Computer Algebra Independent Integration Tests

Summer 2024

8-Special-functions/358-8.9

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**420**]. This is test number [358].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	84.52 (355)	15.48 (65)
Mathematica	84.52 (355)	15.48 (65)
Maple	58.10 (244)	41.90 (176)
Fricas	26.67 (112)	73.33 (308)
Sympy	19.52 (82)	80.48 (338)
Reduce	7.38 (31)	92.62 (389)
Maxima	4.05 (17)	95.95 (403)
Mupad	2.86 (12)	97.14 (408)
Giac	2.86 (12)	97.14 (408)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

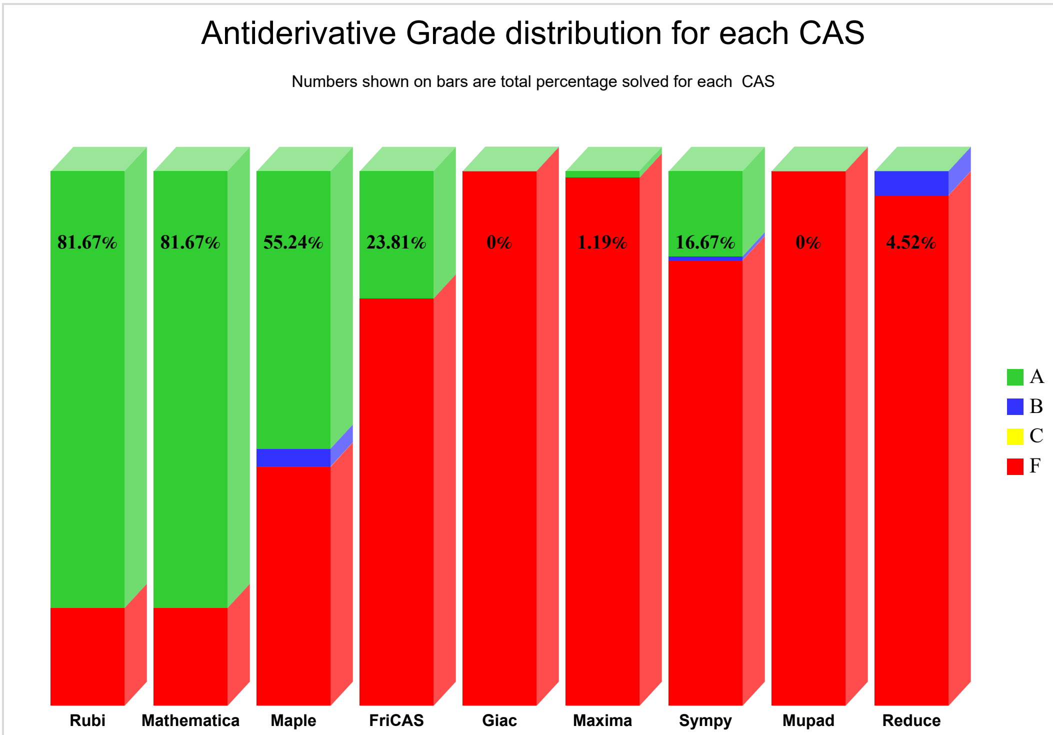
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

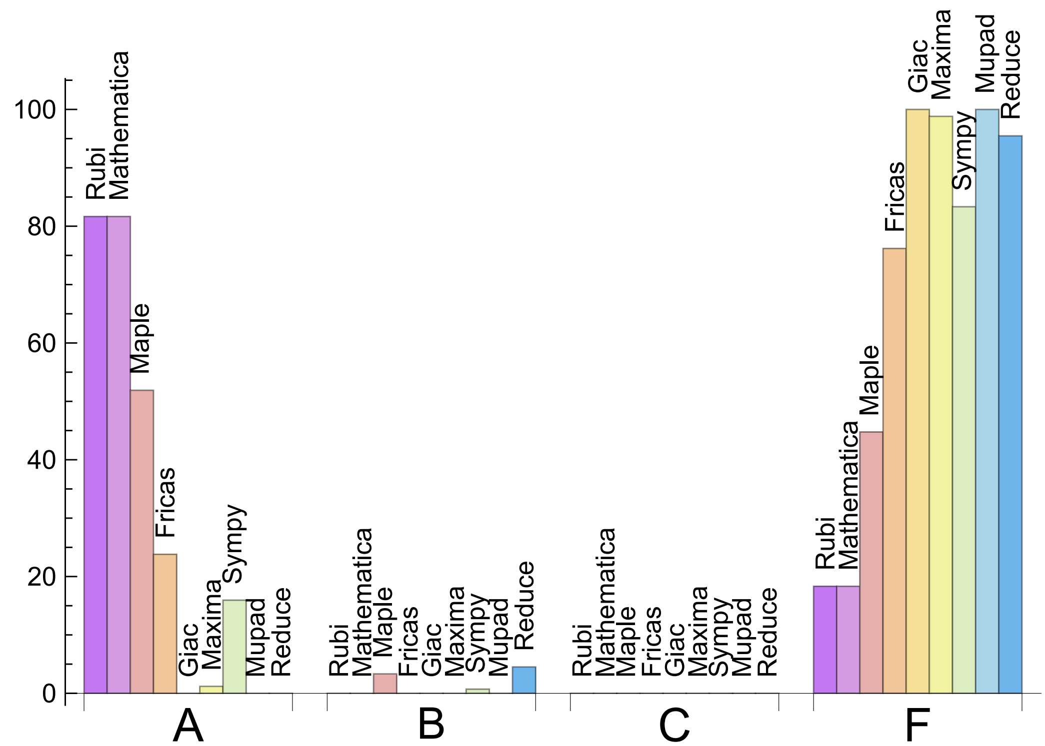
System	% A grade	% B grade	% C grade	% F grade
Rubi	81.667	0.000	0.000	18.333
Mathematica	81.667	0.000	0.000	18.333
Maple	51.905	3.333	0.000	44.762
Fricas	23.810	0.000	0.000	76.190
Sympy	15.952	0.714	0.000	83.333
Maxima	1.190	0.000	0.000	98.810
Giac	0.000	0.000	0.000	100.000
Mupad	0.000	0.000	0.000	100.000
Reduce	0.000	4.524	0.000	95.476

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	65	100.00	0.00	0.00
Mathematica	65	100.00	0.00	0.00
Maple	176	100.00	0.00	0.00
Fricas	308	99.35	0.00	0.65
Sympy	338	92.60	7.40	0.00
Reduce	389	100.00	0.00	0.00
Maxima	403	100.00	0.00	0.00
Mupad	408	0.00	100.00	0.00
Giac	408	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Mathematica	0.04
Maxima	0.05
Fricas	0.08
Mupad	0.12
Giac	0.13
Maple	0.18
Reduce	0.31
Sympy	0.48
Rubi	0.49

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	15.50	1.08	16.00	1.06
Giac	16.50	1.15	17.00	1.12
Maxima	21.59	1.15	18.00	1.06
Sympy	26.98	0.98	20.00	0.91
Reduce	34.81	1.68	30.00	1.07
Fricas	37.25	1.07	28.50	0.93
Mathematica	54.50	0.90	47.00	1.00
Rubi	70.24	1.03	55.00	1.00
Maple	75.45	1.14	41.50	1.10

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

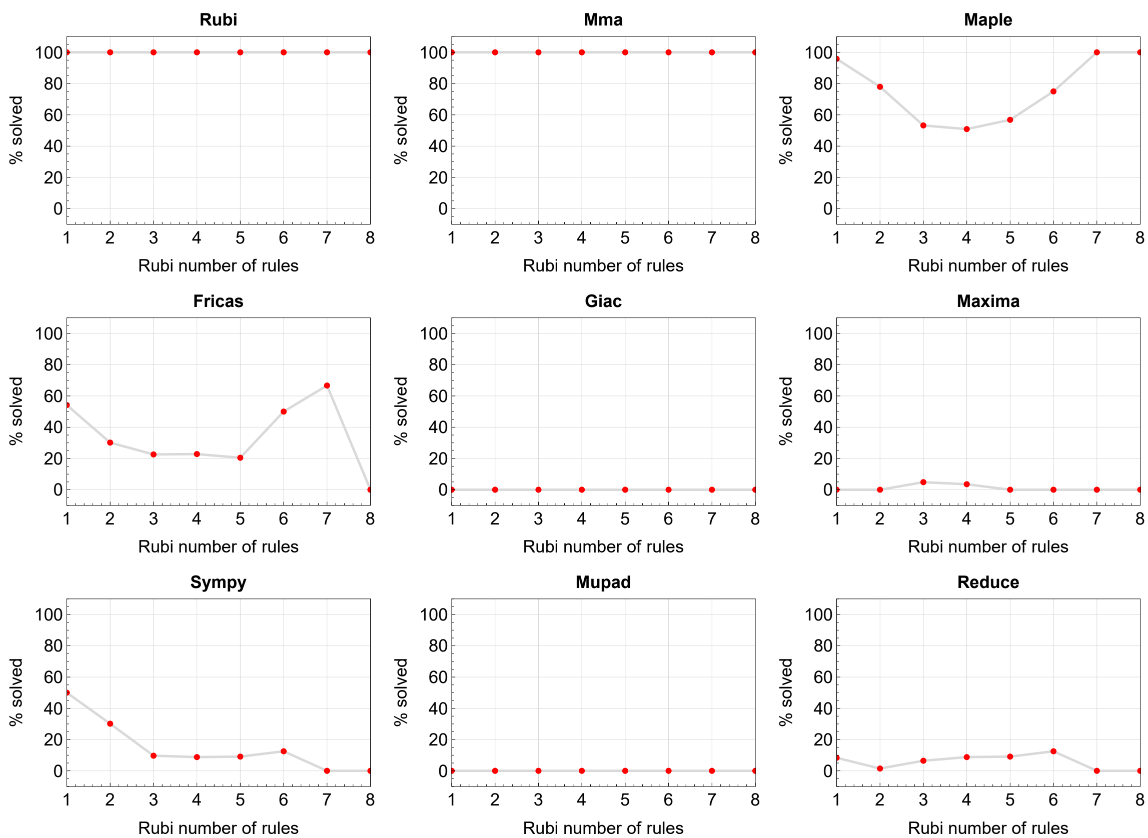


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

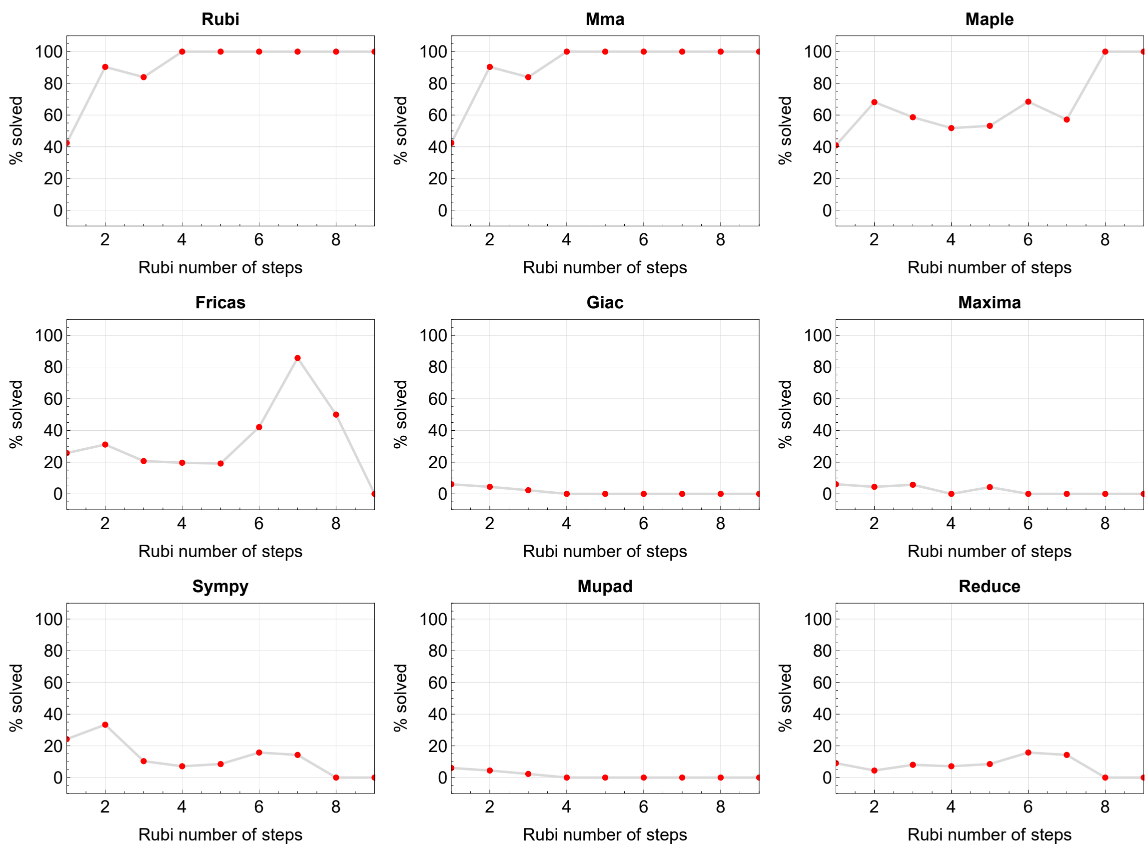


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

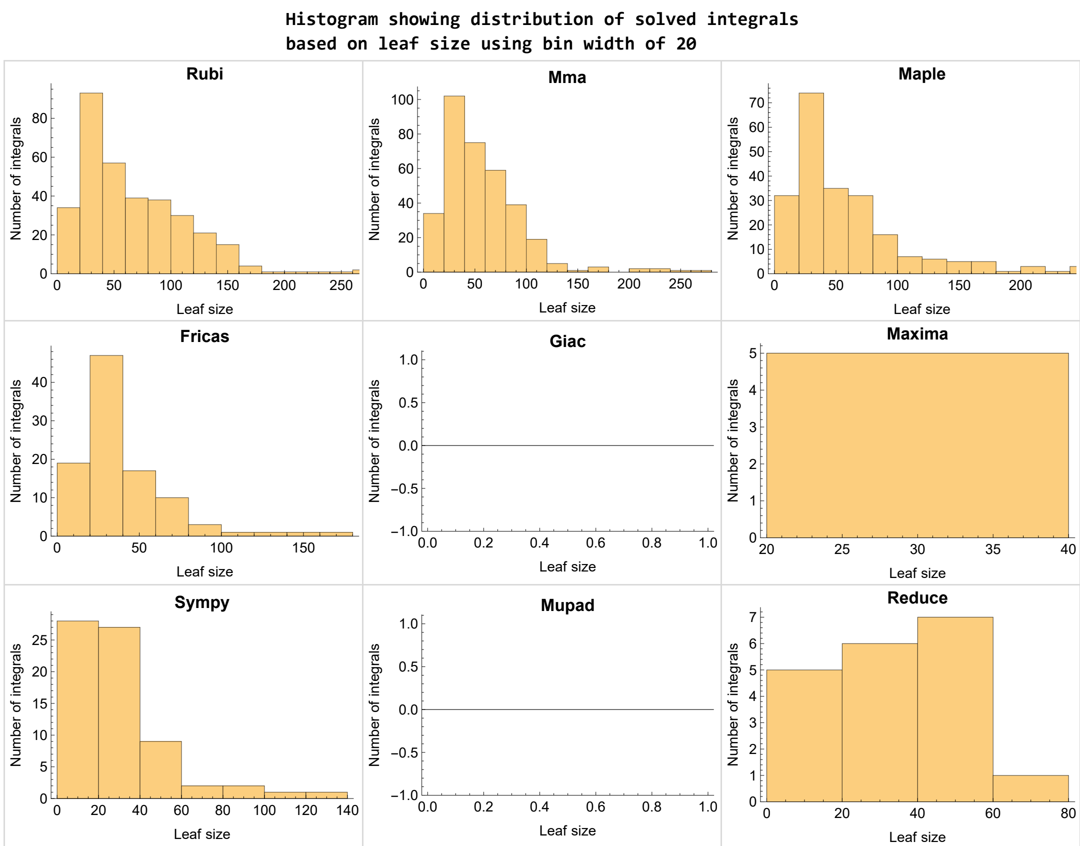


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

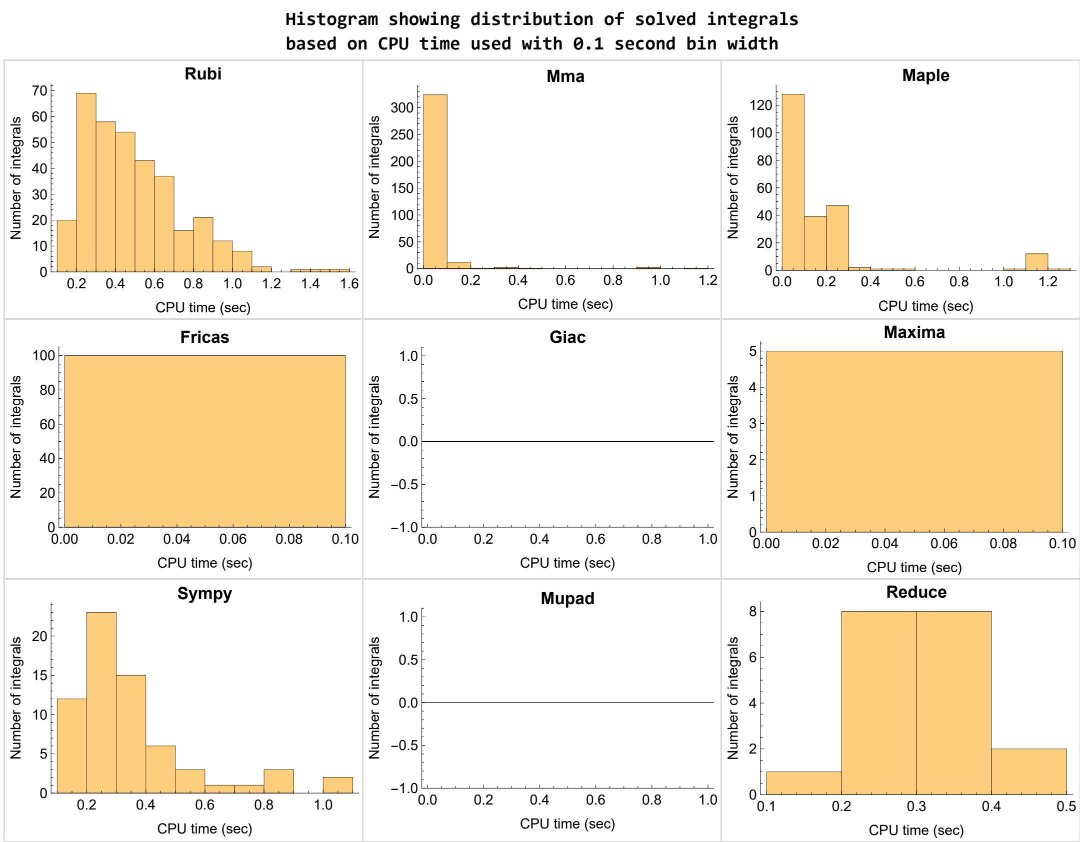


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

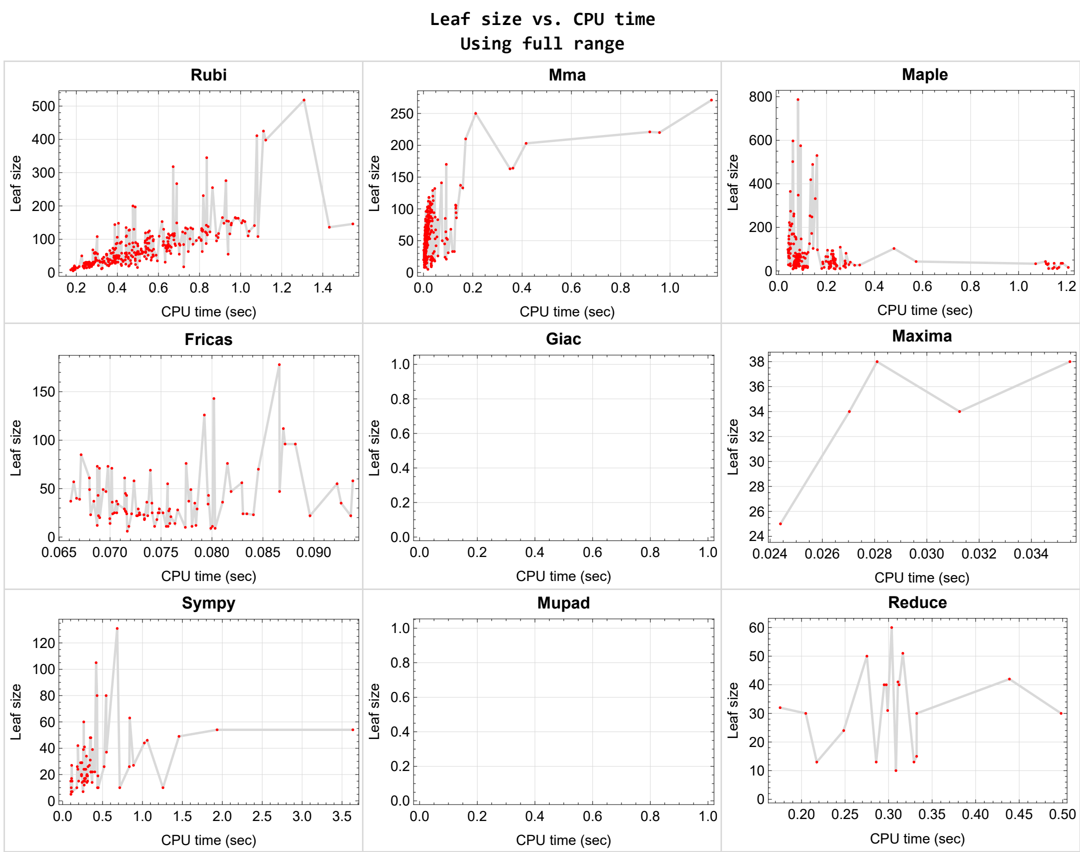


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{389, 390, 395, 396, 401, 402, 407, 408, 413, 414, 419, 420}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {324, 325, 326, 329, 330}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

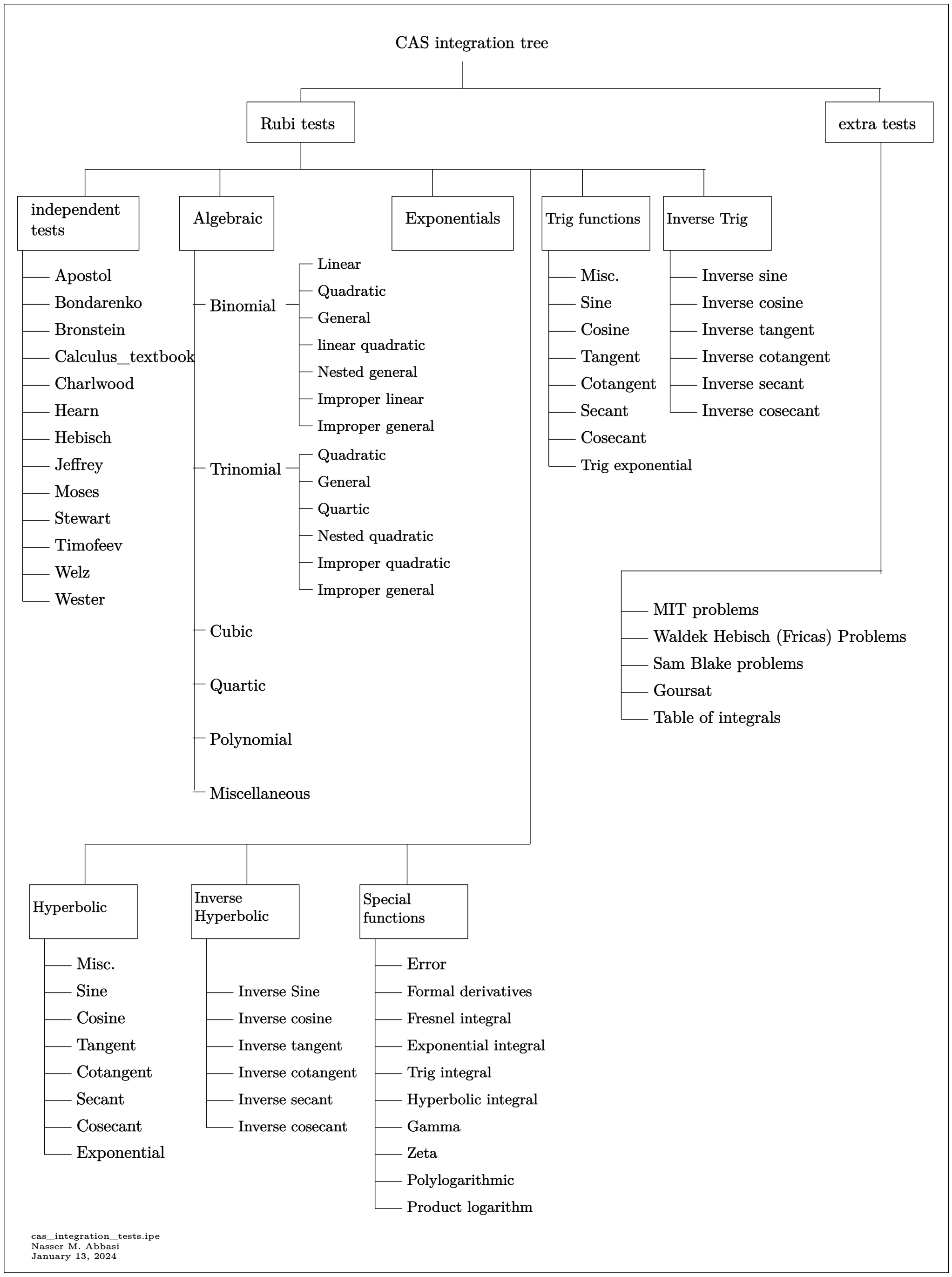
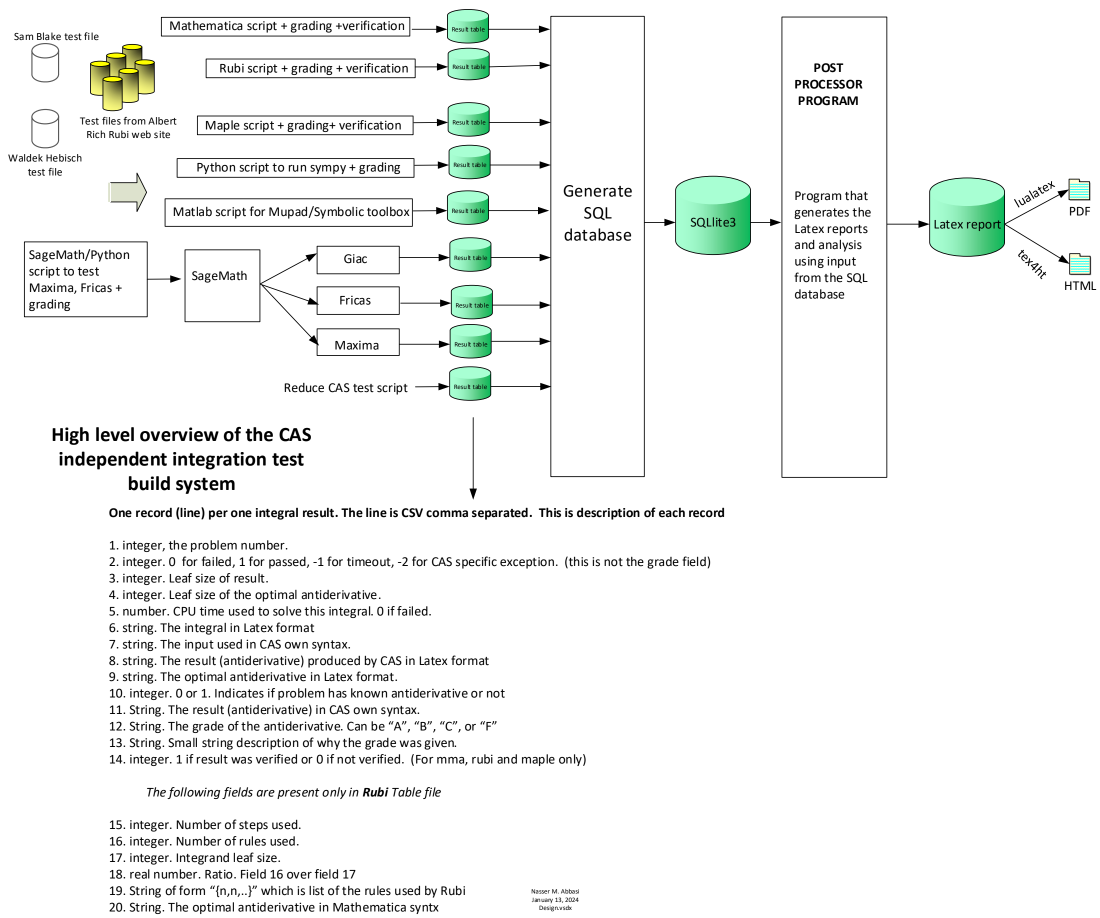


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 104, 105, 106, 107, 108, 109, 110, 117, 118, 119, 120, 121, 122, 123, 130, 131, 132, 133, 134, 135, 136, 142, 143, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 287, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 338, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 391, 392, 393, 394, 397, 398, 399, 400, 403, 404, 405, 406, 409, 410, 411, 412, 415, 416, 417, 418 }

B grade { }

C grade { }

F normal fail { 99, 100, 101, 102, 103, 111, 112, 113, 114, 115, 116, 124, 125, 126, 127, 128, 129, 137, 138, 139, 140, 141, 148, 149, 150, 151, 152, 153, 165, 166, 177, 178, 179, 184, 185, 186, 243, 244, 245, 246, 247, 285, 286, 288, 289, 292, 293, 294, 312, 313, 314, 315, 316, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 357, 360 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 104, 105, 106, 107, 108, 109, 110, 117, 118, 119, 120, 121, 122, 123, 130, 131, 132, 133, 134, 135, 136, 142, 143, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 287, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 338, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 391, 392, 393, 394, 397, 398, 399, 400, 403, 404, 405, 406, 409, 410, 411, 412, 415, 416, 417, 418 }

B grade { }

C grade { }

F normal fail { 99, 100, 101, 102, 103, 111, 112, 113, 114, 115, 116, 124, 125, 126, 127, 128, 129, 137, 138, 139, 140, 141, 148, 149, 150, 151, 152, 153, 165, 166, 177, 178, 179, 184, 185, 186, 243, 244, 245, 246, 247, 285, 286, 288, 289, 292, 293, 294, 312, 313, 314, 315, 316, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 357, 360 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 70, 71, 72, 73, 74, 75, 88, 94, 95, 105, 106, 107, 117, 118, 119, 120, 132, 133, 134, 143, 145, 146, 156, 157, 169, 170, 182, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 211, 212, 219, 220, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 248, 249, 250, 251, 252, 287, 290, 295, 298, 299, 300, 301, 302, 303, 304, 305, 308, 309, 317, 318, 319, 320, 321, 322, 323, 327, 328, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 377, 378, 379, 380, 381, 382, 385, 386, 387, 388, 391, 392, 393, 394, 397, 398, 399, 400, 403, 404, 405, 406, 409, 410, 411, 412, 416, 417, 418 }

B grade { 66, 67, 68, 69, 76, 77, 78, 79, 291, 374, 375, 376, 383, 415 }

C grade { }

F normal fail { 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 96, 97, 98, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 114, 115, 116, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 135, 136, 137, 138, 139, 140, 141, 142, 144, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 186, 207, 208, 209, 210, 213, 214, 215, 216, 217, 218, 221, 222, 223, 224, 226, 227, 243, 244, 245, 246, 247, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 292, 293, 294, 296, 297, 306, 307, 310, 311, 312, 313, 314, 315, 316, 324, 325, 326, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 357, 360, 384 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 2, 3, 4, 5, 11, 12, 13, 14, 15, 21, 22, 23, 24, 25, 26, 33, 34, 35, 37, 42, 43, 44, 47, 51, 52, 53, 57, 92, 93, 94, 95, 104, 105, 106, 107, 117, 118, 119, 120, 130, 131, 132, 134, 142, 143, 146, 192, 193, 194, 195, 201, 202, 203, 204, 205, 228, 229, 230, 231, 232, 233, 234, 248, 249, 250, 251, 252, 299, 300, 301, 302, 306, 307, 308, 309, 320, 321, 322, 323, 327, 328, 329, 330, 342, 343, 344, 345, 346, 347, 348, 361, 362, 363, 364, 387, 388, 393, 394, 417, 418 }

B grade { }

C grade { }

F normal fail { 1, 6, 7, 8, 9, 10, 16, 17, 18, 19, 20, 27, 28, 29, 30, 31, 32, 36, 38, 39, 40, 41, 45, 46, 48, 49, 50, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 96, 97, 98, 99, 100, 101, 102, 103, 108, 109, 110, 111, 112, 113, 114, 115, 116, 121, 122, 123, 124, 125, 126, 127, 128, 129, 133, 135, 136, 137, 138, 139, 140, 141, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 196, 197, 198, 199, 200, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 303, 304, 305, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 324, 325, 326, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 391, 392, 397, 398, 399, 400, 403, 404, 405, 406, 409, 410, 411, 412, 415, 416 }

F(-1) timedout fail { }

F(-2) exception fail { 84, 85 }

Maxima

A grade { 4, 94, 193, 364, 388 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 391, 392, 393, 394, 397, 398, 399, 400, 403, 404, 405, 406, 409, 410, 411, 412, 415, 416, 417, 418 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 391, 392, 393, 394, 397, 398, 399, 400, 403, 404, 405, 406, 409, 410, 411, 412, 415, 416, 417, 418 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 391, 392, 393, 394, 397, 398, 399, 400, 403, 404, 405, 406, 409, 410, 411, 412, 415, 416, 417, 418 }

F(-2) exception fail { }

Sympy

A grade { 4, 5, 13, 14, 15, 23, 24, 25, 26, 35, 37, 44, 47, 53, 57, 75, 94, 95, 105, 106, 107, 117, 118, 119, 120, 132, 134, 143, 146, 164, 170, 176, 192, 193, 201, 202, 203, 211, 219, 231, 232, 233, 234, 248, 249, 250, 251, 252, 301, 302, 308, 309, 320, 321, 327, 328, 344, 345, 346, 347, 348, 361, 362, 363, 364, 388, 394 }

B grade { 290, 358, 418 }

C grade { }

F normal fail { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 16, 17, 18, 19, 20, 21, 22, 27, 28, 29, 30, 31, 32, 33, 34, 36, 38, 39, 40, 41, 42, 43, 45, 46, 48, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 97, 98, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 114, 115, 116, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 198, 199, 200, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 257, 258, 266, 271, 272, 273, 278, 279, 280, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 303, 304, 305, 306, 307, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 322, 323, 324, 325, 326, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 349, 350, 351, 352, 353, 354, 355, 356, 357, 360, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 391, 392, 393, 397, 398, 399, 400, 403, 404, 405, 406, 409, 410, 411, 412, 415, 416, 417 }

F(-1) timedout fail { 253, 254, 255, 256, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 274, 275, 276, 277, 281, 282, 283, 284, 291, 359 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 4, 13, 23, 94, 105, 117, 193, 203, 301, 308, 321, 328, 361, 362, 363, 364, 388, 394, 418 }

C grade { }

F normal fail { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 302, 303, 304, 305, 306, 307, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 391, 392, 393, 397, 398, 399, 400, 403, 404, 405, 406, 409, 410, 411, 412, 415, 416, 417 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	86	71	82	0	0	0	0	10	0
N.S.	1	1.21	1.00	1.15	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.680	0.007	0.109	0.000	0.000	0.000	0.000	0.252	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	68	58	68	0	61	0	0	10	0
N.S.	1	1.17	1.00	1.17	0.00	1.05	0.00	0.00	0.17	0.00
time (sec)	N/A	0.553	0.007	0.082	0.000	0.071	0.000	0.000	0.379	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	50	45	54	0	49	0	0	8	0
N.S.	1	1.11	1.00	1.20	0.00	1.09	0.00	0.00	0.18	0.00
time (sec)	N/A	0.409	0.004	0.079	0.000	0.068	0.000	0.000	0.309	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	14	26	25	27	15	0	24	0
N.S.	1	1.00	0.78	1.44	1.39	1.50	0.83	0.00	1.33	0.00
time (sec)	N/A	0.274	0.031	0.062	0.024	0.076	0.201	0.000	0.248	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	0	22	12	0	22	0
N.S.	1	1.00	1.00	0.93	0.00	1.47	0.80	0.00	1.47	0.00
time (sec)	N/A	0.264	0.014	0.102	0.000	0.074	0.270	0.000	0.220	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	24	0	0	0	0	39	0
N.S.	1	1.00	1.00	1.26	0.00	0.00	0.00	0.00	2.05	0.00
time (sec)	N/A	0.261	0.004	0.076	0.000	0.000	0.000	0.000	0.348	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	0	0	0	0	46	0
N.S.	1	1.00	1.00	1.18	0.00	0.00	0.00	0.00	2.09	0.00
time (sec)	N/A	0.276	0.004	0.080	0.000	0.000	0.000	0.000	0.203	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	42	0	0	0	0	46	0
N.S.	1	1.00	1.00	1.08	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.388	0.005	0.073	0.000	0.000	0.000	0.000	0.284	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	56	52	56	0	0	0	0	46	0
N.S.	1	1.08	1.00	1.08	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.506	0.006	0.076	0.000	0.000	0.000	0.000	0.328	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	76	65	70	0	0	0	0	46	0
N.S.	1	1.17	1.00	1.08	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.645	0.007	0.076	0.000	0.000	0.000	0.000	0.218	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	86	71	82	0	73	0	0	12	0
N.S.	1	1.21	1.00	1.15	0.00	1.03	0.00	0.00	0.17	0.00
time (sec)	N/A	0.701	0.008	0.070	0.000	0.069	0.000	0.000	0.277	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	63	58	68	0	61	0	0	10	0
N.S.	1	1.09	1.00	1.17	0.00	1.05	0.00	0.00	0.17	0.00
time (sec)	N/A	0.813	0.005	0.066	0.000	0.068	0.000	0.000	0.283	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	31	24	37	0	39	29	0	32	0
N.S.	1	1.11	0.86	1.32	0.00	1.39	1.04	0.00	1.14	0.00
time (sec)	N/A	0.633	0.033	0.065	0.000	0.067	0.240	0.000	0.175	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	0	19	15	0	22	0
N.S.	1	1.00	1.00	0.86	0.00	0.90	0.71	0.00	1.05	0.00
time (sec)	N/A	0.458	0.012	0.108	0.000	0.070	0.276	0.000	0.275	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	19	0	20	17	0	91	0
N.S.	1	1.00	1.00	0.90	0.00	0.95	0.81	0.00	4.33	0.00
time (sec)	N/A	0.420	0.004	0.066	0.000	0.069	0.112	0.000	0.229	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	30	0	0	0	0	54	0
N.S.	1	1.00	1.00	1.20	0.00	0.00	0.00	0.00	2.16	0.00
time (sec)	N/A	0.363	0.004	0.072	0.000	0.000	0.000	0.000	0.177	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	30	0	0	0	0	54	0
N.S.	1	1.00	1.00	1.25	0.00	0.00	0.00	0.00	2.25	0.00
time (sec)	N/A	0.489	0.004	0.081	0.000	0.000	0.000	0.000	0.312	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	43	0	0	0	0	54	0
N.S.	1	1.00	1.00	1.19	0.00	0.00	0.00	0.00	1.50	0.00
time (sec)	N/A	0.426	0.005	0.083	0.000	0.000	0.000	0.000	0.259	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	60	54	58	0	0	0	0	54	0
N.S.	1	1.11	1.00	1.07	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.538	0.006	0.082	0.000	0.000	0.000	0.000	0.253	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	74	63	72	0	0	0	0	54	0
N.S.	1	1.17	1.00	1.14	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.635	0.008	0.083	0.000	0.000	0.000	0.000	0.285	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	99	84	96	0	85	0	0	12	0
N.S.	1	1.18	1.00	1.14	0.00	1.01	0.00	0.00	0.14	0.00
time (sec)	N/A	0.807	0.008	0.088	0.000	0.067	0.000	0.000	0.274	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	84	71	82	0	73	0	0	10	0
N.S.	1	1.18	1.00	1.15	0.00	1.03	0.00	0.00	0.14	0.00
time (sec)	N/A	0.625	0.006	0.100	0.000	0.070	0.000	0.000	0.310	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	42	30	47	0	49	39	0	40	0
N.S.	1	1.14	0.81	1.27	0.00	1.32	1.05	0.00	1.08	0.00
time (sec)	N/A	0.453	0.039	0.075	0.000	0.069	0.260	0.000	0.295	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	0	19	15	0	12	0
N.S.	1	1.00	1.00	0.86	0.00	0.90	0.71	0.00	0.57	0.00
time (sec)	N/A	0.265	0.013	0.116	0.000	0.073	0.313	0.000	0.283	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	35	32	27	0	29	27	0	94	0
N.S.	1	1.09	1.00	0.84	0.00	0.91	0.84	0.00	2.94	0.00
time (sec)	N/A	0.375	0.005	0.081	0.000	0.075	0.113	0.000	0.204	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	0	24	26	0	56	0
N.S.	1	1.00	1.00	0.85	0.00	0.89	0.96	0.00	2.07	0.00
time (sec)	N/A	0.285	0.004	0.088	0.000	0.072	0.184	0.000	0.268	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	30	0	0	0	0	56	0
N.S.	1	1.00	1.00	1.20	0.00	0.00	0.00	0.00	2.24	0.00
time (sec)	N/A	0.278	0.004	0.073	0.000	0.000	0.000	0.000	0.267	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	30	0	0	0	0	56	0
N.S.	1	1.00	1.00	1.25	0.00	0.00	0.00	0.00	2.33	0.00
time (sec)	N/A	0.283	0.004	0.080	0.000	0.000	0.000	0.000	0.318	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	42	41	44	0	0	0	0	56	0
N.S.	1	1.02	1.00	1.07	0.00	0.00	0.00	0.00	1.37	0.00
time (sec)	N/A	0.395	0.006	0.097	0.000	0.000	0.000	0.000	0.262	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	53	50	58	0	0	0	0	56	0
N.S.	1	1.06	1.00	1.16	0.00	0.00	0.00	0.00	1.12	0.00
time (sec)	N/A	0.490	0.006	0.111	0.000	0.000	0.000	0.000	0.292	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	78	67	72	0	0	0	0	56	0
N.S.	1	1.16	1.00	1.07	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.641	0.007	0.084	0.000	0.000	0.000	0.000	0.336	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	81	66	76	0	0	0	0	12	0
N.S.	1	1.23	1.00	1.15	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.642	0.007	0.076	0.000	0.000	0.000	0.000	0.218	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	63	53	62	0	47	0	0	12	0
N.S.	1	1.19	1.00	1.17	0.00	0.89	0.00	0.00	0.23	0.00
time (sec)	N/A	0.502	0.006	0.072	0.000	0.070	0.000	0.000	0.241	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	45	40	48	0	37	0	0	12	0
N.S.	1	1.12	1.00	1.20	0.00	0.92	0.00	0.00	0.30	0.00
time (sec)	N/A	0.383	0.005	0.072	0.000	0.066	0.000	0.000	0.322	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	0	23	20	0	10	0
N.S.	1	1.00	1.00	0.89	0.00	0.85	0.74	0.00	0.37	0.00
time (sec)	N/A	0.259	0.004	0.068	0.000	0.073	0.238	0.000	0.280	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	0	0	0	0	28	0
N.S.	1	1.00	1.00	1.39	0.00	0.00	0.00	0.00	1.56	0.00
time (sec)	N/A	0.232	0.034	0.073	0.000	0.000	0.000	0.000	0.237	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	0	22	14	0	12	0
N.S.	1	1.00	1.00	1.07	0.00	1.57	1.00	0.00	0.86	0.00
time (sec)	N/A	0.250	0.009	0.121	0.000	0.069	0.244	0.000	0.320	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	34	33	34	0	0	0	0	30	0
N.S.	1	1.03	1.00	1.03	0.00	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	0.335	0.004	0.079	0.000	0.000	0.000	0.000	0.305	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	50	0	0	0	0	34	0
N.S.	1	1.00	1.00	1.09	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.452	0.005	0.082	0.000	0.000	0.000	0.000	0.239	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	65	59	64	0	0	0	0	34	0
N.S.	1	1.10	1.00	1.08	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.568	0.007	0.093	0.000	0.000	0.000	0.000	0.337	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	81	66	76	0	0	0	0	12	0
N.S.	1	1.23	1.00	1.15	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.622	0.007	0.079	0.000	0.000	0.000	0.000	0.315	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	63	53	62	0	49	0	0	12	0
N.S.	1	1.19	1.00	1.17	0.00	0.92	0.00	0.00	0.23	0.00
time (sec)	N/A	0.745	0.006	0.084	0.000	0.078	0.000	0.000	0.317	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	45	40	48	0	37	0	0	12	0
N.S.	1	1.12	1.00	1.20	0.00	0.92	0.00	0.00	0.30	0.00
time (sec)	N/A	0.567	0.005	0.070	0.000	0.071	0.000	0.000	0.321	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	0	25	24	0	12	0
N.S.	1	1.00	1.00	0.89	0.00	0.93	0.89	0.00	0.44	0.00
time (sec)	N/A	0.368	0.005	0.073	0.000	0.075	0.267	0.000	0.334	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	30	0	0	0	0	10	0
N.S.	1	1.00	1.00	1.20	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.418	0.003	0.070	0.000	0.000	0.000	0.000	0.319	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	26	0	0	0	0	8	0
N.S.	1	1.00	1.00	1.30	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.358	0.035	0.073	0.000	0.000	0.000	0.000	0.311	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	0	18	17	0	12	0
N.S.	1	1.00	1.00	0.89	0.00	0.95	0.89	0.00	0.63	0.00
time (sec)	N/A	0.455	0.013	0.101	0.000	0.073	0.283	0.000	0.328	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	49	46	48	0	0	0	0	170	0
N.S.	1	1.07	1.00	1.04	0.00	0.00	0.00	0.00	3.70	0.00
time (sec)	N/A	0.575	0.009	0.088	0.000	0.000	0.000	0.000	0.330	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	64	59	64	0	0	0	0	185	0
N.S.	1	1.08	1.00	1.08	0.00	0.00	0.00	0.00	3.14	0.00
time (sec)	N/A	0.640	0.007	0.091	0.000	0.000	0.000	0.000	0.332	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	81	66	76	0	0	0	0	12	0
N.S.	1	1.23	1.00	1.15	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.635	0.009	0.089	0.000	0.000	0.000	0.000	0.330	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	63	53	62	0	47	0	0	12	0
N.S.	1	1.19	1.00	1.17	0.00	0.89	0.00	0.00	0.23	0.00
time (sec)	N/A	0.498	0.006	0.072	0.000	0.082	0.000	0.000	0.329	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	45	40	48	0	37	0	0	12	0
N.S.	1	1.12	1.00	1.20	0.00	0.92	0.00	0.00	0.30	0.00
time (sec)	N/A	0.383	0.005	0.075	0.000	0.068	0.000	0.000	0.292	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	0	25	24	0	12	0
N.S.	1	1.00	1.00	0.89	0.00	0.93	0.89	0.00	0.44	0.00
time (sec)	N/A	0.273	0.004	0.076	0.000	0.071	0.290	0.000	0.297	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	30	0	0	0	0	12	0
N.S.	1	1.00	1.00	1.20	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.275	0.004	0.073	0.000	0.000	0.000	0.000	0.170	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	30	0	0	0	0	10	0
N.S.	1	1.00	1.00	1.25	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.260	0.004	0.073	0.000	0.000	0.000	0.000	0.192	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	33	36	0	0	0	0	8	0
N.S.	1	1.00	0.94	1.03	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.313	0.044	0.075	0.000	0.000	0.000	0.000	0.242	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	0	18	20	0	12	0
N.S.	1	1.00	1.00	0.81	0.00	0.86	0.95	0.00	0.57	0.00
time (sec)	N/A	0.266	0.015	0.113	0.000	0.075	0.304	0.000	0.317	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	70	59	62	0	0	0	0	226	0
N.S.	1	1.19	1.00	1.05	0.00	0.00	0.00	0.00	3.83	0.00
time (sec)	N/A	0.576	0.007	0.091	0.000	0.000	0.000	0.000	0.257	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	82	72	78	0	0	0	0	247	0
N.S.	1	1.14	1.00	1.08	0.00	0.00	0.00	0.00	3.43	0.00
time (sec)	N/A	0.705	0.008	0.086	0.000	0.000	0.000	0.000	0.436	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	148	112	147	0	0	0	0	14	0
N.S.	1	1.13	0.85	1.12	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.955	0.020	0.094	0.000	0.000	0.000	0.000	0.269	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	132	104	124	0	0	0	0	14	0
N.S.	1	1.11	0.87	1.04	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.810	0.022	0.063	0.000	0.000	0.000	0.000	0.287	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	108	90	102	0	0	0	0	12	0
N.S.	1	1.09	0.91	1.03	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.561	0.018	0.050	0.000	0.000	0.000	0.000	0.337	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	66	54	82	0	0	0	0	10	0
N.S.	1	1.05	0.86	1.30	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.363	0.093	0.041	0.000	0.000	0.000	0.000	0.385	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	27	0	0	0	0	14	0
N.S.	1	1.00	0.67	0.90	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.329	0.043	0.338	0.000	0.000	0.000	0.000	0.322	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	51	66	0	0	0	0	55	0
N.S.	1	1.00	1.11	1.43	0.00	0.00	0.00	0.00	1.20	0.00
time (sec)	N/A	0.334	0.009	0.043	0.000	0.000	0.000	0.000	0.239	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	80	73	122	0	0	0	0	61	0
N.S.	1	1.01	0.92	1.54	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.500	0.019	0.046	0.000	0.000	0.000	0.000	0.218	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	103	86	170	0	0	0	0	61	0
N.S.	1	1.04	0.87	1.72	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.651	0.021	0.057	0.000	0.000	0.000	0.000	0.242	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	124	90	206	0	0	0	0	61	0
N.S.	1	1.10	0.80	1.82	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.815	0.019	0.052	0.000	0.000	0.000	0.000	0.350	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	153	106	262	0	0	0	0	61	0
N.S.	1	1.10	0.76	1.88	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	1.021	0.030	0.064	0.000	0.000	0.000	0.000	0.252	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	156	118	146	0	0	0	0	23	0
N.S.	1	1.12	0.85	1.05	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	1.012	0.022	0.073	0.000	0.000	0.000	0.000	0.322	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	124	98	125	0	0	0	0	23	0
N.S.	1	1.12	0.88	1.13	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.041	0.015	0.052	0.000	0.000	0.000	0.000	0.322	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	108	89	102	0	0	0	0	23	0
N.S.	1	1.09	0.90	1.03	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.084	0.018	0.048	0.000	0.000	0.000	0.000	0.326	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	84	71	80	0	0	0	0	21	0
N.S.	1	1.06	0.90	1.01	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.722	0.017	0.049	0.000	0.000	0.000	0.000	0.320	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	59	0	0	0	0	20	0
N.S.	1	1.00	0.96	1.26	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.441	0.086	0.046	0.000	0.000	0.000	0.000	0.300	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	18	26	0	0	27	0	23	0
N.S.	1	1.00	0.64	0.93	0.00	0.00	0.96	0.00	0.82	0.00
time (sec)	N/A	0.391	0.046	0.317	0.000	0.000	0.336	0.000	0.321	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	71	48	107	0	0	0	0	23	0
N.S.	1	1.01	0.69	1.53	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.541	0.011	0.049	0.000	0.000	0.000	0.000	0.400	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	105	71	168	0	0	0	0	23	0
N.S.	1	1.06	0.72	1.70	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.642	0.018	0.056	0.000	0.000	0.000	0.000	0.305	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	128	77	216	0	0	0	0	23	0
N.S.	1	1.08	0.65	1.82	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.815	0.019	0.056	0.000	0.000	0.000	0.000	0.373	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	149	81	252	0	0	0	0	23	0
N.S.	1	1.12	0.61	1.89	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	1.005	0.018	0.063	0.000	0.000	0.000	0.000	0.342	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	109	66	0	0	0	0	0	10	0
N.S.	1	0.96	0.58	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.570	0.020	0.000	0.000	0.000	0.000	0.000	0.284	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	111	67	0	0	0	0	0	12	0
N.S.	1	0.97	0.58	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.655	0.018	0.000	0.000	0.000	0.000	0.000	0.301	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	101	64	0	0	0	0	0	58	0
N.S.	1	0.96	0.61	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.754	0.091	0.000	0.000	0.000	0.000	0.000	0.322	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	109	60	0	0	0	0	0	12	0
N.S.	1	0.96	0.53	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.603	0.015	0.000	0.000	0.000	0.000	0.000	0.352	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	130	85	0	0	0	0	0	14	0
N.S.	1	0.97	0.63	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.625	0.036	0.000	0.000	0.000	0.000	0.000	0.317	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	130	87	0	0	0	0	0	23	0
N.S.	1	0.97	0.65	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.480	0.047	0.000	0.000	0.000	0.000	0.000	0.333	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	67	0	0	0	0	0	16	0
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.445	0.020	0.000	0.000	0.000	0.000	0.000	0.339	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	52	0	0	0	0	0	14	0
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.399	0.018	0.000	0.000	0.000	0.000	0.000	0.324	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	36	0	0	0	0	16	0
N.S.	1	1.00	0.81	1.12	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.326	0.059	0.218	0.000	0.000	0.000	0.000	0.284	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	49	0	0	0	0	0	58	0
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.414	0.013	0.000	0.000	0.000	0.000	0.000	0.366	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	64	0	0	0	0	0	65	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.443	0.017	0.000	0.000	0.000	0.000	0.000	0.298	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	127	81	0	0	0	0	0	16	0
N.S.	1	0.97	0.62	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.456	0.025	0.000	0.000	0.000	0.000	0.000	0.304	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	81	66	0	0	71	0	0	12	0
N.S.	1	1.23	1.00	0.00	0.00	1.08	0.00	0.00	0.18	0.00
time (sec)	N/A	0.683	0.015	0.000	0.000	0.070	0.000	0.000	0.334	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	61	51	0	0	57	0	0	12	0
N.S.	1	1.20	1.00	0.00	0.00	1.12	0.00	0.00	0.24	0.00
time (sec)	N/A	0.526	0.012	0.000	0.000	0.066	0.000	0.000	0.298	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	37	34	40	29	0	31	0
N.S.	1	1.00	0.92	1.03	0.94	1.11	0.81	0.00	0.86	0.00
time (sec)	N/A	0.385	0.011	0.075	0.027	0.067	0.240	0.000	0.299	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	0	26	17	0	24	0
N.S.	1	1.00	1.00	0.87	0.00	1.13	0.74	0.00	1.04	0.00
time (sec)	N/A	0.268	0.036	0.121	0.000	0.072	0.273	0.000	0.330	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	0	0	0	51	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.82	0.00
time (sec)	N/A	0.262	0.004	0.000	0.000	0.000	0.000	0.000	0.324	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	0	0	0	0	0	55	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.83	0.00
time (sec)	N/A	0.282	0.004	0.000	0.000	0.000	0.000	0.000	0.293	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	50	45	0	0	0	0	0	55	0
N.S.	1	1.11	1.00	0.00	0.00	0.00	0.00	0.00	1.22	0.00
time (sec)	N/A	0.383	0.006	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	0	0	0	0	0	0	0	12	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.308	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	0	0	0	0	0	0	0	12	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.353	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	0	0	0	0	0	0	0	8	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	45	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.363	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	0	0	0	0	0	0	0	55	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	76	66	0	0	71	0	0	14	0
N.S.	1	1.15	1.00	0.00	0.00	1.08	0.00	0.00	0.21	0.00
time (sec)	N/A	0.662	0.010	0.000	0.000	0.069	0.000	0.000	0.259	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	57	43	52	0	56	41	0	41	0
N.S.	1	1.27	0.96	1.16	0.00	1.24	0.91	0.00	0.91	0.00
time (sec)	N/A	0.520	0.011	0.084	0.000	0.083	0.274	0.000	0.311	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	0	23	19	0	24	0
N.S.	1	1.00	1.00	0.88	0.00	0.92	0.76	0.00	0.96	0.00
time (sec)	N/A	0.283	0.030	0.116	0.000	0.068	0.303	0.000	0.311	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	0	24	24	0	114	0
N.S.	1	1.00	1.00	0.85	0.00	0.89	0.89	0.00	4.22	0.00
time (sec)	N/A	0.284	0.004	0.050	0.000	0.070	0.191	0.000	0.330	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	0	0	0	64	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.00	0.00
time (sec)	N/A	0.292	0.004	0.000	0.000	0.000	0.000	0.000	0.297	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	0	0	0	0	0	64	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.13	0.00
time (sec)	N/A	0.289	0.004	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0	64	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.42	0.00
time (sec)	N/A	0.406	0.006	0.000	0.000	0.000	0.000	0.000	0.315	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	0	0	0	0	0	0	0	14	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.300	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	0	0	0	0	0	0	0	10	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.342	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	0	0	0	0	0	0	0	54	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	0	0	0	0	0	0	64	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	0	0	0	0	0	0	0	64	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.285	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	64	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.309	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	75	43	66	0	70	60	0	51	0
N.S.	1	1.21	0.69	1.06	0.00	1.13	0.97	0.00	0.82	0.00
time (sec)	N/A	0.647	0.012	0.072	0.000	0.085	0.265	0.000	0.316	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	0	23	19	0	14	0
N.S.	1	1.00	1.00	0.88	0.00	0.92	0.76	0.00	0.56	0.00
time (sec)	N/A	0.272	0.033	0.109	0.000	0.073	0.312	0.000	0.397	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	47	44	33	0	35	42	0	116	0
N.S.	1	1.07	1.00	0.75	0.00	0.80	0.95	0.00	2.64	0.00
time (sec)	N/A	0.392	0.007	0.049	0.000	0.078	0.191	0.000	0.438	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	0	28	29	0	66	0
N.S.	1	1.00	1.00	0.87	0.00	0.90	0.94	0.00	2.13	0.00
time (sec)	N/A	0.403	0.005	0.044	0.000	0.077	0.225	0.000	0.349	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	0	0	0	66	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.06	0.00
time (sec)	N/A	0.381	0.004	0.000	0.000	0.000	0.000	0.000	0.349	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	0	0	0	66	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.06	0.00
time (sec)	N/A	0.394	0.005	0.000	0.000	0.000	0.000	0.000	0.290	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	52	47	0	0	0	0	0	66	0
N.S.	1	1.11	1.00	0.00	0.00	0.00	0.00	0.00	1.40	0.00
time (sec)	N/A	0.589	0.008	0.000	0.000	0.000	0.000	0.000	0.298	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	0	0	0	0	0	0	0	14	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0	10	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.358	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	56	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.354	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	0	0	0	0	0	0	0	66	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.385	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	0	0	0	0	0	0	66	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.96	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.426	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	0	0	0	0	0	0	0	66	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.439	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	71	61	0	0	55	0	0	14	0
N.S.	1	1.16	1.00	0.00	0.00	0.90	0.00	0.00	0.23	0.00
time (sec)	N/A	0.516	0.006	0.000	0.000	0.076	0.000	0.000	0.465	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	51	46	0	0	43	0	0	14	0
N.S.	1	1.11	1.00	0.00	0.00	0.93	0.00	0.00	0.30	0.00
time (sec)	N/A	0.395	0.005	0.000	0.000	0.080	0.000	0.000	0.541	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	0	27	24	0	14	0
N.S.	1	1.00	1.00	0.90	0.00	0.87	0.77	0.00	0.45	0.00
time (sec)	N/A	0.283	0.004	0.041	0.000	0.071	0.279	0.000	0.440	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	32	0	0	0	0	38	0
N.S.	1	1.00	1.00	1.07	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.255	0.003	0.075	0.000	0.000	0.000	0.000	0.449	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	0	29	20	0	14	0
N.S.	1	1.00	1.00	0.88	0.00	1.21	0.83	0.00	0.58	0.00
time (sec)	N/A	0.253	0.016	0.104	0.000	0.071	0.236	0.000	0.417	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	38	37	0	0	0	0	0	36	0
N.S.	1	1.03	1.00	0.00	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.438	0.010	0.000	0.000	0.000	0.000	0.000	0.475	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	36	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.550	0.011	0.000	0.000	0.000	0.000	0.000	0.419	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0	14	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.417	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	0	0	0	0	0	0	0	14	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.433	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	0	0	0	0	0	0	0	10	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.383	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	0	0	0	0	0	0	0	32	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.409	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.493	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	51	46	0	0	43	0	0	14	0
N.S.	1	1.11	1.00	0.00	0.00	0.93	0.00	0.00	0.30	0.00
time (sec)	N/A	0.403	0.005	0.000	0.000	0.069	0.000	0.000	0.359	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	0	29	26	0	14	0
N.S.	1	1.00	1.00	0.90	0.00	0.94	0.84	0.00	0.45	0.00
time (sec)	N/A	0.281	0.004	0.046	0.000	0.076	0.318	0.000	0.389	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	0	0	0	14	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.289	0.005	0.000	0.000	0.000	0.000	0.000	0.413	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	33	0	0	0	0	54	0
N.S.	1	1.00	1.00	1.22	0.00	0.00	0.00	0.00	2.00	0.00
time (sec)	N/A	0.342	0.004	0.066	0.000	0.000	0.000	0.000	0.391	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	0	22	22	0	14	0
N.S.	1	1.00	1.00	0.84	0.00	0.88	0.88	0.00	0.56	0.00
time (sec)	N/A	0.441	0.021	0.105	0.000	0.073	0.373	0.000	0.384	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	55	52	0	0	0	0	0	217	0
N.S.	1	1.06	1.00	0.00	0.00	0.00	0.00	0.00	4.17	0.00
time (sec)	N/A	0.939	0.010	0.000	0.000	0.000	0.000	0.000	0.397	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0	14	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.303	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0	14	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.417	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	0	0	0	0	0	0	0	14	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.392	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	0	0	0	0	0	0	0	10	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.381	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	204	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.29	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.431	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	221	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.17	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.357	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	142	118	0	0	0	0	0	16	0
N.S.	1	1.10	0.91	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.833	0.023	0.000	0.000	0.000	0.000	0.000	0.280	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	116	102	0	0	0	0	0	16	0
N.S.	1	1.08	0.95	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.649	0.021	0.000	0.000	0.000	0.000	0.000	0.413	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	81	66	95	0	0	0	0	78	0
N.S.	1	1.07	0.87	1.25	0.00	0.00	0.00	0.00	1.03	0.00
time (sec)	N/A	0.442	0.015	0.204	0.000	0.000	0.000	0.000	0.403	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	30	0	0	0	0	16	0
N.S.	1	1.00	0.75	0.94	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.332	0.087	0.278	0.000	0.000	0.000	0.000	0.386	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	51	0	0	0	0	0	69	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	1.33	0.00
time (sec)	N/A	0.348	0.015	0.000	0.000	0.000	0.000	0.000	0.447	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	86	83	0	0	0	0	0	73	0
N.S.	1	1.01	0.98	0.00	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.498	0.018	0.000	0.000	0.000	0.000	0.000	0.455	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	111	96	0	0	0	0	0	73	0
N.S.	1	1.04	0.90	0.00	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.677	0.023	0.000	0.000	0.000	0.000	0.000	0.416	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	119	67	0	0	0	0	0	16	0
N.S.	1	1.12	0.63	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.834	0.013	0.000	0.000	0.000	0.000	0.000	0.332	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	93	57	0	0	0	0	0	16	0
N.S.	1	1.11	0.68	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.624	0.012	0.000	0.000	0.000	0.000	0.000	0.391	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	67	47	0	0	0	0	0	16	0
N.S.	1	1.08	0.76	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.472	0.011	0.000	0.000	0.000	0.000	0.000	0.398	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	0	0	0	34	0	12	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	1.10	0.00	0.39	0.00
time (sec)	N/A	0.264	0.007	0.000	0.000	0.000	0.298	0.000	0.387	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	0	0	0	0	0	0	0	64	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.427	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0	73	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.511	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	116	101	0	0	0	0	0	27	0
N.S.	1	1.08	0.94	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.670	0.018	0.000	0.000	0.000	0.000	0.000	0.388	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	90	79	0	0	0	0	0	27	0
N.S.	1	1.06	0.93	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.489	0.018	0.000	0.000	0.000	0.000	0.000	0.390	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	66	0	0	0	0	57	0
N.S.	1	1.00	0.95	1.18	0.00	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	0.303	0.009	0.197	0.000	0.000	0.000	0.000	0.384	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	21	29	0	0	31	0	27	0
N.S.	1	1.00	0.68	0.94	0.00	0.00	1.00	0.00	0.87	0.00
time (sec)	N/A	0.316	0.093	0.293	0.000	0.000	0.346	0.000	0.437	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	77	61	0	0	0	0	0	27	0
N.S.	1	1.01	0.80	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.479	0.010	0.000	0.000	0.000	0.000	0.000	0.422	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	113	81	0	0	0	0	0	27	0
N.S.	1	1.06	0.76	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.665	0.016	0.000	0.000	0.000	0.000	0.000	0.428	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	138	91	0	0	0	0	0	27	0
N.S.	1	1.07	0.71	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.842	0.020	0.000	0.000	0.000	0.000	0.000	0.352	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	93	57	0	0	0	0	0	27	0
N.S.	1	1.11	0.68	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.646	0.014	0.000	0.000	0.000	0.000	0.000	0.339	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	67	47	0	0	0	0	0	27	0
N.S.	1	1.08	0.76	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.557	0.010	0.000	0.000	0.000	0.000	0.000	0.392	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	30	0	0	0	39	0	27	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.98	0.00	0.68	0.00
time (sec)	N/A	0.419	0.008	0.000	0.000	0.000	0.374	0.000	0.315	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0	24	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.397	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0	27	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.395	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	0	0	0	0	0	0	0	27	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.481	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	143	60	0	0	0	0	0	18	0
N.S.	1	1.13	0.48	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.954	0.026	0.000	0.000	0.000	0.000	0.000	0.549	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	59	56	0	0	0	0	0	86	0
N.S.	1	0.98	0.93	0.00	0.00	0.00	0.00	0.00	1.43	0.00
time (sec)	N/A	0.494	0.055	0.000	0.000	0.000	0.000	0.000	0.371	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	33	44	0	0	0	0	18	0
N.S.	1	1.00	0.79	1.05	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.407	0.118	0.217	0.000	0.000	0.000	0.000	0.416	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	95	61	0	0	0	0	0	74	0
N.S.	1	0.88	0.56	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.574	0.020	0.000	0.000	0.000	0.000	0.000	0.390	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	0	0	0	0	0	0	0	18	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.250	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	0	0	0	0	0	0	0	69	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.361	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	0	0	0	0	0	0	0	78	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.403	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	85	75	81	0	0	0	0	47	0
N.S.	1	1.13	1.00	1.08	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.648	0.010	0.281	0.000	0.000	0.000	0.000	0.261	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	63	60	65	0	0	0	0	47	0
N.S.	1	1.05	1.00	1.08	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.521	0.008	0.205	0.000	0.000	0.000	0.000	0.342	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	46	45	49	0	0	0	0	45	0
N.S.	1	1.02	1.00	1.09	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.394	0.005	0.226	0.000	0.000	0.000	0.000	0.384	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	31	0	0	0	0	43	0
N.S.	1	1.00	1.00	1.29	0.00	0.00	0.00	0.00	1.79	0.00
time (sec)	N/A	0.264	0.003	0.237	0.000	0.000	0.000	0.000	0.430	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	27	0	0	0	0	42	0
N.S.	1	1.00	1.00	1.29	0.00	0.00	0.00	0.00	2.00	0.00
time (sec)	N/A	0.308	0.004	0.217	0.000	0.000	0.000	0.000	0.384	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	0	22	14	0	12	0
N.S.	1	1.00	1.00	0.95	0.00	1.05	0.67	0.00	0.57	0.00
time (sec)	N/A	0.274	0.031	0.100	0.000	0.073	0.369	0.000	0.459	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	21	31	34	34	19	0	30	0
N.S.	1	1.00	0.75	1.11	1.21	1.21	0.68	0.00	1.07	0.00
time (sec)	N/A	0.389	0.013	0.064	0.031	0.071	0.236	0.000	0.498	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	56	51	61	0	47	0	0	12	0
N.S.	1	1.10	1.00	1.20	0.00	0.92	0.00	0.00	0.24	0.00
time (sec)	N/A	0.510	0.006	0.230	0.000	0.087	0.000	0.000	0.518	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	76	66	77	0	58	0	0	12	0
N.S.	1	1.15	1.00	1.17	0.00	0.88	0.00	0.00	0.18	0.00
time (sec)	N/A	0.646	0.008	0.220	0.000	0.094	0.000	0.000	0.369	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	96	81	93	0	0	0	0	12	0
N.S.	1	1.19	1.00	1.15	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.784	0.010	0.223	0.000	0.000	0.000	0.000	0.492	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	67	62	67	0	0	0	0	55	0
N.S.	1	1.08	1.00	1.08	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.516	0.010	0.227	0.000	0.000	0.000	0.000	0.398	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	50	0	0	0	0	55	0
N.S.	1	1.00	1.00	1.16	0.00	0.00	0.00	0.00	1.28	0.00
time (sec)	N/A	0.392	0.005	0.236	0.000	0.000	0.000	0.000	0.432	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	35	0	0	0	0	53	0
N.S.	1	1.00	1.00	1.30	0.00	0.00	0.00	0.00	1.96	0.00
time (sec)	N/A	0.278	0.005	0.234	0.000	0.000	0.000	0.000	0.431	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	35	0	0	0	0	51	0
N.S.	1	1.00	1.00	1.17	0.00	0.00	0.00	0.00	1.70	0.00
time (sec)	N/A	0.259	0.004	0.229	0.000	0.000	0.000	0.000	0.501	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	0	22	15	0	156	0
N.S.	1	1.00	1.00	1.05	0.00	1.10	0.75	0.00	7.80	0.00
time (sec)	N/A	0.244	0.003	0.218	0.000	0.094	0.118	0.000	0.456	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	0	23	17	0	14	0
N.S.	1	1.00	1.00	0.88	0.00	0.92	0.68	0.00	0.56	0.00
time (sec)	N/A	0.274	0.033	0.105	0.000	0.073	0.286	0.000	0.459	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	48	41	43	0	45	29	0	42	0
N.S.	1	1.12	0.95	1.00	0.00	1.05	0.67	0.00	0.98	0.00
time (sec)	N/A	0.532	0.015	0.066	0.000	0.072	0.234	0.000	0.439	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	71	66	77	0	58	0	0	14	0
N.S.	1	1.08	1.00	1.17	0.00	0.88	0.00	0.00	0.21	0.00
time (sec)	N/A	0.641	0.007	0.228	0.000	0.072	0.000	0.000	0.404	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	96	81	93	0	69	0	0	14	0
N.S.	1	1.19	1.00	1.15	0.00	0.85	0.00	0.00	0.17	0.00
time (sec)	N/A	0.808	0.009	0.226	0.000	0.074	0.000	0.000	0.416	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	116	96	109	0	0	0	0	14	0
N.S.	1	1.21	1.00	1.14	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.947	0.010	0.257	0.000	0.000	0.000	0.000	0.362	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	105	88	0	0	0	0	0	64	0
N.S.	1	1.12	0.94	0.00	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.890	0.013	0.000	0.000	0.000	0.000	0.000	0.431	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	87	79	0	0	0	0	0	62	0
N.S.	1	1.05	0.95	0.00	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	0.832	0.015	0.000	0.000	0.000	0.000	0.000	0.449	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	67	62	0	0	0	0	0	61	0
N.S.	1	1.02	0.94	0.00	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.554	0.010	0.000	0.000	0.000	0.000	0.000	0.289	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	0	0	0	60	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.88	0.00
time (sec)	N/A	0.357	0.006	0.000	0.000	0.000	0.000	0.000	0.387	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	0	0	22	0	13	0
N.S.	1	1.00	0.81	0.81	0.00	0.00	0.81	0.00	0.48	0.00
time (sec)	N/A	0.467	0.038	0.273	0.000	0.000	0.403	0.000	0.392	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	58	56	48	0	0	0	0	113	0
N.S.	1	1.04	1.00	0.86	0.00	0.00	0.00	0.00	2.02	0.00
time (sec)	N/A	0.458	0.013	0.227	0.000	0.000	0.000	0.000	0.460	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	95	79	0	0	0	0	0	13	0
N.S.	1	1.12	0.93	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.535	0.020	0.000	0.000	0.000	0.000	0.000	0.307	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	117	102	0	0	0	0	0	13	0
N.S.	1	1.15	1.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.677	0.025	0.000	0.000	0.000	0.000	0.000	0.389	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	127	103	0	0	0	0	0	21	0
N.S.	1	1.14	0.93	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.810	0.016	0.000	0.000	0.000	0.000	0.000	0.444	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	109	95	0	0	0	0	0	21	0
N.S.	1	1.09	0.95	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.675	0.016	0.000	0.000	0.000	0.000	0.000	0.433	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	89	83	0	0	0	0	0	19	0
N.S.	1	1.07	1.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.475	0.014	0.000	0.000	0.000	0.000	0.000	0.397	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	53	47	0	0	0	0	0	18	0
N.S.	1	1.02	0.90	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.445	0.010	0.000	0.000	0.000	0.000	0.000	0.429	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	20	22	0	0	19	0	21	0
N.S.	1	1.00	0.80	0.88	0.00	0.00	0.76	0.00	0.84	0.00
time (sec)	N/A	0.279	0.042	0.271	0.000	0.000	0.442	0.000	0.503	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	0	0	0	0	90	0
N.S.	1	1.00	1.00	0.87	0.00	0.00	0.00	0.00	2.31	0.00
time (sec)	N/A	0.362	0.008	0.233	0.000	0.000	0.000	0.000	0.337	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	73	63	0	0	0	0	0	21	0
N.S.	1	1.07	0.93	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.548	0.016	0.000	0.000	0.000	0.000	0.000	0.433	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	95	79	0	0	0	0	0	21	0
N.S.	1	1.12	0.93	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.883	0.022	0.000	0.000	0.000	0.000	0.000	0.398	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	73	0	0	0	0	0	68	0
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.851	0.019	0.000	0.000	0.000	0.000	0.000	0.435	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	73	0	0	0	0	0	67	0
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.829	0.016	0.000	0.000	0.000	0.000	0.000	0.434	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	31	44	0	0	0	0	18	0
N.S.	1	1.00	0.82	1.16	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.365	0.100	0.198	0.000	0.000	0.000	0.000	0.363	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	54	0	0	0	0	0	308	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	5.60	0.00
time (sec)	N/A	0.347	0.018	0.000	0.000	0.000	0.000	0.000	0.411	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	62	0	0	0	0	0	18	0
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.459	0.015	0.000	0.000	0.000	0.000	0.000	0.392	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	35	0	24	0	0	58	0
N.S.	1	1.00	1.00	1.25	0.00	0.86	0.00	0.00	2.07	0.00
time (sec)	N/A	0.239	0.004	1.177	0.000	0.072	0.000	0.000	0.506	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	35	0	24	0	0	58	0
N.S.	1	1.00	1.00	1.25	0.00	0.86	0.00	0.00	2.07	0.00
time (sec)	N/A	0.244	0.006	1.148	0.000	0.083	0.000	0.000	0.425	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	35	0	24	0	0	61	0
N.S.	1	1.00	1.00	1.25	0.00	0.86	0.00	0.00	2.18	0.00
time (sec)	N/A	0.247	0.005	1.183	0.000	0.083	0.000	0.000	0.518	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	0	22	15	0	156	0
N.S.	1	1.00	1.00	1.05	0.00	1.10	0.75	0.00	7.80	0.00
time (sec)	N/A	0.241	0.000	0.241	0.000	0.090	0.102	0.000	0.416	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	35	0	23	22	0	9	0
N.S.	1	1.00	1.00	1.25	0.00	0.82	0.79	0.00	0.32	0.00
time (sec)	N/A	0.238	0.005	0.280	0.000	0.084	0.351	0.000	0.400	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	35	0	25	26	0	10	0
N.S.	1	1.00	1.00	1.25	0.00	0.89	0.93	0.00	0.36	0.00
time (sec)	N/A	0.249	0.005	0.278	0.000	0.075	0.520	0.000	0.421	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	35	0	25	26	0	10	0
N.S.	1	1.00	1.00	1.25	0.00	0.89	0.93	0.00	0.36	0.00
time (sec)	N/A	0.245	0.005	0.279	0.000	0.073	0.836	0.000	0.468	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	0	0	0	0	58	0
N.S.	1	1.00	1.00	1.10	0.00	0.00	0.00	0.00	1.93	0.00
time (sec)	N/A	0.246	0.004	1.124	0.000	0.000	0.000	0.000	0.428	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	0	0	0	0	58	0
N.S.	1	1.00	1.00	1.10	0.00	0.00	0.00	0.00	1.93	0.00
time (sec)	N/A	0.249	0.004	1.115	0.000	0.000	0.000	0.000	0.461	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	0	0	0	0	56	0
N.S.	1	1.00	1.00	1.10	0.00	0.00	0.00	0.00	1.87	0.00
time (sec)	N/A	0.250	0.004	1.071	0.000	0.000	0.000	0.000	0.450	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	0	0	0	0	49	0
N.S.	1	1.00	1.00	1.04	0.00	0.00	0.00	0.00	1.75	0.00
time (sec)	N/A	0.245	0.003	1.120	0.000	0.000	0.000	0.000	0.419	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	26	0	0	0	0	8	0
N.S.	1	1.00	1.00	1.30	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.234	0.004	0.066	0.000	0.000	0.000	0.000	0.503	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	0	0	0	0	9	0
N.S.	1	1.00	1.00	1.10	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.244	0.004	0.211	0.000	0.000	0.000	0.000	0.413	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	0	0	0	0	10	0
N.S.	1	1.00	1.00	1.10	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.258	0.004	0.185	0.000	0.000	0.000	0.000	0.464	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	0	0	0	0	10	0
N.S.	1	1.00	1.00	1.10	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.251	0.004	0.217	0.000	0.000	0.000	0.000	0.424	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	186	0	0	0	0	0	0	0	161	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.438	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	0	0	0	0	0	0	0	147	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.280	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	0	0	0	0	0	0	0	62	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.347	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.419	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.508	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	30	43	0	35	54	0	82	0
N.S.	1	1.00	0.73	1.05	0.00	0.85	1.32	0.00	2.00	0.00
time (sec)	N/A	0.327	0.008	1.112	0.000	0.093	3.636	0.000	0.270	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	30	43	0	35	54	0	78	0
N.S.	1	1.00	0.73	1.05	0.00	0.85	1.32	0.00	1.90	0.00
time (sec)	N/A	0.353	0.008	0.573	0.000	0.074	1.935	0.000	0.378	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	40	0	29	46	0	127	0
N.S.	1	1.00	0.69	1.14	0.00	0.83	1.31	0.00	3.63	0.00
time (sec)	N/A	0.509	0.008	0.299	0.000	0.079	1.060	0.000	0.383	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	30	41	0	34	44	0	47	0
N.S.	1	1.00	0.73	1.00	0.00	0.83	1.07	0.00	1.15	0.00
time (sec)	N/A	0.541	0.008	0.184	0.000	0.080	1.024	0.000	0.372	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	30	41	0	36	49	0	19	0
N.S.	1	1.00	0.73	1.00	0.00	0.88	1.20	0.00	0.46	0.00
time (sec)	N/A	0.487	0.008	0.207	0.000	0.081	1.458	0.000	0.422	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	146	108	0	0	0	0	0	87	0
N.S.	1	1.05	0.78	0.00	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	1.546	0.027	0.000	0.000	0.000	0.000	0.000	0.487	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	117	96	0	0	0	0	0	87	0
N.S.	1	1.04	0.86	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.893	0.023	0.000	0.000	0.000	0.000	0.000	0.536	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	88	81	0	0	0	0	0	85	0
N.S.	1	1.04	0.95	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.553	0.028	0.000	0.000	0.000	0.000	0.000	0.481	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	56	0	0	0	0	0	75	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	1.25	0.00
time (sec)	N/A	0.380	0.022	0.000	0.000	0.000	0.000	0.000	0.458	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	0	0	0	0	0	75	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	1.29	0.00
time (sec)	N/A	0.373	0.026	0.000	0.000	0.000	0.000	0.000	0.415	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	90	63	0	0	0	0	0	32	0
N.S.	1	1.01	0.71	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.535	0.016	0.000	0.000	0.000	0.000	0.000	0.437	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	122	76	0	0	0	0	0	32	0
N.S.	1	1.05	0.66	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.718	0.020	0.000	0.000	0.000	0.000	0.000	0.368	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	154	86	0	0	0	0	0	32	0
N.S.	1	1.08	0.60	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.941	0.024	0.000	0.000	0.000	0.000	0.000	0.452	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	165	120	0	0	0	0	0	97	0
N.S.	1	1.09	0.79	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.974	0.038	0.000	0.000	0.000	0.000	0.000	0.490	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	134	108	0	0	0	0	0	97	0
N.S.	1	1.07	0.86	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.757	0.035	0.000	0.000	0.000	0.000	0.000	0.451	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	103	96	0	0	0	0	0	97	0
N.S.	1	1.05	0.98	0.00	0.00	0.00	0.00	0.00	0.99	0.00
time (sec)	N/A	0.556	0.030	0.000	0.000	0.000	0.000	0.000	0.454	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	66	0	0	0	0	0	95	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	1.30	0.00
time (sec)	N/A	0.379	0.033	0.000	0.000	0.000	0.000	0.000	0.410	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	0	0	0	0	0	85	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.385	0.032	0.000	0.000	0.000	0.000	0.000	0.434	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	99	90	0	0	0	0	0	85	0
N.S.	1	1.01	0.92	0.00	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	0.544	0.022	0.000	0.000	0.000	0.000	0.000	0.441	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	131	88	0	0	0	0	0	34	0
N.S.	1	1.05	0.70	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.742	0.030	0.000	0.000	0.000	0.000	0.000	0.435	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	163	98	0	0	0	0	0	34	0
N.S.	1	1.07	0.64	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.978	0.033	0.000	0.000	0.000	0.000	0.000	0.438	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	136	100	0	0	0	0	0	30	0
N.S.	1	1.03	0.76	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.432	0.027	0.000	0.000	0.000	0.000	0.000	0.421	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	110	86	0	0	0	0	0	26	0
N.S.	1	1.03	0.80	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.036	0.024	0.000	0.000	0.000	0.000	0.000	0.375	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	85	69	0	0	0	0	0	19	0
N.S.	1	1.04	0.84	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.768	0.018	0.000	0.000	0.000	0.000	0.000	0.470	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	56	0	0	0	0	0	30	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.444	0.011	0.000	0.000	0.000	0.000	0.000	0.463	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	0	0	0	0	0	30	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.369	0.013	0.000	0.000	0.000	0.000	0.000	0.405	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	88	68	0	0	0	0	0	30	0
N.S.	1	1.01	0.78	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.545	0.018	0.000	0.000	0.000	0.000	0.000	0.414	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	118	94	0	0	0	0	0	30	0
N.S.	1	1.05	0.84	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.730	0.027	0.000	0.000	0.000	0.000	0.000	0.409	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	148	108	0	0	0	0	0	30	0
N.S.	1	1.08	0.79	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.919	0.031	0.000	0.000	0.000	0.000	0.000	0.422	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	165	129	0	0	0	0	0	28	0
N.S.	1	1.09	0.85	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.911	0.037	0.000	0.000	0.000	0.000	0.000	0.516	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	134	111	0	0	0	0	0	21	0
N.S.	1	1.07	0.89	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.732	0.027	0.000	0.000	0.000	0.000	0.000	0.406	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	103	86	0	0	0	0	0	32	0
N.S.	1	1.05	0.88	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.548	0.026	0.000	0.000	0.000	0.000	0.000	0.621	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	69	0	0	0	0	0	32	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.392	0.021	0.000	0.000	0.000	0.000	0.000	0.430	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	0	0	0	0	0	32	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.387	0.020	0.000	0.000	0.000	0.000	0.000	0.436	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	99	95	0	0	0	0	0	32	0
N.S.	1	1.01	0.97	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.575	0.034	0.000	0.000	0.000	0.000	0.000	0.418	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	131	113	0	0	0	0	0	32	0
N.S.	1	1.05	0.90	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.767	0.040	0.000	0.000	0.000	0.000	0.000	0.492	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	163	132	0	0	0	0	0	32	0
N.S.	1	1.07	0.87	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.988	0.046	0.000	0.000	0.000	0.000	0.000	0.321	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	0	0	0	0	0	0	0	73	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.438	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	0	0	0	0	0	0	0	69	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.451	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	33	46	0	0	0	0	18	0
N.S.	1	1.00	0.79	1.10	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.352	0.125	0.277	0.000	0.000	0.000	0.000	0.404	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	0	0	0	0	0	0	0	78	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.355	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	183	0	0	0	0	0	0	0	81	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.361	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	25	69	0	0	63	0	21	0
N.S.	1	1.00	0.64	1.77	0.00	0.00	1.62	0.00	0.54	0.00
time (sec)	N/A	0.295	0.008	0.063	0.000	0.000	0.840	0.000	0.286	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	97	0	0	0	0	184	0
N.S.	1	1.00	0.89	2.20	0.00	0.00	0.00	0.00	4.18	0.00
time (sec)	N/A	0.294	0.014	0.162	0.000	0.000	0.000	0.000	0.288	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	201	0	0	0	0	0	0	0	204	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.01	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.266	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0	264	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.11	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0	98	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.24	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.342	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	65	43	103	0	0	0	0	28	0
N.S.	1	0.98	0.65	1.56	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.654	0.018	0.481	0.000	0.000	0.000	0.000	0.314	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	111	63	0	0	0	0	0	30	0
N.S.	1	1.09	0.62	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.882	0.029	0.000	0.000	0.000	0.000	0.000	0.290	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	155	81	0	0	0	0	0	30	0
N.S.	1	1.11	0.58	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.932	0.036	0.000	0.000	0.000	0.000	0.000	0.283	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	63	53	62	0	0	0	0	14	0
N.S.	1	1.19	1.00	1.17	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.538	0.008	0.076	0.000	0.000	0.000	0.000	0.252	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	45	40	48	0	37	0	0	14	0
N.S.	1	1.12	1.00	1.20	0.00	0.92	0.00	0.00	0.35	0.00
time (sec)	N/A	0.400	0.006	0.069	0.000	0.078	0.000	0.000	0.234	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	34	0	25	0	0	12	0
N.S.	1	1.00	1.00	1.26	0.00	0.93	0.00	0.00	0.44	0.00
time (sec)	N/A	0.274	0.004	0.069	0.000	0.075	0.000	0.000	0.388	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	0	9	7	0	10	0
N.S.	1	1.00	1.00	1.12	0.00	1.12	0.88	0.00	1.25	0.00
time (sec)	N/A	0.173	0.010	0.060	0.000	0.080	0.256	0.000	0.308	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	0	6	7	0	12	0
N.S.	1	1.00	1.00	1.20	0.00	1.20	1.40	0.00	2.40	0.00
time (sec)	N/A	0.184	0.018	0.101	0.000	0.072	0.120	0.000	0.261	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	18	0	0	0	0	50	0
N.S.	1	1.00	1.00	1.12	0.00	0.00	0.00	0.00	3.12	0.00
time (sec)	N/A	0.264	0.006	0.068	0.000	0.000	0.000	0.000	0.286	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	35	0	0	0	0	54	0
N.S.	1	1.00	1.00	1.25	0.00	0.00	0.00	0.00	1.93	0.00
time (sec)	N/A	0.374	0.005	0.073	0.000	0.000	0.000	0.000	0.323	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	47	46	50	0	0	0	0	54	0
N.S.	1	1.02	1.00	1.09	0.00	0.00	0.00	0.00	1.17	0.00
time (sec)	N/A	0.475	0.008	0.076	0.000	0.000	0.000	0.000	0.272	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	55	46	0	0	43	0	0	16	0
N.S.	1	1.20	1.00	0.00	0.00	0.93	0.00	0.00	0.35	0.00
time (sec)	N/A	0.501	0.015	0.000	0.000	0.072	0.000	0.000	0.253	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	31	0	0	29	0	0	16	0
N.S.	1	1.13	1.00	0.00	0.00	0.94	0.00	0.00	0.52	0.00
time (sec)	N/A	0.356	0.009	0.000	0.000	0.073	0.000	0.000	0.243	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	0	14	15	0	13	0
N.S.	1	1.00	1.00	0.93	0.00	0.93	1.00	0.00	0.87	0.00
time (sec)	N/A	0.237	0.009	0.063	0.000	0.076	0.261	0.000	0.218	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	10	10	0	14	0
N.S.	1	1.00	1.00	0.91	0.00	0.91	0.91	0.00	1.27	0.00
time (sec)	N/A	0.192	0.031	0.100	0.000	0.077	0.107	0.000	0.326	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	0	0	0	0	60	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.73	0.00
time (sec)	N/A	0.351	0.010	0.000	0.000	0.000	0.000	0.000	0.273	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	34	0	0	0	0	0	60	0
N.S.	1	1.06	1.00	0.00	0.00	0.00	0.00	0.00	1.76	0.00
time (sec)	N/A	0.464	0.011	0.000	0.000	0.000	0.000	0.000	0.273	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	0	0	0	0	0	0	0	16	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.363	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0	16	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	0	0	0	0	0	0	0	12	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.387	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	0	0	0	0	0	0	0	56	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.278	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	0	0	0	0	0	0	60	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.355	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	53	52	57	0	0	0	0	16	0
N.S.	1	1.02	1.00	1.10	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.574	0.009	0.227	0.000	0.000	0.000	0.000	0.271	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	40	0	0	0	0	14	0
N.S.	1	1.00	1.00	1.21	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.428	0.007	0.230	0.000	0.000	0.000	0.000	0.190	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	19	0	0	0	0	12	0
N.S.	1	1.00	1.00	1.46	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.326	0.006	0.211	0.000	0.000	0.000	0.000	0.330	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	12	7	0	14	0
N.S.	1	1.00	1.00	1.11	0.00	1.33	0.78	0.00	1.56	0.00
time (sec)	N/A	0.194	0.032	0.102	0.000	0.078	0.109	0.000	0.386	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	0	14	14	0	13	0
N.S.	1	1.00	1.00	1.08	0.00	1.08	1.08	0.00	1.00	0.00
time (sec)	N/A	0.238	0.007	0.067	0.000	0.070	0.304	0.000	0.286	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	39	0	25	0	0	18	0
N.S.	1	1.00	1.00	1.26	0.00	0.81	0.00	0.00	0.58	0.00
time (sec)	N/A	0.332	0.005	0.234	0.000	0.074	0.000	0.000	0.326	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	51	46	55	0	36	0	0	18	0
N.S.	1	1.11	1.00	1.20	0.00	0.78	0.00	0.00	0.39	0.00
time (sec)	N/A	0.468	0.007	0.237	0.000	0.070	0.000	0.000	0.336	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	57	52	0	0	0	0	0	16	0
N.S.	1	1.10	1.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.660	0.015	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	37	35	0	0	0	0	0	16	0
N.S.	1	1.06	1.00	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.564	0.009	0.000	0.000	0.000	0.000	0.000	0.287	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	17	22	0	0	0	0	0	14	0
N.S.	1	0.77	1.00	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.723	0.009	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	12	10	0	14	0
N.S.	1	1.00	1.00	0.91	0.00	1.09	0.91	0.00	1.27	0.00
time (sec)	N/A	0.323	0.033	0.101	0.000	0.069	0.178	0.000	0.237	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	0	14	22	0	13	0
N.S.	1	1.00	1.00	0.93	0.00	0.93	1.47	0.00	0.87	0.00
time (sec)	N/A	0.498	0.009	0.073	0.000	0.076	0.355	0.000	0.329	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	31	0	0	25	0	0	18	0
N.S.	1	1.13	1.00	0.00	0.00	0.81	0.00	0.00	0.58	0.00
time (sec)	N/A	0.595	0.008	0.000	0.000	0.070	0.000	0.000	0.198	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	55	46	0	0	36	0	0	18	0
N.S.	1	1.20	1.00	0.00	0.00	0.78	0.00	0.00	0.39	0.00
time (sec)	N/A	0.701	0.010	0.000	0.000	0.074	0.000	0.000	0.237	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	16	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	0	0	0	0	0	0	16	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.327	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	0	0	0	0	0	0	0	12	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.273	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0	18	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.307	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	0	0	0	0	0	0	0	18	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.238	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	102	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.17	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.311	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	102	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.17	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.377	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	50	50	0	0	0	0	0	88	0
N.S.	1	0.96	0.96	0.00	0.00	0.00	0.00	0.00	1.69	0.00
time (sec)	N/A	0.227	0.076	0.000	0.000	0.000	0.000	0.000	0.280	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	0	0	0	0	0	0	0	100	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.69	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.258	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	0	0	0	0	0	0	0	100	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.19	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	0	0	0	0	0	0	0	102	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.19	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	0	11	0	0	21	0
N.S.	1	1.00	1.00	0.92	0.00	0.92	0.00	0.00	1.75	0.00
time (sec)	N/A	0.194	0.005	1.161	0.000	0.076	0.000	0.000	0.212	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	0	11	0	0	21	0
N.S.	1	1.00	1.00	0.92	0.00	0.92	0.00	0.00	1.75	0.00
time (sec)	N/A	0.193	0.005	1.125	0.000	0.076	0.000	0.000	0.354	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	0	11	10	0	23	0
N.S.	1	1.00	1.00	0.92	0.00	0.92	0.83	0.00	1.92	0.00
time (sec)	N/A	0.187	0.004	1.142	0.000	0.078	0.437	0.000	0.297	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	0	9	5	0	21	0
N.S.	1	1.00	1.00	1.12	0.00	1.12	0.62	0.00	2.62	0.00
time (sec)	N/A	0.189	0.004	0.236	0.000	0.080	0.104	0.000	0.321	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	0	11	10	0	17	0
N.S.	1	1.00	1.00	0.92	0.00	0.92	0.83	0.00	1.42	0.00
time (sec)	N/A	0.191	0.005	0.256	0.000	0.074	0.445	0.000	0.212	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	0	11	10	0	21	0
N.S.	1	1.00	1.00	0.92	0.00	0.92	0.83	0.00	1.75	0.00
time (sec)	N/A	0.187	0.005	0.180	0.000	0.080	0.715	0.000	0.261	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	0	11	10	0	21	0
N.S.	1	1.00	1.00	0.92	0.00	0.92	0.83	0.00	1.75	0.00
time (sec)	N/A	0.189	0.006	0.210	0.000	0.072	1.257	0.000	0.296	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	0	0	0	0	21	0
N.S.	1	1.00	1.00	1.00	0.00	0.00	0.00	0.00	1.31	0.00
time (sec)	N/A	0.192	0.004	1.168	0.000	0.000	0.000	0.000	0.315	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	0	0	0	0	21	0
N.S.	1	1.00	1.00	1.00	0.00	0.00	0.00	0.00	1.31	0.00
time (sec)	N/A	0.191	0.004	1.148	0.000	0.000	0.000	0.000	0.264	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	0	0	0	0	23	0
N.S.	1	1.00	1.00	1.00	0.00	0.00	0.00	0.00	1.44	0.00
time (sec)	N/A	0.189	0.004	1.207	0.000	0.000	0.000	0.000	0.221	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	0	0	0	0	19	0
N.S.	1	1.00	1.00	0.92	0.00	0.00	0.00	0.00	1.58	0.00
time (sec)	N/A	0.181	0.003	0.231	0.000	0.000	0.000	0.000	0.251	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	14	0	0	0	0	15	0
N.S.	1	1.00	1.00	1.56	0.00	0.00	0.00	0.00	1.67	0.00
time (sec)	N/A	0.183	0.004	0.073	0.000	0.000	0.000	0.000	0.343	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	0	0	0	0	19	0
N.S.	1	1.00	1.00	1.00	0.00	0.00	0.00	0.00	1.19	0.00
time (sec)	N/A	0.191	0.005	0.216	0.000	0.000	0.000	0.000	0.300	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	0	0	0	0	21	0
N.S.	1	1.00	1.00	1.00	0.00	0.00	0.00	0.00	1.31	0.00
time (sec)	N/A	0.194	0.005	0.190	0.000	0.000	0.000	0.000	0.183	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	0	0	0	0	21	0
N.S.	1	1.00	1.00	1.00	0.00	0.00	0.00	0.00	1.31	0.00
time (sec)	N/A	0.194	0.005	0.279	0.000	0.000	0.000	0.000	0.240	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0	122	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.54	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.278	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	25	0	0	27	0	39	0
N.S.	1	1.00	1.00	1.79	0.00	0.00	1.93	0.00	2.79	0.00
time (sec)	N/A	0.206	0.007	0.243	0.000	0.000	0.888	0.000	0.313	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	31	0	0	0	0	1281	0
N.S.	1	1.00	1.00	1.72	0.00	0.00	0.00	0.00	71.17	0.00
time (sec)	N/A	0.214	0.008	0.237	0.000	0.000	0.000	0.000	0.623	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	0	0	0	0	0	0	0	1043	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.54	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.388	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	98	63	85	0	126	131	0	60	0
N.S.	1	1.08	0.69	0.93	0.00	1.38	1.44	0.00	0.66	0.00
time (sec)	N/A	0.675	0.041	0.123	0.000	0.079	0.684	0.000	0.304	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	78	45	70	0	112	105	0	50	0
N.S.	1	1.07	0.62	0.96	0.00	1.53	1.44	0.00	0.68	0.00
time (sec)	N/A	0.538	0.008	0.091	0.000	0.087	0.419	0.000	0.275	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	58	45	55	0	96	80	0	40	0
N.S.	1	1.05	0.82	1.00	0.00	1.75	1.45	0.00	0.73	0.00
time (sec)	N/A	0.421	0.008	0.076	0.000	0.087	0.545	0.000	0.297	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	35	39	38	76	48	0	30	0
N.S.	1	1.00	0.97	1.08	1.06	2.11	1.33	0.00	0.83	0.00
time (sec)	N/A	0.302	0.005	0.076	0.035	0.077	0.345	0.000	0.205	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	32	0	0	0	0	34	0
N.S.	1	1.00	0.90	1.10	0.00	0.00	0.00	0.00	1.17	0.00
time (sec)	N/A	0.253	0.010	0.076	0.000	0.000	0.000	0.000	0.343	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	35	33	0	0	0	0	10	0
N.S.	1	1.00	1.13	1.06	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.260	0.008	0.085	0.000	0.000	0.000	0.000	0.321	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	48	0	0	0	0	10	0
N.S.	1	1.00	0.91	0.87	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.348	0.011	0.089	0.000	0.000	0.000	0.000	0.320	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	80	67	63	0	0	0	0	10	0
N.S.	1	1.07	0.89	0.84	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.655	0.049	0.104	0.000	0.000	0.000	0.000	0.322	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	105	83	78	0	0	0	0	10	0
N.S.	1	1.11	0.87	0.82	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.798	0.058	0.117	0.000	0.000	0.000	0.000	0.200	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	141	101	158	0	0	0	0	24	0
N.S.	1	1.03	0.74	1.15	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.067	0.131	0.068	0.000	0.000	0.000	0.000	0.235	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	113	86	128	0	0	0	0	20	0
N.S.	1	1.03	0.78	1.16	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.838	0.134	0.042	0.000	0.000	0.000	0.000	0.294	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	86	68	98	0	0	0	0	12	0
N.S.	1	1.04	0.82	1.18	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.458	0.021	0.044	0.000	0.000	0.000	0.000	0.346	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	68	0	0	0	0	24	0
N.S.	1	1.00	0.93	1.17	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.291	0.017	0.043	0.000	0.000	0.000	0.000	0.208	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	52	102	0	0	0	0	24	0
N.S.	1	1.00	0.91	1.79	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.291	0.097	0.046	0.000	0.000	0.000	0.000	0.373	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	89	68	160	0	0	0	0	24	0
N.S.	1	1.01	0.77	1.82	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.415	0.111	0.043	0.000	0.000	0.000	0.000	0.342	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	121	94	222	0	0	0	0	24	0
N.S.	1	1.05	0.82	1.93	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.550	0.132	0.045	0.000	0.000	0.000	0.000	0.353	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	149	102	154	0	0	0	0	25	0
N.S.	1	1.05	0.72	1.08	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.689	0.025	0.072	0.000	0.000	0.000	0.000	0.292	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	119	87	123	0	0	0	0	22	0
N.S.	1	1.04	0.76	1.08	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.537	0.020	0.049	0.000	0.000	0.000	0.000	0.328	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	88	69	92	0	0	0	0	13	0
N.S.	1	1.02	0.80	1.07	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.399	0.019	0.043	0.000	0.000	0.000	0.000	0.348	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	55	61	0	0	0	0	26	0
N.S.	1	1.00	0.92	1.02	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.290	0.017	0.043	0.000	0.000	0.000	0.000	0.248	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	88	0	0	0	0	25	0
N.S.	1	1.00	0.92	1.49	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.295	0.018	0.041	0.000	0.000	0.000	0.000	0.216	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	92	69	146	0	0	0	0	26	0
N.S.	1	1.01	0.76	1.60	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.420	0.020	0.043	0.000	0.000	0.000	0.000	0.251	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	125	95	210	0	0	0	0	25	0
N.S.	1	1.05	0.80	1.76	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.556	0.034	0.048	0.000	0.000	0.000	0.000	0.265	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	53	0	0	0	0	0	14	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.293	0.047	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	318	210	419	0	0	0	0	12	0
N.S.	1	0.87	0.58	1.15	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.672	0.171	0.134	0.000	0.000	0.000	0.000	0.308	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	200	141	253	0	0	0	0	12	0
N.S.	1	0.88	0.62	1.11	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.476	0.073	0.131	0.000	0.000	0.000	0.000	0.333	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	108	73	125	0	143	0	0	10	0
N.S.	1	0.89	0.60	1.03	0.00	1.18	0.00	0.00	0.08	0.00
time (sec)	N/A	0.302	0.032	0.129	0.000	0.080	0.000	0.000	0.341	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	35	39	38	76	48	0	30	0
N.S.	1	1.00	0.97	1.08	1.06	2.11	1.33	0.00	0.83	0.00
time (sec)	N/A	0.487	0.001	0.082	0.028	0.082	0.354	0.000	0.332	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	13	13	8	13	24	12
N.S.	1	1.00	1.20	1.00	1.30	1.30	0.80	1.30	2.40	1.20
time (sec)	N/A	0.286	0.011	0.016	0.076	0.075	0.353	0.105	0.289	0.126

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	13	13	10	13	47	12
N.S.	1	1.00	1.20	1.00	1.30	1.30	1.00	1.30	4.70	1.20
time (sec)	N/A	0.276	0.012	0.017	0.083	0.079	0.400	0.097	0.333	0.118

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	456	398	250	530	0	0	0	0	14	0
N.S.	1	0.87	0.55	1.16	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	1.122	0.212	0.160	0.000	0.000	0.000	0.000	0.279	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	255	170	332	0	0	0	0	14	0
N.S.	1	0.87	0.58	1.14	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.863	0.093	0.153	0.000	0.000	0.000	0.000	0.273	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	144	94	172	0	178	0	0	12	0
N.S.	1	0.88	0.58	1.06	0.00	1.09	0.00	0.00	0.07	0.00
time (sec)	N/A	0.388	0.043	0.138	0.000	0.087	0.000	0.000	0.307	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	58	45	55	0	96	80	0	40	0
N.S.	1	1.05	0.82	1.00	0.00	1.75	1.45	0.00	0.73	0.00
time (sec)	N/A	0.438	0.003	0.080	0.000	0.088	0.431	0.000	0.312	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	15	15	10	15	24	14
N.S.	1	1.00	1.17	1.00	1.25	1.25	0.83	1.25	2.00	1.17
time (sec)	N/A	0.232	0.015	0.014	0.052	0.073	0.398	0.105	0.235	0.117

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	15	15	12	15	107	14
N.S.	1	1.00	1.17	1.00	1.25	1.25	1.00	1.25	8.92	1.17
time (sec)	N/A	0.202	0.019	0.020	0.051	0.076	0.397	0.106	0.323	0.116

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	568	518	271	787	0	0	0	0	16	0
N.S.	1	0.91	0.48	1.39	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	1.309	1.168	0.081	0.000	0.000	0.000	0.000	0.261	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	345	203	502	0	0	0	0	16	0
N.S.	1	0.92	0.54	1.33	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.835	0.416	0.059	0.000	0.000	0.000	0.000	0.307	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	197	137	274	0	0	0	0	14	0
N.S.	1	0.92	0.64	1.28	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.487	0.150	0.050	0.000	0.000	0.000	0.000	0.306	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	86	68	98	0	0	0	0	12	0
N.S.	1	1.04	0.82	1.18	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.394	0.013	0.039	0.000	0.000	0.000	0.000	0.182	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	17	17	14	17	16	16
N.S.	1	1.00	1.12	0.88	1.06	1.06	0.88	1.06	1.00	1.00
time (sec)	N/A	0.299	0.198	0.053	0.053	0.069	0.516	0.141	0.385	0.118

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	17	17	15	17	67	16
N.S.	1	1.00	1.12	0.88	1.06	1.06	0.94	1.06	4.19	1.00
time (sec)	N/A	0.301	0.195	0.028	0.053	0.067	0.764	0.145	0.339	0.121

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	411	220	597	0	0	0	0	27	0
N.S.	1	0.92	0.49	1.33	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.079	0.957	0.060	0.000	0.000	0.000	0.000	0.348	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	267	163	365	0	0	0	0	27	0
N.S.	1	0.92	0.56	1.26	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.689	0.351	0.049	0.000	0.000	0.000	0.000	0.319	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	148	105	188	0	0	0	0	25	0
N.S.	1	0.93	0.66	1.18	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.405	0.132	0.047	0.000	0.000	0.000	0.000	0.332	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	68	0	0	0	0	24	0
N.S.	1	1.00	0.93	1.17	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.287	0.011	0.042	0.000	0.000	0.000	0.000	0.237	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	17	29	15	17	27	16
N.S.	1	1.00	1.12	0.88	1.06	1.81	0.94	1.06	1.69	1.00
time (sec)	N/A	0.297	0.148	0.034	0.054	0.066	0.729	0.176	0.340	0.121

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	17	29	17	17	27	16
N.S.	1	1.00	1.12	0.88	1.06	1.81	1.06	1.06	1.69	1.00
time (sec)	N/A	0.299	0.141	0.027	0.051	0.066	1.159	0.176	0.230	0.117

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	425	221	575	0	0	0	0	29	0
N.S.	1	0.92	0.48	1.24	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.111	0.918	0.092	0.000	0.000	0.000	0.000	0.258	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	276	164	348	0	0	0	0	29	0
N.S.	1	0.92	0.55	1.16	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.929	0.363	0.082	0.000	0.000	0.000	0.000	0.305	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	153	106	170	0	0	0	0	27	0
N.S.	1	0.93	0.65	1.04	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.618	0.130	0.063	0.000	0.000	0.000	0.000	0.343	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	55	61	0	0	0	0	26	0
N.S.	1	1.00	0.92	1.02	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.483	0.009	0.044	0.000	0.000	0.000	0.000	0.306	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	15	18	31	17	18	29	17
N.S.	1	1.00	1.12	0.88	1.06	1.82	1.00	1.06	1.71	1.00
time (sec)	N/A	0.506	0.133	0.029	0.054	0.072	0.735	0.127	0.329	0.117

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	15	18	31	19	18	29	17
N.S.	1	1.00	1.12	0.88	1.06	1.82	1.12	1.06	1.71	1.00
time (sec)	N/A	0.504	0.127	0.026	0.060	0.064	1.080	0.123	0.347	0.116

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	231	133	489	0	0	0	0	20	0
N.S.	1	0.90	0.52	1.90	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.818	0.159	0.142	0.000	0.000	0.000	0.000	0.309	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	135	85	248	0	0	0	0	20	0
N.S.	1	0.91	0.57	1.66	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.443	0.088	0.139	0.000	0.000	0.000	0.000	0.332	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	68	42	103	0	55	0	0	18	0
N.S.	1	0.93	0.58	1.41	0.00	0.75	0.00	0.00	0.25	0.00
time (sec)	N/A	0.281	0.051	0.144	0.000	0.092	0.000	0.000	0.311	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	0	21	37	0	15	0
N.S.	1	1.00	1.00	1.05	0.00	1.05	1.85	0.00	0.75	0.00
time (sec)	N/A	0.191	0.029	0.085	0.000	0.076	0.549	0.000	0.332	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	19	18	14	19	18	18
N.S.	1	1.00	1.12	1.00	1.19	1.12	0.88	1.19	1.12	1.12
time (sec)	N/A	0.290	0.033	0.027	0.053	0.074	0.923	0.106	0.340	0.127

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	19	22	17	19	59	18
N.S.	1	1.00	1.12	1.00	1.19	1.38	1.06	1.19	3.69	1.12
time (sec)	N/A	0.295	0.066	0.034	0.051	0.078	0.842	0.107	0.313	0.127

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [23] had the largest ratio of [.833332999999999990]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	6	1.21	8	0.750
2	A	5	5	1.17	8	0.625
3	A	4	4	1.11	6	0.667
4	A	3	3	1.00	4	0.750
5	A	2	2	1.00	8	0.250
6	A	2	2	1.00	8	0.250
7	A	2	2	1.00	8	0.250
8	A	3	3	1.00	8	0.375
9	A	4	4	1.08	8	0.500
10	A	5	5	1.17	8	0.625
11	A	6	6	1.21	10	0.600
12	A	5	5	1.09	8	0.625
13	A	4	4	1.11	6	0.667
14	A	2	2	1.00	10	0.200
15	A	2	2	1.00	10	0.200
16	A	2	2	1.00	10	0.200
17	A	2	2	1.00	10	0.200
18	A	3	3	1.00	10	0.300
19	A	4	4	1.11	10	0.400
20	A	5	5	1.17	10	0.500
21	A	7	7	1.18	10	0.700

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	6	6	1.18	8	0.750
23	A	5	5	1.14	6	0.833
24	A	2	2	1.00	10	0.200
25	A	3	3	1.09	10	0.300
26	A	2	2	1.00	10	0.200
27	A	2	2	1.00	10	0.200
28	A	2	2	1.00	10	0.200
29	A	3	3	1.02	10	0.300
30	A	4	4	1.06	10	0.400
31	A	5	5	1.16	10	0.500
32	A	5	5	1.23	10	0.500
33	A	4	4	1.19	10	0.400
34	A	3	3	1.12	10	0.300
35	A	2	2	1.00	8	0.250
36	A	2	2	1.00	6	0.333
37	A	2	2	1.00	10	0.200
38	A	3	3	1.03	10	0.300
39	A	4	4	1.00	10	0.400
40	A	5	5	1.10	10	0.500
41	A	5	5	1.23	10	0.500
42	A	4	4	1.19	10	0.400
43	A	3	3	1.12	10	0.300
44	A	2	2	1.00	10	0.200
45	A	2	2	1.00	8	0.250
46	A	2	2	1.00	6	0.333
47	A	2	2	1.00	10	0.200
48	A	4	4	1.07	10	0.400
49	A	5	5	1.08	10	0.500
50	A	5	5	1.23	10	0.500
51	A	4	4	1.19	10	0.400
52	A	3	3	1.12	10	0.300
53	A	2	2	1.00	10	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	10	0.200
55	A	2	2	1.00	8	0.250
56	A	3	3	1.00	6	0.500
57	A	2	2	1.00	10	0.200
58	A	5	5	1.19	10	0.500
59	A	6	6	1.14	10	0.600
60	A	6	6	1.13	14	0.429
61	A	5	5	1.11	14	0.357
62	A	4	4	1.09	12	0.333
63	A	3	3	1.05	10	0.300
64	A	2	2	1.00	14	0.143
65	A	2	2	1.00	14	0.143
66	A	3	3	1.01	14	0.214
67	A	4	4	1.04	14	0.286
68	A	5	5	1.10	14	0.357
69	A	6	6	1.10	14	0.429
70	A	6	6	1.12	14	0.429
71	A	5	5	1.12	14	0.357
72	A	4	4	1.09	14	0.286
73	A	3	3	1.06	12	0.250
74	A	2	2	1.00	10	0.200
75	A	2	2	1.00	14	0.143
76	A	3	3	1.01	14	0.214
77	A	4	4	1.06	14	0.286
78	A	5	5	1.08	14	0.357
79	A	6	6	1.12	14	0.429
80	A	2	2	0.96	8	0.250
81	A	2	2	0.97	10	0.200
82	A	3	3	0.96	10	0.300
83	A	2	2	0.96	10	0.200
84	A	2	2	0.97	14	0.143
85	A	2	2	0.97	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.00	12	0.167
87	A	2	2	1.00	10	0.200
88	A	2	2	1.00	12	0.167
89	A	2	2	1.00	12	0.167
90	A	2	2	1.00	12	0.167
91	A	2	2	0.97	12	0.167
92	A	7	6	1.23	10	0.600
93	A	6	5	1.20	10	0.500
94	A	5	4	1.00	8	0.500
95	A	2	2	1.00	10	0.200
96	A	2	2	1.00	10	0.200
97	A	2	2	1.00	10	0.200
98	A	3	3	1.11	10	0.300
99	F	0	0	N/A	0.000	N/A
100	F	0	0	N/A	0.000	N/A
101	F	0	0	N/A	0.000	N/A
102	F	0	0	N/A	0.000	N/A
103	F	0	0	N/A	0.000	N/A
104	A	7	6	1.15	12	0.500
105	A	6	5	1.27	10	0.500
106	A	2	2	1.00	12	0.167
107	A	2	2	1.00	12	0.167
108	A	2	2	1.00	12	0.167
109	A	2	2	1.00	12	0.167
110	A	3	3	1.00	12	0.250
111	F	0	0	N/A	0.000	N/A
112	F	0	0	N/A	0.000	N/A
113	F	0	0	N/A	0.000	N/A
114	F	0	0	N/A	0.000	N/A
115	F	0	0	N/A	0.000	N/A
116	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
117	A	7	6	1.21	10	0.600
118	A	2	2	1.00	12	0.167
119	A	3	3	1.07	12	0.250
120	A	2	2	1.00	12	0.167
121	A	2	2	1.00	12	0.167
122	A	2	2	1.00	12	0.167
123	A	3	3	1.11	12	0.250
124	F	0	0	N/A	0.000	N/A
125	F	0	0	N/A	0.000	N/A
126	F	0	0	N/A	0.000	N/A
127	F	0	0	N/A	0.000	N/A
128	F	0	0	N/A	0.000	N/A
129	F	0	0	N/A	0.000	N/A
130	A	4	4	1.16	12	0.333
131	A	3	3	1.11	12	0.250
132	A	2	2	1.00	12	0.167
133	A	2	2	1.00	10	0.200
134	A	2	2	1.00	12	0.167
135	A	5	4	1.03	12	0.333
136	A	6	5	1.00	12	0.417
137	F	0	0	N/A	0.000	N/A
138	F	0	0	N/A	0.000	N/A
139	F	0	0	N/A	0.000	N/A
140	F	0	0	N/A	0.000	N/A
141	F	0	0	N/A	0.000	N/A
142	A	3	3	1.11	12	0.250
143	A	2	2	1.00	12	0.167
144	A	2	2	1.00	12	0.167
145	A	2	2	1.00	10	0.200
146	A	2	2	1.00	12	0.167
147	A	6	5	1.06	12	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
148	F	0	0	N/A	0.000	N/A
149	F	0	0	N/A	0.000	N/A
150	F	0	0	N/A	0.000	N/A
151	F	0	0	N/A	0.000	N/A
152	F	0	0	N/A	0.000	N/A
153	F	0	0	N/A	0.000	N/A
154	A	5	5	1.10	16	0.312
155	A	4	4	1.08	16	0.250
156	A	3	3	1.07	14	0.214
157	A	2	2	1.00	16	0.125
158	A	2	2	1.00	16	0.125
159	A	3	3	1.01	16	0.188
160	A	4	4	1.04	16	0.250
161	A	5	5	1.12	16	0.312
162	A	4	4	1.11	16	0.250
163	A	3	3	1.08	16	0.188
164	A	2	2	1.00	12	0.167
165	F	0	0	N/A	0.000	N/A
166	F	0	0	N/A	0.000	N/A
167	A	4	4	1.08	16	0.250
168	A	3	3	1.06	16	0.188
169	A	2	2	1.00	14	0.143
170	A	2	2	1.00	16	0.125
171	A	3	3	1.01	16	0.188
172	A	4	4	1.06	16	0.250
173	A	5	5	1.07	16	0.312
174	A	4	4	1.11	16	0.250
175	A	3	3	1.08	16	0.188
176	A	2	2	1.00	16	0.125
177	F	0	0	N/A	0.000	N/A
178	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
179	F	0	0	N/A	0.000	N/A
180	A	5	4	1.13	14	0.286
181	A	4	3	0.98	12	0.250
182	A	2	2	1.00	14	0.143
183	A	5	4	0.88	14	0.286
184	F	0	0	N/A	0.000	N/A
185	F	0	0	N/A	0.000	N/A
186	F	0	0	N/A	0.000	N/A
187	A	5	5	1.13	10	0.500
188	A	4	4	1.05	10	0.400
189	A	3	3	1.02	10	0.300
190	A	2	2	1.00	8	0.250
191	A	4	3	1.00	6	0.500
192	A	2	2	1.00	10	0.200
193	A	5	4	1.00	10	0.400
194	A	6	5	1.10	10	0.500
195	A	7	6	1.15	10	0.600
196	A	8	7	1.19	10	0.700
197	A	4	4	1.08	12	0.333
198	A	3	3	1.00	12	0.250
199	A	2	2	1.00	12	0.167
200	A	2	2	1.00	10	0.200
201	A	2	2	1.00	8	0.250
202	A	2	2	1.00	12	0.167
203	A	6	5	1.12	12	0.417
204	A	7	6	1.08	12	0.500
205	A	8	7	1.19	12	0.583
206	A	9	8	1.21	12	0.667
207	A	5	5	1.12	14	0.357
208	A	4	4	1.05	14	0.286
209	A	3	3	1.02	12	0.250
210	A	2	2	1.00	10	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
211	A	2	2	1.00	14	0.143
212	A	3	3	1.04	14	0.214
213	A	4	4	1.12	14	0.286
214	A	5	5	1.15	14	0.357
215	A	6	6	1.14	14	0.429
216	A	5	5	1.09	14	0.357
217	A	4	4	1.07	12	0.333
218	A	5	4	1.02	10	0.400
219	A	2	2	1.00	14	0.143
220	A	2	2	1.00	14	0.143
221	A	3	3	1.07	14	0.214
222	A	4	4	1.12	14	0.286
223	A	4	3	1.00	14	0.214
224	A	4	3	1.00	12	0.250
225	A	2	2	1.00	14	0.143
226	A	4	3	1.00	14	0.214
227	A	4	3	1.00	14	0.214
228	A	2	2	1.00	10	0.200
229	A	2	2	1.00	10	0.200
230	A	2	2	1.00	10	0.200
231	A	2	2	1.00	8	0.250
232	A	2	2	1.00	10	0.200
233	A	2	2	1.00	10	0.200
234	A	2	2	1.00	10	0.200
235	A	2	2	1.00	10	0.200
236	A	2	2	1.00	10	0.200
237	A	2	2	1.00	10	0.200
238	A	2	2	1.00	8	0.250
239	A	2	2	1.00	6	0.333
240	A	2	2	1.00	10	0.200
241	A	2	2	1.00	10	0.200
242	A	2	2	1.00	10	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
243	F	0	0	N/A	0.000	N/A
244	F	0	0	N/A	0.000	N/A
245	F	0	0	N/A	0.000	N/A
246	F	0	0	N/A	0.000	N/A
247	F	0	0	N/A	0.000	N/A
248	A	2	2	1.00	16	0.125
249	A	2	2	1.00	16	0.125
250	A	2	2	1.00	16	0.125
251	A	2	2	1.00	16	0.125
252	A	2	2	1.00	16	0.125
253	A	5	5	1.05	20	0.250
254	A	4	4	1.04	20	0.200
255	A	3	3	1.04	20	0.150
256	A	2	2	1.00	20	0.100
257	A	2	2	1.00	20	0.100
258	A	3	3	1.01	20	0.150
259	A	4	4	1.05	20	0.200
260	A	5	5	1.08	20	0.250
261	A	5	5	1.09	20	0.250
262	A	4	4	1.07	20	0.200
263	A	3	3	1.05	20	0.150
264	A	2	2	1.00	20	0.100
265	A	2	2	1.00	20	0.100
266	A	3	3	1.01	20	0.150
267	A	4	4	1.05	20	0.200
268	A	5	5	1.07	20	0.250
269	A	5	5	1.03	18	0.278
270	A	4	4	1.03	18	0.222
271	A	3	3	1.04	18	0.167
272	A	2	2	1.00	18	0.111
273	A	2	2	1.00	18	0.111
274	A	3	3	1.01	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
275	A	4	4	1.05	18	0.222
276	A	5	5	1.08	18	0.278
277	A	5	5	1.09	20	0.250
278	A	4	4	1.07	20	0.200
279	A	3	3	1.05	20	0.150
280	A	2	2	1.00	20	0.100
281	A	2	2	1.00	20	0.100
282	A	3	3	1.01	20	0.150
283	A	4	4	1.05	20	0.200
284	A	5	5	1.07	20	0.250
285	F	0	0	N/A	0.000	N/A
286	F	0	0	N/A	0.000	N/A
287	A	2	2	1.00	14	0.143
288	F	0	0	N/A	0.000	N/A
289	F	0	0	N/A	0.000	N/A
290	A	2	2	1.00	14	0.143
291	A	2	2	1.00	14	0.143
292	F	0	0	N/A	0.000	N/A
293	F	0	0	N/A	0.000	N/A
294	F	0	0	N/A	0.000	N/A
295	A	2	2	0.98	22	0.091
296	A	3	3	1.09	22	0.136
297	A	4	4	1.11	22	0.182
298	A	4	4	1.19	12	0.333
299	A	3	3	1.12	12	0.250
300	A	2	2	1.00	10	0.200
301	A	1	1	1.00	8	0.125
302	A	1	1	1.00	12	0.083
303	A	2	2	1.00	12	0.167
304	A	3	3	1.00	12	0.250
305	A	4	4	1.02	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	5	4	1.20	14	0.286
307	A	4	3	1.13	14	0.214
308	A	3	2	1.00	12	0.167
309	A	1	1	1.00	14	0.071
310	A	4	3	1.00	14	0.214
311	A	5	4	1.06	14	0.286
312	F	0	0	N/A	0.000	N/A
313	F	0	0	N/A	0.000	N/A
314	F	0	0	N/A	0.000	N/A
315	F	0	0	N/A	0.000	N/A
316	F	0	0	N/A	0.000	N/A
317	A	6	5	1.02	14	0.357
318	A	5	4	1.00	12	0.333
319	A	4	3	1.00	10	0.300
320	A	1	1	1.00	14	0.071
321	A	3	2	1.00	14	0.143
322	A	4	3	1.00	14	0.214
323	A	5	4	1.11	14	0.286
324	A	7	6	1.10	14	0.429
325	A	6	5	1.06	14	0.357
326	A	5	4	0.77	12	0.333
327	A	1	1	1.00	14	0.071
328	A	4	3	1.00	14	0.214
329	A	5	4	1.13	14	0.286
330	A	6	5	1.20	14	0.357
331	F	0	0	N/A	0.000	N/A
332	F	0	0	N/A	0.000	N/A
333	F	0	0	N/A	0.000	N/A
334	F	0	0	N/A	0.000	N/A
335	F	0	0	N/A	0.000	N/A
336	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
337	F	0	0	N/A	0.000	N/A
338	A	1	1	0.96	14	0.071
339	F	0	0	N/A	0.000	N/A
340	F	0	0	N/A	0.000	N/A
341	F	0	0	N/A	0.000	N/A
342	A	1	1	1.00	23	0.043
343	A	1	1	1.00	23	0.043
344	A	1	1	1.00	23	0.043
345	A	1	1	1.00	19	0.053
346	A	1	1	1.00	23	0.043
347	A	1	1	1.00	23	0.043
348	A	1	1	1.00	23	0.043
349	A	1	1	1.00	23	0.043
350	A	1	1	1.00	23	0.043
351	A	1	1	1.00	23	0.043
352	A	1	1	1.00	17	0.059
353	A	1	1	1.00	15	0.067
354	A	1	1	1.00	23	0.043
355	A	1	1	1.00	23	0.043
356	A	1	1	1.00	23	0.043
357	F	0	0	N/A	0.000	N/A
358	A	1	1	1.00	25	0.040
359	A	1	1	1.00	31	0.032
360	F	0	0	N/A	0.000	N/A
361	A	6	6	1.08	8	0.750
362	A	5	5	1.07	8	0.625
363	A	4	4	1.05	8	0.500
364	A	3	3	1.00	6	0.500
365	A	2	2	1.00	8	0.250
366	A	2	2	1.00	8	0.250
367	A	3	3	1.00	8	0.375
368	A	4	4	1.07	8	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
369	A	5	5	1.11	8	0.625
370	A	5	5	1.03	12	0.417
371	A	4	4	1.03	12	0.333
372	A	3	3	1.04	12	0.250
373	A	2	2	1.00	12	0.167
374	A	2	2	1.00	12	0.167
375	A	3	3	1.01	12	0.250
376	A	4	4	1.05	12	0.333
377	A	5	5	1.05	13	0.385
378	A	4	4	1.04	13	0.308
379	A	3	3	1.02	13	0.231
380	A	2	2	1.00	13	0.154
381	A	2	2	1.00	13	0.154
382	A	3	3	1.01	13	0.231
383	A	4	4	1.05	13	0.308
384	A	2	2	1.00	10	0.200
385	A	3	2	0.87	10	0.200
386	A	3	2	0.88	10	0.200
387	A	3	2	0.89	8	0.250
388	A	3	3	1.00	6	0.500
389	N/A	1	0	1.00	10	0.000
390	N/A	1	0	1.00	10	0.000
391	A	3	2	0.87	12	0.167
392	A	3	2	0.87	12	0.167
393	A	3	2	0.88	10	0.200
394	A	4	4	1.05	8	0.500
395	N/A	1	0	1.00	12	0.000
396	N/A	1	0	1.00	12	0.000
397	A	3	2	0.91	16	0.125
398	A	3	2	0.92	16	0.125
399	A	3	2	0.92	14	0.143
400	A	3	3	1.04	12	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
401	N/A	2	0	1.00	16	0.000
402	N/A	2	0	1.00	16	0.000
403	A	3	2	0.92	16	0.125
404	A	3	2	0.92	16	0.125
405	A	3	2	0.93	14	0.143
406	A	2	2	1.00	12	0.167
407	N/A	2	0	1.00	16	0.000
408	N/A	2	0	1.00	16	0.000
409	A	3	2	0.92	17	0.118
410	A	3	2	0.92	17	0.118
411	A	3	2	0.93	15	0.133
412	A	2	2	1.00	13	0.154
413	N/A	2	0	1.00	17	0.000
414	N/A	2	0	1.00	17	0.000
415	A	3	2	0.90	16	0.125
416	A	3	2	0.91	16	0.125
417	A	3	2	0.93	14	0.143
418	A	1	1	1.00	12	0.083
419	N/A	3	0	1.00	16	0.000
420	N/A	3	0	1.00	16	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3 W(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 71

$$\int x^3 W(ax)\,dx = -\frac{x^4}{16}-\frac{3x^4}{512W(ax)^4}+\frac{3x^4}{128W(ax)^3}-\frac{3x^4}{64W(ax)^2}+\frac{x^4}{16W(ax)}+\frac{1}{4}x^4W(ax)$$

output
```
-1/16*x^4-3/512*x^4/LambertW(a*x)^4+3/128*x^4/LambertW(a*x)^3-3/64*x^4/Lam
bertW(a*x)^2+1/16*x^4/LambertW(a*x)+1/4*x^4*LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int x^3 W(ax)\,dx = -\frac{x^4}{16}-\frac{3x^4}{512W(ax)^4}+\frac{3x^4}{128W(ax)^3}-\frac{3x^4}{64W(ax)^2}+\frac{x^4}{16W(ax)}+\frac{1}{4}x^4W(ax)$$

input
```
Integrate[x^3*ProductLog[a*x],x]
```

output
```
-1/16*x^4 - (3*x^4)/(512*ProductLog[a*x]^4) + (3*x^4)/(128*ProductLog[a*x]
^3) - (3*x^4)/(64*ProductLog[a*x]^2) + x^4/(16*ProductLog[a*x]) + (x^4*Pro
ductLog[a*x])/4
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {7172, 7205, 7194, 7205, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 W(ax)\,dx$$

$$\Big\downarrow \text{ 7172}$$

$$\frac{1}{4}x^4W(ax)-\frac{1}{4}\int \frac{x^3W(ax)}{W(ax)+1}dx$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{4}\left(\int \frac{x^3}{W(ax)+1}dx-\frac{x^4}{4}\right)+\frac{1}{4}x^4W(ax)$$

$$\Big\downarrow \text{ 7194}$$

$$\frac{1}{4}\left(-\frac{3}{4}\int \frac{x^3}{W(ax)(W(ax)+1)}dx+\frac{x^4}{4W(ax)}-\frac{x^4}{4}\right)+\frac{1}{4}x^4W(ax)$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{4}\left(-\frac{3}{4}\left(\frac{x^4}{4W(ax)^2}-\frac{1}{2}\int \frac{x^3}{W(ax)^2(W(ax)+1)}dx\right)+\frac{x^4}{4W(ax)}-\frac{x^4}{4}\right)+\frac{1}{4}x^4W(ax)$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{4}\left(-\frac{3}{4}\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{x^3}{W(ax)^3(W(ax)+1)}dx-\frac{x^4}{4W(ax)^3}\right)+\frac{x^4}{4W(ax)^2}\right)+\frac{x^4}{4W(ax)}-\frac{x^4}{4}\right)+\frac{1}{4}x^4W(ax)$$

$$\Big\downarrow \text{ 7201}$$

$$\frac{1}{4}x^4W(ax)+\frac{1}{4}\left(\frac{x^4}{4W(ax)}-\frac{3}{4}\left(\frac{x^4}{4W(ax)^2}+\frac{1}{2}\left(\frac{x^4}{16W(ax)^4}-\frac{x^4}{4W(ax)^3}\right)\right)-\frac{x^4}{4}\right)$$

input
```
Int[x^3*ProductLog[a*x],x]
```

output
```
(-1/4*x^4 - (3*((x^4/(16*ProductLog[a*x]^4) - x^4/(4*ProductLog[a*x]^3))/2
 + x^4/(4*ProductLog[a*x]^2)))/4 + x^4/(4*ProductLog[a*x]))/4 + (x^4*Produ
ctLog[a*x])/4
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.15

method	result	size
derivativedivides	$\frac{-\frac{a^4x^4}{16}+\frac{x^4a^4}{16\,\mathrm{LambertW}(xa)}-\frac{3x^4a^4}{64\,\mathrm{LambertW}(xa)^2}+\frac{3x^4a^4}{128\,\mathrm{LambertW}(xa)^3}-\frac{3x^4a^4}{512\,\mathrm{LambertW}(xa)^4}+\frac{\mathrm{LambertW}(xa)x^4a^4}{4}}{a^4}$	82
default	$\frac{-\frac{a^4x^4}{16}+\frac{x^4a^4}{16\,\mathrm{LambertW}(xa)}-\frac{3x^4a^4}{64\,\mathrm{LambertW}(xa)^2}+\frac{3x^4a^4}{128\,\mathrm{LambertW}(xa)^3}-\frac{3x^4a^4}{512\,\mathrm{LambertW}(xa)^4}+\frac{\mathrm{LambertW}(xa)x^4a^4}{4}}{a^4}$	82

input
```
int(x^3*LambertW(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(-1/16*a^4*x^4+1/16/LambertW(x*a)*x^4*a^4-3/64*x^4*a^4/LambertW(x*a)
^2+3/128/LambertW(x*a)^3*x^4*a^4-3/512*x^4*a^4/LambertW(x*a)^4+1/4*Lambert
W(x*a)*x^4*a^4)
```

Fricas [F]

$$\int x^3W(ax)\,dx=\int x^3\,\mathrm{W}(ax)\,dx$$

input
```
integrate(x^3*lambert_w(a*x),x, algorithm="fricas")
```

output
```
integral(x^3*lambert_w(a*x), x)
```

Sympy [F]

$$\int x^3W(ax)\,dx=\int x^3W(ax)\,dx$$

input
```
integrate(x**3*LambertW(a*x),x)
```

output
```
Integral(x**3*LambertW(a*x), x)
```

Maxima [F]

$$\int x^3 W(ax)\,dx = \int x^3\,\mathrm{W}(ax)\,dx$$

input
```
integrate(x^3*lambert_w(a*x),x, algorithm="maxima")
```

output
```
integrate(x^3*lambert_w(a*x), x)
```

Giac [F]

$$\int x^3 W(ax)\,dx = \int x^3\,\mathrm{W}(ax)\,dx$$

input
```
integrate(x^3*lambert_w(a*x),x, algorithm="giac")
```

output
```
integrate(x^3*lambert_w(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 W(ax)\,dx = \int x^3\,\mathrm{LambertW}(a\,x)\,dx$$

input
```
int(x^3*LambertW(a*x),x)
```

output
```
int(x^3*LambertW(a*x), x)
```

Reduce [F]

$$\int x^3W(ax)\,dx = \int lambert\_w(ax)\,x^3dx$$

input
```
int(x^3*Lambert_W(a*x),x)
```

output
```
int(lambert_w(a*x)*x**3,x)
```

3.2 $\int x^2 W(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 58

$$\int x^2 W(ax)\,dx = -\frac{x^3}{9} + \frac{2x^3}{81W(ax)^3} - \frac{2x^3}{27W(ax)^2} + \frac{x^3}{9W(ax)} + \frac{1}{3}x^3 W(ax)$$

output
```
-1/9*x^3+2/81*x^3/LambertW(a*x)^3-2/27*x^3/LambertW(a*x)^2+1/9*x^3/Lambert
W(a*x)+1/3*x^3*LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int x^2 W(ax)\,dx = -\frac{x^3}{9} + \frac{2x^3}{81W(ax)^3} - \frac{2x^3}{27W(ax)^2} + \frac{x^3}{9W(ax)} + \frac{1}{3}x^3 W(ax)$$

input
```
Integrate[x^2*ProductLog[a*x],x]
```

output
```
-1/9*x^3 + (2*x^3)/(81*ProductLog[a*x]^3) - (2*x^3)/(27*ProductLog[a*x]^2)
 + x^3/(9*ProductLog[a*x]) + (x^3*ProductLog[a*x])/3
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {7172, 7205, 7194, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 W(ax)\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{3}x^3 W(ax)-\frac{1}{3}\int \frac{x^2 W(ax)}{W(ax)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{3}\left(\int \frac{x^2}{W(ax)+1}dx-\frac{x^3}{3}\right)+\frac{1}{3}x^3 W(ax)$$

$$\Big\downarrow \text{7194}$$

$$\frac{1}{3}\left(-\frac{2}{3}\int \frac{x^2}{W(ax)(W(ax)+1)}dx+\frac{x^3}{3W(ax)}-\frac{x^3}{3}\right)+\frac{1}{3}x^3 W(ax)$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{3}\left(-\frac{2}{3}\left(\frac{x^3}{3W(ax)^2}-\frac{1}{3}\int \frac{x^2}{W(ax)^2(W(ax)+1)}dx\right)+\frac{x^3}{3W(ax)}-\frac{x^3}{3}\right)+\frac{1}{3}x^3 W(ax)$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{3}x^3 W(ax)+\frac{1}{3}\left(\frac{x^3}{3W(ax)}-\frac{2}{3}\left(\frac{x^3}{3W(ax)^2}-\frac{x^3}{9W(ax)^3}\right)-\frac{x^3}{3}\right)$$

input
```
Int[x^2*ProductLog[a*x],x]
```

output
```
(-1/3*x^3 - (2*(-1/9*x^3/ProductLog[a*x]^3 + x^3/(3*ProductLog[a*x]^2)))/3
 + x^3/(3*ProductLog[a*x]))/3 + (x^3*ProductLog[a*x])/3
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{-\frac{x^3a^3}{9}+\frac{x^3a^3}{9\,\mathrm{LambertW}(xa)}-\frac{2x^3a^3}{27\,\mathrm{LambertW}(xa)^2}+\frac{2x^3a^3}{81\,\mathrm{LambertW}(xa)^3}+\frac{\mathrm{LambertW}(xa)x^3a^3}{3}}{a^3}$	68
default	$\frac{-\frac{x^3a^3}{9}+\frac{x^3a^3}{9\,\mathrm{LambertW}(xa)}-\frac{2x^3a^3}{27\,\mathrm{LambertW}(xa)^2}+\frac{2x^3a^3}{81\,\mathrm{LambertW}(xa)^3}+\frac{\mathrm{LambertW}(xa)x^3a^3}{3}}{a^3}$	68

input
```
int(x^2*LambertW(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/9*x^3*a^3+1/9/LambertW(x*a)*x^3*a^3-2/27/LambertW(x*a)^2*x^3*a^3
+2/81*x^3*a^3/LambertW(x*a)^3+1/3*LambertW(x*a)*x^3*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int x^2W(ax)\,dx = \frac{27\,x^3\,\mathrm{W}(ax)^4 - 9\,x^3\,\mathrm{W}(ax)^3 + 9\,x^3\,\mathrm{W}(ax)^2 - 6\,x^3\,\mathrm{W}(ax) + 2\,x^3}{81\,\mathrm{W}(ax)^3}$$

input
```
integrate(x^2*lambert_w(a*x),x, algorithm="fricas")
```

output
```
1/81*(27*x^3*lambert_w(a*x)^4 - 9*x^3*lambert_w(a*x)^3 + 9*x^3*lambert_w(a
*x)^2 - 6*x^3*lambert_w(a*x) + 2*x^3)/lambert_w(a*x)^3
```

Sympy [F]

$$\int x^2W(ax)\,dx = \int x^2W(ax)\,dx$$

input
```
integrate(x**2*LambertW(a*x),x)
```

output
```
Integral(x**2*LambertW(a*x), x)
```

Maxima [F]

$$\int x^2W(ax)\,dx = \int x^2\,\mathrm{W}(ax)\,dx$$

input
```
integrate(x^2*lambert_w(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2*lambert_w(a*x), x)
```

Giac [F]

$$\int x^2 W(ax)\, dx = \int x^2\, \mathrm{W}(ax)\, dx$$

input
```
integrate(x^2*lambert_w(a*x),x, algorithm="giac")
```

output
```
integrate(x^2*lambert_w(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 W(ax)\, dx = \int x^2\, \mathrm{LambertW}(a\, x)\, dx$$

input
```
int(x^2*LambertW(a*x),x)
```

output
```
int(x^2*LambertW(a*x), x)
```

Reduce [F]

$$\int x^2 W(ax)\, dx = \int lambert\_w(ax)\, x^2 dx$$

input
```
int(x^2*Lambert_W(a*x),x)
```

output
```
int(lambert_w(a*x)*x**2,x)
```

3.3 $\int xW(ax)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 45

$$\int xW(ax)\,dx = -\frac{x^2}{4} - \frac{x^2}{8W(ax)^2} + \frac{x^2}{4W(ax)} + \frac{1}{2}x^2W(ax)$$

output
```
-1/4*x^2-1/8*x^2/LambertW(a*x)^2+1/4*x^2/LambertW(a*x)+1/2*x^2*LambertW(a*
x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int xW(ax)\,dx = -\frac{x^2}{4} - \frac{x^2}{8W(ax)^2} + \frac{x^2}{4W(ax)} + \frac{1}{2}x^2W(ax)$$

input
```
Integrate[x*ProductLog[a*x],x]
```

output
```
-1/4*x^2 - x^2/(8*ProductLog[a*x]^2) + x^2/(4*ProductLog[a*x]) + (x^2*Prod
uctLog[a*x])/2
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {7172, 7205, 7194, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xW(ax)\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{2}x^2W(ax)-\frac{1}{2}\int \frac{xW(ax)}{W(ax)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{2}\left(\int \frac{x}{W(ax)+1}dx-\frac{x^2}{2}\right)+\frac{1}{2}x^2W(ax)$$

$$\Big\downarrow \text{7194}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{x}{W(ax)(W(ax)+1)}dx+\frac{x^2}{2W(ax)}-\frac{x^2}{2}\right)+\frac{1}{2}x^2W(ax)$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{2}x^2W(ax)+\frac{1}{2}\left(\frac{x^2}{2W(ax)}-\frac{x^2}{4W(ax)^2}-\frac{x^2}{2}\right)$$

input
```
Int[x*ProductLog[a*x],x]
```

output
```
(-1/2*x^2 - x^2/(4*ProductLog[a*x]^2) + x^2/(2*ProductLog[a*x]))/2 + (x^2*
ProductLog[a*x])/2
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{-\frac{a^2x^2}{4}+\frac{x^2a^2}{4\,\mathrm{LambertW}(xa)}-\frac{x^2a^2}{8\,\mathrm{LambertW}(xa)^2}+\frac{\mathrm{LambertW}(xa)x^2a^2}{2}}{a^2}$	54
default	$\frac{-\frac{a^2x^2}{4}+\frac{x^2a^2}{4\,\mathrm{LambertW}(xa)}-\frac{x^2a^2}{8\,\mathrm{LambertW}(xa)^2}+\frac{\mathrm{LambertW}(xa)x^2a^2}{2}}{a^2}$	54

input
```
int(x*LambertW(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/4*a^2*x^2+1/4/LambertW(x*a)*x^2*a^2-1/8*x^2*a^2/LambertW(x*a)^2+
1/2*LambertW(x*a)*x^2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int xW(ax)\,dx = \frac{4\,x^2\,\mathrm{W}(ax)^3 - 2\,x^2\,\mathrm{W}(ax)^2 + 2\,x^2\,\mathrm{W}(ax) - x^2}{8\,\mathrm{W}(ax)^2}$$

input
```
integrate(x*lambert_w(a*x),x, algorithm="fricas")
```

output
```
1/8*(4*x^2*lambert_w(a*x)^3 - 2*x^2*lambert_w(a*x)^2 + 2*x^2*lambert_w(a*x
) - x^2)/lambert_w(a*x)^2
```

Sympy [F]

$$\int xW(ax)\,dx = \int xW(ax)\;dx$$

input
```
integrate(x*LambertW(a*x),x)
```

output
```
Integral(x*LambertW(a*x), x)
```

Maxima [F]

$$\int xW(ax)\,dx = \int x\,\mathrm{W}(ax)\,dx$$

input
```
integrate(x*lambert_w(a*x),x, algorithm="maxima")
```

output
```
integrate(x*lambert_w(a*x), x)
```

Giac [F]

$$\int xW(ax)\,dx = \int x\,\mathrm{W}(ax)\,dx$$

input
```
integrate(x*lambert_w(a*x),x, algorithm="giac")
```

output
```
integrate(x*lambert_w(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int xW(ax)\,dx = \int x\,\mathrm{LambertW}(a\,x)\;dx$$

input
```
int(x*LambertW(a*x),x)
```

output
```
int(x*LambertW(a*x), x)
```

Reduce [F]

$$\int xW(ax)\,dx = \int lambert\_w(ax)\,xdx$$

input
```
int(x*Lambert_W(a*x),x)
```

output
```
int(lambert_w(a*x)*x,x)
```

3.4 $\int W(ax)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 18

$$\int W(ax)\,dx = -x + \frac{x}{W(ax)} + xW(ax)$$

output
```
-x+x/LambertW(a*x)+x*LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int W(ax)\,dx = x\left(-1 + \frac{1}{W(ax)} + W(ax)\right)$$

input
```
Integrate[ProductLog[a*x],x]
```

output
```
x*(-1 + ProductLog[a*x]^(-1) + ProductLog[a*x])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {7167, 7177, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W(ax)\,dx$$

$$\Big\downarrow \text{7167}$$

$$xW(ax)-\int \frac{W(ax)}{W(ax)+1}dx$$

$$\Big\downarrow \text{7177}$$

$$\int \frac{1}{W(ax)+1}dx+xW(ax)-x$$

$$\Big\downarrow \text{7176}$$

$$xW(ax)+\frac{x}{W(ax)}-x$$

input
```
Int[ProductLog[a*x],x]
```

output
```
-x + x/ProductLog[a*x] + x*ProductLog[a*x]
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7177
```
Int[ProductLog[(a_.) + (b_.)*(x_)]/((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(
x_)]), x_Symbol] :> Simp[d*x, x] - Int[1/(d + d*ProductLog[a + b*x]), x] /;
 FreeQ[{a, b, d}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

method	result	size
derivativedivides	$\frac{-xa+\frac{xa}{\mathrm{LambertW}(xa)}+xa\,\mathrm{LambertW}(xa)}{a}$	26
default	$\frac{-xa+\frac{xa}{\mathrm{LambertW}(xa)}+xa\,\mathrm{LambertW}(xa)}{a}$	26
parallelrisch	$-\frac{-x\,\mathrm{LambertW}(xa)^{2}+x\,\mathrm{LambertW}(xa)-x}{\mathrm{LambertW}(xa)}$	28

input
```
int(LambertW(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-x*a+x*a/LambertW(x*a)+x*a*LambertW(x*a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int W(ax)\,dx = \frac{x\,\mathrm{W}(ax)^2 - x\,\mathrm{W}(ax) + x}{\mathrm{W}(ax)}$$

input
```
integrate(lambert_w(a*x),x, algorithm="fricas")
```

output
```
(x*lambert_w(a*x)^2 - x*lambert_w(a*x) + x)/lambert_w(a*x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int W(ax)\,dx = \begin{cases} xW(ax) - x + \frac{x}{W(ax)} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(LambertW(a*x),x)
```

output
```
Piecewise((x*LambertW(a*x) - x + x/LambertW(a*x), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int W(ax)\,dx = \frac{(\mathrm{W}(ax)^2 - \mathrm{W}(ax) + 1)x}{\mathrm{W}(ax)}$$

input
```
integrate(lambert_w(a*x),x, algorithm="maxima")
```

output
```
(lambert_w(a*x)^2 - lambert_w(a*x) + 1)*x/lambert_w(a*x)
```

Giac [F]

$$\int W(ax)\,dx = \int \mathrm{W}(ax)\,dx$$

input
```
integrate(lambert_w(a*x),x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int W(ax)\,dx = \int \mathrm{LambertW}(a\,x)\;dx$$

input
```
int(LambertW(a*x),x)
```

output
```
int(LambertW(a*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int W(ax)\,dx = \frac{e^{lambert\_w(ax)}\left(lambert\_w(ax)^2 - lambert\_w(ax) + 1\right)}{a}$$

input
```
int(Lambert_W(a*x),x)
```

output
```
(e**lambert_w(a*x)*(lambert_w(a*x)**2 - lambert_w(a*x) + 1))/a
```

3.5 $\int \frac{W(ax)}{x} dx$

Optimal result

Integrand size = 8, antiderivative size = 15

$$\int \frac{W(ax)}{x} dx = W(ax) + \frac{1}{2}W(ax)^2$$

output
```
LambertW(a*x)+1/2*LambertW(a*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)}{x} dx = W(ax) + \frac{1}{2}W(ax)^2$$

input
```
Integrate[ProductLog[a*x]/x,x]
```

output
```
ProductLog[a*x] + ProductLog[a*x]^2/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)}{x} dx$$

$$\Big\downarrow \; 7173$$

$$\int \frac{W(ax)^2}{x(W(ax)+1)}dx + W(ax)$$

$$\Big\downarrow \; 7200$$

$$\frac{1}{2}W(ax)^2 + W(ax)$$

input
```
Int[ProductLog[a*x]/x,x]
```

output
```
ProductLog[a*x] + ProductLog[a*x]^2/2
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\mathrm{LambertW}(xa)+\frac{\mathrm{LambertW}(xa)^2}{2}$	14
default	$\mathrm{LambertW}(xa)+\frac{\mathrm{LambertW}(xa)^2}{2}$	14

input
```
int(LambertW(x*a)/x,x,method=_RETURNVERBOSE)
```

output
```
LambertW(x*a)+1/2*LambertW(x*a)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{W(ax)}{x}\,dx = \frac{1}{2}\,\mathrm{W}(ax)^2 + \log(x) - \log(-\,\mathrm{W}(ax))$$

input
```
integrate(lambert_w(a*x)/x,x, algorithm="fricas")
```

output
```
1/2*lambert_w(a*x)^2 + log(x) - log(-lambert_w(a*x))
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{W(ax)}{x}\,dx = \frac{W^2(ax)}{2} + W(ax)$$

input
```
integrate(LambertW(a*x)/x,x)
```

output
```
LambertW(a*x)**2/2 + LambertW(a*x)
```

Maxima [F]

$$\int \frac{W(ax)}{x}\,dx = \int \frac{\mathrm{W}(ax)}{x}\,dx$$

input
```
integrate(lambert_w(a*x)/x,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)/x, x)
```

Giac [F]

$$\int \frac{W(ax)}{x}\,dx = \int \frac{\mathrm{W}(ax)}{x}\,dx$$

input
```
integrate(lambert_w(a*x)/x,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)}{x}\,dx = \int \frac{\mathrm{LambertW}(a\,x)}{x}\,dx$$

input
```
int(LambertW(a*x)/x,x)
```

output
```
int(LambertW(a*x)/x, x)
```

Reduce [F]

$$\int \frac{W(ax)}{x}\,dx = \int \frac{lambert\_w(ax)}{x}dx - 2\left(\int \frac{1}{x}dx\right) + 2\log(x)$$

input
```
int(Lambert_W(a*x)/x,x)
```

output
```
int(lambert_w(a*x)/x,x) - 2*int(1/x,x) + 2*log(x)
```

3.6 $\int \frac{W(ax)}{x^2} dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int \frac{W(ax)}{x^2} dx = a \operatorname{ExpIntegralEi}(-W(ax)) - \frac{W(ax)}{x}$$

output
```
a*Ei(-LambertW(a*x))-LambertW(a*x)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)}{x^2} dx = a \operatorname{ExpIntegralEi}(-W(ax)) - \frac{W(ax)}{x}$$

input
```
Integrate[ProductLog[a*x]/x^2,x]
```

output
```
a*ExpIntegralEi[-ProductLog[a*x]] - ProductLog[a*x]/x
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7172, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)}{x^2} dx$$

$$\Big\downarrow \; 7172$$

$$\int \frac{W(ax)}{x^2(W(ax)+1)} dx - \frac{W(ax)}{x}$$

$$\Big\downarrow \; 7202$$

$$a \,\text{ExpIntegralEi}(-W(ax)) - \frac{W(ax)}{x}$$

input
```
Int[ProductLog[a*x]/x^2,x]
```

output
```
a*ExpIntegralEi[-ProductLog[a*x]] - ProductLog[a*x]/x
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$a\left(-\frac{\mathrm{LambertW}(xa)}{xa}-\mathrm{expIntegral}_1\left(\mathrm{LambertW}\left(xa\right)\right)\right)$	24
default	$a\left(-\frac{\mathrm{LambertW}(xa)}{xa}-\mathrm{expIntegral}_1\left(\mathrm{LambertW}\left(xa\right)\right)\right)$	24

input
```
int(LambertW(x*a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/x/a*LambertW(x*a)-Ei(1,LambertW(x*a)))
```

Fricas [F]

$$\int \frac{W(ax)}{x^2} dx = \int \frac{\mathrm{W}(ax)}{x^2} dx$$

input
```
integrate(lambert_w(a*x)/x^2,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)/x^2, x)
```

Sympy [F]

$$\int \frac{W(ax)}{x^2} dx = \int \frac{W(ax)}{x^2} dx$$

input
```
integrate(LambertW(a*x)/x**2,x)
```

output
```
Integral(LambertW(a*x)/x**2, x)
```

Maxima [F]

$$\int \frac{W(ax)}{x^2}\,dx = \int \frac{\mathrm{W}(ax)}{x^2}\,dx$$

input
```
integrate(lambert_w(a*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)/x^2, x)
```

Giac [F]

$$\int \frac{W(ax)}{x^2}\,dx = \int \frac{\mathrm{W}(ax)}{x^2}\,dx$$

input
```
integrate(lambert_w(a*x)/x^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)}{x^2}\,dx = \int \frac{\mathrm{LambertW}(a\,x)}{x^2}\,dx$$

input
```
int(LambertW(a*x)/x^2,x)
```

output
```
int(LambertW(a*x)/x^2, x)
```

Reduce [F]

$$\int \frac{W(ax)}{x^2}\,dx = \frac{\left(\int \frac{1}{e^{lambert\_w(ax)}lambert\_w(ax)x+e^{lambert\_w(ax)}x}dx\right)ax - lambert\_w(ax)}{x}$$

input
```
int(Lambert_W(a*x)/x^2,x)
```

output
```
(int(1/(e**lambert_w(a*x)*lambert_w(a*x)*x + e**lambert_w(a*x)*x),x)*a*x -
 lambert_w(a*x))/x
```

3.7 $\int \frac{W(ax)}{x^3}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 22

$$\int \frac{W(ax)}{x^3}\,dx = -a^2\,\text{ExpIntegralEi}(-2W(ax)) - \frac{W(ax)}{x^2}$$

output
```
-a^2*Ei(-2*LambertW(a*x))-LambertW(a*x)/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)}{x^3}\,dx = -a^2\,\text{ExpIntegralEi}(-2W(ax)) - \frac{W(ax)}{x^2}$$

input
```
Integrate[ProductLog[a*x]/x^3,x]
```

output
```
-(a^2*ExpIntegralEi[-2*ProductLog[a*x]]) - ProductLog[a*x]/x^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7173, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)}{x^3}dx$$

$$\Big\downarrow \text{7173}$$

$$-\int \frac{W(ax)^2}{x^3(W(ax)+1)}dx-\frac{W(ax)}{x^2}$$

$$\Big\downarrow \text{7202}$$

$$a^2(-\text{ExpIntegralEi}(-2W(ax)))-\frac{W(ax)}{x^2}$$

input
```
Int[ProductLog[a*x]/x^3,x]
```

output
```
-(a^2*ExpIntegralEi[-2*ProductLog[a*x]]) - ProductLog[a*x]/x^2
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$a^2\left(-\frac{\mathrm{LambertW}(xa)}{x^2a^2}+\mathrm{expIntegral}_1\left(2\,\mathrm{LambertW}\left(xa\right)\right)\right)$	26
default	$a^2\left(-\frac{\mathrm{LambertW}(xa)}{x^2a^2}+\mathrm{expIntegral}_1\left(2\,\mathrm{LambertW}\left(xa\right)\right)\right)$	26

input
```
int(LambertW(x*a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-LambertW(x*a)/x^2/a^2+Ei(1,2*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{W(ax)}{x^3}\,dx=\int \frac{W(ax)}{x^3}\,dx$$

input
```
integrate(lambert_w(a*x)/x^3,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)/x^3, x)
```

Sympy [F]

$$\int \frac{W(ax)}{x^3}\,dx=\int \frac{W(ax)}{x^3}\,dx$$

input
```
integrate(LambertW(a*x)/x**3,x)
```

output
```
Integral(LambertW(a*x)/x**3, x)
```

Maxima [F]

$$\int \frac{W(ax)}{x^3}\,dx = \int \frac{\mathrm{W}(ax)}{x^3}\,dx$$

input
```
integrate(lambert_w(a*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)/x^3, x)
```

Giac [F]

$$\int \frac{W(ax)}{x^3}\,dx = \int \frac{\mathrm{W}(ax)}{x^3}\,dx$$

input
```
integrate(lambert_w(a*x)/x^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)}{x^3}\,dx = \int \frac{\mathrm{LambertW}(a\,x)}{x^3}\,dx$$

input
```
int(LambertW(a*x)/x^3,x)
```

output
```
int(LambertW(a*x)/x^3, x)
```

Reduce [F]

$$\int \frac{W(ax)}{x^3}\,dx = \frac{\left(\int \frac{1}{e^{lambert\_w(ax)}lambert\_w(ax)x^2+e^{lambert\_w(ax)}x^2}dx\right)a\,x^2 - lambert\_w(ax)}{2x^2}$$

input
```
int(Lambert_W(a*x)/x^3,x)
```

output
```
(int(1/(e**lambert_w(a*x)*lambert_w(a*x)*x**2 + e**lambert_w(a*x)*x**2),x)
*a*x**2 - lambert_w(a*x))/(2*x**2)
```

3.8 $\int \frac{W(ax)}{x^4} dx$

Optimal result

Integrand size = 8, antiderivative size = 39

$$\int \frac{W(ax)}{x^4} dx = \frac{3}{2}a^3 \text{ExpIntegralEi}(-3W(ax)) - \frac{W(ax)}{2x^3} + \frac{W(ax)^2}{2x^3}$$

output
```
3/2*a^3*Ei(-3*LambertW(a*x))-1/2*LambertW(a*x)/x^3+1/2*LambertW(a*x)^2/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)}{x^4} dx = \frac{3}{2}a^3 \text{ExpIntegralEi}(-3W(ax)) - \frac{W(ax)}{2x^3} + \frac{W(ax)^2}{2x^3}$$

input
```
Integrate[ProductLog[a*x]/x^4,x]
```

output
```
(3*a^3*ExpIntegralEi[-3*ProductLog[a*x]])/2 - ProductLog[a*x]/(2*x^3) + Pr
oductLog[a*x]^2/(2*x^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {7173, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)}{x^4} \, dx$$

$$\Big\downarrow \text{7173}$$

$$-\frac{1}{2}\int \frac{W(ax)^2}{x^4(W(ax)+1)}dx - \frac{W(ax)}{2x^3}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{2}\left(3\int \frac{W(ax)^3}{x^4(W(ax)+1)}dx + \frac{W(ax)^2}{x^3}\right) - \frac{W(ax)}{2x^3}$$

$$\Big\downarrow \text{7202}$$

$$\frac{1}{2}\left(3a^3\,\text{ExpIntegralEi}(-3W(ax)) + \frac{W(ax)^2}{x^3}\right) - \frac{W(ax)}{2x^3}$$

input
```
Int[ProductLog[a*x]/x^4,x]
```

output
```
-1/2*ProductLog[a*x]/x^3 + (3*a^3*ExpIntegralEi[-3*ProductLog[a*x]] + Prod
uctLog[a*x]^2/x^3)/2
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$a^{3}\left(-\frac{\operatorname{LambertW}\left(x a\right)}{2 x^{3} a^{3}}+\frac{\operatorname{LambertW}\left(x a\right)^{2}}{2 x^{3} a^{3}}-\frac{3 \,\operatorname{expIntegral}_{1}\left(3 \operatorname{LambertW}\left(x a\right)\right)}{2}\right)$	42
default	$a^{3}\left(-\frac{\operatorname{LambertW}\left(x a\right)}{2 x^{3} a^{3}}+\frac{\operatorname{LambertW}\left(x a\right)^{2}}{2 x^{3} a^{3}}-\frac{3 \,\operatorname{expIntegral}_{1}\left(3 \operatorname{LambertW}\left(x a\right)\right)}{2}\right)$	42

input
```
int(LambertW(x*a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/2*LambertW(x*a)/x^3/a^3+1/2*LambertW(x*a)^2/x^3/a^3-3/2*Ei(1,3*Lam
bertW(x*a)))
```

Fricas [F]

$$\int \frac{W(ax)}{x^4}\,dx=\int \frac{\mathrm{W}(ax)}{x^4}\,dx$$

input
```
integrate(lambert_w(a*x)/x^4,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)/x^4, x)
```

Sympy [F]

$$\int \frac{W(ax)}{x^4}\,dx = \int \frac{W(ax)}{x^4}\,dx$$

input
```
integrate(LambertW(a*x)/x**4,x)
```

output
```
Integral(LambertW(a*x)/x**4, x)
```

Maxima [F]

$$\int \frac{W(ax)}{x^4}\,dx = \int \frac{\mathrm{W}(ax)}{x^4}\,dx$$

input
```
integrate(lambert_w(a*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)/x^4, x)
```

Giac [F]

$$\int \frac{W(ax)}{x^4}\,dx = \int \frac{\mathrm{W}(ax)}{x^4}\,dx$$

input
```
integrate(lambert_w(a*x)/x^4,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)}{x^4}\,dx = \int \frac{\mathrm{LambertW}(a\,x)}{x^4}\,dx$$

input
```
int(LambertW(a*x)/x^4,x)
```

output
```
int(LambertW(a*x)/x^4, x)
```

Reduce [F]

$$\int \frac{W(ax)}{x^4}\,dx = \frac{\left(\int \frac{1}{e^{lambert\_w(ax)}lambert\_w(ax)x^3+e^{lambert\_w(ax)}x^3}dx\right)a\,x^3 - lambert\_w(ax)}{3x^3}$$

input
```
int(Lambert_W(a*x)/x^4,x)
```

output
```
(int(1/(e**lambert_w(a*x)*lambert_w(a*x)*x**3 + e**lambert_w(a*x)*x**3),x)
*a*x**3 - lambert_w(a*x))/(3*x**3)
```

3.9 $\int \frac{W(ax)}{x^5} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 52

$$\int \frac{W(ax)}{x^5} \, dx = -\frac{8}{3}a^4 \operatorname{ExpIntegralEi}(-4W(ax)) - \frac{W(ax)}{3x^4} + \frac{W(ax)^2}{6x^4} - \frac{2W(ax)^3}{3x^4}$$

output
```
-8/3*a^4*Ei(-4*LambertW(a*x))-1/3*LambertW(a*x)/x^4+1/6*LambertW(a*x)^2/x^
4-2/3*LambertW(a*x)^3/x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)}{x^5} \, dx = -\frac{8}{3}a^4 \operatorname{ExpIntegralEi}(-4W(ax)) - \frac{W(ax)}{3x^4} + \frac{W(ax)^2}{6x^4} - \frac{2W(ax)^3}{3x^4}$$

input
```
Integrate[ProductLog[a*x]/x^5,x]
```

output
```
(-8*a^4*ExpIntegralEi[-4*ProductLog[a*x]])/3 - ProductLog[a*x]/(3*x^4) + P
roductLog[a*x]^2/(6*x^4) - (2*ProductLog[a*x]^3)/(3*x^4)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7173, 7206, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)}{x^5} dx$$

$$\Big\downarrow \text{7173}$$

$$-\frac{1}{3} \int \frac{W(ax)^2}{x^5(W(ax)+1)} dx - \frac{W(ax)}{3x^4}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{3} \left(2 \int \frac{W(ax)^3}{x^5(W(ax)+1)} dx + \frac{W(ax)^2}{2x^4}\right) - \frac{W(ax)}{3x^4}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{3} \left(2 \left(-4 \int \frac{W(ax)^4}{x^5(W(ax)+1)} dx - \frac{W(ax)^3}{x^4}\right) + \frac{W(ax)^2}{2x^4}\right) - \frac{W(ax)}{3x^4}$$

$$\Big\downarrow \text{7202}$$

$$\frac{1}{3} \left(2 \left(-4a^4 \text{ExpIntegralEi}(-4W(ax)) - \frac{W(ax)^3}{x^4}\right) + \frac{W(ax)^2}{2x^4}\right) - \frac{W(ax)}{3x^4}$$

input
```
Int[ProductLog[a*x]/x^5,x]
```

output
```
-1/3*ProductLog[a*x]/x^4 + (ProductLog[a*x]^2/(2*x^4) + 2*(-4*a^4*ExpInteg
ralEi[-4*ProductLog[a*x]] - ProductLog[a*x]^3/x^4))/3
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$a^4\left(-\frac{\mathrm{LambertW}(xa)}{3x^4a^4}+\frac{\mathrm{LambertW}(xa)^2}{6x^4a^4}-\frac{2\,\mathrm{LambertW}(xa)^3}{3x^4a^4}+\frac{8\,\mathrm{expIntegral}_1(4\,\mathrm{LambertW}(xa))}{3}\right)$	56
default	$a^4\left(-\frac{\mathrm{LambertW}(xa)}{3x^4a^4}+\frac{\mathrm{LambertW}(xa)^2}{6x^4a^4}-\frac{2\,\mathrm{LambertW}(xa)^3}{3x^4a^4}+\frac{8\,\mathrm{expIntegral}_1(4\,\mathrm{LambertW}(xa))}{3}\right)$	56

input
```
int(LambertW(x*a)/x^5,x,method=_RETURNVERBOSE)
```

output
```
a^4*(-1/3*LambertW(x*a)/x^4/a^4+1/6*LambertW(x*a)^2/x^4/a^4-2/3*LambertW(x
*a)^3/x^4/a^4+8/3*Ei(1,4*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{W(ax)}{x^5} \, dx = \int \frac{\mathrm{W}(ax)}{x^5} \, dx$$

input
```
integrate(lambert_w(a*x)/x^5,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)/x^5, x)
```

Sympy [F]

$$\int \frac{W(ax)}{x^5} \, dx = \int \frac{W(ax)}{x^5} \, dx$$

input
```
integrate(LambertW(a*x)/x**5,x)
```

output
```
Integral(LambertW(a*x)/x**5, x)
```

Maxima [F]

$$\int \frac{W(ax)}{x^5} \, dx = \int \frac{\mathrm{W}(ax)}{x^5} \, dx$$

input
```
integrate(lambert_w(a*x)/x^5,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)/x^5, x)
```

Giac [F]

$$\int \frac{W(ax)}{x^5}\,dx = \int \frac{\mathrm{W}(ax)}{x^5}\,dx$$

input
```
integrate(lambert_w(a*x)/x^5,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)}{x^5}\,dx = \int \frac{\mathrm{LambertW}(a\,x)}{x^5}\,dx$$

input
```
int(LambertW(a*x)/x^5,x)
```

output
```
int(LambertW(a*x)/x^5, x)
```

Reduce [F]

$$\int \frac{W(ax)}{x^5}\,dx = \frac{\left(\int \frac{1}{e^{lambert\_w(ax)}lambert\_w(ax)x^4+e^{lambert\_w(ax)}x^4}dx\right)a\,x^4 - lambert\_w(ax)}{4x^4}$$

input
```
int(Lambert_W(a*x)/x^5,x)
```

output
```
(int(1/(e**lambert_w(a*x)*lambert_w(a*x)*x**4 + e**lambert_w(a*x)*x**4),x)
*a*x**4 - lambert_w(a*x))/(4*x**4)
```

3.10 $\int \frac{W(ax)}{x^6} dx$

Optimal result

Integrand size = 8, antiderivative size = 65

$$\int \frac{W(ax)}{x^6} dx = \frac{125}{24} a^5 \text{ExpIntegralEi}(-5W(ax)) - \frac{W(ax)}{4x^5} + \frac{W(ax)^2}{12x^5} - \frac{5W(ax)^3}{24x^5} + \frac{25W(ax)^4}{24x^5}$$

output
```
125/24*a^5*Ei(-5*LambertW(a*x))-1/4*LambertW(a*x)/x^5+1/12*LambertW(a*x)^2
/x^5-5/24*LambertW(a*x)^3/x^5+25/24*LambertW(a*x)^4/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)}{x^6} dx = \frac{125}{24} a^5 \text{ExpIntegralEi}(-5W(ax)) - \frac{W(ax)}{4x^5} + \frac{W(ax)^2}{12x^5} - \frac{5W(ax)^3}{24x^5} + \frac{25W(ax)^4}{24x^5}$$

input
```
Integrate[ProductLog[a*x]/x^6,x]
```

output
```
(125*a^5*ExpIntegralEi[-5*ProductLog[a*x]])/24 - ProductLog[a*x]/(4*x^5) +
 ProductLog[a*x]^2/(12*x^5) - (5*ProductLog[a*x]^3)/(24*x^5) + (25*Product
Log[a*x]^4)/(24*x^5)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {7173, 7206, 7206, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)}{x^6}dx$$

$$\Big\downarrow \text{ 7173}$$

$$-\frac{1}{4}\int \frac{W(ax)^2}{x^6(W(ax)+1)}dx-\frac{W(ax)}{4x^5}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{1}{4}\left(\frac{5}{3}\int \frac{W(ax)^3}{x^6(W(ax)+1)}dx+\frac{W(ax)^2}{3x^5}\right)-\frac{W(ax)}{4x^5}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{1}{4}\left(\frac{5}{3}\left(-\frac{5}{2}\int \frac{W(ax)^4}{x^6(W(ax)+1)}dx-\frac{W(ax)^3}{2x^5}\right)+\frac{W(ax)^2}{3x^5}\right)-\frac{W(ax)}{4x^5}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{1}{4}\left(\frac{5}{3}\left(-\frac{5}{2}\left(-5\int \frac{W(ax)^5}{x^6(W(ax)+1)}dx-\frac{W(ax)^4}{x^5}\right)-\frac{W(ax)^3}{2x^5}\right)+\frac{W(ax)^2}{3x^5}\right)-\frac{W(ax)}{4x^5}$$

$$\Big\downarrow \text{ 7202}$$

$$\frac{1}{4}\left(\frac{5}{3}\left(-\frac{5}{2}\left(-5a^5\,\text{ExpIntegralEi}(-5W(ax))-\frac{W(ax)^4}{x^5}\right)-\frac{W(ax)^3}{2x^5}\right)+\frac{W(ax)^2}{3x^5}\right)-\frac{W(ax)}{4x^5}$$

input
```
Int[ProductLog[a*x]/x^6,x]
```

output
```
-1/4*ProductLog[a*x]/x^5 + (ProductLog[a*x]^2/(3*x^5) + (5*(-1/2*ProductLo
g[a*x]^3/x^5 - (5*(-5*a^5*ExpIntegralEi[-5*ProductLog[a*x]] - ProductLog[a
*x]^4/x^5))/2))/3)/4
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

method	result
derivativedivides	$a^5\left(-\frac{\text{LambertW}(xa)}{4x^5a^5}+\frac{\text{LambertW}(xa)^2}{12x^5a^5}-\frac{5\,\text{LambertW}(xa)^3}{24x^5a^5}+\frac{25\,\text{LambertW}(xa)^4}{24x^5a^5}-\frac{125\ \text{expIntegral}_1(5\,\text{I}}{24}\right.$
default	$a^5\left(-\frac{\text{LambertW}(xa)}{4x^5a^5}+\frac{\text{LambertW}(xa)^2}{12x^5a^5}-\frac{5\,\text{LambertW}(xa)^3}{24x^5a^5}+\frac{25\,\text{LambertW}(xa)^4}{24x^5a^5}-\frac{125\ \text{expIntegral}_1(5\,\text{I}}{24}\right.$

input
```
int(LambertW(x*a)/x^6,x,method=_RETURNVERBOSE)
```

output
```
a^5*(-1/4*LambertW(x*a)/x^5/a^5+1/12*LambertW(x*a)^2/x^5/a^5-5/24*LambertW
(x*a)^3/x^5/a^5+25/24*LambertW(x*a)^4/x^5/a^5-125/24*Ei(1,5*LambertW(x*a))
)
```

Fricas [F]

$$\int \frac{W(ax)}{x^6}\,dx = \int \frac{\mathrm{W}(ax)}{x^6}\,dx$$

input
```
integrate(lambert_w(a*x)/x^6,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)/x^6, x)
```

Sympy [F]

$$\int \frac{W(ax)}{x^6}\,dx = \int \frac{W(ax)}{x^6}\,dx$$

input
```
integrate(LambertW(a*x)/x**6,x)
```

output
```
Integral(LambertW(a*x)/x**6, x)
```

Maxima [F]

$$\int \frac{W(ax)}{x^6}\,dx = \int \frac{\mathrm{W}(ax)}{x^6}\,dx$$

input
```
integrate(lambert_w(a*x)/x^6,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)/x^6, x)
```

Giac [F]

$$\int \frac{W(ax)}{x^6}\,dx = \int \frac{\mathrm{W}(ax)}{x^6}\,dx$$

input
```
integrate(lambert_w(a*x)/x^6,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)}{x^6}\,dx = \int \frac{\mathrm{LambertW}(a\,x)}{x^6}\,dx$$

input
```
int(LambertW(a*x)/x^6,x)
```

output
```
int(LambertW(a*x)/x^6, x)
```

Reduce [F]

$$\int \frac{W(ax)}{x^6}\,dx = \frac{\left(\int \frac{1}{e^{lambert\_w(ax)}lambert\_w(ax)x^5+e^{lambert\_w(ax)}x^5}dx\right)a\,x^5 - lambert\_w(ax)}{5x^5}$$

input
```
int(Lambert_W(a*x)/x^6,x)
```

output
```
(int(1/(e**lambert_w(a*x)*lambert_w(a*x)*x**5 + e**lambert_w(a*x)*x**5),x)
*a*x**5 - lambert_w(a*x))/(5*x**5)
```

3.11 $\int x^2 W(ax)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 71

$$\int x^2 W(ax)^2\,dx=\frac{8x^3}{27}-\frac{16x^3}{243W(ax)^3}+\frac{16x^3}{81W(ax)^2}-\frac{8x^3}{27W(ax)}-\frac{2}{9}x^3W(ax)+\frac{1}{3}x^3W(ax)^2$$

output
```
8/27*x^3-16/243*x^3/LambertW(a*x)^3+16/81*x^3/LambertW(a*x)^2-8/27*x^3/Lam
bertW(a*x)-2/9*x^3*LambertW(a*x)+1/3*x^3*LambertW(a*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int x^2 W(ax)^2\,dx=\frac{8x^3}{27}-\frac{16x^3}{243W(ax)^3}+\frac{16x^3}{81W(ax)^2}-\frac{8x^3}{27W(ax)}-\frac{2}{9}x^3W(ax)+\frac{1}{3}x^3W(ax)^2$$

input
```
Integrate[x^2*ProductLog[a*x]^2,x]
```

output
```
(8*x^3)/27 - (16*x^3)/(243*ProductLog[a*x]^3) + (16*x^3)/(81*ProductLog[a*
x]^2) - (8*x^3)/(27*ProductLog[a*x]) - (2*x^3*ProductLog[a*x])/9 + (x^3*Pr
oductLog[a*x]^2)/3
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {7172, 7205, 7205, 7194, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 W(ax)^2\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{3}x^3 W(ax)^2-\frac{2}{3}\int \frac{x^2 W(ax)^2}{W(ax)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{3}x^3 W(ax)^2-\frac{2}{3}\left(\frac{1}{3}x^3 W(ax)-\frac{4}{3}\int \frac{x^2 W(ax)}{W(ax)+1}dx\right)$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{3}x^3 W(ax)^2-\frac{2}{3}\left(\frac{1}{3}x^3 W(ax)-\frac{4}{3}\left(\frac{x^3}{3}-\int \frac{x^2}{W(ax)+1}dx\right)\right)$$

$$\Big\downarrow \text{7194}$$

$$\frac{1}{3}x^3 W(ax)^2-\frac{2}{3}\left(\frac{1}{3}x^3 W(ax)-\frac{4}{3}\left(\frac{2}{3}\int \frac{x^2}{W(ax)(W(ax)+1)}dx-\frac{x^3}{3W(ax)}+\frac{x^3}{3}\right)\right)$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{3}x^3 W(ax)^2-\frac{2}{3}\left(\frac{1}{3}x^3 W(ax)-\frac{4}{3}\left(\frac{2}{3}\left(\frac{x^3}{3W(ax)^2}-\frac{1}{3}\int \frac{x^2}{W(ax)^2(W(ax)+1)}dx\right)-\frac{x^3}{3W(ax)}+\frac{x^3}{3}\right)\right)$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{3}x^3 W(ax)^2-\frac{2}{3}\left(\frac{1}{3}x^3 W(ax)-\frac{4}{3}\left(-\frac{x^3}{3W(ax)}+\frac{2}{3}\left(\frac{x^3}{3W(ax)^2}-\frac{x^3}{9W(ax)^3}\right)+\frac{x^3}{3}\right)\right)$$

input

```
Int[x^2*ProductLog[a*x]^2,x]
```

output
```
(x^3*ProductLog[a*x]^2)/3 - (2*((-4*(x^3/3 + (2*(-1/9*x^3/ProductLog[a*x]^
3 + x^3/(3*ProductLog[a*x]^2)))/3 - x^3/(3*ProductLog[a*x])))/3 + (x^3*Pro
ductLog[a*x])/3))/3
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.15

method	result	siz
derivativedivides	$\frac{-\frac{2\,\mathrm{LambertW}(xa)x^3a^3}{9}+\frac{8x^3a^3}{27}-\frac{8x^3a^3}{27\,\mathrm{LambertW}(xa)}+\frac{16x^3a^3}{81\,\mathrm{LambertW}(xa)^2}-\frac{16x^3a^3}{243\,\mathrm{LambertW}(xa)^3}+\frac{\mathrm{LambertW}(xa)^2x^3a^3}{3}}{a^3}$	82
default	$\frac{-\frac{2\,\mathrm{LambertW}(xa)x^3a^3}{9}+\frac{8x^3a^3}{27}-\frac{8x^3a^3}{27\,\mathrm{LambertW}(xa)}+\frac{16x^3a^3}{81\,\mathrm{LambertW}(xa)^2}-\frac{16x^3a^3}{243\,\mathrm{LambertW}(xa)^3}+\frac{\mathrm{LambertW}(xa)^2x^3a^3}{3}}{a^3}$	82

input
```
int(x^2*LambertW(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-2/9*LambertW(x*a)*x^3*a^3+8/27*x^3*a^3-8/27/LambertW(x*a)*x^3*a^3+
16/81/LambertW(x*a)^2*x^3*a^3-16/243*x^3*a^3/LambertW(x*a)^3+1/3*LambertW(
x*a)^2*x^3*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int x^2W(ax)^2\,dx = \frac{81\,x^3\,\mathrm{W}(ax)^5-54\,x^3\,\mathrm{W}(ax)^4+72\,x^3\,\mathrm{W}(ax)^3-72\,x^3\,\mathrm{W}(ax)^2+48\,x^3\,\mathrm{W}(ax)-16\,x^3}{243\,\mathrm{W}(ax)^3}$$

input
```
integrate(x^2*lambert_w(a*x)^2,x, algorithm="fricas")
```

output
```
1/243*(81*x^3*lambert_w(a*x)^5 - 54*x^3*lambert_w(a*x)^4 + 72*x^3*lambert_
w(a*x)^3 - 72*x^3*lambert_w(a*x)^2 + 48*x^3*lambert_w(a*x) - 16*x^3)/lambe
rt_w(a*x)^3
```

Sympy [F]

$$\int x^2 W(ax)^2\,dx = \int x^2 W^2(ax)\,dx$$

input
```
integrate(x**2*LambertW(a*x)**2,x)
```

output
```
Integral(x**2*LambertW(a*x)**2, x)
```

Maxima [F]

$$\int x^2 W(ax)^2\,dx = \int x^2\,\mathrm{W}(ax)^2\,dx$$

input
```
integrate(x^2*lambert_w(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^2*lambert_w(a*x)^2, x)
```

Giac [F]

$$\int x^2 W(ax)^2\,dx = \int x^2\,\mathrm{W}(ax)^2\,dx$$

input
```
integrate(x^2*lambert_w(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2*lambert_w(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 W(ax)^2\,dx = \int x^2\,\mathrm{LambertW}(a\,x)^2\,dx$$

input
```
int(x^2*LambertW(a*x)^2,x)
```

output
```
int(x^2*LambertW(a*x)^2, x)
```

Reduce [F]

$$\int x^2 W(ax)^2\,dx = \int lambert\_w(ax)^2\,x^2 dx$$

input
```
int(x^2*Lambert_W(a*x)^2,x)
```

output
```
int(lambert_w(a*x)**2*x**2,x)
```

3.12 $\int xW(ax)^2 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 58

$$\int xW(ax)^2 \, dx = \frac{3x^2}{4} + \frac{3x^2}{8W(ax)^2} - \frac{3x^2}{4W(ax)} - \frac{1}{2}x^2W(ax) + \frac{1}{2}x^2W(ax)^2$$

output
```
3/4*x^2+3/8*x^2/LambertW(a*x)^2-3/4*x^2/LambertW(a*x)-1/2*x^2*LambertW(a*x
)+1/2*x^2*LambertW(a*x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int xW(ax)^2 \, dx = \frac{3x^2}{4} + \frac{3x^2}{8W(ax)^2} - \frac{3x^2}{4W(ax)} - \frac{1}{2}x^2W(ax) + \frac{1}{2}x^2W(ax)^2$$

input
```
Integrate[x*ProductLog[a*x]^2,x]
```

output
```
(3*x^2)/4 + (3*x^2)/(8*ProductLog[a*x]^2) - (3*x^2)/(4*ProductLog[a*x]) -
(x^2*ProductLog[a*x])/2 + (x^2*ProductLog[a*x]^2)/2
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {7172, 7205, 7205, 7194, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xW(ax)^2\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{2}x^2W(ax)^2-\int \frac{xW(ax)^2}{W(ax)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{3}{2}\int \frac{xW(ax)}{W(ax)+1}dx+\frac{1}{2}x^2W(ax)^2-\frac{1}{2}x^2W(ax)$$

$$\Big\downarrow \text{7205}$$

$$\frac{3}{2}\left(\frac{x^2}{2}-\int \frac{x}{W(ax)+1}dx\right)+\frac{1}{2}x^2W(ax)^2-\frac{1}{2}x^2W(ax)$$

$$\Big\downarrow \text{7194}$$

$$\frac{3}{2}\left(\frac{1}{2}\int \frac{x}{W(ax)(W(ax)+1)}dx-\frac{x^2}{2W(ax)}+\frac{x^2}{2}\right)+\frac{1}{2}x^2W(ax)^2-\frac{1}{2}x^2W(ax)$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{2}x^2W(ax)^2-\frac{1}{2}x^2W(ax)+\frac{3}{2}\left(-\frac{x^2}{2W(ax)}+\frac{x^2}{4W(ax)^2}+\frac{x^2}{2}\right)$$

input
```
Int[x*ProductLog[a*x]^2,x]
```

output
```
(3*(x^2/2 + x^2/(4*ProductLog[a*x]^2) - x^2/(2*ProductLog[a*x])))/2 - (x^2
*ProductLog[a*x])/2 + (x^2*ProductLog[a*x]^2)/2
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{-\frac{\operatorname{LambertW}(xa)x^2a^2}{2}+\frac{3a^2x^2}{4}-\frac{3x^2a^2}{4\operatorname{LambertW}(xa)}+\frac{3x^2a^2}{8\operatorname{LambertW}(xa)^2}+\frac{\operatorname{LambertW}(xa)^2x^2a^2}{2}}{a^2}$	68
default	$\frac{-\frac{\operatorname{LambertW}(xa)x^2a^2}{2}+\frac{3a^2x^2}{4}-\frac{3x^2a^2}{4\operatorname{LambertW}(xa)}+\frac{3x^2a^2}{8\operatorname{LambertW}(xa)^2}+\frac{\operatorname{LambertW}(xa)^2x^2a^2}{2}}{a^2}$	68

input
```
int(x*LambertW(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/2*LambertW(x*a)*x^2*a^2+3/4*a^2*x^2-3/4/LambertW(x*a)*x^2*a^2+3/
8*x^2*a^2/LambertW(x*a)^2+1/2*LambertW(x*a)^2*x^2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int xW(ax)^2\,dx = \frac{4\,x^2\,\mathrm{W}(ax)^4 - 4\,x^2\,\mathrm{W}(ax)^3 + 6\,x^2\,\mathrm{W}(ax)^2 - 6\,x^2\,\mathrm{W}(ax) + 3\,x^2}{8\,\mathrm{W}(ax)^2}$$

input
```
integrate(x*lambert_w(a*x)^2,x, algorithm="fricas")
```

output
```
1/8*(4*x^2*lambert_w(a*x)^4 - 4*x^2*lambert_w(a*x)^3 + 6*x^2*lambert_w(a*x
)^2 - 6*x^2*lambert_w(a*x) + 3*x^2)/lambert_w(a*x)^2
```

Sympy [F]

$$\int xW(ax)^2\,dx = \int xW^2(ax)\;dx$$

input
```
integrate(x*LambertW(a*x)**2,x)
```

output
```
Integral(x*LambertW(a*x)**2, x)
```

Maxima [F]

$$\int xW(ax)^2\,dx = \int x\,\mathrm{W}(ax)^2\,dx$$

input
```
integrate(x*lambert_w(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x*lambert_w(a*x)^2, x)
```

Giac [F]

$$\int xW(ax)^2\,dx = \int x\,\mathrm{W}(ax)^2\,dx$$

input
```
integrate(x*lambert_w(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x*lambert_w(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int xW(ax)^2\,dx = \int x\,\mathrm{LambertW}(a\,x)^2\,dx$$

input
```
int(x*LambertW(a*x)^2,x)
```

output
```
int(x*LambertW(a*x)^2, x)
```

Reduce [F]

$$\int xW(ax)^2\,dx = \int lambert\_w(ax)^2\,xdx$$

input
```
int(x*Lambert_W(a*x)^2,x)
```

output
```
int(lambert_w(a*x)**2*x,x)
```

3.13 $\int W(ax)^2\,dx$

Optimal result

Integrand size = 6, antiderivative size = 28

$$\int W(ax)^2\,dx = 4x - \frac{4x}{W(ax)} - 2xW(ax) + xW(ax)^2$$

output
```
4*x-4*x/LambertW(a*x)-2*x*LambertW(a*x)+x*LambertW(a*x)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int W(ax)^2\,dx = x\left(4 - \frac{4}{W(ax)} - 2W(ax) + W(ax)^2\right)$$

input
```
Integrate[ProductLog[a*x]^2,x]
```

output
```
x*(4 - 4/ProductLog[a*x] - 2*ProductLog[a*x] + ProductLog[a*x]^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {7167, 7178, 7177, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W(ax)^2\,dx$$

$$\Big\downarrow \text{7167}$$

$$xW(ax)^2-2\int \frac{W(ax)^2}{W(ax)+1}dx$$

$$\Big\downarrow \text{7178}$$

$$xW(ax)^2-2\left(xW(ax)-2\int \frac{W(ax)}{W(ax)+1}dx\right)$$

$$\Big\downarrow \text{7177}$$

$$xW(ax)^2-2\left(xW(ax)-2\left(x-\int \frac{1}{W(ax)+1}dx\right)\right)$$

$$\Big\downarrow \text{7176}$$

$$xW(ax)^2-2\left(xW(ax)-2\left(x-\frac{x}{W(ax)}\right)\right)$$

input
```
Int[ProductLog[a*x]^2,x]
```

output
```
x*ProductLog[a*x]^2 - 2*(-2*(x - x/ProductLog[a*x]) + x*ProductLog[a*x])
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7177
```
Int[ProductLog[(a_.) + (b_.)*(x_)]/((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(
x_)]), x_Symbol] :> Simp[d*x, x] - Int[1/(d + d*ProductLog[a + b*x]), x] /;
 FreeQ[{a, b, d}, x]
```

rule 7178
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[c*(a + b*x)*((c*ProductLog[a + b*x])^
(p - 1)/(b*d)), x] - Simp[c*p   Int[(c*ProductLog[a + b*x])^(p - 1)/(d + d*
ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{\mathrm{LambertW}(xa)^2 xa - 2xa\,\mathrm{LambertW}(xa) + 4xa - \frac{4xa}{\mathrm{LambertW}(xa)}}{a}$	37
default	$\frac{\mathrm{LambertW}(xa)^2 xa - 2xa\,\mathrm{LambertW}(xa) + 4xa - \frac{4xa}{\mathrm{LambertW}(xa)}}{a}$	37
parallelrisch	$-\frac{-x\,\mathrm{LambertW}(xa)^3 + 2x\,\mathrm{LambertW}(xa)^2 - 4x\,\mathrm{LambertW}(xa) + 4x}{\mathrm{LambertW}(xa)}$	38

input
```
int(LambertW(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(LambertW(x*a)^2*x*a-2*x*a*LambertW(x*a)+4*x*a-4*x*a/LambertW(x*a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int W(ax)^2\,dx = \frac{x\,\mathrm{W}(ax)^3 - 2\,x\,\mathrm{W}(ax)^2 + 4\,x\,\mathrm{W}(ax) - 4\,x}{\mathrm{W}(ax)}$$

input
```
integrate(lambert_w(a*x)^2,x, algorithm="fricas")
```

output
```
(x*lambert_w(a*x)^3 - 2*x*lambert_w(a*x)^2 + 4*x*lambert_w(a*x) - 4*x)/lam
bert_w(a*x)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int W(ax)^2\,dx = \begin{cases} xW^2(ax) - 2xW(ax) + 4x - \frac{4x}{W(ax)} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(LambertW(a*x)**2,x)
```

output
```
Piecewise((x*LambertW(a*x)**2 - 2*x*LambertW(a*x) + 4*x - 4*x/LambertW(a*x
), Ne(a, 0)), (0, True))
```

Maxima [F]

$$\int W(ax)^2\,dx = \int \mathrm{W}(ax)^2\,dx$$

input
```
integrate(lambert_w(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^2, x)
```

Giac [F]

$$\int W(ax)^2\,dx = \int \mathrm{W}(ax)^2\,dx$$

input
```
integrate(lambert_w(a*x)^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int W(ax)^2\,dx = \int \mathrm{LambertW}(a\,x)^2\,dx$$

input
```
int(LambertW(a*x)^2,x)
```

output
```
int(LambertW(a*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int W(ax)^2\,dx = \frac{e^{lambert\_w(ax)}\left(lambert\_w(ax)^3 - 2lambert\_w(ax)^2 + 4lambert\_w(ax) - 4\right)}{a}$$

input
```
int(Lambert_W(a*x)^2,x)
```

output
```
(e**lambert_w(a*x)*(lambert_w(a*x)**3 - 2*lambert_w(a*x)**2 + 4*lambert_w(
a*x) - 4))/a
```

