/a^101*(-48145208015596674800
7633880476899108122537146653367093423140625/536870912*(x*a)^(145/2)/exp(x*a
)+1339928042684663702626654216353741866828222367324552136009000175903930
37397395620020133980897190881330248533785934355559588023275706870249406853
90271844007692919747253644648194313049316406255/070602400912917605986812821
504*Pi^(1/2)*erf((x*a)^(1/2))-13399280426846637026266542163537418668282223
6732455213600900017590393037397395620020133980897190881330248533785934355
559588023275706870249406853902718440076929197472536446481943130493164062 5/25
353012004564588029934064107 52*(x*a)^(1/2)/exp(x*a)-21645666573352512727725
98790366307082218216489362932419713454203208066804438875765168487009365012
55208105181593184682267383344749723361758291666539176857361675405502319335
9375/15474250491067253436239052 8*(x*a)^(29/2)/exp(x*a)-448521237216300747
70648232099297351402638604113644017792168316182066770143839918326024069734
17510097227214294433593 75/36028797018963968*(x*a)^(93/2)/exp(x*a)-52807561
35183187623678204625058584456195236679697610045159230094685100008110657484
42743933970429745790785601791638234099662743402296870134357958489036560058
59375/60446290980731458735308 8*(x*a)^(45/2)/exp(x*a)-20465381695240335899
64909098246082727925626897391951221975288291426389417598349668832623339618
61830147452263038623045050485994188828211500164176919618912718866498287916
1834716796875/4951760157141521099596496896*(x*a)^(19/2)/exp(x*a)-974541985...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(25) = 50$.

Time = 0.51 (sec) , antiderivative size = 848, normalized size of antiderivative = 29.24

$$\int x^{100}\Gamma\left(\frac{1}{2},ax\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^100*pi^(1/2)*erfc((a*x)^(1/2)),x, algorithm="fricas")
```

output
```
1/2560654212461023391023340474859 52*(2535301200456458802993406410752*sqrt(
pi)*a^101*x^101 - sqrt(pi)*(2535301200456458802993406410752*a^101*x^101 -
1339928042684663702626654216353741866828222367324552136009000175903930 3739
7395620020133980897190881330248533785934355559588023275706870249406853 9027
1844007692919747253644648194313049316406 25)*erf(sqrt(a*x)) - 2*(1267650600
228229401496703205376*a^100*x^100 + 127398885322937054850418672140288*a^99
*x^99 + 12676189089632236957616657877958656*a^98*x^98 + 124860462532877534
0325240800978927616*a^97*x^97 + 121738950969555595681710978095445442560*a^
96*x^96 + 11747808768562114983285109386210485207040*a^95*x^95 + 1121915737
3976819809037279463831013372723 20*a^94*x^94 + 106021037184080947195402290 9
33203076372234240*a^93*x^93 + 99129669767115685627701142022544876408039014
40*a^92*x^92 + 916949445345820092056235563708540106774360883200*a^91*x^91
+ 839008742491425384231455540793314197698540208 12800*a^90*x^90 + 759302911
954739972729467264417949348917178888 3558400*a^89*x^89 + 679576106199492275
592873201654064667280875105078476800*a^88*x^88 + 601424853986550663899692 7
8346384723054357446799445196800*a^87*x^87 + 526246747238231830912231185530
866326725627659495145472 0000*a^86*x^86 + 4552034363610705337390799754841 99
37261766792546330083328 0000*a^85*x^85 + 389198938088715306346913379038990 4
63588106076271122212454 40000*a^84*x^84 + 3288731026849644338631418052879 46
94173194963444909826952396 80000*a^83*x^83 + 274609040741945302275723407...
```

Sympy [F(-1)]

Timed out.

$$\int x^{100}\Gamma\left(\frac{1}{2},ax\right)\,dx=\text{Timed out}$$

input
```
integrate(x**100*pi**(1/2)*erfc((a*x)**(1/2)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(25) = 50$.

Time = 0.04 (sec) , antiderivative size = 755, normalized size of antiderivative = 26.03

$$\int x^{100}\Gamma\left(\frac{1}{2}, ax\right)\, dx = \text{Too large to display}$$

input
```
integrate(x^100*pi^(1/2)*erfc((a*x)^(1/2)),x, algorithm="maxima")
```

output
```
1/25606542124610233910233404748595 2*sqrt(pi)*(2535301200456458802993406410
752*a^101*x^101*erfc(sqrt(a*x)) - (2*(1267650600228229401496703205376*(a*x
)^(201/2) + 127398885322937054850418672140288*(a*x)^(199/2) + 126761890896
32236957616657877958656*(a*x)^(197/2) + 1248604625328775340325240800978927
616*(a*x)^(195/2) + 121738950969555595681710978095445442560*(a*x)^(193/2)
+ 11747808768562114983285109386210485207040*(a*x)^(191/2) + 11219157373976
81980903727946383101337272320*(a*x)^(189/2) + 106021037184080947195402290 9
3320307637223424 0*(a*x)^(187/2) + 99129669767115685627701142022544876408 03
901440*(a*x)^(185/2) + 9169494453458200920562355637085401067743608832 00*(a
*x)^(183/2) + 839008742491425384231455540793314197698540208128 00*(a*x)^(18
1/2) + 7593029119547399727294672644179493489171788883558400*(a*x)^(179/2)
+ 679576106199492275592873201654064667280875105078476800*(a*x)^(177/2) + 6
0142485398655066389969278346384723054357446799445196800*(a*x)^(175/2) + 52
6246747238231830912231185530866326725627659495145472 0000*(a*x)^(173/2) + 4
5520343636107053373907997548419937261766792546330083328 0000*(a*x)^(171/2)
+ 3891989380887153063469133790389904635881060762711222124 5440000*(a*x)^(16
9/2) + 328873102684964433863141805287946941731949634449098269523 9680000*(a
*x)^(167/2) + 2746090407419453022757234074154356963461779447649970550525 13
280000*(a*x)^(165/2) + 226552458612104874377471811117734449485596804431122
5704183234560 0000*(a*x)^(163/2) + 1846402537688654726176395260609535763...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. $2(25) = 50$.

Time = 0.12 (sec) , antiderivative size = 1362, normalized size of antiderivative = 46.97

$$\int x^{100}\Gamma\left(\frac{1}{2},ax\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^100*pi^(1/2)*erfc((a*x)^(1/2)),x, algorithm="giac")
```

output
```
1/256065421246102339102334047485952*sqrt(pi)*(2535301200456458802993406410
752*a*x^101 - (2535301200456458802993406410752*a^101*x^101*erf(sqrt(a*x))
+ (2*(1267650600228229401496703205376*sqrt(a*x)*a^100*x^100 + 127398885322
937054850418672140288*sqrt(a*x)*a^99*x^99 + 126761890896322369576166578779
58656*sqrt(a*x)*a^98*x^98 + 1248604625328775340325240800978927616*sqrt(a*x
)*a^97*x^97 + 121738950969555595681710978095445442560*sqrt(a*x)*a^96*x^96
+ 11747808768562114983285109386210485207040*sqrt(a*x)*a^95*x^95 + 11219157
37397681980903727946383101337272320*sqrt(a*x)*a^94*x^94 + 1060210371840809
47195402290933203076372234240*sqrt(a*x)*a^93*x^93 + 9912966976711568562770
114202254487640803901440*sqrt(a*x)*a^92*x^92 + 916949445345820092056235563
708540106774360883200*sqrt(a*x)*a^91*x^91 + 839008742491425384231455540793
31419769854020812800*sqrt(a*x)*a^90*x^90 + 7593029119547399727294672644179
493489171788883558400*sqrt(a*x)*a^89*x^89 + 679576106199492275592873201654
064667280875105078476800*sqrt(a*x)*a^88*x^88 + 601424853986550663899692783
46384723054357446799445196800*sqrt(a*x)*a^87*x^87 + 5262467472382318309122
311855308663267256276594951454720000*sqrt(a*x)*a^86*x^86 + 455203436361070
533739079975484199372617667925463300833280000*sqrt(a*x)*a^85*x^85 + 389198
93808871530634691337903899046358810607627112221245440000*sqrt(a*x)*a^84*x^
84 + 3288731026849644338631418052879469417319496344490982695239680000*sqrt
(a*x)*a^83*x^83 + 27460904074194530227572340741543569634617794476499705...
```

Mupad [F(-1)]

Timed out.

$$\int x^{100}\Gamma\left(\frac{1}{2},ax\right)\,dx=\int\sqrt{\Pi}\,x^{100}\,\mathrm{erfc}\left(\sqrt{a\,x}\right)\,dx$$

input
```
int(Pi^(1/2)*x^100*erfc((a*x)^(1/2)),x)
```

output
```
int(Pi^(1/2)*x^100*erfc((a*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 1266, normalized size of antiderivative = 43.66

$$\int x^{100}\Gamma\left(\frac{1}{2},ax\right)\,dx=\text{Too large to display}$$

input
```
int(x^100*Pi^(1/2)*erfc((a*x)^(1/2)),x)
```

output

```
( - 25353012004564588029934064107520*sqrt(pi)*e**(a*x)*erf(sqrt(x)*sqrt(a))
*a**101*x**101 + 133992804268466370262665421635374186682822236732455213600
90001759039303739739562002013398089719088133024853378593435555958802327570
68702494068539027184400769291974725364464819431304931640625*sqrt(pi)*e**(a
*x)*erf(sqrt(x)*sqrt(a)) - 25353012004564588029934064107520*sqrt(x)*sqrt(a)
*a**100*x**100 - 254797770645874109700837344280576*sqrt(x)*sqrt(a)*a**99*x
**99 - 25352378179264473915233315755917312*sqrt(x)*sqrt(a)*a**98*x**98 - 2
497209250657550680650481601957855232*sqrt(x)*sqrt(a)*a**97*x**97 - 2434779
0193911191363421956190890885120*sqrt(x)*sqrt(a)*a**96*x**96 - 23495617537
124229966570218772420970414080*sqrt(x)*sqrt(a)*a**95*x**95 - 2243831474795
363961807455892766202674544640*sqrt(x)*sqrt(a)*a**94*x**94 - 2120420743681
61894390804581866406152744468480*sqrt(x)*sqrt(a)*a**93*x**93 - 19825933953
423137125540228404508975281607802880*sqrt(x)*sqrt(a)*a**92*x**92 - 1833898
890691640184112471127417080213548721766400*sqrt(x)*sqrt(a)*a**91*x**91 - 1
67801748498285076846291108158662839539708041625600*sqrt(x)*sqrt(a)*a**90*x
**90 - 15186058239094799454589345288358986978343577767116800*sqrt(x)*sqrt(
a)*a**89*x**89 - 1359152212398984551185746403308129334561750210156953600*s
qrt(x)*sqrt(a)*a**88*x**88 - 120284970797310132779938556692769446108714893
598890393600*sqrt(x)*sqrt(a)*a**87*x**87 - 1052493494476463661824462371061
7326534512553189902909440000*sqrt(x)*sqrt(a)*a**86*x**86 - 910406872722...
```

3.61 $\int x^2\Gamma\left(\frac{1}{2},ax\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int x^2\Gamma\left(\frac{1}{2},ax\right)\,dx=\frac{1}{3}x^3\Gamma\left(\frac{1}{2},ax\right)-\frac{\Gamma\left(\frac{7}{2},ax\right)}{3a^3}$$

output
```
1/3*x^3*Pi^(1/2)*erfc((a*x)^(1/2))-1/3*((a*x)^(5/2)*exp(-a*x)+5/2*(a*x)^(3
/2)*exp(-a*x)+15/4*(a*x)^(1/2)*exp(-a*x)+15/8*Pi^(1/2)*erfc((a*x)^(1/2)))/
a^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^2\Gamma\left(\frac{1}{2},ax\right)\,dx=\frac{1}{3}x^3\Gamma\left(\frac{1}{2},ax\right)-\frac{\Gamma\left(\frac{7}{2},ax\right)}{3a^3}$$

input
```
Integrate[x^2*Gamma[1/2, a*x],x]
```

output
```
(x^3*Gamma[1/2, a*x])/3 - Gamma[7/2, a*x]/(3*a^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\Gamma\left(\frac{1}{2},ax\right)\,dx$$

$$\Big\downarrow \text{ 7116}$$

$$\frac{1}{3}x^3\Gamma\left(\frac{1}{2},ax\right)-\frac{\Gamma\left(\frac{7}{2},ax\right)}{3a^3}$$

input
```
Int[x^2*Gamma[1/2, a*x],x]
```

output
```
(x^3*Gamma[1/2, a*x])/3 - Gamma[7/2, a*x]/(3*a^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.48

method	result	size
parts	$\frac{x^{3}\sqrt{\pi}\,\operatorname{erfc}\left(\sqrt{x a}\right)}{3}+\frac{-\frac{\left(x a\right)^{\frac{5}{2}} {\mathrm e}^{-x a}}{3}-\frac{5 \left(x a\right)^{\frac{3}{2}} {\mathrm e}^{-x a}}{6}-\frac{5 \sqrt{x a}\, {\mathrm e}^{-x a}}{4}+\frac{5 \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{x a}\right)}{8}}{a^{3}}$	72
derivativedivides	$\frac{2 \sqrt{\pi}\, \left(\frac{x^{3} a^{3} \operatorname{erfc}\left(\sqrt{x a}\right)}{6}+\frac{-\frac{\left(x a\right)^{\frac{5}{2}} {\mathrm e}^{-x a}}{2}-\frac{5 \left(x a\right)^{\frac{3}{2}} {\mathrm e}^{-x a}}{4}-\frac{15 \sqrt{x a}\, {\mathrm e}^{-x a}}{8}+\frac{15 \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{x a}\right)}{16}}{3 \sqrt{\pi}}\right)}{a^{3}}$	80
default	$\frac{2 \sqrt{\pi}\, \left(\frac{x^{3} a^{3} \operatorname{erfc}\left(\sqrt{x a}\right)}{6}+\frac{-\frac{\left(x a\right)^{\frac{5}{2}} {\mathrm e}^{-x a}}{2}-\frac{5 \left(x a\right)^{\frac{3}{2}} {\mathrm e}^{-x a}}{4}-\frac{15 \sqrt{x a}\, {\mathrm e}^{-x a}}{8}+\frac{15 \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{x a}\right)}{16}}{3 \sqrt{\pi}}\right)}{a^{3}}$	80

input
```
int(x^2*Pi^(1/2)*erfc((x*a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*Pi^(1/2)*erfc((x*a)^(1/2))+2/3/a^3*(-1/2*(x*a)^(5/2)/exp(x*a)-5/4*
(x*a)^(3/2)/exp(x*a)-15/8*(x*a)^(1/2)/exp(x*a)+15/16*Pi^(1/2)*erf((x*a)^(1
/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.21

$$\int x^2\Gamma\left(\frac{1}{2},ax\right)\,dx = \frac{8\,\sqrt{\pi}a^3x^3-\sqrt{\pi}(8\,a^3x^3-15)\operatorname{erf}\left(\sqrt{ax}\right)-2\,(4\,a^2x^2+10\,ax+15)\sqrt{ax}e^{(-ax)}}{24\,a^3}$$

input
```
integrate(x^2*pi^(1/2)*erfc((a*x)^(1/2)),x, algorithm="fricas")
```

output
```
1/24*(8*sqrt(pi)*a^3*x^3 - sqrt(pi)*(8*a^3*x^3 - 15)*erf(sqrt(a*x)) - 2*(4
*a^2*x^2 + 10*a*x + 15)*sqrt(a*x)*e^(-a*x))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.79

$$\int x^2\Gamma\left(\frac{1}{2},ax\right)\,dx$$
$$=\sqrt{\pi}\left(\begin{cases}\frac{x^3\operatorname{erfc}\left(\sqrt{ax}\right)}{3}-\frac{x^2\sqrt{ax}e^{-ax}}{3\sqrt{\pi}a}-\frac{5x\sqrt{ax}e^{-ax}}{6\sqrt{\pi}a^2}-\frac{5\sqrt{ax}e^{-ax}}{4\sqrt{\pi}a^3}-\frac{5\operatorname{erfc}\left(\sqrt{ax}\right)}{8a^3} & \text{for } a\neq 0\\ \frac{x^3}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**2*pi**(1/2)*erfc((a*x)**(1/2)),x)
```

output
```
sqrt(pi)*Piecewise((x**3*erfc(sqrt(a*x))/3 - x**2*sqrt(a*x)*exp(-a*x)/(3*s
qrt(pi)*a) - 5*x*sqrt(a*x)*exp(-a*x)/(6*sqrt(pi)*a**2) - 5*sqrt(a*x)*exp(-
a*x)/(4*sqrt(pi)*a**3) - 5*erfc(sqrt(a*x))/(8*a**3), Ne(a, 0)), (x**3/3, T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.38

$$\int x^2\Gamma\left(\frac{1}{2},ax\right)\,dx$$
$$=\frac{\sqrt{\pi}\left(8\,a^3x^3\operatorname{erfc}\left(\sqrt{ax}\right)-\frac{2\left(4\,(ax)^{\frac{5}{2}}+10\,(ax)^{\frac{3}{2}}+15\,\sqrt{ax}\right)e^{(-ax)}-15\,\sqrt{\pi}\operatorname{erf}\left(\sqrt{ax}\right)}{\sqrt{\pi}}\right)}{24\,a^3}$$

input
```
integrate(x^2*pi^(1/2)*erfc((a*x)^(1/2)),x, algorithm="maxima")
```

output
```
1/24*sqrt(pi)*(8*a^3*x^3*erfc(sqrt(a*x)) - (2*(4*(a*x)^(5/2) + 10*(a*x)^(3
/2) + 15*sqrt(a*x))*e^(-a*x) - 15*sqrt(pi)*erf(sqrt(a*x)))/sqrt(pi))/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.03

$$\int x^2\Gamma\left(\frac{1}{2},ax\right)\,dx = \frac{\sqrt{\pi}\left(8\,ax^3 - \frac{8\,a^3x^3\,\mathrm{erf}\left(\sqrt{ax}\right)+\frac{2\left(4\,\sqrt{ax}a^2x^2+10\,\sqrt{ax}ax+15\,\sqrt{ax}\right)e^{(-ax)}-15\,\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{ax}\right)}{\sqrt{\pi}}}{a^2}\right)}{24\,a}$$

input
```
integrate(x^2*pi^(1/2)*erfc((a*x)^(1/2)),x, algorithm="giac")
```

output
```
1/24*sqrt(pi)*(8*a*x^3 - (8*a^3*x^3*erf(sqrt(a*x)) + (2*(4*sqrt(a*x)*a^2*x
^2 + 10*sqrt(a*x)*a*x + 15*sqrt(a*x))*e^(-a*x) - 15*sqrt(pi)*erf(sqrt(a*x)
))/sqrt(pi))/a^2)/a
```

Mupad [F(-1)]

Timed out.

$$\int x^2\Gamma\left(\frac{1}{2},ax\right)\,dx = \int \sqrt{\Pi}\,x^2\,\mathrm{erfc}\left(\sqrt{a\,x}\right)\,dx$$

input
```
int(Pi^(1/2)*x^2*erfc((a*x)^(1/2)),x)
```

output
```
int(Pi^(1/2)*x^2*erfc((a*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.10

$$\int x^2\Gamma\left(\frac{1}{2},ax\right)\,dx = \frac{-8\sqrt{\pi}\,e^{ax}\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right)a^3x^3+15\sqrt{\pi}\,e^{ax}\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right)-8\sqrt{x}\,\sqrt{a}\,a^2x^2-20\sqrt{x}\,\sqrt{a}\,ax-30\sqrt{x}\,\sqrt{a}+8\sqrt{\pi}\,e}{24e^{ax}a^3}$$

input
```
int(x^2*Pi^(1/2)*erfc((a*x)^(1/2)),x)
```

output
```
( - 8*sqrt(pi)*e**(a*x)*erf(sqrt(x)*sqrt(a))*a**3*x**3 + 15*sqrt(pi)*e**(a
*x)*erf(sqrt(x)*sqrt(a)) - 8*sqrt(x)*sqrt(a)*a**2*x**2 - 20*sqrt(x)*sqrt(a
)*a*x - 30*sqrt(x)*sqrt(a) + 8*sqrt(pi)*e**(a*x)*a**3*x**3)/(24*e**(a*x)*a
**3)
```

3.62 $\int x\Gamma\left(\frac{1}{2},ax\right)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 29

$$\int x\Gamma\left(\frac{1}{2},ax\right)\,dx=\frac{1}{2}x^2\Gamma\left(\frac{1}{2},ax\right)-\frac{\Gamma\left(\frac{5}{2},ax\right)}{2a^2}$$

output
```
1/2*x^2*Pi^(1/2)*erfc((a*x)^(1/2))-1/2*((a*x)^(3/2)*exp(-a*x)+3/2*(a*x)^(1
/2)*exp(-a*x)+3/4*Pi^(1/2)*erfc((a*x)^(1/2)))/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x\Gamma\left(\frac{1}{2},ax\right)\,dx=\frac{1}{2}x^2\Gamma\left(\frac{1}{2},ax\right)-\frac{\Gamma\left(\frac{5}{2},ax\right)}{2a^2}$$

input
```
Integrate[x*Gamma[1/2, a*x],x]
```

output
```
(x^2*Gamma[1/2, a*x])/2 - Gamma[5/2, a*x]/(2*a^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\Gamma\left(\frac{1}{2},ax\right)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{2}x^2\Gamma\left(\frac{1}{2},ax\right)-\frac{\Gamma\left(\frac{5}{2},ax\right)}{2a^2}$$

input
```
Int[x*Gamma[1/2, a*x],x]
```

output
```
(x^2*Gamma[1/2, a*x])/2 - Gamma[5/2, a*x]/(2*a^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.00

method	result	size
parts	$\frac{x^{2} \sqrt{\pi}\, \operatorname{erfc}\left(\sqrt{x a}\right)}{2}+\frac{-\frac{\left(x a\right)^{\frac{3}{2}} {\mathrm e}^{-x a}}{2}-\frac{3 \sqrt{x a}\, {\mathrm e}^{-x a}}{4}+\frac{3 \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{x a}\right)}{8}}{a^{2}}$	58
derivativedivides	$\frac{2 \sqrt{\pi}\, \left(\frac{x^{2} a^{2} \operatorname{erfc}\left(\sqrt{x a}\right)}{4}+\frac{-\frac{\left(x a\right)^{\frac{3}{2}} {\mathrm e}^{-x a}}{2}-\frac{3 \sqrt{x a}\, {\mathrm e}^{-x a}}{4}+\frac{3 \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{x a}\right)}{8}}{2 \sqrt{\pi}}\right)}{a^{2}}$	67
default	$\frac{2 \sqrt{\pi}\, \left(\frac{x^{2} a^{2} \operatorname{erfc}\left(\sqrt{x a}\right)}{4}+\frac{-\frac{\left(x a\right)^{\frac{3}{2}} {\mathrm e}^{-x a}}{2}-\frac{3 \sqrt{x a}\, {\mathrm e}^{-x a}}{4}+\frac{3 \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{x a}\right)}{8}}{2 \sqrt{\pi}}\right)}{a^{2}}$	67

input
```
int(x*Pi^(1/2)*erfc((x*a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*Pi^(1/2)*erfc((x*a)^(1/2))+1/a^2*(-1/2*(x*a)^(3/2)/exp(x*a)-3/4*(x
*a)^(1/2)/exp(x*a)+3/8*Pi^(1/2)*erf((x*a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.93

$$\int x\Gamma\left(\frac{1}{2},ax\right)\,dx=\frac{4\,\sqrt{\pi}a^{2}x^{2}-\sqrt{\pi}(4\,a^{2}x^{2}-3)\operatorname{erf}\left(\sqrt{ax}\right)-2\,(2\,ax+3)\sqrt{ax}e^{(-ax)}}{8\,a^{2}}$$

input
```
integrate(x*pi^(1/2)*erfc((a*x)^(1/2)),x, algorithm="fricas")
```

output
```
1/8*(4*sqrt(pi)*a^2*x^2 - sqrt(pi)*(4*a^2*x^2 - 3)*erf(sqrt(a*x)) - 2*(2*a
*x + 3)*sqrt(a*x)*e^(-a*x))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.86

$$\int x\Gamma\left(\frac{1}{2},ax\right)\,dx=\sqrt{\pi}\left(\begin{cases}\frac{x^2\operatorname{erfc}\left(\sqrt{ax}\right)}{2}-\frac{x\sqrt{ax}e^{-ax}}{2\sqrt{\pi}a}-\frac{3\sqrt{ax}e^{-ax}}{4\sqrt{\pi}a^2}-\frac{3\operatorname{erfc}\left(\sqrt{ax}\right)}{8a^2} & \text{for } a\neq 0\\ \frac{x^2}{2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x*pi**(1/2)*erfc((a*x)**(1/2)),x)
```

output
```
sqrt(pi)*Piecewise((x**2*erfc(sqrt(a*x))/2 - x*sqrt(a*x)*exp(-a*x)/(2*sqrt
(pi)*a) - 3*sqrt(a*x)*exp(-a*x)/(4*sqrt(pi)*a**2) - 3*erfc(sqrt(a*x))/(8*a
**2), Ne(a, 0)), (x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.14

$$\int x\Gamma\left(\frac{1}{2},ax\right)\,dx=\frac{\sqrt{\pi}\left(4\,a^2x^2\operatorname{erfc}\left(\sqrt{ax}\right)-\frac{2\left(2\,(ax)^{\frac{3}{2}}+3\,\sqrt{ax}\right)e^{(-ax)}-3\,\sqrt{\pi}\operatorname{erf}\left(\sqrt{ax}\right)}{\sqrt{\pi}}\right)}{8\,a^2}$$

input
```
integrate(x*pi^(1/2)*erfc((a*x)^(1/2)),x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*(4*a^2*x^2*erfc(sqrt(a*x)) - (2*(2*(a*x)^(3/2) + 3*sqrt(a*x))
*e^(-a*x) - 3*sqrt(pi)*erf(sqrt(a*x)))/sqrt(pi))/a^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.59

$$\int x\Gamma\left(\frac{1}{2},ax\right)\,dx=\frac{\sqrt{\pi}\left(4\,ax^2-\frac{4\,a^2x^2\operatorname{erf}\left(\sqrt{ax}\right)+\frac{2\left(2\,\sqrt{ax}ax+3\,\sqrt{ax}\right)e^{(-ax)}-3\,\sqrt{\pi}\operatorname{erf}\left(\sqrt{ax}\right)}{\sqrt{\pi}}}{a}\right)}{8\,a}$$

input
```
integrate(x*pi^(1/2)*erfc((a*x)^(1/2)),x, algorithm="giac")
```

output
```
1/8*sqrt(pi)*(4*a*x^2 - (4*a^2*x^2*erf(sqrt(a*x)) + (2*(2*sqrt(a*x)*a*x +
3*sqrt(a*x))*e^(-a*x) - 3*sqrt(pi)*erf(sqrt(a*x)))/sqrt(pi))/a)/a
```

Mupad [F(-1)]

Timed out.

$$\int x\Gamma\left(\frac{1}{2},ax\right)\,dx=\int\sqrt{\Pi}\,x\,\mathrm{erfc}\left(\sqrt{a\,x}\right)\,dx$$

input
```
int(Pi^(1/2)*x*erfc((a*x)^(1/2)),x)
```

output
```
int(Pi^(1/2)*x*erfc((a*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.69

$$\int x\Gamma\left(\frac{1}{2},ax\right)\,dx=\frac{-4\sqrt{\pi}\,e^{ax}\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right)a^2x^2+3\sqrt{\pi}\,e^{ax}\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right)-4\sqrt{x}\,\sqrt{a}\,ax-6\sqrt{x}\,\sqrt{a}+4\sqrt{\pi}\,e^{ax}a^2x^2}{8e^{ax}a^2}$$

input
```
int(x*Pi^(1/2)*erfc((a*x)^(1/2)),x)
```

output
```
( - 4*sqrt(pi)*e**(a*x)*erf(sqrt(x)*sqrt(a))*a**2*x**2 + 3*sqrt(pi)*e**(a*
x)*erf(sqrt(x)*sqrt(a)) - 4*sqrt(x)*sqrt(a)*a*x - 6*sqrt(x)*sqrt(a) + 4*sq
rt(pi)*e**(a*x)*a**2*x**2)/(8*e**(a*x)*a**2)
```

3.63 $\int \Gamma\left(\frac{1}{2}, ax\right) dx$

Optimal result

Integrand size = 7, antiderivative size = 22

$$\int \Gamma\left(\frac{1}{2}, ax\right) dx = x\Gamma\left(\frac{1}{2}, ax\right) - \frac{\Gamma\left(\frac{3}{2}, ax\right)}{a}$$

output
```
x*Pi^(1/2)*erfc((a*x)^(1/2))-((a*x)^(1/2)*exp(-a*x)+1/2*Pi^(1/2)*erfc((a*x
)^(1/2)))/a
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 50 vs. 2(22) = 44.

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \Gamma\left(\frac{1}{2}, ax\right) dx = \frac{2\left(-\frac{1}{2}e^{-ax}\sqrt{ax} + \frac{1}{4}\sqrt{\pi}\operatorname{erf}\left(\sqrt{ax}\right)\right)}{a} + x\Gamma\left(\frac{1}{2}, ax\right)$$

input
```
Integrate[Gamma[1/2, a*x],x]
```

output
```
(2*(-1/2*Sqrt[a*x]/E^(a*x) + (Sqrt[Pi]*Erf[Sqrt[a*x]])/4))/a + x*Gamma[1/2
, a*x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma\left(\frac{1}{2},ax\right) dx$$

$$\Big\downarrow \text{ 7111}$$

$$x\Gamma\left(\frac{1}{2},ax\right)-\frac{\Gamma\left(\frac{3}{2},ax\right)}{a}$$

input
```
Int[Gamma[1/2, a*x],x]
```

output
```
x*Gamma[1/2, a*x] - Gamma[3/2, a*x]/a
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.95

method	result	size
parts	$x\sqrt{\pi}\,\operatorname{erfc}\left(\sqrt{xa}\right)+\frac{-\sqrt{xa}\,{\mathrm e}^{-xa}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{2}}{a}$	43
derivativedivides	$\frac{2\sqrt{\pi}\left(\frac{xa\,\operatorname{erfc}\left(\sqrt{xa}\right)}{2}+\frac{-\frac{\sqrt{xa}\,{\mathrm e}^{-xa}}{2}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{4}}{\sqrt{\pi}}\right)}{a}$	49
default	$\frac{2\sqrt{\pi}\left(\frac{xa\,\operatorname{erfc}\left(\sqrt{xa}\right)}{2}+\frac{-\frac{\sqrt{xa}\,{\mathrm e}^{-xa}}{2}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{4}}{\sqrt{\pi}}\right)}{a}$	49

input
```
int(Pi^(1/2)*erfc((x*a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*Pi^(1/2)*erfc((x*a)^(1/2))+2/a*(-1/2*(x*a)^(1/2)/exp(x*a)+1/4*Pi^(1/2)*e
rf((x*a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \Gamma\left(\frac{1}{2},ax\right)\,dx=\frac{2\,\sqrt{\pi}ax-\sqrt{\pi}(2\,ax-1)\operatorname{erf}\left(\sqrt{ax}\right)-2\,\sqrt{ax}e^{(-ax)}}{2\,a}$$

input
```
integrate(pi^(1/2)*erfc((a*x)^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(pi)*a*x - sqrt(pi)*(2*a*x - 1)*erf(sqrt(a*x)) - 2*sqrt(a*x)*e^
(-a*x))/a
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \Gamma\left(\frac{1}{2},ax\right)\,dx=\sqrt{\pi}\left(\begin{cases} x\operatorname{erfc}\left(\sqrt{ax}\right)-\frac{\sqrt{ax}e^{-ax}}{\sqrt{\pi}a}-\frac{\operatorname{erfc}\left(\sqrt{ax}\right)}{2a} & \text{for } a\neq 0\\ x & \text{otherwise}\end{cases}\right)$$

input
```
integrate(pi**(1/2)*erfc((a*x)**(1/2)),x)
```

output
```
sqrt(pi)*Piecewise((x*erfc(sqrt(a*x)) - sqrt(a*x)*exp(-a*x)/(sqrt(pi)*a) -
 erfc(sqrt(a*x))/(2*a), Ne(a, 0)), (x, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \Gamma\left(\frac{1}{2},ax\right)\,dx=\frac{\sqrt{\pi}\left(2\,ax\operatorname{erfc}\left(\sqrt{ax}\right)+\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{ax}\right)-2\,\sqrt{ax}e^{(-ax)}}{\sqrt{\pi}}\right)}{2\,a}$$

input
```
integrate(pi^(1/2)*erfc((a*x)^(1/2)),x, algorithm="maxima")
```

output
```
1/2*sqrt(pi)*(2*a*x*erfc(sqrt(a*x)) + (sqrt(pi)*erf(sqrt(a*x)) - 2*sqrt(a*
x)*e^(-a*x))/sqrt(pi))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \Gamma\left(\frac{1}{2},ax\right)\,dx=\frac{1}{2}\,\sqrt{\pi}\left(2\,x-\frac{2\,ax\operatorname{erf}\left(\sqrt{ax}\right)-\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{ax}\right)-2\,\sqrt{ax}e^{(-ax)}}{\sqrt{\pi}}}{a}\right)$$

input
```
integrate(pi^(1/2)*erfc((a*x)^(1/2)),x, algorithm="giac")
```

output
```
1/2*sqrt(pi)*(2*x - (2*a*x*erf(sqrt(a*x)) - (sqrt(pi)*erf(sqrt(a*x)) - 2*s
qrt(a*x)*e^(-a*x))/sqrt(pi))/a)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.14

$$\int \Gamma\left(\frac{1}{2}, ax\right) dx = \sqrt{\Pi}\, x \operatorname{erfc}\left(\sqrt{a\,x}\right) - \frac{\sqrt{\Pi} \operatorname{erfc}\left(\sqrt{a\,x}\right)}{2\,a} - \frac{\sqrt{\Pi}\, \mathrm{e}^{-a\,x}\, \sqrt{a\,x}}{a\,\sqrt{\pi}}$$

input
```
int(Pi^(1/2)*erfc((a*x)^(1/2)),x)
```

output
```
Pi^(1/2)*x*erfc((a*x)^(1/2)) - (Pi^(1/2)*erfc((a*x)^(1/2)))/(2*a) - (Pi^(1
/2)*exp(-a*x)*(a*x)^(1/2))/(a*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.77

$$\int \Gamma\left(\frac{1}{2}, ax\right) dx = \frac{-2\sqrt{\pi}\, e^{ax} \operatorname{erf}\left(\sqrt{x}\,\sqrt{a}\right) ax + \sqrt{\pi}\, e^{ax} \operatorname{erf}\left(\sqrt{x}\,\sqrt{a}\right) - 2\sqrt{x}\,\sqrt{a} + 2\sqrt{\pi}\, e^{ax} ax}{2e^{ax} a}$$

input
```
int(Pi^(1/2)*erfc((a*x)^(1/2)),x)
```

output
```
( - 2*sqrt(pi)*e**(a*x)*erf(sqrt(x)*sqrt(a))*a*x + sqrt(pi)*e**(a*x)*erf(s
qrt(x)*sqrt(a)) - 2*sqrt(x)*sqrt(a) + 2*sqrt(pi)*e**(a*x)*a*x)/(2*e**(a*x)
*a)
```

3.64 $\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x} dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x} dx = -4\sqrt{ax}\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-ax\right)+\sqrt{\pi}\log(x)$$

output
```
-4*(a*x)^(1/2)*hypergeom([1/2, 1/2],[3/2, 3/2],-a*x)+Pi^(1/2)*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x} dx = -4\sqrt{ax}\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-ax\right)+\left(\sqrt{\pi}\text{erf}\left(\sqrt{ax}\right)+\Gamma\left(\frac{1}{2},ax\right)\right)\log(ax)$$

input
```
Integrate[Gamma[1/2, a*x]/x,x]
```

output
```
-4*Sqrt[a*x]*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, -(a*x)] + (Sqrt[Pi]
*Erf[Sqrt[a*x]] + Gamma[1/2, a*x])*Log[a*x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7115}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x}\,dx$$

$$\Big\downarrow \ 7115$$

$$\sqrt{\pi}\log(x)-4\sqrt{ax}\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-ax\right)$$

input
```
Int[Gamma[1/2, a*x]/x,x]
```

output
```
-4*Sqrt[a*x]*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, -(a*x)] + Sqrt[Pi]*
Log[x]
```

Defintions of rubi rules used

rule 7115
```
Int[Gamma[n_, (b_.)*(x_)]/(x_), x_Symbol] :> Simp[Gamma[n]*Log[x], x] - Sim
p[((b*x)^n/n^2)*HypergeometricPFQ[{n, n}, {1 + n, 1 + n}, (-b)*x], x] /; Fr
eeQ[{b, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{\sqrt{\pi}\,\operatorname{erfc}\left(\sqrt{xa}\right)}{x}dx$$

input
```
int(Pi^(1/2)*erfc((x*a)^(1/2))/x,x)
```

output
```
int(Pi^(1/2)*erfc((x*a)^(1/2))/x,x)
```

Fricas [F]

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x} \, dx = \int \frac{\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{ax}\right)}{x} \, dx$$

input
```
integrate(pi^(1/2)*erfc((a*x)^(1/2))/x,x, algorithm="fricas")
```

output
```
integral(-(sqrt(pi)*erf(sqrt(a*x)) - sqrt(pi))/x, x)
```

Sympy [F]

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x} \, dx = \sqrt{\pi} \int \frac{\mathrm{erfc}\left(\sqrt{ax}\right)}{x} \, dx$$

input
```
integrate(pi**(1/2)*erfc((a*x)**(1/2))/x,x)
```

output
```
sqrt(pi)*Integral(erfc(sqrt(a*x))/x, x)
```

Maxima [F]

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x} \, dx = \int \frac{\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{ax}\right)}{x} \, dx$$

input
```
integrate(pi^(1/2)*erfc((a*x)^(1/2))/x,x, algorithm="maxima")
```

output
```
sqrt(pi)*integrate(erfc(sqrt(a*x))/x, x)
```

Giac [F]

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x}\, dx = \int \frac{\sqrt{\pi}\, \operatorname{erfc}\left(\sqrt{ax}\right)}{x}\, dx$$

input
```
integrate(pi^(1/2)*erfc((a*x)^(1/2))/x,x, algorithm="giac")
```

output
```
integrate(sqrt(pi)*erfc(sqrt(a*x))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x}\, dx = \int \frac{\sqrt{\Pi}\, \operatorname{erfc}\left(\sqrt{a\, x}\right)}{x}\, dx$$

input
```
int((Pi^(1/2)*erfc((a*x)^(1/2)))/x,x)
```

output
```
int((Pi^(1/2)*erfc((a*x)^(1/2)))/x, x)
```

Reduce [F]

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x}\, dx = \sqrt{\pi}\left(-\left(\int \frac{\operatorname{erf}\left(\sqrt{x}\,\sqrt{a}\right)}{x} dx\right) + \log(x)\right)$$

input
```
int(Pi^(1/2)*erfc((a*x)^(1/2))/x,x)
```

output
```
sqrt(pi)*( - int(erf(sqrt(x)*sqrt(a))/x,x) + log(x))
```

3.65 $\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x^2}\,dx = a\Gamma\left(-\frac{1}{2},ax\right) - \frac{\Gamma\left(\frac{1}{2},ax\right)}{x}$$

output
```
a*(-2*Pi^(1/2)*erfc((a*x)^(1/2))+2/(a*x)^(1/2)*exp(-a*x))-Pi^(1/2)*erfc((a
*x)^(1/2))/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x^2}\,dx = a\Gamma\left(-\frac{1}{2},ax\right) - \frac{\Gamma\left(\frac{1}{2},ax\right)}{x}$$

input
```
Integrate[Gamma[1/2, a*x]/x^2,x]
```

output
```
a*Gamma[-1/2, a*x] - Gamma[1/2, a*x]/x
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x^2}dx$$

$$\Big\downarrow \text{7116}$$

$$a\Gamma\left(-\frac{1}{2},ax\right)-\frac{\Gamma\left(\frac{1}{2},ax\right)}{x}$$

input
```
Int[Gamma[1/2, a*x]/x^2,x]
```

output
```
a*Gamma[-1/2, a*x] - Gamma[1/2, a*x]/x
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

method	result	size
parts	$-\frac{\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{xa}\right)}{x}-a\left(-\frac{2\,\mathrm{e}^{-xa}}{\sqrt{xa}}-2\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{xa}\right)\right)$	44
derivativedivides	$2\sqrt{\pi}\,a\left(-\frac{\mathrm{erfc}\left(\sqrt{xa}\right)}{2xa}-\frac{-\frac{\mathrm{e}^{-xa}}{\sqrt{xa}}-\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{xa}\right)}{\sqrt{\pi}}\right)$	52
default	$2\sqrt{\pi}\,a\left(-\frac{\mathrm{erfc}\left(\sqrt{xa}\right)}{2xa}-\frac{-\frac{\mathrm{e}^{-xa}}{\sqrt{xa}}-\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{xa}\right)}{\sqrt{\pi}}\right)$	52

input
```
int(Pi^(1/2)*erfc((x*a)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-Pi^(1/2)*erfc((x*a)^(1/2))/x-a*(-2/exp(x*a)/(x*a)^(1/2)-2*Pi^(1/2)*erf((x
*a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x^2}\,dx=\frac{\sqrt{\pi}(2\,ax+1)\,\mathrm{erf}\left(\sqrt{ax}\right)+2\,\sqrt{ax}e^{(-ax)}-\sqrt{\pi}}{x}$$

input
```
integrate(pi^(1/2)*erfc((a*x)^(1/2))/x^2,x, algorithm="fricas")
```

output
```
(sqrt(pi)*(2*a*x + 1)*erf(sqrt(a*x)) + 2*sqrt(a*x)*e^(-a*x) - sqrt(pi))/x
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x^2} \, dx = \sqrt{\pi}\left(-2a \operatorname{erfc}\left(\sqrt{ax}\right) + \frac{2\sqrt{ax}e^{-ax}}{\sqrt{\pi}x} - \frac{\operatorname{erfc}\left(\sqrt{ax}\right)}{x}\right)$$

input
```
integrate(pi**(1/2)*erfc((a*x)**(1/2))/x**2,x)
```

output
```
sqrt(pi)*(-2*a*erfc(sqrt(a*x)) + 2*sqrt(a*x)*exp(-a*x)/(sqrt(pi)*x) - erfc
(sqrt(a*x))/x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x^2} \, dx = \sqrt{\pi}a\left(\frac{\Gamma\left(-\frac{1}{2}, ax\right)}{\sqrt{\pi}} - \frac{\operatorname{erfc}\left(\sqrt{ax}\right)}{ax}\right)$$

input
```
integrate(pi^(1/2)*erfc((a*x)^(1/2))/x^2,x, algorithm="maxima")
```

output
```
sqrt(pi)*a*(gamma(-1/2, a*x)/sqrt(pi) - erfc(sqrt(a*x))/(a*x))
```

Giac [F]

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x^2} \, dx = \int \frac{\sqrt{\pi} \operatorname{erfc}\left(\sqrt{ax}\right)}{x^2} \, dx$$

input
```
integrate(pi^(1/2)*erfc((a*x)^(1/2))/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(pi)*erfc(sqrt(a*x))/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x^2} \, dx = \int \frac{\sqrt{\Pi}\,\mathrm{erfc}\left(\sqrt{a\,x}\right)}{x^2} \, dx$$

input
```
int((Pi^(1/2)*erfc((a*x)^(1/2)))/x^2,x)
```

output
```
int((Pi^(1/2)*erfc((a*x)^(1/2)))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.64

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x^2} \, dx = \frac{2\sqrt{\pi}\, e^{ax}\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right) ax + \sqrt{\pi}\, e^{ax}\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right) + 2\sqrt{x}\,\sqrt{a} - \sqrt{\pi}\, e^{ax}}{e^{ax}x}$$

input
```
int(Pi^(1/2)*erfc((a*x)^(1/2))/x^2,x)
```

output
```
(2*sqrt(pi)*e**(a*x)*erf(sqrt(x)*sqrt(a))*a*x + sqrt(pi)*e**(a*x)*erf(sqrt
(x)*sqrt(a)) + 2*sqrt(x)*sqrt(a) - sqrt(pi)*e**(a*x))/(e**(a*x)*x)
```

3.66 $\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x^3}\,dx=\frac{1}{2}a^2\Gamma\left(-\frac{3}{2},ax\right)-\frac{\Gamma\left(\frac{1}{2},ax\right)}{2x^2}$$

output
```
1/2*a^2*(4/3*Pi^(1/2)*erfc((a*x)^(1/2))-4/3/(a*x)^(1/2)*exp(-a*x)+2/3/(a*x
)^(3/2)*exp(-a*x))-1/2*Pi^(1/2)*erfc((a*x)^(1/2))/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x^3}\,dx=\frac{1}{2}a^2\Gamma\left(-\frac{3}{2},ax\right)-\frac{\Gamma\left(\frac{1}{2},ax\right)}{2x^2}$$

input
```
Integrate[Gamma[1/2, a*x]/x^3,x]
```

output
```
(a^2*Gamma[-3/2, a*x])/2 - Gamma[1/2, a*x]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x^3}\,dx$$

$$\Big\downarrow \text{ 7116}$$

$$\frac{1}{2}a^2\Gamma\left(-\frac{3}{2},ax\right)-\frac{\Gamma\left(\frac{1}{2},ax\right)}{2x^2}$$

input
```
Int[Gamma[1/2, a*x]/x^3,x]
```

output
```
(a^2*Gamma[-3/2, a*x])/2 - Gamma[1/2, a*x]/(2*x^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.03

method	result	size
parts	$-\frac{\sqrt{\pi}\,\operatorname{erfc}\left(\sqrt{xa}\right)}{2x^{2}}-a^{2}\left(-\frac{\mathrm{e}^{-xa}}{3\left(xa\right)^{\frac{3}{2}}}+\frac{2\,\mathrm{e}^{-xa}}{3\sqrt{xa}}+\frac{2\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{3}\right)$	59
derivativedivides	$2\sqrt{\pi}\,a^{2}\left(-\frac{\operatorname{erfc}\left(\sqrt{xa}\right)}{4x^{2}a^{2}}-\frac{-\frac{\mathrm{e}^{-xa}}{3\left(xa\right)^{\frac{3}{2}}}+\frac{2\,\mathrm{e}^{-xa}}{3\sqrt{xa}}+\frac{2\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{3}}{2\sqrt{\pi}}\right)$	67
default	$2\sqrt{\pi}\,a^{2}\left(-\frac{\operatorname{erfc}\left(\sqrt{xa}\right)}{4x^{2}a^{2}}-\frac{-\frac{\mathrm{e}^{-xa}}{3\left(xa\right)^{\frac{3}{2}}}+\frac{2\,\mathrm{e}^{-xa}}{3\sqrt{xa}}+\frac{2\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{3}}{2\sqrt{\pi}}\right)$	67

input
```
int(Pi^(1/2)*erfc((x*a)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*Pi^(1/2)*erfc((x*a)^(1/2))/x^2-a^2*(-1/3/exp(x*a)/(x*a)^(3/2)+2/3/exp
(x*a)/(x*a)^(1/2)+2/3*Pi^(1/2)*erf((x*a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x^{3}}\,dx=-\frac{\sqrt{\pi}(4\,a^{2}x^{2}-3)\operatorname{erf}\left(\sqrt{ax}\right)+2\,(2\,ax-1)\sqrt{ax}e^{(-ax)}+3\,\sqrt{\pi}}{6\,x^{2}}$$

input
```
integrate(pi^(1/2)*erfc((a*x)^(1/2))/x^3,x, algorithm="fricas")
```

output
```
-1/6*(sqrt(pi)*(4*a^2*x^2 - 3)*erf(sqrt(a*x)) + 2*(2*a*x - 1)*sqrt(a*x)*e^
(-a*x) + 3*sqrt(pi))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.69

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x^3} \, dx = \sqrt{\pi}\left(\frac{2a^2 \operatorname{erfc}\left(\sqrt{ax}\right)}{3} - \frac{2a\sqrt{ax}e^{-ax}}{3\sqrt{\pi}x} + \frac{\sqrt{ax}e^{-ax}}{3\sqrt{\pi}x^2} - \frac{\operatorname{erfc}\left(\sqrt{ax}\right)}{2x^2}\right)$$

input
```
integrate(pi**(1/2)*erfc((a*x)**(1/2))/x**3,x)
```

output
```
sqrt(pi)*(2*a**2*erfc(sqrt(a*x))/3 - 2*a*sqrt(a*x)*exp(-a*x)/(3*sqrt(pi)*x
) + sqrt(a*x)*exp(-a*x)/(3*sqrt(pi)*x**2) - erfc(sqrt(a*x))/(2*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x^3} \, dx = \frac{1}{2}\sqrt{\pi}a^2\left(\frac{\Gamma\left(-\frac{3}{2}, ax\right)}{\sqrt{\pi}} - \frac{\operatorname{erfc}\left(\sqrt{ax}\right)}{a^2x^2}\right)$$

input
```
integrate(pi^(1/2)*erfc((a*x)^(1/2))/x^3,x, algorithm="maxima")
```

output
```
1/2*sqrt(pi)*a^2*(gamma(-3/2, a*x)/sqrt(pi) - erfc(sqrt(a*x))/(a^2*x^2))
```

Giac [F]

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x^3} \, dx = \int \frac{\sqrt{\pi} \operatorname{erfc}\left(\sqrt{ax}\right)}{x^3} \, dx$$

input
```
integrate(pi^(1/2)*erfc((a*x)^(1/2))/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(pi)*erfc(sqrt(a*x))/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x^3} \, dx = \int \frac{\sqrt{\Pi}\, \mathrm{erfc}\left(\sqrt{a\,x}\right)}{x^3} \, dx$$

input
```
int((Pi^(1/2)*erfc((a*x)^(1/2)))/x^3,x)
```

output
```
int((Pi^(1/2)*erfc((a*x)^(1/2)))/x^3, x)
```

Reduce [F]

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x^3} \, dx = \frac{3\sqrt{\pi}\, e^{ax}\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right) + 2e^{ax}\sqrt{a}\left(\int \frac{\sqrt{x}}{e^{ax}x^2}dx\right) a\,x^2 + 2\sqrt{x}\,\sqrt{a} - 3\sqrt{\pi}\, e^{ax}}{6e^{ax}x^2}$$

input
```
int(Pi^(1/2)*erfc((a*x)^(1/2))/x^3,x)
```

output
```
(3*sqrt(pi)*e**(a*x)*erf(sqrt(x)*sqrt(a)) + 2*e**(a*x)*sqrt(a)*int(sqrt(x)
/(e**(a*x)*x**2),x)*a*x**2 + 2*sqrt(x)*sqrt(a) - 3*sqrt(pi)*e**(a*x))/(6*e
**(a*x)*x**2)
```

3.67 $\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x^4} dx = \frac{1}{3}a^3\Gamma\left(-\frac{5}{2},ax\right) - \frac{\Gamma\left(\frac{1}{2},ax\right)}{3x^3}$$

output
```
1/3*a^3*(-8/15*Pi^(1/2)*erfc((a*x)^(1/2))+8/15/(a*x)^(1/2)*exp(-a*x)-4/15/
(a*x)^(3/2)*exp(-a*x)+2/5/(a*x)^(5/2)*exp(-a*x))-1/3*Pi^(1/2)*erfc((a*x)^(
1/2))/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x^4} dx = \frac{1}{3}a^3\Gamma\left(-\frac{5}{2},ax\right) - \frac{\Gamma\left(\frac{1}{2},ax\right)}{3x^3}$$

input
```
Integrate[Gamma[1/2, a*x]/x^4,x]
```

output
```
(a^3*Gamma[-5/2, a*x])/3 - Gamma[1/2, a*x]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x^4}\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{3}a^3\Gamma\left(-\frac{5}{2},ax\right)-\frac{\Gamma\left(\frac{1}{2},ax\right)}{3x^3}$$

input
```
Int[Gamma[1/2, a*x]/x^4,x]
```

output
```
(a^3*Gamma[-5/2, a*x])/3 - Gamma[1/2, a*x]/(3*x^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.48

method	result	size
parts	$-\frac{\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{xa}\right)}{3x^{3}}-\frac{2a^{3}\left(-\frac{\mathrm{e}^{-xa}}{5(xa)^{\frac{5}{2}}}+\frac{2\,\mathrm{e}^{-xa}}{15(xa)^{\frac{3}{2}}}-\frac{4\,\mathrm{e}^{-xa}}{15\sqrt{xa}}-\frac{4\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{xa}\right)}{15}\right)}{3}$	72
derivativedivides	$2\sqrt{\pi}\,a^{3}\left(-\frac{\mathrm{erfc}\left(\sqrt{xa}\right)}{6x^{3}a^{3}}-\frac{-\frac{\mathrm{e}^{-xa}}{5(xa)^{\frac{5}{2}}}+\frac{2\,\mathrm{e}^{-xa}}{15(xa)^{\frac{3}{2}}}-\frac{4\,\mathrm{e}^{-xa}}{15\sqrt{xa}}-\frac{4\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{xa}\right)}{15}}{3\sqrt{\pi}}\right)$	80
default	$2\sqrt{\pi}\,a^{3}\left(-\frac{\mathrm{erfc}\left(\sqrt{xa}\right)}{6x^{3}a^{3}}-\frac{-\frac{\mathrm{e}^{-xa}}{5(xa)^{\frac{5}{2}}}+\frac{2\,\mathrm{e}^{-xa}}{15(xa)^{\frac{3}{2}}}-\frac{4\,\mathrm{e}^{-xa}}{15\sqrt{xa}}-\frac{4\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{xa}\right)}{15}}{3\sqrt{\pi}}\right)$	80

input
```
int(Pi^(1/2)*erfc((x*a)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*Pi^(1/2)*erfc((x*a)^(1/2))/x^3-2/3*a^3*(-1/5/exp(x*a)/(x*a)^(5/2)+2/1
5/exp(x*a)/(x*a)^(3/2)-4/15/exp(x*a)/(x*a)^(1/2)-4/15*Pi^(1/2)*erf((x*a)^(
1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.97

$$\int \frac{\Gamma\left(\frac{1}{2},ax\right)}{x^4}\,dx=\frac{\sqrt{\pi}(8\,a^3x^3+15)\,\mathrm{erf}\left(\sqrt{ax}\right)+2\,(4\,a^2x^2-2\,ax+3)\sqrt{ax}e^{(-ax)}-15\,\sqrt{\pi}}{45\,x^3}$$

input
```
integrate(pi^(1/2)*erfc((a*x)^(1/2))/x^4,x, algorithm="fricas")
```

output
```
1/45*(sqrt(pi)*(8*a^3*x^3 + 15)*erf(sqrt(a*x)) + 2*(4*a^2*x^2 - 2*a*x + 3)
*sqrt(a*x)*e^(-a*x) - 15*sqrt(pi))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.69

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x^4}\, dx = \sqrt{\pi}\left(-\frac{8a^3 \operatorname{erfc}\left(\sqrt{ax}\right)}{45} + \frac{8a^2\sqrt{ax}e^{-ax}}{45\sqrt{\pi}x} - \frac{4a\sqrt{ax}e^{-ax}}{45\sqrt{\pi}x^2} + \frac{2\sqrt{ax}e^{-ax}}{15\sqrt{\pi}x^3} - \frac{\operatorname{erfc}\left(\sqrt{ax}\right)}{3x^3}\right)$$

input
```
integrate(pi**(1/2)*erfc((a*x)**(1/2))/x**4,x)
```

output
```
sqrt(pi)*(-8*a**3*erfc(sqrt(a*x))/45 + 8*a**2*sqrt(a*x)*exp(-a*x)/(45*sqrt
(pi)*x) - 4*a*sqrt(a*x)*exp(-a*x)/(45*sqrt(pi)*x**2) + 2*sqrt(a*x)*exp(-a*
x)/(15*sqrt(pi)*x**3) - erfc(sqrt(a*x))/(3*x**3))
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x^4}\, dx = \frac{1}{3}\sqrt{\pi}a^3\left(\frac{\Gamma\left(-\frac{5}{2}, ax\right)}{\sqrt{\pi}} - \frac{\operatorname{erfc}\left(\sqrt{ax}\right)}{a^3x^3}\right)$$

input
```
integrate(pi^(1/2)*erfc((a*x)^(1/2))/x^4,x, algorithm="maxima")
```

output
```
1/3*sqrt(pi)*a^3*(gamma(-5/2, a*x)/sqrt(pi) - erfc(sqrt(a*x))/(a^3*x^3))
```

Giac [F]

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x^4} \, dx = \int \frac{\sqrt{\pi}\,\operatorname{erfc}\left(\sqrt{ax}\right)}{x^4} \, dx$$

input
```
integrate(pi^(1/2)*erfc((a*x)^(1/2))/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(pi)*erfc(sqrt(a*x))/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x^4} \, dx = \int \frac{\sqrt{\Pi}\,\operatorname{erfc}\left(\sqrt{a\,x}\right)}{x^4} \, dx$$

input
```
int((Pi^(1/2)*erfc((a*x)^(1/2)))/x^4,x)
```

output
```
int((Pi^(1/2)*erfc((a*x)^(1/2)))/x^4, x)
```

Reduce [F]

$$\int \frac{\Gamma\left(\frac{1}{2}, ax\right)}{x^4} \, dx = \frac{15\sqrt{x}\,\sqrt{\pi}\,e^{ax}\operatorname{erf}\left(\sqrt{x}\,\sqrt{a}\right) - 4\sqrt{x}\,e^{ax}\sqrt{a}\left(\int \frac{\sqrt{x}}{e^{ax}x^2}dx\right)a^2x^3 - 4\sqrt{a}\,a\,x^2 + 6\sqrt{a}\,x - 15\sqrt{x}\,\sqrt{\pi}\,e^{ax}}{45\sqrt{x}\,e^{ax}x^3}$$

input
```
int(Pi^(1/2)*erfc((a*x)^(1/2))/x^4,x)
```

output
```
(15*sqrt(x)*sqrt(pi)*e**(a*x)*erf(sqrt(x)*sqrt(a)) - 4*sqrt(x)*e**(a*x)*sq
rt(a)*int(sqrt(x)/(e**(a*x)*x**2),x)*a**2*x**3 - 4*sqrt(a)*a*x**2 + 6*sqrt
(a)*x - 15*sqrt(x)*sqrt(pi)*e**(a*x))/(45*sqrt(x)*e**(a*x)*x**3)
```

3.68 $\int x^{100}\Gamma\left(\frac{3}{2}, ax\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int x^{100}\Gamma\left(\frac{3}{2},ax\right)\,dx=\frac{1}{101}x^{101}\Gamma\left(\frac{3}{2},ax\right)-\frac{\Gamma\left(\frac{205}{2},ax\right)}{101a^{101}}$$

output
```
1/101*x^101*((a*x)^(1/2)*exp(-a*x)+1/2*Pi^(1/2)*erfc((a*x)^(1/2)))-1/101*G
AMMA(205/2,a*x)/a^101
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^{100}\Gamma\left(\frac{3}{2},ax\right)\,dx=\frac{1}{101}x^{101}\Gamma\left(\frac{3}{2},ax\right)-\frac{\Gamma\left(\frac{205}{2},ax\right)}{101a^{101}}$$

input
```
Integrate[x^100*Gamma[3/2, a*x],x]
```

output
```
(x^101*Gamma[3/2, a*x])/101 - Gamma[205/2, a*x]/(101*a^101)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{100}\Gamma\left(\frac{3}{2},ax\right)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{101}x^{101}\Gamma\left(\frac{3}{2},ax\right)-\frac{\Gamma\left(\frac{205}{2},ax\right)}{101a^{101}}$$

input
```
Int[x^100*Gamma[3/2, a*x],x]
```

output
```
(x^101*Gamma[3/2, a*x])/101 - Gamma[205/2, a*x]/(101*a^101)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2678 vs. 2(39) = 78.

Time = 0.80 (sec) , antiderivative size = 2679, normalized size of antiderivative = 92.38

method	result	size
derivativedivides	Expression too large to display	2679
default	Expression too large to display	2679
parts	Expression too large to display	2684

input
```
int(x^100*((x*a)^(1/2)*exp(-x*a)+1/2*Pi^(1/2)*erfc((x*a)^(1/2))),x,method=
_RETURNVERBOSE)
```

output
```
1/a^101*(-481452080155966748007633880476899108122537146653367093423140625/
268435456*(x*a)^(145/2)/exp(x*a)+13399280426846637026266542163537418668282
22367324552136009000175903930373973956200201339808971908813302485337859343
55559588023275706870249406853902718440076929197472536446481943130493164062
5/2535301200456458802993406410752*Pi^(1/2)*erf((x*a)^(1/2))-13399280426846
63702626654216353741866828222367324552136009000175903930373973956200201339
80897190881330248533785934355559588023275706870249406853902718440076929197
4725364464819431304931640625/1267650600228229401496703205376*(x*a)^(1/2)/e
xp(x*a)-216456657335251272772598790366307082218216489362932419713454203208
06680443887576516848700936501255208105181593184682267383344749723361758291
6665391768573616754055023193359375/77371252455336267181195264*(x*a)^(29/2)
/exp(x*a)-4485212372163007477064823209929735140263860411364401779216831618
20667701438399183260240697341751009722721429443359375/18014398509481984*(x
*a)^(93/2)/exp(x*a)-528075613518318762367820462505858445619523667969761004
51592300946851000081106574844274393397042974579078560179163823409966274340
229687013435795848903656005859375/302231454903657293676544*(x*a)^(45/2)/ex
p(x*a)-2046538169524033589964909098246082727925626897391951221975288291426
38941759834966883262333961861830147452263038623045050485994188828211500164
1769196189127188664982879161834716796875/2475880078570760549798248448*(x*a
)^(19/2)/exp(x*a)-97454198548763504284043290392670606091696518923426248...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(39) = 78$.

Time = 0.48 (sec) , antiderivative size = 848, normalized size of antiderivative = 29.24

$$\int x^{100}\Gamma\left(\frac{3}{2}, ax\right) dx = \text{Too large to display}$$

input
```
integrate(x^100*((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2))),x,
algorithm="fricas")
```

output
```
1/5121308424922046782046680949719O4*(2535301200456458802993406410752*sqrt(
pi)*a^101*x^101 - sqrt(pi)*(2535301200456458802993406410752*a^101*x^101 -
27200539266498673163321080591980959896612914056688408360982703570849786591
67131086408719812212974891004045235854467417859636872496849466062959134225
18433356166270869248986358344554901123046875)*erf(sqrt(a*x)) - 406*(126765
0600228229401496703205376*a^100*x^100 + 1273988853229370548504186721402 88*
a^99*x^99 + 1267618908963223695761665787795 8656*a^98*x^98 + 12486046253287
75340325240800978927616*a^97*x^97 + 12173895096955559568171097809544544256
0*a^96*x^96 + 11747808768562114983285109386210485207040*a^95*x^95 + 112191
57373976819809037279463831013372723 20*a^94*x^94 + 106021037184080947195402
2909332030763722 34240*a^93*x^93 + 99129669767115685627701142022544876408 03
901440*a^92*x^92 + 91694944534582009205623556370854010677436 0883200*a^91*x
^91 + 83900874249142538423145554079331419769854020812800*a^90*x^90 + 75930
29119547399727294672644179493489171788883558400*a^89*x^89 + 67957610619949
2275592873201654064667280875105078476800*a^88*x^88 + 601424853986550663899
6927834638472305435744679944519 6800*a^87*x^87 + 526246747238231830912231 18
55308663267256276594951454720000*a^86*x^86 + 4552034363610705337390799754 8
4199372617667925463300833280000*a^85*x^85 + 3891989380887153063469133790 38
99046358810607627112221245440000*a^84*x^84 + 328873102684964433863141805 28
79469417319496344490982695239680000*a^83*x^83 + 27460904074194530227572...
```

Sympy [F(-1)]

Timed out.

$$\int x^{100}\Gamma\left(\frac{3}{2},ax\right)\,dx=\text{Timed out}$$

input
```
integrate(x**100*((a*x)**(1/2)*exp(-a*x)+1/2*pi**(1/2)*erfc((a*x)**(1/2)))
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{100}\Gamma\left(\frac{3}{2},ax\right)\,dx=\int\frac{1}{2}\left(\sqrt{\pi}\operatorname{erfc}\left(\sqrt{ax}\right)+2\sqrt{ax}e^{(-ax)}\right)x^{100}\,dx$$

input
```
integrate(x^100*((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2))),x,
algorithm="maxima")
```

output
```
1/2*integrate((sqrt(pi)*erfc(sqrt(a*x)) + 2*sqrt(a*x)*e^(-a*x))*x^100, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2693 vs. $2(39) = 78$.

Time = 0.14 (sec) , antiderivative size = 2693, normalized size of antiderivative = 92.86

$$\int x^{100}\Gamma\left(\frac{3}{2},ax\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^100*((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2))),x,
algorithm="giac")
```

output
```
1/5121308424922046782046680949719O4*(2535301200456458802993406410752*sqrt(
pi)*a*x^101 - sqrt(pi)*(2535301200456458802993406410752*a^101*x^101*erf(sq
rt(a*x)) + (2*(1267650600228229401496703205376*sqrt(a*x)*a^100*x^100 + 127
3988853229370548504186721402888*sqrt(a*x)*a^99*x^99 + 12676189089632236957
616657877958656*sqrt(a*x)*a^98*x^98 + 1248604625328775340325240800978927616
*sqrt(a*x)*a^97*x^97 + 121738950969555595681710978095445442560*sqrt(a*x)*a
^96*x^96 + 11747808768562114983285109386210485207040*sqrt(a*x)*a^95*x^95 +
 1121915737397681980903727946383101337272320*sqrt(a*x)*a^94*x^94 + 1060210
37184080947195402290933203076372234240*sqrt(a*x)*a^93*x^93 + 9912966976711
5685627701142022544876408039014440*sqrt(a*x)*a^92*x^92 + 916949445345820092
05623556370854010677436088320O*sqrt(a*x)*a^91*x^91 + 839008742491425384231
4555407933141976985402081280O*sqrt(a*x)*a^90*x^90 + 759302911954739972729 4
672644179493489171788883558400*sqrt(a*x)*a^89*x^89 + 679576106199492275592
8732016540646672808751050784768OO*sqrt(a*x)*a^88*x^88 + 601424853986550663
8996927834638472305435744679944519680O*sqrt(a*x)*a^87*x^87 + 5262467472382
3183091223118553086632672562765949514547200O0*sqrt(a*x)*a^86*x^86 + 455203
4363610705337390799754841993726176679254633008332800O0*sqrt(a*x)*a^85*x^85
 + 3891989380887153063469133790389904635881060762711222124544OOOO*sqrt(a*x
)*a^84*x^84 + 328873102684964433863141805287946941731949634449098269523968
0000*sqrt(a*x)*a^83*x^83 + 274609040741945302275723407415435696346177 94...
```

Mupad [F(-1)]

Timed out.

$$\int x^{100}\Gamma\left(\frac{3}{2},ax\right)\,dx=\int x^{100}\left(\frac{\sqrt{\Pi}\,\mathrm{erfc}\left(\sqrt{a\,x}\right)}{2}+\mathrm{e}^{-a\,x}\,\sqrt{a\,x}\right)\,dx$$

input
```
int(x^100*((Pi^(1/2)*erfc((a*x)^(1/2)))/2 + exp(-a*x)*(a*x)^(1/2)),x)
```

output
```
int(x^100*((Pi^(1/2)*erfc((a*x)^(1/2)))/2 + exp(-a*x)*(a*x)^(1/2)), x)
```

Reduce [F]

$$\int x^{100}\Gamma\left(\frac{3}{2},ax\right)\,dx=\text{too large to display}$$

input
```
int(x^100*((a*x)^(1/2)*exp(-a*x)+1/2*Pi^(1/2)*erfc((a*x)^(1/2))),x)
```

output
```
( - 2535301200456458802993406410752*sqrt(pi)*e**(a*x)*erf(sqrt(x)*sqrt(a))
*a**101*x**101 + 133992804268466370262665421635374186682822367324552136009
00017590393037397395620020133980897190881330248533785934355559588023275706
87024940685390271844007692919747253644648194313049316406258*sqrt(pi)*e**(a
*x)*erf(sqrt(x)*sqrt(a)) + 270665464622302067930584151703455857099300918199
55953147381803553259393554273915244067064141232558028710203824758739823036
78070169278779038018448834912489553969788945236218935251235961914062508*e*
*(a*x)*sqrt(a)*int(sqrt(x)/(e**(a*x)*x),x) - 51466614369266113700766150138
2656*sqrt(x)*sqrt(a)*a**100*x**100 - 5172394744111244426926998088895692 8*s
qrt(x)*sqrt(a)*a**99*x**99 - 5146532770390688204792363098451214336*sqrt(x)
*sqrt(a)*a**98*x**98 - 50693347788348278817204776519744461 2096*sqrt(x)*sqr
t(a)*a**97*x**97 - 494260140936395718467746571067508496 79360*sqrt(x)*sqrt(
a)*a**96*x**96 - 476961036003621868321375441080145699405 8240*sqrt(x)*sqrt(
a)*a**95*x**95 - 45549778938345888424691354623153914293 2561920*sqrt(x)*sqr
t(a)*a**94*x**94 - 430445410967368645613333301188804490071 27101440*sqrt(x)
*sqrt(a)*a**93*x**93 - 4024664592544896836484666366115321982166383984640*s
qrt(x)*sqrt(a)*a**92*x**92 - 3722814748104029573748316388656672833503905 18
579200*sqrt(x)*sqrt(a)*a**91*x**91 - 340637549451518705997970949562085564 2
6560732449996800*sqrt(x)*sqrt(a)*a**90*x**90 - 308276982253624428928163709
35368743566037462867247 10400*sqrt(x)*sqrt(a)*a**89*x**89 - 275907899116...
```

3.69 $\int x^2\Gamma\left(\frac{3}{2},ax\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int x^2\Gamma\left(\frac{3}{2},ax\right)\,dx=\frac{1}{3}x^3\Gamma\left(\frac{3}{2},ax\right)-\frac{\Gamma\left(\frac{9}{2},ax\right)}{3a^3}$$

output
```
1/3*x^3*((a*x)^(1/2)*exp(-a*x)+1/2*Pi^(1/2)*erfc((a*x)^(1/2)))-1/3*((a*x)^
(7/2)*exp(-a*x)+7/2*(a*x)^(5/2)*exp(-a*x)+35/4*(a*x)^(3/2)*exp(-a*x)+105/8
*(a*x)^(1/2)*exp(-a*x)+105/16*Pi^(1/2)*erfc((a*x)^(1/2)))/a^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^2\Gamma\left(\frac{3}{2},ax\right)\,dx=\frac{1}{3}x^3\Gamma\left(\frac{3}{2},ax\right)-\frac{\Gamma\left(\frac{9}{2},ax\right)}{3a^3}$$

input
```
Integrate[x^2*Gamma[3/2, a*x],x]
```

output
```
(x^3*Gamma[3/2, a*x])/3 - Gamma[9/2, a*x]/(3*a^3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \Gamma\left(\frac{3}{2}, ax\right) dx$$
$$\Big\downarrow \text{7116}$$
$$\frac{1}{3} x^3 \Gamma\left(\frac{3}{2}, ax\right) - \frac{\Gamma\left(\frac{9}{2}, ax\right)}{3a^3}$$

input
```
Int[x^2*Gamma[3/2, a*x],x]
```

output
```
(x^3*Gamma[3/2, a*x])/3 - Gamma[9/2, a*x]/(3*a^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.45

method	result
derivativedivides	$\frac{8\sqrt{\pi}\,\operatorname{erfc}\left(\sqrt{xa}\right)x^3a^3-56(xa)^{\frac{5}{2}}\mathrm{e}^{-xa}-140(xa)^{\frac{3}{2}}\mathrm{e}^{-xa}-105\sqrt{\pi}\,\operatorname{erfc}\left(\sqrt{xa}\right)-210\sqrt{xa}\,\mathrm{e}^{-xa}}{48a^3}$
default	$\frac{\sqrt{\pi}\left(\frac{x^3a^3\operatorname{erfc}\left(\sqrt{xa}\right)}{6}+\frac{-\frac{(xa)^{\frac{5}{2}}\mathrm{e}^{-xa}}{2}-\frac{5(xa)^{\frac{3}{2}}\mathrm{e}^{-xa}}{4}-\frac{15\sqrt{xa}\,\mathrm{e}^{-xa}}{8}+\frac{15\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{16}}{3\sqrt{\pi}}\right)-(xa)^{\frac{5}{2}}\mathrm{e}^{-xa}-\frac{5(xa)^{\frac{3}{2}}\mathrm{e}^{-xa}}{2}-\frac{15\sqrt{xa}}{4}}{a^3}$
parts	$\frac{-(xa)^{\frac{5}{2}}\mathrm{e}^{-xa}-\frac{5(xa)^{\frac{3}{2}}\mathrm{e}^{-xa}}{2}-\frac{15\sqrt{xa}\,\mathrm{e}^{-xa}}{4}+\frac{15\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{8}}{a^3}+\frac{\sqrt{\pi}\left(\frac{x^3a^3\operatorname{erfc}\left(\sqrt{xa}\right)}{6}+\frac{-\frac{(xa)^{\frac{5}{2}}\mathrm{e}^{-xa}}{2}-\frac{5(xa)^{\frac{3}{2}}\mathrm{e}^{-xa}}{4}-15\sqrt{}}{3\sqrt{\pi}}\right.}{a^3}$

input
```
int(x^2*((x*a)^(1/2)*exp(-x*a)+1/2*Pi^(1/2)*erfc((x*a)^(1/2))),x,method=_R
ETURNVERBOSE)
```

output
```
1/48*(8*Pi^(1/2)*erfc((x*a)^(1/2))*x^3*a^3-56*(x*a)^(5/2)*exp(-x*a)-140*(x
*a)^(3/2)*exp(-x*a)-105*Pi^(1/2)*erfc((x*a)^(1/2))-210*(x*a)^(1/2)*exp(-x*
a))/a^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.21

$$\int x^2\Gamma\left(\frac{3}{2},ax\right)\,dx = \frac{8\sqrt{\pi}a^3x^3-\sqrt{\pi}(8\,a^3x^3-105)\operatorname{erf}\left(\sqrt{ax}\right)-14\,(4\,a^2x^2+10\,ax+15)\sqrt{ax}e^{(-ax)}}{48\,a^3}$$

input
```
integrate(x^2*((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2))),x, al
gorithm="fricas")
```

output
```
1/48*(8*sqrt(pi)*a^3*x^3 - sqrt(pi)*(8*a^3*x^3 - 105)*erf(sqrt(a*x)) - 14*
(4*a^2*x^2 + 10*a*x + 15)*sqrt(a*x)*e^(-a*x))/a^3
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.69

$$\int x^2\Gamma\left(\frac{3}{2},ax\right)\,dx$$
$$=\begin{cases}\frac{\sqrt{\pi}x^3\operatorname{erfc}\left(\sqrt{ax}\right)}{6}-\frac{7x^2\sqrt{ax}e^{-ax}}{6a}-\frac{35x\sqrt{ax}e^{-ax}}{12a^2}-\frac{35\sqrt{ax}e^{-ax}}{8a^3}-\frac{35\sqrt{\pi}\operatorname{erfc}\left(\sqrt{ax}\right)}{16a^3} & \text{for } a\neq 0\\ \frac{\sqrt{\pi}x^3}{6} & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*((a*x)**(1/2)*exp(-a*x)+1/2*pi**(1/2)*erfc((a*x)**(1/2))),x
)
```

output
```
Piecewise((sqrt(pi)*x**3*erfc(sqrt(a*x))/6 - 7*x**2*sqrt(a*x)*exp(-a*x)/(6
*a) - 35*x*sqrt(a*x)*exp(-a*x)/(12*a**2) - 35*sqrt(a*x)*exp(-a*x)/(8*a**3)
 - 35*sqrt(pi)*erfc(sqrt(a*x))/(16*a**3), Ne(a, 0)), (sqrt(pi)*x**3/6, Tru
e))
```

Maxima [F]

$$\int x^2\Gamma\left(\frac{3}{2},ax\right)\,dx=\int\frac{1}{2}\left(\sqrt{\pi}\operatorname{erfc}\left(\sqrt{ax}\right)+2\sqrt{ax}e^{(-ax)}\right)x^2\,dx$$

input
```
integrate(x^2*((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2))),x, al
gorithm="maxima")
```

output
```
1/2*integrate((sqrt(pi)*erfc(sqrt(a*x)) + 2*sqrt(a*x)*e^(-a*x))*x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 5.00

$$\int x^2\Gamma\left(\frac{3}{2},ax\right)\,dx$$
$$=\frac{8\sqrt{\pi}ax^3-\frac{\sqrt{\pi}\left(8a^3x^3\operatorname{erf}\left(\sqrt{ax}\right)+\frac{2\left(4\sqrt{ax}a^2x^2+10\sqrt{ax}ax+15\sqrt{ax}\right)e^{(-ax)}-15\sqrt{\pi}\operatorname{erf}\left(\sqrt{ax}\right)}{\sqrt{\pi}}\right)}{a^2}-\frac{6\left(2\left(4\sqrt{ax}a^2x^2+10\sqrt{ax}ax+15\sqrt{ax}\right)e^{(-c}}{a^2}}{48a}$$

input
```
integrate(x^2*((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2))),x, al
gorithm="giac")
```

output
```
1/48*(8*sqrt(pi)*a*x^3 - sqrt(pi)*(8*a^3*x^3*erf(sqrt(a*x)) + (2*(4*sqrt(a
*x)*a^2*x^2 + 10*sqrt(a*x)*a*x + 15*sqrt(a*x))*e^(-a*x) - 15*sqrt(pi)*erf(
sqrt(a*x)))/sqrt(pi))/a^2 - 6*(2*(4*sqrt(a*x)*a^2*x^2 + 10*sqrt(a*x)*a*x +
 15*sqrt(a*x))*e^(-a*x) - 15*sqrt(pi)*erf(sqrt(a*x)))/a^2)/a
```

Mupad [F(-1)]

Timed out.

$$\int x^2\Gamma\left(\frac{3}{2},ax\right)\,dx=\int x^2\left(\frac{\sqrt{\Pi}\operatorname{erfc}\left(\sqrt{a\,x}\right)}{2}+\mathrm{e}^{-a\,x}\sqrt{a\,x}\right)\,dx$$

input
```
int(x^2*((Pi^(1/2)*erfc((a*x)^(1/2)))/2 + exp(-a*x)*(a*x)^(1/2)),x)
```

output
```
int(x^2*((Pi^(1/2)*erfc((a*x)^(1/2)))/2 + exp(-a*x)*(a*x)^(1/2)), x)
```

Reduce [F]

$$\int x^2\Gamma\left(\frac{3}{2},ax\right)\,dx$$
$$=\frac{-8\sqrt{\pi}\,e^{ax}\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right)a^3x^3+15\sqrt{\pi}\,e^{ax}\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right)+90e^{ax}\sqrt{a}\left(\int\frac{\sqrt{x}}{e^{ax}x}dx\right)-56\sqrt{x}\,\sqrt{a}\,a^2x^2-140\sqrt{x}\,\sqrt{}}{48e^{ax}a^3}$$

input
```
int(x^2*((a*x)^(1/2)*exp(-a*x)+1/2*Pi^(1/2)*erfc((a*x)^(1/2))),x)
```

output
```
( - 8*sqrt(pi)*e**(a*x)*erf(sqrt(x)*sqrt(a))*a**3*x**3 + 15*sqrt(pi)*e**(a
*x)*erf(sqrt(x)*sqrt(a)) + 90*e**(a*x)*sqrt(a)*int(sqrt(x)/(e**(a*x)*x),x)
 - 56*sqrt(x)*sqrt(a)*a**2*x**2 - 140*sqrt(x)*sqrt(a)*a*x - 210*sqrt(x)*sq
rt(a) + 8*sqrt(pi)*e**(a*x)*a**3*x**3)/(48*e**(a*x)*a**3)
```

3.70 $\int x\Gamma\left(\frac{3}{2},ax\right) dx$

Optimal result

Integrand size = 9, antiderivative size = 29

$$\int x\Gamma\left(\frac{3}{2},ax\right) dx = \frac{1}{2}x^2\Gamma\left(\frac{3}{2},ax\right) - \frac{\Gamma\left(\frac{7}{2},ax\right)}{2a^2}$$

output
```
1/2*x^2*((a*x)^(1/2)*exp(-a*x)+1/2*Pi^(1/2)*erfc((a*x)^(1/2)))-1/2*((a*x)^
(5/2)*exp(-a*x)+5/2*(a*x)^(3/2)*exp(-a*x)+15/4*(a*x)^(1/2)*exp(-a*x)+15/8*
Pi^(1/2)*erfc((a*x)^(1/2)))/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x\Gamma\left(\frac{3}{2},ax\right) dx = \frac{1}{2}x^2\Gamma\left(\frac{3}{2},ax\right) - \frac{\Gamma\left(\frac{7}{2},ax\right)}{2a^2}$$

input
```
Integrate[x*Gamma[3/2, a*x],x]
```

output
```
(x^2*Gamma[3/2, a*x])/2 - Gamma[7/2, a*x]/(2*a^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\Gamma\left(\frac{3}{2},ax\right)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{2}x^2\Gamma\left(\frac{3}{2},ax\right)-\frac{\Gamma\left(\frac{7}{2},ax\right)}{2a^2}$$

input
```
Int[x*Gamma[3/2, a*x],x]
```

output
```
(x^2*Gamma[3/2, a*x])/2 - Gamma[7/2, a*x]/(2*a^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.03

method	result	si
derivativedivides	$\frac{4\sqrt{\pi}\,\operatorname{erfc}\left(\sqrt{xa}\right)x^{2}a^{2}-20(xa)^{\frac{3}{2}}\mathrm{e}^{-xa}-15\sqrt{\pi}\,\operatorname{erfc}\left(\sqrt{xa}\right)-30\sqrt{xa}\,\mathrm{e}^{-xa}}{16a^{2}}$	5
default	$\frac{\sqrt{\pi}\left(\frac{x^{2}a^{2}\operatorname{erfc}\left(\sqrt{xa}\right)}{4}+\frac{-\frac{(xa)^{\frac{3}{2}}\mathrm{e}^{-xa}}{2}-\frac{3\sqrt{xa}\,\mathrm{e}^{-xa}}{4}+\frac{3\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{8}}{2\sqrt{\pi}}\right)-(xa)^{\frac{3}{2}}\mathrm{e}^{-xa}-\frac{3\sqrt{xa}\,\mathrm{e}^{-xa}}{2}+\frac{3\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{4}}{a^{2}}$	1
parts	$\frac{-(xa)^{\frac{3}{2}}\mathrm{e}^{-xa}-\frac{3\sqrt{xa}\,\mathrm{e}^{-xa}}{2}+\frac{3\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{4}}{a^{2}}+\frac{\sqrt{\pi}\left(\frac{x^{2}a^{2}\operatorname{erfc}\left(\sqrt{xa}\right)}{4}+\frac{-\frac{(xa)^{\frac{3}{2}}\mathrm{e}^{-xa}}{2}-\frac{3\sqrt{xa}\,\mathrm{e}^{-xa}}{4}+\frac{3\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{8}}{2\sqrt{\pi}}\right)}{a^{2}}$	1

input
```
int(x*((x*a)^(1/2)*exp(-x*a)+1/2*Pi^(1/2)*erfc((x*a)^(1/2))),x,method=_RET
URNVERBOSE)
```

output
```
1/16*(4*Pi^(1/2)*erfc((x*a)^(1/2))*x^2*a^2-20*(x*a)^(3/2)*exp(-x*a)-15*Pi^
(1/2)*erfc((x*a)^(1/2))-30*(x*a)^(1/2)*exp(-x*a))/a^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.93

$$\int x\Gamma\left(\frac{3}{2},ax\right)\,dx=\frac{4\,\sqrt{\pi}a^{2}x^{2}-\sqrt{\pi}(4\,a^{2}x^{2}-15)\operatorname{erf}\left(\sqrt{ax}\right)-10\,(2\,ax+3)\sqrt{ax}e^{(-ax)}}{16\,a^{2}}$$

input
```
integrate(x*((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2))),x, algo
rithm="fricas")
```

output
```
1/16*(4*sqrt(pi)*a^2*x^2 - sqrt(pi)*(4*a^2*x^2 - 15)*erf(sqrt(a*x)) - 10*(
2*a*x + 3)*sqrt(a*x)*e^(-a*x))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.93

$$\int x\Gamma\left(\frac{3}{2},ax\right)\,dx=\begin{cases}\frac{\sqrt{\pi}x^2\operatorname{erfc}\left(\sqrt{ax}\right)}{4}-\frac{5x\sqrt{ax}e^{-ax}}{4a}-\frac{15\sqrt{ax}e^{-ax}}{8a^2}-\frac{15\sqrt{\pi}\operatorname{erfc}\left(\sqrt{ax}\right)}{16a^2} & \text{for } a\neq 0\\ \frac{\sqrt{\pi}x^2}{4} & \text{otherwise}\end{cases}$$

input
```
integrate(x*((a*x)**(1/2)*exp(-a*x)+1/2*pi**(1/2)*erfc((a*x)**(1/2))),x)
```

output
```
Piecewise((sqrt(pi)*x**2*erfc(sqrt(a*x))/4 - 5*x*sqrt(a*x)*exp(-a*x)/(4*a)
 - 15*sqrt(a*x)*exp(-a*x)/(8*a**2) - 15*sqrt(pi)*erfc(sqrt(a*x))/(16*a**2)
, Ne(a, 0)), (sqrt(pi)*x**2/4, True))
```

Maxima [F]

$$\int x\Gamma\left(\frac{3}{2},ax\right)\,dx=\int\frac{1}{2}\left(\sqrt{\pi}\operatorname{erfc}\left(\sqrt{ax}\right)+2\sqrt{ax}e^{(-ax)}\right)x\,dx$$

input
```
integrate(x*((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2))),x, algo
rithm="maxima")
```

output
```
1/2*integrate((sqrt(pi)*erfc(sqrt(a*x)) + 2*sqrt(a*x)*e^(-a*x))*x, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 4.10

$$\int x\Gamma\left(\frac{3}{2},ax\right)\,dx$$
$$=\frac{4\sqrt{\pi}ax^2-\frac{\sqrt{\pi}\left(4a^2x^2\operatorname{erf}\left(\sqrt{ax}\right)+\frac{2\left(2\sqrt{ax}ax+3\sqrt{ax}\right)e^{(-ax)}-3\sqrt{\pi}\operatorname{erf}\left(\sqrt{ax}\right)}{\sqrt{\pi}}\right)}{a}-\frac{4\left(2\left(2\sqrt{ax}ax+3\sqrt{ax}\right)e^{(-ax)}-3\sqrt{\pi}\operatorname{erf}\left(\sqrt{ax}\right)\right)}{a}}{16a}$$

input
```
integrate(x*((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2))),x, algo
rithm="giac")
```

output
```
1/16*(4*sqrt(pi)*a*x^2 - sqrt(pi)*(4*a^2*x^2*erf(sqrt(a*x)) + (2*(2*sqrt(a
*x)*a*x + 3*sqrt(a*x))*e^(-a*x) - 3*sqrt(pi)*erf(sqrt(a*x)))/sqrt(pi))/a -
 4*(2*(2*sqrt(a*x)*a*x + 3*sqrt(a*x))*e^(-a*x) - 3*sqrt(pi)*erf(sqrt(a*x))
)/a)/a
```

Mupad [F(-1)]

Timed out.

$$\int x\Gamma\left(\frac{3}{2},ax\right)\,dx=\int x\left(\frac{\sqrt{\Pi}\,\mathrm{erfc}\left(\sqrt{a\,x}\right)}{2}+\mathrm{e}^{-a\,x}\,\sqrt{a\,x}\right)\,dx$$

input
```
int(x*((Pi^(1/2)*erfc((a*x)^(1/2)))/2 + exp(-a*x)*(a*x)^(1/2)),x)
```

output
```
int(x*((Pi^(1/2)*erfc((a*x)^(1/2)))/2 + exp(-a*x)*(a*x)^(1/2)), x)
```

Reduce [F]

$$\int x\Gamma\left(\frac{3}{2},ax\right)\,dx$$
$$=\frac{-4\sqrt{\pi}\,e^{ax}\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right)a^2x^2+3\sqrt{\pi}\,e^{ax}\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right)+12e^{ax}\sqrt{a}\left(\int\frac{\sqrt{x}}{e^{ax}x}dx\right)-20\sqrt{x}\,\sqrt{a}\,ax-30\sqrt{x}\,\sqrt{a}+}{16e^{ax}a^2}$$

input
```
int(x*((a*x)^(1/2)*exp(-a*x)+1/2*Pi^(1/2)*erfc((a*x)^(1/2))),x)
```

output
```
( - 4*sqrt(pi)*e**(a*x)*erf(sqrt(x)*sqrt(a))*a**2*x**2 + 3*sqrt(pi)*e**(a*
x)*erf(sqrt(x)*sqrt(a)) + 12*e**(a*x)*sqrt(a)*int(sqrt(x)/(e**(a*x)*x),x)
- 20*sqrt(x)*sqrt(a)*a*x - 30*sqrt(x)*sqrt(a) + 4*sqrt(pi)*e**(a*x)*a**2*x
**2)/(16*e**(a*x)*a**2)
```

3.71 $\int \Gamma\left(\frac{3}{2}, ax\right) dx$

Optimal result

Integrand size = 7, antiderivative size = 22

$$\int \Gamma\left(\frac{3}{2}, ax\right) dx = x\Gamma\left(\frac{3}{2}, ax\right) - \frac{\Gamma\left(\frac{5}{2}, ax\right)}{a}$$

output
```
x*((a*x)^(1/2)*exp(-a*x)+1/2*Pi^(1/2)*erfc((a*x)^(1/2)))-((a*x)^(3/2)*exp(
-a*x)+3/2*(a*x)^(1/2)*exp(-a*x)+3/4*Pi^(1/2)*erfc((a*x)^(1/2)))/a
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 54 vs. 2(22) = 44.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \Gamma\left(\frac{3}{2}, ax\right) dx = \frac{-2e^{-ax}\sqrt{ax}(3 + 2ax) + 3\sqrt{\pi}\operatorname{erf}\left(\sqrt{ax}\right)}{4a} + x\Gamma\left(\frac{3}{2}, ax\right)$$

input
```
Integrate[Gamma[3/2, a*x],x]
```

output
```
((-2*Sqrt[a*x]*(3 + 2*a*x))/E^(a*x) + 3*Sqrt[Pi]*Erf[Sqrt[a*x]])/(4*a) + x
*Gamma[3/2, a*x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma\left(\frac{3}{2},ax\right)dx$$

$$\Big\downarrow \; 7111$$

$$x\Gamma\left(\frac{3}{2},ax\right)-\frac{\Gamma\left(\frac{5}{2},ax\right)}{a}$$

input
```
Int[Gamma[3/2, a*x],x]
```

output
```
x*Gamma[3/2, a*x] - Gamma[5/2, a*x]/a
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.95

method	result	size
derivativedivides	$\frac{2\sqrt{\pi}\,xa\,\operatorname{erfc}\left(\sqrt{xa}\right)-3\sqrt{\pi}\,\operatorname{erfc}\left(\sqrt{xa}\right)-6\sqrt{xa}\,\mathrm{e}^{-xa}}{4a}$	43
default	$\frac{\sqrt{\pi}\left(\frac{xa\,\operatorname{erfc}\left(\sqrt{xa}\right)}{2}+\frac{-\frac{\sqrt{xa}\,\mathrm{e}^{-xa}}{2}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{4}}{\sqrt{\pi}}\right)}{a}+\frac{-\sqrt{xa}\,\mathrm{e}^{-xa}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{2}}{a}$	79
parts	$\frac{\sqrt{\pi}\left(\frac{xa\,\operatorname{erfc}\left(\sqrt{xa}\right)}{2}+\frac{-\frac{\sqrt{xa}\,\mathrm{e}^{-xa}}{2}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{4}}{\sqrt{\pi}}\right)}{a}+\frac{-\sqrt{xa}\,\mathrm{e}^{-xa}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{2}}{a}$	79

input
```
int((x*a)^(1/2)*exp(-x*a)+1/2*Pi^(1/2)*erfc((x*a)^(1/2)),x,method=_RETURNV
ERBOSE)
```

output
```
1/4*(2*Pi^(1/2)*x*a*erfc((x*a)^(1/2))-3*Pi^(1/2)*erfc((x*a)^(1/2))-6*(x*a)
^(1/2)*exp(-x*a))/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \Gamma\left(\frac{3}{2},ax\right)\,dx=\frac{2\,\sqrt{\pi}ax-\sqrt{\pi}(2\,ax-3)\,\operatorname{erf}\left(\sqrt{ax}\right)-6\,\sqrt{ax}e^{(-ax)}}{4\,a}$$

input
```
integrate((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2)),x, algorith
m="fricas")
```

output
```
1/4*(2*sqrt(pi)*a*x - sqrt(pi)*(2*a*x - 3)*erf(sqrt(a*x)) - 6*sqrt(a*x)*e^
(-a*x))/a
```

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.77

$$\int \Gamma\left(\frac{3}{2},ax\right)\,dx = \frac{\sqrt{\pi}\left(\begin{cases} x\operatorname{erfc}\left(\sqrt{ax}\right) - \frac{\sqrt{ax}e^{-ax}}{\sqrt{\pi}a} - \frac{\operatorname{erfc}\left(\sqrt{ax}\right)}{2a} & \text{for}\ a\neq 0 \\ x & \text{otherwise} \end{cases}\right)}{2} + \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{a}\sqrt{x}\right)}{2a} - \frac{\sqrt{x}e^{-ax}}{\sqrt{a}}$$

input
```
integrate((a*x)**(1/2)*exp(-a*x)+1/2*pi**(1/2)*erfc((a*x)**(1/2)),x)
```

output
```
sqrt(pi)*Piecewise((x*erfc(sqrt(a*x)) - sqrt(a*x)*exp(-a*x)/(sqrt(pi)*a) -
 erfc(sqrt(a*x))/(2*a), Ne(a, 0)), (x, True))/2 + sqrt(pi)*erf(sqrt(a)*sqr
t(x))/(2*a) - sqrt(x)*exp(-a*x)/sqrt(a)
```

Maxima [F]

$$\int \Gamma\left(\frac{3}{2},ax\right)\,dx = \int \frac{1}{2}\sqrt{\pi}\operatorname{erfc}\left(\sqrt{ax}\right) + \sqrt{ax}e^{(-ax)}\,dx$$

input
```
integrate((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2)),x, algorith
m="maxima")
```

output
```
1/2*sqrt(pi)*integrate(erfc(sqrt(a*x)), x) + 1/2*(sqrt(pi)*erf(sqrt(a*x))
- 2*sqrt(a*x)*e^(-a*x))/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.73

$$\int \Gamma\left(\frac{3}{2},ax\right)\,dx = \frac{1}{4}\sqrt{\pi}\left(2\,x - \frac{2\,ax\,\mathrm{erf}\left(\sqrt{ax}\right) - \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{ax}\right)-2\,\sqrt{ax}e^{(-ax)}}{\sqrt{\pi}}}{a}\right) + \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{ax}\right) - 2\,\sqrt{ax}e^{(-ax)}}{2\,a}$$

input
```
integrate((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2)),x, algorith
m="giac")
```

output
```
1/4*sqrt(pi)*(2*x - (2*a*x*erf(sqrt(a*x)) - (sqrt(pi)*erf(sqrt(a*x)) - 2*s
qrt(a*x)*e^(-a*x))/sqrt(pi))/a) + 1/2*(sqrt(pi)*erf(sqrt(a*x)) - 2*sqrt(a*
x)*e^(-a*x))/a
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.50

$$\int \Gamma\left(\frac{3}{2},ax\right)\,dx = \frac{\sqrt{\Pi}\,x\,\mathrm{erfc}\left(\sqrt{a\,x}\right)}{2} - \frac{\mathrm{e}^{-a\,x}\,\sqrt{a\,x}}{a} - \frac{\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{a\,x}\right)}{2\,a} - \frac{\sqrt{\Pi}\,\mathrm{erfc}\left(\sqrt{a\,x}\right)}{4\,a} - \frac{\sqrt{\Pi}\,\mathrm{e}^{-a\,x}\,\sqrt{a\,x}}{2\,a\,\sqrt{\pi}}$$

input
```
int((Pi^(1/2)*erfc((a*x)^(1/2)))/2 + exp(-a*x)*(a*x)^(1/2),x)
```

output
```
(Pi^(1/2)*x*erfc((a*x)^(1/2)))/2 - (exp(-a*x)*(a*x)^(1/2))/a - (pi^(1/2)*e
rfc((a*x)^(1/2)))/(2*a) - (Pi^(1/2)*erfc((a*x)^(1/2)))/(4*a) - (Pi^(1/2)*e
xp(-a*x)*(a*x)^(1/2))/(2*a*pi^(1/2))
```

Reduce [F]

$$\int \Gamma\left(\frac{3}{2}, ax\right) dx$$
$$= \frac{-2\sqrt{\pi}\, e^{ax} \operatorname{erf}\left(\sqrt{x}\,\sqrt{a}\right) ax + \sqrt{\pi}\, e^{ax} \operatorname{erf}\left(\sqrt{x}\,\sqrt{a}\right) + 2e^{ax}\sqrt{a}\left(\int \frac{\sqrt{x}}{e^{ax}x} dx\right) - 6\sqrt{x}\,\sqrt{a} + 2\sqrt{\pi}\, e^{ax} ax}{4e^{ax}a}$$

input
```
int((a*x)^(1/2)*exp(-a*x)+1/2*Pi^(1/2)*erfc((a*x)^(1/2)),x)
```

output
```
( - 2*sqrt(pi)*e**(a*x)*erf(sqrt(x)*sqrt(a))*a*x + sqrt(pi)*e**(a*x)*erf(s
qrt(x)*sqrt(a)) + 2*e**(a*x)*sqrt(a)*int(sqrt(x)/(e**(a*x)*x),x) - 6*sqrt(
x)*sqrt(a) + 2*sqrt(pi)*e**(a*x)*a*x)/(4*e**(a*x)*a)
```

3.72 $\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x}\,dx = -\frac{4}{9}(ax)^{3/2}\,{}_2F_2\left(\frac{3}{2},\frac{3}{2};\frac{5}{2},\frac{5}{2};-ax\right)+\frac{1}{2}\sqrt{\pi}\log(x)$$

output
```
-4/9*(a*x)^(3/2)*hypergeom([3/2, 3/2],[5/2, 5/2],-a*x)+1/2*Pi^(1/2)*ln(x)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x}\,dx = -\frac{4}{9}(ax)^{3/2}\,{}_2F_2\left(\frac{3}{2},\frac{3}{2};\frac{5}{2},\frac{5}{2};-ax\right) + \frac{1}{2}\left(-2e^{-ax}\sqrt{ax}+\sqrt{\pi}\operatorname{erf}\left(\sqrt{ax}\right)+2\Gamma\left(\frac{3}{2},ax\right)\right)\log(ax)$$

input
```
Integrate[Gamma[3/2, a*x]/x,x]
```

output
```
(-4*(a*x)^(3/2)*HypergeometricPFQ[{3/2, 3/2}, {5/2, 5/2}, -(a*x)])/9 + (((
-2*Sqrt[a*x])/E^(a*x) + Sqrt[Pi]*Erf[Sqrt[a*x]] + 2*Gamma[3/2, a*x])*Log[a
*x])/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7115}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x}\,dx$$

$$\Big\downarrow \text{7115}$$

$$\frac{1}{2}\sqrt{\pi}\log(x)-\frac{4}{9}(ax)^{3/2}\,{}_2F_2\left(\frac{3}{2},\frac{3}{2};\frac{5}{2},\frac{5}{2};-ax\right)$$

input
```
Int[Gamma[3/2, a*x]/x,x]
```

output
```
(-4*(a*x)^(3/2)*HypergeometricPFQ[{3/2, 3/2}, {5/2, 5/2}, -(a*x)])/9 + (Sq
rt[Pi]*Log[x])/2
```

Defintions of rubi rules used

rule 7115
```
Int[Gamma[n_, (b_.)*(x_)]/(x_), x_Symbol] :> Simp[Gamma[n]*Log[x], x] - Sim
p[((b*x)^n/n^2)*HypergeometricPFQ[{n, n}, {1 + n, 1 + n}, (-b)*x], x] /; Fr
eeQ[{b, n}, x] && !IntegerQ[n]
```

Maple [F]

$$\int \frac{\sqrt{xa}\,\mathrm{e}^{-xa}+\frac{\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{xa}\right)}{2}}{x}dx$$

input
```
int(((x*a)^(1/2)*exp(-x*a)+1/2*Pi^(1/2)*erfc((x*a)^(1/2)))/x,x)
```

output
```
int(((x*a)^(1/2)*exp(-x*a)+1/2*Pi^(1/2)*erfc((x*a)^(1/2)))/x,x)
```

Fricas [F]

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x}\,dx=\int \frac{\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{ax}\right)+2\,\sqrt{ax}e^{(-ax)}}{2\,x}\,dx$$

input
```
integrate(((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2)))/x,x, algo
rithm="fricas")
```

output
```
integral(-1/2*(sqrt(pi)*erf(sqrt(a*x)) - 2*sqrt(a*x)*e^(-a*x) - sqrt(pi))/
x, x)
```

Sympy [F]

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x}\,dx=\frac{\int \frac{\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{ax}\right)}{x}\,dx+\int \frac{2\sqrt{ax}e^{-ax}}{x}\,dx}{2}$$

input
```
integrate(((a*x)**(1/2)*exp(-a*x)+1/2*pi**(1/2)*erfc((a*x)**(1/2)))/x,x)
```

output
```
(Integral(sqrt(pi)*erfc(sqrt(a*x))/x, x) + Integral(2*sqrt(a*x)*exp(-a*x)/
x, x))/2
```

Maxima [F]

$$\int \frac{\Gamma\left(\frac{3}{2}, ax\right)}{x}\, dx = \int \frac{\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{ax}\right) + 2\sqrt{ax}e^{(-ax)}}{2\,x}\, dx$$

input
```
integrate(((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2)))/x,x, algo
rithm="maxima")
```

output
```
1/2*integrate((sqrt(pi)*erfc(sqrt(a*x)) + 2*sqrt(a*x)*e^(-a*x))/x, x)
```

Giac [F]

$$\int \frac{\Gamma\left(\frac{3}{2}, ax\right)}{x}\, dx = \int \frac{\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{ax}\right) + 2\sqrt{ax}e^{(-ax)}}{2\,x}\, dx$$

input
```
integrate(((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2)))/x,x, algo
rithm="giac")
```

output
```
integrate(1/2*(sqrt(pi)*erfc(sqrt(a*x)) + 2*sqrt(a*x)*e^(-a*x))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma\left(\frac{3}{2}, ax\right)}{x}\, dx = \int \frac{\frac{\sqrt{\Pi}\,\mathrm{erfc}\left(\sqrt{a\,x}\right)}{2} + \mathrm{e}^{-a\,x}\,\sqrt{a\,x}}{x}\, dx$$

input
```
int(((Pi^(1/2)*erfc((a*x)^(1/2)))/2 + exp(-a*x)*(a*x)^(1/2))/x,x)
```

output
```
int(((Pi^(1/2)*erfc((a*x)^(1/2)))/2 + exp(-a*x)*(a*x)^(1/2))/x, x)
```

Reduce [F]

$$\int \frac{\Gamma\left(\frac{3}{2}, ax\right)}{x}\,dx = \frac{\sqrt{\pi}\left(2\,\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right) - \left(\int \frac{\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right)}{x}dx\right) + \log(x)\right)}{2}$$

input
```
int(((a*x)^(1/2)*exp(-a*x)+1/2*Pi^(1/2)*erfc((a*x)^(1/2)))/x,x)
```

output
```
(sqrt(pi)*(2*erf(sqrt(x)*sqrt(a)) - int(erf(sqrt(x)*sqrt(a))/x,x) + log(x)
))/2
```

3.73 $\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^2}\,dx = a\Gamma\left(\frac{1}{2},ax\right) - \frac{\Gamma\left(\frac{3}{2},ax\right)}{x}$$

output
```
a*Pi^(1/2)*erfc((a*x)^(1/2))-((a*x)^(1/2)*exp(-a*x)+1/2*Pi^(1/2)*erfc((a*x
)^(1/2)))/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^2}\,dx = a\Gamma\left(\frac{1}{2},ax\right) - \frac{\Gamma\left(\frac{3}{2},ax\right)}{x}$$

input
```
Integrate[Gamma[3/2, a*x]/x^2,x]
```

output
```
a*Gamma[1/2, a*x] - Gamma[3/2, a*x]/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^2}\,dx$$

$$\Big\downarrow \text{ 7116}$$

$$a\Gamma\left(\frac{1}{2},ax\right)-\frac{\Gamma\left(\frac{3}{2},ax\right)}{x}$$

input
```
Int[Gamma[3/2, a*x]/x^2,x]
```

output
```
a*Gamma[1/2, a*x] - Gamma[3/2, a*x]/x
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

method	result	size
derivativedivides	$\frac{2\sqrt{\pi}\,a^{2}x\,\operatorname{erfc}\left(\sqrt{xa}\right)-a\sqrt{\pi}\,\operatorname{erfc}\left(\sqrt{xa}\right)-2\sqrt{xa}\,a\,\mathrm{e}^{-xa}}{2xa}$	50
default	$a\left(\sqrt{\pi}\left(-\frac{\operatorname{erfc}\left(\sqrt{xa}\right)}{2xa}-\frac{-\frac{\mathrm{e}^{-xa}}{\sqrt{xa}}-\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{\sqrt{\pi}}\right)-\frac{2\,\mathrm{e}^{-xa}}{\sqrt{xa}}-2\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)\right)$	77
parts	$2a\left(-\frac{\mathrm{e}^{-xa}}{\sqrt{xa}}-\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)\right)+\sqrt{\pi}\,a\left(-\frac{\operatorname{erfc}\left(\sqrt{xa}\right)}{2xa}-\frac{-\frac{\mathrm{e}^{-xa}}{\sqrt{xa}}-\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{\sqrt{\pi}}\right)$	80

input
```
int(((x*a)^(1/2)*exp(-x*a)+1/2*Pi^(1/2)*erfc((x*a)^(1/2)))/x^2,x,method=_R
ETURNVERBOSE)
```

output
```
1/2*(2*Pi^(1/2)*a^2*x*erfc((x*a)^(1/2))-a*Pi^(1/2)*erfc((x*a)^(1/2))-2*(x*
a)^(1/2)*a*exp(-x*a))/x/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^2}\,dx=-\frac{\sqrt{\pi}(2\,ax-1)\operatorname{erf}\left(\sqrt{ax}\right)+2\,\sqrt{ax}e^{(-ax)}+\sqrt{\pi}}{2\,x}$$

input
```
integrate(((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2)))/x^2,x, al
gorithm="fricas")
```

output
```
-1/2*(sqrt(pi)*(2*a*x - 1)*erf(sqrt(a*x)) + 2*sqrt(a*x)*e^(-a*x) + sqrt(pi
))/x
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{\Gamma\left(\frac{3}{2}, ax\right)}{x^2} \, dx = \sqrt{\pi} a \operatorname{erfc}\left(\sqrt{ax}\right) - \frac{\sqrt{ax} e^{-ax}}{x} - \frac{\sqrt{\pi} \operatorname{erfc}\left(\sqrt{ax}\right)}{2x}$$

input
```
integrate(((a*x)**(1/2)*exp(-a*x)+1/2*pi**(1/2)*erfc((a*x)**(1/2)))/x**2,x
)
```

output
```
sqrt(pi)*a*erfc(sqrt(a*x)) - sqrt(a*x)*exp(-a*x)/x - sqrt(pi)*erfc(sqrt(a*
x))/(2*x)
```

Maxima [F]

$$\int \frac{\Gamma\left(\frac{3}{2}, ax\right)}{x^2} \, dx = \int \frac{\sqrt{\pi} \operatorname{erfc}\left(\sqrt{ax}\right) + 2 \sqrt{ax} e^{(-ax)}}{2 x^2} \, dx$$

input
```
integrate(((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2)))/x^2,x, al
gorithm="maxima")
```

output
```
1/2*integrate((sqrt(pi)*erfc(sqrt(a*x)) + 2*sqrt(a*x)*e^(-a*x))/x^2, x)
```

Giac [F]

$$\int \frac{\Gamma\left(\frac{3}{2}, ax\right)}{x^2} \, dx = \int \frac{\sqrt{\pi} \operatorname{erfc}\left(\sqrt{ax}\right) + 2 \sqrt{ax} e^{(-ax)}}{2 x^2} \, dx$$

input
```
integrate(((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2)))/x^2,x, al
gorithm="giac")
```

output
```
integrate(1/2*(sqrt(pi)*erfc(sqrt(a*x)) + 2*sqrt(a*x)*e^(-a*x))/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma\left(\frac{3}{2}, ax\right)}{x^2}\,dx = \int \frac{\frac{\sqrt{\Pi}\,\mathrm{erfc}\left(\sqrt{a\,x}\right)}{2} + \mathrm{e}^{-a\,x}\,\sqrt{a\,x}}{x^2}\,dx$$

input
```
int(((Pi^(1/2)*erfc((a*x)^(1/2)))/2 + exp(-a*x)*(a*x)^(1/2))/x^2,x)
```

output
```
int(((Pi^(1/2)*erfc((a*x)^(1/2)))/2 + exp(-a*x)*(a*x)^(1/2))/x^2, x)
```

Reduce [F]

$$\int \frac{\Gamma\left(\frac{3}{2}, ax\right)}{x^2}\,dx = \frac{2\sqrt{\pi}\,e^{ax}\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right)ax + \sqrt{\pi}\,e^{ax}\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right) + 2e^{ax}\sqrt{a}\left(\int \frac{\sqrt{x}}{e^{ax}x^2}dx\right)x + 2\sqrt{x}\,\sqrt{a} - \sqrt{\pi}\,e^{ax}}{2e^{ax}x}$$

input
```
int(((a*x)^(1/2)*exp(-a*x)+1/2*Pi^(1/2)*erfc((a*x)^(1/2)))/x^2,x)
```

output
```
(2*sqrt(pi)*e**(a*x)*erf(sqrt(x)*sqrt(a))*a*x + sqrt(pi)*e**(a*x)*erf(sqrt
(x)*sqrt(a)) + 2*e**(a*x)*sqrt(a)*int(sqrt(x)/(e**(a*x)*x**2),x)*x + 2*sqr
t(x)*sqrt(a) - sqrt(pi)*e**(a*x))/(2*e**(a*x)*x)
```

3.74 $\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^3}\,dx=\frac{1}{2}a^2\Gamma\left(-\frac{1}{2},ax\right)-\frac{\Gamma\left(\frac{3}{2},ax\right)}{2x^2}$$

output
```
1/2*a^2*(-2*Pi^(1/2)*erfc((a*x)^(1/2))+2/(a*x)^(1/2)*exp(-a*x))-1/2*((a*x)
^(1/2)*exp(-a*x)+1/2*Pi^(1/2)*erfc((a*x)^(1/2)))/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^3}\,dx=\frac{1}{2}a^2\Gamma\left(-\frac{1}{2},ax\right)-\frac{\Gamma\left(\frac{3}{2},ax\right)}{2x^2}$$

input
```
Integrate[Gamma[3/2, a*x]/x^3,x]
```

output
```
(a^2*Gamma[-1/2, a*x])/2 - Gamma[3/2, a*x]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^3}\,dx$$

$$\Big\downarrow \ 7116$$

$$\frac{1}{2}a^2\Gamma\left(-\frac{1}{2},ax\right)-\frac{\Gamma\left(\frac{3}{2},ax\right)}{2x^2}$$

input
```
Int[Gamma[3/2, a*x]/x^3,x]
```

output
```
(a^2*Gamma[-1/2, a*x])/2 - Gamma[3/2, a*x]/(2*x^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.41

method	result
derivativedivides	$-\frac{4\sqrt{\pi}\,a^4 \operatorname{erfc}\left(\sqrt{xa}\right)x^2-4a^2\,\mathrm{e}^{-xa}(xa)^{\frac{3}{2}}+\sqrt{\pi}\,a^2 \operatorname{erfc}\left(\sqrt{xa}\right)+2\sqrt{xa}\,a^2\,\mathrm{e}^{-xa}}{4x^2a^2}$
default	$a^2\left(\sqrt{\pi}\left(-\frac{\operatorname{erfc}\left(\sqrt{xa}\right)}{4x^2a^2}-\frac{-\frac{\mathrm{e}^{-xa}}{3(xa)^{\frac{3}{2}}}+\frac{2\,\mathrm{e}^{-xa}}{3\sqrt{xa}}+\frac{2\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{3}}{2\sqrt{\pi}}\right)-\frac{2\,\mathrm{e}^{-xa}}{3(xa)^{\frac{3}{2}}}+\frac{4\,\mathrm{e}^{-xa}}{3\sqrt{xa}}+\frac{4\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{3}\right)$
parts	$2a^2\left(-\frac{\mathrm{e}^{-xa}}{3(xa)^{\frac{3}{2}}}+\frac{2\,\mathrm{e}^{-xa}}{3\sqrt{xa}}+\frac{2\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{3}\right)+\sqrt{\pi}\,a^2\left(-\frac{\operatorname{erfc}\left(\sqrt{xa}\right)}{4x^2a^2}-\frac{-\frac{\mathrm{e}^{-xa}}{3(xa)^{\frac{3}{2}}}+\frac{2\,\mathrm{e}^{-xa}}{3\sqrt{xa}}+\frac{2\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{3}}{2\sqrt{\pi}}\right.$

input
```
int(((x*a)^(1/2)*exp(-x*a)+1/2*Pi^(1/2)*erfc((x*a)^(1/2)))/x^3,x,method=_R
ETURNVERBOSE)
```

output
```
-1/4*(4*Pi^(1/2)*a^4*erfc((x*a)^(1/2))*x^2-4*a^2*exp(-x*a)*(x*a)^(3/2)+Pi^
(1/2)*a^2*erfc((x*a)^(1/2))+2*(x*a)^(1/2)*a^2*exp(-x*a))/x^2/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^3}\,dx=\frac{\sqrt{\pi}(4\,a^2x^2+1)\operatorname{erf}\left(\sqrt{ax}\right)+2\,(2\,ax-1)\sqrt{ax}e^{(-ax)}-\sqrt{\pi}}{4\,x^2}$$

input
```
integrate(((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2)))/x^3,x, al
gorithm="fricas")
```

output
```
1/4*(sqrt(pi)*(4*a^2*x^2 + 1)*erf(sqrt(a*x)) + 2*(2*a*x - 1)*sqrt(a*x)*e^(
-a*x) - sqrt(pi))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.28

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^3}\,dx = -\sqrt{\pi}a^2\,\mathrm{erfc}\left(\sqrt{ax}\right) + \frac{a\sqrt{ax}e^{-ax}}{x} - \frac{\sqrt{ax}e^{-ax}}{2x^2} - \frac{\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{ax}\right)}{4x^2}$$

input
```
integrate(((a*x)**(1/2)*exp(-a*x)+1/2*pi**(1/2)*erfc((a*x)**(1/2)))/x**3,x
)
```

output
```
-sqrt(pi)*a**2*erfc(sqrt(a*x)) + a*sqrt(a*x)*exp(-a*x)/x - sqrt(a*x)*exp(-
a*x)/(2*x**2) - sqrt(pi)*erfc(sqrt(a*x))/(4*x**2)
```

Maxima [F]

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^3}\,dx = \int \frac{\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{ax}\right) + 2\,\sqrt{ax}e^{(-ax)}}{2\,x^3}\,dx$$

input
```
integrate(((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2)))/x^3,x, al
gorithm="maxima")
```

output
```
1/2*integrate((sqrt(pi)*erfc(sqrt(a*x)) + 2*sqrt(a*x)*e^(-a*x))/x^3, x)
```

Giac [F]

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^3}\,dx = \int \frac{\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{ax}\right) + 2\,\sqrt{ax}e^{(-ax)}}{2\,x^3}\,dx$$

input
```
integrate(((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2)))/x^3,x, al
gorithm="giac")
```

output
```
integrate(1/2*(sqrt(pi)*erfc(sqrt(a*x)) + 2*sqrt(a*x)*e^(-a*x))/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^3}\,dx = \int \frac{\frac{\sqrt{\Pi}\,\mathrm{erfc}\left(\sqrt{a\,x}\right)}{2} + \mathrm{e}^{-a\,x}\,\sqrt{a\,x}}{x^3}\,dx$$

input
```
int(((Pi^(1/2)*erfc((a*x)^(1/2)))/2 + exp(-a*x)*(a*x)^(1/2))/x^3,x)
```

output
```
int(((Pi^(1/2)*erfc((a*x)^(1/2)))/2 + exp(-a*x)*(a*x)^(1/2))/x^3, x)
```

Reduce [F]

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^3}\,dx = \frac{\sqrt{x}\,\sqrt{\pi}\,e^{ax}\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right) - 2\sqrt{x}\,e^{ax}\sqrt{a}\left(\int \frac{\sqrt{x}}{e^{ax}x^2}dx\right)a\,x^2 - 2\sqrt{a}\,x - \sqrt{x}\,\sqrt{\pi}\,e^{ax}}{4\sqrt{x}\,e^{ax}x^2}$$

input
```
int(((a*x)^(1/2)*exp(-a*x)+1/2*Pi^(1/2)*erfc((a*x)^(1/2)))/x^3,x)
```

output
```
(sqrt(x)*sqrt(pi)*e**(a*x)*erf(sqrt(x)*sqrt(a)) - 2*sqrt(x)*e**(a*x)*sqrt(
a)*int(sqrt(x)/(e**(a*x)*x**2),x)*a*x**2 - 2*sqrt(a)*x - sqrt(x)*sqrt(pi)*
e**(a*x))/(4*sqrt(x)*e**(a*x)*x**2)
```

3.75 $\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^4}\,dx=\frac{1}{3}a^3\Gamma\left(-\frac{3}{2},ax\right)-\frac{\Gamma\left(\frac{3}{2},ax\right)}{3x^3}$$

output
```
1/3*a^3*(4/3*Pi^(1/2)*erfc((a*x)^(1/2))-4/3/(a*x)^(1/2)*exp(-a*x)+2/3/(a*x
)^(3/2)*exp(-a*x))-1/3*((a*x)^(1/2)*exp(-a*x)+1/2*Pi^(1/2)*erfc((a*x)^(1/2
)))/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^4}\,dx=\frac{1}{3}a^3\Gamma\left(-\frac{3}{2},ax\right)-\frac{\Gamma\left(\frac{3}{2},ax\right)}{3x^3}$$

input
```
Integrate[Gamma[3/2, a*x]/x^4,x]
```

output
```
(a^3*Gamma[-3/2, a*x])/3 - Gamma[3/2, a*x]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^4}\,dx$$

$$\Big\downarrow \ 7116$$

$$\frac{1}{3}a^3\Gamma\left(-\frac{3}{2},ax\right)-\frac{\Gamma\left(\frac{3}{2},ax\right)}{3x^3}$$

input
```
Int[Gamma[3/2, a*x]/x^4,x]
```

output
```
(a^3*Gamma[-3/2, a*x])/3 - Gamma[3/2, a*x]/(3*x^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.97

method	result
derivativedivides	$\frac{8\sqrt{\pi}\,a^6\operatorname{erfc}\left(\sqrt{xa}\right)x^3-8a^3\mathrm{e}^{-xa}(xa)^{\frac{5}{2}}+4a^3\mathrm{e}^{-xa}(xa)^{\frac{3}{2}}-3\sqrt{\pi}\,a^3\operatorname{erfc}\left(\sqrt{xa}\right)-6a^3\mathrm{e}^{-xa}\sqrt{xa}}{18x^3a^3}$
default	$a^3\left(\sqrt{\pi}\left(-\frac{\operatorname{erfc}\left(\sqrt{xa}\right)}{6x^3a^3}-\frac{-\frac{\mathrm{e}^{-xa}}{5(xa)^{\frac{5}{2}}}+\frac{2\,\mathrm{e}^{-xa}}{15(xa)^{\frac{3}{2}}}-\frac{4\,\mathrm{e}^{-xa}}{15\sqrt{xa}}-\frac{4\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{15}}{3\sqrt{\pi}}\right)-\frac{2\,\mathrm{e}^{-xa}}{5(xa)^{\frac{5}{2}}}+\frac{4\,\mathrm{e}^{-xa}}{15(xa)^{\frac{3}{2}}}-\frac{8\,\mathrm{e}^{-xa}}{15\sqrt{xa}}$
parts	$2a^3\left(-\frac{\mathrm{e}^{-xa}}{5(xa)^{\frac{5}{2}}}+\frac{2\,\mathrm{e}^{-xa}}{15(xa)^{\frac{3}{2}}}-\frac{4\,\mathrm{e}^{-xa}}{15\sqrt{xa}}-\frac{4\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{xa}\right)}{15}\right)+\sqrt{\pi}\,a^3\left(-\frac{\operatorname{erfc}\left(\sqrt{xa}\right)}{6x^3a^3}-\frac{-\frac{\mathrm{e}^{-xa}}{5(xa)^{\frac{5}{2}}}+\frac{2\,\mathrm{e}^{-xa}}{15(xa)^{\frac{3}{2}}}}{}\right.$

input
```
int(((x*a)^(1/2)*exp(-x*a)+1/2*Pi^(1/2)*erfc((x*a)^(1/2)))/x^4,x,method=_R
ETURNVERBOSE)
```

output
```
1/18*(8*Pi^(1/2)*a^6*erfc((x*a)^(1/2))*x^3-8*a^3*exp(-x*a)*(x*a)^(5/2)+4*a
^3*exp(-x*a)*(x*a)^(3/2)-3*Pi^(1/2)*a^3*erfc((x*a)^(1/2))-6*a^3*exp(-x*a)*
(x*a)^(1/2))/x^3/a^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.97

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^4}\,dx=-\frac{\sqrt{\pi}(8\,a^3x^3-3)\operatorname{erf}\left(\sqrt{ax}\right)+2\left(4\,a^2x^2-2\,ax+3\right)\sqrt{ax}e^{(-ax)}+3\sqrt{\pi}}{18\,x^3}$$

input
```
integrate(((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2)))/x^4,x, al
gorithm="fricas")
```

output
```
-1/18*(sqrt(pi)*(8*a^3*x^3 - 3)*erf(sqrt(a*x)) + 2*(4*a^2*x^2 - 2*a*x + 3)
*sqrt(a*x)*e^(-a*x) + 3*sqrt(pi))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.28

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^4}\,dx = \frac{4\sqrt{\pi}a^3\operatorname{erfc}\left(\sqrt{ax}\right)}{9} - \frac{4a^2\sqrt{ax}e^{-ax}}{9x} + \frac{2a\sqrt{ax}e^{-ax}}{9x^2} - \frac{\sqrt{ax}e^{-ax}}{3x^3} - \frac{\sqrt{\pi}\operatorname{erfc}\left(\sqrt{ax}\right)}{6x^3}$$

input
```
integrate(((a*x)**(1/2)*exp(-a*x)+1/2*pi**(1/2)*erfc((a*x)**(1/2)))/x**4,x
)
```

output
```
4*sqrt(pi)*a**3*erfc(sqrt(a*x))/9 - 4*a**2*sqrt(a*x)*exp(-a*x)/(9*x) + 2*a
*sqrt(a*x)*exp(-a*x)/(9*x**2) - sqrt(a*x)*exp(-a*x)/(3*x**3) - sqrt(pi)*er
fc(sqrt(a*x))/(6*x**3)
```

Maxima [F]

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^4}\,dx = \int \frac{\sqrt{\pi}\operatorname{erfc}\left(\sqrt{ax}\right) + 2\sqrt{ax}e^{(-ax)}}{2x^4}\,dx$$

input
```
integrate(((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2)))/x^4,x, al
gorithm="maxima")
```

output
```
1/2*integrate((sqrt(pi)*erfc(sqrt(a*x)) + 2*sqrt(a*x)*e^(-a*x))/x^4, x)
```

Giac [F]

$$\int \frac{\Gamma\left(\frac{3}{2},ax\right)}{x^4}\,dx = \int \frac{\sqrt{\pi}\operatorname{erfc}\left(\sqrt{ax}\right) + 2\sqrt{ax}e^{(-ax)}}{2x^4}\,dx$$

input
```
integrate(((a*x)^(1/2)*exp(-a*x)+1/2*pi^(1/2)*erfc((a*x)^(1/2)))/x^4,x, al
gorithm="giac")
```

output
```
integrate(1/2*(sqrt(pi)*erfc(sqrt(a*x)) + 2*sqrt(a*x)*e^(-a*x))/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma\left(\frac{3}{2}, ax\right)}{x^4} dx = \int \frac{\frac{\sqrt{\Pi}\,\mathrm{erfc}\left(\sqrt{a\,x}\right)}{2} + \mathrm{e}^{-a\,x}\,\sqrt{a\,x}}{x^4} dx$$

input
```
int(((Pi^(1/2)*erfc((a*x)^(1/2)))/2 + exp(-a*x)*(a*x)^(1/2))/x^4,x)
```

output
```
int(((Pi^(1/2)*erfc((a*x)^(1/2)))/2 + exp(-a*x)*(a*x)^(1/2))/x^4, x)
```

Reduce [F]

$$\int \frac{\Gamma\left(\frac{3}{2}, ax\right)}{x^4} dx = \frac{3\sqrt{x}\,\sqrt{\pi}\,e^{ax}\mathrm{erf}\left(\sqrt{x}\,\sqrt{a}\right) + 4\sqrt{x}\,e^{ax}\sqrt{a}\left(\int \frac{\sqrt{x}}{e^{ax}x^2}dx\right)a^2x^3 + 4\sqrt{a}\,a\,x^2 - 6\sqrt{a}\,x - 3\sqrt{x}\,\sqrt{\pi}\,e^{ax}}{18\sqrt{x}\,e^{ax}x^3}$$

input
```
int(((a*x)^(1/2)*exp(-a*x)+1/2*Pi^(1/2)*erfc((a*x)^(1/2)))/x^4,x)
```

output
```
(3*sqrt(x)*sqrt(pi)*e**(a*x)*erf(sqrt(x)*sqrt(a)) + 4*sqrt(x)*e**(a*x)*sqr
t(a)*int(sqrt(x)/(e**(a*x)*x**2),x)*a**2*x**3 + 4*sqrt(a)*a*x**2 - 6*sqrt(
a)*x - 3*sqrt(x)*sqrt(pi)*e**(a*x))/(18*sqrt(x)*e**(a*x)*x**3)
```

3.76 $\int (dx)^m\Gamma(3,bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 51

$$\int (dx)^m\Gamma(3,bx)\,dx=\frac{(dx)^{1+m}\Gamma(3,bx)}{d(1+m)}-\frac{(bx)^{-m}(dx)^m\Gamma(4+m,bx)}{b(1+m)}$$

output
```
2*(d*x)^(1+m)*exp(-b*x)*(1+b*x+1/2*b^2*x^2)/d/(1+m)-(d*x)^m*GAMMA(4+m,b*x)
/b/(1+m)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int (dx)^m\Gamma(3,bx)\,dx=\frac{(dx)^m\left(bx\Gamma(3,bx)-(bx)^{-m}\Gamma(4+m,bx)\right)}{b(1+m)}$$

input
```
Integrate[(d*x)^m*Gamma[3, b*x],x]
```

output
```
((d*x)^m*(b*x*Gamma[3, b*x] - Gamma[4 + m, b*x]/(b*x)^m))/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(3,bx)(dx)^m\,dx$$

$$\Big\downarrow \ 7116$$

$$\frac{\Gamma(3,bx)(dx)^{m+1}}{d(m+1)}-\frac{(bx)^{-m}(dx)^m\Gamma(m+4,bx)}{b(m+1)}$$

input
```
Int[(d*x)^m*Gamma[3, b*x],x]
```

output
```
((d*x)^(1 + m)*Gamma[3, b*x])/(d*(1 + m)) - ((d*x)^m*Gamma[4 + m, b*x])/(b
*(1 + m)*(b*x)^m)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.72 (sec) , antiderivative size = 228, normalized size of antiderivative = 4.47

method	result
meijerg	$\frac{2(dx)^m(bx)^{-\frac{m}{2}}\mathrm{e}^{-\frac{bx}{2}}\,\mathrm{WhittakerM}\left(\frac{m}{2},\frac{1}{2}+\frac{m}{2},bx\right)}{b(1+m)}+2(dx)^m x^{-m}b^{-1-m}\Bigg(x^m b^m(bx)^{-\frac{m}{2}}\mathrm{e}^{-\frac{bx}{2}}\,\mathrm{WhittakerM}\left(\frac{m}{2},\right.$

input
```
int(2*(d*x)^m*exp(-b*x)*(1+b*x+1/2*b^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
2*(d*x)^m/b/(1+m)*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1/2*m,1/2+1/2*m,
b*x)+2*(d*x)^m*x^(-m)*b^(-1-m)*(x^m*b^m*(b*x)^(-1/2*m)*exp(-1/2*b*x)*Whitt
akerM(1/2*m,1/2+1/2*m,b*x)+1/(2+m)*x^m*b^m*(-2-m)*(b*x)^(-1/2*m)*exp(-1/2*
b*x)*WhittakerM(1+1/2*m,1/2+1/2*m,b*x))+(d*x)^m*x^(-m)*b^(-1-m)*(x^m*b^m*(
2+m)*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1/2*m,1/2+1/2*m,b*x)-x^m*b^m*
(b*x+m+2)*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1+1/2*m,1/2+1/2*m,b*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.96

$$\int (dx)^m\Gamma(3,bx)\,dx$$
$$= \frac{\left(bx\Gamma(3,bx)-(b^3x^3+(b^2m+3\,b^2)x^2+(bm^2+5\,bm+6\,b)x)e^{(-bx)}\right)(dx)^m-\frac{(m^3+6\,m^2+11\,m+6)\Gamma(m+1,bx)}{\left(\frac{b}{d}\right)^m}}{bm+b}$$

input
```
integrate((d*x)^m*gamma(3,b*x),x, algorithm="fricas")
```

output
```
((b*x*gamma(3, b*x) - (b^3*x^3 + (b^2*m + 3*b^2)*x^2 + (b*m^2 + 5*b*m + 6*
b)*x)*e^(-b*x))*(d*x)^m - (m^3 + 6*m^2 + 11*m + 6)*gamma(m + 1, b*x)/(b/d)
^m)/(b*m + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(53) = 106$.

Time = 3.01 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.63

$$\int (dx)^m\Gamma(3,bx)\,dx = \frac{b^2b^{-m-3}d^m m\Gamma(m+3)\,\gamma(m+3,bx)}{\Gamma(m+4)} + \frac{3b^2b^{-m-3}d^m\Gamma(m+3)\,\gamma(m+3,bx)}{\Gamma(m+4)} + \frac{2bb^{-m-2}d^m m\Gamma(m+2)\,\gamma(m+2,bx)}{\Gamma(m+3)} + \frac{4bb^{-m-2}d^m\Gamma(m+2)\,\gamma(m+2,bx)}{\Gamma(m+3)} + \frac{2b^{-m-1}d^m m\Gamma(m+1)\,\gamma(m+1,bx)}{\Gamma(m+2)} + \frac{2b^{-m-1}d^m\Gamma(m+1)\,\gamma(m+1,bx)}{\Gamma(m+2)}$$

input
```
integrate((d*x)**m*uppergamma(3,b*x),x)
```

output
```
b**2*b**(-m - 3)*d**m*m*gamma(m + 3)*lowergamma(m + 3, b*x)/gamma(m + 4) +
 3*b**2*b**(-m - 3)*d**m*gamma(m + 3)*lowergamma(m + 3, b*x)/gamma(m + 4)
+ 2*b*b**(-m - 2)*d**m*m*gamma(m + 2)*lowergamma(m + 2, b*x)/gamma(m + 3)
+ 4*b*b**(-m - 2)*d**m*gamma(m + 2)*lowergamma(m + 2, b*x)/gamma(m + 3) +
2*b**(-m - 1)*d**m*m*gamma(m + 1)*lowergamma(m + 1, b*x)/gamma(m + 2) + 2*
b**(-m - 1)*d**m*gamma(m + 1)*lowergamma(m + 1, b*x)/gamma(m + 2)
```

Maxima [F]

$$\int (dx)^m\Gamma(3,bx)\,dx = \int (dx)^m\,\Gamma(3,bx)\,dx$$

input
```
integrate((d*x)^m*gamma(3,b*x),x, algorithm="maxima")
```

output
```
integrate((d*x)^m*gamma(3, b*x), x)
```

Giac [F]

$$\int (dx)^m\Gamma(3,bx)\,dx=\int (dx)^m\,\Gamma(3,bx)\,dx$$

input
```
integrate((d*x)^m*gamma(3,b*x),x, algorithm="giac")
```

output
```
integrate((d*x)^m*gamma(3, b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m\Gamma(3,bx)\,dx=\int 2\,\mathrm{e}^{-b\,x}\,(d\,x)^m\,\left(\frac{b^2\,x^2}{2}+b\,x+1\right)\,dx$$

input
```
int(2*exp(-b*x)*(d*x)^m*(b*x + (b^2*x^2)/2 + 1),x)
```

output
```
int(2*exp(-b*x)*(d*x)^m*(b*x + (b^2*x^2)/2 + 1), x)
```

Reduce [F]

$$\int (dx)^m\Gamma(3,bx)\,dx$$
$$=\frac{d^m\left(e^{bx}\left(\int\frac{x^m}{e^{bx}x}dx\right)m^3+5e^{bx}\left(\int\frac{x^m}{e^{bx}x}dx\right)m^2+6e^{bx}\left(\int\frac{x^m}{e^{bx}x}dx\right)m-x^mb^2x^2-x^mbmx-4x^mbx-x^mm^2-\right.}{e^{bx}b}$$

input
```
int(2*(d*x)^m*exp(-b*x)*(1+b*x+1/2*b^2*x^2),x)
```

output
```
(d**m*(e**(b*x)*int(x**m/(e**(b*x)*x),x)*m**3 + 5*e**(b*x)*int(x**m/(e**(b
*x)*x),x)*m**2 + 6*e**(b*x)*int(x**m/(e**(b*x)*x),x)*m - x**m*b**2*x**2 -
x**m*b*m*x - 4*x**m*b*x - x**m*m**2 - 5*x**m*m - 6*x**m))/(e**(b*x)*b)
```

3.77 $\int (dx)^m\Gamma(2,bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 51

$$\int (dx)^m\Gamma(2,bx)\,dx=\frac{(dx)^{1+m}\Gamma(2,bx)}{d(1+m)}-\frac{(bx)^{-m}(dx)^m\Gamma(3+m,bx)}{b(1+m)}$$

output
```
(d*x)^(1+m)*exp(-b*x)*(b*x+1)/d/(1+m)-(d*x)^m*GAMMA(3+m,b*x)/b/(1+m)/((b*x
)^m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int (dx)^m\Gamma(2,bx)\,dx=\frac{(dx)^m\left(bx\Gamma(2,bx)-(bx)^{-m}\Gamma(3+m,bx)\right)}{b(1+m)}$$

input
```
Integrate[(d*x)^m*Gamma[2, b*x],x]
```

output
```
((d*x)^m*(b*x*Gamma[2, b*x] - Gamma[3 + m, b*x]/(b*x)^m))/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(2,bx)(dx)^m\,dx$$

$$\Big\downarrow \; 7116$$

$$\frac{\Gamma(2,bx)(dx)^{m+1}}{d(m+1)}-\frac{(bx)^{-m}(dx)^m\Gamma(m+3,bx)}{b(m+1)}$$

input
```
Int[(d*x)^m*Gamma[2, b*x],x]
```

output
```
((d*x)^(1 + m)*Gamma[2, b*x])/(d*(1 + m)) - ((d*x)^m*Gamma[3 + m, b*x])/(b
*(1 + m)*(b*x)^m)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.61

method	result
meijerg	$(dx)^m x^{-m} b^{-1-m}\left(x^m b^m (bx)^{-\frac{m}{2}} e^{-\frac{bx}{2}} \text{WhittakerM}\left(\frac{m}{2},\frac{1}{2}+\frac{m}{2},bx\right)+\frac{x^m b^m(-2-m)(bx)^{-\frac{m}{2}} e^{-\frac{bx}{2}} \text{Whitta}}{2+m}\right.$

input
```
int((d*x)^m*exp(-b*x)*(b*x+1),x,method=_RETURNVERBOSE)
```

output
```
(d*x)^m*x^(-m)*b^(-1-m)*(x^m*b^m*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1
/2*m,1/2+1/2*m,b*x)+1/(2+m)*x^m*b^m*(-2-m)*(b*x)^(-1/2*m)*exp(-1/2*b*x)*Wh
ittakerM(1+1/2*m,1/2+1/2*m,b*x))+(d*x)^m/b/(1+m)*(b*x)^(-1/2*m)*exp(-1/2*b
*x)*WhittakerM(1/2*m,1/2+1/2*m,b*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

$$\int (dx)^m\Gamma(2,bx)\,dx = \frac{\left(bx\Gamma(2,bx)-(b^2x^2+(bm+2\,b)x)e^{(-bx)}\right)(dx)^m-\frac{(m^2+3\,m+2)\Gamma(m+1,bx)}{\left(\frac{b}{d}\right)^m}}{bm+b}$$

input
```
integrate((d*x)^m*gamma(2,b*x),x, algorithm="fricas")
```

output
```
((b*x*gamma(2, b*x) - (b^2*x^2 + (b*m + 2*b)*x)*e^(-b*x))*(d*x)^m - (m^2 +
 3*m + 2)*gamma(m + 1, b*x)/(b/d)^m)/(b*m + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(42) = 84.

Time = 2.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.27

$$\int (dx)^m\Gamma(2,bx)\,dx = \frac{bb^{-m-2}d^m m\Gamma(m+2)\,\gamma(m+2,bx)}{\Gamma(m+3)} + \frac{2bb^{-m-2}d^m\Gamma(m+2)\,\gamma(m+2,bx)}{\Gamma(m+3)} + \frac{b^{-m-1}d^m m\Gamma(m+1)\,\gamma(m+1,bx)}{\Gamma(m+2)} + \frac{b^{-m-1}d^m\Gamma(m+1)\,\gamma(m+1,bx)}{\Gamma(m+2)}$$

input
```
integrate((d*x)**m*uppergamma(2,b*x),x)
```

output
```
b*b**(-m - 2)*d**m*m*gamma(m + 2)*lowergamma(m + 2, b*x)/gamma(m + 3) + 2*
b*b**(-m - 2)*d**m*gamma(m + 2)*lowergamma(m + 2, b*x)/gamma(m + 3) + b**(
-m - 1)*d**m*m*gamma(m + 1)*lowergamma(m + 1, b*x)/gamma(m + 2) + b**(-m -
 1)*d**m*gamma(m + 1)*lowergamma(m + 1, b*x)/gamma(m + 2)
```

Maxima [F]

$$\int (dx)^m\Gamma(2,bx)\,dx = \int (dx)^m\,\Gamma(2,bx)\,dx$$

input
```
integrate((d*x)^m*gamma(2,b*x),x, algorithm="maxima")
```

output
```
integrate((d*x)^m*gamma(2, b*x), x)
```

Giac [F]

$$\int (dx)^m\Gamma(2,bx)\,dx = \int (dx)^m\,\Gamma(2,bx)\,dx$$

input
```
integrate((d*x)^m*gamma(2,b*x),x, algorithm="giac")
```

output
```
integrate((d*x)^m*gamma(2, b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \Gamma(2,bx)\,dx = \int e^{-b\,x}\,(d\,x)^m\,(b\,x+1)\,dx$$

input
```
int(exp(-b*x)*(d*x)^m*(b*x + 1),x)
```

output
```
int(exp(-b*x)*(d*x)^m*(b*x + 1), x)
```

Reduce [F]

$$\int (dx)^m \Gamma(2,bx)\,dx = \frac{d^m \left(e^{bx} \left(\int \frac{x^m}{e^{bx}x} dx\right) m^2 + 2e^{bx} \left(\int \frac{x^m}{e^{bx}x} dx\right) m - x^m bx - x^m m - 2x^m\right)}{e^{bx} b}$$

input
```
int((d*x)^m*exp(-b*x)*(b*x+1),x)
```

output
```
(d**m*(e**(b*x)*int(x**m/(e**(b*x)*x),x)*m**2 + 2*e**(b*x)*int(x**m/(e**(b
*x)*x),x)*m - x**m*b*x - x**m*m - 2*x**m))/(e**(b*x)*b)
```

3.78 $\int e^{-bx}(dx)^m\,dx$

Optimal result

Integrand size = 12, antiderivative size = 24

$$\int e^{-bx}(dx)^m\,dx = -\frac{(bx)^{-m}(dx)^m\Gamma(1+m,bx)}{b}$$

output
```
-(d*x)^m*GAMMA(1+m,b*x)/b/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int e^{-bx}(dx)^m\,dx = -\frac{(bx)^{-m}(dx)^m\Gamma(1+m,bx)}{b}$$

input
```
Integrate[(d*x)^m/E^(b*x),x]
```

output
```
-(((d*x)^m*Gamma[1 + m, b*x])/(b*(b*x)^m))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-bx}(dx)^m \, dx$$

$$\Big\downarrow \text{2612}$$

$$-\frac{(bx)^{-m}(dx)^m\Gamma(m+1,bx)}{b}$$

input
```
Int[(d*x)^m/E^(b*x),x]
```

output
```
-(((d*x)^m*Gamma[1 + m, b*x])/(b*(b*x)^m))
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

method	result	size
meijerg	$\frac{(dx)^m (bx)^{-\frac{m}{2}} \mathrm{e}^{-\frac{bx}{2}} \operatorname{WhittakerM}\left(\frac{m}{2},\frac{1}{2}+\frac{m}{2},bx\right)}{b(1+m)}$	39

input
```
int((d*x)^m/exp(b*x),x,method=_RETURNVERBOSE)
```

output
```
(d*x)^m/b/(1+m)*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1/2*m,1/2+1/2*m,b*
x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int e^{-bx}(dx)^m\,dx = -\frac{\Gamma(m+1,bx)}{b\left(\frac{b}{d}\right)^m}$$

input
```
integrate((d*x)^m/exp(b*x),x, algorithm="fricas")
```

output
```
-gamma(m + 1, b*x)/(b*(b/d)^m)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(19) = 38.