3.14 $\int \frac{W(ax)^2}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \frac{W(ax)^2}{x}\,dx = \frac{1}{2}W(ax)^2 + \frac{1}{3}W(ax)^3$$

output
```
1/2*LambertW(a*x)^2+1/3*LambertW(a*x)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)^2}{x}\,dx = \frac{1}{2}W(ax)^2 + \frac{1}{3}W(ax)^3$$

input
```
Integrate[ProductLog[a*x]^2/x,x]
```

output
```
ProductLog[a*x]^2/2 + ProductLog[a*x]^3/3
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)^2}{x} \, dx$$

$$\Big\downarrow \; 7173$$

$$\int \frac{W(ax)^3}{x(W(ax)+1)}dx + \frac{1}{2}W(ax)^2$$

$$\Big\downarrow \; 7200$$

$$\frac{1}{3}W(ax)^3 + \frac{1}{2}W(ax)^2$$

input
```
Int[ProductLog[a*x]^2/x,x]
```

output
```
ProductLog[a*x]^2/2 + ProductLog[a*x]^3/3
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\text{LambertW}(xa)^2}{2}+\frac{\text{LambertW}(xa)^3}{3}$	18
default	$\frac{\text{LambertW}(xa)^2}{2}+\frac{\text{LambertW}(xa)^3}{3}$	18

input
```
int(LambertW(x*a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*LambertW(x*a)^2+1/3*LambertW(x*a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{W(ax)^2}{x}\,dx = \frac{1}{3}\ \mathrm{W}(ax)^3 + \frac{1}{2}\ \mathrm{W}(ax)^2$$

input
```
integrate(lambert_w(a*x)^2/x,x, algorithm="fricas")
```

output
```
1/3*lambert_w(a*x)^3 + 1/2*lambert_w(a*x)^2
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{W(ax)^2}{x}\,dx = \frac{W^3(ax)}{3} + \frac{W^2(ax)}{2}$$

input
```
integrate(LambertW(a*x)**2/x,x)
```

output
```
LambertW(a*x)**3/3 + LambertW(a*x)**2/2
```

Maxima [F]

$$\int \frac{W(ax)^2}{x}\,dx = \int \frac{\mathrm{W}(ax)^2}{x}\,dx$$

input
```
integrate(lambert_w(a*x)^2/x,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^2/x, x)
```

Giac [F]

$$\int \frac{W(ax)^2}{x}\,dx = \int \frac{\mathrm{W}(ax)^2}{x}\,dx$$

input
```
integrate(lambert_w(a*x)^2/x,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)^2}{x}\,dx = \int \frac{\mathrm{LambertW}(a\,x)^2}{x}\,dx$$

input
```
int(LambertW(a*x)^2/x,x)
```

output
```
int(LambertW(a*x)^2/x, x)
```

Reduce [F]

$$\int \frac{W(ax)^2}{x} dx = \int \frac{lambert\_w(ax)^2}{x} dx - \left(\int \frac{1}{x} dx\right) + \log(x)$$

input
```
int(Lambert_W(a*x)^2/x,x)
```

output
```
int(lambert_w(a*x)**2/x,x) - int(1/x,x) + log(x)
```

3.15 $\int \frac{W(ax)^2}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \frac{W(ax)^2}{x^2}\,dx = -\frac{2W(ax)}{x} - \frac{W(ax)^2}{x}$$

output
```
-2*LambertW(a*x)/x-LambertW(a*x)^2/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)^2}{x^2}\,dx = -\frac{2W(ax)}{x} - \frac{W(ax)^2}{x}$$

input
```
Integrate[ProductLog[a*x]^2/x^2,x]
```

output
```
(-2*ProductLog[a*x])/x - ProductLog[a*x]^2/x
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7172, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)^2}{x^2} \, dx$$

$$\Big\downarrow \text{ 7172}$$

$$2\int \frac{W(ax)^2}{x^2(W(ax)+1)}dx - \frac{W(ax)^2}{x}$$

$$\Big\downarrow \text{ 7201}$$

$$-\frac{W(ax)^2}{x} - \frac{2W(ax)}{x}$$

input
```
Int[ProductLog[a*x]^2/x^2,x]
```

output
```
(-2*ProductLog[a*x])/x - ProductLog[a*x]^2/x
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$-\frac{\mathrm{LambertW}(xa)^{2}+2\,\mathrm{LambertW}(xa)}{x}$	19
derivativedivides	$a\left(-\frac{2\,\mathrm{LambertW}(xa)}{xa}-\frac{\mathrm{LambertW}(xa)^{2}}{xa}\right)$	30
default	$a\left(-\frac{2\,\mathrm{LambertW}(xa)}{xa}-\frac{\mathrm{LambertW}(xa)^{2}}{xa}\right)$	30

input
```
int(LambertW(x*a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*(LambertW(x*a)^2+2*LambertW(x*a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{W(ax)^2}{x^2}\,dx = -\frac{\mathrm{W}(ax)^2+2\ \mathrm{W}(ax)}{x}$$

input
```
integrate(lambert_w(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
-(lambert_w(a*x)^2 + 2*lambert_w(a*x))/x
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{W(ax)^2}{x^2}\,dx = -\frac{W^2(ax)}{x}-\frac{2W(ax)}{x}$$

input
```
integrate(LambertW(a*x)**2/x**2,x)
```

output
```
-LambertW(a*x)**2/x - 2*LambertW(a*x)/x
```

Maxima [F]

$$\int \frac{W(ax)^2}{x^2}\,dx = \int \frac{\mathrm{W}(ax)^2}{x^2}\,dx$$

input
```
integrate(lambert_w(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^2/x^2, x)
```

Giac [F]

$$\int \frac{W(ax)^2}{x^2}\,dx = \int \frac{\mathrm{W}(ax)^2}{x^2}\,dx$$

input
```
integrate(lambert_w(a*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)^2}{x^2}\,dx = \int \frac{\mathrm{LambertW}(a\,x)^2}{x^2}\,dx$$

input
```
int(LambertW(a*x)^2/x^2,x)
```

output
```
int(LambertW(a*x)^2/x^2, x)
```

Reduce [F]

$$\int \frac{W(ax)^2}{x^2}\,dx$$
$$= \frac{2\left(\int \frac{lambert\_w(ax)}{e^{lambert\_w(ax)}lambert\_w(ax)x+e^{lambert\_w(ax)}x}dx\right)ax + 2\left(\int \frac{lambert\_w(ax)}{lambert\_w(ax)x^2+x^2}dx\right)x + 2\left(\int \frac{1}{lambert\_w}\right.}{x}$$

input
```
int(Lambert_W(a*x)^2/x^2,x)
```

output
```
(2*int(lambert_w(a*x)/(e**lambert_w(a*x)*lambert_w(a*x)*x + e**lambert_w(a
*x)*x),x)*a*x + 2*int(lambert_w(a*x)/(lambert_w(a*x)*x**2 + x**2),x)*x + 2
*int(1/(lambert_w(a*x)*x**2 + x**2),x)*x - lambert_w(a*x)**2 + 2)/x
```

3.16 $\int \frac{W(ax)^2}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \frac{W(ax)^2}{x^3} dx = a^2 \operatorname{ExpIntegralEi}(-2W(ax)) - \frac{W(ax)^2}{2x^2}$$

output
```
a^2*Ei(-2*LambertW(a*x))-1/2*LambertW(a*x)^2/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)^2}{x^3} dx = a^2 \operatorname{ExpIntegralEi}(-2W(ax)) - \frac{W(ax)^2}{2x^2}$$

input
```
Integrate[ProductLog[a*x]^2/x^3,x]
```

output
```
a^2*ExpIntegralEi[-2*ProductLog[a*x]] - ProductLog[a*x]^2/(2*x^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7172, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)^2}{x^3} \, dx$$
$$\Big\downarrow \text{7172}$$
$$\int \frac{W(ax)^2}{x^3(W(ax)+1)}dx - \frac{W(ax)^2}{2x^2}$$
$$\Big\downarrow \text{7202}$$
$$a^2 \operatorname{ExpIntegralEi}(-2W(ax)) - \frac{W(ax)^2}{2x^2}$$

input
```
Int[ProductLog[a*x]^2/x^3,x]
```

output
```
a^2*ExpIntegralEi[-2*ProductLog[a*x]] - ProductLog[a*x]^2/(2*x^2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$a^{2}\left(-\frac{\mathrm{LambertW}(xa)^{2}}{2x^{2}a^{2}}-\mathrm{expIntegral}_{1}\left(2\,\mathrm{LambertW}\left(xa\right)\right)\right)$	30
default	$a^{2}\left(-\frac{\mathrm{LambertW}(xa)^{2}}{2x^{2}a^{2}}-\mathrm{expIntegral}_{1}\left(2\,\mathrm{LambertW}\left(xa\right)\right)\right)$	30

input
```
int(LambertW(x*a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/x^2/a^2*LambertW(x*a)^2-Ei(1,2*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{W(ax)^2}{x^3}\,dx=\int \frac{\mathrm{W}(ax)^2}{x^3}\,dx$$

input
```
integrate(lambert_w(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)^2/x^3, x)
```

Sympy [F]

$$\int \frac{W(ax)^2}{x^3}\,dx=\int \frac{W^{2}(ax)}{x^3}\,dx$$

input
```
integrate(LambertW(a*x)**2/x**3,x)
```

output
```
Integral(LambertW(a*x)**2/x**3, x)
```

Maxima [F]

$$\int \frac{W(ax)^2}{x^3}\,dx = \int \frac{\mathrm{W}(ax)^2}{x^3}\,dx$$

input
```
integrate(lambert_w(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^2/x^3, x)
```

Giac [F]

$$\int \frac{W(ax)^2}{x^3}\,dx = \int \frac{\mathrm{W}(ax)^2}{x^3}\,dx$$

input
```
integrate(lambert_w(a*x)^2/x^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)^2}{x^3}\,dx = \int \frac{\mathrm{LambertW}(a\,x)^2}{x^3}\,dx$$

input
```
int(LambertW(a*x)^2/x^3,x)
```

output
```
int(LambertW(a*x)^2/x^3, x)
```

Reduce [F]

$$\int \frac{W(ax)^2}{x^3}\,dx = \frac{2\left(\int \frac{lambert\_w(ax)}{e^{lambert\_w(ax)}lambert\_w(ax)x^2+e^{lambert\_w(ax)}x^2}dx\right)a\,x^2 - lambert\_w(ax)^2}{2x^2}$$

input
```
int(Lambert_W(a*x)^2/x^3,x)
```

output
```
(2*int(lambert_w(a*x)/(e**lambert_w(a*x)*lambert_w(a*x)*x**2 + e**lambert_
w(a*x)*x**2),x)*a*x**2 - lambert_w(a*x)**2)/(2*x**2)
```

3.17 $\int \frac{W(ax)^2}{x^4} dx$

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \frac{W(ax)^2}{x^4} dx = -2a^3 \operatorname{ExpIntegralEi}(-3W(ax)) - \frac{W(ax)^2}{x^3}$$

output
```
-2*a^3*Ei(-3*LambertW(a*x))-LambertW(a*x)^2/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)^2}{x^4} dx = -2a^3 \operatorname{ExpIntegralEi}(-3W(ax)) - \frac{W(ax)^2}{x^3}$$

input
```
Integrate[ProductLog[a*x]^2/x^4,x]
```

output
```
-2*a^3*ExpIntegralEi[-3*ProductLog[a*x]] - ProductLog[a*x]^2/x^3
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7173, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)^2}{x^4} dx$$

$$\Big\downarrow \text{ 7173}$$

$$-2\int \frac{W(ax)^3}{x^4(W(ax)+1)}dx - \frac{W(ax)^2}{x^3}$$

$$\Big\downarrow \text{ 7202}$$

$$-2a^3 \operatorname{ExpIntegralEi}(-3W(ax)) - \frac{W(ax)^2}{x^3}$$

input
```
Int[ProductLog[a*x]^2/x^4,x]
```

output
```
-2*a^3*ExpIntegralEi[-3*ProductLog[a*x]] - ProductLog[a*x]^2/x^3
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$a^{3}\left(-\frac{\operatorname{LambertW}(xa)^{2}}{x^{3}a^{3}}+2\operatorname{expIntegral}_{1}\left(3\operatorname{LambertW}\left(xa\right)\right)\right)$	30
default	$a^{3}\left(-\frac{\operatorname{LambertW}(xa)^{2}}{x^{3}a^{3}}+2\operatorname{expIntegral}_{1}\left(3\operatorname{LambertW}\left(xa\right)\right)\right)$	30

input
```
int(LambertW(x*a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-LambertW(x*a)^2/x^3/a^3+2*Ei(1,3*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{W(ax)^2}{x^4}\,dx=\int \frac{\mathrm{W}(ax)^2}{x^4}\,dx$$

input
```
integrate(lambert_w(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)^2/x^4, x)
```

Sympy [F]

$$\int \frac{W(ax)^2}{x^4}\,dx=\int \frac{W^2(ax)}{x^4}\,dx$$

input
```
integrate(LambertW(a*x)**2/x**4,x)
```

output
```
Integral(LambertW(a*x)**2/x**4, x)
```

Maxima [F]

$$\int \frac{W(ax)^2}{x^4} \, dx = \int \frac{\mathrm{W}(ax)^2}{x^4} \, dx$$

input
```
integrate(lambert_w(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^2/x^4, x)
```

Giac [F]

$$\int \frac{W(ax)^2}{x^4} \, dx = \int \frac{\mathrm{W}(ax)^2}{x^4} \, dx$$

input
```
integrate(lambert_w(a*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)^2}{x^4} \, dx = \int \frac{\mathrm{LambertW}(a\,x)^2}{x^4} \, dx$$

input
```
int(LambertW(a*x)^2/x^4,x)
```

output
```
int(LambertW(a*x)^2/x^4, x)
```

Reduce [F]

$$\int \frac{W(ax)^2}{x^4}\,dx = \frac{2\left(\int \frac{lambert\_w(ax)}{e^{lambert\_w(ax)} lambert\_w(ax)x^3+e^{lambert\_w(ax)}x^3}dx\right) a\,x^3 - lambert\_w(ax)^2}{3x^3}$$

input
```
int(Lambert_W(a*x)^2/x^4,x)
```

output
```
(2*int(lambert_w(a*x)/(e**lambert_w(a*x)*lambert_w(a*x)*x**3 + e**lambert_
w(a*x)*x**3),x)*a*x**3 - lambert_w(a*x)**2)/(3*x**3)
```

3.18 $\int \frac{W(ax)^2}{x^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \frac{W(ax)^2}{x^5}\,dx = 4a^4\,\text{ExpIntegralEi}(-4W(ax)) - \frac{W(ax)^2}{2x^4} + \frac{W(ax)^3}{x^4}$$

output
```
4*a^4*Ei(-4*LambertW(a*x))-1/2*LambertW(a*x)^2/x^4+LambertW(a*x)^3/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)^2}{x^5}\,dx = 4a^4\,\text{ExpIntegralEi}(-4W(ax)) - \frac{W(ax)^2}{2x^4} + \frac{W(ax)^3}{x^4}$$

input
```
Integrate[ProductLog[a*x]^2/x^5,x]
```

output
```
4*a^4*ExpIntegralEi[-4*ProductLog[a*x]] - ProductLog[a*x]^2/(2*x^4) + Prod
uctLog[a*x]^3/x^4
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {7173, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)^2}{x^5}dx$$

$$\Big\downarrow \text{ 7173}$$

$$-\int \frac{W(ax)^3}{x^5(W(ax)+1)}dx - \frac{W(ax)^2}{2x^4}$$

$$\Big\downarrow \text{ 7206}$$

$$4\int \frac{W(ax)^4}{x^5(W(ax)+1)}dx + \frac{W(ax)^3}{x^4} - \frac{W(ax)^2}{2x^4}$$

$$\Big\downarrow \text{ 7202}$$

$$4a^4 \text{ExpIntegralEi}(-4W(ax)) + \frac{W(ax)^3}{x^4} - \frac{W(ax)^2}{2x^4}$$

input
```
Int[ProductLog[a*x]^2/x^5,x]
```

output
```
4*a^4*ExpIntegralEi[-4*ProductLog[a*x]] - ProductLog[a*x]^2/(2*x^4) + Prod
uctLog[a*x]^3/x^4
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$a^4\left(-\frac{\mathrm{LambertW}(xa)^2}{2x^4a^4}+\frac{\mathrm{LambertW}(xa)^3}{x^4a^4}-4\,\mathrm{expIntegral}_1\left(4\,\mathrm{LambertW}\left(xa\right)\right)\right)$	43
default	$a^4\left(-\frac{\mathrm{LambertW}(xa)^2}{2x^4a^4}+\frac{\mathrm{LambertW}(xa)^3}{x^4a^4}-4\,\mathrm{expIntegral}_1\left(4\,\mathrm{LambertW}\left(xa\right)\right)\right)$	43

input
```
int(LambertW(x*a)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
a^4*(-1/2*LambertW(x*a)^2/x^4/a^4+LambertW(x*a)^3/x^4/a^4-4*Ei(1,4*Lambert
W(x*a)))
```

Fricas [F]

$$\int \frac{W(ax)^2}{x^5}\,dx=\int \frac{\mathrm{W}(ax)^2}{x^5}\,dx$$

input
```
integrate(lambert_w(a*x)^2/x^5,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)^2/x^5, x)
```

Sympy [F]

$$\int \frac{W(ax)^2}{x^5}\,dx = \int \frac{W^2(ax)}{x^5}\,dx$$

input
```
integrate(LambertW(a*x)**2/x**5,x)
```

output
```
Integral(LambertW(a*x)**2/x**5, x)
```

Maxima [F]

$$\int \frac{W(ax)^2}{x^5}\,dx = \int \frac{\mathrm{W}(ax)^2}{x^5}\,dx$$

input
```
integrate(lambert_w(a*x)^2/x^5,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^2/x^5, x)
```

Giac [F]

$$\int \frac{W(ax)^2}{x^5}\,dx = \int \frac{\mathrm{W}(ax)^2}{x^5}\,dx$$

input
```
integrate(lambert_w(a*x)^2/x^5,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)^2}{x^5}\,dx = \int \frac{\mathrm{LambertW}(a\,x)^2}{x^5}\,dx$$

input
```
int(LambertW(a*x)^2/x^5,x)
```

output
```
int(LambertW(a*x)^2/x^5, x)
```

Reduce [F]

$$\int \frac{W(ax)^2}{x^5}\,dx = \frac{2\left(\int \frac{lambert\_w(ax)}{e^{lambert\_w(ax)} lambert\_w(ax)x^4+e^{lambert\_w(ax)}x^4}dx\right)a\,x^4 - lambert\_w(ax)^2}{4x^4}$$

input
```
int(Lambert_W(a*x)^2/x^5,x)
```

output
```
(2*int(lambert_w(a*x)/(e**lambert_w(a*x)*lambert_w(a*x)*x**4 + e**lambert_
w(a*x)*x**4),x)*a*x**4 - lambert_w(a*x)**2)/(4*x**4)
```

3.19 $\int \frac{W(ax)^2}{x^6}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{W(ax)^2}{x^6}\,dx = -\frac{25}{3}a^5\,\text{ExpIntegralEi}(-5W(ax)) - \frac{W(ax)^2}{3x^5} + \frac{W(ax)^3}{3x^5} - \frac{5W(ax)^4}{3x^5}$$

output
```
-25/3*a^5*Ei(-5*LambertW(a*x))-1/3*LambertW(a*x)^2/x^5+1/3*LambertW(a*x)^3
/x^5-5/3*LambertW(a*x)^4/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)^2}{x^6}\,dx = -\frac{25}{3}a^5\,\text{ExpIntegralEi}(-5W(ax)) - \frac{W(ax)^2}{3x^5} + \frac{W(ax)^3}{3x^5} - \frac{5W(ax)^4}{3x^5}$$

input
```
Integrate[ProductLog[a*x]^2/x^6,x]
```

output
```
(-25*a^5*ExpIntegralEi[-5*ProductLog[a*x]])/3 - ProductLog[a*x]^2/(3*x^5)
+ ProductLog[a*x]^3/(3*x^5) - (5*ProductLog[a*x]^4)/(3*x^5)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {7173, 7206, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)^2}{x^6} \, dx$$

$$\Big\downarrow \text{7173}$$

$$-\frac{2}{3} \int \frac{W(ax)^3}{x^6(W(ax)+1)}dx - \frac{W(ax)^2}{3x^5}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{2}{3}\left(-\frac{5}{2} \int \frac{W(ax)^4}{x^6(W(ax)+1)}dx - \frac{W(ax)^3}{2x^5}\right) - \frac{W(ax)^2}{3x^5}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{2}{3}\left(-\frac{5}{2}\left(-5 \int \frac{W(ax)^5}{x^6(W(ax)+1)}dx - \frac{W(ax)^4}{x^5}\right) - \frac{W(ax)^3}{2x^5}\right) - \frac{W(ax)^2}{3x^5}$$

$$\Big\downarrow \text{7202}$$

$$-\frac{2}{3}\left(-\frac{5}{2}\left(-5a^5 \operatorname{ExpIntegralEi}(-5W(ax)) - \frac{W(ax)^4}{x^5}\right) - \frac{W(ax)^3}{2x^5}\right) - \frac{W(ax)^2}{3x^5}$$

input
```
Int[ProductLog[a*x]^2/x^6,x]
```

output
```
-1/3*ProductLog[a*x]^2/x^5 - (2*(-1/2*ProductLog[a*x]^3/x^5 - (5*(-5*a^5*E
xpIntegralEi[-5*ProductLog[a*x]] - ProductLog[a*x]^4/x^5))/2))/3
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

method	result	siz
derivativedivides	$a^5\left(-\frac{\mathrm{LambertW}(xa)^2}{3x^5a^5}+\frac{\mathrm{LambertW}(xa)^3}{3x^5a^5}-\frac{5\,\mathrm{LambertW}(xa)^4}{3x^5a^5}+\frac{25\ \mathrm{expIntegral}_1(5\,\mathrm{LambertW}(xa))}{3}\right)$	58
default	$a^5\left(-\frac{\mathrm{LambertW}(xa)^2}{3x^5a^5}+\frac{\mathrm{LambertW}(xa)^3}{3x^5a^5}-\frac{5\,\mathrm{LambertW}(xa)^4}{3x^5a^5}+\frac{25\ \mathrm{expIntegral}_1(5\,\mathrm{LambertW}(xa))}{3}\right)$	58

input
```
int(LambertW(x*a)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
a^5*(-1/3*LambertW(x*a)^2/x^5/a^5+1/3*LambertW(x*a)^3/x^5/a^5-5/3*LambertW
(x*a)^4/x^5/a^5+25/3*Ei(1,5*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{W(ax)^2}{x^6}\,dx = \int \frac{\mathrm{W}(ax)^2}{x^6}\,dx$$

input
```
integrate(lambert_w(a*x)^2/x^6,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)^2/x^6, x)
```

Sympy [F]

$$\int \frac{W(ax)^2}{x^6}\,dx = \int \frac{W^2(ax)}{x^6}\,dx$$

input
```
integrate(LambertW(a*x)**2/x**6,x)
```

output
```
Integral(LambertW(a*x)**2/x**6, x)
```

Maxima [F]

$$\int \frac{W(ax)^2}{x^6}\,dx = \int \frac{\mathrm{W}(ax)^2}{x^6}\,dx$$

input
```
integrate(lambert_w(a*x)^2/x^6,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^2/x^6, x)
```

Giac [F]

$$\int \frac{W(ax)^2}{x^6} \, dx = \int \frac{W(ax)^2}{x^6} \, dx$$

input
```
integrate(lambert_w(a*x)^2/x^6,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^2/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)^2}{x^6} \, dx = \int \frac{\mathrm{LambertW}(a\,x)^2}{x^6} \, dx$$

input
```
int(LambertW(a*x)^2/x^6,x)
```

output
```
int(LambertW(a*x)^2/x^6, x)
```

Reduce [F]

$$\int \frac{W(ax)^2}{x^6} \, dx = \frac{2\left(\int \frac{lambert\_w(ax)}{e^{lambert\_w(ax)} lambert\_w(ax)x^5 + e^{lambert\_w(ax)} x^5} dx\right) a\,x^5 - lambert\_w(ax)^2}{5x^5}$$

input
```
int(Lambert_W(a*x)^2/x^6,x)
```

output
```
(2*int(lambert_w(a*x)/(e**lambert_w(a*x)*lambert_w(a*x)*x**5 + e**lambert_
w(a*x)*x**5),x)*a*x**5 - lambert_w(a*x)**2)/(5*x**5)
```

3.20 $\int \frac{W(ax)^2}{x^7} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 63

$$\int \frac{W(ax)^2}{x^7} \, dx = 18a^6 \operatorname{ExpIntegralEi}(-6W(ax)) - \frac{W(ax)^2}{4x^6} + \frac{W(ax)^3}{6x^6} - \frac{W(ax)^4}{2x^6} + \frac{3W(ax)^5}{x^6}$$

output
```
18*a^6*Ei(-6*LambertW(a*x))-1/4*LambertW(a*x)^2/x^6+1/6*LambertW(a*x)^3/x^
6-1/2*LambertW(a*x)^4/x^6+3*LambertW(a*x)^5/x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)^2}{x^7} \, dx = 18a^6 \operatorname{ExpIntegralEi}(-6W(ax)) - \frac{W(ax)^2}{4x^6} + \frac{W(ax)^3}{6x^6} - \frac{W(ax)^4}{2x^6} + \frac{3W(ax)^5}{x^6}$$

input
```
Integrate[ProductLog[a*x]^2/x^7,x]
```

output
```
18*a^6*ExpIntegralEi[-6*ProductLog[a*x]] - ProductLog[a*x]^2/(4*x^6) + Pro
ductLog[a*x]^3/(6*x^6) - ProductLog[a*x]^4/(2*x^6) + (3*ProductLog[a*x]^5)
/x^6
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {7173, 7206, 7206, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)^2}{x^7}dx$$

$$\Big\downarrow \text{7173}$$

$$-\frac{1}{2}\int \frac{W(ax)^3}{x^7(W(ax)+1)}dx-\frac{W(ax)^2}{4x^6}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{2}\left(2\int \frac{W(ax)^4}{x^7(W(ax)+1)}dx+\frac{W(ax)^3}{3x^6}\right)-\frac{W(ax)^2}{4x^6}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{2}\left(2\left(-3\int \frac{W(ax)^5}{x^7(W(ax)+1)}dx-\frac{W(ax)^4}{2x^6}\right)+\frac{W(ax)^3}{3x^6}\right)-\frac{W(ax)^2}{4x^6}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{2}\left(2\left(-3\left(-6\int \frac{W(ax)^6}{x^7(W(ax)+1)}dx-\frac{W(ax)^5}{x^6}\right)-\frac{W(ax)^4}{2x^6}\right)+\frac{W(ax)^3}{3x^6}\right)-\frac{W(ax)^2}{4x^6}$$

$$\Big\downarrow \text{7202}$$

$$\frac{1}{2}\left(2\left(-3\left(-6a^6\,\text{ExpIntegralEi}(-6W(ax))-\frac{W(ax)^5}{x^6}\right)-\frac{W(ax)^4}{2x^6}\right)+\frac{W(ax)^3}{3x^6}\right)-\frac{W(ax)^2}{4x^6}$$

input
```
Int[ProductLog[a*x]^2/x^7,x]
```

output
```
-1/4*ProductLog[a*x]^2/x^6 + (ProductLog[a*x]^3/(3*x^6) + 2*(-1/2*ProductL
og[a*x]^4/x^6 - 3*(-6*a^6*ExpIntegralEi[-6*ProductLog[a*x]] - ProductLog[a
*x]^5/x^6)))/2
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

method	result
derivativedivides	$a^6\left(-\frac{\mathrm{LambertW}(xa)^2}{4x^6a^6}+\frac{\mathrm{LambertW}(xa)^3}{6x^6a^6}-\frac{\mathrm{LambertW}(xa)^4}{2x^6a^6}+\frac{3\,\mathrm{LambertW}(xa)^5}{x^6a^6}-18\ \mathrm{expIntegral}_1\right.$
default	$a^6\left(-\frac{\mathrm{LambertW}(xa)^2}{4x^6a^6}+\frac{\mathrm{LambertW}(xa)^3}{6x^6a^6}-\frac{\mathrm{LambertW}(xa)^4}{2x^6a^6}+\frac{3\,\mathrm{LambertW}(xa)^5}{x^6a^6}-18\ \mathrm{expIntegral}_1\right.$

input
```
int(LambertW(x*a)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
a^6*(-1/4*LambertW(x*a)^2/x^6/a^6+1/6*LambertW(x*a)^3/x^6/a^6-1/2*LambertW
(x*a)^4/x^6/a^6+3*LambertW(x*a)^5/x^6/a^6-18*Ei(1,6*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{W(ax)^2}{x^7} \, dx = \int \frac{\mathrm{W}(ax)^2}{x^7} \, dx$$

input
```
integrate(lambert_w(a*x)^2/x^7,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)^2/x^7, x)
```

Sympy [F]

$$\int \frac{W(ax)^2}{x^7} \, dx = \int \frac{W^2(ax)}{x^7} \, dx$$

input
```
integrate(LambertW(a*x)**2/x**7,x)
```

output
```
Integral(LambertW(a*x)**2/x**7, x)
```

Maxima [F]

$$\int \frac{W(ax)^2}{x^7} \, dx = \int \frac{\mathrm{W}(ax)^2}{x^7} \, dx$$

input
```
integrate(lambert_w(a*x)^2/x^7,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^2/x^7, x)
```

Giac [F]

$$\int \frac{W(ax)^2}{x^7}\,dx = \int \frac{\mathrm{W}(ax)^2}{x^7}\,dx$$

input
```
integrate(lambert_w(a*x)^2/x^7,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^2/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)^2}{x^7}\,dx = \int \frac{\mathrm{LambertW}(a\,x)^2}{x^7}\,dx$$

input
```
int(LambertW(a*x)^2/x^7,x)
```

output
```
int(LambertW(a*x)^2/x^7, x)
```

Reduce [F]

$$\int \frac{W(ax)^2}{x^7}\,dx = \frac{2\left(\int \frac{lambert\_w(ax)}{e^{lambert\_w(ax)}lambert\_w(ax)x^6+e^{lambert\_w(ax)}x^6}dx\right)a\,x^6 - lambert\_w(ax)^2}{6x^6}$$

input
```
int(Lambert_W(a*x)^2/x^7,x)
```

output
```
(2*int(lambert_w(a*x)/(e**lambert_w(a*x)*lambert_w(a*x)*x**6 + e**lambert_
w(a*x)*x**6),x)*a*x**6 - lambert_w(a*x)**2)/(6*x**6)
```

3.21 $\int x^2 W(ax)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 84

$$\int x^2 W(ax)^3 \, dx = -\frac{20x^3}{27} + \frac{40x^3}{243W(ax)^3} - \frac{40x^3}{81W(ax)^2} + \frac{20x^3}{27W(ax)} + \frac{5}{9}x^3W(ax) - \frac{1}{3}x^3W(ax)^2 + \frac{1}{3}x^3W(ax)^3$$

output
```
-20/27*x^3+40/243*x^3/LambertW(a*x)^3-40/81*x^3/LambertW(a*x)^2+20/27*x^3/
LambertW(a*x)+5/9*x^3*LambertW(a*x)-1/3*x^3*LambertW(a*x)^2+1/3*x^3*Lamber
tW(a*x)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int x^2 W(ax)^3 \, dx = -\frac{20x^3}{27} + \frac{40x^3}{243W(ax)^3} - \frac{40x^3}{81W(ax)^2} + \frac{20x^3}{27W(ax)} + \frac{5}{9}x^3W(ax) - \frac{1}{3}x^3W(ax)^2 + \frac{1}{3}x^3W(ax)^3$$

input
```
Integrate[x^2*ProductLog[a*x]^3,x]
```

output
```
(-20*x^3)/27 + (40*x^3)/(243*ProductLog[a*x]^3) - (40*x^3)/(81*ProductLog[
a*x]^2) + (20*x^3)/(27*ProductLog[a*x]) + (5*x^3*ProductLog[a*x])/9 - (x^3
*ProductLog[a*x]^2)/3 + (x^3*ProductLog[a*x]^3)/3
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {7172, 7205, 7205, 7205, 7194, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 W(ax)^3 \, dx$$

$$\Big\downarrow \text{ 7172}$$

$$\frac{1}{3}x^3 W(ax)^3 - \int \frac{x^2 W(ax)^3}{W(ax)+1}dx$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{5}{3}\int \frac{x^2 W(ax)^2}{W(ax)+1}dx + \frac{1}{3}x^3 W(ax)^3 - \frac{1}{3}x^3 W(ax)^2$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{5}{3}\left(\frac{1}{3}x^3 W(ax) - \frac{4}{3}\int \frac{x^2 W(ax)}{W(ax)+1}dx\right) + \frac{1}{3}x^3 W(ax)^3 - \frac{1}{3}x^3 W(ax)^2$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{5}{3}\left(\frac{1}{3}x^3 W(ax) - \frac{4}{3}\left(\frac{x^3}{3} - \int \frac{x^2}{W(ax)+1}dx\right)\right) + \frac{1}{3}x^3 W(ax)^3 - \frac{1}{3}x^3 W(ax)^2$$

$$\Big\downarrow \text{ 7194}$$

$$\frac{5}{3}\left(\frac{1}{3}x^3 W(ax) - \frac{4}{3}\left(\frac{2}{3}\int \frac{x^2}{W(ax)(W(ax)+1)}dx - \frac{x^3}{3W(ax)} + \frac{x^3}{3}\right)\right) + \frac{1}{3}x^3 W(ax)^3 - \frac{1}{3}x^3 W(ax)^2$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{5}{3}\left(\frac{1}{3}x^3W(ax)-\frac{4}{3}\left(\frac{2}{3}\left(\frac{x^3}{3W(ax)^2}-\frac{1}{3}\int\frac{x^2}{W(ax)^2(W(ax)+1)}dx\right)-\frac{x^3}{3W(ax)}+\frac{x^3}{3}\right)\right)+\frac{1}{3}x^3W(ax)^3-\frac{1}{3}x^3W(ax)^2$$

$\big\downarrow$ 7201

$$\frac{1}{3}x^3W(ax)^3-\frac{1}{3}x^3W(ax)^2+\frac{5}{3}\left(\frac{1}{3}x^3W(ax)-\frac{4}{3}\left(-\frac{x^3}{3W(ax)}+\frac{2}{3}\left(\frac{x^3}{3W(ax)^2}-\frac{x^3}{9W(ax)^3}\right)+\frac{x^3}{3}\right)\right)$$

input

```
Int[x^2*ProductLog[a*x]^3,x]
```

output

```
-1/3*(x^3*ProductLog[a*x]^2) + (x^3*ProductLog[a*x]^3)/3 + (5*((-4*(x^3/3
+ (2*(-1/9*x^3/ProductLog[a*x]^3 + x^3/(3*ProductLog[a*x]^2)))/3 - x^3/(3*
ProductLog[a*x])))/3 + (x^3*ProductLog[a*x])/3))/3
```

Defintions of rubi rules used

rule 7172

```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7194

```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7201

```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{-\frac{\mathrm{LambertW}(xa)^2x^3a^3}{3}+\frac{5\,\mathrm{LambertW}(xa)x^3a^3}{9}-\frac{20x^3a^3}{27}+\frac{20x^3a^3}{27\,\mathrm{LambertW}(xa)}-\frac{40x^3a^3}{81\,\mathrm{LambertW}(xa)^2}+\frac{40x^3a^3}{243\,\mathrm{LambertW}(xa)^3}+\mathrm{Lam}}{a^3}$
default	$\frac{-\frac{\mathrm{LambertW}(xa)^2x^3a^3}{3}+\frac{5\,\mathrm{LambertW}(xa)x^3a^3}{9}-\frac{20x^3a^3}{27}+\frac{20x^3a^3}{27\,\mathrm{LambertW}(xa)}-\frac{40x^3a^3}{81\,\mathrm{LambertW}(xa)^2}+\frac{40x^3a^3}{243\,\mathrm{LambertW}(xa)^3}+\mathrm{Lam}}{a^3}$

input
```
int(x^2*LambertW(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/3*LambertW(x*a)^2*x^3*a^3+5/9*LambertW(x*a)*x^3*a^3-20/27*x^3*a^
3+20/27/LambertW(x*a)*x^3*a^3-40/81/LambertW(x*a)^2*x^3*a^3+40/243*x^3*a^3
/LambertW(x*a)^3+1/3*LambertW(x*a)^3*x^3*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

$$\int x^2W(ax)^3\,dx$$
$$=\frac{81\,x^3\,\mathrm{W}(ax)^6-81\,x^3\,\mathrm{W}(ax)^5+135\,x^3\,\mathrm{W}(ax)^4-180\,x^3\,\mathrm{W}(ax)^3+180\,x^3\,\mathrm{W}(ax)^2-120\,x^3\,\mathrm{W}(ax)+40}{243\,\mathrm{W}(ax)^3}$$

input
```
integrate(x^2*lambert_w(a*x)^3,x, algorithm="fricas")
```

output
```
1/243*(81*x^3*lambert_w(a*x)^6 - 81*x^3*lambert_w(a*x)^5 + 135*x^3*lambert
_w(a*x)^4 - 180*x^3*lambert_w(a*x)^3 + 180*x^3*lambert_w(a*x)^2 - 120*x^3*
lambert_w(a*x) + 40*x^3)/lambert_w(a*x)^3
```

Sympy [F]

$$\int x^2 W(ax)^3 \, dx = \int x^2 W^3(ax) \, dx$$

input
```
integrate(x**2*LambertW(a*x)**3,x)
```

output
```
Integral(x**2*LambertW(a*x)**3, x)
```

Maxima [F]

$$\int x^2 W(ax)^3 \, dx = \int x^2 \, \mathrm{W}(ax)^3 \, dx$$

input
```
integrate(x^2*lambert_w(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^2*lambert_w(a*x)^3, x)
```

Giac [F]

$$\int x^2 W(ax)^3 \, dx = \int x^2 \, \mathrm{W}(ax)^3 \, dx$$

input
```
integrate(x^2*lambert_w(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^2*lambert_w(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 W(ax)^3 \, dx = \int x^2 \operatorname{LambertW}(a\,x)^3 \, dx$$

input
```
int(x^2*LambertW(a*x)^3,x)
```

output
```
int(x^2*LambertW(a*x)^3, x)
```

Reduce [F]

$$\int x^2 W(ax)^3 \, dx = \int lambert\_w(ax)^3 x^2 dx$$

input
```
int(x^2*Lambert_W(a*x)^3,x)
```

output
```
int(lambert_w(a*x)**3*x**2,x)
```

3.22 $\int xW(ax)^3\,dx$

Optimal result

Integrand size = 8, antiderivative size = 71

$$\int xW(ax)^3\,dx = -\frac{9x^2}{4} - \frac{9x^2}{8W(ax)^2} + \frac{9x^2}{4W(ax)} + \frac{3}{2}x^2W(ax) - \frac{3}{4}x^2W(ax)^2 + \frac{1}{2}x^2W(ax)^3$$

output
```
-9/4*x^2-9/8*x^2/LambertW(a*x)^2+9/4*x^2/LambertW(a*x)+3/2*x^2*LambertW(a*
x)-3/4*x^2*LambertW(a*x)^2+1/2*x^2*LambertW(a*x)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int xW(ax)^3\,dx = -\frac{9x^2}{4} - \frac{9x^2}{8W(ax)^2} + \frac{9x^2}{4W(ax)} + \frac{3}{2}x^2W(ax) - \frac{3}{4}x^2W(ax)^2 + \frac{1}{2}x^2W(ax)^3$$

input
```
Integrate[x*ProductLog[a*x]^3,x]
```

output
```
(-9*x^2)/4 - (9*x^2)/(8*ProductLog[a*x]^2) + (9*x^2)/(4*ProductLog[a*x]) +
 (3*x^2*ProductLog[a*x])/2 - (3*x^2*ProductLog[a*x]^2)/4 + (x^2*ProductLog
[a*x]^3)/2
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {7172, 7205, 7205, 7205, 7194, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xW(ax)^3\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{2}x^2W(ax)^3-\frac{3}{2}\int \frac{xW(ax)^3}{W(ax)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{2}x^2W(ax)^3-\frac{3}{2}\left(\frac{1}{2}x^2W(ax)^2-2\int \frac{xW(ax)^2}{W(ax)+1}dx\right)$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{2}x^2W(ax)^3-\frac{3}{2}\left(\frac{1}{2}x^2W(ax)^2-2\left(\frac{1}{2}x^2W(ax)-\frac{3}{2}\int \frac{xW(ax)}{W(ax)+1}dx\right)\right)$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{2}x^2W(ax)^3-\frac{3}{2}\left(\frac{1}{2}x^2W(ax)^2-2\left(\frac{1}{2}x^2W(ax)-\frac{3}{2}\left(\frac{x^2}{2}-\int \frac{x}{W(ax)+1}dx\right)\right)\right)$$

$$\Big\downarrow \text{7194}$$

$$\frac{1}{2}x^2W(ax)^3-\frac{3}{2}\left(\frac{1}{2}x^2W(ax)^2-2\left(\frac{1}{2}x^2W(ax)-\frac{3}{2}\left(\frac{1}{2}\int \frac{x}{W(ax)(W(ax)+1)}dx-\frac{x^2}{2W(ax)}+\frac{x^2}{2}\right)\right)\right)$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{2}x^2W(ax)^3-\frac{3}{2}\left(\frac{1}{2}x^2W(ax)^2-2\left(\frac{1}{2}x^2W(ax)-\frac{3}{2}\left(-\frac{x^2}{2W(ax)}+\frac{x^2}{4W(ax)^2}+\frac{x^2}{2}\right)\right)\right)$$

input
```
Int[x*ProductLog[a*x]^3,x]
```

output
```
(x^2*ProductLog[a*x]^3)/2 - (3*((x^2*ProductLog[a*x]^2)/2 - 2*((-3*(x^2/2
+ x^2/(4*ProductLog[a*x]^2) - x^2/(2*ProductLog[a*x])))/2 + (x^2*ProductLo
g[a*x])/2)))/2
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.15

method	result	si
derivativedivides	$\frac{-\frac{3 \operatorname{LambertW}(xa)^2 x^2 a^2}{4}+\frac{3 \operatorname{LambertW}(xa) x^2 a^2}{2}-\frac{9a^2x^2}{4}+\frac{9x^2a^2}{4 \operatorname{LambertW}(xa)}-\frac{9x^2a^2}{8 \operatorname{LambertW}(xa)^2}+\frac{\operatorname{LambertW}(xa)^3 x^2 a^2}{2}}{a^2}$	82
default	$\frac{-\frac{3 \operatorname{LambertW}(xa)^2 x^2 a^2}{4}+\frac{3 \operatorname{LambertW}(xa) x^2 a^2}{2}-\frac{9a^2x^2}{4}+\frac{9x^2a^2}{4 \operatorname{LambertW}(xa)}-\frac{9x^2a^2}{8 \operatorname{LambertW}(xa)^2}+\frac{\operatorname{LambertW}(xa)^3 x^2 a^2}{2}}{a^2}$	82

input
```
int(x*LambertW(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-3/4*LambertW(x*a)^2*x^2*a^2+3/2*LambertW(x*a)*x^2*a^2-9/4*a^2*x^2+
9/4/LambertW(x*a)*x^2*a^2-9/8*x^2*a^2/LambertW(x*a)^2+1/2*LambertW(x*a)^3*
x^2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int xW(ax)^3\,dx = \frac{4\,x^2\,\mathrm{W}(ax)^5-6\,x^2\,\mathrm{W}(ax)^4+12\,x^2\,\mathrm{W}(ax)^3-18\,x^2\,\mathrm{W}(ax)^2+18\,x^2\,\mathrm{W}(ax)-9\,x^2}{8\,\mathrm{W}(ax)^2}$$

input
```
integrate(x*lambert_w(a*x)^3,x, algorithm="fricas")
```

output
```
1/8*(4*x^2*lambert_w(a*x)^5 - 6*x^2*lambert_w(a*x)^4 + 12*x^2*lambert_w(a*
x)^3 - 18*x^2*lambert_w(a*x)^2 + 18*x^2*lambert_w(a*x) - 9*x^2)/lambert_w(
a*x)^2
```

Sympy [F]

$$\int xW(ax)^3\,dx = \int xW^3(ax)\;dx$$

input
```
integrate(x*LambertW(a*x)**3,x)
```

output
```
Integral(x*LambertW(a*x)**3, x)
```

Maxima [F]

$$\int xW(ax)^3\,dx = \int x\,\mathrm{W}(ax)^3\,dx$$

input
```
integrate(x*lambert_w(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x*lambert_w(a*x)^3, x)
```

Giac [F]

$$\int xW(ax)^3\,dx = \int x\,\mathrm{W}(ax)^3\,dx$$

input
```
integrate(x*lambert_w(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x*lambert_w(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int xW(ax)^3\,dx = \int x\,\mathrm{LambertW}(a\,x)^3\,dx$$

input
```
int(x*LambertW(a*x)^3,x)
```

output
```
int(x*LambertW(a*x)^3, x)
```

Reduce [F]

$$\int xW(ax)^3\,dx = \int lambert\_w(ax)^3\,xdx$$

input
```
int(x*Lambert_W(a*x)^3,x)
```

output
```
int(lambert_w(a*x)**3*x,x)
```

3.23 $\int W(ax)^3\,dx$

Optimal result

Integrand size = 6, antiderivative size = 37

$$\int W(ax)^3\,dx = -18x + \frac{18x}{W(ax)} + 9xW(ax) - 3xW(ax)^2 + xW(ax)^3$$

output
```
-18*x+18*x/LambertW(a*x)+9*x*LambertW(a*x)-3*x*LambertW(a*x)^2+x*LambertW(
a*x)^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int W(ax)^3\,dx = \frac{x(6 + W(ax)^2)\,(3 - 3W(ax) + W(ax)^2)}{W(ax)}$$

input
```
Integrate[ProductLog[a*x]^3,x]
```

output
```
(x*(6 + ProductLog[a*x]^2)*(3 - 3*ProductLog[a*x] + ProductLog[a*x]^2))/Pr
oductLog[a*x]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {7167, 7178, 7178, 7177, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W(ax)^3\,dx$$

$$\Big\downarrow \text{7167}$$

$$xW(ax)^3-3\int \frac{W(ax)^3}{W(ax)+1}dx$$

$$\Big\downarrow \text{7178}$$

$$xW(ax)^3-3\left(xW(ax)^2-3\int \frac{W(ax)^2}{W(ax)+1}dx\right)$$

$$\Big\downarrow \text{7178}$$

$$xW(ax)^3-3\left(xW(ax)^2-3\left(xW(ax)-2\int \frac{W(ax)}{W(ax)+1}dx\right)\right)$$

$$\Big\downarrow \text{7177}$$

$$xW(ax)^3-3\left(xW(ax)^2-3\left(xW(ax)-2\left(x-\int \frac{1}{W(ax)+1}dx\right)\right)\right)$$

$$\Big\downarrow \text{7176}$$

$$xW(ax)^3-3\left(xW(ax)^2-3\left(xW(ax)-2\left(x-\frac{x}{W(ax)}\right)\right)\right)$$

input
```
Int[ProductLog[a*x]^3,x]
```

output
```
x*ProductLog[a*x]^3 - 3*(x*ProductLog[a*x]^2 - 3*(-2*(x - x/ProductLog[a*x
]) + x*ProductLog[a*x]))
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7177
```
Int[ProductLog[(a_.) + (b_.)*(x_)]/((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(
x_)]), x_Symbol] :> Simp[d*x, x] - Int[1/(d + d*ProductLog[a + b*x]), x] /;
 FreeQ[{a, b, d}, x]
```

rule 7178
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[c*(a + b*x)*((c*ProductLog[a + b*x])^
(p - 1)/(b*d)), x] - Simp[c*p   Int[(c*ProductLog[a + b*x])^(p - 1)/(d + d*
ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$\frac{-3\,\mathrm{LambertW}(xa)^2 xa+9xa\,\mathrm{LambertW}(xa)-18xa+\frac{18xa}{\mathrm{LambertW}(xa)}+\mathrm{LambertW}(xa)^3 xa}{a}$	47
default	$\frac{-3\,\mathrm{LambertW}(xa)^2 xa+9xa\,\mathrm{LambertW}(xa)-18xa+\frac{18xa}{\mathrm{LambertW}(xa)}+\mathrm{LambertW}(xa)^3 xa}{a}$	47
parallelrisch	$-\frac{-\mathrm{LambertW}(xa)^4 x+3x\,\mathrm{LambertW}(xa)^3-9x\,\mathrm{LambertW}(xa)^2+18x\,\mathrm{LambertW}(xa)-18x}{\mathrm{LambertW}(xa)}$	47

input
```
int(LambertW(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-3*LambertW(x*a)^2*x*a+9*x*a*LambertW(x*a)-18*x*a+18*x*a/LambertW(x*a
)+LambertW(x*a)^3*x*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int W(ax)^3\,dx = \frac{x\,\mathrm{W}(ax)^4 - 3\,x\,\mathrm{W}(ax)^3 + 9\,x\,\mathrm{W}(ax)^2 - 18\,x\,\mathrm{W}(ax) + 18\,x}{\mathrm{W}(ax)}$$

input
```
integrate(lambert_w(a*x)^3,x, algorithm="fricas")
```

output
```
(x*lambert_w(a*x)^4 - 3*x*lambert_w(a*x)^3 + 9*x*lambert_w(a*x)^2 - 18*x*l
ambert_w(a*x) + 18*x)/lambert_w(a*x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int W(ax)^3\,dx = \begin{cases} xW^3(ax) - 3xW^2(ax) + 9xW(ax) - 18x + \frac{18x}{W(ax)} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(LambertW(a*x)**3,x)
```

output
```
Piecewise((x*LambertW(a*x)**3 - 3*x*LambertW(a*x)**2 + 9*x*LambertW(a*x) -
 18*x + 18*x/LambertW(a*x), Ne(a, 0)), (0, True))
```

Maxima [F]

$$\int W(ax)^3\,dx = \int \mathrm{W}(ax)^3\,dx$$

input
```
integrate(lambert_w(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^3, x)
```

Giac [F]

$$\int W(ax)^3\,dx = \int \mathrm{W}(ax)^3\,dx$$

input
```
integrate(lambert_w(a*x)^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int W(ax)^3\,dx = \int \mathrm{LambertW}(a\,x)^3\,dx$$

input
```
int(LambertW(a*x)^3,x)
```

output
```
int(LambertW(a*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int W(ax)^3\,dx = \frac{e^{lambert\_w(ax)}\left(lambert\_w(ax)^4 - 3lambert\_w(ax)^3 + 9lambert\_w(ax)^2 - 18lambert\_w(ax) + 18\right)}{a}$$

input
```
int(Lambert_W(a*x)^3,x)
```

output
```
(e**lambert_w(a*x)*(lambert_w(a*x)**4 - 3*lambert_w(a*x)**3 + 9*lambert_w(
a*x)**2 - 18*lambert_w(a*x) + 18))/a
```

3.24 $\int \frac{W(ax)^3}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \frac{W(ax)^3}{x}\,dx = \frac{1}{3}W(ax)^3 + \frac{1}{4}W(ax)^4$$

output
```
1/3*LambertW(a*x)^3+1/4*LambertW(a*x)^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)^3}{x}\,dx = \frac{1}{3}W(ax)^3 + \frac{1}{4}W(ax)^4$$

input
```
Integrate[ProductLog[a*x]^3/x,x]
```

output
```
ProductLog[a*x]^3/3 + ProductLog[a*x]^4/4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)^3}{x} \, dx$$

$$\Big\downarrow \text{ 7173}$$

$$\int \frac{W(ax)^4}{x(W(ax)+1)}dx + \frac{1}{3}W(ax)^3$$

$$\Big\downarrow \text{ 7200}$$

$$\frac{1}{4}W(ax)^4 + \frac{1}{3}W(ax)^3$$

input
```
Int[ProductLog[a*x]^3/x,x]
```

output
```
ProductLog[a*x]^3/3 + ProductLog[a*x]^4/4
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\text{LambertW}(xa)^3}{3}+\frac{\text{LambertW}(xa)^4}{4}$	18
default	$\frac{\text{LambertW}(xa)^3}{3}+\frac{\text{LambertW}(xa)^4}{4}$	18

input
```
int(LambertW(x*a)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*LambertW(x*a)^3+1/4*LambertW(x*a)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{W(ax)^3}{x}\,dx = \frac{1}{4}\,\mathrm{W}(ax)^4 + \frac{1}{3}\,\mathrm{W}(ax)^3$$

input
```
integrate(lambert_w(a*x)^3/x,x, algorithm="fricas")
```

output
```
1/4*lambert_w(a*x)^4 + 1/3*lambert_w(a*x)^3
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{W(ax)^3}{x}\,dx = \frac{W^4(ax)}{4} + \frac{W^3(ax)}{3}$$

input
```
integrate(LambertW(a*x)**3/x,x)
```

output
```
LambertW(a*x)**4/4 + LambertW(a*x)**3/3
```

Maxima [F]

$$\int \frac{W(ax)^3}{x}\,dx = \int \frac{\mathrm{W}(ax)^3}{x}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^3/x, x)
```

Giac [F]

$$\int \frac{W(ax)^3}{x}\,dx = \int \frac{\mathrm{W}(ax)^3}{x}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)^3}{x}\,dx = \int \frac{\mathrm{LambertW}(a\,x)^3}{x}\,dx$$

input
```
int(LambertW(a*x)^3/x,x)
```

output
```
int(LambertW(a*x)^3/x, x)
```

Reduce [F]

$$\int \frac{W(ax)^3}{x} dx = \int \frac{lambert\_w(ax)^3}{x} dx$$

input
```
int(Lambert_W(a*x)^3/x,x)
```

output
```
int(lambert_w(a*x)**3/x,x)
```

3.25 $\int \frac{W(ax)^3}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 32

$$\int \frac{W(ax)^3}{x^2}\,dx = -\frac{3W(ax)}{x} - \frac{3W(ax)^2}{x} - \frac{W(ax)^3}{x}$$

output
```
-3*LambertW(a*x)/x-3*LambertW(a*x)^2/x-LambertW(a*x)^3/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)^3}{x^2}\,dx = -\frac{3W(ax)}{x} - \frac{3W(ax)^2}{x} - \frac{W(ax)^3}{x}$$

input
```
Integrate[ProductLog[a*x]^3/x^2,x]
```

output
```
(-3*ProductLog[a*x])/x - (3*ProductLog[a*x]^2)/x - ProductLog[a*x]^3/x
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7172, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)^3}{x^2}\,dx$$

$$\Big\downarrow \text{7172}$$

$$3\int \frac{W(ax)^3}{x^2(W(ax)+1)}dx-\frac{W(ax)^3}{x}$$

$$\Big\downarrow \text{7205}$$

$$3\left(\int \frac{W(ax)^2}{x^2(W(ax)+1)}dx-\frac{W(ax)^2}{x}\right)-\frac{W(ax)^3}{x}$$

$$\Big\downarrow \text{7201}$$

$$3\left(-\frac{W(ax)^2}{x}-\frac{W(ax)}{x}\right)-\frac{W(ax)^3}{x}$$

input
```
Int[ProductLog[a*x]^3/x^2,x]
```

output
```
-(ProductLog[a*x]^3/x) + 3*(-(ProductLog[a*x]/x) - ProductLog[a*x]^2/x)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$-\frac{\mathrm{LambertW}(xa)^3+3\,\mathrm{LambertW}(xa)^2+3\,\mathrm{LambertW}(xa)}{x}$	27
derivativedivides	$a\left(-\frac{3\,\mathrm{LambertW}(xa)^2}{xa}-\frac{3\,\mathrm{LambertW}(xa)}{xa}-\frac{\mathrm{LambertW}(xa)^3}{xa}\right)$	44
default	$a\left(-\frac{3\,\mathrm{LambertW}(xa)^2}{xa}-\frac{3\,\mathrm{LambertW}(xa)}{xa}-\frac{\mathrm{LambertW}(xa)^3}{xa}\right)$	44

input
```
int(LambertW(x*a)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*(LambertW(x*a)^3+3*LambertW(x*a)^2+3*LambertW(x*a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{W(ax)^3}{x^2}\,dx = -\frac{\mathrm{W}(ax)^3+3\,\mathrm{W}(ax)^2+3\,\mathrm{W}(ax)}{x}$$

input
```
integrate(lambert_w(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
-(lambert_w(a*x)^3 + 3*lambert_w(a*x)^2 + 3*lambert_w(a*x))/x
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{W(ax)^3}{x^2} dx = -\frac{W^3(ax)}{x} - \frac{3W^2(ax)}{x} - \frac{3W(ax)}{x}$$

input
```
integrate(LambertW(a*x)**3/x**2,x)
```

output
```
-LambertW(a*x)**3/x - 3*LambertW(a*x)**2/x - 3*LambertW(a*x)/x
```

Maxima [F]

$$\int \frac{W(ax)^3}{x^2} dx = \int \frac{\mathrm{W}(ax)^3}{x^2} dx$$

input
```
integrate(lambert_w(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^3/x^2, x)
```

Giac [F]

$$\int \frac{W(ax)^3}{x^2} dx = \int \frac{\mathrm{W}(ax)^3}{x^2} dx$$

input
```
integrate(lambert_w(a*x)^3/x^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)^3}{x^2}\,dx = \int \frac{\mathrm{LambertW}(a\,x)^3}{x^2}\,dx$$

input
```
int(LambertW(a*x)^3/x^2,x)
```

output
```
int(LambertW(a*x)^3/x^2, x)
```

Reduce [F]

$$\int \frac{W(ax)^3}{x^2}\,dx$$
$$= \frac{12\left(\int \frac{lambert\_w(ax)^2}{e^{lambert\_w(ax)}lambert\_w(ax)x+e^{lambert\_w(ax)}x}dx\right)ax - 9\left(\int \frac{lambert\_w(ax)}{lambert\_w(ax)x^2+x^2}dx\right)x - 9\left(\int \frac{}{lambert\_}\right.}{4x}$$

input
```
int(Lambert_W(a*x)^3/x^2,x)
```

output
```
(12*int(lambert_w(a*x)**2/(e**lambert_w(a*x)*lambert_w(a*x)*x + e**lambert
_w(a*x)*x),x)*a*x - 9*int(lambert_w(a*x)/(lambert_w(a*x)*x**2 + x**2),x)*x
 - 9*int(1/(lambert_w(a*x)*x**2 + x**2),x)*x - 4*lambert_w(a*x)**3 - 9)/(4
*x)
```

3.26 $\int \frac{W(ax)^3}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \frac{W(ax)^3}{x^3} dx = -\frac{3W(ax)^2}{4x^2} - \frac{W(ax)^3}{2x^2}$$

output
```
-3/4*LambertW(a*x)^2/x^2-1/2*LambertW(a*x)^3/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)^3}{x^3} dx = -\frac{3W(ax)^2}{4x^2} - \frac{W(ax)^3}{2x^2}$$

input
```
Integrate[ProductLog[a*x]^3/x^3,x]
```

output
```
(-3*ProductLog[a*x]^2)/(4*x^2) - ProductLog[a*x]^3/(2*x^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7172, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)^3}{x^3} \, dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{3}{2} \int \frac{W(ax)^3}{x^3(W(ax)+1)}dx - \frac{W(ax)^3}{2x^2}$$

$$\Big\downarrow \text{7201}$$

$$-\frac{W(ax)^3}{2x^2} - \frac{3W(ax)^2}{4x^2}$$

input
```
Int[ProductLog[a*x]^3/x^3,x]
```

output
```
(-3*ProductLog[a*x]^2)/(4*x^2) - ProductLog[a*x]^3/(2*x^2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$-\frac{2\,\mathrm{LambertW}(xa)^{3}+3\,\mathrm{LambertW}(xa)^{2}}{4x^{2}}$	23
derivativedivides	$a^{2}\left(-\frac{3\,\mathrm{LambertW}(xa)^{2}}{4x^{2}a^{2}}-\frac{\mathrm{LambertW}(xa)^{3}}{2x^{2}a^{2}}\right)$	34
default	$a^{2}\left(-\frac{3\,\mathrm{LambertW}(xa)^{2}}{4x^{2}a^{2}}-\frac{\mathrm{LambertW}(xa)^{3}}{2x^{2}a^{2}}\right)$	34

input
```
int(LambertW(x*a)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^2*(2*LambertW(x*a)^3+3*LambertW(x*a)^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{W(ax)^3}{x^3}\,dx = -\frac{2\ \mathrm{W}(ax)^3 + 3\ \mathrm{W}(ax)^2}{4\,x^2}$$

input
```
integrate(lambert_w(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*lambert_w(a*x)^3 + 3*lambert_w(a*x)^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{W(ax)^3}{x^3}\,dx = -\frac{W^3(ax)}{2x^2} - \frac{3W^2(ax)}{4x^2}$$

input
```
integrate(LambertW(a*x)**3/x**3,x)
```

output
```
-LambertW(a*x)**3/(2*x**2) - 3*LambertW(a*x)**2/(4*x**2)
```

Maxima [F]

$$\int \frac{W(ax)^3}{x^3}\,dx = \int \frac{\mathrm{W}(ax)^3}{x^3}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^3/x^3, x)
```

Giac [F]

$$\int \frac{W(ax)^3}{x^3}\,dx = \int \frac{\mathrm{W}(ax)^3}{x^3}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)^3}{x^3}\,dx = \int \frac{\mathrm{LambertW}(a\,x)^3}{x^3}\,dx$$

input
```
int(LambertW(a*x)^3/x^3,x)
```

output
```
int(LambertW(a*x)^3/x^3, x)
```

Reduce [F]

$$\int \frac{W(ax)^3}{x^3}\,dx = \frac{3\left(\int \frac{lambert\_w(ax)^2}{e^{lambert\_w(ax)}lambert\_w(ax)x^2+e^{lambert\_w(ax)}x^2}dx\right)a\,x^2 - lambert\_w(ax)^3}{2x^2}$$

input
```
int(Lambert_W(a*x)^3/x^3,x)
```

output
```
(3*int(lambert_w(a*x)**2/(e**lambert_w(a*x)*lambert_w(a*x)*x**2 + e**lambe
rt_w(a*x)*x**2),x)*a*x**2 - lambert_w(a*x)**3)/(2*x**2)
```

3.27 $\int \frac{W(ax)^3}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \frac{W(ax)^3}{x^4}\,dx = a^3\,\text{ExpIntegralEi}(-3W(ax)) - \frac{W(ax)^3}{3x^3}$$

output
```
a^3*Ei(-3*LambertW(a*x))-1/3*LambertW(a*x)^3/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)^3}{x^4}\,dx = a^3\,\text{ExpIntegralEi}(-3W(ax)) - \frac{W(ax)^3}{3x^3}$$

input
```
Integrate[ProductLog[a*x]^3/x^4,x]
```

output
```
a^3*ExpIntegralEi[-3*ProductLog[a*x]] - ProductLog[a*x]^3/(3*x^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7172, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)^3}{x^4} dx$$
$$\Big\downarrow \text{ 7172}$$
$$\int \frac{W(ax)^3}{x^4(W(ax)+1)}dx - \frac{W(ax)^3}{3x^3}$$
$$\Big\downarrow \text{ 7202}$$
$$a^3 \text{ExpIntegralEi}(-3W(ax)) - \frac{W(ax)^3}{3x^3}$$

input
```
Int[ProductLog[a*x]^3/x^4,x]
```

output
```
a^3*ExpIntegralEi[-3*ProductLog[a*x]] - ProductLog[a*x]^3/(3*x^3)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$a^3\left(-\frac{\text{LambertW}(xa)^3}{3x^3a^3}-\text{expIntegral}_1\left(3\,\text{LambertW}\left(xa\right)\right)\right)$	30
default	$a^3\left(-\frac{\text{LambertW}(xa)^3}{3x^3a^3}-\text{expIntegral}_1\left(3\,\text{LambertW}\left(xa\right)\right)\right)$	30

input
```
int(LambertW(x*a)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3/x^3/a^3*LambertW(x*a)^3-Ei(1,3*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{W(ax)^3}{x^4}\,dx = \int \frac{\text{W}(ax)^3}{x^4}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)^3/x^4, x)
```

Sympy [F]

$$\int \frac{W(ax)^3}{x^4}\,dx = \int \frac{W^3(ax)}{x^4}\,dx$$

input
```
integrate(LambertW(a*x)**3/x**4,x)
```

output
```
Integral(LambertW(a*x)**3/x**4, x)
```

Maxima [F]

$$\int \frac{W(ax)^3}{x^4}\,dx = \int \frac{\mathrm{W}(ax)^3}{x^4}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^3/x^4, x)
```

Giac [F]

$$\int \frac{W(ax)^3}{x^4}\,dx = \int \frac{\mathrm{W}(ax)^3}{x^4}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^4,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^3/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)^3}{x^4}\,dx = \int \frac{\mathrm{LambertW}(a\,x)^3}{x^4}\,dx$$

input
```
int(LambertW(a*x)^3/x^4,x)
```

output
```
int(LambertW(a*x)^3/x^4, x)
```

Reduce [F]

$$\int \frac{W(ax)^3}{x^4}\,dx$$
$$= \frac{3\left(\int \frac{lambert\_w(ax)^2}{e^{lambert\_w(ax)}lambert\_w(ax)x^3+e^{lambert\_w(ax)}x^3}dx\right)a\,x^3 - lambert\_w(ax)^3}{3x^3}$$

input
```
int(Lambert_W(a*x)^3/x^4,x)
```

output
```
(3*int(lambert_w(a*x)**2/(e**lambert_w(a*x)*lambert_w(a*x)*x**3 + e**lambe
rt_w(a*x)*x**3),x)*a*x**3 - lambert_w(a*x)**3)/(3*x**3)
```

3.28 $\int \frac{W(ax)^3}{x^5} dx$

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \frac{W(ax)^3}{x^5} dx = -3a^4 \text{ExpIntegralEi}(-4W(ax)) - \frac{W(ax)^3}{x^4}$$

output
```
-3*a^4*Ei(-4*LambertW(a*x))-LambertW(a*x)^3/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)^3}{x^5} dx = -3a^4 \text{ExpIntegralEi}(-4W(ax)) - \frac{W(ax)^3}{x^4}$$

input
```
Integrate[ProductLog[a*x]^3/x^5,x]
```

output
```
-3*a^4*ExpIntegralEi[-4*ProductLog[a*x]] - ProductLog[a*x]^3/x^4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7173, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)^3}{x^5} dx$$

$$\Big\downarrow \text{ 7173}$$

$$-3\int \frac{W(ax)^4}{x^5(W(ax)+1)}dx - \frac{W(ax)^3}{x^4}$$

$$\Big\downarrow \text{ 7202}$$

$$-3a^4 \operatorname{ExpIntegralEi}(-4W(ax)) - \frac{W(ax)^3}{x^4}$$

input
```
Int[ProductLog[a*x]^3/x^5,x]
```

output
```
-3*a^4*ExpIntegralEi[-4*ProductLog[a*x]] - ProductLog[a*x]^3/x^4
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$a^4\left(-\frac{\text{LambertW}(xa)^3}{x^4a^4}+3\,\text{expIntegral}_1\left(4\,\text{LambertW}\left(xa\right)\right)\right)$	30
default	$a^4\left(-\frac{\text{LambertW}(xa)^3}{x^4a^4}+3\,\text{expIntegral}_1\left(4\,\text{LambertW}\left(xa\right)\right)\right)$	30

input
```
int(LambertW(x*a)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
a^4*(-LambertW(x*a)^3/x^4/a^4+3*Ei(1,4*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{W(ax)^3}{x^5}\,dx=\int \frac{\text{W}(ax)^3}{x^5}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^5,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)^3/x^5, x)
```

Sympy [F]

$$\int \frac{W(ax)^3}{x^5}\,dx=\int \frac{W^3(ax)}{x^5}\,dx$$

input
```
integrate(LambertW(a*x)**3/x**5,x)
```

output
```
Integral(LambertW(a*x)**3/x**5, x)
```

Maxima [F]

$$\int \frac{W(ax)^3}{x^5}\,dx = \int \frac{\mathrm{W}(ax)^3}{x^5}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^5,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^3/x^5, x)
```

Giac [F]

$$\int \frac{W(ax)^3}{x^5}\,dx = \int \frac{\mathrm{W}(ax)^3}{x^5}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^5,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^3/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)^3}{x^5}\,dx = \int \frac{\mathrm{LambertW}(a\,x)^3}{x^5}\,dx$$

input
```
int(LambertW(a*x)^3/x^5,x)
```

output
```
int(LambertW(a*x)^3/x^5, x)
```

Reduce [F]

$$\int \frac{W(ax)^3}{x^5}\,dx = \frac{3\left(\int \frac{lambert\_w(ax)^2}{e^{lambert\_w(ax)}lambert\_w(ax)x^4+e^{lambert\_w(ax)}x^4}dx\right)a\,x^4 - lambert\_w(ax)^3}{4x^4}$$

input
```
int(Lambert_W(a*x)^3/x^5,x)
```

output
```
(3*int(lambert_w(a*x)**2/(e**lambert_w(a*x)*lambert_w(a*x)*x**4 + e**lambe
rt_w(a*x)*x**4),x)*a*x**4 - lambert_w(a*x)**3)/(4*x**4)
```

3.29 $\int \frac{W(ax)^3}{x^6} dx$

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \frac{W(ax)^3}{x^6} dx = \frac{15}{2}a^5 \text{ExpIntegralEi}(-5W(ax)) - \frac{W(ax)^3}{2x^5} + \frac{3W(ax)^4}{2x^5}$$

output
```
15/2*a^5*Ei(-5*LambertW(a*x))-1/2*LambertW(a*x)^3/x^5+3/2*LambertW(a*x)^4/
x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)^3}{x^6} dx = \frac{15}{2}a^5 \text{ExpIntegralEi}(-5W(ax)) - \frac{W(ax)^3}{2x^5} + \frac{3W(ax)^4}{2x^5}$$

input
```
Integrate[ProductLog[a*x]^3/x^6,x]
```

output
```
(15*a^5*ExpIntegralEi[-5*ProductLog[a*x]])/2 - ProductLog[a*x]^3/(2*x^5) +
 (3*ProductLog[a*x]^4)/(2*x^5)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7173, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)^3}{x^6} dx$$

$$\Big\downarrow \text{ 7173}$$

$$-\frac{3}{2} \int \frac{W(ax)^4}{x^6(W(ax)+1)} dx - \frac{W(ax)^3}{2x^5}$$

$$\Big\downarrow \text{ 7206}$$

$$-\frac{3}{2}\left(-5 \int \frac{W(ax)^5}{x^6(W(ax)+1)} dx - \frac{W(ax)^4}{x^5}\right) - \frac{W(ax)^3}{2x^5}$$

$$\Big\downarrow \text{ 7202}$$

$$-\frac{3}{2}\left(-5a^5 \operatorname{ExpIntegralEi}(-5W(ax)) - \frac{W(ax)^4}{x^5}\right) - \frac{W(ax)^3}{2x^5}$$

input
```
Int[ProductLog[a*x]^3/x^6,x]
```

output
```
-1/2*ProductLog[a*x]^3/x^5 - (3*(-5*a^5*ExpIntegralEi[-5*ProductLog[a*x]]
- ProductLog[a*x]^4/x^5))/2
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$a^{5}\left(-\frac{\operatorname{LambertW}(xa)^{3}}{2x^{5}a^{5}}+\frac{3\operatorname{LambertW}(xa)^{4}}{2x^{5}a^{5}}-\frac{15\operatorname{expIntegral}_{1}(5\operatorname{LambertW}(xa))}{2}\right)$	44
default	$a^{5}\left(-\frac{\operatorname{LambertW}(xa)^{3}}{2x^{5}a^{5}}+\frac{3\operatorname{LambertW}(xa)^{4}}{2x^{5}a^{5}}-\frac{15\operatorname{expIntegral}_{1}(5\operatorname{LambertW}(xa))}{2}\right)$	44

input
```
int(LambertW(x*a)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
a^5*(-1/2*LambertW(x*a)^3/x^5/a^5+3/2*LambertW(x*a)^4/x^5/a^5-15/2*Ei(1,5*
LambertW(x*a)))
```

Fricas [F]

$$\int \frac{W(ax)^3}{x^6}\,dx=\int \frac{\mathrm{W}(ax)^3}{x^6}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^6,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)^3/x^6, x)
```

Sympy [F]

$$\int \frac{W(ax)^3}{x^6}\,dx = \int \frac{W^3(ax)}{x^6}\,dx$$

input
```
integrate(LambertW(a*x)**3/x**6,x)
```

output
```
Integral(LambertW(a*x)**3/x**6, x)
```

Maxima [F]

$$\int \frac{W(ax)^3}{x^6}\,dx = \int \frac{\mathrm{W}(ax)^3}{x^6}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^6,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^3/x^6, x)
```

Giac [F]

$$\int \frac{W(ax)^3}{x^6}\,dx = \int \frac{\mathrm{W}(ax)^3}{x^6}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^6,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^3/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)^3}{x^6}\,dx = \int \frac{\mathrm{LambertW}(a\,x)^3}{x^6}\,dx$$

input
```
int(LambertW(a*x)^3/x^6,x)
```

output
```
int(LambertW(a*x)^3/x^6, x)
```

Reduce [F]

$$\int \frac{W(ax)^3}{x^6}\,dx = \frac{3\left(\int \frac{lambert\_w(ax)^2}{e^{lambert\_w(ax)}lambert\_w(ax)x^5+e^{lambert\_w(ax)}x^5}dx\right)a\,x^5 - lambert\_w(ax)^3}{5x^5}$$

input
```
int(Lambert_W(a*x)^3/x^6,x)
```

output
```
(3*int(lambert_w(a*x)**2/(e**lambert_w(a*x)*lambert_w(a*x)*x**5 + e**lambe
rt_w(a*x)*x**5),x)*a*x**5 - lambert_w(a*x)**3)/(5*x**5)
```

3.30 $\int \frac{W(ax)^3}{x^7} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 50

$$\int \frac{W(ax)^3}{x^7} \, dx = -18a^6 \operatorname{ExpIntegralEi}(-6W(ax)) - \frac{W(ax)^3}{3x^6} + \frac{W(ax)^4}{2x^6} - \frac{3W(ax)^5}{x^6}$$

output
```
-18*a^6*Ei(-6*LambertW(a*x))-1/3*LambertW(a*x)^3/x^6+1/2*LambertW(a*x)^4/x
^6-3*LambertW(a*x)^5/x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)^3}{x^7} \, dx = -18a^6 \operatorname{ExpIntegralEi}(-6W(ax)) - \frac{W(ax)^3}{3x^6} + \frac{W(ax)^4}{2x^6} - \frac{3W(ax)^5}{x^6}$$

input
```
Integrate[ProductLog[a*x]^3/x^7,x]
```

output
```
-18*a^6*ExpIntegralEi[-6*ProductLog[a*x]] - ProductLog[a*x]^3/(3*x^6) + Pr
oductLog[a*x]^4/(2*x^6) - (3*ProductLog[a*x]^5)/x^6
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {7173, 7206, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)^3}{x^7} dx$$

$$\Big\downarrow \text{7173}$$

$$-\int \frac{W(ax)^4}{x^7(W(ax)+1)}dx - \frac{W(ax)^3}{3x^6}$$

$$\Big\downarrow \text{7206}$$

$$3\int \frac{W(ax)^5}{x^7(W(ax)+1)}dx + \frac{W(ax)^4}{2x^6} - \frac{W(ax)^3}{3x^6}$$

$$\Big\downarrow \text{7206}$$

$$3\left(-6\int \frac{W(ax)^6}{x^7(W(ax)+1)}dx - \frac{W(ax)^5}{x^6}\right) + \frac{W(ax)^4}{2x^6} - \frac{W(ax)^3}{3x^6}$$

$$\Big\downarrow \text{7202}$$

$$3\left(-6a^6 \text{ExpIntegralEi}(-6W(ax)) - \frac{W(ax)^5}{x^6}\right) + \frac{W(ax)^4}{2x^6} - \frac{W(ax)^3}{3x^6}$$

input
```
Int[ProductLog[a*x]^3/x^7,x]
```

output
```
-1/3*ProductLog[a*x]^3/x^6 + ProductLog[a*x]^4/(2*x^6) + 3*(-6*a^6*ExpInte
gralEi[-6*ProductLog[a*x]] - ProductLog[a*x]^5/x^6)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16

method	result
derivativedivides	$a^6\left(-\frac{\mathrm{LambertW}(xa)^3}{3x^6a^6}+\frac{\mathrm{LambertW}(xa)^4}{2x^6a^6}-\frac{3\,\mathrm{LambertW}(xa)^5}{x^6a^6}+18\ \mathrm{expIntegral}_1\left(6\,\mathrm{LambertW}\left(\right.\right.\right.$
default	$a^6\left(-\frac{\mathrm{LambertW}(xa)^3}{3x^6a^6}+\frac{\mathrm{LambertW}(xa)^4}{2x^6a^6}-\frac{3\,\mathrm{LambertW}(xa)^5}{x^6a^6}+18\ \mathrm{expIntegral}_1\left(6\,\mathrm{LambertW}\left(\right.\right.\right.$

input
```
int(LambertW(x*a)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
a^6*(-1/3*LambertW(x*a)^3/x^6/a^6+1/2*LambertW(x*a)^4/x^6/a^6-3*LambertW(x
*a)^5/x^6/a^6+18*Ei(1,6*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{W(ax)^3}{x^7}\,dx = \int \frac{\mathrm{W}(ax)^3}{x^7}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^7,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)^3/x^7, x)
```

Sympy [F]

$$\int \frac{W(ax)^3}{x^7}\,dx = \int \frac{W^3(ax)}{x^7}\,dx$$

input
```
integrate(LambertW(a*x)**3/x**7,x)
```

output
```
Integral(LambertW(a*x)**3/x**7, x)
```

Maxima [F]

$$\int \frac{W(ax)^3}{x^7}\,dx = \int \frac{\mathrm{W}(ax)^3}{x^7}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^7,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^3/x^7, x)
```

Giac [F]

$$\int \frac{W(ax)^3}{x^7}\,dx = \int \frac{W(ax)^3}{x^7}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^7,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^3/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)^3}{x^7}\,dx = \int \frac{\mathrm{LambertW}(a\,x)^3}{x^7}\,dx$$

input
```
int(LambertW(a*x)^3/x^7,x)
```

output
```
int(LambertW(a*x)^3/x^7, x)
```

Reduce [F]

$$\int \frac{W(ax)^3}{x^7}\,dx = \frac{3\left(\int \frac{lambert\_w(ax)^2}{e^{lambert\_w(ax)}lambert\_w(ax)x^6+e^{lambert\_w(ax)}x^6}dx\right)a\,x^6 - lambert\_w(ax)^3}{6x^6}$$

input
```
int(Lambert_W(a*x)^3/x^7,x)
```

output
```
(3*int(lambert_w(a*x)**2/(e**lambert_w(a*x)*lambert_w(a*x)*x**6 + e**lambe
rt_w(a*x)*x**6),x)*a*x**6 - lambert_w(a*x)**3)/(6*x**6)
```

3.31 $\int \frac{W(ax)^3}{x^8}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 67

$$\int \frac{W(ax)^3}{x^8}\,dx = \frac{343}{8}a^7\,\text{ExpIntegralEi}(-7W(ax)) - \frac{W(ax)^3}{4x^7} + \frac{W(ax)^4}{4x^7} - \frac{7W(ax)^5}{8x^7} + \frac{49W(ax)^6}{8x^7}$$

output
```
343/8*a^7*Ei(-7*LambertW(a*x))-1/4*LambertW(a*x)^3/x^7+1/4*LambertW(a*x)^4
/x^7-7/8*LambertW(a*x)^5/x^7+49/8*LambertW(a*x)^6/x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{W(ax)^3}{x^8}\,dx = \frac{343}{8}a^7\,\text{ExpIntegralEi}(-7W(ax)) - \frac{W(ax)^3}{4x^7} + \frac{W(ax)^4}{4x^7} - \frac{7W(ax)^5}{8x^7} + \frac{49W(ax)^6}{8x^7}$$

input
```
Integrate[ProductLog[a*x]^3/x^8,x]
```

output
```
(343*a^7*ExpIntegralEi[-7*ProductLog[a*x]])/8 - ProductLog[a*x]^3/(4*x^7)
+ ProductLog[a*x]^4/(4*x^7) - (7*ProductLog[a*x]^5)/(8*x^7) + (49*ProductL
og[a*x]^6)/(8*x^7)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {7173, 7206, 7206, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax)^3}{x^8}dx$$

$$\Big\downarrow \text{7173}$$

$$-\frac{3}{4}\int \frac{W(ax)^4}{x^8(W(ax)+1)}dx-\frac{W(ax)^3}{4x^7}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{3}{4}\left(-\frac{7}{3}\int \frac{W(ax)^5}{x^8(W(ax)+1)}dx-\frac{W(ax)^4}{3x^7}\right)-\frac{W(ax)^3}{4x^7}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{3}{4}\left(-\frac{7}{3}\left(-\frac{7}{2}\int \frac{W(ax)^6}{x^8(W(ax)+1)}dx-\frac{W(ax)^5}{2x^7}\right)-\frac{W(ax)^4}{3x^7}\right)-\frac{W(ax)^3}{4x^7}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{3}{4}\left(-\frac{7}{3}\left(-\frac{7}{2}\left(-7\int \frac{W(ax)^7}{x^8(W(ax)+1)}dx-\frac{W(ax)^6}{x^7}\right)-\frac{W(ax)^5}{2x^7}\right)-\frac{W(ax)^4}{3x^7}\right)-\frac{W(ax)^3}{4x^7}$$

$$\Big\downarrow \text{7202}$$

$$-\frac{3}{4}\left(-\frac{7}{3}\left(-\frac{7}{2}\left(-7a^7\,\text{ExpIntegralEi}(-7W(ax))-\frac{W(ax)^6}{x^7}\right)-\frac{W(ax)^5}{2x^7}\right)-\frac{W(ax)^4}{3x^7}\right)-\frac{W(ax)^3}{4x^7}$$

input
```
Int[ProductLog[a*x]^3/x^8,x]
```

output
```
-1/4*ProductLog[a*x]^3/x^7 - (3*(-1/3*ProductLog[a*x]^4/x^7 - (7*(-1/2*Pro
ductLog[a*x]^5/x^7 - (7*(-7*a^7*ExpIntegralEi[-7*ProductLog[a*x]] - Produc
tLog[a*x]^6/x^7))/2))/3))/4
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

method	result
derivativedivides	$a^7\left(-\frac{\mathrm{LambertW}(xa)^3}{4x^7a^7}+\frac{\mathrm{LambertW}(xa)^4}{4x^7a^7}-\frac{7\,\mathrm{LambertW}(xa)^5}{8x^7a^7}+\frac{49\,\mathrm{LambertW}(xa)^6}{8x^7a^7}-\frac{343\ \mathrm{expIntegral}_1(7}{8}\right.$
default	$a^7\left(-\frac{\mathrm{LambertW}(xa)^3}{4x^7a^7}+\frac{\mathrm{LambertW}(xa)^4}{4x^7a^7}-\frac{7\,\mathrm{LambertW}(xa)^5}{8x^7a^7}+\frac{49\,\mathrm{LambertW}(xa)^6}{8x^7a^7}-\frac{343\ \mathrm{expIntegral}_1(7}{8}\right.$

input
```
int(LambertW(x*a)^3/x^8,x,method=_RETURNVERBOSE)
```

output
```
a^7*(-1/4*LambertW(x*a)^3/x^7/a^7+1/4*LambertW(x*a)^4/x^7/a^7-7/8*LambertW
(x*a)^5/x^7/a^7+49/8*LambertW(x*a)^6/x^7/a^7-343/8*Ei(1,7*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{W(ax)^3}{x^8}\,dx = \int \frac{\mathrm{W}(ax)^3}{x^8}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^8,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)^3/x^8, x)
```

Sympy [F]

$$\int \frac{W(ax)^3}{x^8}\,dx = \int \frac{W^3(ax)}{x^8}\,dx$$

input
```
integrate(LambertW(a*x)**3/x**8,x)
```

output
```
Integral(LambertW(a*x)**3/x**8, x)
```

Maxima [F]

$$\int \frac{W(ax)^3}{x^8}\,dx = \int \frac{\mathrm{W}(ax)^3}{x^8}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^8,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^3/x^8, x)
```

Giac [F]

$$\int \frac{W(ax)^3}{x^8}\,dx = \int \frac{\mathrm{W}(ax)^3}{x^8}\,dx$$

input
```
integrate(lambert_w(a*x)^3/x^8,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^3/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax)^3}{x^8}\,dx = \int \frac{\mathrm{LambertW}(a\,x)^3}{x^8}\,dx$$

input
```
int(LambertW(a*x)^3/x^8,x)
```

output
```
int(LambertW(a*x)^3/x^8, x)
```

Reduce [F]

$$\int \frac{W(ax)^3}{x^8}\,dx = \frac{3\left(\int \frac{lambert\_w(ax)^2}{e^{lambert\_w(ax)}lambert\_w(ax)x^7+e^{lambert\_w(ax)}x^7}dx\right)a\,x^7 - lambert\_w(ax)^3}{7x^7}$$

input
```
int(Lambert_W(a*x)^3/x^8,x)
```

output
```
(3*int(lambert_w(a*x)**2/(e**lambert_w(a*x)*lambert_w(a*x)*x**7 + e**lambe
rt_w(a*x)*x**7),x)*a*x**7 - lambert_w(a*x)**3)/(7*x**7)
```

3.32 $\int \frac{x^4}{W(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 66

$$\int \frac{x^4}{W(ax)}\,dx=-\frac{6x^5}{3125W(ax)^5}+\frac{6x^5}{625W(ax)^4}-\frac{3x^5}{125W(ax)^3}+\frac{x^5}{25W(ax)^2}+\frac{x^5}{5W(ax)}$$

output
```
-6/3125*x^5/LambertW(a*x)^5+6/625*x^5/LambertW(a*x)^4-3/125*x^5/LambertW(a
*x)^3+1/25*x^5/LambertW(a*x)^2+1/5*x^5/LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{W(ax)}\,dx=-\frac{6x^5}{3125W(ax)^5}+\frac{6x^5}{625W(ax)^4}-\frac{3x^5}{125W(ax)^3}+\frac{x^5}{25W(ax)^2}+\frac{x^5}{5W(ax)}$$

input
```
Integrate[x^4/ProductLog[a*x],x]
```

output
```
(-6*x^5)/(3125*ProductLog[a*x]^5) + (6*x^5)/(625*ProductLog[a*x]^4) - (3*x
^5)/(125*ProductLog[a*x]^3) + x^5/(25*ProductLog[a*x]^2) + x^5/(5*ProductL
og[a*x])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7172, 7205, 7205, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{W(ax)}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{5}\int \frac{x^4}{W(ax)(W(ax)+1)}dx+\frac{x^5}{5W(ax)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{5}\left(\frac{x^5}{5W(ax)^2}-\frac{3}{5}\int \frac{x^4}{W(ax)^2(W(ax)+1)}dx\right)+\frac{x^5}{5W(ax)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{5}\left(\frac{x^5}{5W(ax)^2}-\frac{3}{5}\left(\frac{x^5}{5W(ax)^3}-\frac{2}{5}\int \frac{x^4}{W(ax)^3(W(ax)+1)}dx\right)\right)+\frac{x^5}{5W(ax)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{5}\left(\frac{x^5}{5W(ax)^2}-\frac{3}{5}\left(\frac{x^5}{5W(ax)^3}-\frac{2}{5}\left(\frac{x^5}{5W(ax)^4}-\frac{1}{5}\int \frac{x^4}{W(ax)^4(W(ax)+1)}dx\right)\right)\right)+\frac{x^5}{5W(ax)}$$

$$\Big\downarrow \text{7201}$$

$$\frac{x^5}{5W(ax)}+\frac{1}{5}\left(\frac{x^5}{5W(ax)^2}-\frac{3}{5}\left(\frac{x^5}{5W(ax)^3}-\frac{2}{5}\left(\frac{x^5}{5W(ax)^4}-\frac{x^5}{25W(ax)^5}\right)\right)\right)$$

input
```
Int[x^4/ProductLog[a*x],x]
```

output
```
((-3*((-2*(-1/25*x^5/ProductLog[a*x]^5 + x^5/(5*ProductLog[a*x]^4)))/5 + x
^5/(5*ProductLog[a*x]^3)))/5 + x^5/(5*ProductLog[a*x]^2))/5 + x^5/(5*Produ
ctLog[a*x])
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

method	result	size
derivativedivides	$\frac{\frac{x^5a^5}{25\,\mathrm{LambertW}(xa)^2}-\frac{3x^5a^5}{125\,\mathrm{LambertW}(xa)^3}+\frac{6x^5a^5}{625\,\mathrm{LambertW}(xa)^4}-\frac{6x^5a^5}{3125\,\mathrm{LambertW}(xa)^5}+\frac{x^5a^5}{5\,\mathrm{LambertW}(xa)}}{a^5}$	76
default	$\frac{\frac{x^5a^5}{25\,\mathrm{LambertW}(xa)^2}-\frac{3x^5a^5}{125\,\mathrm{LambertW}(xa)^3}+\frac{6x^5a^5}{625\,\mathrm{LambertW}(xa)^4}-\frac{6x^5a^5}{3125\,\mathrm{LambertW}(xa)^5}+\frac{x^5a^5}{5\,\mathrm{LambertW}(xa)}}{a^5}$	76

input
```
int(x^4/LambertW(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/25/LambertW(x*a)^2*x^5*a^5-3/125/LambertW(x*a)^3*x^5*a^5+6/625/La
mbertW(x*a)^4*x^5*a^5-6/3125*x^5*a^5/LambertW(x*a)^5+1/5/LambertW(x*a)*x^5
*a^5)
```

Fricas [F]

$$\int \frac{x^4}{W(ax)}\,dx = \int \frac{x^4}{\mathrm{W}(ax)}\,dx$$

input
```
integrate(x^4/lambert_w(a*x),x, algorithm="fricas")
```

output
```
integral(x^4/lambert_w(a*x), x)
```

Sympy [F]

$$\int \frac{x^4}{W(ax)}\,dx = \int \frac{x^4}{W\,(ax)}\,dx$$

input
```
integrate(x**4/LambertW(a*x),x)
```

output
```
Integral(x**4/LambertW(a*x), x)
```

Maxima [F]

$$\int \frac{x^4}{W(ax)}\,dx = \int \frac{x^4}{\mathrm{W}(ax)}\,dx$$

input
```
integrate(x^4/lambert_w(a*x),x, algorithm="maxima")
```

output
```
integrate(x^4/lambert_w(a*x), x)
```

Giac [F]

$$\int \frac{x^4}{W(ax)}\,dx = \int \frac{x^4}{\mathrm{W}(ax)}\,dx$$

input
```
integrate(x^4/lambert_w(a*x),x, algorithm="giac")
```

output
```
integrate(x^4/lambert_w(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{W(ax)}\,dx = \int \frac{x^4}{\mathrm{LambertW}\,(a\,x)}\,dx$$

input
```
int(x^4/LambertW(a*x),x)
```

output
```
int(x^4/LambertW(a*x), x)
```

Reduce [F]

$$\int \frac{x^4}{W(ax)}\,dx = \int \frac{x^4}{lambert\_w\,(ax)}dx$$

input
```
int(x^4/Lambert_W(a*x),x)
```

output
```
int(x**4/lambert_w(a*x),x)
```

3.33 $\int \frac{x^3}{W(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \frac{x^3}{W(ax)}\,dx=\frac{x^4}{128W(ax)^4}-\frac{x^4}{32W(ax)^3}+\frac{x^4}{16W(ax)^2}+\frac{x^4}{4W(ax)}$$

output
```
1/128*x^4/LambertW(a*x)^4-1/32*x^4/LambertW(a*x)^3+1/16*x^4/LambertW(a*x)^
2+1/4*x^4/LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{W(ax)}\,dx=\frac{x^4}{128W(ax)^4}-\frac{x^4}{32W(ax)^3}+\frac{x^4}{16W(ax)^2}+\frac{x^4}{4W(ax)}$$

input
```
Integrate[x^3/ProductLog[a*x],x]
```

output
```
x^4/(128*ProductLog[a*x]^4) - x^4/(32*ProductLog[a*x]^3) + x^4/(16*Product
Log[a*x]^2) + x^4/(4*ProductLog[a*x])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {7172, 7205, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{W(ax)}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{4}\int \frac{x^3}{W(ax)(W(ax)+1)}dx+\frac{x^4}{4W(ax)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{4}\left(\frac{x^4}{4W(ax)^2}-\frac{1}{2}\int \frac{x^3}{W(ax)^2(W(ax)+1)}dx\right)+\frac{x^4}{4W(ax)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{x^3}{W(ax)^3(W(ax)+1)}dx-\frac{x^4}{4W(ax)^3}\right)+\frac{x^4}{4W(ax)^2}\right)+\frac{x^4}{4W(ax)}$$

$$\Big\downarrow \text{7201}$$

$$\frac{x^4}{4W(ax)}+\frac{1}{4}\left(\frac{x^4}{4W(ax)^2}+\frac{1}{2}\left(\frac{x^4}{16W(ax)^4}-\frac{x^4}{4W(ax)^3}\right)\right)$$

input
```
Int[x^3/ProductLog[a*x],x]
```

output
```
((x^4/(16*ProductLog[a*x]^4) - x^4/(4*ProductLog[a*x]^3))/2 + x^4/(4*Produ
ctLog[a*x]^2))/4 + x^4/(4*ProductLog[a*x])
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{\frac{x^4a^4}{4\,\mathrm{LambertW}(xa)}+\frac{x^4a^4}{16\,\mathrm{LambertW}(xa)^2}-\frac{x^4a^4}{32\,\mathrm{LambertW}(xa)^3}+\frac{x^4a^4}{128\,\mathrm{LambertW}(xa)^4}}{a^4}$	62
default	$\frac{\frac{x^4a^4}{4\,\mathrm{LambertW}(xa)}+\frac{x^4a^4}{16\,\mathrm{LambertW}(xa)^2}-\frac{x^4a^4}{32\,\mathrm{LambertW}(xa)^3}+\frac{x^4a^4}{128\,\mathrm{LambertW}(xa)^4}}{a^4}$	62

input
```
int(x^3/LambertW(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/4/LambertW(x*a)*x^4*a^4+1/16*x^4*a^4/LambertW(x*a)^2-1/32/Lambert
W(x*a)^3*x^4*a^4+1/128*x^4*a^4/LambertW(x*a)^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{W(ax)}\,dx = \frac{32\,x^4\,\mathrm{W}(ax)^3 + 8\,x^4\,\mathrm{W}(ax)^2 - 4\,x^4\,\mathrm{W}(ax) + x^4}{128\,\mathrm{W}(ax)^4}$$

input
```
integrate(x^3/lambert_w(a*x),x, algorithm="fricas")
```

output
```
1/128*(32*x^4*lambert_w(a*x)^3 + 8*x^4*lambert_w(a*x)^2 - 4*x^4*lambert_w(
a*x) + x^4)/lambert_w(a*x)^4
```

Sympy [F]

$$\int \frac{x^3}{W(ax)}\,dx = \int \frac{x^3}{W(ax)}\,dx$$

input
```
integrate(x**3/LambertW(a*x),x)
```

output
```
Integral(x**3/LambertW(a*x), x)
```

Maxima [F]

$$\int \frac{x^3}{W(ax)}\,dx = \int \frac{x^3}{\mathrm{W}(ax)}\,dx$$

input
```
integrate(x^3/lambert_w(a*x),x, algorithm="maxima")
```

output
```
integrate(x^3/lambert_w(a*x), x)
```

Giac [F]

$$\int \frac{x^3}{W(ax)}\,dx = \int \frac{x^3}{\mathrm{W}(ax)}\,dx$$

input
```
integrate(x^3/lambert_w(a*x),x, algorithm="giac")
```

output
```
integrate(x^3/lambert_w(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{W(ax)}\,dx = \int \frac{x^3}{\mathrm{LambertW}\,(a\,x)}\,dx$$

input
```
int(x^3/LambertW(a*x),x)
```

output
```
int(x^3/LambertW(a*x), x)
```

Reduce [F]

$$\int \frac{x^3}{W(ax)}\,dx = \int \frac{x^3}{lambert\_w\,(ax)}dx$$

input
```
int(x^3/Lambert_W(a*x),x)
```

output
```
int(x**3/lambert_w(a*x),x)
```

3.34 $\int \frac{x^2}{W(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 40

$$\int \frac{x^2}{W(ax)}\,dx = -\frac{x^3}{27W(ax)^3} + \frac{x^3}{9W(ax)^2} + \frac{x^3}{3W(ax)}$$

output
```
-1/27*x^3/LambertW(a*x)^3+1/9*x^3/LambertW(a*x)^2+1/3*x^3/LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{W(ax)}\,dx = -\frac{x^3}{27W(ax)^3} + \frac{x^3}{9W(ax)^2} + \frac{x^3}{3W(ax)}$$

input
```
Integrate[x^2/ProductLog[a*x],x]
```

output
```
-1/27*x^3/ProductLog[a*x]^3 + x^3/(9*ProductLog[a*x]^2) + x^3/(3*ProductLo
g[a*x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7172, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{W(ax)}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{3}\int \frac{x^2}{W(ax)(W(ax)+1)}dx+\frac{x^3}{3W(ax)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{3}\left(\frac{x^3}{3W(ax)^2}-\frac{1}{3}\int \frac{x^2}{W(ax)^2(W(ax)+1)}dx\right)+\frac{x^3}{3W(ax)}$$

$$\Big\downarrow \text{7201}$$

$$\frac{x^3}{3W(ax)}+\frac{1}{3}\left(\frac{x^3}{3W(ax)^2}-\frac{x^3}{9W(ax)^3}\right)$$

input
```
Int[x^2/ProductLog[a*x],x]
```

output
```
(-1/9*x^3/ProductLog[a*x]^3 + x^3/(3*ProductLog[a*x]^2))/3 + x^3/(3*Produc
tLog[a*x])
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{\frac{x^3a^3}{9\,\mathrm{LambertW}(xa)^2}-\frac{x^3a^3}{27\,\mathrm{LambertW}(xa)^3}+\frac{x^3a^3}{3\,\mathrm{LambertW}(xa)}}{a^3}$	48
default	$\frac{\frac{x^3a^3}{9\,\mathrm{LambertW}(xa)^2}-\frac{x^3a^3}{27\,\mathrm{LambertW}(xa)^3}+\frac{x^3a^3}{3\,\mathrm{LambertW}(xa)}}{a^3}$	48

input
```
int(x^2/LambertW(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/9/LambertW(x*a)^2*x^3*a^3-1/27*x^3*a^3/LambertW(x*a)^3+1/3/Lamber
tW(x*a)*x^3*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{W(ax)}\,dx = \frac{9\,x^3\,\mathrm{W}(ax)^2+3\,x^3\,\mathrm{W}(ax)-x^3}{27\,\mathrm{W}(ax)^3}$$

input
```
integrate(x^2/lambert_w(a*x),x, algorithm="fricas")
```

output
```
1/27*(9*x^3*lambert_w(a*x)^2 + 3*x^3*lambert_w(a*x) - x^3)/lambert_w(a*x)^
3
```

Sympy [F]

$$\int \frac{x^2}{W(ax)}\,dx = \int \frac{x^2}{W(ax)}\,dx$$

input
```
integrate(x**2/LambertW(a*x),x)
```

output
```
Integral(x**2/LambertW(a*x), x)
```

Maxima [F]

$$\int \frac{x^2}{W(ax)}\,dx = \int \frac{x^2}{\mathrm{W}(ax)}\,dx$$

input
```
integrate(x^2/lambert_w(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/lambert_w(a*x), x)
```

Giac [F]

$$\int \frac{x^2}{W(ax)}\,dx = \int \frac{x^2}{\mathrm{W}(ax)}\,dx$$

input
```
integrate(x^2/lambert_w(a*x),x, algorithm="giac")
```

output
```
integrate(x^2/lambert_w(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{W(ax)}\,dx = \int \frac{x^2}{\mathrm{LambertW}\left(a\,x\right)}\,dx$$

input
```
int(x^2/LambertW(a*x),x)
```

output
```
int(x^2/LambertW(a*x), x)
```

Reduce [F]

$$\int \frac{x^2}{W(ax)}\,dx = \int \frac{x^2}{lambert\_w\left(ax\right)}dx$$

input
```
int(x^2/Lambert_W(a*x),x)
```

output
```
int(x**2/lambert_w(a*x),x)
```

3.35 $\int \frac{x}{W(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \frac{x}{W(ax)}\,dx = \frac{x^2}{4W(ax)^2} + \frac{x^2}{2W(ax)}$$

output
```
1/4*x^2/LambertW(a*x)^2+1/2*x^2/LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x}{W(ax)}\,dx = \frac{x^2}{4W(ax)^2} + \frac{x^2}{2W(ax)}$$

input
```
Integrate[x/ProductLog[a*x],x]
```

output
```
x^2/(4*ProductLog[a*x]^2) + x^2/(2*ProductLog[a*x])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7172, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{W(ax)}dx$$
$$\Big\downarrow \text{7172}$$
$$\frac{1}{2}\int \frac{x}{W(ax)(W(ax)+1)}dx+\frac{x^2}{2W(ax)}$$
$$\Big\downarrow \text{7201}$$
$$\frac{x^2}{2W(ax)}+\frac{x^2}{4W(ax)^2}$$

input
```
Int[x/ProductLog[a*x],x]
```

output
```
x^2/(4*ProductLog[a*x]^2) + x^2/(2*ProductLog[a*x])
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$-\frac{-2x^2 \operatorname{LambertW}(xa)-x^2}{4 \operatorname{LambertW}(xa)^2}$	24
derivativedivides	$\frac{\frac{x^2a^2}{4 \operatorname{LambertW}(xa)^2}+\frac{x^2a^2}{2 \operatorname{LambertW}(xa)}}{a^2}$	34
default	$\frac{\frac{x^2a^2}{4 \operatorname{LambertW}(xa)^2}+\frac{x^2a^2}{2 \operatorname{LambertW}(xa)}}{a^2}$	34

input
```
int(x/LambertW(x*a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*x^2*LambertW(x*a)-x^2)/LambertW(x*a)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x}{W(ax)}\,dx = \frac{2\,x^2\,\mathrm{W}(ax)+x^2}{4\,\mathrm{W}(ax)^2}$$

input
```
integrate(x/lambert_w(a*x),x, algorithm="fricas")
```

output
```
1/4*(2*x^2*lambert_w(a*x) + x^2)/lambert_w(a*x)^2
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x}{W(ax)}\,dx = \frac{x^2}{2W(ax)} + \frac{x^2}{4W^2(ax)}$$

input
```
integrate(x/LambertW(a*x),x)
```

output
```
x**2/(2*LambertW(a*x)) + x**2/(4*LambertW(a*x)**2)
```

Maxima [F]

$$\int \frac{x}{W(ax)}\,dx = \int \frac{x}{\mathrm{W}(ax)}\,dx$$

input
```
integrate(x/lambert_w(a*x),x, algorithm="maxima")
```

output
```
integrate(x/lambert_w(a*x), x)
```

Giac [F]

$$\int \frac{x}{W(ax)}\,dx = \int \frac{x}{\mathrm{W}(ax)}\,dx$$

input
```
integrate(x/lambert_w(a*x),x, algorithm="giac")
```

output
```
integrate(x/lambert_w(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{W(ax)}\,dx = \int \frac{x}{\mathrm{LambertW}\,(a\,x)}\,dx$$

input
```
int(x/LambertW(a*x),x)
```

output
```
int(x/LambertW(a*x), x)
```

Reduce [F]

$$\int \frac{x}{W(ax)}\,dx = \int \frac{x}{lambert\_w\,(ax)}dx$$

input
```
int(x/Lambert_W(a*x),x)
```

output
```
int(x/lambert_w(a*x),x)
```

3.36 $\int \frac{1}{W(ax)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 18

$$\int \frac{1}{W(ax)}\,dx = \frac{\text{ExpIntegralEi}(W(ax))}{a} + \frac{x}{W(ax)}$$

output
```
Ei(LambertW(a*x))/a+x/LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{W(ax)}\,dx = \frac{\text{ExpIntegralEi}(W(ax))}{a} + \frac{x}{W(ax)}$$

input
```
Integrate[ProductLog[a*x]^(-1),x]
```

output
```
ExpIntegralEi[ProductLog[a*x]]/a + x/ProductLog[a*x]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7167, 7179}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W(ax)}dx$$

$$\Big\downarrow \text{7167}$$

$$\int \frac{1}{W(ax)(W(ax)+1)}dx+\frac{x}{W(ax)}$$

$$\Big\downarrow \text{7179}$$

$$\frac{\text{ExpIntegralEi}(W(ax))}{a}+\frac{x}{W(ax)}$$

input
```
Int[ProductLog[a*x]^(-1),x]
```

output
```
ExpIntegralEi[ProductLog[a*x]]/a + x/ProductLog[a*x]
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7179
```
Int[1/(ProductLog[(a_.) + (b_.)*(x_)]*((d_) + (d_.)*ProductLog[(a_.) + (b_.
)*(x_)])), x_Symbol] :> Simp[ExpIntegralEi[ProductLog[a + b*x]]/(b*d), x] /
; FreeQ[{a, b, d}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

method	result	size
derivativedivides	$\frac{-\text{expIntegral}_1(-\text{LambertW}(xa))+\frac{xa}{\text{LambertW}(xa)}}{a}$	25
default	$\frac{-\text{expIntegral}_1(-\text{LambertW}(xa))+\frac{xa}{\text{LambertW}(xa)}}{a}$	25

input
```
int(1/LambertW(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-Ei(1,-LambertW(x*a))+x*a/LambertW(x*a))
```

Fricas [F]

$$\int \frac{1}{W(ax)}\,dx = \int \frac{1}{\mathrm{W}(ax)}\,dx$$

input
```
integrate(1/lambert_w(a*x),x, algorithm="fricas")
```

output
```
integral(1/lambert_w(a*x), x)
```

Sympy [F]

$$\int \frac{1}{W(ax)}\,dx = \int \frac{1}{W(ax)}\,dx$$

input
```
integrate(1/LambertW(a*x),x)
```

output
```
Integral(1/LambertW(a*x), x)
```

Maxima [F]

$$\int \frac{1}{W(ax)}\,dx = \int \frac{1}{\mathrm{W}(ax)}\,dx$$

input
```
integrate(1/lambert_w(a*x),x, algorithm="maxima")
```

output
```
integrate(1/lambert_w(a*x), x)
```

Giac [F]

$$\int \frac{1}{W(ax)}\,dx = \int \frac{1}{\mathrm{W}(ax)}\,dx$$

input
```
integrate(1/lambert_w(a*x),x, algorithm="giac")
```

output
```
integrate(1/lambert_w(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W(ax)}\,dx = \int \frac{1}{\mathrm{LambertW}\left(a\,x\right)}\,dx$$

input
```
int(1/LambertW(a*x),x)
```

output
```
int(1/LambertW(a*x), x)
```

Reduce [F]

$$\int \frac{1}{W(ax)}\,dx = \frac{e^{lambert\_w(ax)} + \left(\int \frac{1}{lambert\_w(ax)^2+lambert\_w(ax)}dx\right)a}{a}$$

input
```
int(1/Lambert_W(a*x),x)
```

output
```
(e**lambert_w(a*x) + int(1/(lambert_w(a*x)**2 + lambert_w(a*x)),x)*a)/a
```

3.37 $\int \frac{1}{xW(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 14

$$\int \frac{1}{xW(ax)}\,dx = \log(W(ax)) - \frac{1}{W(ax)}$$

output
```
ln(LambertW(a*x))-1/LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{xW(ax)}\,dx = \log(W(ax)) - \frac{1}{W(ax)}$$

input
```
Integrate[1/(x*ProductLog[a*x]),x]
```

output
```
Log[ProductLog[a*x]] - ProductLog[a*x]^(-1)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7173, 7195}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{xW(ax)}dx$$

$$\Big\downarrow \; 7173$$

$$\int \frac{1}{x(W(ax)+1)}dx-\frac{1}{W(ax)}$$

$$\Big\downarrow \; 7195$$

$$\log(W(ax))-\frac{1}{W(ax)}$$

input
```
Int[1/(x*ProductLog[a*x]),x]
```

output
```
Log[ProductLog[a*x]] - ProductLog[a*x]^(-1)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7195
```
Int[1/((x_)*((d_) + (d_.)*ProductLog[(a_.)*(x_)])), x_Symbol] :> Simp[Log[P
roductLog[a*x]]/d, x] /; FreeQ[{a, d}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\ln(\text{LambertW}(xa)) - \frac{1}{\text{LambertW}(xa)}$	15
default	$\ln(\text{LambertW}(xa)) - \frac{1}{\text{LambertW}(xa)}$	15
parallelrisch	$-\frac{1-\ln(x)\,\text{LambertW}(xa)+\text{LambertW}(xa)^2}{\text{LambertW}(xa)}$	25

input
```
int(1/x/LambertW(x*a),x,method=_RETURNVERBOSE)
```

output
```
ln(LambertW(x*a))-1/LambertW(x*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{1}{xW(ax)}\,dx = \frac{\mathrm{W}(ax)\log\left(\mathrm{W}(ax)\right) - 1}{\mathrm{W}(ax)}$$

input
```
integrate(1/x/lambert_w(a*x),x, algorithm="fricas")
```

output
```
(lambert_w(a*x)*log(lambert_w(a*x)) - 1)/lambert_w(a*x)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{xW(ax)}\,dx = \log(x) - W(ax) - \frac{1}{W(ax)}$$

input
```
integrate(1/x/LambertW(a*x),x)
```

output
```
log(x) - LambertW(a*x) - 1/LambertW(a*x)
```

Maxima [F]

$$\int \frac{1}{xW(ax)}\,dx = \int \frac{1}{x\,\mathrm{W}(ax)}\,dx$$

input
```
integrate(1/x/lambert_w(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(x*lambert_w(a*x)), x)
```

Giac [F]

$$\int \frac{1}{xW(ax)}\,dx = \int \frac{1}{x\,\mathrm{W}(ax)}\,dx$$

input
```
integrate(1/x/lambert_w(a*x),x, algorithm="giac")
```

output
```
integrate(1/(x*lambert_w(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{xW(ax)}\,dx = \int \frac{1}{x\,\mathrm{LambertW}\,(a\,x)}\,dx$$

input
```
int(1/(x*LambertW(a*x)),x)
```

output
```
int(1/(x*LambertW(a*x)), x)
```

Reduce [F]

$$\int \frac{1}{xW(ax)}\,dx = \int \frac{1}{lambert\_w(ax)\,x}dx$$

input
```
int(1/x/Lambert_W(a*x),x)
```

output
```
int(1/(lambert_w(a*x)*x),x)
```

3.38 $\int \frac{1}{x^2 W(ax)} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 33

$$\int \frac{1}{x^2 W(ax)} \, dx = -\frac{1}{2x} - \frac{1}{2} a \, \text{ExpIntegralEi}(-W(ax)) - \frac{1}{2xW(ax)}$$

output
```
-1/2/x-1/2*a*Ei(-LambertW(a*x))-1/2/x/LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 W(ax)} \, dx = -\frac{1}{2x} - \frac{1}{2} a \, \text{ExpIntegralEi}(-W(ax)) - \frac{1}{2xW(ax)}$$

input
```
Integrate[1/(x^2*ProductLog[a*x]),x]
```

output
```
-1/2*1/x - (a*ExpIntegralEi[-ProductLog[a*x]])/2 - 1/(2*x*ProductLog[a*x])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7173, 7196, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 W(ax)} dx$$
$$\Big\downarrow \text{ 7173}$$
$$\frac{1}{2}\int \frac{1}{x^2(W(ax)+1)}dx - \frac{1}{2xW(ax)}$$
$$\Big\downarrow \text{ 7196}$$
$$\frac{1}{2}\left(-\int \frac{W(ax)}{x^2(W(ax)+1)}dx - \frac{1}{x}\right) - \frac{1}{2xW(ax)}$$
$$\Big\downarrow \text{ 7202}$$
$$\frac{1}{2}\left(-a\,\text{ExpIntegralEi}(-W(ax)) - \frac{1}{x}\right) - \frac{1}{2xW(ax)}$$

input
```
Int[1/(x^2*ProductLog[a*x]),x]
```

output
```
(-x^(-1) - a*ExpIntegralEi[-ProductLog[a*x]])/2 - 1/(2*x*ProductLog[a*x])
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$a\left(-\frac{1}{2\,\mathrm{LambertW}(xa)xa}-\frac{1}{2ax}+\frac{\mathrm{expIntegral}_1(\mathrm{LambertW}(xa))}{2}\right)$	34
default	$a\left(-\frac{1}{2\,\mathrm{LambertW}(xa)xa}-\frac{1}{2ax}+\frac{\mathrm{expIntegral}_1(\mathrm{LambertW}(xa))}{2}\right)$	34

input
```
int(1/x^2/LambertW(x*a),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/LambertW(x*a)/x/a-1/2/a/x+1/2*Ei(1,LambertW(x*a)))
```

Fricas [F]

$$\int \frac{1}{x^2W(ax)}\,dx=\int \frac{1}{x^2\,\mathrm{W}(ax)}\,dx$$

input
```
integrate(1/x^2/lambert_w(a*x),x, algorithm="fricas")
```

output
```
integral(1/(x^2*lambert_w(a*x)), x)
```

Sympy [F]

$$\int \frac{1}{x^2 W(ax)}\,dx = \int \frac{1}{x^2 W(ax)}\,dx$$

input
```
integrate(1/x**2/LambertW(a*x),x)
```

output
```
Integral(1/(x**2*LambertW(a*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 W(ax)}\,dx = \int \frac{1}{x^2\,\mathrm{W}(ax)}\,dx$$

input
```
integrate(1/x^2/lambert_w(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*lambert_w(a*x)), x)
```

Giac [F]

$$\int \frac{1}{x^2 W(ax)}\,dx = \int \frac{1}{x^2\,\mathrm{W}(ax)}\,dx$$

input
```
integrate(1/x^2/lambert_w(a*x),x, algorithm="giac")
```

output
```
integrate(1/(x^2*lambert_w(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2W(ax)}\,dx = \int \frac{1}{x^2\,\mathrm{LambertW}\left(a\,x\right)}\,dx$$

input
```
int(1/(x^2*LambertW(a*x)),x)
```

output
```
int(1/(x^2*LambertW(a*x)), x)
```

Reduce [F]

$$\int \frac{1}{x^2W(ax)}\,dx = \frac{-\left(\int \frac{1}{x^2}dx\right)x + 2\left(\int \frac{1}{lambert\_w(ax)x^2}dx\right)x - 1}{2x}$$

input
```
int(1/x^2/Lambert_W(a*x),x)
```

output
```
( - int(1/x**2,x)*x + 2*int(1/(lambert_w(a*x)*x**2),x)*x - 1)/(2*x)
```

3.39 $\int \frac{1}{x^3 W(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{1}{x^3 W(ax)}\,dx = -\frac{1}{6x^2} + \frac{2}{3}a^2 \operatorname{ExpIntegralEi}(-2W(ax)) - \frac{1}{3x^2 W(ax)} + \frac{W(ax)}{3x^2}$$

output
```
-1/6/x^2+2/3*a^2*Ei(-2*LambertW(a*x))-1/3/x^2/LambertW(a*x)+1/3*LambertW(a
*x)/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 W(ax)}\,dx = -\frac{1}{6x^2} + \frac{2}{3}a^2 \operatorname{ExpIntegralEi}(-2W(ax)) - \frac{1}{3x^2 W(ax)} + \frac{W(ax)}{3x^2}$$

input
```
Integrate[1/(x^3*ProductLog[a*x]),x]
```

output
```
-1/6*1/x^2 + (2*a^2*ExpIntegralEi[-2*ProductLog[a*x]])/3 - 1/(3*x^2*Produc
tLog[a*x]) + ProductLog[a*x]/(3*x^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {7173, 7196, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 W(ax)} \, dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{1}{3}\int \frac{1}{x^3(W(ax)+1)}dx-\frac{1}{3x^2W(ax)}$$

$$\Big\downarrow \text{7196}$$

$$\frac{1}{3}\left(-\int \frac{W(ax)}{x^3(W(ax)+1)}dx-\frac{1}{2x^2}\right)-\frac{1}{3x^2W(ax)}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{3}\left(2\int \frac{W(ax)^2}{x^3(W(ax)+1)}dx+\frac{W(ax)}{x^2}-\frac{1}{2x^2}\right)-\frac{1}{3x^2W(ax)}$$

$$\Big\downarrow \text{7202}$$

$$\frac{1}{3}\left(2a^2\,\text{ExpIntegralEi}(-2W(ax))+\frac{W(ax)}{x^2}-\frac{1}{2x^2}\right)-\frac{1}{3x^2W(ax)}$$

input
```
Int[1/(x^3*ProductLog[a*x]),x]
```

output
```
-1/3*1/(x^2*ProductLog[a*x]) + (-1/2*1/x^2 + 2*a^2*ExpIntegralEi[-2*Produc
tLog[a*x]] + ProductLog[a*x]/x^2)/3
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$a^{2}\left(-\frac{1}{3\,\mathrm{LambertW}(xa)\,x^{2}a^{2}}-\frac{1}{6x^{2}a^{2}}+\frac{\mathrm{LambertW}(xa)}{3x^{2}a^{2}}-\frac{2\,\mathrm{expIntegral}_{1}(2\,\mathrm{LambertW}(xa))}{3}\right)$	50
default	$a^{2}\left(-\frac{1}{3\,\mathrm{LambertW}(xa)\,x^{2}a^{2}}-\frac{1}{6x^{2}a^{2}}+\frac{\mathrm{LambertW}(xa)}{3x^{2}a^{2}}-\frac{2\,\mathrm{expIntegral}_{1}(2\,\mathrm{LambertW}(xa))}{3}\right)$	50

input
```
int(1/x^3/LambertW(x*a),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/3/LambertW(x*a)/x^2/a^2-1/6/x^2/a^2+1/3*LambertW(x*a)/x^2/a^2-2/3*
Ei(1,2*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{1}{x^3W(ax)}\,dx = \int \frac{1}{x^3\,\mathrm{W}(ax)}\,dx$$

input
```
integrate(1/x^3/lambert_w(a*x),x, algorithm="fricas")
```

output
```
integral(1/(x^3*lambert_w(a*x)), x)
```

Sympy [F]

$$\int \frac{1}{x^3W(ax)}\,dx = \int \frac{1}{x^3W\left(ax\right)}\,dx$$

input
```
integrate(1/x**3/LambertW(a*x),x)
```

output
```
Integral(1/(x**3*LambertW(a*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^3W(ax)}\,dx = \int \frac{1}{x^3\,\mathrm{W}(ax)}\,dx$$

input
```
integrate(1/x^3/lambert_w(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(x^3*lambert_w(a*x)), x)
```

Giac [F]

$$\int \frac{1}{x^3W(ax)}\,dx = \int \frac{1}{x^3\,\mathrm{W}(ax)}\,dx$$

input
```
integrate(1/x^3/lambert_w(a*x),x, algorithm="giac")
```

output
```
integrate(1/(x^3*lambert_w(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3W(ax)}\,dx = \int \frac{1}{x^3\,\mathrm{LambertW}\,(a\,x)}\,dx$$

input
```
int(1/(x^3*LambertW(a*x)),x)
```

output
```
int(1/(x^3*LambertW(a*x)), x)
```

Reduce [F]

$$\int \frac{1}{x^3W(ax)}\,dx = \frac{-2\left(\int \frac{1}{x^3}dx\right)x^2+4\left(\int \frac{1}{lambert\_w(ax)\,x^3}dx\right)x^2-1}{4x^2}$$

input
```
int(1/x^3/Lambert_W(a*x),x)
```

output
```
( - 2*int(1/x**3,x)*x**2 + 4*int(1/(lambert_w(a*x)*x**3),x)*x**2 - 1)/(4*x
**2)
```

3.40 $\int \frac{1}{x^4 W(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 59

$$\int \frac{1}{x^4 W(ax)}\,dx = -\frac{1}{12x^3} - \frac{9}{8}a^3 \operatorname{ExpIntegralEi}(-3W(ax)) - \frac{1}{4x^3 W(ax)} + \frac{W(ax)}{8x^3} - \frac{3W(ax)^2}{8x^3}$$

output
```
-1/12/x^3-9/8*a^3*Ei(-3*LambertW(a*x))-1/4/x^3/LambertW(a*x)+1/8*LambertW(
a*x)/x^3-3/8*LambertW(a*x)^2/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 W(ax)}\,dx = -\frac{1}{12x^3} - \frac{9}{8}a^3 \operatorname{ExpIntegralEi}(-3W(ax)) - \frac{1}{4x^3 W(ax)} + \frac{W(ax)}{8x^3} - \frac{3W(ax)^2}{8x^3}$$

input
```
Integrate[1/(x^4*ProductLog[a*x]),x]
```

output
```
-1/12*1/x^3 - (9*a^3*ExpIntegralEi[-3*ProductLog[a*x]])/8 - 1/(4*x^3*Produ
ctLog[a*x]) + ProductLog[a*x]/(8*x^3) - (3*ProductLog[a*x]^2)/(8*x^3)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7173, 7196, 7206, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 W(ax)} \, dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{1}{4} \int \frac{1}{x^4 (W(ax)+1)} dx - \frac{1}{4x^3 W(ax)}$$

$$\Big\downarrow \text{7196}$$

$$\frac{1}{4} \left(-\int \frac{W(ax)}{x^4 (W(ax)+1)} dx - \frac{1}{3x^3} \right) - \frac{1}{4x^3 W(ax)}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{4} \left(\frac{3}{2} \int \frac{W(ax)^2}{x^4 (W(ax)+1)} dx + \frac{W(ax)}{2x^3} - \frac{1}{3x^3} \right) - \frac{1}{4x^3 W(ax)}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{4} \left(\frac{3}{2} \left(-3 \int \frac{W(ax)^3}{x^4 (W(ax)+1)} dx - \frac{W(ax)^2}{x^3} \right) + \frac{W(ax)}{2x^3} - \frac{1}{3x^3} \right) - \frac{1}{4x^3 W(ax)}$$

$$\Big\downarrow \text{7202}$$

$$\frac{1}{4} \left(\frac{3}{2} \left(-3a^3 \operatorname{ExpIntegralEi}(-3W(ax)) - \frac{W(ax)^2}{x^3} \right) + \frac{W(ax)}{2x^3} - \frac{1}{3x^3} \right) - \frac{1}{4x^3 W(ax)}$$

input
```
Int[1/(x^4*ProductLog[a*x]),x]
```

output
```
-1/4*1/(x^3*ProductLog[a*x]) + (-1/3*1/x^3 + ProductLog[a*x]/(2*x^3) + (3*
(-3*a^3*ExpIntegralEi[-3*ProductLog[a*x]] - ProductLog[a*x]^2/x^3))/2)/4
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

method	result
derivativedivides	$a^3\left(-\frac{1}{4\,\text{LambertW}(xa)x^3a^3}-\frac{1}{12x^3a^3}+\frac{\text{LambertW}(xa)}{8x^3a^3}-\frac{3\,\text{LambertW}(xa)^2}{8x^3a^3}+\frac{9\ \text{expIntegral}_1(3\,\text{LambertW}$
default	$a^3\left(-\frac{1}{4\,\text{LambertW}(xa)x^3a^3}-\frac{1}{12x^3a^3}+\frac{\text{LambertW}(xa)}{8x^3a^3}-\frac{3\,\text{LambertW}(xa)^2}{8x^3a^3}+\frac{9\ \text{expIntegral}_1(3\,\text{LambertW}$

input
```
int(1/x^4/LambertW(x*a),x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/4/LambertW(x*a)/x^3/a^3-1/12/x^3/a^3+1/8*LambertW(x*a)/x^3/a^3-3/8
*LambertW(x*a)^2/x^3/a^3+9/8*Ei(1,3*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{1}{x^4W(ax)}\,dx=\int \frac{1}{x^4\,\text{W}(ax)}\,dx$$

input
```
integrate(1/x^4/lambert_w(a*x),x, algorithm="fricas")
```

output
```
integral(1/(x^4*lambert_w(a*x)), x)
```

Sympy [F]

$$\int \frac{1}{x^4W(ax)}\,dx=\int \frac{1}{x^4W\left(ax\right)}\,dx$$

input
```
integrate(1/x**4/LambertW(a*x),x)
```

output
```
Integral(1/(x**4*LambertW(a*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 W(ax)}\,dx = \int \frac{1}{x^4\,\mathrm{W}(ax)}\,dx$$

input
```
integrate(1/x^4/lambert_w(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(x^4*lambert_w(a*x)), x)
```

Giac [F]

$$\int \frac{1}{x^4 W(ax)}\,dx = \int \frac{1}{x^4\,\mathrm{W}(ax)}\,dx$$

input
```
integrate(1/x^4/lambert_w(a*x),x, algorithm="giac")
```

output
```
integrate(1/(x^4*lambert_w(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 W(ax)}\,dx = \int \frac{1}{x^4\,\mathrm{LambertW}\,(a\,x)}\,dx$$

input
```
int(1/(x^4*LambertW(a*x)),x)
```

output
```
int(1/(x^4*LambertW(a*x)), x)
```

Reduce [F]

$$\int \frac{1}{x^4W(ax)}\,dx = \frac{-3\left(\int \frac{1}{x^4}dx\right)x^3 + 6\left(\int \frac{1}{lambert\_w(ax)\,x^4}dx\right)x^3 - 1}{6x^3}$$

input
```
int(1/x^4/Lambert_W(a*x),x)
```

output
```
( - 3*int(1/x**4,x)*x**3 + 6*int(1/(lambert_w(a*x)*x**4),x)*x**3 - 1)/(6*x
**3)
```

3.41 $\int \frac{x^5}{W(ax)^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 66

$$\int \frac{x^5}{W(ax)^2} \, dx = -\frac{x^6}{648W(ax)^6} + \frac{x^6}{108W(ax)^5} - \frac{x^6}{36W(ax)^4} + \frac{x^6}{18W(ax)^3} + \frac{x^6}{6W(ax)^2}$$

output
```
-1/648*x^6/LambertW(a*x)^6+1/108*x^6/LambertW(a*x)^5-1/36*x^6/LambertW(a*x
)^4+1/18*x^6/LambertW(a*x)^3+1/6*x^6/LambertW(a*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{W(ax)^2} \, dx = -\frac{x^6}{648W(ax)^6} + \frac{x^6}{108W(ax)^5} - \frac{x^6}{36W(ax)^4} + \frac{x^6}{18W(ax)^3} + \frac{x^6}{6W(ax)^2}$$

input
```
Integrate[x^5/ProductLog[a*x]^2,x]
```

output
```
-1/648*x^6/ProductLog[a*x]^6 + x^6/(108*ProductLog[a*x]^5) - x^6/(36*Produ
ctLog[a*x]^4) + x^6/(18*ProductLog[a*x]^3) + x^6/(6*ProductLog[a*x]^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7172, 7205, 7205, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{W(ax)^2}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{3}\int \frac{x^5}{W(ax)^2(W(ax)+1)}dx+\frac{x^6}{6W(ax)^2}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{3}\left(\frac{x^6}{6W(ax)^3}-\frac{1}{2}\int \frac{x^5}{W(ax)^3(W(ax)+1)}dx\right)+\frac{x^6}{6W(ax)^2}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{1}{3}\int \frac{x^5}{W(ax)^4(W(ax)+1)}dx-\frac{x^6}{6W(ax)^4}\right)+\frac{x^6}{6W(ax)^3}\right)+\frac{x^6}{6W(ax)^2}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{1}{3}\left(\frac{x^6}{6W(ax)^5}-\frac{1}{6}\int \frac{x^5}{W(ax)^5(W(ax)+1)}dx\right)-\frac{x^6}{6W(ax)^4}\right)+\frac{x^6}{6W(ax)^3}\right)+\frac{x^6}{6W(ax)^2}$$

$$\Big\downarrow \text{7201}$$

$$\frac{x^6}{6W(ax)^2}+\frac{1}{3}\left(\frac{x^6}{6W(ax)^3}+\frac{1}{2}\left(\frac{1}{3}\left(\frac{x^6}{6W(ax)^5}-\frac{x^6}{36W(ax)^6}\right)-\frac{x^6}{6W(ax)^4}\right)\right)$$

input
```
Int[x^5/ProductLog[a*x]^2,x]
```

output
```
(((-1/36*x^6/ProductLog[a*x]^6 + x^6/(6*ProductLog[a*x]^5))/3 - x^6/(6*Pro
ductLog[a*x]^4))/2 + x^6/(6*ProductLog[a*x]^3))/3 + x^6/(6*ProductLog[a*x]
^2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

method	result	size
derivativedivides	$\frac{\frac{x^6a^6}{18\,\mathrm{LambertW}(xa)^3}-\frac{x^6a^6}{36\,\mathrm{LambertW}(xa)^4}+\frac{x^6a^6}{108\,\mathrm{LambertW}(xa)^5}-\frac{x^6a^6}{648\,\mathrm{LambertW}(xa)^6}+\frac{x^6a^6}{6\,\mathrm{LambertW}(xa)^2}}{a^6}$	76
default	$\frac{\frac{x^6a^6}{18\,\mathrm{LambertW}(xa)^3}-\frac{x^6a^6}{36\,\mathrm{LambertW}(xa)^4}+\frac{x^6a^6}{108\,\mathrm{LambertW}(xa)^5}-\frac{x^6a^6}{648\,\mathrm{LambertW}(xa)^6}+\frac{x^6a^6}{6\,\mathrm{LambertW}(xa)^2}}{a^6}$	76

input
```
int(x^5/LambertW(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^6*(1/18/LambertW(x*a)^3*x^6*a^6-1/36/LambertW(x*a)^4*x^6*a^6+1/108/Lam
bertW(x*a)^5*x^6*a^6-1/648*x^6*a^6/LambertW(x*a)^6+1/6/LambertW(x*a)^2*x^6
*a^6)
```

Fricas [F]

$$\int \frac{x^5}{W(ax)^2}\,dx = \int \frac{x^5}{\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(x^5/lambert_w(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^5/lambert_w(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^5}{W(ax)^2}\,dx = \int \frac{x^5}{W^2(ax)}\,dx$$

input
```
integrate(x**5/LambertW(a*x)**2,x)
```

output
```
Integral(x**5/LambertW(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^5}{W(ax)^2}\,dx = \int \frac{x^5}{\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(x^5/lambert_w(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^5/lambert_w(a*x)^2, x)
```

Giac [F]

$$\int \frac{x^5}{W(ax)^2}\,dx = \int \frac{x^5}{\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(x^5/lambert_w(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^5/lambert_w(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{W(ax)^2}\,dx = \int \frac{x^5}{\mathrm{LambertW}\left(a\,x\right)^2}\,dx$$

input
```
int(x^5/LambertW(a*x)^2,x)
```

output
```
int(x^5/LambertW(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x^5}{W(ax)^2}\,dx = \int \frac{x^5}{lambert\_w(ax)^2}dx$$

input
```
int(x^5/Lambert_W(a*x)^2,x)
```

output
```
int(x**5/lambert_w(a*x)**2,x)
```

3.42 $\int \frac{x^4}{W(ax)^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \frac{x^4}{W(ax)^2} dx = \frac{4x^5}{625W(ax)^5} - \frac{4x^5}{125W(ax)^4} + \frac{2x^5}{25W(ax)^3} + \frac{x^5}{5W(ax)^2}$$

output
```
4/625*x^5/LambertW(a*x)^5-4/125*x^5/LambertW(a*x)^4+2/25*x^5/LambertW(a*x)
^3+1/5*x^5/LambertW(a*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{W(ax)^2} dx = \frac{4x^5}{625W(ax)^5} - \frac{4x^5}{125W(ax)^4} + \frac{2x^5}{25W(ax)^3} + \frac{x^5}{5W(ax)^2}$$

input
```
Integrate[x^4/ProductLog[a*x]^2,x]
```

output
```
(4*x^5)/(625*ProductLog[a*x]^5) - (4*x^5)/(125*ProductLog[a*x]^4) + (2*x^5
)/(25*ProductLog[a*x]^3) + x^5/(5*ProductLog[a*x]^2)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {7172, 7205, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{W(ax)^2}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{2}{5}\int \frac{x^4}{W(ax)^2(W(ax)+1)}dx+\frac{x^5}{5W(ax)^2}$$

$$\Big\downarrow \text{7205}$$

$$\frac{2}{5}\left(\frac{x^5}{5W(ax)^3}-\frac{2}{5}\int \frac{x^4}{W(ax)^3(W(ax)+1)}dx\right)+\frac{x^5}{5W(ax)^2}$$

$$\Big\downarrow \text{7205}$$

$$\frac{2}{5}\left(\frac{x^5}{5W(ax)^3}-\frac{2}{5}\left(\frac{x^5}{5W(ax)^4}-\frac{1}{5}\int \frac{x^4}{W(ax)^4(W(ax)+1)}dx\right)\right)+\frac{x^5}{5W(ax)^2}$$

$$\Big\downarrow \text{7201}$$

$$\frac{x^5}{5W(ax)^2}+\frac{2}{5}\left(\frac{x^5}{5W(ax)^3}-\frac{2}{5}\left(\frac{x^5}{5W(ax)^4}-\frac{x^5}{25W(ax)^5}\right)\right)$$

input
```
Int[x^4/ProductLog[a*x]^2,x]
```

output
```
(2*((-2*(-1/25*x^5/ProductLog[a*x]^5 + x^5/(5*ProductLog[a*x]^4)))/5 + x^5
/(5*ProductLog[a*x]^3)))/5 + x^5/(5*ProductLog[a*x]^2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{\frac{2x^5a^5}{25\,\mathrm{LambertW}(xa)^3}-\frac{4x^5a^5}{125\,\mathrm{LambertW}(xa)^4}+\frac{4x^5a^5}{625\,\mathrm{LambertW}(xa)^5}+\frac{x^5a^5}{5\,\mathrm{LambertW}(xa)^2}}{a^5}$	62
default	$\frac{\frac{2x^5a^5}{25\,\mathrm{LambertW}(xa)^3}-\frac{4x^5a^5}{125\,\mathrm{LambertW}(xa)^4}+\frac{4x^5a^5}{625\,\mathrm{LambertW}(xa)^5}+\frac{x^5a^5}{5\,\mathrm{LambertW}(xa)^2}}{a^5}$	62

input
```
int(x^4/LambertW(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(2/25/LambertW(x*a)^3*x^5*a^5-4/125/LambertW(x*a)^4*x^5*a^5+4/625*x^
5*a^5/LambertW(x*a)^5+1/5/LambertW(x*a)^2*x^5*a^5)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{W(ax)^2}\,dx = \frac{125\,x^5\,\mathrm{W}(ax)^3 + 50\,x^5\,\mathrm{W}(ax)^2 - 20\,x^5\,\mathrm{W}(ax) + 4\,x^5}{625\,\mathrm{W}(ax)^5}$$

input
```
integrate(x^4/lambert_w(a*x)^2,x, algorithm="fricas")
```

output
```
1/625*(125*x^5*lambert_w(a*x)^3 + 50*x^5*lambert_w(a*x)^2 - 20*x^5*lambert
_w(a*x) + 4*x^5)/lambert_w(a*x)^5
```

Sympy [F]

$$\int \frac{x^4}{W(ax)^2}\,dx = \int \frac{x^4}{W^2(ax)}\,dx$$

input
```
integrate(x**4/LambertW(a*x)**2,x)
```

output
```
Integral(x**4/LambertW(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^4}{W(ax)^2}\,dx = \int \frac{x^4}{\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(x^4/lambert_w(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^4/lambert_w(a*x)^2, x)
```

Giac [F]

$$\int \frac{x^4}{W(ax)^2}\,dx = \int \frac{x^4}{\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(x^4/lambert_w(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^4/lambert_w(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{W(ax)^2}\,dx = \int \frac{x^4}{\mathrm{LambertW}\left(a\,x\right)^2}\,dx$$

input
```
int(x^4/LambertW(a*x)^2,x)
```

output
```
int(x^4/LambertW(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x^4}{W(ax)^2}\,dx = \int \frac{x^4}{lambert\_w(ax)^2}dx$$

input
```
int(x^4/Lambert_W(a*x)^2,x)
```

output
```
int(x**4/lambert_w(a*x)**2,x)
```

3.43 $\int \frac{x^3}{W(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 40

$$\int \frac{x^3}{W(ax)^2}\,dx = -\frac{x^4}{32W(ax)^4} + \frac{x^4}{8W(ax)^3} + \frac{x^4}{4W(ax)^2}$$

output
```
-1/32*x^4/LambertW(a*x)^4+1/8*x^4/LambertW(a*x)^3+1/4*x^4/LambertW(a*x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{W(ax)^2}\,dx = -\frac{x^4}{32W(ax)^4} + \frac{x^4}{8W(ax)^3} + \frac{x^4}{4W(ax)^2}$$

input
```
Integrate[x^3/ProductLog[a*x]^2,x]
```

output
```
-1/32*x^4/ProductLog[a*x]^4 + x^4/(8*ProductLog[a*x]^3) + x^4/(4*ProductLo
g[a*x]^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {7172, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{W(ax)^2}dx$$
$$\Big\downarrow \text{7172}$$
$$\frac{1}{2}\int \frac{x^3}{W(ax)^2(W(ax)+1)}dx+\frac{x^4}{4W(ax)^2}$$
$$\Big\downarrow \text{7205}$$
$$\frac{1}{2}\left(\frac{x^4}{4W(ax)^3}-\frac{1}{4}\int \frac{x^3}{W(ax)^3(W(ax)+1)}dx\right)+\frac{x^4}{4W(ax)^2}$$
$$\Big\downarrow \text{7201}$$
$$\frac{x^4}{4W(ax)^2}+\frac{1}{2}\left(\frac{x^4}{4W(ax)^3}-\frac{x^4}{16W(ax)^4}\right)$$

input
```
Int[x^3/ProductLog[a*x]^2,x]
```

output
```
(-1/16*x^4/ProductLog[a*x]^4 + x^4/(4*ProductLog[a*x]^3))/2 + x^4/(4*Produ
ctLog[a*x]^2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{\frac{x^4 a^4}{8\,\mathrm{LambertW}(xa)^3}-\frac{x^4 a^4}{32\,\mathrm{LambertW}(xa)^4}+\frac{x^4 a^4}{4\,\mathrm{LambertW}(xa)^2}}{a^4}$	48
default	$\frac{\frac{x^4 a^4}{8\,\mathrm{LambertW}(xa)^3}-\frac{x^4 a^4}{32\,\mathrm{LambertW}(xa)^4}+\frac{x^4 a^4}{4\,\mathrm{LambertW}(xa)^2}}{a^4}$	48

input
```
int(x^3/LambertW(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/8/LambertW(x*a)^3*x^4*a^4-1/32*x^4*a^4/LambertW(x*a)^4+1/4*x^4*a^
4/LambertW(x*a)^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{W(ax)^2}\,dx = \frac{8\,x^4\,\mathrm{W}(ax)^2 + 4\,x^4\,\mathrm{W}(ax) - x^4}{32\,\mathrm{W}(ax)^4}$$

input
```
integrate(x^3/lambert_w(a*x)^2,x, algorithm="fricas")
```

output
```
1/32*(8*x^4*lambert_w(a*x)^2 + 4*x^4*lambert_w(a*x) - x^4)/lambert_w(a*x)^
4
```

Sympy [F]

$$\int \frac{x^3}{W(ax)^2}\,dx = \int \frac{x^3}{W^2\left(ax\right)}\,dx$$

input
```
integrate(x**3/LambertW(a*x)**2,x)
```

output
```
Integral(x**3/LambertW(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^3}{W(ax)^2}\,dx = \int \frac{x^3}{\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(x^3/lambert_w(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^3/lambert_w(a*x)^2, x)
```

Giac [F]

$$\int \frac{x^3}{W(ax)^2}\,dx = \int \frac{x^3}{\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(x^3/lambert_w(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^3/lambert_w(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{W(ax)^2}\,dx = \int \frac{x^3}{\mathrm{LambertW}\left(a\,x\right)^2}\,dx$$

input
```
int(x^3/LambertW(a*x)^2,x)
```

output
```
int(x^3/LambertW(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x^3}{W(ax)^2}\,dx = \int \frac{x^3}{lambert\_w(ax)^2}dx$$

input
```
int(x^3/Lambert_W(a*x)^2,x)
```

output
```
int(x**3/lambert_w(a*x)**2,x)
```

3.44 $\int \frac{x^2}{W(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \frac{x^2}{W(ax)^2}\,dx=\frac{2x^3}{9W(ax)^3}+\frac{x^3}{3W(ax)^2}$$

output
```
2/9*x^3/LambertW(a*x)^3+1/3*x^3/LambertW(a*x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{W(ax)^2}\,dx=\frac{2x^3}{9W(ax)^3}+\frac{x^3}{3W(ax)^2}$$

input
```
Integrate[x^2/ProductLog[a*x]^2,x]
```

output
```
(2*x^3)/(9*ProductLog[a*x]^3) + x^3/(3*ProductLog[a*x]^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7172, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{W(ax)^2} \, dx$$

$$\Big\downarrow \text{ 7172}$$

$$\frac{2}{3} \int \frac{x^2}{W(ax)^2(W(ax)+1)}dx + \frac{x^3}{3W(ax)^2}$$

$$\Big\downarrow \text{ 7201}$$

$$\frac{x^3}{3W(ax)^2} + \frac{2x^3}{9W(ax)^3}$$

input
```
Int[x^2/ProductLog[a*x]^2,x]
```

output
```
(2*x^3)/(9*ProductLog[a*x]^3) + x^3/(3*ProductLog[a*x]^2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$-\frac{-3x^{3}\operatorname{LambertW}(xa)-2x^{3}}{9\operatorname{LambertW}(xa)^{3}}$	24
derivativedivides	$\frac{\frac{2x^{3}a^{3}}{9\operatorname{LambertW}(xa)^{3}}+\frac{x^{3}a^{3}}{3\operatorname{LambertW}(xa)^{2}}}{a^{3}}$	34
default	$\frac{\frac{2x^{3}a^{3}}{9\operatorname{LambertW}(xa)^{3}}+\frac{x^{3}a^{3}}{3\operatorname{LambertW}(xa)^{2}}}{a^{3}}$	34

input
```
int(x^2/LambertW(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/9*(-3*x^3*LambertW(x*a)-2*x^3)/LambertW(x*a)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{W(ax)^2}\,dx=\frac{3\,x^3\,\mathrm{W}(ax)+2\,x^3}{9\,\mathrm{W}(ax)^3}$$

input
```
integrate(x^2/lambert_w(a*x)^2,x, algorithm="fricas")
```

output
```
1/9*(3*x^3*lambert_w(a*x) + 2*x^3)/lambert_w(a*x)^3
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{W(ax)^2}\,dx = \frac{x^3}{3W^2(ax)} + \frac{2x^3}{9W^3(ax)}$$

input
```
integrate(x**2/LambertW(a*x)**2,x)
```

output
```
x**3/(3*LambertW(a*x)**2) + 2*x**3/(9*LambertW(a*x)**3)
```

Maxima [F]

$$\int \frac{x^2}{W(ax)^2}\,dx = \int \frac{x^2}{\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(x^2/lambert_w(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^2/lambert_w(a*x)^2, x)
```

Giac [F]

$$\int \frac{x^2}{W(ax)^2}\,dx = \int \frac{x^2}{\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(x^2/lambert_w(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2/lambert_w(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{W(ax)^2}\,dx = \int \frac{x^2}{\mathrm{LambertW}\left(a\,x\right)^2}\,dx$$

input
```
int(x^2/LambertW(a*x)^2,x)
```

output
```
int(x^2/LambertW(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x^2}{W(ax)^2}\,dx = \int \frac{x^2}{lambert\_w\left(ax\right)^2}dx$$

input
```
int(x^2/Lambert_W(a*x)^2,x)
```

output
```
int(x**2/lambert_w(a*x)**2,x)
```

3.45 $\int \frac{x}{W(ax)^2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \frac{x}{W(ax)^2}\,dx = \frac{\text{ExpIntegralEi}(2W(ax))}{a^2} + \frac{x^2}{2W(ax)^2}$$

output
```
Ei(2*LambertW(a*x))/a^2+1/2*x^2/LambertW(a*x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x}{W(ax)^2}\,dx = \frac{\text{ExpIntegralEi}(2W(ax))}{a^2} + \frac{x^2}{2W(ax)^2}$$

input
```
Integrate[x/ProductLog[a*x]^2,x]
```

output
```
ExpIntegralEi[2*ProductLog[a*x]]/a^2 + x^2/(2*ProductLog[a*x]^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7172, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{W(ax)^2}\,dx$$
$$\Big\downarrow \text{7172}$$
$$\int \frac{x}{W(ax)^2(W(ax)+1)}dx + \frac{x^2}{2W(ax)^2}$$
$$\Big\downarrow \text{7202}$$
$$\frac{\text{ExpIntegralEi}(2W(ax))}{a^2} + \frac{x^2}{2W(ax)^2}$$

input
```
Int[x/ProductLog[a*x]^2,x]
```

output
```
ExpIntegralEi[2*ProductLog[a*x]]/a^2 + x^2/(2*ProductLog[a*x]^2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{\frac{x^2a^2}{2\,\mathrm{LambertW}(xa)^2}-\mathrm{expIntegral}_1(-2\,\mathrm{LambertW}(xa))}{a^2}$	30
default	$\frac{\frac{x^2a^2}{2\,\mathrm{LambertW}(xa)^2}-\mathrm{expIntegral}_1(-2\,\mathrm{LambertW}(xa))}{a^2}$	30

input
```
int(x/LambertW(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/2*x^2*a^2/LambertW(x*a)^2-Ei(1,-2*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{x}{W(ax)^2}\,dx = \int \frac{x}{\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(x/lambert_w(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x/lambert_w(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x}{W(ax)^2}\,dx = \int \frac{x}{W^2(ax)}\,dx$$

input
```
integrate(x/LambertW(a*x)**2,x)
```

output
```
Integral(x/LambertW(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x}{W(ax)^2}\,dx = \int \frac{x}{\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(x/lambert_w(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x/lambert_w(a*x)^2, x)
```

Giac [F]

$$\int \frac{x}{W(ax)^2}\,dx = \int \frac{x}{\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(x/lambert_w(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x/lambert_w(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{W(ax)^2}\,dx = \int \frac{x}{\mathrm{LambertW}\,(a\,x)^2}\,dx$$

input
```
int(x/LambertW(a*x)^2,x)
```

output
```
int(x/LambertW(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x}{W(ax)^2}\,dx = \int \frac{x}{lambert\_w(ax)^2}dx$$

input
```
int(x/Lambert_W(a*x)^2,x)
```

output
```
int(x/lambert_w(a*x)**2,x)
```

3.46 $\int \frac{1}{W(ax)^2}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 20

$$\int \frac{1}{W(ax)^2}\,dx = \frac{2\,\text{ExpIntegralEi}(W(ax))}{a} - \frac{x}{W(ax)^2}$$

output
```
2*Ei(LambertW(a*x))/a-x/LambertW(a*x)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{W(ax)^2}\,dx = \frac{2\,\text{ExpIntegralEi}(W(ax))}{a} - \frac{x}{W(ax)^2}$$

input
```
Integrate[ProductLog[a*x]^(-2),x]
```

output
```
(2*ExpIntegralEi[ProductLog[a*x]])/a - x/ProductLog[a*x]^2
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7166, 7179}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W(ax)^2} \, dx$$

$$\Big\downarrow \ 7166$$

$$2\int \frac{1}{W(ax)(W(ax)+1)}dx - \frac{x}{W(ax)^2}$$

$$\Big\downarrow \ 7179$$

$$\frac{2\,\text{ExpIntegralEi}(W(ax))}{a} - \frac{x}{W(ax)^2}$$

input
```
Int[ProductLog[a*x]^(-2),x]
```

output
```
(2*ExpIntegralEi[ProductLog[a*x]])/a - x/ProductLog[a*x]^2
```

Defintions of rubi rules used

rule 7166
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_), x_Symbol] :> Simp[(a + b*x
)*((c*ProductLog[a + b*x])^p/(b*(p + 1))), x] + Simp[p/(c*(p + 1))   Int[(c
*ProductLog[a + b*x])^(p + 1)/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a
, b, c}, x] && LtQ[p, -1]
```

rule 7179
```
Int[1/(ProductLog[(a_.) + (b_.)*(x_)]*((d_) + (d_.)*ProductLog[(a_.) + (b_.
)*(x_)])), x_Symbol] :> Simp[ExpIntegralEi[ProductLog[a + b*x]]/(b*d), x] /
; FreeQ[{a, b, d}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$\frac{-\frac{xa}{\text{LambertW}(xa)^2}-2\,\text{expIntegral}_1(-\text{LambertW}(xa))}{a}$	26
default	$\frac{-\frac{xa}{\text{LambertW}(xa)^2}-2\,\text{expIntegral}_1(-\text{LambertW}(xa))}{a}$	26

input
```
int(1/LambertW(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/LambertW(x*a)^2*x*a-2*Ei(1,-LambertW(x*a)))
```

Fricas [F]

$$\int \frac{1}{W(ax)^2}\,dx = \int \frac{1}{\text{W}(ax)^2}\,dx$$

input
```
integrate(1/lambert_w(a*x)^2,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)^(-2), x)
```

Sympy [F]

$$\int \frac{1}{W(ax)^2}\,dx = \int \frac{1}{W^2(ax)}\,dx$$

input
```
integrate(1/LambertW(a*x)**2,x)
```

output
```
Integral(LambertW(a*x)**(-2), x)
```

Maxima [F]

$$\int \frac{1}{W(ax)^2} dx = \int \frac{1}{\mathrm{W}(ax)^2} dx$$

input
```
integrate(1/lambert_w(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^(-2), x)
```

Giac [F]

$$\int \frac{1}{W(ax)^2} dx = \int \frac{1}{\mathrm{W}(ax)^2} dx$$

input
```
integrate(1/lambert_w(a*x)^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W(ax)^2} dx = \int \frac{1}{\mathrm{LambertW}(a\,x)^2} dx$$

input
```
int(1/LambertW(a*x)^2,x)
```

output
```
int(1/LambertW(a*x)^2, x)
```

Reduce [F]

$$\int \frac{1}{W(ax)^2} \, dx = \int \frac{1}{lambert\_w(ax)^2} dx$$

input
```
int(1/Lambert_W(a*x)^2,x)
```

output
```
int(1/lambert_w(a*x)**2,x)
```

3.47 $\int \frac{1}{xW(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{1}{xW(ax)^2}\,dx = -\frac{1}{2W(ax)^2} - \frac{1}{W(ax)}$$

output
```
-1/2/LambertW(a*x)^2-1/LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{xW(ax)^2}\,dx = -\frac{1}{2W(ax)^2} - \frac{1}{W(ax)}$$

input
```
Integrate[1/(x*ProductLog[a*x]^2),x]
```

output
```
-1/2*1/ProductLog[a*x]^2 - ProductLog[a*x]^(-1)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{xW(ax)^2}\,dx$$

$$\Big\downarrow \text{7173}$$

$$\int \frac{1}{xW(ax)(W(ax)+1)}dx - \frac{1}{2W(ax)^2}$$

$$\Big\downarrow \text{7200}$$

$$-\frac{1}{W(ax)} - \frac{1}{2W(ax)^2}$$

input
```
Int[1/(x*ProductLog[a*x]^2),x]
```

output
```
-1/2*1/ProductLog[a*x]^2 - ProductLog[a*x]^(-1)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$-\frac{1+2\,\mathrm{LambertW}(xa)}{2\,\mathrm{LambertW}(xa)^{2}}$	17
derivativedivides	$-\frac{1}{2\,\mathrm{LambertW}(xa)^{2}}-\frac{1}{\mathrm{LambertW}(xa)}$	18
default	$-\frac{1}{2\,\mathrm{LambertW}(xa)^{2}}-\frac{1}{\mathrm{LambertW}(xa)}$	18

input
```
int(1/x/LambertW(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/LambertW(x*a)^2*(1+2*LambertW(x*a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{1}{xW(ax)^2}\,dx=-\frac{2\ \mathrm{W}(ax)+1}{2\ \mathrm{W}(ax)^2}$$

input
```
integrate(1/x/lambert_w(a*x)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*lambert_w(a*x) + 1)/lambert_w(a*x)^2
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{xW(ax)^2}\,dx=-\frac{1}{W\left(ax\right)}-\frac{1}{2W^{2}\left(ax\right)}$$

input
```
integrate(1/x/LambertW(a*x)**2,x)
```

output
```
-1/LambertW(a*x) - 1/(2*LambertW(a*x)**2)
```

Maxima [F]

$$\int \frac{1}{xW(ax)^2}\,dx = \int \frac{1}{x\,\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(1/x/lambert_w(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/(x*lambert_w(a*x)^2), x)
```

Giac [F]

$$\int \frac{1}{xW(ax)^2}\,dx = \int \frac{1}{x\,\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(1/x/lambert_w(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/(x*lambert_w(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{xW(ax)^2}\,dx = \int \frac{1}{x\,\mathrm{LambertW}\left(a\,x\right)^2}\,dx$$

input
```
int(1/(x*LambertW(a*x)^2),x)
```

output
```
int(1/(x*LambertW(a*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{xW(ax)^2} dx = \int \frac{1}{lambert\_w(ax)^2 x} dx$$

input
```
int(1/x/Lambert_W(a*x)^2,x)
```

output
```
int(1/(lambert_w(a*x)**2*x),x)
```

3.48 $\int \frac{1}{x^2 W(ax)^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{1}{x^2 W(ax)^2} dx = \frac{1}{3x} + \frac{1}{3} a \operatorname{ExpIntegralEi}(-W(ax)) - \frac{1}{3xW(ax)^2} - \frac{1}{3xW(ax)}$$

output
```
1/3/x+1/3*a*Ei(-LambertW(a*x))-1/3/x/LambertW(a*x)^2-1/3/x/LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 W(ax)^2} dx = \frac{1}{3x} + \frac{1}{3} a \operatorname{ExpIntegralEi}(-W(ax)) - \frac{1}{3xW(ax)^2} - \frac{1}{3xW(ax)}$$

input
```
Integrate[1/(x^2*ProductLog[a*x]^2),x]
```

output
```
1/(3*x) + (a*ExpIntegralEi[-ProductLog[a*x]])/3 - 1/(3*x*ProductLog[a*x]^2
) - 1/(3*x*ProductLog[a*x])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {7173, 7206, 7196, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 W(ax)^2} \, dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{2}{3} \int \frac{1}{x^2 W(ax)(W(ax)+1)} dx - \frac{1}{3x W(ax)^2}$$

$$\Big\downarrow \text{7206}$$

$$\frac{2}{3} \left(-\frac{1}{2} \int \frac{1}{x^2 (W(ax)+1)} dx - \frac{1}{2x W(ax)} \right) - \frac{1}{3x W(ax)^2}$$

$$\Big\downarrow \text{7196}$$

$$\frac{2}{3} \left(\frac{1}{2} \left(\int \frac{W(ax)}{x^2 (W(ax)+1)} dx + \frac{1}{x} \right) - \frac{1}{2x W(ax)} \right) - \frac{1}{3x W(ax)^2}$$

$$\Big\downarrow \text{7202}$$

$$\frac{2}{3} \left(\frac{1}{2} \left(a \,\text{ExpIntegralEi}(-W(ax)) + \frac{1}{x} \right) - \frac{1}{2x W(ax)} \right) - \frac{1}{3x W(ax)^2}$$

input
```
Int[1/(x^2*ProductLog[a*x]^2),x]
```

output
```
(2*((x^(-1) + a*ExpIntegralEi[-ProductLog[a*x]])/2 - 1/(2*x*ProductLog[a*x
])))/3 - 1/(3*x*ProductLog[a*x]^2)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$a\left(-\frac{1}{3\,\mathrm{LambertW}(xa)^{2} xa}-\frac{1}{3\,\mathrm{LambertW}(xa) xa}+\frac{1}{3ax}-\frac{\mathrm{expIntegral}_{1}(\mathrm{LambertW}(xa))}{3}\right)$	48
default	$a\left(-\frac{1}{3\,\mathrm{LambertW}(xa)^{2} xa}-\frac{1}{3\,\mathrm{LambertW}(xa) xa}+\frac{1}{3ax}-\frac{\mathrm{expIntegral}_{1}(\mathrm{LambertW}(xa))}{3}\right)$	48

input
```
int(1/x^2/LambertW(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3/LambertW(x*a)^2/x/a-1/3/LambertW(x*a)/x/a+1/3/a/x-1/3*Ei(1,Lambert
W(x*a)))
```

Fricas [F]

$$\int \frac{1}{x^2 W(ax)^2} \, dx = \int \frac{1}{x^2 \, \mathrm{W}(ax)^2} \, dx$$

input
```
integrate(1/x^2/lambert_w(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/(x^2*lambert_w(a*x)^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 W(ax)^2} \, dx = \int \frac{1}{x^2 W^2 (ax)} \, dx$$

input
```
integrate(1/x**2/LambertW(a*x)**2,x)
```

output
```
Integral(1/(x**2*LambertW(a*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2 W(ax)^2} \, dx = \int \frac{1}{x^2 \, \mathrm{W}(ax)^2} \, dx$$

input
```
integrate(1/x^2/lambert_w(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/(x^2*lambert_w(a*x)^2), x)
```

Giac [F]

$$\int \frac{1}{x^2W(ax)^2}\,dx = \int \frac{1}{x^2\,\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(1/x^2/lambert_w(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/(x^2*lambert_w(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2W(ax)^2}\,dx = \int \frac{1}{x^2\,\mathrm{LambertW}\left(a\,x\right)^2}\,dx$$

input
```
int(1/(x^2*LambertW(a*x)^2),x)
```

output
```
int(1/(x^2*LambertW(a*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^2W(ax)^2}\,dx = \frac{\left(\int \frac{lambert\_w(ax)}{lambert\_w(ax)x^2+x^2}dx\right) lambert\_w(ax)\,x + 4\left(\int \frac{1}{lambert\_w(ax)^3x^2+lambert\_w(ax)^2x^2}dx\right) lambert\_w(\ldots}{\ldots}$$

input
```
int(1/x^2/Lambert_W(a*x)^2,x)
```

output
```
(int(lambert_w(a*x)/(lambert_w(a*x)*x**2 + x**2),x)*lambert_w(a*x)*x + 4*i
nt(1/(lambert_w(a*x)**3*x**2 + lambert_w(a*x)**2*x**2),x)*lambert_w(a*x)*x
 + 2*int(1/(lambert_w(a*x)**2*x**2 + lambert_w(a*x)*x**2),x)*lambert_w(a*x
)*x - 2*int(1/(e**lambert_w(a*x)*lambert_w(a*x)**3*x + e**lambert_w(a*x)*l
ambert_w(a*x)**2*x),x)*lambert_w(a*x)*a*x - int(1/(lambert_w(a*x)*x**2 + x
**2),x)*lambert_w(a*x)*x + lambert_w(a*x) - 2)/(4*lambert_w(a*x)*x)
```

3.49 $\int \frac{1}{x^3W(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 59

$$\int \frac{1}{x^3W(ax)^2}\,dx = \frac{1}{6x^2} - \frac{2}{3}a^2\,\text{ExpIntegralEi}(-2W(ax)) - \frac{1}{4x^2W(ax)^2} - \frac{1}{6x^2W(ax)} - \frac{W(ax)}{3x^2}$$

output
```
1/6/x^2-2/3*a^2*Ei(-2*LambertW(a*x))-1/4/x^2/LambertW(a*x)^2-1/6/x^2/Lambe
rtW(a*x)-1/3*LambertW(a*x)/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3W(ax)^2}\,dx = \frac{1}{6x^2} - \frac{2}{3}a^2\,\text{ExpIntegralEi}(-2W(ax)) - \frac{1}{4x^2W(ax)^2} - \frac{1}{6x^2W(ax)} - \frac{W(ax)}{3x^2}$$

input
```
Integrate[1/(x^3*ProductLog[a*x]^2),x]
```

output
```
1/(6*x^2) - (2*a^2*ExpIntegralEi[-2*ProductLog[a*x]])/3 - 1/(4*x^2*Product
Log[a*x]^2) - 1/(6*x^2*ProductLog[a*x]) - ProductLog[a*x]/(3*x^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {7173, 7206, 7196, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 W(ax)^2} \, dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{1}{2} \int \frac{1}{x^3 W(ax)(W(ax)+1)}dx - \frac{1}{4x^2 W(ax)^2}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{2} \left(-\frac{2}{3} \int \frac{1}{x^3 (W(ax)+1)}dx - \frac{1}{3x^2 W(ax)}\right) - \frac{1}{4x^2 W(ax)^2}$$

$$\Big\downarrow \text{7196}$$

$$\frac{1}{2} \left(-\frac{2}{3} \left(-\int \frac{W(ax)}{x^3 (W(ax)+1)}dx - \frac{1}{2x^2}\right) - \frac{1}{3x^2 W(ax)}\right) - \frac{1}{4x^2 W(ax)^2}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{2} \left(-\frac{2}{3} \left(2\int \frac{W(ax)^2}{x^3 (W(ax)+1)}dx + \frac{W(ax)}{x^2} - \frac{1}{2x^2}\right) - \frac{1}{3x^2 W(ax)}\right) - \frac{1}{4x^2 W(ax)^2}$$

$$\Big\downarrow \text{7202}$$

$$\frac{1}{2} \left(-\frac{2}{3} \left(2a^2 \text{ExpIntegralEi}(-2W(ax)) + \frac{W(ax)}{x^2} - \frac{1}{2x^2}\right) - \frac{1}{3x^2 W(ax)}\right) - \frac{1}{4x^2 W(ax)^2}$$

input
```
Int[1/(x^3*ProductLog[a*x]^2),x]
```

output
```
-1/4*1/(x^2*ProductLog[a*x]^2) + (-1/3*1/(x^2*ProductLog[a*x]) - (2*(-1/2*
1/x^2 + 2*a^2*ExpIntegralEi[-2*ProductLog[a*x]] + ProductLog[a*x]/x^2))/3)
/2
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

method	result
derivativedivides	$a^2\left(-\frac{1}{4\,\mathrm{LambertW}(xa)^2 x^2 a^2}-\frac{1}{6\,\mathrm{LambertW}(xa)x^2a^2}+\frac{1}{6x^2a^2}-\frac{\mathrm{LambertW}(xa)}{3x^2a^2}+\frac{2\ \mathrm{expIntegral}_1(2\,\mathrm{Lambe}}{3}\right.$
default	$a^2\left(-\frac{1}{4\,\mathrm{LambertW}(xa)^2 x^2 a^2}-\frac{1}{6\,\mathrm{LambertW}(xa)x^2a^2}+\frac{1}{6x^2a^2}-\frac{\mathrm{LambertW}(xa)}{3x^2a^2}+\frac{2\ \mathrm{expIntegral}_1(2\,\mathrm{Lambe}}{3}\right.$

input
```
int(1/x^3/LambertW(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/4/LambertW(x*a)^2/x^2/a^2-1/6/LambertW(x*a)/x^2/a^2+1/6/x^2/a^2-1/
3*LambertW(x*a)/x^2/a^2+2/3*Ei(1,2*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{1}{x^3W(ax)^2}\,dx=\int \frac{1}{x^3\,\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(1/x^3/lambert_w(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/(x^3*lambert_w(a*x)^2), x)
```

Sympy [F]

$$\int \frac{1}{x^3W(ax)^2}\,dx=\int \frac{1}{x^3W^2\left(ax\right)}\,dx$$

input
```
integrate(1/x**3/LambertW(a*x)**2,x)
```

output
```
Integral(1/(x**3*LambertW(a*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^3 W(ax)^2}\,dx = \int \frac{1}{x^3\,\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(1/x^3/lambert_w(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/(x^3*lambert_w(a*x)^2), x)
```

Giac [F]

$$\int \frac{1}{x^3 W(ax)^2}\,dx = \int \frac{1}{x^3\,\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(1/x^3/lambert_w(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/(x^3*lambert_w(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 W(ax)^2}\,dx = \int \frac{1}{x^3\,\mathrm{LambertW}\,(a\,x)^2}\,dx$$

input
```
int(1/(x^3*LambertW(a*x)^2),x)
```

output
```
int(1/(x^3*LambertW(a*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^3 W(ax)^2}\,dx$$
$$= \frac{2\left(\int \frac{lambert\_w(ax)}{lambert\_w(ax)x^3+x^3}dx\right) lambert\_w(ax)\,x^2 + 8\left(\int \frac{1}{lambert\_w(ax)^3x^3+lambert\_w(ax)^2x^3}dx\right) lambert\_}{}$$

input
```
int(1/x^3/Lambert_W(a*x)^2,x)
```

output
```
(2*int(lambert_w(a*x)/(lambert_w(a*x)*x**3 + x**3),x)*lambert_w(a*x)*x**2
+ 8*int(1/(lambert_w(a*x)**3*x**3 + lambert_w(a*x)**2*x**3),x)*lambert_w(a
*x)*x**2 + 4*int(1/(lambert_w(a*x)**2*x**3 + lambert_w(a*x)*x**3),x)*lambe
rt_w(a*x)*x**2 - 2*int(1/(e**lambert_w(a*x)*lambert_w(a*x)**3*x**2 + e**la
mbert_w(a*x)*lambert_w(a*x)**2*x**2),x)*lambert_w(a*x)*a*x**2 - 2*int(1/(l
ambert_w(a*x)*x**3 + x**3),x)*lambert_w(a*x)*x**2 + lambert_w(a*x) - 2)/(8
*lambert_w(a*x)*x**2)
```

3.50 $\int \frac{x^6}{W(ax)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 66

$$\int \frac{x^6}{W(ax)^3}\,dx=-\frac{18x^7}{16807W(ax)^7}+\frac{18x^7}{2401W(ax)^6}-\frac{9x^7}{343W(ax)^5}+\frac{3x^7}{49W(ax)^4}+\frac{x^7}{7W(ax)^3}$$

output
```
-18/16807*x^7/LambertW(a*x)^7+18/2401*x^7/LambertW(a*x)^6-9/343*x^7/Lamber
tW(a*x)^5+3/49*x^7/LambertW(a*x)^4+1/7*x^7/LambertW(a*x)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{W(ax)^3}\,dx=-\frac{18x^7}{16807W(ax)^7}+\frac{18x^7}{2401W(ax)^6}-\frac{9x^7}{343W(ax)^5}+\frac{3x^7}{49W(ax)^4}+\frac{x^7}{7W(ax)^3}$$

input
```
Integrate[x^6/ProductLog[a*x]^3,x]
```

output
```
(-18*x^7)/(16807*ProductLog[a*x]^7) + (18*x^7)/(2401*ProductLog[a*x]^6) -
(9*x^7)/(343*ProductLog[a*x]^5) + (3*x^7)/(49*ProductLog[a*x]^4) + x^7/(7*
ProductLog[a*x]^3)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7172, 7205, 7205, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{W(ax)^3}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{3}{7}\int \frac{x^6}{W(ax)^3(W(ax)+1)}dx+\frac{x^7}{7W(ax)^3}$$

$$\Big\downarrow \text{7205}$$

$$\frac{3}{7}\left(\frac{x^7}{7W(ax)^4}-\frac{3}{7}\int \frac{x^6}{W(ax)^4(W(ax)+1)}dx\right)+\frac{x^7}{7W(ax)^3}$$

$$\Big\downarrow \text{7205}$$

$$\frac{3}{7}\left(\frac{x^7}{7W(ax)^4}-\frac{3}{7}\left(\frac{x^7}{7W(ax)^5}-\frac{2}{7}\int \frac{x^6}{W(ax)^5(W(ax)+1)}dx\right)\right)+\frac{x^7}{7W(ax)^3}$$

$$\Big\downarrow \text{7205}$$

$$\frac{3}{7}\left(\frac{x^7}{7W(ax)^4}-\frac{3}{7}\left(\frac{x^7}{7W(ax)^5}-\frac{2}{7}\left(\frac{x^7}{7W(ax)^6}-\frac{1}{7}\int \frac{x^6}{W(ax)^6(W(ax)+1)}dx\right)\right)\right)+\frac{x^7}{7W(ax)^3}$$

$$\Big\downarrow \text{7201}$$

$$\frac{x^7}{7W(ax)^3}+\frac{3}{7}\left(\frac{x^7}{7W(ax)^4}-\frac{3}{7}\left(\frac{x^7}{7W(ax)^5}-\frac{2}{7}\left(\frac{x^7}{7W(ax)^6}-\frac{x^7}{49W(ax)^7}\right)\right)\right)$$

input
```
Int[x^6/ProductLog[a*x]^3,x]
```

output
```
(3*((-3*((-2*(-1/49*x^7/ProductLog[a*x]^7 + x^7/(7*ProductLog[a*x]^6)))/7
+ x^7/(7*ProductLog[a*x]^5)))/7 + x^7/(7*ProductLog[a*x]^4)))/7 + x^7/(7*P
roductLog[a*x]^3)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

method	result	size
derivativedivides	$\frac{\frac{3x^7a^7}{49\,\mathrm{LambertW}(xa)^4}-\frac{9x^7a^7}{343\,\mathrm{LambertW}(xa)^5}+\frac{18x^7a^7}{2401\,\mathrm{LambertW}(xa)^6}-\frac{18x^7a^7}{16807\,\mathrm{LambertW}(xa)^7}+\frac{x^7a^7}{7\,\mathrm{LambertW}(xa)^3}}{a^7}$	76
default	$\frac{\frac{3x^7a^7}{49\,\mathrm{LambertW}(xa)^4}-\frac{9x^7a^7}{343\,\mathrm{LambertW}(xa)^5}+\frac{18x^7a^7}{2401\,\mathrm{LambertW}(xa)^6}-\frac{18x^7a^7}{16807\,\mathrm{LambertW}(xa)^7}+\frac{x^7a^7}{7\,\mathrm{LambertW}(xa)^3}}{a^7}$	76

input
```
int(x^6/LambertW(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^7*(3/49/LambertW(x*a)^4*x^7*a^7-9/343/LambertW(x*a)^5*x^7*a^7+18/2401/
LambertW(x*a)^6*x^7*a^7-18/16807*x^7*a^7/LambertW(x*a)^7+1/7/LambertW(x*a)
^3*x^7*a^7)
```

Fricas [F]

$$\int \frac{x^6}{W(ax)^3}\, dx = \int \frac{x^6}{\mathrm{W}(ax)^3}\, dx$$

input
```
integrate(x^6/lambert_w(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^6/lambert_w(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x^6}{W(ax)^3}\, dx = \int \frac{x^6}{W^3(ax)}\, dx$$

input
```
integrate(x**6/LambertW(a*x)**3,x)
```

output
```
Integral(x**6/LambertW(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x^6}{W(ax)^3}\, dx = \int \frac{x^6}{\mathrm{W}(ax)^3}\, dx$$

input
```
integrate(x^6/lambert_w(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^6/lambert_w(a*x)^3, x)
```

Giac [F]

$$\int \frac{x^6}{W(ax)^3} dx = \int \frac{x^6}{\mathrm{W}(ax)^3} dx$$

input
```
integrate(x^6/lambert_w(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^6/lambert_w(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{W(ax)^3} dx = \int \frac{x^6}{\mathrm{LambertW}\,(a\,x)^3} dx$$

input
```
int(x^6/LambertW(a*x)^3,x)
```

output
```
int(x^6/LambertW(a*x)^3, x)
```

Reduce [F]

$$\int \frac{x^6}{W(ax)^3} dx = \int \frac{x^6}{lambert\_w(ax)^3} dx$$

input
```
int(x^6/Lambert_W(a*x)^3,x)
```

output
```
int(x**6/lambert_w(a*x)**3,x)
```

3.51 $\int \frac{x^5}{W(ax)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \frac{x^5}{W(ax)^3}\,dx=\frac{x^6}{216W(ax)^6}-\frac{x^6}{36W(ax)^5}+\frac{x^6}{12W(ax)^4}+\frac{x^6}{6W(ax)^3}$$

output
```
1/216*x^6/LambertW(a*x)^6-1/36*x^6/LambertW(a*x)^5+1/12*x^6/LambertW(a*x)^
4+1/6*x^6/LambertW(a*x)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{W(ax)^3}\,dx=\frac{x^6}{216W(ax)^6}-\frac{x^6}{36W(ax)^5}+\frac{x^6}{12W(ax)^4}+\frac{x^6}{6W(ax)^3}$$

input
```
Integrate[x^5/ProductLog[a*x]^3,x]
```

output
```
x^6/(216*ProductLog[a*x]^6) - x^6/(36*ProductLog[a*x]^5) + x^6/(12*Product
Log[a*x]^4) + x^6/(6*ProductLog[a*x]^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {7172, 7205, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{W(ax)^3}dx$$

$$\Big\downarrow \text{ 7172}$$

$$\frac{1}{2}\int \frac{x^5}{W(ax)^3(W(ax)+1)}dx+\frac{x^6}{6W(ax)^3}$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{2}\left(\frac{x^6}{6W(ax)^4}-\frac{1}{3}\int \frac{x^5}{W(ax)^4(W(ax)+1)}dx\right)+\frac{x^6}{6W(ax)^3}$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{6}\int \frac{x^5}{W(ax)^5(W(ax)+1)}dx-\frac{x^6}{6W(ax)^5}\right)+\frac{x^6}{6W(ax)^4}\right)+\frac{x^6}{6W(ax)^3}$$

$$\Big\downarrow \text{ 7201}$$

$$\frac{x^6}{6W(ax)^3}+\frac{1}{2}\left(\frac{x^6}{6W(ax)^4}+\frac{1}{3}\left(\frac{x^6}{36W(ax)^6}-\frac{x^6}{6W(ax)^5}\right)\right)$$

input
```
Int[x^5/ProductLog[a*x]^3,x]
```

output
```
((x^6/(36*ProductLog[a*x]^6) - x^6/(6*ProductLog[a*x]^5))/3 + x^6/(6*Produ
ctLog[a*x]^4))/2 + x^6/(6*ProductLog[a*x]^3)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{\frac{x^6a^6}{12\,\mathrm{LambertW}(xa)^4}-\frac{x^6a^6}{36\,\mathrm{LambertW}(xa)^5}+\frac{x^6a^6}{216\,\mathrm{LambertW}(xa)^6}+\frac{x^6a^6}{6\,\mathrm{LambertW}(xa)^3}}{a^6}$	62
default	$\frac{\frac{x^6a^6}{12\,\mathrm{LambertW}(xa)^4}-\frac{x^6a^6}{36\,\mathrm{LambertW}(xa)^5}+\frac{x^6a^6}{216\,\mathrm{LambertW}(xa)^6}+\frac{x^6a^6}{6\,\mathrm{LambertW}(xa)^3}}{a^6}$	62

input
```
int(x^5/LambertW(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^6*(1/12/LambertW(x*a)^4*x^6*a^6-1/36/LambertW(x*a)^5*x^6*a^6+1/216*x^6
*a^6/LambertW(x*a)^6+1/6/LambertW(x*a)^3*x^6*a^6)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{W(ax)^3}\,dx = \frac{36\,x^6\,\mathrm{W}(ax)^3 + 18\,x^6\,\mathrm{W}(ax)^2 - 6\,x^6\,\mathrm{W}(ax) + x^6}{216\,\mathrm{W}(ax)^6}$$

input
```
integrate(x^5/lambert_w(a*x)^3,x, algorithm="fricas")
```

output
```
1/216*(36*x^6*lambert_w(a*x)^3 + 18*x^6*lambert_w(a*x)^2 - 6*x^6*lambert_w
(a*x) + x^6)/lambert_w(a*x)^6
```

Sympy [F]

$$\int \frac{x^5}{W(ax)^3}\,dx = \int \frac{x^5}{W^3(ax)}\,dx$$

input
```
integrate(x**5/LambertW(a*x)**3,x)
```

output
```
Integral(x**5/LambertW(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x^5}{W(ax)^3}\,dx = \int \frac{x^5}{\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(x^5/lambert_w(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^5/lambert_w(a*x)^3, x)
```

Giac [F]

$$\int \frac{x^5}{W(ax)^3} \, dx = \int \frac{x^5}{\mathrm{W}(ax)^3} \, dx$$

input
```
integrate(x^5/lambert_w(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^5/lambert_w(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{W(ax)^3} \, dx = \int \frac{x^5}{\mathrm{LambertW}\left(a\,x\right)^3} \, dx$$

input
```
int(x^5/LambertW(a*x)^3,x)
```

output
```
int(x^5/LambertW(a*x)^3, x)
```

Reduce [F]

$$\int \frac{x^5}{W(ax)^3} \, dx = \int \frac{x^5}{lambert\_w(ax)^3} dx$$

input
```
int(x^5/Lambert_W(a*x)^3,x)
```

output
```
int(x**5/lambert_w(a*x)**3,x)
```

3.52 $\int \frac{x^4}{W(ax)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 40

$$\int \frac{x^4}{W(ax)^3}\,dx=-\frac{3x^5}{125W(ax)^5}+\frac{3x^5}{25W(ax)^4}+\frac{x^5}{5W(ax)^3}$$

output
```
-3/125*x^5/LambertW(a*x)^5+3/25*x^5/LambertW(a*x)^4+1/5*x^5/LambertW(a*x)^
3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{W(ax)^3}\,dx=-\frac{3x^5}{125W(ax)^5}+\frac{3x^5}{25W(ax)^4}+\frac{x^5}{5W(ax)^3}$$

input
```
Integrate[x^4/ProductLog[a*x]^3,x]
```

output
```
(-3*x^5)/(125*ProductLog[a*x]^5) + (3*x^5)/(25*ProductLog[a*x]^4) + x^5/(5
*ProductLog[a*x]^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7172, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{W(ax)^3}dx$$

$$\Big\downarrow \ 7172$$

$$\frac{3}{5}\int \frac{x^4}{W(ax)^3(W(ax)+1)}dx+\frac{x^5}{5W(ax)^3}$$

$$\Big\downarrow \ 7205$$

$$\frac{3}{5}\left(\frac{x^5}{5W(ax)^4}-\frac{1}{5}\int \frac{x^4}{W(ax)^4(W(ax)+1)}dx\right)+\frac{x^5}{5W(ax)^3}$$

$$\Big\downarrow \ 7201$$

$$\frac{x^5}{5W(ax)^3}+\frac{3}{5}\left(\frac{x^5}{5W(ax)^4}-\frac{x^5}{25W(ax)^5}\right)$$

input
```
Int[x^4/ProductLog[a*x]^3,x]
```

output
```
(3*(-1/25*x^5/ProductLog[a*x]^5 + x^5/(5*ProductLog[a*x]^4)))/5 + x^5/(5*P
roductLog[a*x]^3)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))    In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{\frac{3x^5a^5}{25\,\mathrm{LambertW}(xa)^4}-\frac{3x^5a^5}{125\,\mathrm{LambertW}(xa)^5}+\frac{x^5a^5}{5\,\mathrm{LambertW}(xa)^3}}{a^5}$	48
default	$\frac{\frac{3x^5a^5}{25\,\mathrm{LambertW}(xa)^4}-\frac{3x^5a^5}{125\,\mathrm{LambertW}(xa)^5}+\frac{x^5a^5}{5\,\mathrm{LambertW}(xa)^3}}{a^5}$	48

input
```
int(x^4/LambertW(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(3/25/LambertW(x*a)^4*x^5*a^5-3/125*x^5*a^5/LambertW(x*a)^5+1/5/Lamb
ertW(x*a)^3*x^5*a^5)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{W(ax)^3}\,dx = \frac{25\,x^5\,\mathrm{W}(ax)^2 + 15\,x^5\,\mathrm{W}(ax) - 3\,x^5}{125\,\mathrm{W}(ax)^5}$$

input
```
integrate(x^4/lambert_w(a*x)^3,x, algorithm="fricas")
```

output
```
1/125*(25*x^5*lambert_w(a*x)^2 + 15*x^5*lambert_w(a*x) - 3*x^5)/lambert_w(
a*x)^5
```

Sympy [F]

$$\int \frac{x^4}{W(ax)^3}\,dx = \int \frac{x^4}{W^3\,(ax)}\,dx$$

input
```
integrate(x**4/LambertW(a*x)**3,x)
```

output
```
Integral(x**4/LambertW(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x^4}{W(ax)^3}\,dx = \int \frac{x^4}{\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(x^4/lambert_w(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^4/lambert_w(a*x)^3, x)
```

Giac [F]

$$\int \frac{x^4}{W(ax)^3}\,dx = \int \frac{x^4}{\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(x^4/lambert_w(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^4/lambert_w(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{W(ax)^3}\,dx = \int \frac{x^4}{\mathrm{LambertW}\left(a\,x\right)^3}\,dx$$

input
```
int(x^4/LambertW(a*x)^3,x)
```

output
```
int(x^4/LambertW(a*x)^3, x)
```

Reduce [F]

$$\int \frac{x^4}{W(ax)^3}\,dx = \int \frac{x^4}{lambert\_w(ax)^3}dx$$

input
```
int(x^4/Lambert_W(a*x)^3,x)
```

output
```
int(x**4/lambert_w(a*x)**3,x)
```

3.53 $\int \frac{x^3}{W(ax)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \frac{x^3}{W(ax)^3}\,dx = \frac{3x^4}{16W(ax)^4} + \frac{x^4}{4W(ax)^3}$$

output
```
3/16*x^4/LambertW(a*x)^4+1/4*x^4/LambertW(a*x)^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{W(ax)^3}\,dx = \frac{3x^4}{16W(ax)^4} + \frac{x^4}{4W(ax)^3}$$

input
```
Integrate[x^3/ProductLog[a*x]^3,x]
```

output
```
(3*x^4)/(16*ProductLog[a*x]^4) + x^4/(4*ProductLog[a*x]^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7172, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{W(ax)^3} dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{3}{4}\int \frac{x^3}{W(ax)^3(W(ax)+1)}dx + \frac{x^4}{4W(ax)^3}$$

$$\Big\downarrow \text{7201}$$

$$\frac{x^4}{4W(ax)^3} + \frac{3x^4}{16W(ax)^4}$$

input
```
Int[x^3/ProductLog[a*x]^3,x]
```

output
```
(3*x^4)/(16*ProductLog[a*x]^4) + x^4/(4*ProductLog[a*x]^3)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$-\frac{-4x^{4} \operatorname{LambertW}(xa)-3x^{4}}{16 \operatorname{LambertW}(xa)^{4}}$	24
derivativedivides	$\frac{\frac{3x^{4}a^{4}}{16 \operatorname{LambertW}(xa)^{4}}+\frac{x^{4}a^{4}}{4 \operatorname{LambertW}(xa)^{3}}}{a^{4}}$	34
default	$\frac{\frac{3x^{4}a^{4}}{16 \operatorname{LambertW}(xa)^{4}}+\frac{x^{4}a^{4}}{4 \operatorname{LambertW}(xa)^{3}}}{a^{4}}$	34

input
```
int(x^3/LambertW(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-4*x^4*LambertW(x*a)-3*x^4)/LambertW(x*a)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{W(ax)^3} dx = \frac{4 x^4 W(ax) + 3 x^4}{16 W(ax)^4}$$

input
```
integrate(x^3/lambert_w(a*x)^3,x, algorithm="fricas")
```

output
```
1/16*(4*x^4*lambert_w(a*x) + 3*x^4)/lambert_w(a*x)^4
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{W(ax)^3}\,dx = \frac{x^4}{4W^3(ax)} + \frac{3x^4}{16W^4(ax)}$$

input
```
integrate(x**3/LambertW(a*x)**3,x)
```

output
```
x**4/(4*LambertW(a*x)**3) + 3*x**4/(16*LambertW(a*x)**4)
```

Maxima [F]

$$\int \frac{x^3}{W(ax)^3}\,dx = \int \frac{x^3}{\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(x^3/lambert_w(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^3/lambert_w(a*x)^3, x)
```

Giac [F]

$$\int \frac{x^3}{W(ax)^3}\,dx = \int \frac{x^3}{\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(x^3/lambert_w(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^3/lambert_w(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{W(ax)^3} \, dx = \int \frac{x^3}{\mathrm{LambertW}\left(a\,x\right)^3} \, dx$$

input
```
int(x^3/LambertW(a*x)^3,x)
```

output
```
int(x^3/LambertW(a*x)^3, x)
```

Reduce [F]

$$\int \frac{x^3}{W(ax)^3} \, dx = \int \frac{x^3}{lambert\_w(ax)^3} dx$$

input
```
int(x^3/Lambert_W(a*x)^3,x)
```

output
```
int(x**3/lambert_w(a*x)**3,x)
```

3.54 $\int \frac{x^2}{W(ax)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \frac{x^2}{W(ax)^3}\,dx = \frac{\text{ExpIntegralEi}(3W(ax))}{a^3} + \frac{x^3}{3W(ax)^3}$$

output
```
Ei(3*LambertW(a*x))/a^3+1/3*x^3/LambertW(a*x)^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{W(ax)^3}\,dx = \frac{\text{ExpIntegralEi}(3W(ax))}{a^3} + \frac{x^3}{3W(ax)^3}$$

input
```
Integrate[x^2/ProductLog[a*x]^3,x]
```

output
```
ExpIntegralEi[3*ProductLog[a*x]]/a^3 + x^3/(3*ProductLog[a*x]^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7172, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{W(ax)^3}\,dx$$

$$\Big\downarrow \text{ 7172}$$

$$\int \frac{x^2}{W(ax)^3(W(ax)+1)}dx + \frac{x^3}{3W(ax)^3}$$

$$\Big\downarrow \text{ 7202}$$

$$\frac{\text{ExpIntegralEi}(3W(ax))}{a^3} + \frac{x^3}{3W(ax)^3}$$

input
```
Int[x^2/ProductLog[a*x]^3,x]
```

output
```
ExpIntegralEi[3*ProductLog[a*x]]/a^3 + x^3/(3*ProductLog[a*x]^3)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{\frac{x^3a^3}{3\,\mathrm{LambertW}(xa)^3}-\mathrm{expIntegral}_1(-3\,\mathrm{LambertW}(xa))}{a^3}$	30
default	$\frac{\frac{x^3a^3}{3\,\mathrm{LambertW}(xa)^3}-\mathrm{expIntegral}_1(-3\,\mathrm{LambertW}(xa))}{a^3}$	30

input
```
int(x^2/LambertW(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/3*x^3*a^3/LambertW(x*a)^3-Ei(1,-3*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{x^2}{W(ax)^3}\,dx=\int \frac{x^2}{\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(x^2/lambert_w(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^2/lambert_w(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x^2}{W(ax)^3}\,dx = \int \frac{x^2}{W^3(ax)}\,dx$$

input
```
integrate(x**2/LambertW(a*x)**3,x)
```

output
```
Integral(x**2/LambertW(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x^2}{W(ax)^3}\,dx = \int \frac{x^2}{\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(x^2/lambert_w(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^2/lambert_w(a*x)^3, x)
```

Giac [F]

$$\int \frac{x^2}{W(ax)^3}\,dx = \int \frac{x^2}{\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(x^2/lambert_w(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^2/lambert_w(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{W(ax)^3} \, dx = \int \frac{x^2}{\mathrm{LambertW}\left(a\,x\right)^3} \, dx$$

input
```
int(x^2/LambertW(a*x)^3,x)
```

output
```
int(x^2/LambertW(a*x)^3, x)
```

Reduce [F]

$$\int \frac{x^2}{W(ax)^3} \, dx = \int \frac{x^2}{lambert\_w(ax)^3} dx$$

input
```
int(x^2/Lambert_W(a*x)^3,x)
```

output
```
int(x**2/lambert_w(a*x)**3,x)
```

3.55 $\int \frac{x}{W(ax)^3} dx$

Optimal result

Integrand size = 8, antiderivative size = 24

$$\int \frac{x}{W(ax)^3} dx = \frac{3 \operatorname{ExpIntegralEi}(2W(ax))}{a^2} - \frac{x^2}{W(ax)^3}$$

output
```
3*Ei(2*LambertW(a*x))/a^2-x^2/LambertW(a*x)^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x}{W(ax)^3} dx = \frac{3 \operatorname{ExpIntegralEi}(2W(ax))}{a^2} - \frac{x^2}{W(ax)^3}$$

input
```
Integrate[x/ProductLog[a*x]^3,x]
```

output
```
(3*ExpIntegralEi[2*ProductLog[a*x]])/a^2 - x^2/ProductLog[a*x]^3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7173, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{W(ax)^3}\,dx$$

$$\Big\downarrow \text{ 7173}$$

$$3\int \frac{x}{W(ax)^2(W(ax)+1)}dx-\frac{x^2}{W(ax)^3}$$

$$\Big\downarrow \text{ 7202}$$

$$\frac{3\,\text{ExpIntegralEi}(2W(ax))}{a^2}-\frac{x^2}{W(ax)^3}$$

input
```
Int[x/ProductLog[a*x]^3,x]
```

output
```
(3*ExpIntegralEi[2*ProductLog[a*x]])/a^2 - x^2/ProductLog[a*x]^3
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{-\frac{x^2 a^2}{\mathrm{LambertW}(xa)^3}-3\,\mathrm{expIntegral}_1(-2\,\mathrm{LambertW}(xa))}{a^2}$	30
default	$\frac{-\frac{x^2 a^2}{\mathrm{LambertW}(xa)^3}-3\,\mathrm{expIntegral}_1(-2\,\mathrm{LambertW}(xa))}{a^2}$	30

input
```
int(x/LambertW(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/LambertW(x*a)^3*x^2*a^2-3*Ei(1,-2*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{x}{W(ax)^3}\,dx=\int \frac{x}{\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(x/lambert_w(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x/lambert_w(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x}{W(ax)^3}\,dx = \int \frac{x}{W^3(ax)}\,dx$$

input
```
integrate(x/LambertW(a*x)**3,x)
```

output
```
Integral(x/LambertW(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x}{W(ax)^3}\,dx = \int \frac{x}{\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(x/lambert_w(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x/lambert_w(a*x)^3, x)
```

Giac [F]

$$\int \frac{x}{W(ax)^3}\,dx = \int \frac{x}{\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(x/lambert_w(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x/lambert_w(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{W(ax)^3} \, dx = \int \frac{x}{\mathrm{LambertW}\left(a\,x\right)^3} \,dx$$

input
```
int(x/LambertW(a*x)^3,x)
```

output
```
int(x/LambertW(a*x)^3, x)
```

Reduce [F]

$$\int \frac{x}{W(ax)^3} \, dx = \int \frac{x}{lambert\_w(ax)^3} dx$$

input
```
int(x/Lambert_W(a*x)^3,x)
```

output
```
int(x/lambert_w(a*x)**3,x)
```

3.56 $\int \frac{1}{W(ax)^3} \, dx$

Optimal result

Integrand size = 6, antiderivative size = 35

$$\int \frac{1}{W(ax)^3} \, dx = \frac{3\,\text{ExpIntegralEi}(W(ax))}{2a} - \frac{x}{2W(ax)^3} - \frac{3x}{2W(ax)^2}$$

output
```
3/2*Ei(LambertW(a*x))/a-1/2*x/LambertW(a*x)^3-3/2*x/LambertW(a*x)^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{W(ax)^3} \, dx = \frac{1}{2}\left(\frac{3\,\text{ExpIntegralEi}(W(ax))}{a} - \frac{x}{W(ax)^3} - \frac{3x}{W(ax)^2}\right)$$

input
```
Integrate[ProductLog[a*x]^(-3),x]
```

output
```
((3*ExpIntegralEi[ProductLog[a*x]])/a - x/ProductLog[a*x]^3 - (3*x)/Produc
tLog[a*x]^2)/2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {7166, 7182, 7179}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W(ax)^3} \, dx$$

$$\Big\downarrow \text{ 7166}$$

$$\frac{3}{2} \int \frac{1}{W(ax)^2 (W(ax)+1)} dx - \frac{x}{2W(ax)^3}$$

$$\Big\downarrow \text{ 7182}$$

$$\frac{3}{2} \left(\int \frac{1}{W(ax)(W(ax)+1)} dx - \frac{x}{W(ax)^2} \right) - \frac{x}{2W(ax)^3}$$

$$\Big\downarrow \text{ 7179}$$

$$\frac{3}{2} \left(\frac{\text{ExpIntegralEi}(W(ax))}{a} - \frac{x}{W(ax)^2} \right) - \frac{x}{2W(ax)^3}$$

input
```
Int[ProductLog[a*x]^(-3),x]
```

output
```
(3*(ExpIntegralEi[ProductLog[a*x]]/a - x/ProductLog[a*x]^2))/2 - x/(2*Prod
uctLog[a*x]^3)
```

Defintions of rubi rules used

rule 7166
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_), x_Symbol] :> Simp[(a + b*x
)*((c*ProductLog[a + b*x])^p/(b*(p + 1))), x] + Simp[p/(c*(p + 1))   Int[(c
*ProductLog[a + b*x])^(p + 1)/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a
, b, c}, x] && LtQ[p, -1]
```

rule 7179
```
Int[1/(ProductLog[(a_.) + (b_.)*(x_)]*((d_) + (d_.)*ProductLog[(a_.) + (b_.
)*(x_)])), x_Symbol] :> Simp[ExpIntegralEi[ProductLog[a + b*x]]/(b*d), x] /
; FreeQ[{a, b, d}, x]
```

rule 7182
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[(a + b*x)*((c*ProductLog[a + b*x])^p/
(b*d*(p + 1))), x] - Simp[1/(c*(p + 1))   Int[(c*ProductLog[a + b*x])^(p +
1)/(d + d*ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p,
-1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{-\frac{3xa}{2\,\mathrm{LambertW}(xa)^2}-\frac{3\,\mathrm{expIntegral}_1(-\mathrm{LambertW}(xa))}{2}-\frac{xa}{2\,\mathrm{LambertW}(xa)^3}}{a}$	36
default	$\frac{-\frac{3xa}{2\,\mathrm{LambertW}(xa)^2}-\frac{3\,\mathrm{expIntegral}_1(-\mathrm{LambertW}(xa))}{2}-\frac{xa}{2\,\mathrm{LambertW}(xa)^3}}{a}$	36

input
```
int(1/LambertW(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-3/2/LambertW(x*a)^2*x*a-3/2*Ei(1,-LambertW(x*a))-1/2/LambertW(x*a)^3
*x*a)
```

Fricas [F]

$$\int \frac{1}{W(ax)^3}\,dx = \int \frac{1}{\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(1/lambert_w(a*x)^3,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)^(-3), x)
```

Sympy [F]

$$\int \frac{1}{W(ax)^3}\,dx = \int \frac{1}{W^3\left(ax\right)}\,dx$$

input
```
integrate(1/LambertW(a*x)**3,x)
```

output
```
Integral(LambertW(a*x)**(-3), x)
```

Maxima [F]

$$\int \frac{1}{W(ax)^3}\,dx = \int \frac{1}{\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(1/lambert_w(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^(-3), x)
```

Giac [F]

$$\int \frac{1}{W(ax)^3}\,dx = \int \frac{1}{\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(1/lambert_w(a*x)^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^(-3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W(ax)^3} \, dx = \int \frac{1}{\mathrm{LambertW}\left(a\,x\right)^3} \, dx$$

input
```
int(1/LambertW(a*x)^3,x)
```

output
```
int(1/LambertW(a*x)^3, x)
```

Reduce [F]

$$\int \frac{1}{W(ax)^3} \, dx = \int \frac{1}{lambert\_w(ax)^3} dx$$

input
```
int(1/Lambert_W(a*x)^3,x)
```

output
```
int(1/lambert_w(a*x)**3,x)
```

3.57 $\int \frac{1}{xW(ax)^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \frac{1}{xW(ax)^3} dx = -\frac{1}{3W(ax)^3} - \frac{1}{2W(ax)^2}$$

output
```
-1/3/LambertW(a*x)^3-1/2/LambertW(a*x)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{xW(ax)^3} dx = -\frac{1}{3W(ax)^3} - \frac{1}{2W(ax)^2}$$

input
```
Integrate[1/(x*ProductLog[a*x]^3),x]
```

output
```
-1/3*1/ProductLog[a*x]^3 - 1/(2*ProductLog[a*x]^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{xW(ax)^3}dx$$

$$\Big\downarrow \text{7173}$$

$$\int \frac{1}{xW(ax)^2(W(ax)+1)}dx-\frac{1}{3W(ax)^3}$$

$$\Big\downarrow \text{7200}$$

$$-\frac{1}{2W(ax)^2}-\frac{1}{3W(ax)^3}$$

input
```
Int[1/(x*ProductLog[a*x]^3),x]
```

output
```
-1/3*1/ProductLog[a*x]^3 - 1/(2*ProductLog[a*x]^2)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$-\frac{2+3\,\mathrm{LambertW}(xa)}{6\,\mathrm{LambertW}(xa)^{3}}$	17
derivativedivides	$-\frac{1}{3\,\mathrm{LambertW}(xa)^{3}}-\frac{1}{2\,\mathrm{LambertW}(xa)^{2}}$	18
default	$-\frac{1}{3\,\mathrm{LambertW}(xa)^{3}}-\frac{1}{2\,\mathrm{LambertW}(xa)^{2}}$	18

input
```
int(1/x/LambertW(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6/LambertW(x*a)^3*(2+3*LambertW(x*a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{1}{xW(ax)^3}\,dx=-\frac{3\ \mathrm{W}(ax)+2}{6\ \mathrm{W}(ax)^3}$$

input
```
integrate(1/x/lambert_w(a*x)^3,x, algorithm="fricas")
```

output
```
-1/6*(3*lambert_w(a*x) + 2)/lambert_w(a*x)^3
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{xW(ax)^3}\,dx=-\frac{1}{2W^2\left(ax\right)}-\frac{1}{3W^3\left(ax\right)}$$

input
```
integrate(1/x/LambertW(a*x)**3,x)
```

output
```
-1/(2*LambertW(a*x)**2) - 1/(3*LambertW(a*x)**3)
```

Maxima [F]

$$\int \frac{1}{xW(ax)^3}\,dx = \int \frac{1}{x\,\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(1/x/lambert_w(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/(x*lambert_w(a*x)^3), x)
```

Giac [F]

$$\int \frac{1}{xW(ax)^3}\,dx = \int \frac{1}{x\,\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(1/x/lambert_w(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/(x*lambert_w(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{xW(ax)^3}\,dx = \int \frac{1}{x\,\mathrm{LambertW}\left(a\,x\right)^3}\,dx$$

input
```
int(1/(x*LambertW(a*x)^3),x)
```

output
```
int(1/(x*LambertW(a*x)^3), x)
```

Reduce [F]

$$\int \frac{1}{xW(ax)^3} \, dx = \int \frac{1}{lambert\_w(ax)^3 x} dx$$

input
```
int(1/x/Lambert_W(a*x)^3,x)
```

output
```
int(1/(lambert_w(a*x)**3*x),x)
```

3.58 $\int \frac{1}{x^2 W(ax)^3} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 59

$$\int \frac{1}{x^2 W(ax)^3} \, dx = -\frac{1}{8x} - \frac{1}{8} a \operatorname{ExpIntegralEi}(-W(ax)) - \frac{1}{4xW(ax)^3} - \frac{1}{4xW(ax)^2} + \frac{1}{8xW(ax)}$$

output
```
-1/8/x-1/8*a*Ei(-LambertW(a*x))-1/4/x/LambertW(a*x)^3-1/4/x/LambertW(a*x)^
2+1/8/x/LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 W(ax)^3} \, dx = -\frac{1}{8x} - \frac{1}{8} a \operatorname{ExpIntegralEi}(-W(ax)) - \frac{1}{4xW(ax)^3} - \frac{1}{4xW(ax)^2} + \frac{1}{8xW(ax)}$$

input
```
Integrate[1/(x^2*ProductLog[a*x]^3),x]
```

output
```
-1/8*1/x - (a*ExpIntegralEi[-ProductLog[a*x]])/8 - 1/(4*x*ProductLog[a*x]^
3) - 1/(4*x*ProductLog[a*x]^2) + 1/(8*x*ProductLog[a*x])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7173, 7206, 7206, 7196, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 W(ax)^3} \, dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{3}{4} \int \frac{1}{x^2 W(ax)^2 (W(ax)+1)} dx - \frac{1}{4x W(ax)^3}$$

$$\Big\downarrow \text{7206}$$

$$\frac{3}{4} \left(-\frac{1}{3} \int \frac{1}{x^2 W(ax)(W(ax)+1)} dx - \frac{1}{3x W(ax)^2} \right) - \frac{1}{4x W(ax)^3}$$

$$\Big\downarrow \text{7206}$$

$$\frac{3}{4} \left(\frac{1}{3} \left(\frac{1}{2} \int \frac{1}{x^2 (W(ax)+1)} dx + \frac{1}{2x W(ax)} \right) - \frac{1}{3x W(ax)^2} \right) - \frac{1}{4x W(ax)^3}$$

$$\Big\downarrow \text{7196}$$

$$\frac{3}{4} \left(\frac{1}{3} \left(\frac{1}{2} \left(-\int \frac{W(ax)}{x^2 (W(ax)+1)} dx - \frac{1}{x} \right) + \frac{1}{2x W(ax)} \right) - \frac{1}{3x W(ax)^2} \right) - \frac{1}{4x W(ax)^3}$$

$$\Big\downarrow \text{7202}$$

$$\frac{3}{4} \left(\frac{1}{3} \left(\frac{1}{2} \left(-a \, \text{ExpIntegralEi}(-W(ax)) - \frac{1}{x} \right) + \frac{1}{2x W(ax)} \right) - \frac{1}{3x W(ax)^2} \right) - \frac{1}{4x W(ax)^3}$$

input
```
Int[1/(x^2*ProductLog[a*x]^3),x]
```

output
```
(3*(((-x^(-1) - a*ExpIntegralEi[-ProductLog[a*x]])/2 + 1/(2*x*ProductLog[a
*x]))/3 - 1/(3*x*ProductLog[a*x]^2)))/4 - 1/(4*x*ProductLog[a*x]^3)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

method	result
derivativedivides	$a\left(-\frac{1}{4\,\mathrm{LambertW}(xa)^{3}xa}-\frac{1}{4\,\mathrm{LambertW}(xa)^{2}xa}+\frac{1}{8\,\mathrm{LambertW}(xa)xa}-\frac{1}{8ax}+\frac{\mathrm{expIntegral}_{1}(\mathrm{LambertW}(x$
default	$a\left(-\frac{1}{4\,\mathrm{LambertW}(xa)^{3}xa}-\frac{1}{4\,\mathrm{LambertW}(xa)^{2}xa}+\frac{1}{8\,\mathrm{LambertW}(xa)xa}-\frac{1}{8ax}+\frac{\mathrm{expIntegral}_{1}(\mathrm{LambertW}(x$

input
```
int(1/x^2/LambertW(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/4/LambertW(x*a)^3/x/a-1/4/LambertW(x*a)^2/x/a+1/8/LambertW(x*a)/x/a-
1/8/a/x+1/8*Ei(1,LambertW(x*a)))
```

Fricas [F]

$$\int \frac{1}{x^2W(ax)^3}\,dx=\int \frac{1}{x^2\,\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(1/x^2/lambert_w(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/(x^2*lambert_w(a*x)^3), x)
```

Sympy [F]

$$\int \frac{1}{x^2W(ax)^3}\,dx=\int \frac{1}{x^2W^3\,(ax)}\,dx$$

input
```
integrate(1/x**2/LambertW(a*x)**3,x)
```

output
```
Integral(1/(x**2*LambertW(a*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{x^2W(ax)^3}\,dx = \int \frac{1}{x^2\,\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(1/x^2/lambert_w(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/(x^2*lambert_w(a*x)^3), x)
```

Giac [F]

$$\int \frac{1}{x^2W(ax)^3}\,dx = \int \frac{1}{x^2\,\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(1/x^2/lambert_w(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/(x^2*lambert_w(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2W(ax)^3}\,dx = \int \frac{1}{x^2\,\mathrm{LambertW}\left(a\,x\right)^3}\,dx$$

input
```
int(1/(x^2*LambertW(a*x)^3),x)
```

output
```
int(1/(x^2*LambertW(a*x)^3), x)
```

Reduce [F]

$$\int \frac{1}{x^2 W(ax)^3} \, dx$$

$$= \frac{2\left(\int \frac{1}{lambert\_w(ax)^4 x^2 + lambert\_w(ax)^3 x^2} dx\right) lambert\_w(ax)^2 x - \left(\int \frac{1}{lambert\_w(ax)^3 x^2 + lambert\_w(ax)^2 x^2}\right.}{}$$

input
```
int(1/x^2/Lambert_W(a*x)^3,x)
```

output
```
(2*int(1/(lambert_w(a*x)**4*x**2 + lambert_w(a*x)**3*x**2),x)*lambert_w(a*
x)**2*x - int(1/(lambert_w(a*x)**3*x**2 + lambert_w(a*x)**2*x**2),x)*lambe
rt_w(a*x)**2*x - 2*int(1/(lambert_w(a*x)**2*x**2 + lambert_w(a*x)*x**2),x)
*lambert_w(a*x)**2*x - 6*int(1/(e**lambert_w(a*x)*lambert_w(a*x)**4*x + e*
*lambert_w(a*x)*lambert_w(a*x)**3*x),x)*lambert_w(a*x)**2*a*x + int(1/(e**
lambert_w(a*x)*lambert_w(a*x)**3*x + e**lambert_w(a*x)*lambert_w(a*x)**2*x
),x)*lambert_w(a*x)**2*a*x + int(1/(lambert_w(a*x)*x**2 + x**2),x)*lambert
_w(a*x)**2*x + lambert_w(a*x) - 3)/(2*lambert_w(a*x)**2*x)
```

3.59 $\int \frac{1}{x^3 W(ax)^3} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 72

$$\int \frac{1}{x^3 W(ax)^3} \, dx = -\frac{1}{10x^2} + \frac{2}{5} a^2 \, \text{ExpIntegralEi}(-2W(ax)) - \frac{1}{5x^2 W(ax)^3} - \frac{3}{20x^2 W(ax)^2} + \frac{1}{10x^2 W(ax)} + \frac{W(ax)}{5x^2}$$

output
```
-1/10/x^2+2/5*a^2*Ei(-2*LambertW(a*x))-1/5/x^2/LambertW(a*x)^3-3/20/x^2/La
mbertW(a*x)^2+1/10/x^2/LambertW(a*x)+1/5*LambertW(a*x)/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 W(ax)^3} \, dx = -\frac{1}{10x^2} + \frac{2}{5} a^2 \, \text{ExpIntegralEi}(-2W(ax)) - \frac{1}{5x^2 W(ax)^3} - \frac{3}{20x^2 W(ax)^2} + \frac{1}{10x^2 W(ax)} + \frac{W(ax)}{5x^2}$$

input
```
Integrate[1/(x^3*ProductLog[a*x]^3),x]
```

output
```
-1/10*1/x^2 + (2*a^2*ExpIntegralEi[-2*ProductLog[a*x]])/5 - 1/(5*x^2*Produ
ctLog[a*x]^3) - 3/(20*x^2*ProductLog[a*x]^2) + 1/(10*x^2*ProductLog[a*x])
+ ProductLog[a*x]/(5*x^2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {7173, 7206, 7206, 7196, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 W(ax)^3} \, dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{3}{5} \int \frac{1}{x^3 W(ax)^2 (W(ax)+1)} dx - \frac{1}{5x^2 W(ax)^3}$$

$$\Big\downarrow \text{7206}$$

$$\frac{3}{5} \left(-\frac{1}{2} \int \frac{1}{x^3 W(ax)(W(ax)+1)} dx - \frac{1}{4x^2 W(ax)^2}\right) - \frac{1}{5x^2 W(ax)^3}$$

$$\Big\downarrow \text{7206}$$

$$\frac{3}{5} \left(\frac{1}{2} \left(\frac{2}{3} \int \frac{1}{x^3 (W(ax)+1)} dx + \frac{1}{3x^2 W(ax)}\right) - \frac{1}{4x^2 W(ax)^2}\right) - \frac{1}{5x^2 W(ax)^3}$$

$$\Big\downarrow \text{7196}$$

$$\frac{3}{5} \left(\frac{1}{2} \left(\frac{2}{3} \left(-\int \frac{W(ax)}{x^3 (W(ax)+1)} dx - \frac{1}{2x^2}\right) + \frac{1}{3x^2 W(ax)}\right) - \frac{1}{4x^2 W(ax)^2}\right) - \frac{1}{5x^2 W(ax)^3}$$

$$\Big\downarrow \text{7206}$$

$$\frac{3}{5} \left(\frac{1}{2} \left(\frac{2}{3} \left(2 \int \frac{W(ax)^2}{x^3 (W(ax)+1)} dx + \frac{W(ax)}{x^2} - \frac{1}{2x^2}\right) + \frac{1}{3x^2 W(ax)}\right) - \frac{1}{4x^2 W(ax)^2}\right) - \frac{1}{5x^2 W(ax)^3}$$

$$\Big\downarrow \text{7202}$$

$$\frac{3}{5}\left(\frac{1}{2}\left(\frac{2}{3}\left(2a^2\,\text{ExpIntegralEi}(-2W(ax))+\frac{W(ax)}{x^2}-\frac{1}{2x^2}\right)+\frac{1}{3x^2W(ax)}\right)-\frac{1}{4x^2W(ax)^2}\right)-\frac{1}{5x^2W(ax)^3}$$

input
```
Int[1/(x^3*ProductLog[a*x]^3),x]
```

output
```
-1/5*1/(x^2*ProductLog[a*x]^3) + (3*(-1/4*1/(x^2*ProductLog[a*x]^2) + (1/(
3*x^2*ProductLog[a*x]) + (2*(-1/2*1/x^2 + 2*a^2*ExpIntegralEi[-2*ProductLo
g[a*x]] + ProductLog[a*x]/x^2))/3)/2))/5
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08

method	result
derivativedivides	$a^2\left(-\frac{1}{5\,\mathrm{LambertW}(xa)^3x^2a^2}-\frac{3}{20\,\mathrm{LambertW}(xa)^2x^2a^2}+\frac{1}{10\,\mathrm{LambertW}(xa)x^2a^2}-\frac{1}{10x^2a^2}+\frac{\mathrm{LambertW}(}{5x^2a^2}\right.$
default	$a^2\left(-\frac{1}{5\,\mathrm{LambertW}(xa)^3x^2a^2}-\frac{3}{20\,\mathrm{LambertW}(xa)^2x^2a^2}+\frac{1}{10\,\mathrm{LambertW}(xa)x^2a^2}-\frac{1}{10x^2a^2}+\frac{\mathrm{LambertW}(}{5x^2a^2}\right.$

input
```
int(1/x^3/LambertW(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/5/LambertW(x*a)^3/x^2/a^2-3/20/LambertW(x*a)^2/x^2/a^2+1/10/Lamber
tW(x*a)/x^2/a^2-1/10/x^2/a^2+1/5*LambertW(x*a)/x^2/a^2-2/5*Ei(1,2*LambertW
(x*a)))
```

Fricas [F]

$$\int \frac{1}{x^3W(ax)^3}\,dx = \int \frac{1}{x^3\,\mathrm{W}(ax)^3}\,dx$$

input
```
integrate(1/x^3/lambert_w(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/(x^3*lambert_w(a*x)^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3W(ax)^3}\,dx = \int \frac{1}{x^3W^3\left(ax\right)}\,dx$$

input
```
integrate(1/x**3/LambertW(a*x)**3,x)
```

output
```
Integral(1/(x**3*LambertW(a*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{x^3 W(ax)^3} \, dx = \int \frac{1}{x^3 \, \mathrm{W}(ax)^3} \, dx$$

input
```
integrate(1/x^3/lambert_w(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/(x^3*lambert_w(a*x)^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 W(ax)^3} \, dx = \int \frac{1}{x^3 \, \mathrm{W}(ax)^3} \, dx$$

input
```
integrate(1/x^3/lambert_w(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/(x^3*lambert_w(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 W(ax)^3} \, dx = \int \frac{1}{x^3 \, \mathrm{LambertW}\left(a\,x\right)^3} \, dx$$

input
```
int(1/(x^3*LambertW(a*x)^3),x)
```

output
```
int(1/(x^3*LambertW(a*x)^3), x)
```

Reduce [F]

$$\int \frac{1}{x^3 W(ax)^3} \, dx$$
$$= \frac{4\left(\int \frac{1}{lambert\_w(ax)^4 x^3 + lambert\_w(ax)^3 x^3} dx\right) lambert\_w(ax)^2 x^2 - 2\left(\int \frac{1}{lambert\_w(ax)^3 x^3 + lambert\_w(ax)^2 \cdots}\right.}{}$$

input
```
int(1/x^3/Lambert_W(a*x)^3,x)
```

output
```
(4*int(1/(lambert_w(a*x)**4*x**3 + lambert_w(a*x)**3*x**3),x)*lambert_w(a*
x)**2*x**2 - 2*int(1/(lambert_w(a*x)**3*x**3 + lambert_w(a*x)**2*x**3),x)*
lambert_w(a*x)**2*x**2 - 4*int(1/(lambert_w(a*x)**2*x**3 + lambert_w(a*x)*
x**3),x)*lambert_w(a*x)**2*x**2 - 6*int(1/(e**lambert_w(a*x)*lambert_w(a*x
)**4*x**2 + e**lambert_w(a*x)*lambert_w(a*x)**3*x**2),x)*lambert_w(a*x)**2
*a*x**2 + int(1/(e**lambert_w(a*x)*lambert_w(a*x)**3*x**2 + e**lambert_w(a
*x)*lambert_w(a*x)**2*x**2),x)*lambert_w(a*x)**2*a*x**2 + 2*int(1/(lambert
_w(a*x)*x**3 + x**3),x)*lambert_w(a*x)**2*x**2 + lambert_w(a*x) - 3)/(4*la
mbert_w(a*x)**2*x**2)
```

3.60 $\int x^3\sqrt{cW(ax)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 131

$$\int x^3\sqrt{cW(ax)}\,dx = -\frac{105\sqrt{c}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2\sqrt{cW(ax)}}{\sqrt{c}}\right)}{65536a^4}+\frac{105c^4x^4}{16384(cW(ax))^{7/2}} -\frac{35c^3x^4}{2048(cW(ax))^{5/2}}+\frac{7c^2x^4}{256(cW(ax))^{3/2}} -\frac{cx^4}{32\sqrt{cW(ax)}}+\frac{1}{4}x^4\sqrt{cW(ax)}$$

output
```
-105/65536*c^(1/2)*Pi^(1/2)*erfi(2*(c*LambertW(a*x))^(1/2)/c^(1/2))/a^4+10
5/16384*c^4*x^4/(c*LambertW(a*x))^(7/2)-35/2048*c^3*x^4/(c*LambertW(a*x))^
(5/2)+7/256*c^2*x^4/(c*LambertW(a*x))^(3/2)-1/32*c*x^4/(c*LambertW(a*x))^(
1/2)+1/4*x^4*(c*LambertW(a*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^3\sqrt{cW(ax)}\,dx = \frac{\sqrt{cW(ax)}\left(420a^4x^4-1120a^4x^4W(ax)+1792a^4x^4W(ax)^2-2048a^4x^4W(ax)^3-105\sqrt{\pi}\text{erfi}\left(2\sqrt{W(ax}\right.\right.}{65536a^4W(ax)^4}$$

input
```
Integrate[x^3*Sqrt[c*ProductLog[a*x]],x]
```

output
```
(Sqrt[c*ProductLog[a*x]]*(420*a^4*x^4 - 1120*a^4*x^4*ProductLog[a*x] + 179
2*a^4*x^4*ProductLog[a*x]^2 - 2048*a^4*x^4*ProductLog[a*x]^3 - 105*Sqrt[Pi
]*Erfi[2*Sqrt[ProductLog[a*x]]]*ProductLog[a*x]^(7/2) + 16384*a^4*x^4*Prod
uctLog[a*x]^4))/(65536*a^4*ProductLog[a*x]^4)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {7172, 7205, 7205, 7205, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{cW(ax)}\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{4}x^4\sqrt{cW(ax)}-\frac{1}{8}\int \frac{x^3\sqrt{cW(ax)}}{W(ax)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{8}\left(\frac{7}{8}c\int \frac{x^3}{\sqrt{cW(ax)}(W(ax)+1)}dx-\frac{cx^4}{4\sqrt{cW(ax)}}\right)+\frac{1}{4}x^4\sqrt{cW(ax)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{8}\left(\frac{7}{8}c\left(\frac{cx^4}{4(cW(ax))^{3/2}}-\frac{5}{8}c\int\frac{x^3}{(cW(ax))^{3/2}(W(ax)+1)}dx\right)-\frac{cx^4}{4\sqrt{cW(ax)}}\right)+\frac{1}{4}x^4\sqrt{cW(ax)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{8}\left(\frac{7}{8}c\left(\frac{cx^4}{4(cW(ax))^{3/2}}-\frac{5}{8}c\left(\frac{cx^4}{4(cW(ax))^{5/2}}-\frac{3}{8}c\int\frac{x^3}{(cW(ax))^{5/2}(W(ax)+1)}dx\right)\right)-\frac{cx^4}{4\sqrt{cW(ax)}}\right)+\frac{1}{4}x^4\sqrt{cW(ax)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{8}\left(\frac{7}{8}c\left(\frac{cx^4}{4(cW(ax))^{3/2}}-\frac{5}{8}c\left(\frac{cx^4}{4(cW(ax))^{5/2}}-\frac{3}{8}c\left(\frac{cx^4}{4(cW(ax))^{7/2}}-\frac{1}{8}c\int\frac{x^3}{(cW(ax))^{7/2}(W(ax)+1)}dx\right)\right)\right)-\frac{cx^4}{4\sqrt{cW(ax)}}\right)+\frac{1}{4}x^4\sqrt{cW(ax)}$$

$$\Big\downarrow \text{7204}$$

$$\frac{1}{8}\left(\frac{7}{8}c\left(\frac{cx^4}{4(cW(ax))^{3/2}}-\frac{5}{8}c\left(\frac{cx^4}{4(cW(ax))^{5/2}}-\frac{3}{8}c\left(\frac{cx^4}{4(cW(ax))^{7/2}}-\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{cW(ax)}}{\sqrt{c}}\right)}{16a^4c^{5/2}}\right)\right)\right)-\frac{cx^4}{4\sqrt{cW(ax)}}\right)+\frac{1}{4}x^4\sqrt{cW(ax)}$$

input
```
Int[x^3*Sqrt[c*ProductLog[a*x]],x]
```

output
```
(x^4*Sqrt[c*ProductLog[a*x]])/4 + (-1/4*(c*x^4)/Sqrt[c*ProductLog[a*x]] +
(7*c*((c*x^4)/(4*(c*ProductLog[a*x])^(3/2)) - (5*c*((c*x^4)/(4*(c*ProductL
og[a*x])^(5/2)) - (3*c*(-1/16*(Sqrt[Pi]*Erfi[(2*Sqrt[c*ProductLog[a*x]])/S
qrt[c]])/(a^4*c^(5/2)) + (c*x^4)/(4*(c*ProductLog[a*x])^(7/2))))/8))/8))/8
)/8
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.12

method	result
default	$\frac{\frac{c(c\,\mathrm{LambertW}(xa))^{\frac{9}{2}}\mathrm{e}^{4\,\mathrm{LambertW}(xa)}}{4}-\frac{c\left(\frac{c(c\,\mathrm{LambertW}(xa))^{\frac{7}{2}}\mathrm{e}^{4\,\mathrm{LambertW}(xa)}}{8}-\frac{7c\left(\frac{c(c\,\mathrm{LambertW}(xa))^{\frac{5}{2}}\mathrm{e}^{4\,\mathrm{LambertW}(xa)}}{8}-\frac{5c\left(\frac{c(c\,\mathrm{Lam}}{}\right.}{}\right.}{}\right.}{}}{a^{4}c^{5}}$

input
```
int(x^3*(c*LambertW(x*a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/a^4/c^5*(1/8*c*(c*LambertW(x*a))^(9/2)*exp(4*LambertW(x*a))-1/8*c*(1/8*c
*(c*LambertW(x*a))^(7/2)*exp(4*LambertW(x*a))-7/8*c*(1/8*c*(c*LambertW(x*a
))^(5/2)*exp(4*LambertW(x*a))-5/8*c*(1/8*c*(c*LambertW(x*a))^(3/2)*exp(4*L
ambertW(x*a))-3/8*c*(1/8*c*(c*LambertW(x*a))^(1/2)*exp(4*LambertW(x*a))-1/
32*c*Pi^(1/2)/(-1/c)^(1/2)*erf(2*(-1/c)^(1/2)*(c*LambertW(x*a))^(1/2))))))
)
```

Fricas [F]

$$\int x^3\sqrt{cW(ax)}\,dx=\int \sqrt{c\,\mathrm{W}(ax)}x^3\,dx$$

input
```
integrate(x^3*(c*lambert_w(a*x))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))*x^3, x)
```

Sympy [F]

$$\int x^3\sqrt{cW(ax)}\,dx = \int x^3\sqrt{cW\left(ax\right)}\,dx$$

input
```
integrate(x**3*(c*LambertW(a*x))**(1/2),x)
```

output
```
Integral(x**3*sqrt(c*LambertW(a*x)), x)
```

Maxima [F]

$$\int x^3\sqrt{cW(ax)}\,dx = \int \sqrt{c\,\mathrm{W}(ax)}x^3\,dx$$

input
```
integrate(x^3*(c*lambert_w(a*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x))*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{cW(ax)}\,dx = \int \sqrt{c\,\mathrm{W}(ax)}x^3\,dx$$

input
```
integrate(x^3*(c*lambert_w(a*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x))*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{cW(ax)}\,dx = \int x^3\,\sqrt{c\,\mathrm{LambertW}\,(a\,x)}\,dx$$

input
```
int(x^3*(c*LambertW(a*x))^(1/2),x)
```

output
```
int(x^3*(c*LambertW(a*x))^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{cW(ax)}\,dx = \sqrt{c}\left(\int \sqrt{lambert\_w(ax)}\,x^3dx\right)$$

input
```
int(x^3*(c*Lambert_W(a*x))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a*x))*x**3,x)
```

3.61 $\int x^2\sqrt{cW(ax)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 119

$$\int x^2\sqrt{cW(ax)}\,dx=\frac{5\sqrt{c}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{cW(ax)}}{\sqrt{c}}\right)}{432a^3}-\frac{5c^3x^3}{216(cW(ax))^{5/2}}+\frac{5c^2x^3}{108(cW(ax))^{3/2}}-\frac{cx^3}{18\sqrt{cW(ax)}}+\frac{1}{3}x^3\sqrt{cW(ax)}$$

output
```
5/1296*c^(1/2)*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(c*LambertW(a*x))^(1/2)/c^(1/
2))/a^3-5/216*c^3*x^3/(c*LambertW(a*x))^(5/2)+5/108*c^2*x^3/(c*LambertW(a*
x))^(3/2)-1/18*c*x^3/(c*LambertW(a*x))^(1/2)+1/3*x^3*(c*LambertW(a*x))^(1/
2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int x^2\sqrt{cW(ax)}\,dx$$
$$=\frac{\sqrt{cW(ax)}\left(-30a^3x^3+60a^3x^3W(ax)-72a^3x^3W(ax)^2+5\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{W(ax)}\right)W(ax)^{5/2}+432a^3x^3\right.}{1296a^3W(ax)^3}$$

input
```
Integrate[x^2*Sqrt[c*ProductLog[a*x]],x]
```

output
```
(Sqrt[c*ProductLog[a*x]]*(-30*a^3*x^3 + 60*a^3*x^3*ProductLog[a*x] - 72*a^
3*x^3*ProductLog[a*x]^2 + 5*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ProductLog[a*x]]]
*ProductLog[a*x]^(5/2) + 432*a^3*x^3*ProductLog[a*x]^3))/(1296*a^3*Product
Log[a*x]^3)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {7172, 7205, 7205, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{cW(ax)}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{3}x^3\sqrt{cW(ax)}-\frac{1}{6}\int \frac{x^2\sqrt{cW(ax)}}{W(ax)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{6}\left(\frac{5}{6}c\int \frac{x^2}{\sqrt{cW(ax)}(W(ax)+1)}dx-\frac{cx^3}{3\sqrt{cW(ax)}}\right)+\frac{1}{3}x^3\sqrt{cW(ax)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{6}\left(\frac{5}{6}c\left(\frac{cx^3}{3(cW(ax))^{3/2}}-\frac{1}{2}c\int \frac{x^2}{(cW(ax))^{3/2}(W(ax)+1)}dx\right)-\frac{cx^3}{3\sqrt{cW(ax)}}\right)+\frac{1}{3}x^3\sqrt{cW(ax)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{6}\left(\frac{5}{6}c\left(\frac{cx^3}{3(cW(ax))^{3/2}}-\frac{1}{2}c\left(\frac{cx^3}{3(cW(ax))^{5/2}}-\frac{1}{6}c\int \frac{x^2}{(cW(ax))^{5/2}(W(ax)+1)}dx\right)\right)-\frac{cx^3}{3\sqrt{cW(ax)}}\right)+\frac{1}{3}x^3\sqrt{cW(ax)}$$

$$\Big\downarrow \text{7204}$$

$$\frac{1}{6}\left(\frac{5}{6}c\left(\frac{cx^3}{3(cW(ax))^{3/2}}-\frac{1}{2}c\left(\frac{cx^3}{3(cW(ax))^{5/2}}-\frac{\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{cW(ax)}}{\sqrt{c}}\right)}{6a^3c^{3/2}}\right)\right)-\frac{cx^3}{3\sqrt{cW(ax)}}\right)+\frac{1}{3}x^3\sqrt{cW(ax)}$$

input
```
Int[x^2*Sqrt[c*ProductLog[a*x]],x]
```

output
```
(x^3*Sqrt[c*ProductLog[a*x]])/3 + (-1/3*(c*x^3)/Sqrt[c*ProductLog[a*x]] +
(5*c*((c*x^3)/(3*(c*ProductLog[a*x])^(3/2)) - (c*(-1/6*(Sqrt[Pi/3]*Erfi[(S
qrt[3]*Sqrt[c*ProductLog[a*x]])/Sqrt[c]])/(a^3*c^(3/2)) + (c*x^3)/(3*(c*Pr
oductLog[a*x])^(5/2))))/2))/6)/6
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04

method	result
default	$\frac{\frac{c(c\,\mathrm{LambertW}(xa))^{\frac{7}{2}}\mathrm{e}^{3\,\mathrm{LambertW}(xa)}}{3}-\frac{c\left(\frac{c(c\,\mathrm{LambertW}(xa))^{\frac{5}{2}}\mathrm{e}^{3\,\mathrm{LambertW}(xa)}}{6}-\frac{5c\left(\frac{c(c\,\mathrm{LambertW}(xa))^{\frac{3}{2}}\mathrm{e}^{3\,\mathrm{LambertW}(xa)}}{6}-\frac{c\left(\frac{c\sqrt{c\,\mathrm{Lam}\ldots}}{}\right.}{}\right.}{}\right.}{3}}{a^{3}c^{4}}$

input
```
int(x^2*(c*LambertW(x*a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/a^3/c^4*(1/6*c*(c*LambertW(x*a))^(7/2)*exp(3*LambertW(x*a))-1/6*c*(1/6*c
*(c*LambertW(x*a))^(5/2)*exp(3*LambertW(x*a))-5/6*c*(1/6*c*(c*LambertW(x*a
))^(3/2)*exp(3*LambertW(x*a))-1/2*c*(1/6*c*(c*LambertW(x*a))^(1/2)*exp(3*L
ambertW(x*a))-1/12*c*Pi^(1/2)/(-3/c)^(1/2)*erf((-3/c)^(1/2)*(c*LambertW(x*
a))^(1/2)))))
```

Fricas [F]

$$\int x^2\sqrt{cW(ax)}\,dx=\int\sqrt{c\,\mathrm{W}(ax)}x^2\,dx$$

input
```
integrate(x^2*(c*lambert_w(a*x))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))*x^2, x)
```

Sympy [F]

$$\int x^2 \sqrt{cW(ax)}\, dx = \int x^2 \sqrt{cW\left(ax\right)}\, dx$$

input
```
integrate(x**2*(c*LambertW(a*x))**(1/2),x)
```

output
```
Integral(x**2*sqrt(c*LambertW(a*x)), x)
```

Maxima [F]

$$\int x^2 \sqrt{cW(ax)}\, dx = \int \sqrt{c\,\mathrm{W}(ax)}x^2\, dx$$

input
```
integrate(x^2*(c*lambert_w(a*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x))*x^2, x)
```

Giac [F]

$$\int x^2 \sqrt{cW(ax)}\, dx = \int \sqrt{c\,\mathrm{W}(ax)}x^2\, dx$$

input
```
integrate(x^2*(c*lambert_w(a*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x))*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{cW(ax)}\,dx = \int x^2\,\sqrt{c\,\mathrm{LambertW}\,(a\,x)}\,dx$$

input
```
int(x^2*(c*LambertW(a*x))^(1/2),x)
```

output
```
int(x^2*(c*LambertW(a*x))^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{cW(ax)}\,dx = \sqrt{c}\left(\int \sqrt{lambert\_w\,(ax)}\,x^2dx\right)$$

input
```
int(x^2*(c*Lambert_W(a*x))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a*x))*x**2,x)
```

3.62 $\int x\sqrt{cW(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 99

$$\int x\sqrt{cW(ax)}\,dx=-\frac{3\sqrt{c}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax)}}{\sqrt{c}}\right)}{64a^2}+\frac{3c^2x^2}{32(cW(ax))^{3/2}}-\frac{cx^2}{8\sqrt{cW(ax)}}+\frac{1}{2}x^2\sqrt{cW(ax)}$$

output
```
-3/128*c^(1/2)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(a*x))^(1/2)/c^(1/
2))/a^2+3/32*c^2*x^2/(c*LambertW(a*x))^(3/2)-1/8*c*x^2/(c*LambertW(a*x))^(
1/2)+1/2*x^2*(c*LambertW(a*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int x\sqrt{cW(ax)}\,dx=\frac{\sqrt{cW(ax)}\left(12a^2x^2-16a^2x^2W(ax)-3\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{W(ax)}\right)W(ax)^{3/2}+64a^2x^2W(ax)^2\right)}{128a^2W(ax)^2}$$

input
```
Integrate[x*Sqrt[c*ProductLog[a*x]],x]
```

output
```
(Sqrt[c*ProductLog[a*x]]*(12*a^2*x^2 - 16*a^2*x^2*ProductLog[a*x] - 3*Sqrt
[2*Pi]*Erfi[Sqrt[2]*Sqrt[ProductLog[a*x]]]*ProductLog[a*x]^(3/2) + 64*a^2*
x^2*ProductLog[a*x]^2))/(128*a^2*ProductLog[a*x]^2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7172, 7205, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{cW(ax)}\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{2}x^2\sqrt{cW(ax)}-\frac{1}{4}\int \frac{x\sqrt{cW(ax)}}{W(ax)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{4}\left(\frac{3}{4}c\int \frac{x}{\sqrt{cW(ax)}(W(ax)+1)}dx-\frac{cx^2}{2\sqrt{cW(ax)}}\right)+\frac{1}{2}x^2\sqrt{cW(ax)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{4}\left(\frac{3}{4}c\left(\frac{cx^2}{2(cW(ax))^{3/2}}-\frac{1}{4}c\int \frac{x}{(cW(ax))^{3/2}(W(ax)+1)}dx\right)-\frac{cx^2}{2\sqrt{cW(ax)}}\right)+\frac{1}{2}x^2\sqrt{cW(ax)}$$

$$\Big\downarrow \text{7204}$$

$$\frac{1}{4}\left(\frac{3}{4}c\left(\frac{cx^2}{2(cW(ax))^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax)}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}\right)-\frac{cx^2}{2\sqrt{cW(ax)}}\right)+\frac{1}{2}x^2\sqrt{cW(ax)}$$

input
```
Int[x*Sqrt[c*ProductLog[a*x]],x]
```

output
```
(x^2*Sqrt[c*ProductLog[a*x]])/2 + (-1/2*(c*x^2)/Sqrt[c*ProductLog[a*x]] +
(3*c*(-1/4*(Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[c*ProductLog[a*x]])/Sqrt[c]])/(a
^2*Sqrt[c]) + (c*x^2)/(2*(c*ProductLog[a*x])^(3/2))))/4)/4
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

method	result
default	$\frac{\frac{c(c\,\mathrm{LambertW}(xa))^{\frac{5}{2}}\,\mathrm{e}^{2\,\mathrm{LambertW}(xa)}}{2}-\frac{c\left(\frac{c(c\,\mathrm{LambertW}(xa))^{\frac{3}{2}}\,\mathrm{e}^{2\,\mathrm{LambertW}(xa)}}{4}-\frac{3c\left(\frac{c\sqrt{c\,\mathrm{LambertW}(xa)}\,\mathrm{e}^{2\,\mathrm{LambertW}(xa)}}{4}-\frac{c\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\cdots}\right)}{\cdots}\right)}{4}\right)}{2}}{a^2c^3}$

input
```
int(x*(c*LambertW(x*a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/a^2/c^3*(1/4*c*(c*LambertW(x*a))^(5/2)*exp(2*LambertW(x*a))-1/4*c*(1/4*c
*(c*LambertW(x*a))^(3/2)*exp(2*LambertW(x*a))-3/4*c*(1/4*c*(c*LambertW(x*a
))^(1/2)*exp(2*LambertW(x*a))-1/8*c*Pi^(1/2)/(-2/c)^(1/2)*erf((-2/c)^(1/2)
*(c*LambertW(x*a))^(1/2)))))
```

Fricas [F]

$$\int x\sqrt{cW(ax)}\,dx = \int \sqrt{c\,\mathrm{W}(ax)}x\,dx$$

input
```
integrate(x*(c*lambert_w(a*x))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))*x, x)
```

Sympy [F]

$$\int x\sqrt{cW(ax)}\,dx = \int x\sqrt{cW\left(ax\right)}\,dx$$

input
```
integrate(x*(c*LambertW(a*x))**(1/2),x)
```

output
```
Integral(x*sqrt(c*LambertW(a*x)), x)
```

Maxima [F]

$$\int x\sqrt{cW(ax)}\,dx = \int \sqrt{c\,\mathrm{W}(ax)}x\,dx$$

input
```
integrate(x*(c*lambert_w(a*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x))*x, x)
```

Giac [F]

$$\int x\sqrt{cW(ax)}\,dx = \int \sqrt{c\,\mathrm{W}(ax)}x\,dx$$

input
```
integrate(x*(c*lambert_w(a*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x))*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{cW(ax)}\,dx = \int x\,\sqrt{c\,\mathrm{LambertW}\,(a\,x)}\,dx$$

input
```
int(x*(c*LambertW(a*x))^(1/2),x)
```

output
```
int(x*(c*LambertW(a*x))^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{cW(ax)}\,dx = \sqrt{c}\left(\int \sqrt{lambert\_w(ax)}\,xdx\right)$$

input
```
int(x*(c*Lambert_W(a*x))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a*x))*x,x)
```

3.63 $\int \sqrt{cW(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 63

$$\int \sqrt{cW(ax)}\,dx = \frac{\sqrt{c}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{cW(ax)}}{\sqrt{c}}\right)}{4a} - \frac{cx}{2\sqrt{cW(ax)}} + x\sqrt{cW(ax)}$$

output
```
1/4*c^(1/2)*Pi^(1/2)*erfi((c*LambertW(a*x))^(1/2)/c^(1/2))/a-1/2*c*x/(c*La
mbertW(a*x))^(1/2)+x*(c*LambertW(a*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \sqrt{cW(ax)}\,dx = \frac{c\left(-2ax + \sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{W(ax)}\right)\sqrt{W(ax)} + 4axW(ax)\right)}{4a\sqrt{cW(ax)}}$$

input
```
Integrate[Sqrt[c*ProductLog[a*x]],x]
```

output
```
(c*(-2*a*x + Sqrt[Pi]*Erfi[Sqrt[ProductLog[a*x]]]*Sqrt[ProductLog[a*x]] +
4*a*x*ProductLog[a*x]))/(4*a*Sqrt[c*ProductLog[a*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {7167, 7178, 7180}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cW(ax)}\,dx$$

$$\Big\downarrow \text{ 7167}$$

$$x\sqrt{cW(ax)}-\frac{1}{2}\int \frac{\sqrt{cW(ax)}}{W(ax)+1}dx$$

$$\Big\downarrow \text{ 7178}$$

$$\frac{1}{2}\left(\frac{1}{2}c\int \frac{1}{\sqrt{cW(ax)}(W(ax)+1)}dx-\frac{cx}{\sqrt{cW(ax)}}\right)+x\sqrt{cW(ax)}$$

$$\Big\downarrow \text{ 7180}$$

$$\frac{1}{2}\left(\frac{\sqrt{\pi}\sqrt{c}\text{erfi}\left(\frac{\sqrt{cW(ax)}}{\sqrt{c}}\right)}{2a}-\frac{cx}{\sqrt{cW(ax)}}\right)+x\sqrt{cW(ax)}$$

input
```
Int[Sqrt[c*ProductLog[a*x]],x]
```

output
```
x*Sqrt[c*ProductLog[a*x]] + ((Sqrt[c]*Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a*x]
]/Sqrt[c]])/(2*a) - (c*x)/Sqrt[c*ProductLog[a*x]])/2
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7178
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[c*(a + b*x)*((c*ProductLog[a + b*x])^
(p - 1)/(b*d)), x] - Simp[c*p   Int[(c*ProductLog[a + b*x])^(p - 1)/(d + d*
ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0]
```

rule 7180
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[Pi*c, 2]*(Erfi[Sqrt[c*ProductLo
g[a + b*x]]/Rt[c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && PosQ[c]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

method	result	size
default	$\frac{\frac{(c\,\mathrm{LambertW}(xa))^{\frac{3}{2}}\,xac}{\mathrm{LambertW}(xa)}-c\left(\frac{c\sqrt{c\,\mathrm{LambertW}(xa)}\,xa}{2\,\mathrm{LambertW}(xa)}-\frac{c\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{c}}\,\sqrt{c\,\mathrm{LambertW}(xa)}\right)}{4\sqrt{-\frac{1}{c}}}\right)}{a\,c^{2}}$	82

input
```
int((c*LambertW(x*a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/a/c^2*(1/2*(c*LambertW(x*a))^(3/2)*x*a/LambertW(x*a)*c-1/2*c*(1/2*c*(c*L
ambertW(x*a))^(1/2)*x*a/LambertW(x*a)-1/4*c*Pi^(1/2)/(-1/c)^(1/2)*erf((-1/
c)^(1/2)*(c*LambertW(x*a))^(1/2))))
```

Fricas [F]

$$\int \sqrt{cW(ax)}\,dx = \int \sqrt{c\,\mathrm{W}(ax)}\,dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x)), x)
```

Sympy [F]

$$\int \sqrt{cW(ax)}\,dx = \int \sqrt{cW\left(ax\right)}\,dx$$

input
```
integrate((c*LambertW(a*x))**(1/2),x)
```

output
```
Integral(sqrt(c*LambertW(a*x)), x)
```

Maxima [F]

$$\int \sqrt{cW(ax)}\,dx = \int \sqrt{c\,\mathrm{W}(ax)}\,dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x)), x)
```

Giac [F]

$$\int \sqrt{cW(ax)}\,dx = \int \sqrt{c\,\mathrm{W}(ax)}\,dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cW(ax)}\,dx = \int \sqrt{c\,\mathrm{LambertW}\,(a\,x)}\,dx$$

input
```
int((c*LambertW(a*x))^(1/2),x)
```

output
```
int((c*LambertW(a*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{cW(ax)}\,dx = \sqrt{c}\left(\int \sqrt{lambert\_w(ax)}dx\right)$$

input
```
int((c*Lambert_W(a*x))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a*x)),x)
```

3.64 $\int \frac{\sqrt{cW(ax)}}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 30

$$\int \frac{\sqrt{cW(ax)}}{x} dx = 2\sqrt{cW(ax)} + \frac{2(cW(ax))^{3/2}}{3c}$$

output
```
2*(c*LambertW(a*x))^(1/2)+2/3*(c*LambertW(a*x))^(3/2)/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{cW(ax)}}{x} dx = \frac{2}{3}\sqrt{cW(ax)}(3 + W(ax))$$

input
```
Integrate[Sqrt[c*ProductLog[a*x]]/x,x]
```

output
```
(2*Sqrt[c*ProductLog[a*x]]*(3 + ProductLog[a*x]))/3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cW(ax)}}{x} \, dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{\int \frac{(cW(ax))^{3/2}}{x(W(ax)+1)}dx}{c} + 2\sqrt{cW(ax)}$$

$$\Big\downarrow \text{7200}$$

$$\frac{2(cW(ax))^{3/2}}{3c} + 2\sqrt{cW(ax)}$$

input
```
Int[Sqrt[c*ProductLog[a*x]]/x,x]
```

output
```
2*Sqrt[c*ProductLog[a*x]] + (2*(c*ProductLog[a*x])^(3/2))/(3*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\frac{2(c \operatorname{LambertW}(xa))^{\frac{3}{2}}}{3}+2c\sqrt{c \operatorname{LambertW}(xa)}}{c}$	27
default	$\frac{\frac{2(c \operatorname{LambertW}(xa))^{\frac{3}{2}}}{3}+2c\sqrt{c \operatorname{LambertW}(xa)}}{c}$	27

input
```
int((c*LambertW(x*a))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/c*(1/3*(c*LambertW(x*a))^(3/2)+c*(c*LambertW(x*a))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{cW(ax)}}{x}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax)}}{x}\,dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))/x, x)
```

Sympy [F]

$$\int \frac{\sqrt{cW(ax)}}{x}\,dx = \int \frac{\sqrt{cW\left(ax\right)}}{x}\,dx$$

input
```
integrate((c*LambertW(a*x))**(1/2)/x,x)
```

output
```
Integral(sqrt(c*LambertW(a*x))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{cW(ax)}}{x}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax)}}{x}\,dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x))/x, x)
```

Giac [F]

$$\int \frac{\sqrt{cW(ax)}}{x}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax)}}{x}\,dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cW(ax)}}{x}\,dx = \int \frac{\sqrt{c\,\mathrm{LambertW}\,(a\,x)}}{x}\,dx$$

input
```
int((c*LambertW(a*x))^(1/2)/x,x)
```

output
```
int((c*LambertW(a*x))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{cW(ax)}}{x}\,dx = \sqrt{c}\left(\int \frac{\sqrt{lambert\_w(ax)}}{x}dx\right)$$

input
```
int((c*Lambert_W(a*x))^(1/2)/x,x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a*x))/x,x)
```

3.65 $\int \frac{\sqrt{cW(ax)}}{x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{\sqrt{cW(ax)}}{x^2} dx = -a\sqrt{c}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{cW(ax)}}{\sqrt{c}}\right) - \frac{2\sqrt{cW(ax)}}{x}$$

output
```
-a*c^(1/2)*Pi^(1/2)*erf((c*LambertW(a*x))^(1/2)/c^(1/2))-2*(c*LambertW(a*x
))^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{cW(ax)}}{x^2} dx = -\frac{2\sqrt{cW(ax)}}{x} - \frac{a\sqrt{\pi}\operatorname{erf}\left(\sqrt{W(ax)}\right)\sqrt{cW(ax)}}{\sqrt{W(ax)}}$$

input
```
Integrate[Sqrt[c*ProductLog[a*x]]/x^2,x]
```

output
```
(-2*Sqrt[c*ProductLog[a*x]])/x - (a*Sqrt[Pi]*Erf[Sqrt[ProductLog[a*x]]]*Sq
rt[c*ProductLog[a*x]])/Sqrt[ProductLog[a*x]]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7173, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cW(ax)}}{x^2} \, dx$$

$$\Big\downarrow \text{7173}$$

$$-\frac{\int \frac{(cW(ax))^{3/2}}{x^2(W(ax)+1)}dx}{c} - \frac{2\sqrt{cW(ax)}}{x}$$

$$\Big\downarrow \text{7203}$$

$$\sqrt{\pi}(-a)\sqrt{c}\text{erf}\left(\frac{\sqrt{cW(ax)}}{\sqrt{c}}\right) - \frac{2\sqrt{cW(ax)}}{x}$$

input
```
Int[Sqrt[c*ProductLog[a*x]]/x^2,x]
```

output
```
-(a*Sqrt[c]*Sqrt[Pi]*Erf[Sqrt[c*ProductLog[a*x]]/Sqrt[c]]) - (2*Sqrt[c*Pro
ductLog[a*x]])/x
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

method	result	size
default	$a\left(\sqrt{\pi}\,\sqrt{c}\,\operatorname{erf}\left(\frac{\sqrt{c\,\mathrm{LambertW}(xa)}}{\sqrt{c}}\right)+2c\left(-\frac{\mathrm{e}^{-\mathrm{LambertW}(xa)}}{\sqrt{c\,\mathrm{LambertW}(xa)}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{c\,\mathrm{LambertW}(xa)}}{\sqrt{c}}\right)}{\sqrt{c}}\right)\right)$	66

input
```
int((c*LambertW(x*a))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(Pi^(1/2)*c^(1/2)*erf((c*LambertW(x*a))^(1/2)/c^(1/2))+2*c*(-1/(c*Lamber
tW(x*a))^(1/2)*exp(-LambertW(x*a))-1/c^(1/2)*Pi^(1/2)*erf((c*LambertW(x*a)
)^(1/2)/c^(1/2))))
```

Fricas [F]

$$\int \frac{\sqrt{cW(ax)}}{x^2}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax)}}{x^2}\,dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{cW(ax)}}{x^2}\,dx = \int \frac{\sqrt{cW\left(ax\right)}}{x^2}\,dx$$

input
```
integrate((c*LambertW(a*x))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(c*LambertW(a*x))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{cW(ax)}}{x^2}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax)}}{x^2}\,dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x))/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{cW(ax)}}{x^2}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax)}}{x^2}\,dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x))/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cW(ax)}}{x^2}\,dx = \int \frac{\sqrt{c\,\mathrm{LambertW}\left(a\,x\right)}}{x^2}\,dx$$

input

```
int((c*LambertW(a*x))^(1/2)/x^2,x)
```

output

```
int((c*LambertW(a*x))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{cW(ax)}}{x^2}\,dx = \frac{\sqrt{c}\left(-2\sqrt{lambert\_w(ax)}+\left(\int \frac{\sqrt{lambert\_w(ax)}}{e^{lambert\_w(ax)}lambert\_w(ax)^2x+e^{lambert\_w(ax)}lambert\_w(ax)x}dx\right)ax\right)}{2x}$$

input

```
int((c*Lambert_W(a*x))^(1/2)/x^2,x)
```

output

```
(sqrt(c)*( - 2*sqrt(lambert_w(a*x)) + int(sqrt(lambert_w(a*x))/(e**lambert
_w(a*x)*lambert_w(a*x)**2*x + e**lambert_w(a*x)*lambert_w(a*x)*x),x)*a*x))
/(2*x)
```

3.66 $\int \frac{\sqrt{cW(ax)}}{x^3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int \frac{\sqrt{cW(ax)}}{x^3} \, dx = \frac{2}{3}a^2\sqrt{c}\sqrt{2\pi}\text{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax)}}{\sqrt{c}}\right) - \frac{2\sqrt{cW(ax)}}{3x^2} + \frac{2(cW(ax))^{3/2}}{3cx^2}$$

output
```
2/3*a^2*c^(1/2)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(c*LambertW(a*x))^(1/2)/c^(1/
2))-2/3*(c*LambertW(a*x))^(1/2)/x^2+2/3*(c*LambertW(a*x))^(3/2)/c/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{cW(ax)}}{x^3} \, dx = \frac{2\sqrt{cW(ax)}\left(a^2\sqrt{2\pi}x^2\text{erf}\left(\sqrt{2}\sqrt{W(ax)}\right) - \sqrt{W(ax)} + W(ax)^{3/2}\right)}{3x^2\sqrt{W(ax)}}$$

input
```
Integrate[Sqrt[c*ProductLog[a*x]]/x^3,x]
```

output
```
(2*Sqrt[c*ProductLog[a*x]]*(a^2*Sqrt[2*Pi]*x^2*Erf[Sqrt[2]*Sqrt[ProductLog
[a*x]]] - Sqrt[ProductLog[a*x]] + ProductLog[a*x]^(3/2)))/(3*x^2*Sqrt[Prod
uctLog[a*x]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {7173, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cW(ax)}}{x^3} \, dx$$

$$\Big\downarrow \text{7173}$$

$$-\frac{\int \frac{(cW(ax))^{3/2}}{x^3(W(ax)+1)}dx}{3c} - \frac{2\sqrt{cW(ax)}}{3x^2}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{-\frac{4\int \frac{(cW(ax))^{5/2}}{x^3(W(ax)+1)}dx}{c} - \frac{2(cW(ax))^{3/2}}{x^2}}{3c} - \frac{2\sqrt{cW(ax)}}{3x^2}$$

$$\Big\downarrow \text{7203}$$

$$-\frac{-2\sqrt{2\pi}a^2c^{3/2}\text{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax)}}{\sqrt{c}}\right) - \frac{2(cW(ax))^{3/2}}{x^2}}{3c} - \frac{2\sqrt{cW(ax)}}{3x^2}$$

input
```
Int[Sqrt[c*ProductLog[a*x]]/x^3,x]
```

output
```
(-2*Sqrt[c*ProductLog[a*x]])/(3*x^2) - (-2*a^2*c^(3/2)*Sqrt[2*Pi]*Erf[(Sqr
t[2]*Sqrt[c*ProductLog[a*x]])/Sqrt[c]] - (2*(c*ProductLog[a*x])^(3/2))/x^2
)/(3*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(60) = 120$.

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.54

method	result
default	$2a^2c\left(-\frac{e^{-2\,\mathrm{LambertW}(xa)}}{\sqrt{c\,\mathrm{LambertW}(xa)}}-\frac{\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\frac{\sqrt{2}\,\sqrt{c\,\mathrm{LambertW}(xa)}}{\sqrt{c}}\right)}{\sqrt{c}}+c\left(-\frac{e^{-2\,\mathrm{LambertW}(xa)}}{3(c\,\mathrm{LambertW}(xa))^{\frac{3}{2}}}-\frac{4\left(-\frac{e^{-2\,\mathrm{LambertW}(xa}}{\sqrt{c\,\mathrm{LambertW}(xa}}\right.}{}\right.\right.$

input
```
int((c*LambertW(x*a))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2*a^2*c*(-1/(c*LambertW(x*a))^(1/2)*exp(-2*LambertW(x*a))-1/c^(1/2)*Pi^(1/
2)*2^(1/2)*erf(2^(1/2)*(c*LambertW(x*a))^(1/2)/c^(1/2))+c*(-1/3/(c*Lambert
W(x*a))^(3/2)*exp(-2*LambertW(x*a))-4/3/c*(-1/(c*LambertW(x*a))^(1/2)*exp(
-2*LambertW(x*a))-1/c^(1/2)*Pi^(1/2)*2^(1/2)*erf(2^(1/2)*(c*LambertW(x*a))
^(1/2)/c^(1/2)))))
```

Fricas [F]

$$\int \frac{\sqrt{cW(ax)}}{x^3}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax)}}{x^3}\,dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{cW(ax)}}{x^3}\,dx = \int \frac{\sqrt{cW\left(ax\right)}}{x^3}\,dx$$

input
```
integrate((c*LambertW(a*x))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(c*LambertW(a*x))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{cW(ax)}}{x^3}\,dx=\int \frac{\sqrt{c\,\mathrm{W}(ax)}}{x^3}\,dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x))/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{cW(ax)}}{x^3}\,dx=\int \frac{\sqrt{c\,\mathrm{W}(ax)}}{x^3}\,dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x))/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cW(ax)}}{x^3}\,dx=\int \frac{\sqrt{c\,\mathrm{LambertW}\left(a\,x\right)}}{x^3}\,dx$$

input
```
int((c*LambertW(a*x))^(1/2)/x^3,x)
```

output
```
int((c*LambertW(a*x))^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{cW(ax)}}{x^3}\,dx$$
$$= \frac{\sqrt{c}\left(-2\sqrt{lambert\_w(ax)} + \left(\int \frac{\sqrt{lambert\_w(ax)}}{e^{lambert\_w(ax)} lambert\_w(ax)^2 x^2 + e^{lambert\_w(ax)} lambert\_w(ax) x^2}dx\right) a\,x^2\right)}{4x^2}$$

input
```
int((c*Lambert_W(a*x))^(1/2)/x^3,x)
```

output
```
(sqrt(c)*( - 2*sqrt(lambert_w(a*x)) + int(sqrt(lambert_w(a*x))/(e**lambert
_w(a*x)*lambert_w(a*x)**2*x**2 + e**lambert_w(a*x)*lambert_w(a*x)*x**2),x)
*a*x**2))/(4*x**2)
```

3.67 $\int \frac{\sqrt{cW(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 99

$$\int \frac{\sqrt{cW(ax)}}{x^4}\,dx = -\frac{4}{5}a^3\sqrt{c}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{cW(ax)}}{\sqrt{c}}\right) - \frac{2\sqrt{cW(ax)}}{5x^3} + \frac{2(cW(ax))^{3/2}}{15cx^3} - \frac{4(cW(ax))^{5/2}}{5c^2x^3}$$

output
```
-4/5*a^3*c^(1/2)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(c*LambertW(a*x))^(1/2)/c^(1
/2))-2/5*(c*LambertW(a*x))^(1/2)/x^3+2/15*(c*LambertW(a*x))^(3/2)/c/x^3-4/
5*(c*LambertW(a*x))^(5/2)/c^2/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{cW(ax)}}{x^4}\,dx = -\frac{2\sqrt{cW(ax)}\left(6a^3\sqrt{3\pi}x^3\text{erf}\left(\sqrt{3}\sqrt{W(ax)}\right) + 3\sqrt{W(ax)} - W(ax)^{3/2} + 6W(ax)^{5/2}\right)}{15x^3\sqrt{W(ax)}}$$

input
```
Integrate[Sqrt[c*ProductLog[a*x]]/x^4,x]
```

output
```
(-2*Sqrt[c*ProductLog[a*x]]*(6*a^3*Sqrt[3*Pi]*x^3*Erf[Sqrt[3]*Sqrt[Product
Log[a*x]]] + 3*Sqrt[ProductLog[a*x]] - ProductLog[a*x]^(3/2) + 6*ProductLo
g[a*x]^(5/2)))/(15*x^3*Sqrt[ProductLog[a*x]])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {7173, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cW(ax)}}{x^4} \, dx$$

$$\Big\downarrow \text{ 7173}$$

$$-\frac{\int \frac{(cW(ax))^{3/2}}{x^4(W(ax)+1)}dx}{5c} - \frac{2\sqrt{cW(ax)}}{5x^3}$$

$$\Big\downarrow \text{ 7206}$$

$$-\frac{-\frac{2\int \frac{(cW(ax))^{5/2}}{x^4(W(ax)+1)}dx}{c} - \frac{2(cW(ax))^{3/2}}{3x^3}}{5c} - \frac{2\sqrt{cW(ax)}}{5x^3}$$

$$\Big\downarrow \text{ 7206}$$

$$-\frac{-\frac{2\left(-\frac{6\int \frac{(cW(ax))^{7/2}}{x^4(W(ax)+1)}dx}{c} - \frac{2(cW(ax))^{5/2}}{x^3}\right)}{c} - \frac{2(cW(ax))^{3/2}}{3x^3}}{5c} - \frac{2\sqrt{cW(ax)}}{5x^3}$$

$$\Big\downarrow \text{ 7203}$$

$$-\frac{-\frac{2\left(-2\sqrt{3\pi}a^3c^{5/2}\text{erf}\left(\frac{\sqrt{3}\sqrt{cW(ax)}}{\sqrt{c}}\right) - \frac{2(cW(ax))^{5/2}}{x^3}\right)}{c} - \frac{2(cW(ax))^{3/2}}{3x^3}}{5c} - \frac{2\sqrt{cW(ax)}}{5x^3}$$

input
```
Int[Sqrt[c*ProductLog[a*x]]/x^4,x]
```

output
```
(-2*Sqrt[c*ProductLog[a*x]])/(5*x^3) - ((-2*(c*ProductLog[a*x])^(3/2))/(3*
x^3) - (2*(-2*a^3*c^(5/2)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[c*ProductLog[a*x]])
/Sqrt[c]] - (2*(c*ProductLog[a*x])^(5/2))/x^3))/c)/(5*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(76) = 152$.

Time = 0.06 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.72

method	result
default	$2a^3c^2\left(-\frac{\mathrm{e}^{-3\,\mathrm{LambertW}(xa)}}{3(c\,\mathrm{LambertW}(xa))^{\frac{3}{2}}}-\frac{2\left(-\frac{\mathrm{e}^{-3\,\mathrm{LambertW}(xa)}}{\sqrt{c\,\mathrm{LambertW}(xa)}}-\frac{\sqrt{\pi}\,\sqrt{3}\,\mathrm{erf}\left(\frac{\sqrt{3}\,\sqrt{c\,\mathrm{LambertW}(xa)}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{c}+c\left(-\frac{\mathrm{e}^{-3\,\mathrm{LambertW}(xa)}}{5(c\,\mathrm{LambertW}(xa)}\right.\right.$

input
```
int((c*LambertW(x*a))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2*a^3*c^2*(-1/3/(c*LambertW(x*a))^(3/2)*exp(-3*LambertW(x*a))-2/c*(-1/(c*L
ambertW(x*a))^(1/2)*exp(-3*LambertW(x*a))-1/c^(1/2)*Pi^(1/2)*3^(1/2)*erf(3
^(1/2)*(c*LambertW(x*a))^(1/2)/c^(1/2)))+c*(-1/5/(c*LambertW(x*a))^(5/2)*e
xp(-3*LambertW(x*a))-6/5/c*(-1/3/(c*LambertW(x*a))^(3/2)*exp(-3*LambertW(x
*a))-2/c*(-1/(c*LambertW(x*a))^(1/2)*exp(-3*LambertW(x*a))-1/c^(1/2)*Pi^(1
/2)*3^(1/2)*erf(3^(1/2)*(c*LambertW(x*a))^(1/2)/c^(1/2))))))
```

Fricas [F]

$$\int \frac{\sqrt{cW(ax)}}{x^4}\,dx=\int \frac{\sqrt{c\,\mathrm{W}(ax)}}{x^4}\,dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{cW(ax)}}{x^4} dx = \int \frac{\sqrt{cW\left(ax\right)}}{x^4} dx$$

input
```
integrate((c*LambertW(a*x))**(1/2)/x**4,x)
```

output
```
Integral(sqrt(c*LambertW(a*x))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{cW(ax)}}{x^4} dx = \int \frac{\sqrt{c\,\mathrm{W}(ax)}}{x^4} dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x))/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{cW(ax)}}{x^4} dx = \int \frac{\sqrt{c\,\mathrm{W}(ax)}}{x^4} dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x))/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cW(ax)}}{x^4}\,dx = \int \frac{\sqrt{c\,\mathrm{LambertW}\left(a\,x\right)}}{x^4}\,dx$$

input
```
int((c*LambertW(a*x))^(1/2)/x^4,x)
```

output
```
int((c*LambertW(a*x))^(1/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{cW(ax)}}{x^4}\,dx = \frac{\sqrt{c}\left(-2\sqrt{lambert\_w\left(ax\right)}+\left(\int \frac{\sqrt{lambert\_w(ax)}}{e^{lambert\_w(ax)}lambert\_w(ax)^2x^3+e^{lambert\_w(ax)}lambert\_w(ax)x^3}dx\right)a\,x^3\right)}{6x^3}$$

input
```
int((c*Lambert_W(a*x))^(1/2)/x^4,x)
```

output
```
(sqrt(c)*( - 2*sqrt(lambert_w(a*x)) + int(sqrt(lambert_w(a*x))/(e**lambert
_w(a*x)*lambert_w(a*x)**2*x**3 + e**lambert_w(a*x)*lambert_w(a*x)*x**3),x)
*a*x**3))/(6*x**3)
```

3.68 $\int \frac{\sqrt{cW(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 113

$$\int \frac{\sqrt{cW(ax)}}{x^5}\,dx = \frac{256}{105}a^4\sqrt{c}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{cW(ax)}}{\sqrt{c}}\right)-\frac{2\sqrt{cW(ax)}}{7x^4}+\frac{2(cW(ax))^{3/2}}{35cx^4}-\frac{16(cW(ax))^{5/2}}{105c^2x^4}+\frac{128(cW(ax))^{7/2}}{105c^3x^4}$$

output
```
256/105*a^4*c^(1/2)*Pi^(1/2)*erf(2*(c*LambertW(a*x))^(1/2)/c^(1/2))-2/7*(c
*LambertW(a*x))^(1/2)/x^4+2/35*(c*LambertW(a*x))^(3/2)/c/x^4-16/105*(c*Lam
bertW(a*x))^(5/2)/c^2/x^4+128/105*(c*LambertW(a*x))^(7/2)/c^3/x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{cW(ax)}}{x^5}\,dx = \frac{2\sqrt{cW(ax)}\left(128a^4\sqrt{\pi}x^4\operatorname{erf}\left(2\sqrt{W(ax)}\right)-15\sqrt{W(ax)}+3W(ax)^{3/2}-8W(ax)^{5/2}+64W(ax)^{7/2}\right)}{105x^4\sqrt{W(ax)}}$$

input
```
Integrate[Sqrt[c*ProductLog[a*x]]/x^5,x]
```

output
```
(2*Sqrt[c*ProductLog[a*x]]*(128*a^4*Sqrt[Pi]*x^4*Erf[2*Sqrt[ProductLog[a*x
]]] - 15*Sqrt[ProductLog[a*x]] + 3*ProductLog[a*x]^(3/2) - 8*ProductLog[a*
x]^(5/2) + 64*ProductLog[a*x]^(7/2)))/(105*x^4*Sqrt[ProductLog[a*x]])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {7173, 7206, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cW(ax)}}{x^5}dx$$

$$\Big\downarrow \text{7173}$$

$$-\frac{\int \frac{(cW(ax))^{3/2}}{x^5(W(ax)+1)}dx}{7c}-\frac{2\sqrt{cW(ax)}}{7x^4}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{-\frac{8\int \frac{(cW(ax))^{5/2}}{x^5(W(ax)+1)}dx}{5c}-\frac{2(cW(ax))^{3/2}}{5x^4}}{7c}-\frac{2\sqrt{cW(ax)}}{7x^4}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{-\frac{8\left(-\frac{8\int \frac{(cW(ax))^{7/2}}{x^5(W(ax)+1)}dx}{3c}-\frac{2(cW(ax))^{5/2}}{3x^4}\right)}{5c}-\frac{2(cW(ax))^{3/2}}{5x^4}}{7c}-\frac{2\sqrt{cW(ax)}}{7x^4}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{-\frac{8\left(-\frac{8\left(-\frac{8\int \frac{(cW(ax))^{9/2}}{x^5(W(ax)+1)}dx}{c}-\frac{2(cW(ax))^{7/2}}{x^4}\right)}{3c}-\frac{2(cW(ax))^{5/2}}{3x^4}\right)}{5c}-\frac{2(cW(ax))^{3/2}}{5x^4}}{7c}-\frac{2\sqrt{cW(ax)}}{7x^4}$$

$$\Big\downarrow \text{7203}$$

$$-\frac{-\frac{8\left(-\frac{8\left(-4\sqrt{\pi}a^4c^{7/2}\text{erf}\left(\frac{2\sqrt{cW(ax)}}{\sqrt{c}}\right)-\frac{2(cW(ax))^{7/2}}{x^4}\right)}{3c}-\frac{2(cW(ax))^{5/2}}{3x^4}\right)}{5c}-\frac{2(cW(ax))^{3/2}}{5x^4}}{7c}-\frac{2\sqrt{cW(ax)}}{7x^4}$$

input
```
Int[Sqrt[c*ProductLog[a*x]]/x^5,x]
```

output
```
(-2*Sqrt[c*ProductLog[a*x]])/(7*x^4) - ((-2*(c*ProductLog[a*x])^(3/2))/(5*
x^4) - (8*((-2*(c*ProductLog[a*x])^(5/2))/(3*x^4) - (8*(-4*a^4*c^(7/2)*Sqr
t[Pi]*Erf[(2*Sqrt[c*ProductLog[a*x]])/Sqrt[c]] - (2*(c*ProductLog[a*x])^(7
/2))/x^4))/(3*c)))/(5*c))/(7*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(87) = 174$.

Time = 0.05 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.82

method	result
default	$2a^4c^3\left(-\frac{e^{-4\,\mathrm{LambertW}(xa)}}{5(c\,\mathrm{LambertW}(xa))^{\frac{5}{2}}}-\frac{8\left(-\frac{e^{-4\,\mathrm{LambertW}(xa)}}{3(c\,\mathrm{LambertW}(xa))^{\frac{3}{2}}}-\frac{8\left(-\frac{e^{-4\,\mathrm{LambertW}(xa)}}{\sqrt{c\,\mathrm{LambertW}(xa)}}-\frac{2\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\sqrt{c\,\mathrm{LambertW}(xa)}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{3c}\right)}{5c}+c\right.$

input
```
int((c*LambertW(x*a))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2*a^4*c^3*(-1/5/(c*LambertW(x*a))^(5/2)*exp(-4*LambertW(x*a))-8/5/c*(-1/3/
(c*LambertW(x*a))^(3/2)*exp(-4*LambertW(x*a))-8/3/c*(-1/(c*LambertW(x*a))^
(1/2)*exp(-4*LambertW(x*a))-2/c^(1/2)*Pi^(1/2)*erf(2*(c*LambertW(x*a))^(1/
2)/c^(1/2))))+c*(-1/7/(c*LambertW(x*a))^(7/2)*exp(-4*LambertW(x*a))-8/7/c*
(-1/5/(c*LambertW(x*a))^(5/2)*exp(-4*LambertW(x*a))-8/5/c*(-1/3/(c*Lambert
W(x*a))^(3/2)*exp(-4*LambertW(x*a))-8/3/c*(-1/(c*LambertW(x*a))^(1/2)*exp(
-4*LambertW(x*a))-2/c^(1/2)*Pi^(1/2)*erf(2*(c*LambertW(x*a))^(1/2)/c^(1/2)
)))))
```

Fricas [F]

$$\int \frac{\sqrt{cW(ax)}}{x^5} \, dx = \int \frac{\sqrt{c\,\mathrm{W}(ax)}}{x^5} \, dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))/x^5, x)
```

Sympy [F]

$$\int \frac{\sqrt{cW(ax)}}{x^5} \, dx = \int \frac{\sqrt{cW(ax)}}{x^5} \, dx$$

input
```
integrate((c*LambertW(a*x))**(1/2)/x**5,x)
```

output
```
Integral(sqrt(c*LambertW(a*x))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{cW(ax)}}{x^5} \, dx = \int \frac{\sqrt{c\,\mathrm{W}(ax)}}{x^5} \, dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x))/x^5, x)
```

Giac [F]

$$\int \frac{\sqrt{cW(ax)}}{x^5}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax)}}{x^5}\,dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x^5,x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x))/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cW(ax)}}{x^5}\,dx = \int \frac{\sqrt{c\,\mathrm{LambertW}\left(a\,x\right)}}{x^5}\,dx$$

input
```
int((c*LambertW(a*x))^(1/2)/x^5,x)
```

output
```
int((c*LambertW(a*x))^(1/2)/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{cW(ax)}}{x^5}\,dx = \frac{\sqrt{c}\left(-2\sqrt{lambert\_w(ax)} + \left(\int \frac{\sqrt{lambert\_w(ax)}}{e^{lambert\_w(ax)}\,lambert\_w(ax)^2 x^4 + e^{lambert\_w(ax)}\,lambert\_w(ax)x^4}dx\right) a\,x^4\right)}{8x^4}$$

input
```
int((c*Lambert_W(a*x))^(1/2)/x^5,x)
```

output
```
(sqrt(c)*( - 2*sqrt(lambert_w(a*x)) + int(sqrt(lambert_w(a*x))/(e**lambert
_w(a*x)*lambert_w(a*x)**2*x**4 + e**lambert_w(a*x)*lambert_w(a*x)*x**4),x)
*a*x**4))/(8*x**4)
```

3.69 $\int \frac{\sqrt{cW(ax)}}{x^6} dx$

Optimal result

Integrand size = 14, antiderivative size = 139

$$\int \frac{\sqrt{cW(ax)}}{x^6} dx = -\frac{400}{189}a^5\sqrt{c}\sqrt{5\pi}\text{erf}\left(\frac{\sqrt{5}\sqrt{cW(ax)}}{\sqrt{c}}\right) - \frac{2\sqrt{cW(ax)}}{9x^5} + \frac{2(cW(ax))^{3/2}}{63cx^5} - \frac{4(cW(ax))^{5/2}}{63c^2x^5} + \frac{40(cW(ax))^{7/2}}{189c^3x^5} - \frac{400(cW(ax))^{9/2}}{189c^4x^5}$$

output
```
-400/189*a^5*c^(1/2)*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*(c*LambertW(a*x))^(1/2)/
c^(1/2))-2/9*(c*LambertW(a*x))^(1/2)/x^5+2/63*(c*LambertW(a*x))^(3/2)/c/x^
5-4/63*(c*LambertW(a*x))^(5/2)/c^2/x^5+40/189*(c*LambertW(a*x))^(7/2)/c^3/
x^5-400/189*(c*LambertW(a*x))^(9/2)/c^4/x^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{cW(ax)}}{x^6} dx = -\frac{2\sqrt{cW(ax)}\left(200a^5\sqrt{5\pi}x^5\text{erf}\left(\sqrt{5}\sqrt{W(ax)}\right) + 21\sqrt{W(ax)} - 3W(ax)^{3/2} + 6W(ax)^{5/2} - 20W(ax)^{7/2}\right.}{189x^5\sqrt{W(ax)}}$$

input
```
Integrate[Sqrt[c*ProductLog[a*x]]/x^6,x]
```

output
```
(-2*Sqrt[c*ProductLog[a*x]]*(200*a^5*Sqrt[5*Pi]*x^5*Erf[Sqrt[5]*Sqrt[Produ
ctLog[a*x]]] + 21*Sqrt[ProductLog[a*x]] - 3*ProductLog[a*x]^(3/2) + 6*Prod
uctLog[a*x]^(5/2) - 20*ProductLog[a*x]^(7/2) + 200*ProductLog[a*x]^(9/2)))
/(189*x^5*Sqrt[ProductLog[a*x]])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {7173, 7206, 7206, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cW(ax)}}{x^6}dx$$

$$\Big\downarrow \text{7173}$$

$$-\frac{\int \frac{(cW(ax))^{3/2}}{x^6(W(ax)+1)}dx}{9c}-\frac{2\sqrt{cW(ax)}}{9x^5}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{-\frac{10\int \frac{(cW(ax))^{5/2}}{x^6(W(ax)+1)}dx}{7c}-\frac{2(cW(ax))^{3/2}}{7x^5}}{9c}-\frac{2\sqrt{cW(ax)}}{9x^5}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{-\frac{10\left(-\frac{2\int \frac{(cW(ax))^{7/2}}{x^6(W(ax)+1)}dx}{c}-\frac{2(cW(ax))^{5/2}}{5x^5}\right)}{7c}-\frac{2(cW(ax))^{3/2}}{7x^5}}{9c}-\frac{2\sqrt{cW(ax)}}{9x^5}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{-\frac{10\left(-\frac{2\left(-\frac{10\int \frac{(cW(ax))^{9/2}}{x^6(W(ax)+1)}dx}{3c}-\frac{2(cW(ax))^{7/2}}{3x^5}\right)}{c}-\frac{2(cW(ax))^{5/2}}{5x^5}\right)}{7c}-\frac{2(cW(ax))^{3/2}}{7x^5}}{9c}-\frac{2\sqrt{cW(ax)}}{9x^5}$$

$$\Big\downarrow \; 7206$$

$$-\frac{-\frac{10\left(-\frac{2\left(-\frac{10\left(-\frac{10\int \frac{(cW(ax))^{11/2}}{x^6(W(ax)+1)}dx}{c}-\frac{2(cW(ax))^{9/2}}{x^5}\right)}{3c}-\frac{2(cW(ax))^{7/2}}{3x^5}\right)}{c}-\frac{2(cW(ax))^{5/2}}{5x^5}\right)}{7c}-\frac{2(cW(ax))^{3/2}}{7x^5}}{9c}-\frac{2\sqrt{cW(ax)}}{9x^5}$$

$$\Big\downarrow \; 7203$$

$$-\frac{-\frac{10\left(-\frac{2\left(-\frac{10\left(-2\sqrt{5\pi}a^5c^{9/2}\text{erf}\left(\frac{\sqrt{5}\sqrt{cW(ax)}}{\sqrt{c}}\right)-\frac{2(cW(ax))^{9/2}}{x^5}\right)}{3c}-\frac{2(cW(ax))^{7/2}}{3x^5}\right)}{c}-\frac{2(cW(ax))^{5/2}}{5x^5}\right)}{7c}-\frac{2(cW(ax))^{3/2}}{7x^5}}{9c}-\frac{2\sqrt{cW(ax)}}{9x^5}$$

input
```
Int[Sqrt[c*ProductLog[a*x]]/x^6,x]
```

output
```
(-2*Sqrt[c*ProductLog[a*x]])/(9*x^5) - ((-2*(c*ProductLog[a*x])^(3/2))/(7*
x^5) - (10*((-2*(c*ProductLog[a*x])^(5/2))/(5*x^5) - (2*((-2*(c*ProductLog
[a*x])^(7/2))/(3*x^5) - (10*(-2*a^5*c^(9/2)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[c
*ProductLog[a*x]])/Sqrt[c]] - (2*(c*ProductLog[a*x])^(9/2))/x^5))/(3*c)))/
c))/(7*c))/(9*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(108) = 216$.

Time = 0.06 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.88

method	result
default	$2a^5c^4\left(-\frac{e^{-5\,\mathrm{LambertW}(xa)}}{7(c\,\mathrm{LambertW}(xa))^{\frac{7}{2}}}-\frac{10\left(-\frac{e^{-5\,\mathrm{LambertW}(xa)}}{5(c\,\mathrm{LambertW}(xa))^{\frac{5}{2}}}-\frac{2\left(-\frac{e^{-5\,\mathrm{LambertW}(xa)}}{3(c\,\mathrm{LambertW}(xa))^{\frac{3}{2}}}-\frac{10\left(-\frac{e^{-5\,\mathrm{LambertW}(xa)}}{\sqrt{c\,\mathrm{LambertW}(xa)}}-\frac{\sqrt{\pi}\,\sqrt{5}\,\mathrm{erf}\left(\right.}{}\right.}{3c}\right.}{c}\right.}{7c}\right.$

input
```
int((c*LambertW(x*a))^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
2*a^5*c^4*(-1/7/(c*LambertW(x*a))^(7/2)*exp(-5*LambertW(x*a))-10/7/c*(-1/5
/(c*LambertW(x*a))^(5/2)*exp(-5*LambertW(x*a))-2/c*(-1/3/(c*LambertW(x*a))
^(3/2)*exp(-5*LambertW(x*a))-10/3/c*(-1/(c*LambertW(x*a))^(1/2)*exp(-5*Lam
bertW(x*a))-1/c^(1/2)*Pi^(1/2)*5^(1/2)*erf(5^(1/2)*(c*LambertW(x*a))^(1/2)
/c^(1/2)))))+c*(-1/9/(c*LambertW(x*a))^(9/2)*exp(-5*LambertW(x*a))-10/9/c*
(-1/7/(c*LambertW(x*a))^(7/2)*exp(-5*LambertW(x*a))-10/7/c*(-1/5/(c*Lamber
tW(x*a))^(5/2)*exp(-5*LambertW(x*a))-2/c*(-1/3/(c*LambertW(x*a))^(3/2)*exp
(-5*LambertW(x*a))-10/3/c*(-1/(c*LambertW(x*a))^(1/2)*exp(-5*LambertW(x*a)
)-1/c^(1/2)*Pi^(1/2)*5^(1/2)*erf(5^(1/2)*(c*LambertW(x*a))^(1/2)/c^(1/2)))
)))))
```

Fricas [F]

$$\int \frac{\sqrt{cW(ax)}}{x^6}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax)}}{x^6}\,dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x^6,x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))/x^6, x)
```

Sympy [F]

$$\int \frac{\sqrt{cW(ax)}}{x^6}\,dx = \int \frac{\sqrt{cW(ax)}}{x^6}\,dx$$

input
```
integrate((c*LambertW(a*x))**(1/2)/x**6,x)
```

output
```
Integral(sqrt(c*LambertW(a*x))/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{cW(ax)}}{x^6} \, dx = \int \frac{\sqrt{c \operatorname{W}(ax)}}{x^6} \, dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x))/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{cW(ax)}}{x^6} \, dx = \int \frac{\sqrt{c \operatorname{W}(ax)}}{x^6} \, dx$$

input
```
integrate((c*lambert_w(a*x))^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x))/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cW(ax)}}{x^6} \, dx = \int \frac{\sqrt{c \operatorname{LambertW}(a\,x)}}{x^6} \, dx$$

input
```
int((c*LambertW(a*x))^(1/2)/x^6,x)
```

output
```
int((c*LambertW(a*x))^(1/2)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{cW(ax)}}{x^6}\,dx$$
$$= \frac{\sqrt{c}\left(-2\sqrt{lambert\_w(ax)} + \left(\int \frac{\sqrt{lambert\_w(ax)}}{e^{lambert\_w(ax)}lambert\_w(ax)^2x^5 + e^{lambert\_w(ax)}lambert\_w(ax)x^5}dx\right)a\,x^5\right)}{10x^5}$$

input
```
int((c*Lambert_W(a*x))^(1/2)/x^6,x)
```

output
```
(sqrt(c)*( - 2*sqrt(lambert_w(a*x)) + int(sqrt(lambert_w(a*x))/(e**lambert
_w(a*x)*lambert_w(a*x)**2*x**5 + e**lambert_w(a*x)*lambert_w(a*x)*x**5),x)
*a*x**5))/(10*x**5)
```

3.70 $\int \frac{x^4}{\sqrt{cW(ax)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 139

$$\int \frac{x^4}{\sqrt{cW(ax)}}\,dx=\frac{21\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{cW(ax)}}{\sqrt{c}}\right)}{20000a^5\sqrt{c}}-\frac{21c^4x^5}{10000(cW(ax))^{9/2}}+\frac{7c^3x^5}{1000(cW(ax))^{7/2}}-\frac{7c^2x^5}{500(cW(ax))^{5/2}}+\frac{cx^5}{50(cW(ax))^{3/2}}+\frac{x^5}{5\sqrt{cW(ax)}}$$

output
```
21/100000*5^(1/2)*Pi^(1/2)*erfi(5^(1/2)*(c*LambertW(a*x))^(1/2)/c^(1/2))/a
^5/c^(1/2)-21/10000*c^4*x^5/(c*LambertW(a*x))^(9/2)+7/1000*c^3*x^5/(c*Lamb
ertW(a*x))^(7/2)-7/500*c^2*x^5/(c*LambertW(a*x))^(5/2)+1/50*c*x^5/(c*Lambe
rtW(a*x))^(3/2)+1/5*x^5/(c*LambertW(a*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{\sqrt{cW(ax)}}\,dx$$

$$=\frac{-210a^5x^5+700a^5x^5W(ax)-1400a^5x^5W(ax)^2+2000a^5x^5W(ax)^3+20000a^5x^5W(ax)^4+21\sqrt{5\pi}\operatorname{erfi}\left(\right.}{100000a^5W(ax)^4\sqrt{cW(ax)}}$$

input
```
Integrate[x^4/Sqrt[c*ProductLog[a*x]],x]
```

output
```
(-210*a^5*x^5 + 700*a^5*x^5*ProductLog[a*x] - 1400*a^5*x^5*ProductLog[a*x]
^2 + 2000*a^5*x^5*ProductLog[a*x]^3 + 20000*a^5*x^5*ProductLog[a*x]^4 + 21
*Sqrt[5*Pi]*Erfi[Sqrt[5]*Sqrt[ProductLog[a*x]]]*ProductLog[a*x]^(9/2))/(10
0000*a^5*ProductLog[a*x]^4*Sqrt[c*ProductLog[a*x]])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {7172, 7205, 7205, 7205, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{cW(ax)}}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{10}\int \frac{x^4}{\sqrt{cW(ax)}(W(ax)+1)}dx+\frac{x^5}{5\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{10}\left(\frac{cx^5}{5(cW(ax))^{3/2}}-\frac{7}{10}c\int \frac{x^4}{(cW(ax))^{3/2}(W(ax)+1)}dx\right)+\frac{x^5}{5\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{10}\left(\frac{cx^5}{5(cW(ax))^{3/2}}-\frac{7}{10}c\left(\frac{cx^5}{5(cW(ax))^{5/2}}-\frac{1}{2}c\int \frac{x^4}{(cW(ax))^{5/2}(W(ax)+1)}dx\right)\right)+\frac{x^5}{5\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{10}\left(\frac{cx^5}{5(cW(ax))^{3/2}}-\frac{7}{10}c\left(\frac{cx^5}{5(cW(ax))^{5/2}}-\frac{1}{2}c\left(\frac{cx^5}{5(cW(ax))^{7/2}}-\frac{3}{10}c\int\frac{x^4}{(cW(ax))^{7/2}(W(ax)+1)}dx\right)\right)\right)+\frac{x^5}{5\sqrt{cW(ax)}}$$

$\Big\downarrow$ 7205

$$\frac{1}{10}\left(\frac{cx^5}{5(cW(ax))^{3/2}}-\frac{7}{10}c\left(\frac{cx^5}{5(cW(ax))^{5/2}}-\frac{1}{2}c\left(\frac{cx^5}{5(cW(ax))^{7/2}}-\frac{3}{10}c\left(\frac{cx^5}{5(cW(ax))^{9/2}}-\frac{1}{10}c\int\frac{x^4}{(cW(ax))^{9/2}(W\right.\right.\right.\right.$$
$$\frac{x^5}{5\sqrt{cW(ax)}}$$

$\Big\downarrow$ 7204

$$\frac{1}{10}\left(\frac{cx^5}{5(cW(ax))^{3/2}}-\frac{7}{10}c\left(\frac{cx^5}{5(cW(ax))^{5/2}}-\frac{1}{2}c\left(\frac{cx^5}{5(cW(ax))^{7/2}}-\frac{3}{10}c\left(\frac{cx^5}{5(cW(ax))^{9/2}}-\frac{\sqrt{\frac{\pi}{5}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{cW(ax)}}{\sqrt{c}}\right)}{10a^5c^{7/2}}\right.\right.\right.\right.$$
$$\frac{x^5}{5\sqrt{cW(ax)}}$$

input
```
Int[x^4/Sqrt[c*ProductLog[a*x]],x]
```

output
```
x^5/(5*Sqrt[c*ProductLog[a*x]]) + ((c*x^5)/(5*(c*ProductLog[a*x])^(3/2)) -
 (7*c*((c*x^5)/(5*(c*ProductLog[a*x])^(5/2)) - (c*((c*x^5)/(5*(c*ProductLo
g[a*x])^(7/2)) - (3*c*(-1/10*(Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[c*ProductLog[a
*x]])/Sqrt[c]])/(a^5*c^(7/2)) + (c*x^5)/(5*(c*ProductLog[a*x])^(9/2))))/10
))/2))/10)/10
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.05

method	result
default	$\frac{\frac{c(c\,\mathrm{LambertW}(xa))^{\frac{9}{2}}\mathrm{e}^{5\,\mathrm{LambertW}(xa)}}{5}+\frac{c\left(\frac{c(c\,\mathrm{LambertW}(xa))^{\frac{7}{2}}\mathrm{e}^{5\,\mathrm{LambertW}(xa)}}{10}-\frac{7c\left(\frac{c(c\,\mathrm{LambertW}(xa))^{\frac{5}{2}}\mathrm{e}^{5\,\mathrm{LambertW}(xa)}}{10}-\frac{c\left(\frac{c(c\,\mathrm{Lamb}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}}{a^5c^6}$

input
```
int(x^4/(c*LambertW(x*a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/a^5/c^6*(1/10*c*(c*LambertW(x*a))^(9/2)*exp(5*LambertW(x*a))+1/10*c*(1/1
0*c*(c*LambertW(x*a))^(7/2)*exp(5*LambertW(x*a))-7/10*c*(1/10*c*(c*Lambert
W(x*a))^(5/2)*exp(5*LambertW(x*a))-1/2*c*(1/10*c*(c*LambertW(x*a))^(3/2)*e
xp(5*LambertW(x*a))-3/10*c*(1/10*c*(c*LambertW(x*a))^(1/2)*exp(5*LambertW(
x*a))-1/20*c*Pi^(1/2)/(-5/c)^(1/2)*erf((-5/c)^(1/2)*(c*LambertW(x*a))^(1/2
)))))))
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{cW(ax)}}\,dx = \int \frac{x^4}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(x^4/(c*lambert_w(a*x))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))*x^4/(c*lambert_w(a*x)), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{cW(ax)}}\,dx = \int \frac{x^4}{\sqrt{cW\left(ax\right)}}\,dx$$

input
```
integrate(x**4/(c*LambertW(a*x))**(1/2),x)
```

output
```
Integral(x**4/sqrt(c*LambertW(a*x)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{cW(ax)}}\,dx = \int \frac{x^4}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(x^4/(c*lambert_w(a*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(c*lambert_w(a*x)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{cW(ax)}}\,dx = \int \frac{x^4}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(x^4/(c*lambert_w(a*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(c*lambert_w(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{cW(ax)}}\,dx = \int \frac{x^4}{\sqrt{c\,\mathrm{LambertW}\,(a\,x)}}\,dx$$

input
```
int(x^4/(c*LambertW(a*x))^(1/2),x)
```

output
```
int(x^4/(c*LambertW(a*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{cW(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(ax)}\,x^4}{lambert\_w(ax)}dx\right)}{c}$$

input
```
int(x^4/(c*Lambert_W(a*x))^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(lambert_w(a*x))*x**4)/lambert_w(a*x),x))/c
```

3.71 $\int \frac{x^3}{\sqrt{cW(ax)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 111

$$\int \frac{x^3}{\sqrt{cW(ax)}}\,dx = -\frac{15\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{cW(ax)}}{\sqrt{c}}\right)}{8192a^4\sqrt{c}} + \frac{15c^3x^4}{2048(cW(ax))^{7/2}} - \frac{5c^2x^4}{256(cW(ax))^{5/2}} + \frac{cx^4}{32(cW(ax))^{3/2}} + \frac{x^4}{4\sqrt{cW(ax)}}$$

output

```
-15/8192*Pi^(1/2)*erfi(2*(c*LambertW(a*x))^(1/2)/c^(1/2))/a^4/c^(1/2)+15/2
048*c^3*x^4/(c*LambertW(a*x))^(7/2)-5/256*c^2*x^4/(c*LambertW(a*x))^(5/2)+
1/32*c*x^4/(c*LambertW(a*x))^(3/2)+1/4*x^4/(c*LambertW(a*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{\sqrt{cW(ax)}}\,dx = \frac{60a^4x^4 - 160a^4x^4W(ax) + 256a^4x^4W(ax)^2 + 2048a^4x^4W(ax)^3 - 15\sqrt{\pi}\,\text{erfi}\left(2\sqrt{W(ax)}\right)W(ax)^{7/2}}{8192a^4W(ax)^3\sqrt{cW(ax)}}$$

input
```
Integrate[x^3/Sqrt[c*ProductLog[a*x]],x]
```

output
```
(60*a^4*x^4 - 160*a^4*x^4*ProductLog[a*x] + 256*a^4*x^4*ProductLog[a*x]^2
+ 2048*a^4*x^4*ProductLog[a*x]^3 - 15*Sqrt[Pi]*Erfi[2*Sqrt[ProductLog[a*x]
]]*ProductLog[a*x]^(7/2))/(8192*a^4*ProductLog[a*x]^3*Sqrt[c*ProductLog[a*
x]])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {7172, 7205, 7205, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{cW(ax)}}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{8}\int \frac{x^3}{\sqrt{cW(ax)}(W(ax)+1)}dx+\frac{x^4}{4\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{8}\left(\frac{cx^4}{4(cW(ax))^{3/2}}-\frac{5}{8}c\int \frac{x^3}{(cW(ax))^{3/2}(W(ax)+1)}dx\right)+\frac{x^4}{4\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{8}\left(\frac{cx^4}{4(cW(ax))^{3/2}}-\frac{5}{8}c\left(\frac{cx^4}{4(cW(ax))^{5/2}}-\frac{3}{8}c\int \frac{x^3}{(cW(ax))^{5/2}(W(ax)+1)}dx\right)\right)+\frac{x^4}{4\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{8}\left(\frac{cx^4}{4(cW(ax))^{3/2}}-\frac{5}{8}c\left(\frac{cx^4}{4(cW(ax))^{5/2}}-\frac{3}{8}c\left(\frac{cx^4}{4(cW(ax))^{7/2}}-\frac{1}{8}c\int \frac{x^3}{(cW(ax))^{7/2}(W(ax)+1)}dx\right)\right)\right)+\frac{x^4}{4\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7204}$$

$$\frac{1}{8}\left(\frac{cx^4}{4(cW(ax))^{3/2}}-\frac{5}{8}c\left(\frac{cx^4}{4(cW(ax))^{5/2}}-\frac{3}{8}c\left(\frac{cx^4}{4(cW(ax))^{7/2}}-\frac{\sqrt{\pi}\text{erfi}\left(\frac{2\sqrt{cW(ax)}}{\sqrt{c}}\right)}{16a^4c^{5/2}}\right)\right)\right)+\frac{x^4}{4\sqrt{cW(ax)}}$$

input
```
Int[x^3/Sqrt[c*ProductLog[a*x]],x]
```

output
```
x^4/(4*Sqrt[c*ProductLog[a*x]]) + ((c*x^4)/(4*(c*ProductLog[a*x])^(3/2)) -
 (5*c*((c*x^4)/(4*(c*ProductLog[a*x])^(5/2)) - (3*c*(-1/16*(Sqrt[Pi]*Erfi[
(2*Sqrt[c*ProductLog[a*x]])/Sqrt[c]])/(a^4*c^(5/2)) + (c*x^4)/(4*(c*Produc
tLog[a*x])^(7/2))))/8))/8)/8
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.13

method	result
default	$\frac{\frac{c(c\,\mathrm{LambertW}(xa))^{\frac{7}{2}}\,\mathrm{e}^{4\,\mathrm{LambertW}(xa)}}{4}+\frac{c\left(\frac{c(c\,\mathrm{LambertW}(xa))^{\frac{5}{2}}\,\mathrm{e}^{4\,\mathrm{LambertW}(xa)}}{8}-\frac{5c\left(\frac{c(c\,\mathrm{LambertW}(xa))^{\frac{3}{2}}\,\mathrm{e}^{4\,\mathrm{LambertW}(xa)}}{8}-\frac{3c\left(\frac{c\sqrt{c\,\mathrm{Lar}}}{}\right.}{}\right.}{}\right.}{4}}{a^{4}c^{5}}$

input
```
int(x^3/(c*LambertW(x*a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/a^4/c^5*(1/8*c*(c*LambertW(x*a))^(7/2)*exp(4*LambertW(x*a))+1/8*c*(1/8*c
*(c*LambertW(x*a))^(5/2)*exp(4*LambertW(x*a))-5/8*c*(1/8*c*(c*LambertW(x*a
))^(3/2)*exp(4*LambertW(x*a))-3/8*c*(1/8*c*(c*LambertW(x*a))^(1/2)*exp(4*L
ambertW(x*a))-1/32*c*Pi^(1/2)/(-1/c)^(1/2)*erf(2*(-1/c)^(1/2)*(c*LambertW(
x*a))^(1/2))))))
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{cW(ax)}}\,dx = \int \frac{x^3}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(x^3/(c*lambert_w(a*x))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))*x^3/(c*lambert_w(a*x)), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{cW(ax)}}\,dx = \int \frac{x^3}{\sqrt{cW\left(ax\right)}}\,dx$$

input
```
integrate(x**3/(c*LambertW(a*x))**(1/2),x)
```

output
```
Integral(x**3/sqrt(c*LambertW(a*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{cW(ax)}}\,dx = \int \frac{x^3}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(x^3/(c*lambert_w(a*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(c*lambert_w(a*x)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{cW(ax)}}\,dx = \int \frac{x^3}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(x^3/(c*lambert_w(a*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(c*lambert_w(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{cW(ax)}}\,dx = \int \frac{x^3}{\sqrt{c\,\mathrm{LambertW}\,(a\,x)}}\,dx$$

input
```
int(x^3/(c*LambertW(a*x))^(1/2),x)
```

output
```
int(x^3/(c*LambertW(a*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{cW(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(ax)}\,x^3}{lambert\_w(ax)}dx\right)}{c}$$

input
```
int(x^3/(c*Lambert_W(a*x))^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(lambert_w(a*x))*x**3)/lambert_w(a*x),x))/c
```

3.72 $\int \frac{x^2}{\sqrt{cW(ax)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 99

$$\int \frac{x^2}{\sqrt{cW(ax)}}\,dx=\frac{\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{cW(ax)}}{\sqrt{c}}\right)}{72a^3\sqrt{c}}-\frac{c^2x^3}{36(cW(ax))^{5/2}}+\frac{cx^3}{18(cW(ax))^{3/2}}+\frac{x^3}{3\sqrt{cW(ax)}}$$

output
```
1/216*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(c*LambertW(a*x))^(1/2)/c^(1/2))/a^3/c
^(1/2)-1/36*c^2*x^3/(c*LambertW(a*x))^(5/2)+1/18*c*x^3/(c*LambertW(a*x))^(
3/2)+1/3*x^3/(c*LambertW(a*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{\sqrt{cW(ax)}}\,dx =\frac{-6a^3x^3+12a^3x^3W(ax)+72a^3x^3W(ax)^2+\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{W(ax)}\right)W(ax)^{5/2}}{216a^3W(ax)^2\sqrt{cW(ax)}}$$

input
```
Integrate[x^2/Sqrt[c*ProductLog[a*x]],x]
```

output
```
(-6*a^3*x^3 + 12*a^3*x^3*ProductLog[a*x] + 72*a^3*x^3*ProductLog[a*x]^2 +
Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ProductLog[a*x]]]*ProductLog[a*x]^(5/2))/(216
*a^3*ProductLog[a*x]^2*Sqrt[c*ProductLog[a*x]])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {7172, 7205, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{cW(ax)}}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{6}\int \frac{x^2}{\sqrt{cW(ax)}(W(ax)+1)}dx+\frac{x^3}{3\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{6}\left(\frac{cx^3}{3(cW(ax))^{3/2}}-\frac{1}{2}c\int \frac{x^2}{(cW(ax))^{3/2}(W(ax)+1)}dx\right)+\frac{x^3}{3\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{6}\left(\frac{cx^3}{3(cW(ax))^{3/2}}-\frac{1}{2}c\left(\frac{cx^3}{3(cW(ax))^{5/2}}-\frac{1}{6}c\int \frac{x^2}{(cW(ax))^{5/2}(W(ax)+1)}dx\right)\right)+\frac{x^3}{3\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7204}$$

$$\frac{1}{6}\left(\frac{cx^3}{3(cW(ax))^{3/2}}-\frac{1}{2}c\left(\frac{cx^3}{3(cW(ax))^{5/2}}-\frac{\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{cW(ax)}}{\sqrt{c}}\right)}{6a^3c^{3/2}}\right)\right)+\frac{x^3}{3\sqrt{cW(ax)}}$$

input
```
Int[x^2/Sqrt[c*ProductLog[a*x]],x]
```

output
```
x^3/(3*Sqrt[c*ProductLog[a*x]]) + ((c*x^3)/(3*(c*ProductLog[a*x])^(3/2)) -
 (c*(-1/6*(Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[c*ProductLog[a*x]])/Sqrt[c]])/(a^
3*c^(3/2)) + (c*x^3)/(3*(c*ProductLog[a*x])^(5/2))))/2)/6
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

method	result
default	$\frac{\frac{c(c\,\text{LambertW}(xa))^{\frac{5}{2}}\,\mathrm{e}^{3\,\text{LambertW}(xa)}}{3}+\frac{c\left(\frac{c(c\,\text{LambertW}(xa))^{\frac{3}{2}}\,\mathrm{e}^{3\,\text{LambertW}(xa)}}{6}-\frac{c\left(\frac{c\sqrt{c\,\text{LambertW}(xa)}\,\mathrm{e}^{3\,\text{LambertW}(xa)}}{6}-\frac{c\sqrt{\pi}\,\text{erf}\left(\sqrt{-\frac{3}{c}\cdots}\right)}{1\cdots}\right.}{2}\right.}{3}}{a^3c^4}$

input
```
int(x^2/(c*LambertW(x*a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/a^3/c^4*(1/6*c*(c*LambertW(x*a))^(5/2)*exp(3*LambertW(x*a))+1/6*c*(1/6*c
*(c*LambertW(x*a))^(3/2)*exp(3*LambertW(x*a))-1/2*c*(1/6*c*(c*LambertW(x*a
))^(1/2)*exp(3*LambertW(x*a))-1/12*c*Pi^(1/2)/(-3/c)^(1/2)*erf((-3/c)^(1/2
)*(c*LambertW(x*a))^(1/2)))))
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{cW(ax)}}\,dx = \int \frac{x^2}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(x^2/(c*lambert_w(a*x))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))*x^2/(c*lambert_w(a*x)), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{cW(ax)}}\,dx = \int \frac{x^2}{\sqrt{cW\left(ax\right)}}\,dx$$

input
```
integrate(x**2/(c*LambertW(a*x))**(1/2),x)
```

output
```
Integral(x**2/sqrt(c*LambertW(a*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{cW(ax)}}\,dx = \int \frac{x^2}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(x^2/(c*lambert_w(a*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(c*lambert_w(a*x)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{cW(ax)}}\,dx = \int \frac{x^2}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(x^2/(c*lambert_w(a*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(c*lambert_w(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{cW(ax)}}\,dx = \int \frac{x^2}{\sqrt{c\,\mathrm{LambertW}\left(a\,x\right)}}\,dx$$

input
```
int(x^2/(c*LambertW(a*x))^(1/2),x)
```

output
```
int(x^2/(c*LambertW(a*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{cW(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(ax)}\,x^2}{lambert\_w(ax)}dx\right)}{c}$$

input
```
int(x^2/(c*Lambert_W(a*x))^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(lambert_w(a*x))*x**2)/lambert_w(a*x),x))/c
```

3.73 $\int \frac{x}{\sqrt{cW(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int \frac{x}{\sqrt{cW(ax)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax)}}{\sqrt{c}}\right)}{16a^2\sqrt{c}}+\frac{cx^2}{8(cW(ax))^{3/2}}+\frac{x^2}{2\sqrt{cW(ax)}}$$

output
```
-1/32*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(a*x))^(1/2)/c^(1/2))/a^2/c
^(1/2)+1/8*c*x^2/(c*LambertW(a*x))^(3/2)+1/2*x^2/(c*LambertW(a*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{x}{\sqrt{cW(ax)}}\,dx = \frac{c\left(4a^2x^2+16a^2x^2W(ax)-\sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{W(ax)}\right)W(ax)^{3/2}\right)}{32a^2(cW(ax))^{3/2}}$$

input
```
Integrate[x/Sqrt[c*ProductLog[a*x]],x]
```

output
```
(c*(4*a^2*x^2 + 16*a^2*x^2*ProductLog[a*x] - Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[
ProductLog[a*x]]]*ProductLog[a*x]^(3/2)))/(32*a^2*(c*ProductLog[a*x])^(3/2
))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7172, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{cW(ax)}}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{4}\int \frac{x}{\sqrt{cW(ax)}(W(ax)+1)}dx+\frac{x^2}{2\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{4}\left(\frac{cx^2}{2(cW(ax))^{3/2}}-\frac{1}{4}c\int \frac{x}{(cW(ax))^{3/2}(W(ax)+1)}dx\right)+\frac{x^2}{2\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7204}$$

$$\frac{1}{4}\left(\frac{cx^2}{2(cW(ax))^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax)}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}\right)+\frac{x^2}{2\sqrt{cW(ax)}}$$

input
```
Int[x/Sqrt[c*ProductLog[a*x]],x]
```

output
```
x^2/(2*Sqrt[c*ProductLog[a*x]]) + (-1/4*(Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[c*P
roductLog[a*x]])/Sqrt[c]])/(a^2*Sqrt[c]) + (c*x^2)/(2*(c*ProductLog[a*x])^
(3/2)))/4
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{\frac{c(c\,\mathrm{LambertW}(xa))^{\frac{3}{2}}\,\mathrm{e}^{2\,\mathrm{LambertW}(xa)}}{2}+\frac{c\left(\frac{c\sqrt{c\,\mathrm{LambertW}(xa)}\,\mathrm{e}^{2\,\mathrm{LambertW}(xa)}}{4}-\frac{c\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{2}{c}}\,\sqrt{c\,\mathrm{LambertW}(xa)}\right)}{8\sqrt{-\frac{2}{c}}}\right)}{2}}{a^{2}c^{3}}$	80

input
```
int(x/(c*LambertW(x*a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/a^2/c^3*(1/4*c*(c*LambertW(x*a))^(3/2)*exp(2*LambertW(x*a))+1/4*c*(1/4*c
*(c*LambertW(x*a))^(1/2)*exp(2*LambertW(x*a))-1/8*c*Pi^(1/2)/(-2/c)^(1/2)*
erf((-2/c)^(1/2)*(c*LambertW(x*a))^(1/2))))
```

Fricas [F]

$$\int \frac{x}{\sqrt{cW(ax)}}\,dx = \int \frac{x}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(x/(c*lambert_w(a*x))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))*x/(c*lambert_w(a*x)), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{cW(ax)}}\,dx = \int \frac{x}{\sqrt{cW\left(ax\right)}}\,dx$$

input
```
integrate(x/(c*LambertW(a*x))**(1/2),x)
```

output
```
Integral(x/sqrt(c*LambertW(a*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{cW(ax)}}\,dx = \int \frac{x}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(x/(c*lambert_w(a*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(c*lambert_w(a*x)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{cW(ax)}}\,dx = \int \frac{x}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(x/(c*lambert_w(a*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(c*lambert_w(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{cW(ax)}}\,dx = \int \frac{x}{\sqrt{c\,\mathrm{LambertW}\,(a\,x)}}\,dx$$

input
```
int(x/(c*LambertW(a*x))^(1/2),x)
```

output
```
int(x/(c*LambertW(a*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{cW(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(ax)}\,x}{lambert\_w(ax)}dx\right)}{c}$$

input
```
int(x/(c*Lambert_W(a*x))^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(lambert_w(a*x))*x)/lambert_w(a*x),x))/c
```

3.74 $\int \frac{1}{\sqrt{cW(ax)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int \frac{1}{\sqrt{cW(ax)}}\,dx=\frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{cW(ax)}}{\sqrt{c}}\right)}{2a\sqrt{c}}+\frac{x}{\sqrt{cW(ax)}}$$

output
```
1/2*Pi^(1/2)*erfi((c*LambertW(a*x))^(1/2)/c^(1/2))/a/c^(1/2)+x/(c*LambertW
(a*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{cW(ax)}}\,dx=\frac{2ax+\sqrt{\pi}\operatorname{erfi}\left(\sqrt{W(ax)}\right)\sqrt{W(ax)}}{2a\sqrt{cW(ax)}}$$

input
```
Integrate[1/Sqrt[c*ProductLog[a*x]],x]
```

output
```
(2*a*x + Sqrt[Pi]*Erfi[Sqrt[ProductLog[a*x]]]*Sqrt[ProductLog[a*x]])/(2*a*
Sqrt[c*ProductLog[a*x]])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7167, 7180}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cW(ax)}}dx$$

$$\Big\downarrow \text{ 7167}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{cW(ax)}(W(ax)+1)}dx+\frac{x}{\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{ 7180}$$

$$\frac{\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{cW(ax)}}{\sqrt{c}}\right)}{2a\sqrt{c}}+\frac{x}{\sqrt{cW(ax)}}$$

input
```
Int[1/Sqrt[c*ProductLog[a*x]],x]
```

output
```
(Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a*x]]/Sqrt[c]])/(2*a*Sqrt[c]) + x/Sqrt[c*
ProductLog[a*x]]
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7180
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[Pi*c, 2]*(Erfi[Sqrt[c*ProductLo
g[a + b*x]]/Rt[c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && PosQ[c]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\frac{c\sqrt{c\,\text{LambertW}(xa)}\,xa}{\text{LambertW}(xa)}+\frac{c\sqrt{\pi}\,\text{erf}\left(\sqrt{-\frac{1}{c}}\,\sqrt{c\,\text{LambertW}(xa)}\right)}{2\sqrt{-\frac{1}{c}}}}{a\,c^{2}}$	59

input
```
int(1/(c*LambertW(x*a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/a/c^2*(1/2*c*(c*LambertW(x*a))^(1/2)*x*a/LambertW(x*a)+1/4*c*Pi^(1/2)/(-
1/c)^(1/2)*erf((-1/c)^(1/2)*(c*LambertW(x*a))^(1/2)))
```

Fricas [F]

$$\int \frac{1}{\sqrt{cW(ax)}}\,dx=\int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(1/(c*lambert_w(a*x))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))/(c*lambert_w(a*x)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{cW(ax)}}\,dx=\int \frac{1}{\sqrt{cW\left(ax\right)}}\,dx$$

input
```
integrate(1/(c*LambertW(a*x))**(1/2),x)
```

output
```
Integral(1/sqrt(c*LambertW(a*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(1/(c*lambert_w(a*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*lambert_w(a*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(1/(c*lambert_w(a*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*lambert_w(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{LambertW}\left(a\,x\right)}}\,dx$$

input
```
int(1/(c*LambertW(a*x))^(1/2),x)
```

output
```
int(1/(c*LambertW(a*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{cW(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(ax)}}{lambert\_w(ax)}dx\right)}{c}$$

input
```
int(1/(c*Lambert_W(a*x))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a*x))/lambert_w(a*x),x))/c
```

3.75 $\int \frac{1}{x\sqrt{cW(ax)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 28

$$\int \frac{1}{x\sqrt{cW(ax)}}\,dx=-\frac{2}{\sqrt{cW(ax)}}+\frac{2\sqrt{cW(ax)}}{c}$$

output
```
-2/(c*LambertW(a*x))^(1/2)+2*(c*LambertW(a*x))^(1/2)/c
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.64

$$\int \frac{1}{x\sqrt{cW(ax)}}\,dx=\frac{2(-1+W(ax))}{\sqrt{cW(ax)}}$$

input
```
Integrate[1/(x*Sqrt[c*ProductLog[a*x]]),x]
```

output
```
(2*(-1 + ProductLog[a*x]))/Sqrt[c*ProductLog[a*x]]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{cW(ax)}}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{\int \frac{\sqrt{cW(ax)}}{x(W(ax)+1)}dx}{c} - \frac{2}{\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7200}$$

$$\frac{2\sqrt{cW(ax)}}{c} - \frac{2}{\sqrt{cW(ax)}}$$

input
```
Int[1/(x*Sqrt[c*ProductLog[a*x]]),x]
```

output
```
-2/Sqrt[c*ProductLog[a*x]] + (2*Sqrt[c*ProductLog[a*x]])/c
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{2\sqrt{c\,\mathrm{LambertW}(xa)}-\frac{2c}{\sqrt{c\,\mathrm{LambertW}(xa)}}}{c}$	26
default	$\frac{2\sqrt{c\,\mathrm{LambertW}(xa)}-\frac{2c}{\sqrt{c\,\mathrm{LambertW}(xa)}}}{c}$	26

input
```
int(1/x/(c*LambertW(x*a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/c*((c*LambertW(x*a))^(1/2)-c/(c*LambertW(x*a))^(1/2))
```

Fricas [F]

$$\int \frac{1}{x\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}x}\,dx$$

input
```
integrate(1/x/(c*lambert_w(a*x))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))/(c*x*lambert_w(a*x)), x)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\sqrt{cW(ax)}}\,dx = \frac{2W(ax)}{\sqrt{cW(ax)}} - \frac{2}{\sqrt{cW(ax)}}$$

input
```
integrate(1/x/(c*LambertW(a*x))**(1/2),x)
```

output
```
2*LambertW(a*x)/sqrt(c*LambertW(a*x)) - 2/sqrt(c*LambertW(a*x))
```

Maxima [F]

$$\int \frac{1}{x\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}x}\,dx$$

input
```
integrate(1/x/(c*lambert_w(a*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x))*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}x}\,dx$$

input
```
integrate(1/x/(c*lambert_w(a*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x))*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{cW(ax)}}\,dx = \int \frac{1}{x\,\sqrt{c\,\mathrm{LambertW}\,(a\,x)}}\,dx$$

input
```
int(1/(x*(c*LambertW(a*x))^(1/2)),x)
```

output
```
int(1/(x*(c*LambertW(a*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{cW(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(ax)}}{lambert\_w(ax)x}dx\right)}{c}$$

input
```
int(1/x/(c*Lambert_W(a*x))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a*x))/(lambert_w(a*x)*x),x))/c
```

3.76 $\int \frac{1}{x^2\sqrt{cW(ax)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 70

$$\int \frac{1}{x^2\sqrt{cW(ax)}}\,dx=-\frac{2a\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{cW(ax)}}{\sqrt{c}}\right)}{3\sqrt{c}}-\frac{2}{3x\sqrt{cW(ax)}}-\frac{2\sqrt{cW(ax)}}{3cx}$$

output
```
-2/3*a*Pi^(1/2)*erf((c*LambertW(a*x))^(1/2)/c^(1/2))/c^(1/2)-2/3/x/(c*Lamb
ertW(a*x))^(1/2)-2/3*(c*LambertW(a*x))^(1/2)/c/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^2\sqrt{cW(ax)}}\,dx=-\frac{2\left(1+a\sqrt{\pi}x\operatorname{erf}\left(\sqrt{W(ax)}\right)\sqrt{W(ax)}+W(ax)\right)}{3x\sqrt{cW(ax)}}$$

input
```
Integrate[1/(x^2*Sqrt[c*ProductLog[a*x]]),x]
```

output
```
(-2*(1 + a*Sqrt[Pi]*x*Erf[Sqrt[ProductLog[a*x]]]*Sqrt[ProductLog[a*x]] + P
roductLog[a*x]))/(3*x*Sqrt[c*ProductLog[a*x]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {7173, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{cW(ax)}}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{\int \frac{\sqrt{cW(ax)}}{x^2(W(ax)+1)}dx}{3c}-\frac{2}{3x\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{-\frac{2\int \frac{(cW(ax))^{3/2}}{x^2(W(ax)+1)}dx}{c}-\frac{2\sqrt{cW(ax)}}{x}}{3c}-\frac{2}{3x\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7203}$$

$$\frac{-2\sqrt{\pi}a\sqrt{c}\text{erf}\left(\frac{\sqrt{cW(ax)}}{\sqrt{c}}\right)-\frac{2\sqrt{cW(ax)}}{x}}{3c}-\frac{2}{3x\sqrt{cW(ax)}}$$

input
```
Int[1/(x^2*Sqrt[c*ProductLog[a*x]]),x]
```

output
```
-2/(3*x*Sqrt[c*ProductLog[a*x]]) + (-2*a*Sqrt[c]*Sqrt[Pi]*Erf[Sqrt[c*Produ
ctLog[a*x]]/Sqrt[c]] - (2*Sqrt[c*ProductLog[a*x]])/x)/(3*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(52) = 104$.

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.53

method	result
default	$a\left(-\frac{2\,\mathrm{e}^{-\mathrm{LambertW}(xa)}}{\sqrt{c\,\mathrm{LambertW}(xa)}}-\frac{2\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{c\,\mathrm{LambertW}(xa)}}{\sqrt{c}}\right)}{\sqrt{c}}+2c\left(-\frac{\mathrm{e}^{-\mathrm{LambertW}(xa)}}{3(c\,\mathrm{LambertW}(xa))^{\frac{3}{2}}}-\frac{2\left(-\frac{\mathrm{e}^{-\mathrm{LambertW}(xa)}}{\sqrt{c\,\mathrm{LambertW}(xa)}}-\frac{\sqrt{\pi}\,\mathrm{er}}{}\right.}{3c}\right.\right.$

input
```
int(1/x^2/(c*LambertW(x*a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-2/(c*LambertW(x*a))^(1/2)*exp(-LambertW(x*a))-2/c^(1/2)*Pi^(1/2)*erf((
c*LambertW(x*a))^(1/2)/c^(1/2))+2*c*(-1/3/(c*LambertW(x*a))^(3/2)*exp(-Lam
bertW(x*a))-2/3/c*(-1/(c*LambertW(x*a))^(1/2)*exp(-LambertW(x*a))-1/c^(1/2
)*Pi^(1/2)*erf((c*LambertW(x*a))^(1/2)/c^(1/2)))))
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}x^2}\,dx$$

input
```
integrate(1/x^2/(c*lambert_w(a*x))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))/(c*x^2*lambert_w(a*x)), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{cW(ax)}}\,dx = \int \frac{1}{x^2\sqrt{cW\left(ax\right)}}\,dx$$

input
```
integrate(1/x**2/(c*LambertW(a*x))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(c*LambertW(a*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}x^2}\,dx$$

input
```
integrate(1/x^2/(c*lambert_w(a*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x))*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}x^2}\,dx$$

input
```
integrate(1/x^2/(c*lambert_w(a*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x))*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{cW(ax)}}\,dx = \int \frac{1}{x^2\,\sqrt{c\,\mathrm{LambertW}\,(a\,x)}}\,dx$$

input
```
int(1/(x^2*(c*LambertW(a*x))^(1/2)),x)
```

output
```
int(1/(x^2*(c*LambertW(a*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{cW(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(ax)}}{lambert\_w(ax)x^2}dx\right)}{c}$$

input
```
int(1/x^2/(c*Lambert_W(a*x))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a*x))/(lambert_w(a*x)*x**2),x))/c
```

3.77 $\int \frac{1}{x^3\sqrt{cW(ax)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 99

$$\int \frac{1}{x^3\sqrt{cW(ax)}}\,dx = \frac{8a^2\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax)}}{\sqrt{c}}\right)}{15\sqrt{c}} - \frac{2}{5x^2\sqrt{cW(ax)}} - \frac{2\sqrt{cW(ax)}}{15cx^2} + \frac{8(cW(ax))^{3/2}}{15c^2x^2}$$

output
```
8/15*a^2*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(c*LambertW(a*x))^(1/2)/c^(1/2))/c^(
1/2)-2/5/x^2/(c*LambertW(a*x))^(1/2)-2/15*(c*LambertW(a*x))^(1/2)/c/x^2+8/
15*(c*LambertW(a*x))^(3/2)/c^2/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^3\sqrt{cW(ax)}}\,dx = \frac{2\left(-3+4a^2\sqrt{2\pi}x^2\,\mathrm{erf}\left(\sqrt{2}\sqrt{W(ax)}\right)\sqrt{W(ax)} - W(ax) + 4W(ax)^2\right)}{15x^2\sqrt{cW(ax)}}$$

input
```
Integrate[1/(x^3*Sqrt[c*ProductLog[a*x]]),x]
```

output
```
(2*(-3 + 4*a^2*Sqrt[2*Pi]*x^2*Erf[Sqrt[2]*Sqrt[ProductLog[a*x]]]*Sqrt[Prod
uctLog[a*x]] - ProductLog[a*x] + 4*ProductLog[a*x]^2))/(15*x^2*Sqrt[c*Prod
uctLog[a*x]])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {7173, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{cW(ax)}}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{\int \frac{\sqrt{cW(ax)}}{x^3(W(ax)+1)}dx}{5c}-\frac{2}{5x^2\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{-\frac{4\int \frac{(cW(ax))^{3/2}}{x^3(W(ax)+1)}dx}{3c}-\frac{2\sqrt{cW(ax)}}{3x^2}}{5c}-\frac{2}{5x^2\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{-\frac{4\left(-\frac{4\int \frac{(cW(ax))^{5/2}}{x^3(W(ax)+1)}dx}{c}-\frac{2(cW(ax))^{3/2}}{x^2}\right)}{3c}-\frac{2\sqrt{cW(ax)}}{3x^2}}{5c}-\frac{2}{5x^2\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7203}$$

$$\frac{-\frac{4\left(-2\sqrt{2\pi}a^2c^{3/2}\text{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax)}}{\sqrt{c}}\right)-\frac{2(cW(ax))^{3/2}}{x^2}\right)}{3c}-\frac{2\sqrt{cW(ax)}}{3x^2}}{5c}-\frac{2}{5x^2\sqrt{cW(ax)}}$$

input
```
Int[1/(x^3*Sqrt[c*ProductLog[a*x]]),x]
```

output
```
-2/(5*x^2*Sqrt[c*ProductLog[a*x]]) + ((-2*Sqrt[c*ProductLog[a*x]])/(3*x^2)
 - (4*(-2*a^2*c^(3/2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[c*ProductLog[a*x]])/Sqr
t[c]] - (2*(c*ProductLog[a*x])^(3/2))/x^2))/(3*c))/(5*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(76) = 152.

Time = 0.06 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.70

method	result
default	$2a^2c\left(-\frac{e^{-2\,\mathrm{LambertW}(xa)}}{3(c\,\mathrm{LambertW}(xa))^{\frac{3}{2}}}-\frac{4\left(-\frac{e^{-2\,\mathrm{LambertW}(xa)}}{\sqrt{c\,\mathrm{LambertW}(xa)}}-\frac{\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\frac{\sqrt{2}\,\sqrt{c\,\mathrm{LambertW}(xa)}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{3c}+c\left(-\frac{e^{-2\,\mathrm{LambertW}(xa)}}{5(c\,\mathrm{LambertW}(xa))\right.\right.$

input
```
int(1/x^3/(c*LambertW(x*a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^2*c*(-1/3/(c*LambertW(x*a))^(3/2)*exp(-2*LambertW(x*a))-4/3/c*(-1/(c*L
ambertW(x*a))^(1/2)*exp(-2*LambertW(x*a))-1/c^(1/2)*Pi^(1/2)*2^(1/2)*erf(2
^(1/2)*(c*LambertW(x*a))^(1/2)/c^(1/2)))+c*(-1/5/(c*LambertW(x*a))^(5/2)*e
xp(-2*LambertW(x*a))-4/5/c*(-1/3/(c*LambertW(x*a))^(3/2)*exp(-2*LambertW(x
*a))-4/3/c*(-1/(c*LambertW(x*a))^(1/2)*exp(-2*LambertW(x*a))-1/c^(1/2)*Pi^
(1/2)*2^(1/2)*erf(2^(1/2)*(c*LambertW(x*a))^(1/2)/c^(1/2))))))
```

Fricas [F]

$$\int \frac{1}{x^3\sqrt{cW(ax)}}\,dx=\int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}x^3}\,dx$$

input
```
integrate(1/x^3/(c*lambert_w(a*x))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))/(c*x^3*lambert_w(a*x)), x)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{cW(ax)}}\,dx = \int \frac{1}{x^3\sqrt{cW\left(ax\right)}}\,dx$$

input
```
integrate(1/x**3/(c*LambertW(a*x))**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(c*LambertW(a*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}x^3}\,dx$$

input
```
integrate(1/x^3/(c*lambert_w(a*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x))*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}x^3}\,dx$$

input
```
integrate(1/x^3/(c*lambert_w(a*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x))*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{cW(ax)}}\,dx = \int \frac{1}{x^3\,\sqrt{c\,\mathrm{LambertW}\,(a\,x)}}\,dx$$

input
```
int(1/(x^3*(c*LambertW(a*x))^(1/2)),x)
```

output
```
int(1/(x^3*(c*LambertW(a*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{cW(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(ax)}}{lambert\_w(ax)x^3}dx\right)}{c}$$

input
```
int(1/x^3/(c*Lambert_W(a*x))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a*x))/(lambert_w(a*x)*x**3),x))/c
```

3.78 $\int \frac{1}{x^4\sqrt{cW(ax)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 119

$$\int \frac{1}{x^4\sqrt{cW(ax)}}\,dx = -\frac{24a^3\sqrt{3\pi}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{cW(ax)}}{\sqrt{c}}\right)}{35\sqrt{c}} - \frac{2}{7x^3\sqrt{cW(ax)}} - \frac{2\sqrt{cW(ax)}}{35cx^3} + \frac{4(cW(ax))^{3/2}}{35c^2x^3} - \frac{24(cW(ax))^{5/2}}{35c^3x^3}$$

output
```
-24/35*a^3*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(c*LambertW(a*x))^(1/2)/c^(1/2))/c
^(1/2)-2/7/x^3/(c*LambertW(a*x))^(1/2)-2/35*(c*LambertW(a*x))^(1/2)/c/x^3+
4/35*(c*LambertW(a*x))^(3/2)/c^2/x^3-24/35*(c*LambertW(a*x))^(5/2)/c^3/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^4\sqrt{cW(ax)}}\,dx = -\frac{2\left(5+12a^3\sqrt{3\pi}x^3\,\mathrm{erf}\left(\sqrt{3}\sqrt{W(ax)}\right)\sqrt{W(ax)}+W(ax)-2W(ax)^2+12W(ax)^3\right)}{35x^3\sqrt{cW(ax)}}$$

input
```
Integrate[1/(x^4*Sqrt[c*ProductLog[a*x]]),x]
```

output
```
(-2*(5 + 12*a^3*Sqrt[3*Pi]*x^3*Erf[Sqrt[3]*Sqrt[ProductLog[a*x]]]*Sqrt[Pro
ductLog[a*x]] + ProductLog[a*x] - 2*ProductLog[a*x]^2 + 12*ProductLog[a*x]
^3))/(35*x^3*Sqrt[c*ProductLog[a*x]])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {7173, 7206, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{cW(ax)}}dx$$

$$\Big\downarrow \text{ 7173}$$

$$\frac{\int \frac{\sqrt{cW(ax)}}{x^4(W(ax)+1)}dx}{7c}-\frac{2}{7x^3\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{-\frac{6\int \frac{(cW(ax))^{3/2}}{x^4(W(ax)+1)}dx}{5c}-\frac{2\sqrt{cW(ax)}}{5x^3}}{7c}-\frac{2}{7x^3\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{-\frac{6\left(-\frac{2\int \frac{(cW(ax))^{5/2}}{x^4(W(ax)+1)}dx}{c}-\frac{2(cW(ax))^{3/2}}{3x^3}\right)}{5c}-\frac{2\sqrt{cW(ax)}}{5x^3}}{7c}-\frac{2}{7x^3\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{ 7206}$$

$$-\frac{6\left(-\frac{2\left(-\frac{6\int \frac{(cW(ax))^{7/2}}{x^4(W(ax)+1)}dx}{c}-\frac{2(cW(ax))^{5/2}}{x^3}\right)}{c}-\frac{2(cW(ax))^{3/2}}{3x^3}\right)}{5c}-\frac{2\sqrt{cW(ax)}}{5x^3}}{7c}-\frac{2}{7x^3\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7203}$$

$$-\frac{6\left(-\frac{2\left(-2\sqrt{3\pi}a^3c^{5/2}\text{erf}\left(\frac{\sqrt{3}\sqrt{cW(ax)}}{\sqrt{c}}\right)-\frac{2(cW(ax))^{5/2}}{x^3}\right)}{c}-\frac{2(cW(ax))^{3/2}}{3x^3}\right)}{5c}-\frac{2\sqrt{cW(ax)}}{5x^3}}{7c}-\frac{2}{7x^3\sqrt{cW(ax)}}$$

input
```
Int[1/(x^4*Sqrt[c*ProductLog[a*x]]),x]
```

output
```
-2/(7*x^3*Sqrt[c*ProductLog[a*x]]) + ((-2*Sqrt[c*ProductLog[a*x]])/(5*x^3)
 - (6*((-2*(c*ProductLog[a*x])^(3/2))/(3*x^3) - (2*(-2*a^3*c^(5/2)*Sqrt[3*
Pi]*Erf[(Sqrt[3]*Sqrt[c*ProductLog[a*x]])/Sqrt[c]] - (2*(c*ProductLog[a*x]
)^(5/2))/x^3))/c))/(5*c))/(7*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(92) = 184$.

Time = 0.06 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.82

method	result
default	$2a^3c^2\left(-\frac{e^{-3\,\mathrm{LambertW}(xa)}}{5(c\,\mathrm{LambertW}(xa))^{\frac{5}{2}}}-\frac{6\left(-\frac{e^{-3\,\mathrm{LambertW}(xa)}}{3(c\,\mathrm{LambertW}(xa))^{\frac{3}{2}}}-\frac{2\left(-\frac{e^{-3\,\mathrm{LambertW}(xa)}}{\sqrt{c\,\mathrm{LambertW}(xa)}}-\frac{\sqrt{\pi}\,\sqrt{3}\,\mathrm{erf}\left(\frac{\sqrt{3}\,\sqrt{c\,\mathrm{LambertW}(xa)}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{c}\right)}{5c}+\right.$

input
```
int(1/x^4/(c*LambertW(x*a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^3*c^2*(-1/5/(c*LambertW(x*a))^(5/2)*exp(-3*LambertW(x*a))-6/5/c*(-1/3/
(c*LambertW(x*a))^(3/2)*exp(-3*LambertW(x*a))-2/c*(-1/(c*LambertW(x*a))^(1
/2)*exp(-3*LambertW(x*a))-1/c^(1/2)*Pi^(1/2)*3^(1/2)*erf(3^(1/2)*(c*Lamber
tW(x*a))^(1/2)/c^(1/2))))+c*(-1/7/(c*LambertW(x*a))^(7/2)*exp(-3*LambertW(
x*a))-6/7/c*(-1/5/(c*LambertW(x*a))^(5/2)*exp(-3*LambertW(x*a))-6/5/c*(-1/
3/(c*LambertW(x*a))^(3/2)*exp(-3*LambertW(x*a))-2/c*(-1/(c*LambertW(x*a))^
(1/2)*exp(-3*LambertW(x*a))-1/c^(1/2)*Pi^(1/2)*3^(1/2)*erf(3^(1/2)*(c*Lamb
ertW(x*a))^(1/2)/c^(1/2)))))))
```

Fricas [F]

$$\int \frac{1}{x^4\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}x^4}\,dx$$

input
```
integrate(1/x^4/(c*lambert_w(a*x))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))/(c*x^4*lambert_w(a*x)), x)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{cW(ax)}}\,dx = \int \frac{1}{x^4\sqrt{cW\left(ax\right)}}\,dx$$

input
```
integrate(1/x**4/(c*LambertW(a*x))**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(c*LambertW(a*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}x^4}\,dx$$

input
```
integrate(1/x^4/(c*lambert_w(a*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x))*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}x^4}\,dx$$

input
```
integrate(1/x^4/(c*lambert_w(a*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x))*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{cW(ax)}}\,dx = \int \frac{1}{x^4\,\sqrt{c\,\mathrm{LambertW}\left(a\,x\right)}}\,dx$$

input
```
int(1/(x^4*(c*LambertW(a*x))^(1/2)),x)
```

output
```
int(1/(x^4*(c*LambertW(a*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{cW(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(ax)}}{lambert\_w(ax)x^4}dx\right)}{c}$$

input
```
int(1/x^4/(c*Lambert_W(a*x))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a*x))/(lambert_w(a*x)*x**4),x))/c
```

3.79 $\int \frac{1}{x^5\sqrt{cW(ax)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 133

$$\int \frac{1}{x^5\sqrt{cW(ax)}}\,dx = \frac{2048a^4\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{cW(ax)}}{\sqrt{c}}\right)}{945\sqrt{c}} - \frac{2}{9x^4\sqrt{cW(ax)}} - \frac{2\sqrt{cW(ax)}}{63cx^4} + \frac{16(cW(ax))^{3/2}}{315c^2x^4} - \frac{128(cW(ax))^{5/2}}{945c^3x^4} + \frac{1024(cW(ax))^{7/2}}{945c^4x^4}$$

output
```
2048/945*a^4*Pi^(1/2)*erf(2*(c*LambertW(a*x))^(1/2)/c^(1/2))/c^(1/2)-2/9/x
^4/(c*LambertW(a*x))^(1/2)-2/63*(c*LambertW(a*x))^(1/2)/c/x^4+16/315*(c*La
mbertW(a*x))^(3/2)/c^2/x^4-128/945*(c*LambertW(a*x))^(5/2)/c^3/x^4+1024/94
5*(c*LambertW(a*x))^(7/2)/c^4/x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^5\sqrt{cW(ax)}}\,dx = \frac{2\left(-105+1024a^4\sqrt{\pi}x^4\operatorname{erf}\left(2\sqrt{W(ax)}\right)\sqrt{W(ax)}-15W(ax)+24W(ax)^2-64W(ax)^3+512W(ax)^4\right)}{945x^4\sqrt{cW(ax)}}$$

input
```
Integrate[1/(x^5*Sqrt[c*ProductLog[a*x]]),x]
```

output
```
(2*(-105 + 1024*a^4*Sqrt[Pi]*x^4*Erf[2*Sqrt[ProductLog[a*x]]]*Sqrt[Product
Log[a*x]] - 15*ProductLog[a*x] + 24*ProductLog[a*x]^2 - 64*ProductLog[a*x]
^3 + 512*ProductLog[a*x]^4))/(945*x^4*Sqrt[c*ProductLog[a*x]])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {7173, 7206, 7206, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{cW(ax)}}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{\int \frac{\sqrt{cW(ax)}}{x^5(W(ax)+1)}dx}{9c}-\frac{2}{9x^4\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{-\frac{8\int \frac{(cW(ax))^{3/2}}{x^5(W(ax)+1)}dx}{7c}-\frac{2\sqrt{cW(ax)}}{7x^4}}{9c}-\frac{2}{9x^4\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{-\frac{8\left(-\frac{8\int \frac{(cW(ax))^{5/2}}{x^5(W(ax)+1)}dx}{5c}-\frac{2(cW(ax))^{3/2}}{5x^4}\right)}{7c}-\frac{2\sqrt{cW(ax)}}{7x^4}}{9c}-\frac{2}{9x^4\sqrt{cW(ax)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{-\frac{8\left(-\frac{8\left(-\frac{8\int \frac{(cW(ax))^{7/2}}{x^5(W(ax)+1)}dx}{3c}-\frac{2(cW(ax))^{5/2}}{3x^4}\right)}{5c}-\frac{2(cW(ax))^{3/2}}{5x^4}\right)}{7c}-\frac{2\sqrt{cW(ax)}}{7x^4}}{9c}-\frac{2}{9x^4\sqrt{cW(ax)}}$$

$\downarrow$ 7206

$$\frac{-\frac{8\left(-\frac{8\left(-\frac{8\left(-\frac{8\int \frac{(cW(ax))^{9/2}}{x^5(W(ax)+1)}dx}{c}-\frac{2(cW(ax))^{7/2}}{x^4}\right)}{3c}-\frac{2(cW(ax))^{5/2}}{3x^4}\right)}{5c}-\frac{2(cW(ax))^{3/2}}{5x^4}\right)}{7c}-\frac{2\sqrt{cW(ax)}}{7x^4}}{9c}-\frac{2}{9x^4\sqrt{cW(ax)}}$$

$\downarrow$ 7203

$$\frac{-\frac{8\left(-\frac{8\left(-\frac{8\left(-4\sqrt{\pi}a^4c^{7/2}\text{erf}\left(\frac{2\sqrt{cW(ax)}}{\sqrt{c}}\right)-\frac{2(cW(ax))^{7/2}}{x^4}\right)}{3c}-\frac{2(cW(ax))^{5/2}}{3x^4}\right)}{5c}-\frac{2(cW(ax))^{3/2}}{5x^4}\right)}{7c}-\frac{2\sqrt{cW(ax)}}{7x^4}}{9c}-\frac{2}{9x^4\sqrt{cW(ax)}}$$

input
```
Int[1/(x^5*Sqrt[c*ProductLog[a*x]]),x]
```

output
```
-2/(9*x^4*Sqrt[c*ProductLog[a*x]]) + ((-2*Sqrt[c*ProductLog[a*x]])/(7*x^4)
 - (8*((-2*(c*ProductLog[a*x])^(3/2))/(5*x^4) - (8*((-2*(c*ProductLog[a*x]
)^(5/2))/(3*x^4) - (8*(-4*a^4*c^(7/2)*Sqrt[Pi]*Erf[(2*Sqrt[c*ProductLog[a*
x]])/Sqrt[c]] - (2*(c*ProductLog[a*x])^(7/2))/x^4))/(3*c)))/(5*c)))/(7*c))
/(9*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(103) = 206$.

Time = 0.06 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.89

method	result
default	$2a^4c^3\left(-\frac{e^{-4\,\mathrm{LambertW}(xa)}}{7(c\,\mathrm{LambertW}(xa))^{\frac{7}{2}}}-\frac{8\left(-\frac{e^{-4\,\mathrm{LambertW}(xa)}}{5(c\,\mathrm{LambertW}(xa))^{\frac{5}{2}}}-\frac{8\left(-\frac{e^{-4\,\mathrm{LambertW}(xa)}}{3(c\,\mathrm{LambertW}(xa))^{\frac{3}{2}}}-\frac{8\left(-\frac{e^{-4\,\mathrm{LambertW}(xa)}}{\sqrt{c\,\mathrm{LambertW}(xa)}}-\frac{2\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\sqrt{c}}{}\right.}{}\right.}{3c}\right.}{5c}\right.}{7c}\right.$

input
```
int(1/x^5/(c*LambertW(x*a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^4*c^3*(-1/7/(c*LambertW(x*a))^(7/2)*exp(-4*LambertW(x*a))-8/7/c*(-1/5/
(c*LambertW(x*a))^(5/2)*exp(-4*LambertW(x*a))-8/5/c*(-1/3/(c*LambertW(x*a)
)^(3/2)*exp(-4*LambertW(x*a))-8/3/c*(-1/(c*LambertW(x*a))^(1/2)*exp(-4*Lam
bertW(x*a))-2/c^(1/2)*Pi^(1/2)*erf(2*(c*LambertW(x*a))^(1/2)/c^(1/2)))))+c
*(-1/9/(c*LambertW(x*a))^(9/2)*exp(-4*LambertW(x*a))-8/9/c*(-1/7/(c*Lamber
tW(x*a))^(7/2)*exp(-4*LambertW(x*a))-8/7/c*(-1/5/(c*LambertW(x*a))^(5/2)*e
xp(-4*LambertW(x*a))-8/5/c*(-1/3/(c*LambertW(x*a))^(3/2)*exp(-4*LambertW(x
*a))-8/3/c*(-1/(c*LambertW(x*a))^(1/2)*exp(-4*LambertW(x*a))-2/c^(1/2)*Pi^
(1/2)*erf(2*(c*LambertW(x*a))^(1/2)/c^(1/2)))))))
```

Fricas [F]

$$\int \frac{1}{x^5\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}x^5}\,dx$$

input
```
integrate(1/x^5/(c*lambert_w(a*x))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x))/(c*x^5*lambert_w(a*x)), x)
```

Sympy [F]

$$\int \frac{1}{x^5\sqrt{cW(ax)}}\,dx = \int \frac{1}{x^5\sqrt{cW\left(ax\right)}}\,dx$$

input
```
integrate(1/x**5/(c*LambertW(a*x))**(1/2),x)
```

output
```
Integral(1/(x**5*sqrt(c*LambertW(a*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^5\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}x^5}\,dx$$

input
```
integrate(1/x^5/(c*lambert_w(a*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x))*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5\sqrt{cW(ax)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax)}x^5}\,dx$$

input
```
integrate(1/x^5/(c*lambert_w(a*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x))*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{cW(ax)}}\,dx = \int \frac{1}{x^5\,\sqrt{c\,\mathrm{LambertW}\,(a\,x)}}\,dx$$

input
```
int(1/(x^5*(c*LambertW(a*x))^(1/2)),x)
```

output
```
int(1/(x^5*(c*LambertW(a*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^5\sqrt{cW(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(ax)}}{lambert\_w(ax)x^5}dx\right)}{c}$$

input
```
int(1/x^5/(c*Lambert_W(a*x))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a*x))/(lambert_w(a*x)*x**5),x))/c
```

3.80 $\int x^m W(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 113

$$\int x^m W(ax)\,dx = \frac{e^{-mW(ax)}x^m\Gamma(3+m,(-1-m)W(ax))W(ax)^2((-1-m)W(ax))^{-2-m}}{a(1+m)} + \frac{e^{-mW(ax)}x^m\Gamma(2+m,(-1-m)W(ax))W(ax)((-1-m)W(ax))^{-1-m}}{a(1+m)}$$

output
```
x^m*GAMMA(3+m,(-1-m)*LambertW(a*x))*LambertW(a*x)^2*((-1-m)*LambertW(a*x))
^(-2-m)/a/exp(m*LambertW(a*x))/(1+m)+x^m*GAMMA(2+m,(-1-m)*LambertW(a*x))*L
ambertW(a*x)*((-1-m)*LambertW(a*x))^(-1-m)/a/exp(m*LambertW(a*x))/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.58

$$\int x^m W(ax)\,dx = \frac{e^{-mW(ax)}x^m(-((1+m)\Gamma(2+m,-((1+m)W(ax)))) + \Gamma(3+m,-((1+m)W(ax))))(-((1+m)W(a}{a(1+m)^3}$$

input
```
Integrate[x^m*ProductLog[a*x],x]
```

output
```
(x^m*(-((1 + m)*Gamma[2 + m, -((1 + m)*ProductLog[a*x])]) + Gamma[3 + m, -
((1 + m)*ProductLog[a*x])]))/(a*E^(m*ProductLog[a*x])*(1 + m)^3*(-((1 + m)
*ProductLog[a*x]))^m)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7174, 7207}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m W(ax)\,dx$$
$$\Big\downarrow \text{7174}$$
$$\int \frac{x^m W(ax)}{W(ax)+1}dx+\int \frac{x^m W(ax)^2}{W(ax)+1}dx$$
$$\Big\downarrow \text{7207}$$
$$\frac{x^m W(ax)^2 e^{-mW(ax)}(-((m+1)W(ax)))^{-m-2}\Gamma(m+3,-((m+1)W(ax)))}{a(m+1)}+\frac{x^m W(ax)e^{-mW(ax)}(-((m+1)W(ax)))^{-m-1}\Gamma(m+2,-((m+1)W(ax)))}{a(m+1)}$$

input
```
Int[x^m*ProductLog[a*x],x]
```

output
```
(x^m*Gamma[3 + m, -((1 + m)*ProductLog[a*x])]*ProductLog[a*x]^2*(-((1 + m)
*ProductLog[a*x]))^(-2 - m))/(a*E^(m*ProductLog[a*x])*(1 + m)) + (x^m*Gamm
a[2 + m, -((1 + m)*ProductLog[a*x])]*ProductLog[a*x]*(-((1 + m)*ProductLog
[a*x]))^(-1 - m))/(a*E^(m*ProductLog[a*x])*(1 + m))
```

Defintions of rubi rules used

rule 7174
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.), x_Symbol] :> Int[x^m*(
(c*ProductLog[a*x])^p/(1 + ProductLog[a*x])), x] + Simp[1/c   Int[x^m*((c*P
roductLog[a*x])^(p + 1)/(1 + ProductLog[a*x])), x], x] /; FreeQ[{a, c, m},
x]
```

rule 7207
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.))/((d_) + (d_.)*Product
Log[(a_.)*(x_)]), x_Symbol] :> Simp[x^m*Gamma[m + p + 1, (-(m + 1))*Product
Log[a*x]]*((c*ProductLog[a*x])^p/(a*d*(m + 1)*E^(m*ProductLog[a*x])*((-(m +
 1))*ProductLog[a*x])^(m + p))), x] /; FreeQ[{a, c, d, m, p}, x] && NeQ[m,
-1]
```

Maple [F]

$$\int x^m \operatorname{LambertW}(xa)\,dx$$

input
```
int(x^m*LambertW(x*a),x)
```

output
```
int(x^m*LambertW(x*a),x)
```

Fricas [F]

$$\int x^m W(ax)\,dx = \int x^m \operatorname{W}(ax)\,dx$$

input
```
integrate(x^m*lambert_w(a*x),x, algorithm="fricas")
```

output
```
integral(x^m*lambert_w(a*x), x)
```

Sympy [F]

$$\int x^m W(ax)\,dx = \int x^m W(ax)\;dx$$

input
```
integrate(x**m*LambertW(a*x),x)
```

output
```
Integral(x**m*LambertW(a*x), x)
```

Maxima [F]

$$\int x^m W(ax)\,dx = \int x^m\,\mathrm{W}(ax)\,dx$$

input
```
integrate(x^m*lambert_w(a*x),x, algorithm="maxima")
```

output
```
integrate(x^m*lambert_w(a*x), x)
```

Giac [F]

$$\int x^m W(ax)\,dx = \int x^m\,\mathrm{W}(ax)\,dx$$

input
```
integrate(x^m*lambert_w(a*x),x, algorithm="giac")
```

output
```
integrate(x^m*lambert_w(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m W(ax)\,dx = \int x^m \,\mathrm{LambertW}(a\,x)\;dx$$

input
```
int(x^m*LambertW(a*x),x)
```

output
```
int(x^m*LambertW(a*x), x)
```

Reduce [F]

$$\int x^m W(ax)\,dx = \int x^m lambert\_w(ax)\,dx$$

input
```
int(x^m*Lambert_W(a*x),x)
```

output
```
int(x**m*lambert_w(a*x),x)
```

3.81 $\int x^m W(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 115

$$\int x^m W(ax)^2 \, dx = \frac{e^{-mW(ax)} x^m \Gamma(4+m,(-1-m)W(ax)) W(ax)^3 ((-1-m)W(ax))^{-3-m}}{a(1+m)} + \frac{e^{-mW(ax)} x^m \Gamma(3+m,(-1-m)W(ax)) W(ax)^2 ((-1-m)W(ax))^{-2-m}}{a(1+m)}$$

output
```
x^m*GAMMA(4+m,(-1-m)*LambertW(a*x))*LambertW(a*x)^3*((-1-m)*LambertW(a*x))
^(-3-m)/a/exp(m*LambertW(a*x))/(1+m)+x^m*GAMMA(3+m,(-1-m)*LambertW(a*x))*L
ambertW(a*x)^2*((-1-m)*LambertW(a*x))^(-2-m)/a/exp(m*LambertW(a*x))/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.58

$$\int x^m W(ax)^2 \, dx = \frac{e^{-mW(ax)} x^m ((1+m)\Gamma(3+m,-((1+m)W(ax))) - \Gamma(4+m,-((1+m)W(ax))))(-((1+m)W(ax)))}{a(1+m)^4}$$

input
```
Integrate[x^m*ProductLog[a*x]^2,x]
```

output
```
(x^m*((1 + m)*Gamma[3 + m, -((1 + m)*ProductLog[a*x])] - Gamma[4 + m, -((1
 + m)*ProductLog[a*x])]))/(a*E^(m*ProductLog[a*x])*(1 + m)^4*(-((1 + m)*Pr
oductLog[a*x]))^m)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7174, 7207}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m W(ax)^2 \, dx$$

$$\Big\downarrow \text{ 7174}$$

$$\int \frac{x^m W(ax)^2}{W(ax)+1}dx+\int \frac{x^m W(ax)^3}{W(ax)+1}dx$$

$$\Big\downarrow \text{ 7207}$$

$$\frac{x^m W(ax)^3 e^{-mW(ax)}(-((m+1)W(ax)))^{-m-3}\Gamma(m+4,-((m+1)W(ax)))}{a(m+1)}+\frac{x^m W(ax)^2 e^{-mW(ax)}(-((m+1)W(ax)))^{-m-2}\Gamma(m+3,-((m+1)W(ax)))}{a(m+1)}$$

input
```
Int[x^m*ProductLog[a*x]^2,x]
```

output
```
(x^m*Gamma[4 + m, -((1 + m)*ProductLog[a*x])]*ProductLog[a*x]^3*(-((1 + m)
*ProductLog[a*x]))^(-3 - m))/(a*E^(m*ProductLog[a*x])*(1 + m)) + (x^m*Gamm
a[3 + m, -((1 + m)*ProductLog[a*x])]*ProductLog[a*x]^2*(-((1 + m)*ProductL
og[a*x]))^(-2 - m))/(a*E^(m*ProductLog[a*x])*(1 + m))
```

Defintions of rubi rules used

rule 7174
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.), x_Symbol] :> Int[x^m*(
(c*ProductLog[a*x])^p/(1 + ProductLog[a*x])), x] + Simp[1/c   Int[x^m*((c*P
roductLog[a*x])^(p + 1)/(1 + ProductLog[a*x])), x], x] /; FreeQ[{a, c, m},
x]
```

rule 7207
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.))/((d_) + (d_.)*Product
Log[(a_.)*(x_)]), x_Symbol] :> Simp[x^m*Gamma[m + p + 1, (-(m + 1))*Product
Log[a*x]]*((c*ProductLog[a*x])^p/(a*d*(m + 1)*E^(m*ProductLog[a*x])*((-(m +
 1))*ProductLog[a*x])^(m + p))), x] /; FreeQ[{a, c, d, m, p}, x] && NeQ[m,
-1]
```

Maple [F]

$$\int x^m \operatorname{LambertW}(xa)^2 dx$$

input
```
int(x^m*LambertW(x*a)^2,x)
```

output
```
int(x^m*LambertW(x*a)^2,x)
```

Fricas [F]

$$\int x^m W(ax)^2 dx = \int x^m \operatorname{W}(ax)^2 dx$$

input
```
integrate(x^m*lambert_w(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^m*lambert_w(a*x)^2, x)
```

Sympy [F]

$$\int x^m W(ax)^2\,dx = \int x^m W^2(ax)\,dx$$

input
```
integrate(x**m*LambertW(a*x)**2,x)
```

output
```
Integral(x**m*LambertW(a*x)**2, x)
```

Maxima [F]

$$\int x^m W(ax)^2\,dx = \int x^m\,\mathrm{W}(ax)^2\,dx$$

input
```
integrate(x^m*lambert_w(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*lambert_w(a*x)^2, x)
```

Giac [F]

$$\int x^m W(ax)^2\,dx = \int x^m\,\mathrm{W}(ax)^2\,dx$$

input
```
integrate(x^m*lambert_w(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^m*lambert_w(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m W(ax)^2 \, dx = \int x^m \, \mathrm{LambertW}(a\,x)^2 \, dx$$

input
```
int(x^m*LambertW(a*x)^2,x)
```

output
```
int(x^m*LambertW(a*x)^2, x)
```

Reduce [F]

$$\int x^m W(ax)^2 \, dx = \int x^m lambert\_w(ax)^2 \, dx$$

input
```
int(x^m*Lambert_W(a*x)^2,x)
```

output
```
int(x**m*lambert_w(a*x)**2,x)
```

3.82 $\int \frac{x^m}{W(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 105

$$\int \frac{x^m}{W(ax)}\,dx = \frac{e^{-mW(ax)}x^m\Gamma(m,(-1-m)W(ax))((-1-m)W(ax))^{1-m}}{a(1+m)W(ax)} + \frac{e^{-mW(ax)}x^m\Gamma(1+m,(-1-m)W(ax))((-1-m)W(ax))^{-m}}{a(1+m)}$$

output
```
x^m*GAMMA(m,(-1-m)*LambertW(a*x))*((-1-m)*LambertW(a*x))^(1-m)/a/exp(m*Lam
bertW(a*x))/(1+m)/LambertW(a*x)+x^m*GAMMA(1+m,(-1-m)*LambertW(a*x))/a/exp(
m*LambertW(a*x))/(1+m)/(((-1-m)*LambertW(a*x))^m)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.61

$$\int \frac{x^m}{W(ax)}\,dx = \frac{e^{-mW(ax)}x^m(-((1+m)\Gamma(m,-((1+m)W(ax)))) + \Gamma(1+m,-((1+m)W(ax))))(-((1+m)W(ax)))}{a(1+m)}$$

input
```
Integrate[x^m/ProductLog[a*x],x]
```

output
```
(x^m*(-((1 + m)*Gamma[m, -((1 + m)*ProductLog[a*x])]) + Gamma[1 + m, -((1
+ m)*ProductLog[a*x])]))/(a*E^(m*ProductLog[a*x])*(1 + m)*(-((1 + m)*Produ
ctLog[a*x]))^m)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {7174, 7197, 7207}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{W(ax)}\,dx$$

$$\Big\downarrow \text{7174}$$

$$\int \frac{x^m}{W(ax)+1}dx+\int \frac{x^m}{W(ax)(W(ax)+1)}dx$$

$$\Big\downarrow \text{7197}$$

$$\int \frac{x^m}{W(ax)(W(ax)+1)}dx+\frac{x^m e^{-mW(ax)}(-((m+1)W(ax)))^{-m}\Gamma(m+1,-((m+1)W(ax)))}{a(m+1)}$$

$$\Big\downarrow \text{7207}$$

$$\frac{x^m e^{-mW(ax)}(-((m+1)W(ax)))^{1-m}\Gamma(m,-((m+1)W(ax)))}{a(m+1)W(ax)}+\frac{x^m e^{-mW(ax)}(-((m+1)W(ax)))^{-m}\Gamma(m+1,-((m+1)W(ax)))}{a(m+1)}$$

input
```
Int[x^m/ProductLog[a*x],x]
```

output
```
(x^m*Gamma[m, -((1 + m)*ProductLog[a*x])]*(-((1 + m)*ProductLog[a*x]))^(1
- m))/(a*E^(m*ProductLog[a*x])*(1 + m)*ProductLog[a*x]) + (x^m*Gamma[1 + m
, -((1 + m)*ProductLog[a*x])])/(a*E^(m*ProductLog[a*x])*(1 + m)*(-((1 + m)
*ProductLog[a*x]))^m)
```

Defintions of rubi rules used

rule 7174
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.), x_Symbol] :> Int[x^m*(
(c*ProductLog[a*x])^p/(1 + ProductLog[a*x])), x] + Simp[1/c   Int[x^m*((c*P
roductLog[a*x])^(p + 1)/(1 + ProductLog[a*x])), x], x] /; FreeQ[{a, c, m},
x]
```

rule 7197
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^m
*(Gamma[m + 1, (-(m + 1))*ProductLog[a*x]]/(a*d*(m + 1)*E^(m*ProductLog[a*x
])*((-(m + 1))*ProductLog[a*x])^m)), x] /; FreeQ[{a, d, m}, x] &&  !Integer
Q[m]
```

rule 7207
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.))/((d_) + (d_.)*Product
Log[(a_.)*(x_)]), x_Symbol] :> Simp[x^m*Gamma[m + p + 1, (-(m + 1))*Product
Log[a*x]]*((c*ProductLog[a*x])^p/(a*d*(m + 1)*E^(m*ProductLog[a*x])*((-(m +
 1))*ProductLog[a*x])^(m + p))), x] /; FreeQ[{a, c, d, m, p}, x] && NeQ[m,
-1]
```

Maple [F]

$$\int \frac{x^m}{\mathrm{LambertW}(xa)}dx$$

input
```
int(x^m/LambertW(x*a),x)
```

output
```
int(x^m/LambertW(x*a),x)
```

Fricas [F]

$$\int \frac{x^m}{W(ax)}\,dx = \int \frac{x^m}{\mathrm{W}(ax)}\,dx$$

input
```
integrate(x^m/lambert_w(a*x),x, algorithm="fricas")
```

output
```
integral(x^m/lambert_w(a*x), x)
```

Sympy [F]

$$\int \frac{x^m}{W(ax)}\,dx = \int \frac{x^m}{W\left(ax\right)}\,dx$$

input
```
integrate(x**m/LambertW(a*x),x)
```

output
```
Integral(x**m/LambertW(a*x), x)
```

Maxima [F]

$$\int \frac{x^m}{W(ax)}\,dx = \int \frac{x^m}{\mathrm{W}(ax)}\,dx$$

input
```
integrate(x^m/lambert_w(a*x),x, algorithm="maxima")
```

output
```
integrate(x^m/lambert_w(a*x), x)
```

Giac [F]

$$\int \frac{x^m}{W(ax)}\,dx = \int \frac{x^m}{\mathrm{W}(ax)}\,dx$$

input
```
integrate(x^m/lambert_w(a*x),x, algorithm="giac")
```

output
```
integrate(x^m/lambert_w(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{W(ax)}\,dx = \int \frac{x^m}{\mathrm{LambertW}\left(a\,x\right)}\,dx$$

input
```
int(x^m/LambertW(a*x),x)
```

output
```
int(x^m/LambertW(a*x), x)
```

Reduce [F]

$$\int \frac{x^m}{W(ax)}\,dx$$
$$= \frac{x^m x - \left(\int x^m dx\right) m - \left(\int x^m dx\right) + 2\left(\int \frac{x^m}{lambert\_w(ax)} dx\right) m + 2\left(\int \frac{x^m}{lambert\_w(ax)} dx\right)}{2m+2}$$

input
```
int(x^m/Lambert_W(a*x),x)
```

output
```
(x**m*x - int(x**m,x)*m - int(x**m,x) + 2*int(x**m/lambert_w(a*x),x)*m + 2
*int(x**m/lambert_w(a*x),x))/(2*(m + 1))
```

3.83 $\int \frac{x^m}{W(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 113

$$\int \frac{x^m}{W(ax)^2}\,dx = \frac{e^{-mW(ax)}x^m\Gamma(m,(-1-m)W(ax))((-1-m)W(ax))^{1-m}}{a(1+m)W(ax)} + \frac{e^{-mW(ax)}x^m\Gamma(-1+m,(-1-m)W(ax))((-1-m)W(ax))^{2-m}}{a(1+m)W(ax)^2}$$

output
```
x^m*GAMMA(m,(-1-m)*LambertW(a*x))*((-1-m)*LambertW(a*x))^(1-m)/a/exp(m*Lam
bertW(a*x))/(1+m)/LambertW(a*x)+x^m*GAMMA(-1+m,(-1-m)*LambertW(a*x))*((-1-
m)*LambertW(a*x))^(2-m)/a/exp(m*LambertW(a*x))/(1+m)/LambertW(a*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.53

$$\int \frac{x^m}{W(ax)^2}\,dx = \frac{e^{-mW(ax)}x^m((1+m)\Gamma(-1+m,-((1+m)W(ax)))-\Gamma(m,-((1+m)W(ax))))(-((1+m)W(ax)))^{-m}}{a}$$

input
```
Integrate[x^m/ProductLog[a*x]^2,x]
```

output
```
(x^m*((1 + m)*Gamma[-1 + m, -((1 + m)*ProductLog[a*x])] - Gamma[m, -((1 +
m)*ProductLog[a*x])]))/(a*E^(m*ProductLog[a*x])*(-((1 + m)*ProductLog[a*x]
))^m)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7174, 7207}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{W(ax)^2}dx$$

$$\Big\downarrow \text{7174}$$

$$\int \frac{x^m}{W(ax)^2(W(ax)+1)}dx+\int \frac{x^m}{W(ax)(W(ax)+1)}dx$$

$$\Big\downarrow \text{7207}$$

$$\frac{x^m e^{-mW(ax)}(-((m+1)W(ax)))^{1-m}\Gamma(m,-((m+1)W(ax)))}{a(m+1)W(ax)}+\frac{x^m e^{-mW(ax)}(-((m+1)W(ax)))^{2-m}\Gamma(m-1,-((m+1)W(ax)))}{a(m+1)W(ax)^2}$$

input
```
Int[x^m/ProductLog[a*x]^2,x]
```

output
```
(x^m*Gamma[m, -((1 + m)*ProductLog[a*x])]*(-((1 + m)*ProductLog[a*x]))^(1
- m))/(a*E^(m*ProductLog[a*x])*(1 + m)*ProductLog[a*x]) + (x^m*Gamma[-1 +
m, -((1 + m)*ProductLog[a*x])]*(-((1 + m)*ProductLog[a*x]))^(2 - m))/(a*E^
(m*ProductLog[a*x])*(1 + m)*ProductLog[a*x]^2)
```

Defintions of rubi rules used

rule 7174
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.), x_Symbol] :> Int[x^m*(
(c*ProductLog[a*x])^p/(1 + ProductLog[a*x])), x] + Simp[1/c   Int[x^m*((c*P
roductLog[a*x])^(p + 1)/(1 + ProductLog[a*x])), x], x] /; FreeQ[{a, c, m},
x]
```

rule 7207
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.))/((d_) + (d_.)*Product
Log[(a_.)*(x_)]), x_Symbol] :> Simp[x^m*Gamma[m + p + 1, (-(m + 1))*Product
Log[a*x]]*((c*ProductLog[a*x])^p/(a*d*(m + 1)*E^(m*ProductLog[a*x])*((-(m +
 1))*ProductLog[a*x])^(m + p))), x] /; FreeQ[{a, c, d, m, p}, x] && NeQ[m,
-1]
```

Maple [F]

$$\int \frac{x^m}{\text{LambertW}(xa)^2} dx$$

input
```
int(x^m/LambertW(x*a)^2,x)
```

output
```
int(x^m/LambertW(x*a)^2,x)
```

Fricas [F]

$$\int \frac{x^m}{W(ax)^2} dx = \int \frac{x^m}{\text{W}(ax)^2} dx$$

input
```
integrate(x^m/lambert_w(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^m/lambert_w(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^m}{W(ax)^2} \, dx = \int \frac{x^m}{W^2(ax)} \, dx$$

input
```
integrate(x**m/LambertW(a*x)**2,x)
```

output
```
Integral(x**m/LambertW(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^m}{W(ax)^2} \, dx = \int \frac{x^m}{\mathrm{W}(ax)^2} \, dx$$

input
```
integrate(x^m/lambert_w(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^m/lambert_w(a*x)^2, x)
```

Giac [F]

$$\int \frac{x^m}{W(ax)^2} \, dx = \int \frac{x^m}{\mathrm{W}(ax)^2} \, dx$$

input
```
integrate(x^m/lambert_w(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^m/lambert_w(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{W(ax)^2}\,dx = \int \frac{x^m}{\mathrm{LambertW}\left(a\,x\right)^2}\,dx$$

input
```
int(x^m/LambertW(a*x)^2,x)
```

output
```
int(x^m/LambertW(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x^m}{W(ax)^2}\,dx = \int \frac{x^m}{lambert\_w(ax)^2}dx$$

input
```
int(x^m/Lambert_W(a*x)^2,x)
```

output
```
int(x**m/lambert_w(a*x)**2,x)
```

3.84 $\int x^m \sqrt{cW(ax)}\, dx$

Optimal result

Integrand size = 14, antiderivative size = 134

$$\int x^m \sqrt{cW(ax)}\, dx = \frac{e^{-mW(ax)}x^m\Gamma\left(\frac{5}{2}+m,(-1-m)W(ax)\right)(cW(ax))^{3/2}((-1-m)W(ax))^{-\frac{3}{2}-m}}{ac(1+m)} + \frac{e^{-mW(ax)}x^m\Gamma\left(\frac{3}{2}+m,(-1-m)W(ax)\right)\sqrt{cW(ax)}((-1-m)W(ax))^{-\frac{1}{2}-m}}{a(1+m)}$$

output
```
x^m*GAMMA(5/2+m,(-1-m)*LambertW(a*x))*(c*LambertW(a*x))^(3/2)*((-1-m)*Lamb
ertW(a*x))^(-3/2-m)/a/c/exp(m*LambertW(a*x))/(1+m)+x^m*GAMMA(3/2+m,(-1-m)*
LambertW(a*x))*(c*LambertW(a*x))^(1/2)*((-1-m)*LambertW(a*x))^(-1/2-m)/a/e
xp(m*LambertW(a*x))/(1+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.63

$$\int x^m \sqrt{cW(ax)}\,dx$$
$$= \frac{e^{-mW(ax)}x^m\left((1+m)\Gamma\left(\frac{3}{2}+m,-((1+m)W(ax))\right)-\Gamma\left(\frac{5}{2}+m,-((1+m)W(ax))\right)\right)\sqrt{cW(ax)}(-((1+}{a(1+m)^2}$$

input
```
Integrate[x^m*Sqrt[c*ProductLog[a*x]],x]
```

output
```
(x^m*((1 + m)*Gamma[3/2 + m, -((1 + m)*ProductLog[a*x])] - Gamma[5/2 + m,
-((1 + m)*ProductLog[a*x])])*Sqrt[c*ProductLog[a*x]]*(-((1 + m)*ProductLog
[a*x]))^(-1/2 - m))/(a*E^(m*ProductLog[a*x])*(1 + m)^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7174, 7207}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{cW(ax)}\,dx$$
$$\Big\downarrow \text{7174}$$
$$\int \frac{x^m\sqrt{cW(ax)}}{W(ax)+1}dx+\frac{\int \frac{x^m(cW(ax))^{3/2}}{W(ax)+1}dx}{c}$$
$$\Big\downarrow \text{7207}$$
$$\frac{x^m(cW(ax))^{3/2}e^{-mW(ax)}(-((m+1)W(ax)))^{-m-\frac{3}{2}}\Gamma\left(m+\frac{5}{2},-((m+1)W(ax))\right)}{ac(m+1)}+$$
$$\frac{x^m\sqrt{cW(ax)}e^{-mW(ax)}(-((m+1)W(ax)))^{-m-\frac{1}{2}}\Gamma\left(m+\frac{3}{2},-((m+1)W(ax))\right)}{a(m+1)}$$

input
```
Int[x^m*Sqrt[c*ProductLog[a*x]],x]
```

output
```
(x^m*Gamma[5/2 + m, -((1 + m)*ProductLog[a*x])]*(c*ProductLog[a*x])^(3/2)*
(-((1 + m)*ProductLog[a*x]))^(-3/2 - m))/(a*c*E^(m*ProductLog[a*x])*(1 + m
)) + (x^m*Gamma[3/2 + m, -((1 + m)*ProductLog[a*x])]*Sqrt[c*ProductLog[a*x
]]*(-((1 + m)*ProductLog[a*x]))^(-1/2 - m))/(a*E^(m*ProductLog[a*x])*(1 +
m))
```

Defintions of rubi rules used

rule 7174
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.), x_Symbol] :> Int[x^m*(
(c*ProductLog[a*x])^p/(1 + ProductLog[a*x])), x] + Simp[1/c   Int[x^m*((c*P
roductLog[a*x])^(p + 1)/(1 + ProductLog[a*x])), x], x] /; FreeQ[{a, c, m},
x]
```

rule 7207
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.))/((d_) + (d_.)*Product
Log[(a_.)*(x_)]), x_Symbol] :> Simp[x^m*Gamma[m + p + 1, (-(m + 1))*Product
Log[a*x]]*((c*ProductLog[a*x])^p/(a*d*(m + 1)*E^(m*ProductLog[a*x])*((-(m +
 1))*ProductLog[a*x])^(m + p))), x] /; FreeQ[{a, c, d, m, p}, x] && NeQ[m,
-1]
```

Maple [F]

$$\int x^m \sqrt{c \,\mathrm{LambertW}\left(xa\right)}dx$$

input
```
int(x^m*(c*LambertW(x*a))^(1/2),x)
```

output
```
int(x^m*(c*LambertW(x*a))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \sqrt{cW(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(c*lambert_w(a*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   loge
xtint: unhandled kernel
```

Sympy [F]

$$\int x^m \sqrt{cW(ax)}\,dx = \int x^m \sqrt{cW(ax)}\,dx$$

input
```
integrate(x**m*(c*LambertW(a*x))**(1/2),x)
```

output
```
Integral(x**m*sqrt(c*LambertW(a*x)), x)
```

Maxima [F]

$$\int x^m \sqrt{cW(ax)}\,dx = \int \sqrt{c\,\mathrm{W}(ax)}x^m\,dx$$

input
```
integrate(x^m*(c*lambert_w(a*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x))*x^m, x)
```

Giac [F]

$$\int x^m \sqrt{cW(ax)}\, dx = \int \sqrt{c\,\mathrm{W}(ax)} x^m\, dx$$

input
```
integrate(x^m*(c*lambert_w(a*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x))*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sqrt{cW(ax)}\, dx = \int x^m \sqrt{c\,\mathrm{LambertW}\,(a\,x)}\, dx$$

input
```
int(x^m*(c*LambertW(a*x))^(1/2),x)
```

output
```
int(x^m*(c*LambertW(a*x))^(1/2), x)
```

Reduce [F]

$$\int x^m \sqrt{cW(ax)}\, dx = \sqrt{c}\left(\int x^m \sqrt{lambert\_w(ax)} dx\right)$$

input
```
int(x^m*(c*Lambert_W(a*x))^(1/2),x)
```

output
```
sqrt(c)*int(x**m*sqrt(lambert_w(a*x)),x)
```

3.85 $\int \frac{x^m}{\sqrt{cW(ax)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 134

$$\int \frac{x^m}{\sqrt{cW(ax)}}\,dx = \frac{e^{-mW(ax)}x^m\Gamma\left(\frac{3}{2}+m,(-1-m)W(ax)\right)\sqrt{cW(ax)}((-1-m)W(ax))^{-\frac{1}{2}-m}}{ac(1+m)} + \frac{e^{-mW(ax)}x^m\Gamma\left(\frac{1}{2}+m,(-1-m)W(ax)\right)((-1-m)W(ax))^{\frac{1}{2}-m}}{a(1+m)\sqrt{cW(ax)}}$$

output
```
x^m*GAMMA(3/2+m,(-1-m)*LambertW(a*x))*(c*LambertW(a*x))^(1/2)*((-1-m)*Lamb
ertW(a*x))^(-1/2-m)/a/c/exp(m*LambertW(a*x))/(1+m)+x^m*GAMMA(1/2+m,(-1-m)*
LambertW(a*x))*((-1-m)*LambertW(a*x))^(1/2-m)/a/exp(m*LambertW(a*x))/(1+m)
/(c*LambertW(a*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.65

$$\int \frac{x^m}{\sqrt{cW(ax)}}\,dx = \frac{e^{-mW(ax)}x^m\left(-\left((1+m)\Gamma\left(\frac{1}{2}+m,-((1+m)W(ax))\right)\right)+\Gamma\left(\frac{3}{2}+m,-((1+m)W(ax))\right)\right)\sqrt{cW(ax)}(-(\cdots}{ac(1+m)}$$

input
```
Integrate[x^m/Sqrt[c*ProductLog[a*x]],x]
```

output
```
(x^m*(-((1 + m)*Gamma[1/2 + m, -((1 + m)*ProductLog[a*x])]) + Gamma[3/2 +
m, -((1 + m)*ProductLog[a*x])])*Sqrt[c*ProductLog[a*x]]*(-((1 + m)*Product
Log[a*x]))^(-1/2 - m))/(a*c*E^(m*ProductLog[a*x])*(1 + m))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {7174, 7207}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{cW(ax)}}\,dx$$

$$\Big\downarrow \text{7174}$$

$$\int \frac{x^m}{\sqrt{cW(ax)}(W(ax)+1)}dx+\frac{\int \frac{x^m\sqrt{cW(ax)}}{W(ax)+1}dx}{c}$$

$$\Big\downarrow \text{7207}$$

$$\frac{x^m\sqrt{cW(ax)}e^{-mW(ax)}(-((m+1)W(ax)))^{-m-\frac{1}{2}}\Gamma\left(m+\frac{3}{2},-((m+1)W(ax))\right)}{ac(m+1)}+\frac{x^m e^{-mW(ax)}(-((m+1)W(ax)))^{\frac{1}{2}-m}\Gamma\left(m+\frac{1}{2},-((m+1)W(ax))\right)}{a(m+1)\sqrt{cW(ax)}}$$

input
```
Int[x^m/Sqrt[c*ProductLog[a*x]],x]
```

output
```
(x^m*Gamma[3/2 + m, -((1 + m)*ProductLog[a*x])]*Sqrt[c*ProductLog[a*x]]*(-
((1 + m)*ProductLog[a*x]))^(-1/2 - m))/(a*c*E^(m*ProductLog[a*x])*(1 + m))
 + (x^m*Gamma[1/2 + m, -((1 + m)*ProductLog[a*x])]*(-((1 + m)*ProductLog[a
*x]))^(1/2 - m))/(a*E^(m*ProductLog[a*x])*(1 + m)*Sqrt[c*ProductLog[a*x]])
```

Defintions of rubi rules used

rule 7174
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.), x_Symbol] :> Int[x^m*(
(c*ProductLog[a*x])^p/(1 + ProductLog[a*x])), x] + Simp[1/c   Int[x^m*((c*P
roductLog[a*x])^(p + 1)/(1 + ProductLog[a*x])), x], x] /; FreeQ[{a, c, m},
x]
```

rule 7207
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.))/((d_) + (d_.)*Product
Log[(a_.)*(x_)]), x_Symbol] :> Simp[x^m*Gamma[m + p + 1, (-(m + 1))*Product
Log[a*x]]*((c*ProductLog[a*x])^p/(a*d*(m + 1)*E^(m*ProductLog[a*x])*((-(m +
 1))*ProductLog[a*x])^(m + p))), x] /; FreeQ[{a, c, d, m, p}, x] && NeQ[m,
-1]
```

Maple [F]

$$\int \frac{x^m}{\sqrt{c\,\mathrm{LambertW}\left(xa\right)}}dx$$

input
```
int(x^m/(c*LambertW(x*a))^(1/2),x)
```

output
```
int(x^m/(c*LambertW(x*a))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{cW(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/(c*lambert_w(a*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   loge
xtint: unhandled kernel
```

Sympy [F]

$$\int \frac{x^m}{\sqrt{cW(ax)}}\,dx = \int \frac{x^m}{\sqrt{cW\left(ax\right)}}\,dx$$

input
```
integrate(x**m/(c*LambertW(a*x))**(1/2),x)
```

output
```
Integral(x**m/sqrt(c*LambertW(a*x)), x)
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{cW(ax)}}\,dx = \int \frac{x^m}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(x^m/(c*lambert_w(a*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(c*lambert_w(a*x)), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{cW(ax)}}\,dx = \int \frac{x^m}{\sqrt{c\,\mathrm{W}(ax)}}\,dx$$

input
```
integrate(x^m/(c*lambert_w(a*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(c*lambert_w(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{cW(ax)}}\,dx = \int \frac{x^m}{\sqrt{c\,\mathrm{LambertW}\,(a\,x)}}\,dx$$

input
```
int(x^m/(c*LambertW(a*x))^(1/2),x)
```

output
```
int(x^m/(c*LambertW(a*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{cW(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^m\sqrt{lambert\_w(ax)}}{lambert\_w(ax)}dx\right)}{c}$$

input
```
int(x^m/(c*Lambert_W(a*x))^(1/2),x)
```

output
```
(sqrt(c)*int((x**m*sqrt(lambert_w(a*x)))/lambert_w(a*x),x))/c
```

3.86 $\int x^2 (cW(ax))^p\, dx$

Optimal result

Integrand size = 12, antiderivative size = 110

$$\int x^2 (cW(ax))^p\, dx = \frac{3^{-3-p} e^{-2W(ax)} x^2 \Gamma(3+p, -3W(ax)) (-W(ax))^{-2-p} (cW(ax))^p}{a} + \frac{3^{-4-p} e^{-2W(ax)} x^2 \Gamma(4+p, -3W(ax)) (-W(ax))^{-3-p} (cW(ax))^{1+p}}{ac}$$

output
```
3^(-3-p)*x^2*GAMMA(3+p,-3*LambertW(a*x))*(-LambertW(a*x))^(-2-p)*(c*Lamber
tW(a*x))^p/a/exp(2*LambertW(a*x))+3^(-4-p)*x^2*GAMMA(4+p,-3*LambertW(a*x))
*(-LambertW(a*x))^(-3-p)*(c*LambertW(a*x))^(p+1)/a/c/exp(2*LambertW(a*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.61

$$\int x^2 (cW(ax))^p\, dx = \frac{3^{-4-p} e^{-2W(ax)} x^2 (3\Gamma(3+p, -3W(ax)) - \Gamma(4+p, -3W(ax))) (-W(ax))^{-2-p} (cW(ax))^p}{a}$$

input
```
Integrate[x^2*(c*ProductLog[a*x])^p,x]
```

output
```
(3^(-4 - p)*x^2*(3*Gamma[3 + p, -3*ProductLog[a*x]] - Gamma[4 + p, -3*Prod
uctLog[a*x]])*(-ProductLog[a*x])^(-2 - p)*(c*ProductLog[a*x])^p)/(a*E^(2*P
roductLog[a*x]))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7174, 7207}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (cW(ax))^p \, dx$$

$$\Big\downarrow \text{7174}$$

$$\int \frac{x^2 (cW(ax))^p}{W(ax)+1} dx + \frac{\int \frac{x^2 (cW(ax))^{p+1}}{W(ax)+1} dx}{c}$$

$$\Big\downarrow \text{7207}$$

$$\frac{3^{-p-4} x^2 e^{-2W(ax)} (-W(ax))^{-p-3} (cW(ax))^{p+1} \Gamma(p+4,-3W(ax))}{ac} + \frac{3^{-p-3} x^2 e^{-2W(ax)} (-W(ax))^{-p-2} (cW(ax))^p \Gamma(p+3,-3W(ax))}{a}$$

input
```
Int[x^2*(c*ProductLog[a*x])^p,x]
```

output
```
(3^(-3 - p)*x^2*Gamma[3 + p, -3*ProductLog[a*x]]*(-ProductLog[a*x])^(-2 -
p)*(c*ProductLog[a*x])^p)/(a*E^(2*ProductLog[a*x])) + (3^(-4 - p)*x^2*Gamm
a[4 + p, -3*ProductLog[a*x]]*(-ProductLog[a*x])^(-3 - p)*(c*ProductLog[a*x
])^(1 + p))/(a*c*E^(2*ProductLog[a*x]))
```

Defintions of rubi rules used

rule 7174
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.), x_Symbol] :> Int[x^m*(
(c*ProductLog[a*x])^p/(1 + ProductLog[a*x])), x] + Simp[1/c   Int[x^m*((c*P
roductLog[a*x])^(p + 1)/(1 + ProductLog[a*x])), x], x] /; FreeQ[{a, c, m},
x]
```

rule 7207
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.))/((d_) + (d_.)*Product
Log[(a_.)*(x_)]), x_Symbol] :> Simp[x^m*Gamma[m + p + 1, (-(m + 1))*Product
Log[a*x]]*((c*ProductLog[a*x])^p/(a*d*(m + 1)*E^(m*ProductLog[a*x])*((-(m +
 1))*ProductLog[a*x])^(m + p))), x] /; FreeQ[{a, c, d, m, p}, x] && NeQ[m,
-1]
```

Maple [F]

$$\int x^{2}(c\,\mathrm{LambertW}\,(xa))^{p}\,dx$$

input
```
int(x^2*(c*LambertW(x*a))^p,x)
```

output
```
int(x^2*(c*LambertW(x*a))^p,x)
```

Fricas [F]

$$\int x^2(cW(ax))^p\,dx=\int (c\,\mathrm{W}(ax))^p\,x^2\,dx$$

input
```
integrate(x^2*(c*lambert_w(a*x))^p,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x))^p*x^2, x)
```

Sympy [F]

$$\int x^2(cW(ax))^p\,dx = \int x^2(cW(ax))^p\;dx$$

input
```
integrate(x**2*(c*LambertW(a*x))**p,x)
```

output
```
Integral(x**2*(c*LambertW(a*x))**p, x)
```

Maxima [F]

$$\int x^2(cW(ax))^p\,dx = \int (c\,\mathrm{W}(ax))^p\,x^2\,dx$$

input
```
integrate(x^2*(c*lambert_w(a*x))^p,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x))^p*x^2, x)
```

Giac [F]

$$\int x^2(cW(ax))^p\,dx = \int (c\,\mathrm{W}(ax))^p\,x^2\,dx$$

input
```
integrate(x^2*(c*lambert_w(a*x))^p,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x))^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(cW(ax))^p\,dx = \int x^2\,(c\,\mathrm{LambertW}(a\,x))^p\,dx$$

input
```
int(x^2*(c*LambertW(a*x))^p,x)
```

output
```
int(x^2*(c*LambertW(a*x))^p, x)
```

Reduce [F]

$$\int x^2(cW(ax))^p\,dx = c^p\left(\int lambert\_w(ax)^p\,x^2dx\right)$$

input
```
int(x^2*(c*Lambert_W(a*x))^p,x)
```

output
```
c**p*int(lambert_w(a*x)**p*x**2,x)
```

3.87 $\int x(cW(ax))^p \, dx$

Optimal result

Integrand size = 10, antiderivative size = 106

$$\int x(cW(ax))^p \, dx = \frac{2^{-2-p}e^{-W(ax)}x\Gamma(2+p,-2W(ax))(-W(ax))^{-1-p}(cW(ax))^p}{a} + \frac{2^{-3-p}e^{-W(ax)}x\Gamma(3+p,-2W(ax))(-W(ax))^{-2-p}(cW(ax))^{1+p}}{ac}$$

output
```
2^(-2-p)*x*GAMMA(2+p,-2*LambertW(a*x))*(-LambertW(a*x))^(-1-p)*(c*LambertW
(a*x))^p/a/exp(LambertW(a*x))+2^(-3-p)*x*GAMMA(3+p,-2*LambertW(a*x))*(-Lam
bertW(a*x))^(-2-p)*(c*LambertW(a*x))^(p+1)/a/c/exp(LambertW(a*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.49

$$\int x(cW(ax))^p \, dx = \frac{2^{-3-p}(-2\Gamma(2+p,-2W(ax))+\Gamma(3+p,-2W(ax)))(-W(ax))^{-p}(cW(ax))^p}{a^2}$$

input
```
Integrate[x*(c*ProductLog[a*x])^p,x]
```

output
```
(2^(-3 - p)*(-2*Gamma[2 + p, -2*ProductLog[a*x]] + Gamma[3 + p, -2*Product
Log[a*x]])*(c*ProductLog[a*x])^p)/(a^2*(-ProductLog[a*x])^p)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7174, 7207}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(cW(ax))^p\,dx$$

$$\Big\downarrow \text{7174}$$

$$\int \frac{x(cW(ax))^p}{W(ax)+1}dx + \frac{\int \frac{x(cW(ax))^{p+1}}{W(ax)+1}dx}{c}$$

$$\Big\downarrow \text{7207}$$

$$\frac{2^{-p-3}xe^{-W(ax)}(-W(ax))^{-p-2}(cW(ax))^{p+1}\Gamma(p+3,-2W(ax))}{ac} + \frac{2^{-p-2}xe^{-W(ax)}(-W(ax))^{-p-1}(cW(ax))^{p}\Gamma(p+2,-2W(ax))}{a}$$

input
```
Int[x*(c*ProductLog[a*x])^p,x]
```

output
```
(2^(-2 - p)*x*Gamma[2 + p, -2*ProductLog[a*x]]*(-ProductLog[a*x])^(-1 - p)
*(c*ProductLog[a*x])^p)/(a*E^ProductLog[a*x]) + (2^(-3 - p)*x*Gamma[3 + p,
 -2*ProductLog[a*x]]*(-ProductLog[a*x])^(-2 - p)*(c*ProductLog[a*x])^(1 +
p))/(a*c*E^ProductLog[a*x])
```

Defintions of rubi rules used

rule 7174
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.), x_Symbol] :> Int[x^m*(
(c*ProductLog[a*x])^p/(1 + ProductLog[a*x])), x] + Simp[1/c   Int[x^m*((c*P
roductLog[a*x])^(p + 1)/(1 + ProductLog[a*x])), x], x] /; FreeQ[{a, c, m},
x]
```

rule 7207
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.))/((d_) + (d_.)*Product
Log[(a_.)*(x_)]), x_Symbol] :> Simp[x^m*Gamma[m + p + 1, (-(m + 1))*Product
Log[a*x]]*((c*ProductLog[a*x])^p/(a*d*(m + 1)*E^(m*ProductLog[a*x])*((-(m +
 1))*ProductLog[a*x])^(m + p))), x] /; FreeQ[{a, c, d, m, p}, x] && NeQ[m,
-1]
```

Maple [F]

$$\int x(c\,\mathrm{LambertW}\,(xa))^p\,dx$$

input
```
int(x*(c*LambertW(x*a))^p,x)
```

output
```
int(x*(c*LambertW(x*a))^p,x)
```

Fricas [F]

$$\int x(cW(ax))^p\,dx = \int (c\,\mathrm{W}(ax))^p\,x\,dx$$

input
```
integrate(x*(c*lambert_w(a*x))^p,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x))^p*x, x)
```

Sympy [F]

$$\int x(cW(ax))^p\,dx = \int x(cW(ax))^p\;dx$$

input
```
integrate(x*(c*LambertW(a*x))**p,x)
```

output
```
Integral(x*(c*LambertW(a*x))**p, x)
```

Maxima [F]

$$\int x(cW(ax))^p\,dx = \int (c\,\mathrm{W}(ax))^p\,x\,dx$$

input
```
integrate(x*(c*lambert_w(a*x))^p,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x))^p*x, x)
```

Giac [F]

$$\int x(cW(ax))^p\,dx = \int (c\,\mathrm{W}(ax))^p\,x\,dx$$

input
```
integrate(x*(c*lambert_w(a*x))^p,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x))^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(cW(ax))^p\,dx = \int x\,(c\,\mathrm{LambertW}(a\,x))^p\,dx$$

input
```
int(x*(c*LambertW(a*x))^p,x)
```

output
```
int(x*(c*LambertW(a*x))^p, x)
```

Reduce [F]

$$\int x(cW(ax))^p\,dx = c^p\left(\int lambert\_w(ax)^p\,xdx\right)$$

input
```
int(x*(c*Lambert_W(a*x))^p,x)
```

output
```
c**p*int(lambert_w(a*x)**p*x,x)
```

3.88 $\int \frac{(cW(ax))^p}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{(cW(ax))^p}{x}\,dx = \frac{(cW(ax))^p}{p} + \frac{(cW(ax))^{1+p}}{c(1+p)}$$

output
```
(c*LambertW(a*x))^p/p+(c*LambertW(a*x))^(p+1)/c/(p+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{(cW(ax))^p}{x}\,dx = \frac{(cW(ax))^p(1+p+pW(ax))}{p(1+p)}$$

input
```
Integrate[(c*ProductLog[a*x])^p/x,x]
```

output
```
((c*ProductLog[a*x])^p*(1 + p + p*ProductLog[a*x]))/(p*(1 + p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cW(ax))^p}{x} \, dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{\int \frac{(cW(ax))^{p+1}}{x(W(ax)+1)}dx}{c} + \frac{(cW(ax))^p}{p}$$

$$\Big\downarrow \text{7200}$$

$$\frac{(cW(ax))^p}{p} + \frac{(cW(ax))^{p+1}}{c(p+1)}$$

input
```
Int[(c*ProductLog[a*x])^p/x,x]
```

output
```
(c*ProductLog[a*x])^p/p + (c*ProductLog[a*x])^(1 + p)/(c*(1 + p))
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\frac{e^{p\ln(c\,\mathrm{LambertW}(xa))}}{p}+\frac{\mathrm{LambertW}(xa)e^{p\ln(c\,\mathrm{LambertW}(xa))}}{p+1}$	36
default	$\frac{e^{p\ln(c\,\mathrm{LambertW}(xa))}}{p}+\frac{\mathrm{LambertW}(xa)e^{p\ln(c\,\mathrm{LambertW}(xa))}}{p+1}$	36

input
```
int((c*LambertW(x*a))^p/x,x,method=_RETURNVERBOSE)
```

output
```
1/p*exp(p*ln(c*LambertW(x*a)))+1/(p+1)*LambertW(x*a)*exp(p*ln(c*LambertW(x
*a)))
```

Fricas [F]

$$\int \frac{(cW(ax))^p}{x}\,dx=\int \frac{(c\,\mathrm{W}(ax))^p}{x}\,dx$$

input
```
integrate((c*lambert_w(a*x))^p/x,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x))^p/x, x)
```

Sympy [F]

$$\int \frac{(cW(ax))^p}{x}\,dx=\int \frac{(cW(ax))^p}{x}\,dx$$

input
```
integrate((c*LambertW(a*x))**p/x,x)
```

output
```
Integral((c*LambertW(a*x))**p/x, x)
```

Maxima [F]

$$\int \frac{(cW(ax))^p}{x}\,dx = \int \frac{(c\,\mathrm{W}(ax))^p}{x}\,dx$$

input
```
integrate((c*lambert_w(a*x))^p/x,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x))^p/x, x)
```