Time = 0.43 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int e^{-bx}(dx)^m\,dx = \frac{b^{-m-1}d^m m\Gamma(m+1)\,\gamma(m+1,bx)}{\Gamma\left(m+2\right)} + \frac{b^{-m-1}d^m\Gamma(m+1)\,\gamma(m+1,bx)}{\Gamma\left(m+2\right)}$$

input
```
integrate((d*x)**m/exp(b*x),x)
```

output
```
b**(-m - 1)*d**m*m*gamma(m + 1)*lowergamma(m + 1, b*x)/gamma(m + 2) + b**(
-m - 1)*d**m*gamma(m + 1)*lowergamma(m + 1, b*x)/gamma(m + 2)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int e^{-bx}(dx)^m\,dx = -\frac{(dx)^{m+1}\,E_{-m}(bx)}{d}$$

input
```
integrate((d*x)^m/exp(b*x),x, algorithm="maxima")
```

output
```
-(d*x)^(m + 1)*exp_integral_e(-m, b*x)/d
```

Giac [F]

$$\int e^{-bx}(dx)^m\,dx = \int (dx)^m\,e^{(-bx)}\,dx$$

input
```
integrate((d*x)^m/exp(b*x),x, algorithm="giac")
```

output
```
integrate((d*x)^m*e^(-b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-bx}(dx)^m\,dx = \int \mathrm{e}^{-b\,x}\,{\left(d\,x\right)}^m\,dx$$

input
```
int(exp(-b*x)*(d*x)^m,x)
```

output
```
int(exp(-b*x)*(d*x)^m, x)
```

Reduce [F]

$$\int e^{-bx}(dx)^m\,dx = \frac{d^m\left(e^{bx}\left(\int \frac{x^m}{e^{bx}x}dx\right)m - x^m\right)}{e^{bx}b}$$

input
```
int((d*x)^m/exp(b*x),x)
```

output
```
(d**m*(e**(b*x)*int(x**m/(e**(b*x)*x),x)*m - x**m))/(e**(b*x)*b)
```

3.79 $\int (dx)^m \Gamma(0, bx) \, dx$

Optimal result

Integrand size = 11, antiderivative size = 51

$$\int (dx)^m \Gamma(0, bx) \, dx = \frac{(dx)^{1+m}\Gamma(0, bx)}{d(1+m)} - \frac{(bx)^{-m}(dx)^m\Gamma(1+m, bx)}{b(1+m)}$$

output
```
(d*x)^(1+m)*Ei(1,b*x)/d/(1+m)-(d*x)^m*GAMMA(1+m,b*x)/b/(1+m)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int (dx)^m \Gamma(0, bx) \, dx = \frac{(dx)^m \left(bx\Gamma(0, bx) - (bx)^{-m}\Gamma(1+m, bx)\right)}{b(1+m)}$$

input
```
Integrate[(d*x)^m*Gamma[0, b*x],x]
```

output
```
((d*x)^m*(b*x*Gamma[0, b*x] - Gamma[1 + m, b*x]/(b*x)^m))/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(0,bx)(dx)^m\,dx$$

$$\Big\downarrow \; 7116$$

$$\frac{\Gamma(0,bx)(dx)^{m+1}}{d(m+1)}-\frac{(bx)^{-m}(dx)^m\Gamma(m+1,bx)}{b(m+1)}$$

input
```
Int[(d*x)^m*Gamma[0, b*x],x]
```

output
```
((d*x)^(1 + m)*Gamma[0, b*x])/(d*(1 + m)) - ((d*x)^m*Gamma[1 + m, b*x])/(b
*(1 + m)*(b*x)^m)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.69

method	result
meijerg	$(dx)^m x^{-m} b^{-1-m}\left(-\frac{(\Psi(1+m)+\gamma-\Psi(2+m)+\ln(x)+\ln(b))x^{1+m}b^{1+m}}{1+m}+\frac{x^{2+m}b^{2+m}\,\text{hypergeom}([1,1,2+m],[2,2,3+m],-b}{2+m}\right.$

input
```
int((d*x)^m*Ei(1,b*x),x,method=_RETURNVERBOSE)
```

output
```
(d*x)^m*x^(-m)*b^(-1-m)*(-(Psi(1+m)+gamma-Psi(2+m)+ln(x)+ln(b))*x^(1+m)*b^
(1+m)/(1+m)+1/(2+m)*x^(2+m)*b^(2+m)*hypergeom([1,1,2+m],[2,2,3+m],-b*x))
```

Fricas [F(-2)]

Exception generated.

$$\int (dx)^m \Gamma(0, bx) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*exp_integral_e(1,b*x),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated (((
d)*(x))^(m))*(exp_integral_e(((1)::EXPR INT),(b)*(x))):   There are no lib
rary operations named exp_integral_e       Use HyperDoc Browse or issue
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 2358, normalized size of antiderivative = 46.24

$$\int (dx)^m \Gamma(0, bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*expint(1,b*x),x)
```

output
```
-(-1)**m*b**2*b**m*b**(-m - 1)*d**m*m*x**2*x**m*exp(b*x)*log(b*x)*gamma(m
+ 3)/((-1)**m*b*m**2*x*exp(b*x)*gamma(m + 3) + 2*(-1)**m*b*m*x*exp(b*x)*ga
mma(m + 3) + (-1)**m*b*x*exp(b*x)*gamma(m + 3)) - (-1)**m*EulerGamma*b**2*
b**m*b**(-m - 1)*d**m*m*x**2*x**m*exp(b*x)*gamma(m + 3)/((-1)**m*b*m**2*x*
exp(b*x)*gamma(m + 3) + 2*(-1)**m*b*m*x*exp(b*x)*gamma(m + 3) + (-1)**m*b*
x*exp(b*x)*gamma(m + 3)) - (-1)**m*b**2*b**m*b**(-m - 1)*d**m*x**2*x**m*ex
p(b*x)*log(b*x)*gamma(m + 3)/((-1)**m*b*m**2*x*exp(b*x)*gamma(m + 3) + 2*(
-1)**m*b*m*x*exp(b*x)*gamma(m + 3) + (-1)**m*b*x*exp(b*x)*gamma(m + 3)) -
(-1)**m*EulerGamma*b**2*b**m*b**(-m - 1)*d**m*x**2*x**m*exp(b*x)*gamma(m +
 3)/((-1)**m*b*m**2*x*exp(b*x)*gamma(m + 3) + 2*(-1)**m*b*m*x*exp(b*x)*gam
ma(m + 3) + (-1)**m*b*x*exp(b*x)*gamma(m + 3)) + (-1)**m*b**2*b**m*b**(-m
- 1)*d**m*x**2*x**m*exp(b*x)*gamma(m + 3)/((-1)**m*b*m**2*x*exp(b*x)*gamma
(m + 3) + 2*(-1)**m*b*m*x*exp(b*x)*gamma(m + 3) + (-1)**m*b*x*exp(b*x)*gam
ma(m + 3)) + (-1)**m*b**(-m - 1)*b**(m + 2)*d**m*m**2*x**(m + 2)*exp(b*x)*
log(b*x)*gamma(m + 2)/((-1)**m*b*m**2*x*exp(b*x)*gamma(m + 3) + 2*(-1)**m*
b*m*x*exp(b*x)*gamma(m + 3) + (-1)**m*b*x*exp(b*x)*gamma(m + 3)) + (-1)**m
*b**(-m - 1)*b**(m + 2)*d**m*m**2*x**(m + 2)*exp(b*x)*expint(1, b*x)*gamma
(m + 2)/((-1)**m*b*m**2*x*exp(b*x)*gamma(m + 3) + 2*(-1)**m*b*m*x*exp(b*x)
*gamma(m + 3) + (-1)**m*b*x*exp(b*x)*gamma(m + 3)) + (-1)**m*EulerGamma*b*
*(-m - 1)*b**(m + 2)*d**m*m**2*x**(m + 2)*exp(b*x)*gamma(m + 2)/((-1)**...
```

Maxima [F]

$$\int (dx)^m\Gamma(0,bx)\,dx=\int (dx)^m\,E_1(bx)\,dx$$

input
```
integrate((d*x)^m*exp_integral_e(1,b*x),x, algorithm="maxima")
```

output
```
integrate((d*x)^m*exp_integral_e(1, b*x), x)
```

Giac [F]

$$\int (dx)^m\Gamma(0,bx)\,dx = \int (dx)^m\, E_1(bx)\, dx$$

input
```
integrate((d*x)^m*exp_integral_e(1,b*x),x, algorithm="giac")
```

output
```
integrate((d*x)^m*exp_integral_e(1, b*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.24

$$\int (dx)^m\Gamma(0,bx)\,dx = \int (d\,x)^m\,\mathrm{expint}(b\,x)\, dx$$

input
```
int((d*x)^m*expint(b*x),x)
```

output
```
int((d*x)^m*expint(b*x), x)
```

Reduce [F]

$$\int (dx)^m\Gamma(0,bx)\,dx = d^m\left(\int x^m\, ei(1,bx)\,dx\right)$$

input
```
int((d*x)^m*Ei(1,b*x),x)
```

output
```
d**m*int(x**m*ei(1,b*x),x)
```

3.80 $\int (dx)^m\Gamma(-1,bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int (dx)^m\Gamma(-1,bx)\,dx=\frac{(dx)^{1+m}\Gamma(-1,bx)}{d(1+m)}-\frac{(bx)^{-m}(dx)^m\Gamma(m,bx)}{b(1+m)}$$

output
```
(d*x)^(1+m)/b/x*Ei(2,b*x)/d/(1+m)-(d*x)^m*GAMMA(m,b*x)/b/(1+m)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int (dx)^m\Gamma(-1,bx)\,dx=\frac{(dx)^m\left(bx\Gamma(-1,bx)-(bx)^{-m}\Gamma(m,bx)\right)}{b(1+m)}$$

input
```
Integrate[(d*x)^m*Gamma[-1, b*x],x]
```

output
```
((d*x)^m*(b*x*Gamma[-1, b*x] - Gamma[m, b*x]/(b*x)^m))/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(-1,bx)(dx)^m\,dx$$

$$\Big\downarrow \; 7116$$

$$\frac{\Gamma(-1,bx)(dx)^{m+1}}{d(m+1)}-\frac{(bx)^{-m}(dx)^m\Gamma(m,bx)}{b(m+1)}$$

input
```
Int[(d*x)^m*Gamma[-1, b*x],x]
```

output
```
((d*x)^(1 + m)*Gamma[-1, b*x])/(d*(1 + m)) - ((d*x)^m*Gamma[m, b*x])/(b*(1
 + m)*(b*x)^m)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.98

method	result
meijerg	$b^{-1-m}(dx)^m\,x^{-m}\left(\frac{x^m b^m}{m}+\frac{(\Psi(1+m)+\gamma-1-\Psi(2+m)+\ln(x)+\ln(b))x^{1+m}b^{1+m}}{1+m}-\frac{x^{2+m}b^{2+m}\,\text{hypergeom}([1,1,2+m],[2\right.$

input
```
int((d*x)^m/b/x*Ei(2,b*x),x,method=_RETURNVERBOSE)
```

output
```
b^(-1-m)*(d*x)^m*x^(-m)*(x^m*b^m/m+(Psi(1+m)+gamma-1-Psi(2+m)+ln(x)+ln(b))
*x^(1+m)*b^(1+m)/(1+m)-1/2/(2+m)*x^(2+m)*b^(2+m)*hypergeom([1,1,2+m],[2,3,
3+m],-b*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int (dx)^m\Gamma(-1,bx)\,dx=\frac{\left(bmx\Gamma(-1,bx)+e^{(-bx)}\right)(dx)^m-\frac{\Gamma(m+1,bx)}{\left(\frac{b}{d}\right)^m}}{bm^2+bm}$$

input
```
integrate((d*x)^m*gamma(-1,b*x),x, algorithm="fricas")
```

output
```
((b*m*x*gamma(-1, b*x) + e^(-b*x))*(d*x)^m - gamma(m + 1, b*x)/(b/d)^m)/(b
*m^2 + b*m)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 4357, normalized size of antiderivative = 88.92

$$\int (dx)^m\Gamma(-1,bx)\,dx=\text{Too large to display}$$

input
```
integrate((d*x)**m*uppergamma(-1,b*x),x)
```

output
```
((-1)**m*b**3*b**m*d**m*m**2*x**3*x**m*exp(b*x)*log(b*x)*gamma(m + 3)/((-1
)**m*b**2*b**m*m**3*x**2*exp(b*x)*gamma(m + 3) + 2*(-1)**m*b**2*b**m*m**2*
x**2*exp(b*x)*gamma(m + 3) + (-1)**m*b**2*b**m*m*x**2*exp(b*x)*gamma(m + 3
)) - (-1)**m*b**3*b**m*d**m*m**2*x**3*x**m*exp(b*x)*gamma(m + 3)/((-1)**m*
b**2*b**m*m**3*x**2*exp(b*x)*gamma(m + 3) + 2*(-1)**m*b**2*b**m*m**2*x**2*
exp(b*x)*gamma(m + 3) + (-1)**m*b**2*b**m*m*x**2*exp(b*x)*gamma(m + 3)) +
(-1)**m*EulerGamma*b**3*b**m*d**m*m**2*x**3*x**m*exp(b*x)*gamma(m + 3)/((-
1)**m*b**2*b**m*m**3*x**2*exp(b*x)*gamma(m + 3) + 2*(-1)**m*b**2*b**m*m**2
*x**2*exp(b*x)*gamma(m + 3) + (-1)**m*b**2*b**m*m*x**2*exp(b*x)*gamma(m +
3)) + (-1)**m*b**3*b**m*d**m*m*x**3*x**m*exp(b*x)*log(b*x)*gamma(m + 3)/((
-1)**m*b**2*b**m*m**3*x**2*exp(b*x)*gamma(m + 3) + 2*(-1)**m*b**2*b**m*m**
2*x**2*exp(b*x)*gamma(m + 3) + (-1)**m*b**2*b**m*m*x**2*exp(b*x)*gamma(m +
 3)) - 2*(-1)**m*b**3*b**m*d**m*m*x**3*x**m*exp(b*x)*gamma(m + 3)/((-1)**m
*b**2*b**m*m**3*x**2*exp(b*x)*gamma(m + 3) + 2*(-1)**m*b**2*b**m*m**2*x**2
*exp(b*x)*gamma(m + 3) + (-1)**m*b**2*b**m*m*x**2*exp(b*x)*gamma(m + 3)) +
 (-1)**m*EulerGamma*b**3*b**m*d**m*m*x**3*x**m*exp(b*x)*gamma(m + 3)/((-1)
**m*b**2*b**m*m**3*x**2*exp(b*x)*gamma(m + 3) + 2*(-1)**m*b**2*b**m*m**2*x
**2*exp(b*x)*gamma(m + 3) + (-1)**m*b**2*b**m*m*x**2*exp(b*x)*gamma(m + 3)
) + (-1)**m*b**2*b**m*d**m*m**2*x**2*x**m*exp(b*x)*gamma(m + 3)/((-1)**m*b
**2*b**m*m**3*x**2*exp(b*x)*gamma(m + 3) + 2*(-1)**m*b**2*b**m*m**2*x**...
```

Maxima [F]

$$\int (dx)^m\Gamma(-1,bx)\,dx=\int (dx)^m\,\Gamma(-1,bx)\,dx$$

input
```
integrate((d*x)^m*gamma(-1,b*x),x, algorithm="maxima")
```

output
```
integrate((d*x)^m*gamma(-1, b*x), x)
```

Giac [F]

$$\int (dx)^m\Gamma(-1,bx)\,dx=\int (dx)^m\,\Gamma(-1,bx)\;dx$$

input
```
integrate((d*x)^m*gamma(-1,b*x),x, algorithm="giac")
```

output
```
integrate((d*x)^m*gamma(-1, b*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.39

$$\int (dx)^m\Gamma(-1,bx)\,dx=\int \frac{(d\,x)^m\,\mathrm{expint}(2,b\,x)}{b\,x}\,dx$$

input
```
int(((d*x)^m*expint(2, b*x))/(b*x),x)
```

output
```
int(((d*x)^m*expint(2, b*x))/(b*x), x)
```

Reduce [F]

$$\int (dx)^m\Gamma(-1,bx)\,dx=\frac{d^m\left(\int \frac{x^m\,ei(2,bx)}{x}dx\right)}{b}$$

input
```
int((d*x)^m/b/x*Ei(2,b*x),x)
```

output
```
(d**m*int((x**m*ei(2,b*x))/x,x))/b
```

3.81 $\int (dx)^m\Gamma(-2,bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 51

$$\int (dx)^m\Gamma(-2,bx)\,dx = \frac{(dx)^{1+m}\Gamma(-2,bx)}{d(1+m)} - \frac{(bx)^{-m}(dx)^m\Gamma(-1+m,bx)}{b(1+m)}$$

output
```
(d*x)^(1+m)/b^2/x^2*Ei(3,b*x)/d/(1+m)-(d*x)^m*GAMMA(-1+m,b*x)/b/(1+m)/((b*
x)^m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int (dx)^m\Gamma(-2,bx)\,dx = \frac{(dx)^m\left(bx\Gamma(-2,bx) - (bx)^{-m}\Gamma(-1+m,bx)\right)}{b(1+m)}$$

input
```
Integrate[(d*x)^m*Gamma[-2, b*x],x]
```

output
```
((d*x)^m*(b*x*Gamma[-2, b*x] - Gamma[-1 + m, b*x]/(b*x)^m))/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(-2,bx)(dx)^m\,dx$$

$$\Big\downarrow \text{ 7116}$$

$$\frac{\Gamma(-2,bx)(dx)^{m+1}}{d(m+1)}-\frac{(bx)^{-m}(dx)^m\Gamma(m-1,bx)}{b(m+1)}$$

input
```
Int[(d*x)^m*Gamma[-2, b*x],x]
```

output
```
((d*x)^(1 + m)*Gamma[-2, b*x])/(d*(1 + m)) - ((d*x)^m*Gamma[-1 + m, b*x])/
(b*(1 + m)*(b*x)^m)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.27

method	result
meijerg	$b^{-1-m}(dx)^m\,x^{-m}\left(\frac{x^{-1+m}b^{-1+m}}{-2+2m}-\frac{x^m b^m}{m}-\frac{\left(\Psi(1+m)+\gamma-\frac{3}{2}-\Psi(2+m)+\ln(x)+\ln(b)\right)x^{1+m}b^{1+m}}{2(1+m)}+\frac{x^{2+m}b^{2+m}\,\text{hype}}{}\right.$

input
```
int((d*x)^m/b^2/x^2*Ei(3,b*x),x,method=_RETURNVERBOSE)
```

output
```
b^(-1-m)*(d*x)^m*x^(-m)*(1/2*x^(-1+m)*b^(-1+m)/(-1+m)-x^m*b^m/m-1/2*(Psi(1
+m)+gamma-3/2-Psi(2+m)+ln(x)+ln(b))*x^(1+m)*b^(1+m)/(1+m)+1/6/(2+m)*x^(2+m
)*b^(2+m)*hypergeom([1,1,2+m],[2,4,3+m],-b*x))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.61

$$\int (dx)^m\Gamma(-2,bx)\,dx = \frac{\left((b^2m^2-b^2m)x^2\Gamma(-2,bx)+(bx+m)e^{(-bx)}\right)(dx)^m-\frac{bx\Gamma(m+1,bx)}{\left(\frac{b}{d}\right)^m}}{(b^2m^3-b^2m)x}$$

input
```
integrate((d*x)^m*gamma(-2,b*x),x, algorithm="fricas")
```

output
```
(((b^2*m^2 - b^2*m)*x^2*gamma(-2, b*x) + (b*x + m)*e^(-b*x))*(d*x)^m - b*x
*gamma(m + 1, b*x)/(b/d)^m)/((b^2*m^3 - b^2*m)*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.43 (sec) , antiderivative size = 8162, normalized size of antiderivative = 160.04

$$\int (dx)^m\Gamma(-2,bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*uppergamma(-2,b*x),x)
```

output
```
(-2*(-1)**m*b**4*b**m*b**(1 - m)*d**m*m**3*x**4*x**m*exp(b*x)*log(b*x)*gam
ma(m + 3)/(4*(-1)**m*b**3*m**4*x**3*exp(b*x)*gamma(m + 3) + 4*(-1)**m*b**3
*m**3*x**3*exp(b*x)*gamma(m + 3) - 4*(-1)**m*b**3*m**2*x**3*exp(b*x)*gamma
(m + 3) - 4*(-1)**m*b**3*m*x**3*exp(b*x)*gamma(m + 3)) - 2*(-1)**m*EulerGa
mma*b**4*b**m*b**(1 - m)*d**m*m**3*x**4*x**m*exp(b*x)*gamma(m + 3)/(4*(-1)
**m*b**3*m**4*x**3*exp(b*x)*gamma(m + 3) + 4*(-1)**m*b**3*m**3*x**3*exp(b*
x)*gamma(m + 3) - 4*(-1)**m*b**3*m**2*x**3*exp(b*x)*gamma(m + 3) - 4*(-1)*
*m*b**3*m*x**3*exp(b*x)*gamma(m + 3)) + 3*(-1)**m*b**4*b**m*b**(1 - m)*d**
m*m**3*x**4*x**m*exp(b*x)*gamma(m + 3)/(4*(-1)**m*b**3*m**4*x**3*exp(b*x)*
gamma(m + 3) + 4*(-1)**m*b**3*m**3*x**3*exp(b*x)*gamma(m + 3) - 4*(-1)**m*
b**3*m**2*x**3*exp(b*x)*gamma(m + 3) - 4*(-1)**m*b**3*m*x**3*exp(b*x)*gamm
a(m + 3)) + 2*(-1)**m*b**4*b**m*b**(1 - m)*d**m*m**2*x**4*x**m*exp(b*x)*ga
mma(m + 3)/(4*(-1)**m*b**3*m**4*x**3*exp(b*x)*gamma(m + 3) + 4*(-1)**m*b**
3*m**3*x**3*exp(b*x)*gamma(m + 3) - 4*(-1)**m*b**3*m**2*x**3*exp(b*x)*gamm
a(m + 3) - 4*(-1)**m*b**3*m*x**3*exp(b*x)*gamma(m + 3)) + 2*(-1)**m*b**4*b
**m*b**(1 - m)*d**m*m*x**4*x**m*exp(b*x)*log(b*x)*gamma(m + 3)/(4*(-1)**m*
b**3*m**4*x**3*exp(b*x)*gamma(m + 3) + 4*(-1)**m*b**3*m**3*x**3*exp(b*x)*g
amma(m + 3) - 4*(-1)**m*b**3*m**2*x**3*exp(b*x)*gamma(m + 3) - 4*(-1)**m*b
**3*m*x**3*exp(b*x)*gamma(m + 3)) - 5*(-1)**m*b**4*b**m*b**(1 - m)*d**m*m*
x**4*x**m*exp(b*x)*gamma(m + 3)/(4*(-1)**m*b**3*m**4*x**3*exp(b*x)*gamm...
```

Maxima [F]

$$\int (dx)^m \Gamma(-2, bx) \, dx = \int (dx)^m \, \Gamma(-2, bx) \, dx$$

input
```
integrate((d*x)^m*gamma(-2,b*x),x, algorithm="maxima")
```

output
```
integrate((d*x)^m*gamma(-2, b*x), x)
```

Giac [F]

$$\int (dx)^m \Gamma(-2,bx)\,dx = \int (dx)^m\,\Gamma(-2,bx)\ dx$$

input
```
integrate((d*x)^m*gamma(-2,b*x),x, algorithm="giac")
```

output
```
integrate((d*x)^m*gamma(-2, b*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.37

$$\int (dx)^m \Gamma(-2,bx)\,dx = \int \frac{(d\,x)^m\,\mathrm{expint}(3,b\,x)}{b^2\,x^2}\,dx$$

input
```
int(((d*x)^m*expint(3, b*x))/(b^2*x^2),x)
```

output
```
int(((d*x)^m*expint(3, b*x))/(b^2*x^2), x)
```

Reduce [F]

$$\int (dx)^m \Gamma(-2,bx)\,dx = \frac{d^m \left(\int \frac{x^m ei(3,bx)}{x^2}dx\right)}{b^2}$$

input
```
int((d*x)^m/b^2/x^2*Ei(3,b*x),x)
```

output
```
(d**m*int((x**m*ei(3,b*x))/x**2,x))/b**2
```

3.82 $\int x^m \Gamma(n,x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 28

$$\int x^m \Gamma(n,x)\,dx = \frac{x^{1+m}\Gamma(n,x)}{1+m} - \frac{\Gamma(1+m+n,x)}{1+m}$$

output
```
x^(1+m)*GAMMA(n,x)/(1+m)-GAMMA(1+m+n,x)/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int x^m \Gamma(n,x)\,dx = \frac{x^{1+m}\Gamma(n,x) - \Gamma(1+m+n,x)}{1+m}$$

input
```
Integrate[x^m*Gamma[n, x],x]
```

output
```
(x^(1 + m)*Gamma[n, x] - Gamma[1 + m + n, x])/(1 + m)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \Gamma(n,x) \, dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{x^{m+1}\Gamma(n,x)}{m+1} - \frac{\Gamma(m+n+1,x)}{m+1}$$

input
```
Int[x^m*Gamma[n, x],x]
```

output
```
(x^(1 + m)*Gamma[n, x])/(1 + m) - Gamma[1 + m + n, x]/(1 + m)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.18

method	result	size
meijerg	$\frac{\pi x^{1+m} \csc(\pi n)}{(1+m)\Gamma(-n+1)} - \frac{x^{m+n+1} \text{hypergeom}([n,m+n+1],[1+n,2+m+n],-x)}{n(m+n+1)}$	61

input
```
int(x^m*GAMMA(n,x),x,method=_RETURNVERBOSE)
```

output
```
Pi*x^(1+m)*csc(Pi*n)/(1+m)/GAMMA(-n+1)-1/n/(m+n+1)*x^(m+n+1)*hypergeom([n,
m+n+1],[1+n,2+m+n],-x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int x^m \Gamma(n,x)\,dx = -\frac{xx^m x^{n-1}e^{(-x)} - xx^m\Gamma(n,x) + (m+n)\Gamma(m+n,x)}{m+1}$$

input
```
integrate(x^m*gamma(n,x),x, algorithm="fricas")
```

output
```
-(x*x^m*x^(n - 1)*e^(-x) - x*x^m*gamma(n, x) + (m + n)*gamma(m + n, x))/(m
 + 1)
```

Sympy [F]

$$\int x^m \Gamma(n,x)\,dx = \int x^m \Gamma(n,x)\,dx$$

input
```
integrate(x**m*uppergamma(n,x),x)
```

output
```
Integral(x**m*uppergamma(n, x), x)
```

Maxima [F]

$$\int x^m\Gamma(n,x)\,dx = \int x^m\Gamma(n,x)\;dx$$

input
```
integrate(x^m*gamma(n,x),x, algorithm="maxima")
```

output
```
integrate(x^m*gamma(n, x), x)
```

Giac [F]

$$\int x^m\Gamma(n,x)\,dx = \int x^m\Gamma(n,x)\;dx$$

input
```
integrate(x^m*gamma(n,x),x, algorithm="giac")
```

output
```
integrate(x^m*gamma(n, x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m\Gamma(n,x)\,dx = \int x^m\,\Gamma(n,x)\;dx$$

input
```
int(x^m*igamma(n, x),x)
```

output
```
int(x^m*igamma(n, x), x)
```

Reduce [F]

$$\int x^m \Gamma(n,x)\,dx = \int x^m \gamma(n,x)\,dx$$

input
```
int(x^m*GAMMA(n,x),x)
```

output
```
int(x**m*gamma(n,x),x)
```

3.83 $\int x^m \Gamma(n, bx)\, dx$

Optimal result

Integrand size = 9, antiderivative size = 45

$$\int x^m \Gamma(n, bx)\, dx = \frac{x^{1+m}\Gamma(n, bx)}{1+m} - \frac{x^m (bx)^{-m}\Gamma(1+m+n, bx)}{b(1+m)}$$

output
```
x^(1+m)*GAMMA(n,b*x)/(1+m)-x^m*GAMMA(1+m+n,b*x)/b/(1+m)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int x^m \Gamma(n, bx)\, dx = \frac{x^m (bx\Gamma(n, bx) - (bx)^{-m}\Gamma(1+m+n, bx))}{b(1+m)}$$

input
```
Integrate[x^m*Gamma[n, b*x],x]
```

output
```
(x^m*(b*x*Gamma[n, b*x] - Gamma[1 + m + n, b*x]/(b*x)^m))/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\Gamma(n,bx)\,dx$$

$$\Big\downarrow \text{ 7116}$$

$$\frac{x^{m+1}\Gamma(n,bx)}{m+1}-\frac{x^m(bx)^{-m}\Gamma(m+n+1,bx)}{b(m+1)}$$

input
```
Int[x^m*Gamma[n, b*x],x]
```

output
```
(x^(1 + m)*Gamma[n, b*x])/(1 + m) - (x^m*Gamma[1 + m + n, b*x])/(b*(1 + m)
*(b*x)^m)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.80

method	result	size
meijerg	$b^{-1-m}\left(\frac{\pi\, x^{1+m} b^{1+m}\csc(\pi n)}{(1+m)\Gamma(-n+1)}-\frac{x^{m+n+1} b^{m+n+1}\,\text{hypergeom}([n,m+n+1],[1+n,2+m+n],-bx)}{n(m+n+1)}\right)$	81

input
```
int(x^m*GAMMA(n,b*x),x,method=_RETURNVERBOSE)
```

output
```
b^(-1-m)*(Pi*x^(1+m)*b^(1+m)*csc(Pi*n)/(1+m)/GAMMA(-n+1)-1/n/(m+n+1)*x^(m+
n+1)*b^(m+n+1)*hypergeom([n,m+n+1],[1+n,2+m+n],-b*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

$$\int x^m\Gamma(n,bx)\,dx = -\frac{bxx^m e^{(-bx+(n-1)\log(b)+(n-1)\log(x))} - bxx^m\Gamma(n,bx) + \frac{(m+n)\Gamma(m+n,bx)}{b^m}}{bm+b}$$

input
```
integrate(x^m*gamma(n,b*x),x, algorithm="fricas")
```

output
```
-(b*x*x^m*e^(-b*x + (n - 1)*log(b) + (n - 1)*log(x)) - b*x*x^m*gamma(n, b*
x) + (m + n)*gamma(m + n, b*x)/b^m)/(b*m + b)
```

Sympy [F]

$$\int x^m\Gamma(n,bx)\,dx = \int x^m\Gamma(n,bx)\;dx$$

input
```
integrate(x**m*uppergamma(n,b*x),x)
```

output
```
Integral(x**m*uppergamma(n, b*x), x)
```

Maxima [F]

$$\int x^m\Gamma(n,bx)\,dx = \int x^m\Gamma(n,bx)\;dx$$

input
```
integrate(x^m*gamma(n,b*x),x, algorithm="maxima")
```

output
```
integrate(x^m*gamma(n, b*x), x)
```

Giac [F]

$$\int x^m\Gamma(n,bx)\,dx = \int x^m\Gamma(n,bx)\;dx$$

input
```
integrate(x^m*gamma(n,b*x),x, algorithm="giac")
```

output
```
integrate(x^m*gamma(n, b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m\Gamma(n,bx)\,dx = \int x^m\,\Gamma(n,b\,x)\;dx$$

input
```
int(x^m*igamma(n, b*x),x)
```

output
```
int(x^m*igamma(n, b*x), x)
```

Reduce [F]

$$\int x^m \Gamma(n, bx)\, dx = \int x^m \gamma(n, bx)\, dx$$

input
```
int(x^m*GAMMA(n,b*x),x)
```

output
```
int(x**m*gamma(n,b*x),x)
```

3.84 $\int (dx)^m\Gamma(n,x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 43

$$\int (dx)^m\Gamma(n,x)\,dx=\frac{(dx)^{1+m}\Gamma(n,x)}{d(1+m)}-\frac{x^{-m}(dx)^m\Gamma(1+m+n,x)}{1+m}$$

output
```
(d*x)^(1+m)*GAMMA(n,x)/d/(1+m)-(d*x)^m*GAMMA(1+m+n,x)/(1+m)/(x^m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int (dx)^m\Gamma(n,x)\,dx=\frac{x^{-m}(dx)^m\left(x^{1+m}\Gamma(n,x)-\Gamma(1+m+n,x)\right)}{1+m}$$

input
```
Integrate[(d*x)^m*Gamma[n, x],x]
```

output
```
((d*x)^m*(x^(1 + m)*Gamma[n, x] - Gamma[1 + m + n, x]))/((1 + m)*x^m)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \Gamma(n,x)\,dx$$

$$\Big\downarrow \; 7116$$

$$\frac{(dx)^{m+1}\Gamma(n,x)}{d(m+1)}-\frac{x^{-m}(dx)^m\Gamma(m+n+1,x)}{m+1}$$

input
```
Int[(d*x)^m*Gamma[n, x],x]
```

output
```
((d*x)^(1 + m)*Gamma[n, x])/(d*(1 + m)) - ((d*x)^m*Gamma[1 + m + n, x])/((
1 + m)*x^m)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.67

method	result	size
meijerg	$(dx)^m\, x^{-m}\left(\frac{\pi\, x^{1+m}\csc(\pi n)}{(1+m)\Gamma(-n+1)}-\frac{x^{m+n+1}\,\mathrm{hypergeom}([n,m+n+1],[1+n,2+m+n],-x)}{n(m+n+1)}\right)$	72

input
```
int((d*x)^m*GAMMA(n,x),x,method=_RETURNVERBOSE)
```

output
```
(d*x)^m*x^(-m)*(Pi*x^(1+m)*csc(Pi*n)/(1+m)/GAMMA(-n+1)-1/n/(m+n+1)*x^(m+n+
1)*hypergeom([n,m+n+1],[1+n,2+m+n],-x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int (dx)^m\Gamma(n,x)\,dx = -\frac{xx^{n-1}e^{(m\log(d)+m\log(x)-x)} + d^m(m+n)\Gamma(m+n,x) - xe^{(m\log(d)+m\log(x))}\Gamma(n,x)}{m+1}$$

input
```
integrate((d*x)^m*gamma(n,x),x, algorithm="fricas")
```

output
```
-(x*x^(n - 1)*e^(m*log(d) + m*log(x) - x) + d^m*(m + n)*gamma(m + n, x) -
x*e^(m*log(d) + m*log(x))*gamma(n, x))/(m + 1)
```

Sympy [F]

$$\int (dx)^m\Gamma(n,x)\,dx = \int (dx)^m\,\Gamma(n,x)\,dx$$

input
```
integrate((d*x)**m*uppergamma(n,x),x)
```

output
```
Integral((d*x)**m*uppergamma(n, x), x)
```

Maxima [F]

$$\int (dx)^m\Gamma(n,x)\,dx = \int (dx)^m\,\Gamma(n,x)\,dx$$

input
```
integrate((d*x)^m*gamma(n,x),x, algorithm="maxima")
```

output
```
integrate((d*x)^m*gamma(n, x), x)
```

Giac [F]

$$\int (dx)^m\Gamma(n,x)\,dx = \int (dx)^m\,\Gamma(n,x)\,dx$$

input
```
integrate((d*x)^m*gamma(n,x),x, algorithm="giac")
```

output
```
integrate((d*x)^m*gamma(n, x), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m\Gamma(n,x)\,dx = \int (d\,x)^m\,\Gamma(n,x)\,dx$$

input
```
int((d*x)^m*igamma(n, x),x)
```

output
```
int((d*x)^m*igamma(n, x), x)
```

Reduce [F]

$$\int (dx)^m \Gamma(n,x)\, dx = d^m \left(\int x^m \gamma(n,x)\, dx \right)$$

input
```
int((d*x)^m*GAMMA(n,x),x)
```

output
```
d**m*int(x**m*gamma(n,x),x)
```

3.85 $\int (dx)^m \Gamma(n,bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 52

$$\int (dx)^m \Gamma(n,bx)\,dx = \frac{(dx)^{1+m}\Gamma(n,bx)}{d(1+m)} - \frac{(bx)^{-m}(dx)^m\Gamma(1+m+n,bx)}{b(1+m)}$$

output
```
(d*x)^(1+m)*GAMMA(n,b*x)/d/(1+m)-(d*x)^m*GAMMA(1+m+n,b*x)/b/(1+m)/((b*x)^m
)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int (dx)^m \Gamma(n,bx)\,dx = \frac{(dx)^m\left(bx\Gamma(n,bx) - (bx)^{-m}\Gamma(1+m+n,bx)\right)}{b(1+m)}$$

input
```
Integrate[(d*x)^m*Gamma[n, b*x],x]
```

output
```
((d*x)^m*(b*x*Gamma[n, b*x] - Gamma[1 + m + n, b*x]/(b*x)^m))/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m\Gamma(n,bx)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{(dx)^{m+1}\Gamma(n,bx)}{d(m+1)}-\frac{(bx)^{-m}(dx)^m\Gamma(m+n+1,bx)}{b(m+1)}$$

input
```
Int[(d*x)^m*Gamma[n, b*x],x]
```

output
```
((d*x)^(1 + m)*Gamma[n, b*x])/(d*(1 + m)) - ((d*x)^m*Gamma[1 + m + n, b*x]
)/(b*(1 + m)*(b*x)^m)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.75

method	result	size
meijerg	$(dx)^m x^{-m} b^{-1-m}\left(\frac{\pi x^{1+m} b^{1+m}\csc(\pi n)}{(1+m)\Gamma(-n+1)}-\frac{x^{m+n+1} b^{m+n+1}\,\text{hypergeom}([n,m+n+1],[1+n,2+m+n],-bx)}{n(m+n+1)}\right)$	91

input
```
int((d*x)^m*GAMMA(n,b*x),x,method=_RETURNVERBOSE)
```

output
```
(d*x)^m*x^(-m)*b^(-1-m)*(Pi*x^(1+m)*b^(1+m)*csc(Pi*n)/(1+m)/GAMMA(-n+1)-1/
n/(m+n+1)*x^(m+n+1)*b^(m+n+1)*hypergeom([n,m+n+1],[1+n,2+m+n],-b*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.60

$$\int (dx)^m\Gamma(n,bx)\,dx = -\frac{(bx)^{n-1}\,bxe^{\left(-bx+m\log(bx)+m\log\left(\frac{d}{b}\right)\right)} - bxe^{\left(m\log(bx)+m\log\left(\frac{d}{b}\right)\right)}\Gamma(n,bx) + (m+n)\left(\frac{d}{b}\right)^m\Gamma(m+n,bx)}{bm+b}$$

input
```
integrate((d*x)^m*gamma(n,b*x),x, algorithm="fricas")
```

output
```
-((b*x)^(n - 1)*b*x*e^(-b*x + m*log(b*x) + m*log(d/b)) - b*x*e^(m*log(b*x)
 + m*log(d/b))*gamma(n, b*x) + (m + n)*(d/b)^m*gamma(m + n, b*x))/(b*m + b
)
```

Sympy [F]

$$\int (dx)^m\Gamma(n,bx)\,dx = \int (dx)^m\,\Gamma(n,bx)\;dx$$

input
```
integrate((d*x)**m*uppergamma(n,b*x),x)
```

output
```
Integral((d*x)**m*uppergamma(n, b*x), x)
```

Maxima [F]

$$\int (dx)^m\Gamma(n,bx)\,dx=\int (dx)^m\,\Gamma(n,bx)\,\,dx$$

input
```
integrate((d*x)^m*gamma(n,b*x),x, algorithm="maxima")
```

output
```
integrate((d*x)^m*gamma(n, b*x), x)
```

Giac [F]

$$\int (dx)^m\Gamma(n,bx)\,dx=\int (dx)^m\,\Gamma(n,bx)\,\,dx$$

input
```
integrate((d*x)^m*gamma(n,b*x),x, algorithm="giac")
```

output
```
integrate((d*x)^m*gamma(n, b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m\Gamma(n,bx)\,dx=\int \Gamma(n,b\,x)\,\,(d\,x)^m\,dx$$

input
```
int(igamma(n, b*x)*(d*x)^m,x)
```

output
```
int(igamma(n, b*x)*(d*x)^m, x)
```

Reduce [F]

$$\int (dx)^m\Gamma(n,bx)\,dx = d^m\left(\int x^m\gamma(n,bx)\,dx\right)$$

input
```
int((d*x)^m*GAMMA(n,b*x),x)
```

output
```
d**m*int(x**m*gamma(n,b*x),x)
```

3.86 $\int x^{100}\Gamma(n,ax)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 27

$$\int x^{100}\Gamma(n,ax)\,dx=\frac{1}{101}x^{101}\Gamma(n,ax)-\frac{\Gamma(101+n,ax)}{101a^{101}}$$

output
```
1/101*x^101*GAMMA(n,a*x)-1/101*GAMMA(101+n,a*x)/a^101
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^{100}\Gamma(n,ax)\,dx=\frac{1}{101}x^{101}\Gamma(n,ax)-\frac{\Gamma(101+n,ax)}{101a^{101}}$$

input
```
Integrate[x^100*Gamma[n, a*x],x]
```

output
```
(x^101*Gamma[n, a*x])/101 - Gamma[101 + n, a*x]/(101*a^101)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{100}\Gamma(n,ax)\,dx$$

$$\Big\downarrow \text{ 7116}$$

$$\frac{1}{101}x^{101}\Gamma(n,ax)-\frac{\Gamma(n+101,ax)}{101a^{101}}$$

input
```
Int[x^100*Gamma[n, a*x],x]
```

output
```
(x^101*Gamma[n, a*x])/101 - Gamma[101 + n, a*x]/(101*a^101)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.37

$$\frac{\frac{\pi \csc(\pi n) x^{101} a^{101}}{101\Gamma(-n+1)}-\frac{x^{101+n} a^{101+n}\,\text{hypergeom}([n,101+n],[1+n,102+n],-xa)}{n(101+n)}}{a^{101}}$$

input
```
int(x^100*GAMMA(n,x*a),x)
```

output
```
1/a^101*(1/101*Pi*csc(Pi*n)/GAMMA(-n+1)*x^101*a^101-1/n/(101+n)*x^(101+n)*
a^(101+n)*hypergeom([n,101+n],[1+n,102+n],-x*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41441 vs. 2(23) = 46.

Time = 3.03 (sec) , antiderivative size = 41441, normalized size of antiderivative = 1534.85

$$\int x^{100}\Gamma(n,ax)\,dx = \text{Too large to display}$$

input
```
integrate(x^100*gamma(n,a*x),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int x^{100}\Gamma(n,ax)\,dx = \text{Timed out}$$

input
```
integrate(x**100*uppergamma(n,a*x),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{100}\Gamma(n,ax)\,dx = \int x^{100}\Gamma(n,ax)\,dx$$

input
```
integrate(x^100*gamma(n,a*x),x, algorithm="maxima")
```

output
```
integrate(x^100*gamma(n, a*x), x)
```

Giac [F]

$$\int x^{100}\Gamma(n,ax)\,dx = \int x^{100}\Gamma(n,ax)\,dx$$

input
```
integrate(x^100*gamma(n,a*x),x, algorithm="giac")
```

output
```
integrate(x^100*gamma(n, a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{100}\Gamma(n,ax)\,dx = \int x^{100}\,\Gamma(n,a\,x)\,dx$$

input
```
int(x^100*igamma(n, a*x),x)
```

output
```
int(x^100*igamma(n, a*x), x)
```

Reduce [F]

$$\int x^{100}\Gamma(n,ax)\,dx=\int \gamma(n,ax)\,x^{100}dx$$

input
```
int(x^100*GAMMA(n,a*x),x)
```

output
```
int(gamma(n,a*x)*x**100,x)
```

3.87 $\int x^2\Gamma(n,ax)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 27

$$\int x^2\Gamma(n,ax)\,dx=\frac{1}{3}x^3\Gamma(n,ax)-\frac{\Gamma(3+n,ax)}{3a^3}$$

output
```
1/3*x^3*GAMMA(n,a*x)-1/3*GAMMA(3+n,a*x)/a^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^2\Gamma(n,ax)\,dx=\frac{1}{3}x^3\Gamma(n,ax)-\frac{\Gamma(3+n,ax)}{3a^3}$$

input
```
Integrate[x^2*Gamma[n, a*x],x]
```

output
```
(x^3*Gamma[n, a*x])/3 - Gamma[3 + n, a*x]/(3*a^3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\Gamma(n,ax)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{3}x^3\Gamma(n,ax)-\frac{\Gamma(n+3,ax)}{3a^3}$$

input
```
Int[x^2*Gamma[n, a*x],x]
```

output
```
(x^3*Gamma[n, a*x])/3 - Gamma[3 + n, a*x]/(3*a^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.37

method	result	size
meijerg	$\frac{\frac{\pi \csc(\pi n) x^3 a^3}{3\Gamma(-n+1)}-\frac{x^{3+n} a^{3+n}\,\text{hypergeom}([n,3+n],[1+n,4+n],-xa)}{n(3+n)}}{a^3}$	64

input
```
int(x^2*GAMMA(n,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/3*Pi*csc(Pi*n)/GAMMA(-n+1)*x^3*a^3-1/n/(3+n)*x^(3+n)*a^(3+n)*hype
rgeom([n,3+n],[1+n,4+n],-x*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(23) = 46$.

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.15

$$\int x^2\Gamma(n,ax)\,dx = -\frac{(a^3x^3+(a^2n+2\,a^2)x^2+(an^2+3\,an+2\,a)x)(ax)^{n-1}\,e^{(-ax)}-(a^3x^3-n^3-3\,n^2-2\,n)\Gamma(n,ax)}{3\,a^3}$$

input
```
integrate(x^2*gamma(n,a*x),x, algorithm="fricas")
```

output
```
-1/3*((a^3*x^3 + (a^2*n + 2*a^2)*x^2 + (a*n^2 + 3*a*n + 2*a)*x)*(a*x)^(n -
 1)*e^(-a*x) - (a^3*x^3 - n^3 - 3*n^2 - 2*n)*gamma(n, a*x))/a^3
```

Sympy [F]

$$\int x^2\Gamma(n,ax)\,dx = \int x^2\Gamma(n,ax)\,dx$$

input
```
integrate(x**2*uppergamma(n,a*x),x)
```

output
```
Integral(x**2*uppergamma(n, a*x), x)
```

Maxima [F]

$$\int x^2\Gamma(n,ax)\,dx = \int x^2\Gamma(n,ax)\;dx$$

input
```
integrate(x^2*gamma(n,a*x),x, algorithm="maxima")
```

output
```
integrate(x^2*gamma(n, a*x), x)
```

Giac [F]

$$\int x^2\Gamma(n,ax)\,dx = \int x^2\Gamma(n,ax)\;dx$$

input
```
integrate(x^2*gamma(n,a*x),x, algorithm="giac")
```

output
```
integrate(x^2*gamma(n, a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\Gamma(n,ax)\,dx = \int x^2\,\Gamma(n,a\,x)\;dx$$

input
```
int(x^2*igamma(n, a*x),x)
```

output
```
int(x^2*igamma(n, a*x), x)
```

Reduce [F]

$$\int x^2\Gamma(n,ax)\,dx = \int \gamma(n,ax)\,x^2dx$$

input
```
int(x^2*GAMMA(n,a*x),x)
```

output
```
int(gamma(n,a*x)*x**2,x)
```

3.88 $\int x\Gamma(n,ax)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 27

$$\int x\Gamma(n,ax)\,dx = \frac{1}{2}x^2\Gamma(n,ax) - \frac{\Gamma(2+n,ax)}{2a^2}$$

output
```
1/2*x^2*GAMMA(n,a*x)-1/2*GAMMA(2+n,a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x\Gamma(n,ax)\,dx = \frac{1}{2}x^2\Gamma(n,ax) - \frac{\Gamma(2+n,ax)}{2a^2}$$

input
```
Integrate[x*Gamma[n, a*x],x]
```

output
```
(x^2*Gamma[n, a*x])/2 - Gamma[2 + n, a*x]/(2*a^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\Gamma(n,ax)\,dx$$

$$\Big\downarrow \; 7116$$

$$\frac{1}{2}x^2\Gamma(n,ax)-\frac{\Gamma(n+2,ax)}{2a^2}$$

input
```
Int[x*Gamma[n, a*x],x]
```

output
```
(x^2*Gamma[n, a*x])/2 - Gamma[2 + n, a*x]/(2*a^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.37

method	result	size
meijerg	$\frac{\frac{\pi \csc(\pi n)x^2a^2}{2\Gamma(-n+1)}-\frac{x^{2+n}a^{2+n}\operatorname{hypergeom}([n,2+n],[1+n,3+n],-xa)}{n(2+n)}}{a^2}$	64

input
```
int(x*GAMMA(n,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/2*Pi*csc(Pi*n)/GAMMA(-n+1)*x^2*a^2-1/n/(2+n)*x^(2+n)*a^(2+n)*hype
rgeom([n,2+n],[1+n,3+n],-x*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(23) = 46$.

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int x\Gamma(n,ax)\,dx = -\frac{(a^2x^2 + (an+a)x)(ax)^{n-1}\,e^{(-ax)} - (a^2x^2 - n^2 - n)\Gamma(n,ax)}{2\,a^2}$$

input
```
integrate(x*gamma(n,a*x),x, algorithm="fricas")
```

output
```
-1/2*((a^2*x^2 + (a*n + a)*x)*(a*x)^(n - 1)*e^(-a*x) - (a^2*x^2 - n^2 - n)
*gamma(n, a*x))/a^2
```

Sympy [F]

$$\int x\Gamma(n,ax)\,dx = \int x\Gamma(n,ax)\,dx$$

input
```
integrate(x*uppergamma(n,a*x),x)
```

output
```
Integral(x*uppergamma(n, a*x), x)
```

Maxima [F]

$$\int x\Gamma(n,ax)\,dx = \int x\Gamma(n,ax)\,dx$$

input
```
integrate(x*gamma(n,a*x),x, algorithm="maxima")
```

output
```
integrate(x*gamma(n, a*x), x)
```

Giac [F]

$$\int x\Gamma(n,ax)\,dx = \int x\Gamma(n,ax)\,dx$$

input
```
integrate(x*gamma(n,a*x),x, algorithm="giac")
```

output
```
integrate(x*gamma(n, a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\Gamma(n,ax)\,dx = \int x\,\Gamma(n,a\,x)\,dx$$

input
```
int(x*igamma(n, a*x),x)
```

output
```
int(x*igamma(n, a*x), x)
```

Reduce [F]

$$\int x\Gamma(n,ax)\,dx=\int \gamma(n,ax)\,xdx$$

input
```
int(x*GAMMA(n,a*x),x)
```

output
```
int(gamma(n,a*x)*x,x)
```

3.89 $\int \Gamma(n, ax)\, dx$

Optimal result

Integrand size = 5, antiderivative size = 20

$$\int \Gamma(n, ax)\, dx = x\Gamma(n, ax) - \frac{\Gamma(1+n, ax)}{a}$$

output
```
x*GAMMA(n,a*x)-GAMMA(1+n,a*x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \Gamma(n, ax)\, dx = x\Gamma(n, ax) - \frac{\Gamma(1+n, ax)}{a}$$

input
```
Integrate[Gamma[n, a*x],x]
```

output
```
x*Gamma[n, a*x] - Gamma[1 + n, a*x]/a
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(n,ax)\,dx$$

$$\Big\downarrow \text{7111}$$

$$x\Gamma(n,ax)-\frac{\Gamma(n+1,ax)}{a}$$

input
```
Int[Gamma[n, a*x],x]
```

output
```
x*Gamma[n, a*x] - Gamma[1 + n, a*x]/a
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 5.30

method	result	size
meijerg	$\frac{\frac{\pi \csc(\pi n) x a}{\Gamma(-n+1)}-\frac{x^{n} a^{n} (x a-n) (x a)^{-\frac{n}{2}} {\mathrm e}^{-\frac{x a}{2}} \operatorname{WhittakerM}\left(\frac{n}{2}, \frac{n}{2}+\frac{1}{2}, x a\right)}{n (1+n)}-\frac{x^{n} a^{n} (x a)^{-\frac{n}{2}} {\mathrm e}^{-\frac{x a}{2}} \operatorname{WhittakerM}\left(\frac{n}{2}+1, \frac{n}{2}+\frac{1}{2}, x a\right)}{n}}{a}$	106

input
```
int(GAMMA(n,x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(Pi*csc(Pi*n)/GAMMA(-n+1)*x*a-1/n/(1+n)*x^n*a^n*(a*x-n)*(x*a)^(-1/2*n)
*exp(-1/2*x*a)*WhittakerM(1/2*n,1/2*n+1/2,x*a)-1/n*x^n*a^n*(x*a)^(-1/2*n)*
exp(-1/2*x*a)*WhittakerM(1/2*n+1,1/2*n+1/2,x*a))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \Gamma(n,ax)\,dx = -\frac{(ax)^{n-1}\,axe^{(-ax)} - (ax-n)\Gamma(n,ax)}{a}$$

input
```
integrate(gamma(n,a*x),x, algorithm="fricas")
```

output
```
-((a*x)^(n - 1)*a*x*e^(-a*x) - (a*x - n)*gamma(n, a*x))/a
```

Sympy [F]

$$\int \Gamma(n,ax)\,dx = \int \Gamma(n,ax)\,dx$$

input
```
integrate(uppergamma(n,a*x),x)
```

output
```
Integral(uppergamma(n, a*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \Gamma(n,ax)\,dx = \frac{ax\Gamma(n,ax) - \Gamma(n+1,ax)}{a}$$

input
```
integrate(gamma(n,a*x),x, algorithm="maxima")
```

output
```
(a*x*gamma(n, a*x) - gamma(n + 1, a*x))/a
```

Giac [F]

$$\int \Gamma(n,ax)\,dx = \int \Gamma(n,ax)\;dx$$

input
```
integrate(gamma(n,a*x),x, algorithm="giac")
```

output
```
integrate(gamma(n, a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \Gamma(n,ax)\,dx = \int \Gamma(n,a\,x)\;dx$$

input
```
int(igamma(n, a*x),x)
```

output
```
int(igamma(n, a*x), x)
```

Reduce [F]

$$\int \Gamma(n,ax)\,dx = \int \gamma(n,ax)\,dx$$

input
```
int(GAMMA(n,a*x),x)
```

output
```
int(gamma(n,a*x),x)
```

3.90 $\int \frac{\Gamma(n,ax)}{x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 31

$$\int \frac{\Gamma(n,ax)}{x} \, dx = -\frac{(ax)^n \, {}_2F_2(n,n;1+n,1+n;-ax)}{n^2} + \text{Gamma}(n)\log(x)$$

output
```
-(a*x)^n*hypergeom([n, n],[1+n, 1+n],-a*x)/n^2+GAMMA(n)*ln(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\Gamma(n,ax)}{x} \, dx = -\frac{(ax)^n \, {}_2F_2(n,n;1+n,1+n;-ax)}{n^2} + (\Gamma(n,ax)+\Gamma(n,0,ax))\log(x)$$

input
```
Integrate[Gamma[n, a*x]/x,x]
```

output
```
-(((a*x)^n*HypergeometricPFQ[{n, n}, {1 + n, 1 + n}, -(a*x)])/n^2) + (Gamm
a[n, a*x] + Gamma[n, 0, a*x])*Log[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7115}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(n,ax)}{x}\,dx$$

$$\Big\downarrow \text{ 7115}$$

$$\text{Gamma}(n)\log(x)-\frac{(ax)^n\,{}_2F_2(n,n;n+1,n+1;-ax)}{n^2}$$

input
```
Int[Gamma[n, a*x]/x,x]
```

output
```
-(((a*x)^n*HypergeometricPFQ[{n, n}, {1 + n, 1 + n}, -(a*x)])/n^2) + Gamma
[n]*Log[x]
```

Defintions of rubi rules used

rule 7115
```
Int[Gamma[n_, (b_.)*(x_)]/(x_), x_Symbol] :> Simp[Gamma[n]*Log[x], x] - Sim
p[((b*x)^n/n^2)*HypergeometricPFQ[{n, n}, {1 + n, 1 + n}, (-b)*x], x] /; Fr
eeQ[{b, n}, x] &&  !IntegerQ[n]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

method	result	size
meijerg	$\frac{(-\Psi(-n+1)+\pi\cot(\pi n)+\ln(x)+\ln(a))\pi\csc(\pi n)}{\Gamma(-n+1)}-\frac{x^n a^n\,\text{hypergeom}([n,n],[1+n,1+n],-xa)}{n^2}$	61

input
```
int(GAMMA(n,x*a)/x,x,method=_RETURNVERBOSE)
```

output
```
(-Psi(-n+1)+Pi*cot(Pi*n)+ln(x)+ln(a))*Pi*csc(Pi*n)/GAMMA(-n+1)-1/n^2*x^n*a
^n*hypergeom([n,n],[1+n,1+n],-x*a)
```

Fricas [F]

$$\int \frac{\Gamma(n, ax)}{x} dx = \int \frac{\Gamma(n, ax)}{x} dx$$

input
```
integrate(gamma(n,a*x)/x,x, algorithm="fricas")
```

output
```
integral(gamma(n, a*x)/x, x)
```

Sympy [F]

$$\int \frac{\Gamma(n, ax)}{x} dx = \int \frac{\Gamma(n, ax)}{x} dx$$

input
```
integrate(uppergamma(n,a*x)/x,x)
```

output
```
Integral(uppergamma(n, a*x)/x, x)
```

Maxima [F]

$$\int \frac{\Gamma(n, ax)}{x} dx = \int \frac{\Gamma(n, ax)}{x} dx$$

input
```
integrate(gamma(n,a*x)/x,x, algorithm="maxima")
```

output
```
integrate(gamma(n, a*x)/x, x)
```

Giac [F]

$$\int \frac{\Gamma(n,ax)}{x}\,dx = \int \frac{\Gamma(n,ax)}{x}\,dx$$

input
```
integrate(gamma(n,a*x)/x,x, algorithm="giac")
```

output
```
integrate(gamma(n, a*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(n,ax)}{x}\,dx = \int \frac{\Gamma(n,a\,x)}{x}\,dx$$

input
```
int(igamma(n, a*x)/x,x)
```

output
```
int(igamma(n, a*x)/x, x)
```

Reduce [F]

$$\int \frac{\Gamma(n,ax)}{x}\,dx = \int \frac{\gamma(n,ax)}{x}dx$$

input
```
int(GAMMA(n,a*x)/x,x)
```

output
```
int(gamma(n,a*x)/x,x)
```

3.91 $\int \frac{\Gamma(n,ax)}{x^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \frac{\Gamma(n,ax)}{x^2}\,dx = a\Gamma(-1+n,ax) - \frac{\Gamma(n,ax)}{x}$$

output
```
a*GAMMA(-1+n,a*x)-GAMMA(n,a*x)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(n,ax)}{x^2}\,dx = a\Gamma(-1+n,ax) - \frac{\Gamma(n,ax)}{x}$$

input
```
Integrate[Gamma[n, a*x]/x^2,x]
```

output
```
a*Gamma[-1 + n, a*x] - Gamma[n, a*x]/x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(n,ax)}{x^2}\,dx$$

$$\Big\downarrow \text{7116}$$

$$a\Gamma(n-1,ax)-\frac{\Gamma(n,ax)}{x}$$

input
```
Int[Gamma[n, a*x]/x^2,x]
```

output
```
a*Gamma[-1 + n, a*x] - Gamma[n, a*x]/x
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 7.40

method	result
meijerg	$a\left(-\frac{\pi\csc(\pi n)}{\Gamma(-n+1)xa}-\frac{\pi\csc(\pi n)}{\Gamma(-n+2)}-\frac{x^{-1+n}a^{-1+n}(xa-n+1)(xa)^{-\frac{n}{2}}\mathrm{e}^{-\frac{xa}{2}}\,\text{WhittakerM}\left(\frac{n}{2},\frac{n}{2}+\frac{1}{2},xa\right)}{n(-1+n)(1+n)}-\frac{x^{n-2}a^{n-2}(xa+1)(xa)}{}\right.$

input
```
int(GAMMA(n,x*a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-Pi*csc(Pi*n)/GAMMA(-n+1)/x/a-Pi*csc(Pi*n)/GAMMA(-n+2)-1/n/(-1+n)*x^(-1
+n)*a^(-1+n)*(a*x-n+1)/(1+n)*(x*a)^(-1/2*n)*exp(-1/2*x*a)*WhittakerM(1/2*n
,1/2*n+1/2,x*a)-1/n/(-1+n)*x^(n-2)*a^(n-2)*(a*x+1)*(x*a)^(-1/2*n)*exp(-1/2
*x*a)*WhittakerM(1/2*n+1,1/2*n+1/2,x*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

$$\int \frac{\Gamma(n,ax)}{x^2}\,dx = -\frac{(ax)^{n-1}\,axe^{(-ax)} - (ax-n+1)\Gamma(n,ax)}{(n-1)x}$$

input
```
integrate(gamma(n,a*x)/x^2,x, algorithm="fricas")
```

output
```
-((a*x)^(n - 1)*a*x*e^(-a*x) - (a*x - n + 1)*gamma(n, a*x))/((n - 1)*x)
```

Sympy [F]

$$\int \frac{\Gamma(n,ax)}{x^2}\,dx = \int \frac{\Gamma(n,ax)}{x^2}\,dx$$

input
```
integrate(uppergamma(n,a*x)/x**2,x)
```

output
```
Integral(uppergamma(n, a*x)/x**2, x)
```

Maxima [F]

$$\int \frac{\Gamma(n, ax)}{x^2} \, dx = \int \frac{\Gamma(n, ax)}{x^2} \, dx$$

input
```
integrate(gamma(n,a*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(gamma(n, a*x)/x^2, x)
```

Giac [F]

$$\int \frac{\Gamma(n, ax)}{x^2} \, dx = \int \frac{\Gamma(n, ax)}{x^2} \, dx$$

input
```
integrate(gamma(n,a*x)/x^2,x, algorithm="giac")
```

output
```
integrate(gamma(n, a*x)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(n, ax)}{x^2} \, dx = \int \frac{\Gamma(n, a\,x)}{x^2} \, dx$$

input
```
int(igamma(n, a*x)/x^2,x)
```

output
```
int(igamma(n, a*x)/x^2, x)
```

Reduce [F]

$$\int \frac{\Gamma(n, ax)}{x^2} \, dx = \int \frac{\gamma(n, ax)}{x^2} dx$$

input
```
int(GAMMA(n,a*x)/x^2,x)
```

output
```
int(gamma(n,a*x)/x**2,x)
```

3.92 $\int \frac{\Gamma(n,ax)}{x^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 27

$$\int \frac{\Gamma(n,ax)}{x^3} dx = \frac{1}{2}a^2\Gamma(-2+n,ax) - \frac{\Gamma(n,ax)}{2x^2}$$

output
```
1/2*a^2*GAMMA(-2+n,a*x)-1/2*GAMMA(n,a*x)/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(n,ax)}{x^3} dx = \frac{1}{2}a^2\Gamma(-2+n,ax) - \frac{\Gamma(n,ax)}{2x^2}$$

input
```
Integrate[Gamma[n, a*x]/x^3,x]
```

output
```
(a^2*Gamma[-2 + n, a*x])/2 - Gamma[n, a*x]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(n,ax)}{x^3}\,dx$$
$$\Big\downarrow \text{ 7116}$$
$$\frac{1}{2}a^2\Gamma(n-2,ax)-\frac{\Gamma(n,ax)}{2x^2}$$

input
```
Int[Gamma[n, a*x]/x^3,x]
```

output
```
(a^2*Gamma[-2 + n, a*x])/2 - Gamma[n, a*x]/(2*x^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.93

method	result	size
meijerg	$a^2\left(-\frac{\pi\csc(\pi n)}{2\Gamma(-n+1)x^2a^2}+\frac{\pi\csc(\pi n)}{2\Gamma(-n+3)}-\frac{x^{n-2}a^{n-2}\,\mathrm{hypergeom}([n,n-2],[-1+n,1+n],-xa)}{n(n-2)}\right)$	79

input
```
int(GAMMA(n,x*a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2*Pi*csc(Pi*n)/GAMMA(-n+1)/x^2/a^2+1/2*Pi*csc(Pi*n)/GAMMA(-n+3)-1/
n/(n-2)*x^(n-2)*a^(n-2)*hypergeom([n,n-2],[-1+n,1+n],-x*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(23) = 46$.

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.59

$$\int \frac{\Gamma(n,ax)}{x^3}\,dx = -\frac{(a^2x^2 + (an-a)x)(ax)^{n-1}\,e^{(-ax)} - (a^2x^2 - n^2 + 3\,n - 2)\Gamma(n,ax)}{2\,(n^2 - 3\,n + 2)x^2}$$

input
```
integrate(gamma(n,a*x)/x^3,x, algorithm="fricas")
```

output
```
-1/2*((a^2*x^2 + (a*n - a)*x)*(a*x)^(n - 1)*e^(-a*x) - (a^2*x^2 - n^2 + 3*
n - 2)*gamma(n, a*x))/((n^2 - 3*n + 2)*x^2)
```

Sympy [F]

$$\int \frac{\Gamma(n,ax)}{x^3}\,dx = \int \frac{\Gamma(n,ax)}{x^3}\,dx$$

input
```
integrate(uppergamma(n,a*x)/x**3,x)
```

output
```
Integral(uppergamma(n, a*x)/x**3, x)
```

Maxima [F]

$$\int \frac{\Gamma(n,ax)}{x^3}\,dx = \int \frac{\Gamma(n,ax)}{x^3}\,dx$$

input
```
integrate(gamma(n,a*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(gamma(n, a*x)/x^3, x)
```

Giac [F]

$$\int \frac{\Gamma(n,ax)}{x^3}\,dx = \int \frac{\Gamma(n,ax)}{x^3}\,dx$$

input
```
integrate(gamma(n,a*x)/x^3,x, algorithm="giac")
```

output
```
integrate(gamma(n, a*x)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(n,ax)}{x^3}\,dx = \int \frac{\Gamma(n,a\,x)}{x^3}\,dx$$

input
```
int(igamma(n, a*x)/x^3,x)
```

output
```
int(igamma(n, a*x)/x^3, x)
```

Reduce [F]

$$\int \frac{\Gamma(n, ax)}{x^3} \, dx = \int \frac{\gamma(n, ax)}{x^3} dx$$

input
```
int(GAMMA(n,a*x)/x^3,x)
```

output
```
int(gamma(n,a*x)/x**3,x)
```

3.93 $\int \frac{\Gamma(n,ax)}{x^4} dx$

Optimal result

Integrand size = 9, antiderivative size = 27

$$\int \frac{\Gamma(n,ax)}{x^4} dx = \frac{1}{3}a^3\Gamma(-3+n,ax) - \frac{\Gamma(n,ax)}{3x^3}$$

output
```
1/3*a^3*GAMMA(-3+n,a*x)-1/3*GAMMA(n,a*x)/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(n,ax)}{x^4} dx = \frac{1}{3}a^3\Gamma(-3+n,ax) - \frac{\Gamma(n,ax)}{3x^3}$$

input
```
Integrate[Gamma[n, a*x]/x^4,x]
```

output
```
(a^3*Gamma[-3 + n, a*x])/3 - Gamma[n, a*x]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(n, ax)}{x^4} dx$$
$$\Big\downarrow \; 7116$$
$$\frac{1}{3} a^3 \Gamma(n-3, ax) - \frac{\Gamma(n, ax)}{3x^3}$$

input
```
Int[Gamma[n, a*x]/x^4,x]
```

output
```
(a^3*Gamma[-3 + n, a*x])/3 - Gamma[n, a*x]/(3*x^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.63 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.93

method	result	size
meijerg	$a^3 \left(-\frac{\pi \csc(\pi n)}{3\Gamma(-n+1) x^3 a^3} - \frac{\pi \csc(\pi n)}{3\Gamma(-n+4)} - \frac{x^{n-3} a^{n-3} \operatorname{hypergeom}([n,n-3],[1+n,n-2],-xa)}{n(n-3)} \right)$	79

input
```
int(GAMMA(n,x*a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3*Pi*csc(Pi*n)/GAMMA(-n+1)/x^3/a^3-1/3*Pi*csc(Pi*n)/GAMMA(-n+4)-1/
n/(n-3)*x^(n-3)*a^(n-3)*hypergeom([n,n-3],[1+n,n-2],-x*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(23) = 46.

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.74

$$\int \frac{\Gamma(n, ax)}{x^4} dx = -\frac{(a^3x^3 + (a^2n - a^2)x^2 + (an^2 - 3\,an + 2\,a)x)(ax)^{n-1}\,e^{(-ax)} - (a^3x^3 - n^3 + 6\,n^2 - 11\,n + 6)\Gamma(n, ax)}{3\,(n^3 - 6\,n^2 + 11\,n - 6)x^3}$$

input
```
integrate(gamma(n,a*x)/x^4,x, algorithm="fricas")
```

output
```
-1/3*((a^3*x^3 + (a^2*n - a^2)*x^2 + (a*n^2 - 3*a*n + 2*a)*x)*(a*x)^(n - 1
)*e^(-a*x) - (a^3*x^3 - n^3 + 6*n^2 - 11*n + 6)*gamma(n, a*x))/((n^3 - 6*n
^2 + 11*n - 6)*x^3)
```

Sympy [F]

$$\int \frac{\Gamma(n, ax)}{x^4} dx = \int \frac{\Gamma(n, ax)}{x^4} dx$$

input
```
integrate(uppergamma(n,a*x)/x**4,x)
```

output
```
Integral(uppergamma(n, a*x)/x**4, x)
```

Maxima [F]

$$\int \frac{\Gamma(n,ax)}{x^4}\,dx=\int \frac{\Gamma(n,ax)}{x^4}\,dx$$

input
```
integrate(gamma(n,a*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(gamma(n, a*x)/x^4, x)
```

Giac [F]

$$\int \frac{\Gamma(n,ax)}{x^4}\,dx=\int \frac{\Gamma(n,ax)}{x^4}\,dx$$

input
```
integrate(gamma(n,a*x)/x^4,x, algorithm="giac")
```

output
```
integrate(gamma(n, a*x)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(n,ax)}{x^4}\,dx=\int \frac{\Gamma(n,a\,x)}{x^4}\,dx$$

input
```
int(igamma(n, a*x)/x^4,x)
```

output
```
int(igamma(n, a*x)/x^4, x)
```

Reduce [F]

$$\int \frac{\Gamma(n,ax)}{x^4}\,dx = \int \frac{\gamma(n,ax)}{x^4}dx$$

input
```
int(GAMMA(n,a*x)/x^4,x)
```

output
```
int(gamma(n,a*x)/x**4,x)
```

3.94 $\int x^{100}\Gamma(n,2x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int x^{100}\Gamma(n,2x)\,dx=\frac{1}{101}x^{101}\Gamma(n,2x)-\frac{\Gamma(101+n,2x)}{2560654212461023391023340474859520}$$

output
```
1/101*x^101*GAMMA(n,2*x)-1/2560654212461023391023340474859520*GAMMA(101+n,2
*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^{100}\Gamma(n,2x)\,dx=\frac{1}{101}x^{101}\Gamma(n,2x)-\frac{\Gamma(101+n,2x)}{2560654212461023391023340474859520}$$

input
```
Integrate[x^100*Gamma[n, 2*x],x]
```

output
```
(x^101*Gamma[n, 2*x])/101 - Gamma[101 + n, 2*x]/256065421246102339102334047
4859520
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{100}\Gamma(n,2x)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{101}x^{101}\Gamma(n,2x)-\frac{\Gamma(n+101,2x)}{2560654212461023391023340474859 52}$$

input
```
Int[x^100*Gamma[n, 2*x],x]
```

output
```
(x^101*Gamma[n, 2*x])/101 - Gamma[101 + n, 2*x]/2560654212461023391023340474859 52
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\frac{\pi \csc(\pi n)\,x^{101}}{101\Gamma(-n+1)}-\frac{x^{101+n}2^{n}\,\text{hypergeom}\left(\left[n,101+n\right],\left[1+n,102+n\right],-2x\right)}{n\,(101+n)}$$

input
```
int(x^100*GAMMA(n,2*x),x)
```

output
```
1/101*Pi*csc(Pi*n)/GAMMA(-n+1)*x^101-1/n/(101+n)*x^(101+n)*2^n*hypergeom([
n,101+n],[1+n,102+n],-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26081 vs. 2(20) = 40.

Time = 2.64 (sec) , antiderivative size = 26081, normalized size of antiderivative = 1086.71

$$\int x^{100}\Gamma(n,2x)\,dx = \text{Too large to display}$$

input
```
integrate(x^100*gamma(n,2*x),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int x^{100}\Gamma(n,2x)\,dx = \text{Timed out}$$

input
```
integrate(x**100*uppergamma(n,2*x),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{100}\Gamma(n,2x)\,dx = \int x^{100}\Gamma(n,2\,x)\ dx$$

input
```
integrate(x^100*gamma(n,2*x),x, algorithm="maxima")
```

output
```
integrate(x^100*gamma(n, 2*x), x)
```

Giac [F]

$$\int x^{100}\Gamma(n,2x)\,dx = \int x^{100}\Gamma(n,2\,x)\ dx$$

input
```
integrate(x^100*gamma(n,2*x),x, algorithm="giac")
```

output
```
integrate(x^100*gamma(n, 2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{100}\Gamma(n,2x)\,dx = \int x^{100}\,\Gamma(n,2\,x)\ dx$$

input
```
int(x^100*igamma(n, 2*x),x)
```

output
```
int(x^100*igamma(n, 2*x), x)
```

Reduce [F]

$$\int x^{100}\Gamma(n,2x)\,dx=\int \gamma(n,2x)\,x^{100}dx$$

input
```
int(x^100*GAMMA(n,2*x),x)
```

output
```
int(gamma(n,2*x)*x**100,x)
```

3.95 $\int x^2\Gamma(n,2x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int x^2\Gamma(n,2x)\,dx = \frac{1}{3}x^3\Gamma(n,2x) - \frac{1}{24}\Gamma(3+n,2x)$$

output
```
1/3*x^3*GAMMA(n,2*x)-1/24*GAMMA(3+n,2*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\Gamma(n,2x)\,dx = \frac{1}{3}x^3\Gamma(n,2x) - \frac{1}{24}\Gamma(3+n,2x)$$

input
```
Integrate[x^2*Gamma[n, 2*x],x]
```

output
```
(x^3*Gamma[n, 2*x])/3 - Gamma[3 + n, 2*x]/24
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\Gamma(n,2x)\,dx$$

$$\Big\downarrow \text{7116}$$

$$\frac{1}{3}x^3\Gamma(n,2x)-\frac{1}{24}\Gamma(n+3,2x)$$

input
```
Int[x^2*Gamma[n, 2*x],x]
```

output
```
(x^3*Gamma[n, 2*x])/3 - Gamma[3 + n, 2*x]/24
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

method	result	size
meijerg	$\frac{\pi \csc(\pi n) x^3}{3\Gamma(-n+1)}-\frac{x^{3+n} 2^n \operatorname{hypergeom}([n,3+n],[1+n,4+n],-2x)}{n(3+n)}$	54

input
```
int(x^2*GAMMA(n,2*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*Pi*csc(Pi*n)/GAMMA(-n+1)*x^3-1/n/(3+n)*x^(3+n)*2^n*hypergeom([n,3+n],[
1+n,4+n],-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(20) = 40$.

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int x^2\Gamma(n,2x)\,dx = -\frac{1}{12}\left(2\,(n+2)x^2+4\,x^3+\left(n^2+3\,n+2\right)x\right)(2\,x)^{n-1}\,e^{(-2\,x)} - \frac{1}{24}\left(n^3-8\,x^3+3\,n^2+2\,n\right)\Gamma(n,2\,x)$$

input
```
integrate(x^2*gamma(n,2*x),x, algorithm="fricas")
```

output
```
-1/12*(2*(n + 2)*x^2 + 4*x^3 + (n^2 + 3*n + 2)*x)*(2*x)^(n - 1)*e^(-2*x) -
 1/24*(n^3 - 8*x^3 + 3*n^2 + 2*n)*gamma(n, 2*x)
```

Sympy [F]

$$\int x^2\Gamma(n,2x)\,dx = \int x^2\Gamma(n,2x)\,dx$$

input
```
integrate(x**2*uppergamma(n,2*x),x)
```

output
```
Integral(x**2*uppergamma(n, 2*x), x)
```

Maxima [F]

$$\int x^2\Gamma(n,2x)\,dx = \int x^2\Gamma(n,2\,x)\,dx$$

input
```
integrate(x^2*gamma(n,2*x),x, algorithm="maxima")
```

output
```
integrate(x^2*gamma(n, 2*x), x)
```

Giac [F]

$$\int x^2\Gamma(n,2x)\,dx = \int x^2\Gamma(n,2\,x)\,dx$$

input
```
integrate(x^2*gamma(n,2*x),x, algorithm="giac")
```

output
```
integrate(x^2*gamma(n, 2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\Gamma(n,2x)\,dx = \int x^2\,\Gamma(n,2\,x)\,dx$$

input
```
int(x^2*igamma(n, 2*x),x)
```

output
```
int(x^2*igamma(n, 2*x), x)
```

Reduce [F]

$$\int x^2\Gamma(n,2x)\,dx = \int \gamma(n,2x)\,x^2dx$$

input
```
int(x^2*GAMMA(n,2*x),x)
```

output
```
int(gamma(n,2*x)*x**2,x)
```

3.96 $\int x\Gamma(n,2x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 24

$$\int x\Gamma(n,2x)\,dx=\frac{1}{2}x^2\Gamma(n,2x)-\frac{1}{8}\Gamma(2+n,2x)$$

output
```
1/2*x^2*GAMMA(n,2*x)-1/8*GAMMA(2+n,2*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x\Gamma(n,2x)\,dx=\frac{1}{2}x^2\Gamma(n,2x)-\frac{1}{8}\Gamma(2+n,2x)$$

input
```
Integrate[x*Gamma[n, 2*x],x]
```

output
```
(x^2*Gamma[n, 2*x])/2 - Gamma[2 + n, 2*x]/8
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\Gamma(n,2x)\,dx$$

$$\Big\downarrow \text{ 7116}$$

$$\frac{1}{2}x^2\Gamma(n,2x)-\frac{1}{8}\Gamma(n+2,2x)$$

input
```
Int[x*Gamma[n, 2*x],x]
```

output
```
(x^2*Gamma[n, 2*x])/2 - Gamma[2 + n, 2*x]/8
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

method	result	size
meijerg	$\frac{\pi \csc(\pi n) x^2}{2\Gamma(-n+1)}-\frac{x^{2+n} 2^n \operatorname{hypergeom}([n,2+n],[1+n,3+n],-2x)}{n(2+n)}$	54

input
```
int(x*GAMMA(n,2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*Pi*csc(Pi*n)/GAMMA(-n+1)*x^2-1/n/(2+n)*x^(2+n)*2^n*hypergeom([n,2+n],[
1+n,3+n],-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int x\Gamma(n,2x)\,dx = -\frac{1}{4}\left((n+1)x+2\,x^2\right)(2\,x)^{n-1}\,e^{(-2\,x)} - \frac{1}{8}\left(n^2-4\,x^2+n\right)\Gamma(n,2\,x)$$

input
```
integrate(x*gamma(n,2*x),x, algorithm="fricas")
```

output
```
-1/4*((n + 1)*x + 2*x^2)*(2*x)^(n - 1)*e^(-2*x) - 1/8*(n^2 - 4*x^2 + n)*ga
mma(n, 2*x)
```

Sympy [F]

$$\int x\Gamma(n,2x)\,dx = \int x\Gamma(n,2x)\;dx$$

input
```
integrate(x*uppergamma(n,2*x),x)
```

output
```
Integral(x*uppergamma(n, 2*x), x)
```

Maxima [F]

$$\int x\Gamma(n,2x)\,dx = \int x\Gamma(n,2\,x)\,\,dx$$

input
```
integrate(x*gamma(n,2*x),x, algorithm="maxima")
```

output
```
integrate(x*gamma(n, 2*x), x)
```

Giac [F]

$$\int x\Gamma(n,2x)\,dx = \int x\Gamma(n,2\,x)\,\,dx$$

input
```
integrate(x*gamma(n,2*x),x, algorithm="giac")
```

output
```
integrate(x*gamma(n, 2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\Gamma(n,2x)\,dx = \int x\,\Gamma(n,2\,x)\,\,dx$$

input
```
int(x*igamma(n, 2*x),x)
```

output
```
int(x*igamma(n, 2*x), x)
```

Reduce [F]

$$\int x\Gamma(n,2x)\,dx = \int \gamma(n,2x)\,x dx$$

input
```
int(x*GAMMA(n,2*x),x)
```

output
```
int(gamma(n,2*x)*x,x)
```

3.97 $\int \Gamma(n,2x)\,dx$

Optimal result

Integrand size = 5, antiderivative size = 19

$$\int \Gamma(n,2x)\,dx = x\Gamma(n,2x) - \frac{1}{2}\Gamma(1+n,2x)$$

output
```
x*GAMMA(n,2*x)-1/2*GAMMA(1+n,2*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \Gamma(n,2x)\,dx = x\Gamma(n,2x) - \frac{1}{2}\Gamma(1+n,2x)$$

input
```
Integrate[Gamma[n, 2*x],x]
```

output
```
x*Gamma[n, 2*x] - Gamma[1 + n, 2*x]/2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(n,2x)\,dx$$

$$\Big\downarrow \text{7111}$$

$$x\Gamma(n,2x)-\frac{1}{2}\Gamma(n+1,2x)$$

input
```
Int[Gamma[n, 2*x],x]
```

output
```
x*Gamma[n, 2*x] - Gamma[1 + n, 2*x]/2
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 5.11

method	result	size
meijerg	$\frac{\pi \csc(\pi n) x}{\Gamma(-n+1)}-\frac{2^{-1+\frac{n}{2}} x^{\frac{n}{2}} (2x-n) {\mathrm e}^{-x} \operatorname{WhittakerM}\left(\frac{n}{2}, \frac{n}{2}+\frac{1}{2}, 2x\right)}{n(1+n)}-\frac{2^{-1+\frac{n}{2}} x^{\frac{n}{2}} {\mathrm e}^{-x} \operatorname{WhittakerM}\left(\frac{n}{2}+1, \frac{n}{2}+\frac{1}{2}, 2x\right)}{n}$	97

input
```
int(GAMMA(n,2*x),x,method=_RETURNVERBOSE)
```

output
```
Pi*csc(Pi*n)/GAMMA(-n+1)*x-2^(-1+1/2*n)/n/(1+n)*x^(1/2*n)*(2*x-n)*exp(-x)*
WhittakerM(1/2*n,1/2*n+1/2,2*x)-2^(-1+1/2*n)/n*x^(1/2*n)*exp(-x)*Whittaker
M(1/2*n+1,1/2*n+1/2,2*x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \Gamma(n,2x)\,dx = -(2\,x)^{n-1}\,xe^{(-2\,x)} - \frac{1}{2}\,(n-2\,x)\Gamma(n,2\,x)$$

input
```
integrate(gamma(n,2*x),x, algorithm="fricas")
```

output
```
-(2*x)^(n - 1)*x*e^(-2*x) - 1/2*(n - 2*x)*gamma(n, 2*x)
```

Sympy [F]

$$\int \Gamma(n,2x)\,dx = \int \Gamma(n,2x)\;dx$$

input
```
integrate(uppergamma(n,2*x),x)
```

output
```
Integral(uppergamma(n, 2*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \Gamma(n,2x)\,dx = x\Gamma(n,2\,x) - \frac{1}{2}\,\Gamma(n+1,2\,x)$$

input
```
integrate(gamma(n,2*x),x, algorithm="maxima")
```

output
```
x*gamma(n, 2*x) - 1/2*gamma(n + 1, 2*x)
```

Giac [F]

$$\int \Gamma(n,2x)\,dx = \int \Gamma(n,2\,x)\,\,dx$$

input
```
integrate(gamma(n,2*x),x, algorithm="giac")
```

output
```
integrate(gamma(n, 2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \Gamma(n,2x)\,dx = \int \Gamma(n,2\,x)\,\,dx$$

input
```
int(igamma(n, 2*x),x)
```

output
```
int(igamma(n, 2*x), x)
```

Reduce [F]

$$\int \Gamma(n,2x)\,dx = \int \gamma(n,2x)\,dx$$

input
```
int(GAMMA(n,2*x),x)
```

output
```
int(gamma(n,2*x),x)
```

3.98 $\int \frac{\Gamma(n,2x)}{x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 31

$$\int \frac{\Gamma(n,2x)}{x} \, dx = -\frac{2^n x^n \, {}_2F_2(n,n;1+n,1+n;-2x)}{n^2} + \text{Gamma}(n)\log(x)$$

output
```
-2^n*x^n*hypergeom([n, n],[1+n, 1+n],-2*x)/n^2+GAMMA(n)*ln(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int \frac{\Gamma(n,2x)}{x} \, dx = -\frac{2^n x^n \, {}_2F_2(n,n;1+n,1+n;-2x)}{n^2} + (\Gamma(n,2x) + \Gamma(n,0,2x))\log(2x)$$

input
```
Integrate[Gamma[n, 2*x]/x,x]
```

output
```
-((2^n*x^n*HypergeometricPFQ[{n, n}, {1 + n, 1 + n}, -2*x])/n^2) + (Gamma[
n, 2*x] + Gamma[n, 0, 2*x])*Log[2*x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7115}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(n,2x)}{x} \, dx$$

$$\Big\downarrow \text{ 7115}$$

$$\text{Gamma}(n)\log(x) - \frac{2^n x^n \, {}_2F_2(n,n;n+1,n+1;-2x)}{n^2}$$

input
```
Int[Gamma[n, 2*x]/x,x]
```

output
```
-((2^n*x^n*HypergeometricPFQ[{n, n}, {1 + n, 1 + n}, -2*x])/n^2) + Gamma[n
]*Log[x]
```

Defintions of rubi rules used

rule 7115
```
Int[Gamma[n_, (b_.)*(x_)]/(x_), x_Symbol] :> Simp[Gamma[n]*Log[x], x] - Sim
p[((b*x)^n/n^2)*HypergeometricPFQ[{n, n}, {1 + n, 1 + n}, (-b)*x], x] /; Fr
eeQ[{b, n}, x] && !IntegerQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

method	result	size
meijerg	$\frac{(-\Psi(-n+1)+\pi\cot(\pi n)+\ln(x)+\ln(2))\pi\csc(\pi n)}{\Gamma(-n+1)} - \frac{2^n x^n \operatorname{hypergeom}([n,n],[1+n,1+n],-2x)}{n^2}$	60

input
```
int(GAMMA(n,2*x)/x,x,method=_RETURNVERBOSE)
```

output
```
(-Psi(-n+1)+Pi*cot(Pi*n)+ln(x)+ln(2))*Pi*csc(Pi*n)/GAMMA(-n+1)-2^n*x^n*hyp
ergeom([n,n],[1+n,1+n],-2*x)/n^2
```

Fricas [F]

$$\int \frac{\Gamma(n,2x)}{x}\,dx = \int \frac{\Gamma(n,2\,x)}{x}\,dx$$

input
```
integrate(gamma(n,2*x)/x,x, algorithm="fricas")
```

output
```
integral(gamma(n, 2*x)/x, x)
```

Sympy [F]

$$\int \frac{\Gamma(n,2x)}{x}\,dx = \int \frac{\Gamma(n,2x)}{x}\,dx$$

input
```
integrate(uppergamma(n,2*x)/x,x)
```

output
```
Integral(uppergamma(n, 2*x)/x, x)
```

Maxima [F]

$$\int \frac{\Gamma(n,2x)}{x}\,dx = \int \frac{\Gamma(n,2\,x)}{x}\,dx$$

input
```
integrate(gamma(n,2*x)/x,x, algorithm="maxima")
```

output
```
integrate(gamma(n, 2*x)/x, x)
```

Giac [F]

$$\int \frac{\Gamma(n,2x)}{x}\,dx = \int \frac{\Gamma(n,2\,x)}{x}\,dx$$

input
```
integrate(gamma(n,2*x)/x,x, algorithm="giac")
```

output
```
integrate(gamma(n, 2*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(n,2x)}{x}\,dx = \int \frac{\Gamma(n,2\,x)}{x}\,dx$$

input
```
int(igamma(n, 2*x)/x,x)
```

output
```
int(igamma(n, 2*x)/x, x)
```

Reduce [F]

$$\int \frac{\Gamma(n,2x)}{x}\,dx = \int \frac{\gamma(n,2x)}{x}dx$$

input
```
int(GAMMA(n,2*x)/x,x)
```

output
```
int(gamma(n,2*x)/x,x)
```

3.99 $\int \frac{\Gamma(n,2x)}{x^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \frac{\Gamma(n,2x)}{x^2} dx = 2\Gamma(-1+n,2x) - \frac{\Gamma(n,2x)}{x}$$

output
```
2*GAMMA(-1+n,2*x)-GAMMA(n,2*x)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(n,2x)}{x^2} dx = 2\Gamma(-1+n,2x) - \frac{\Gamma(n,2x)}{x}$$

input
```
Integrate[Gamma[n, 2*x]/x^2,x]
```

output
```
2*Gamma[-1 + n, 2*x] - Gamma[n, 2*x]/x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(n,2x)}{x^2} \, dx$$

$$\Big\downarrow \; 7116$$

$$2\Gamma(n-1,2x)-\frac{\Gamma(n,2x)}{x}$$

input
```
Int[Gamma[n, 2*x]/x^2,x]
```

output
```
2*Gamma[-1 + n, 2*x] - Gamma[n, 2*x]/x
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 6.65

method	result
meijerg	$-\frac{\pi \csc(\pi n)}{x\Gamma(-n+1)}-\frac{2\pi \csc(\pi n)}{\Gamma(-n+2)}-\frac{x^{-1+\frac{n}{2}} 2^{\frac{n}{2}} (2x-n+1) \mathrm{e}^{-x} \operatorname{WhittakerM}\left(\frac{n}{2},\frac{n}{2}+\frac{1}{2},2x\right)}{n(-1+n)(1+n)}-\frac{2^{-1+\frac{n}{2}} x^{\frac{n}{2}-2} (1+2x) \mathrm{e}^{-x} \operatorname{WhittakerM}(\ldots}{n(-1+n)}$

input
```
int(GAMMA(n,2*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*Pi*csc(Pi*n)/GAMMA(-n+1)-2*Pi*csc(Pi*n)/GAMMA(-n+2)-1/n/(-1+n)*x^(-1+
1/2*n)*2^(1/2*n)*(2*x-n+1)/(1+n)*exp(-x)*WhittakerM(1/2*n,1/2*n+1/2,2*x)-2
^(-1+1/2*n)/n/(-1+n)*x^(1/2*n-2)*(1+2*x)*exp(-x)*WhittakerM(1/2*n+1,1/2*n+
1/2,2*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{\Gamma(n,2x)}{x^2}\,dx = -\frac{2\,(2\,x)^{n-1}\,xe^{(-2\,x)} + (n-2\,x-1)\Gamma(n,2\,x)}{(n-1)x}$$

input
```
integrate(gamma(n,2*x)/x^2,x, algorithm="fricas")
```

output
```
-(2*(2*x)^(n - 1)*x*e^(-2*x) + (n - 2*x - 1)*gamma(n, 2*x))/((n - 1)*x)
```

Sympy [F]

$$\int \frac{\Gamma(n,2x)}{x^2}\,dx = \int \frac{\Gamma(n,2x)}{x^2}\,dx$$

input
```
integrate(uppergamma(n,2*x)/x**2,x)
```

output
```
Integral(uppergamma(n, 2*x)/x**2, x)
```

Maxima [F]

$$\int \frac{\Gamma(n,2x)}{x^2}\,dx = \int \frac{\Gamma(n,2\,x)}{x^2}\,dx$$

input
```
integrate(gamma(n,2*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(gamma(n, 2*x)/x^2, x)
```

Giac [F]

$$\int \frac{\Gamma(n,2x)}{x^2}\,dx = \int \frac{\Gamma(n,2\,x)}{x^2}\,dx$$

input
```
integrate(gamma(n,2*x)/x^2,x, algorithm="giac")
```

output
```
integrate(gamma(n, 2*x)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(n,2x)}{x^2}\,dx = \int \frac{\Gamma(n,2\,x)}{x^2}\,dx$$

input
```
int(igamma(n, 2*x)/x^2,x)
```

output
```
int(igamma(n, 2*x)/x^2, x)
```

Reduce [F]

$$\int \frac{\Gamma(n, 2x)}{x^2} \, dx = \int \frac{\gamma(n, 2x)}{x^2} dx$$

input
```
int(GAMMA(n,2*x)/x^2,x)
```

output
```
int(gamma(n,2*x)/x**2,x)
```

3.100 $\int \frac{\Gamma(n,2x)}{x^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 22

$$\int \frac{\Gamma(n,2x)}{x^3} dx = 2\Gamma(-2+n,2x) - \frac{\Gamma(n,2x)}{2x^2}$$

output
```
2*GAMMA(-2+n,2*x)-1/2*GAMMA(n,2*x)/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(n,2x)}{x^3} dx = 2\Gamma(-2+n,2x) - \frac{\Gamma(n,2x)}{2x^2}$$

input
```
Integrate[Gamma[n, 2*x]/x^3,x]
```

output
```
2*Gamma[-2 + n, 2*x] - Gamma[n, 2*x]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(n,2x)}{x^3} \, dx$$
$$\Big\downarrow \text{7116}$$
$$2\Gamma(n-2,2x)-\frac{\Gamma(n,2x)}{2x^2}$$

input
```
Int[Gamma[n, 2*x]/x^3,x]
```

output
```
2*Gamma[-2 + n, 2*x] - Gamma[n, 2*x]/(2*x^2)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.14

method	result	size
meijerg	$-\frac{\pi \csc(\pi n)}{2x^2\Gamma(-n+1)}+\frac{2\pi \csc(\pi n)}{\Gamma(-n+3)}-\frac{x^{n-2}2^n \operatorname{hypergeom}([n,n-2],[-1+n,1+n],-2x)}{n(n-2)}$	69

input
```
int(GAMMA(n,2*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2*Pi*csc(Pi*n)/GAMMA(-n+1)+2*Pi*csc(Pi*n)/GAMMA(-n+3)-1/n/(n-2)*x^(
n-2)*2^n*hypergeom([n,n-2],[-1+n,1+n],-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(20) = 40$.

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.68

$$\int \frac{\Gamma(n,2x)}{x^3}\,dx = -\frac{2\,((n-1)x+2\,x^2)(2\,x)^{n-1}\,e^{(-2\,x)}+(n^2-4\,x^2-3\,n+2)\Gamma(n,2\,x)}{2\,(n^2-3\,n+2)x^2}$$

input
```
integrate(gamma(n,2*x)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(2*((n - 1)*x + 2*x^2)*(2*x)^(n - 1)*e^(-2*x) + (n^2 - 4*x^2 - 3*n +
2)*gamma(n, 2*x))/((n^2 - 3*n + 2)*x^2)
```

Sympy [F]

$$\int \frac{\Gamma(n,2x)}{x^3}\,dx = \int \frac{\Gamma(n,2x)}{x^3}\,dx$$

input
```
integrate(uppergamma(n,2*x)/x**3,x)
```

output
```
Integral(uppergamma(n, 2*x)/x**3, x)
```

Maxima [F]

$$\int \frac{\Gamma(n,2x)}{x^3}\,dx = \int \frac{\Gamma(n,2\,x)}{x^3}\,dx$$

input
```
integrate(gamma(n,2*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(gamma(n, 2*x)/x^3, x)
```

Giac [F]

$$\int \frac{\Gamma(n,2x)}{x^3}\,dx = \int \frac{\Gamma(n,2\,x)}{x^3}\,dx$$

input
```
integrate(gamma(n,2*x)/x^3,x, algorithm="giac")
```

output
```
integrate(gamma(n, 2*x)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(n,2x)}{x^3}\,dx = \int \frac{\Gamma(n,2\,x)}{x^3}\,dx$$

input
```
int(igamma(n, 2*x)/x^3,x)
```

output
```
int(igamma(n, 2*x)/x^3, x)
```

Reduce [F]

$$\int \frac{\Gamma(n, 2x)}{x^3} \, dx = \int \frac{\gamma(n, 2x)}{x^3} dx$$

input
```
int(GAMMA(n,2*x)/x^3,x)
```

output
```
int(gamma(n,2*x)/x**3,x)
```

3.101 $\int \frac{\Gamma(n,2x)}{x^4} dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{\Gamma(n,2x)}{x^4} dx = \frac{8}{3}\Gamma(-3+n,2x) - \frac{\Gamma(n,2x)}{3x^3}$$

output
```
8/3*GAMMA(-3+n,2*x)-1/3*GAMMA(n,2*x)/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(n,2x)}{x^4} dx = \frac{8}{3}\Gamma(-3+n,2x) - \frac{\Gamma(n,2x)}{3x^3}$$

input
```
Integrate[Gamma[n, 2*x]/x^4,x]
```

output
```
(8*Gamma[-3 + n, 2*x])/3 - Gamma[n, 2*x]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7116}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(n,2x)}{x^4}\,dx$$
$$\Big\downarrow \; 7116$$
$$\frac{8}{3}\Gamma(n-3,2x)-\frac{\Gamma(n,2x)}{3x^3}$$

input
```
Int[Gamma[n, 2*x]/x^4,x]
```

output
```
(8*Gamma[-3 + n, 2*x])/3 - Gamma[n, 2*x]/(3*x^3)
```

Defintions of rubi rules used

rule 7116
```
Int[Gamma[n_, (b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m +
1)*(Gamma[n, b*x]/(d*(m + 1))), x] - Simp[(d*x)^m*(Gamma[m + n + 1, b*x]/(b
*(m + 1)*(b*x)^m)), x] /; FreeQ[{b, d, m, n}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.88

method	result	size
meijerg	$-\frac{\pi\csc(\pi n)}{3x^3\Gamma(-n+1)}-\frac{8\pi\csc(\pi n)}{3\Gamma(-n+4)}-\frac{x^{n-3}2^n\,\text{hypergeom}([n,n-3],[1+n,n-2],-2x)}{n(n-3)}$	69

input
```
int(GAMMA(n,2*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3*Pi*csc(Pi*n)/GAMMA(-n+1)-8/3*Pi*csc(Pi*n)/GAMMA(-n+4)-1/n/(n-3)*x
^(n-3)*2^n*hypergeom([n,n-3],[1+n,n-2],-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(20) = 40$.

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.42

$$\int \frac{\Gamma(n,2x)}{x^4}\,dx = -\frac{2\left(2\,(n-1)x^2+4\,x^3+(n^2-3\,n+2)x\right)(2\,x)^{n-1}\,e^{(-2\,x)}+(n^3-8\,x^3-6\,n^2+11\,n-6)\Gamma(n,2\,x)}{3\,(n^3-6\,n^2+11\,n-6)x^3}$$

input
```
integrate(gamma(n,2*x)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(2*(2*(n - 1)*x^2 + 4*x^3 + (n^2 - 3*n + 2)*x)*(2*x)^(n - 1)*e^(-2*x)
 + (n^3 - 8*x^3 - 6*n^2 + 11*n - 6)*gamma(n, 2*x))/((n^3 - 6*n^2 + 11*n -
6)*x^3)
```

Sympy [F]

$$\int \frac{\Gamma(n,2x)}{x^4}\,dx = \int \frac{\Gamma(n,2x)}{x^4}\,dx$$

input
```
integrate(uppergamma(n,2*x)/x**4,x)
```

output
```
Integral(uppergamma(n, 2*x)/x**4, x)
```

Maxima [F]

$$\int \frac{\Gamma(n,2x)}{x^4}\,dx = \int \frac{\Gamma(n,2\,x)}{x^4}\,dx$$

input
```
integrate(gamma(n,2*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(gamma(n, 2*x)/x^4, x)
```

Giac [F]

$$\int \frac{\Gamma(n,2x)}{x^4}\,dx = \int \frac{\Gamma(n,2\,x)}{x^4}\,dx$$

input
```
integrate(gamma(n,2*x)/x^4,x, algorithm="giac")
```

output
```
integrate(gamma(n, 2*x)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(n,2x)}{x^4}\,dx = \int \frac{\Gamma(n,2\,x)}{x^4}\,dx$$

input
```
int(igamma(n, 2*x)/x^4,x)
```

output
```
int(igamma(n, 2*x)/x^4, x)
```

Reduce [F]

$$\int \frac{\Gamma(n, 2x)}{x^4} \, dx = \int \frac{\gamma(n, 2x)}{x^4} dx$$

input
```
int(GAMMA(n,2*x)/x^4,x)
```

output
```
int(gamma(n,2*x)/x**4,x)
```

3.102 $\int (c+dx)^3\Gamma(0,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 194

$$\int (c+dx)^3\Gamma(0,a+bx)\,dx = -\frac{(bc-ad)^3e^{-a-bx}}{4b^4} - \frac{(bc-ad)^4\Gamma(0,a+bx)}{4b^4d} + \frac{(c+dx)^4\Gamma(0,a+bx)}{4d} - \frac{d(bc-ad)^2e^{-a+\frac{bc}{d}}\Gamma\left(2,\frac{b(c+dx)}{d}\right)}{4b^4} - \frac{d^2(bc-ad)e^{-a+\frac{bc}{d}}\Gamma\left(3,\frac{b(c+dx)}{d}\right)}{4b^4} - \frac{d^3e^{-a+\frac{bc}{d}}\Gamma\left(4,\frac{b(c+dx)}{d}\right)}{4b^4}$$

output
```
-1/4*(-a*d+b*c)^3*exp(-b*x-a)/b^4-1/4*(-a*d+b*c)^4*Ei(1,b*x+a)/b^4/d+1/4*(
d*x+c)^4*Ei(1,b*x+a)/d-1/4*d*(-a*d+b*c)^2*exp(-a+b*c/d)*exp(-b*(d*x+c)/d)*
(1+b*(d*x+c)/d)/b^4-1/2*d^2*(-a*d+b*c)*exp(-a+b*c/d)*exp(-b*(d*x+c)/d)*(1+
b*(d*x+c)/d+1/2*b^2*(d*x+c)^2/d^2)/b^4-3/2*d^3*exp(-a+b*c/d)*exp(-b*(d*x+c
)/d)*(1+b*(d*x+c)/d+1/2*b^2*(d*x+c)^2/d^2+1/6*b^3*(d*x+c)^3/d^3)/b^4
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.54

$$\int (c+dx)^3\Gamma(0,a+bx)\,dx$$
$$= \frac{e^{-a-bx}\left(-4b^3c^3-6b^2c^2d+6ab^2c^2d-8bcd^2+4abcd^2-4a^2bcd^2-6d^3+2ad^3-a^2d^3+a^3d^3-6b^3c^2dx-\right.}{}$$

input
```
Integrate[(c + d*x)^3*Gamma[0, a + b*x],x]
```

output
```
(E^(-a - b*x)*(-4*b^3*c^3 - 6*b^2*c^2*d + 6*a*b^2*c^2*d - 8*b*c*d^2 + 4*a*
b*c*d^2 - 4*a^2*b*c*d^2 - 6*d^3 + 2*a*d^3 - a^2*d^3 + a^3*d^3 - 6*b^3*c^2*
d*x - 8*b^2*c*d^2*x + 4*a*b^2*c*d^2*x - 6*b*d^3*x + 2*a*b*d^3*x - a^2*b*d^
3*x - 4*b^3*c*d^2*x^2 - 3*b^2*d^3*x^2 + a*b^2*d^3*x^2 - b^3*d^3*x^3 + a*(-
4*b^3*c^3 + 6*a*b^2*c^2*d - 4*a^2*b*c*d^2 + a^3*d^3)*E^(a + b*x)*ExpIntegr
alEi[-a - b*x] + b^4*E^(a + b*x)*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*
x^3)*Gamma[0, a + b*x]))/(4*b^4)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.63, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\Gamma(0,a+bx)\,dx$$
$$\Big\downarrow \text{7119}$$
$$\frac{b\int \frac{e^{-a-bx}(c+dx)^4}{a+bx}dx}{4d}+\frac{(c+dx)^4\Gamma(0,a+bx)}{4d}$$
$$\Big\downarrow \text{2629}$$

$$\frac{b\int\left(\frac{e^{-a-bx}(bc-ad)^4}{b^4(a+bx)}+\frac{de^{-a-bx}(bc-ad)^3}{b^4}+\frac{de^{-a-bx}(c+dx)(bc-ad)^2}{b^3}+\frac{de^{-a-bx}(c+dx)^2(bc-ad)}{b^2}+\frac{de^{-a-bx}(c+dx)^3}{b}\right)dx}{4d}+\frac{(c+dx)^4\Gamma(0,a+bx)}{4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{2d^3e^{-a-bx}(bc-ad)}{b^5}-\frac{d^2e^{-a-bx}(bc-ad)^2}{b^5}+\frac{(bc-ad)^4\,\text{ExpIntegralEi}(-a-bx)}{b^5}-\frac{de^{-a-bx}(bc-ad)^3}{b^5}-\frac{6d^4e^{-a-bx}}{b^5}-\frac{6d^3e^{-a-bx}(c+}{b^4}\right.}{4d}$$

$$\frac{(c+dx)^4\Gamma(0,a+bx)}{4d}$$

input
```
Int[(c + d*x)^3*Gamma[0, a + b*x],x]
```

output
```
(b*((-6*d^4*E^(-a - b*x))/b^5 - (2*d^3*(b*c - a*d)*E^(-a - b*x))/b^5 - (d^
2*(b*c - a*d)^2*E^(-a - b*x))/b^5 - (d*(b*c - a*d)^3*E^(-a - b*x))/b^5 - (
6*d^3*E^(-a - b*x)*(c + d*x))/b^4 - (2*d^2*(b*c - a*d)*E^(-a - b*x)*(c + d
*x))/b^4 - (d*(b*c - a*d)^2*E^(-a - b*x)*(c + d*x))/b^4 - (3*d^2*E^(-a - b
*x)*(c + d*x)^2)/b^3 - (d*(b*c - a*d)*E^(-a - b*x)*(c + d*x)^2)/b^3 - (d*E
^(-a - b*x)*(c + d*x)^3)/b^2 + ((b*c - a*d)^4*ExpIntegralEi[-a - b*x])/b^5
))/(4*d) + ((c + d*x)^4*Gamma[0, a + b*x])/(4*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. 2(259) = 518.

Time = 1.37 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.76

method	result
parallelrisch	$\frac{4x^3 \operatorname{expIntegral}_1(bx+a)a\,b^4c\,d^2+6x^2 \operatorname{expIntegral}_1(bx+a)a\,b^4c^2d-4x^2\mathrm{e}^{-bx-a}a\,b^3c\,d^2-6x\,\mathrm{e}^{-bx-a}a\,b^3c^2d-8x\,\mathrm{e}^{-bx-a}}{}$
parts	$\frac{\operatorname{expIntegral}_1(bx+a)d^3x^4}{4}+\operatorname{expIntegral}_1(bx+a)\,d^2c\,x^3+\frac{3\operatorname{expIntegral}_1(bx+a)d\,c^2x^2}{2}+\operatorname{expInteg}$
derivativedivides	$\frac{\frac{d^3\operatorname{expIntegral}_1(bx+a)a^4}{4b^3}-\frac{d^2\operatorname{expIntegral}_1(bx+a)a^3c}{b^2}-\frac{d^3\operatorname{expIntegral}_1(bx+a)a^3(bx+a)}{b^3}+\frac{3d\operatorname{expIntegral}_1(bx+a)a^2c^2}{2b}+\frac{3d^2\operatorname{expI}}{}}{}$
default	$\frac{\frac{d^3\operatorname{expIntegral}_1(bx+a)a^4}{4b^3}-\frac{d^2\operatorname{expIntegral}_1(bx+a)a^3c}{b^2}-\frac{d^3\operatorname{expIntegral}_1(bx+a)a^3(bx+a)}{b^3}+\frac{3d\operatorname{expIntegral}_1(bx+a)a^2c^2}{2b}+\frac{3d^2\operatorname{expI}}{}}{}$

input
```
int((d*x+c)^3*Ei(1,b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*x^3*Ei(1,b*x+a)*a*b^4*c*d^2+6*x^2*Ei(1,b*x+a)*a*b^4*c^2*d-4*x^2*exp
(-b*x-a)*a*b^3*c*d^2-6*x*exp(-b*x-a)*a*b^3*c^2*d-8*x*exp(-b*x-a)*a*b^2*c*d
^2+4*x*exp(-b*x-a)*a^2*b^2*c*d^2-Ei(1,b*x+a)*a^5*d^3+exp(-b*x-a)*a^4*d^3-e
xp(-b*x-a)*a^3*d^3+2*exp(-b*x-a)*a^2*d^3-6*exp(-b*x-a)*a*d^3+4*Ei(1,b*x+a)
*a^2*b^3*c^3-4*exp(-b*x-a)*a*b^3*c^3+x^4*Ei(1,b*x+a)*a*b^4*d^3-x^3*exp(-b*
x-a)*a*b^3*d^3+x^2*exp(-b*x-a)*a^2*b^2*d^3+4*x*Ei(1,b*x+a)*a*b^4*c^3-3*x^2
*exp(-b*x-a)*a*b^2*d^3-x*exp(-b*x-a)*a^3*b*d^3+4*Ei(1,b*x+a)*a^4*b*c*d^2-6
*Ei(1,b*x+a)*a^3*b^2*c^2*d+2*x*exp(-b*x-a)*a^2*b*d^3-4*exp(-b*x-a)*a^3*b*c
*d^2+6*exp(-b*x-a)*a^2*b^2*c^2*d-6*x*exp(-b*x-a)*a*b*d^3+4*exp(-b*x-a)*a^2
*b*c*d^2-6*exp(-b*x-a)*a*b^2*c^2*d-8*exp(-b*x-a)*a*b*c*d^2)/b^4/a
```

Fricas [F(-2)]

Exception generated.

$$\int (c+dx)^3\Gamma(0,a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^3*exp_integral_e(1,b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated (((
(d)*(x))+(c))^(((3)::EXPR INT)))*(exp_integral_e(((1)::EXPR INT),((b)*(x))
+(a))):   There are no library operations named exp_integral_e       Use H
yperDoc
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(233) = 466$.

Time = 46.68 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.93

$$\int (c+dx)^3\Gamma(0,a+bx)\,dx$$
$$= \begin{cases} -\frac{a^4d^3\,\mathrm{E}_1(a+bx)}{4b^4} + \frac{a^3cd^2\,\mathrm{E}_1(a+bx)}{b^3} + \frac{a^3d^3e^{-a}e^{-bx}}{4b^4} - \frac{3a^2c^2d\,\mathrm{E}_1(a+bx)}{2b^2} - \frac{a^2cd^2e^{-a}e^{-bx}}{b^3} - \frac{a^2d^3xe^{-a}e^{-bx}}{4b^3} - \frac{a^2d^3e^{-a}e^{-bx}}{4b^4} + \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\mathrm{E}_1(a) \end{cases}$$

input
```
integrate((d*x+c)**3*expint(1,b*x+a),x)
```

output
```
Piecewise((-a**4*d**3*expint(1, a + b*x)/(4*b**4) + a**3*c*d**2*expint(1,
a + b*x)/b**3 + a**3*d**3*exp(-a)*exp(-b*x)/(4*b**4) - 3*a**2*c**2*d*expin
t(1, a + b*x)/(2*b**2) - a**2*c*d**2*exp(-a)*exp(-b*x)/b**3 - a**2*d**3*x*
exp(-a)*exp(-b*x)/(4*b**3) - a**2*d**3*exp(-a)*exp(-b*x)/(4*b**4) + a*c**3
*expint(1, a + b*x)/b + 3*a*c**2*d*exp(-a)*exp(-b*x)/(2*b**2) + a*c*d**2*x
*exp(-a)*exp(-b*x)/b**2 + a*d**3*x**2*exp(-a)*exp(-b*x)/(4*b**2) + a*c*d**
2*exp(-a)*exp(-b*x)/b**3 + a*d**3*x*exp(-a)*exp(-b*x)/(2*b**3) + a*d**3*ex
p(-a)*exp(-b*x)/(2*b**4) + c**3*x*expint(1, a + b*x) + 3*c**2*d*x**2*expin
t(1, a + b*x)/2 + c*d**2*x**3*expint(1, a + b*x) + d**3*x**4*expint(1, a +
 b*x)/4 - c**3*exp(-a)*exp(-b*x)/b - 3*c**2*d*x*exp(-a)*exp(-b*x)/(2*b) -
c*d**2*x**2*exp(-a)*exp(-b*x)/b - d**3*x**3*exp(-a)*exp(-b*x)/(4*b) - 3*c*
*2*d*exp(-a)*exp(-b*x)/(2*b**2) - 2*c*d**2*x*exp(-a)*exp(-b*x)/b**2 - 3*d*
*3*x**2*exp(-a)*exp(-b*x)/(4*b**2) - 2*c*d**2*exp(-a)*exp(-b*x)/b**3 - 3*d
**3*x*exp(-a)*exp(-b*x)/(2*b**3) - 3*d**3*exp(-a)*exp(-b*x)/(2*b**4), Ne(b
, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*expint(1, a
), True))
```

Maxima [F]

$$\int (c+dx)^3\Gamma(0,a+bx)\,dx = \int (dx+c)^3E_1(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*exp_integral_e(1,b*x+a),x, algorithm="maxima")
```

output
```
-c^3*exp_integral_e(2, b*x + a)/b + integrate(d^3*x^3*exp_integral_e(1, b*
x + a) + 3*c*d^2*x^2*exp_integral_e(1, b*x + a) + 3*c^2*d*x*exp_integral_e
(1, b*x + a), x)
```

Giac [F]

$$\int (c+dx)^3\Gamma(0,a+bx)\,dx = \int (dx+c)^3E_1(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*exp_integral_e(1,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*exp_integral_e(1, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.08

$$\int (c+dx)^3\Gamma(0,a+bx)\,dx = \int \mathrm{expint}(a+b\,x)\,{(c+d\,x)}^3\,dx$$

input
```
int(expint(a + b*x)*(c + d*x)^3,x)
```

output
```
int(expint(a + b*x)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3\Gamma(0,a+bx)\,dx = \left(\int ei(1,bx+a)\,dx\right)c^3 + \left(\int ei(1,bx+a)\,x^3dx\right)d^3 + 3\left(\int ei(1,bx+a)\,x^2dx\right)c\,d^2 + 3\left(\int ei(1,bx+a)\,xdx\right)c^2d$$

input
```
int((d*x+c)^3*Ei(1,b*x+a),x)
```

output
```
int(ei(1,a + b*x),x)*c**3 + int(ei(1,a + b*x)*x**3,x)*d**3 + 3*int(ei(1,a
+ b*x)*x**2,x)*c*d**2 + 3*int(ei(1,a + b*x)*x,x)*c**2*d
```

3.103 $\int (c+dx)^2\Gamma(0,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 150

$$\int (c+dx)^2\Gamma(0,a+bx)\,dx = -\frac{(bc-ad)^2e^{-a-bx}}{3b^3} - \frac{(bc-ad)^3\Gamma(0,a+bx)}{3b^3d} + \frac{(c+dx)^3\Gamma(0,a+bx)}{3d} - \frac{d(bc-ad)e^{-a+\frac{bc}{d}}\Gamma\left(2,\frac{b(c+dx)}{d}\right)}{3b^3} - \frac{d^2e^{-a+\frac{bc}{d}}\Gamma\left(3,\frac{b(c+dx)}{d}\right)}{3b^3}$$

output

```
-1/3*(-a*d+b*c)^2*exp(-b*x-a)/b^3-1/3*(-a*d+b*c)^3*Ei(1,b*x+a)/b^3/d+1/3*(
d*x+c)^3*Ei(1,b*x+a)/d-1/3*d*(-a*d+b*c)*exp(-a+b*c/d)*exp(-b*(d*x+c)/d)*(1
+b*(d*x+c)/d)/b^3-2/3*d^2*exp(-a+b*c/d)*exp(-b*(d*x+c)/d)*(1+b*(d*x+c)/d+1
/2*b^2*(d*x+c)^2/d^2)/b^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11

$$\int (c+dx)^2\Gamma(0,a+bx)\,dx$$
$$= \frac{e^{-a-bx}\left(-3b^2c^2-3bcd+3abcd-2d^2+ad^2-a^2d^2-3b^2cdx-2bd^2x+abd^2x-b^2d^2x^2-a(3b^2c^2-3ab\right.}{3b^3}$$

input
```
Integrate[(c + d*x)^2*Gamma[0, a + b*x],x]
```

output
```
(E^(-a - b*x)*(-3*b^2*c^2 - 3*b*c*d + 3*a*b*c*d - 2*d^2 + a*d^2 - a^2*d^2
- 3*b^2*c*d*x - 2*b*d^2*x + a*b*d^2*x - b^2*d^2*x^2 - a*(3*b^2*c^2 - 3*a*b
*c*d + a^2*d^2)*E^(a + b*x)*ExpIntegralEi[-a - b*x] + b^3*E^(a + b*x)*x*(3
*c^2 + 3*c*d*x + d^2*x^2)*Gamma[0, a + b*x]))/(3*b^3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.33, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\Gamma(0,a+bx)\,dx$$
$$\Big\downarrow \; 7119$$
$$\frac{b\int \frac{e^{-a-bx}(c+dx)^3}{a+bx}dx}{3d}+\frac{(c+dx)^3\Gamma(0,a+bx)}{3d}$$
$$\Big\downarrow \; 2629$$
$$\frac{b\int \left(\frac{e^{-a-bx}(bc-ad)^3}{b^3(a+bx)}+\frac{de^{-a-bx}(bc-ad)^2}{b^3}+\frac{de^{-a-bx}(c+dx)(bc-ad)}{b^2}+\frac{de^{-a-bx}(c+dx)^2}{b}\right)dx}{3d}+$$
$$\frac{(c+dx)^3\Gamma(0,a+bx)}{3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{d^2e^{-a-bx}(bc-ad)}{b^4}+\frac{(bc-ad)^3\,\text{ExpIntegralEi}(-a-bx)}{b^4}-\frac{de^{-a-bx}(bc-ad)^2}{b^4}-\frac{2d^3e^{-a-bx}}{b^4}-\frac{2d^2e^{-a-bx}(c+dx)}{b^3}-\frac{de^{-a-bx}(c+dx)(b}{b^3}\right.}{3d}$$
$$\frac{(c+dx)^3\Gamma(0,a+bx)}{3d}$$

input
```
Int[(c + d*x)^2*Gamma[0, a + b*x],x]
```

output
```
(b*((-2*d^3*E^(-a - b*x))/b^4 - (d^2*(b*c - a*d)*E^(-a - b*x))/b^4 - (d*(b
*c - a*d)^2*E^(-a - b*x))/b^4 - (2*d^2*E^(-a - b*x)*(c + d*x))/b^3 - (d*(b
*c - a*d)*E^(-a - b*x)*(c + d*x))/b^3 - (d*E^(-a - b*x)*(c + d*x)^2)/b^2 +
 ((b*c - a*d)^3*ExpIntegralEi[-a - b*x])/b^4))/(3*d) + ((c + d*x)^3*Gamma[
0, a + b*x])/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.88

method	result
parallelrisch	$\frac{x^3 \operatorname{expIntegral}_1(bx+a)a\,b^3d^2+3x^2 \operatorname{expIntegral}_1(bx+a)a\,b^3cd-x^2\mathrm{e}^{-bx-a}a\,b^2d^2+3x\operatorname{expIntegral}_1(bx+a)a\,b^3c^2+x\,\mathrm{e}^{-b}}{}$
parts	$\frac{\operatorname{expIntegral}_1(bx+a)d^2x^3}{3}+\operatorname{expIntegral}_1(bx+a)\,dc\,x^2+\operatorname{expIntegral}_1(bx+a)\,c^2x+\frac{\operatorname{expInteg}}{}$
derivativedivides	$\frac{-\frac{d^2\operatorname{expIntegral}_1(bx+a)a^3}{3b^2}+\frac{d\operatorname{expIntegral}_1(bx+a)a^2c}{b}+\frac{d^2\operatorname{expIntegral}_1(bx+a)a^2(bx+a)}{b^2}-\operatorname{expIntegral}_1(bx+a)a\,c^2-\frac{2d\operatorname{expInt}}{}}{}$
default	$\frac{-\frac{d^2\operatorname{expIntegral}_1(bx+a)a^3}{3b^2}+\frac{d\operatorname{expIntegral}_1(bx+a)a^2c}{b}+\frac{d^2\operatorname{expIntegral}_1(bx+a)a^2(bx+a)}{b^2}-\operatorname{expIntegral}_1(bx+a)a\,c^2-\frac{2d\operatorname{expInt}}{}}{}$

input
```
int((d*x+c)^2*Ei(1,b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^3*Ei(1,b*x+a)*a*b^3*d^2+3*x^2*Ei(1,b*x+a)*a*b^3*c*d-x^2*exp(-b*x-a)
*a*b^2*d^2+3*x*Ei(1,b*x+a)*a*b^3*c^2+x*exp(-b*x-a)*a^2*b*d^2-3*x*exp(-b*x-
a)*a*b^2*c*d+Ei(1,b*x+a)*a^4*d^2-3*Ei(1,b*x+a)*a^3*b*c*d+3*Ei(1,b*x+a)*a^2
*b^2*c^2-2*x*exp(-b*x-a)*a*b*d^2-exp(-b*x-a)*a^3*d^2+3*exp(-b*x-a)*a^2*b*c
*d-3*exp(-b*x-a)*a*b^2*c^2+exp(-b*x-a)*a^2*d^2-3*exp(-b*x-a)*a*b*c*d-2*exp
(-b*x-a)*a*d^2)/a/b^3
```

Fricas [F(-2)]

Exception generated.

$$\int (c+dx)^2\Gamma(0,a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^2*exp_integral_e(1,b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated (((
(d)*(x))+(c))^(((2)::EXPR INT)))*(exp_integral_e(((1)::EXPR INT),((b)*(x))
+(a))):   There are no library operations named exp_integral_e      Use H
yperDoc
```

Sympy [A] (verification not implemented)

Time = 25.33 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.94

$$\int (c+dx)^2\Gamma(0,a+bx)\,dx$$
$$= \begin{cases} \frac{a^3d^2\operatorname{E}_1(a+bx)}{3b^3} - \frac{a^2cd\operatorname{E}_1(a+bx)}{b^2} - \frac{a^2d^2e^{-a}e^{-bx}}{3b^3} + \frac{ac^2\operatorname{E}_1(a+bx)}{b} + \frac{acde^{-a}e^{-bx}}{b^2} + \frac{ad^2xe^{-a}e^{-bx}}{3b^2} + \frac{ad^2e^{-a}e^{-bx}}{3b^3} + c^2x\operatorname{E}_1\left(\right. \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\operatorname{E}_1(a) \end{cases}$$

input
```
integrate((d*x+c)**2*expint(1,b*x+a),x)
```

output
```
Piecewise((a**3*d**2*expint(1, a + b*x)/(3*b**3) - a**2*c*d*expint(1, a +
b*x)/b**2 - a**2*d**2*exp(-a)*exp(-b*x)/(3*b**3) + a*c**2*expint(1, a + b*
x)/b + a*c*d*exp(-a)*exp(-b*x)/b**2 + a*d**2*x*exp(-a)*exp(-b*x)/(3*b**2)
+ a*d**2*exp(-a)*exp(-b*x)/(3*b**3) + c**2*x*expint(1, a + b*x) + c*d*x**2
*expint(1, a + b*x) + d**2*x**3*expint(1, a + b*x)/3 - c**2*exp(-a)*exp(-b
*x)/b - c*d*x*exp(-a)*exp(-b*x)/b - d**2*x**2*exp(-a)*exp(-b*x)/(3*b) - c*
d*exp(-a)*exp(-b*x)/b**2 - 2*d**2*x*exp(-a)*exp(-b*x)/(3*b**2) - 2*d**2*ex
p(-a)*exp(-b*x)/(3*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*ex
pint(1, a), True))
```

Maxima [F]

$$\int (c+dx)^2\Gamma(0,a+bx)\,dx = \int (dx+c)^2E_1(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*exp_integral_e(1,b*x+a),x, algorithm="maxima")
```

output
```
-c^2*exp_integral_e(2, b*x + a)/b + integrate(d^2*x^2*exp_integral_e(1, b*
x + a) + 2*c*d*x*exp_integral_e(1, b*x + a), x)
```

Giac [F]

$$\int (c+dx)^2\Gamma(0,a+bx)\,dx = \int (dx+c)^2 E_1(bx+a)\;dx$$

input
```
integrate((d*x+c)^2*exp_integral_e(1,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*exp_integral_e(1, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.84

$$\int (c+dx)^2\Gamma(0,a+bx)\,dx$$
$$= \frac{\frac{\mathrm{expint}(a+b\,x)\,\left(\frac{a^4\,d^2}{3}-a^3\,b\,c\,d+a^2\,b^2\,c^2\right)}{b^3}-x^2\,\mathrm{e}^{-a-b\,x}\,\left(c\,d+\frac{2\,d^2}{3\,b}\right)-x\,\mathrm{e}^{-a-b\,x}\,\left(\frac{\frac{a\,d^2}{3}+\frac{2\,d^2}{3}+b\,c\,d}{b^2}+c^2\right)-\frac{\mathrm{e}^{-a-b\,x}\,\left(\frac{2\,a\,d^2}{3}+b\right.}{}}{}$$

input
```
int(expint(a + b*x)*(c + d*x)^2,x)
```

output
```
((expint(a + b*x)*((a^4*d^2)/3 + a^2*b^2*c^2 - a^3*b*c*d))/b^3 - x^2*exp(-
 a - b*x)*(c*d + (2*d^2)/(3*b)) - x*exp(- a - b*x)*(((a*d^2)/3 + (2*d^2)/3
 + b*c*d)/b^2 + c^2) - (exp(- a - b*x)*((2*a*d^2)/3 + b*(a*c*d - a^2*c*d)
- (a^2*d^2)/3 + (a^3*d^2)/3 + a*b^2*c^2))/b^3 + x*expint(a + b*x)*(2*a*c^2
 + ((a^3*d^2)/3 - a^2*b*c*d)/b^2) - (d^2*x^3*exp(- a - b*x))/3 + x^2*expin
t(a + b*x)*(b*c^2 + a*c*d) + x^3*expint(a + b*x)*((a*d^2)/3 + b*c*d) + (b*
d^2*x^4*expint(a + b*x))/3)/(a + b*x)
```

Reduce [F]

$$\int (c+dx)^2\Gamma(0,a+bx)\,dx = \left(\int ei(1,bx+a)\,dx\right)c^2 + \left(\int ei(1,bx+a)\,x^2dx\right)d^2 + 2\left(\int ei(1,bx+a)\,xdx\right)cd$$

input
```
int((d*x+c)^2*Ei(1,b*x+a),x)
```

output
```
int(ei(1,a + b*x),x)*c**2 + int(ei(1,a + b*x)*x**2,x)*d**2 + 2*int(ei(1,a
+ b*x)*x,x)*c*d
```

3.104 $\int (c+dx)\Gamma(0,a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 106

$$\int (c+dx)\Gamma(0,a+bx)\,dx = -\frac{(bc-ad)e^{-a-bx}}{2b^2} - \frac{(bc-ad)^2\Gamma(0,a+bx)}{2b^2d} + \frac{(c+dx)^2\Gamma(0,a+bx)}{2d} - \frac{de^{-a+\frac{bc}{d}}\Gamma\left(2,\frac{b(c+dx)}{d}\right)}{2b^2}$$

output
```
-1/2*(-a*d+b*c)*exp(-b*x-a)/b^2-1/2*(-a*d+b*c)^2*Ei(1,b*x+a)/b^2/d+1/2*(d*
x+c)^2*Ei(1,b*x+a)/d-1/2*d*exp(-a+b*c/d)*exp(-b*(d*x+c)/d)*(1+b*(d*x+c)/d)
/b^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.80

$$\int (c+dx)\Gamma(0,a+bx)\,dx = \frac{e^{-a-bx}\left(-2bc-d+ad-bdx+a(-2bc+ad)e^{a+bx}\,\text{ExpIntegralEi}(-a-bx)+b^2e^{a+bx}x(2c+dx)\Gamma(0,a+\right.}{2b^2}$$

input
```
Integrate[(c + d*x)*Gamma[0, a + b*x],x]
```

output
```
(E^(-a - b*x)*(-2*b*c - d + a*d - b*d*x + a*(-2*b*c + a*d)*E^(a + b*x)*Exp
IntegralEi[-a - b*x] + b^2*E^(a + b*x)*x*(2*c + d*x)*Gamma[0, a + b*x]))/(
2*b^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\Gamma(0,a+bx)\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{b\int \frac{e^{-a-bx}(c+dx)^2}{a+bx}dx}{2d}+\frac{(c+dx)^2\Gamma(0,a+bx)}{2d}$$

$$\Big\downarrow \text{2629}$$

$$\frac{b\int \left(\frac{e^{-a-bx}(bc-ad)^2}{b^2(a+bx)}+\frac{de^{-a-bx}(bc-ad)}{b^2}+\frac{de^{-a-bx}(c+dx)}{b}\right)dx}{2d}+\frac{(c+dx)^2\Gamma(0,a+bx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{(bc-ad)^2\text{ExpIntegralEi}(-a-bx)}{b^3}-\frac{de^{-a-bx}(bc-ad)}{b^3}-\frac{d^2e^{-a-bx}}{b^3}-\frac{de^{-a-bx}(c+dx)}{b^2}\right)}{2d}+\frac{(c+dx)^2\Gamma(0,a+bx)}{2d}$$

input
```
Int[(c + d*x)*Gamma[0, a + b*x],x]
```

output
```
(b*(-((d^2*E^(-a - b*x))/b^3) - (d*(b*c - a*d)*E^(-a - b*x))/b^3 - (d*E^(-
a - b*x)*(c + d*x))/b^2 + ((b*c - a*d)^2*ExpIntegralEi[-a - b*x])/b^3))/(2
*d) + ((c + d*x)^2*Gamma[0, a + b*x])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.13

method	result
parts	$\frac{\operatorname{expIntegral}_1(bx+a)d\,x^2}{2}+\operatorname{expIntegral}_1(bx+a)\,cx+\frac{d\left((-bx-a)\mathrm{e}^{-bx-a}-\mathrm{e}^{-bx-a}\right)}{2b^2}-\frac{d\,a^2\operatorname{expIntegral}_1}{2b^2}$
derivativedivides	$\frac{-\frac{\operatorname{expIntegral}_1(bx+a)da(bx+a)}{b}+\operatorname{expIntegral}_1(bx+a)c(bx+a)+\frac{\operatorname{expIntegral}_1(bx+a)d(bx+a)^2}{2b}-\frac{-2\,\mathrm{e}^{-bx-a}ad+2\,\mathrm{e}^{-bx-a}bc-d}{2b}}{b}$
default	$\frac{-\frac{\operatorname{expIntegral}_1(bx+a)da(bx+a)}{b}+\operatorname{expIntegral}_1(bx+a)c(bx+a)+\frac{\operatorname{expIntegral}_1(bx+a)d(bx+a)^2}{2b}-\frac{-2\,\mathrm{e}^{-bx-a}ad+2\,\mathrm{e}^{-bx-a}bc-d}{2b}}{b}$
parallelrisch	$\frac{x^2\operatorname{expIntegral}_1(bx+a)a\,b^2d+2x\operatorname{expIntegral}_1(bx+a)a\,b^2c-x\,\mathrm{e}^{-bx-a}abd-\operatorname{expIntegral}_1(bx+a)a^3d+2\operatorname{expIntegral}_1(b}{2a\,b^2}$

input
```
int((d*x+c)*Ei(1,b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*Ei(1,b*x+a)*d*x^2+Ei(1,b*x+a)*c*x+1/2*d/b^2*((-b*x-a)*exp(-b*x-a)-exp(
-b*x-a))-1/2*d/b^2*a^2*Ei(1,b*x+a)-exp(-b*x-a)*c/b+exp(-b*x-a)*d/b^2*a+c/b
*a*Ei(1,b*x+a)
```

Fricas [F(-2)]

Exception generated.

$$\int (c+dx)\Gamma(0,a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)*exp_integral_e(1,b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated (((
d)*(x))+(c))*(exp_integral_e(((1)::EXPR INT),((b)*(x))+(a))):   There are
no library operations named exp_integral_e       Use HyperDoc Browse or is
sue
```

Sympy [A] (verification not implemented)

Time = 12.46 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.17

$$\int (c+dx)\Gamma(0,a+bx)\,dx$$
$$= \begin{cases} -\frac{a^2 d \operatorname{E}_1(a+bx)}{2b^2} + \frac{ac \operatorname{E}_1(a+bx)}{b} + \frac{ade^{-a}e^{-bx}}{2b^2} + cx \operatorname{E}_1(a+bx) + \frac{dx^2 \operatorname{E}_1(a+bx)}{2} - \frac{ce^{-a}e^{-bx}}{b} - \frac{dxe^{-a}e^{-bx}}{2b} - \frac{de^{-a}e^{-bx}}{2b^2} \\ \left(cx + \frac{dx^2}{2}\right) \operatorname{E}_1(a) \end{cases}$$

input
```
integrate((d*x+c)*expint(1,b*x+a),x)
```

output
```
Piecewise((-a**2*d*expint(1, a + b*x)/(2*b**2) + a*c*expint(1, a + b*x)/b
+ a*d*exp(-a)*exp(-b*x)/(2*b**2) + c*x*expint(1, a + b*x) + d*x**2*expint(
1, a + b*x)/2 - c*exp(-a)*exp(-b*x)/b - d*x*exp(-a)*exp(-b*x)/(2*b) - d*ex
p(-a)*exp(-b*x)/(2*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*expint(1, a), True)
)
```

Maxima [F]

$$\int (c+dx)\Gamma(0,a+bx)\,dx = \int (dx+c)E_1(bx+a)\,dx$$

input
```
integrate((d*x+c)*exp_integral_e(1,b*x+a),x, algorithm="maxima")
```

output
```
d*integrate(x*exp_integral_e(1, b*x + a), x) - c*exp_integral_e(2, b*x + a
)/b
```

Giac [F]

$$\int (c+dx)\Gamma(0,a+bx)\,dx = \int (dx+c)E_1(bx+a)\,dx$$

input
```
integrate((d*x+c)*exp_integral_e(1,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*exp_integral_e(1, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05

$$\int (c+dx)\Gamma(0,a+bx)\,dx = c\,x\,\mathrm{expint}(a+b\,x) - \frac{d\,\mathrm{e}^{-a-b\,x}}{2\,b^2} - \frac{c\,\mathrm{e}^{-a-b\,x}}{b} + \frac{d\,x^2\,\mathrm{expint}(a+b\,x)}{2} + \frac{a\,c\,\mathrm{expint}(a+b\,x)}{b} - \frac{a^2\,d\,\mathrm{expint}(a+b\,x)}{2\,b^2} + \frac{a\,d\,\mathrm{e}^{-a-b\,x}}{2\,b^2} - \frac{d\,x\,\mathrm{e}^{-a-b\,x}}{2\,b}$$

input
```
int(expint(a + b*x)*(c + d*x),x)
```

output
```
c*x*expint(a + b*x) - (d*exp(- a - b*x))/(2*b^2) - (c*exp(- a - b*x))/b +
(d*x^2*expint(a + b*x))/2 + (a*c*expint(a + b*x))/b - (a^2*d*expint(a + b*
x))/(2*b^2) + (a*d*exp(- a - b*x))/(2*b^2) - (d*x*exp(- a - b*x))/(2*b)
```

Reduce [F]

$$\int (c+dx)\Gamma(0,a+bx)\,dx = \left(\int ei(1,bx+a)\,dx\right)c + \left(\int ei(1,bx+a)\,xdx\right)d$$

input
```
int((d*x+c)*Ei(1,b*x+a),x)
```

output
```
int(ei(1,a + b*x),x)*c + int(ei(1,a + b*x)*x,x)*d
```

3.105 $\int \Gamma(0, a+bx)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 32

$$\int \Gamma(0,a+bx)\,dx=-\frac{e^{-a-bx}}{b}+\frac{(a+bx)\Gamma(0,a+bx)}{b}$$

output
```
-exp(-b*x-a)/b+(b*x+a)*Ei(1,b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \Gamma(0,a+bx)\,dx=-\frac{e^{-a-bx}}{b}-\frac{a\,\text{ExpIntegralEi}(-a-bx)}{b}+x\Gamma(0,a+bx)$$

input
```
Integrate[Gamma[0, a + b*x],x]
```

output
```
-(E^(-a - b*x)/b) - (a*ExpIntegralEi[-a - b*x])/b + x*Gamma[0, a + b*x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(0, a + bx) \, dx$$

$$\Big\downarrow \; 7111$$

$$\frac{(a + bx)\Gamma(0, a + bx)}{b} - \frac{e^{-a-bx}}{b}$$

input
```
Int[Gamma[0, a + b*x],x]
```

output
```
-(E^(-a - b*x)/b) + ((a + b*x)*Gamma[0, a + b*x])/b
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{(bx+a)\,\mathrm{expIntegral}_1(bx+a)-\mathrm{e}^{-bx-a}}{b}$	30
default	$\frac{(bx+a)\,\mathrm{expIntegral}_1(bx+a)-\mathrm{e}^{-bx-a}}{b}$	30
parts	$x\ \mathrm{expIntegral}_1\left(bx+a\right)-\frac{-a\ \mathrm{expIntegral}_1(bx+a)+\mathrm{e}^{-bx-a}}{b}$	36
parallelrisch	$\frac{x\ \mathrm{expIntegral}_1(bx+a)ab+\mathrm{expIntegral}_1(bx+a)a^2-a\,\mathrm{e}^{-bx-a}}{ab}$	43

input
```
int(Ei(1,b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*((b*x+a)*Ei(1,b*x+a)-exp(-b*x-a))
```

Fricas [F(-2)]

Exception generated.

$$\int \Gamma(0, a + bx)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp_integral_e(1,b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated exp
_integral_e(((1)::EXPR INT),((b)*(x))+(a)):   There are no library operati
ons named exp_integral_e       Use HyperDoc Browse or issue
```

Sympy [A] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \Gamma(0, a + bx)\, dx = \begin{cases} \frac{a \operatorname{E}_1(a+bx)}{b} + x \operatorname{E}_1(a + bx) - \frac{e^{-a} e^{-bx}}{b} & \text{for } b \neq 0 \\ x \operatorname{E}_1(a) & \text{otherwise} \end{cases}$$

input
```
integrate(expint(1,b*x+a),x)
```

output
```
Piecewise((a*expint(1, a + b*x)/b + x*expint(1, a + b*x) - exp(-a)*exp(-b*
x)/b, Ne(b, 0)), (x*expint(1, a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.38

$$\int \Gamma(0,a+bx)\,dx=-\frac{E_2(bx+a)}{b}$$

input
```
integrate(exp_integral_e(1,b*x+a),x, algorithm="maxima")
```

output
```
-exp_integral_e(2, b*x + a)/b
```

Giac [F]

$$\int \Gamma(0,a+bx)\,dx=\int E_1(bx+a)\,dx$$

input
```
integrate(exp_integral_e(1,b*x+a),x, algorithm="giac")
```

output
```
integrate(exp_integral_e(1, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.19

$$\int \Gamma(0,a+bx)\,dx = \frac{\frac{a^2\,\mathrm{expint}(a+b\,x)}{b}-x\,\mathrm{e}^{-a-b\,x}-\frac{a\,\mathrm{e}^{-a-b\,x}}{b}+2\,a\,x\,\mathrm{expint}(a+b\,x)+b\,x^2\,\mathrm{expint}(a+b\,x)}{a+b\,x}$$

input
```
int(expint(a + b*x),x)
```

output
```
((a^2*expint(a + b*x))/b - x*exp(- a - b*x) - (a*exp(- a - b*x))/b + 2*a*x
*expint(a + b*x) + b*x^2*expint(a + b*x))/(a + b*x)
```

Reduce [F]

$$\int \Gamma(0, a+bx)\,dx = \int ei(1, bx+a)\,dx$$

input
```
int(Ei(1,b*x+a),x)
```

output
```
int(ei(1,a + b*x),x)
```

3.106 $\int \frac{\Gamma(0,a+bx)}{c+dx} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\Gamma(0,a+bx)}{c+dx} dx = \text{Int}\left(\frac{\Gamma(0,a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(Ei(1,b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(0,a+bx)}{c+dx} dx = \int \frac{\Gamma(0,a+bx)}{c+dx} dx$$

input
```
Integrate[Gamma[0, a + b*x]/(c + d*x),x]
```

output
```
Integrate[Gamma[0, a + b*x]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(0,a+bx)}{c+dx}\,dx$$
$$\Big\downarrow \; 7120$$
$$\int \frac{\Gamma(0,a+bx)}{c+dx}dx$$

input
```
Int[Gamma[0, a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\text{expIntegral}_1\,(bx+a)}{dx+c}dx$$

input
```
int(Ei(1,b*x+a)/(d*x+c),x)
```

output
```
int(Ei(1,b*x+a)/(d*x+c),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\Gamma(0,a+bx)}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp_integral_e(1,b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated (((
(d)*(x))+(c))^(((-1)::EXPR INT)))*(exp_integral_e(((1)::EXPR INT),((b)*(x)
)+(a))):   There are no library operations named exp_integral_e       Use
HyperDo
```

Sympy [N/A]

Not integrable

Time = 21.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{\Gamma(0,a+bx)}{c+dx}\,dx = \int \frac{\mathrm{E}_1\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(expint(1,b*x+a)/(d*x+c),x)
```

output
```
Integral(expint(1, a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(0,a+bx)}{c+dx}\,dx = \int \frac{E_1(bx+a)}{dx+c}\,dx$$

input
```
integrate(exp_integral_e(1,b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(exp_integral_e(1, b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(0, a + bx)}{c + dx} \, dx = \int \frac{E_1(bx + a)}{dx + c} \, dx$$

input
```
integrate(exp_integral_e(1,b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(exp_integral_e(1, b*x + a)/(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{\Gamma(0, a + bx)}{c + dx} \, dx = \int \frac{\mathrm{expint}(a + b\,x)}{c + d\,x} \, dx$$

input
```
int(expint(a + b*x)/(c + d*x),x)
```

output
```
int(expint(a + b*x)/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(0, a + bx)}{c + dx} \, dx = \int \frac{ei(1, bx + a)}{dx + c} dx$$

input
```
int(Ei(1,b*x+a)/(d*x+c),x)
```

output
```
int(ei(1,a + b*x)/(c + d*x),x)
```

3.107 $\int \frac{\Gamma(0,a+bx)}{(c+dx)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 82

$$\int \frac{\Gamma(0,a+bx)}{(c+dx)^2} \, dx = \frac{b\Gamma(0,a+bx)}{d(bc-ad)} - \frac{\Gamma(0,a+bx)}{d(c+dx)} - \frac{be^{-a+\frac{bc}{d}}\Gamma\left(0,\frac{b(c+dx)}{d}\right)}{d(bc-ad)}$$

output
```
b*Ei(1,b*x+a)/d/(-a*d+b*c)-Ei(1,b*x+a)/d/(d*x+c)-b*exp(-a+b*c/d)*Ei(1,b*(d
*x+c)/d)/d/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{\Gamma(0,a+bx)}{(c+dx)^2} \, dx = \frac{\frac{b \operatorname{ExpIntegralEi}(-a-bx)}{-bc+ad} + \frac{be^{-a+\frac{bc}{d}} \operatorname{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{bc-ad} - \frac{\Gamma(0,a+bx)}{c+dx}}{d}$$

input
```
Integrate[Gamma[0, a + b*x]/(c + d*x)^2,x]
```

output
```
((b*ExpIntegralEi[-a - b*x])/(-(b*c) + a*d) + (b*E^(-a + (b*c)/d)*ExpInteg
ralEi[-((b*(c + d*x))/d)])/(b*c - a*d) - Gamma[0, a + b*x]/(c + d*x))/d
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7119, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(0,a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{ 7119}$$

$$-\frac{b\int \frac{e^{-a-bx}}{(a+bx)(c+dx)}dx}{d}-\frac{\Gamma(0,a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{ 7293}$$

$$-\frac{b\int \left(\frac{be^{-a-bx}}{(bc-ad)(a+bx)}-\frac{de^{-a-bx}}{(bc-ad)(c+dx)}\right)dx}{d}-\frac{\Gamma(0,a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b\left(\frac{\text{ExpIntegralEi}(-a-bx)}{bc-ad}-\frac{e^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{bc-ad}\right)}{d}-\frac{\Gamma(0,a+bx)}{d(c+dx)}$$

input
```
Int[Gamma[0, a + b*x]/(c + d*x)^2,x]
```

output
```
-((b*(ExpIntegralEi[-a - b*x]/(b*c - a*d) - (E^(-a + (b*c)/d)*ExpIntegralE
i[-((b*(c + d*x))/d)])/(b*c - a*d)))/d) - Gamma[0, a + b*x]/(d*(c + d*x))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.13

method	result	size
parts	$-\frac{\text{expIntegral}_1(bx+a)}{d(dx+c)}-\frac{\frac{b\,\text{expIntegral}_1(bx+a)}{ad-cb}-\frac{b\,\mathrm{e}^{-\frac{ad-cb}{d}}\,\text{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{ad-cb}}{d}$	93
derivativedivides	$\frac{\frac{b^2\,\text{expIntegral}_1(bx+a)}{(ad-cb-d(bx+a))d}+\frac{b^2\left(-\frac{\text{expIntegral}_1(bx+a)}{ad-cb}+\frac{\mathrm{e}^{-\frac{ad-cb}{d}}\,\text{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{ad-cb}\right)}{d}}{b}$	110
default	$\frac{\frac{b^2\,\text{expIntegral}_1(bx+a)}{(ad-cb-d(bx+a))d}+\frac{b^2\left(-\frac{\text{expIntegral}_1(bx+a)}{ad-cb}+\frac{\mathrm{e}^{-\frac{ad-cb}{d}}\,\text{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{ad-cb}\right)}{d}}{b}$	110

input
```
int(Ei(1,b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-Ei(1,b*x+a)/d/(d*x+c)-1/d*(b/(a*d-b*c)*Ei(1,b*x+a)-b/(a*d-b*c)*exp(-(a*d-
b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\Gamma(0, a + bx)}{(c + dx)^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp_integral_e(1,b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated (((
(d)*(x))+(c))^(((-2)::EXPR INT)))*(exp_integral_e(((1)::EXPR INT),((b)*(x)
)+(a))):   There are no library operations named exp_integral_e       Use
HyperDo
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\Gamma(0, a + bx)}{(c + dx)^2} \, dx = \text{Timed out}$$

input
```
integrate(expint(1,b*x+a)/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\Gamma(0, a + bx)}{(c + dx)^2} \, dx = \int \frac{E_1(bx + a)}{(dx + c)^2} \, dx$$

input
```
integrate(exp_integral_e(1,b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(exp_integral_e(1, b*x + a)/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{\Gamma(0,a+bx)}{(c+dx)^2}\,dx = \int \frac{E_1(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(exp_integral_e(1,b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(exp_integral_e(1, b*x + a)/(d*x + c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.20

$$\int \frac{\Gamma(0,a+bx)}{(c+dx)^2}\,dx = \int \frac{\mathrm{expint}(a+b\,x)}{(c+d\,x)^2}\,dx$$

input
```
int(expint(a + b*x)/(c + d*x)^2,x)
```

output
```
int(expint(a + b*x)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\Gamma(0,a+bx)}{(c+dx)^2}\,dx = \int \frac{ei(1,bx+a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(Ei(1,b*x+a)/(d*x+c)^2,x)
```

output
```
int(ei(1,a + b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.108 $\int \frac{\Gamma(0,a+bx)}{(c+dx)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 137

$$\int \frac{\Gamma(0,a+bx)}{(c+dx)^3} dx = -\frac{b^2e^{-a+\frac{bc}{d}}\Gamma\left(-1,\frac{b(c+dx)}{d}\right)}{2d^2(bc-ad)}+\frac{b^2\Gamma(0,a+bx)}{2d(bc-ad)^2} -\frac{\Gamma(0,a+bx)}{2d(c+dx)^2}-\frac{b^2e^{-a+\frac{bc}{d}}\Gamma\left(0,\frac{b(c+dx)}{d}\right)}{2d(bc-ad)^2}$$

output
```
-1/2*b*exp(-a+b*c/d)/(d*x+c)/d*Ei(2,b*(d*x+c)/d)/(-a*d+b*c)+1/2*b^2*Ei(1,b
*x+a)/d/(-a*d+b*c)^2-1/2*Ei(1,b*x+a)/d/(d*x+c)^2-1/2*b^2*exp(-a+b*c/d)*Ei(
1,b*(d*x+c)/d)/d/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int \frac{\Gamma(0,a+bx)}{(c+dx)^3} dx = -\frac{\frac{b^2d\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^2}+\frac{b^2(bc-(1+a)d)e^{-a+\frac{bc}{d}}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{(bc-ad)^2}+\frac{d\left(\frac{be^{-a-bx}(c+dx)}{bc-ad}+\Gamma(0,a+bx)\right)}{(c+dx)^2}}{2d^2}$$

input
```
Integrate[Gamma[0, a + b*x]/(c + d*x)^3,x]
```

output
```
-1/2*((b^2*d*ExpIntegralEi[-a - b*x])/(b*c - a*d)^2 + (b^2*(b*c - (1 + a)*
d)*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/(b*c - a*d)^2 + (d*
((b*E^(-a - b*x)*(c + d*x))/(b*c - a*d) + Gamma[0, a + b*x]))/(c + d*x)^2)
/d^2
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(0,a+bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{7119}$$

$$-\frac{b\int \frac{e^{-a-bx}}{(a+bx)(c+dx)^2}dx}{2d}-\frac{\Gamma(0,a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\int \left(\frac{e^{-a-bx}b^2}{(bc-ad)^2(a+bx)}-\frac{de^{-a-bx}b}{(bc-ad)^2(c+dx)}-\frac{de^{-a-bx}}{(bc-ad)(c+dx)^2}\right)dx}{2d}-\frac{\Gamma(0,a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{b\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^2}+\frac{be^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d(bc-ad)}-\frac{be^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{(bc-ad)^2}+\frac{e^{-a-bx}}{(c+dx)(bc-ad)}\right)}{2d}-\frac{\Gamma(0,a+bx)}{2d(c+dx)^2}$$

input
```
Int[Gamma[0, a + b*x]/(c + d*x)^3,x]
```

output

```
-1/2*(b*(E^(-a - b*x)/((b*c - a*d)*(c + d*x)) + (b*ExpIntegralEi[-a - b*x]
)/(b*c - a*d)^2 - (b*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/(
b*c - a*d)^2 + (b*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/(d*(
b*c - a*d))))/d - Gamma[0, a + b*x]/(2*d*(c + d*x)^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7119

```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

rule 7293

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.35

method	result
parts	$-\frac{\operatorname{expIntegral}_1(bx+a)}{2d(dx+c)^2}-\frac{\frac{b^2 e^{-\frac{ad-cb}{d}}\operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{(ad-cb)^2}-\frac{b^2\left(-\frac{e^{-bx-a}}{-bx-a+\frac{ad-cb}{d}}-e^{-\frac{ad-cb}{d}}\operatorname{expIntegral}_1\left(bx+a-\right.\right.}{(ad-cb)d}}{2d}$
derivativedivides	$\frac{-\frac{b^3\operatorname{expIntegral}_1(bx+a)}{2(ad-cb-d(bx+a))^2 d}-\frac{b^3\left(-\frac{-\frac{e^{-bx-a}}{-bx-a+\frac{ad-cb}{d}}-e^{-\frac{ad-cb}{d}}\operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{(ad-cb)d}-\frac{\operatorname{expIntegral}_1(bx+a)}{(ad-cb)^2}+\frac{e^{-\frac{ad-cb}{d}}\operatorname{ex}}{}\right.}{2d}}{b}$
default	$\frac{-\frac{b^3\operatorname{expIntegral}_1(bx+a)}{2(ad-cb-d(bx+a))^2 d}-\frac{b^3\left(-\frac{-\frac{e^{-bx-a}}{-bx-a+\frac{ad-cb}{d}}-e^{-\frac{ad-cb}{d}}\operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{(ad-cb)d}-\frac{\operatorname{expIntegral}_1(bx+a)}{(ad-cb)^2}+\frac{e^{-\frac{ad-cb}{d}}\operatorname{ex}}{}\right.}{2d}}{b}$

input
```
int(Ei(1,b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*Ei(1,b*x+a)/d/(d*x+c)^2-1/2/d*(b^2/(a*d-b*c)^2*exp(-(a*d-b*c)/d)*Ei(1
,b*x+a-(a*d-b*c)/d)-b^2/(a*d-b*c)/d*(-exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-exp
(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))-1/(a*d-b*c)^2*b^2*Ei(1,b*x+a))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\Gamma(0, a + bx)}{(c + dx)^3} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp_integral_e(1,b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated (((
(d)*(x))+(c))^(((-3)::EXPR INT)))*(exp_integral_e(((1)::EXPR INT),((b)*(x)
)+(a))):   There are no library operations named exp_integral_e       Use
HyperDo
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\Gamma(0, a + bx)}{(c + dx)^3} \, dx = \text{Timed out}$$

input
```
integrate(expint(1,b*x+a)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\Gamma(0, a+bx)}{(c+dx)^3} \, dx = \int \frac{E_1(bx+a)}{(dx+c)^3} \, dx$$

input
```
integrate(exp_integral_e(1,b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(exp_integral_e(1, b*x + a)/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{\Gamma(0, a+bx)}{(c+dx)^3} \, dx = \int \frac{E_1(bx+a)}{(dx+c)^3} \, dx$$

input
```
integrate(exp_integral_e(1,b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(exp_integral_e(1, b*x + a)/(d*x + c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.12

$$\int \frac{\Gamma(0, a+bx)}{(c+dx)^3} \, dx = \int \frac{\mathrm{expint}(a+b\,x)}{(c+d\,x)^3} \, dx$$

input
```
int(expint(a + b*x)/(c + d*x)^3,x)
```

output
```
int(expint(a + b*x)/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\Gamma(0,a+bx)}{(c+dx)^3}\,dx=\int \frac{ei(1,bx+a)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int(Ei(1,b*x+a)/(d*x+c)^3,x)
```

output
```
int(ei(1,a + b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)
```

3.109 $\int \frac{\Gamma(0,a+bx)}{(c+dx)^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 181

$$\int \frac{\Gamma(0,a+bx)}{(c+dx)^4} dx = -\frac{b^3 e^{-a+\frac{bc}{d}}\Gamma\left(-2,\frac{b(c+dx)}{d}\right)}{3d^3(bc-ad)} - \frac{b^3 e^{-a+\frac{bc}{d}}\Gamma\left(-1,\frac{b(c+dx)}{d}\right)}{3d^2(bc-ad)^2} + \frac{b^3\Gamma(0,a+bx)}{3d(bc-ad)^3} - \frac{\Gamma(0,a+bx)}{3d(c+dx)^3} - \frac{b^3 e^{-a+\frac{bc}{d}}\Gamma\left(0,\frac{b(c+dx)}{d}\right)}{3d(bc-ad)^3}$$

output
```
-1/3*b*exp(-a+b*c/d)/(d*x+c)^2/d*Ei(3,b*(d*x+c)/d)/(-a*d+b*c)-1/3*b^2*exp(
-a+b*c/d)/(d*x+c)/d*Ei(2,b*(d*x+c)/d)/(-a*d+b*c)^2+1/3*b^3*Ei(1,b*x+a)/d/(
-a*d+b*c)^3-1/3*Ei(1,b*x+a)/d/(d*x+c)^3-1/3*b^3*exp(-a+b*c/d)*Ei(1,b*(d*x+
c)/d)/d/(-a*d+b*c)^3
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.99

$$\int \frac{\Gamma(0,a+bx)}{(c+dx)^4} dx = \frac{\frac{2b^3d^2\,\text{ExpIntegralEi}(-a-bx)}{(-bc+ad)^3} + \frac{b^3\left(b^2c^2-2(1+a)bcd+\left(2+2a+a^2\right)d^2\right)e^{-a+\frac{bc}{d}}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{(bc-ad)^3} + d\left(\frac{be^{-a-bx}(c+dx)\left(ad^2+b^2c(c+}{(bc-ad}\right.}{6d^3}$$

input
```
Integrate[Gamma[0, a + b*x]/(c + d*x)^4,x]
```

output
```
((2*b^3*d^2*ExpIntegralEi[-a - b*x])/(-(b*c) + a*d)^3 + (b^3*(b^2*c^2 - 2*
(1 + a)*b*c*d + (2 + 2*a + a^2)*d^2)*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(
c + d*x))/d)])/(b*c - a*d)^3 + (d*((b*E^(-a - b*x)*(c + d*x)*(a*d^2 + b^2*
c*(c + d*x) - b*d*((3 + a)*c + (2 + a)*d*x)))/(b*c - a*d)^2 - 2*d*Gamma[0,
 a + b*x]))/(c + d*x)^3)/(6*d^3)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.51, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7119, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(0,a+bx)}{(c+dx)^4}dx$$

$$\Big\downarrow \text{7119}$$

$$-\frac{b\int \frac{e^{-a-bx}}{(a+bx)(c+dx)^3}dx}{3d}-\frac{\Gamma(0,a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\int \left(\frac{e^{-a-bx}b^3}{(bc-ad)^3(a+bx)}-\frac{de^{-a-bx}b^2}{(bc-ad)^3(c+dx)}-\frac{de^{-a-bx}b}{(bc-ad)^2(c+dx)^2}-\frac{de^{-a-bx}}{(bc-ad)(c+dx)^3}\right)dx}{3d}-\frac{\Gamma(0,a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{b^2e^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{2d^2(bc-ad)}+\frac{b^2\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^3}+\frac{b^2e^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d(bc-ad)^2}-\frac{b^2e^{\frac{bc}{d}-a}\text{ExpIntegralEi}(}{(bc-ad)^3}\right.}{3d}$$

$$\frac{\Gamma(0,a+bx)}{3d(c+dx)^3}$$

input
```
Int[Gamma[0, a + b*x]/(c + d*x)^4,x]
```

output
```
-1/3*(b*(E^(-a - b*x)/(2*(b*c - a*d)*(c + d*x)^2) + (b*E^(-a - b*x))/((b*c
 - a*d)^2*(c + d*x)) - (b*E^(-a - b*x))/(2*d*(b*c - a*d)*(c + d*x)) + (b^2
*ExpIntegralEi[-a - b*x])/(b*c - a*d)^3 - (b^2*E^(-a + (b*c)/d)*ExpIntegra
lEi[-((b*(c + d*x))/d)])/(b*c - a*d)^3 + (b^2*E^(-a + (b*c)/d)*ExpIntegral
Ei[-((b*(c + d*x))/d)])/(d*(b*c - a*d)^2) - (b^2*E^(-a + (b*c)/d)*ExpInteg
ralEi[-((b*(c + d*x))/d)])/(2*d^2*(b*c - a*d))))/d - Gamma[0, a + b*x]/(3*
d*(c + d*x)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.68

method	result
parts	$-\frac{\mathrm{expIntegral}_1(bx+a)}{3d(dx+c)^3} - \frac{\frac{b^3\,\mathrm{expIntegral}_1(bx+a)}{(ad-cb)^3} - \frac{b^3 e^{-\frac{ad-cb}{d}}\,\mathrm{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{(ad-cb)^3} + \frac{b^3\left(-\frac{e^{-bx-a}}{-bx-a+\frac{ad-cb}{d}} - e^{-\frac{ad}{}}\right.}{(a}}{}$
derivativedivides	$\frac{\frac{b^4\,\mathrm{expIntegral}_1(bx+a)}{3(ad-cb-d(bx+a))^3 d} + \frac{b^4\left(-\frac{-\frac{e^{-bx-a}}{2\left(-bx-a+\frac{ad-cb}{d}\right)^2} - \frac{e^{-bx-a}}{2\left(-bx-a+\frac{ad-cb}{d}\right)} - \frac{e^{-\frac{ad-cb}{d}}\,\mathrm{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{2}}{(ad-cb)d^2} + \frac{e^{-\frac{ad-c}{d}}}{}\right.}{}}{b}$
default	$\frac{\frac{b^4\,\mathrm{expIntegral}_1(bx+a)}{3(ad-cb-d(bx+a))^3 d} + \frac{b^4\left(-\frac{-\frac{e^{-bx-a}}{2\left(-bx-a+\frac{ad-cb}{d}\right)^2} - \frac{e^{-bx-a}}{2\left(-bx-a+\frac{ad-cb}{d}\right)} - \frac{e^{-\frac{ad-cb}{d}}\,\mathrm{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{2}}{(ad-cb)d^2} + \frac{e^{-\frac{ad-c}{d}}}{}\right.}{}}{b}$

input
```
int(Ei(1,b*x+a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*Ei(1,b*x+a)/d/(d*x+c)^3-1/3/d*(b^3/(a*d-b*c)^3*Ei(1,b*x+a)-b^3/(a*d-b
*c)^3*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+b^3/(a*d-b*c)^2/d*(-exp(-b
*x-a)/(-b*x-a+(a*d-b*c)/d)-exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))+b^3/
(a*d-b*c)/d^2*(-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/2*exp(-b*x-a)/(-b
*x-a+(a*d-b*c)/d)-1/2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\Gamma(0,a+bx)}{(c+dx)^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp_integral_e(1,b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated (((
(d)*(x))+(c))^(((-4)::EXPR INT)))*(exp_integral_e(((1)::EXPR INT),((b)*(x)
)+(a))):   There are no library operations named exp_integral_e       Use
HyperDo
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\Gamma(0, a+bx)}{(c+dx)^4}\, dx = \text{Timed out}$$

input
```
integrate(expint(1,b*x+a)/(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\Gamma(0, a+bx)}{(c+dx)^4}\, dx = \int \frac{E_1(bx+a)}{(dx+c)^4}\, dx$$

input
```
integrate(exp_integral_e(1,b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate(exp_integral_e(1, b*x + a)/(d*x + c)^4, x)
```

Giac [F]

$$\int \frac{\Gamma(0, a+bx)}{(c+dx)^4}\, dx = \int \frac{E_1(bx+a)}{(dx+c)^4}\, dx$$

input
```
integrate(exp_integral_e(1,b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate(exp_integral_e(1, b*x + a)/(d*x + c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.09

$$\int \frac{\Gamma(0,a+bx)}{(c+dx)^4}\,dx=\int \frac{\mathrm{expint}(a+b\,x)}{{\left(c+d\,x\right)}^4}\,dx$$

input
```
int(expint(a + b*x)/(c + d*x)^4,x)
```

output
```
int(expint(a + b*x)/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{\Gamma(0,a+bx)}{(c+dx)^4}\,dx=\int \frac{ei(1,bx+a)}{d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+c^4}dx$$

input
```
int(Ei(1,b*x+a)/(d*x+c)^4,x)
```

output
```
int(ei(1,a + b*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 +
d**4*x**4),x)
```

3.110 $\int e^{-a-bx}(c+dx)^4\,dx$

Optimal result

Integrand size = 18, antiderivative size = 112

$$\int e^{-a-bx}(c+dx)^4\,dx=-\frac{24d^4e^{-a-bx}}{b^5}-\frac{24d^3e^{-a-bx}(c+dx)}{b^4}-\frac{12d^2e^{-a-bx}(c+dx)^2}{b^3}-\frac{4de^{-a-bx}(c+dx)^3}{b^2}-\frac{e^{-a-bx}(c+dx)^4}{b}$$

output
```
-24*d^4*exp(-b*x-a)/b^5-24*d^3*exp(-b*x-a)*(d*x+c)/b^4-12*d^2*exp(-b*x-a)*
(d*x+c)^2/b^3-4*d*exp(-b*x-a)*(d*x+c)^3/b^2-exp(-b*x-a)*(d*x+c)^4/b
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.63

$$\int e^{-a-bx}(c+dx)^4\,dx=\frac{e^{-a-bx}(-24d^4-24bd^3(c+dx)-12b^2d^2(c+dx)^2-4b^3d(c+dx)^3-b^4(c+dx)^4)}{b^5}$$

input
```
Integrate[E^(-a - b*x)*(c + d*x)^4,x]
```

output
```
(E^(-a - b*x)*(-24*d^4 - 24*b*d^3*(c + d*x) - 12*b^2*d^2*(c + d*x)^2 - 4*b
^3*d*(c + d*x)^3 - b^4*(c + d*x)^4))/b^5
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2607, 2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-a-bx}(c+dx)^4\,dx$$

$$\Big\downarrow \text{2607}$$

$$\frac{4d\int e^{-a-bx}(c+dx)^3dx}{b}-\frac{e^{-a-bx}(c+dx)^4}{b}$$

$$\Big\downarrow \text{2607}$$

$$\frac{4d\left(\frac{3d\int e^{-a-bx}(c+dx)^2dx}{b}-\frac{e^{-a-bx}(c+dx)^3}{b}\right)}{b}-\frac{e^{-a-bx}(c+dx)^4}{b}$$

$$\Big\downarrow \text{2607}$$

$$\frac{4d\left(\frac{3d\left(\frac{2d\int e^{-a-bx}(c+dx)dx}{b}-\frac{e^{-a-bx}(c+dx)^2}{b}\right)}{b}-\frac{e^{-a-bx}(c+dx)^3}{b}\right)}{b}-\frac{e^{-a-bx}(c+dx)^4}{b}$$

$$\Big\downarrow \text{2607}$$

$$\frac{4d\left(\frac{3d\left(\frac{2d\left(\frac{d\int e^{-a-bx}dx}{b}-\frac{e^{-a-bx}(c+dx)}{b}\right)}{b}-\frac{e^{-a-bx}(c+dx)^2}{b}\right)}{b}-\frac{e^{-a-bx}(c+dx)^3}{b}\right)}{b}-\frac{e^{-a-bx}(c+dx)^4}{b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{4d\left(\frac{3d\left(\frac{2d\left(-\frac{de^{-a-bx}}{b^2}-\frac{e^{-a-bx}(c+dx)}{b}\right)}{b}-\frac{e^{-a-bx}(c+dx)^2}{b}\right)}{b}-\frac{e^{-a-bx}(c+dx)^3}{b}\right)}{b}-\frac{e^{-a-bx}(c+dx)^4}{b}$$

input
```
Int[E^(-a - b*x)*(c + d*x)^4,x]
```

output
```
-((E^(-a - b*x)*(c + d*x)^4)/b) + (4*d*(-((E^(-a - b*x)*(c + d*x)^3)/b) +
(3*d*(-((E^(-a - b*x)*(c + d*x)^2)/b) + (2*d*(-((d*E^(-a - b*x))/b^2) - (E
^(-a - b*x)*(c + d*x))/b))/b))/b))/b
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.46

method	result
gosper	$-\frac{(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^3d^4x^3+4b^4c^3dx+12b^3c\,d^3x^2+c^4b^4+12b^3c^2d^2x+12b^2d^4x^2+4b^3c^3d+24b^2c\,d^3x \cdots}{b^5}$
risch	$-\frac{(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^3d^4x^3+4b^4c^3dx+12b^3c\,d^3x^2+c^4b^4+12b^3c^2d^2x+12b^2d^4x^2+4b^3c^3d+24b^2c\,d^3x \cdots}{b^5}$
orering	$-\frac{(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^3d^4x^3+4b^4c^3dx+12b^3c\,d^3x^2+c^4b^4+12b^3c^2d^2x+12b^2d^4x^2+4b^3c^3d+24b^2c\,d^3x \cdots}{b^5}$
meijerg	$\frac{e^{-a}d^4\left(24-\frac{(5b^4x^4+20b^3x^3+60b^2x^2+120bx+120)e^{-bx}}{5}\right)}{b^5}+\frac{4\,e^{-a}d^3c\left(6-\frac{(4b^3x^3+12b^2x^2+24bx+24)e^{-bx}}{4}\right)}{b^4}+\frac{6\,e^{-a}d^2 \cdots}{\cdots}$
norman	$-\frac{(c^4b^4+4b^3c^3d+12b^2c^2d^2+24bc\,d^3+24d^4)e^{-bx-a}}{b^5}-\frac{d^4x^4e^{-bx-a}}{b}-\frac{4d(b^3c^3+3b^2c^2d+6d^2cb+6d^3)x\,e^{-bx-a}}{b^4}- \cdots$
parallelrisch	$-\frac{x^4e^{-bx-a}d^4b^4+4x^3e^{-bx-a}b^4c\,d^3+4x^3e^{-bx-a}b^3d^4+6x^2e^{-bx-a}b^4c^2d^2+12x^2e^{-bx-a}b^3c\,d^3+4x\,e^{-bx-a}b^4c^3d+12x \cdots}{\cdots}$
parts	$-\frac{d^4x^4e^{-bx-a}}{b}-\frac{4\,e^{-bx-a}c\,d^3x^3}{b}-\frac{6\,e^{-bx-a}c^2d^2x^2}{b}-\frac{4\,e^{-bx-a}c^3dx}{b}-\frac{e^{-bx-a}c^4}{b}-\frac{4d\left(e^{-bx-a}c^3-\frac{e^{-bx-a}\cdots}{b^3}\cdots\right)}{\cdots}$
derivativedivides	$-\frac{e^{-bx-a}c^4+\frac{e^{-bx-a}d^4a^4}{b^4}+\frac{d^4\left(e^{-bx-a}(-bx-a)^4-4\,e^{-bx-a}(-bx-a)^3+12(-bx-a)^2e^{-bx-a}-24(-bx-a)e^{-bx-a}+24\,e^{-bx-}\cdots\right)}{b^4}\cdots}{\cdots}$
default	$-\frac{e^{-bx-a}c^4+\frac{e^{-bx-a}d^4a^4}{b^4}+\frac{d^4\left(e^{-bx-a}(-bx-a)^4-4\,e^{-bx-a}(-bx-a)^3+12(-bx-a)^2e^{-bx-a}-24(-bx-a)e^{-bx-a}+24\,e^{-bx-}\cdots\right)}{b^4}\cdots}{\cdots}$

input
```
int(exp(-b*x-a)*(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-(b^4*d^4*x^4+4*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^3*d^4*x^3+4*b^4*c^3*d*
x+12*b^3*c*d^3*x^2+b^4*c^4+12*b^3*c^2*d^2*x+12*b^2*d^4*x^2+4*b^3*c^3*d+24*
b^2*c*d^3*x+12*b^2*c^2*d^2+24*b*d^4*x+24*b*c*d^3+24*d^4)*exp(-b*x-a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.39

$$\int e^{-a-bx}(c+dx)^4\,dx =$$

$$-\frac{(b^4d^4x^4+b^4c^4+4\,b^3c^3d+12\,b^2c^2d^2+24\,bcd^3+24\,d^4+4\,(b^4cd^3+b^3d^4)x^3+6\,(b^4c^2d^2+2\,b^3cd^3+2\,b^{\cdots}}{b^5}$$

input
```
integrate(exp(-b*x-a)*(d*x+c)^4,x, algorithm="fricas")
```

output
```
-(b^4*d^4*x^4 + b^4*c^4 + 4*b^3*c^3*d + 12*b^2*c^2*d^2 + 24*b*c*d^3 + 24*d
^4 + 4*(b^4*c*d^3 + b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 + 2*b^3*c*d^3 + 2*b^2*d^
4)*x^2 + 4*(b^4*c^3*d + 3*b^3*c^2*d^2 + 6*b^2*c*d^3 + 6*b*d^4)*x)*e^(-b*x
- a)/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(102) = 204$.