Giac [F]

$$\int \frac{(cW(ax))^p}{x}\,dx = \int \frac{(c\,\mathrm{W}(ax))^p}{x}\,dx$$

input
```
integrate((c*lambert_w(a*x))^p/x,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x))^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cW(ax))^p}{x}\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x))^p}{x}\,dx$$

input
```
int((c*LambertW(a*x))^p/x,x)
```

output
```
int((c*LambertW(a*x))^p/x, x)
```

Reduce [F]

$$\int \frac{(cW(ax))^p}{x} \, dx = c^p \left(\int \frac{lambert\_w(ax)^p}{x} dx \right)$$

input
```
int((c*Lambert_W(a*x))^p/x,x)
```

output
```
c**p*int(lambert_w(a*x)**p/x,x)
```

3.89 $\int \frac{(cW(ax))^p}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{(cW(ax))^p}{x^2}\,dx = -\frac{e^{2W(ax)}\Gamma(-1+p,W(ax))W(ax)^{2-p}(cW(ax))^p}{ax^2} - \frac{e^{2W(ax)}\Gamma(p,W(ax))W(ax)^{1-p}(cW(ax))^{1+p}}{acx^2}$$

output
```
-exp(2*LambertW(a*x))*GAMMA(-1+p,LambertW(a*x))*LambertW(a*x)^(2-p)*(c*Lam
bertW(a*x))^p/a/x^2-exp(2*LambertW(a*x))*GAMMA(p,LambertW(a*x))*LambertW(a
*x)^(1-p)*(c*LambertW(a*x))^(p+1)/a/c/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.56

$$\int \frac{(cW(ax))^p}{x^2}\,dx = -\frac{e^{2W(ax)}(\Gamma(-1+p,W(ax))+\Gamma(p,W(ax)))W(ax)^{2-p}(cW(ax))^p}{ax^2}$$

input
```
Integrate[(c*ProductLog[a*x])^p/x^2,x]
```

output
```
-((E^(2*ProductLog[a*x])*(Gamma[-1 + p, ProductLog[a*x]] + Gamma[p, Produc
tLog[a*x]])*ProductLog[a*x]^(2 - p)*(c*ProductLog[a*x])^p)/(a*x^2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7174, 7207}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cW(ax))^p}{x^2}dx$$

$$\Big\downarrow \text{7174}$$

$$\int \frac{(cW(ax))^p}{x^2(W(ax)+1)}dx+\frac{\int \frac{(cW(ax))^{p+1}}{x^2(W(ax)+1)}dx}{c}$$

$$\Big\downarrow \text{7207}$$

$$-\frac{e^{2W(ax)}W(ax)^{1-p}(cW(ax))^{p+1}\Gamma(p,W(ax))}{acx^2}-\frac{e^{2W(ax)}W(ax)^{2-p}(cW(ax))^p\Gamma(p-1,W(ax))}{ax^2}$$

input
```
Int[(c*ProductLog[a*x])^p/x^2,x]
```

output
```
-((E^(2*ProductLog[a*x])*Gamma[-1 + p, ProductLog[a*x]]*ProductLog[a*x]^(2
 - p)*(c*ProductLog[a*x])^p)/(a*x^2)) - (E^(2*ProductLog[a*x])*Gamma[p, Pr
oductLog[a*x]]*ProductLog[a*x]^(1 - p)*(c*ProductLog[a*x])^(1 + p))/(a*c*x
^2)
```

Defintions of rubi rules used

rule 7174
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.), x_Symbol] :> Int[x^m*(
(c*ProductLog[a*x])^p/(1 + ProductLog[a*x])), x] + Simp[1/c   Int[x^m*((c*P
roductLog[a*x])^(p + 1)/(1 + ProductLog[a*x])), x], x] /; FreeQ[{a, c, m},
x]
```

rule 7207
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.))/((d_) + (d_.)*Product
Log[(a_.)*(x_)]), x_Symbol] :> Simp[x^m*Gamma[m + p + 1, (-(m + 1))*Product
Log[a*x]]*((c*ProductLog[a*x])^p/(a*d*(m + 1)*E^(m*ProductLog[a*x])*((-(m +
 1))*ProductLog[a*x])^(m + p))), x] /; FreeQ[{a, c, d, m, p}, x] && NeQ[m,
-1]
```

Maple [F]

$$\int \frac{(c\,\mathrm{LambertW}\,(xa))^p}{x^2}dx$$

input
```
int((c*LambertW(x*a))^p/x^2,x)
```

output
```
int((c*LambertW(x*a))^p/x^2,x)
```

Fricas [F]

$$\int \frac{(cW(ax))^p}{x^2}\,dx = \int \frac{(c\,\mathrm{W}(ax))^p}{x^2}\,dx$$

input
```
integrate((c*lambert_w(a*x))^p/x^2,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x))^p/x^2, x)
```

Sympy [F]

$$\int \frac{(cW(ax))^p}{x^2} dx = \int \frac{(cW(ax))^p}{x^2} dx$$

input
```
integrate((c*LambertW(a*x))**p/x**2,x)
```

output
```
Integral((c*LambertW(a*x))**p/x**2, x)
```

Maxima [F]

$$\int \frac{(cW(ax))^p}{x^2} dx = \int \frac{(c\,\mathrm{W}(ax))^p}{x^2} dx$$

input
```
integrate((c*lambert_w(a*x))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x))^p/x^2, x)
```

Giac [F]

$$\int \frac{(cW(ax))^p}{x^2} dx = \int \frac{(c\,\mathrm{W}(ax))^p}{x^2} dx$$

input
```
integrate((c*lambert_w(a*x))^p/x^2,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x))^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cW(ax))^p}{x^2}\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x))^p}{x^2}\,dx$$

input
```
int((c*LambertW(a*x))^p/x^2,x)
```

output
```
int((c*LambertW(a*x))^p/x^2, x)
```

Reduce [F]

$$\int \frac{(cW(ax))^p}{x^2}\,dx = \frac{c^p\left(-lambert\_w(ax)^p + \left(\int \frac{lambert\_w(ax)^p}{e^{lambert\_w(ax)}lambert\_w(ax)^2x + e^{lambert\_w(ax)}lambert\_w(ax)x}dx\right)apx\right)}{x}$$

input
```
int((c*Lambert_W(a*x))^p/x^2,x)
```

output
```
(c**p*( - lambert_w(a*x)**p + int(lambert_w(a*x)**p/(e**lambert_w(a*x)*lam
bert_w(a*x)**2*x + e**lambert_w(a*x)*lambert_w(a*x)*x),x)*a*p*x))/x
```

3.90 $\int \frac{(cW(ax))^p}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 108

$$\int \frac{(cW(ax))^p}{x^3}\,dx = -\frac{2^{2-p}e^{3W(ax)}\Gamma(-2+p,2W(ax))W(ax)^{3-p}(cW(ax))^p}{ax^3} - \frac{2^{1-p}e^{3W(ax)}\Gamma(-1+p,2W(ax))W(ax)^{2-p}(cW(ax))^{1+p}}{acx^3}$$

output
```
-2^(2-p)*exp(3*LambertW(a*x))*GAMMA(-2+p,2*LambertW(a*x))*LambertW(a*x)^(3
-p)*(c*LambertW(a*x))^p/a/x^3-2^(1-p)*exp(3*LambertW(a*x))*GAMMA(-1+p,2*La
mbertW(a*x))*LambertW(a*x)^(2-p)*(c*LambertW(a*x))^(p+1)/a/c/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int \frac{(cW(ax))^p}{x^3}\,dx = -\frac{2^{1-p}e^{3W(ax)}(2\Gamma(-2+p,2W(ax))+\Gamma(-1+p,2W(ax)))W(ax)^{3-p}(cW(ax))^p}{ax^3}$$

input
```
Integrate[(c*ProductLog[a*x])^p/x^3,x]
```

output
```
-((2^(1 - p)*E^(3*ProductLog[a*x])*(2*Gamma[-2 + p, 2*ProductLog[a*x]] + G
amma[-1 + p, 2*ProductLog[a*x]])*ProductLog[a*x]^(3 - p)*(c*ProductLog[a*x
])^p)/(a*x^3))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7174, 7207}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cW(ax))^p}{x^3}dx$$

$$\Big\downarrow \text{ 7174}$$

$$\int \frac{(cW(ax))^p}{x^3(W(ax)+1)}dx+\frac{\int \frac{(cW(ax))^{p+1}}{x^3(W(ax)+1)}dx}{c}$$

$$\Big\downarrow \text{ 7207}$$

$$-\frac{2^{1-p}e^{3W(ax)}W(ax)^{2-p}(cW(ax))^{p+1}\Gamma(p-1,2W(ax))}{acx^3}-\frac{2^{2-p}e^{3W(ax)}W(ax)^{3-p}(cW(ax))^p\Gamma(p-2,2W(ax))}{ax^3}$$

input
```
Int[(c*ProductLog[a*x])^p/x^3,x]
```

output
```
-((2^(2 - p)*E^(3*ProductLog[a*x])*Gamma[-2 + p, 2*ProductLog[a*x]]*Produc
tLog[a*x]^(3 - p)*(c*ProductLog[a*x])^p)/(a*x^3)) - (2^(1 - p)*E^(3*Produc
tLog[a*x])*Gamma[-1 + p, 2*ProductLog[a*x]]*ProductLog[a*x]^(2 - p)*(c*Pro
ductLog[a*x])^(1 + p))/(a*c*x^3)
```

Defintions of rubi rules used

rule 7174
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.), x_Symbol] :> Int[x^m*(
(c*ProductLog[a*x])^p/(1 + ProductLog[a*x])), x] + Simp[1/c   Int[x^m*((c*P
roductLog[a*x])^(p + 1)/(1 + ProductLog[a*x])), x], x] /; FreeQ[{a, c, m},
x]
```

rule 7207
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.))/((d_) + (d_.)*Product
Log[(a_.)*(x_)]), x_Symbol] :> Simp[x^m*Gamma[m + p + 1, (-(m + 1))*Product
Log[a*x]]*((c*ProductLog[a*x])^p/(a*d*(m + 1)*E^(m*ProductLog[a*x])*((-(m +
 1))*ProductLog[a*x])^(m + p))), x] /; FreeQ[{a, c, d, m, p}, x] && NeQ[m,
-1]
```

Maple [F]

$$\int \frac{(c\,\mathrm{LambertW}\,(xa))^{p}}{x^{3}}dx$$

input
```
int((c*LambertW(x*a))^p/x^3,x)
```

output
```
int((c*LambertW(x*a))^p/x^3,x)
```

Fricas [F]

$$\int \frac{(cW(ax))^p}{x^3}\,dx = \int \frac{(c\,\mathrm{W}(ax))^{p}}{x^{3}}\,dx$$

input
```
integrate((c*lambert_w(a*x))^p/x^3,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x))^p/x^3, x)
```

Sympy [F]

$$\int \frac{(cW(ax))^p}{x^3}\,dx = \int \frac{(cW(ax))^p}{x^3}\,dx$$

input
```
integrate((c*LambertW(a*x))**p/x**3,x)
```

output
```
Integral((c*LambertW(a*x))**p/x**3, x)
```

Maxima [F]

$$\int \frac{(cW(ax))^p}{x^3}\,dx = \int \frac{(c\,\mathrm{W}(ax))^p}{x^3}\,dx$$

input
```
integrate((c*lambert_w(a*x))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x))^p/x^3, x)
```

Giac [F]

$$\int \frac{(cW(ax))^p}{x^3}\,dx = \int \frac{(c\,\mathrm{W}(ax))^p}{x^3}\,dx$$

input
```
integrate((c*lambert_w(a*x))^p/x^3,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x))^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cW(ax))^p}{x^3}\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x))^p}{x^3}\,dx$$

input
```
int((c*LambertW(a*x))^p/x^3,x)
```

output
```
int((c*LambertW(a*x))^p/x^3, x)
```

Reduce [F]

$$\int \frac{(cW(ax))^p}{x^3}\,dx = \frac{c^p\left(-lambert\_w(ax)^p + \left(\int \frac{lambert\_w(ax)^p}{e^{lambert\_w(ax)}lambert\_w(ax)^2x^2+e^{lambert\_w(ax)}lambert\_w(ax)x^2}dx\right)ap\,x^2\right)}{2x^2}$$

input
```
int((c*Lambert_W(a*x))^p/x^3,x)
```

output
```
(c**p*( - lambert_w(a*x)**p + int(lambert_w(a*x)**p/(e**lambert_w(a*x)*lam
bert_w(a*x)**2*x**2 + e**lambert_w(a*x)*lambert_w(a*x)*x**2),x)*a*p*x**2))
/(2*x**2)
```

3.91 $\int x^m (cW(ax))^p \, dx$

Optimal result

Integrand size = 12, antiderivative size = 131

$$\int x^m (cW(ax))^p \, dx = \frac{e^{-mW(ax)} x^m \Gamma(2+m+p,(-1-m)W(ax))(cW(ax))^{1+p}((-1-m)W(ax))^{-1-m-p}}{ac(1+m)} + \frac{e^{-mW(ax)} x^m \Gamma(1+m+p,(-1-m)W(ax))(cW(ax))^p((-1-m)W(ax))^{-m-p}}{a(1+m)}$$

output
```
x^m*GAMMA(2+m+p,(-1-m)*LambertW(a*x))*(c*LambertW(a*x))^(p+1)*((-1-m)*Lamb
ertW(a*x))^(-1-m-p)/a/c/exp(m*LambertW(a*x))/(1+m)+x^m*GAMMA(1+m+p,(-1-m)*
LambertW(a*x))*(c*LambertW(a*x))^p*((-1-m)*LambertW(a*x))^(-m-p)/a/exp(m*L
ambertW(a*x))/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.62

$$\int x^m (cW(ax))^p \, dx = \frac{e^{-mW(ax)} x^m ((1+m)\Gamma(1+m+p,-((1+m)W(ax))) - \Gamma(2+m+p,-((1+m)W(ax))))(cW(ax))^p(-$$

$$a(1+m)^2$$

input
```
Integrate[x^m*(c*ProductLog[a*x])^p,x]
```

output
```
(x^m*((1 + m)*Gamma[1 + m + p, -((1 + m)*ProductLog[a*x])] - Gamma[2 + m +
 p, -((1 + m)*ProductLog[a*x])])*(c*ProductLog[a*x])^p*(-((1 + m)*ProductL
og[a*x]))^(-m - p))/(a*E^(m*ProductLog[a*x])*(1 + m)^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7174, 7207}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (cW(ax))^p \, dx$$

$$\Big\downarrow \text{7174}$$

$$\int \frac{x^m (cW(ax))^p}{W(ax)+1} dx + \frac{\int \frac{x^m (cW(ax))^{p+1}}{W(ax)+1} dx}{c}$$

$$\Big\downarrow \text{7207}$$

$$\frac{x^m e^{-mW(ax)} (cW(ax))^{p+1} (-((m+1)W(ax)))^{-m-p-1} \Gamma(m+p+2, -((m+1)W(ax)))}{ac(m+1)} + \frac{x^m e^{-mW(ax)} (cW(ax))^p (-((m+1)W(ax)))^{-m-p} \Gamma(m+p+1, -((m+1)W(ax)))}{a(m+1)}$$

input
```
Int[x^m*(c*ProductLog[a*x])^p,x]
```

output
```
(x^m*Gamma[2 + m + p, -((1 + m)*ProductLog[a*x])]*(c*ProductLog[a*x])^(1 +
 p)*(-((1 + m)*ProductLog[a*x]))^(-1 - m - p))/(a*c*E^(m*ProductLog[a*x])*
(1 + m)) + (x^m*Gamma[1 + m + p, -((1 + m)*ProductLog[a*x])]*(c*ProductLog
[a*x])^p*(-((1 + m)*ProductLog[a*x]))^(-m - p))/(a*E^(m*ProductLog[a*x])*(
1 + m))
```

Defintions of rubi rules used

rule 7174
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.), x_Symbol] :> Int[x^m*(
(c*ProductLog[a*x])^p/(1 + ProductLog[a*x])), x] + Simp[1/c   Int[x^m*((c*P
roductLog[a*x])^(p + 1)/(1 + ProductLog[a*x])), x], x] /; FreeQ[{a, c, m},
x]
```

rule 7207
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.))/((d_) + (d_.)*Product
Log[(a_.)*(x_)]), x_Symbol] :> Simp[x^m*Gamma[m + p + 1, (-(m + 1))*Product
Log[a*x]]*((c*ProductLog[a*x])^p/(a*d*(m + 1)*E^(m*ProductLog[a*x])*((-(m +
 1))*ProductLog[a*x])^(m + p))), x] /; FreeQ[{a, c, d, m, p}, x] && NeQ[m,
-1]
```

Maple [F]

$$\int x^m(c\,\mathrm{LambertW}\,(xa))^p\,dx$$

input
```
int(x^m*(c*LambertW(x*a))^p,x)
```

output
```
int(x^m*(c*LambertW(x*a))^p,x)
```

Fricas [F]

$$\int x^m(cW(ax))^p\,dx = \int (c\,\mathrm{W}(ax))^p\,x^m\,dx$$

input
```
integrate(x^m*(c*lambert_w(a*x))^p,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x))^p*x^m, x)
```

Sympy [F]

$$\int x^m (cW(ax))^p \, dx = \int x^m (cW(ax))^p \, dx$$

input
```
integrate(x**m*(c*LambertW(a*x))**p,x)
```

output
```
Integral(x**m*(c*LambertW(a*x))**p, x)
```

Maxima [F]

$$\int x^m (cW(ax))^p \, dx = \int (c \, \mathrm{W}(ax))^p x^m \, dx$$

input
```
integrate(x^m*(c*lambert_w(a*x))^p,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x))^p*x^m, x)
```

Giac [F]

$$\int x^m (cW(ax))^p \, dx = \int (c \, \mathrm{W}(ax))^p x^m \, dx$$

input
```
integrate(x^m*(c*lambert_w(a*x))^p,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x))^p*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m(cW(ax))^p\,dx = \int x^m\,(c\,\mathrm{LambertW}(a\,x))^p\,dx$$

input
```
int(x^m*(c*LambertW(a*x))^p,x)
```

output
```
int(x^m*(c*LambertW(a*x))^p, x)
```

Reduce [F]

$$\int x^m(cW(ax))^p\,dx = c^p\left(\int x^m lambert\_w(ax)^p\,dx\right)$$

input
```
int(x^m*(c*Lambert_W(a*x))^p,x)
```

output
```
c**p*int(x**m*lambert_w(a*x)**p,x)
```

3.92 $\int x^5 W(ax^2)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 66

$$\int x^5 W(ax^2)\, dx = -\frac{x^6}{18} + \frac{x^6}{81W(ax^2)^3} - \frac{x^6}{27W(ax^2)^2} + \frac{x^6}{18W(ax^2)} + \frac{1}{6}x^6 W(ax^2)$$

output
```
-1/18*x^6+1/81*x^6/LambertW(a*x^2)^3-1/27*x^6/LambertW(a*x^2)^2+1/18*x^6/L
ambertW(a*x^2)+1/6*x^6*LambertW(a*x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int x^5 W(ax^2)\, dx = -\frac{x^6}{18} + \frac{x^6}{81W(ax^2)^3} - \frac{x^6}{27W(ax^2)^2} + \frac{x^6}{18W(ax^2)} + \frac{1}{6}x^6 W(ax^2)$$

input
```
Integrate[x^5*ProductLog[a*x^2],x]
```

output
```
-1/18*x^6 + x^6/(81*ProductLog[a*x^2]^3) - x^6/(27*ProductLog[a*x^2]^2) +
x^6/(18*ProductLog[a*x^2]) + (x^6*ProductLog[a*x^2])/6
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {7172, 7205, 7283, 7194, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 W\left(ax^2\right) \, dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{6}x^6 W\left(ax^2\right)-\frac{1}{3}\int \frac{x^5 W\left(ax^2\right)}{W\left(ax^2\right)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{3}\left(\int \frac{x^5}{W\left(ax^2\right)+1}dx-\frac{x^6}{6}\right)+\frac{1}{6}x^6 W\left(ax^2\right)$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{x^4}{W\left(ax^2\right)+1}dx^2-\frac{x^6}{6}\right)+\frac{1}{6}x^6 W\left(ax^2\right)$$

$$\Big\downarrow \text{7194}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{x^6}{3W\left(ax^2\right)}-\frac{2}{3}\int \frac{x^4}{W\left(ax^2\right)\left(W\left(ax^2\right)+1\right)}dx^2\right)-\frac{x^6}{6}\right)+\frac{1}{6}x^6 W\left(ax^2\right)$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{x^6}{3W\left(ax^2\right)}-\frac{2}{3}\left(\frac{x^6}{3W\left(ax^2\right)^2}-\frac{1}{3}\int \frac{x^4}{W\left(ax^2\right)^2\left(W\left(ax^2\right)+1\right)}dx^2\right)\right)-\frac{x^6}{6}\right)+\frac{1}{6}x^6 W\left(ax^2\right)$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{6}x^6 W\left(ax^2\right)+\frac{1}{3}\left(\frac{1}{2}\left(\frac{x^6}{3W\left(ax^2\right)}-\frac{2}{3}\left(\frac{x^6}{3W\left(ax^2\right)^2}-\frac{x^6}{9W\left(ax^2\right)^3}\right)\right)-\frac{x^6}{6}\right)$$

input
```
Int[x^5*ProductLog[a*x^2],x]
```

output
```
(-1/6*x^6 + ((-2*(-1/9*x^6/ProductLog[a*x^2]^3 + x^6/(3*ProductLog[a*x^2]^
2)))/3 + x^6/(3*ProductLog[a*x^2]))/2)/3 + (x^6*ProductLog[a*x^2])/6
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int x^5 \operatorname{LambertW}\left(a\,x^2\right) dx$$

input
```
int(x^5*LambertW(a*x^2),x)
```

output
```
int(x^5*LambertW(a*x^2),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int x^5 W\left(ax^2\right)\,dx = \frac{27\,x^6\,\mathrm{W}(ax^2)^4 - 9\,x^6\,\mathrm{W}(ax^2)^3 + 9\,x^6\,\mathrm{W}(ax^2)^2 - 6\,x^6\,\mathrm{W}(ax^2) + 2\,x^6}{162\,\mathrm{W}(ax^2)^3}$$

input
```
integrate(x^5*lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
1/162*(27*x^6*lambert_w(a*x^2)^4 - 9*x^6*lambert_w(a*x^2)^3 + 9*x^6*lamber
t_w(a*x^2)^2 - 6*x^6*lambert_w(a*x^2) + 2*x^6)/lambert_w(a*x^2)^3
```

Sympy [F]

$$\int x^5 W\left(ax^2\right)\,dx = \int x^5 W\left(ax^2\right)\,dx$$

input
```
integrate(x**5*LambertW(a*x**2),x)
```

output
```
Integral(x**5*LambertW(a*x**2), x)
```

Maxima [F]

$$\int x^5W(ax^2)\ dx = \int x^5\,\mathrm{W}(ax^2)\,dx$$

input
```
integrate(x^5*lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(x^5*lambert_w(a*x^2), x)
```

Giac [F]

$$\int x^5W(ax^2)\ dx = \int x^5\,\mathrm{W}(ax^2)\,dx$$

input
```
integrate(x^5*lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(x^5*lambert_w(a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5W(ax^2)\ dx = \int x^5\,\mathrm{LambertW}\left(a\,x^2\right)\,dx$$

input
```
int(x^5*LambertW(a*x^2),x)
```

output
```
int(x^5*LambertW(a*x^2), x)
```

Reduce [F]

$$\int x^5 W\left(ax^2\right)\,dx = \int lambert\_w\left(a\,x^2\right)x^5 dx$$

input
```
int(x^5*Lambert_W(a*x^2),x)
```

output
```
int(lambert_w(a*x**2)*x**5,x)
```

3.93 $\int x^3 W(ax^2)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 51

$$\int x^3 W(ax^2)\,dx = -\frac{x^4}{8} - \frac{x^4}{16W(ax^2)^2} + \frac{x^4}{8W(ax^2)} + \frac{1}{4}x^4 W(ax^2)$$

output
```
-1/8*x^4-1/16*x^4/LambertW(a*x^2)^2+1/8*x^4/LambertW(a*x^2)+1/4*x^4*Lamber
tW(a*x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^3 W(ax^2)\,dx = -\frac{x^4}{8} - \frac{x^4}{16W(ax^2)^2} + \frac{x^4}{8W(ax^2)} + \frac{1}{4}x^4 W(ax^2)$$

input
```
Integrate[x^3*ProductLog[a*x^2],x]
```

output
```
-1/8*x^4 - x^4/(16*ProductLog[a*x^2]^2) + x^4/(8*ProductLog[a*x^2]) + (x^4
*ProductLog[a*x^2])/4
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7172, 7205, 7283, 7194, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 W\left(ax^2\right) dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{4}x^4 W\left(ax^2\right)-\frac{1}{2}\int \frac{x^3 W\left(ax^2\right)}{W\left(ax^2\right)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{2}\left(\int \frac{x^3}{W\left(ax^2\right)+1}dx-\frac{x^4}{4}\right)+\frac{1}{4}x^4 W\left(ax^2\right)$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{x^2}{W\left(ax^2\right)+1}dx^2-\frac{x^4}{4}\right)+\frac{1}{4}x^4 W\left(ax^2\right)$$

$$\Big\downarrow \text{7194}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{x^4}{2W\left(ax^2\right)}-\frac{1}{2}\int \frac{x^2}{W\left(ax^2\right)\left(W\left(ax^2\right)+1\right)}dx^2\right)-\frac{x^4}{4}\right)+\frac{1}{4}x^4 W\left(ax^2\right)$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{4}x^4 W\left(ax^2\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{x^4}{2W\left(ax^2\right)}-\frac{x^4}{4W\left(ax^2\right)^2}\right)-\frac{x^4}{4}\right)$$

input
```
Int[x^3*ProductLog[a*x^2],x]
```

output
```
(-1/4*x^4 + (-1/4*x^4/ProductLog[a*x^2]^2 + x^4/(2*ProductLog[a*x^2]))/2)/
2 + (x^4*ProductLog[a*x^2])/4
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int x^3 \operatorname{LambertW}\left(a\,x^2\right) dx$$

input
```
int(x^3*LambertW(a*x^2),x)
```

output
```
int(x^3*LambertW(a*x^2),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int x^3 W\left(ax^2\right)\,dx = \frac{4\,x^4\,\mathrm{W}(ax^2)^3 - 2\,x^4\,\mathrm{W}(ax^2)^2 + 2\,x^4\,\mathrm{W}(ax^2) - x^4}{16\,\mathrm{W}(ax^2)^2}$$

input
```
integrate(x^3*lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
1/16*(4*x^4*lambert_w(a*x^2)^3 - 2*x^4*lambert_w(a*x^2)^2 + 2*x^4*lambert_
w(a*x^2) - x^4)/lambert_w(a*x^2)^2
```

Sympy [F]

$$\int x^3 W\left(ax^2\right)\,dx = \int x^3 W\left(ax^2\right)\,dx$$

input
```
integrate(x**3*LambertW(a*x**2),x)
```

output
```
Integral(x**3*LambertW(a*x**2), x)
```

Maxima [F]

$$\int x^3 W(ax^2)\ dx = \int x^3\,\mathrm{W}(ax^2)\,dx$$

input
```
integrate(x^3*lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(x^3*lambert_w(a*x^2), x)
```

Giac [F]

$$\int x^3 W(ax^2)\ dx = \int x^3\,\mathrm{W}(ax^2)\,dx$$

input
```
integrate(x^3*lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(x^3*lambert_w(a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 W(ax^2)\ dx = \int x^3\,\mathrm{LambertW}\left(a\,x^2\right)\ dx$$

input
```
int(x^3*LambertW(a*x^2),x)
```

output
```
int(x^3*LambertW(a*x^2), x)
```

Reduce [F]

$$\int x^3 W\left(ax^2\right)\, dx = \int lambert\_w\left(a\,x^2\right) x^3 dx$$

input
```
int(x^3*Lambert_W(a*x^2),x)
```

output
```
int(lambert_w(a*x**2)*x**3,x)
```

3.94 $\int xW(ax^2)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 36

$$\int xW(ax^2)\,dx = -\frac{x^2}{2} + \frac{x^2}{2W(ax^2)} + \frac{1}{2}x^2W(ax^2)$$

output
```
-1/2*x^2+1/2*x^2/LambertW(a*x^2)+1/2*x^2*LambertW(a*x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int xW(ax^2)\,dx = \frac{x^2\left(1 - W(ax^2) + W(ax^2)^2\right)}{2W(ax^2)}$$

input
```
Integrate[x*ProductLog[a*x^2],x]
```

output
```
(x^2*(1 - ProductLog[a*x^2] + ProductLog[a*x^2]^2))/(2*ProductLog[a*x^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {7172, 7205, 7266, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xW(ax^2)\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{2}x^2W(ax^2) - \int \frac{xW(ax^2)}{W(ax^2)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\int \frac{x}{W(ax^2)+1}dx + \frac{1}{2}x^2W(ax^2) - \frac{x^2}{2}$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}\int \frac{1}{W(ax^2)+1}dx^2 + \frac{1}{2}x^2W(ax^2) - \frac{x^2}{2}$$

$$\Big\downarrow \text{7176}$$

$$\frac{1}{2}x^2W(ax^2) + \frac{x^2}{2W(ax^2)} - \frac{x^2}{2}$$

input
```
Int[x*ProductLog[a*x^2],x]
```

output
```
-1/2*x^2 + x^2/(2*ProductLog[a*x^2]) + (x^2*ProductLog[a*x^2])/2
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{-a\,x^{2}+\frac{a\,x^{2}}{\operatorname{LambertW}\left(a\,x^{2}\right)}+a\,x^{2} \operatorname{LambertW}\left(a\,x^{2}\right)}{2a}$	37
default	$\frac{-a\,x^{2}+\frac{a\,x^{2}}{\operatorname{LambertW}\left(a\,x^{2}\right)}+a\,x^{2} \operatorname{LambertW}\left(a\,x^{2}\right)}{2a}$	37
parallelrisch	$-\frac{-x^{2} \operatorname{LambertW}\left(a\,x^{2}\right)^{2}+x^{2} \operatorname{LambertW}\left(a\,x^{2}\right)-x^{2}}{2 \operatorname{LambertW}\left(a\,x^{2}\right)}$	40

input
```
int(x*LambertW(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(-a*x^2+a*x^2/LambertW(a*x^2)+a*x^2*LambertW(a*x^2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int xW\left(ax^2\right)\,dx = \frac{x^2\,\mathrm{W}(ax^2)^2 - x^2\,\mathrm{W}(ax^2) + x^2}{2\,\mathrm{W}(ax^2)}$$

input
```
integrate(x*lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
1/2*(x^2*lambert_w(a*x^2)^2 - x^2*lambert_w(a*x^2) + x^2)/lambert_w(a*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int xW\left(ax^2\right)\,dx = \begin{cases} \frac{x^2W\left(ax^2\right)}{2} - \frac{x^2}{2} + \frac{x^2}{2W\left(ax^2\right)} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*LambertW(a*x**2),x)
```

output
```
Piecewise((x**2*LambertW(a*x**2)/2 - x**2/2 + x**2/(2*LambertW(a*x**2)), N
e(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int xW\left(ax^2\right)\,dx = \frac{(\mathrm{W}(ax^2)^2 - \mathrm{W}(ax^2) + 1)x^2}{2\,\mathrm{W}(ax^2)}$$

input
```
integrate(x*lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
1/2*(lambert_w(a*x^2)^2 - lambert_w(a*x^2) + 1)*x^2/lambert_w(a*x^2)
```

Giac [F]

$$\int xW\left(ax^2\right)\,dx = \int x\,\mathrm{W}\left(ax^2\right)dx$$

input
```
integrate(x*lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(x*lambert_w(a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int xW\left(ax^2\right)\,dx = \int x\,\mathrm{LambertW}\left(a\,x^2\right)\,dx$$

input
```
int(x*LambertW(a*x^2),x)
```

output
```
int(x*LambertW(a*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int xW(ax^2)\,dx = \frac{e^{lambert\_w(a\,x^2)}\left(lambert\_w(a\,x^2)^2 - lambert\_w(a\,x^2) + 1\right)}{2a}$$

input

```
int(x*Lambert_W(a*x^2),x)
```

output

```
(e**lambert_w(a*x**2)*(lambert_w(a*x**2)**2 - lambert_w(a*x**2) + 1))/(2*a
)
```

3.95 $\int \frac{W(ax^2)}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 23

$$\int \frac{W(ax^2)}{x} dx = \frac{1}{2}W(ax^2) + \frac{1}{4}W(ax^2)^2$$

output
```
1/2*LambertW(a*x^2)+1/4*LambertW(a*x^2)^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{W(ax^2)}{x} dx = \frac{1}{2}W(ax^2) + \frac{1}{4}W(ax^2)^2$$

input
```
Integrate[ProductLog[a*x^2]/x,x]
```

output
```
ProductLog[a*x^2]/2 + ProductLog[a*x^2]^2/4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)}{x}\,dx$$

$$\Big\downarrow \text{7173}$$

$$\int \frac{W(ax^2)^2}{x\left(W(ax^2)+1\right)}dx+\frac{1}{2}W(ax^2)$$

$$\Big\downarrow \text{7200}$$

$$\frac{1}{4}W(ax^2)^2+\frac{1}{2}W(ax^2)$$

input
```
Int[ProductLog[a*x^2]/x,x]
```

output
```
ProductLog[a*x^2]/2 + ProductLog[a*x^2]^2/4
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\operatorname{LambertW}(a\,x^2)}{2}+\frac{\operatorname{LambertW}(a\,x^2)^2}{4}$	20
default	$\frac{\operatorname{LambertW}(a\,x^2)}{2}+\frac{\operatorname{LambertW}(a\,x^2)^2}{4}$	20

input
```
int(LambertW(a*x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*LambertW(a*x^2)+1/4*LambertW(a*x^2)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{W(ax^2)}{x}\,dx = \frac{1}{4}\,\mathrm{W}(ax^2)^2 + \log(x) - \frac{1}{2}\,\log\left(-\,\mathrm{W}(ax^2)\right)$$

input
```
integrate(lambert_w(a*x^2)/x,x, algorithm="fricas")
```

output
```
1/4*lambert_w(a*x^2)^2 + log(x) - 1/2*log(-lambert_w(a*x^2))
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{W(ax^2)}{x}\,dx = \frac{W^2(ax^2)}{4} + \frac{W(ax^2)}{2}$$

input
```
integrate(LambertW(a*x**2)/x,x)
```

output
```
LambertW(a*x**2)**2/4 + LambertW(a*x**2)/2
```

Maxima [F]

$$\int \frac{W(ax^2)}{x}\,dx = \int \frac{\mathrm{W}(ax^2)}{x}\,dx$$

input
```
integrate(lambert_w(a*x^2)/x,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)/x, x)
```

Giac [F]

$$\int \frac{W(ax^2)}{x}\,dx = \int \frac{\mathrm{W}(ax^2)}{x}\,dx$$

input
```
integrate(lambert_w(a*x^2)/x,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)}{x}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)}{x}\,dx$$

input
```
int(LambertW(a*x^2)/x,x)
```

output
```
int(LambertW(a*x^2)/x, x)
```

Reduce [F]

$$\int \frac{W(ax^2)}{x}\,dx = \int \frac{lambert\_w(a\,x^2)}{x}dx - 2\left(\int \frac{1}{x}dx\right) + 2\log(x)$$

input
```
int(Lambert_W(a*x^2)/x,x)
```

output
```
int(lambert_w(a*x**2)/x,x) - 2*int(1/x,x) + 2*log(x)
```

3.96 $\int \frac{W(ax^2)}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 28

$$\int \frac{W(ax^2)}{x^3} dx = \frac{1}{2}a \,\text{ExpIntegralEi}\left(-W\left(ax^2\right)\right) - \frac{W(ax^2)}{2x^2}$$

output
```
1/2*a*Ei(-LambertW(a*x^2))-1/2*LambertW(a*x^2)/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{W(ax^2)}{x^3} dx = \frac{1}{2}a \,\text{ExpIntegralEi}\left(-W\left(ax^2\right)\right) - \frac{W(ax^2)}{2x^2}$$

input
```
Integrate[ProductLog[a*x^2]/x^3,x]
```

output
```
(a*ExpIntegralEi[-ProductLog[a*x^2]])/2 - ProductLog[a*x^2]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7172, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)}{x^3} \, dx$$
$$\Big\downarrow \text{7172}$$
$$\int \frac{W(ax^2)}{x^3 (W(ax^2)+1)} dx - \frac{W(ax^2)}{2x^2}$$
$$\Big\downarrow \text{7202}$$
$$\frac{1}{2} a \operatorname{ExpIntegralEi}\left(-W(ax^2)\right) - \frac{W(ax^2)}{2x^2}$$

input
```
Int[ProductLog[a*x^2]/x^3,x]
```

output
```
(a*ExpIntegralEi[-ProductLog[a*x^2]])/2 - ProductLog[a*x^2]/(2*x^2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [F]

$$\int \frac{\text{LambertW}\left(a\,x^{2}\right)}{x^{3}}dx$$

input
```
int(LambertW(a*x^2)/x^3,x)
```

output
```
int(LambertW(a*x^2)/x^3,x)
```

Fricas [F]

$$\int \frac{W(ax^2)}{x^3}\,dx = \int \frac{\mathrm{W}(ax^2)}{x^3}\,dx$$

input
```
integrate(lambert_w(a*x^2)/x^3,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)/x^3, x)
```

Sympy [F]

$$\int \frac{W(ax^2)}{x^3}\,dx = \int \frac{W(ax^2)}{x^3}\,dx$$

input
```
integrate(LambertW(a*x**2)/x**3,x)
```

output
```
Integral(LambertW(a*x**2)/x**3, x)
```

Maxima [F]

$$\int \frac{W(ax^2)}{x^3}\,dx = \int \frac{\mathrm{W}(ax^2)}{x^3}\,dx$$

input
```
integrate(lambert_w(a*x^2)/x^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)/x^3, x)
```

Giac [F]

$$\int \frac{W(ax^2)}{x^3}\,dx = \int \frac{\mathrm{W}(ax^2)}{x^3}\,dx$$

input
```
integrate(lambert_w(a*x^2)/x^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)}{x^3}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)}{x^3}\,dx$$

input
```
int(LambertW(a*x^2)/x^3,x)
```

output
```
int(LambertW(a*x^2)/x^3, x)
```

Reduce [F]

$$\int \frac{W(ax^2)}{x^3}\,dx$$
$$= \frac{2\left(\int \frac{1}{e^{lambert\_w\left(a\,x^2\right)}lambert\_w(a\,x^2)x+e^{lambert\_w\left(a\,x^2\right)}x}dx\right)a\,x^2 - lambert\_w(a\,x^2)}{2x^2}$$

input
```
int(Lambert_W(a*x^2)/x^3,x)
```

output
```
(2*int(1/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x + e**lambert_w(a*x**2)*
x),x)*a*x**2 - lambert_w(a*x**2))/(2*x**2)
```

3.97 $\int \frac{W(ax^2)}{x^5} dx$

Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{W(ax^2)}{x^5} dx = -\frac{1}{2}a^2 \text{ExpIntegralEi}\left(-2W\left(ax^2\right)\right) - \frac{W(ax^2)}{2x^4}$$

output
```
-1/2*a^2*Ei(-2*LambertW(a*x^2))-1/2*LambertW(a*x^2)/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{W(ax^2)}{x^5} dx = -\frac{1}{2}a^2 \text{ExpIntegralEi}\left(-2W\left(ax^2\right)\right) - \frac{W(ax^2)}{2x^4}$$

input
```
Integrate[ProductLog[a*x^2]/x^5,x]
```

output
```
-1/2*(a^2*ExpIntegralEi[-2*ProductLog[a*x^2]]) - ProductLog[a*x^2]/(2*x^4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7173, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(ax^2\right)}{x^5} \, dx$$

$$\Big\downarrow \text{ 7173}$$

$$-\int \frac{W\left(ax^2\right)^2}{x^5\left(W\left(ax^2\right)+1\right)}dx-\frac{W\left(ax^2\right)}{2x^4}$$

$$\Big\downarrow \text{ 7202}$$

$$-\frac{1}{2}a^2\,\text{ExpIntegralEi}\left(-2W\left(ax^2\right)\right)-\frac{W\left(ax^2\right)}{2x^4}$$

input
```
Int[ProductLog[a*x^2]/x^5,x]
```

output
```
-1/2*(a^2*ExpIntegralEi[-2*ProductLog[a*x^2]]) - ProductLog[a*x^2]/(2*x^4)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [F]

$$\int \frac{\text{LambertW}\left(a\,x^2\right)}{x^5}dx$$

input
```
int(LambertW(a*x^2)/x^5,x)
```

output
```
int(LambertW(a*x^2)/x^5,x)
```

Fricas [F]

$$\int \frac{W(ax^2)}{x^5}\,dx = \int \frac{\mathrm{W}(ax^2)}{x^5}\,dx$$

input
```
integrate(lambert_w(a*x^2)/x^5,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)/x^5, x)
```

Sympy [F]

$$\int \frac{W(ax^2)}{x^5}\,dx = \int \frac{W(ax^2)}{x^5}\,dx$$

input
```
integrate(LambertW(a*x**2)/x**5,x)
```

output
```
Integral(LambertW(a*x**2)/x**5, x)
```

Maxima [F]

$$\int \frac{W(ax^2)}{x^5}\,dx = \int \frac{\mathrm{W}(ax^2)}{x^5}\,dx$$

input
```
integrate(lambert_w(a*x^2)/x^5,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)/x^5, x)
```

Giac [F]

$$\int \frac{W(ax^2)}{x^5}\,dx = \int \frac{\mathrm{W}(ax^2)}{x^5}\,dx$$

input
```
integrate(lambert_w(a*x^2)/x^5,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)}{x^5}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)}{x^5}\,dx$$

input
```
int(LambertW(a*x^2)/x^5,x)
```

output
```
int(LambertW(a*x^2)/x^5, x)
```

Reduce [F]

$$\int \frac{W(ax^2)}{x^5} \, dx$$
$$= \frac{2\left(\int \frac{1}{e^{lambert\_w(a\,x^2)}lambert\_w(a\,x^2)x^3+e^{lambert\_w(a\,x^2)}x^3}dx\right)a\,x^4 - lambert\_w(a\,x^2)}{4x^4}$$

input
```
int(Lambert_W(a*x^2)/x^5,x)
```

output
```
(2*int(1/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x**3 + e**lambert_w(a*x**
2)*x**3),x)*a*x**4 - lambert_w(a*x**2))/(4*x**4)
```

3.98 $\int \frac{W(ax^2)}{x^7} dx$

Optimal result

Integrand size = 10, antiderivative size = 45

$$\int \frac{W(ax^2)}{x^7} dx = \frac{3}{4}a^3 \operatorname{ExpIntegralEi}\left(-3W(ax^2)\right) - \frac{W(ax^2)}{4x^6} + \frac{W(ax^2)^2}{4x^6}$$

output
```
3/4*a^3*Ei(-3*LambertW(a*x^2))-1/4*LambertW(a*x^2)/x^6+1/4*LambertW(a*x^2)
^2/x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{W(ax^2)}{x^7} dx = \frac{3}{4}a^3 \operatorname{ExpIntegralEi}\left(-3W(ax^2)\right) - \frac{W(ax^2)}{4x^6} + \frac{W(ax^2)^2}{4x^6}$$

input
```
Integrate[ProductLog[a*x^2]/x^7,x]
```

output
```
(3*a^3*ExpIntegralEi[-3*ProductLog[a*x^2]])/4 - ProductLog[a*x^2]/(4*x^6)
+ ProductLog[a*x^2]^2/(4*x^6)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7173, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)}{x^7}\,dx$$

$$\Big\downarrow \text{ 7173}$$

$$-\frac{1}{2}\int \frac{W(ax^2)^2}{x^7\left(W(ax^2)+1\right)}dx-\frac{W(ax^2)}{4x^6}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{1}{2}\left(3\int \frac{W(ax^2)^3}{x^7\left(W(ax^2)+1\right)}dx+\frac{W(ax^2)^2}{2x^6}\right)-\frac{W(ax^2)}{4x^6}$$

$$\Big\downarrow \text{ 7202}$$

$$\frac{1}{2}\left(\frac{3}{2}a^3\,\text{ExpIntegralEi}\left(-3W(ax^2)\right)+\frac{W(ax^2)^2}{2x^6}\right)-\frac{W(ax^2)}{4x^6}$$

input
```
Int[ProductLog[a*x^2]/x^7,x]
```

output
```
-1/4*ProductLog[a*x^2]/x^6 + ((3*a^3*ExpIntegralEi[-3*ProductLog[a*x^2]])/
2 + ProductLog[a*x^2]^2/(2*x^6))/2
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{\text{LambertW}(a\,x^2)}{x^7}dx$$

input
```
int(LambertW(a*x^2)/x^7,x)
```

output
```
int(LambertW(a*x^2)/x^7,x)
```

Fricas [F]

$$\int \frac{W(ax^2)}{x^7}\,dx = \int \frac{\mathrm{W}(ax^2)}{x^7}\,dx$$

input
```
integrate(lambert_w(a*x^2)/x^7,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)/x^7, x)
```

Sympy [F]

$$\int \frac{W(ax^2)}{x^7}\,dx = \int \frac{W(ax^2)}{x^7}\,dx$$

input
```
integrate(LambertW(a*x**2)/x**7,x)
```

output
```
Integral(LambertW(a*x**2)/x**7, x)
```

Maxima [F]

$$\int \frac{W(ax^2)}{x^7}\,dx = \int \frac{\mathrm{W}(ax^2)}{x^7}\,dx$$

input
```
integrate(lambert_w(a*x^2)/x^7,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)/x^7, x)
```

Giac [F]

$$\int \frac{W(ax^2)}{x^7}\,dx = \int \frac{\mathrm{W}(ax^2)}{x^7}\,dx$$

input
```
integrate(lambert_w(a*x^2)/x^7,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)}{x^7}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)}{x^7}\,dx$$

input
```
int(LambertW(a*x^2)/x^7,x)
```

output
```
int(LambertW(a*x^2)/x^7, x)
```

Reduce [F]

$$\int \frac{W(ax^2)}{x^7}\,dx = \frac{2\left(\int \frac{1}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)x^5+e^{lambert\_w(a\,x^2)}x^5}dx\right)a\,x^6 - lambert\_w(a\,x^2)}{6x^6}$$

input
```
int(Lambert_W(a*x^2)/x^7,x)
```

output
```
(2*int(1/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x**5 + e**lambert_w(a*x**
2)*x**5),x)*a*x**6 - lambert_w(a*x**2))/(6*x**6)
```

3.99 $\int x^4 W(ax^2)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 114

$$\int x^4 W(ax^2)\,dx = -\frac{2x^5}{25} - \frac{6x^5}{125W(ax^2)^2} + \frac{3\sqrt{\frac{2}{5}}e^{-\frac{3}{2}W(ax^2)}x^3\Gamma\left(\frac{1}{2},-\frac{5}{2}W(ax^2)\right)\sqrt{-W(ax^2)}}{125aW(ax^2)^2} + \frac{2x^5}{25W(ax^2)} + \frac{1}{5}x^5W(ax^2)$$

output
```
-2/25*x^5-6/125*x^5/LambertW(a*x^2)^2+3/625*10^(1/2)*x^3*Pi^(1/2)*erfc(1/2
*(-10*LambertW(a*x^2))^(1/2))*(-LambertW(a*x^2))^(1/2)/a/exp(3/2*LambertW(
a*x^2))/LambertW(a*x^2)^2+2/25*x^5/LambertW(a*x^2)+1/5*x^5*LambertW(a*x^2)
```

Mathematica [F]

$$\int x^4 W(ax^2)\,dx = \int x^4 W(ax^2)\,dx$$

input
```
Integrate[x^4*ProductLog[a*x^2],x]
```

output
```
Integrate[x^4*ProductLog[a*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 W\left(ax^2\right)\, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int x^4 W\left(ax^2\right) dx$$

input
```
Int[x^4*ProductLog[a*x^2],x]
```

output
```
$Aborted
```

Maple [F]

$$\int x^{4} \operatorname{LambertW}\left(a\,x^{2}\right) dx$$

input
```
int(x^4*LambertW(a*x^2),x)
```

output
```
int(x^4*LambertW(a*x^2),x)
```

Fricas [F]

$$\int x^4W(ax^2)\,dx = \int x^4\,\mathrm{W}(ax^2)\,dx$$

input
```
integrate(x^4*lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
integral(x^4*lambert_w(a*x^2), x)
```

Sympy [F]

$$\int x^4W(ax^2)\,dx = \int x^4W(ax^2)\,dx$$

input
```
integrate(x**4*LambertW(a*x**2),x)
```

output
```
Integral(x**4*LambertW(a*x**2), x)
```

Maxima [F]

$$\int x^4W(ax^2)\,dx = \int x^4\,\mathrm{W}(ax^2)\,dx$$

input
```
integrate(x^4*lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(x^4*lambert_w(a*x^2), x)
```

Giac [F]

$$\int x^4W\left(ax^2\right)\,dx=\int x^4\,\mathrm{W}(ax^2)\,dx$$

input
```
integrate(x^4*lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(x^4*lambert_w(a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4W\left(ax^2\right)\,dx=\int x^4\,\mathrm{LambertW}\left(a\,x^2\right)\,dx$$

input
```
int(x^4*LambertW(a*x^2),x)
```

output
```
int(x^4*LambertW(a*x^2), x)
```

Reduce [F]

$$\int x^4W\left(ax^2\right)\,dx=\int lambert\_w(a\,x^2)\,x^4dx$$

input
```
int(x^4*Lambert_W(a*x^2),x)
```

output
```
int(lambert_w(a*x**2)*x**4,x)
```

3.100 $\int x^2W(ax^2)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 97

$$\int x^2W(ax^2)\,dx = -\frac{2x^3}{9}+\frac{2x^3}{9W(ax^2)} - \frac{\sqrt{\frac{2}{3}}e^{-\frac{1}{2}W(ax^2)}x\Gamma\left(\frac{1}{2},-\frac{3}{2}W(ax^2)\right)\sqrt{-W(ax^2)}}{9aW(ax^2)}+\frac{1}{3}x^3W(ax^2)$$

output
```
-2/9*x^3+2/9*x^3/LambertW(a*x^2)-1/27*6^(1/2)*x*Pi^(1/2)*erfc(1/2*(-6*Lamb
ertW(a*x^2))^(1/2))*(-LambertW(a*x^2))^(1/2)/a/exp(1/2*LambertW(a*x^2))/La
mbertW(a*x^2)+1/3*x^3*LambertW(a*x^2)
```

Mathematica [F]

$$\int x^2W(ax^2)\,dx = \int x^2W(ax^2)\,dx$$

input
```
Integrate[x^2*ProductLog[a*x^2],x]
```

output
```
Integrate[x^2*ProductLog[a*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 W\left(ax^2\right)\, dx$$

$$\Big\downarrow \; 7299$$

$$\int x^2 W\left(ax^2\right) dx$$

input
```
Int[x^2*ProductLog[a*x^2],x]
```

output
```
$Aborted
```

Maple [F]

$$\int x^2 \operatorname{LambertW}\left(a\, x^2\right) dx$$

input
```
int(x^2*LambertW(a*x^2),x)
```

output
```
int(x^2*LambertW(a*x^2),x)
```

Fricas [F]

$$\int x^2 W\left(ax^2\right)\, dx = \int x^2\, \mathrm{W}(ax^2)\, dx$$

input
```
integrate(x^2*lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
integral(x^2*lambert_w(a*x^2), x)
```

Sympy [F]

$$\int x^2W\left(ax^2\right)\,dx=\int x^2W\left(ax^2\right)\,dx$$

input
```
integrate(x**2*LambertW(a*x**2),x)
```

output
```
Integral(x**2*LambertW(a*x**2), x)
```

Maxima [F]

$$\int x^2W\left(ax^2\right)\,dx=\int x^2\,\mathrm{W}(ax^2)\,dx$$

input
```
integrate(x^2*lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(x^2*lambert_w(a*x^2), x)
```

Giac [F]

$$\int x^2W\left(ax^2\right)\,dx=\int x^2\,\mathrm{W}(ax^2)\,dx$$

input
```
integrate(x^2*lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(x^2*lambert_w(a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2W\left(ax^2\right)\,dx=\int x^2\,\mathrm{LambertW}\left(a\,x^2\right)\,dx$$

input
```
int(x^2*LambertW(a*x^2),x)
```

output
```
int(x^2*LambertW(a*x^2), x)
```

Reduce [F]

$$\int x^2W\left(ax^2\right)\,dx=\int \mathit{lambert\_w}\left(a\,x^2\right)x^2dx$$

input
```
int(x^2*Lambert_W(a*x^2),x)
```

output
```
int(lambert_w(a*x**2)*x**2,x)
```

3.101 $\int W(ax^2)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 62

$$\int W(ax^2)\,dx = -2x + \frac{\sqrt{2}e^{\frac{1}{2}W(ax^2)}\Gamma\left(\frac{1}{2},-\frac{1}{2}W(ax^2)\right)\sqrt{-W(ax^2)}}{ax} + xW(ax^2)$$

output
```
-2*x+2^(1/2)*exp(1/2*LambertW(a*x^2))*Pi^(1/2)*erfc(1/2*(-2*LambertW(a*x^2
))^(1/2))*(-LambertW(a*x^2))^(1/2)/a/x+x*LambertW(a*x^2)
```

Mathematica [F]

$$\int W(ax^2)\,dx = \int W(ax^2)\,dx$$

input
```
Integrate[ProductLog[a*x^2],x]
```

output
```
Integrate[ProductLog[a*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W\left(ax^2\right) \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int W\left(ax^2\right) dx$$

input
```
Int[ProductLog[a*x^2],x]
```

output
```
$Aborted
```

Maple [F]

$$\int \text{LambertW}\left(a\,x^2\right) dx$$

input
```
int(LambertW(a*x^2),x)
```

output
```
int(LambertW(a*x^2),x)
```

Fricas [F]

$$\int W\left(ax^2\right) \, dx = \int \mathrm{W}(ax^2)\, dx$$

input
```
integrate(lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2), x)
```

Sympy [F]

$$\int W(ax^2)\ dx = \int W(ax^2)\ dx$$

input
```
integrate(LambertW(a*x**2),x)
```

output
```
Integral(LambertW(a*x**2), x)
```

Maxima [F]

$$\int W(ax^2)\ dx = \int \mathrm{W}(ax^2)\,dx$$

input
```
integrate(lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2), x)
```

Giac [F]

$$\int W(ax^2)\ dx = \int \mathrm{W}(ax^2)\,dx$$

input
```
integrate(lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int W\left(ax^2\right)\,dx = \int \mathrm{LambertW}\left(a\,x^2\right)\,dx$$

input
```
int(LambertW(a*x^2),x)
```

output
```
int(LambertW(a*x^2), x)
```

Reduce [F]

$$\int W\left(ax^2\right)\,dx = \int lambert\_w\left(a\,x^2\right)dx$$

input
```
int(Lambert_W(a*x^2),x)
```

output
```
int(lambert_w(a*x**2),x)
```

3.102 $\int \frac{W(ax^2)}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \frac{W(ax^2)}{x^2} dx = -\frac{W(ax^2)}{x} - \frac{\sqrt{2}e^{\frac{3}{2}W(ax^2)}\Gamma\left(\frac{1}{2}, \frac{1}{2}W(ax^2)\right) W(ax^2)^{3/2}}{ax^3}$$

output
```
-LambertW(a*x^2)/x-2^(1/2)*exp(3/2*LambertW(a*x^2))*Pi^(1/2)*erfc(1/2*2^(1
/2)*LambertW(a*x^2)^(1/2))*LambertW(a*x^2)^(3/2)/a/x^3
```

Mathematica [F]

$$\int \frac{W(ax^2)}{x^2} dx = \int \frac{W(ax^2)}{x^2} dx$$

input
```
Integrate[ProductLog[a*x^2]/x^2,x]
```

output
```
Integrate[ProductLog[a*x^2]/x^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)}{x^2} \, dx$$

$$\big\downarrow \text{ 7299}$$

$$\int \frac{W(ax^2)}{x^2} dx$$

input
```
Int[ProductLog[a*x^2]/x^2,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{\mathrm{LambertW}\left(a\,x^2\right)}{x^2}dx$$

input
```
int(LambertW(a*x^2)/x^2,x)
```

output
```
int(LambertW(a*x^2)/x^2,x)
```

Fricas [F]

$$\int \frac{W(ax^2)}{x^2} \, dx = \int \frac{\mathrm{W}(ax^2)}{x^2} \, dx$$

input
```
integrate(lambert_w(a*x^2)/x^2,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)/x^2, x)
```

Sympy [F]

$$\int \frac{W(ax^2)}{x^2}\,dx = \int \frac{W(ax^2)}{x^2}\,dx$$

input
```
integrate(LambertW(a*x**2)/x**2,x)
```

output
```
Integral(LambertW(a*x**2)/x**2, x)
```

Maxima [F]

$$\int \frac{W(ax^2)}{x^2}\,dx = \int \frac{\mathrm{W}(ax^2)}{x^2}\,dx$$

input
```
integrate(lambert_w(a*x^2)/x^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)/x^2, x)
```

Giac [F]

$$\int \frac{W(ax^2)}{x^2}\,dx = \int \frac{\mathrm{W}(ax^2)}{x^2}\,dx$$

input
```
integrate(lambert_w(a*x^2)/x^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)}{x^2}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)}{x^2}\,dx$$

input
```
int(LambertW(a*x^2)/x^2,x)
```

output
```
int(LambertW(a*x^2)/x^2, x)
```

Reduce [F]

$$\int \frac{W(ax^2)}{x^2}\,dx = \frac{2\left(\int \frac{1}{e^{lambert\_w\left(a\,x^2\right)}\,lambert\_w(a\,x^2)+e^{lambert\_w\left(a\,x^2\right)}}dx\right)ax - lambert\_w(a\,x^2)}{x}$$

input
```
int(Lambert_W(a*x^2)/x^2,x)
```

output
```
(2*int(1/(e**lambert_w(a*x**2)*lambert_w(a*x**2) + e**lambert_w(a*x**2)),x
)*a*x - lambert_w(a*x**2))/x
```

3.103 $\int \frac{W(ax^2)}{x^4} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 111

$$\int \frac{W(ax^2)}{x^4} \, dx = -\frac{W(ax^2)}{x^3} + \frac{\sqrt{\frac{3}{2}} e^{\frac{5}{2} W(ax^2)} \Gamma\left(\frac{1}{2}, \frac{3}{2} W(ax^2)\right) W(ax^2)^{5/2}}{a x^5} - \frac{e^{\frac{5}{2} W(ax^2)} \Gamma\left(\frac{1}{2}, \frac{3}{2} W(ax^2)\right) W(ax^2)^{5/2}}{\sqrt{6} a x^5}$$

output
```
-LambertW(a*x^2)/x^3+1/3*6^(1/2)*exp(5/2*LambertW(a*x^2))*Pi^(1/2)*erfc(1/
2*6^(1/2)*LambertW(a*x^2)^(1/2))*LambertW(a*x^2)^(5/2)/a/x^5
```

Mathematica [F]

$$\int \frac{W(ax^2)}{x^4} \, dx = \int \frac{W(ax^2)}{x^4} \, dx$$

input
```
Integrate[ProductLog[a*x^2]/x^4,x]
```

output
```
Integrate[ProductLog[a*x^2]/x^4, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)}{x^4} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{W(ax^2)}{x^4}dx$$

input
```
Int[ProductLog[a*x^2]/x^4,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{\mathrm{LambertW}\left(a\,x^{2}\right)}{x^{4}}dx$$

input
```
int(LambertW(a*x^2)/x^4,x)
```

output
```
int(LambertW(a*x^2)/x^4,x)
```

Fricas [F]

$$\int \frac{W(ax^2)}{x^4} \, dx = \int \frac{\mathrm{W}(ax^2)}{x^4} \, dx$$

input
```
integrate(lambert_w(a*x^2)/x^4,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)/x^4, x)
```

Sympy [F]

$$\int \frac{W(ax^2)}{x^4}\,dx = \int \frac{W(ax^2)}{x^4}\,dx$$

input
```
integrate(LambertW(a*x**2)/x**4,x)
```

output
```
Integral(LambertW(a*x**2)/x**4, x)
```

Maxima [F]

$$\int \frac{W(ax^2)}{x^4}\,dx = \int \frac{\mathrm{W}(ax^2)}{x^4}\,dx$$

input
```
integrate(lambert_w(a*x^2)/x^4,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)/x^4, x)
```

Giac [F]

$$\int \frac{W(ax^2)}{x^4}\,dx = \int \frac{\mathrm{W}(ax^2)}{x^4}\,dx$$

input
```
integrate(lambert_w(a*x^2)/x^4,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)}{x^4}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)}{x^4}\,dx$$

input
```
int(LambertW(a*x^2)/x^4,x)
```

output
```
int(LambertW(a*x^2)/x^4, x)
```

Reduce [F]

$$\int \frac{W(ax^2)}{x^4}\,dx = \frac{2\left(\int \frac{1}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)x^2+e^{lambert\_w(a\,x^2)}x^2}dx\right)a\,x^3 - lambert\_w(a\,x^2)}{3x^3}$$

input
```
int(Lambert_W(a*x^2)/x^4,x)
```

output
```
(2*int(1/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x**2 + e**lambert_w(a*x**
2)*x**2),x)*a*x**3 - lambert_w(a*x**2))/(3*x**3)
```

3.104 $\int x^3 W(ax^2)^2 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int x^3 W(ax^2)^2 \, dx = \frac{3x^4}{8} + \frac{3x^4}{16W(ax^2)^2} - \frac{3x^4}{8W(ax^2)} - \frac{1}{4}x^4 W(ax^2) + \frac{1}{4}x^4 W(ax^2)^2$$

output
```
3/8*x^4+3/16*x^4/LambertW(a*x^2)^2-3/8*x^4/LambertW(a*x^2)-1/4*x^4*Lambert
W(a*x^2)+1/4*x^4*LambertW(a*x^2)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int x^3 W(ax^2)^2 \, dx = \frac{3x^4}{8} + \frac{3x^4}{16W(ax^2)^2} - \frac{3x^4}{8W(ax^2)} - \frac{1}{4}x^4 W(ax^2) + \frac{1}{4}x^4 W(ax^2)^2$$

input
```
Integrate[x^3*ProductLog[a*x^2]^2,x]
```

output
```
(3*x^4)/8 + (3*x^4)/(16*ProductLog[a*x^2]^2) - (3*x^4)/(8*ProductLog[a*x^2
]) - (x^4*ProductLog[a*x^2])/4 + (x^4*ProductLog[a*x^2]^2)/4
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7172, 7205, 7205, 7283, 7194, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 W\left(ax^2\right)^2 dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{4}x^4 W\left(ax^2\right)^2 - \int \frac{x^3 W\left(ax^2\right)^2}{W\left(ax^2\right)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{3}{2}\int \frac{x^3 W\left(ax^2\right)}{W\left(ax^2\right)+1}dx + \frac{1}{4}x^4 W\left(ax^2\right)^2 - \frac{1}{4}x^4 W\left(ax^2\right)$$

$$\Big\downarrow \text{7205}$$

$$\frac{3}{2}\left(\frac{x^4}{4} - \int \frac{x^3}{W\left(ax^2\right)+1}dx\right) + \frac{1}{4}x^4 W\left(ax^2\right)^2 - \frac{1}{4}x^4 W\left(ax^2\right)$$

$$\Big\downarrow \text{7283}$$

$$\frac{3}{2}\left(\frac{x^4}{4} - \frac{1}{2}\int \frac{x^2}{W\left(ax^2\right)+1}dx^2\right) + \frac{1}{4}x^4 W\left(ax^2\right)^2 - \frac{1}{4}x^4 W\left(ax^2\right)$$

$$\Big\downarrow \text{7194}$$

$$\frac{3}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{x^2}{W\left(ax^2\right)\left(W\left(ax^2\right)+1\right)}dx^2 - \frac{x^4}{2W\left(ax^2\right)}\right) + \frac{x^4}{4}\right) + \frac{1}{4}x^4 W\left(ax^2\right)^2 - \frac{1}{4}x^4 W\left(ax^2\right)$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{4}x^4 W\left(ax^2\right)^2 - \frac{1}{4}x^4 W\left(ax^2\right) + \frac{3}{2}\left(\frac{1}{2}\left(\frac{x^4}{4W\left(ax^2\right)^2} - \frac{x^4}{2W\left(ax^2\right)}\right) + \frac{x^4}{4}\right)$$

input
```
Int[x^3*ProductLog[a*x^2]^2,x]
```

output
```
(3*(x^4/4 + (x^4/(4*ProductLog[a*x^2]^2) - x^4/(2*ProductLog[a*x^2]))/2))/
2 - (x^4*ProductLog[a*x^2])/4 + (x^4*ProductLog[a*x^2]^2)/4
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int x^3 \operatorname{LambertW}\left(a\,x^2\right)^2 dx$$

input
```
int(x^3*LambertW(a*x^2)^2,x)
```

output
```
int(x^3*LambertW(a*x^2)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int x^3 W\left(ax^2\right)^2 \, dx = \frac{4\,x^4\,\mathrm{W}(ax^2)^4 - 4\,x^4\,\mathrm{W}(ax^2)^3 + 6\,x^4\,\mathrm{W}(ax^2)^2 - 6\,x^4\,\mathrm{W}(ax^2) + 3\,x^4}{16\,\mathrm{W}(ax^2)^2}$$

input
```
integrate(x^3*lambert_w(a*x^2)^2,x, algorithm="fricas")
```

output
```
1/16*(4*x^4*lambert_w(a*x^2)^4 - 4*x^4*lambert_w(a*x^2)^3 + 6*x^4*lambert_
w(a*x^2)^2 - 6*x^4*lambert_w(a*x^2) + 3*x^4)/lambert_w(a*x^2)^2
```

Sympy [F]

$$\int x^3 W\left(ax^2\right)^2 \, dx = \int x^3 W^2\left(ax^2\right) \, dx$$

input
```
integrate(x**3*LambertW(a*x**2)**2,x)
```

output
```
Integral(x**3*LambertW(a*x**2)**2, x)
```

Maxima [F]

$$\int x^3 W\left(ax^2\right)^2 \, dx = \int x^3 \,\mathrm{W}(ax^2)^2 \, dx$$

input
```
integrate(x^3*lambert_w(a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*lambert_w(a*x^2)^2, x)
```

Giac [F]

$$\int x^3 W\left(ax^2\right)^2 \, dx = \int x^3 \,\mathrm{W}(ax^2)^2 \, dx$$

input
```
integrate(x^3*lambert_w(a*x^2)^2,x, algorithm="giac")
```

output
```
integrate(x^3*lambert_w(a*x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 W\left(ax^2\right)^2 \, dx = \int x^3 \,\mathrm{LambertW}\left(a\,x^2\right)^2 \, dx$$

input
```
int(x^3*LambertW(a*x^2)^2,x)
```

output
```
int(x^3*LambertW(a*x^2)^2, x)
```

Reduce [F]

$$\int x^3 W\left(ax^2\right)^2 \, dx = \int lambert\_w\left(a\,x^2\right)^2 x^3 dx$$

input
```
int(x^3*Lambert_W(a*x^2)^2,x)
```

output
```
int(lambert_w(a*x**2)**2*x**3,x)
```

3.105 $\int xW(ax^2)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 45

$$\int xW(ax^2)^2\,dx = 2x^2 - \frac{2x^2}{W(ax^2)} - x^2W(ax^2) + \frac{1}{2}x^2W(ax^2)^2$$

output
```
2*x^2-2*x^2/LambertW(a*x^2)-x^2*LambertW(a*x^2)+1/2*x^2*LambertW(a*x^2)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int xW(ax^2)^2\,dx = \frac{x^2\left(-4+4W(ax^2)-2W(ax^2)^2+W(ax^2)^3\right)}{2W(ax^2)}$$

input
```
Integrate[x*ProductLog[a*x^2]^2,x]
```

output
```
(x^2*(-4 + 4*ProductLog[a*x^2] - 2*ProductLog[a*x^2]^2 + ProductLog[a*x^2]
^3))/(2*ProductLog[a*x^2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7172, 7205, 7205, 7266, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xW\left(ax^2\right)^2 dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{2}x^2W\left(ax^2\right)^2-2\int \frac{xW\left(ax^2\right)^2}{W\left(ax^2\right)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{2}x^2W\left(ax^2\right)^2-2\left(\frac{1}{2}x^2W\left(ax^2\right)-2\int \frac{xW\left(ax^2\right)}{W\left(ax^2\right)+1}dx\right)$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{2}x^2W\left(ax^2\right)^2-2\left(\frac{1}{2}x^2W\left(ax^2\right)-2\left(\frac{x^2}{2}-\int \frac{x}{W\left(ax^2\right)+1}dx\right)\right)$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}x^2W\left(ax^2\right)^2-2\left(\frac{1}{2}x^2W\left(ax^2\right)-2\left(\frac{x^2}{2}-\frac{1}{2}\int \frac{1}{W\left(ax^2\right)+1}dx^2\right)\right)$$

$$\Big\downarrow \text{7176}$$

$$\frac{1}{2}x^2W\left(ax^2\right)^2-2\left(\frac{1}{2}x^2W\left(ax^2\right)-2\left(\frac{x^2}{2}-\frac{x^2}{2W\left(ax^2\right)}\right)\right)$$

input
```
Int[x*ProductLog[a*x^2]^2,x]
```

output
```
(x^2*ProductLog[a*x^2]^2)/2 - 2*(-2*(x^2/2 - x^2/(2*ProductLog[a*x^2])) +
(x^2*ProductLog[a*x^2])/2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{\operatorname{LambertW}\left(a\,x^{2}\right)^{2} a\,x^{2}-2 a\,x^{2} \operatorname{LambertW}\left(a\,x^{2}\right)+4 a\,x^{2}-\frac{4 a\,x^{2}}{\operatorname{LambertW}\left(a\,x^{2}\right)}}{2 a}$	52
default	$\frac{\operatorname{LambertW}\left(a\,x^{2}\right)^{2} a\,x^{2}-2 a\,x^{2} \operatorname{LambertW}\left(a\,x^{2}\right)+4 a\,x^{2}-\frac{4 a\,x^{2}}{\operatorname{LambertW}\left(a\,x^{2}\right)}}{2 a}$	52
parallelrisch	$-\frac{-x^{2} \operatorname{LambertW}\left(a\,x^{2}\right)^{3}+2 x^{2} \operatorname{LambertW}\left(a\,x^{2}\right)^{2}-4 x^{2} \operatorname{LambertW}\left(a\,x^{2}\right)+4 x^{2}}{2 \operatorname{LambertW}\left(a\,x^{2}\right)}$	54

input
```
int(x*LambertW(a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(LambertW(a*x^2)^2*a*x^2-2*a*x^2*LambertW(a*x^2)+4*a*x^2-4*a*x^2/Lam
bertW(a*x^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int xW(ax^2)^2\,dx = \frac{x^2\,\mathrm{W}(ax^2)^3 - 2\,x^2\,\mathrm{W}(ax^2)^2 + 4\,x^2\,\mathrm{W}(ax^2) - 4\,x^2}{2\,\mathrm{W}(ax^2)}$$

input
```
integrate(x*lambert_w(a*x^2)^2,x, algorithm="fricas")
```

output
```
1/2*(x^2*lambert_w(a*x^2)^3 - 2*x^2*lambert_w(a*x^2)^2 + 4*x^2*lambert_w(a
*x^2) - 4*x^2)/lambert_w(a*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int xW(ax^2)^2\,dx = \begin{cases} \frac{x^2W^2(ax^2)}{2} - x^2W(ax^2) + 2x^2 - \frac{2x^2}{W(ax^2)} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*LambertW(a*x**2)**2,x)
```

output
```
Piecewise((x**2*LambertW(a*x**2)**2/2 - x**2*LambertW(a*x**2) + 2*x**2 - 2
*x**2/LambertW(a*x**2), Ne(a, 0)), (0, True))
```

Maxima [F]

$$\int xW(ax^2)^2\,dx = \int x\,\mathrm{W}(ax^2)^2\,dx$$

input
```
integrate(x*lambert_w(a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate(x*lambert_w(a*x^2)^2, x)
```

Giac [F]

$$\int xW(ax^2)^2\,dx = \int x\,\mathrm{W}(ax^2)^2\,dx$$

input
```
integrate(x*lambert_w(a*x^2)^2,x, algorithm="giac")
```

output
```
integrate(x*lambert_w(a*x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int xW(ax^2)^2\,dx = \int x\,\mathrm{LambertW}\left(a\,x^2\right)^2\,dx$$

input
```
int(x*LambertW(a*x^2)^2,x)
```

output
```
int(x*LambertW(a*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int xW\left(ax^2\right)^2\,dx$$
$$= \frac{e^{lambert\_w(a\,x^2)}\left(lambert\_w(a\,x^2)^3-2lambert\_w(a\,x^2)^2+4lambert\_w(a\,x^2)-4\right)}{2a}$$

input
```
int(x*Lambert_W(a*x^2)^2,x)
```

output
```
(e**lambert_w(a*x**2)*(lambert_w(a*x**2)**3 - 2*lambert_w(a*x**2)**2 + 4*l
ambert_w(a*x**2) - 4))/(2*a)
```

3.106 $\int \frac{W(ax^2)^2}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{W(ax^2)^2}{x} dx = \frac{1}{4}W(ax^2)^2 + \frac{1}{6}W(ax^2)^3$$

output
```
1/4*LambertW(a*x^2)^2+1/6*LambertW(a*x^2)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{W(ax^2)^2}{x} dx = \frac{1}{4}W(ax^2)^2 + \frac{1}{6}W(ax^2)^3$$

input
```
Integrate[ProductLog[a*x^2]^2/x,x]
```

output
```
ProductLog[a*x^2]^2/4 + ProductLog[a*x^2]^3/6
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)^2}{x} dx$$
$$\Big\downarrow \text{ 7173}$$
$$\int \frac{W(ax^2)^3}{x(W(ax^2)+1)}dx + \frac{1}{4}W(ax^2)^2$$
$$\Big\downarrow \text{ 7200}$$
$$\frac{1}{6}W(ax^2)^3 + \frac{1}{4}W(ax^2)^2$$

input
```
Int[ProductLog[a*x^2]^2/x,x]
```

output
```
ProductLog[a*x^2]^2/4 + ProductLog[a*x^2]^3/6
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\mathrm{LambertW}\left(a\,x^{2}\right)^{2}}{4}+\frac{\mathrm{LambertW}\left(a\,x^{2}\right)^{3}}{6}$	22
default	$\frac{\mathrm{LambertW}\left(a\,x^{2}\right)^{2}}{4}+\frac{\mathrm{LambertW}\left(a\,x^{2}\right)^{3}}{6}$	22

input
```
int(LambertW(a*x^2)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*LambertW(a*x^2)^2+1/6*LambertW(a*x^2)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{W(ax^2)^2}{x}\,dx = \frac{1}{6}\,\mathrm{W}(ax^2)^3 + \frac{1}{4}\,\mathrm{W}(ax^2)^2$$

input
```
integrate(lambert_w(a*x^2)^2/x,x, algorithm="fricas")
```

output
```
1/6*lambert_w(a*x^2)^3 + 1/4*lambert_w(a*x^2)^2
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{W(ax^2)^2}{x}\,dx = \frac{W^3(ax^2)}{6} + \frac{W^2(ax^2)}{4}$$

input
```
integrate(LambertW(a*x**2)**2/x,x)
```

output
```
LambertW(a*x**2)**3/6 + LambertW(a*x**2)**2/4
```

Maxima [F]

$$\int \frac{W(ax^2)^2}{x}\,dx = \int \frac{\mathrm{W}(ax^2)^2}{x}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^2/x, x)
```

Giac [F]

$$\int \frac{W(ax^2)^2}{x}\,dx = \int \frac{\mathrm{W}(ax^2)^2}{x}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^2}{x}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)^2}{x}\,dx$$

input
```
int(LambertW(a*x^2)^2/x,x)
```

output
```
int(LambertW(a*x^2)^2/x, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^2}{x}\,dx = \int \frac{lambert\_w(a\,x^2)^2}{x}dx - \left(\int \frac{1}{x}dx\right) + \log(x)$$

input
```
int(Lambert_W(a*x^2)^2/x,x)
```

output
```
int(lambert_w(a*x**2)**2/x,x) - int(1/x,x) + log(x)
```

3.107 $\int \frac{W(ax^2)^2}{x^3} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{W(ax^2)^2}{x^3} \, dx = -\frac{W(ax^2)}{x^2} - \frac{W(ax^2)^2}{2x^2}$$

output
```
-LambertW(a*x^2)/x^2-1/2*LambertW(a*x^2)^2/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{W(ax^2)^2}{x^3} \, dx = -\frac{W(ax^2)}{x^2} - \frac{W(ax^2)^2}{2x^2}$$

input
```
Integrate[ProductLog[a*x^2]^2/x^3,x]
```

output
```
-(ProductLog[a*x^2]/x^2) - ProductLog[a*x^2]^2/(2*x^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7172, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)^2}{x^3} dx$$

$$\Big\downarrow \text{7172}$$

$$2\int \frac{W(ax^2)^2}{x^3 (W(ax^2)+1)} dx - \frac{W(ax^2)^2}{2x^2}$$

$$\Big\downarrow \text{7201}$$

$$-\frac{W(ax^2)^2}{2x^2} - \frac{W(ax^2)}{x^2}$$

input
```
Int[ProductLog[a*x^2]^2/x^3,x]
```

output
```
-(ProductLog[a*x^2]/x^2) - ProductLog[a*x^2]^2/(2*x^2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$-\frac{\text{LambertW}\left(a\,x^{2}\right)^{2}+2\,\text{LambertW}\left(a\,x^{2}\right)}{2 x^{2}}$	23

input
```
int(LambertW(a*x^2)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2*(LambertW(a*x^2)^2+2*LambertW(a*x^2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{W(ax^2)^2}{x^3}\,dx = -\frac{\mathrm{W}(ax^2)^2 + 2\ \mathrm{W}(ax^2)}{2\,x^2}$$

input
```
integrate(lambert_w(a*x^2)^2/x^3,x, algorithm="fricas")
```

output
```
-1/2*(lambert_w(a*x^2)^2 + 2*lambert_w(a*x^2))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{W(ax^2)^2}{x^3}\,dx = -\frac{W^2(ax^2)}{2x^2} - \frac{W(ax^2)}{x^2}$$

input
```
integrate(LambertW(a*x**2)**2/x**3,x)
```

output
```
-LambertW(a*x**2)**2/(2*x**2) - LambertW(a*x**2)/x**2
```

Maxima [F]

$$\int \frac{W(ax^2)^2}{x^3}\,dx = \int \frac{\mathrm{W}(ax^2)^2}{x^3}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^2/x^3, x)
```

Giac [F]

$$\int \frac{W(ax^2)^2}{x^3}\,dx = \int \frac{\mathrm{W}(ax^2)^2}{x^3}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^2}{x^3}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)^2}{x^3}\,dx$$

input
```
int(LambertW(a*x^2)^2/x^3,x)
```

output
```
int(LambertW(a*x^2)^2/x^3, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^2}{x^3}\,dx$$
$$= \frac{4\left(\int \frac{lambert\_w(a\,x^2)}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)x+e^{lambert\_w(a\,x^2)}x}dx\right)a\,x^2+4\left(\int \frac{lambert\_w(a\,x^2)}{lambert\_w(a\,x^2)x^3+x^3}dx\right)x^2+4\left(\int \frac{}{la\cdots}\right.}{2x^2}$$

input
```
int(Lambert_W(a*x^2)^2/x^3,x)
```

output
```
(4*int(lambert_w(a*x**2)/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x + e**la
mbert_w(a*x**2)*x),x)*a*x**2 + 4*int(lambert_w(a*x**2)/(lambert_w(a*x**2)*
x**3 + x**3),x)*x**2 + 4*int(1/(lambert_w(a*x**2)*x**3 + x**3),x)*x**2 - l
ambert_w(a*x**2)**2 + 2)/(2*x**2)
```

3.108 $\int \frac{W(ax^2)^2}{x^5} dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{W(ax^2)^2}{x^5} dx = \frac{1}{2}a^2 \operatorname{ExpIntegralEi}\left(-2W\left(ax^2\right)\right) - \frac{W(ax^2)^2}{4x^4}$$

output
```
1/2*a^2*Ei(-2*LambertW(a*x^2))-1/4*LambertW(a*x^2)^2/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{W(ax^2)^2}{x^5} dx = \frac{1}{2}a^2 \operatorname{ExpIntegralEi}\left(-2W\left(ax^2\right)\right) - \frac{W(ax^2)^2}{4x^4}$$

input
```
Integrate[ProductLog[a*x^2]^2/x^5,x]
```

output
```
(a^2*ExpIntegralEi[-2*ProductLog[a*x^2]])/2 - ProductLog[a*x^2]^2/(4*x^4)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7172, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)^2}{x^5}dx$$
$$\Big\downarrow \text{ 7172}$$
$$\int \frac{W(ax^2)^2}{x^5 (W(ax^2)+1)}dx-\frac{W(ax^2)^2}{4x^4}$$
$$\Big\downarrow \text{ 7202}$$
$$\frac{1}{2}a^2 \text{ExpIntegralEi}\left(-2W\left(ax^2\right)\right)-\frac{W(ax^2)^2}{4x^4}$$

input
```
Int[ProductLog[a*x^2]^2/x^5,x]
```

output
```
(a^2*ExpIntegralEi[-2*ProductLog[a*x^2]])/2 - ProductLog[a*x^2]^2/(4*x^4)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [F]

$$\int \frac{\text{LambertW}\left(a\,x^{2}\right)^{2}}{x^{5}}dx$$

input
```
int(LambertW(a*x^2)^2/x^5,x)
```

output
```
int(LambertW(a*x^2)^2/x^5,x)
```

Fricas [F]

$$\int \frac{W(ax^2)^2}{x^5}\,dx = \int \frac{\mathrm{W}(ax^2)^2}{x^5}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^5,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^2/x^5, x)
```

Sympy [F]

$$\int \frac{W(ax^2)^2}{x^5}\,dx = \int \frac{W^2(ax^2)}{x^5}\,dx$$

input
```
integrate(LambertW(a*x**2)**2/x**5,x)
```

output
```
Integral(LambertW(a*x**2)**2/x**5, x)
```

Maxima [F]

$$\int \frac{W(ax^2)^2}{x^5} dx = \int \frac{\mathrm{W}(ax^2)^2}{x^5} dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^5,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^2/x^5, x)
```

Giac [F]

$$\int \frac{W(ax^2)^2}{x^5} dx = \int \frac{\mathrm{W}(ax^2)^2}{x^5} dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^5,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^2}{x^5} dx = \int \frac{\mathrm{LambertW}(a\,x^2)^2}{x^5} dx$$

input
```
int(LambertW(a*x^2)^2/x^5,x)
```

output
```
int(LambertW(a*x^2)^2/x^5, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^2}{x^5}\,dx$$
$$= \frac{4\left(\int \frac{lambert\_w(a\,x^2)}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)x^3+e^{lambert\_w(a\,x^2)}x^3}dx\right)a\,x^4 - lambert\_w(a\,x^2)^2}{4x^4}$$

input
```
int(Lambert_W(a*x^2)^2/x^5,x)
```

output
```
(4*int(lambert_w(a*x**2)/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x**3 + e*
*lambert_w(a*x**2)*x**3),x)*a*x**4 - lambert_w(a*x**2)**2)/(4*x**4)
```

3.109 $\int \frac{W(ax^2)^2}{x^7} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{W(ax^2)^2}{x^7} \, dx = -a^3 \text{ExpIntegralEi}\left(-3W\left(ax^2\right)\right) - \frac{W(ax^2)^2}{2x^6}$$

output
```
-a^3*Ei(-3*LambertW(a*x^2))-1/2*LambertW(a*x^2)^2/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{W(ax^2)^2}{x^7} \, dx = -a^3 \text{ExpIntegralEi}\left(-3W\left(ax^2\right)\right) - \frac{W(ax^2)^2}{2x^6}$$

input
```
Integrate[ProductLog[a*x^2]^2/x^7,x]
```

output
```
-(a^3*ExpIntegralEi[-3*ProductLog[a*x^2]]) - ProductLog[a*x^2]^2/(2*x^6)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7173, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)^2}{x^7} \, dx$$
$$\Big\downarrow \text{7173}$$
$$-2\int \frac{W(ax^2)^3}{x^7 (W(ax^2)+1)}dx - \frac{W(ax^2)^2}{2x^6}$$
$$\Big\downarrow \text{7202}$$
$$a^3(-\text{ExpIntegralEi}(-3W(ax^2))) - \frac{W(ax^2)^2}{2x^6}$$

input
```
Int[ProductLog[a*x^2]^2/x^7,x]
```

output
```
-(a^3*ExpIntegralEi[-3*ProductLog[a*x^2]]) - ProductLog[a*x^2]^2/(2*x^6)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [F]

$$\int \frac{\mathrm{LambertW}\left(a\,x^{2}\right)^{2}}{x^{7}}dx$$

input
```
int(LambertW(a*x^2)^2/x^7,x)
```

output
```
int(LambertW(a*x^2)^2/x^7,x)
```

Fricas [F]

$$\int \frac{W(ax^2)^2}{x^7}\,dx = \int \frac{\mathrm{W}(ax^2)^2}{x^7}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^7,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^2/x^7, x)
```

Sympy [F]

$$\int \frac{W(ax^2)^2}{x^7}\,dx = \int \frac{W^2(ax^2)}{x^7}\,dx$$

input
```
integrate(LambertW(a*x**2)**2/x**7,x)
```

output
```
Integral(LambertW(a*x**2)**2/x**7, x)
```

Maxima [F]

$$\int \frac{W(ax^2)^2}{x^7}\,dx = \int \frac{\mathrm{W}(ax^2)^2}{x^7}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^7,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^2/x^7, x)
```

Giac [F]

$$\int \frac{W(ax^2)^2}{x^7}\,dx = \int \frac{\mathrm{W}(ax^2)^2}{x^7}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^7,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^2/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^2}{x^7}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)^2}{x^7}\,dx$$

input
```
int(LambertW(a*x^2)^2/x^7,x)
```

output
```
int(LambertW(a*x^2)^2/x^7, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^2}{x^7}\,dx = \frac{4\left(\int \frac{lambert\_w(a\,x^2)}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)x^5+e^{lambert\_w(a\,x^2)}x^5}dx\right)a\,x^6-lambert\_w(a\,x^2)^2}{6x^6}$$

input

```
int(Lambert_W(a*x^2)^2/x^7,x)
```

output

```
(4*int(lambert_w(a*x**2)/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x**5 + e*
*lambert_w(a*x**2)*x**5),x)*a*x**6 - lambert_w(a*x**2)**2)/(6*x**6)
```

3.110 $\int \frac{W(ax^2)^2}{x^9} dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int \frac{W(ax^2)^2}{x^9} dx = 2a^4 \operatorname{ExpIntegralEi}\left(-4W\left(ax^2\right)\right) - \frac{W(ax^2)^2}{4x^8} + \frac{W(ax^2)^3}{2x^8}$$

output
```
2*a^4*Ei(-4*LambertW(a*x^2))-1/4*LambertW(a*x^2)^2/x^8+1/2*LambertW(a*x^2)
^3/x^8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{W(ax^2)^2}{x^9} dx = 2a^4 \operatorname{ExpIntegralEi}\left(-4W\left(ax^2\right)\right) - \frac{W(ax^2)^2}{4x^8} + \frac{W(ax^2)^3}{2x^8}$$

input
```
Integrate[ProductLog[a*x^2]^2/x^9,x]
```

output
```
2*a^4*ExpIntegralEi[-4*ProductLog[a*x^2]] - ProductLog[a*x^2]^2/(4*x^8) +
ProductLog[a*x^2]^3/(2*x^8)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7173, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)^2}{x^9}dx$$

$$\Big\downarrow \text{ 7173}$$

$$-\int \frac{W(ax^2)^3}{x^9(W(ax^2)+1)}dx - \frac{W(ax^2)^2}{4x^8}$$

$$\Big\downarrow \text{ 7206}$$

$$4\int \frac{W(ax^2)^4}{x^9(W(ax^2)+1)}dx + \frac{W(ax^2)^3}{2x^8} - \frac{W(ax^2)^2}{4x^8}$$

$$\Big\downarrow \text{ 7202}$$

$$2a^4 \text{ExpIntegralEi}(-4W(ax^2)) + \frac{W(ax^2)^3}{2x^8} - \frac{W(ax^2)^2}{4x^8}$$

input
```
Int[ProductLog[a*x^2]^2/x^9,x]
```

output
```
2*a^4*ExpIntegralEi[-4*ProductLog[a*x^2]] - ProductLog[a*x^2]^2/(4*x^8) +
ProductLog[a*x^2]^3/(2*x^8)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{\text{LambertW}\left(a\,x^{2}\right)^{2}}{x^{9}}dx$$

input
```
int(LambertW(a*x^2)^2/x^9,x)
```

output
```
int(LambertW(a*x^2)^2/x^9,x)
```

Fricas [F]

$$\int \frac{W(ax^2)^2}{x^9}\,dx = \int \frac{\mathrm{W}(ax^2)^2}{x^9}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^9,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^2/x^9, x)
```

Sympy [F]

$$\int \frac{W(ax^2)^2}{x^9} \, dx = \int \frac{W^2(ax^2)}{x^9} \, dx$$

input
```
integrate(LambertW(a*x**2)**2/x**9,x)
```

output
```
Integral(LambertW(a*x**2)**2/x**9, x)
```

Maxima [F]

$$\int \frac{W(ax^2)^2}{x^9} \, dx = \int \frac{\mathrm{W}(ax^2)^2}{x^9} \, dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^9,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^2/x^9, x)
```

Giac [F]

$$\int \frac{W(ax^2)^2}{x^9} \, dx = \int \frac{\mathrm{W}(ax^2)^2}{x^9} \, dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^9,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^2/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^2}{x^9}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)^2}{x^9}\,dx$$

input
```
int(LambertW(a*x^2)^2/x^9,x)
```

output
```
int(LambertW(a*x^2)^2/x^9, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^2}{x^9}\,dx = \frac{4\left(\int \frac{lambert\_w(a\,x^2)}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)x^7+e^{lambert\_w(a\,x^2)}x^7}dx\right)a\,x^8 - lambert\_w(a\,x^2)^2}{8x^8}$$

input
```
int(Lambert_W(a*x^2)^2/x^9,x)
```

output
```
(4*int(lambert_w(a*x**2)/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x**7 + e*
*lambert_w(a*x**2)*x**7),x)*a*x**8 - lambert_w(a*x**2)**2)/(8*x**8)
```

3.111 $\int x^2 W(ax^2)^2 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 112

$$\int x^2 W(ax^2)^2 \, dx = \frac{20x^3}{27} - \frac{20x^3}{27W(ax^2)} + \frac{10\sqrt{\frac{2}{3}}e^{-\frac{1}{2}W(ax^2)}x\Gamma\left(\frac{1}{2}, -\frac{3}{2}W(ax^2)\right)\sqrt{-W(ax^2)}}{27aW(ax^2)} - \frac{4}{9}x^3W(ax^2) + \frac{1}{3}x^3W(ax^2)^2$$

output
```
20/27*x^3-20/27*x^3/LambertW(a*x^2)+10/81*6^(1/2)*x*Pi^(1/2)*erfc(1/2*(-6*
LambertW(a*x^2))^(1/2))*(-LambertW(a*x^2))^(1/2)/a/exp(1/2*LambertW(a*x^2)
)/LambertW(a*x^2)-4/9*x^3*LambertW(a*x^2)+1/3*x^3*LambertW(a*x^2)^2
```

Mathematica [F]

$$\int x^2 W(ax^2)^2 \, dx = \int x^2 W(ax^2)^2 \, dx$$

input
```
Integrate[x^2*ProductLog[a*x^2]^2,x]
```

output
```
Integrate[x^2*ProductLog[a*x^2]^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 W\left(ax^2\right)^2 \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int x^2 W\left(ax^2\right)^2 dx$$

input
```
Int[x^2*ProductLog[a*x^2]^2,x]
```

output
```
$Aborted
```

Maple [F]

$$\int x^2 \operatorname{LambertW}\left(a\,x^2\right)^2 dx$$

input
```
int(x^2*LambertW(a*x^2)^2,x)
```

output
```
int(x^2*LambertW(a*x^2)^2,x)
```

Fricas [F]

$$\int x^2 W(ax^2)^2 \, dx = \int x^2 \mathrm{W}(ax^2)^2 \, dx$$

input
```
integrate(x^2*lambert_w(a*x^2)^2,x, algorithm="fricas")
```

output
```
integral(x^2*lambert_w(a*x^2)^2, x)
```

Sympy [F]

$$\int x^2 W(ax^2)^2 \, dx = \int x^2 W^2(ax^2) \, dx$$

input
```
integrate(x**2*LambertW(a*x**2)**2,x)
```

output
```
Integral(x**2*LambertW(a*x**2)**2, x)
```

Maxima [F]

$$\int x^2 W(ax^2)^2 \, dx = \int x^2 \mathrm{W}(ax^2)^2 \, dx$$

input
```
integrate(x^2*lambert_w(a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate(x^2*lambert_w(a*x^2)^2, x)
```

Giac [F]

$$\int x^2W\left(ax^2\right)^2\,dx=\int x^2\,\mathrm{W}(ax^2)^2\,dx$$

input
```
integrate(x^2*lambert_w(a*x^2)^2,x, algorithm="giac")
```

output
```
integrate(x^2*lambert_w(a*x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2W\left(ax^2\right)^2\,dx=\int x^2\,\mathrm{LambertW}\left(a\,x^2\right)^2\,dx$$

input
```
int(x^2*LambertW(a*x^2)^2,x)
```

output
```
int(x^2*LambertW(a*x^2)^2, x)
```

Reduce [F]

$$\int x^2W\left(ax^2\right)^2\,dx=\int lambert\_w\left(a\,x^2\right)^2x^2dx$$

input
```
int(x^2*Lambert_W(a*x^2)^2,x)
```

output
```
int(lambert_w(a*x**2)**2*x**2,x)
```

3.112 $\int W(ax^2)^2 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 74

$$\int W\left(ax^2\right)^2 \, dx = 12x - \frac{6\sqrt{2}e^{\frac{1}{2}W(ax^2)}\Gamma\left(\frac{1}{2}, -\frac{1}{2}W(ax^2)\right)\sqrt{-W\left(ax^2\right)}}{ax} - 4xW\left(ax^2\right) + xW\left(ax^2\right)^2$$

output
```
12*x-6*2^(1/2)*exp(1/2*LambertW(a*x^2))*Pi^(1/2)*erfc(1/2*(-2*LambertW(a*x
^2))^(1/2))*(-LambertW(a*x^2))^(1/2)/a/x-4*x*LambertW(a*x^2)+x*LambertW(a*
x^2)^2
```

Mathematica [F]

$$\int W\left(ax^2\right)^2 \, dx = \int W\left(ax^2\right)^2 \, dx$$

input
```
Integrate[ProductLog[a*x^2]^2,x]
```

output
```
Integrate[ProductLog[a*x^2]^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W(ax^2)^2\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int W(ax^2)^2\,dx$$

input
```
Int[ProductLog[a*x^2]^2,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \mathrm{LambertW}\left(a\,x^2\right)^2 dx$$

input
```
int(LambertW(a*x^2)^2,x)
```

output
```
int(LambertW(a*x^2)^2,x)
```

Fricas [F]

$$\int W(ax^2)^2\,dx = \int \mathrm{W}(ax^2)^2\,dx$$

input
```
integrate(lambert_w(a*x^2)^2,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^2, x)
```

Sympy [F]

$$\int W(ax^2)^2\,dx = \int W^2(ax^2)\,dx$$

input
```
integrate(LambertW(a*x**2)**2,x)
```

output
```
Integral(LambertW(a*x**2)**2, x)
```

Maxima [F]

$$\int W(ax^2)^2\,dx = \int \mathrm{W}(ax^2)^2\,dx$$

input
```
integrate(lambert_w(a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^2, x)
```

Giac [F]

$$\int W(ax^2)^2\,dx = \int \mathrm{W}(ax^2)^2\,dx$$

input
```
integrate(lambert_w(a*x^2)^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int W(ax^2)^2 \, dx = \int \mathrm{LambertW}\left(a\,x^2\right)^2 \, dx$$

input
```
int(LambertW(a*x^2)^2,x)
```

output
```
int(LambertW(a*x^2)^2, x)
```

Reduce [F]

$$\int W(ax^2)^2 \, dx = \int lambert\_w\left(a\,x^2\right)^2 dx$$

input
```
int(Lambert_W(a*x^2)^2,x)
```

output
```
int(lambert_w(a*x**2)**2,x)
```

3.113 $\int \frac{W(ax^2)^2}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 74

$$\int \frac{W(ax^2)^2}{x^2}\,dx = -\frac{4W(ax^2)}{x} - \frac{2\sqrt{2}e^{\frac{3}{2}W(ax^2)}\Gamma\left(\frac{1}{2},\frac{1}{2}W(ax^2)\right)W(ax^2)^{3/2}}{ax^3} - \frac{W(ax^2)^2}{x}$$

output
```
-4*LambertW(a*x^2)/x-2*2^(1/2)*exp(3/2*LambertW(a*x^2))*Pi^(1/2)*erfc(1/2*
2^(1/2)*LambertW(a*x^2)^(1/2))*LambertW(a*x^2)^(3/2)/a/x^3-LambertW(a*x^2)
^2/x
```

Mathematica [F]

$$\int \frac{W(ax^2)^2}{x^2}\,dx = \int \frac{W(ax^2)^2}{x^2}\,dx$$

input
```
Integrate[ProductLog[a*x^2]^2/x^2,x]
```

output
```
Integrate[ProductLog[a*x^2]^2/x^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(ax^2\right)^2}{x^2}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{W\left(ax^2\right)^2}{x^2}dx$$

input
```
Int[ProductLog[a*x^2]^2/x^2,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{\mathrm{LambertW}\left(a\,x^2\right)^2}{x^2}dx$$

input
```
int(LambertW(a*x^2)^2/x^2,x)
```

output
```
int(LambertW(a*x^2)^2/x^2,x)
```

Fricas [F]

$$\int \frac{W\left(ax^2\right)^2}{x^2}\,dx = \int \frac{\mathrm{W}\left(ax^2\right)^2}{x^2}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^2,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^2/x^2, x)
```

Sympy [F]

$$\int \frac{W(ax^2)^2}{x^2}\,dx = \int \frac{W^2(ax^2)}{x^2}\,dx$$

input
```
integrate(LambertW(a*x**2)**2/x**2,x)
```

output
```
Integral(LambertW(a*x**2)**2/x**2, x)
```

Maxima [F]

$$\int \frac{W(ax^2)^2}{x^2}\,dx = \int \frac{\mathrm{W}(ax^2)^2}{x^2}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^2/x^2, x)
```

Giac [F]

$$\int \frac{W(ax^2)^2}{x^2}\,dx = \int \frac{\mathrm{W}(ax^2)^2}{x^2}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^2}{x^2}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)^2}{x^2}\,dx$$

input
```
int(LambertW(a*x^2)^2/x^2,x)
```

output
```
int(LambertW(a*x^2)^2/x^2, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^2}{x^2}\,dx = \frac{4\left(\int \frac{lambert\_w(a\,x^2)}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)+e^{lambert\_w(a\,x^2)}}dx\right)ax - lambert\_w(a\,x^2)^2}{x}$$

input
```
int(Lambert_W(a*x^2)^2/x^2,x)
```

output
```
(4*int(lambert_w(a*x**2)/(e**lambert_w(a*x**2)*lambert_w(a*x**2) + e**lamb
ert_w(a*x**2)),x)*a*x - lambert_w(a*x**2)**2)/x
```

3.114 $\int \frac{W(ax^2)^2}{x^4} dx$

Optimal result

Integrand size = 12, antiderivative size = 69

$$\int \frac{W(ax^2)^2}{x^4} dx = -\frac{W(ax^2)^2}{3x^3} - \frac{2\sqrt{\frac{2}{3}}e^{\frac{5}{2}W(ax^2)}\Gamma\left(\frac{1}{2},\frac{3}{2}W(ax^2)\right)W(ax^2)^{5/2}}{3ax^5}$$

output
```
-1/3*LambertW(a*x^2)^2/x^3-2/9*6^(1/2)*exp(5/2*LambertW(a*x^2))*Pi^(1/2)*e
rfc(1/2*6^(1/2)*LambertW(a*x^2)^(1/2))*LambertW(a*x^2)^(5/2)/a/x^5
```

Mathematica [F]

$$\int \frac{W(ax^2)^2}{x^4} dx = \int \frac{W(ax^2)^2}{x^4} dx$$

input
```
Integrate[ProductLog[a*x^2]^2/x^4,x]
```

output
```
Integrate[ProductLog[a*x^2]^2/x^4, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)^2}{x^4} dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{W(ax^2)^2}{x^4} dx$$

input
```
Int[ProductLog[a*x^2]^2/x^4,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{\mathrm{LambertW}\left(a\,x^{2}\right)^{2}}{x^{4}}dx$$

input
```
int(LambertW(a*x^2)^2/x^4,x)
```

output
```
int(LambertW(a*x^2)^2/x^4,x)
```

Fricas [F]

$$\int \frac{W(ax^2)^2}{x^4} dx = \int \frac{\mathrm{W}(ax^2)^2}{x^4} dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^4,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^2/x^4, x)
```

Sympy [F]

$$\int \frac{W(ax^2)^2}{x^4} \, dx = \int \frac{W^2(ax^2)}{x^4} \, dx$$

input
```
integrate(LambertW(a*x**2)**2/x**4,x)
```

output
```
Integral(LambertW(a*x**2)**2/x**4, x)
```

Maxima [F]

$$\int \frac{W(ax^2)^2}{x^4} \, dx = \int \frac{\mathrm{W}(ax^2)^2}{x^4} \, dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^4,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^2/x^4, x)
```

Giac [F]

$$\int \frac{W(ax^2)^2}{x^4} \, dx = \int \frac{\mathrm{W}(ax^2)^2}{x^4} \, dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^4,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^2}{x^4}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)^2}{x^4}\,dx$$

input
```
int(LambertW(a*x^2)^2/x^4,x)
```

output
```
int(LambertW(a*x^2)^2/x^4, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^2}{x^4}\,dx = \frac{4\left(\int \frac{lambert\_w(a\,x^2)}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)x^2+e^{lambert\_w(a\,x^2)}x^2}dx\right)a\,x^3 - lambert\_w(a\,x^2)^2}{3x^3}$$

input
```
int(Lambert_W(a*x^2)^2/x^4,x)
```

output
```
(4*int(lambert_w(a*x**2)/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x**2 + e*
*lambert_w(a*x**2)*x**2),x)*a*x**3 - lambert_w(a*x**2)**2)/(3*x**3)
```

3.115 $\int \frac{W(ax^2)^2}{x^6} dx$

Optimal result

Integrand size = 12, antiderivative size = 113

$$\int \frac{W(ax^2)^2}{x^6} dx = -\frac{W(ax^2)^2}{x^5} + \frac{\sqrt{\frac{5}{2}} e^{\frac{7}{2}W(ax^2)} \Gamma\left(\frac{1}{2}, \frac{5}{2}W(ax^2)\right) W(ax^2)^{7/2}}{ax^7} - \frac{e^{\frac{7}{2}W(ax^2)} \Gamma\left(\frac{1}{2}, \frac{5}{2}W(ax^2)\right) W(ax^2)^{7/2}}{\sqrt{10}ax^7}$$

output
```
-LambertW(a*x^2)^2/x^5+2/5*10^(1/2)*exp(7/2*LambertW(a*x^2))*Pi^(1/2)*erfc
(1/2*10^(1/2)*LambertW(a*x^2)^(1/2))*LambertW(a*x^2)^(7/2)/a/x^7
```

Mathematica [F]

$$\int \frac{W(ax^2)^2}{x^6} dx = \int \frac{W(ax^2)^2}{x^6} dx$$

input
```
Integrate[ProductLog[a*x^2]^2/x^6,x]
```

output
```
Integrate[ProductLog[a*x^2]^2/x^6, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(ax^2\right)^2}{x^6}\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{W\left(ax^2\right)^2}{x^6}dx$$

input
```
Int[ProductLog[a*x^2]^2/x^6,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{\operatorname{LambertW}\left(a\,x^{2}\right)^{2}}{x^{6}}dx$$

input
```
int(LambertW(a*x^2)^2/x^6,x)
```

output
```
int(LambertW(a*x^2)^2/x^6,x)
```

Fricas [F]

$$\int \frac{W\left(ax^2\right)^2}{x^6}\,dx = \int \frac{\mathrm{W}(ax^2)^2}{x^6}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^6,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^2/x^6, x)
```

Sympy [F]

$$\int \frac{W(ax^2)^2}{x^6}\,dx = \int \frac{W^2(ax^2)}{x^6}\,dx$$

input
```
integrate(LambertW(a*x**2)**2/x**6,x)
```

output
```
Integral(LambertW(a*x**2)**2/x**6, x)
```

Maxima [F]

$$\int \frac{W(ax^2)^2}{x^6}\,dx = \int \frac{\mathrm{W}(ax^2)^2}{x^6}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^6,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^2/x^6, x)
```

Giac [F]

$$\int \frac{W(ax^2)^2}{x^6}\,dx = \int \frac{\mathrm{W}(ax^2)^2}{x^6}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^6,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^2/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^2}{x^6} \, dx = \int \frac{\mathrm{LambertW}(a\,x^2)^2}{x^6} \, dx$$

input
```
int(LambertW(a*x^2)^2/x^6,x)
```

output
```
int(LambertW(a*x^2)^2/x^6, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^2}{x^6} \, dx = \frac{4\left(\int \frac{lambert\_w(a\,x^2)}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)x^4+e^{lambert\_w(a\,x^2)}x^4}dx\right)a\,x^5 - lambert\_w(a\,x^2)^2}{5x^5}$$

input
```
int(Lambert_W(a*x^2)^2/x^6,x)
```

output
```
(4*int(lambert_w(a*x**2)/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x**4 + e*
*lambert_w(a*x**2)*x**4),x)*a*x**5 - lambert_w(a*x**2)**2)/(5*x**5)
```

3.116 $\int \frac{W(ax^2)^2}{x^8} dx$

Optimal result

Integrand size = 12, antiderivative size = 82

$$\int \frac{W(ax^2)^2}{x^8} dx = -\frac{W(ax^2)^2}{3x^7} + \frac{4W(ax^2)^3}{3x^7} - \frac{2\sqrt{14}e^{\frac{9}{2}W(ax^2)}\Gamma\left(\frac{1}{2}, \frac{7}{2}W(ax^2)\right) W(ax^2)^{9/2}}{3ax^9}$$

output
```
-1/3*LambertW(a*x^2)^2/x^7+4/3*LambertW(a*x^2)^3/x^7-2/3*14^(1/2)*exp(9/2*
LambertW(a*x^2))*Pi^(1/2)*erfc(1/2*14^(1/2)*LambertW(a*x^2)^(1/2))*Lambert
W(a*x^2)^(9/2)/a/x^9
```

Mathematica [F]

$$\int \frac{W(ax^2)^2}{x^8} dx = \int \frac{W(ax^2)^2}{x^8} dx$$

input
```
Integrate[ProductLog[a*x^2]^2/x^8,x]
```

output
```
Integrate[ProductLog[a*x^2]^2/x^8, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)^2}{x^8} dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{W(ax^2)^2}{x^8} dx$$

input
```
Int[ProductLog[a*x^2]^2/x^8,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{\mathrm{LambertW}\left(a\,x^2\right)^2}{x^8} dx$$

input
```
int(LambertW(a*x^2)^2/x^8,x)
```

output
```
int(LambertW(a*x^2)^2/x^8,x)
```

Fricas [F]

$$\int \frac{W(ax^2)^2}{x^8} dx = \int \frac{\mathrm{W}(ax^2)^2}{x^8} dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^8,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^2/x^8, x)
```

Sympy [F]

$$\int \frac{W(ax^2)^2}{x^8}\,dx = \int \frac{W^2(ax^2)}{x^8}\,dx$$

input
```
integrate(LambertW(a*x**2)**2/x**8,x)
```

output
```
Integral(LambertW(a*x**2)**2/x**8, x)
```

Maxima [F]

$$\int \frac{W(ax^2)^2}{x^8}\,dx = \int \frac{\mathrm{W}(ax^2)^2}{x^8}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^8,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^2/x^8, x)
```

Giac [F]

$$\int \frac{W(ax^2)^2}{x^8}\,dx = \int \frac{\mathrm{W}(ax^2)^2}{x^8}\,dx$$

input
```
integrate(lambert_w(a*x^2)^2/x^8,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^2/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^2}{x^8}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)^2}{x^8}\,dx$$

input
```
int(LambertW(a*x^2)^2/x^8,x)
```

output
```
int(LambertW(a*x^2)^2/x^8, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^2}{x^8}\,dx = \frac{4\left(\int \frac{lambert\_w(a\,x^2)}{e^{lambert\_w(a\,x^2)}lambert\_w(a\,x^2)x^6+e^{lambert\_w(a\,x^2)}x^6}dx\right)a\,x^7 - lambert\_w(a\,x^2)^2}{7x^7}$$

input
```
int(Lambert_W(a*x^2)^2/x^8,x)
```

output
```
(4*int(lambert_w(a*x**2)/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x**6 + e*
*lambert_w(a*x**2)*x**6),x)*a*x**7 - lambert_w(a*x**2)**2)/(7*x**7)
```

3.117 $\int xW(ax^2)^3\,dx$

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int xW\left(ax^2\right)^3\,dx=-9x^2+\frac{9x^2}{W\left(ax^2\right)}+\frac{9}{2}x^2W\left(ax^2\right)-\frac{3}{2}x^2W\left(ax^2\right)^2+\frac{1}{2}x^2W\left(ax^2\right)^3$$

output
```
-9*x^2+9*x^2/LambertW(a*x^2)+9/2*x^2*LambertW(a*x^2)-3/2*x^2*LambertW(a*x^
2)^2+1/2*x^2*LambertW(a*x^2)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.69

$$\int xW\left(ax^2\right)^3\,dx=\frac{x^2\left(6+W(ax^2)^2\right)\left(3-3W(ax^2)+W(ax^2)^2\right)}{2W\left(ax^2\right)}$$

input
```
Integrate[x*ProductLog[a*x^2]^3,x]
```

output
```
(x^2*(6 + ProductLog[a*x^2]^2)*(3 - 3*ProductLog[a*x^2] + ProductLog[a*x^2
]^2))/(2*ProductLog[a*x^2])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {7172, 7205, 7205, 7205, 7266, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xW\left(ax^2\right)^3 \, dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{2}x^2W\left(ax^2\right)^3-3\int \frac{xW\left(ax^2\right)^3}{W\left(ax^2\right)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{2}x^2W\left(ax^2\right)^3-3\left(\frac{1}{2}x^2W\left(ax^2\right)^2-3\int \frac{xW\left(ax^2\right)^2}{W\left(ax^2\right)+1}dx\right)$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{2}x^2W\left(ax^2\right)^3-3\left(\frac{1}{2}x^2W\left(ax^2\right)^2-3\left(\frac{1}{2}x^2W\left(ax^2\right)-2\int \frac{xW\left(ax^2\right)}{W\left(ax^2\right)+1}dx\right)\right)$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{2}x^2W\left(ax^2\right)^3-3\left(\frac{1}{2}x^2W\left(ax^2\right)^2-3\left(\frac{1}{2}x^2W\left(ax^2\right)-2\left(\frac{x^2}{2}-\int \frac{x}{W\left(ax^2\right)+1}dx\right)\right)\right)$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}x^2W\left(ax^2\right)^3-3\left(\frac{1}{2}x^2W\left(ax^2\right)^2-3\left(\frac{1}{2}x^2W\left(ax^2\right)-2\left(\frac{x^2}{2}-\frac{1}{2}\int \frac{1}{W\left(ax^2\right)+1}dx^2\right)\right)\right)$$

$$\Big\downarrow \text{7176}$$

$$\frac{1}{2}x^2W\left(ax^2\right)^3-3\left(\frac{1}{2}x^2W\left(ax^2\right)^2-3\left(\frac{1}{2}x^2W\left(ax^2\right)-2\left(\frac{x^2}{2}-\frac{x^2}{2W\left(ax^2\right)}\right)\right)\right)$$

input
```
Int[x*ProductLog[a*x^2]^3,x]
```

output
```
(x^2*ProductLog[a*x^2]^3)/2 - 3*((x^2*ProductLog[a*x^2]^2)/2 - 3*(-2*(x^2/
2 - x^2/(2*ProductLog[a*x^2])) + (x^2*ProductLog[a*x^2])/2))
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{-3\,\mathrm{LambertW}\left(a\,x^{2}\right)^{2} a\,x^{2}+9 a\,x^{2}\,\mathrm{LambertW}\left(a\,x^{2}\right)-18 a\,x^{2}+\frac{18 a\,x^{2}}{\mathrm{LambertW}\left(a\,x^{2}\right)}+\mathrm{LambertW}\left(a\,x^{2}\right)^{3} a\,x^{2}}{2 a}$	66
default	$\frac{-3\,\mathrm{LambertW}\left(a\,x^{2}\right)^{2} a\,x^{2}+9 a\,x^{2}\,\mathrm{LambertW}\left(a\,x^{2}\right)-18 a\,x^{2}+\frac{18 a\,x^{2}}{\mathrm{LambertW}\left(a\,x^{2}\right)}+\mathrm{LambertW}\left(a\,x^{2}\right)^{3} a\,x^{2}}{2 a}$	66
parallelrisch	$-\frac{-\,\mathrm{LambertW}\left(a\,x^{2}\right)^{4} x^{2}+3 x^{2}\,\mathrm{LambertW}\left(a\,x^{2}\right)^{3}-9 x^{2}\,\mathrm{LambertW}\left(a\,x^{2}\right)^{2}+18 x^{2}\,\mathrm{LambertW}\left(a\,x^{2}\right)-18 x^{2}}{2\,\mathrm{LambertW}\left(a\,x^{2}\right)}$	67

input
```
int(x*LambertW(a*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(-3*LambertW(a*x^2)^2*a*x^2+9*a*x^2*LambertW(a*x^2)-18*a*x^2+18*a*x^
2/LambertW(a*x^2)+LambertW(a*x^2)^3*a*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int xW\left(ax^2\right)^3\,dx=\frac{x^2\,\mathrm{W}(ax^2)^4-3\,x^2\,\mathrm{W}(ax^2)^3+9\,x^2\,\mathrm{W}(ax^2)^2-18\,x^2\,\mathrm{W}(ax^2)+18\,x^2}{2\,\mathrm{W}(ax^2)}$$

input
```
integrate(x*lambert_w(a*x^2)^3,x, algorithm="fricas")
```

output
```
1/2*(x^2*lambert_w(a*x^2)^4 - 3*x^2*lambert_w(a*x^2)^3 + 9*x^2*lambert_w(a
*x^2)^2 - 18*x^2*lambert_w(a*x^2) + 18*x^2)/lambert_w(a*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int xW\left(ax^2\right)^3\,dx=\begin{cases}\frac{x^2W^3(ax^2)}{2}-\frac{3x^2W^2(ax^2)}{2}+\frac{9x^2W(ax^2)}{2}-9x^2+\frac{9x^2}{W(ax^2)} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(x*LambertW(a*x**2)**3,x)
```

output
```
Piecewise((x**2*LambertW(a*x**2)**3/2 - 3*x**2*LambertW(a*x**2)**2/2 + 9*x
**2*LambertW(a*x**2)/2 - 9*x**2 + 9*x**2/LambertW(a*x**2), Ne(a, 0)), (0,
True))
```

Maxima [F]

$$\int xW\left(ax^2\right)^3\,dx=\int x\,\mathrm{W}(ax^2)^3\,dx$$

input
```
integrate(x*lambert_w(a*x^2)^3,x, algorithm="maxima")
```

output
```
integrate(x*lambert_w(a*x^2)^3, x)
```

Giac [F]

$$\int xW\left(ax^2\right)^3\,dx=\int x\,\mathrm{W}(ax^2)^3\,dx$$

input
```
integrate(x*lambert_w(a*x^2)^3,x, algorithm="giac")
```

output
```
integrate(x*lambert_w(a*x^2)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int xW\left(ax^2\right)^3\,dx=\int x\,\mathrm{LambertW}\left(a\,x^2\right)^3\,dx$$

input
```
int(x*LambertW(a*x^2)^3,x)
```

output
```
int(x*LambertW(a*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int xW\left(ax^2\right)^3\,dx = \frac{e^{lambert\_w(a\,x^2)}\left(lambert\_w(a\,x^2)^4-3lambert\_w(a\,x^2)^3+9lambert\_w(a\,x^2)^2-18lambert\_w(a\,x^2)+18\right)}{2a}$$

input
```
int(x*Lambert_W(a*x^2)^3,x)
```

output
```
(e**lambert_w(a*x**2)*(lambert_w(a*x**2)**4 - 3*lambert_w(a*x**2)**3 + 9*l
ambert_w(a*x**2)**2 - 18*lambert_w(a*x**2) + 18))/(2*a)
```

3.118 $\int \frac{W(ax^2)^3}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{W(ax^2)^3}{x} dx = \frac{1}{6}W(ax^2)^3 + \frac{1}{8}W(ax^2)^4$$

output
```
1/6*LambertW(a*x^2)^3+1/8*LambertW(a*x^2)^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{W(ax^2)^3}{x} dx = \frac{1}{6}W(ax^2)^3 + \frac{1}{8}W(ax^2)^4$$

input
```
Integrate[ProductLog[a*x^2]^3/x,x]
```

output
```
ProductLog[a*x^2]^3/6 + ProductLog[a*x^2]^4/8
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)^3}{x} dx$$
$$\Big\downarrow \ 7173$$
$$\int \frac{W(ax^2)^4}{x(W(ax^2)+1)}dx+\frac{1}{6}W(ax^2)^3$$
$$\Big\downarrow \ 7200$$
$$\frac{1}{8}W(ax^2)^4+\frac{1}{6}W(ax^2)^3$$

input
```
Int[ProductLog[a*x^2]^3/x,x]
```

output
```
ProductLog[a*x^2]^3/6 + ProductLog[a*x^2]^4/8
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\operatorname{LambertW}(a\,x^2)^3}{6}+\frac{\operatorname{LambertW}(a\,x^2)^4}{8}$	22
default	$\frac{\operatorname{LambertW}(a\,x^2)^3}{6}+\frac{\operatorname{LambertW}(a\,x^2)^4}{8}$	22

input
```
int(LambertW(a*x^2)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/6*LambertW(a*x^2)^3+1/8*LambertW(a*x^2)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{W(ax^2)^3}{x}\,dx = \frac{1}{8}\,\mathrm{W}(ax^2)^4 + \frac{1}{6}\,\mathrm{W}(ax^2)^3$$

input
```
integrate(lambert_w(a*x^2)^3/x,x, algorithm="fricas")
```

output
```
1/8*lambert_w(a*x^2)^4 + 1/6*lambert_w(a*x^2)^3
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{W(ax^2)^3}{x}\,dx = \frac{W^4(ax^2)}{8} + \frac{W^3(ax^2)}{6}$$

input
```
integrate(LambertW(a*x**2)**3/x,x)
```

output
```
LambertW(a*x**2)**4/8 + LambertW(a*x**2)**3/6
```

Maxima [F]

$$\int \frac{W(ax^2)^3}{x}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^3/x, x)
```

Giac [F]

$$\int \frac{W(ax^2)^3}{x}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^3}{x}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)^3}{x}\,dx$$

input
```
int(LambertW(a*x^2)^3/x,x)
```

output
```
int(LambertW(a*x^2)^3/x, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^3}{x} \, dx = \int \frac{lambert\_w(a\,x^2)^3}{x} dx$$

input
```
int(Lambert_W(a*x^2)^3/x,x)
```

output
```
int(lambert_w(a*x**2)**3/x,x)
```

3.119 $\int \frac{W(ax^2)^3}{x^3} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 44

$$\int \frac{W(ax^2)^3}{x^3} \, dx = -\frac{3W(ax^2)}{2x^2} - \frac{3W(ax^2)^2}{2x^2} - \frac{W(ax^2)^3}{2x^2}$$

output
```
-3/2*LambertW(a*x^2)/x^2-3/2*LambertW(a*x^2)^2/x^2-1/2*LambertW(a*x^2)^3/x
^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{W(ax^2)^3}{x^3} \, dx = -\frac{3W(ax^2)}{2x^2} - \frac{3W(ax^2)^2}{2x^2} - \frac{W(ax^2)^3}{2x^2}$$

input
```
Integrate[ProductLog[a*x^2]^3/x^3,x]
```

output
```
(-3*ProductLog[a*x^2])/(2*x^2) - (3*ProductLog[a*x^2]^2)/(2*x^2) - Product
Log[a*x^2]^3/(2*x^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7172, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)^3}{x^3}dx$$

$$\Big\downarrow \text{ 7172}$$

$$3\int \frac{W(ax^2)^3}{x^3(W(ax^2)+1)}dx - \frac{W(ax^2)^3}{2x^2}$$

$$\Big\downarrow \text{ 7205}$$

$$3\left(\int \frac{W(ax^2)^2}{x^3(W(ax^2)+1)}dx - \frac{W(ax^2)^2}{2x^2}\right) - \frac{W(ax^2)^3}{2x^2}$$

$$\Big\downarrow \text{ 7201}$$

$$3\left(-\frac{W(ax^2)^2}{2x^2} - \frac{W(ax^2)}{2x^2}\right) - \frac{W(ax^2)^3}{2x^2}$$

input
```
Int[ProductLog[a*x^2]^3/x^3,x]
```

output
```
-1/2*ProductLog[a*x^2]^3/x^2 + 3*(-1/2*ProductLog[a*x^2]/x^2 - ProductLog[
a*x^2]^2/(2*x^2))
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

method	result	size
parallelrisch	$-\frac{\operatorname{LambertW}\left(a\,x^{2}\right)^{3}+3\operatorname{LambertW}\left(a\,x^{2}\right)^{2}+3\operatorname{LambertW}\left(a\,x^{2}\right)}{2x^{2}}$	33

input
```
int(LambertW(a*x^2)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2*(LambertW(a*x^2)^3+3*LambertW(a*x^2)^2+3*LambertW(a*x^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{W(ax^2)^3}{x^3}\,dx = -\frac{\mathrm{W}(ax^2)^3 + 3\,\mathrm{W}(ax^2)^2 + 3\,\mathrm{W}(ax^2)}{2\,x^2}$$

input
```
integrate(lambert_w(a*x^2)^3/x^3,x, algorithm="fricas")
```

output
```
-1/2*(lambert_w(a*x^2)^3 + 3*lambert_w(a*x^2)^2 + 3*lambert_w(a*x^2))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{W(ax^2)^3}{x^3}\,dx = -\frac{W^3(ax^2)}{2x^2} - \frac{3W^2(ax^2)}{2x^2} - \frac{3W(ax^2)}{2x^2}$$

input
```
integrate(LambertW(a*x**2)**3/x**3,x)
```

output
```
-LambertW(a*x**2)**3/(2*x**2) - 3*LambertW(a*x**2)**2/(2*x**2) - 3*Lambert
W(a*x**2)/(2*x**2)
```

Maxima [F]

$$\int \frac{W(ax^2)^3}{x^3}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x^3}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^3/x^3, x)
```

Giac [F]

$$\int \frac{W(ax^2)^3}{x^3}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x^3}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^3}{x^3}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)^3}{x^3}\,dx$$

input
```
int(LambertW(a*x^2)^3/x^3,x)
```

output
```
int(LambertW(a*x^2)^3/x^3, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^3}{x^3}\,dx = \frac{24\left(\int \frac{lambert\_w(a\,x^2)^2}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)x+e^{lambert\_w(a\,x^2)}x}dx\right)a\,x^2-18\left(\int \frac{lambert\_w(a\,x^2)}{lambert\_w(a\,x^2)x^3+x^3}dx\right)x^2-18\Big(}{8x^2}$$

input
```
int(Lambert_W(a*x^2)^3/x^3,x)
```

output
```
(24*int(lambert_w(a*x**2)**2/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x + e
**lambert_w(a*x**2)*x),x)*a*x**2 - 18*int(lambert_w(a*x**2)/(lambert_w(a*x
**2)*x**3 + x**3),x)*x**2 - 18*int(1/(lambert_w(a*x**2)*x**3 + x**3),x)*x*
*2 - 4*lambert_w(a*x**2)**3 - 9)/(8*x**2)
```

3.120 $\int \frac{W(ax^2)^3}{x^5} dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{W(ax^2)^3}{x^5} dx = -\frac{3W(ax^2)^2}{8x^4} - \frac{W(ax^2)^3}{4x^4}$$

output
```
-3/8*LambertW(a*x^2)^2/x^4-1/4*LambertW(a*x^2)^3/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{W(ax^2)^3}{x^5} dx = -\frac{3W(ax^2)^2}{8x^4} - \frac{W(ax^2)^3}{4x^4}$$

input
```
Integrate[ProductLog[a*x^2]^3/x^5,x]
```

output
```
(-3*ProductLog[a*x^2]^2)/(8*x^4) - ProductLog[a*x^2]^3/(4*x^4)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7172, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)^3}{x^5} dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{3}{2} \int \frac{W(ax^2)^3}{x^5 (W(ax^2)+1)} dx - \frac{W(ax^2)^3}{4x^4}$$

$$\Big\downarrow \text{7201}$$

$$-\frac{W(ax^2)^3}{4x^4} - \frac{3W(ax^2)^2}{8x^4}$$

input
```
Int[ProductLog[a*x^2]^3/x^5,x]
```

output
```
(-3*ProductLog[a*x^2]^2)/(8*x^4) - ProductLog[a*x^2]^3/(4*x^4)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
parallelrisch	$\frac{-2\,\mathrm{LambertW}\left(a\,x^{2}\right)^{3}-3\,\mathrm{LambertW}\left(a\,x^{2}\right)^{2}}{8 x^{4}}$	27

input
```
int(LambertW(a*x^2)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/8/x^4*(-2*LambertW(a*x^2)^3-3*LambertW(a*x^2)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{W(ax^2)^3}{x^5} \, dx = -\frac{2 \; \mathrm{W}(ax^2)^3 + 3 \; \mathrm{W}(ax^2)^2}{8 \, x^4}$$

input
```
integrate(lambert_w(a*x^2)^3/x^5,x, algorithm="fricas")
```

output
```
-1/8*(2*lambert_w(a*x^2)^3 + 3*lambert_w(a*x^2)^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{W(ax^2)^3}{x^5}\,dx = -\frac{W^3(ax^2)}{4x^4} - \frac{3W^2(ax^2)}{8x^4}$$

input
```
integrate(LambertW(a*x**2)**3/x**5,x)
```

output
```
-LambertW(a*x**2)**3/(4*x**4) - 3*LambertW(a*x**2)**2/(8*x**4)
```

Maxima [F]

$$\int \frac{W(ax^2)^3}{x^5}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x^5}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^5,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^3/x^5, x)
```

Giac [F]

$$\int \frac{W(ax^2)^3}{x^5}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x^5}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^5,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^3/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^3}{x^5}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)^3}{x^5}\,dx$$

input
```
int(LambertW(a*x^2)^3/x^5,x)
```

output
```
int(LambertW(a*x^2)^3/x^5, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^3}{x^5}\,dx = \frac{6\left(\int \frac{lambert\_w(a\,x^2)^2}{e^{lambert\_w(a\,x^2)}lambert\_w(a\,x^2)x^3+e^{lambert\_w(a\,x^2)}x^3}dx\right)a\,x^4-lambert\_w(a\,x^2)^3}{4x^4}$$

input
```
int(Lambert_W(a*x^2)^3/x^5,x)
```

output
```
(6*int(lambert_w(a*x**2)**2/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x**3 +
 e**lambert_w(a*x**2)*x**3),x)*a*x**4 - lambert_w(a*x**2)**3)/(4*x**4)
```

3.121 $\int \frac{W(ax^2)^3}{x^7} dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{W(ax^2)^3}{x^7} dx = \frac{1}{2}a^3 \text{ExpIntegralEi}\left(-3W\left(ax^2\right)\right) - \frac{W(ax^2)^3}{6x^6}$$

output
```
1/2*a^3*Ei(-3*LambertW(a*x^2))-1/6*LambertW(a*x^2)^3/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{W(ax^2)^3}{x^7} dx = \frac{1}{2}a^3 \text{ExpIntegralEi}\left(-3W\left(ax^2\right)\right) - \frac{W(ax^2)^3}{6x^6}$$

input
```
Integrate[ProductLog[a*x^2]^3/x^7,x]
```

output
```
(a^3*ExpIntegralEi[-3*ProductLog[a*x^2]])/2 - ProductLog[a*x^2]^3/(6*x^6)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7172, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)^3}{x^7} dx$$

$$\Big\downarrow \text{7172}$$

$$\int \frac{W(ax^2)^3}{x^7 (W(ax^2)+1)} dx - \frac{W(ax^2)^3}{6x^6}$$

$$\Big\downarrow \text{7202}$$

$$\frac{1}{2} a^3 \text{ExpIntegralEi}\left(-3W(ax^2)\right) - \frac{W(ax^2)^3}{6x^6}$$

input
```
Int[ProductLog[a*x^2]^3/x^7,x]
```

output
```
(a^3*ExpIntegralEi[-3*ProductLog[a*x^2]])/2 - ProductLog[a*x^2]^3/(6*x^6)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [F]

$$\int \frac{\text{LambertW}\left(a\,x^{2}\right)^{3}}{x^{7}}dx$$

input
```
int(LambertW(a*x^2)^3/x^7,x)
```

output
```
int(LambertW(a*x^2)^3/x^7,x)
```

Fricas [F]

$$\int \frac{W\left(ax^2\right)^3}{x^7}\,dx = \int \frac{\mathrm{W}\left(ax^2\right)^3}{x^7}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^7,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^3/x^7, x)
```

Sympy [F]

$$\int \frac{W\left(ax^2\right)^3}{x^7}\,dx = \int \frac{W^{3}\left(ax^{2}\right)}{x^{7}}\,dx$$

input
```
integrate(LambertW(a*x**2)**3/x**7,x)
```

output
```
Integral(LambertW(a*x**2)**3/x**7, x)
```

Maxima [F]

$$\int \frac{W(ax^2)^3}{x^7}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x^7}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^7,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^3/x^7, x)
```

Giac [F]

$$\int \frac{W(ax^2)^3}{x^7}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x^7}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^7,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^3/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^3}{x^7}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)^3}{x^7}\,dx$$

input
```
int(LambertW(a*x^2)^3/x^7,x)
```

output
```
int(LambertW(a*x^2)^3/x^7, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^3}{x^7} dx$$
$$= \frac{6\left(\int \frac{lambert\_w(a x^2)^2}{e^{lambert\_w(a x^2)} lambert\_w(a x^2) x^5 + e^{lambert\_w(a x^2)} x^5} dx\right) a x^6 - lambert\_w(a x^2)^3}{6x^6}$$

input
```
int(Lambert_W(a*x^2)^3/x^7,x)
```

output
```
(6*int(lambert_w(a*x**2)**2/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x**5 +
 e**lambert_w(a*x**2)*x**5),x)*a*x**6 - lambert_w(a*x**2)**3)/(6*x**6)
```

3.122 $\int \frac{W(ax^2)^3}{x^9} dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{W(ax^2)^3}{x^9} dx = -\frac{3}{2}a^4 \text{ExpIntegralEi}\left(-4W\left(ax^2\right)\right) - \frac{W(ax^2)^3}{2x^8}$$

output
```
-3/2*a^4*Ei(-4*LambertW(a*x^2))-1/2*LambertW(a*x^2)^3/x^8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{W(ax^2)^3}{x^9} dx = -\frac{3}{2}a^4 \text{ExpIntegralEi}\left(-4W\left(ax^2\right)\right) - \frac{W(ax^2)^3}{2x^8}$$

input
```
Integrate[ProductLog[a*x^2]^3/x^9,x]
```

output
```
(-3*a^4*ExpIntegralEi[-4*ProductLog[a*x^2]])/2 - ProductLog[a*x^2]^3/(2*x^
8)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7173, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)^3}{x^9} dx$$

$$\Big\downarrow \text{7173}$$

$$-3\int \frac{W(ax^2)^4}{x^9 (W(ax^2)+1)}dx - \frac{W(ax^2)^3}{2x^8}$$

$$\Big\downarrow \text{7202}$$

$$-\frac{3}{2}a^4 \,\text{ExpIntegralEi}\left(-4W(ax^2)\right) - \frac{W(ax^2)^3}{2x^8}$$

input
```
Int[ProductLog[a*x^2]^3/x^9,x]
```

output
```
(-3*a^4*ExpIntegralEi[-4*ProductLog[a*x^2]])/2 - ProductLog[a*x^2]^3/(2*x^
8)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [F]

$$\int \frac{\mathrm{LambertW}\left(a\,x^{2}\right)^{3}}{x^{9}}dx$$

input
```
int(LambertW(a*x^2)^3/x^9,x)
```

output
```
int(LambertW(a*x^2)^3/x^9,x)
```

Fricas [F]

$$\int \frac{W(ax^2)^3}{x^9}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x^9}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^9,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^3/x^9, x)
```

Sympy [F]

$$\int \frac{W(ax^2)^3}{x^9}\,dx = \int \frac{W^3(ax^2)}{x^9}\,dx$$

input
```
integrate(LambertW(a*x**2)**3/x**9,x)
```

output
```
Integral(LambertW(a*x**2)**3/x**9, x)
```

Maxima [F]

$$\int \frac{W(ax^2)^3}{x^9} dx = \int \frac{\mathrm{W}(ax^2)^3}{x^9} dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^9,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^3/x^9, x)
```

Giac [F]

$$\int \frac{W(ax^2)^3}{x^9} dx = \int \frac{\mathrm{W}(ax^2)^3}{x^9} dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^9,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^3/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^3}{x^9} dx = \int \frac{\mathrm{LambertW}(a\,x^2)^3}{x^9} dx$$

input
```
int(LambertW(a*x^2)^3/x^9,x)
```

output
```
int(LambertW(a*x^2)^3/x^9, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^3}{x^9}\,dx = \frac{6\left(\int \frac{lambert\_w(a\,x^2)^2}{e^{lambert\_w(a\,x^2)}lambert\_w(a\,x^2)x^7+e^{lambert\_w(a\,x^2)}x^7}dx\right)a\,x^8-lambert\_w(a\,x^2)^3}{8x^8}$$

input
```
int(Lambert_W(a*x^2)^3/x^9,x)
```

output
```
(6*int(lambert_w(a*x**2)**2/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x**7 +
 e**lambert_w(a*x**2)*x**7),x)*a*x**8 - lambert_w(a*x**2)**3)/(8*x**8)
```

3.123 $\int \frac{W(ax^2)^3}{x^{11}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 47

$$\int \frac{W(ax^2)^3}{x^{11}} \, dx = \frac{15}{4} a^5 \operatorname{ExpIntegralEi}\left(-5W(ax^2)\right) - \frac{W(ax^2)^3}{4x^{10}} + \frac{3W(ax^2)^4}{4x^{10}}$$

output
```
15/4*a^5*Ei(-5*LambertW(a*x^2))-1/4*LambertW(a*x^2)^3/x^10+3/4*LambertW(a*
x^2)^4/x^10
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{W(ax^2)^3}{x^{11}} \, dx = \frac{15}{4} a^5 \operatorname{ExpIntegralEi}\left(-5W(ax^2)\right) - \frac{W(ax^2)^3}{4x^{10}} + \frac{3W(ax^2)^4}{4x^{10}}$$

input
```
Integrate[ProductLog[a*x^2]^3/x^11,x]
```

output
```
(15*a^5*ExpIntegralEi[-5*ProductLog[a*x^2]])/4 - ProductLog[a*x^2]^3/(4*x^
10) + (3*ProductLog[a*x^2]^4)/(4*x^10)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7173, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)^3}{x^{11}} \, dx$$

$$\Big\downarrow \text{ 7173}$$

$$-\frac{3}{2}\int \frac{W(ax^2)^4}{x^{11}(W(ax^2)+1)}dx-\frac{W(ax^2)^3}{4x^{10}}$$

$$\Big\downarrow \text{ 7206}$$

$$-\frac{3}{2}\left(-5\int \frac{W(ax^2)^5}{x^{11}(W(ax^2)+1)}dx-\frac{W(ax^2)^4}{2x^{10}}\right)-\frac{W(ax^2)^3}{4x^{10}}$$

$$\Big\downarrow \text{ 7202}$$

$$-\frac{3}{2}\left(-\frac{5}{2}a^5\,\text{ExpIntegralEi}\left(-5W(ax^2)\right)-\frac{W(ax^2)^4}{2x^{10}}\right)-\frac{W(ax^2)^3}{4x^{10}}$$

input
```
Int[ProductLog[a*x^2]^3/x^11,x]
```

output
```
-1/4*ProductLog[a*x^2]^3/x^10 - (3*((-5*a^5*ExpIntegralEi[-5*ProductLog[a*
x^2]])/2 - ProductLog[a*x^2]^4/(2*x^10)))/2
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{\text{LambertW}\left(a\,x^2\right)^3}{x^{11}}dx$$

input
```
int(LambertW(a*x^2)^3/x^11,x)
```

output
```
int(LambertW(a*x^2)^3/x^11,x)
```

Fricas [F]

$$\int \frac{W(ax^2)^3}{x^{11}}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x^{11}}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^11,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^3/x^11, x)
```

Sympy [F]

$$\int \frac{W(ax^2)^3}{x^{11}}\,dx = \int \frac{W^3(ax^2)}{x^{11}}\,dx$$

input
```
integrate(LambertW(a*x**2)**3/x**11,x)
```

output
```
Integral(LambertW(a*x**2)**3/x**11, x)
```

Maxima [F]

$$\int \frac{W(ax^2)^3}{x^{11}}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x^{11}}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^11,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^3/x^11, x)
```

Giac [F]

$$\int \frac{W(ax^2)^3}{x^{11}}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x^{11}}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^11,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^3/x^11, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^3}{x^{11}}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)^3}{x^{11}}\,dx$$

input
```
int(LambertW(a*x^2)^3/x^11,x)
```

output
```
int(LambertW(a*x^2)^3/x^11, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^3}{x^{11}}\,dx = \frac{6\left(\int \frac{lambert\_w(a\,x^2)^2}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)x^9+e^{lambert\_w(a\,x^2)}x^9}dx\right)a\,x^{10}-lambert\_w(a\,x^2)^3}{10x^{10}}$$

input
```
int(Lambert_W(a*x^2)^3/x^11,x)
```

output
```
(6*int(lambert_w(a*x**2)**2/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x**9 +
 e**lambert_w(a*x**2)*x**9),x)*a*x**10 - lambert_w(a*x**2)**3)/(10*x**10)
```

3.124 $\int x^2 W(ax^2)^3 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 127

$$\int x^2 W\left(ax^2\right)^3 \, dx = -\frac{70x^3}{27} + \frac{70x^3}{27W\left(ax^2\right)} - \frac{35\sqrt{\frac{2}{3}}e^{-\frac{1}{2}W\left(ax^2\right)}x\Gamma\left(\frac{1}{2},-\frac{3}{2}W\left(ax^2\right)\right)\sqrt{-W\left(ax^2\right)}}{27aW\left(ax^2\right)} + \frac{14}{9}x^3W\left(ax^2\right) - \frac{2}{3}x^3W\left(ax^2\right)^2 + \frac{1}{3}x^3W\left(ax^2\right)^3$$

output
```
-70/27*x^3+70/27*x^3/LambertW(a*x^2)-35/81*6^(1/2)*x*Pi^(1/2)*erfc(1/2*(-6
*LambertW(a*x^2))^(1/2))*(-LambertW(a*x^2))^(1/2)/a/exp(1/2*LambertW(a*x^2
))/LambertW(a*x^2)+14/9*x^3*LambertW(a*x^2)-2/3*x^3*LambertW(a*x^2)^2+1/3*
x^3*LambertW(a*x^2)^3
```

Mathematica [F]

$$\int x^2 W\left(ax^2\right)^3 \, dx = \int x^2 W\left(ax^2\right)^3 \, dx$$

input
```
Integrate[x^2*ProductLog[a*x^2]^3,x]
```

output
```
Integrate[x^2*ProductLog[a*x^2]^3, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 W\left(ax^2\right)^3 \, dx$$
$$\Big\downarrow \; 7299$$
$$\int x^2 W\left(ax^2\right)^3 dx$$

input
```
Int[x^2*ProductLog[a*x^2]^3,x]
```

output
```
$Aborted
```

Maple [F]

$$\int x^2 \operatorname{LambertW}\left(a\,x^2\right)^3 dx$$

input
```
int(x^2*LambertW(a*x^2)^3,x)
```

output
```
int(x^2*LambertW(a*x^2)^3,x)
```

Fricas [F]

$$\int x^2 W\left(ax^2\right)^3 \, dx = \int x^2 \operatorname{W}(ax^2)^3 \, dx$$

input
```
integrate(x^2*lambert_w(a*x^2)^3,x, algorithm="fricas")
```

output
```
integral(x^2*lambert_w(a*x^2)^3, x)
```

Sympy [F]

$$\int x^2 W\left(ax^2\right)^3 \, dx = \int x^2 W^3\left(ax^2\right) \, dx$$

input
```
integrate(x**2*LambertW(a*x**2)**3,x)
```

output
```
Integral(x**2*LambertW(a*x**2)**3, x)
```

Maxima [F]

$$\int x^2 W\left(ax^2\right)^3 \, dx = \int x^2 \operatorname{W}(ax^2)^3 \, dx$$

input
```
integrate(x^2*lambert_w(a*x^2)^3,x, algorithm="maxima")
```

output
```
integrate(x^2*lambert_w(a*x^2)^3, x)
```

Giac [F]

$$\int x^2 W(ax^2)^3 \, dx = \int x^2 \, \mathrm{W}(ax^2)^3 \, dx$$

input
```
integrate(x^2*lambert_w(a*x^2)^3,x, algorithm="giac")
```

output
```
integrate(x^2*lambert_w(a*x^2)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 W(ax^2)^3 \, dx = \int x^2 \, \mathrm{LambertW}\left(a\,x^2\right)^3 \, dx$$

input
```
int(x^2*LambertW(a*x^2)^3,x)
```

output
```
int(x^2*LambertW(a*x^2)^3, x)
```

Reduce [F]

$$\int x^2 W(ax^2)^3 \, dx = \int lambert\_w\left(a\,x^2\right)^3 x^2 dx$$

input
```
int(x^2*Lambert_W(a*x^2)^3,x)
```

output
```
int(lambert_w(a*x**2)**3*x**2,x)
```

3.125 $\int W(ax^2)^3 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 85

$$\int W(ax^2)^3 \, dx = -90x + \frac{45\sqrt{2}e^{\frac{1}{2}W(ax^2)}\Gamma\left(\frac{1}{2}, -\frac{1}{2}W(ax^2)\right)\sqrt{-W(ax^2)}}{ax} + 30xW(ax^2) - 6xW(ax^2)^2 + xW(ax^2)^3$$

output
```
-90*x+45*2^(1/2)*exp(1/2*LambertW(a*x^2))*Pi^(1/2)*erfc(1/2*(-2*LambertW(a
*x^2))^(1/2))*(-LambertW(a*x^2))^(1/2)/a/x+30*x*LambertW(a*x^2)-6*x*Lamber
tW(a*x^2)^2+x*LambertW(a*x^2)^3
```

Mathematica [F]

$$\int W(ax^2)^3 \, dx = \int W(ax^2)^3 \, dx$$

input
```
Integrate[ProductLog[a*x^2]^3,x]
```

output
```
Integrate[ProductLog[a*x^2]^3, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W\left(ax^2\right)^3 \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int W\left(ax^2\right)^3 dx$$

input
```
Int[ProductLog[a*x^2]^3,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \operatorname{LambertW}\left(a\,x^2\right)^3 dx$$

input
```
int(LambertW(a*x^2)^3,x)
```

output
```
int(LambertW(a*x^2)^3,x)
```

Fricas [F]

$$\int W\left(ax^2\right)^3 \, dx = \int \mathrm{W}(ax^2)^3 \, dx$$

input
```
integrate(lambert_w(a*x^2)^3,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^3, x)
```

Sympy [F]

$$\int W\left(ax^2\right)^3 \, dx = \int W^3\left(ax^2\right) \, dx$$

input
```
integrate(LambertW(a*x**2)**3,x)
```

output
```
Integral(LambertW(a*x**2)**3, x)
```

Maxima [F]

$$\int W\left(ax^2\right)^3 \, dx = \int \mathrm{W}(ax^2)^3 \, dx$$

input
```
integrate(lambert_w(a*x^2)^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^3, x)
```

Giac [F]

$$\int W\left(ax^2\right)^3 \, dx = \int \mathrm{W}(ax^2)^3 \, dx$$

input
```
integrate(lambert_w(a*x^2)^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int W\left(ax^2\right)^3\,dx = \int \mathrm{LambertW}\left(a\,x^2\right)^3\,dx$$

input
```
int(LambertW(a*x^2)^3,x)
```

output
```
int(LambertW(a*x^2)^3, x)
```

Reduce [F]

$$\int W\left(ax^2\right)^3\,dx = \int lambert\_w\left(a\,x^2\right)^3 dx$$

input
```
int(Lambert_W(a*x^2)^3,x)
```

output
```
int(lambert_w(a*x**2)**3,x)
```

3.126 $\int \frac{W(ax^2)^3}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 87

$$\int \frac{W(ax^2)^3}{x^2}\,dx = -\frac{18W(ax^2)}{x} - \frac{9\sqrt{2}e^{\frac{3}{2}W(ax^2)}\Gamma\left(\frac{1}{2},\frac{1}{2}W(ax^2)\right)W(ax^2)^{3/2}}{ax^3} - \frac{6W(ax^2)^2}{x} - \frac{W(ax^2)^3}{x}$$

output
```
-18*LambertW(a*x^2)/x-9*2^(1/2)*exp(3/2*LambertW(a*x^2))*Pi^(1/2)*erfc(1/2
*2^(1/2)*LambertW(a*x^2)^(1/2))*LambertW(a*x^2)^(3/2)/a/x^3-6*LambertW(a*x
^2)^2/x-LambertW(a*x^2)^3/x
```

Mathematica [F]

$$\int \frac{W(ax^2)^3}{x^2}\,dx = \int \frac{W(ax^2)^3}{x^2}\,dx$$

input
```
Integrate[ProductLog[a*x^2]^3/x^2,x]
```

output
```
Integrate[ProductLog[a*x^2]^3/x^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)^3}{x^2} dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{W(ax^2)^3}{x^2} dx$$

input
```
Int[ProductLog[a*x^2]^3/x^2,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{\text{LambertW}\left(a\,x^2\right)^3}{x^2} dx$$

input
```
int(LambertW(a*x^2)^3/x^2,x)
```

output
```
int(LambertW(a*x^2)^3/x^2,x)
```

Fricas [F]

$$\int \frac{W(ax^2)^3}{x^2} dx = \int \frac{\mathrm{W}(ax^2)^3}{x^2} dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^2,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^3/x^2, x)
```

Sympy [F]

$$\int \frac{W(ax^2)^3}{x^2} \, dx = \int \frac{W^3(ax^2)}{x^2} \, dx$$

input
```
integrate(LambertW(a*x**2)**3/x**2,x)
```

output
```
Integral(LambertW(a*x**2)**3/x**2, x)
```

Maxima [F]

$$\int \frac{W(ax^2)^3}{x^2} \, dx = \int \frac{\mathrm{W}(ax^2)^3}{x^2} \, dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^3/x^2, x)
```

Giac [F]

$$\int \frac{W(ax^2)^3}{x^2} \, dx = \int \frac{\mathrm{W}(ax^2)^3}{x^2} \, dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^3}{x^2}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)^3}{x^2}\,dx$$

input
```
int(LambertW(a*x^2)^3/x^2,x)
```

output
```
int(LambertW(a*x^2)^3/x^2, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^3}{x^2}\,dx = \frac{6\left(\int \frac{lambert\_w(a\,x^2)^2}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)+e^{lambert\_w(a\,x^2)}}dx\right)ax - lambert\_w(a\,x^2)^3}{x}$$

input
```
int(Lambert_W(a*x^2)^3/x^2,x)
```

output
```
(6*int(lambert_w(a*x**2)**2/(e**lambert_w(a*x**2)*lambert_w(a*x**2) + e**l
ambert_w(a*x**2)),x)*a*x - lambert_w(a*x**2)**3)/x
```

3.127 $\int \frac{W(ax^2)^3}{x^4} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 84

$$\int \frac{W(ax^2)^3}{x^4} \, dx = -\frac{2W(ax^2)^2}{3x^3} - \frac{\sqrt{\frac{2}{3}} e^{\frac{5}{2}W(ax^2)} \Gamma\left(\frac{1}{2}, \frac{3}{2}W(ax^2)\right) W(ax^2)^{5/2}}{3ax^5} - \frac{W(ax^2)^3}{3x^3}$$

output
```
-2/3*LambertW(a*x^2)^2/x^3-1/9*6^(1/2)*exp(5/2*LambertW(a*x^2))*Pi^(1/2)*e
rfc(1/2*6^(1/2)*LambertW(a*x^2)^(1/2))*LambertW(a*x^2)^(5/2)/a/x^5-1/3*Lam
bertW(a*x^2)^3/x^3
```

Mathematica [F]

$$\int \frac{W(ax^2)^3}{x^4} \, dx = \int \frac{W(ax^2)^3}{x^4} \, dx$$

input
```
Integrate[ProductLog[a*x^2]^3/x^4,x]
```

output
```
Integrate[ProductLog[a*x^2]^3/x^4, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(ax^2\right)^3}{x^4} \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{W\left(ax^2\right)^3}{x^4}dx$$

input
```
Int[ProductLog[a*x^2]^3/x^4,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{\mathrm{LambertW}\left(a\,x^2\right)^3}{x^4}dx$$

input
```
int(LambertW(a*x^2)^3/x^4,x)
```

output
```
int(LambertW(a*x^2)^3/x^4,x)
```

Fricas [F]

$$\int \frac{W\left(ax^2\right)^3}{x^4} \, dx = \int \frac{\mathrm{W}\left(ax^2\right)^3}{x^4} \, dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^4,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^3/x^4, x)
```

Sympy [F]

$$\int \frac{W(ax^2)^3}{x^4}\,dx = \int \frac{W^3(ax^2)}{x^4}\,dx$$

input
```
integrate(LambertW(a*x**2)**3/x**4,x)
```

output
```
Integral(LambertW(a*x**2)**3/x**4, x)
```

Maxima [F]

$$\int \frac{W(ax^2)^3}{x^4}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x^4}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^4,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^3/x^4, x)
```

Giac [F]

$$\int \frac{W(ax^2)^3}{x^4}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x^4}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^4,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^3/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^3}{x^4}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)^3}{x^4}\,dx$$

input
```
int(LambertW(a*x^2)^3/x^4,x)
```

output
```
int(LambertW(a*x^2)^3/x^4, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^3}{x^4}\,dx = \frac{6\left(\int \frac{lambert\_w(a\,x^2)^2}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)x^2+e^{lambert\_w(a\,x^2)}x^2}dx\right)a\,x^3-lambert\_w(a\,x^2)^3}{3x^3}$$

input
```
int(Lambert_W(a*x^2)^3/x^4,x)
```

output
```
(6*int(lambert_w(a*x**2)**2/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x**2 +
 e**lambert_w(a*x**2)*x**2),x)*a*x**3 - lambert_w(a*x**2)**3)/(3*x**3)
```

3.128 $\int \frac{W(ax^2)^3}{x^6} dx$

Optimal result

Integrand size = 12, antiderivative size = 69

$$\int \frac{W(ax^2)^3}{x^6} dx = -\frac{W(ax^2)^3}{5x^5} - \frac{3\sqrt{\frac{2}{5}}e^{\frac{7}{2}W(ax^2)}\Gamma\left(\frac{1}{2}, \frac{5}{2}W(ax^2)\right) W(ax^2)^{7/2}}{5ax^7}$$

output
```
-1/5*LambertW(a*x^2)^3/x^5-3/25*10^(1/2)*exp(7/2*LambertW(a*x^2))*Pi^(1/2)
*erfc(1/2*10^(1/2)*LambertW(a*x^2)^(1/2))*LambertW(a*x^2)^(7/2)/a/x^7
```

Mathematica [F]

$$\int \frac{W(ax^2)^3}{x^6} dx = \int \frac{W(ax^2)^3}{x^6} dx$$

input
```
Integrate[ProductLog[a*x^2]^3/x^6,x]
```

output
```
Integrate[ProductLog[a*x^2]^3/x^6, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(ax^2\right)^3}{x^6} dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{W\left(ax^2\right)^3}{x^6} dx$$

input
```
Int[ProductLog[a*x^2]^3/x^6,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{\text{LambertW}\left(a\,x^2\right)^3}{x^6} dx$$

input
```
int(LambertW(a*x^2)^3/x^6,x)
```

output
```
int(LambertW(a*x^2)^3/x^6,x)
```

Fricas [F]

$$\int \frac{W(ax^2)^3}{x^6} dx = \int \frac{\text{W}(ax^2)^3}{x^6} dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^6,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^3/x^6, x)
```

Sympy [F]

$$\int \frac{W(ax^2)^3}{x^6}\,dx = \int \frac{W^3(ax^2)}{x^6}\,dx$$

input
```
integrate(LambertW(a*x**2)**3/x**6,x)
```

output
```
Integral(LambertW(a*x**2)**3/x**6, x)
```

Maxima [F]

$$\int \frac{W(ax^2)^3}{x^6}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x^6}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^6,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^3/x^6, x)
```

Giac [F]

$$\int \frac{W(ax^2)^3}{x^6}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x^6}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^6,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^3/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^3}{x^6}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)^3}{x^6}\,dx$$

input
```
int(LambertW(a*x^2)^3/x^6,x)
```

output
```
int(LambertW(a*x^2)^3/x^6, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^3}{x^6}\,dx = \frac{6\left(\int \frac{lambert\_w(a\,x^2)^2}{e^{lambert\_w(a\,x^2)}lambert\_w(a\,x^2)x^4+e^{lambert\_w(a\,x^2)}x^4}dx\right)a\,x^5-lambert\_w(a\,x^2)^3}{5x^5}$$

input
```
int(Lambert_W(a*x^2)^3/x^6,x)
```

output
```
(6*int(lambert_w(a*x**2)**2/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x**4 +
 e**lambert_w(a*x**2)*x**4),x)*a*x**5 - lambert_w(a*x**2)**3)/(5*x**5)
```

3.129 $\int \frac{W(ax^2)^3}{x^8} dx$

Optimal result

Integrand size = 12, antiderivative size = 113

$$\int \frac{W(ax^2)^3}{x^8} dx = -\frac{W(ax^2)^3}{x^7} + \frac{\sqrt{\frac{7}{2}} e^{\frac{9}{2}W(ax^2)} \Gamma\left(\frac{1}{2}, \frac{7}{2}W(ax^2)\right) W(ax^2)^{9/2}}{ax^9} - \frac{e^{\frac{9}{2}W(ax^2)} \Gamma\left(\frac{1}{2}, \frac{7}{2}W(ax^2)\right) W(ax^2)^{9/2}}{\sqrt{14}ax^9}$$

output
```
-LambertW(a*x^2)^3/x^7+3/7*14^(1/2)*exp(9/2*LambertW(a*x^2))*Pi^(1/2)*erfc
(1/2*14^(1/2)*LambertW(a*x^2)^(1/2))*LambertW(a*x^2)^(9/2)/a/x^9
```

Mathematica [F]

$$\int \frac{W(ax^2)^3}{x^8} dx = \int \frac{W(ax^2)^3}{x^8} dx$$

input
```
Integrate[ProductLog[a*x^2]^3/x^8,x]
```

output
```
Integrate[ProductLog[a*x^2]^3/x^8, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^2)^3}{x^8} dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{W(ax^2)^3}{x^8} dx$$

input
```
Int[ProductLog[a*x^2]^3/x^8,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{\text{LambertW}\left(a\,x^2\right)^3}{x^8} dx$$

input
```
int(LambertW(a*x^2)^3/x^8,x)
```

output
```
int(LambertW(a*x^2)^3/x^8,x)
```

Fricas [F]

$$\int \frac{W(ax^2)^3}{x^8} dx = \int \frac{\text{W}(ax^2)^3}{x^8} dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^8,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^3/x^8, x)
```

Sympy [F]

$$\int \frac{W(ax^2)^3}{x^8}\,dx = \int \frac{W^3(ax^2)}{x^8}\,dx$$

input
```
integrate(LambertW(a*x**2)**3/x**8,x)
```

output
```
Integral(LambertW(a*x**2)**3/x**8, x)
```

Maxima [F]

$$\int \frac{W(ax^2)^3}{x^8}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x^8}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^8,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^3/x^8, x)
```

Giac [F]

$$\int \frac{W(ax^2)^3}{x^8}\,dx = \int \frac{\mathrm{W}(ax^2)^3}{x^8}\,dx$$

input
```
integrate(lambert_w(a*x^2)^3/x^8,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^3/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^2)^3}{x^8}\,dx = \int \frac{\mathrm{LambertW}(a\,x^2)^3}{x^8}\,dx$$

input
```
int(LambertW(a*x^2)^3/x^8,x)
```

output
```
int(LambertW(a*x^2)^3/x^8, x)
```

Reduce [F]

$$\int \frac{W(ax^2)^3}{x^8}\,dx = \frac{6\left(\int \frac{lambert\_w(a\,x^2)^2}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)x^6+e^{lambert\_w(a\,x^2)}x^6}dx\right)a\,x^7-lambert\_w(a\,x^2)^3}{7x^7}$$

input
```
int(Lambert_W(a*x^2)^3/x^8,x)
```

output
```
(6*int(lambert_w(a*x**2)**2/(e**lambert_w(a*x**2)*lambert_w(a*x**2)*x**6 +
 e**lambert_w(a*x**2)*x**6),x)*a*x**7 - lambert_w(a*x**2)**3)/(7*x**7)
```

3.130 $\int \frac{x^7}{W(ax^2)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 61

$$\int \frac{x^7}{W(ax^2)}\,dx = \frac{x^8}{256W(ax^2)^4} - \frac{x^8}{64W(ax^2)^3} + \frac{x^8}{32W(ax^2)^2} + \frac{x^8}{8W(ax^2)}$$

output
```
1/256*x^8/LambertW(a*x^2)^4-1/64*x^8/LambertW(a*x^2)^3+1/32*x^8/LambertW(a
*x^2)^2+1/8*x^8/LambertW(a*x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{W(ax^2)}\,dx = \frac{x^8}{256W(ax^2)^4} - \frac{x^8}{64W(ax^2)^3} + \frac{x^8}{32W(ax^2)^2} + \frac{x^8}{8W(ax^2)}$$

input
```
Integrate[x^7/ProductLog[a*x^2],x]
```

output
```
x^8/(256*ProductLog[a*x^2]^4) - x^8/(64*ProductLog[a*x^2]^3) + x^8/(32*Pro
ductLog[a*x^2]^2) + x^8/(8*ProductLog[a*x^2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7172, 7205, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{W(ax^2)}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{4}\int \frac{x^7}{W(ax^2)(W(ax^2)+1)}dx+\frac{x^8}{8W(ax^2)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{4}\left(\frac{x^8}{8W(ax^2)^2}-\frac{1}{2}\int \frac{x^7}{W(ax^2)^2(W(ax^2)+1)}dx\right)+\frac{x^8}{8W(ax^2)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{x^7}{W(ax^2)^3(W(ax^2)+1)}dx-\frac{x^8}{8W(ax^2)^3}\right)+\frac{x^8}{8W(ax^2)^2}\right)+\frac{x^8}{8W(ax^2)}$$

$$\Big\downarrow \text{7201}$$

$$\frac{x^8}{8W(ax^2)}+\frac{1}{4}\left(\frac{x^8}{8W(ax^2)^2}+\frac{1}{2}\left(\frac{x^8}{32W(ax^2)^4}-\frac{x^8}{8W(ax^2)^3}\right)\right)$$

input
```
Int[x^7/ProductLog[a*x^2],x]
```

output
```
((x^8/(32*ProductLog[a*x^2]^4) - x^8/(8*ProductLog[a*x^2]^3))/2 + x^8/(8*P
roductLog[a*x^2]^2))/4 + x^8/(8*ProductLog[a*x^2])
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int \frac{x^7}{\text{LambertW}\left(a\,x^2\right)}dx$$

input
```
int(x^7/LambertW(a*x^2),x)
```

output
```
int(x^7/LambertW(a*x^2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{x^7}{W(ax^2)}\,dx = \frac{32\,x^8\,\mathrm{W}(ax^2)^3 + 8\,x^8\,\mathrm{W}(ax^2)^2 - 4\,x^8\,\mathrm{W}(ax^2) + x^8}{256\,\mathrm{W}(ax^2)^4}$$

input
```
integrate(x^7/lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
1/256*(32*x^8*lambert_w(a*x^2)^3 + 8*x^8*lambert_w(a*x^2)^2 - 4*x^8*lamber
t_w(a*x^2) + x^8)/lambert_w(a*x^2)^4
```

Sympy [F]

$$\int \frac{x^7}{W(ax^2)}\,dx = \int \frac{x^7}{W(ax^2)}\,dx$$

input
```
integrate(x**7/LambertW(a*x**2),x)
```

output
```
Integral(x**7/LambertW(a*x**2), x)
```

Maxima [F]

$$\int \frac{x^7}{W(ax^2)}\,dx = \int \frac{x^7}{\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(x^7/lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(x^7/lambert_w(a*x^2), x)
```

Giac [F]

$$\int \frac{x^7}{W(ax^2)}\,dx = \int \frac{x^7}{\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(x^7/lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(x^7/lambert_w(a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{W(ax^2)}\,dx = \int \frac{x^7}{\mathrm{LambertW}(a\,x^2)}\,dx$$

input
```
int(x^7/LambertW(a*x^2),x)
```

output
```
int(x^7/LambertW(a*x^2), x)
```

Reduce [F]

$$\int \frac{x^7}{W(ax^2)}\,dx = \int \frac{x^7}{lambert\_w(a\,x^2)}dx$$

input
```
int(x^7/Lambert_W(a*x^2),x)
```

output
```
int(x**7/lambert_w(a*x**2),x)
```

3.131 $\int \frac{x^5}{W(ax^2)} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{x^5}{W(ax^2)} \, dx = -\frac{x^6}{54W(ax^2)^3} + \frac{x^6}{18W(ax^2)^2} + \frac{x^6}{6W(ax^2)}$$

output
```
-1/54*x^6/LambertW(a*x^2)^3+1/18*x^6/LambertW(a*x^2)^2+1/6*x^6/LambertW(a*
x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{W(ax^2)} \, dx = -\frac{x^6}{54W(ax^2)^3} + \frac{x^6}{18W(ax^2)^2} + \frac{x^6}{6W(ax^2)}$$

input
```
Integrate[x^5/ProductLog[a*x^2],x]
```

output
```
-1/54*x^6/ProductLog[a*x^2]^3 + x^6/(18*ProductLog[a*x^2]^2) + x^6/(6*Prod
uctLog[a*x^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7172, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{W\left(ax^2\right)}dx$$
$$\Big\downarrow \text{7172}$$
$$\frac{1}{3}\int \frac{x^5}{W\left(ax^2\right)\left(W\left(ax^2\right)+1\right)}dx+\frac{x^6}{6W\left(ax^2\right)}$$
$$\Big\downarrow \text{7205}$$
$$\frac{1}{3}\left(\frac{x^6}{6W\left(ax^2\right)^2}-\frac{1}{3}\int \frac{x^5}{W\left(ax^2\right)^2\left(W\left(ax^2\right)+1\right)}dx\right)+\frac{x^6}{6W\left(ax^2\right)}$$
$$\Big\downarrow \text{7201}$$
$$\frac{x^6}{6W\left(ax^2\right)}+\frac{1}{3}\left(\frac{x^6}{6W\left(ax^2\right)^2}-\frac{x^6}{18W\left(ax^2\right)^3}\right)$$

input
```
Int[x^5/ProductLog[a*x^2],x]
```

output
```
(-1/18*x^6/ProductLog[a*x^2]^3 + x^6/(6*ProductLog[a*x^2]^2))/3 + x^6/(6*P
roductLog[a*x^2])
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int \frac{x^5}{\mathrm{LambertW}\left(a\,x^2\right)}dx$$

input
```
int(x^5/LambertW(a*x^2),x)
```

output
```
int(x^5/LambertW(a*x^2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{W\left(ax^2\right)}\,dx = \frac{9\,x^6\,\mathrm{W}(ax^2)^2 + 3\,x^6\,\mathrm{W}(ax^2) - x^6}{54\,\mathrm{W}(ax^2)^3}$$

input
```
integrate(x^5/lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
1/54*(9*x^6*lambert_w(a*x^2)^2 + 3*x^6*lambert_w(a*x^2) - x^6)/lambert_w(a
*x^2)^3
```

Sympy [F]

$$\int \frac{x^5}{W(ax^2)} \, dx = \int \frac{x^5}{W(ax^2)} \, dx$$

input
```
integrate(x**5/LambertW(a*x**2),x)
```

output
```
Integral(x**5/LambertW(a*x**2), x)
```

Maxima [F]

$$\int \frac{x^5}{W(ax^2)} \, dx = \int \frac{x^5}{\mathrm{W}(ax^2)} \, dx$$

input
```
integrate(x^5/lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(x^5/lambert_w(a*x^2), x)
```

Giac [F]

$$\int \frac{x^5}{W(ax^2)} \, dx = \int \frac{x^5}{\mathrm{W}(ax^2)} \, dx$$

input
```
integrate(x^5/lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(x^5/lambert_w(a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{W(ax^2)}\,dx = \int \frac{x^5}{\mathrm{LambertW}(a\,x^2)}\,dx$$

input
```
int(x^5/LambertW(a*x^2),x)
```

output
```
int(x^5/LambertW(a*x^2), x)
```

Reduce [F]

$$\int \frac{x^5}{W(ax^2)}\,dx = \int \frac{x^5}{lambert\_w(a\,x^2)}dx$$

input
```
int(x^5/Lambert_W(a*x^2),x)
```

output
```
int(x**5/lambert_w(a*x**2),x)
```

3.132 $\int \frac{x^3}{W(ax^2)} dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{x^3}{W(ax^2)} dx = \frac{x^4}{8W(ax^2)^2} + \frac{x^4}{4W(ax^2)}$$

output
```
1/8*x^4/LambertW(a*x^2)^2+1/4*x^4/LambertW(a*x^2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{W(ax^2)} dx = \frac{x^4}{8W(ax^2)^2} + \frac{x^4}{4W(ax^2)}$$

input
```
Integrate[x^3/ProductLog[a*x^2],x]
```

output
```
x^4/(8*ProductLog[a*x^2]^2) + x^4/(4*ProductLog[a*x^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7172, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{W(ax^2)}dx$$

$$\Big\downarrow \text{ 7172}$$

$$\frac{1}{2}\int \frac{x^3}{W(ax^2)(W(ax^2)+1)}dx+\frac{x^4}{4W(ax^2)}$$

$$\Big\downarrow \text{ 7201}$$

$$\frac{x^4}{4W(ax^2)}+\frac{x^4}{8W(ax^2)^2}$$

input
```
Int[x^3/ProductLog[a*x^2],x]
```

output
```
x^4/(8*ProductLog[a*x^2]^2) + x^4/(4*ProductLog[a*x^2])
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$-\frac{-2x^4 \operatorname{LambertW}(a\,x^2)-x^4}{8\operatorname{LambertW}(a\,x^2)^2}$	28

input
```
int(x^3/LambertW(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-2*x^4*LambertW(a*x^2)-x^4)/LambertW(a*x^2)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{W(ax^2)}\,dx = \frac{2\,x^4\,\mathrm{W}(ax^2) + x^4}{8\,\mathrm{W}(ax^2)^2}$$

input
```
integrate(x^3/lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
1/8*(2*x^4*lambert_w(a*x^2) + x^4)/lambert_w(a*x^2)^2
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{W(ax^2)}\,dx = \frac{x^4}{4W(ax^2)} + \frac{x^4}{8W^2(ax^2)}$$

input
```
integrate(x**3/LambertW(a*x**2),x)
```

output
```
x**4/(4*LambertW(a*x**2)) + x**4/(8*LambertW(a*x**2)**2)
```

Maxima [F]

$$\int \frac{x^3}{W(ax^2)}\,dx = \int \frac{x^3}{\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(x^3/lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(x^3/lambert_w(a*x^2), x)
```

Giac [F]

$$\int \frac{x^3}{W(ax^2)}\,dx = \int \frac{x^3}{\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(x^3/lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(x^3/lambert_w(a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{W(ax^2)}\,dx = \int \frac{x^3}{\mathrm{LambertW}\left(a\,x^2\right)}\,dx$$

input
```
int(x^3/LambertW(a*x^2),x)
```

output
```
int(x^3/LambertW(a*x^2), x)
```

Reduce [F]

$$\int \frac{x^3}{W(ax^2)}\,dx = \int \frac{x^3}{lambert\_w\left(a\,x^2\right)}dx$$

input
```
int(x^3/Lambert_W(a*x^2),x)
```

output
```
int(x**3/lambert_w(a*x**2),x)
```

3.133 $\int \frac{x}{W(ax^2)} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{x}{W(ax^2)} \, dx = \frac{\text{ExpIntegralEi}(W(ax^2))}{2a} + \frac{x^2}{2W(ax^2)}$$

output
```
1/2*Ei(LambertW(a*x^2))/a+1/2*x^2/LambertW(a*x^2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{x}{W(ax^2)} \, dx = \frac{\text{ExpIntegralEi}(W(ax^2))}{2a} + \frac{x^2}{2W(ax^2)}$$

input
```
Integrate[x/ProductLog[a*x^2],x]
```

output
```
ExpIntegralEi[ProductLog[a*x^2]]/(2*a) + x^2/(2*ProductLog[a*x^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7172, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{W(ax^2)}\,dx$$
$$\Big\downarrow \text{ 7172}$$
$$\int \frac{x}{W(ax^2)(W(ax^2)+1)}dx + \frac{x^2}{2W(ax^2)}$$
$$\Big\downarrow \text{ 7202}$$
$$\frac{\text{ExpIntegralEi}\left(W(ax^2)\right)}{2a} + \frac{x^2}{2W(ax^2)}$$

input
```
Int[x/ProductLog[a*x^2],x]
```

output
```
ExpIntegralEi[ProductLog[a*x^2]]/(2*a) + x^2/(2*ProductLog[a*x^2])
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{-\operatorname{expIntegral}_1\left(-\operatorname{LambertW}\left(a\,x^2\right)\right)+\frac{a\,x^2}{\operatorname{LambertW}\left(a\,x^2\right)}}{2a}$	32
default	$\frac{-\operatorname{expIntegral}_1\left(-\operatorname{LambertW}\left(a\,x^2\right)\right)+\frac{a\,x^2}{\operatorname{LambertW}\left(a\,x^2\right)}}{2a}$	32

input
```
int(x/LambertW(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(-Ei(1,-LambertW(a*x^2))+a*x^2/LambertW(a*x^2))
```

Fricas [F]

$$\int \frac{x}{W\left(ax^2\right)}\,dx = \int \frac{x}{\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(x/lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
integral(x/lambert_w(a*x^2), x)
```

Sympy [F]

$$\int \frac{x}{W(ax^2)}\,dx = \int \frac{x}{W(ax^2)}\,dx$$

input
```
integrate(x/LambertW(a*x**2),x)
```

output
```
Integral(x/LambertW(a*x**2), x)
```

Maxima [F]

$$\int \frac{x}{W(ax^2)}\,dx = \int \frac{x}{\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(x/lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(x/lambert_w(a*x^2), x)
```

Giac [F]

$$\int \frac{x}{W(ax^2)}\,dx = \int \frac{x}{\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(x/lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(x/lambert_w(a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{W(ax^2)}\,dx = \int \frac{x}{\mathrm{LambertW}\left(a\,x^2\right)}\,dx$$

input
```
int(x/LambertW(a*x^2),x)
```

output
```
int(x/LambertW(a*x^2), x)
```

Reduce [F]

$$\int \frac{x}{W(ax^2)}\,dx = \frac{e^{lambert\_w(a\,x^2)} + 2\left(\int \frac{x}{lambert\_w(a\,x^2)^2 + lambert\_w(a\,x^2)}dx\right)a}{2a}$$

input
```
int(x/Lambert_W(a*x^2),x)
```

output
```
(e**lambert_w(a*x**2) + 2*int(x/(lambert_w(a*x**2)**2 + lambert_w(a*x**2))
,x)*a)/(2*a)
```

3.134 $\int \frac{1}{xW(ax^2)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 24

$$\int \frac{1}{xW(ax^2)}\,dx = \frac{1}{2}\log\left(W(ax^2)\right) - \frac{1}{2W(ax^2)}$$

output
```
1/2*ln(LambertW(a*x^2))-1/2/LambertW(a*x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{xW(ax^2)}\,dx = \frac{1}{2}\log\left(W(ax^2)\right) - \frac{1}{2W(ax^2)}$$

input
```
Integrate[1/(x*ProductLog[a*x^2]),x]
```

output
```
Log[ProductLog[a*x^2]]/2 - 1/(2*ProductLog[a*x^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7173, 7198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{xW(ax^2)}dx$$
$$\Big\downarrow \text{ 7173}$$
$$\int \frac{1}{x(W(ax^2)+1)}dx - \frac{1}{2W(ax^2)}$$
$$\Big\downarrow \text{ 7198}$$
$$\frac{1}{2}\log\left(W(ax^2)\right) - \frac{1}{2W(ax^2)}$$

input
```
Int[1/(x*ProductLog[a*x^2]),x]
```

output
```
Log[ProductLog[a*x^2]]/2 - 1/(2*ProductLog[a*x^2])
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7198
```
Int[1/((x_)*((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp
[Log[ProductLog[a*x^n]]/(d*n), x] /; FreeQ[{a, d, n}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\ln(\mathrm{LambertW}(a\,x^2))}{2}-\frac{1}{2\,\mathrm{LambertW}(a\,x^2)}$	21
default	$\frac{\ln(\mathrm{LambertW}(a\,x^2))}{2}-\frac{1}{2\,\mathrm{LambertW}(a\,x^2)}$	21
parallelrisch	$-\frac{1-2\ln(x)\,\mathrm{LambertW}(a\,x^2)+\mathrm{LambertW}(a\,x^2)^2}{2\,\mathrm{LambertW}(a\,x^2)}$	31

input
```
int(1/x/LambertW(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(LambertW(a*x^2))-1/2/LambertW(a*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{xW(ax^2)}\,dx = \frac{\mathrm{W}(ax^2)\log\left(\mathrm{W}(ax^2)\right)-1}{2\ \mathrm{W}(ax^2)}$$

input
```
integrate(1/x/lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
1/2*(lambert_w(a*x^2)*log(lambert_w(a*x^2)) - 1)/lambert_w(a*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{xW(ax^2)}\,dx = \log(x)-\frac{W(ax^2)}{2}-\frac{1}{2W(ax^2)}$$

input
```
integrate(1/x/LambertW(a*x**2),x)
```

output
```
log(x) - LambertW(a*x**2)/2 - 1/(2*LambertW(a*x**2))
```

Maxima [F]

$$\int \frac{1}{xW\left(ax^2\right)}\,dx = \int \frac{1}{x\,\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(1/x/lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(1/(x*lambert_w(a*x^2)), x)
```

Giac [F]

$$\int \frac{1}{xW\left(ax^2\right)}\,dx = \int \frac{1}{x\,\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(1/x/lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(1/(x*lambert_w(a*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{xW\left(ax^2\right)}\,dx = \int \frac{1}{x\,\mathrm{LambertW}\left(a\,x^2\right)}\,dx$$

input
```
int(1/(x*LambertW(a*x^2)),x)
```

output
```
int(1/(x*LambertW(a*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{xW(ax^2)}\,dx = \int \frac{1}{lambert\_w(a\,x^2)\,x}dx$$

input
```
int(1/x/Lambert_W(a*x^2),x)
```

output
```
int(1/(lambert_w(a*x**2)*x),x)
```

3.135 $\int \frac{1}{x^3 W(ax^2)} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{1}{x^3 W(ax^2)} \, dx = -\frac{1}{4x^2} - \frac{1}{4} a \, \text{ExpIntegralEi}\left(-W\left(ax^2\right)\right) - \frac{1}{4x^2 W(ax^2)}$$

output
```
-1/4/x^2-1/4*a*Ei(-LambertW(a*x^2))-1/4/x^2/LambertW(a*x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 W(ax^2)} \, dx = -\frac{1}{4x^2} - \frac{1}{4} a \, \text{ExpIntegralEi}\left(-W\left(ax^2\right)\right) - \frac{1}{4x^2 W(ax^2)}$$

input
```
Integrate[1/(x^3*ProductLog[a*x^2]),x]
```

output
```
-1/4*1/x^2 - (a*ExpIntegralEi[-ProductLog[a*x^2]])/4 - 1/(4*x^2*ProductLog
[a*x^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7173, 7283, 7196, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3W(ax^2)}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{1}{2}\int \frac{1}{x^3(W(ax^2)+1)}dx-\frac{1}{4x^2W(ax^2)}$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{4}\int \frac{1}{x^4(W(ax^2)+1)}dx^2-\frac{1}{4x^2W(ax^2)}$$

$$\Big\downarrow \text{7196}$$

$$\frac{1}{4}\left(-\int \frac{W(ax^2)}{x^4(W(ax^2)+1)}dx^2-\frac{1}{x^2}\right)-\frac{1}{4x^2W(ax^2)}$$

$$\Big\downarrow \text{7202}$$

$$\frac{1}{4}\left(-a\,\text{ExpIntegralEi}(-W(ax^2))-\frac{1}{x^2}\right)-\frac{1}{4x^2W(ax^2)}$$

input
```
Int[1/(x^3*ProductLog[a*x^2]),x]
```

output
```
(-x^(-2) - a*ExpIntegralEi[-ProductLog[a*x^2]])/4 - 1/(4*x^2*ProductLog[a*
x^2])
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{1}{x^3 \operatorname{LambertW}(a x^2)} dx$$

input
```
int(1/x^3/LambertW(a*x^2),x)
```

output
```
int(1/x^3/LambertW(a*x^2),x)
```

Fricas [F]

$$\int \frac{1}{x^3 W(ax^2)} \, dx = \int \frac{1}{x^3 \operatorname{W}(ax^2)} \, dx$$

input
```
integrate(1/x^3/lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
integral(1/(x^3*lambert_w(a*x^2)), x)
```

Sympy [F]

$$\int \frac{1}{x^3 W(ax^2)} \, dx = \int \frac{1}{x^3 W(ax^2)} \, dx$$

input
```
integrate(1/x**3/LambertW(a*x**2),x)
```

output
```
Integral(1/(x**3*LambertW(a*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 W(ax^2)} \, dx = \int \frac{1}{x^3 \operatorname{W}(ax^2)} \, dx$$

input
```
integrate(1/x^3/lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(1/(x^3*lambert_w(a*x^2)), x)
```

Giac [F]

$$\int \frac{1}{x^3W(ax^2)}\,dx = \int \frac{1}{x^3\,\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(1/x^3/lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(1/(x^3*lambert_w(a*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3W(ax^2)}\,dx = \int \frac{1}{x^3\,\mathrm{LambertW}\left(a\,x^2\right)}\,dx$$

input
```
int(1/(x^3*LambertW(a*x^2)),x)
```

output
```
int(1/(x^3*LambertW(a*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3W(ax^2)}\,dx = \frac{-2\left(\int \frac{1}{x^3}dx\right)x^2+4\left(\int \frac{1}{lambert\_w(a\,x^2)x^3}dx\right)x^2-1}{4x^2}$$

input
```
int(1/x^3/Lambert_W(a*x^2),x)
```

output
```
( - 2*int(1/x**3,x)*x**2 + 4*int(1/(lambert_w(a*x**2)*x**3),x)*x**2 - 1)/(
4*x**2)
```

3.136 $\int \frac{1}{x^5 W(ax^2)} dx$

Optimal result

Integrand size = 12, antiderivative size = 52

$$\int \frac{1}{x^5 W(ax^2)} dx = -\frac{1}{12x^4} + \frac{1}{3}a^2 \operatorname{ExpIntegralEi}\left(-2W\left(ax^2\right)\right) - \frac{1}{6x^4 W(ax^2)} + \frac{W(ax^2)}{6x^4}$$

output
```
-1/12/x^4+1/3*a^2*Ei(-2*LambertW(a*x^2))-1/6/x^4/LambertW(a*x^2)+1/6*Lambe
rtW(a*x^2)/x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 W(ax^2)} dx = -\frac{1}{12x^4} + \frac{1}{3}a^2 \operatorname{ExpIntegralEi}\left(-2W\left(ax^2\right)\right) - \frac{1}{6x^4 W(ax^2)} + \frac{W(ax^2)}{6x^4}$$

input
```
Integrate[1/(x^5*ProductLog[a*x^2]),x]
```

output
```
-1/12*1/x^4 + (a^2*ExpIntegralEi[-2*ProductLog[a*x^2]])/3 - 1/(6*x^4*Produ
ctLog[a*x^2]) + ProductLog[a*x^2]/(6*x^4)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {7173, 7283, 7196, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 W(ax^2)} \, dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{1}{3} \int \frac{1}{x^5 (W(ax^2)+1)} dx - \frac{1}{6x^4 W(ax^2)}$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{6} \int \frac{1}{x^6 (W(ax^2)+1)} dx^2 - \frac{1}{6x^4 W(ax^2)}$$

$$\Big\downarrow \text{7196}$$

$$\frac{1}{6} \left(-\int \frac{W(ax^2)}{x^6 (W(ax^2)+1)} dx^2 - \frac{1}{2x^4}\right) - \frac{1}{6x^4 W(ax^2)}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{6} \left(2\int \frac{W(ax^2)^2}{x^6 (W(ax^2)+1)} dx^2 + \frac{W(ax^2)}{x^4} - \frac{1}{2x^4}\right) - \frac{1}{6x^4 W(ax^2)}$$

$$\Big\downarrow \text{7202}$$

$$\frac{1}{6} \left(2a^2 \text{ExpIntegralEi}\left(-2W(ax^2)\right) + \frac{W(ax^2)}{x^4} - \frac{1}{2x^4}\right) - \frac{1}{6x^4 W(ax^2)}$$

input
```
Int[1/(x^5*ProductLog[a*x^2]),x]
```

output
```
-1/6*1/(x^4*ProductLog[a*x^2]) + (-1/2*1/x^4 + 2*a^2*ExpIntegralEi[-2*Prod
uctLog[a*x^2]] + ProductLog[a*x^2]/x^4)/6
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{1}{x^5 \,\mathrm{LambertW}\left(a\, x^2\right)}dx$$

input
```
int(1/x^5/LambertW(a*x^2),x)
```

output
```
int(1/x^5/LambertW(a*x^2),x)
```

Fricas [F]

$$\int \frac{1}{x^5 W\left(ax^2\right)}\, dx = \int \frac{1}{x^5\, \mathrm{W}(ax^2)}\, dx$$

input
```
integrate(1/x^5/lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
integral(1/(x^5*lambert_w(a*x^2)), x)
```

Sympy [F]

$$\int \frac{1}{x^5 W\left(ax^2\right)}\, dx = \int \frac{1}{x^5 W\left(ax^2\right)}\, dx$$

input
```
integrate(1/x**5/LambertW(a*x**2),x)
```

output
```
Integral(1/(x**5*LambertW(a*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 W(ax^2)} \, dx = \int \frac{1}{x^5 \, \mathrm{W}(ax^2)} \, dx$$

input
```
integrate(1/x^5/lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(1/(x^5*lambert_w(a*x^2)), x)
```

Giac [F]

$$\int \frac{1}{x^5 W(ax^2)} \, dx = \int \frac{1}{x^5 \, \mathrm{W}(ax^2)} \, dx$$

input
```
integrate(1/x^5/lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(1/(x^5*lambert_w(a*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 W(ax^2)} \, dx = \int \frac{1}{x^5 \, \mathrm{LambertW}\left(a\,x^2\right)} \, dx$$

input
```
int(1/(x^5*LambertW(a*x^2)),x)
```

output
```
int(1/(x^5*LambertW(a*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{x^5 W(ax^2)}\,dx = \frac{-4\left(\int \frac{1}{x^5}dx\right)x^4 + 8\left(\int \frac{1}{lambert\_w(a\,x^2)x^5}dx\right)x^4 - 1}{8x^4}$$

input
```
int(1/x^5/Lambert_W(a*x^2),x)
```

output
```
( - 4*int(1/x**5,x)*x**4 + 8*int(1/(lambert_w(a*x**2)*x**5),x)*x**4 - 1)/(
8*x**4)
```

3.137 $\int \frac{x^4}{W(ax^2)} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 94

$$\int \frac{x^4}{W(ax^2)} \, dx = \frac{2x^5}{25W(ax^2)^2} - \frac{\sqrt{\frac{2}{5}}e^{-\frac{3}{2}W(ax^2)}x^3\Gamma\left(\frac{1}{2}, -\frac{5}{2}W(ax^2)\right)\sqrt{-W(ax^2)}}{25aW(ax^2)^2} + \frac{x^5}{5W(ax^2)}$$

output
```
2/25*x^5/LambertW(a*x^2)^2-1/125*10^(1/2)*x^3*Pi^(1/2)*erfc(1/2*(-10*Lambe
rtW(a*x^2))^(1/2))*(-LambertW(a*x^2))^(1/2)/a/exp(3/2*LambertW(a*x^2))/Lam
bertW(a*x^2)^2+1/5*x^5/LambertW(a*x^2)
```

Mathematica [F]

$$\int \frac{x^4}{W(ax^2)} \, dx = \int \frac{x^4}{W(ax^2)} \, dx$$

input
```
Integrate[x^4/ProductLog[a*x^2],x]
```

output
```
Integrate[x^4/ProductLog[a*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{W(ax^2)} \, dx$$

$$\big\downarrow \text{ 7299}$$

$$\int \frac{x^4}{W(ax^2)} dx$$

input
```
Int[x^4/ProductLog[a*x^2],x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^4}{\text{LambertW}\left(a\,x^2\right)} dx$$

input
```
int(x^4/LambertW(a*x^2),x)
```

output
```
int(x^4/LambertW(a*x^2),x)
```

Fricas [F]

$$\int \frac{x^4}{W(ax^2)} \, dx = \int \frac{x^4}{\mathrm{W}(ax^2)} \, dx$$

input
```
integrate(x^4/lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
integral(x^4/lambert_w(a*x^2), x)
```

Sympy [F]

$$\int \frac{x^4}{W(ax^2)}\,dx = \int \frac{x^4}{W(ax^2)}\,dx$$

input
```
integrate(x**4/LambertW(a*x**2),x)
```

output
```
Integral(x**4/LambertW(a*x**2), x)
```

Maxima [F]

$$\int \frac{x^4}{W(ax^2)}\,dx = \int \frac{x^4}{\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(x^4/lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(x^4/lambert_w(a*x^2), x)
```

Giac [F]

$$\int \frac{x^4}{W(ax^2)}\,dx = \int \frac{x^4}{\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(x^4/lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(x^4/lambert_w(a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{W(ax^2)}\,dx = \int \frac{x^4}{\mathrm{LambertW}\left(a\,x^2\right)}\,dx$$

input
```
int(x^4/LambertW(a*x^2),x)
```

output
```
int(x^4/LambertW(a*x^2), x)
```

Reduce [F]

$$\int \frac{x^4}{W(ax^2)}\,dx = \int \frac{x^4}{lambert\_w\left(a\,x^2\right)}dx$$

input
```
int(x^4/Lambert_W(a*x^2),x)
```

output
```
int(x**4/lambert_w(a*x**2),x)
```

3.138 $\int \frac{x^2}{W(ax^2)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int \frac{x^2}{W(ax^2)}\,dx = \frac{x^3}{3W(ax^2)} + \frac{\sqrt{\frac{2}{3}}e^{-\frac{1}{2}W(ax^2)}x\Gamma\left(\frac{1}{2},-\frac{3}{2}W(ax^2)\right)\sqrt{-W(ax^2)}}{3aW(ax^2)}$$

output
```
1/3*x^3/LambertW(a*x^2)+1/9*6^(1/2)*x*Pi^(1/2)*erfc(1/2*(-6*LambertW(a*x^2
))^(1/2))*(-LambertW(a*x^2))^(1/2)/a/exp(1/2*LambertW(a*x^2))/LambertW(a*x
^2)
```

Mathematica [F]

$$\int \frac{x^2}{W(ax^2)}\,dx = \int \frac{x^2}{W(ax^2)}\,dx$$

input
```
Integrate[x^2/ProductLog[a*x^2],x]
```

output
```
Integrate[x^2/ProductLog[a*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{W(ax^2)}\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{x^2}{W(ax^2)}dx$$

input
```
Int[x^2/ProductLog[a*x^2],x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^2}{\text{LambertW}\left(a\,x^2\right)}dx$$

input
```
int(x^2/LambertW(a*x^2),x)
```

output
```
int(x^2/LambertW(a*x^2),x)
```

Fricas [F]

$$\int \frac{x^2}{W(ax^2)}\,dx = \int \frac{x^2}{\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(x^2/lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
integral(x^2/lambert_w(a*x^2), x)
```

Sympy [F]

$$\int \frac{x^2}{W(ax^2)}\,dx = \int \frac{x^2}{W(ax^2)}\,dx$$

input
```
integrate(x**2/LambertW(a*x**2),x)
```

output
```
Integral(x**2/LambertW(a*x**2), x)
```

Maxima [F]

$$\int \frac{x^2}{W(ax^2)}\,dx = \int \frac{x^2}{\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(x^2/lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(x^2/lambert_w(a*x^2), x)
```

Giac [F]

$$\int \frac{x^2}{W(ax^2)}\,dx = \int \frac{x^2}{\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(x^2/lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(x^2/lambert_w(a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{W(ax^2)}\,dx = \int \frac{x^2}{\mathrm{LambertW}\left(a\,x^2\right)}\,dx$$

input
```
int(x^2/LambertW(a*x^2),x)
```

output
```
int(x^2/LambertW(a*x^2), x)
```

Reduce [F]

$$\int \frac{x^2}{W(ax^2)}\,dx = \int \frac{x^2}{lambert\_w\left(a\,x^2\right)}dx$$

input
```
int(x^2/Lambert_W(a*x^2),x)
```

output
```
int(x**2/lambert_w(a*x**2),x)
```

3.139 $\int \frac{1}{W(ax^2)} dx$

Optimal result

Integrand size = 8, antiderivative size = 62

$$\int \frac{1}{W(ax^2)} dx = \frac{\sqrt{2}e^{\frac{1}{2}W(ax^2)}\Gamma\left(\frac{1}{2}, -\frac{1}{2}W(ax^2)\right)\sqrt{-W(ax^2)}}{ax} - \frac{x}{W(ax^2)}$$

output
```
2^(1/2)*exp(1/2*LambertW(a*x^2))*Pi^(1/2)*erfc(1/2*(-2*LambertW(a*x^2))^(1
/2))*(-LambertW(a*x^2))^(1/2)/a/x-x/LambertW(a*x^2)
```

Mathematica [F]

$$\int \frac{1}{W(ax^2)} dx = \int \frac{1}{W(ax^2)} dx$$

input
```
Integrate[ProductLog[a*x^2]^(-1),x]
```

output
```
Integrate[ProductLog[a*x^2]^(-1), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W(ax^2)} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{1}{W(ax^2)}dx$$

input
```
Int[ProductLog[a*x^2]^(-1),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{1}{\mathrm{LambertW}\left(a\,x^{2}\right)}dx$$

input
```
int(1/LambertW(a*x^2),x)
```

output
```
int(1/LambertW(a*x^2),x)
```

Fricas [F]

$$\int \frac{1}{W(ax^2)} \, dx = \int \frac{1}{\mathrm{W}(ax^2)} \, dx$$

input
```
integrate(1/lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
integral(1/lambert_w(a*x^2), x)
```

Sympy [F]

$$\int \frac{1}{W\left(ax^2\right)}\,dx = \int \frac{1}{W\left(ax^2\right)}\,dx$$

input
```
integrate(1/LambertW(a*x**2),x)
```

output
```
Integral(1/LambertW(a*x**2), x)
```

Maxima [F]

$$\int \frac{1}{W\left(ax^2\right)}\,dx = \int \frac{1}{\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(1/lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(1/lambert_w(a*x^2), x)
```

Giac [F]

$$\int \frac{1}{W\left(ax^2\right)}\,dx = \int \frac{1}{\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(1/lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(1/lambert_w(a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W\left(ax^2\right)}\,dx = \int \frac{1}{\mathrm{LambertW}\left(a\,x^2\right)}\,dx$$

input
```
int(1/LambertW(a*x^2),x)
```

output
```
int(1/LambertW(a*x^2), x)
```

Reduce [F]

$$\int \frac{1}{W\left(ax^2\right)}\,dx = \int \frac{1}{lambert\_w\left(a\,x^2\right)}dx$$

input
```
int(1/Lambert_W(a*x^2),x)
```

output
```
int(1/lambert_w(a*x**2),x)
```

3.140 $\int \frac{1}{x^2W(ax^2)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 74

$$\int \frac{1}{x^2W(ax^2)}\,dx = -\frac{2}{3x} - \frac{1}{3xW(ax^2)} + \frac{\sqrt{2}e^{\frac{3}{2}W(ax^2)}\Gamma\left(\frac{1}{2},\frac{1}{2}W(ax^2)\right)W(ax^2)^{3/2}}{3ax^3}$$

output
```
-2/3/x-1/3/x/LambertW(a*x^2)+1/3*2^(1/2)*exp(3/2*LambertW(a*x^2))*Pi^(1/2)
*erfc(1/2*2^(1/2)*LambertW(a*x^2)^(1/2))*LambertW(a*x^2)^(3/2)/a/x^3
```

Mathematica [F]

$$\int \frac{1}{x^2W(ax^2)}\,dx = \int \frac{1}{x^2W(ax^2)}\,dx$$

input
```
Integrate[1/(x^2*ProductLog[a*x^2]),x]
```

output
```
Integrate[1/(x^2*ProductLog[a*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 W\left(a x^2\right)} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{1}{x^2 W\left(a x^2\right)}dx$$

input
```
Int[1/(x^2*ProductLog[a*x^2]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{1}{x^{2} \operatorname{LambertW}\left(a\,x^{2}\right)}dx$$

input
```
int(1/x^2/LambertW(a*x^2),x)
```

output
```
int(1/x^2/LambertW(a*x^2),x)
```

Fricas [F]

$$\int \frac{1}{x^2 W\left(a x^2\right)} \, dx=\int { \frac {1}{x^{2} \operatorname{W}(a x^{2})} \, dx }$$

input
```
integrate(1/x^2/lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
integral(1/(x^2*lambert_w(a*x^2)), x)
```

Sympy [F]

$$\int \frac{1}{x^2W(ax^2)}\,dx = \int \frac{1}{x^2W(ax^2)}\,dx$$

input
```
integrate(1/x**2/LambertW(a*x**2),x)
```

output
```
Integral(1/(x**2*LambertW(a*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2W(ax^2)}\,dx = \int \frac{1}{x^2\,\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(1/x^2/lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*lambert_w(a*x^2)), x)
```

Giac [F]

$$\int \frac{1}{x^2W(ax^2)}\,dx = \int \frac{1}{x^2\,\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(1/x^2/lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(1/(x^2*lambert_w(a*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2W(ax^2)}\,dx = \int \frac{1}{x^2\,\mathrm{LambertW}\left(a\,x^2\right)}\,dx$$

input
```
int(1/(x^2*LambertW(a*x^2)),x)
```

output
```
int(1/(x^2*LambertW(a*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2W(ax^2)}\,dx = \frac{-\left(\int \frac{1}{x^2}dx\right)x+2\left(\int \frac{1}{lambert\_w(a\,x^2)\,x^2}dx\right)x-1}{2x}$$

input
```
int(1/x^2/Lambert_W(a*x^2),x)
```

output
```
( - int(1/x**2,x)*x + 2*int(1/(lambert_w(a*x**2)*x**2),x)*x - 1)/(2*x)
```

3.141 $\int \frac{1}{x^4 W(ax^2)} dx$

Optimal result

Integrand size = 12, antiderivative size = 87

$$\int \frac{1}{x^4 W(ax^2)} dx = -\frac{2}{15x^3} - \frac{1}{5x^3 W(ax^2)} + \frac{2W(ax^2)}{5x^3} - \frac{\sqrt{6}e^{\frac{5}{2}W(ax^2)}\Gamma\left(\frac{1}{2}, \frac{3}{2}W(ax^2)\right) W(ax^2)^{5/2}}{5ax^5}$$

output
```
-2/15/x^3-1/5/x^3/LambertW(a*x^2)+2/5*LambertW(a*x^2)/x^3-1/5*6^(1/2)*exp(
5/2*LambertW(a*x^2))*Pi^(1/2)*erfc(1/2*6^(1/2)*LambertW(a*x^2)^(1/2))*Lamb
ertW(a*x^2)^(5/2)/a/x^5
```

Mathematica [F]

$$\int \frac{1}{x^4 W(ax^2)} dx = \int \frac{1}{x^4 W(ax^2)} dx$$

input
```
Integrate[1/(x^4*ProductLog[a*x^2]),x]
```

output
```
Integrate[1/(x^4*ProductLog[a*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 W(ax^2)} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{1}{x^4 W(ax^2)} dx$$

input
```
Int[1/(x^4*ProductLog[a*x^2]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{1}{x^4 \operatorname{LambertW}(a\,x^2)} dx$$

input
```
int(1/x^4/LambertW(a*x^2),x)
```

output
```
int(1/x^4/LambertW(a*x^2),x)
```

Fricas [F]

$$\int \frac{1}{x^4 W(ax^2)} \, dx = \int \frac{1}{x^4 \, \mathrm{W}(ax^2)} \, dx$$

input
```
integrate(1/x^4/lambert_w(a*x^2),x, algorithm="fricas")
```

output
```
integral(1/(x^4*lambert_w(a*x^2)), x)
```

Sympy [F]

$$\int \frac{1}{x^4W(ax^2)}\,dx = \int \frac{1}{x^4W(ax^2)}\,dx$$

input
```
integrate(1/x**4/LambertW(a*x**2),x)
```

output
```
Integral(1/(x**4*LambertW(a*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4W(ax^2)}\,dx = \int \frac{1}{x^4\,\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(1/x^4/lambert_w(a*x^2),x, algorithm="maxima")
```

output
```
integrate(1/(x^4*lambert_w(a*x^2)), x)
```

Giac [F]

$$\int \frac{1}{x^4W(ax^2)}\,dx = \int \frac{1}{x^4\,\mathrm{W}(ax^2)}\,dx$$

input
```
integrate(1/x^4/lambert_w(a*x^2),x, algorithm="giac")
```

output
```
integrate(1/(x^4*lambert_w(a*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4W(ax^2)}\,dx = \int \frac{1}{x^4\,\mathrm{LambertW}(a\,x^2)}\,dx$$

input
```
int(1/(x^4*LambertW(a*x^2)),x)
```

output
```
int(1/(x^4*LambertW(a*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4W(ax^2)}\,dx = \frac{-3\left(\int \frac{1}{x^4}dx\right)x^3+6\left(\int \frac{1}{\mathit{lambert\_w}(a\,x^2)\,x^4}dx\right)x^3-1}{6x^3}$$

input
```
int(1/x^4/Lambert_W(a*x^2),x)
```

output
```
( - 3*int(1/x**4,x)*x**3 + 6*int(1/(lambert_w(a*x**2)*x**4),x)*x**3 - 1)/(
6*x**3)
```

3.142 $\int \frac{x^7}{W(ax^2)^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{x^7}{W(ax^2)^2} dx = -\frac{x^8}{64W(ax^2)^4} + \frac{x^8}{16W(ax^2)^3} + \frac{x^8}{8W(ax^2)^2}$$

output
```
-1/64*x^8/LambertW(a*x^2)^4+1/16*x^8/LambertW(a*x^2)^3+1/8*x^8/LambertW(a*
x^2)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{W(ax^2)^2} dx = -\frac{x^8}{64W(ax^2)^4} + \frac{x^8}{16W(ax^2)^3} + \frac{x^8}{8W(ax^2)^2}$$

input
```
Integrate[x^7/ProductLog[a*x^2]^2,x]
```

output
```
-1/64*x^8/ProductLog[a*x^2]^4 + x^8/(16*ProductLog[a*x^2]^3) + x^8/(8*Prod
uctLog[a*x^2]^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7172, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{W(ax^2)^2}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{2}\int \frac{x^7}{W(ax^2)^2(W(ax^2)+1)}dx+\frac{x^8}{8W(ax^2)^2}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{2}\left(\frac{x^8}{8W(ax^2)^3}-\frac{1}{4}\int \frac{x^7}{W(ax^2)^3(W(ax^2)+1)}dx\right)+\frac{x^8}{8W(ax^2)^2}$$

$$\Big\downarrow \text{7201}$$

$$\frac{x^8}{8W(ax^2)^2}+\frac{1}{2}\left(\frac{x^8}{8W(ax^2)^3}-\frac{x^8}{32W(ax^2)^4}\right)$$

input
```
Int[x^7/ProductLog[a*x^2]^2,x]
```

output
```
(-1/32*x^8/ProductLog[a*x^2]^4 + x^8/(8*ProductLog[a*x^2]^3))/2 + x^8/(8*P
roductLog[a*x^2]^2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int \frac{x^7}{\mathrm{LambertW}\left(a\,x^2\right)^2}dx$$

input
```
int(x^7/LambertW(a*x^2)^2,x)
```

output
```
int(x^7/LambertW(a*x^2)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{x^7}{W\left(ax^2\right)^2}\,dx = \frac{8\,x^8\,\mathrm{W}(ax^2)^2 + 4\,x^8\,\mathrm{W}(ax^2) - x^8}{64\,\mathrm{W}(ax^2)^4}$$

input
```
integrate(x^7/lambert_w(a*x^2)^2,x, algorithm="fricas")
```

output
```
1/64*(8*x^8*lambert_w(a*x^2)^2 + 4*x^8*lambert_w(a*x^2) - x^8)/lambert_w(a
*x^2)^4
```

Sympy [F]

$$\int \frac{x^7}{W(ax^2)^2}\,dx = \int \frac{x^7}{W^2(ax^2)}\,dx$$

input
```
integrate(x**7/LambertW(a*x**2)**2,x)
```

output
```
Integral(x**7/LambertW(a*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^7}{W(ax^2)^2}\,dx = \int \frac{x^7}{\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(x^7/lambert_w(a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate(x^7/lambert_w(a*x^2)^2, x)
```

Giac [F]

$$\int \frac{x^7}{W(ax^2)^2}\,dx = \int \frac{x^7}{\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(x^7/lambert_w(a*x^2)^2,x, algorithm="giac")
```

output
```
integrate(x^7/lambert_w(a*x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{W(ax^2)^2}\,dx = \int \frac{x^7}{\mathrm{LambertW}\left(a\,x^2\right)^2}\,dx$$

input
```
int(x^7/LambertW(a*x^2)^2,x)
```

output
```
int(x^7/LambertW(a*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^7}{W(ax^2)^2}\,dx = \int \frac{x^7}{lambert\_w\left(a\,x^2\right)^2}dx$$

input
```
int(x^7/Lambert_W(a*x^2)^2,x)
```

output
```
int(x**7/lambert_w(a*x**2)**2,x)
```

3.143 $\int \frac{x^5}{W(ax^2)^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{x^5}{W(ax^2)^2} \, dx = \frac{x^6}{9W(ax^2)^3} + \frac{x^6}{6W(ax^2)^2}$$

output
```
1/9*x^6/LambertW(a*x^2)^3+1/6*x^6/LambertW(a*x^2)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{W(ax^2)^2} \, dx = \frac{x^6}{9W(ax^2)^3} + \frac{x^6}{6W(ax^2)^2}$$

input
```
Integrate[x^5/ProductLog[a*x^2]^2,x]
```

output
```
x^6/(9*ProductLog[a*x^2]^3) + x^6/(6*ProductLog[a*x^2]^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7172, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{W\left(ax^2\right)^2}dx$$

$$\Big\downarrow \text{ 7172}$$

$$\frac{2}{3}\int \frac{x^5}{W\left(ax^2\right)^2\left(W\left(ax^2\right)+1\right)}dx+\frac{x^6}{6W\left(ax^2\right)^2}$$

$$\Big\downarrow \text{ 7201}$$

$$\frac{x^6}{6W\left(ax^2\right)^2}+\frac{x^6}{9W\left(ax^2\right)^3}$$

input
```
Int[x^5/ProductLog[a*x^2]^2,x]
```

output
```
x^6/(9*ProductLog[a*x^2]^3) + x^6/(6*ProductLog[a*x^2]^2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$\frac{3x^6 \operatorname{LambertW}(a\,x^2)+2x^6}{18 \operatorname{LambertW}(a\,x^2)^3}$	28

input
```
int(x^5/LambertW(a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/18*(3*x^6*LambertW(a*x^2)+2*x^6)/LambertW(a*x^2)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{W(ax^2)^2}\,dx = \frac{3\,x^6\,\mathrm{W}(ax^2) + 2\,x^6}{18\,\mathrm{W}(ax^2)^3}$$

input
```
integrate(x^5/lambert_w(a*x^2)^2,x, algorithm="fricas")
```

output
```
1/18*(3*x^6*lambert_w(a*x^2) + 2*x^6)/lambert_w(a*x^2)^3
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{W(ax^2)^2}\,dx = \frac{x^6}{6W^2(ax^2)} + \frac{x^6}{9W^3(ax^2)}$$

input
```
integrate(x**5/LambertW(a*x**2)**2,x)
```

output
```
x**6/(6*LambertW(a*x**2)**2) + x**6/(9*LambertW(a*x**2)**3)
```

Maxima [F]

$$\int \frac{x^5}{W(ax^2)^2}\,dx = \int \frac{x^5}{\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(x^5/lambert_w(a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate(x^5/lambert_w(a*x^2)^2, x)
```

Giac [F]

$$\int \frac{x^5}{W(ax^2)^2}\,dx = \int \frac{x^5}{\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(x^5/lambert_w(a*x^2)^2,x, algorithm="giac")
```

output
```
integrate(x^5/lambert_w(a*x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{W\left(ax^2\right)^2}\,dx = \int \frac{x^5}{\mathrm{LambertW}\left(a\,x^2\right)^2}\,dx$$

input
```
int(x^5/LambertW(a*x^2)^2,x)
```

output
```
int(x^5/LambertW(a*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^5}{W\left(ax^2\right)^2}\,dx = \int \frac{x^5}{lambert\_w\left(a\,x^2\right)^2}dx$$

input
```
int(x^5/Lambert_W(a*x^2)^2,x)
```

output
```
int(x**5/lambert_w(a*x**2)**2,x)
```

3.144 $\int \frac{x^3}{W(ax^2)^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{x^3}{W(ax^2)^2} \, dx = \frac{\text{ExpIntegralEi}(2W(ax^2))}{2a^2} + \frac{x^4}{4W(ax^2)^2}$$

output
```
1/2*Ei(2*LambertW(a*x^2))/a^2+1/4*x^4/LambertW(a*x^2)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{W(ax^2)^2} \, dx = \frac{\text{ExpIntegralEi}(2W(ax^2))}{2a^2} + \frac{x^4}{4W(ax^2)^2}$$

input
```
Integrate[x^3/ProductLog[a*x^2]^2,x]
```

output
```
ExpIntegralEi[2*ProductLog[a*x^2]]/(2*a^2) + x^4/(4*ProductLog[a*x^2]^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7172, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{W(ax^2)^2}dx$$
$$\Big\downarrow \text{7172}$$
$$\int \frac{x^3}{W(ax^2)^2 (W(ax^2)+1)}dx+\frac{x^4}{4W(ax^2)^2}$$
$$\Big\downarrow \text{7202}$$
$$\frac{\text{ExpIntegralEi}(2W(ax^2))}{2a^2}+\frac{x^4}{4W(ax^2)^2}$$

input
```
Int[x^3/ProductLog[a*x^2]^2,x]
```

output
```
ExpIntegralEi[2*ProductLog[a*x^2]]/(2*a^2) + x^4/(4*ProductLog[a*x^2]^2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [F]

$$\int \frac{x^3}{\text{LambertW}\,(a\,x^2)^2}dx$$

input
```
int(x^3/LambertW(a*x^2)^2,x)
```

output
```
int(x^3/LambertW(a*x^2)^2,x)
```

Fricas [F]

$$\int \frac{x^3}{W\,(ax^2)^2}\,dx = \int \frac{x^3}{\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(x^3/lambert_w(a*x^2)^2,x, algorithm="fricas")
```

output
```
integral(x^3/lambert_w(a*x^2)^2, x)
```

Sympy [F]

$$\int \frac{x^3}{W\,(ax^2)^2}\,dx = \int \frac{x^3}{W^2\,(ax^2)}\,dx$$

input
```
integrate(x**3/LambertW(a*x**2)**2,x)
```

output
```
Integral(x**3/LambertW(a*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^3}{W\left(a x^2\right)^2} \, dx = \int \frac{x^3}{\mathrm{W}(a x^2)^2} \, dx$$

input
```
integrate(x^3/lambert_w(a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate(x^3/lambert_w(a*x^2)^2, x)
```

Giac [F]

$$\int \frac{x^3}{W\left(a x^2\right)^2} \, dx = \int \frac{x^3}{\mathrm{W}(a x^2)^2} \, dx$$

input
```
integrate(x^3/lambert_w(a*x^2)^2,x, algorithm="giac")
```

output
```
integrate(x^3/lambert_w(a*x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{W\left(a x^2\right)^2} \, dx = \int \frac{x^3}{\mathrm{LambertW}\left(a\,x^2\right)^2} \, dx$$

input
```
int(x^3/LambertW(a*x^2)^2,x)
```

output
```
int(x^3/LambertW(a*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^3}{W(ax^2)^2}\,dx = \int \frac{x^3}{lambert\_w(a\,x^2)^2}dx$$

input
```
int(x^3/Lambert_W(a*x^2)^2,x)
```

output
```
int(x**3/lambert_w(a*x**2)**2,x)
```

3.145 $\int \frac{x}{W(ax^2)^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \frac{x}{W(ax^2)^2} dx = \frac{\text{ExpIntegralEi}(W(ax^2))}{a} - \frac{x^2}{2W(ax^2)^2}$$

output
```
Ei(LambertW(a*x^2))/a-1/2*x^2/LambertW(a*x^2)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x}{W(ax^2)^2} dx = \frac{\text{ExpIntegralEi}(W(ax^2))}{a} - \frac{x^2}{2W(ax^2)^2}$$

input
```
Integrate[x/ProductLog[a*x^2]^2,x]
```

output
```
ExpIntegralEi[ProductLog[a*x^2]]/a - x^2/(2*ProductLog[a*x^2]^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7173, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{W(ax^2)^2} dx$$

$$\Big\downarrow \text{ 7173}$$

$$2\int \frac{x}{W(ax^2)(W(ax^2)+1)}dx - \frac{x^2}{2W(ax^2)^2}$$

$$\Big\downarrow \text{ 7202}$$

$$\frac{\text{ExpIntegralEi}\left(W\left(ax^2\right)\right)}{a} - \frac{x^2}{2W(ax^2)^2}$$

input
```
Int[x/ProductLog[a*x^2]^2,x]
```

output
```
ExpIntegralEi[ProductLog[a*x^2]]/a - x^2/(2*ProductLog[a*x^2]^2)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{-\frac{a\,x^{2}}{\operatorname{LambertW}\left(a\,x^{2}\right)^{2}}-2\,\operatorname{expIntegral}_{1}\left(-\operatorname{LambertW}\left(a\,x^{2}\right)\right)}{2a}$	33
default	$\frac{-\frac{a\,x^{2}}{\operatorname{LambertW}\left(a\,x^{2}\right)^{2}}-2\,\operatorname{expIntegral}_{1}\left(-\operatorname{LambertW}\left(a\,x^{2}\right)\right)}{2a}$	33

input
```
int(x/LambertW(a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(-1/LambertW(a*x^2)^2*a*x^2-2*Ei(1,-LambertW(a*x^2)))
```

Fricas [F]

$$\int \frac{x}{W\left(ax^2\right)^2}\,dx = \int \frac{x}{\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(x/lambert_w(a*x^2)^2,x, algorithm="fricas")
```

output
```
integral(x/lambert_w(a*x^2)^2, x)
```

Sympy [F]

$$\int \frac{x}{W\left(ax^2\right)^2}\,dx = \int \frac{x}{W^2\left(ax^2\right)}\,dx$$

input
```
integrate(x/LambertW(a*x**2)**2,x)
```

output
```
Integral(x/LambertW(a*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x}{W\left(ax^2\right)^2}\,dx = \int \frac{x}{\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(x/lambert_w(a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate(x/lambert_w(a*x^2)^2, x)
```

Giac [F]

$$\int \frac{x}{W\left(ax^2\right)^2}\,dx = \int \frac{x}{\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(x/lambert_w(a*x^2)^2,x, algorithm="giac")
```

output
```
integrate(x/lambert_w(a*x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{W(ax^2)^2} \, dx = \int \frac{x}{\mathrm{LambertW}(a\,x^2)^2} \, dx$$

input
```
int(x/LambertW(a*x^2)^2,x)
```

output
```
int(x/LambertW(a*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x}{W(ax^2)^2} \, dx = \frac{e^{lambert\_w(a\,x^2)} + 4\left(\int \frac{x}{lambert\_w(a\,x^2)^3 + lambert\_w(a\,x^2)^2} dx\right) lambert\_w(a\,x^2)\,a}{2lambert\_w(a\,x^2)\,a}$$

input
```
int(x/Lambert_W(a*x^2)^2,x)
```

output
```
(e**lambert_w(a*x**2) + 4*int(x/(lambert_w(a*x**2)**3 + lambert_w(a*x**2)*
*2),x)*lambert_w(a*x**2)*a)/(2*lambert_w(a*x**2)*a)
```

3.146 $\int \frac{1}{xW(ax^2)^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{1}{xW(ax^2)^2} dx = -\frac{1}{4W(ax^2)^2} - \frac{1}{2W(ax^2)}$$

output
```
-1/4/LambertW(a*x^2)^2-1/2/LambertW(a*x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{xW(ax^2)^2} dx = -\frac{1}{4W(ax^2)^2} - \frac{1}{2W(ax^2)}$$

input
```
Integrate[1/(x*ProductLog[a*x^2]^2),x]
```

output
```
-1/4*1/ProductLog[a*x^2]^2 - 1/(2*ProductLog[a*x^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{xW(ax^2)^2}dx$$

$$\Big\downarrow \text{7173}$$

$$\int \frac{1}{xW(ax^2)(W(ax^2)+1)}dx - \frac{1}{4W(ax^2)^2}$$

$$\Big\downarrow \text{7200}$$

$$-\frac{1}{2W(ax^2)} - \frac{1}{4W(ax^2)^2}$$

input
```
Int[1/(x*ProductLog[a*x^2]^2),x]
```

output
```
-1/4*1/ProductLog[a*x^2]^2 - 1/(2*ProductLog[a*x^2])
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$-\frac{1+2\,\mathrm{LambertW}(a\,x^2)}{4\,\mathrm{LambertW}(a\,x^2)^2}$	21
derivativedivides	$-\frac{1}{4\,\mathrm{LambertW}(a\,x^2)^2}-\frac{1}{2\,\mathrm{LambertW}(a\,x^2)}$	22
default	$-\frac{1}{4\,\mathrm{LambertW}(a\,x^2)^2}-\frac{1}{2\,\mathrm{LambertW}(a\,x^2)}$	22

input
```
int(1/x/LambertW(a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/LambertW(a*x^2)^2*(1+2*LambertW(a*x^2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{xW(ax^2)^2}\,dx = -\frac{2\;\mathrm{W}(ax^2)+1}{4\;\mathrm{W}(ax^2)^2}$$

input
```
integrate(1/x/lambert_w(a*x^2)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*lambert_w(a*x^2) + 1)/lambert_w(a*x^2)^2
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{xW(ax^2)^2}\,dx = -\frac{1}{2W(ax^2)}-\frac{1}{4W^2(ax^2)}$$

input
```
integrate(1/x/LambertW(a*x**2)**2,x)
```

output
```
-1/(2*LambertW(a*x**2)) - 1/(4*LambertW(a*x**2)**2)
```

Maxima [F]

$$\int \frac{1}{xW\left(ax^2\right)^2}\,dx = \int \frac{1}{x\,\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(1/x/lambert_w(a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate(1/(x*lambert_w(a*x^2)^2), x)
```

Giac [F]

$$\int \frac{1}{xW\left(ax^2\right)^2}\,dx = \int \frac{1}{x\,\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(1/x/lambert_w(a*x^2)^2,x, algorithm="giac")
```

output
```
integrate(1/(x*lambert_w(a*x^2)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{xW\left(ax^2\right)^2}\,dx = \int \frac{1}{x\,\mathrm{LambertW}\left(a\,x^2\right)^2}\,dx$$

input
```
int(1/(x*LambertW(a*x^2)^2),x)
```

output
```
int(1/(x*LambertW(a*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{xW\left(ax^2\right)^2}\,dx = \int \frac{1}{lambert\_w\left(a\,x^2\right)^2 x}dx$$

input
```
int(1/x/Lambert_W(a*x^2)^2,x)
```

output
```
int(1/(lambert_w(a*x**2)**2*x),x)
```

3.147 $\int \frac{1}{x^3 W\left(ax^2\right)^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 52

$$\int \frac{1}{x^3 W\left(ax^2\right)^2} dx = \frac{1}{6x^2} + \frac{1}{6} a \operatorname{ExpIntegralEi}\left(-W\left(ax^2\right)\right) - \frac{1}{6x^2 W\left(ax^2\right)^2} - \frac{1}{6x^2 W\left(ax^2\right)}$$

output
```
1/6/x^2+1/6*a*Ei(-LambertW(a*x^2))-1/6/x^2/LambertW(a*x^2)^2-1/6/x^2/Lambe
rtW(a*x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 W\left(ax^2\right)^2} dx = \frac{1}{6x^2} + \frac{1}{6} a \operatorname{ExpIntegralEi}\left(-W\left(ax^2\right)\right) - \frac{1}{6x^2 W\left(ax^2\right)^2} - \frac{1}{6x^2 W\left(ax^2\right)}$$

input
```
Integrate[1/(x^3*ProductLog[a*x^2]^2),x]
```

output
```
1/(6*x^2) + (a*ExpIntegralEi[-ProductLog[a*x^2]])/6 - 1/(6*x^2*ProductLog[
a*x^2]^2) - 1/(6*x^2*ProductLog[a*x^2])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {7173, 7206, 7283, 7196, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 W(ax^2)^2} dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{2}{3} \int \frac{1}{x^3 W(ax^2)(W(ax^2)+1)} dx - \frac{1}{6x^2 W(ax^2)^2}$$

$$\Big\downarrow \text{7206}$$

$$\frac{2}{3} \left(-\frac{1}{2} \int \frac{1}{x^3 (W(ax^2)+1)} dx - \frac{1}{4x^2 W(ax^2)}\right) - \frac{1}{6x^2 W(ax^2)^2}$$

$$\Big\downarrow \text{7283}$$

$$\frac{2}{3} \left(-\frac{1}{4} \int \frac{1}{x^4 (W(ax^2)+1)} dx^2 - \frac{1}{4x^2 W(ax^2)}\right) - \frac{1}{6x^2 W(ax^2)^2}$$

$$\Big\downarrow \text{7196}$$

$$\frac{2}{3} \left(\frac{1}{4} \left(\int \frac{W(ax^2)}{x^4 (W(ax^2)+1)} dx^2 + \frac{1}{x^2}\right) - \frac{1}{4x^2 W(ax^2)}\right) - \frac{1}{6x^2 W(ax^2)^2}$$

$$\Big\downarrow \text{7202}$$

$$\frac{2}{3} \left(\frac{1}{4} \left(a \,\text{ExpIntegralEi}(-W(ax^2)) + \frac{1}{x^2}\right) - \frac{1}{4x^2 W(ax^2)}\right) - \frac{1}{6x^2 W(ax^2)^2}$$

input
```
Int[1/(x^3*ProductLog[a*x^2]^2),x]
```

output
```
(2*((x^(-2) + a*ExpIntegralEi[-ProductLog[a*x^2]])/4 - 1/(4*x^2*ProductLog
[a*x^2])))/3 - 1/(6*x^2*ProductLog[a*x^2]^2)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{1}{x^3 \operatorname{LambertW}(a\,x^2)^2}dx$$

input
```
int(1/x^3/LambertW(a*x^2)^2,x)
```

output
```
int(1/x^3/LambertW(a*x^2)^2,x)
```

Fricas [F]

$$\int \frac{1}{x^3 W(ax^2)^2}\,dx = \int \frac{1}{x^3\,\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(1/x^3/lambert_w(a*x^2)^2,x, algorithm="fricas")
```

output
```
integral(1/(x^3*lambert_w(a*x^2)^2), x)
```

Sympy [F]

$$\int \frac{1}{x^3 W(ax^2)^2}\,dx = \int \frac{1}{x^3 W^2(ax^2)}\,dx$$

input
```
integrate(1/x**3/LambertW(a*x**2)**2,x)
```

output
```
Integral(1/(x**3*LambertW(a*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^3W\left(ax^2\right)^2}\,dx = \int \frac{1}{x^3\,\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(1/x^3/lambert_w(a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate(1/(x^3*lambert_w(a*x^2)^2), x)
```

Giac [F]

$$\int \frac{1}{x^3W\left(ax^2\right)^2}\,dx = \int \frac{1}{x^3\,\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(1/x^3/lambert_w(a*x^2)^2,x, algorithm="giac")
```

output
```
integrate(1/(x^3*lambert_w(a*x^2)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3W\left(ax^2\right)^2}\,dx = \int \frac{1}{x^3\,\mathrm{LambertW}\left(a\,x^2\right)^2}\,dx$$

input
```
int(1/(x^3*LambertW(a*x^2)^2),x)
```

output
```
int(1/(x^3*LambertW(a*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^3 W\left(ax^2\right)^2} \, dx$$
$$= \frac{2\left(\int \frac{lambert\_w(a\,x^2)}{lambert\_w(a\,x^2)x^3+x^3}dx\right) lambert\_w(a\,x^2)\,x^2 + 8\left(\int \frac{1}{lambert\_w(a\,x^2)^3x^3+lambert\_w(a\,x^2)^2x^3}dx\right) lamb}{}$$

input
```
int(1/x^3/Lambert_W(a*x^2)^2,x)
```

output
```
(2*int(lambert_w(a*x**2)/(lambert_w(a*x**2)*x**3 + x**3),x)*lambert_w(a*x*
*2)*x**2 + 8*int(1/(lambert_w(a*x**2)**3*x**3 + lambert_w(a*x**2)**2*x**3)
,x)*lambert_w(a*x**2)*x**2 + 4*int(1/(lambert_w(a*x**2)**2*x**3 + lambert_
w(a*x**2)*x**3),x)*lambert_w(a*x**2)*x**2 - 4*int(1/(e**lambert_w(a*x**2)*
lambert_w(a*x**2)**3*x + e**lambert_w(a*x**2)*lambert_w(a*x**2)**2*x),x)*l
ambert_w(a*x**2)*a*x**2 - 2*int(1/(lambert_w(a*x**2)*x**3 + x**3),x)*lambe
rt_w(a*x**2)*x**2 + lambert_w(a*x**2) - 2)/(8*lambert_w(a*x**2)*x**2)
```

3.148 $\int \frac{x^6}{W(ax^2)^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 94

$$\int \frac{x^6}{W(ax^2)^2}\,dx = \frac{4x^7}{49W(ax^2)^3} - \frac{2\sqrt{\frac{2}{7}}e^{-\frac{5}{2}W(ax^2)}x^5\Gamma\left(\frac{1}{2},-\frac{7}{2}W(ax^2)\right)\sqrt{-W(ax^2)}}{49aW(ax^2)^3} + \frac{x^7}{7W(ax^2)^2}$$

output
```
4/49*x^7/LambertW(a*x^2)^3-2/343*14^(1/2)*x^5*Pi^(1/2)*erfc(1/2*(-14*Lambe
rtW(a*x^2))^(1/2))*(-LambertW(a*x^2))^(1/2)/a/exp(5/2*LambertW(a*x^2))/Lam
bertW(a*x^2)^3+1/7*x^7/LambertW(a*x^2)^2
```

Mathematica [F]

$$\int \frac{x^6}{W(ax^2)^2}\,dx = \int \frac{x^6}{W(ax^2)^2}\,dx$$

input
```
Integrate[x^6/ProductLog[a*x^2]^2,x]
```

output
```
Integrate[x^6/ProductLog[a*x^2]^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{W(ax^2)^2} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{x^6}{W(ax^2)^2} dx$$

input
```
Int[x^6/ProductLog[a*x^2]^2,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^6}{\text{LambertW}\left(a\,x^2\right)^2} dx$$

input
```
int(x^6/LambertW(a*x^2)^2,x)
```

output
```
int(x^6/LambertW(a*x^2)^2,x)
```

Fricas [F]

$$\int \frac{x^6}{W(ax^2)^2} \, dx = \int \frac{x^6}{\text{W}(ax^2)^2} \, dx$$

input
```
integrate(x^6/lambert_w(a*x^2)^2,x, algorithm="fricas")
```

output
```
integral(x^6/lambert_w(a*x^2)^2, x)
```

Sympy [F]

$$\int \frac{x^6}{W(ax^2)^2} \, dx = \int \frac{x^6}{W^2(ax^2)} \, dx$$

input
```
integrate(x**6/LambertW(a*x**2)**2,x)
```

output
```
Integral(x**6/LambertW(a*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^6}{W(ax^2)^2} \, dx = \int \frac{x^6}{\mathrm{W}(ax^2)^2} \, dx$$

input
```
integrate(x^6/lambert_w(a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate(x^6/lambert_w(a*x^2)^2, x)
```

Giac [F]

$$\int \frac{x^6}{W(ax^2)^2} \, dx = \int \frac{x^6}{\mathrm{W}(ax^2)^2} \, dx$$

input
```
integrate(x^6/lambert_w(a*x^2)^2,x, algorithm="giac")
```

output
```
integrate(x^6/lambert_w(a*x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{W(ax^2)^2}\,dx = \int \frac{x^6}{\mathrm{LambertW}\left(a\,x^2\right)^2}\,dx$$

input
```
int(x^6/LambertW(a*x^2)^2,x)
```

output
```
int(x^6/LambertW(a*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^6}{W(ax^2)^2}\,dx = \int \frac{x^6}{lambert\_w\left(a\,x^2\right)^2}dx$$

input
```
int(x^6/Lambert_W(a*x^2)^2,x)
```

output
```
int(x**6/lambert_w(a*x**2)**2,x)
```

3.149 $\int \frac{x^4}{W(ax^2)^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int \frac{x^4}{W(ax^2)^2} dx = \frac{x^5}{5W(ax^2)^2} + \frac{2\sqrt{\frac{2}{5}}e^{-\frac{3}{2}W(ax^2)}x^3\Gamma\left(\frac{1}{2}, -\frac{5}{2}W(ax^2)\right)\sqrt{-W(ax^2)}}{5aW(ax^2)^2}$$

output
```
1/5*x^5/LambertW(a*x^2)^2+2/25*10^(1/2)*x^3*Pi^(1/2)*erfc(1/2*(-10*Lambert
W(a*x^2))^(1/2))*(-LambertW(a*x^2))^(1/2)/a/exp(3/2*LambertW(a*x^2))/Lambe
rtW(a*x^2)^2
```

Mathematica [F]

$$\int \frac{x^4}{W(ax^2)^2} dx = \int \frac{x^4}{W(ax^2)^2} dx$$

input
```
Integrate[x^4/ProductLog[a*x^2]^2,x]
```

output
```
Integrate[x^4/ProductLog[a*x^2]^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{W(ax^2)^2}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{x^4}{W(ax^2)^2}dx$$

input
```
Int[x^4/ProductLog[a*x^2]^2,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^4}{\mathrm{LambertW}\left(a\,x^2\right)^2}dx$$

input
```
int(x^4/LambertW(a*x^2)^2,x)
```

output
```
int(x^4/LambertW(a*x^2)^2,x)
```

Fricas [F]

$$\int \frac{x^4}{W(ax^2)^2}\,dx = \int \frac{x^4}{\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(x^4/lambert_w(a*x^2)^2,x, algorithm="fricas")
```

output
```
integral(x^4/lambert_w(a*x^2)^2, x)
```

Sympy [F]

$$\int \frac{x^4}{W(ax^2)^2}\,dx = \int \frac{x^4}{W^2(ax^2)}\,dx$$

input
```
integrate(x**4/LambertW(a*x**2)**2,x)
```

output
```
Integral(x**4/LambertW(a*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^4}{W(ax^2)^2}\,dx = \int \frac{x^4}{\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(x^4/lambert_w(a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate(x^4/lambert_w(a*x^2)^2, x)
```

Giac [F]

$$\int \frac{x^4}{W(ax^2)^2}\,dx = \int \frac{x^4}{\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(x^4/lambert_w(a*x^2)^2,x, algorithm="giac")
```

output
```
integrate(x^4/lambert_w(a*x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{W(ax^2)^2}\,dx = \int \frac{x^4}{\mathrm{LambertW}\left(a\,x^2\right)^2}\,dx$$

input
```
int(x^4/LambertW(a*x^2)^2,x)
```

output
```
int(x^4/LambertW(a*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^4}{W(ax^2)^2}\,dx = \int \frac{x^4}{\mathit{lambert\_w}\left(a\,x^2\right)^2}dx$$

input
```
int(x^4/Lambert_W(a*x^2)^2,x)
```

output
```
int(x**4/lambert_w(a*x**2)**2,x)
```

3.150 $\int \frac{x^2}{W(ax^2)^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 128

$$\int \frac{x^2}{W(ax^2)^2}\,dx = -\frac{x^3}{W(ax^2)^2} + \frac{\sqrt{\frac{3}{2}}e^{-\frac{1}{2}W(ax^2)}x\Gamma\left(\frac{1}{2},-\frac{3}{2}W(ax^2)\right)\sqrt{-W(ax^2)}}{aW(ax^2)} + \frac{e^{-\frac{1}{2}W(ax^2)}x\Gamma\left(\frac{1}{2},-\frac{3}{2}W(ax^2)\right)\sqrt{-W(ax^2)}}{\sqrt{6}aW(ax^2)}$$

output
```
-x^3/LambertW(a*x^2)^2+2/3*6^(1/2)*x*Pi^(1/2)*erfc(1/2*(-6*LambertW(a*x^2)
)^(1/2))*(-LambertW(a*x^2))^(1/2)/a/exp(1/2*LambertW(a*x^2))/LambertW(a*x^
2)
```

Mathematica [F]

$$\int \frac{x^2}{W(ax^2)^2}\,dx = \int \frac{x^2}{W(ax^2)^2}\,dx$$

input
```
Integrate[x^2/ProductLog[a*x^2]^2,x]
```

output
```
Integrate[x^2/ProductLog[a*x^2]^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{W\left(ax^2\right)^2}\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{x^2}{W\left(ax^2\right)^2}dx$$

input
```
Int[x^2/ProductLog[a*x^2]^2,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^2}{\text{LambertW}\left(a\,x^2\right)^2}dx$$

input
```
int(x^2/LambertW(a*x^2)^2,x)
```

output
```
int(x^2/LambertW(a*x^2)^2,x)
```

Fricas [F]

$$\int \frac{x^2}{W(ax^2)^2}\,dx = \int \frac{x^2}{\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(x^2/lambert_w(a*x^2)^2,x, algorithm="fricas")
```

output
```
integral(x^2/lambert_w(a*x^2)^2, x)
```

Sympy [F]

$$\int \frac{x^2}{W(ax^2)^2}\,dx = \int \frac{x^2}{W^2(ax^2)}\,dx$$

input
```
integrate(x**2/LambertW(a*x**2)**2,x)
```

output
```
Integral(x**2/LambertW(a*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^2}{W(ax^2)^2}\,dx = \int \frac{x^2}{\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(x^2/lambert_w(a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate(x^2/lambert_w(a*x^2)^2, x)
```

Giac [F]

$$\int \frac{x^2}{W(ax^2)^2} dx = \int \frac{x^2}{\mathrm{W}(ax^2)^2} dx$$

input
```
integrate(x^2/lambert_w(a*x^2)^2,x, algorithm="giac")
```

output
```
integrate(x^2/lambert_w(a*x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{W(ax^2)^2} dx = \int \frac{x^2}{\mathrm{LambertW}\left(a\,x^2\right)^2} dx$$

input
```
int(x^2/LambertW(a*x^2)^2,x)
```

output
```
int(x^2/LambertW(a*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^2}{W(ax^2)^2} dx = \int \frac{x^2}{lambert\_w\left(a\,x^2\right)^2} dx$$

input
```
int(x^2/Lambert_W(a*x^2)^2,x)
```

output
```
int(x**2/lambert_w(a*x**2)**2,x)
```

3.151 $\int \frac{1}{W(ax^2)^2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 80

$$\int \frac{1}{W(ax^2)^2}\,dx = \frac{2\sqrt{2}e^{\frac{1}{2}W(ax^2)}\Gamma\left(\frac{1}{2},-\frac{1}{2}W(ax^2)\right)\sqrt{-W(ax^2)}}{3ax} - \frac{x}{3W(ax^2)^2} - \frac{4x}{3W(ax^2)}$$

output
```
2/3*2^(1/2)*exp(1/2*LambertW(a*x^2))*Pi^(1/2)*erfc(1/2*(-2*LambertW(a*x^2)
)^(1/2))*(-LambertW(a*x^2))^(1/2)/a/x-1/3*x/LambertW(a*x^2)^2-4/3*x/Lamber
tW(a*x^2)
```

Mathematica [F]

$$\int \frac{1}{W(ax^2)^2}\,dx = \int \frac{1}{W(ax^2)^2}\,dx$$

input
```
Integrate[ProductLog[a*x^2]^(-2),x]
```

output
```
Integrate[ProductLog[a*x^2]^(-2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W\left(ax^2\right)^2}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{1}{W\left(ax^2\right)^2}dx$$

input
```
Int[ProductLog[a*x^2]^(-2),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{1}{\mathrm{LambertW}\left(a\,x^2\right)^2}dx$$

input
```
int(1/LambertW(a*x^2)^2,x)
```

output
```
int(1/LambertW(a*x^2)^2,x)
```

Fricas [F]

$$\int \frac{1}{W\left(ax^2\right)^2}\,dx = \int \frac{1}{\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(1/lambert_w(a*x^2)^2,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^2)^(-2), x)
```

Sympy [F]

$$\int \frac{1}{W(ax^2)^2} \, dx = \int \frac{1}{W^2(ax^2)} \, dx$$

input
```
integrate(1/LambertW(a*x**2)**2,x)
```

output
```
Integral(LambertW(a*x**2)**(-2), x)
```

Maxima [F]

$$\int \frac{1}{W(ax^2)^2} \, dx = \int \frac{1}{\mathrm{W}(ax^2)^2} \, dx$$

input
```
integrate(1/lambert_w(a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^2)^(-2), x)
```

Giac [F]

$$\int \frac{1}{W(ax^2)^2} \, dx = \int \frac{1}{\mathrm{W}(ax^2)^2} \, dx$$

input
```
integrate(1/lambert_w(a*x^2)^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^2)^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W(ax^2)^2} \, dx = \int \frac{1}{\mathrm{LambertW}\left(a\,x^2\right)^2} \, dx$$

input
```
int(1/LambertW(a*x^2)^2,x)
```

output
```
int(1/LambertW(a*x^2)^2, x)
```

Reduce [F]

$$\int \frac{1}{W(ax^2)^2} \, dx = \int \frac{1}{lambert\_w\left(a\,x^2\right)^2}dx$$

input
```
int(1/Lambert_W(a*x^2)^2,x)
```

output
```
int(1/lambert_w(a*x**2)**2,x)
```

3.152 $\int \frac{1}{x^2 W\left(a x^2\right)^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 89

$$\int \frac{1}{x^2 W\left(a x^2\right)^2} \, dx = \frac{4}{15x} - \frac{1}{5xW\left(ax^2\right)^2} - \frac{4}{15xW\left(ax^2\right)} - \frac{2\sqrt{2}e^{\frac{3}{2}W\left(ax^2\right)}\Gamma\left(\frac{1}{2}, \frac{1}{2}W\left(ax^2\right)\right) W\left(ax^2\right)^{3/2}}{15ax^3}$$

output
```
4/15/x-1/5/x/LambertW(a*x^2)^2-4/15/x/LambertW(a*x^2)-2/15*2^(1/2)*exp(3/2
*LambertW(a*x^2))*Pi^(1/2)*erfc(1/2*2^(1/2)*LambertW(a*x^2)^(1/2))*Lambert
W(a*x^2)^(3/2)/a/x^3
```

Mathematica [F]

$$\int \frac{1}{x^2 W\left(a x^2\right)^2} \, dx = \int \frac{1}{x^2 W\left(a x^2\right)^2} \, dx$$

input
```
Integrate[1/(x^2*ProductLog[a*x^2]^2),x]
```

output
```
Integrate[1/(x^2*ProductLog[a*x^2]^2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 W\left(a x^2\right)^2} \, dx$$

$$\big\downarrow \text{ 7299}$$

$$\int \frac{1}{x^2 W\left(a x^2\right)^2}dx$$

input
```
Int[1/(x^2*ProductLog[a*x^2]^2),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{1}{x^{2} \operatorname{LambertW}\left(a \,x^{2}\right)^{2}}d x$$

input
```
int(1/x^2/LambertW(a*x^2)^2,x)
```

output
```
int(1/x^2/LambertW(a*x^2)^2,x)
```

Fricas [F]

$$\int \frac{1}{x^2 W\left(a x^2\right)^2} \, dx = \int \frac{1}{x^{2} \mathrm{W}\left(a x^{2}\right)^{2}} \, dx$$

input
```
integrate(1/x^2/lambert_w(a*x^2)^2,x, algorithm="fricas")
```

output
```
integral(1/(x^2*lambert_w(a*x^2)^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 W\left(ax^2\right)^2}\, dx = \int \frac{1}{x^2 W^2\left(ax^2\right)}\, dx$$

input
```
integrate(1/x**2/LambertW(a*x**2)**2,x)
```

output
```
Integral(1/(x**2*LambertW(a*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2 W\left(ax^2\right)^2}\, dx = \int \frac{1}{x^2\, \mathrm{W}(ax^2)^2}\, dx$$

input
```
integrate(1/x^2/lambert_w(a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate(1/(x^2*lambert_w(a*x^2)^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 W\left(ax^2\right)^2}\, dx = \int \frac{1}{x^2\, \mathrm{W}(ax^2)^2}\, dx$$

input
```
integrate(1/x^2/lambert_w(a*x^2)^2,x, algorithm="giac")
```

output
```
integrate(1/(x^2*lambert_w(a*x^2)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2W\left(ax^2\right)^2}\,dx=\int \frac{1}{x^2\,\mathrm{LambertW}\left(a\,x^2\right)^2}\,dx$$

input
```
int(1/(x^2*LambertW(a*x^2)^2),x)
```

output
```
int(1/(x^2*LambertW(a*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^2W\left(ax^2\right)^2}\,dx = \frac{\left(\int \frac{lambert\_w(a\,x^2)}{lambert\_w(a\,x^2)x^2+x^2}dx\right) lambert\_w(a\,x^2)\,x+4\left(\int \frac{1}{lambert\_w(a\,x^2)^3x^2+lambert\_w(a\,x^2)^2x^2}dx\right) lambert}{}$$

input
```
int(1/x^2/Lambert_W(a*x^2)^2,x)
```

output
```
(int(lambert_w(a*x**2)/(lambert_w(a*x**2)*x**2 + x**2),x)*lambert_w(a*x**2
)*x + 4*int(1/(lambert_w(a*x**2)**3*x**2 + lambert_w(a*x**2)**2*x**2),x)*l
ambert_w(a*x**2)*x + 2*int(1/(lambert_w(a*x**2)**2*x**2 + lambert_w(a*x**2
)*x**2),x)*lambert_w(a*x**2)*x - 4*int(1/(e**lambert_w(a*x**2)*lambert_w(a
*x**2)**3 + e**lambert_w(a*x**2)*lambert_w(a*x**2)**2),x)*lambert_w(a*x**2
)*a*x - int(1/(lambert_w(a*x**2)*x**2 + x**2),x)*lambert_w(a*x**2)*x + lam
bert_w(a*x**2) - 2)/(4*lambert_w(a*x**2)*x)
```

3.153 $\int \frac{1}{x^4W(ax^2)^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 102

$$\int \frac{1}{x^4W(ax^2)^2} dx = \frac{4}{35x^3} - \frac{1}{7x^3W(ax^2)^2} - \frac{4}{35x^3W(ax^2)} - \frac{12W(ax^2)}{35x^3} + \frac{6\sqrt{6}e^{\frac{5}{2}W(ax^2)}\Gamma\left(\frac{1}{2},\frac{3}{2}W(ax^2)\right)W(ax^2)^{5/2}}{35ax^5}$$

output
```
4/35/x^3-1/7/x^3/LambertW(a*x^2)^2-4/35/x^3/LambertW(a*x^2)-12/35*LambertW
(a*x^2)/x^3+6/35*6^(1/2)*exp(5/2*LambertW(a*x^2))*Pi^(1/2)*erfc(1/2*6^(1/2
)*LambertW(a*x^2)^(1/2))*LambertW(a*x^2)^(5/2)/a/x^5
```

Mathematica [F]

$$\int \frac{1}{x^4W(ax^2)^2} dx = \int \frac{1}{x^4W(ax^2)^2} dx$$

input
```
Integrate[1/(x^4*ProductLog[a*x^2]^2),x]
```

output
```
Integrate[1/(x^4*ProductLog[a*x^2]^2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 W\left(ax^2\right)^2}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{1}{x^4 W\left(ax^2\right)^2}dx$$

input
```
Int[1/(x^4*ProductLog[a*x^2]^2),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{1}{x^4\,\mathrm{LambertW}\left(a\,x^2\right)^2}dx$$

input
```
int(1/x^4/LambertW(a*x^2)^2,x)
```

output
```
int(1/x^4/LambertW(a*x^2)^2,x)
```

Fricas [F]

$$\int \frac{1}{x^4 W\left(ax^2\right)^2}\,dx = \int \frac{1}{x^4\,\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(1/x^4/lambert_w(a*x^2)^2,x, algorithm="fricas")
```

output
```
integral(1/(x^4*lambert_w(a*x^2)^2), x)
```

Sympy [F]

$$\int \frac{1}{x^4 W\left(ax^2\right)^2}\,dx = \int \frac{1}{x^4 W^2\left(ax^2\right)}\,dx$$

input
```
integrate(1/x**4/LambertW(a*x**2)**2,x)
```

output
```
Integral(1/(x**4*LambertW(a*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^4 W\left(ax^2\right)^2}\,dx = \int \frac{1}{x^4\,\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(1/x^4/lambert_w(a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate(1/(x^4*lambert_w(a*x^2)^2), x)
```

Giac [F]

$$\int \frac{1}{x^4 W\left(ax^2\right)^2}\,dx = \int \frac{1}{x^4\,\mathrm{W}(ax^2)^2}\,dx$$

input
```
integrate(1/x^4/lambert_w(a*x^2)^2,x, algorithm="giac")
```

output
```
integrate(1/(x^4*lambert_w(a*x^2)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 W(ax^2)^2}\,dx = \int \frac{1}{x^4\,\mathrm{LambertW}\left(a\,x^2\right)^2}\,dx$$

input
```
int(1/(x^4*LambertW(a*x^2)^2),x)
```

output
```
int(1/(x^4*LambertW(a*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^4 W(ax^2)^2}\,dx$$
$$= \frac{3\left(\int \frac{lambert\_w(a\,x^2)}{lambert\_w(a\,x^2)x^4+x^4}dx\right) lambert\_w(a\,x^2)\,x^3 + 12\left(\int \frac{1}{lambert\_w(a\,x^2)^3x^4+lambert\_w(a\,x^2)^2x^4}dx\right) lam}{}$$

input
```
int(1/x^4/Lambert_W(a*x^2)^2,x)
```

output
```
(3*int(lambert_w(a*x**2)/(lambert_w(a*x**2)*x**4 + x**4),x)*lambert_w(a*x*
*2)*x**3 + 12*int(1/(lambert_w(a*x**2)**3*x**4 + lambert_w(a*x**2)**2*x**4
),x)*lambert_w(a*x**2)*x**3 + 6*int(1/(lambert_w(a*x**2)**2*x**4 + lambert
_w(a*x**2)*x**4),x)*lambert_w(a*x**2)*x**3 - 4*int(1/(e**lambert_w(a*x**2)
*lambert_w(a*x**2)**3*x**2 + e**lambert_w(a*x**2)*lambert_w(a*x**2)**2*x**
2),x)*lambert_w(a*x**2)*a*x**3 - 3*int(1/(lambert_w(a*x**2)*x**4 + x**4),x
)*lambert_w(a*x**2)*x**3 + lambert_w(a*x**2) - 2)/(12*lambert_w(a*x**2)*x*
*3)
```

3.154 $\int x^5 \sqrt{cW(ax^2)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 129

$$\int x^5 \sqrt{cW(ax^2)}\,dx = \frac{5\sqrt{c}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{cW(ax^2)}}{\sqrt{c}}\right)}{864a^3} - \frac{5c^3x^6}{432\,(cW(ax^2))^{5/2}} + \frac{5c^2x^6}{216\,(cW(ax^2))^{3/2}} - \frac{cx^6}{36\sqrt{cW(ax^2)}} + \frac{1}{6}x^6\sqrt{cW(ax^2)}$$

output
```
5/2592*c^(1/2)*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(c*LambertW(a*x^2))^(1/2)/c^(
1/2))/a^3-5/432*c^3*x^6/(c*LambertW(a*x^2))^(5/2)+5/216*c^2*x^6/(c*Lambert
W(a*x^2))^(3/2)-1/36*c*x^6/(c*LambertW(a*x^2))^(1/2)+1/6*x^6*(c*LambertW(a
*x^2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

$$\int x^5 \sqrt{cW(ax^2)}\,dx = \frac{\sqrt{cW(ax^2)}\left(-30a^3x^6 + 60a^3x^6W(ax^2) - 72a^3x^6W(ax^2)^2 + 5\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{W(ax^2)}\right)W(ax^2)^{5/2} + 43\right.}{2592a^3W(ax^2)^3}$$

input
```
Integrate[x^5*Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(Sqrt[c*ProductLog[a*x^2]]*(-30*a^3*x^6 + 60*a^3*x^6*ProductLog[a*x^2] - 7
2*a^3*x^6*ProductLog[a*x^2]^2 + 5*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ProductLog[
a*x^2]]]*ProductLog[a*x^2]^(5/2) + 432*a^3*x^6*ProductLog[a*x^2]^3))/(2592
*a^3*ProductLog[a*x^2]^3)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {7172, 7205, 7205, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{cW\left(ax^2\right)}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{6}x^6\sqrt{cW\left(ax^2\right)}-\frac{1}{6}\int \frac{x^5\sqrt{cW\left(ax^2\right)}}{W\left(ax^2\right)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{6}\left(\frac{5}{6}c\int \frac{x^5}{\sqrt{cW\left(ax^2\right)}\left(W\left(ax^2\right)+1\right)}dx-\frac{cx^6}{6\sqrt{cW\left(ax^2\right)}}\right)+\frac{1}{6}x^6\sqrt{cW\left(ax^2\right)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{6}\left(\frac{5}{6}c\left(\frac{cx^6}{6\left(cW\left(ax^2\right)\right)^{3/2}}-\frac{1}{2}c\int \frac{x^5}{\left(cW\left(ax^2\right)\right)^{3/2}\left(W\left(ax^2\right)+1\right)}dx\right)-\frac{cx^6}{6\sqrt{cW\left(ax^2\right)}}\right)+\frac{1}{6}x^6\sqrt{cW\left(ax^2\right)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{6}\left(\frac{5}{6}c\left(\frac{cx^6}{6\left(cW\left(ax^2\right)\right)^{3/2}}-\frac{1}{2}c\left(\frac{cx^6}{6\left(cW\left(ax^2\right)\right)^{5/2}}-\frac{1}{6}c\int \frac{x^5}{\left(cW\left(ax^2\right)\right)^{5/2}\left(W\left(ax^2\right)+1\right)}dx\right)\right)-\frac{cx^6}{6\sqrt{cW\left(ax^2\right)}}\right)+\frac{1}{6}x^6\sqrt{cW\left(ax^2\right)}$$

$$\Big\downarrow \text{7204}$$

$$\frac{1}{6}\left(\frac{5}{6}c\left(\frac{cx^6}{6\,(cW\,(ax^2))^{3/2}}-\frac{1}{2}c\left(\frac{cx^6}{6\,(cW\,(ax^2))^{5/2}}-\frac{\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{cW(ax^2)}}{\sqrt{c}}\right)}{12a^3c^{3/2}}\right)\right)-\frac{cx^6}{6\sqrt{cW\,(ax^2)}}\right)+\frac{1}{6}x^6\sqrt{cW\,(ax^2)}$$

input
```
Int[x^5*Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(x^6*Sqrt[c*ProductLog[a*x^2]])/6 + (-1/6*(c*x^6)/Sqrt[c*ProductLog[a*x^2]
] + (5*c*((c*x^6)/(6*(c*ProductLog[a*x^2])^(3/2)) - (c*(-1/12*(Sqrt[Pi/3]*
Erfi[(Sqrt[3]*Sqrt[c*ProductLog[a*x^2]])/Sqrt[c]])/(a^3*c^(3/2)) + (c*x^6)
/(6*(c*ProductLog[a*x^2])^(5/2))))/2))/6)/6
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^5 \sqrt{c \,\mathrm{LambertW}\left(a\,x^2\right)}dx$$

input
```
int(x^5*(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^5*(c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int x^5 \sqrt{cW\left(ax^2\right)}\,dx = \int \sqrt{c\,\mathrm{W}(ax^2)}x^5\,dx$$

input
```
integrate(x^5*(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))*x^5, x)
```

Sympy [F]

$$\int x^5 \sqrt{cW\left(ax^2\right)}\,dx = \int x^5 \sqrt{cW\left(ax^2\right)}\,dx$$

input
```
integrate(x**5*(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(x**5*sqrt(c*LambertW(a*x**2)), x)
```

Maxima [F]

$$\int x^5\sqrt{cW\left(ax^2\right)}\,dx=\int \sqrt{c\,\mathrm{W}(ax^2)}x^5\,dx$$

input
```
integrate(x^5*(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))*x^5, x)
```

Giac [F]

$$\int x^5\sqrt{cW\left(ax^2\right)}\,dx=\int \sqrt{c\,\mathrm{W}(ax^2)}x^5\,dx$$

input
```
integrate(x^5*(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{cW\left(ax^2\right)}\,dx=\int x^5\,\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}\,dx$$

input
```
int(x^5*(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^5*(c*LambertW(a*x^2))^(1/2), x)
```

Reduce [F]

$$\int x^5\sqrt{cW\left(ax^2\right)}\,dx=\sqrt{c}\left(\int\sqrt{lambert\_w\left(a\,x^2\right)}\,x^5dx\right)$$

input
```
int(x^5*(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a*x**2))*x**5,x)
```

3.155 $\int x^3 \sqrt{cW(ax^2)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 107

$$\int x^3 \sqrt{cW(ax^2)}\,dx = -\frac{3\sqrt{c}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^2)}}{\sqrt{c}}\right)}{128a^2} + \frac{3c^2x^4}{64\left(cW(ax^2)\right)^{3/2}} - \frac{cx^4}{16\sqrt{cW(ax^2)}} + \frac{1}{4}x^4\sqrt{cW(ax^2)}$$

output
```
-3/256*c^(1/2)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(a*x^2))^(1/2)/c^(
1/2))/a^2+3/64*c^2*x^4/(c*LambertW(a*x^2))^(3/2)-1/16*c*x^4/(c*LambertW(a*
x^2))^(1/2)+1/4*x^4*(c*LambertW(a*x^2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95

$$\int x^3 \sqrt{cW(ax^2)}\,dx = \frac{\sqrt{cW(ax^2)}\left(12a^2x^4 - 16a^2x^4W(ax^2) - 3\sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{W(ax^2)}\right)W(ax^2)^{3/2} + 64a^2x^4W(ax^2)^2\right)}{256a^2W(ax^2)^2}$$

input
```
Integrate[x^3*Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(Sqrt[c*ProductLog[a*x^2]]*(12*a^2*x^4 - 16*a^2*x^4*ProductLog[a*x^2] - 3*
Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ProductLog[a*x^2]]]*ProductLog[a*x^2]^(3/2) +
 64*a^2*x^4*ProductLog[a*x^2]^2))/(256*a^2*ProductLog[a*x^2]^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7172, 7205, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{cW\left(ax^2\right)}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{4}x^4\sqrt{cW\left(ax^2\right)}-\frac{1}{4}\int \frac{x^3\sqrt{cW\left(ax^2\right)}}{W\left(ax^2\right)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{4}\left(\frac{3}{4}c\int \frac{x^3}{\sqrt{cW\left(ax^2\right)}\left(W\left(ax^2\right)+1\right)}dx-\frac{cx^4}{4\sqrt{cW\left(ax^2\right)}}\right)+\frac{1}{4}x^4\sqrt{cW\left(ax^2\right)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{4}\left(\frac{3}{4}c\left(\frac{cx^4}{4\left(cW\left(ax^2\right)\right)^{3/2}}-\frac{1}{4}c\int \frac{x^3}{\left(cW\left(ax^2\right)\right)^{3/2}\left(W\left(ax^2\right)+1\right)}dx\right)-\frac{cx^4}{4\sqrt{cW\left(ax^2\right)}}\right)+\frac{1}{4}x^4\sqrt{cW\left(ax^2\right)}$$

$$\Big\downarrow \text{7204}$$

$$\frac{1}{4}\left(\frac{3}{4}c\left(\frac{cx^4}{4\left(cW\left(ax^2\right)\right)^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW\left(ax^2\right)}}{\sqrt{c}}\right)}{8a^2\sqrt{c}}\right)-\frac{cx^4}{4\sqrt{cW\left(ax^2\right)}}\right)+\frac{1}{4}x^4\sqrt{cW\left(ax^2\right)}$$

input
```
Int[x^3*Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(x^4*Sqrt[c*ProductLog[a*x^2]])/4 + (-1/4*(c*x^4)/Sqrt[c*ProductLog[a*x^2]
] + (3*c*(-1/8*(Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[c*ProductLog[a*x^2]])/Sqrt[c
]])/(a^2*Sqrt[c]) + (c*x^4)/(4*(c*ProductLog[a*x^2])^(3/2))))/4)/4
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^3 \sqrt{c \,\mathrm{LambertW}\left(a\,x^2\right)}dx$$

input
```
int(x^3*(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^3*(c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int x^3\sqrt{cW\left(ax^2\right)}\,dx=\int \sqrt{c\,\mathrm{W}(ax^2)}x^3\,dx$$

input
```
integrate(x^3*(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))*x^3, x)
```

Sympy [F]

$$\int x^3\sqrt{cW\left(ax^2\right)}\,dx=\int x^3\sqrt{cW\left(ax^2\right)}\,dx$$

input
```
integrate(x**3*(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(x**3*sqrt(c*LambertW(a*x**2)), x)
```

Maxima [F]

$$\int x^3\sqrt{cW\left(ax^2\right)}\,dx=\int \sqrt{c\,\mathrm{W}(ax^2)}x^3\,dx$$

input
```
integrate(x^3*(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{cW\left(ax^2\right)}\,dx=\int \sqrt{c\,\mathrm{W}(ax^2)}x^3\,dx$$

input
```
integrate(x^3*(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{cW\left(ax^2\right)}\,dx=\int x^3\,\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}\,dx$$

input
```
int(x^3*(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^3*(c*LambertW(a*x^2))^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{cW\left(ax^2\right)}\,dx=\sqrt{c}\left(\int\sqrt{lambert\_w\left(a\,x^2\right)}\,x^3dx\right)$$

input
```
int(x^3*(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a*x**2))*x**3,x)
```

3.156 $\int x\sqrt{cW\left(ax^2\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 76

$$\int x\sqrt{cW\left(ax^2\right)}\,dx=\frac{\sqrt{c}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{cW(ax^2)}}{\sqrt{c}}\right)}{8a}-\frac{cx^2}{4\sqrt{cW\left(ax^2\right)}}+\frac{1}{2}x^2\sqrt{cW\left(ax^2\right)}$$

output
```
1/8*c^(1/2)*Pi^(1/2)*erfi((c*LambertW(a*x^2))^(1/2)/c^(1/2))/a-1/4*c*x^2/(
c*LambertW(a*x^2))^(1/2)+1/2*x^2*(c*LambertW(a*x^2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int x\sqrt{cW\left(ax^2\right)}\,dx=\frac{c\left(-2ax^2+\sqrt{\pi}\,\text{erfi}\left(\sqrt{W\left(ax^2\right)}\right)\sqrt{W\left(ax^2\right)}+4ax^2W(ax^2)\right)}{8a\sqrt{cW\left(ax^2\right)}}$$

input
```
Integrate[x*Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(c*(-2*a*x^2 + Sqrt[Pi]*Erfi[Sqrt[ProductLog[a*x^2]]]*Sqrt[ProductLog[a*x^
2]] + 4*a*x^2*ProductLog[a*x^2]))/(8*a*Sqrt[c*ProductLog[a*x^2]])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {7172, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{cW\left(ax^2\right)}dx$$

$$\Big\downarrow \ 7172$$

$$\frac{1}{2}x^2\sqrt{cW\left(ax^2\right)}-\frac{1}{2}\int \frac{x\sqrt{cW\left(ax^2\right)}}{W\left(ax^2\right)+1}dx$$

$$\Big\downarrow \ 7205$$

$$\frac{1}{2}\left(\frac{1}{2}c\int \frac{x}{\sqrt{cW\left(ax^2\right)}\left(W\left(ax^2\right)+1\right)}dx-\frac{cx^2}{2\sqrt{cW\left(ax^2\right)}}\right)+\frac{1}{2}x^2\sqrt{cW\left(ax^2\right)}$$

$$\Big\downarrow \ 7204$$

$$\frac{1}{2}\left(\frac{\sqrt{\pi}\sqrt{c}\operatorname{erfi}\left(\frac{\sqrt{cW\left(ax^2\right)}}{\sqrt{c}}\right)}{4a}-\frac{cx^2}{2\sqrt{cW\left(ax^2\right)}}\right)+\frac{1}{2}x^2\sqrt{cW\left(ax^2\right)}$$

input
```
Int[x*Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(x^2*Sqrt[c*ProductLog[a*x^2]])/2 + ((Sqrt[c]*Sqrt[Pi]*Erfi[Sqrt[c*Product
Log[a*x^2]]/Sqrt[c]])/(4*a) - (c*x^2)/(2*Sqrt[c*ProductLog[a*x^2]]))/2
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{\frac{\left(c\,\mathrm{LambertW}\left(a\,x^{2}\right)\right)^{\frac{3}{2}} a\,x^{2} c}{2\,\mathrm{LambertW}\left(a\,x^{2}\right)}-\frac{c\left(\frac{c\sqrt{c\,\mathrm{LambertW}\left(a\,x^{2}\right)}\,a\,x^{2}}{2\,\mathrm{LambertW}\left(a\,x^{2}\right)}-\frac{c\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{c}}\,\sqrt{c\,\mathrm{LambertW}\left(a\,x^{2}\right)}\right)}{4\sqrt{-\frac{1}{c}}}\right)}{2}}{a\,c^{2}}$	95
default	$\frac{\frac{\left(c\,\mathrm{LambertW}\left(a\,x^{2}\right)\right)^{\frac{3}{2}} a\,x^{2} c}{2\,\mathrm{LambertW}\left(a\,x^{2}\right)}-\frac{c\left(\frac{c\sqrt{c\,\mathrm{LambertW}\left(a\,x^{2}\right)}\,a\,x^{2}}{2\,\mathrm{LambertW}\left(a\,x^{2}\right)}-\frac{c\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{c}}\,\sqrt{c\,\mathrm{LambertW}\left(a\,x^{2}\right)}\right)}{4\sqrt{-\frac{1}{c}}}\right)}{2}}{a\,c^{2}}$	95

input
```
int(x*(c*LambertW(a*x^2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/c^2*(1/2*(c*LambertW(a*x^2))^(3/2)*a*x^2/LambertW(a*x^2)*c-1/2*c*(1/2*
c*(c*LambertW(a*x^2))^(1/2)*a*x^2/LambertW(a*x^2)-1/4*c*Pi^(1/2)/(-1/c)^(1
/2)*erf((-1/c)^(1/2)*(c*LambertW(a*x^2))^(1/2))))
```

Fricas [F]

$$\int x\sqrt{cW\left(ax^2\right)}\,dx=\int\sqrt{c\,\mathrm{W}(ax^2)}x\,dx$$

input
```
integrate(x*(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))*x, x)
```

Sympy [F]

$$\int x\sqrt{cW\left(ax^2\right)}\,dx=\int x\sqrt{cW\left(ax^2\right)}\,dx$$

input
```
integrate(x*(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(x*sqrt(c*LambertW(a*x**2)), x)
```

Maxima [F]

$$\int x\sqrt{cW\left(ax^2\right)}\,dx=\int\sqrt{c\,\mathrm{W}(ax^2)}x\,dx$$

input
```
integrate(x*(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))*x, x)
```

Giac [F]

$$\int x\sqrt{cW(ax^2)}\,dx = \int \sqrt{c\,\mathrm{W}(ax^2)}x\,dx$$

input
```
integrate(x*(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{cW(ax^2)}\,dx = \int x\,\sqrt{c\,\mathrm{LambertW}(a\,x^2)}\,dx$$

input
```
int(x*(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x*(c*LambertW(a*x^2))^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{cW(ax^2)}\,dx = \frac{\sqrt{c}\left(2e^{lambert\_w(a\,x^2)}\sqrt{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2) - e^{lambert\_w(a\,x^2)}\sqrt{lambert\_w(a\,x^2)} + \left(\int \frac{}{lam}\right.\right.}{4a}$$

input
```
int(x*(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
(sqrt(c)*(2*e**lambert_w(a*x**2)*sqrt(lambert_w(a*x**2))*lambert_w(a*x**2)
 - e**lambert_w(a*x**2)*sqrt(lambert_w(a*x**2)) + int((sqrt(lambert_w(a*x*
*2))*x)/(lambert_w(a*x**2)**2 + lambert_w(a*x**2)),x)*a))/(4*a)
```

3.157 $\int \frac{\sqrt{cW(ax^2)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{\sqrt{cW(ax^2)}}{x}\,dx = \sqrt{cW(ax^2)} + \frac{(cW(ax^2))^{3/2}}{3c}$$

output
```
(c*LambertW(a*x^2))^(1/2)+1/3*(c*LambertW(a*x^2))^(3/2)/c
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{cW(ax^2)}}{x}\,dx = \frac{1}{3}\sqrt{cW(ax^2)}(3 + W(ax^2))$$

input
```
Integrate[Sqrt[c*ProductLog[a*x^2]]/x,x]
```

output
```
(Sqrt[c*ProductLog[a*x^2]]*(3 + ProductLog[a*x^2]))/3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cW(ax^2)}}{x} dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{\int \frac{(cW(ax^2))^{3/2}}{x(W(ax^2)+1)}dx}{c} + \sqrt{cW(ax^2)}$$

$$\Big\downarrow \text{7200}$$

$$\frac{(cW(ax^2))^{3/2}}{3c} + \sqrt{cW(ax^2)}$$

input
```
Int[Sqrt[c*ProductLog[a*x^2]]/x,x]
```

output
```
Sqrt[c*ProductLog[a*x^2]] + (c*ProductLog[a*x^2])^(3/2)/(3*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\frac{\left(c\,\mathrm{LambertW}\left(a\,x^2\right)\right)^{\frac{3}{2}}}{3}+c\sqrt{c\,\mathrm{LambertW}(a\,x^2)}}{c}$	30
default	$\frac{\frac{\left(c\,\mathrm{LambertW}\left(a\,x^2\right)\right)^{\frac{3}{2}}}{3}+c\sqrt{c\,\mathrm{LambertW}(a\,x^2)}}{c}$	30

input
```
int((c*LambertW(a*x^2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/3*(c*LambertW(a*x^2))^(3/2)+c*(c*LambertW(a*x^2))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{cW\left(ax^2\right)}}{x}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))/x, x)
```

Sympy [F]

$$\int \frac{\sqrt{cW\left(ax^2\right)}}{x}\,dx = \int \frac{\sqrt{cW\left(ax^2\right)}}{x}\,dx$$

input
```
integrate((c*LambertW(a*x**2))**(1/2)/x,x)
```

output
```
Integral(sqrt(c*LambertW(a*x**2))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))/x, x)
```

Giac [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cW(ax^2)}}{x}\,dx = \int \frac{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}{x}\,dx$$

input
```
int((c*LambertW(a*x^2))^(1/2)/x,x)
```

output
```
int((c*LambertW(a*x^2))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x}\,dx = \sqrt{c}\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}}{x}dx\right)$$

input
```
int((c*Lambert_W(a*x^2))^(1/2)/x,x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a*x**2))/x,x)
```

3.158 $\int \frac{\sqrt{cW(ax^2)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 52

$$\int \frac{\sqrt{cW(ax^2)}}{x^3}\,dx = -\frac{1}{2}a\sqrt{c}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{cW(ax^2)}}{\sqrt{c}}\right) - \frac{\sqrt{cW(ax^2)}}{x^2}$$

output
```
-1/2*a*c^(1/2)*Pi^(1/2)*erf((c*LambertW(a*x^2))^(1/2)/c^(1/2))-(c*LambertW
(a*x^2))^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{cW(ax^2)}}{x^3}\,dx = \frac{1}{2}\left(-\frac{2}{x^2} - \frac{a\sqrt{\pi}\operatorname{erf}\left(\sqrt{W(ax^2)}\right)}{\sqrt{W(ax^2)}}\right)\sqrt{cW(ax^2)}$$

input
```
Integrate[Sqrt[c*ProductLog[a*x^2]]/x^3,x]
```

output
```
((-2/x^2 - (a*Sqrt[Pi]*Erf[Sqrt[ProductLog[a*x^2]]])/Sqrt[ProductLog[a*x^2
]])*Sqrt[c*ProductLog[a*x^2]])/2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7173, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cW(ax^2)}}{x^3}dx$$

$$\Big\downarrow \text{7173}$$

$$-\frac{\int \frac{(cW(ax^2))^{3/2}}{x^3(W(ax^2)+1)}dx}{c}-\frac{\sqrt{cW(ax^2)}}{x^2}$$

$$\Big\downarrow \text{7203}$$

$$-\frac{1}{2}\sqrt{\pi}a\sqrt{c}\text{erf}\left(\frac{\sqrt{cW(ax^2)}}{\sqrt{c}}\right)-\frac{\sqrt{cW(ax^2)}}{x^2}$$

input
```
Int[Sqrt[c*ProductLog[a*x^2]]/x^3,x]
```

output
```
-1/2*(a*Sqrt[c]*Sqrt[Pi]*Erf[Sqrt[c*ProductLog[a*x^2]]/Sqrt[c]]) - Sqrt[c*
ProductLog[a*x^2]]/x^2
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

Maple [F]

$$\int \frac{\sqrt{c \,\mathrm{LambertW}\left(a \,x^{2}\right)}}{x^{3}}d x$$

input
```
int((c*LambertW(a*x^2))^(1/2)/x^3,x)
```

output
```
int((c*LambertW(a*x^2))^(1/2)/x^3,x)
```

Fricas [F]

$$\int \frac{\sqrt{cW\left(ax^2\right)}}{x^3}\,dx=\int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x^3}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{cW\left(ax^2\right)}}{x^3}\,dx=\int \frac{\sqrt{cW\left(ax^2\right)}}{x^3}\,dx$$

input
```
integrate((c*LambertW(a*x**2))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(c*LambertW(a*x**2))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^3}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x^3}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^3}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x^3}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cW(ax^2)}}{x^3}\,dx = \int \frac{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}{x^3}\,dx$$

input
```
int((c*LambertW(a*x^2))^(1/2)/x^3,x)
```

output
```
int((c*LambertW(a*x^2))^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{cW\left(ax^2\right)}}{x^3}\,dx$$
$$= \frac{\sqrt{c}\left(-\sqrt{lambert\_w\left(a\,x^2\right)}+\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}}{e^{lambert\_w\left(a\,x^2\right)}\,lambert\_w(a\,x^2)^2x+e^{lambert\_w\left(a\,x^2\right)}\,lambert\_w(a\,x^2)x}dx\right)a\,x^2}{2x^2}$$

input
```
int((c*Lambert_W(a*x^2))^(1/2)/x^3,x)
```

output
```
(sqrt(c)*( - sqrt(lambert_w(a*x**2)) + int(sqrt(lambert_w(a*x**2))/(e**lam
bert_w(a*x**2)*lambert_w(a*x**2)**2*x + e**lambert_w(a*x**2)*lambert_w(a*x
**2)*x),x)*a*x**2))/(2*x**2)
```

3.159 $\int \frac{\sqrt{cW(ax^2)}}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{\sqrt{cW(ax^2)}}{x^5}\,dx=\frac{1}{3}a^2\sqrt{c}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^2)}}{\sqrt{c}}\right)-\frac{\sqrt{cW(ax^2)}}{3x^4}+\frac{(cW(ax^2))^{3/2}}{3cx^4}$$

output
```
1/3*a^2*c^(1/2)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(c*LambertW(a*x^2))^(1/2)/c^(
1/2))-1/3*(c*LambertW(a*x^2))^(1/2)/x^4+1/3*(c*LambertW(a*x^2))^(3/2)/c/x^
4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{cW(ax^2)}}{x^5}\,dx=\frac{\sqrt{cW(ax^2)}\left(a^2\sqrt{2\pi}x^4\,\mathrm{erf}\left(\sqrt{2}\sqrt{W(ax^2)}\right)-\sqrt{W(ax^2)}+W(ax^2)^{3/2}\right)}{3x^4\sqrt{W(ax^2)}}$$

input
```
Integrate[Sqrt[c*ProductLog[a*x^2]]/x^5,x]
```

output
```
(Sqrt[c*ProductLog[a*x^2]]*(a^2*Sqrt[2*Pi]*x^4*Erf[Sqrt[2]*Sqrt[ProductLog
[a*x^2]]] - Sqrt[ProductLog[a*x^2]] + ProductLog[a*x^2]^(3/2)))/(3*x^4*Sqr
t[ProductLog[a*x^2]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {7173, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cW(ax^2)}}{x^5} \, dx$$

$$\Big\downarrow \text{7173}$$

$$-\frac{\int \frac{(cW(ax^2))^{3/2}}{x^5(W(ax^2)+1)}dx}{3c} - \frac{\sqrt{cW(ax^2)}}{3x^4}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{-\frac{4\int \frac{\left(cW\left(ax^2\right)\right)^{5/2}}{x^5\left(W\left(ax^2\right)+1\right)}dx}{c} - \frac{(cW(ax^2))^{3/2}}{x^4}}{3c} - \frac{\sqrt{cW(ax^2)}}{3x^4}$$

$$\Big\downarrow \text{7203}$$

$$-\frac{\sqrt{2\pi}a^2\left(-c^{3/2}\right)\text{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^2)}}{\sqrt{c}}\right) - \frac{(cW(ax^2))^{3/2}}{x^4}}{3c} - \frac{\sqrt{cW(ax^2)}}{3x^4}$$

input
```
Int[Sqrt[c*ProductLog[a*x^2]]/x^5,x]
```

output
```
-1/3*Sqrt[c*ProductLog[a*x^2]]/x^4 - (-(a^2*c^(3/2)*Sqrt[2*Pi]*Erf[(Sqrt[2
]*Sqrt[c*ProductLog[a*x^2]])/Sqrt[c]]) - (c*ProductLog[a*x^2])^(3/2)/x^4)/
(3*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{\sqrt{c \,\mathrm{LambertW}\left(a \,x^{2}\right)}}{x^{5}}d x$$

input
```
int((c*LambertW(a*x^2))^(1/2)/x^5,x)
```

output
```
int((c*LambertW(a*x^2))^(1/2)/x^5,x)
```

Fricas [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^5}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x^5}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))/x^5, x)
```

Sympy [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^5}\,dx = \int \frac{\sqrt{cW(ax^2)}}{x^5}\,dx$$

input
```
integrate((c*LambertW(a*x**2))**(1/2)/x**5,x)
```

output
```
Integral(sqrt(c*LambertW(a*x**2))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^5}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x^5}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))/x^5, x)
```

Giac [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^5}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x^5}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x^5,x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cW(ax^2)}}{x^5}\,dx = \int \frac{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}{x^5}\,dx$$

input
```
int((c*LambertW(a*x^2))^(1/2)/x^5,x)
```

output
```
int((c*LambertW(a*x^2))^(1/2)/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^5}\,dx$$
$$= \frac{\sqrt{c}\left(-\sqrt{lambert\_w\left(a\,x^2\right)} + \left(\int \frac{\sqrt{lambert\_w(a\,x^2)}}{e^{lambert\_w\left(a\,x^2\right)}\,lambert\_w(a\,x^2)^2 x^3 + e^{lambert\_w\left(a\,x^2\right)}\,lambert\_w(a\,x^2) x^3}dx\right) a\,x}{4x^4}$$

input
```
int((c*Lambert_W(a*x^2))^(1/2)/x^5,x)
```

output
```
(sqrt(c)*( - sqrt(lambert_w(a*x**2)) + int(sqrt(lambert_w(a*x**2))/(e**lam
bert_w(a*x**2)*lambert_w(a*x**2)**2*x**3 + e**lambert_w(a*x**2)*lambert_w(
a*x**2)*x**3),x)*a*x**4))/(4*x**4)
```

3.160 $\int \frac{\sqrt{cW(ax^2)}}{x^7}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 107

$$\int \frac{\sqrt{cW(ax^2)}}{x^7}\,dx = -\frac{2}{5}a^3\sqrt{c}\sqrt{3\pi}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{cW(ax^2)}}{\sqrt{c}}\right) - \frac{\sqrt{cW(ax^2)}}{5x^6} + \frac{(cW(ax^2))^{3/2}}{15cx^6} - \frac{2(cW(ax^2))^{5/2}}{5c^2x^6}$$

output
```
-2/5*a^3*c^(1/2)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(c*LambertW(a*x^2))^(1/2)/c^
(1/2))-1/5*(c*LambertW(a*x^2))^(1/2)/x^6+1/15*(c*LambertW(a*x^2))^(3/2)/c/
x^6-2/5*(c*LambertW(a*x^2))^(5/2)/c^2/x^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{cW(ax^2)}}{x^7}\,dx = \frac{\sqrt{cW(ax^2)}\left(-6a^3\sqrt{3\pi}x^6\mathrm{erf}\left(\sqrt{3}\sqrt{W(ax^2)}\right) - 3\sqrt{W(ax^2)} + W(ax^2)^{3/2} - 6W(ax^2)^{5/2}\right)}{15x^6\sqrt{W(ax^2)}}$$

input
```
Integrate[Sqrt[c*ProductLog[a*x^2]]/x^7,x]
```

output
```
(Sqrt[c*ProductLog[a*x^2]]*(-6*a^3*Sqrt[3*Pi]*x^6*Erf[Sqrt[3]*Sqrt[Product
Log[a*x^2]]] - 3*Sqrt[ProductLog[a*x^2]] + ProductLog[a*x^2]^(3/2) - 6*Pro
ductLog[a*x^2]^(5/2)))/(15*x^6*Sqrt[ProductLog[a*x^2]])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7173, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cW\left(ax^2\right)}}{x^7}dx$$

$$\Big\downarrow \text{7173}$$

$$-\frac{\int \frac{\left(cW\left(ax^2\right)\right)^{3/2}}{x^7\left(W\left(ax^2\right)+1\right)}dx}{5c}-\frac{\sqrt{cW\left(ax^2\right)}}{5x^6}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{-\frac{2\int \frac{\left(cW\left(ax^2\right)\right)^{5/2}}{x^7\left(W\left(ax^2\right)+1\right)}dx}{c}-\frac{\left(cW\left(ax^2\right)\right)^{3/2}}{3x^6}}{5c}-\frac{\sqrt{cW\left(ax^2\right)}}{5x^6}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{-\frac{2\left(-\frac{6\int \frac{\left(cW\left(ax^2\right)\right)^{7/2}}{x^7\left(W\left(ax^2\right)+1\right)}dx}{c}-\frac{\left(cW\left(ax^2\right)\right)^{5/2}}{x^6}\right)}{c}-\frac{\left(cW\left(ax^2\right)\right)^{3/2}}{3x^6}}{5c}-\frac{\sqrt{cW\left(ax^2\right)}}{5x^6}$$

$$\Big\downarrow \text{7203}$$

$$-\frac{-\frac{2\left(\sqrt{3\pi}a^3\left(-c^{5/2}\right)\text{erf}\left(\frac{\sqrt{3}\sqrt{cW\left(ax^2\right)}}{\sqrt{c}}\right)-\frac{\left(cW\left(ax^2\right)\right)^{5/2}}{x^6}\right)}{c}-\frac{\left(cW\left(ax^2\right)\right)^{3/2}}{3x^6}}{5c}-\frac{\sqrt{cW\left(ax^2\right)}}{5x^6}$$

input
```
Int[Sqrt[c*ProductLog[a*x^2]]/x^7,x]
```

output
```
-1/5*Sqrt[c*ProductLog[a*x^2]]/x^6 - (-1/3*(c*ProductLog[a*x^2])^(3/2)/x^6
 - (2*(-(a^3*c^(5/2)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[c*ProductLog[a*x^2]])/Sq
rt[c]]) - (c*ProductLog[a*x^2])^(5/2)/x^6))/c)/(5*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{\sqrt{c\,\mathrm{LambertW}\left(a\,x^{2}\right)}}{x^{7}}dx$$

input
```
int((c*LambertW(a*x^2))^(1/2)/x^7,x)
```

output
```
int((c*LambertW(a*x^2))^(1/2)/x^7,x)
```

Fricas [F]

$$\int \frac{\sqrt{cW\left(ax^2\right)}}{x^7}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x^7}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x^7,x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))/x^7, x)
```

Sympy [F]

$$\int \frac{\sqrt{cW\left(ax^2\right)}}{x^7}\,dx = \int \frac{\sqrt{cW\left(ax^2\right)}}{x^7}\,dx$$

input
```
integrate((c*LambertW(a*x**2))**(1/2)/x**7,x)
```

output
```
Integral(sqrt(c*LambertW(a*x**2))/x**7, x)
```

Maxima [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^7}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x^7}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x^7,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))/x^7, x)
```

Giac [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^7}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x^7}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x^7,x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cW(ax^2)}}{x^7}\,dx = \int \frac{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}{x^7}\,dx$$

input
```
int((c*LambertW(a*x^2))^(1/2)/x^7,x)
```

output
```
int((c*LambertW(a*x^2))^(1/2)/x^7, x)
```

Reduce [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^7}\,dx$$

$$= \frac{\sqrt{c}\left(-\sqrt{lambert\_w(a\,x^2)}+\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)^2 x^5+e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)\,x^5}dx\right) a\,x}{6x^6}$$

input
```
int((c*Lambert_W(a*x^2))^(1/2)/x^7,x)
```

output
```
(sqrt(c)*( - sqrt(lambert_w(a*x**2)) + int(sqrt(lambert_w(a*x**2))/(e**lam
bert_w(a*x**2)*lambert_w(a*x**2)**2*x**5 + e**lambert_w(a*x**2)*lambert_w(
a*x**2)*x**5),x)*a*x**6))/(6*x**6)
```

3.161 $\int x^6 \sqrt{cW(ax^2)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 106

$$\int x^6\sqrt{cW(ax^2)}\,dx = \frac{48c^4x^7}{16807\,(cW(ax^2))^{7/2}} - \frac{24c^3x^7}{2401\,(cW(ax^2))^{5/2}} + \frac{6c^2x^7}{343\,(cW(ax^2))^{3/2}} - \frac{cx^7}{49\sqrt{cW(ax^2)}} + \frac{1}{7}x^7\sqrt{cW(ax^2)}$$

output
```
48/16807*c^4*x^7/(c*LambertW(a*x^2))^(7/2)-24/2401*c^3*x^7/(c*LambertW(a*x
^2))^(5/2)+6/343*c^2*x^7/(c*LambertW(a*x^2))^(3/2)-1/49*c*x^7/(c*LambertW(
a*x^2))^(1/2)+1/7*x^7*(c*LambertW(a*x^2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

$$\int x^6\sqrt{cW(ax^2)}\,dx = \frac{x^7\sqrt{cW(ax^2)}\left(48-168W(ax^2)+294W(ax^2)^2-343W(ax^2)^3+2401W(ax^2)^4\right)}{16807W(ax^2)^4}$$

input
```
Integrate[x^6*Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(x^7*Sqrt[c*ProductLog[a*x^2]]*(48 - 168*ProductLog[a*x^2] + 294*ProductLo
g[a*x^2]^2 - 343*ProductLog[a*x^2]^3 + 2401*ProductLog[a*x^2]^4))/(16807*P
roductLog[a*x^2]^4)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {7172, 7205, 7205, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6\sqrt{cW\left(ax^2\right)}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{7}x^7\sqrt{cW\left(ax^2\right)}-\frac{1}{7}\int \frac{x^6\sqrt{cW\left(ax^2\right)}}{W\left(ax^2\right)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{7}\left(\frac{6}{7}c\int \frac{x^6}{\sqrt{cW\left(ax^2\right)}\left(W\left(ax^2\right)+1\right)}dx-\frac{cx^7}{7\sqrt{cW\left(ax^2\right)}}\right)+\frac{1}{7}x^7\sqrt{cW\left(ax^2\right)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{7}\left(\frac{6}{7}c\left(\frac{cx^7}{7\left(cW\left(ax^2\right)\right)^{3/2}}-\frac{4}{7}c\int \frac{x^6}{\left(cW\left(ax^2\right)\right)^{3/2}\left(W\left(ax^2\right)+1\right)}dx\right)-\frac{cx^7}{7\sqrt{cW\left(ax^2\right)}}\right)+\frac{1}{7}x^7\sqrt{cW\left(ax^2\right)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{7}\left(\frac{6}{7}c\left(\frac{cx^7}{7\left(cW\left(ax^2\right)\right)^{3/2}}-\frac{4}{7}c\left(\frac{cx^7}{7\left(cW\left(ax^2\right)\right)^{5/2}}-\frac{2}{7}c\int \frac{x^6}{\left(cW\left(ax^2\right)\right)^{5/2}\left(W\left(ax^2\right)+1\right)}dx\right)\right)-\frac{cx^7}{7\sqrt{cW\left(ax^2\right)}}\right)+\frac{1}{7}x^7\sqrt{cW\left(ax^2\right)}$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{7}\left(\frac{6}{7}c\left(\frac{cx^7}{7\,(cW\,(ax^2))^{3/2}}-\frac{4}{7}c\left(\frac{cx^7}{7\,(cW\,(ax^2))^{5/2}}-\frac{2c^2x^7}{49\,(cW\,(ax^2))^{7/2}}\right)\right)-\frac{cx^7}{7\sqrt{cW\,(ax^2)}}\right)+\frac{1}{7}x^7\sqrt{cW\,(ax^2)}$$

input
```
Int[x^6*Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(x^7*Sqrt[c*ProductLog[a*x^2]])/7 + (-1/7*(c*x^7)/Sqrt[c*ProductLog[a*x^2]
] + (6*c*((c*x^7)/(7*(c*ProductLog[a*x^2])^(3/2)) - (4*c*((-2*c^2*x^7)/(49
*(c*ProductLog[a*x^2])^(7/2)) + (c*x^7)/(7*(c*ProductLog[a*x^2])^(5/2))))/
7))/7)/7
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^6 \sqrt{c \,\mathrm{LambertW}\left(a\,x^2\right)}dx$$

input
```
int(x^6*(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^6*(c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int x^6 \sqrt{cW\left(ax^2\right)}\, dx = \int \sqrt{c\,\mathrm{W}(ax^2)}x^6\, dx$$

input
```
integrate(x^6*(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))*x^6, x)
```

Sympy [F]

$$\int x^6 \sqrt{cW\left(ax^2\right)}\, dx = \int x^6 \sqrt{cW\left(ax^2\right)}\, dx$$

input
```
integrate(x**6*(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(x**6*sqrt(c*LambertW(a*x**2)), x)
```

Maxima [F]

$$\int x^6\sqrt{cW\left(ax^2\right)}\,dx=\int\sqrt{c\,\mathrm{W}(ax^2)}x^6\,dx$$

input
```
integrate(x^6*(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))*x^6, x)
```

Giac [F]

$$\int x^6\sqrt{cW\left(ax^2\right)}\,dx=\int\sqrt{c\,\mathrm{W}(ax^2)}x^6\,dx$$

input
```
integrate(x^6*(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6\sqrt{cW\left(ax^2\right)}\,dx=\int x^6\,\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}\,dx$$

input
```
int(x^6*(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^6*(c*LambertW(a*x^2))^(1/2), x)
```

Reduce [F]

$$\int x^6\sqrt{cW\left(ax^2\right)}\,dx=\sqrt{c}\left(\int\sqrt{lambert\_w\left(a\,x^2\right)}\,x^6dx\right)$$

input
```
int(x^6*(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a*x**2))*x**6,x)
```

3.162 $\int x^4 \sqrt{cW(ax^2)}\, dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int x^4 \sqrt{cW(ax^2)}\, dx = -\frac{8c^3x^5}{625(cW(ax^2))^{5/2}} + \frac{4c^2x^5}{125(cW(ax^2))^{3/2}} - \frac{cx^5}{25\sqrt{cW(ax^2)}} + \frac{1}{5}x^5\sqrt{cW(ax^2)}$$

output
```
-8/625*c^3*x^5/(c*LambertW(a*x^2))^(5/2)+4/125*c^2*x^5/(c*LambertW(a*x^2))
^(3/2)-1/25*c*x^5/(c*LambertW(a*x^2))^(1/2)+1/5*x^5*(c*LambertW(a*x^2))^(1
/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.68

$$\int x^4 \sqrt{cW(ax^2)}\, dx = \frac{x^5\sqrt{cW(ax^2)}\left(-8 + 20W(ax^2) - 25W(ax^2)^2 + 125W(ax^2)^3\right)}{625W(ax^2)^3}$$

input
```
Integrate[x^4*Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(x^5*Sqrt[c*ProductLog[a*x^2]]*(-8 + 20*ProductLog[a*x^2] - 25*ProductLog[
a*x^2]^2 + 125*ProductLog[a*x^2]^3))/(625*ProductLog[a*x^2]^3)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7172, 7205, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sqrt{cW\left(ax^2\right)}\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{5}x^5\sqrt{cW\left(ax^2\right)}-\frac{1}{5}\int \frac{x^4\sqrt{cW\left(ax^2\right)}}{W\left(ax^2\right)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{5}\left(\frac{4}{5}c\int \frac{x^4}{\sqrt{cW\left(ax^2\right)}\left(W\left(ax^2\right)+1\right)}dx-\frac{cx^5}{5\sqrt{cW\left(ax^2\right)}}\right)+\frac{1}{5}x^5\sqrt{cW\left(ax^2\right)}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{5}\left(\frac{4}{5}c\left(\frac{cx^5}{5\left(cW\left(ax^2\right)\right)^{3/2}}-\frac{2}{5}c\int \frac{x^4}{\left(cW\left(ax^2\right)\right)^{3/2}\left(W\left(ax^2\right)+1\right)}dx\right)-\frac{cx^5}{5\sqrt{cW\left(ax^2\right)}}\right)+\frac{1}{5}x^5\sqrt{cW\left(ax^2\right)}$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{5}\left(\frac{4}{5}c\left(\frac{cx^5}{5\left(cW\left(ax^2\right)\right)^{3/2}}-\frac{2c^2x^5}{25\left(cW\left(ax^2\right)\right)^{5/2}}\right)-\frac{cx^5}{5\sqrt{cW\left(ax^2\right)}}\right)+\frac{1}{5}x^5\sqrt{cW\left(ax^2\right)}$$

input
```
Int[x^4*Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(x^5*Sqrt[c*ProductLog[a*x^2]])/5 + (-1/5*(c*x^5)/Sqrt[c*ProductLog[a*x^2]
] + (4*c*((-2*c^2*x^5)/(25*(c*ProductLog[a*x^2])^(5/2)) + (c*x^5)/(5*(c*Pr
oductLog[a*x^2])^(3/2))))/5)/5
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^4 \sqrt{c \operatorname{LambertW}\left(a\,x^2\right)}dx$$

input
```
int(x^4*(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^4*(c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int x^4 \sqrt{cW\left(ax^2\right)}\,dx = \int \sqrt{c\,\mathrm{W}(ax^2)}x^4\,dx$$

input
```
integrate(x^4*(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))*x^4, x)
```

Sympy [F]

$$\int x^4 \sqrt{cW\left(ax^2\right)}\,dx = \int x^4 \sqrt{cW\left(ax^2\right)}\,dx$$

input
```
integrate(x**4*(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(x**4*sqrt(c*LambertW(a*x**2)), x)
```

Maxima [F]

$$\int x^4 \sqrt{cW\left(ax^2\right)}\,dx = \int \sqrt{c\,\mathrm{W}(ax^2)}x^4\,dx$$

input
```
integrate(x^4*(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{cW(ax^2)}\,dx=\int\sqrt{c\,\mathrm{W}(ax^2)}x^4\,dx$$

input
```
integrate(x^4*(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{cW(ax^2)}\,dx=\int x^4\,\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}\,dx$$

input
```
int(x^4*(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^4*(c*LambertW(a*x^2))^(1/2), x)
```

Reduce [F]

$$\int x^4\sqrt{cW(ax^2)}\,dx=\sqrt{c}\left(\int\sqrt{lambert\_w\left(a\,x^2\right)}\,x^4dx\right)$$

input
```
int(x^4*(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a*x**2))*x***4,x)
```

3.163 $\int x^2\sqrt{cW\left(ax^2\right)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int x^2\sqrt{cW\left(ax^2\right)}\,dx=\frac{2c^2x^3}{27\left(cW\left(ax^2\right)\right)^{3/2}}-\frac{cx^3}{9\sqrt{cW\left(ax^2\right)}}+\frac{1}{3}x^3\sqrt{cW\left(ax^2\right)}$$

output
```
2/27*c^2*x^3/(c*LambertW(a*x^2))^(3/2)-1/9*c*x^3/(c*LambertW(a*x^2))^(1/2)
+1/3*x^3*(c*LambertW(a*x^2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int x^2\sqrt{cW\left(ax^2\right)}\,dx=\frac{x^3\sqrt{cW\left(ax^2\right)}\left(2-3W\left(ax^2\right)+9W\left(ax^2\right)^2\right)}{27W\left(ax^2\right)^2}$$

input
```
Integrate[x^2*Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(x^3*Sqrt[c*ProductLog[a*x^2]]*(2 - 3*ProductLog[a*x^2] + 9*ProductLog[a*x
^2]^2))/(27*ProductLog[a*x^2]^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {7172, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{cW(ax^2)}\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{3}x^3\sqrt{cW(ax^2)} - \frac{1}{3}\int \frac{x^2\sqrt{cW(ax^2)}}{W(ax^2)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{3}\left(\frac{2}{3}c\int \frac{x^2}{\sqrt{cW(ax^2)}\,(W(ax^2)+1)}dx - \frac{cx^3}{3\sqrt{cW(ax^2)}}\right) + \frac{1}{3}x^3\sqrt{cW(ax^2)}$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{3}\left(\frac{2c^2x^3}{9\,(cW(ax^2))^{3/2}} - \frac{cx^3}{3\sqrt{cW(ax^2)}}\right) + \frac{1}{3}x^3\sqrt{cW(ax^2)}$$

input
```
Int[x^2*Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(x^3*Sqrt[c*ProductLog[a*x^2]])/3 + ((2*c^2*x^3)/(9*(c*ProductLog[a*x^2])^
(3/2)) - (c*x^3)/(3*Sqrt[c*ProductLog[a*x^2]]))/3
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^{2} \sqrt{c \,\mathrm{LambertW}\left(a \,x^{2}\right)}dx$$

input
```
int(x^2*(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^2*(c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int x^2 \sqrt{cW\left(ax^2\right)}\, dx = \int \sqrt{c\,\mathrm{W}(ax^2)}x^2\, dx$$

input
```
integrate(x^2*(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))*x^2, x)
```

Sympy [F]

$$\int x^2\sqrt{cW\left(ax^2\right)}\,dx=\int x^2\sqrt{cW\left(ax^2\right)}\,dx$$

input
```
integrate(x**2*(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(x**2*sqrt(c*LambertW(a*x**2)), x)
```

Maxima [F]

$$\int x^2\sqrt{cW\left(ax^2\right)}\,dx=\int \sqrt{c\,\mathrm{W}(ax^2)}x^2\,dx$$

input
```
integrate(x^2*(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{cW\left(ax^2\right)}\,dx=\int \sqrt{c\,\mathrm{W}(ax^2)}x^2\,dx$$

input
```
integrate(x^2*(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt{cW\left(ax^2\right)}\,dx = \int x^2\,\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}\,dx$$

input
```
int(x^2*(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^2*(c*LambertW(a*x^2))^(1/2), x)
```

Reduce [F]

$$\int x^2 \sqrt{cW\left(ax^2\right)}\,dx = \sqrt{c}\left(\int \sqrt{lambert\_w\left(a\,x^2\right)}\,x^2dx\right)$$

input
```
int(x^2*(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a*x**2))*x**2,x)
```

3.164 $\int \sqrt{cW(ax^2)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \sqrt{cW(ax^2)}\,dx = -\frac{cx}{\sqrt{cW(ax^2)}} + x\sqrt{cW(ax^2)}$$

output
```
-c*x/(c*LambertW(a*x^2))^(1/2)+x*(c*LambertW(a*x^2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \sqrt{cW(ax^2)}\,dx = \frac{cx(-1+W(ax^2))}{\sqrt{cW(ax^2)}}$$

input
```
Integrate[Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(c*x*(-1 + ProductLog[a*x^2]))/Sqrt[c*ProductLog[a*x^2]]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7169, 7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cW(ax^2)}\,dx$$

$$\Big\downarrow \text{ 7169}$$

$$x\sqrt{cW(ax^2)} - \int \frac{\sqrt{cW(ax^2)}}{W(ax^2)+1}dx$$

$$\Big\downarrow \text{ 7187}$$

$$x\sqrt{cW(ax^2)} - \frac{cx}{\sqrt{cW(ax^2)}}$$

input
```
Int[Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
-((c*x)/Sqrt[c*ProductLog[a*x^2]]) + x*Sqrt[c*ProductLog[a*x^2]]
```

Defintions of rubi rules used

rule 7169
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*(c*Produ
ctLog[a*x^n])^p, x] - Simp[n*p   Int[(c*ProductLog[a*x^n])^p/(1 + ProductLo
g[a*x^n]), x], x] /; FreeQ[{a, c, n, p}, x] && (EqQ[n*(p - 1), -1] || (Inte
gerQ[p - 1/2] && EqQ[n*(p - 1/2), -1]))
```

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [F]

$$\int \sqrt{c\,\mathrm{LambertW}\left(a\,x^{2}\right)}dx$$

input
```
int((c*LambertW(a*x^2))^(1/2),x)
```

output
```
int((c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int \sqrt{cW\left(ax^{2}\right)}\,dx=\int \sqrt{c\,\mathrm{W}(ax^{2})}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2)), x)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \sqrt{cW\left(ax^{2}\right)}\,dx=\begin{cases} x\sqrt{cW\left(ax^{2}\right)}-\frac{x\sqrt{cW(ax^{2})}}{W(ax^{2})} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate((c*LambertW(a*x**2))**(1/2),x)
```

output
```
Piecewise((x*sqrt(c*LambertW(a*x**2)) - x*sqrt(c*LambertW(a*x**2))/Lambert
W(a*x**2), Ne(a, 0)), (0, True))
```

Maxima [F]

$$\int \sqrt{cW\left(ax^2\right)}\,dx = \int \sqrt{c\,\mathrm{W}(ax^2)}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2)), x)
```

Giac [F]

$$\int \sqrt{cW\left(ax^2\right)}\,dx = \int \sqrt{c\,\mathrm{W}(ax^2)}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cW\left(ax^2\right)}\,dx = \int \sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}\,dx$$

input
```
int((c*LambertW(a*x^2))^(1/2),x)
```

output
```
int((c*LambertW(a*x^2))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{cW\left(ax^2\right)}\,dx = \sqrt{c}\left(\int \sqrt{lambert\_w(a\,x^2)}dx\right)$$

input
```
int((c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a*x**2)),x)
```

3.165 $\int \frac{\sqrt{cW(ax^2)}}{x^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 70

$$\int \frac{\sqrt{cW(ax^2)}}{x^2} \, dx = -\frac{\sqrt{cW(ax^2)}}{x} - \frac{e^{\frac{3}{2}W(ax^2)}\Gamma\left(0, \frac{1}{2}W(ax^2)\right)W(ax^2)\sqrt{cW(ax^2)}}{2ax^3}$$

output
```
-(c*LambertW(a*x^2))^(1/2)/x-1/2*exp(3/2*LambertW(a*x^2))*Ei(1,1/2*Lambert
W(a*x^2))*LambertW(a*x^2)*(c*LambertW(a*x^2))^(1/2)/a/x^3
```

Mathematica [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^2} \, dx = \int \frac{\sqrt{cW(ax^2)}}{x^2} \, dx$$

input
```
Integrate[Sqrt[c*ProductLog[a*x^2]]/x^2,x]
```

output
```
Integrate[Sqrt[c*ProductLog[a*x^2]]/x^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cW(ax^2)}}{x^2}dx$$
$$\Big\downarrow \text{ 7271}$$
$$\frac{\sqrt{cW(ax^2)}\int \frac{\sqrt{W(ax^2)}}{x^2}dx}{\sqrt{W(ax^2)}}$$
$$\Big\downarrow \text{ 7299}$$
$$\frac{\sqrt{cW(ax^2)}\int \frac{\sqrt{W(ax^2)}}{x^2}dx}{\sqrt{W(ax^2)}}$$

input
```
Int[Sqrt[c*ProductLog[a*x^2]]/x^2,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}{x^2}dx$$

input
```
int((c*LambertW(a*x^2))^(1/2)/x^2,x)
```

output
```
int((c*LambertW(a*x^2))^(1/2)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^2}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x^2}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^2}\,dx = \int \frac{\sqrt{cW(ax^2)}}{x^2}\,dx$$

input
```
integrate((c*LambertW(a*x**2))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(c*LambertW(a*x**2))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^2}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x^2}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^2}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x^2}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cW(ax^2)}}{x^2}\,dx = \int \frac{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}{x^2}\,dx$$

input
```
int((c*LambertW(a*x^2))^(1/2)/x^2,x)
```

output
```
int((c*LambertW(a*x^2))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^2}\,dx = \frac{\sqrt{c}\left(-\sqrt{lambert\_w\left(a\,x^2\right)}+\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}}{e^{lambert\_w\left(a\,x^2\right)}\,lambert\_w(a\,x^2)^2+e^{lambert\_w\left(a\,x^2\right)}\,lambert\_w(a\,x^2)}dx\right)ax\right)}{x}$$

input
```
int((c*Lambert_W(a*x^2))^(1/2)/x^2,x)
```

output
```
(sqrt(c)*( - sqrt(lambert_w(a*x**2)) + int(sqrt(lambert_w(a*x**2))/(e**lam
bert_w(a*x**2)*lambert_w(a*x**2)**2 + e**lambert_w(a*x**2)*lambert_w(a*x**
2)),x)*a*x))/x
```

3.166 $\int \frac{\sqrt{cW(ax^2)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int \frac{\sqrt{cW(ax^2)}}{x^4}\,dx = -\frac{\sqrt{cW(ax^2)}}{2x^3} + \frac{e^{\frac{5}{2}W(ax^2)}\Gamma\left(0,\frac{3}{2}W(ax^2)\right)W(ax^2)\,(cW(ax^2))^{3/2}}{4acx^5}$$

output
```
-1/2*(c*LambertW(a*x^2))^(1/2)/x^3+1/4*exp(5/2*LambertW(a*x^2))*Ei(1,3/2*L
ambertW(a*x^2))*LambertW(a*x^2)*(c*LambertW(a*x^2))^(3/2)/a/c/x^5
```

Mathematica [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^4}\,dx = \int \frac{\sqrt{cW(ax^2)}}{x^4}\,dx$$

input
```
Integrate[Sqrt[c*ProductLog[a*x^2]]/x^4,x]
```

output
```
Integrate[Sqrt[c*ProductLog[a*x^2]]/x^4, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cW(ax^2)}}{x^4} dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\sqrt{cW(ax^2)} \int \frac{\sqrt{W(ax^2)}}{x^4} dx}{\sqrt{W(ax^2)}}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\sqrt{cW(ax^2)} \int \frac{\sqrt{W(ax^2)}}{x^4} dx}{\sqrt{W(ax^2)}}$$

input
```
Int[Sqrt[c*ProductLog[a*x^2]]/x^4,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}{x^4} dx$$

input
```
int((c*LambertW(a*x^2))^(1/2)/x^4,x)
```

output
```
int((c*LambertW(a*x^2))^(1/2)/x^4,x)
```

Fricas [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^4}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x^4}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^4}\,dx = \int \frac{\sqrt{cW(ax^2)}}{x^4}\,dx$$

input
```
integrate((c*LambertW(a*x**2))**(1/2)/x**4,x)
```

output
```
Integral(sqrt(c*LambertW(a*x**2))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^4}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x^4}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^4}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(ax^2)}}{x^4}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^2))/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cW(ax^2)}}{x^4}\,dx = \int \frac{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}{x^4}\,dx$$

input
```
int((c*LambertW(a*x^2))^(1/2)/x^4,x)
```

output
```
int((c*LambertW(a*x^2))^(1/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{cW(ax^2)}}{x^4}\,dx$$
$$= \frac{\sqrt{c}\left(-\sqrt{lambert\_w\left(a\,x^2\right)}+\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}}{e^{lambert\_w\left(a\,x^2\right)}\,lambert\_w(a\,x^2)^2x^2+e^{lambert\_w\left(a\,x^2\right)}\,lambert\_w(a\,x^2)x^2}dx\right)a\,x}{3x^3}$$

input
```
int((c*Lambert_W(a*x^2))^(1/2)/x^4,x)
```

output
```
(sqrt(c)*( - sqrt(lambert_w(a*x**2)) + int(sqrt(lambert_w(a*x**2))/(e**lam
bert_w(a*x**2)*lambert_w(a*x**2)**2*x**2 + e**lambert_w(a*x**2)*lambert_w(
a*x**2)*x**2),x)*a*x**3))/(3*x**3)
```

3.167 $\int \frac{x^5}{\sqrt{cW(ax^2)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 107

$$\int \frac{x^5}{\sqrt{cW(ax^2)}}\,dx = \frac{\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{cW(ax^2)}}{\sqrt{c}}\right)}{144a^3\sqrt{c}} - \frac{c^2x^6}{72\left(cW(ax^2)\right)^{5/2}} + \frac{cx^6}{36\left(cW(ax^2)\right)^{3/2}} + \frac{x^6}{6\sqrt{cW(ax^2)}}$$

output
```
1/432*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(c*LambertW(a*x^2))^(1/2)/c^(1/2))/a^3
/c^(1/2)-1/72*c^2*x^6/(c*LambertW(a*x^2))^(5/2)+1/36*c*x^6/(c*LambertW(a*x
^2))^(3/2)+1/6*x^6/(c*LambertW(a*x^2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{\sqrt{cW(ax^2)}}\,dx = \frac{-6a^3x^6 + 12a^3x^6W(ax^2) + 72a^3x^6W(ax^2)^2 + \sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{W(ax^2)}\right)W(ax^2)^{5/2}}{432a^3W(ax^2)^2\sqrt{cW(ax^2)}}$$

input
```
Integrate[x^5/Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(-6*a^3*x^6 + 12*a^3*x^6*ProductLog[a*x^2] + 72*a^3*x^6*ProductLog[a*x^2]^
2 + Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ProductLog[a*x^2]]]*ProductLog[a*x^2]^(5/
2))/(432*a^3*ProductLog[a*x^2]^2*Sqrt[c*ProductLog[a*x^2]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7172, 7205, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{cW(ax^2)}}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{6}\int \frac{x^5}{\sqrt{cW(ax^2)}(W(ax^2)+1)}dx+\frac{x^6}{6\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{6}\left(\frac{cx^6}{6(cW(ax^2))^{3/2}}-\frac{1}{2}c\int \frac{x^5}{(cW(ax^2))^{3/2}(W(ax^2)+1)}dx\right)+\frac{x^6}{6\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{6}\left(\frac{cx^6}{6(cW(ax^2))^{3/2}}-\frac{1}{2}c\left(\frac{cx^6}{6(cW(ax^2))^{5/2}}-\frac{1}{6}c\int \frac{x^5}{(cW(ax^2))^{5/2}(W(ax^2)+1)}dx\right)\right)+\frac{x^6}{6\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{7204}$$

$$\frac{1}{6}\left(\frac{cx^6}{6(cW(ax^2))^{3/2}}-\frac{1}{2}c\left(\frac{cx^6}{6(cW(ax^2))^{5/2}}-\frac{\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{cW(ax^2)}}{\sqrt{c}}\right)}{12a^3c^{3/2}}\right)\right)+\frac{x^6}{6\sqrt{cW(ax^2)}}$$

input
```
Int[x^5/Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
x^6/(6*Sqrt[c*ProductLog[a*x^2]]) + ((c*x^6)/(6*(c*ProductLog[a*x^2])^(3/2
)) - (c*(-1/12*(Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[c*ProductLog[a*x^2]])/Sqrt[c
]])/(a^3*c^(3/2)) + (c*x^6)/(6*(c*ProductLog[a*x^2])^(5/2))))/2)/6
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int \frac{x^5}{\sqrt{c\,\text{LambertW}\,(a\,x^2)}}dx$$

input
```
int(x^5/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^5/(c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^5}{\sqrt{cW\,(ax^2)}}\,dx = \int \frac{x^5}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x^5/(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))*x^5/(c*lambert_w(a*x^2)), x)
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{cW\,(ax^2)}}\,dx = \int \frac{x^5}{\sqrt{cW\,(ax^2)}}\,dx$$

input
```
integrate(x**5/(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(x**5/sqrt(c*LambertW(a*x**2)), x)
```

Maxima [F]

$$\int \frac{x^5}{\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{x^5}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x^5/(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/sqrt(c*lambert_w(a*x^2)), x)
```

Giac [F]

$$\int \frac{x^5}{\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{x^5}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x^5/(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^5/sqrt(c*lambert_w(a*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{x^5}{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}\,dx$$

input
```
int(x^5/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^5/(c*LambertW(a*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^5}{\sqrt{cW(ax^2)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}\,x^5}{lambert\_w(a\,x^2)}dx\right)}{c}$$

input
```
int(x^5/(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(lambert_w(a*x**2))*x**5)/lambert_w(a*x**2),x))/c
```

3.168 $\int \frac{x^3}{\sqrt{cW(ax^2)}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{x^3}{\sqrt{cW(ax^2)}} \, dx = -\frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^2)}}{\sqrt{c}}\right)}{32a^2\sqrt{c}} + \frac{cx^4}{16\left(cW(ax^2)\right)^{3/2}} + \frac{x^4}{4\sqrt{cW(ax^2)}}$$

output
```
-1/64*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(a*x^2))^(1/2)/c^(1/2))/a^2
/c^(1/2)+1/16*c*x^4/(c*LambertW(a*x^2))^(3/2)+1/4*x^4/(c*LambertW(a*x^2))^
(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{\sqrt{cW(ax^2)}} \, dx = \frac{c\left(4a^2x^4 + 16a^2x^4W(ax^2) - \sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{W(ax^2)}\right)W(ax^2)^{3/2}\right)}{64a^2\left(cW(ax^2)\right)^{3/2}}$$

input
```
Integrate[x^3/Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(c*(4*a^2*x^4 + 16*a^2*x^4*ProductLog[a*x^2] - Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqr
t[ProductLog[a*x^2]]]*ProductLog[a*x^2]^(3/2)))/(64*a^2*(c*ProductLog[a*x^
2])^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {7172, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{cW(ax^2)}}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{4}\int \frac{x^3}{\sqrt{cW(ax^2)}(W(ax^2)+1)}dx+\frac{x^4}{4\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{4}\left(\frac{cx^4}{4(cW(ax^2))^{3/2}}-\frac{1}{4}c\int \frac{x^3}{(cW(ax^2))^{3/2}(W(ax^2)+1)}dx\right)+\frac{x^4}{4\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{7204}$$

$$\frac{1}{4}\left(\frac{cx^4}{4(cW(ax^2))^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^2)}}{\sqrt{c}}\right)}{8a^2\sqrt{c}}\right)+\frac{x^4}{4\sqrt{cW(ax^2)}}$$

input
```
Int[x^3/Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
x^4/(4*Sqrt[c*ProductLog[a*x^2]]) + (-1/8*(Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[c
*ProductLog[a*x^2]])/Sqrt[c]])/(a^2*Sqrt[c]) + (c*x^4)/(4*(c*ProductLog[a*
x^2])^(3/2)))/4
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int \frac{x^3}{\sqrt{c \,\mathrm{LambertW}\left(a\,x^2\right)}}dx$$

input
```
int(x^3/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^3/(c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{cW(ax^2)}}\,dx = \int \frac{x^3}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x^3/(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))*x^3/(c*lambert_w(a*x^2)), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{cW(ax^2)}}\,dx = \int \frac{x^3}{\sqrt{cW(ax^2)}}\,dx$$

input
```
integrate(x**3/(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(x**3/sqrt(c*LambertW(a*x**2)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{cW(ax^2)}}\,dx = \int \frac{x^3}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x^3/(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(c*lambert_w(a*x^2)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{cW(ax^2)}}\,dx = \int \frac{x^3}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x^3/(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(c*lambert_w(a*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{cW(ax^2)}}\,dx = \int \frac{x^3}{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}\,dx$$

input
```
int(x^3/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^3/(c*LambertW(a*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{cW(ax^2)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}\,x^3}{lambert\_w(a\,x^2)}dx\right)}{c}$$

input
```
int(x^3/(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(lambert_w(a*x**2))*x**3)/lambert_w(a*x**2),x))/c
```

3.169 $\int \frac{x}{\sqrt{cW(ax^2)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 56

$$\int \frac{x}{\sqrt{cW(ax^2)}}\,dx=\frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{cW(ax^2)}}{\sqrt{c}}\right)}{4a\sqrt{c}}+\frac{x^2}{2\sqrt{cW(ax^2)}}$$

output
```
1/4*Pi^(1/2)*erfi((c*LambertW(a*x^2))^(1/2)/c^(1/2))/a/c^(1/2)+1/2*x^2/(c*
LambertW(a*x^2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{x}{\sqrt{cW(ax^2)}}\,dx=\frac{2ax^2+\sqrt{\pi}\operatorname{erfi}\left(\sqrt{W(ax^2)}\right)\sqrt{W(ax^2)}}{4a\sqrt{cW(ax^2)}}$$

input
```
Integrate[x/Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(2*a*x^2 + Sqrt[Pi]*Erfi[Sqrt[ProductLog[a*x^2]]]*Sqrt[ProductLog[a*x^2]])
/(4*a*Sqrt[c*ProductLog[a*x^2]])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {7172, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{cW(ax^2)}}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{2}\int \frac{x}{\sqrt{cW(ax^2)}(W(ax^2)+1)}dx+\frac{x^2}{2\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{7204}$$

$$\frac{\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{cW(ax^2)}}{\sqrt{c}}\right)}{4a\sqrt{c}}+\frac{x^2}{2\sqrt{cW(ax^2)}}$$

input
```
Int[x/Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a*x^2]]/Sqrt[c]])/(4*a*Sqrt[c]) + x^2/(2*
Sqrt[c*ProductLog[a*x^2]])
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$\frac{\frac{c\sqrt{c\,\mathrm{LambertW}\left(a\,x^{2}\right)}\,a\,x^{2}}{2\,\mathrm{LambertW}\left(a\,x^{2}\right)}+\frac{c\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{c}}\,\sqrt{c\,\mathrm{LambertW}\left(a\,x^{2}\right)}\right)}{4\sqrt{-\frac{1}{c}}}}{a\,c^{2}}$	66
default	$\frac{\frac{c\sqrt{c\,\mathrm{LambertW}\left(a\,x^{2}\right)}\,a\,x^{2}}{2\,\mathrm{LambertW}\left(a\,x^{2}\right)}+\frac{c\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{c}}\,\sqrt{c\,\mathrm{LambertW}\left(a\,x^{2}\right)}\right)}{4\sqrt{-\frac{1}{c}}}}{a\,c^{2}}$	66

input
```
int(x/(c*LambertW(a*x^2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/c^2*(1/2*c*(c*LambertW(a*x^2))^(1/2)*a*x^2/LambertW(a*x^2)+1/4*c*Pi^(1
/2)/(-1/c)^(1/2)*erf((-1/c)^(1/2)*(c*LambertW(a*x^2))^(1/2)))
```

Fricas [F]

$$\int \frac{x}{\sqrt{cW\left(ax^{2}\right)}}\,dx=\int \frac{x}{\sqrt{c\,\mathrm{W}(ax^{2})}}\,dx$$

input
```
integrate(x/(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))*x/(c*lambert_w(a*x^2)), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{cW(ax^2)}}\,dx = \int \frac{x}{\sqrt{cW(ax^2)}}\,dx$$

input
```
integrate(x/(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(x/sqrt(c*LambertW(a*x**2)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{cW(ax^2)}}\,dx = \int \frac{x}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x/(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(c*lambert_w(a*x^2)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{cW(ax^2)}}\,dx = \int \frac{x}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x/(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(c*lambert_w(a*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{x}{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}\,dx$$

input
```
int(x/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x/(c*LambertW(a*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{cW\left(ax^2\right)}}\,dx = \frac{\sqrt{c}\left(e^{lambert\_w\left(a\,x^2\right)}\sqrt{lambert\_w\left(a\,x^2\right)}+\left(\int \frac{\sqrt{lambert\_w\left(a\,x^2\right)}\,x}{lambert\_w\left(a\,x^2\right)^2+lambert\_w\left(a\,x^2\right)}dx\right)a\right)}{2ac}$$

input
```
int(x/(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
(sqrt(c)*(e**lambert_w(a*x**2)*sqrt(lambert_w(a*x**2)) + int((sqrt(lambert
_w(a*x**2))*x)/(lambert_w(a*x**2)**2 + lambert_w(a*x**2)),x)*a))/(2*a*c)
```

3.170 $\int \frac{1}{x\sqrt{cW(ax^2)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 31

$$\int \frac{1}{x\sqrt{cW(ax^2)}}\,dx = -\frac{1}{\sqrt{cW(ax^2)}} + \frac{\sqrt{cW(ax^2)}}{c}$$

output
```
-1/(c*LambertW(a*x^2))^(1/2)+(c*LambertW(a*x^2))^(1/2)/c
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{x\sqrt{cW(ax^2)}}\,dx = \frac{-1+W(ax^2)}{\sqrt{cW(ax^2)}}$$

input
```
Integrate[1/(x*Sqrt[c*ProductLog[a*x^2]]),x]
```

output
```
(-1 + ProductLog[a*x^2])/Sqrt[c*ProductLog[a*x^2]]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{cW(ax^2)}}dx$$
$$\Big\downarrow \text{ 7173}$$
$$\frac{\int \frac{\sqrt{cW(ax^2)}}{x(W(ax^2)+1)}dx}{c}-\frac{1}{\sqrt{cW(ax^2)}}$$
$$\Big\downarrow \text{ 7200}$$
$$\frac{\sqrt{cW(ax^2)}}{c}-\frac{1}{\sqrt{cW(ax^2)}}$$

input
```
Int[1/(x*Sqrt[c*ProductLog[a*x^2]]),x]
```

output
```
-(1/Sqrt[c*ProductLog[a*x^2]]) + Sqrt[c*ProductLog[a*x^2]]/c
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\sqrt{c \,\mathrm{LambertW}\left(a \,x^{2}\right)}-\frac{c}{\sqrt{c \,\mathrm{LambertW}\left(a \,x^{2}\right)}}}{c}$	29
default	$\frac{\sqrt{c \,\mathrm{LambertW}\left(a \,x^{2}\right)}-\frac{c}{\sqrt{c \,\mathrm{LambertW}\left(a \,x^{2}\right)}}}{c}$	29

input
```
int(1/x/(c*LambertW(a*x^2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/c*((c*LambertW(a*x^2))^(1/2)-c/(c*LambertW(a*x^2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{x\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x}\,dx$$

input
```
integrate(1/x/(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))/(c*x*lambert_w(a*x^2)), x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{cW\left(ax^2\right)}}\,dx = \frac{W(ax^2)}{\sqrt{cW\left(ax^2\right)}} - \frac{1}{\sqrt{cW\left(ax^2\right)}}$$

input
```
integrate(1/x/(c*LambertW(a*x**2))**(1/2),x)
```

output
```
LambertW(a*x**2)/sqrt(c*LambertW(a*x**2)) - 1/sqrt(c*LambertW(a*x**2))
```

Maxima [F]

$$\int \frac{1}{x\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x}\,dx$$

input
```
integrate(1/x/(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x^2))*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x}\,dx$$

input
```
integrate(1/x/(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x^2))*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{x\,\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}\,dx$$

input
```
int(1/(x*(c*LambertW(a*x^2))^(1/2)),x)
```

output
```
int(1/(x*(c*LambertW(a*x^2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{cW(ax^2)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}}{lambert\_w(a\,x^2)x}dx\right)}{c}$$

input
```
int(1/x/(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a*x**2))/(lambert_w(a*x**2)*x),x))/c
```

3.171 $\int \frac{1}{x^3\sqrt{cW(ax^2)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \frac{1}{x^3\sqrt{cW(ax^2)}}\,dx=-\frac{a\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{cW(ax^2)}}{\sqrt{c}}\right)}{3\sqrt{c}}-\frac{1}{3x^2\sqrt{cW(ax^2)}}-\frac{\sqrt{cW(ax^2)}}{3cx^2}$$

output
```
-1/3*a*Pi^(1/2)*erf((c*LambertW(a*x^2))^(1/2)/c^(1/2))/c^(1/2)-1/3/x^2/(c*
LambertW(a*x^2))^(1/2)-1/3*(c*LambertW(a*x^2))^(1/2)/c/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^3\sqrt{cW(ax^2)}}\,dx=\frac{-1-a\sqrt{\pi}x^2\text{erf}\left(\sqrt{W(ax^2)}\right)\sqrt{W(ax^2)}-W(ax^2)}{3x^2\sqrt{cW(ax^2)}}$$

input
```
Integrate[1/(x^3*Sqrt[c*ProductLog[a*x^2]]),x]
```

output
```
(-1 - a*Sqrt[Pi]*x^2*Erf[Sqrt[ProductLog[a*x^2]]]*Sqrt[ProductLog[a*x^2]]
- ProductLog[a*x^2])/(3*x^2*Sqrt[c*ProductLog[a*x^2]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {7173, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{cW(ax^2)}}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{\int \frac{\sqrt{cW(ax^2)}}{x^3(W(ax^2)+1)}dx}{3c}-\frac{1}{3x^2\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{-\frac{2\int \frac{\left(cW\left(ax^2\right)\right)^{3/2}}{x^3\left(W\left(ax^2\right)+1\right)}dx}{c}-\frac{\sqrt{cW(ax^2)}}{x^2}}{3c}-\frac{1}{3x^2\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{7203}$$

$$\frac{\sqrt{\pi}(-a)\sqrt{c}\text{erf}\left(\frac{\sqrt{cW(ax^2)}}{\sqrt{c}}\right)-\frac{\sqrt{cW(ax^2)}}{x^2}}{3c}-\frac{1}{3x^2\sqrt{cW(ax^2)}}$$

input
```
Int[1/(x^3*Sqrt[c*ProductLog[a*x^2]]),x]
```

output
```
-1/3*1/(x^2*Sqrt[c*ProductLog[a*x^2]]) + (-(a*Sqrt[c]*Sqrt[Pi]*Erf[Sqrt[c*
ProductLog[a*x^2]]/Sqrt[c]]) - Sqrt[c*ProductLog[a*x^2]]/x^2)/(3*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{1}{x^{3}\sqrt{c\,\mathrm{LambertW}\left(a\,x^{2}\right)}}dx$$

input
```
int(1/x^3/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(1/x^3/(c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^3\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x^3}\,dx$$

input
```
integrate(1/x^3/(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))/(c*x^3*lambert_w(a*x^2)), x)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{1}{x^3\sqrt{cW\left(ax^2\right)}}\,dx$$

input
```
integrate(1/x**3/(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(c*LambertW(a*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x^3}\,dx$$

input
```
integrate(1/x^3/(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x^2))*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x^3}\,dx$$

input
```
integrate(1/x^3/(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x^2))*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{x^3\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}\,dx$$

input
```
int(1/(x^3*(c*LambertW(a*x^2))^(1/2)),x)
```

output
```
int(1/(x^3*(c*LambertW(a*x^2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{cW(ax^2)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}}{lambert\_w(a\,x^2)x^3}dx\right)}{c}$$

input
```
int(1/x^3/(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a*x**2))/(lambert_w(a*x**2)*x**3),x))/c
```

3.172 $\int \frac{1}{x^5\sqrt{cW(ax^2)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 107

$$\int \frac{1}{x^5\sqrt{cW(ax^2)}}\,dx = \frac{4a^2\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^2)}}{\sqrt{c}}\right)}{15\sqrt{c}} - \frac{1}{5x^4\sqrt{cW(ax^2)}} - \frac{\sqrt{cW(ax^2)}}{15cx^4} + \frac{4(cW(ax^2))^{3/2}}{15c^2x^4}$$

output
```
4/15*a^2*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(c*LambertW(a*x^2))^(1/2)/c^(1/2))/c
^(1/2)-1/5/x^4/(c*LambertW(a*x^2))^(1/2)-1/15*(c*LambertW(a*x^2))^(1/2)/c/
x^4+4/15*(c*LambertW(a*x^2))^(3/2)/c^2/x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^5\sqrt{cW(ax^2)}}\,dx = \frac{-3+4a^2\sqrt{2\pi}x^4\mathrm{erf}\left(\sqrt{2}\sqrt{W(ax^2)}\right)\sqrt{W(ax^2)} - W(ax^2) + 4W(ax^2)^2}{15x^4\sqrt{cW(ax^2)}}$$

input
```
Integrate[1/(x^5*Sqrt[c*ProductLog[a*x^2]]),x]
```

output
```
(-3 + 4*a^2*Sqrt[2*Pi]*x^4*Erf[Sqrt[2]*Sqrt[ProductLog[a*x^2]]]*Sqrt[Produ
ctLog[a*x^2]] - ProductLog[a*x^2] + 4*ProductLog[a*x^2]^2)/(15*x^4*Sqrt[c*
ProductLog[a*x^2]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7173, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{cW(ax^2)}}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{\int \frac{\sqrt{cW(ax^2)}}{x^5(W(ax^2)+1)}dx}{5c}-\frac{1}{5x^4\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{-\frac{4\int \frac{(cW(ax^2))^{3/2}}{x^5(W(ax^2)+1)}dx}{3c}-\frac{\sqrt{cW(ax^2)}}{3x^4}}{5c}-\frac{1}{5x^4\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{-\frac{4\left(-\frac{4\int \frac{(cW(ax^2))^{5/2}}{x^5(W(ax^2)+1)}dx}{c}-\frac{(cW(ax^2))^{3/2}}{x^4}\right)}{3c}-\frac{\sqrt{cW(ax^2)}}{3x^4}}{5c}-\frac{1}{5x^4\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{7203}$$

$$-\frac{-\frac{4\left(\sqrt{2\pi}a^2(-c^{3/2})\text{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^2)}}{\sqrt{c}}\right)-\frac{(cW(ax^2))^{3/2}}{x^4}\right)}{3c}-\frac{\sqrt{cW(ax^2)}}{3x^4}}{5c}-\frac{1}{5x^4\sqrt{cW(ax^2)}}$$

input
```
Int[1/(x^5*Sqrt[c*ProductLog[a*x^2]]),x]
```

output
```
-1/5*1/(x^4*Sqrt[c*ProductLog[a*x^2]]) + (-1/3*Sqrt[c*ProductLog[a*x^2]]/x
^4 - (4*(-(a^2*c^(3/2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[c*ProductLog[a*x^2]])/
Sqrt[c]]) - (c*ProductLog[a*x^2])^(3/2)/x^4))/(3*c))/(5*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{1}{x^{5} \sqrt{c \,\mathrm{LambertW}\left(a \,x^{2}\right)}}d x$$

input
```
int(1/x^5/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(1/x^5/(c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^5 \sqrt{cW\left(ax^2\right)}} \, dx = \int { \frac{1}{\sqrt{c \mathrm{W}\left(a x^{2}\right)} x^{5}} \, dx }$$

input
```
integrate(1/x^5/(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))/(c*x^5*lambert_w(a*x^2)), x)
```

Sympy [F]

$$\int \frac{1}{x^5 \sqrt{cW\left(ax^2\right)}} \, dx = \int \frac{1}{x^{5} \sqrt{c W\left(a x^{2}\right)}}\, dx$$

input
```
integrate(1/x**5/(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(1/(x**5*sqrt(c*LambertW(a*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^5\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x^5}\,dx$$

input
```
integrate(1/x^5/(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x^2))*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x^5}\,dx$$

input
```
integrate(1/x^5/(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x^2))*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{x^5\,\sqrt{c\,\mathrm{LambertW}\,(a\,x^2)}}\,dx$$

input
```
int(1/(x^5*(c*LambertW(a*x^2))^(1/2)),x)
```

output
```
int(1/(x^5*(c*LambertW(a*x^2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^5\sqrt{cW(ax^2)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}}{lambert\_w(a\,x^2)x^5}dx\right)}{c}$$

input
```
int(1/x^5/(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a*x**2))/(lambert_w(a*x**2)*x**5),x))/c
```

3.173 $\int \frac{1}{x^7\sqrt{cW(ax^2)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 129

$$\int \frac{1}{x^7\sqrt{cW(ax^2)}}\,dx = -\frac{12a^3\sqrt{3\pi}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{cW(ax^2)}}{\sqrt{c}}\right)}{35\sqrt{c}} - \frac{1}{7x^6\sqrt{cW(ax^2)}} - \frac{\sqrt{cW(ax^2)}}{35cx^6} + \frac{2(cW(ax^2))^{3/2}}{35c^2x^6} - \frac{12(cW(ax^2))^{5/2}}{35c^3x^6}$$

output
```
-12/35*a^3*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(c*LambertW(a*x^2))^(1/2)/c^(1/2))
/c^(1/2)-1/7/x^6/(c*LambertW(a*x^2))^(1/2)-1/35*(c*LambertW(a*x^2))^(1/2)/
c/x^6+2/35*(c*LambertW(a*x^2))^(3/2)/c^2/x^6-12/35*(c*LambertW(a*x^2))^(5/
2)/c^3/x^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^7\sqrt{cW(ax^2)}}\,dx = \frac{-5-12a^3\sqrt{3\pi}x^6\,\mathrm{erf}\left(\sqrt{3}\sqrt{W(ax^2)}\right)\sqrt{W(ax^2)} - W(ax^2) + 2W(ax^2)^2 - 12W(ax^2)^3}{35x^6\sqrt{cW(ax^2)}}$$

input
```
Integrate[1/(x^7*Sqrt[c*ProductLog[a*x^2]]),x]
```

output
```
(-5 - 12*a^3*Sqrt[3*Pi]*x^6*Erf[Sqrt[3]*Sqrt[ProductLog[a*x^2]]]*Sqrt[Prod
uctLog[a*x^2]] - ProductLog[a*x^2] + 2*ProductLog[a*x^2]^2 - 12*ProductLog
[a*x^2]^3)/(35*x^6*Sqrt[c*ProductLog[a*x^2]])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {7173, 7206, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{cW(ax^2)}}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{\int \frac{\sqrt{cW(ax^2)}}{x^7(W(ax^2)+1)}dx}{7c}-\frac{1}{7x^6\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{-\frac{6\int \frac{\left(cW\left(ax^2\right)\right)^{3/2}}{x^7\left(W\left(ax^2\right)+1\right)}dx}{5c}-\frac{\sqrt{cW(ax^2)}}{5x^6}}{7c}-\frac{1}{7x^6\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{-\frac{6\left(-\frac{2\int \frac{\left(cW\left(ax^2\right)\right)^{5/2}}{x^7\left(W\left(ax^2\right)+1\right)}dx}{c}-\frac{\left(cW\left(ax^2\right)\right)^{3/2}}{3x^6}\right)}{5c}-\frac{\sqrt{cW(ax^2)}}{5x^6}}{7c}-\frac{1}{7x^6\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{6\left(-\frac{2\left(-\frac{6\int \frac{\left(cW\left(ax^2\right)\right)^{7/2}}{x^7\left(W\left(ax^2\right)+1\right)}dx}{c}-\frac{\left(cW\left(ax^2\right)\right)^{5/2}}{x^6}\right)}{c}-\frac{\left(cW\left(ax^2\right)\right)^{3/2}}{3x^6}\right)}{5c}-\frac{\sqrt{cW(ax^2)}}{5x^6}}{7c}-\frac{1}{7x^6\sqrt{cW\left(ax^2\right)}}$$

$$\Big\downarrow \text{7203}$$

$$\frac{-\frac{6\left(-\frac{2\left(\sqrt{3\pi}a^3\left(-c^{5/2}\right)\text{erf}\left(\frac{\sqrt{3}\sqrt{cW\left(ax^2\right)}}{\sqrt{c}}\right)-\frac{\left(cW\left(ax^2\right)\right)^{5/2}}{x^6}\right)}{c}-\frac{\left(cW\left(ax^2\right)\right)^{3/2}}{3x^6}\right)}{5c}-\frac{\sqrt{cW(ax^2)}}{5x^6}}{7c}-\frac{1}{7x^6\sqrt{cW\left(ax^2\right)}}$$

input
```
Int[1/(x^7*Sqrt[c*ProductLog[a*x^2]]),x]
```

output
```
-1/7*1/(x^6*Sqrt[c*ProductLog[a*x^2]]) + (-1/5*Sqrt[c*ProductLog[a*x^2]]/x
^6 - (6*(-1/3*(c*ProductLog[a*x^2])^(3/2)/x^6 - (2*(-(a^3*c^(5/2)*Sqrt[3*P
i]*Erf[(Sqrt[3]*Sqrt[c*ProductLog[a*x^2]])/Sqrt[c]]) - (c*ProductLog[a*x^2
])^(5/2)/x^6))/c))/(5*c))/(7*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203

```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206

```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{1}{x^{7} \sqrt{c \,\mathrm{LambertW}\left(a \,x^{2}\right)}}dx$$

input

```
int(1/x^7/(c*LambertW(a*x^2))^(1/2),x)
```

output

```
int(1/x^7/(c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^7\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x^7}\,dx$$

input

```
integrate(1/x^7/(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output

```
integral(sqrt(c*lambert_w(a*x^2))/(c*x^7*lambert_w(a*x^2)), x)
```

Sympy [F]

$$\int \frac{1}{x^7\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{x^7\sqrt{cW(ax^2)}}\,dx$$

input
```
integrate(1/x**7/(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(1/(x**7*sqrt(c*LambertW(a*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^7\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x^7}\,dx$$

input
```
integrate(1/x^7/(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x^2))*x^7), x)
```

Giac [F]

$$\int \frac{1}{x^7\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x^7}\,dx$$

input
```
integrate(1/x^7/(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x^2))*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7\sqrt{cW\left(ax^2\right)}}\,dx=\int \frac{1}{x^7\,\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}\,dx$$

input
```
int(1/(x^7*(c*LambertW(a*x^2))^(1/2)),x)
```

output
```
int(1/(x^7*(c*LambertW(a*x^2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^7\sqrt{cW\left(ax^2\right)}}\,dx=\frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}}{lambert\_w(a\,x^2)x^7}dx\right)}{c}$$

input
```
int(1/x^7/(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a*x**2))/(lambert_w(a*x**2)*x**7),x))/c
```

3.174 $\int \frac{x^6}{\sqrt{cW(ax^2)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int \frac{x^6}{\sqrt{cW(ax^2)}}\,dx = \frac{8c^3x^7}{2401(cW(ax^2))^{7/2}} - \frac{4c^2x^7}{343(cW(ax^2))^{5/2}} + \frac{cx^7}{49(cW(ax^2))^{3/2}} + \frac{x^7}{7\sqrt{cW(ax^2)}}$$

output
```
8/2401*c^3*x^7/(c*LambertW(a*x^2))^(7/2)-4/343*c^2*x^7/(c*LambertW(a*x^2))
^(5/2)+1/49*c*x^7/(c*LambertW(a*x^2))^(3/2)+1/7*x^7/(c*LambertW(a*x^2))^(1
/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.68

$$\int \frac{x^6}{\sqrt{cW(ax^2)}}\,dx = \frac{x^7\left(8-28W(ax^2)+49W(ax^2)^2+343W(ax^2)^3\right)}{2401W(ax^2)^3\sqrt{cW(ax^2)}}$$

input
```
Integrate[x^6/Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(x^7*(8 - 28*ProductLog[a*x^2] + 49*ProductLog[a*x^2]^2 + 343*ProductLog[a
*x^2]^3))/(2401*ProductLog[a*x^2]^3*Sqrt[c*ProductLog[a*x^2]])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7172, 7205, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{cW\left(ax^2\right)}}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{7}\int \frac{x^6}{\sqrt{cW\left(ax^2\right)}\left(W\left(ax^2\right)+1\right)}dx+\frac{x^7}{7\sqrt{cW\left(ax^2\right)}}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{7}\left(\frac{cx^7}{7\left(cW\left(ax^2\right)\right)^{3/2}}-\frac{4}{7}c\int \frac{x^6}{\left(cW\left(ax^2\right)\right)^{3/2}\left(W\left(ax^2\right)+1\right)}dx\right)+\frac{x^7}{7\sqrt{cW\left(ax^2\right)}}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{7}\left(\frac{cx^7}{7\left(cW\left(ax^2\right)\right)^{3/2}}-\frac{4}{7}c\left(\frac{cx^7}{7\left(cW\left(ax^2\right)\right)^{5/2}}-\frac{2}{7}c\int \frac{x^6}{\left(cW\left(ax^2\right)\right)^{5/2}\left(W\left(ax^2\right)+1\right)}dx\right)\right)+\frac{x^7}{7\sqrt{cW\left(ax^2\right)}}$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{7}\left(\frac{cx^7}{7\left(cW\left(ax^2\right)\right)^{3/2}}-\frac{4}{7}c\left(\frac{cx^7}{7\left(cW\left(ax^2\right)\right)^{5/2}}-\frac{2c^2x^7}{49\left(cW\left(ax^2\right)\right)^{7/2}}\right)\right)+\frac{x^7}{7\sqrt{cW\left(ax^2\right)}}$$

input
```
Int[x^6/Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
x^7/(7*Sqrt[c*ProductLog[a*x^2]]) + ((c*x^7)/(7*(c*ProductLog[a*x^2])^(3/2
)) - (4*c*((-2*c^2*x^7)/(49*(c*ProductLog[a*x^2])^(7/2)) + (c*x^7)/(7*(c*P
roductLog[a*x^2])^(5/2))))/7)/7
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int \frac{x^6}{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}dx$$

input
```
int(x^6/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^6/(c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^6}{\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{x^6}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x^6/(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))*x^6/(c*lambert_w(a*x^2)), x)
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{x^6}{\sqrt{cW\left(ax^2\right)}}\,dx$$

input
```
integrate(x**6/(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(x**6/sqrt(c*LambertW(a*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{x^6}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x^6/(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/sqrt(c*lambert_w(a*x^2)), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{cW(ax^2)}}\,dx = \int \frac{x^6}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x^6/(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt(c*lambert_w(a*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{cW(ax^2)}}\,dx = \int \frac{x^6}{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}\,dx$$

input
```
int(x^6/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^6/(c*LambertW(a*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{cW(ax^2)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}\,x^6}{lambert\_w(a\,x^2)}dx\right)}{c}$$

input
```
int(x^6/(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(lambert_w(a*x**2))*x**6)/lambert_w(a*x**2),x))/c
```

3.175 $\int \frac{x^4}{\sqrt{cW(ax^2)}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{x^4}{\sqrt{cW(ax^2)}} \, dx = -\frac{2c^2x^5}{125(cW(ax^2))^{5/2}} + \frac{cx^5}{25(cW(ax^2))^{3/2}} + \frac{x^5}{5\sqrt{cW(ax^2)}}$$

output
```
-2/125*c^2*x^5/(c*LambertW(a*x^2))^(5/2)+1/25*c*x^5/(c*LambertW(a*x^2))^(3
/2)+1/5*x^5/(c*LambertW(a*x^2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{\sqrt{cW(ax^2)}} \, dx = \frac{x^5 \left(-2 + 5W(ax^2) + 25W(ax^2)^2\right)}{125W(ax^2)^2 \sqrt{cW(ax^2)}}$$

input
```
Integrate[x^4/Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(x^5*(-2 + 5*ProductLog[a*x^2] + 25*ProductLog[a*x^2]^2))/(125*ProductLog[
a*x^2]^2*Sqrt[c*ProductLog[a*x^2]])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {7172, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{cW(ax^2)}}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{5}\int \frac{x^4}{\sqrt{cW(ax^2)}\,(W(ax^2)+1)}dx+\frac{x^5}{5\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{5}\left(\frac{cx^5}{5(cW(ax^2))^{3/2}}-\frac{2}{5}c\int \frac{x^4}{(cW(ax^2))^{3/2}(W(ax^2)+1)}dx\right)+\frac{x^5}{5\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{5}\left(\frac{cx^5}{5(cW(ax^2))^{3/2}}-\frac{2c^2x^5}{25(cW(ax^2))^{5/2}}\right)+\frac{x^5}{5\sqrt{cW(ax^2)}}$$

input
```
Int[x^4/Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
x^5/(5*Sqrt[c*ProductLog[a*x^2]]) + ((-2*c^2*x^5)/(25*(c*ProductLog[a*x^2]
)^(5/2)) + (c*x^5)/(5*(c*ProductLog[a*x^2])^(3/2)))/5
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int \frac{x^4}{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}dx$$

input
```
int(x^4/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^4/(c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{cW(ax^2)}}\,dx = \int \frac{x^4}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x^4/(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))*x^4/(c*lambert_w(a*x^2)), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{cW(ax^2)}}\,dx = \int \frac{x^4}{\sqrt{cW(ax^2)}}\,dx$$

input
```
integrate(x**4/(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(x**4/sqrt(c*LambertW(a*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{cW(ax^2)}}\,dx = \int \frac{x^4}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x^4/(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(c*lambert_w(a*x^2)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{cW(ax^2)}}\,dx = \int \frac{x^4}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x^4/(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(c*lambert_w(a*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{cW(ax^2)}}\,dx = \int \frac{x^4}{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}\,dx$$

input
```
int(x^4/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^4/(c*LambertW(a*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{cW(ax^2)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}\,x^4}{lambert\_w(a\,x^2)}dx\right)}{c}$$

input
```
int(x^4/(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(lambert_w(a*x**2))*x**4)/lambert_w(a*x**2),x))/c
```

3.176 $\int \frac{x^2}{\sqrt{cW(ax^2)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{x^2}{\sqrt{cW(ax^2)}}\,dx = \frac{cx^3}{9(cW(ax^2))^{3/2}} + \frac{x^3}{3\sqrt{cW(ax^2)}}$$

output
```
1/9*c*x^3/(c*LambertW(a*x^2))^(3/2)+1/3*x^3/(c*LambertW(a*x^2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\sqrt{cW(ax^2)}}\,dx = \frac{cx^3(1+3W(ax^2))}{9(cW(ax^2))^{3/2}}$$

input
```
Integrate[x^2/Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(c*x^3*(1 + 3*ProductLog[a*x^2]))/(9*(c*ProductLog[a*x^2])^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7172, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{cW\left(ax^2\right)}}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{3}\int \frac{x^2}{\sqrt{cW\left(ax^2\right)}\left(W\left(ax^2\right)+1\right)}dx+\frac{x^3}{3\sqrt{cW\left(ax^2\right)}}$$

$$\Big\downarrow \text{7201}$$

$$\frac{x^3}{3\sqrt{cW\left(ax^2\right)}}+\frac{cx^3}{9\left(cW\left(ax^2\right)\right)^{3/2}}$$

input
```
Int[x^2/Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
(c*x^3)/(9*(c*ProductLog[a*x^2])^(3/2)) + x^3/(3*Sqrt[c*ProductLog[a*x^2]]
)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}dx$$

input
```
int(x^2/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^2/(c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{x^2}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x^2/(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))*x^2/(c*lambert_w(a*x^2)), x)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{\sqrt{cW\left(ax^2\right)}}\,dx = \frac{x^3}{3\sqrt{cW\left(ax^2\right)}} + \frac{x^3}{9\sqrt{cW\left(ax^2\right)}W\left(ax^2\right)}$$

input
```
integrate(x**2/(c*LambertW(a*x**2))**(1/2),x)
```

output
```
x**3/(3*sqrt(c*LambertW(a*x**2))) + x**3/(9*sqrt(c*LambertW(a*x**2))*Lambe
rtW(a*x**2))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{x^2}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x^2/(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(c*lambert_w(a*x^2)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{x^2}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(x^2/(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(c*lambert_w(a*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{x^2}{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}\,dx$$

input
```
int(x^2/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(x^2/(c*LambertW(a*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{cW(ax^2)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}\,x^2}{lambert\_w(a\,x^2)}dx\right)}{c}$$

input
```
int(x^2/(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(lambert_w(a*x**2))*x**2)/lambert_w(a*x**2),x))/c
```

3.177 $\int \frac{1}{\sqrt{cW\left(ax^2\right)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 67

$$\int \frac{1}{\sqrt{cW\left(ax^2\right)}}\,dx=\frac{x}{\sqrt{cW\left(ax^2\right)}}-\frac{e^{\frac{1}{2}W\left(ax^2\right)}\Gamma\left(0,-\frac{1}{2}W\left(ax^2\right)\right)W\left(ax^2\right)}{2ax\sqrt{cW\left(ax^2\right)}}$$

output
```
x/(c*LambertW(a*x^2))^(1/2)-1/2*exp(1/2*LambertW(a*x^2))*Ei(1,-1/2*Lambert
W(a*x^2))*LambertW(a*x^2)/a/x/(c*LambertW(a*x^2))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\sqrt{cW\left(ax^2\right)}}\,dx=\int \frac{1}{\sqrt{cW\left(ax^2\right)}}\,dx$$

input
```
Integrate[1/Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
Integrate[1/Sqrt[c*ProductLog[a*x^2]], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cW(ax^2)}}dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sqrt{W(ax^2)}\int \frac{1}{\sqrt{W(ax^2)}}dx}{\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\sqrt{W(ax^2)}\int \frac{1}{\sqrt{W(ax^2)}}dx}{\sqrt{cW(ax^2)}}$$

input
```
Int[1/Sqrt[c*ProductLog[a*x^2]],x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{1}{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}dx$$

input
```
int(1/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(1/(c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(1/(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))/(c*lambert_w(a*x^2)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{1}{\sqrt{cW\left(ax^2\right)}}\,dx$$

input
```
integrate(1/(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(1/sqrt(c*LambertW(a*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(1/(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*lambert_w(a*x^2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}}\,dx$$

input
```
integrate(1/(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*lambert_w(a*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}\,dx$$

input
```
int(1/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(1/(c*LambertW(a*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{cW\left(ax^2\right)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}}{lambert\_w(a\,x^2)}dx\right)}{c}$$

input
```
int(1/(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a*x**2))/lambert_w(a*x**2),x))/c
```

3.178 $\int \frac{1}{x^2\sqrt{cW(ax^2)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int \frac{1}{x^2\sqrt{cW(ax^2)}}\,dx = -\frac{1}{2x\sqrt{cW(ax^2)}} - \frac{e^{\frac{3}{2}W(ax^2)}\Gamma\left(0,\frac{1}{2}W(ax^2)\right)W(ax^2)\sqrt{cW(ax^2)}}{4acx^3}$$

output
```
-1/2/x/(c*LambertW(a*x^2))^(1/2)-1/4*exp(3/2*LambertW(a*x^2))*Ei(1,1/2*Lam
bertW(a*x^2))*LambertW(a*x^2)*(c*LambertW(a*x^2))^(1/2)/a/c/x^3
```

Mathematica [F]

$$\int \frac{1}{x^2\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{x^2\sqrt{cW(ax^2)}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[c*ProductLog[a*x^2]]),x]
```

output
```
Integrate[1/(x^2*Sqrt[c*ProductLog[a*x^2]]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{cW(ax^2)}}dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sqrt{W(ax^2)}\int \frac{1}{x^2\sqrt{W(ax^2)}}dx}{\sqrt{cW(ax^2)}}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\sqrt{W(ax^2)}\int \frac{1}{x^2\sqrt{W(ax^2)}}dx}{\sqrt{cW(ax^2)}}$$

input
```
Int[1/(x^2*Sqrt[c*ProductLog[a*x^2]]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{1}{x^2\sqrt{c\,\mathrm{LambertW}(a\,x^2)}}dx$$

input
```
int(1/x^2/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(1/x^2/(c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x^2}\,dx$$

input
```
integrate(1/x^2/(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))/(c*x^2*lambert_w(a*x^2)), x)
```

Sympy [F]

$$\int \frac{1}{x^2 \sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{1}{x^2 \sqrt{cW\left(ax^2\right)}}\,dx$$

input
```
integrate(1/x**2/(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(c*LambertW(a*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \sqrt{cW\left(ax^2\right)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x^2}\,dx$$

input
```
integrate(1/x^2/(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x^2))*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x^2}\,dx$$

input
```
integrate(1/x^2/(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x^2))*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{x^2\,\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}\,dx$$

input
```
int(1/(x^2*(c*LambertW(a*x^2))^(1/2)),x)
```

output
```
int(1/(x^2*(c*LambertW(a*x^2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{cW(ax^2)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}}{lambert\_w(a\,x^2)x^2}dx\right)}{c}$$

input
```
int(1/x^2/(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a*x**2))/(lambert_w(a*x**2)*x**2),x))/c
```

3.179 $\int \frac{1}{x^4\sqrt{cW(ax^2)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 97

$$\int \frac{1}{x^4\sqrt{cW(ax^2)}}\,dx = -\frac{1}{4x^3\sqrt{cW(ax^2)}} - \frac{\sqrt{cW(ax^2)}}{8cx^3} + \frac{3e^{\frac{5}{2}W(ax^2)}\Gamma\left(0,\frac{3}{2}W(ax^2)\right)W(ax^2)\left(cW(ax^2)\right)^{3/2}}{16ac^2x^5}$$

output
```
-1/4/x^3/(c*LambertW(a*x^2))^(1/2)-1/8*(c*LambertW(a*x^2))^(1/2)/c/x^3+3/1
6*exp(5/2*LambertW(a*x^2))*Ei(1,3/2*LambertW(a*x^2))*LambertW(a*x^2)*(c*La
mbertW(a*x^2))^(3/2)/a/c^2/x^5
```

Mathematica [F]

$$\int \frac{1}{x^4\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{x^4\sqrt{cW(ax^2)}}\,dx$$

input
```
Integrate[1/(x^4*Sqrt[c*ProductLog[a*x^2]]),x]
```

output
```
Integrate[1/(x^4*Sqrt[c*ProductLog[a*x^2]]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \sqrt{cW\left(ax^2\right)}} \, dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sqrt{W\left(ax^2\right)} \int \frac{1}{x^4 \sqrt{W\left(ax^2\right)}}dx}{\sqrt{cW\left(ax^2\right)}}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\sqrt{W\left(ax^2\right)} \int \frac{1}{x^4 \sqrt{W\left(ax^2\right)}}dx}{\sqrt{cW\left(ax^2\right)}}$$

input
```
Int[1/(x^4*Sqrt[c*ProductLog[a*x^2]]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{1}{x^4 \sqrt{c \,\mathrm{LambertW}\left(a \,x^2\right)}}dx$$

input
```
int(1/x^4/(c*LambertW(a*x^2))^(1/2),x)
```

output
```
int(1/x^4/(c*LambertW(a*x^2))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^4\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x^4}\,dx$$

input
```
integrate(1/x^4/(c*lambert_w(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^2))/(c*x^4*lambert_w(a*x^2)), x)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{x^4\sqrt{cW(ax^2)}}\,dx$$

input
```
integrate(1/x**4/(c*LambertW(a*x**2))**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(c*LambertW(a*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{cW(ax^2)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x^4}\,dx$$

input
```
integrate(1/x^4/(c*lambert_w(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x^2))*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{cW(ax^2)}}\,dx=\int \frac{1}{\sqrt{c\,\mathrm{W}(ax^2)}x^4}\,dx$$

input
```
integrate(1/x^4/(c*lambert_w(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*lambert_w(a*x^2))*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{cW(ax^2)}}\,dx=\int \frac{1}{x^4\,\sqrt{c\,\mathrm{LambertW}\left(a\,x^2\right)}}\,dx$$

input
```
int(1/(x^4*(c*LambertW(a*x^2))^(1/2)),x)
```

output
```
int(1/(x^4*(c*LambertW(a*x^2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{cW(ax^2)}}\,dx=\frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(a\,x^2)}}{lambert\_w(a\,x^2)x^4}dx\right)}{c}$$

input
```
int(1/x^4/(c*Lambert_W(a*x^2))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a*x**2))/(lambert_w(a*x**2)*x**4),x))/c
```

3.180 $\int x^3(cW(ax^2))^p\,dx$

Optimal result

Integrand size = 14, antiderivative size = 126

$$\int x^3(cW(ax^2))^p\,dx = \frac{2^{-3-p}e^{-W(ax^2)}x^2\Gamma(2+p,-2W(ax^2))\left(-W(ax^2)\right)^{-1-p}(cW(ax^2))^p}{a} + \frac{2^{-4-p}e^{-W(ax^2)}x^2\Gamma(3+p,-2W(ax^2))\left(-W(ax^2)\right)^{-2-p}(cW(ax^2))^{1+p}}{ac}$$

output
```
2^(-3-p)*x^2*GAMMA(2+p,-2*LambertW(a*x^2))*(-LambertW(a*x^2))^(-1-p)*(c*La
mbertW(a*x^2))^p/a/exp(LambertW(a*x^2))+2^(-4-p)*x^2*GAMMA(3+p,-2*LambertW
(a*x^2))*(-LambertW(a*x^2))^(-2-p)*(c*LambertW(a*x^2))^(p+1)/a/c/exp(Lambe
rtW(a*x^2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

$$\int x^3(cW(ax^2))^p\,dx = \frac{2^{-4-p}(-2\Gamma(2+p,-2W(ax^2))+\Gamma(3+p,-2W(ax^2)))\left(-W(ax^2)\right)^{-p}(cW(ax^2))^p}{a^2}$$

input
```
Integrate[x^3*(c*ProductLog[a*x^2])^p,x]
```

output
```
(2^(-4 - p)*(-2*Gamma[2 + p, -2*ProductLog[a*x^2]] + Gamma[3 + p, -2*Produ
ctLog[a*x^2]])*(c*ProductLog[a*x^2])^p)/(a^2*(-ProductLog[a*x^2])^p)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {7271, 7283, 7174, 7207}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(cW\left(ax^2\right)\right)^p dx$$

$$\Big\downarrow \text{7271}$$

$$W\left(ax^2\right)^{-p} \left(cW\left(ax^2\right)\right)^p \int x^3 W\left(ax^2\right)^p dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2} W\left(ax^2\right)^{-p} \left(cW\left(ax^2\right)\right)^p \int x^2 W\left(ax^2\right)^p dx^2$$

$$\Big\downarrow \text{7174}$$

$$\frac{1}{2} W\left(ax^2\right)^{-p} \left(cW\left(ax^2\right)\right)^p \left(\int \frac{x^2 W\left(ax^2\right)^p}{W\left(ax^2\right)+1} dx^2 + \int \frac{x^2 W\left(ax^2\right)^{p+1}}{W\left(ax^2\right)+1} dx^2\right)$$

$$\Big\downarrow \text{7207}$$

$$\frac{1}{2} W\left(ax^2\right)^{-p} \left(cW\left(ax^2\right)\right)^p \left(\frac{2^{-p-3} x^2 e^{-W\left(ax^2\right)} W\left(ax^2\right)^{p+1} \left(-W\left(ax^2\right)\right)^{-p-2} \Gamma\left(p+3,-2W\left(ax^2\right)\right)}{a} + \frac{2^{-p-2} x^2 e^{-W}}{}\right.$$

input
```
Int[x^3*(c*ProductLog[a*x^2])^p,x]
```

output
```
((c*ProductLog[a*x^2])^p*((2^(-2 - p)*x^2*Gamma[2 + p, -2*ProductLog[a*x^2
]]*(-ProductLog[a*x^2])^(-1 - p)*ProductLog[a*x^2]^p)/(a*E^ProductLog[a*x^
2]) + (2^(-3 - p)*x^2*Gamma[3 + p, -2*ProductLog[a*x^2]]*(-ProductLog[a*x^
2])^(-2 - p)*ProductLog[a*x^2]^(1 + p))/(a*E^ProductLog[a*x^2])))/(2*Produ
ctLog[a*x^2]^p)
```

Defintions of rubi rules used

rule 7174
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.), x_Symbol] :> Int[x^m*(
(c*ProductLog[a*x])^p/(1 + ProductLog[a*x])), x] + Simp[1/c   Int[x^m*((c*P
roductLog[a*x])^(p + 1)/(1 + ProductLog[a*x])), x], x] /; FreeQ[{a, c, m},
x]
```

rule 7207
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.))/((d_) + (d_.)*Product
Log[(a_.)*(x_)]), x_Symbol] :> Simp[x^m*Gamma[m + p + 1, (-(m + 1))*Product
Log[a*x]]*((c*ProductLog[a*x])^p/(a*d*(m + 1)*E^(m*ProductLog[a*x])*((-(m +
 1))*ProductLog[a*x])^(m + p))), x] /; FreeQ[{a, c, d, m, p}, x] && NeQ[m,
-1]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int x^3 (c \,\mathrm{LambertW}\left(a\,x^2\right))^p dx$$

input
```
int(x^3*(c*LambertW(a*x^2))^p,x)
```

output
```
int(x^3*(c*LambertW(a*x^2))^p,x)
```

Fricas [F]

$$\int x^3 (cW(ax^2))^p \, dx = \int (c\,\mathrm{W}(ax^2))^p \, x^3 \, dx$$

input
```
integrate(x^3*(c*lambert_w(a*x^2))^p,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^2))^p*x^3, x)
```

Sympy [F]

$$\int x^3 (cW(ax^2))^p \, dx = \int x^3 (cW(ax^2))^p \, dx$$

input
```
integrate(x**3*(c*LambertW(a*x**2))**p,x)
```

output
```
Integral(x**3*(c*LambertW(a*x**2))**p, x)
```

Maxima [F]

$$\int x^3 (cW(ax^2))^p \, dx = \int (c\,\mathrm{W}(ax^2))^p \, x^3 \, dx$$

input
```
integrate(x^3*(c*lambert_w(a*x^2))^p,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^2))^p*x^3, x)
```

Giac [F]

$$\int x^3 (cW(ax^2))^p \, dx = \int (c\,\mathrm{W}(ax^2))^p \, x^3 \, dx$$

input
```
integrate(x^3*(c*lambert_w(a*x^2))^p,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^2))^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (cW(ax^2))^p \, dx = \int x^3 \left(c\,\mathrm{LambertW}\left(a\,x^2\right)\right)^p \, dx$$

input
```
int(x^3*(c*LambertW(a*x^2))^p,x)
```

output
```
int(x^3*(c*LambertW(a*x^2))^p, x)
```

Reduce [F]

$$\int x^3 \left(cW\left(ax^2\right)\right)^p \, dx = c^p \left(\int lambert\_w\left(a\, x^2\right)^p x^3 dx \right)$$

input
```
int(x^3*(c*Lambert_W(a*x^2))^p,x)
```

output
```
c**p*int(lambert_w(a*x**2)**p*x**3,x)
```

3.181 $\int x(cW(ax^2))^p \, dx$

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int x(cW(ax^2))^p \, dx = \frac{1}{2}x^2(cW(ax^2))^p - \frac{p\Gamma(1+p,-W(ax^2))\,(-W(ax^2))^{-p}\,(cW(ax^2))^p}{2a}$$

output
```
1/2*x^2*(c*LambertW(a*x^2))^p-1/2*p*GAMMA(p+1,-LambertW(a*x^2))*(c*Lambert
W(a*x^2))^p/a/((-LambertW(a*x^2))^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int x(cW(ax^2))^p \, dx = \frac{(\Gamma(1+p,-W(ax^2))-\Gamma(2+p,-W(ax^2)))\,(-W(ax^2))^{-p}\,(cW(ax^2))^p}{2a}$$

input
```
Integrate[x*(c*ProductLog[a*x^2])^p,x]
```

output
```
((Gamma[1 + p, -ProductLog[a*x^2]] - Gamma[2 + p, -ProductLog[a*x^2]])*(c*
ProductLog[a*x^2])^p)/(2*a*(-ProductLog[a*x^2])^p)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7266, 7167, 7183}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(cW\left(ax^2\right)\right)^p\,dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}\int \left(cW\left(ax^2\right)\right)^p\,dx^2$$

$$\Big\downarrow \text{7167}$$

$$\frac{1}{2}\left(x^2\left(cW\left(ax^2\right)\right)^p-p\int \frac{\left(cW\left(ax^2\right)\right)^p}{W\left(ax^2\right)+1}dx^2\right)$$

$$\Big\downarrow \text{7183}$$

$$\frac{1}{2}\left(x^2\left(cW\left(ax^2\right)\right)^p-\frac{p\left(-W\left(ax^2\right)\right)^{-p}\left(cW\left(ax^2\right)\right)^p\Gamma\left(p+1,-W\left(ax^2\right)\right)}{a}\right)$$

input
```
Int[x*(c*ProductLog[a*x^2])^p,x]
```

output
```
(x^2*(c*ProductLog[a*x^2])^p - (p*Gamma[1 + p, -ProductLog[a*x^2]]*(c*Prod
uctLog[a*x^2])^p)/(a*(-ProductLog[a*x^2])^p))/2
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] && !LtQ[p, -1]
```

rule 7183
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.)/((d_) + (d_.)*ProductLog[(
a_.) + (b_.)*(x_)]), x_Symbol] :> Simp[Gamma[p + 1, -ProductLog[a + b*x]]*(
(c*ProductLog[a + b*x])^p/(b*d*(-ProductLog[a + b*x])^p)), x] /; FreeQ[{a,
b, c, d, p}, x]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [F]

$$\int x\left(c \,\mathrm{LambertW}\left(a \,x^{2}\right)\right)^{p}d x$$

input
```
int(x*(c*LambertW(a*x^2))^p,x)
```

output
```
int(x*(c*LambertW(a*x^2))^p,x)
```

Fricas [F]

$$\int x\left(cW\left(ax^2\right)\right)^p\,dx=\int \left(c\,\mathrm{W}(ax^2)\right)^p\,x\,dx$$

input
```
integrate(x*(c*lambert_w(a*x^2))^p,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^2))^p*x, x)
```

Sympy [F]

$$\int x\left(cW\left(ax^2\right)\right)^p\,dx=\int x\left(cW\left(ax^2\right)\right)^p\,dx$$

input
```
integrate(x*(c*LambertW(a*x**2))**p,x)
```

output
```
Integral(x*(c*LambertW(a*x**2))**p, x)
```

Maxima [F]

$$\int x\left(cW\left(ax^2\right)\right)^p\,dx=\int \left(c\,\mathrm{W}\left(ax^2\right)\right)^p x\,dx$$

input
```
integrate(x*(c*lambert_w(a*x^2))^p,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^2))^p*x, x)
```

Giac [F]

$$\int x\left(cW\left(ax^2\right)\right)^p\,dx=\int \left(c\,\mathrm{W}\left(ax^2\right)\right)^p x\,dx$$

input
```
integrate(x*(c*lambert_w(a*x^2))^p,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^2))^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(cW(ax^2))^p \, dx = \int x \left(c \,\mathrm{LambertW}\left(a\,x^2\right)\right)^p dx$$

input
```
int(x*(c*LambertW(a*x^2))^p,x)
```

output
```
int(x*(c*LambertW(a*x^2))^p, x)
```

Reduce [F]

$$\int x(cW(ax^2))^p \, dx = \frac{c^p \left(e^{lambert\_w(a\,x^2)}\, lambert\_w(a\,x^2)^p \, lambert\_w(a\,x^2) - e^{lambert\_w(a\,x^2)}\, lambert\_w(a\,x^2)^p \, p + 2\left(\int \frac{l}{lambert\_}\right.\right.}{2a}$$

input
```
int(x*(c*Lambert_W(a*x^2))^p,x)
```

output
```
(c**p*(e**lambert_w(a*x**2)*lambert_w(a*x**2)**p*lambert_w(a*x**2) - e**la
mbert_w(a*x**2)*lambert_w(a*x**2)**p*p + 2*int((lambert_w(a*x**2)**p*x)/(l
ambert_w(a*x**2)**2 + lambert_w(a*x**2)),x)*a*p**2))/(2*a)
```

3.182 $\int \frac{(cW(ax^2))^p}{x} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 42

$$\int \frac{(cW(ax^2))^p}{x} \, dx = \frac{(cW(ax^2))^p}{2p} + \frac{(cW(ax^2))^{1+p}}{2c(1+p)}$$

output
```
1/2*(c*LambertW(a*x^2))^p/p+1/2*(c*LambertW(a*x^2))^(p+1)/c/(p+1)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{(cW(ax^2))^p}{x} \, dx = \frac{(cW(ax^2))^p (1+p+pW(ax^2))}{2p(1+p)}$$

input
```
Integrate[(c*ProductLog[a*x^2])^p/x,x]
```

output
```
((c*ProductLog[a*x^2])^p*(1 + p + p*ProductLog[a*x^2]))/(2*p*(1 + p))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cW(ax^2))^p}{x} \, dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{\int \frac{(cW(ax^2))^{p+1}}{x(W(ax^2)+1)}dx}{c}+\frac{(cW(ax^2))^p}{2p}$$

$$\Big\downarrow \text{7200}$$

$$\frac{(cW(ax^2))^p}{2p}+\frac{(cW(ax^2))^{p+1}}{2c(p+1)}$$

input
```
Int[(c*ProductLog[a*x^2])^p/x,x]
```

output
```
(c*ProductLog[a*x^2])^p/(2*p) + (c*ProductLog[a*x^2])^(1 + p)/(2*c*(1 + p)
)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{e^{p\ln\left(c\,\mathrm{LambertW}\left(a\,x^2\right)\right)}}{2p}+\frac{\mathrm{LambertW}\left(a\,x^2\right)e^{p\ln\left(c\,\mathrm{LambertW}\left(a\,x^2\right)\right)}}{2p+2}$	44
default	$\frac{e^{p\ln\left(c\,\mathrm{LambertW}\left(a\,x^2\right)\right)}}{2p}+\frac{\mathrm{LambertW}\left(a\,x^2\right)e^{p\ln\left(c\,\mathrm{LambertW}\left(a\,x^2\right)\right)}}{2p+2}$	44

input
```
int((c*LambertW(a*x^2))^p/x,x,method=_RETURNVERBOSE)
```

output
```
1/2/p*exp(p*ln(c*LambertW(a*x^2)))+1/2/(p+1)*LambertW(a*x^2)*exp(p*ln(c*La
mbertW(a*x^2)))
```

Fricas [F]

$$\int \frac{(cW(ax^2))^p}{x}\,dx = \int \frac{(c\,\mathrm{W}(ax^2))^p}{x}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^p/x,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^2))^p/x, x)
```

Sympy [F]

$$\int \frac{(cW(ax^2))^p}{x} \, dx = \int \frac{(cW(ax^2))^p}{x} \, dx$$

input
```
integrate((c*LambertW(a*x**2))**p/x,x)
```

output
```
Integral((c*LambertW(a*x**2))**p/x, x)
```

Maxima [F]

$$\int \frac{(cW(ax^2))^p}{x} \, dx = \int \frac{(c\,\mathrm{W}(ax^2))^p}{x} \, dx$$

input
```
integrate((c*lambert_w(a*x^2))^p/x,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^2))^p/x, x)
```

Giac [F]

$$\int \frac{(cW(ax^2))^p}{x} \, dx = \int \frac{(c\,\mathrm{W}(ax^2))^p}{x} \, dx$$

input
```
integrate((c*lambert_w(a*x^2))^p/x,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^2))^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cW(ax^2))^p}{x}\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x^2))^p}{x}\,dx$$

input
```
int((c*LambertW(a*x^2))^p/x,x)
```

output
```
int((c*LambertW(a*x^2))^p/x, x)
```

Reduce [F]

$$\int \frac{(cW(ax^2))^p}{x}\,dx = c^p\left(\int \frac{lambert\_w(a\,x^2)^p}{x}dx\right)$$

input
```
int((c*Lambert_W(a*x^2))^p/x,x)
```

output
```
c**p*int(lambert_w(a*x**2)**p/x,x)
```

3.183 $\int \frac{(cW(ax^2))^p}{x^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 108

$$\int \frac{(cW(ax^2))^p}{x^3} dx = -\frac{e^{2W(ax^2)}\Gamma(-1+p, W(ax^2))\, W(ax^2)^{2-p}\, (cW(ax^2))^p}{2ax^4} - \frac{e^{2W(ax^2)}\Gamma(p, W(ax^2))\, W(ax^2)^{1-p}\, (cW(ax^2))^{1+p}}{2acx^4}$$

output
```
-1/2*exp(2*LambertW(a*x^2))*GAMMA(-1+p,LambertW(a*x^2))*LambertW(a*x^2)^(2
-p)*(c*LambertW(a*x^2))^p/a/x^4-1/2*exp(2*LambertW(a*x^2))*GAMMA(p,Lambert
W(a*x^2))*LambertW(a*x^2)^(1-p)*(c*LambertW(a*x^2))^(p+1)/a/c/x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.56

$$\int \frac{(cW(ax^2))^p}{x^3} dx = -\frac{e^{2W(ax^2)}(\Gamma(-1+p, W(ax^2)) + \Gamma(p, W(ax^2)))\, W(ax^2)^{2-p}\, (cW(ax^2))^p}{2ax^4}$$

input
```
Integrate[(c*ProductLog[a*x^2])^p/x^3,x]
```

output
```
-1/2*(E^(2*ProductLog[a*x^2])*(Gamma[-1 + p, ProductLog[a*x^2]] + Gamma[p,
 ProductLog[a*x^2]])*ProductLog[a*x^2]^(2 - p)*(c*ProductLog[a*x^2])^p)/(a
*x^4)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {7271, 7283, 7174, 7207}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cW(ax^2))^p}{x^3} \, dx$$

$$\Big\downarrow \text{7271}$$

$$W(ax^2)^{-p}(cW(ax^2))^p \int \frac{W(ax^2)^p}{x^3} dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}W(ax^2)^{-p}(cW(ax^2))^p \int \frac{W(ax^2)^p}{x^4} dx^2$$

$$\Big\downarrow \text{7174}$$

$$\frac{1}{2}W(ax^2)^{-p}(cW(ax^2))^p \left(\int \frac{W(ax^2)^p}{x^4(W(ax^2)+1)} dx^2 + \int \frac{W(ax^2)^{p+1}}{x^4(W(ax^2)+1)} dx^2\right)$$

$$\Big\downarrow \text{7207}$$

$$\frac{1}{2}W(ax^2)^{-p}(cW(ax^2))^p \left(-\frac{e^{2W(ax^2)}W(ax^2)^2\Gamma(p-1,W(ax^2))}{ax^4} - \frac{e^{2W(ax^2)}W(ax^2)^2\Gamma(p,W(ax^2))}{ax^4}\right)$$

input
```
Int[(c*ProductLog[a*x^2])^p/x^3,x]
```

output
```
((c*ProductLog[a*x^2])^p*(-((E^(2*ProductLog[a*x^2])*Gamma[-1 + p, Product
Log[a*x^2]]*ProductLog[a*x^2]^2)/(a*x^4)) - (E^(2*ProductLog[a*x^2])*Gamma
[p, ProductLog[a*x^2]]*ProductLog[a*x^2]^2)/(a*x^4)))/(2*ProductLog[a*x^2]
^p)
```

Defintions of rubi rules used

rule 7174
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.), x_Symbol] :> Int[x^m*(
(c*ProductLog[a*x])^p/(1 + ProductLog[a*x])), x] + Simp[1/c   Int[x^m*((c*P
roductLog[a*x])^(p + 1)/(1 + ProductLog[a*x])), x], x] /; FreeQ[{a, c, m},
x]
```

rule 7207
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.))/((d_) + (d_.)*Product
Log[(a_.)*(x_)]), x_Symbol] :> Simp[x^m*Gamma[m + p + 1, (-(m + 1))*Product
Log[a*x]]*((c*ProductLog[a*x])^p/(a*d*(m + 1)*E^(m*ProductLog[a*x])*((-(m +
 1))*ProductLog[a*x])^(m + p))), x] /; FreeQ[{a, c, d, m, p}, x] && NeQ[m,
-1]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{(c\,\mathrm{LambertW}\,(a\,x^2))^p}{x^3}dx$$

input
```
int((c*LambertW(a*x^2))^p/x^3,x)
```

output
```
int((c*LambertW(a*x^2))^p/x^3,x)
```

Fricas [F]

$$\int \frac{(cW(ax^2))^p}{x^3}\,dx = \int \frac{(c\,\mathrm{W}(ax^2))^p}{x^3}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^p/x^3,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^2))^p/x^3, x)
```

Sympy [F]

$$\int \frac{(cW(ax^2))^p}{x^3}\,dx = \int \frac{(cW(ax^2))^p}{x^3}\,dx$$

input
```
integrate((c*LambertW(a*x**2))**p/x**3,x)
```

output
```
Integral((c*LambertW(a*x**2))**p/x**3, x)
```

Maxima [F]

$$\int \frac{(cW(ax^2))^p}{x^3}\,dx = \int \frac{(c\,\mathrm{W}(ax^2))^p}{x^3}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^2))^p/x^3, x)
```

Giac [F]

$$\int \frac{(cW(ax^2))^p}{x^3}\,dx = \int \frac{(c\,\mathrm{W}(ax^2))^p}{x^3}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^p/x^3,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^2))^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cW(ax^2))^p}{x^3}\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x^2))^p}{x^3}\,dx$$

input
```
int((c*LambertW(a*x^2))^p/x^3,x)
```

output
```
int((c*LambertW(a*x^2))^p/x^3, x)
```

Reduce [F]

$$\int \frac{(cW(ax^2))^p}{x^3}\,dx$$
$$= \frac{c^p\left(-lambert\_w(a\,x^2)^p + 2\left(\int \frac{lambert\_w(a\,x^2)^p}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)^2x+e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)x}dx\right)ap\,x^2\right)}{2x^2}$$

input
```
int((c*Lambert_W(a*x^2))^p/x^3,x)
```

output
```
(c**p*( - lambert_w(a*x**2)**p + 2*int(lambert_w(a*x**2)**p/(e**lambert_w(
a*x**2)*lambert_w(a*x**2)**2*x + e**lambert_w(a*x**2)*lambert_w(a*x**2)*x)
,x)*a*p*x**2))/(2*x**2)
```

3.184 $\int x^2(cW(ax^2))^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 156

$$\int x^2 (cW(ax^2))^p \, dx = \frac{2^{\frac{1}{2}+p}3^{-\frac{3}{2}-p}e^{-\frac{1}{2}W(ax^2)}x\Gamma\left(\frac{3}{2}+p,-\frac{3}{2}W(ax^2)\right)\left(-W(ax^2)\right)^{-\frac{1}{2}-p}(cW(ax^2))^p}{a} + \frac{2^{\frac{3}{2}+p}3^{-\frac{5}{2}-p}e^{-\frac{1}{2}W(ax^2)}x\Gamma\left(\frac{5}{2}+p,-\frac{3}{2}W(ax^2)\right)\left(-W(ax^2)\right)^{-\frac{3}{2}-p}(cW(ax^2))^{1+p}}{ac}$$

output
```
2^(1/2+p)*3^(-3/2-p)*x*GAMMA(3/2+p,-3/2*LambertW(a*x^2))*(-LambertW(a*x^2)
)^(-1/2-p)*(c*LambertW(a*x^2))^p/a/exp(1/2*LambertW(a*x^2))+2^(3/2+p)*3^(-
5/2-p)*x*GAMMA(5/2+p,-3/2*LambertW(a*x^2))*(-LambertW(a*x^2))^(-3/2-p)*(c*
LambertW(a*x^2))^(p+1)/a/c/exp(1/2*LambertW(a*x^2))
```

Mathematica [F]

$$\int x^2 \left(cW\left(ax^2\right)\right)^p \, dx = \int x^2 \left(cW\left(ax^2\right)\right)^p \, dx$$

input
```
Integrate[x^2*(c*ProductLog[a*x^2])^p,x]
```

output
```
Integrate[x^2*(c*ProductLog[a*x^2])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(cW\left(ax^2\right)\right)^p \, dx$$

$$\Big\downarrow \text{ 7271}$$

$$W\left(ax^2\right)^{-p} \left(cW\left(ax^2\right)\right)^p \int x^2 W\left(ax^2\right)^p dx$$

$$\Big\downarrow \text{ 7299}$$

$$W\left(ax^2\right)^{-p} \left(cW\left(ax^2\right)\right)^p \int x^2 W\left(ax^2\right)^p dx$$

input
```
Int[x^2*(c*ProductLog[a*x^2])^p,x]
```

output
```
$Aborted
```

Maple [F]

$$\int x^2 (c \operatorname{LambertW}(a x^2))^p dx$$

input
```
int(x^2*(c*LambertW(a*x^2))^p,x)
```

output
```
int(x^2*(c*LambertW(a*x^2))^p,x)
```

Fricas [F]

$$\int x^2 (cW(ax^2))^p \, dx = \int (c\,\mathrm{W}(ax^2))^p \, x^2 \, dx$$

input
```
integrate(x^2*(c*lambert_w(a*x^2))^p,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^2))^p*x^2, x)
```

Sympy [F]

$$\int x^2 (cW(ax^2))^p \, dx = \int x^2 (cW(ax^2))^p \, dx$$

input
```
integrate(x**2*(c*LambertW(a*x**2))**p,x)
```

output
```
Integral(x**2*(c*LambertW(a*x**2))**p, x)
```

Maxima [F]

$$\int x^2\left(cW\left(ax^2\right)\right)^p\,dx=\int \left(c\,\mathrm{W}(ax^2)\right)^p x^2\,dx$$

input
```
integrate(x^2*(c*lambert_w(a*x^2))^p,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^2))^p*x^2, x)
```

Giac [F]

$$\int x^2\left(cW\left(ax^2\right)\right)^p\,dx=\int \left(c\,\mathrm{W}(ax^2)\right)^p x^2\,dx$$

input
```
integrate(x^2*(c*lambert_w(a*x^2))^p,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^2))^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(cW\left(ax^2\right)\right)^p\,dx=\int x^2\,\left(c\,\mathrm{LambertW}\left(a\,x^2\right)\right)^p\,dx$$

input
```
int(x^2*(c*LambertW(a*x^2))^p,x)
```

output
```
int(x^2*(c*LambertW(a*x^2))^p, x)
```

Reduce [F]

$$\int x^2 (cW(ax^2))^p \, dx = c^p \left(\int lambert\_w(a\,x^2)^p \, x^2 dx \right)$$

input
```
int(x^2*(c*Lambert_W(a*x^2))^p,x)
```

output
```
c**p*int(lambert_w(a*x**2)**p*x**2,x)
```

3.185 $\int \frac{(cW(ax^2))^p}{x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 140

$$\int \frac{(cW(ax^2))^p}{x^2} dx = -\frac{2^{-\frac{3}{2}+p}e^{\frac{3}{2}W(ax^2)}\Gamma\left(-\frac{1}{2}+p,\frac{1}{2}W(ax^2)\right)W(ax^2)^{\frac{3}{2}-p}(cW(ax^2))^p}{ax^3} - \frac{2^{-\frac{1}{2}+p}e^{\frac{3}{2}W(ax^2)}\Gamma\left(\frac{1}{2}+p,\frac{1}{2}W(ax^2)\right)W(ax^2)^{\frac{1}{2}-p}(cW(ax^2))^{1+p}}{acx^3}$$

output
```
-2^(-3/2+p)*exp(3/2*LambertW(a*x^2))*GAMMA(-1/2+p,1/2*LambertW(a*x^2))*Lam
bertW(a*x^2)^(3/2-p)*(c*LambertW(a*x^2))^p/a/x^3-2^(-1/2+p)*exp(3/2*Lamber
tW(a*x^2))*GAMMA(1/2+p,1/2*LambertW(a*x^2))*LambertW(a*x^2)^(1/2-p)*(c*Lam
bertW(a*x^2))^(p+1)/a/c/x^3
```

Mathematica [F]

$$\int \frac{(cW(ax^2))^p}{x^2} dx = \int \frac{(cW(ax^2))^p}{x^2} dx$$

input
```
Integrate[(c*ProductLog[a*x^2])^p/x^2,x]
```

output
```
Integrate[(c*ProductLog[a*x^2])^p/x^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cW(ax^2))^p}{x^2}\,dx$$
$$\Big\downarrow \text{ 7271}$$
$$W(ax^2)^{-p}(cW(ax^2))^p\int \frac{W(ax^2)^p}{x^2}dx$$
$$\Big\downarrow \text{ 7299}$$
$$W(ax^2)^{-p}(cW(ax^2))^p\int \frac{W(ax^2)^p}{x^2}dx$$

input
```
Int[(c*ProductLog[a*x^2])^p/x^2,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(c\,\mathrm{LambertW}(a\,x^2))^p}{x^2}dx$$

input
```
int((c*LambertW(a*x^2))^p/x^2,x)
```

output
```
int((c*LambertW(a*x^2))^p/x^2,x)
```

Fricas [F]

$$\int \frac{(cW(ax^2))^p}{x^2} \, dx = \int \frac{(c\,\mathrm{W}(ax^2))^p}{x^2} \, dx$$

input
```
integrate((c*lambert_w(a*x^2))^p/x^2,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^2))^p/x^2, x)
```

Sympy [F]

$$\int \frac{(cW(ax^2))^p}{x^2} \, dx = \int \frac{(cW(ax^2))^p}{x^2} \, dx$$

input
```
integrate((c*LambertW(a*x**2))**p/x**2,x)
```

output
```
Integral((c*LambertW(a*x**2))**p/x**2, x)
```

Maxima [F]

$$\int \frac{(cW(ax^2))^p}{x^2} \, dx = \int \frac{(c\,\mathrm{W}(ax^2))^p}{x^2} \, dx$$

input
```
integrate((c*lambert_w(a*x^2))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^2))^p/x^2, x)
```

Giac [F]

$$\int \frac{(cW(ax^2))^p}{x^2}\,dx = \int \frac{(c\,\mathrm{W}(ax^2))^p}{x^2}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^p/x^2,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^2))^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cW(ax^2))^p}{x^2}\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x^2))^p}{x^2}\,dx$$

input
```
int((c*LambertW(a*x^2))^p/x^2,x)
```

output
```
int((c*LambertW(a*x^2))^p/x^2, x)
```

Reduce [F]

$$\int \frac{(cW(ax^2))^p}{x^2}\,dx = \frac{c^p\left(-lambert\_w(a\,x^2)^p+2\left(\int \frac{lambert\_w(a\,x^2)^p}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)^2+e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)}dx\right)apx\right)}{x}$$

input
```
int((c*Lambert_W(a*x^2))^p/x^2,x)
```

output
```
(c**p*( - lambert_w(a*x**2)**p + 2*int(lambert_w(a*x**2)**p/(e**lambert_w(
a*x**2)*lambert_w(a*x**2)**2 + e**lambert_w(a*x**2)*lambert_w(a*x**2)),x)*
a*p*x))/x
```

3.186 $\int \frac{(cW(ax^2))^p}{x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 158

$$\int \frac{(cW(ax^2))^p}{x^4} \, dx = -\frac{2^{-\frac{5}{2}+p}3^{\frac{3}{2}-p}e^{\frac{5}{2}W(ax^2)}\Gamma\left(-\frac{3}{2}+p,\frac{3}{2}W(ax^2)\right)W(ax^2)^{\frac{5}{2}-p}(cW(ax^2))^p}{ax^5} - \frac{2^{-\frac{3}{2}+p}3^{\frac{1}{2}-p}e^{\frac{5}{2}W(ax^2)}\Gamma\left(-\frac{1}{2}+p,\frac{3}{2}W(ax^2)\right)W(ax^2)^{\frac{3}{2}-p}(cW(ax^2))^{1+p}}{acx^5}$$

output
```
-2^(-5/2+p)*3^(3/2-p)*exp(5/2*LambertW(a*x^2))*GAMMA(-3/2+p,3/2*LambertW(a
*x^2))*LambertW(a*x^2)^(5/2-p)*(c*LambertW(a*x^2))^p/a/x^5-2^(-3/2+p)*3^(1
/2-p)*exp(5/2*LambertW(a*x^2))*GAMMA(-1/2+p,3/2*LambertW(a*x^2))*LambertW(
a*x^2)^(3/2-p)*(c*LambertW(a*x^2))^(p+1)/a/c/x^5
```

Mathematica [F]

$$\int \frac{(cW(ax^2))^p}{x^4}\,dx = \int \frac{(cW(ax^2))^p}{x^4}\,dx$$

input
```
Integrate[(c*ProductLog[a*x^2])^p/x^4,x]
```

output
```
Integrate[(c*ProductLog[a*x^2])^p/x^4, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cW(ax^2))^p}{x^4}\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$W(ax^2)^{-p}(cW(ax^2))^p \int \frac{W(ax^2)^p}{x^4}dx$$

$$\Big\downarrow \text{ 7299}$$

$$W(ax^2)^{-p}(cW(ax^2))^p \int \frac{W(ax^2)^p}{x^4}dx$$

input
```
Int[(c*ProductLog[a*x^2])^p/x^4,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(c\,\mathrm{LambertW}\,(a\,x^2))^p}{x^4}dx$$

input
```
int((c*LambertW(a*x^2))^p/x^4,x)
```

output
```
int((c*LambertW(a*x^2))^p/x^4,x)
```

Fricas [F]

$$\int \frac{(cW(ax^2))^p}{x^4}\,dx = \int \frac{(c\,\mathrm{W}(ax^2))^p}{x^4}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^p/x^4,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^2))^p/x^4, x)
```

Sympy [F]

$$\int \frac{(cW(ax^2))^p}{x^4}\,dx = \int \frac{(cW(ax^2))^p}{x^4}\,dx$$

input
```
integrate((c*LambertW(a*x**2))**p/x**4,x)
```

output
```
Integral((c*LambertW(a*x**2))**p/x**4, x)
```

Maxima [F]

$$\int \frac{(cW(ax^2))^p}{x^4}\,dx = \int \frac{(c\,\mathrm{W}(ax^2))^p}{x^4}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^p/x^4,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^2))^p/x^4, x)
```

Giac [F]

$$\int \frac{(cW(ax^2))^p}{x^4}\,dx = \int \frac{(c\,\mathrm{W}(ax^2))^p}{x^4}\,dx$$

input
```
integrate((c*lambert_w(a*x^2))^p/x^4,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^2))^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cW(ax^2))^p}{x^4}\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x^2))^p}{x^4}\,dx$$

input
```
int((c*LambertW(a*x^2))^p/x^4,x)
```

output
```
int((c*LambertW(a*x^2))^p/x^4, x)
```

Reduce [F]

$$\int \frac{(cW(ax^2))^p}{x^4}\,dx$$
$$= \frac{c^p\left(-lambert\_w(a\,x^2)^p + 2\left(\int \frac{lambert\_w(a\,x^2)^p}{e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)^2 x^2 + e^{lambert\_w(a\,x^2)}\,lambert\_w(a\,x^2)\,x^2}dx\right) ap\,x^3\right)}{3x^3}$$

input
```
int((c*Lambert_W(a*x^2))^p/x^4,x)
```

output
```
(c**p*( - lambert_w(a*x**2)**p + 2*int(lambert_w(a*x**2)**p/(e**lambert_w(
a*x**2)*lambert_w(a*x**2)**2*x**2 + e**lambert_w(a*x**2)*lambert_w(a*x**2)
*x**2),x)*a*p*x**3))/(3*x**3)
```