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.99

$$\int e^{-a-bx}(c+dx)^4\,dx$$
$$= \begin{cases} \frac{(-b^4c^4-4b^4c^3dx-6b^4c^2d^2x^2-4b^4cd^3x^3-b^4d^4x^4-4b^3c^3d-12b^3c^2d^2x-12b^3cd^3x^2-4b^3d^4x^3-12b^2c^2d^2-24b^2cd^3x-12b^2d^4x^2-24bcd^3-24bc}{b^5} \\ c^4x+2c^3dx^2+2c^2d^2x^3+cd^3x^4+\frac{d^4x^5}{5} \end{cases}$$

input
```
integrate(exp(-b*x-a)*(d*x+c)**4,x)
```

output
```
Piecewise(((-b**4*c**4 - 4*b**4*c**3*d*x - 6*b**4*c**2*d**2*x**2 - 4*b**4*
c*d**3*x**3 - b**4*d**4*x**4 - 4*b**3*c**3*d - 12*b**3*c**2*d**2*x - 12*b*
*3*c*d**3*x**2 - 4*b**3*d**4*x**3 - 12*b**2*c**2*d**2 - 24*b**2*c*d**3*x -
 12*b**2*d**4*x**2 - 24*b*c*d**3 - 24*b*d**4*x - 24*d**4)*exp(-a - b*x)/b*
*5, Ne(b**5, 0)), (c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4
 + d**4*x**5/5, True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.42

$$\int e^{-a-bx}(c+dx)^4\,dx = -\frac{c^4e^{(-bx-a)}}{b} - \frac{4\,(bx+1)c^3de^{(-bx-a)}}{b^2} - \frac{6\,(b^2x^2+2\,bx+2)c^2d^2e^{(-bx-a)}}{b^3} - \frac{4\,(b^3x^3+3\,b^2x^2+6\,bx+6)cd^3e^{(-bx-a)}}{b^4} - \frac{(b^4x^4+4\,b^3x^3+12\,b^2x^2+24\,bx+24)d^4e^{(-bx-a)}}{b^5}$$

input
```
integrate(exp(-b*x-a)*(d*x+c)^4,x, algorithm="maxima")
```

output
```
-c^4*e^(-b*x - a)/b - 4*(b*x + 1)*c^3*d*e^(-b*x - a)/b^2 - 6*(b^2*x^2 + 2*
b*x + 2)*c^2*d^2*e^(-b*x - a)/b^3 - 4*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*c*
d^3*e^(-b*x - a)/b^4 - (b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*d^
4*e^(-b*x - a)/b^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.46

$$\int e^{-a-bx}(c+dx)^4\,dx = -\frac{(b^4d^4x^4+4\,b^4cd^3x^3+6\,b^4c^2d^2x^2+4\,b^3d^4x^3+4\,b^4c^3dx+12\,b^3cd^3x^2+b^4c^4+12\,b^3c^2d^2x+12\,b^2d^4x^2+}{b^5}$$

input
```
integrate(exp(-b*x-a)*(d*x+c)^4,x, algorithm="giac")
```

output
```
-(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^3*d^4*x^3 + 4*b^
4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4 + 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^2
 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 + 24*b*d^4*x + 24*b*c*d^3
 + 24*d^4)*e^(-b*x - a)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.46

$$\int e^{-a-bx}(c+dx)^4\,dx = -\frac{e^{-a-b\,x}\,(b^4\,c^4+4\,b^4\,c^3\,d\,x+6\,b^4\,c^2\,d^2\,x^2+4\,b^4\,c\,d^3\,x^3+b^4\,d^4\,x^4+4\,b^3\,c^3\,d+12\,b^3\,c^2\,d^2\,x+12\,b^3\,c\,d^3\,x^2}{b^5}$$

input
```
int(exp(- a - b*x)*(c + d*x)^4,x)
```

output
```
-(exp(- a - b*x)*(24*d^4 + b^4*c^4 + 4*b^3*c^3*d + 12*b^2*c^2*d^2 + 12*b^2
*d^4*x^2 + 4*b^3*d^4*x^3 + b^4*d^4*x^4 + 24*b*c*d^3 + 24*b*d^4*x + 12*b^3*
c^2*d^2*x + 12*b^3*c*d^3*x^2 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 24*b^
2*c*d^3*x + 4*b^4*c^3*d*x))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.46

$$\int e^{-a-bx}(c+dx)^4\,dx = \frac{-b^4d^4x^4-4b^4c\,d^3x^3-6b^4c^2d^2x^2-4b^3d^4x^3-4b^4c^3dx-12b^3c\,d^3x^2-b^4c^4-12b^3c^2d^2x-12b^2d^4x^2-4b}{e^{bx+a}b^5}$$

input
```
int(exp(-b*x-a)*(d*x+c)^4,x)
```

output
```
( - b**4*c**4 - 4*b**4*c**3*d*x - 6*b**4*c**2*d**2*x**2 - 4*b**4*c*d**3*x*
*3 - b**4*d**4*x**4 - 4*b**3*c**3*d - 12*b**3*c**2*d**2*x - 12*b**3*c*d**3
*x**2 - 4*b**3*d**4*x**3 - 12*b**2*c**2*d**2 - 24*b**2*c*d**3*x - 12*b**2*
d**4*x**2 - 24*b*c*d**3 - 24*b*d**4*x - 24*d**4)/(e**(a + b*x)*b**5)
```

3.111 $\int e^{-a-bx}(c+dx)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 87

$$\int e^{-a-bx}(c+dx)^3\,dx=-\frac{6d^3e^{-a-bx}}{b^4}-\frac{6d^2e^{-a-bx}(c+dx)}{b^3}-\frac{3de^{-a-bx}(c+dx)^2}{b^2}-\frac{e^{-a-bx}(c+dx)^3}{b}$$

output
```
-6*d^3*exp(-b*x-a)/b^4-6*d^2*exp(-b*x-a)*(d*x+c)/b^3-3*d*exp(-b*x-a)*(d*x+
c)^2/b^2-exp(-b*x-a)*(d*x+c)^3/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int e^{-a-bx}(c+dx)^3\,dx=\frac{e^{-a-bx}(-6d^3-6bd^2(c+dx)-3b^2d(c+dx)^2-b^3(c+dx)^3)}{b^4}$$

input
```
Integrate[E^(-a - b*x)*(c + d*x)^3,x]
```

output
```
(E^(-a - b*x)*(-6*d^3 - 6*b*d^2*(c + d*x) - 3*b^2*d*(c + d*x)^2 - b^3*(c +
 d*x)^3))/b^4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-a-bx}(c+dx)^3\,dx$$

$$\Big\downarrow \text{2607}$$

$$\frac{3d\int e^{-a-bx}(c+dx)^2dx}{b}-\frac{e^{-a-bx}(c+dx)^3}{b}$$

$$\Big\downarrow \text{2607}$$

$$\frac{3d\left(\frac{2d\int e^{-a-bx}(c+dx)dx}{b}-\frac{e^{-a-bx}(c+dx)^2}{b}\right)}{b}-\frac{e^{-a-bx}(c+dx)^3}{b}$$

$$\Big\downarrow \text{2607}$$

$$\frac{3d\left(\frac{2d\left(\frac{d\int e^{-a-bx}dx}{b}-\frac{e^{-a-bx}(c+dx)}{b}\right)}{b}-\frac{e^{-a-bx}(c+dx)^2}{b}\right)}{b}-\frac{e^{-a-bx}(c+dx)^3}{b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{3d\left(\frac{2d\left(-\frac{de^{-a-bx}}{b^2}-\frac{e^{-a-bx}(c+dx)}{b}\right)}{b}-\frac{e^{-a-bx}(c+dx)^2}{b}\right)}{b}-\frac{e^{-a-bx}(c+dx)^3}{b}$$

input
```
Int[E^(-a - b*x)*(c + d*x)^3,x]
```

output
```
-((E^(-a - b*x)*(c + d*x)^3)/b) + (3*d*(-((E^(-a - b*x)*(c + d*x)^2)/b) +
(2*d*(-((d*E^(-a - b*x))/b^2) - (E^(-a - b*x)*(c + d*x))/b))/b))/b
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

method	result
gosper	$-\frac{\left(b^3 d^3 x^3+3 b^3 c\, d^2 x^2+3 b^3 c^2 d x+3 b^2 d^3 x^2+b^3 c^3+6 b^2 c\, d^2 x+3 b^2 c^2 d+6 b\, d^3 x+6 d^2 c b+6 d^3\right) \mathrm{e}^{-b x-a}}{b^4}$
risch	$-\frac{\left(b^3 d^3 x^3+3 b^3 c\, d^2 x^2+3 b^3 c^2 d x+3 b^2 d^3 x^2+b^3 c^3+6 b^2 c\, d^2 x+3 b^2 c^2 d+6 b\, d^3 x+6 d^2 c b+6 d^3\right) \mathrm{e}^{-b x-a}}{b^4}$
orering	$-\frac{\left(b^3 d^3 x^3+3 b^3 c\, d^2 x^2+3 b^3 c^2 d x+3 b^2 d^3 x^2+b^3 c^3+6 b^2 c\, d^2 x+3 b^2 c^2 d+6 b\, d^3 x+6 d^2 c b+6 d^3\right) \mathrm{e}^{-b x-a}}{b^4}$
norman	$-\frac{\left(b^3 c^3+3 b^2 c^2 d+6 d^2 c b+6 d^3\right) \mathrm{e}^{-b x-a}}{b^4}-\frac{d^3 x^3 \mathrm{e}^{-b x-a}}{b}-\frac{3 d \left(b^2 c^2+2 b c d+2 d^2\right) x\, \mathrm{e}^{-b x-a}}{b^3}-\frac{3 d^2 (c b+d) x^2 \mathrm{e}^{-b x-a}}{b^2}$
meijerg	$\frac{\mathrm{e}^{-a} d^3 \left(6-\frac{\left(4 b^3 x^3+12 b^2 x^2+24 b x+24\right) \mathrm{e}^{-b x}}{4}\right)}{b^4}+\frac{3\, \mathrm{e}^{-a} d^2 c \left(2-\frac{\left(3 b^2 x^2+6 b x+6\right) \mathrm{e}^{-b x}}{3}\right)}{b^3}+\frac{3\, \mathrm{e}^{-a} d\, c^2 \left(1-\frac{(2 b x+2) \mathrm{e}^{-b x}}{2}\right)}{b^2}$
parallelrisch	$-\frac{x^3 \mathrm{e}^{-b x-a} d^3 b^3+3 x^2 \mathrm{e}^{-b x-a} b^3 c\, d^2+3 x^2 \mathrm{e}^{-b x-a} b^2 d^3+3 x\, \mathrm{e}^{-b x-a} b^3 c^2 d+6 x\, \mathrm{e}^{-b x-a} b^2 c\, d^2+\mathrm{e}^{-b x-a} b^3 c^3+6 x\, \mathrm{e}^{-b x-a} b}{b^4}$
parts	$-\frac{d^3 x^3 \mathrm{e}^{-b x-a}}{b}-\frac{3\, \mathrm{e}^{-b x-a} d^2 c\, x^2}{b}-\frac{3\, \mathrm{e}^{-b x-a} d\, c^2 x}{b}-\frac{\mathrm{e}^{-b x-a} c^3}{b}-\frac{3 d \left(\mathrm{e}^{-b x-a} c^2+\frac{\mathrm{e}^{-b x-a} d^2 a^2}{b^2}+\frac{d^2 \left((-b x-a)^2 \mathrm{e}\right.}{}\right.}{}$
derivativedivides	$-\frac{\mathrm{e}^{-b x-a} c^3-\frac{\mathrm{e}^{-b x-a} d^3 a^3}{b^3}-\frac{d^3 \left(\mathrm{e}^{-b x-a} (-b x-a)^3-3 (-b x-a)^2 \mathrm{e}^{-b x-a}+6 (-b x-a) \mathrm{e}^{-b x-a}-6\, \mathrm{e}^{-b x-a}\right)}{b^3}-\frac{3\, \mathrm{e}^{-b x-a} d a\, c^2}{b}-}{}$
default	$-\frac{\mathrm{e}^{-b x-a} c^3-\frac{\mathrm{e}^{-b x-a} d^3 a^3}{b^3}-\frac{d^3 \left(\mathrm{e}^{-b x-a} (-b x-a)^3-3 (-b x-a)^2 \mathrm{e}^{-b x-a}+6 (-b x-a) \mathrm{e}^{-b x-a}-6\, \mathrm{e}^{-b x-a}\right)}{b^3}-\frac{3\, \mathrm{e}^{-b x-a} d a\, c^2}{b}-}{}$

input
```
int(exp(-b*x-a)*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+3*b^2*d^3*x^2+b^3*c^3+6*b^2*c*
d^2*x+3*b^2*c^2*d+6*b*d^3*x+6*b*c*d^2+6*d^3)*exp(-b*x-a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16

$$\int e^{-a-bx}(c+dx)^3\,dx = -\frac{(b^3d^3x^3+b^3c^3+3\,b^2c^2d+6\,bcd^2+6\,d^3+3\,(b^3cd^2+b^2d^3)x^2+3\,(b^3c^2d+2\,b^2cd^2+2\,bd^3)x)e^{(-bx-a)}}{b^4}$$

input
```
integrate(exp(-b*x-a)*(d*x+c)^3,x, algorithm="fricas")
```

output
```
-(b^3*d^3*x^3 + b^3*c^3 + 3*b^2*c^2*d + 6*b*c*d^2 + 6*d^3 + 3*(b^3*c*d^2 +
 b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2 + 2*b*d^3)*x)*e^(-b*x - a)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.68

$$\int e^{-a-bx}(c+dx)^3\,dx = \begin{cases} \frac{(-b^3c^3-3b^3c^2dx-3b^3cd^2x^2-b^3d^3x^3-3b^2c^2d-6b^2cd^2x-3b^2d^3x^2-6bcd^2-6bd^3x-6d^3)e^{-a-bx}}{b^4} & \text{for } b^4\neq 0 \\ c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(-b*x-a)*(d*x+c)**3,x)
```

output
```
Piecewise(((-b**3*c**3 - 3*b**3*c**2*d*x - 3*b**3*c*d**2*x**2 - b**3*d**3*
x**3 - 3*b**2*c**2*d - 6*b**2*c*d**2*x - 3*b**2*d**3*x**2 - 6*b*c*d**2 - 6
*b*d**3*x - 6*d**3)*exp(-a - b*x)/b**4, Ne(b**4, 0)), (c**3*x + 3*c**2*d*x
**2/2 + c*d**2*x**3 + d**3*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.26

$$\int e^{-a-bx}(c+dx)^3\,dx=-\frac{c^3e^{(-bx-a)}}{b}-\frac{3\,(bx+1)c^2de^{(-bx-a)}}{b^2}-\frac{3\,(b^2x^2+2\,bx+2)cd^2e^{(-bx-a)}}{b^3}-\frac{(b^3x^3+3\,b^2x^2+6\,bx+6)d^3e^{(-bx-a)}}{b^4}$$

input
```
integrate(exp(-b*x-a)*(d*x+c)^3,x, algorithm="maxima")
```

output
```
-c^3*e^(-b*x - a)/b - 3*(b*x + 1)*c^2*d*e^(-b*x - a)/b^2 - 3*(b^2*x^2 + 2*
b*x + 2)*c*d^2*e^(-b*x - a)/b^3 - (b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*d^3*e^
(-b*x - a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.18

$$\int e^{-a-bx}(c+dx)^3\,dx=-\frac{(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+3\,b^2d^3x^2+b^3c^3+6\,b^2cd^2x+3\,b^2c^2d+6\,bd^3x+6\,bcd^2+6\,d^3)e^{(-bx-a}}{b^4}$$

input
```
integrate(exp(-b*x-a)*(d*x+c)^3,x, algorithm="giac")
```

output
```
-(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*b^2*d^3*x^2 + b^3*c^3
+ 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 6*d^3)*e^(-b*x - a
)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.18

$$\int e^{-a-bx}(c+dx)^3\,dx = -\frac{e^{-a-b\,x}\,\left(b^3\,c^3+3\,b^3\,c^2\,d\,x+3\,b^3\,c\,d^2\,x^2+b^3\,d^3\,x^3+3\,b^2\,c^2\,d+6\,b^2\,c\,d^2\,x+3\,b^2\,d^3\,x^2+6\,b\,c\,d^2+6\,b\,d^3\,x}{b^4}$$

input
```
int(exp(- a - b*x)*(c + d*x)^3,x)
```

output
```
-(exp(- a - b*x)*(6*d^3 + b^3*c^3 + 3*b^2*c^2*d + 3*b^2*d^3*x^2 + b^3*d^3*
x^3 + 6*b*c*d^2 + 6*b*d^3*x + 3*b^3*c*d^2*x^2 + 6*b^2*c*d^2*x + 3*b^3*c^2*
d*x))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int e^{-a-bx}(c+dx)^3\,dx = \frac{-b^3d^3x^3-3b^3c\,d^2x^2-3b^3c^2dx-3b^2d^3x^2-b^3c^3-6b^2c\,d^2x-3b^2c^2d-6b\,d^3x-6bc\,d^2-6d^3}{e^{bx+a}b^4}$$

input
```
int(exp(-b*x-a)*(d*x+c)^3,x)
```

output
```
( - b**3*c**3 - 3*b**3*c**2*d*x - 3*b**3*c*d**2*x**2 - b**3*d**3*x**3 - 3*
b**2*c**2*d - 6*b**2*c*d**2*x - 3*b**2*d**3*x**2 - 6*b*c*d**2 - 6*b*d**3*x
 - 6*d**3)/(e**(a + b*x)*b**4)
```

3.112 $\int e^{-a-bx}(c+dx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int e^{-a-bx}(c+dx)^2\,dx=-\frac{2d^2e^{-a-bx}}{b^3}-\frac{2de^{-a-bx}(c+dx)}{b^2}-\frac{e^{-a-bx}(c+dx)^2}{b}$$

output
```
-2*d^2*exp(-b*x-a)/b^3-2*d*exp(-b*x-a)*(d*x+c)/b^2-exp(-b*x-a)*(d*x+c)^2/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

$$\int e^{-a-bx}(c+dx)^2\,dx=\frac{e^{-a-bx}(-2d^2-2bd(c+dx)-b^2(c+dx)^2)}{b^3}$$

input
```
Integrate[E^(-a - b*x)*(c + d*x)^2,x]
```

output
```
(E^(-a - b*x)*(-2*d^2 - 2*b*d*(c + d*x) - b^2*(c + d*x)^2))/b^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-a-bx}(c+dx)^2\,dx$$

$$\Big\downarrow \text{2607}$$

$$\frac{2d\int e^{-a-bx}(c+dx)dx}{b}-\frac{e^{-a-bx}(c+dx)^2}{b}$$

$$\Big\downarrow \text{2607}$$

$$\frac{2d\left(\frac{d\int e^{-a-bx}dx}{b}-\frac{e^{-a-bx}(c+dx)}{b}\right)}{b}-\frac{e^{-a-bx}(c+dx)^2}{b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{2d\left(-\frac{de^{-a-bx}}{b^2}-\frac{e^{-a-bx}(c+dx)}{b}\right)}{b}-\frac{e^{-a-bx}(c+dx)^2}{b}$$

input
```
Int[E^(-a - b*x)*(c + d*x)^2,x]
```

output
```
-((E^(-a - b*x)*(c + d*x)^2)/b) + (2*d*(-((d*E^(-a - b*x))/b^2) - (E^(-a -
 b*x)*(c + d*x))/b))/b
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] && !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{\left(b^{2} d^{2} x^{2}+2 b^{2} c d x+b^{2} c^{2}+2 b \,d^{2} x+2 b c d+2 d^{2}\right) {\mathrm e}^{-b x -a}}{b^{3}}$
risch	$-\frac{\left(b^{2} d^{2} x^{2}+2 b^{2} c d x+b^{2} c^{2}+2 b \,d^{2} x+2 b c d+2 d^{2}\right) {\mathrm e}^{-b x -a}}{b^{3}}$
orering	$-\frac{\left(b^{2} d^{2} x^{2}+2 b^{2} c d x+b^{2} c^{2}+2 b \,d^{2} x+2 b c d+2 d^{2}\right) {\mathrm e}^{-b x -a}}{b^{3}}$
norman	$-\frac{\left(b^{2} c^{2}+2 b c d+2 d^{2}\right) {\mathrm e}^{-b x -a}}{b^{3}}-\frac{d^{2} x^{2} {\mathrm e}^{-b x -a}}{b}-\frac{2 d \left(c b +d \right) x \,{\mathrm e}^{-b x -a}}{b^{2}}$
meijerg	$\frac{{\mathrm e}^{-a} d^{2} \left(2-\frac{\left(3 b^{2} x^{2}+6 b x +6\right) {\mathrm e}^{-b x}}{3}\right)}{b^{3}}+\frac{2 \,{\mathrm e}^{-a} d c \left(1-\frac{\left(2 b x +2\right) {\mathrm e}^{-b x}}{2}\right)}{b^{2}}+\frac{{\mathrm e}^{-a} c^{2} \left(1-{\mathrm e}^{-b x}\right)}{b}$
parallelrisch	$-\frac{x^{2} {\mathrm e}^{-b x -a} d^{2} b^{2}+2 x \,{\mathrm e}^{-b x -a} b^{2} c d +2 x \,{\mathrm e}^{-b x -a} b \,d^{2}+{\mathrm e}^{-b x -a} b^{2} c^{2}+2 \,{\mathrm e}^{-b x -a} b c d +2 \,{\mathrm e}^{-b x -a} d^{2}}{b^{3}}$
parts	$-\frac{d^{2} x^{2} {\mathrm e}^{-b x -a}}{b}-\frac{2 \,{\mathrm e}^{-b x -a} c d x}{b}-\frac{{\mathrm e}^{-b x -a} c^{2}}{b}-\frac{2 d \left({\mathrm e}^{-b x -a} c -\frac{{\mathrm e}^{-b x -a} d a}{b}-\frac{d \left(\left(-b x -a \right) {\mathrm e}^{-b x -a}-{\mathrm e}^{-b x -a}\right)}{b}\right)}{b^{2}}$
derivativedivides	$-\frac{{\mathrm e}^{-b x -a} c^{2}+\frac{{\mathrm e}^{-b x -a} d^{2} a^{2}}{b^{2}}+\frac{d^{2} \left(\left(-b x -a \right)^{2} {\mathrm e}^{-b x -a}-2 \left(-b x -a \right) {\mathrm e}^{-b x -a}+2 \,{\mathrm e}^{-b x -a}\right)}{b^{2}}-\frac{2 \,{\mathrm e}^{-b x -a} d a c}{b}-\frac{2 d c \left(\left(-b x -a \right) {\mathrm e}^{-b x -a}-\cdots\right)}{b}\cdots}{b}$
default	$-\frac{{\mathrm e}^{-b x -a} c^{2}+\frac{{\mathrm e}^{-b x -a} d^{2} a^{2}}{b^{2}}+\frac{d^{2} \left(\left(-b x -a \right)^{2} {\mathrm e}^{-b x -a}-2 \left(-b x -a \right) {\mathrm e}^{-b x -a}+2 \,{\mathrm e}^{-b x -a}\right)}{b^{2}}-\frac{2 \,{\mathrm e}^{-b x -a} d a c}{b}-\frac{2 d c \left(\left(-b x -a \right) {\mathrm e}^{-b x -a}-\cdots\right)}{b}\cdots}{b}$

input
```
int(exp(-b*x-a)*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2+2*b*d^2*x+2*b*c*d+2*d^2)*exp(-b*x-a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int e^{-a-bx}(c+dx)^2\,dx = -\frac{(b^2d^2x^2+b^2c^2+2\,bcd+2\,d^2+2\,(b^2cd+bd^2)x)e^{(-bx-a)}}{b^3}$$

input
```
integrate(exp(-b*x-a)*(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*d^2 + 2*(b^2*c*d + b*d^2)*x)*e^(-b*x
 - a)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int e^{-a-bx}(c+dx)^2\,dx = \begin{cases} \frac{(-b^2c^2-2b^2cdx-b^2d^2x^2-2bcd-2bd^2x-2d^2)e^{-a-bx}}{b^3} & \text{for } b^3 \neq 0 \\ c^2x+cdx^2+\frac{d^2x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(-b*x-a)*(d*x+c)**2,x)
```

output
```
Piecewise(((-b**2*c**2 - 2*b**2*c*d*x - b**2*d**2*x**2 - 2*b*c*d - 2*b*d**
2*x - 2*d**2)*exp(-a - b*x)/b**3, Ne(b**3, 0)), (c**2*x + c*d*x**2 + d**2*
x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int e^{-a-bx}(c+dx)^2\,dx = -\frac{c^2e^{(-bx-a)}}{b} - \frac{2\,(bx+1)cde^{(-bx-a)}}{b^2} - \frac{(b^2x^2+2\,bx+2)d^2e^{(-bx-a)}}{b^3}$$

input
```
integrate(exp(-b*x-a)*(d*x+c)^2,x, algorithm="maxima")
```

output
```
-c^2*e^(-b*x - a)/b - 2*(b*x + 1)*c*d*e^(-b*x - a)/b^2 - (b^2*x^2 + 2*b*x
+ 2)*d^2*e^(-b*x - a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int e^{-a-bx}(c+dx)^2\,dx = -\frac{(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + 2\,bd^2x + 2\,bcd + 2\,d^2)e^{(-bx-a)}}{b^3}$$

input
```
integrate(exp(-b*x-a)*(d*x+c)^2,x, algorithm="giac")
```

output
```
-(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*b*d^2*x + 2*b*c*d + 2*d^2)*e^(-b
*x - a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int e^{-a-bx}(c+dx)^2\,dx = -\frac{e^{-a-b\,x}\,(b^2\,c^2 + 2\,b^2\,c\,d\,x + b^2\,d^2\,x^2 + 2\,b\,c\,d + 2\,b\,d^2\,x + 2\,d^2)}{b^3}$$

input
```
int(exp(- a - b*x)*(c + d*x)^2,x)
```

output
```
-(exp(- a - b*x)*(2*d^2 + b^2*c^2 + 2*b*c*d + b^2*d^2*x^2 + 2*b*d^2*x + 2*
b^2*c*d*x))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int e^{-a-bx}(c+dx)^2\,dx = \frac{-b^2d^2x^2-2b^2cdx-b^2c^2-2b\,d^2x-2bcd-2d^2}{e^{bx+a}b^3}$$

input
```
int(exp(-b*x-a)*(d*x+c)^2,x)
```

output
```
( - b**2*c**2 - 2*b**2*c*d*x - b**2*d**2*x**2 - 2*b*c*d - 2*b*d**2*x - 2*d
**2)/(e**(a + b*x)*b**3)
```

3.113 $\int e^{-a-bx}(c+dx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 37

$$\int e^{-a-bx}(c+dx)\,dx=-\frac{de^{-a-bx}}{b^2}-\frac{e^{-a-bx}(c+dx)}{b}$$

output
```
-d*exp(-b*x-a)/b^2-exp(-b*x-a)*(d*x+c)/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int e^{-a-bx}(c+dx)\,dx=-\frac{e^{-a-bx}(d+b(c+dx))}{b^2}$$

input
```
Integrate[E^(-a - b*x)*(c + d*x),x]
```

output
```
-((E^(-a - b*x)*(d + b*(c + d*x)))/b^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-a-bx}(c+dx)\,dx$$

$$\Big\downarrow \text{2607}$$

$$\frac{d\int e^{-a-bx}dx}{b}-\frac{e^{-a-bx}(c+dx)}{b}$$

$$\Big\downarrow \text{2624}$$

$$-\frac{de^{-a-bx}}{b^2}-\frac{e^{-a-bx}(c+dx)}{b}$$

input
```
Int[E^(-a - b*x)*(c + d*x),x]
```

output
```
-((d*E^(-a - b*x))/b^2) - (E^(-a - b*x)*(c + d*x))/b
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{(bdx+cb+d)\,\mathrm{e}^{-bx-a}}{b^{2}}$	24
risch	$-\frac{(bdx+cb+d)\,\mathrm{e}^{-bx-a}}{b^{2}}$	24
orering	$-\frac{(bdx+cb+d)\,\mathrm{e}^{-bx-a}}{b^{2}}$	24
norman	$-\frac{(cb+d)\,\mathrm{e}^{-bx-a}}{b^{2}}-\frac{dx\,\mathrm{e}^{-bx-a}}{b}$	37
parallelrisch	$-\frac{\mathrm{e}^{-bx-a}bdx+\mathrm{e}^{-bx-a}bc+\mathrm{e}^{-bx-a}d}{b^{2}}$	43
meijerg	$\frac{\mathrm{e}^{-a}d\left(1-\frac{(2bx+2)\,\mathrm{e}^{-bx}}{2}\right)}{b^{2}}+\frac{\mathrm{e}^{-a}c\,(1-\mathrm{e}^{-bx})}{b}$	44
parts	$-\frac{dx\,\mathrm{e}^{-bx-a}}{b}-\frac{\mathrm{e}^{-bx-a}c}{b}-\frac{d\,\mathrm{e}^{-bx-a}}{b^{2}}$	48
derivativedivides	$-\frac{\mathrm{e}^{-bx-a}c-\frac{\mathrm{e}^{-bx-a}da}{b}-\frac{d\left((-bx-a)\,\mathrm{e}^{-bx-a}-\mathrm{e}^{-bx-a}\right)}{b}}{b}$	70
default	$-\frac{\mathrm{e}^{-bx-a}c-\frac{\mathrm{e}^{-bx-a}da}{b}-\frac{d\left((-bx-a)\,\mathrm{e}^{-bx-a}-\mathrm{e}^{-bx-a}\right)}{b}}{b}$	70

input
```
int(exp(-b*x-a)*(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-(b*d*x+b*c+d)*exp(-b*x-a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int e^{-a-bx}(c+dx)\,dx=-\frac{(bdx+bc+d)e^{(-bx-a)}}{b^2}$$

input
```
integrate(exp(-b*x-a)*(d*x+c),x, algorithm="fricas")
```

output
```
-(b*d*x + b*c + d)*e^(-b*x - a)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int e^{-a-bx}(c+dx)\,dx=\begin{cases}\frac{(-bc-bdx-d)e^{-a-bx}}{b^2} & \text{for } b^2\neq 0\\ cx+\frac{dx^2}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(-b*x-a)*(d*x+c),x)
```

output
```
Piecewise(((-b*c - b*d*x - d)*exp(-a - b*x)/b**2, Ne(b**2, 0)), (c*x + d*x
**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int e^{-a-bx}(c+dx)\,dx=-\frac{ce^{(-bx-a)}}{b}-\frac{(bx+1)de^{(-bx-a)}}{b^2}$$

input
```
integrate(exp(-b*x-a)*(d*x+c),x, algorithm="maxima")
```

output
```
-c*e^(-b*x - a)/b - (b*x + 1)*d*e^(-b*x - a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int e^{-a-bx}(c+dx)\,dx=-\frac{(bdx+bc+d)e^{(-bx-a)}}{b^2}$$

input
```
integrate(exp(-b*x-a)*(d*x+c),x, algorithm="giac")
```

output
```
-(b*d*x + b*c + d)*e^(-b*x - a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int e^{-a-bx}(c+dx)\,dx=-\frac{\mathrm{e}^{-a-b\,x}\,\left(d+b\,c+b\,d\,x\right)}{b^2}$$

input
```
int(exp(- a - b*x)*(c + d*x),x)
```

output
```
-(exp(- a - b*x)*(d + b*c + b*d*x))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int e^{-a-bx}(c+dx)\,dx=\frac{-bdx-bc-d}{e^{bx+a}b^2}$$

input
```
int(exp(-b*x-a)*(d*x+c),x)
```

output
```
( - (b*c + b*d*x + d))/(e**(a + b*x)*b**2)
```

3.114 $\int e^{-a-bx} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int e^{-a-bx} \, dx = -\frac{e^{-a-bx}}{b}$$

output
```
-exp(-b*x-a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int e^{-a-bx} \, dx = -\frac{e^{-a-bx}}{b}$$

input
```
Integrate[E^(-a - b*x),x]
```

output
```
-(E^(-a - b*x)/b)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-a-bx}\,dx$$

$$\Big\downarrow \text{2624}$$

$$-\frac{e^{-a-bx}}{b}$$

input
```
Int[E^(-a - b*x),x]
```

output
```
-(E^(-a - b*x)/b)
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{e^{-bx-a}}{b}$	15
derivativedivides	$-\frac{e^{-bx-a}}{b}$	15
default	$-\frac{e^{-bx-a}}{b}$	15
norman	$-\frac{e^{-bx-a}}{b}$	15
risch	$-\frac{e^{-bx-a}}{b}$	15
parallelrisch	$-\frac{e^{-bx-a}}{b}$	15
orering	$-\frac{e^{-bx-a}}{b}$	15
meijerg	$\frac{e^{-a}(1-e^{-bx})}{b}$	18

input
```
int(exp(-b*x-a),x,method=_RETURNVERBOSE)
```

output
```
-exp(-b*x-a)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int e^{-a-bx}\,dx=-\frac{e^{(-bx-a)}}{b}$$

input
```
integrate(exp(-b*x-a),x, algorithm="fricas")
```

output
```
-e^(-b*x - a)/b
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int e^{-a-bx}\,dx = \begin{cases} -\frac{e^{-a-bx}}{b} & \text{for } b \neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(exp(-b*x-a),x)
```

output
```
Piecewise((-exp(-a - b*x)/b, Ne(b, 0)), (x, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int e^{-a-bx}\,dx = -\frac{e^{(-bx-a)}}{b}$$

input
```
integrate(exp(-b*x-a),x, algorithm="maxima")
```

output
```
-e^(-b*x - a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int e^{-a-bx}\,dx = -\frac{e^{(-bx-a)}}{b}$$

input
```
integrate(exp(-b*x-a),x, algorithm="giac")
```

output
```
-e^(-b*x - a)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int e^{-a-bx}\,dx = -\frac{\mathrm{e}^{-a-b\,x}}{b}$$

input
```
int(exp(- a - b*x),x)
```

output
```
-exp(- a - b*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int e^{-a-bx}\,dx = -\frac{1}{e^{bx+a}b}$$

input
```
int(exp(-b*x-a),x)
```

output
```
( - 1)/(e**(a + b*x)*b)
```

3.115 $\int \frac{e^{-a-bx}}{c+dx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{e^{-a-bx}}{c+dx}\,dx = \frac{e^{-a+\frac{bc}{d}}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d}$$

output
```
exp(-a+b*c/d)*Ei(-b*(d*x+c)/d)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{e^{-a-bx}}{c+dx}\,dx = \frac{e^{-a+\frac{bc}{d}}\,\text{ExpIntegralEi}\left(-\frac{bc}{d}-bx\right)}{d}$$

input
```
Integrate[E^(-a - b*x)/(c + d*x),x]
```

output
```
(E^(-a + (b*c)/d)*ExpIntegralEi[-((b*c)/d) - b*x])/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-a-bx}}{c+dx}\,dx$$

$$\Big\downarrow \text{2609}$$

$$\frac{e^{\frac{bc}{d}-a}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d}$$

input
```
Int[E^(-a - b*x)/(c + d*x),x]
```

output
```
(E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

method	result	size
derivativedivides	$-\frac{e^{-\frac{ad-cb}{d}} \operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{d}$	40
default	$-\frac{e^{-\frac{ad-cb}{d}} \operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{d}$	40
risch	$-\frac{e^{-\frac{ad-cb}{d}} \operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{d}$	40

input
```
int(exp(-b*x-a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{e^{-a-bx}}{c+dx}\,dx = \frac{\operatorname{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{d}$$

input
```
integrate(exp(-b*x-a)/(d*x+c),x, algorithm="fricas")
```

output
```
Ei(-(b*d*x + b*c)/d)*e^((b*c - a*d)/d)/d
```

Sympy [F]

$$\int \frac{e^{-a-bx}}{c+dx}\,dx = e^{-a}\int \frac{1}{ce^{bx}+dxe^{bx}}\,dx$$

input
```
integrate(exp(-b*x-a)/(d*x+c),x)
```

output
```
exp(-a)*Integral(1/(c*exp(b*x) + d*x*exp(b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{e^{-a-bx}}{c+dx}\,dx = -\frac{e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{d}$$

input
```
integrate(exp(-b*x-a)/(d*x+c),x, algorithm="maxima")
```

output
```
-e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{e^{-a-bx}}{c+dx}\,dx = \frac{\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}}{d}$$

input
```
integrate(exp(-b*x-a)/(d*x+c),x, algorithm="giac")
```

output
```
Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-a-bx}}{c+dx}\,dx = \int \frac{\mathrm{e}^{-a-b\,x}}{c+d\,x}\,dx$$

input
```
int(exp(- a - b*x)/(c + d*x),x)
```

output
```
int(exp(- a - b*x)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{e^{-a-bx}}{c+dx}\,dx = \frac{\int \frac{1}{e^{bx}c+e^{bx}dx}dx}{e^{a}}$$

input
```
int(exp(-b*x-a)/(d*x+c),x)
```

output
```
int(1/(e**(b*x)*c + e**(b*x)*d*x),x)/e**a
```

3.116 $\int \frac{e^{-a-bx}}{(c+dx)^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{e^{-a-bx}}{(c+dx)^2} \, dx = -\frac{e^{-a-bx}}{d(c+dx)} - \frac{be^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^2}$$

output
```
-exp(-b*x-a)/d/(d*x+c)-b*exp(-a+b*c/d)*Ei(-b*(d*x+c)/d)/d^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{e^{-a-bx}}{(c+dx)^2} \, dx = -\frac{e^{-a-bx}}{d(c+dx)} - \frac{be^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{bc}{d} - bx\right)}{d^2}$$

input
```
Integrate[E^(-a - b*x)/(c + d*x)^2,x]
```

output
```
-(E^(-a - b*x)/(d*(c + d*x))) - (b*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*c)/
d) - b*x])/d^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-a-bx}}{(c+dx)^2}dx$$
$$\Big\downarrow \text{2608}$$
$$-\frac{b\int \frac{e^{-a-bx}}{c+dx}dx}{d}-\frac{e^{-a-bx}}{d(c+dx)}$$
$$\Big\downarrow \text{2609}$$
$$-\frac{be^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^2}-\frac{e^{-a-bx}}{d(c+dx)}$$

input
```
Int[E^(-a - b*x)/(c + d*x)^2,x]
```

output
```
-(E^(-a - b*x)/(d*(c + d*x))) - (b*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c
+ d*x))/d)])/d^2
```

Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

method	result	size
risch	$\frac{b\,\mathrm{e}^{-bx-a}}{d^{2}\left(-bx-\frac{cb}{d}\right)}+\frac{b\,\mathrm{e}^{-\frac{ad-cb}{d}}\,\mathrm{expIntegral}_{1}\left(bx+a-\frac{ad-cb}{d}\right)}{d^{2}}$	69
derivativedivides	$-\frac{b\left(-\frac{\mathrm{e}^{-bx-a}}{-bx-a+\frac{ad-cb}{d}}-\mathrm{e}^{-\frac{ad-cb}{d}}\,\mathrm{expIntegral}_{1}\left(bx+a-\frac{ad-cb}{d}\right)\right)}{d^{2}}$	77
default	$-\frac{b\left(-\frac{\mathrm{e}^{-bx-a}}{-bx-a+\frac{ad-cb}{d}}-\mathrm{e}^{-\frac{ad-cb}{d}}\,\mathrm{expIntegral}_{1}\left(bx+a-\frac{ad-cb}{d}\right)\right)}{d^{2}}$	77

input
```
int(exp(-b*x-a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b/d^2*exp(-b*x-a)/(-b*x-c*b/d)+b/d^2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c
)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \frac{e^{-a-bx}}{(c+dx)^2}\,dx = -\frac{(bdx+bc)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}+de^{(-bx-a)}}{d^3x+cd^2}$$

input
```
integrate(exp(-b*x-a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-((b*d*x + b*c)*Ei(-(b*d*x + b*c)/d)*e^((b*c - a*d)/d) + d*e^(-b*x - a))/(
d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{e^{-a-bx}}{(c+dx)^2}\,dx = e^{-a}\int \frac{1}{c^2e^{bx}+2cdxe^{bx}+d^2x^2e^{bx}}\,dx$$

input
```
integrate(exp(-b*x-a)/(d*x+c)**2,x)
```

output
```
exp(-a)*Integral(1/(c**2*exp(b*x) + 2*c*d*x*exp(b*x) + d**2*x**2*exp(b*x))
, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int \frac{e^{-a-bx}}{(c+dx)^2}\,dx = -\frac{e^{\left(-a+\frac{bc}{d}\right)}E_2\left(\frac{(dx+c)b}{d}\right)}{(dx+c)d}$$

input
```
integrate(exp(-b*x-a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(51) = 102.

Time = 0.12 (sec) , antiderivative size = 307, normalized size of antiderivative = 5.79

$$\int \frac{e^{-a-bx}}{(c+dx)^2}\,dx =$$

$$-\frac{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}+b^3c\mathrm{Ei}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right.\right.}{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right.\right.}$$

input
```
integrate(exp(-b*x-a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(-((d*x + c)*(b - b*
c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) + b^3*c*Ei(
-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c -
a*d)/d) - a*b^2*d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c
 - a*d)/d)*e^((b*c - a*d)/d) + b^2*d*e^(-(d*x + c)*(b - b*c/(d*x + c) + a*
d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^4 +
 b*c*d^4 - a*d^5)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-a-bx}}{(c+dx)^2}\,dx=\int \frac{\mathrm{e}^{-a-b\,x}}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(exp(- a - b*x)/(c + d*x)^2,x)
```

output
```
int(exp(- a - b*x)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{e^{-a-bx}}{(c+dx)^2}\,dx=\frac{\int \frac{1}{e^{bx}c^2+2e^{bx}cdx+e^{bx}d^2x^2}dx}{e^a}$$

input
```
int(exp(-b*x-a)/(d*x+c)^2,x)
```

output
```
int(1/(e**(b*x)*c**2 + 2*e**(b*x)*c*d*x + e**(b*x)*d**2*x**2),x)/e**a
```

3.117 $\int \frac{e^{-a-bx}}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 84

$$\int \frac{e^{-a-bx}}{(c+dx)^3}\,dx=-\frac{e^{-a-bx}}{2d(c+dx)^2}+\frac{be^{-a-bx}}{2d^2(c+dx)}+\frac{b^2e^{-a+\frac{bc}{d}}\operatorname{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{2d^3}$$

output
```
-1/2*exp(-b*x-a)/d/(d*x+c)^2+1/2*b*exp(-b*x-a)/d^2/(d*x+c)+1/2*b^2*exp(-a+
b*c/d)*Ei(-b*(d*x+c)/d)/d^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{e^{-a-bx}}{(c+dx)^3}\,dx = \frac{e^{-a-bx}\left(d(-d+b(c+dx))+b^2e^{b\left(\frac{c}{d}+x\right)}(c+dx)^2\operatorname{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)\right)}{2d^3(c+dx)^2}$$

input
```
Integrate[E^(-a - b*x)/(c + d*x)^3,x]
```

output
```
(E^(-a - b*x)*(d*(-d + b*(c + d*x)) + b^2*E^(b*(c/d + x))*(c + d*x)^2*ExpI
ntegralEi[-((b*(c + d*x))/d)]))/(2*d^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2608, 2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-a-bx}}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{2608}$$

$$-\frac{b\int \frac{e^{-a-bx}}{(c+dx)^2}dx}{2d}-\frac{e^{-a-bx}}{2d(c+dx)^2}$$

$$\Big\downarrow \text{2608}$$

$$-\frac{b\left(-\frac{b\int \frac{e^{-a-bx}}{c+dx}dx}{d}-\frac{e^{-a-bx}}{d(c+dx)}\right)}{2d}-\frac{e^{-a-bx}}{2d(c+dx)^2}$$

$$\Big\downarrow \text{2609}$$

$$-\frac{b\left(-\frac{be^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^2}-\frac{e^{-a-bx}}{d(c+dx)}\right)}{2d}-\frac{e^{-a-bx}}{2d(c+dx)^2}$$

input
```
Int[E^(-a - b*x)/(c + d*x)^3,x]
```

output
```
-1/2*E^(-a - b*x)/(d*(c + d*x)^2) - (b*(-(E^(-a - b*x)/(d*(c + d*x))) - (b
*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^2))/(2*d)
```

Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

method	result	size
risch	$-\frac{b^{2} {\mathrm e}^{-b x -a}}{2 d^{3} \left(-b x -\frac{c b}{d}\right)^{2}}-\frac{b^{2} {\mathrm e}^{-b x -a}}{2 d^{3} \left(-b x -\frac{c b}{d}\right)}-\frac{b^{2} {\mathrm e}^{-\frac{a d -c b}{d}} \operatorname{expIntegral}_{1}\left(b x +a -\frac{a d -c b}{d}\right)}{2 d^{3}}$	106
derivativedivides	$\frac{b^{2} \left(-\frac{{\mathrm e}^{-b x -a}}{2 \left(-b x -a +\frac{a d -c b}{d}\right)^{2}}-\frac{{\mathrm e}^{-b x -a}}{2 \left(-b x -a +\frac{a d -c b}{d}\right)}-\frac{{\mathrm e}^{-\frac{a d -c b}{d}} \operatorname{expIntegral}_{1}\left(b x +a -\frac{a d -c b}{d}\right)}{2}\right)}{d^{3}}$	111
default	$\frac{b^{2} \left(-\frac{{\mathrm e}^{-b x -a}}{2 \left(-b x -a +\frac{a d -c b}{d}\right)^{2}}-\frac{{\mathrm e}^{-b x -a}}{2 \left(-b x -a +\frac{a d -c b}{d}\right)}-\frac{{\mathrm e}^{-\frac{a d -c b}{d}} \operatorname{expIntegral}_{1}\left(b x +a -\frac{a d -c b}{d}\right)}{2}\right)}{d^{3}}$	111

input
```
int(exp(-b*x-a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2/d^3*exp(-b*x-a)/(-b*x-c*b/d)^2-1/2*b^2/d^3*exp(-b*x-a)/(-b*x-c*b/
d)-1/2*b^2/d^3*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int \frac{e^{-a-bx}}{(c+dx)^3}\,dx = \frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}+(bd^2x+bcd-d^2)e^{(-bx-a)}}{2\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

input
```
integrate(exp(-b*x-a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-(b*d*x + b*c)/d)*e^((b*c -
a*d)/d) + (b*d^2*x + b*c*d - d^2)*e^(-b*x - a))/(d^5*x^2 + 2*c*d^4*x + c^2
*d^3)
```

Sympy [F]

$$\int \frac{e^{-a-bx}}{(c+dx)^3}\,dx = e^{-a}\int \frac{1}{c^3e^{bx}+3c^2dxe^{bx}+3cd^2x^2e^{bx}+d^3x^3e^{bx}}\,dx$$

input
```
integrate(exp(-b*x-a)/(d*x+c)**3,x)
```

output
```
exp(-a)*Integral(1/(c**3*exp(b*x) + 3*c**2*d*x*exp(b*x) + 3*c*d**2*x**2*ex
p(b*x) + d**3*x**3*exp(b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.42

$$\int \frac{e^{-a-bx}}{(c+dx)^3}\,dx = -\frac{e^{\left(-a+\frac{bc}{d}\right)}E_3\left(\frac{(dx+c)b}{d}\right)}{(dx+c)^2d}$$

input
```
integrate(exp(-b*x-a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-e^(-a + b*c/d)*exp_integral_e(3, (d*x + c)*b/d)/((d*x + c)^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(75) = 150.

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.01

$$\int \frac{e^{-a-bx}}{(c+dx)^3}\,dx = \frac{b^2d^2x^2\text{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)} + 2\,b^2cdx\text{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)} + b^2c^2\text{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)} + bd^2xe^{(-bx-a)} -}{2\,(d^5x^2 + 2\,cd^4x + c^2d^3)}$$

input
```
integrate(exp(-b*x-a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/2*(b^2*d^2*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 2*b^2*c*d*x*Ei(-(b*
d*x + b*c)/d)*e^(-a + b*c/d) + b^2*c^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d)
 + b*d^2*x*e^(-b*x - a) + b*c*d*e^(-b*x - a) - d^2*e^(-b*x - a))/(d^5*x^2
+ 2*c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-a-bx}}{(c+dx)^3}\,dx = \int \frac{\mathrm{e}^{-a-b\,x}}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int(exp(- a - b*x)/(c + d*x)^3,x)
```

output
```
int(exp(- a - b*x)/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{e^{-a-bx}}{(c+dx)^3}\,dx = \frac{\int \frac{1}{e^{bx}c^3+3e^{bx}c^2dx+3e^{bx}c\,d^2x^2+e^{bx}d^3x^3}dx}{e^a}$$

input
```
int(exp(-b*x-a)/(d*x+c)^3,x)
```

output
```
int(1/(e**(b*x)*c**3 + 3*e**(b*x)*c**2*d*x + 3*e**(b*x)*c*d**2*x**2 + e**(
b*x)*d**3*x**3),x)/e**a
```

3.118 $\int \frac{e^{-a-bx}}{(c+dx)^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 111

$$\int \frac{e^{-a-bx}}{(c+dx)^4} dx = -\frac{e^{-a-bx}}{3d(c+dx)^3} + \frac{be^{-a-bx}}{6d^2(c+dx)^2} - \frac{b^2e^{-a-bx}}{6d^3(c+dx)} - \frac{b^3e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{6d^4}$$

output
```
-1/3*exp(-b*x-a)/d/(d*x+c)^3+1/6*b*exp(-b*x-a)/d^2/(d*x+c)^2-1/6*b^2*exp(-
b*x-a)/d^3/(d*x+c)-1/6*b^3*exp(-a+b*c/d)*Ei(-b*(d*x+c)/d)/d^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int \frac{e^{-a-bx}}{(c+dx)^4} dx = \frac{e^{-a-bx}\left(-2d^3 + bd^2(c+dx) - b^2d(c+dx)^2 - b^3e^{b\left(\frac{c}{d}+x\right)}(c+dx)^3 \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)\right)}{6d^4(c+dx)^3}$$

input
```
Integrate[E^(-a - b*x)/(c + d*x)^4,x]
```

output
```
(E^(-a - b*x)*(-2*d^3 + b*d^2*(c + d*x) - b^2*d*(c + d*x)^2 - b^3*E^(b*(c/
d + x))*(c + d*x)^3*ExpIntegralEi[-((b*(c + d*x))/d)]))/(6*d^4*(c + d*x)^3
)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2608, 2608, 2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-a-bx}}{(c+dx)^4}\,dx$$

$$\Big\downarrow \text{ 2608}$$

$$-\frac{b\int \frac{e^{-a-bx}}{(c+dx)^3}dx}{3d}-\frac{e^{-a-bx}}{3d(c+dx)^3}$$

$$\Big\downarrow \text{ 2608}$$

$$-\frac{b\left(-\frac{b\int \frac{e^{-a-bx}}{(c+dx)^2}dx}{2d}-\frac{e^{-a-bx}}{2d(c+dx)^2}\right)}{3d}-\frac{e^{-a-bx}}{3d(c+dx)^3}$$

$$\Big\downarrow \text{ 2608}$$

$$-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{e^{-a-bx}}{c+dx}dx}{d}-\frac{e^{-a-bx}}{d(c+dx)}\right)}{2d}-\frac{e^{-a-bx}}{2d(c+dx)^2}\right)}{3d}-\frac{e^{-a-bx}}{3d(c+dx)^3}$$

$$\Big\downarrow \text{ 2609}$$

$$-\frac{b\left(-\frac{b\left(-\frac{be^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^2}-\frac{e^{-a-bx}}{d(c+dx)}\right)}{2d}-\frac{e^{-a-bx}}{2d(c+dx)^2}\right)}{3d}-\frac{e^{-a-bx}}{3d(c+dx)^3}$$

input
```
Int[E^(-a - b*x)/(c + d*x)^4,x]
```

output
```
-1/3*E^(-a - b*x)/(d*(c + d*x)^3) - (b*(-1/2*E^(-a - b*x)/(d*(c + d*x)^2)
- (b*(-(E^(-a - b*x)/(d*(c + d*x))) - (b*E^(-a + (b*c)/d)*ExpIntegralEi[-(
(b*(c + d*x))/d)])/d^2))/(2*d)))/(3*d)
```

Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.23

method	result	siz
risch	$\frac{b^3 e^{-bx-a}}{3d^4\left(-bx-\frac{cb}{d}\right)^3}+\frac{b^3 e^{-bx-a}}{6d^4\left(-bx-\frac{cb}{d}\right)^2}+\frac{b^3 e^{-bx-a}}{6d^4\left(-bx-\frac{cb}{d}\right)}+\frac{b^3 e^{-\frac{ad-cb}{d}}\operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{6d^4}$	137
derivativedivides	$-\frac{b^3\left(-\frac{e^{-bx-a}}{3\left(-bx-a+\frac{ad-cb}{d}\right)^3}-\frac{e^{-bx-a}}{6\left(-bx-a+\frac{ad-cb}{d}\right)^2}-\frac{e^{-bx-a}}{6\left(-bx-a+\frac{ad-cb}{d}\right)}-\frac{e^{-\frac{ad-cb}{d}}\operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{6}\right)}{d^4}$	145
default	$-\frac{b^3\left(-\frac{e^{-bx-a}}{3\left(-bx-a+\frac{ad-cb}{d}\right)^3}-\frac{e^{-bx-a}}{6\left(-bx-a+\frac{ad-cb}{d}\right)^2}-\frac{e^{-bx-a}}{6\left(-bx-a+\frac{ad-cb}{d}\right)}-\frac{e^{-\frac{ad-cb}{d}}\operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{6}\right)}{d^4}$	145

input
```
int(exp(-b*x-a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^3/d^4*exp(-b*x-a)/(-b*x-c*b/d)^3+1/6*b^3/d^4*exp(-b*x-a)/(-b*x-c*b/d
)^2+1/6*b^3/d^4*exp(-b*x-a)/(-b*x-c*b/d)+1/6*b^3/d^4*exp(-(a*d-b*c)/d)*Ei(
1,b*x+a-(a*d-b*c)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.49

$$\int \frac{e^{-a-bx}}{(c+dx)^4}\,dx = -\frac{(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(\frac{bc-ad}{d}\right)} + (b^2d^3x^2 + b^2c^2d - bcd^2 + 2\,d^3 + (2\,b^2cd^2}{6\,(d^7x^3 + 3\,cd^6x^2 + 3\,c^2d^5x + c^3d^4)}$$

input
```
integrate(exp(-b*x-a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/6*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*Ei(-(b*d*x
 + b*c)/d)*e^((b*c - a*d)/d) + (b^2*d^3*x^2 + b^2*c^2*d - b*c*d^2 + 2*d^3
+ (2*b^2*c*d^2 - b*d^3)*x)*e^(-b*x - a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^
5*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{e^{-a-bx}}{(c+dx)^4}\,dx = e^{-a}\int \frac{1}{c^4e^{bx} + 4c^3dxe^{bx} + 6c^2d^2x^2e^{bx} + 4cd^3x^3e^{bx} + d^4x^4e^{bx}}\,dx$$

input
```
integrate(exp(-b*x-a)/(d*x+c)**4,x)
```

output
```
exp(-a)*Integral(1/(c**4*exp(b*x) + 4*c**3*d*x*exp(b*x) + 6*c**2*d**2*x**2
*exp(b*x) + 4*c*d**3*x**3*exp(b*x) + d**4*x**4*exp(b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.32

$$\int \frac{e^{-a-bx}}{(c+dx)^4}\,dx = -\frac{e^{\left(-a+\frac{bc}{d}\right)}E_4\left(\frac{(dx+c)b}{d}\right)}{(dx+c)^3 d}$$

input
```
integrate(exp(-b*x-a)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-e^(-a + b*c/d)*exp_integral_e(4, (d*x + c)*b/d)/((d*x + c)^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(99) = 198.

Time = 0.11 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.50

$$\int \frac{e^{-a-bx}}{(c+dx)^4}\,dx =$$

$$-\frac{b^3d^3x^3\text{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+3\,b^3cd^2x^2\text{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+3\,b^3c^2dx\text{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+b^2d^3x}{6\,(d^7x^3+}$$

input
```
integrate(exp(-b*x-a)/(d*x+c)^4,x, algorithm="giac")
```

output
```
-1/6*(b^3*d^3*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 3*b^3*c*d^2*x^2*Ei
(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 3*b^3*c^2*d*x*Ei(-(b*d*x + b*c)/d)*e^(
-a + b*c/d) + b^2*d^3*x^2*e^(-b*x - a) + b^3*c^3*Ei(-(b*d*x + b*c)/d)*e^(-
a + b*c/d) + 2*b^2*c*d^2*x*e^(-b*x - a) + b^2*c^2*d*e^(-b*x - a) - b*d^3*x
*e^(-b*x - a) - b*c*d^2*e^(-b*x - a) + 2*d^3*e^(-b*x - a))/(d^7*x^3 + 3*c*
d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-a-bx}}{(c+dx)^4}\,dx = \int \frac{\mathrm{e}^{-a-b\,x}}{(c+d\,x)^4}\,dx$$

input
```
int(exp(- a - b*x)/(c + d*x)^4,x)
```

output
```
int(exp(- a - b*x)/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{e^{-a-bx}}{(c+dx)^4}\,dx = \frac{\int \frac{1}{e^{bx}c^4+4e^{bx}c^3dx+6e^{bx}c^2d^2x^2+4e^{bx}c\,d^3x^3+e^{bx}d^4x^4}dx}{e^a}$$

input
```
int(exp(-b*x-a)/(d*x+c)^4,x)
```

output
```
int(1/(e**(b*x)*c**4 + 4*e**(b*x)*c**3*d*x + 6*e**(b*x)*c**2*d**2*x**2 + 4
*e**(b*x)*c*d**3*x**3 + e**(b*x)*d**4*x**4),x)/e**a
```

3.119 $\int (c+dx)^3\Gamma(2,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 98

$$\int (c+dx)^3\Gamma(2,a+bx)\,dx = \frac{(c+dx)^4\Gamma(2,a+bx)}{4d} + \frac{d^2(bc-ad)e^{-a+\frac{bc}{d}}\Gamma\left(5,\frac{b(c+dx)}{d}\right)}{4b^4} - \frac{d^3e^{-a+\frac{bc}{d}}\Gamma\left(6,\frac{b(c+dx)}{d}\right)}{4b^4}$$

output

```
1/4*(d*x+c)^4*exp(-b*x-a)*(b*x+a+1)/d+6*d^2*(-a*d+b*c)*exp(-a+b*c/d)*exp(-
b*(d*x+c)/d)*(1+b*(d*x+c)/d+1/2*b^2*(d*x+c)^2/d^2+1/6*b^3*(d*x+c)^3/d^3+1/
24*b^4*(d*x+c)^4/d^4)/b^4-30*d^3*exp(-a+b*c/d)*exp(-b*(d*x+c)/d)*(1+b*(d*x
+c)/d+1/2*b^2*(d*x+c)^2/d^2+1/6*b^3*(d*x+c)^3/d^3+1/24*b^4*(d*x+c)^4/d^4+1
/120*b^5*(d*x+c)^5/d^5)/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 223 vs. 2(98) = 196.

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.28

$$\int (c+dx)^3\Gamma(2,a+bx)\,dx$$

$$= \frac{e^{-a-bx}\left(-24(5+a)d^3 - 4b^3(c+dx)^2((2+a)c+(5+a)dx) - 12b^2d(c+dx)((3+a)c+(5+a)dx) - 24\right.}{}$$

input
```
Integrate[(c + d*x)^3*Gamma[2, a + b*x],x]
```

output
```
(E^(-a - b*x)*(-24*(5 + a)*d^3 - 4*b^3*(c + d*x)^2*((2 + a)*c + (5 + a)*d*
x) - 12*b^2*d*(c + d*x)*((3 + a)*c + (5 + a)*d*x) - 24*b*d^2*((4 + a)*c +
(5 + a)*d*x) - b^5*x^2*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - b^4*x
*(4*(2 + a)*c^3 + 6*(3 + a)*c^2*d*x + 4*(4 + a)*c*d^2*x^2 + (5 + a)*d^3*x^
3) + b^4*E^(a + b*x)*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Gamma[2
, a + b*x]))/(4*b^4)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 314 vs. $2(98) = 196$.

Time = 1.15 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\Gamma(2,a+bx)\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{b\int e^{-a-bx}(a+bx)(c+dx)^4dx}{4d}+\frac{(c+dx)^4\Gamma(2,a+bx)}{4d}$$

$$\Big\downarrow \text{2626}$$

$$\frac{b\int \left(\frac{be^{-a-bx}(c+dx)^5}{d}+\frac{(ad-bc)e^{-a-bx}(c+dx)^4}{d}\right)dx}{4d}+\frac{(c+dx)^4\Gamma(2,a+bx)}{4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{24d^3e^{-a-bx}(bc-ad)}{b^5}-\frac{120d^4e^{-a-bx}}{b^5}-\frac{120d^3e^{-a-bx}(c+dx)}{b^4}+\frac{24d^2e^{-a-bx}(c+dx)(bc-ad)}{b^4}-\frac{60d^2e^{-a-bx}(c+dx)^2}{b^3}+\frac{12de^{-a-bx}(c+d}{b^3}\right.}{4d}$$

$$\frac{(c+dx)^4\Gamma(2,a+bx)}{4d}$$

input
```
Int[(c + d*x)^3*Gamma[2, a + b*x],x]
```

output
```
(b*((-120*d^4*E^(-a - b*x))/b^5 + (24*d^3*(b*c - a*d)*E^(-a - b*x))/b^5 -
(120*d^3*E^(-a - b*x)*(c + d*x))/b^4 + (24*d^2*(b*c - a*d)*E^(-a - b*x)*(c
 + d*x))/b^4 - (60*d^2*E^(-a - b*x)*(c + d*x)^2)/b^3 + (12*d*(b*c - a*d)*E
^(-a - b*x)*(c + d*x)^2)/b^3 - (20*d*E^(-a - b*x)*(c + d*x)^3)/b^2 + (4*(b
*c - a*d)*E^(-a - b*x)*(c + d*x)^3)/b^2 - (5*E^(-a - b*x)*(c + d*x)^4)/b +
 ((b*c - a*d)*E^(-a - b*x)*(c + d*x)^4)/(b*d) - (E^(-a - b*x)*(c + d*x)^5)
/d))/(4*d) + ((c + d*x)^4*Gamma[2, a + b*x])/(4*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2626
```
Int[(F_)^(v_)*(Px_), x_Symbol] :> Int[ExpandIntegrand[F^v, Px, x], x] /; Fr
eeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.45

method	result
norman	$-d^3x^4\mathrm{e}^{-bx-a}-\frac{(a\,b^3c^3+3c^2da\,b^2+2b^3c^3+6abc\,d^2+9b^2c^2d+6a\,d^3+24d^2cb+30d^3)\mathrm{e}^{-bx-a}}{b^4}-\frac{(3c^2da\,b^2+b^3c^3+}{}$
gosper	$-\frac{(b^4d^3x^4+a\,b^3d^3x^3+3b^4c\,d^2x^3+3a\,b^3c\,d^2x^2+3b^4c^2d\,x^2+5b^3d^3x^3+3a\,b^3c^2dx+3a\,b^2d^3x^2+b^4c^3x+12b^3c\,d^2x^2+a\,b^3c}{}$
risch	$-\frac{(b^4d^3x^4+a\,b^3d^3x^3+3b^4c\,d^2x^3+3a\,b^3c\,d^2x^2+3b^4c^2d\,x^2+5b^3d^3x^3+3a\,b^3c^2dx+3a\,b^2d^3x^2+b^4c^3x+12b^3c\,d^2x^2+a\,b^3c}{}$
orering	$-\frac{(b^4d^3x^4+a\,b^3d^3x^3+3b^4c\,d^2x^3+3a\,b^3c\,d^2x^2+3b^4c^2d\,x^2+5b^3d^3x^3+3a\,b^3c^2dx+3a\,b^2d^3x^2+b^4c^3x+12b^3c\,d^2x^2+a\,b^3c}{}$
meijerg	$\frac{d^3\mathrm{e}^{-a}\left(24-\frac{(5b^4x^4+20b^3x^3+60b^2x^2+120bx+120)\mathrm{e}^{-bx}}{5}\right)}{b^4}+\frac{d^3\mathrm{e}^{-a}a\left(6-\frac{(4b^3x^3+12b^2x^2+24bx+24)\mathrm{e}^{-bx}}{4}\right)}{b^4}+\frac{\mathrm{e}^{-a}d^3(}{}$
parallelrisch	$-\frac{3x^2\mathrm{e}^{-bx-a}a\,b^3c\,d^2+3x\,\mathrm{e}^{-bx-a}a\,b^3c^2d+6x\,\mathrm{e}^{-bx-a}a\,b^2c\,d^2+5x^3\mathrm{e}^{-bx-a}d^3b^3+15x^2\mathrm{e}^{-bx-a}b^2d^3+30x\,\mathrm{e}^{-bx-a}b\,d^3+6}{}$
parts	$-d^3x^4\mathrm{e}^{-bx-a}-\frac{\mathrm{e}^{-bx-a}a\,d^3x^3}{b}-3\,\mathrm{e}^{-bx-a}c\,d^2x^3-\frac{3\,\mathrm{e}^{-bx-a}ac\,d^2x^2}{b}-3\,\mathrm{e}^{-bx-a}c^2d\,x^2-\frac{d^3x^3\mathrm{e}^{-b}}{b}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^3*exp(-b*x-a)*(b*x+a+1),x,method=_RETURNVERBOSE)
```

output
```
-d^3*x^4*exp(-b*x-a)-(a*b^3*c^3+3*a*b^2*c^2*d+2*b^3*c^3+6*a*b*c*d^2+9*b^2*
c^2*d+6*a*d^3+24*b*c*d^2+30*d^3)/b^4*exp(-b*x-a)-(3*a*b^2*c^2*d+b^3*c^3+6*
a*b*c*d^2+9*b^2*c^2*d+6*a*d^3+24*b*c*d^2+30*d^3)/b^3*x*exp(-b*x-a)-3*d*(a*
b*c*d+b^2*c^2+a*d^2+4*b*c*d+5*d^2)/b^2*x^2*exp(-b*x-a)-d^2*(a*d+3*b*c+5*d)
/b*x^3*exp(-b*x-a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(90) = 180$.

Time = 0.09 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.85

$$\int (c+dx)^3\Gamma(2,a+bx)\,dx =$$

$$-\frac{(b^5d^3x^5+4\,(a+2)b^3c^3+12\,(a+3)b^2c^2d+24\,(a+4)bcd^2+(4\,b^5cd^2+(a+5)b^4d^3)x^4+24\,(a+5)d^3}{}$$

input
```
integrate((d*x+c)^3*gamma(2,b*x+a),x, algorithm="fricas")
```

output
```
-1/4*((b^5*d^3*x^5 + 4*(a + 2)*b^3*c^3 + 12*(a + 3)*b^2*c^2*d + 24*(a + 4)
*b*c*d^2 + (4*b^5*c*d^2 + (a + 5)*b^4*d^3)*x^4 + 24*(a + 5)*d^3 + 2*(3*b^5
*c^2*d + 2*(a + 4)*b^4*c*d^2 + 2*(a + 5)*b^3*d^3)*x^3 + 2*(2*b^5*c^3 + 3*(
a + 3)*b^4*c^2*d + 6*(a + 4)*b^3*c*d^2 + 6*(a + 5)*b^2*d^3)*x^2 + 4*((a +
2)*b^4*c^3 + 3*(a + 3)*b^3*c^2*d + 6*(a + 4)*b^2*c*d^2 + 6*(a + 5)*b*d^3)*
x)*e^(-b*x - a) - (b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 + 4*b^4
*c^3*x)*gamma(2, b*x + a))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 374, normalized size of antiderivative = 3.82

$$\int (c+dx)^3\Gamma(2,a+bx)\,dx$$
$$= \begin{cases} \frac{(-ab^3c^3-3ab^3c^2dx-3ab^3cd^2x^2-ab^3d^3x^3-3ab^2c^2d-6ab^2cd^2x-3ab^2d^3x^2-6abcd^2-6abd^3x-6ad^3-b^4c^3x-3b^4c^2dx^2-3b^4cd^2x^3-b^4d^3x^4-2}{b^4} \\ \frac{bd^3x^5}{5}+x^4\left(\frac{ad^3}{4}+\frac{3bcd^2}{4}+\frac{d^3}{4}\right)+x^3(acd^2+bc^2d+cd^2)+x^2\cdot\left(\frac{3ac^2d}{2}+\frac{bc^3}{2}+\frac{3c^2d}{2}\right)+x(ac^3+c^3) \end{cases}$$

input
```
integrate((d*x+c)**3*uppergamma(2,b*x+a),x)
```

output
```
Piecewise(((-a*b**3*c**3 - 3*a*b**3*c**2*d*x - 3*a*b**3*c*d**2*x**2 - a*b*
*3*d**3*x**3 - 3*a*b**2*c**2*d - 6*a*b**2*c*d**2*x - 3*a*b**2*d**3*x**2 -
6*a*b*c*d**2 - 6*a*b*d**3*x - 6*a*d**3 - b**4*c**3*x - 3*b**4*c**2*d*x**2
- 3*b**4*c*d**2*x**3 - b**4*d**3*x**4 - 2*b**3*c**3 - 9*b**3*c**2*d*x - 12
*b**3*c*d**2*x**2 - 5*b**3*d**3*x**3 - 9*b**2*c**2*d - 24*b**2*c*d**2*x -
15*b**2*d**3*x**2 - 24*b*c*d**2 - 30*b*d**3*x - 30*d**3)*exp(-a - b*x)/b**
4, Ne(b**4, 0)), (b*d**3*x**5/5 + x**4*(a*d**3/4 + 3*b*c*d**2/4 + d**3/4)
+ x**3*(a*c*d**2 + b*c**2*d + c*d**2) + x**2*(3*a*c**2*d/2 + b*c**3/2 + 3*
c**2*d/2) + x*(a*c**3 + c**3), True))
```

Maxima [F]

$$\int (c+dx)^3\Gamma(2,a+bx)\,dx = \int (dx+c)^3\Gamma(2,bx+a)\,dx$$

input
```
integrate((d*x+c)^3*gamma(2,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(2, b*x + a) - gamma(3, b*x + a))*c^3/b + integrate(d^3*x^
3*gamma(2, b*x + a) + 3*c*d^2*x^2*gamma(2, b*x + a) + 3*c^2*d*x*gamma(2, b
*x + a), x)
```

Giac [F]

$$\int (c+dx)^3\Gamma(2,a+bx)\,dx = \int (dx+c)^3\Gamma(2,bx+a)\,dx$$

input
```
integrate((d*x+c)^3*gamma(2,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*gamma(2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.53

$$\begin{aligned}\int (c+dx)^3\Gamma(2,a+bx)\,dx &= -x^3\,\mathrm{e}^{-a-b\,x}\left(\frac{a\,d^3+5\,d^3}{b}+3\,c\,d^2\right) \\ &\quad - x\,\mathrm{e}^{-a-b\,x}\left(\frac{6\,a\,d^3+b\,(24\,c\,d^2+6\,a\,c\,d^2)+30\,d^3+b^2\,(9\,c^2\,d+3\,a\,c^2\,d)}{b^3}+c^3\right) \\ &\quad - \frac{\mathrm{e}^{-a-b\,x}\,(b^3\,(a\,c^3+2\,c^3)+6\,a\,d^3+b\,(24\,c\,d^2+6\,a\,c\,d^2)+30\,d^3+b^2\,(9\,c^2\,d+3\,a\,c^2\,d))}{b^4} \\ &\quad - x^2\,\mathrm{e}^{-a-b\,x}\left(3\,c^2\,d+\frac{3\,a\,d^3+b\,(12\,c\,d^2+3\,a\,c\,d^2)+15\,d^3}{b^2}\right) - d^3\,x^4\,\mathrm{e}^{-a-b\,x}\end{aligned}$$

input
```
int(exp(- a - b*x)*(c + d*x)^3*(a + b*x + 1),x)
```

output
```
- x^3*exp(- a - b*x)*((a*d^3 + 5*d^3)/b + 3*c*d^2) - x*exp(- a - b*x)*((6*
a*d^3 + b*(24*c*d^2 + 6*a*c*d^2) + 30*d^3 + b^2*(9*c^2*d + 3*a*c^2*d))/b^3
 + c^3) - (exp(- a - b*x)*(b^3*(a*c^3 + 2*c^3) + 6*a*d^3 + b*(24*c*d^2 + 6
*a*c*d^2) + 30*d^3 + b^2*(9*c^2*d + 3*a*c^2*d)))/b^4 - x^2*exp(- a - b*x)*
(3*c^2*d + (3*a*d^3 + b*(12*c*d^2 + 3*a*c*d^2) + 15*d^3)/b^2) - d^3*x^4*ex
p(- a - b*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.53

$$\int (c+dx)^3\Gamma(2,a+bx)\,dx = \frac{-b^4d^3x^4 - a\,b^3d^3x^3 - 3b^4c\,d^2x^3 - 3a\,b^3c\,d^2x^2 - 3b^4c^2d\,x^2 - 5b^3d^3x^3 - 3a\,b^3c^2dx - 3a\,b^2d^3x^2 - b^4c^3x - \cdots}{}$$

input
```
int((d*x+c)^3*exp(-b*x-a)*(b*x+a+1),x)
```

output
```
( - a*b**3*c**3 - 3*a*b**3*c**2*d*x - 3*a*b**3*c*d**2*x**2 - a*b**3*d**3*x
**3 - 3*a*b**2*c**2*d - 6*a*b**2*c*d**2*x - 3*a*b**2*d**3*x**2 - 6*a*b*c*d
**2 - 6*a*b*d**3*x - 6*a*d**3 - b**4*c**3*x - 3*b**4*c**2*d*x**2 - 3*b**4*
c*d**2*x**3 - b**4*d**3*x**4 - 2*b**3*c**3 - 9*b**3*c**2*d*x - 12*b**3*c*d
**2*x**2 - 5*b**3*d**3*x**3 - 9*b**2*c**2*d - 24*b**2*c*d**2*x - 15*b**2*d
**3*x**2 - 24*b*c*d**2 - 30*b*d**3*x - 30*d**3)/(e**(a + b*x)*b**4)
```

3.120 $\int (c+dx)^2\Gamma(2,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 96

$$\int (c+dx)^2\Gamma(2,a+bx)\,dx=\frac{(c+dx)^3\Gamma(2,a+bx)}{3d}+\frac{d(bc-ad)e^{-a+\frac{bc}{d}}\Gamma\left(4,\frac{b(c+dx)}{d}\right)}{3b^3}-\frac{d^2e^{-a+\frac{bc}{d}}\Gamma\left(5,\frac{b(c+dx)}{d}\right)}{3b^3}$$

output
```
1/3*(d*x+c)^3*exp(-b*x-a)*(b*x+a+1)/d+2*d*(-a*d+b*c)*exp(-a+b*c/d)*exp(-b*
(d*x+c)/d)*(1+b*(d*x+c)/d+1/2*b^2*(d*x+c)^2/d^2+1/6*b^3*(d*x+c)^3/d^3)/b^3
-8*d^2*exp(-a+b*c/d)*exp(-b*(d*x+c)/d)*(1+b*(d*x+c)/d+1/2*b^2*(d*x+c)^2/d^
2+1/6*b^3*(d*x+c)^3/d^3+1/24*b^4*(d*x+c)^4/d^4)/b^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.67

$$\int (c+dx)^2\Gamma(2,a+bx)\,dx$$
$$=\frac{e^{-a-bx}\left(-6(4+a)d^2-3b^2(c+dx)((2+a)c+(4+a)dx)-6bd((3+a)c+(4+a)dx)-b^4x^2(3c^2+3cd\right.}{3b}$$

input
```
Integrate[(c + d*x)^2*Gamma[2, a + b*x],x]
```

output
```
(E^(-a - b*x)*(-6*(4 + a)*d^2 - 3*b^2*(c + d*x)*((2 + a)*c + (4 + a)*d*x)
- 6*b*d*((3 + a)*c + (4 + a)*d*x) - b^4*x^2*(3*c^2 + 3*c*d*x + d^2*x^2) -
b^3*x*(3*(2 + a)*c^2 + 3*(3 + a)*c*d*x + (4 + a)*d^2*x^2) + b^3*E^(a + b*x
)*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Gamma[2, a + b*x]))/(3*b^3)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 256 vs. 2(96) = 192.

Time = 0.92 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7119, 2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\Gamma(2,a+bx)\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{b\int e^{-a-bx}(a+bx)(c+dx)^3dx}{3d}+\frac{(c+dx)^3\Gamma(2,a+bx)}{3d}$$

$$\Big\downarrow \text{2626}$$

$$\frac{b\int \left(\frac{be^{-a-bx}(c+dx)^4}{d}+\frac{(ad-bc)e^{-a-bx}(c+dx)^3}{d}\right)dx}{3d}+\frac{(c+dx)^3\Gamma(2,a+bx)}{3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{6d^2e^{-a-bx}(bc-ad)}{b^4}-\frac{24d^3e^{-a-bx}}{b^4}-\frac{24d^2e^{-a-bx}(c+dx)}{b^3}+\frac{6de^{-a-bx}(c+dx)(bc-ad)}{b^3}-\frac{12de^{-a-bx}(c+dx)^2}{b^2}+\frac{3e^{-a-bx}(c+dx)^2(bc-a}{b^2}\right.}{3d}$$
$$\frac{(c+dx)^3\Gamma(2,a+bx)}{3d}$$

input
```
Int[(c + d*x)^2*Gamma[2, a + b*x],x]
```

output
```
(b*((-24*d^3*E^(-a - b*x))/b^4 + (6*d^2*(b*c - a*d)*E^(-a - b*x))/b^4 - (2
4*d^2*E^(-a - b*x)*(c + d*x))/b^3 + (6*d*(b*c - a*d)*E^(-a - b*x)*(c + d*x
))/b^3 - (12*d*E^(-a - b*x)*(c + d*x)^2)/b^2 + (3*(b*c - a*d)*E^(-a - b*x)
*(c + d*x)^2)/b^2 - (4*E^(-a - b*x)*(c + d*x)^3)/b + ((b*c - a*d)*E^(-a -
b*x)*(c + d*x)^3)/(b*d) - (E^(-a - b*x)*(c + d*x)^4)/d))/(3*d) + ((c + d*x
)^3*Gamma[2, a + b*x])/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2626
```
Int[(F_)^(v_)*(Px_), x_Symbol] :> Int[ExpandIntegrand[F^v, Px, x], x] /; Fr
eeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.42

method	result
gosper	$-\frac{\left(b^3d^2x^3+a\,b^2d^2x^2+2b^3cd\,x^2+2a\,b^2cdx+c^2x\,b^3+4b^2d^2x^2+a\,c^2b^2+2ab\,d^2x+6b^2cdx+2abcd+2b^2c^2+8b\,d^2x+2a\,d^2+}{b^3}$
risch	$-\frac{\left(b^3d^2x^3+a\,b^2d^2x^2+2b^3cd\,x^2+2a\,b^2cdx+c^2x\,b^3+4b^2d^2x^2+a\,c^2b^2+2ab\,d^2x+6b^2cdx+2abcd+2b^2c^2+8b\,d^2x+2a\,d^2+}{b^3}$
orering	$-\frac{\left(b^3d^2x^3+a\,b^2d^2x^2+2b^3cd\,x^2+2a\,b^2cdx+c^2x\,b^3+4b^2d^2x^2+a\,c^2b^2+2ab\,d^2x+6b^2cdx+2abcd+2b^2c^2+8b\,d^2x+2a\,d^2+}{b^3}$
norman	$-d^2x^3\mathrm{e}^{-bx-a}-\frac{\left(a\,c^2b^2+2abcd+2b^2c^2+2a\,d^2+6bcd+8d^2\right)\mathrm{e}^{-bx-a}}{b^3}-\frac{\left(2abcd+b^2c^2+2a\,d^2+6bcd+8d^2\right)x\,\mathrm{e}^{-bx-a}}{b^2}$
parallelrisch	$-\frac{d^2\mathrm{e}^{-bx-a}x^3b^3+x^2\mathrm{e}^{-bx-a}a\,b^2d^2+2x^2\mathrm{e}^{-bx-a}b^3cd+4x^2\mathrm{e}^{-bx-a}d^2b^2+2x\,\mathrm{e}^{-bx-a}a\,b^2cd+x\,\mathrm{e}^{-bx-a}b^3c^2+2x\,\mathrm{e}^{-bx-a}c}{}$
meijerg	$\frac{d^2\mathrm{e}^{-a}\left(6-\frac{\left(4b^3x^3+12b^2x^2+24bx+24\right)\mathrm{e}^{-bx}}{4}\right)}{b^3}+\frac{d^2\mathrm{e}^{-a}a\left(2-\frac{\left(3b^2x^2+6bx+6\right)\mathrm{e}^{-bx}}{3}\right)}{b^3}+\frac{\mathrm{e}^{-a}d^2\left(2-\frac{\left(3b^2x^2+6bx+6\right)\mathrm{e}^{-b}}{3}\right)}{b^3}$
parts	$-d^2x^3\mathrm{e}^{-bx-a}-\frac{\mathrm{e}^{-bx-a}a\,d^2x^2}{b}-2\,\mathrm{e}^{-bx-a}cd\,x^2-\frac{2\,\mathrm{e}^{-bx-a}xadc}{b}-\mathrm{e}^{-bx-a}c^2x-\frac{d^2x^2\mathrm{e}^{-bx-a}}{b}-\mathrm{e}$
derivativedivides	$-\frac{\mathrm{e}^{-bx-a}c^2+\frac{\mathrm{e}^{-bx-a}d^2a^2}{b^2}+\frac{d^2\left((-bx-a)^2\mathrm{e}^{-bx-a}-2(-bx-a)\mathrm{e}^{-bx-a}+2\,\mathrm{e}^{-bx-a}\right)}{b^2}-c^2\left((-bx-a)\mathrm{e}^{-bx-a}-\mathrm{e}^{-bx-a}\right)-\frac{d^2(}{}}{}$
default	$-\frac{\mathrm{e}^{-bx-a}c^2+\frac{\mathrm{e}^{-bx-a}d^2a^2}{b^2}+\frac{d^2\left((-bx-a)^2\mathrm{e}^{-bx-a}-2(-bx-a)\mathrm{e}^{-bx-a}+2\,\mathrm{e}^{-bx-a}\right)}{b^2}-c^2\left((-bx-a)\mathrm{e}^{-bx-a}-\mathrm{e}^{-bx-a}\right)-\frac{d^2(}{}}{}$

input
```
int((d*x+c)^2*exp(-b*x-a)*(b*x+a+1),x,method=_RETURNVERBOSE)
```

output
```
-(b^3*d^2*x^3+a*b^2*d^2*x^2+2*b^3*c*d*x^2+2*a*b^2*c*d*x+b^3*c^2*x+4*b^2*d^
2*x^2+a*b^2*c^2+2*a*b*d^2*x+6*b^2*c*d*x+2*a*b*c*d+2*b^2*c^2+8*b*d^2*x+2*a*
d^2+6*b*c*d+8*d^2)*exp(-b*x-a)/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(88) = 176$.

Time = 0.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.88

$$\int (c+dx)^2\Gamma(2,a+bx)\,dx =$$
$$-\frac{\left(b^4d^2x^4+3\,(a+2)b^2c^2+6\,(a+3)bcd+\left(3\,b^4cd+(a+4)b^3d^2\right)x^3+6\,(a+4)d^2+3\,\left(b^4c^2+(a+3)b^3c\right.}{}$$

input
```
integrate((d*x+c)^2*gamma(2,b*x+a),x, algorithm="fricas")
```

output
```
-1/3*((b^4*d^2*x^4 + 3*(a + 2)*b^2*c^2 + 6*(a + 3)*b*c*d + (3*b^4*c*d + (a
 + 4)*b^3*d^2)*x^3 + 6*(a + 4)*d^2 + 3*(b^4*c^2 + (a + 3)*b^3*c*d + (a + 4
)*b^2*d^2)*x^2 + 3*((a + 2)*b^3*c^2 + 2*(a + 3)*b^2*c*d + 2*(a + 4)*b*d^2)
*x)*e^(-b*x - a) - (b^3*d^2*x^3 + 3*b^3*c*d*x^2 + 3*b^3*c^2*x)*gamma(2, b*
x + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.25

$$\int (c+dx)^2\Gamma(2,a+bx)\,dx = \begin{cases} \frac{(-ab^2c^2-2ab^2cdx-ab^2d^2x^2-2abcd-2abd^2x-2ad^2-b^3c^2x-2b^3cdx^2-b^3d^2x^3-2b^2c^2-6b^2cdx-4b^2d^2x^2-6bcd-8bd^2x-8d^2)e^{-a-bx}}{b^3} & \text{for} \\ \frac{bd^2x^4}{4}+x^3\left(\frac{ad^2}{3}+\frac{2bcd}{3}+\frac{d^2}{3}\right)+x^2\left(acd+\frac{bc^2}{2}+cd\right)+x(ac^2+c^2) & \text{oth} \end{cases}$$

input
```
integrate((d*x+c)**2*uppergamma(2,b*x+a),x)
```

output
```
Piecewise(((-a*b**2*c**2 - 2*a*b**2*c*d*x - a*b**2*d**2*x**2 - 2*a*b*c*d -
 2*a*b*d**2*x - 2*a*d**2 - b**3*c**2*x - 2*b**3*c*d*x**2 - b**3*d**2*x**3
- 2*b**2*c**2 - 6*b**2*c*d*x - 4*b**2*d**2*x**2 - 6*b*c*d - 8*b*d**2*x - 8
*d**2)*exp(-a - b*x)/b**3, Ne(b**3, 0)), (b*d**2*x**4/4 + x**3*(a*d**2/3 +
 2*b*c*d/3 + d**2/3) + x**2*(a*c*d + b*c**2/2 + c*d) + x*(a*c**2 + c**2),
True))
```

Maxima [F]

$$\int (c+dx)^2\Gamma(2,a+bx)\,dx = \int (dx+c)^2\Gamma(2,bx+a)\,dx$$

input
```
integrate((d*x+c)^2*gamma(2,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(2, b*x + a) - gamma(3, b*x + a))*c^2/b + integrate(d^2*x^
2*gamma(2, b*x + a) + 2*c*d*x*gamma(2, b*x + a), x)
```

Giac [F]

$$\int (c+dx)^2\Gamma(2,a+bx)\,dx = \int (dx+c)^2\Gamma(2,bx+a)\,dx$$

input
```
integrate((d*x+c)^2*gamma(2,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*gamma(2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.55

$$\int (c+dx)^2\Gamma(2,a+bx)\,dx = -x^2\,e^{-a-b\,x}\left(2\,c\,d+\frac{a\,d^2+4\,d^2}{b}\right)-x\,e^{-a-b\,x}\left(c^2+\frac{2\,a\,d^2+8\,d^2+b\,(6\,c\,d+2\,a\,c\,d)}{b^2}\right) - d^2\,x^3\,e^{-a-b\,x}-\frac{e^{-a-b\,x}\,\left(b^2\,\left(a\,c^2+2\,c^2\right)+2\,a\,d^2+8\,d^2+b\,(6\,c\,d+2\,a\,c\,d)\right)}{b^3}$$

input
```
int(exp(- a - b*x)*(c + d*x)^2*(a + b*x + 1),x)
```

output
```
- x^2*exp(- a - b*x)*(2*c*d + (a*d^2 + 4*d^2)/b) - x*exp(- a - b*x)*(c^2 +
 (2*a*d^2 + 8*d^2 + b*(6*c*d + 2*a*c*d))/b^2) - d^2*x^3*exp(- a - b*x) - (
exp(- a - b*x)*(b^2*(a*c^2 + 2*c^2) + 2*a*d^2 + 8*d^2 + b*(6*c*d + 2*a*c*d
)))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.44

$$\int (c+dx)^2\Gamma(2,a+bx)\,dx$$
$$= \frac{-b^3d^2x^3 - a\,b^2d^2x^2 - 2b^3cd\,x^2 - 2a\,b^2cdx - b^3c^2x - 4b^2d^2x^2 - a\,b^2c^2 - 2ab\,d^2x - 6b^2cdx - 2abcd - 2b^2}{e^{bx+a}b^3}$$

input
```
int((d*x+c)^2*exp(-b*x-a)*(b*x+a+1),x)
```

output
```
( - a*b**2*c**2 - 2*a*b**2*c*d*x - a*b**2*d**2*x**2 - 2*a*b*c*d - 2*a*b*d*
*2*x - 2*a*d**2 - b**3*c**2*x - 2*b**3*c*d*x**2 - b**3*d**2*x**3 - 2*b**2*
c**2 - 6*b**2*c*d*x - 4*b**2*d**2*x**2 - 6*b*c*d - 8*b*d**2*x - 8*d**2)/(e
**(a + b*x)*b**3)
```

3.121 $\int (c+dx)\Gamma(2,a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 93

$$\int (c+dx)\Gamma(2,a+bx)\,dx = \frac{(c+dx)^2\Gamma(2,a+bx)}{2d} + \frac{(bc-ad)e^{-a+\frac{bc}{d}}\Gamma\left(3,\frac{b(c+dx)}{d}\right)}{2b^2} - \frac{de^{-a+\frac{bc}{d}}\Gamma\left(4,\frac{b(c+dx)}{d}\right)}{2b^2}$$

output
```
1/2*(d*x+c)^2*exp(-b*x-a)*(b*x+a+1)/d+(-a*d+b*c)*exp(-a+b*c/d)*exp(-b*(d*x
+c)/d)*(1+b*(d*x+c)/d+1/2*b^2*(d*x+c)^2/d^2)/b^2-3*d*exp(-a+b*c/d)*exp(-b*
(d*x+c)/d)*(1+b*(d*x+c)/d+1/2*b^2*(d*x+c)^2/d^2+1/6*b^3*(d*x+c)^3/d^3)/b^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06

$$\int (c+dx)\Gamma(2,a+bx)\,dx = \frac{e^{-a-bx}\left(-2(3+a)d - b^3x^2(2c+dx) - 2b((2+a)c+(3+a)dx) - b^2x(2(2+a)c+(3+a)dx) + b^2e^{a+bx}\right.}{2b^2}$$

input
```
Integrate[(c + d*x)*Gamma[2, a + b*x],x]
```

output
```
(E^(-a - b*x)*(-2*(3 + a)*d - b^3*x^2*(2*c + d*x) - 2*b*((2 + a)*c + (3 +
a)*d*x) - b^2*x*(2*(2 + a)*c + (3 + a)*d*x) + b^2*E^(a + b*x)*x*(2*c + d*x
)*Gamma[2, a + b*x]))/(2*b^2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 198 vs. $2(93) = 186$.

Time = 0.58 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7119, 2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\Gamma(2,a+bx)\,dx$$

$$\Big\downarrow \text{ 7119}$$

$$\frac{b\int e^{-a-bx}(a+bx)(c+dx)^2dx}{2d}+\frac{(c+dx)^2\Gamma(2,a+bx)}{2d}$$

$$\Big\downarrow \text{ 2626}$$

$$\frac{b\int \left(\frac{be^{-a-bx}(c+dx)^3}{d}+\frac{(ad-bc)e^{-a-bx}(c+dx)^2}{d}\right)dx}{2d}+\frac{(c+dx)^2\Gamma(2,a+bx)}{2d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\left(\frac{2de^{-a-bx}(bc-ad)}{b^3}-\frac{6d^2e^{-a-bx}}{b^3}-\frac{6de^{-a-bx}(c+dx)}{b^2}+\frac{2e^{-a-bx}(c+dx)(bc-ad)}{b^2}-\frac{e^{-a-bx}(c+dx)^3}{d}+\frac{e^{-a-bx}(c+dx)^2(bc-ad)}{bd}-\frac{3e^{-}\cdots}{\cdots}\right.}{2d}$$
$$\frac{(c+dx)^2\Gamma(2,a+bx)}{2d}$$

input
```
Int[(c + d*x)*Gamma[2, a + b*x],x]
```

output
```
(b*((-6*d^2*E^(-a - b*x))/b^3 + (2*d*(b*c - a*d)*E^(-a - b*x))/b^3 - (6*d*
E^(-a - b*x)*(c + d*x))/b^2 + (2*(b*c - a*d)*E^(-a - b*x)*(c + d*x))/b^2 -
 (3*E^(-a - b*x)*(c + d*x)^2)/b + ((b*c - a*d)*E^(-a - b*x)*(c + d*x)^2)/(
b*d) - (E^(-a - b*x)*(c + d*x)^3)/d))/(2*d) + ((c + d*x)^2*Gamma[2, a + b*
x])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2626
```
Int[(F_)^(v_)*(Px_), x_Symbol] :> Int[ExpandIntegrand[F^v, Px, x], x] /; Fr
eeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.58

method	result
gosper	$-\frac{(b^2 d\,x^2+abdx+b^2cx+abc+3bdx+ad+2cb+3d)\,\mathrm{e}^{-bx-a}}{b^2}$
risch	$-\frac{(b^2 d\,x^2+abdx+b^2cx+abc+3bdx+ad+2cb+3d)\,\mathrm{e}^{-bx-a}}{b^2}$
orering	$-\frac{(b^2 d\,x^2+abdx+b^2cx+abc+3bdx+ad+2cb+3d)\,\mathrm{e}^{-bx-a}}{b^2}$
norman	$-d\,x^2\mathrm{e}^{-bx-a}-\frac{(abc+ad+2cb+3d)\mathrm{e}^{-bx-a}}{b^2}-\frac{(ad+cb+3d)x\,\mathrm{e}^{-bx-a}}{b}$
parallelrisch	$-\frac{d\,x^2\mathrm{e}^{-bx-a}b^2+x\,\mathrm{e}^{-bx-a}abd+x\,\mathrm{e}^{-bx-a}b^2c+3\,\mathrm{e}^{-bx-a}bdx+\mathrm{e}^{-bx-a}abc+\mathrm{e}^{-bx-a}ad+2\,\mathrm{e}^{-bx-a}bc+3\,\mathrm{e}^{-bx-a}d}{b^2}$
meijerg	$\frac{d\,\mathrm{e}^{-a}\left(2-\frac{(3b^2x^2+6bx+6)\mathrm{e}^{-bx}}{3}\right)}{b^2}+\frac{d\,\mathrm{e}^{-a}a\left(1-\frac{(2bx+2)\mathrm{e}^{-bx}}{2}\right)}{b^2}+\frac{\mathrm{e}^{-a}d\left(1-\frac{(2bx+2)\mathrm{e}^{-bx}}{2}\right)}{b^2}+\frac{c\,\mathrm{e}^{-a}\left(1-\frac{(2bx+2)\mathrm{e}^{-}}{2}\right.}{b}$
parts	$-d\,x^2\mathrm{e}^{-bx-a}-\frac{\mathrm{e}^{-bx-a}dxa}{b}-\mathrm{e}^{-bx-a}cx-\frac{\mathrm{e}^{-bx-a}ac}{b}-\frac{dx\,\mathrm{e}^{-bx-a}}{b}-\frac{\mathrm{e}^{-bx-a}c}{b}-\frac{\mathrm{e}^{-bx-a}d+\mathrm{e}^{-bx-a}bc}{}$
derivativedivides	$-\frac{\mathrm{e}^{-bx-a}c+\frac{d\left((-bx-a)^2\mathrm{e}^{-bx-a}-2(-bx-a)\mathrm{e}^{-bx-a}+2\,\mathrm{e}^{-bx-a}\right)}{b}+\frac{da\left((-bx-a)\mathrm{e}^{-bx-a}-\mathrm{e}^{-bx-a}\right)}{b}-c\left((-bx-a)\mathrm{e}^{-bx-a}-\mathrm{e}\right.}{b}$
default	$-\frac{\mathrm{e}^{-bx-a}c+\frac{d\left((-bx-a)^2\mathrm{e}^{-bx-a}-2(-bx-a)\mathrm{e}^{-bx-a}+2\,\mathrm{e}^{-bx-a}\right)}{b}+\frac{da\left((-bx-a)\mathrm{e}^{-bx-a}-\mathrm{e}^{-bx-a}\right)}{b}-c\left((-bx-a)\mathrm{e}^{-bx-a}-\mathrm{e}\right.}{b}$

input
```
int((d*x+c)*exp(-b*x-a)*(b*x+a+1),x,method=_RETURNVERBOSE)
```

output
```
-(b^2*d*x^2+a*b*d*x+b^2*c*x+a*b*c+3*b*d*x+a*d+2*b*c+3*d)*exp(-b*x-a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.08

$$\int (c+dx)\Gamma(2,a+bx)\,dx = -\frac{(b^3dx^3+2\,(a+2)bc+(2\,b^3c+(a+3)b^2d)x^2+2\,(a+3)d+2\,((a+2)b^2c+(a+3)bd)x)e^{(-bx-a)}-(b}{2\,b^2}$$

input
```
integrate((d*x+c)*gamma(2,b*x+a),x, algorithm="fricas")
```

output
```
-1/2*((b^3*d*x^3 + 2*(a + 2)*b*c + (2*b^3*c + (a + 3)*b^2*d)*x^2 + 2*(a +
3)*d + 2*((a + 2)*b^2*c + (a + 3)*b*d)*x)*e^(-b*x - a) - (b^2*d*x^2 + 2*b^
2*c*x)*gamma(2, b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int (c+dx)\Gamma(2,a+bx)\,dx = \begin{cases} \frac{(-abc-abdx-ad-b^2cx-b^2dx^2-2bc-3bdx-3d)e^{-a-bx}}{b^2} & \text{for } b^2 \neq 0 \\ \frac{bdx^3}{3}+x^2\left(\frac{ad}{2}+\frac{bc}{2}+\frac{d}{2}\right)+x(ac+c) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*uppergamma(2,b*x+a),x)
```

output
```
Piecewise(((-a*b*c - a*b*d*x - a*d - b**2*c*x - b**2*d*x**2 - 2*b*c - 3*b*
d*x - 3*d)*exp(-a - b*x)/b**2, Ne(b**2, 0)), (b*d*x**3/3 + x**2*(a*d/2 + b
*c/2 + d/2) + x*(a*c + c), True))
```

Maxima [F]

$$\int (c+dx)\Gamma(2,a+bx)\,dx = \int (dx+c)\Gamma(2,bx+a)\,dx$$

input
```
integrate((d*x+c)*gamma(2,b*x+a),x, algorithm="maxima")
```

output
```
d*integrate(x*gamma(2, b*x + a), x) + ((b*x + a)*gamma(2, b*x + a) - gamma
(3, b*x + a))*c/b
```

Giac [F]

$$\int (c+dx)\Gamma(2,a+bx)\,dx = \int (dx+c)\Gamma(2,bx+a)\,dx$$

input
```
integrate((d*x+c)*gamma(2,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*gamma(2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.75

$$\int (c+dx)\Gamma(2,a+bx)\,dx = -\mathrm{e}^{-a-b\,x}\left(d\,x^2+c\,x\right) - \frac{\mathrm{e}^{-a-b\,x}\,(3\,d+a\,d)+b\,\mathrm{e}^{-a-b\,x}\,(2\,c+a\,c+3\,d\,x+a\,d\,x)}{b^2}$$

input
```
int(exp(- a - b*x)*(c + d*x)*(a + b*x + 1),x)
```

output
```
- exp(- a - b*x)*(c*x + d*x^2) - (exp(- a - b*x)*(3*d + a*d) + b*exp(- a -
 b*x)*(2*c + a*c + 3*d*x + a*d*x))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int (c+dx)\Gamma(2,a+bx)\,dx = \frac{-b^2d\,x^2-abdx-b^2cx-abc-3bdx-ad-2bc-3d}{e^{bx+a}b^2}$$

input
```
int((d*x+c)*exp(-b*x-a)*(b*x+a+1),x)
```

output
```
( - a*b*c - a*b*d*x - a*d - b**2*c*x - b**2*d*x**2 - 2*b*c - 3*b*d*x - 3*d
)/(e**(a + b*x)*b**2)
```

3.122 $\int \Gamma(2, a + bx)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 29

$$\int \Gamma(2, a + bx)\,dx = \frac{(a + bx)\Gamma(2, a + bx)}{b} - \frac{\Gamma(3, a + bx)}{b}$$

output
```
(b*x+a)*exp(-b*x-a)*(b*x+a+1)/b-2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int \Gamma(2, a + bx)\,dx = e^{-bx}\left(-\frac{(2 + a)e^{-a}}{b} - (2 + a)e^{-a}x - be^{-a}x^2\right) + x\Gamma(2, a + bx)$$

input
```
Integrate[Gamma[2, a + b*x],x]
```

output
```
(-((2 + a)/(b*E^a)) - ((2 + a)*x)/E^a - (b*x^2)/E^a)/E^(b*x) + x*Gamma[2,
a + b*x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{7111\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(2, a+bx)\,dx$$

$$\Big\downarrow \; 7111$$

$$\frac{(a+bx)\Gamma(2,a+bx)}{b} - \frac{\Gamma(3,a+bx)}{b}$$

input
```
Int[Gamma[2, a + b*x],x]
```

output
```
((a + b*x)*Gamma[2, a + b*x])/b - Gamma[3, a + b*x]/b
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

method	result	size
gosper	$-\frac{(bx+a+2)\,\mathrm{e}^{-bx-a}}{b}$	21
risch	$-\frac{(bx+a+2)\,\mathrm{e}^{-bx-a}}{b}$	21
orering	$-\frac{(bx+a+2)\,\mathrm{e}^{-bx-a}}{b}$	21
norman	$-x\,\mathrm{e}^{-bx-a}-\frac{(2+a)\,\mathrm{e}^{-bx-a}}{b}$	31
derivativedivides	$-\frac{-(-bx-a)\,\mathrm{e}^{-bx-a}+2\,\mathrm{e}^{-bx-a}}{b}$	37
default	$-\frac{-(-bx-a)\,\mathrm{e}^{-bx-a}+2\,\mathrm{e}^{-bx-a}}{b}$	37
parallelrisch	$-\frac{x\,\mathrm{e}^{-bx-a}b+a\,\mathrm{e}^{-bx-a}+2\,\mathrm{e}^{-bx-a}}{b}$	41
parts	$-x\,\mathrm{e}^{-bx-a}-\frac{\mathrm{e}^{-bx-a}a}{b}-\frac{2\,\mathrm{e}^{-bx-a}}{b}$	43
meijerg	$\frac{\mathrm{e}^{-a}\left(1-\frac{(2bx+2)\,\mathrm{e}^{-bx}}{2}\right)}{b}+\frac{\mathrm{e}^{-a}a\left(1-\mathrm{e}^{-bx}\right)}{b}+\frac{\mathrm{e}^{-a}\left(1-\mathrm{e}^{-bx}\right)}{b}$	60

input
```
int(exp(-b*x-a)*(b*x+a+1),x,method=_RETURNVERBOSE)
```

output
```
-(b*x+a+2)*exp(-b*x-a)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \Gamma(2,a+bx)\,dx=\frac{bx\Gamma(2,bx+a)-(b^2x^2+(a+2)bx+a+2)e^{(-bx-a)}}{b}$$

input
```
integrate(gamma(2,b*x+a),x, algorithm="fricas")
```

output
```
(b*x*gamma(2, b*x + a) - (b^2*x^2 + (a + 2)*b*x + a + 2)*e^(-b*x - a))/b
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \Gamma(2, a + bx)\, dx = \begin{cases} \frac{(-a-bx-2)e^{-a-bx}}{b} & \text{for } b \neq 0 \\ \frac{bx^2}{2} + x(a+1) & \text{otherwise} \end{cases}$$

input
```
integrate(uppergamma(2,b*x+a),x)
```

output
```
Piecewise(((-a - b*x - 2)*exp(-a - b*x)/b, Ne(b, 0)), (b*x**2/2 + x*(a + 1
), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \Gamma(2, a + bx)\, dx = \frac{(bx + a)\Gamma(2, bx + a) - \Gamma(3, bx + a)}{b}$$

input
```
integrate(gamma(2,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(2, b*x + a) - gamma(3, b*x + a))/b
```

Giac [F]

$$\int \Gamma(2, a + bx)\, dx = \int \Gamma(2, bx + a)\, dx$$

input
```
integrate(gamma(2,b*x+a),x, algorithm="giac")
```

output
```
integrate(gamma(2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \Gamma(2, a + bx) \, dx = -\frac{e^{-a-b\,x}\,(a + b\,x + 2)}{b}$$

input
```
int(exp(- a - b*x)*(a + b*x + 1),x)
```

output
```
-(exp(- a - b*x)*(a + b*x + 2))/b
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \Gamma(2, a + bx) \, dx = \frac{-bx - a - 2}{e^{bx+a} b}$$

input
```
int(exp(-b*x-a)*(b*x+a+1),x)
```

output
```
( - a - b*x - 2)/(e**(a + b*x)*b)
```

3.123 $\int \frac{\Gamma(2,a+bx)}{c+dx} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \frac{\Gamma(2,a+bx)}{c+dx} \, dx = -\frac{e^{-a-bx}}{d} + \frac{e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d} - \frac{(bc-ad)e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^2}$$

output
```
-exp(-b*x-a)/d+exp(-a+b*c/d)*Ei(-b*(d*x+c)/d)/d-(-a*d+b*c)*exp(-a+b*c/d)*E
i(-b*(d*x+c)/d)/d^2
```

Mathematica [F]

$$\int \frac{\Gamma(2,a+bx)}{c+dx} \, dx = \int \frac{\Gamma(2,a+bx)}{c+dx} \, dx$$

input
```
Integrate[Gamma[2, a + b*x]/(c + d*x),x]
```

output
```
Integrate[Gamma[2, a + b*x]/(c + d*x), x]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {7118, 2609, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{7118}$$

$$\int \frac{e^{-a-bx}}{c+dx}dx+\int \frac{e^{-a-bx}(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{2609}$$

$$\int \frac{e^{-a-bx}(a+bx)}{c+dx}dx+\frac{e^{\frac{bc}{d}-a}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d}$$

$$\Big\downarrow \text{2629}$$

$$\int \left(\frac{e^{-a-bx}b}{d}+\frac{(ad-bc)e^{-a-bx}}{d(c+dx)}\right)dx+\frac{e^{\frac{bc}{d}-a}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)e^{\frac{bc}{d}-a}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^2}+\frac{e^{\frac{bc}{d}-a}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d}-\frac{e^{-a-bx}}{d}$$

input
```
Int[Gamma[2, a + b*x]/(c + d*x),x]
```

output
```
-(E^(-a - b*x)/d) + (E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d
 - ((b*c - a*d)*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]
```

rule 7118
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Int[(a
 + b*x)^(n - 1)/((c + d*x)*E^(a + b*x)), x] + Simp[(n - 1)   Int[Gamma[n -
1, a + b*x]/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 1]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.32

method	result
derivativedivides	$-\frac{\frac{b\,e^{-\frac{ad-cb}{d}}\,\text{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{d}+\frac{b\,e^{-bx-a}}{d}+\frac{(ad-cb)b\,e^{-\frac{ad-cb}{d}}\,\text{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{d^2}}{b}$
default	$-\frac{\frac{b\,e^{-\frac{ad-cb}{d}}\,\text{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{d}+\frac{b\,e^{-bx-a}}{d}+\frac{(ad-cb)b\,e^{-\frac{ad-cb}{d}}\,\text{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{d^2}}{b}$
risch	$-\frac{e^{-\frac{ad-cb}{d}}\,\text{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{d}-\frac{e^{-bx-a}}{d}-\frac{e^{-\frac{ad-cb}{d}}\,\text{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)a}{d}+\frac{b\,e^{-\frac{ad-cb}{d}}\,\text{exp}}{}$

input
```
int(exp(-b*x-a)*(b*x+a+1)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(b/d*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+b/d*exp(-b*x-a)+(a*d-b
*c)*b/d^2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))
```

Fricas [F]

$$\int \frac{\Gamma(2, a + bx)}{c + dx} \, dx = \int \frac{\Gamma(2, bx + a)}{dx + c} \, dx$$

input
```
integrate(gamma(2,b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(gamma(2, b*x + a)/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\Gamma(2, a + bx)}{c + dx} \, dx = \left(\int \frac{a}{ce^{bx} + dxe^{bx}} \, dx + \int \frac{bx}{ce^{bx} + dxe^{bx}} \, dx + \int \frac{1}{ce^{bx} + dxe^{bx}} \, dx \right) e^{-a}$$

input
```
integrate(uppergamma(2,b*x+a)/(d*x+c),x)
```

output
```
(Integral(a/(c*exp(b*x) + d*x*exp(b*x)), x) + Integral(b*x/(c*exp(b*x) + d
*x*exp(b*x)), x) + Integral(1/(c*exp(b*x) + d*x*exp(b*x)), x))*exp(-a)
```

Maxima [F]

$$\int \frac{\Gamma(2, a + bx)}{c + dx} \, dx = \int \frac{\Gamma(2, bx + a)}{dx + c} \, dx$$

input
```
integrate(gamma(2,b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(gamma(2, b*x + a)/(d*x + c), x)
```

Giac [F]

$$\int \frac{\Gamma(2,a+bx)}{c+dx}\,dx = \int \frac{\Gamma(2,bx+a)}{dx+c}\,dx$$

input
```
integrate(gamma(2,b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(gamma(2, b*x + a)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(2,a+bx)}{c+dx}\,dx = \int \frac{e^{-a-b\,x}\,(a+b\,x+1)}{c+d\,x}\,dx$$

input
```
int((exp(- a - b*x)*(a + b*x + 1))/(c + d*x),x)
```

output
```
int((exp(- a - b*x)*(a + b*x + 1))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\Gamma(2,a+bx)}{c+dx}\,dx = \frac{e^{bx}\left(\int \frac{1}{e^{bx}c+e^{bx}dx}dx\right)ad - e^{bx}\left(\int \frac{1}{e^{bx}c+e^{bx}dx}dx\right)bc + e^{bx}\left(\int \frac{1}{e^{bx}c+e^{bx}dx}dx\right)d - 1}{e^{bx+a}d}$$

input
```
int(exp(-b*x-a)*(b*x+a+1)/(d*x+c),x)
```

output
```
(e**(b*x)*int(1/(e**(b*x)*c + e**(b*x)*d*x),x)*a*d - e**(b*x)*int(1/(e**(b
*x)*c + e**(b*x)*d*x),x)*b*c + e**(b*x)*int(1/(e**(b*x)*c + e**(b*x)*d*x),
x)*d - 1)/(e**(a + b*x)*d)
```

3.124 $\int \frac{\Gamma(2,a+bx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^2}\,dx=\frac{be^{-a-bx}}{d^2}-\frac{b(bc-ad)e^{-a+\frac{bc}{d}}\Gamma\left(0,\frac{b(c+dx)}{d}\right)}{d^3}-\frac{\Gamma(2,a+bx)}{d(c+dx)}$$

output
```
b*exp(-b*x-a)/d^2-b*(-a*d+b*c)*exp(-a+b*c/d)*Ei(1,b*(d*x+c)/d)/d^3-exp(-b*
x-a)*(b*x+a+1)/d/(d*x+c)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^2}\,dx$$
$$=\frac{e^{-a-bx}\left(b(bc-ad)e^{b\left(\frac{c}{d}+x\right)}(c+dx)\,\mathrm{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)+d\left(b(c+dx)-de^{a+bx}\Gamma(2,a+bx)\right)\right)}{d^3(c+dx)}$$

input
```
Integrate[Gamma[2, a + b*x]/(c + d*x)^2,x]
```

output
```
(E^(-a - b*x)*(b*(b*c - a*d)*E^(b*(c/d + x))*(c + d*x)*ExpIntegralEi[-((b*
(c + d*x))/d)] + d*(b*(c + d*x) - d*E^(a + b*x)*Gamma[2, a + b*x])))/(d^3*
(c + d*x))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{7119}$$

$$-\frac{b\int \frac{e^{-a-bx}(a+bx)}{c+dx}dx}{d}-\frac{\Gamma(2,a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{2629}$$

$$-\frac{b\int \left(\frac{e^{-a-bx}b}{d}+\frac{(ad-bc)e^{-a-bx}}{d(c+dx)}\right)dx}{d}-\frac{\Gamma(2,a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{(bc-ad)e^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^2}-\frac{e^{-a-bx}}{d}\right)}{d}-\frac{\Gamma(2,a+bx)}{d(c+dx)}$$

input
```
Int[Gamma[2, a + b*x]/(c + d*x)^2,x]
```

output
```
-((b*(-(E^(-a - b*x)/d) - ((b*c - a*d)*E^(-a + (b*c)/d)*ExpIntegralEi[-((b
*(c + d*x))/d)])/d^2))/d) - Gamma[2, a + b*x]/(d*(c + d*x))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(79) = 158$.

Time = 0.72 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.38

method	result
risch	$\frac{b\,\mathrm{e}^{-bx-a}}{d^{2}\left(-bx-\frac{cb}{d}\right)}+\frac{b\,\mathrm{e}^{-bx-a}a}{d^{2}\left(-bx-\frac{cb}{d}\right)}-\frac{b^{2}\mathrm{e}^{-bx-a}c}{d^{3}\left(-bx-\frac{cb}{d}\right)}+\frac{b\,\mathrm{e}^{-\frac{ad-cb}{d}}\,\mathrm{expIntegral}_{1}\left(bx+a-\frac{ad-cb}{d}\right)a}{d^{2}}-\frac{b^{2}\mathrm{e}^{-\frac{ad-cb}{d}}\,\mathrm{expI}\cdots}{\cdots}$
derivativedivides	$-\frac{\frac{b^{2}\left(-\frac{\mathrm{e}^{-bx-a}}{-bx-a+\frac{ad-cb}{d}}-\mathrm{e}^{-\frac{ad-cb}{d}}\,\mathrm{expIntegral}_{1}\left(bx+a-\frac{ad-cb}{d}\right)\right)}{d^{2}}+\frac{b^{2}\mathrm{e}^{-\frac{ad-cb}{d}}\,\mathrm{expIntegral}_{1}\left(bx+a-\frac{ad-cb}{d}\right)}{d^{2}}+\frac{b^{2}(ad-cb)\left(-\cdots\right.}{\cdots}}{b}$
default	$-\frac{\frac{b^{2}\left(-\frac{\mathrm{e}^{-bx-a}}{-bx-a+\frac{ad-cb}{d}}-\mathrm{e}^{-\frac{ad-cb}{d}}\,\mathrm{expIntegral}_{1}\left(bx+a-\frac{ad-cb}{d}\right)\right)}{d^{2}}+\frac{b^{2}\mathrm{e}^{-\frac{ad-cb}{d}}\,\mathrm{expIntegral}_{1}\left(bx+a-\frac{ad-cb}{d}\right)}{d^{2}}+\frac{b^{2}(ad-cb)\left(-\cdots\right.}{\cdots}}{b}$

input
```
int(exp(-b*x-a)*(b*x+a+1)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b/d^2*exp(-b*x-a)/(-b*x-c*b/d)+b/d^2*exp(-b*x-a)/(-b*x-c*b/d)*a-b^2/d^3*ex
p(-b*x-a)/(-b*x-c*b/d)*c+b/d^2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)*a
-b^2/d^3*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)*c
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.45

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^2}\,dx = \frac{(b^2c^2 - abcd + (b^2cd - abd^2)x)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(\frac{bc-ad}{d}\right)} - d^2\Gamma(2, bx+a) + (bd^2x + bcd)e^{(-bx-a)}}{d^4x + cd^3}$$

input
```
integrate(gamma(2,b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
((b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x)*Ei(-(b*d*x + b*c)/d)*e^((b*c
- a*d)/d) - d^2*gamma(2, b*x + a) + (b*d^2*x + b*c*d)*e^(-b*x - a))/(d^4*x
 + c*d^3)
```

Sympy [F]

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^2}\,dx = \left(\int \frac{a}{c^2e^{bx} + 2cdxe^{bx} + d^2x^2e^{bx}}\,dx + \int \frac{bx}{c^2e^{bx} + 2cdxe^{bx} + d^2x^2e^{bx}}\,dx + \int \frac{1}{c^2e^{bx} + 2cdxe^{bx} + d^2x^2e^{bx}}\,dx\right)e^{-a}$$

input
```
integrate(uppergamma(2,b*x+a)/(d*x+c)**2,x)
```

output
```
(Integral(a/(c**2*exp(b*x) + 2*c*d*x*exp(b*x) + d**2*x**2*exp(b*x)), x) +
Integral(b*x/(c**2*exp(b*x) + 2*c*d*x*exp(b*x) + d**2*x**2*exp(b*x)), x) +
 Integral(1/(c**2*exp(b*x) + 2*c*d*x*exp(b*x) + d**2*x**2*exp(b*x)), x))*e
xp(-a)
```

Maxima [F]

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^2} \, dx = \int \frac{\Gamma(2, bx+a)}{(dx+c)^2} \, dx$$

input
```
integrate(gamma(2,b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(gamma(2, b*x + a)/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^2} \, dx = \int \frac{\Gamma(2, bx+a)}{(dx+c)^2} \, dx$$

input
```
integrate(gamma(2,b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(gamma(2, b*x + a)/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^2} \, dx = \int \frac{e^{-a-b\,x}\,(a+b\,x+1)}{(c+d\,x)^2} \, dx$$

input
```
int((exp(- a - b*x)*(a + b*x + 1))/(c + d*x)^2,x)
```

output
```
int((exp(- a - b*x)*(a + b*x + 1))/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{-e^{bx}\left(\int \frac{x}{e^{bx}b\,c^3+2e^{bx}b\,c^2dx+e^{bx}bc\,d^2x^2+e^{bx}c^2d+2e^{bx}c\,d^2x+e^{bx}d^3x^2}dx\right)a\,b^2c^2d-e^{bx}\left(\int \frac{x}{e^{bx}b\,c^3+2e^{bx}b\,c^2dx+e^{bx}bc\,d^2x^2+e^{bx}c^2d\cdots}\right.}{}$$

input
```
int(exp(-b*x-a)*(b*x+a+1)/(d*x+c)^2,x)
```

output
```
( - e**(b*x)*int(x/(e**(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b*c
*d**2*x**2 + e**(b*x)*c**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x)*d**3*x**2),x
)*a*b**2*c**2*d - e**(b*x)*int(x/(e**(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x
+ e**(b*x)*b*c*d**2*x**2 + e**(b*x)*c**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x
)*d**3*x**2),x)*a*b**2*c*d**2*x - e**(b*x)*int(x/(e**(b*x)*b*c**3 + 2*e**(
b*x)*b*c**2*d*x + e**(b*x)*b*c*d**2*x**2 + e**(b*x)*c**2*d + 2*e**(b*x)*c*
d**2*x + e**(b*x)*d**3*x**2),x)*a*b*c*d**2 - e**(b*x)*int(x/(e**(b*x)*b*c*
*3 + 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b*c*d**2*x**2 + e**(b*x)*c**2*d + 2*
e**(b*x)*c*d**2*x + e**(b*x)*d**3*x**2),x)*a*b*d**3*x + e**(b*x)*int(x/(e*
*(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b*c*d**2*x**2 + e**(b*x)*
c**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x)*d**3*x**2),x)*b**3*c**3 + e**(b*x)
*int(x/(e**(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b*c*d**2*x**2 +
 e**(b*x)*c**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x)*d**3*x**2),x)*b**3*c**2*
d*x + e**(b*x)*int(x/(e**(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b
*c*d**2*x**2 + e**(b*x)*c**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x)*d**3*x**2)
,x)*b**2*c**2*d + e**(b*x)*int(x/(e**(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x
+ e**(b*x)*b*c*d**2*x**2 + e**(b*x)*c**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x
)*d**3*x**2),x)*b**2*c*d**2*x - a - 1)/(e**(a + b*x)*(b*c**2 + b*c*d*x + c
*d + d**2*x))
```

3.125 $\int \frac{\Gamma(2,a+bx)}{(c+dx)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 98

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^3} dx = -\frac{b^2(bc-ad)e^{-a+\frac{bc}{d}}\Gamma\left(-1,\frac{b(c+dx)}{d}\right)}{2d^4} + \frac{b^2e^{-a+\frac{bc}{d}}\Gamma\left(0,\frac{b(c+dx)}{d}\right)}{2d^3} - \frac{\Gamma(2,a+bx)}{2d(c+dx)^2}$$

output
```
-1/2*b*(-a*d+b*c)*exp(-a+b*c/d)/(d*x+c)/d^3*Ei(2,b*(d*x+c)/d)+1/2*b^2*exp(
-a+b*c/d)*Ei(1,b*(d*x+c)/d)/d^3-1/2*exp(-b*x-a)*(b*x+a+1)/d/(d*x+c)^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^3} dx = \frac{-b^2(bc+d-ad)e^{-a+\frac{bc}{d}}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right) + \frac{d\left(-b(bc-ad)e^{-a-bx}(c+dx)-d^2\Gamma(2,a+bx)\right)}{(c+dx)^2}}{2d^4}$$

input
```
Integrate[Gamma[2, a + b*x]/(c + d*x)^3,x]
```

output
```
(-(b^2*(b*c + d - a*d)*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])
 + (d*(-(b*(b*c - a*d)*E^(-a - b*x)*(c + d*x)) - d^2*Gamma[2, a + b*x]))/(
c + d*x)^2)/(2*d^4)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^3} \, dx$$

$$\Big\downarrow \text{ 7119}$$

$$-\frac{b\int \frac{e^{-a-bx}(a+bx)}{(c+dx)^2}dx}{2d}-\frac{\Gamma(2,a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 2629}$$

$$-\frac{b\int \left(\frac{e^{-a-bx}b}{d(c+dx)}+\frac{(ad-bc)e^{-a-bx}}{d(c+dx)^2}\right)dx}{2d}-\frac{\Gamma(2,a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b\left(\frac{be^{\frac{bc}{d}-a}(bc-ad)\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^3}+\frac{be^{\frac{bc}{d}-a}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^2}+\frac{e^{-a-bx}(bc-ad)}{d^2(c+dx)}\right)}{2d}-\frac{\Gamma(2,a+bx)}{2d(c+dx)^2}$$

input
```
Int[Gamma[2, a + b*x]/(c + d*x)^3,x]
```

output
```
-1/2*(b*(((b*c - a*d)*E^(-a - b*x))/(d^2*(c + d*x)) + (b*E^(-a + (b*c)/d)*
ExpIntegralEi[-((b*(c + d*x))/d)])/d^2 + (b*(b*c - a*d)*E^(-a + (b*c)/d)*E
xpIntegralEi[-((b*(c + d*x))/d)])/d^3))/d - Gamma[2, a + b*x]/(2*d*(c + d*
x)^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(103) = 206$.

Time = 0.71 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.20

method	result
derivativedivides	$-\frac{-\frac{b^{3}\left(-\frac{{\mathrm e}^{-b x -a}}{2\left(-b x -a +\frac{a d -c b}{d}\right)^{2}}-\frac{{\mathrm e}^{-b x -a}}{2\left(-b x -a +\frac{a d -c b}{d}\right)}-\frac{{\mathrm e}^{-\frac{a d -c b}{d}} \operatorname{expIntegral}_{1}\left(b x +a -\frac{a d -c b}{d}\right)}{2}\right)}{d^{3}}+\frac{b^{3}\left(-\frac{{\mathrm e}^{-b x -a}}{-b x -a +\frac{a d -c b}{d}}-{\mathrm e}^{-\frac{a}{}}\right.}{}}{}$
default	$-\frac{-\frac{b^{3}\left(-\frac{{\mathrm e}^{-b x -a}}{2\left(-b x -a +\frac{a d -c b}{d}\right)^{2}}-\frac{{\mathrm e}^{-b x -a}}{2\left(-b x -a +\frac{a d -c b}{d}\right)}-\frac{{\mathrm e}^{-\frac{a d -c b}{d}} \operatorname{expIntegral}_{1}\left(b x +a -\frac{a d -c b}{d}\right)}{2}\right)}{d^{3}}+\frac{b^{3}\left(-\frac{{\mathrm e}^{-b x -a}}{-b x -a +\frac{a d -c b}{d}}-{\mathrm e}^{-\frac{a}{}}\right.}{}}{}$
risch	$-\frac{b^{2} {\mathrm e}^{-b x -a}}{2 d^{3} \left(-b x -\frac{c b}{d}\right)^{2}}+\frac{b^{2} {\mathrm e}^{-b x -a}}{2 d^{3} \left(-b x -\frac{c b}{d}\right)}+\frac{b^{2} {\mathrm e}^{-\frac{a d -c b}{d}} \operatorname{expIntegral}_{1}\left(b x +a -\frac{a d -c b}{d}\right)}{2 d^{3}}-\frac{b^{2} {\mathrm e}^{-b x -a} a}{2 d^{3} \left(-b x -\frac{c b}{d}\right)^{2}}+\frac{b^{3} {\mathrm e}^{-b}}{2 d^{4} \left(-b\right.}$

input
```
int(exp(-b*x-a)*(b*x+a+1)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(-b^3/d^3*(-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/2*exp(-b*x-a)/(-
b*x-a+(a*d-b*c)/d)-1/2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))+b^3/d^3*
(-exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/
d))-b^3*(a*d-b*c)/d^4*(-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/2*exp(-b*
x-a)/(-b*x-a+(a*d-b*c)/d)-1/2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.83

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^3}\,dx =$$

$$-\frac{d^3\Gamma(2,bx+a)+(b^3c^3-(a-1)b^2c^2d+(b^3cd^2-(a-1)b^2d^3)x^2+2\,(b^3c^2d-(a-1)b^2cd^2)x)\mathrm{Ei}\left(-\frac{bdx}{d}\right.}{2\,(d^6x^2+2\,cd^5x+c^2d^4)}$$

input
```
integrate(gamma(2,b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(d^3*gamma(2, b*x + a) + (b^3*c^3 - (a - 1)*b^2*c^2*d + (b^3*c*d^2 -
(a - 1)*b^2*d^3)*x^2 + 2*(b^3*c^2*d - (a - 1)*b^2*c*d^2)*x)*Ei(-(b*d*x + b
*c)/d)*e^((b*c - a*d)/d) + (b^2*c^2*d - a*b*c*d^2 + (b^2*c*d^2 - a*b*d^3)*
x)*e^(-b*x - a))/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate(uppergamma(2,b*x+a)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^3} \, dx = \int \frac{\Gamma(2, bx+a)}{(dx+c)^3} \, dx$$

input
```
integrate(gamma(2,b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(gamma(2, b*x + a)/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^3} \, dx = \int \frac{\Gamma(2, bx+a)}{(dx+c)^3} \, dx$$

input
```
integrate(gamma(2,b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(gamma(2, b*x + a)/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^3} \, dx = \int \frac{e^{-a-b\,x}\,(a+b\,x+1)}{(c+d\,x)^3} \, dx$$

input
```
int((exp(- a - b*x)*(a + b*x + 1))/(c + d*x)^3,x)
```

output
```
int((exp(- a - b*x)*(a + b*x + 1))/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^3} \, dx = \text{too large to display}$$

input
```
int(exp(-b*x-a)*(b*x+a+1)/(d*x+c)^3,x)
```

output
```
( - e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b*x)*b*c**3*d*x + 3*e**(b*x)*b
*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 + 2*e**(b*x)*c**3*d + 6*e**(b*x)*
c**2*d**2*x + 6*e**(b*x)*c*d**3*x**2 + 2*e**(b*x)*d**4*x**3),x)*a*b**2*c**
3*d - 2*e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b*x)*b*c**3*d*x + 3*e**(b*
x)*b*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 + 2*e**(b*x)*c**3*d + 6*e**(b
*x)*c**2*d**2*x + 6*e**(b*x)*c*d**3*x**2 + 2*e**(b*x)*d**4*x**3),x)*a*b**2
*c**2*d**2*x - e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b*x)*b*c**3*d*x + 3
*e**(b*x)*b*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 + 2*e**(b*x)*c**3*d +
6*e**(b*x)*c**2*d**2*x + 6*e**(b*x)*c*d**3*x**2 + 2*e**(b*x)*d**4*x**3),x)
*a*b**2*c*d**3*x**2 - 2*e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b*x)*b*c**
3*d*x + 3*e**(b*x)*b*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 + 2*e**(b*x)*
c**3*d + 6*e**(b*x)*c**2*d**2*x + 6*e**(b*x)*c*d**3*x**2 + 2*e**(b*x)*d**4
*x**3),x)*a*b*c**2*d**2 - 4*e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b*x)*b
*c**3*d*x + 3*e**(b*x)*b*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 + 2*e**(b
*x)*c**3*d + 6*e**(b*x)*c**2*d**2*x + 6*e**(b*x)*c*d**3*x**2 + 2*e**(b*x)*
d**4*x**3),x)*a*b*c*d**3*x - 2*e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b*x
)*b*c**3*d*x + 3*e**(b*x)*b*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 + 2*e*
*(b*x)*c**3*d + 6*e**(b*x)*c**2*d**2*x + 6*e**(b*x)*c*d**3*x**2 + 2*e**(b*
x)*d**4*x**3),x)*a*b*d**4*x**2 + e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b
*x)*b*c**3*d*x + 3*e**(b*x)*b*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 +...
```

3.126 $\int \frac{\Gamma(2,a+bx)}{(c+dx)^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 98

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^4} dx = -\frac{b^3(bc-ad)e^{-a+\frac{bc}{d}}\Gamma\left(-2,\frac{b(c+dx)}{d}\right)}{3d^5} + \frac{b^3e^{-a+\frac{bc}{d}}\Gamma\left(-1,\frac{b(c+dx)}{d}\right)}{3d^4} - \frac{\Gamma(2,a+bx)}{3d(c+dx)^3}$$

output
```
-1/3*b*(-a*d+b*c)*exp(-a+b*c/d)/(d*x+c)^2/d^3*Ei(3,b*(d*x+c)/d)+1/3*b^2*ex
p(-a+b*c/d)/(d*x+c)/d^3*Ei(2,b*(d*x+c)/d)-1/3*exp(-b*x-a)*(b*x+a+1)/d/(d*x
+c)^3
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.19

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^4} dx = \frac{b^3(bc-(-2+a)d)e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right) + \frac{d\left(be^{-a-bx}(c+dx)\left(ad^2+b^2c(c+dx)+bd(c-ac-(-2+a)dx)\right)-2d^3\Gamma(}{(c+dx)^3}}{6d^5}$$

input
```
Integrate[Gamma[2, a + b*x]/(c + d*x)^4,x]
```

output
```
(b^3*(b*c - (-2 + a)*d)*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)]
 + (d*(b*E^(-a - b*x)*(c + d*x)*(a*d^2 + b^2*c*(c + d*x) + b*d*(c - a*c -
(-2 + a)*d*x)) - 2*d^3*Gamma[2, a + b*x]))/(c + d*x)^3)/(6*d^5)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^4}dx$$

$$\Big\downarrow \text{7119}$$

$$-\frac{b\int \frac{e^{-a-bx}(a+bx)}{(c+dx)^3}dx}{3d}-\frac{\Gamma(2,a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{2629}$$

$$-\frac{b\int \left(\frac{e^{-a-bx}b}{d(c+dx)^2}+\frac{(ad-bc)e^{-a-bx}}{d(c+dx)^3}\right)dx}{3d}-\frac{\Gamma(2,a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{b^2(bc-ad)e^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{2d^4}-\frac{b^2e^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^3}-\frac{be^{-a-bx}(bc-ad)}{2d^3(c+dx)}-\frac{be^{-a-bx}}{d^2(c+dx)}+\frac{e^{-a-bx}(bc-}{2d^2(c+dx}\right.}{3d}$$
$$\frac{\Gamma(2,a+bx)}{3d(c+dx)^3}$$

input
```
Int[Gamma[2, a + b*x]/(c + d*x)^4,x]
```

output
```
-1/3*(b*(((b*c - a*d)*E^(-a - b*x))/(2*d^2*(c + d*x)^2) - (b*E^(-a - b*x))
/(d^2*(c + d*x)) - (b*(b*c - a*d)*E^(-a - b*x))/(2*d^3*(c + d*x)) - (b^2*E
^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^3 - (b^2*(b*c - a*d)*
E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/(2*d^4)))/d - Gamma[2,
 a + b*x]/(3*d*(c + d*x)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(110) = 220$.

Time = 0.76 (sec) , antiderivative size = 412, normalized size of antiderivative = 4.20

method	result
derivativedivides	$-\frac{b^4\left(-\frac{e^{-bx-a}}{3\left(-bx-a+\frac{ad-cb}{d}\right)^3}-\frac{e^{-bx-a}}{6\left(-bx-a+\frac{ad-cb}{d}\right)^2}-\frac{e^{-bx-a}}{6\left(-bx-a+\frac{ad-cb}{d}\right)}-\frac{e^{-\frac{ad-cb}{d}}\operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{6}\right)}{d^4}+\frac{b^4(ad-}{}$
default	$-\frac{b^4\left(-\frac{e^{-bx-a}}{3\left(-bx-a+\frac{ad-cb}{d}\right)^3}-\frac{e^{-bx-a}}{6\left(-bx-a+\frac{ad-cb}{d}\right)^2}-\frac{e^{-bx-a}}{6\left(-bx-a+\frac{ad-cb}{d}\right)}-\frac{e^{-\frac{ad-cb}{d}}\operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{6}\right)}{d^4}+\frac{b^4(ad-}{}$
risch	$\frac{b^3e^{-bx-a}}{3d^4\left(-bx-\frac{cb}{d}\right)^3}-\frac{b^3e^{-bx-a}}{3d^4\left(-bx-\frac{cb}{d}\right)^2}-\frac{b^3e^{-bx-a}}{3d^4\left(-bx-\frac{cb}{d}\right)}-\frac{b^3e^{-\frac{ad-cb}{d}}\operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{3d^4}+\frac{b^3e^{-bx-}}{3d^4(-bx-}$

input
```
int(exp(-b*x-a)*(b*x+a+1)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(b^4/d^4*(-1/3*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^3-1/6*exp(-b*x-a)/(-b
*x-a+(a*d-b*c)/d)^2-1/6*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/6*exp(-(a*d-b*c
)/d)*Ei(1,b*x+a-(a*d-b*c)/d))+b^4*(a*d-b*c)/d^5*(-1/3*exp(-b*x-a)/(-b*x-a+
(a*d-b*c)/d)^3-1/6*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/6*exp(-b*x-a)/(-b*
x-a+(a*d-b*c)/d)-1/6*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))-b^4/d^4*(-
1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d
)-1/2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(90) = 180$.