3.187 $\int x^4 W\left(\frac{a}{x}\right) dx$

Optimal result

Integrand size = 10, antiderivative size = 75

$$\int x^4 W\left(\frac{a}{x}\right) dx = -\frac{125}{24} a^5 \operatorname{ExpIntegralEi}\left(-5W\left(\frac{a}{x}\right)\right) + \frac{1}{4} x^5 W\left(\frac{a}{x}\right) - \frac{1}{12} x^5 W\left(\frac{a}{x}\right)^2 + \frac{5}{24} x^5 W\left(\frac{a}{x}\right)^3 - \frac{25}{24} x^5 W\left(\frac{a}{x}\right)^4$$

output
```
-125/24*a^5*Ei(-5*LambertW(a/x))+1/4*x^5*LambertW(a/x)-1/12*x^5*LambertW(a
/x)^2+5/24*x^5*LambertW(a/x)^3-25/24*x^5*LambertW(a/x)^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x^4 W\left(\frac{a}{x}\right) dx = -\frac{125}{24} a^5 \operatorname{ExpIntegralEi}\left(-5W\left(\frac{a}{x}\right)\right) + \frac{1}{4} x^5 W\left(\frac{a}{x}\right) - \frac{1}{12} x^5 W\left(\frac{a}{x}\right)^2 + \frac{5}{24} x^5 W\left(\frac{a}{x}\right)^3 - \frac{25}{24} x^5 W\left(\frac{a}{x}\right)^4$$

input
```
Integrate[x^4*ProductLog[a/x],x]
```

output
```
(-125*a^5*ExpIntegralEi[-5*ProductLog[a/x]])/24 + (x^5*ProductLog[a/x])/4
- (x^5*ProductLog[a/x]^2)/12 + (5*x^5*ProductLog[a/x]^3)/24 - (25*x^5*Prod
uctLog[a/x]^4)/24
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {7173, 7206, 7206, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 W\left(\frac{a}{x}\right) \, dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{1}{4}x^5 W\left(\frac{a}{x}\right)-\frac{1}{4}\int \frac{x^4 W\left(\frac{a}{x}\right)^2}{W\left(\frac{a}{x}\right)+1}dx$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{4}\left(\frac{5}{3}\int \frac{x^4 W\left(\frac{a}{x}\right)^3}{W\left(\frac{a}{x}\right)+1}dx-\frac{1}{3}x^5 W\left(\frac{a}{x}\right)^2\right)+\frac{1}{4}x^5 W\left(\frac{a}{x}\right)$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{4}\left(\frac{5}{3}\left(\frac{1}{2}x^5 W\left(\frac{a}{x}\right)^3-\frac{5}{2}\int \frac{x^4 W\left(\frac{a}{x}\right)^4}{W\left(\frac{a}{x}\right)+1}dx\right)-\frac{1}{3}x^5 W\left(\frac{a}{x}\right)^2\right)+\frac{1}{4}x^5 W\left(\frac{a}{x}\right)$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{4}\left(\frac{5}{3}\left(\frac{1}{2}x^5 W\left(\frac{a}{x}\right)^3-\frac{5}{2}\left(x^5 W\left(\frac{a}{x}\right)^4-5\int \frac{x^4 W\left(\frac{a}{x}\right)^5}{W\left(\frac{a}{x}\right)+1}dx\right)\right)-\frac{1}{3}x^5 W\left(\frac{a}{x}\right)^2\right)+\frac{1}{4}x^5 W\left(\frac{a}{x}\right)$$

$$\Big\downarrow \text{7202}$$

$$\frac{1}{4}\left(\frac{5}{3}\left(\frac{1}{2}x^5 W\left(\frac{a}{x}\right)^3-\frac{5}{2}\left(5a^5 \operatorname{ExpIntegralEi}\left(-5W\left(\frac{a}{x}\right)\right)+x^5 W\left(\frac{a}{x}\right)^4\right)\right)-\frac{1}{3}x^5 W\left(\frac{a}{x}\right)^2\right)+\frac{1}{4}x^5 W\left(\frac{a}{x}\right)$$

input
```
Int[x^4*ProductLog[a/x],x]
```

output
```
(x^5*ProductLog[a/x])/4 + (-1/3*(x^5*ProductLog[a/x]^2) + (5*((x^5*Product
Log[a/x]^3)/2 - (5*(5*a^5*ExpIntegralEi[-5*ProductLog[a/x]] + x^5*ProductL
og[a/x]^4))/2))/3)/4
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-a^5\left(-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)x^5}{4a^5}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^2x^5}{12a^5}-\frac{5\,\mathrm{LambertW}\left(\frac{a}{x}\right)^3x^5}{24a^5}+\frac{25\,\mathrm{LambertW}\left(\frac{a}{x}\right)^4x^5}{24a^5}-\frac{125\ \mathrm{expI}}{}\right.$
default	$-a^5\left(-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)x^5}{4a^5}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^2x^5}{12a^5}-\frac{5\,\mathrm{LambertW}\left(\frac{a}{x}\right)^3x^5}{24a^5}+\frac{25\,\mathrm{LambertW}\left(\frac{a}{x}\right)^4x^5}{24a^5}-\frac{125\ \mathrm{expI}}{}\right.$

input
```
int(x^4*LambertW(a/x),x,method=_RETURNVERBOSE)
```

output
```
-a^5*(-1/4*LambertW(a/x)*x^5/a^5+1/12*LambertW(a/x)^2*x^5/a^5-5/24*Lambert
W(a/x)^3*x^5/a^5+25/24*LambertW(a/x)^4*x^5/a^5-125/24*Ei(1,5*LambertW(a/x)
))
```

Fricas [F]

$$\int x^4W\left(\frac{a}{x}\right)\,dx=\int x^4\,\mathrm{W}\left(\frac{a}{x}\right)dx$$

input
```
integrate(x^4*lambert_w(a/x),x, algorithm="fricas")
```

output
```
integral(x^4*lambert_w(a/x), x)
```

Sympy [F]

$$\int x^4W\left(\frac{a}{x}\right)\,dx=\int x^4W\left(\frac{a}{x}\right)\,dx$$

input
```
integrate(x**4*LambertW(a/x),x)
```

output
```
Integral(x**4*LambertW(a/x), x)
```

Maxima [F]

$$\int x^4 W\left(\frac{a}{x}\right)\,dx = \int x^4\,\mathrm{W}(\frac{a}{x})\,dx$$

input
```
integrate(x^4*lambert_w(a/x),x, algorithm="maxima")
```

output
```
integrate(x^4*lambert_w(a/x), x)
```

Giac [F]

$$\int x^4 W\left(\frac{a}{x}\right)\,dx = \int x^4\,\mathrm{W}(\frac{a}{x})\,dx$$

input
```
integrate(x^4*lambert_w(a/x),x, algorithm="giac")
```

output
```
integrate(x^4*lambert_w(a/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 W\left(\frac{a}{x}\right)\,dx = \int x^4\,\mathrm{LambertW}\left(\frac{a}{x}\right)\,dx$$

input
```
int(x^4*LambertW(a/x),x)
```

output
```
int(x^4*LambertW(a/x), x)
```

Reduce [F]

$$\int x^4 W\left(\frac{a}{x}\right) dx = \frac{\left(\int \frac{x^3}{e^{lambert\_w\left(\frac{a}{x}\right)} lambert\_w\left(\frac{a}{x}\right)+e^{lambert\_w\left(\frac{a}{x}\right)}}dx\right) a}{5} + \frac{lambert\_w\left(\frac{a}{x}\right) x^5}{5}$$

input
```
int(x^4*Lambert_W(a/x),x)
```

output
```
(int(x**3/(e**lambert_w(a/x)*lambert_w(a/x) + e**lambert_w(a/x)),x)*a + la
mbert_w(a/x)*x**5)/5
```

3.188 $\int x^3 W\left(\frac{a}{x}\right)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int x^3 W\left(\frac{a}{x}\right)\, dx = \frac{8}{3}a^4 \operatorname{ExpIntegralEi}\left(-4W\left(\frac{a}{x}\right)\right) + \frac{1}{3}x^4 W\left(\frac{a}{x}\right) - \frac{1}{6}x^4 W\left(\frac{a}{x}\right)^2 + \frac{2}{3}x^4 W\left(\frac{a}{x}\right)^3$$

output
```
8/3*a^4*Ei(-4*LambertW(a/x))+1/3*x^4*LambertW(a/x)-1/6*x^4*LambertW(a/x)^2
+2/3*x^4*LambertW(a/x)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int x^3 W\left(\frac{a}{x}\right)\, dx = \frac{8}{3}a^4 \operatorname{ExpIntegralEi}\left(-4W\left(\frac{a}{x}\right)\right) + \frac{1}{3}x^4 W\left(\frac{a}{x}\right) - \frac{1}{6}x^4 W\left(\frac{a}{x}\right)^2 + \frac{2}{3}x^4 W\left(\frac{a}{x}\right)^3$$

input
```
Integrate[x^3*ProductLog[a/x],x]
```

output
```
(8*a^4*ExpIntegralEi[-4*ProductLog[a/x]])/3 + (x^4*ProductLog[a/x])/3 - (x
^4*ProductLog[a/x]^2)/6 + (2*x^4*ProductLog[a/x]^3)/3
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {7173, 7206, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 W\left(\frac{a}{x}\right) dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{1}{3}x^4 W\left(\frac{a}{x}\right) - \frac{1}{3}\int \frac{x^3 W\left(\frac{a}{x}\right)^2}{W\left(\frac{a}{x}\right)+1}dx$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{3}\left(2\int \frac{x^3 W\left(\frac{a}{x}\right)^3}{W\left(\frac{a}{x}\right)+1}dx - \frac{1}{2}x^4 W\left(\frac{a}{x}\right)^2\right) + \frac{1}{3}x^4 W\left(\frac{a}{x}\right)$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{3}\left(2\left(x^4 W\left(\frac{a}{x}\right)^3 - 4\int \frac{x^3 W\left(\frac{a}{x}\right)^4}{W\left(\frac{a}{x}\right)+1}dx\right) - \frac{1}{2}x^4 W\left(\frac{a}{x}\right)^2\right) + \frac{1}{3}x^4 W\left(\frac{a}{x}\right)$$

$$\Big\downarrow \text{7202}$$

$$\frac{1}{3}\left(2\left(4a^4 \text{ExpIntegralEi}\left(-4W\left(\frac{a}{x}\right)\right) + x^4 W\left(\frac{a}{x}\right)^3\right) - \frac{1}{2}x^4 W\left(\frac{a}{x}\right)^2\right) + \frac{1}{3}x^4 W\left(\frac{a}{x}\right)$$

input
```
Int[x^3*ProductLog[a/x],x]
```

output
```
(x^4*ProductLog[a/x])/3 + (-1/2*(x^4*ProductLog[a/x]^2) + 2*(4*a^4*ExpInte
gralEi[-4*ProductLog[a/x]] + x^4*ProductLog[a/x]^3))/3
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-a^4\left(-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)x^4}{3a^4}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^2x^4}{6a^4}-\frac{2\,\mathrm{LambertW}\left(\frac{a}{x}\right)^3x^4}{3a^4}+\frac{8\,\mathrm{expIntegral}_1\left(4\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)}{3}\right.$
default	$-a^4\left(-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)x^4}{3a^4}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^2x^4}{6a^4}-\frac{2\,\mathrm{LambertW}\left(\frac{a}{x}\right)^3x^4}{3a^4}+\frac{8\,\mathrm{expIntegral}_1\left(4\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)}{3}\right.$

input
```
int(x^3*LambertW(a/x),x,method=_RETURNVERBOSE)
```

output
```
-a^4*(-1/3*LambertW(a/x)*x^4/a^4+1/6*LambertW(a/x)^2*x^4/a^4-2/3*LambertW(
a/x)^3*x^4/a^4+8/3*Ei(1,4*LambertW(a/x)))
```

Fricas [F]

$$\int x^3 W\left(\frac{a}{x}\right) dx = \int x^3 \operatorname{W}\left(\frac{a}{x}\right) dx$$

input
```
integrate(x^3*lambert_w(a/x),x, algorithm="fricas")
```

output
```
integral(x^3*lambert_w(a/x), x)
```

Sympy [F]

$$\int x^3 W\left(\frac{a}{x}\right) dx = \int x^3 W\left(\frac{a}{x}\right) dx$$

input
```
integrate(x**3*LambertW(a/x),x)
```

output
```
Integral(x**3*LambertW(a/x), x)
```

Maxima [F]

$$\int x^3 W\left(\frac{a}{x}\right) dx = \int x^3 \operatorname{W}\left(\frac{a}{x}\right) dx$$

input
```
integrate(x^3*lambert_w(a/x),x, algorithm="maxima")
```

output
```
integrate(x^3*lambert_w(a/x), x)
```

Giac [F]

$$\int x^3W\left(\frac{a}{x}\right)\,dx=\int x^3\,\mathrm{W}(\frac{a}{x})\,dx$$

input
```
integrate(x^3*lambert_w(a/x),x, algorithm="giac")
```

output
```
integrate(x^3*lambert_w(a/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3W\left(\frac{a}{x}\right)\,dx=\int x^3\,\mathrm{LambertW}\left(\frac{a}{x}\right)\,dx$$

input
```
int(x^3*LambertW(a/x),x)
```

output
```
int(x^3*LambertW(a/x), x)
```

Reduce [F]

$$\int x^3W\left(\frac{a}{x}\right)\,dx=\frac{\left(\int\frac{x^2}{e^{lambert\_w\left(\frac{a}{x}\right)}lambert\_w\left(\frac{a}{x}\right)+e^{lambert\_w\left(\frac{a}{x}\right)}}dx\right)a}{4}+\frac{lambert\_w\left(\frac{a}{x}\right)x^4}{4}$$

input
```
int(x^3*Lambert_W(a/x),x)
```

output
```
(int(x**2/(e**lambert_w(a/x)*lambert_w(a/x) + e**lambert_w(a/x)),x)*a + la
mbert_w(a/x)*x**4)/4
```

3.189 $\int x^2 W\left(\frac{a}{x}\right)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 45

$$\int x^2 W\left(\frac{a}{x}\right)\,dx=-\frac{3}{2}a^3\,\text{ExpIntegralEi}\left(-3W\left(\frac{a}{x}\right)\right)+\frac{1}{2}x^3W\left(\frac{a}{x}\right)-\frac{1}{2}x^3W\left(\frac{a}{x}\right)^2$$

output
```
-3/2*a^3*Ei(-3*LambertW(a/x))+1/2*x^3*LambertW(a/x)-1/2*x^3*LambertW(a/x)^
2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int x^2 W\left(\frac{a}{x}\right)\,dx=-\frac{3}{2}a^3\,\text{ExpIntegralEi}\left(-3W\left(\frac{a}{x}\right)\right)+\frac{1}{2}x^3W\left(\frac{a}{x}\right)-\frac{1}{2}x^3W\left(\frac{a}{x}\right)^2$$

input
```
Integrate[x^2*ProductLog[a/x],x]
```

output
```
(-3*a^3*ExpIntegralEi[-3*ProductLog[a/x]])/2 + (x^3*ProductLog[a/x])/2 - (
x^3*ProductLog[a/x]^2)/2
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7173, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 W\left(\frac{a}{x}\right) \, dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{1}{2}x^3 W\left(\frac{a}{x}\right)-\frac{1}{2}\int \frac{x^2 W\left(\frac{a}{x}\right)^2}{W\left(\frac{a}{x}\right)+1}dx$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{2}\left(3\int \frac{x^2 W\left(\frac{a}{x}\right)^3}{W\left(\frac{a}{x}\right)+1}dx-x^3 W\left(\frac{a}{x}\right)^2\right)+\frac{1}{2}x^3 W\left(\frac{a}{x}\right)$$

$$\Big\downarrow \text{7202}$$

$$\frac{1}{2}\left(-3a^3 \text{ExpIntegralEi}\left(-3W\left(\frac{a}{x}\right)\right)-x^3 W\left(\frac{a}{x}\right)^2\right)+\frac{1}{2}x^3 W\left(\frac{a}{x}\right)$$

input
```
Int[x^2*ProductLog[a/x],x]
```

output
```
(x^3*ProductLog[a/x])/2 + (-3*a^3*ExpIntegralEi[-3*ProductLog[a/x]] - x^3*
ProductLog[a/x]^2)/2
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-a^{3}\left(-\frac{\operatorname{LambertW}\left(\frac{a}{x}\right) x^{3}}{2 a^{3}}+\frac{\operatorname{LambertW}\left(\frac{a}{x}\right)^{2} x^{3}}{2 a^{3}}-\frac{3 \operatorname{expIntegral}_{1}\left(3 \operatorname{LambertW}\left(\frac{a}{x}\right)\right)}{2}\right)$	49
default	$-a^{3}\left(-\frac{\operatorname{LambertW}\left(\frac{a}{x}\right) x^{3}}{2 a^{3}}+\frac{\operatorname{LambertW}\left(\frac{a}{x}\right)^{2} x^{3}}{2 a^{3}}-\frac{3 \operatorname{expIntegral}_{1}\left(3 \operatorname{LambertW}\left(\frac{a}{x}\right)\right)}{2}\right)$	49

input
```
int(x^2*LambertW(a/x),x,method=_RETURNVERBOSE)
```

output
```
-a^3*(-1/2*LambertW(a/x)*x^3/a^3+1/2*LambertW(a/x)^2*x^3/a^3-3/2*Ei(1,3*La
mbertW(a/x)))
```

Fricas [F]

$$\int x^2 W\left(\frac{a}{x}\right)\,dx=\int x^2\,\mathrm{W}\left(\frac{a}{x}\right)dx$$

input
```
integrate(x^2*lambert_w(a/x),x, algorithm="fricas")
```

output
```
integral(x^2*lambert_w(a/x), x)
```

Sympy [F]

$$\int x^2 W\left(\frac{a}{x}\right)\, dx = \int x^2 W\left(\frac{a}{x}\right)\, dx$$

input
```
integrate(x**2*LambertW(a/x),x)
```

output
```
Integral(x**2*LambertW(a/x), x)
```

Maxima [F]

$$\int x^2 W\left(\frac{a}{x}\right)\, dx = \int x^2\, \mathrm{W}\left(\frac{a}{x}\right) dx$$

input
```
integrate(x^2*lambert_w(a/x),x, algorithm="maxima")
```

output
```
integrate(x^2*lambert_w(a/x), x)
```

Giac [F]

$$\int x^2 W\left(\frac{a}{x}\right)\, dx = \int x^2\, \mathrm{W}\left(\frac{a}{x}\right) dx$$

input
```
integrate(x^2*lambert_w(a/x),x, algorithm="giac")
```

output
```
integrate(x^2*lambert_w(a/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2W\left(\frac{a}{x}\right)\,dx=\int x^2\,\mathrm{LambertW}\left(\frac{a}{x}\right)\,dx$$

input
```
int(x^2*LambertW(a/x),x)
```

output
```
int(x^2*LambertW(a/x), x)
```

Reduce [F]

$$\int x^2W\left(\frac{a}{x}\right)\,dx=\frac{\left(\int\frac{x}{e^{lambert\_w\left(\frac{a}{x}\right)}lambert\_w\left(\frac{a}{x}\right)+e^{lambert\_w\left(\frac{a}{x}\right)}}dx\right)a}{3}+\frac{lambert\_w\left(\frac{a}{x}\right)x^3}{3}$$

input
```
int(x^2*Lambert_W(a/x),x)
```

output
```
(int(x/(e**lambert_w(a/x)*lambert_w(a/x) + e**lambert_w(a/x)),x)*a + lambe
rt_w(a/x)*x**3)/3
```

3.190 $\int xW\left(\frac{a}{x}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 24

$$\int xW\left(\frac{a}{x}\right)\,dx=a^2\,\text{ExpIntegralEi}\left(-2W\left(\frac{a}{x}\right)\right)+x^2W\left(\frac{a}{x}\right)$$

output
```
a^2*Ei(-2*LambertW(a/x))+x^2*LambertW(a/x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int xW\left(\frac{a}{x}\right)\,dx=a^2\,\text{ExpIntegralEi}\left(-2W\left(\frac{a}{x}\right)\right)+x^2W\left(\frac{a}{x}\right)$$

input
```
Integrate[x*ProductLog[a/x],x]
```

output
```
a^2*ExpIntegralEi[-2*ProductLog[a/x]] + x^2*ProductLog[a/x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7173, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xW\left(\frac{a}{x}\right) dx$$
$$\Big\downarrow \text{ 7173}$$
$$x^2W\left(\frac{a}{x}\right) - \int \frac{xW\left(\frac{a}{x}\right)^2}{W\left(\frac{a}{x}\right)+1}dx$$
$$\Big\downarrow \text{ 7202}$$
$$a^2 \text{ExpIntegralEi}\left(-2W\left(\frac{a}{x}\right)\right) + x^2W\left(\frac{a}{x}\right)$$

input
```
Int[x*ProductLog[a/x],x]
```

output
```
a^2*ExpIntegralEi[-2*ProductLog[a/x]] + x^2*ProductLog[a/x]
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

method	result	size
derivativedivides	$-a^{2}\left(-\frac{\operatorname{LambertW}\left(\frac{a}{x}\right) x^{2}}{a^{2}}+\operatorname{expIntegral}_{1}\left(2 \operatorname{LambertW}\left(\frac{a}{x}\right)\right)\right)$	31
default	$-a^{2}\left(-\frac{\operatorname{LambertW}\left(\frac{a}{x}\right) x^{2}}{a^{2}}+\operatorname{expIntegral}_{1}\left(2 \operatorname{LambertW}\left(\frac{a}{x}\right)\right)\right)$	31

input
```
int(x*LambertW(a/x),x,method=_RETURNVERBOSE)
```

output
```
-a^2*(-LambertW(a/x)*x^2/a^2+Ei(1,2*LambertW(a/x)))
```

Fricas [F]

$$\int xW\left(\frac{a}{x}\right)\,dx=\int x\,\mathrm{W}(\frac{a}{x})\,dx$$

input
```
integrate(x*lambert_w(a/x),x, algorithm="fricas")
```

output
```
integral(x*lambert_w(a/x), x)
```

Sympy [F]

$$\int xW\left(\frac{a}{x}\right)\,dx=\int xW\left(\frac{a}{x}\right)\,dx$$

input
```
integrate(x*LambertW(a/x),x)
```

output
```
Integral(x*LambertW(a/x), x)
```

Maxima [F]

$$\int xW\left(\frac{a}{x}\right)\,dx = \int x\,\mathrm{W}\left(\frac{a}{x}\right)dx$$

input
```
integrate(x*lambert_w(a/x),x, algorithm="maxima")
```

output
```
integrate(x*lambert_w(a/x), x)
```

Giac [F]

$$\int xW\left(\frac{a}{x}\right)\,dx = \int x\,\mathrm{W}\left(\frac{a}{x}\right)dx$$

input
```
integrate(x*lambert_w(a/x),x, algorithm="giac")
```

output
```
integrate(x*lambert_w(a/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int xW\left(\frac{a}{x}\right)\,dx = \int x\,\mathrm{LambertW}\left(\frac{a}{x}\right)\,dx$$

input
```
int(x*LambertW(a/x),x)
```

output
```
int(x*LambertW(a/x), x)
```

Reduce [F]

$$\int xW\left(\frac{a}{x}\right)\,dx=\frac{\left(\int \frac{1}{e^{lambert\_w\left(\frac{a}{x}\right)}lambert\_w\left(\frac{a}{x}\right)+e^{lambert\_w\left(\frac{a}{x}\right)}}dx\right)a}{2}+\frac{lambert\_w\left(\frac{a}{x}\right)x^{2}}{2}$$

input
```
int(x*Lambert_W(a/x),x)
```

output
```
(int(1/(e**lambert_w(a/x)*lambert_w(a/x) + e**lambert_w(a/x)),x)*a + lambe
rt_w(a/x)*x**2)/2
```

3.191 $\int W\left(\frac{a}{x}\right)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 21

$$\int W\left(\frac{a}{x}\right)\,dx = -a\,\text{ExpIntegralEi}\left(-W\left(\frac{a}{x}\right)\right) + xW\left(\frac{a}{x}\right)$$

output
```
-a*Ei(-LambertW(a/x))+x*LambertW(a/x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int W\left(\frac{a}{x}\right)\,dx = -a\,\text{ExpIntegralEi}\left(-W\left(\frac{a}{x}\right)\right) + xW\left(\frac{a}{x}\right)$$

input
```
Integrate[ProductLog[a/x],x]
```

output
```
-(a*ExpIntegralEi[-ProductLog[a/x]]) + x*ProductLog[a/x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7171, 7172, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W\left(\frac{a}{x}\right)dx$$

$$\Big\downarrow \text{7171}$$

$$-\int x^2 W\left(\frac{a}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{7172}$$

$$xW\left(\frac{a}{x}\right)-\int \frac{x^2 W\left(\frac{a}{x}\right)}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}$$

$$\Big\downarrow \text{7202}$$

$$xW\left(\frac{a}{x}\right)-a\,\text{ExpIntegralEi}\left(-W\left(\frac{a}{x}\right)\right)$$

input
```
Int[ProductLog[a/x],x]
```

output
```
-(a*ExpIntegralEi[-ProductLog[a/x]]) + x*ProductLog[a/x]
```

Defintions of rubi rules used

rule 7171
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subst[Int[(c*P
roductLog[a/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, c, p}, x] && ILtQ[n, 0]
```

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

method	result	size
derivativedivides	$-a\left(-\frac{x\,\mathrm{LambertW}\left(\frac{a}{x}\right)}{a}-\mathrm{expIntegral}_1\left(\mathrm{LambertW}\left(\frac{a}{x}\right)\right)\right)$	27
default	$-a\left(-\frac{x\,\mathrm{LambertW}\left(\frac{a}{x}\right)}{a}-\mathrm{expIntegral}_1\left(\mathrm{LambertW}\left(\frac{a}{x}\right)\right)\right)$	27

input
```
int(LambertW(a/x),x,method=_RETURNVERBOSE)
```

output
```
-a*(-x/a*LambertW(a/x)-Ei(1,LambertW(a/x)))
```

Fricas [F]

$$\int W\left(\frac{a}{x}\right)\,dx=\int \mathrm{W}\left(\frac{a}{x}\right)dx$$

input
```
integrate(lambert_w(a/x),x, algorithm="fricas")
```

output
```
integral(lambert_w(a/x), x)
```

Sympy [F]

$$\int W\left(\frac{a}{x}\right)\,dx = \int W\left(\frac{a}{x}\right)\,dx$$

input
```
integrate(LambertW(a/x),x)
```

output
```
Integral(LambertW(a/x), x)
```

Maxima [F]

$$\int W\left(\frac{a}{x}\right)\,dx = \int \mathrm{W}(\frac{a}{x})\,dx$$

input
```
integrate(lambert_w(a/x),x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x), x)
```

Giac [F]

$$\int W\left(\frac{a}{x}\right)\,dx = \int \mathrm{W}(\frac{a}{x})\,dx$$

input
```
integrate(lambert_w(a/x),x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int W\left(\frac{a}{x}\right)\,dx=\int \mathrm{LambertW}\left(\frac{a}{x}\right)\,dx$$

input
```
int(LambertW(a/x),x)
```

output
```
int(LambertW(a/x), x)
```

Reduce [F]

$$\int W\left(\frac{a}{x}\right)\,dx=\left(\int \frac{1}{e^{lambert\_w\left(\frac{a}{x}\right)}lambert\_w\left(\frac{a}{x}\right)x+e^{lambert\_w\left(\frac{a}{x}\right)}x}dx\right)a+lambert\_w\left(\frac{a}{x}\right)x$$

input
```
int(Lambert_W(a/x),x)
```

output
```
int(1/(e**lambert_w(a/x)*lambert_w(a/x)*x + e**lambert_w(a/x)*x),x)*a + la
mbert_w(a/x)*x
```

3.192 $\int \frac{W\left(\frac{a}{x}\right)}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \frac{W\left(\frac{a}{x}\right)}{x}\,dx = -W\left(\frac{a}{x}\right) - \frac{1}{2}W\left(\frac{a}{x}\right)^2$$

output
```
-LambertW(a/x)-1/2*LambertW(a/x)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{W\left(\frac{a}{x}\right)}{x}\,dx = -W\left(\frac{a}{x}\right) - \frac{1}{2}W\left(\frac{a}{x}\right)^2$$

input
```
Integrate[ProductLog[a/x]/x,x]
```

output
```
-ProductLog[a/x] - ProductLog[a/x]^2/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{x}\right)}{x} \, dx$$
$$\Big\downarrow \; 7173$$
$$\int \frac{W\left(\frac{a}{x}\right)^2}{x\left(W\left(\frac{a}{x}\right)+1\right)}dx - W\left(\frac{a}{x}\right)$$
$$\Big\downarrow \; 7200$$
$$-\frac{1}{2}W\left(\frac{a}{x}\right)^2 - W\left(\frac{a}{x}\right)$$

input
```
Int[ProductLog[a/x]/x,x]
```

output
```
-ProductLog[a/x] - ProductLog[a/x]^2/2
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\mathrm{LambertW}\left(\frac{a}{x}\right)-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^{2}}{2}$	20
default	$-\mathrm{LambertW}\left(\frac{a}{x}\right)-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^{2}}{2}$	20

input
```
int(LambertW(a/x)/x,x,method=_RETURNVERBOSE)
```

output
```
-LambertW(a/x)-1/2*LambertW(a/x)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{W\left(\frac{a}{x}\right)}{x}\,dx=-\frac{1}{2}\,\mathrm{W}(\frac{a}{x})^{2}+\log\left(x\right)+\log\left(\mathrm{W}(\frac{a}{x})\right)$$

input
```
integrate(lambert_w(a/x)/x,x, algorithm="fricas")
```

output
```
-1/2*lambert_w(a/x)^2 + log(x) + log(lambert_w(a/x))
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{W\left(\frac{a}{x}\right)}{x}\,dx=-\frac{W^{2}\left(\frac{a}{x}\right)}{2}-W\left(\frac{a}{x}\right)$$

input
```
integrate(LambertW(a/x)/x,x)
```

output
```
-LambertW(a/x)**2/2 - LambertW(a/x)
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)}{x}\,dx$$

input
```
integrate(lambert_w(a/x)/x,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x)/x, x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)}{x}\,dx$$

input
```
integrate(lambert_w(a/x)/x,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\mathrm{LambertW}\left(\frac{a}{x}\right)}{x}\,dx$$

input
```
int(LambertW(a/x)/x,x)
```

output
```
int(LambertW(a/x)/x, x)
```

Reduce [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{lambert\_w\left(\frac{a}{x}\right)}{x}dx$$

input
```
int(Lambert_W(a/x)/x,x)
```

output
```
int(lambert_w(a/x)/x,x)
```

3.193 $\int \frac{W\left(\frac{a}{x}\right)}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 28

$$\int \frac{W\left(\frac{a}{x}\right)}{x^2} dx = \frac{1}{x} - \frac{1}{xW\left(\frac{a}{x}\right)} - \frac{W\left(\frac{a}{x}\right)}{x}$$

output
```
1/x-1/x/LambertW(a/x)-LambertW(a/x)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{W\left(\frac{a}{x}\right)}{x^2} dx = -\frac{-1 + \frac{1}{W\left(\frac{a}{x}\right)} + W\left(\frac{a}{x}\right)}{x}$$

input
```
Integrate[ProductLog[a/x]/x^2,x]
```

output
```
-((-1 + ProductLog[a/x]^(-1) + ProductLog[a/x])/x)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {7172, 7205, 7199, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{x}\right)}{x^2}dx$$

$$\Big\downarrow \text{ 7172}$$

$$-\int \frac{W\left(\frac{a}{x}\right)}{x^2\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{W\left(\frac{a}{x}\right)}{x}$$

$$\Big\downarrow \text{ 7205}$$

$$\int \frac{1}{x^2\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{W\left(\frac{a}{x}\right)}{x}+\frac{1}{x}$$

$$\Big\downarrow \text{ 7199}$$

$$-\int \frac{1}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}-\frac{W\left(\frac{a}{x}\right)}{x}+\frac{1}{x}$$

$$\Big\downarrow \text{ 7176}$$

$$-\frac{W\left(\frac{a}{x}\right)}{x}-\frac{1}{xW\left(\frac{a}{x}\right)}+\frac{1}{x}$$

input
```
Int[ProductLog[a/x]/x^2,x]
```

output
```
x^(-1) - 1/(x*ProductLog[a/x]) - ProductLog[a/x]/x
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
parallelrisch	$\frac{-1-\operatorname{LambertW}\left(\frac{a}{x}\right)^{2}+\operatorname{LambertW}\left(\frac{a}{x}\right)}{x \operatorname{LambertW}\left(\frac{a}{x}\right)}$	31
derivativedivides	$-\frac{-\frac{a}{x}+\frac{a}{x \operatorname{LambertW}\left(\frac{a}{x}\right)}+\frac{a \operatorname{LambertW}\left(\frac{a}{x}\right)}{x}}{a}$	37
default	$-\frac{-\frac{a}{x}+\frac{a}{x \operatorname{LambertW}\left(\frac{a}{x}\right)}+\frac{a \operatorname{LambertW}\left(\frac{a}{x}\right)}{x}}{a}$	37

input
```
int(LambertW(a/x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x*(-1-LambertW(a/x)^2+LambertW(a/x))/LambertW(a/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{W\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{\mathrm{W}\left(\frac{a}{x}\right)^2 - \mathrm{W}\left(\frac{a}{x}\right) + 1}{x\,\mathrm{W}\left(\frac{a}{x}\right)}$$

input
```
integrate(lambert_w(a/x)/x^2,x, algorithm="fricas")
```

output
```
-(lambert_w(a/x)^2 - lambert_w(a/x) + 1)/(x*lambert_w(a/x))
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{W\left(\frac{a}{x}\right)}{x^2}\,dx = \begin{cases} -\frac{W\left(\frac{a}{x}\right)}{x} + \frac{1}{x} - \frac{1}{xW\left(\frac{a}{x}\right)} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(LambertW(a/x)/x**2,x)
```

output
```
Piecewise((-LambertW(a/x)/x + 1/x - 1/(x*LambertW(a/x)), Ne(a, 0)), (0, Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{W\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{W\left(\frac{a}{x}\right)^2 - W\left(\frac{a}{x}\right) + 1}{x\,W\left(\frac{a}{x}\right)}$$

input
```
integrate(lambert_w(a/x)/x^2,x, algorithm="maxima")
```

output
```
-(lambert_w(a/x)^2 - lambert_w(a/x) + 1)/(x*lambert_w(a/x))
```

Giac [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x^2}\,dx = \int \frac{W\left(\frac{a}{x}\right)}{x^2}\,dx$$

input
```
integrate(lambert_w(a/x)/x^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{x}\right)}{x^2}\,dx = \int \frac{\mathrm{LambertW}\left(\frac{a}{x}\right)}{x^2}\,dx$$

input
```
int(LambertW(a/x)/x^2,x)
```

output
```
int(LambertW(a/x)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{W\left(\frac{a}{x}\right)}{x^2}\,dx = \frac{e^{lambert\_w\left(\frac{a}{x}\right)}\left(-lambert\_w\left(\frac{a}{x}\right)^2 + lambert\_w\left(\frac{a}{x}\right) - 1\right)}{a}$$

input
```
int(Lambert_W(a/x)/x^2,x)
```

output
```
(e**lambert_w(a/x)*( - lambert_w(a/x)**2 + lambert_w(a/x) - 1))/a
```

3.194 $\int \frac{W\left(\frac{a}{x}\right)}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 51

$$\int \frac{W\left(\frac{a}{x}\right)}{x^3} dx = \frac{1}{4x^2} + \frac{1}{8x^2W\left(\frac{a}{x}\right)^2} - \frac{1}{4x^2W\left(\frac{a}{x}\right)} - \frac{W\left(\frac{a}{x}\right)}{2x^2}$$

output
```
1/4/x^2+1/8/x^2/LambertW(a/x)^2-1/4/x^2/LambertW(a/x)-1/2*LambertW(a/x)/x^
2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{W\left(\frac{a}{x}\right)}{x^3} dx = \frac{1}{4x^2} + \frac{1}{8x^2W\left(\frac{a}{x}\right)^2} - \frac{1}{4x^2W\left(\frac{a}{x}\right)} - \frac{W\left(\frac{a}{x}\right)}{2x^2}$$

input
```
Integrate[ProductLog[a/x]/x^3,x]
```

output
```
1/(4*x^2) + 1/(8*x^2*ProductLog[a/x]^2) - 1/(4*x^2*ProductLog[a/x]) - Prod
uctLog[a/x]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7172, 7205, 7199, 7194, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{x}\right)}{x^3} \, dx$$

$$\Big\downarrow \text{ 7172}$$

$$-\frac{1}{2} \int \frac{W\left(\frac{a}{x}\right)}{x^3 \left(W\left(\frac{a}{x}\right)+1\right)}dx - \frac{W\left(\frac{a}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{2}\left(\int \frac{1}{x^3 \left(W\left(\frac{a}{x}\right)+1\right)}dx + \frac{1}{2x^2}\right) - \frac{W\left(\frac{a}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 7199}$$

$$\frac{1}{2}\left(\frac{1}{2x^2} - \int \frac{1}{x \left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}\right) - \frac{W\left(\frac{a}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 7194}$$

$$\frac{1}{2}\left(\frac{1}{2} \int \frac{1}{xW\left(\frac{a}{x}\right) \left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x} - \frac{1}{2x^2W\left(\frac{a}{x}\right)} + \frac{1}{2x^2}\right) - \frac{W\left(\frac{a}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 7201}$$

$$\frac{1}{2}\left(-\frac{1}{2x^2W\left(\frac{a}{x}\right)} + \frac{1}{4x^2W\left(\frac{a}{x}\right)^2} + \frac{1}{2x^2}\right) - \frac{W\left(\frac{a}{x}\right)}{2x^2}$$

input
```
Int[ProductLog[a/x]/x^3,x]
```

output
```
(1/(2*x^2) + 1/(4*x^2*ProductLog[a/x]^2) - 1/(2*x^2*ProductLog[a/x]))/2 -
ProductLog[a/x]/(2*x^2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$-\frac{-\frac{a^{2}}{4x^{2}}+\frac{a^{2}}{4\,\mathrm{LambertW}\left(\frac{a}{x}\right)x^{2}}-\frac{a^{2}}{8x^{2}\,\mathrm{LambertW}\left(\frac{a}{x}\right)^{2}}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)a^{2}}{2x^{2}}}{a^{2}}$	61
default	$-\frac{-\frac{a^{2}}{4x^{2}}+\frac{a^{2}}{4\,\mathrm{LambertW}\left(\frac{a}{x}\right)x^{2}}-\frac{a^{2}}{8x^{2}\,\mathrm{LambertW}\left(\frac{a}{x}\right)^{2}}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)a^{2}}{2x^{2}}}{a^{2}}$	61

input
```
int(LambertW(a/x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(-1/4*a^2/x^2+1/4/LambertW(a/x)/x^2*a^2-1/8/x^2*a^2/LambertW(a/x)^2
+1/2*LambertW(a/x)/x^2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{W\left(\frac{a}{x}\right)}{x^{3}}\,dx=-\frac{4\ \mathrm{W}(\frac{a}{x})^{3}-2\ \mathrm{W}(\frac{a}{x})^{2}+2\ \mathrm{W}(\frac{a}{x})-1}{8\,x^{2}\,\mathrm{W}(\frac{a}{x})^{2}}$$

input
```
integrate(lambert_w(a/x)/x^3,x, algorithm="fricas")
```

output
```
-1/8*(4*lambert_w(a/x)^3 - 2*lambert_w(a/x)^2 + 2*lambert_w(a/x) - 1)/(x^2
*lambert_w(a/x)^2)
```

Sympy [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x^3}\,dx = \int \frac{W\left(\frac{a}{x}\right)}{x^3}\,dx$$

input
```
integrate(LambertW(a/x)/x**3,x)
```

output
```
Integral(LambertW(a/x)/x**3, x)
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x^3}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)}{x^3}\,dx$$

input
```
integrate(lambert_w(a/x)/x^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x)/x^3, x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x^3}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)}{x^3}\,dx$$

input
```
integrate(lambert_w(a/x)/x^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{x}\right)}{x^3}\,dx = \int \frac{\mathrm{LambertW}\left(\frac{a}{x}\right)}{x^3}\,dx$$

input
```
int(LambertW(a/x)/x^3,x)
```

output
```
int(LambertW(a/x)/x^3, x)
```

Reduce [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x^3}\,dx = \int \frac{lambert\_w\left(\frac{a}{x}\right)}{x^3}\,dx$$

input
```
int(Lambert_W(a/x)/x^3,x)
```

output
```
int(lambert_w(a/x)/x**3,x)
```

3.195 $\int \frac{W\left(\frac{a}{x}\right)}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 66

$$\int \frac{W\left(\frac{a}{x}\right)}{x^4}\,dx=\frac{1}{9x^3}-\frac{2}{81x^3W\left(\frac{a}{x}\right)^3}+\frac{2}{27x^3W\left(\frac{a}{x}\right)^2}-\frac{1}{9x^3W\left(\frac{a}{x}\right)}-\frac{W\left(\frac{a}{x}\right)}{3x^3}$$

output
```
1/9/x^3-2/81/x^3/LambertW(a/x)^3+2/27/x^3/LambertW(a/x)^2-1/9/x^3/LambertW
(a/x)-1/3*LambertW(a/x)/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{W\left(\frac{a}{x}\right)}{x^4}\,dx=\frac{1}{9x^3}-\frac{2}{81x^3W\left(\frac{a}{x}\right)^3}+\frac{2}{27x^3W\left(\frac{a}{x}\right)^2}-\frac{1}{9x^3W\left(\frac{a}{x}\right)}-\frac{W\left(\frac{a}{x}\right)}{3x^3}$$

input
```
Integrate[ProductLog[a/x]/x^4,x]
```

output
```
1/(9*x^3) - 2/(81*x^3*ProductLog[a/x]^3) + 2/(27*x^3*ProductLog[a/x]^2) -
1/(9*x^3*ProductLog[a/x]) - ProductLog[a/x]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {7172, 7205, 7199, 7194, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{x}\right)}{x^4}dx$$

$$\Big\downarrow \text{7172}$$

$$-\frac{1}{3}\int \frac{W\left(\frac{a}{x}\right)}{x^4\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{W\left(\frac{a}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{3}\left(\int \frac{1}{x^4\left(W\left(\frac{a}{x}\right)+1\right)}dx+\frac{1}{3x^3}\right)-\frac{W\left(\frac{a}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{7199}$$

$$\frac{1}{3}\left(\frac{1}{3x^3}-\int \frac{1}{x^2\left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}\right)-\frac{W\left(\frac{a}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{7194}$$

$$\frac{1}{3}\left(\frac{2}{3}\int \frac{1}{x^2W\left(\frac{a}{x}\right)\left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}-\frac{1}{3x^3W\left(\frac{a}{x}\right)}+\frac{1}{3x^3}\right)-\frac{W\left(\frac{a}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{3}\left(\frac{2}{3}\left(\frac{1}{3x^3W\left(\frac{a}{x}\right)^2}-\frac{1}{3}\int \frac{1}{x^2W\left(\frac{a}{x}\right)^2\left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}\right)-\frac{1}{3x^3W\left(\frac{a}{x}\right)}+\frac{1}{3x^3}\right)-\frac{W\left(\frac{a}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{3}\left(\frac{2}{3}\left(\frac{1}{3x^3W\left(\frac{a}{x}\right)^2}-\frac{1}{9x^3W\left(\frac{a}{x}\right)^3}\right)-\frac{1}{3x^3W\left(\frac{a}{x}\right)}+\frac{1}{3x^3}\right)-\frac{W\left(\frac{a}{x}\right)}{3x^3}$$

input
```
Int[ProductLog[a/x]/x^4,x]
```

output
```
(1/(3*x^3) + (2*(-1/9*1/(x^3*ProductLog[a/x]^3) + 1/(3*x^3*ProductLog[a/x]
^2)))/3 - 1/(3*x^3*ProductLog[a/x]))/3 - ProductLog[a/x]/(3*x^3)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$-\frac{-\frac{a^3}{9x^3}+\frac{a^3}{9\,\mathrm{LambertW}\left(\frac{a}{x}\right)x^3}-\frac{2a^3}{27\,\mathrm{LambertW}\left(\frac{a}{x}\right)^2x^3}+\frac{2a^3}{81x^3\,\mathrm{LambertW}\left(\frac{a}{x}\right)^3}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)a^3}{3x^3}}{a^3}$	77
default	$-\frac{-\frac{a^3}{9x^3}+\frac{a^3}{9\,\mathrm{LambertW}\left(\frac{a}{x}\right)x^3}-\frac{2a^3}{27\,\mathrm{LambertW}\left(\frac{a}{x}\right)^2x^3}+\frac{2a^3}{81x^3\,\mathrm{LambertW}\left(\frac{a}{x}\right)^3}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)a^3}{3x^3}}{a^3}$	77

input
```
int(LambertW(a/x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*(-1/9*a^3/x^3+1/9/LambertW(a/x)/x^3*a^3-2/27/LambertW(a/x)^2/x^3*a^
3+2/81/x^3*a^3/LambertW(a/x)^3+1/3*LambertW(a/x)/x^3*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \frac{W\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{27\ \mathrm{W}(\frac{a}{x})^4 - 9\ \mathrm{W}(\frac{a}{x})^3 + 9\ \mathrm{W}(\frac{a}{x})^2 - 6\ \mathrm{W}(\frac{a}{x}) + 2}{81\,x^3\,\mathrm{W}(\frac{a}{x})^3}$$

input
```
integrate(lambert_w(a/x)/x^4,x, algorithm="fricas")
```

output
```
-1/81*(27*lambert_w(a/x)^4 - 9*lambert_w(a/x)^3 + 9*lambert_w(a/x)^2 - 6*l
ambert_w(a/x) + 2)/(x^3*lambert_w(a/x)^3)
```

Sympy [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x^4}\,dx = \int \frac{W\left(\frac{a}{x}\right)}{x^4}\,dx$$

input
```
integrate(LambertW(a/x)/x**4,x)
```

output
```
Integral(LambertW(a/x)/x**4, x)
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x^4}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)}{x^4}\,dx$$

input
```
integrate(lambert_w(a/x)/x^4,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x)/x^4, x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x^4}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)}{x^4}\,dx$$

input
```
integrate(lambert_w(a/x)/x^4,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{x}\right)}{x^4} \, dx = \int \frac{\mathrm{LambertW}\left(\frac{a}{x}\right)}{x^4} \, dx$$

input
```
int(LambertW(a/x)/x^4,x)
```

output
```
int(LambertW(a/x)/x^4, x)
```

Reduce [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x^4} \, dx = \int \frac{lambert\_w\left(\frac{a}{x}\right)}{x^4} dx$$

input
```
int(Lambert_W(a/x)/x^4,x)
```

output
```
int(lambert_w(a/x)/x**4,x)
```

3.196 $\int \frac{W\left(\frac{a}{x}\right)}{x^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 81

$$\int \frac{W\left(\frac{a}{x}\right)}{x^5}\,dx = \frac{1}{16x^4}+\frac{3}{512x^4W\left(\frac{a}{x}\right)^4}-\frac{3}{128x^4W\left(\frac{a}{x}\right)^3}+\frac{3}{64x^4W\left(\frac{a}{x}\right)^2}-\frac{1}{16x^4W\left(\frac{a}{x}\right)}-\frac{W\left(\frac{a}{x}\right)}{4x^4}$$

output
```
1/16/x^4+3/512/x^4/LambertW(a/x)^4-3/128/x^4/LambertW(a/x)^3+3/64/x^4/Lamb
ertW(a/x)^2-1/16/x^4/LambertW(a/x)-1/4*LambertW(a/x)/x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{W\left(\frac{a}{x}\right)}{x^5}\,dx = \frac{1}{16x^4}+\frac{3}{512x^4W\left(\frac{a}{x}\right)^4}-\frac{3}{128x^4W\left(\frac{a}{x}\right)^3}+\frac{3}{64x^4W\left(\frac{a}{x}\right)^2}-\frac{1}{16x^4W\left(\frac{a}{x}\right)}-\frac{W\left(\frac{a}{x}\right)}{4x^4}$$

input
```
Integrate[ProductLog[a/x]/x^5,x]
```

output
```
1/(16*x^4) + 3/(512*x^4*ProductLog[a/x]^4) - 3/(128*x^4*ProductLog[a/x]^3)
 + 3/(64*x^4*ProductLog[a/x]^2) - 1/(16*x^4*ProductLog[a/x]) - ProductLog[
a/x]/(4*x^4)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {7172, 7205, 7199, 7194, 7205, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{x}\right)}{x^5}dx$$

$$\Big\downarrow \text{ 7172}$$

$$-\frac{1}{4}\int \frac{W\left(\frac{a}{x}\right)}{x^5\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{W\left(\frac{a}{x}\right)}{4x^4}$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{4}\left(\int \frac{1}{x^5\left(W\left(\frac{a}{x}\right)+1\right)}dx+\frac{1}{4x^4}\right)-\frac{W\left(\frac{a}{x}\right)}{4x^4}$$

$$\Big\downarrow \text{ 7199}$$

$$\frac{1}{4}\left(\frac{1}{4x^4}-\int \frac{1}{x^3\left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}\right)-\frac{W\left(\frac{a}{x}\right)}{4x^4}$$

$$\Big\downarrow \text{ 7194}$$

$$\frac{1}{4}\left(\frac{3}{4}\int \frac{1}{x^3W\left(\frac{a}{x}\right)\left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}-\frac{1}{4x^4W\left(\frac{a}{x}\right)}+\frac{1}{4x^4}\right)-\frac{W\left(\frac{a}{x}\right)}{4x^4}$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\frac{1}{4x^4W\left(\frac{a}{x}\right)^2}-\frac{1}{2}\int \frac{1}{x^3W\left(\frac{a}{x}\right)^2\left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}\right)-\frac{1}{4x^4W\left(\frac{a}{x}\right)}+\frac{1}{4x^4}\right)-\frac{W\left(\frac{a}{x}\right)}{4x^4}$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{x^3W\left(\frac{a}{x}\right)^3\left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}-\frac{1}{4x^4W\left(\frac{a}{x}\right)^3}\right)+\frac{1}{4x^4W\left(\frac{a}{x}\right)^2}\right)-\frac{1}{4x^4W\left(\frac{a}{x}\right)}+\frac{1}{4x^4}\right)-\frac{W\left(\frac{a}{x}\right)}{4x^4}$$

$\big\downarrow$ 7201

$$\frac{1}{4}\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{1}{16x^4W\left(\frac{a}{x}\right)^4}-\frac{1}{4x^4W\left(\frac{a}{x}\right)^3}\right)+\frac{1}{4x^4W\left(\frac{a}{x}\right)^2}\right)-\frac{1}{4x^4W\left(\frac{a}{x}\right)}+\frac{1}{4x^4}\right)-\frac{W\left(\frac{a}{x}\right)}{4x^4}$$

input
```
Int[ProductLog[a/x]/x^5,x]
```

output
```
(1/(4*x^4) + (3*((1/(16*x^4*ProductLog[a/x]^4) - 1/(4*x^4*ProductLog[a/x]^
3))/2 + 1/(4*x^4*ProductLog[a/x]^2)))/4 - 1/(4*x^4*ProductLog[a/x]))/4 - P
roductLog[a/x]/(4*x^4)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.15

method	result
derivativedivides	$-\frac{-\frac{a^4}{16x^4}+\frac{a^4}{16\,\mathrm{LambertW}\left(\frac{a}{x}\right)x^4}-\frac{3a^4}{64x^4\,\mathrm{LambertW}\left(\frac{a}{x}\right)^2}+\frac{3a^4}{128\,\mathrm{LambertW}\left(\frac{a}{x}\right)^3x^4}-\frac{3a^4}{512x^4\,\mathrm{LambertW}\left(\frac{a}{x}\right)^4}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)a^4}{4x^4}}{a^4}$
default	$-\frac{-\frac{a^4}{16x^4}+\frac{a^4}{16\,\mathrm{LambertW}\left(\frac{a}{x}\right)x^4}-\frac{3a^4}{64x^4\,\mathrm{LambertW}\left(\frac{a}{x}\right)^2}+\frac{3a^4}{128\,\mathrm{LambertW}\left(\frac{a}{x}\right)^3x^4}-\frac{3a^4}{512x^4\,\mathrm{LambertW}\left(\frac{a}{x}\right)^4}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)a^4}{4x^4}}{a^4}$

input
```
int(LambertW(a/x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/a^4*(-1/16/x^4*a^4+1/16/LambertW(a/x)/x^4*a^4-3/64/x^4*a^4/LambertW(a/x
)^2+3/128/LambertW(a/x)^3/x^4*a^4-3/512/x^4*a^4/LambertW(a/x)^4+1/4*Lamber
tW(a/x)/x^4*a^4)
```

Fricas [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x^5}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)}{x^5}\,dx$$

input
```
integrate(lambert_w(a/x)/x^5,x, algorithm="fricas")
```

output
```
integral(lambert_w(a/x)/x^5, x)
```

Sympy [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x^5}\,dx = \int \frac{W\left(\frac{a}{x}\right)}{x^5}\,dx$$

input
```
integrate(LambertW(a/x)/x**5,x)
```

output
```
Integral(LambertW(a/x)/x**5, x)
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x^5}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)}{x^5}\,dx$$

input
```
integrate(lambert_w(a/x)/x^5,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x)/x^5, x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x^5} \, dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)}{x^5} \, dx$$

input
```
integrate(lambert_w(a/x)/x^5,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{x}\right)}{x^5} \, dx = \int \frac{\mathrm{LambertW}\left(\frac{a}{x}\right)}{x^5} \, dx$$

input
```
int(LambertW(a/x)/x^5,x)
```

output
```
int(LambertW(a/x)/x^5, x)
```

Reduce [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{x^5} \, dx = \int \frac{lambert\_w\left(\frac{a}{x}\right)}{x^5} dx$$

input
```
int(Lambert_W(a/x)/x^5,x)
```

output
```
int(lambert_w(a/x)/x**5,x)
```

3.197 $\int x^4 W\left(\frac{a}{x}\right)^2 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 62

$$\int x^4 W\left(\frac{a}{x}\right)^2 \, dx = \frac{25}{3} a^5 \operatorname{ExpIntegralEi}\left(-5W\left(\frac{a}{x}\right)\right) + \frac{1}{3} x^5 W\left(\frac{a}{x}\right)^2 - \frac{1}{3} x^5 W\left(\frac{a}{x}\right)^3 + \frac{5}{3} x^5 W\left(\frac{a}{x}\right)^4$$

output
```
25/3*a^5*Ei(-5*LambertW(a/x))+1/3*x^5*LambertW(a/x)^2-1/3*x^5*LambertW(a/x
)^3+5/3*x^5*LambertW(a/x)^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int x^4 W\left(\frac{a}{x}\right)^2 \, dx = \frac{25}{3} a^5 \operatorname{ExpIntegralEi}\left(-5W\left(\frac{a}{x}\right)\right) + \frac{1}{3} x^5 W\left(\frac{a}{x}\right)^2 - \frac{1}{3} x^5 W\left(\frac{a}{x}\right)^3 + \frac{5}{3} x^5 W\left(\frac{a}{x}\right)^4$$

input
```
Integrate[x^4*ProductLog[a/x]^2,x]
```

output
```
(25*a^5*ExpIntegralEi[-5*ProductLog[a/x]])/3 + (x^5*ProductLog[a/x]^2)/3 -
 (x^5*ProductLog[a/x]^3)/3 + (5*x^5*ProductLog[a/x]^4)/3
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7173, 7206, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 W\left(\frac{a}{x}\right)^2 dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{1}{3}x^5 W\left(\frac{a}{x}\right)^2 - \frac{2}{3}\int \frac{x^4 W\left(\frac{a}{x}\right)^3}{W\left(\frac{a}{x}\right)+1}dx$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{3}x^5 W\left(\frac{a}{x}\right)^2 - \frac{2}{3}\left(\frac{1}{2}x^5 W\left(\frac{a}{x}\right)^3 - \frac{5}{2}\int \frac{x^4 W\left(\frac{a}{x}\right)^4}{W\left(\frac{a}{x}\right)+1}dx\right)$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{3}x^5 W\left(\frac{a}{x}\right)^2 - \frac{2}{3}\left(\frac{1}{2}x^5 W\left(\frac{a}{x}\right)^3 - \frac{5}{2}\left(x^5 W\left(\frac{a}{x}\right)^4 - 5\int \frac{x^4 W\left(\frac{a}{x}\right)^5}{W\left(\frac{a}{x}\right)+1}dx\right)\right)$$

$$\Big\downarrow \text{7202}$$

$$\frac{1}{3}x^5 W\left(\frac{a}{x}\right)^2 - \frac{2}{3}\left(\frac{1}{2}x^5 W\left(\frac{a}{x}\right)^3 - \frac{5}{2}\left(5a^5 \operatorname{ExpIntegralEi}\left(-5W\left(\frac{a}{x}\right)\right) + x^5 W\left(\frac{a}{x}\right)^4\right)\right)$$

input
```
Int[x^4*ProductLog[a/x]^2,x]
```

output
```
(x^5*ProductLog[a/x]^2)/3 - (2*((x^5*ProductLog[a/x]^3)/2 - (5*(5*a^5*ExpI
ntegralEi[-5*ProductLog[a/x]] + x^5*ProductLog[a/x]^4))/2))/3
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-a^5\left(-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^2 x^5}{3a^5}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^3 x^5}{3a^5}-\frac{5\,\mathrm{LambertW}\left(\frac{a}{x}\right)^4 x^5}{3a^5}+\frac{25\ \mathrm{expIntegral}_1\left(5\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)}{3}\right.$
default	$-a^5\left(-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^2 x^5}{3a^5}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^3 x^5}{3a^5}-\frac{5\,\mathrm{LambertW}\left(\frac{a}{x}\right)^4 x^5}{3a^5}+\frac{25\ \mathrm{expIntegral}_1\left(5\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)}{3}\right.$

input
```
int(x^4*LambertW(a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^5*(-1/3*LambertW(a/x)^2*x^5/a^5+1/3*LambertW(a/x)^3*x^5/a^5-5/3*Lambert
W(a/x)^4*x^5/a^5+25/3*Ei(1,5*LambertW(a/x)))
```

Fricas [F]

$$\int x^4W\left(\frac{a}{x}\right)^2\,dx=\int x^4\,\mathrm{W}(\frac{a}{x})^2\,dx$$

input
```
integrate(x^4*lambert_w(a/x)^2,x, algorithm="fricas")
```

output
```
integral(x^4*lambert_w(a/x)^2, x)
```

Sympy [F]

$$\int x^4W\left(\frac{a}{x}\right)^2\,dx=\int x^4W^2\left(\frac{a}{x}\right)\,dx$$

input
```
integrate(x**4*LambertW(a/x)**2,x)
```

output
```
Integral(x**4*LambertW(a/x)**2, x)
```

Maxima [F]

$$\int x^4W\left(\frac{a}{x}\right)^2\,dx=\int x^4\,\mathrm{W}(\frac{a}{x})^2\,dx$$

input
```
integrate(x^4*lambert_w(a/x)^2,x, algorithm="maxima")
```

output
```
integrate(x^4*lambert_w(a/x)^2, x)
```

Giac [F]

$$\int x^4 W\left(\frac{a}{x}\right)^2 dx = \int x^4 \operatorname{W}(\frac{a}{x})^2 dx$$

input
```
integrate(x^4*lambert_w(a/x)^2,x, algorithm="giac")
```

output
```
integrate(x^4*lambert_w(a/x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 W\left(\frac{a}{x}\right)^2 dx = \int x^4 \operatorname{LambertW}\left(\frac{a}{x}\right)^2 dx$$

input
```
int(x^4*LambertW(a/x)^2,x)
```

output
```
int(x^4*LambertW(a/x)^2, x)
```

Reduce [F]

$$\int x^4 W\left(\frac{a}{x}\right)^2 dx = \frac{2\left(\int \frac{lambert\_w\left(\frac{a}{x}\right)x^3}{e^{lambert\_w\left(\frac{a}{x}\right)}lambert\_w\left(\frac{a}{x}\right)+e^{lambert\_w\left(\frac{a}{x}\right)}}dx\right)a}{5} + \frac{lambert\_w\left(\frac{a}{x}\right)^2 x^5}{5}$$

input
```
int(x^4*Lambert_W(a/x)^2,x)
```

output
```
(2*int((lambert_w(a/x)*x**3)/(e**lambert_w(a/x)*lambert_w(a/x) + e**lamber
t_w(a/x)),x)*a + lambert_w(a/x)**2*x**5)/5
```

3.198 $\int x^3 W\left(\frac{a}{x}\right)^2 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int x^3 W\left(\frac{a}{x}\right)^2 \, dx = -4a^4 \operatorname{ExpIntegralEi}\left(-4W\left(\frac{a}{x}\right)\right) + \frac{1}{2}x^4 W\left(\frac{a}{x}\right)^2 - x^4 W\left(\frac{a}{x}\right)^3$$

output
```
-4*a^4*Ei(-4*LambertW(a/x))+1/2*x^4*LambertW(a/x)^2-x^4*LambertW(a/x)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^3 W\left(\frac{a}{x}\right)^2 \, dx = -4a^4 \operatorname{ExpIntegralEi}\left(-4W\left(\frac{a}{x}\right)\right) + \frac{1}{2}x^4 W\left(\frac{a}{x}\right)^2 - x^4 W\left(\frac{a}{x}\right)^3$$

input
```
Integrate[x^3*ProductLog[a/x]^2,x]
```

output
```
-4*a^4*ExpIntegralEi[-4*ProductLog[a/x]] + (x^4*ProductLog[a/x]^2)/2 - x^4
*ProductLog[a/x]^3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7173, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 W\left(\frac{a}{x}\right)^2 dx$$
$$\Big\downarrow \; 7173$$
$$\frac{1}{2}x^4 W\left(\frac{a}{x}\right)^2 - \int \frac{x^3 W\left(\frac{a}{x}\right)^3}{W\left(\frac{a}{x}\right)+1}dx$$
$$\Big\downarrow \; 7206$$
$$4\int \frac{x^3 W\left(\frac{a}{x}\right)^4}{W\left(\frac{a}{x}\right)+1}dx + x^4\left(-W\left(\frac{a}{x}\right)^3\right) + \frac{1}{2}x^4 W\left(\frac{a}{x}\right)^2$$
$$\Big\downarrow \; 7202$$
$$-4a^4 \operatorname{ExpIntegralEi}\left(-4W\left(\frac{a}{x}\right)\right) - x^4 W\left(\frac{a}{x}\right)^3 + \frac{1}{2}x^4 W\left(\frac{a}{x}\right)^2$$

input
```
Int[x^3*ProductLog[a/x]^2,x]
```

output
```
-4*a^4*ExpIntegralEi[-4*ProductLog[a/x]] + (x^4*ProductLog[a/x]^2)/2 - x^4
*ProductLog[a/x]^3
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$-a^4\left(-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^2 x^4}{2a^4}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^3 x^4}{a^4}-4\,\mathrm{expIntegral}_1\left(4\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)\right)$	50
default	$-a^4\left(-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^2 x^4}{2a^4}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^3 x^4}{a^4}-4\,\mathrm{expIntegral}_1\left(4\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)\right)$	50

input
```
int(x^3*LambertW(a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^4*(-1/2*LambertW(a/x)^2*x^4/a^4+LambertW(a/x)^3*x^4/a^4-4*Ei(1,4*Lamber
tW(a/x)))
```

Fricas [F]

$$\int x^3 W\left(\frac{a}{x}\right)^2\,dx=\int x^3\,\mathrm{W}\left(\frac{a}{x}\right)^2\,dx$$

input
```
integrate(x^3*lambert_w(a/x)^2,x, algorithm="fricas")
```

output
```
integral(x^3*lambert_w(a/x)^2, x)
```

Sympy [F]

$$\int x^3W\left(\frac{a}{x}\right)^2\,dx=\int x^3W^2\left(\frac{a}{x}\right)\,dx$$

input
```
integrate(x**3*LambertW(a/x)**2,x)
```

output
```
Integral(x**3*LambertW(a/x)**2, x)
```

Maxima [F]

$$\int x^3W\left(\frac{a}{x}\right)^2\,dx=\int x^3\,\mathrm{W}(\frac{a}{x})^2\,dx$$

input
```
integrate(x^3*lambert_w(a/x)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*lambert_w(a/x)^2, x)
```

Giac [F]

$$\int x^3W\left(\frac{a}{x}\right)^2\,dx=\int x^3\,\mathrm{W}(\frac{a}{x})^2\,dx$$

input
```
integrate(x^3*lambert_w(a/x)^2,x, algorithm="giac")
```

output
```
integrate(x^3*lambert_w(a/x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 W\left(\frac{a}{x}\right)^2 dx = \int x^3 \operatorname{LambertW}\left(\frac{a}{x}\right)^2 dx$$

input
```
int(x^3*LambertW(a/x)^2,x)
```

output
```
int(x^3*LambertW(a/x)^2, x)
```

Reduce [F]

$$\int x^3 W\left(\frac{a}{x}\right)^2 dx = \frac{\left(\int \frac{lambert\_w\left(\frac{a}{x}\right)x^2}{e^{lambert\_w\left(\frac{a}{x}\right)} lambert\_w\left(\frac{a}{x}\right)+e^{lambert\_w\left(\frac{a}{x}\right)}}dx\right)a}{2} + \frac{lambert\_w\left(\frac{a}{x}\right)^2 x^4}{4}$$

input
```
int(x^3*Lambert_W(a/x)^2,x)
```

output
```
(2*int((lambert_w(a/x)*x**2)/(e**lambert_w(a/x)*lambert_w(a/x) + e**lamber
t_w(a/x)),x)*a + lambert_w(a/x)**2*x**4)/4
```

3.199 $\int x^2 W\left(\frac{a}{x}\right)^2 dx$

Optimal result

Integrand size = 12, antiderivative size = 27

$$\int x^2 W\left(\frac{a}{x}\right)^2 dx = 2a^3 \operatorname{ExpIntegralEi}\left(-3W\left(\frac{a}{x}\right)\right) + x^3 W\left(\frac{a}{x}\right)^2$$

output
```
2*a^3*Ei(-3*LambertW(a/x))+x^3*LambertW(a/x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^2 W\left(\frac{a}{x}\right)^2 dx = 2a^3 \operatorname{ExpIntegralEi}\left(-3W\left(\frac{a}{x}\right)\right) + x^3 W\left(\frac{a}{x}\right)^2$$

input
```
Integrate[x^2*ProductLog[a/x]^2,x]
```

output
```
2*a^3*ExpIntegralEi[-3*ProductLog[a/x]] + x^3*ProductLog[a/x]^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7173, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 W\left(\frac{a}{x}\right)^2 dx$$

$$\Big\downarrow \text{7173}$$

$$x^3 W\left(\frac{a}{x}\right)^2 - 2\int \frac{x^2 W\left(\frac{a}{x}\right)^3}{W\left(\frac{a}{x}\right)+1}dx$$

$$\Big\downarrow \text{7202}$$

$$2a^3 \text{ExpIntegralEi}\left(-3W\left(\frac{a}{x}\right)\right) + x^3 W\left(\frac{a}{x}\right)^2$$

input
```
Int[x^2*ProductLog[a/x]^2,x]
```

output
```
2*a^3*ExpIntegralEi[-3*ProductLog[a/x]] + x^3*ProductLog[a/x]^2
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$-a^{3}\left(-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^{2} x^{3}}{a^{3}}+2\,\mathrm{expIntegral}_{1}\left(3\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)\right)$	35
default	$-a^{3}\left(-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^{2} x^{3}}{a^{3}}+2\,\mathrm{expIntegral}_{1}\left(3\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)\right)$	35

input
```
int(x^2*LambertW(a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^3*(-LambertW(a/x)^2*x^3/a^3+2*Ei(1,3*LambertW(a/x)))
```

Fricas [F]

$$\int x^2 W\left(\frac{a}{x}\right)^2 \, dx = \int x^2\, \mathrm{W}\left(\frac{a}{x}\right)^2 \, dx$$

input
```
integrate(x^2*lambert_w(a/x)^2,x, algorithm="fricas")
```

output
```
integral(x^2*lambert_w(a/x)^2, x)
```

Sympy [F]

$$\int x^2 W\left(\frac{a}{x}\right)^2 \, dx = \int x^{2} W^{2}\left(\frac{a}{x}\right)\, dx$$

input
```
integrate(x**2*LambertW(a/x)**2,x)
```

output
```
Integral(x**2*LambertW(a/x)**2, x)
```

Maxima [F]

$$\int x^2 W\left(\frac{a}{x}\right)^2 dx = \int x^2 \mathrm{W}(\frac{a}{x})^2 dx$$

input
```
integrate(x^2*lambert_w(a/x)^2,x, algorithm="maxima")
```

output
```
integrate(x^2*lambert_w(a/x)^2, x)
```

Giac [F]

$$\int x^2 W\left(\frac{a}{x}\right)^2 dx = \int x^2 \mathrm{W}(\frac{a}{x})^2 dx$$

input
```
integrate(x^2*lambert_w(a/x)^2,x, algorithm="giac")
```

output
```
integrate(x^2*lambert_w(a/x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 W\left(\frac{a}{x}\right)^2 dx = \int x^2 \mathrm{LambertW}\left(\frac{a}{x}\right)^2 dx$$

input
```
int(x^2*LambertW(a/x)^2,x)
```

output
```
int(x^2*LambertW(a/x)^2, x)
```

Reduce [F]

$$\int x^2 W\left(\frac{a}{x}\right)^2 dx$$
$$= \frac{2\left(\int \frac{lambert\_w(\frac{a}{x})x}{e^{lambert\_w(\frac{a}{x})}lambert\_w(\frac{a}{x})+e^{lambert\_w(\frac{a}{x})}}dx\right)a}{3} + \frac{lambert\_w(\frac{a}{x})^2 x^3}{3}$$

input
```
int(x^2*Lambert_W(a/x)^2,x)
```

output
```
(2*int((lambert_w(a/x)*x)/(e**lambert_w(a/x)*lambert_w(a/x) + e**lambert_w
(a/x)),x)*a + lambert_w(a/x)**2*x**3)/3
```

3.200 $\int xW\left(\frac{a}{x}\right)^2 dx$

Optimal result

Integrand size = 10, antiderivative size = 30

$$\int xW\left(\frac{a}{x}\right)^2 dx = -a^2 \operatorname{ExpIntegralEi}\left(-2W\left(\frac{a}{x}\right)\right) + \frac{1}{2}x^2W\left(\frac{a}{x}\right)^2$$

output
```
-a^2*Ei(-2*LambertW(a/x))+1/2*x^2*LambertW(a/x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int xW\left(\frac{a}{x}\right)^2 dx = -a^2 \operatorname{ExpIntegralEi}\left(-2W\left(\frac{a}{x}\right)\right) + \frac{1}{2}x^2W\left(\frac{a}{x}\right)^2$$

input
```
Integrate[x*ProductLog[a/x]^2,x]
```

output
```
-(a^2*ExpIntegralEi[-2*ProductLog[a/x]]) + (x^2*ProductLog[a/x]^2)/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7172, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xW\left(\frac{a}{x}\right)^2 dx$$

$$\Big\downarrow \text{7172}$$

$$\int \frac{xW\left(\frac{a}{x}\right)^2}{W\left(\frac{a}{x}\right)+1}dx+\frac{1}{2}x^2W\left(\frac{a}{x}\right)^2$$

$$\Big\downarrow \text{7202}$$

$$\frac{1}{2}x^2W\left(\frac{a}{x}\right)^2-a^2\,\text{ExpIntegralEi}\left(-2W\left(\frac{a}{x}\right)\right)$$

input
```
Int[x*ProductLog[a/x]^2,x]
```

output
```
-(a^2*ExpIntegralEi[-2*ProductLog[a/x]]) + (x^2*ProductLog[a/x]^2)/2
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$-a^2\left(-\frac{x^2 \operatorname{LambertW}\left(\frac{a}{x}\right)^2}{2a^2}-\operatorname{expIntegral}_1\left(2 \operatorname{LambertW}\left(\frac{a}{x}\right)\right)\right)$	35
default	$-a^2\left(-\frac{x^2 \operatorname{LambertW}\left(\frac{a}{x}\right)^2}{2a^2}-\operatorname{expIntegral}_1\left(2 \operatorname{LambertW}\left(\frac{a}{x}\right)\right)\right)$	35

input
```
int(x*LambertW(a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2*(-1/2*x^2/a^2*LambertW(a/x)^2-Ei(1,2*LambertW(a/x)))
```

Fricas [F]

$$\int xW\left(\frac{a}{x}\right)^2\,dx = \int x\,\mathrm{W}\left(\frac{a}{x}\right)^2\,dx$$

input
```
integrate(x*lambert_w(a/x)^2,x, algorithm="fricas")
```

output
```
integral(x*lambert_w(a/x)^2, x)
```

Sympy [F]

$$\int xW\left(\frac{a}{x}\right)^2\,dx = \int xW^2\left(\frac{a}{x}\right)\,dx$$

input
```
integrate(x*LambertW(a/x)**2,x)
```

output
```
Integral(x*LambertW(a/x)**2, x)
```

Maxima [F]

$$\int xW\left(\frac{a}{x}\right)^2\,dx = \int x\,\mathrm{W}(\frac{a}{x})^2\,dx$$

input
```
integrate(x*lambert_w(a/x)^2,x, algorithm="maxima")
```

output
```
integrate(x*lambert_w(a/x)^2, x)
```

Giac [F]

$$\int xW\left(\frac{a}{x}\right)^2\,dx = \int x\,\mathrm{W}(\frac{a}{x})^2\,dx$$

input
```
integrate(x*lambert_w(a/x)^2,x, algorithm="giac")
```

output
```
integrate(x*lambert_w(a/x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int xW\left(\frac{a}{x}\right)^2\,dx = \int x\,\mathrm{LambertW}\left(\frac{a}{x}\right)^2\,dx$$

input
```
int(x*LambertW(a/x)^2,x)
```

output
```
int(x*LambertW(a/x)^2, x)
```

Reduce [F]

$$\int xW\left(\frac{a}{x}\right)^2 dx = \left(\int \frac{lambert\_w(\frac{a}{x})}{e^{lambert\_w(\frac{a}{x})}lambert\_w\left(\frac{a}{x}\right) + e^{lambert\_w(\frac{a}{x})}}dx\right)a + \frac{lambert\_w(\frac{a}{x})^2 x^2}{2}$$

input
```
int(x*Lambert_W(a/x)^2,x)
```

output
```
(2*int(lambert_w(a/x)/(e**lambert_w(a/x)*lambert_w(a/x) + e**lambert_w(a/x
)),x)*a + lambert_w(a/x)**2*x**2)/2
```

3.201 $\int W\left(\frac{a}{x}\right)^2 dx$

Optimal result

Integrand size = 8, antiderivative size = 20

$$\int W\left(\frac{a}{x}\right)^2 dx = 2xW\left(\frac{a}{x}\right) + xW\left(\frac{a}{x}\right)^2$$

output
```
2*x*LambertW(a/x)+x*LambertW(a/x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int W\left(\frac{a}{x}\right)^2 dx = 2xW\left(\frac{a}{x}\right) + xW\left(\frac{a}{x}\right)^2$$

input
```
Integrate[ProductLog[a/x]^2,x]
```

output
```
2*x*ProductLog[a/x] + x*ProductLog[a/x]^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7169, 7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W\left(\frac{a}{x}\right)^2 dx$$

$$\Big\downarrow \text{7169}$$

$$2\int \frac{W\left(\frac{a}{x}\right)^2}{W\left(\frac{a}{x}\right)+1}dx + xW\left(\frac{a}{x}\right)^2$$

$$\Big\downarrow \text{7187}$$

$$xW\left(\frac{a}{x}\right)^2 + 2xW\left(\frac{a}{x}\right)$$

input
```
Int[ProductLog[a/x]^2,x]
```

output
```
2*x*ProductLog[a/x] + x*ProductLog[a/x]^2
```

Defintions of rubi rules used

rule 7169
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*(c*Produ
ctLog[a*x^n])^p, x] - Simp[n*p   Int[(c*ProductLog[a*x^n])^p/(1 + ProductLo
g[a*x^n]), x], x] /; FreeQ[{a, c, n, p}, x] && (EqQ[n*(p - 1), -1] || (Inte
gerQ[p - 1/2] && EqQ[n*(p - 1/2), -1]))
```

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$2x\,\mathrm{LambertW}\left(\frac{a}{x}\right)+x\,\mathrm{LambertW}\left(\frac{a}{x}\right)^{2}$	21
derivativedivides	$-a\left(-\frac{2x\,\mathrm{LambertW}\left(\frac{a}{x}\right)}{a}-\frac{x\,\mathrm{LambertW}\left(\frac{a}{x}\right)^{2}}{a}\right)$	31
default	$-a\left(-\frac{2x\,\mathrm{LambertW}\left(\frac{a}{x}\right)}{a}-\frac{x\,\mathrm{LambertW}\left(\frac{a}{x}\right)^{2}}{a}\right)$	31

input
```
int(LambertW(a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
2*x*LambertW(a/x)+x*LambertW(a/x)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int W\left(\frac{a}{x}\right)^2\,dx = x\,\mathrm{W}(\frac{a}{x})^2 + 2\,x\,\mathrm{W}(\frac{a}{x})$$

input
```
integrate(lambert_w(a/x)^2,x, algorithm="fricas")
```

output
```
x*lambert_w(a/x)^2 + 2*x*lambert_w(a/x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int W\left(\frac{a}{x}\right)^2\,dx = xW^2\left(\frac{a}{x}\right) + 2xW\left(\frac{a}{x}\right)$$

input
```
integrate(LambertW(a/x)**2,x)
```

output
```
x*LambertW(a/x)**2 + 2*x*LambertW(a/x)
```

Maxima [F]

$$\int W\left(\frac{a}{x}\right)^2 dx = \int W(\frac{a}{x})^2 dx$$

input
```
integrate(lambert_w(a/x)^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x)^2, x)
```

Giac [F]

$$\int W\left(\frac{a}{x}\right)^2 dx = \int W(\frac{a}{x})^2 dx$$

input
```
integrate(lambert_w(a/x)^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int W\left(\frac{a}{x}\right)^2 dx = \int \mathrm{LambertW}\left(\frac{a}{x}\right)^2 dx$$

input
```
int(LambertW(a/x)^2,x)
```

output
```
int(LambertW(a/x)^2, x)
```

Reduce [F]

$$\int W\left(\frac{a}{x}\right)^2 dx = 2\left(\int \frac{lambert\_w\left(\frac{a}{x}\right)^2}{lambert\_w\left(\frac{a}{x}\right)+1}dx\right) - 2\left(\int \frac{lambert\_w\left(\frac{a}{x}\right)}{lambert\_w\left(\frac{a}{x}\right)+1}dx\right) + 2\left(\int \frac{lambert\_w\left(\frac{a}{x}\right)}{e^{lambert\_w\left(\frac{a}{x}\right)} lambert\_w\left(\frac{a}{x}\right) x + e^{lambert\_w\left(\frac{a}{x}\right)} x}dx\right) a - 4\left(\int \frac{1}{lambert\_w\left(\frac{a}{x}\right)+1}dx\right) - 2\left(\int \frac{1}{e^{lambert\_w\left(\frac{a}{x}\right)} lambert\_w\left(\frac{a}{x}\right) x + e^{lambert\_w\left(\frac{a}{x}\right)} x}dx\right) a + lambert\_w\left(\frac{a}{x}\right)^2 x - 2lambert\_w\left(\frac{a}{x}\right) x + 4x$$

input
```
int(Lambert_W(a/x)^2,x)
```

output
```
2*int(lambert_w(a/x)**2/(lambert_w(a/x) + 1),x) - 2*int(lambert_w(a/x)/(la
mbert_w(a/x) + 1),x) + 2*int(lambert_w(a/x)/(e**lambert_w(a/x)*lambert_w(a
/x)*x + e**lambert_w(a/x)*x),x)*a - 4*int(1/(lambert_w(a/x) + 1),x) - 2*in
t(1/(e**lambert_w(a/x)*lambert_w(a/x)*x + e**lambert_w(a/x)*x),x)*a + lamb
ert_w(a/x)**2*x - 2*lambert_w(a/x)*x + 4*x
```

3.202 $\int \frac{W\left(\frac{a}{x}\right)^2}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x} \, dx = -\frac{1}{2} W\left(\frac{a}{x}\right)^2 - \frac{1}{3} W\left(\frac{a}{x}\right)^3$$

output
```
-1/2*LambertW(a/x)^2-1/3*LambertW(a/x)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x} \, dx = -\frac{1}{2} W\left(\frac{a}{x}\right)^2 - \frac{1}{3} W\left(\frac{a}{x}\right)^3$$

input
```
Integrate[ProductLog[a/x]^2/x,x]
```

output
```
-1/2*ProductLog[a/x]^2 - ProductLog[a/x]^3/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x}dx$$
$$\Big\downarrow \text{7173}$$
$$\int \frac{W\left(\frac{a}{x}\right)^3}{x\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{1}{2}W\left(\frac{a}{x}\right)^2$$
$$\Big\downarrow \text{7200}$$
$$-\frac{1}{3}W\left(\frac{a}{x}\right)^3-\frac{1}{2}W\left(\frac{a}{x}\right)^2$$

input
```
Int[ProductLog[a/x]^2/x,x]
```

output
```
-1/2*ProductLog[a/x]^2 - ProductLog[a/x]^3/3
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^{2}}{2}-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^{3}}{3}$	22
default	$-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^{2}}{2}-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^{3}}{3}$	22

input
```
int(LambertW(a/x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*LambertW(a/x)^2-1/3*LambertW(a/x)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x}\,dx = -\frac{1}{3}\,\mathrm{W}\left(\frac{a}{x}\right)^3 - \frac{1}{2}\,\mathrm{W}\left(\frac{a}{x}\right)^2$$

input
```
integrate(lambert_w(a/x)^2/x,x, algorithm="fricas")
```

output
```
-1/3*lambert_w(a/x)^3 - 1/2*lambert_w(a/x)^2
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x}\,dx = -\frac{W^3\left(\frac{a}{x}\right)}{3} - \frac{W^2\left(\frac{a}{x}\right)}{2}$$

input
```
integrate(LambertW(a/x)**2/x,x)
```

output
```
-LambertW(a/x)**3/3 - LambertW(a/x)**2/2
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x}\,dx = \int \frac{W(\frac{a}{x})^2}{x}\,dx$$

input
```
integrate(lambert_w(a/x)^2/x,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x)^2/x, x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x}\,dx = \int \frac{W(\frac{a}{x})^2}{x}\,dx$$

input
```
integrate(lambert_w(a/x)^2/x,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x}\,dx = \int \frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^2}{x}\,dx$$

input
```
int(LambertW(a/x)^2/x,x)
```

output
```
int(LambertW(a/x)^2/x, x)
```

Reduce [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x}\, dx = \int \frac{lambert\_w\left(\frac{a}{x}\right)^2}{x} dx$$

input
```
int(Lambert_W(a/x)^2/x,x)
```

output
```
int(lambert_w(a/x)**2/x,x)
```

3.203 $\int \frac{W\left(\frac{a}{x}\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^2}\,dx = -\frac{4}{x}+\frac{4}{xW\left(\frac{a}{x}\right)}+\frac{2W\left(\frac{a}{x}\right)}{x}-\frac{W\left(\frac{a}{x}\right)^2}{x}$$

output
```
-4/x+4/x/LambertW(a/x)+2*LambertW(a/x)/x-LambertW(a/x)^2/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^2}\,dx = -\frac{-4+4W\left(\frac{a}{x}\right)-2W\left(\frac{a}{x}\right)^2+W\left(\frac{a}{x}\right)^3}{xW\left(\frac{a}{x}\right)}$$

input
```
Integrate[ProductLog[a/x]^2/x^2,x]
```

output
```
-((-4 + 4*ProductLog[a/x] - 2*ProductLog[a/x]^2 + ProductLog[a/x]^3)/(x*Pr
oductLog[a/x]))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {7172, 7205, 7205, 7199, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^2}dx$$

$$\Big\downarrow \text{7172}$$

$$-2\int \frac{W\left(\frac{a}{x}\right)^2}{x^2\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{W\left(\frac{a}{x}\right)^2}{x}$$

$$\Big\downarrow \text{7205}$$

$$-2\left(-2\int \frac{W\left(\frac{a}{x}\right)}{x^2\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{W\left(\frac{a}{x}\right)}{x}\right)-\frac{W\left(\frac{a}{x}\right)^2}{x}$$

$$\Big\downarrow \text{7205}$$

$$-2\left(-2\left(-\int \frac{1}{x^2\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{1}{x}\right)-\frac{W\left(\frac{a}{x}\right)}{x}\right)-\frac{W\left(\frac{a}{x}\right)^2}{x}$$

$$\Big\downarrow \text{7199}$$

$$-2\left(-2\left(\int \frac{1}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}-\frac{1}{x}\right)-\frac{W\left(\frac{a}{x}\right)}{x}\right)-\frac{W\left(\frac{a}{x}\right)^2}{x}$$

$$\Big\downarrow \text{7176}$$

$$-\frac{W\left(\frac{a}{x}\right)^2}{x}-2\left(-2\left(\frac{1}{xW\left(\frac{a}{x}\right)}-\frac{1}{x}\right)-\frac{W\left(\frac{a}{x}\right)}{x}\right)$$

input
```
Int[ProductLog[a/x]^2/x^2,x]
```

output
```
-(ProductLog[a/x]^2/x) - 2*(-2*(-x^(-1) + 1/(x*ProductLog[a/x])) - Product
Log[a/x]/x)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{4-\operatorname{LambertW}\left(\frac{a}{x}\right)^{3}+2\operatorname{LambertW}\left(\frac{a}{x}\right)^{2}-4\operatorname{LambertW}\left(\frac{a}{x}\right)}{x\operatorname{LambertW}\left(\frac{a}{x}\right)}$	43
derivativedivides	$-\frac{\frac{\operatorname{LambertW}\left(\frac{a}{x}\right)^{2}a}{x}-\frac{2a\operatorname{LambertW}\left(\frac{a}{x}\right)}{x}+\frac{4a}{x}-\frac{4a}{x\operatorname{LambertW}\left(\frac{a}{x}\right)}}{a}$	52
default	$-\frac{\frac{\operatorname{LambertW}\left(\frac{a}{x}\right)^{2}a}{x}-\frac{2a\operatorname{LambertW}\left(\frac{a}{x}\right)}{x}+\frac{4a}{x}-\frac{4a}{x\operatorname{LambertW}\left(\frac{a}{x}\right)}}{a}$	52

input
```
int(LambertW(a/x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x*(4-LambertW(a/x)^3+2*LambertW(a/x)^2-4*LambertW(a/x))/LambertW(a/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^2}\,dx = -\frac{\mathrm{W}(\frac{a}{x})^3 - 2\ \mathrm{W}(\frac{a}{x})^2 + 4\ \mathrm{W}(\frac{a}{x}) - 4}{x\,\mathrm{W}(\frac{a}{x})}$$

input
```
integrate(lambert_w(a/x)^2/x^2,x, algorithm="fricas")
```

output
```
-(lambert_w(a/x)^3 - 2*lambert_w(a/x)^2 + 4*lambert_w(a/x) - 4)/(x*lambert
_w(a/x))
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^2}\,dx = \begin{cases} -\frac{W^2\left(\frac{a}{x}\right)}{x} + \frac{2W\left(\frac{a}{x}\right)}{x} - \frac{4}{x} + \frac{4}{xW\left(\frac{a}{x}\right)} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(LambertW(a/x)**2/x**2,x)
```

output
```
Piecewise((-LambertW(a/x)**2/x + 2*LambertW(a/x)/x - 4/x + 4/(x*LambertW(a
/x)), Ne(a, 0)), (0, True))
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^2}\,dx = \int \frac{\mathrm{W}(\frac{a}{x})^2}{x^2}\,dx$$

input
```
integrate(lambert_w(a/x)^2/x^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x)^2/x^2, x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^2}\,dx = \int \frac{\mathrm{W}(\frac{a}{x})^2}{x^2}\,dx$$

input
```
integrate(lambert_w(a/x)^2/x^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^2}\,dx = \int \frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^2}{x^2}\,dx$$

input
```
int(LambertW(a/x)^2/x^2,x)
```

output
```
int(LambertW(a/x)^2/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^2}\,dx = \frac{e^{lambert\_w\left(\frac{a}{x}\right)}\left(-lambert\_w\left(\frac{a}{x}\right)^3+2lambert\_w\left(\frac{a}{x}\right)^2-4lambert\_w\left(\frac{a}{x}\right)+4\right)}{a}$$

input
```
int(Lambert_W(a/x)^2/x^2,x)
```

output
```
(e**lambert_w(a/x)*( - lambert_w(a/x)**3 + 2*lambert_w(a/x)**2 - 4*lambert
_w(a/x) + 4))/a
```

3.204 $\int \frac{W\left(\frac{a}{x}\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^3}\,dx = -\frac{3}{4x^2} - \frac{3}{8x^2W\left(\frac{a}{x}\right)^2} + \frac{3}{4x^2W\left(\frac{a}{x}\right)} + \frac{W\left(\frac{a}{x}\right)}{2x^2} - \frac{W\left(\frac{a}{x}\right)^2}{2x^2}$$

output
```
-3/4/x^2-3/8/x^2/LambertW(a/x)^2+3/4/x^2/LambertW(a/x)+1/2*LambertW(a/x)/x
^2-1/2*LambertW(a/x)^2/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^3}\,dx = -\frac{3}{4x^2} - \frac{3}{8x^2W\left(\frac{a}{x}\right)^2} + \frac{3}{4x^2W\left(\frac{a}{x}\right)} + \frac{W\left(\frac{a}{x}\right)}{2x^2} - \frac{W\left(\frac{a}{x}\right)^2}{2x^2}$$

input
```
Integrate[ProductLog[a/x]^2/x^3,x]
```

output
```
-3/(4*x^2) - 3/(8*x^2*ProductLog[a/x]^2) + 3/(4*x^2*ProductLog[a/x]) + Pro
ductLog[a/x]/(2*x^2) - ProductLog[a/x]^2/(2*x^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7172, 7205, 7205, 7199, 7194, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^3}dx$$

$$\Big\downarrow \text{7172}$$

$$-\int \frac{W\left(\frac{a}{x}\right)^2}{x^3\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{W\left(\frac{a}{x}\right)^2}{2x^2}$$

$$\Big\downarrow \text{7205}$$

$$\frac{3}{2}\int \frac{W\left(\frac{a}{x}\right)}{x^3\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{W\left(\frac{a}{x}\right)^2}{2x^2}+\frac{W\left(\frac{a}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{7205}$$

$$\frac{3}{2}\left(-\int \frac{1}{x^3\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{1}{2x^2}\right)-\frac{W\left(\frac{a}{x}\right)^2}{2x^2}+\frac{W\left(\frac{a}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{7199}$$

$$\frac{3}{2}\left(\int \frac{1}{x\left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}-\frac{1}{2x^2}\right)-\frac{W\left(\frac{a}{x}\right)^2}{2x^2}+\frac{W\left(\frac{a}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{7194}$$

$$\frac{3}{2}\left(-\frac{1}{2}\int \frac{1}{xW\left(\frac{a}{x}\right)\left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}+\frac{1}{2x^2W\left(\frac{a}{x}\right)}-\frac{1}{2x^2}\right)-\frac{W\left(\frac{a}{x}\right)^2}{2x^2}+\frac{W\left(\frac{a}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{7201}$$

$$-\frac{W\left(\frac{a}{x}\right)^2}{2x^2}+\frac{W\left(\frac{a}{x}\right)}{2x^2}+\frac{3}{2}\left(\frac{1}{2x^2W\left(\frac{a}{x}\right)}-\frac{1}{4x^2W\left(\frac{a}{x}\right)^2}-\frac{1}{2x^2}\right)$$

input
```
Int[ProductLog[a/x]^2/x^3,x]
```

output
```
(3*(-1/2*1/x^2 - 1/(4*x^2*ProductLog[a/x]^2) + 1/(2*x^2*ProductLog[a/x])))
/2 + ProductLog[a/x]/(2*x^2) - ProductLog[a/x]^2/(2*x^2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$-\frac{-\frac{\operatorname{LambertW}\left(\frac{a}{x}\right)a^{2}}{2x^{2}}+\frac{3a^{2}}{4x^{2}}-\frac{3a^{2}}{4\operatorname{LambertW}\left(\frac{a}{x}\right)x^{2}}+\frac{3a^{2}}{8x^{2}\operatorname{LambertW}\left(\frac{a}{x}\right)^{2}}+\frac{\operatorname{LambertW}\left(\frac{a}{x}\right)^{2}a^{2}}{2x^{2}}}{a^{2}}$	77
default	$-\frac{-\frac{\operatorname{LambertW}\left(\frac{a}{x}\right)a^{2}}{2x^{2}}+\frac{3a^{2}}{4x^{2}}-\frac{3a^{2}}{4\operatorname{LambertW}\left(\frac{a}{x}\right)x^{2}}+\frac{3a^{2}}{8x^{2}\operatorname{LambertW}\left(\frac{a}{x}\right)^{2}}+\frac{\operatorname{LambertW}\left(\frac{a}{x}\right)^{2}a^{2}}{2x^{2}}}{a^{2}}$	77

input
```
int(LambertW(a/x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(-1/2*LambertW(a/x)/x^2*a^2+3/4*a^2/x^2-3/4/LambertW(a/x)/x^2*a^2+3
/8/x^2*a^2/LambertW(a/x)^2+1/2*LambertW(a/x)^2/x^2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^3}\,dx = -\frac{4\;\mathrm{W}(\frac{a}{x})^4-4\;\mathrm{W}(\frac{a}{x})^3+6\;\mathrm{W}(\frac{a}{x})^2-6\;\mathrm{W}(\frac{a}{x})+3}{8\,x^2\;\mathrm{W}(\frac{a}{x})^2}$$

input
```
integrate(lambert_w(a/x)^2/x^3,x, algorithm="fricas")
```

output
```
-1/8*(4*lambert_w(a/x)^4 - 4*lambert_w(a/x)^3 + 6*lambert_w(a/x)^2 - 6*lam
bert_w(a/x) + 3)/(x^2*lambert_w(a/x)^2)
```

Sympy [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^3}\,dx = \int \frac{W^2\left(\frac{a}{x}\right)}{x^3}\,dx$$

input
```
integrate(LambertW(a/x)**2/x**3,x)
```

output
```
Integral(LambertW(a/x)**2/x**3, x)
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^3}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)^2}{x^3}\,dx$$

input
```
integrate(lambert_w(a/x)^2/x^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x)^2/x^3, x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^3}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)^2}{x^3}\,dx$$

input
```
integrate(lambert_w(a/x)^2/x^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^3}\,dx = \int \frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^2}{x^3}\,dx$$

input
```
int(LambertW(a/x)^2/x^3,x)
```

output
```
int(LambertW(a/x)^2/x^3, x)
```

Reduce [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^3}\,dx = \int \frac{lambert\_w\left(\frac{a}{x}\right)^2}{x^3}dx$$

input
```
int(Lambert_W(a/x)^2/x^3,x)
```

output
```
int(lambert_w(a/x)**2/x**3,x)
```

3.205 $\int \frac{W\left(\frac{a}{x}\right)^2}{x^4} dx$

Optimal result

Integrand size = 12, antiderivative size = 81

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^4} dx = -\frac{8}{27x^3} + \frac{16}{243x^3W\left(\frac{a}{x}\right)^3} - \frac{16}{81x^3W\left(\frac{a}{x}\right)^2} + \frac{8}{27x^3W\left(\frac{a}{x}\right)} + \frac{2W\left(\frac{a}{x}\right)}{9x^3} - \frac{W\left(\frac{a}{x}\right)^2}{3x^3}$$

output
```
-8/27/x^3+16/243/x^3/LambertW(a/x)^3-16/81/x^3/LambertW(a/x)^2+8/27/x^3/La
mbertW(a/x)+2/9*LambertW(a/x)/x^3-1/3*LambertW(a/x)^2/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^4} dx = -\frac{8}{27x^3} + \frac{16}{243x^3W\left(\frac{a}{x}\right)^3} - \frac{16}{81x^3W\left(\frac{a}{x}\right)^2} + \frac{8}{27x^3W\left(\frac{a}{x}\right)} + \frac{2W\left(\frac{a}{x}\right)}{9x^3} - \frac{W\left(\frac{a}{x}\right)^2}{3x^3}$$

input
```
Integrate[ProductLog[a/x]^2/x^4,x]
```

output
```
-8/(27*x^3) + 16/(243*x^3*ProductLog[a/x]^3) - 16/(81*x^3*ProductLog[a/x]^
2) + 8/(27*x^3*ProductLog[a/x]) + (2*ProductLog[a/x])/(9*x^3) - ProductLog
[a/x]^2/(3*x^3)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {7172, 7205, 7205, 7199, 7194, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^4}dx$$

$$\Big\downarrow \text{ 7172}$$

$$-\frac{2}{3}\int \frac{W\left(\frac{a}{x}\right)^2}{x^4 \left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{W\left(\frac{a}{x}\right)^2}{3x^3}$$

$$\Big\downarrow \text{ 7205}$$

$$-\frac{2}{3}\left(-\frac{4}{3}\int \frac{W\left(\frac{a}{x}\right)}{x^4 \left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{W\left(\frac{a}{x}\right)}{3x^3}\right)-\frac{W\left(\frac{a}{x}\right)^2}{3x^3}$$

$$\Big\downarrow \text{ 7205}$$

$$-\frac{2}{3}\left(-\frac{4}{3}\left(-\int \frac{1}{x^4 \left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{1}{3x^3}\right)-\frac{W\left(\frac{a}{x}\right)}{3x^3}\right)-\frac{W\left(\frac{a}{x}\right)^2}{3x^3}$$

$$\Big\downarrow \text{ 7199}$$

$$-\frac{2}{3}\left(-\frac{4}{3}\left(\int \frac{1}{x^2 \left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}-\frac{1}{3x^3}\right)-\frac{W\left(\frac{a}{x}\right)}{3x^3}\right)-\frac{W\left(\frac{a}{x}\right)^2}{3x^3}$$

$$\Big\downarrow \text{ 7194}$$

$$-\frac{2}{3}\left(-\frac{4}{3}\left(-\frac{2}{3}\int \frac{1}{x^2 W\left(\frac{a}{x}\right) \left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}+\frac{1}{3x^3 W\left(\frac{a}{x}\right)}-\frac{1}{3x^3}\right)-\frac{W\left(\frac{a}{x}\right)}{3x^3}\right)-\frac{W\left(\frac{a}{x}\right)^2}{3x^3}$$

$$\Big\downarrow \text{ 7205}$$

$$-\frac{2}{3}\left(-\frac{4}{3}\left(-\frac{2}{3}\left(\frac{1}{3x^3 W\left(\frac{a}{x}\right)^2}-\frac{1}{3}\int \frac{1}{x^2 W\left(\frac{a}{x}\right)^2 \left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}\right)+\frac{1}{3x^3 W\left(\frac{a}{x}\right)}-\frac{1}{3x^3}\right)-\frac{W\left(\frac{a}{x}\right)}{3x^3}\right)-\frac{W\left(\frac{a}{x}\right)^2}{3x^3}$$

$$\Big\downarrow \text{ 7201}$$

$$-\frac{W\left(\frac{a}{x}\right)^2}{3x^3}-\frac{2}{3}\left(-\frac{4}{3}\left(-\frac{2}{3}\left(\frac{1}{3x^3W\left(\frac{a}{x}\right)^2}-\frac{1}{9x^3W\left(\frac{a}{x}\right)^3}\right)+\frac{1}{3x^3W\left(\frac{a}{x}\right)}-\frac{1}{3x^3}\right)-\frac{W\left(\frac{a}{x}\right)}{3x^3}\right)$$

input
```
Int[ProductLog[a/x]^2/x^4,x]
```

output
```
-1/3*ProductLog[a/x]^2/x^3 - (2*((-4*(-1/3*1/x^3 - (2*(-1/9*1/(x^3*Product
Log[a/x]^3) + 1/(3*x^3*ProductLog[a/x]^2)))/3 + 1/(3*x^3*ProductLog[a/x]))
)/3 - ProductLog[a/x]/(3*x^3)))/3
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.15

method	result
derivativedivides	$-\frac{-\frac{2\,\mathrm{LambertW}\left(\frac{a}{x}\right)a^{3}}{9x^{3}}+\frac{8a^{3}}{27x^{3}}-\frac{8a^{3}}{27\,\mathrm{LambertW}\left(\frac{a}{x}\right)x^{3}}+\frac{16a^{3}}{81\,\mathrm{LambertW}\left(\frac{a}{x}\right)^{2}x^{3}}-\frac{16a^{3}}{243x^{3}\,\mathrm{LambertW}\left(\frac{a}{x}\right)^{3}}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^{2}a^{3}}{3x^{3}}}{a^{3}}$
default	$-\frac{-\frac{2\,\mathrm{LambertW}\left(\frac{a}{x}\right)a^{3}}{9x^{3}}+\frac{8a^{3}}{27x^{3}}-\frac{8a^{3}}{27\,\mathrm{LambertW}\left(\frac{a}{x}\right)x^{3}}+\frac{16a^{3}}{81\,\mathrm{LambertW}\left(\frac{a}{x}\right)^{2}x^{3}}-\frac{16a^{3}}{243x^{3}\,\mathrm{LambertW}\left(\frac{a}{x}\right)^{3}}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^{2}a^{3}}{3x^{3}}}{a^{3}}$

input
```
int(LambertW(a/x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*(-2/9*LambertW(a/x)/x^3*a^3+8/27*a^3/x^3-8/27/LambertW(a/x)/x^3*a^3
+16/81/LambertW(a/x)^2/x^3*a^3-16/243/x^3*a^3/LambertW(a/x)^3+1/3*LambertW
(a/x)^2/x^3*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^4}\,dx=-\frac{81\;\mathrm{W}(\frac{a}{x})^5-54\;\mathrm{W}(\frac{a}{x})^4+72\;\mathrm{W}(\frac{a}{x})^3-72\;\mathrm{W}(\frac{a}{x})^2+48\;\mathrm{W}(\frac{a}{x})-16}{243\,x^3\,\mathrm{W}(\frac{a}{x})^3}$$

input
```
integrate(lambert_w(a/x)^2/x^4,x, algorithm="fricas")
```

output
```
-1/243*(81*lambert_w(a/x)^5 - 54*lambert_w(a/x)^4 + 72*lambert_w(a/x)^3 -
72*lambert_w(a/x)^2 + 48*lambert_w(a/x) - 16)/(x^3*lambert_w(a/x)^3)
```

Sympy [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^4}\,dx = \int \frac{W^2\left(\frac{a}{x}\right)}{x^4}\,dx$$

input
```
integrate(LambertW(a/x)**2/x**4,x)
```

output
```
Integral(LambertW(a/x)**2/x**4, x)
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^4}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)^2}{x^4}\,dx$$

input
```
integrate(lambert_w(a/x)^2/x^4,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x)^2/x^4, x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^4}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)^2}{x^4}\,dx$$

input
```
integrate(lambert_w(a/x)^2/x^4,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^4} \, dx = \int \frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^2}{x^4} \, dx$$

input
```
int(LambertW(a/x)^2/x^4,x)
```

output
```
int(LambertW(a/x)^2/x^4, x)
```

Reduce [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^4} \, dx = \int \frac{lambert\_w\left(\frac{a}{x}\right)^2}{x^4} dx$$

input
```
int(Lambert_W(a/x)^2/x^4,x)
```

output
```
int(lambert_w(a/x)**2/x**4,x)
```

3.206 $\int \frac{W\left(\frac{a}{x}\right)^2}{x^5}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 96

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^5}\,dx = -\frac{5}{32x^4} - \frac{15}{1024x^4W\left(\frac{a}{x}\right)^4} + \frac{15}{256x^4W\left(\frac{a}{x}\right)^3} - \frac{15}{128x^4W\left(\frac{a}{x}\right)^2} + \frac{5}{32x^4W\left(\frac{a}{x}\right)} + \frac{W\left(\frac{a}{x}\right)}{8x^4} - \frac{W\left(\frac{a}{x}\right)^2}{4x^4}$$

output
```
-5/32/x^4-15/1024/x^4/LambertW(a/x)^4+15/256/x^4/LambertW(a/x)^3-15/128/x^
4/LambertW(a/x)^2+5/32/x^4/LambertW(a/x)+1/8*LambertW(a/x)/x^4-1/4*Lambert
W(a/x)^2/x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^5}\,dx = -\frac{5}{32x^4} - \frac{15}{1024x^4W\left(\frac{a}{x}\right)^4} + \frac{15}{256x^4W\left(\frac{a}{x}\right)^3} - \frac{15}{128x^4W\left(\frac{a}{x}\right)^2} + \frac{5}{32x^4W\left(\frac{a}{x}\right)} + \frac{W\left(\frac{a}{x}\right)}{8x^4} - \frac{W\left(\frac{a}{x}\right)^2}{4x^4}$$

input
```
Integrate[ProductLog[a/x]^2/x^5,x]
```

output
```
-5/(32*x^4) - 15/(1024*x^4*ProductLog[a/x]^4) + 15/(256*x^4*ProductLog[a/x
]^3) - 15/(128*x^4*ProductLog[a/x]^2) + 5/(32*x^4*ProductLog[a/x]) + Produ
ctLog[a/x]/(8*x^4) - ProductLog[a/x]^2/(4*x^4)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {7172, 7205, 7205, 7199, 7194, 7205, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^5}dx$$

$$\Big\downarrow \text{ 7172}$$

$$-\frac{1}{2}\int \frac{W\left(\frac{a}{x}\right)^2}{x^5\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{W\left(\frac{a}{x}\right)^2}{4x^4}$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{2}\left(\frac{5}{4}\int \frac{W\left(\frac{a}{x}\right)}{x^5\left(W\left(\frac{a}{x}\right)+1\right)}dx+\frac{W\left(\frac{a}{x}\right)}{4x^4}\right)-\frac{W\left(\frac{a}{x}\right)^2}{4x^4}$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{2}\left(\frac{5}{4}\left(-\int \frac{1}{x^5\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{1}{4x^4}\right)+\frac{W\left(\frac{a}{x}\right)}{4x^4}\right)-\frac{W\left(\frac{a}{x}\right)^2}{4x^4}$$

$$\Big\downarrow \text{ 7199}$$

$$\frac{1}{2}\left(\frac{5}{4}\left(\int \frac{1}{x^3\left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}-\frac{1}{4x^4}\right)+\frac{W\left(\frac{a}{x}\right)}{4x^4}\right)-\frac{W\left(\frac{a}{x}\right)^2}{4x^4}$$

$$\Big\downarrow \text{ 7194}$$

$$\frac{1}{2}\left(\frac{5}{4}\left(-\frac{3}{4}\int \frac{1}{x^3 W\left(\frac{a}{x}\right)\left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}+\frac{1}{4x^4 W\left(\frac{a}{x}\right)}-\frac{1}{4x^4}\right)+\frac{W\left(\frac{a}{x}\right)}{4x^4}\right)-\frac{W\left(\frac{a}{x}\right)^2}{4x^4}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{2}\left(\frac{5}{4}\left(-\frac{3}{4}\left(\frac{1}{4x^4 W\left(\frac{a}{x}\right)^2}-\frac{1}{2}\int \frac{1}{x^3 W\left(\frac{a}{x}\right)^2\left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}\right)+\frac{1}{4x^4 W\left(\frac{a}{x}\right)}-\frac{1}{4x^4}\right)+\frac{W\left(\frac{a}{x}\right)}{4x^4}\right)-\frac{W\left(\frac{a}{x}\right)^2}{4x^4}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{2}\left(\frac{5}{4}\left(-\frac{3}{4}\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{x^3 W\left(\frac{a}{x}\right)^3\left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}-\frac{1}{4x^4 W\left(\frac{a}{x}\right)^3}\right)+\frac{1}{4x^4 W\left(\frac{a}{x}\right)^2}\right)+\frac{1}{4x^4 W\left(\frac{a}{x}\right)}-\frac{1}{4x^4}\right)+\frac{W\left(\frac{a}{x}\right)}{4x^4}$$
$$\frac{W\left(\frac{a}{x}\right)^2}{4x^4}$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{2}\left(\frac{5}{4}\left(-\frac{3}{4}\left(\frac{1}{2}\left(\frac{1}{16x^4 W\left(\frac{a}{x}\right)^4}-\frac{1}{4x^4 W\left(\frac{a}{x}\right)^3}\right)+\frac{1}{4x^4 W\left(\frac{a}{x}\right)^2}\right)+\frac{1}{4x^4 W\left(\frac{a}{x}\right)}-\frac{1}{4x^4}\right)+\frac{W\left(\frac{a}{x}\right)}{4x^4}\right)-\frac{W\left(\frac{a}{x}\right)^2}{4x^4}$$

input
```
Int[ProductLog[a/x]^2/x^5,x]
```

output
```
-1/4*ProductLog[a/x]^2/x^4 + ((5*(-1/4*1/x^4 - (3*((1/(16*x^4*ProductLog[a
/x]^4) - 1/(4*x^4*ProductLog[a/x]^3))/2 + 1/(4*x^4*ProductLog[a/x]^2)))/4
+ 1/(4*x^4*ProductLog[a/x])))/4 + ProductLog[a/x]/(4*x^4))/2
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.14

method	result
derivativedivides	$-\frac{-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)a^{4}}{8x^{4}}+\frac{5a^{4}}{32x^{4}}-\frac{5a^{4}}{32\,\mathrm{LambertW}\left(\frac{a}{x}\right)x^{4}}+\frac{15a^{4}}{128x^{4}\,\mathrm{LambertW}\left(\frac{a}{x}\right)^{2}}-\frac{15a^{4}}{256\,\mathrm{LambertW}\left(\frac{a}{x}\right)^{3}x^{4}}+\frac{15a^{4}}{1024x^{4}\,\mathrm{LambertW}\left(\frac{a}{x}\right.}}{a^{4}}$
default	$-\frac{-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)a^{4}}{8x^{4}}+\frac{5a^{4}}{32x^{4}}-\frac{5a^{4}}{32\,\mathrm{LambertW}\left(\frac{a}{x}\right)x^{4}}+\frac{15a^{4}}{128x^{4}\,\mathrm{LambertW}\left(\frac{a}{x}\right)^{2}}-\frac{15a^{4}}{256\,\mathrm{LambertW}\left(\frac{a}{x}\right)^{3}x^{4}}+\frac{15a^{4}}{1024x^{4}\,\mathrm{LambertW}\left(\frac{a}{x}\right.}}{a^{4}}$

input
```
int(LambertW(a/x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/a^4*(-1/8*LambertW(a/x)/x^4*a^4+5/32/x^4*a^4-5/32/LambertW(a/x)/x^4*a^4
+15/128/x^4*a^4/LambertW(a/x)^2-15/256/LambertW(a/x)^3/x^4*a^4+15/1024/x^4
*a^4/LambertW(a/x)^4+1/4*LambertW(a/x)^2/x^4*a^4)
```

Fricas [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^5}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)^2}{x^5}\,dx$$

input
```
integrate(lambert_w(a/x)^2/x^5,x, algorithm="fricas")
```

output
```
integral(lambert_w(a/x)^2/x^5, x)
```

Sympy [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^5}\,dx = \int \frac{W^2\left(\frac{a}{x}\right)}{x^5}\,dx$$

input
```
integrate(LambertW(a/x)**2/x**5,x)
```

output
```
Integral(LambertW(a/x)**2/x**5, x)
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^5}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)^2}{x^5}\,dx$$

input
```
integrate(lambert_w(a/x)^2/x^5,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x)^2/x^5, x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^5}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)^2}{x^5}\,dx$$

input
```
integrate(lambert_w(a/x)^2/x^5,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^5}\,dx = \int \frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^2}{x^5}\,dx$$

input
```
int(LambertW(a/x)^2/x^5,x)
```

output
```
int(LambertW(a/x)^2/x^5, x)
```

Reduce [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{x^5} \, dx = \int \frac{lambert\_w\left(\frac{a}{x}\right)^2}{x^5} dx$$

input
```
int(Lambert_W(a/x)^2/x^5,x)
```

output
```
int(lambert_w(a/x)**2/x**5,x)
```

3.207 $\int x^3\sqrt{W\left(\frac{a}{x}\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 94

$$\int x^3\sqrt{W\left(\frac{a}{x}\right)}\,dx = -\frac{256}{105}a^4\sqrt{\pi}\operatorname{erf}\left(2\sqrt{W\left(\frac{a}{x}\right)}\right)+\frac{2}{7}x^4\sqrt{W\left(\frac{a}{x}\right)} -\frac{2}{35}x^4W\left(\frac{a}{x}\right)^{3/2}+\frac{16}{105}x^4W\left(\frac{a}{x}\right)^{5/2}-\frac{128}{105}x^4W\left(\frac{a}{x}\right)^{7/2}$$

output
```
-256/105*a^4*Pi^(1/2)*erf(2*LambertW(a/x)^(1/2))+2/7*x^4*LambertW(a/x)^(1/
2)-2/35*x^4*LambertW(a/x)^(3/2)+16/105*x^4*LambertW(a/x)^(5/2)-128/105*x^4
*LambertW(a/x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int x^3\sqrt{W\left(\frac{a}{x}\right)}\,dx = -\frac{2}{105}\left(128a^4\sqrt{\pi}\operatorname{erf}\left(2\sqrt{W\left(\frac{a}{x}\right)}\right)-15x^4\sqrt{W\left(\frac{a}{x}\right)} +3x^4W\left(\frac{a}{x}\right)^{3/2}-8x^4W\left(\frac{a}{x}\right)^{5/2}+64x^4W\left(\frac{a}{x}\right)^{7/2}\right)$$

input
```
Integrate[x^3*Sqrt[ProductLog[a/x]],x]
```

output
```
(-2*(128*a^4*Sqrt[Pi]*Erf[2*Sqrt[ProductLog[a/x]]] - 15*x^4*Sqrt[ProductLo
g[a/x]] + 3*x^4*ProductLog[a/x]^(3/2) - 8*x^4*ProductLog[a/x]^(5/2) + 64*x
^4*ProductLog[a/x]^(7/2)))/105
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {7173, 7206, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{W\left(\frac{a}{x}\right)}dx$$

$$\Big\downarrow \text{ 7173}$$

$$\frac{2}{7}x^4\sqrt{W\left(\frac{a}{x}\right)}-\frac{1}{7}\int \frac{x^3W\left(\frac{a}{x}\right)^{3/2}}{W\left(\frac{a}{x}\right)+1}dx$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{1}{7}\left(\frac{8}{5}\int \frac{x^3W\left(\frac{a}{x}\right)^{5/2}}{W\left(\frac{a}{x}\right)+1}dx-\frac{2}{5}x^4W\left(\frac{a}{x}\right)^{3/2}\right)+\frac{2}{7}x^4\sqrt{W\left(\frac{a}{x}\right)}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{1}{7}\left(\frac{8}{5}\left(\frac{2}{3}x^4W\left(\frac{a}{x}\right)^{5/2}-\frac{8}{3}\int \frac{x^3W\left(\frac{a}{x}\right)^{7/2}}{W\left(\frac{a}{x}\right)+1}dx\right)-\frac{2}{5}x^4W\left(\frac{a}{x}\right)^{3/2}\right)+\frac{2}{7}x^4\sqrt{W\left(\frac{a}{x}\right)}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{1}{7}\left(\frac{8}{5}\left(\frac{2}{3}x^4W\left(\frac{a}{x}\right)^{5/2}-\frac{8}{3}\left(2x^4W\left(\frac{a}{x}\right)^{7/2}-8\int \frac{x^3W\left(\frac{a}{x}\right)^{9/2}}{W\left(\frac{a}{x}\right)+1}dx\right)\right)-\frac{2}{5}x^4W\left(\frac{a}{x}\right)^{3/2}\right)+\frac{2}{7}x^4\sqrt{W\left(\frac{a}{x}\right)}$$

$$\Big\downarrow \text{ 7203}$$

$$\frac{1}{7}\left(\frac{8}{5}\left(\frac{2}{3}x^4W\left(\frac{a}{x}\right)^{5/2}-\frac{8}{3}\left(4\sqrt{\pi}a^4\text{erf}\left(2\sqrt{W\left(\frac{a}{x}\right)}\right)+2x^4W\left(\frac{a}{x}\right)^{7/2}\right)\right)-\frac{2}{5}x^4W\left(\frac{a}{x}\right)^{3/2}\right)+\frac{2}{7}x^4\sqrt{W\left(\frac{a}{x}\right)}$$

input

```
Int[x^3*Sqrt[ProductLog[a/x]],x]
```

output

```
(2*x^4*Sqrt[ProductLog[a/x]])/7 + ((-2*x^4*ProductLog[a/x]^(3/2))/5 + (8*(
(2*x^4*ProductLog[a/x]^(5/2))/3 - (8*(4*a^4*Sqrt[Pi]*Erf[2*Sqrt[ProductLog
[a/x]]] + 2*x^4*ProductLog[a/x]^(7/2)))/3))/5)/7
```

Defintions of rubi rules used

rule 7173

```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203

```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206

```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int x^3\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}dx$$

input
```
int(x^3*LambertW(a/x)^(1/2),x)
```

output
```
int(x^3*LambertW(a/x)^(1/2),x)
```

Fricas [F]

$$\int x^3\sqrt{W\left(\frac{a}{x}\right)}\,dx=\int x^3\sqrt{\mathrm{W}(\frac{a}{x})}\,dx$$

input
```
integrate(x^3*lambert_w(a/x)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^3*sqrt(lambert_w(a/x)), x)
```

Sympy [F]

$$\int x^3\sqrt{W\left(\frac{a}{x}\right)}\,dx=\int x^3\sqrt{W\left(\frac{a}{x}\right)}\,dx$$

input
```
integrate(x**3*LambertW(a/x)**(1/2),x)
```

output
```
Integral(x**3*sqrt(LambertW(a/x)), x)
```

Maxima [F]

$$\int x^3 \sqrt{W\left(\frac{a}{x}\right)}\,dx = \int x^3 \sqrt{\mathrm{W}(\frac{a}{x})}\,dx$$

input
```
integrate(x^3*lambert_w(a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3*sqrt(lambert_w(a/x)), x)
```

Giac [F]

$$\int x^3 \sqrt{W\left(\frac{a}{x}\right)}\,dx = \int x^3 \sqrt{\mathrm{W}(\frac{a}{x})}\,dx$$

input
```
integrate(x^3*lambert_w(a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3*sqrt(lambert_w(a/x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \sqrt{W\left(\frac{a}{x}\right)}\,dx = \int x^3 \sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}\,dx$$

input
```
int(x^3*LambertW(a/x)^(1/2),x)
```

output
```
int(x^3*LambertW(a/x)^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{W\left(\frac{a}{x}\right)}\,dx = \frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}\,x^4}{4} + \frac{\left(\int \frac{\sqrt{lambert\_w(\frac{a}{x})}\,x^2}{e^{lambert\_w\left(\frac{a}{x}\right)}lambert\_w(\frac{a}{x})^2+e^{lambert\_w\left(\frac{a}{x}\right)}lambert\_w(\frac{a}{x})}dx\right)a}{8}$$

input
```
int(x^3*Lambert_W(a/x)^(1/2),x)
```

output
```
(2*sqrt(lambert_w(a/x))*x**4 + int((sqrt(lambert_w(a/x))*x**2)/(e**lambert
_w(a/x)*lambert_w(a/x)**2 + e**lambert_w(a/x)*lambert_w(a/x)),x)*a)/8
```

3.208 $\int x^2 \sqrt{W\left(\frac{a}{x}\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 83

$$\int x^2\sqrt{W\left(\frac{a}{x}\right)}\,dx=\frac{4}{5}a^3\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{W\left(\frac{a}{x}\right)}\right)+\frac{2}{5}x^3\sqrt{W\left(\frac{a}{x}\right)}-\frac{2}{15}x^3W\left(\frac{a}{x}\right)^{3/2}+\frac{4}{5}x^3W\left(\frac{a}{x}\right)^{5/2}$$

output
```
4/5*a^3*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*LambertW(a/x)^(1/2))+2/5*x^3*LambertW
(a/x)^(1/2)-2/15*x^3*LambertW(a/x)^(3/2)+4/5*x^3*LambertW(a/x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int x^2\sqrt{W\left(\frac{a}{x}\right)}\,dx=\frac{2}{15}\left(6a^3\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{W\left(\frac{a}{x}\right)}\right)+3x^3\sqrt{W\left(\frac{a}{x}\right)}-x^3W\left(\frac{a}{x}\right)^{3/2}+6x^3W\left(\frac{a}{x}\right)^{5/2}\right)$$

input
```
Integrate[x^2*Sqrt[ProductLog[a/x]],x]
```

output
```
(2*(6*a^3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ProductLog[a/x]]] + 3*x^3*Sqrt[Produ
ctLog[a/x]] - x^3*ProductLog[a/x]^(3/2) + 6*x^3*ProductLog[a/x]^(5/2)))/15
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7173, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{W\left(\frac{a}{x}\right)}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{2}{5}x^3\sqrt{W\left(\frac{a}{x}\right)}-\frac{1}{5}\int \frac{x^2W\left(\frac{a}{x}\right)^{3/2}}{W\left(\frac{a}{x}\right)+1}dx$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{5}\left(2\int \frac{x^2W\left(\frac{a}{x}\right)^{5/2}}{W\left(\frac{a}{x}\right)+1}dx-\frac{2}{3}x^3W\left(\frac{a}{x}\right)^{3/2}\right)+\frac{2}{5}x^3\sqrt{W\left(\frac{a}{x}\right)}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{5}\left(2\left(2x^3W\left(\frac{a}{x}\right)^{5/2}-6\int \frac{x^2W\left(\frac{a}{x}\right)^{7/2}}{W\left(\frac{a}{x}\right)+1}dx\right)-\frac{2}{3}x^3W\left(\frac{a}{x}\right)^{3/2}\right)+\frac{2}{5}x^3\sqrt{W\left(\frac{a}{x}\right)}$$

$$\Big\downarrow \text{7203}$$

$$\frac{1}{5}\left(2\left(2\sqrt{3\pi}a^3\text{erf}\left(\sqrt{3}\sqrt{W\left(\frac{a}{x}\right)}\right)+2x^3W\left(\frac{a}{x}\right)^{5/2}\right)-\frac{2}{3}x^3W\left(\frac{a}{x}\right)^{3/2}\right)+\frac{2}{5}x^3\sqrt{W\left(\frac{a}{x}\right)}$$

input
```
Int[x^2*Sqrt[ProductLog[a/x]],x]
```

output
```
(2*x^3*Sqrt[ProductLog[a/x]])/5 + ((-2*x^3*ProductLog[a/x]^(3/2))/3 + 2*(2
*a^3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ProductLog[a/x]]] + 2*x^3*ProductLog[a/x]
^(5/2)))/5
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int x^2 \sqrt{\text{LambertW}\left(\frac{a}{x}\right)}dx$$

input
```
int(x^2*LambertW(a/x)^(1/2),x)
```

output
```
int(x^2*LambertW(a/x)^(1/2),x)
```

Fricas [F]

$$\int x^2 \sqrt{W\left(\frac{a}{x}\right)}\, dx = \int x^2 \sqrt{\mathrm{W}(\frac{a}{x})}\, dx$$

input
```
integrate(x^2*lambert_w(a/x)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2*sqrt(lambert_w(a/x)), x)
```

Sympy [F]

$$\int x^2 \sqrt{W\left(\frac{a}{x}\right)}\, dx = \int x^2 \sqrt{W\left(\frac{a}{x}\right)}\, dx$$

input
```
integrate(x**2*LambertW(a/x)**(1/2),x)
```

output
```
Integral(x**2*sqrt(LambertW(a/x)), x)
```

Maxima [F]

$$\int x^2 \sqrt{W\left(\frac{a}{x}\right)}\, dx = \int x^2 \sqrt{\mathrm{W}(\frac{a}{x})}\, dx$$

input
```
integrate(x^2*lambert_w(a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*sqrt(lambert_w(a/x)), x)
```

Giac [F]

$$\int x^2\sqrt{W\left(\frac{a}{x}\right)}\,dx=\int x^2\sqrt{\mathrm{W}(\frac{a}{x})}\,dx$$

input
```
integrate(x^2*lambert_w(a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*sqrt(lambert_w(a/x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{W\left(\frac{a}{x}\right)}\,dx=\int x^2\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}\,dx$$

input
```
int(x^2*LambertW(a/x)^(1/2),x)
```

output
```
int(x^2*LambertW(a/x)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{W\left(\frac{a}{x}\right)}\,dx=\frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}\,x^3}{3}+\frac{\left(\int\frac{\sqrt{lambert\_w(\frac{a}{x})}\,x}{e^{lambert\_w(\frac{a}{x})}\,lambert\_w(\frac{a}{x})^2+e^{lambert\_w(\frac{a}{x})}\,lambert\_w(\frac{a}{x})}dx\right)a}{6}$$

input
```
int(x^2*Lambert_W(a/x)^(1/2),x)
```

output
```
(2*sqrt(lambert_w(a/x))*x**3 + int((sqrt(lambert_w(a/x))*x)/(e**lambert_w(
a/x)*lambert_w(a/x)**2 + e**lambert_w(a/x)*lambert_w(a/x)),x)*a)/6
```

3.209 $\int x\sqrt{W\left(\frac{a}{x}\right)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int x\sqrt{W\left(\frac{a}{x}\right)}\,dx=-\frac{2}{3}a^2\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{W\left(\frac{a}{x}\right)}\right)+\frac{2}{3}x^2\sqrt{W\left(\frac{a}{x}\right)}-\frac{2}{3}x^2W\left(\frac{a}{x}\right)^{3/2}$$

output
```
-2/3*a^2*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*LambertW(a/x)^(1/2))+2/3*x^2*Lambert
W(a/x)^(1/2)-2/3*x^2*LambertW(a/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int x\sqrt{W\left(\frac{a}{x}\right)}\,dx=-\frac{2}{3}\left(a^2\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{W\left(\frac{a}{x}\right)}\right)-x^2\sqrt{W\left(\frac{a}{x}\right)}+x^2W\left(\frac{a}{x}\right)^{3/2}\right)$$

input
```
Integrate[x*Sqrt[ProductLog[a/x]],x]
```

output
```
(-2*(a^2*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ProductLog[a/x]]] - x^2*Sqrt[ProductL
og[a/x]] + x^2*ProductLog[a/x]^(3/2)))/3
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7173, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{W\left(\frac{a}{x}\right)}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{2}{3}x^2\sqrt{W\left(\frac{a}{x}\right)}-\frac{1}{3}\int \frac{xW\left(\frac{a}{x}\right)^{3/2}}{W\left(\frac{a}{x}\right)+1}dx$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{3}\left(4\int \frac{xW\left(\frac{a}{x}\right)^{5/2}}{W\left(\frac{a}{x}\right)+1}dx-2x^2W\left(\frac{a}{x}\right)^{3/2}\right)+\frac{2}{3}x^2\sqrt{W\left(\frac{a}{x}\right)}$$

$$\Big\downarrow \text{7203}$$

$$\frac{1}{3}\left(-2\sqrt{2\pi}a^2\text{erf}\left(\sqrt{2}\sqrt{W\left(\frac{a}{x}\right)}\right)-2x^2W\left(\frac{a}{x}\right)^{3/2}\right)+\frac{2}{3}x^2\sqrt{W\left(\frac{a}{x}\right)}$$

input
```
Int[x*Sqrt[ProductLog[a/x]],x]
```

output
```
(2*x^2*Sqrt[ProductLog[a/x]])/3 + (-2*a^2*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[Prod
uctLog[a/x]]] - 2*x^2*ProductLog[a/x]^(3/2))/3
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int x\sqrt{\text{LambertW}\left(\frac{a}{x}\right)}dx$$

input
```
int(x*LambertW(a/x)^(1/2),x)
```

output
```
int(x*LambertW(a/x)^(1/2),x)
```

Fricas [F]

$$\int x\sqrt{W\left(\frac{a}{x}\right)}\,dx = \int x\sqrt{\mathrm{W}(\frac{a}{x})}\,dx$$

input
```
integrate(x*lambert_w(a/x)^(1/2),x, algorithm="fricas")
```

output
```
integral(x*sqrt(lambert_w(a/x)), x)
```

Sympy [F]

$$\int x\sqrt{W\left(\frac{a}{x}\right)}\,dx = \int x\sqrt{W\left(\frac{a}{x}\right)}\,dx$$

input
```
integrate(x*LambertW(a/x)**(1/2),x)
```

output
```
Integral(x*sqrt(LambertW(a/x)), x)
```

Maxima [F]

$$\int x\sqrt{W\left(\frac{a}{x}\right)}\,dx = \int x\sqrt{\mathrm{W}(\frac{a}{x})}\,dx$$

input
```
integrate(x*lambert_w(a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(lambert_w(a/x)), x)
```

Giac [F]

$$\int x\sqrt{W\left(\frac{a}{x}\right)}\,dx = \int x\sqrt{\mathrm{W}(\frac{a}{x})}\,dx$$

input
```
integrate(x*lambert_w(a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sqrt(lambert_w(a/x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{W\left(\frac{a}{x}\right)}\,dx = \int x\,\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}\,dx$$

input
```
int(x*LambertW(a/x)^(1/2),x)
```

output
```
int(x*LambertW(a/x)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{W\left(\frac{a}{x}\right)}\,dx = \frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}\,x^2}{2} + \frac{\left(\int \frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}}{e^{lambert\_w\left(\frac{a}{x}\right)}\,lambert\_w\left(\frac{a}{x}\right)^2+e^{lambert\_w\left(\frac{a}{x}\right)}\,lambert\_w\left(\frac{a}{x}\right)}dx\right)a}{4}$$

input
```
int(x*Lambert_W(a/x)^(1/2),x)
```

output
```
(2*sqrt(lambert_w(a/x))*x**2 + int(sqrt(lambert_w(a/x))/(e**lambert_w(a/x)
*lambert_w(a/x)**2 + e**lambert_w(a/x)*lambert_w(a/x)),x)*a)/4
```

3.210 $\int \sqrt{W\left(\frac{a}{x}\right)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 32

$$\int \sqrt{W\left(\frac{a}{x}\right)}\,dx = a\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{W\left(\frac{a}{x}\right)}\right) + 2x\sqrt{W\left(\frac{a}{x}\right)}$$

output
```
a*Pi^(1/2)*erf(LambertW(a/x)^(1/2))+2*x*LambertW(a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \sqrt{W\left(\frac{a}{x}\right)}\,dx = a\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{W\left(\frac{a}{x}\right)}\right) + 2x\sqrt{W\left(\frac{a}{x}\right)}$$

input
```
Integrate[Sqrt[ProductLog[a/x]],x]
```

output
```
a*Sqrt[Pi]*Erf[Sqrt[ProductLog[a/x]]] + 2*x*Sqrt[ProductLog[a/x]]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7170, 7190}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{W\left(\frac{a}{x}\right)}dx$$
$$\Big\downarrow \text{ 7170}$$
$$2x\sqrt{W\left(\frac{a}{x}\right)}-\int \frac{W\left(\frac{a}{x}\right)^{3/2}}{W\left(\frac{a}{x}\right)+1}dx$$
$$\Big\downarrow \text{ 7190}$$
$$\sqrt{\pi}a\text{erf}\left(\sqrt{W\left(\frac{a}{x}\right)}\right)+2x\sqrt{W\left(\frac{a}{x}\right)}$$

input
```
Int[Sqrt[ProductLog[a/x]],x]
```

output
```
a*Sqrt[Pi]*Erf[Sqrt[ProductLog[a/x]]] + 2*x*Sqrt[ProductLog[a/x]]
```

Defintions of rubi rules used

rule 7170
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*((c*Prod
uctLog[a*x^n])^p/(n*p + 1)), x] + Simp[n*(p/(c*(n*p + 1)))   Int[(c*Product
Log[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]), x], x] /; FreeQ[{a, c, n}, x]
&& ((IntegerQ[p] && EqQ[n*(p + 1), -1]) || (IntegerQ[p - 1/2] && EqQ[n*(p +
 1/2), -1]))
```

rule 7190
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_)/((d_) + (d_.)*ProductLog[(a_.
)*(x_)^(n_.)]), x_Symbol] :> Simp[(Rt[(-Pi)*c*n, 2]/(d*n*a^(1/n)*c^(1/n)))*
Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[(-c)*n, 2]], x] /; FreeQ[{a, c, d}, x] &&
IntegerQ[1/n] && EqQ[p, 1/2 - 1/n] && NegQ[c*n]
```

Maple [F]

$$\int \sqrt{\text{LambertW}\left(\frac{a}{x}\right)}dx$$

input
```
int(LambertW(a/x)^(1/2),x)
```

output
```
int(LambertW(a/x)^(1/2),x)
```

Fricas [F]

$$\int \sqrt{W\left(\frac{a}{x}\right)}\,dx = \int \sqrt{\text{W}(\frac{a}{x})}\,dx$$

input
```
integrate(lambert_w(a/x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(lambert_w(a/x)), x)
```

Sympy [F]

$$\int \sqrt{W\left(\frac{a}{x}\right)}\,dx = \int \sqrt{W\left(\frac{a}{x}\right)}\,dx$$

input
```
integrate(LambertW(a/x)**(1/2),x)
```

output
```
Integral(sqrt(LambertW(a/x)), x)
```

Maxima [F]

$$\int \sqrt{W\left(\frac{a}{x}\right)}\,dx = \int \sqrt{\mathrm{W}(\frac{a}{x})}\,dx$$

input
```
integrate(lambert_w(a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(lambert_w(a/x)), x)
```

Giac [F]

$$\int \sqrt{W\left(\frac{a}{x}\right)}\,dx = \int \sqrt{\mathrm{W}(\frac{a}{x})}\,dx$$

input
```
integrate(lambert_w(a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(lambert_w(a/x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{W\left(\frac{a}{x}\right)}\,dx = \int \sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}\,dx$$

input
```
int(LambertW(a/x)^(1/2),x)
```

output
```
int(LambertW(a/x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{W\left(\frac{a}{x}\right)}\,dx = \sqrt{lambert\_w\left(\frac{a}{x}\right)}\,x + \frac{\left(\int \frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}}{e^{lambert\_w\left(\frac{a}{x}\right)}\,lambert\_w\left(\frac{a}{x}\right)^{2}x+e^{lambert\_w\left(\frac{a}{x}\right)}\,lambert\_w\left(\frac{a}{x}\right)x}dx\right)a}{2}$$

input
```
int(Lambert_W(a/x)^(1/2),x)
```

output
```
(2*sqrt(lambert_w(a/x))*x + int(sqrt(lambert_w(a/x))/(e**lambert_w(a/x)*la
mbert_w(a/x)**2*x + e**lambert_w(a/x)*lambert_w(a/x)*x),x)*a)/2
```

3.211 $\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x}\,dx=-2\sqrt{W\left(\frac{a}{x}\right)}-\frac{2}{3}W\left(\frac{a}{x}\right)^{3/2}$$

output
```
-2*LambertW(a/x)^(1/2)-2/3*LambertW(a/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x}\,dx=-\frac{2}{3}\sqrt{W\left(\frac{a}{x}\right)}\left(3+W\left(\frac{a}{x}\right)\right)$$

input
```
Integrate[Sqrt[ProductLog[a/x]]/x,x]
```

output
```
(-2*Sqrt[ProductLog[a/x]]*(3 + ProductLog[a/x]))/3
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x}dx$$

$$\Big\downarrow \text{7173}$$

$$\int \frac{W\left(\frac{a}{x}\right)^{3/2}}{x\left(W\left(\frac{a}{x}\right)+1\right)}dx-2\sqrt{W\left(\frac{a}{x}\right)}$$

$$\Big\downarrow \text{7200}$$

$$-\frac{2}{3}W\left(\frac{a}{x}\right)^{3/2}-2\sqrt{W\left(\frac{a}{x}\right)}$$

input
```
Int[Sqrt[ProductLog[a/x]]/x,x]
```

output
```
-2*Sqrt[ProductLog[a/x]] - (2*ProductLog[a/x]^(3/2))/3
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-2\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}-\frac{2\,\mathrm{LambertW}\left(\frac{a}{x}\right)^{\frac{3}{2}}}{3}$	22
default	$-2\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}-\frac{2\,\mathrm{LambertW}\left(\frac{a}{x}\right)^{\frac{3}{2}}}{3}$	22

input
```
int(LambertW(a/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2*LambertW(a/x)^(1/2)-2/3*LambertW(a/x)^(3/2)
```

Fricas [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x}\,dx=\int \frac{\sqrt{\mathrm{W}\left(\frac{a}{x}\right)}}{x}\,dx$$

input
```
integrate(lambert_w(a/x)^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(lambert_w(a/x))/x, x)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x}\,dx=-\frac{2W^{\frac{3}{2}}\left(\frac{a}{x}\right)}{3}-2\sqrt{W\left(\frac{a}{x}\right)}$$

input
```
integrate(LambertW(a/x)**(1/2)/x,x)
```

output
```
-2*LambertW(a/x)**(3/2)/3 - 2*sqrt(LambertW(a/x))
```

Maxima [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x}\,dx = \int \frac{\sqrt{\mathrm{W}(\frac{a}{x})}}{x}\,dx$$

input
```
integrate(lambert_w(a/x)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(lambert_w(a/x))/x, x)
```

Giac [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x}\,dx = \int \frac{\sqrt{\mathrm{W}(\frac{a}{x})}}{x}\,dx$$

input
```
integrate(lambert_w(a/x)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(lambert_w(a/x))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x}\,dx = \int \frac{\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}}{x}\,dx$$

input
```
int(LambertW(a/x)^(1/2)/x,x)
```

output
```
int(LambertW(a/x)^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x}\,dx = \int \frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}}{x}dx$$

input
```
int(Lambert_W(a/x)^(1/2)/x,x)
```

output
```
int(sqrt(lambert_w(a/x))/x,x)
```

3.212 $\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 56

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^2}\,dx=-\frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{W\left(\frac{a}{x}\right)}\right)}{4a}+\frac{1}{2x\sqrt{W\left(\frac{a}{x}\right)}}-\frac{\sqrt{W\left(\frac{a}{x}\right)}}{x}$$

output
```
-1/4*Pi^(1/2)*erfi(LambertW(a/x)^(1/2))/a+1/2/x/LambertW(a/x)^(1/2)-Lamber
tW(a/x)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^2}\,dx=-\frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{W\left(\frac{a}{x}\right)}\right)}{4a}+\frac{1}{2x\sqrt{W\left(\frac{a}{x}\right)}}-\frac{\sqrt{W\left(\frac{a}{x}\right)}}{x}$$

input
```
Integrate[Sqrt[ProductLog[a/x]]/x^2,x]
```

output
```
-1/4*(Sqrt[Pi]*Erfi[Sqrt[ProductLog[a/x]]])/a + 1/(2*x*Sqrt[ProductLog[a/x
]]) - Sqrt[ProductLog[a/x]]/x
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {7172, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^2}dx$$

$$\Big\downarrow \text{7172}$$

$$-\frac{1}{2}\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^2\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{\sqrt{W\left(\frac{a}{x}\right)}}{x}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{W\left(\frac{a}{x}\right)}\left(W\left(\frac{a}{x}\right)+1\right)}dx+\frac{1}{x\sqrt{W\left(\frac{a}{x}\right)}}\right)-\frac{\sqrt{W\left(\frac{a}{x}\right)}}{x}$$

$$\Big\downarrow \text{7204}$$

$$\frac{1}{2}\left(\frac{1}{x\sqrt{W\left(\frac{a}{x}\right)}}-\frac{\sqrt{\pi}\text{erfi}\left(\sqrt{W\left(\frac{a}{x}\right)}\right)}{2a}\right)-\frac{\sqrt{W\left(\frac{a}{x}\right)}}{x}$$

input
```
Int[Sqrt[ProductLog[a/x]]/x^2,x]
```

output
```
(-1/2*(Sqrt[Pi]*Erfi[Sqrt[ProductLog[a/x]]])/a + 1/(x*Sqrt[ProductLog[a/x]
]))/2 - Sqrt[ProductLog[a/x]]/x
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{-\frac{a}{2\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}\,x}+\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}\right)}{4}+\frac{\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}\,a}{x}}{a}$	48
default	$-\frac{-\frac{a}{2\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}\,x}+\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}\right)}{4}+\frac{\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}\,a}{x}}{a}$	48

input
```
int(LambertW(a/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a*(-1/2/LambertW(a/x)^(1/2)*a/x+1/4*Pi^(1/2)*erfi(LambertW(a/x)^(1/2))+
LambertW(a/x)^(1/2)*a/x)
```

Fricas [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^2}\,dx = \int \frac{\sqrt{\mathrm{W}(\frac{a}{x})}}{x^2}\,dx$$

input
```
integrate(lambert_w(a/x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(lambert_w(a/x))/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^2}\,dx = \int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^2}\,dx$$

input
```
integrate(LambertW(a/x)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(LambertW(a/x))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^2}\,dx = \int \frac{\sqrt{\mathrm{W}(\frac{a}{x})}}{x^2}\,dx$$

input
```
integrate(lambert_w(a/x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(lambert_w(a/x))/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^2}\,dx = \int \frac{\sqrt{\mathrm{W}\left(\frac{a}{x}\right)}}{x^2}\,dx$$

input
```
integrate(lambert_w(a/x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(lambert_w(a/x))/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^2}\,dx = \int \frac{\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}}{x^2}\,dx$$

input
```
int(LambertW(a/x)^(1/2)/x^2,x)
```

output
```
int(LambertW(a/x)^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^2}\,dx = \frac{-18e^{lambert\_w\left(\frac{a}{x}\right)}\sqrt{lambert\_w\left(\frac{a}{x}\right)}\,lambert\_w\left(\frac{a}{x}\right) + 8e^{lambert\_w\left(\frac{a}{x}\right)}\sqrt{lambert\_w\left(\frac{a}{x}\right)} + 4\left(\int \frac{\sqrt{lam}}{lambert\_w\left(\frac{a}{x}\right)}\right.}{18a}$$

input
```
int(Lambert_W(a/x)^(1/2)/x^2,x)
```

output

```
( - 18*e**lambert_w(a/x)*sqrt(lambert_w(a/x))*lambert_w(a/x) + 8*e**lamber
t_w(a/x)*sqrt(lambert_w(a/x)) + 4*int(sqrt(lambert_w(a/x))/(lambert_w(a/x)
**2*x**2 + lambert_w(a/x)*x**2),x)*a - int(sqrt(lambert_w(a/x))/(lambert_w
(a/x)*x**2 + x**2),x)*a)/(18*a)
```

3.213 $\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^3}\,dx=\frac{3\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{W\left(\frac{a}{x}\right)}\right)}{64a^2}-\frac{3}{32x^2W\left(\frac{a}{x}\right)^{3/2}}+\frac{1}{8x^2\sqrt{W\left(\frac{a}{x}\right)}}-\frac{\sqrt{W\left(\frac{a}{x}\right)}}{2x^2}$$

output
```
3/128*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*LambertW(a/x)^(1/2))/a^2-3/32/x^2/Lamb
ertW(a/x)^(3/2)+1/8/x^2/LambertW(a/x)^(1/2)-1/2*LambertW(a/x)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^3}\,dx=\frac{1}{128}\left(\frac{3\sqrt{2\pi}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{W\left(\frac{a}{x}\right)}\right)}{a^2}-\frac{12}{x^2W\left(\frac{a}{x}\right)^{3/2}}+\frac{16}{x^2\sqrt{W\left(\frac{a}{x}\right)}}-\frac{64\sqrt{W\left(\frac{a}{x}\right)}}{x^2}\right)$$

input
```
Integrate[Sqrt[ProductLog[a/x]]/x^3,x]
```

output
```
((3*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ProductLog[a/x]]])/a^2 - 12/(x^2*ProductL
og[a/x]^(3/2)) + 16/(x^2*Sqrt[ProductLog[a/x]]) - (64*Sqrt[ProductLog[a/x]
])/x^2)/128
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {7172, 7205, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^3}dx$$

$$\Big\downarrow \text{7172}$$

$$-\frac{1}{4}\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^3\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{\sqrt{W\left(\frac{a}{x}\right)}}{2x^2}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{4}\left(\frac{3}{4}\int \frac{1}{x^3\sqrt{W\left(\frac{a}{x}\right)}\left(W\left(\frac{a}{x}\right)+1\right)}dx+\frac{1}{2x^2\sqrt{W\left(\frac{a}{x}\right)}}\right)-\frac{\sqrt{W\left(\frac{a}{x}\right)}}{2x^2}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(-\frac{1}{4}\int \frac{1}{x^3W\left(\frac{a}{x}\right)^{3/2}\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{1}{2x^2W\left(\frac{a}{x}\right)^{3/2}}\right)+\frac{1}{2x^2\sqrt{W\left(\frac{a}{x}\right)}}\right)-\frac{\sqrt{W\left(\frac{a}{x}\right)}}{2x^2}$$

$$\Big\downarrow \text{7204}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{W\left(\frac{a}{x}\right)}\right)}{4a^2}-\frac{1}{2x^2W\left(\frac{a}{x}\right)^{3/2}}\right)+\frac{1}{2x^2\sqrt{W\left(\frac{a}{x}\right)}}\right)-\frac{\sqrt{W\left(\frac{a}{x}\right)}}{2x^2}$$

input
```
Int[Sqrt[ProductLog[a/x]]/x^3,x]
```

output
```
((3*((Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ProductLog[a/x]]])/(4*a^2) - 1/(2*x^2*P
roductLog[a/x]^(3/2))))/4 + 1/(2*x^2*Sqrt[ProductLog[a/x]]))/4 - Sqrt[Prod
uctLog[a/x]]/(2*x^2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int \frac{\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}}{x^3}dx$$

input
```
int(LambertW(a/x)^(1/2)/x^3,x)
```

output
```
int(LambertW(a/x)^(1/2)/x^3,x)
```

Fricas [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^3}\,dx = \int \frac{\sqrt{\mathrm{W}(\frac{a}{x})}}{x^3}\,dx$$

input
```
integrate(lambert_w(a/x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(lambert_w(a/x))/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^3}\,dx = \int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^3}\,dx$$

input
```
integrate(LambertW(a/x)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(LambertW(a/x))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^3}\,dx = \int \frac{\sqrt{\mathrm{W}(\frac{a}{x})}}{x^3}\,dx$$

input
```
integrate(lambert_w(a/x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(lambert_w(a/x))/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^3}\,dx = \int \frac{\sqrt{\mathrm{W}(\frac{a}{x})}}{x^3}\,dx$$

input
```
integrate(lambert_w(a/x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(lambert_w(a/x))/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^3}\,dx = \int \frac{\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}}{x^3}\,dx$$

input
```
int(LambertW(a/x)^(1/2)/x^3,x)
```

output
```
int(LambertW(a/x)^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^3}\,dx = \int \frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}}{x^3}dx$$

input
```
int(Lambert_W(a/x)^(1/2)/x^3,x)
```

output
```
int(sqrt(lambert_w(a/x))/x**3,x)
```

3.214 $\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 102

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^4}\,dx = -\frac{5\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{W\left(\frac{a}{x}\right)}\right)}{432a^3}+\frac{5}{216x^3W\left(\frac{a}{x}\right)^{5/2}} -\frac{5}{108x^3W\left(\frac{a}{x}\right)^{3/2}}+\frac{1}{18x^3\sqrt{W\left(\frac{a}{x}\right)}}-\frac{\sqrt{W\left(\frac{a}{x}\right)}}{3x^3}$$

output
```
-5/1296*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*LambertW(a/x)^(1/2))/a^3+5/216/x^3/L
ambertW(a/x)^(5/2)-5/108/x^3/LambertW(a/x)^(3/2)+1/18/x^3/LambertW(a/x)^(1
/2)-1/3*LambertW(a/x)^(1/2)/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^4}\,dx = -\frac{5\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{W\left(\frac{a}{x}\right)}\right)}{432a^3} + \frac{5}{216x^3W\left(\frac{a}{x}\right)^{5/2}} - \frac{5}{108x^3W\left(\frac{a}{x}\right)^{3/2}} + \frac{1}{18x^3\sqrt{W\left(\frac{a}{x}\right)}} - \frac{\sqrt{W\left(\frac{a}{x}\right)}}{3x^3}$$

input
```
Integrate[Sqrt[ProductLog[a/x]]/x^4,x]
```

output
```
(-5*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ProductLog[a/x]]])/(432*a^3) + 5/(216*x^3
*ProductLog[a/x]^(5/2)) - 5/(108*x^3*ProductLog[a/x]^(3/2)) + 1/(18*x^3*Sq
rt[ProductLog[a/x]]) - Sqrt[ProductLog[a/x]]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {7172, 7205, 7205, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^4}dx$$

$$\Big\downarrow \text{ 7172}$$

$$-\frac{1}{6}\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^4\left(W\left(\frac{a}{x}\right)+1\right)}dx - \frac{\sqrt{W\left(\frac{a}{x}\right)}}{3x^3}$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{6}\left(\frac{5}{6}\int \frac{1}{x^4\sqrt{W\left(\frac{a}{x}\right)}\left(W\left(\frac{a}{x}\right)+1\right)}dx+\frac{1}{3x^3\sqrt{W\left(\frac{a}{x}\right)}}\right)-\frac{\sqrt{W\left(\frac{a}{x}\right)}}{3x^3}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{6}\left(\frac{5}{6}\left(-\frac{1}{2}\int \frac{1}{x^4W\left(\frac{a}{x}\right)^{3/2}\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{1}{3x^3W\left(\frac{a}{x}\right)^{3/2}}\right)+\frac{1}{3x^3\sqrt{W\left(\frac{a}{x}\right)}}\right)-\frac{\sqrt{W\left(\frac{a}{x}\right)}}{3x^3}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{6}\left(\frac{5}{6}\left(\frac{1}{2}\left(\frac{1}{6}\int \frac{1}{x^4W\left(\frac{a}{x}\right)^{5/2}\left(W\left(\frac{a}{x}\right)+1\right)}dx+\frac{1}{3x^3W\left(\frac{a}{x}\right)^{5/2}}\right)-\frac{1}{3x^3W\left(\frac{a}{x}\right)^{3/2}}\right)+\frac{1}{3x^3\sqrt{W\left(\frac{a}{x}\right)}}\right)-\frac{\sqrt{W\left(\frac{a}{x}\right)}}{3x^3}$$

$$\Big\downarrow \text{7204}$$

$$\frac{1}{6}\left(\frac{5}{6}\left(\frac{1}{2}\left(\frac{1}{3x^3W\left(\frac{a}{x}\right)^{5/2}}-\frac{\sqrt{\frac{\pi}{3}}\text{erfi}\left(\sqrt{3}\sqrt{W\left(\frac{a}{x}\right)}\right)}{6a^3}\right)-\frac{1}{3x^3W\left(\frac{a}{x}\right)^{3/2}}\right)+\frac{1}{3x^3\sqrt{W\left(\frac{a}{x}\right)}}\right)-\frac{\sqrt{W\left(\frac{a}{x}\right)}}{3x^3}$$

input
```
Int[Sqrt[ProductLog[a/x]]/x^4,x]
```

output
```
((5*((-1/6*(Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ProductLog[a/x]]])/a^3 + 1/(3*x^3
*ProductLog[a/x]^(5/2)))/2 - 1/(3*x^3*ProductLog[a/x]^(3/2))))/6 + 1/(3*x^
3*Sqrt[ProductLog[a/x]]))/6 - Sqrt[ProductLog[a/x]]/(3*x^3)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int \frac{\sqrt{\text{LambertW}\left(\frac{a}{x}\right)}}{x^4}dx$$

input
```
int(LambertW(a/x)^(1/2)/x^4,x)
```

output
```
int(LambertW(a/x)^(1/2)/x^4,x)
```

Fricas [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^4}\,dx = \int \frac{\sqrt{\mathrm{W}(\frac{a}{x})}}{x^4}\,dx$$

input
```
integrate(lambert_w(a/x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(lambert_w(a/x))/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^4}\,dx = \int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^4}\,dx$$

input
```
integrate(LambertW(a/x)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(LambertW(a/x))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^4}\,dx = \int \frac{\sqrt{\mathrm{W}(\frac{a}{x})}}{x^4}\,dx$$

input
```
integrate(lambert_w(a/x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(lambert_w(a/x))/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^4}\,dx = \int \frac{\sqrt{\mathrm{W}\left(\frac{a}{x}\right)}}{x^4}\,dx$$

input
```
integrate(lambert_w(a/x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(lambert_w(a/x))/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^4}\,dx = \int \frac{\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}}{x^4}\,dx$$

input
```
int(LambertW(a/x)^(1/2)/x^4,x)
```

output
```
int(LambertW(a/x)^(1/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x^4}\,dx = \int \frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}}{x^4}dx$$

input
```
int(Lambert_W(a/x)^(1/2)/x^4,x)
```

output
```
int(sqrt(lambert_w(a/x))/x**4,x)
```

3.215 $\int \frac{x^3}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 111

$$\int \frac{x^3}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = -\frac{2048}{945}a^4\sqrt{\pi}\operatorname{erf}\left(2\sqrt{W\left(\frac{a}{x}\right)}\right)+\frac{2x^4}{9\sqrt{W\left(\frac{a}{x}\right)}}+\frac{2}{63}x^4\sqrt{W\left(\frac{a}{x}\right)} -\frac{16}{315}x^4W\left(\frac{a}{x}\right)^{3/2}+\frac{128}{945}x^4W\left(\frac{a}{x}\right)^{5/2}-\frac{1024}{945}x^4W\left(\frac{a}{x}\right)^{7/2}$$

output
```
-2048/945*a^4*Pi^(1/2)*erf(2*LambertW(a/x)^(1/2))+2/9*x^4/LambertW(a/x)^(1
/2)+2/63*x^4*LambertW(a/x)^(1/2)-16/315*x^4*LambertW(a/x)^(3/2)+128/945*x^
4*LambertW(a/x)^(5/2)-1024/945*x^4*LambertW(a/x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = -\frac{2\left(-105x^4+1024a^4\sqrt{\pi}\operatorname{erf}\left(2\sqrt{W\left(\frac{a}{x}\right)}\right)\sqrt{W\left(\frac{a}{x}\right)}-15x^4W\left(\frac{a}{x}\right)+24x^4W\left(\frac{a}{x}\right)^2-64x^4W\left(\frac{a}{x}\right)^3+512x^4\right.}{945\sqrt{W\left(\frac{a}{x}\right)}}$$

input
```
Integrate[x^3/Sqrt[ProductLog[a/x]],x]
```

output
```
(-2*(-105*x^4 + 1024*a^4*Sqrt[Pi]*Erf[2*Sqrt[ProductLog[a/x]]]*Sqrt[Produc
tLog[a/x]] - 15*x^4*ProductLog[a/x] + 24*x^4*ProductLog[a/x]^2 - 64*x^4*Pr
oductLog[a/x]^3 + 512*x^4*ProductLog[a/x]^4))/(945*Sqrt[ProductLog[a/x]])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {7173, 7206, 7206, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{W\left(\frac{a}{x}\right)}}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{1}{9}\int \frac{x^3\sqrt{W\left(\frac{a}{x}\right)}}{W\left(\frac{a}{x}\right)+1}dx+\frac{2x^4}{9\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{9}\left(\frac{2}{7}x^4\sqrt{W\left(\frac{a}{x}\right)}-\frac{8}{7}\int \frac{x^3W\left(\frac{a}{x}\right)^{3/2}}{W\left(\frac{a}{x}\right)+1}dx\right)+\frac{2x^4}{9\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{9}\left(\frac{2}{7}x^4\sqrt{W\left(\frac{a}{x}\right)}-\frac{8}{7}\left(\frac{2}{5}x^4W\left(\frac{a}{x}\right)^{3/2}-\frac{8}{5}\int \frac{x^3W\left(\frac{a}{x}\right)^{5/2}}{W\left(\frac{a}{x}\right)+1}dx\right)\right)+\frac{2x^4}{9\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{9}\left(\frac{2}{7}x^4\sqrt{W\left(\frac{a}{x}\right)}-\frac{8}{7}\left(\frac{2}{5}x^4W\left(\frac{a}{x}\right)^{3/2}-\frac{8}{5}\left(\frac{2}{3}x^4W\left(\frac{a}{x}\right)^{5/2}-\frac{8}{3}\int\frac{x^3W\left(\frac{a}{x}\right)^{7/2}}{W\left(\frac{a}{x}\right)+1}dx\right)\right)\right)+\frac{2x^4}{9\sqrt{W\left(\frac{a}{x}\right)}}$$

$\big\downarrow$ 7206

$$\frac{1}{9}\left(\frac{2}{7}x^4\sqrt{W\left(\frac{a}{x}\right)}-\frac{8}{7}\left(\frac{2}{5}x^4W\left(\frac{a}{x}\right)^{3/2}-\frac{8}{5}\left(\frac{2}{3}x^4W\left(\frac{a}{x}\right)^{5/2}-\frac{8}{3}\left(2x^4W\left(\frac{a}{x}\right)^{7/2}-8\int\frac{x^3W\left(\frac{a}{x}\right)^{9/2}}{W\left(\frac{a}{x}\right)+1}dx\right)\right)\right)\right)+\frac{2x^4}{9\sqrt{W\left(\frac{a}{x}\right)}}$$

$\big\downarrow$ 7203

$$\frac{1}{9}\left(\frac{2}{7}x^4\sqrt{W\left(\frac{a}{x}\right)}-\frac{8}{7}\left(\frac{2}{5}x^4W\left(\frac{a}{x}\right)^{3/2}-\frac{8}{5}\left(\frac{2}{3}x^4W\left(\frac{a}{x}\right)^{5/2}-\frac{8}{3}\left(4\sqrt{\pi}a^4\text{erf}\left(2\sqrt{W\left(\frac{a}{x}\right)}\right)+2x^4W\left(\frac{a}{x}\right)^{7/2}\right)\right)\right)\right)\frac{2x^4}{9\sqrt{W\left(\frac{a}{x}\right)}}$$

input

```
Int[x^3/Sqrt[ProductLog[a/x]],x]
```

output

```
(2*x^4)/(9*Sqrt[ProductLog[a/x]]) + ((2*x^4*Sqrt[ProductLog[a/x]])/7 - (8*
((2*x^4*ProductLog[a/x]^(3/2))/5 - (8*((2*x^4*ProductLog[a/x]^(5/2))/3 - (
8*(4*a^4*Sqrt[Pi]*Erf[2*Sqrt[ProductLog[a/x]]] + 2*x^4*ProductLog[a/x]^(7/
2)))/3))/5))/7)/9
```

Defintions of rubi rules used

rule 7173

```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203

```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206

```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{x^3}{\sqrt{\operatorname{LambertW}\left(\frac{a}{x}\right)}}dx$$

input

```
int(x^3/LambertW(a/x)^(1/2),x)
```

output

```
int(x^3/LambertW(a/x)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{x^3}{\sqrt{\mathrm{W}(\frac{a}{x})}}\,dx$$

input

```
integrate(x^3/lambert_w(a/x)^(1/2),x, algorithm="fricas")
```

output

```
integral(x^3/sqrt(lambert_w(a/x)), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{x^3}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx$$

input
```
integrate(x**3/LambertW(a/x)**(1/2),x)
```

output
```
Integral(x**3/sqrt(LambertW(a/x)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{x^3}{\sqrt{W(\frac{a}{x})}}\,dx$$

input
```
integrate(x^3/lambert_w(a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(lambert_w(a/x)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{x^3}{\sqrt{W(\frac{a}{x})}}\,dx$$

input
```
integrate(x^3/lambert_w(a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(lambert_w(a/x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{x^3}{\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}}\,dx$$

input
```
int(x^3/LambertW(a/x)^(1/2),x)
```

output
```
int(x^3/LambertW(a/x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}\,x^3}{lambert\_w\left(\frac{a}{x}\right)}dx$$

input
```
int(x^3/Lambert_W(a/x)^(1/2),x)
```

output
```
int((sqrt(lambert_w(a/x))*x**3)/lambert_w(a/x),x)
```

3.216 $\int \frac{x^2}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 100

$$\int \frac{x^2}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \frac{24}{35}a^3\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{W\left(\frac{a}{x}\right)}\right) + \frac{2x^3}{7\sqrt{W\left(\frac{a}{x}\right)}} + \frac{2}{35}x^3\sqrt{W\left(\frac{a}{x}\right)} - \frac{4}{35}x^3W\left(\frac{a}{x}\right)^{3/2} + \frac{24}{35}x^3W\left(\frac{a}{x}\right)^{5/2}$$

output
```
24/35*a^3*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*LambertW(a/x)^(1/2))+2/7*x^3/Lamber
tW(a/x)^(1/2)+2/35*x^3*LambertW(a/x)^(1/2)-4/35*x^3*LambertW(a/x)^(3/2)+24
/35*x^3*LambertW(a/x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \frac{2\left(5x^3+12a^3\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{W\left(\frac{a}{x}\right)}\right)\sqrt{W\left(\frac{a}{x}\right)}+x^3W\left(\frac{a}{x}\right)-2x^3W\left(\frac{a}{x}\right)^2+12x^3W\left(\frac{a}{x}\right)^3\right)}{35\sqrt{W\left(\frac{a}{x}\right)}}$$

input
```
Integrate[x^2/Sqrt[ProductLog[a/x]],x]
```

output
```
(2*(5*x^3 + 12*a^3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ProductLog[a/x]]]*Sqrt[Prod
uctLog[a/x]] + x^3*ProductLog[a/x] - 2*x^3*ProductLog[a/x]^2 + 12*x^3*Prod
uctLog[a/x]^3))/(35*Sqrt[ProductLog[a/x]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {7173, 7206, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{W\left(\frac{a}{x}\right)}}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{1}{7}\int \frac{x^2\sqrt{W\left(\frac{a}{x}\right)}}{W\left(\frac{a}{x}\right)+1}dx+\frac{2x^3}{7\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{7}\left(\frac{2}{5}x^3\sqrt{W\left(\frac{a}{x}\right)}-\frac{6}{5}\int \frac{x^2W\left(\frac{a}{x}\right)^{3/2}}{W\left(\frac{a}{x}\right)+1}dx\right)+\frac{2x^3}{7\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{7}\left(\frac{2}{5}x^3\sqrt{W\left(\frac{a}{x}\right)}-\frac{6}{5}\left(\frac{2}{3}x^3W\left(\frac{a}{x}\right)^{3/2}-2\int \frac{x^2W\left(\frac{a}{x}\right)^{5/2}}{W\left(\frac{a}{x}\right)+1}dx\right)\right)+\frac{2x^3}{7\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{7}\left(\frac{2}{5}x^3\sqrt{W\left(\frac{a}{x}\right)}-\frac{6}{5}\left(\frac{2}{3}x^3W\left(\frac{a}{x}\right)^{3/2}-2\left(2x^3W\left(\frac{a}{x}\right)^{5/2}-6\int\frac{x^2W\left(\frac{a}{x}\right)^{7/2}}{W\left(\frac{a}{x}\right)+1}dx\right)\right)\right)+\frac{2x^3}{7\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \; 7203$$

$$\frac{1}{7}\left(\frac{2}{5}x^3\sqrt{W\left(\frac{a}{x}\right)}-\frac{6}{5}\left(\frac{2}{3}x^3W\left(\frac{a}{x}\right)^{3/2}-2\left(2\sqrt{3\pi}a^3\text{erf}\left(\sqrt{3}\sqrt{W\left(\frac{a}{x}\right)}\right)+2x^3W\left(\frac{a}{x}\right)^{5/2}\right)\right)\right)+\frac{2x^3}{7\sqrt{W\left(\frac{a}{x}\right)}}$$

input
```
Int[x^2/Sqrt[ProductLog[a/x]],x]
```

output
```
(2*x^3)/(7*Sqrt[ProductLog[a/x]]) + ((2*x^3*Sqrt[ProductLog[a/x]])/5 - (6*
((2*x^3*ProductLog[a/x]^(3/2))/3 - 2*(2*a^3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[Pr
oductLog[a/x]]] + 2*x^3*ProductLog[a/x]^(5/2))))/5)/7
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}}dx$$

input
```
int(x^2/LambertW(a/x)^(1/2),x)
```

output
```
int(x^2/LambertW(a/x)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{x^2}{\sqrt{\mathrm{W}(\frac{a}{x})}}\,dx$$

input
```
integrate(x^2/lambert_w(a/x)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2/sqrt(lambert_w(a/x)), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{x^2}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx$$

input
```
integrate(x**2/LambertW(a/x)**(1/2),x)
```

output
```
Integral(x**2/sqrt(LambertW(a/x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{x^2}{\sqrt{\mathrm{W}\left(\frac{a}{x}\right)}}\,dx$$

input
```
integrate(x^2/lambert_w(a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(lambert_w(a/x)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{x^2}{\sqrt{\mathrm{W}\left(\frac{a}{x}\right)}}\,dx$$

input
```
integrate(x^2/lambert_w(a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(lambert_w(a/x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{x^2}{\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}}\,dx$$

input
```
int(x^2/LambertW(a/x)^(1/2),x)
```

output
```
int(x^2/LambertW(a/x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}\,x^2}{lambert\_w\left(\frac{a}{x}\right)}dx$$

input
```
int(x^2/Lambert_W(a/x)^(1/2),x)
```

output
```
int((sqrt(lambert_w(a/x))*x**2)/lambert_w(a/x),x)
```

3.217 $\int \frac{x}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 83

$$\int \frac{x}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx=-\frac{8}{15}a^2\sqrt{2\pi}\,\text{erf}\left(\sqrt{2}\sqrt{W\left(\frac{a}{x}\right)}\right)+\frac{2x^2}{5\sqrt{W\left(\frac{a}{x}\right)}}+\frac{2}{15}x^2\sqrt{W\left(\frac{a}{x}\right)}-\frac{8}{15}x^2W\left(\frac{a}{x}\right)^{3/2}$$

output
```
-8/15*a^2*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*LambertW(a/x)^(1/2))+2/5*x^2/Lamber
tW(a/x)^(1/2)+2/15*x^2*LambertW(a/x)^(1/2)-8/15*x^2*LambertW(a/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx=-\frac{8}{15}a^2\sqrt{2\pi}\,\text{erf}\left(\sqrt{2}\sqrt{W\left(\frac{a}{x}\right)}\right)+\frac{2x^2}{5\sqrt{W\left(\frac{a}{x}\right)}}+\frac{2}{15}x^2\sqrt{W\left(\frac{a}{x}\right)}-\frac{8}{15}x^2W\left(\frac{a}{x}\right)^{3/2}$$

input
```
Integrate[x/Sqrt[ProductLog[a/x]],x]
```

output
```
(-8*a^2*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ProductLog[a/x]]])/15 + (2*x^2)/(5*Sqr
t[ProductLog[a/x]]) + (2*x^2*Sqrt[ProductLog[a/x]])/15 - (8*x^2*ProductLog
[a/x]^(3/2))/15
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7173, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{W\left(\frac{a}{x}\right)}}dx$$

$$\Big\downarrow \text{ 7173}$$

$$\frac{1}{5}\int \frac{x\sqrt{W\left(\frac{a}{x}\right)}}{W\left(\frac{a}{x}\right)+1}dx+\frac{2x^2}{5\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{1}{5}\left(\frac{2}{3}x^2\sqrt{W\left(\frac{a}{x}\right)}-\frac{4}{3}\int \frac{xW\left(\frac{a}{x}\right)^{3/2}}{W\left(\frac{a}{x}\right)+1}dx\right)+\frac{2x^2}{5\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{1}{5}\left(\frac{2}{3}x^2\sqrt{W\left(\frac{a}{x}\right)}-\frac{4}{3}\left(2x^2W\left(\frac{a}{x}\right)^{3/2}-4\int \frac{xW\left(\frac{a}{x}\right)^{5/2}}{W\left(\frac{a}{x}\right)+1}dx\right)\right)+\frac{2x^2}{5\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{ 7203}$$

$$\frac{1}{5}\left(\frac{2}{3}x^2\sqrt{W\left(\frac{a}{x}\right)}-\frac{4}{3}\left(2\sqrt{2\pi}a^2\text{erf}\left(\sqrt{2}\sqrt{W\left(\frac{a}{x}\right)}\right)+2x^2W\left(\frac{a}{x}\right)^{3/2}\right)\right)+\frac{2x^2}{5\sqrt{W\left(\frac{a}{x}\right)}}$$

input
```
Int[x/Sqrt[ProductLog[a/x]],x]
```

output
```
(2*x^2)/(5*Sqrt[ProductLog[a/x]]) + ((2*x^2*Sqrt[ProductLog[a/x]])/3 - (4*
(2*a^2*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ProductLog[a/x]]] + 2*x^2*ProductLog[a/
x]^(3/2)))/3)/5
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{x}{\sqrt{\text{LambertW}\left(\frac{a}{x}\right)}}dx$$

input
```
int(x/LambertW(a/x)^(1/2),x)
```

output
```
int(x/LambertW(a/x)^(1/2),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{x}{\sqrt{W(\frac{a}{x})}}\,dx$$

input
```
integrate(x/lambert_w(a/x)^(1/2),x, algorithm="fricas")
```

output
```
integral(x/sqrt(lambert_w(a/x)), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{x}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx$$

input
```
integrate(x/LambertW(a/x)**(1/2),x)
```

output
```
Integral(x/sqrt(LambertW(a/x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{x}{\sqrt{W(\frac{a}{x})}}\,dx$$

input
```
integrate(x/lambert_w(a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(lambert_w(a/x)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{x}{\sqrt{\mathrm{W}\left(\frac{a}{x}\right)}}\,dx$$

input
```
integrate(x/lambert_w(a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(lambert_w(a/x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{x}{\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}}\,dx$$

input
```
int(x/LambertW(a/x)^(1/2),x)
```

output
```
int(x/LambertW(a/x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}\,x}{lambert\_w\left(\frac{a}{x}\right)}dx$$

input
```
int(x/Lambert_W(a/x)^(1/2),x)
```

output
```
int((sqrt(lambert_w(a/x))*x)/lambert_w(a/x),x)
```

3.218 $\int \frac{1}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 52

$$\int \frac{1}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx=\frac{2}{3}a\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{W\left(\frac{a}{x}\right)}\right)+\frac{2x}{3\sqrt{W\left(\frac{a}{x}\right)}}+\frac{2}{3}x\sqrt{W\left(\frac{a}{x}\right)}$$

output
```
2/3*a*Pi^(1/2)*erf(LambertW(a/x)^(1/2))+2/3*x/LambertW(a/x)^(1/2)+2/3*x*La
mbertW(a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx=\frac{2}{3}\left(a\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{W\left(\frac{a}{x}\right)}\right)+\frac{x}{\sqrt{W\left(\frac{a}{x}\right)}}+x\sqrt{W\left(\frac{a}{x}\right)}\right)$$

input
```
Integrate[1/Sqrt[ProductLog[a/x]],x]
```

output
```
(2*(a*Sqrt[Pi]*Erf[Sqrt[ProductLog[a/x]]] + x/Sqrt[ProductLog[a/x]] + x*Sq
rt[ProductLog[a/x]]))/3
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {7171, 7173, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{W\left(\frac{a}{x}\right)}}dx$$

$$\Big\downarrow \text{ 7171}$$

$$-\int \frac{x^2}{\sqrt{W\left(\frac{a}{x}\right)}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 7173}$$

$$\frac{2x}{3\sqrt{W\left(\frac{a}{x}\right)}}-\frac{1}{3}\int \frac{x^2\sqrt{W\left(\frac{a}{x}\right)}}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{1}{3}\left(2\int \frac{x^2W\left(\frac{a}{x}\right)^{3/2}}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}+2x\sqrt{W\left(\frac{a}{x}\right)}\right)+\frac{2x}{3\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{ 7203}$$

$$\frac{1}{3}\left(2\sqrt{\pi}a\text{erf}\left(\sqrt{W\left(\frac{a}{x}\right)}\right)+2x\sqrt{W\left(\frac{a}{x}\right)}\right)+\frac{2x}{3\sqrt{W\left(\frac{a}{x}\right)}}$$

input
```
Int[1/Sqrt[ProductLog[a/x]],x]
```

output
```
(2*a*Sqrt[Pi]*Erf[Sqrt[ProductLog[a/x]]] + 2*x*Sqrt[ProductLog[a/x]])/3 +
(2*x)/(3*Sqrt[ProductLog[a/x]])
```

Defintions of rubi rules used

rule 7171
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subst[Int[(c*P
roductLog[a/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, c, p}, x] && ILtQ[n, 0]
```

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{1}{\sqrt{\operatorname{LambertW}\left(\frac{a}{x}\right)}}dx$$

input
```
int(1/LambertW(a/x)^(1/2),x)
```

output
```
int(1/LambertW(a/x)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{\sqrt{\mathrm{W}(\frac{a}{x})}}\,dx$$

input
```
integrate(1/lambert_w(a/x)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(lambert_w(a/x)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx$$

input
```
integrate(1/LambertW(a/x)**(1/2),x)
```

output
```
Integral(1/sqrt(LambertW(a/x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{\sqrt{\mathrm{W}(\frac{a}{x})}}\,dx$$

input
```
integrate(1/lambert_w(a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(lambert_w(a/x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{\sqrt{W(\frac{a}{x})}}\,dx$$

input
```
integrate(1/lambert_w(a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(lambert_w(a/x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}}\,dx$$

input
```
int(1/LambertW(a/x)^(1/2),x)
```

output
```
int(1/LambertW(a/x)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}}{lambert\_w\left(\frac{a}{x}\right)}\,dx$$

input
```
int(1/Lambert_W(a/x)^(1/2),x)
```

output
```
int(sqrt(lambert_w(a/x))/lambert_w(a/x),x)
```

3.219 $\int \frac{1}{x\sqrt{W\left(\frac{a}{x}\right)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{1}{x\sqrt{W\left(\frac{a}{x}\right)}}\,dx=\frac{2}{\sqrt{W\left(\frac{a}{x}\right)}}-2\sqrt{W\left(\frac{a}{x}\right)}$$

output
```
2/LambertW(a/x)^(1/2)-2*LambertW(a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\sqrt{W\left(\frac{a}{x}\right)}}\,dx=-\frac{2\left(-1+W\left(\frac{a}{x}\right)\right)}{\sqrt{W\left(\frac{a}{x}\right)}}$$

input
```
Integrate[1/(x*Sqrt[ProductLog[a/x]]),x]
```

output
```
(-2*(-1 + ProductLog[a/x]))/Sqrt[ProductLog[a/x]]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{W\left(\frac{a}{x}\right)}}dx$$

$$\Big\downarrow \text{7173}$$

$$\int \frac{\sqrt{W\left(\frac{a}{x}\right)}}{x\left(W\left(\frac{a}{x}\right)+1\right)}dx+\frac{2}{\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{7200}$$

$$\frac{2}{\sqrt{W\left(\frac{a}{x}\right)}}-2\sqrt{W\left(\frac{a}{x}\right)}$$

input
```
Int[1/(x*Sqrt[ProductLog[a/x]]),x]
```

output
```
2/Sqrt[ProductLog[a/x]] - 2*Sqrt[ProductLog[a/x]]
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{2}{\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}}-2\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}$	22
default	$\frac{2}{\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}}-2\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}$	22

input
```
int(1/x/LambertW(a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/LambertW(a/x)^(1/2)-2*LambertW(a/x)^(1/2)
```

Fricas [F]

$$\int \frac{1}{x\sqrt{W\left(\frac{a}{x}\right)}}\,dx=\int \frac{1}{x\sqrt{\mathrm{W}\left(\frac{a}{x}\right)}}\,dx$$

input
```
integrate(1/x/lambert_w(a/x)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/(x*sqrt(lambert_w(a/x))), x)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{W\left(\frac{a}{x}\right)}}\,dx = -2\sqrt{W\left(\frac{a}{x}\right)} + \frac{2}{\sqrt{W\left(\frac{a}{x}\right)}}$$

input
```
integrate(1/x/LambertW(a/x)**(1/2),x)
```

output
```
-2*sqrt(LambertW(a/x)) + 2/sqrt(LambertW(a/x))
```

Maxima [F]

$$\int \frac{1}{x\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x\sqrt{\mathrm{W}\left(\frac{a}{x}\right)}}\,dx$$

input
```
integrate(1/x/lambert_w(a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt(lambert_w(a/x))), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x\sqrt{\mathrm{W}\left(\frac{a}{x}\right)}}\,dx$$

input
```
integrate(1/x/lambert_w(a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x*sqrt(lambert_w(a/x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x\,\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}}\,dx$$

input
```
int(1/(x*LambertW(a/x)^(1/2)),x)
```

output
```
int(1/(x*LambertW(a/x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}}{lambert\_w\left(\frac{a}{x}\right)x}dx$$

input
```
int(1/x/Lambert_W(a/x)^(1/2),x)
```

output
```
int(sqrt(lambert_w(a/x))/(lambert_w(a/x)*x),x)
```

3.220 $\int \frac{1}{x^2\sqrt{W\left(\frac{a}{x}\right)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{1}{x^2\sqrt{W\left(\frac{a}{x}\right)}}\,dx=-\frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{W\left(\frac{a}{x}\right)}\right)}{2a}-\frac{1}{x\sqrt{W\left(\frac{a}{x}\right)}}$$

output
```
-1/2*Pi^(1/2)*erfi(LambertW(a/x)^(1/2))/a-1/x/LambertW(a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{W\left(\frac{a}{x}\right)}}\,dx=-\frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{W\left(\frac{a}{x}\right)}\right)}{2a}-\frac{1}{x\sqrt{W\left(\frac{a}{x}\right)}}$$

input
```
Integrate[1/(x^2*Sqrt[ProductLog[a/x]]),x]
```

output
```
-1/2*(Sqrt[Pi]*Erfi[Sqrt[ProductLog[a/x]]])/a - 1/(x*Sqrt[ProductLog[a/x]]
)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {7172, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{W\left(\frac{a}{x}\right)}}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{W\left(\frac{a}{x}\right)}\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{1}{x\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{7204}$$

$$-\frac{\sqrt{\pi}\text{erfi}\left(\sqrt{W\left(\frac{a}{x}\right)}\right)}{2a}-\frac{1}{x\sqrt{W\left(\frac{a}{x}\right)}}$$

input
```
Int[1/(x^2*Sqrt[ProductLog[a/x]]),x]
```

output
```
-1/2*(Sqrt[Pi]*Erfi[Sqrt[ProductLog[a/x]]])/a - 1/(x*Sqrt[ProductLog[a/x]]
)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$-\frac{\frac{a}{\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}\, x}+\frac{\sqrt{\pi}\, \mathrm{erfi}\left(\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}\right)}{2}}{a}$	34
default	$-\frac{\frac{a}{\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}\, x}+\frac{\sqrt{\pi}\, \mathrm{erfi}\left(\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}\right)}{2}}{a}$	34

input
```
int(1/x^2/LambertW(a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a*(1/LambertW(a/x)^(1/2)*a/x+1/2*Pi^(1/2)*erfi(LambertW(a/x)^(1/2)))
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt{W\left(\frac{a}{x}\right)}}\,dx=\int \frac{1}{x^2\sqrt{\mathrm{W}(\frac{a}{x})}}\,dx$$

input
```
integrate(1/x^2/lambert_w(a/x)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/(x^2*sqrt(lambert_w(a/x))), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x^2\sqrt{W\left(\frac{a}{x}\right)}}\,dx$$

input
```
integrate(1/x**2/LambertW(a/x)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(LambertW(a/x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x^2\sqrt{\mathrm{W}\left(\frac{a}{x}\right)}}\,dx$$

input
```
integrate(1/x^2/lambert_w(a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*sqrt(lambert_w(a/x))), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x^2\sqrt{\mathrm{W}\left(\frac{a}{x}\right)}}\,dx$$

input
```
integrate(1/x^2/lambert_w(a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x^2*sqrt(lambert_w(a/x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x^2\,\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}}\,dx$$

input
```
int(1/(x^2*LambertW(a/x)^(1/2)),x)
```

output
```
int(1/(x^2*LambertW(a/x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \frac{-10e^{lambert\_w\left(\frac{a}{x}\right)}\sqrt{lambert\_w\left(\frac{a}{x}\right)}+4\left(\int \frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}}{lambert\_w\left(\frac{a}{x}\right)^2x^2+lambert\_w\left(\frac{a}{x}\right)x^2}dx\right)a-\left(\int \frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}}{lambert\_w\left(\frac{a}{x}\right)x^2}\right.}{9a}$$

input
```
int(1/x^2/Lambert_W(a/x)^(1/2),x)
```

output
```
( - 10*e**lambert_w(a/x)*sqrt(lambert_w(a/x)) + 4*int(sqrt(lambert_w(a/x))
/(lambert_w(a/x)**2*x**2 + lambert_w(a/x)*x**2),x)*a - int(sqrt(lambert_w(
a/x))/(lambert_w(a/x)*x**2 + x**2),x)*a)/(9*a)
```

3.221 $\int \frac{1}{x^3\sqrt{W\left(\frac{a}{x}\right)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 68

$$\int \frac{1}{x^3\sqrt{W\left(\frac{a}{x}\right)}}\,dx=\frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{W\left(\frac{a}{x}\right)}\right)}{16a^2}-\frac{1}{8x^2W\left(\frac{a}{x}\right)^{3/2}}-\frac{1}{2x^2\sqrt{W\left(\frac{a}{x}\right)}}$$

output
```
1/32*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*LambertW(a/x)^(1/2))/a^2-1/8/x^2/Lamber
tW(a/x)^(3/2)-1/2/x^2/LambertW(a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3\sqrt{W\left(\frac{a}{x}\right)}}\,dx=\frac{1}{32}\left(\frac{\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{W\left(\frac{a}{x}\right)}\right)}{a^2}-\frac{4}{x^2W\left(\frac{a}{x}\right)^{3/2}}-\frac{16}{x^2\sqrt{W\left(\frac{a}{x}\right)}}\right)$$

input
```
Integrate[1/(x^3*Sqrt[ProductLog[a/x]]),x]
```

output
```
((Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ProductLog[a/x]]])/a^2 - 4/(x^2*ProductLog[
a/x]^(3/2)) - 16/(x^2*Sqrt[ProductLog[a/x]]))/32
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {7172, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{W\left(\frac{a}{x}\right)}}dx$$

$$\Big\downarrow \text{ 7172}$$

$$\frac{1}{4}\int \frac{1}{x^3\sqrt{W\left(\frac{a}{x}\right)}\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{1}{2x^2\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{4}\left(-\frac{1}{4}\int \frac{1}{x^3W\left(\frac{a}{x}\right)^{3/2}\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{1}{2x^2W\left(\frac{a}{x}\right)^{3/2}}\right)-\frac{1}{2x^2\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{ 7204}$$

$$\frac{1}{4}\left(\frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{W\left(\frac{a}{x}\right)}\right)}{4a^2}-\frac{1}{2x^2W\left(\frac{a}{x}\right)^{3/2}}\right)-\frac{1}{2x^2\sqrt{W\left(\frac{a}{x}\right)}}$$

input
```
Int[1/(x^3*Sqrt[ProductLog[a/x]]),x]
```

output
```
((Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ProductLog[a/x]]])/(4*a^2) - 1/(2*x^2*Produ
ctLog[a/x]^(3/2)))/4 - 1/(2*x^2*Sqrt[ProductLog[a/x]])
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int \frac{1}{x^3\sqrt{\text{LambertW}\left(\frac{a}{x}\right)}}dx$$

input
```
int(1/x^3/LambertW(a/x)^(1/2),x)
```

output
```
int(1/x^3/LambertW(a/x)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^3\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x^3\sqrt{\mathrm{W}(\frac{a}{x})}}\,dx$$

input
```
integrate(1/x^3/lambert_w(a/x)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/(x^3*sqrt(lambert_w(a/x))), x)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x^3\sqrt{W\left(\frac{a}{x}\right)}}\,dx$$

input
```
integrate(1/x**3/LambertW(a/x)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(LambertW(a/x))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x^3\sqrt{\mathrm{W}(\frac{a}{x})}}\,dx$$

input
```
integrate(1/x^3/lambert_w(a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x^3*sqrt(lambert_w(a/x))), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x^3\sqrt{\mathrm{W}(\frac{a}{x})}}\,dx$$

input
```
integrate(1/x^3/lambert_w(a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x^3*sqrt(lambert_w(a/x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x^3\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}}\,dx$$

input
```
int(1/(x^3*LambertW(a/x)^(1/2)),x)
```

output
```
int(1/(x^3*LambertW(a/x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}}{lambert\_w\left(\frac{a}{x}\right)x^3}dx$$

input
```
int(1/x^3/Lambert_W(a/x)^(1/2),x)
```

output
```
int(sqrt(lambert_w(a/x))/(lambert_w(a/x)*x**3),x)
```

3.222 $\int \frac{1}{x^4\sqrt{W\left(\frac{a}{x}\right)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \frac{1}{x^4\sqrt{W\left(\frac{a}{x}\right)}}\,dx = -\frac{\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{W\left(\frac{a}{x}\right)}\right)}{72a^3}+\frac{1}{36x^3W\left(\frac{a}{x}\right)^{5/2}}-\frac{1}{18x^3W\left(\frac{a}{x}\right)^{3/2}}-\frac{1}{3x^3\sqrt{W\left(\frac{a}{x}\right)}}$$

output
```
-1/216*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*LambertW(a/x)^(1/2))/a^3+1/36/x^3/Lam
bertW(a/x)^(5/2)-1/18/x^3/LambertW(a/x)^(3/2)-1/3/x^3/LambertW(a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \frac{1}{216}\left(-\frac{\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{W\left(\frac{a}{x}\right)}\right)}{a^3}+\frac{6}{x^3W\left(\frac{a}{x}\right)^{5/2}}-\frac{12}{x^3W\left(\frac{a}{x}\right)^{3/2}}-\frac{72}{x^3\sqrt{W\left(\frac{a}{x}\right)}}\right)$$

input
```
Integrate[1/(x^4*Sqrt[ProductLog[a/x]]),x]
```

output
```
(-((Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ProductLog[a/x]]])/a^3) + 6/(x^3*ProductL
og[a/x]^(5/2)) - 12/(x^3*ProductLog[a/x]^(3/2)) - 72/(x^3*Sqrt[ProductLog[
a/x]]))/216
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7172, 7205, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{W\left(\frac{a}{x}\right)}}dx$$

$$\Big\downarrow \text{ 7172}$$

$$\frac{1}{6}\int \frac{1}{x^4\sqrt{W\left(\frac{a}{x}\right)}\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{1}{3x^3\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{6}\left(-\frac{1}{2}\int \frac{1}{x^4W\left(\frac{a}{x}\right)^{3/2}\left(W\left(\frac{a}{x}\right)+1\right)}dx-\frac{1}{3x^3W\left(\frac{a}{x}\right)^{3/2}}\right)-\frac{1}{3x^3\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{6}\int \frac{1}{x^4W\left(\frac{a}{x}\right)^{5/2}\left(W\left(\frac{a}{x}\right)+1\right)}dx+\frac{1}{3x^3W\left(\frac{a}{x}\right)^{5/2}}\right)-\frac{1}{3x^3W\left(\frac{a}{x}\right)^{3/2}}\right)-\frac{1}{3x^3\sqrt{W\left(\frac{a}{x}\right)}}$$

$$\Big\downarrow \text{ 7204}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{3x^3W\left(\frac{a}{x}\right)^{5/2}}-\frac{\sqrt{\frac{\pi}{3}}\text{erfi}\left(\sqrt{3}\sqrt{W\left(\frac{a}{x}\right)}\right)}{6a^3}\right)-\frac{1}{3x^3W\left(\frac{a}{x}\right)^{3/2}}\right)-\frac{1}{3x^3\sqrt{W\left(\frac{a}{x}\right)}}$$

input
```
Int[1/(x^4*Sqrt[ProductLog[a/x]]),x]
```

output
```
((-1/6*(Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ProductLog[a/x]]])/a^3 + 1/(3*x^3*Pro
ductLog[a/x]^(5/2)))/2 - 1/(3*x^3*ProductLog[a/x]^(3/2)))/6 - 1/(3*x^3*Sqr
t[ProductLog[a/x]])
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int \frac{1}{x^4\sqrt{\operatorname{LambertW}\left(\frac{a}{x}\right)}}dx$$

input
```
int(1/x^4/LambertW(a/x)^(1/2),x)
```

output
```
int(1/x^4/LambertW(a/x)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^4\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x^4\sqrt{\mathrm{W}(\frac{a}{x})}}\,dx$$

input
```
integrate(1/x^4/lambert_w(a/x)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/(x^4*sqrt(lambert_w(a/x))), x)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x^4\sqrt{W\left(\frac{a}{x}\right)}}\,dx$$

input
```
integrate(1/x**4/LambertW(a/x)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(LambertW(a/x))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x^4\sqrt{\mathrm{W}(\frac{a}{x})}}\,dx$$

input
```
integrate(1/x^4/lambert_w(a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x^4*sqrt(lambert_w(a/x))), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x^4\sqrt{\mathrm{W}(\frac{a}{x})}}\,dx$$

input
```
integrate(1/x^4/lambert_w(a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x^4*sqrt(lambert_w(a/x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{1}{x^4\,\sqrt{\mathrm{LambertW}\left(\frac{a}{x}\right)}}\,dx$$

input
```
int(1/(x^4*LambertW(a/x)^(1/2)),x)
```

output
```
int(1/(x^4*LambertW(a/x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{W\left(\frac{a}{x}\right)}}\,dx = \int \frac{\sqrt{lambert\_w\left(\frac{a}{x}\right)}}{lambert\_w\left(\frac{a}{x}\right)x^4}dx$$

input
```
int(1/x^4/Lambert_W(a/x)^(1/2),x)
```

output
```
int(sqrt(lambert_w(a/x))/(lambert_w(a/x)*x**4),x)
```

3.223 $\int x^2 \left(cW\left(\frac{a}{x}\right)\right)^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 122

$$\int x^2 \left(cW\left(\frac{a}{x}\right)\right)^p \, dx = \frac{3^{3-p} e^{4W\left(\frac{a}{x}\right)} x^4 \Gamma\left(-3+p, 3W\left(\frac{a}{x}\right)\right) W\left(\frac{a}{x}\right)^{4-p} \left(cW\left(\frac{a}{x}\right)\right)^p}{a} + \frac{3^{2-p} e^{4W\left(\frac{a}{x}\right)} x^4 \Gamma\left(-2+p, 3W\left(\frac{a}{x}\right)\right) W\left(\frac{a}{x}\right)^{3-p} \left(cW\left(\frac{a}{x}\right)\right)^{1+p}}{ac}$$

output
```
3^(3-p)*exp(4*LambertW(a/x))*x^4*GAMMA(-3+p,3*LambertW(a/x))*LambertW(a/x)
^(4-p)*(c*LambertW(a/x))^p/a+3^(2-p)*exp(4*LambertW(a/x))*x^4*GAMMA(-2+p,3
*LambertW(a/x))*LambertW(a/x)^(3-p)*(c*LambertW(a/x))^(p+1)/a/c
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.60

$$\int x^2 \left(cW\left(\frac{a}{x}\right)\right)^p \, dx = \frac{3^{2-p} e^{4W\left(\frac{a}{x}\right)} x^4 \left(3\Gamma\left(-3+p, 3W\left(\frac{a}{x}\right)\right) + \Gamma\left(-2+p, 3W\left(\frac{a}{x}\right)\right)\right) W\left(\frac{a}{x}\right)^{4-p} \left(cW\left(\frac{a}{x}\right)\right)^p}{a}$$

input
```
Integrate[x^2*(c*ProductLog[a/x])^p,x]
```

output
```
(3^(2 - p)*E^(4*ProductLog[a/x])*x^4*(3*Gamma[-3 + p, 3*ProductLog[a/x]] +
 Gamma[-2 + p, 3*ProductLog[a/x]])*ProductLog[a/x]^(4 - p)*(c*ProductLog[a
/x])^p)/a
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {7175, 7174, 7207}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(cW\left(\frac{a}{x}\right)\right)^p dx$$
$$\Big\downarrow \text{ 7175}$$
$$-\int x^4 \left(cW\left(\frac{a}{x}\right)\right)^p d\frac{1}{x}$$
$$\Big\downarrow \text{ 7174}$$
$$-\int \frac{x^4 \left(cW\left(\frac{a}{x}\right)\right)^p}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}-\frac{\int \frac{x^4 \left(cW\left(\frac{a}{x}\right)\right)^{p+1}}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}}{c}$$
$$\Big\downarrow \text{ 7207}$$
$$\frac{3^{2-p}x^4e^{4W\left(\frac{a}{x}\right)}W\left(\frac{a}{x}\right)^{3-p}\left(cW\left(\frac{a}{x}\right)\right)^{p+1}\Gamma\left(p-2,3W\left(\frac{a}{x}\right)\right)}{ac}+\frac{3^{3-p}x^4e^{4W\left(\frac{a}{x}\right)}W\left(\frac{a}{x}\right)^{4-p}\left(cW\left(\frac{a}{x}\right)\right)^p\Gamma\left(p-3,3W\left(\frac{a}{x}\right)\right)}{a}$$

input
```
Int[x^2*(c*ProductLog[a/x])^p,x]
```

output
```
(3^(3 - p)*E^(4*ProductLog[a/x])*x^4*Gamma[-3 + p, 3*ProductLog[a/x]]*Prod
uctLog[a/x]^(4 - p)*(c*ProductLog[a/x])^p)/a + (3^(2 - p)*E^(4*ProductLog[
a/x])*x^4*Gamma[-2 + p, 3*ProductLog[a/x]]*ProductLog[a/x]^(3 - p)*(c*Prod
uctLog[a/x])^(1 + p))/(a*c)
```

Defintions of rubi rules used

rule 7174
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.), x_Symbol] :> Int[x^m*(
(c*ProductLog[a*x])^p/(1 + ProductLog[a*x])), x] + Simp[1/c   Int[x^m*((c*P
roductLog[a*x])^(p + 1)/(1 + ProductLog[a*x])), x], x] /; FreeQ[{a, c, m},
x]
```

rule 7175
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Sub
st[Int[(c*ProductLog[a/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, c, p}, x
] && ILtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

rule 7207
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.))/((d_) + (d_.)*Product
Log[(a_.)*(x_)]), x_Symbol] :> Simp[x^m*Gamma[m + p + 1, (-(m + 1))*Product
Log[a*x]]*((c*ProductLog[a*x])^p/(a*d*(m + 1)*E^(m*ProductLog[a*x])*((-(m +
 1))*ProductLog[a*x])^(m + p))), x] /; FreeQ[{a, c, d, m, p}, x] && NeQ[m,
-1]
```

Maple [F]

$$\int x^2 \left(c\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)^p dx$$

input
```
int(x^2*(c*LambertW(a/x))^p,x)
```

output
```
int(x^2*(c*LambertW(a/x))^p,x)
```

Fricas [F]

$$\int x^2 \left(cW\left(\frac{a}{x}\right)\right)^p dx = \int \left(c\,\mathrm{W}\left(\frac{a}{x}\right)\right)^p x^2\, dx$$

input
```
integrate(x^2*(c*lambert_w(a/x))^p,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a/x))^p*x^2, x)
```

Sympy [F]

$$\int x^2 \left(cW\left(\frac{a}{x}\right)\right)^p\,dx = \int x^2 \left(cW\left(\frac{a}{x}\right)\right)^p\,dx$$

input
```
integrate(x**2*(c*LambertW(a/x))**p,x)
```

output
```
Integral(x**2*(c*LambertW(a/x))**p, x)
```

Maxima [F]

$$\int x^2 \left(cW\left(\frac{a}{x}\right)\right)^p\,dx = \int \left(c\,\mathrm{W}\left(\frac{a}{x}\right)\right)^p x^2\,dx$$

input
```
integrate(x^2*(c*lambert_w(a/x))^p,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a/x))^p*x^2, x)
```

Giac [F]

$$\int x^2 \left(cW\left(\frac{a}{x}\right)\right)^p\,dx = \int \left(c\,\mathrm{W}\left(\frac{a}{x}\right)\right)^p x^2\,dx$$

input
```
integrate(x^2*(c*lambert_w(a/x))^p,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a/x))^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(cW\left(\frac{a}{x}\right)\right)^p \, dx = \int x^2 \left(c\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)^p \, dx$$

input
```
int(x^2*(c*LambertW(a/x))^p,x)
```

output
```
int(x^2*(c*LambertW(a/x))^p, x)
```

Reduce [F]

$$\int x^2 \left(cW\left(\frac{a}{x}\right)\right)^p \, dx = \frac{c^p \left(lambert\_w\left(\frac{a}{x}\right)^p x^3 + \left(\int \frac{lambert\_w\left(\frac{a}{x}\right)^p x}{e^{lambert\_w\left(\frac{a}{x}\right)} lambert\_w\left(\frac{a}{x}\right)^2 + e^{lambert\_w\left(\frac{a}{x}\right)} lambert\_w\left(\frac{a}{x}\right)} dx\right) ap\right)}{3}$$

input
```
int(x^2*(c*Lambert_W(a/x))^p,x)
```

output
```
(c**p*(lambert_w(a/x)**p*x**3 + int((lambert_w(a/x)**p*x)/(e**lambert_w(a/
x)*lambert_w(a/x)**2 + e**lambert_w(a/x)*lambert_w(a/x)),x)*a*p))/3
```

3.224 $\int x\left(cW\left(\frac{a}{x}\right)\right)^p\,dx$

Optimal result

Integrand size = 12, antiderivative size = 122

$$\int x\left(cW\left(\frac{a}{x}\right)\right)^p\,dx=\frac{2^{2-p}e^{3W\left(\frac{a}{x}\right)}x^3\Gamma\left(-2+p,2W\left(\frac{a}{x}\right)\right)W\left(\frac{a}{x}\right)^{3-p}\left(cW\left(\frac{a}{x}\right)\right)^p}{a}+\frac{2^{1-p}e^{3W\left(\frac{a}{x}\right)}x^3\Gamma\left(-1+p,2W\left(\frac{a}{x}\right)\right)W\left(\frac{a}{x}\right)^{2-p}\left(cW\left(\frac{a}{x}\right)\right)^{1+p}}{ac}$$

output
```
2^(2-p)*exp(3*LambertW(a/x))*x^3*GAMMA(-2+p,2*LambertW(a/x))*LambertW(a/x)
^(3-p)*(c*LambertW(a/x))^p/a+2^(1-p)*exp(3*LambertW(a/x))*x^3*GAMMA(-1+p,2
*LambertW(a/x))*LambertW(a/x)^(2-p)*(c*LambertW(a/x))^(p+1)/a/c
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.60

$$\int x\left(cW\left(\frac{a}{x}\right)\right)^p\,dx=\frac{2^{1-p}e^{3W\left(\frac{a}{x}\right)}x^3\left(2\Gamma\left(-2+p,2W\left(\frac{a}{x}\right)\right)+\Gamma\left(-1+p,2W\left(\frac{a}{x}\right)\right)\right)W\left(\frac{a}{x}\right)^{3-p}\left(cW\left(\frac{a}{x}\right)\right)^p}{a}$$

input
```
Integrate[x*(c*ProductLog[a/x])^p,x]
```

output
```
(2^(1 - p)*E^(3*ProductLog[a/x])*x^3*(2*Gamma[-2 + p, 2*ProductLog[a/x]] +
 Gamma[-1 + p, 2*ProductLog[a/x]])*ProductLog[a/x]^(3 - p)*(c*ProductLog[a
/x])^p)/a
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7175, 7174, 7207}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(cW\left(\frac{a}{x}\right)\right)^p dx$$

$$\Big\downarrow \text{7175}$$

$$-\int x^3\left(cW\left(\frac{a}{x}\right)\right)^p d\frac{1}{x}$$

$$\Big\downarrow \text{7174}$$

$$-\int \frac{x^3\left(cW\left(\frac{a}{x}\right)\right)^p}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}-\frac{\int \frac{x^3\left(cW\left(\frac{a}{x}\right)\right)^{p+1}}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{7207}$$

$$\frac{2^{1-p}x^3e^{3W\left(\frac{a}{x}\right)}W\left(\frac{a}{x}\right)^{2-p}\left(cW\left(\frac{a}{x}\right)\right)^{p+1}\Gamma\left(p-1,2W\left(\frac{a}{x}\right)\right)}{ac}+\frac{2^{2-p}x^3e^{3W\left(\frac{a}{x}\right)}W\left(\frac{a}{x}\right)^{3-p}\left(cW\left(\frac{a}{x}\right)\right)^p\Gamma\left(p-2,2W\left(\frac{a}{x}\right)\right)}{a}$$

input
```
Int[x*(c*ProductLog[a/x])^p,x]
```

output
```
(2^(2 - p)*E^(3*ProductLog[a/x])*x^3*Gamma[-2 + p, 2*ProductLog[a/x]]*Prod
uctLog[a/x]^(3 - p)*(c*ProductLog[a/x])^p)/a + (2^(1 - p)*E^(3*ProductLog[
a/x])*x^3*Gamma[-1 + p, 2*ProductLog[a/x]]*ProductLog[a/x]^(2 - p)*(c*Prod
uctLog[a/x])^(1 + p))/(a*c)
```

Defintions of rubi rules used

rule 7174
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.), x_Symbol] :> Int[x^m*(
(c*ProductLog[a*x])^p/(1 + ProductLog[a*x])), x] + Simp[1/c    Int[x^m*((c*P
roductLog[a*x])^(p + 1)/(1 + ProductLog[a*x])), x], x] /; FreeQ[{a, c, m},
x]
```

rule 7175
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Sub
st[Int[(c*ProductLog[a/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, c, p}, x
] && ILtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

rule 7207
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.))/((d_) + (d_.)*Product
Log[(a_.)*(x_)]), x_Symbol] :> Simp[x^m*Gamma[m + p + 1, (-(m + 1))*Product
Log[a*x]]*((c*ProductLog[a*x])^p/(a*d*(m + 1)*E^(m*ProductLog[a*x])*((-(m +
 1))*ProductLog[a*x])^(m + p))), x] /; FreeQ[{a, c, d, m, p}, x] && NeQ[m,
-1]
```

Maple [F]

$$\int x\left(c\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)^{p}dx$$

input
```
int(x*(c*LambertW(a/x))^p,x)
```

output
```
int(x*(c*LambertW(a/x))^p,x)
```

Fricas [F]

$$\int x\left(cW\left(\frac{a}{x}\right)\right)^{p}\,dx=\int\left(c\,\mathrm{W}\left(\frac{a}{x}\right)\right)^{p}x\,dx$$

input
```
integrate(x*(c*lambert_w(a/x))^p,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a/x))^p*x, x)
```

Sympy [F]

$$\int x\left(cW\left(\frac{a}{x}\right)\right)^p\,dx=\int x\left(cW\left(\frac{a}{x}\right)\right)^p\,dx$$

input
```
integrate(x*(c*LambertW(a/x))**p,x)
```

output
```
Integral(x*(c*LambertW(a/x))**p, x)
```

Maxima [F]

$$\int x\left(cW\left(\frac{a}{x}\right)\right)^p\,dx=\int\left(c\,\mathrm{W}\left(\frac{a}{x}\right)\right)^p x\,dx$$

input
```
integrate(x*(c*lambert_w(a/x))^p,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a/x))^p*x, x)
```

Giac [F]

$$\int x\left(cW\left(\frac{a}{x}\right)\right)^p\,dx=\int\left(c\,\mathrm{W}\left(\frac{a}{x}\right)\right)^p x\,dx$$

input
```
integrate(x*(c*lambert_w(a/x))^p,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a/x))^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(cW\left(\frac{a}{x}\right)\right)^p\,dx=\int x\left(c\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)^p\,dx$$

input
```
int(x*(c*LambertW(a/x))^p,x)
```

output
```
int(x*(c*LambertW(a/x))^p, x)
```

Reduce [F]

$$\int x\left(cW\left(\frac{a}{x}\right)\right)^p\,dx$$
$$=\frac{c^p\left(lambert\_w\left(\frac{a}{x}\right)^p x^2+\left(\int\frac{lambert\_w\left(\frac{a}{x}\right)^p}{e^{lambert\_w\left(\frac{a}{x}\right)}lambert\_w\left(\frac{a}{x}\right)^2+e^{lambert\_w\left(\frac{a}{x}\right)}lambert\_w\left(\frac{a}{x}\right)}dx\right)ap\right)}{2}$$

input
```
int(x*(c*Lambert_W(a/x))^p,x)
```

output
```
(c**p*(lambert_w(a/x)**p*x**2 + int(lambert_w(a/x)**p/(e**lambert_w(a/x)*l
ambert_w(a/x)**2 + e**lambert_w(a/x)*lambert_w(a/x)),x)*a*p))/2
```

3.225 $\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 38

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x} \, dx = -\frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{p} - \frac{\left(cW\left(\frac{a}{x}\right)\right)^{1+p}}{c(1+p)}$$

output
```
-(c*LambertW(a/x))^p/p-(c*LambertW(a/x))^(p+1)/c/(p+1)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x} \, dx = -\frac{\left(cW\left(\frac{a}{x}\right)\right)^p \left(1+p+pW\left(\frac{a}{x}\right)\right)}{p(1+p)}$$

input
```
Integrate[(c*ProductLog[a/x])^p/x,x]
```

output
```
-(((c*ProductLog[a/x])^p*(1 + p + p*ProductLog[a/x]))/(p*(1 + p)))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^{p+1}}{x\left(W\left(\frac{a}{x}\right)+1\right)}dx}{c}-\frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{p}$$

$$\Big\downarrow \text{7200}$$

$$-\frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{p}-\frac{\left(cW\left(\frac{a}{x}\right)\right)^{p+1}}{c(p+1)}$$

input
```
Int[(c*ProductLog[a/x])^p/x,x]
```

output
```
-((c*ProductLog[a/x])^p/p) - (c*ProductLog[a/x])^(1 + p)/(c*(1 + p))
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$-\frac{e^{p\ln\left(c\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)}}{p}-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)e^{p\ln\left(c\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)}}{p+1}$	44
default	$-\frac{e^{p\ln\left(c\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)}}{p}-\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)e^{p\ln\left(c\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)}}{p+1}$	44

input
```
int((c*LambertW(a/x))^p/x,x,method=_RETURNVERBOSE)
```

output
```
-1/p*exp(p*ln(c*LambertW(a/x)))-1/(p+1)*LambertW(a/x)*exp(p*ln(c*LambertW(
a/x)))
```

Fricas [F]

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x}\,dx = \int \frac{\left(c\,\mathrm{W}\left(\frac{a}{x}\right)\right)^p}{x}\,dx$$

input
```
integrate((c*lambert_w(a/x))^p/x,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a/x))^p/x, x)
```

Sympy [F]

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x}\,dx = \int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x}\,dx$$

input
```
integrate((c*LambertW(a/x))**p/x,x)
```

output
```
Integral((c*LambertW(a/x))**p/x, x)
```

Maxima [F]

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x}\,dx = \int \frac{\left(c\,\mathrm{W}\left(\frac{a}{x}\right)\right)^p}{x}\,dx$$

input
```
integrate((c*lambert_w(a/x))^p/x,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a/x))^p/x, x)
```

Giac [F]

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x}\,dx = \int \frac{\left(c\,\mathrm{W}\left(\frac{a}{x}\right)\right)^p}{x}\,dx$$

input
```
integrate((c*lambert_w(a/x))^p/x,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a/x))^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x}\,dx=\int \frac{\left(c\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)^p}{x}\,dx$$

input
```
int((c*LambertW(a/x))^p/x,x)
```

output
```
int((c*LambertW(a/x))^p/x, x)
```

Reduce [F]

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x}\,dx=c^p\left(\int \frac{lambert\_w\left(\frac{a}{x}\right)^p}{x}dx\right)$$

input
```
int((c*Lambert_W(a/x))^p/x,x)
```

output
```
c**p*int(lambert_w(a/x)**p/x,x)
```

3.226 $\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^2}\,dx = -\frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x}+\frac{p\Gamma\left(1+p,-W\left(\frac{a}{x}\right)\right)\left(-W\left(\frac{a}{x}\right)\right)^{-p}\left(cW\left(\frac{a}{x}\right)\right)^p}{a}$$

output
```
-(c*LambertW(a/x))^p/x+p*GAMMA(p+1,-LambertW(a/x))*(c*LambertW(a/x))^p/a/(
(-LambertW(a/x))^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^2}\,dx = -\frac{\left(\Gamma\left(1+p,-W\left(\frac{a}{x}\right)\right)-\Gamma\left(2+p,-W\left(\frac{a}{x}\right)\right)\right)\left(-W\left(\frac{a}{x}\right)\right)^{-p}\left(cW\left(\frac{a}{x}\right)\right)^p}{a}$$

input
```
Integrate[(c*ProductLog[a/x])^p/x^2,x]
```

output
```
-(((Gamma[1 + p, -ProductLog[a/x]] - Gamma[2 + p, -ProductLog[a/x]])*(c*Pr
oductLog[a/x])^p)/(a*(-ProductLog[a/x])^p))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {7175, 7167, 7183}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^2} \, dx$$
$$\Big\downarrow \text{7175}$$
$$-\int \left(cW\left(\frac{a}{x}\right)\right)^p d\frac{1}{x}$$
$$\Big\downarrow \text{7167}$$
$$p\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}-\frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x}$$
$$\Big\downarrow \text{7183}$$
$$\frac{p\left(-W\left(\frac{a}{x}\right)\right)^{-p}\left(cW\left(\frac{a}{x}\right)\right)^p\Gamma\left(p+1,-W\left(\frac{a}{x}\right)\right)}{a}-\frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x}$$

input
```
Int[(c*ProductLog[a/x])^p/x^2,x]
```

output
```
-((c*ProductLog[a/x])^p/x) + (p*Gamma[1 + p, -ProductLog[a/x]]*(c*ProductL
og[a/x])^p)/(a*(-ProductLog[a/x])^p)
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7175
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Sub
st[Int[(c*ProductLog[a/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, c, p}, x
] && ILtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

rule 7183
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.)/((d_) + (d_.)*ProductLog[(
a_.) + (b_.)*(x_)]), x_Symbol] :> Simp[Gamma[p + 1, -ProductLog[a + b*x]]*(
(c*ProductLog[a + b*x])^p/(b*d*(-ProductLog[a + b*x])^p)), x] /; FreeQ[{a,
b, c, d, p}, x]
```

Maple [F]

$$\int \frac{\left(c \,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)^{p}}{x^{2}}dx$$

input
```
int((c*LambertW(a/x))^p/x^2,x)
```

output
```
int((c*LambertW(a/x))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^2}\,dx=\int \frac{\left(c\,\mathrm{W}\left(\frac{a}{x}\right)\right)^{p}}{x^{2}}\,dx$$

input
```
integrate((c*lambert_w(a/x))^p/x^2,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a/x))^p/x^2, x)
```

Sympy [F]

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^2}\,dx = \int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^2}\,dx$$

input
```
integrate((c*LambertW(a/x))**p/x**2,x)
```

output
```
Integral((c*LambertW(a/x))**p/x**2, x)
```

Maxima [F]

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^2}\,dx = \int \frac{\left(c\,\mathrm{W}\left(\frac{a}{x}\right)\right)^p}{x^2}\,dx$$

input
```
integrate((c*lambert_w(a/x))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a/x))^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^2}\,dx = \int \frac{\left(c\,\mathrm{W}\left(\frac{a}{x}\right)\right)^p}{x^2}\,dx$$

input
```
integrate((c*lambert_w(a/x))^p/x^2,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a/x))^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^2}\,dx = \int \frac{\left(c\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)^p}{x^2}\,dx$$

input
```
int((c*LambertW(a/x))^p/x^2,x)
```

output
```
int((c*LambertW(a/x))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^2}\,dx = \frac{c^p\left(-e^{lambert\_w\left(\frac{a}{x}\right)}\,lambert\_w\left(\frac{a}{x}\right)^p\,lambert\_w\left(\frac{a}{x}\right)p - e^{lambert\_w\left(\frac{a}{x}\right)}\,lambert\_w\left(\frac{a}{x}\right)^p\,lambert\_w\left(\frac{a}{x}\right) + e^{lambe}\right.}{}$$

input
```
int((c*Lambert_W(a/x))^p/x^2,x)
```

output
```
(c**p*( - e**lambert_w(a/x)*lambert_w(a/x)**p*lambert_w(a/x)*p - e**lamber
t_w(a/x)*lambert_w(a/x)**p*lambert_w(a/x) + e**lambert_w(a/x)*lambert_w(a/
x)**p*p + int(lambert_w(a/x)**p/(lambert_w(a/x)**2*p*x**2 + lambert_w(a/x)
**2*x**2 + lambert_w(a/x)*p*x**2 + lambert_w(a/x)*x**2),x)*a*p**3 + int(la
mbert_w(a/x)**p/(lambert_w(a/x)**2*p*x**2 + lambert_w(a/x)**2*x**2 + lambe
rt_w(a/x)*p*x**2 + lambert_w(a/x)*x**2),x)*a*p**2 - int(lambert_w(a/x)**p/
(lambert_w(a/x)*p*x**2 + lambert_w(a/x)*x**2 + p*x**2 + x**2),x)*a*p**3 -
int(lambert_w(a/x)**p/(lambert_w(a/x)*p*x**2 + lambert_w(a/x)*x**2 + p*x**
2 + x**2),x)*a*p**2))/(a*(p + 1))
```

3.227 $\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 128

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^3}\,dx = -\frac{2^{-2-p}e^{-W\left(\frac{a}{x}\right)}\Gamma\left(2+p,-2W\left(\frac{a}{x}\right)\right)\left(-W\left(\frac{a}{x}\right)\right)^{-1-p}\left(cW\left(\frac{a}{x}\right)\right)^p}{ax} - \frac{2^{-3-p}e^{-W\left(\frac{a}{x}\right)}\Gamma\left(3+p,-2W\left(\frac{a}{x}\right)\right)\left(-W\left(\frac{a}{x}\right)\right)^{-2-p}\left(cW\left(\frac{a}{x}\right)\right)^{1+p}}{acx}$$

output
```
-2^(-2-p)*GAMMA(2+p,-2*LambertW(a/x))*(-LambertW(a/x))^(-1-p)*(c*LambertW(
a/x))^p/a/exp(LambertW(a/x))/x-2^(-3-p)*GAMMA(3+p,-2*LambertW(a/x))*(-Lamb
ertW(a/x))^(-2-p)*(c*LambertW(a/x))^(p+1)/a/c/exp(LambertW(a/x))/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.48

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^3}\,dx = \frac{2^{-3-p}\left(2\Gamma\left(2+p,-2W\left(\frac{a}{x}\right)\right)-\Gamma\left(3+p,-2W\left(\frac{a}{x}\right)\right)\right)\left(-W\left(\frac{a}{x}\right)\right)^{-p}\left(cW\left(\frac{a}{x}\right)\right)^p}{a^2}$$

input
```
Integrate[(c*ProductLog[a/x])^p/x^3,x]
```

output
```
(2^(-3 - p)*(2*Gamma[2 + p, -2*ProductLog[a/x]] - Gamma[3 + p, -2*ProductL
og[a/x]])*(c*ProductLog[a/x])^p)/(a^2*(-ProductLog[a/x])^p)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {7175, 7174, 7207}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^3}dx$$

$$\Big\downarrow \text{7175}$$

$$-\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{7174}$$

$$-\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x\left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}-\frac{\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^{p+1}}{x\left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{7207}$$

$$-\frac{2^{-p-3}e^{-W\left(\frac{a}{x}\right)}\left(-W\left(\frac{a}{x}\right)\right)^{-p-2}\left(cW\left(\frac{a}{x}\right)\right)^{p+1}\Gamma\left(p+3,-2W\left(\frac{a}{x}\right)\right)}{acx}-\frac{2^{-p-2}e^{-W\left(\frac{a}{x}\right)}\left(-W\left(\frac{a}{x}\right)\right)^{-p-1}\left(cW\left(\frac{a}{x}\right)\right)^p\Gamma\left(p+2,-2W\left(\frac{a}{x}\right)\right)}{ax}$$

input
```
Int[(c*ProductLog[a/x])^p/x^3,x]
```

output
```
-((2^(-2 - p)*Gamma[2 + p, -2*ProductLog[a/x]]*(-ProductLog[a/x])^(-1 - p)
*(c*ProductLog[a/x])^p)/(a*E^ProductLog[a/x]*x)) - (2^(-3 - p)*Gamma[3 + p
, -2*ProductLog[a/x]]*(-ProductLog[a/x])^(-2 - p)*(c*ProductLog[a/x])^(1 +
 p))/(a*c*E^ProductLog[a/x]*x)
```

Defintions of rubi rules used

rule 7174
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.), x_Symbol] :> Int[x^m*(
(c*ProductLog[a*x])^p/(1 + ProductLog[a*x])), x] + Simp[1/c   Int[x^m*((c*P
roductLog[a*x])^(p + 1)/(1 + ProductLog[a*x])), x], x] /; FreeQ[{a, c, m},
x]
```

rule 7175
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Sub
st[Int[(c*ProductLog[a/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, c, p}, x
] && ILtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

rule 7207
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)])^(p_.))/((d_) + (d_.)*Product
Log[(a_.)*(x_)]), x_Symbol] :> Simp[x^m*Gamma[m + p + 1, (-(m + 1))*Product
Log[a*x]]*((c*ProductLog[a*x])^p/(a*d*(m + 1)*E^(m*ProductLog[a*x])*((-(m +
 1))*ProductLog[a*x])^(m + p))), x] /; FreeQ[{a, c, d, m, p}, x] && NeQ[m,
-1]
```

Maple [F]

$$\int \frac{\left(c\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)^{p}}{x^{3}}dx$$

input
```
int((c*LambertW(a/x))^p/x^3,x)
```

output
```
int((c*LambertW(a/x))^p/x^3,x)
```

Fricas [F]

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^{p}}{x^{3}}\,dx=\int \frac{\left(c\,\mathrm{W}\left(\frac{a}{x}\right)\right)^{p}}{x^{3}}\,dx$$

input
```
integrate((c*lambert_w(a/x))^p/x^3,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a/x))^p/x^3, x)
```

Sympy [F]

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^3}\,dx = \int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^3}\,dx$$

input
```
integrate((c*LambertW(a/x))**p/x**3,x)
```

output
```
Integral((c*LambertW(a/x))**p/x**3, x)
```

Maxima [F]

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^3}\,dx = \int \frac{\left(c\,\mathrm{W}\left(\frac{a}{x}\right)\right)^p}{x^3}\,dx$$

input
```
integrate((c*lambert_w(a/x))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a/x))^p/x^3, x)
```

Giac [F]

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^3}\,dx = \int \frac{\left(c\,\mathrm{W}\left(\frac{a}{x}\right)\right)^p}{x^3}\,dx$$

input
```
integrate((c*lambert_w(a/x))^p/x^3,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a/x))^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^3}\,dx = \int \frac{\left(c\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)^p}{x^3}\,dx$$

input
```
int((c*LambertW(a/x))^p/x^3,x)
```

output
```
int((c*LambertW(a/x))^p/x^3, x)
```

Reduce [F]

$$\int \frac{\left(cW\left(\frac{a}{x}\right)\right)^p}{x^3}\,dx = c^p\left(\int \frac{lambert\_w\left(\frac{a}{x}\right)^p}{x^3}dx\right)$$

input
```
int((c*Lambert_W(a/x))^p/x^3,x)
```

output
```
c**p*int(lambert_w(a/x)**p/x**3,x)
```

3.228 $\int W\left(\frac{a}{\sqrt[4]{x}}\right)^5 dx$

Optimal result

Integrand size = 10, antiderivative size = 28

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^5 dx = \frac{5}{4}xW\left(\frac{a}{\sqrt[4]{x}}\right)^4 + xW\left(\frac{a}{\sqrt[4]{x}}\right)^5$$

output
```
5/4*x*LambertW(a/x^(1/4))^4+x*LambertW(a/x^(1/4))^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^5 dx = \frac{5}{4}xW\left(\frac{a}{\sqrt[4]{x}}\right)^4 + xW\left(\frac{a}{\sqrt[4]{x}}\right)^5$$

input
```
Integrate[ProductLog[a/x^(1/4)]^5,x]
```

output
```
(5*x*ProductLog[a/x^(1/4)]^4)/4 + x*ProductLog[a/x^(1/4)]^5
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7169, 7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^5dx$$

$$\Big\downarrow \text{ 7169}$$

$$\frac{5}{4}\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^5}{W\left(\frac{a}{\sqrt[4]{x}}\right)+1}dx+xW\left(\frac{a}{\sqrt[4]{x}}\right)^5$$

$$\Big\downarrow \text{ 7187}$$

$$xW\left(\frac{a}{\sqrt[4]{x}}\right)^5+\frac{5}{4}xW\left(\frac{a}{\sqrt[4]{x}}\right)^4$$

input
```
Int[ProductLog[a/x^(1/4)]^5,x]
```

output
```
(5*x*ProductLog[a/x^(1/4)]^4)/4 + x*ProductLog[a/x^(1/4)]^5
```

Defintions of rubi rules used

rule 7169
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*(c*Produ
ctLog[a*x^n])^p, x] - Simp[n*p   Int[(c*ProductLog[a*x^n])^p/(1 + ProductLo
g[a*x^n]), x], x] /; FreeQ[{a, c, n, p}, x] && (EqQ[n*(p - 1), -1] || (Inte
gerQ[p - 1/2] && EqQ[n*(p - 1/2), -1]))
```

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$-4a^4\left(-\frac{5x\,\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{4}}}\right)^4}{16a^4}-\frac{x\,\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{4}}}\right)^5}{4a^4}\right)$	35
default	$-4a^4\left(-\frac{5x\,\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{4}}}\right)^4}{16a^4}-\frac{x\,\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{4}}}\right)^5}{4a^4}\right)$	35

input
```
int(LambertW(a/x^(1/4))^5,x,method=_RETURNVERBOSE)
```

output
```
-4*a^4*(-5/16*x/a^4*LambertW(a/x^(1/4))^4-1/4*x/a^4*LambertW(a/x^(1/4))^5)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^5\,dx = x\,\mathrm{W}(\frac{a}{x^{\frac{1}{4}}})^5+\frac{5}{4}\,x\,\mathrm{W}(\frac{a}{x^{\frac{1}{4}}})^4$$

input
```
integrate(lambert_w(a/x^(1/4))^5,x, algorithm="fricas")
```

output
```
x*lambert_w(a/x^(1/4))^5 + 5/4*x*lambert_w(a/x^(1/4))^4
```

Sympy [F]

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^5 dx = \int W^5\left(\frac{a}{\sqrt[4]{x}}\right) dx$$

input
```
integrate(LambertW(a/x**(1/4))**5,x)
```

output
```
Integral(LambertW(a/x**(1/4))**5, x)
```

Maxima [F]

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^5 dx = \int \mathrm{W}\left(\frac{a}{x^{\frac{1}{4}}}\right)^5 dx$$

input
```
integrate(lambert_w(a/x^(1/4))^5,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x^(1/4))^5, x)
```

Giac [F]

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^5 dx = \int \mathrm{W}\left(\frac{a}{x^{\frac{1}{4}}}\right)^5 dx$$

input
```
integrate(lambert_w(a/x^(1/4))^5,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x^(1/4))^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^5 dx = \int \mathrm{LambertW}\left(\frac{a}{x^{1/4}}\right)^5 dx$$

input
```
int(LambertW(a/x^(1/4))^5,x)
```

output
```
int(LambertW(a/x^(1/4))^5, x)
```

Reduce [F]

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^5 dx = \frac{5\left(\int \frac{lambert\_w\left(\frac{a}{x^{\frac{1}{4}}}\right)^4}{x^{\frac{1}{4}} e^{lambert\_w\left(\frac{a}{x^{\frac{1}{4}}}\right)} lambert\_w\left(\frac{a}{x^{\frac{1}{4}}}\right)+x^{\frac{1}{4}} e^{lambert\_w\left(\frac{a}{x^{\frac{1}{4}}}\right)}}dx\right) a}{4} + lambert\_w\left(\frac{a}{x^{\frac{1}{4}}}\right)^5 x$$

input
```
int(Lambert_W(a/x^(1/4))^5,x)
```

output
```
(5*int(lambert_w(a/x**(1/4))**4/(x**(1/4)*e**lambert_w(a/x**(1/4))*lambert
_w(a/x**(1/4)) + x**(1/4)*e**lambert_w(a/x**(1/4))),x)*a + 4*lambert_w(a/x
**(1/4))**5*x)/4
```

3.229 $\int W\left(\frac{a}{\sqrt[3]{x}}\right)^4 dx$

Optimal result

Integrand size = 10, antiderivative size = 28

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^4 dx = \frac{4}{3}xW\left(\frac{a}{\sqrt[3]{x}}\right)^3 + xW\left(\frac{a}{\sqrt[3]{x}}\right)^4$$

output
```
4/3*x*LambertW(a/x^(1/3))^3+x*LambertW(a/x^(1/3))^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^4 dx = \frac{4}{3}xW\left(\frac{a}{\sqrt[3]{x}}\right)^3 + xW\left(\frac{a}{\sqrt[3]{x}}\right)^4$$

input
```
Integrate[ProductLog[a/x^(1/3)]^4,x]
```

output
```
(4*x*ProductLog[a/x^(1/3)]^3)/3 + x*ProductLog[a/x^(1/3)]^4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7169, 7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^4dx$$
$$\Big\downarrow \text{ 7169}$$
$$\frac{4}{3}\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^4}{W\left(\frac{a}{\sqrt[3]{x}}\right)+1}dx+xW\left(\frac{a}{\sqrt[3]{x}}\right)^4$$
$$\Big\downarrow \text{ 7187}$$
$$xW\left(\frac{a}{\sqrt[3]{x}}\right)^4+\frac{4}{3}xW\left(\frac{a}{\sqrt[3]{x}}\right)^3$$

input
```
Int[ProductLog[a/x^(1/3)]^4,x]
```

output
```
(4*x*ProductLog[a/x^(1/3)]^3)/3 + x*ProductLog[a/x^(1/3)]^4
```

Defintions of rubi rules used

rule 7169
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*(c*Produ
ctLog[a*x^n])^p, x] - Simp[n*p   Int[(c*ProductLog[a*x^n])^p/(1 + ProductLo
g[a*x^n]), x], x] /; FreeQ[{a, c, n, p}, x] && (EqQ[n*(p - 1), -1] || (Inte
gerQ[p - 1/2] && EqQ[n*(p - 1/2), -1]))
```

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$-3a^3\left(-\frac{4x\,\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{3}}}\right)^3}{9a^3}-\frac{x\,\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{3}}}\right)^4}{3a^3}\right)$	35
default	$-3a^3\left(-\frac{4x\,\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{3}}}\right)^3}{9a^3}-\frac{x\,\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{3}}}\right)^4}{3a^3}\right)$	35

input
```
int(LambertW(a/x^(1/3))^4,x,method=_RETURNVERBOSE)
```

output
```
-3*a^3*(-4/9*x/a^3*LambertW(a/x^(1/3))^3-1/3*x/a^3*LambertW(a/x^(1/3))^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^4 dx = x\,\mathrm{W}(\frac{a}{x^{\frac{1}{3}}})^4 + \frac{4}{3}\,x\,\mathrm{W}(\frac{a}{x^{\frac{1}{3}}})^3$$

input
```
integrate(lambert_w(a/x^(1/3))^4,x, algorithm="fricas")
```

output
```
x*lambert_w(a/x^(1/3))^4 + 4/3*x*lambert_w(a/x^(1/3))^3
```

Sympy [F]

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^4 dx = \int W^4\left(\frac{a}{\sqrt[3]{x}}\right) dx$$

input
```
integrate(LambertW(a/x**(1/3))**4,x)
```

output
```
Integral(LambertW(a/x**(1/3))**4, x)
```

Maxima [F]

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^4 dx = \int \mathrm{W}\left(\frac{a}{x^{\frac{1}{3}}}\right)^4 dx$$

input
```
integrate(lambert_w(a/x^(1/3))^4,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x^(1/3))^4, x)
```

Giac [F]

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^4 dx = \int \mathrm{W}\left(\frac{a}{x^{\frac{1}{3}}}\right)^4 dx$$

input
```
integrate(lambert_w(a/x^(1/3))^4,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x^(1/3))^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^4 dx = \int \mathrm{LambertW}\left(\frac{a}{x^{1/3}}\right)^4 dx$$

input
```
int(LambertW(a/x^(1/3))^4,x)
```

output
```
int(LambertW(a/x^(1/3))^4, x)
```

Reduce [F]

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^4 dx = \frac{4\left(\int \frac{lambert\_w\left(\frac{a}{x^{\frac{1}{3}}}\right)^3}{x^{\frac{1}{3}} e^{lambert\_w\left(\frac{a}{x^{\frac{1}{3}}}\right)} lambert\_w\left(\frac{a}{x^{\frac{1}{3}}}\right)+x^{\frac{1}{3}} e^{lambert\_w\left(\frac{a}{x^{\frac{1}{3}}}\right)}}dx\right) a}{3} + lambert\_w\left(\frac{a}{x^{\frac{1}{3}}}\right)^4 x$$

input
```
int(Lambert_W(a/x^(1/3))^4,x)
```

output
```
(4*int(lambert_w(a/x**(1/3))**3/(x**(1/3)*e**lambert_w(a/x**(1/3))*lambert
_w(a/x**(1/3)) + x**(1/3)*e**lambert_w(a/x**(1/3))),x)*a + 3*lambert_w(a/x
**(1/3))**4*x)/3
```

3.230 $\int W\left(\frac{a}{\sqrt{x}}\right)^3 dx$

Optimal result

Integrand size = 10, antiderivative size = 28

$$\int W\left(\frac{a}{\sqrt{x}}\right)^3 dx = \frac{3}{2}xW\left(\frac{a}{\sqrt{x}}\right)^2 + xW\left(\frac{a}{\sqrt{x}}\right)^3$$

output
```
3/2*x*LambertW(a/x^(1/2))^2+x*LambertW(a/x^(1/2))^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int W\left(\frac{a}{\sqrt{x}}\right)^3 dx = \frac{3}{2}xW\left(\frac{a}{\sqrt{x}}\right)^2 + xW\left(\frac{a}{\sqrt{x}}\right)^3$$

input
```
Integrate[ProductLog[a/Sqrt[x]]^3,x]
```

output
```
(3*x*ProductLog[a/Sqrt[x]]^2)/2 + x*ProductLog[a/Sqrt[x]]^3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7169, 7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W\left(\frac{a}{\sqrt{x}}\right)^3 dx$$

$$\Big\downarrow \text{ 7169}$$

$$\frac{3}{2}\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^3}{W\left(\frac{a}{\sqrt{x}}\right)+1}dx + xW\left(\frac{a}{\sqrt{x}}\right)^3$$

$$\Big\downarrow \text{ 7187}$$

$$xW\left(\frac{a}{\sqrt{x}}\right)^3 + \frac{3}{2}xW\left(\frac{a}{\sqrt{x}}\right)^2$$

input
```
Int[ProductLog[a/Sqrt[x]]^3,x]
```

output
```
(3*x*ProductLog[a/Sqrt[x]]^2)/2 + x*ProductLog[a/Sqrt[x]]^3
```

Defintions of rubi rules used

rule 7169
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*(c*Produ
ctLog[a*x^n])^p, x] - Simp[n*p   Int[(c*ProductLog[a*x^n])^p/(1 + ProductLo
g[a*x^n]), x], x] /; FreeQ[{a, c, n, p}, x] && (EqQ[n*(p - 1), -1] || (Inte
gerQ[p - 1/2] && EqQ[n*(p - 1/2), -1]))
```

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$-2a^2\left(-\frac{3x\,\mathrm{LambertW}\left(\frac{a}{\sqrt{x}}\right)^2}{4a^2}-\frac{x\,\mathrm{LambertW}\left(\frac{a}{\sqrt{x}}\right)^3}{2a^2}\right)$	35
default	$-2a^2\left(-\frac{3x\,\mathrm{LambertW}\left(\frac{a}{\sqrt{x}}\right)^2}{4a^2}-\frac{x\,\mathrm{LambertW}\left(\frac{a}{\sqrt{x}}\right)^3}{2a^2}\right)$	35

input
```
int(LambertW(a/x^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*a^2*(-3/4*x/a^2*LambertW(a/x^(1/2))^2-1/2*x/a^2*LambertW(a/x^(1/2))^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int W\left(\frac{a}{\sqrt{x}}\right)^3\,dx = x\,\mathrm{W}(\frac{a}{\sqrt{x}})^3+\frac{3}{2}\,x\,\mathrm{W}(\frac{a}{\sqrt{x}})^2$$

input
```
integrate(lambert_w(a/x^(1/2))^3,x, algorithm="fricas")
```

output
```
x*lambert_w(a/sqrt(x))^3 + 3/2*x*lambert_w(a/sqrt(x))^2
```

Sympy [F]

$$\int W\left(\frac{a}{\sqrt{x}}\right)^3\,dx = \int W^3\left(\frac{a}{\sqrt{x}}\right)\,dx$$

input
```
integrate(LambertW(a/x**(1/2))**3,x)
```

output
```
Integral(LambertW(a/sqrt(x))**3, x)
```

Maxima [F]

$$\int W\left(\frac{a}{\sqrt{x}}\right)^3 dx = \int \mathrm{W}\left(\frac{a}{\sqrt{x}}\right)^3 dx$$

input
```
integrate(lambert_w(a/x^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/sqrt(x))^3, x)
```

Giac [F]

$$\int W\left(\frac{a}{\sqrt{x}}\right)^3 dx = \int \mathrm{W}\left(\frac{a}{\sqrt{x}}\right)^3 dx$$

input
```
integrate(lambert_w(a/x^(1/2))^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/sqrt(x))^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int W\left(\frac{a}{\sqrt{x}}\right)^3 dx = \int \mathrm{LambertW}\left(\frac{a}{\sqrt{x}}\right)^3 dx$$

input
```
int(LambertW(a/x^(1/2))^3,x)
```

output
```
int(LambertW(a/x^(1/2))^3, x)
```

Reduce [F]

$$\int W\left(\frac{a}{\sqrt{x}}\right)^3 dx = \frac{3\left(\int \frac{lambert\_w\left(\frac{a}{\sqrt{x}}\right)^2}{\sqrt{x}\, e^{lambert\_w\left(\frac{a}{\sqrt{x}}\right)} lambert\_w\left(\frac{a}{\sqrt{x}}\right)+\sqrt{x}\, e^{lambert\_w\left(\frac{a}{\sqrt{x}}\right)}}dx\right)a}{2} + lambert\_w\left(\frac{a}{\sqrt{x}}\right)^3 x$$

input
```
int(Lambert_W(a/x^(1/2))^3,x)
```

output
```
(3*int(lambert_w(a/sqrt(x))**2/(sqrt(x)*e**lambert_w(a/sqrt(x))*lambert_w(
a/sqrt(x)) + sqrt(x)*e**lambert_w(a/sqrt(x))),x)*a + 2*lambert_w(a/sqrt(x)
)**3*x)/2
```

3.231 $\int W\left(\frac{a}{x}\right)^2 dx$

Optimal result

Integrand size = 8, antiderivative size = 20

$$\int W\left(\frac{a}{x}\right)^2 dx = 2xW\left(\frac{a}{x}\right) + xW\left(\frac{a}{x}\right)^2$$

output
```
2*x*LambertW(a/x)+x*LambertW(a/x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int W\left(\frac{a}{x}\right)^2 dx = 2xW\left(\frac{a}{x}\right) + xW\left(\frac{a}{x}\right)^2$$

input
```
Integrate[ProductLog[a/x]^2,x]
```

output
```
2*x*ProductLog[a/x] + x*ProductLog[a/x]^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7169, 7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W\left(\frac{a}{x}\right)^2 dx$$

$$\Big\downarrow \text{7169}$$

$$2\int \frac{W\left(\frac{a}{x}\right)^2}{W\left(\frac{a}{x}\right)+1}dx+xW\left(\frac{a}{x}\right)^2$$

$$\Big\downarrow \text{7187}$$

$$xW\left(\frac{a}{x}\right)^2+2xW\left(\frac{a}{x}\right)$$

input
```
Int[ProductLog[a/x]^2,x]
```

output
```
2*x*ProductLog[a/x] + x*ProductLog[a/x]^2
```

Defintions of rubi rules used

rule 7169
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*(c*Produ
ctLog[a*x^n])^p, x] - Simp[n*p   Int[(c*ProductLog[a*x^n])^p/(1 + ProductLo
g[a*x^n]), x], x] /; FreeQ[{a, c, n, p}, x] && (EqQ[n*(p - 1), -1] || (Inte
gerQ[p - 1/2] && EqQ[n*(p - 1/2), -1]))
```

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$2x \operatorname{LambertW}\left(\frac{a}{x}\right)+x \operatorname{LambertW}\left(\frac{a}{x}\right)^{2}$	21
derivativedivides	$-a\left(-\frac{2x \operatorname{LambertW}\left(\frac{a}{x}\right)}{a}-\frac{x \operatorname{LambertW}\left(\frac{a}{x}\right)^{2}}{a}\right)$	31
default	$-a\left(-\frac{2x \operatorname{LambertW}\left(\frac{a}{x}\right)}{a}-\frac{x \operatorname{LambertW}\left(\frac{a}{x}\right)^{2}}{a}\right)$	31

input
```
int(LambertW(a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
2*x*LambertW(a/x)+x*LambertW(a/x)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int W\left(\frac{a}{x}\right)^2\,dx = x\,\mathrm{W}(\frac{a}{x})^2 + 2\,x\,\mathrm{W}(\frac{a}{x})$$

input
```
integrate(lambert_w(a/x)^2,x, algorithm="fricas")
```

output
```
x*lambert_w(a/x)^2 + 2*x*lambert_w(a/x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int W\left(\frac{a}{x}\right)^2\,dx = xW^2\left(\frac{a}{x}\right) + 2xW\left(\frac{a}{x}\right)$$

input
```
integrate(LambertW(a/x)**2,x)
```

output
```
x*LambertW(a/x)**2 + 2*x*LambertW(a/x)
```

Maxima [F]

$$\int W\left(\frac{a}{x}\right)^2\,dx = \int \mathrm{W}(\frac{a}{x})^2\,dx$$

input
```
integrate(lambert_w(a/x)^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x)^2, x)
```

Giac [F]

$$\int W\left(\frac{a}{x}\right)^2\,dx = \int \mathrm{W}(\frac{a}{x})^2\,dx$$

input
```
integrate(lambert_w(a/x)^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int W\left(\frac{a}{x}\right)^2\,dx = \int \mathrm{LambertW}\left(\frac{a}{x}\right)^2\,dx$$

input
```
int(LambertW(a/x)^2,x)
```

output
```
int(LambertW(a/x)^2, x)
```

Reduce [F]

$$\int W\left(\frac{a}{x}\right)^2 dx = 2\left(\int \frac{lambert\_w\left(\frac{a}{x}\right)^2}{lambert\_w\left(\frac{a}{x}\right)+1}dx\right) - 2\left(\int \frac{lambert\_w\left(\frac{a}{x}\right)}{lambert\_w\left(\frac{a}{x}\right)+1}dx\right) + 2\left(\int \frac{lambert\_w\left(\frac{a}{x}\right)}{e^{lambert\_w\left(\frac{a}{x}\right)} lambert\_w\left(\frac{a}{x}\right)x + e^{lambert\_w\left(\frac{a}{x}\right)}x}dx\right)a - 4\left(\int \frac{1}{lambert\_w\left(\frac{a}{x}\right)+1}dx\right) - 2\left(\int \frac{1}{e^{lambert\_w\left(\frac{a}{x}\right)} lambert\_w\left(\frac{a}{x}\right)x + e^{lambert\_w\left(\frac{a}{x}\right)}x}dx\right)a + lambert\_w\left(\frac{a}{x}\right)^2 x - 2lambert\_w\left(\frac{a}{x}\right)x + 4x$$

input
```
int(Lambert_W(a/x)^2,x)
```

output
```
2*int(lambert_w(a/x)**2/(lambert_w(a/x) + 1),x) - 2*int(lambert_w(a/x)/(la
mbert_w(a/x) + 1),x) + 2*int(lambert_w(a/x)/(e**lambert_w(a/x)*lambert_w(a
/x)*x + e**lambert_w(a/x)*x),x)*a - 4*int(1/(lambert_w(a/x) + 1),x) - 2*in
t(1/(e**lambert_w(a/x)*lambert_w(a/x)*x + e**lambert_w(a/x)*x),x)*a + lamb
ert_w(a/x)**2*x - 2*lambert_w(a/x)*x + 4*x
```

3.232 $\int \frac{1}{W\left(a\sqrt{x}\right)} dx$

Optimal result

Integrand size = 10, antiderivative size = 28

$$\int \frac{1}{W\left(a\sqrt{x}\right)} dx = \frac{x}{2W\left(a\sqrt{x}\right)^2} + \frac{x}{W\left(a\sqrt{x}\right)}$$

output
```
1/2*x/LambertW(a*x^(1/2))^2+x/LambertW(a*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{W\left(a\sqrt{x}\right)} dx = \frac{x}{2W\left(a\sqrt{x}\right)^2} + \frac{x}{W\left(a\sqrt{x}\right)}$$

input
```
Integrate[ProductLog[a*Sqrt[x]]^(-1),x]
```

output
```
x/(2*ProductLog[a*Sqrt[x]]^2) + x/ProductLog[a*Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7169, 7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W\left(a\sqrt{x}\right)}dx$$

$$\Big\downarrow \text{ 7169}$$

$$\frac{1}{2}\int \frac{1}{W\left(a\sqrt{x}\right)\left(W\left(a\sqrt{x}\right)+1\right)}dx+\frac{x}{W\left(a\sqrt{x}\right)}$$

$$\Big\downarrow \text{ 7187}$$

$$\frac{x}{W\left(a\sqrt{x}\right)}+\frac{x}{2W\left(a\sqrt{x}\right)^2}$$

input
```
Int[ProductLog[a*Sqrt[x]]^(-1),x]
```

output
```
x/(2*ProductLog[a*Sqrt[x]]^2) + x/ProductLog[a*Sqrt[x]]
```

Defintions of rubi rules used

rule 7169
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*(c*Produ
ctLog[a*x^n])^p, x] - Simp[n*p   Int[(c*ProductLog[a*x^n])^p/(1 + ProductLo
g[a*x^n]), x], x] /; FreeQ[{a, c, n, p}, x] && (EqQ[n*(p - 1), -1] || (Inte
gerQ[p - 1/2] && EqQ[n*(p - 1/2), -1]))
```

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{\frac{x a^2}{2 \operatorname{LambertW}(a\sqrt{x})^2}+\frac{x a^2}{\operatorname{LambertW}(a\sqrt{x})}}{a^2}$	35
default	$\frac{\frac{x a^2}{2 \operatorname{LambertW}(a\sqrt{x})^2}+\frac{x a^2}{\operatorname{LambertW}(a\sqrt{x})}}{a^2}$	35

input
```
int(1/LambertW(a*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/a^2*(1/4/LambertW(a*x^(1/2))^2*x*a^2+1/2/LambertW(a*x^(1/2))*x*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{1}{W\left(a\sqrt{x}\right)}\,dx = \frac{2\,x\,\mathrm{W}(a\sqrt{x}) + x}{2\,\mathrm{W}(a\sqrt{x})^2}$$

input
```
integrate(1/lambert_w(a*x^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(2*x*lambert_w(a*sqrt(x)) + x)/lambert_w(a*sqrt(x))^2
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{1}{W\left(a\sqrt{x}\right)}\,dx = \frac{x}{W\left(a\sqrt{x}\right)} + \frac{x}{2W^2\left(a\sqrt{x}\right)}$$

input
```
integrate(1/LambertW(a*x**(1/2)),x)
```

output
```
x/LambertW(a*sqrt(x)) + x/(2*LambertW(a*sqrt(x))**2)
```

Maxima [F]

$$\int \frac{1}{W\left(a\sqrt{x}\right)}\,dx = \int \frac{1}{\mathrm{W}(a\sqrt{x})}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/lambert_w(a*sqrt(x)), x)
```

Giac [F]

$$\int \frac{1}{W\left(a\sqrt{x}\right)}\,dx = \int \frac{1}{\mathrm{W}(a\sqrt{x})}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/2)),x, algorithm="giac")
```

output
```
integrate(1/lambert_w(a*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W\left(a\sqrt{x}\right)}\,dx = \int \frac{1}{\mathrm{LambertW}\left(a\,\sqrt{x}\right)}\,dx$$

input
```
int(1/LambertW(a*x^(1/2)),x)
```

output
```
int(1/LambertW(a*x^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{W\left(a\sqrt{x}\right)}\,dx = \int \frac{1}{lambert\_w\left(\sqrt{x}\,a\right)}dx$$

input
```
int(1/Lambert_W(a*x^(1/2)),x)
```

output
```
int(1/lambert_w(sqrt(x)*a),x)
```

3.233 $\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 28

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2}\,dx=\frac{2x}{3W\left(a\sqrt[3]{x}\right)^3}+\frac{x}{W\left(a\sqrt[3]{x}\right)^2}$$

output
```
2/3*x/LambertW(a*x^(1/3))^3+x/LambertW(a*x^(1/3))^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2}\,dx=\frac{2x}{3W\left(a\sqrt[3]{x}\right)^3}+\frac{x}{W\left(a\sqrt[3]{x}\right)^2}$$

input
```
Integrate[ProductLog[a*x^(1/3)]^(-2),x]
```

output
```
(2*x)/(3*ProductLog[a*x^(1/3)]^3) + x/ProductLog[a*x^(1/3)]^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7169, 7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2}dx$$

$$\Big\downarrow \text{7169}$$

$$\frac{2}{3}\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2\left(W\left(a\sqrt[3]{x}\right)+1\right)}dx+\frac{x}{W\left(a\sqrt[3]{x}\right)^2}$$

$$\Big\downarrow \text{7187}$$

$$\frac{x}{W\left(a\sqrt[3]{x}\right)^2}+\frac{2x}{3W\left(a\sqrt[3]{x}\right)^3}$$

input
```
Int[ProductLog[a*x^(1/3)]^(-2),x]
```

output
```
(2*x)/(3*ProductLog[a*x^(1/3)]^3) + x/ProductLog[a*x^(1/3)]^2
```

Defintions of rubi rules used

rule 7169
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*(c*Produ
ctLog[a*x^n])^p, x] - Simp[n*p   Int[(c*ProductLog[a*x^n])^p/(1 + ProductLo
g[a*x^n]), x], x] /; FreeQ[{a, c, n, p}, x] && (EqQ[n*(p - 1), -1] || (Inte
gerQ[p - 1/2] && EqQ[n*(p - 1/2), -1]))
```

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{\frac{2x\,a^3}{3\,\mathrm{LambertW}\left(a\,x^{\frac{1}{3}}\right)^3}+\frac{x\,a^3}{\mathrm{LambertW}\left(a\,x^{\frac{1}{3}}\right)^2}}{a^3}$	35
default	$\frac{\frac{2x\,a^3}{3\,\mathrm{LambertW}\left(a\,x^{\frac{1}{3}}\right)^3}+\frac{x\,a^3}{\mathrm{LambertW}\left(a\,x^{\frac{1}{3}}\right)^2}}{a^3}$	35

input
```
int(1/LambertW(a*x^(1/3))^2,x,method=_RETURNVERBOSE)
```

output
```
3/a^3*(2/9*x*a^3/LambertW(a*x^(1/3))^3+1/3/LambertW(a*x^(1/3))^2*x*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2}\,dx = \frac{3\,x\,\mathrm{W}(ax^{\frac{1}{3}})+2\,x}{3\,\mathrm{W}(ax^{\frac{1}{3}})^3}$$

input
```
integrate(1/lambert_w(a*x^(1/3))^2,x, algorithm="fricas")
```

output
```
1/3*(3*x*lambert_w(a*x^(1/3)) + 2*x)/lambert_w(a*x^(1/3))^3
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2}\,dx = \frac{x}{W^2\left(a\sqrt[3]{x}\right)}+\frac{2x}{3W^3\left(a\sqrt[3]{x}\right)}$$

input
```
integrate(1/LambertW(a*x**(1/3))**2,x)
```

output
```
x/LambertW(a*x**(1/3))**2 + 2*x/(3*LambertW(a*x**(1/3))**3)
```

Maxima [F]

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2}\,dx = \int \frac{1}{W(ax^{\frac{1}{3}})^2}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/3))^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^(1/3))^(-2), x)
```

Giac [F]

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2}\,dx = \int \frac{1}{W(ax^{\frac{1}{3}})^2}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/3))^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^(1/3))^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2}\,dx = \int \frac{1}{\mathrm{LambertW}\left(a\,x^{1/3}\right)^2}\,dx$$

input
```
int(1/LambertW(a*x^(1/3))^2,x)
```

output
```
int(1/LambertW(a*x^(1/3))^2, x)
```

Reduce [F]

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2}\,dx = \int \frac{1}{lambert\_w\left(x^{\frac{1}{3}}a\right)^2}dx$$

input
```
int(1/Lambert_W(a*x^(1/3))^2,x)
```

output
```
int(1/lambert_w(x**(1/3)*a)**2,x)
```

3.234 $\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 28

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3}\,dx=\frac{3x}{4W\left(a\sqrt[4]{x}\right)^4}+\frac{x}{W\left(a\sqrt[4]{x}\right)^3}$$

output
```
3/4*x/LambertW(a*x^(1/4))^4+x/LambertW(a*x^(1/4))^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3}\,dx=\frac{3x}{4W\left(a\sqrt[4]{x}\right)^4}+\frac{x}{W\left(a\sqrt[4]{x}\right)^3}$$

input
```
Integrate[ProductLog[a*x^(1/4)]^(-3),x]
```

output
```
(3*x)/(4*ProductLog[a*x^(1/4)]^4) + x/ProductLog[a*x^(1/4)]^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7169, 7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3}dx$$
$$\Big\downarrow \text{ 7169}$$
$$\frac{3}{4}\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3\left(W\left(a\sqrt[4]{x}\right)+1\right)}dx+\frac{x}{W\left(a\sqrt[4]{x}\right)^3}$$
$$\Big\downarrow \text{ 7187}$$
$$\frac{x}{W\left(a\sqrt[4]{x}\right)^3}+\frac{3x}{4W\left(a\sqrt[4]{x}\right)^4}$$

input
```
Int[ProductLog[a*x^(1/4)]^(-3),x]
```

output
```
(3*x)/(4*ProductLog[a*x^(1/4)]^4) + x/ProductLog[a*x^(1/4)]^3
```

Defintions of rubi rules used

rule 7169
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*(c*Produ
ctLog[a*x^n])^p, x] - Simp[n*p   Int[(c*ProductLog[a*x^n])^p/(1 + ProductLo
g[a*x^n]), x], x] /; FreeQ[{a, c, n, p}, x] && (EqQ[n*(p - 1), -1] || (Inte
gerQ[p - 1/2] && EqQ[n*(p - 1/2), -1]))
```

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{\frac{3x\,a^4}{4\,\mathrm{LambertW}\left(a\,x^{\frac{1}{4}}\right)^4}+\frac{x\,a^4}{\mathrm{LambertW}\left(a\,x^{\frac{1}{4}}\right)^3}}{a^4}$	35
default	$\frac{\frac{3x\,a^4}{4\,\mathrm{LambertW}\left(a\,x^{\frac{1}{4}}\right)^4}+\frac{x\,a^4}{\mathrm{LambertW}\left(a\,x^{\frac{1}{4}}\right)^3}}{a^4}$	35

input
```
int(1/LambertW(a*x^(1/4))^3,x,method=_RETURNVERBOSE)
```

output
```
4/a^4*(3/16*x*a^4/LambertW(a*x^(1/4))^4+1/4/LambertW(a*x^(1/4))^3*x*a^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3}\,dx = \frac{4\,x\,\mathrm{W}(ax^{\frac{1}{4}})+3\,x}{4\,\mathrm{W}(ax^{\frac{1}{4}})^4}$$

input
```
integrate(1/lambert_w(a*x^(1/4))^3,x, algorithm="fricas")
```

output
```
1/4*(4*x*lambert_w(a*x^(1/4)) + 3*x)/lambert_w(a*x^(1/4))^4
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3}\,dx = \frac{x}{W^3\left(a\sqrt[4]{x}\right)}+\frac{3x}{4W^4\left(a\sqrt[4]{x}\right)}$$

input
```
integrate(1/LambertW(a*x**(1/4))**3,x)
```

output
```
x/LambertW(a*x**(1/4))**3 + 3*x/(4*LambertW(a*x**(1/4))**4)
```

Maxima [F]

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3}\,dx = \int \frac{1}{\mathrm{W}(ax^{\frac{1}{4}})^3}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/4))^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^(1/4))^(-3), x)
```

Giac [F]

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3}\,dx = \int \frac{1}{\mathrm{W}(ax^{\frac{1}{4}})^3}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/4))^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^(1/4))^(-3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3}\,dx = \int \frac{1}{\mathrm{LambertW}\left(a\,x^{1/4}\right)^3}\,dx$$

input
```
int(1/LambertW(a*x^(1/4))^3,x)
```

output
```
int(1/LambertW(a*x^(1/4))^3, x)
```

Reduce [F]

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3}\,dx = \int \frac{1}{lambert\_w\left(x^{\frac{1}{4}}a\right)^3}dx$$

input
```
int(1/Lambert_W(a*x^(1/4))^3,x)
```

output
```
int(1/lambert_w(x**(1/4)*a)**3,x)
```

3.235 $\int W\left(\frac{a}{\sqrt[5]{x}}\right)^4 dx$

Optimal result

Integrand size = 10, antiderivative size = 30

$$\int W\left(\frac{a}{\sqrt[5]{x}}\right)^4 dx = 20a^5\,\text{ExpIntegralEi}\left(-5W\left(\frac{a}{\sqrt[5]{x}}\right)\right) + 5xW\left(\frac{a}{\sqrt[5]{x}}\right)^4$$

output
```
20*a^5*Ei(-5*LambertW(a/x^(1/5)))+5*x*LambertW(a/x^(1/5))^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int W\left(\frac{a}{\sqrt[5]{x}}\right)^4 dx = 20a^5\,\text{ExpIntegralEi}\left(-5W\left(\frac{a}{\sqrt[5]{x}}\right)\right) + 5xW\left(\frac{a}{\sqrt[5]{x}}\right)^4$$

input
```
Integrate[ProductLog[a/x^(1/5)]^4,x]
```

output
```
20*a^5*ExpIntegralEi[-5*ProductLog[a/x^(1/5)]] + 5*x*ProductLog[a/x^(1/5)]
^4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7170, 7188}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W\left(\frac{a}{\sqrt[5]{x}}\right)^4dx$$

$$\Big\downarrow \text{7170}$$

$$5xW\left(\frac{a}{\sqrt[5]{x}}\right)^4-4\int \frac{W\left(\frac{a}{\sqrt[5]{x}}\right)^5}{W\left(\frac{a}{\sqrt[5]{x}}\right)+1}dx$$

$$\Big\downarrow \text{7188}$$

$$20a^5\,\text{ExpIntegralEi}\left(-5W\left(\frac{a}{\sqrt[5]{x}}\right)\right)+5xW\left(\frac{a}{\sqrt[5]{x}}\right)^4$$

input
```
Int[ProductLog[a/x^(1/5)]^4,x]
```

output
```
20*a^5*ExpIntegralEi[-5*ProductLog[a/x^(1/5)]] + 5*x*ProductLog[a/x^(1/5)]
^4
```

Defintions of rubi rules used

rule 7170
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*((c*Prod
uctLog[a*x^n])^p/(n*p + 1)), x] + Simp[n*(p/(c*(n*p + 1)))   Int[(c*Product
Log[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]), x], x] /; FreeQ[{a, c, n}, x]
&& ((IntegerQ[p] && EqQ[n*(p + 1), -1]) || (IntegerQ[p - 1/2] && EqQ[n*(p +
 1/2), -1]))
```

rule 7188
```
Int[ProductLog[(a_.)*(x_)^(n_.)]^(p_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^
(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[a*x^n]]/(d*n)
), x] /; FreeQ[{a, d}, x] && IntegerQ[p] && EqQ[n*p, -1]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$-5a^5\left(-\frac{\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{5}}}\right)^4 x}{a^5}+4\,\mathrm{expIntegral}_1\left(5\,\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{5}}}\right)\right)\right)$	33
default	$-5a^5\left(-\frac{\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{5}}}\right)^4 x}{a^5}+4\,\mathrm{expIntegral}_1\left(5\,\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{5}}}\right)\right)\right)$	33

input
```
int(LambertW(a/x^(1/5))^4,x,method=_RETURNVERBOSE)
```

output
```
-5*a^5*(-LambertW(a/x^(1/5))^4*x/a^5+4*Ei(1,5*LambertW(a/x^(1/5))))
```

Fricas [F]

$$\int W\left(\frac{a}{\sqrt[5]{x}}\right)^4 dx = \int \mathrm{W}\left(\frac{a}{x^{\frac{1}{5}}}\right)^4 dx$$

input
```
integrate(lambert_w(a/x^(1/5))^4,x, algorithm="fricas")
```

output
```
integral(lambert_w(a/x^(1/5))^4, x)
```

Sympy [F]

$$\int W\left(\frac{a}{\sqrt[5]{x}}\right)^4 dx = \int W^4\left(\frac{a}{\sqrt[5]{x}}\right) dx$$

input
```
integrate(LambertW(a/x**(1/5))**4,x)
```

output
```
Integral(LambertW(a/x**(1/5))**4, x)
```

Maxima [F]

$$\int W\left(\frac{a}{\sqrt[5]{x}}\right)^4 dx = \int \mathrm{W}\left(\frac{a}{x^{\frac{1}{5}}}\right)^4 dx$$

input
```
integrate(lambert_w(a/x^(1/5))^4,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x^(1/5))^4, x)
```

Giac [F]

$$\int W\left(\frac{a}{\sqrt[5]{x}}\right)^4 dx = \int \mathrm{W}\left(\frac{a}{x^{\frac{1}{5}}}\right)^4 dx$$

input
```
integrate(lambert_w(a/x^(1/5))^4,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x^(1/5))^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int W\left(\frac{a}{\sqrt[5]{x}}\right)^4\,dx=\int \text{LambertW}\left(\frac{a}{x^{1/5}}\right)^4\,dx$$

input
```
int(LambertW(a/x^(1/5))^4,x)
```

output
```
int(LambertW(a/x^(1/5))^4, x)
```

Reduce [F]

$$\int W\left(\frac{a}{\sqrt[5]{x}}\right)^4\,dx=\frac{4\left(\int \frac{lambert\_w\left(\frac{a}{x^{\frac{1}{5}}}\right)^3}{x^{\frac{1}{5}}e^{lambert\_w\left(\frac{a}{x^{\frac{1}{5}}}\right)}lambert\_w\left(\frac{a}{x^{\frac{1}{5}}}\right)+x^{\frac{1}{5}}e^{lambert\_w\left(\frac{a}{x^{\frac{1}{5}}}\right)}}dx\right)a}{5}+lambert\_w\left(\frac{a}{x^{\frac{1}{5}}}\right)^4x$$

input
```
int(Lambert_W(a/x^(1/5))^4,x)
```

output
```
(4*int(lambert_w(a/x**(1/5))**3/(x**(1/5)*e**lambert_w(a/x**(1/5))*lambert
_w(a/x**(1/5)) + x**(1/5)*e**lambert_w(a/x**(1/5))),x)*a + 5*lambert_w(a/x
**(1/5))**4*x)/5
```

3.236 $\int W\left(\frac{a}{\sqrt[4]{x}}\right)^3 dx$

Optimal result

Integrand size = 10, antiderivative size = 30

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^3 dx = 12a^4 \operatorname{ExpIntegralEi}\left(-4W\left(\frac{a}{\sqrt[4]{x}}\right)\right) + 4xW\left(\frac{a}{\sqrt[4]{x}}\right)^3$$

output
```
12*a^4*Ei(-4*LambertW(a/x^(1/4)))+4*x*LambertW(a/x^(1/4))^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^3 dx = 12a^4 \operatorname{ExpIntegralEi}\left(-4W\left(\frac{a}{\sqrt[4]{x}}\right)\right) + 4xW\left(\frac{a}{\sqrt[4]{x}}\right)^3$$

input
```
Integrate[ProductLog[a/x^(1/4)]^3,x]
```

output
```
12*a^4*ExpIntegralEi[-4*ProductLog[a/x^(1/4)]] + 4*x*ProductLog[a/x^(1/4)]
^3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7170, 7188}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^3 dx$$
$$\Big\downarrow \text{ 7170}$$
$$4xW\left(\frac{a}{\sqrt[4]{x}}\right)^3 - 3\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^4}{W\left(\frac{a}{\sqrt[4]{x}}\right)+1}dx$$
$$\Big\downarrow \text{ 7188}$$
$$12a^4 \operatorname{ExpIntegralEi}\left(-4W\left(\frac{a}{\sqrt[4]{x}}\right)\right) + 4xW\left(\frac{a}{\sqrt[4]{x}}\right)^3$$

input
```
Int[ProductLog[a/x^(1/4)]^3,x]
```

output
```
12*a^4*ExpIntegralEi[-4*ProductLog[a/x^(1/4)]] + 4*x*ProductLog[a/x^(1/4)]
^3
```

Defintions of rubi rules used

rule 7170
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*((c*Prod
uctLog[a*x^n])^p/(n*p + 1)), x] + Simp[n*(p/(c*(n*p + 1)))   Int[(c*Product
Log[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]), x], x] /; FreeQ[{a, c, n}, x]
&& ((IntegerQ[p] && EqQ[n*(p + 1), -1]) || (IntegerQ[p - 1/2] && EqQ[n*(p +
 1/2), -1]))
```

rule 7188
```
Int[ProductLog[(a_.)*(x_)^(n_.)]^(p_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^
(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[a*x^n]]/(d*n)
), x] /; FreeQ[{a, d}, x] && IntegerQ[p] && EqQ[n*p, -1]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$-4a^4\left(-\frac{\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{4}}}\right)^3 x}{a^4}+3\,\mathrm{expIntegral}_1\left(4\,\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{4}}}\right)\right)\right)$	33
default	$-4a^4\left(-\frac{\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{4}}}\right)^3 x}{a^4}+3\,\mathrm{expIntegral}_1\left(4\,\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{4}}}\right)\right)\right)$	33

input
```
int(LambertW(a/x^(1/4))^3,x,method=_RETURNVERBOSE)
```

output
```
-4*a^4*(-LambertW(a/x^(1/4))^3*x/a^4+3*Ei(1,4*LambertW(a/x^(1/4))))
```

Fricas [F]

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^3 dx = \int \mathrm{W}\left(\frac{a}{x^{\frac{1}{4}}}\right)^3 dx$$

input
```
integrate(lambert_w(a/x^(1/4))^3,x, algorithm="fricas")
```

output
```
integral(lambert_w(a/x^(1/4))^3, x)
```

Sympy [F]

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^3 dx = \int W^3\left(\frac{a}{\sqrt[4]{x}}\right) dx$$

input
```
integrate(LambertW(a/x**(1/4))**3,x)
```

output
```
Integral(LambertW(a/x**(1/4))**3, x)
```

Maxima [F]

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^3 dx = \int \mathrm{W}\left(\frac{a}{x^{\frac{1}{4}}}\right)^3 dx$$

input
```
integrate(lambert_w(a/x^(1/4))^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x^(1/4))^3, x)
```

Giac [F]

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^3 dx = \int \mathrm{W}\left(\frac{a}{x^{\frac{1}{4}}}\right)^3 dx$$

input
```
integrate(lambert_w(a/x^(1/4))^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x^(1/4))^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^3 dx = \int \mathrm{LambertW}\left(\frac{a}{x^{1/4}}\right)^3 dx$$

input
```
int(LambertW(a/x^(1/4))^3,x)
```

output
```
int(LambertW(a/x^(1/4))^3, x)
```

Reduce [F]

$$\int W\left(\frac{a}{\sqrt[4]{x}}\right)^3 dx = \frac{3\left(\int \frac{lambert\_w\left(\frac{a}{x^{\frac{1}{4}}}\right)^2}{x^{\frac{1}{4}} e^{lambert\_w\left(\frac{a}{x^{\frac{1}{4}}}\right)} lambert\_w\left(\frac{a}{x^{\frac{1}{4}}}\right)+x^{\frac{1}{4}} e^{lambert\_w\left(\frac{a}{x^{\frac{1}{4}}}\right)}}dx\right) a}{4} + lambert\_w\left(\frac{a}{x^{\frac{1}{4}}}\right)^3 x$$

input
```
int(Lambert_W(a/x^(1/4))^3,x)
```

output
```
(3*int(lambert_w(a/x**(1/4))**2/(x**(1/4)*e**lambert_w(a/x**(1/4))*lambert
_w(a/x**(1/4)) + x**(1/4)*e**lambert_w(a/x**(1/4))),x)*a + 4*lambert_w(a/x
**(1/4))**3*x)/4
```

3.237 $\int W\left(\frac{a}{\sqrt[3]{x}}\right)^2 dx$

Optimal result

Integrand size = 10, antiderivative size = 30

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^2 dx = 6a^3 \operatorname{ExpIntegralEi}\left(-3W\left(\frac{a}{\sqrt[3]{x}}\right)\right) + 3xW\left(\frac{a}{\sqrt[3]{x}}\right)^2$$

output
```
6*a^3*Ei(-3*LambertW(a/x^(1/3)))+3*x*LambertW(a/x^(1/3))^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^2 dx = 6a^3 \operatorname{ExpIntegralEi}\left(-3W\left(\frac{a}{\sqrt[3]{x}}\right)\right) + 3xW\left(\frac{a}{\sqrt[3]{x}}\right)^2$$

input
```
Integrate[ProductLog[a/x^(1/3)]^2,x]
```

output
```
6*a^3*ExpIntegralEi[-3*ProductLog[a/x^(1/3)]] + 3*x*ProductLog[a/x^(1/3)]^
2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7170, 7188}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^2 dx$$

$$\Big\downarrow \text{ 7170}$$

$$3xW\left(\frac{a}{\sqrt[3]{x}}\right)^2 - 2\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^3}{W\left(\frac{a}{\sqrt[3]{x}}\right)+1}dx$$

$$\Big\downarrow \text{ 7188}$$

$$6a^3 \text{ExpIntegralEi}\left(-3W\left(\frac{a}{\sqrt[3]{x}}\right)\right) + 3xW\left(\frac{a}{\sqrt[3]{x}}\right)^2$$

input
```
Int[ProductLog[a/x^(1/3)]^2,x]
```

output
```
6*a^3*ExpIntegralEi[-3*ProductLog[a/x^(1/3)]] + 3*x*ProductLog[a/x^(1/3)]^
2
```

Defintions of rubi rules used

rule 7170
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*((c*Prod
uctLog[a*x^n])^p/(n*p + 1)), x] + Simp[n*(p/(c*(n*p + 1)))   Int[(c*Product
Log[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]), x], x] /; FreeQ[{a, c, n}, x]
&& ((IntegerQ[p] && EqQ[n*(p + 1), -1]) || (IntegerQ[p - 1/2] && EqQ[n*(p +
 1/2), -1]))
```

rule 7188
```
Int[ProductLog[(a_.)*(x_)^(n_.)]^(p_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^
(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[a*x^n]]/(d*n)
), x] /; FreeQ[{a, d}, x] && IntegerQ[p] && EqQ[n*p, -1]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$-3a^3\left(-\frac{\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{3}}}\right)^2 x}{a^3}+2\,\mathrm{expIntegral}_1\left(3\,\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{3}}}\right)\right)\right)$	33
default	$-3a^3\left(-\frac{\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{3}}}\right)^2 x}{a^3}+2\,\mathrm{expIntegral}_1\left(3\,\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{3}}}\right)\right)\right)$	33

input
```
int(LambertW(a/x^(1/3))^2,x,method=_RETURNVERBOSE)
```

output
```
-3*a^3*(-LambertW(a/x^(1/3))^2*x/a^3+2*Ei(1,3*LambertW(a/x^(1/3))))
```

Fricas [F]

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^2 dx = \int \mathrm{W}\left(\frac{a}{x^{\frac{1}{3}}}\right)^2 dx$$

input
```
integrate(lambert_w(a/x^(1/3))^2,x, algorithm="fricas")
```

output
```
integral(lambert_w(a/x^(1/3))^2, x)
```

Sympy [F]

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^2 dx = \int W^2\left(\frac{a}{\sqrt[3]{x}}\right) dx$$

input
```
integrate(LambertW(a/x**(1/3))**2,x)
```

output
```
Integral(LambertW(a/x**(1/3))**2, x)
```

Maxima [F]

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^2 dx = \int \mathrm{W}\left(\frac{a}{x^{\frac{1}{3}}}\right)^2 dx$$

input
```
integrate(lambert_w(a/x^(1/3))^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x^(1/3))^2, x)
```

Giac [F]

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^2 dx = \int \mathrm{W}\left(\frac{a}{x^{\frac{1}{3}}}\right)^2 dx$$

input
```
integrate(lambert_w(a/x^(1/3))^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x^(1/3))^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^2 dx = \int \mathrm{LambertW}\left(\frac{a}{x^{1/3}}\right)^2 dx$$

input
```
int(LambertW(a/x^(1/3))^2,x)
```

output
```
int(LambertW(a/x^(1/3))^2, x)
```

Reduce [F]

$$\int W\left(\frac{a}{\sqrt[3]{x}}\right)^2 dx = \frac{2\left(\int \frac{lambert\_w\left(\frac{a}{x^{\frac{1}{3}}}\right)}{x^{\frac{1}{3}} e^{lambert\_w\left(\frac{a}{x^{\frac{1}{3}}}\right)} lambert\_w\left(\frac{a}{x^{\frac{1}{3}}}\right)+x^{\frac{1}{3}} e^{lambert\_w\left(\frac{a}{x^{\frac{1}{3}}}\right)}}dx\right)a}{3} + lambert\_w\left(\frac{a}{x^{\frac{1}{3}}}\right)^2 x$$

input
```
int(Lambert_W(a/x^(1/3))^2,x)
```

output
```
(2*int(lambert_w(a/x**(1/3))/(x**(1/3)*e**lambert_w(a/x**(1/3))*lambert_w(
a/x**(1/3)) + x**(1/3)*e**lambert_w(a/x**(1/3))),x)*a + 3*lambert_w(a/x**(
1/3))**2*x)/3
```

3.238 $\int W\left(\frac{a}{\sqrt{x}}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 28

$$\int W\left(\frac{a}{\sqrt{x}}\right)\,dx=2a^2\,\text{ExpIntegralEi}\left(-2W\left(\frac{a}{\sqrt{x}}\right)\right)+2xW\left(\frac{a}{\sqrt{x}}\right)$$

output
```
2*a^2*Ei(-2*LambertW(a/x^(1/2)))+2*x*LambertW(a/x^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int W\left(\frac{a}{\sqrt{x}}\right)\,dx=2a^2\,\text{ExpIntegralEi}\left(-2W\left(\frac{a}{\sqrt{x}}\right)\right)+2xW\left(\frac{a}{\sqrt{x}}\right)$$

input
```
Integrate[ProductLog[a/Sqrt[x]],x]
```

output
```
2*a^2*ExpIntegralEi[-2*ProductLog[a/Sqrt[x]]] + 2*x*ProductLog[a/Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7170, 7188}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W\left(\frac{a}{\sqrt{x}}\right) dx$$

$$\Big\downarrow \text{ 7170}$$

$$2xW\left(\frac{a}{\sqrt{x}}\right) - \int \frac{W\left(\frac{a}{\sqrt{x}}\right)^2}{W\left(\frac{a}{\sqrt{x}}\right)+1}dx$$

$$\Big\downarrow \text{ 7188}$$

$$2a^2 \operatorname{ExpIntegralEi}\left(-2W\left(\frac{a}{\sqrt{x}}\right)\right) + 2xW\left(\frac{a}{\sqrt{x}}\right)$$

input
```
Int[ProductLog[a/Sqrt[x]],x]
```

output
```
2*a^2*ExpIntegralEi[-2*ProductLog[a/Sqrt[x]]] + 2*x*ProductLog[a/Sqrt[x]]
```

Defintions of rubi rules used

rule 7170
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*((c*Prod
uctLog[a*x^n])^p/(n*p + 1)), x] + Simp[n*(p/(c*(n*p + 1)))   Int[(c*Product
Log[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]), x], x] /; FreeQ[{a, c, n}, x]
&& ((IntegerQ[p] && EqQ[n*(p + 1), -1]) || (IntegerQ[p - 1/2] && EqQ[n*(p +
 1/2), -1]))
```

rule 7188
```
Int[ProductLog[(a_.)*(x_)^(n_.)]^(p_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^
(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[a*x^n]]/(d*n)
), x] /; FreeQ[{a, d}, x] && IntegerQ[p] && EqQ[n*p, -1]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$-2a^2\left(-\frac{\mathrm{LambertW}\left(\frac{a}{\sqrt{x}}\right)x}{a^2}+\mathrm{expIntegral}_1\left(2\,\mathrm{LambertW}\left(\frac{a}{\sqrt{x}}\right)\right)\right)$	29
default	$-2a^2\left(-\frac{\mathrm{LambertW}\left(\frac{a}{\sqrt{x}}\right)x}{a^2}+\mathrm{expIntegral}_1\left(2\,\mathrm{LambertW}\left(\frac{a}{\sqrt{x}}\right)\right)\right)$	29

input
```
int(LambertW(a/x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2*a^2*(-LambertW(a/x^(1/2))*x/a^2+Ei(1,2*LambertW(a/x^(1/2))))
```

Fricas [F]

$$\int W\left(\frac{a}{\sqrt{x}}\right)\,dx=\int \mathrm{W}\left(\frac{a}{\sqrt{x}}\right)\,dx$$

input
```
integrate(lambert_w(a/x^(1/2)),x, algorithm="fricas")
```

output
```
integral(lambert_w(a/sqrt(x)), x)
```

Sympy [F]

$$\int W\left(\frac{a}{\sqrt{x}}\right)\,dx=\int W\left(\frac{a}{\sqrt{x}}\right)\,dx$$

input
```
integrate(LambertW(a/x**(1/2)),x)
```

output
```
Integral(LambertW(a/sqrt(x)), x)
```

Maxima [F]

$$\int W\left(\frac{a}{\sqrt{x}}\right) dx = \int W(\frac{a}{\sqrt{x}}) dx$$

input
```
integrate(lambert_w(a/x^(1/2)),x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/sqrt(x)), x)
```

Giac [F]

$$\int W\left(\frac{a}{\sqrt{x}}\right) dx = \int W(\frac{a}{\sqrt{x}}) dx$$

input
```
integrate(lambert_w(a/x^(1/2)),x, algorithm="giac")
```

output
```
integrate(lambert_w(a/sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int W\left(\frac{a}{\sqrt{x}}\right) dx = \int \mathrm{LambertW}\left(\frac{a}{\sqrt{x}}\right) dx$$

input
```
int(LambertW(a/x^(1/2)),x)
```

output
```
int(LambertW(a/x^(1/2)), x)
```

Reduce [F]

$$\int W\left(\frac{a}{\sqrt{x}}\right)\,dx = \frac{\left(\int \frac{1}{\sqrt{x}\,e^{lambert\_w\left(\frac{a}{\sqrt{x}}\right)}lambert\_w\left(\frac{a}{\sqrt{x}}\right)+\sqrt{x}\,e^{lambert\_w\left(\frac{a}{\sqrt{x}}\right)}}dx\right)a}{2} + lambert\_w\left(\frac{a}{\sqrt{x}}\right)x$$

input

```
int(Lambert_W(a/x^(1/2)),x)
```

output

```
(int(1/(sqrt(x)*e**lambert_w(a/sqrt(x))*lambert_w(a/sqrt(x)) + sqrt(x)*e**
lambert_w(a/sqrt(x))),x)*a + 2*lambert_w(a/sqrt(x))*x)/2
```

3.239 $\int \frac{1}{W(ax)^2}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 20

$$\int \frac{1}{W(ax)^2}\,dx = \frac{2\,\text{ExpIntegralEi}(W(ax))}{a} - \frac{x}{W(ax)^2}$$

output
```
2*Ei(LambertW(a*x))/a-x/LambertW(a*x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{W(ax)^2}\,dx = \frac{2\,\text{ExpIntegralEi}(W(ax))}{a} - \frac{x}{W(ax)^2}$$

input
```
Integrate[ProductLog[a*x]^(-2),x]
```

output
```
(2*ExpIntegralEi[ProductLog[a*x]])/a - x/ProductLog[a*x]^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7166, 7179}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W(ax)^2}dx$$

$$\Big\downarrow \text{7166}$$

$$2\int \frac{1}{W(ax)(W(ax)+1)}dx-\frac{x}{W(ax)^2}$$

$$\Big\downarrow \text{7179}$$

$$\frac{2\,\text{ExpIntegralEi}(W(ax))}{a}-\frac{x}{W(ax)^2}$$

input
```
Int[ProductLog[a*x]^(-2),x]
```

output
```
(2*ExpIntegralEi[ProductLog[a*x]])/a - x/ProductLog[a*x]^2
```

Defintions of rubi rules used

rule 7166
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_), x_Symbol] :> Simp[(a + b*x
)*((c*ProductLog[a + b*x])^p/(b*(p + 1))), x] + Simp[p/(c*(p + 1))   Int[(c
*ProductLog[a + b*x])^(p + 1)/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a
, b, c}, x] && LtQ[p, -1]
```

rule 7179
```
Int[1/(ProductLog[(a_.) + (b_.)*(x_)]*((d_) + (d_.)*ProductLog[(a_.) + (b_.
)*(x_)])), x_Symbol] :> Simp[ExpIntegralEi[ProductLog[a + b*x]]/(b*d), x] /
; FreeQ[{a, b, d}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$\frac{-\frac{xa}{\mathrm{LambertW}(xa)^2}-2\,\mathrm{expIntegral}_1(-\mathrm{LambertW}(xa))}{a}$	26
default	$\frac{-\frac{xa}{\mathrm{LambertW}(xa)^2}-2\,\mathrm{expIntegral}_1(-\mathrm{LambertW}(xa))}{a}$	26

input
```
int(1/LambertW(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/LambertW(x*a)^2*x*a-2*Ei(1,-LambertW(x*a)))
```

Fricas [F]

$$\int \frac{1}{W(ax)^2}\,dx = \int \frac{1}{\mathrm{W}(ax)^2}\,dx$$

input
```
integrate(1/lambert_w(a*x)^2,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x)^(-2), x)
```

Sympy [F]

$$\int \frac{1}{W(ax)^2}\,dx = \int \frac{1}{W^2(ax)}\,dx$$

input
```
integrate(1/LambertW(a*x)**2,x)
```

output
```
Integral(LambertW(a*x)**(-2), x)
```

Maxima [F]

$$\int \frac{1}{W(ax)^2} \, dx = \int \frac{1}{\mathrm{W}(ax)^2} \, dx$$

input
```
integrate(1/lambert_w(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x)^(-2), x)
```

Giac [F]

$$\int \frac{1}{W(ax)^2} \, dx = \int \frac{1}{\mathrm{W}(ax)^2} \, dx$$

input
```
integrate(1/lambert_w(a*x)^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x)^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W(ax)^2} \, dx = \int \frac{1}{\mathrm{LambertW}\left(a\,x\right)^2} \, dx$$

input
```
int(1/LambertW(a*x)^2,x)
```

output
```
int(1/LambertW(a*x)^2, x)
```

Reduce [F]

$$\int \frac{1}{W(ax)^2}\,dx = \int \frac{1}{lambert\_w\,(ax)^2}dx$$

input
```
int(1/Lambert_W(a*x)^2,x)
```

output
```
int(1/lambert_w(a*x)**2,x)
```

3.240 $\int \frac{1}{W(a\sqrt{x})^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{1}{W(a\sqrt{x})^3}\,dx=\frac{6\,\text{ExpIntegralEi}\left(2W\left(a\sqrt{x}\right)\right)}{a^2}-\frac{2x}{W(a\sqrt{x})^3}$$

output
```
6*Ei(2*LambertW(a*x^(1/2)))/a^2-2*x/LambertW(a*x^(1/2))^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{W(a\sqrt{x})^3}\,dx=\frac{6\,\text{ExpIntegralEi}\left(2W\left(a\sqrt{x}\right)\right)}{a^2}-\frac{2x}{W(a\sqrt{x})^3}$$

input
```
Integrate[ProductLog[a*Sqrt[x]]^(-3),x]
```

output
```
(6*ExpIntegralEi[2*ProductLog[a*Sqrt[x]]])/a^2 - (2*x)/ProductLog[a*Sqrt[x
]]^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7170, 7188}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W\left(a\sqrt{x}\right)^3}dx$$

$$\Big\downarrow \text{7170}$$

$$3\int \frac{1}{W\left(a\sqrt{x}\right)^2\left(W\left(a\sqrt{x}\right)+1\right)}dx-\frac{2x}{W\left(a\sqrt{x}\right)^3}$$

$$\Big\downarrow \text{7188}$$

$$\frac{6\,\text{ExpIntegralEi}\left(2W\left(a\sqrt{x}\right)\right)}{a^2}-\frac{2x}{W\left(a\sqrt{x}\right)^3}$$

input
```
Int[ProductLog[a*Sqrt[x]]^(-3),x]
```

output
```
(6*ExpIntegralEi[2*ProductLog[a*Sqrt[x]]])/a^2 - (2*x)/ProductLog[a*Sqrt[x
]]^3
```

Defintions of rubi rules used

rule 7170
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*((c*Prod
uctLog[a*x^n])^p/(n*p + 1)), x] + Simp[n*(p/(c*(n*p + 1)))   Int[(c*Product
Log[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]), x], x] /; FreeQ[{a, c, n}, x]
&& ((IntegerQ[p] && EqQ[n*(p + 1), -1]) || (IntegerQ[p - 1/2] && EqQ[n*(p +
 1/2), -1]))
```

rule 7188
```
Int[ProductLog[(a_.)*(x_)^(n_.)]^(p_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^
(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[a*x^n]]/(d*n)
), x] /; FreeQ[{a, d}, x] && IntegerQ[p] && EqQ[n*p, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{-\frac{2x\,a^{2}}{\operatorname{LambertW}\left(a\sqrt{x}\right)^{3}}-6\,\operatorname{expIntegral}_{1}\left(-2\operatorname{LambertW}\left(a\sqrt{x}\right)\right)}{a^{2}}$	33
default	$\frac{-\frac{2x\,a^{2}}{\operatorname{LambertW}\left(a\sqrt{x}\right)^{3}}-6\,\operatorname{expIntegral}_{1}\left(-2\operatorname{LambertW}\left(a\sqrt{x}\right)\right)}{a^{2}}$	33

input
```
int(1/LambertW(a*x^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/a^2*(-1/LambertW(a*x^(1/2))^3*x*a^2-3*Ei(1,-2*LambertW(a*x^(1/2))))
```

Fricas [F]

$$\int \frac{1}{W\left(a\sqrt{x}\right)^3}\,dx = \int \frac{1}{\mathrm{W}(a\sqrt{x})^3}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/2))^3,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*sqrt(x))^(-3), x)
```

Sympy [F]

$$\int \frac{1}{W\left(a\sqrt{x}\right)^3}\,dx = \int \frac{1}{W^{3}\left(a\sqrt{x}\right)}\,dx$$

input
```
integrate(1/LambertW(a*x**(1/2))**3,x)
```

output
```
Integral(LambertW(a*sqrt(x))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{W\left(a\sqrt{x}\right)^3}\,dx = \int \frac{1}{\mathrm{W}(a\sqrt{x})^3}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*sqrt(x))^(-3), x)
```

Giac [F]

$$\int \frac{1}{W\left(a\sqrt{x}\right)^3}\,dx = \int \frac{1}{\mathrm{W}(a\sqrt{x})^3}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/2))^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*sqrt(x))^(-3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W\left(a\sqrt{x}\right)^3}\,dx = \int \frac{1}{\mathrm{LambertW}\left(a\,\sqrt{x}\right)^3}\,dx$$

input
```
int(1/LambertW(a*x^(1/2))^3,x)
```

output
```
int(1/LambertW(a*x^(1/2))^3, x)
```

Reduce [F]

$$\int \frac{1}{W\left(a\sqrt{x}\right)^3}\,dx = \int \frac{1}{lambert\_w\left(\sqrt{x}\,a\right)^3}dx$$

input
```
int(1/Lambert_W(a*x^(1/2))^3,x)
```

output
```
int(1/lambert_w(sqrt(x)*a)**3,x)
```

3.241 $\int \frac{1}{W\left(a\sqrt[3]{x}\right)^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^4}\,dx=\frac{12\,\text{ExpIntegralEi}\left(3W\left(a\sqrt[3]{x}\right)\right)}{a^3}-\frac{3x}{W\left(a\sqrt[3]{x}\right)^4}$$

output
```
12*Ei(3*LambertW(a*x^(1/3)))/a^3-3*x/LambertW(a*x^(1/3))^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^4}\,dx=\frac{12\,\text{ExpIntegralEi}\left(3W\left(a\sqrt[3]{x}\right)\right)}{a^3}-\frac{3x}{W\left(a\sqrt[3]{x}\right)^4}$$

input
```
Integrate[ProductLog[a*x^(1/3)]^(-4),x]
```

output
```
(12*ExpIntegralEi[3*ProductLog[a*x^(1/3)]])/a^3 - (3*x)/ProductLog[a*x^(1/
3)]^4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7170, 7188}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^4}dx$$
$$\Big\downarrow \text{7170}$$
$$4\int \frac{1}{W\left(a\sqrt[3]{x}\right)^3\left(W\left(a\sqrt[3]{x}\right)+1\right)}dx-\frac{3x}{W\left(a\sqrt[3]{x}\right)^4}$$
$$\Big\downarrow \text{7188}$$
$$\frac{12\,\text{ExpIntegralEi}\left(3W\left(a\sqrt[3]{x}\right)\right)}{a^3}-\frac{3x}{W\left(a\sqrt[3]{x}\right)^4}$$

input
```
Int[ProductLog[a*x^(1/3)]^(-4),x]
```

output
```
(12*ExpIntegralEi[3*ProductLog[a*x^(1/3)]])/a^3 - (3*x)/ProductLog[a*x^(1/
3)]^4
```

Defintions of rubi rules used

rule 7170
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*((c*Prod
uctLog[a*x^n])^p/(n*p + 1)), x] + Simp[n*(p/(c*(n*p + 1)))   Int[(c*Product
Log[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]), x], x] /; FreeQ[{a, c, n}, x]
&& ((IntegerQ[p] && EqQ[n*(p + 1), -1]) || (IntegerQ[p - 1/2] && EqQ[n*(p +
 1/2), -1]))
```

rule 7188
```
Int[ProductLog[(a_.)*(x_)^(n_.)]^(p_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^
(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[a*x^n]]/(d*n)
), x] /; FreeQ[{a, d}, x] && IntegerQ[p] && EqQ[n*p, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{-\frac{3x\,a^{3}}{\mathrm{LambertW}\left(a\,x^{\frac{1}{3}}\right)^{4}}-12\,\mathrm{expIntegral}_{1}\left(-3\,\mathrm{LambertW}\left(a\,x^{\frac{1}{3}}\right)\right)}{a^{3}}$	33
default	$\frac{-\frac{3x\,a^{3}}{\mathrm{LambertW}\left(a\,x^{\frac{1}{3}}\right)^{4}}-12\,\mathrm{expIntegral}_{1}\left(-3\,\mathrm{LambertW}\left(a\,x^{\frac{1}{3}}\right)\right)}{a^{3}}$	33

input
```
int(1/LambertW(a*x^(1/3))^4,x,method=_RETURNVERBOSE)
```

output
```
3/a^3*(-1/LambertW(a*x^(1/3))^4*x*a^3-4*Ei(1,-3*LambertW(a*x^(1/3))))
```

Fricas [F]

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^{4}}\,dx = \int \frac{1}{\mathrm{W}(ax^{\frac{1}{3}})^{4}}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/3))^4,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^(1/3))^(-4), x)
```

Sympy [F]

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^{4}}\,dx = \int \frac{1}{W^{4}\left(a\sqrt[3]{x}\right)}\,dx$$

input
```
integrate(1/LambertW(a*x**(1/3))**4,x)
```

output
```
Integral(LambertW(a*x**(1/3))**(-4), x)
```

Maxima [F]

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^4}\,dx = \int \frac{1}{\mathrm{W}(ax^{\frac{1}{3}})^4}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/3))^4,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^(1/3))^(-4), x)
```

Giac [F]

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^4}\,dx = \int \frac{1}{\mathrm{W}(ax^{\frac{1}{3}})^4}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/3))^4,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^(1/3))^(-4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^4}\,dx = \int \frac{1}{\mathrm{LambertW}\left(a\,x^{1/3}\right)^4}\,dx$$

input
```
int(1/LambertW(a*x^(1/3))^4,x)
```

output
```
int(1/LambertW(a*x^(1/3))^4, x)
```

Reduce [F]

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^4}\,dx = \int \frac{1}{lambert\_w\left(x^{\frac{1}{3}}a\right)^4}dx$$

input
```
int(1/Lambert_W(a*x^(1/3))^4,x)
```

output
```
int(1/lambert_w(x**(1/3)*a)**4,x)
```

3.242 $\int \frac{1}{W\left(a\sqrt[4]{x}\right)^5} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^5} \, dx = \frac{20\,\text{ExpIntegralEi}\left(4W\left(a\sqrt[4]{x}\right)\right)}{a^4} - \frac{4x}{W\left(a\sqrt[4]{x}\right)^5}$$

output
```
20*Ei(4*LambertW(a*x^(1/4)))/a^4-4*x/LambertW(a*x^(1/4))^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^5} \, dx = \frac{20\,\text{ExpIntegralEi}\left(4W\left(a\sqrt[4]{x}\right)\right)}{a^4} - \frac{4x}{W\left(a\sqrt[4]{x}\right)^5}$$

input
```
Integrate[ProductLog[a*x^(1/4)]^(-5),x]
```

output
```
(20*ExpIntegralEi[4*ProductLog[a*x^(1/4)]])/a^4 - (4*x)/ProductLog[a*x^(1/
4)]^5
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7170, 7188}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^5} \, dx$$

$$\Big\downarrow \text{7170}$$

$$5\int \frac{1}{W\left(a\sqrt[4]{x}\right)^4 \left(W\left(a\sqrt[4]{x}\right)+1\right)}dx-\frac{4x}{W\left(a\sqrt[4]{x}\right)^5}$$

$$\Big\downarrow \text{7188}$$

$$\frac{20\,\text{ExpIntegralEi}\left(4W\left(a\sqrt[4]{x}\right)\right)}{a^4}-\frac{4x}{W\left(a\sqrt[4]{x}\right)^5}$$

input
```
Int[ProductLog[a*x^(1/4)]^(-5),x]
```

output
```
(20*ExpIntegralEi[4*ProductLog[a*x^(1/4)]])/a^4 - (4*x)/ProductLog[a*x^(1/
4)]^5
```

Defintions of rubi rules used

rule 7170
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*((c*Prod
uctLog[a*x^n])^p/(n*p + 1)), x] + Simp[n*(p/(c*(n*p + 1)))   Int[(c*Product
Log[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]), x], x] /; FreeQ[{a, c, n}, x]
&& ((IntegerQ[p] && EqQ[n*(p + 1), -1]) || (IntegerQ[p - 1/2] && EqQ[n*(p +
 1/2), -1]))
```

rule 7188
```
Int[ProductLog[(a_.)*(x_)^(n_.)]^(p_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^
(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[a*x^n]]/(d*n)
), x] /; FreeQ[{a, d}, x] && IntegerQ[p] && EqQ[n*p, -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{-\frac{4x\,a^4}{\mathrm{LambertW}\left(a\,x^{\frac{1}{4}}\right)^5}-20\;\mathrm{expIntegral}_1\left(-4\,\mathrm{LambertW}\left(a\,x^{\frac{1}{4}}\right)\right)}{a^4}$	33
default	$\frac{-\frac{4x\,a^4}{\mathrm{LambertW}\left(a\,x^{\frac{1}{4}}\right)^5}-20\;\mathrm{expIntegral}_1\left(-4\,\mathrm{LambertW}\left(a\,x^{\frac{1}{4}}\right)\right)}{a^4}$	33

input
```
int(1/LambertW(a*x^(1/4))^5,x,method=_RETURNVERBOSE)
```

output
```
4/a^4*(-1/LambertW(a*x^(1/4))^5*x*a^4-5*Ei(1,-4*LambertW(a*x^(1/4))))
```

Fricas [F]

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^5}\,dx=\int \frac{1}{\mathrm{W}(ax^{\frac{1}{4}})^5}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/4))^5,x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^(1/4))^(-5), x)
```

Sympy [F]

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^5}\,dx=\int \frac{1}{W^5\left(a\sqrt[4]{x}\right)}\,dx$$

input
```
integrate(1/LambertW(a*x**(1/4))**5,x)
```

output
```
Integral(LambertW(a*x**(1/4))**(-5), x)
```

Maxima [F]

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^5}\,dx = \int \frac{1}{\mathrm{W}(ax^{\frac{1}{4}})^5}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/4))^5,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^(1/4))^(-5), x)
```

Giac [F]

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^5}\,dx = \int \frac{1}{\mathrm{W}(ax^{\frac{1}{4}})^5}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/4))^5,x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^(1/4))^(-5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^5}\,dx = \int \frac{1}{\mathrm{LambertW}\left(a\,x^{1/4}\right)^5}\,dx$$

input
```
int(1/LambertW(a*x^(1/4))^5,x)
```

output
```
int(1/LambertW(a*x^(1/4))^5, x)
```

Reduce [F]

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^5}\,dx = \int \frac{1}{lambert\_w\left(x^{\frac{1}{4}}a\right)^5}dx$$

input
```
int(1/Lambert_W(a*x^(1/4))^5,x)
```

output
```
int(1/lambert_w(x**(1/4)*a)**5,x)
```

3.243 $\int x^m W(ax^n)^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 186

$$\int x^m W(ax^n)^2\,dx = \frac{e^{\left(1-\frac{1+m}{n}\right)W(ax^n)}x^{1+m-n}\Gamma\left(3+\frac{1+m}{n},-\frac{(1+m)W(ax^n)}{n}\right)W(ax^n)^3\left(-\frac{(1+m)W(ax^n)}{n}\right)^{-2-\frac{1+m}{n}}}{a(1+m)} + \frac{e^{\left(1-\frac{1+m}{n}\right)W(ax^n)}x^{1+m-n}\Gamma\left(2+\frac{1+m}{n},-\frac{(1+m)W(ax^n)}{n}\right)W(ax^n)^2\left(-\frac{(1+m)W(ax^n)}{n}\right)^{-\frac{1+m+n}{n}}}{a(1+m)}$$

output
```
exp((1-(1+m)/n)*LambertW(a*x^n))*x^(1+m-n)*GAMMA(3+(1+m)/n,-(1+m)*LambertW
(a*x^n)/n)*LambertW(a*x^n)^3*(-(1+m)*LambertW(a*x^n)/n)^(-2-(1+m)/n)/a/(1+
m)+exp((1-(1+m)/n)*LambertW(a*x^n))*x^(1+m-n)*GAMMA(2+(1+m)/n,-(1+m)*Lambe
rtW(a*x^n)/n)*LambertW(a*x^n)^2/a/(1+m)/((-(1+m)*LambertW(a*x^n)/n)^((1+m+
n)/n))
```

Mathematica [F]

$$\int x^m W(ax^n)^2 \, dx = \int x^m W(ax^n)^2 \, dx$$

input
```
Integrate[x^m*ProductLog[a*x^n]^2,x]
```

output
```
Integrate[x^m*ProductLog[a*x^n]^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m W(ax^n)^2 \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int x^m W(ax^n)^2 dx$$

input
```
Int[x^m*ProductLog[a*x^n]^2,x]
```

output
```
$Aborted
```

Maple [F]

$$\int x^m \operatorname{LambertW}(a\, x^n)^2 \, dx$$

input
```
int(x^m*LambertW(a*x^n)^2,x)
```

output
```
int(x^m*LambertW(a*x^n)^2,x)
```

Fricas [F]

$$\int x^m W(ax^n)^2\,dx = \int x^m\,\mathrm{W}(ax^n)^2\,dx$$

input
```
integrate(x^m*lambert_w(a*x^n)^2,x, algorithm="fricas")
```

output
```
integral(x^m*lambert_w(a*x^n)^2, x)
```

Sympy [F]

$$\int x^m W(ax^n)^2\,dx = \int x^m W^2(ax^n)\,dx$$

input
```
integrate(x**m*LambertW(a*x**n)**2,x)
```

output
```
Integral(x**m*LambertW(a*x**n)**2, x)
```

Maxima [F]

$$\int x^m W(ax^n)^2\,dx = \int x^m\,\mathrm{W}(ax^n)^2\,dx$$

input
```
integrate(x^m*lambert_w(a*x^n)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*lambert_w(a*x^n)^2, x)
```

Giac [F]

$$\int x^m W(ax^n)^2 \, dx = \int x^m \mathrm{W}(ax^n)^2 \, dx$$

input
```
integrate(x^m*lambert_w(a*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x^m*lambert_w(a*x^n)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m W(ax^n)^2 \, dx = \int x^m \operatorname{LambertW}(a\,x^n)^2 \, dx$$

input
```
int(x^m*LambertW(a*x^n)^2,x)
```

output
```
int(x^m*LambertW(a*x^n)^2, x)
```

Reduce [F]

$$\int x^m W(ax^n)^2 \, dx$$
$$= \frac{x^m lambert\_w(x^n a)^2 x - 2\left(\int \frac{x^{m+n} lambert\_w(x^n a)}{e^{lambert\_w(x^n a)} lambert\_w(x^n a) m + e^{lambert\_w(x^n a)} lambert\_w(x^n a) + e^{lambert\_w(x^n a)} m}\right.}{}$$

input
```
int(x^m*Lambert_W(a*x^n)^2,x)
```

output
```
(x**m*lambert_w(x**n*a)**2*x - 2*int((x**(m + n)*lambert_w(x**n*a))/(e**la
mbert_w(x**n*a)*lambert_w(x**n*a)*m + e**lambert_w(x**n*a)*lambert_w(x**n*
a) + e**lambert_w(x**n*a)*m + e**lambert_w(x**n*a)),x)*a*m*n - 2*int((x**(
m + n)*lambert_w(x**n*a))/(e**lambert_w(x**n*a)*lambert_w(x**n*a)*m + e**l
ambert_w(x**n*a)*lambert_w(x**n*a) + e**lambert_w(x**n*a)*m + e**lambert_w
(x**n*a)),x)*a*n)/(m + 1)
```

3.244 $\int x^m W(ax^n)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 181

$$\int x^m W(ax^n)\, dx = \frac{e^{\left(1-\frac{1+m}{n}\right)W(ax^n)} x^{1+m-n}\Gamma\left(\frac{1+m+n}{n}, -\frac{(1+m)W(ax^n)}{n}\right) W(ax^n) \left(-\frac{(1+m)W(ax^n)}{n}\right)^{-\frac{1+m}{n}}}{a(1+m)} + \frac{e^{\left(1-\frac{1+m}{n}\right)W(ax^n)} x^{1+m-n}\Gamma\left(2+\frac{1+m}{n}, -\frac{(1+m)W(ax^n)}{n}\right) W(ax^n)^2 \left(-\frac{(1+m)W(ax^n)}{n}\right)^{-\frac{1+m+n}{n}}}{a(1+m)}$$

output
```
exp((1-(1+m)/n)*LambertW(a*x^n))*x^(1+m-n)*GAMMA((1+m+n)/n,-(1+m)*LambertW
(a*x^n)/n)*LambertW(a*x^n)/a/(1+m)/((-(1+m)*LambertW(a*x^n)/n)^((1+m)/n))+
exp((1-(1+m)/n)*LambertW(a*x^n))*x^(1+m-n)*GAMMA(2+(1+m)/n,-(1+m)*LambertW
(a*x^n)/n)*LambertW(a*x^n)^2/a/(1+m)/((-(1+m)*LambertW(a*x^n)/n)^((1+m+n)/
n))
```

Mathematica [F]

$$\int x^m W(ax^n)\,dx = \int x^m W(ax^n)\,dx$$

input
```
Integrate[x^m*ProductLog[a*x^n],x]
```

output
```
Integrate[x^m*ProductLog[a*x^n], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m W(ax^n)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int x^m W(ax^n)dx$$

input
```
Int[x^m*ProductLog[a*x^n],x]
```

output
```
$Aborted
```

Maple [F]

$$\int x^m \operatorname{LambertW}(a\,x^n)\,dx$$

input
```
int(x^m*LambertW(a*x^n),x)
```

output
```
int(x^m*LambertW(a*x^n),x)
```

Fricas [F]

$$\int x^m W(ax^n)\,dx = \int x^m \mathrm{W}(ax^n)\,dx$$

input
```
integrate(x^m*lambert_w(a*x^n),x, algorithm="fricas")
```

output
```
integral(x^m*lambert_w(a*x^n), x)
```

Sympy [F]

$$\int x^m W(ax^n)\,dx = \int x^m W(ax^n)\,dx$$

input
```
integrate(x**m*LambertW(a*x**n),x)
```

output
```
Integral(x**m*LambertW(a*x**n), x)
```

Maxima [F]

$$\int x^m W(ax^n)\,dx = \int x^m \mathrm{W}(ax^n)\,dx$$

input
```
integrate(x^m*lambert_w(a*x^n),x, algorithm="maxima")
```

output
```
integrate(x^m*lambert_w(a*x^n), x)
```

Giac [F]

$$\int x^m W(ax^n)\,dx=\int x^m\,\mathrm{W}(ax^n)\,dx$$

input
```
integrate(x^m*lambert_w(a*x^n),x, algorithm="giac")
```

output
```
integrate(x^m*lambert_w(a*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m W(ax^n)\,dx=\int x^m\,\mathrm{LambertW}(a\,x^n)\,dx$$

input
```
int(x^m*LambertW(a*x^n),x)
```

output
```
int(x^m*LambertW(a*x^n), x)
```

Reduce [F]

$$\int x^m W(ax^n)\,dx$$
$$=\frac{x^m lambert\_w(x^n a)\,x-\left(\int \frac{x^{m+n}}{e^{lambert\_w(x^n a)}lambert\_w(x^n a)m+e^{lambert\_w(x^n a)}lambert\_w(x^n a)+e^{lambert\_w(x^n a)}m+e$$

input
```
int(x^m*Lambert_W(a*x^n),x)
```

output
```
(x**m*lambert_w(x**n*a)*x - int(x**(m + n)/(e**lambert_w(x**n*a)*lambert_w
(x**n*a)*m + e**lambert_w(x**n*a)*lambert_w(x**n*a) + e**lambert_w(x**n*a)
*m + e**lambert_w(x**n*a)),x)*a*m*n - int(x**(m + n)/(e**lambert_w(x**n*a)
*lambert_w(x**n*a)*m + e**lambert_w(x**n*a)*lambert_w(x**n*a) + e**lambert
_w(x**n*a)*m + e**lambert_w(x**n*a)),x)*a*n)/(m + 1)
```

3.245 $\int \frac{x^m}{W(ax^n)} dx$

Optimal result

Integrand size = 12, antiderivative size = 177

$$\int \frac{x^m}{W(ax^n)} dx = \frac{e^{\left(1-\frac{1+m}{n}\right)W(ax^n)} x^{1+m-n} \Gamma\left(\frac{1+m}{n}, -\frac{(1+m)W(ax^n)}{n}\right)\left(-\frac{(1+m)W(ax^n)}{n}\right)^{1-\frac{1+m}{n}}}{a(1+m)} + \frac{e^{\left(1-\frac{1+m}{n}\right)W(ax^n)} x^{1+m-n} \Gamma\left(-1+\frac{1+m}{n}, -\frac{(1+m)W(ax^n)}{n}\right)\left(-\frac{(1+m)W(ax^n)}{n}\right)^{2-\frac{1+m}{n}}}{a(1+m)W(ax^n)}$$

output
```
exp((1-(1+m)/n)*LambertW(a*x^n))*x^(1+m-n)*GAMMA((1+m)/n,-(1+m)*LambertW(a
*x^n)/n)*(-(1+m)*LambertW(a*x^n)/n)^(1-(1+m)/n)/a/(1+m)+exp((1-(1+m)/n)*La
mbertW(a*x^n))*x^(1+m-n)*GAMMA(-1+(1+m)/n,-(1+m)*LambertW(a*x^n)/n)*(-(1+m
)*LambertW(a*x^n)/n)^(2-(1+m)/n)/a/(1+m)/LambertW(a*x^n)
```

Mathematica [F]

$$\int \frac{x^m}{W(ax^n)}\,dx = \int \frac{x^m}{W(ax^n)}\,dx$$

input
```
Integrate[x^m/ProductLog[a*x^n],x]
```

output
```
Integrate[x^m/ProductLog[a*x^n], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{W(ax^n)}\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{x^m}{W(ax^n)}dx$$

input
```
Int[x^m/ProductLog[a*x^n],x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^m}{\mathrm{LambertW}(a\,x^n)}dx$$

input
```
int(x^m/LambertW(a*x^n),x)
```

output
```
int(x^m/LambertW(a*x^n),x)
```

Fricas [F]

$$\int \frac{x^m}{W(ax^n)}\,dx = \int \frac{x^m}{\mathrm{W}(ax^n)}\,dx$$

input
```
integrate(x^m/lambert_w(a*x^n),x, algorithm="fricas")
```

output
```
integral(x^m/lambert_w(a*x^n), x)
```

Sympy [F]

$$\int \frac{x^m}{W(ax^n)}\,dx = \int \frac{x^m}{W(ax^n)}\,dx$$

input
```
integrate(x**m/LambertW(a*x**n),x)
```

output
```
Integral(x**m/LambertW(a*x**n), x)
```

Maxima [F]

$$\int \frac{x^m}{W(ax^n)}\,dx = \int \frac{x^m}{\mathrm{W}(ax^n)}\,dx$$

input
```
integrate(x^m/lambert_w(a*x^n),x, algorithm="maxima")
```

output
```
integrate(x^m/lambert_w(a*x^n), x)
```

Giac [F]

$$\int \frac{x^m}{W(ax^n)}\,dx = \int \frac{x^m}{\mathrm{W}(ax^n)}\,dx$$

input
```
integrate(x^m/lambert_w(a*x^n),x, algorithm="giac")
```

output
```
integrate(x^m/lambert_w(a*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{W(ax^n)}\,dx = \int \frac{x^m}{\mathrm{LambertW}(a\,x^n)}\,dx$$

input
```
int(x^m/LambertW(a*x^n),x)
```

output
```
int(x^m/LambertW(a*x^n), x)
```

Reduce [F]

$$\int \frac{x^m}{W(ax^n)}\,dx = \frac{x^m x - \left(\int x^m dx\right) m - \left(\int x^m dx\right) + 2\left(\int \frac{x^m}{lambert\_w(x^n a)}dx\right) m + 2\left(\int \frac{x^m}{lambert\_w(x^n a)}dx\right)}{2m+2}$$

input
```
int(x^m/Lambert_W(a*x^n),x)
```

output
```
(x**m*x - int(x**m,x)*m - int(x**m,x) + 2*int(x**m/lambert_w(x**n*a),x)*m
+ 2*int(x**m/lambert_w(x**n*a),x))/(2*(m + 1))
```

3.246 $\int \frac{x^m}{W(ax^n)^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 187

$$\int \frac{x^m}{W\left(ax^n\right)^2}\,dx = \frac{e^{\left(1-\frac{1+m}{n}\right)W(ax^n)}x^{1+m-n}\Gamma\left(-1+\frac{1+m}{n},-\frac{(1+m)W(ax^n)}{n}\right)\left(-\frac{(1+m)W(ax^n)}{n}\right)^{2-\frac{1+m}{n}}}{a(1+m)W\left(ax^n\right)} + \frac{e^{\left(1-\frac{1+m}{n}\right)W(ax^n)}x^{1+m-n}\Gamma\left(-2+\frac{1+m}{n},-\frac{(1+m)W(ax^n)}{n}\right)\left(-\frac{(1+m)W(ax^n)}{n}\right)^{3-\frac{1+m}{n}}}{a(1+m)W\left(ax^n\right)^2}$$

output
```
exp((1-(1+m)/n)*LambertW(a*x^n))*x^(1+m-n)*GAMMA(-1+(1+m)/n,-(1+m)*Lambert
W(a*x^n)/n)*(-(1+m)*LambertW(a*x^n)/n)^(2-(1+m)/n)/a/(1+m)/LambertW(a*x^n)
+exp((1-(1+m)/n)*LambertW(a*x^n))*x^(1+m-n)*GAMMA(-2+(1+m)/n,-(1+m)*Lamber
tW(a*x^n)/n)*(-(1+m)*LambertW(a*x^n)/n)^(3-(1+m)/n)/a/(1+m)/LambertW(a*x^n
)^2
```

Mathematica [F]

$$\int \frac{x^m}{W(ax^n)^2} \, dx = \int \frac{x^m}{W(ax^n)^2} \, dx$$

input
```
Integrate[x^m/ProductLog[a*x^n]^2,x]
```

output
```
Integrate[x^m/ProductLog[a*x^n]^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{W(ax^n)^2} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{x^m}{W(ax^n)^2} dx$$

input
```
Int[x^m/ProductLog[a*x^n]^2,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^m}{\mathrm{LambertW}(a\,x^n)^2} dx$$

input
```
int(x^m/LambertW(a*x^n)^2,x)
```

output
```
int(x^m/LambertW(a*x^n)^2,x)
```

Fricas [F]

$$\int \frac{x^m}{W(ax^n)^2} \, dx = \int \frac{x^m}{\mathrm{W}(ax^n)^2} \, dx$$

input
```
integrate(x^m/lambert_w(a*x^n)^2,x, algorithm="fricas")
```

output
```
integral(x^m/lambert_w(a*x^n)^2, x)
```

Sympy [F]

$$\int \frac{x^m}{W(ax^n)^2} \, dx = \int \frac{x^m}{W^2(ax^n)} \, dx$$

input
```
integrate(x**m/LambertW(a*x**n)**2,x)
```

output
```
Integral(x**m/LambertW(a*x**n)**2, x)
```

Maxima [F]

$$\int \frac{x^m}{W(ax^n)^2} \, dx = \int \frac{x^m}{\mathrm{W}(ax^n)^2} \, dx$$

input
```
integrate(x^m/lambert_w(a*x^n)^2,x, algorithm="maxima")
```

output
```
integrate(x^m/lambert_w(a*x^n)^2, x)
```

Giac [F]

$$\int \frac{x^m}{W(ax^n)^2} \, dx = \int \frac{x^m}{\mathrm{W}(ax^n)^2} \, dx$$

input
```
integrate(x^m/lambert_w(a*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x^m/lambert_w(a*x^n)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{W(ax^n)^2} \, dx = \int \frac{x^m}{\mathrm{LambertW}(a\,x^n)^2} \, dx$$

input
```
int(x^m/LambertW(a*x^n)^2,x)
```

output
```
int(x^m/LambertW(a*x^n)^2, x)
```

Reduce [F]

$$\int \frac{x^m}{W(ax^n)^2} \, dx = \text{too large to display}$$

input
```
int(x^m/Lambert_W(a*x^n)^2,x)
```

output

```
( - 3*x**m*lambert_w(x**n*a)*x + 6*x**m*x + 4*int(x**(m + n)/(e**lambert_w
(x**n*a)*lambert_w(x**n*a)**3*m**2 + 2*e**lambert_w(x**n*a)*lambert_w(x**n
*a)**3*m + e**lambert_w(x**n*a)*lambert_w(x**n*a)**3 + e**lambert_w(x**n*a
)*lambert_w(x**n*a)**2*m**2 + 2*e**lambert_w(x**n*a)*lambert_w(x**n*a)**2*
m + e**lambert_w(x**n*a)*lambert_w(x**n*a)**2),x)*lambert_w(x**n*a)*a*m**2
*n + 8*int(x**(m + n)/(e**lambert_w(x**n*a)*lambert_w(x**n*a)**3*m**2 + 2*
e**lambert_w(x**n*a)*lambert_w(x**n*a)**3*m + e**lambert_w(x**n*a)*lambert
_w(x**n*a)**3 + e**lambert_w(x**n*a)*lambert_w(x**n*a)**2*m**2 + 2*e**lamb
ert_w(x**n*a)*lambert_w(x**n*a)**2*m + e**lambert_w(x**n*a)*lambert_w(x**n
*a)**2),x)*lambert_w(x**n*a)*a*m*n + 4*int(x**(m + n)/(e**lambert_w(x**n*a
)*lambert_w(x**n*a)**3*m**2 + 2*e**lambert_w(x**n*a)*lambert_w(x**n*a)**3*
m + e**lambert_w(x**n*a)*lambert_w(x**n*a)**3 + e**lambert_w(x**n*a)*lambe
rt_w(x**n*a)**2*m**2 + 2*e**lambert_w(x**n*a)*lambert_w(x**n*a)**2*m + e**
lambert_w(x**n*a)*lambert_w(x**n*a)**2),x)*lambert_w(x**n*a)*a*n + 2*int(x
**(m + n)/(e**lambert_w(x**n*a)*lambert_w(x**n*a)**3*m + e**lambert_w(x**n
*a)*lambert_w(x**n*a)**3 + e**lambert_w(x**n*a)*lambert_w(x**n*a)**2*m + e
**lambert_w(x**n*a)*lambert_w(x**n*a)**2),x)*lambert_w(x**n*a)*a*m*n + 2*i
nt(x**(m + n)/(e**lambert_w(x**n*a)*lambert_w(x**n*a)**3*m + e**lambert_w(
x**n*a)*lambert_w(x**n*a)**3 + e**lambert_w(x**n*a)*lambert_w(x**n*a)**2*m
 + e**lambert_w(x**n*a)*lambert_w(x**n*a)**2),x)*lambert_w(x**n*a)*a*n ...
```

3.247 $\int \frac{x^m}{W(ax^n)^3} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 187

$$\int \frac{x^m}{W(ax^n)^3} \, dx = \frac{e^{\left(1-\frac{1+m}{n}\right)W(ax^n)}x^{1+m-n}\Gamma\left(-2+\frac{1+m}{n},-\frac{(1+m)W(ax^n)}{n}\right)\left(-\frac{(1+m)W(ax^n)}{n}\right)^{3-\frac{1+m}{n}}}{a(1+m)W(ax^n)^2} + \frac{e^{\left(1-\frac{1+m}{n}\right)W(ax^n)}x^{1+m-n}\Gamma\left(-3+\frac{1+m}{n},-\frac{(1+m)W(ax^n)}{n}\right)\left(-\frac{(1+m)W(ax^n)}{n}\right)^{4-\frac{1+m}{n}}}{a(1+m)W(ax^n)^3}$$

output
```
exp((1-(1+m)/n)*LambertW(a*x^n))*x^(1+m-n)*GAMMA(-2+(1+m)/n,-(1+m)*Lambert
W(a*x^n)/n)*(-(1+m)*LambertW(a*x^n)/n)^(3-(1+m)/n)/a/(1+m)/LambertW(a*x^n)
^2+exp((1-(1+m)/n)*LambertW(a*x^n))*x^(1+m-n)*GAMMA(-3+(1+m)/n,-(1+m)*Lamb
ertW(a*x^n)/n)*(-(1+m)*LambertW(a*x^n)/n)^(4-(1+m)/n)/a/(1+m)/LambertW(a*x
^n)^3
```

Mathematica [F]

$$\int \frac{x^m}{W(ax^n)^3} dx = \int \frac{x^m}{W(ax^n)^3} dx$$

input
```
Integrate[x^m/ProductLog[a*x^n]^3,x]
```

output
```
Integrate[x^m/ProductLog[a*x^n]^3, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{W(ax^n)^3} dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{x^m}{W(ax^n)^3} dx$$

input
```
Int[x^m/ProductLog[a*x^n]^3,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^m}{\mathrm{LambertW}(a\,x^n)^3} dx$$

input
```
int(x^m/LambertW(a*x^n)^3,x)
```

output
```
int(x^m/LambertW(a*x^n)^3,x)
```

Fricas [F]

$$\int \frac{x^m}{W(ax^n)^3} \, dx = \int \frac{x^m}{\mathrm{W}(ax^n)^3} \, dx$$

input
```
integrate(x^m/lambert_w(a*x^n)^3,x, algorithm="fricas")
```

output
```
integral(x^m/lambert_w(a*x^n)^3, x)
```

Sympy [F]

$$\int \frac{x^m}{W(ax^n)^3} \, dx = \int \frac{x^m}{W^3(ax^n)} \, dx$$

input
```
integrate(x**m/LambertW(a*x**n)**3,x)
```

output
```
Integral(x**m/LambertW(a*x**n)**3, x)
```

Maxima [F]

$$\int \frac{x^m}{W(ax^n)^3} \, dx = \int \frac{x^m}{\mathrm{W}(ax^n)^3} \, dx$$

input
```
integrate(x^m/lambert_w(a*x^n)^3,x, algorithm="maxima")
```

output
```
integrate(x^m/lambert_w(a*x^n)^3, x)
```

Giac [F]

$$\int \frac{x^m}{W(ax^n)^3}\,dx = \int \frac{x^m}{\mathrm{W}(ax^n)^3}\,dx$$

input
```
integrate(x^m/lambert_w(a*x^n)^3,x, algorithm="giac")
```

output
```
integrate(x^m/lambert_w(a*x^n)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{W(ax^n)^3}\,dx = \int \frac{x^m}{\mathrm{LambertW}\,(a\,x^n)^3}\,dx$$

input
```
int(x^m/LambertW(a*x^n)^3,x)
```

output
```
int(x^m/LambertW(a*x^n)^3, x)
```

Reduce [F]

$$\int \frac{x^m}{W(ax^n)^3}\,dx = \text{too large to display}$$

input
```
int(x^m/Lambert_W(a*x^n)^3,x)
```

output
```
( - x**m*lambert_w(x**n*a)**2*x + x**m*lambert_w(x**n*a)*x + 4*x**m*x + 8*
int(x**(m + n)/(e**lambert_w(x**n*a)*lambert_w(x**n*a)**4*m**2 + 2*e**lamb
ert_w(x**n*a)*lambert_w(x**n*a)**4*m + e**lambert_w(x**n*a)*lambert_w(x**n
*a)**4 + e**lambert_w(x**n*a)*lambert_w(x**n*a)**3*m**2 + 2*e**lambert_w(x
**n*a)*lambert_w(x**n*a)**3*m + e**lambert_w(x**n*a)*lambert_w(x**n*a)**3)
,x)*lambert_w(x**n*a)**2*a*m**2*n + 16*int(x**(m + n)/(e**lambert_w(x**n*a
)*lambert_w(x**n*a)**4*m**2 + 2*e**lambert_w(x**n*a)*lambert_w(x**n*a)**4*
m + e**lambert_w(x**n*a)*lambert_w(x**n*a)**4 + e**lambert_w(x**n*a)*lambe
rt_w(x**n*a)**3*m**2 + 2*e**lambert_w(x**n*a)*lambert_w(x**n*a)**3*m + e**
lambert_w(x**n*a)*lambert_w(x**n*a)**3),x)*lambert_w(x**n*a)**2*a*m*n + 8*
int(x**(m + n)/(e**lambert_w(x**n*a)*lambert_w(x**n*a)**4*m**2 + 2*e**lamb
ert_w(x**n*a)*lambert_w(x**n*a)**4*m + e**lambert_w(x**n*a)*lambert_w(x**n
*a)**4 + e**lambert_w(x**n*a)*lambert_w(x**n*a)**3*m**2 + 2*e**lambert_w(x
**n*a)*lambert_w(x**n*a)**3*m + e**lambert_w(x**n*a)*lambert_w(x**n*a)**3)
,x)*lambert_w(x**n*a)**2*a*n + int(x**(m + n)/(e**lambert_w(x**n*a)*lamber
t_w(x**n*a)**3*m + e**lambert_w(x**n*a)*lambert_w(x**n*a)**3 + e**lambert_
w(x**n*a)*lambert_w(x**n*a)**2*m + e**lambert_w(x**n*a)*lambert_w(x**n*a)*
*2),x)*lambert_w(x**n*a)**2*a*m*n + int(x**(m + n)/(e**lambert_w(x**n*a)*l
ambert_w(x**n*a)**3*m + e**lambert_w(x**n*a)*lambert_w(x**n*a)**3 + e**lam
bert_w(x**n*a)*lambert_w(x**n*a)**2*m + e**lambert_w(x**n*a)*lambert_w(...
```

3.248 $\int x^{-1-3n} W(ax^n)^4 \, dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int x^{-1-3n} W(ax^n)^4 \, dx = -\frac{4x^{-3n} W(ax^n)^3}{9n} - \frac{x^{-3n} W(ax^n)^4}{3n}$$

output
```
-4/9*LambertW(a*x^n)^3/n/(x^(3*n))-1/3*LambertW(a*x^n)^4/n/(x^(3*n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int x^{-1-3n} W(ax^n)^4 \, dx = -\frac{x^{-3n} W(ax^n)^3 (4 + 3W(ax^n))}{9n}$$

input
```
Integrate[x^(-1 - 3*n)*ProductLog[a*x^n]^4,x]
```

output
```
-1/9*(ProductLog[a*x^n]^3*(4 + 3*ProductLog[a*x^n]))/(n*x^(3*n))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7172, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3n-1}W(ax^n)^4\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{4}{3}\int \frac{x^{-3n-1}W(ax^n)^4}{W(ax^n)+1}dx-\frac{x^{-3n}W(ax^n)^4}{3n}$$

$$\Big\downarrow \text{7201}$$

$$-\frac{x^{-3n}W(ax^n)^4}{3n}-\frac{4x^{-3n}W(ax^n)^3}{9n}$$

input
```
Int[x^(-1 - 3*n)*ProductLog[a*x^n]^4,x]
```

output
```
(-4*ProductLog[a*x^n]^3)/(9*n*x^(3*n)) - ProductLog[a*x^n]^4/(3*n*x^(3*n))
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$-\frac{3 \operatorname{LambertW}(a\,x^{n})^{4} x\,x^{-1-3 n}+4 \operatorname{LambertW}(a\,x^{n})^{3} x\,x^{-1-3 n}}{9 n}$	43

input
```
int(x^(-1-3*n)*LambertW(a*x^n)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/9*(3*LambertW(a*x^n)^4*x*x^(-1-3*n)+4*LambertW(a*x^n)^3*x*x^(-1-3*n))/n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int x^{-1-3n} W(ax^n)^4 \, dx = -\frac{3 \operatorname{W}(ax^n)^4 + 4 \operatorname{W}(ax^n)^3}{9\, n x^{3\, n}}$$

input
```
integrate(x^(-1-3*n)*lambert_w(a*x^n)^4,x, algorithm="fricas")
```

output
```
-1/9*(3*lambert_w(a*x^n)^4 + 4*lambert_w(a*x^n)^3)/(n*x^(3*n))
```

Sympy [A] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int x^{-1-3n} W(ax^n)^4 \, dx = \begin{cases} -\frac{x x^{-3n-1} W^{4}(ax^n)}{3n} - \frac{4 x x^{-3n-1} W^{3}(ax^n)}{9n} & \text{for}\: n \neq 0 \\ \log(x) W^{4}(a) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-3*n)*LambertW(a*x**n)**4,x)
```

output
```
Piecewise((-x*x**(-3*n - 1)*LambertW(a*x**n)**4/(3*n) - 4*x*x**(-3*n - 1)*
LambertW(a*x**n)**3/(9*n), Ne(n, 0)), (log(x)*LambertW(a)**4, True))
```

Maxima [F]

$$\int x^{-1-3n}W(ax^n)^4\,dx = \int x^{-3\,n-1}\,\mathrm{W}(ax^n)^4\,dx$$

input
```
integrate(x^(-1-3*n)*lambert_w(a*x^n)^4,x, algorithm="maxima")
```

output
```
integrate(x^(-3*n - 1)*lambert_w(a*x^n)^4, x)
```

Giac [F]

$$\int x^{-1-3n}W(ax^n)^4\,dx = \int x^{-3\,n-1}\,\mathrm{W}(ax^n)^4\,dx$$

input
```
integrate(x^(-1-3*n)*lambert_w(a*x^n)^4,x, algorithm="giac")
```

output
```
integrate(x^(-3*n - 1)*lambert_w(a*x^n)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-3n}W(ax^n)^4\,dx = \int \frac{\mathrm{LambertW}(a\,x^n)^4}{x^{3\,n+1}}\,dx$$

input
```
int(LambertW(a*x^n)^4/x^(3*n + 1),x)
```

output
```
int(LambertW(a*x^n)^4/x^(3*n + 1), x)
```

Reduce [F]

$$\int x^{-1-3n}W(ax^n)^4\,dx$$
$$= \frac{4x^{3n}\left(\int \frac{lambert\_w(x^n a)^3}{x^{2n}e^{lambert\_w(x^n a)}lambert\_w(x^n a)x+x^{2n}e^{lambert\_w(x^n a)}x}dx\right)an - lambert\_w(x^n a)^4}{3x^{3n}n}$$

input
```
int(x^(-1-3*n)*Lambert_W(a*x^n)^4,x)
```

output
```
(4*x**(3*n)*int(lambert_w(x**n*a)**3/(x**(2*n)*e**lambert_w(x**n*a)*lamber
t_w(x**n*a)*x + x**(2*n)*e**lambert_w(x**n*a)*x),x)*a*n - lambert_w(x**n*a
)**4)/(3*x**(3*n)*n)
```

3.249 $\int x^{-1-2n}W(ax^n)^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int x^{-1-2n}W(ax^n)^3\,dx=-\frac{3x^{-2n}W(ax^n)^2}{4n}-\frac{x^{-2n}W(ax^n)^3}{2n}$$

output
```
-3/4*LambertW(a*x^n)^2/n/(x^(2*n))-1/2*LambertW(a*x^n)^3/n/(x^(2*n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int x^{-1-2n}W(ax^n)^3\,dx=-\frac{x^{-2n}W(ax^n)^2(3+2W(ax^n))}{4n}$$

input
```
Integrate[x^(-1 - 2*n)*ProductLog[a*x^n]^3,x]
```

output
```
-1/4*(ProductLog[a*x^n]^2*(3 + 2*ProductLog[a*x^n]))/(n*x^(2*n))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7172, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}W(ax^n)^3\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{3}{2}\int \frac{x^{-2n-1}W(ax^n)^3}{W(ax^n)+1}dx-\frac{x^{-2n}W(ax^n)^3}{2n}$$

$$\Big\downarrow \text{7201}$$

$$-\frac{x^{-2n}W(ax^n)^3}{2n}-\frac{3x^{-2n}W(ax^n)^2}{4n}$$

input
```
Int[x^(-1 - 2*n)*ProductLog[a*x^n]^3,x]
```

output
```
(-3*ProductLog[a*x^n]^2)/(4*n*x^(2*n)) - ProductLog[a*x^n]^3/(2*n*x^(2*n))
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$-\frac{2\,\mathrm{LambertW}(a\,x^{n})^{3}x\,x^{-1-2n}+3\,\mathrm{LambertW}(a\,x^{n})^{2}x\,x^{-1-2n}}{4n}$	43

input
```
int(x^(-1-2*n)*LambertW(a*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*LambertW(a*x^n)^3*x*x^(-1-2*n)+3*LambertW(a*x^n)^2*x*x^(-1-2*n))/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int x^{-1-2n}W(ax^n)^3\,dx=-\frac{2\,\mathrm{W}(ax^n)^3+3\,\mathrm{W}(ax^n)^2}{4\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*lambert_w(a*x^n)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*lambert_w(a*x^n)^3 + 3*lambert_w(a*x^n)^2)/(n*x^(2*n))
```

Sympy [A] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int x^{-1-2n}W(ax^n)^3\,dx=\begin{cases}-\frac{xx^{-2n-1}W^3(ax^n)}{2n}-\frac{3xx^{-2n-1}W^2(ax^n)}{4n} & \text{for } n\neq 0\\ \log(x)W^3(a) & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1-2*n)*LambertW(a*x**n)**3,x)
```

output
```
Piecewise((-x*x**(-2*n - 1)*LambertW(a*x**n)**3/(2*n) - 3*x*x**(-2*n - 1)*
LambertW(a*x**n)**2/(4*n), Ne(n, 0)), (log(x)*LambertW(a)**3, True))
```

Maxima [F]

$$\int x^{-1-2n}W(ax^n)^3\,dx = \int x^{-2n-1}\,\mathrm{W}(ax^n)^3\,dx$$

input
```
integrate(x^(-1-2*n)*lambert_w(a*x^n)^3,x, algorithm="maxima")
```

output
```
integrate(x^(-2*n - 1)*lambert_w(a*x^n)^3, x)
```

Giac [F]

$$\int x^{-1-2n}W(ax^n)^3\,dx = \int x^{-2n-1}\,\mathrm{W}(ax^n)^3\,dx$$

input
```
integrate(x^(-1-2*n)*lambert_w(a*x^n)^3,x, algorithm="giac")
```

output
```
integrate(x^(-2*n - 1)*lambert_w(a*x^n)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}W(ax^n)^3\,dx = \int \frac{\mathrm{LambertW}(a\,x^n)^3}{x^{2\,n+1}}\,dx$$

input
```
int(LambertW(a*x^n)^3/x^(2*n + 1),x)
```

output
```
int(LambertW(a*x^n)^3/x^(2*n + 1), x)
```

Reduce [F]

$$\int x^{-1-2n}W(ax^n)^3\,dx$$
$$= \frac{3x^{2n}\left(\int \frac{lambert\_w(x^n a)^2}{x^n e^{lambert\_w(x^n a)} lambert\_w(x^n a)x + x^n e^{lambert\_w(x^n a)} x}dx\right)an - lambert\_w(x^n a)^3}{2x^{2n}n}$$

input
```
int(x^(-1-2*n)*Lambert_W(a*x^n)^3,x)
```

output
```
(3*x**(2*n)*int(lambert_w(x**n*a)**2/(x**n*e**lambert_w(x**n*a)*lambert_w(
x**n*a)*x + x**n*e**lambert_w(x**n*a)*x),x)*a*n - lambert_w(x**n*a)**3)/(2
*x**(2*n)*n)
```

3.250 $\int x^{-1-n}W(ax^n)^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 35

$$\int x^{-1-n}W(ax^n)^2\,dx = -\frac{2x^{-n}W(ax^n)}{n} - \frac{x^{-n}W(ax^n)^2}{n}$$

output
```
-2*LambertW(a*x^n)/n/(x^n)-LambertW(a*x^n)^2/n/(x^n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int x^{-1-n}W(ax^n)^2\,dx = -\frac{x^{-n}W(ax^n)\,(2+W(ax^n))}{n}$$

input
```
Integrate[x^(-1 - n)*ProductLog[a*x^n]^2,x]
```

output
```
-((ProductLog[a*x^n]*(2 + ProductLog[a*x^n]))/(n*x^n))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7172, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}W(ax^n)^2\,dx$$

$$\Big\downarrow \text{7172}$$

$$2\int \frac{x^{-n-1}W(ax^n)^2}{W(ax^n)+1}dx-\frac{x^{-n}W(ax^n)^2}{n}$$

$$\Big\downarrow \text{7201}$$

$$-\frac{x^{-n}W(ax^n)^2}{n}-\frac{2x^{-n}W(ax^n)}{n}$$

input
```
Int[x^(-1 - n)*ProductLog[a*x^n]^2,x]
```

output
```
(-2*ProductLog[a*x^n])/(n*x^n) - ProductLog[a*x^n]^2/(n*x^n)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

method	result	size
parallelrisch	$-\frac{\operatorname{LambertW}(a\,x^{n})^{2} x\,x^{-1-n}+2\operatorname{LambertW}(a\,x^{n}) x\,x^{-1-n}}{n}$	40

input
```
int(x^(-1-n)*LambertW(a*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-(LambertW(a*x^n)^2*x*x^(-1-n)+2*LambertW(a*x^n)*x*x^(-1-n))/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x^{-1-n}W(ax^n)^2\,dx = -\frac{\mathrm{W}(ax^n)^2 + 2\,\mathrm{W}(ax^n)}{nx^n}$$

input
```
integrate(x^(-1-n)*lambert_w(a*x^n)^2,x, algorithm="fricas")
```

output
```
-(lambert_w(a*x^n)^2 + 2*lambert_w(a*x^n))/(n*x^n)
```

Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int x^{-1-n}W(ax^n)^2\,dx = \begin{cases} -\frac{xx^{-n-1}W^2(ax^n)}{n} - \frac{2xx^{-n-1}W(ax^n)}{n} & \text{for } n \neq 0 \\ \log(x)W^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-n)*LambertW(a*x**n)**2,x)
```

output
```
Piecewise((-x*x**(-n - 1)*LambertW(a*x**n)**2/n - 2*x*x**(-n - 1)*LambertW
(a*x**n)/n, Ne(n, 0)), (log(x)*LambertW(a)**2, True))
```

Maxima [F]

$$\int x^{-1-n}W(ax^n)^2\,dx = \int x^{-n-1}\,\mathrm{W}(ax^n)^2\,dx$$

input
```
integrate(x^(-1-n)*lambert_w(a*x^n)^2,x, algorithm="maxima")
```

output
```
integrate(x^(-n - 1)*lambert_w(a*x^n)^2, x)
```

Giac [F]

$$\int x^{-1-n}W(ax^n)^2\,dx = \int x^{-n-1}\,\mathrm{W}(ax^n)^2\,dx$$

input
```
integrate(x^(-1-n)*lambert_w(a*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)*lambert_w(a*x^n)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}W(ax^n)^2\,dx = \int \frac{\mathrm{LambertW}(a\,x^n)^2}{x^{n+1}}\,dx$$

input
```
int(LambertW(a*x^n)^2/x^(n + 1),x)
```

output
```
int(LambertW(a*x^n)^2/x^(n + 1), x)
```

Reduce [F]

$$\int x^{-1-n}W(ax^n)^2\,dx$$
$$=\frac{2x^n\left(\int\frac{lambert\_w(x^na)}{e^{lambert\_w(x^na)}lambert\_w(x^na)x+e^{lambert\_w(x^na)}x}dx\right)an+2x^n\left(\int\frac{lambert\_w(x^na)}{x^nlambert\_w(x^na)x+x^nx}dx\right)n+2x^n}{x^nn}$$

input
```
int(x^(-1-n)*Lambert_W(a*x^n)^2,x)
```

output
```
(2*x**n*int(lambert_w(x**n*a)/(e**lambert_w(x**n*a)*lambert_w(x**n*a)*x +
e**lambert_w(x**n*a)*x),x)*a*n + 2*x**n*int(lambert_w(x**n*a)/(x**n*lamber
t_w(x**n*a)*x + x**n*x),x)*n + 2*x**n*int(1/(x**n*lambert_w(x**n*a)*x + x*
*n*x),x)*n - lambert_w(x**n*a)**2 + 2)/(x**n*n)
```

3.251 $\int \frac{x^{-1+2n}}{W(ax^n)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{x^{-1+2n}}{W(ax^n)}\,dx = \frac{x^{2n}}{4nW(ax^n)^2} + \frac{x^{2n}}{2nW(ax^n)}$$

output
```
1/4*x^(2*n)/n/LambertW(a*x^n)^2+1/2*x^(2*n)/n/LambertW(a*x^n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{x^{-1+2n}}{W(ax^n)}\,dx = \frac{x^{2n}(1 + 2W(ax^n))}{4nW(ax^n)^2}$$

input
```
Integrate[x^(-1 + 2*n)/ProductLog[a*x^n],x]
```

output
```
(x^(2*n)*(1 + 2*ProductLog[a*x^n]))/(4*n*ProductLog[a*x^n]^2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {7172, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{W(ax^n)} dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{2} \int \frac{x^{2n-1}}{W(ax^n)(W(ax^n)+1)} dx + \frac{x^{2n}}{2nW(ax^n)}$$

$$\Big\downarrow \text{7201}$$

$$\frac{x^{2n}}{2nW(ax^n)} + \frac{x^{2n}}{4nW(ax^n)^2}$$

input
```
Int[x^(-1 + 2*n)/ProductLog[a*x^n],x]
```

output
```
x^(2*n)/(4*n*ProductLog[a*x^n]^2) + x^(2*n)/(2*n*ProductLog[a*x^n])
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$-\frac{-2\,\mathrm{LambertW}(a\,x^{n})x\,x^{-1+2n}-x\,x^{-1+2n}}{4n\,\mathrm{LambertW}(a\,x^{n})^{2}}$	41

input
```
int(x^(-1+2*n)/LambertW(a*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*LambertW(a*x^n)*x*x^(-1+2*n)-x*x^(-1+2*n))/n/LambertW(a*x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+2n}}{W(ax^n)}\,dx = \frac{2\,x^{2\,n}\,\mathrm{W}(ax^n) + x^{2\,n}}{4\,n\,\mathrm{W}(ax^n)^2}$$

input
```
integrate(x^(-1+2*n)/lambert_w(a*x^n),x, algorithm="fricas")
```

output
```
1/4*(2*x^(2*n)*lambert_w(a*x^n) + x^(2*n))/(n*lambert_w(a*x^n)^2)
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{x^{-1+2n}}{W(ax^n)}\,dx = \begin{cases} \frac{xx^{2n-1}}{2nW(ax^n)} + \frac{xx^{2n-1}}{4nW^2(ax^n)} & \text{for } n \neq 0 \\ \frac{\log(x)}{W(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+2*n)/LambertW(a*x**n),x)
```

output
```
Piecewise((x*x**(2*n - 1)/(2*n*LambertW(a*x**n)) + x*x**(2*n - 1)/(4*n*Lam
bertW(a*x**n)**2), Ne(n, 0)), (log(x)/LambertW(a), True))
```

Maxima [F]

$$\int \frac{x^{-1+2n}}{W(ax^n)}\,dx = \int \frac{x^{2n-1}}{\mathrm{W}(ax^n)}\,dx$$

input
```
integrate(x^(-1+2*n)/lambert_w(a*x^n),x, algorithm="maxima")
```

output
```
integrate(x^(2*n - 1)/lambert_w(a*x^n), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{W(ax^n)}\,dx = \int \frac{x^{2n-1}}{\mathrm{W}(ax^n)}\,dx$$

input
```
integrate(x^(-1+2*n)/lambert_w(a*x^n),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/lambert_w(a*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{W(ax^n)}\,dx = \int \frac{x^{2\,n-1}}{\mathrm{LambertW}\left(a\,x^n\right)}\,dx$$

input
```
int(x^(2*n - 1)/LambertW(a*x^n),x)
```

output
```
int(x^(2*n - 1)/LambertW(a*x^n), x)
```

Reduce [F]

$$\int \frac{x^{-1+2n}}{W(ax^n)}\,dx = \frac{x^{2n}+4\left(\int \frac{x^{2n}}{lambert\_w(x^n a)x}dx\right)n-2\left(\int \frac{x^{2n}}{x}dx\right)n}{4n}$$

input
```
int(x^(-1+2*n)/Lambert_W(a*x^n),x)
```

output
```
(x**(2*n) + 4*int(x**(2*n)/(lambert_w(x**n*a)*x),x)*n - 2*int(x**(2*n)/x,x
)*n)/(4*n)
```

3.252 $\int \frac{x^{-1+3n}}{W(ax^n)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{x^{-1+3n}}{W(ax^n)^2} dx = \frac{2x^{3n}}{9nW(ax^n)^3} + \frac{x^{3n}}{3nW(ax^n)^2}$$

output
```
2/9*x^(3*n)/n/LambertW(a*x^n)^3+1/3*x^(3*n)/n/LambertW(a*x^n)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{x^{-1+3n}}{W(ax^n)^2} dx = \frac{x^{3n}(2 + 3W(ax^n))}{9nW(ax^n)^3}$$

input
```
Integrate[x^(-1 + 3*n)/ProductLog[a*x^n]^2,x]
```

output
```
(x^(3*n)*(2 + 3*ProductLog[a*x^n]))/(9*n*ProductLog[a*x^n]^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7172, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{W(ax^n)^2}dx$$

$$\Big\downarrow \text{ 7172}$$

$$\frac{2}{3}\int \frac{x^{3n-1}}{W(ax^n)^2(W(ax^n)+1)}dx+\frac{x^{3n}}{3nW(ax^n)^2}$$

$$\Big\downarrow \text{ 7201}$$

$$\frac{x^{3n}}{3nW(ax^n)^2}+\frac{2x^{3n}}{9nW(ax^n)^3}$$

input
```
Int[x^(-1 + 3*n)/ProductLog[a*x^n]^2,x]
```

output
```
(2*x^(3*n))/(9*n*ProductLog[a*x^n]^3) + x^(3*n)/(3*n*ProductLog[a*x^n]^2)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$-\frac{-3 \operatorname{LambertW}(a\,x^{n}) x\,x^{-1+3 n}-2 x\,x^{-1+3 n}}{9 n \operatorname{LambertW}(a\,x^{n})^{3}}$	41

input
```
int(x^(-1+3*n)/LambertW(a*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/9*(-3*LambertW(a*x^n)*x*x^(-1+3*n)-2*x*x^(-1+3*n))/n/LambertW(a*x^n)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x^{-1+3n}}{W\left(ax^n\right)^2}\,dx=\frac{3\,x^{3\,n}\,\mathrm{W}(ax^n)+2\,x^{3\,n}}{9\,n\,\mathrm{W}(ax^n)^3}$$

input
```
integrate(x^(-1+3*n)/lambert_w(a*x^n)^2,x, algorithm="fricas")
```

output
```
1/9*(3*x^(3*n)*lambert_w(a*x^n) + 2*x^(3*n))/(n*lambert_w(a*x^n)^3)
```

Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{x^{-1+3n}}{W(ax^n)^2} \, dx = \begin{cases} \frac{xx^{3n-1}}{3nW^2(ax^n)} + \frac{2xx^{3n-1}}{9nW^3(ax^n)} & \text{for } n \neq 0 \\ \frac{\log(x)}{W^2(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+3*n)/LambertW(a*x**n)**2,x)
```

output
```
Piecewise((x*x**(3*n - 1)/(3*n*LambertW(a*x**n)**2) + 2*x*x**(3*n - 1)/(9*
n*LambertW(a*x**n)**3), Ne(n, 0)), (log(x)/LambertW(a)**2, True))
```

Maxima [F]

$$\int \frac{x^{-1+3n}}{W(ax^n)^2} \, dx = \int \frac{x^{3n-1}}{W(ax^n)^2} \, dx$$

input
```
integrate(x^(-1+3*n)/lambert_w(a*x^n)^2,x, algorithm="maxima")
```

output
```
integrate(x^(3*n - 1)/lambert_w(a*x^n)^2, x)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{W(ax^n)^2} \, dx = \int \frac{x^{3n-1}}{W(ax^n)^2} \, dx$$

input
```
integrate(x^(-1+3*n)/lambert_w(a*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/lambert_w(a*x^n)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{W(ax^n)^2} \, dx = \int \frac{x^{3\,n-1}}{\mathrm{LambertW}\left(a\,x^n\right)^2} \, dx$$

input
```
int(x^(3*n - 1)/LambertW(a*x^n)^2,x)
```

output
```
int(x^(3*n - 1)/LambertW(a*x^n)^2, x)
```

Reduce [F]

$$\int \frac{x^{-1+3n}}{W(ax^n)^2} \, dx = \int \frac{x^{3n}}{lambert\_w(x^n a)^2 \, x} dx$$

input
```
int(x^(-1+3*n)/Lambert_W(a*x^n)^2,x)
```

output
```
int(x**(3*n)/(lambert_w(x**n*a)**2*x),x)
```

3.253 $\int x^{-1-n}(cW(ax^n))^{9/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 139

$$\int x^{-1-n}(cW(ax^n))^{9/2}\,dx = \frac{135ac^{9/2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{16n} - \frac{135c^3x^{-n}(cW(ax^n))^{3/2}}{8n} - \frac{45c^2x^{-n}(cW(ax^n))^{5/2}}{4n} - \frac{9cx^{-n}(cW(ax^n))^{7/2}}{2n} - \frac{x^{-n}(cW(ax^n))^{9/2}}{n}$$

output
```
135/16*a*c^(9/2)*Pi^(1/2)*erf((c*LambertW(a*x^n))^(1/2)/c^(1/2))/n-135/8*c
^3*(c*LambertW(a*x^n))^(3/2)/n/(x^n)-45/4*c^2*(c*LambertW(a*x^n))^(5/2)/n/
(x^n)-9/2*c*(c*LambertW(a*x^n))^(7/2)/n/(x^n)-(c*LambertW(a*x^n))^(9/2)/n/
(x^n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

$$\int x^{-1-n}(cW(ax^n))^{9/2}\,dx = -\frac{c^4x^{-n}\sqrt{cW(ax^n)}\left(-135a\sqrt{\pi}x^n\mathrm{erf}\left(\sqrt{W(ax^n)}\right) + 270W(ax^n)^{3/2} + 180W(ax^n)^{5/2} + 72W(ax^n)^{7/2} + 16\right.}{16n\sqrt{W(ax^n)}}$$

input
```
Integrate[x^(-1 - n)*(c*ProductLog[a*x^n])^(9/2),x]
```

output
```
-1/16*(c^4*Sqrt[c*ProductLog[a*x^n]]*(-135*a*Sqrt[Pi]*x^n*Erf[Sqrt[Product
Log[a*x^n]]] + 270*ProductLog[a*x^n]^(3/2) + 180*ProductLog[a*x^n]^(5/2) +
 72*ProductLog[a*x^n]^(7/2) + 16*ProductLog[a*x^n]^(9/2)))/(n*x^n*Sqrt[Pro
ductLog[a*x^n]])
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7172, 7205, 7205, 7205, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}(cW(ax^n))^{9/2}\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{9}{2}\int \frac{x^{-n-1}(cW(ax^n))^{9/2}}{W(ax^n)+1}dx-\frac{x^{-n}(cW(ax^n))^{9/2}}{n}$$

$$\Big\downarrow \text{7205}$$

$$\frac{9}{2}\left(\frac{5}{2}c\int \frac{x^{-n-1}(cW(ax^n))^{7/2}}{W(ax^n)+1}dx-\frac{cx^{-n}(cW(ax^n))^{7/2}}{n}\right)-\frac{x^{-n}(cW(ax^n))^{9/2}}{n}$$

$$\Big\downarrow \text{7205}$$

$$\frac{9}{2}\left(\frac{5}{2}c\left(\frac{3}{2}c\int \frac{x^{-n-1}(cW(ax^n))^{5/2}}{W(ax^n)+1}dx-\frac{cx^{-n}(cW(ax^n))^{5/2}}{n}\right)-\frac{cx^{-n}(cW(ax^n))^{7/2}}{n}\right)-\frac{x^{-n}(cW(ax^n))^{9/2}}{n}$$

$$\Big\downarrow \text{7205}$$

$$\frac{9}{2}\left(\frac{5}{2}c\left(\frac{3}{2}c\left(\frac{1}{2}c\int\frac{x^{-n-1}(cW(ax^n))^{3/2}}{W(ax^n)+1}dx-\frac{cx^{-n}(cW(ax^n))^{3/2}}{n}\right)-\frac{cx^{-n}(cW(ax^n))^{5/2}}{n}\right)-\frac{cx^{-n}(cW(ax^n))^{7/2}}{n}\right.$$
$$\frac{x^{-n}(cW(ax^n))^{9/2}}{n}$$

$$\Big\downarrow \text{7203}$$

$$\frac{9}{2}\left(\frac{5}{2}c\left(\frac{3}{2}c\left(\frac{\sqrt{\pi}ac^{5/2}\text{erf}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{2n}-\frac{cx^{-n}(cW(ax^n))^{3/2}}{n}\right)-\frac{cx^{-n}(cW(ax^n))^{5/2}}{n}\right)-\frac{cx^{-n}(cW(ax^n))^{7/2}}{n}\right)-$$
$$\frac{x^{-n}(cW(ax^n))^{9/2}}{n}$$

input
```
Int[x^(-1 - n)*(c*ProductLog[a*x^n])^(9/2),x]
```

output
```
-((c*ProductLog[a*x^n])^(9/2)/(n*x^n)) + (9*(-((c*(c*ProductLog[a*x^n])^(7
/2))/(n*x^n)) + (5*c*(-((c*(c*ProductLog[a*x^n])^(5/2))/(n*x^n)) + (3*c*((
a*c^(5/2)*Sqrt[Pi]*Erf[Sqrt[c*ProductLog[a*x^n]]/Sqrt[c]])/(2*n) - (c*(c*P
roductLog[a*x^n])^(3/2))/(n*x^n)))/2))/2))/2
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^{-1-n}(c\,\mathrm{LambertW}\left(a\,x^{n}\right))^{\frac{9}{2}}dx$$

input
```
int(x^(-1-n)*(c*LambertW(a*x^n))^(9/2),x)
```

output
```
int(x^(-1-n)*(c*LambertW(a*x^n))^(9/2),x)
```

Fricas [F]

$$\int x^{-1-n}(cW(ax^n))^{9/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{9}{2}}\,x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(c*lambert_w(a*x^n))^(9/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*c^4*x^(-n - 1)*lambert_w(a*x^n)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-n}(cW(ax^n))^{9/2}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n)*(c*LambertW(a*x**n))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-n}(cW(ax^n))^{9/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{9}{2}}\,x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(c*lambert_w(a*x^n))^(9/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^(9/2)*x^(-n - 1), x)
```

Giac [F]

$$\int x^{-1-n}(cW(ax^n))^{9/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{9}{2}}\,x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(c*lambert_w(a*x^n))^(9/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^(9/2)*x^(-n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}(cW(ax^n))^{9/2}\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x^n))^{9/2}}{x^{n+1}}\,dx$$

input
```
int((c*LambertW(a*x^n))^(9/2)/x^(n + 1),x)
```

output
```
int((c*LambertW(a*x^n))^(9/2)/x^(n + 1), x)
```

Reduce [F]

$$\int x^{-1-n}(cW(ax^n))^{9/2}\,dx = \frac{\sqrt{c}\,c^4\left(-2\sqrt{lambert\_w(x^na)}\,lambert\_w(x^na)^4+9x^n\left(\int \frac{\sqrt{lambert\_w(x}}{e^{lambert\_w(x^na)}\,lamber}\right.\right.}{2x^nn}$$

input
```
int(x^(-1-n)*(c*Lambert_W(a*x^n))^(9/2),x)
```

output
```
(sqrt(c)*c**4*( - 2*sqrt(lambert_w(x**n*a))*lambert_w(x**n*a)**4 + 9*x**n*
int((sqrt(lambert_w(x**n*a))*lambert_w(x**n*a)**3)/(e**lambert_w(x**n*a)*l
ambert_w(x**n*a)*x + e**lambert_w(x**n*a)*x),x)*a*n))/(2*x**n*n)
```

3.254 $\int x^{-1-n}(cW(ax^n))^{7/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int x^{-1-n}(cW(ax^n))^{7/2}\,dx = \frac{21ac^{7/2}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{8n} - \frac{21c^2x^{-n}(cW(ax^n))^{3/2}}{4n} - \frac{7cx^{-n}(cW(ax^n))^{5/2}}{2n} - \frac{x^{-n}(cW(ax^n))^{7/2}}{n}$$

output
```
21/8*a*c^(7/2)*Pi^(1/2)*erf((c*LambertW(a*x^n))^(1/2)/c^(1/2))/n-21/4*c^2*
(c*LambertW(a*x^n))^(3/2)/n/(x^n)-7/2*c*(c*LambertW(a*x^n))^(5/2)/n/(x^n)-
(c*LambertW(a*x^n))^(7/2)/n/(x^n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int x^{-1-n}(cW(ax^n))^{7/2}\,dx = -\frac{c^3x^{-n}\sqrt{cW(ax^n)}\left(-21a\sqrt{\pi}x^n\text{erf}\left(\sqrt{W(ax^n)}\right) + 42W(ax^n)^{3/2} + 28W(ax^n)^{5/2} + 8W(ax^n)^{7/2}\right)}{8n\sqrt{W(ax^n)}}$$

input
```
Integrate[x^(-1 - n)*(c*ProductLog[a*x^n])^(7/2),x]
```

output
```
-1/8*(c^3*Sqrt[c*ProductLog[a*x^n]]*(-21*a*Sqrt[Pi]*x^n*Erf[Sqrt[ProductLo
g[a*x^n]]] + 42*ProductLog[a*x^n]^(3/2) + 28*ProductLog[a*x^n]^(5/2) + 8*P
roductLog[a*x^n]^(7/2)))/(n*x^n*Sqrt[ProductLog[a*x^n]])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7172, 7205, 7205, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}(cW(ax^n))^{7/2} dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{7}{2}\int \frac{x^{-n-1}(cW(ax^n))^{7/2}}{W(ax^n)+1}dx-\frac{x^{-n}(cW(ax^n))^{7/2}}{n}$$

$$\Big\downarrow \text{7205}$$

$$\frac{7}{2}\left(\frac{3}{2}c\int \frac{x^{-n-1}(cW(ax^n))^{5/2}}{W(ax^n)+1}dx-\frac{cx^{-n}(cW(ax^n))^{5/2}}{n}\right)-\frac{x^{-n}(cW(ax^n))^{7/2}}{n}$$

$$\Big\downarrow \text{7205}$$

$$\frac{7}{2}\left(\frac{3}{2}c\left(\frac{1}{2}c\int \frac{x^{-n-1}(cW(ax^n))^{3/2}}{W(ax^n)+1}dx-\frac{cx^{-n}(cW(ax^n))^{3/2}}{n}\right)-\frac{cx^{-n}(cW(ax^n))^{5/2}}{n}\right)-\frac{x^{-n}(cW(ax^n))^{7/2}}{n}$$

$$\Big\downarrow \text{7203}$$

$$\frac{7}{2}\left(\frac{3}{2}c\left(\frac{\sqrt{\pi}ac^{5/2}\text{erf}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{2n}-\frac{cx^{-n}(cW(ax^n))^{3/2}}{n}\right)-\frac{cx^{-n}(cW(ax^n))^{5/2}}{n}\right)-\frac{x^{-n}(cW(ax^n))^{7/2}}{n}$$

input
```
Int[x^(-1 - n)*(c*ProductLog[a*x^n])^(7/2),x]
```

output
```
-((c*ProductLog[a*x^n])^(7/2)/(n*x^n)) + (7*(-((c*(c*ProductLog[a*x^n])^(5
/2))/(n*x^n)) + (3*c*((a*c^(5/2)*Sqrt[Pi]*Erf[Sqrt[c*ProductLog[a*x^n]]/Sq
rt[c]])/(2*n) - (c*(c*ProductLog[a*x^n])^(3/2))/(n*x^n)))/2))/2
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^{-1-n} (c \operatorname{LambertW}(a x^n))^{\frac{7}{2}} dx$$

input
```
int(x^(-1-n)*(c*LambertW(a*x^n))^(7/2),x)
```

output
```
int(x^(-1-n)*(c*LambertW(a*x^n))^(7/2),x)
```

Fricas [F]

$$\int x^{-1-n}(cW(ax^n))^{7/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{7}{2}}\,x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(c*lambert_w(a*x^n))^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*c^3*x^(-n - 1)*lambert_w(a*x^n)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-n}(cW(ax^n))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n)*(c*LambertW(a*x**n))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-n}(cW(ax^n))^{7/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{7}{2}}\,x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(c*lambert_w(a*x^n))^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^(7/2)*x^(-n - 1), x)
```

Giac [F]

$$\int x^{-1-n}(cW(ax^n))^{7/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{7}{2}}\,x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(c*lambert_w(a*x^n))^(7/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^(7/2)*x^(-n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}(cW(ax^n))^{7/2}\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x^n))^{7/2}}{x^{n+1}}\,dx$$

input
```
int((c*LambertW(a*x^n))^(7/2)/x^(n + 1),x)
```

output
```
int((c*LambertW(a*x^n))^(7/2)/x^(n + 1), x)
```

Reduce [F]

$$\int x^{-1-n}(cW(ax^n))^{7/2}\,dx = \frac{\sqrt{c}\,c^3\left(-2\sqrt{lambert\_w(x^n a)}\,lambert\_w(x^n a)^3 + 7x^n\left(\int \frac{\sqrt{lambert\_w(x}}{e^{lambert\_w(x^n a)}lamber}\right.\right.}{2x^n n}$$

input
```
int(x^(-1-n)*(c*Lambert_W(a*x^n))^(7/2),x)
```

output
```
(sqrt(c)*c**3*( - 2*sqrt(lambert_w(x**n*a))*lambert_w(x**n*a)**3 + 7*x**n*
int((sqrt(lambert_w(x**n*a))*lambert_w(x**n*a)**2)/(e**lambert_w(x**n*a)*l
ambert_w(x**n*a)*x + e**lambert_w(x**n*a)*x),x)*a*n))/(2*x**n*n)
```

3.255 $\int x^{-1-n}(cW(ax^n))^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int x^{-1-n}(cW(ax^n))^{5/2}\,dx=\frac{5ac^{5/2}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{4n}-\frac{5cx^{-n}(cW(ax^n))^{3/2}}{2n}-\frac{x^{-n}(cW(ax^n))^{5/2}}{n}$$

output
```
5/4*a*c^(5/2)*Pi^(1/2)*erf((c*LambertW(a*x^n))^(1/2)/c^(1/2))/n-5/2*c*(c*L
ambertW(a*x^n))^(3/2)/n/(x^n)-(c*LambertW(a*x^n))^(5/2)/n/(x^n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int x^{-1-n}(cW(ax^n))^{5/2}\,dx=-\frac{x^{-n}(cW(ax^n))^{5/2}\left(-5a\sqrt{\pi}x^n\operatorname{erf}\left(\sqrt{W(ax^n)}\right)+10W(ax^n)^{3/2}+4W(ax^n)^{5/2}\right)}{4nW(ax^n)^{5/2}}$$

input
```
Integrate[x^(-1 - n)*(c*ProductLog[a*x^n])^(5/2),x]
```

output
```
-1/4*((c*ProductLog[a*x^n])^(5/2)*(-5*a*Sqrt[Pi]*x^n*Erf[Sqrt[ProductLog[a
*x^n]]] + 10*ProductLog[a*x^n]^(3/2) + 4*ProductLog[a*x^n]^(5/2)))/(n*x^n*
ProductLog[a*x^n]^(5/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7172, 7205, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}(cW(ax^n))^{5/2}\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{5}{2}\int \frac{x^{-n-1}(cW(ax^n))^{5/2}}{W(ax^n)+1}dx - \frac{x^{-n}(cW(ax^n))^{5/2}}{n}$$

$$\Big\downarrow \text{7205}$$

$$\frac{5}{2}\left(\frac{1}{2}c\int \frac{x^{-n-1}(cW(ax^n))^{3/2}}{W(ax^n)+1}dx - \frac{cx^{-n}(cW(ax^n))^{3/2}}{n}\right) - \frac{x^{-n}(cW(ax^n))^{5/2}}{n}$$

$$\Big\downarrow \text{7203}$$

$$\frac{5}{2}\left(\frac{\sqrt{\pi}ac^{5/2}\text{erf}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{2n} - \frac{cx^{-n}(cW(ax^n))^{3/2}}{n}\right) - \frac{x^{-n}(cW(ax^n))^{5/2}}{n}$$

input
```
Int[x^(-1 - n)*(c*ProductLog[a*x^n])^(5/2),x]
```

output
```
-((c*ProductLog[a*x^n])^(5/2)/(n*x^n)) + (5*((a*c^(5/2)*Sqrt[Pi]*Erf[Sqrt[
c*ProductLog[a*x^n]]/Sqrt[c]])/(2*n) - (c*(c*ProductLog[a*x^n])^(3/2))/(n*
x^n)))/2
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^{-1-n} (c \,\text{LambertW}\left(a \,x^{n}\right))^{\frac{5}{2}}d x$$

input
```
int(x^(-1-n)*(c*LambertW(a*x^n))^(5/2),x)
```

output
```
int(x^(-1-n)*(c*LambertW(a*x^n))^(5/2),x)
```

Fricas [F]

$$\int x^{-1-n}(cW(ax^n))^{5/2}\,dx=\int (c\,\mathrm{W}(ax^n))^{\frac{5}{2}}\,x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(c*lambert_w(a*x^n))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*c^2*x^(-n - 1)*lambert_w(a*x^n)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-n}(cW(ax^n))^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(x**(-1-n)*(c*LambertW(a*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-n}(cW(ax^n))^{5/2}\,dx=\int (c\,\mathrm{W}(ax^n))^{\frac{5}{2}}\,x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(c*lambert_w(a*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^(5/2)*x^(-n - 1), x)
```

Giac [F]

$$\int x^{-1-n}(cW(ax^n))^{5/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{5}{2}}\,x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(c*lambert_w(a*x^n))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^(5/2)*x^(-n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}(cW(ax^n))^{5/2}\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x^n))^{5/2}}{x^{n+1}}\,dx$$

input
```
int((c*LambertW(a*x^n))^(5/2)/x^(n + 1),x)
```

output
```
int((c*LambertW(a*x^n))^(5/2)/x^(n + 1), x)
```

Reduce [F]

$$\int x^{-1-n}(cW(ax^n))^{5/2}\,dx = \frac{\sqrt{c}\,c^2\left(-2\sqrt{lambert\_w(x^n a)}\,lambert\_w(x^n a)^2 + 5x^n\left(\int \frac{\sqrt{lambert\_w(x}}{e^{lambert\_w(x^n a)}lamber}\right.\right.}{2x^n n}$$

input
```
int(x^(-1-n)*(c*Lambert_W(a*x^n))^(5/2),x)
```

output
```
(sqrt(c)*c**2*( - 2*sqrt(lambert_w(x**n*a))*lambert_w(x**n*a)**2 + 5*x**n*
int((sqrt(lambert_w(x**n*a))*lambert_w(x**n*a))/(e**lambert_w(x**n*a)*lamb
ert_w(x**n*a)*x + e**lambert_w(x**n*a)*x),x)*a*n))/(2*x**n*n)
```

3.256 $\int x^{-1-n}(cW(ax^n))^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int x^{-1-n}(cW(ax^n))^{3/2}\,dx=\frac{3ac^{3/2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{2n}-\frac{x^{-n}(cW(ax^n))^{3/2}}{n}$$

output

```
3/2*a*c^(3/2)*Pi^(1/2)*erf((c*LambertW(a*x^n))^(1/2)/c^(1/2))/n-(c*Lambert
W(a*x^n))^(3/2)/n/(x^n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int x^{-1-n}(cW(ax^n))^{3/2}\,dx=\frac{\left(-2x^{-n}+\frac{3a\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{W(ax^n)}\right)}{W(ax^n)^{3/2}}\right)(cW(ax^n))^{3/2}}{2n}$$

input

```
Integrate[x^(-1 - n)*(c*ProductLog[a*x^n])^(3/2),x]
```

output

```
((-2/x^n + (3*a*Sqrt[Pi]*Erf[Sqrt[ProductLog[a*x^n]]])/ProductLog[a*x^n]^(
3/2))*(c*ProductLog[a*x^n])^(3/2))/(2*n)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7172, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}(cW(ax^n))^{3/2}\,dx$$

$$\Big\downarrow \text{ 7172}$$

$$\frac{3}{2}\int \frac{x^{-n-1}(cW(ax^n))^{3/2}}{W(ax^n)+1}dx - \frac{x^{-n}(cW(ax^n))^{3/2}}{n}$$

$$\Big\downarrow \text{ 7203}$$

$$\frac{3\sqrt{\pi}ac^{3/2}\text{erf}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{2n} - \frac{x^{-n}(cW(ax^n))^{3/2}}{n}$$

input
```
Int[x^(-1 - n)*(c*ProductLog[a*x^n])^(3/2),x]
```

output
```
(3*a*c^(3/2)*Sqrt[Pi]*Erf[Sqrt[c*ProductLog[a*x^n]]/Sqrt[c]])/(2*n) - (c*P
roductLog[a*x^n])^(3/2)/(n*x^n)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

Maple [F]

$$\int x^{-1-n}(c\,\mathrm{LambertW}\,(a\,x^{n}))^{\frac{3}{2}}dx$$

input
```
int(x^(-1-n)*(c*LambertW(a*x^n))^(3/2),x)
```

output
```
int(x^(-1-n)*(c*LambertW(a*x^n))^(3/2),x)
```

Fricas [F]

$$\int x^{-1-n}(cW(ax^n))^{3/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{3}{2}}\,x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(c*lambert_w(a*x^n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*c*x^(-n - 1)*lambert_w(a*x^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-n}(cW(ax^n))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n)*(c*LambertW(a*x**n))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-n}(cW(ax^n))^{3/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{3}{2}}\,x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(c*lambert_w(a*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^(3/2)*x^(-n - 1), x)
```

Giac [F]

$$\int x^{-1-n}(cW(ax^n))^{3/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{3}{2}}\,x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(c*lambert_w(a*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^(3/2)*x^(-n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}(cW(ax^n))^{3/2}\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x^n))^{3/2}}{x^{n+1}}\,dx$$

input
```
int((c*LambertW(a*x^n))^(3/2)/x^(n + 1),x)
```

output
```
int((c*LambertW(a*x^n))^(3/2)/x^(n + 1), x)
```

Reduce [F]

$$\int x^{-1-n}(cW(ax^n))^{3/2}\,dx = \frac{\sqrt{c}\,c\left(-2\sqrt{lambert\_w(x^n a)}\,lambert\_w(x^n a) + 3x^n\left(\int \frac{\sqrt{lamber}}{e^{lambert\_w(x^n a)}\,lambert\_}\right.\right.}{2x^n n}$$

input
```
int(x^(-1-n)*(c*Lambert_W(a*x^n))^(3/2),x)
```

output
```
(sqrt(c)*c*( - 2*sqrt(lambert_w(x**n*a))*lambert_w(x**n*a) + 3*x**n*int(sq
rt(lambert_w(x**n*a))/(e**lambert_w(x**n*a)*lambert_w(x**n*a)*x + e**lambe
rt_w(x**n*a)*x),x)*a*n))/(2*x**n*n)
```

3.257 $\int x^{-1-n}\sqrt{cW(ax^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int x^{-1-n}\sqrt{cW(ax^n)}\,dx=-\frac{a\sqrt{c}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{n}-\frac{2x^{-n}\sqrt{cW(ax^n)}}{n}$$

output
```
-a*c^(1/2)*Pi^(1/2)*erf((c*LambertW(a*x^n))^(1/2)/c^(1/2))/n-2*(c*LambertW
(a*x^n))^(1/2)/n/(x^n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int x^{-1-n}\sqrt{cW(ax^n)}\,dx=-\frac{\left(2x^{-n}+\frac{a\sqrt{\pi}\,\text{erf}\left(\sqrt{W(ax^n)}\right)}{\sqrt{W(ax^n)}}\right)\sqrt{cW(ax^n)}}{n}$$

input
```
Integrate[x^(-1 - n)*Sqrt[c*ProductLog[a*x^n]],x]
```

output
```
-(((2/x^n + (a*Sqrt[Pi]*Erf[Sqrt[ProductLog[a*x^n]]])/Sqrt[ProductLog[a*x^
n]])*Sqrt[c*ProductLog[a*x^n]])/n)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7173, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}\sqrt{cW(ax^n)}dx$$

$$\Big\downarrow \text{7173}$$

$$-\frac{\int \frac{x^{-n-1}(cW(ax^n))^{3/2}}{W(ax^n)+1}dx}{c}-\frac{2x^{-n}\sqrt{cW(ax^n)}}{n}$$

$$\Big\downarrow \text{7203}$$

$$-\frac{\sqrt{\pi}a\sqrt{c}\text{erf}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{n}-\frac{2x^{-n}\sqrt{cW(ax^n)}}{n}$$

input
```
Int[x^(-1 - n)*Sqrt[c*ProductLog[a*x^n]],x]
```

output
```
-((a*Sqrt[c]*Sqrt[Pi]*Erf[Sqrt[c*ProductLog[a*x^n]]/Sqrt[c]])/n) - (2*Sqrt
[c*ProductLog[a*x^n]])/(n*x^n)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

Maple [F]

$$\int x^{-1-n}\sqrt{c\,\mathrm{LambertW}\left(a\,x^{n}\right)}dx$$

input
```
int(x^(-1-n)*(c*LambertW(a*x^n))^(1/2),x)
```

output
```
int(x^(-1-n)*(c*LambertW(a*x^n))^(1/2),x)
```

Fricas [F]

$$\int x^{-1-n}\sqrt{cW\left(ax^{n}\right)}\,dx=\int\sqrt{c\,\mathrm{W}(ax^{n})}x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(c*lambert_w(a*x^n))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(-n - 1), x)
```

Sympy [F]

$$\int x^{-1-n}\sqrt{cW\left(ax^{n}\right)}\,dx=\int x^{-n-1}\sqrt{cW\left(ax^{n}\right)}\,dx$$

input
```
integrate(x**(-1-n)*(c*LambertW(a*x**n))**(1/2),x)
```

output
```
Integral(x**(-n - 1)*sqrt(c*LambertW(a*x**n)), x)
```

Maxima [F]

$$\int x^{-1-n}\sqrt{cW(ax^n)}\,dx = \int \sqrt{c\,\mathrm{W}(ax^n)}x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(c*lambert_w(a*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^n))*x^(-n - 1), x)
```

Giac [F]

$$\int x^{-1-n}\sqrt{cW(ax^n)}\,dx = \int \sqrt{c\,\mathrm{W}(ax^n)}x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(c*lambert_w(a*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^n))*x^(-n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\sqrt{cW(ax^n)}\,dx = \int \frac{\sqrt{c\,\mathrm{LambertW}\left(a\,x^n\right)}}{x^{n+1}}\,dx$$

input
```
int((c*LambertW(a*x^n))^(1/2)/x^(n + 1),x)
```

output
```
int((c*LambertW(a*x^n))^(1/2)/x^(n + 1), x)
```

Reduce [F]

$$\int x^{-1-n}\sqrt{cW\left(ax^n\right)}\,dx$$
$$= \frac{\sqrt{c}\left(-2\sqrt{lambert\_w\left(x^n a\right)}+x^n\left(\int \frac{\sqrt{lambert\_w(x^n a)}}{e^{lambert\_w(x^n a)}lambert\_w(x^n a)^2 x+e^{lambert\_w(x^n a)}lambert\_w(x^n a)x}dx\right) an\right)}{2x^n n}$$

input
```
int(x^(-1-n)*(c*Lambert_W(a*x^n))^(1/2),x)
```

output
```
(sqrt(c)*( - 2*sqrt(lambert_w(x**n*a)) + x**n*int(sqrt(lambert_w(x**n*a))/
(e**lambert_w(x**n*a)*lambert_w(x**n*a)**2*x + e**lambert_w(x**n*a)*lamber
t_w(x**n*a)*x),x)*a*n))/(2*x**n*n)
```

3.258 $\int \frac{x^{-1-n}}{\sqrt{cW(ax^n)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{x^{-1-n}}{\sqrt{cW(ax^n)}}\,dx=-\frac{2a\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{3\sqrt{c}n}-\frac{2x^{-n}}{3n\sqrt{cW(ax^n)}}-\frac{2x^{-n}\sqrt{cW(ax^n)}}{3cn}$$

output
```
-2/3*a*Pi^(1/2)*erf((c*LambertW(a*x^n))^(1/2)/c^(1/2))/c^(1/2)/n-2/3/n/(x^
n)/(c*LambertW(a*x^n))^(1/2)-2/3*(c*LambertW(a*x^n))^(1/2)/c/n/(x^n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

$$\int \frac{x^{-1-n}}{\sqrt{cW(ax^n)}}\,dx=-\frac{2x^{-n}\left(1+a\sqrt{\pi}x^n\mathrm{erf}\left(\sqrt{W(ax^n)}\right)\sqrt{W(ax^n)}+W(ax^n)\right)}{3n\sqrt{cW(ax^n)}}$$

input
```
Integrate[x^(-1 - n)/Sqrt[c*ProductLog[a*x^n]],x]
```

output
```
(-2*(1 + a*Sqrt[Pi]*x^n*Erf[Sqrt[ProductLog[a*x^n]]]*Sqrt[ProductLog[a*x^n
]] + ProductLog[a*x^n]))/(3*n*x^n*Sqrt[c*ProductLog[a*x^n]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7173, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n-1}}{\sqrt{cW(ax^n)}}dx$$

$$\Big\downarrow \text{ 7173}$$

$$\frac{\int \frac{x^{-n-1}\sqrt{cW(ax^n)}}{W(ax^n)+1}dx}{3c}-\frac{2x^{-n}}{3n\sqrt{cW(ax^n)}}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{-\frac{2\int \frac{x^{-n-1}(cW(ax^n))^{3/2}}{W(ax^n)+1}dx}{c}-\frac{2x^{-n}\sqrt{cW(ax^n)}}{n}}{3c}-\frac{2x^{-n}}{3n\sqrt{cW(ax^n)}}$$

$$\Big\downarrow \text{ 7203}$$

$$\frac{-\frac{2\sqrt{\pi}a\sqrt{c}\text{erf}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{n}-\frac{2x^{-n}\sqrt{cW(ax^n)}}{n}}{3c}-\frac{2x^{-n}}{3n\sqrt{cW(ax^n)}}$$

input
```
Int[x^(-1 - n)/Sqrt[c*ProductLog[a*x^n]],x]
```

output
```
-2/(3*n*x^n*Sqrt[c*ProductLog[a*x^n]]) + ((-2*a*Sqrt[c]*Sqrt[Pi]*Erf[Sqrt[
c*ProductLog[a*x^n]]/Sqrt[c]])/n - (2*Sqrt[c*ProductLog[a*x^n]])/(n*x^n))/
(3*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{x^{-1-n}}{\sqrt{c\,\mathrm{LambertW}\left(a\,x^{n}\right)}}dx$$

input
```
int(x^(-1-n)/(c*LambertW(a*x^n))^(1/2),x)
```

output
```
int(x^(-1-n)/(c*LambertW(a*x^n))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^{-1-n}}{\sqrt{cW\left(ax^n\right)}}\,dx=\int \frac{x^{-n-1}}{\sqrt{c\,\mathrm{W}(ax^n)}}\,dx$$

input
```
integrate(x^(-1-n)/(c*lambert_w(a*x^n))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(-n - 1)/(c*lambert_w(a*x^n)), x)
```

Sympy [F]

$$\int \frac{x^{-1-n}}{\sqrt{cW\left(ax^n\right)}}\,dx=\int \frac{x^{-n-1}}{\sqrt{cW\left(ax^n\right)}}\,dx$$

input
```
integrate(x**(-1-n)/(c*LambertW(a*x**n))**(1/2),x)
```

output
```
Integral(x**(-n - 1)/sqrt(c*LambertW(a*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1-n}}{\sqrt{cW\left(ax^n\right)}}\,dx=\int \frac{x^{-n-1}}{\sqrt{c\,\mathrm{W}(ax^n)}}\,dx$$

input
```
integrate(x^(-1-n)/(c*lambert_w(a*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(-n - 1)/sqrt(c*lambert_w(a*x^n)), x)
```

Giac [F]

$$\int \frac{x^{-1-n}}{\sqrt{cW(ax^n)}}\,dx = \int \frac{x^{-n-1}}{\sqrt{c\,\mathrm{W}(ax^n)}}\,dx$$

input
```
integrate(x^(-1-n)/(c*lambert_w(a*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)/sqrt(c*lambert_w(a*x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{\sqrt{cW(ax^n)}}\,dx = \int \frac{1}{x^{n+1}\sqrt{c\,\mathrm{LambertW}(a\,x^n)}}\,dx$$

input
```
int(1/(x^(n + 1)*(c*LambertW(a*x^n))^(1/2)),x)
```

output
```
int(1/(x^(n + 1)*(c*LambertW(a*x^n))^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{-1-n}}{\sqrt{cW(ax^n)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(x^n a)}}{x^n lambert\_w(x^n a)x}dx\right)}{c}$$

input
```
int(x^(-1-n)/(c*Lambert_W(a*x^n))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(x**n*a))/(x**n*lambert_w(x**n*a)*x),x))/c
```

3.259 $\int \frac{x^{-1-n}}{(cW(ax^n))^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 116

$$\int \frac{x^{-1-n}}{(cW(ax^n))^{3/2}}\,dx = \frac{4a\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{5c^{3/2}n} - \frac{2x^{-n}}{5n(cW(ax^n))^{3/2}} - \frac{2x^{-n}}{5cn\sqrt{cW(ax^n)}} + \frac{4x^{-n}\sqrt{cW(ax^n)}}{5c^2n}$$

output
```
4/5*a*Pi^(1/2)*erf((c*LambertW(a*x^n))^(1/2)/c^(1/2))/c^(3/2)/n-2/5/n/(x^n
)/(c*LambertW(a*x^n))^(3/2)-2/5/c/n/(x^n)/(c*LambertW(a*x^n))^(1/2)+4/5*(c
*LambertW(a*x^n))^(1/2)/c^2/n/(x^n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

$$\int \frac{x^{-1-n}}{(cW(ax^n))^{3/2}}\,dx = \frac{x^{-n}\left(-2-2W(ax^n)+4a\sqrt{\pi}x^n\text{erf}\left(\sqrt{W(ax^n)}\right)W(ax^n)^{3/2}+4W(ax^n)^2\right)}{5n(cW(ax^n))^{3/2}}$$

input
```
Integrate[x^(-1 - n)/(c*ProductLog[a*x^n])^(3/2),x]
```

output
```
(-2 - 2*ProductLog[a*x^n] + 4*a*Sqrt[Pi]*x^n*Erf[Sqrt[ProductLog[a*x^n]]]*
ProductLog[a*x^n]^(3/2) + 4*ProductLog[a*x^n]^2)/(5*n*x^n*(c*ProductLog[a*
x^n])^(3/2))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7173, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n-1}}{(cW(ax^n))^{3/2}}dx$$

$$\Big\downarrow \text{ 7173}$$

$$\frac{3\int \frac{x^{-n-1}}{\sqrt{cW(ax^n)}(W(ax^n)+1)}dx}{5c}-\frac{2x^{-n}}{5n(cW(ax^n))^{3/2}}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{3\left(-\frac{2\int \frac{x^{-n-1}\sqrt{cW(ax^n)}}{W(ax^n)+1}dx}{3c}-\frac{2x^{-n}}{3n\sqrt{cW(ax^n)}}\right)}{5c}-\frac{2x^{-n}}{5n(cW(ax^n))^{3/2}}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{3\left(-\frac{2\left(-\frac{2\int \frac{x^{-n-1}(cW(ax^n))^{3/2}}{W(ax^n)+1}dx}{c}-\frac{2x^{-n}\sqrt{cW(ax^n)}}{n}\right)}{3c}-\frac{2x^{-n}}{3n\sqrt{cW(ax^n)}}\right)}{5c}-\frac{2x^{-n}}{5n(cW(ax^n))^{3/2}}$$

$$\Big\downarrow \text{ 7203}$$

$$\frac{3\left(-\frac{2\left(-\frac{\frac{2\sqrt{\pi}a\sqrt{c}\operatorname{erf}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{n}}{}-\frac{2x^{-n}\sqrt{cW(ax^n)}}{n}\right)}{3c}-\frac{2x^{-n}}{3n\sqrt{cW(ax^n)}}\right)}{5c}-\frac{2x^{-n}}{5n\left(cW\left(ax^n\right)\right)^{3/2}}$$

input
```
Int[x^(-1 - n)/(c*ProductLog[a*x^n])^(3/2),x]
```

output
```
-2/(5*n*x^n*(c*ProductLog[a*x^n])^(3/2)) + (3*(-2/(3*n*x^n*Sqrt[c*ProductL
og[a*x^n]]) - (2*((-2*a*Sqrt[c]*Sqrt[Pi]*Erf[Sqrt[c*ProductLog[a*x^n]]/Sqr
t[c]])/n - (2*Sqrt[c*ProductLog[a*x^n]])/(n*x^n)))/(3*c)))/(5*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{x^{-1-n}}{(c \,\mathrm{LambertW}\,(a\,x^n))^{\frac{3}{2}}}dx$$

input
```
int(x^(-1-n)/(c*LambertW(a*x^n))^(3/2),x)
```

output
```
int(x^(-1-n)/(c*LambertW(a*x^n))^(3/2),x)
```

Fricas [F]

$$\int \frac{x^{-1-n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{x^{-n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1-n)/(c*lambert_w(a*x^n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(-n - 1)/(c^2*lambert_w(a*x^n)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{(cW(ax^n))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n)/(c*LambertW(a*x**n))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{x^{-n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1-n)/(c*lambert_w(a*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(-n - 1)/(c*lambert_w(a*x^n))^(3/2), x)
```