Time = 0.12 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.82

$$\int \frac{\Gamma(2, a + bx)}{(c + dx)^4} dx =$$

$$-\frac{2\, d^4\Gamma(2, bx + a) - (b^4c^4 - (a - 2)b^3c^3d + (b^4cd^3 - (a - 2)b^3d^4)x^3 + 3\,(b^4c^2d^2 - (a - 2)b^3cd^3)x^2 + 3\,(}{}$$

input
```
integrate(gamma(2,b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/6*(2*d^4*gamma(2, b*x + a) - (b^4*c^4 - (a - 2)*b^3*c^3*d + (b^4*c*d^3
- (a - 2)*b^3*d^4)*x^3 + 3*(b^4*c^2*d^2 - (a - 2)*b^3*c*d^3)*x^2 + 3*(b^4*
c^3*d - (a - 2)*b^3*c^2*d^2)*x)*Ei(-(b*d*x + b*c)/d)*e^((b*c - a*d)/d) - (
b^3*c^3*d - (a - 1)*b^2*c^2*d^2 + a*b*c*d^3 + (b^3*c*d^3 - (a - 2)*b^2*d^4
)*x^2 + (2*b^3*c^2*d^2 - (2*a - 3)*b^2*c*d^3 + a*b*d^4)*x)*e^(-b*x - a))/(
d^8*x^3 + 3*c*d^7*x^2 + 3*c^2*d^6*x + c^3*d^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^4} \, dx = \text{Timed out}$$

input
```
integrate(uppergamma(2,b*x+a)/(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^4} \, dx = \int \frac{\Gamma(2, bx+a)}{(dx+c)^4} \, dx$$

input
```
integrate(gamma(2,b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate(gamma(2, b*x + a)/(d*x + c)^4, x)
```

Giac [F]

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^4} \, dx = \int \frac{\Gamma(2, bx+a)}{(dx+c)^4} \, dx$$

input
```
integrate(gamma(2,b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate(gamma(2, b*x + a)/(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^4}\,dx=\int \frac{\mathrm{e}^{-a-b\,x}\,(a+b\,x+1)}{(c+d\,x)^4}\,dx$$

input
```
int((exp(- a - b*x)*(a + b*x + 1))/(c + d*x)^4,x)
```

output
```
int((exp(- a - b*x)*(a + b*x + 1))/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^4}\,dx=\text{too large to display}$$

input
```
int(exp(-b*x-a)*(b*x+a+1)/(d*x+c)^4,x)
```

output
```
( - e**(b*x)*int(x/(e**(b*x)*b*c**5 + 4*e**(b*x)*b*c**4*d*x + 6*e**(b*x)*b
*c**3*d**2*x**2 + 4*e**(b*x)*b*c**2*d**3*x**3 + e**(b*x)*b*c*d**4*x**4 + 3
*e**(b*x)*c**4*d + 12*e**(b*x)*c**3*d**2*x + 18*e**(b*x)*c**2*d**3*x**2 +
12*e**(b*x)*c*d**4*x**3 + 3*e**(b*x)*d**5*x**4),x)*a*b**2*c**4*d - 3*e**(b
*x)*int(x/(e**(b*x)*b*c**5 + 4*e**(b*x)*b*c**4*d*x + 6*e**(b*x)*b*c**3*d**
2*x**2 + 4*e**(b*x)*b*c**2*d**3*x**3 + e**(b*x)*b*c*d**4*x**4 + 3*e**(b*x)
*c**4*d + 12*e**(b*x)*c**3*d**2*x + 18*e**(b*x)*c**2*d**3*x**2 + 12*e**(b*
x)*c*d**4*x**3 + 3*e**(b*x)*d**5*x**4),x)*a*b**2*c**3*d**2*x - 3*e**(b*x)*
int(x/(e**(b*x)*b*c**5 + 4*e**(b*x)*b*c**4*d*x + 6*e**(b*x)*b*c**3*d**2*x*
*2 + 4*e**(b*x)*b*c**2*d**3*x**3 + e**(b*x)*b*c*d**4*x**4 + 3*e**(b*x)*c**
4*d + 12*e**(b*x)*c**3*d**2*x + 18*e**(b*x)*c**2*d**3*x**2 + 12*e**(b*x)*c
*d**4*x**3 + 3*e**(b*x)*d**5*x**4),x)*a*b**2*c**2*d**3*x**2 - e**(b*x)*int
(x/(e**(b*x)*b*c**5 + 4*e**(b*x)*b*c**4*d*x + 6*e**(b*x)*b*c**3*d**2*x**2
+ 4*e**(b*x)*b*c**2*d**3*x**3 + e**(b*x)*b*c*d**4*x**4 + 3*e**(b*x)*c**4*d
 + 12*e**(b*x)*c**3*d**2*x + 18*e**(b*x)*c**2*d**3*x**2 + 12*e**(b*x)*c*d*
*4*x**3 + 3*e**(b*x)*d**5*x**4),x)*a*b**2*c*d**4*x**3 - 3*e**(b*x)*int(x/(
e**(b*x)*b*c**5 + 4*e**(b*x)*b*c**4*d*x + 6*e**(b*x)*b*c**3*d**2*x**2 + 4*
e**(b*x)*b*c**2*d**3*x**3 + e**(b*x)*b*c*d**4*x**4 + 3*e**(b*x)*c**4*d + 1
2*e**(b*x)*c**3*d**2*x + 18*e**(b*x)*c**2*d**3*x**2 + 12*e**(b*x)*c*d**4*x
**3 + 3*e**(b*x)*d**5*x**4),x)*a*b*c**3*d**2 - 9*e**(b*x)*int(x/(e**(b*...
```

3.127 $\int \frac{\Gamma(2,a+bx)}{(c+dx)^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 98

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^5} \, dx = -\frac{b^4(bc-ad)e^{-a+\frac{bc}{d}}\Gamma\left(-3,\frac{b(c+dx)}{d}\right)}{4d^6} + \frac{b^4e^{-a+\frac{bc}{d}}\Gamma\left(-2,\frac{b(c+dx)}{d}\right)}{4d^5} - \frac{\Gamma(2,a+bx)}{4d(c+dx)^4}$$

output
```
-1/4*b*(-a*d+b*c)*exp(-a+b*c/d)/(d*x+c)^3/d^3*Ei(4,b*(d*x+c)/d)+1/4*b^2*ex
p(-a+b*c/d)/(d*x+c)^2/d^3*Ei(3,b*(d*x+c)/d)-1/4*exp(-b*x-a)*(b*x+a+1)/d/(d
*x+c)^4
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.95

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^5} \, dx = -\frac{\frac{bde^{-a-bx}\left(2d^2(bc-ad)-bd(bc-(-3+a)d)(c+dx)+b^2(bc-(-3+a)d)(c+dx)^2\right)}{(c+dx)^3} + b^5ce^{-a+\frac{bc}{d}}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right) + 3b^4}{24d^6}$$

input
```
Integrate[Gamma[2, a + b*x]/(c + d*x)^5,x]
```

output
```
-1/24*((b*d*E^(-a - b*x)*(2*d^2*(b*c - a*d) - b*d*(b*c - (-3 + a)*d)*(c +
d*x) + b^2*(b*c - (-3 + a)*d)*(c + d*x)^2))/(c + d*x)^3 + b^5*c*E^(-a + (b
*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)] + 3*b^4*d*E^(-a + (b*c)/d)*ExpInt
egralEi[-((b*(c + d*x))/d)] - a*b^4*d*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*
(c + d*x))/d)] + (6*d^5*Gamma[2, a + b*x])/(c + d*x)^4)/d^6
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(98) = 196.

Time = 0.65 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^5}\,dx$$

$$\Big\downarrow \text{7119}$$

$$-\frac{b\int \frac{e^{-a-bx}(a+bx)}{(c+dx)^4}dx}{4d}-\frac{\Gamma(2,a+bx)}{4d(c+dx)^4}$$

$$\Big\downarrow \text{2629}$$

$$-\frac{b\int \left(\frac{e^{-a-bx}b}{d(c+dx)^3}+\frac{(ad-bc)e^{-a-bx}}{d(c+dx)^4}\right)dx}{4d}-\frac{\Gamma(2,a+bx)}{4d(c+dx)^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{b^3(bc-ad)e^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{6d^5}+\frac{b^3e^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{2d^4}+\frac{b^2e^{-a-bx}(bc-ad)}{6d^4(c+dx)}+\frac{b^2e^{-a-bx}}{2d^3(c+dx)}-\frac{be^{-a-bx}(bc}{6d^3(c+dx}\right.}{4d}$$

$$\frac{\Gamma(2,a+bx)}{4d(c+dx)^4}$$

input
```
Int[Gamma[2, a + b*x]/(c + d*x)^5,x]
```

output
```
-1/4*(b*(((b*c - a*d)*E^(-a - b*x))/(3*d^2*(c + d*x)^3) - (b*E^(-a - b*x))
/(2*d^2*(c + d*x)^2) - (b*(b*c - a*d)*E^(-a - b*x))/(6*d^3*(c + d*x)^2) +
(b^2*E^(-a - b*x))/(2*d^3*(c + d*x)) + (b^2*(b*c - a*d)*E^(-a - b*x))/(6*d
^4*(c + d*x)) + (b^3*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/(
2*d^4) + (b^3*(b*c - a*d)*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d
)])/(6*d^5)))/d - Gamma[2, a + b*x]/(4*d*(c + d*x)^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. 2(110) = 220.

Time = 0.80 (sec) , antiderivative size = 510, normalized size of antiderivative = 5.20

method	result
risch	$-\frac{b^4 \mathrm{e}^{-bx-a}}{4d^5\left(-bx-\frac{cb}{d}\right)^4}+\frac{b^4 \mathrm{e}^{-bx-a}}{4d^5\left(-bx-\frac{cb}{d}\right)^3}+\frac{b^4 \mathrm{e}^{-bx-a}}{8d^5\left(-bx-\frac{cb}{d}\right)^2}+\frac{b^4 \mathrm{e}^{-bx-a}}{8d^5\left(-bx-\frac{cb}{d}\right)}+\frac{b^4 \mathrm{e}^{-\frac{ad-cb}{d}} \mathrm{expIntegral}_1\left(bx+a-\right.}{8d^5}$
derivativedivides	$-\frac{-\frac{b^5\left(-\frac{\mathrm{e}^{-bx-a}}{4\left(-bx-a+\frac{ad-cb}{d}\right)^4}-\frac{\mathrm{e}^{-bx-a}}{12\left(-bx-a+\frac{ad-cb}{d}\right)^3}-\frac{\mathrm{e}^{-bx-a}}{24\left(-bx-a+\frac{ad-cb}{d}\right)^2}-\frac{\mathrm{e}^{-bx-a}}{24\left(-bx-a+\frac{ad-cb}{d}\right)}-\frac{\mathrm{e}^{-\frac{ad-cb}{d}} \mathrm{expIntegral}_1}{24}\right.}{d^5}}{}$
default	$-\frac{-\frac{b^5\left(-\frac{\mathrm{e}^{-bx-a}}{4\left(-bx-a+\frac{ad-cb}{d}\right)^4}-\frac{\mathrm{e}^{-bx-a}}{12\left(-bx-a+\frac{ad-cb}{d}\right)^3}-\frac{\mathrm{e}^{-bx-a}}{24\left(-bx-a+\frac{ad-cb}{d}\right)^2}-\frac{\mathrm{e}^{-bx-a}}{24\left(-bx-a+\frac{ad-cb}{d}\right)}-\frac{\mathrm{e}^{-\frac{ad-cb}{d}} \mathrm{expIntegral}_1}{24}\right.}{d^5}}{}$

input
```
int(exp(-b*x-a)*(b*x+a+1)/(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^4/d^5*exp(-b*x-a)/(-b*x-c*b/d)^4+1/4*b^4/d^5*exp(-b*x-a)/(-b*x-c*b/
d)^3+1/8*b^4/d^5*exp(-b*x-a)/(-b*x-c*b/d)^2+1/8*b^4/d^5*exp(-b*x-a)/(-b*x-
c*b/d)+1/8*b^4/d^5*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-1/4*b^4/d^5*e
xp(-b*x-a)/(-b*x-c*b/d)^4*a+1/4*b^5/d^6*exp(-b*x-a)/(-b*x-c*b/d)^4*c-1/12*
b^4/d^5*exp(-b*x-a)/(-b*x-c*b/d)^3*a+1/12*b^5/d^6*exp(-b*x-a)/(-b*x-c*b/d)
^3*c-1/24*b^4/d^5*exp(-b*x-a)/(-b*x-c*b/d)^2*a+1/24*b^5/d^6*exp(-b*x-a)/(-
b*x-c*b/d)^2*c-1/24*b^4/d^5*exp(-b*x-a)/(-b*x-c*b/d)*a+1/24*b^5/d^6*exp(-b
*x-a)/(-b*x-c*b/d)*c-1/24*b^4/d^5*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d
)*a+1/24*b^5/d^6*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)*c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(90) = 180.

Time = 0.15 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.92

$$\int \frac{\Gamma(2,a+bx)}{(c+dx)^5}\,dx =$$

$$-\frac{6\,d^5\Gamma(2,bx+a)+(b^5c^5-(a-3)b^4c^4d+(b^5cd^4-(a-3)b^4d^5)x^4+4\,(b^5c^2d^3-(a-3)b^4cd^4)x^3+6\,(}{}$$

input
```
integrate(gamma(2,b*x+a)/(d*x+c)^5,x, algorithm="fricas")
```

output
```
-1/24*(6*d^5*gamma(2, b*x + a) + (b^5*c^5 - (a - 3)*b^4*c^4*d + (b^5*c*d^4
 - (a - 3)*b^4*d^5)*x^4 + 4*(b^5*c^2*d^3 - (a - 3)*b^4*c*d^4)*x^3 + 6*(b^5
*c^3*d^2 - (a - 3)*b^4*c^2*d^3)*x^2 + 4*(b^5*c^4*d - (a - 3)*b^4*c^3*d^2)*
x)*Ei(-(b*d*x + b*c)/d)*e^((b*c - a*d)/d) + (b^4*c^4*d - (a - 2)*b^3*c^3*d
^2 + (a - 1)*b^2*c^2*d^3 - 2*a*b*c*d^4 + (b^4*c*d^4 - (a - 3)*b^3*d^5)*x^3
 + (3*b^4*c^2*d^3 - (3*a - 8)*b^3*c*d^4 + (a - 3)*b^2*d^5)*x^2 + (3*b^4*c^
3*d^2 - (3*a - 7)*b^3*c^2*d^3 + 2*(a - 2)*b^2*c*d^4 - 2*a*b*d^5)*x)*e^(-b*
x - a))/(d^10*x^4 + 4*c*d^9*x^3 + 6*c^2*d^8*x^2 + 4*c^3*d^7*x + c^4*d^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^5} \, dx = \text{Timed out}$$

input
```
integrate(uppergamma(2,b*x+a)/(d*x+c)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^5} \, dx = \int \frac{\Gamma(2, bx+a)}{(dx+c)^5} \, dx$$

input
```
integrate(gamma(2,b*x+a)/(d*x+c)^5,x, algorithm="maxima")
```

output
```
integrate(gamma(2, b*x + a)/(d*x + c)^5, x)
```

Giac [F]

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^5}\, dx = \int \frac{\Gamma(2, bx+a)}{(dx+c)^5}\, dx$$

input
```
integrate(gamma(2,b*x+a)/(d*x+c)^5,x, algorithm="giac")
```

output
```
integrate(gamma(2, b*x + a)/(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^5}\, dx = \int \frac{e^{-a-b\,x}\,(a+b\,x+1)}{(c+d\,x)^5}\, dx$$

input
```
int((exp(- a - b*x)*(a + b*x + 1))/(c + d*x)^5,x)
```

output
```
int((exp(- a - b*x)*(a + b*x + 1))/(c + d*x)^5, x)
```

Reduce [F]

$$\int \frac{\Gamma(2, a+bx)}{(c+dx)^5}\, dx = \text{too large to display}$$

input
```
int(exp(-b*x-a)*(b*x+a+1)/(d*x+c)^5,x)
```

output
```
( - e**(b*x)*int(x/(e**(b*x)*b*c**6 + 5*e**(b*x)*b*c**5*d*x + 10*e**(b*x)*
b*c**4*d**2*x**2 + 10*e**(b*x)*b*c**3*d**3*x**3 + 5*e**(b*x)*b*c**2*d**4*x
**4 + e**(b*x)*b*c*d**5*x**5 + 4*e**(b*x)*c**5*d + 20*e**(b*x)*c**4*d**2*x
 + 40*e**(b*x)*c**3*d**3*x**2 + 40*e**(b*x)*c**2*d**4*x**3 + 20*e**(b*x)*c
*d**5*x**4 + 4*e**(b*x)*d**6*x**5),x)*a*b**2*c**5*d - 4*e**(b*x)*int(x/(e*
*(b*x)*b*c**6 + 5*e**(b*x)*b*c**5*d*x + 10*e**(b*x)*b*c**4*d**2*x**2 + 10*
e**(b*x)*b*c**3*d**3*x**3 + 5*e**(b*x)*b*c**2*d**4*x**4 + e**(b*x)*b*c*d**
5*x**5 + 4*e**(b*x)*c**5*d + 20*e**(b*x)*c**4*d**2*x + 40*e**(b*x)*c**3*d*
*3*x**2 + 40*e**(b*x)*c**2*d**4*x**3 + 20*e**(b*x)*c*d**5*x**4 + 4*e**(b*x
)*d**6*x**5),x)*a*b**2*c**4*d**2*x - 6*e**(b*x)*int(x/(e**(b*x)*b*c**6 + 5
*e**(b*x)*b*c**5*d*x + 10*e**(b*x)*b*c**4*d**2*x**2 + 10*e**(b*x)*b*c**3*d
**3*x**3 + 5*e**(b*x)*b*c**2*d**4*x**4 + e**(b*x)*b*c*d**5*x**5 + 4*e**(b*
x)*c**5*d + 20*e**(b*x)*c**4*d**2*x + 40*e**(b*x)*c**3*d**3*x**2 + 40*e**(
b*x)*c**2*d**4*x**3 + 20*e**(b*x)*c*d**5*x**4 + 4*e**(b*x)*d**6*x**5),x)*a
*b**2*c**3*d**3*x**2 - 4*e**(b*x)*int(x/(e**(b*x)*b*c**6 + 5*e**(b*x)*b*c*
*5*d*x + 10*e**(b*x)*b*c**4*d**2*x**2 + 10*e**(b*x)*b*c**3*d**3*x**3 + 5*e
**(b*x)*b*c**2*d**4*x**4 + e**(b*x)*b*c*d**5*x**5 + 4*e**(b*x)*c**5*d + 20
*e**(b*x)*c**4*d**2*x + 40*e**(b*x)*c**3*d**3*x**2 + 40*e**(b*x)*c**2*d**4
*x**3 + 20*e**(b*x)*c*d**5*x**4 + 4*e**(b*x)*d**6*x**5),x)*a*b**2*c**2*d**
4*x**3 - e**(b*x)*int(x/(e**(b*x)*b*c**6 + 5*e**(b*x)*b*c**5*d*x + 10*e...
```

3.128 $\int (c+dx)^3\Gamma(3,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 140

$$\int (c+dx)^3\Gamma(3,a+bx)\,dx = \frac{(c+dx)^4\Gamma(3,a+bx)}{4d} - \frac{d(bc-ad)^2e^{-a+\frac{bc}{d}}\Gamma\left(5,\frac{b(c+dx)}{d}\right)}{4b^4} + \frac{d^2(bc-ad)e^{-a+\frac{bc}{d}}\Gamma\left(6,\frac{b(c+dx)}{d}\right)}{2b^4} - \frac{d^3e^{-a+\frac{bc}{d}}\Gamma\left(7,\frac{b(c+dx)}{d}\right)}{4b^4}$$

output
```
1/2*(d*x+c)^4*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/d-6*d*(-a*d+b*c)^2*exp(-
a+b*c/d)*exp(-b*(d*x+c)/d)*(1+b*(d*x+c)/d+1/2*b^2*(d*x+c)^2/d^2+1/6*b^3*(d
*x+c)^3/d^3+1/24*b^4*(d*x+c)^4/d^4)/b^4+60*d^2*(-a*d+b*c)*exp(-a+b*c/d)*ex
p(-b*(d*x+c)/d)*(1+b*(d*x+c)/d+1/2*b^2*(d*x+c)^2/d^2+1/6*b^3*(d*x+c)^3/d^3
+1/24*b^4*(d*x+c)^4/d^4+1/120*b^5*(d*x+c)^5/d^5)/b^4-180*d^3*exp(-a+b*c/d)
*exp(-b*(d*x+c)/d)*(1+b*(d*x+c)/d+1/2*b^2*(d*x+c)^2/d^2+1/6*b^3*(d*x+c)^3/
d^3+1/24*b^4*(d*x+c)^4/d^4+1/120*b^5*(d*x+c)^5/d^5+1/720*b^6*(d*x+c)^6/d^6
)/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 361 vs. 2(140) = 280.

Time = 0.15 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.58

$$\int (c+dx)^3\Gamma(3,a+bx)\,dx$$
$$= \frac{e^{-a-bx}\left(-24(30+10a+a^2)\,d^3-24bd^2\left((20+8a+a^2)\,c+(30+10a+a^2)\,dx\right)-4b^3(c+dx)\left((6+4a+\right.\right.}{}$$

input
```
Integrate[(c + d*x)^3*Gamma[3, a + b*x],x]
```

output
```
(E^(-a - b*x)*(-24*(30 + 10*a + a^2)*d^3 - 24*b*d^2*((20 + 8*a + a^2)*c +
(30 + 10*a + a^2)*d*x) - 4*b^3*(c + d*x)*((6 + 4*a + a^2)*c^2 + 2*(15 + 7*
a + a^2)*c*d*x + (30 + 10*a + a^2)*d^2*x^2) - 12*b^2*d*((12 + 6*a + a^2)*c
^2 + 2*(20 + 8*a + a^2)*c*d*x + (30 + 10*a + a^2)*d^2*x^2) - b^6*x^3*(4*c^
3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 2*b^5*x^2*((6 + 4*a)*c^3 + 6*(2 +
 a)*c^2*d*x + 2*(5 + 2*a)*c*d^2*x^2 + (3 + a)*d^3*x^3) - b^4*x*(4*(6 + 4*a
 + a^2)*c^3 + 6*(12 + 6*a + a^2)*c^2*d*x + 4*(20 + 8*a + a^2)*c*d^2*x^2 +
(30 + 10*a + a^2)*d^3*x^3) + b^4*E^(a + b*x)*x*(4*c^3 + 6*c^2*d*x + 4*c*d^
2*x^2 + d^3*x^3)*Gamma[3, a + b*x]))/(4*b^4)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 529 vs. 2(140) = 280.

Time = 1.17 (sec) , antiderivative size = 529, normalized size of antiderivative = 3.78, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7119, 2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\Gamma(3,a+bx)\,dx$$
$$\Big\downarrow \; 7119$$

$$\frac{b\int e^{-a-bx}(a+bx)^2(c+dx)^4dx}{4d}+\frac{(c+dx)^4\Gamma(3,a+bx)}{4d}$$

$$\Big\downarrow \text{2626}$$

$$\frac{b\int \left(\frac{b^2e^{-a-bx}(c+dx)^6}{d^2}-\frac{2b(bc-ad)e^{-a-bx}(c+dx)^5}{d^2}+\frac{(ad-bc)^2e^{-a-bx}(c+dx)^4}{d^2}\right)dx}{4d}+\frac{(c+dx)^4\Gamma(3,a+bx)}{4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{240d^3e^{-a-bx}(bc-ad)}{b^5}-\frac{24d^2e^{-a-bx}(bc-ad)^2}{b^5}-\frac{720d^4e^{-a-bx}}{b^5}-\frac{720d^3e^{-a-bx}(c+dx)}{b^4}+\frac{240d^2e^{-a-bx}(c+dx)(bc-ad)}{b^4}-\frac{24de^{-a-bx}(c\right.}{}$$

$$\frac{(c+dx)^4\Gamma(3,a+bx)}{4d}$$

input
```
Int[(c + d*x)^3*Gamma[3, a + b*x],x]
```

output
```
(b*((-720*d^4*E^(-a - b*x))/b^5 + (240*d^3*(b*c - a*d)*E^(-a - b*x))/b^5 -
 (24*d^2*(b*c - a*d)^2*E^(-a - b*x))/b^5 - (720*d^3*E^(-a - b*x)*(c + d*x)
)/b^4 + (240*d^2*(b*c - a*d)*E^(-a - b*x)*(c + d*x))/b^4 - (24*d*(b*c - a*
d)^2*E^(-a - b*x)*(c + d*x))/b^4 - (360*d^2*E^(-a - b*x)*(c + d*x)^2)/b^3
+ (120*d*(b*c - a*d)*E^(-a - b*x)*(c + d*x)^2)/b^3 - (12*(b*c - a*d)^2*E^(
-a - b*x)*(c + d*x)^2)/b^3 - (120*d*E^(-a - b*x)*(c + d*x)^3)/b^2 + (40*(b
*c - a*d)*E^(-a - b*x)*(c + d*x)^3)/b^2 - (4*(b*c - a*d)^2*E^(-a - b*x)*(c
 + d*x)^3)/(b^2*d) - (30*E^(-a - b*x)*(c + d*x)^4)/b + (10*(b*c - a*d)*E^(
-a - b*x)*(c + d*x)^4)/(b*d) - ((b*c - a*d)^2*E^(-a - b*x)*(c + d*x)^4)/(b
*d^2) - (6*E^(-a - b*x)*(c + d*x)^5)/d + (2*(b*c - a*d)*E^(-a - b*x)*(c +
d*x)^5)/d^2 - (b*E^(-a - b*x)*(c + d*x)^6)/d^2))/(4*d) + ((c + d*x)^4*Gamm
a[3, a + b*x])/(4*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2626
```
Int[(F_)^(v_)*(Px_), x_Symbol] :> Int[ExpandIntegrand[F^v, Px, x], x] /; Fr
eeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.01

method	result
norman	$\left(-2a\,d^{3}-3d^{2}cb-7d^{3}\right)x^{4}\mathrm{e}^{-bx-a}-\frac{\left(a^{2}c^{3}b^{3}+3a^{2}b^{2}c^{2}d+4a\,b^{3}c^{3}+6a^{2}bc\,d^{2}+18c^{2}da\,b^{2}+6b^{3}c^{3}+6a^{2}d^{3}+48\right.}{b^{4}}$
gosper	$-\frac{\mathrm{e}^{-bx-a}\left(d^{3}b^{5}x^{5}+2a\,b^{4}d^{3}x^{4}+3b^{5}c\,d^{2}x^{4}+a^{2}b^{3}d^{3}x^{3}+6a\,b^{4}c\,d^{2}x^{3}+3b^{5}c^{2}d\,x^{3}+7b^{4}d^{3}x^{4}+3a^{2}b^{3}c\,d^{2}x^{2}+6a\,b^{4}c^{2}d\,x^{2}+1\right.}{}$
risch	$-\frac{\mathrm{e}^{-bx-a}\left(d^{3}b^{5}x^{5}+2a\,b^{4}d^{3}x^{4}+3b^{5}c\,d^{2}x^{4}+a^{2}b^{3}d^{3}x^{3}+6a\,b^{4}c\,d^{2}x^{3}+3b^{5}c^{2}d\,x^{3}+7b^{4}d^{3}x^{4}+3a^{2}b^{3}c\,d^{2}x^{2}+6a\,b^{4}c^{2}d\,x^{2}+1\right.}{}$
orering	$-\frac{2\left(d^{3}b^{5}x^{5}+2a\,b^{4}d^{3}x^{4}+3b^{5}c\,d^{2}x^{4}+a^{2}b^{3}d^{3}x^{3}+6a\,b^{4}c\,d^{2}x^{3}+3b^{5}c^{2}d\,x^{3}+7b^{4}d^{3}x^{4}+3a^{2}b^{3}c\,d^{2}x^{2}+6a\,b^{4}c^{2}d\,x^{2}+10a\,b^{3}d\right.}{}$
meijerg	$\frac{2\,\mathrm{e}^{-a}d^{3}\left(6-\frac{\left(4b^{3}x^{3}+12b^{2}x^{2}+24bx+24\right)\mathrm{e}^{-bx}}{4}\right)}{b^{4}}+\frac{2d^{3}\mathrm{e}^{-a}a\left(6-\frac{\left(4b^{3}x^{3}+12b^{2}x^{2}+24bx+24\right)\mathrm{e}^{-bx}}{4}\right)}{b^{4}}+\frac{d^{3}\mathrm{e}^{-a}a^{2}\left(6-\frac{\left(4b\right.}{}\right.}{}$
parallelrisch	$-\frac{24x^{2}\mathrm{e}^{-bx-a}a\,b^{3}c\,d^{2}+18x\,\mathrm{e}^{-bx-a}a\,b^{3}c^{2}d+48x\,\mathrm{e}^{-bx-a}a\,b^{2}c\,d^{2}+6x\,\mathrm{e}^{-bx-a}a^{2}b^{2}c\,d^{2}+6x^{3}\mathrm{e}^{-bx-a}a\,b^{4}c\,d^{2}+3x^{2}\mathrm{e}^{-bx-}}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(2*(d*x+c)^3*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2),x,method=_RETURNVERBOS
E)
```

output
```
(-2*a*d^3-3*b*c*d^2-7*d^3)*x^4*exp(-b*x-a)-(a^2*b^3*c^3+3*a^2*b^2*c^2*d+4*
a*b^3*c^3+6*a^2*b*c*d^2+18*a*b^2*c^2*d+6*b^3*c^3+6*a^2*d^3+48*a*b*c*d^2+36
*b^2*c^2*d+60*a*d^3+120*b*c*d^2+180*d^3)/b^4*exp(-b*x-a)-d^3*b*x^5*exp(-b*
x-a)-(3*a^2*b*c*d^2+6*a*b^2*c^2*d+b^3*c^3+3*a^2*d^3+24*a*b*c*d^2+15*b^2*c^
2*d+30*a*d^3+60*b*c*d^2+90*d^3)/b^2*x^2*exp(-b*x-a)-(3*a^2*b^2*c^2*d+2*a*b
^3*c^3+6*a^2*b*c*d^2+18*a*b^2*c^2*d+4*b^3*c^3+6*a^2*d^3+48*a*b*c*d^2+36*b^
2*c^2*d+60*a*d^3+120*b*c*d^2+180*d^3)/b^3*x*exp(-b*x-a)-d*(a^2*d^2+6*a*b*c
*d+3*b^2*c^2+10*a*d^2+18*b*c*d+30*d^2)/b*x^3*exp(-b*x-a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(129) = 258$.

Time = 0.10 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.86

$$\int (c+dx)^3\Gamma(3,a+bx)\,dx = -\frac{(b^6d^3x^6+4\,(a^2+4\,a+6)b^3c^3+12\,(a^2+6\,a+12)b^2c^2d+2\,(2\,b^6cd^2+(a+3)b^5d^3)x^5+24\,(a^2+8\,a+$$

input
```
integrate((d*x+c)^3*gamma(3,b*x+a),x, algorithm="fricas")
```

output
```
-1/4*((b^6*d^3*x^6 + 4*(a^2 + 4*a + 6)*b^3*c^3 + 12*(a^2 + 6*a + 12)*b^2*c
^2*d + 2*(2*b^6*c*d^2 + (a + 3)*b^5*d^3)*x^5 + 24*(a^2 + 8*a + 20)*b*c*d^2
 + (6*b^6*c^2*d + 4*(2*a + 5)*b^5*c*d^2 + (a^2 + 10*a + 30)*b^4*d^3)*x^4 +
 24*(a^2 + 10*a + 30)*d^3 + 4*(b^6*c^3 + 3*(a + 2)*b^5*c^2*d + (a^2 + 8*a
+ 20)*b^4*c*d^2 + (a^2 + 10*a + 30)*b^3*d^3)*x^3 + 2*(2*(2*a + 3)*b^5*c^3
+ 3*(a^2 + 6*a + 12)*b^4*c^2*d + 6*(a^2 + 8*a + 20)*b^3*c*d^2 + 6*(a^2 + 1
0*a + 30)*b^2*d^3)*x^2 + 4*((a^2 + 4*a + 6)*b^4*c^3 + 3*(a^2 + 6*a + 12)*b
^3*c^2*d + 6*(a^2 + 8*a + 20)*b^2*c*d^2 + 6*(a^2 + 10*a + 30)*b*d^3)*x)*e^
(-b*x - a) - (b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 + 4*b^4*c^3*
x)*gamma(3, b*x + a))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(342) = 684$.

Time = 0.20 (sec) , antiderivative size = 741, normalized size of antiderivative = 5.29

$$\int (c+dx)^3\Gamma(3,a+bx)\,dx$$

$$= \begin{cases} \frac{(-a^2b^3c^3-3a^2b^3c^2dx-3a^2b^3cd^2x^2-a^2b^3d^3x^3-3a^2b^2c^2d-6a^2b^2cd^2x-3a^2b^2d^3x^2-6a^2bcd^2-6a^2bd^3x-6a^2d^3-2ab^4c^3x-6ab^4c^2dx^2-6ab^4c}{} \\ \frac{b^2d^3x^6}{6}+x^5\cdot\left(\frac{2abd^3}{5}+\frac{3b^2cd^2}{5}+\frac{2bd^3}{5}\right)+x^4\left(\frac{a^2d^3}{4}+\frac{3abcd^2}{2}+\frac{ad^3}{2}+\frac{3b^2c^2d}{4}+\frac{3bcd^2}{2}+\frac{d^3}{2}\right)+x^3\Big(a^2cd^2+2abc \end{cases}$$

input
```
integrate((d*x+c)**3*uppergamma(3,b*x+a),x)
```

output
```
Piecewise(((-a**2*b**3*c**3 - 3*a**2*b**3*c**2*d*x - 3*a**2*b**3*c*d**2*x*
*2 - a**2*b**3*d**3*x**3 - 3*a**2*b**2*c**2*d - 6*a**2*b**2*c*d**2*x - 3*a
**2*b**2*d**3*x**2 - 6*a**2*b*c*d**2 - 6*a**2*b*d**3*x - 6*a**2*d**3 - 2*a
*b**4*c**3*x - 6*a*b**4*c**2*d*x**2 - 6*a*b**4*c*d**2*x**3 - 2*a*b**4*d**3
*x**4 - 4*a*b**3*c**3 - 18*a*b**3*c**2*d*x - 24*a*b**3*c*d**2*x**2 - 10*a*
b**3*d**3*x**3 - 18*a*b**2*c**2*d - 48*a*b**2*c*d**2*x - 30*a*b**2*d**3*x*
*2 - 48*a*b*c*d**2 - 60*a*b*d**3*x - 60*a*d**3 - b**5*c**3*x**2 - 3*b**5*c
**2*d*x**3 - 3*b**5*c*d**2*x**4 - b**5*d**3*x**5 - 4*b**4*c**3*x - 15*b**4
*c**2*d*x**2 - 18*b**4*c*d**2*x**3 - 7*b**4*d**3*x**4 - 6*b**3*c**3 - 36*b
**3*c**2*d*x - 60*b**3*c*d**2*x**2 - 30*b**3*d**3*x**3 - 36*b**2*c**2*d -
120*b**2*c*d**2*x - 90*b**2*d**3*x**2 - 120*b*c*d**2 - 180*b*d**3*x - 180*
d**3)*exp(-a - b*x)/b**4, Ne(b**4, 0)), (b**2*d**3*x**6/6 + x**5*(2*a*b*d*
*3/5 + 3*b**2*c*d**2/5 + 2*b*d**3/5) + x**4*(a**2*d**3/4 + 3*a*b*c*d**2/2
+ a*d**3/2 + 3*b**2*c**2*d/4 + 3*b*c*d**2/2 + d**3/2) + x**3*(a**2*c*d**2
+ 2*a*b*c**2*d + 2*a*c*d**2 + b**2*c**3/3 + 2*b*c**2*d + 2*c*d**2) + x**2*
(3*a**2*c**2*d/2 + a*b*c**3 + 3*a*c**2*d + b*c**3 + 3*c**2*d) + x*(a**2*c*
*3 + 2*a*c**3 + 2*c**3), True))
```

Maxima [F]

$$\int (c+dx)^3\Gamma(3,a+bx)\,dx = \int (dx+c)^3\Gamma(3,bx+a)\,dx$$

input
```
integrate((d*x+c)^3*gamma(3,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(3, b*x + a) - gamma(4, b*x + a))*c^3/b + integrate(d^3*x^
3*gamma(3, b*x + a) + 3*c*d^2*x^2*gamma(3, b*x + a) + 3*c^2*d*x*gamma(3, b
*x + a), x)
```

Giac [F]

$$\int (c+dx)^3\Gamma(3,a+bx)\,dx = \int (dx+c)^3\Gamma(3,bx+a)\,dx$$

input
```
integrate((d*x+c)^3*gamma(3,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*gamma(3, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.94

$$\int (c+dx)^3\Gamma(3,a+bx)\,dx$$
$$= -x^3\,\mathrm{e}^{-a-b\,x}\left(\frac{a^2\,d^3+10\,a\,d^3+30\,d^3}{b}+18\,c\,d^2+6\,a\,c\,d^2+3\,b\,c^2\,d\right)$$
$$-\frac{\mathrm{e}^{-a-b\,x}\left(b^3\left(a^2\,c^3+4\,a\,c^3+6\,c^3\right)+b^2\left(3\,d\,a^2\,c^2+18\,d\,a\,c^2+36\,d\,c^2\right)+60\,a\,d^3+180\,d^3+b\left(6\,c\,a^2\,d^2+\right.\right.}{b^4}$$
$$-b\,d^3\,x^5\,\mathrm{e}^{-a-b\,x}-d^2\,x^4\,\mathrm{e}^{-a-b\,x}\left(7\,d+2\,a\,d+3\,b\,c\right)$$
$$-\frac{x\,\mathrm{e}^{-a-b\,x}\left(3\,a^2\,b^2\,c^2\,d+6\,a^2\,b\,c\,d^2+6\,a^2\,d^3+2\,a\,b^3\,c^3+18\,a\,b^2\,c^2\,d+48\,a\,b\,c\,d^2+60\,a\,d^3+4\,b^3\,c^3+36\right.}{b^3}$$
$$-\frac{x^2\,\mathrm{e}^{-a-b\,x}\left(3\,a^2\,b\,c\,d^2+3\,a^2\,d^3+6\,a\,b^2\,c^2\,d+24\,a\,b\,c\,d^2+30\,a\,d^3+b^3\,c^3+15\,b^2\,c^2\,d+60\,b\,c\,d^2+90\,d^3\right)}{b^2}$$

input
```
int(2*exp(- a - b*x)*(c + d*x)^3*(a + b*x + (a + b*x)^2/2 + 1),x)
```

output
```
- x^3*exp(- a - b*x)*((10*a*d^3 + 30*d^3 + a^2*d^3)/b + 18*c*d^2 + 6*a*c*d
^2 + 3*b*c^2*d) - (exp(- a - b*x)*(b^3*(4*a*c^3 + 6*c^3 + a^2*c^3) + b^2*(
36*c^2*d + 3*a^2*c^2*d + 18*a*c^2*d) + 60*a*d^3 + 180*d^3 + b*(120*c*d^2 +
 6*a^2*c*d^2 + 48*a*c*d^2) + 6*a^2*d^3))/b^4 - b*d^3*x^5*exp(- a - b*x) -
d^2*x^4*exp(- a - b*x)*(7*d + 2*a*d + 3*b*c) - (x*exp(- a - b*x)*(60*a*d^3
 + 180*d^3 + 6*a^2*d^3 + 4*b^3*c^3 + 2*a*b^3*c^3 + 36*b^2*c^2*d + 120*b*c*
d^2 + 3*a^2*b^2*c^2*d + 48*a*b*c*d^2 + 18*a*b^2*c^2*d + 6*a^2*b*c*d^2))/b^
3 - (x^2*exp(- a - b*x)*(30*a*d^3 + 90*d^3 + 3*a^2*d^3 + b^3*c^3 + 15*b^2*
c^2*d + 60*b*c*d^2 + 24*a*b*c*d^2 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 462, normalized size of antiderivative = 3.30

$$\int (c+dx)^3\Gamma(3,a+bx)\,dx$$
$$= \frac{-b^5d^3x^5 - 2a\,b^4d^3x^4 - 3b^5c\,d^2x^4 - a^2b^3d^3x^3 - 6a\,b^4c\,d^2x^3 - 3b^5c^2d\,x^3 - 7b^4d^3x^4 - 3a^2b^3c\,d^2x^2 - 6a\,b^4c^2}{}$$

input
```
int(2*(d*x+c)^3*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2),x)
```

output
```
( - a**2*b**3*c**3 - 3*a**2*b**3*c**2*d*x - 3*a**2*b**3*c*d**2*x**2 - a**2
*b**3*d**3*x**3 - 3*a**2*b**2*c**2*d - 6*a**2*b**2*c*d**2*x - 3*a**2*b**2*
d**3*x**2 - 6*a**2*b*c*d**2 - 6*a**2*b*d**3*x - 6*a**2*d**3 - 2*a*b**4*c**
3*x - 6*a*b**4*c**2*d*x**2 - 6*a*b**4*c*d**2*x**3 - 2*a*b**4*d**3*x**4 - 4
*a*b**3*c**3 - 18*a*b**3*c**2*d*x - 24*a*b**3*c*d**2*x**2 - 10*a*b**3*d**3
*x**3 - 18*a*b**2*c**2*d - 48*a*b**2*c*d**2*x - 30*a*b**2*d**3*x**2 - 48*a
*b*c*d**2 - 60*a*b*d**3*x - 60*a*d**3 - b**5*c**3*x**2 - 3*b**5*c**2*d*x**
3 - 3*b**5*c*d**2*x**4 - b**5*d**3*x**5 - 4*b**4*c**3*x - 15*b**4*c**2*d*x
**2 - 18*b**4*c*d**2*x**3 - 7*b**4*d**3*x**4 - 6*b**3*c**3 - 36*b**3*c**2*
d*x - 60*b**3*c*d**2*x**2 - 30*b**3*d**3*x**3 - 36*b**2*c**2*d - 120*b**2*
c*d**2*x - 90*b**2*d**3*x**2 - 120*b*c*d**2 - 180*b*d**3*x - 180*d**3)/(e*
*(a + b*x)*b**4)
```

3.129 $\int (c+dx)^2\Gamma(3,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 137

$$\int (c+dx)^2\Gamma(3,a+bx)\,dx=\frac{(c+dx)^3\Gamma(3,a+bx)}{3d}-\frac{(bc-ad)^2e^{-a+\frac{bc}{d}}\Gamma\left(4,\frac{b(c+dx)}{d}\right)}{3b^3}+\frac{2d(bc-ad)e^{-a+\frac{bc}{d}}\Gamma\left(5,\frac{b(c+dx)}{d}\right)}{3b^3}-\frac{d^2e^{-a+\frac{bc}{d}}\Gamma\left(6,\frac{b(c+dx)}{d}\right)}{3b^3}$$

output
```
2/3*(d*x+c)^3*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/d-2*(-a*d+b*c)^2*exp(-a+
b*c/d)*exp(-b*(d*x+c)/d)*(1+b*(d*x+c)/d+1/2*b^2*(d*x+c)^2/d^2+1/6*b^3*(d*x
+c)^3/d^3)/b^3+16*d*(-a*d+b*c)*exp(-a+b*c/d)*exp(-b*(d*x+c)/d)*(1+b*(d*x+c
)/d+1/2*b^2*(d*x+c)^2/d^2+1/6*b^3*(d*x+c)^3/d^3+1/24*b^4*(d*x+c)^4/d^4)/b^
3-40*d^2*exp(-a+b*c/d)*exp(-b*(d*x+c)/d)*(1+b*(d*x+c)/d+1/2*b^2*(d*x+c)^2/
d^2+1/6*b^3*(d*x+c)^3/d^3+1/24*b^4*(d*x+c)^4/d^4+1/120*b^5*(d*x+c)^5/d^5)/
b^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.84

$$\int (c+dx)^2\Gamma(3,a+bx)\,dx$$
$$=\frac{e^{-a-bx}\left(-6(20+8a+a^2)\,d^2-6bd((12+6a+a^2)\,c+(20+8a+a^2)\,dx)-b^5x^3(3c^2+3cdx+d^2x^2)-b\right.}{}$$

input
```
Integrate[(c + d*x)^2*Gamma[3, a + b*x],x]
```

output
```
(E^(-a - b*x)*(-6*(20 + 8*a + a^2)*d^2 - 6*b*d*((12 + 6*a + a^2)*c + (20 +
 8*a + a^2)*d*x) - b^5*x^3*(3*c^2 + 3*c*d*x + d^2*x^2) - b^4*x^2*((9 + 6*a
)*c^2 + 6*(2 + a)*c*d*x + (5 + 2*a)*d^2*x^2) - 3*b^2*((6 + 4*a + a^2)*c^2
+ 2*(12 + 6*a + a^2)*c*d*x + (20 + 8*a + a^2)*d^2*x^2) - b^3*x*(3*(6 + 4*a
 + a^2)*c^2 + 3*(12 + 6*a + a^2)*c*d*x + (20 + 8*a + a^2)*d^2*x^2) + b^3*E
^(a + b*x)*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Gamma[3, a + b*x]))/(3*b^3)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 436 vs. 2(137) = 274.

Time = 0.95 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7119, 2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\Gamma(3,a+bx)\,dx$$
$$\Big\downarrow \text{7119}$$
$$\frac{b\int e^{-a-bx}(a+bx)^2(c+dx)^3dx}{3d}+\frac{(c+dx)^3\Gamma(3,a+bx)}{3d}$$
$$\Big\downarrow \text{2626}$$

$$\frac{b\int \left(\frac{b^2e^{-a-bx}(c+dx)^5}{d^2}-\frac{2b(bc-ad)e^{-a-bx}(c+dx)^4}{d^2}+\frac{(ad-bc)^2e^{-a-bx}(c+dx)^3}{d^2}\right)dx}{3d}+\frac{(c+dx)^3\Gamma(3,a+bx)}{3d}$$

$\big\downarrow$ 2009

$$\frac{b\left(\frac{48d^2e^{-a-bx}(bc-ad)}{b^4}-\frac{6de^{-a-bx}(bc-ad)^2}{b^4}-\frac{120d^3e^{-a-bx}}{b^4}-\frac{120d^2e^{-a-bx}(c+dx)}{b^3}-\frac{6e^{-a-bx}(c+dx)(bc-ad)^2}{b^3}+\frac{48de^{-a-bx}(c+dx)(}{b^3}\right.}{}$$

$$\frac{(c+dx)^3\Gamma(3,a+bx)}{3d}$$

input
```
Int[(c + d*x)^2*Gamma[3, a + b*x],x]
```

output
```
(b*((-120*d^3*E^(-a - b*x))/b^4 + (48*d^2*(b*c - a*d)*E^(-a - b*x))/b^4 -
(6*d*(b*c - a*d)^2*E^(-a - b*x))/b^4 - (120*d^2*E^(-a - b*x)*(c + d*x))/b^
3 + (48*d*(b*c - a*d)*E^(-a - b*x)*(c + d*x))/b^3 - (6*(b*c - a*d)^2*E^(-a
 - b*x)*(c + d*x))/b^3 - (60*d*E^(-a - b*x)*(c + d*x)^2)/b^2 + (24*(b*c -
a*d)*E^(-a - b*x)*(c + d*x)^2)/b^2 - (3*(b*c - a*d)^2*E^(-a - b*x)*(c + d*
x)^2)/(b^2*d) - (20*E^(-a - b*x)*(c + d*x)^3)/b + (8*(b*c - a*d)*E^(-a - b
*x)*(c + d*x)^3)/(b*d) - ((b*c - a*d)^2*E^(-a - b*x)*(c + d*x)^3)/(b*d^2)
- (5*E^(-a - b*x)*(c + d*x)^4)/d + (2*(b*c - a*d)*E^(-a - b*x)*(c + d*x)^4
)/d^2 - (b*E^(-a - b*x)*(c + d*x)^5)/d^2))/(3*d) + ((c + d*x)^3*Gamma[3, a
 + b*x])/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2626
```
Int[(F_)^(v_)*(Px_), x_Symbol] :> Int[ExpandIntegrand[F^v, Px, x], x] /; Fr
eeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.84

method	result
norman	$\left(-2a\,d^{2}-2bcd-6d^{2}\right)x^{3}\mathrm{e}^{-bx-a}-\frac{\left(a^{2}c^{2}b^{2}+2a^{2}bcd+4a\,c^{2}b^{2}+2a^{2}d^{2}+12abcd+6b^{2}c^{2}+16a\,d^{2}+24bcd+40\cdots\right.}{b^{3}}$
gosper	$-\frac{\mathrm{e}^{-bx-a}\left(b^{4}d^{2}x^{4}+2a\,b^{3}d^{2}x^{3}+2b^{4}cd\,x^{3}+a^{2}b^{2}d^{2}x^{2}+4a\,b^{3}cd\,x^{2}+b^{4}c^{2}x^{2}+6b^{3}d^{2}x^{3}+2a^{2}b^{2}cdx+2a\,b^{3}c^{2}x+8a\,b^{2}d^{2}x^{2}+\cdots\right.}{\cdots}$
risch	$-\frac{\mathrm{e}^{-bx-a}\left(b^{4}d^{2}x^{4}+2a\,b^{3}d^{2}x^{3}+2b^{4}cd\,x^{3}+a^{2}b^{2}d^{2}x^{2}+4a\,b^{3}cd\,x^{2}+b^{4}c^{2}x^{2}+6b^{3}d^{2}x^{3}+2a^{2}b^{2}cdx+2a\,b^{3}c^{2}x+8a\,b^{2}d^{2}x^{2}+\cdots\right.}{\cdots}$
orering	$-\frac{2\left(b^{4}d^{2}x^{4}+2a\,b^{3}d^{2}x^{3}+2b^{4}cd\,x^{3}+a^{2}b^{2}d^{2}x^{2}+4a\,b^{3}cd\,x^{2}+b^{4}c^{2}x^{2}+6b^{3}d^{2}x^{3}+2a^{2}b^{2}cdx+2a\,b^{3}c^{2}x+8a\,b^{2}d^{2}x^{2}+10b^{3}c\cdots\right.}{\cdots}$
parallelrisch	$-\frac{12x\,\mathrm{e}^{-bx-a}a\,b^{2}cd+4x^{2}\mathrm{e}^{-bx-a}a\,b^{3}cd+2x\,\mathrm{e}^{-bx-a}a^{2}b^{2}cd+10x^{2}\mathrm{e}^{-bx-a}b^{3}cd+20x^{2}\mathrm{e}^{-bx-a}d^{2}b^{2}+40x\,\mathrm{e}^{-bx-a}b\,d^{2}+24\cdots}{\cdots}$
meijerg	$\frac{2d^{2}\mathrm{e}^{-a}\left(2-\frac{\left(3b^{2}x^{2}+6bx+6\right)\mathrm{e}^{-bx}}{3}\right)}{b^{3}}+\frac{2d^{2}\mathrm{e}^{-a}a\left(2-\frac{\left(3b^{2}x^{2}+6bx+6\right)\mathrm{e}^{-bx}}{3}\right)}{b^{3}}+\frac{d^{2}\mathrm{e}^{-a}a^{2}\left(2-\frac{\left(3b^{2}x^{2}+6bx+6\right)\mathrm{e}^{-bx}}{3}\right)}{b^{3}}\cdots$
parts	$-b\,d^{2}x^{4}\mathrm{e}^{-bx-a}-2\,\mathrm{e}^{-bx-a}a\,d^{2}x^{3}-2\,\mathrm{e}^{-bx-a}bcd\,x^{3}-\frac{\mathrm{e}^{-bx-a}d^{2}x^{2}a^{2}}{b}-4\,\mathrm{e}^{-bx-a}acd\,x^{2}-\mathrm{e}^{\cdots}$
derivativedivides	$-\frac{c^{2}\left(\left(-bx-a\right)^{2}\mathrm{e}^{-bx-a}-2\left(-bx-a\right)\mathrm{e}^{-bx-a}+2\,\mathrm{e}^{-bx-a}\right)+\frac{d^{2}\left(\mathrm{e}^{-bx-a}\left(-bx-a\right)^{4}-4\,\mathrm{e}^{-bx-a}\left(-bx-a\right)^{3}+12\left(-bx-a\right)^{2}\mathrm{e}^{-bx-\cdots}\right.}{b^{2}}}{\cdots}$
default	$-\frac{c^{2}\left(\left(-bx-a\right)^{2}\mathrm{e}^{-bx-a}-2\left(-bx-a\right)\mathrm{e}^{-bx-a}+2\,\mathrm{e}^{-bx-a}\right)+\frac{d^{2}\left(\mathrm{e}^{-bx-a}\left(-bx-a\right)^{4}-4\,\mathrm{e}^{-bx-a}\left(-bx-a\right)^{3}+12\left(-bx-a\right)^{2}\mathrm{e}^{-bx-\cdots}\right.}{b^{2}}}{\cdots}$

input
```
int(2*(d*x+c)^2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2),x,method=_RETURNVERBOS
E)
```

output
```
(-2*a*d^2-2*b*c*d-6*d^2)*x^3*exp(-b*x-a)-(a^2*b^2*c^2+2*a^2*b*c*d+4*a*b^2*
c^2+2*a^2*d^2+12*a*b*c*d+6*b^2*c^2+16*a*d^2+24*b*c*d+40*d^2)/b^3*exp(-b*x-
a)-b*d^2*x^4*exp(-b*x-a)-(a^2*d^2+4*a*b*c*d+b^2*c^2+8*a*d^2+10*b*c*d+20*d^
2)/b*x^2*exp(-b*x-a)-2*(a^2*b*c*d+a*b^2*c^2+a^2*d^2+6*a*b*c*d+2*b^2*c^2+8*
a*d^2+12*b*c*d+20*d^2)/b^2*x*exp(-b*x-a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(126) = 252$.

Time = 0.09 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.93

$$\int (c+dx)^2\Gamma(3,a+bx)\,dx =$$
$$-\frac{(b^5d^2x^5+3\,(a^2+4\,a+6)b^2c^2+(3\,b^5cd+(2\,a+5)b^4d^2)x^4+6\,(a^2+6\,a+12)bcd+(3\,b^5c^2+6\,(a+2$$

input
```
integrate((d*x+c)^2*gamma(3,b*x+a),x, algorithm="fricas")
```

output
```
-1/3*((b^5*d^2*x^5 + 3*(a^2 + 4*a + 6)*b^2*c^2 + (3*b^5*c*d + (2*a + 5)*b^
4*d^2)*x^4 + 6*(a^2 + 6*a + 12)*b*c*d + (3*b^5*c^2 + 6*(a + 2)*b^4*c*d + (
a^2 + 8*a + 20)*b^3*d^2)*x^3 + 6*(a^2 + 8*a + 20)*d^2 + 3*((2*a + 3)*b^4*c
^2 + (a^2 + 6*a + 12)*b^3*c*d + (a^2 + 8*a + 20)*b^2*d^2)*x^2 + 3*((a^2 +
4*a + 6)*b^3*c^2 + 2*(a^2 + 6*a + 12)*b^2*c*d + 2*(a^2 + 8*a + 20)*b*d^2)*
x)*e^(-b*x - a) - (b^3*d^2*x^3 + 3*b^3*c*d*x^2 + 3*b^3*c^2*x)*gamma(3, b*x
 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 447, normalized size of antiderivative = 3.26

$$\int (c+dx)^2\Gamma(3,a+bx)\,dx$$
$$= \begin{cases} \frac{(-a^2b^2c^2-2a^2b^2cdx-a^2b^2d^2x^2-2a^2bcd-2a^2bd^2x-2a^2d^2-2ab^3c^2x-4ab^3cdx^2-2ab^3d^2x^3-4ab^2c^2-12ab^2cdx-8ab^2d^2x^2-12abcd-16abd^2}{b^3} \\ \frac{b^2d^2x^5}{5}+x^4\left(\frac{abd^2}{2}+\frac{b^2cd}{2}+\frac{bd^2}{2}\right)+x^3\left(\frac{a^2d^2}{3}+\frac{4abcd}{3}+\frac{2ad^2}{3}+\frac{b^2c^2}{3}+\frac{4bcd}{3}+\frac{2d^2}{3}\right)+x^2(a^2cd+abc^2+2acd\end{cases}$$

input
```
integrate((d*x+c)**2*uppergamma(3,b*x+a),x)
```

output
```
Piecewise(((-a**2*b**2*c**2 - 2*a**2*b**2*c*d*x - a**2*b**2*d**2*x**2 - 2*
a**2*b*c*d - 2*a**2*b*d**2*x - 2*a**2*d**2 - 2*a*b**3*c**2*x - 4*a*b**3*c*
d*x**2 - 2*a*b**3*d**2*x**3 - 4*a*b**2*c**2 - 12*a*b**2*c*d*x - 8*a*b**2*d
**2*x**2 - 12*a*b*c*d - 16*a*b*d**2*x - 16*a*d**2 - b**4*c**2*x**2 - 2*b**
4*c*d*x**3 - b**4*d**2*x**4 - 4*b**3*c**2*x - 10*b**3*c*d*x**2 - 6*b**3*d*
*2*x**3 - 6*b**2*c**2 - 24*b**2*c*d*x - 20*b**2*d**2*x**2 - 24*b*c*d - 40*
b*d**2*x - 40*d**2)*exp(-a - b*x)/b**3, Ne(b**3, 0)), (b**2*d**2*x**5/5 +
x**4*(a*b*d**2/2 + b**2*c*d/2 + b*d**2/2) + x**3*(a**2*d**2/3 + 4*a*b*c*d/
3 + 2*a*d**2/3 + b**2*c**2/3 + 4*b*c*d/3 + 2*d**2/3) + x**2*(a**2*c*d + a*
b*c**2 + 2*a*c*d + b*c**2 + 2*c*d) + x*(a**2*c**2 + 2*a*c**2 + 2*c**2), Tr
ue))
```

Maxima [F]

$$\int (c+dx)^2\Gamma(3,a+bx)\,dx=\int (dx+c)^2\Gamma(3,bx+a)\,dx$$

input
```
integrate((d*x+c)^2*gamma(3,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(3, b*x + a) - gamma(4, b*x + a))*c^2/b + integrate(d^2*x^
2*gamma(3, b*x + a) + 2*c*d*x*gamma(3, b*x + a), x)
```

Giac [F]

$$\int (c+dx)^2\Gamma(3,a+bx)\,dx=\int (dx+c)^2\Gamma(3,bx+a)\,dx$$

input
```
integrate((d*x+c)^2*gamma(3,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*gamma(3, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.74

$$\int (c+dx)^2\Gamma(3,a+bx)\,dx$$
$$= -x\,\mathrm{e}^{-a-b\,x}\left(2\,a\,c^2+\frac{16\,a\,d^2+b\,(2\,c\,d\,a^2+12\,c\,d\,a+24\,c\,d)+40\,d^2+2\,a^2\,d^2}{b^2}+4\,c^2\right)$$
$$-\frac{\mathrm{e}^{-a-b\,x}\,(b^2\,(a^2\,c^2+4\,a\,c^2+6\,c^2)+16\,a\,d^2+b\,(2\,c\,d\,a^2+12\,c\,d\,a+24\,c\,d)+40\,d^2+2\,a^2\,d^2)}{b^3}$$
$$-x^2\,\mathrm{e}^{-a-b\,x}\left(\frac{a^2\,d^2+8\,a\,d^2+20\,d^2}{b}+10\,c\,d+b\,c^2+4\,a\,c\,d\right)$$
$$-b\,d^2\,x^4\,\mathrm{e}^{-a-b\,x}-2\,d\,x^3\,\mathrm{e}^{-a-b\,x}\,(3\,d+a\,d+b\,c)$$

input
```
int(2*exp(- a - b*x)*(c + d*x)^2*(a + b*x + (a + b*x)^2/2 + 1),x)
```

output
```
- x*exp(- a - b*x)*(2*a*c^2 + (16*a*d^2 + b*(24*c*d + 12*a*c*d + 2*a^2*c*d
) + 40*d^2 + 2*a^2*d^2)/b^2 + 4*c^2) - (exp(- a - b*x)*(b^2*(4*a*c^2 + 6*c
^2 + a^2*c^2) + 16*a*d^2 + b*(24*c*d + 12*a*c*d + 2*a^2*c*d) + 40*d^2 + 2*
a^2*d^2))/b^3 - x^2*exp(- a - b*x)*((8*a*d^2 + 20*d^2 + a^2*d^2)/b + 10*c*
d + b*c^2 + 4*a*c*d) - b*d^2*x^4*exp(- a - b*x) - 2*d*x^3*exp(- a - b*x)*(
3*d + a*d + b*c)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.93

$$\int (c+dx)^2\Gamma(3,a+bx)\,dx$$
$$= \frac{-b^4d^2x^4-2a\,b^3d^2x^3-2b^4cd\,x^3-a^2b^2d^2x^2-4a\,b^3cd\,x^2-b^4c^2x^2-6b^3d^2x^3-2a^2b^2cdx-2a\,b^3c^2x-8a}{}$$

input
```
int(2*(d*x+c)^2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2),x)
```

output
```
( - a**2*b**2*c**2 - 2*a**2*b**2*c*d*x - a**2*b**2*d**2*x**2 - 2*a**2*b*c*
d - 2*a**2*b*d**2*x - 2*a**2*d**2 - 2*a*b**3*c**2*x - 4*a*b**3*c*d*x**2 -
2*a*b**3*d**2*x**3 - 4*a*b**2*c**2 - 12*a*b**2*c*d*x - 8*a*b**2*d**2*x**2
- 12*a*b*c*d - 16*a*b*d**2*x - 16*a*d**2 - b**4*c**2*x**2 - 2*b**4*c*d*x**
3 - b**4*d**2*x**4 - 4*b**3*c**2*x - 10*b**3*c*d*x**2 - 6*b**3*d**2*x**3 -
 6*b**2*c**2 - 24*b**2*c*d*x - 20*b**2*d**2*x**2 - 24*b*c*d - 40*b*d**2*x
- 40*d**2)/(e**(a + b*x)*b**3)
```

3.130 $\int (c+dx)\Gamma(3,a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int (c+dx)\Gamma(3,a+bx)\,dx = -\frac{(bc-ad)^2\Gamma(3,a+bx)}{2b^2d}+\frac{(c+dx)^2\Gamma(3,a+bx)}{2d} - \frac{(bc-ad)\Gamma(4,a+bx)}{b^2}-\frac{d\Gamma(5,a+bx)}{2b^2}$$

output
```
-(-a*d+b*c)^2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/b^2/d+(d*x+c)^2*exp(-b*x
-a)*(1+b*x+a+1/2*(b*x+a)^2)/d-6*(-a*d+b*c)*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a
)^2+1/6*(b*x+a)^3)/b^2-12*d*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2+1/6*(b*x+a)
^3+1/24*(b*x+a)^4)/b^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.74

$$\int (c+dx)\Gamma(3,a+bx)\,dx$$
$$= \frac{e^{-a-bx}\left(-2(12+6a+a^2)\,d-b^4x^3(2c+dx)-2b^3x^2((3+2a)c+(2+a)dx)-2b((6+4a+a^2)\,c+(12\right.}{2b^2}$$

input
```
Integrate[(c + d*x)*Gamma[3, a + b*x],x]
```

output
```
(E^(-a - b*x)*(-2*(12 + 6*a + a^2)*d - b^4*x^3*(2*c + d*x) - 2*b^3*x^2*((3
 + 2*a)*c + (2 + a)*d*x) - 2*b*((6 + 4*a + a^2)*c + (12 + 6*a + a^2)*d*x)
- b^2*x*(2*(6 + 4*a + a^2)*c + (12 + 6*a + a^2)*d*x) + b^2*E^(a + b*x)*x*(
2*c + d*x)*Gamma[3, a + b*x]))/(2*b^2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 349 vs. $2(84) = 168$.

Time = 0.81 (sec) , antiderivative size = 349, normalized size of antiderivative = 4.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7119, 2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\Gamma(3,a+bx)\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{b\int e^{-a-bx}(a+bx)^2(c+dx)^2dx}{2d}+\frac{(c+dx)^2\Gamma(3,a+bx)}{2d}$$

$$\Big\downarrow \text{2626}$$

$$\frac{b\int \left(\frac{d^2e^{-a-bx}(a+bx)^4}{b^2}+\frac{2d(bc-ad)e^{-a-bx}(a+bx)^3}{b^2}+\frac{(bc-ad)^2e^{-a-bx}(a+bx)^2}{b^2}\right)dx}{2d}+$$
$$\frac{(c+dx)^2\Gamma(3,a+bx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$b\Big(-\frac{2de^{-a-bx}(a+bx)^3(bc-ad)}{b^3}-\frac{e^{-a-bx}(a+bx)^2(bc-ad)^2}{b^3}-\frac{6de^{-a-bx}(a+bx)^2(bc-ad)}{b^3}-\frac{2e^{-a-bx}(a+bx)(bc-ad)^2}{b^3}-\frac{12de^{-a-bx}(a+bx}{b^3}$$

$$\frac{(c+dx)^2\Gamma(3,a+bx)}{2d}$$

input
```
Int[(c + d*x)*Gamma[3, a + b*x],x]
```

output
```
(b*((-24*d^2*E^(-a - b*x))/b^3 - (12*d*(b*c - a*d)*E^(-a - b*x))/b^3 - (2*
(b*c - a*d)^2*E^(-a - b*x))/b^3 - (24*d^2*E^(-a - b*x)*(a + b*x))/b^3 - (1
2*d*(b*c - a*d)*E^(-a - b*x)*(a + b*x))/b^3 - (2*(b*c - a*d)^2*E^(-a - b*x
)*(a + b*x))/b^3 - (12*d^2*E^(-a - b*x)*(a + b*x)^2)/b^3 - (6*d*(b*c - a*d
)*E^(-a - b*x)*(a + b*x)^2)/b^3 - ((b*c - a*d)^2*E^(-a - b*x)*(a + b*x)^2)
/b^3 - (4*d^2*E^(-a - b*x)*(a + b*x)^3)/b^3 - (2*d*(b*c - a*d)*E^(-a - b*x
)*(a + b*x)^3)/b^3 - (d^2*E^(-a - b*x)*(a + b*x)^4)/b^3))/(2*d) + ((c + d*
x)^2*Gamma[3, a + b*x])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2626
```
Int[(F_)^(v_)*(Px_), x_Symbol] :> Int[ExpandIntegrand[F^v, Px, x], x] /; Fr
eeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.32

method	result
gosper	$-\frac{e^{-bx-a}(b^3dx^3+2ab^2dx^2+b^3cx^2+ba^2dx+2ab^2cx+5b^2dx^2+ba^2c+6abdx+4b^2cx+da^2+4abc+12bdx+6ad+6cb+}{b^2}$
risch	$-\frac{e^{-bx-a}(b^3dx^3+2ab^2dx^2+b^3cx^2+ba^2dx+2ab^2cx+5b^2dx^2+ba^2c+6abdx+4b^2cx+da^2+4abc+12bdx+6ad+6cb+}{b^2}$
norman	$(-2ad-cb-5d)x^2e^{-bx-a}-\frac{(ba^2c+da^2+4abc+6ad+6cb+12d)e^{-bx-a}}{b^2}-bdx^3e^{-bx-a}-\frac{(da^2+2a}{}$
orering	$-\frac{2(b^3dx^3+2ab^2dx^2+b^3cx^2+ba^2dx+2ab^2cx+5b^2dx^2+ba^2c+6abdx+4b^2cx+da^2+4abc+12bdx+6ad+6cb+12d)e^{}}{b^2(b^2x^2+2bxa+a^2+2bx+2a+2)}$
parallelrisch	$-\frac{b^3de^{-bx-a}x^3+2x^2e^{-bx-a}ab^2d+x^2e^{-bx-a}b^3c+5dx^2e^{-bx-a}b^2+xe^{-bx-a}a^2bd+2xe^{-bx-a}ab^2c+6xe^{-bx-a}abd+}{b^2}$
meijerg	$\frac{2de^{-a}\left(1-\frac{(2bx+2)e^{-bx}}{2}\right)}{b^2}+\frac{2de^{-a}a\left(1-\frac{(2bx+2)e^{-bx}}{2}\right)}{b^2}+\frac{de^{-a}a^2\left(1-\frac{(2bx+2)e^{-bx}}{2}\right)}{b^2}+\frac{de^{-a}\left(6-\frac{(4b^3x^3+12b^2}{}\right.}{b^2}$
derivativedivides	$-\frac{c\left((-bx-a)^2e^{-bx-a}-2(-bx-a)e^{-bx-a}+2e^{-bx-a}\right)+2e^{-bx-a}c-2c\left((-bx-a)e^{-bx-a}-e^{-bx-a}\right)-\frac{2e^{-bx-a}da}{b}-\frac{2d}{}}{}$
default	$-\frac{c\left((-bx-a)^2e^{-bx-a}-2(-bx-a)e^{-bx-a}+2e^{-bx-a}\right)+2e^{-bx-a}c-2c\left((-bx-a)e^{-bx-a}-e^{-bx-a}\right)-\frac{2e^{-bx-a}da}{b}-\frac{2d}{}}{}$
parts	$-bdx^3e^{-bx-a}-2e^{-bx-a}adx^2-e^{-bx-a}bcx^2-\frac{e^{-bx-a}a^2dx}{b}-2e^{-bx-a}acx-2e^{-bx-a}dx^2$

input
```
int(2*(d*x+c)*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
-exp(-b*x-a)*(b^3*d*x^3+2*a*b^2*d*x^2+b^3*c*x^2+a^2*b*d*x+2*a*b^2*c*x+5*b^
2*d*x^2+a^2*b*c+6*a*b*d*x+4*b^2*c*x+a^2*d+4*a*b*c+12*b*d*x+6*a*d+6*b*c+12*
d)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.77

$$\int (c+dx)\Gamma(3,a+bx)\,dx =$$

$$-\frac{(b^4dx^4+2\,(b^4c+(a+2)b^3d)x^3+2\,(a^2+4\,a+6)bc+(2\,(2\,a+3)b^3c+(a^2+6\,a+12)b^2d)x^2+2\,(a^2}{2\,b^2}$$

input
```
integrate((d*x+c)*gamma(3,b*x+a),x, algorithm="fricas")
```

output
```
-1/2*((b^4*d*x^4 + 2*(b^4*c + (a + 2)*b^3*d)*x^3 + 2*(a^2 + 4*a + 6)*b*c +
 (2*(2*a + 3)*b^3*c + (a^2 + 6*a + 12)*b^2*d)*x^2 + 2*(a^2 + 6*a + 12)*d +
 2*((a^2 + 4*a + 6)*b^2*c + (a^2 + 6*a + 12)*b*d)*x)*e^(-b*x - a) - (b^2*d
*x^2 + 2*b^2*c*x)*gamma(3, b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.40

$$\int (c+dx)\Gamma(3,a+bx)\,dx = \begin{cases} \frac{(-a^2bc-a^2bdx-a^2d-2ab^2cx-2ab^2dx^2-4abc-6abdx-6ad-b^3cx^2-b^3dx^3-4b^2cx-5b^2dx^2-6bc-12bdx-12d)e^{-a-bx}}{b^2} & \text{for } b^2\neq 0 \\ \frac{b^2dx^4}{4}+x^3\cdot\left(\frac{2abd}{3}+\frac{b^2c}{3}+\frac{2bd}{3}\right)+x^2\left(\frac{a^2d}{2}+abc+ad+bc+d\right)+x(a^2c+2ac+2c) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*uppergamma(3,b*x+a),x)
```

output
```
Piecewise(((-a**2*b*c - a**2*b*d*x - a**2*d - 2*a*b**2*c*x - 2*a*b**2*d*x*
*2 - 4*a*b*c - 6*a*b*d*x - 6*a*d - b**3*c*x**2 - b**3*d*x**3 - 4*b**2*c*x
- 5*b**2*d*x**2 - 6*b*c - 12*b*d*x - 12*d)*exp(-a - b*x)/b**2, Ne(b**2, 0)
), (b**2*d*x**4/4 + x**3*(2*a*b*d/3 + b**2*c/3 + 2*b*d/3) + x**2*(a**2*d/2
 + a*b*c + a*d + b*c + d) + x*(a**2*c + 2*a*c + 2*c), True))
```

Maxima [F]

$$\int (c+dx)\Gamma(3,a+bx)\,dx = \int (dx+c)\Gamma(3,bx+a)\,dx$$

input
```
integrate((d*x+c)*gamma(3,b*x+a),x, algorithm="maxima")
```

output
```
d*integrate(x*gamma(3, b*x + a), x) + ((b*x + a)*gamma(3, b*x + a) - gamma
(4, b*x + a))*c/b
```

Giac [F]

$$\int (c+dx)\Gamma(3,a+bx)\,dx = \int (dx+c)\Gamma(3,bx+a)\,dx$$

input
```
integrate((d*x+c)*gamma(3,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*gamma(3, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.44

$$\int (c+dx)\Gamma(3,a+bx)\,dx = -\frac{e^{-a-b\,x}\,(12\,d+6\,a\,d+a^2\,d+b\,(c\,a^2+4\,c\,a+6\,c))}{b^2} - x^2\,e^{-a-b\,x}\,(5\,d+2\,a\,d+b\,c) - x\,e^{-a-b\,x}\left(4\,c+2\,a\,c+\frac{d\,a^2+6\,d\,a+12\,d}{b}\right) - b\,d\,x^3\,e^{-a-b\,x}$$

input
```
int(2*exp(- a - b*x)*(c + d*x)*(a + b*x + (a + b*x)^2/2 + 1),x)
```

output
```
- (exp(- a - b*x)*(12*d + 6*a*d + a^2*d + b*(6*c + 4*a*c + a^2*c)))/b^2 -
x^2*exp(- a - b*x)*(5*d + 2*a*d + b*c) - x*exp(- a - b*x)*(4*c + 2*a*c + (
12*d + 6*a*d + a^2*d)/b) - b*d*x^3*exp(- a - b*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.36

$$\int (c+dx)\Gamma(3,a+bx)\,dx = \frac{-b^3d\,x^3-2a\,b^2d\,x^2-b^3c\,x^2-a^2bdx-2a\,b^2cx-5b^2d\,x^2-a^2bc-6abdx-4b^2cx-a^2d-4abc-12bdx}{e^{bx+a}b^2}$$

input
```
int(2*(d*x+c)*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2),x)
```

output
```
( - a**2*b*c - a**2*b*d*x - a**2*d - 2*a*b**2*c*x - 2*a*b**2*d*x**2 - 4*a*
b*c - 6*a*b*d*x - 6*a*d - b**3*c*x**2 - b**3*d*x**3 - 4*b**2*c*x - 5*b**2*
d*x**2 - 6*b*c - 12*b*d*x - 12*d)/(e**(a + b*x)*b**2)
```

3.131 $\int \Gamma(3,a+bx)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 29

$$\int \Gamma(3,a+bx)\,dx=\frac{(a+bx)\Gamma(3,a+bx)}{b}-\frac{\Gamma(4,a+bx)}{b}$$

output
```
2*(b*x+a)*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/b-6*exp(-b*x-a)*(1+b*x+a+1/2
*(b*x+a)^2+1/6*(b*x+a)^3)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 81 vs. 2(29) = 58.

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.79

$$\int \Gamma(3,a+bx)\,dx=e^{-bx}\left(-\frac{(6+4a+a^2)\,e^{-a}}{b}-(6+4a+a^2)\,e^{-a}x-(3+2a)be^{-a}x^2-b^2e^{-a}x^3\right)+x\Gamma(3,a+bx)$$

input
```
Integrate[Gamma[3, a + b*x],x]
```

output
```
(-((6 + 4*a + a^2)/(b*E^a)) - ((6 + 4*a + a^2)*x)/E^a - ((3 + 2*a)*b*x^2)/
E^a - (b^2*x^3)/E^a)/E^(b*x) + x*Gamma[3, a + b*x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(3,a+bx)\,dx$$

$$\Big\downarrow \text{7111}$$

$$\frac{(a+bx)\Gamma(3,a+bx)}{b}-\frac{\Gamma(4,a+bx)}{b}$$

input
```
Int[Gamma[3, a + b*x],x]
```

output
```
((a + b*x)*Gamma[3, a + b*x])/b - Gamma[4, a + b*x]/b
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

method	result
gosper	$-\frac{{\mathrm e}^{-b x -a} \left(b^{2} x^{2}+2 b x a +a^{2}+4 b x +4 a +6\right)}{b}$
risch	$-\frac{{\mathrm e}^{-b x -a} \left(b^{2} x^{2}+2 b x a +a^{2}+4 b x +4 a +6\right)}{b}$
norman	$\left(-4-2 a \right) x \,{\mathrm e}^{-b x -a}-b \,x^{2} {\mathrm e}^{-b x -a}-\frac{\left(a^{2}+4 a +6\right) {\mathrm e}^{-b x -a}}{b}$
derivativedivides	$-\frac{\left(-b x -a \right)^{2} {\mathrm e}^{-b x -a}-4 \left(-b x -a \right) {\mathrm e}^{-b x -a}+6 \,{\mathrm e}^{-b x -a}}{b}$
default	$-\frac{\left(-b x -a \right)^{2} {\mathrm e}^{-b x -a}-4 \left(-b x -a \right) {\mathrm e}^{-b x -a}+6 \,{\mathrm e}^{-b x -a}}{b}$
orering	$-\frac{2 \left(b^{2} x^{2}+2 b x a +a^{2}+4 b x +4 a +6\right) {\mathrm e}^{-b x -a} \left(1+b x +a +\frac{\left(b x +a \right)^{2}}{2}\right)}{b \left(b^{2} x^{2}+2 b x a +a^{2}+2 b x +2 a +2\right)}$
parallelrisch	$-\frac{x^{2} {\mathrm e}^{-b x -a} b^{2}+2 x \,{\mathrm e}^{-b x -a} a b +4 x \,{\mathrm e}^{-b x -a} b +{\mathrm e}^{-b x -a} a^{2}+4 a \,{\mathrm e}^{-b x -a}+6 \,{\mathrm e}^{-b x -a}}{b}$
meijerg	$\frac{2 \,{\mathrm e}^{-a} \left(1-{\mathrm e}^{-b x}\right)}{b}+\frac{2 \,{\mathrm e}^{-a} a \left(1-{\mathrm e}^{-b x}\right)}{b}+\frac{{\mathrm e}^{-a} a^{2} \left(1-{\mathrm e}^{-b x}\right)}{b}+\frac{{\mathrm e}^{-a} \left(2-\frac{\left(3 b^{2} x^{2}+6 b x +6\right) {\mathrm e}^{-b x}}{3}\right)}{b}+\frac{2 \,{\mathrm e}^{-a} \left(a b +b \right) \left(1-\frac{(2}{}\right.}{b^{2}}$
parts	$-b \,x^{2} {\mathrm e}^{-b x -a}-2 \,{\mathrm e}^{-b x -a} x a -\frac{{\mathrm e}^{-b x -a} a^{2}}{b}-2 x \,{\mathrm e}^{-b x -a}-\frac{2 \,{\mathrm e}^{-b x -a} a}{b}-\frac{2 \,{\mathrm e}^{-b x -a}}{b}-\frac{2 \left(b \,{\mathrm e}^{-b x -a}-b \left(\left(-\right.\right.\right.}{}$

input
```
int(2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
-exp(-b*x-a)*(b^2*x^2+2*a*b*x+a^2+4*b*x+4*a+6)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(29) = 58.

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.21

$$\int \Gamma(3, a + bx)\, dx = \frac{bx\Gamma(3, bx + a) - (b^3x^3 + (2\,a + 3)b^2x^2 + (a^2 + 4\,a + 6)bx + a^2 + 4\,a + 6)e^{(-bx-a)}}{b}$$

input
```
integrate(gamma(3,b*x+a),x, algorithm="fricas")
```

output
```
(b*x*gamma(3, b*x + a) - (b^3*x^3 + (2*a + 3)*b^2*x^2 + (a^2 + 4*a + 6)*b*
x + a^2 + 4*a + 6)*e^(-b*x - a))/b
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.24

$$\int \Gamma(3, a + bx)\, dx = \begin{cases} \frac{(-a^2-2abx-4a-b^2x^2-4bx-6)e^{-a-bx}}{b} & \text{for } b \neq 0 \\ \frac{b^2x^3}{3} + x^2(ab + b) + x(a^2 + 2a + 2) & \text{otherwise} \end{cases}$$

input
```
integrate(uppergamma(3,b*x+a),x)
```

output
```
Piecewise(((-a**2 - 2*a*b*x - 4*a - b**2*x**2 - 4*b*x - 6)*exp(-a - b*x)/b
, Ne(b, 0)), (b**2*x**3/3 + x**2*(a*b + b) + x*(a**2 + 2*a + 2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \Gamma(3, a+bx)\,dx = \frac{(bx+a)\Gamma(3, bx+a) - \Gamma(4, bx+a)}{b}$$

input
```
integrate(gamma(3,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(3, b*x + a) - gamma(4, b*x + a))/b
```

Giac [F]

$$\int \Gamma(3, a+bx)\,dx = \int \Gamma(3, bx+a)\,dx$$

input
```
integrate(gamma(3,b*x+a),x, algorithm="giac")
```

output
```
integrate(gamma(3, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int \Gamma(3, a+bx)\,dx = -\frac{e^{-a-b\,x}\,(a^2+4\,a+6)}{b} - x\,e^{-a-b\,x}\,(2\,a+b\,x+4)$$

input
```
int(2*exp(- a - b*x)*(a + b*x + (a + b*x)^2/2 + 1),x)
```

output
```
- (exp(- a - b*x)*(4*a + a^2 + 6))/b - x*exp(- a - b*x)*(2*a + b*x + 4)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \Gamma(3, a + bx) \, dx = \frac{-b^2x^2 - 2abx - a^2 - 4bx - 4a - 6}{e^{bx+a}b}$$

input
```
int(2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2),x)
```

output
```
( - a**2 - 2*a*b*x - 4*a - b**2*x**2 - 4*b*x - 6)/(e**(a + b*x)*b)
```

3.132 $\int \frac{\Gamma(3,a+bx)}{c+dx} dx$

Optimal result

Integrand size = 15, antiderivative size = 162

$$\int \frac{\Gamma(3,a+bx)}{c+dx} dx = -\frac{3e^{-a-bx}}{d} + \frac{(bc-ad)e^{-a-bx}}{d^2} - \frac{e^{-a-bx}(a+bx)}{d} + \frac{2e^{-a+\frac{bc}{d}} \operatorname{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d} - \frac{2(bc-ad)e^{-a+\frac{bc}{d}} \operatorname{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^2} + \frac{(bc-ad)^2e^{-a+\frac{bc}{d}} \operatorname{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^3}$$

output
```
-3*exp(-b*x-a)/d+(-a*d+b*c)*exp(-b*x-a)/d^2-exp(-b*x-a)*(b*x+a)/d+2*exp(-a
+b*c/d)*Ei(-b*(d*x+c)/d)/d-2*(-a*d+b*c)*exp(-a+b*c/d)*Ei(-b*(d*x+c)/d)/d^2
+(-a*d+b*c)^2*exp(-a+b*c/d)*Ei(-b*(d*x+c)/d)/d^3
```

Mathematica [F]

$$\int \frac{\Gamma(3,a+bx)}{c+dx}\,dx=\int \frac{\Gamma(3,a+bx)}{c+dx}\,dx$$

input
```
Integrate[Gamma[3, a + b*x]/(c + d*x),x]
```

output
```
Integrate[Gamma[3, a + b*x]/(c + d*x), x]
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {7118, 2629, 2009, 7118, 2609, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(3,a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{7118}$$

$$\int \frac{e^{-a-bx}(a+bx)^2}{c+dx}dx+2\int \frac{\Gamma(2,a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{2629}$$

$$\int \left(\frac{e^{-a-bx}(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)e^{-a-bx}}{d^2}+\frac{be^{-a-bx}(a+bx)}{d}\right)dx+2\int \frac{\Gamma(2,a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{2009}$$

$$2\int \frac{\Gamma(2,a+bx)}{c+dx}dx+\frac{e^{\frac{bc}{d}-a}(bc-ad)^2\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^3}+\frac{e^{-a-bx}(bc-ad)}{d^2}-\frac{e^{-a-bx}}{d}-\frac{e^{-a-bx}(a+bx)}{d}$$

$$\Big\downarrow \text{7118}$$

$$2\left(\int \frac{e^{-a-bx}}{c+dx}dx+\int \frac{e^{-a-bx}(a+bx)}{c+dx}dx\right)+\frac{e^{\frac{bc}{d}-a}(bc-ad)^2 \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^3}+\frac{e^{-a-bx}(bc-ad)}{d^2}-\frac{e^{-a-bx}}{d}-\frac{e^{-a-bx}(a+bx)}{d}$$

$$\Big\downarrow \text{2609}$$

$$2\left(\int \frac{e^{-a-bx}(a+bx)}{c+dx}dx+\frac{e^{\frac{bc}{d}-a} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d}\right)+\frac{e^{\frac{bc}{d}-a}(bc-ad)^2 \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^3}+\frac{e^{-a-bx}(bc-ad)}{d^2}-\frac{e^{-a-bx}}{d}-\frac{e^{-a-bx}(a+bx)}{d}$$

$$\Big\downarrow \text{2629}$$

$$2\left(\int \left(\frac{e^{-a-bx}b}{d}+\frac{(ad-bc)e^{-a-bx}}{d(c+dx)}\right)dx+\frac{e^{\frac{bc}{d}-a} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d}\right)+\frac{e^{\frac{bc}{d}-a}(bc-ad)^2 \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^3}+\frac{e^{-a-bx}(bc-ad)}{d^2}-\frac{e^{-a-bx}}{d}-\frac{e^{-a-bx}(a+bx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{\frac{bc}{d}-a}(bc-ad)^2 \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^3}+2\left(-\frac{(bc-ad)e^{\frac{bc}{d}-a} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^2}+\frac{e^{\frac{bc}{d}-a} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d}-\frac{e^{-a-bx}}{d}\right)+\frac{e^{-a-bx}(bc-ad)}{d^2}-\frac{e^{-a-bx}}{d}-\frac{e^{-a-bx}(a+bx)}{d}$$

input
```
Int[Gamma[3, a + b*x]/(c + d*x),x]
```

output
```
-(E^(-a - b*x)/d) + ((b*c - a*d)*E^(-a - b*x))/d^2 - (E^(-a - b*x)*(a + b*
x))/d + ((b*c - a*d)^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])
/d^3 + 2*(-(E^(-a - b*x)/d) + (E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*
x))/d)])/d - ((b*c - a*d)*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d
)])/d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7118
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Int[(a
 + b*x)^(n - 1)/((c + d*x)*E^(a + b*x)), x] + Simp[(n - 1)   Int[Gamma[n -
1, a + b*x]/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 1]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.48

method	result
derivativedivides	$-\frac{\frac{ba\,\mathrm{e}^{-bx-a}}{d}-\frac{b^{2}c\,\mathrm{e}^{-bx-a}}{d^{2}}-\frac{b\left(\left(-bx-a\right)\mathrm{e}^{-bx-a}-\mathrm{e}^{-bx-a}\right)}{d}+\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)b\,\mathrm{e}^{-\frac{ad-cb}{d}}\,\mathrm{expIntegral}_{1}\left(bx+a-\frac{ad-cb}{d}\right)}{d^{3}}+}{b}$
default	$-\frac{\frac{ba\,\mathrm{e}^{-bx-a}}{d}-\frac{b^{2}c\,\mathrm{e}^{-bx-a}}{d^{2}}-\frac{b\left(\left(-bx-a\right)\mathrm{e}^{-bx-a}-\mathrm{e}^{-bx-a}\right)}{d}+\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)b\,\mathrm{e}^{-\frac{ad-cb}{d}}\,\mathrm{expIntegral}_{1}\left(bx+a-\frac{ad-cb}{d}\right)}{d^{3}}+}{b}$
risch	$-\frac{2a\,\mathrm{e}^{-bx-a}}{d}+\frac{bc\,\mathrm{e}^{-bx-a}}{d^{2}}-\frac{b\,\mathrm{e}^{-bx-a}x}{d}-\frac{3\,\mathrm{e}^{-bx-a}}{d}-\frac{\mathrm{e}^{-\frac{ad-cb}{d}}\,\mathrm{expIntegral}_{1}\left(bx+a-\frac{ad-cb}{d}\right)a^{2}}{d}+\frac{2b\,\mathrm{e}^{-\frac{ad-c}{d}}}{}$

input
```
int(2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(b/d*a*exp(-b*x-a)-b^2/d^2*c*exp(-b*x-a)-1/d*b*((-b*x-a)*exp(-b*x-a)-
exp(-b*x-a))+(a^2*d^2-2*a*b*c*d+b^2*c^2)*b/d^3*exp(-(a*d-b*c)/d)*Ei(1,b*x+
a-(a*d-b*c)/d)+2*b/d*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+2*b/d*exp(-
b*x-a)+2*(a*d-b*c)*b/d^2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))
```

Fricas [F]

$$\int \frac{\Gamma(3, a+bx)}{c+dx}\,dx = \int \frac{\Gamma(3, bx+a)}{dx+c}\,dx$$

input
```
integrate(gamma(3,b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(gamma(3, b*x + a)/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\Gamma(3, a+bx)}{c+dx}\,dx = \left(\int \frac{2a}{ce^{bx}+dxe^{bx}}\,dx + \int \frac{a^2}{ce^{bx}+dxe^{bx}}\,dx + \int \frac{2bx}{ce^{bx}+dxe^{bx}}\,dx + \int \frac{b^2x^2}{ce^{bx}+dxe^{bx}}\,dx + \int \frac{2abx}{ce^{bx}+dxe^{bx}}\,dx + \int \frac{2}{ce^{bx}+dxe^{bx}}\,dx\right)e^{-a}$$

input
```
integrate(uppergamma(3,b*x+a)/(d*x+c),x)
```

output
```
(Integral(2*a/(c*exp(b*x) + d*x*exp(b*x)), x) + Integral(a**2/(c*exp(b*x)
+ d*x*exp(b*x)), x) + Integral(2*b*x/(c*exp(b*x) + d*x*exp(b*x)), x) + Int
egral(b**2*x**2/(c*exp(b*x) + d*x*exp(b*x)), x) + Integral(2*a*b*x/(c*exp(
b*x) + d*x*exp(b*x)), x) + Integral(2/(c*exp(b*x) + d*x*exp(b*x)), x))*exp
(-a)
```

Maxima [F]

$$\int \frac{\Gamma(3,a+bx)}{c+dx}\,dx=\int \frac{\Gamma(3,bx+a)}{dx+c}\,dx$$

input
```
integrate(gamma(3,b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(gamma(3, b*x + a)/(d*x + c), x)
```

Giac [F]

$$\int \frac{\Gamma(3,a+bx)}{c+dx}\,dx=\int \frac{\Gamma(3,bx+a)}{dx+c}\,dx$$

input
```
integrate(gamma(3,b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(gamma(3, b*x + a)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(3,a+bx)}{c+dx}\,dx=\int \frac{2\,\mathrm{e}^{-a-b\,x}\,\left(a+b\,x+\frac{(a+b\,x)^2}{2}+1\right)}{c+d\,x}\,dx$$

input
```
int((2*exp(- a - b*x)*(a + b*x + (a + b*x)^2/2 + 1))/(c + d*x),x)
```

output
```
int((2*exp(- a - b*x)*(a + b*x + (a + b*x)^2/2 + 1))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\Gamma(3, a+bx)}{c+dx}\,dx$$
$$= \frac{e^{bx}\left(\int \frac{1}{e^{bx}c+e^{bx}dx}dx\right)a^2d^2 - 2e^{bx}\left(\int \frac{1}{e^{bx}c+e^{bx}dx}dx\right)abcd + 2e^{bx}\left(\int \frac{1}{e^{bx}c+e^{bx}dx}dx\right)a\,d^2 + e^{bx}\left(\int \frac{1}{e^{bx}c+e^{bx}dx}dx\right)}{e^{bx+a}d^2}$$

input
```
int(2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/(d*x+c),x)
```

output
```
(e**(b*x)*int(1/(e**(b*x)*c + e**(b*x)*d*x),x)*a**2*d**2 - 2*e**(b*x)*int(
1/(e**(b*x)*c + e**(b*x)*d*x),x)*a*b*c*d + 2*e**(b*x)*int(1/(e**(b*x)*c +
e**(b*x)*d*x),x)*a*d**2 + e**(b*x)*int(1/(e**(b*x)*c + e**(b*x)*d*x),x)*b*
*2*c**2 - 2*e**(b*x)*int(1/(e**(b*x)*c + e**(b*x)*d*x),x)*b*c*d + 2*e**(b*
x)*int(1/(e**(b*x)*c + e**(b*x)*d*x),x)*d**2 - 2*a*d + b*c - b*d*x - 3*d)/
(e**(a + b*x)*d**2)
```

3.133 $\int \frac{\Gamma(3,a+bx)}{(c+dx)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^2} dx = -\frac{b(bc-ad)e^{-a-bx}}{d^3} + \frac{b(bc-ad)^2e^{-a+\frac{bc}{d}}\Gamma\left(0,\frac{b(c+dx)}{d}\right)}{d^4} + \frac{b\Gamma(2,a+bx)}{d^2} - \frac{\Gamma(3,a+bx)}{d(c+dx)}$$

output
```
-b*(-a*d+b*c)*exp(-b*x-a)/d^3+b*(-a*d+b*c)^2*exp(-a+b*c/d)*Ei(1,b*(d*x+c)/
d)/d^4+b*exp(-b*x-a)*(b*x+a+1)/d^2-2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/d
/(d*x+c)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^2} dx = \frac{bde^{-a-bx}(-bc+d+2ad+bdx) - b(bc-ad)^2e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right) - \frac{d^3\Gamma(3,a+bx)}{c+dx}}{d^4}$$

input
```
Integrate[Gamma[3, a + b*x]/(c + d*x)^2,x]
```

output
```
(b*d*E^(-a - b*x)*(-(b*c) + d + 2*a*d + b*d*x) - b*(b*c - a*d)^2*E^(-a + (
b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)] - (d^3*Gamma[3, a + b*x])/(c + d
*x))/d^4
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.28, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{7119}$$

$$-\frac{b\int \frac{e^{-a-bx}(a+bx)^2}{c+dx}dx}{d}-\frac{\Gamma(3,a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{2629}$$

$$-\frac{b\int \left(\frac{e^{-a-bx}(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)e^{-a-bx}}{d^2}+\frac{be^{-a-bx}(a+bx)}{d}\right)dx}{d}-\frac{\Gamma(3,a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{e^{\frac{bc}{d}-a}(bc-ad)^2\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^3}+\frac{e^{-a-bx}(bc-ad)}{d^2}-\frac{e^{-a-bx}}{d}-\frac{e^{-a-bx}(a+bx)}{d}\right)}{d}-\frac{\Gamma(3,a+bx)}{d(c+dx)}$$

input
```
Int[Gamma[3, a + b*x]/(c + d*x)^2,x]
```

output
```
-((b*(-(E^(-a - b*x)/d) + ((b*c - a*d)*E^(-a - b*x))/d^2 - (E^(-a - b*x)*(
a + b*x))/d + ((b*c - a*d)^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x)
)/d)])/d^3))/d) - Gamma[3, a + b*x]/(d*(c + d*x))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(118) = 236$.

Time = 0.85 (sec) , antiderivative size = 335, normalized size of antiderivative = 3.53

method	result
risch	$-\frac{b\,\mathrm{e}^{-bx-a}}{d^{2}}+\frac{b\,\mathrm{e}^{-bx-a} a^{2}}{d^{2}\left(-bx-\frac{cb}{d}\right)}-\frac{2 b^{2} \mathrm{e}^{-bx-a} a c}{d^{3}\left(-bx-\frac{cb}{d}\right)}+\frac{b^{3} \mathrm{e}^{-bx-a} c^{2}}{d^{4}\left(-bx-\frac{cb}{d}\right)}+\frac{b\,\mathrm{e}^{-\frac{ad-cb}{d}} \operatorname{expIntegral}_{1}\left(bx+a-\frac{ad-cb}{d}\right) a^{2}}{d^{2}}-$
derivativedivides	$-\frac{b^{2} \mathrm{e}^{-bx-a}}{d^{2}}+\frac{2(ad-cb) b^{2} \mathrm{e}^{-\frac{ad-cb}{d}} \operatorname{expIntegral}_{1}\left(bx+a-\frac{ad-cb}{d}\right)}{d^{3}}+\frac{\left(a^{2} d^{2}-2abcd+b^{2} c^{2}\right) b^{2}\left(-\frac{\mathrm{e}^{-bx-a}}{-bx-a+\frac{ad-cb}{d}}-\mathrm{e}^{-\frac{ad-cb}{d}} \operatorname{expl}\right.}{d^{4}}$
default	$-\frac{b^{2} \mathrm{e}^{-bx-a}}{d^{2}}+\frac{2(ad-cb) b^{2} \mathrm{e}^{-\frac{ad-cb}{d}} \operatorname{expIntegral}_{1}\left(bx+a-\frac{ad-cb}{d}\right)}{d^{3}}+\frac{\left(a^{2} d^{2}-2abcd+b^{2} c^{2}\right) b^{2}\left(-\frac{\mathrm{e}^{-bx-a}}{-bx-a+\frac{ad-cb}{d}}-\mathrm{e}^{-\frac{ad-cb}{d}} \operatorname{expl}\right.}{d^{4}}$

input
```
int(2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/(d*x+c)^2,x,method=_RETURNVERBOS
E)
```

output
```
-b*exp(-b*x-a)/d^2+b/d^2*exp(-b*x-a)/(-b*x-c*b/d)*a^2-2*b^2/d^3*exp(-b*x-a
)/(-b*x-c*b/d)*a*c+b^3/d^4*exp(-b*x-a)/(-b*x-c*b/d)*c^2+b/d^2*exp(-(a*d-b*
c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)*a^2-2*b^2/d^3*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(
a*d-b*c)/d)*a*c+b^3/d^4*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)*c^2+2*b/
d^2*exp(-b*x-a)/(-b*x-c*b/d)+2*b/d^2*exp(-b*x-a)/(-b*x-c*b/d)*a-2*b^2/d^3*
exp(-b*x-a)/(-b*x-c*b/d)*c
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.73

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^2}\,dx = -\frac{d^3\Gamma(3,bx+a) + (b^3c^3 - 2\,ab^2c^2d + a^2bcd^2 + (b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)x)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(\frac{bc-ad}{d}\right)} - (b^2d}{d^5x + cd^4}$$

input
```
integrate(gamma(3,b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(d^3*gamma(3, b*x + a) + (b^3*c^3 - 2*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^
2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)*Ei(-(b*d*x + b*c)/d)*e^((b*c - a*d)/d)
 - (b^2*d^3*x^2 + (2*a + 1)*b*d^3*x - b^2*c^2*d + (2*a + 1)*b*c*d^2)*e^(-b
*x - a))/(d^5*x + c*d^4)
```

Sympy [F]

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^2}\,dx = \left(\int \frac{2a}{c^2e^{bx} + 2cdxe^{bx} + d^2x^2e^{bx}}\,dx + \int \frac{a^2}{c^2e^{bx} + 2cdxe^{bx} + d^2x^2e^{bx}}\,dx + \int \frac{2bx}{c^2e^{bx} + 2cdxe^{bx} + d^2x^2e^{bx}}\,dx + \int \frac{b^2x^2}{c^2e^{bx} + 2cdxe^{bx} + d^2x^2e^{bx}}\,dx + \int \frac{2abx}{c^2e^{bx} + 2cdxe^{bx} + d^2x^2e^{bx}}\,dx + \int \frac{2}{c^2e^{bx} + 2cdxe^{bx} + d^2x^2e^{bx}}\,dx\right)e^{-a}$$

input
```
integrate(uppergamma(3,b*x+a)/(d*x+c)**2,x)
```

output
```
(Integral(2*a/(c**2*exp(b*x) + 2*c*d*x*exp(b*x) + d**2*x**2*exp(b*x)), x)
+ Integral(a**2/(c**2*exp(b*x) + 2*c*d*x*exp(b*x) + d**2*x**2*exp(b*x)), x
) + Integral(2*b*x/(c**2*exp(b*x) + 2*c*d*x*exp(b*x) + d**2*x**2*exp(b*x))
, x) + Integral(b**2*x**2/(c**2*exp(b*x) + 2*c*d*x*exp(b*x) + d**2*x**2*ex
p(b*x)), x) + Integral(2*a*b*x/(c**2*exp(b*x) + 2*c*d*x*exp(b*x) + d**2*x*
*2*exp(b*x)), x) + Integral(2/(c**2*exp(b*x) + 2*c*d*x*exp(b*x) + d**2*x**
2*exp(b*x)), x))*exp(-a)
```

Maxima [F]

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^2}\,dx = \int \frac{\Gamma(3,bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(gamma(3,b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(gamma(3, b*x + a)/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^2}\,dx = \int \frac{\Gamma(3,bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(gamma(3,b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(gamma(3, b*x + a)/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^2}\,dx = \int \frac{2\,\mathrm{e}^{-a-b\,x}\,\left(a+b\,x+\frac{{\left(a+b\,x\right)}^2}{2}+1\right)}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((2*exp(- a - b*x)*(a + b*x + (a + b*x)^2/2 + 1))/(c + d*x)^2,x)
```

output
```
int((2*exp(- a - b*x)*(a + b*x + (a + b*x)^2/2 + 1))/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/(d*x+c)^2,x)
```

output

```
( - e**(b*x)*int(x/(e**(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b*c
*d**2*x**2 + e**(b*x)*c**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x)*d**3*x**2),x
)*a**2*b**2*c**2*d**2 - e**(b*x)*int(x/(e**(b*x)*b*c**3 + 2*e**(b*x)*b*c**
2*d*x + e**(b*x)*b*c*d**2*x**2 + e**(b*x)*c**2*d + 2*e**(b*x)*c*d**2*x + e
**(b*x)*d**3*x**2),x)*a**2*b**2*c*d**3*x - e**(b*x)*int(x/(e**(b*x)*b*c**3
 + 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b*c*d**2*x**2 + e**(b*x)*c**2*d + 2*e*
*(b*x)*c*d**2*x + e**(b*x)*d**3*x**2),x)*a**2*b*c*d**3 - e**(b*x)*int(x/(e
**(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b*c*d**2*x**2 + e**(b*x)
*c**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x)*d**3*x**2),x)*a**2*b*d**4*x + 2*e
**(b*x)*int(x/(e**(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b*c*d**2
*x**2 + e**(b*x)*c**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x)*d**3*x**2),x)*a*b
**3*c**3*d + 2*e**(b*x)*int(x/(e**(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x + e
**(b*x)*b*c*d**2*x**2 + e**(b*x)*c**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x)*d
**3*x**2),x)*a*b**3*c**2*d**2*x + 2*e**(b*x)*int(x/(e**(b*x)*b*c**3 + 2*e*
*(b*x)*b*c**2*d*x + e**(b*x)*b*c*d**2*x**2 + e**(b*x)*c**2*d + 2*e**(b*x)*
c*d**2*x + e**(b*x)*d**3*x**2),x)*a*b**2*c**2*d**2 + 2*e**(b*x)*int(x/(e**
(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b*c*d**2*x**2 + e**(b*x)*c
**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x)*d**3*x**2),x)*a*b**2*c*d**3*x - e**
(b*x)*int(x/(e**(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b*c*d**2*x
**2 + e**(b*x)*c**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x)*d**3*x**2),x)*b*...
```

3.134 $\int \frac{\Gamma(3,a+bx)}{(c+dx)^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^3} \, dx = \frac{b^2e^{-a-bx}}{2d^3} + \frac{b^2(bc-ad)^2e^{-a+\frac{bc}{d}}\Gamma\left(-1,\frac{b(c+dx)}{d}\right)}{2d^5} - \frac{b^2(bc-ad)e^{-a+\frac{bc}{d}}\Gamma\left(0,\frac{b(c+dx)}{d}\right)}{d^4} - \frac{\Gamma(3,a+bx)}{2d(c+dx)^2}$$

output
```
1/2*b^2*exp(-b*x-a)/d^3+1/2*b*(-a*d+b*c)^2*exp(-a+b*c/d)/(d*x+c)/d^4*Ei(2,
b*(d*x+c)/d)-b^2*(-a*d+b*c)*exp(-a+b*c/d)*Ei(1,b*(d*x+c)/d)/d^4-exp(-b*x-a
)*(1+b*x+a+1/2*(b*x+a)^2)/d/(d*x+c)^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^3} \, dx = \frac{b^2(b^2c^2-2(-1+a)bcd+(-2+a)ad^2)\,e^{-a+\frac{bc}{d}}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right) + \frac{d\left(be^{-a-bx}(c+dx)(b^2c^2+a^2d^2+bd(c-\right.}{(c+dx)^2}}{2d^5}$$

input
```
Integrate[Gamma[3, a + b*x]/(c + d*x)^3,x]
```

output
```
(b^2*(b^2*c^2 - 2*(-1 + a)*b*c*d + (-2 + a)*a*d^2)*E^(-a + (b*c)/d)*ExpInt
egralEi[-((b*(c + d*x))/d)] + (d*(b*E^(-a - b*x)*(c + d*x)*(b^2*c^2 + a^2*
d^2 + b*d*(c - 2*a*c + d*x)) - d^3*Gamma[3, a + b*x]))/(c + d*x)^2)/(2*d^5
)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.25, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^3} \, dx$$

$$\Big\downarrow \text{ 7119}$$

$$-\frac{b\int \frac{e^{-a-bx}(a+bx)^2}{(c+dx)^2}dx}{2d}-\frac{\Gamma(3,a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 2629}$$

$$-\frac{b\int \left(\frac{e^{-a-bx}b^2}{d^2}-\frac{2(bc-ad)e^{-a-bx}b}{d^2(c+dx)}+\frac{(ad-bc)^2e^{-a-bx}}{d^2(c+dx)^2}\right)dx}{2d}-\frac{\Gamma(3,a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b\left(-\frac{be^{\frac{bc}{d}-a}(bc-ad)^2 \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^4}-\frac{2be^{\frac{bc}{d}-a}(bc-ad) \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^3}-\frac{e^{-a-bx}(bc-ad)^2}{d^3(c+dx)}-\frac{be^{-a-bx}}{d^2}\right)}{2d}-\frac{\Gamma(3,a+bx)}{2d(c+dx)^2}$$

input
```
Int[Gamma[3, a + b*x]/(c + d*x)^3,x]
```

output
```
-1/2*(b*(-((b*E^(-a - b*x))/d^2) - ((b*c - a*d)^2*E^(-a - b*x))/(d^3*(c +
d*x)) - (2*b*(b*c - a*d)*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)
])/d^3 - (b*(b*c - a*d)^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d
)])/d^4))/d - Gamma[3, a + b*x]/(2*d*(c + d*x)^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(139) = 278$.

Time = 0.81 (sec) , antiderivative size = 563, normalized size of antiderivative = 4.47

method	result
derivativedivides	$-\frac{b^3 e^{-\frac{ad-cb}{d}} \operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{d^3} - \frac{b^3(ad-cb)^2\left(-\frac{e^{-bx-a}}{2\left(-bx-a+\frac{ad-cb}{d}\right)^2} - \frac{e^{-bx-a}}{2\left(-bx-a+\frac{ad-cb}{d}\right)} - \frac{e^{-\frac{ad-cb}{d}} \operatorname{expIntegral}}{2}\right.}{d^5}$
default	$-\frac{b^3 e^{-\frac{ad-cb}{d}} \operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{d^3} - \frac{b^3(ad-cb)^2\left(-\frac{e^{-bx-a}}{2\left(-bx-a+\frac{ad-cb}{d}\right)^2} - \frac{e^{-bx-a}}{2\left(-bx-a+\frac{ad-cb}{d}\right)} - \frac{e^{-\frac{ad-cb}{d}} \operatorname{expIntegral}}{2}\right.}{d^5}$
risch	$\frac{b^3 e^{-bx-a} ac}{d^4\left(-bx-\frac{cb}{d}\right)^2} + \frac{b^3 e^{-bx-a} ac}{d^4\left(-bx-\frac{cb}{d}\right)} + \frac{b^3 e^{-\frac{ad-cb}{d}} \operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right) ac}{d^4} - \frac{b^2 e^{-bx-a} a}{d^3\left(-bx-\frac{cb}{d}\right)^2} + \frac{b^3 e^{-bx-a}}{d^4\left(-bx-\frac{cb}{d}\right.}$

input
```
int(2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/(d*x+c)^3,x,method=_RETURNVERBOS
E)
```

output
```
-1/b*(b^3/d^3*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-b^3*(a*d-b*c)^2/d^
5*(-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*
c)/d)-1/2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))+2*b^3*(a*d-b*c)/d^4*(
-exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d
))-2*b^3/d^3*(-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/2*exp(-b*x-a)/(-b*
x-a+(a*d-b*c)/d)-1/2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))-2*b^3*(a*d
-b*c)/d^4*(-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/2*exp(-b*x-a)/(-b*x-a
+(a*d-b*c)/d)-1/2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))+2*b^3/d^3*(-e
xp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(117) = 234.

Time = 0.09 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.17

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^3}\,dx =$$

$$-\frac{d^4\Gamma(3,bx+a) - (b^4c^4 - 2\,(a-1)b^3c^3d + (a^2-2\,a)b^2c^2d^2 + (b^4c^2d^2 - 2\,(a-1)b^3cd^3 + (a^2-2\,a)b^2d^4}{}$$

input
```
integrate(gamma(3,b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(d^4*gamma(3, b*x + a) - (b^4*c^4 - 2*(a - 1)*b^3*c^3*d + (a^2 - 2*a)
*b^2*c^2*d^2 + (b^4*c^2*d^2 - 2*(a - 1)*b^3*c*d^3 + (a^2 - 2*a)*b^2*d^4)*x
^2 + 2*(b^4*c^3*d - 2*(a - 1)*b^3*c^2*d^2 + (a^2 - 2*a)*b^2*c*d^3)*x)*Ei(-
(b*d*x + b*c)/d)*e^((b*c - a*d)/d) - (b^2*d^4*x^2 + b^3*c^3*d - (2*a - 1)*
b^2*c^2*d^2 + a^2*b*c*d^3 + (b^3*c^2*d^2 - 2*(a - 1)*b^2*c*d^3 + a^2*b*d^4
)*x)*e^(-b*x - a))/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\Gamma(3, a + bx)}{(c + dx)^3} \, dx = \text{Timed out}$$

input
```
integrate(uppergamma(3,b*x+a)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\Gamma(3, a + bx)}{(c + dx)^3} \, dx = \int \frac{\Gamma(3, bx + a)}{(dx + c)^3} \, dx$$

input
```
integrate(gamma(3,b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(gamma(3, b*x + a)/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{\Gamma(3, a + bx)}{(c + dx)^3} \, dx = \int \frac{\Gamma(3, bx + a)}{(dx + c)^3} \, dx$$

input
```
integrate(gamma(3,b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(gamma(3, b*x + a)/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^3}\,dx = \int \frac{2\,\mathrm{e}^{-a-b\,x}\,\left(a+b\,x+\frac{(a+b\,x)^2}{2}+1\right)}{(c+d\,x)^3}\,dx$$

input
```
int((2*exp(- a - b*x)*(a + b*x + (a + b*x)^2/2 + 1))/(c + d*x)^3,x)
```

output
```
int((2*exp(- a - b*x)*(a + b*x + (a + b*x)^2/2 + 1))/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^3}\,dx = \text{too large to display}$$

input
```
int(2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/(d*x+c)^3,x)
```

output
```
( - e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b*x)*b*c**3*d*x + 3*e**(b*x)*b
*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 + 2*e**(b*x)*c**3*d + 6*e**(b*x)*
c**2*d**2*x + 6*e**(b*x)*c*d**3*x**2 + 2*e**(b*x)*d**4*x**3),x)*a**2*b**2*
c**3*d**2 - 2*e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b*x)*b*c**3*d*x + 3*
e**(b*x)*b*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 + 2*e**(b*x)*c**3*d + 6
*e**(b*x)*c**2*d**2*x + 6*e**(b*x)*c*d**3*x**2 + 2*e**(b*x)*d**4*x**3),x)*
a**2*b**2*c**2*d**3*x - e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b*x)*b*c**
3*d*x + 3*e**(b*x)*b*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 + 2*e**(b*x)*
c**3*d + 6*e**(b*x)*c**2*d**2*x + 6*e**(b*x)*c*d**3*x**2 + 2*e**(b*x)*d**4
*x**3),x)*a**2*b**2*c*d**4*x**2 - 2*e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e*
*(b*x)*b*c**3*d*x + 3*e**(b*x)*b*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 +
 2*e**(b*x)*c**3*d + 6*e**(b*x)*c**2*d**2*x + 6*e**(b*x)*c*d**3*x**2 + 2*e
**(b*x)*d**4*x**3),x)*a**2*b*c**2*d**3 - 4*e**(b*x)*int(x/(e**(b*x)*b*c**4
 + 3*e**(b*x)*b*c**3*d*x + 3*e**(b*x)*b*c**2*d**2*x**2 + e**(b*x)*b*c*d**3
*x**3 + 2*e**(b*x)*c**3*d + 6*e**(b*x)*c**2*d**2*x + 6*e**(b*x)*c*d**3*x**
2 + 2*e**(b*x)*d**4*x**3),x)*a**2*b*c*d**4*x - 2*e**(b*x)*int(x/(e**(b*x)*
b*c**4 + 3*e**(b*x)*b*c**3*d*x + 3*e**(b*x)*b*c**2*d**2*x**2 + e**(b*x)*b*
c*d**3*x**3 + 2*e**(b*x)*c**3*d + 6*e**(b*x)*c**2*d**2*x + 6*e**(b*x)*c*d*
*3*x**2 + 2*e**(b*x)*d**4*x**3),x)*a**2*b*d**5*x**2 + 2*e**(b*x)*int(x/(e*
*(b*x)*b*c**4 + 3*e**(b*x)*b*c**3*d*x + 3*e**(b*x)*b*c**2*d**2*x**2 + e...
```

3.135 $\int \frac{\Gamma(3,a+bx)}{(c+dx)^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 142

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^4} \, dx = \frac{b^3(bc-ad)^2 e^{-a+\frac{bc}{d}}\Gamma\left(-2,\frac{b(c+dx)}{d}\right)}{3d^6} - \frac{2b^3(bc-ad)e^{-a+\frac{bc}{d}}\Gamma\left(-1,\frac{b(c+dx)}{d}\right)}{3d^5} + \frac{b^3e^{-a+\frac{bc}{d}}\Gamma\left(0,\frac{b(c+dx)}{d}\right)}{3d^4} - \frac{\Gamma(3,a+bx)}{3d(c+dx)^3}$$

output
```
1/3*b*(-a*d+b*c)^2*exp(-a+b*c/d)/(d*x+c)^2/d^4*Ei(3,b*(d*x+c)/d)-2/3*b^2*(
-a*d+b*c)*exp(-a+b*c/d)/(d*x+c)/d^4*Ei(2,b*(d*x+c)/d)+1/3*b^3*exp(-a+b*c/d
)*Ei(1,b*(d*x+c)/d)/d^4-2/3*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/d/(d*x+c)^
3
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^4}\,dx = \frac{-b^3(b^2c^2-2(-2+a)bcd+(2-4a+a^2)\,d^2)\,e^{-a+\frac{bc}{d}}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)+\frac{d(-b(bc-ad)e^{-a-bx}(c+dx)(a}{}}{6d^6}$$

input
```
Integrate[Gamma[3, a + b*x]/(c + d*x)^4,x]
```

output
```
(-(b^3*(b^2*c^2 - 2*(-2 + a)*b*c*d + (2 - 4*a + a^2)*d^2)*E^(-a + (b*c)/d)
*ExpIntegralEi[-((b*(c + d*x))/d)]) + (d*(-(b*(b*c - a*d)*E^(-a - b*x)*(c
+ d*x)*(a*d^2 + b^2*c*(c + d*x) - b*d*((-3 + a)*c + (-4 + a)*d*x))) - 2*d^
4*Gamma[3, a + b*x]))/(c + d*x)^3)/(6*d^6)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.73, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^4}\,dx$$

$$\Big\downarrow \ 7119$$

$$-\frac{b\int \frac{e^{-a-bx}(a+bx)^2}{(c+dx)^3}dx}{3d}-\frac{\Gamma(3,a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \ 2629$$

$$-\frac{b\int \left(\frac{e^{-a-bx}b^2}{d^2(c+dx)}-\frac{2(bc-ad)e^{-a-bx}b}{d^2(c+dx)^2}+\frac{(ad-bc)^2e^{-a-bx}}{d^2(c+dx)^3}\right)dx}{3d}-\frac{\Gamma(3,a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \ 2009$$

$$-\frac{b\left(\frac{b^2(bc-ad)^2e^{\frac{bc}{d}-a}\operatorname{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{2d^5}+\frac{2b^2(bc-ad)e^{\frac{bc}{d}-a}\operatorname{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^4}+\frac{b^2e^{\frac{bc}{d}-a}\operatorname{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^3}+}{3d}$$

$$\frac{\Gamma(3,a+bx)}{3d(c+dx)^3}$$

input
```
Int[Gamma[3, a + b*x]/(c + d*x)^4,x]
```

output
```
-1/3*(b*(-1/2*((b*c - a*d)^2*E^(-a - b*x))/(d^3*(c + d*x)^2) + (2*b*(b*c -
 a*d)*E^(-a - b*x))/(d^3*(c + d*x)) + (b*(b*c - a*d)^2*E^(-a - b*x))/(2*d^
4*(c + d*x)) + (b^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^
3 + (2*b^2*(b*c - a*d)*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])
/d^4 + (b^2*(b*c - a*d)^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d
)])/(2*d^5)))/d - Gamma[3, a + b*x]/(3*d*(c + d*x)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(160) = 320$.

Time = 0.88 (sec) , antiderivative size = 763, normalized size of antiderivative = 5.37

method	result
derivativedivides	$-\frac{b^4\left(-\frac{e^{-bx-a}}{-bx-a+\frac{ad-cb}{d}}-e^{-\frac{ad-cb}{d}}\,\text{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)\right)}{d^4}-\frac{2b^4(ad-cb)\left(-\frac{e^{-bx-a}}{2\left(-bx-a+\frac{ad-cb}{d}\right)^2}-\frac{e^{-bx-a}}{2\left(-bx-a+\frac{ad-cb}{d}\right)}\right.}{d^5}$
default	$-\frac{b^4\left(-\frac{e^{-bx-a}}{-bx-a+\frac{ad-cb}{d}}-e^{-\frac{ad-cb}{d}}\,\text{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)\right)}{d^4}-\frac{2b^4(ad-cb)\left(-\frac{e^{-bx-a}}{2\left(-bx-a+\frac{ad-cb}{d}\right)^2}-\frac{e^{-bx-a}}{2\left(-bx-a+\frac{ad-cb}{d}\right)}\right.}{d^5}$
risch	$-\frac{2b^4e^{-bx-a}ac}{3d^5\left(-bx-\frac{cb}{d}\right)^3}-\frac{b^4e^{-bx-a}ac}{3d^5\left(-bx-\frac{cb}{d}\right)^2}-\frac{b^4e^{-bx-a}ac}{3d^5\left(-bx-\frac{cb}{d}\right)}-\frac{b^4e^{-\frac{ad-cb}{d}}\,\text{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)ac}{3d^5}+\frac{2b^3e}{3d^4(-}$

input
```
int(2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/(d*x+c)^4,x,method=_RETURNVERBOS
E)
```

output
```
-1/b*(b^4/d^4*(-exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-exp(-(a*d-b*c)/d)*Ei(1,b*
x+a-(a*d-b*c)/d))-2*b^4*(a*d-b*c)/d^5*(-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/
d)^2-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a
-(a*d-b*c)/d))+b^4*(a*d-b*c)^2/d^6*(-1/3*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^
3-1/6*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/6*exp(-b*x-a)/(-b*x-a+(a*d-b*c)
/d)-1/6*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))+2*b^4/d^4*(-1/3*exp(-b*
x-a)/(-b*x-a+(a*d-b*c)/d)^3-1/6*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/6*exp
(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/6*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d
))-2*b^4/d^4*(-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/2*exp(-b*x-a)/(-b*
x-a+(a*d-b*c)/d)-1/2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))+2*b^4*(a*d
-b*c)/d^5*(-1/3*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^3-1/6*exp(-b*x-a)/(-b*x-a
+(a*d-b*c)/d)^2-1/6*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/6*exp(-(a*d-b*c)/d)
*Ei(1,b*x+a-(a*d-b*c)/d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(131) = 262$.

Time = 0.11 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.86

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^4}\,dx =$$
$$-\frac{2\,d^5\Gamma(3,bx+a)+(b^5c^5-2\,(a-2)b^4c^4d+(a^2-4\,a+2)b^3c^3d^2+(b^5c^2d^3-2\,(a-2)b^4cd^4+(a^2-4\,a}{}$$

input
```
integrate(gamma(3,b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/6*(2*d^5*gamma(3, b*x + a) + (b^5*c^5 - 2*(a - 2)*b^4*c^4*d + (a^2 - 4*
a + 2)*b^3*c^3*d^2 + (b^5*c^2*d^3 - 2*(a - 2)*b^4*c*d^4 + (a^2 - 4*a + 2)*
b^3*d^5)*x^3 + 3*(b^5*c^3*d^2 - 2*(a - 2)*b^4*c^2*d^3 + (a^2 - 4*a + 2)*b^
3*c*d^4)*x^2 + 3*(b^5*c^4*d - 2*(a - 2)*b^4*c^3*d^2 + (a^2 - 4*a + 2)*b^3*
c^2*d^3)*x)*Ei(-(b*d*x + b*c)/d)*e^((b*c - a*d)/d) + (b^4*c^4*d - (2*a - 3
)*b^3*c^3*d^2 + (a^2 - 2*a)*b^2*c^2*d^3 - a^2*b*c*d^4 + (b^4*c^2*d^3 - 2*(
a - 2)*b^3*c*d^4 + (a^2 - 4*a)*b^2*d^5)*x^2 + (2*b^4*c^3*d^2 - (4*a - 7)*b
^3*c^2*d^3 + 2*(a^2 - 3*a)*b^2*c*d^4 - a^2*b*d^5)*x)*e^(-b*x - a))/(d^9*x^
3 + 3*c*d^8*x^2 + 3*c^2*d^7*x + c^3*d^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^4}\,dx = \text{Timed out}$$

input
```
integrate(uppergamma(3,b*x+a)/(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^4}\,dx=\int \frac{\Gamma(3,bx+a)}{(dx+c)^4}\,dx$$

input
```
integrate(gamma(3,b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate(gamma(3, b*x + a)/(d*x + c)^4, x)
```

Giac [F]

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^4}\,dx=\int \frac{\Gamma(3,bx+a)}{(dx+c)^4}\,dx$$

input
```
integrate(gamma(3,b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate(gamma(3, b*x + a)/(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^4}\,dx=\int \frac{2\,\mathrm{e}^{-a-b\,x}\,\left(a+b\,x+\frac{(a+b\,x)^2}{2}+1\right)}{(c+d\,x)^4}\,dx$$

input
```
int((2*exp(- a - b*x)*(a + b*x + (a + b*x)^2/2 + 1))/(c + d*x)^4,x)
```

output
```
int((2*exp(- a - b*x)*(a + b*x + (a + b*x)^2/2 + 1))/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^4}\,dx = \text{too large to display}$$

input
```
int(2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/(d*x+c)^4,x)
```

output
```
( - e**(b*x)*int(x/(e**(b*x)*b*c**5 + 4*e**(b*x)*b*c**4*d*x + 6*e**(b*x)*b
*c**3*d**2*x**2 + 4*e**(b*x)*b*c**2*d**3*x**3 + e**(b*x)*b*c*d**4*x**4 + 3
*e**(b*x)*c**4*d + 12*e**(b*x)*c**3*d**2*x + 18*e**(b*x)*c**2*d**3*x**2 +
12*e**(b*x)*c*d**4*x**3 + 3*e**(b*x)*d**5*x**4),x)*a**2*b**2*c**4*d**2 - 3
*e**(b*x)*int(x/(e**(b*x)*b*c**5 + 4*e**(b*x)*b*c**4*d*x + 6*e**(b*x)*b*c*
*3*d**2*x**2 + 4*e**(b*x)*b*c**2*d**3*x**3 + e**(b*x)*b*c*d**4*x**4 + 3*e*
*(b*x)*c**4*d + 12*e**(b*x)*c**3*d**2*x + 18*e**(b*x)*c**2*d**3*x**2 + 12*
e**(b*x)*c*d**4*x**3 + 3*e**(b*x)*d**5*x**4),x)*a**2*b**2*c**3*d**3*x - 3*
e**(b*x)*int(x/(e**(b*x)*b*c**5 + 4*e**(b*x)*b*c**4*d*x + 6*e**(b*x)*b*c**
3*d**2*x**2 + 4*e**(b*x)*b*c**2*d**3*x**3 + e**(b*x)*b*c*d**4*x**4 + 3*e**
(b*x)*c**4*d + 12*e**(b*x)*c**3*d**2*x + 18*e**(b*x)*c**2*d**3*x**2 + 12*e
**(b*x)*c*d**4*x**3 + 3*e**(b*x)*d**5*x**4),x)*a**2*b**2*c**2*d**4*x**2 -
e**(b*x)*int(x/(e**(b*x)*b*c**5 + 4*e**(b*x)*b*c**4*d*x + 6*e**(b*x)*b*c**
3*d**2*x**2 + 4*e**(b*x)*b*c**2*d**3*x**3 + e**(b*x)*b*c*d**4*x**4 + 3*e**
(b*x)*c**4*d + 12*e**(b*x)*c**3*d**2*x + 18*e**(b*x)*c**2*d**3*x**2 + 12*e
**(b*x)*c*d**4*x**3 + 3*e**(b*x)*d**5*x**4),x)*a**2*b**2*c*d**5*x**3 - 3*e
**(b*x)*int(x/(e**(b*x)*b*c**5 + 4*e**(b*x)*b*c**4*d*x + 6*e**(b*x)*b*c**3
*d**2*x**2 + 4*e**(b*x)*b*c**2*d**3*x**3 + e**(b*x)*b*c*d**4*x**4 + 3*e**(
b*x)*c**4*d + 12*e**(b*x)*c**3*d**2*x + 18*e**(b*x)*c**2*d**3*x**2 + 12*e*
*(b*x)*c*d**4*x**3 + 3*e**(b*x)*d**5*x**4),x)*a**2*b*c**3*d**3 - 9*e**(...
```

3.136 $\int \frac{\Gamma(3,a+bx)}{(c+dx)^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 142

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^5}\,dx = \frac{b^4(bc-ad)^2e^{-a+\frac{bc}{d}}\Gamma\left(-3,\frac{b(c+dx)}{d}\right)}{4d^7} - \frac{b^4(bc-ad)e^{-a+\frac{bc}{d}}\Gamma\left(-2,\frac{b(c+dx)}{d}\right)}{2d^6} + \frac{b^4e^{-a+\frac{bc}{d}}\Gamma\left(-1,\frac{b(c+dx)}{d}\right)}{4d^5} - \frac{\Gamma(3,a+bx)}{4d(c+dx)^4}$$

output
```
1/4*b*(-a*d+b*c)^2*exp(-a+b*c/d)/(d*x+c)^3/d^4*Ei(4,b*(d*x+c)/d)-1/2*b^2*(
-a*d+b*c)*exp(-a+b*c/d)/(d*x+c)^2/d^4*Ei(3,b*(d*x+c)/d)+1/4*b^3*exp(-a+b*c
/d)/(d*x+c)/d^4*Ei(2,b*(d*x+c)/d)-1/2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/
d/(d*x+c)^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 328 vs. 2(142) = 284.

Time = 0.18 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.31

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^5}\,dx$$
$$= \frac{\frac{bde^{-a-bx}(2d^2(bc-ad)^2-bd(b^2c^2-2(-3+a)bcd+(-6+a)ad^2)(c+dx)+b^2(b^2c^2-2(-3+a)bcd+(6-6a+a^2)d^2)(c+dx)^2)}{(c+dx)^3}+b^6c^2e^{-a+\frac{bc}{d}}\text{Ex}}{}$$

input
```
Integrate[Gamma[3, a + b*x]/(c + d*x)^5,x]
```

output
```
((b*d*E^(-a - b*x)*(2*d^2*(b*c - a*d)^2 - b*d*(b^2*c^2 - 2*(-3 + a)*b*c*d
+ (-6 + a)*a*d^2)*(c + d*x) + b^2*(b^2*c^2 - 2*(-3 + a)*b*c*d + (6 - 6*a +
 a^2)*d^2)*(c + d*x)^2))/(c + d*x)^3 + b^6*c^2*E^(-a + (b*c)/d)*ExpIntegra
lEi[-((b*(c + d*x))/d)] + 6*b^5*c*d*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c
 + d*x))/d)] - 2*a*b^5*c*d*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/
d)] + 6*b^4*d^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)] - 6*a*b
^4*d^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)] + a^2*b^4*d^2*E^
(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)] - (6*d^6*Gamma[3, a + b*x
])/(c + d*x)^4)/(24*d^7)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 341 vs. 2(142) = 284.

Time = 0.82 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.40, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(3,a+bx)}{(c+dx)^5}\,dx$$
$$\Big\downarrow \; 7119$$

$$-\frac{b\int \frac{e^{-a-bx}(a+bx)^2}{(c+dx)^4}dx}{4d}-\frac{\Gamma(3,a+bx)}{4d(c+dx)^4}$$

$$\Big\downarrow \text{2629}$$

$$-\frac{b\int \left(\frac{e^{-a-bx}b^2}{d^2(c+dx)^2}-\frac{2(bc-ad)e^{-a-bx}b}{d^2(c+dx)^3}+\frac{(ad-bc)^2e^{-a-bx}}{d^2(c+dx)^4}\right)dx}{4d}-\frac{\Gamma(3,a+bx)}{4d(c+dx)^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{b^3(bc-ad)^2e^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{6d^6}-\frac{b^3(bc-ad)e^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^5}-\frac{b^3e^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^4}\right.}{4d}$$

$$\frac{\Gamma(3,a+bx)}{4d(c+dx)^4}$$

input
```
Int[Gamma[3, a + b*x]/(c + d*x)^5,x]
```

output
```
-1/4*(b*(-1/3*((b*c - a*d)^2*E^(-a - b*x))/(d^3*(c + d*x)^3) + (b*(b*c - a
*d)*E^(-a - b*x))/(d^3*(c + d*x)^2) + (b*(b*c - a*d)^2*E^(-a - b*x))/(6*d^
4*(c + d*x)^2) - (b^2*E^(-a - b*x))/(d^3*(c + d*x)) - (b^2*(b*c - a*d)*E^(
-a - b*x))/(d^4*(c + d*x)) - (b^2*(b*c - a*d)^2*E^(-a - b*x))/(6*d^5*(c +
d*x)) - (b^3*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^4 - (b^
3*(b*c - a*d)*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^5 - (b
^3*(b*c - a*d)^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/(6*d^
6)))/d - Gamma[3, a + b*x]/(4*d*(c + d*x)^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 962 vs. $2(167) = 334$.

Time = 0.89 (sec) , antiderivative size = 963, normalized size of antiderivative = 6.78

method	result
derivativedivides	$-\frac{-\frac{b^5\left(-\frac{e^{-bx-a}}{2\left(-bx-a+\frac{ad-cb}{d}\right)^2}-\frac{e^{-bx-a}}{2\left(-bx-a+\frac{ad-cb}{d}\right)}-\frac{e^{-\frac{ad-cb}{d}}\operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{2}\right)}{d^5}+\frac{2b^5(ad-cb)\left(-\frac{e^{-bx-}}{3\left(-bx-a+\frac{a}{}\right.}\right.}{}\cdots}{}$
default	$-\frac{-\frac{b^5\left(-\frac{e^{-bx-a}}{2\left(-bx-a+\frac{ad-cb}{d}\right)^2}-\frac{e^{-bx-a}}{2\left(-bx-a+\frac{ad-cb}{d}\right)}-\frac{e^{-\frac{ad-cb}{d}}\operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{2}\right)}{d^5}+\frac{2b^5(ad-cb)\left(-\frac{e^{-bx-}}{3\left(-bx-a+\frac{a}{}\right.}\right.}{}\cdots}{}$
risch	Expression too large to display

input
```
int(2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/(d*x+c)^5,x,method=_RETURNVERBOS
E)
```

output
```
-1/b*(-b^5/d^5*(-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/2*exp(-b*x-a)/(-
b*x-a+(a*d-b*c)/d)-1/2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))+2*b^5*(a
*d-b*c)/d^6*(-1/3*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^3-1/6*exp(-b*x-a)/(-b*x
-a+(a*d-b*c)/d)^2-1/6*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/6*exp(-(a*d-b*c)/
d)*Ei(1,b*x+a-(a*d-b*c)/d))-b^5*(a*d-b*c)^2/d^7*(-1/4*exp(-b*x-a)/(-b*x-a+
(a*d-b*c)/d)^4-1/12*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^3-1/24*exp(-b*x-a)/(-
b*x-a+(a*d-b*c)/d)^2-1/24*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/24*exp(-(a*d-
b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))-2*b^5/d^5*(-1/4*exp(-b*x-a)/(-b*x-a+(a*d-
b*c)/d)^4-1/12*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^3-1/24*exp(-b*x-a)/(-b*x-a
+(a*d-b*c)/d)^2-1/24*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/24*exp(-(a*d-b*c)/
d)*Ei(1,b*x+a-(a*d-b*c)/d))+2*b^5/d^5*(-1/3*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/
d)^3-1/6*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/6*exp(-b*x-a)/(-b*x-a+(a*d-b
*c)/d)-1/6*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))-2*(a*d-b*c)/d^6*b^5*
(-1/4*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^4-1/12*exp(-b*x-a)/(-b*x-a+(a*d-b*c
)/d)^3-1/24*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/24*exp(-b*x-a)/(-b*x-a+(a
*d-b*c)/d)-1/24*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(131) = 262$.

Time = 0.15 (sec) , antiderivative size = 572, normalized size of antiderivative = 4.03

$$\int \frac{\Gamma(3, a+bx)}{(c+dx)^5}\,dx =$$

$$-\frac{6\,d^6\Gamma(3, bx+a) - (b^6c^6 - 2\,(a-3)b^5c^5d + (a^2-6\,a+6)b^4c^4d^2 + (b^6c^2d^4 - 2\,(a-3)b^5cd^5 + (a^2-6\,a

output
```
-1/24*(6*d^6*gamma(3, b*x + a) - (b^6*c^6 - 2*(a - 3)*b^5*c^5*d + (a^2 - 6
*a + 6)*b^4*c^4*d^2 + (b^6*c^2*d^4 - 2*(a - 3)*b^5*c*d^5 + (a^2 - 6*a + 6)
*b^4*d^6)*x^4 + 4*(b^6*c^3*d^3 - 2*(a - 3)*b^5*c^2*d^4 + (a^2 - 6*a + 6)*b
^4*c*d^5)*x^3 + 6*(b^6*c^4*d^2 - 2*(a - 3)*b^5*c^3*d^3 + (a^2 - 6*a + 6)*b
^4*c^2*d^4)*x^2 + 4*(b^6*c^5*d - 2*(a - 3)*b^5*c^4*d^2 + (a^2 - 6*a + 6)*b
^4*c^3*d^3)*x)*Ei(-(b*d*x + b*c)/d)*e^((b*c - a*d)/d) - (b^5*c^5*d - (2*a
- 5)*b^4*c^4*d^2 + (a^2 - 4*a + 2)*b^3*c^3*d^3 - (a^2 - 2*a)*b^2*c^2*d^4 +
 2*a^2*b*c*d^5 + (b^5*c^2*d^4 - 2*(a - 3)*b^4*c*d^5 + (a^2 - 6*a + 6)*b^3*
d^6)*x^3 + (3*b^5*c^3*d^3 - (6*a - 17)*b^4*c^2*d^4 + (3*a^2 - 16*a + 12)*b
^3*c*d^5 - (a^2 - 6*a)*b^2*d^6)*x^2 + (3*b^5*c^4*d^2 - 2*(3*a - 8)*b^4*c^3
*d^3 + (3*a^2 - 14*a + 8)*b^3*c^2*d^4 - 2*(a^2 - 4*a)*b^2*c*d^5 + 2*a^2*b*
d^6)*x)*e^(-b*x - a))/(d^11*x^4 + 4*c*d^10*x^3 + 6*c^2*d^9*x^2 + 4*c^3*d^8
*x + c^4*d^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\Gamma(3, a + bx)}{(c + dx)^5} \, dx = \text{Timed out}$$

input
```
integrate(uppergamma(3,b*x+a)/(d*x+c)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\Gamma(3, a + bx)}{(c + dx)^5} \, dx = \int \frac{\Gamma(3, bx + a)}{(dx + c)^5} \, dx$$

input
```
integrate(gamma(3,b*x+a)/(d*x+c)^5,x, algorithm="maxima")
```

output
```
integrate(gamma(3, b*x + a)/(d*x + c)^5, x)
```

Giac [F]

$$\int \frac{\Gamma(3, a+bx)}{(c+dx)^5}\,dx = \int \frac{\Gamma(3, bx+a)}{(dx+c)^5}\,dx$$

input
```
integrate(gamma(3,b*x+a)/(d*x+c)^5,x, algorithm="giac")
```

output
```
integrate(gamma(3, b*x + a)/(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(3, a+bx)}{(c+dx)^5}\,dx = \int \frac{2\,\mathrm{e}^{-a-b\,x}\,\left(a+b\,x+\frac{(a+b\,x)^2}{2}+1\right)}{(c+d\,x)^5}\,dx$$

input
```
int((2*exp(- a - b*x)*(a + b*x + (a + b*x)^2/2 + 1))/(c + d*x)^5,x)
```

output
```
int((2*exp(- a - b*x)*(a + b*x + (a + b*x)^2/2 + 1))/(c + d*x)^5, x)
```

Reduce [F]

$$\int \frac{\Gamma(3, a+bx)}{(c+dx)^5}\,dx = \text{too large to display}$$

input
```
int(2*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/(d*x+c)^5,x)
```

output
```
( - e**(b*x)*int(x/(e**(b*x)*b*c**6 + 5*e**(b*x)*b*c**5*d*x + 10*e**(b*x)*
b*c**4*d**2*x**2 + 10*e**(b*x)*b*c**3*d**3*x**3 + 5*e**(b*x)*b*c**2*d**4*x
**4 + e**(b*x)*b*c*d**5*x**5 + 4*e**(b*x)*c**5*d + 20*e**(b*x)*c**4*d**2*x
 + 40*e**(b*x)*c**3*d**3*x**2 + 40*e**(b*x)*c**2*d**4*x**3 + 20*e**(b*x)*c
*d**5*x**4 + 4*e**(b*x)*d**6*x**5),x)*a**2*b**2*c**5*d**2 - 4*e**(b*x)*int
(x/(e**(b*x)*b*c**6 + 5*e**(b*x)*b*c**5*d*x + 10*e**(b*x)*b*c**4*d**2*x**2
 + 10*e**(b*x)*b*c**3*d**3*x**3 + 5*e**(b*x)*b*c**2*d**4*x**4 + e**(b*x)*b
*c*d**5*x**5 + 4*e**(b*x)*c**5*d + 20*e**(b*x)*c**4*d**2*x + 40*e**(b*x)*c
**3*d**3*x**2 + 40*e**(b*x)*c**2*d**4*x**3 + 20*e**(b*x)*c*d**5*x**4 + 4*e
**(b*x)*d**6*x**5),x)*a**2*b**2*c**4*d**3*x - 6*e**(b*x)*int(x/(e**(b*x)*b
*c**6 + 5*e**(b*x)*b*c**5*d*x + 10*e**(b*x)*b*c**4*d**2*x**2 + 10*e**(b*x)
*b*c**3*d**3*x**3 + 5*e**(b*x)*b*c**2*d**4*x**4 + e**(b*x)*b*c*d**5*x**5 +
 4*e**(b*x)*c**5*d + 20*e**(b*x)*c**4*d**2*x + 40*e**(b*x)*c**3*d**3*x**2
+ 40*e**(b*x)*c**2*d**4*x**3 + 20*e**(b*x)*c*d**5*x**4 + 4*e**(b*x)*d**6*x
**5),x)*a**2*b**2*c**3*d**4*x**2 - 4*e**(b*x)*int(x/(e**(b*x)*b*c**6 + 5*e
**(b*x)*b*c**5*d*x + 10*e**(b*x)*b*c**4*d**2*x**2 + 10*e**(b*x)*b*c**3*d**
3*x**3 + 5*e**(b*x)*b*c**2*d**4*x**4 + e**(b*x)*b*c*d**5*x**5 + 4*e**(b*x)
*c**5*d + 20*e**(b*x)*c**4*d**2*x + 40*e**(b*x)*c**3*d**3*x**2 + 40*e**(b*
x)*c**2*d**4*x**3 + 20*e**(b*x)*c*d**5*x**4 + 4*e**(b*x)*d**6*x**5),x)*a**
2*b**2*c**2*d**5*x**3 - e**(b*x)*int(x/(e**(b*x)*b*c**6 + 5*e**(b*x)*b*...
```

3.137 $\int (c+dx)^3\Gamma(-1,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 139

$$\int (c+dx)^3\Gamma(-1,a+bx)\,dx = -\frac{3d(bc-ad)^2e^{-a-bx}}{2b^4} - \frac{(bc-ad)^4\Gamma(-1,a+bx)}{4b^4d} + \frac{(c+dx)^4\Gamma(-1,a+bx)}{4d} - \frac{(bc-ad)^3\Gamma(0,a+bx)}{b^4} - \frac{d^2(bc-ad)\Gamma(2,a+bx)}{b^4} - \frac{d^3\Gamma(3,a+bx)}{4b^4}$$

output
```
-3/2*d*(-a*d+b*c)^2*exp(-b*x-a)/b^4-1/4*(-a*d+b*c)^4/(b*x+a)*Ei(2,b*x+a)/b
^4/d+1/4*(d*x+c)^4/(b*x+a)*Ei(2,b*x+a)/d-(-a*d+b*c)^3*Ei(1,b*x+a)/b^4-d^2*
(-a*d+b*c)*exp(-b*x-a)*(b*x+a+1)/b^4-1/2*d^3*exp(-b*x-a)*(1+b*x+a+1/2*(b*x
+a)^2)/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 282 vs. 2(139) = 278.

Time = 0.33 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.03

$$\int (c+dx)^3\Gamma(-1,a+bx)\,dx$$

$$= \frac{(4(1+a)b^3c^3 - 6a(2+a)b^2c^2d + 4a^2(3+a)bcd^2 - a^3(4+a)d^3)\,\text{ExpIntegralEi}(-a-bx) + \frac{e^{-a-bx}(a^3(4bc\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(c + d*x)^3*Gamma[-1, a + b*x],x]
```

output
```
((4*(1 + a)*b^3*c^3 - 6*a*(2 + a)*b^2*c^2*d + 4*a^2*(3 + a)*b*c*d^2 - a^3*
(4 + a)*d^3)*ExpIntegralEi[-a - b*x] + (E^(-a - b*x)*(a^3*(4*b*c - 3*d)*d^
2 - a^4*d^3 + a^2*d*(-6*b^2*c^2 + 2*d^2 + b*d*(8*c - d*x)) + a*(4*b^3*c^3
- 4*b*c*d^2 - 2*d^3 + b^2*d*(-6*c^2 + 4*c*d*x + d^2*x^2)) - b*d*x*(2*d^2 +
 2*b*d*(2*c + d*x) + b^2*(6*c^2 + 4*c*d*x + d^2*x^2)) + b^4*E^(a + b*x)*x*
(a + b*x)*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Gamma[-1, a + b*x]))
/(a + b*x))/(4*b^4)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\Gamma(-1,a+bx)\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{b\int \frac{e^{-a-bx}(c+dx)^4}{(a+bx)^2}dx}{4d}+\frac{(c+dx)^4\Gamma(-1,a+bx)}{4d}$$

$$\Big\downarrow \text{2629}$$

$$\frac{b\int \left(\frac{e^{-a-bx}(a+bx)^2d^4}{b^4}+\frac{4(bc-ad)e^{-a-bx}(a+bx)d^3}{b^4}+\frac{6(bc-ad)^2e^{-a-bx}d^2}{b^4}+\frac{4(bc-ad)^3e^{-a-bx}d}{b^4(a+bx)}+\frac{(bc-ad)^4e^{-a-bx}}{b^4(a+bx)^2}\right)dx}{4d}+\frac{(c+dx)^4\Gamma(-1,a+bx)}{4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{4d^3e^{-a-bx}(bc-ad)}{b^5}-\frac{4d^3e^{-a-bx}(a+bx)(bc-ad)}{b^5}-\frac{6d^2e^{-a-bx}(bc-ad)^2}{b^5}+\frac{4d(bc-ad)^3\,\text{ExpIntegralEi}(-a-bx)}{b^5}-\frac{(bc-ad)^4\,\text{ExpInteg}}{b^5}\right.}{4d}$$

$$\frac{(c+dx)^4\Gamma(-1,a+bx)}{4d}$$

input
```
Int[(c + d*x)^3*Gamma[-1, a + b*x],x]
```

output
```
(b*((-2*d^4*E^(-a - b*x))/b^5 - (4*d^3*(b*c - a*d)*E^(-a - b*x))/b^5 - (6*
d^2*(b*c - a*d)^2*E^(-a - b*x))/b^5 - ((b*c - a*d)^4*E^(-a - b*x))/(b^5*(a
 + b*x)) - (2*d^4*E^(-a - b*x)*(a + b*x))/b^5 - (4*d^3*(b*c - a*d)*E^(-a -
 b*x)*(a + b*x))/b^5 - (d^4*E^(-a - b*x)*(a + b*x)^2)/b^5 + (4*d*(b*c - a*
d)^3*ExpIntegralEi[-a - b*x])/b^5 - ((b*c - a*d)^4*ExpIntegralEi[-a - b*x]
)/b^5))/(4*d) + ((c + d*x)^4*Gamma[-1, a + b*x])/(4*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [F]

$$\int \frac{(dx+c)^3 \operatorname{expIntegral}_2(bx+a)}{bx+a}dx$$

input
```
int((d*x+c)^3/(b*x+a)*Ei(2,b*x+a),x)
```

output
```
int((d*x+c)^3/(b*x+a)*Ei(2,b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(130) = 260$.

Time = 0.10 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.59

$$\int (c+dx)^3\Gamma(-1,a+bx)\,dx = -\frac{(b^3d^3x^3+4\,b^3c^3-6\,ab^2c^2d+4\,(a^2+a)bcd^2-(a^3+2\,a^2-2\,a)d^3+(4\,b^3cd^2-(a-2)b^2d^3)x^2+(6\,b^3c}{}$$

input
```
integrate((d*x+c)^3*gamma(-1,b*x+a),x, algorithm="fricas")
```

output
```
-1/4*((b^3*d^3*x^3 + 4*b^3*c^3 - 6*a*b^2*c^2*d + 4*(a^2 + a)*b*c*d^2 - (a^
3 + 2*a^2 - 2*a)*d^3 + (4*b^3*c*d^2 - (a - 2)*b^2*d^3)*x^2 + (6*b^3*c^2*d
- 4*(a - 1)*b^2*c*d^2 + (a^2 + 2)*b*d^3)*x)*e^(-b*x - a) - (b^5*d^3*x^5 +
4*(a^2 + a)*b^3*c^3 - 6*(a^3 + 2*a^2)*b^2*c^2*d + 4*(a^4 + 3*a^3)*b*c*d^2
+ (4*b^5*c*d^2 + a*b^4*d^3)*x^4 - (a^5 + 4*a^4)*d^3 + 2*(3*b^5*c^2*d + 2*a
*b^4*c*d^2)*x^3 + 2*(2*b^5*c^3 + 3*a*b^4*c^2*d)*x^2 + (4*(2*a + 1)*b^4*c^3
 - 6*(a^2 + 2*a)*b^3*c^2*d + 4*(a^3 + 3*a^2)*b^2*c*d^2 - (a^4 + 4*a^3)*b*d
^3)*x)*gamma(-1, b*x + a))/(b^5*x + a*b^4)
```

Sympy [F]

$$\int (c+dx)^3\Gamma(-1,a+bx)\,dx = \int \frac{(c+dx)^3\,\mathrm{E}_2\,(a+bx)}{a+bx}\,dx$$

input
```
integrate((d*x+c)**3*uppergamma(-1,b*x+a),x)
```

output
```
Integral((c + d*x)**3*expint(2, a + b*x)/(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^3\Gamma(-1,a+bx)\,dx=\int (dx+c)^3\Gamma(-1,bx+a)\,dx$$

input
```
integrate((d*x+c)^3*gamma(-1,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(-1, b*x + a) + Ei(-b*x - a))*c^3/b + integrate(d^3*x^3*ga
mma(-1, b*x + a) + 3*c*d^2*x^2*gamma(-1, b*x + a) + 3*c^2*d*x*gamma(-1, b*
x + a), x)
```

Giac [F]

$$\int (c+dx)^3\Gamma(-1,a+bx)\,dx=\int (dx+c)^3\Gamma(-1,bx+a)\,dx$$

input
```
integrate((d*x+c)^3*gamma(-1,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*gamma(-1, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.17

$$\int (c+dx)^3\Gamma(-1,a+bx)\,dx=\int \frac{\mathrm{expint}(2,a+b\,x)\,\left(c+d\,x\right)^3}{a+b\,x}\,dx$$

input
```
int((expint(2, a + b*x)*(c + d*x)^3)/(a + b*x),x)
```

output
```
int((expint(2, a + b*x)*(c + d*x)^3)/(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^3\Gamma(-1,a+bx)\,dx = \left(\int \frac{ei(2,bx+a)}{bx+a}dx\right)c^3 + \left(\int \frac{ei(2,bx+a)\,x^3}{bx+a}dx\right)d^3 + 3\left(\int \frac{ei(2,bx+a)\,x^2}{bx+a}dx\right)c\,d^2 + 3\left(\int \frac{ei(2,bx+a)\,x}{bx+a}dx\right)c^2d$$

input
```
int((d*x+c)^3/(b*x+a)*Ei(2,b*x+a),x)
```

output
```
int(ei(2,a + b*x)/(a + b*x),x)*c**3 + int((ei(2,a + b*x)*x**3)/(a + b*x),x
)*d**3 + 3*int((ei(2,a + b*x)*x**2)/(a + b*x),x)*c*d**2 + 3*int((ei(2,a +
b*x)*x)/(a + b*x),x)*c**2*d
```

3.138 $\int (c+dx)^2\Gamma(-1,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 118

$$\int (c+dx)^2\Gamma(-1,a+bx)\,dx = -\frac{d(3bc-2ad)e^{-a-bx}}{3b^3}-\frac{(bc-ad)^3\Gamma(-1,a+bx)}{3b^3d}+\frac{(c+dx)^3\Gamma(-1,a+bx)}{3d}-\frac{(bc-ad)^2\Gamma(0,a+bx)}{b^3}-\frac{d^2e^{-a}\Gamma(2,bx)}{3b^3}$$

output
```
-1/3*d*(-2*a*d+3*b*c)*exp(-b*x-a)/b^3-1/3*(-a*d+b*c)^3/(b*x+a)*Ei(2,b*x+a)
/b^3/d+1/3*(d*x+c)^3/(b*x+a)*Ei(2,b*x+a)/d-(-a*d+b*c)^2*Ei(1,b*x+a)/b^3-1/
3*d^2*exp(-b*x)*(b*x+1)/b^3/exp(a)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.45

$$\int (c+dx)^2\Gamma(-1,a+bx)\,dx$$
$$= \frac{(3(1+a)b^2c^2-3a(2+a)bcd+a^2(3+a)d^2)\,\text{ExpIntegralEi}(-a-bx)+\frac{e^{-a-bx}(a^3d^2+a^2d(-3bc+2d)-bdx(3bc+d}{}}{3b^3}$$

input
```
Integrate[(c + d*x)^2*Gamma[-1, a + b*x],x]
```

output
```
((3*(1 + a)*b^2*c^2 - 3*a*(2 + a)*b*c*d + a^2*(3 + a)*d^2)*ExpIntegralEi[-
a - b*x] + (E^(-a - b*x)*(a^3*d^2 + a^2*d*(-3*b*c + 2*d) - b*d*x*(3*b*c +
d + b*d*x) + a*(3*b^2*c^2 - d^2 + b*d*(-3*c + d*x)) + b^3*E^(a + b*x)*x*(a
 + b*x)*(3*c^2 + 3*c*d*x + d^2*x^2)*Gamma[-1, a + b*x]))/(a + b*x))/(3*b^3
)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.49, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\Gamma(-1,a+bx)\,dx$$

$$\Big\downarrow \text{ 7119}$$

$$\frac{b\int \frac{e^{-a-bx}(c+dx)^3}{(a+bx)^2}dx}{3d}+\frac{(c+dx)^3\Gamma(-1,a+bx)}{3d}$$

$$\Big\downarrow \text{ 2629}$$

$$\frac{b\int \left(\frac{e^{-a-bx}xd^3}{b^2}+\frac{(3bc-2ad)e^{-a-bx}d^2}{b^3}+\frac{3(bc-ad)^2e^{-a-bx}d}{b^3(a+bx)}+\frac{(bc-ad)^3e^{-a-bx}}{b^3(a+bx)^2}\right)dx}{3d}+\frac{(c+dx)^3\Gamma(-1,a+bx)}{3d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\left(-\frac{d^2e^{-a-bx}(3bc-2ad)}{b^4}+\frac{3d(bc-ad)^2\text{ExpIntegralEi}(-a-bx)}{b^4}-\frac{(bc-ad)^3\text{ExpIntegralEi}(-a-bx)}{b^4}-\frac{e^{-a-bx}(bc-ad)^3}{b^4(a+bx)}-\frac{d^3e^{-a-bx}}{b^4}-\right.}{3d}$$
$$\frac{(c+dx)^3\Gamma(-1,a+bx)}{3d}$$

input
```
Int[(c + d*x)^2*Gamma[-1, a + b*x],x]
```

output
```
(b*(-((d^3*E^(-a - b*x))/b^4) - (d^2*(3*b*c - 2*a*d)*E^(-a - b*x))/b^4 - (
d^3*E^(-a - b*x)*x)/b^3 - ((b*c - a*d)^3*E^(-a - b*x))/(b^4*(a + b*x)) + (
3*d*(b*c - a*d)^2*ExpIntegralEi[-a - b*x])/b^4 - ((b*c - a*d)^3*ExpIntegra
lEi[-a - b*x])/b^4))/(3*d) + ((c + d*x)^3*Gamma[-1, a + b*x])/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [F]

$$\int \frac{(dx + c)^2 \operatorname{expIntegral}_2(bx + a)}{bx + a} dx$$

input
```
int((d*x+c)^2/(b*x+a)*Ei(2,b*x+a),x)
```

output
```
int((d*x+c)^2/(b*x+a)*Ei(2,b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(108) = 216$.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.89

$$\int (c+dx)^2\Gamma(-1,a+bx)\,dx = -\frac{(b^2d^2x^2+3\,b^2c^2-3\,abcd+(a^2+a)d^2+(3\,b^2cd-(a-1)bd^2)x)e^{(-bx-a)}-(b^4d^2x^4+3\,(a^2+a)b^2c^2-$$

input
```
integrate((d*x+c)^2*gamma(-1,b*x+a),x, algorithm="fricas")
```

output
```
-1/3*((b^2*d^2*x^2 + 3*b^2*c^2 - 3*a*b*c*d + (a^2 + a)*d^2 + (3*b^2*c*d -
(a - 1)*b*d^2)*x)*e^(-b*x - a) - (b^4*d^2*x^4 + 3*(a^2 + a)*b^2*c^2 - 3*(a
^3 + 2*a^2)*b*c*d + (3*b^4*c*d + a*b^3*d^2)*x^3 + (a^4 + 3*a^3)*d^2 + 3*(b
^4*c^2 + a*b^3*c*d)*x^2 + (3*(2*a + 1)*b^3*c^2 - 3*(a^2 + 2*a)*b^2*c*d + (
a^3 + 3*a^2)*b*d^2)*x)*gamma(-1, b*x + a))/(b^4*x + a*b^3)
```

Sympy [F]

$$\int (c+dx)^2\Gamma(-1,a+bx)\,dx = \int \frac{(c+dx)^2\,\mathrm{E}_2\,(a+bx)}{a+bx}\,dx$$

input
```
integrate((d*x+c)**2*uppergamma(-1,b*x+a),x)
```

output
```
Integral((c + d*x)**2*expint(2, a + b*x)/(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^2\Gamma(-1,a+bx)\,dx = \int (dx+c)^2\Gamma(-1,bx+a)\,dx$$

input
```
integrate((d*x+c)^2*gamma(-1,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(-1, b*x + a) + Ei(-b*x - a))*c^2/b + integrate(d^2*x^2*ga
mma(-1, b*x + a) + 2*c*d*x*gamma(-1, b*x + a), x)
```

Giac [F]

$$\int (c+dx)^2\Gamma(-1,a+bx)\,dx = \int (dx+c)^2\Gamma(-1,bx+a)\,dx$$

input
```
integrate((d*x+c)^2*gamma(-1,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*gamma(-1, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.20

$$\int (c+dx)^2\Gamma(-1,a+bx)\,dx = \int \frac{\mathrm{expint}(2,a+b\,x)\,\left(c+d\,x\right)^2}{a+b\,x}\,dx$$

input
```
int((expint(2, a + b*x)*(c + d*x)^2)/(a + b*x),x)
```

output
```
int((expint(2, a + b*x)*(c + d*x)^2)/(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^2\Gamma(-1,a+bx)\,dx = \left(\int \frac{ei(2,bx+a)}{bx+a}dx\right)c^2 + \left(\int \frac{ei(2,bx+a)\,x^2}{bx+a}dx\right)d^2 + 2\left(\int \frac{ei(2,bx+a)\,x}{bx+a}dx\right)cd$$

input
```
int((d*x+c)^2/(b*x+a)*Ei(2,b*x+a),x)
```

output
```
int(ei(2,a + b*x)/(a + b*x),x)*c**2 + int((ei(2,a + b*x)*x**2)/(a + b*x),x
)*d**2 + 2*int((ei(2,a + b*x)*x)/(a + b*x),x)*c*d
```

3.139 $\int (c+dx)\Gamma(-1,a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int (c+dx)\Gamma(-1,a+bx)\,dx = -\frac{de^{-a-bx}}{2b^2} - \frac{(bc-ad)^2\Gamma(-1,a+bx)}{2b^2d} + \frac{(c+dx)^2\Gamma(-1,a+bx)}{2d} - \frac{(bc-ad)\Gamma(0,a+bx)}{b^2}$$

output
```
-1/2*d*exp(-b*x-a)/b^2-1/2*(-a*d+b*c)^2/(b*x+a)*Ei(2,b*x+a)/b^2/d+1/2*(d*x
+c)^2/(b*x+a)*Ei(2,b*x+a)/d-(-a*d+b*c)*Ei(1,b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.30

$$\int (c+dx)\Gamma(-1,a+bx)\,dx = \frac{e^{-a-bx}\left(2abc-ad-a^2d-bdx+(2(1+a)bc-a(2+a)d)e^{a+bx}(a+bx)\,\text{ExpIntegralEi}(-a-bx)+b^2e^{a}\right.}{2b^2(a+bx)}$$

input
```
Integrate[(c + d*x)*Gamma[-1, a + b*x],x]
```

output
```
(E^(-a - b*x)*(2*a*b*c - a*d - a^2*d - b*d*x + (2*(1 + a)*b*c - a*(2 + a)*
d)*E^(a + b*x)*(a + b*x)*ExpIntegralEi[-a - b*x] + b^2*E^(a + b*x)*x*(a +
b*x)*(2*c + d*x)*Gamma[-1, a + b*x]))/(2*b^2*(a + b*x))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.47, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\Gamma(-1,a+bx)\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{b\int \frac{e^{-a-bx}(c+dx)^2}{(a+bx)^2}dx}{2d}+\frac{(c+dx)^2\Gamma(-1,a+bx)}{2d}$$

$$\Big\downarrow \text{2629}$$

$$\frac{b\int \left(\frac{e^{-a-bx}d^2}{b^2}+\frac{2(bc-ad)e^{-a-bx}d}{b^2(a+bx)}+\frac{(bc-ad)^2e^{-a-bx}}{b^2(a+bx)^2}\right)dx}{2d}+\frac{(c+dx)^2\Gamma(-1,a+bx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{2d(bc-ad)\operatorname{ExpIntegralEi}(-a-bx)}{b^3}-\frac{(bc-ad)^2\operatorname{ExpIntegralEi}(-a-bx)}{b^3}-\frac{e^{-a-bx}(bc-ad)^2}{b^3(a+bx)}-\frac{d^2e^{-a-bx}}{b^3}\right)}{2d}+\frac{(c+dx)^2\Gamma(-1,a+bx)}{2d}$$

input
```
Int[(c + d*x)*Gamma[-1, a + b*x],x]
```

output
```
(b*(-((d^2*E^(-a - b*x))/b^3) - ((b*c - a*d)^2*E^(-a - b*x))/(b^3*(a + b*x
)) + (2*d*(b*c - a*d)*ExpIntegralEi[-a - b*x])/b^3 - ((b*c - a*d)^2*ExpInt
egralEi[-a - b*x])/b^3))/(2*d) + ((c + d*x)^2*Gamma[-1, a + b*x])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [F]

$$\int \frac{(dx+c)\,\text{expIntegral}_2\,(bx+a)}{bx+a}dx$$

input
```
int((d*x+c)/(b*x+a)*Ei(2,b*x+a),x)
```

output
```
int((d*x+c)/(b*x+a)*Ei(2,b*x+a),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.38

$$\int (c+dx)\Gamma(-1,a+bx)\,dx = -\frac{(bdx+2\,bc-ad)e^{(-bx-a)}-(b^3dx^3+2\,(a^2+a)bc+(2\,b^3c+ab^2d)x^2-(a^3+2\,a^2)d+(2\,(2\,a+1)b^2c}{2\,(b^3x+ab^2)}$$

input
```
integrate((d*x+c)*gamma(-1,b*x+a),x, algorithm="fricas")
```

output
```
-1/2*((b*d*x + 2*b*c - a*d)*e^(-b*x - a) - (b^3*d*x^3 + 2*(a^2 + a)*b*c +
(2*b^3*c + a*b^2*d)*x^2 - (a^3 + 2*a^2)*d + (2*(2*a + 1)*b^2*c - (a^2 + 2*
a)*b*d)*x)*gamma(-1, b*x + a))/(b^3*x + a*b^2)
```

Sympy [F]

$$\int (c+dx)\Gamma(-1,a+bx)\,dx=\int \frac{(c+dx)\,\mathrm{E}_2\,(a+bx)}{a+bx}\,dx$$

input
```
integrate((d*x+c)*uppergamma(-1,b*x+a),x)
```

output
```
Integral((c + d*x)*expint(2, a + b*x)/(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)\Gamma(-1,a+bx)\,dx=\int (dx+c)\Gamma(-1,bx+a)\;dx$$

input
```
integrate((d*x+c)*gamma(-1,b*x+a),x, algorithm="maxima")
```

output
```
d*integrate(x*gamma(-1, b*x + a), x) + ((b*x + a)*gamma(-1, b*x + a) + Ei(
-b*x - a))*c/b
```

Giac [F]

$$\int (c+dx)\Gamma(-1,a+bx)\,dx=\int (dx+c)\Gamma(-1,bx+a)\;dx$$

input
```
integrate((d*x+c)*gamma(-1,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*gamma(-1, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.25

$$\int (c+dx)\Gamma(-1,a+bx)\,dx=\int \frac{\mathrm{expint}(2,a+b\,x)\ (c+d\,x)}{a+b\,x}\,dx$$

input
```
int((expint(2, a + b*x)*(c + d*x))/(a + b*x),x)
```

output
```
int((expint(2, a + b*x)*(c + d*x))/(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)\Gamma(-1,a+bx)\,dx=\left(\int \frac{ei(2,bx+a)}{bx+a}dx\right)c+\left(\int \frac{ei(2,bx+a)\,x}{bx+a}dx\right)d$$

input
```
int((d*x+c)/(b*x+a)*Ei(2,b*x+a),x)
```

output
```
int(ei(2,a + b*x)/(a + b*x),x)*c + int((ei(2,a + b*x)*x)/(a + b*x),x)*d
```

3.140 $\int \Gamma(-1, a + bx)\, dx$

Optimal result

Integrand size = 7, antiderivative size = 29

$$\int \Gamma(-1, a + bx)\, dx = \frac{(a + bx)\Gamma(-1, a + bx)}{b} - \frac{\Gamma(0, a + bx)}{b}$$

output
```
Ei(2,b*x+a)/b-Ei(1,b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

$$\int \Gamma(-1, a + bx)\, dx = \frac{ae^{-a-bx}}{b(a + bx)} + \frac{(1 + a)\,\text{ExpIntegralEi}(-a - bx)}{b} + x\Gamma(-1, a + bx)$$

input
```
Integrate[Gamma[-1, a + b*x],x]
```

output
```
(a*E^(-a - b*x))/(b*(a + b*x)) + ((1 + a)*ExpIntegralEi[-a - b*x])/b + x*G
amma[-1, a + b*x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(-1, a + bx) \, dx$$

$$\Big\downarrow \; 7111$$

$$\frac{(a + bx)\Gamma(-1, a + bx)}{b} - \frac{\Gamma(0, a + bx)}{b}$$

input
```
Int[Gamma[-1, a + b*x],x]
```

output
```
((a + b*x)*Gamma[-1, a + b*x])/b - Gamma[0, a + b*x]/b
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [F]

$$\int \frac{\text{expIntegral}_2\left(bx + a\right)}{bx + a} dx$$

input
```
int(1/(b*x+a)*Ei(2,b*x+a),x)
```

output
```
int(1/(b*x+a)*Ei(2,b*x+a),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.79

$$\int \Gamma(-1, a+bx)\,dx = \frac{(b^2x^2 + (2\,a+1)bx + a^2 + a)\Gamma(-1, bx+a) - e^{(-bx-a)}}{b^2x + ab}$$

input
```
integrate(gamma(-1,b*x+a),x, algorithm="fricas")
```

output
```
((b^2*x^2 + (2*a + 1)*b*x + a^2 + a)*gamma(-1, b*x + a) - e^(-b*x - a))/(b
^2*x + a*b)
```

Sympy [F]

$$\int \Gamma(-1, a+bx)\,dx = \int \frac{E_2(a+bx)}{a+bx}\,dx$$

input
```
integrate(uppergamma(-1,b*x+a),x)
```

output
```
Integral(expint(2, a + b*x)/(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \Gamma(-1, a+bx)\,dx = \frac{(bx+a)\Gamma(-1, bx+a) + \mathrm{Ei}(-bx-a)}{b}$$

input
```
integrate(gamma(-1,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(-1, b*x + a) + Ei(-b*x - a))/b
```

Giac [F]

$$\int \Gamma(-1, a + bx)\, dx = \int \Gamma(-1, bx + a)\, dx$$

input
```
integrate(gamma(-1,b*x+a),x, algorithm="giac")
```

output
```
integrate(gamma(-1, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \Gamma(-1, a + bx)\, dx = -\frac{\mathrm{expint}(a + b\, x) - \mathrm{expint}(2, a + b\, x)}{b}$$

input
```
int(expint(2, a + b*x)/(a + b*x),x)
```

output
```
-(expint(a + b*x) - expint(2, a + b*x))/b
```

Reduce [F]

$$\int \Gamma(-1, a + bx)\, dx = \int \frac{ei(2, bx + a)}{bx + a} dx$$

input
```
int(1/(b*x+a)*Ei(2,b*x+a),x)
```

output
```
int(ei(2,a + b*x)/(a + b*x),x)
```

3.141 $\int \frac{\Gamma(-1,a+bx)}{c+dx} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\Gamma(-1,a+bx)}{c+dx} \, dx = \text{Int}\left(\frac{\Gamma(-1,a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(1/(b*x+a)*Ei(2,b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(-1,a+bx)}{c+dx} \, dx = \int \frac{\Gamma(-1,a+bx)}{c+dx} \, dx$$

input
```
Integrate[Gamma[-1, a + b*x]/(c + d*x),x]
```

output
```
Integrate[Gamma[-1, a + b*x]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-1,a+bx)}{c+dx}\,dx$$

$$\big\downarrow \text{ 7120}$$

$$\int \frac{\Gamma(-1,a+bx)}{c+dx}dx$$

input
```
Int[Gamma[-1, a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{\operatorname{expIntegral}_{2}\left(bx+a\right)}{\left(bx+a\right)\left(dx+c\right)}dx$$

input
```
int(1/(b*x+a)*Ei(2,b*x+a)/(d*x+c),x)
```

output
```
int(1/(b*x+a)*Ei(2,b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(-1,a+bx)}{c+dx}\,dx = \int \frac{\Gamma(-1,bx+a)}{dx+c}\,dx$$

input
```
integrate(gamma(-1,b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(gamma(-1, b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(-1,a+bx)}{c+dx}\,dx = \int \frac{\mathrm{E}_2\left(a+bx\right)}{(a+bx)\,(c+dx)}\,dx$$

input
```
integrate(uppergamma(-1,b*x+a)/(d*x+c),x)
```

output
```
Integral(expint(2, a + b*x)/((a + b*x)*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(-1,a+bx)}{c+dx}\,dx = \int \frac{\Gamma(-1,bx+a)}{dx+c}\,dx$$

input
```
integrate(gamma(-1,b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(gamma(-1, b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(-1,a+bx)}{c+dx}\,dx=\int \frac{\Gamma(-1,bx+a)}{dx+c}\,dx$$

input
```
integrate(gamma(-1,b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(gamma(-1, b*x + a)/(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{\Gamma(-1,a+bx)}{c+dx}\,dx=\int \frac{\mathrm{expint}(2,a+b\,x)}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(expint(2, a + b*x)/((a + b*x)*(c + d*x)),x)
```

output
```
int(expint(2, a + b*x)/((a + b*x)*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

$$\int \frac{\Gamma(-1,a+bx)}{c+dx}\,dx=\int \frac{ei(2,bx+a)}{bd\,x^2+adx+bcx+ac}dx$$

input
```
int(1/(b*x+a)*Ei(2,b*x+a)/(d*x+c),x)
```

output
```
int(ei(2,a + b*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)
```

3.142 $\int \frac{\Gamma(-1,a+bx)}{(c+dx)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 98

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^2} dx = \frac{b\Gamma(-1,a+bx)}{d(bc-ad)} - \frac{\Gamma(-1,a+bx)}{d(c+dx)} - \frac{b\Gamma(0,a+bx)}{(bc-ad)^2} + \frac{be^{-a+\frac{bc}{d}}\Gamma\left(0,\frac{b(c+dx)}{d}\right)}{(bc-ad)^2}$$

output
```
b/(b*x+a)*Ei(2,b*x+a)/d/(-a*d+b*c)-1/(b*x+a)*Ei(2,b*x+a)/d/(d*x+c)-b*Ei(1,
b*x+a)/(-a*d+b*c)^2+b*exp(-a+b*c/d)*Ei(1,b*(d*x+c)/d)/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.23

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^2} dx = \frac{b(bc+d-ad)\,\text{ExpIntegralEi}(-a-bx)}{d(bc-ad)^2} - \frac{be^{-a+\frac{bc}{d}}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{(bc-ad)^2} + \frac{\frac{be^{-a-bx}}{(bc-ad)(a+bx)} - \frac{\Gamma(-1,a+bx)}{c+dx}}{d}$$

input
```
Integrate[Gamma[-1, a + b*x]/(c + d*x)^2,x]
```

output
```
(b*(b*c + d - a*d)*ExpIntegralEi[-a - b*x])/(d*(b*c - a*d)^2) - (b*E^(-a +
 (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/(b*c - a*d)^2 + ((b*E^(-a - b
*x))/((b*c - a*d)*(a + b*x)) - Gamma[-1, a + b*x]/(c + d*x))/d
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.38, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{7119}$$

$$-\frac{b\int \frac{e^{-a-bx}}{(a+bx)^2(c+dx)}dx}{d}-\frac{\Gamma(-1,a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\int \left(\frac{e^{-a-bx}d^2}{(bc-ad)^2(c+dx)}-\frac{be^{-a-bx}d}{(bc-ad)^2(a+bx)}+\frac{be^{-a-bx}}{(bc-ad)(a+bx)^2}\right)dx}{d}-\frac{\Gamma(-1,a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{\text{ExpIntegralEi}(-a-bx)}{bc-ad}-\frac{d\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^2}+\frac{de^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{(bc-ad)^2}-\frac{e^{-a-bx}}{(a+bx)(bc-ad)}\right)}{d}-\frac{\Gamma(-1,a+bx)}{d(c+dx)}$$

input
```
Int[Gamma[-1, a + b*x]/(c + d*x)^2,x]
```

output
```
-((b*(-(E^(-a - b*x)/((b*c - a*d)*(a + b*x))) - (d*ExpIntegralEi[-a - b*x]
)/(b*c - a*d)^2 - ExpIntegralEi[-a - b*x]/(b*c - a*d) + (d*E^(-a + (b*c)/d
)*ExpIntegralEi[-((b*(c + d*x))/d)])/(b*c - a*d)^2))/d) - Gamma[-1, a + b*
x]/(d*(c + d*x))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\text{expIntegral}_2(bx+a)}{(bx+a)(dx+c)^2}dx$$

input
```
int(1/(b*x+a)*Ei(2,b*x+a)/(d*x+c)^2,x)
```

output
```
int(1/(b*x+a)*Ei(2,b*x+a)/(d*x+c)^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(97) = 194$.

Time = 0.13 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.56

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^2}\,dx = -\frac{(b^2d^2x^2+abcd+(b^2cd+abd^2)x)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}-(ab^2c^2-(a^2-a)bcd+(b^3cd-(a-1)b^2d^2)x}{ab^2c^3d-2\,a^2bc^2d^2+}$$

input
```
integrate(gamma(-1,b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-((b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*Ei(-(b*d*x + b*c)/d)*e^(
(b*c - a*d)/d) - (a*b^2*c^2 - (a^2 - a)*b*c*d + (b^3*c*d - (a - 1)*b^2*d^2
)*x^2 + (b^3*c^2 + b^2*c*d - (a^2 - a)*b*d^2)*x)*Ei(-b*x - a) - (b^2*c^2 -
 a*b*c*d + (b^2*c*d - a*b*d^2)*x)*e^(-b*x - a) + (a*b^2*c^2 - 2*a^2*b*c*d
+ a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x)*gamma(-1, b*x + a))/(a*
b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a
^2*b*d^4)*x^2 + (b^3*c^3*d - a*b^2*c^2*d^2 - a^2*b*c*d^3 + a^3*d^4)*x)
```

Sympy [F]

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^2}\,dx = \int \frac{\mathrm{E}_2\left(a+bx\right)}{\left(a+bx\right)\left(c+dx\right)^2}\,dx$$

input
```
integrate(uppergamma(-1,b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(expint(2, a + b*x)/((a + b*x)*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^2}\,dx = \int \frac{\Gamma(-1,bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(gamma(-1,b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(gamma(-1, b*x + a)/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^2}\,dx = \int \frac{\Gamma(-1,bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(gamma(-1,b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(gamma(-1, b*x + a)/(d*x + c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.24

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^2}\,dx = \int \frac{\mathrm{expint}(2,a+b\,x)}{(a+b\,x)\,(c+d\,x)^2}\,dx$$

input
```
int(expint(2, a + b*x)/((a + b*x)*(c + d*x)^2),x)
```

output
```
int(expint(2, a + b*x)/((a + b*x)*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{\Gamma(-1, a + bx)}{(c + dx)^2} \, dx = \int \frac{ei(2, bx + a)}{b\, d^2x^3 + a\, d^2x^2 + 2bcd\, x^2 + 2acdx + b\, c^2x + a\, c^2} dx$$

input
```
int(1/(b*x+a)*Ei(2,b*x+a)/(d*x+c)^2,x)
```

output
```
int(ei(2,a + b*x)/(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b*c**2*x + 2*b*c*d*x
**2 + b*d**2*x**3),x)
```

3.143 $\int \frac{\Gamma(-1,a+bx)}{(c+dx)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 153

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^3} dx = \frac{b^2\Gamma(-1,a+bx)}{2d(bc-ad)^2} - \frac{\Gamma(-1,a+bx)}{2d(c+dx)^2} + \frac{b^2e^{-a+\frac{bc}{d}}\Gamma\left(-1,\frac{b(c+dx)}{d}\right)}{2d(bc-ad)^2} - \frac{b^2\Gamma(0,a+bx)}{(bc-ad)^3} + \frac{b^2e^{-a+\frac{bc}{d}}\Gamma\left(0,\frac{b(c+dx)}{d}\right)}{(bc-ad)^3}$$

output
```
1/2*b^2/(b*x+a)*Ei(2,b*x+a)/d/(-a*d+b*c)^2-1/2/(b*x+a)*Ei(2,b*x+a)/d/(d*x+
c)^2+1/2*b*exp(-a+b*c/d)/(d*x+c)*Ei(2,b*(d*x+c)/d)/(-a*d+b*c)^2-b^2*Ei(1,b
*x+a)/(-a*d+b*c)^3+b^2*exp(-a+b*c/d)*Ei(1,b*(d*x+c)/d)/(-a*d+b*c)^3
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^3}\,dx = \frac{1}{2}\left(\frac{b^2e^{-a-bx}}{d(bc-ad)^2(a+bx)} + \frac{be^{-a-bx}}{(bc-ad)^2(c+dx)} + \frac{b^2(-bc+(-2+a)d)\,\text{ExpIntegralEi}(-a-bx)}{d(-bc+ad)^3} + \frac{b^2(bc-(2+a)d)e^{-a+\frac{bc}{d}}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d(bc-ad)^3} - \frac{\Gamma(-1,a+bx)}{d(c+dx)^2}\right)$$

input
```
Integrate[Gamma[-1, a + b*x]/(c + d*x)^3,x]
```

output
```
((b^2*E^(-a - b*x))/(d*(b*c - a*d)^2*(a + b*x)) + (b*E^(-a - b*x))/((b*c -
 a*d)^2*(c + d*x)) + (b^2*(-(b*c) + (-2 + a)*d)*ExpIntegralEi[-a - b*x])/(
d*(-(b*c) + a*d)^3) + (b^2*(b*c - (2 + a)*d)*E^(-a + (b*c)/d)*ExpIntegralE
i[-((b*(c + d*x))/d)])/(d*(b*c - a*d)^3) - Gamma[-1, a + b*x]/(d*(c + d*x)
^2))/2
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.38, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{ 7119}$$

$$-\frac{b\int \frac{e^{-a-bx}}{(a+bx)^2(c+dx)^2}dx}{2d}-\frac{\Gamma(-1,a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 7293}$$

$$-\frac{b\int \left(-\frac{2de^{-a-bx}b^2}{(bc-ad)^3(a+bx)}+\frac{e^{-a-bx}b^2}{(bc-ad)^2(a+bx)^2}+\frac{2d^2e^{-a-bx}b}{(bc-ad)^3(c+dx)}+\frac{d^2e^{-a-bx}}{(bc-ad)^2(c+dx)^2}\right)dx}{2d}-\frac{\Gamma(-1,a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b\left(-\frac{b\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^2}-\frac{2bd\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^3}-\frac{be^{\frac{bc}{d}-a}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{(bc-ad)^2}+\frac{2bde^{\frac{bc}{d}-a}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right.}{(bc-ad)^3}\right.}{2d}$$

$$\frac{\Gamma(-1,a+bx)}{2d(c+dx)^2}$$

input
```
Int[Gamma[-1, a + b*x]/(c + d*x)^3,x]
```

output
```
-1/2*(b*(-((b*E^(-a - b*x))/((b*c - a*d)^2*(a + b*x))) - (d*E^(-a - b*x))/
((b*c - a*d)^2*(c + d*x)) - (2*b*d*ExpIntegralEi[-a - b*x])/(b*c - a*d)^3
- (b*ExpIntegralEi[-a - b*x])/(b*c - a*d)^2 + (2*b*d*E^(-a + (b*c)/d)*ExpI
ntegralEi[-((b*(c + d*x))/d)])/(b*c - a*d)^3 - (b*E^(-a + (b*c)/d)*ExpInte
gralEi[-((b*(c + d*x))/d)])/(b*c - a*d)^2))/d - Gamma[-1, a + b*x]/(2*d*(c
 + d*x)^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\text{expIntegral}_2(bx+a)}{(bx+a)(dx+c)^3} dx$$

input
```
int(1/(b*x+a)*Ei(2,b*x+a)/(d*x+c)^3,x)
```

output
```
int(1/(b*x+a)*Ei(2,b*x+a)/(d*x+c)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(145) = 290$.

Time = 0.11 (sec) , antiderivative size = 680, normalized size of antiderivative = 4.44

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^3} dx$$
$$= \frac{(ab^3c^3 - (a^2+2a)b^2c^2d + (b^4cd^2 - (a+2)b^3d^3)x^3 + (2b^4c^2d - (a+4)b^3cd^2 - (a^2+2a)b^2d^3)x^2 + (b^4c^3}{}$$

input
```
integrate(gamma(-1,b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*((a*b^3*c^3 - (a^2 + 2*a)*b^2*c^2*d + (b^4*c*d^2 - (a + 2)*b^3*d^3)*x^
3 + (2*b^4*c^2*d - (a + 4)*b^3*c*d^2 - (a^2 + 2*a)*b^2*d^3)*x^2 + (b^4*c^3
 + (a - 2)*b^3*c^2*d - 2*(a^2 + 2*a)*b^2*c*d^2)*x)*Ei(-(b*d*x + b*c)/d)*e^
((b*c - a*d)/d) + (a*b^3*c^3 - (a^2 - 2*a)*b^2*c^2*d + (b^4*c*d^2 - (a - 2
)*b^3*d^3)*x^3 + (2*b^4*c^2*d - (a - 4)*b^3*c*d^2 - (a^2 - 2*a)*b^2*d^3)*x
^2 + (b^4*c^3 + (a + 2)*b^3*c^2*d - 2*(a^2 - 2*a)*b^2*c*d^2)*x)*Ei(-b*x -
a) + (b^3*c^3 - a^2*b*c*d^2 + 2*(b^3*c*d^2 - a*b^2*d^3)*x^2 + (3*b^3*c^2*d
 - 2*a*b^2*c*d^2 - a^2*b*d^3)*x)*e^(-b*x - a) - (a*b^3*c^3 - 3*a^2*b^2*c^2
*d + 3*a^3*b*c*d^2 - a^4*d^3 + (b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2
- a^3*b*d^3)*x)*gamma(-1, b*x + a))/(a*b^3*c^5*d - 3*a^2*b^2*c^4*d^2 + 3*a
^3*b*c^3*d^3 - a^4*c^2*d^4 + (b^4*c^3*d^3 - 3*a*b^3*c^2*d^4 + 3*a^2*b^2*c*
d^5 - a^3*b*d^6)*x^3 + (2*b^4*c^4*d^2 - 5*a*b^3*c^3*d^3 + 3*a^2*b^2*c^2*d^
4 + a^3*b*c*d^5 - a^4*d^6)*x^2 + (b^4*c^5*d - a*b^3*c^4*d^2 - 3*a^2*b^2*c^
3*d^3 + 5*a^3*b*c^2*d^4 - 2*a^4*c*d^5)*x)
```

Sympy [F]

$$\int \frac{\Gamma(-1, a+bx)}{(c+dx)^3}\,dx = \int \frac{\mathrm{E}_2(a+bx)}{(a+bx)(c+dx)^3}\,dx$$

input
```
integrate(uppergamma(-1,b*x+a)/(d*x+c)**3,x)
```

output
```
Integral(expint(2, a + b*x)/((a + b*x)*(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\Gamma(-1, a+bx)}{(c+dx)^3}\,dx = \int \frac{\Gamma(-1, bx+a)}{(dx+c)^3}\,dx$$

input
```
integrate(gamma(-1,b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(gamma(-1, b*x + a)/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^3}\,dx = \int \frac{\Gamma(-1,bx+a)}{(dx+c)^3}\,dx$$

input
```
integrate(gamma(-1,b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(gamma(-1, b*x + a)/(d*x + c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.16

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^3}\,dx = \int \frac{\mathrm{expint}(2,a+b\,x)}{(a+b\,x)\,(c+d\,x)^3}\,dx$$

input
```
int(expint(2, a + b*x)/((a + b*x)*(c + d*x)^3),x)
```

output
```
int(expint(2, a + b*x)/((a + b*x)*(c + d*x)^3), x)
```

Reduce [F]

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^3}\,dx = \int \frac{ei(2,bx+a)}{b\,d^3x^4+a\,d^3x^3+3bc\,d^2x^3+3ac\,d^2x^2+3b\,c^2d\,x^2+3a\,c^2dx+b\,c^3x+a\,c^3}dx$$

input
```
int(1/(b*x+a)*Ei(2,b*x+a)/(d*x+c)^3,x)
```

output
```
int(ei(2,a + b*x)/(a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 +
 b*c**3*x + 3*b*c**2*d*x**2 + 3*b*c*d**2*x**3 + b*d**3*x**4),x)
```

3.144 $\int \frac{\Gamma(-1,a+bx)}{(c+dx)^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 197

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^4} dx = \frac{b^3 e^{-a+\frac{bc}{d}} \Gamma\left(-2,\frac{b(c+dx)}{d}\right)}{3d^2(bc-ad)^2} + \frac{b^3\Gamma(-1,a+bx)}{3d(bc-ad)^3} - \frac{\Gamma(-1,a+bx)}{3d(c+dx)^3} + \frac{2b^3 e^{-a+\frac{bc}{d}} \Gamma\left(-1,\frac{b(c+dx)}{d}\right)}{3d(bc-ad)^3} - \frac{b^3\Gamma(0,a+bx)}{(bc-ad)^4} + \frac{b^3 e^{-a+\frac{bc}{d}} \Gamma\left(0,\frac{b(c+dx)}{d}\right)}{(bc-ad)^4}$$

output
```
1/3*b*exp(-a+b*c/d)/(d*x+c)^2*Ei(3,b*(d*x+c)/d)/(-a*d+b*c)^2+1/3*b^3/(b*x+
a)*Ei(2,b*x+a)/d/(-a*d+b*c)^3-1/3/(b*x+a)*Ei(2,b*x+a)/d/(d*x+c)^3+2/3*b^2*
exp(-a+b*c/d)/(d*x+c)*Ei(2,b*(d*x+c)/d)/(-a*d+b*c)^3-b^3*Ei(1,b*x+a)/(-a*d
+b*c)^4+b^3*exp(-a+b*c/d)*Ei(1,b*(d*x+c)/d)/(-a*d+b*c)^4
```

Mathematica [F]

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^4}\,dx=\int \frac{\Gamma(-1,a+bx)}{(c+dx)^4}\,dx$$

input
```
Integrate[Gamma[-1, a + b*x]/(c + d*x)^4,x]
```

output
```
Integrate[Gamma[-1, a + b*x]/(c + d*x)^4, x]
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.68, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7119, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^4}\,dx$$

$$\Big\downarrow \text{7119}$$

$$-\frac{b\int \frac{e^{-a-bx}}{(a+bx)^2(c+dx)^3}dx}{3d}-\frac{\Gamma(-1,a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\int \left(-\frac{3de^{-a-bx}b^3}{(bc-ad)^4(a+bx)}+\frac{e^{-a-bx}b^3}{(bc-ad)^3(a+bx)^2}+\frac{3d^2e^{-a-bx}b^2}{(bc-ad)^4(c+dx)}+\frac{2d^2e^{-a-bx}b}{(bc-ad)^3(c+dx)^2}+\frac{d^2e^{-a-bx}}{(bc-ad)^2(c+dx)^3}\right)dx}{3d}-\frac{\Gamma(-1,a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{b^2\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^3}-\frac{3b^2d\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^4}+\frac{b^2e^{\frac{bc}{d}-a}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{2d(bc-ad)^2}-\frac{2b^2e^{\frac{bc}{d}-a}\,\text{ExpIntegralEi}\left(-\frac{b(c+}{d}\right.}{(bc-ad)^3}\right.}{3d}$$

$$\frac{\Gamma(-1,a+bx)}{3d(c+dx)^3}$$

input
```
Int[Gamma[-1, a + b*x]/(c + d*x)^4,x]
```

output
```
-1/3*(b*(-((b^2*E^(-a - b*x))/((b*c - a*d)^3*(a + b*x))) - (d*E^(-a - b*x)
)/(2*(b*c - a*d)^2*(c + d*x)^2) - (2*b*d*E^(-a - b*x))/((b*c - a*d)^3*(c +
 d*x)) + (b*E^(-a - b*x))/(2*(b*c - a*d)^2*(c + d*x)) - (3*b^2*d*ExpIntegr
alEi[-a - b*x])/(b*c - a*d)^4 - (b^2*ExpIntegralEi[-a - b*x])/(b*c - a*d)^
3 + (3*b^2*d*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/(b*c - a*
d)^4 - (2*b^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/(b*c - a
*d)^3 + (b^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/(2*d*(b*c
 - a*d)^2)))/d - Gamma[-1, a + b*x]/(3*d*(c + d*x)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\text{expIntegral}_2(bx+a)}{(bx+a)(dx+c)^4}dx$$

input
```
int(1/(b*x+a)*Ei(2,b*x+a)/(d*x+c)^4,x)
```

output
```
int(1/(b*x+a)*Ei(2,b*x+a)/(d*x+c)^4,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1218 vs. $2(186) = 372$.

Time = 0.14 (sec) , antiderivative size = 1218, normalized size of antiderivative = 6.18

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(gamma(-1,b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/6*((a*b^5*c^5 - 2*(a^2 + 2*a)*b^4*c^4*d + (a^3 + 4*a^2 + 6*a)*b^3*c^3*d
^2 + (b^6*c^2*d^3 - 2*(a + 2)*b^5*c*d^4 + (a^2 + 4*a + 6)*b^4*d^5)*x^4 + (
3*b^6*c^3*d^2 - (5*a + 12)*b^5*c^2*d^3 + (a^2 + 8*a + 18)*b^4*c*d^4 + (a^3
 + 4*a^2 + 6*a)*b^3*d^5)*x^3 + 3*(b^6*c^4*d - (a + 4)*b^5*c^3*d^2 - (a^2 -
 6)*b^4*c^2*d^3 + (a^3 + 4*a^2 + 6*a)*b^3*c*d^4)*x^2 + (b^6*c^5 + (a - 4)*
b^5*c^4*d - (5*a^2 + 8*a - 6)*b^4*c^3*d^2 + 3*(a^3 + 4*a^2 + 6*a)*b^3*c^2*
d^3)*x)*Ei(-(b*d*x + b*c)/d)*e^((b*c - a*d)/d) - 2*(a*b^4*c^4*d - (a^2 - 3
*a)*b^3*c^3*d^2 + (b^5*c*d^4 - (a - 3)*b^4*d^5)*x^4 + (3*b^5*c^2*d^3 - (2*
a - 9)*b^4*c*d^4 - (a^2 - 3*a)*b^3*d^5)*x^3 + 3*(b^5*c^3*d^2 + 3*b^4*c^2*d
^3 - (a^2 - 3*a)*b^3*c*d^4)*x^2 + (b^5*c^4*d + (2*a + 3)*b^4*c^3*d^2 - 3*(
a^2 - 3*a)*b^3*c^2*d^3)*x)*Ei(-b*x - a) + ((a - 2)*b^4*c^4*d - (2*a^2 + 3*
a)*b^3*c^3*d^2 - a^3*b*c*d^4 + (a^3 + 6*a^2)*b^2*c^2*d^3 + (b^5*c^2*d^3 -
2*(a + 3)*b^4*c*d^4 + (a^2 + 6*a)*b^3*d^5)*x^3 + (2*b^5*c^3*d^2 - 3*(a + 5
)*b^4*c^2*d^3 + 12*a*b^3*c*d^4 + (a^3 + 3*a^2)*b^2*d^5)*x^2 + (b^5*c^4*d -
 11*b^4*c^3*d^2 - 3*(a^2 - a)*b^3*c^2*d^3 - a^3*b*d^5 + (2*a^3 + 9*a^2)*b^
2*c*d^4)*x)*e^(-b*x - a) + 2*(a*b^4*c^4*d - 4*a^2*b^3*c^3*d^2 + 6*a^3*b^2*
c^2*d^3 - 4*a^4*b*c*d^4 + a^5*d^5 + (b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b
^3*c^2*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)*gamma(-1, b*x + a))/(a*b^4*c^
7*d^2 - 4*a^2*b^3*c^6*d^3 + 6*a^3*b^2*c^5*d^4 - 4*a^4*b*c^4*d^5 + a^5*c^3*
d^6 + (b^5*c^4*d^5 - 4*a*b^4*c^3*d^6 + 6*a^2*b^3*c^2*d^7 - 4*a^3*b^2*c*...
```

Sympy [F]

$$\int \frac{\Gamma(-1,a+bx)}{(c+dx)^4}\,dx=\int \frac{\mathrm{E}_2\left(a+bx\right)}{(a+bx)\left(c+dx\right)^4}\,dx$$

input
```
integrate(uppergamma(-1,b*x+a)/(d*x+c)**4,x)
```

output
```
Integral(expint(2, a + b*x)/((a + b*x)*(c + d*x)**4), x)
```

Maxima [F]

$$\int \frac{\Gamma(-1, a + bx)}{(c + dx)^4}\, dx = \int \frac{\Gamma(-1, bx + a)}{(dx + c)^4}\, dx$$

input
```
integrate(gamma(-1,b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate(gamma(-1, b*x + a)/(d*x + c)^4, x)
```

Giac [F]

$$\int \frac{\Gamma(-1, a + bx)}{(c + dx)^4}\, dx = \int \frac{\Gamma(-1, bx + a)}{(dx + c)^4}\, dx$$

input
```
integrate(gamma(-1,b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate(gamma(-1, b*x + a)/(d*x + c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.12

$$\int \frac{\Gamma(-1, a + bx)}{(c + dx)^4}\, dx = \int \frac{\mathrm{expint}(2, a + b\,x)}{(a + b\,x)\,(c + d\,x)^4}\, dx$$

input
```
int(expint(2, a + b*x)/((a + b*x)*(c + d*x)^4),x)
```

output
```
int(expint(2, a + b*x)/((a + b*x)*(c + d*x)^4), x)
```

Reduce [F]

$$\int \frac{\Gamma(-1, a+bx)}{(c+dx)^4} \, dx$$
$$= \int \frac{ei(2, bx+a)}{b\,d^4x^5 + a\,d^4x^4 + 4bc\,d^3x^4 + 4ac\,d^3x^3 + 6b\,c^2d^2x^3 + 6a\,c^2d^2x^2 + 4b\,c^3d\,x^2 + 4a\,c^3dx + b\,c^4x + a\,c^4} dx$$

input
```
int(1/(b*x+a)*Ei(2,b*x+a)/(d*x+c)^4,x)
```

output
```
int(ei(2,a + b*x)/(a*c**4 + 4*a*c**3*d*x + 6*a*c**2*d**2*x**2 + 4*a*c*d**3
*x**3 + a*d**4*x**4 + b*c**4*x + 4*b*c**3*d*x**2 + 6*b*c**2*d**2*x**3 + 4*
b*c*d**3*x**4 + b*d**4*x**5),x)
```

3.145 $\int (c+dx)^3\Gamma(-2,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 145

$$\int (c+dx)^3\Gamma(-2,a+bx)\,dx = -\frac{d^2(4bc-3ad)e^{-a-bx}}{4b^4} - \frac{(bc-ad)^4\Gamma(-2,a+bx)}{4b^4d} + \frac{(c+dx)^4\Gamma(-2,a+bx)}{4d} - \frac{(bc-ad)^3\Gamma(-1,a+bx)}{b^4} - \frac{3d(bc-ad)^2\Gamma(0,a+bx)}{2b^4} - \frac{d^3e^{-a}\Gamma(2,bx)}{4b^4}$$

output
```
-1/4*d^2*(-3*a*d+4*b*c)*exp(-b*x-a)/b^4-1/4*(-a*d+b*c)^4/(b*x+a)^2*Ei(3,b*
x+a)/b^4/d+1/4*(d*x+c)^4/(b*x+a)^2*Ei(3,b*x+a)/d-(-a*d+b*c)^3/(b*x+a)*Ei(2
,b*x+a)/b^4-3/2*d*(-a*d+b*c)^2*Ei(1,b*x+a)/b^4-1/4*d^3*exp(-b*x)*(b*x+1)/b
^4/exp(a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 398 vs. 2(145) = 290.

Time = 0.35 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.74

$$\int (c+dx)^3\Gamma(-2,a+bx)\,dx$$

$$= \frac{1}{8}\left(\frac{e^{-a-bx}\left(2d^2(-4bc+(-1+3a)d)-2bd^3x-\frac{a(-4b^3c^3+6ab^2c^2d-4a^2bcd^2+a^3d^3)}{(a+bx)^2}+\frac{-4(2+a)b^3c^3+6a(4+a)b^2c^2d-4a^2}{a+bx}\right.}{b^4}\right.$$
$$-\frac{8c^3\,\text{ExpIntegralEi}(-a-bx)}{b}-\frac{4ac^3\,\text{ExpIntegralEi}(-a-bx)}{b}$$
$$+\frac{12c^2d\,\text{ExpIntegralEi}(-a-bx)}{b^2}+\frac{24ac^2d\,\text{ExpIntegralEi}(-a-bx)}{b^2}$$
$$+\frac{6a^2c^2d\,\text{ExpIntegralEi}(-a-bx)}{b^2}-\frac{24acd^2\,\text{ExpIntegralEi}(-a-bx)}{b^3}$$
$$-\frac{24a^2cd^2\,\text{ExpIntegralEi}(-a-bx)}{b^3}-\frac{4a^3cd^2\,\text{ExpIntegralEi}(-a-bx)}{b^3}$$
$$+\frac{12a^2d^3\,\text{ExpIntegralEi}(-a-bx)}{b^4}+\frac{8a^3d^3\,\text{ExpIntegralEi}(-a-bx)}{b^4}$$
$$\left.+\frac{a^4d^3\,\text{ExpIntegralEi}(-a-bx)}{b^4}+2x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)\Gamma(-2,a+bx)\right)$$

input
```
Integrate[(c + d*x)^3*Gamma[-2, a + b*x],x]
```

output
```
((E^(-a - b*x)*(2*d^2*(-4*b*c + (-1 + 3*a)*d) - 2*b*d^3*x - (a*(-4*b^3*c^3
 + 6*a*b^2*c^2*d - 4*a^2*b*c*d^2 + a^3*d^3))/(a + b*x)^2 + (-4*(2 + a)*b^3
*c^3 + 6*a*(4 + a)*b^2*c^2*d - 4*a^2*(6 + a)*b*c*d^2 + a^3*(8 + a)*d^3)/(a
 + b*x)))/b^4 - (8*c^3*ExpIntegralEi[-a - b*x])/b - (4*a*c^3*ExpIntegralEi
[-a - b*x])/b + (12*c^2*d*ExpIntegralEi[-a - b*x])/b^2 + (24*a*c^2*d*ExpIn
tegralEi[-a - b*x])/b^2 + (6*a^2*c^2*d*ExpIntegralEi[-a - b*x])/b^2 - (24*
a*c*d^2*ExpIntegralEi[-a - b*x])/b^3 - (24*a^2*c*d^2*ExpIntegralEi[-a - b*
x])/b^3 - (4*a^3*c*d^2*ExpIntegralEi[-a - b*x])/b^3 + (12*a^2*d^3*ExpInteg
ralEi[-a - b*x])/b^4 + (8*a^3*d^3*ExpIntegralEi[-a - b*x])/b^4 + (a^4*d^3*
ExpIntegralEi[-a - b*x])/b^4 + 2*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*
x^3)*Gamma[-2, a + b*x])/8
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\Gamma(-2,a+bx)\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{b\int \frac{e^{-a-bx}(c+dx)^4}{(a+bx)^3}dx}{4d}+\frac{(c+dx)^4\Gamma(-2,a+bx)}{4d}$$

$$\Big\downarrow \text{2629}$$

$$\frac{b\int \left(\frac{e^{-a-bx}xd^4}{b^3}+\frac{(4bc-3ad)e^{-a-bx}d^3}{b^4}+\frac{6(bc-ad)^2e^{-a-bx}d^2}{b^4(a+bx)}+\frac{4(bc-ad)^3e^{-a-bx}d}{b^4(a+bx)^2}+\frac{(bc-ad)^4e^{-a-bx}}{b^4(a+bx)^3}\right)dx}{4d}+\frac{(c+dx)^4\Gamma(-2,a+bx)}{4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{d^3e^{-a-bx}(4bc-3ad)}{b^5}+\frac{6d^2(bc-ad)^2\text{ExpIntegralEi}(-a-bx)}{b^5}-\frac{4d(bc-ad)^3\text{ExpIntegralEi}(-a-bx)}{b^5}+\frac{(bc-ad)^4\text{ExpIntegralEi}(-a-bx)}{2b^5}\right.}{4d}$$

$$\frac{(c+dx)^4\Gamma(-2,a+bx)}{4d}$$

input
```
Int[(c + d*x)^3*Gamma[-2, a + b*x],x]
```

output
```
(b*(-((d^4*E^(-a - b*x))/b^5) - (d^3*(4*b*c - 3*a*d)*E^(-a - b*x))/b^5 - (
d^4*E^(-a - b*x)*x)/b^4 - ((b*c - a*d)^4*E^(-a - b*x))/(2*b^5*(a + b*x)^2)
 - (4*d*(b*c - a*d)^3*E^(-a - b*x))/(b^5*(a + b*x)) + ((b*c - a*d)^4*E^(-a
 - b*x))/(2*b^5*(a + b*x)) + (6*d^2*(b*c - a*d)^2*ExpIntegralEi[-a - b*x])
/b^5 - (4*d*(b*c - a*d)^3*ExpIntegralEi[-a - b*x])/b^5 + ((b*c - a*d)^4*Ex
pIntegralEi[-a - b*x])/(2*b^5)))/(4*d) + ((c + d*x)^4*Gamma[-2, a + b*x])/
(4*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [F]

$$\int \frac{(dx+c)^3 \operatorname{expIntegral}_3(bx+a)}{(bx+a)^2}dx$$

input
```
int((d*x+c)^3/(b*x+a)^2*Ei(3,b*x+a),x)
```

output
```
int((d*x+c)^3/(b*x+a)^2*Ei(3,b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. 2(133) = 266.

Time = 0.12 (sec) , antiderivative size = 506, normalized size of antiderivative = 3.49

$$\int (c+dx)^3\Gamma(-2,a+bx)\,dx =$$
$$-\frac{(b^3d^3x^3 + 4\,b^3c^3 - 6\,(a+1)b^2c^2d + 4\,(a^2+3\,a)bcd^2 - (a^3+5\,a^2)d^3 + (4\,b^3cd^2 - (a-1)b^2d^3)x^2 + (6\,b}{}$$

input
```
integrate((d*x+c)^3*gamma(-2,b*x+a),x, algorithm="fricas")
```

output
```
-1/4*((b^3*d^3*x^3 + 4*b^3*c^3 - 6*(a + 1)*b^2*c^2*d + 4*(a^2 + 3*a)*b*c*d
^2 - (a^3 + 5*a^2)*d^3 + (4*b^3*c*d^2 - (a - 1)*b^2*d^3)*x^2 + (6*b^3*c^2*
d - 4*a*b^2*c*d^2 + (a^2 + 2*a)*b*d^3)*x)*e^(-b*x - a) - (b^6*d^3*x^6 + 4*
(a^3 + 2*a^2)*b^3*c^3 - 6*(a^4 + 4*a^3 + 2*a^2)*b^2*c^2*d + 2*(2*b^6*c*d^2
 + a*b^5*d^3)*x^5 + 4*(a^5 + 6*a^4 + 6*a^3)*b*c*d^2 + (6*b^6*c^2*d + 8*a*b
^5*c*d^2 + a^2*b^4*d^3)*x^4 - (a^6 + 8*a^5 + 12*a^4)*d^3 + 4*(b^6*c^3 + 3*
a*b^5*c^2*d + a^2*b^4*c*d^2)*x^3 + (4*(3*a + 2)*b^5*c^3 - 12*(2*a + 1)*b^4
*c^2*d + 4*(a^3 + 6*a^2 + 6*a)*b^3*c*d^2 - (a^4 + 8*a^3 + 12*a^2)*b^2*d^3)
*x^2 + 2*(2*(3*a^2 + 4*a)*b^4*c^3 - 6*(a^3 + 4*a^2 + 2*a)*b^3*c^2*d + 4*(a
^4 + 6*a^3 + 6*a^2)*b^2*c*d^2 - (a^5 + 8*a^4 + 12*a^3)*b*d^3)*x)*gamma(-2,
 b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [F]

$$\int (c+dx)^3\Gamma(-2,a+bx)\,dx = \int \frac{(c+dx)^3\,\mathrm{E}_3\,(a+bx)}{(a+bx)^2}\,dx$$

input
```
integrate((d*x+c)**3*uppergamma(-2,b*x+a),x)
```

output
```
Integral((c + d*x)**3*expint(3, a + b*x)/(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)^3\Gamma(-2,a+bx)\,dx = \int (dx+c)^3\Gamma(-2,bx+a)\,dx$$

input
```
integrate((d*x+c)^3*gamma(-2,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(-2, b*x + a) - gamma(-1, b*x + a))*c^3/b + integrate(d^3*
x^3*gamma(-2, b*x + a) + 3*c*d^2*x^2*gamma(-2, b*x + a) + 3*c^2*d*x*gamma(
-2, b*x + a), x)
```

Giac [F]

$$\int (c+dx)^3\Gamma(-2,a+bx)\,dx = \int (dx+c)^3\Gamma(-2,bx+a)\,dx$$

input
```
integrate((d*x+c)^3*gamma(-2,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*gamma(-2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.17

$$\int (c+dx)^3\Gamma(-2,a+bx)\,dx = \int \frac{\mathrm{expint}(3,a+b\,x)\,\left(c+d\,x\right)^3}{\left(a+b\,x\right)^2}\,dx$$

input
```
int((expint(3, a + b*x)*(c + d*x)^3)/(a + b*x)^2,x)
```

output
```
int((expint(3, a + b*x)*(c + d*x)^3)/(a + b*x)^2, x)
```

Reduce [F]

$$\begin{aligned}\int (c+dx)^3\Gamma(-2,a+bx)\,dx = {} & \left(\int \frac{ei(3,bx+a)}{b^2x^2+2abx+a^2}dx\right)c^3 \\ & + \left(\int \frac{ei(3,bx+a)\,x^3}{b^2x^2+2abx+a^2}dx\right)d^3 \\ & + 3\left(\int \frac{ei(3,bx+a)\,x^2}{b^2x^2+2abx+a^2}dx\right)c\,d^2 \\ & + 3\left(\int \frac{ei(3,bx+a)\,x}{b^2x^2+2abx+a^2}dx\right)c^2d\end{aligned}$$

input
```
int((d*x+c)^3/(b*x+a)^2*Ei(3,b*x+a),x)
```

output
```
int(ei(3,a + b*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*c**3 + int((ei(3,a + b*x
)*x**3)/(a**2 + 2*a*b*x + b**2*x**2),x)*d**3 + 3*int((ei(3,a + b*x)*x**2)/
(a**2 + 2*a*b*x + b**2*x**2),x)*c*d**2 + 3*int((ei(3,a + b*x)*x)/(a**2 + 2
*a*b*x + b**2*x**2),x)*c**2*d
```

3.146 $\int (c+dx)^2\Gamma(-2,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 112

$$\int (c+dx)^2\Gamma(-2,a+bx)\,dx=-\frac{d^2e^{-a-bx}}{3b^3}-\frac{(bc-ad)^3\Gamma(-2,a+bx)}{3b^3d}+\frac{(c+dx)^3\Gamma(-2,a+bx)}{3d}-\frac{(bc-ad)^2\Gamma(-1,a+bx)}{b^3}-\frac{d(bc-ad)\Gamma(0,a+bx)}{b^3}$$

output
```
-1/3*d^2*exp(-b*x-a)/b^3-1/3*(-a*d+b*c)^3/(b*x+a)^2*Ei(3,b*x+a)/b^3/d+1/3*
(d*x+c)^3/(b*x+a)^2*Ei(3,b*x+a)/d-(-a*d+b*c)^2/(b*x+a)*Ei(2,b*x+a)/b^3-d*(
-a*d+b*c)*Ei(1,b*x+a)/b^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 229 vs. 2(112) = 224.

Time = 0.24 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.04

$$\int (c+dx)^2\Gamma(-2,a+bx)\,dx=\frac{-((3(2+a)b^2c^2-3(2+4a+a^2)\,bcd+a(6+6a+a^2)\,d^2)\,\text{ExpIntegralEi}(-a-bx))+\frac{e^{-a-bx}(-a^4d^2-a^3d(\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(c + d*x)^2*Gamma[-2, a + b*x],x]
```

output
```
(-((3*(2 + a)*b^2*c^2 - 3*(2 + 4*a + a^2)*b*c*d + a*(6 + 6*a + a^2)*d^2)*E
xpIntegralEi[-a - b*x]) + (E^(-a - b*x)*(-(a^4*d^2) - a^3*d*(-3*b*c + 5*d
+ b*d*x) - 2*b^2*x*(3*b*c^2 + d^2*x) - a*b*(3*b^2*c^2*x + 4*d^2*x + 3*b*c*
(c - 4*d*x)) + a^2*(-2*d^2 + 3*b*d*(3*c - 2*d*x) - 3*b^2*c*(c - d*x)) + 2*
b^3*E^(a + b*x)*x*(a + b*x)^2*(3*c^2 + 3*c*d*x + d^2*x^2)*Gamma[-2, a + b*
x]))/(a + b*x)^2)/(6*b^3)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 226 vs. $2(112) = 224$.

Time = 0.65 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\Gamma(-2,a+bx)\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{b\int \frac{e^{-a-bx}(c+dx)^3}{(a+bx)^3}dx}{3d}+\frac{(c+dx)^3\Gamma(-2,a+bx)}{3d}$$

$$\Big\downarrow \text{2629}$$

$$\frac{b\int \left(\frac{e^{-a-bx}d^3}{b^3}+\frac{3(bc-ad)e^{-a-bx}d^2}{b^3(a+bx)}+\frac{3(bc-ad)^2e^{-a-bx}d}{b^3(a+bx)^2}+\frac{(bc-ad)^3e^{-a-bx}}{b^3(a+bx)^3}\right)dx}{3d}+\frac{(c+dx)^3\Gamma(-2,a+bx)}{3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{3d^2(bc-ad)\,\text{ExpIntegralEi}(-a-bx)}{b^4}-\frac{3d(bc-ad)^2\,\text{ExpIntegralEi}(-a-bx)}{b^4}+\frac{(bc-ad)^3\,\text{ExpIntegralEi}(-a-bx)}{2b^4}-\frac{3de^{-a-bx}(bc-ad)^2}{b^4(a+bx)}+\right.}{3d}$$

$$\frac{(c+dx)^3\Gamma(-2,a+bx)}{3d}$$

input
```
Int[(c + d*x)^2*Gamma[-2, a + b*x],x]
```

output
```
(b*(-((d^3*E^(-a - b*x))/b^4) - ((b*c - a*d)^3*E^(-a - b*x))/(2*b^4*(a + b
*x)^2) - (3*d*(b*c - a*d)^2*E^(-a - b*x))/(b^4*(a + b*x)) + ((b*c - a*d)^3
*E^(-a - b*x))/(2*b^4*(a + b*x)) + (3*d^2*(b*c - a*d)*ExpIntegralEi[-a - b
*x])/b^4 - (3*d*(b*c - a*d)^2*ExpIntegralEi[-a - b*x])/b^4 + ((b*c - a*d)^
3*ExpIntegralEi[-a - b*x])/(2*b^4)))/(3*d) + ((c + d*x)^3*Gamma[-2, a + b*
x])/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [F]

$$\int \frac{(dx+c)^2 \operatorname{expIntegral}_3(bx+a)}{(bx+a)^2}dx$$

input
```
int((d*x+c)^2/(b*x+a)^2*Ei(3,b*x+a),x)
```

output
```
int((d*x+c)^2/(b*x+a)^2*Ei(3,b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(105) = 210$.

Time = 0.10 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.91

$$\int (c+dx)^2\Gamma(-2,a+bx)\,dx = -\frac{(b^2d^2x^2+3\,b^2c^2-3\,(a+1)bcd+(a^2+3\,a)d^2+(3\,b^2cd-abd^2)x)e^{(-bx-a)}-(b^5d^2x^5+3\,(a^3+2\,a^2)b^2}{}$$

input
```
integrate((d*x+c)^2*gamma(-2,b*x+a),x, algorithm="fricas")
```

output
```
-1/3*((b^2*d^2*x^2 + 3*b^2*c^2 - 3*(a + 1)*b*c*d + (a^2 + 3*a)*d^2 + (3*b^
2*c*d - a*b*d^2)*x)*e^(-b*x - a) - (b^5*d^2*x^5 + 3*(a^3 + 2*a^2)*b^2*c^2
+ (3*b^5*c*d + 2*a*b^4*d^2)*x^4 - 3*(a^4 + 4*a^3 + 2*a^2)*b*c*d + (3*b^5*c
^2 + 6*a*b^4*c*d + a^2*b^3*d^2)*x^3 + (a^5 + 6*a^4 + 6*a^3)*d^2 + (3*(3*a
+ 2)*b^4*c^2 - 6*(2*a + 1)*b^3*c*d + (a^3 + 6*a^2 + 6*a)*b^2*d^2)*x^2 + (3
*(3*a^2 + 4*a)*b^3*c^2 - 6*(a^3 + 4*a^2 + 2*a)*b^2*c*d + 2*(a^4 + 6*a^3 +
6*a^2)*b*d^2)*x)*gamma(-2, b*x + a))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)
```

Sympy [F]

$$\int (c+dx)^2\Gamma(-2,a+bx)\,dx = \int \frac{(c+dx)^2\,\mathrm{E}_3\,(a+bx)}{(a+bx)^2}\,dx$$

input
```
integrate((d*x+c)**2*uppergamma(-2,b*x+a),x)
```

output
```
Integral((c + d*x)**2*expint(3, a + b*x)/(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)^2\Gamma(-2,a+bx)\,dx = \int (dx+c)^2\Gamma(-2,bx+a)\,dx$$

input
```
integrate((d*x+c)^2*gamma(-2,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(-2, b*x + a) - gamma(-1, b*x + a))*c^2/b + integrate(d^2*
x^2*gamma(-2, b*x + a) + 2*c*d*x*gamma(-2, b*x + a), x)
```

Giac [F]

$$\int (c+dx)^2\Gamma(-2,a+bx)\,dx = \int (dx+c)^2\Gamma(-2,bx+a)\,dx$$

input
```
integrate((d*x+c)^2*gamma(-2,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*gamma(-2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.21

$$\int (c+dx)^2\Gamma(-2,a+bx)\,dx = \int \frac{\mathrm{expint}(3,a+b\,x)\,\left(c+d\,x\right)^2}{\left(a+b\,x\right)^2}\,dx$$

input
```
int((expint(3, a + b*x)*(c + d*x)^2)/(a + b*x)^2,x)
```

output
```
int((expint(3, a + b*x)*(c + d*x)^2)/(a + b*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2\Gamma(-2,a+bx)\,dx = \left(\int \frac{ei(3,bx+a)}{b^2x^2+2abx+a^2}dx\right)c^2 + \left(\int \frac{ei(3,bx+a)\,x^2}{b^2x^2+2abx+a^2}dx\right)d^2 + 2\left(\int \frac{ei(3,bx+a)\,x}{b^2x^2+2abx+a^2}dx\right)cd$$

input

```
int((d*x+c)^2/(b*x+a)^2*Ei(3,b*x+a),x)
```

output

```
int(ei(3,a + b*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*c**2 + int((ei(3,a + b*x
)*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*d**2 + 2*int((ei(3,a + b*x)*x)/(a*
*2 + 2*a*b*x + b**2*x**2),x)*c*d
```

3.147 $\int (c+dx)\Gamma(-2,a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int (c+dx)\Gamma(-2,a+bx)\,dx = -\frac{(bc-ad)^2\Gamma(-2,a+bx)}{2b^2d}+\frac{(c+dx)^2\Gamma(-2,a+bx)}{2d} - \frac{(bc-ad)\Gamma(-1,a+bx)}{b^2}-\frac{d\Gamma(0,a+bx)}{2b^2}$$

output
```
-1/2*(-a*d+b*c)^2/(b*x+a)^2*Ei(3,b*x+a)/b^2/d+1/2*(d*x+c)^2/(b*x+a)^2*Ei(3
,b*x+a)/d-(-a*d+b*c)/(b*x+a)*Ei(2,b*x+a)/b^2-1/2*d*Ei(1,b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.69

$$\int (c+dx)\Gamma(-2,a+bx)\,dx$$
$$= \frac{-\left(\left(2(2+a)bc-\left(2+4a+a^2\right)d\right)\text{ExpIntegralEi}(-a-bx)\right)+\frac{e^{-a-bx}\left(-2abc-2a^2bc+3a^2d+a^3d-4b^2cx-2ab^2cx+4abd\right.}{(a+bx)}}{4b^2}$$

input
```
Integrate[(c + d*x)*Gamma[-2, a + b*x],x]
```

output
```
(-((2*(2 + a)*b*c - (2 + 4*a + a^2)*d)*ExpIntegralEi[-a - b*x]) + (E^(-a -
 b*x)*(-2*a*b*c - 2*a^2*b*c + 3*a^2*d + a^3*d - 4*b^2*c*x - 2*a*b^2*c*x +
4*a*b*d*x + a^2*b*d*x + 2*b^2*E^(a + b*x)*x*(a + b*x)^2*(2*c + d*x)*Gamma[
-2, a + b*x]))/(a + b*x)^2)/(4*b^2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 195 vs. 2(84) = 168.

Time = 0.59 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.32, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\Gamma(-2,a+bx)\,dx$$

$$\Big\downarrow \; 7119$$

$$\frac{b\int \frac{e^{-a-bx}(c+dx)^2}{(a+bx)^3}dx}{2d}+\frac{(c+dx)^2\Gamma(-2,a+bx)}{2d}$$

$$\Big\downarrow \; 2629$$

$$\frac{b\int \left(\frac{e^{-a-bx}d^2}{b^2(a+bx)}+\frac{2(bc-ad)e^{-a-bx}d}{b^2(a+bx)^2}+\frac{(bc-ad)^2e^{-a-bx}}{b^2(a+bx)^3}\right)dx}{2d}+\frac{(c+dx)^2\Gamma(-2,a+bx)}{2d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\left(-\frac{2d(bc-ad)\,\text{ExpIntegralEi}(-a-bx)}{b^3}+\frac{(bc-ad)^2\,\text{ExpIntegralEi}(-a-bx)}{2b^3}-\frac{2de^{-a-bx}(bc-ad)}{b^3(a+bx)}+\frac{e^{-a-bx}(bc-ad)^2}{2b^3(a+bx)}-\frac{e^{-a-bx}(bc-ad)^2}{2b^3(a+bx)^2}\right.}{2d}$$

$$\frac{(c+dx)^2\Gamma(-2,a+bx)}{2d}$$

input
```
Int[(c + d*x)*Gamma[-2, a + b*x],x]
```

output
```
(b*(-1/2*((b*c - a*d)^2*E^(-a - b*x))/(b^3*(a + b*x)^2) - (2*d*(b*c - a*d)
*E^(-a - b*x))/(b^3*(a + b*x)) + ((b*c - a*d)^2*E^(-a - b*x))/(2*b^3*(a +
b*x)) + (d^2*ExpIntegralEi[-a - b*x])/b^3 - (2*d*(b*c - a*d)*ExpIntegralEi
[-a - b*x])/b^3 + ((b*c - a*d)^2*ExpIntegralEi[-a - b*x])/(2*b^3)))/(2*d)
+ ((c + d*x)^2*Gamma[-2, a + b*x])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [F]

$$\int \frac{(dx+c)\,\mathrm{expIntegral}_3\,(bx+a)}{(bx+a)^2}dx$$

input
```
int((d*x+c)/(b*x+a)^2*Ei(3,b*x+a),x)
```

output
```
int((d*x+c)/(b*x+a)^2*Ei(3,b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(78) = 156$.

Time = 0.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.12

$$\int (c+dx)\Gamma(-2,a+bx)\,dx = -\frac{(bdx+2\,bc-(a+1)d)e^{(-bx-a)}-(b^4dx^4+2\,(b^4c+ab^3d)x^3+2\,(a^3+2\,a^2)bc+2\,((3\,a+2)b^3c-(2\,a}{2\,(b^4x^2+2\,ab^3x}$$

input
```
integrate((d*x+c)*gamma(-2,b*x+a),x, algorithm="fricas")
```

output
```
-1/2*((b*d*x + 2*b*c - (a + 1)*d)*e^(-b*x - a) - (b^4*d*x^4 + 2*(b^4*c + a
*b^3*d)*x^3 + 2*(a^3 + 2*a^2)*b*c + 2*((3*a + 2)*b^3*c - (2*a + 1)*b^2*d)*
x^2 - (a^4 + 4*a^3 + 2*a^2)*d + 2*((3*a^2 + 4*a)*b^2*c - (a^3 + 4*a^2 + 2*
a)*b*d)*x)*gamma(-2, b*x + a))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [F]

$$\int (c+dx)\Gamma(-2,a+bx)\,dx = \int \frac{(c+dx)\,\mathrm{E}_3\,(a+bx)}{(a+bx)^2}\,dx$$

input
```
integrate((d*x+c)*uppergamma(-2,b*x+a),x)
```

output
```
Integral((c + d*x)*expint(3, a + b*x)/(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)\Gamma(-2,a+bx)\,dx = \int (dx+c)\Gamma(-2,bx+a)\,dx$$

input
```
integrate((d*x+c)*gamma(-2,b*x+a),x, algorithm="maxima")
```

output
```
d*integrate(x*gamma(-2, b*x + a), x) + ((b*x + a)*gamma(-2, b*x + a) - gam
ma(-1, b*x + a))*c/b
```

Giac [F]

$$\int (c+dx)\Gamma(-2,a+bx)\,dx = \int (dx+c)\Gamma(-2,bx+a)\,dx$$

input
```
integrate((d*x+c)*gamma(-2,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*gamma(-2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.26

$$\int (c+dx)\Gamma(-2,a+bx)\,dx = \int \frac{\mathrm{expint}(3,a+b\,x)\,(c+d\,x)}{(a+b\,x)^2}\,dx$$

input
```
int((expint(3, a + b*x)*(c + d*x))/(a + b*x)^2,x)
```

output
```
int((expint(3, a + b*x)*(c + d*x))/(a + b*x)^2, x)
```

Reduce [F]

$$\int (c+dx)\Gamma(-2,a+bx)\,dx = \left(\int \frac{ei(3,bx+a)}{b^2x^2+2abx+a^2}dx\right)c + \left(\int \frac{ei(3,bx+a)\,x}{b^2x^2+2abx+a^2}dx\right)d$$

input
```
int((d*x+c)/(b*x+a)^2*Ei(3,b*x+a),x)
```

output
```
int(ei(3,a + b*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*c + int((ei(3,a + b*x)*x
)/(a**2 + 2*a*b*x + b**2*x**2),x)*d
```

3.148 $\int \Gamma(-2, a+bx)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 29

$$\int \Gamma(-2, a+bx)\,dx = \frac{(a+bx)\Gamma(-2, a+bx)}{b} - \frac{\Gamma(-1, a+bx)}{b}$$

output
```
1/(b*x+a)*Ei(3,b*x+a)/b-1/(b*x+a)*Ei(2,b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 77 vs. 2(29) = 58.

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.66

$$\int \Gamma(-2, a+bx)\,dx = \frac{e^{-a-bx}(a-(2+a)(a+bx))}{2b(a+bx)^2} - \frac{\text{ExpIntegralEi}(-a-bx)}{b} - \frac{a\,\text{ExpIntegralEi}(-a-bx)}{2b} + x\Gamma(-2, a+bx)$$

input
```
Integrate[Gamma[-2, a + b*x],x]
```

output
```
(E^(-a - b*x)*(a - (2 + a)*(a + b*x)))/(2*b*(a + b*x)^2) - ExpIntegralEi[-
a - b*x]/b - (a*ExpIntegralEi[-a - b*x])/(2*b) + x*Gamma[-2, a + b*x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(-2,a+bx)\,dx$$

$$\Big\downarrow \; 7111$$

$$\frac{(a+bx)\Gamma(-2,a+bx)}{b}-\frac{\Gamma(-1,a+bx)}{b}$$

input
```
Int[Gamma[-2, a + b*x],x]
```

output
```
((a + b*x)*Gamma[-2, a + b*x])/b - Gamma[-1, a + b*x]/b
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [F]

$$\int \frac{\text{expIntegral}_3\left(bx+a\right)}{\left(bx+a\right)^2}dx$$

input
```
int(1/(b*x+a)^2*Ei(3,b*x+a),x)
```

output
```
int(1/(b*x+a)^2*Ei(3,b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.86

$$\int \Gamma(-2, a+bx)\,dx = \frac{(b^3x^3 + (3\,a+2)b^2x^2 + a^3 + (3\,a^2 + 4\,a)bx + 2\,a^2)\Gamma(-2, bx+a) - e^{(-bx-a)}}{b^3x^2 + 2\,ab^2x + a^2b}$$

input
```
integrate(gamma(-2,b*x+a),x, algorithm="fricas")
```

output
```
((b^3*x^3 + (3*a + 2)*b^2*x^2 + a^3 + (3*a^2 + 4*a)*b*x + 2*a^2)*gamma(-2,
 b*x + a) - e^(-b*x - a))/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [F]

$$\int \Gamma(-2, a+bx)\,dx = \int \frac{E_3(a+bx)}{(a+bx)^2}\,dx$$

input
```
integrate(uppergamma(-2,b*x+a),x)
```

output
```
Integral(expint(3, a + b*x)/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \Gamma(-2, a+bx)\,dx = \frac{(bx+a)\Gamma(-2, bx+a) - \Gamma(-1, bx+a)}{b}$$

input
```
integrate(gamma(-2,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(-2, b*x + a) - gamma(-1, b*x + a))/b
```

Giac [F]

$$\int \Gamma(-2, a+bx)\,dx = \int \Gamma(-2, bx+a)\;dx$$

input
```
integrate(gamma(-2,b*x+a),x, algorithm="giac")
```

output
```
integrate(gamma(-2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \Gamma(-2, a+bx)\,dx = -\frac{\mathrm{expint}(2, a+b\,x) - \mathrm{expint}(3, a+b\,x)}{b\;(a+b\,x)}$$

input
```
int(expint(3, a + b*x)/(a + b*x)^2,x)
```

output
```
-(expint(2, a + b*x) - expint(3, a + b*x))/(b*(a + b*x))
```

Reduce [F]

$$\int \Gamma(-2, a+bx)\,dx = \int \frac{ei(3, bx+a)}{b^2x^2+2abx+a^2}dx$$

input
```
int(1/(b*x+a)^2*Ei(3,b*x+a),x)
```

output
```
int(ei(3,a + b*x)/(a**2 + 2*a*b*x + b**2*x**2),x)
```

3.149 $\int \frac{\Gamma(-2,a+bx)}{c+dx} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\Gamma(-2,a+bx)}{c+dx} dx = \text{Int}\left(\frac{\Gamma(-2,a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(1/(b*x+a)^2*Ei(3,b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(-2,a+bx)}{c+dx} dx = \int \frac{\Gamma(-2,a+bx)}{c+dx} dx$$

input
```
Integrate[Gamma[-2, a + b*x]/(c + d*x),x]
```

output
```
Integrate[Gamma[-2, a + b*x]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-2,a+bx)}{c+dx} \, dx$$
$$\Big\downarrow \; 7120$$
$$\int \frac{\Gamma(-2,a+bx)}{c+dx}dx$$

input
```
Int[Gamma[-2, a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{\text{expIntegral}_3\left(bx+a\right)}{\left(bx+a\right)^2\left(dx+c\right)}dx$$

input
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/(d*x+c),x)
```

output
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(-2, a+bx)}{c+dx} \, dx = \int \frac{\Gamma(-2, bx+a)}{dx+c} \, dx$$

input
```
integrate(gamma(-2,b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(gamma(-2, b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{\Gamma(-2, a+bx)}{c+dx} \, dx = \int \frac{\mathrm{E}_3\,(a+bx)}{(a+bx)^2\,(c+dx)} \, dx$$

input
```
integrate(uppergamma(-2,b*x+a)/(d*x+c),x)
```

output
```
Integral(expint(3, a + b*x)/((a + b*x)**2*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(-2, a+bx)}{c+dx} \, dx = \int \frac{\Gamma(-2, bx+a)}{dx+c} \, dx$$

input
```
integrate(gamma(-2,b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(gamma(-2, b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(-2, a+bx)}{c+dx}\,dx = \int \frac{\Gamma(-2, bx+a)}{dx+c}\,dx$$

input
```
integrate(gamma(-2,b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(gamma(-2, b*x + a)/(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{\Gamma(-2, a+bx)}{c+dx}\,dx = \int \frac{\mathrm{expint}(3, a+b\,x)}{(a+b\,x)^2\,(c+d\,x)}\,dx$$

input
```
int(expint(3, a + b*x)/((a + b*x)^2*(c + d*x)),x)
```

output
```
int(expint(3, a + b*x)/((a + b*x)^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.60

$$\int \frac{\Gamma(-2, a+bx)}{c+dx}\,dx = \int \frac{ei(3, bx+a)}{b^2 d\,x^3 + 2abd\,x^2 + b^2 c\,x^2 + a^2 dx + 2abcx + a^2 c} dx$$

input
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/(d*x+c),x)
```

output
```
int(ei(3,a + b*x)/(a**2*c + a**2*d*x + 2*a*b*c*x + 2*a*b*d*x**2 + b**2*c*x
**2 + b**2*d*x**3),x)
```

3.150 $\int \frac{\Gamma(-2,a+bx)}{(c+dx)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 120

$$\int \frac{\Gamma(-2,a+bx)}{(c+dx)^2}\,dx = \frac{b\Gamma(-2,a+bx)}{d(bc-ad)} - \frac{\Gamma(-2,a+bx)}{d(c+dx)} - \frac{b\Gamma(-1,a+bx)}{(bc-ad)^2} + \frac{bd\Gamma(0,a+bx)}{(bc-ad)^3} - \frac{bde^{-a+\frac{bc}{d}}\Gamma\left(0,\frac{b(c+dx)}{d}\right)}{(bc-ad)^3}$$

output
```
b/(b*x+a)^2*Ei(3,b*x+a)/d/(-a*d+b*c)-1/(b*x+a)^2*Ei(3,b*x+a)/d/(d*x+c)-b/(
b*x+a)*Ei(2,b*x+a)/(-a*d+b*c)^2+b*d*Ei(1,b*x+a)/(-a*d+b*c)^3-b*d*exp(-a+b*
c/d)*Ei(1,b*(d*x+c)/d)/(-a*d+b*c)^3
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.65

$$\int \frac{\Gamma(-2,a+bx)}{(c+dx)^2}\,dx = -\frac{be^{-a-bx}(-1+a+bx)}{2d(bc-ad)(a+bx)^2} - \frac{be^{-a-bx}}{(bc-ad)^2(a+bx)} - \frac{b\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^2} + \frac{bd\,\text{ExpIntegralEi}(-a-bx)}{(-bc+ad)^3} - \frac{b\,\text{ExpIntegralEi}(-a-bx)}{2bcd-2ad^2} + \frac{bde^{-a+\frac{bc}{d}}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{(bc-ad)^3} - \frac{\Gamma(-2,a+bx)}{d(c+dx)}$$

input
```
Integrate[Gamma[-2, a + b*x]/(c + d*x)^2,x]
```

output
```
-1/2*(b*E^(-a - b*x)*(-1 + a + b*x))/(d*(b*c - a*d)*(a + b*x)^2) - (b*E^(-
a - b*x))/((b*c - a*d)^2*(a + b*x)) - (b*ExpIntegralEi[-a - b*x])/(b*c - a
*d)^2 + (b*d*ExpIntegralEi[-a - b*x])/(-(b*c) + a*d)^3 - (b*ExpIntegralEi[
-a - b*x])/(2*b*c*d - 2*a*d^2) + (b*d*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*
(c + d*x))/d)])/(b*c - a*d)^3 - Gamma[-2, a + b*x]/(d*(c + d*x))
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.87, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-2,a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{7119}$$

$$-\frac{b\int \frac{e^{-a-bx}}{(a+bx)^3(c+dx)}dx}{d} - \frac{\Gamma(-2,a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\int \left(-\frac{e^{-a-bx}d^3}{(bc-ad)^3(c+dx)}+\frac{be^{-a-bx}d^2}{(bc-ad)^3(a+bx)}-\frac{be^{-a-bx}d}{(bc-ad)^2(a+bx)^2}+\frac{be^{-a-bx}}{(bc-ad)(a+bx)^3}\right)dx}{d}-\frac{\Gamma(-2,a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{d^2\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^3}-\frac{d^2e^{\frac{bc}{d}-a}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{(bc-ad)^3}+\frac{d\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^2}+\frac{\text{ExpIntegralEi}(-a-bx)}{2(bc-ad)}+\frac{de^{-a\cdots}}{(a+bx)(b\cdots}\right.}{d}$$

$$\frac{\Gamma(-2,a+bx)}{d(c+dx)}$$

input
```
Int[Gamma[-2, a + b*x]/(c + d*x)^2,x]
```

output
```
-((b*(-1/2*E^(-a - b*x)/((b*c - a*d)*(a + b*x)^2) + (d*E^(-a - b*x))/((b*c
 - a*d)^2*(a + b*x)) + E^(-a - b*x)/(2*(b*c - a*d)*(a + b*x)) + (d^2*ExpIn
tegralEi[-a - b*x])/(b*c - a*d)^3 + (d*ExpIntegralEi[-a - b*x])/(b*c - a*d
)^2 + ExpIntegralEi[-a - b*x]/(2*(b*c - a*d)) - (d^2*E^(-a + (b*c)/d)*ExpI
ntegralEi[-((b*(c + d*x))/d)])/(b*c - a*d)^3))/d) - Gamma[-2, a + b*x]/(d*
(c + d*x))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\text{expIntegral}_3\left(bx+a\right)}{\left(bx+a\right)^2\left(dx+c\right)^2}dx$$

input
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/(d*x+c)^2,x)
```

output
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/(d*x+c)^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 821 vs. 2(119) = 238.

Time = 0.11 (sec) , antiderivative size = 821, normalized size of antiderivative = 6.84

$$\int \frac{\Gamma(-2,a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(gamma(-2,b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(2*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b
^2*c*d^2 + a^2*b*d^3)*x)*Ei(-(b*d*x + b*c)/d)*e^((b*c - a*d)/d) - (a^2*b^3
*c^3 - 2*(a^3 - a^2)*b^2*c^2*d + (a^4 - 2*a^3 + 2*a^2)*b*c*d^2 + (b^5*c^2*
d - 2*(a - 1)*b^4*c*d^2 + (a^2 - 2*a + 2)*b^3*d^3)*x^3 + (b^5*c^3 + 2*b^4*
c^2*d - (3*a^2 - 2*a - 2)*b^3*c*d^2 + 2*(a^3 - 2*a^2 + 2*a)*b^2*d^3)*x^2 +
 (2*a*b^4*c^3 - (3*a^2 - 4*a)*b^3*c^2*d - 2*(a^2 - 2*a)*b^2*c*d^2 + (a^4 -
 2*a^3 + 2*a^2)*b*d^3)*x)*Ei(-b*x - a) - ((a - 1)*b^3*c^3 - 2*(a^2 - 2*a)*
b^2*c^2*d + (a^3 - 3*a^2)*b*c*d^2 + (b^4*c^2*d - 2*(a - 1)*b^3*c*d^2 + (a^
2 - 2*a)*b^2*d^3)*x^2 + (b^4*c^3 - (a - 1)*b^3*c^2*d - (a^2 - 2*a)*b^2*c*d
^2 + (a^3 - 3*a^2)*b*d^3)*x)*e^(-b*x - a) - 2*(a^2*b^3*c^3 - 3*a^3*b^2*c^2
*d + 3*a^4*b*c*d^2 - a^5*d^3 + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2
- a^3*b^2*d^3)*x^2 + 2*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^
4*b*d^3)*x)*gamma(-2, b*x + a))/(a^2*b^3*c^4*d - 3*a^3*b^2*c^3*d^2 + 3*a^4
*b*c^2*d^3 - a^5*c*d^4 + (b^5*c^3*d^2 - 3*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4
- a^3*b^2*d^5)*x^3 + (b^5*c^4*d - a*b^4*c^3*d^2 - 3*a^2*b^3*c^2*d^3 + 5*a^
3*b^2*c*d^4 - 2*a^4*b*d^5)*x^2 + (2*a*b^4*c^4*d - 5*a^2*b^3*c^3*d^2 + 3*a^
3*b^2*c^2*d^3 + a^4*b*c*d^4 - a^5*d^5)*x)
```

Sympy [F]

$$\int \frac{\Gamma(-2, a+bx)}{(c+dx)^2}\,dx = \int \frac{\mathrm{E}_3\,(a+bx)}{(a+bx)^2\,(c+dx)^2}\,dx$$

input
```
integrate(uppergamma(-2,b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(expint(3, a + b*x)/((a + b*x)**2*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\Gamma(-2, a+bx)}{(c+dx)^2}\,dx = \int \frac{\Gamma(-2, bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(gamma(-2,b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(gamma(-2, b*x + a)/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{\Gamma(-2, a+bx)}{(c+dx)^2}\,dx = \int \frac{\Gamma(-2, bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(gamma(-2,b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(gamma(-2, b*x + a)/(d*x + c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.20

$$\int \frac{\Gamma(-2, a+bx)}{(c+dx)^2} \, dx = \int \frac{\mathrm{expint}(3, a+b\,x)}{(a+b\,x)^2\,(c+d\,x)^2} \, dx$$

input
```
int(expint(3, a + b*x)/((a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(expint(3, a + b*x)/((a + b*x)^2*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{\Gamma(-2, a+bx)}{(c+dx)^2} \, dx = \int \frac{ei(3, bx+a)}{b^2d^2x^4+2ab\,d^2x^3+2b^2cd\,x^3+a^2d^2x^2+4abcd\,x^2+b^2c^2x^2+2a^2cdx+2ab\,c^2x+a^2c^2}dx$$

input
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/(d*x+c)^2,x)
```

output
```
int(ei(3,a + b*x)/(a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2 + 2*a*b*c**2*
x + 4*a*b*c*d*x**2 + 2*a*b*d**2*x**3 + b**2*c**2*x**2 + 2*b**2*c*d*x**3 +
b**2*d**2*x**4),x)
```

3.151 $\int \frac{\Gamma(-2,a+bx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 179

$$\begin{aligned}\int \frac{\Gamma(-2,a+bx)}{(c+dx)^3}\,dx &= \frac{b^2\Gamma(-2,a+bx)}{2d(bc-ad)^2} - \frac{\Gamma(-2,a+bx)}{2d(c+dx)^2} \\ &\quad - \frac{b^2\Gamma(-1,a+bx)}{(bc-ad)^3} - \frac{b^2e^{-a+\frac{bc}{d}}\Gamma\left(-1,\frac{b(c+dx)}{d}\right)}{2(bc-ad)^3} \\ &\quad + \frac{3b^2d\Gamma(0,a+bx)}{2(bc-ad)^4} - \frac{3b^2de^{-a+\frac{bc}{d}}\Gamma\left(0,\frac{b(c+dx)}{d}\right)}{2(bc-ad)^4}\end{aligned}$$

output
```
1/2*b^2/(b*x+a)^2*Ei(3,b*x+a)/d/(-a*d+b*c)^2-1/2/(b*x+a)^2*Ei(3,b*x+a)/d/(
d*x+c)^2-b^2/(b*x+a)*Ei(2,b*x+a)/(-a*d+b*c)^3-1/2*b*exp(-a+b*c/d)/(d*x+c)*
d*Ei(2,b*(d*x+c)/d)/(-a*d+b*c)^3+3/2*b^2*d*Ei(1,b*x+a)/(-a*d+b*c)^4-3/2*b^
2*d*exp(-a+b*c/d)*Ei(1,b*(d*x+c)/d)/(-a*d+b*c)^4
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.73

$$\int \frac{\Gamma(-2,a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{\frac{b^2(b^2c^2-2(-2+a)bcd+(6-4a+a^2)d^2)\,\text{ExpIntegralEi}(-a-bx)}{d}+2b^2(bc-(3+a)d)e^{-a+\frac{bc}{d}}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)\cdots}{\cdots}$$

input
```
Integrate[Gamma[-2, a + b*x]/(c + d*x)^3,x]
```

output
```
-1/4*((b^2*(b^2*c^2 - 2*(-2 + a)*b*c*d + (6 - 4*a + a^2)*d^2)*ExpIntegralE
i[-a - b*x])/d + 2*b^2*(b*c - (3 + a)*d)*E^(-a + (b*c)/d)*ExpIntegralEi[-(
(b*(c + d*x))/d)] + ((b*c - a*d)*(2*b*d^2*E^(-a - b*x)*(a + b*x)^2*(c + d*
x) - b^3*c*E^(-a - b*x)*(c + d*x)^2 + a*b^2*d*E^(-a - b*x)*(c + d*x)^2 + b
^3*c*E^(-a - b*x)*(a + b*x)*(c + d*x)^2 + 4*b^2*d*E^(-a - b*x)*(a + b*x)*(
c + d*x)^2 - a*b^2*d*E^(-a - b*x)*(a + b*x)*(c + d*x)^2 + 2*(b*c - a*d)^3*
(a + b*x)^2*Gamma[-2, a + b*x]))/(d*(a + b*x)^2*(c + d*x)^2))/(b*c - a*d)^
4
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.71, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-2,a+bx)}{(c+dx)^3}\,dx$$
$$\Big\downarrow \text{7119}$$
$$-\frac{b\int \frac{e^{-a-bx}}{(a+bx)^3(c+dx)^2}dx}{2d}-\frac{\Gamma(-2,a+bx)}{2d(c+dx)^2}$$
$$\Big\downarrow \text{7293}$$

$$-\frac{b\int \left(-\frac{3be^{-a-bx}d^3}{(bc-ad)^4(c+dx)}-\frac{e^{-a-bx}d^3}{(bc-ad)^3(c+dx)^2}+\frac{3b^2e^{-a-bx}d^2}{(bc-ad)^4(a+bx)}-\frac{2b^2e^{-a-bx}d}{(bc-ad)^3(a+bx)^2}+\frac{b^2e^{-a-bx}}{(bc-ad)^2(a+bx)^3}\right)dx}{2d}-\frac{\Gamma(-2,a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{b\left(\frac{3bd^2\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^4}-\frac{3bd^2e^{\frac{bc}{d}-a}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{(bc-ad)^4}+\frac{d^2e^{-a-bx}}{(c+dx)(bc-ad)^3}+\frac{2bd\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^3}+\frac{bde^{\frac{bc}{d}-a}\,\text{E}}{}\right.}{2d}-\frac{\Gamma(-2,a+bx)}{2d(c+dx)^2}$$

input
```
Int[Gamma[-2, a + b*x]/(c + d*x)^3,x]
```

output
```
-1/2*(b*(-1/2*(b*E^(-a - b*x))/((b*c - a*d)^2*(a + b*x)^2) + (2*b*d*E^(-a
- b*x))/((b*c - a*d)^3*(a + b*x)) + (b*E^(-a - b*x))/(2*(b*c - a*d)^2*(a +
 b*x)) + (d^2*E^(-a - b*x))/((b*c - a*d)^3*(c + d*x)) + (3*b*d^2*ExpIntegr
alEi[-a - b*x])/(b*c - a*d)^4 + (2*b*d*ExpIntegralEi[-a - b*x])/(b*c - a*d
)^3 + (b*ExpIntegralEi[-a - b*x])/(2*(b*c - a*d)^2) - (3*b*d^2*E^(-a + (b*
c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/(b*c - a*d)^4 + (b*d*E^(-a + (b*c
)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/(b*c - a*d)^3))/d - Gamma[-2, a +
b*x]/(2*d*(c + d*x)^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\text{expIntegral}_3\left(bx+a\right)}{\left(bx+a\right)^2\left(dx+c\right)^3}dx$$

input
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/(d*x+c)^3,x)
```

output
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/(d*x+c)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1284 vs. $2(167) = 334$.

Time = 0.13 (sec) , antiderivative size = 1284, normalized size of antiderivative = 7.17

$$\int \frac{\Gamma(-2, a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(gamma(-2,b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*(a^2*b^3*c^3*d - (a^3 + 3*a^2)*b^2*c^2*d^2 + (b^5*c*d^3 - (a + 3)*
b^4*d^4)*x^4 + 2*(b^5*c^2*d^2 - 3*b^4*c*d^3 - (a^2 + 3*a)*b^3*d^4)*x^3 + (
b^5*c^3*d + 3*(a - 1)*b^4*c^2*d^2 - 3*(a^2 + 4*a)*b^3*c*d^3 - (a^3 + 3*a^2
)*b^2*d^4)*x^2 + 2*(a*b^4*c^3*d - 3*a*b^3*c^2*d^2 - (a^3 + 3*a^2)*b^2*c*d^
3)*x)*Ei(-(b*d*x + b*c)/d)*e^((b*c - a*d)/d) + (a^2*b^4*c^4 - 2*(a^3 - 2*a
^2)*b^3*c^3*d + (a^4 - 4*a^3 + 6*a^2)*b^2*c^2*d^2 + (b^6*c^2*d^2 - 2*(a -
2)*b^5*c*d^3 + (a^2 - 4*a + 6)*b^4*d^4)*x^4 + 2*(b^6*c^3*d - (a - 4)*b^5*c
^2*d^2 - (a^2 - 6)*b^4*c*d^3 + (a^3 - 4*a^2 + 6*a)*b^3*d^4)*x^3 + (b^6*c^4
 + 2*(a + 2)*b^5*c^3*d - 6*(a^2 - 2*a - 1)*b^4*c^2*d^2 + 2*(a^3 - 6*a^2 +
12*a)*b^3*c*d^3 + (a^4 - 4*a^3 + 6*a^2)*b^2*d^4)*x^2 + 2*(a*b^5*c^4 - (a^2
 - 4*a)*b^4*c^3*d - (a^3 - 6*a)*b^3*c^2*d^2 + (a^4 - 4*a^3 + 6*a^2)*b^2*c*
d^3)*x)*Ei(-b*x - a) + ((a - 1)*b^4*c^4 - 2*(a^2 - 3*a)*b^3*c^3*d - 2*a^3*
b*c*d^3 + (a^3 - 3*a^2)*b^2*c^2*d^2 + (b^5*c^2*d^2 - 2*(a - 3)*b^4*c*d^3 +
 (a^2 - 6*a)*b^3*d^4)*x^3 + (2*b^5*c^3*d - 3*(a - 3)*b^4*c^2*d^2 + (a^3 -
9*a^2)*b^2*d^4)*x^2 + (b^5*c^4 + 2*b^4*c^3*d - 3*(a^2 - 4*a)*b^3*c^2*d^2 -
 2*a^3*b*d^4 + 2*(a^3 - 6*a^2)*b^2*c*d^3)*x)*e^(-b*x - a) + 2*(a^2*b^4*c^4
 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4 + (b^6*c^
4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^2
 + 2*(a*b^5*c^4 - 4*a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 +
a^5*b*d^4)*x)*gamma(-2, b*x + a))/(a^2*b^4*c^6*d - 4*a^3*b^3*c^5*d^2 + ...
```

Sympy [F]

$$\int \frac{\Gamma(-2,a+bx)}{(c+dx)^3}\,dx=\int \frac{\mathrm{E}_3\left(a+bx\right)}{\left(a+bx\right)^2\left(c+dx\right)^3}\,dx$$

input
```
integrate(uppergamma(-2,b*x+a)/(d*x+c)**3,x)
```

output
```
Integral(expint(3, a + b*x)/((a + b*x)**2*(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\Gamma(-2, a+bx)}{(c+dx)^3} \, dx = \int \frac{\Gamma(-2, bx+a)}{(dx+c)^3} \, dx$$

input
```
integrate(gamma(-2,b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(gamma(-2, b*x + a)/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{\Gamma(-2, a+bx)}{(c+dx)^3} \, dx = \int \frac{\Gamma(-2, bx+a)}{(dx+c)^3} \, dx$$

input
```
integrate(gamma(-2,b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(gamma(-2, b*x + a)/(d*x + c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 19.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.13

$$\int \frac{\Gamma(-2, a+bx)}{(c+dx)^3} \, dx = \int \frac{\mathrm{expint}(3, a+b\,x)}{(a+b\,x)^2\,(c+d\,x)^3} \, dx$$

input
```
int(expint(3, a + b*x)/((a + b*x)^2*(c + d*x)^3),x)
```

output
```
int(expint(3, a + b*x)/((a + b*x)^2*(c + d*x)^3), x)
```

Reduce [F]

$$\int \frac{\Gamma(-2, a+bx)}{(c+dx)^3} \, dx$$
$$= \int \frac{ei(3, bx+a)}{b^2d^3x^5 + 2ab\,d^3x^4 + 3b^2c\,d^2x^4 + a^2d^3x^3 + 6abc\,d^2x^3 + 3b^2c^2d\,x^3 + 3a^2c\,d^2x^2 + 6ab\,c^2d\,x^2 + b^2c^3x^2 + 3}$$

input
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/(d*x+c)^3,x)
```

output
```
int(ei(3,a + b*x)/(a**2*c**3 + 3*a**2*c**2*d*x + 3*a**2*c*d**2*x**2 + a**2
*d**3*x**3 + 2*a*b*c**3*x + 6*a*b*c**2*d*x**2 + 6*a*b*c*d**2*x**3 + 2*a*b*
d**3*x**4 + b**2*c**3*x**2 + 3*b**2*c**2*d*x**3 + 3*b**2*c*d**2*x**4 + b**
2*d**3*x**5),x)
```

3.152 $\int \frac{\Gamma(-2,a+bx)}{(c+dx)^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 217

$$\int \frac{\Gamma(-2,a+bx)}{(c+dx)^4}\,dx = \frac{b^3\Gamma(-2,a+bx)}{3d(bc-ad)^3} - \frac{\Gamma(-2,a+bx)}{3d(c+dx)^3} - \frac{b^3e^{-a+\frac{bc}{d}}\Gamma\left(-2,\frac{b(c+dx)}{d}\right)}{3d(bc-ad)^3} - \frac{b^3\Gamma(-1,a+bx)}{(bc-ad)^4} - \frac{b^3e^{-a+\frac{bc}{d}}\Gamma\left(-1,\frac{b(c+dx)}{d}\right)}{(bc-ad)^4} + \frac{2b^3d\Gamma(0,a+bx)}{(bc-ad)^5} - \frac{2b^3de^{-a+\frac{bc}{d}}\Gamma\left(0,\frac{b(c+dx)}{d}\right)}{(bc-ad)^5}$$

output

```
1/3*b^3/(b*x+a)^2*Ei(3,b*x+a)/d/(-a*d+b*c)^3-1/3/(b*x+a)^2*Ei(3,b*x+a)/d/(
d*x+c)^3-1/3*b*exp(-a+b*c/d)/(d*x+c)^2*d*Ei(3,b*(d*x+c)/d)/(-a*d+b*c)^3-b^
3/(b*x+a)*Ei(2,b*x+a)/(-a*d+b*c)^4-b^2*exp(-a+b*c/d)/(d*x+c)*d*Ei(2,b*(d*x
+c)/d)/(-a*d+b*c)^4+2*b^3*d*Ei(1,b*x+a)/(-a*d+b*c)^5-2*b^3*d*exp(-a+b*c/d)
*Ei(1,b*(d*x+c)/d)/(-a*d+b*c)^5
```

Mathematica [F]

$$\int \frac{\Gamma(-2,a+bx)}{(c+dx)^4}\,dx = \int \frac{\Gamma(-2,a+bx)}{(c+dx)^4}\,dx$$

input
```
Integrate[Gamma[-2, a + b*x]/(c + d*x)^4,x]
```

output
```
Integrate[Gamma[-2, a + b*x]/(c + d*x)^4, x]
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-2,a+bx)}{(c+dx)^4}\,dx$$

$$\Big\downarrow \text{7119}$$

$$-\frac{b\int \frac{e^{-a-bx}}{(a+bx)^3(c+dx)^3}dx}{3d}-\frac{\Gamma(-2,a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\int \left(\frac{6d^2e^{-a-bx}b^3}{(bc-ad)^5(a+bx)}-\frac{3de^{-a-bx}b^3}{(bc-ad)^4(a+bx)^2}+\frac{e^{-a-bx}b^3}{(bc-ad)^3(a+bx)^3}-\frac{6d^3e^{-a-bx}b^2}{(bc-ad)^5(c+dx)}-\frac{3d^3e^{-a-bx}b}{(bc-ad)^4(c+dx)^2}-\frac{d^3e^{-a-bx}}{(bc-ad)^3(c+dx)^3}\right)dx}{3d}-\frac{\Gamma(-2,a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{6b^2d^2\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^5}-\frac{6b^2d^2e^{\frac{bc}{d}-a}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{(bc-ad)^5}+\frac{b^2\,\text{ExpIntegralEi}(-a-bx)}{2(bc-ad)^3}+\frac{3b^2d\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^4}\right.}{}$$

$$\frac{\Gamma(-2,a+bx)}{3d(c+dx)^3}$$

input
```
Int[Gamma[-2, a + b*x]/(c + d*x)^4,x]
```

output
```
-1/3*(b*(-1/2*(b^2*E^(-a - b*x))/((b*c - a*d)^3*(a + b*x)^2) + (3*b^2*d*E^
(-a - b*x))/((b*c - a*d)^4*(a + b*x)) + (b^2*E^(-a - b*x))/(2*(b*c - a*d)^
3*(a + b*x)) + (d^2*E^(-a - b*x))/(2*(b*c - a*d)^3*(c + d*x)^2) + (3*b*d^2
*E^(-a - b*x))/((b*c - a*d)^4*(c + d*x)) - (b*d*E^(-a - b*x))/(2*(b*c - a*
d)^3*(c + d*x)) + (6*b^2*d^2*ExpIntegralEi[-a - b*x])/(b*c - a*d)^5 + (3*b
^2*d*ExpIntegralEi[-a - b*x])/(b*c - a*d)^4 + (b^2*ExpIntegralEi[-a - b*x]
)/(2*(b*c - a*d)^3) - (6*b^2*d^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c +
d*x))/d)])/(b*c - a*d)^5 + (3*b^2*d*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c
 + d*x))/d)])/(b*c - a*d)^4 - (b^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c
+ d*x))/d)])/(2*(b*c - a*d)^3)))/d - Gamma[-2, a + b*x]/(3*d*(c + d*x)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\text{expIntegral}_3(bx+a)}{(bx+a)^2 (dx+c)^4} dx$$

input
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/(d*x+c)^4,x)
```

output
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/(d*x+c)^4,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1987 vs. $2(208) = 416$.

Time = 0.18 (sec) , antiderivative size = 1987, normalized size of antiderivative = 9.16

$$\int \frac{\Gamma(-2, a+bx)}{(c+dx)^4} dx = \text{Too large to display}$$

input
```
integrate(gamma(-2,b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*((a^2*b^5*c^5 - 2*(a^3 + 3*a^2)*b^4*c^4*d + (a^4 + 6*a^3 + 12*a^2)*b^3
*c^3*d^2 + (b^7*c^2*d^3 - 2*(a + 3)*b^6*c*d^4 + (a^2 + 6*a + 12)*b^5*d^5)*
x^5 + (3*b^7*c^3*d^2 - 2*(2*a + 9)*b^6*c^2*d^3 - (a^2 - 6*a - 36)*b^5*c*d^
4 + 2*(a^3 + 6*a^2 + 12*a)*b^4*d^5)*x^4 + (3*b^7*c^4*d - 18*b^6*c^3*d^2 -
2*(4*a^2 + 9*a - 18)*b^5*c^2*d^3 + 2*(2*a^3 + 15*a^2 + 36*a)*b^4*c*d^4 + (
a^4 + 6*a^3 + 12*a^2)*b^3*d^5)*x^3 + (b^7*c^5 + 2*(2*a - 3)*b^6*c^4*d - 2*
(4*a^2 + 15*a - 6)*b^5*c^3*d^2 + 18*(a^2 + 4*a)*b^4*c^2*d^3 + 3*(a^4 + 6*a
^3 + 12*a^2)*b^3*c*d^4)*x^2 + (2*a*b^6*c^5 - (a^2 + 12*a)*b^5*c^4*d - 2*(2
*a^3 + 3*a^2 - 12*a)*b^4*c^3*d^2 + 3*(a^4 + 6*a^3 + 12*a^2)*b^3*c^2*d^3)*x
)*Ei(-(b*d*x + b*c)/d)*e^((b*c - a*d)/d) - (a^2*b^5*c^5 - 2*(a^3 - 3*a^2)*
b^4*c^4*d + (a^4 - 6*a^3 + 12*a^2)*b^3*c^3*d^2 + (b^7*c^2*d^3 - 2*(a - 3)*
b^6*c*d^4 + (a^2 - 6*a + 12)*b^5*d^5)*x^5 + (3*b^7*c^3*d^2 - 2*(2*a - 9)*b
^6*c^2*d^3 - (a^2 + 6*a - 36)*b^5*c*d^4 + 2*(a^3 - 6*a^2 + 12*a)*b^4*d^5)*
x^4 + (3*b^7*c^4*d + 18*b^6*c^3*d^2 - 2*(4*a^2 - 9*a - 18)*b^5*c^2*d^3 + 2
*(2*a^3 - 15*a^2 + 36*a)*b^4*c*d^4 + (a^4 - 6*a^3 + 12*a^2)*b^3*d^5)*x^3 +
 (b^7*c^5 + 2*(2*a + 3)*b^6*c^4*d - 2*(4*a^2 - 15*a - 6)*b^5*c^3*d^2 - 18*
(a^2 - 4*a)*b^4*c^2*d^3 + 3*(a^4 - 6*a^3 + 12*a^2)*b^3*c*d^4)*x^2 + (2*a*b
^6*c^5 - (a^2 - 12*a)*b^5*c^4*d - 2*(2*a^3 - 3*a^2 - 12*a)*b^4*c^3*d^2 + 3
*(a^4 - 6*a^3 + 12*a^2)*b^3*c^2*d^3)*x)*Ei(-b*x - a) - ((a - 1)*b^5*c^5 +
3*a^3*b^3*c^3*d^2 - (3*a^2 - 8*a)*b^4*c^4*d + a^4*b*c*d^4 - (a^4 + 8*a^...
```

Sympy [F]

$$\int \frac{\Gamma(-2, a+bx)}{(c+dx)^4}\,dx = \int \frac{\mathrm{E}_3\left(a+bx\right)}{\left(a+bx\right)^2\left(c+dx\right)^4}\,dx$$

input
```
integrate(uppergamma(-2,b*x+a)/(d*x+c)**4,x)
```

output
```
Integral(expint(3, a + b*x)/((a + b*x)**2*(c + d*x)**4), x)
```

Maxima [F]

$$\int \frac{\Gamma(-2, a+bx)}{(c+dx)^4}\,dx = \int \frac{\Gamma(-2, bx+a)}{(dx+c)^4}\,dx$$

input
```
integrate(gamma(-2,b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate(gamma(-2, b*x + a)/(d*x + c)^4, x)
```

Giac [F]

$$\int \frac{\Gamma(-2, a+bx)}{(c+dx)^4}\,dx = \int \frac{\Gamma(-2, bx+a)}{(dx+c)^4}\,dx$$

input
```
integrate(gamma(-2,b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate(gamma(-2, b*x + a)/(d*x + c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 61.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.11

$$\int \frac{\Gamma(-2, a+bx)}{(c+dx)^4}\,dx = \int \frac{\mathrm{expint}(3, a+b\,x)}{(a+b\,x)^2\,(c+d\,x)^4}\,dx$$

input
```
int(expint(3, a + b*x)/((a + b*x)^2*(c + d*x)^4),x)
```

output
```
int(expint(3, a + b*x)/((a + b*x)^2*(c + d*x)^4), x)
```

Reduce [F]

$$\int \frac{\Gamma(-2, a + bx)}{(c + dx)^4} \, dx$$

$$= \int \frac{ei(3, bx + a)}{b^2d^4x^6 + 2ab\,d^4x^5 + 4b^2c\,d^3x^5 + a^2d^4x^4 + 8abc\,d^3x^4 + 6b^2c^2d^2x^4 + 4a^2c\,d^3x^3 + 12ab\,c^2d^2x^3 + 4b^2c^3d\,x}$$

input
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/(d*x+c)^4,x)
```

output
```
int(ei(3,a + b*x)/(a**2*c**4 + 4*a**2*c**3*d*x + 6*a**2*c**2*d**2*x**2 + 4
*a**2*c*d**3*x**3 + a**2*d**4*x**4 + 2*a*b*c**4*x + 8*a*b*c**3*d*x**2 + 12
*a*b*c**2*d**2*x**3 + 8*a*b*c*d**3*x**4 + 2*a*b*d**4*x**5 + b**2*c**4*x**2
 + 4*b**2*c**3*d*x**3 + 6*b**2*c**2*d**2*x**4 + 4*b**2*c*d**3*x**5 + b**2*
d**4*x**6),x)
```

3.153 $\int (c+dx)^3\Gamma(-3,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 139

$$\int (c+dx)^3\Gamma(-3,a+bx)\,dx = -\frac{d^3e^{-a-bx}}{4b^4}-\frac{(bc-ad)^4\Gamma(-3,a+bx)}{4b^4d}+\frac{(c+dx)^4\Gamma(-3,a+bx)}{4d}-\frac{(bc-ad)^3\Gamma(-2,a+bx)}{b^4}-\frac{3d(bc-ad)^2\Gamma(-1,a+bx)}{2b^4}-\frac{d^2(bc-ad)\Gamma(0,a+bx)}{b^4}$$

output
```
-1/4*d^3*exp(-b*x-a)/b^4-1/4*(-a*d+b*c)^4/(b*x+a)^3*Ei(4,b*x+a)/b^4/d+1/4*
(d*x+c)^4/(b*x+a)^3*Ei(4,b*x+a)/d-(-a*d+b*c)^3/(b*x+a)^2*Ei(3,b*x+a)/b^4-3
/2*d*(-a*d+b*c)^2/(b*x+a)*Ei(2,b*x+a)/b^4-d^2*(-a*d+b*c)*Ei(1,b*x+a)/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 482 vs. $2(139) = 278$.

Time = 0.32 (sec) , antiderivative size = 482, normalized size of antiderivative = 3.47

$$\int (c+dx)^3\Gamma(-3,a+bx)\,dx$$
$$= \frac{1}{24}\left(\frac{e^{-a-bx}\left(-6d^3-\frac{2a(-4b^3c^3+6ab^2c^2d-4a^2bcd^2+a^3d^3)}{(a+bx)^3}+\frac{-4(3+a)b^3c^3+6a(6+a)b^2c^2d-4a^2(9+a)bcd^2+a^3(12+a)d^3}{(a+bx)^2}+\frac{4(3+a)}{}\right.}{b^4}\right.$$
$$+\frac{12c^3\,\text{ExpIntegralEi}(-a-bx)}{b}+\frac{4ac^3\,\text{ExpIntegralEi}(-a-bx)}{b}$$
$$-\frac{36c^2d\,\text{ExpIntegralEi}(-a-bx)}{b^2}-\frac{36ac^2d\,\text{ExpIntegralEi}(-a-bx)}{b^2}$$
$$-\frac{6a^2c^2d\,\text{ExpIntegralEi}(-a-bx)}{b^2}+\frac{24cd^2\,\text{ExpIntegralEi}(-a-bx)}{b^3}$$
$$+\frac{72acd^2\,\text{ExpIntegralEi}(-a-bx)}{b^3}+\frac{36a^2cd^2\,\text{ExpIntegralEi}(-a-bx)}{b^3}$$
$$+\frac{4a^3cd^2\,\text{ExpIntegralEi}(-a-bx)}{b^3}-\frac{24ad^3\,\text{ExpIntegralEi}(-a-bx)}{b^4}$$
$$-\frac{36a^2d^3\,\text{ExpIntegralEi}(-a-bx)}{b^4}-\frac{12a^3d^3\,\text{ExpIntegralEi}(-a-bx)}{b^4}$$
$$\left.-\frac{a^4d^3\,\text{ExpIntegralEi}(-a-bx)}{b^4}+6x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)\Gamma(-3,a+bx)\right)$$

input
```
Integrate[(c + d*x)^3*Gamma[-3, a + b*x],x]
```

output
```
((E^(-a - b*x)*(-6*d^3 - (2*a*(-4*b^3*c^3 + 6*a*b^2*c^2*d - 4*a^2*b*c*d^2
+ a^3*d^3))/(a + b*x)^3 + (-4*(3 + a)*b^3*c^3 + 6*a*(6 + a)*b^2*c^2*d - 4*
a^2*(9 + a)*b*c*d^2 + a^3*(12 + a)*d^3)/(a + b*x)^2 + (4*(3 + a)*b^3*c^3 -
 6*(6 + 6*a + a^2)*b^2*c^2*d + 4*a*(18 + 9*a + a^2)*b*c*d^2 - a^2*(6 + a)^
2*d^3)/(a + b*x)))/b^4 + (12*c^3*ExpIntegralEi[-a - b*x])/b + (4*a*c^3*Exp
IntegralEi[-a - b*x])/b - (36*c^2*d*ExpIntegralEi[-a - b*x])/b^2 - (36*a*c
^2*d*ExpIntegralEi[-a - b*x])/b^2 - (6*a^2*c^2*d*ExpIntegralEi[-a - b*x])/
b^2 + (24*c*d^2*ExpIntegralEi[-a - b*x])/b^3 + (72*a*c*d^2*ExpIntegralEi[-
a - b*x])/b^3 + (36*a^2*c*d^2*ExpIntegralEi[-a - b*x])/b^3 + (4*a^3*c*d^2*
ExpIntegralEi[-a - b*x])/b^3 - (24*a*d^3*ExpIntegralEi[-a - b*x])/b^4 - (3
6*a^2*d^3*ExpIntegralEi[-a - b*x])/b^4 - (12*a^3*d^3*ExpIntegralEi[-a - b*
x])/b^4 - (a^4*d^3*ExpIntegralEi[-a - b*x])/b^4 + 6*x*(4*c^3 + 6*c^2*d*x +
 4*c*d^2*x^2 + d^3*x^3)*Gamma[-3, a + b*x])/24
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 355 vs. $2(139) = 278$.

Time = 0.86 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.55, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\Gamma(-3,a+bx)\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{b\int \frac{e^{-a-bx}(c+dx)^4}{(a+bx)^4}dx}{4d}+\frac{(c+dx)^4\Gamma(-3,a+bx)}{4d}$$

$$\Big\downarrow \text{2629}$$

$$\frac{b\int \left(\frac{e^{-a-bx}d^4}{b^4}+\frac{4(bc-ad)e^{-a-bx}d^3}{b^4(a+bx)}+\frac{6(bc-ad)^2e^{-a-bx}d^2}{b^4(a+bx)^2}+\frac{4(bc-ad)^3e^{-a-bx}d}{b^4(a+bx)^3}+\frac{(bc-ad)^4e^{-a-bx}}{b^4(a+bx)^4}\right)dx}{4d}+\frac{(c+dx)^4\Gamma(-3,a+bx)}{4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{4d^3(bc-ad)\,\text{ExpIntegralEi}(-a-bx)}{b^5}-\frac{6d^2(bc-ad)^2\,\text{ExpIntegralEi}(-a-bx)}{b^5}-\frac{6d^2e^{-a-bx}(bc-ad)^2}{b^5(a+bx)}+\frac{2d(bc-ad)^3\,\text{ExpIntegralEi}(-a-bx)}{b^5}\right.}{}$$

$$\frac{(c+dx)^4\Gamma(-3,a+bx)}{4d}$$

input
```
Int[(c + d*x)^3*Gamma[-3, a + b*x],x]
```

output
```
(b*(-((d^4*E^(-a - b*x))/b^5) - ((b*c - a*d)^4*E^(-a - b*x))/(3*b^5*(a + b
*x)^3) - (2*d*(b*c - a*d)^3*E^(-a - b*x))/(b^5*(a + b*x)^2) + ((b*c - a*d)
^4*E^(-a - b*x))/(6*b^5*(a + b*x)^2) - (6*d^2*(b*c - a*d)^2*E^(-a - b*x))/
(b^5*(a + b*x)) + (2*d*(b*c - a*d)^3*E^(-a - b*x))/(b^5*(a + b*x)) - ((b*c
 - a*d)^4*E^(-a - b*x))/(6*b^5*(a + b*x)) + (4*d^3*(b*c - a*d)*ExpIntegral
Ei[-a - b*x])/b^5 - (6*d^2*(b*c - a*d)^2*ExpIntegralEi[-a - b*x])/b^5 + (2
*d*(b*c - a*d)^3*ExpIntegralEi[-a - b*x])/b^5 - ((b*c - a*d)^4*ExpIntegral
Ei[-a - b*x])/(6*b^5)))/(4*d) + ((c + d*x)^4*Gamma[-3, a + b*x])/(4*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [F]

$$\int \frac{(dx+c)^3 \operatorname{expIntegral}_4(bx+a)}{(bx+a)^3}dx$$

input
```
int((d*x+c)^3/(b*x+a)^3*Ei(4,b*x+a),x)
```

output
```
int((d*x+c)^3/(b*x+a)^3*Ei(4,b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(130) = 260$.

Time = 0.09 (sec) , antiderivative size = 662, normalized size of antiderivative = 4.76

$$\int (c+dx)^3\Gamma(-3,a+bx)\,dx = -\frac{(b^3d^3x^3+4\,b^3c^3-6\,(a+2)b^2c^2d+4\,(a^2+5\,a+2)bcd^2-(a^3+8\,a^2+8\,a)d^3+(4\,b^3cd^2-ab^2d^3)x^2+}{}$$

input
```
integrate((d*x+c)^3*gamma(-3,b*x+a),x, algorithm="fricas")
```

output
```
-1/4*((b^3*d^3*x^3 + 4*b^3*c^3 - 6*(a + 2)*b^2*c^2*d + 4*(a^2 + 5*a + 2)*b
*c*d^2 - (a^3 + 8*a^2 + 8*a)*d^3 + (4*b^3*c*d^2 - a*b^2*d^3)*x^2 + (6*b^3*
c^2*d - 4*(a + 1)*b^2*c*d^2 + (a^2 + 4*a)*b*d^3)*x)*e^(-b*x - a) - (b^7*d^
3*x^7 + 4*(a^4 + 3*a^3)*b^3*c^3 + (4*b^7*c*d^2 + 3*a*b^6*d^3)*x^6 - 6*(a^5
 + 6*a^4 + 6*a^3)*b^2*c^2*d + 3*(2*b^7*c^2*d + 4*a*b^6*c*d^2 + a^2*b^5*d^3
)*x^5 + 4*(a^6 + 9*a^5 + 18*a^4 + 6*a^3)*b*c*d^2 + (4*b^7*c^3 + 18*a*b^6*c
^2*d + 12*a^2*b^5*c*d^2 + a^3*b^4*d^3)*x^4 - (a^7 + 12*a^6 + 36*a^5 + 24*a
^4)*d^3 + (4*(4*a + 3)*b^6*c^3 + 12*(a^2 - 3*a - 3)*b^5*c^2*d + 4*(2*a^3 +
 9*a^2 + 18*a + 6)*b^4*c*d^2 - (a^4 + 12*a^3 + 36*a^2 + 24*a)*b^3*d^3)*x^3
 + 3*(4*(2*a^2 + 3*a)*b^5*c^3 - 4*(a^3 + 9*a^2 + 9*a)*b^4*c^2*d + 4*(a^4 +
 9*a^3 + 18*a^2 + 6*a)*b^3*c*d^2 - (a^5 + 12*a^4 + 36*a^3 + 24*a^2)*b^2*d^
3)*x^2 + (4*(4*a^3 + 9*a^2)*b^4*c^3 - 18*(a^4 + 6*a^3 + 6*a^2)*b^3*c^2*d +
 12*(a^5 + 9*a^4 + 18*a^3 + 6*a^2)*b^2*c*d^2 - 3*(a^6 + 12*a^5 + 36*a^4 +
24*a^3)*b*d^3)*x)*gamma(-3, b*x + a))/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x
 + a^3*b^4)
```

Sympy [F]

$$\int (c+dx)^3\Gamma(-3,a+bx)\,dx = \int \frac{(c+dx)^3\,\mathrm{E}_4\,(a+bx)}{(a+bx)^3}\,dx$$

input
```
integrate((d*x+c)**3*uppergamma(-3,b*x+a),x)
```

output
```
Integral((c + d*x)**3*expint(4, a + b*x)/(a + b*x)**3, x)
```

Maxima [F]

$$\int (c+dx)^3\Gamma(-3,a+bx)\,dx=\int (dx+c)^3\Gamma(-3,bx+a)\,dx$$

input
```
integrate((d*x+c)^3*gamma(-3,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(-3, b*x + a) - gamma(-2, b*x + a))*c^3/b + integrate(d^3*
x^3*gamma(-3, b*x + a) + 3*c*d^2*x^2*gamma(-3, b*x + a) + 3*c^2*d*x*gamma(
-3, b*x + a), x)
```

Giac [F]

$$\int (c+dx)^3\Gamma(-3,a+bx)\,dx=\int (dx+c)^3\Gamma(-3,bx+a)\,dx$$

input
```
integrate((d*x+c)^3*gamma(-3,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*gamma(-3, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.17

$$\int (c+dx)^3\Gamma(-3,a+bx)\,dx=\int \frac{\mathrm{expint}(4,a+b\,x)\,\left(c+d\,x\right)^3}{\left(a+b\,x\right)^3}\,dx$$

input
```
int((expint(4, a + b*x)*(c + d*x)^3)/(a + b*x)^3,x)
```

output
```
int((expint(4, a + b*x)*(c + d*x)^3)/(a + b*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3\Gamma(-3,a+bx)\,dx = \left(\int \frac{ei(4,bx+a)}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}dx\right)c^3 + \left(\int \frac{ei(4,bx+a)\,x^3}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}dx\right)d^3 + 3\left(\int \frac{ei(4,bx+a)\,x^2}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}dx\right)c\,d^2 + 3\left(\int \frac{ei(4,bx+a)\,x}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}dx\right)c^2d$$

input
```
int((d*x+c)^3/(b*x+a)^3*Ei(4,b*x+a),x)
```

output
```
int(ei(4,a + b*x)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*c**3
+ int((ei(4,a + b*x)*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)
,x)*d**3 + 3*int((ei(4,a + b*x)*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 +
 b**3*x**3),x)*c*d**2 + 3*int((ei(4,a + b*x)*x)/(a**3 + 3*a**2*b*x + 3*a*b
**2*x**2 + b**3*x**3),x)*c**2*d
```

3.154 $\int (c+dx)^2\Gamma(-3,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int (c+dx)^2\Gamma(-3,a+bx)\,dx = -\frac{(bc-ad)^3\Gamma(-3,a+bx)}{3b^3d}+\frac{(c+dx)^3\Gamma(-3,a+bx)}{3d} - \frac{(bc-ad)^2\Gamma(-2,a+bx)}{b^3} - \frac{d(bc-ad)\Gamma(-1,a+bx)}{b^3}-\frac{d^2\Gamma(0,a+bx)}{3b^3}$$

output
```
-1/3*(-a*d+b*c)^3/(b*x+a)^3*Ei(4,b*x+a)/b^3/d+1/3*(d*x+c)^3/(b*x+a)^3*Ei(4
,b*x+a)/d-(-a*d+b*c)^2/(b*x+a)^2*Ei(3,b*x+a)/b^3-d*(-a*d+b*c)/(b*x+a)*Ei(2
,b*x+a)/b^3-1/3*d^2*Ei(1,b*x+a)/b^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 351 vs. 2(109) = 218.

Time = 0.23 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.22

$$\int (c+dx)^2\Gamma(-3,a+bx)\,dx = \frac{1}{18}\left(\frac{2a(3b^2c^2-3abcd+a^2d^2)\,e^{-a-bx}}{b^3(a+bx)^3} + \frac{(-3(3+a)b^2c^2+3a(6+a)bcd-a^2(9+a)d^2)\,e^{-a-bx}}{b^3(a+bx)^2} + \frac{(3(3+a)b^2c^2-3(6+6a+a^2)\,bcd+a(18+9a+a^2)\,d^2)\,e^{-a-bx}}{b^3(a+bx)} + \frac{9c^2\,\text{ExpIntegralEi}(-a-bx)}{b} + \frac{3ac^2\,\text{ExpIntegralEi}(-a-bx)}{b} - \frac{18cd\,\text{ExpIntegralEi}(-a-bx)}{b^2} - \frac{18acd\,\text{ExpIntegralEi}(-a-bx)}{b^2} - \frac{3a^2cd\,\text{ExpIntegralEi}(-a-bx)}{b^2} + \frac{6d^2\,\text{ExpIntegralEi}(-a-bx)}{b^3} + \frac{18ad^2\,\text{ExpIntegralEi}(-a-bx)}{b^3} + \frac{9a^2d^2\,\text{ExpIntegralEi}(-a-bx)}{b^3} + \frac{a^3d^2\,\text{ExpIntegralEi}(-a-bx)}{b^3} + 6x\left(3c^2+3cdx+d^2x^2\right)\Gamma(-3,a+bx)\right)$$

input
```
Integrate[(c + d*x)^2*Gamma[-3, a + b*x],x]
```

output
```
((2*a*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*E^(-a - b*x))/(b^3*(a + b*x)^3) +
((-3*(3 + a)*b^2*c^2 + 3*a*(6 + a)*b*c*d - a^2*(9 + a)*d^2)*E^(-a - b*x))/
(b^3*(a + b*x)^2) + ((3*(3 + a)*b^2*c^2 - 3*(6 + 6*a + a^2)*b*c*d + a*(18
+ 9*a + a^2)*d^2)*E^(-a - b*x))/(b^3*(a + b*x)) + (9*c^2*ExpIntegralEi[-a
- b*x])/b + (3*a*c^2*ExpIntegralEi[-a - b*x])/b - (18*c*d*ExpIntegralEi[-a
 - b*x])/b^2 - (18*a*c*d*ExpIntegralEi[-a - b*x])/b^2 - (3*a^2*c*d*ExpInte
gralEi[-a - b*x])/b^2 + (6*d^2*ExpIntegralEi[-a - b*x])/b^3 + (18*a*d^2*Ex
pIntegralEi[-a - b*x])/b^3 + (9*a^2*d^2*ExpIntegralEi[-a - b*x])/b^3 + (a^
3*d^2*ExpIntegralEi[-a - b*x])/b^3 + 6*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Gamma
[-3, a + b*x])/18
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 330 vs. $2(109) = 218$.

Time = 0.82 (sec) , antiderivative size = 330, normalized size of antiderivative = 3.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\Gamma(-3,a+bx)\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{b\int \frac{e^{-a-bx}(c+dx)^3}{(a+bx)^4}dx}{3d}+\frac{(c+dx)^3\Gamma(-3,a+bx)}{3d}$$

$$\Big\downarrow \text{2629}$$

$$\frac{b\int \left(\frac{e^{-a-bx}d^3}{b^3(a+bx)}+\frac{3(bc-ad)e^{-a-bx}d^2}{b^3(a+bx)^2}+\frac{3(bc-ad)^2e^{-a-bx}d}{b^3(a+bx)^3}+\frac{(bc-ad)^3e^{-a-bx}}{b^3(a+bx)^4}\right)dx}{3d}+\frac{(c+dx)^3\Gamma(-3,a+bx)}{3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{3d^2(bc-ad)\operatorname{ExpIntegralEi}(-a-bx)}{b^4}-\frac{3d^2e^{-a-bx}(bc-ad)}{b^4(a+bx)}+\frac{3d(bc-ad)^2\operatorname{ExpIntegralEi}(-a-bx)}{2b^4}-\frac{(bc-ad)^3\operatorname{ExpIntegralEi}(-a-bx)}{6b^4}\cdots\right.}{3d}+\frac{(c+dx)^3\Gamma(-3,a+bx)}{3d}$$

input
```
Int[(c + d*x)^2*Gamma[-3, a + b*x],x]
```

output
```
(b*(-1/3*((b*c - a*d)^3*E^(-a - b*x))/(b^4*(a + b*x)^3) - (3*d*(b*c - a*d)
^2*E^(-a - b*x))/(2*b^4*(a + b*x)^2) + ((b*c - a*d)^3*E^(-a - b*x))/(6*b^4
*(a + b*x)^2) - (3*d^2*(b*c - a*d)*E^(-a - b*x))/(b^4*(a + b*x)) + (3*d*(b
*c - a*d)^2*E^(-a - b*x))/(2*b^4*(a + b*x)) - ((b*c - a*d)^3*E^(-a - b*x))
/(6*b^4*(a + b*x)) + (d^3*ExpIntegralEi[-a - b*x])/b^4 - (3*d^2*(b*c - a*d
)*ExpIntegralEi[-a - b*x])/b^4 + (3*d*(b*c - a*d)^2*ExpIntegralEi[-a - b*x
])/(2*b^4) - ((b*c - a*d)^3*ExpIntegralEi[-a - b*x])/(6*b^4)))/(3*d) + ((c
 + d*x)^3*Gamma[-3, a + b*x])/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [F]

$$\int \frac{(dx+c)^2 \operatorname{expIntegral}_4(bx+a)}{(bx+a)^3}\,dx$$

input
```
int((d*x+c)^2/(b*x+a)^3*Ei(4,b*x+a),x)
```

output
```
int((d*x+c)^2/(b*x+a)^3*Ei(4,b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(103) = 206$.

Time = 0.11 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.89

$$\int (c+dx)^2\Gamma(-3,a+bx)\,dx = -\frac{(b^2d^2x^2+3\,b^2c^2-3\,(a+2)bcd+(a^2+5\,a+2)d^2+(3\,b^2cd-(a+1)bd^2)x)e^{(-bx-a)}-(b^6d^2x^6+3\,(b^6}{}$$

input
```
integrate((d*x+c)^2*gamma(-3,b*x+a),x, algorithm="fricas")
```

output
```
-1/3*((b^2*d^2*x^2 + 3*b^2*c^2 - 3*(a + 2)*b*c*d + (a^2 + 5*a + 2)*d^2 + (
3*b^2*c*d - (a + 1)*b*d^2)*x)*e^(-b*x - a) - (b^6*d^2*x^6 + 3*(b^6*c*d + a
*b^5*d^2)*x^5 + 3*(a^4 + 3*a^3)*b^2*c^2 + 3*(b^6*c^2 + 3*a*b^5*c*d + a^2*b
^4*d^2)*x^4 - 3*(a^5 + 6*a^4 + 6*a^3)*b*c*d + (3*(4*a + 3)*b^5*c^2 + 6*(a^
2 - 3*a - 3)*b^4*c*d + (2*a^3 + 9*a^2 + 18*a + 6)*b^3*d^2)*x^3 + (a^6 + 9*
a^5 + 18*a^4 + 6*a^3)*d^2 + 3*(3*(2*a^2 + 3*a)*b^4*c^2 - 2*(a^3 + 9*a^2 +
9*a)*b^3*c*d + (a^4 + 9*a^3 + 18*a^2 + 6*a)*b^2*d^2)*x^2 + 3*((4*a^3 + 9*a
^2)*b^3*c^2 - 3*(a^4 + 6*a^3 + 6*a^2)*b^2*c*d + (a^5 + 9*a^4 + 18*a^3 + 6*
a^2)*b*d^2)*x)*gamma(-3, b*x + a))/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x +
a^3*b^3)
```

Sympy [F]

$$\int (c+dx)^2\Gamma(-3,a+bx)\,dx = \int \frac{(c+dx)^2\,\mathrm{E}_4\,(a+bx)}{(a+bx)^3}\,dx$$

input
```
integrate((d*x+c)**2*uppergamma(-3,b*x+a),x)
```

output
```
Integral((c + d*x)**2*expint(4, a + b*x)/(a + b*x)**3, x)
```

Maxima [F]

$$\int (c+dx)^2\Gamma(-3,a+bx)\,dx = \int (dx+c)^2\Gamma(-3,bx+a)\,dx$$

input
```
integrate((d*x+c)^2*gamma(-3,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(-3, b*x + a) - gamma(-2, b*x + a))*c^2/b + integrate(d^2*
x^2*gamma(-3, b*x + a) + 2*c*d*x*gamma(-3, b*x + a), x)
```

Giac [F]

$$\int (c+dx)^2\Gamma(-3,a+bx)\,dx = \int (dx+c)^2\Gamma(-3,bx+a)\,dx$$

input
```
integrate((d*x+c)^2*gamma(-3,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*gamma(-3, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.22

$$\int (c+dx)^2\Gamma(-3,a+bx)\,dx = \int \frac{\mathrm{expint}(4,a+b\,x)\,{(c+d\,x)}^2}{{(a+b\,x)}^3}\,dx$$

input
```
int((expint(4, a + b*x)*(c + d*x)^2)/(a + b*x)^3,x)
```

output
```
int((expint(4, a + b*x)*(c + d*x)^2)/(a + b*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^2\Gamma(-3,a+bx)\,dx = \left(\int \frac{ei(4,bx+a)}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}dx\right)c^2 + \left(\int \frac{ei(4,bx+a)\,x^2}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}dx\right)d^2 + 2\left(\int \frac{ei(4,bx+a)\,x}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}dx\right)cd$$

input
```
int((d*x+c)^2/(b*x+a)^3*Ei(4,b*x+a),x)
```

output
```
int(ei(4,a + b*x)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*c**2
+ int((ei(4,a + b*x)*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)
,x)*d**2 + 2*int((ei(4,a + b*x)*x)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b*
*3*x**3),x)*c*d
```

3.155 $\int (c+dx)\Gamma(-3,a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int (c+dx)\Gamma(-3,a+bx)\,dx = -\frac{(bc-ad)^2\Gamma(-3,a+bx)}{2b^2d}+\frac{(c+dx)^2\Gamma(-3,a+bx)}{2d} -\frac{(bc-ad)\Gamma(-2,a+bx)}{b^2}-\frac{d\Gamma(-1,a+bx)}{2b^2}$$

output
```
-1/2*(-a*d+b*c)^2/(b*x+a)^3*Ei(4,b*x+a)/b^2/d+1/2*(d*x+c)^2/(b*x+a)^3*Ei(4
,b*x+a)/d-(-a*d+b*c)/(b*x+a)^2*Ei(3,b*x+a)/b^2-1/2*d/(b*x+a)*Ei(2,b*x+a)/b
^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 270 vs. $2(84) = 168$.

Time = 0.13 (sec) , antiderivative size = 270, normalized size of antiderivative = 3.21

$$\int (c+dx)\Gamma(-3,a+bx)\,dx = de^{-bx}\left(-\frac{a^2e^{-a}}{6b^2(a+bx)^3}+\frac{a(6+a)e^{-a}}{12b^2(a+bx)^2}-\frac{(6+6a+a^2)\,e^{-a}}{12b^2(a+bx)}\right)+ce^{-bx}\left(\frac{ae^{-a}}{3b(a+bx)^3}-\frac{(3+a)e^{-a}}{6b(a+bx)^2}+\frac{(3+a)e^{-a}}{6b(a+bx)}\right)+\frac{c\,\text{ExpIntegralEi}(-a-bx)}{2b}+\frac{ac\,\text{ExpIntegralEi}(-a-bx)}{6b}-\frac{d\,\text{ExpIntegralEi}(-a-bx)}{2b^2}-\frac{ad\,\text{ExpIntegralEi}(-a-bx)}{2b^2}-\frac{a^2d\,\text{ExpIntegralEi}(-a-bx)}{12b^2}+cx\Gamma(-3,a+bx)+\frac{1}{2}dx^2\Gamma(-3,a+bx)$$

input
```
Integrate[(c + d*x)*Gamma[-3, a + b*x],x]
```

output
```
(d*(-1/6*a^2/(b^2*E^a*(a + b*x)^3) + (a*(6 + a))/(12*b^2*E^a*(a + b*x)^2)
- (6 + 6*a + a^2)/(12*b^2*E^a*(a + b*x))))/E^(b*x) + (c*(a/(3*b*E^a*(a + b
*x)^3) - (3 + a)/(6*b*E^a*(a + b*x)^2) + (3 + a)/(6*b*E^a*(a + b*x))))/E^(
b*x) + (c*ExpIntegralEi[-a - b*x])/(2*b) + (a*c*ExpIntegralEi[-a - b*x])/(
6*b) - (d*ExpIntegralEi[-a - b*x])/(2*b^2) - (a*d*ExpIntegralEi[-a - b*x])
/(2*b^2) - (a^2*d*ExpIntegralEi[-a - b*x])/(12*b^2) + c*x*Gamma[-3, a + b*
x] + (d*x^2*Gamma[-3, a + b*x])/2
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 284 vs. 2(84) = 168.

Time = 0.71 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.38, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\Gamma(-3,a+bx)\,dx$$

$$\Big\downarrow \text{ 7119}$$

$$\frac{b\int \frac{e^{-a-bx}(c+dx)^2}{(a+bx)^4}dx}{2d}+\frac{(c+dx)^2\Gamma(-3,a+bx)}{2d}$$

$$\Big\downarrow \text{ 2629}$$

$$\frac{b\int \left(\frac{e^{-a-bx}d^2}{b^2(a+bx)^2}+\frac{2(bc-ad)e^{-a-bx}d}{b^2(a+bx)^3}+\frac{(bc-ad)^2e^{-a-bx}}{b^2(a+bx)^4}\right)dx}{2d}+\frac{(c+dx)^2\Gamma(-3,a+bx)}{2d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\left(\frac{d(bc-ad)\text{ExpIntegralEi}(-a-bx)}{b^3}-\frac{(bc-ad)^2\text{ExpIntegralEi}(-a-bx)}{6b^3}+\frac{de^{-a-bx}(bc-ad)}{b^3(a+bx)}-\frac{de^{-a-bx}(bc-ad)}{b^3(a+bx)^2}-\frac{e^{-a-bx}(bc-ad)^2}{6b^3(a+bx)}+\frac{e^{\cdot}}{}\right.}{2d}$$

$$\frac{(c+dx)^2\Gamma(-3,a+bx)}{2d}$$

input
```
Int[(c + d*x)*Gamma[-3, a + b*x],x]
```

output
```
(b*(-1/3*((b*c - a*d)^2*E^(-a - b*x))/(b^3*(a + b*x)^3) - (d*(b*c - a*d)*E
^(-a - b*x))/(b^3*(a + b*x)^2) + ((b*c - a*d)^2*E^(-a - b*x))/(6*b^3*(a +
b*x)^2) - (d^2*E^(-a - b*x))/(b^3*(a + b*x)) + (d*(b*c - a*d)*E^(-a - b*x)
)/(b^3*(a + b*x)) - ((b*c - a*d)^2*E^(-a - b*x))/(6*b^3*(a + b*x)) - (d^2*
ExpIntegralEi[-a - b*x])/b^3 + (d*(b*c - a*d)*ExpIntegralEi[-a - b*x])/b^3
 - ((b*c - a*d)^2*ExpIntegralEi[-a - b*x])/(6*b^3)))/(2*d) + ((c + d*x)^2*
Gamma[-3, a + b*x])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [F]

$$\int \frac{(dx+c)\,\mathrm{expIntegral}_4\,(bx+a)}{(bx+a)^3}dx$$

input
```
int((d*x+c)/(b*x+a)^3*Ei(4,b*x+a),x)
```

output
```
int((d*x+c)/(b*x+a)^3*Ei(4,b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(78) = 156.

Time = 0.15 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.80

$$\int (c+dx)\Gamma(-3,a+bx)\,dx = -\frac{(bdx+2\,bc-(a+2)d)e^{(-bx-a)}-(b^5dx^5+(2\,b^5c+3\,ab^4d)x^4+2\,((4\,a+3)b^4c+(a^2-3\,a-3)b^3d)x^3}{}$$

input
```
integrate((d*x+c)*gamma(-3,b*x+a),x, algorithm="fricas")
```

output
```
-1/2*((b*d*x + 2*b*c - (a + 2)*d)*e^(-b*x - a) - (b^5*d*x^5 + (2*b^5*c + 3
*a*b^4*d)*x^4 + 2*((4*a + 3)*b^4*c + (a^2 - 3*a - 3)*b^3*d)*x^3 + 2*(a^4 +
 3*a^3)*b*c + 2*(3*(2*a^2 + 3*a)*b^3*c - (a^3 + 9*a^2 + 9*a)*b^2*d)*x^2 -
(a^5 + 6*a^4 + 6*a^3)*d + (2*(4*a^3 + 9*a^2)*b^2*c - 3*(a^4 + 6*a^3 + 6*a^
2)*b*d)*x)*gamma(-3, b*x + a))/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*
b^2)
```

Sympy [F]

$$\int (c+dx)\Gamma(-3,a+bx)\,dx = \int \frac{(c+dx)\,\mathrm{E}_4\,(a+bx)}{(a+bx)^3}\,dx$$

input
```
integrate((d*x+c)*uppergamma(-3,b*x+a),x)
```

output
```
Integral((c + d*x)*expint(4, a + b*x)/(a + b*x)**3, x)
```

Maxima [F]

$$\int (c+dx)\Gamma(-3,a+bx)\,dx = \int (dx+c)\Gamma(-3,bx+a)\,dx$$

input
```
integrate((d*x+c)*gamma(-3,b*x+a),x, algorithm="maxima")
```

output
```
d*integrate(x*gamma(-3, b*x + a), x) + ((b*x + a)*gamma(-3, b*x + a) - gam
ma(-2, b*x + a))*c/b
```

Giac [F]

$$\int (c+dx)\Gamma(-3,a+bx)\,dx = \int (dx+c)\Gamma(-3,bx+a)\,dx$$

input
```
integrate((d*x+c)*gamma(-3,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*gamma(-3, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.26

$$\int (c+dx)\Gamma(-3,a+bx)\,dx = \int \frac{\mathrm{expint}(4,a+b\,x)\,\left(c+d\,x\right)}{{\left(a+b\,x\right)}^3}\,dx$$

input
```
int((expint(4, a + b*x)*(c + d*x))/(a + b*x)^3,x)
```

output
```
int((expint(4, a + b*x)*(c + d*x))/(a + b*x)^3, x)
```

Reduce [F]

$$\int (c+dx)\Gamma(-3,a+bx)\,dx = \left(\int \frac{ei(4,bx+a)}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}dx\right)c + \left(\int \frac{ei(4,bx+a)\,x}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}dx\right)d$$

input
```
int((d*x+c)/(b*x+a)^3*Ei(4,b*x+a),x)
```

output
```
int(ei(4,a + b*x)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*c + i
nt((ei(4,a + b*x)*x)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*d
```

3.156 $\int \Gamma(-3, a + bx)\, dx$

Optimal result

Integrand size = 7, antiderivative size = 29

$$\int \Gamma(-3, a + bx)\, dx = \frac{(a + bx)\Gamma(-3, a + bx)}{b} - \frac{\Gamma(-2, a + bx)}{b}$$

output
```
1/(b*x+a)^2*Ei(4,b*x+a)/b-1/(b*x+a)^2*Ei(3,b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 89 vs. 2(29) = 58.

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.07

$$\int \Gamma(-3, a + bx)\, dx = \frac{1}{6}\left(\frac{e^{-a-bx}(2a - (3 + a)(a + bx) + (3 + a)(a + bx)^2)}{b(a + bx)^3} + \frac{3\,\text{ExpIntegralEi}(-a - bx)}{b} + \frac{a\,\text{ExpIntegralEi}(-a - bx)}{b} + 6x\Gamma(-3, a + bx)\right)$$

input
```
Integrate[Gamma[-3, a + b*x],x]
```

output
```
((E^(-a - b*x)*(2*a - (3 + a)*(a + b*x) + (3 + a)*(a + b*x)^2))/(b*(a + b*
x)^3) + (3*ExpIntegralEi[-a - b*x])/b + (a*ExpIntegralEi[-a - b*x])/b + 6*
x*Gamma[-3, a + b*x])/6
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{7111\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(-3, a + bx) \, dx$$

$$\Big\downarrow \text{7111}$$

$$\frac{(a + bx)\Gamma(-3, a + bx)}{b} - \frac{\Gamma(-2, a + bx)}{b}$$

input
```
Int[Gamma[-3, a + b*x],x]
```

output
```
((a + b*x)*Gamma[-3, a + b*x])/b - Gamma[-2, a + b*x]/b
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [F]

$$\int \frac{\mathrm{expIntegral}_4\left(bx+a\right)}{\left(bx+a\right)^3}dx$$

input
```
int(1/(b*x+a)^3*Ei(4,b*x+a),x)
```

output
```
int(1/(b*x+a)^3*Ei(4,b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(29) = 58.

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.90

$$\int \Gamma(-3,a+bx)\,dx = \frac{(b^4x^4+(4\,a+3)b^3x^3+3\,(2\,a^2+3\,a)b^2x^2+a^4+3\,a^3+(4\,a^3+9\,a^2)bx)\Gamma(-3,bx+a)-e^{(-bx-a)}}{b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b}$$

input
```
integrate(gamma(-3,b*x+a),x, algorithm="fricas")
```

output
```
((b^4*x^4 + (4*a + 3)*b^3*x^3 + 3*(2*a^2 + 3*a)*b^2*x^2 + a^4 + 3*a^3 + (4
*a^3 + 9*a^2)*b*x)*gamma(-3, b*x + a) - e^(-b*x - a))/(b^4*x^3 + 3*a*b^3*x
^2 + 3*a^2*b^2*x + a^3*b)
```

Sympy [F]

$$\int \Gamma(-3, a+bx)\, dx = \int \frac{\mathrm{E}_4\left(a+bx\right)}{\left(a+bx\right)^3}\, dx$$

input
```
integrate(uppergamma(-3,b*x+a),x)
```

output
```
Integral(expint(4, a + b*x)/(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \Gamma(-3, a+bx)\, dx = \frac{(bx+a)\Gamma(-3, bx+a) - \Gamma(-2, bx+a)}{b}$$

input
```
integrate(gamma(-3,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(-3, b*x + a) - gamma(-2, b*x + a))/b
```

Giac [F]

$$\int \Gamma(-3, a+bx)\, dx = \int \Gamma(-3, bx+a)\, dx$$

input
```
integrate(gamma(-3,b*x+a),x, algorithm="giac")
```

output
```
integrate(gamma(-3, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \Gamma(-3,a+bx)\,dx = -\frac{\mathrm{expint}(3,a+b\,x)-\mathrm{expint}(4,a+b\,x)}{b\,(a+b\,x)^2}$$

input
```
int(expint(4, a + b*x)/(a + b*x)^3,x)
```

output
```
-(expint(3, a + b*x) - expint(4, a + b*x))/(b*(a + b*x)^2)
```

Reduce [F]

$$\int \Gamma(-3,a+bx)\,dx = \int \frac{ei(4,bx+a)}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}dx$$

input
```
int(1/(b*x+a)^3*Ei(4,b*x+a),x)
```

output
```
int(ei(4,a + b*x)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)
```

3.157 $\int \frac{\Gamma(-3,a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\Gamma(-3,a+bx)}{c+dx}\,dx = \operatorname{Int}\left(\frac{\Gamma(-3,a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(1/(b*x+a)^3*Ei(4,b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(-3,a+bx)}{c+dx}\,dx = \int \frac{\Gamma(-3,a+bx)}{c+dx}\,dx$$

input
```
Integrate[Gamma[-3, a + b*x]/(c + d*x),x]
```

output
```
Integrate[Gamma[-3, a + b*x]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-3, a+bx)}{c+dx} \, dx$$
$$\Big\downarrow \ 7120$$
$$\int \frac{\Gamma(-3, a+bx)}{c+dx} dx$$

input
```
Int[Gamma[-3, a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{\text{expIntegral}_4\,(bx+a)}{(bx+a)^3\,(dx+c)} dx$$

input
```
int(1/(b*x+a)^3*Ei(4,b*x+a)/(d*x+c),x)
```

output
```
int(1/(b*x+a)^3*Ei(4,b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(-3, a+bx)}{c+dx} \, dx = \int \frac{\Gamma(-3, bx+a)}{dx+c} \, dx$$

input
```
integrate(gamma(-3,b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(gamma(-3, b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{\Gamma(-3, a+bx)}{c+dx} \, dx = \int \frac{\mathrm{E}_4(a+bx)}{(a+bx)^3 (c+dx)} \, dx$$

input
```
integrate(uppergamma(-3,b*x+a)/(d*x+c),x)
```

output
```
Integral(expint(4, a + b*x)/((a + b*x)**3*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(-3, a+bx)}{c+dx} \, dx = \int \frac{\Gamma(-3, bx+a)}{dx+c} \, dx$$

input
```
integrate(gamma(-3,b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(gamma(-3, b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(-3, a + bx)}{c + dx} \, dx = \int \frac{\Gamma(-3, bx + a)}{dx + c} \, dx$$

input
```
integrate(gamma(-3,b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(gamma(-3, b*x + a)/(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{\Gamma(-3, a + bx)}{c + dx} \, dx = \int \frac{\mathrm{expint}(4, a + b\,x)}{(a + b\,x)^3\,(c + d\,x)} \, dx$$

input
```
int(expint(4, a + b*x)/((a + b*x)^3*(c + d*x)),x)
```

output
```
int(expint(4, a + b*x)/((a + b*x)^3*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 5.20

$$\int \frac{\Gamma(-3, a+bx)}{c+dx}\,dx$$
$$= \int \frac{ei(4, bx+a)}{b^3d\,x^4 + 3a\,b^2d\,x^3 + b^3c\,x^3 + 3a^2bd\,x^2 + 3a\,b^2c\,x^2 + a^3dx + 3a^2bcx + a^3c}dx$$

input
```
int(1/(b*x+a)^3*Ei(4,b*x+a)/(d*x+c),x)
```

output
```
int(ei(4,a + b*x)/(a**3*c + a**3*d*x + 3*a**2*b*c*x + 3*a**2*b*d*x**2 + 3*
a*b**2*c*x**2 + 3*a*b**2*d*x**3 + b**3*c*x**3 + b**3*d*x**4),x)
```

3.158 $\int \frac{\Gamma(-3,a+bx)}{(c+dx)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 144

$$\int \frac{\Gamma(-3,a+bx)}{(c+dx)^2} dx = \frac{b\Gamma(-3,a+bx)}{d(bc-ad)} - \frac{\Gamma(-3,a+bx)}{d(c+dx)} - \frac{b\Gamma(-2,a+bx)}{(bc-ad)^2} + \frac{bd\Gamma(-1,a+bx)}{(bc-ad)^3} - \frac{bd^2\Gamma(0,a+bx)}{(bc-ad)^4} + \frac{bd^2e^{-a+\frac{bc}{d}}\Gamma\left(0,\frac{b(c+dx)}{d}\right)}{(bc-ad)^4}$$

output
```
b/(b*x+a)^3*Ei(4,b*x+a)/d/(-a*d+b*c)-1/(b*x+a)^3*Ei(4,b*x+a)/d/(d*x+c)-b/(
b*x+a)^2*Ei(3,b*x+a)/(-a*d+b*c)^2+b*d/(b*x+a)*Ei(2,b*x+a)/(-a*d+b*c)^3-b*d
^2*Ei(1,b*x+a)/(-a*d+b*c)^4+b*d^2*exp(-a+b*c/d)*Ei(1,b*(d*x+c)/d)/(-a*d+b*
c)^4
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.90

$$\int \frac{\Gamma(-3,a+bx)}{(c+dx)^2}\,dx = \frac{1}{6}\left(\frac{3be^{-a-bx}(-1+a+bx)}{(bc-ad)^2(a+bx)^2}+\frac{6bde^{-a-bx}}{(bc-ad)^3(a+bx)} + \frac{be^{-a-bx}(2-a-bx+(a+bx)^2)}{d(bc-ad)(a+bx)^3} + \frac{6bd^2\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^4}+\frac{6bd\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^3} + \frac{3b\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^2}+\frac{b\,\text{ExpIntegralEi}(-a-bx)}{bcd-ad^2} - \frac{6bd^2e^{-a+\frac{bc}{d}}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{(bc-ad)^4}-\frac{6\Gamma(-3,a+bx)}{d(c+dx)}\right)$$

input
```
Integrate[Gamma[-3, a + b*x]/(c + d*x)^2,x]
```

output
```
((3*b*E^(-a - b*x)*(-1 + a + b*x))/((b*c - a*d)^2*(a + b*x)^2) + (6*b*d*E^
(-a - b*x))/((b*c - a*d)^3*(a + b*x)) + (b*E^(-a - b*x)*(2 - a - b*x + (a
+ b*x)^2))/(d*(b*c - a*d)*(a + b*x)^3) + (6*b*d^2*ExpIntegralEi[-a - b*x])
/(b*c - a*d)^4 + (6*b*d*ExpIntegralEi[-a - b*x])/(b*c - a*d)^3 + (3*b*ExpI
ntegralEi[-a - b*x])/(b*c - a*d)^2 + (b*ExpIntegralEi[-a - b*x])/(b*c*d -
a*d^2) - (6*b*d^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/(b*c
 - a*d)^4 - (6*Gamma[-3, a + b*x])/(d*(c + d*x)))/6
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 349 vs. $2(144) = 288$.

Time = 1.02 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.42, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-3,a+bx)}{(c+dx)^2}dx$$

$\Big\downarrow$ 7119

$$-\frac{b\int \frac{e^{-a-bx}}{(a+bx)^4(c+dx)}dx}{d}-\frac{\Gamma(-3,a+bx)}{d(c+dx)}$$

$\Big\downarrow$ 7293

$$-\frac{b\int \left(\frac{e^{-a-bx}d^4}{(bc-ad)^4(c+dx)}-\frac{be^{-a-bx}d^3}{(bc-ad)^4(a+bx)}+\frac{be^{-a-bx}d^2}{(bc-ad)^3(a+bx)^2}-\frac{be^{-a-bx}d}{(bc-ad)^2(a+bx)^3}+\frac{be^{-a-bx}}{(bc-ad)(a+bx)^4}\right)dx}{d}-\frac{\Gamma(-3,a+bx)}{d(c+dx)}$$

$\Big\downarrow$ 2009

$$-\frac{b\left(-\frac{d^3\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^4}+\frac{d^3e^{\frac{bc}{d}-a}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{(bc-ad)^4}-\frac{d^2\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^3}-\frac{d^2e^{-a-bx}}{(a+bx)(bc-ad)^3}-\frac{d\,\text{ExpIntegra}}{2(bc-}\right.}{}$$

$$\frac{\Gamma(-3,a+bx)}{d(c+dx)}$$

input
```
Int[Gamma[-3, a + b*x]/(c + d*x)^2,x]
```

output
```
-((b*(-1/3*E^(-a - b*x)/((b*c - a*d)*(a + b*x)^3) + (d*E^(-a - b*x))/(2*(b
*c - a*d)^2*(a + b*x)^2) + E^(-a - b*x)/(6*(b*c - a*d)*(a + b*x)^2) - (d^2
*E^(-a - b*x))/((b*c - a*d)^3*(a + b*x)) - (d*E^(-a - b*x))/(2*(b*c - a*d)
^2*(a + b*x)) - E^(-a - b*x)/(6*(b*c - a*d)*(a + b*x)) - (d^3*ExpIntegralE
i[-a - b*x])/(b*c - a*d)^4 - (d^2*ExpIntegralEi[-a - b*x])/(b*c - a*d)^3 -
 (d*ExpIntegralEi[-a - b*x])/(2*(b*c - a*d)^2) - ExpIntegralEi[-a - b*x]/(
6*(b*c - a*d)) + (d^3*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/
(b*c - a*d)^4))/d) - Gamma[-3, a + b*x]/(d*(c + d*x))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\text{expIntegral}_4\left(bx+a\right)}{\left(bx+a\right)^3\left(dx+c\right)^2}dx$$

input
```
int(1/(b*x+a)^3*Ei(4,b*x+a)/(d*x+c)^2,x)
```

output
```
int(1/(b*x+a)^3*Ei(4,b*x+a)/(d*x+c)^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1517 vs. $2(143) = 286$.

Time = 0.14 (sec) , antiderivative size = 1517, normalized size of antiderivative = 10.53

$$\int \frac{\Gamma(-3,a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(gamma(-3,b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/6*(6*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*
b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*Ei(-(b*d*x
 + b*c)/d)*e^((b*c - a*d)/d) - (a^3*b^4*c^4 - 3*(a^4 - a^3)*b^3*c^3*d + 3*
(a^5 - 2*a^4 + 2*a^3)*b^2*c^2*d^2 - (a^6 - 3*a^5 + 6*a^4 - 6*a^3)*b*c*d^3
+ (b^7*c^3*d - 3*(a - 1)*b^6*c^2*d^2 + 3*(a^2 - 2*a + 2)*b^5*c*d^3 - (a^3
- 3*a^2 + 6*a - 6)*b^4*d^4)*x^4 + (b^7*c^4 + 3*b^6*c^3*d - 3*(2*a^2 - a -
2)*b^5*c^2*d^2 + (8*a^3 - 15*a^2 + 12*a + 6)*b^4*c*d^3 - 3*(a^4 - 3*a^3 +
6*a^2 - 6*a)*b^3*d^4)*x^3 + 3*(a*b^6*c^4 - (2*a^2 - 3*a)*b^5*c^3*d - 3*(a^
2 - 2*a)*b^4*c^2*d^2 + (2*a^4 - 3*a^3 + 6*a)*b^3*c*d^3 - (a^5 - 3*a^4 + 6*
a^3 - 6*a^2)*b^2*d^4)*x^2 + (3*a^2*b^5*c^4 - (8*a^3 - 9*a^2)*b^4*c^3*d + 3
*(2*a^4 - 5*a^3 + 6*a^2)*b^3*c^2*d^2 + 3*(a^4 - 4*a^3 + 6*a^2)*b^2*c*d^3 -
 (a^6 - 3*a^5 + 6*a^4 - 6*a^3)*b*d^4)*x)*Ei(-b*x - a) - ((a^2 - a + 2)*b^4
*c^4 - 3*(a^3 - 2*a^2 + 3*a)*b^3*c^3*d + 3*(a^4 - 3*a^3 + 6*a^2)*b^2*c^2*d
^2 - (a^5 - 4*a^4 + 11*a^3)*b*c*d^3 + (b^6*c^3*d - 3*(a - 1)*b^5*c^2*d^2 +
 3*(a^2 - 2*a + 2)*b^4*c*d^3 - (a^3 - 3*a^2 + 6*a)*b^3*d^4)*x^3 + (b^6*c^4
 - (a - 2)*b^5*c^3*d - 3*(a^2 - a - 1)*b^4*c^2*d^2 + (5*a^3 - 12*a^2 + 12*
a)*b^3*c*d^3 - (2*a^4 - 7*a^3 + 15*a^2)*b^2*d^4)*x^2 + ((2*a - 1)*b^5*c^4
- (5*a^2 - 8*a + 1)*b^4*c^3*d + 3*(a^3 - 3*a^2 + 3*a)*b^3*c^2*d^2 + (a^4 -
 2*a^3 + 3*a^2)*b^2*c*d^3 - (a^5 - 4*a^4 + 11*a^3)*b*d^4)*x)*e^(-b*x - a)
+ 6*(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 ...
```

Sympy [F]

$$\int \frac{\Gamma(-3, a + bx)}{(c + dx)^2} \, dx = \int \frac{\mathrm{E}_4\left(a + bx\right)}{\left(a + bx\right)^3 \left(c + dx\right)^2} \, dx$$

input
```
integrate(uppergamma(-3,b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(expint(4, a + b*x)/((a + b*x)**3*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\Gamma(-3, a+bx)}{(c+dx)^2}\,dx = \int \frac{\Gamma(-3, bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(gamma(-3,b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(gamma(-3, b*x + a)/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{\Gamma(-3, a+bx)}{(c+dx)^2}\,dx = \int \frac{\Gamma(-3, bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(gamma(-3,b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(gamma(-3, b*x + a)/(d*x + c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.17

$$\int \frac{\Gamma(-3, a+bx)}{(c+dx)^2}\,dx = \int \frac{\mathrm{expint}(4, a+b\,x)}{(a+b\,x)^3\,(c+d\,x)^2}\,dx$$

input
```
int(expint(4, a + b*x)/((a + b*x)^3*(c + d*x)^2),x)
```

output
```
int(expint(4, a + b*x)/((a + b*x)^3*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{\Gamma(-3,a+bx)}{(c+dx)^2}\,dx$$

$$= \int \frac{ei(4,bx+a)}{b^3d^2x^5+3a\,b^2d^2x^4+2b^3cd\,x^4+3a^2b\,d^2x^3+6a\,b^2cd\,x^3+b^3c^2x^3+a^3d^2x^2+6a^2bcd\,x^2+3a\,b^2c^2x^2+2}$$

input
```
int(1/(b*x+a)^3*Ei(4,b*x+a)/(d*x+c)^2,x)
```

output
```
int(ei(4,a + b*x)/(a**3*c**2 + 2*a**3*c*d*x + a**3*d**2*x**2 + 3*a**2*b*c*
*2*x + 6*a**2*b*c*d*x**2 + 3*a**2*b*d**2*x**3 + 3*a*b**2*c**2*x**2 + 6*a*b
**2*c*d*x**3 + 3*a*b**2*d**2*x**4 + b**3*c**2*x**3 + 2*b**3*c*d*x**4 + b**
3*d**2*x**5),x)
```

3.159 $\int \frac{\Gamma(-3,a+bx)}{(c+dx)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 205

$$\int \frac{\Gamma(-3,a+bx)}{(c+dx)^3} dx = \frac{b^2\Gamma(-3,a+bx)}{2d(bc-ad)^2} - \frac{\Gamma(-3,a+bx)}{2d(c+dx)^2} - \frac{b^2\Gamma(-2,a+bx)}{(bc-ad)^3} + \frac{3b^2d\Gamma(-1,a+bx)}{2(bc-ad)^4} + \frac{b^2de^{-a+\frac{bc}{d}}\Gamma\left(-1,\frac{b(c+dx)}{d}\right)}{2(bc-ad)^4} - \frac{2b^2d^2\Gamma(0,a+bx)}{(bc-ad)^5} + \frac{2b^2d^2e^{-a+\frac{bc}{d}}\Gamma\left(0,\frac{b(c+dx)}{d}\right)}{(bc-ad)^5}$$

output
```
1/2*b^2/(b*x+a)^3*Ei(4,b*x+a)/d/(-a*d+b*c)^2-1/2/(b*x+a)^3*Ei(4,b*x+a)/d/(
d*x+c)^2-b^2/(b*x+a)^2*Ei(3,b*x+a)/(-a*d+b*c)^3+3/2*b^2*d/(b*x+a)*Ei(2,b*x
+a)/(-a*d+b*c)^4+1/2*b*d^2*exp(-a+b*c/d)/(d*x+c)*Ei(2,b*(d*x+c)/d)/(-a*d+b
*c)^4-2*b^2*d^2*Ei(1,b*x+a)/(-a*d+b*c)^5+2*b^2*d^2*exp(-a+b*c/d)*Ei(1,b*(d
*x+c)/d)/(-a*d+b*c)^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 877 vs. $2(205) = 410$.

Time = 6.91 (sec) , antiderivative size = 877, normalized size of antiderivative = 4.28

$$\int \frac{\Gamma(-3, a + bx)}{(c + dx)^3} \, dx = \text{Too large to display}$$

input
```
Integrate[Gamma[-3, a + b*x]/(c + d*x)^3,x]
```

output
```
((2*((E^(-a - b*x)*(2*a - (3 + a)*(a + b*x) + (3 + a)*(a + b*x)^2))/(b*(a
+ b*x)^3) + (3*ExpIntegralEi[-a - b*x])/b + (a*ExpIntegralEi[-a - b*x])/b
+ 6*x*Gamma[-3, a + b*x]))/(c + d*x)^3 + d*(E^(-a - b*x)*((-2*b^2*(b*c - 3
*a*d))/(d^2*(-(b*c) + a*d)^3*(a + b*x)^3) - (b^2*(b^2*c^2 - 4*a*b*c*d + 3*
(-4 + a)*a*d^2))/(d^2*(b*c - a*d)^4*(a + b*x)^2) - (b^2*(b^3*c^3 - 5*a*b^2
*c^2*d + a*(-6 + 7*a)*b*c*d^2 - 3*a*(8 - 2*a + a^2)*d^3))/(d^2*(-(b*c) + a
*d)^5*(a + b*x)) + (2*((3 + a)*b^2*c^2 + (3 - 5*a - 2*a^2)*b*c*d + a*(-1 +
 2*a + a^2)*d^2))/(b*(b*c - a*d)^3*(c + d*x)^3) + (2*((3 + a)*b^2*c^2 - 2*
(-3 + 2*a + a^2)*b*c*d + a^2*(1 + a)*d^2))/((b*c - a*d)^4*(c + d*x)^2) + (
2*b*((3 + a)*b^2*c^2 + (12 - 3*a - 2*a^2)*b*c*d + a^3*d^2))/((b*c - a*d)^5
*(c + d*x))) + (18*b^3*c*ExpIntegralEi[-a - b*x])/(b*c - a*d)^5 - (12*a*b^
3*c*ExpIntegralEi[-a - b*x])/(b*c - a*d)^5 + (3*a^2*b^3*c*ExpIntegralEi[-a
 - b*x])/(b*c - a*d)^5 + (b^5*c^3*ExpIntegralEi[-a - b*x])/(d^2*(b*c - a*d
)^5) + (6*b^4*c^2*ExpIntegralEi[-a - b*x])/(d*(b*c - a*d)^5) + (24*b^2*d*E
xpIntegralEi[-a - b*x])/(b*c - a*d)^5 + (3*a*b^4*c^2*ExpIntegralEi[-a - b*
x])/(d*(-(b*c) + a*d)^5) + (18*a*b^2*d*ExpIntegralEi[-a - b*x])/(-(b*c) +
a*d)^5 - (6*a^2*b^2*d*ExpIntegralEi[-a - b*x])/(-(b*c) + a*d)^5 + (a^3*b^2
*d*ExpIntegralEi[-a - b*x])/(-(b*c) + a*d)^5 - (2*(3 + a)*ExpIntegralEi[-a
 - b*x])/(b*d*(c + d*x)^3) + (6*b^3*c*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*
(c + d*x))/d)])/(b*c - a*d)^5 - (24*b^2*d*E^(-a + (b*c)/d)*ExpIntegralE...
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 430 vs. $2(205) = 410$.

Time = 1.17 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-3,a+bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{7119}$$

$$-\frac{b\int \frac{e^{-a-bx}}{(a+bx)^4(c+dx)^2}dx}{2d}-\frac{\Gamma(-3,a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\int \left(\frac{4be^{-a-bx}d^4}{(bc-ad)^5(c+dx)}+\frac{e^{-a-bx}d^4}{(bc-ad)^4(c+dx)^2}-\frac{4b^2e^{-a-bx}d^3}{(bc-ad)^5(a+bx)}+\frac{3b^2e^{-a-bx}d^2}{(bc-ad)^4(a+bx)^2}-\frac{2b^2e^{-a-bx}d}{(bc-ad)^3(a+bx)^3}+\frac{b^2e^{-a-bx}}{(bc-ad)^2(a+bx)^4}\right)dx}{2d}-\frac{\Gamma(-3,a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{4bd^3\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^5}+\frac{4bd^3e^{\frac{bc}{d}-a}\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{(bc-ad)^5}-\frac{d^3e^{-a-bx}}{(c+dx)(bc-ad)^4}-\frac{3bd^2\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^4}-\frac{bd^2e^{\frac{bc}{d}}\cdots}{\cdots}\right.}{\cdots}-\frac{\Gamma(-3,a+bx)}{2d(c+dx)^2}$$

input
```
Int[Gamma[-3, a + b*x]/(c + d*x)^3,x]
```

output
```
-1/2*(b*(-1/3*(b*E^(-a - b*x))/((b*c - a*d)^2*(a + b*x)^3) + (b*d*E^(-a -
b*x))/((b*c - a*d)^3*(a + b*x)^2) + (b*E^(-a - b*x))/(6*(b*c - a*d)^2*(a +
 b*x)^2) - (3*b*d^2*E^(-a - b*x))/((b*c - a*d)^4*(a + b*x)) - (b*d*E^(-a -
 b*x))/((b*c - a*d)^3*(a + b*x)) - (b*E^(-a - b*x))/(6*(b*c - a*d)^2*(a +
b*x)) - (d^3*E^(-a - b*x))/((b*c - a*d)^4*(c + d*x)) - (4*b*d^3*ExpIntegra
lEi[-a - b*x])/(b*c - a*d)^5 - (3*b*d^2*ExpIntegralEi[-a - b*x])/(b*c - a*
d)^4 - (b*d*ExpIntegralEi[-a - b*x])/(b*c - a*d)^3 - (b*ExpIntegralEi[-a -
 b*x])/(6*(b*c - a*d)^2) + (4*b*d^3*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c
 + d*x))/d)])/(b*c - a*d)^5 - (b*d^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(
c + d*x))/d)])/(b*c - a*d)^4))/d - Gamma[-3, a + b*x]/(2*d*(c + d*x)^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\text{expIntegral}_4(bx+a)}{(bx+a)^3(dx+c)^3}dx$$

input
```
int(1/(b*x+a)^3*Ei(4,b*x+a)/(d*x+c)^3,x)
```

output
```
int(1/(b*x+a)^3*Ei(4,b*x+a)/(d*x+c)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2285 vs. $2(195) = 390$.

Time = 0.21 (sec) , antiderivative size = 2285, normalized size of antiderivative = 11.15

$$\int \frac{\Gamma(-3, a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(gamma(-3,b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/12*(6*(a^3*b^3*c^3*d^2 - (a^4 + 4*a^3)*b^2*c^2*d^3 + (b^6*c*d^4 - (a + 4
)*b^5*d^5)*x^5 + (2*b^6*c^2*d^3 + (a - 8)*b^5*c*d^4 - 3*(a^2 + 4*a)*b^4*d^
5)*x^4 + (b^6*c^3*d^2 + (5*a - 4)*b^5*c^2*d^3 - 3*(a^2 + 8*a)*b^4*c*d^4 -
3*(a^3 + 4*a^2)*b^3*d^5)*x^3 + (3*a*b^5*c^3*d^2 + 3*(a^2 - 4*a)*b^4*c^2*d^
3 - (5*a^3 + 24*a^2)*b^3*c*d^4 - (a^4 + 4*a^3)*b^2*d^5)*x^2 + (3*a^2*b^4*c
^3*d^2 - (a^3 + 12*a^2)*b^3*c^2*d^3 - 2*(a^4 + 4*a^3)*b^2*c*d^4)*x)*Ei(-(b
*d*x + b*c)/d)*e^((b*c - a*d)/d) + (a^3*b^5*c^5 - 3*(a^4 - 2*a^3)*b^4*c^4*
d + 3*(a^5 - 4*a^4 + 6*a^3)*b^3*c^3*d^2 - (a^6 - 6*a^5 + 18*a^4 - 24*a^3)*
b^2*c^2*d^3 + (b^8*c^3*d^2 - 3*(a - 2)*b^7*c^2*d^3 + 3*(a^2 - 4*a + 6)*b^6
*c*d^4 - (a^3 - 6*a^2 + 18*a - 24)*b^5*d^5)*x^5 + (2*b^8*c^4*d - 3*(a - 4)
*b^7*c^3*d^2 - 3*(a^2 + 2*a - 12)*b^6*c^2*d^3 + (7*a^3 - 24*a^2 + 18*a + 4
8)*b^5*c*d^4 - 3*(a^4 - 6*a^3 + 18*a^2 - 24*a)*b^4*d^5)*x^4 + (b^8*c^5 + 3
*(a + 2)*b^7*c^4*d - 6*(2*a^2 - 4*a - 3)*b^6*c^3*d^2 + 2*(4*a^3 - 24*a^2 +
 45*a + 12)*b^5*c^2*d^3 + 3*(a^4 - 18*a^2 + 48*a)*b^4*c*d^4 - 3*(a^5 - 6*a
^4 + 18*a^3 - 24*a^2)*b^3*d^5)*x^3 + (3*a*b^7*c^5 - 3*(a^2 - 6*a)*b^6*c^4*
d - 2*(4*a^3 - 27*a)*b^5*c^3*d^2 + 6*(2*a^4 - 8*a^3 + 9*a^2 + 12*a)*b^4*c^
2*d^3 - 3*(a^5 - 8*a^4 + 30*a^3 - 48*a^2)*b^3*c*d^4 - (a^6 - 6*a^5 + 18*a^
4 - 24*a^3)*b^2*d^5)*x^2 + (3*a^2*b^6*c^5 - (7*a^3 - 18*a^2)*b^5*c^4*d + 3
*(a^4 - 8*a^3 + 18*a^2)*b^4*c^3*d^2 + 3*(a^5 - 2*a^4 - 6*a^3 + 24*a^2)*b^3
*c^2*d^3 - 2*(a^6 - 6*a^5 + 18*a^4 - 24*a^3)*b^2*c*d^4)*x)*Ei(-b*x - a)...
```

Sympy [F]

$$\int \frac{\Gamma(-3, a + bx)}{(c + dx)^3}\, dx = \int \frac{\mathrm{E}_4\left(a + bx\right)}{\left(a + bx\right)^3\left(c + dx\right)^3}\, dx$$

input
```
integrate(uppergamma(-3,b*x+a)/(d*x+c)**3,x)
```

output
```
Integral(expint(4, a + b*x)/((a + b*x)**3*(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\Gamma(-3, a + bx)}{(c + dx)^3}\, dx = \int \frac{\Gamma(-3, bx + a)}{\left(dx + c\right)^3}\, dx$$

input
```
integrate(gamma(-3,b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(gamma(-3, b*x + a)/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{\Gamma(-3, a + bx)}{(c + dx)^3}\, dx = \int \frac{\Gamma(-3, bx + a)}{\left(dx + c\right)^3}\, dx$$

input
```
integrate(gamma(-3,b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(gamma(-3, b*x + a)/(d*x + c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.12

$$\int \frac{\Gamma(-3,a+bx)}{(c+dx)^3}\,dx=\int \frac{\mathrm{expint}(4,a+b\,x)}{{(a+b\,x)}^3\,{(c+d\,x)}^3}\,dx$$

input
```
int(expint(4, a + b*x)/((a + b*x)^3*(c + d*x)^3),x)
```

output
```
int(expint(4, a + b*x)/((a + b*x)^3*(c + d*x)^3), x)
```

Reduce [F]

$$\int \frac{\Gamma(-3,a+bx)}{(c+dx)^3}\,dx$$
$$=\int \frac{ei(4,bx+a)}{b^3d^3x^6+3a\,b^2d^3x^5+3b^3c\,d^2x^5+3a^2b\,d^3x^4+9a\,b^2c\,d^2x^4+3b^3c^2d\,x^4+a^3d^3x^3+9a^2bc\,d^2x^3+9a\,b^2c^2}$$

input
```
int(1/(b*x+a)^3*Ei(4,b*x+a)/(d*x+c)^3,x)
```

output
```
int(ei(4,a + b*x)/(a**3*c**3 + 3*a**3*c**2*d*x + 3*a**3*c*d**2*x**2 + a**3
*d**3*x**3 + 3*a**2*b*c**3*x + 9*a**2*b*c**2*d*x**2 + 9*a**2*b*c*d**2*x**3
 + 3*a**2*b*d**3*x**4 + 3*a*b**2*c**3*x**2 + 9*a*b**2*c**2*d*x**3 + 9*a*b*
*2*c*d**2*x**4 + 3*a*b**2*d**3*x**5 + b**3*c**3*x**3 + 3*b**3*c**2*d*x**4
+ 3*b**3*c*d**2*x**5 + b**3*d**3*x**6),x)
```

3.160 $\int \frac{\Gamma(-3,a+bx)}{(c+dx)^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 248

$$\begin{aligned}\int \frac{\Gamma(-3,a+bx)}{(c+dx)^4}\,dx &= \frac{b^3\Gamma(-3,a+bx)}{3d(bc-ad)^3}-\frac{\Gamma(-3,a+bx)}{3d(c+dx)^3}\\ &\quad-\frac{b^3\Gamma(-2,a+bx)}{(bc-ad)^4}+\frac{b^3e^{-a+\frac{bc}{d}}\Gamma\left(-2,\frac{b(c+dx)}{d}\right)}{3(bc-ad)^4}\\ &\quad+\frac{2b^3d\Gamma(-1,a+bx)}{(bc-ad)^5}+\frac{4b^3de^{-a+\frac{bc}{d}}\Gamma\left(-1,\frac{b(c+dx)}{d}\right)}{3(bc-ad)^5}\\ &\quad-\frac{10b^3d^2\Gamma(0,a+bx)}{3(bc-ad)^6}+\frac{10b^3d^2e^{-a+\frac{bc}{d}}\Gamma\left(0,\frac{b(c+dx)}{d}\right)}{3(bc-ad)^6}\end{aligned}$$

output
```
1/3*b^3/(b*x+a)^3*Ei(4,b*x+a)/d/(-a*d+b*c)^3-1/3/(b*x+a)^3*Ei(4,b*x+a)/d/(
d*x+c)^3-b^3/(b*x+a)^2*Ei(3,b*x+a)/(-a*d+b*c)^4+1/3*b*exp(-a+b*c/d)/(d*x+c
)^2*d^2*Ei(3,b*(d*x+c)/d)/(-a*d+b*c)^4+2*b^3*d/(b*x+a)*Ei(2,b*x+a)/(-a*d+b
*c)^5+4/3*b^2*d^2*exp(-a+b*c/d)/(d*x+c)*Ei(2,b*(d*x+c)/d)/(-a*d+b*c)^5-10/
3*b^3*d^2*Ei(1,b*x+a)/(-a*d+b*c)^6+10/3*b^3*d^2*exp(-a+b*c/d)*Ei(1,b*(d*x+
c)/d)/(-a*d+b*c)^6
```

Mathematica [F]

$$\int \frac{\Gamma(-3,a+bx)}{(c+dx)^4}\,dx=\int \frac{\Gamma(-3,a+bx)}{(c+dx)^4}\,dx$$

input
```
Integrate[Gamma[-3, a + b*x]/(c + d*x)^4,x]
```

output
```
Integrate[Gamma[-3, a + b*x]/(c + d*x)^4, x]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 573 vs. 2(248) = 496.

Time = 1.37 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.31, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7119, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-3,a+bx)}{(c+dx)^4}\,dx$$

$$\Big\downarrow \text{7119}$$

$$-\frac{b\int \frac{e^{-a-bx}}{(a+bx)^4(c+dx)^3}dx}{3d}-\frac{\Gamma(-3,a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\int \left(\frac{10b^2e^{-a-bx}d^4}{(bc-ad)^6(c+dx)}+\frac{4be^{-a-bx}d^4}{(bc-ad)^5(c+dx)^2}+\frac{e^{-a-bx}d^4}{(bc-ad)^4(c+dx)^3}-\frac{10b^3e^{-a-bx}d^3}{(bc-ad)^6(a+bx)}+\frac{6b^3e^{-a-bx}d^2}{(bc-ad)^5(a+bx)^2}-\frac{3b^3e^{-a-bx}d}{(bc-ad)^4(a+bx)^3}+\frac{b^3 \cdots}{(bc-\cdots}\right.}{3d}$$

$$\frac{\Gamma(-3,a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{10b^2d^3\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^6}+\frac{10b^2d^3e^{\frac{bc}{d}-a}\,\text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{(bc-ad)^6}-\frac{6b^2d^2\,\text{ExpIntegralEi}(-a-bx)}{(bc-ad)^5}-\frac{4b^2d^2e^{\frac{bc}{d}-a}\,\text{ExpInteg}}{(bc-a}\right.}{}$$

$$\frac{\Gamma(-3,a+bx)}{3d(c+dx)^3}$$

input
```
Int[Gamma[-3, a + b*x]/(c + d*x)^4,x]
```

output
```
-1/3*(b*(-1/3*(b^2*E^(-a - b*x))/((b*c - a*d)^3*(a + b*x)^3) + (3*b^2*d*E^
(-a - b*x))/(2*(b*c - a*d)^4*(a + b*x)^2) + (b^2*E^(-a - b*x))/(6*(b*c - a
*d)^3*(a + b*x)^2) - (6*b^2*d^2*E^(-a - b*x))/((b*c - a*d)^5*(a + b*x)) -
(3*b^2*d*E^(-a - b*x))/(2*(b*c - a*d)^4*(a + b*x)) - (b^2*E^(-a - b*x))/(6
*(b*c - a*d)^3*(a + b*x)) - (d^3*E^(-a - b*x))/(2*(b*c - a*d)^4*(c + d*x)^
2) - (4*b*d^3*E^(-a - b*x))/((b*c - a*d)^5*(c + d*x)) + (b*d^2*E^(-a - b*x
))/(2*(b*c - a*d)^4*(c + d*x)) - (10*b^2*d^3*ExpIntegralEi[-a - b*x])/(b*c
 - a*d)^6 - (6*b^2*d^2*ExpIntegralEi[-a - b*x])/(b*c - a*d)^5 - (3*b^2*d*E
xpIntegralEi[-a - b*x])/(2*(b*c - a*d)^4) - (b^2*ExpIntegralEi[-a - b*x])/
(6*(b*c - a*d)^3) + (10*b^2*d^3*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d
*x))/d)])/(b*c - a*d)^6 - (4*b^2*d^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(
c + d*x))/d)])/(b*c - a*d)^5 + (b^2*d*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*
(c + d*x))/d)])/(2*(b*c - a*d)^4)))/d - Gamma[-3, a + b*x]/(3*d*(c + d*x)^
3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\text{expIntegral}_4\left(bx+a\right)}{\left(bx+a\right)^3\left(dx+c\right)^4}dx$$

input
```
int(1/(b*x+a)^3*Ei(4,b*x+a)/(d*x+c)^4,x)
```

output
```
int(1/(b*x+a)^3*Ei(4,b*x+a)/(d*x+c)^4,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3116 vs. $2(233) = 466$.

Time = 0.35 (sec) , antiderivative size = 3116, normalized size of antiderivative = 12.56

$$\int \frac{\Gamma(-3,a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(gamma(-3,b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\Gamma(-3,a+bx)}{(c+dx)^4}\,dx = \int \frac{E_4\left(a+bx\right)}{\left(a+bx\right)^3\left(c+dx\right)^4}\,dx$$

input
```
integrate(uppergamma(-3,b*x+a)/(d*x+c)**4,x)
```

output
```
Integral(expint(4, a + b*x)/((a + b*x)**3*(c + d*x)**4), x)
```

Maxima [F]

$$\int \frac{\Gamma(-3,a+bx)}{(c+dx)^4}\,dx = \int \frac{\Gamma(-3,bx+a)}{(dx+c)^4}\,dx$$

input
```
integrate(gamma(-3,b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate(gamma(-3, b*x + a)/(d*x + c)^4, x)
```

Giac [F]

$$\int \frac{\Gamma(-3,a+bx)}{(c+dx)^4}\,dx = \int \frac{\Gamma(-3,bx+a)}{(dx+c)^4}\,dx$$

input
```
integrate(gamma(-3,b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate(gamma(-3, b*x + a)/(d*x + c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.10

$$\int \frac{\Gamma(-3,a+bx)}{(c+dx)^4}\,dx = \int \frac{\mathrm{expint}(4,a+b\,x)}{(a+b\,x)^3\,(c+d\,x)^4}\,dx$$

input
```
int(expint(4, a + b*x)/((a + b*x)^3*(c + d*x)^4),x)
```

output
```
int(expint(4, a + b*x)/((a + b*x)^3*(c + d*x)^4), x)
```

Reduce [F]

$$\int \frac{\Gamma(-3, a+bx)}{(c+dx)^4}\,dx$$

$$= \int \frac{}{b^3d^4x^7 + 3a\,b^2d^4x^6 + 4b^3c\,d^3x^6 + 3a^2b\,d^4x^5 + 12a\,b^2c\,d^3x^5 + 6b^3c^2d^2x^5 + a^3d^4x^4 + 12a^2bc\,d^3x^4 + 18a}$$

input
```
int(1/(b*x+a)^3*Ei(4,b*x+a)/(d*x+c)^4,x)
```

output
```
int(ei(4,a + b*x)/(a**3*c**4 + 4*a**3*c**3*d*x + 6*a**3*c**2*d**2*x**2 + 4
*a**3*c*d**3*x**3 + a**3*d**4*x**4 + 3*a**2*b*c**4*x + 12*a**2*b*c**3*d*x*
*2 + 18*a**2*b*c**2*d**2*x**3 + 12*a**2*b*c*d**3*x**4 + 3*a**2*b*d**4*x**5
 + 3*a*b**2*c**4*x**2 + 12*a*b**2*c**3*d*x**3 + 18*a*b**2*c**2*d**2*x**4 +
 12*a*b**2*c*d**3*x**5 + 3*a*b**2*d**4*x**6 + b**3*c**4*x**3 + 4*b**3*c**3
*d*x**4 + 6*b**3*c**2*d**2*x**5 + 4*b**3*c*d**3*x**6 + b**3*d**4*x**7),x)
```

3.161 $\int x^{5/2}\Gamma(2,a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int x^{5/2}\Gamma(2,a+bx)\,dx=\frac{2}{7}x^{7/2}\Gamma(2,a+bx)-\frac{2ae^{-a}\sqrt{x}\Gamma\left(\frac{9}{2},bx\right)}{7b^3\sqrt{bx}}-\frac{2e^{-a}\sqrt{x}\Gamma\left(\frac{11}{2},bx\right)}{7b^3\sqrt{bx}}$$

output
```
2/7*x^(7/2)*exp(-b*x-a)*(b*x+a+1)-2/7*a*x^(1/2)*((b*x)^(7/2)*exp(-b*x)+7/2
*(b*x)^(5/2)*exp(-b*x)+35/4*(b*x)^(3/2)*exp(-b*x)+105/8*(b*x)^(1/2)*exp(-b
*x)+105/16*Pi^(1/2)*erfc((b*x)^(1/2)))/b^3/exp(a)/(b*x)^(1/2)-2/7*x^(1/2)*
(1048576/6183686925497065825762484O625*GAMMA(51/2,b*x)-1048576/61836869254
970658257624840625*(b*x)^(49/2)*exp(-b*x)-524288/1261976923570829760359690
625*(b*x)^(47/2)*exp(-b*x)-262144/26850572841932548092759375*(b*x)^(45/2)*
exp(-b*x)-131072/596679396487389957616875*(b*x)^(43/2)*exp(-b*x)-65536/138
76265034590464130625*(b*x)^(41/2)*exp(-b*x)-32768/338445488648547905625*(b
*x)^(39/2)*exp(-b*x)-16384/8678089452526869375*(b*x)^(37/2)*exp(-b*x)-8192
/234542958176401875*(b*x)^(35/2)*exp(-b*x)-4096/6701227376468625*(b*x)^(33
/2)*exp(-b*x)-2048/203067496256625*(b*x)^(31/2)*exp(-b*x)-1024/6550564395
375*(b*x)^(29/2)*exp(-b*x)-512/225881530875*(b*x)^(27/2)*exp(-b*x)-256/8365
982625*(b*x)^(25/2)*exp(-b*x)-128/334639305*(b*x)^(23/2)*exp(-b*x)-64/1454
9535*(b*x)^(21/2)*exp(-b*x)-32/692835*(b*x)^(19/2)*exp(-b*x)-16/36465*(b*x
)^(17/2)*exp(-b*x)-8/2145*(b*x)^(15/2)*exp(-b*x)-4/143*(b*x)^(13/2)*exp(-b
*x)-2/11*(b*x)^(11/2)*exp(-b*x))/b^3/exp(a)/(b*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.68

$$\int x^{5/2}\Gamma(2,a+bx)\,dx=\frac{e^{-a}\left(105(9+2a)\sqrt{\pi}\text{erf}\left(\sqrt{b}\sqrt{x}\right)-2\sqrt{b}e^{-bx}\sqrt{x}(945+630bx+252b^2x^2+72b^3x^3+16b^4x^4+2a}{112b^{7/2}}$$

input
```
Integrate[x^(5/2)*Gamma[2, a + b*x],x]
```

output
```
(105*(9 + 2*a)*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]] - (2*Sqrt[b]*Sqrt[x]*(945 + 6
30*b*x + 252*b^2*x^2 + 72*b^3*x^3 + 16*b^4*x^4 + 2*a*(105 + 70*b*x + 28*b^
2*x^2 + 8*b^3*x^3) - 16*b^3*E^(a + b*x)*x^3*Gamma[2, a + b*x]))/E^(b*x))/(
112*b^(7/2)*E^a)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 281 vs. 2(80) = 160.

Time = 0.68 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.51, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\Gamma(2,a+bx)\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{2}{7}b\int e^{-a-bx}x^{7/2}(a+bx)dx+\frac{2}{7}x^{7/2}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{2629}$$

$$\frac{2}{7}b\int \left(be^{-a-bx}x^{9/2}+ae^{-a-bx}x^{7/2}\right)dx+\frac{2}{7}x^{7/2}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}b\left(\frac{105\sqrt{\pi}ae^{-a}\text{erf}\left(\sqrt{b}\sqrt{x}\right)}{16b^{9/2}}+\frac{945\sqrt{\pi}e^{-a}\text{erf}\left(\sqrt{b}\sqrt{x}\right)}{32b^{9/2}}-\frac{105a\sqrt{x}e^{-a-bx}}{8b^4}-\frac{945\sqrt{x}e^{-a-bx}}{16b^4}-\frac{35ax^{3/2}e^{-a-bx}}{4b^3}-3\right.$$
$$\frac{2}{7}x^{7/2}\Gamma(2,a+bx)$$

input
```
Int[x^(5/2)*Gamma[2, a + b*x],x]
```

output
```
(2*b*((-945*E^(-a - b*x)*Sqrt[x])/(16*b^4) - (105*a*E^(-a - b*x)*Sqrt[x])/
(8*b^4) - (315*E^(-a - b*x)*x^(3/2))/(8*b^3) - (35*a*E^(-a - b*x)*x^(3/2))
/(4*b^3) - (63*E^(-a - b*x)*x^(5/2))/(4*b^2) - (7*a*E^(-a - b*x)*x^(5/2))/
(2*b^2) - (9*E^(-a - b*x)*x^(7/2))/(2*b) - (a*E^(-a - b*x)*x^(7/2))/b - E^
(-a - b*x)*x^(9/2) + (945*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/(32*b^(9/2)*E^a)
+ (105*a*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/(16*b^(9/2)*E^a)))/7 + (2*x^(7/2)*
Gamma[2, a + b*x])/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.98

method	result
meijerg	$\frac{e^{-a}\left(-\frac{\sqrt{x}\,\sqrt{b}\,\left(72b^{3}x^{3}+252b^{2}x^{2}+630bx+945\right)e^{-bx}}{72}+\frac{105\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{16}\right)}{b^{\frac{7}{2}}}+\frac{e^{-a}a\left(-\frac{\sqrt{x}\,\sqrt{b}\,\left(28b^{2}x^{2}+70bx+105\right)e^{-bx}}{28}+\cdots\right.}{b^{\frac{7}{2}}}$
derivativedivides	$2e^{-a}\left(-\frac{x^{\frac{5}{2}}e^{-bx}}{2b}+\frac{-\frac{5x^{\frac{3}{2}}e^{-bx}}{4b}+\frac{5\left(-\frac{3\sqrt{x}\,e^{-bx}}{4b}+\frac{3\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{8b^{\frac{3}{2}}}\right)}{2b}}{b}\right)+2e^{-a}a\left(-\frac{x^{\frac{5}{2}}e^{-bx}}{2b}+\frac{-\frac{5x^{\frac{3}{2}}e^{-bx}}{4b}\cdots}{}\right.$
default	$2e^{-a}\left(-\frac{x^{\frac{5}{2}}e^{-bx}}{2b}+\frac{-\frac{5x^{\frac{3}{2}}e^{-bx}}{4b}+\frac{5\left(-\frac{3\sqrt{x}\,e^{-bx}}{4b}+\frac{3\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{8b^{\frac{3}{2}}}\right)}{2b}}{b}\right)+2e^{-a}a\left(-\frac{x^{\frac{5}{2}}e^{-bx}}{2b}+\frac{-\frac{5x^{\frac{3}{2}}e^{-bx}}{4b}\cdots}{}\right.$

input
```
int(x^(5/2)*exp(-b*x-a)*(b*x+a+1),x,method=_RETURNVERBOSE)
```

output
```
1/b^(7/2)*exp(-a)*(-1/72*x^(1/2)*b^(1/2)*(72*b^3*x^3+252*b^2*x^2+630*b*x+9
45)*exp(-b*x)+105/16*Pi^(1/2)*erf(b^(1/2)*x^(1/2)))+1/b^(7/2)*exp(-a)*a*(-
1/28*x^(1/2)*b^(1/2)*(28*b^2*x^2+70*b*x+105)*exp(-b*x)+15/8*Pi^(1/2)*erf(b
^(1/2)*x^(1/2)))+1/b^(7/2)*exp(-a)*(-1/28*x^(1/2)*b^(1/2)*(28*b^2*x^2+70*b
*x+105)*exp(-b*x)+15/8*Pi^(1/2)*erf(b^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.46

$$\int x^{5/2}\Gamma(2,a+bx)\,dx = \frac{105\,\sqrt{\pi}(2\,a+9)\sqrt{b}\,\mathrm{erf}\left(\sqrt{b}\sqrt{x}\right)e^{(-a)} + 2\left(16\,b^4x^3\Gamma(2,bx+a) - (16\,b^5x^4 + 8\,(2\,a+9)b^4x^3 + 28\ldots\right.}{112\,b^4}$$

input
```
integrate(x^(5/2)*gamma(2,b*x+a),x, algorithm="fricas")
```

output
```
1/112*(105*sqrt(pi)*(2*a + 9)*sqrt(b)*erf(sqrt(b)*sqrt(x))*e^(-a) + 2*(16*
b^4*x^3*gamma(2, b*x + a) - (16*b^5*x^4 + 8*(2*a + 9)*b^4*x^3 + 28*(2*a +
9)*b^3*x^2 + 70*(2*a + 9)*b^2*x + 105*(2*a + 9)*b)*e^(-b*x - a))*sqrt(x))/
b^4
```

Sympy [F(-1)]

Timed out.

$$\int x^{5/2}\Gamma(2,a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*uppergamma(2,b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{5/2}\Gamma(2,a+bx)\,dx = \int x^{\frac{5}{2}}\Gamma(2,bx+a)\,dx$$

input
```
integrate(x^(5/2)*gamma(2,b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)*gamma(2, b*x + a), x)
```

Giac [F]

$$\int x^{5/2}\Gamma(2, a+bx)\,dx = \int x^{\frac{5}{2}}\Gamma(2, bx+a)\,dx$$

input
```
integrate(x^(5/2)*gamma(2,b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(5/2)*gamma(2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.09

$$\int x^{5/2}\Gamma(2, a+bx)\,dx = -x^{7/2}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x} - \frac{135\,\sqrt{x}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{8\,b^3} - \frac{45\,x^{3/2}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{4\,b^2} - \frac{9\,x^{5/2}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2\,b} - \frac{15\,a\,\sqrt{x}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{4\,b^3} - \frac{5\,a\,x^{3/2}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2\,b^2} - \frac{a\,x^{5/2}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{b} - \frac{135\,x^{7/2}\,\sqrt{\pi}\,\mathrm{e}^{-a}\,\mathrm{erfc}\left(\sqrt{b\,x}\right)}{16\,(b\,x)^{7/2}} - \frac{15\,a\,x^{7/2}\,\sqrt{\pi}\,\mathrm{e}^{-a}\,\mathrm{erfc}\left(\sqrt{b\,x}\right)}{8\,(b\,x)^{7/2}}$$

input
```
int(x^(5/2)*exp(- a - b*x)*(a + b*x + 1),x)
```

output
```
- x^(7/2)*exp(-a)*exp(-b*x) - (135*x^(1/2)*exp(-a)*exp(-b*x))/(8*b^3) - (4
5*x^(3/2)*exp(-a)*exp(-b*x))/(4*b^2) - (9*x^(5/2)*exp(-a)*exp(-b*x))/(2*b)
 - (15*a*x^(1/2)*exp(-a)*exp(-b*x))/(4*b^3) - (5*a*x^(3/2)*exp(-a)*exp(-b*
x))/(2*b^2) - (a*x^(5/2)*exp(-a)*exp(-b*x))/b - (135*x^(7/2)*pi^(1/2)*exp(
-a)*erfc((b*x)^(1/2)))/(16*(b*x)^(7/2)) - (15*a*x^(7/2)*pi^(1/2)*exp(-a)*e
rfc((b*x)^(1/2)))/(8*(b*x)^(7/2))
```

Reduce [F]

$$\int x^{5/2}\Gamma(2,a+bx)\,dx = \frac{30e^{bx}\left(\int \frac{\sqrt{x}}{e^{bx}x}dx\right)a+135e^{bx}\left(\int \frac{\sqrt{x}}{e^{bx}x}dx\right)-16\sqrt{x}\,a\,b^2x^2-40\sqrt{x}\,abx-60\sqrt{x}\,a-16\sqrt{x}\,b^3x^3-72}{16e^{bx+a}b^3}$$

input
```
int(x^(5/2)*exp(-b*x-a)*(b*x+a+1),x)
```

output
```
(30*e**(b*x)*int(sqrt(x)/(e**(b*x)*x),x)*a + 135*e**(b*x)*int(sqrt(x)/(e**
(b*x)*x),x) - 16*sqrt(x)*a*b**2*x**2 - 40*sqrt(x)*a*b*x - 60*sqrt(x)*a - 1
6*sqrt(x)*b**3*x**3 - 72*sqrt(x)*b**2*x**2 - 180*sqrt(x)*b*x - 270*sqrt(x)
)/(16*e**(a + b*x)*b**3)
```

3.162 $\int x^{3/2}\Gamma(2,a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int x^{3/2}\Gamma(2,a+bx)\,dx=\frac{2}{5}x^{5/2}\Gamma(2,a+bx)-\frac{2ae^{-a}\sqrt{x}\,\Gamma\left(\frac{7}{2},bx\right)}{5b^2\sqrt{bx}}-\frac{2e^{-a}\sqrt{x}\,\Gamma\left(\frac{9}{2},bx\right)}{5b^2\sqrt{bx}}$$

output
```
2/5*x^(5/2)*exp(-b*x-a)*(b*x+a+1)-2/5*a*x^(1/2)*((b*x)^(5/2)*exp(-b*x)+5/2
*(b*x)^(3/2)*exp(-b*x)+15/4*(b*x)^(1/2)*exp(-b*x)+15/8*Pi^(1/2)*erfc((b*x)
^(1/2)))/b^2/exp(a)/(b*x)^(1/2)-2/5*x^(1/2)*((b*x)^(7/2)*exp(-b*x)+7/2*(b*
x)^(5/2)*exp(-b*x)+35/4*(b*x)^(3/2)*exp(-b*x)+105/8*(b*x)^(1/2)*exp(-b*x)+
105/16*Pi^(1/2)*erfc((b*x)^(1/2)))/b^2/exp(a)/(b*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.46

$$\int x^{3/2}\Gamma(2,a+bx)\,dx=\frac{e^{-a}\left(15(7+2a)\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}\sqrt{x}\right)-2\sqrt{b}e^{-bx}\sqrt{x}\left(105+70bx+28b^2x^2+8b^3x^3+a(30+20bx+8b\right.\right.}{40b^{5/2}}$$

input
```
Integrate[x^(3/2)*Gamma[2, a + b*x],x]
```

output
```
(15*(7 + 2*a)*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]] - (2*Sqrt[b]*Sqrt[x]*(105 + 70
*b*x + 28*b^2*x^2 + 8*b^3*x^3 + a*(30 + 20*b*x + 8*b^2*x^2) - 8*b^2*E^(a +
 b*x)*x^2*Gamma[2, a + b*x]))/E^(b*x))/(40*b^(5/2)*E^a)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 236 vs. 2(80) = 160.

Time = 0.56 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\Gamma(2,a+bx)\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{2}{5}b\int e^{-a-bx}x^{5/2}(a+bx)dx+\frac{2}{5}x^{5/2}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{2629}$$

$$\frac{2}{5}b\int \left(be^{-a-bx}x^{7/2}+ae^{-a-bx}x^{5/2}\right)dx+\frac{2}{5}x^{5/2}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}b\left(\frac{15\sqrt{\pi}ae^{-a}\text{erf}\left(\sqrt{b}\sqrt{x}\right)}{8b^{7/2}}+\frac{105\sqrt{\pi}e^{-a}\text{erf}\left(\sqrt{b}\sqrt{x}\right)}{16b^{7/2}}-\frac{15a\sqrt{x}e^{-a-bx}}{4b^3}-\frac{105\sqrt{x}e^{-a-bx}}{8b^3}-\frac{5ax^{3/2}e^{-a-bx}}{2b^2}-\frac{35x^3}{}\right.$$
$$\frac{2}{5}x^{5/2}\Gamma(2,a+bx)$$

input
```
Int[x^(3/2)*Gamma[2, a + b*x],x]
```

output
```
(2*b*((-105*E^(-a - b*x)*Sqrt[x])/(8*b^3) - (15*a*E^(-a - b*x)*Sqrt[x])/(4
*b^3) - (35*E^(-a - b*x)*x^(3/2))/(4*b^2) - (5*a*E^(-a - b*x)*x^(3/2))/(2*
b^2) - (7*E^(-a - b*x)*x^(5/2))/(2*b) - (a*E^(-a - b*x)*x^(5/2))/b - E^(-a
 - b*x)*x^(7/2) + (105*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/(16*b^(7/2)*E^a) + (
15*a*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/(8*b^(7/2)*E^a)))/5 + (2*x^(5/2)*Gamma
[2, a + b*x])/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.68

method	result
meijerg	$\frac{e^{-a}\left(-\frac{\sqrt{x}\,\sqrt{b}\,(28b^2x^2+70bx+105)e^{-bx}}{28}+\frac{15\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{8}\right)}{b^{\frac{5}{2}}}+\frac{e^{-a}a\left(-\frac{\sqrt{x}\,\sqrt{b}\,(10bx+15)e^{-bx}}{10}+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{4}\right)}{b^{\frac{5}{2}}}+$
derivativedivides	$2\,e^{-a}\left(-\frac{x^{\frac{3}{2}}e^{-bx}}{2b}+\frac{-\frac{3\sqrt{x}\,e^{-bx}}{4b}+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{8b^{\frac{3}{2}}}}{b}\right)+2\,e^{-a}a\left(-\frac{x^{\frac{3}{2}}e^{-bx}}{2b}+\frac{-\frac{3\sqrt{x}\,e^{-bx}}{4b}+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{8b^{\frac{3}{2}}}}{b}\right.$
default	$2\,e^{-a}\left(-\frac{x^{\frac{3}{2}}e^{-bx}}{2b}+\frac{-\frac{3\sqrt{x}\,e^{-bx}}{4b}+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{8b^{\frac{3}{2}}}}{b}\right)+2\,e^{-a}a\left(-\frac{x^{\frac{3}{2}}e^{-bx}}{2b}+\frac{-\frac{3\sqrt{x}\,e^{-bx}}{4b}+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{8b^{\frac{3}{2}}}}{b}\right.$

input
```
int(x^(3/2)*exp(-b*x-a)*(b*x+a+1),x,method=_RETURNVERBOSE)
```

output
```
1/b^(5/2)*exp(-a)*(-1/28*x^(1/2)*b^(1/2)*(28*b^2*x^2+70*b*x+105)*exp(-b*x)
+15/8*Pi^(1/2)*erf(b^(1/2)*x^(1/2)))+1/b^(5/2)*exp(-a)*a*(-1/10*x^(1/2)*b^
(1/2)*(10*b*x+15)*exp(-b*x)+3/4*Pi^(1/2)*erf(b^(1/2)*x^(1/2)))+1/b^(5/2)*e
xp(-a)*(-1/10*x^(1/2)*b^(1/2)*(10*b*x+15)*exp(-b*x)+3/4*Pi^(1/2)*erf(b^(1/
2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.30

$$\int x^{3/2}\Gamma(2,a+bx)\,dx=\frac{15\,\sqrt{\pi}(2\,a+7)\sqrt{b}\,\mathrm{erf}\left(\sqrt{b}\sqrt{x}\right)e^{(-a)}+2\left(8\,b^3x^2\Gamma(2,bx+a)-(8\,b^4x^3+4\,(2\,a+7)b^3x^2+10\,(2\,a\right.}{40\,b^3}$$

input
```
integrate(x^(3/2)*gamma(2,b*x+a),x, algorithm="fricas")
```

output
```
1/40*(15*sqrt(pi)*(2*a + 7)*sqrt(b)*erf(sqrt(b)*sqrt(x))*e^(-a) + 2*(8*b^3
*x^2*gamma(2, b*x + a) - (8*b^4*x^3 + 4*(2*a + 7)*b^3*x^2 + 10*(2*a + 7)*b
^2*x + 15*(2*a + 7)*b)*e^(-b*x - a))*sqrt(x))/b^3
```

Sympy [A] (verification not implemented)

Time = 28.69 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.06

$$\int x^{3/2}\Gamma(2,a+bx)\,dx = \left(-\frac{ax^{\frac{3}{2}}\left(-\sqrt{bx}\left(-bx-\frac{3}{2}\right)e^{-bx}+\frac{3\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{bx}\right)}{4}\right)}{b\,(bx)^{\frac{3}{2}}} - \frac{x^{\frac{5}{2}}\left(\sqrt{bx}\left(b^2x^2+\frac{5bx}{2}+\frac{15}{4}\right)e^{-bx}+\frac{15\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{bx}\right)}{8}\right)}{(bx)^{\frac{5}{2}}} - \frac{x^{\frac{3}{2}}\left(-\sqrt{bx}\left(-bx-\frac{3}{2}\right)e^{-bx}+\frac{3\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{bx}\right)}{4}\right)}{b\,(bx)^{\frac{3}{2}}}\right)e^{-a}$$

input
```
integrate(x**(3/2)*uppergamma(2,b*x+a),x)
```

output
```
(-a*x**(3/2)*(-sqrt(b*x)*(-b*x - 3/2)*exp(-b*x) + 3*sqrt(pi)*erfc(sqrt(b*x
))/4)/(b*(b*x)**(3/2)) - x**(5/2)*(sqrt(b*x)*(b**2*x**2 + 5*b*x/2 + 15/4)*
exp(-b*x) + 15*sqrt(pi)*erfc(sqrt(b*x))/8)/(b*x)**(5/2) - x**(3/2)*(-sqrt(
b*x)*(-b*x - 3/2)*exp(-b*x) + 3*sqrt(pi)*erfc(sqrt(b*x))/4)/(b*(b*x)**(3/2
)))*exp(-a)
```

Maxima [F]

$$\int x^{3/2}\Gamma(2,a+bx)\,dx = \int x^{\frac{3}{2}}\Gamma(2,bx+a)\,dx$$

input
```
integrate(x^(3/2)*gamma(2,b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)*gamma(2, b*x + a), x)
```

Giac [F]

$$\int x^{3/2}\Gamma(2,a+bx)\,dx=\int x^{\frac{3}{2}}\Gamma(2,bx+a)\;dx$$

input
```
integrate(x^(3/2)*gamma(2,b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(3/2)*gamma(2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.65

$$\int x^{3/2}\Gamma(2,a+bx)\,dx=-x^{5/2}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}-\frac{21\,\sqrt{x}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{4\,b^2}-\frac{7\,x^{3/2}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2\,b}-\frac{3\,a\,\sqrt{x}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2\,b^2}-\frac{a\,x^{3/2}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{b}-\frac{21\,x^{5/2}\,\sqrt{\pi}\,\mathrm{e}^{-a}\,\mathrm{erfc}\left(\sqrt{b\,x}\right)}{8\,{\left(b\,x\right)}^{5/2}}-\frac{3\,a\,x^{5/2}\,\sqrt{\pi}\,\mathrm{e}^{-a}\,\mathrm{erfc}\left(\sqrt{b\,x}\right)}{4\,{\left(b\,x\right)}^{5/2}}$$

input
```
int(x^(3/2)*exp(- a - b*x)*(a + b*x + 1),x)
```

output
```
- x^(5/2)*exp(-a)*exp(-b*x) - (21*x^(1/2)*exp(-a)*exp(-b*x))/(4*b^2) - (7*
x^(3/2)*exp(-a)*exp(-b*x))/(2*b) - (3*a*x^(1/2)*exp(-a)*exp(-b*x))/(2*b^2)
 - (a*x^(3/2)*exp(-a)*exp(-b*x))/b - (21*x^(5/2)*pi^(1/2)*exp(-a)*erfc((b*
x)^(1/2)))/(8*(b*x)^(5/2)) - (3*a*x^(5/2)*pi^(1/2)*exp(-a)*erfc((b*x)^(1/2
)))/(4*(b*x)^(5/2))
```

Reduce [F]

$$\int x^{3/2}\Gamma(2, a+bx)\, dx = \frac{6e^{bx}\left(\int \frac{\sqrt{x}}{e^{bx}x}dx\right)a + 21e^{bx}\left(\int \frac{\sqrt{x}}{e^{bx}x}dx\right) - 8\sqrt{x}\, abx - 12\sqrt{x}\, a - 8\sqrt{x}\, b^2x^2 - 28\sqrt{x}\, bx - 42\sqrt{x}}{8e^{bx+a}b^2}$$

input

```
int(x^(3/2)*exp(-b*x-a)*(b*x+a+1),x)
```

output

```
(6*e**(b*x)*int(sqrt(x)/(e**(b*x)*x),x)*a + 21*e**(b*x)*int(sqrt(x)/(e**(b
*x)*x),x) - 8*sqrt(x)*a*b*x - 12*sqrt(x)*a - 8*sqrt(x)*b**2*x**2 - 28*sqrt
(x)*b*x - 42*sqrt(x))/(8*e**(a + b*x)*b**2)
```

3.163 $\int \sqrt{x}\Gamma(2,a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \sqrt{x}\Gamma(2,a+bx)\,dx=\frac{2}{3}x^{3/2}\Gamma(2,a+bx)-\frac{2ae^{-a}\sqrt{x}\Gamma\left(\frac{5}{2},bx\right)}{3b\sqrt{bx}}-\frac{2e^{-a}\sqrt{x}\Gamma\left(\frac{7}{2},bx\right)}{3b\sqrt{bx}}$$

output
```
2/3*x^(3/2)*exp(-b*x-a)*(b*x+a+1)-2/3*a*x^(1/2)*((b*x)^(3/2)*exp(-b*x)+3/2
*(b*x)^(1/2)*exp(-b*x)+3/4*Pi^(1/2)*erfc((b*x)^(1/2)))/b/exp(a)/(b*x)^(1/2
)-2/3*x^(1/2)*((b*x)^(5/2)*exp(-b*x)+5/2*(b*x)^(3/2)*exp(-b*x)+15/4*(b*x)^
(1/2)*exp(-b*x)+15/8*Pi^(1/2)*erfc((b*x)^(1/2)))/b/exp(a)/(b*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.21

$$\int \sqrt{x}\Gamma(2,a+bx)\,dx =\frac{e^{-a}\left(3(5+2a)\sqrt{\pi}\text{erf}\left(\sqrt{b}\sqrt{x}\right)-2\sqrt{b}e^{-bx}\sqrt{x}\left(15+6a+10bx+4abx+4b^2x^2-4be^{a+bx}x\Gamma(2,a+bx)\right)\right)}{12b^{3/2}}$$

input
```
Integrate[Sqrt[x]*Gamma[2, a + b*x],x]
```

output
```
(3*(5 + 2*a)*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]] - (2*Sqrt[b]*Sqrt[x]*(15 + 6*a
+ 10*b*x + 4*a*b*x + 4*b^2*x^2 - 4*b*E^(a + b*x)*x*Gamma[2, a + b*x]))/E^(
b*x))/(12*b^(3/2)*E^a)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 191 vs. 2(80) = 160.

Time = 0.51 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.39, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\Gamma(2, a+bx)\, dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{2}{3}b\int e^{-a-bx}x^{3/2}(a+bx)dx+\frac{2}{3}x^{3/2}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{2629}$$

$$\frac{2}{3}b\int \left(be^{-a-bx}x^{5/2}+ae^{-a-bx}x^{3/2}\right)dx+\frac{2}{3}x^{3/2}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}b\left(\frac{3\sqrt{\pi}ae^{-a}\text{erf}\left(\sqrt{b}\sqrt{x}\right)}{4b^{5/2}}+\frac{15\sqrt{\pi}e^{-a}\text{erf}\left(\sqrt{b}\sqrt{x}\right)}{8b^{5/2}}-\frac{3a\sqrt{x}e^{-a-bx}}{2b^2}-\frac{15\sqrt{x}e^{-a-bx}}{4b^2}+x^{5/2}\left(-e^{-a-bx}\right)-\frac{ax^{3/2}e^{-}}{b}\right.$$
$$\frac{2}{3}x^{3/2}\Gamma(2,a+bx)$$

input
```
Int[Sqrt[x]*Gamma[2, a + b*x],x]
```

output
```
(2*b*((-15*E^(-a - b*x)*Sqrt[x])/(4*b^2) - (3*a*E^(-a - b*x)*Sqrt[x])/(2*b
^2) - (5*E^(-a - b*x)*x^(3/2))/(2*b) - (a*E^(-a - b*x)*x^(3/2))/b - E^(-a
- b*x)*x^(5/2) + (15*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/(8*b^(5/2)*E^a) + (3*a
*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/(4*b^(5/2)*E^a)))/3 + (2*x^(3/2)*Gamma[2,
a + b*x])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

method	result
meijerg	$\frac{e^{-a}\left(-\frac{\sqrt{x}\,\sqrt{b}\,(10bx+15)\,e^{-bx}}{10}+\frac{3\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{4}\right)}{b^{\frac{3}{2}}}+\frac{e^{-a}a\left(-\sqrt{x}\,\sqrt{b}\,e^{-bx}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{2}\right)}{b^{\frac{3}{2}}}+\frac{e^{-a}\left(-\sqrt{x}\,\sqrt{b}\,e^{-bx}+\right.}{b^{\frac{3}{2}}}$
derivativedivides	$2\,e^{-a}\left(-\frac{\sqrt{x}\,e^{-bx}}{2b}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{4b^{\frac{3}{2}}}\right)+2\,e^{-a}a\left(-\frac{\sqrt{x}\,e^{-bx}}{2b}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{4b^{\frac{3}{2}}}\right)+2\,e^{-a}b\left(-\frac{x^{\frac{3}{2}}e}{2}\right.$
default	$2\,e^{-a}\left(-\frac{\sqrt{x}\,e^{-bx}}{2b}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{4b^{\frac{3}{2}}}\right)+2\,e^{-a}a\left(-\frac{\sqrt{x}\,e^{-bx}}{2b}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{4b^{\frac{3}{2}}}\right)+2\,e^{-a}b\left(-\frac{x^{\frac{3}{2}}e}{2}\right.$

input
```
int(x^(1/2)*exp(-b*x-a)*(b*x+a+1),x,method=_RETURNVERBOSE)
```

output
```
1/b^(3/2)*exp(-a)*(-1/10*x^(1/2)*b^(1/2)*(10*b*x+15)*exp(-b*x)+3/4*Pi^(1/2
)*erf(b^(1/2)*x^(1/2)))+1/b^(3/2)*exp(-a)*a*(-x^(1/2)*b^(1/2)*exp(-b*x)+1/
2*Pi^(1/2)*erf(b^(1/2)*x^(1/2)))+1/b^(3/2)*exp(-a)*(-x^(1/2)*b^(1/2)*exp(-
b*x)+1/2*Pi^(1/2)*erf(b^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

$$\int \sqrt{x}\Gamma(2,a+bx)\,dx$$
$$= \frac{3\sqrt{\pi}(2a+5)\sqrt{b}\operatorname{erf}\left(\sqrt{b}\sqrt{x}\right)e^{(-a)}+2\left(4b^2x\Gamma(2,bx+a)-(4b^3x^2+2(2a+5)b^2x+3(2a+5)b)e^{(-bx}\right.}{12b^2}$$

input
```
integrate(x^(1/2)*gamma(2,b*x+a),x, algorithm="fricas")
```

output
```
1/12*(3*sqrt(pi)*(2*a + 5)*sqrt(b)*erf(sqrt(b)*sqrt(x))*e^(-a) + 2*(4*b^2*
x*gamma(2, b*x + a) - (4*b^3*x^2 + 2*(2*a + 5)*b^2*x + 3*(2*a + 5)*b)*e^(-
b*x - a))*sqrt(x))/b^2
```

Sympy [A] (verification not implemented)

Time = 8.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.70

$$\int \sqrt{x}\Gamma(2,a+bx)\,dx = \left(-\frac{a\sqrt{x}\left(\sqrt{bx}e^{-bx}+\frac{\sqrt{\pi}\operatorname{erfc}\left(\sqrt{bx}\right)}{2}\right)}{b\sqrt{bx}} - \frac{x^{\frac{3}{2}}\left(-\sqrt{bx}\left(-bx-\frac{3}{2}\right)e^{-bx}+\frac{3\sqrt{\pi}\operatorname{erfc}\left(\sqrt{bx}\right)}{4}\right)}{(bx)^{\frac{3}{2}}} - \frac{\sqrt{x}\left(\sqrt{bx}e^{-bx}+\frac{\sqrt{\pi}\operatorname{erfc}\left(\sqrt{bx}\right)}{2}\right)}{b\sqrt{bx}}\right)e^{-a}$$

input
```
integrate(x**(1/2)*uppergamma(2,b*x+a),x)
```

output
```
(-a*sqrt(x)*(sqrt(b*x)*exp(-b*x) + sqrt(pi)*erfc(sqrt(b*x))/2)/(b*sqrt(b*x
)) - x**(3/2)*(-sqrt(b*x)*(-b*x - 3/2)*exp(-b*x) + 3*sqrt(pi)*erfc(sqrt(b*
x))/4)/(b*x)**(3/2) - sqrt(x)*(sqrt(b*x)*exp(-b*x) + sqrt(pi)*erfc(sqrt(b*
x))/2)/(b*sqrt(b*x)))*exp(-a)
```

Maxima [F]

$$\int \sqrt{x}\Gamma(2,a+bx)\,dx = \int \sqrt{x}\Gamma(2,bx+a)\,dx$$

input
```
integrate(x^(1/2)*gamma(2,b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)*gamma(2, b*x + a), x)
```

Giac [F]

$$\int \sqrt{x}\Gamma(2, a + bx)\, dx = \int \sqrt{x}\Gamma(2, bx + a)\, dx$$

input
```
integrate(x^(1/2)*gamma(2,b*x+a),x, algorithm="giac")
```

output
```
integrate(sqrt(x)*gamma(2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.68

$$\int \sqrt{x}\Gamma(2, a + bx)\, dx = -\frac{\sqrt{x}\,e^{-a-b\,x}}{b} - \frac{a\,\sqrt{x}\,e^{-a-b\,x}}{b} - \frac{x^{3/2}\,e^{-a}\left(\frac{3\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{b\,x}\right)}{4} + e^{-b\,x}\left(\frac{3\,\sqrt{b\,x}}{2} + (b\,x)^{3/2}\right)\right)}{(b\,x)^{3/2}} - \frac{\sqrt{x}\,\sqrt{\pi}\,e^{-a}\,\mathrm{erfc}\left(\sqrt{b\,x}\right)}{2\,b\,\sqrt{b\,x}} - \frac{a\,\sqrt{x}\,\sqrt{\pi}\,e^{-a}\,\mathrm{erfc}\left(\sqrt{b\,x}\right)}{2\,b\,\sqrt{b\,x}}$$

input
```
int(x^(1/2)*exp(- a - b*x)*(a + b*x + 1),x)
```

output
```
- (x^(1/2)*exp(- a - b*x))/b - (a*x^(1/2)*exp(- a - b*x))/b - (x^(3/2)*exp
(-a)*((3*pi^(1/2)*erfc((b*x)^(1/2)))/4 + exp(-b*x)*((3*(b*x)^(1/2))/2 + (b
*x)^(3/2))))/(b*x)^(3/2) - (x^(1/2)*pi^(1/2)*exp(-a)*erfc((b*x)^(1/2)))/(2
*b*(b*x)^(1/2)) - (a*x^(1/2)*pi^(1/2)*exp(-a)*erfc((b*x)^(1/2)))/(2*b*(b*x
)^(1/2))
```

Reduce [F]

$$\int \sqrt{x}\Gamma(2,a+bx)\,dx = \frac{2e^{bx}\left(\int \frac{\sqrt{x}}{e^{bx}x}dx\right)a+5e^{bx}\left(\int \frac{\sqrt{x}}{e^{bx}x}dx\right)-4\sqrt{x}\,a-4\sqrt{x}\,bx-10\sqrt{x}}{4e^{bx+a}b}$$

input
```
int(x^(1/2)*exp(-b*x-a)*(b*x+a+1),x)
```

output
```
(2*e**(b*x)*int(sqrt(x)/(e**(b*x)*x),x)*a + 5*e**(b*x)*int(sqrt(x)/(e**(b*
x)*x),x) - 4*sqrt(x)*a - 4*sqrt(x)*b*x - 10*sqrt(x))/(4*e**(a + b*x)*b)
```

3.164 $\int \frac{\Gamma(2,a+bx)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{\Gamma(2,a+bx)}{\sqrt{x}}\,dx=-\frac{2ae^{-a}\sqrt{x}\Gamma\left(\frac{3}{2},bx\right)}{\sqrt{bx}}+2\sqrt{x}\Gamma(2,a+bx)-\frac{2e^{-a}\sqrt{x}\Gamma\left(\frac{5}{2},bx\right)}{\sqrt{bx}}$$

output
```
-2*a*x^(1/2)*((b*x)^(1/2)*exp(-b*x)+1/2*Pi^(1/2)*erfc((b*x)^(1/2)))/exp(a)
/(b*x)^(1/2)+2*x^(1/2)*exp(-b*x-a)*(b*x+a+1)-2*x^(1/2)*((b*x)^(3/2)*exp(-b
*x)+3/2*(b*x)^(1/2)*exp(-b*x)+3/4*Pi^(1/2)*erfc((b*x)^(1/2)))/exp(a)/(b*x)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \frac{\Gamma(2,a+bx)}{\sqrt{x}}\,dx=\frac{1}{2}e^{-a}\left(\frac{(3+2a)\sqrt{\pi}\text{erf}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}-2e^{-bx}\sqrt{x}\left(3+2a+2bx-2e^{a+bx}\Gamma(2,a+bx)\right)\right)$$

input
```
Integrate[Gamma[2, a + b*x]/Sqrt[x],x]
```

output
```
(((3 + 2*a)*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/Sqrt[b] - (2*Sqrt[x]*(3 + 2*a +
 2*b*x - 2*E^(a + b*x)*Gamma[2, a + b*x]))/E^(b*x))/(2*E^a)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 142 vs. 2(68) = 136.

Time = 0.45 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,a+bx)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{7119}$$

$$2b\int e^{-a-bx}\sqrt{x}(a+bx)dx+2\sqrt{x}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{2629}$$

$$2b\int \left(be^{-a-bx}x^{3/2}+ae^{-a-bx}\sqrt{x}\right)dx+2\sqrt{x}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{2009}$$

$$2b\left(\frac{\sqrt{\pi}ae^{-a}\text{erf}\left(\sqrt{b}\sqrt{x}\right)}{2b^{3/2}}+\frac{3\sqrt{\pi}e^{-a}\text{erf}\left(\sqrt{b}\sqrt{x}\right)}{4b^{3/2}}+x^{3/2}\left(-e^{-a-bx}\right)-\frac{a\sqrt{x}e^{-a-bx}}{b}-\frac{3\sqrt{x}e^{-a-bx}}{2b}\right)+2\sqrt{x}\Gamma(2,a+bx)$$

input
```
Int[Gamma[2, a + b*x]/Sqrt[x],x]
```

output
```
2*b*((-3*E^(-a - b*x)*Sqrt[x])/(2*b) - (a*E^(-a - b*x)*Sqrt[x])/b - E^(-a
- b*x)*x^(3/2) + (3*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/(4*b^(3/2)*E^a) + (a*Sq
rt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/(2*b^(3/2)*E^a)) + 2*Sqrt[x]*Gamma[2, a + b*x
]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

method	result	size
meijerg	$\frac{e^{-a}\left(-\sqrt{x}\,\sqrt{b}\,e^{-bx}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{2}\right)}{\sqrt{b}}+\frac{e^{-a}a\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}}+\frac{e^{-a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}}$	76
derivativedivides	$\frac{e^{-a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}}+\frac{e^{-a}a\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}}+2\,e^{-a}b\left(-\frac{\sqrt{x}\,e^{-bx}}{2b}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{4b^{\frac{3}{2}}}\right)$	78
default	$\frac{e^{-a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}}+\frac{e^{-a}a\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}}+2\,e^{-a}b\left(-\frac{\sqrt{x}\,e^{-bx}}{2b}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)}{4b^{\frac{3}{2}}}\right)$	78

input
```
int(exp(-b*x-a)*(b*x+a+1)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b^(1/2)*exp(-a)*(-x^(1/2)*b^(1/2)*exp(-b*x)+1/2*Pi^(1/2)*erf(b^(1/2)*x^(
1/2)))+exp(-a)*a/b^(1/2)*Pi^(1/2)*erf(b^(1/2)*x^(1/2))+1/b^(1/2)*exp(-a)*P
i^(1/2)*erf(b^(1/2)*x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \frac{\Gamma(2,a+bx)}{\sqrt{x}}\,dx = \frac{\sqrt{\pi}(2\,a+3)\sqrt{b}\,\mathrm{erf}\left(\sqrt{b}\sqrt{x}\right)e^{(-a)} - 2\left(\left(2\,b^2x+(2\,a+3)b\right)e^{(-bx-a)} - 2\,b\Gamma(2,bx+a)\right)\sqrt{x}}{2\,b}$$

input
```
integrate(gamma(2,b*x+a)/x^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(pi)*(2*a + 3)*sqrt(b)*erf(sqrt(b)*sqrt(x))*e^(-a) - 2*((2*b^2*x
+ (2*a + 3)*b)*e^(-b*x - a) - 2*b*gamma(2, b*x + a))*sqrt(x))/b
```

Sympy [A] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.46

$$\int \frac{\Gamma(2,a+bx)}{\sqrt{x}}\,dx = \left(-\frac{\sqrt{\pi}a\sqrt{bx}\,\mathrm{erfc}\left(\sqrt{bx}\right)}{b\sqrt{x}} - \frac{\sqrt{x}\left(\sqrt{bx}e^{-bx} + \frac{\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{bx}\right)}{2}\right)}{\sqrt{bx}} - \frac{\sqrt{\pi}\sqrt{bx}\,\mathrm{erfc}\left(\sqrt{bx}\right)}{b\sqrt{x}}\right)e^{-a}$$

input
```
integrate(uppergamma(2,b*x+a)/x**(1/2),x)
```

output
```
(-sqrt(pi)*a*sqrt(b*x)*erfc(sqrt(b*x))/(b*sqrt(x)) - sqrt(x)*(sqrt(b*x)*ex
p(-b*x) + sqrt(pi)*erfc(sqrt(b*x))/2)/sqrt(b*x) - sqrt(pi)*sqrt(b*x)*erfc(
sqrt(b*x))/(b*sqrt(x)))*exp(-a)
```

Maxima [F]

$$\int \frac{\Gamma(2, a+bx)}{\sqrt{x}}\,dx = \int \frac{\Gamma(2, bx+a)}{\sqrt{x}}\,dx$$

input
```
integrate(gamma(2,b*x+a)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate(gamma(2, b*x + a)/sqrt(x), x)
```

Giac [F]

$$\int \frac{\Gamma(2, a+bx)}{\sqrt{x}}\,dx = \int \frac{\Gamma(2, bx+a)}{\sqrt{x}}\,dx$$

input
```
integrate(gamma(2,b*x+a)/x^(1/2),x, algorithm="giac")
```

output
```
integrate(gamma(2, b*x + a)/sqrt(x), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.28

$$\int \frac{\Gamma(2, a+bx)}{\sqrt{x}}\,dx = -\sqrt{x}\,e^{-a-b\,x} - \frac{e^{-a}\,\mathrm{erfc}\left(\sqrt{b\,x}\right)\,\sqrt{\pi\,b\,x}}{b\,\sqrt{x}} - \frac{\sqrt{x}\,\sqrt{\pi}\,e^{-a}\,\mathrm{erfc}\left(\sqrt{b\,x}\right)}{2\,\sqrt{b\,x}} - \frac{a\,e^{-a}\,\mathrm{erfc}\left(\sqrt{b\,x}\right)\,\sqrt{\pi\,b\,x}}{b\,\sqrt{x}}$$

input
```
int((exp(- a - b*x)*(a + b*x + 1))/x^(1/2),x)
```

output
```
- x^(1/2)*exp(- a - b*x) - (exp(-a)*erfc((b*x)^(1/2))*(b*x*pi)^(1/2))/(b*x
^(1/2)) - (x^(1/2)*pi^(1/2)*exp(-a)*erfc((b*x)^(1/2)))/(2*(b*x)^(1/2)) - (
a*exp(-a)*erfc((b*x)^(1/2))*(b*x*pi)^(1/2))/(b*x^(1/2))
```

Reduce [F]

$$\int \frac{\Gamma(2,a+bx)}{\sqrt{x}}\,dx = \frac{2e^{bx}\left(\int \frac{\sqrt{x}}{e^{bx}x}dx\right)a + 3e^{bx}\left(\int \frac{\sqrt{x}}{e^{bx}x}dx\right) - 2\sqrt{x}}{2e^{bx+a}}$$

input
```
int(exp(-b*x-a)*(b*x+a+1)/x^(1/2),x)
```

output
```
(2*e**(b*x)*int(sqrt(x)/(e**(b*x)*x),x)*a + 3*e**(b*x)*int(sqrt(x)/(e**(b*
x)*x),x) - 2*sqrt(x))/(2*e**(a + b*x))
```

3.165 $\int \frac{\Gamma(2,a+bx)}{x^{3/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{\Gamma(2,a+bx)}{x^{3/2}} dx = \frac{2ae^{-a}\sqrt{bx}\Gamma\left(\frac{1}{2},bx\right)}{\sqrt{x}} + \frac{2be^{-a}\sqrt{x}\Gamma\left(\frac{3}{2},bx\right)}{\sqrt{bx}} - \frac{2\Gamma(2,a+bx)}{\sqrt{x}}$$

output
```
2*a*(b*x)^(1/2)*Pi^(1/2)*erfc((b*x)^(1/2))/exp(a)/x^(1/2)+2*b*x^(1/2)*((b*
x)^(1/2)*exp(-b*x)+1/2*Pi^(1/2)*erfc((b*x)^(1/2)))/exp(a)/(b*x)^(1/2)-2*ex
p(-b*x-a)*(b*x+a+1)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{\Gamma(2,a+bx)}{x^{3/2}} dx = -\left((1+2a)\sqrt{b}e^{-a}\sqrt{\pi}\text{erf}\left(\sqrt{b}\sqrt{x}\right)\right) + \frac{2\left(be^{-a-bx}x - \Gamma(2,a+bx)\right)}{\sqrt{x}}$$

input
```
Integrate[Gamma[2, a + b*x]/x^(3/2),x]
```

output
```
-(((1 + 2*a)*Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/E^a) + (2*(b*E^(-a - b
*x)*x - Gamma[2, a + b*x]))/Sqrt[x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.39, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,a+bx)}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{7119}$$

$$-2b\int \frac{e^{-a-bx}(a+bx)}{\sqrt{x}}dx-\frac{2\Gamma(2,a+bx)}{\sqrt{x}}$$

$$\Big\downarrow \text{2629}$$

$$-2b\int \left(\frac{e^{-a-bx}a}{\sqrt{x}}+be^{-a-bx}\sqrt{x}\right)dx-\frac{2\Gamma(2,a+bx)}{\sqrt{x}}$$

$$\Big\downarrow \text{2009}$$

$$-2b\left(\frac{\sqrt{\pi}ae^{-a}\text{erf}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}+\frac{\sqrt{\pi}e^{-a}\text{erf}\left(\sqrt{b}\sqrt{x}\right)}{2\sqrt{b}}-\sqrt{x}e^{-a-bx}\right)-\frac{2\Gamma(2,a+bx)}{\sqrt{x}}$$

input
```
Int[Gamma[2, a + b*x]/x^(3/2),x]
```

output
```
-2*b*(-(E^(-a - b*x)*Sqrt[x]) + (Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/(2*Sqrt[b]
*E^a) + (a*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/(Sqrt[b]*E^a)) - (2*Gamma[2, a +
 b*x])/Sqrt[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\sqrt{b}\,\mathrm{e}^{-a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)+2\,\mathrm{e}^{-a}\left(-\frac{\mathrm{e}^{-bx}}{\sqrt{x}}-\sqrt{b}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)\right)+2\,\mathrm{e}^{-a}a\left(-\frac{\mathrm{e}^{-bx}}{\sqrt{x}}-\sqrt{}\right.$
default	$\sqrt{b}\,\mathrm{e}^{-a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)+2\,\mathrm{e}^{-a}\left(-\frac{\mathrm{e}^{-bx}}{\sqrt{x}}-\sqrt{b}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)\right)+2\,\mathrm{e}^{-a}a\left(-\frac{\mathrm{e}^{-bx}}{\sqrt{x}}-\sqrt{}\right.$
meijerg	$\sqrt{b}\,\mathrm{e}^{-a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)+\mathrm{e}^{-a}a\sqrt{b}\left(-\frac{2\,\mathrm{e}^{-bx}}{\sqrt{x}\,\sqrt{b}}-2\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,\sqrt{x}\right)\right)+\sqrt{b}\,\mathrm{e}^{-a}\left(-\frac{2\,\mathrm{e}^{-bx}}{\sqrt{x}\,\sqrt{b}}\right.$

input
```
int(exp(-b*x-a)*(b*x+a+1)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/exp(a)*b^(1/2)*Pi^(1/2)*erf(b^(1/2)*x^(1/2))+2/exp(a)*(-1/x^(1/2)*exp(-b
*x)-b^(1/2)*Pi^(1/2)*erf(b^(1/2)*x^(1/2)))+2/exp(a)*a*(-1/x^(1/2)*exp(-b*x
)-b^(1/2)*Pi^(1/2)*erf(b^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{\Gamma(2, a+bx)}{x^{3/2}}\, dx = -\frac{\sqrt{\pi}(2\,a+1)\sqrt{b}x\,\mathrm{erf}\left(\sqrt{b}\sqrt{x}\right)e^{(-a)} - 2\left(bxe^{(-bx-a)} - \Gamma(2, bx+a)\right)\sqrt{x}}{x}$$

input
```
integrate(gamma(2,b*x+a)/x^(3/2),x, algorithm="fricas")
```

output
```
-(sqrt(pi)*(2*a + 1)*sqrt(b)*x*erf(sqrt(b)*sqrt(x))*e^(-a) - 2*(b*x*e^(-b*
x - a) - gamma(2, b*x + a))*sqrt(x))/x
```

Sympy [A] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.68

$$\int \frac{\Gamma(2, a+bx)}{x^{3/2}}\, dx = \left(-\frac{a(bx)^{\frac{3}{2}}\left(-2\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{bx}\right) + \frac{2e^{-bx}}{\sqrt{bx}}\right)}{bx^{\frac{3}{2}}} - \frac{\sqrt{\pi}\sqrt{bx}\,\mathrm{erfc}\left(\sqrt{bx}\right)}{\sqrt{x}} - \frac{(bx)^{\frac{3}{2}}\left(-2\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{bx}\right) + \frac{2e^{-bx}}{\sqrt{bx}}\right)}{bx^{\frac{3}{2}}}\right)e^{-a}$$

input
```
integrate(uppergamma(2,b*x+a)/x**(3/2),x)
```

output
```
(-a*(b*x)**(3/2)*(-2*sqrt(pi)*erfc(sqrt(b*x)) + 2*exp(-b*x)/sqrt(b*x))/(b*
x**(3/2)) - sqrt(pi)*sqrt(b*x)*erfc(sqrt(b*x))/sqrt(x) - (b*x)**(3/2)*(-2*
sqrt(pi)*erfc(sqrt(b*x)) + 2*exp(-b*x)/sqrt(b*x))/(b*x**(3/2)))*exp(-a)
```

Maxima [F]

$$\int \frac{\Gamma(2, a+bx)}{x^{3/2}}\, dx = \int \frac{\Gamma(2, bx+a)}{x^{\frac{3}{2}}}\, dx$$

input
```
integrate(gamma(2,b*x+a)/x^(3/2),x, algorithm="maxima")
```

output
```
integrate(gamma(2, b*x + a)/x^(3/2), x)
```

Giac [F]

$$\int \frac{\Gamma(2, a+bx)}{x^{3/2}}\, dx = \int \frac{\Gamma(2, bx+a)}{x^{\frac{3}{2}}}\, dx$$

input
```
integrate(gamma(2,b*x+a)/x^(3/2),x, algorithm="giac")
```

output
```
integrate(gamma(2, b*x + a)/x^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.42

$$\int \frac{\Gamma(2, a+bx)}{x^{3/2}}\, dx = \frac{2\sqrt{\pi}\, e^{-a}\, \mathrm{erfc}\left(\sqrt{b\,x}\right)\,\sqrt{b\,x}}{\sqrt{x}} - \frac{2\,a\, e^{-a-b\,x}}{\sqrt{x}} - \frac{e^{-a}\, \mathrm{erfc}\left(\sqrt{b\,x}\right)\,\sqrt{\pi\, b\,x}}{\sqrt{x}} - \frac{2\, e^{-a-b\,x}}{\sqrt{x}} + \frac{2\,a\,\sqrt{\pi}\, e^{-a}\, \mathrm{erfc}\left(\sqrt{b\,x}\right)\,\sqrt{b\,x}}{\sqrt{x}}$$

input
```
int((exp(- a - b*x)*(a + b*x + 1))/x^(3/2),x)
```

output
```
(2*pi^(1/2)*exp(-a)*erfc((b*x)^(1/2))*(b*x)^(1/2))/x^(1/2) - (2*a*exp(- a
- b*x))/x^(1/2) - (exp(-a)*erfc((b*x)^(1/2))*(b*x*pi)^(1/2))/x^(1/2) - (2*
exp(- a - b*x))/x^(1/2) + (2*a*pi^(1/2)*exp(-a)*erfc((b*x)^(1/2))*(b*x)^(1
/2))/x^(1/2)
```

Reduce [F]

$$\int \frac{\Gamma(2, a+bx)}{x^{3/2}}\,dx = \frac{2\sqrt{x}\,e^{bx}\left(\int \frac{\sqrt{x}}{e^{bx}x^2}dx\right)a + \sqrt{x}\,e^{bx}\left(\int \frac{\sqrt{x}}{e^{bx}x^2}dx\right) - 2}{2\sqrt{x}\,e^{bx+a}}$$

input
```
int(exp(-b*x-a)*(b*x+a+1)/x^(3/2),x)
```

output
```
(2*sqrt(x)*e**(b*x)*int(sqrt(x)/(e**(b*x)*x**2),x)*a + sqrt(x)*e**(b*x)*in
t(sqrt(x)/(e**(b*x)*x**2),x) - 2)/(2*sqrt(x)*e**(a + b*x))
```

3.166 $\int \frac{\Gamma(2,a+bx)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 76

$$\int \frac{\Gamma(2,a+bx)}{x^{5/2}}\,dx = \frac{2abe^{-a}\sqrt{bx}\,\Gamma\left(-\frac{1}{2},bx\right)}{3\sqrt{x}} + \frac{2be^{-a}\sqrt{bx}\,\Gamma\left(\frac{1}{2},bx\right)}{3\sqrt{x}} - \frac{2\Gamma(2,a+bx)}{3x^{3/2}}$$

output
```
2/3*a*b*(b*x)^(1/2)*(-2*Pi^(1/2)*erfc((b*x)^(1/2))+2/(b*x)^(1/2)*exp(-b*x)
)/exp(a)/x^(1/2)+2/3*b*(b*x)^(1/2)*Pi^(1/2)*erfc((b*x)^(1/2))/exp(a)/x^(1/
2)-2/3*exp(-b*x-a)*(b*x+a+1)/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \frac{\Gamma(2,a+bx)}{x^{5/2}}\,dx = \frac{2}{3}\left((-1+2a)b^{3/2}e^{-a}\sqrt{\pi}\,\text{erf}\left(\sqrt{b}\sqrt{x}\right) + \frac{2abe^{-a-bx}x - \Gamma(2,a+bx)}{x^{3/2}}\right)$$

input
```
Integrate[Gamma[2, a + b*x]/x^(5/2),x]
```

output
```
(2*(((-1 + 2*a)*b^(3/2)*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/E^a + (2*a*b*E^(-a
- b*x)*x - Gamma[2, a + b*x])/x^(3/2)))/3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,a+bx)}{x^{5/2}}\,dx$$

$$\Big\downarrow \; 7119$$

$$-\frac{2}{3}b\int \frac{e^{-a-bx}(a+bx)}{x^{3/2}}dx-\frac{2\Gamma(2,a+bx)}{3x^{3/2}}$$

$$\Big\downarrow \; 2629$$

$$-\frac{2}{3}b\int \left(\frac{e^{-a-bx}a}{x^{3/2}}+\frac{be^{-a-bx}}{\sqrt{x}}\right)dx-\frac{2\Gamma(2,a+bx)}{3x^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2}{3}b\left(-2\sqrt{\pi}e^{-a}a\sqrt{b}\text{erf}\left(\sqrt{b}\sqrt{x}\right)+\sqrt{\pi}e^{-a}\sqrt{b}\text{erf}\left(\sqrt{b}\sqrt{x}\right)-\frac{2ae^{-a-bx}}{\sqrt{x}}\right)-\frac{2\Gamma(2,a+bx)}{3x^{3/2}}$$

input
```
Int[Gamma[2, a + b*x]/x^(5/2),x]
```

output
```
(-2*b*((-2*a*E^(-a - b*x))/Sqrt[x] + (Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]
])/E^a - (2*a*Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/E^a))/3 - (2*Gamma[2,
 a + b*x])/(3*x^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.58

method	result
meijerg	$b^{\frac{3}{2}} e^{-a} \left(-\frac{2 e^{-b x}}{\sqrt{x}\, \sqrt{b}}-2 \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{b}\, \sqrt{x}\right)\right)+e^{-a} a\, b^{\frac{3}{2}} \left(-\frac{2 (-2 b x+1) e^{-b x}}{3 x^{\frac{3}{2}} b^{\frac{3}{2}}}+\frac{4 \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{b}\, \sqrt{x}\right)}{3}\right)+b$
derivativedivides	$2 e^{-a} \left(-\frac{e^{-b x}}{3 x^{\frac{3}{2}}}-\frac{2 b \left(-\frac{e^{-b x}}{\sqrt{x}}-\sqrt{b}\, \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{b}\, \sqrt{x}\right)\right)}{3}\right)+2 e^{-a} a \left(-\frac{e^{-b x}}{3 x^{\frac{3}{2}}}-\frac{2 b (-\frac{e^{-b x}}{\sqrt{x}}-\sqrt{b}\, \sqrt{\pi}\, \operatorname{erf}(\sqrt{b}\, \sqrt{x}}{3}$
default	$2 e^{-a} \left(-\frac{e^{-b x}}{3 x^{\frac{3}{2}}}-\frac{2 b \left(-\frac{e^{-b x}}{\sqrt{x}}-\sqrt{b}\, \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{b}\, \sqrt{x}\right)\right)}{3}\right)+2 e^{-a} a \left(-\frac{e^{-b x}}{3 x^{\frac{3}{2}}}-\frac{2 b (-\frac{e^{-b x}}{\sqrt{x}}-\sqrt{b}\, \sqrt{\pi}\, \operatorname{erf}(\sqrt{b}\, \sqrt{x}}{3}$

input
```
int(exp(-b*x-a)*(b*x+a+1)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
b^(3/2)*exp(-a)*(-2/x^(1/2)/b^(1/2)*exp(-b*x)-2*Pi^(1/2)*erf(b^(1/2)*x^(1/
2)))+exp(-a)*a*b^(3/2)*(-2/3/x^(3/2)/b^(3/2)*(-2*b*x+1)*exp(-b*x)+4/3*Pi^(
1/2)*erf(b^(1/2)*x^(1/2)))+b^(3/2)*exp(-a)*(-2/3/x^(3/2)/b^(3/2)*(-2*b*x+1
)*exp(-b*x)+4/3*Pi^(1/2)*erf(b^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int \frac{\Gamma(2,a+bx)}{x^{5/2}}\,dx = \frac{2\left(\sqrt{\pi}(2\,a-1)b^{\frac{3}{2}}x^2\,\mathrm{erf}\left(\sqrt{b}\sqrt{x}\right)e^{(-a)} + \left(2\,abxe^{(-bx-a)} - \Gamma(2,bx+a)\right)\sqrt{x}\right)}{3\,x^2}$$

input
```
integrate(gamma(2,b*x+a)/x^(5/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(pi)*(2*a - 1)*b^(3/2)*x^2*erf(sqrt(b)*sqrt(x))*e^(-a) + (2*a*b*x
*e^(-b*x - a) - gamma(2, b*x + a))*sqrt(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 9.99 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.99

$$\int \frac{\Gamma(2,a+bx)}{x^{5/2}}\,dx = \left(-\frac{a(bx)^{\frac{5}{2}}\cdot\left(\frac{4\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{bx}\right)}{3} - \frac{\left(\frac{4bx}{3}-\frac{2}{3}\right)e^{-bx}}{(bx)^{\frac{3}{2}}}\right)}{bx^{\frac{5}{2}}} - \frac{(bx)^{\frac{3}{2}}\left(-2\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{bx}\right) + \frac{2e^{-bx}}{\sqrt{bx}}\right)}{x^{\frac{3}{2}}} - \frac{(bx)^{\frac{5}{2}}\cdot\left(\frac{4\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{bx}\right)}{3} - \frac{\left(\frac{4bx}{3}-\frac{2}{3}\right)e^{-bx}}{(bx)^{\frac{3}{2}}}\right)}{bx^{\frac{5}{2}}}\right)e^{-a}$$

input
```
integrate(uppergamma(2,b*x+a)/x**(5/2),x)
```

output
```
(-a*(b*x)**(5/2)*(4*sqrt(pi)*erfc(sqrt(b*x))/3 - (4*b*x/3 - 2/3)*exp(-b*x)
/(b*x)**(3/2))/(b*x**(5/2)) - (b*x)**(3/2)*(-2*sqrt(pi)*erfc(sqrt(b*x)) +
2*exp(-b*x)/sqrt(b*x))/x**(3/2) - (b*x)**(5/2)*(4*sqrt(pi)*erfc(sqrt(b*x))
/3 - (4*b*x/3 - 2/3)*exp(-b*x)/(b*x)**(3/2))/(b*x**(5/2)))*exp(-a)
```

Maxima [F]

$$\int \frac{\Gamma(2,a+bx)}{x^{5/2}}\,dx=\int \frac{\Gamma(2,bx+a)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate(gamma(2,b*x+a)/x^(5/2),x, algorithm="maxima")
```

output
```
integrate(gamma(2, b*x + a)/x^(5/2), x)
```

Giac [F]

$$\int \frac{\Gamma(2,a+bx)}{x^{5/2}}\,dx=\int \frac{\Gamma(2,bx+a)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate(gamma(2,b*x+a)/x^(5/2),x, algorithm="giac")
```

output
```
integrate(gamma(2, b*x + a)/x^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.91

$$\int \frac{\Gamma(2,a+bx)}{x^{5/2}}\,dx=\frac{2\,b\,\sqrt{\pi}\,\mathrm{e}^{-a}\,\mathrm{erfc}\left(\sqrt{b\,x}\right)\,\sqrt{b\,x}}{\sqrt{x}} - \frac{\mathrm{e}^{-a}\,{(b\,x)}^{5/2}\,\left(\frac{4\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{b\,x}\right)}{3}+\mathrm{e}^{-b\,x}\,\left(\frac{2}{3\,{(b\,x)}^{3/2}}-\frac{4\,b\,x}{3\,{(b\,x)}^{3/2}}\right)\right)}{b\,x^{5/2}} - \frac{2\,b\,\mathrm{e}^{-a-b\,x}}{\sqrt{x}} - \frac{a\,\mathrm{e}^{-a}\,{(b\,x)}^{5/2}\,\left(\frac{4\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{b\,x}\right)}{3}+\mathrm{e}^{-b\,x}\,\left(\frac{2}{3\,{(b\,x)}^{3/2}}-\frac{4\,b\,x}{3\,{(b\,x)}^{3/2}}\right)\right)}{b\,x^{5/2}}$$

input
```
int((exp(- a - b*x)*(a + b*x + 1))/x^(5/2),x)
```

output
```
(2*b*pi^(1/2)*exp(-a)*erfc((b*x)^(1/2))*(b*x)^(1/2))/x^(1/2) - (exp(-a)*(b
*x)^(5/2)*((4*pi^(1/2)*erfc((b*x)^(1/2)))/3 + exp(-b*x)*(2/(3*(b*x)^(3/2))
 - (4*b*x)/(3*(b*x)^(3/2)))))/(b*x^(5/2)) - (2*b*exp(- a - b*x))/x^(1/2) -
 (a*exp(-a)*(b*x)^(5/2)*((4*pi^(1/2)*erfc((b*x)^(1/2)))/3 + exp(-b*x)*(2/(
3*(b*x)^(3/2)) - (4*b*x)/(3*(b*x)^(3/2)))))/(b*x^(5/2))
```

Reduce [F]

$$\int \frac{\Gamma(2, a+bx)}{x^{5/2}}\,dx = \frac{-2\sqrt{x}\,e^{bx}\left(\int \frac{\sqrt{x}}{e^{bx}x^2}dx\right)abx + \sqrt{x}\,e^{bx}\left(\int \frac{\sqrt{x}}{e^{bx}x^2}dx\right)bx - 2a - 2}{3\sqrt{x}\,e^{bx+a}x}$$

input
```
int(exp(-b*x-a)*(b*x+a+1)/x^(5/2),x)
```

output
```
( - 2*sqrt(x)*e**(b*x)*int(sqrt(x)/(e**(b*x)*x**2),x)*a*b*x + sqrt(x)*e**(
b*x)*int(sqrt(x)/(e**(b*x)*x**2),x)*b*x - 2*a - 2)/(3*sqrt(x)*e**(a + b*x)
*x)
```

3.167 $\int \frac{\Gamma(2,a+bx)}{x^{7/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{\Gamma(2,a+bx)}{x^{7/2}}\,dx = \frac{2ab^2e^{-a}\sqrt{bx}\,\Gamma\left(-\frac{3}{2},bx\right)}{5\sqrt{x}} + \frac{2b^2e^{-a}\sqrt{bx}\,\Gamma\left(-\frac{1}{2},bx\right)}{5\sqrt{x}} - \frac{2\Gamma(2,a+bx)}{5x^{5/2}}$$

output
```
2/5*a*b^2*(b*x)^(1/2)*(4/3*Pi^(1/2)*erfc((b*x)^(1/2))-4/3/(b*x)^(1/2)*exp(
-b*x)+2/3/(b*x)^(3/2)*exp(-b*x))/exp(a)/x^(1/2)+2/5*b^2*(b*x)^(1/2)*(-2*Pi
^(1/2)*erfc((b*x)^(1/2))+2/(b*x)^(1/2)*exp(-b*x))/exp(a)/x^(1/2)-2/5*exp(-
b*x-a)*(b*x+a+1)/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(2,a+bx)}{x^{7/2}}\,dx = \frac{2}{15}\left(-2(-3+2a)b^{5/2}e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\sqrt{x}\right) + \frac{2be^{-a-bx}x(a+3bx-2abx)-3\Gamma(2,a+bx)}{x^{5/2}}\right.$$

input
```
Integrate[Gamma[2, a + b*x]/x^(7/2),x]
```

output
```
(2*((-2*(-3 + 2*a)*b^(5/2)*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/E^a + (2*b*E^(-a
 - b*x)*x*(a + 3*b*x - 2*a*b*x) - 3*Gamma[2, a + b*x])/x^(5/2)))/15
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.79, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,a+bx)}{x^{7/2}}dx$$

$$\Big\downarrow \text{7119}$$

$$-\frac{2}{5}b\int \frac{e^{-a-bx}(a+bx)}{x^{5/2}}dx-\frac{2\Gamma(2,a+bx)}{5x^{5/2}}$$

$$\Big\downarrow \text{2629}$$

$$-\frac{2}{5}b\int \left(\frac{e^{-a-bx}a}{x^{5/2}}+\frac{be^{-a-bx}}{x^{3/2}}\right)dx-\frac{2\Gamma(2,a+bx)}{5x^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{5}b\left(\frac{4}{3}\sqrt{\pi}ae^{-a}b^{3/2}\text{erf}\left(\sqrt{b}\sqrt{x}\right)-2\sqrt{\pi}e^{-a}b^{3/2}\text{erf}\left(\sqrt{b}\sqrt{x}\right)-\frac{2ae^{-a-bx}}{3x^{3/2}}+\frac{4abe^{-a-bx}}{3\sqrt{x}}-\frac{2be^{-a-bx}}{\sqrt{x}}\right)-\frac{2\Gamma(2,a+bx)}{5x^{5/2}}$$

input
```
Int[Gamma[2, a + b*x]/x^(7/2),x]
```

output
```
(-2*b*((-2*a*E^(-a - b*x))/(3*x^(3/2)) - (2*b*E^(-a - b*x))/Sqrt[x] + (4*a
*b*E^(-a - b*x))/(3*Sqrt[x]) - (2*b^(3/2)*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/E
^a + (4*a*b^(3/2)*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/(3*E^a)))/5 - (2*Gamma[2,
 a + b*x])/(5*x^(5/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.78

method	result
meijerg	$b^{\frac{5}{2}} \mathrm{e}^{-a} \left(-\frac{2(-2bx+1)\mathrm{e}^{-bx}}{3x^{\frac{3}{2}} b^{\frac{3}{2}}} + \frac{4\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\sqrt{x}\right)}{3}\right) + \mathrm{e}^{-a} a\, b^{\frac{5}{2}} \left(-\frac{2\left(\frac{4}{3}b^2x^2-\frac{2}{3}bx+1\right)\mathrm{e}^{-bx}}{5x^{\frac{5}{2}} b^{\frac{5}{2}}} - \frac{8\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\sqrt{\right.}\right.}{15}\right.$
derivativedivides	$2\,\mathrm{e}^{-a}\left(-\frac{\mathrm{e}^{-bx}}{5x^{\frac{5}{2}}} - \frac{2b\left(-\frac{\mathrm{e}^{-bx}}{3x^{\frac{3}{2}}} - \frac{2b\left(-\frac{\mathrm{e}^{-bx}}{\sqrt{x}} - \sqrt{b}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\sqrt{x}\right)\right)}{3}\right)}{5}\right) + 2\,\mathrm{e}^{-a} a \left(-\frac{\mathrm{e}^{-bx}}{5x^{\frac{5}{2}}} - \frac{2b\left(-\frac{\mathrm{e}^{-bx}}{3x^{\frac{3}{2}}} - \frac{2b(}{}\right.}{}\right.$
default	$2\,\mathrm{e}^{-a}\left(-\frac{\mathrm{e}^{-bx}}{5x^{\frac{5}{2}}} - \frac{2b\left(-\frac{\mathrm{e}^{-bx}}{3x^{\frac{3}{2}}} - \frac{2b\left(-\frac{\mathrm{e}^{-bx}}{\sqrt{x}} - \sqrt{b}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\sqrt{x}\right)\right)}{3}\right)}{5}\right) + 2\,\mathrm{e}^{-a} a \left(-\frac{\mathrm{e}^{-bx}}{5x^{\frac{5}{2}}} - \frac{2b\left(-\frac{\mathrm{e}^{-bx}}{3x^{\frac{3}{2}}} - \frac{2b(}{}\right.}{}\right.$

input
```
int(exp(-b*x-a)*(b*x+a+1)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
b^(5/2)*exp(-a)*(-2/3/x^(3/2)/b^(3/2)*(-2*b*x+1)*exp(-b*x)+4/3*Pi^(1/2)*er
f(b^(1/2)*x^(1/2)))+exp(-a)*a*b^(5/2)*(-2/5/x^(5/2)/b^(5/2)*(4/3*b^2*x^2-2
/3*b*x+1)*exp(-b*x)-8/15*Pi^(1/2)*erf(b^(1/2)*x^(1/2)))+b^(5/2)*exp(-a)*(-
2/5/x^(5/2)/b^(5/2)*(4/3*b^2*x^2-2/3*b*x+1)*exp(-b*x)-8/15*Pi^(1/2)*erf(b^
(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{\Gamma(2,a+bx)}{x^{7/2}}\,dx = -\frac{2\left(2\sqrt{\pi}(2a-3)b^{\frac{5}{2}}x^3\operatorname{erf}\left(\sqrt{b}\sqrt{x}\right)e^{(-a)}+\left(2\left((2a-3)b^2x^2-abx\right)e^{(-bx-a)}+3\Gamma(2,bx+a)\right)\sqrt{x}\right)}{15x^3}$$

input
```
integrate(gamma(2,b*x+a)/x^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(2*sqrt(pi)*(2*a - 3)*b^(5/2)*x^3*erf(sqrt(b)*sqrt(x))*e^(-a) + (2*(
(2*a - 3)*b^2*x^2 - a*b*x)*e^(-b*x - a) + 3*gamma(2, b*x + a))*sqrt(x))/x^
3
```

Sympy [A] (verification not implemented)

Time = 102.34 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.31

$$\int \frac{\Gamma(2,a+bx)}{x^{7/2}}\,dx = \left(-\frac{a(bx)^{\frac{7}{2}}\left(-\frac{8\sqrt{\pi}\operatorname{erfc}\left(\sqrt{bx}\right)}{15}-\frac{\left(-\frac{8b^2x^2}{15}+\frac{4bx}{15}-\frac{2}{5}\right)e^{-bx}}{(bx)^{\frac{5}{2}}}\right)}{bx^{\frac{7}{2}}} - \frac{(bx)^{\frac{5}{2}}\cdot\left(\frac{4\sqrt{\pi}\operatorname{erfc}\left(\sqrt{bx}\right)}{3}-\frac{\left(\frac{4bx}{3}-\frac{2}{3}\right)e^{-bx}}{(bx)^{\frac{3}{2}}}\right)}{x^{\frac{5}{2}}} - \frac{(bx)^{\frac{7}{2}}\left(-\frac{8\sqrt{\pi}\operatorname{erfc}\left(\sqrt{bx}\right)}{15}-\frac{\left(-\frac{8b^2x^2}{15}+\frac{4bx}{15}-\frac{2}{5}\right)e^{-bx}}{(bx)^{\frac{5}{2}}}\right)}{bx^{\frac{7}{2}}}\right)e^{-a}$$

input
```
integrate(uppergamma(2,b*x+a)/x**(7/2),x)
```

output
```
(-a*(b*x)**(7/2)*(-8*sqrt(pi)*erfc(sqrt(b*x))/15 - (-8*b**2*x**2/15 + 4*b*
x/15 - 2/5)*exp(-b*x)/(b*x)**(5/2))/(b*x**(7/2)) - (b*x)**(5/2)*(4*sqrt(pi
)*erfc(sqrt(b*x))/3 - (4*b*x/3 - 2/3)*exp(-b*x)/(b*x)**(3/2))/x**(5/2) - (
b*x)**(7/2)*(-8*sqrt(pi)*erfc(sqrt(b*x))/15 - (-8*b**2*x**2/15 + 4*b*x/15
- 2/5)*exp(-b*x)/(b*x)**(5/2))/(b*x**(7/2)))*exp(-a)
```

Maxima [F]

$$\int \frac{\Gamma(2, a+bx)}{x^{7/2}}\, dx = \int \frac{\Gamma(2, bx+a)}{x^{\frac{7}{2}}}\, dx$$

input
```
integrate(gamma(2,b*x+a)/x^(7/2),x, algorithm="maxima")
```

output
```
integrate(gamma(2, b*x + a)/x^(7/2), x)
```

Giac [F]

$$\int \frac{\Gamma(2, a+bx)}{x^{7/2}}\, dx = \int \frac{\Gamma(2, bx+a)}{x^{\frac{7}{2}}}\, dx$$

input
```
integrate(gamma(2,b*x+a)/x^(7/2),x, algorithm="giac")
```

output
```
integrate(gamma(2, b*x + a)/x^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.55

$$\int \frac{\Gamma(2,a+bx)}{x^{7/2}}\,dx =$$

$$-\frac{2\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}\left(3\,b\,x+3\,b^2\,x^2-6\,b^3\,x^3-2\,a\,b^2\,x^2+4\,a\,b^3\,x^3+3\,a\,b\,x-4\,\sqrt{\pi}\,\mathrm{e}^{b\,x}\,\mathrm{erfc}\left(\sqrt{b\,x}\right)\,{\left(b\,x\right)}^{7/2}-4\,a\,\sqrt{}\right.}{15\,b\,x^{7/2}}$$

input
```
int((exp(- a - b*x)*(a + b*x + 1))/x^(7/2),x)
```

output
```
-(2*exp(-a)*exp(-b*x)*(3*b*x + 3*b^2*x^2 - 6*b^3*x^3 - 2*a*b^2*x^2 + 4*a*b
^3*x^3 + 3*a*b*x - 4*pi^(1/2)*exp(b*x)*erfc((b*x)^(1/2))*(b*x)^(7/2) - 4*a
*pi^(1/2)*exp(b*x)*erfc((b*x)^(1/2))*(b*x)^(7/2) + 10*b*x*pi^(1/2)*exp(b*x
)*erfc((b*x)^(1/2))*(b*x)^(5/2)))/(15*b*x^(7/2))
```

Reduce [F]

$$\int \frac{\Gamma(2,a+bx)}{x^{7/2}}\,dx = \frac{\frac{4\sqrt{x}\,e^{bx}\left(\int \frac{\sqrt{x}}{e^{bx}x^2}dx\right)a\,b^2x^2}{15}-\frac{2\sqrt{x}\,e^{bx}\left(\int \frac{\sqrt{x}}{e^{bx}x^2}dx\right)b^2x^2}{5}+\frac{4abx}{15}-\frac{2a}{5}-\frac{2bx}{5}-\frac{2}{5}}{\sqrt{x}\,e^{bx+a}x^2}$$

input
```
int(exp(-b*x-a)*(b*x+a+1)/x^(7/2),x)
```

output
```
(2*(2*sqrt(x)*e**(b*x)*int(sqrt(x)/(e**(b*x)*x**2),x)*a*b**2*x**2 - 3*sqrt
(x)*e**(b*x)*int(sqrt(x)/(e**(b*x)*x**2),x)*b**2*x**2 + 2*a*b*x - 3*a - 3*
b*x - 3))/(15*sqrt(x)*e**(a + b*x)*x**2)
```

3.168 $\int \frac{\Gamma(2,a+bx)}{x^{9/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{\Gamma(2,a+bx)}{x^{9/2}} \, dx = \frac{2ab^3e^{-a}\sqrt{bx}\Gamma\left(-\frac{5}{2},bx\right)}{7\sqrt{x}} + \frac{2b^3e^{-a}\sqrt{bx}\Gamma\left(-\frac{3}{2},bx\right)}{7\sqrt{x}} - \frac{2\Gamma(2,a+bx)}{7x^{7/2}}$$

output
```
2/7*a*b^3*(b*x)^(1/2)*(-8/15*Pi^(1/2)*erfc((b*x)^(1/2))+8/15/(b*x)^(1/2)*e
xp(-b*x)-4/15/(b*x)^(3/2)*exp(-b*x)+2/5/(b*x)^(5/2)*exp(-b*x))/exp(a)/x^(1
/2)+2/7*b^3*(b*x)^(1/2)*(4/3*Pi^(1/2)*erfc((b*x)^(1/2))-4/3/(b*x)^(1/2)*ex
p(-b*x)+2/3/(b*x)^(3/2)*exp(-b*x))/exp(a)/x^(1/2)-2/7*exp(-b*x-a)*(b*x+a+1
)/x^(7/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20

$$\int \frac{\Gamma(2,a+bx)}{x^{9/2}} \, dx = \frac{2}{105}\left(4(-5+2a)b^{7/2}e^{-a}\sqrt{\pi}\text{erf}\left(\sqrt{b}\sqrt{x}\right) + \frac{2be^{-a-bx}x(5bx(1-2bx)+a(3-2bx+4b^2x}{x^{7/2}}\right.$$

input
```
Integrate[Gamma[2, a + b*x]/x^(9/2),x]
```

output
```
(2*((4*(-5 + 2*a)*b^(7/2)*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/E^a + (2*b*E^(-a
- b*x)*x*(5*b*x*(1 - 2*b*x) + a*(3 - 2*b*x + 4*b^2*x^2)) - 15*Gamma[2, a +
 b*x])/x^(7/2)))/105
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 192 vs. $2(80) = 160$.

Time = 0.51 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.40, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,a+bx)}{x^{9/2}}\,dx$$

$$\Big\downarrow \text{7119}$$

$$-\frac{2}{7}b\int \frac{e^{-a-bx}(a+bx)}{x^{7/2}}dx-\frac{2\Gamma(2,a+bx)}{7x^{7/2}}$$

$$\Big\downarrow \text{2629}$$

$$-\frac{2}{7}b\int \left(\frac{e^{-a-bx}a}{x^{7/2}}+\frac{be^{-a-bx}}{x^{5/2}}\right)dx-\frac{2\Gamma(2,a+bx)}{7x^{7/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{7}b\left(-\frac{8}{15}\sqrt{\pi}ae^{-a}b^{5/2}\text{erf}\left(\sqrt{b}\sqrt{x}\right)+\frac{4}{3}\sqrt{\pi}e^{-a}b^{5/2}\text{erf}\left(\sqrt{b}\sqrt{x}\right)-\frac{8ab^2e^{-a-bx}}{15\sqrt{x}}+\frac{4b^2e^{-a-bx}}{3\sqrt{x}}+\frac{4abe^{-a-bx}}{15x^{3/2}}-\frac{2be^{-a}}{3x^{3}}\right.$$
$$\frac{2\Gamma(2,a+bx)}{7x^{7/2}}$$

input
```
Int[Gamma[2, a + b*x]/x^(9/2),x]
```

output
```
(-2*b*((-2*a*E^(-a - b*x))/(5*x^(5/2)) - (2*b*E^(-a - b*x))/(3*x^(3/2)) +
(4*a*b*E^(-a - b*x))/(15*x^(3/2)) + (4*b^2*E^(-a - b*x))/(3*Sqrt[x]) - (8*
a*b^2*E^(-a - b*x))/(15*Sqrt[x]) + (4*b^(5/2)*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]
])/(3*E^a) - (8*a*b^(5/2)*Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/(15*E^a)))/7 - (2
*Gamma[2, a + b*x])/(7*x^(7/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.08

method	result
meijerg	$b^{\frac{7}{2}} \mathrm{e}^{-a} \left(-\frac{2(\frac{4}{3}b^2x^2-\frac{2}{3}bx+1)\mathrm{e}^{-bx}}{5x^{\frac{5}{2}}b^{\frac{5}{2}}}-\frac{8\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\sqrt{x}\right)}{15}\right)+\mathrm{e}^{-a}a\,b^{\frac{7}{2}}\left(-\frac{2(-\frac{8}{15}b^3x^3+\frac{4}{15}b^2x^2-\frac{2}{5}bx+1)\mathrm{e}^{-bx}}{7x^{\frac{7}{2}}b^{\frac{7}{2}}}\right.$
derivativedivides	$2\,\mathrm{e}^{-a}\left(-\frac{\mathrm{e}^{-bx}}{7x^{\frac{7}{2}}}-\frac{2b\left(-\frac{\mathrm{e}^{-bx}}{5x^{\frac{5}{2}}}-\frac{2b\left(-\frac{\mathrm{e}^{-bx}}{3x^{\frac{3}{2}}}-\frac{2b\left(-\frac{\mathrm{e}^{-bx}}{\sqrt{x}}-\sqrt{b}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\sqrt{x}\right)\right)}{3}\right)}{5}\right)}{7}\right)+2\,\mathrm{e}^{-a}a\left(-\frac{\mathrm{e}^{-bx}}{7x^{\frac{7}{2}}}-\frac{2b}{}\right.$
default	$2\,\mathrm{e}^{-a}\left(-\frac{\mathrm{e}^{-bx}}{7x^{\frac{7}{2}}}-\frac{2b\left(-\frac{\mathrm{e}^{-bx}}{5x^{\frac{5}{2}}}-\frac{2b\left(-\frac{\mathrm{e}^{-bx}}{3x^{\frac{3}{2}}}-\frac{2b\left(-\frac{\mathrm{e}^{-bx}}{\sqrt{x}}-\sqrt{b}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\sqrt{x}\right)\right)}{3}\right)}{5}\right)}{7}\right)+2\,\mathrm{e}^{-a}a\left(-\frac{\mathrm{e}^{-bx}}{7x^{\frac{7}{2}}}-\frac{2b}{}\right.$

input
```
int(exp(-b*x-a)*(b*x+a+1)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
b^(7/2)*exp(-a)*(-2/5/x^(5/2)/b^(5/2)*(4/3*b^2*x^2-2/3*b*x+1)*exp(-b*x)-8/
15*Pi^(1/2)*erf(b^(1/2)*x^(1/2)))+exp(-a)*a*b^(7/2)*(-2/7/x^(7/2)/b^(7/2)*
(-8/15*b^3*x^3+4/15*b^2*x^2-2/5*b*x+1)*exp(-b*x)+16/105*Pi^(1/2)*erf(b^(1/
2)*x^(1/2)))+b^(7/2)*exp(-a)*(-2/7/x^(7/2)/b^(7/2)*(-8/15*b^3*x^3+4/15*b^2
*x^2-2/5*b*x+1)*exp(-b*x)+16/105*Pi^(1/2)*erf(b^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.14

$$\int \frac{\Gamma(2, a+bx)}{x^{9/2}}\,dx = \frac{2\left(4\sqrt{\pi}(2a-5)b^{\frac{7}{2}}x^4\,\mathrm{erf}\left(\sqrt{b}\sqrt{x}\right)e^{(-a)}+\left(2\left(2\left(2a-5\right)b^3x^3-(2a-5)b^2x^2+3\,abx\right.\right.\right.}{105\,x^4}$$

input
```
integrate(gamma(2,b*x+a)/x^(9/2),x, algorithm="fricas")
```

output
```
2/105*(4*sqrt(pi)*(2*a - 5)*b^(7/2)*x^4*erf(sqrt(b)*sqrt(x))*e^(-a) + (2*(
2*(2*a - 5)*b^3*x^3 - (2*a - 5)*b^2*x^2 + 3*a*b*x)*e^(-b*x - a) - 15*gamma
(2, b*x + a))*sqrt(x))/x^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\Gamma(2, a + bx)}{x^{9/2}}\, dx = \text{Timed out}$$

input
```
integrate(uppergamma(2,b*x+a)/x**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\Gamma(2, a + bx)}{x^{9/2}}\, dx = \int \frac{\Gamma(2, bx + a)}{x^{\frac{9}{2}}}\, dx$$

input
```
integrate(gamma(2,b*x+a)/x^(9/2),x, algorithm="maxima")
```

output
```
integrate(gamma(2, b*x + a)/x^(9/2), x)
```

Giac [F]

$$\int \frac{\Gamma(2, a + bx)}{x^{9/2}}\, dx = \int \frac{\Gamma(2, bx + a)}{x^{\frac{9}{2}}}\, dx$$

input
```
integrate(gamma(2,b*x+a)/x^(9/2),x, algorithm="giac")
```

output
```
integrate(gamma(2, b*x + a)/x^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.42

$$\int \frac{\Gamma(2,a+bx)}{x^{9/2}}\,dx =$$
$$-\frac{30\,b^2\,x^2\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}-20\,b^3\,x^3\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}+40\,b^4\,x^4\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}+30\,b\,x\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}-12\,a\,b^2\,x^2\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}+8\,a\,b^3\,x^3\,\mathrm{e}}{}$$

input
```
int((exp(- a - b*x)*(a + b*x + 1))/x^(9/2),x)
```

output
```
-(30*b^2*x^2*exp(-a)*exp(-b*x) - 20*b^3*x^3*exp(-a)*exp(-b*x) + 40*b^4*x^4
*exp(-a)*exp(-b*x) + 30*b*x*exp(-a)*exp(-b*x) - 12*a*b^2*x^2*exp(-a)*exp(-
b*x) + 8*a*b^3*x^3*exp(-a)*exp(-b*x) - 16*a*b^4*x^4*exp(-a)*exp(-b*x) + 30
*a*b*x*exp(-a)*exp(-b*x) - 40*b^4*x^4*pi^(1/2)*exp(-a)*erfc((b*x)^(1/2))*(
b*x)^(1/2) + 16*a*b^4*x^4*pi^(1/2)*exp(-a)*erfc((b*x)^(1/2))*(b*x)^(1/2))/
(105*b*x^(9/2))
```

Reduce [F]

$$\int \frac{\Gamma(2,a+bx)}{x^{9/2}}\,dx = \frac{-\frac{8\sqrt{x}\,e^{bx}\left(\int \frac{1}{\sqrt{x}\,e^{bx}x}dx\right)a\,b^3x^3}{105}+\frac{4\sqrt{x}\,e^{bx}\left(\int \frac{1}{\sqrt{x}\,e^{bx}x}dx\right)b^3x^3}{21}-\frac{8a\,b^2x^2}{105}+\frac{4abx}{35}-\frac{2a}{7}+\frac{4b^2x^2}{21}-\frac{2bx}{7}-}{\sqrt{x}\,e^{bx+a}x^3}$$

input
```
int(exp(-b*x-a)*(b*x+a+1)/x^(9/2),x)
```

output
```
(2*( - 4*sqrt(x)*e**(b*x)*int(1/(sqrt(x)*e**(b*x)*x),x)*a*b**3*x**3 + 10*s
qrt(x)*e**(b*x)*int(1/(sqrt(x)*e**(b*x)*x),x)*b**3*x**3 - 4*a*b**2*x**2 +
6*a*b*x - 15*a + 10*b**2*x**2 - 15*b*x - 15))/(105*sqrt(x)*e**(a + b*x)*x**
*3)
```

3.169 $\int x^{3/2}\Gamma(-2,a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int x^{3/2}\Gamma(-2,a+bx)\,dx = \text{Int}\big(x^{3/2}\Gamma(-2,a+bx),x\big)$$

output
```
Defer(Int)(x^(3/2)/(b*x+a)^2*Ei(3,b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 4.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int x^{3/2}\Gamma(-2,a+bx)\,dx = \int x^{3/2}\Gamma(-2,a+bx)\,dx$$

input
```
Integrate[x^(3/2)*Gamma[-2, a + b*x],x]
```

output
```
Integrate[x^(3/2)*Gamma[-2, a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\Gamma(-2,a+bx)\,dx$$

$$\Big\downarrow \ 7120$$

$$\int x^{3/2}\Gamma(-2,a+bx)dx$$

input
```
Int[x^(3/2)*Gamma[-2, a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{x^{\frac{3}{2}} \operatorname{expIntegral}_{3}\left(bx+a\right)}{\left(bx+a\right)^{2}}dx$$

input
```
int(x^(3/2)/(b*x+a)^2*Ei(3,b*x+a),x)
```

output
```
int(x^(3/2)/(b*x+a)^2*Ei(3,b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int x^{3/2}\Gamma(-2,a+bx)\,dx=\int x^{\frac{3}{2}}\Gamma(-2,bx+a)\,dx$$

input
```
integrate(x^(3/2)*gamma(-2,b*x+a),x, algorithm="fricas")
```

output
```
integral(x^(3/2)*gamma(-2, b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int x^{3/2}\Gamma(-2,a+bx)\,dx=\int \frac{x^{\frac{3}{2}}\operatorname{E}_{3}\left(a+bx\right)}{\left(a+bx\right)^{2}}\,dx$$

input
```
integrate(x**(3/2)*uppergamma(-2,b*x+a),x)
```

output
```
Integral(x**(3/2)*expint(3, a + b*x)/(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int x^{3/2}\Gamma(-2,a+bx)\,dx=\int x^{\frac{3}{2}}\Gamma(-2,bx+a)\,dx$$

input
```
integrate(x^(3/2)*gamma(-2,b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)*gamma(-2, b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int x^{3/2}\Gamma(-2,a+bx)\,dx=\int x^{\frac{3}{2}}\Gamma(-2,bx+a)\;dx$$

input
```
integrate(x^(3/2)*gamma(-2,b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(3/2)*gamma(-2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int x^{3/2}\Gamma(-2,a+bx)\,dx=\int \frac{x^{3/2}\,\mathrm{expint}(3,a+b\,x)}{{(a+b\,x)}^2}\,dx$$

input
```
int((x^(3/2)*expint(3, a + b*x))/(a + b*x)^2,x)
```

output
```
int((x^(3/2)*expint(3, a + b*x))/(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.38

$$\int x^{3/2}\Gamma(-2,a+bx)\,dx=\int \frac{\sqrt{x}\,ei(3,bx+a)\,x}{b^2x^2+2abx+a^2}dx$$

input
```
int(x^(3/2)/(b*x+a)^2*Ei(3,b*x+a),x)
```

output
```
int((sqrt(x)*ei(3,a + b*x)*x)/(a**2 + 2*a*b*x + b**2*x**2),x)
```

3.170 $\int \sqrt{x}\Gamma(-2,a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \sqrt{x}\Gamma(-2,a+bx)\,dx = \text{Int}\left(\sqrt{x}\Gamma(-2,a+bx),x\right)$$

output
```
Defer(Int)(x^(1/2)/(b*x+a)^2*Ei(3,b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 4.48 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sqrt{x}\Gamma(-2,a+bx)\,dx = \int \sqrt{x}\Gamma(-2,a+bx)\,dx$$

input
```
Integrate[Sqrt[x]*Gamma[-2, a + b*x],x]
```

output
```
Integrate[Sqrt[x]*Gamma[-2, a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\Gamma(-2,a+bx)\,dx$$

$$\Big\downarrow \text{ 7120}$$

$$\int \sqrt{x}\Gamma(-2,a+bx)dx$$

input
```
Int[Sqrt[x]*Gamma[-2, a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{x}\ \mathrm{expIntegral}_3\left(bx+a\right)}{\left(bx+a\right)^2}dx$$

input
```
int(x^(1/2)/(b*x+a)^2*Ei(3,b*x+a),x)
```

output
```
int(x^(1/2)/(b*x+a)^2*Ei(3,b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{x}\Gamma(-2,a+bx)\,dx = \int \sqrt{x}\Gamma(-2,bx+a)\,dx$$

input
```
integrate(x^(1/2)*gamma(-2,b*x+a),x, algorithm="fricas")
```

output
```
integral(sqrt(x)*gamma(-2, b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \sqrt{x}\Gamma(-2,a+bx)\,dx = \int \frac{\sqrt{x}\,\mathrm{E}_3\,(a+bx)}{(a+bx)^2}\,dx$$

input
```
integrate(x**(1/2)*uppergamma(-2,b*x+a),x)
```

output
```
Integral(sqrt(x)*expint(3, a + b*x)/(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{x}\Gamma(-2,a+bx)\,dx = \int \sqrt{x}\Gamma(-2,bx+a)\,dx$$

input
```
integrate(x^(1/2)*gamma(-2,b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)*gamma(-2, b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{x}\Gamma(-2,a+bx)\,dx=\int \sqrt{x}\Gamma(-2,bx+a)\;dx$$

input
```
integrate(x^(1/2)*gamma(-2,b*x+a),x, algorithm="giac")
```

output
```
integrate(sqrt(x)*gamma(-2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \sqrt{x}\Gamma(-2,a+bx)\,dx=\int \frac{\sqrt{x}\,\mathrm{expint}(3,a+b\,x)}{{\left(a+b\,x\right)}^2}\,dx$$

input
```
int((x^(1/2)*expint(3, a + b*x))/(a + b*x)^2,x)
```

output
```
int((x^(1/2)*expint(3, a + b*x))/(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.31

$$\int \sqrt{x}\Gamma(-2, a + bx) \, dx = \int \frac{\sqrt{x} \, ei(3, bx + a)}{b^2x^2 + 2abx + a^2} dx$$

input
```
int(x^(1/2)/(b*x+a)^2*Ei(3,b*x+a),x)
```

output
```
int((sqrt(x)*ei(3,a + b*x))/(a**2 + 2*a*b*x + b**2*x**2),x)
```

3.171 $\int \frac{\Gamma(-2,a+bx)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{\Gamma(-2,a+bx)}{\sqrt{x}}\,dx = \text{Int}\left(\frac{\Gamma(-2,a+bx)}{\sqrt{x}},x\right)$$

output
```
Defer(Int)(1/(b*x+a)^2*Ei(3,b*x+a)/x^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\Gamma(-2,a+bx)}{\sqrt{x}}\,dx = \int \frac{\Gamma(-2,a+bx)}{\sqrt{x}}\,dx$$

input
```
Integrate[Gamma[-2, a + b*x]/Sqrt[x],x]
```

output
```
Integrate[Gamma[-2, a + b*x]/Sqrt[x], x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-2,a+bx)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 7120}$$

$$\int \frac{\Gamma(-2,a+bx)}{\sqrt{x}}dx$$

input
```
Int[Gamma[-2, a + b*x]/Sqrt[x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{\text{expIntegral}_3\left(bx+a\right)}{\left(bx+a\right)^2\sqrt{x}}dx$$

input
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/x^(1/2),x)
```

output
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/x^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(-2,a+bx)}{\sqrt{x}}\,dx = \int \frac{\Gamma(-2,bx+a)}{\sqrt{x}}\,dx$$

input
```
integrate(gamma(-2,b*x+a)/x^(1/2),x, algorithm="fricas")
```

output
```
integral(gamma(-2, b*x + a)/sqrt(x), x)
```

Sympy [N/A]

Not integrable

Time = 2.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{\Gamma(-2,a+bx)}{\sqrt{x}}\,dx = \int \frac{\mathrm{E}_3\left(a+bx\right)}{\sqrt{x}\left(a+bx\right)^2}\,dx$$

input
```
integrate(uppergamma(-2,b*x+a)/x**(1/2),x)
```

output
```
Integral(expint(3, a + b*x)/(sqrt(x)*(a + b*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(-2,a+bx)}{\sqrt{x}}\,dx = \int \frac{\Gamma(-2,bx+a)}{\sqrt{x}}\,dx$$

input
```
integrate(gamma(-2,b*x+a)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate(gamma(-2, b*x + a)/sqrt(x), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(-2, a + bx)}{\sqrt{x}} \, dx = \int \frac{\Gamma(-2, bx + a)}{\sqrt{x}} \, dx$$

input
```
integrate(gamma(-2,b*x+a)/x^(1/2),x, algorithm="giac")
```

output
```
integrate(gamma(-2, b*x + a)/sqrt(x), x)
```

Mupad [B] (verification not implemented)

Time = 12.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{\Gamma(-2, a + bx)}{\sqrt{x}} \, dx = \int \frac{\mathrm{expint}(3, a + b\,x)}{\sqrt{x}\,(a + b\,x)^2} \, dx$$

input
```
int(expint(3, a + b*x)/(x^(1/2)*(a + b*x)^2),x)
```

output
```
int(expint(3, a + b*x)/(x^(1/2)*(a + b*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.69

$$\int \frac{\Gamma(-2, a + bx)}{\sqrt{x}}\,dx = \int \frac{ei(3, bx + a)}{\sqrt{x}\,a^2 + 2\sqrt{x}\,abx + \sqrt{x}\,b^2x^2}dx$$

input
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/x^(1/2),x)
```

output
```
int(ei(3,a + b*x)/(sqrt(x)*a**2 + 2*sqrt(x)*a*b*x + sqrt(x)*b**2*x**2),x)
```

3.172 $\int \frac{\Gamma(-2,a+bx)}{x^{3/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{\Gamma(-2,a+bx)}{x^{3/2}} dx = \text{Int}\left(\frac{\Gamma(-2,a+bx)}{x^{3/2}},x\right)$$

output
```
Defer(Int)(1/(b*x+a)^2*Ei(3,b*x+a)/x^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\Gamma(-2,a+bx)}{x^{3/2}} dx = \int \frac{\Gamma(-2,a+bx)}{x^{3/2}} dx$$

input
```
Integrate[Gamma[-2, a + b*x]/x^(3/2),x]
```

output
```
Integrate[Gamma[-2, a + b*x]/x^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(-2,a+bx)}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{ 7120}$$

$$\int \frac{\Gamma(-2,a+bx)}{x^{3/2}}dx$$

input
```
Int[Gamma[-2, a + b*x]/x^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{\text{expIntegral}_3\left(bx+a\right)}{\left(bx+a\right)^2 x^{\frac{3}{2}}}dx$$

input
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/x^(3/2),x)
```

output
```
int(1/(b*x+a)^2*Ei(3,b*x+a)/x^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(-2, a+bx)}{x^{3/2}} \, dx = \int \frac{\Gamma(-2, bx+a)}{x^{\frac{3}{2}}} \, dx$$

input
```
integrate(gamma(-2,b*x+a)/x^(3/2),x, algorithm="fricas")
```

output
```
integral(gamma(-2, b*x + a)/x^(3/2), x)
```

Sympy [N/A]

Not integrable

Time = 3.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{\Gamma(-2, a+bx)}{x^{3/2}} \, dx = \int \frac{\mathrm{E}_3\,(a+bx)}{x^{\frac{3}{2}}\,(a+bx)^2} \, dx$$

input
```
integrate(uppergamma(-2,b*x+a)/x**(3/2),x)
```

output
```
Integral(expint(3, a + b*x)/(x**(3/2)*(a + b*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(-2, a+bx)}{x^{3/2}} \, dx = \int \frac{\Gamma(-2, bx+a)}{x^{\frac{3}{2}}} \, dx$$

input
```
integrate(gamma(-2,b*x+a)/x^(3/2),x, algorithm="maxima")
```

output
```
integrate(gamma(-2, b*x + a)/x^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(-2, a+bx)}{x^{3/2}}\,dx = \int \frac{\Gamma(-2, bx+a)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(gamma(-2,b*x+a)/x^(3/2),x, algorithm="giac")
```

output
```
integrate(gamma(-2, b*x + a)/x^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 12.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{\Gamma(-2, a+bx)}{x^{3/2}}\,dx = \int \frac{\mathrm{expint}(3, a+b\,x)}{x^{3/2}\,{(a+b\,x)}^2}\,dx$$

input
```
int(expint(3, a + b*x)/(x^(3/2)*(a + b*x)^2),x)
```

output
```
int(expint(3, a + b*x)/(x^(3/2)*(a + b*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.92

$$\int \frac{\Gamma(-2, a+bx)}{x^{3/2}} \, dx = \int \frac{ei(3, bx+a)}{\sqrt{x}\, a^2x + 2\sqrt{x}\, ab\, x^2 + \sqrt{x}\, b^2x^3} dx$$

input

```
int(1/(b*x+a)^2*Ei(3,b*x+a)/x^(3/2),x)
```

output

```
int(ei(3,a + b*x)/(sqrt(x)*a**2*x + 2*sqrt(x)*a*b*x**2 + sqrt(x)*b**2*x**3
),x)
```

3.173 $\int x^{4/3}\Gamma(2,a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int x^{4/3}\Gamma(2,a+bx)\,dx=\frac{3}{7}x^{7/3}\Gamma(2,a+bx)-\frac{3ae^{-a}\sqrt[3]{x}\Gamma\left(\frac{10}{3},bx\right)}{7b^2\sqrt[3]{bx}}-\frac{3e^{-a}\sqrt[3]{x}\Gamma\left(\frac{13}{3},bx\right)}{7b^2\sqrt[3]{bx}}$$

output
```
3/7*x^(7/3)*exp(-b*x-a)*(b*x+a+1)-3/7*a*x^(1/3)*GAMMA(10/3,b*x)/b^2/exp(a)
/(b*x)^(1/3)-3/7*x^(1/3)*GAMMA(13/3,b*x)/b^2/exp(a)/(b*x)^(1/3)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.49

$$\int x^{4/3}\Gamma(2,a+bx)\,dx = -\frac{e^{-a-bx}\sqrt[3]{x}\left(28(10+3a)e^{bx}\Gamma\left(\frac{1}{3},bx\right)+3\sqrt[3]{bx}\left(280+210bx+90b^2x^2+27b^3x^3+3a(28+21bx+9b^2x^2)-\right.\right.}{189b^2\sqrt[3]{bx}}$$

input
```
Integrate[x^(4/3)*Gamma[2, a + b*x],x]
```

output
```
-1/189*(E^(-a - b*x)*x^(1/3)*(28*(10 + 3*a)*E^(b*x)*Gamma[1/3, b*x] + 3*(b
*x)^(1/3)*(280 + 210*b*x + 90*b^2*x^2 + 27*b^3*x^3 + 3*a*(28 + 21*b*x + 9*
b^2*x^2) - 27*b^2*E^(a + b*x)*x^2*Gamma[2, a + b*x])))/(b^2*(b*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{4/3}\Gamma(2,a+bx)\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{3}{7}b\int e^{-a-bx}x^{7/3}(a+bx)dx+\frac{3}{7}x^{7/3}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{2629}$$

$$\frac{3}{7}b\int \left(be^{-a-bx}x^{10/3}+ae^{-a-bx}x^{7/3}\right)dx+\frac{3}{7}x^{7/3}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{7}b\left(-\frac{ae^{-a}\sqrt[3]{x}\Gamma\left(\frac{10}{3},bx\right)}{b^3\sqrt[3]{bx}}-\frac{e^{-a}\sqrt[3]{x}\Gamma\left(\frac{13}{3},bx\right)}{b^3\sqrt[3]{bx}}\right)+\frac{3}{7}x^{7/3}\Gamma(2,a+bx)$$

input
```
Int[x^(4/3)*Gamma[2, a + b*x],x]
```

output
```
(3*x^(7/3)*Gamma[2, a + b*x])/7 + (3*b*(-((a*x^(1/3)*Gamma[10/3, b*x])/(b^
3*E^a*(b*x)^(1/3))) - (x^(1/3)*Gamma[13/3, b*x])/(b^3*E^a*(b*x)^(1/3))))/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.22

method	result
meijerg	$\frac{e^{-a}\left(\frac{7x^{\frac{1}{3}}b^{\frac{1}{3}}e^{-\frac{bx}{2}}\text{WhittakerM}\left(\frac{1}{6},\frac{2}{3},bx\right)}{3(bx)^{\frac{1}{6}}}-\frac{x^{\frac{1}{3}}b^{\frac{1}{3}}(30bx+70)e^{-\frac{bx}{2}}\text{WhittakerM}\left(\frac{7}{6},\frac{2}{3},bx\right)}{30(bx)^{\frac{1}{6}}}\right)}{b^{\frac{7}{3}}}+\frac{e^{-a}a\left(\frac{x^{\frac{1}{3}}b^{\frac{1}{3}}e^{-\frac{bx}{2}}\text{WhittakerM}\left(\frac{1}{6},\frac{2}{3},bx\right)}{(bx)^{\frac{1}{6}}}-\right.}{b^{\frac{7}{3}}}$

input
```
int(x^(4/3)*exp(-b*x-a)*(b*x+a+1),x,method=_RETURNVERBOSE)
```

output
```
1/b^(7/3)*exp(-a)*(7/3*x^(1/3)*b^(1/3)/(b*x)^(1/6)*exp(-1/2*b*x)*Whittaker
M(1/6,2/3,b*x)-1/30*x^(1/3)*b^(1/3)*(30*b*x+70)/(b*x)^(1/6)*exp(-1/2*b*x)*
WhittakerM(7/6,2/3,b*x))+1/b^(7/3)*exp(-a)*a*(x^(1/3)*b^(1/3)/(b*x)^(1/6)*
exp(-1/2*b*x)*WhittakerM(1/6,2/3,b*x)-x^(1/3)*b^(1/3)/(b*x)^(1/6)*exp(-1/2
*b*x)*WhittakerM(7/6,2/3,b*x))+1/b^(7/3)*exp(-a)*(x^(1/3)*b^(1/3)/(b*x)^(1
/6)*exp(-1/2*b*x)*WhittakerM(1/6,2/3,b*x)-x^(1/3)*b^(1/3)/(b*x)^(1/6)*exp(
-1/2*b*x)*WhittakerM(7/6,2/3,b*x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22

$$\int x^{4/3}\Gamma(2,a+bx)\,dx = -\frac{28\,(3\,a+10)b^{\frac{2}{3}}e^{(-a)}\Gamma\left(\frac{1}{3},bx\right)-3\left(27\,b^3x^2\Gamma(2,bx+a)-(27\,b^4x^3+9\,(3\,a+10)b^3x^2+21\,(3\,a+10)b^2x+\right.}{189\,b^3}$$

input
```
integrate(x^(4/3)*gamma(2,b*x+a),x, algorithm="fricas")
```

output
```
-1/189*(28*(3*a + 10)*b^(2/3)*e^(-a)*gamma(1/3, b*x) - 3*(27*b^3*x^2*gamma
(2, b*x + a) - (27*b^4*x^3 + 9*(3*a + 10)*b^3*x^2 + 21*(3*a + 10)*b^2*x +
28*(3*a + 10)*b)*e^(-b*x - a))*x^(1/3))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int x^{4/3}\Gamma(2,a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**(4/3)*uppergamma(2,b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{4/3}\Gamma(2,a+bx)\,dx = \int x^{\frac{4}{3}}\Gamma(2,bx+a)\,dx$$

input
```
integrate(x^(4/3)*gamma(2,b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(4/3)*gamma(2, b*x + a), x)
```

Giac [F]

$$\int x^{4/3}\Gamma(2,a+bx)\,dx=\int x^{\frac{4}{3}}\Gamma(2,bx+a)\ dx$$

input
```
integrate(x^(4/3)*gamma(2,b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(4/3)*gamma(2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.55

$$\int x^{4/3}\Gamma(2,a+bx)\,dx=-x^{7/3}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}-\frac{40\,x^{7/3}\,\mathrm{e}^{-a}\,\Gamma\left(\frac{1}{3},b\,x\right)}{27\,{(b\,x)}^{7/3}}-\frac{40\,x^{1/3}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{9\,b^2}-\frac{10\,x^{4/3}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{3\,b}-\frac{4\,a\,x^{1/3}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{3\,b^2}-\frac{a\,x^{4/3}\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{b}-\frac{4\,a\,x^{7/3}\,\mathrm{e}^{-a}\,\Gamma\left(\frac{1}{3},b\,x\right)}{9\,{(b\,x)}^{7/3}}$$

input
```
int(x^(4/3)*exp(- a - b*x)*(a + b*x + 1),x)
```

output
```
- x^(7/3)*exp(-a)*exp(-b*x) - (40*x^(7/3)*exp(-a)*igamma(1/3, b*x))/(27*(b
*x)^(7/3)) - (40*x^(1/3)*exp(-a)*exp(-b*x))/(9*b^2) - (10*x^(4/3)*exp(-a)*
exp(-b*x))/(3*b) - (4*a*x^(1/3)*exp(-a)*exp(-b*x))/(3*b^2) - (a*x^(4/3)*ex
p(-a)*exp(-b*x))/b - (4*a*x^(7/3)*exp(-a)*igamma(1/3, b*x))/(9*(b*x)^(7/3)
)
```

Reduce [F]

$$\int x^{4/3}\Gamma(2,a+bx)\,dx=\frac{-24x^{\frac{2}{3}}e^{bx}\left(\int \frac{1}{x^{\frac{5}{3}}e^{bx}}dx\right)a-80x^{\frac{2}{3}}e^{bx}\left(\int \frac{1}{x^{\frac{5}{3}}e^{bx}}dx\right)-81a\,b^2x^2-108abx-36a-81b^3x^3-270b^2x^2}{81x^{\frac{2}{3}}e^{bx+a}b^3}$$

input
```
int(x^(4/3)*exp(-b*x-a)*(b*x+a+1),x)
```

output
```
( - 24*x**(2/3)*e**(b*x)*int(1/(x**(2/3)*e**(b*x)*x),x)*a - 80*x**(2/3)*e*
*(b*x)*int(1/(x**(2/3)*e**(b*x)*x),x) - 81*a*b**2*x**2 - 108*a*b*x - 36*a
- 81*b**3*x**3 - 270*b**2*x**2 - 360*b*x - 120)/(81*x**(2/3)*e**(a + b*x)*
b**3)
```

3.174 $\int x^{2/3}\Gamma(2,a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int x^{2/3}\Gamma(2,a+bx)\,dx=\frac{3}{5}x^{5/3}\Gamma(2,a+bx)-\frac{3ae^{-a}x^{2/3}\Gamma\left(\frac{8}{3},bx\right)}{5b(bx)^{2/3}}-\frac{3e^{-a}x^{2/3}\Gamma\left(\frac{11}{3},bx\right)}{5b(bx)^{2/3}}$$

output
```
3/5*x^(5/3)*exp(-b*x-a)*(b*x+a+1)-3/5*a*x^(2/3)*GAMMA(8/3,b*x)/b/exp(a)/(b
*x)^(2/3)-3/5*x^(2/3)*GAMMA(11/3,b*x)/b/exp(a)/(b*x)^(2/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19

$$\int x^{2/3}\Gamma(2,a+bx)\,dx = -\frac{e^{-a-bx}x^{5/3}\left(10(8+3a)e^{bx}\Gamma\left(\frac{2}{3},bx\right)+3(bx)^{2/3}\left(40+15a+24bx+9abx+9b^2x^2-9be^{a+bx}x\Gamma(2,a+bx)\right)\right)}{45(bx)^{5/3}}$$

input
```
Integrate[x^(2/3)*Gamma[2, a + b*x],x]
```

output
```
-1/45*(E^(-a - b*x)*x^(5/3)*(10*(8 + 3*a)*E^(b*x)*Gamma[2/3, b*x] + 3*(b*x
)^(2/3)*(40 + 15*a + 24*b*x + 9*a*b*x + 9*b^2*x^2 - 9*b*E^(a + b*x)*x*Gamm
a[2, a + b*x])))/(b*x)^(5/3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2/3}\Gamma(2,a+bx)\,dx$$

$$\Big\downarrow \text{ 7119}$$

$$\frac{3}{5}b\int e^{-a-bx}x^{5/3}(a+bx)dx+\frac{3}{5}x^{5/3}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{ 2629}$$

$$\frac{3}{5}b\int \left(be^{-a-bx}x^{8/3}+ae^{-a-bx}x^{5/3}\right)dx+\frac{3}{5}x^{5/3}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{5}b\left(-\frac{ae^{-a}x^{2/3}\Gamma\left(\frac{8}{3},bx\right)}{b^2(bx)^{2/3}}-\frac{e^{-a}x^{2/3}\Gamma\left(\frac{11}{3},bx\right)}{b^2(bx)^{2/3}}\right)+\frac{3}{5}x^{5/3}\Gamma(2,a+bx)$$

input
```
Int[x^(2/3)*Gamma[2, a + b*x],x]
```

output
```
(3*x^(5/3)*Gamma[2, a + b*x])/5 + (3*b*(-((a*x^(2/3)*Gamma[8/3, b*x])/(b^2
*E^a*(b*x)^(2/3))) - (x^(2/3)*Gamma[11/3, b*x])/(b^2*E^a*(b*x)^(2/3))))/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.44

method	result
meijerg	$\frac{e^{-a}\left(\frac{x^{\frac{2}{3}} b^{\frac{2}{3}} e^{-\frac{bx}{2}} \operatorname{WhittakerM}\left(\frac{1}{3},\frac{5}{6},bx\right)}{(bx)^{\frac{1}{3}}}-\frac{x^{\frac{2}{3}} b^{\frac{2}{3}} e^{-\frac{bx}{2}} \operatorname{WhittakerM}\left(\frac{4}{3},\frac{5}{6},bx\right)}{(bx)^{\frac{1}{3}}}\right)}{b^{\frac{5}{3}}}+\frac{3\, e^{-a-\frac{bx}{2}} a\, x^{\frac{2}{3}} \operatorname{WhittakerM}\left(\frac{1}{3},\frac{5}{6},bx\right)}{5b(bx)^{\frac{1}{3}}}+\frac{3\, e^{-a-\frac{bx}{2}}}{}$

input
```
int(x^(2/3)*exp(-b*x-a)*(b*x+a+1),x,method=_RETURNVERBOSE)
```

output
```
1/b^(5/3)*exp(-a)*(x^(2/3)*b^(2/3)/(b*x)^(1/3)*exp(-1/2*b*x)*WhittakerM(1/
3,5/6,b*x)-x^(2/3)*b^(2/3)/(b*x)^(1/3)*exp(-1/2*b*x)*WhittakerM(4/3,5/6,b*
x))+3/5/b*exp(-a-1/2*b*x)*a*x^(2/3)/(b*x)^(1/3)*WhittakerM(1/3,5/6,b*x)+3/
5/b*exp(-a-1/2*b*x)*x^(2/3)/(b*x)^(1/3)*WhittakerM(1/3,5/6,b*x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int x^{2/3}\Gamma(2,a+bx)\,dx = -\frac{10\,(3\,a+8)b^{\frac{1}{3}}e^{(-a)}\Gamma\left(\frac{2}{3},bx\right)-3\left(9\,b^2x\Gamma(2,bx+a)-(9\,b^3x^2+3\,(3\,a+8)b^2x+5\,(3\,a+8)b)e^{(-bx-a)}\right)x^{\frac{2}{3}}}{45\,b^2}$$

input
```
integrate(x^(2/3)*gamma(2,b*x+a),x, algorithm="fricas")
```

output
```
-1/45*(10*(3*a + 8)*b^(1/3)*e^(-a)*gamma(2/3, b*x) - 3*(9*b^2*x*gamma(2, b
*x + a) - (9*b^3*x^2 + 3*(3*a + 8)*b^2*x + 5*(3*a + 8)*b)*e^(-b*x - a))*x^
(2/3))/b^2
```

Sympy [A] (verification not implemented)

Time = 28.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

$$\int x^{2/3}\Gamma(2,a+bx)\,dx = \left(-\frac{ax^{\frac{2}{3}}\Gamma\left(\frac{5}{3},bx\right)}{b\,(bx)^{\frac{2}{3}}}-\frac{x^{\frac{5}{3}}\Gamma\left(\frac{8}{3},bx\right)}{(bx)^{\frac{5}{3}}}-\frac{x^{\frac{2}{3}}\Gamma\left(\frac{5}{3},bx\right)}{b\,(bx)^{\frac{2}{3}}}\right)e^{-a}$$

input
```
integrate(x**(2/3)*uppergamma(2,b*x+a),x)
```

output
```
(-a*x**(2/3)*uppergamma(5/3, b*x)/(b*(b*x)**(2/3)) - x**(5/3)*uppergamma(8
/3, b*x)/(b*x)**(5/3) - x**(2/3)*uppergamma(5/3, b*x)/(b*(b*x)**(2/3)))*ex
p(-a)
```

Maxima [F]

$$\int x^{2/3}\Gamma(2,a+bx)\,dx=\int x^{\frac{2}{3}}\Gamma(2,bx+a)\,dx$$

input
```
integrate(x^(2/3)*gamma(2,b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(2/3)*gamma(2, b*x + a), x)
```

Giac [F]

$$\int x^{2/3}\Gamma(2,a+bx)\,dx=\int x^{\frac{2}{3}}\Gamma(2,bx+a)\,dx$$

input
```
integrate(x^(2/3)*gamma(2,b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(2/3)*gamma(2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int x^{2/3}\Gamma(2,a+bx)\,dx=-x^{5/3}\,\mathrm{e}^{-a-b\,x}-\frac{8\,x^{2/3}\,\mathrm{e}^{-a-b\,x}}{3\,b}-\frac{a\,x^{2/3}\,\mathrm{e}^{-a-b\,x}}{b}-\frac{10\,x^{5/3}\,\mathrm{e}^{-a}\,\Gamma\left(\frac{2}{3},b\,x\right)}{9\,{(b\,x)}^{5/3}}-\frac{2\,x^{2/3}\,\mathrm{e}^{-a}\,\Gamma\left(\frac{2}{3},b\,x\right)}{3\,b\,{(b\,x)}^{2/3}}-\frac{2\,a\,x^{2/3}\,\mathrm{e}^{-a}\,\Gamma\left(\frac{2}{3},b\,x\right)}{3\,b\,{(b\,x)}^{2/3}}$$

input
```
int(x^(2/3)*exp(- a - b*x)*(a + b*x + 1),x)
```

output
```
- x^(5/3)*exp(- a - b*x) - (8*x^(2/3)*exp(- a - b*x))/(3*b) - (a*x^(2/3)*e
xp(- a - b*x))/b - (10*x^(5/3)*exp(-a)*igamma(2/3, b*x))/(9*(b*x)^(5/3)) -
 (2*x^(2/3)*exp(-a)*igamma(2/3, b*x))/(3*b*(b*x)^(2/3)) - (2*a*x^(2/3)*exp
(-a)*igamma(2/3, b*x))/(3*b*(b*x)^(2/3))
```

Reduce [F]

$$\int x^{2/3}\Gamma(2,a+bx)\,dx = \frac{-6x^{\frac{1}{3}}e^{bx}\left(\int \frac{1}{x^{\frac{4}{3}}e^{bx}}dx\right)a - 16x^{\frac{1}{3}}e^{bx}\left(\int \frac{1}{x^{\frac{4}{3}}e^{bx}}dx\right) - 27abx - 18a - 27b^2x^2 - 72bx - 48}{27x^{\frac{1}{3}}e^{bx+a}b^2}$$

input
```
int(x^(2/3)*exp(-b*x-a)*(b*x+a+1),x)
```

output
```
( - 6*x**(1/3)*e**(b*x)*int(1/(x**(1/3)*e**(b*x)*x),x)*a - 16*x**(1/3)*e**
(b*x)*int(1/(x**(1/3)*e**(b*x)*x),x) - 27*a*b*x - 18*a - 27*b**2*x**2 - 72
*b*x - 48)/(27*x**(1/3)*e**(a + b*x)*b**2)
```

3.175 $\int \sqrt[3]{x}\Gamma(2, a+bx)\, dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \sqrt[3]{x}\Gamma(2,a+bx)\,dx = \frac{3}{4}x^{4/3}\Gamma(2,a+bx) - \frac{3ae^{-a}\sqrt[3]{x}\Gamma\left(\frac{7}{3},bx\right)}{4b\sqrt[3]{bx}} - \frac{3e^{-a}\sqrt[3]{x}\Gamma\left(\frac{10}{3},bx\right)}{4b\sqrt[3]{bx}}$$

output
```
3/4*x^(4/3)*exp(-b*x-a)*(b*x+a+1)-3/4*a*x^(1/3)*GAMMA(7/3,b*x)/b/exp(a)/(b
*x)^(1/3)-3/4*x^(1/3)*GAMMA(10/3,b*x)/b/exp(a)/(b*x)^(1/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19

$$\int \sqrt[3]{x}\Gamma(2,a+bx)\,dx = -\frac{e^{-a-bx}x^{4/3}\left(4(7+3a)e^{bx}\Gamma\left(\frac{1}{3},bx\right)+3\sqrt[3]{bx}\left(28+12a+21bx+9abx+9b^2x^2-9be^{a+bx}x\Gamma(2,a+bx)\right)\right)}{36(bx)^{4/3}}$$

input
```
Integrate[x^(1/3)*Gamma[2, a + b*x],x]
```

output
```
-1/36*(E^(-a - b*x)*x^(4/3)*(4*(7 + 3*a)*E^(b*x)*Gamma[1/3, b*x] + 3*(b*x)
^(1/3)*(28 + 12*a + 21*b*x + 9*a*b*x + 9*b^2*x^2 - 9*b*E^(a + b*x)*x*Gamma
[2, a + b*x])))/(b*x)^(4/3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{x}\Gamma(2,a+bx)\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{3}{4}b\int e^{-a-bx}x^{4/3}(a+bx)dx+\frac{3}{4}x^{4/3}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{2629}$$

$$\frac{3}{4}b\int \left(be^{-a-bx}x^{7/3}+ae^{-a-bx}x^{4/3}\right)dx+\frac{3}{4}x^{4/3}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{4}b\left(-\frac{ae^{-a}\sqrt[3]{x}\Gamma\left(\frac{7}{3},bx\right)}{b^2\sqrt[3]{bx}}-\frac{e^{-a}\sqrt[3]{x}\Gamma\left(\frac{10}{3},bx\right)}{b^2\sqrt[3]{bx}}\right)+\frac{3}{4}x^{4/3}\Gamma(2,a+bx)$$

input
```
Int[x^(1/3)*Gamma[2, a + b*x],x]
```

output
```
(3*x^(4/3)*Gamma[2, a + b*x])/4 + (3*b*(-((a*x^(1/3)*Gamma[7/3, b*x])/(b^2
*E^a*(b*x)^(1/3))) - (x^(1/3)*Gamma[10/3, b*x])/(b^2*E^a*(b*x)^(1/3))))/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.44

method	result
meijerg	$\frac{e^{-a}\left(\frac{x^{\frac{1}{3}} b^{\frac{1}{3}} e^{-\frac{bx}{2}} \operatorname{WhittakerM}\left(\frac{1}{6},\frac{2}{3},bx\right)}{(bx)^{\frac{1}{6}}}-\frac{x^{\frac{1}{3}} b^{\frac{1}{3}} e^{-\frac{bx}{2}} \operatorname{WhittakerM}\left(\frac{7}{6},\frac{2}{3},bx\right)}{(bx)^{\frac{1}{6}}}\right)}{b^{\frac{4}{3}}}+\frac{3\,e^{-a-\frac{bx}{2}}\, a\, x^{\frac{1}{3}} \operatorname{WhittakerM}\left(\frac{1}{6},\frac{2}{3},bx\right)}{4b(bx)^{\frac{1}{6}}}+\frac{3\,e^{-a-\frac{bx}{2}}}{}$

input
```
int(x^(1/3)*exp(-b*x-a)*(b*x+a+1),x,method=_RETURNVERBOSE)
```

output
```
1/b^(4/3)*exp(-a)*(x^(1/3)*b^(1/3)/(b*x)^(1/6)*exp(-1/2*b*x)*WhittakerM(1/
6,2/3,b*x)-x^(1/3)*b^(1/3)/(b*x)^(1/6)*exp(-1/2*b*x)*WhittakerM(7/6,2/3,b*
x))+3/4/b*exp(-a-1/2*b*x)*a*x^(1/3)/(b*x)^(1/6)*WhittakerM(1/6,2/3,b*x)+3/
4/b*exp(-a-1/2*b*x)*x^(1/3)/(b*x)^(1/6)*WhittakerM(1/6,2/3,b*x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \sqrt[3]{x}\Gamma(2,a+bx)\,dx = -\frac{4\,(3\,a+7)b^{\frac{2}{3}}e^{(-a)}\Gamma\left(\frac{1}{3},bx\right)-3\left(9\,b^2x\Gamma(2,bx+a)-\left(9\,b^3x^2+3\,(3\,a+7)b^2x+4\,(3\,a+7)b\right)e^{(-bx-a)}\right)x^{\frac{1}{3}}}{36\,b^2}$$

input
```
integrate(x^(1/3)*gamma(2,b*x+a),x, algorithm="fricas")
```

output
```
-1/36*(4*(3*a + 7)*b^(2/3)*e^(-a)*gamma(1/3, b*x) - 3*(9*b^2*x*gamma(2, b*
x + a) - (9*b^3*x^2 + 3*(3*a + 7)*b^2*x + 4*(3*a + 7)*b)*e^(-b*x - a))*x^(
1/3))/b^2
```

Sympy [A] (verification not implemented)

Time = 9.78 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

$$\int \sqrt[3]{x}\Gamma(2,a+bx)\,dx = \left(-\frac{a\sqrt[3]{x}\Gamma\left(\frac{4}{3},bx\right)}{b\sqrt[3]{bx}}-\frac{x^{\frac{4}{3}}\Gamma\left(\frac{7}{3},bx\right)}{(bx)^{\frac{4}{3}}}-\frac{\sqrt[3]{x}\Gamma\left(\frac{4}{3},bx\right)}{b\sqrt[3]{bx}}\right)e^{-a}$$

input
```
integrate(x**(1/3)*uppergamma(2,b*x+a),x)
```

output
```
(-a*x**(1/3)*uppergamma(4/3, b*x)/(b*(b*x)**(1/3)) - x**(4/3)*uppergamma(7
/3, b*x)/(b*x)**(4/3) - x**(1/3)*uppergamma(4/3, b*x)/(b*(b*x)**(1/3)))*ex
p(-a)
```

Maxima [F]

$$\int \sqrt[3]{x}\Gamma(2, a+bx)\,dx = \int x^{\frac{1}{3}}\Gamma(2, bx+a)\,dx$$

input
```
integrate(x^(1/3)*gamma(2,b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(1/3)*gamma(2, b*x + a), x)
```

Giac [F]

$$\int \sqrt[3]{x}\Gamma(2, a+bx)\,dx = \int x^{\frac{1}{3}}\Gamma(2, bx+a)\,dx$$

input
```
integrate(x^(1/3)*gamma(2,b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(1/3)*gamma(2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int \sqrt[3]{x}\Gamma(2, a+bx)\,dx = -x^{4/3}\,\mathrm{e}^{-a-b\,x} - \frac{7\,x^{1/3}\,\mathrm{e}^{-a-b\,x}}{3\,b} - \frac{a\,x^{1/3}\,\mathrm{e}^{-a-b\,x}}{b} - \frac{4\,x^{4/3}\,\mathrm{e}^{-a}\,\Gamma\left(\frac{1}{3}, b\,x\right)}{9\,(b\,x)^{4/3}} - \frac{x^{1/3}\,\mathrm{e}^{-a}\,\Gamma\left(\frac{1}{3}, b\,x\right)}{3\,b\,(b\,x)^{1/3}} - \frac{a\,x^{1/3}\,\mathrm{e}^{-a}\,\Gamma\left(\frac{1}{3}, b\,x\right)}{3\,b\,(b\,x)^{1/3}}$$

input
```
int(x^(1/3)*exp(- a - b*x)*(a + b*x + 1),x)
```

output
```
- x^(4/3)*exp(- a - b*x) - (7*x^(1/3)*exp(- a - b*x))/(3*b) - (a*x^(1/3)*e
xp(- a - b*x))/b - (4*x^(4/3)*exp(-a)*igamma(1/3, b*x))/(9*(b*x)^(4/3)) -
(x^(1/3)*exp(-a)*igamma(1/3, b*x))/(3*b*(b*x)^(1/3)) - (a*x^(1/3)*exp(-a)*
igamma(1/3, b*x))/(3*b*(b*x)^(1/3))
```

Reduce [F]

$$\int \sqrt[3]{x}\Gamma(2,a+bx)\,dx$$
$$= \frac{-6x^{\frac{2}{3}}e^{bx}\left(\int \frac{1}{x^{\frac{5}{3}}e^{bx}}dx\right)a - 14x^{\frac{2}{3}}e^{bx}\left(\int \frac{1}{x^{\frac{5}{3}}e^{bx}}dx\right) - 27abx - 9a - 27b^2x^2 - 63bx - 21}{27x^{\frac{2}{3}}e^{bx+a}b^2}$$

input
```
int(x^(1/3)*exp(-b*x-a)*(b*x+a+1),x)
```

output
```
( - 6*x**(2/3)*e**(b*x)*int(1/(x**(2/3)*e**(b*x)*x),x)*a - 14*x**(2/3)*e**
(b*x)*int(1/(x**(2/3)*e**(b*x)*x),x) - 27*a*b*x - 9*a - 27*b**2*x**2 - 63*
b*x - 21)/(27*x**(2/3)*e**(a + b*x)*b**2)
```

3.176 $\int \frac{\Gamma(2,a+bx)}{\sqrt[3]{x}} dx$

Optimal result

Integrand size = 13, antiderivative size = 74

$$\int \frac{\Gamma(2,a+bx)}{\sqrt[3]{x}} dx = -\frac{3ae^{-a}x^{2/3}\Gamma\left(\frac{5}{3},bx\right)}{2(bx)^{2/3}} + \frac{3}{2}x^{2/3}\Gamma(2,a+bx) - \frac{3e^{-a}x^{2/3}\Gamma\left(\frac{8}{3},bx\right)}{2(bx)^{2/3}}$$

output
```
-3/2*a*x^(2/3)*GAMMA(5/3,b*x)/exp(a)/(b*x)^(2/3)+3/2*x^(2/3)*exp(-b*x-a)*(
b*x+a+1)-3/2*x^(2/3)*GAMMA(8/3,b*x)/exp(a)/(b*x)^(2/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{\Gamma(2,a+bx)}{\sqrt[3]{x}} dx = -\frac{e^{-a-bx}x^{2/3}\left(2(5+3a)e^{bx}\Gamma\left(\frac{2}{3},bx\right) + 3(bx)^{2/3}\left(5+3a+3bx-3e^{a+bx}\Gamma(2,a+bx)\right)\right)}{6(bx)^{2/3}}$$

input
```
Integrate[Gamma[2, a + b*x]/x^(1/3),x]
```

output
```
-1/6*(E^(-a - b*x)*x^(2/3)*(2*(5 + 3*a)*E^(b*x)*Gamma[2/3, b*x] + 3*(b*x)^
(2/3)*(5 + 3*a + 3*b*x - 3*E^(a + b*x)*Gamma[2, a + b*x])))/(b*x)^(2/3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,a+bx)}{\sqrt[3]{x}}\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{3}{2}b\int e^{-a-bx}x^{2/3}(a+bx)dx+\frac{3}{2}x^{2/3}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{2629}$$

$$\frac{3}{2}b\int \left(be^{-a-bx}x^{5/3}+ae^{-a-bx}x^{2/3}\right)dx+\frac{3}{2}x^{2/3}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}x^{2/3}\Gamma(2,a+bx)+\frac{3}{2}b\left(-\frac{ae^{-a}x^{2/3}\Gamma\left(\frac{5}{3},bx\right)}{b(bx)^{2/3}}-\frac{e^{-a}x^{2/3}\Gamma\left(\frac{8}{3},bx\right)}{b(bx)^{2/3}}\right)$$

input
```
Int[Gamma[2, a + b*x]/x^(1/3),x]
```

output
```
(3*x^(2/3)*Gamma[2, a + b*x])/2 + (3*b*(-((a*x^(2/3)*Gamma[5/3, b*x])/(b*E
^a*(b*x)^(2/3))) - (x^(2/3)*Gamma[8/3, b*x])/(b*E^a*(b*x)^(2/3))))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.92

method	result
meijerg	$\frac{3\,\mathrm{e}^{-a-\frac{bx}{2}} x^{\frac{2}{3}} \operatorname{WhittakerM}\left(\frac{1}{3},\frac{5}{6},bx\right)}{5(bx)^{\frac{1}{3}}}+\frac{\mathrm{e}^{-a} a \left(\frac{9x^{\frac{2}{3}} b^{\frac{2}{3}} \mathrm{e}^{-\frac{bx}{2}} \operatorname{WhittakerM}\left(\frac{1}{3},\frac{5}{6},bx\right)}{10(bx)^{\frac{1}{3}}}+\frac{3\,\mathrm{e}^{-\frac{bx}{2}} \operatorname{WhittakerM}\left(\frac{4}{3},\frac{5}{6},bx\right)}{2x^{\frac{1}{3}} b^{\frac{1}{3}} (bx)^{\frac{1}{3}}}\right)}{b^{\frac{2}{3}}}+\frac{\mathrm{e}^{-a} \left(\frac{9x^{\frac{2}{3}} b^{\frac{2}{3}} \mathrm{e}\cdots\right.}{}$

input
```
int(exp(-b*x-a)*(b*x+a+1)/x^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/5*exp(-a-1/2*b*x)*x^(2/3)/(b*x)^(1/3)*WhittakerM(1/3,5/6,b*x)+exp(-a)*a/
b^(2/3)*(9/10*x^(2/3)*b^(2/3)/(b*x)^(1/3)*exp(-1/2*b*x)*WhittakerM(1/3,5/6
,b*x)+3/2/x^(1/3)/b^(1/3)/(b*x)^(1/3)*exp(-1/2*b*x)*WhittakerM(4/3,5/6,b*x
))+1/b^(2/3)*exp(-a)*(9/10*x^(2/3)*b^(2/3)/(b*x)^(1/3)*exp(-1/2*b*x)*Whitt
akerM(1/3,5/6,b*x)+3/2/x^(1/3)/b^(1/3)/(b*x)^(1/3)*exp(-1/2*b*x)*Whittaker
M(4/3,5/6,b*x))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{\Gamma(2,a+bx)}{\sqrt[3]{x}}\,dx = -\frac{2\,(3\,a+5)b^{\frac{1}{3}}e^{(-a)}\Gamma\left(\frac{2}{3},bx\right)+3\,\left((3\,b^{2}x+(3\,a+5)b)e^{(-bx-a)}-3\,b\Gamma(2,bx+a)\right)x^{\frac{2}{3}}}{6\,b}$$

input
```
integrate(gamma(2,b*x+a)/x^(1/3),x, algorithm="fricas")
```

output
```
-1/6*(2*(3*a + 5)*b^(1/3)*e^(-a)*gamma(2/3, b*x) + 3*((3*b^2*x + (3*a + 5)
*b)*e^(-b*x - a) - 3*b*gamma(2, b*x + a))*x^(2/3))/b
```

Sympy [A] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{\Gamma(2,a+bx)}{\sqrt[3]{x}}\,dx = \left(-\frac{a\sqrt[3]{bx}\Gamma\left(\frac{2}{3},bx\right)}{b\sqrt[3]{x}}-\frac{x^{\frac{2}{3}}\Gamma\left(\frac{5}{3},bx\right)}{(bx)^{\frac{2}{3}}}-\frac{\sqrt[3]{bx}\Gamma\left(\frac{2}{3},bx\right)}{b\sqrt[3]{x}}\right)e^{-a}$$

input
```
integrate(uppergamma(2,b*x+a)/x**(1/3),x)
```

output
```
(-a*(b*x)**(1/3)*uppergamma(2/3, b*x)/(b*x**(1/3)) - x**(2/3)*uppergamma(5
/3, b*x)/(b*x)**(2/3) - (b*x)**(1/3)*uppergamma(2/3, b*x)/(b*x**(1/3)))*ex
p(-a)
```

Maxima [F]

$$\int \frac{\Gamma(2, a+bx)}{\sqrt[3]{x}}\,dx = \int \frac{\Gamma(2, bx+a)}{x^{\frac{1}{3}}}\,dx$$

input
```
integrate(gamma(2,b*x+a)/x^(1/3),x, algorithm="maxima")
```

output
```
integrate(gamma(2, b*x + a)/x^(1/3), x)
```

Giac [F]

$$\int \frac{\Gamma(2, a+bx)}{\sqrt[3]{x}}\,dx = \int \frac{\Gamma(2, bx+a)}{x^{\frac{1}{3}}}\,dx$$

input
```
integrate(gamma(2,b*x+a)/x^(1/3),x, algorithm="giac")
```

output
```
integrate(gamma(2, b*x + a)/x^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07

$$\int \frac{\Gamma(2, a+bx)}{\sqrt[3]{x}}\,dx = -x^{2/3}\,\mathrm{e}^{-a-b\,x} - \frac{2\,x^{2/3}\,\mathrm{e}^{-a}\,\Gamma\left(\frac{2}{3}, b\,x\right)}{3\,{(b\,x)}^{2/3}} - \frac{\mathrm{e}^{-a}\,{(b\,x)}^{1/3}\,\Gamma\left(\frac{2}{3}, b\,x\right)}{b\,x^{1/3}} - \frac{a\,\mathrm{e}^{-a}\,{(b\,x)}^{1/3}\,\Gamma\left(\frac{2}{3}, b\,x\right)}{b\,x^{1/3}}$$

input
```
int((exp(- a - b*x)*(a + b*x + 1))/x^(1/3),x)
```

output
```
- x^(2/3)*exp(- a - b*x) - (2*x^(2/3)*exp(-a)*igamma(2/3, b*x))/(3*(b*x)^(
2/3)) - (exp(-a)*(b*x)^(1/3)*igamma(2/3, b*x))/(b*x^(1/3)) - (a*exp(-a)*(b
*x)^(1/3)*igamma(2/3, b*x))/(b*x^(1/3))
```

Reduce [F]

$$\int \frac{\Gamma(2, a+bx)}{\sqrt[3]{x}}\,dx = \frac{-3x^{\frac{1}{3}}e^{bx}\left(\int \frac{1}{x^{\frac{4}{3}}e^{bx}}dx\right)a - 5x^{\frac{1}{3}}e^{bx}\left(\int \frac{1}{x^{\frac{4}{3}}e^{bx}}dx\right) - 9a - 9bx - 15}{9x^{\frac{1}{3}}e^{bx+a}b}$$

input
```
int(exp(-b*x-a)*(b*x+a+1)/x^(1/3),x)
```

output
```
( - 3*x**(1/3)*e**(b*x)*int(1/(x**(1/3)*e**(b*x)*x),x)*a - 5*x**(1/3)*e**(
b*x)*int(1/(x**(1/3)*e**(b*x)*x),x) - 9*a - 9*b*x - 15)/(9*x**(1/3)*e**(a
+ b*x)*b)
```

3.177 $\int \frac{\Gamma(2,a+bx)}{x^{2/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{\Gamma(2,a+bx)}{x^{2/3}}\,dx=-\frac{3ae^{-a}\sqrt[3]{x}\Gamma\left(\frac{4}{3},bx\right)}{\sqrt[3]{bx}}+3\sqrt[3]{x}\Gamma(2,a+bx)-\frac{3e^{-a}\sqrt[3]{x}\Gamma\left(\frac{7}{3},bx\right)}{\sqrt[3]{bx}}$$

output
```
-3*a*x^(1/3)*GAMMA(4/3,b*x)/exp(a)/(b*x)^(1/3)+3*x^(1/3)*exp(-b*x-a)*(b*x+
a+1)-3*x^(1/3)*GAMMA(7/3,b*x)/exp(a)/(b*x)^(1/3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{\Gamma(2,a+bx)}{x^{2/3}}\,dx=\frac{1}{3}e^{-a}\sqrt[3]{x}\left(-\frac{(4+3a)\Gamma\left(\frac{1}{3},bx\right)}{\sqrt[3]{bx}}-3e^{-bx}\left(4+3a+3bx-3e^{a+bx}\Gamma(2,a+bx)\right)\right)$$

input
```
Integrate[Gamma[2, a + b*x]/x^(2/3),x]
```

output
```
(x^(1/3)*(-(((4 + 3*a)*Gamma[1/3, b*x])/(b*x)^(1/3)) - (3*(4 + 3*a + 3*b*x
 - 3*E^(a + b*x)*Gamma[2, a + b*x]))/E^(b*x)))/(3*E^a)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,a+bx)}{x^{2/3}}\,dx$$

$$\Big\downarrow \text{ 7119}$$

$$3b\int e^{-a-bx}\sqrt[3]{x}(a+bx)dx+3\sqrt[3]{x}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{ 2629}$$

$$3b\int \left(be^{-a-bx}x^{4/3}+ae^{-a-bx}\sqrt[3]{x}\right)dx+3\sqrt[3]{x}\Gamma(2,a+bx)$$

$$\Big\downarrow \text{ 2009}$$

$$3\sqrt[3]{x}\Gamma(2,a+bx)+3b\left(-\frac{ae^{-a}\sqrt[3]{x}\Gamma\left(\frac{4}{3},bx\right)}{b\sqrt[3]{bx}}-\frac{e^{-a}\sqrt[3]{x}\Gamma\left(\frac{7}{3},bx\right)}{b\sqrt[3]{bx}}\right)$$

input
```
Int[Gamma[2, a + b*x]/x^(2/3),x]
```

output
```
3*x^(1/3)*Gamma[2, a + b*x] + 3*b*(-((a*x^(1/3)*Gamma[4/3, b*x])/(b*E^a*(b
*x)^(1/3))) - (x^(1/3)*Gamma[7/3, b*x])/(b*E^a*(b*x)^(1/3)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.41 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.09

method	result
meijerg	$\frac{3\,e^{-a-\frac{bx}{2}}\,x^{\frac{1}{3}}\,\mathrm{WhittakerM}\left(\frac{1}{6},\frac{2}{3},bx\right)}{4(bx)^{\frac{1}{6}}}+\frac{e^{-a}a\left(\frac{9x^{\frac{1}{3}}b^{\frac{1}{3}}e^{-\frac{bx}{2}}\,\mathrm{WhittakerM}\left(\frac{1}{6},\frac{2}{3},bx\right)}{4(bx)^{\frac{1}{6}}}+\frac{3\,e^{-\frac{bx}{2}}\,\mathrm{WhittakerM}\left(\frac{7}{6},\frac{2}{3},bx\right)}{x^{\frac{2}{3}}b^{\frac{2}{3}}(bx)^{\frac{1}{6}}}\right)}{b^{\frac{1}{3}}}+\frac{e^{-a}\left(\frac{9x^{\frac{1}{3}}b^{\frac{1}{3}}e\cdots\right.}{\cdots}$

input
```
int(exp(-b*x-a)*(b*x+a+1)/x^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/4*exp(-a-1/2*b*x)*x^(1/3)/(b*x)^(1/6)*WhittakerM(1/6,2/3,b*x)+exp(-a)*a/
b^(1/3)*(9/4*x^(1/3)*b^(1/3)/(b*x)^(1/6)*exp(-1/2*b*x)*WhittakerM(1/6,2/3,
b*x)+3/x^(2/3)/b^(2/3)/(b*x)^(1/6)*exp(-1/2*b*x)*WhittakerM(7/6,2/3,b*x))+
1/b^(1/3)*exp(-a)*(9/4*x^(1/3)*b^(1/3)/(b*x)^(1/6)*exp(-1/2*b*x)*Whittaker
M(1/6,2/3,b*x)+3/x^(2/3)/b^(2/3)/(b*x)^(1/6)*exp(-1/2*b*x)*WhittakerM(7/6,
2/3,b*x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{\Gamma(2,a+bx)}{x^{2/3}}\,dx = -\frac{(3\,a+4)b^{\frac{2}{3}}e^{(-a)}\Gamma\left(\frac{1}{3},bx\right)+3\left((3\,b^2x+(3\,a+4)b)e^{(-bx-a)}-3\,b\Gamma(2,bx+a)\right)x^{\frac{1}{3}}}{3\,b}$$

input
```
integrate(gamma(2,b*x+a)/x^(2/3),x, algorithm="fricas")
```

output
```
-1/3*((3*a + 4)*b^(2/3)*e^(-a)*gamma(1/3, b*x) + 3*((3*b^2*x + (3*a + 4)*b
)*e^(-b*x - a) - 3*b*gamma(2, b*x + a))*x^(1/3))/b
```

Sympy [A] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{\Gamma(2,a+bx)}{x^{2/3}}\,dx = \left(-\frac{a(bx)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},bx\right)}{bx^{\frac{2}{3}}}-\frac{\sqrt[3]{x}\Gamma\left(\frac{4}{3},bx\right)}{\sqrt[3]{bx}}-\frac{(bx)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},bx\right)}{bx^{\frac{2}{3}}}\right)e^{-a}$$

input
```
integrate(uppergamma(2,b*x+a)/x**(2/3),x)
```

output
```
(-a*(b*x)**(2/3)*uppergamma(1/3, b*x)/(b*x**(2/3)) - x**(1/3)*uppergamma(4
/3, b*x)/(b*x)**(1/3) - (b*x)**(2/3)*uppergamma(1/3, b*x)/(b*x**(2/3)))*ex
p(-a)
```

Maxima [F]

$$\int \frac{\Gamma(2, a+bx)}{x^{2/3}} \, dx = \int \frac{\Gamma(2, bx+a)}{x^{\frac{2}{3}}} \, dx$$

input
```
integrate(gamma(2,b*x+a)/x^(2/3),x, algorithm="maxima")
```

output
```
integrate(gamma(2, b*x + a)/x^(2/3), x)
```

Giac [F]

$$\int \frac{\Gamma(2, a+bx)}{x^{2/3}} \, dx = \int \frac{\Gamma(2, bx+a)}{x^{\frac{2}{3}}} \, dx$$

input
```
integrate(gamma(2,b*x+a)/x^(2/3),x, algorithm="giac")
```

output
```
integrate(gamma(2, b*x + a)/x^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.16

$$\int \frac{\Gamma(2, a+bx)}{x^{2/3}} \, dx = -x^{1/3} \, e^{-a-b\,x} - \frac{x^{1/3} \, e^{-a} \, \Gamma\left(\frac{1}{3}, b\,x\right)}{3 \, (b\,x)^{1/3}} - \frac{e^{-a} \, (b\,x)^{2/3} \, \Gamma\left(\frac{1}{3}, b\,x\right)}{b\,x^{2/3}} - \frac{a\,e^{-a} \, (b\,x)^{2/3} \, \Gamma\left(\frac{1}{3}, b\,x\right)}{b\,x^{2/3}}$$

input
```
int((exp(- a - b*x)*(a + b*x + 1))/x^(2/3),x)
```

output
```
- x^(1/3)*exp(- a - b*x) - (x^(1/3)*exp(-a)*igamma(1/3, b*x))/(3*(b*x)^(1/
3)) - (exp(-a)*(b*x)^(2/3)*igamma(1/3, b*x))/(b*x^(2/3)) - (a*exp(-a)*(b*x
)^(2/3)*igamma(1/3, b*x))/(b*x^(2/3))
```

Reduce [F]

$$\int \frac{\Gamma(2,a+bx)}{x^{2/3}}\,dx = \frac{-6x^{\frac{2}{3}}e^{bx}\left(\int \frac{1}{x^{\frac{5}{3}}e^{bx}}dx\right)a - 8x^{\frac{2}{3}}e^{bx}\left(\int \frac{1}{x^{\frac{5}{3}}e^{bx}}dx\right) - 9a - 9bx - 12}{9x^{\frac{2}{3}}e^{bx+a}b}$$

input
```
int(exp(-b*x-a)*(b*x+a+1)/x^(2/3),x)
```

output
```
( - 6*x**(2/3)*e**(b*x)*int(1/(x**(2/3)*e**(b*x)*x),x)*a - 8*x**(2/3)*e**(
b*x)*int(1/(x**(2/3)*e**(b*x)*x),x) - 9*a - 9*b*x - 12)/(9*x**(2/3)*e**(a
+ b*x)*b)
```

3.178 $\int \frac{\Gamma(2,a+bx)}{x^{4/3}} dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{\Gamma(2,a+bx)}{x^{4/3}} dx = \frac{3ae^{-a}\sqrt[3]{bx}\Gamma\left(\frac{2}{3},bx\right)}{\sqrt[3]{x}} + \frac{3be^{-a}x^{2/3}\Gamma\left(\frac{5}{3},bx\right)}{(bx)^{2/3}} - \frac{3\Gamma(2,a+bx)}{\sqrt[3]{x}}$$

output
```
3*a*(b*x)^(1/3)*GAMMA(2/3,b*x)/exp(a)/x^(1/3)+3*b*x^(2/3)*GAMMA(5/3,b*x)/e
xp(a)/(b*x)^(2/3)-3*exp(-b*x-a)*(b*x+a+1)/x^(1/3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

$$\int \frac{\Gamma(2,a+bx)}{x^{4/3}} dx = \frac{e^{-a}\left(3be^{-bx}x + (2+3a)\sqrt[3]{bx}\Gamma\left(\frac{2}{3},bx\right) - 3e^a\Gamma(2,a+bx)\right)}{\sqrt[3]{x}}$$

input
```
Integrate[Gamma[2, a + b*x]/x^(4/3),x]
```

output
```
((3*b*x)/E^(b*x) + (2 + 3*a)*(b*x)^(1/3)*Gamma[2/3, b*x] - 3*E^a*Gamma[2,
a + b*x])/(E^a*x^(1/3))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(2,a+bx)}{x^{4/3}} \, dx$$

$$\Big\downarrow \text{7119}$$

$$-3b\int \frac{e^{-a-bx}(a+bx)}{\sqrt[3]{x}}dx - \frac{3\Gamma(2,a+bx)}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2629}$$

$$-3b\int \left(\frac{e^{-a-bx}a}{\sqrt[3]{x}} + be^{-a-bx}x^{2/3}\right)dx - \frac{3\Gamma(2,a+bx)}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2009}$$

$$-3b\left(-\frac{e^{-a}x^{2/3}\Gamma\left(\frac{5}{3},bx\right)}{(bx)^{2/3}} - \frac{ae^{-a}\sqrt[3]{bx}\Gamma\left(\frac{2}{3},bx\right)}{b\sqrt[3]{x}}\right) - \frac{3\Gamma(2,a+bx)}{\sqrt[3]{x}}$$

input
```
Int[Gamma[2, a + b*x]/x^(4/3),x]
```

output
```
-3*b*(-((a*(b*x)^(1/3)*Gamma[2/3, b*x])/(b*E^a*x^(1/3))) - (x^(2/3)*Gamma[
5/3, b*x])/(E^a*(b*x)^(2/3))) - (3*Gamma[2, a + b*x])/x^(1/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.70

method	result
meijerg	$b^{\frac{1}{3}} \mathrm{e}^{-a} \left(\frac{9 x^{\frac{2}{3}} b^{\frac{2}{3}} \mathrm{e}^{-\frac{b x}{2}} \operatorname{WhittakerM}\left(\frac{1}{3},\frac{5}{6},b x\right)}{10 (b x)^{\frac{1}{3}}}+\frac{3 \,\mathrm{e}^{-\frac{b x}{2}} \operatorname{WhittakerM}\left(\frac{4}{3},\frac{5}{6},b x\right)}{2 x^{\frac{1}{3}} b^{\frac{1}{3}} (b x)^{\frac{1}{3}}}\right)+\mathrm{e}^{-a} a \,b^{\frac{1}{3}} \left(-\frac{27 x^{\frac{2}{3}} b^{\frac{2}{3}} \mathrm{e}^{-\frac{b x}{2}} \operatorname{WhittakerM}(}{10 (b x)^{\frac{1}{3}}}\right.$

input
```
int(exp(-b*x-a)*(b*x+a+1)/x^(4/3),x,method=_RETURNVERBOSE)
```

output
```
b^(1/3)*exp(-a)*(9/10*x^(2/3)*b^(2/3)/(b*x)^(1/3)*exp(-1/2*b*x)*WhittakerM
(1/3,5/6,b*x)+3/2/x^(1/3)/b^(1/3)/(b*x)^(1/3)*exp(-1/2*b*x)*WhittakerM(4/3
,5/6,b*x))+exp(-a)*a*b^(1/3)*(-27/10*x^(2/3)*b^(2/3)/(b*x)^(1/3)*exp(-1/2*
b*x)*WhittakerM(1/3,5/6,b*x)-3/2/x^(4/3)/b^(4/3)*(3*b*x+2)/(b*x)^(1/3)*exp
(-1/2*b*x)*WhittakerM(4/3,5/6,b*x))+b^(1/3)*exp(-a)*(-27/10*x^(2/3)*b^(2/3
)/(b*x)^(1/3)*exp(-1/2*b*x)*WhittakerM(1/3,5/6,b*x)-3/2/x^(4/3)/b^(4/3)*(3
*b*x+2)/(b*x)^(1/3)*exp(-1/2*b*x)*WhittakerM(4/3,5/6,b*x))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int \frac{\Gamma(2,a+bx)}{x^{4/3}}\,dx=\frac{(3\,a+2)b^{\frac{1}{3}}xe^{(-a)}\Gamma\left(\frac{2}{3},bx\right)+3\left(bxe^{(-bx-a)}-\Gamma(2,bx+a)\right)x^{\frac{2}{3}}}{x}$$

input
```
integrate(gamma(2,b*x+a)/x^(4/3),x, algorithm="fricas")
```

output
```
((3*a + 2)*b^(1/3)*x*e^(-a)*gamma(2/3, b*x) + 3*(b*x*e^(-b*x - a) - gamma(
2, b*x + a))*x^(2/3))/x
```

Sympy [A] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{\Gamma(2, a+bx)}{x^{4/3}}\, dx = \left(-\frac{a(bx)^{\frac{4}{3}}\,\Gamma\left(-\frac{1}{3}, bx\right)}{bx^{\frac{4}{3}}} - \frac{\sqrt[3]{bx}\,\Gamma\left(\frac{2}{3}, bx\right)}{\sqrt[3]{x}} - \frac{(bx)^{\frac{4}{3}}\,\Gamma\left(-\frac{1}{3}, bx\right)}{bx^{\frac{4}{3}}}\right)e^{-a}$$

input
```
integrate(uppergamma(2,b*x+a)/x**(4/3),x)
```

output
```
(-a*(b*x)**(4/3)*uppergamma(-1/3, b*x)/(b*x**(4/3)) - (b*x)**(1/3)*upperga
mma(2/3, b*x)/x**(1/3) - (b*x)**(4/3)*uppergamma(-1/3, b*x)/(b*x**(4/3)))*
exp(-a)
```

Maxima [F]

$$\int \frac{\Gamma(2, a+bx)}{x^{4/3}}\, dx = \int \frac{\Gamma(2, bx+a)}{x^{\frac{4}{3}}}\, dx$$

input
```
integrate(gamma(2,b*x+a)/x^(4/3),x, algorithm="maxima")
```

output
```
integrate(gamma(2, b*x + a)/x^(4/3), x)
```

Giac [F]

$$\int \frac{\Gamma(2,a+bx)}{x^{4/3}}\,dx = \int \frac{\Gamma(2,bx+a)}{x^{\frac{4}{3}}}\,dx$$

input
```
integrate(gamma(2,b*x+a)/x^(4/3),x, algorithm="giac")
```

output
```
integrate(gamma(2, b*x + a)/x^(4/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(2,a+bx)}{x^{4/3}}\,dx = \frac{2\,\mathrm{e}^{-a}\,{(b\,x)}^{1/3}\,\Gamma\left(\frac{2}{3},b\,x\right)}{x^{1/3}} - \frac{3\,a\,\mathrm{e}^{-a-b\,x}}{x^{1/3}} - \frac{3\,\mathrm{e}^{-a-b\,x}}{x^{1/3}} + \frac{3\,a\,\mathrm{e}^{-a}\,{(b\,x)}^{1/3}\,\Gamma\left(\frac{2}{3},b\,x\right)}{x^{1/3}}$$

input
```
int((exp(- a - b*x)*(a + b*x + 1))/x^(4/3),x)
```

output
```
(2*exp(-a)*(b*x)^(1/3)*igamma(2/3, b*x))/x^(1/3) - (3*a*exp(- a - b*x))/x^
(1/3) - (3*exp(- a - b*x))/x^(1/3) + (3*a*exp(-a)*(b*x)^(1/3)*igamma(2/3,
b*x))/x^(1/3)
```

Reduce [F]

$$\int \frac{\Gamma(2,a+bx)}{x^{4/3}}\,dx = \frac{3x^{\frac{1}{3}}e^{bx}\left(\int \frac{1}{x^{\frac{4}{3}}e^{bx}}dx\right)a + 2x^{\frac{1}{3}}e^{bx}\left(\int \frac{1}{x^{\frac{4}{3}}e^{bx}}dx\right) - 3}{3x^{\frac{1}{3}}e^{bx+a}}$$

input
```
int(exp(-b*x-a)*(b*x+a+1)/x^(4/3),x)
```

output
```
(3*x**(1/3)*e**(b*x)*int(1/(x**(1/3)*e**(b*x)*x),x)*a + 2*x**(1/3)*e**(b*x
)*int(1/(x**(1/3)*e**(b*x)*x),x) - 3)/(3*x**(1/3)*e**(a + b*x))
```

3.179 $\int (c+dx)^m \Gamma(3,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 221

$$\begin{aligned}\int (c+dx)^m \Gamma(3,a+bx)\,dx &= \frac{(c+dx)^{1+m}\Gamma(3,a+bx)}{d(1+m)} \\ &\quad - \frac{(bc-ad)^2 e^{-a+\frac{bc}{d}}(c+dx)^m \left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(2+m,\frac{b(c+dx)}{d}\right)}{bd^2(1+m)} \\ &\quad + \frac{2(bc-ad)e^{-a+\frac{bc}{d}}(c+dx)^m \left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(3+m,\frac{b(c+dx)}{d}\right)}{bd(1+m)} \\ &\quad - \frac{e^{-a+\frac{bc}{d}}(c+dx)^m \left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(4+m,\frac{b(c+dx)}{d}\right)}{b(1+m)}\end{aligned}$$

output
```
2*(d*x+c)^(1+m)*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2)/d/(1+m)-(-a*d+b*c)^2*e
xp(-a+b*c/d)*(d*x+c)^m*GAMMA(2+m,b*(d*x+c)/d)/b/d^2/(1+m)/((b*(d*x+c)/d)^m
)+2*(-a*d+b*c)*exp(-a+b*c/d)*(d*x+c)^m*GAMMA(3+m,b*(d*x+c)/d)/b/d/(1+m)/((
b*(d*x+c)/d)^m)-exp(-a+b*c/d)*(d*x+c)^m*GAMMA(4+m,b*(d*x+c)/d)/b/(1+m)/((b
*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.67

$$\int (c+dx)^m\Gamma(3,a+bx)\,dx$$
$$=\frac{e^{-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\left(bde^a\left(b\left(\frac{c}{d}+x\right)\right)^m(c+dx)\Gamma(3,a+bx)-e^{\frac{bc}{d}}\left((bc-ad)^2\Gamma\left(2+m,\frac{b(c+dx)}{d}\right)+\right.\right.}{bd^2(1+m)}$$

input
```
Integrate[(c + d*x)^m*Gamma[3, a + b*x],x]
```

output
```
((c + d*x)^m*(b*d*E^a*(b*(c/d + x))^m*(c + d*x)*Gamma[3, a + b*x] - E^((b*
c)/d)*((b*c - a*d)^2*Gamma[2 + m, (b*(c + d*x))/d] + d*((-2*b*c + 2*a*d)*G
amma[3 + m, (b*(c + d*x))/d] + d*Gamma[4 + m, (b*(c + d*x))/d]))))/(b*d^2*
E^a*(1 + m)*((b*(c + d*x))/d)^m)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(3,a+bx)(c+dx)^m\,dx$$
$$\Big\downarrow \text{7119}$$
$$\frac{b\int e^{-a-bx}(a+bx)^2(c+dx)^{m+1}dx}{d(m+1)}+\frac{\Gamma(3,a+bx)(c+dx)^{m+1}}{d(m+1)}$$
$$\Big\downarrow \text{2629}$$
$$\frac{b\int\left(\frac{(ad-bc)^2e^{-a-bx}(c+dx)^{m+1}}{d^2}-\frac{2b(bc-ad)e^{-a-bx}(c+dx)^{m+2}}{d^2}+\frac{b^2e^{-a-bx}(c+dx)^{m+3}}{d^2}\right)dx}{d(m+1)}+$$
$$\frac{\Gamma(3,a+bx)(c+dx)^{m+1}}{d(m+1)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{(bc-ad)^2e^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+2,\frac{b(c+dx)}{d}\right)}{b^2d}+\frac{2(bc-ad)e^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+3,\frac{b(c+dx)}{d}\right)}{b^2}-\frac{de^{\frac{bc}{d}-a}(c+dx)}{}\right.}{d(m+1)}$$
$$\frac{\Gamma(3,a+bx)(c+dx)^{m+1}}{d(m+1)}$$

input
```
Int[(c + d*x)^m*Gamma[3, a + b*x],x]
```

output
```
((c + d*x)^(1 + m)*Gamma[3, a + b*x])/(d*(1 + m)) + (b*(-(((b*c - a*d)^2*E
^(-a + (b*c)/d)*(c + d*x)^m*Gamma[2 + m, (b*(c + d*x))/d])/(b^2*d*((b*(c +
 d*x))/d)^m)) + (2*(b*c - a*d)*E^(-a + (b*c)/d)*(c + d*x)^m*Gamma[3 + m, (
b*(c + d*x))/d])/(b^2*((b*(c + d*x))/d)^m) - (d*E^(-a + (b*c)/d)*(c + d*x)
^m*Gamma[4 + m, (b*(c + d*x))/d])/(b^2*((b*(c + d*x))/d)^m)))/(d*(1 + m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [F]

$$\int 2(dx+c)^m \mathrm{e}^{-bx-a}\left(1+bx+a+\frac{(bx+a)^2}{2}\right)dx$$

input

```
int(2*(d*x+c)^m*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2),x)
```

output

```
int(2*(d*x+c)^m*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.42

$$\int (c+dx)^m\Gamma(3,a+bx)\,dx =$$

$$-\frac{(d^2m^3+b^2c^2-2\,(a+2)bcd+(a^2+4\,a+6)d^2-2\,(bcd-(a+3)d^2)m^2+(b^2c^2-2\,(a+3)bcd+(a^2$$

input

```
integrate((d*x+c)^m*gamma(3,b*x+a),x, algorithm="fricas")
```

output

```
-((d^2*m^3 + b^2*c^2 - 2*(a + 2)*b*c*d + (a^2 + 4*a + 6)*d^2 - 2*(b*c*d -
(a + 3)*d^2)*m^2 + (b^2*c^2 - 2*(a + 3)*b*c*d + (a^2 + 6*a + 11)*d^2)*m)*e
^(-(d*m*log(b/d) - b*c + a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d) + ((b^3*d^2
*x^3 + b*c*d*m^2 - b^2*c^2 + (a^2 + 4*a + 6)*b*c*d + (b^3*c*d + (2*a + 3)*
b^2*d^2 + b^2*d^2*m)*x^2 - (b^2*c^2 - (2*a + 5)*b*c*d)*m + (2*(a + 1)*b^2*
c*d + (2*a + 5)*b*d^2*m + b*d^2*m^2 + (a^2 + 4*a + 6)*b*d^2)*x)*e^(-b*x -
a) - (b*d^2*x + b*c*d)*gamma(3, b*x + a))*(d*x + c)^m)/(b*d^2*m + b*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m\Gamma(3,a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*uppergamma(3,b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^m\Gamma(3,a+bx)\,dx = \int (dx+c)^m\Gamma(3,bx+a)\;dx$$

input
```
integrate((d*x+c)^m*gamma(3,b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*gamma(3, b*x + a), x)
```

Giac [F]

$$\int (c+dx)^m\Gamma(3,a+bx)\,dx = \int (dx+c)^m\Gamma(3,bx+a)\;dx$$

input
```
integrate((d*x+c)^m*gamma(3,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*gamma(3, b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m\Gamma(3,a+bx)\,dx=\int 2\,\mathrm{e}^{-a-b\,x}\,(c+d\,x)^m\,\left(a+b\,x+\frac{(a+b\,x)^2}{2}+1\right)\,dx$$

input
```
int(2*exp(- a - b*x)*(c + d*x)^m*(a + b*x + (a + b*x)^2/2 + 1),x)
```

output
```
int(2*exp(- a - b*x)*(c + d*x)^m*(a + b*x + (a + b*x)^2/2 + 1), x)
```

Reduce [F]

$$\int (c+dx)^m\Gamma(3,a+bx)\,dx$$
$$=\frac{-(dx+c)^m\,a^2d-2(dx+c)^m\,abdx-2(dx+c)^m\,adm-4(dx+c)^m\,ad-(dx+c)^m\,b^2d\,x^2+(dx+c)^m\,b}{}$$

input
```
int(2*(d*x+c)^m*exp(-b*x-a)*(1+b*x+a+1/2*(b*x+a)^2),x)
```

output
```
( - (c + d*x)**m*a**2*d - 2*(c + d*x)**m*a*b*d*x - 2*(c + d*x)**m*a*d*m -
4*(c + d*x)**m*a*d - (c + d*x)**m*b**2*d*x**2 + (c + d*x)**m*b*c*m - (c +
d*x)**m*b*d*m*x - 4*(c + d*x)**m*b*d*x - (c + d*x)**m*d*m**2 - 5*(c + d*x)
**m*d*m - 6*(c + d*x)**m*d + e**(b*x)*int((c + d*x)**m/(e**(b*x)*c + e**(b
*x)*d*x),x)*a**2*d**2*m - 2*e**(b*x)*int((c + d*x)**m/(e**(b*x)*c + e**(b*
x)*d*x),x)*a*b*c*d*m + 2*e**(b*x)*int((c + d*x)**m/(e**(b*x)*c + e**(b*x)*
d*x),x)*a*d**2*m**2 + 4*e**(b*x)*int((c + d*x)**m/(e**(b*x)*c + e**(b*x)*d
*x),x)*a*d**2*m + e**(b*x)*int((c + d*x)**m/(e**(b*x)*c + e**(b*x)*d*x),x)
*b**2*c**2*m - 2*e**(b*x)*int((c + d*x)**m/(e**(b*x)*c + e**(b*x)*d*x),x)*
b*c*d*m**2 - 4*e**(b*x)*int((c + d*x)**m/(e**(b*x)*c + e**(b*x)*d*x),x)*b*
c*d*m + e**(b*x)*int((c + d*x)**m/(e**(b*x)*c + e**(b*x)*d*x),x)*d**2*m**3
 + 5*e**(b*x)*int((c + d*x)**m/(e**(b*x)*c + e**(b*x)*d*x),x)*d**2*m**2 +
6*e**(b*x)*int((c + d*x)**m/(e**(b*x)*c + e**(b*x)*d*x),x)*d**2*m)/(e**(a
+ b*x)*b*d)
```

3.180 $\int (c+dx)^m\Gamma(2,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 150

$$\int (c+dx)^m\Gamma(2,a+bx)\,dx = \frac{(c+dx)^{1+m}\Gamma(2,a+bx)}{d(1+m)}+\frac{(bc-ad)e^{-a+\frac{bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(2+m,\frac{b(c+dx)}{d}\right)}{bd(1+m)} - \frac{e^{-a+\frac{bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(3+m,\frac{b(c+dx)}{d}\right)}{b(1+m)}$$

output
```
(d*x+c)^(1+m)*exp(-b*x-a)*(b*x+a+1)/d/(1+m)+(-a*d+b*c)*exp(-a+b*c/d)*(d*x+
c)^m*GAMMA(2+m,b*(d*x+c)/d)/b/d/(1+m)/((b*(d*x+c)/d)^m)-exp(-a+b*c/d)*(d*x
+c)^m*GAMMA(3+m,b*(d*x+c)/d)/b/(1+m)/((b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.78

$$\int (c+dx)^m\Gamma(2,a+bx)\,dx$$
$$=\frac{e^{-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\left(be^a\left(b\left(\frac{c}{d}+x\right)\right)^m(c+dx)\Gamma(2,a+bx)+e^{\frac{bc}{d}}\left((bc-ad)\Gamma\left(2+m,\frac{b(c+dx)}{d}\right)-d\Gamma\left(3+m,\frac{b(c+dx)}{d}\right)\right)\right)}{bd(1+m)}$$

input
```
Integrate[(c + d*x)^m*Gamma[2, a + b*x],x]
```

output
```
((c + d*x)^m*(b*E^a*(b*(c/d + x))^m*(c + d*x)*Gamma[2, a + b*x] + E^((b*c)
/d)*((b*c - a*d)*Gamma[2 + m, (b*(c + d*x))/d] - d*Gamma[3 + m, (b*(c + d*
x))/d])))/(b*d*E^a*(1 + m)*((b*(c + d*x))/d)^m)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(2,a+bx)(c+dx)^m\,dx$$
$$\Big\downarrow \text{7119}$$
$$\frac{b\int e^{-a-bx}(a+bx)(c+dx)^{m+1}dx}{d(m+1)}+\frac{\Gamma(2,a+bx)(c+dx)^{m+1}}{d(m+1)}$$
$$\Big\downarrow \text{2629}$$
$$\frac{b\int\left(\frac{(ad-bc)e^{-a-bx}(c+dx)^{m+1}}{d}+\frac{be^{-a-bx}(c+dx)^{m+2}}{d}\right)dx}{d(m+1)}+\frac{\Gamma(2,a+bx)(c+dx)^{m+1}}{d(m+1)}$$
$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{(bc-ad)e^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+2,\frac{b(c+dx)}{d}\right)}{b^2}-\frac{de^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+3,\frac{b(c+dx)}{d}\right)}{b^2}\right)}{d(m+1)}+\frac{\Gamma(2,a+bx)(c+dx)^{m+1}}{d(m+1)}$$

input
```
Int[(c + d*x)^m*Gamma[2, a + b*x],x]
```

output
```
((c + d*x)^(1 + m)*Gamma[2, a + b*x])/(d*(1 + m)) + (b*(((b*c - a*d)*E^(-a
 + (b*c)/d)*(c + d*x)^m*Gamma[2 + m, (b*(c + d*x))/d])/(b^2*((b*(c + d*x))
/d)^m) - (d*E^(-a + (b*c)/d)*(c + d*x)^m*Gamma[3 + m, (b*(c + d*x))/d])/(b
^2*((b*(c + d*x))/d)^m)))/(d*(1 + m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [F]

$$\int (dx+c)^m \, \mathrm{e}^{-bx-a}(bx+a+1)\, dx$$

input
```
int((d*x+c)^m*exp(-b*x-a)*(b*x+a+1),x)
```

output
```
int((d*x+c)^m*exp(-b*x-a)*(b*x+a+1),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03

$$\int (c+dx)^m\Gamma(2,a+bx)\, dx =$$

$$-\frac{(dm^2-bc+(a+2)d-(bc-(a+3)d)m)e^{\left(-\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)}\Gamma\left(m+1,\frac{bdx+bc}{d}\right)+\left((b^2dx^2+(a+2)bc\right.}{bdm+bd}$$

input
```
integrate((d*x+c)^m*gamma(2,b*x+a),x, algorithm="fricas")
```

output
```
-((d*m^2 - b*c + (a + 2)*d - (b*c - (a + 3)*d)*m)*e^(-(d*m*log(b/d) - b*c
+ a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d) + ((b^2*d*x^2 + (a + 2)*b*c + b*c*
m + (b^2*c + (a + 2)*b*d + b*d*m)*x)*e^(-b*x - a) - (b*d*x + b*c)*gamma(2,
 b*x + a))*(d*x + c)^m)/(b*d*m + b*d)
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m\Gamma(2,a+bx)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*uppergamma(2,b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [F]

$$\int (c+dx)^m\Gamma(2,a+bx)\,dx=\int (dx+c)^m\Gamma(2,bx+a)\,dx$$

input
```
integrate((d*x+c)^m*gamma(2,b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*gamma(2, b*x + a), x)
```

Giac [F]

$$\int (c+dx)^m\Gamma(2,a+bx)\,dx=\int (dx+c)^m\Gamma(2,bx+a)\,dx$$

input
```
integrate((d*x+c)^m*gamma(2,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*gamma(2, b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m\Gamma(2,a+bx)\,dx=\int e^{-a-b\,x}\,(c+d\,x)^m\,(a+b\,x+1)\,dx$$

input
```
int(exp(- a - b*x)*(c + d*x)^m*(a + b*x + 1),x)
```

output
```
int(exp(- a - b*x)*(c + d*x)^m*(a + b*x + 1), x)
```

Reduce [F]

$$\int (c+dx)^m\Gamma(2,a+bx)\,dx$$
$$= \frac{-(dx+c)^m a-(dx+c)^m bx-(dx+c)^m m-2(dx+c)^m+e^{bx}\left(\int \frac{(dx+c)^m}{e^{bx}c+e^{bx}dx}dx\right)adm-e^{bx}\left(\int \frac{(dx+c)^m}{e^{bx}c+e^{bx}dx}\right.}{e^{bx+a}b}$$

input
```
int((d*x+c)^m*exp(-b*x-a)*(b*x+a+1),x)
```

output
```
( - (c + d*x)**m*a - (c + d*x)**m*b*x - (c + d*x)**m*m - 2*(c + d*x)**m +
e**(b*x)*int((c + d*x)**m/(e**(b*x)*c + e**(b*x)*d*x),x)*a*d*m - e**(b*x)*
int((c + d*x)**m/(e**(b*x)*c + e**(b*x)*d*x),x)*b*c*m + e**(b*x)*int((c +
d*x)**m/(e**(b*x)*c + e**(b*x)*d*x),x)*d*m**2 + 2*e**(b*x)*int((c + d*x)**
m/(e**(b*x)*c + e**(b*x)*d*x),x)*d*m)/(e**(a + b*x)*b)
```

3.181 $\int e^{-a-bx}(c+dx)^m\,dx$

Optimal result

Integrand size = 18, antiderivative size = 52

$$\int e^{-a-bx}(c+dx)^m\,dx=-\frac{e^{-a+\frac{bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{b(c+dx)}{d}\right)}{b}$$

output
```
-exp(-a+b*c/d)*(d*x+c)^m*GAMMA(1+m,b*(d*x+c)/d)/b/((b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int e^{-a-bx}(c+dx)^m\,dx=-\frac{e^{-a+\frac{bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{b(c+dx)}{d}\right)}{b}$$

input
```
Integrate[E^(-a - b*x)*(c + d*x)^m,x]
```

output
```
-((E^(-a + (b*c)/d)*(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))/d])/(b*((b*(c + 
d*x))/d)^m))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-a-bx}(c+dx)^m\,dx$$

$$\Big\downarrow \ 2612$$

$$-\frac{e^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{b(c+dx)}{d}\right)}{b}$$

input
```
Int[E^(-a - b*x)*(c + d*x)^m,x]
```

output
```
-((E^(-a + (b*c)/d)*(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))/d])/(b*((b*(c +
 d*x))/d)^m))
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Maple [F]

$$\int e^{-bx-a}(dx+c)^m\,dx$$

input
```
int(exp(-b*x-a)*(d*x+c)^m,x)
```

output
```
int(exp(-b*x-a)*(d*x+c)^m,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int e^{-a-bx}(c+dx)^m\,dx = -\frac{e^{\left(-\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)}\Gamma\left(m+1,\frac{bdx+bc}{d}\right)}{b}$$

input
```
integrate(exp(-b*x-a)*(d*x+c)^m,x, algorithm="fricas")
```

output
```
-e^(-(d*m*log(b/d) - b*c + a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d)/b
```

Sympy [F(-2)]

Exception generated.

$$\int e^{-a-bx}(c+dx)^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(-b*x-a)*(d*x+c)**m,x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int e^{-a-bx}(c+dx)^m\,dx = -\frac{(dx+c)^{m+1}e^{\left(-a+\frac{bc}{d}\right)}E_{-m}\left(\frac{(dx+c)b}{d}\right)}{d}$$

input
```
integrate(exp(-b*x-a)*(d*x+c)^m,x, algorithm="maxima")
```

output
```
-(d*x + c)^(m + 1)*e^(-a + b*c/d)*exp_integral_e(-m, (d*x + c)*b/d)/d
```

Giac [F]

$$\int e^{-a-bx}(c+dx)^m\,dx = \int (dx+c)^m e^{(-bx-a)}\,dx$$

input
```
integrate(exp(-b*x-a)*(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*e^(-b*x - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-a-bx}(c+dx)^m\,dx = \int \mathrm{e}^{-a-b\,x}\,{\left(c+d\,x\right)}^m\,dx$$

input
```
int(exp(- a - b*x)*(c + d*x)^m,x)
```

output
```
int(exp(- a - b*x)*(c + d*x)^m, x)
```

Reduce [F]

$$\int e^{-a-bx}(c+dx)^m\,dx = \frac{-(dx+c)^m + e^{bx}\left(\int \frac{(dx+c)^m}{e^{bx}c+e^{bx}dx}dx\right)dm}{e^{bx+a}b}$$

input
```
int(exp(-b*x-a)*(d*x+c)^m,x)
```

output
```
( - (c + d*x)**m + e**(b*x)*int((c + d*x)**m/(e**(b*x)*c + e**(b*x)*d*x),x
)*d*m)/(e**(a + b*x)*b)
```

3.182 $\int (c+dx)^m\Gamma(0,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int (c+dx)^m\Gamma(0,a+bx)\,dx=\text{Int}((c+dx)^m\Gamma(0,a+bx),x)$$

output
```
Defer(Int)((d*x+c)^m*Ei(1,b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(0,a+bx)\,dx=\int (c+dx)^m\Gamma(0,a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Gamma[0, a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Gamma[0, a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(0,a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{ 7120}$$

$$\int \Gamma(0,a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Gamma[0, a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \operatorname{expIntegral}_1(bx+a)\,dx$$

input
```
int((d*x+c)^m*Ei(1,b*x+a),x)
```

output
```
int((d*x+c)^m*Ei(1,b*x+a),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (c+dx)^m\Gamma(0,a+bx)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^m*exp_integral_e(1,b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> An error occurred when FriCAS evaluated (((
(d)*(x))+(c))^(m))*(exp_integral_e(((1)::EXPR INT),((b)*(x))+(a))):   Ther
e are no library operations named exp_integral_e       Use HyperDoc Browse
 or iss
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m\Gamma(0,a+bx)\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)**m*expint(1,b*x+a),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(0,a+bx)\,dx=\int (dx+c)^mE_1(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*exp_integral_e(1,b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*exp_integral_e(1, b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(0,a+bx)\,dx = \int (dx+c)^m E_1(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*exp_integral_e(1,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*exp_integral_e(1, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int (c+dx)^m\Gamma(0,a+bx)\,dx = \int \mathrm{expint}(a+b\,x)\,(c+d\,x)^m\,dx$$

input
```
int(expint(a + b*x)*(c + d*x)^m,x)
```

output
```
int(expint(a + b*x)*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(0,a+bx)\,dx = \int (dx+c)^m\,ei(1,bx+a)\,dx$$

input
```
int((d*x+c)^m*Ei(1,b*x+a),x)
```

output
```
int((c + d*x)**m*ei(1,a + b*x),x)
```

3.183 $\int (c+dx)^m\Gamma(-1,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int (c+dx)^m\Gamma(-1,a+bx)\,dx=\text{Int}((c+dx)^m\Gamma(-1,a+bx),x)$$

output
```
Defer(Int)((d*x+c)^m/(b*x+a)*Ei(2,b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(-1,a+bx)\,dx=\int (c+dx)^m\Gamma(-1,a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Gamma[-1, a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Gamma[-1, a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(-1,a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{7120}$$

$$\int \Gamma(-1,a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Gamma[-1, a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{(dx+c)^m \operatorname{expIntegral}_2(bx+a)}{bx+a}dx$$

input
```
int((d*x+c)^m/(b*x+a)*Ei(2,b*x+a),x)
```

output
```
int((d*x+c)^m/(b*x+a)*Ei(2,b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(-1,a+bx)\,dx = \int (dx+c)^m\Gamma(-1,bx+a)\,dx$$

input
```
integrate((d*x+c)^m*gamma(-1,b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*gamma(-1, b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int (c+dx)^m\Gamma(-1,a+bx)\,dx = \int \frac{(c+dx)^m \operatorname{E}_2(a+bx)}{a+bx}\,dx$$

input
```
integrate((d*x+c)**m*uppergamma(-1,b*x+a),x)
```

output
```
Integral((c + d*x)**m*expint(2, a + b*x)/(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(-1,a+bx)\,dx = \int (dx+c)^m\Gamma(-1,bx+a)\,dx$$

input
```
integrate((d*x+c)^m*gamma(-1,b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*gamma(-1, b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(-1,a+bx)\,dx=\int (dx+c)^m\Gamma(-1,bx+a)\,dx$$

input
```
integrate((d*x+c)^m*gamma(-1,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*gamma(-1, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int (c+dx)^m\Gamma(-1,a+bx)\,dx=\int \frac{\mathrm{expint}(2,a+b\,x)\,\left(c+d\,x\right)^m}{a+b\,x}\,dx$$

input
```
int((expint(2, a + b*x)*(c + d*x)^m)/(a + b*x),x)
```

output
```
int((expint(2, a + b*x)*(c + d*x)^m)/(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int (c+dx)^m\Gamma(-1,a+bx)\,dx=\int \frac{(dx+c)^m\,ei(2,bx+a)}{bx+a}dx$$

input
```
int((d*x+c)^m/(b*x+a)*Ei(2,b*x+a),x)
```

output
```
int(((c + d*x)**m*ei(2,a + b*x))/(a + b*x),x)
```

3.184 $\int (c+dx)^m\Gamma(-2,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int (c+dx)^m\Gamma(-2,a+bx)\,dx = \text{Int}((c+dx)^m\Gamma(-2,a+bx),x)$$

output
```
Defer(Int)((d*x+c)^m/(b*x+a)^2*Ei(3,b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(-2,a+bx)\,dx = \int (c+dx)^m\Gamma(-2,a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Gamma[-2, a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Gamma[-2, a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(-2,a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{ 7120}$$

$$\int \Gamma(-2,a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Gamma[-2, a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{(dx+c)^m \operatorname{expIntegral}_3(bx+a)}{(bx+a)^2}dx$$

input
```
int((d*x+c)^m/(b*x+a)^2*Ei(3,b*x+a),x)
```

output
```
int((d*x+c)^m/(b*x+a)^2*Ei(3,b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(-2,a+bx)\,dx=\int (dx+c)^m\Gamma(-2,bx+a)\ dx$$

input
```
integrate((d*x+c)^m*gamma(-2,b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*gamma(-2, b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int (c+dx)^m\Gamma(-2,a+bx)\,dx=\int \frac{(c+dx)^m\,\mathrm{E}_3\left(a+bx\right)}{\left(a+bx\right)^2}\,dx$$

input
```
integrate((d*x+c)**m*uppergamma(-2,b*x+a),x)
```

output
```
Integral((c + d*x)**m*expint(3, a + b*x)/(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(-2,a+bx)\,dx=\int (dx+c)^m\Gamma(-2,bx+a)\ dx$$

input
```
integrate((d*x+c)^m*gamma(-2,b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*gamma(-2, b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(-2,a+bx)\,dx=\int (dx+c)^m\Gamma(-2,bx+a)\,dx$$

input
```
integrate((d*x+c)^m*gamma(-2,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*gamma(-2, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int (c+dx)^m\Gamma(-2,a+bx)\,dx=\int \frac{\mathrm{expint}(3,a+b\,x)\ (c+d\,x)^m}{(a+b\,x)^2}\,dx$$

input
```
int((expint(3, a + b*x)*(c + d*x)^m)/(a + b*x)^2,x)
```

output
```
int((expint(3, a + b*x)*(c + d*x)^m)/(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int (c+dx)^m\Gamma(-2,a+bx)\,dx=\int \frac{(dx+c)^m\ ei(3,bx+a)}{b^2x^2+2abx+a^2}dx$$

input
```
int((d*x+c)^m/(b*x+a)^2*Ei(3,b*x+a),x)
```

output
```
int(((c + d*x)**m*ei(3,a + b*x))/(a**2 + 2*a*b*x + b**2*x**2),x)
```

3.185 $\int (c+dx)^m\Gamma(-3,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int (c+dx)^m\Gamma(-3,a+bx)\,dx = \text{Int}((c+dx)^m\Gamma(-3,a+bx),x)$$

output
```
Defer(Int)((d*x+c)^m/(b*x+a)^3*Ei(4,b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(-3,a+bx)\,dx = \int (c+dx)^m\Gamma(-3,a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Gamma[-3, a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Gamma[-3, a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(-3, a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \ 7120$$

$$\int \Gamma(-3, a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Gamma[-3, a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{(dx+c)^m \operatorname{expIntegral}_4(bx+a)}{(bx+a)^3}dx$$

input
```
int((d*x+c)^m/(b*x+a)^3*Ei(4,b*x+a),x)
```

output
```
int((d*x+c)^m/(b*x+a)^3*Ei(4,b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(-3,a+bx)\,dx = \int (dx+c)^m\Gamma(-3,bx+a)\,dx$$

input
```
integrate((d*x+c)^m*gamma(-3,b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*gamma(-3, b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int (c+dx)^m\Gamma(-3,a+bx)\,dx = \int \frac{(c+dx)^m \operatorname{E}_4(a+bx)}{(a+bx)^3}\,dx$$

input
```
integrate((d*x+c)**m*uppergamma(-3,b*x+a),x)
```

output
```
Integral((c + d*x)**m*expint(4, a + b*x)/(a + b*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(-3,a+bx)\,dx = \int (dx+c)^m\Gamma(-3,bx+a)\,dx$$

input
```
integrate((d*x+c)^m*gamma(-3,b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*gamma(-3, b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(-3,a+bx)\,dx=\int (dx+c)^m\Gamma(-3,bx+a)\,dx$$

input
```
integrate((d*x+c)^m*gamma(-3,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*gamma(-3, b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int (c+dx)^m\Gamma(-3,a+bx)\,dx=\int \frac{\mathrm{expint}(4,a+b\,x)\ (c+d\,x)^m}{(a+b\,x)^3}\,dx$$

input
```
int((expint(4, a + b*x)*(c + d*x)^m)/(a + b*x)^3,x)
```

output
```
int((expint(4, a + b*x)*(c + d*x)^m)/(a + b*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.07

$$\int (c+dx)^m\Gamma(-3,a+bx)\,dx=\int \frac{(dx+c)^m\,ei(4,bx+a)}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}dx$$

input
```
int((d*x+c)^m/(b*x+a)^3*Ei(4,b*x+a),x)
```

output
```
int(((c + d*x)**m*ei(4,a + b*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3
*x**3),x)
```

3.186 $\int x^m \Gamma(n, a+bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int x^m \Gamma(n, a+bx)\,dx = \text{Int}(x^m \Gamma(n, a+bx), x)$$

output
```
Defer(Int)(x^m*GAMMA(n,b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int x^m \Gamma(n, a+bx)\,dx = \int x^m \Gamma(n, a+bx)\,dx$$

input
```
Integrate[x^m*Gamma[n, a + b*x],x]
```

output
```
Integrate[x^m*Gamma[n, a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \Gamma(n, a + bx) \, dx$$

$$\Big\downarrow \text{7120}$$

$$\int x^m \Gamma(n, a + bx) dx$$

input
```
Int[x^m*Gamma[n, a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int x^m \Gamma(n, bx + a) \, dx$$

input
```
int(x^m*GAMMA(n,b*x+a),x)
```

output
```
int(x^m*GAMMA(n,b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int x^m\Gamma(n,a+bx)\,dx=\int x^m\Gamma(n,bx+a)\,dx$$

input
```
integrate(x^m*gamma(n,b*x+a),x, algorithm="fricas")
```

output
```
integral(x^m*gamma(n, b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m\Gamma(n,a+bx)\,dx=\text{Timed out}$$

input
```
integrate(x**m*uppergamma(n,b*x+a),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int x^m\Gamma(n,a+bx)\,dx=\int x^m\Gamma(n,bx+a)\,dx$$

input
```
integrate(x^m*gamma(n,b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^m*gamma(n, b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int x^m\Gamma(n,a+bx)\,dx=\int x^m\Gamma(n,bx+a)\,dx$$

input
```
integrate(x^m*gamma(n,b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*gamma(n, b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int x^m\Gamma(n,a+bx)\,dx=\int x^m\,\Gamma(n,a+b\,x)\,dx$$

input
```
int(x^m*igamma(n, a + b*x),x)
```

output
```
int(x^m*igamma(n, a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int x^m\Gamma(n,a+bx)\,dx=\int x^m\gamma(n,bx+a)\,dx$$

input
```
int(x^m*GAMMA(n,b*x+a),x)
```

output

```
int(x**m*gamma(n,a + b*x),x)
```

3.187 $\int (c+dx)^m\Gamma(n,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int (c+dx)^m\Gamma(n,a+bx)\,dx = \text{Int}((c+dx)^m\Gamma(n,a+bx),x)$$

output
```
Defer(Int)((d*x+c)^m*GAMMA(n,b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(n,a+bx)\,dx = \int (c+dx)^m\Gamma(n,a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Gamma[n, a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Gamma[n, a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m\Gamma(n,a+bx)\,dx$$

$$\Big\downarrow \text{7120}$$

$$\int (c+dx)^m\Gamma(n,a+bx)dx$$

input
```
Int[(c + d*x)^m*Gamma[n, a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(dx+c\right)^{m} \Gamma\left(n,bx+a\right) dx$$

input
```
int((d*x+c)^m*GAMMA(n,b*x+a),x)
```

output
```
int((d*x+c)^m*GAMMA(n,b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(n,a+bx)\,dx=\int (dx+c)^m\Gamma(n,bx+a)\;dx$$

input
```
integrate((d*x+c)^m*gamma(n,b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*gamma(n, b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m\Gamma(n,a+bx)\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)**m*uppergamma(n,b*x+a),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(n,a+bx)\,dx=\int (dx+c)^m\Gamma(n,bx+a)\;dx$$

input
```
integrate((d*x+c)^m*gamma(n,b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*gamma(n, b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(n,a+bx)\,dx = \int (dx+c)^m\Gamma(n,bx+a)\,dx$$

input
```
integrate((d*x+c)^m*gamma(n,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*gamma(n, b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(n,a+bx)\,dx = \int \Gamma(n,a+b\,x)\,\left(c+d\,x\right)^m\,dx$$

input
```
int(igamma(n, a + b*x)*(c + d*x)^m,x)
```

output
```
int(igamma(n, a + b*x)*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\Gamma(n,a+bx)\,dx = \int (dx+c)^m\,\gamma(n,bx+a)\,dx$$

input
```
int((d*x+c)^m*GAMMA(n,b*x+a),x)
```

output
```
int((c + d*x)**m*gamma(n,a + b*x),x)
```

3.188 $\int (c+dx)^4\Gamma(n,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 169

$$\int (c+dx)^4\Gamma(n,a+bx)\,dx = -\frac{(bc-ad)^5\Gamma(n,a+bx)}{5b^5d}+\frac{(c+dx)^5\Gamma(n,a+bx)}{5d} - \frac{(bc-ad)^4\Gamma(1+n,a+bx)}{b^5} - \frac{2d(bc-ad)^3\Gamma(2+n,a+bx)}{b^5} - \frac{2d^2(bc-ad)^2\Gamma(3+n,a+bx)}{b^5} - \frac{d^3(bc-ad)\Gamma(4+n,a+bx)}{b^5} - \frac{d^4\Gamma(5+n,a+bx)}{5b^5}$$

output
```
-1/5*(-a*d+b*c)^5*GAMMA(n,b*x+a)/b^5/d+1/5*(d*x+c)^5*GAMMA(n,b*x+a)/d-(-a*
d+b*c)^4*GAMMA(1+n,b*x+a)/b^5-2*d*(-a*d+b*c)^3*GAMMA(2+n,b*x+a)/b^5-2*d^2*
(-a*d+b*c)^2*GAMMA(3+n,b*x+a)/b^5-d^3*(-a*d+b*c)*GAMMA(4+n,b*x+a)/b^5-1/5*
d^4*GAMMA(5+n,b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.28

$$\int (c+dx)^4\Gamma(n,a+bx)\,dx$$
$$=\frac{(5ab^4c^4-10a^2b^3c^3d+10a^3b^2c^2d^2-5a^4bcd^3+a^5d^4+b^5x(5c^4+10c^3dx+10c^2d^2x^2+5cd^3x^3+d^4x^4))\,\Gamma}{}$$

input
```
Integrate[(c + d*x)^4*Gamma[n, a + b*x],x]
```

output
```
((5*a*b^4*c^4 - 10*a^2*b^3*c^3*d + 10*a^3*b^2*c^2*d^2 - 5*a^4*b*c*d^3 + a^
5*d^4 + b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4
))*Gamma[n, a + b*x] - 5*(b*c - a*d)^4*Gamma[1 + n, a + b*x] + d*(-10*(b*c
 - a*d)^3*Gamma[2 + n, a + b*x] - d*(10*(b*c - a*d)^2*Gamma[3 + n, a + b*x
] + d*(5*(b*c - a*d)*Gamma[4 + n, a + b*x] + d*Gamma[5 + n, a + b*x]))))/(
5*b^5)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\Gamma(n,a+bx)\,dx$$
$$\Big\downarrow \text{7119}$$
$$\frac{b\int e^{-a-bx}(a+bx)^{n-1}(c+dx)^5dx}{5d}+\frac{(c+dx)^5\Gamma(n,a+bx)}{5d}$$
$$\Big\downarrow \text{2629}$$

$$\frac{b\int \left(\frac{(bc-ad)^5e^{-a-bx}(a+bx)^{n-1}}{b^5}+\frac{5d(bc-ad)^4e^{-a-bx}(a+bx)^n}{b^5}+\frac{10d^2(bc-ad)^3e^{-a-bx}(a+bx)^{n+1}}{b^5}+\frac{10d^3(bc-ad)^2e^{-a-bx}(a+bx)^{n+2}}{b^5}+\cdots}{5d}$$

$$\frac{(c+dx)^5\Gamma(n,a+bx)}{5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{5d^4(bc-ad)\Gamma(n+4,a+bx)}{b^6}-\frac{10d^3(bc-ad)^2\Gamma(n+3,a+bx)}{b^6}-\frac{10d^2(bc-ad)^3\Gamma(n+2,a+bx)}{b^6}-\frac{5d(bc-ad)^4\Gamma(n+1,a+bx)}{b^6}-\frac{(bc-ad)^5\Gamma(n,a\cdots}{b^6}\right.}{5d}$$

$$\frac{(c+dx)^5\Gamma(n,a+bx)}{5d}$$

input
```
Int[(c + d*x)^4*Gamma[n, a + b*x],x]
```

output
```
((c + d*x)^5*Gamma[n, a + b*x])/(5*d) + (b*(-(((b*c - a*d)^5*Gamma[n, a +
b*x])/b^6) - (5*d*(b*c - a*d)^4*Gamma[1 + n, a + b*x])/b^6 - (10*d^2*(b*c
- a*d)^3*Gamma[2 + n, a + b*x])/b^6 - (10*d^3*(b*c - a*d)^2*Gamma[3 + n, a
 + b*x])/b^6 - (5*d^4*(b*c - a*d)*Gamma[4 + n, a + b*x])/b^6 - (d^5*Gamma[
5 + n, a + b*x])/b^6))/(5*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3716 vs. $2(163) = 326$.

Time = 17.76 (sec) , antiderivative size = 3717, normalized size of antiderivative = 21.99

method	result	size
parallelrisch	Expression too large to display	3717

input
```
int((d*x+c)^4*GAMMA(n,b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(5*x^4*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^5*c*d^3+x^4*exp(-b*x-a)*(b*x+a)
^(-1+n)*a*b^4*d^4*n-x^3*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b^3*d^4*n+10*x^3*ex
p(-b*x-a)*(b*x+a)^(-1+n)*a*b^5*c^2*d^2+x^3*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^
3*d^4*n^2+15*x^3*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^4*c*d^3+7*x^3*exp(-b*x-a)*
(b*x+a)^(-1+n)*a*b^3*d^4*n+x^2*exp(-b*x-a)*(b*x+a)^(-1+n)*a^3*b^2*d^4*n-2*
x^2*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b^2*d^4*n^2+10*x^2*exp(-b*x-a)*(b*x+a)^
(-1+n)*a*b^5*c^3*d+x^2*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^2*d^4*n^3+5*x^2*exp(
-b*x-a)*(b*x+a)^(-1+n)*a^2*b^3*c*d^3+50*x*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b*d
^4*n+55*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b*c*d^3*n-35*exp(-b*x-a)*(b*x+a)^(-
1+n)*a^3*b*c*d^3*n+30*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b^2*c^2*d^2*n+30*exp(
-b*x-a)*(b*x+a)^(-1+n)*a^2*b*c*d^3*n^2+30*x*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b
^2*c*d^3-4*x^2*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b^2*d^4*n+20*x^2*exp(-b*x-a)
*(b*x+a)^(-1+n)*a*b^4*c^2*d^2+9*x^2*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^2*d^4*n
^2-x*exp(-b*x-a)*(b*x+a)^(-1+n)*a^4*b*d^4*n+3*x*exp(-b*x-a)*(b*x+a)^(-1+n)
*a^3*b*d^4*n^2-3*x*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b*d^4*n^3+x*exp(-b*x-a)*
(b*x+a)^(-1+n)*a*b*d^4*n^4+30*x^2*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^3*c*d^3+2
6*x^2*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^2*d^4*n-5*x*exp(-b*x-a)*(b*x+a)^(-1+n
)*a^3*b^2*c*d^3+x*exp(-b*x-a)*(b*x+a)^(-1+n)*a^3*b*d^4*n+10*x*exp(-b*x-a)*
(b*x+a)^(-1+n)*a^2*b^3*c^2*d^2-12*x*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b*d^4*n
^2+10*x*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^4*c^3*d+10*x*exp(-b*x-a)*(b*x+a)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1080 vs. $2(163) = 326$.

Time = 0.13 (sec) , antiderivative size = 1080, normalized size of antiderivative = 6.39

$$\int (c+dx)^4\Gamma(n,a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*gamma(n,b*x+a),x, algorithm="fricas")
```

output
```
-1/5*((b^5*d^4*x^5 + 5*a*b^4*c^4 + a*d^4*n^4 - 10*(a^2 - a)*b^3*c^3*d + 10
*(a^3 - a^2 + 2*a)*b^2*c^2*d^2 - 5*(a^4 - a^3 + 2*a^2 - 6*a)*b*c*d^3 + (a^
5 - a^4 + 2*a^3 - 6*a^2 + 24*a)*d^4 + (5*b^5*c*d^3 + b^4*d^4*n + 4*b^4*d^4
)*x^4 + (5*a*b*c*d^3 - 2*(2*a^2 - 5*a)*d^4)*n^3 + (10*b^5*c^2*d^2 + b^3*d^
4*n^2 + 15*b^4*c*d^3 + (a + 12)*b^3*d^4 + (5*b^4*c*d^3 - (a - 7)*b^3*d^4)*
n)*x^3 + (10*a*b^2*c^2*d^2 - 15*(a^2 - 2*a)*b*c*d^3 + (6*a^3 - 21*a^2 + 35
*a)*d^4)*n^2 + (10*b^5*c^3*d + b^2*d^4*n^3 + 20*b^4*c^2*d^2 + 5*(a + 6)*b^
3*c*d^3 - (a^2 - 6*a - 24)*b^2*d^4 + (5*b^3*c*d^3 - (2*a - 9)*b^2*d^4)*n^2
 + (10*b^4*c^2*d^2 - 5*(a - 5)*b^3*c*d^3 + (a^2 - 4*a + 26)*b^2*d^4)*n)*x^
2 + (10*a*b^3*c^3*d - 10*(2*a^2 - 3*a)*b^2*c^2*d^2 + 5*(3*a^3 - 7*a^2 + 11
*a)*b*c*d^3 - (4*a^4 - 12*a^3 + 29*a^2 - 50*a)*d^4)*n + (5*b^5*c^4 + b*d^4
*n^4 + 10*b^4*c^3*d + 10*(a + 2)*b^3*c^2*d^2 - 5*(a^2 - 4*a - 6)*b^2*c*d^3
 + (a^3 - 4*a^2 + 18*a + 24)*b*d^4 + (5*b^2*c*d^3 - (3*a - 10)*b*d^4)*n^3
+ (10*b^3*c^2*d^2 - 10*(a - 3)*b^2*c*d^3 + (3*a^2 - 12*a + 35)*b*d^4)*n^2
+ (10*b^4*c^3*d - 10*(a - 3)*b^3*c^2*d^2 + 5*(a^2 - 2*a + 11)*b^2*c*d^3 -
(a^3 - a^2 + 3*a - 50)*b*d^4)*n)*x)*(b*x + a)^(n - 1)*e^(-b*x - a) - (b^5*
d^4*x^5 + 5*b^5*c*d^3*x^4 + 10*b^5*c^2*d^2*x^3 + 10*b^5*c^3*d*x^2 + 5*b^5*
c^4*x + 5*a*b^4*c^4 - 10*a^2*b^3*c^3*d + 10*a^3*b^2*c^2*d^2 - 5*a^4*b*c*d^
3 + a^5*d^4 - d^4*n^5 - 5*(b*c*d^3 - (a - 2)*d^4)*n^4 - 5*(2*b^2*c^2*d^2 -
 2*(2*a - 3)*b*c*d^3 + (2*a^2 - 6*a + 7)*d^4)*n^3 - 5*(2*b^3*c^3*d - 6*...
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^4\Gamma(n,a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**4*uppergamma(n,b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^4\Gamma(n,a+bx)\,dx = \int (dx+c)^4\Gamma(n,bx+a)\,dx$$

input
```
integrate((d*x+c)^4*gamma(n,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(n, b*x + a) - gamma(n + 1, b*x + a))*c^4/b + integrate(d^
4*x^4*gamma(n, b*x + a) + 4*c*d^3*x^3*gamma(n, b*x + a) + 6*c^2*d^2*x^2*ga
mma(n, b*x + a) + 4*c^3*d*x*gamma(n, b*x + a), x)
```

Giac [F]

$$\int (c+dx)^4\Gamma(n,a+bx)\,dx = \int (dx+c)^4\Gamma(n,bx+a)\,dx$$

input
```
integrate((d*x+c)^4*gamma(n,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*gamma(n, b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4\Gamma(n,a+bx)\,dx=\int \Gamma(n,a+b\,x)\;\left(c+d\,x\right)^4\,dx$$

input
```
int(igamma(n, a + b*x)*(c + d*x)^4,x)
```

output
```
int(igamma(n, a + b*x)*(c + d*x)^4, x)
```

Reduce [F]

$$\int (c+dx)^4\Gamma(n,a+bx)\,dx=\left(\int \gamma(n,bx+a)\,dx\right)c^4+\left(\int \gamma(n,bx+a)\,x^4dx\right)d^4$$
$$+4\left(\int \gamma(n,bx+a)\,x^3dx\right)c\,d^3$$
$$+6\left(\int \gamma(n,bx+a)\,x^2dx\right)c^2d^2$$
$$+4\left(\int \gamma(n,bx+a)\,xdx\right)c^3d$$

input
```
int((d*x+c)^4*GAMMA(n,b*x+a),x)
```

output
```
int(gamma(n,a + b*x),x)*c**4 + int(gamma(n,a + b*x)*x**4,x)*d**4 + 4*int(g
amma(n,a + b*x)*x**3,x)*c*d**3 + 6*int(gamma(n,a + b*x)*x**2,x)*c**2*d**2
+ 4*int(gamma(n,a + b*x)*x,x)*c**3*d
```

3.189 $\int (c+dx)^3\Gamma(n,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 144

$$\begin{aligned}\int (c+dx)^3\Gamma(n,a+bx)\,dx = &-\frac{(bc-ad)^4\Gamma(n,a+bx)}{4b^4d}+\frac{(c+dx)^4\Gamma(n,a+bx)}{4d}\\&-\frac{(bc-ad)^3\Gamma(1+n,a+bx)}{b^4}\\&-\frac{3d(bc-ad)^2\Gamma(2+n,a+bx)}{2b^4}\\&-\frac{d^2(bc-ad)\Gamma(3+n,a+bx)}{b^4}-\frac{d^3\Gamma(4+n,a+bx)}{4b^4}\end{aligned}$$

output
```
-1/4*(-a*d+b*c)^4*GAMMA(n,b*x+a)/b^4/d+1/4*(d*x+c)^4*GAMMA(n,b*x+a)/d-(-a*
d+b*c)^3*GAMMA(1+n,b*x+a)/b^4-3/2*d*(-a*d+b*c)^2*GAMMA(2+n,b*x+a)/b^4-d^2*
(-a*d+b*c)*GAMMA(3+n,b*x+a)/b^4-1/4*d^3*GAMMA(4+n,b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.17

$$\int (c+dx)^3\Gamma(n,a+bx)\,dx$$
$$= \frac{(4ab^3c^3-6a^2b^2c^2d+4a^3bcd^2-a^4d^3+b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3))\,\Gamma(n,a+bx)-4(bc-ad)^3\Gamma(}{4b^4}$$

input
```
Integrate[(c + d*x)^3*Gamma[n, a + b*x],x]
```

output
```
((4*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 4*a^3*b*c*d^2 - a^4*d^3 + b^4*x*(4*c^3 +
 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))*Gamma[n, a + b*x] - 4*(b*c - a*d)^3*G
amma[1 + n, a + b*x] - d*(6*(b*c - a*d)^2*Gamma[2 + n, a + b*x] + d*(4*(b*
c - a*d)*Gamma[3 + n, a + b*x] + d*Gamma[4 + n, a + b*x])))/(4*b^4)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\Gamma(n,a+bx)\,dx$$
$$\Big\downarrow \; 7119$$
$$\frac{b\int e^{-a-bx}(a+bx)^{n-1}(c+dx)^4dx}{4d}+\frac{(c+dx)^4\Gamma(n,a+bx)}{4d}$$
$$\Big\downarrow \; 2629$$
$$\frac{b\int \left(\frac{(bc-ad)^4e^{-a-bx}(a+bx)^{n-1}}{b^4}+\frac{4d(bc-ad)^3e^{-a-bx}(a+bx)^n}{b^4}+\frac{6d^2(bc-ad)^2e^{-a-bx}(a+bx)^{n+1}}{b^4}+\frac{4d^3(bc-ad)e^{-a-bx}(a+bx)^{n+2}}{b^4}+\frac{d^4}{}\right.}{4d}$$
$$\frac{(c+dx)^4\Gamma(n,a+bx)}{4d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\left(-\frac{4d^3(bc-ad)\Gamma(n+3,a+bx)}{b^5}-\frac{6d^2(bc-ad)^2\Gamma(n+2,a+bx)}{b^5}-\frac{4d(bc-ad)^3\Gamma(n+1,a+bx)}{b^5}-\frac{(bc-ad)^4\Gamma(n,a+bx)}{b^5}-\frac{d^4\Gamma(n+4,a+bx)}{b^5}\right)}{4d}+\frac{(c+dx)^4\Gamma(n,a+bx)}{4d}$$

input
```
Int[(c + d*x)^3*Gamma[n, a + b*x],x]
```

output
```
((c + d*x)^4*Gamma[n, a + b*x])/(4*d) + (b*(-(((b*c - a*d)^4*Gamma[n, a +
b*x])/b^5) - (4*d*(b*c - a*d)^3*Gamma[1 + n, a + b*x])/b^5 - (6*d^2*(b*c -
 a*d)^2*Gamma[2 + n, a + b*x])/b^5 - (4*d^3*(b*c - a*d)*Gamma[3 + n, a + b
*x])/b^5 - (d^4*Gamma[4 + n, a + b*x])/b^5))/(4*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1928 vs. $2(136) = 272$.

Time = 5.84 (sec) , antiderivative size = 1929, normalized size of antiderivative = 13.40

method	result	size
parallelrisch	Expression too large to display	1929

input
```
int((d*x+c)^3*GAMMA(n,b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(8*x^2*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^3*c*d^2+5*x^2*exp(-b*x-a)*(b*x+
a)^(-1+n)*a*b^2*d^3*n+4*x^3*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^4*c*d^2+x^3*exp
(-b*x-a)*(b*x+a)^(-1+n)*a*b^3*d^3*n-x^2*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b^2
*d^3*n+6*x^2*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^4*c^2*d+x^2*exp(-b*x-a)*(b*x+a
)^(-1+n)*a*b^2*d^3*n^2+x*exp(-b*x-a)*(b*x+a)^(-1+n)*a^3*b*d^3*n-2*x*exp(-b
*x-a)*(b*x+a)^(-1+n)*a^2*b*d^3*n^2+x*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b*d^3*n^
3+4*x*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b^2*c*d^2-2*x*exp(-b*x-a)*(b*x+a)^(-1
+n)*a^2*b*d^3*n+6*x*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^3*c^2*d+6*x*exp(-b*x-a)
*(b*x+a)^(-1+n)*a*b*d^3*n^2-8*exp(-b*x-a)*(b*x+a)^(-1+n)*a^3*b*c*d^2*n+6*e
xp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b^2*c^2*d*n+4*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2
*b*c*d^2*n^2+8*x*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^2*c*d^2+11*x*exp(-b*x-a)*(
b*x+a)^(-1+n)*a*b*d^3*n+12*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b*c*d^2*n-x^4*GA
MMA(n,b*x+a)*a*b^4*d^3-4*x*GAMMA(n,b*x+a)*a*b^4*c^3+3*exp(-b*x-a)*(b*x+a)^
(-1+n)*a^4*d^3*n-3*exp(-b*x-a)*(b*x+a)^(-1+n)*a^3*d^3*n^2+4*exp(-b*x-a)*(b
*x+a)^(-1+n)*a^2*b^3*c^3+exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*d^3*n^3+4*x^2*exp(
-b*x-a)*(b*x+a)^(-1+n)*a*b^3*c*d^2*n-4*x*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b^
2*c*d^2*n+6*x*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^3*c^2*d*n+4*x*exp(-b*x-a)*(b*
x+a)^(-1+n)*a*b^2*c*d^2*n^2+12*x*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^2*c*d^2*n+
x^2*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b^2*d^3-6*x^2*GAMMA(n,b*x+a)*a*b^4*c^2*
d+4*x*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^4*c^3+6*x^2*exp(-b*x-a)*(b*x+a)^(-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(136) = 272$.

Time = 0.14 (sec) , antiderivative size = 618, normalized size of antiderivative = 4.29

$$\int (c+dx)^3\Gamma(n,a+bx)\,dx = -\frac{(b^4d^3x^4 + 4\,ab^3c^3 + ad^3n^3 - 6\,(a^2-a)b^2c^2d + 4\,(a^3-a^2+2\,a)bcd^2 - (a^4-a^3+2\,a^2-6\,a)d^3 + (4\,b^4}{}$$

input
```
integrate((d*x+c)^3*gamma(n,b*x+a),x, algorithm="fricas")
```

output
```
-1/4*((b^4*d^3*x^4 + 4*a*b^3*c^3 + a*d^3*n^3 - 6*(a^2 - a)*b^2*c^2*d + 4*(
a^3 - a^2 + 2*a)*b*c*d^2 - (a^4 - a^3 + 2*a^2 - 6*a)*d^3 + (4*b^4*c*d^2 +
b^3*d^3*n + 3*b^3*d^3)*x^3 + (4*a*b*c*d^2 - 3*(a^2 - 2*a)*d^3)*n^2 + (6*b^
4*c^2*d + b^2*d^3*n^2 + 8*b^3*c*d^2 + (a + 6)*b^2*d^3 + (4*b^3*c*d^2 - (a
- 5)*b^2*d^3)*n)*x^2 + (6*a*b^2*c^2*d - 4*(2*a^2 - 3*a)*b*c*d^2 + (3*a^3 -
 7*a^2 + 11*a)*d^3)*n + (4*b^4*c^3 + b*d^3*n^3 + 6*b^3*c^2*d + 4*(a + 2)*b
^2*c*d^2 - (a^2 - 4*a - 6)*b*d^3 + 2*(2*b^2*c*d^2 - (a - 3)*b*d^3)*n^2 + (
6*b^3*c^2*d - 4*(a - 3)*b^2*c*d^2 + (a^2 - 2*a + 11)*b*d^3)*n)*x)*(b*x + a
)^(n - 1)*e^(-b*x - a) - (b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2
+ 4*b^4*c^3*x + 4*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 4*a^3*b*c*d^2 - a^4*d^3 -
d^3*n^4 - 2*(2*b*c*d^2 - (2*a - 3)*d^3)*n^3 - (6*b^2*c^2*d - 12*(a - 1)*b*
c*d^2 + (6*a^2 - 12*a + 11)*d^3)*n^2 - 2*(2*b^3*c^3 - 3*(2*a - 1)*b^2*c^2*
d + 2*(3*a^2 - 3*a + 2)*b*c*d^2 - (2*a^3 - 3*a^2 + 4*a - 3)*d^3)*n)*gamma(
n, b*x + a))/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3\Gamma(n,a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*uppergamma(n,b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^3\Gamma(n,a+bx)\,dx = \int (dx+c)^3\Gamma(n,bx+a)\,dx$$

input
```
integrate((d*x+c)^3*gamma(n,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(n, b*x + a) - gamma(n + 1, b*x + a))*c^3/b + integrate(d^
3*x^3*gamma(n, b*x + a) + 3*c*d^2*x^2*gamma(n, b*x + a) + 3*c^2*d*x*gamma(
n, b*x + a), x)
```

Giac [F]

$$\int (c+dx)^3\Gamma(n,a+bx)\,dx = \int (dx+c)^3\Gamma(n,bx+a)\,dx$$

input
```
integrate((d*x+c)^3*gamma(n,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*gamma(n, b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\Gamma(n,a+bx)\,dx = \int \Gamma(n,a+b\,x)\,\left(c+d\,x\right)^3\,dx$$

input
```
int(igamma(n, a + b*x)*(c + d*x)^3,x)
```

output
```
int(igamma(n, a + b*x)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3\Gamma(n,a+bx)\,dx = \left(\int \gamma(n,bx+a)\,dx\right)c^3 + \left(\int \gamma(n,bx+a)\,x^3dx\right)d^3 + 3\left(\int \gamma(n,bx+a)\,x^2dx\right)c\,d^2 + 3\left(\int \gamma(n,bx+a)\,xdx\right)c^2d$$

input
```
int((d*x+c)^3*GAMMA(n,b*x+a),x)
```

output
```
int(gamma(n,a + b*x),x)*c**3 + int(gamma(n,a + b*x)*x**3,x)*d**3 + 3*int(g
amma(n,a + b*x)*x**2,x)*c*d**2 + 3*int(gamma(n,a + b*x)*x,x)*c**2*d
```

3.190 $\int (c+dx)^2\Gamma(n,a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 115

$$\int (c+dx)^2\Gamma(n,a+bx)\,dx = -\frac{(bc-ad)^3\Gamma(n,a+bx)}{3b^3d}+\frac{(c+dx)^3\Gamma(n,a+bx)}{3d} - \frac{(bc-ad)^2\Gamma(1+n,a+bx)}{b^3} - \frac{d(bc-ad)\Gamma(2+n,a+bx)}{b^3}-\frac{d^2\Gamma(3+n,a+bx)}{3b^3}$$

output
```
-1/3*(-a*d+b*c)^3*GAMMA(n,b*x+a)/b^3/d+1/3*(d*x+c)^3*GAMMA(n,b*x+a)/d-(-a*
d+b*c)^2*GAMMA(1+n,b*x+a)/b^3-d*(-a*d+b*c)*GAMMA(2+n,b*x+a)/b^3-1/3*d^2*GA
MMA(3+n,b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int (c+dx)^2\Gamma(n,a+bx)\,dx = \frac{(3ab^2c^2-3a^2bcd+a^3d^2+b^3x(3c^2+3cdx+d^2x^2))\,\Gamma(n,a+bx)-3(bc-ad)^2\Gamma(1+n,a+bx)+d((-3}{3b^3}$$

input
```
Integrate[(c + d*x)^2*Gamma[n, a + b*x],x]
```

output
```
((3*a*b^2*c^2 - 3*a^2*b*c*d + a^3*d^2 + b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))
*Gamma[n, a + b*x] - 3*(b*c - a*d)^2*Gamma[1 + n, a + b*x] + d*((-3*b*c +
3*a*d)*Gamma[2 + n, a + b*x] - d*Gamma[3 + n, a + b*x]))/(3*b^3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\Gamma(n,a+bx)\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{b\int e^{-a-bx}(a+bx)^{n-1}(c+dx)^3dx}{3d}+\frac{(c+dx)^3\Gamma(n,a+bx)}{3d}$$

$$\Big\downarrow \text{2629}$$

$$\frac{b\int \left(\frac{(bc-ad)^3e^{-a-bx}(a+bx)^{n-1}}{b^3}+\frac{3d(bc-ad)^2e^{-a-bx}(a+bx)^n}{b^3}+\frac{3d^2(bc-ad)e^{-a-bx}(a+bx)^{n+1}}{b^3}+\frac{d^3e^{-a-bx}(a+bx)^{n+2}}{b^3}\right)dx}{3d}+\frac{(c+dx)^3\Gamma(n,a+bx)}{3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{3d^2(bc-ad)\Gamma(n+2,a+bx)}{b^4}-\frac{3d(bc-ad)^2\Gamma(n+1,a+bx)}{b^4}-\frac{(bc-ad)^3\Gamma(n,a+bx)}{b^4}-\frac{d^3\Gamma(n+3,a+bx)}{b^4}\right)}{3d}+\frac{(c+dx)^3\Gamma(n,a+bx)}{3d}$$

input
```
Int[(c + d*x)^2*Gamma[n, a + b*x],x]
```

output
```
((c + d*x)^3*Gamma[n, a + b*x])/(3*d) + (b*(-(((b*c - a*d)^3*Gamma[n, a +
b*x])/b^4) - (3*d*(b*c - a*d)^2*Gamma[1 + n, a + b*x])/b^4 - (3*d^2*(b*c -
 a*d)*Gamma[2 + n, a + b*x])/b^4 - (d^3*Gamma[3 + n, a + b*x])/b^4))/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(109) = 218$.

Time = 1.77 (sec) , antiderivative size = 875, normalized size of antiderivative = 7.61

method	result
parallelrisch	$-\frac{3x\,\mathrm{e}^{-bx-a}(bx+a)^{-1+n}a\,b^{2}cdn+3x\,\mathrm{e}^{-bx-a}(bx+a)^{-1+n}ab\,d^{2}n+3\,\mathrm{e}^{-bx-a}(bx+a)^{-1+n}a^{2}bcdn+3x\,\mathrm{e}^{-bx-a}(bx+a)^{-1+n}a\,b}{}$

input
```
int((d*x+c)^2*GAMMA(n,b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*x*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^2*c*d*n+3*x*exp(-b*x-a)*(b*x+a)^(
-1+n)*a*b*d^2*n+3*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b*c*d*n+3*x*exp(-b*x-a)*(
b*x+a)^(-1+n)*a*b^2*c*d+3*x^2*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^3*c*d+x^2*exp
(-b*x-a)*(b*x+a)^(-1+n)*a*b^2*d^2*n-x*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b*d^2
*n+x*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b*d^2*n^2+x^3*exp(-b*x-a)*(b*x+a)^(-1+n)
*a*b^3*d^2+2*x^2*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^2*d^2-3*x^2*GAMMA(n,b*x+a)
*a*b^3*c*d+3*x*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^3*c^2+x*exp(-b*x-a)*(b*x+a)^
(-1+n)*a^2*b*d^2-3*exp(-b*x-a)*(b*x+a)^(-1+n)*a^3*b*c*d+2*x*exp(-b*x-a)*(b
*x+a)^(-1+n)*a*b*d^2+3*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b*c*d-6*GAMMA(n,b*x+
a)*a^2*b*c*d*n+3*GAMMA(n,b*x+a)*a*b*c*d*n^2+3*GAMMA(n,b*x+a)*a*b*c*d*n-x^3
*GAMMA(n,b*x+a)*a*b^3*d^2-3*x*GAMMA(n,b*x+a)*a*b^3*c^2-2*exp(-b*x-a)*(b*x+
a)^(-1+n)*a^3*d^2*n+3*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b^2*c^2+exp(-b*x-a)*(
b*x+a)^(-1+n)*a^2*d^2*n^2+3*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*d^2*n+3*GAMMA(n
,b*x+a)*a^3*b*c*d+3*GAMMA(n,b*x+a)*a*b^2*c^2*n+exp(-b*x-a)*(b*x+a)^(-1+n)*
a^4*d^2-exp(-b*x-a)*(b*x+a)^(-1+n)*a^3*d^2+3*GAMMA(n,b*x+a)*a^3*d^2*n-3*GA
MMA(n,b*x+a)*a^2*b^2*c^2-3*GAMMA(n,b*x+a)*a^2*d^2*n^2+GAMMA(n,b*x+a)*a*d^2
*n^3+2*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*d^2-3*GAMMA(n,b*x+a)*a^2*d^2*n+3*GAM
MA(n,b*x+a)*a*d^2*n^2+2*GAMMA(n,b*x+a)*a*d^2*n-GAMMA(n,b*x+a)*a^4*d^2)/a/b
^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(109) = 218$.

Time = 0.10 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.70

$$\int (c+dx)^2\Gamma(n,a+bx)\,dx = -\frac{(b^3d^2x^3 + 3\,ab^2c^2 + ad^2n^2 - 3\,(a^2-a)bcd + (a^3-a^2+2\,a)d^2 + (3\,b^3cd + b^2d^2n + 2\,b^2d^2)x^2 + (3\,abcd$$

input
```
integrate((d*x+c)^2*gamma(n,b*x+a),x, algorithm="fricas")
```

output
```
-1/3*((b^3*d^2*x^3 + 3*a*b^2*c^2 + a*d^2*n^2 - 3*(a^2 - a)*b*c*d + (a^3 -
a^2 + 2*a)*d^2 + (3*b^3*c*d + b^2*d^2*n + 2*b^2*d^2)*x^2 + (3*a*b*c*d - (2
*a^2 - 3*a)*d^2)*n + (3*b^3*c^2 + b*d^2*n^2 + 3*b^2*c*d + (a + 2)*b*d^2 +
(3*b^2*c*d - (a - 3)*b*d^2)*n)*x)*(b*x + a)^(n - 1)*e^(-b*x - a) - (b^3*d^
2*x^3 + 3*b^3*c*d*x^2 + 3*b^3*c^2*x + 3*a*b^2*c^2 - 3*a^2*b*c*d + a^3*d^2
- d^2*n^3 - 3*(b*c*d - (a - 1)*d^2)*n^2 - (3*b^2*c^2 - 3*(2*a - 1)*b*c*d +
 (3*a^2 - 3*a + 2)*d^2)*n)*gamma(n, b*x + a))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^2\Gamma(n,a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*uppergamma(n,b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^2\Gamma(n,a+bx)\,dx = \int (dx+c)^2\Gamma(n,bx+a)\,dx$$

input
```
integrate((d*x+c)^2*gamma(n,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(n, b*x + a) - gamma(n + 1, b*x + a))*c^2/b + integrate(d^
2*x^2*gamma(n, b*x + a) + 2*c*d*x*gamma(n, b*x + a), x)
```

Giac [F]

$$\int (c+dx)^2\Gamma(n,a+bx)\,dx = \int (dx+c)^2\Gamma(n,bx+a)\,dx$$

input
```
integrate((d*x+c)^2*gamma(n,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*gamma(n, b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\Gamma(n,a+bx)\,dx = \int \Gamma(n,a+b\,x)\,\left(c+d\,x\right)^2\,dx$$

input
```
int(igamma(n, a + b*x)*(c + d*x)^2,x)
```

output
```
int(igamma(n, a + b*x)*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2\Gamma(n,a+bx)\,dx = \left(\int \gamma(n,bx+a)\,dx\right)c^2 + \left(\int \gamma(n,bx+a)\,x^2dx\right)d^2 + 2\left(\int \gamma(n,bx+a)\,xdx\right)cd$$

input
```
int((d*x+c)^2*GAMMA(n,b*x+a),x)
```

output
```
int(gamma(n,a + b*x),x)*c**2 + int(gamma(n,a + b*x)*x**2,x)*d**2 + 2*int(g
amma(n,a + b*x)*x,x)*c*d
```

3.191 $\int (c+dx)\Gamma(n,a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 88

$$\int (c+dx)\Gamma(n,a+bx)\,dx=-\frac{(bc-ad)^2\Gamma(n,a+bx)}{2b^2d}+\frac{(c+dx)^2\Gamma(n,a+bx)}{2d}-\frac{(bc-ad)\Gamma(1+n,a+bx)}{b^2}-\frac{d\Gamma(2+n,a+bx)}{2b^2}$$

output
```
-1/2*(-a*d+b*c)^2*GAMMA(n,b*x+a)/b^2/d+1/2*(d*x+c)^2*GAMMA(n,b*x+a)/d-(-a*
d+b*c)*GAMMA(1+n,b*x+a)/b^2-1/2*d*GAMMA(2+n,b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int (c+dx)\Gamma(n,a+bx)\,dx=-\frac{(a+bx)(ad-b(2c+dx))\Gamma(n,a+bx)+2(bc-ad)\Gamma(1+n,a+bx)+d\Gamma(2+n,a+bx)}{2b^2}$$

input
```
Integrate[(c + d*x)*Gamma[n, a + b*x],x]
```

output
```
-1/2*((a + b*x)*(a*d - b*(2*c + d*x))*Gamma[n, a + b*x] + 2*(b*c - a*d)*Ga
mma[1 + n, a + b*x] + d*Gamma[2 + n, a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7119, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\Gamma(n,a+bx)\,dx$$

$$\Big\downarrow \text{7119}$$

$$\frac{b\int e^{-a-bx}(a+bx)^{n-1}(c+dx)^2dx}{2d}+\frac{(c+dx)^2\Gamma(n,a+bx)}{2d}$$

$$\Big\downarrow \text{2629}$$

$$\frac{b\int \left(\frac{(bc-ad)^2e^{-a-bx}(a+bx)^{n-1}}{b^2}+\frac{2d(bc-ad)e^{-a-bx}(a+bx)^n}{b^2}+\frac{d^2e^{-a-bx}(a+bx)^{n+1}}{b^2}\right)dx}{2d}+\frac{(c+dx)^2\Gamma(n,a+bx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{2d(bc-ad)\Gamma(n+1,a+bx)}{b^3}-\frac{(bc-ad)^2\Gamma(n,a+bx)}{b^3}-\frac{d^2\Gamma(n+2,a+bx)}{b^3}\right)}{2d}+\frac{(c+dx)^2\Gamma(n,a+bx)}{2d}$$

input
```
Int[(c + d*x)*Gamma[n, a + b*x],x]
```

output
```
((c + d*x)^2*Gamma[n, a + b*x])/(2*d) + (b*(-(((b*c - a*d)^2*Gamma[n, a +
b*x])/b^3) - (2*d*(b*c - a*d)*Gamma[1 + n, a + b*x])/b^3 - (d^2*Gamma[2 +
n, a + b*x])/b^3))/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

rule 7119
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Block[{$UseGamma = True}, Simp[(c + d*x)^(m + 1)*(Gamma[n, a + b*x]/(d*(m +
 1))), x] + Simp[b/(d*(m + 1))   Int[(c + d*x)^(m + 1)*((a + b*x)^(n - 1)/E
^(a + b*x)), x], x]] /; FreeQ[{a, b, c, d, m, n}, x] && (IGtQ[m, 0] || IGtQ
[n, 0] || IntegersQ[m, n]) && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(82) = 164$.

Time = 0.74 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.58

method	result
parallelrisch	$-\frac{x^2 \mathrm{e}^{-bx-a}(bx+a)^{-1+n} a\, b^2 d - x^2 \Gamma(n,bx+a) a\, b^2 d + 2x\, \mathrm{e}^{-bx-a}(bx+a)^{-1+n} a\, b^2 c + x\, \mathrm{e}^{-bx-a}(bx+a)^{-1+n} abdn + x\, \mathrm{e}^{-bx-a}(bx}{}$

input
```
int((d*x+c)*GAMMA(n,b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x^2*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^2*d-x^2*GAMMA(n,b*x+a)*a*b^2*d+2*
x*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b^2*c+x*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b*d*n+
x*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b*d-2*x*GAMMA(n,b*x+a)*a*b^2*c-exp(-b*x-a)*
(b*x+a)^(-1+n)*a^3*d+2*exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*b*c+exp(-b*x-a)*(b*x
+a)^(-1+n)*a^2*d*n+exp(-b*x-a)*(b*x+a)^(-1+n)*a^2*d+GAMMA(n,b*x+a)*a^3*d-2
*GAMMA(n,b*x+a)*a^2*b*c-2*GAMMA(n,b*x+a)*a^2*d*n+2*GAMMA(n,b*x+a)*a*b*c*n+
GAMMA(n,b*x+a)*a*d*n^2+GAMMA(n,b*x+a)*a*d*n)/a/b^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.44

$$\int (c+dx)\Gamma(n,a+bx)\,dx =$$

$$-\frac{(b^2dx^2+2\,abc+adn-(a^2-a)d+(2\,b^2c+bdn+bd)x)(bx+a)^{n-1}e^{(-bx-a)}-(b^2dx^2+2\,b^2cx+2\,ab}{2\,b^2}$$

input
```
integrate((d*x+c)*gamma(n,b*x+a),x, algorithm="fricas")
```

output
```
-1/2*((b^2*d*x^2 + 2*a*b*c + a*d*n - (a^2 - a)*d + (2*b^2*c + b*d*n + b*d)
*x)*(b*x + a)^(n - 1)*e^(-b*x - a) - (b^2*d*x^2 + 2*b^2*c*x + 2*a*b*c - a^
2*d - d*n^2 - (2*b*c - (2*a - 1)*d)*n)*gamma(n, b*x + a))/b^2
```

Sympy [F]

$$\int (c+dx)\Gamma(n,a+bx)\,dx = \int (c+dx)\,\Gamma(n,a+bx)\;dx$$

input
```
integrate((d*x+c)*uppergamma(n,b*x+a),x)
```

output
```
Integral((c + d*x)*uppergamma(n, a + b*x), x)
```

Maxima [F]

$$\int (c+dx)\Gamma(n,a+bx)\,dx = \int (dx+c)\Gamma(n,bx+a)\;dx$$

input
```
integrate((d*x+c)*gamma(n,b*x+a),x, algorithm="maxima")
```

output
```
d*integrate(x*gamma(n, b*x + a), x) + ((b*x + a)*gamma(n, b*x + a) - gamma
(n + 1, b*x + a))*c/b
```

Giac [F]

$$\int (c+dx)\Gamma(n,a+bx)\,dx=\int (dx+c)\Gamma(n,bx+a)\,dx$$

input
```
integrate((d*x+c)*gamma(n,b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*gamma(n, b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\Gamma(n,a+bx)\,dx=\int \Gamma(n,a+b\,x)\,(c+d\,x)\,dx$$

input
```
int(igamma(n, a + b*x)*(c + d*x),x)
```

output
```
int(igamma(n, a + b*x)*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\Gamma(n,a+bx)\,dx=\left(\int \gamma(n,bx+a)\,dx\right)c+\left(\int \gamma(n,bx+a)\,xdx\right)d$$

input
```
int((d*x+c)*GAMMA(n,b*x+a),x)
```

output
```
int(gamma(n,a + b*x),x)*c + int(gamma(n,a + b*x)*x,x)*d
```

3.192 $\int \Gamma(n, a + bx) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 31

$$\int \Gamma(n, a + bx) \, dx = \frac{(a + bx)\Gamma(n, a + bx)}{b} - \frac{\Gamma(1 + n, a + bx)}{b}$$

output
```
(b*x+a)*GAMMA(n,b*x+a)/b-GAMMA(1+n,b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \Gamma(n, a + bx) \, dx = \frac{a\Gamma(n, a + bx)}{b} + x\Gamma(n, a + bx) - \frac{\Gamma(1 + n, a + bx)}{b}$$

input
```
Integrate[Gamma[n, a + b*x],x]
```

output
```
(a*Gamma[n, a + b*x])/b + x*Gamma[n, a + b*x] - Gamma[1 + n, a + b*x]/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(n,a+bx)\,dx$$
$$\Big\downarrow \text{7111}$$
$$\frac{(a+bx)\Gamma(n,a+bx)}{b}-\frac{\Gamma(n+1,a+bx)}{b}$$

input
```
Int[Gamma[n, a + b*x],x]
```

output
```
((a + b*x)*Gamma[n, a + b*x])/b - Gamma[1 + n, a + b*x]/b
```

Defintions of rubi rules used

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(31) = 62.

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.84

method	result	size
parallelrisch	$-\frac{x\,{\mathrm e}^{-bx-a}(bx+a)^{-1+n}ab-x\Gamma(n,bx+a)ab+a^{2}(bx+a)^{-1+n}{\mathrm e}^{-bx-a}-\Gamma(n,bx+a)a^{2}+\Gamma(n,bx+a)an}{ab}$	88

input
```
int(GAMMA(n,b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(x*exp(-b*x-a)*(b*x+a)^(-1+n)*a*b-x*GAMMA(n,b*x+a)*a*b+a^2*(b*x+a)^(-1+n)
*exp(-b*x-a)-GAMMA(n,b*x+a)*a^2+GAMMA(n,b*x+a)*a*n)/a/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \Gamma(n,a+bx)\,dx = -\frac{(bx+a)(bx+a)^{n-1}e^{(-bx-a)} - (bx+a-n)\Gamma(n,bx+a)}{b}$$

input
```
integrate(gamma(n,b*x+a),x, algorithm="fricas")
```

output
```
-((b*x + a)*(b*x + a)^(n - 1)*e^(-b*x - a) - (b*x + a - n)*gamma(n, b*x +
a))/b
```

Sympy [F]

$$\int \Gamma(n,a+bx)\,dx = \int \Gamma(n,a+bx)\,dx$$

input
```
integrate(uppergamma(n,b*x+a),x)
```

output
```
Integral(uppergamma(n, a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \Gamma(n,a+bx)\,dx = \frac{(bx+a)\Gamma(n,bx+a) - \Gamma(n+1,bx+a)}{b}$$

input
```
integrate(gamma(n,b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*gamma(n, b*x + a) - gamma(n + 1, b*x + a))/b
```

Giac [F]

$$\int \Gamma(n, a + bx) \, dx = \int \Gamma(n, bx + a) \, dx$$

input
```
integrate(gamma(n,b*x+a),x, algorithm="giac")
```

output
```
integrate(gamma(n, b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \Gamma(n, a + bx) \, dx = \int \Gamma(n, a + b\,x) \, dx$$

input
```
int(igamma(n, a + b*x),x)
```

output
```
int(igamma(n, a + b*x), x)
```

Reduce [F]

$$\int \Gamma(n, a + bx) \, dx = \int \gamma(n, bx + a) \, dx$$

input
```
int(GAMMA(n,b*x+a),x)
```

output
```
int(gamma(n,a + b*x),x)
```

3.193 $\int \frac{\Gamma(n,a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\Gamma(n,a+bx)}{c+dx}\,dx = \operatorname{Int}\left(\frac{\Gamma(n,a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(GAMMA(n,b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(n,a+bx)}{c+dx}\,dx = \int \frac{\Gamma(n,a+bx)}{c+dx}\,dx$$

input
```
Integrate[Gamma[n, a + b*x]/(c + d*x),x]
```

output
```
Integrate[Gamma[n, a + b*x]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(n, a+bx)}{c+dx} \, dx$$
$$\Big\downarrow \; 7120$$
$$\int \frac{\Gamma(n, a+bx)}{c+dx} dx$$

input
```
Int[Gamma[n, a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(n, bx+a)}{dx+c} dx$$

input
```
int(GAMMA(n,b*x+a)/(d*x+c),x)
```

output
```
int(GAMMA(n,b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(n,a+bx)}{c+dx}\,dx=\int \frac{\Gamma(n,bx+a)}{dx+c}\,dx$$

input
```
integrate(gamma(n,b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(gamma(n, b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 51.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{\Gamma(n,a+bx)}{c+dx}\,dx=\int \frac{\Gamma(n,a+bx)}{c+dx}\,dx$$

input
```
integrate(uppergamma(n,b*x+a)/(d*x+c),x)
```

output
```
Integral(uppergamma(n, a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(n,a+bx)}{c+dx}\,dx=\int \frac{\Gamma(n,bx+a)}{dx+c}\,dx$$

input
```
integrate(gamma(n,b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(gamma(n, b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(n,a+bx)}{c+dx}\,dx = \int \frac{\Gamma(n,bx+a)}{dx+c}\,dx$$

input
```
integrate(gamma(n,b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(gamma(n, b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(n,a+bx)}{c+dx}\,dx = \int \frac{\Gamma(n,a+b\,x)}{c+d\,x}\,dx$$

input
```
int(igamma(n, a + b*x)/(c + d*x),x)
```

output
```
int(igamma(n, a + b*x)/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(n, a+bx)}{c+dx} \, dx = \int \frac{\gamma(n, bx+a)}{dx+c} dx$$

input

```
int(GAMMA(n,b*x+a)/(d*x+c),x)
```

output

```
int(gamma(n,a + b*x)/(c + d*x),x)
```

3.194 $\int \frac{\Gamma(n,a+bx)}{(c+dx)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\Gamma(n,a+bx)}{(c+dx)^2} \, dx = \text{Int}\left(\frac{\Gamma(n,a+bx)}{(c+dx)^2},x\right)$$

output
```
Defer(Int)(GAMMA(n,b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(n,a+bx)}{(c+dx)^2} \, dx = \int \frac{\Gamma(n,a+bx)}{(c+dx)^2} \, dx$$

input
```
Integrate[Gamma[n, a + b*x]/(c + d*x)^2,x]
```

output
```
Integrate[Gamma[n, a + b*x]/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(n,a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \; 7120$$

$$\int \frac{\Gamma(n,a+bx)}{(c+dx)^2}dx$$

input
```
Int[Gamma[n, a + b*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(n,bx+a)}{\left(dx+c\right)^{2}}dx$$

input
```
int(GAMMA(n,b*x+a)/(d*x+c)^2,x)
```

output
```
int(GAMMA(n,b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{\Gamma(n, a+bx)}{(c+dx)^2}\,dx = \int \frac{\Gamma(n, bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(gamma(n,b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(gamma(n, b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\Gamma(n, a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(uppergamma(n,b*x+a)/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(n, a+bx)}{(c+dx)^2}\,dx = \int \frac{\Gamma(n, bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(gamma(n,b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(gamma(n, b*x + a)/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(n, a+bx)}{(c+dx)^2} \, dx = \int \frac{\Gamma(n, bx+a)}{(dx+c)^2} \, dx$$

input
```
integrate(gamma(n,b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(gamma(n, b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(n, a+bx)}{(c+dx)^2} \, dx = \int \frac{\Gamma(n, a+b\,x)}{(c+d\,x)^2} \, dx$$

input
```
int(igamma(n, a + b*x)/(c + d*x)^2,x)
```

output
```
int(igamma(n, a + b*x)/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{\Gamma(n, a+bx)}{(c+dx)^2} \, dx = \int \frac{\gamma(n, bx+a)}{d^2x^2+2cdx+c^2} dx$$

input
```
int(GAMMA(n,b*x+a)/(d*x+c)^2,x)
```

output
```
int(gamma(n,a + b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.195 $\int \frac{\Gamma(n,a+bx)}{(c+dx)^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\Gamma(n,a+bx)}{(c+dx)^3} \, dx = \text{Int}\left(\frac{\Gamma(n,a+bx)}{(c+dx)^3},x\right)$$

output
```
Defer(Int)(GAMMA(n,b*x+a)/(d*x+c)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(n,a+bx)}{(c+dx)^3} \, dx = \int \frac{\Gamma(n,a+bx)}{(c+dx)^3} \, dx$$

input
```
Integrate[Gamma[n, a + b*x]/(c + d*x)^3,x]
```

output
```
Integrate[Gamma[n, a + b*x]/(c + d*x)^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(n,a+bx)}{(c+dx)^3} \, dx$$
$$\Big\downarrow \; 7120$$
$$\int \frac{\Gamma(n,a+bx)}{(c+dx)^3} dx$$

input
```
Int[Gamma[n, a + b*x]/(c + d*x)^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\Gamma(n,bx+a)}{(dx+c)^3} dx$$

input
```
int(GAMMA(n,b*x+a)/(d*x+c)^3,x)
```

output
```
int(GAMMA(n,b*x+a)/(d*x+c)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{\Gamma(n, a+bx)}{(c+dx)^3} \, dx = \int \frac{\Gamma(n, bx+a)}{(dx+c)^3} \, dx$$

input
```
integrate(gamma(n,b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(gamma(n, b*x + a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\Gamma(n, a+bx)}{(c+dx)^3} \, dx = \text{Timed out}$$

input
```
integrate(uppergamma(n,b*x+a)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(n, a+bx)}{(c+dx)^3} \, dx = \int \frac{\Gamma(n, bx+a)}{(dx+c)^3} \, dx$$

input
```
integrate(gamma(n,b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(gamma(n, b*x + a)/(d*x + c)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(n, a+bx)}{(c+dx)^3} \, dx = \int \frac{\Gamma(n, bx+a)}{(dx+c)^3} \, dx$$

input
```
integrate(gamma(n,b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(gamma(n, b*x + a)/(d*x + c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\Gamma(n, a+bx)}{(c+dx)^3} \, dx = \int \frac{\Gamma(n, a+b\,x)}{(c+d\,x)^3} \, dx$$

input
```
int(igamma(n, a + b*x)/(c + d*x)^3,x)
```

output
```
int(igamma(n, a + b*x)/(c + d*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{\Gamma(n, a+bx)}{(c+dx)^3} \, dx = \int \frac{\gamma(n, bx+a)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3} dx$$

input
```
int(GAMMA(n,b*x+a)/(d*x+c)^3,x)
```

output
```
int(gamma(n,a + b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)
```

3.196 $\int x^2\Gamma(p,d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 121

$$\int x^2\Gamma(p,d(a+b\log(cx^n)))\,dx=\frac{1}{3}x^3\Gamma(p,d(a+b\log(cx^n)))-\frac{1}{3}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\Gamma\left(p,-\frac{(3-bdn)(a+b\log(cx^n))}{bn}\right)(d(a+b\log(cx^n)))^p\left(-\frac{(3-bdn)(a+b\log(cx^n))}{bn}\right)^{-p}$$

output
```
1/3*x^3*GAMMA(p,d*(a+b*ln(c*x^n)))-1/3*x^3*GAMMA(p,-(-b*d*n+3)*(a+b*ln(c*x
^n))/b/n)*(d*(a+b*ln(c*x^n)))^p/exp(3*a/b/n)/((c*x^n)^(3/n))/((-(-b*d*n+3)
*(a+b*ln(c*x^n))/b/n)^p)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int x^2\Gamma(p,d(a+b\log(cx^n)))\,dx=\frac{1}{3}x^3\left(\Gamma(p,d(a+b\log(cx^n)))-e^{-\frac{3a}{bn}}(cx^n)^{-3/n}\Gamma\left(p,\frac{(-3+bdn)(a+b\log(cx^n))}{bn}\right)(d(a+b\log(cx^n)))^p\left(\frac{(-3+bdn)(a+b\log(cx^n))}{bn}\right)\right)$$

input
```
Integrate[x^2*Gamma[p, d*(a + b*Log[c*x^n])],x]
```

output
```
(x^3*(Gamma[p, d*(a + b*Log[c*x^n])] - (Gamma[p, ((-3 + b*d*n)*(a + b*Log[
c*x^n]))/(b*n)]*(d*(a + b*Log[c*x^n]))^p)/(E^((3*a)/(b*n))*(c*x^n)^(3/n)*(
((-3 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))^p)))/3
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {7132, 7271, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\Gamma(p,d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{7132}$$

$$\frac{1}{3}bdne^{-ad}x^{bdn}(cx^n)^{-bd}\int x^{2-bdn}(d(a+b\log(cx^n)))^{p-1}dx+\frac{1}{3}x^3\Gamma(p,d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{7271}$$

$$\frac{1}{3}bne^{-ad}x^{bdn}(cx^n)^{-bd}(a+b\log(cx^n))^{-p}(d(a+b\log(cx^n)))^p\int x^{2-bdn}(a+b\log(cx^n))^{p-1}dx+\frac{1}{3}x^3\Gamma(p,d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{2747}$$

$$\frac{1}{3}bx^3e^{-ad}(cx^n)^{-3/n}(a+b\log(cx^n))^{-p}(d(a+b\log(cx^n)))^p\int (cx^n)^{\frac{3-bdn}{n}}(a+b\log(cx^n))^{p-1}d\log(cx^n)+\frac{1}{3}x^3\Gamma(p,d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{2612}$$

$$\frac{1}{3}x^3\Gamma(p,d(a+b\log(cx^n)))-\frac{1}{3}x^3(cx^n)^{-3/n}e^{a\left(d-\frac{3}{bn}\right)-ad}(d(a+b\log(cx^n)))^p\left(-\frac{(3-bdn)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p,-\frac{(3-bdn)(a+b\log(cx$$

input
```
Int[x^2*Gamma[p, d*(a + b*Log[c*x^n])],x]
```

output
```
(x^3*Gamma[p, d*(a + b*Log[c*x^n])])/3 - (E^(-(a*d) + a*(d - 3/(b*n)))*x^3
*Gamma[p, -(((3 - b*d*n)*(a + b*Log[c*x^n]))/(b*n))]*(d*(a + b*Log[c*x^n])
)^p)/(3*(c*x^n)^(3/n)*(-(((3 - b*d*n)*(a + b*Log[c*x^n]))/(b*n)))^p)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 7132
```
Int[Gamma[p_, ((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*(Gamma[p, d*(a + b*Log[c*x^n])]/(e*(m +
 1))), x] + Simp[(b*d*n*((e*x)^(b*d*n)/((m + 1)*(c*x^n)^(b*d))))/E^(a*d)
Int[(e*x)^(m - b*d*n)*(d*(a + b*Log[c*x^n]))^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int x^2\Gamma(p,d(a+b\ln(c\,x^n)))\,dx$$

input
```
int(x^2*GAMMA(p,d*(a+b*ln(c*x^n))),x)
```

output
```
int(x^2*GAMMA(p,d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x^2\Gamma(p,d(a+b\log(cx^n)))\,dx=\int x^2\Gamma(p,(b\log(cx^n)+a)d)\,dx$$

input
```
integrate(x^2*gamma(p,d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(x^2*gamma(p, b*d*log(c*x^n) + a*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\Gamma(p,d(a+b\log(cx^n)))\,dx=\text{Timed out}$$

input
```
integrate(x**2*uppergamma(p,d*(a+b*ln(c*x**n))),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2\Gamma(p,d(a+b\log{(cx^n)}))\,dx=\int x^2\Gamma(p,(b\log{(cx^n)}+a)d)\,dx$$

input
```
integrate(x^2*gamma(p,d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x^2*gamma(p, (b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x^2\Gamma(p,d(a+b\log{(cx^n)}))\,dx=\int x^2\Gamma(p,(b\log{(cx^n)}+a)d)\,dx$$

input
```
integrate(x^2*gamma(p,d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x^2*gamma(p, (b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\Gamma(p,d(a+b\log{(cx^n)}))\,dx=\int x^2\,\Gamma(p,d\,(a+b\,\ln{(c\,x^n)}))\,dx$$

input
```
int(x^2*igamma(p, d*(a + b*log(c*x^n))),x)
```

output
```
int(x^2*igamma(p, d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x^2\Gamma(p, d(a + b\log(cx^n)))\,dx = \int \gamma(p, \log(x^n c)\,bd + ad)\,x^2 dx$$

input
```
int(x^2*GAMMA(p,d*(a+b*log(c*x^n))),x)
```

output
```
int(gamma(p,log(x**n*c)*b*d + a*d)*x**2,x)
```

3.197 $\int x\Gamma(p, d(a + b\log(cx^n)))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 121

$$\int x\Gamma(p, d(a + b\log(cx^n)))\,dx = \frac{1}{2}x^2\Gamma(p, d(a + b\log(cx^n))) - \frac{1}{2}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\Gamma\left(p, -\frac{(2 - bdn)(a + b\log(cx^n))}{bn}\right)(d(a + b\log(cx^n)))^p\left(-\frac{(2 - bdn)(a + b\log(cx^n))}{bn}\right)^{-p}$$

output
```
1/2*x^2*GAMMA(p,d*(a+b*ln(c*x^n)))-1/2*x^2*GAMMA(p,-(-b*d*n+2)*(a+b*ln(c*x
^n))/b/n)*(d*(a+b*ln(c*x^n)))^p/exp(2*a/b/n)/((c*x^n)^(2/n))/((-(-b*d*n+2)
*(a+b*ln(c*x^n))/b/n)^p)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int x\Gamma(p, d(a + b\log(cx^n)))\,dx = \frac{1}{2}x^2\Bigg(\Gamma(p, d(a + b\log(cx^n))) - e^{-\frac{2a}{bn}}(cx^n)^{-2/n}\Gamma\left(p, \frac{(-2 + bdn)(a + b\log(cx^n))}{bn}\right)(d(a+b\log(cx^n)))^p\left(\frac{(-2 + bdn)(a + b\log(cx^n))}{bn}\right.$$

input
```
Integrate[x*Gamma[p, d*(a + b*Log[c*x^n])],x]
```

output
```
(x^2*(Gamma[p, d*(a + b*Log[c*x^n])] - (Gamma[p, ((-2 + b*d*n)*(a + b*Log[
c*x^n]))/(b*n)]*(d*(a + b*Log[c*x^n]))^p)/(E^((2*a)/(b*n))*(c*x^n)^(2/n)*(
((-2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))^p)))/2
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7132, 7271, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\Gamma(p,d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{7132}$$

$$\frac{1}{2}bdne^{-ad}x^{bdn}(cx^n)^{-bd}\int x^{1-bdn}(d(a+b\log(cx^n)))^{p-1}dx+\frac{1}{2}x^2\Gamma(p,d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{7271}$$

$$\frac{1}{2}bne^{-ad}x^{bdn}(cx^n)^{-bd}(a+b\log(cx^n))^{-p}(d(a+b\log(cx^n)))^p\int x^{1-bdn}(a+b\log(cx^n))^{p-1}dx+\frac{1}{2}x^2\Gamma(p,d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{2747}$$

$$\frac{1}{2}bx^2e^{-ad}(cx^n)^{-2/n}(a+b\log(cx^n))^{-p}(d(a+b\log(cx^n)))^p\int (cx^n)^{\frac{2-bdn}{n}}(a+b\log(cx^n))^{p-1}d\log(cx^n)+\frac{1}{2}x^2\Gamma(p,d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{2612}$$

$$\frac{1}{2}x^2\Gamma(p,d(a+b\log(cx^n)))-\frac{1}{2}x^2(cx^n)^{-2/n}e^{a\left(d-\frac{2}{bn}\right)-ad}(d(a+b\log(cx^n)))^p\left(-\frac{(2-bdn)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p,-\frac{(2-bdn)(a+b\log(cx$$

input
```
Int[x*Gamma[p, d*(a + b*Log[c*x^n])],x]
```

output
```
(x^2*Gamma[p, d*(a + b*Log[c*x^n])])/2 - (E^(-(a*d) + a*(d - 2/(b*n)))*x^2
*Gamma[p, -(((2 - b*d*n)*(a + b*Log[c*x^n]))/(b*n))]*(d*(a + b*Log[c*x^n])
)^p)/(2*(c*x^n)^(2/n)*(-(((2 - b*d*n)*(a + b*Log[c*x^n]))/(b*n)))^p)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 7132
```
Int[Gamma[p_, ((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*(Gamma[p, d*(a + b*Log[c*x^n])]/(e*(m +
 1))), x] + Simp[(b*d*n*((e*x)^(b*d*n)/((m + 1)*(c*x^n)^(b*d))))/E^(a*d)
Int[(e*x)^(m - b*d*n)*(d*(a + b*Log[c*x^n]))^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int x\Gamma(p, d(a + b\ln(c\,x^n)))\,dx$$

input
```
int(x*GAMMA(p,d*(a+b*ln(c*x^n))),x)
```

output
```
int(x*GAMMA(p,d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x\Gamma(p, d(a + b\log(cx^n)))\ dx = \int x\Gamma(p, (b\log(cx^n) + a)d)\ dx$$

input
```
integrate(x*gamma(p,d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(x*gamma(p, b*d*log(c*x^n) + a*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x\Gamma(p, d(a + b\log(cx^n)))\ dx = \text{Timed out}$$

input
```
integrate(x*uppergamma(p,d*(a+b*ln(c*x**n))),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\Gamma(p, d(a + b\log(cx^n)))\,dx = \int x\Gamma(p, (b\log(cx^n) + a)d)\,dx$$

input
```
integrate(x*gamma(p,d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x*gamma(p, (b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x\Gamma(p, d(a + b\log(cx^n)))\,dx = \int x\Gamma(p, (b\log(cx^n) + a)d)\,dx$$

input
```
integrate(x*gamma(p,d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x*gamma(p, (b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\Gamma(p, d(a + b\log(cx^n)))\,dx = \int x\,\Gamma(p, d\,(a + b\,\ln(c\,x^n)))\,dx$$

input
```
int(x*igamma(p, d*(a + b*log(c*x^n))),x)
```

output
```
int(x*igamma(p, d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x\Gamma(p, d(a+b\log(cx^n)))\ dx = \int \gamma(p, \log(x^n c)\, bd + ad)\, x dx$$

input
```
int(x*GAMMA(p,d*(a+b*log(c*x^n))),x)
```

output
```
int(gamma(p,log(x**n*c)*b*d + a*d)*x,x)
```

3.198 $\int \Gamma(p, d(a + b\log(cx^n)))\, dx$

Optimal result

Integrand size = 14, antiderivative size = 112

$$\int \Gamma(p, d(a + b\log(cx^n)))\, dx = x\Gamma(p, d(a + b\log(cx^n))) - e^{-\frac{a}{bn}} x (cx^n)^{-1/n} \Gamma\left(p, -\frac{(1 - bdn)(a + b\log(cx^n))}{bn}\right) (d(a + b\log(cx^n)))^p \left(-\frac{(1 - bdn)(a + b\log(cx^n))}{bn}\right)^{-p}$$

output
```
x*GAMMA(p,d*(a+b*ln(c*x^n)))-x*GAMMA(p,-(-b*d*n+1)*(a+b*ln(c*x^n))/b/n)*(d
*(a+b*ln(c*x^n)))^p/exp(a/b/n)/((c*x^n)^(1/n))/((-(-b*d*n+1)*(a+b*ln(c*x^n
))/b/n)^p)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \Gamma(p, d(a + b\log(cx^n)))\, dx = x\Bigg(\Gamma(p, d(a + b\log(cx^n))) - e^{-\frac{a}{bn}} (cx^n)^{-1/n} \Gamma\left(p, \frac{(-1 + bdn)(a + b\log(cx^n))}{bn}\right) (d(a+b\log(cx^n)))^p \left(\frac{(-1 + bdn)(a + b\log(cx^n))}{bn}\right.$$

input
```
Integrate[Gamma[p, d*(a + b*Log[c*x^n])],x]
```

output
```
x*(Gamma[p, d*(a + b*Log[c*x^n])] - (Gamma[p, ((-1 + b*d*n)*(a + b*Log[c*x
^n]))/(b*n)]*(d*(a + b*Log[c*x^n]))^p)/(E^(a/(b*n))*(c*x^n)^n^(-1)*(((-1 +
 b*d*n)*(a + b*Log[c*x^n]))/(b*n))^p))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {7130, 34, 7271, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \Gamma(p, d(a + b\log(cx^n)))\, dx$$

$$\Big\downarrow \text{7130}$$

$$bdne^{-ad}\int (cx^n)^{-bd}(d(a + b\log(cx^n)))^{p-1}dx + x\Gamma(p, d(a + b\log(cx^n)))$$

$$\Big\downarrow \text{34}$$

$$bdne^{-ad}x^{bdn}(cx^n)^{-bd}\int x^{-bdn}(d(a + b\log(cx^n)))^{p-1}dx + x\Gamma(p, d(a + b\log(cx^n)))$$

$$\Big\downarrow \text{7271}$$

$$bne^{-ad}x^{bdn}(cx^n)^{-bd}(a + b\log(cx^n))^{-p}(d(a + b\log(cx^n)))^p\int x^{-bdn}(a + b\log(cx^n))^{p-1}dx + x\Gamma(p, d(a + b\log(cx^n)))$$

$$\Big\downarrow \text{2747}$$

$$bxe^{-ad}(cx^n)^{-1/n}(a + b\log(cx^n))^{-p}(d(a + b\log(cx^n)))^p\int (cx^n)^{\frac{1-bdn}{n}}(a + b\log(cx^n))^{p-1}d\log(cx^n) + x\Gamma(p, d(a + b\log(cx^n)))$$

$$\Big\downarrow \text{2612}$$

$$x\Gamma(p, d(a + b\log(cx^n))) - x(cx^n)^{-1/n}e^{a\left(d-\frac{1}{bn}\right)-ad}(d(a + b\log(cx^n)))^p\left(-\frac{(1-bdn)(a + b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p, -\frac{(1-bdn)(a + b\log(cx^n))}{bn}\right.$$

input
```
Int[Gamma[p, d*(a + b*Log[c*x^n])],x]
```

output
```
x*Gamma[p, d*(a + b*Log[c*x^n])] - (E^(-(a*d) + a*(d - 1/(b*n)))*x*Gamma[p
, -(((1 - b*d*n)*(a + b*Log[c*x^n]))/(b*n))]*(d*(a + b*Log[c*x^n]))^p)/((c
*x^n)^n^(-1)*(-(((1 - b*d*n)*(a + b*Log[c*x^n]))/(b*n)))^p)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 7130
```
Int[Gamma[p_, ((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Si
mp[x*Gamma[p, d*(a + b*Log[c*x^n])], x] + Simp[(b*d*n)/E^(a*d)   Int[(d*(a
+ b*Log[c*x^n]))^(p - 1)/(c*x^n)^(b*d), x], x] /; FreeQ[{a, b, c, d, n, p},
 x]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int \Gamma(p, d(a + b\ln(c\,x^n)))\,dx$$

input
```
int(GAMMA(p,d*(a+b*ln(c*x^n))),x)
```

output
```
int(GAMMA(p,d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int \Gamma(p, d(a + b\log(cx^n)))\ dx = \int \Gamma(p, (b\log(cx^n) + a)d)\ dx$$

input
```
integrate(gamma(p,d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(gamma(p, b*d*log(c*x^n) + a*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \Gamma(p, d(a + b\log(cx^n)))\ dx = \text{Timed out}$$

input
```
integrate(uppergamma(p,d*(a+b*ln(c*x**n))),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \Gamma(p, d(a + b\log(cx^n)))\,dx = \int \Gamma(p, (b\log(cx^n) + a)d)\,dx$$

input
```
integrate(gamma(p,d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(gamma(p, (b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int \Gamma(p, d(a + b\log(cx^n)))\,dx = \int \Gamma(p, (b\log(cx^n) + a)d)\,dx$$

input
```
integrate(gamma(p,d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(gamma(p, (b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \Gamma(p, d(a + b\log(cx^n)))\,dx = \int \Gamma(p, d\,(a + b\,\ln(c\,x^n)))\,dx$$

input
```
int(igamma(p, d*(a + b*log(c*x^n))),x)
```

output
```
int(igamma(p, d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int \Gamma(p, d(a + b\log{(cx^n)}))\ dx = \int \gamma(p, \log(x^n c)\, bd + ad)\, dx$$

input
```
int(GAMMA(p,d*(a+b*log(c*x^n))),x)
```

output
```
int(gamma(p,log(x**n*c)*b*d + a*d),x)
```

3.199 $\int \frac{\Gamma(p,d(a+b\log(cx^n)))}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \frac{\Gamma(p,d(a+b\log(cx^n)))}{x}\,dx = -\frac{\Gamma(1+p,ad+bd\log(cx^n))}{bdn} + \frac{\Gamma(p,ad+bd\log(cx^n))\,(a+b\log(cx^n))}{bn}$$

output
```
-GAMMA(p+1,a*d+b*d*ln(c*x^n))/b/d/n+GAMMA(p,a*d+b*d*ln(c*x^n))*(a+b*ln(c*x
^n))/b/n
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{\Gamma(p,d(a+b\log(cx^n)))}{x}\,dx = \frac{-\Gamma(1+p,d(a+b\log(cx^n))) + d\Gamma(p,d(a+b\log(cx^n)))\,(a+b\log(cx^n))}{bdn}$$

input
```
Integrate[Gamma[p, d*(a + b*Log[c*x^n])]/x,x]
```

output
```
(-Gamma[1 + p, d*(a + b*Log[c*x^n])] + d*Gamma[p, d*(a + b*Log[c*x^n])]*(a
 + b*Log[c*x^n]))/(b*d*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3039, 7281, 7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(p, d(a + b\log(cx^n)))}{x} dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \Gamma(p, d(a + b\log(cx^n))) d\log(cx^n)}{n}$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int \Gamma(p, ad + b\log(cx^n) d) d(ad + b\log(cx^n) d)}{bdn}$$

$$\Big\downarrow \text{7111}$$

$$\frac{(ad + bd\log(cx^n))\Gamma(p, ad + b\log(cx^n) d) - \Gamma(p+1, ad + b\log(cx^n) d)}{bdn}$$

input
```
Int[Gamma[p, d*(a + b*Log[c*x^n])]/x,x]
```

output
```
(-Gamma[1 + p, a*d + b*d*Log[c*x^n]] + Gamma[p, a*d + b*d*Log[c*x^n]]*(a*d
 + b*d*Log[c*x^n]))/(b*d*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int \frac{\Gamma(p, d(a+b\ln(c\,x^n)))}{x}dx$$

input
```
int(GAMMA(p,d*(a+b*ln(c*x^n)))/x,x)
```

output
```
int(GAMMA(p,d*(a+b*ln(c*x^n)))/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.66

$$\int \frac{\Gamma(p, d(a+b\log(cx^n)))}{x}\,dx = -\frac{(bdn\log(x)+bd\log(c)+ad)(bdn\log(x)+bd\log(c)+ad)^{p-1}e^{(-bdn\log(x)-bd\log(c)-ad)} - (bdn\log(x)+}{bdn}$$

input
```
integrate(gamma(p,d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

output
```
-((b*d*n*log(x) + b*d*log(c) + a*d)*(b*d*n*log(x) + b*d*log(c) + a*d)^(p -
 1)*e^(-b*d*n*log(x) - b*d*log(c) - a*d) - (b*d*n*log(x) + b*d*log(c) + a*
d - p)*gamma(p, b*d*log(c*x^n) + a*d))/(b*d*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\Gamma(p, d(a + b\log(cx^n)))}{x} \, dx = \text{Timed out}$$

input
```
integrate(uppergamma(p,d*(a+b*ln(c*x**n)))/x,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{\Gamma(p, d(a + b\log(cx^n)))}{x} \, dx = \frac{(b\log(cx^n) + a)d\Gamma(p, (b\log(cx^n) + a)d) - \Gamma(p + 1, (b\log(cx^n) + a)d)}{bdn}$$

input
```
integrate(gamma(p,d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

output
```
((b*log(c*x^n) + a)*d*gamma(p, (b*log(c*x^n) + a)*d) - gamma(p + 1, (b*log
(c*x^n) + a)*d))/(b*d*n)
```

Giac [F]

$$\int \frac{\Gamma(p, d(a + b\log(cx^n)))}{x} \, dx = \int \frac{\Gamma(p, (b\log(cx^n) + a)d)}{x} \, dx$$

input
```
integrate(gamma(p,d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

output
```
integrate(gamma(p, (b*log(c*x^n) + a)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(p, d(a + b\log(cx^n)))}{x} \, dx = \int \frac{\Gamma(p, d\,(a + b\,\ln(c\,x^n)))}{x} \, dx$$

input
```
int(igamma(p, d*(a + b*log(c*x^n)))/x,x)
```

output
```
int(igamma(p, d*(a + b*log(c*x^n)))/x, x)
```

Reduce [F]

$$\int \frac{\Gamma(p, d(a + b\log(cx^n)))}{x} \, dx = \int \frac{\gamma(p, \log(x^n c)\,bd + ad)}{x} dx$$

input
```
int(GAMMA(p,d*(a+b*log(c*x^n)))/x,x)
```

output
```
int(gamma(p,log(x**n*c)*b*d + a*d)/x,x)
```

3.200 $\int \frac{\Gamma(p,d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 109

$$\int \frac{\Gamma(p,d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{\Gamma(p,d(a+b\log(cx^n)))}{x} + \frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\Gamma\left(p,\frac{(1+bdn)(a+b\log(cx^n))}{bn}\right)(d(a+b\log(cx^n)))^p\left(\frac{(1+bdn)(a+b\log(cx^n))}{bn}\right)^{-p}}{x}$$

output
```
-GAMMA(p,d*(a+b*ln(c*x^n)))/x+exp(a/b/n)*(c*x^n)^(1/n)*GAMMA(p,(b*d*n+1)*(
a+b*ln(c*x^n))/b/n)*(d*(a+b*ln(c*x^n)))^p/x/(((b*d*n+1)*(a+b*ln(c*x^n))/b/
n)^p)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.98

$$\int \frac{\Gamma(p,d(a+b\log(cx^n)))}{x^2}\,dx = \frac{-\Gamma(p,d(a+b\log(cx^n))) + e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\Gamma\left(p,\frac{(1+bdn)(a+b\log(cx^n))}{bn}\right)(d(a+b\log(cx^n)))^p\left(\frac{(1+bdn)(a+b\log(cx^n))}{bn}\right)}{x}$$

input
```
Integrate[Gamma[p, d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
(-Gamma[p, d*(a + b*Log[c*x^n])] + (E^(a/(b*n))*(c*x^n)^n^(-1)*Gamma[p, ((
1 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)]*(d*(a + b*Log[c*x^n]))^p)/(((1 + b*d
*n)*(a + b*Log[c*x^n]))/(b*n))^p)/x
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {7132, 7271, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(p, d(a + b\log(cx^n)))}{x^2} dx$$

$$\Big\downarrow \text{7132}$$

$$-bdne^{-ad}x^{bdn}(cx^n)^{-bd}\int x^{-bdn-2}(d(a + b\log(cx^n)))^{p-1}dx - \frac{\Gamma(p, d(a + b\log(cx^n)))}{x}$$

$$\Big\downarrow \text{7271}$$

$$-bne^{-ad}x^{bdn}(cx^n)^{-bd}(a + b\log(cx^n))^{-p}(d(a + b\log(cx^n)))^p\int x^{-bdn-2}(a + b\log(cx^n))^{p-1}dx - \frac{\Gamma(p, d(a + b\log(cx^n)))}{x}$$

$$\Big\downarrow \text{2747}$$

$$-\frac{be^{-ad}(cx^n)^{\frac{1}{n}}(a + b\log(cx^n))^{-p}(d(a + b\log(cx^n)))^p\int (cx^n)^{-\frac{bdn+1}{n}}(a + b\log(cx^n))^{p-1}d\log(cx^n)}{x} - \frac{\Gamma(p, d(a + b\log(cx^n)))}{x}$$

$$\Big\downarrow \text{2612}$$

$$\frac{(cx^n)^{\frac{1}{n}} e^{a\left(\frac{1}{bn}+d\right)-ad}(d(a+b\log(cx^n)))^p \left(\frac{(bdn+1)(a+b\log(cx^n))}{bn}\right)^{-p} \Gamma\left(p, \frac{(bdn+1)(a+b\log(cx^n))}{bn}\right)}{x} - \frac{\Gamma(p, d(a+b\log(cx^n)))}{x}$$

input
```
Int[Gamma[p, d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
-(Gamma[p, d*(a + b*Log[c*x^n])]/x) + (E^(-(a*d) + a*(d + 1/(b*n)))*(c*x^n
)^n^(-1)*Gamma[p, ((1 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)]*(d*(a + b*Log[c*
x^n]))^p)/(x*(((1 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))^p)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 7132
```
Int[Gamma[p_, ((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*(Gamma[p, d*(a + b*Log[c*x^n])]/(e*(m +
 1))), x] + Simp[(b*d*n*((e*x)^(b*d*n)/((m + 1)*(c*x^n)^(b*d))))/E^(a*d)
Int[(e*x)^(m - b*d*n)*(d*(a + b*Log[c*x^n]))^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int \frac{\Gamma(p, d(a + b \ln (c\, x^n)))}{x^2} dx$$

input
```
int(GAMMA(p,d*(a+b*ln(c*x^n)))/x^2,x)
```

output
```
int(GAMMA(p,d*(a+b*ln(c*x^n)))/x^2,x)
```

Fricas [F]

$$\int \frac{\Gamma(p, d(a + b \log (cx^n)))}{x^2} \, dx = \int \frac{\Gamma(p, (b \log (cx^n) + a)d)}{x^2} \, dx$$

input
```
integrate(gamma(p,d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

output
```
integral(gamma(p, b*d*log(c*x^n) + a*d)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\Gamma(p, d(a + b \log (cx^n)))}{x^2} \, dx = \text{Timed out}$$

input
```
integrate(uppergamma(p,d*(a+b*ln(c*x**n)))/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\Gamma(p, d(a + b \log(cx^n)))}{x^2} dx = \int \frac{\Gamma(p, (b \log(cx^n) + a)d)}{x^2} dx$$

input
```
integrate(gamma(p,d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

output
```
integrate(gamma(p, (b*log(c*x^n) + a)*d)/x^2, x)
```

Giac [F]

$$\int \frac{\Gamma(p, d(a + b \log(cx^n)))}{x^2} dx = \int \frac{\Gamma(p, (b \log(cx^n) + a)d)}{x^2} dx$$

input
```
integrate(gamma(p,d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

output
```
integrate(gamma(p, (b*log(c*x^n) + a)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(p, d(a + b \log(cx^n)))}{x^2} dx = \int \frac{\Gamma(p, d(a + b \ln(c x^n)))}{x^2} dx$$

input
```
int(igamma(p, d*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int(igamma(p, d*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [F]

$$\int \frac{\Gamma(p, d(a + b\log(cx^n)))}{x^2} \, dx = \int \frac{\gamma(p, \log(x^n c)\, bd + ad)}{x^2} dx$$

input
```
int(GAMMA(p,d*(a+b*log(c*x^n)))/x^2,x)
```

output
```
int(gamma(p,log(x**n*c)*b*d + a*d)/x**2,x)
```

3.201 $\int \frac{\Gamma(p,d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 117

$$\int \frac{\Gamma(p,d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{\Gamma(p,d(a+b\log(cx^n)))}{2x^2} + \frac{e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\Gamma\left(p,\frac{(2+bdn)(a+b\log(cx^n))}{bn}\right)(d(a+b\log(cx^n)))^p\left(\frac{(2+bdn)(a+b\log(cx^n))}{bn}\right)^{-p}}{2x^2}$$

output
```
-1/2*GAMMA(p,d*(a+b*ln(c*x^n)))/x^2+1/2*exp(2*a/b/n)*(c*x^n)^(2/n)*GAMMA(p
,(b*d*n+2)*(a+b*ln(c*x^n))/b/n)*(d*(a+b*ln(c*x^n)))^p/x^2/(((b*d*n+2)*(a+b
*ln(c*x^n))/b/n)^p)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int \frac{\Gamma(p,d(a+b\log(cx^n)))}{x^3}\,dx = \frac{-\Gamma(p,d(a+b\log(cx^n))) + e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\Gamma\left(p,\frac{(2+bdn)(a+b\log(cx^n))}{bn}\right)(d(a+b\log(cx^n)))^p\left(\frac{(2+bdn)(a+b\log(cx^n))}{bn}\right)}{2x^2}$$

input
```
Integrate[Gamma[p, d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
(-Gamma[p, d*(a + b*Log[c*x^n])] + (E^((2*a)/(b*n))*(c*x^n)^(2/n)*Gamma[p,
 ((2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)]*(d*(a + b*Log[c*x^n]))^p)/(((2 +
b*d*n)*(a + b*Log[c*x^n]))/(b*n))^p)/(2*x^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {7132, 7271, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\Gamma(p,d(a+b\log(cx^n)))}{x^3}dx$$

$$\Big\downarrow \text{7132}$$

$$-\frac{1}{2}bdne^{-ad}x^{bdn}(cx^n)^{-bd}\int x^{-bdn-3}(d(a+b\log(cx^n)))^{p-1}dx-\frac{\Gamma(p,d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{7271}$$

$$-\frac{1}{2}bne^{-ad}x^{bdn}(cx^n)^{-bd}(a+b\log(cx^n))^{-p}(d(a+b\log(cx^n)))^p\int x^{-bdn-3}(a+b\log(cx^n))^{p-1}dx-\frac{\Gamma(p,d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{2747}$$

$$-\frac{be^{-ad}(cx^n)^{2/n}(a+b\log(cx^n))^{-p}(d(a+b\log(cx^n)))^p\int (cx^n)^{-\frac{bdn+2}{n}}(a+b\log(cx^n))^{p-1}d\log(cx^n)}{2x^2}-\frac{\Gamma(p,d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{2612}$$

$$\frac{(cx^n)^{2/n}\,e^{a\left(\frac{2}{bn}+d\right)-ad}(d(a+b\log(cx^n)))^p\left(\frac{(bdn+2)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p,\frac{(bdn+2)(a+b\log(cx^n))}{bn}\right)}{2x^2}-\frac{\Gamma(p,d(a+b\log(cx^n)))}{2x^2}$$

input
```
Int[Gamma[p, d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
-1/2*Gamma[p, d*(a + b*Log[c*x^n])]/x^2 + (E^(-(a*d) + a*(d + 2/(b*n)))*(c
*x^n)^(2/n)*Gamma[p, ((2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)]*(d*(a + b*Log
[c*x^n]))^p)/(2*x^2*(((2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))^p)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 7132
```
Int[Gamma[p_, ((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*(Gamma[p, d*(a + b*Log[c*x^n])]/(e*(m +
 1))), x] + Simp[(b*d*n*((e*x)^(b*d*n)/((m + 1)*(c*x^n)^(b*d))))/E^(a*d)
Int[(e*x)^(m - b*d*n)*(d*(a + b*Log[c*x^n]))^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int \frac{\Gamma(p, d(a+b\ln(c\,x^n)))}{x^3}dx$$

input
```
int(GAMMA(p,d*(a+b*ln(c*x^n)))/x^3,x)
```

output
```
int(GAMMA(p,d*(a+b*ln(c*x^n)))/x^3,x)
```

Fricas [F]

$$\int \frac{\Gamma(p, d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\Gamma(p, (b\log(cx^n)+a)d)}{x^3}\,dx$$

input
```
integrate(gamma(p,d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

output
```
integral(gamma(p, b*d*log(c*x^n) + a*d)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\Gamma(p, d(a+b\log(cx^n)))}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(uppergamma(p,d*(a+b*ln(c*x**n)))/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\Gamma(p, d(a + b\log(cx^n)))}{x^3} dx = \int \frac{\Gamma(p, (b\log(cx^n) + a)d)}{x^3} dx$$

input
```
integrate(gamma(p,d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

output
```
integrate(gamma(p, (b*log(c*x^n) + a)*d)/x^3, x)
```

Giac [F]

$$\int \frac{\Gamma(p, d(a + b\log(cx^n)))}{x^3} dx = \int \frac{\Gamma(p, (b\log(cx^n) + a)d)}{x^3} dx$$

input
```
integrate(gamma(p,d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

output
```
integrate(gamma(p, (b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\Gamma(p, d(a + b\log(cx^n)))}{x^3} dx = \int \frac{\Gamma(p, d(a + b\ln(cx^n)))}{x^3} dx$$

input
```
int(igamma(p, d*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int(igamma(p, d*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [F]

$$\int \frac{\Gamma(p, d(a + b\log(cx^n)))}{x^3} dx = \int \frac{\gamma(p, \log(x^n c) bd + ad)}{x^3} dx$$

input
```
int(GAMMA(p,d*(a+b*log(c*x^n)))/x^3,x)
```

output
```
int(gamma(p,log(x**n*c)*b*d + a*d)/x**3,x)
```

3.202 $\int (ex)^m\Gamma(p,d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 144

$$\int (ex)^m\Gamma(p,d(a+b\log(cx^n)))\,dx=\frac{(ex)^{1+m}\Gamma(p,d(a+b\log(cx^n)))}{e(1+m)} -\frac{e^{-\frac{a(1+m)}{bn}}x(ex)^m(cx^n)^{-\frac{1+m}{n}}\Gamma\left(p,-\frac{(1+m-bdn)(a+b\log(cx^n))}{bn}\right)(d(a+b\log(cx^n)))^p\left(-\frac{(1+m-bdn)(a+b\log(cx^n))}{bn}\right)}{1+m}$$

output
```
(e*x)^(1+m)*GAMMA(p,d*(a+b*ln(c*x^n)))/e/(1+m)-x*(e*x)^m*GAMMA(p,-(-b*d*n+
m+1)*(a+b*ln(c*x^n))/b/n)*(d*(a+b*ln(c*x^n)))^p/exp(a*(1+m)/b/n)/(1+m)/((c
*x^n)^((1+m)/n))/((-(-b*d*n+m+1)*(a+b*ln(c*x^n))/b/n)^p)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int (ex)^m\Gamma(p,d(a+b\log(cx^n)))\,dx =\frac{(ex)^m\left(x\Gamma(p,d(a+b\log(cx^n)))-e^{-\frac{(1+m)(a-bn\log(x)+b\log(cx^n))}{bn}}x^{-m}\Gamma\left(p,\frac{(-1-m+bdn)(a+b\log(cx^n))}{bn}\right)(d(a+b\log}{1+m}$$

input
```
Integrate[(e*x)^m*Gamma[p, d*(a + b*Log[c*x^n])],x]
```

output
```
((e*x)^m*(x*Gamma[p, d*(a + b*Log[c*x^n])] - (Gamma[p, ((-1 - m + b*d*n)*(
a + b*Log[c*x^n]))/(b*n)]*(d*(a + b*Log[c*x^n]))^p)/(E^(((1 + m)*(a - b*n*
Log[x] + b*Log[c*x^n]))/(b*n))*x^m*(((-1 - m + b*d*n)*(a + b*Log[c*x^n]))/
(b*n))^p)))/(1 + m)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7132, 7271, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m\Gamma(p,d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \; 7132$$

$$\frac{bdne^{-ad}(cx^n)^{-bd}(ex)^{bdn}\int (ex)^{m-bdn}(d(a+b\log(cx^n)))^{p-1}dx}{m+1}+\frac{(ex)^{m+1}\Gamma(p,d(a+b\log(cx^n)))}{e(m+1)}$$

$$\Big\downarrow \; 7271$$

$$\frac{bne^{-ad}(cx^n)^{-bd}(ex)^{bdn}(a+b\log(cx^n))^{-p}(d(a+b\log(cx^n)))^{p}\int (ex)^{m-bdn}(a+b\log(cx^n))^{p-1}dx}{m+1}+\frac{(ex)^{m+1}\Gamma(p,d(a+b\log(cx^n)))}{e(m+1)}$$

$$\Big\downarrow \; 2747$$

$$\frac{be^{-ad}(ex)^{m+1}(cx^n)^{-\frac{-bdn+m+1}{n}-bd}(a+b\log(cx^n))^{-p}(d(a+b\log(cx^n)))^{p}\int (cx^n)^{\frac{m-bdn+1}{n}}(a+b\log(cx^n))^{p-1}d\,\mathrm{lo}}{e(m+1)}$$
$$\frac{(ex)^{m+1}\Gamma(p,d(a+b\log(cx^n)))}{e(m+1)}$$

$$\Big\downarrow \; 2612$$

$$\frac{(ex)^{m+1}\Gamma(p,d(a+b\log(cx^n)))}{e(m+1)} - \frac{(ex)^{m+1}e^{-\frac{a(-bdn+m+1)}{bn}-ad}(cx^n)^{-\frac{-bdn+m+1}{n}-bd}(d(a+b\log(cx^n)))^p\left(-\frac{(-bdn+m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p,-\frac{(m-bdn+1}{}\right.}{e(m+1)}$$

input
```
Int[(e*x)^m*Gamma[p, d*(a + b*Log[c*x^n])],x]
```

output
```
((e*x)^(1 + m)*Gamma[p, d*(a + b*Log[c*x^n])])/(e*(1 + m)) - (E^(-(a*d) -
(a*(1 + m - b*d*n))/(b*n))*(e*x)^(1 + m)*(c*x^n)^(-(b*d) - (1 + m - b*d*n)
/n)*Gamma[p, -(((1 + m - b*d*n)*(a + b*Log[c*x^n]))/(b*n))]*(d*(a + b*Log[
c*x^n]))^p)/(e*(1 + m)*(-(((1 + m - b*d*n)*(a + b*Log[c*x^n]))/(b*n)))^p)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 7132
```
Int[Gamma[p_, ((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*(Gamma[p, d*(a + b*Log[c*x^n])]/(e*(m +
 1))), x] + Simp[(b*d*n*((e*x)^(b*d*n)/((m + 1)*(c*x^n)^(b*d))))/E^(a*d)
Int[(e*x)^(m - b*d*n)*(d*(a + b*Log[c*x^n]))^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int (ex)^m \Gamma(p, d(a + b \ln (c\, x^n)))\, dx$$

input
```
int((e*x)^m*GAMMA(p,d*(a+b*ln(c*x^n))),x)
```

output
```
int((e*x)^m*GAMMA(p,d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int (ex)^m \Gamma(p, d(a + b \log (cx^n)))\, dx = \int (ex)^m\, \Gamma(p, (b \log (cx^n) + a)d)\, dx$$

input
```
integrate((e*x)^m*gamma(p,d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral((e*x)^m*gamma(p, b*d*log(c*x^n) + a*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \Gamma(p, d(a + b \log (cx^n)))\, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*uppergamma(p,d*(a+b*ln(c*x**n))),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m\Gamma(p,d(a+b\log(cx^n)))\,dx = \int (ex)^m\,\Gamma(p,(b\log(cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*gamma(p,d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate((e*x)^m*gamma(p, (b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int (ex)^m\Gamma(p,d(a+b\log(cx^n)))\,dx = \int (ex)^m\,\Gamma(p,(b\log(cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*gamma(p,d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate((e*x)^m*gamma(p, (b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m\Gamma(p,d(a+b\log(cx^n)))\,dx = \int (e\,x)^m\,\Gamma(p,d\,(a+b\,\ln(c\,x^n)))\,dx$$

input
```
int((e*x)^m*igamma(p, d*(a + b*log(c*x^n))),x)
```

output
```
int((e*x)^m*igamma(p, d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int (ex)^m \Gamma(p, d(a + b \log(cx^n))) \, dx = e^m \left(\int x^m \gamma(p, \log(x^n c) \, bd + ad) \, dx \right)$$

input
```
int((e*x)^m*GAMMA(p,d*(a+b*log(c*x^n))),x)
```

output
```
e**m*int(x**m*gamma(p,log(x**n*c)*b*d + a*d),x)
```

3.203 $\int (c+dx)^3 \log\Gamma(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 74

$$\int (c+dx)^3 \log\Gamma(a+bx)\,dx = -\frac{6d^3\psi^{(-5)}(a+bx)}{b^4}+\frac{6d^2(c+dx)\psi^{(-4)}(a+bx)}{b^3} - \frac{3d(c+dx)^2\psi^{(-3)}(a+bx)}{b^2}+\frac{(c+dx)^3\psi^{(-2)}(a+bx)}{b}$$

output
```
-6*d^3*Psi(-5,b*x+a)/b^4+6*d^2*(d*x+c)*Psi(-4,b*x+a)/b^3-3*d*(d*x+c)^2*Psi
(-3,b*x+a)/b^2+(d*x+c)^3*Psi(-2,b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int (c+dx)^3 \log\Gamma(a+bx)\,dx = \frac{-6d^3\psi^{(-5)}(a+bx)+b(c+dx)\left(6d^2\psi^{(-4)}(a+bx)+b(c+dx)(-3d\psi^{(-3)}(a+bx)+b(c+dx)\psi^{(-2)}(a+b\right.}{b^4}$$

input
```
Integrate[(c + d*x)^3*LogGamma[a + b*x],x]
```

output
```
(-6*d^3*PolyGamma[-5, a + b*x] + b*(c + d*x)*(6*d^2*PolyGamma[-4, a + b*x]
 + b*(c + d*x)*(-3*d*PolyGamma[-3, a + b*x] + b*(c + d*x)*PolyGamma[-2, a
+ b*x])))/b^4
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {7122, 7125, 7125, 7124}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \log\Gamma(a+bx)\,dx$$

$$\Big\downarrow \text{7122}$$

$$\frac{(c+dx)^3\psi^{(-2)}(a+bx)}{b}-\frac{3d\int (c+dx)^2\psi^{(-2)}(a+bx)dx}{b}$$

$$\Big\downarrow \text{7125}$$

$$\frac{(c+dx)^3\psi^{(-2)}(a+bx)}{b}-\frac{3d\left(\frac{(c+dx)^2\psi^{(-3)}(a+bx)}{b}-\frac{2d\int (c+dx)\psi^{(-3)}(a+bx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{7125}$$

$$\frac{(c+dx)^3\psi^{(-2)}(a+bx)}{b}-\frac{3d\left(\frac{(c+dx)^2\psi^{(-3)}(a+bx)}{b}-\frac{2d\left(\frac{(c+dx)\psi^{(-4)}(a+bx)}{b}-\frac{d\int \psi^{(-4)}(a+bx)dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{7124}$$

$$\frac{(c+dx)^3\psi^{(-2)}(a+bx)}{b}-\frac{3d\left(\frac{(c+dx)^2\psi^{(-3)}(a+bx)}{b}-\frac{2d\left(\frac{(c+dx)\psi^{(-4)}(a+bx)}{b}-\frac{d\psi^{(-5)}(a+bx)}{b^2}\right)}{b}\right)}{b}$$

input
```
Int[(c + d*x)^3*LogGamma[a + b*x],x]
```

output
```
(-3*d*((-2*d*(-((d*PolyGamma[-5, a + b*x])/b^2) + ((c + d*x)*PolyGamma[-4,
 a + b*x])/b))/b + ((c + d*x)^2*PolyGamma[-3, a + b*x])/b))/b + ((c + d*x)
^3*PolyGamma[-2, a + b*x])/b
```

Defintions of rubi rules used

rule 7122
```
Int[LogGamma[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^m*(PolyGamma[-2, a + b*x]/b), x] - Simp[d*(m/b)   Int[(c + d*
x)^(m - 1)*PolyGamma[-2, a + b*x], x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[m, 0]
```

rule 7124
```
Int[PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[PolyGamma[n - 1, a
 + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

rule 7125
```
Int[((c_.) + (d_.)*(x_))^(m_.)*PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[(c + d*x)^m*(PolyGamma[n - 1, a + b*x]/b), x] - Simp[d*(m/b)   Int
[(c + d*x)^(m - 1)*PolyGamma[n - 1, a + b*x], x], x] /; FreeQ[{a, b, c, d,
n}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{\Psi(-2,bx+a)(dx+c)^3-\frac{3d\left(\Psi(-3,bx+a)\left(\frac{d(bx+a)}{b}-\frac{ad}{b}+c\right)^2-\frac{2d\left(\Psi(-4,bx+a)\left(\frac{d(bx+a)}{b}-\frac{ad}{b}+c\right)-\frac{d\Psi(-5,bx+a)}{b}\right)}{b}\right)}{b}+\frac{\ln(2\pi \ldots}{\ldots}}{b}$
default	$\frac{\Psi(-2,bx+a)(dx+c)^3-\frac{3d\left(\Psi(-3,bx+a)\left(\frac{d(bx+a)}{b}-\frac{ad}{b}+c\right)^2-\frac{2d\left(\Psi(-4,bx+a)\left(\frac{d(bx+a)}{b}-\frac{ad}{b}+c\right)-\frac{d\Psi(-5,bx+a)}{b}\right)}{b}\right)}{b}+\frac{\ln(2\pi \ldots}{\ldots}}{b}$
parts	$\frac{d^3\,\mathrm{lnGAMMA}(bx+a)x^4}{4}+d^2\,\mathrm{lnGAMMA}\,(bx+a)\,c\,x^3+\frac{3d\,\mathrm{lnGAMMA}(bx+a)c^2x^2}{2}+\mathrm{lnGAMMA}\ldots$

input
```
int((d*x+c)^3*lnGAMMA(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(Psi(-2,b*x+a)*(d*x+c)^3-3*d/b*(Psi(-3,b*x+a)*(d/b*(b*x+a)-a*d/b+c)^2-
2*d/b*(Psi(-4,b*x+a)*(d/b*(b*x+a)-a*d/b+c)-d/b*Psi(-5,b*x+a)))+1/8*ln(2*Pi
)*(d/b*(b*x+a)-a*d/b+c)^4/d*b)
```

Fricas [F]

$$\int (c+dx)^3 \log\Gamma(a+bx)\,dx = \int (dx+c)^3 \mathrm{lngamma}(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*lngamma(b*x+a),x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*lngamma(b*x + a), x)
```

Sympy [F]

$$\int (c+dx)^3 \log\Gamma(a+bx)\,dx = \int (c+dx)^3 \,\mathrm{lnGAMMA}\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*lnGAMMA(b*x+a),x)
```

output
```
Integral((c + d*x)**3*lnGAMMA(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^3 \log\Gamma(a+bx)\,dx = \int (dx+c)^3 \mathrm{lngamma}(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*lngamma(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3*lngamma(b*x + a), x)
```

Giac [F]

$$\int (c+dx)^3 \log\Gamma(a+bx)\,dx = \int (dx+c)^3 \mathrm{lngamma}(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*lngamma(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*lngamma(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \log\Gamma(a+bx)\,dx = \int \mathrm{lnGAMMA}(a+b\,x)\,(c+d\,x)^3\,dx$$

input
```
int(lnGAMMA(a + b*x)*(c + d*x)^3,x)
```

output
```
int(lnGAMMA(a + b*x)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \log\Gamma(a+bx)\,dx = \int (dx+c)^3\, lnGAMMA(bx+a)\,dx$$

input
```
int((d*x+c)^3*lnGAMMA(b*x+a),x)
```

output
```
int((d*x+c)^3*lnGAMMA(b*x+a),x)
```

3.204 $\int (c+dx)^2 \log\Gamma(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int (c+dx)^2 \log\Gamma(a+bx)\,dx = \frac{2d^2\psi^{(-4)}(a+bx)}{b^3} - \frac{2d(c+dx)\psi^{(-3)}(a+bx)}{b^2} + \frac{(c+dx)^2\psi^{(-2)}(a+bx)}{b}$$

output
```
2*d^2*Psi(-4,b*x+a)/b^3-2*d*(d*x+c)*Psi(-3,b*x+a)/b^2+(d*x+c)^2*Psi(-2,b*x
+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int (c+dx)^2 \log\Gamma(a+bx)\,dx = \frac{2d^2\psi^{(-4)}(a+bx)+b(c+dx)(-2d\psi^{(-3)}(a+bx)+b(c+dx)\psi^{(-2)}(a+bx))}{b^3}$$

input
```
Integrate[(c + d*x)^2*LogGamma[a + b*x],x]
```

output
```
(2*d^2*PolyGamma[-4, a + b*x] + b*(c + d*x)*(-2*d*PolyGamma[-3, a + b*x] +
 b*(c + d*x)*PolyGamma[-2, a + b*x]))/b^3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {7122, 7125, 7124}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\log\Gamma(a+bx)\,dx$$

$$\Big\downarrow \text{7122}$$

$$\frac{(c+dx)^2\psi^{(-2)}(a+bx)}{b}-\frac{2d\int (c+dx)\psi^{(-2)}(a+bx)dx}{b}$$

$$\Big\downarrow \text{7125}$$

$$\frac{(c+dx)^2\psi^{(-2)}(a+bx)}{b}-\frac{2d\left(\frac{(c+dx)\psi^{(-3)}(a+bx)}{b}-\frac{d\int \psi^{(-3)}(a+bx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{7124}$$

$$\frac{(c+dx)^2\psi^{(-2)}(a+bx)}{b}-\frac{2d\left(\frac{(c+dx)\psi^{(-3)}(a+bx)}{b}-\frac{d\psi^{(-4)}(a+bx)}{b^2}\right)}{b}$$

input
```
Int[(c + d*x)^2*LogGamma[a + b*x],x]
```

output
```
(-2*d*(-((d*PolyGamma[-4, a + b*x])/b^2) + ((c + d*x)*PolyGamma[-3, a + b*
x])/b))/b + ((c + d*x)^2*PolyGamma[-2, a + b*x])/b
```

Defintions of rubi rules used

rule 7122
```
Int[LogGamma[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^m*(PolyGamma[-2, a + b*x]/b), x] - Simp[d*(m/b)   Int[(c + d*
x)^(m - 1)*PolyGamma[-2, a + b*x], x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[m, 0]
```

rule 7124
```
Int[PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[PolyGamma[n - 1, a
 + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

rule 7125
```
Int[((c_.) + (d_.)*(x_))^(m_.)*PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[(c + d*x)^m*(PolyGamma[n - 1, a + b*x]/b), x] - Simp[d*(m/b)   Int
[(c + d*x)^(m - 1)*PolyGamma[n - 1, a + b*x], x], x] /; FreeQ[{a, b, c, d,
n}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.90

method	result
derivativedivides	$\frac{\Psi(-2,bx+a)(dx+c)^2-\frac{2d\left(\Psi(-3,bx+a)\left(\frac{d(bx+a)}{b}-\frac{ad}{b}+c\right)-\frac{d\Psi(-4,bx+a)}{b}\right)}{b}+\frac{\ln(2\pi)\left(\frac{d(bx+a)}{b}-\frac{ad}{b}+c\right)^3 b}{6d}}{b}$
default	$\frac{\Psi(-2,bx+a)(dx+c)^2-\frac{2d\left(\Psi(-3,bx+a)\left(\frac{d(bx+a)}{b}-\frac{ad}{b}+c\right)-\frac{d\Psi(-4,bx+a)}{b}\right)}{b}+\frac{\ln(2\pi)\left(\frac{d(bx+a)}{b}-\frac{ad}{b}+c\right)^3 b}{6d}}{b}$
parts	$\frac{\mathrm{lnGAMMA}(bx+a)d^2x^3}{3}+\mathrm{lnGAMMA}\left(bx+a\right)dc\,x^2+\mathrm{lnGAMMA}\left(bx+a\right)c^2x+\frac{\mathrm{lnGAMMA}}{3d}$

input
```
int((d*x+c)^2*lnGAMMA(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(Psi(-2,b*x+a)*(d*x+c)^2-2*d/b*(Psi(-3,b*x+a)*(d/b*(b*x+a)-a*d/b+c)-d/
b*Psi(-4,b*x+a))+1/6*ln(2*Pi)*(d/b*(b*x+a)-a*d/b+c)^3/d*b)
```

Fricas [F]

$$\int (c+dx)^2\log\Gamma(a+bx)\,dx = \int (dx+c)^2 \mathrm{lngamma}(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*lngamma(b*x+a),x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*lngamma(b*x + a), x)
```

Sympy [F]

$$\int (c+dx)^2\log\Gamma(a+bx)\,dx = \int (c+dx)^2 \,\mathrm{lnGAMMA}\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*lnGAMMA(b*x+a),x)
```

output
```
Integral((c + d*x)**2*lnGAMMA(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^2\log\Gamma(a+bx)\,dx = \int (dx+c)^2 \mathrm{lngamma}(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*lngamma(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*lngamma(b*x + a), x)
```

Giac [F]

$$\int (c+dx)^2 \log\Gamma(a+bx)\,dx = \int (dx+c)^2 \text{lngamma}(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*lngamma(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*lngamma(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \log\Gamma(a+bx)\,dx = \int \text{lnGAMMA}(a+b\,x)\;(c+d\,x)^2\,dx$$

input
```
int(lnGAMMA(a + b*x)*(c + d*x)^2,x)
```

output
```
int(lnGAMMA(a + b*x)*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \log\Gamma(a+bx)\,dx = \int (dx+c)^2\, lnGAMMA(bx+a)\,dx$$

input
```
int((d*x+c)^2*lnGAMMA(b*x+a),x)
```

output
```
int((d*x+c)^2*lnGAMMA(b*x+a),x)
```

3.205 $\int (c+dx)\log\Gamma(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int (c+dx)\log\Gamma(a+bx)\,dx=-\frac{d\psi^{(-3)}(a+bx)}{b^2}+\frac{(c+dx)\psi^{(-2)}(a+bx)}{b}$$

output
```
-d*Psi(-3,b*x+a)/b^2+(d*x+c)*Psi(-2,b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int (c+dx)\log\Gamma(a+bx)\,dx=\frac{c\psi^{(-2)}(a+bx)}{b}+\frac{d\left(-\frac{\psi^{(-3)}(a+bx)}{b}+x\psi^{(-2)}(a+bx)\right)}{b}$$

input
```
Integrate[(c + d*x)*LogGamma[a + b*x],x]
```

output
```
(c*PolyGamma[-2, a + b*x])/b + (d*(-(PolyGamma[-3, a + b*x]/b) + x*PolyGam
ma[-2, a + b*x]))/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7122, 7124}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\log\Gamma(a+bx)\,dx$$

$$\Big\downarrow \text{7122}$$

$$\frac{(c+dx)\psi^{(-2)}(a+bx)}{b}-\frac{d\int \psi^{(-2)}(a+bx)dx}{b}$$

$$\Big\downarrow \text{7124}$$

$$\frac{(c+dx)\psi^{(-2)}(a+bx)}{b}-\frac{d\psi^{(-3)}(a+bx)}{b^2}$$

input
```
Int[(c + d*x)*LogGamma[a + b*x],x]
```

output
```
-((d*PolyGamma[-3, a + b*x])/b^2) + ((c + d*x)*PolyGamma[-2, a + b*x])/b
```

Defintions of rubi rules used

rule 7122
```
Int[LogGamma[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^m*(PolyGamma[-2, a + b*x]/b), x] - Simp[d*(m/b)   Int[(c + d*
x)^(m - 1)*PolyGamma[-2, a + b*x], x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[m, 0]
```

rule 7124
```
Int[PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[PolyGamma[n - 1, a
 + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(30) = 60$.

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.30

method	result	size
derivativedivides	$\frac{\Psi(-2,bx+a)(dx+c)-\frac{d\Psi(-3,bx+a)}{b}-\frac{\ln(2\pi)\left(da(bx+a)-cb(bx+a)-\frac{d(bx+a)^2}{2}\right)}{2b}}{b}$	69
default	$\frac{\Psi(-2,bx+a)(dx+c)-\frac{d\Psi(-3,bx+a)}{b}-\frac{\ln(2\pi)\left(da(bx+a)-cb(bx+a)-\frac{d(bx+a)^2}{2}\right)}{2b}}{b}$	69

input
```
int((d*x+c)*lnGAMMA(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(Psi(-2,b*x+a)*(d*x+c)-d/b*Psi(-3,b*x+a)-1/2*ln(2*Pi)/b*(d*a*(b*x+a)-c
*b*(b*x+a)-1/2*d*(b*x+a)^2))
```

Fricas [F]

$$\int (c+dx)\log\Gamma(a+bx)\,dx = \int (dx+c)\text{lngamma}(bx+a)\,dx$$

input
```
integrate((d*x+c)*lngamma(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)*lngamma(b*x + a), x)
```

Sympy [F]

$$\int (c+dx)\log\Gamma(a+bx)\,dx = \int (c+dx)\,\mathrm{lnGAMMA}\,(a+bx)\,dx$$

input
```
integrate((d*x+c)*lnGAMMA(b*x+a),x)
```

output
```
Integral((c + d*x)*lnGAMMA(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)\log\Gamma(a+bx)\,dx = \int (dx+c)\mathrm{lngamma}(bx+a)\,dx$$

input
```
integrate((d*x+c)*lngamma(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*lngamma(b*x + a), x)
```

Giac [F]

$$\int (c+dx)\log\Gamma(a+bx)\,dx = \int (dx+c)\mathrm{lngamma}(bx+a)\,dx$$

input
```
integrate((d*x+c)*lngamma(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*lngamma(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\log\Gamma(a+bx)\,dx = \int \mathrm{lnGAMMA}(a+b\,x)\,\left(c+d\,x\right)\,dx$$

input
```
int(lnGAMMA(a + b*x)*(c + d*x),x)
```

output
```
int(lnGAMMA(a + b*x)*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\log\Gamma(a+bx)\,dx = \int (dx+c)\,lnGAMMA(bx+a)\,dx$$

input
```
int((d*x+c)*lnGAMMA(b*x+a),x)
```

output
```
int((d*x+c)*lnGAMMA(b*x+a),x)
```

3.206 $\int \log\Gamma(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \log\Gamma(a+bx)\,dx = \frac{\psi^{(-2)}(a+bx)}{b}$$

output
```
Psi(-2,b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \log\Gamma(a+bx)\,dx = \frac{\psi^{(-2)}(a+bx)}{b}$$

input
```
Integrate[LogGamma[a + b*x],x]
```

output
```
PolyGamma[-2, a + b*x]/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7121}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\Gamma(a+bx)\,dx$$
$$\Big\downarrow \ 7121$$
$$\frac{\psi^{(-2)}(a+bx)}{b}$$

input
```
Int[LogGamma[a + b*x],x]
```

output
```
PolyGamma[-2, a + b*x]/b
```

Defintions of rubi rules used

rule 7121
```
Int[LogGamma[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[PolyGamma[-2, a + b*x]/
b, x] /; FreeQ[{a, b}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

method	result	size
derivativedivides	$\frac{\Psi(-2,bx+a)+\frac{(bx+a)\ln(2\pi)}{2}}{b}$	24
default	$\frac{\Psi(-2,bx+a)+\frac{(bx+a)\ln(2\pi)}{2}}{b}$	24
parts	$x\,\mathrm{lnGAMMA}\left(bx+a\right)-\frac{\Psi(-1,bx+a)bx-\Psi(-2,bx+a)}{b}$	35

input
```
int(lnGAMMA(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(Psi(-2,b*x+a)+1/2*(b*x+a)*ln(2*Pi))
```

Fricas [F]

$$\int \log\Gamma(a+bx)\,dx = \int \text{lngamma}(bx+a)\,dx$$

input
```
integrate(lngamma(b*x+a),x, algorithm="fricas")
```

output
```
integral(lngamma(b*x + a), x)
```

Sympy [F]

$$\int \log\Gamma(a+bx)\,dx = \int \text{lnGAMMA}\,(a+bx)\,dx$$

input
```
integrate(lnGAMMA(b*x+a),x)
```

output
```
Integral(lnGAMMA(a + b*x), x)
```

Maxima [F]

$$\int \log\Gamma(a+bx)\,dx = \int \text{lngamma}(bx+a)\,dx$$

input
```
integrate(lngamma(b*x+a),x, algorithm="maxima")
```

output
```
integrate(lngamma(b*x + a), x)
```

Giac [F]

$$\int \log\Gamma(a+bx)\,dx=\int \text{lngamma}(bx+a)\,dx$$

input
```
integrate(lngamma(b*x+a),x, algorithm="giac")
```

output
```
integrate(lngamma(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\Gamma(a+bx)\,dx=\int \text{lnGAMMA}(a+b\,x)\,dx$$

input
```
int(lnGAMMA(a + b*x),x)
```

output
```
int(lnGAMMA(a + b*x), x)
```

Reduce [F]

$$\int \log\Gamma(a+bx)\,dx=\int lnGAMMA(bx+a)\,dx$$

input
```
int(lnGAMMA(b*x+a),x)
```

output
```
int(lnGAMMA(b*x+a),x)
```

3.207 $\int \frac{\log\Gamma(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\log\Gamma(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\log\Gamma(a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(lnGAMMA(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\log\Gamma(a+bx)}{c+dx}\,dx = \int \frac{\log\Gamma(a+bx)}{c+dx}\,dx$$

input
```
Integrate[LogGamma[a + b*x]/(c + d*x),x]
```

output
```
Integrate[LogGamma[a + b*x]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\Gamma(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \text{7123}$$

$$\int \frac{\log\Gamma(a+bx)}{c+dx}dx$$

input
```
Int[LogGamma[a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{lnGAMMA}\,(bx+a)}{dx+c}dx$$

input
```
int(lnGAMMA(b*x+a)/(d*x+c),x)
```

output
```
int(lnGAMMA(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\log\Gamma(a+bx)}{c+dx}\,dx = \int \frac{\mathrm{lngamma}(bx+a)}{dx+c}\,dx$$

input
```
integrate(lngamma(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(lngamma(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\log\Gamma(a+bx)}{c+dx}\,dx = \int \frac{\mathrm{lnGAMMA}\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(lnGAMMA(b*x+a)/(d*x+c),x)
```

output
```
Integral(lnGAMMA(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\log\Gamma(a+bx)}{c+dx}\,dx = \int \frac{\mathrm{lngamma}(bx+a)}{dx+c}\,dx$$

input
```
integrate(lngamma(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(lngamma(b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\log\Gamma(a+bx)}{c+dx}\,dx = \int \frac{\mathrm{lngamma}(bx+a)}{dx+c}\,dx$$

input
```
integrate(lngamma(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(lngamma(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\log\Gamma(a+bx)}{c+dx}\,dx = \int \frac{\mathrm{lnGAMMA}(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(lnGAMMA(a + b*x)/(c + d*x),x)
```

output
```
int(lnGAMMA(a + b*x)/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\log\Gamma(a+bx)}{c+dx}\,dx = \int \frac{lnGAMMA(bx+a)}{dx+c}dx$$

input
```
int(lnGAMMA(b*x+a)/(d*x+c),x)
```

output
```
int(lnGAMMA(b*x+a)/(d*x+c),x)
```

3.208 $\int \frac{\log\Gamma(a+bx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\log\Gamma(a+bx)}{(c+dx)^2}\,dx = \operatorname{Int}\left(\frac{\log\Gamma(a+bx)}{(c+dx)^2},x\right)$$

output
```
Defer(Int)(lnGAMMA(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\log\Gamma(a+bx)}{(c+dx)^2}\,dx = \int \frac{\log\Gamma(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[LogGamma[a + b*x]/(c + d*x)^2,x]
```

output
```
Integrate[LogGamma[a + b*x]/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\Gamma(a+bx)}{(c+dx)^2} \, dx$$
$$\Big\downarrow \; 7123$$
$$\int \frac{\log\Gamma(a+bx)}{(c+dx)^2}dx$$

input
```
Int[LogGamma[a + b*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\ln\text{GAMMA}\left(bx+a\right)}{\left(dx+c\right)^{2}}dx$$

input
```
int(lnGAMMA(b*x+a)/(d*x+c)^2,x)
```

output
```
int(lnGAMMA(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\log\Gamma(a+bx)}{(c+dx)^2}\,dx = \int \frac{\mathrm{lngamma}(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(lngamma(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(lngamma(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\log\Gamma(a+bx)}{(c+dx)^2}\,dx = \int \frac{\mathrm{lnGAMMA}\,(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(lnGAMMA(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(lnGAMMA(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\log\Gamma(a+bx)}{(c+dx)^2}\,dx = \int \frac{\mathrm{lngamma}(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(lngamma(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(lngamma(b*x + a)/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\log\Gamma(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{lngamma}(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(lngamma(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(lngamma(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\log\Gamma(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{lnGAMMA}(a+b\,x)}{(c+d\,x)^2}\,dx$$

input
```
int(lnGAMMA(a + b*x)/(c + d*x)^2,x)
```

output
```
int(lnGAMMA(a + b*x)/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\log\Gamma(a+bx)}{(c+dx)^2}\,dx = \int \frac{lnGAMMA(bx+a)}{(dx+c)^2}dx$$

input
```
int(lnGAMMA(b*x+a)/(d*x+c)^2,x)
```

output
```
int(lnGAMMA(b*x+a)/(d*x+c)^2,x)
```

3.209 $\int (c+dx)^{3/2}\log\Gamma(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (c+dx)^{3/2}\log\Gamma(a+bx)\,dx = \text{Int}\left((c+dx)^{3/2}\log\Gamma(a+bx),x\right)$$

output
```
Defer(Int)((d*x+c)^(3/2)*lnGAMMA(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^{3/2}\log\Gamma(a+bx)\,dx = \int (c+dx)^{3/2}\log\Gamma(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^(3/2)*LogGamma[a + b*x],x]
```

output
```
Integrate[(c + d*x)^(3/2)*LogGamma[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\text{log}\Gamma(a+bx)\,dx$$

$$\Big\downarrow \ 7123$$

$$\int (c+dx)^{3/2}\text{log}\Gamma(a+bx)dx$$

input
```
Int[(c + d*x)^(3/2)*LogGamma[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (dx+c)^{\frac{3}{2}} \ln\text{GAMMA}\,(bx+a)\,dx$$

input
```
int((d*x+c)^(3/2)*lnGAMMA(b*x+a),x)
```

output
```
int((d*x+c)^(3/2)*lnGAMMA(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (c+dx)^{3/2}\log\Gamma(a+bx)\,dx = \int (dx+c)^{\frac{3}{2}}\mathrm{lngamma}(bx+a)\,dx$$

input
```
integrate((d*x+c)^(3/2)*lngamma(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^(3/2)*lngamma(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (c+dx)^{3/2}\log\Gamma(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\,\mathrm{lnGAMMA}\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*lnGAMMA(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*lnGAMMA(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (c+dx)^{3/2}\log\Gamma(a+bx)\,dx = \int (dx+c)^{\frac{3}{2}}\mathrm{lngamma}(bx+a)\,dx$$

input
```
integrate((d*x+c)^(3/2)*lngamma(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/2)*lngamma(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (c+dx)^{3/2}\log\Gamma(a+bx)\,dx=\int (dx+c)^{\frac{3}{2}}\text{lngamma}(bx+a)\,dx$$

input
```
integrate((d*x+c)^(3/2)*lngamma(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)*lngamma(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (c+dx)^{3/2}\log\Gamma(a+bx)\,dx=\int \text{lnGAMMA}(a+b\,x)\ (c+d\,x)^{3/2}\,dx$$

input
```
int(lnGAMMA(a + b*x)*(c + d*x)^(3/2),x)
```

output
```
int(lnGAMMA(a + b*x)*(c + d*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (c+dx)^{3/2}\log\Gamma(a+bx)\,dx = \int (dx+c)^{\frac{3}{2}}\, lnGAMMA(bx+a)\,dx$$

input
```
int((d*x+c)^(3/2)*lnGAMMA(b*x+a),x)
```

output
```
int((d*x+c)^(3/2)*lnGAMMA(b*x+a),x)
```

3.210 $\int \sqrt{c+dx}\log\Gamma(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \sqrt{c+dx}\log\Gamma(a+bx)\,dx=\mathrm{Int}\left(\sqrt{c+dx}\log\Gamma(a+bx),x\right)$$

output
```
Defer(Int)((d*x+c)^(1/2)*lnGAMMA(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \sqrt{c+dx}\log\Gamma(a+bx)\,dx=\int \sqrt{c+dx}\log\Gamma(a+bx)\,dx$$

input
```
Integrate[Sqrt[c + d*x]*LogGamma[a + b*x],x]
```

output
```
Integrate[Sqrt[c + d*x]*LogGamma[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\log\Gamma(a+bx)\,dx$$

$$\Big\downarrow \ 7123$$

$$\int \sqrt{c+dx}\log\Gamma(a+bx)dx$$

input
```
Int[Sqrt[c + d*x]*LogGamma[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \sqrt{dx+c}\,\mathrm{lnGAMMA}\,(bx+a)\,dx$$

input
```
int((d*x+c)^(1/2)*lnGAMMA(b*x+a),x)
```

output
```
int((d*x+c)^(1/2)*lnGAMMA(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\log\Gamma(a+bx)\,dx = \int \sqrt{dx+c}\,\mathrm{lngamma}(bx+a)\,dx$$

input
```
integrate((d*x+c)^(1/2)*lngamma(b*x+a),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*lngamma(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{c+dx}\log\Gamma(a+bx)\,dx = \int \sqrt{c+dx}\,\mathrm{lnGAMMA}\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*lnGAMMA(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*lnGAMMA(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\log\Gamma(a+bx)\,dx = \int \sqrt{dx+c}\,\mathrm{lngamma}(bx+a)\,dx$$

input
```
integrate((d*x+c)^(1/2)*lngamma(b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*lngamma(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\log\Gamma(a+bx)\,dx=\int \sqrt{dx+c}\,\mathrm{lngamma}(bx+a)\,dx$$

input
```
integrate((d*x+c)^(1/2)*lngamma(b*x+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*lngamma(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\log\Gamma(a+bx)\,dx=\int \mathrm{lnGAMMA}(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

input
```
int(lnGAMMA(a + b*x)*(c + d*x)^(1/2),x)
```

output
```
int(lnGAMMA(a + b*x)*(c + d*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\log\Gamma(a+bx)\,dx = \int \sqrt{dx+c}\,lnGAMMA(bx+a)\,dx$$

input
```
int((d*x+c)^(1/2)*lnGAMMA(b*x+a),x)
```

output
```
int((d*x+c)^(1/2)*lnGAMMA(b*x+a),x)
```

3.211 $\int \frac{\log\Gamma(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\log\Gamma(a+bx)}{\sqrt{c+dx}}\,dx = \text{Int}\left(\frac{\log\Gamma(a+bx)}{\sqrt{c+dx}},x\right)$$

output
```
Defer(Int)(lnGAMMA(b*x+a)/(d*x+c)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\log\Gamma(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\log\Gamma(a+bx)}{\sqrt{c+dx}}\,dx$$

input
```
Integrate[LogGamma[a + b*x]/Sqrt[c + d*x],x]
```

output
```
Integrate[LogGamma[a + b*x]/Sqrt[c + d*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\Gamma(a+bx)}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{7123}$$

$$\int \frac{\log\Gamma(a+bx)}{\sqrt{c+dx}}dx$$

input
```
Int[LogGamma[a + b*x]/Sqrt[c + d*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\text{lnGAMMA}\,(bx+a)}{\sqrt{dx+c}}dx$$

input
```
int(lnGAMMA(b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(lnGAMMA(b*x+a)/(d*x+c)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\log\Gamma(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\mathrm{lngamma}(bx+a)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(lngamma(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(lngamma(b*x + a)/sqrt(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\log\Gamma(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\mathrm{lnGAMMA}\left(a+bx\right)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(lnGAMMA(b*x+a)/(d*x+c)**(1/2),x)
```

output
```
Integral(lnGAMMA(a + b*x)/sqrt(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\log\Gamma(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\mathrm{lngamma}(bx+a)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(lngamma(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(lngamma(b*x + a)/sqrt(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\log\Gamma(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\mathrm{lngamma}(bx+a)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(lngamma(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(lngamma(b*x + a)/sqrt(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\log\Gamma(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\mathrm{lnGAMMA}(a+b\,x)}{\sqrt{c+d\,x}}\,dx$$

input
```
int(lnGAMMA(a + b*x)/(c + d*x)^(1/2),x)
```

output
```
int(lnGAMMA(a + b*x)/(c + d*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\log\Gamma(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{lnGAMMA(bx+a)}{\sqrt{dx+c}}dx$$

input
```
int(lnGAMMA(b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(lnGAMMA(b*x+a)/(d*x+c)^(1/2),x)
```

3.212 $\int (c+dx)^2 \log(\text{Gamma}(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 93

$$\int (c+dx)^2 \log(\text{Gamma}(a+bx))\,dx = \frac{(c+dx)^3 \log(\text{Gamma}(a+bx))}{3d} - \frac{(c+dx)^3 \log\Gamma(a+bx)}{3d} + \frac{2d^2\psi^{(-4)}(a+bx)}{b^3} - \frac{2d(c+dx)\psi^{(-3)}(a+bx)}{b^2} + \frac{(c+dx)^2\psi^{(-2)}(a+bx)}{b}$$

output
```
1/3*(d*x+c)^3*ln(GAMMA(b*x+a))/d-1/3*(d*x+c)^3*lnGAMMA(b*x+a)/d+2*d^2*Psi(
-4,b*x+a)/b^3-2*d*(d*x+c)*Psi(-3,b*x+a)/b^2+(d*x+c)^2*Psi(-2,b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.24

$$\int (c+dx)^2 \log(\text{Gamma}(a+bx))\,dx = \frac{-b^3x(3c^2+3cdx+d^2x^2)\log\Gamma(a+bx)+6d^2\psi^{(-4)}(a+bx)+b\big(-6d(c+dx)\psi^{(-3)}(a+bx)+b\big(bx(3c^2+}{3b^3}$$

input
```
Integrate[(c + d*x)^2*Log[Gamma[a + b*x]],x]
```

output
```
(-(b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2)*LogGamma[a + b*x]) + 6*d^2*PolyGamma[
-4, a + b*x] + b*(-6*d*(c + d*x)*PolyGamma[-3, a + b*x] + b*(b*x*(3*c^2 +
3*c*d*x + d^2*x^2)*Log[Gamma[a + b*x]] + 3*(c + d*x)^2*PolyGamma[-2, a + b
*x])))/(3*b^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3031, 27, 7125, 7122, 7125, 7124}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \log(\text{Gamma}(a+bx))\,dx$$

$$\Big\downarrow \text{3031}$$

$$\frac{(c+dx)^3\log(\text{Gamma}(a+bx))}{3d}-\frac{\int b(c+dx)^3\psi^{(0)}(a+bx)dx}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{(c+dx)^3\log(\text{Gamma}(a+bx))}{3d}-\frac{b\int (c+dx)^3\psi^{(0)}(a+bx)dx}{3d}$$

$$\Big\downarrow \text{7125}$$

$$\frac{(c+dx)^3\log(\text{Gamma}(a+bx))}{3d}-\frac{b\left(\frac{(c+dx)^3\log\Gamma(a+bx)}{b}-\frac{3d\int (c+dx)^2\log\Gamma(a+bx)dx}{b}\right)}{3d}$$

$$\Big\downarrow \text{7122}$$

$$\frac{(c+dx)^3\log(\text{Gamma}(a+bx))}{3d}-\frac{b\left(\frac{(c+dx)^3\log\Gamma(a+bx)}{b}-\frac{3d\left(\frac{(c+dx)^2\psi^{(-2)}(a+bx)}{b}-\frac{2d\int (c+dx)\psi^{(-2)}(a+bx)dx}{b}\right)}{b}\right)}{3d}$$

$$\Big\downarrow \text{ 7125}$$

$$\frac{(c+dx)^3\log(\text{Gamma}(a+bx))}{3d}-\frac{b\left(\frac{(c+dx)^3\log\Gamma(a+bx)}{b}-\frac{3d\left(\frac{(c+dx)^2\psi^{(-2)}(a+bx)}{b}-\frac{2d\left(\frac{(c+dx)\psi^{(-3)}(a+bx)}{b}-\frac{d\int\psi^{(-3)}(a+bx)dx}{b}\right)}{b}\right)}{b}\right)}{3d}$$

$$\Big\downarrow \text{ 7124}$$

$$\frac{(c+dx)^3\log(\text{Gamma}(a+bx))}{3d}-\frac{b\left(\frac{(c+dx)^3\log\Gamma(a+bx)}{b}-\frac{3d\left(\frac{(c+dx)^2\psi^{(-2)}(a+bx)}{b}-\frac{2d\left(\frac{(c+dx)\psi^{(-3)}(a+bx)}{b}-\frac{d\psi^{(-4)}(a+bx)}{b^2}\right)}{b}\right)}{b}\right)}{3d}$$

input
```
Int[(c + d*x)^2*Log[Gamma[a + b*x]],x]
```

output
```
((c + d*x)^3*Log[Gamma[a + b*x]])/(3*d) - (b*(((c + d*x)^3*LogGamma[a + b*
x])/b - (3*d*((-2*d*(-((d*PolyGamma[-4, a + b*x])/b^2) + ((c + d*x)*PolyGa
mma[-3, a + b*x])/b))/b + ((c + d*x)^2*PolyGamma[-2, a + b*x])/b))/b))/(3*
d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3031
```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Simp[1/(b*(m + 1))   Int[SimplifyIntegrand[(a +
 b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunc
tionFreeQ[u, x] && NeQ[m, -1]
```

rule 7122
```
Int[LogGamma[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^m*(PolyGamma[-2, a + b*x]/b), x] - Simp[d*(m/b)   Int[(c + d*
x)^(m - 1)*PolyGamma[-2, a + b*x], x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[m, 0]
```

rule 7124
```
Int[PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[PolyGamma[n - 1, a
 + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

rule 7125
```
Int[((c_.) + (d_.)*(x_))^(m_.)*PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[(c + d*x)^m*(PolyGamma[n - 1, a + b*x]/b), x] - Simp[d*(m/b)   Int
[(c + d*x)^(m - 1)*PolyGamma[n - 1, a + b*x], x], x] /; FreeQ[{a, b, c, d,
n}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.73

method	result
default	$\frac{\ln(\Gamma(bx+a))d^2x^3}{3}+\ln\left(\Gamma(bx+a)\right)dc\,x^2+\ln\left(\Gamma(bx+a)\right)c^2x+\frac{\ln(\Gamma(bx+a))c^3}{3d}-\frac{\Psi(-1,bx+a)(dx+c)^3-\frac{3d\left(\Psi(\right.}{}}{}$
parts	$\frac{\ln(\Gamma(bx+a))d^2x^3}{3}+\ln\left(\Gamma(bx+a)\right)dc\,x^2+\ln\left(\Gamma(bx+a)\right)c^2x+\frac{\ln(\Gamma(bx+a))c^3}{3d}-\frac{\Psi(-1,bx+a)(dx+c)^3-\frac{3d\left(\Psi(\right.}{}}{}$

input
```
int((d*x+c)^2*ln(GAMMA(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(GAMMA(b*x+a))*d^2*x^3+ln(GAMMA(b*x+a))*d*c*x^2+ln(GAMMA(b*x+a))*c^2
*x+1/3*ln(GAMMA(b*x+a))/d*c^3-1/3/d*(Psi(-1,b*x+a)*(d*x+c)^3-3*d/b*(Psi(-2
,b*x+a)*(d/b*(b*x+a)-a*d/b+c)^2-2*d/b*(Psi(-3,b*x+a)*(d/b*(b*x+a)-a*d/b+c)
-d/b*Psi(-4,b*x+a))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.70

$$\int (c+dx)^2 \log(\text{Gamma}(a+bx))\,dx = \frac{1}{3}\left(d^2x^3+3\,cdx^2+3\,c^2x\right)\log\left(\Gamma(bx+a)\right) - \frac{1}{12}\left(bd^2x^4+4\,bcdx^3+6\,bc^2x^2\right)\psi(bx+a)$$

input
```
integrate((d*x+c)^2*log(gamma(b*x+a)),x, algorithm="fricas")
```

output
```
1/3*(d^2*x^3 + 3*c*d*x^2 + 3*c^2*x)*log(gamma(b*x + a)) - 1/12*(b*d^2*x^4
+ 4*b*c*d*x^3 + 6*b*c^2*x^2)*psi(b*x + a)
```

Sympy [F]

$$\int (c+dx)^2 \log(\text{Gamma}(a+bx))\,dx = \int (c+dx)^2 \log\left(\Gamma(a+bx)\right)dx$$

input
```
integrate((d*x+c)**2*ln(gamma(b*x+a)),x)
```

output
```
Integral((c + d*x)**2*log(gamma(a + b*x)), x)
```

Maxima [F]

$$\int (c+dx)^2 \log(\text{Gamma}(a+bx))\,dx = \int (dx+c)^2 \log\left(\Gamma(bx+a)\right)\,dx$$

input
```
integrate((d*x+c)^2*log(gamma(b*x+a)),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*log(gamma(b*x + a)), x)
```

Giac [F]

$$\int (c+dx)^2 \log(\text{Gamma}(a+bx))\,dx = \int (dx+c)^2 \log\left(\Gamma(bx+a)\right)\,dx$$

input
```
integrate((d*x+c)^2*log(gamma(b*x+a)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*log(gamma(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \log(\text{Gamma}(a+bx))\,dx = \int \ln\left(\Gamma(a+b\,x)\right)\,(c+d\,x)^2\,dx$$

input
```
int(log(gamma(a + b*x))*(c + d*x)^2,x)
```

output
```
int(log(gamma(a + b*x))*(c + d*x)^2, x)
```

Reduce [F]

$$\begin{aligned}\int (c+dx)^2 \log(\text{Gamma}(a+bx))\,dx = &\left(\int \log(\gamma(bx+a))\,dx\right)c^2 \\ &+\left(\int \log(\gamma(bx+a))\,x^2dx\right)d^2 \\ &+2\left(\int \log(\gamma(bx+a))\,xdx\right)cd\end{aligned}$$

input
```
int((d*x+c)^2*log(GAMMA(b*x+a)),x)
```

output
```
int(log(gamma(a + b*x)),x)*c**2 + int(log(gamma(a + b*x))*x**2,x)*d**2 + 2
*int(log(gamma(a + b*x))*x,x)*c*d
```

3.213 $\int (c+dx)\log(\text{Gamma}(a+bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 71

$$\int (c+dx)\log(\text{Gamma}(a+bx))\,dx = \frac{(c+dx)^2\log(\text{Gamma}(a+bx))}{2d} - \frac{(c+dx)^2\log\Gamma(a+bx)}{2d} - \frac{d\psi^{(-3)}(a+bx)}{b^2} + \frac{(c+dx)\psi^{(-2)}(a+bx)}{b}$$

output
```
1/2*(d*x+c)^2*ln(GAMMA(b*x+a))/d-1/2*(d*x+c)^2*lnGAMMA(b*x+a)/d-d*Psi(-3,b
*x+a)/b^2+(d*x+c)*Psi(-2,b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.46

$$\int (c+dx)\log(\text{Gamma}(a+bx))\,dx = \frac{1}{2}dx^2\log(\text{Gamma}(a+bx)) + \frac{c(a+bx)\log(\text{Gamma}(a+bx))}{b} - \frac{c(a+bx)\log\Gamma(a+bx)}{b} + \frac{c\psi^{(-2)}(a+bx)}{b} + \frac{1}{2}d\left(-x^2\log\Gamma(a+bx) - \frac{2\psi^{(-3)}(a+bx)}{b^2} + \frac{2x\psi^{(-2)}(a+bx)}{b}\right)$$

input
```
Integrate[(c + d*x)*Log[Gamma[a + b*x]],x]
```

output
```
(d*x^2*Log[Gamma[a + b*x]])/2 + (c*(a + b*x)*Log[Gamma[a + b*x]])/b - (c*(
a + b*x)*LogGamma[a + b*x])/b + (c*PolyGamma[-2, a + b*x])/b + (d*(-(x^2*L
ogGamma[a + b*x]) - (2*PolyGamma[-3, a + b*x])/b^2 + (2*x*PolyGamma[-2, a
+ b*x])/b))/2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3031, 27, 7125, 7122, 7124}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\log(\text{Gamma}(a+bx))\,dx$$

$$\Big\downarrow \; 3031$$

$$\frac{(c+dx)^2\log(\text{Gamma}(a+bx))}{2d} - \frac{\int b(c+dx)^2\psi^{(0)}(a+bx)dx}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{(c+dx)^2\log(\text{Gamma}(a+bx))}{2d}-\frac{b\int(c+dx)^2\psi^{(0)}(a+bx)dx}{2d}$$

$$\Big\downarrow \text{7125}$$

$$\frac{(c+dx)^2\log(\text{Gamma}(a+bx))}{2d}-\frac{b\left(\frac{(c+dx)^2\log\Gamma(a+bx)}{b}-\frac{2d\int(c+dx)\log\Gamma(a+bx)dx}{b}\right)}{2d}$$

$$\Big\downarrow \text{7122}$$

$$\frac{(c+dx)^2\log(\text{Gamma}(a+bx))}{2d}-\frac{b\left(\frac{(c+dx)^2\log\Gamma(a+bx)}{b}-\frac{2d\left(\frac{(c+dx)\psi^{(-2)}(a+bx)}{b}-\frac{d\int\psi^{(-2)}(a+bx)dx}{b}\right)}{b}\right)}{2d}$$

$$\Big\downarrow \text{7124}$$

$$\frac{(c+dx)^2\log(\text{Gamma}(a+bx))}{2d}-\frac{b\left(\frac{(c+dx)^2\log\Gamma(a+bx)}{b}-\frac{2d\left(\frac{(c+dx)\psi^{(-2)}(a+bx)}{b}-\frac{d\psi^{(-3)}(a+bx)}{b^2}\right)}{b}\right)}{2d}$$

input
```
Int[(c + d*x)*Log[Gamma[a + b*x]],x]
```

output
```
((c + d*x)^2*Log[Gamma[a + b*x]])/(2*d) - (b*(((c + d*x)^2*LogGamma[a + b*
x])/b - (2*d*(-((d*PolyGamma[-3, a + b*x])/b^2) + ((c + d*x)*PolyGamma[-2,
 a + b*x])/b))/b))/(2*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3031
```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Simp[1/(b*(m + 1))   Int[SimplifyIntegrand[(a +
 b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunc
tionFreeQ[u, x] && NeQ[m, -1]
```

rule 7122
```
Int[LogGamma[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^m*(PolyGamma[-2, a + b*x]/b), x] - Simp[d*(m/b)   Int[(c + d*
x)^(m - 1)*PolyGamma[-2, a + b*x], x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[m, 0]
```

rule 7124
```
Int[PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[PolyGamma[n - 1, a
 + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

rule 7125
```
Int[((c_.) + (d_.)*(x_))^(m_.)*PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[(c + d*x)^m*(PolyGamma[n - 1, a + b*x]/b), x] - Simp[d*(m/b)   Int
[(c + d*x)^(m - 1)*PolyGamma[n - 1, a + b*x], x], x] /; FreeQ[{a, b, c, d,
n}, x] && GtQ[m, 0]
```

Maple [F]

$$\int (dx+c)\ln\left(\Gamma(bx+a)\right)dx$$

input
```
int((d*x+c)*ln(GAMMA(b*x+a)),x)
```

output
```
int((d*x+c)*ln(GAMMA(b*x+a)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.59

$$\int (c+dx)\log(\text{Gamma}(a+bx))\,dx = \frac{1}{2}\left(dx^2+2\,cx\right)\log\left(\Gamma(bx+a)\right) - \frac{1}{6}\left(bdx^3+3\,bcx^2\right)\psi(bx+a)$$

input
```
integrate((d*x+c)*log(gamma(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(d*x^2 + 2*c*x)*log(gamma(b*x + a)) - 1/6*(b*d*x^3 + 3*b*c*x^2)*psi(b*
x + a)
```

Sympy [F]

$$\int (c+dx)\log(\text{Gamma}(a+bx))\,dx = \int (c+dx)\log\left(\Gamma(a+bx)\right)dx$$

input
```
integrate((d*x+c)*ln(gamma(b*x+a)),x)
```

output
```
Integral((c + d*x)*log(gamma(a + b*x)), x)
```

Maxima [F]

$$\int (c+dx)\log(\text{Gamma}(a+bx))\,dx = \int (dx+c)\log\left(\Gamma(bx+a)\right)\,dx$$

input
```
integrate((d*x+c)*log(gamma(b*x+a)),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*log(gamma(b*x + a)), x)
```

Giac [F]

$$\int (c+dx)\log(\text{Gamma}(a+bx))\,dx = \int (dx+c)\log\left(\Gamma(bx+a)\right)\,dx$$

input
```
integrate((d*x+c)*log(gamma(b*x+a)),x, algorithm="giac")
```

output
```
integrate((d*x + c)*log(gamma(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\log(\text{Gamma}(a+bx))\,dx = \int \ln\left(\Gamma(a+b\,x)\right)\,(c+d\,x)\,dx$$

input
```
int(log(gamma(a + b*x))*(c + d*x),x)
```

output
```
int(log(gamma(a + b*x))*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\log(\text{Gamma}(a+bx))\,dx = \left(\int \log(\gamma(bx+a))\,dx\right)c + \left(\int \log(\gamma(bx+a))\,xdx\right)d$$

input
```
int((d*x+c)*log(GAMMA(b*x+a)),x)
```

output
```
int(log(gamma(a + b*x)),x)*c + int(log(gamma(a + b*x))*x,x)*d
```

3.214 $\int \log(\text{Gamma}(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 30

$$\int \log(\text{Gamma}(a+bx))\,dx = x\log(\text{Gamma}(a+bx)) - x\log\Gamma(a+bx) + \frac{\psi^{(-2)}(a+bx)}{b}$$

output
```
x*ln(GAMMA(b*x+a))-x*lnGAMMA(b*x+a)+Psi(-2,b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27

$$\int \log(\text{Gamma}(a+bx))\,dx = \frac{(a+bx)\log(\text{Gamma}(a+bx)) - (a+bx)\log\Gamma(a+bx) + \psi^{(-2)}(a+bx)}{b}$$

input
```
Integrate[Log[Gamma[a + b*x]],x]
```

output
```
((a + b*x)*Log[Gamma[a + b*x]] - (a + b*x)*LogGamma[a + b*x] + PolyGamma[-
2, a + b*x])/b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3028, 27, 7125, 7121}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(\text{Gamma}(a+bx))\,dx$$

$$\Big\downarrow \text{3028}$$

$$x\log(\text{Gamma}(a+bx))-\int bx\psi^{(0)}(a+bx)dx$$

$$\Big\downarrow \text{27}$$

$$x\log(\text{Gamma}(a+bx))-b\int x\psi^{(0)}(a+bx)dx$$

$$\Big\downarrow \text{7125}$$

$$x\log(\text{Gamma}(a+bx))-b\left(\frac{x\text{log}\Gamma(a+bx)}{b}-\frac{\int \text{log}\Gamma(a+bx)dx}{b}\right)$$

$$\Big\downarrow \text{7121}$$

$$x\log(\text{Gamma}(a+bx))-b\left(\frac{x\text{log}\Gamma(a+bx)}{b}-\frac{\psi^{(-2)}(a+bx)}{b^2}\right)$$

input
```
Int[Log[Gamma[a + b*x]],x]
```

output
```
x*Log[Gamma[a + b*x]] - b*((x*LogGamma[a + b*x])/b - PolyGamma[-2, a + b*x
]/b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 7121
```
Int[LogGamma[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[PolyGamma[-2, a + b*x]/
b, x] /; FreeQ[{a, b}, x]
```

rule 7125
```
Int[((c_.) + (d_.)*(x_))^(m_.)*PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[(c + d*x)^m*(PolyGamma[n - 1, a + b*x]/b), x] - Simp[d*(m/b)   Int
[(c + d*x)^(m - 1)*PolyGamma[n - 1, a + b*x], x], x] /; FreeQ[{a, b, c, d,
n}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

method	result	size
default	$x \ln\left(\Gamma\left(b x+a\right)\right)-\frac{\Psi\left(-1, b x+a\right) b x-\Psi\left(-2, b x+a\right)}{b}$	36
parts	$x \ln\left(\Gamma\left(b x+a\right)\right)-\frac{\Psi\left(-1, b x+a\right) b x-\Psi\left(-2, b x+a\right)}{b}$	36

input
```
int(ln(GAMMA(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
x*ln(GAMMA(b*x+a))-1/b*(Psi(-1,b*x+a)*b*x-Psi(-2,b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \log(\mathrm{Gamma}(a+bx))\,dx = -\frac{1}{2}bx^2\psi(bx+a) + x\log\left(\Gamma(bx+a)\right)$$

input
```
integrate(log(gamma(b*x+a)),x, algorithm="fricas")
```

output
```
-1/2*b*x^2*psi(b*x + a) + x*log(gamma(b*x + a))
```

Sympy [F]

$$\int \log(\mathrm{Gamma}(a+bx))\,dx = \int \log\left(\Gamma(a+bx)\right)dx$$

input
```
integrate(ln(gamma(b*x+a)),x)
```

output
```
Integral(log(gamma(a + b*x)), x)
```

Maxima [F]

$$\int \log(\mathrm{Gamma}(a+bx))\,dx = \int \log\left(\Gamma(bx+a)\right)\,dx$$

input
```
integrate(log(gamma(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(log(gamma(b*x + a)), x)
```

Giac [F]

$$\int \log(\text{Gamma}(a+bx))\,dx = \int \log\left(\Gamma(bx+a)\right)\,dx$$

input
```
integrate(log(gamma(b*x+a)),x, algorithm="giac")
```

output
```
integrate(log(gamma(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(\text{Gamma}(a+bx))\,dx = \int \ln\left(\Gamma(a+b\,x)\right)\,dx$$

input
```
int(log(gamma(a + b*x)),x)
```

output
```
int(log(gamma(a + b*x)), x)
```

Reduce [F]

$$\int \log(\text{Gamma}(a+bx))\,dx = \int \log(\gamma(bx+a))\,dx$$

input
```
int(log(GAMMA(b*x+a)),x)
```

output
```
int(log(gamma(a + b*x)),x)
```

3.215 $\int \frac{\log(\text{Gamma}(a+bx))}{c+dx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\log(\text{Gamma}(a+bx))}{c+dx}\,dx = \frac{\log(c+dx)(\log(\text{Gamma}(a+bx)) - \log\Gamma(a+bx))}{d} + \text{Int}\left(\frac{\log\Gamma(a+bx)}{c+dx}, x\right)$$

output
```
ln(d*x+c)*(ln(GAMMA(b*x+a))-lnGAMMA(b*x+a))/d+Defer(Int)(lnGAMMA(b*x+a)/(d
*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\log(\text{Gamma}(a+bx))}{c+dx}\,dx = \int \frac{\log(\text{Gamma}(a+bx))}{c+dx}\,dx$$

input
```
Integrate[Log[Gamma[a + b*x]]/(c + d*x),x]
```

output
```
Integrate[Log[Gamma[a + b*x]]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(\text{Gamma}(a+bx))}{c+dx}\,dx$$

$$\Big\downarrow \text{3040}$$

$$\int \frac{\log\Gamma(a+bx)}{c+dx}dx+(\log(\text{Gamma}(a+bx))-\log\Gamma(a+bx))\int \frac{1}{c+dx}dx$$

$$\Big\downarrow \text{16}$$

$$\int \frac{\log\Gamma(a+bx)}{c+dx}dx+\frac{\log(c+dx)(\log(\text{Gamma}(a+bx))-\log\Gamma(a+bx))}{d}$$

$$\Big\downarrow \text{7123}$$

$$\int \frac{\log\Gamma(a+bx)}{c+dx}dx+\frac{\log(c+dx)(\log(\text{Gamma}(a+bx))-\log\Gamma(a+bx))}{d}$$

input
```
Int[Log[Gamma[a + b*x]]/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(\Gamma(bx+a)\right)}{dx+c}dx$$

input
```
int(ln(GAMMA(b*x+a))/(d*x+c),x)
```

output
```
int(ln(GAMMA(b*x+a))/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\log(\mathrm{Gamma}(a+bx))}{c+dx}\,dx = \int \frac{\log\left(\Gamma(bx+a)\right)}{dx+c}\,dx$$

input
```
integrate(log(gamma(b*x+a))/(d*x+c),x, algorithm="fricas")
```

output
```
integral(log(gamma(b*x + a))/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\log(\mathrm{Gamma}(a+bx))}{c+dx}\,dx = \int \frac{\log\left(\Gamma(a+bx)\right)}{c+dx}\,dx$$

input
```
integrate(ln(gamma(b*x+a))/(d*x+c),x)
```

output
```
Integral(log(gamma(a + b*x))/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\log(\text{Gamma}(a+bx))}{c+dx}\,dx = \int \frac{\log\left(\Gamma(bx+a)\right)}{dx+c}\,dx$$

input
```
integrate(log(gamma(b*x+a))/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(log(gamma(b*x + a))/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\log(\text{Gamma}(a+bx))}{c+dx}\,dx = \int \frac{\log\left(\Gamma(bx+a)\right)}{dx+c}\,dx$$

input
```
integrate(log(gamma(b*x+a))/(d*x+c),x, algorithm="giac")
```

output
```
integrate(log(gamma(b*x + a))/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\log(\text{Gamma}(a+bx))}{c+dx}\,dx = \int \frac{\ln\left(\Gamma(a+b\,x)\right)}{c+d\,x}\,dx$$

input
```
int(log(gamma(a + b*x))/(c + d*x),x)
```

output
```
int(log(gamma(a + b*x))/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\log(\mathrm{Gamma}(a+bx))}{c+dx}\,dx = \int \frac{\log(\gamma(bx+a))}{dx+c}dx$$

input
```
int(log(GAMMA(b*x+a))/(d*x+c),x)
```

output
```
int(log(gamma(a + b*x))/(c + d*x),x)
```

3.216 $\int \frac{\log(\text{Gamma}(a+bx))}{(c+dx)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\log(\text{Gamma}(a+bx))}{(c+dx)^2} dx = -\frac{\log(\text{Gamma}(a+bx))}{d(c+dx)} + \frac{b\text{Int}\left(\frac{\psi^{(0)}(a+bx)}{c+dx}, x\right)}{d}$$

output
```
-ln(GAMMA(b*x+a))/d/(d*x+c)+b*Defer(Int)(Psi(b*x+a)/(d*x+c),x)/d
```

Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\log(\text{Gamma}(a+bx))}{(c+dx)^2} dx = \int \frac{\log(\text{Gamma}(a+bx))}{(c+dx)^2} dx$$

input
```
Integrate[Log[Gamma[a + b*x]]/(c + d*x)^2,x]
```

output
```
Integrate[Log[Gamma[a + b*x]]/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(\text{Gamma}(a+bx))}{(c+dx)^2} \, dx$$

$$\Big\downarrow \; 3031$$

$$\frac{\int \frac{b\psi^{(0)}(a+bx)}{c+dx}dx}{d} - \frac{\log(\text{Gamma}(a+bx))}{d(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{b\int \frac{\psi^{(0)}(a+bx)}{c+dx}dx}{d} - \frac{\log(\text{Gamma}(a+bx))}{d(c+dx)}$$

$$\Big\downarrow \; 7127$$

$$\frac{b\int \frac{\psi^{(0)}(a+bx)}{c+dx}dx}{d} - \frac{\log(\text{Gamma}(a+bx))}{d(c+dx)}$$

input
```
Int[Log[Gamma[a + b*x]]/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(\Gamma(bx+a)\right)}{\left(dx+c\right)^2}dx$$

input
```
int(ln(GAMMA(b*x+a))/(d*x+c)^2,x)
```

output
```
int(ln(GAMMA(b*x+a))/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{\log(\mathrm{Gamma}(a+bx))}{(c+dx)^2}\,dx = \int \frac{\log\left(\Gamma(bx+a)\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(log(gamma(b*x+a))/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(log(gamma(b*x + a))/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log(\mathrm{Gamma}(a+bx))}{(c+dx)^2}\,dx = \int \frac{\log\left(\Gamma(a+bx)\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(ln(gamma(b*x+a))/(d*x+c)**2,x)
```

output
```
Integral(log(gamma(a + b*x))/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\log(\text{Gamma}(a+bx))}{(c+dx)^2}\,dx = \int \frac{\log\left(\Gamma(bx+a)\right)}{(dx+c)^2}\,dx$$

input
```
integrate(log(gamma(b*x+a))/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(log(gamma(b*x + a))/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\log(\text{Gamma}(a+bx))}{(c+dx)^2}\,dx = \int \frac{\log\left(\Gamma(bx+a)\right)}{(dx+c)^2}\,dx$$

input
```
integrate(log(gamma(b*x+a))/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(log(gamma(b*x + a))/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\log(\text{Gamma}(a+bx))}{(c+dx)^2}\,dx = \int \frac{\ln\left(\Gamma(a+b\,x)\right)}{\left(c+d\,x\right)^2}\,dx$$

input
```
int(log(gamma(a + b*x))/(c + d*x)^2,x)
```

output
```
int(log(gamma(a + b*x))/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{\log(\text{Gamma}(a+bx))}{(c+dx)^2}\,dx = \int \frac{\log(\gamma(bx+a))}{d^2x^2+2cdx+c^2}dx$$

input
```
int(log(GAMMA(b*x+a))/(d*x+c)^2,x)
```

output
```
int(log(gamma(a + b*x))/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.217 $\int (c+dx)^m \psi^{(n)}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int (c+dx)^m \psi^{(n)}(a+bx)\,dx = \text{Int}\left((c+dx)^m \psi^{(n)}(a+bx),x\right)$$

output
```
Defer(Int)((d*x+c)^m*Psi(n,b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m \psi^{(n)}(a+bx)\,dx = \int (c+dx)^m \psi^{(n)}(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*PolyGamma[n, a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*PolyGamma[n, a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m \psi^{(n)}(a+bx)\,dx$$

$$\Big\downarrow \; 7127$$

$$\int (c+dx)^m \psi^{(n)}(a+bx)dx$$

input
```
Int[(c + d*x)^m*PolyGamma[n, a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \Psi(n, bx+a)\,dx$$

input
```
int((d*x+c)^m*Psi(n,b*x+a),x)
```

output
```
int((d*x+c)^m*Psi(n,b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\psi^{(n)}(a+bx)\,dx = \int (dx+c)^m\Psi(n,bx+a)\,dx$$

input
```
integrate((d*x+c)^m*Psi(n,b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*Psi(n, b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int (c+dx)^m\psi^{(n)}(a+bx)\,dx = \int (c+dx)^m \operatorname{diEulerGamma}(n,a+bx)\,dx$$

input
```
integrate((d*x+c)**m*diEulerGamma(n,b*x+a),x)
```

output
```
Integral((c + d*x)**m*diEulerGamma(n, a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\psi^{(n)}(a+bx)\,dx = \int (dx+c)^m\Psi(n,bx+a)\,dx$$

input
```
integrate((d*x+c)^m*Psi(n,b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*Psi(n, b*x + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int (c+dx)^m \psi^{(n)}(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^m*Psi(n,b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage0:=((((sageVARd)*(sageVARx))+(sageVARc))^(sageVARm))*(Psi(sageVARn
,((sageVARb)*(sageVARx))+(sageVARa))):;OUTPUT:Psi(sageVARn,sageVARb*sageVA
Rx+sageVA
```

Mupad [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m \psi^{(n)}(a+bx)\,dx = \int \psi^{(a+b\,x)}(n)\,\left(c+d\,x\right)^m\,dx$$

input
```
int(psi(a + b*x, n)*(c + d*x)^m,x)
```

output
```
int(psi(a + b*x, n)*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (c+dx)^m\psi^{(n)}(a+bx)\,dx = \int (dx+c)^m\,polygamma(n,bx+a)\,dx$$

input
```
int((d*x+c)^m*Psi(n,b*x+a),x)
```

output
```
int((c + d*x)**m*polygamma(n,a + b*x),x)
```

3.218 $\int (c+dx)^3\psi^{(n)}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 82

$$\int (c+dx)^3\psi^{(n)}(a+bx)\,dx=-\frac{6d^3\psi^{(-4+n)}(a+bx)}{b^4}+\frac{6d^2(c+dx)\psi^{(-3+n)}(a+bx)}{b^3}-\frac{3d(c+dx)^2\psi^{(-2+n)}(a+bx)}{b^2}+\frac{(c+dx)^3\psi^{(-1+n)}(a+bx)}{b}$$

output
```
-6*d^3*Psi(-4+n,b*x+a)/b^4+6*d^2*(d*x+c)*Psi(-3+n,b*x+a)/b^3-3*d*(d*x+c)^2
*Psi(-2+n,b*x+a)/b^2+(d*x+c)^3*Psi(-1+n,b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int (c+dx)^3\psi^{(n)}(a+bx)\,dx=\frac{-6d^3\psi^{(-4+n)}(a+bx)+b(c+dx)\left(6d^2\psi^{(-3+n)}(a+bx)+b(c+dx)(-3d\psi^{(-2+n)}(a+bx)+b(c+dx)\psi^{(-}\right.}{b^4}$$

input
```
Integrate[(c + d*x)^3*PolyGamma[n, a + b*x],x]
```

output
```
(-6*d^3*PolyGamma[-4 + n, a + b*x] + b*(c + d*x)*(6*d^2*PolyGamma[-3 + n,
a + b*x] + b*(c + d*x)*(-3*d*PolyGamma[-2 + n, a + b*x] + b*(c + d*x)*Poly
Gamma[-1 + n, a + b*x])))/b^4
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {7125, 7125, 7125, 7124}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\psi^{(n)}(a+bx)\,dx$$

$$\Big\downarrow \text{7125}$$

$$\frac{(c+dx)^3\psi^{(n-1)}(a+bx)}{b}-\frac{3d\int (c+dx)^2\psi^{(n-1)}(a+bx)dx}{b}$$

$$\Big\downarrow \text{7125}$$

$$\frac{(c+dx)^3\psi^{(n-1)}(a+bx)}{b}-\frac{3d\left(\frac{(c+dx)^2\psi^{(n-2)}(a+bx)}{b}-\frac{2d\int (c+dx)\psi^{(n-2)}(a+bx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{7125}$$

$$\frac{(c+dx)^3\psi^{(n-1)}(a+bx)}{b}-\frac{3d\left(\frac{(c+dx)^2\psi^{(n-2)}(a+bx)}{b}-\frac{2d\left(\frac{(c+dx)\psi^{(n-3)}(a+bx)}{b}-\frac{d\int \psi^{(n-3)}(a+bx)dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{7124}$$

$$\frac{(c+dx)^3\psi^{(n-1)}(a+bx)}{b}-\frac{3d\left(\frac{(c+dx)^2\psi^{(n-2)}(a+bx)}{b}-\frac{2d\left(\frac{(c+dx)\psi^{(n-3)}(a+bx)}{b}-\frac{d\psi^{(n-4)}(a+bx)}{b^2}\right)}{b}\right)}{b}$$

input
```
Int[(c + d*x)^3*PolyGamma[n, a + b*x],x]
```

output
```
(-3*d*((-2*d*(-((d*PolyGamma[-4 + n, a + b*x])/b^2) + ((c + d*x)*PolyGamma
[-3 + n, a + b*x])/b))/b + ((c + d*x)^2*PolyGamma[-2 + n, a + b*x])/b))/b
+ ((c + d*x)^3*PolyGamma[-1 + n, a + b*x])/b
```

Defintions of rubi rules used

rule 7124
```
Int[PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[PolyGamma[n - 1, a
 + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

rule 7125
```
Int[((c_.) + (d_.)*(x_))^(m_.)*PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[(c + d*x)^m*(PolyGamma[n - 1, a + b*x]/b), x] - Simp[d*(m/b)   Int
[(c + d*x)^(m - 1)*PolyGamma[n - 1, a + b*x], x], x] /; FreeQ[{a, b, c, d,
n}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{\Psi(-1+n,bx+a)(dx+c)^3-\frac{3d\left(\Psi(n-2,bx+a)\left(\frac{d(bx+a)}{b}-\frac{ad}{b}+c\right)^2-\frac{2d\left(\Psi(n-3,bx+a)\left(\frac{d(bx+a)}{b}-\frac{ad}{b}+c\right)-\frac{d\Psi(n-4,bx+a)}{b}\right)}{b}\right)}{b}}{b}$
default	$\frac{\Psi(-1+n,bx+a)(dx+c)^3-\frac{3d\left(\Psi(n-2,bx+a)\left(\frac{d(bx+a)}{b}-\frac{ad}{b}+c\right)^2-\frac{2d\left(\Psi(n-3,bx+a)\left(\frac{d(bx+a)}{b}-\frac{ad}{b}+c\right)-\frac{d\Psi(n-4,bx+a)}{b}\right)}{b}\right)}{b}}{b}$

input
```
int((d*x+c)^3*Psi(n,b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(Psi(-1+n,b*x+a)*(d*x+c)^3-3*d/b*(Psi(n-2,b*x+a)*(d/b*(b*x+a)-a*d/b+c)
^2-2*d/b*(Psi(n-3,b*x+a)*(d/b*(b*x+a)-a*d/b+c)-d/b*Psi(n-4,b*x+a))))
```

Fricas [F]

$$\int (c+dx)^3\psi^{(n)}(a+bx)\,dx = \int (dx+c)^3\Psi(n,bx+a)\,dx$$

input
```
integrate((d*x+c)^3*Psi(n,b*x+a),x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*Psi(n, b*x + a), x)
```

Sympy [F]

$$\int (c+dx)^3\psi^{(n)}(a+bx)\,dx = \int (c+dx)^3 \operatorname{diEulerGamma}(n,a+bx)\,dx$$

input
```
integrate((d*x+c)**3*diEulerGamma(n,b*x+a),x)
```

output
```
Integral((c + d*x)**3*diEulerGamma(n, a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^3\psi^{(n)}(a+bx)\,dx = \int (dx+c)^3\Psi(n,bx+a)\,dx$$

input
```
integrate((d*x+c)^3*Psi(n,b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3*Psi(n, b*x + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int (c+dx)^3\psi^{(n)}(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^3*Psi(n,b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage0:=((((sageVARd)*(sageVARx))+(sageVARc))^(3))*(Psi(sageVARn,((sage
VARb)*(sageVARx))+(sageVARa))):;OUTPUT:Psi(sageVARn,sageVARb*sageVARx+sage
VARa)  Er
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\psi^{(n)}(a+bx)\,dx = \int \psi^{(a+b\,x)}(n)\ (c+d\,x)^3\,dx$$

input
```
int(psi(a + b*x, n)*(c + d*x)^3,x)
```

output
```
int(psi(a + b*x, n)*(c + d*x)^3, x)
```

Reduce [F]

$$\begin{aligned}\int (c+dx)^3\psi^{(n)}(a+bx)\,dx &= \left(\int polygamma(n,bx+a)\,dx\right)c^3 \\ &\quad+\left(\int polygamma(n,bx+a)\,x^3dx\right)d^3 \\ &\quad+3\left(\int polygamma(n,bx+a)\,x^2dx\right)c\,d^2 \\ &\quad+3\left(\int polygamma(n,bx+a)\,xdx\right)c^2d\end{aligned}$$

input
```
int((d*x+c)^3*Psi(n,b*x+a),x)
```

output
```
int(polygamma(n,a + b*x),x)*c**3 + int(polygamma(n,a + b*x)*x**3,x)*d**3 +
 3*int(polygamma(n,a + b*x)*x**2,x)*c*d**2 + 3*int(polygamma(n,a + b*x)*x,
x)*c**2*d
```

3.219 $\int (c+dx)^2\psi^{(n)}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int (c+dx)^2\psi^{(n)}(a+bx)\,dx=\frac{2d^2\psi^{(-3+n)}(a+bx)}{b^3}-\frac{2d(c+dx)\psi^{(-2+n)}(a+bx)}{b^2}+\frac{(c+dx)^2\psi^{(-1+n)}(a+bx)}{b}$$

output
```
2*d^2*Psi(-3+n,b*x+a)/b^3-2*d*(d*x+c)*Psi(-2+n,b*x+a)/b^2+(d*x+c)^2*Psi(-1
+n,b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int (c+dx)^2\psi^{(n)}(a+bx)\,dx=\frac{2d^2\psi^{(-3+n)}(a+bx)+b(c+dx)\left(-2d\psi^{(-2+n)}(a+bx)+b(c+dx)\psi^{(-1+n)}(a+bx)\right)}{b^3}$$

input
```
Integrate[(c + d*x)^2*PolyGamma[n, a + b*x],x]
```

output
```
(2*d^2*PolyGamma[-3 + n, a + b*x] + b*(c + d*x)*(-2*d*PolyGamma[-2 + n, a
+ b*x] + b*(c + d*x)*PolyGamma[-1 + n, a + b*x]))/b^3
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7125, 7125, 7124}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\psi^{(n)}(a+bx)\,dx$$

$$\Big\downarrow \text{7125}$$

$$\frac{(c+dx)^2\psi^{(n-1)}(a+bx)}{b}-\frac{2d\int (c+dx)\psi^{(n-1)}(a+bx)dx}{b}$$

$$\Big\downarrow \text{7125}$$

$$\frac{(c+dx)^2\psi^{(n-1)}(a+bx)}{b}-\frac{2d\left(\frac{(c+dx)\psi^{(n-2)}(a+bx)}{b}-\frac{d\int \psi^{(n-2)}(a+bx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{7124}$$

$$\frac{(c+dx)^2\psi^{(n-1)}(a+bx)}{b}-\frac{2d\left(\frac{(c+dx)\psi^{(n-2)}(a+bx)}{b}-\frac{d\psi^{(n-3)}(a+bx)}{b^2}\right)}{b}$$

input
```
Int[(c + d*x)^2*PolyGamma[n, a + b*x],x]
```

output
```
(-2*d*(-((d*PolyGamma[-3 + n, a + b*x])/b^2) + ((c + d*x)*PolyGamma[-2 + n
, a + b*x])/b))/b + ((c + d*x)^2*PolyGamma[-1 + n, a + b*x])/b
```

Defintions of rubi rules used

rule 7124
```
Int[PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[PolyGamma[n - 1, a
 + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

rule 7125
```
Int[((c_.) + (d_.)*(x_))^(m_.)*PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[(c + d*x)^m*(PolyGamma[n - 1, a + b*x]/b), x] - Simp[d*(m/b)   Int
[(c + d*x)^(m - 1)*PolyGamma[n - 1, a + b*x], x], x] /; FreeQ[{a, b, c, d,
n}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28

method	result	size
derivativedivides	$\frac{\Psi(-1+n,bx+a)(dx+c)^2-\frac{2d\left(\Psi(n-2,bx+a)\left(\frac{d(bx+a)}{b}-\frac{ad}{b}+c\right)-\frac{d\Psi(n-3,bx+a)}{b}\right)}{b}}{b}$	74
default	$\frac{\Psi(-1+n,bx+a)(dx+c)^2-\frac{2d\left(\Psi(n-2,bx+a)\left(\frac{d(bx+a)}{b}-\frac{ad}{b}+c\right)-\frac{d\Psi(n-3,bx+a)}{b}\right)}{b}}{b}$	74

input
```
int((d*x+c)^2*Psi(n,b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(Psi(-1+n,b*x+a)*(d*x+c)^2-2*d/b*(Psi(n-2,b*x+a)*(d/b*(b*x+a)-a*d/b+c)
-d/b*Psi(n-3,b*x+a)))
```

Fricas [F]

$$\int (c+dx)^2\psi^{(n)}(a+bx)\,dx=\int (dx+c)^2\Psi(n,bx+a)\,dx$$

input
```
integrate((d*x+c)^2*Psi(n,b*x+a),x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*Psi(n, b*x + a), x)
```

Sympy [F]

$$\int (c+dx)^2\psi^{(n)}(a+bx)\,dx=\int (c+dx)^2\,\text{diEulerGamma}\,(n,a+bx)\,dx$$

input
```
integrate((d*x+c)**2*diEulerGamma(n,b*x+a),x)
```

output
```
Integral((c + d*x)**2*diEulerGamma(n, a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^2\psi^{(n)}(a+bx)\,dx=\int (dx+c)^2\Psi(n,bx+a)\,dx$$

input
```
integrate((d*x+c)^2*Psi(n,b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*Psi(n, b*x + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int (c+dx)^2\psi^{(n)}(a+bx)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^2*Psi(n,b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage0:=((((sageVARd)*(sageVARx))+(sageVARc))^(2))*(Psi(sageVARn,((sage
VARb)*(sageVARx))+(sageVARa))):;OUTPUT:Psi(sageVARn,sageVARb*sageVARx+sage
VARa)  Er
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\psi^{(n)}(a+bx)\,dx=\int \psi^{(a+b\,x)}(n)\,\left(c+d\,x\right)^2\,dx$$

input
```
int(psi(a + b*x, n)*(c + d*x)^2,x)
```

output
```
int(psi(a + b*x, n)*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2\psi^{(n)}(a+bx)\,dx=\left(\int polygamma(n,bx+a)\,dx\right)c^2+\left(\int polygamma(n,bx+a)\,x^2dx\right)d^2+2\left(\int polygamma(n,bx+a)\,xdx\right)cd$$

input
```
int((d*x+c)^2*Psi(n,b*x+a),x)
```

output
```
int(polygamma(n,a + b*x),x)*c**2 + int(polygamma(n,a + b*x)*x**2,x)*d**2 +
 2*int(polygamma(n,a + b*x)*x,x)*c*d
```

3.220 $\int (c+dx)\psi^{(n)}(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int (c+dx)\psi^{(n)}(a+bx)\,dx=-\frac{d\psi^{(-2+n)}(a+bx)}{b^2}+\frac{(c+dx)\psi^{(-1+n)}(a+bx)}{b}$$

output
```
-d*Psi(-2+n,b*x+a)/b^2+(d*x+c)*Psi(-1+n,b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int (c+dx)\psi^{(n)}(a+bx)\,dx=\frac{c\psi^{(-1+n)}(a+bx)}{b}+\frac{d\left(-\frac{\psi^{(-2+n)}(a+bx)}{b}+x\psi^{(-1+n)}(a+bx)\right)}{b}$$

input
```
Integrate[(c + d*x)*PolyGamma[n, a + b*x],x]
```

output
```
(c*PolyGamma[-1 + n, a + b*x])/b + (d*(-(PolyGamma[-2 + n, a + b*x]/b) + x
*PolyGamma[-1 + n, a + b*x]))/b
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {7125, 7124}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\psi^{(n)}(a+bx)\,dx$$

$$\Big\downarrow \text{ 7125}$$

$$\frac{(c+dx)\psi^{(n-1)}(a+bx)}{b}-\frac{d\int \psi^{(n-1)}(a+bx)dx}{b}$$

$$\Big\downarrow \text{ 7124}$$

$$\frac{(c+dx)\psi^{(n-1)}(a+bx)}{b}-\frac{d\psi^{(n-2)}(a+bx)}{b^2}$$

input
```
Int[(c + d*x)*PolyGamma[n, a + b*x],x]
```

output
```
-((d*PolyGamma[-2 + n, a + b*x])/b^2) + ((c + d*x)*PolyGamma[-1 + n, a + b
*x])/b
```

Defintions of rubi rules used

rule 7124
```
Int[PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[PolyGamma[n - 1, a
 + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

rule 7125
```
Int[((c_.) + (d_.)*(x_))^(m_.)*PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[(c + d*x)^m*(PolyGamma[n - 1, a + b*x]/b), x] - Simp[d*(m/b)   Int
[(c + d*x)^(m - 1)*PolyGamma[n - 1, a + b*x], x], x] /; FreeQ[{a, b, c, d,
n}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\Psi(-1+n,bx+a)(dx+c)-\frac{d\Psi(n-2,bx+a)}{b}}{b}$	36
default	$\frac{\Psi(-1+n,bx+a)(dx+c)-\frac{d\Psi(n-2,bx+a)}{b}}{b}$	36

input
```
int((d*x+c)*Psi(n,b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(Psi(-1+n,b*x+a)*(d*x+c)-d/b*Psi(n-2,b*x+a))
```

Fricas [F]

$$\int (c+dx)\psi^{(n)}(a+bx)\,dx=\int (dx+c)\Psi(n,bx+a)\,dx$$

input
```
integrate((d*x+c)*Psi(n,b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)*Psi(n, b*x + a), x)
```

Sympy [F]

$$\int (c+dx)\psi^{(n)}(a+bx)\,dx=\int (c+dx)\,\mathrm{diEulerGamma}\,(n,a+bx)\,dx$$

input
```
integrate((d*x+c)*diEulerGamma(n,b*x+a),x)
```

output
```
Integral((c + d*x)*diEulerGamma(n, a + b*x), x)
```

Maxima [F]

$$\int (c+dx)\psi^{(n)}(a+bx)\,dx = \int (dx+c)\Psi(n,bx+a)\;dx$$

input
```
integrate((d*x+c)*Psi(n,b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*Psi(n, b*x + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int (c+dx)\psi^{(n)}(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)*Psi(n,b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage0:=(((sageVARd)*(sageVARx))+(sageVARc))*(Psi(sageVARn,((sageVARb)*
(sageVARx))+(sageVARa))):;OUTPUT:Psi(sageVARn,sageVARb*sageVARx+sageVARa)
 Error: B
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\psi^{(n)}(a+bx)\,dx = \int \psi^{(a+b\,x)}(n)\;(c+d\,x)\;dx$$

input
```
int(psi(a + b*x, n)*(c + d*x),x)
```

output
```
int(psi(a + b*x, n)*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\psi^{(n)}(a+bx)\,dx = \left(\int polygamma(n,bx+a)\,dx\right)c + \left(\int polygamma(n,bx+a)\,xdx\right)d$$

input
```
int((d*x+c)*Psi(n,b*x+a),x)
```

output
```
int(polygamma(n,a + b*x),x)*c + int(polygamma(n,a + b*x)*x,x)*d
```

3.221 $\int \psi^{(n)}(a+bx)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 13

$$\int \psi^{(n)}(a+bx)\,dx=\frac{\psi^{(-1+n)}(a+bx)}{b}$$

output
```
Psi(-1+n,b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \psi^{(n)}(a+bx)\,dx=\frac{\psi^{(-1+n)}(a+bx)}{b}$$

input
```
Integrate[PolyGamma[n, a + b*x],x]
```

output
```
PolyGamma[-1 + n, a + b*x]/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {7124}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \psi^{(n)}(a+bx)\,dx$$

$$\Big\downarrow \text{7124}$$

$$\frac{\psi^{(n-1)}(a+bx)}{b}$$

input
```
Int[PolyGamma[n, a + b*x],x]
```

output
```
PolyGamma[-1 + n, a + b*x]/b
```

Defintions of rubi rules used

rule 7124
```
Int[PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[PolyGamma[n - 1, a
 + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\Psi(-1+n,bx+a)}{b}$	14
default	$\frac{\Psi(-1+n,bx+a)}{b}$	14

input
```
int(Psi(n,b*x+a),x,method=_RETURNVERBOSE)
```

output
```
Psi(-1+n,b*x+a)/b
```

Fricas [F]

$$\int \psi^{(n)}(a+bx)\,dx=\int \Psi(n,bx+a)\,dx$$

input
```
integrate(Psi(n,b*x+a),x, algorithm="fricas")
```

output
```
integral(Psi(n, b*x + a), x)
```

Sympy [F]

$$\int \psi^{(n)}(a+bx)\,dx=\int \operatorname{diEulerGamma}(n,a+bx)\,dx$$

input
```
integrate(diEulerGamma(n,b*x+a),x)
```

output
```
Integral(diEulerGamma(n, a + b*x), x)
```

Maxima [F]

$$\int \psi^{(n)}(a+bx)\,dx=\int \Psi(n,bx+a)\,dx$$

input
```
integrate(Psi(n,b*x+a),x, algorithm="maxima")
```

output
```
integrate(Psi(n, b*x + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \psi^{(n)}(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(Psi(n,b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage0:=Psi(sageVARn,((sageVARb)*(sageVARx))+(sageVARa)):;OUTPUT:Psi(sa
geVARn,sageVARb*sageVARx+sageVARa)  Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \psi^{(n)}(a+bx)\,dx = \int \psi^{(a+b\,x)}(n)\ dx$$

input
```
int(psi(a + b*x, n),x)
```

output
```
int(psi(a + b*x, n), x)
```

Reduce [F]

$$\int \psi^{(n)}(a+bx)\,dx = \int polygamma(n,bx+a)\,dx$$

input
```
int(Psi(n,b*x+a),x)
```

output
```
int(polygamma(n,a + b*x),x)
```

3.222 $\int \frac{\psi^{(n)}(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\psi^{(n)}(a+bx)}{c+dx}\,dx=\operatorname{Int}\left(\frac{\psi^{(n)}(a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(Psi(n,b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\psi^{(n)}(a+bx)}{c+dx}\,dx=\int \frac{\psi^{(n)}(a+bx)}{c+dx}\,dx$$

input
```
Integrate[PolyGamma[n, a + b*x]/(c + d*x),x]
```

output
```
Integrate[PolyGamma[n, a + b*x]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\psi^{(n)}(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \text{ 7127}$$

$$\int \frac{\psi^{(n)}(a+bx)}{c+dx}dx$$

input
```
Int[PolyGamma[n, a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\Psi(n,bx+a)}{dx+c}dx$$

input
```
int(Psi(n,b*x+a)/(d*x+c),x)
```

output
```
int(Psi(n,b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\psi^{(n)}(a+bx)}{c+dx}\,dx = \int \frac{\Psi(n,bx+a)}{dx+c}\,dx$$

input
```
integrate(Psi(n,b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(Psi(n, b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{\psi^{(n)}(a+bx)}{c+dx}\,dx = \int \frac{\mathrm{diEulerGamma}\left(n,a+bx\right)}{c+dx}\,dx$$

input
```
integrate(diEulerGamma(n,b*x+a)/(d*x+c),x)
```

output
```
Integral(diEulerGamma(n, a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\psi^{(n)}(a+bx)}{c+dx}\,dx = \int \frac{\Psi(n,bx+a)}{dx+c}\,dx$$

input
```
integrate(Psi(n,b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(Psi(n, b*x + a)/(d*x + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\psi^{(n)}(a+bx)}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(Psi(n,b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage0:=((((sageVARd)*(sageVARx))+(sageVARc))^(-1))*(Psi(sageVARn,((sag
eVARb)*(sageVARx))+(sageVARa))):;OUTPUT:Psi(sageVARn,sageVARb*sageVARx+sag
eVARa)  E
```

Mupad [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\psi^{(n)}(a+bx)}{c+dx}\,dx = \int \frac{\psi^{(a+b\,x)}(n)}{c+d\,x}\,dx$$

input
```
int(psi(a + b*x, n)/(c + d*x),x)
```

output
```
int(psi(a + b*x, n)/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\psi^{(n)}(a+bx)}{c+dx}\,dx=\int \frac{polygamma(n,bx+a)}{dx+c}dx$$

input
```
int(Psi(n,b*x+a)/(d*x+c),x)
```

output
```
int(polygamma(n,a + b*x)/(c + d*x),x)
```

3.223 $\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^2} dx = -\frac{\psi^{(n)}(a+bx)}{d(c+dx)} + \frac{b\text{Int}\left(\frac{\psi^{(1+n)}(a+bx)}{c+dx}, x\right)}{d}$$

output
```
-Psi(n,b*x+a)/d/(d*x+c)+b*Defer(Int)(Psi(1+n,b*x+a)/(d*x+c),x)/d
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^2} dx = \int \frac{\psi^{(n)}(a+bx)}{(c+dx)^2} dx$$

input
```
Integrate[PolyGamma[n, a + b*x]/(c + d*x)^2,x]
```

output
```
Integrate[PolyGamma[n, a + b*x]/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 7126}$$

$$\frac{b\int \frac{\psi^{(n+1)}(a+bx)}{c+dx}dx}{d}-\frac{\psi^{(n)}(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{ 7127}$$

$$\frac{b\int \frac{\psi^{(n+1)}(a+bx)}{c+dx}dx}{d}-\frac{\psi^{(n)}(a+bx)}{d(c+dx)}$$

input
```
Int[PolyGamma[n, a + b*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\Psi(n,bx+a)}{(dx+c)^2}dx$$

input
```
int(Psi(n,b*x+a)/(d*x+c)^2,x)
```

output
```
int(Psi(n,b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\Psi(n,bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(Psi(n,b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(Psi(n, b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{diEulerGamma}\,(n,a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(diEulerGamma(n,b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(diEulerGamma(n, a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\Psi(n,bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(Psi(n,b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(Psi(n, b*x + a)/(d*x + c)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(Psi(n,b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage0:=((((sageVARd)*(sageVARx))+(sageVARc))^(-2))*(Psi(sageVARn,((sag
eVARb)*(sageVARx))+(sageVARa))):;OUTPUT:Psi(sageVARn,sageVARb*sageVARx+sag
eVARa)  E
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\psi^{(a+b\,x)}(n)}{(c+d\,x)^2}\,dx$$

input
```
int(psi(a + b*x, n)/(c + d*x)^2,x)
```

output
```
int(psi(a + b*x, n)/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^2}\,dx = \int \frac{polygamma(n,bx+a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(Psi(n,b*x+a)/(d*x+c)^2,x)
```

output
```
int(polygamma(n,a + b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.224 $\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^3} \, dx = -\frac{\psi^{(n)}(a+bx)}{2d(c+dx)^2} - \frac{b\psi^{(1+n)}(a+bx)}{2d^2(c+dx)} + \frac{b^2\mathrm{Int}\left(\frac{\psi^{(2+n)}(a+bx)}{c+dx}, x\right)}{2d^2}$$

output
```
-1/2*Psi(n,b*x+a)/d/(d*x+c)^2-1/2*b*Psi(1+n,b*x+a)/d^2/(d*x+c)+1/2*b^2*Def
er(Int)(Psi(2+n,b*x+a)/(d*x+c),x)/d^2
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^3} \, dx = \int \frac{\psi^{(n)}(a+bx)}{(c+dx)^3} \, dx$$

input
```
Integrate[PolyGamma[n, a + b*x]/(c + d*x)^3,x]
```

output
```
Integrate[PolyGamma[n, a + b*x]/(c + d*x)^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^3} \, dx$$

$$\Big\downarrow \text{ 7126}$$

$$\frac{b \int \frac{\psi^{(n+1)}(a+bx)}{(c+dx)^2}dx}{2d} - \frac{\psi^{(n)}(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 7126}$$

$$\frac{b \left(\frac{b \int \frac{\psi^{(n+2)}(a+bx)}{c+dx}dx}{d} - \frac{\psi^{(n+1)}(a+bx)}{d(c+dx)}\right)}{2d} - \frac{\psi^{(n)}(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 7127}$$

$$\frac{b \left(\frac{b \int \frac{\psi^{(n+2)}(a+bx)}{c+dx}dx}{d} - \frac{\psi^{(n+1)}(a+bx)}{d(c+dx)}\right)}{2d} - \frac{\psi^{(n)}(a+bx)}{2d(c+dx)^2}$$

input
```
Int[PolyGamma[n, a + b*x]/(c + d*x)^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\Psi(n, bx + a)}{(dx + c)^3} dx$$

input
```
int(Psi(n,b*x+a)/(d*x+c)^3,x)
```

output
```
int(Psi(n,b*x+a)/(d*x+c)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{\psi^{(n)}(a + bx)}{(c + dx)^3} dx = \int \frac{\Psi(n, bx + a)}{(dx + c)^3} dx$$

input
```
integrate(Psi(n,b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(Psi(n, b*x + a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\psi^{(n)}(a + bx)}{(c + dx)^3} dx = \int \frac{\mathrm{diEulerGamma}\,(n, a + bx)}{(c + dx)^3} dx$$

input
```
integrate(diEulerGamma(n,b*x+a)/(d*x+c)**3,x)
```

output
```
Integral(diEulerGamma(n, a + b*x)/(c + d*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^3}\,dx = \int \frac{\Psi(n,bx+a)}{(dx+c)^3}\,dx$$

input
```
integrate(Psi(n,b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(Psi(n, b*x + a)/(d*x + c)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(Psi(n,b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage0:=((((sageVARd)*(sageVARx))+(sageVARc))^(-3))*(Psi(sageVARn,((sag
eVARb)*(sageVARx))+(sageVARa))):;OUTPUT:Psi(sageVARn,sageVARb*sageVARx+sag
eVARa)  E
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^3} \, dx = \int \frac{\psi^{(a+b\,x)}(n)}{(c+d\,x)^3} \, dx$$

input
```
int(psi(a + b*x, n)/(c + d*x)^3,x)
```

output
```
int(psi(a + b*x, n)/(c + d*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^3} \, dx = \int \frac{polygamma(n,bx+a)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int(Psi(n,b*x+a)/(d*x+c)^3,x)
```

output
```
int(polygamma(n,a + b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x
)
```

3.225 $\int (c+dx)^{3/2}\psi^{(n)}(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int (c+dx)^{3/2}\psi^{(n)}(a+bx)\,dx = -\frac{3d\sqrt{c+dx}\psi^{(-2+n)}(a+bx)}{2b^2} + \frac{(c+dx)^{3/2}\psi^{(-1+n)}(a+bx)}{b} + \frac{3d^2\text{Int}\left(\frac{\psi^{(-2+n)}(a+bx)}{\sqrt{c+dx}},x\right)}{4b^2}$$

output
```
-3/2*d*(d*x+c)^(1/2)*Psi(-2+n,b*x+a)/b^2+(d*x+c)^(3/2)*Psi(-1+n,b*x+a)/b+3
/4*d^2*Defer(Int)(Psi(-2+n,b*x+a)/(d*x+c)^(1/2),x)/b^2
```

Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int (c+dx)^{3/2}\psi^{(n)}(a+bx)\,dx = \int (c+dx)^{3/2}\psi^{(n)}(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^(3/2)*PolyGamma[n, a + b*x],x]
```

output
```
Integrate[(c + d*x)^(3/2)*PolyGamma[n, a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\psi^{(n)}(a+bx)\,dx$$

$$\Big\downarrow \; 7125$$

$$\frac{(c+dx)^{3/2}\psi^{(n-1)}(a+bx)}{b}-\frac{3d\int \sqrt{c+dx}\psi^{(n-1)}(a+bx)dx}{2b}$$

$$\Big\downarrow \; 7125$$

$$\frac{(c+dx)^{3/2}\psi^{(n-1)}(a+bx)}{b}-\frac{3d\left(\frac{\sqrt{c+dx}\psi^{(n-2)}(a+bx)}{b}-\frac{d\int \frac{\psi^{(n-2)}(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}$$

$$\Big\downarrow \; 7127$$

$$\frac{(c+dx)^{3/2}\psi^{(n-1)}(a+bx)}{b}-\frac{3d\left(\frac{\sqrt{c+dx}\psi^{(n-2)}(a+bx)}{b}-\frac{d\int \frac{\psi^{(n-2)}(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}$$

input
```
Int[(c + d*x)^(3/2)*PolyGamma[n, a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int (dx+c)^{\frac{3}{2}} \Psi(n, bx+a)\, dx$$

input
```
int((d*x+c)^(3/2)*Psi(n,b*x+a),x)
```

output
```
int((d*x+c)^(3/2)*Psi(n,b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (c+dx)^{3/2}\psi^{(n)}(a+bx)\, dx = \int (dx+c)^{\frac{3}{2}} \Psi(n, bx+a)\, dx$$

input
```
integrate((d*x+c)^(3/2)*Psi(n,b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^(3/2)*Psi(n, b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int (c+dx)^{3/2}\psi^{(n)}(a+bx)\, dx = \int (c+dx)^{\frac{3}{2}} \operatorname{diEulerGamma}(n, a+bx)\, dx$$

input
```
integrate((d*x+c)**(3/2)*diEulerGamma(n,b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*diEulerGamma(n, a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (c+dx)^{3/2}\psi^{(n)}(a+bx)\,dx = \int (dx+c)^{\frac{3}{2}}\Psi(n,bx+a)\,dx$$

input
```
integrate((d*x+c)^(3/2)*Psi(n,b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/2)*Psi(n, b*x + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int (c+dx)^{3/2}\psi^{(n)}(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^(3/2)*Psi(n,b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage0:=((((sageVARd)*(sageVARx))+(sageVARc))^(3/2))*(Psi(sageVARn,((sa
geVARb)*(sageVARx))+(sageVARa))):;OUTPUT:Psi(sageVARn,sageVARb*sageVARx+sa
geVARa)
```

Mupad [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (c+dx)^{3/2}\psi^{(n)}(a+bx)\,dx = \int \psi^{(a+b\,x)}(n)\,\left(c+d\,x\right)^{3/2}\,dx$$

input
```
int(psi(a + b*x, n)*(c + d*x)^(3/2),x)
```

output
```
int(psi(a + b*x, n)*(c + d*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.24

$$\int (c+dx)^{3/2}\psi^{(n)}(a+bx)\,dx = \left(\int \sqrt{dx+c}\,polygamma(n,bx+a)\,xdx\right)d + \left(\int \sqrt{dx+c}\,polygamma(n,bx+a)\,dx\right)c$$

input
```
int((d*x+c)^(3/2)*Psi(n,b*x+a),x)
```

output
```
int(sqrt(c + d*x)*polygamma(n,a + b*x)*x,x)*d + int(sqrt(c + d*x)*polygamm
a(n,a + b*x),x)*c
```

3.226 $\int \sqrt{c+dx}\psi^{(n)}(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \sqrt{c+dx}\psi^{(n)}(a+bx)\,dx=\frac{\sqrt{c+dx}\psi^{(-1+n)}(a+bx)}{b}-\frac{d\text{Int}\left(\frac{\psi^{(-1+n)}(a+bx)}{\sqrt{c+dx}},x\right)}{2b}$$

output
```
(d*x+c)^(1/2)*Psi(-1+n,b*x+a)/b-1/2*d*Defer(Int)(Psi(-1+n,b*x+a)/(d*x+c)^(
1/2),x)/b
```

Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sqrt{c+dx}\psi^{(n)}(a+bx)\,dx=\int \sqrt{c+dx}\psi^{(n)}(a+bx)\,dx$$

input
```
Integrate[Sqrt[c + d*x]*PolyGamma[n, a + b*x],x]
```

output
```
Integrate[Sqrt[c + d*x]*PolyGamma[n, a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\psi^{(n)}(a+bx)\,dx$$

$$\Big\downarrow \text{ 7125}$$

$$\frac{\sqrt{c+dx}\psi^{(n-1)}(a+bx)}{b}-\frac{d\int \frac{\psi^{(n-1)}(a+bx)}{\sqrt{c+dx}}dx}{2b}$$

$$\Big\downarrow \text{ 7127}$$

$$\frac{\sqrt{c+dx}\psi^{(n-1)}(a+bx)}{b}-\frac{d\int \frac{\psi^{(n-1)}(a+bx)}{\sqrt{c+dx}}dx}{2b}$$

input
```
Int[Sqrt[c + d*x]*PolyGamma[n, a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{dx+c}\,\Psi(n,bx+a)\,dx$$

input
```
int((d*x+c)^(1/2)*Psi(n,b*x+a),x)
```

output
```
int((d*x+c)^(1/2)*Psi(n,b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\psi^{(n)}(a+bx)\,dx = \int \sqrt{dx+c}\Psi(n,bx+a)\,dx$$

input
```
integrate((d*x+c)^(1/2)*Psi(n,b*x+a),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*Psi(n, b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{c+dx}\psi^{(n)}(a+bx)\,dx = \int \sqrt{c+dx}\,\mathrm{diEulerGamma}\,(n,a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*diEulerGamma(n,b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*diEulerGamma(n, a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\psi^{(n)}(a+bx)\,dx = \int \sqrt{dx+c}\Psi(n,bx+a)\,dx$$

input
```
integrate((d*x+c)^(1/2)*Psi(n,b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*Psi(n, b*x + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+dx}\psi^{(n)}(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^(1/2)*Psi(n,b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage0:=((((sageVARd)*(sageVARx))+(sageVARc))^(1/2))*(Psi(sageVARn,((sa
geVARb)*(sageVARx))+(sageVARa))):;OUTPUT:Psi(sageVARn,sageVARb*sageVARx+sa
geVARa)
```

Mupad [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\psi^{(n)}(a+bx)\,dx = \int \psi^{(a+b\,x)}(n)\,\sqrt{c+d\,x}\,dx$$

input
```
int(psi(a + b*x, n)*(c + d*x)^(1/2),x)
```

output
```
int(psi(a + b*x, n)*(c + d*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \sqrt{c+dx}\psi^{(n)}(a+bx)\,dx=\int \sqrt{dx+c}\,polygamma(n,bx+a)\,dx$$

input
```
int((d*x+c)^(1/2)*Psi(n,b*x+a),x)
```

output
```
int(sqrt(c + d*x)*polygamma(n,a + b*x),x)
```

3.227 $\int \frac{\psi^{(n)}(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{\psi^{(n)}(a+bx)}{\sqrt{c+dx}}\,dx = \text{Int}\left(\frac{\psi^{(n)}(a+bx)}{\sqrt{c+dx}},x\right)$$

output
```
Defer(Int)(Psi(n,b*x+a)/(d*x+c)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{\psi^{(n)}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\psi^{(n)}(a+bx)}{\sqrt{c+dx}}\,dx$$

input
```
Integrate[PolyGamma[n, a + b*x]/Sqrt[c + d*x],x]
```

output
```
Integrate[PolyGamma[n, a + b*x]/Sqrt[c + d*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\psi^{(n)}(a+bx)}{\sqrt{c+dx}} \, dx$$

$$\Big\downarrow \text{ 7127}$$

$$\int \frac{\psi^{(n)}(a+bx)}{\sqrt{c+dx}}dx$$

input
```
Int[PolyGamma[n, a + b*x]/Sqrt[c + d*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\Psi(n, bx+a)}{\sqrt{dx+c}}dx$$

input
```
int(Psi(n,b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(Psi(n,b*x+a)/(d*x+c)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\psi^{(n)}(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\Psi(n,bx+a)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(Psi(n,b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(Psi(n, b*x + a)/sqrt(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\psi^{(n)}(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\operatorname{diEulerGamma}\left(n,a+bx\right)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(diEulerGamma(n,b*x+a)/(d*x+c)**(1/2),x)
```

output
```
Integral(diEulerGamma(n, a + b*x)/sqrt(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\psi^{(n)}(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\Psi(n,bx+a)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(Psi(n,b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(Psi(n, b*x + a)/sqrt(d*x + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\psi^{(n)}(a+bx)}{\sqrt{c+dx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(Psi(n,b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage0:=((((sageVARd)*(sageVARx))+(sageVARc))^(-1/2))*(Psi(sageVARn,((s
ageVARb)*(sageVARx))+(sageVARa))):;OUTPUT:Psi(sageVARn,sageVARb*sageVARx+s
ageVARa)
```

Mupad [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\psi^{(n)}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\psi^{(a+b\,x)}(n)}{\sqrt{c+d\,x}}\,dx$$

input
```
int(psi(a + b*x, n)/(c + d*x)^(1/2),x)
```

output
```
int(psi(a + b*x, n)/(c + d*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\psi^{(n)}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{polygamma(n,bx+a)}{\sqrt{dx+c}}dx$$

input
```
int(Psi(n,b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(polygamma(n,a + b*x)/sqrt(c + d*x),x)
```

3.228 $\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\psi^{(n)}(a+bx)}{d\sqrt{c+dx}} + \frac{2b\text{Int}\left(\frac{\psi^{(1+n)}(a+bx)}{\sqrt{c+dx}},x\right)}{d}$$

output
```
-2*Psi(n,b*x+a)/d/(d*x+c)^(1/2)+2*b*Defer(Int)(Psi(1+n,b*x+a)/(d*x+c)^(1/2
),x)/d
```

Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\psi^{(n)}(a+bx)}{(c+dx)^{3/2}}\,dx$$

input
```
Integrate[PolyGamma[n, a + b*x]/(c + d*x)^(3/2),x]
```

output
```
Integrate[PolyGamma[n, a + b*x]/(c + d*x)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 7126}$$

$$\frac{2b\int \frac{\psi^{(n+1)}(a+bx)}{\sqrt{c+dx}}dx}{d} - \frac{2\psi^{(n)}(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 7127}$$

$$\frac{2b\int \frac{\psi^{(n+1)}(a+bx)}{\sqrt{c+dx}}dx}{d} - \frac{2\psi^{(n)}(a+bx)}{d\sqrt{c+dx}}$$

input
```
Int[PolyGamma[n, a + b*x]/(c + d*x)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\Psi(n, bx+a)}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int(Psi(n,b*x+a)/(d*x+c)^(3/2),x)
```

output
```
int(Psi(n,b*x+a)/(d*x+c)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^{3/2}}\,dx=\int \frac{\Psi(n,bx+a)}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(Psi(n,b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*Psi(n, b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^{3/2}}\,dx=\int \frac{\mathrm{diEulerGamma}\left(n,a+bx\right)}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(diEulerGamma(n,b*x+a)/(d*x+c)**(3/2),x)
```

output
```
Integral(diEulerGamma(n, a + b*x)/(c + d*x)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\Psi(n,bx+a)}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(Psi(n,b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(Psi(n, b*x + a)/(d*x + c)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(Psi(n,b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage0:=((((sageVARd)*(sageVARx))+(sageVARc))^(-3/2))*(Psi(sageVARn,((s
ageVARb)*(sageVARx))+(sageVARa))):;OUTPUT:Psi(sageVARn,sageVARb*sageVARx+s
ageVARa)
```

Mupad [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\psi^{(a+b\,x)}(n)}{(c+d\,x)^{3/2}}\,dx$$

input
```
int(psi(a + b*x, n)/(c + d*x)^(3/2),x)
```

output
```
int(psi(a + b*x, n)/(c + d*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \frac{\psi^{(n)}(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{polygamma(n,bx+a)}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}dx$$

input
```
int(Psi(n,b*x+a)/(d*x+c)^(3/2),x)
```

output
```
int(polygamma(n,a + b*x)/(sqrt(c + d*x)*c + sqrt(c + d*x)*d*x),x)
```

3.229 $\int x^2\psi^{(1)}(a+bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int x^2\psi^{(1)}(a+bx)\,dx=-\frac{2x\log\Gamma(a+bx)}{b^2}+\frac{2\psi^{(-2)}(a+bx)}{b^3}+\frac{x^2\psi^{(0)}(a+bx)}{b}$$

output
```
-2*x*lnGAMMA(b*x+a)/b^2+2*Psi(-2,b*x+a)/b^3+x^2*Psi(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int x^2\psi^{(1)}(a+bx)\,dx=\frac{-\frac{2x\log\Gamma(a+bx)}{b}+\frac{2\psi^{(-2)}(a+bx)}{b^2}+x^2\psi^{(0)}(a+bx)}{b}$$

input
```
Integrate[x^2*PolyGamma[1, a + b*x],x]
```

output
```
((-2*x*LogGamma[a + b*x])/b + (2*PolyGamma[-2, a + b*x])/b^2 + x^2*PolyGam
ma[0, a + b*x])/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {7125, 7125, 7121}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\psi^{(1)}(a+bx)\,dx$$

$$\Big\downarrow \text{7125}$$

$$\frac{x^2\psi^{(0)}(a+bx)}{b}-\frac{2\int x\psi^{(0)}(a+bx)dx}{b}$$

$$\Big\downarrow \text{7125}$$

$$\frac{x^2\psi^{(0)}(a+bx)}{b}-\frac{2\left(\frac{x\log\Gamma(a+bx)}{b}-\frac{\int \log\Gamma(a+bx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{7121}$$

$$\frac{x^2\psi^{(0)}(a+bx)}{b}-\frac{2\left(\frac{x\log\Gamma(a+bx)}{b}-\frac{\psi^{(-2)}(a+bx)}{b^2}\right)}{b}$$

input
```
Int[x^2*PolyGamma[1, a + b*x],x]
```

output
```
(-2*((x*LogGamma[a + b*x])/b - PolyGamma[-2, a + b*x]/b^2))/b + (x^2*PolyG
amma[0, a + b*x])/b
```

Defintions of rubi rules used

rule 7121
```
Int[LogGamma[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[PolyGamma[-2, a + b*x]/
b, x] /; FreeQ[{a, b}, x]
```

rule 7125
```
Int[((c_.) + (d_.)*(x_))^(m_.)*PolyGamma[n_, (a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[(c + d*x)^m*(PolyGamma[n - 1, a + b*x]/b), x] - Simp[d*(m/b)   Int
[(c + d*x)^(m - 1)*PolyGamma[n - 1, a + b*x], x], x] /; FreeQ[{a, b, c, d,
n}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\Psi(bx+a)b^2x^2-2\Psi(-1,bx+a)bx+2\Psi(-2,bx+a)}{b^3}$	39
default	$\frac{\Psi(bx+a)b^2x^2-2\Psi(-1,bx+a)bx+2\Psi(-2,bx+a)}{b^3}$	39

input
```
int(x^2*Psi(1,b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(Psi(b*x+a)*b^2*x^2-2*Psi(-1,b*x+a)*b*x+2*Psi(-2,b*x+a))
```

Fricas [F]

$$\int x^2\psi^{(1)}(a+bx)\,dx=\int x^2\Psi(1,bx+a)\,dx$$

input
```
integrate(x^2*Psi(1,b*x+a),x, algorithm="fricas")
```

output
```
integral(x^2*Psi(1, b*x + a), x)
```

Sympy [F]

$$\int x^2\psi^{(1)}(a+bx)\,dx=\int x^2\,\mathrm{diEulerGamma}\,(1,a+bx)\,dx$$

input
```
integrate(x**2*diEulerGamma(1,b*x+a),x)
```

output
```
Integral(x**2*diEulerGamma(1, a + b*x), x)
```

Maxima [F]

$$\int x^2\psi^{(1)}(a+bx)\,dx=\int x^2\Psi(1,bx+a)\,dx$$

input
```
integrate(x^2*Psi(1,b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^2*Psi(1, b*x + a), x)
```

Giac [F]

$$\int x^2\psi^{(1)}(a+bx)\,dx=\int x^2\Psi(1,bx+a)\,dx$$

input
```
integrate(x^2*Psi(1,b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*Psi(1, b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\psi^{(1)}(a+bx)\,dx=\int x^2\,\psi^{(a+b\,x)}(1)\,dx$$

input
```
int(x^2*psi(a + b*x, 1),x)
```

output
```
int(x^2*psi(a + b*x, 1), x)
```

Reduce [F]

$$\int x^2\psi^{(1)}(a+bx)\,dx=\int polygamma(1,bx+a)\,x^2dx$$

input
```
int(x^2*Psi(1,b*x+a),x)
```

output
```
int(polygamma(1,a + b*x)*x**2,x)
```

3.230 $\int \left(\frac{\psi^{(1)}(a+bx)}{x^2}-\frac{b\psi^{(2)}(a+bx)}{x}\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 12

$$\int \left(\frac{\psi^{(1)}(a+bx)}{x^2}-\frac{b\psi^{(2)}(a+bx)}{x}\right) dx = -\frac{\psi^{(1)}(a+bx)}{x}$$

output
```
-Psi(1,b*x+a)/x
```

Mathematica [F]

$$\int \left(\frac{\psi^{(1)}(a+bx)}{x^2}-\frac{b\psi^{(2)}(a+bx)}{x}\right) dx = \int \left(\frac{\psi^{(1)}(a+bx)}{x^2}-\frac{b\psi^{(2)}(a+bx)}{x}\right) dx$$

input
```
Integrate[PolyGamma[1, a + b*x]/x^2 - (b*PolyGamma[2, a + b*x])/x,x]
```

output
```
Integrate[PolyGamma[1, a + b*x]/x^2 - (b*PolyGamma[2, a + b*x])/x, x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{\psi^{(1)}(a+bx)}{x^2}-\frac{b\psi^{(2)}(a+bx)}{x}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\psi^{(1)}(a+bx)}{x}$$

input
```
Int[PolyGamma[1, a + b*x]/x^2 - (b*PolyGamma[2, a + b*x])/x,x]
```

output
```
-(PolyGamma[1, a + b*x]/x)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{\Psi(1,bx+a)}{x}$	13
parts	$-\frac{\Psi(1,bx+a)}{x}$	13

input
```
int(Psi(1,b*x+a)/x^2-b*Psi(2,b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
-Psi(1,b*x+a)/x
```

Fricas [F]

$$\int \left(\frac{\psi^{(1)}(a+bx)}{x^2}-\frac{b\psi^{(2)}(a+bx)}{x}\right)\,dx=\int -\frac{b\Psi(2,bx+a)}{x}+\frac{\Psi(1,bx+a)}{x^2}\,dx$$

input
```
integrate(Psi(1,b*x+a)/x^2-b*Psi(2,b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(-(b*x*Psi(2, b*x + a) - Psi(1, b*x + a))/x^2, x)
```

Sympy [F]

$$\int \left(\frac{\psi^{(1)}(a+bx)}{x^2}-\frac{b\psi^{(2)}(a+bx)}{x}\right)\,dx=-\int \left(-\frac{\text{diEulerGamma}\,(1,a+bx)}{x^2}\right)\,dx-\int \frac{b\,\text{diEulerGamma}\,(2,a+bx)}{x}\,dx$$

input
```
integrate(diEulerGamma(1,b*x+a)/x**2-b*diEulerGamma(2,b*x+a)/x,x)
```

output
```
-Integral(-diEulerGamma(1, a + b*x)/x**2, x) - Integral(b*diEulerGamma(2,
a + b*x)/x, x)
```

Maxima [F]

$$\int \left(\frac{\psi^{(1)}(a+bx)}{x^2}-\frac{b\psi^{(2)}(a+bx)}{x}\right)\,dx=\int -\frac{b\Psi(2,bx+a)}{x}+\frac{\Psi(1,bx+a)}{x^2}\,dx$$

input
```
integrate(Psi(1,b*x+a)/x^2-b*Psi(2,b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(-b*Psi(2, b*x + a)/x + Psi(1, b*x + a)/x^2, x)
```

Giac [F]

$$\int \left(\frac{\psi^{(1)}(a+bx)}{x^2}-\frac{b\psi^{(2)}(a+bx)}{x}\right)\,dx=\int -\frac{b\Psi(2,bx+a)}{x}+\frac{\Psi(1,bx+a)}{x^2}\,dx$$

input
```
integrate(Psi(1,b*x+a)/x^2-b*Psi(2,b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(-b*Psi(2, b*x + a)/x + Psi(1, b*x + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{\psi^{(1)}(a+bx)}{x^2}-\frac{b\psi^{(2)}(a+bx)}{x}\right)\,dx=\int \frac{\psi^{(a+b\,x)}(1)}{x^2}-\frac{b\,\psi^{(a+b\,x)}(2)}{x}\,dx$$

input
```
int(psi(a + b*x, 1)/x^2 - (b*psi(a + b*x, 2))/x,x)
```

output
```
int(psi(a + b*x, 1)/x^2 - (b*psi(a + b*x, 2))/x, x)
```

Reduce [F]

$$\int \left(\frac{\psi^{(1)}(a+bx)}{x^2}-\frac{b\psi^{(2)}(a+bx)}{x}\right)\,dx=-\left(\int \frac{polygamma(2,bx+a)}{x}dx\right)b+\int \frac{polygamma(1,bx+a)}{x^2}dx$$

input
```
int(Psi(1,b*x+a)/x^2-b*Psi(2,b*x+a)/x,x)
```

output
```
- int(polygamma(2,a + b*x)/x,x)*b + int(polygamma(1,a + b*x)/x**2,x)
```

3.231 $\int \left(\frac{\psi^{(n)}(a+bx)}{x^2}-\frac{b\psi^{(1+n)}(a+bx)}{x}\right) dx$

Optimal result

Integrand size = 27, antiderivative size = 12

$$\int \left(\frac{\psi^{(n)}(a+bx)}{x^2}-\frac{b\psi^{(1+n)}(a+bx)}{x}\right) dx=-\frac{\psi^{(n)}(a+bx)}{x}$$

output
```
-Psi(n,b*x+a)/x
```

Mathematica [F]

$$\int \left(\frac{\psi^{(n)}(a+bx)}{x^2}-\frac{b\psi^{(1+n)}(a+bx)}{x}\right) dx=\int \left(\frac{\psi^{(n)}(a+bx)}{x^2}-\frac{b\psi^{(1+n)}(a+bx)}{x}\right) dx$$

input
```
Integrate[PolyGamma[n, a + b*x]/x^2 - (b*PolyGamma[1 + n, a + b*x])/x,x]
```

output
```
Integrate[PolyGamma[n, a + b*x]/x^2 - (b*PolyGamma[1 + n, a + b*x])/x, x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{\psi^{(n)}(a+bx)}{x^2}-\frac{b\psi^{(n+1)}(a+bx)}{x}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\psi^{(n)}(a+bx)}{x}$$

input
```
Int[PolyGamma[n, a + b*x]/x^2 - (b*PolyGamma[1 + n, a + b*x])/x,x]
```

output
```
-(PolyGamma[n, a + b*x]/x)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{\Psi(n,bx+a)}{x}$	13
parts	$-\frac{\Psi(n,bx+a)}{x}$	13

input
```
int(Psi(n,b*x+a)/x^2-b*Psi(1+n,b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
-Psi(n,b*x+a)/x
```

Fricas [F]

$$\int \left(\frac{\psi^{(n)}(a+bx)}{x^2}-\frac{b\psi^{(1+n)}(a+bx)}{x}\right)\,dx=\int -\frac{b\Psi(n+1,bx+a)}{x}+\frac{\Psi(n,bx+a)}{x^2}\,dx$$

input
```
integrate(Psi(n,b*x+a)/x^2-b*Psi(1+n,b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(-(b*x*Psi(n + 1, b*x + a) - Psi(n, b*x + a))/x^2, x)
```

Sympy [F]

$$\int \left(\frac{\psi^{(n)}(a+bx)}{x^2}-\frac{b\psi^{(1+n)}(a+bx)}{x}\right)\,dx=-\int \left(-\frac{\mathrm{diEulerGamma}\,(n,a+bx)}{x^2}\right)\,dx-\int \frac{b\,\mathrm{diEulerGamma}\,(n+1,a+bx)}{x}\,dx$$

input
```
integrate(diEulerGamma(n,b*x+a)/x**2-b*diEulerGamma(1+n,b*x+a)/x,x)
```

output
```
-Integral(-diEulerGamma(n, a + b*x)/x**2, x) - Integral(b*diEulerGamma(n +
 1, a + b*x)/x, x)
```

Maxima [F]

$$\int \left(\frac{\psi^{(n)}(a+bx)}{x^2}-\frac{b\psi^{(1+n)}(a+bx)}{x}\right)\,dx=\int -\frac{b\Psi(n+1,bx+a)}{x}+\frac{\Psi(n,bx+a)}{x^2}\,dx$$

input
```
integrate(Psi(n,b*x+a)/x^2-b*Psi(1+n,b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(-b*Psi(n + 1, b*x + a)/x + Psi(n, b*x + a)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(\frac{\psi^{(n)}(a+bx)}{x^2}-\frac{b\psi^{(1+n)}(a+bx)}{x}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(Psi(n,b*x+a)/x^2-b*Psi(1+n,b*x+a)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage0:=((sageVARb)*((sageVARx)^(-1))*(Psi((sageVARn)+(1),((sageVARb)*(
sageVARx))+(sageVARa)))*(-1))+(((sageVARx)^(-2))*(Psi(sageVARn,((sageVARb)
*(sageVAR
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{\psi^{(n)}(a+bx)}{x^2}-\frac{b\psi^{(1+n)}(a+bx)}{x}\right)\,dx=\int \frac{\psi^{(a+b\,x)}(n)}{x^2}-\frac{b\,\psi^{(a+b\,x)}(n+1)}{x}\,dx$$

input
```
int(psi(a + b*x, n)/x^2 - (b*psi(a + b*x, n + 1))/x,x)
```

output
```
int(psi(a + b*x, n)/x^2 - (b*psi(a + b*x, n + 1))/x, x)
```

Reduce [F]

$$\int \left(\frac{\psi^{(n)}(a+bx)}{x^2} - \frac{b\psi^{(1+n)}(a+bx)}{x}\right) dx = -\left(\int \frac{polygamma(n+1,bx+a)}{x}dx\right) b + \int \frac{polygamma(n,bx+a)}{x^2}dx$$

input
```
int(Psi(n,b*x+a)/x^2-b*Psi(1+n,b*x+a)/x,x)
```

output
```
 - int(polygamma(n + 1,a + b*x)/x,x)*b + int(polygamma(n,a + b*x)/x**2,x)
```

3.232 $\int \text{Gamma}(a+bx)^n \psi^{(0)}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 15

$$\int \text{Gamma}(a+bx)^n \psi^{(0)}(a+bx)\,dx=\frac{\text{Gamma}(a+bx)^n}{bn}$$

output
```
GAMMA(b*x+a)^n/b/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \text{Gamma}(a+bx)^n \psi^{(0)}(a+bx)\,dx=\frac{\text{Gamma}(a+bx)^n}{bn}$$

input
```
Integrate[Gamma[a + b*x]^n*PolyGamma[0, a + b*x],x]
```

output
```
Gamma[a + b*x]^n/(b*n)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {7128}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \psi^{(0)}(a+bx)\,\text{Gamma}(a+bx)^n\,dx$$

$$\Big\downarrow \ 7128$$

$$\frac{\text{Gamma}(a+bx)^n}{bn}$$

input
```
Int[Gamma[a + b*x]^n*PolyGamma[0, a + b*x],x]
```

output
```
Gamma[a + b*x]^n/(b*n)
```

Defintions of rubi rules used

rule 7128
```
Int[Gamma[(a_.) + (b_.)*(x_)]^(n_.)*PolyGamma[0, (a_.) + (b_.)*(x_)], x_Sym
bol] :> Simp[Gamma[a + b*x]^n/(b*n), x] /; FreeQ[{a, b, n}, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\Gamma(bx+a)^n}{bn}$	16
default	$\frac{\Gamma(bx+a)^n}{bn}$	16

input
```
int(GAMMA(b*x+a)^n*Psi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
GAMMA(b*x+a)^n/b/n
```

Fricas [F]

$$\int \text{Gamma}(a+bx)^n\psi^{(0)}(a+bx)\,dx = \int \Gamma(bx+a)^n\,\Psi(bx+a)\,dx$$

input
```
integrate(gamma(b*x+a)^n*Psi(b*x+a),x, algorithm="fricas")
```

output
```
integral(gamma(b*x + a)^n*Psi(b*x + a), x)
```

Sympy [F]

$$\int \text{Gamma}(a+bx)^n\psi^{(0)}(a+bx)\,dx = \int \text{diEulerGamma}\,(a+bx)\Gamma^n(a+bx)\,dx$$

input
```
integrate(gamma(b*x+a)**n*diEulerGamma(b*x+a),x)
```

output
```
Integral(diEulerGamma(a + b*x)*gamma(a + b*x)**n, x)
```

Maxima [F]

$$\int \text{Gamma}(a+bx)^n\psi^{(0)}(a+bx)\,dx = \int \Gamma(bx+a)^n\,\Psi(bx+a)\,dx$$

input
```
integrate(gamma(b*x+a)^n*Psi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(gamma(b*x + a)^n*Psi(b*x + a), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \text{Gamma}(a+bx)^n\psi^{(0)}(a+bx)\,dx = \frac{\Gamma(bx+a)^n}{bn}$$

input
```
integrate(gamma(b*x+a)^n*Psi(b*x+a),x, algorithm="giac")
```

output
```
gamma(b*x + a)^n/(b*n)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \text{Gamma}(a+bx)^n\psi^{(0)}(a+bx)\,dx = \frac{\Gamma(a+b\,x)^n}{b\,n}$$

input
```
int(gamma(a + b*x)^n*psi(a + b*x),x)
```

output
```
gamma(a + b*x)^n/(b*n)
```

Reduce [F]

$$\int \text{Gamma}(a+bx)^n\psi^{(0)}(a+bx)\,dx = \int \gamma(bx+a)^n\,\psi(bx+a)\,dx$$

input
```
int(GAMMA(b*x+a)^n*Psi(b*x+a),x)
```

output
```
int(gamma(a + b*x)**n*psi(a + b*x),x)
```

3.233 $\int ((a+bx)!)^n \psi^{(0)}(1+a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int ((a+bx)!)^n \psi^{(0)}(1+a+bx)\,dx = \frac{((a+bx)!)^n}{bn}$$

output
```
(b*x+a)!^n/b/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int ((a+bx)!)^n \psi^{(0)}(1+a+bx)\,dx = \frac{((a+bx)!)^n}{bn}$$

input
```
Integrate[(a + b*x)!^n*PolyGamma[0, 1 + a + b*x],x]
```

output
```
(a + b*x)!^n/(b*n)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {7129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \psi^{(0)}(a+bx+1)((a+bx)!)^n\,dx$$

$$\Big\downarrow \text{7129}$$

$$\frac{((a+bx)!)^n}{bn}$$

input
```
Int[(a + b*x)!^n*PolyGamma[0, 1 + a + b*x],x]
```

output
```
(a + b*x)!^n/(b*n)
```

Defintions of rubi rules used

rule 7129
```
Int[((a_.) + (b_.)*(x_))!^(n_.)*PolyGamma[0, (c_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[(a + b*x)!^n/(b*n), x] /; FreeQ[{a, b, c, n}, x] && EqQ[c, a + 1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{(bx+a)!^n}{bn}$	16
default	$\frac{(bx+a)!^n}{bn}$	16

input
```
int((b*x+a)!^n*Psi(b*x+a+1),x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)!^n/b/n
```

Fricas [F]

$$\int ((a+bx)!)^n\psi^{(0)}(1+a+bx)\,dx = \int (bx+a)\,!^n\,\Psi(bx+a+1)\,dx$$

input
```
integrate((b*x+a)!^n*Psi(b*x+a+1),x, algorithm="fricas")
```

output
```
integral(factorial(b*x + a)^n*Psi(b*x + a + 1), x)
```

Sympy [F]

$$\int ((a+bx)!)^n\psi^{(0)}(1+a+bx)\,dx = \int \mathrm{diEulerGamma}\,(a+bx+1)(a+bx)\,!^n\,dx$$

input
```
integrate((b*x+a)!**n*diEulerGamma(b*x+a+1),x)
```

output
```
Integral(diEulerGamma(a + b*x + 1)*factorial(a + b*x)**n, x)
```

Maxima [F]

$$\int ((a+bx)!)^n\psi^{(0)}(1+a+bx)\,dx = \int (bx+a)\,!^n\,\Psi(bx+a+1)\,dx$$

input
```
integrate((b*x+a)!^n*Psi(b*x+a+1),x, algorithm="maxima")
```

output
```
integrate(factorial(b*x + a)^n*Psi(b*x + a + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int ((a+bx)!\,)^n\psi^{(0)}(1+a+bx)\,dx = \frac{(bx+a)\,!^n}{bn}$$

input
```
integrate((b*x+a)!^n*Psi(b*x+a+1),x, algorithm="giac")
```

output
```
factorial(b*x + a)^n/(b*n)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int ((a+bx)!\,)^n\psi^{(0)}(1+a+bx)\,dx = \frac{(a+b\,x)\,!^n}{b\,n}$$

input
```
int(factorial(a + b*x)^n*psi(a + b*x + 1),x)
```

output
```
factorial(a + b*x)^n/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.07

$$\int ((a+bx)!\,)^n\psi^{(0)}(1+a+bx)\,dx = anti$$

input
```
int((b*x+a)!^n*Psi(b*x+a+1),x)
```

output
```
anti
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
           return 1
        else:
           return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
           return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
           if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
              return 1
           else:
              return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
           return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
   if expnType_result <= expnType_optimal:
      if result.has(I):
         if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
               grade = "A"
               grade_annotation =""
            else:
               grade = "B"
               grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
         else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
      else: # result do not contain complex, this assumes optimal do not as well
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =""
         else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
   else:
      grade = "C"
      grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file