Giac [F]

$$\int \frac{x^{-1-n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{x^{-n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1-n)/(c*lambert_w(a*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)/(c*lambert_w(a*x^n))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{1}{x^{n+1}\,(c\,\mathrm{LambertW}\,(a\,x^n))^{3/2}}\,dx$$

input
```
int(1/(x^(n + 1)*(c*LambertW(a*x^n))^(3/2)),x)
```

output
```
int(1/(x^(n + 1)*(c*LambertW(a*x^n))^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^{-1-n}}{(cW(ax^n))^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(x^n a)}}{x^n lambert\_w(x^n a)^2 x}dx\right)}{c^2}$$

input
```
int(x^(-1-n)/(c*Lambert_W(a*x^n))^(3/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(x**n*a))/(x**n*lambert_w(x**n*a)**2*x),x))/c**
2
```

3.260 $\int \frac{x^{-1-n}}{(cW(ax^n))^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \frac{x^{-1-n}}{(cW(ax^n))^{5/2}}\,dx = -\frac{8a\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{21c^{5/2}n} - \frac{2x^{-n}}{7n(cW(ax^n))^{5/2}} - \frac{2x^{-n}}{7cn(cW(ax^n))^{3/2}} + \frac{4x^{-n}}{21c^2n\sqrt{cW(ax^n)}} - \frac{8x^{-n}\sqrt{cW(ax^n)}}{21c^3n}$$

output
```
-8/21*a*Pi^(1/2)*erf((c*LambertW(a*x^n))^(1/2)/c^(1/2))/c^(5/2)/n-2/7/n/(x
^n)/(c*LambertW(a*x^n))^(5/2)-2/7/c/n/(x^n)/(c*LambertW(a*x^n))^(3/2)+4/21
/c^2/n/(x^n)/(c*LambertW(a*x^n))^(1/2)-8/21*(c*LambertW(a*x^n))^(1/2)/c^3/
n/(x^n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.60

$$\int \frac{x^{-1-n}}{(cW(ax^n))^{5/2}}\,dx = -\frac{2x^{-n}\left(3+3W(ax^n)-2W(ax^n)^2+4a\sqrt{\pi}x^n\mathrm{erf}\left(\sqrt{W(ax^n)}\right)W(ax^n)^{5/2}+4W(ax^n)^3\right)}{21n(cW(ax^n))^{5/2}}$$

input
```
Integrate[x^(-1 - n)/(c*ProductLog[a*x^n])^(5/2),x]
```

output
```
(-2*(3 + 3*ProductLog[a*x^n] - 2*ProductLog[a*x^n]^2 + 4*a*Sqrt[Pi]*x^n*Er
f[Sqrt[ProductLog[a*x^n]]]*ProductLog[a*x^n]^(5/2) + 4*ProductLog[a*x^n]^3
))/(21*n*x^n*(c*ProductLog[a*x^n])^(5/2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7173, 7206, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n-1}}{(cW(ax^n))^{5/2}}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{5\int \frac{x^{-n-1}}{(cW(ax^n))^{3/2}(W(ax^n)+1)}dx}{7c}-\frac{2x^{-n}}{7n(cW(ax^n))^{5/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{5\left(-\frac{2\int \frac{x^{-n-1}}{\sqrt{cW(ax^n)}(W(ax^n)+1)}dx}{5c}-\frac{2x^{-n}}{5n(cW(ax^n))^{3/2}}\right)}{7c}-\frac{2x^{-n}}{7n(cW(ax^n))^{5/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{5\left(-\frac{2\left(-\frac{2\int \frac{x^{-n-1}\sqrt{cW(ax^n)}}{W(ax^n)+1}dx}{3c}-\frac{2x^{-n}}{3n\sqrt{cW(ax^n)}}\right)}{5c}-\frac{2x^{-n}}{5n(cW(ax^n))^{3/2}}\right)}{7c}-\frac{2x^{-n}}{7n(cW(ax^n))^{5/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{5\left(-\frac{2\left(-\frac{2\left(-\frac{2\int \frac{x^{-n-1}(cW(ax^n))^{3/2}}{W(ax^n)+1}dx}{c}-\frac{2x^{-n}\sqrt{cW(ax^n)}}{n}\right)}{3c}-\frac{2x^{-n}}{3n\sqrt{cW(ax^n)}}\right)}{5c}-\frac{2x^{-n}}{5n(cW(ax^n))^{3/2}}\right)}{7c}-\frac{2x^{-n}}{7n\left(cW\left(ax^n\right)\right)^{5/2}}$$

$$\Big\downarrow \; 7203$$

$$\frac{5\left(-\frac{2\left(-\frac{2\left(-\frac{2\sqrt{\pi}a\sqrt{c}\,\text{erf}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{n}-\frac{2x^{-n}\sqrt{cW(ax^n)}}{n}\right)}{3c}-\frac{2x^{-n}}{3n\sqrt{cW(ax^n)}}\right)}{5c}-\frac{2x^{-n}}{5n(cW(ax^n))^{3/2}}\right)}{7c}-\frac{2x^{-n}}{7n\left(cW\left(ax^n\right)\right)^{5/2}}$$

input
```
Int[x^(-1 - n)/(c*ProductLog[a*x^n])^(5/2),x]
```

output
```
-2/(7*n*x^n*(c*ProductLog[a*x^n])^(5/2)) + (5*(-2/(5*n*x^n*(c*ProductLog[a
*x^n])^(3/2)) - (2*(-2/(3*n*x^n*Sqrt[c*ProductLog[a*x^n]]) - (2*((-2*a*Sqr
t[c]*Sqrt[Pi]*Erf[Sqrt[c*ProductLog[a*x^n]]/Sqrt[c]])/n - (2*Sqrt[c*Produc
tLog[a*x^n]])/(n*x^n)))/(3*c)))/(5*c)))/(7*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{x^{-1-n}}{(c\,\mathrm{LambertW}\,(a\,x^n))^{\frac{5}{2}}}dx$$

input
```
int(x^(-1-n)/(c*LambertW(a*x^n))^(5/2),x)
```

output
```
int(x^(-1-n)/(c*LambertW(a*x^n))^(5/2),x)
```

Fricas [F]

$$\int \frac{x^{-1-n}}{(cW(ax^n))^{5/2}}\,dx = \int \frac{x^{-n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(-1-n)/(c*lambert_w(a*x^n))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(-n - 1)/(c^3*lambert_w(a*x^n)^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{(cW(ax^n))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n)/(c*LambertW(a*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-n}}{(cW(ax^n))^{5/2}}\,dx = \int \frac{x^{-n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(-1-n)/(c*lambert_w(a*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^(-n - 1)/(c*lambert_w(a*x^n))^(5/2), x)
```

Giac [F]

$$\int \frac{x^{-1-n}}{(cW(ax^n))^{5/2}}\,dx = \int \frac{x^{-n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(-1-n)/(c*lambert_w(a*x^n))^(5/2),x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)/(c*lambert_w(a*x^n))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{(cW(ax^n))^{5/2}}\,dx = \int \frac{1}{x^{n+1}\,(c\,\mathrm{LambertW}\,(a\,x^n))^{5/2}}\,dx$$

input
```
int(1/(x^(n + 1)*(c*LambertW(a*x^n))^(5/2)),x)
```

output
```
int(1/(x^(n + 1)*(c*LambertW(a*x^n))^(5/2)), x)
```

Reduce [F]

$$\int \frac{x^{-1-n}}{(cW(ax^n))^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(x^n a)}}{x^n lambert\_w(x^n a)^3 x}dx\right)}{c^3}$$

input
```
int(x^(-1-n)/(c*Lambert_W(a*x^n))^(5/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(x**n*a))/(x**n*lambert_w(x**n*a)**3*x),x))/c**
3
```

3.261 $\int x^{-1-2n}(cW(ax^n))^{11/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 152

$$\int x^{-1-2n}(cW(ax^n))^{11/2}\,dx = \frac{165a^2c^{11/2}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{256n} - \frac{165c^3x^{-2n}(cW(ax^n))^{5/2}}{128n} - \frac{55c^2x^{-2n}(cW(ax^n))^{7/2}}{32n} - \frac{11cx^{-2n}(cW(ax^n))^{9/2}}{8n} - \frac{x^{-2n}(cW(ax^n))^{11/2}}{2n}$$

output

```
165/512*a^2*c^(11/2)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(c*LambertW(a*x^n))^(1/2
)/c^(1/2))/n-165/128*c^3*(c*LambertW(a*x^n))^(5/2)/n/(x^(2*n))-55/32*c^2*(
c*LambertW(a*x^n))^(7/2)/n/(x^(2*n))-11/8*c*(c*LambertW(a*x^n))^(9/2)/n/(x
^(2*n))-1/2*(c*LambertW(a*x^n))^(11/2)/n/(x^(2*n))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.79

$$\int x^{-1-2n}(cW(ax^n))^{11/2}\,dx =$$

$$-\frac{c^5x^{-2n}\sqrt{cW(ax^n)}\left(-165a^2\sqrt{2\pi}x^{2n}\text{erf}\left(\sqrt{2}\sqrt{W(ax^n)}\right)+660W(ax^n)^{5/2}+880W(ax^n)^{7/2}+704W(ax^n)\right.}{512n\sqrt{W(ax^n)}}$$

input
```
Integrate[x^(-1 - 2*n)*(c*ProductLog[a*x^n])^(11/2),x]
```

output
```
-1/512*(c^5*Sqrt[c*ProductLog[a*x^n]]*(-165*a^2*Sqrt[2*Pi]*x^(2*n)*Erf[Sqr
t[2]*Sqrt[ProductLog[a*x^n]]] + 660*ProductLog[a*x^n]^(5/2) + 880*ProductL
og[a*x^n]^(7/2) + 704*ProductLog[a*x^n]^(9/2) + 256*ProductLog[a*x^n]^(11/
2)))/(n*x^(2*n)*Sqrt[ProductLog[a*x^n]])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7172, 7205, 7205, 7205, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}(cW(ax^n))^{11/2}\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{11}{4}\int \frac{x^{-2n-1}(cW(ax^n))^{11/2}}{W(ax^n)+1}dx-\frac{x^{-2n}(cW(ax^n))^{11/2}}{2n}$$

$$\Big\downarrow \text{7205}$$

$$\frac{11}{4}\left(\frac{5}{4}c\int \frac{x^{-2n-1}(cW(ax^n))^{9/2}}{W(ax^n)+1}dx-\frac{cx^{-2n}(cW(ax^n))^{9/2}}{2n}\right)-\frac{x^{-2n}(cW(ax^n))^{11/2}}{2n}$$

$$\Big\downarrow \text{7205}$$

$$\frac{11}{4}\left(\frac{5}{4}c\left(\frac{3}{4}c\int \frac{x^{-2n-1}(cW(ax^n))^{7/2}}{W(ax^n)+1}dx-\frac{cx^{-2n}(cW(ax^n))^{7/2}}{2n}\right)-\frac{cx^{-2n}(cW(ax^n))^{9/2}}{2n}\right)-\frac{x^{-2n}(cW(ax^n))^{11/2}}{2n}$$

$$\Big\downarrow \text{7205}$$

$$\frac{11}{4}\left(\frac{5}{4}c\left(\frac{3}{4}c\left(\frac{1}{4}c\int \frac{x^{-2n-1}(cW(ax^n))^{5/2}}{W(ax^n)+1}dx-\frac{cx^{-2n}(cW(ax^n))^{5/2}}{2n}\right)-\frac{cx^{-2n}(cW(ax^n))^{7/2}}{2n}\right)-\frac{cx^{-2n}(cW(ax^n}{2n}\right.\ldots\frac{x^{-2n}(cW(ax^n))^{11/2}}{2n}$$

$$\Big\downarrow \text{7203}$$

$$\frac{11}{4}\left(\frac{5}{4}c\left(\frac{3}{4}c\left(\frac{\sqrt{\frac{\pi}{2}}a^2c^{7/2}\text{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{4n}-\frac{cx^{-2n}(cW(ax^n))^{5/2}}{2n}\right)-\frac{cx^{-2n}(cW(ax^n))^{7/2}}{2n}\right)-\frac{cx^{-2n}(cW(ax^n)}{2n}\right.\ldots\frac{x^{-2n}(cW(ax^n))^{11/2}}{2n}$$

input
```
Int[x^(-1 - 2*n)*(c*ProductLog[a*x^n])^(11/2),x]
```

output
```
-1/2*(c*ProductLog[a*x^n])^(11/2)/(n*x^(2*n)) + (11*(-1/2*(c*(c*ProductLog
[a*x^n])^(9/2))/(n*x^(2*n)) + (5*c*(-1/2*(c*(c*ProductLog[a*x^n])^(7/2))/(
n*x^(2*n)) + (3*c*((a^2*c^(7/2)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[c*ProductLog[
a*x^n]])/Sqrt[c]])/(4*n) - (c*(c*ProductLog[a*x^n])^(5/2))/(2*n*x^(2*n))))
/4))/4))/4
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^{-1-2n}(c\,\mathrm{LambertW}\left(a\,x^{n}\right))^{\frac{11}{2}}dx$$

input
```
int(x^(-1-2*n)*(c*LambertW(a*x^n))^(11/2),x)
```

output
```
int(x^(-1-2*n)*(c*LambertW(a*x^n))^(11/2),x)
```

Fricas [F]

$$\int x^{-1-2n}(cW(ax^n))^{11/2}\,dx = \int \left(c\,\mathrm{W}(ax^{n})\right)^{\frac{11}{2}} x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(11/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*c^5*x^(-2*n - 1)*lambert_w(a*x^n)^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-2n}(cW(ax^n))^{11/2}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-2*n)*(c*LambertW(a*x**n))**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-2n}(cW(ax^n))^{11/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{11}{2}}\,x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(11/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^(11/2)*x^(-2*n - 1), x)
```

Giac [F]

$$\int x^{-1-2n}(cW(ax^n))^{11/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{11}{2}}\,x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(11/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^(11/2)*x^(-2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}(cW(ax^n))^{11/2}\,dx=\int\frac{\left(c\,\mathrm{LambertW}(a\,x^n)\right)^{11/2}}{x^{2\,n+1}}\,dx$$

input
```
int((c*LambertW(a*x^n))^(11/2)/x^(2*n + 1),x)
```

output
```
int((c*LambertW(a*x^n))^(11/2)/x^(2*n + 1), x)
```

Reduce [F]

$$\int x^{-1-2n}(cW(ax^n))^{11/2}\,dx=\frac{\sqrt{c}\,c^5\left(-2\sqrt{lambert\_w(x^na)}\,lambert\_w(x^na)^5+11x^{2n}\left(\int\frac{\sqrt{lamber}}{x^ne^{lambert\_w(x^na)}l}\right.\right.}{4x^{2n}n}$$

input
```
int(x^(-1-2*n)*(c*Lambert_W(a*x^n))^(11/2),x)
```

output
```
(sqrt(c)*c**5*( - 2*sqrt(lambert_w(x**n*a))*lambert_w(x**n*a)**5 + 11*x**(
2*n)*int((sqrt(lambert_w(x**n*a))*lambert_w(x**n*a)**4)/(x**n*e**lambert_w
(x**n*a)*lambert_w(x**n*a)*x + x**n*e**lambert_w(x**n*a)*x),x)*a*n))/(4*x*
*(2*n)*n)
```

3.262 $\int x^{-1-2n}(cW(ax^n))^{9/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 125

$$\int x^{-1-2n}(cW(ax^n))^{9/2}\,dx=\frac{27a^2c^{9/2}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{64n}-\frac{27c^2x^{-2n}(cW(ax^n))^{5/2}}{32n}-\frac{9cx^{-2n}(cW(ax^n))^{7/2}}{8n}-\frac{x^{-2n}(cW(ax^n))^{9/2}}{2n}$$

output
```
27/128*a^2*c^(9/2)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(c*LambertW(a*x^n))^(1/2)/
c^(1/2))/n-27/32*c^2*(c*LambertW(a*x^n))^(5/2)/n/(x^(2*n))-9/8*c*(c*Lamber
tW(a*x^n))^(7/2)/n/(x^(2*n))-1/2*(c*LambertW(a*x^n))^(9/2)/n/(x^(2*n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int x^{-1-2n}(cW(ax^n))^{9/2}\,dx = -\frac{c^4x^{-2n}\sqrt{cW(ax^n)}\left(-27a^2\sqrt{2\pi}x^{2n}\operatorname{erf}\left(\sqrt{2}\sqrt{W(ax^n)}\right)+108W(ax^n)^{5/2}+144W(ax^n)^{7/2}+64W(ax^n)^{9/2}\right)}{128n\sqrt{W(ax^n)}}$$

input
```
Integrate[x^(-1 - 2*n)*(c*ProductLog[a*x^n])^(9/2),x]
```

output
```
-1/128*(c^4*Sqrt[c*ProductLog[a*x^n]]*(-27*a^2*Sqrt[2*Pi]*x^(2*n)*Erf[Sqrt
[2]*Sqrt[ProductLog[a*x^n]]] + 108*ProductLog[a*x^n]^(5/2) + 144*ProductLo
g[a*x^n]^(7/2) + 64*ProductLog[a*x^n]^(9/2)))/(n*x^(2*n)*Sqrt[ProductLog[a
*x^n]])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7172, 7205, 7205, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}(cW(ax^n))^{9/2}\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{9}{4}\int \frac{x^{-2n-1}(cW(ax^n))^{9/2}}{W(ax^n)+1}dx-\frac{x^{-2n}(cW(ax^n))^{9/2}}{2n}$$

$$\Big\downarrow \text{7205}$$

$$\frac{9}{4}\left(\frac{3}{4}c\int \frac{x^{-2n-1}(cW(ax^n))^{7/2}}{W(ax^n)+1}dx-\frac{cx^{-2n}(cW(ax^n))^{7/2}}{2n}\right)-\frac{x^{-2n}(cW(ax^n))^{9/2}}{2n}$$

$$\Big\downarrow \text{7205}$$

$$\frac{9}{4}\left(\frac{3}{4}c\left(\frac{1}{4}c\int \frac{x^{-2n-1}(cW(ax^n))^{5/2}}{W(ax^n)+1}dx-\frac{cx^{-2n}(cW(ax^n))^{5/2}}{2n}\right)-\frac{cx^{-2n}(cW(ax^n))^{7/2}}{2n}\right)-\frac{x^{-2n}(cW(ax^n))^{9/2}}{2n}$$

$$\Big\downarrow \text{7203}$$

$$\frac{9}{4}\left(\frac{3}{4}c\left(\frac{\sqrt{\frac{\pi}{2}}a^2c^{7/2}\text{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{4n}-\frac{cx^{-2n}(cW(ax^n))^{5/2}}{2n}\right)-\frac{cx^{-2n}(cW(ax^n))^{7/2}}{2n}\right)-\frac{x^{-2n}(cW(ax^n))^{9/2}}{2n}$$

input
```
Int[x^(-1 - 2*n)*(c*ProductLog[a*x^n])^(9/2),x]
```

output
```
-1/2*(c*ProductLog[a*x^n])^(9/2)/(n*x^(2*n)) + (9*(-1/2*(c*(c*ProductLog[a
*x^n])^(7/2))/(n*x^(2*n)) + (3*c*((a^2*c^(7/2)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqr
t[c*ProductLog[a*x^n]])/Sqrt[c]])/(4*n) - (c*(c*ProductLog[a*x^n])^(5/2))/
(2*n*x^(2*n))))/4))/4
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^{-1-2n}(c\,\mathrm{LambertW}\,(a\,x^{n}))^{\frac{9}{2}}dx$$

input
```
int(x^(-1-2*n)*(c*LambertW(a*x^n))^(9/2),x)
```

output
```
int(x^(-1-2*n)*(c*LambertW(a*x^n))^(9/2),x)
```

Fricas [F]

$$\int x^{-1-2n}(cW(ax^{n}))^{9/2}\,dx = \int (c\,\mathrm{W}(ax^{n}))^{\frac{9}{2}}\,x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(9/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*c^4*x^(-2*n - 1)*lambert_w(a*x^n)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-2n}(cW(ax^{n}))^{9/2}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-2*n)*(c*LambertW(a*x**n))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-2n}(cW(ax^n))^{9/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{9}{2}}\,x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(9/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^(9/2)*x^(-2*n - 1), x)
```

Giac [F]

$$\int x^{-1-2n}(cW(ax^n))^{9/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{9}{2}}\,x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(9/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^(9/2)*x^(-2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}(cW(ax^n))^{9/2}\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x^n))^{9/2}}{x^{2\,n+1}}\,dx$$

input
```
int((c*LambertW(a*x^n))^(9/2)/x^(2*n + 1),x)
```

output
```
int((c*LambertW(a*x^n))^(9/2)/x^(2*n + 1), x)
```

Reduce [F]

$$\int x^{-1-2n}(cW(ax^n))^{9/2}\,dx = \frac{\sqrt{c}\,c^4\left(-2\sqrt{lambert\_w\left(x^n a\right)}\,lambert\_w(x^n a)^4 + 9x^{2n}\left(\int \frac{\sqrt{lambert\_}}{x^n e^{lambert\_w(x^n a)}\,lam}\right.\right.}{4x^{2n}n}$$

input
```
int(x^(-1-2*n)*(c*Lambert_W(a*x^n))^(9/2),x)
```

output
```
(sqrt(c)*c**4*( - 2*sqrt(lambert_w(x**n*a))*lambert_w(x**n*a)**4 + 9*x**(2
*n)*int((sqrt(lambert_w(x**n*a))*lambert_w(x**n*a)**3)/(x**n*e**lambert_w(
x**n*a)*lambert_w(x**n*a)*x + x**n*e**lambert_w(x**n*a)*x),x)*a*n))/(4*x**
(2*n)*n)
```

3.263 $\int x^{-1-2n}(cW(ax^n))^{7/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int x^{-1-2n}(cW(ax^n))^{7/2}\,dx=\frac{7a^2c^{7/2}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{16n}-\frac{7cx^{-2n}(cW(ax^n))^{5/2}}{8n}-\frac{x^{-2n}(cW(ax^n))^{7/2}}{2n}$$

output
```
7/32*a^2*c^(7/2)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(c*LambertW(a*x^n))^(1/2)/c^
(1/2))/n-7/8*c*(c*LambertW(a*x^n))^(5/2)/n/(x^(2*n))-1/2*(c*LambertW(a*x^n
))^(7/2)/n/(x^(2*n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98

$$\int x^{-1-2n}(cW(ax^n))^{7/2}\,dx=-\frac{c^3x^{-2n}\sqrt{cW(ax^n)}\left(-7a^2\sqrt{2\pi}x^{2n}\text{erf}\left(\sqrt{2}\sqrt{W(ax^n)}\right)+28W(ax^n)^{5/2}+16W(ax^n)^{7/2}\right)}{32n\sqrt{W(ax^n)}}$$

input
```
Integrate[x^(-1 - 2*n)*(c*ProductLog[a*x^n])^(7/2),x]
```

output
```
-1/32*(c^3*Sqrt[c*ProductLog[a*x^n]]*(-7*a^2*Sqrt[2*Pi]*x^(2*n)*Erf[Sqrt[2
]*Sqrt[ProductLog[a*x^n]]] + 28*ProductLog[a*x^n]^(5/2) + 16*ProductLog[a*
x^n]^(7/2)))/(n*x^(2*n)*Sqrt[ProductLog[a*x^n]])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7172, 7205, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}(cW(ax^n))^{7/2}\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{7}{4}\int \frac{x^{-2n-1}(cW(ax^n))^{7/2}}{W(ax^n)+1}dx - \frac{x^{-2n}(cW(ax^n))^{7/2}}{2n}$$

$$\Big\downarrow \text{7205}$$

$$\frac{7}{4}\left(\frac{1}{4}c\int \frac{x^{-2n-1}(cW(ax^n))^{5/2}}{W(ax^n)+1}dx - \frac{cx^{-2n}(cW(ax^n))^{5/2}}{2n}\right) - \frac{x^{-2n}(cW(ax^n))^{7/2}}{2n}$$

$$\Big\downarrow \text{7203}$$

$$\frac{7}{4}\left(\frac{\sqrt{\frac{\pi}{2}}a^2c^{7/2}\text{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{4n} - \frac{cx^{-2n}(cW(ax^n))^{5/2}}{2n}\right) - \frac{x^{-2n}(cW(ax^n))^{7/2}}{2n}$$

input
```
Int[x^(-1 - 2*n)*(c*ProductLog[a*x^n])^(7/2),x]
```

output
```
-1/2*(c*ProductLog[a*x^n])^(7/2)/(n*x^(2*n)) + (7*((a^2*c^(7/2)*Sqrt[Pi/2]
*Erf[(Sqrt[2]*Sqrt[c*ProductLog[a*x^n]])/Sqrt[c]])/(4*n) - (c*(c*ProductLo
g[a*x^n])^(5/2))/(2*n*x^(2*n))))/4
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^{-1-2n} (c \operatorname{LambertW}(a x^n))^{\frac{7}{2}} dx$$

input
```
int(x^(-1-2*n)*(c*LambertW(a*x^n))^(7/2),x)
```

output
```
int(x^(-1-2*n)*(c*LambertW(a*x^n))^(7/2),x)
```

Fricas [F]

$$\int x^{-1-2n}(cW(ax^n))^{7/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{7}{2}}\,x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*c^3*x^(-2*n - 1)*lambert_w(a*x^n)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-2n}(cW(ax^n))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-2*n)*(c*LambertW(a*x**n))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-2n}(cW(ax^n))^{7/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{7}{2}}\,x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^(7/2)*x^(-2*n - 1), x)
```

Giac [F]

$$\int x^{-1-2n}(cW(ax^n))^{7/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{7}{2}}x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(7/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^(7/2)*x^(-2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}(cW(ax^n))^{7/2}\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x^n))^{7/2}}{x^{2\,n+1}}\,dx$$

input
```
int((c*LambertW(a*x^n))^(7/2)/x^(2*n + 1),x)
```

output
```
int((c*LambertW(a*x^n))^(7/2)/x^(2*n + 1), x)
```

Reduce [F]

$$\int x^{-1-2n}(cW(ax^n))^{7/2}\,dx = \frac{\sqrt{c}\,c^3\left(-2\sqrt{lambert\_w(x^na)}\,lambert\_w(x^na)^3 + 7x^{2n}\left(\int \frac{\sqrt{lambert\_}}{x^n e^{lambert\_w(x^na)}\,lan}\right.\right.}{4x^{2n}n}$$

input
```
int(x^(-1-2*n)*(c*Lambert_W(a*x^n))^(7/2),x)
```

output
```
(sqrt(c)*c**3*( - 2*sqrt(lambert_w(x**n*a))*lambert_w(x**n*a)**3 + 7*x**(2
*n)*int((sqrt(lambert_w(x**n*a))*lambert_w(x**n*a)**2)/(x**n*e**lambert_w(
x**n*a)*lambert_w(x**n*a)*x + x**n*e**lambert_w(x**n*a)*x),x)*a*n))/(4*x**
(2*n)*n)
```

3.264 $\int x^{-1-2n}(cW(ax^n))^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int x^{-1-2n}(cW(ax^n))^{5/2}\,dx=\frac{5a^2c^{5/2}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{4n}-\frac{x^{-2n}(cW(ax^n))^{5/2}}{2n}$$

output
```
5/8*a^2*c^(5/2)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(c*LambertW(a*x^n))^(1/2)/c^(
1/2))/n-1/2*(c*LambertW(a*x^n))^(5/2)/n/(x^(2*n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int x^{-1-2n}(cW(ax^n))^{5/2}\,dx=\frac{\left(-4x^{-2n}+\frac{5a^2\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{W(ax^n)}\right)}{W(ax^n)^{5/2}}\right)(cW(ax^n))^{5/2}}{8n}$$

input
```
Integrate[x^(-1 - 2*n)*(c*ProductLog[a*x^n])^(5/2),x]
```

output
```
((-4/x^(2*n) + (5*a^2*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ProductLog[a*x^n]]])/Pro
ductLog[a*x^n]^(5/2))*(c*ProductLog[a*x^n])^(5/2))/(8*n)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7172, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}(cW(ax^n))^{5/2}\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{5}{4}\int \frac{x^{-2n-1}(cW(ax^n))^{5/2}}{W(ax^n)+1}dx-\frac{x^{-2n}(cW(ax^n))^{5/2}}{2n}$$

$$\Big\downarrow \text{7203}$$

$$\frac{5\sqrt{\frac{\pi}{2}}a^2c^{5/2}\text{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{4n}-\frac{x^{-2n}(cW(ax^n))^{5/2}}{2n}$$

input
```
Int[x^(-1 - 2*n)*(c*ProductLog[a*x^n])^(5/2),x]
```

output
```
(5*a^2*c^(5/2)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[c*ProductLog[a*x^n]])/Sqrt[c]]
)/(4*n) - (c*ProductLog[a*x^n])^(5/2)/(2*n*x^(2*n))
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

Maple [F]

$$\int x^{-1-2n}(c\,\mathrm{LambertW}\left(a\,x^{n}\right))^{\frac{5}{2}}dx$$

input
```
int(x^(-1-2*n)*(c*LambertW(a*x^n))^(5/2),x)
```

output
```
int(x^(-1-2*n)*(c*LambertW(a*x^n))^(5/2),x)
```

Fricas [F]

$$\int x^{-1-2n}(cW(ax^n))^{5/2}\,dx = \int \left(c\,\mathrm{W}(ax^{n})\right)^{\frac{5}{2}}x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*c^2*x^(-2*n - 1)*lambert_w(a*x^n)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-2n}(cW(ax^n))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-2*n)*(c*LambertW(a*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-2n}(cW(ax^n))^{5/2}\,dx=\int (c\,\mathrm{W}(ax^n))^{\frac{5}{2}}\,x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^(5/2)*x^(-2*n - 1), x)
```

Giac [F]

$$\int x^{-1-2n}(cW(ax^n))^{5/2}\,dx=\int (c\,\mathrm{W}(ax^n))^{\frac{5}{2}}\,x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^(5/2)*x^(-2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}(cW(ax^n))^{5/2}\,dx=\int \frac{(c\,\mathrm{LambertW}(a\,x^n))^{5/2}}{x^{2\,n+1}}\,dx$$

input
```
int((c*LambertW(a*x^n))^(5/2)/x^(2*n + 1),x)
```

output
```
int((c*LambertW(a*x^n))^(5/2)/x^(2*n + 1), x)
```

Reduce [F]

$$\int x^{-1-2n}(cW(ax^n))^{5/2}\,dx = \frac{\sqrt{c}\,c^2\left(-2\sqrt{lambert\_w(x^n a)}\,lambert\_w(x^n a)^2 + 5x^{2n}\left(\int \frac{\sqrt{lambert\_}}{x^n e^{lambert\_w(x^n a)}\,lam}\right.\right.}{4x^{2n}n}$$

input
```
int(x^(-1-2*n)*(c*Lambert_W(a*x^n))^(5/2),x)
```

output
```
(sqrt(c)*c**2*( - 2*sqrt(lambert_w(x**n*a))*lambert_w(x**n*a)**2 + 5*x**(2
*n)*int((sqrt(lambert_w(x**n*a))*lambert_w(x**n*a))/(x**n*e**lambert_w(x**
n*a)*lambert_w(x**n*a)*x + x**n*e**lambert_w(x**n*a)*x),x)*a*n))/(4*x**(2*
n)*n)
```

3.265 $\int x^{-1-2n}(cW(ax^n))^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int x^{-1-2n}(cW(ax^n))^{3/2}\,dx=-\frac{3a^2c^{3/2}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{n}-\frac{2x^{-2n}(cW(ax^n))^{3/2}}{n}$$

output
```
-3/2*a^2*c^(3/2)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(c*LambertW(a*x^n))^(1/2)/c^
(1/2))/n-2*(c*LambertW(a*x^n))^(3/2)/n/(x^(2*n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int x^{-1-2n}(cW(ax^n))^{3/2}\,dx=\frac{\left(-4x^{-2n}-\frac{3a^2\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{W(ax^n)}\right)}{W(ax^n)^{3/2}}\right)(cW(ax^n))^{3/2}}{2n}$$

input
```
Integrate[x^(-1 - 2*n)*(c*ProductLog[a*x^n])^(3/2),x]
```

output
```
((-4/x^(2*n) - (3*a^2*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ProductLog[a*x^n]]])/Pro
ductLog[a*x^n]^(3/2))*(c*ProductLog[a*x^n])^(3/2))/(2*n)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7173, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}(cW(ax^n))^{3/2}\,dx$$
$$\Big\downarrow \text{7173}$$
$$-\frac{3\int \frac{x^{-2n-1}(cW(ax^n))^{5/2}}{W(ax^n)+1}dx}{c}-\frac{2x^{-2n}(cW(ax^n))^{3/2}}{n}$$
$$\Big\downarrow \text{7203}$$
$$-\frac{3\sqrt{\frac{\pi}{2}}a^2c^{3/2}\text{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{n}-\frac{2x^{-2n}(cW(ax^n))^{3/2}}{n}$$

input
```
Int[x^(-1 - 2*n)*(c*ProductLog[a*x^n])^(3/2),x]
```

output
```
(-3*a^2*c^(3/2)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[c*ProductLog[a*x^n]])/Sqrt[c]
])/n - (2*(c*ProductLog[a*x^n])^(3/2))/(n*x^(2*n))
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

Maple [F]

$$\int x^{-1-2n}(c\,\mathrm{LambertW}\,(a\,x^{n}))^{\frac{3}{2}}dx$$

input
```
int(x^(-1-2*n)*(c*LambertW(a*x^n))^(3/2),x)
```

output
```
int(x^(-1-2*n)*(c*LambertW(a*x^n))^(3/2),x)
```

Fricas [F]

$$\int x^{-1-2n}(cW(ax^n))^{3/2}\,dx=\int (c\,\mathrm{W}(ax^n))^{\frac{3}{2}}\,x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*c*x^(-2*n - 1)*lambert_w(a*x^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-2n}(cW(ax^n))^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x**(-1-2*n)*(c*LambertW(a*x**n))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-2n}(cW(ax^n))^{3/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{3}{2}}\,x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^(3/2)*x^(-2*n - 1), x)
```

Giac [F]

$$\int x^{-1-2n}(cW(ax^n))^{3/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{3}{2}}\,x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^(3/2)*x^(-2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}(cW(ax^n))^{3/2}\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x^n))^{3/2}}{x^{2\,n+1}}\,dx$$

input
```
int((c*LambertW(a*x^n))^(3/2)/x^(2*n + 1),x)
```

output
```
int((c*LambertW(a*x^n))^(3/2)/x^(2*n + 1), x)
```

Reduce [F]

$$\int x^{-1-2n}(cW(ax^n))^{3/2}\,dx = \frac{\sqrt{c}\,c\left(-2\sqrt{lambert\_w(x^n a)}\,lambert\_w(x^n a)+3x^{2n}\left(\int \frac{\sqrt{lan}}{x^n e^{lambert\_w(x^n a)}\,lamb}\right.\right.}{4x^{2n}n}$$

input
```
int(x^(-1-2*n)*(c*Lambert_W(a*x^n))^(3/2),x)
```

output
```
(sqrt(c)*c*( - 2*sqrt(lambert_w(x**n*a))*lambert_w(x**n*a) + 3*x**(2*n)*in
t(sqrt(lambert_w(x**n*a))/(x**n*e**lambert_w(x**n*a)*lambert_w(x**n*a)*x +
 x**n*e**lambert_w(x**n*a)*x),x)*a*n))/(4*x**(2*n)*n)
```

3.266 $\int x^{-1-2n}\sqrt{cW(ax^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int x^{-1-2n}\sqrt{cW(ax^n)}\,dx = \frac{2a^2\sqrt{c}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{3n} - \frac{2x^{-2n}\sqrt{cW(ax^n)}}{3n} + \frac{2x^{-2n}(cW(ax^n))^{3/2}}{3cn}$$

output
```
2/3*a^2*c^(1/2)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(c*LambertW(a*x^n))^(1/2)/c^(
1/2))/n-2/3*(c*LambertW(a*x^n))^(1/2)/n/(x^(2*n))+2/3*(c*LambertW(a*x^n))^
(3/2)/c/n/(x^(2*n))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int x^{-1-2n}\sqrt{cW(ax^n)}\,dx = \frac{2x^{-2n}\sqrt{cW(ax^n)}\left(a^2\sqrt{2\pi}x^{2n}\text{erf}\left(\sqrt{2}\sqrt{W(ax^n)}\right) - \sqrt{W(ax^n)} + W(ax^n)^{3/2}\right)}{3n\sqrt{W(ax^n)}}$$

input
```
Integrate[x^(-1 - 2*n)*Sqrt[c*ProductLog[a*x^n]],x]
```

output
```
(2*Sqrt[c*ProductLog[a*x^n]]*(a^2*Sqrt[2*Pi]*x^(2*n)*Erf[Sqrt[2]*Sqrt[Prod
uctLog[a*x^n]]] - Sqrt[ProductLog[a*x^n]] + ProductLog[a*x^n]^(3/2)))/(3*n
*x^(2*n)*Sqrt[ProductLog[a*x^n]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7173, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}\sqrt{cW\left(ax^n\right)}\,dx$$

$$\Big\downarrow \text{ 7173}$$

$$-\frac{\int \frac{x^{-2n-1}(cW(ax^n))^{3/2}}{W(ax^n)+1}dx}{3c}-\frac{2x^{-2n}\sqrt{cW\left(ax^n\right)}}{3n}$$

$$\Big\downarrow \text{ 7206}$$

$$-\frac{-\frac{4\int \frac{x^{-2n-1}(cW(ax^n))^{5/2}}{W(ax^n)+1}dx}{c}-\frac{2x^{-2n}(cW(ax^n))^{3/2}}{n}}{3c}-\frac{2x^{-2n}\sqrt{cW\left(ax^n\right)}}{3n}$$

$$\Big\downarrow \text{ 7203}$$

$$-\frac{-\frac{2\sqrt{2\pi}a^2c^{3/2}\text{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{n}-\frac{2x^{-2n}(cW(ax^n))^{3/2}}{n}}{3c}-\frac{2x^{-2n}\sqrt{cW\left(ax^n\right)}}{3n}$$

input
```
Int[x^(-1 - 2*n)*Sqrt[c*ProductLog[a*x^n]],x]
```

output
```
(-2*Sqrt[c*ProductLog[a*x^n]])/(3*n*x^(2*n)) - ((-2*a^2*c^(3/2)*Sqrt[2*Pi]
*Erf[(Sqrt[2]*Sqrt[c*ProductLog[a*x^n]])/Sqrt[c]])/n - (2*(c*ProductLog[a*
x^n])^(3/2))/(n*x^(2*n)))/(3*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int x^{-1-2n}\sqrt{c\,\mathrm{LambertW}\left(a\,x^{n}\right)}dx$$

input
```
int(x^(-1-2*n)*(c*LambertW(a*x^n))^(1/2),x)
```

output
```
int(x^(-1-2*n)*(c*LambertW(a*x^n))^(1/2),x)
```

Fricas [F]

$$\int x^{-1-2n}\sqrt{cW(ax^n)}\,dx = \int \sqrt{c\,\mathrm{W}(ax^n)}x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(-2*n - 1), x)
```

Sympy [F]

$$\int x^{-1-2n}\sqrt{cW(ax^n)}\,dx = \int x^{-2n-1}\sqrt{cW(ax^n)}\,dx$$

input
```
integrate(x**(-1-2*n)*(c*LambertW(a*x**n))**(1/2),x)
```

output
```
Integral(x**(-2*n - 1)*sqrt(c*LambertW(a*x**n)), x)
```

Maxima [F]

$$\int x^{-1-2n}\sqrt{cW(ax^n)}\,dx = \int \sqrt{c\,\mathrm{W}(ax^n)}x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^n))*x^(-2*n - 1), x)
```

Giac [F]

$$\int x^{-1-2n}\sqrt{cW\left(ax^n\right)}\,dx=\int \sqrt{c\,\mathrm{W}(ax^n)}x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(c*lambert_w(a*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^n))*x^(-2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}\sqrt{cW\left(ax^n\right)}\,dx=\int \frac{\sqrt{c\,\mathrm{LambertW}\left(a\,x^n\right)}}{x^{2\,n+1}}\,dx$$

input
```
int((c*LambertW(a*x^n))^(1/2)/x^(2*n + 1),x)
```

output
```
int((c*LambertW(a*x^n))^(1/2)/x^(2*n + 1), x)
```

Reduce [F]

$$\int x^{-1-2n}\sqrt{cW\left(ax^n\right)}\,dx$$
$$=\frac{\sqrt{c}\left(-2\sqrt{lambert\_w\left(x^na\right)}+x^{2n}\left(\int \frac{\sqrt{lambert\_w(x^na)}}{x^n e^{lambert\_w(x^na)}lambert\_w(x^na)^2x+x^n e^{lambert\_w(x^na)}lambert\_w(x^na)x}dx\right.\right.}{4x^{2n}n}$$

input
```
int(x^(-1-2*n)*(c*Lambert_W(a*x^n))^(1/2),x)
```

output
```
(sqrt(c)*( - 2*sqrt(lambert_w(x**n*a)) + x**(2*n)*int(sqrt(lambert_w(x**n*
a))/(x**n*e**lambert_w(x**n*a)*lambert_w(x**n*a)**2*x + x**n*e**lambert_w(
x**n*a)*lambert_w(x**n*a)*x),x)*a*n))/(4*x**(2*n)*n)
```

3.267 $\int \frac{x^{-1-2n}}{\sqrt{cW(ax^n)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 125

$$\int \frac{x^{-1-2n}}{\sqrt{cW(ax^n)}}\,dx = \frac{8a^2\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{15\sqrt{c}n} - \frac{2x^{-2n}}{5n\sqrt{cW(ax^n)}} - \frac{2x^{-2n}\sqrt{cW(ax^n)}}{15cn} + \frac{8x^{-2n}(cW(ax^n))^{3/2}}{15c^2n}$$

output
```
8/15*a^2*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(c*LambertW(a*x^n))^(1/2)/c^(1/2))/c
^(1/2)/n-2/5/n/(x^(2*n))/(c*LambertW(a*x^n))^(1/2)-2/15*(c*LambertW(a*x^n)
)^(1/2)/c/n/(x^(2*n))+8/15*(c*LambertW(a*x^n))^(3/2)/c^2/n/(x^(2*n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int \frac{x^{-1-2n}}{\sqrt{cW(ax^n)}}\,dx = \frac{2x^{-2n}\left(-3+4a^2\sqrt{2\pi}x^{2n}\operatorname{erf}\left(\sqrt{2}\sqrt{W(ax^n)}\right)\sqrt{W(ax^n)}-W(ax^n)+4W(ax^n)^2\right)}{15n\sqrt{cW(ax^n)}}$$

input
```
Integrate[x^(-1 - 2*n)/Sqrt[c*ProductLog[a*x^n]],x]
```

output
```
(2*(-3 + 4*a^2*Sqrt[2*Pi]*x^(2*n)*Erf[Sqrt[2]*Sqrt[ProductLog[a*x^n]]]*Sqr
t[ProductLog[a*x^n]] - ProductLog[a*x^n] + 4*ProductLog[a*x^n]^2))/(15*n*x
^(2*n)*Sqrt[c*ProductLog[a*x^n]])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7173, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-2n-1}}{\sqrt{cW(ax^n)}}dx$$

$$\Big\downarrow \text{ 7173}$$

$$\frac{\int \frac{x^{-2n-1}\sqrt{cW(ax^n)}}{W(ax^n)+1}dx}{5c}-\frac{2x^{-2n}}{5n\sqrt{cW(ax^n)}}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{-\frac{4\int \frac{x^{-2n-1}(cW(ax^n))^{3/2}}{W(ax^n)+1}dx}{3c}-\frac{2x^{-2n}\sqrt{cW(ax^n)}}{3n}}{5c}-\frac{2x^{-2n}}{5n\sqrt{cW(ax^n)}}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{-\frac{4\left(-\frac{4\int \frac{x^{-2n-1}(cW(ax^n))^{5/2}}{W(ax^n)+1}dx}{c}-\frac{2x^{-2n}(cW(ax^n))^{3/2}}{n}\right)}{3c}-\frac{2x^{-2n}\sqrt{cW(ax^n)}}{3n}}{5c}-\frac{2x^{-2n}}{5n\sqrt{cW(ax^n)}}$$

$$\Big\downarrow \text{ 7203}$$

$$\frac{-\frac{4\left(-\frac{2\sqrt{2\pi}a^2c^{3/2}\text{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{n}-\frac{2x^{-2n}(cW(ax^n))^{3/2}}{n}\right)}{3c}-\frac{2x^{-2n}\sqrt{cW(ax^n)}}{3n}}{5c}-\frac{2x^{-2n}}{5n\sqrt{cW(ax^n)}}$$

input
```
Int[x^(-1 - 2*n)/Sqrt[c*ProductLog[a*x^n]],x]
```

output
```
-2/(5*n*x^(2*n)*Sqrt[c*ProductLog[a*x^n]]) + ((-2*Sqrt[c*ProductLog[a*x^n]
])/(3*n*x^(2*n)) - (4*((-2*a^2*c^(3/2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[c*Prod
uctLog[a*x^n]])/Sqrt[c]])/n - (2*(c*ProductLog[a*x^n])^(3/2))/(n*x^(2*n)))
)/(3*c))/(5*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{x^{-1-2n}}{\sqrt{c\,\mathrm{LambertW}\left(a\,x^{n}\right)}}dx$$

input
```
int(x^(-1-2*n)/(c*LambertW(a*x^n))^(1/2),x)
```

output
```
int(x^(-1-2*n)/(c*LambertW(a*x^n))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^{-1-2n}}{\sqrt{cW\left(ax^{n}\right)}}\,dx = \int \frac{x^{-2\,n-1}}{\sqrt{c\,\mathrm{W}(ax^{n})}}\,dx$$

input
```
integrate(x^(-1-2*n)/(c*lambert_w(a*x^n))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(-2*n - 1)/(c*lambert_w(a*x^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{\sqrt{cW\left(ax^{n}\right)}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-2*n)/(c*LambertW(a*x**n))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-2n}}{\sqrt{cW(ax^n)}}\,dx = \int \frac{x^{-2\,n-1}}{\sqrt{c\,\mathrm{W}(ax^n)}}\,dx$$

input
```
integrate(x^(-1-2*n)/(c*lambert_w(a*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(-2*n - 1)/sqrt(c*lambert_w(a*x^n)), x)
```

Giac [F]

$$\int \frac{x^{-1-2n}}{\sqrt{cW(ax^n)}}\,dx = \int \frac{x^{-2\,n-1}}{\sqrt{c\,\mathrm{W}(ax^n)}}\,dx$$

input
```
integrate(x^(-1-2*n)/(c*lambert_w(a*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(-2*n - 1)/sqrt(c*lambert_w(a*x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{\sqrt{cW(ax^n)}}\,dx = \int \frac{1}{x^{2\,n+1}\,\sqrt{c\,\mathrm{LambertW}\,(a\,x^n)}}\,dx$$

input
```
int(1/(x^(2*n + 1)*(c*LambertW(a*x^n))^(1/2)),x)
```

output
```
int(1/(x^(2*n + 1)*(c*LambertW(a*x^n))^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{-1-2n}}{\sqrt{cW(ax^n)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(x^n a)}}{x^{2n} lambert\_w(x^n a)x}dx\right)}{c}$$

input
```
int(x^(-1-2*n)/(c*Lambert_W(a*x^n))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(x**n*a))/(x**(2*n)*lambert_w(x**n*a)*x),x))/c
```

3.268 $\int \frac{x^{-1-2n}}{(cW(ax^n))^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 152

$$\int \frac{x^{-1-2n}}{(cW(ax^n))^{3/2}}\,dx = -\frac{32a^2\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{35c^{3/2}n} - \frac{2x^{-2n}}{7n\,(cW(ax^n))^{3/2}} - \frac{6x^{-2n}}{35cn\sqrt{cW(ax^n)}} + \frac{8x^{-2n}\sqrt{cW(ax^n)}}{35c^2n} - \frac{32x^{-2n}(cW(ax^n))^{3/2}}{35c^3n}$$

output
```
-32/35*a^2*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(c*LambertW(a*x^n))^(1/2)/c^(1/2))
/c^(3/2)/n-2/7/n/(x^(2*n))/(c*LambertW(a*x^n))^(3/2)-6/35/c/n/(x^(2*n))/(c
*LambertW(a*x^n))^(1/2)+8/35*(c*LambertW(a*x^n))^(1/2)/c^2/n/(x^(2*n))-32/
35*(c*LambertW(a*x^n))^(3/2)/c^3/n/(x^(2*n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

$$\int \frac{x^{-1-2n}}{(cW(ax^n))^{3/2}}\,dx = -\frac{2x^{-2n}\left(5+3W(ax^n)+16a^2\sqrt{2\pi}x^{2n}\text{erf}\left(\sqrt{2}\sqrt{W(ax^n)}\right)W(ax^n)^{3/2}-4W(ax^n)^2+16W(ax^n)^3\right)}{35n\,(cW(ax^n))^{3/2}}$$

input
```
Integrate[x^(-1 - 2*n)/(c*ProductLog[a*x^n])^(3/2),x]
```

output
```
(-2*(5 + 3*ProductLog[a*x^n] + 16*a^2*Sqrt[2*Pi]*x^(2*n)*Erf[Sqrt[2]*Sqrt[
ProductLog[a*x^n]]]*ProductLog[a*x^n]^(3/2) - 4*ProductLog[a*x^n]^2 + 16*P
roductLog[a*x^n]^3))/(35*n*x^(2*n)*(c*ProductLog[a*x^n])^(3/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7173, 7206, 7206, 7206, 7203}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-2n-1}}{(cW(ax^n))^{3/2}}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{3\int \frac{x^{-2n-1}}{\sqrt{cW(ax^n)}(W(ax^n)+1)}dx}{7c}-\frac{2x^{-2n}}{7n(cW(ax^n))^{3/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{3\left(-\frac{4\int \frac{x^{-2n-1}\sqrt{cW(ax^n)}}{W(ax^n)+1}dx}{5c}-\frac{2x^{-2n}}{5n\sqrt{cW(ax^n)}}\right)}{7c}-\frac{2x^{-2n}}{7n(cW(ax^n))^{3/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{3\left(-\frac{4\left(-\frac{4\int \frac{x^{-2n-1}(cW(ax^n))^{3/2}}{W(ax^n)+1}dx}{3c}-\frac{2x^{-2n}\sqrt{cW(ax^n)}}{3n}\right)}{5c}-\frac{2x^{-2n}}{5n\sqrt{cW(ax^n)}}\right)}{7c}-\frac{2x^{-2n}}{7n(cW(ax^n))^{3/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{3\left(-\frac{4\left(-\frac{4\left(-\frac{4\int \frac{x^{-2n-1}(cW(ax^n))^{5/2}}{W(ax^n)+1}dx}{c}-\frac{2x^{-2n}(cW(ax^n))^{3/2}}{n}\right)}{3c}-\frac{2x^{-2n}\sqrt{cW(ax^n)}}{3n}\right)}{5c}-\frac{2x^{-2n}}{5n\sqrt{cW(ax^n)}}\right)}{7c}-\frac{2x^{-2n}}{7n\,(cW\,(ax^n))^{3/2}}$$

$$\Big\downarrow \; 7203$$

$$\frac{3\left(-\frac{4\left(-\frac{4\left(-\frac{2\sqrt{2\pi}a^2c^{3/2}\text{erf}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{n}-\frac{2x^{-2n}(cW(ax^n))^{3/2}}{n}\right)}{3c}-\frac{2x^{-2n}\sqrt{cW(ax^n)}}{3n}\right)}{5c}-\frac{2x^{-2n}}{5n\sqrt{cW(ax^n)}}\right)}{7c}-\frac{2x^{-2n}}{7n\,(cW\,(ax^n))^{3/2}}$$

input
```
Int[x^(-1 - 2*n)/(c*ProductLog[a*x^n])^(3/2),x]
```

output
```
-2/(7*n*x^(2*n)*(c*ProductLog[a*x^n])^(3/2)) + (3*(-2/(5*n*x^(2*n)*Sqrt[c*
ProductLog[a*x^n]]) - (4*((-2*Sqrt[c*ProductLog[a*x^n]])/(3*n*x^(2*n)) - (
4*((-2*a^2*c^(3/2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[c*ProductLog[a*x^n]])/Sqrt
[c]])/n - (2*(c*ProductLog[a*x^n])^(3/2))/(n*x^(2*n))))/(3*c)))/(5*c)))/(7
*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7203
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[P
i*(c/(p - 1/2)), 2]*(Erf[Sqrt[c*ProductLog[a*x^n]]/Rt[c/(p - 1/2), 2]]/(d*n
)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2] && E
qQ[m + n*(p - 1/2), -1] && PosQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{x^{-1-2n}}{\left(c\,\mathrm{LambertW}\left(a\,x^{n}\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(x^(-1-2*n)/(c*LambertW(a*x^n))^(3/2),x)
```

output
```
int(x^(-1-2*n)/(c*LambertW(a*x^n))^(3/2),x)
```

Fricas [F]

$$\int \frac{x^{-1-2n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{x^{-2\,n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1-2*n)/(c*lambert_w(a*x^n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(-2*n - 1)/(c^2*lambert_w(a*x^n)^2), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{(cW(ax^n))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-2*n)/(c*LambertW(a*x**n))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-2n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{x^{-2\,n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1-2*n)/(c*lambert_w(a*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(-2*n - 1)/(c*lambert_w(a*x^n))^(3/2), x)
```

Giac [F]

$$\int \frac{x^{-1-2n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{x^{-2n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1-2*n)/(c*lambert_w(a*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^(-2*n - 1)/(c*lambert_w(a*x^n))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{1}{x^{2n+1}\,(c\,\mathrm{LambertW}\,(a\,x^n))^{3/2}}\,dx$$

input
```
int(1/(x^(2*n + 1)*(c*LambertW(a*x^n))^(3/2)),x)
```

output
```
int(1/(x^(2*n + 1)*(c*LambertW(a*x^n))^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^{-1-2n}}{(cW(ax^n))^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(x^n a)}}{x^{2n}lambert\_w(x^n a)^2 x}dx\right)}{c^2}$$

input
```
int(x^(-1-2*n)/(c*Lambert_W(a*x^n))^(3/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(x**n*a))/(x**(2*n)*lambert_w(x**n*a)**2*x),x))
/c**2
```

3.269 $\int x^{-1+n}(cW(ax^n))^{5/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 132

$$\int x^{-1+n}(cW(ax^n))^{5/2}\,dx = \frac{75c^{5/2}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{16an} - \frac{75c^3x^n}{8n\sqrt{cW(ax^n)}} + \frac{25c^2x^n\sqrt{cW(ax^n)}}{4n} - \frac{5cx^n(cW(ax^n))^{3/2}}{2n} + \frac{x^n(cW(ax^n))^{5/2}}{n}$$

output
```
75/16*c^(5/2)*Pi^(1/2)*erfi((c*LambertW(a*x^n))^(1/2)/c^(1/2))/a/n-75/8*c^
3*x^n/n/(c*LambertW(a*x^n))^(1/2)+25/4*c^2*x^n*(c*LambertW(a*x^n))^(1/2)/n
-5/2*c*x^n*(c*LambertW(a*x^n))^(3/2)/n+x^n*(c*LambertW(a*x^n))^(5/2)/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int x^{-1+n}(cW(ax^n))^{5/2}\,dx = \frac{c^3\left(-150ax^n + 75\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{W(ax^n)}\right)\sqrt{W(ax^n)} + 100ax^nW(ax^n) - 40ax^n\right.}{16an\sqrt{cW(ax^n)}}$$

input
```
Integrate[x^(-1 + n)*(c*ProductLog[a*x^n])^(5/2),x]
```

output
```
(c^3*(-150*a*x^n + 75*Sqrt[Pi]*Erfi[Sqrt[ProductLog[a*x^n]]]*Sqrt[ProductL
og[a*x^n]] + 100*a*x^n*ProductLog[a*x^n] - 40*a*x^n*ProductLog[a*x^n]^2 +
16*a*x^n*ProductLog[a*x^n]^3))/(16*a*n*Sqrt[c*ProductLog[a*x^n]])
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {7172, 7205, 7205, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(cW(ax^n))^{5/2}\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{x^n(cW(ax^n))^{5/2}}{n}-\frac{5}{2}\int \frac{x^{n-1}(cW(ax^n))^{5/2}}{W(ax^n)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{x^n(cW(ax^n))^{5/2}}{n}-\frac{5}{2}\left(\frac{cx^n(cW(ax^n))^{3/2}}{n}-\frac{5}{2}c\int \frac{x^{n-1}(cW(ax^n))^{3/2}}{W(ax^n)+1}dx\right)$$

$$\Big\downarrow \text{7205}$$

$$\frac{x^n(cW(ax^n))^{5/2}}{n}-\frac{5}{2}\left(\frac{cx^n(cW(ax^n))^{3/2}}{n}-\frac{5}{2}c\left(\frac{cx^n\sqrt{cW(ax^n)}}{n}-\frac{3}{2}c\int \frac{x^{n-1}\sqrt{cW(ax^n)}}{W(ax^n)+1}dx\right)\right)$$

$$\Big\downarrow \text{7205}$$

$$\frac{x^n(cW(ax^n))^{5/2}}{n}-\frac{5}{2}\left(\frac{cx^n(cW(ax^n))^{3/2}}{n}-\frac{5}{2}c\left(\frac{cx^n\sqrt{cW(ax^n)}}{n}-\frac{3}{2}c\left(\frac{cx^n}{n\sqrt{cW(ax^n)}}-\frac{1}{2}c\int \frac{x^{n-1}}{\sqrt{cW(ax^n)}(W(ax^n)+1)}dx\right)\right)\right)$$

$$\Big\downarrow \text{7204}$$

$$\frac{x^n(cW(ax^n))^{5/2}}{n} - \frac{5}{2}\left(\frac{cx^n(cW(ax^n))^{3/2}}{n} - \frac{5}{2}c\left(\frac{cx^n\sqrt{cW(ax^n)}}{n} - \frac{3}{2}c\left(\frac{cx^n}{n\sqrt{cW(ax^n)}} - \frac{\sqrt{\pi}\sqrt{c}\,\text{erfi}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{2an}\right)\right)\right)$$

input
```
Int[x^(-1 + n)*(c*ProductLog[a*x^n])^(5/2),x]
```

output
```
(x^n*(c*ProductLog[a*x^n])^(5/2))/n - (5*((c*x^n*(c*ProductLog[a*x^n])^(3/
2))/n - (5*c*((c*x^n*Sqrt[c*ProductLog[a*x^n]])/n - (3*c*(-1/2*(Sqrt[c]*Sq
rt[Pi]*Erfi[Sqrt[c*ProductLog[a*x^n]]/Sqrt[c]])/(a*n) + (c*x^n)/(n*Sqrt[c*
ProductLog[a*x^n]])))/2))/2))/2
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^{-1+n}(c\,\mathrm{LambertW}\,(a\,x^n))^{\frac{5}{2}}dx$$

input
```
int(x^(-1+n)*(c*LambertW(a*x^n))^(5/2),x)
```

output
```
int(x^(-1+n)*(c*LambertW(a*x^n))^(5/2),x)
```

Fricas [F]

$$\int x^{-1+n}(cW(ax^n))^{5/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{5}{2}}\,x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(c*lambert_w(a*x^n))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*c^2*x^(n - 1)*lambert_w(a*x^n)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}(cW(ax^n))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)*(c*LambertW(a*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1+n}(cW(ax^n))^{5/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{5}{2}}\,x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(c*lambert_w(a*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^(5/2)*x^(n - 1), x)
```

Giac [F]

$$\int x^{-1+n}(cW(ax^n))^{5/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{5}{2}}\,x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(c*lambert_w(a*x^n))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^(5/2)*x^(n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+n}(cW(ax^n))^{5/2}\,dx = \int x^{n-1}\,(c\,\mathrm{LambertW}(a\,x^n))^{5/2}\,dx$$

input
```
int(x^(n - 1)*(c*LambertW(a*x^n))^(5/2),x)
```

output
```
int(x^(n - 1)*(c*LambertW(a*x^n))^(5/2), x)
```

Reduce [F]

$$\int x^{-1+n}(cW(ax^n))^{5/2}\,dx = \sqrt{c}\left(\int \frac{x^n\sqrt{lambert\_w(x^n a)}\,lambert\_w(x^n a)^2}{x}dx\right)c^2$$

input
```
int(x^(-1+n)*(c*Lambert_W(a*x^n))^(5/2),x)
```

output
```
sqrt(c)*int((x**n*sqrt(lambert_w(x**n*a))*lambert_w(x**n*a)**2)/x,x)*c**2
```

3.270 $\int x^{-1+n}(cW(ax^n))^{3/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int x^{-1+n}(cW(ax^n))^{3/2}\,dx = -\frac{9c^{3/2}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{8an} + \frac{9c^2x^n}{4n\sqrt{cW(ax^n)}} - \frac{3cx^n\sqrt{cW(ax^n)}}{2n} + \frac{x^n(cW(ax^n))^{3/2}}{n}$$

output
```
-9/8*c^(3/2)*Pi^(1/2)*erfi((c*LambertW(a*x^n))^(1/2)/c^(1/2))/a/n+9/4*c^2*
x^n/n/(c*LambertW(a*x^n))^(1/2)-3/2*c*x^n*(c*LambertW(a*x^n))^(1/2)/n+x^n*
(c*LambertW(a*x^n))^(3/2)/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int x^{-1+n}(cW(ax^n))^{3/2}\,dx = \frac{c^2\left(18ax^n - 9\sqrt{\pi}\operatorname{erfi}\left(\sqrt{W(ax^n)}\right)\sqrt{W(ax^n)} - 12ax^nW(ax^n) + 8ax^nW(ax^n$$
$$8an\sqrt{cW(ax^n)}$$

input
```
Integrate[x^(-1 + n)*(c*ProductLog[a*x^n])^(3/2),x]
```

output
```
(c^2*(18*a*x^n - 9*Sqrt[Pi]*Erfi[Sqrt[ProductLog[a*x^n]]]*Sqrt[ProductLog[
a*x^n]] - 12*a*x^n*ProductLog[a*x^n] + 8*a*x^n*ProductLog[a*x^n]^2))/(8*a*
n*Sqrt[c*ProductLog[a*x^n]])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {7172, 7205, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(cW(ax^n))^{3/2}\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{x^n(cW(ax^n))^{3/2}}{n}-\frac{3}{2}\int \frac{x^{n-1}(cW(ax^n))^{3/2}}{W(ax^n)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{x^n(cW(ax^n))^{3/2}}{n}-\frac{3}{2}\left(\frac{cx^n\sqrt{cW(ax^n)}}{n}-\frac{3}{2}c\int \frac{x^{n-1}\sqrt{cW(ax^n)}}{W(ax^n)+1}dx\right)$$

$$\Big\downarrow \text{7205}$$

$$\frac{x^n(cW(ax^n))^{3/2}}{n}-\frac{3}{2}\left(\frac{cx^n\sqrt{cW(ax^n)}}{n}-\frac{3}{2}c\left(\frac{cx^n}{n\sqrt{cW(ax^n)}}-\frac{1}{2}c\int \frac{x^{n-1}}{\sqrt{cW(ax^n)}(W(ax^n)+1)}dx\right)\right)$$

$$\Big\downarrow \text{7204}$$

$$\frac{x^n(cW(ax^n))^{3/2}}{n}-\frac{3}{2}\left(\frac{cx^n\sqrt{cW(ax^n)}}{n}-\frac{3}{2}c\left(\frac{cx^n}{n\sqrt{cW(ax^n)}}-\frac{\sqrt{\pi}\sqrt{c}\,\text{erfi}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{2an}\right)\right)$$

input
```
Int[x^(-1 + n)*(c*ProductLog[a*x^n])^(3/2),x]
```

output
```
(x^n*(c*ProductLog[a*x^n])^(3/2))/n - (3*((c*x^n*Sqrt[c*ProductLog[a*x^n]]
)/n - (3*c*(-1/2*(Sqrt[c]*Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a*x^n]]/Sqrt[c]]
)/(a*n) + (c*x^n)/(n*Sqrt[c*ProductLog[a*x^n]])))/2))/2
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^{-1+n} (c \operatorname{LambertW}(a x^n))^{\frac{3}{2}} dx$$

input
```
int(x^(-1+n)*(c*LambertW(a*x^n))^(3/2),x)
```

output
```
int(x^(-1+n)*(c*LambertW(a*x^n))^(3/2),x)
```

Fricas [F]

$$\int x^{-1+n}(cW(ax^n))^{3/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{3}{2}}\,x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(c*lambert_w(a*x^n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*c*x^(n - 1)*lambert_w(a*x^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}(cW(ax^n))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)*(c*LambertW(a*x**n))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1+n}(cW(ax^n))^{3/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{3}{2}}\,x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(c*lambert_w(a*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^(3/2)*x^(n - 1), x)
```

Giac [F]

$$\int x^{-1+n}(cW(ax^n))^{3/2}\,dx=\int (c\,\mathrm{W}(ax^n))^{\frac{3}{2}}\,x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(c*lambert_w(a*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^(3/2)*x^(n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+n}(cW(ax^n))^{3/2}\,dx=\int x^{n-1}\,(c\,\mathrm{LambertW}(a\,x^n))^{3/2}\,dx$$

input
```
int(x^(n - 1)*(c*LambertW(a*x^n))^(3/2),x)
```

output
```
int(x^(n - 1)*(c*LambertW(a*x^n))^(3/2), x)
```

Reduce [F]

$$\int x^{-1+n}(cW(ax^n))^{3/2}\,dx=\sqrt{c}\left(\int\frac{x^n\sqrt{lambert\_w(x^na)}\,lambert\_w(x^na)}{x}dx\right)c$$

input
```
int(x^(-1+n)*(c*Lambert_W(a*x^n))^(3/2),x)
```

output
```
sqrt(c)*int((x**n*sqrt(lambert_w(x**n*a))*lambert_w(x**n*a))/x,x)*c
```

3.271 $\int x^{-1+n}\sqrt{cW(ax^n)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 82

$$\int x^{-1+n}\sqrt{cW(ax^n)}\,dx=\frac{\sqrt{c}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{4an}-\frac{cx^n}{2n\sqrt{cW(ax^n)}}+\frac{x^n\sqrt{cW(ax^n)}}{n}$$

output
```
1/4*c^(1/2)*Pi^(1/2)*erfi((c*LambertW(a*x^n))^(1/2)/c^(1/2))/a/n-1/2*c*x^n
/n/(c*LambertW(a*x^n))^(1/2)+x^n*(c*LambertW(a*x^n))^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int x^{-1+n}\sqrt{cW(ax^n)}\,dx=\frac{c\left(-2ax^n+\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{W(ax^n)}\right)\sqrt{W(ax^n)}+4ax^nW(ax^n)\right)}{4an\sqrt{cW(ax^n)}}$$

input
```
Integrate[x^(-1 + n)*Sqrt[c*ProductLog[a*x^n]],x]
```

output
```
(c*(-2*a*x^n + Sqrt[Pi]*Erfi[Sqrt[ProductLog[a*x^n]]]*Sqrt[ProductLog[a*x^
n]] + 4*a*x^n*ProductLog[a*x^n]))/(4*a*n*Sqrt[c*ProductLog[a*x^n]])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7172, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\sqrt{cW(ax^n)}dx$$

$$\Big\downarrow \text{ 7172}$$

$$\frac{x^n\sqrt{cW(ax^n)}}{n}-\frac{1}{2}\int \frac{x^{n-1}\sqrt{cW(ax^n)}}{W(ax^n)+1}dx$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{2}\left(\frac{1}{2}c\int \frac{x^{n-1}}{\sqrt{cW(ax^n)}(W(ax^n)+1)}dx-\frac{cx^n}{n\sqrt{cW(ax^n)}}\right)+\frac{x^n\sqrt{cW(ax^n)}}{n}$$

$$\Big\downarrow \text{ 7204}$$

$$\frac{1}{2}\left(\frac{\sqrt{\pi}\sqrt{c}\text{erfi}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{2an}-\frac{cx^n}{n\sqrt{cW(ax^n)}}\right)+\frac{x^n\sqrt{cW(ax^n)}}{n}$$

input
```
Int[x^(-1 + n)*Sqrt[c*ProductLog[a*x^n]],x]
```

output
```
(x^n*Sqrt[c*ProductLog[a*x^n]])/n + ((Sqrt[c]*Sqrt[Pi]*Erfi[Sqrt[c*Product
Log[a*x^n]]/Sqrt[c]])/(2*a*n) - (c*x^n)/(n*Sqrt[c*ProductLog[a*x^n]]))/2
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^{-1+n} \sqrt{c \,\mathrm{LambertW}\left(a \,x^{n}\right)}dx$$

input
```
int(x^(-1+n)*(c*LambertW(a*x^n))^(1/2),x)
```

output
```
int(x^(-1+n)*(c*LambertW(a*x^n))^(1/2),x)
```

Fricas [F]

$$\int x^{-1+n}\sqrt{cW\left(ax^n\right)}\,dx=\int \sqrt{c\,\mathrm{W}(ax^n)}x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(c*lambert_w(a*x^n))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(n - 1), x)
```

Sympy [F]

$$\int x^{-1+n}\sqrt{cW\left(ax^n\right)}\,dx=\int x^{n-1}\sqrt{cW\left(ax^n\right)}\,dx$$

input
```
integrate(x**(-1+n)*(c*LambertW(a*x**n))**(1/2),x)
```

output
```
Integral(x**(n - 1)*sqrt(c*LambertW(a*x**n)), x)
```

Maxima [F]

$$\int x^{-1+n}\sqrt{cW\left(ax^n\right)}\,dx=\int \sqrt{c\,\mathrm{W}(ax^n)}x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(c*lambert_w(a*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^n))*x^(n - 1), x)
```

Giac [F]

$$\int x^{-1+n}\sqrt{cW(ax^n)}\,dx=\int\sqrt{c\,\mathrm{W}(ax^n)}x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(c*lambert_w(a*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^n))*x^(n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+n}\sqrt{cW(ax^n)}\,dx=\int x^{n-1}\,\sqrt{c\,\mathrm{LambertW}\left(a\,x^n\right)}\,dx$$

input
```
int(x^(n - 1)*(c*LambertW(a*x^n))^(1/2),x)
```

output
```
int(x^(n - 1)*(c*LambertW(a*x^n))^(1/2), x)
```

Reduce [F]

$$\int x^{-1+n}\sqrt{cW(ax^n)}\,dx=\sqrt{c}\left(\int\frac{x^n\sqrt{lambert\_w(x^na)}}{x}dx\right)$$

input
```
int(x^(-1+n)*(c*Lambert_W(a*x^n))^(1/2),x)
```

output
```
sqrt(c)*int((x**n*sqrt(lambert_w(x**n*a)))/x,x)
```

3.272 $\int \frac{x^{-1+n}}{\sqrt{cW(ax^n)}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 59

$$\int \frac{x^{-1+n}}{\sqrt{cW(ax^n)}} \, dx = \frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{2a\sqrt{c}n} + \frac{x^n}{n\sqrt{cW(ax^n)}}$$

output
```
1/2*Pi^(1/2)*erfi((c*LambertW(a*x^n))^(1/2)/c^(1/2))/a/c^(1/2)/n+x^n/n/(c*
LambertW(a*x^n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{x^{-1+n}}{\sqrt{cW(ax^n)}} \, dx = \frac{2ax^n + \sqrt{\pi}\operatorname{erfi}\left(\sqrt{W(ax^n)}\right)\sqrt{W(ax^n)}}{2an\sqrt{cW(ax^n)}}$$

input
```
Integrate[x^(-1 + n)/Sqrt[c*ProductLog[a*x^n]],x]
```

output
```
(2*a*x^n + Sqrt[Pi]*Erfi[Sqrt[ProductLog[a*x^n]]]*Sqrt[ProductLog[a*x^n]])
/(2*a*n*Sqrt[c*ProductLog[a*x^n]])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7172, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{\sqrt{cW(ax^n)}}dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{1}{2}\int \frac{x^{n-1}}{\sqrt{cW(ax^n)}\,(W(ax^n)+1)}dx+\frac{x^n}{n\sqrt{cW(ax^n)}}$$

$$\Big\downarrow \text{7204}$$

$$\frac{\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{2a\sqrt{c}n}+\frac{x^n}{n\sqrt{cW(ax^n)}}$$

input
```
Int[x^(-1 + n)/Sqrt[c*ProductLog[a*x^n]],x]
```

output
```
(Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a*x^n]]/Sqrt[c]])/(2*a*Sqrt[c]*n) + x^n/(
n*Sqrt[c*ProductLog[a*x^n]])
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

Maple [F]

$$\int \frac{x^{-1+n}}{\sqrt{c\,\mathrm{LambertW}\left(a\,x^{n}\right)}}dx$$

input
```
int(x^(-1+n)/(c*LambertW(a*x^n))^(1/2),x)
```

output
```
int(x^(-1+n)/(c*LambertW(a*x^n))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^{-1+n}}{\sqrt{cW\left(ax^{n}\right)}}\,dx=\int \frac{x^{n-1}}{\sqrt{c\,\mathrm{W}(ax^{n})}}\,dx$$

input
```
integrate(x^(-1+n)/(c*lambert_w(a*x^n))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(n - 1)/(c*lambert_w(a*x^n)), x)
```

Sympy [F]

$$\int \frac{x^{-1+n}}{\sqrt{cW\left(ax^{n}\right)}}\,dx=\int \frac{x^{n-1}}{\sqrt{cW\left(ax^{n}\right)}}\,dx$$

input
```
integrate(x**(-1+n)/(c*LambertW(a*x**n))**(1/2),x)
```

output
```
Integral(x**(n - 1)/sqrt(c*LambertW(a*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+n}}{\sqrt{cW\left(ax^n\right)}}\,dx = \int \frac{x^{n-1}}{\sqrt{c\,\mathrm{W}(ax^n)}}\,dx$$

input
```
integrate(x^(-1+n)/(c*lambert_w(a*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(n - 1)/sqrt(c*lambert_w(a*x^n)), x)
```

Giac [F]

$$\int \frac{x^{-1+n}}{\sqrt{cW\left(ax^n\right)}}\,dx = \int \frac{x^{n-1}}{\sqrt{c\,\mathrm{W}(ax^n)}}\,dx$$

input
```
integrate(x^(-1+n)/(c*lambert_w(a*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(n - 1)/sqrt(c*lambert_w(a*x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}}{\sqrt{cW\left(ax^n\right)}}\,dx = \int \frac{x^{n-1}}{\sqrt{c\,\mathrm{LambertW}\left(a\,x^n\right)}}\,dx$$

input
```
int(x^(n - 1)/(c*LambertW(a*x^n))^(1/2),x)
```

output
```
int(x^(n - 1)/(c*LambertW(a*x^n))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+n}}{\sqrt{cW(ax^n)}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^n\sqrt{lambert\_w(x^n a)}}{lambert\_w(x^n a)x}dx\right)}{c}$$

input
```
int(x^(-1+n)/(c*Lambert_W(a*x^n))^(1/2),x)
```

output
```
(sqrt(c)*int((x**n*sqrt(lambert_w(x**n*a)))/(lambert_w(x**n*a)*x),x))/c
```

3.273 $\int \frac{x^{-1+n}}{(cW(ax^n))^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 58

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{3/2}}\,dx=\frac{3\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{ac^{3/2}n}-\frac{2x^n}{n\,(cW(ax^n))^{3/2}}$$

output
```
3*Pi^(1/2)*erfi((c*LambertW(a*x^n))^(1/2)/c^(1/2))/a/c^(3/2)/n-2*x^n/n/(c*
LambertW(a*x^n))^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{3/2}}\,dx=\frac{-2ax^n+3\sqrt{\pi}\,\text{erfi}\left(\sqrt{W(ax^n)}\right)W(ax^n)^{3/2}}{an\,(cW(ax^n))^{3/2}}$$

input
```
Integrate[x^(-1 + n)/(c*ProductLog[a*x^n])^(3/2),x]
```

output
```
(-2*a*x^n + 3*Sqrt[Pi]*Erfi[Sqrt[ProductLog[a*x^n]]]*ProductLog[a*x^n]^(3/
2))/(a*n*(c*ProductLog[a*x^n])^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7173, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{(cW(ax^n))^{3/2}}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{3\int \frac{x^{n-1}}{\sqrt{cW(ax^n)}(W(ax^n)+1)}dx}{c} - \frac{2x^n}{n(cW(ax^n))^{3/2}}$$

$$\Big\downarrow \text{7204}$$

$$\frac{3\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{ac^{3/2}n} - \frac{2x^n}{n(cW(ax^n))^{3/2}}$$

input
```
Int[x^(-1 + n)/(c*ProductLog[a*x^n])^(3/2),x]
```

output
```
(3*Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a*x^n]]/Sqrt[c]])/(a*c^(3/2)*n) - (2*x^
n)/(n*(c*ProductLog[a*x^n])^(3/2))
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

Maple [F]

$$\int \frac{x^{-1+n}}{(c\,\mathrm{LambertW}\,(a\,x^n))^{\frac{3}{2}}}dx$$

input
```
int(x^(-1+n)/(c*LambertW(a*x^n))^(3/2),x)
```

output
```
int(x^(-1+n)/(c*LambertW(a*x^n))^(3/2),x)
```

Fricas [F]

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{x^{n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1+n)/(c*lambert_w(a*x^n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(n - 1)/(c^2*lambert_w(a*x^n)^2), x)
```

Sympy [F]

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{x^{n-1}}{(cW(ax^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(-1+n)/(c*LambertW(a*x**n))**(3/2),x)
```

output
```
Integral(x**(n - 1)/(c*LambertW(a*x**n))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{x^{n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1+n)/(c*lambert_w(a*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(n - 1)/(c*lambert_w(a*x^n))^(3/2), x)
```

Giac [F]

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{x^{n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1+n)/(c*lambert_w(a*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^(n - 1)/(c*lambert_w(a*x^n))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{x^{n-1}}{(c\,\mathrm{LambertW}\,(a\,x^n))^{3/2}}\,dx$$

input
```
int(x^(n - 1)/(c*LambertW(a*x^n))^(3/2),x)
```

output
```
int(x^(n - 1)/(c*LambertW(a*x^n))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+n}}{\left(cW\left(ax^n\right)\right)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^n\sqrt{lambert\_w(x^n a)}}{lambert\_w(x^n a)^2 x}dx\right)}{c^2}$$

input
```
int(x^(-1+n)/(c*Lambert_W(a*x^n))^(3/2),x)
```

output
```
(sqrt(c)*int((x**n*sqrt(lambert_w(x**n*a)))/(lambert_w(x**n*a)**2*x),x))/c
**2
```

3.274 $\int \frac{x^{-1+n}}{(cW(ax^n))^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 87

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{5/2}}\,dx = \frac{10\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{3ac^{5/2}n} - \frac{2x^n}{3n(cW(ax^n))^{5/2}} - \frac{10x^n}{3cn(cW(ax^n))^{3/2}}$$

output
```
10/3*Pi^(1/2)*erfi((c*LambertW(a*x^n))^(1/2)/c^(1/2))/a/c^(5/2)/n-2/3*x^n/
n/(c*LambertW(a*x^n))^(5/2)-10/3*x^n/c/n/(c*LambertW(a*x^n))^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.78

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{5/2}}\,dx = -\frac{2\left(ax^n + 5ax^nW(ax^n) - 5\sqrt{\pi}\operatorname{erfi}\left(\sqrt{W(ax^n)}\right)W(ax^n)^{5/2}\right)}{3an(cW(ax^n))^{5/2}}$$

input
```
Integrate[x^(-1 + n)/(c*ProductLog[a*x^n])^(5/2),x]
```

output
```
(-2*(a*x^n + 5*a*x^n*ProductLog[a*x^n] - 5*Sqrt[Pi]*Erfi[Sqrt[ProductLog[a
*x^n]]]*ProductLog[a*x^n]^(5/2)))/(3*a*n*(c*ProductLog[a*x^n])^(5/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7173, 7206, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{(cW(ax^n))^{5/2}}\,dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{5\int \frac{x^{n-1}}{(cW(ax^n))^{3/2}(W(ax^n)+1)}dx}{3c}-\frac{2x^n}{3n\,(cW(ax^n))^{5/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{5\left(\frac{2\int \frac{x^{n-1}}{\sqrt{cW(ax^n)}(W(ax^n)+1)}dx}{c}-\frac{2x^n}{n(cW(ax^n))^{3/2}}\right)}{3c}-\frac{2x^n}{3n\,(cW(ax^n))^{5/2}}$$

$$\Big\downarrow \text{7204}$$

$$\frac{5\left(\frac{2\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{ac^{3/2}n}-\frac{2x^n}{n(cW(ax^n))^{3/2}}\right)}{3c}-\frac{2x^n}{3n\,(cW(ax^n))^{5/2}}$$

input
```
Int[x^(-1 + n)/(c*ProductLog[a*x^n])^(5/2),x]
```

output
```
(-2*x^n)/(3*n*(c*ProductLog[a*x^n])^(5/2)) + (5*((2*Sqrt[Pi]*Erfi[Sqrt[c*P
roductLog[a*x^n]]/Sqrt[c]])/(a*c^(3/2)*n) - (2*x^n)/(n*(c*ProductLog[a*x^n
])^(3/2))))/(3*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{x^{-1+n}}{\left(c \,\mathrm{LambertW}\left(a \,x^{n}\right)\right)^{\frac{5}{2}}}d x$$

input
```
int(x^(-1+n)/(c*LambertW(a*x^n))^(5/2),x)
```

output
```
int(x^(-1+n)/(c*LambertW(a*x^n))^(5/2),x)
```

Fricas [F]

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{5/2}}\,dx = \int \frac{x^{n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(-1+n)/(c*lambert_w(a*x^n))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(n - 1)/(c^3*lambert_w(a*x^n)^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)/(c*LambertW(a*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{5/2}}\,dx = \int \frac{x^{n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(-1+n)/(c*lambert_w(a*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^(n - 1)/(c*lambert_w(a*x^n))^(5/2), x)
```

Giac [F]

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{5/2}}\,dx = \int \frac{x^{n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(-1+n)/(c*lambert_w(a*x^n))^(5/2),x, algorithm="giac")
```

output
```
integrate(x^(n - 1)/(c*lambert_w(a*x^n))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{5/2}}\,dx = \int \frac{x^{n-1}}{(c\,\mathrm{LambertW}(a\,x^n))^{5/2}}\,dx$$

input
```
int(x^(n - 1)/(c*LambertW(a*x^n))^(5/2),x)
```

output
```
int(x^(n - 1)/(c*LambertW(a*x^n))^(5/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^n\sqrt{lambert\_w(x^n a)}}{lambert\_w(x^n a)^3 x}dx\right)}{c^3}$$

input
```
int(x^(-1+n)/(c*Lambert_W(a*x^n))^(5/2),x)
```

output
```
(sqrt(c)*int((x**n*sqrt(lambert_w(x**n*a)))/(lambert_w(x**n*a)**3*x),x))/c
**3
```

3.275 $\int \frac{x^{-1+n}}{(cW(ax^n))^{7/2}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 112

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{7/2}} \, dx = \frac{28\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{15ac^{7/2}n} - \frac{2x^n}{5n\,(cW(ax^n))^{7/2}} - \frac{14x^n}{15cn\,(cW(ax^n))^{5/2}} - \frac{28x^n}{15c^2n\,(cW(ax^n))^{3/2}}$$

output
```
28/15*Pi^(1/2)*erfi((c*LambertW(a*x^n))^(1/2)/c^(1/2))/a/c^(7/2)/n-2/5*x^n
/n/(c*LambertW(a*x^n))^(7/2)-14/15*x^n/c/n/(c*LambertW(a*x^n))^(5/2)-28/15
*x^n/c^2/n/(c*LambertW(a*x^n))^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.84

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{7/2}} \, dx = -\frac{2\sqrt{cW(ax^n)}\left(3ax^n + 7ax^nW(ax^n) + 14ax^nW(ax^n)^2 - 14\sqrt{\pi}\text{erfi}\left(\sqrt{W(ax^n)}\right)W(ax^n)^{7/2}\right)}{15ac^4nW(ax^n)^4}$$

input
```
Integrate[x^(-1 + n)/(c*ProductLog[a*x^n])^(7/2),x]
```

output
```
(-2*Sqrt[c*ProductLog[a*x^n]]*(3*a*x^n + 7*a*x^n*ProductLog[a*x^n] + 14*a*
x^n*ProductLog[a*x^n]^2 - 14*Sqrt[Pi]*Erfi[Sqrt[ProductLog[a*x^n]]]*Produc
tLog[a*x^n]^(7/2)))/(15*a*c^4*n*ProductLog[a*x^n]^4)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {7173, 7206, 7206, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{(cW(ax^n))^{7/2}}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{7\int \frac{x^{n-1}}{(cW(ax^n))^{5/2}(W(ax^n)+1)}dx}{5c}-\frac{2x^n}{5n(cW(ax^n))^{7/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{7\left(\frac{2\int \frac{x^{n-1}}{(cW(ax^n))^{3/2}(W(ax^n)+1)}dx}{3c}-\frac{2x^n}{3n(cW(ax^n))^{5/2}}\right)}{5c}-\frac{2x^n}{5n(cW(ax^n))^{7/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{7\left(\frac{2\left(\frac{2\int \frac{x^{n-1}}{\sqrt{cW(ax^n)}(W(ax^n)+1)}dx}{c}-\frac{2x^n}{n(cW(ax^n))^{3/2}}\right)}{3c}-\frac{2x^n}{3n(cW(ax^n))^{5/2}}\right)}{5c}-\frac{2x^n}{5n(cW(ax^n))^{7/2}}$$

$$\Big\downarrow \text{7204}$$

$$\frac{7\left(\frac{2\left(\frac{2\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{ac^{3/2}n}-\frac{2x^n}{n(cW(ax^n))^{3/2}}\right)}{3c}-\frac{2x^n}{3n(cW(ax^n))^{5/2}}\right)}{5c}-\frac{2x^n}{5n\,(cW\,(ax^n))^{7/2}}$$

input
```
Int[x^(-1 + n)/(c*ProductLog[a*x^n])^(7/2),x]
```

output
```
(-2*x^n)/(5*n*(c*ProductLog[a*x^n])^(7/2)) + (7*((-2*x^n)/(3*n*(c*ProductL
og[a*x^n])^(5/2)) + (2*((2*Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a*x^n]]/Sqrt[c]
])/(a*c^(3/2)*n) - (2*x^n)/(n*(c*ProductLog[a*x^n])^(3/2))))/(3*c)))/(5*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{x^{-1+n}}{(c\,\text{LambertW}\,(a\,x^n))^{\frac{7}{2}}}dx$$

input
```
int(x^(-1+n)/(c*LambertW(a*x^n))^(7/2),x)
```

output
```
int(x^(-1+n)/(c*LambertW(a*x^n))^(7/2),x)
```

Fricas [F]

$$\int \frac{x^{-1+n}}{(cW\,(ax^n))^{7/2}}\,dx = \int \frac{x^{n-1}}{(c\,W(ax^n))^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^(-1+n)/(c*lambert_w(a*x^n))^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(n - 1)/(c^4*lambert_w(a*x^n)^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}}{(cW\,(ax^n))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)/(c*LambertW(a*x**n))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{7/2}}\,dx = \int \frac{x^{n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^(-1+n)/(c*lambert_w(a*x^n))^(7/2),x, algorithm="maxima")
```

output
```
integrate(x^(n - 1)/(c*lambert_w(a*x^n))^(7/2), x)
```

Giac [F]

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{7/2}}\,dx = \int \frac{x^{n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^(-1+n)/(c*lambert_w(a*x^n))^(7/2),x, algorithm="giac")
```

output
```
integrate(x^(n - 1)/(c*lambert_w(a*x^n))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{7/2}}\,dx = \int \frac{x^{n-1}}{(c\,\mathrm{LambertW}(a\,x^n))^{7/2}}\,dx$$

input
```
int(x^(n - 1)/(c*LambertW(a*x^n))^(7/2),x)
```

output
```
int(x^(n - 1)/(c*LambertW(a*x^n))^(7/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{7/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^n\sqrt{lambert\_w(x^n a)}}{lambert\_w(x^n a)^4 x}dx\right)}{c^4}$$

input
```
int(x^(-1+n)/(c*Lambert_W(a*x^n))^(7/2),x)
```

output
```
(sqrt(c)*int((x**n*sqrt(lambert_w(x**n*a)))/(lambert_w(x**n*a)**4*x),x))/c
**4
```

3.276 $\int \frac{x^{-1+n}}{(cW(ax^n))^{9/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 137

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{9/2}}\,dx=\frac{24\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{35ac^{9/2}n}-\frac{2x^n}{7n(cW(ax^n))^{9/2}}-\frac{18x^n}{35cn(cW(ax^n))^{7/2}}-\frac{12x^n}{35c^2n(cW(ax^n))^{5/2}}-\frac{24x^n}{35c^3n(cW(ax^n))^{3/2}}$$

output
```
24/35*Pi^(1/2)*erfi((c*LambertW(a*x^n))^(1/2)/c^(1/2))/a/c^(9/2)/n-2/7*x^n
/n/(c*LambertW(a*x^n))^(9/2)-18/35*x^n/c/n/(c*LambertW(a*x^n))^(7/2)-12/35
*x^n/c^2/n/(c*LambertW(a*x^n))^(5/2)-24/35*x^n/c^3/n/(c*LambertW(a*x^n))^(
3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{9/2}}\,dx=$$
$$-\frac{2\sqrt{cW(ax^n)}\left(5ax^n+9ax^nW(ax^n)+6ax^nW(ax^n)^2+12ax^nW(ax^n)^3-12\sqrt{\pi}\operatorname{erfi}\left(\sqrt{W(ax^n)}\right)W(ax^n\right.}{35ac^5nW(ax^n)^5}$$

input
```
Integrate[x^(-1 + n)/(c*ProductLog[a*x^n])^(9/2),x]
```

output
```
(-2*Sqrt[c*ProductLog[a*x^n]]*(5*a*x^n + 9*a*x^n*ProductLog[a*x^n] + 6*a*x
^n*ProductLog[a*x^n]^2 + 12*a*x^n*ProductLog[a*x^n]^3 - 12*Sqrt[Pi]*Erfi[S
qrt[ProductLog[a*x^n]]]*ProductLog[a*x^n]^(9/2)))/(35*a*c^5*n*ProductLog[a
*x^n]^5)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {7173, 7206, 7206, 7206, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{(cW(ax^n))^{9/2}}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{9\int \frac{x^{n-1}}{(cW(ax^n))^{7/2}(W(ax^n)+1)}dx}{7c}-\frac{2x^n}{7n(cW(ax^n))^{9/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{9\left(\frac{2\int \frac{x^{n-1}}{(cW(ax^n))^{5/2}(W(ax^n)+1)}dx}{5c}-\frac{2x^n}{5n(cW(ax^n))^{7/2}}\right)}{7c}-\frac{2x^n}{7n(cW(ax^n))^{9/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{9\left(\frac{2\left(\frac{2\int \frac{x^{n-1}}{(cW(ax^n))^{3/2}(W(ax^n)+1)}dx}{3c}-\frac{2x^n}{3n(cW(ax^n))^{5/2}}\right)}{5c}-\frac{2x^n}{5n(cW(ax^n))^{7/2}}\right)}{7c}-\frac{2x^n}{7n(cW(ax^n))^{9/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{9\left(\frac{2\left(\frac{2\left(\frac{2\int \frac{x^{n-1}}{\sqrt{cW(ax^n)}(W(ax^n)+1)}dx}{c}-\frac{2x^n}{n(cW(ax^n))^{3/2}}\right)}{3c}-\frac{2x^n}{3n(cW(ax^n))^{5/2}}\right)}{5c}-\frac{2x^n}{5n(cW(ax^n))^{7/2}}\right)}{7c}-\frac{2x^n}{7n\left(cW\left(ax^n\right)\right)^{9/2}}$$

$$\Big\downarrow \; 7204$$

$$\frac{9\left(\frac{2\left(\frac{2\left(\frac{2\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{ac^{3/2}n}-\frac{2x^n}{n(cW(ax^n))^{3/2}}\right)}{3c}-\frac{2x^n}{3n(cW(ax^n))^{5/2}}\right)}{5c}-\frac{2x^n}{5n(cW(ax^n))^{7/2}}\right)}{7c}-\frac{2x^n}{7n\left(cW\left(ax^n\right)\right)^{9/2}}$$

input
```
Int[x^(-1 + n)/(c*ProductLog[a*x^n])^(9/2),x]
```

output
```
(-2*x^n)/(7*n*(c*ProductLog[a*x^n])^(9/2)) + (9*((-2*x^n)/(5*n*(c*ProductL
og[a*x^n])^(7/2)) + (2*((-2*x^n)/(3*n*(c*ProductLog[a*x^n])^(5/2)) + (2*((
2*Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a*x^n]]/Sqrt[c]])/(a*c^(3/2)*n) - (2*x^n
)/(n*(c*ProductLog[a*x^n])^(3/2))))/(3*c)))/(5*c)))/(7*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{x^{-1+n}}{(c\,\mathrm{LambertW}\,(a\,x^n))^{\frac{9}{2}}}dx$$

input
```
int(x^(-1+n)/(c*LambertW(a*x^n))^(9/2),x)
```

output
```
int(x^(-1+n)/(c*LambertW(a*x^n))^(9/2),x)
```

Fricas [F]

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{9/2}}\,dx = \int \frac{x^{n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(-1+n)/(c*lambert_w(a*x^n))^(9/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(n - 1)/(c^5*lambert_w(a*x^n)^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)/(c*LambertW(a*x**n))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{9/2}}\,dx = \int \frac{x^{n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(-1+n)/(c*lambert_w(a*x^n))^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(n - 1)/(c*lambert_w(a*x^n))^(9/2), x)
```

Giac [F]

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{9/2}}\,dx = \int \frac{x^{n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(-1+n)/(c*lambert_w(a*x^n))^(9/2),x, algorithm="giac")
```

output
```
integrate(x^(n - 1)/(c*lambert_w(a*x^n))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{9/2}}\,dx = \int \frac{x^{n-1}}{(c\,\mathrm{LambertW}(a\,x^n))^{9/2}}\,dx$$

input
```
int(x^(n - 1)/(c*LambertW(a*x^n))^(9/2),x)
```

output
```
int(x^(n - 1)/(c*LambertW(a*x^n))^(9/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+n}}{(cW(ax^n))^{9/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^n\sqrt{lambert\_w(x^n a)}}{lambert\_w(x^n a)^5 x}dx\right)}{c^5}$$

input
```
int(x^(-1+n)/(c*Lambert_W(a*x^n))^(9/2),x)
```

output
```
(sqrt(c)*int((x**n*sqrt(lambert_w(x**n*a)))/(lambert_w(x**n*a)**5*x),x))/c
**5
```

3.277 $\int x^{-1+2n}(cW(ax^n))^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 152

$$\int x^{-1+2n}(cW(ax^n))^{3/2}\,dx=\frac{45c^{3/2}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{256a^2n}-\frac{45c^3x^{2n}}{128n\left(cW\left(ax^n\right)\right)^{3/2}}+\frac{15c^2x^{2n}}{32n\sqrt{cW\left(ax^n\right)}}-\frac{3cx^{2n}\sqrt{cW\left(ax^n\right)}}{8n}+\frac{x^{2n}(cW(ax^n))^{3/2}}{2n}$$

output
```
45/512*c^(3/2)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(a*x^n))^(1/2)/c^(
1/2))/a^2/n-45/128*c^3*x^(2*n)/n/(c*LambertW(a*x^n))^(3/2)+15/32*c^2*x^(2*
n)/n/(c*LambertW(a*x^n))^(1/2)-3/8*c*x^(2*n)*(c*LambertW(a*x^n))^(1/2)/n+1
/2*x^(2*n)*(c*LambertW(a*x^n))^(3/2)/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$\int x^{-1+2n}(cW(ax^n))^{3/2}\,dx=\frac{(cW(ax^n))^{3/2}\left(-180a^2x^{2n}+240a^2x^{2n}W(ax^n)+45\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{W\left(ax^n\right)}\right)\right.}{512a^2nW\left(ax^n\right)^3}$$

input
```
Integrate[x^(-1 + 2*n)*(c*ProductLog[a*x^n])^(3/2),x]
```

output
```
((c*ProductLog[a*x^n])^(3/2)*(-180*a^2*x^(2*n) + 240*a^2*x^(2*n)*ProductLo
g[a*x^n] + 45*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ProductLog[a*x^n]]]*ProductLog[
a*x^n]^(3/2) - 192*a^2*x^(2*n)*ProductLog[a*x^n]^2 + 256*a^2*x^(2*n)*Produ
ctLog[a*x^n]^3))/(512*a^2*n*ProductLog[a*x^n]^3)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7172, 7205, 7205, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}(cW(ax^n))^{3/2}\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{x^{2n}(cW(ax^n))^{3/2}}{2n}-\frac{3}{4}\int \frac{x^{2n-1}(cW(ax^n))^{3/2}}{W(ax^n)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{x^{2n}(cW(ax^n))^{3/2}}{2n}-\frac{3}{4}\left(\frac{cx^{2n}\sqrt{cW(ax^n)}}{2n}-\frac{5}{4}c\int \frac{x^{2n-1}\sqrt{cW(ax^n)}}{W(ax^n)+1}dx\right)$$

$$\Big\downarrow \text{7205}$$

$$\frac{x^{2n}(cW(ax^n))^{3/2}}{2n}-\frac{3}{4}\left(\frac{cx^{2n}\sqrt{cW(ax^n)}}{2n}-\frac{5}{4}c\left(\frac{cx^{2n}}{2n\sqrt{cW(ax^n)}}-\frac{3}{4}c\int \frac{x^{2n-1}}{\sqrt{cW(ax^n)}(W(ax^n)+1)}dx\right)\right)$$

$$\Big\downarrow \text{7205}$$

$$\frac{x^{2n}(cW(ax^n))^{3/2}}{2n}-\frac{3}{4}\left(\frac{cx^{2n}\sqrt{cW(ax^n)}}{2n}-\frac{5}{4}c\left(\frac{cx^{2n}}{2n\sqrt{cW(ax^n)}}-\frac{3}{4}c\left(\frac{cx^{2n}}{2n(cW(ax^n))^{3/2}}-\frac{1}{4}c\int \frac{x^{2n-1}}{(cW(ax^n))^{3/2}(W(ax^n)+1)}dx\right)\right)\right)$$

$$\Big\downarrow \text{7204}$$

$$\frac{x^{2n}(cW(ax^n))^{3/2}}{2n} - \frac{3}{4}\left(\frac{cx^{2n}\sqrt{cW(ax^n)}}{2n} - \frac{5}{4}c\left(\frac{cx^{2n}}{2n\sqrt{cW(ax^n)}} - \frac{3}{4}c\left(\frac{cx^{2n}}{2n(cW(ax^n))^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{4a^2\sqrt{c}n}\right)\right)\right)$$

input
```
Int[x^(-1 + 2*n)*(c*ProductLog[a*x^n])^(3/2),x]
```

output
```
(x^(2*n)*(c*ProductLog[a*x^n])^(3/2))/(2*n) - (3*((c*x^(2*n)*Sqrt[c*Produc
tLog[a*x^n]])/(2*n) - (5*c*((c*x^(2*n))/(2*n*Sqrt[c*ProductLog[a*x^n]]) -
(3*c*(-1/4*(Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[c*ProductLog[a*x^n]])/Sqrt[c]])/
(a^2*Sqrt[c]*n) + (c*x^(2*n))/(2*n*(c*ProductLog[a*x^n])^(3/2))))/4))/4))/
4
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^{-1+2n}(c\,\mathrm{LambertW}\,(a\,x^{n}))^{\frac{3}{2}}dx$$

input
```
int(x^(-1+2*n)*(c*LambertW(a*x^n))^(3/2),x)
```

output
```
int(x^(-1+2*n)*(c*LambertW(a*x^n))^(3/2),x)
```

Fricas [F]

$$\int x^{-1+2n}(cW(ax^n))^{3/2}\,dx=\int (c\,\mathrm{W}(ax^n))^{\frac{3}{2}}\,x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(c*lambert_w(a*x^n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*c*x^(2*n - 1)*lambert_w(a*x^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+2n}(cW(ax^n))^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x**(-1+2*n)*(c*LambertW(a*x**n))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1+2n}(cW(ax^n))^{3/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{3}{2}}\,x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(c*lambert_w(a*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^(3/2)*x^(2*n - 1), x)
```

Giac [F]

$$\int x^{-1+2n}(cW(ax^n))^{3/2}\,dx = \int (c\,\mathrm{W}(ax^n))^{\frac{3}{2}}\,x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(c*lambert_w(a*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^(3/2)*x^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}(cW(ax^n))^{3/2}\,dx = \int x^{2\,n-1}\,(c\,\mathrm{LambertW}(a\,x^n))^{3/2}\,dx$$

input
```
int(x^(2*n - 1)*(c*LambertW(a*x^n))^(3/2),x)
```

output
```
int(x^(2*n - 1)*(c*LambertW(a*x^n))^(3/2), x)
```

Reduce [F]

$$\int x^{-1+2n}(cW(ax^n))^{3/2}\,dx = \sqrt{c}\left(\int \frac{x^{2n}\sqrt{lambert\_w(x^n a)}\,lambert\_w(x^n a)}{x}dx\right)c$$

input
```
int(x^(-1+2*n)*(c*Lambert_W(a*x^n))^(3/2),x)
```

output
```
sqrt(c)*int((x**(2*n)*sqrt(lambert_w(x**n*a))*lambert_w(x**n*a))/x,x)*c
```

3.278 $\int x^{-1+2n}\sqrt{cW(ax^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 125

$$\int x^{-1+2n}\sqrt{cW(ax^n)}\,dx=-\frac{3\sqrt{c}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{64a^2n}+\frac{3c^2x^{2n}}{32n(cW(ax^n))^{3/2}}-\frac{cx^{2n}}{8n\sqrt{cW(ax^n)}}+\frac{x^{2n}\sqrt{cW(ax^n)}}{2n}$$

output
```
-3/128*c^(1/2)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(a*x^n))^(1/2)/c^(
1/2))/a^2/n+3/32*c^2*x^(2*n)/n/(c*LambertW(a*x^n))^(3/2)-1/8*c*x^(2*n)/n/(
c*LambertW(a*x^n))^(1/2)+1/2*x^(2*n)*(c*LambertW(a*x^n))^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.89

$$\int x^{-1+2n}\sqrt{cW(ax^n)}\,dx=\frac{\sqrt{cW(ax^n)}\left(12a^2x^{2n}-16a^2x^{2n}W(ax^n)-3\sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{W(ax^n)}\right)W(ax^n)^{3/2}+64a^2x^{2n}W(ax^n)^2\right)}{128a^2nW(ax^n)^2}$$

input
```
Integrate[x^(-1 + 2*n)*Sqrt[c*ProductLog[a*x^n]],x]
```

output
```
(Sqrt[c*ProductLog[a*x^n]]*(12*a^2*x^(2*n) - 16*a^2*x^(2*n)*ProductLog[a*x
^n] - 3*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ProductLog[a*x^n]]]*ProductLog[a*x^n]
^(3/2) + 64*a^2*x^(2*n)*ProductLog[a*x^n]^2))/(128*a^2*n*ProductLog[a*x^n]
^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7172, 7205, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}\sqrt{cW(ax^n)}\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{x^{2n}\sqrt{cW(ax^n)}}{2n}-\frac{1}{4}\int \frac{x^{2n-1}\sqrt{cW(ax^n)}}{W(ax^n)+1}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{4}\left(\frac{3}{4}c\int \frac{x^{2n-1}}{\sqrt{cW(ax^n)}(W(ax^n)+1)}dx-\frac{cx^{2n}}{2n\sqrt{cW(ax^n)}}\right)+\frac{x^{2n}\sqrt{cW(ax^n)}}{2n}$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{4}\left(\frac{3}{4}c\left(\frac{cx^{2n}}{2n(cW(ax^n))^{3/2}}-\frac{1}{4}c\int \frac{x^{2n-1}}{(cW(ax^n))^{3/2}(W(ax^n)+1)}dx\right)-\frac{cx^{2n}}{2n\sqrt{cW(ax^n)}}\right)+\frac{x^{2n}\sqrt{cW(ax^n)}}{2n}$$

$$\Big\downarrow \text{7204}$$

$$\frac{1}{4}\left(\frac{3}{4}c\left(\frac{cx^{2n}}{2n(cW(ax^n))^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{4a^2\sqrt{c}n}\right)-\frac{cx^{2n}}{2n\sqrt{cW(ax^n)}}\right)+\frac{x^{2n}\sqrt{cW(ax^n)}}{2n}$$

input
```
Int[x^(-1 + 2*n)*Sqrt[c*ProductLog[a*x^n]],x]
```

output
```
(x^(2*n)*Sqrt[c*ProductLog[a*x^n]])/(2*n) + (-1/2*(c*x^(2*n))/(n*Sqrt[c*Pr
oductLog[a*x^n]]) + (3*c*(-1/4*(Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[c*ProductLog
[a*x^n]])/Sqrt[c]])/(a^2*Sqrt[c]*n) + (c*x^(2*n))/(2*n*(c*ProductLog[a*x^n
])^(3/2))))/4)/4
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^{-1+2n}\sqrt{c\,\mathrm{LambertW}\left(a\,x^{n}\right)}dx$$

input
```
int(x^(-1+2*n)*(c*LambertW(a*x^n))^(1/2),x)
```

output
```
int(x^(-1+2*n)*(c*LambertW(a*x^n))^(1/2),x)
```

Fricas [F]

$$\int x^{-1+2n}\sqrt{cW\left(ax^n\right)}\,dx = \int \sqrt{c\,\mathrm{W}(ax^n)}x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(c*lambert_w(a*x^n))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(2*n - 1), x)
```

Sympy [F]

$$\int x^{-1+2n}\sqrt{cW\left(ax^n\right)}\,dx = \int x^{2n-1}\sqrt{cW\left(ax^n\right)}\,dx$$

input
```
integrate(x**(-1+2*n)*(c*LambertW(a*x**n))**(1/2),x)
```

output
```
Integral(x**(2*n - 1)*sqrt(c*LambertW(a*x**n)), x)
```

Maxima [F]

$$\int x^{-1+2n}\sqrt{cW\left(ax^n\right)}\,dx = \int \sqrt{c\,\mathrm{W}(ax^n)}x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(c*lambert_w(a*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(a*x^n))*x^(2*n - 1), x)
```

Giac [F]

$$\int x^{-1+2n}\sqrt{cW(ax^n)}\,dx=\int \sqrt{c\,\mathrm{W}(ax^n)}x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(c*lambert_w(a*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(a*x^n))*x^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\sqrt{cW(ax^n)}\,dx=\int x^{2\,n-1}\,\sqrt{c\,\mathrm{LambertW}\left(a\,x^n\right)}\,dx$$

input
```
int(x^(2*n - 1)*(c*LambertW(a*x^n))^(1/2),x)
```

output
```
int(x^(2*n - 1)*(c*LambertW(a*x^n))^(1/2), x)
```

Reduce [F]

$$\int x^{-1+2n}\sqrt{cW(ax^n)}\,dx=\sqrt{c}\left(\int \frac{x^{2n}\sqrt{lambert\_w(x^n a)}}{x}dx\right)$$

input
```
int(x^(-1+2*n)*(c*Lambert_W(a*x^n))^(1/2),x)
```

output
```
sqrt(c)*int((x**(2*n)*sqrt(lambert_w(x**n*a)))/x,x)
```

3.279 $\int \frac{x^{-1+2n}}{\sqrt{cW(ax^n)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{x^{-1+2n}}{\sqrt{cW(ax^n)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{16a^2\sqrt{c}n} + \frac{cx^{2n}}{8n\,(cW(ax^n))^{3/2}} + \frac{x^{2n}}{2n\sqrt{cW(ax^n)}}$$

output
```
-1/32*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(a*x^n))^(1/2)/c^(1/2))/a^2
/c^(1/2)/n+1/8*c*x^(2*n)/n/(c*LambertW(a*x^n))^(3/2)+1/2*x^(2*n)/n/(c*Lamb
ertW(a*x^n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \frac{x^{-1+2n}}{\sqrt{cW(ax^n)}}\,dx = \frac{c\left(4a^2x^{2n} + 16a^2x^{2n}W(ax^n) - \sqrt{2\pi}\,\text{erfi}\left(\sqrt{2}\sqrt{W(ax^n)}\right)W(ax^n)^{3/2}\right)}{32a^2n\,(cW(ax^n))^{3/2}}$$

input
```
Integrate[x^(-1 + 2*n)/Sqrt[c*ProductLog[a*x^n]],x]
```

output
```
(c*(4*a^2*x^(2*n) + 16*a^2*x^(2*n)*ProductLog[a*x^n] - Sqrt[2*Pi]*Erfi[Sqr
t[2]*Sqrt[ProductLog[a*x^n]]]*ProductLog[a*x^n]^(3/2)))/(32*a^2*n*(c*Produ
ctLog[a*x^n])^(3/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7172, 7205, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{\sqrt{cW(ax^n)}}dx$$

$$\Big\downarrow \text{ 7172}$$

$$\frac{1}{4}\int \frac{x^{2n-1}}{\sqrt{cW(ax^n)}(W(ax^n)+1)}dx+\frac{x^{2n}}{2n\sqrt{cW(ax^n)}}$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{1}{4}\left(\frac{cx^{2n}}{2n(cW(ax^n))^{3/2}}-\frac{1}{4}c\int \frac{x^{2n-1}}{(cW(ax^n))^{3/2}(W(ax^n)+1)}dx\right)+\frac{x^{2n}}{2n\sqrt{cW(ax^n)}}$$

$$\Big\downarrow \text{ 7204}$$

$$\frac{1}{4}\left(\frac{cx^{2n}}{2n(cW(ax^n))^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{4a^2\sqrt{c}n}\right)+\frac{x^{2n}}{2n\sqrt{cW(ax^n)}}$$

input
```
Int[x^(-1 + 2*n)/Sqrt[c*ProductLog[a*x^n]],x]
```

output
```
x^(2*n)/(2*n*Sqrt[c*ProductLog[a*x^n]]) + (-1/4*(Sqrt[Pi/2]*Erfi[(Sqrt[2]*
Sqrt[c*ProductLog[a*x^n]])/Sqrt[c]])/(a^2*Sqrt[c]*n) + (c*x^(2*n))/(2*n*(c
*ProductLog[a*x^n])^(3/2)))/4
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int \frac{x^{-1+2n}}{\sqrt{c\,\mathrm{LambertW}\left(a\,x^{n}\right)}}dx$$

input
```
int(x^(-1+2*n)/(c*LambertW(a*x^n))^(1/2),x)
```

output
```
int(x^(-1+2*n)/(c*LambertW(a*x^n))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^{-1+2n}}{\sqrt{cW(ax^n)}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{c\,\mathrm{W}(ax^n)}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(2*n - 1)/(c*lambert_w(a*x^n)), x)
```

Sympy [F]

$$\int \frac{x^{-1+2n}}{\sqrt{cW(ax^n)}}\,dx = \int \frac{x^{2n-1}}{\sqrt{cW(ax^n)}}\,dx$$

input
```
integrate(x**(-1+2*n)/(c*LambertW(a*x**n))**(1/2),x)
```

output
```
Integral(x**(2*n - 1)/sqrt(c*LambertW(a*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+2n}}{\sqrt{cW(ax^n)}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{c\,\mathrm{W}(ax^n)}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(2*n - 1)/sqrt(c*lambert_w(a*x^n)), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{\sqrt{cW(ax^n)}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{c\,\mathrm{W}(ax^n)}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/sqrt(c*lambert_w(a*x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{\sqrt{cW(ax^n)}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{c\,\mathrm{LambertW}\,(a\,x^n)}}\,dx$$

input
```
int(x^(2*n - 1)/(c*LambertW(a*x^n))^(1/2),x)
```

output
```
int(x^(2*n - 1)/(c*LambertW(a*x^n))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+2n}}{\sqrt{cW(ax^n)}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^{2n}\sqrt{lambert\_w(x^n a)}}{lambert\_w(x^n a)x}dx\right)}{c}$$

input
```
int(x^(-1+2*n)/(c*Lambert_W(a*x^n))^(1/2),x)
```

output
```
(sqrt(c)*int((x**(2*n)*sqrt(lambert_w(x**n*a)))/(lambert_w(x**n*a)*x),x))/
c
```

3.280 $\int \frac{x^{-1+2n}}{(cW(ax^n))^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{3/2}}\,dx = \frac{3\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{4a^2c^{3/2}n} + \frac{x^{2n}}{2n\,(cW(ax^n))^{3/2}}$$

output
```
3/8*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(a*x^n))^(1/2)/c^(1/2))/a^2/c
^(3/2)/n+1/2*x^(2*n)/n/(c*LambertW(a*x^n))^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{3/2}}\,dx = \frac{4a^2x^{2n} + 3\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{W(ax^n)}\right)W(ax^n)^{3/2}}{8a^2n\,(cW(ax^n))^{3/2}}$$

input
```
Integrate[x^(-1 + 2*n)/(c*ProductLog[a*x^n])^(3/2),x]
```

output
```
(4*a^2*x^(2*n) + 3*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ProductLog[a*x^n]]]*Produc
tLog[a*x^n]^(3/2))/(8*a^2*n*(c*ProductLog[a*x^n])^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7172, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(cW(ax^n))^{3/2}}dx$$
$$\Big\downarrow \text{ 7172}$$
$$\frac{3}{4}\int \frac{x^{2n-1}}{(cW(ax^n))^{3/2}(W(ax^n)+1)}dx+\frac{x^{2n}}{2n(cW(ax^n))^{3/2}}$$
$$\Big\downarrow \text{ 7204}$$
$$\frac{3\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{4a^2c^{3/2}n}+\frac{x^{2n}}{2n(cW(ax^n))^{3/2}}$$

input
```
Int[x^(-1 + 2*n)/(c*ProductLog[a*x^n])^(3/2),x]
```

output
```
(3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[c*ProductLog[a*x^n]])/Sqrt[c]])/(4*a^2*c^
(3/2)*n) + x^(2*n)/(2*n*(c*ProductLog[a*x^n])^(3/2))
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

Maple [F]

$$\int \frac{x^{-1+2n}}{(c\,\mathrm{LambertW}(a\,x^{n}))^{\frac{3}{2}}}dx$$

input
```
int(x^(-1+2*n)/(c*LambertW(a*x^n))^(3/2),x)
```

output
```
int(x^(-1+2*n)/(c*LambertW(a*x^n))^(3/2),x)
```

Fricas [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{x^{2\,n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(2*n - 1)/(c^2*lambert_w(a*x^n)^2), x)
```

Sympy [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{x^{2n-1}}{(cW(ax^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(-1+2*n)/(c*LambertW(a*x**n))**(3/2),x)
```

output
```
Integral(x**(2*n - 1)/(c*LambertW(a*x**n))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{x^{2\,n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(2*n - 1)/(c*lambert_w(a*x^n))^(3/2), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{x^{2\,n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/(c*lambert_w(a*x^n))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{3/2}}\,dx = \int \frac{x^{2\,n-1}}{(c\,\mathrm{LambertW}\,(a\,x^n))^{3/2}}\,dx$$

input
```
int(x^(2*n - 1)/(c*LambertW(a*x^n))^(3/2),x)
```

output
```
int(x^(2*n - 1)/(c*LambertW(a*x^n))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^{2n}\sqrt{lambert\_w(x^n a)}}{lambert\_w(x^n a)^2 x}dx\right)}{c^2}$$

input
```
int(x^(-1+2*n)/(c*Lambert_W(a*x^n))^(3/2),x)
```

output
```
(sqrt(c)*int((x**(2*n)*sqrt(lambert_w(x**n*a)))/(lambert_w(x**n*a)**2*x),x
))/c**2
```

3.281 $\int \frac{x^{-1+2n}}{(cW(ax^n))^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{5/2}}\,dx=\frac{5\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{a^2c^{5/2}n}-\frac{2x^{2n}}{n(cW(ax^n))^{5/2}}$$

output
```
5/2*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(a*x^n))^(1/2)/c^(1/2))/a^2/c
^(5/2)/n-2*x^(2*n)/n/(c*LambertW(a*x^n))^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{5/2}}\,dx=\frac{-4a^2x^{2n}+5\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{W(ax^n)}\right)W(ax^n)^{5/2}}{2a^2n(cW(ax^n))^{5/2}}$$

input
```
Integrate[x^(-1 + 2*n)/(c*ProductLog[a*x^n])^(5/2),x]
```

output
```
(-4*a^2*x^(2*n) + 5*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ProductLog[a*x^n]]]*Produ
ctLog[a*x^n]^(5/2))/(2*a^2*n*(c*ProductLog[a*x^n])^(5/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7173, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(cW(ax^n))^{5/2}}dx$$
$$\Big\downarrow \text{7173}$$
$$\frac{5\int \frac{x^{2n-1}}{(cW(ax^n))^{3/2}(W(ax^n)+1)}dx}{c}-\frac{2x^{2n}}{n(cW(ax^n))^{5/2}}$$
$$\Big\downarrow \text{7204}$$
$$\frac{5\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{a^2c^{5/2}n}-\frac{2x^{2n}}{n(cW(ax^n))^{5/2}}$$

input
```
Int[x^(-1 + 2*n)/(c*ProductLog[a*x^n])^(5/2),x]
```

output
```
(5*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[c*ProductLog[a*x^n]])/Sqrt[c]])/(a^2*c^(5
/2)*n) - (2*x^(2*n))/(n*(c*ProductLog[a*x^n])^(5/2))
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

Maple [F]

$$\int \frac{x^{-1+2n}}{(c\,\mathrm{LambertW}\left(a\,x^{n}\right))^{\frac{5}{2}}}dx$$

input
```
int(x^(-1+2*n)/(c*LambertW(a*x^n))^(5/2),x)
```

output
```
int(x^(-1+2*n)/(c*LambertW(a*x^n))^(5/2),x)
```

Fricas [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{5/2}}\,dx = \int \frac{x^{2\,n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(2*n - 1)/(c^3*lambert_w(a*x^n)^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+2*n)/(c*LambertW(a*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{5/2}}\,dx = \int \frac{x^{2n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^(2*n - 1)/(c*lambert_w(a*x^n))^(5/2), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{5/2}}\,dx = \int \frac{x^{2n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(5/2),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/(c*lambert_w(a*x^n))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{5/2}}\,dx = \int \frac{x^{2n-1}}{(c\,\mathrm{LambertW}\,(a\,x^n))^{5/2}}\,dx$$

input
```
int(x^(2*n - 1)/(c*LambertW(a*x^n))^(5/2),x)
```

output
```
int(x^(2*n - 1)/(c*LambertW(a*x^n))^(5/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^{2n}\sqrt{lambert\_w(x^n a)}}{lambert\_w(x^n a)^3 x}dx\right)}{c^3}$$

input
```
int(x^(-1+2*n)/(c*Lambert_W(a*x^n))^(5/2),x)
```

output
```
(sqrt(c)*int((x**(2*n)*sqrt(lambert_w(x**n*a)))/(lambert_w(x**n*a)**3*x),x
))/c**3
```

3.282 $\int \frac{x^{-1+2n}}{(cW(ax^n))^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{7/2}}\,dx = \frac{14\sqrt{2\pi}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{3a^2c^{7/2}n} - \frac{2x^{2n}}{3n\,(cW(ax^n))^{7/2}} - \frac{14x^{2n}}{3cn\,(cW(ax^n))^{5/2}}$$

output
```
14/3*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(a*x^n))^(1/2)/c^(1/2))/a^2/
c^(7/2)/n-2/3*x^(2*n)/n/(c*LambertW(a*x^n))^(7/2)-14/3*x^(2*n)/c/n/(c*Lamb
ertW(a*x^n))^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{7/2}}\,dx = -\frac{2\sqrt{cW(ax^n)}\left(a^2x^{2n} + 7a^2x^{2n}W(ax^n) - 7\sqrt{2\pi}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{W(ax^n)}\right)W(ax^n)^{7/2}\right)}{3a^2c^4nW(ax^n)^4}$$

input
```
Integrate[x^(-1 + 2*n)/(c*ProductLog[a*x^n])^(7/2),x]
```

output
```
(-2*Sqrt[c*ProductLog[a*x^n]]*(a^2*x^(2*n) + 7*a^2*x^(2*n)*ProductLog[a*x^
n] - 7*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ProductLog[a*x^n]]]*ProductLog[a*x^n]^
(7/2)))/(3*a^2*c^4*n*ProductLog[a*x^n]^4)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7173, 7206, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(cW(ax^n))^{7/2}}dx$$

$$\Big\downarrow \text{ 7173}$$

$$\frac{7\int \frac{x^{2n-1}}{(cW(ax^n))^{5/2}(W(ax^n)+1)}dx}{3c}-\frac{2x^{2n}}{3n\,(cW(ax^n))^{7/2}}$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{7\left(\frac{4\int \frac{x^{2n-1}}{(cW(ax^n))^{3/2}(W(ax^n)+1)}dx}{c}-\frac{2x^{2n}}{n(cW(ax^n))^{5/2}}\right)}{3c}-\frac{2x^{2n}}{3n\,(cW(ax^n))^{7/2}}$$

$$\Big\downarrow \text{ 7204}$$

$$\frac{7\left(\frac{2\sqrt{2\pi}\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{a^2c^{5/2}n}-\frac{2x^{2n}}{n(cW(ax^n))^{5/2}}\right)}{3c}-\frac{2x^{2n}}{3n\,(cW(ax^n))^{7/2}}$$

input
```
Int[x^(-1 + 2*n)/(c*ProductLog[a*x^n])^(7/2),x]
```

output
```
(-2*x^(2*n))/(3*n*(c*ProductLog[a*x^n])^(7/2)) + (7*((2*Sqrt[2*Pi]*Erfi[(S
qrt[2]*Sqrt[c*ProductLog[a*x^n]])/Sqrt[c]])/(a^2*c^(5/2)*n) - (2*x^(2*n))/
(n*(c*ProductLog[a*x^n])^(5/2))))/(3*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{x^{-1+2n}}{\left(c\,\mathrm{LambertW}\left(a\,x^{n}\right)\right)^{\frac{7}{2}}}dx$$

input
```
int(x^(-1+2*n)/(c*LambertW(a*x^n))^(7/2),x)
```

output
```
int(x^(-1+2*n)/(c*LambertW(a*x^n))^(7/2),x)
```

Fricas [F]

$$\int \frac{x^{-1+2n}}{\left(cW\left(ax^n\right)\right)^{7/2}}\,dx=\int \frac{x^{2\,n-1}}{\left(c\,\mathrm{W}(ax^n)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(2*n - 1)/(c^4*lambert_w(a*x^n)^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{\left(cW\left(ax^n\right)\right)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**(-1+2*n)/(c*LambertW(a*x**n))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+2n}}{\left(cW\left(ax^n\right)\right)^{7/2}}\,dx=\int \frac{x^{2\,n-1}}{\left(c\,\mathrm{W}(ax^n)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(7/2),x, algorithm="maxima")
```

output
```
integrate(x^(2*n - 1)/(c*lambert_w(a*x^n))^(7/2), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{7/2}}\,dx = \int \frac{x^{2\,n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(7/2),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/(c*lambert_w(a*x^n))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{7/2}}\,dx = \int \frac{x^{2\,n-1}}{(c\,\mathrm{LambertW}\,(a\,x^n))^{7/2}}\,dx$$

input
```
int(x^(2*n - 1)/(c*LambertW(a*x^n))^(7/2),x)
```

output
```
int(x^(2*n - 1)/(c*LambertW(a*x^n))^(7/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{7/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^{2n}\sqrt{lambert\_w(x^n a)}}{lambert\_w(x^n a)^4 x}dx\right)}{c^4}$$

input
```
int(x^(-1+2*n)/(c*Lambert_W(a*x^n))^(7/2),x)
```

output
```
(sqrt(c)*int((x**(2*n)*sqrt(lambert_w(x**n*a)))/(lambert_w(x**n*a)**4*x),x
))/c**4
```

3.283 $\int \frac{x^{-1+2n}}{(cW(ax^n))^{9/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 125

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{9/2}}\,dx = \frac{24\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{5a^2c^{9/2}n} - \frac{2x^{2n}}{5n(cW(ax^n))^{9/2}} - \frac{6x^{2n}}{5cn(cW(ax^n))^{7/2}} - \frac{24x^{2n}}{5c^2n(cW(ax^n))^{5/2}}$$

output
```
24/5*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(a*x^n))^(1/2)/c^(1/2))/a^2/
c^(9/2)/n-2/5*x^(2*n)/n/(c*LambertW(a*x^n))^(9/2)-6/5*x^(2*n)/c/n/(c*Lambe
rtW(a*x^n))^(7/2)-24/5*x^(2*n)/c^2/n/(c*LambertW(a*x^n))^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{9/2}}\,dx = -\frac{2\sqrt{cW(ax^n)}\left(a^2x^{2n} + 3a^2x^{2n}W(ax^n) + 12a^2x^{2n}W(ax^n)^2 - 12\sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{W(ax^n)}\right)W(ax^n)^{9/2}\right)}{5a^2c^5nW(ax^n)^5}$$

input
```
Integrate[x^(-1 + 2*n)/(c*ProductLog[a*x^n])^(9/2),x]
```

output
```
(-2*Sqrt[c*ProductLog[a*x^n]]*(a^2*x^(2*n) + 3*a^2*x^(2*n)*ProductLog[a*x^
n] + 12*a^2*x^(2*n)*ProductLog[a*x^n]^2 - 12*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[
ProductLog[a*x^n]]]*ProductLog[a*x^n]^(9/2)))/(5*a^2*c^5*n*ProductLog[a*x^
n]^5)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7173, 7206, 7206, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(cW(ax^n))^{9/2}}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{9\int \frac{x^{2n-1}}{(cW(ax^n))^{7/2}(W(ax^n)+1)}dx}{5c}-\frac{2x^{2n}}{5n(cW(ax^n))^{9/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{9\left(\frac{4\int \frac{x^{2n-1}}{(cW(ax^n))^{5/2}(W(ax^n)+1)}dx}{3c}-\frac{2x^{2n}}{3n(cW(ax^n))^{7/2}}\right)}{5c}-\frac{2x^{2n}}{5n(cW(ax^n))^{9/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{9\left(\frac{4\left(\frac{4\int \frac{x^{2n-1}}{(cW(ax^n))^{3/2}(W(ax^n)+1)}dx}{c}-\frac{2x^{2n}}{n(cW(ax^n))^{5/2}}\right)}{3c}-\frac{2x^{2n}}{3n(cW(ax^n))^{7/2}}\right)}{5c}-\frac{2x^{2n}}{5n(cW(ax^n))^{9/2}}$$

$$\Big\downarrow \text{7204}$$

$$\frac{9\left(\frac{4\left(\frac{2\sqrt{2\pi}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{a^2c^{5/2}n}-\frac{2x^{2n}}{n(cW(ax^n))^{5/2}}\right)}{3c}-\frac{2x^{2n}}{3n(cW(ax^n))^{7/2}}\right)}{5c}-\frac{2x^{2n}}{5n\left(cW\left(ax^n\right)\right)^{9/2}}$$

input
```
Int[x^(-1 + 2*n)/(c*ProductLog[a*x^n])^(9/2),x]
```

output
```
(-2*x^(2*n))/(5*n*(c*ProductLog[a*x^n])^(9/2)) + (9*((-2*x^(2*n))/(3*n*(c*
ProductLog[a*x^n])^(7/2)) + (4*((2*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[c*Product
Log[a*x^n]])/Sqrt[c]])/(a^2*c^(5/2)*n) - (2*x^(2*n))/(n*(c*ProductLog[a*x^
n])^(5/2))))/(3*c)))/(5*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{x^{-1+2n}}{\left(c\,\mathrm{LambertW}\left(a\,x^{n}\right)\right)^{\frac{9}{2}}}dx$$

input
```
int(x^(-1+2*n)/(c*LambertW(a*x^n))^(9/2),x)
```

output
```
int(x^(-1+2*n)/(c*LambertW(a*x^n))^(9/2),x)
```

Fricas [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{9/2}}\,dx = \int \frac{x^{2\,n-1}}{\left(c\,\mathrm{W}(ax^n)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(9/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(2*n - 1)/(c^5*lambert_w(a*x^n)^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+2*n)/(c*LambertW(a*x**n))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{9/2}}\,dx = \int \frac{x^{2\,n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(2*n - 1)/(c*lambert_w(a*x^n))^(9/2), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{9/2}}\,dx = \int \frac{x^{2\,n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(9/2),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/(c*lambert_w(a*x^n))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{9/2}}\,dx = \int \frac{x^{2\,n-1}}{\left(c\,\mathrm{LambertW}\left(a\,x^n\right)\right)^{9/2}}\,dx$$

input
```
int(x^(2*n - 1)/(c*LambertW(a*x^n))^(9/2),x)
```

output
```
int(x^(2*n - 1)/(c*LambertW(a*x^n))^(9/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{9/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^{2n}\sqrt{lambert\_w(x^n a)}}{lambert\_w(x^n a)^5 x}dx\right)}{c^5}$$

input
```
int(x^(-1+2*n)/(c*Lambert_W(a*x^n))^(9/2),x)
```

output
```
(sqrt(c)*int((x**(2*n)*sqrt(lambert_w(x**n*a)))/(lambert_w(x**n*a)**5*x),x
))/c**5
```

3.284 $\int \frac{x^{-1+2n}}{(cW(ax^n))^{11/2}}\, dx$

Optimal result

Integrand size = 20, antiderivative size = 152

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{11/2}}\, dx = \frac{352\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{105a^2c^{11/2}n} - \frac{2x^{2n}}{7n\,(cW(ax^n))^{11/2}} - \frac{22x^{2n}}{35cn\,(cW(ax^n))^{9/2}} - \frac{88x^{2n}}{105c^2n\,(cW(ax^n))^{7/2}} - \frac{352x^{2n}}{105c^3n\,(cW(ax^n))^{5/2}}$$

output
```
352/105*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(a*x^n))^(1/2)/c^(1/2))/a
^2/c^(11/2)/n-2/7*x^(2*n)/n/(c*LambertW(a*x^n))^(11/2)-22/35*x^(2*n)/c/n/(
c*LambertW(a*x^n))^(9/2)-88/105*x^(2*n)/c^2/n/(c*LambertW(a*x^n))^(7/2)-35
2/105*x^(2*n)/c^3/n/(c*LambertW(a*x^n))^(5/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{11/2}}\, dx =$$
$$-\frac{2\sqrt{cW(ax^n)}\Big(15a^2x^{2n} + 33a^2x^{2n}W(ax^n) + 44a^2x^{2n}W(ax^n)^2 + 176a^2x^{2n}W(ax^n)^3 - 176\sqrt{2\pi}\operatorname{erfi}\Big(\sqrt{2}\sqrt{\cdots}}{105a^2c^6nW(ax^n)^6}$$

input
```
Integrate[x^(-1 + 2*n)/(c*ProductLog[a*x^n])^(11/2),x]
```

output
```
(-2*Sqrt[c*ProductLog[a*x^n]]*(15*a^2*x^(2*n) + 33*a^2*x^(2*n)*ProductLog[
a*x^n] + 44*a^2*x^(2*n)*ProductLog[a*x^n]^2 + 176*a^2*x^(2*n)*ProductLog[a
*x^n]^3 - 176*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ProductLog[a*x^n]]]*ProductLog[
a*x^n]^(11/2)))/(105*a^2*c^6*n*ProductLog[a*x^n]^6)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7173, 7206, 7206, 7206, 7204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(cW(ax^n))^{11/2}}dx$$

$$\Big\downarrow \text{7173}$$

$$\frac{11\int \frac{x^{2n-1}}{(cW(ax^n))^{9/2}(W(ax^n)+1)}dx}{7c}-\frac{2x^{2n}}{7n(cW(ax^n))^{11/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{11\left(\frac{4\int \frac{x^{2n-1}}{(cW(ax^n))^{7/2}(W(ax^n)+1)}dx}{5c}-\frac{2x^{2n}}{5n(cW(ax^n))^{9/2}}\right)}{7c}-\frac{2x^{2n}}{7n(cW(ax^n))^{11/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{11\left(\frac{4\left(\frac{4\int \frac{x^{2n-1}}{(cW(ax^n))^{5/2}(W(ax^n)+1)}dx}{3c}-\frac{2x^{2n}}{3n(cW(ax^n))^{7/2}}\right)}{5c}-\frac{2x^{2n}}{5n(cW(ax^n))^{9/2}}\right)}{7c}-\frac{2x^{2n}}{7n(cW(ax^n))^{11/2}}$$

$$\Big\downarrow \text{7206}$$

$$\frac{11\left(\frac{4\left(\frac{4\left(\frac{4\int \frac{x^{2n-1}}{(cW(ax^n))^{3/2}(W(ax^n)+1)}dx}{c}-\frac{2x^{2n}}{n(cW(ax^n))^{5/2}}\right)}{3c}-\frac{2x^{2n}}{3n(cW(ax^n))^{7/2}}\right)}{5c}-\frac{2x^{2n}}{5n(cW(ax^n))^{9/2}}\right)}{7c}-\frac{2x^{2n}}{7n\left(cW\left(ax^n\right)\right)^{11/2}}$$

$$\Big\downarrow \ 7204$$

$$\frac{11\left(\frac{4\left(\frac{4\left(\frac{2\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{cW(ax^n)}}{\sqrt{c}}\right)}{a^2c^{5/2}n}-\frac{2x^{2n}}{n(cW(ax^n))^{5/2}}\right)}{3c}-\frac{2x^{2n}}{3n(cW(ax^n))^{7/2}}\right)}{5c}-\frac{2x^{2n}}{5n(cW(ax^n))^{9/2}}\right)}{7c}-\frac{2x^{2n}}{7n\left(cW\left(ax^n\right)\right)^{11/2}}$$

input
```
Int[x^(-1 + 2*n)/(c*ProductLog[a*x^n])^(11/2),x]
```

output
```
(-2*x^(2*n))/(7*n*(c*ProductLog[a*x^n])^(11/2)) + (11*((-2*x^(2*n))/(5*n*(
c*ProductLog[a*x^n])^(9/2)) + (4*((-2*x^(2*n))/(3*n*(c*ProductLog[a*x^n])^
(7/2)) + (4*((2*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[c*ProductLog[a*x^n]])/Sqrt[c
]])/(a^2*c^(5/2)*n) - (2*x^(2*n))/(n*(c*ProductLog[a*x^n])^(5/2))))/(3*c))
)/(5*c)))/(7*c)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7204
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_))/((d_) + (d_.)*Pr
oductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^(p - 1/2)*c^(p - 1/2)*Rt[(
-Pi)*(c/(p - 1/2)), 2]*(Erfi[Sqrt[c*ProductLog[a*x^n]]/Rt[-c/(p - 1/2), 2]]
/(d*n)), x] /; FreeQ[{a, c, d, m, n}, x] && NeQ[m, -1] && IntegerQ[p - 1/2]
 && EqQ[m + n*(p - 1/2), -1] && NegQ[c/(p - 1/2)]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [F]

$$\int \frac{x^{-1+2n}}{\left(c\,\mathrm{LambertW}\left(a\,x^{n}\right)\right)^{\frac{11}{2}}}dx$$

input
```
int(x^(-1+2*n)/(c*LambertW(a*x^n))^(11/2),x)
```

output
```
int(x^(-1+2*n)/(c*LambertW(a*x^n))^(11/2),x)
```

Fricas [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{11/2}}\,dx = \int \frac{x^{2\,n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{11}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(11/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(a*x^n))*x^(2*n - 1)/(c^6*lambert_w(a*x^n)^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+2*n)/(c*LambertW(a*x**n))**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{11/2}}\,dx = \int \frac{x^{2\,n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{11}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(11/2),x, algorithm="maxima")
```

output
```
integrate(x^(2*n - 1)/(c*lambert_w(a*x^n))^(11/2), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{11/2}}\,dx = \int \frac{x^{2n-1}}{(c\,\mathrm{W}(ax^n))^{\frac{11}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(c*lambert_w(a*x^n))^(11/2),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/(c*lambert_w(a*x^n))^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{11/2}}\,dx = \int \frac{x^{2n-1}}{(c\,\mathrm{LambertW}(a\,x^n))^{11/2}}\,dx$$

input
```
int(x^(2*n - 1)/(c*LambertW(a*x^n))^(11/2),x)
```

output
```
int(x^(2*n - 1)/(c*LambertW(a*x^n))^(11/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+2n}}{(cW(ax^n))^{11/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^{2n}\sqrt{lambert\_w(x^n a)}}{lambert\_w(x^n a)^6 x}dx\right)}{c^6}$$

input
```
int(x^(-1+2*n)/(c*Lambert_W(a*x^n))^(11/2),x)
```

output
```
(sqrt(c)*int((x**(2*n)*sqrt(lambert_w(x**n*a)))/(lambert_w(x**n*a)**6*x),x
))/c**6
```

3.285 $\int x(cW(ax^n))^p\,dx$

Optimal result

Integrand size = 12, antiderivative size = 189

$$\int x(cW(ax^n))^p\,dx = \frac{2^{-1-\frac{2}{n}-p}e^{-\frac{(2-n)W(ax^n)}{n}}x^{2-n}\Gamma\left(1+\frac{2}{n}+p,-\frac{2W(ax^n)}{n}\right)(cW(ax^n))^{1+p}\left(-\frac{W(ax^n)}{n}\right)^{-\frac{2}{n}-p}}{ac} + \frac{2^{-\frac{2}{n}-p}e^{-\frac{(2-n)W(ax^n)}{n}}x^{2-n}\Gamma\left(\frac{2}{n}+p,-\frac{2W(ax^n)}{n}\right)(cW(ax^n))^{p}\left(-\frac{W(ax^n)}{n}\right)^{1-\frac{2}{n}-p}}{a}$$

output
```
2^(-1-2/n-p)*x^(2-n)*GAMMA(1+2/n+p,-2*LambertW(a*x^n)/n)*(c*LambertW(a*x^n
))^(p+1)*(-LambertW(a*x^n)/n)^(-2/n-p)/a/c/exp((2-n)*LambertW(a*x^n)/n)+2^
(-2/n-p)*x^(2-n)*GAMMA(2/n+p,-2*LambertW(a*x^n)/n)*(c*LambertW(a*x^n))^p*(
-LambertW(a*x^n)/n)^(1-2/n-p)/a/exp((2-n)*LambertW(a*x^n)/n)
```

Mathematica [F]

$$\int x(cW(ax^n))^p\,dx=\int x(cW(ax^n))^p\,dx$$

input
```
Integrate[x*(c*ProductLog[a*x^n])^p,x]
```

output
```
Integrate[x*(c*ProductLog[a*x^n])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(cW(ax^n))^p\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$W(ax^n)^{-p}(cW(ax^n))^p\int xW(ax^n)^p dx$$

$$\Big\downarrow \text{ 7299}$$

$$W(ax^n)^{-p}(cW(ax^n))^p\int xW(ax^n)^p dx$$

input
```
Int[x*(c*ProductLog[a*x^n])^p,x]
```

output
```
$Aborted
```

Maple [F]

$$\int x(c\,\mathrm{LambertW}\,(a\,x^n))^p dx$$

input
```
int(x*(c*LambertW(a*x^n))^p,x)
```

output
```
int(x*(c*LambertW(a*x^n))^p,x)
```

Fricas [F]

$$\int x(cW(ax^n))^p\ dx = \int (c\,\mathrm{W}(ax^n))^p\, x\, dx$$

input
```
integrate(x*(c*lambert_w(a*x^n))^p,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^n))^p*x, x)
```

Sympy [F]

$$\int x(cW(ax^n))^p\ dx = \int x(cW(ax^n))^p\ dx$$

input
```
integrate(x*(c*LambertW(a*x**n))**p,x)
```

output
```
Integral(x*(c*LambertW(a*x**n))**p, x)
```

Maxima [F]

$$\int x(cW(ax^n))^p\,dx = \int (c\,\mathrm{W}(ax^n))^p\,x\,dx$$

input
```
integrate(x*(c*lambert_w(a*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^p*x, x)
```

Giac [F]

$$\int x(cW(ax^n))^p\,dx = \int (c\,\mathrm{W}(ax^n))^p\,x\,dx$$

input
```
integrate(x*(c*lambert_w(a*x^n))^p,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(cW(ax^n))^p\,dx = \int x\,(c\,\mathrm{LambertW}(a\,x^n))^p\,dx$$

input
```
int(x*(c*LambertW(a*x^n))^p,x)
```

output
```
int(x*(c*LambertW(a*x^n))^p, x)
```

Reduce [F]

$$\int x(cW(ax^n))^p\,dx$$
$$= \frac{c^p\left(lambert\_w(x^na)^p\,x^2-\left(\int \frac{x^n lambert\_w(x^na)^p x}{e^{lambert\_w(x^na)} lambert\_w(x^na)^2+e^{lambert\_w(x^na)} lambert\_w(x^na)}dx\right)anp\right)}{2}$$

input
```
int(x*(c*Lambert_W(a*x^n))^p,x)
```

output
```
(c**p*(lambert_w(x**n*a)**p*x**2 - int((x**n*lambert_w(x**n*a)**p*x)/(e**l
ambert_w(x**n*a)*lambert_w(x**n*a)**2 + e**lambert_w(x**n*a)*lambert_w(x**
n*a)),x)*a*n*p))/2
```

3.286 $\int (cW(ax^n))^p \, dx$

Optimal result

Integrand size = 10, antiderivative size = 162

$$\int (cW(ax^n))^p \, dx = \frac{e^{-\frac{(1-n)W(ax^n)}{n}} x^{1-n} \Gamma\left(1+\frac{1}{n}+p,-\frac{W(ax^n)}{n}\right) (cW(ax^n))^{1+p} \left(-\frac{W(ax^n)}{n}\right)^{-\frac{1}{n}-p}}{ac} + \frac{e^{-\frac{(1-n)W(ax^n)}{n}} x^{1-n} \Gamma\left(\frac{1}{n}+p,-\frac{W(ax^n)}{n}\right) (cW(ax^n))^p \left(-\frac{W(ax^n)}{n}\right)^{1-\frac{1}{n}-p}}{a}$$

output
```
x^(1-n)*GAMMA(1+1/n+p,-LambertW(a*x^n)/n)*(c*LambertW(a*x^n))^(p+1)*(-Lamb
ertW(a*x^n)/n)^(-1/n-p)/a/c/exp((1-n)*LambertW(a*x^n)/n)+x^(1-n)*GAMMA(1/n
+p,-LambertW(a*x^n)/n)*(c*LambertW(a*x^n))^p*(-LambertW(a*x^n)/n)^(1-1/n-p
)/a/exp((1-n)*LambertW(a*x^n)/n)
```

Mathematica [F]

$$\int (cW(ax^n))^p \, dx = \int (cW(ax^n))^p \, dx$$

input
```
Integrate[(c*ProductLog[a*x^n])^p,x]
```

output
```
Integrate[(c*ProductLog[a*x^n])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cW(ax^n))^p \, dx$$

$$\Big\downarrow \text{ 7271}$$

$$W(ax^n)^{-p}(cW(ax^n))^p \int W(ax^n)^p dx$$

$$\Big\downarrow \text{ 7299}$$

$$W(ax^n)^{-p}(cW(ax^n))^p \int W(ax^n)^p dx$$

input
```
Int[(c*ProductLog[a*x^n])^p,x]
```

output
```
$Aborted
```

Maple [F]

$$\int (c\,\text{LambertW}\,(a\,x^n))^p dx$$

input
```
int((c*LambertW(a*x^n))^p,x)
```

output
```
int((c*LambertW(a*x^n))^p,x)
```

Fricas [F]

$$\int (cW(ax^n))^p\, dx = \int (c\,\mathrm{W}(ax^n))^p\, dx$$

input
```
integrate((c*lambert_w(a*x^n))^p,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^n))^p, x)
```

Sympy [F]

$$\int (cW(ax^n))^p\, dx = \int (cW(ax^n))^p\, dx$$

input
```
integrate((c*LambertW(a*x**n))**p,x)
```

output
```
Integral((c*LambertW(a*x**n))**p, x)
```

Maxima [F]

$$\int (cW(ax^n))^p \, dx = \int (c\,\mathrm{W}(ax^n))^p \, dx$$

input
```
integrate((c*lambert_w(a*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^p, x)
```

Giac [F]

$$\int (cW(ax^n))^p \, dx = \int (c\,\mathrm{W}(ax^n))^p \, dx$$

input
```
integrate((c*lambert_w(a*x^n))^p,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cW(ax^n))^p \, dx = \int (c\,\mathrm{LambertW}(a\,x^n))^p \, dx$$

input
```
int((c*LambertW(a*x^n))^p,x)
```

output
```
int((c*LambertW(a*x^n))^p, x)
```

Reduce [F]

$$\int (cW(ax^n))^p \, dx$$
$$= c^p \left(lambert\_w(x^n a)^p x - \left(\int \frac{x^n lambert\_w(x^n a)^p}{e^{lambert\_w(x^n a)} lambert\_w(x^n a)^2 + e^{lambert\_w(x^n a)} lambert\_w(x^n a)} dx \right) anp \right)$$

input

```
int((c*Lambert_W(a*x^n))^p,x)
```

output

```
c**p*(lambert_w(x**n*a)**p*x - int((x**n*lambert_w(x**n*a)**p)/(e**lambert
_w(x**n*a)*lambert_w(x**n*a)**2 + e**lambert_w(x**n*a)*lambert_w(x**n*a)),
x)*a*n*p)
```

3.287 $\int \frac{(cW(ax^n))^p}{x} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 42

$$\int \frac{(cW(ax^n))^p}{x} \, dx = \frac{(cW(ax^n))^p}{np} + \frac{(cW(ax^n))^{1+p}}{cn(1+p)}$$

output
```
(c*LambertW(a*x^n))^p/n/p+(c*LambertW(a*x^n))^(p+1)/c/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{(cW(ax^n))^p}{x} \, dx = \frac{(cW(ax^n))^p (1+p+pW(ax^n))}{np(1+p)}$$

input
```
Integrate[(c*ProductLog[a*x^n])^p/x,x]
```

output
```
((c*ProductLog[a*x^n])^p*(1 + p + p*ProductLog[a*x^n]))/(n*p*(1 + p))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7173, 7200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cW(ax^n))^p}{x} dx$$
$$\Big\downarrow \ 7173$$
$$\frac{\int \frac{(cW(ax^n))^{p+1}}{x(W(ax^n)+1)}dx}{c} + \frac{(cW(ax^n))^p}{np}$$
$$\Big\downarrow \ 7200$$
$$\frac{(cW(ax^n))^p}{np} + \frac{(cW(ax^n))^{p+1}}{cn(p+1)}$$

input
```
Int[(c*ProductLog[a*x^n])^p/x,x]
```

output
```
(c*ProductLog[a*x^n])^p/(n*p) + (c*ProductLog[a*x^n])^(1 + p)/(c*n*(1 + p)
)
```

Defintions of rubi rules used

rule 7173
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + n*p + 1)), x] + Simp[n*(p/(c*(m +
 n*p + 1)))   Int[x^m*((c*ProductLog[a*x^n])^(p + 1)/(1 + ProductLog[a*x^n]
)), x], x] /; FreeQ[{a, c, m, n, p}, x] && (EqQ[m, -1] || (IntegerQ[p - 1/2
] && ILtQ[Simplify[p + (m + 1)/n] - 1/2, 0]) || ( !IntegerQ[p - 1/2] && ILt
Q[Simplify[p + (m + 1)/n], 0]))
```

rule 7200
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((x_)*((d_) + (d_.)*ProductL
og[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp[(c*ProductLog[a*x^n])^p/(d*n*p),
x] /; FreeQ[{a, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\frac{e^{p\ln(c\,\mathrm{LambertW}(a\,x^n))}}{p}+\frac{\mathrm{LambertW}(a\,x^n)\,e^{p\ln(c\,\mathrm{LambertW}(a\,x^n))}}{p+1}}{n}$	46
default	$\frac{\frac{e^{p\ln(c\,\mathrm{LambertW}(a\,x^n))}}{p}+\frac{\mathrm{LambertW}(a\,x^n)\,e^{p\ln(c\,\mathrm{LambertW}(a\,x^n))}}{p+1}}{n}$	46

input
```
int((c*LambertW(a*x^n))^p/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/p*exp(p*ln(c*LambertW(a*x^n)))+1/(p+1)*LambertW(a*x^n)*exp(p*ln(c*L
ambertW(a*x^n))))
```

Fricas [F]

$$\int \frac{(cW(ax^n))^p}{x}\,dx = \int \frac{(c\,\mathrm{W}(ax^n))^p}{x}\,dx$$

input
```
integrate((c*lambert_w(a*x^n))^p/x,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^n))^p/x, x)
```

Sympy [F]

$$\int \frac{(cW(ax^n))^p}{x}\,dx = \int \frac{(cW(ax^n))^p}{x}\,dx$$

input
```
integrate((c*LambertW(a*x**n))**p/x,x)
```

output
```
Integral((c*LambertW(a*x**n))**p/x, x)
```

Maxima [F]

$$\int \frac{(cW(ax^n))^p}{x}\,dx = \int \frac{(c\,\mathrm{W}(ax^n))^p}{x}\,dx$$

input
```
integrate((c*lambert_w(a*x^n))^p/x,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^p/x, x)
```

Giac [F]

$$\int \frac{(cW(ax^n))^p}{x}\,dx = \int \frac{(c\,\mathrm{W}(ax^n))^p}{x}\,dx$$

input
```
integrate((c*lambert_w(a*x^n))^p/x,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cW(ax^n))^p}{x} \, dx = \int \frac{(c \, \mathrm{LambertW}(a \, x^n))^p}{x} \, dx$$

input
```
int((c*LambertW(a*x^n))^p/x,x)
```

output
```
int((c*LambertW(a*x^n))^p/x, x)
```

Reduce [F]

$$\int \frac{(cW(ax^n))^p}{x} \, dx = c^p \left(\int \frac{lambert\_w(x^n a)^p}{x} dx \right)$$

input
```
int((c*Lambert_W(a*x^n))^p/x,x)
```

output
```
c**p*int(lambert_w(x**n*a)**p/x,x)
```

3.288 $\int \frac{(cW(ax^n))^p}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 152

$$\int \frac{(cW(ax^n))^p}{x^2}\,dx = -\frac{e^{\left(1+\frac{1}{n}\right)W(ax^n)}x^{-1-n}\Gamma\left(1-\frac{1}{n}+p,\frac{W(ax^n)}{n}\right)(cW(ax^n))^{1+p}\left(\frac{W(ax^n)}{n}\right)^{\frac{1}{n}-p}}{ac} - \frac{e^{\left(1+\frac{1}{n}\right)W(ax^n)}x^{-1-n}\Gamma\left(-\frac{1}{n}+p,\frac{W(ax^n)}{n}\right)(cW(ax^n))^{p}\left(\frac{W(ax^n)}{n}\right)^{1+\frac{1}{n}-p}}{a}$$

output

```
-exp((1+1/n)*LambertW(a*x^n))*x^(-1-n)*GAMMA(1-1/n+p,LambertW(a*x^n)/n)*(c
*LambertW(a*x^n))^(p+1)*(LambertW(a*x^n)/n)^(1/n-p)/a/c-exp((1+1/n)*Lamber
tW(a*x^n))*x^(-1-n)*GAMMA(-1/n+p,LambertW(a*x^n)/n)*(c*LambertW(a*x^n))^p*
(LambertW(a*x^n)/n)^(1+1/n-p)/a
```

Mathematica [F]

$$\int \frac{(cW(ax^n))^p}{x^2} \, dx = \int \frac{(cW(ax^n))^p}{x^2} \, dx$$

input
```
Integrate[(c*ProductLog[a*x^n])^p/x^2,x]
```

output
```
Integrate[(c*ProductLog[a*x^n])^p/x^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cW(ax^n))^p}{x^2} \, dx$$
$$\Big\downarrow \text{ 7271}$$
$$W(ax^n)^{-p}(cW(ax^n))^p \int \frac{W(ax^n)^p}{x^2}dx$$
$$\Big\downarrow \text{ 7299}$$
$$W(ax^n)^{-p}(cW(ax^n))^p \int \frac{W(ax^n)^p}{x^2}dx$$

input
```
Int[(c*ProductLog[a*x^n])^p/x^2,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(c \operatorname{LambertW}(a x^n))^p}{x^2} dx$$

input
```
int((c*LambertW(a*x^n))^p/x^2,x)
```

output
```
int((c*LambertW(a*x^n))^p/x^2,x)
```

Fricas [F]

$$\int \frac{(cW(ax^n))^p}{x^2} dx = \int \frac{(c \operatorname{W}(ax^n))^p}{x^2} dx$$

input
```
integrate((c*lambert_w(a*x^n))^p/x^2,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^n))^p/x^2, x)
```

Sympy [F]

$$\int \frac{(cW(ax^n))^p}{x^2} dx = \int \frac{(cW(ax^n))^p}{x^2} dx$$

input
```
integrate((c*LambertW(a*x**n))**p/x**2,x)
```

output
```
Integral((c*LambertW(a*x**n))**p/x**2, x)
```

Maxima [F]

$$\int \frac{(cW(ax^n))^p}{x^2}\,dx = \int \frac{(c\,\mathrm{W}(ax^n))^p}{x^2}\,dx$$

input
```
integrate((c*lambert_w(a*x^n))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^p/x^2, x)
```

Giac [F]

$$\int \frac{(cW(ax^n))^p}{x^2}\,dx = \int \frac{(c\,\mathrm{W}(ax^n))^p}{x^2}\,dx$$

input
```
integrate((c*lambert_w(a*x^n))^p/x^2,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cW(ax^n))^p}{x^2}\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x^n))^p}{x^2}\,dx$$

input
```
int((c*LambertW(a*x^n))^p/x^2,x)
```

output
```
int((c*LambertW(a*x^n))^p/x^2, x)
```

Reduce [F]

$$\int \frac{(cW(ax^n))^p}{x^2}\,dx$$
$$= \frac{c^p\left(-lambert\_w(x^n a)^p + \left(\int \frac{x^n lambert\_w(x^n a)^p}{e^{lambert\_w(x^n a)} lambert\_w(x^n a)^2 x^2 + e^{lambert\_w(x^n a)} lambert\_w(x^n a) x^2} dx\right) anpx\right)}{x}$$

input
```
int((c*Lambert_W(a*x^n))^p/x^2,x)
```

output
```
(c**p*( - lambert_w(x**n*a)**p + int((x**n*lambert_w(x**n*a)**p)/(e**lambe
rt_w(x**n*a)*lambert_w(x**n*a)**2*x**2 + e**lambert_w(x**n*a)*lambert_w(x*
*n*a)*x**2),x)*a*n*p*x))/x
```

3.289 $\int \frac{(cW(ax^n))^p}{x^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 183

$$\int \frac{(cW(ax^n))^p}{x^3} dx = -\frac{2^{-1+\frac{2}{n}-p}e^{\frac{(2+n)W(ax^n)}{n}}x^{-2-n}\Gamma\left(1-\frac{2}{n}+p,\frac{2W(ax^n)}{n}\right)(cW(ax^n))^{1+p}\left(\frac{W(ax^n)}{n}\right)^{\frac{2}{n}-p}}{ac} - \frac{2^{\frac{2}{n}-p}e^{\frac{(2+n)W(ax^n)}{n}}x^{-2-n}\Gamma\left(-\frac{2}{n}+p,\frac{2W(ax^n)}{n}\right)(cW(ax^n))^{p}\left(\frac{W(ax^n)}{n}\right)^{1+\frac{2}{n}-p}}{a}$$

output

```
-2^(-1+2/n-p)*exp((2+n)*LambertW(a*x^n)/n)*x^(-2-n)*GAMMA(1-2/n+p,2*Lamber
tW(a*x^n)/n)*(c*LambertW(a*x^n))^(p+1)*(LambertW(a*x^n)/n)^(2/n-p)/a/c-2^(
2/n-p)*exp((2+n)*LambertW(a*x^n)/n)*x^(-2-n)*GAMMA(-2/n+p,2*LambertW(a*x^n
)/n)*(c*LambertW(a*x^n))^p*(LambertW(a*x^n)/n)^(1+2/n-p)/a
```

Mathematica [F]

$$\int \frac{(cW(ax^n))^p}{x^3}\,dx = \int \frac{(cW(ax^n))^p}{x^3}\,dx$$

input
```
Integrate[(c*ProductLog[a*x^n])^p/x^3,x]
```

output
```
Integrate[(c*ProductLog[a*x^n])^p/x^3, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cW(ax^n))^p}{x^3}\,dx$$

$$\Big\downarrow \ 7271$$

$$W(ax^n)^{-p}(cW(ax^n))^p\int \frac{W(ax^n)^p}{x^3}dx$$

$$\Big\downarrow \ 7299$$

$$W(ax^n)^{-p}(cW(ax^n))^p\int \frac{W(ax^n)^p}{x^3}dx$$

input
```
Int[(c*ProductLog[a*x^n])^p/x^3,x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{(c\,\mathrm{LambertW}\,(a\,x^n))^p}{x^3}dx$$

input
```
int((c*LambertW(a*x^n))^p/x^3,x)
```

output
```
int((c*LambertW(a*x^n))^p/x^3,x)
```

Fricas [F]

$$\int \frac{(cW(ax^n))^p}{x^3}\,dx = \int \frac{(c\,\mathrm{W}(ax^n))^p}{x^3}\,dx$$

input
```
integrate((c*lambert_w(a*x^n))^p/x^3,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^n))^p/x^3, x)
```

Sympy [F]

$$\int \frac{(cW(ax^n))^p}{x^3}\,dx = \int \frac{(cW(ax^n))^p}{x^3}\,dx$$

input
```
integrate((c*LambertW(a*x**n))**p/x**3,x)
```

output
```
Integral((c*LambertW(a*x**n))**p/x**3, x)
```

Maxima [F]

$$\int \frac{(cW(ax^n))^p}{x^3}\,dx = \int \frac{(c\,\mathrm{W}(ax^n))^p}{x^3}\,dx$$

input
```
integrate((c*lambert_w(a*x^n))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^p/x^3, x)
```

Giac [F]

$$\int \frac{(cW(ax^n))^p}{x^3}\,dx = \int \frac{(c\,\mathrm{W}(ax^n))^p}{x^3}\,dx$$

input
```
integrate((c*lambert_w(a*x^n))^p/x^3,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cW(ax^n))^p}{x^3}\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x^n))^p}{x^3}\,dx$$

input
```
int((c*LambertW(a*x^n))^p/x^3,x)
```

output
```
int((c*LambertW(a*x^n))^p/x^3, x)
```

Reduce [F]

$$\int \frac{(cW(ax^n))^p}{x^3}\,dx$$
$$= \frac{c^p\left(-lambert\_w(x^n a)^p + \left(\int \frac{x^n lambert\_w(x^n a)^p}{e^{lambert\_w(x^n a)} lambert\_w(x^n a)^2 x^3 + e^{lambert\_w(x^n a)} lambert\_w(x^n a)x^3} dx\right) anp\, x^2\right)}{2x^2}$$

input
```
int((c*Lambert_W(a*x^n))^p/x^3,x)
```

output
```
(c**p*( - lambert_w(x**n*a)**p + int((x**n*lambert_w(x**n*a)**p)/(e**lambe
rt_w(x**n*a)*lambert_w(x**n*a)**2*x**3 + e**lambert_w(x**n*a)*lambert_w(x*
*n*a)*x**3),x)*a*n*p*x**2))/(2*x**2)
```

3.290 $\int W(ax^n)^{\frac{-1+n}{n}} dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int W(ax^n)^{\frac{-1+n}{n}} dx = (1-n)xW(ax^n)^{-1/n} + xW(ax^n)^{-\frac{1-n}{n}}$$

output
```
(1-n)*x/(LambertW(a*x^n)^(1/n))+x/(LambertW(a*x^n)^((1-n)/n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int W(ax^n)^{\frac{-1+n}{n}} dx = xW(ax^n)^{-1/n}(1-n+W(ax^n))$$

input
```
Integrate[ProductLog[a*x^n]^((-1 + n)/n),x]
```

output
```
(x*(1 - n + ProductLog[a*x^n]))/ProductLog[a*x^n]^n^(-1)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7169, 7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W(ax^n)^{\frac{n-1}{n}}\,dx$$

$$\Big\downarrow \text{7169}$$

$$(1-n)\int \frac{W(ax^n)^{-\frac{1-n}{n}}}{W(ax^n)+1}dx + xW(ax^n)^{-\frac{1-n}{n}}$$

$$\Big\downarrow \text{7187}$$

$$(1-n)xW(ax^n)^{-1/n} + xW(ax^n)^{-\frac{1-n}{n}}$$

input
```
Int[ProductLog[a*x^n]^((-1 + n)/n),x]
```

output
```
((1 - n)*x)/ProductLog[a*x^n]^n^(-1) + x/ProductLog[a*x^n]^((1 - n)/n)
```

Defintions of rubi rules used

rule 7169
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*(c*Produ
ctLog[a*x^n])^p, x] - Simp[n*p   Int[(c*ProductLog[a*x^n])^p/(1 + ProductLo
g[a*x^n]), x], x] /; FreeQ[{a, c, n, p}, x] && (EqQ[n*(p - 1), -1] || (Inte
gerQ[p - 1/2] && EqQ[n*(p - 1/2), -1]))
```

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.77

method	result	size
parallelrisch	$-\frac{-\operatorname{LambertW}(a\,x^{n})^{\frac{-1+n}{n}} x \operatorname{LambertW}(a\,x^{n})+\operatorname{LambertW}(a\,x^{n})^{\frac{-1+n}{n}} x n-\operatorname{LambertW}(a\,x^{n})^{\frac{-1+n}{n}} x}{\operatorname{LambertW}(a\,x^{n})}$	69

input
```
int(LambertW(a*x^n)^((-1+n)/n),x,method=_RETURNVERBOSE)
```

output
```
-(-LambertW(a*x^n)^((-1+n)/n)*x*LambertW(a*x^n)+LambertW(a*x^n)^((-1+n)/n)
*x*n-LambertW(a*x^n)^((-1+n)/n)*x)/LambertW(a*x^n)
```

Fricas [F]

$$\int W(ax^n)^{\frac{-1+n}{n}}\,dx=\int \mathrm{W}(ax^n)^{\frac{n-1}{n}}\,dx$$

input
```
integrate(lambert_w(a*x^n)^((-1+n)/n),x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^n)^((n - 1)/n), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(27) = 54.

Time = 0.84 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int W(ax^n)^{\frac{-1+n}{n}}\,dx=\begin{cases}-\frac{nxW^{1-\frac{1}{n}}(ax^n)}{W(ax^n)}+xW^{1-\frac{1}{n}}(ax^n)+\frac{xW^{1-\frac{1}{n}}(ax^n)}{W(ax^n)} & \text{for } a\neq 0\\ 0^{\frac{n-1}{n}}x & \text{otherwise}\end{cases}$$

input
```
integrate(LambertW(a*x**n)**((-1+n)/n),x)
```

output
```
Piecewise((-n*x*LambertW(a*x**n)**(1 - 1/n)/LambertW(a*x**n) + x*LambertW(
a*x**n)**(1 - 1/n) + x*LambertW(a*x**n)**(1 - 1/n)/LambertW(a*x**n), Ne(a,
 0)), (0**((n - 1)/n)*x, True))
```

Maxima [F]

$$\int W(ax^n)^{\frac{-1+n}{n}}\,dx = \int \mathrm{W}(ax^n)^{\frac{n-1}{n}}\,dx$$

input
```
integrate(lambert_w(a*x^n)^((-1+n)/n),x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^n)^((n - 1)/n), x)
```

Giac [F]

$$\int W(ax^n)^{\frac{-1+n}{n}}\,dx = \int \mathrm{W}(ax^n)^{\frac{n-1}{n}}\,dx$$

input
```
integrate(lambert_w(a*x^n)^((-1+n)/n),x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^n)^((n - 1)/n), x)
```

Mupad [F(-1)]

Timed out.

$$\int W(ax^n)^{\frac{-1+n}{n}}\,dx = \int \mathrm{LambertW}(a\,x^n)^{\frac{n-1}{n}}\,dx$$

input
```
int(LambertW(a*x^n)^((n - 1)/n),x)
```

output
```
int(LambertW(a*x^n)^((n - 1)/n), x)
```

Reduce [F]

$$\int W(ax^n)^{\frac{-1+n}{n}}\,dx = \int \frac{lambert\_w(x^n a)}{lambert\_w(x^n a)^{\frac{1}{n}}}dx$$

input
```
int(Lambert_W(a*x^n)^((-1+n)/n),x)
```

output
```
int(lambert_w(x**n*a)/lambert_w(x**n*a)**(1/n),x)
```

3.291 $\int W\left(ax^{\frac{1}{1-p}}\right)^p\,dx$

Optimal result

Integrand size = 14, antiderivative size = 44

$$\int W\left(ax^{\frac{1}{1-p}}\right)^p\,dx=-\frac{pxW\left(ax^{\frac{1}{1-p}}\right)^{-1+p}}{1-p}+xW\left(ax^{\frac{1}{1-p}}\right)^p$$

output
```
-p*x*LambertW(a*x^(1/(1-p)))^(-1+p)/(1-p)+x*LambertW(a*x^(1/(1-p)))^p
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int W\left(ax^{\frac{1}{1-p}}\right)^p\,dx=x\left(1+\frac{p}{(-1+p)W\left(ax^{\frac{1}{1-p}}\right)}\right)W\left(ax^{\frac{1}{1-p}}\right)^p$$

input
```
Integrate[ProductLog[a*x^(1 - p)^(-1)]^p,x]
```

output
```
x*(1 + p/((-1 + p)*ProductLog[a*x^(1 - p)^(-1)]))*ProductLog[a*x^(1 - p)^(
-1)]^p
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7169, 7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W\left(ax^{\frac{1}{1-p}}\right)^p dx$$

$$\Big\downarrow \text{ 7169}$$

$$xW\left(ax^{\frac{1}{1-p}}\right)^p - \frac{p\int \frac{W\left(ax^{\frac{1}{1-p}}\right)^p}{W\left(ax^{\frac{1}{1-p}}\right)+1}dx}{1-p}$$

$$\Big\downarrow \text{ 7187}$$

$$xW\left(ax^{\frac{1}{1-p}}\right)^p - \frac{pxW\left(ax^{\frac{1}{1-p}}\right)^{p-1}}{1-p}$$

input
```
Int[ProductLog[a*x^(1 - p)^(-1)]^p,x]
```

output
```
-((p*x*ProductLog[a*x^(1 - p)^(-1)]^(-1 + p))/(1 - p)) + x*ProductLog[a*x^
(1 - p)^(-1)]^p
```

Defintions of rubi rules used

rule 7169
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[x*(c*Produ
ctLog[a*x^n])^p, x] - Simp[n*p   Int[(c*ProductLog[a*x^n])^p/(1 + ProductLo
g[a*x^n]), x], x] /; FreeQ[{a, c, n, p}, x] && (EqQ[n*(p - 1), -1] || (Inte
gerQ[p - 1/2] && EqQ[n*(p - 1/2), -1]))
```

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(44) = 88$.

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.20

method	result
parallelrisch	$\frac{\mathrm{LambertW}\left(a\,x^{\frac{1}{1-p}}\right)^{p} x\,\mathrm{LambertW}\left(a\,x^{\frac{1}{1-p}}\right)p-\mathrm{LambertW}\left(a\,x^{\frac{1}{1-p}}\right)^{p} x\,\mathrm{LambertW}\left(a\,x^{\frac{1}{1-p}}\right)+p\,\mathrm{LambertW}\left(a\,x^{\frac{1}{1-p}}\right)^{p}}{(p-1)\,\mathrm{LambertW}\left(a\,x^{\frac{1}{1-p}}\right)}$

input
```
int(LambertW(a*x^(1/(1-p)))^p,x,method=_RETURNVERBOSE)
```

output
```
(LambertW(a*x^(1/(1-p)))^p*x*LambertW(a*x^(1/(1-p)))*p-LambertW(a*x^(1/(1-
p)))^p*x*LambertW(a*x^(1/(1-p)))+p*LambertW(a*x^(1/(1-p)))^p*x)/(p-1)/Lamb
ertW(a*x^(1/(1-p)))
```

Fricas [F]

$$\int W\left(ax^{\frac{1}{1-p}}\right)^{p}\,dx=\int \mathrm{W}\left(\frac{a}{x^{\left(\frac{1}{p-1}\right)}}\right)^{p}\,dx$$

input
```
integrate(lambert_w(a*x^(1/(1-p)))^p,x, algorithm="fricas")
```

output
```
integral(lambert_w(a/x^(1/(p - 1)))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int W\left(ax^{\frac{1}{1-p}}\right)^p\,dx = \text{Timed out}$$

input
```
integrate(LambertW(a*x**(1/(1-p)))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int W\left(ax^{\frac{1}{1-p}}\right)^p\,dx = \int \mathrm{W}(\frac{a}{x^{\left(\frac{1}{p-1}\right)}})^p\,dx$$

input
```
integrate(lambert_w(a*x^(1/(1-p)))^p,x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x^(1/(p - 1)))^p, x)
```

Giac [F]

$$\int W\left(ax^{\frac{1}{1-p}}\right)^p\,dx = \int \mathrm{W}(\frac{a}{x^{\left(\frac{1}{p-1}\right)}})^p\,dx$$

input
```
integrate(lambert_w(a*x^(1/(1-p)))^p,x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x^(1/(p - 1)))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int W\left(ax^{\frac{1}{1-p}}\right)^p dx = \int \mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{p-1}}}\right)^p dx$$

input
```
int(LambertW(a/x^(1/(p - 1)))^p,x)
```

output
```
int(LambertW(a/x^(1/(p - 1)))^p, x)
```

Reduce [F]

$$\int W\left(ax^{\frac{1}{1-p}}\right)^p dx = lambert\_w\left(\frac{a}{x^{\frac{1}{p-1}}}\right)^p x + \left(\int \frac{lambert\_w\left(\frac{a}{x^{\frac{1}{p-1}}}\right)^p}{x^{\frac{1}{p-1}} e^{lambert\_w\left(\frac{a}{x^{\frac{1}{p-1}}}\right)} lambert\_w\left(\frac{a}{x^{\frac{1}{p-1}}}\right)^2 p - x^{\frac{1}{p-1}} e^{lambert\_w\left(\frac{a}{x^{\frac{1}{p-1}}}\right)} lambert\_w\left(\frac{a}{x^{\frac{1}{p-1}}}\right)^2 + x^{\frac{1}{p-1}} e^{la}}\right.$$

input
```
int(Lambert_W(a*x^(1/(1-p)))^p,x)
```

output
```
lambert_w(a/x**(1/(p - 1)))**p*x + int(lambert_w(a/x**(1/(p - 1)))**p/(x**
(1/(p - 1))*e**lambert_w(a/x**(1/(p - 1)))*lambert_w(a/x**(1/(p - 1)))**2*
p - x**(1/(p - 1))*e**lambert_w(a/x**(1/(p - 1)))*lambert_w(a/x**(1/(p - 1
)))**2 + x**(1/(p - 1))*e**lambert_w(a/x**(1/(p - 1)))*lambert_w(a/x**(1/(
p - 1)))*p - x**(1/(p - 1))*e**lambert_w(a/x**(1/(p - 1)))*lambert_w(a/x**
(1/(p - 1)))),x)*a*p
```

3.292 $\int x^m (cW(ax^n))^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 201

$$\int x^m (cW(ax^n))^p \, dx = \frac{e^{\left(1-\frac{1+m}{n}\right)W(ax^n)} x^{1+m-n} \Gamma\left(\frac{1+m}{n}+p, -\frac{(1+m)W(ax^n)}{n}\right) (cW(ax^n))^p \left(-\frac{(1+m)W(ax^n)}{n}\right)^{1-\frac{1+m}{n}-p}}{a(1+m)} + \frac{e^{\left(1-\frac{1+m}{n}\right)W(ax^n)} x^{1+m-n} \Gamma\left(1+\frac{1+m}{n}+p, -\frac{(1+m)W(ax^n)}{n}\right) (cW(ax^n))^{1+p} \left(-\frac{(1+m)W(ax^n)}{n}\right)^{-\frac{1+m+np}{n}}}{ac(1+m)}$$

output
```
exp((1-(1+m)/n)*LambertW(a*x^n))*x^(1+m-n)*GAMMA((1+m)/n+p,-(1+m)*LambertW
(a*x^n)/n)*(c*LambertW(a*x^n))^p*(-(1+m)*LambertW(a*x^n)/n)^(1-(1+m)/n-p)/
a/(1+m)+exp((1-(1+m)/n)*LambertW(a*x^n))*x^(1+m-n)*GAMMA(1+(1+m)/n+p,-(1+m
)*LambertW(a*x^n)/n)*(c*LambertW(a*x^n))^(p+1)/a/c/(1+m)/((-(1+m)*LambertW
(a*x^n)/n)^((n*p+m+1)/n))
```

Mathematica [F]

$$\int x^m(cW(ax^n))^p\,dx=\int x^m(cW(ax^n))^p\,dx$$

input
```
Integrate[x^m*(c*ProductLog[a*x^n])^p,x]
```

output
```
Integrate[x^m*(c*ProductLog[a*x^n])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m(cW(ax^n))^p\,dx$$

$$\Big\downarrow \text{7271}$$

$$W(ax^n)^{-p}(cW(ax^n))^p\int x^mW(ax^n)^pdx$$

$$\Big\downarrow \text{7299}$$

$$W(ax^n)^{-p}(cW(ax^n))^p\int x^mW(ax^n)^pdx$$

input
```
Int[x^m*(c*ProductLog[a*x^n])^p,x]
```

output
```
$Aborted
```

Maple [F]

$$\int x^m (c\,\mathrm{LambertW}\left(a\,x^n\right))^p dx$$

input
```
int(x^m*(c*LambertW(a*x^n))^p,x)
```

output
```
int(x^m*(c*LambertW(a*x^n))^p,x)
```

Fricas [F]

$$\int x^m (cW(ax^n))^p \, dx = \int (c\,\mathrm{W}(ax^n))^p \, x^m \, dx$$

input
```
integrate(x^m*(c*lambert_w(a*x^n))^p,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^n))^p*x^m, x)
```

Sympy [F]

$$\int x^m (cW(ax^n))^p \, dx = \int x^m (cW(ax^n))^p \, dx$$

input
```
integrate(x**m*(c*LambertW(a*x**n))**p,x)
```

output
```
Integral(x**m*(c*LambertW(a*x**n))**p, x)
```

Maxima [F]

$$\int x^m (cW(ax^n))^p \, dx = \int (c\,\mathrm{W}(ax^n))^p x^m \, dx$$

input
```
integrate(x^m*(c*lambert_w(a*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^p*x^m, x)
```

Giac [F]

$$\int x^m (cW(ax^n))^p \, dx = \int (c\,\mathrm{W}(ax^n))^p x^m \, dx$$

input
```
integrate(x^m*(c*lambert_w(a*x^n))^p,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^p*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m (cW(ax^n))^p \, dx = \int x^m \left(c\,\mathrm{LambertW}(a\,x^n)\right)^p \, dx$$

input
```
int(x^m*(c*LambertW(a*x^n))^p,x)
```

output
```
int(x^m*(c*LambertW(a*x^n))^p, x)
```

Reduce [F]

$$\int x^m (cW(ax^n))^p \, dx$$
$$= \frac{c^p \left(x^m lambert\_w(x^n a)^p x - \left(\int \frac{x^{m+n} lambert\_w(x^n a)^p}{e^{lambert\_w(x^n a)} lambert\_w(x^n a)^2 m + e^{lambert\_w(x^n a)} lambert\_w(x^n a)^2 + e^{lambert\_w(x^n a)} \cdots} \right.\right.}{\cdots}$$

input
```
int(x^m*(c*Lambert_W(a*x^n))^p,x)
```

output
```
(c**p*(x**m*lambert_w(x**n*a)**p*x - int((x**(m + n)*lambert_w(x**n*a)**p)
/(e**lambert_w(x**n*a)*lambert_w(x**n*a)**2*m + e**lambert_w(x**n*a)*lambe
rt_w(x**n*a)**2 + e**lambert_w(x**n*a)*lambert_w(x**n*a)*m + e**lambert_w(
x**n*a)*lambert_w(x**n*a)),x)*a*m*n*p - int((x**(m + n)*lambert_w(x**n*a)*
*p)/(e**lambert_w(x**n*a)*lambert_w(x**n*a)**2*m + e**lambert_w(x**n*a)*la
mbert_w(x**n*a)**2 + e**lambert_w(x**n*a)*lambert_w(x**n*a)*m + e**lambert
_w(x**n*a)*lambert_w(x**n*a)),x)*a*n*p))/(m + 1)
```

3.293 $\int x^{-1-n(1+p)}(cW(ax^n))^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int x^{-1-n(1+p)}(cW(ax^n))^p\,dx = -\frac{x^{-n(1+p)}(cW(ax^n))^p}{n} + \frac{e^{(2+p)W(ax^n)}px^{-n(2+p)}\Gamma(0,(1+p)W(ax^n))\,W(ax^n)\,(cW(ax^n))^{1+p}}{acn}$$

output
```
-(c*LambertW(a*x^n))^p/n/(x^(n*(p+1)))+exp((2+p)*LambertW(a*x^n))*p*Ei(1,(
p+1)*LambertW(a*x^n))*LambertW(a*x^n)*(c*LambertW(a*x^n))^(p+1)/a/c/n/(x^(
n*(2+p)))
```

Mathematica [F]

$$\int x^{-1-n(1+p)}(cW(ax^n))^p\,dx = \int x^{-1-n(1+p)}(cW(ax^n))^p\,dx$$

input
```
Integrate[x^(-1 - n*(1 + p))*(c*ProductLog[a*x^n])^p,x]
```

output
```
Integrate[x^(-1 - n*(1 + p))*(c*ProductLog[a*x^n])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p+1)-1}(cW(ax^n))^p\,dx$$

$$\Big\downarrow \text{ 7173}$$

$$-\frac{p\int \frac{x^{-n(p+1)-1}(cW(ax^n))^{p+1}}{W(ax^n)+1}dx}{c}-\frac{x^{-n(p+1)}(cW(ax^n))^p}{n}$$

$$\Big\downarrow \text{ 7271}$$

$$-pW(ax^n)^{-p}(cW(ax^n))^p\int \frac{x^{-n(p+1)-1}W(ax^n)^{p+1}}{W(ax^n)+1}dx-\frac{x^{-n(p+1)}(cW(ax^n))^p}{n}$$

$$\Big\downarrow \text{ 7299}$$

$$-pW(ax^n)^{-p}(cW(ax^n))^p\int \frac{x^{-n(p+1)-1}W(ax^n)^{p+1}}{W(ax^n)+1}dx-\frac{x^{-n(p+1)}(cW(ax^n))^p}{n}$$

input
```
Int[x^(-1 - n*(1 + p))*(c*ProductLog[a*x^n])^p,x]
```

output
```
$Aborted
```

Maple [F]

$$\int x^{-1-n(p+1)}(c\,\mathrm{LambertW}(a\,x^n))^p dx$$

input
```
int(x^(-1-n*(p+1))*(c*LambertW(a*x^n))^p,x)
```

output
```
int(x^(-1-n*(p+1))*(c*LambertW(a*x^n))^p,x)
```

Fricas [F]

$$\int x^{-1-n(1+p)}(cW(ax^n))^p\,dx = \int (c\,\mathrm{W}(ax^n))^p\,x^{-n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(c*lambert_w(a*x^n))^p,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^n))^p*x^(-n*p - n - 1), x)
```

Sympy [F]

$$\int x^{-1-n(1+p)}(cW(ax^n))^p\,dx = \int x^{-n(p+1)-1}(cW(ax^n))^p\,dx$$

input
```
integrate(x**(-1-n*(p+1))*(c*LambertW(a*x**n))**p,x)
```

output
```
Integral(x**(-n*(p + 1) - 1)*(c*LambertW(a*x**n))**p, x)
```

Maxima [F]

$$\int x^{-1-n(1+p)}(cW(ax^n))^p\,dx = \int (c\,\mathrm{W}(ax^n))^p\,x^{-n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(c*lambert_w(a*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^p*x^(-n*(p + 1) - 1), x)
```

Giac [F]

$$\int x^{-1-n(1+p)}(cW(ax^n))^p \, dx = \int (c\,\mathrm{W}(ax^n))^p \, x^{-n(p+1)-1} \, dx$$

input
```
integrate(x^(-1-n*(p+1))*(c*lambert_w(a*x^n))^p,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^p*x^(-n*(p + 1) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(1+p)}(cW(ax^n))^p \, dx = \int \frac{(c\,\mathrm{LambertW}(a\,x^n))^p}{x^{n\,(p+1)+1}} \, dx$$

input
```
int((c*LambertW(a*x^n))^p/x^(n*(p + 1) + 1),x)
```

output
```
int((c*LambertW(a*x^n))^p/x^(n*(p + 1) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(1+p)}(cW(ax^n))^p \, dx$$
$$= \frac{c^p \left(-lambert\_w(x^n a)^p + x^{np+n} \left(\int \frac{lambert\_\cdots}{x^{np} e^{lambert\_w(x^n a)} lambert\_w(x^n a)^2 px + x^{np} e^{lambert\_w(x^n a)} lambert\_w(x^n a)^2 x + \cdots}\right.\right.}{\cdots}$$

input
```
int(x^(-1-n*(p+1))*(c*Lambert_W(a*x^n))^p,x)
```

output

```
(c**p*( - lambert_w(x**n*a)**p + x**(n*p + n)*int(lambert_w(x**n*a)**p/(x*
*(n*p)*e**lambert_w(x**n*a)*lambert_w(x**n*a)**2*p*x + x**(n*p)*e**lambert
_w(x**n*a)*lambert_w(x**n*a)**2*x + x**(n*p)*e**lambert_w(x**n*a)*lambert_
w(x**n*a)*p*x + x**(n*p)*e**lambert_w(x**n*a)*lambert_w(x**n*a)*x),x)*a*n*
p**2 + x**(n*p + n)*int(lambert_w(x**n*a)**p/(x**(n*p)*e**lambert_w(x**n*a
)*lambert_w(x**n*a)**2*p*x + x**(n*p)*e**lambert_w(x**n*a)*lambert_w(x**n*
a)**2*x + x**(n*p)*e**lambert_w(x**n*a)*lambert_w(x**n*a)*p*x + x**(n*p)*e
**lambert_w(x**n*a)*lambert_w(x**n*a)*x),x)*a*n*p))/(x**(n*p + n)*n*(p + 1
))
```

3.294 $\int x^{-1-np}(cW(ax^n))^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int x^{-1-np}(cW(ax^n))^p \, dx = -\frac{x^{-np}(cW(ax^n))^p}{np} - \frac{e^{(1+p)W(ax^n)}x^{-n(1+p)}\Gamma(0,pW(ax^n))\,W(ax^n)\,(cW(ax^n))^p}{an}$$

output
```
-(c*LambertW(a*x^n))^p/n/p/(x^(n*p))-exp((p+1)*LambertW(a*x^n))*Ei(1,p*Lam
bertW(a*x^n))*LambertW(a*x^n)*(c*LambertW(a*x^n))^p/a/n/(x^(n*(p+1)))
```

Mathematica [F]

$$\int x^{-1-np}(cW(ax^n))^p \, dx = \int x^{-1-np}(cW(ax^n))^p \, dx$$

input
```
Integrate[x^(-1 - n*p)*(c*ProductLog[a*x^n])^p,x]
```

output
```
Integrate[x^(-1 - n*p)*(c*ProductLog[a*x^n])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-np-1}(cW(ax^n))^p\,dx$$

$$\Big\downarrow \text{ 7172}$$

$$\int \frac{x^{-np-1}(cW(ax^n))^p}{W(ax^n)+1}dx - \frac{x^{-np}(cW(ax^n))^p}{np}$$

$$\Big\downarrow \text{ 7271}$$

$$W(ax^n)^{-p}(cW(ax^n))^p\int \frac{x^{-np-1}W(ax^n)^p}{W(ax^n)+1}dx - \frac{x^{-np}(cW(ax^n))^p}{np}$$

$$\Big\downarrow \text{ 7299}$$

$$W(ax^n)^{-p}(cW(ax^n))^p\int \frac{x^{-np-1}W(ax^n)^p}{W(ax^n)+1}dx - \frac{x^{-np}(cW(ax^n))^p}{np}$$

input
```
Int[x^(-1 - n*p)*(c*ProductLog[a*x^n])^p,x]
```

output
```
$Aborted
```

Maple [F]

$$\int x^{-np-1}(c\,\mathrm{LambertW}(a\,x^n))^p dx$$

input
```
int(x^(-n*p-1)*(c*LambertW(a*x^n))^p,x)
```

output
```
int(x^(-n*p-1)*(c*LambertW(a*x^n))^p,x)
```

Fricas [F]

$$\int x^{-1-np}(cW(ax^n))^p\,dx=\int (c\,\mathrm{W}(ax^n))^p\,x^{-np-1}\,dx$$

input
```
integrate(x^(-n*p-1)*(c*lambert_w(a*x^n))^p,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^n))^p*x^(-n*p - 1), x)
```

Sympy [F]

$$\int x^{-1-np}(cW(ax^n))^p\,dx=\int x^{-np-1}(cW(ax^n))^p\,dx$$

input
```
integrate(x**(-n*p-1)*(c*LambertW(a*x**n))**p,x)
```

output
```
Integral(x**(-n*p - 1)*(c*LambertW(a*x**n))**p, x)
```

Maxima [F]

$$\int x^{-1-np}(cW(ax^n))^p\,dx=\int (c\,\mathrm{W}(ax^n))^p\,x^{-np-1}\,dx$$

input
```
integrate(x^(-n*p-1)*(c*lambert_w(a*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^p*x^(-n*p - 1), x)
```

Giac [F]

$$\int x^{-1-np}(cW(ax^n))^p\,dx=\int (c\,\mathrm{W}(ax^n))^p\,x^{-np-1}\,dx$$

input
```
integrate(x^(-n*p-1)*(c*lambert_w(a*x^n))^p,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^p*x^(-n*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-np}(cW(ax^n))^p\,dx=\int \frac{(c\,\mathrm{LambertW}(a\,x^n))^p}{x^{n\,p+1}}\,dx$$

input
```
int((c*LambertW(a*x^n))^p/x^(n*p + 1),x)
```

output
```
int((c*LambertW(a*x^n))^p/x^(n*p + 1), x)
```

Reduce [F]

$$\int x^{-1-np}(cW(ax^n))^p\,dx$$
$$=\frac{c^p\left(-lambert\_w(x^na)^p+x^{np}\left(\int \frac{x^n lambert\_w(x^na)^p}{x^{np}e^{lambert\_w(x^na)}lambert\_w(x^na)^2x+x^{np}e^{lambert\_w(x^na)}lambert\_w(x^na)x}dx\right)a}{x^{np}np}$$

input
```
int(x^(-n*p-1)*(c*Lambert_W(a*x^n))^p,x)
```

output
```
(c**p*( - lambert_w(x**n*a)**p + x**(n*p)*int((x**n*lambert_w(x**n*a)**p)/
(x**(n*p)*e**lambert_w(x**n*a)*lambert_w(x**n*a)**2*x + x**(n*p)*e**lamber
t_w(x**n*a)*lambert_w(x**n*a)*x),x)*a*n*p))/(x**(n*p)*n*p)
```

3.295 $\int x^{-1+n(1-p)}(cW(ax^n))^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int x^{-1+n(1-p)}(cW(ax^n))^p\,dx = -\frac{cpx^{n(1-p)}(cW(ax^n))^{-1+p}}{n(1-p)^2}+\frac{x^{n(1-p)}(cW(ax^n))^p}{n(1-p)}$$

output
```
-c*p*x^(n*(1-p))*(c*LambertW(a*x^n))^(-1+p)/n/(1-p)^2+x^(n*(1-p))*(c*Lambe
rtW(a*x^n))^p/n/(1-p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.65

$$\int x^{-1+n(1-p)}(cW(ax^n))^p\,dx = -\frac{cx^{n-np}(cW(ax^n))^{-1+p}(p+(-1+p)W(ax^n))}{n(-1+p)^2}$$

input
```
Integrate[x^(-1 + n*(1 - p))*(c*ProductLog[a*x^n])^p,x]
```

output
```
-((c*x^(n - n*p)*(c*ProductLog[a*x^n])^(-1 + p)*(p + (-1 + p)*ProductLog[a
*x^n]))/(n*(-1 + p)^2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7172, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n(1-p)-1}(cW(ax^n))^p\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{x^{n(1-p)}(cW(ax^n))^p}{n(1-p)}-\frac{p\int \frac{x^{-pn+n-1}(cW(ax^n))^p}{W(ax^n)+1}dx}{1-p}$$

$$\Big\downarrow \text{7201}$$

$$\frac{x^{n(1-p)}(cW(ax^n))^p}{n(1-p)}-\frac{cpx^{n-np}(cW(ax^n))^{p-1}}{n(1-p)^2}$$

input
```
Int[x^(-1 + n*(1 - p))*(c*ProductLog[a*x^n])^p,x]
```

output
```
-((c*p*x^(n - n*p)*(c*ProductLog[a*x^n])^(-1 + p))/(n*(1 - p)^2)) + (x^(n*
(1 - p))*(c*ProductLog[a*x^n])^p)/(n*(1 - p))
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.56

method	result
parallelrisch	$-\frac{x\,x^{-np+n-1}\,\mathrm{LambertW}(a\,x^{n})(c\,\mathrm{LambertW}(a\,x^{n}))^{p}p-(c\,\mathrm{LambertW}(a\,x^{n}))^{p}\,\mathrm{LambertW}(a\,x^{n})x\,x^{-np+n-1}+p(c\,\mathrm{Lamber}}{\mathrm{LambertW}(a\,x^{n})n(p^{2}-2p+1)}$

input
```
int(x^(-1+n*(1-p))*(c*LambertW(a*x^n))^p,x,method=_RETURNVERBOSE)
```

output
```
-(x*x^(-n*p+n-1)*LambertW(a*x^n)*(c*LambertW(a*x^n))^p*p-(c*LambertW(a*x^n
))^p*LambertW(a*x^n)*x*x^(-n*p+n-1)+p*(c*LambertW(a*x^n))^p*x^(-n*p+n-1)*x
)/LambertW(a*x^n)/n/(p^2-2*p+1)
```

Fricas [F]

$$\int x^{-1+n(1-p)}(cW(ax^n))^p\,dx = \int (c\,\mathrm{W}(ax^n))^p\,x^{-n(p-1)-1}\,dx$$

input
```
integrate(x^(-1+n*(1-p))*(c*lambert_w(a*x^n))^p,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^n))^p*x^(-n*p + n - 1), x)
```

Sympy [F]

$$\int x^{-1+n(1-p)}(cW(ax^n))^p\,dx = \int x^{n(1-p)-1}(cW(ax^n))^p\,dx$$

input
```
integrate(x**(-1+n*(1-p))*(c*LambertW(a*x**n))**p,x)
```

output
```
Integral(x**(n*(1 - p) - 1)*(c*LambertW(a*x**n))**p, x)
```

Maxima [F]

$$\int x^{-1+n(1-p)}(cW(ax^n))^p\,dx = \int (c\,\mathrm{W}(ax^n))^p\,x^{-n(p-1)-1}\,dx$$

input
```
integrate(x^(-1+n*(1-p))*(c*lambert_w(a*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^p*x^(-n*(p - 1) - 1), x)
```

Giac [F]

$$\int x^{-1+n(1-p)}(cW(ax^n))^p\,dx = \int (c\,\mathrm{W}(ax^n))^p\,x^{-n(p-1)-1}\,dx$$

input
```
integrate(x^(-1+n*(1-p))*(c*lambert_w(a*x^n))^p,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^p*x^(-n*(p - 1) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+n(1-p)}(cW(ax^n))^p\,dx=\int \frac{(c\,\mathrm{LambertW}(a\,x^n))^p}{x^{n\,(p-1)+1}}\,dx$$

input
```
int((c*LambertW(a*x^n))^p/x^(n*(p - 1) + 1),x)
```

output
```
int((c*LambertW(a*x^n))^p/x^(n*(p - 1) + 1), x)
```

Reduce [F]

$$\int x^{-1+n(1-p)}(cW(ax^n))^p\,dx=c^p\left(\int \frac{x^n\,lambert\_w(x^na)^p}{x^{np}x}dx\right)$$

input
```
int(x^(-1+n*(1-p))*(c*Lambert_W(a*x^n))^p,x)
```

output
```
c**p*int((x**n*lambert_w(x**n*a)**p)/(x**(n*p)*x),x)
```

3.296 $\int x^{-1+n(2-p)}(cW(ax^n))^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 102

$$\int x^{-1+n(2-p)}(cW(ax^n))^p\,dx = \frac{c^2px^{n(2-p)}(cW(ax^n))^{-2+p}}{n(2-p)^3} - \frac{cpx^{n(2-p)}(cW(ax^n))^{-1+p}}{n(2-p)^2} + \frac{x^{n(2-p)}(cW(ax^n))^p}{n(2-p)}$$

output
```
c^2*p*x^(n*(2-p))*(c*LambertW(a*x^n))^(-2+p)/n/(2-p)^3-c*p*x^(n*(2-p))*(c*
LambertW(a*x^n))^(-1+p)/n/(2-p)^2+x^(n*(2-p))*(c*LambertW(a*x^n))^p/n/(2-p
)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int x^{-1+n(2-p)}(cW(ax^n))^p\,dx = -\frac{x^{-n(-2+p)}(cW(ax^n))^p\left(p+(-2+p)pW(ax^n)+(-2+p)^2W(ax^n)^2\right)}{n(-2+p)^3W(ax^n)^2}$$

input
```
Integrate[x^(-1 + n*(2 - p))*(c*ProductLog[a*x^n])^p,x]
```

output
```
-(((c*ProductLog[a*x^n])^p*(p + (-2 + p)*p*ProductLog[a*x^n] + (-2 + p)^2*
ProductLog[a*x^n]^2))/(n*(-2 + p)^3*x^(n*(-2 + p))*ProductLog[a*x^n]^2))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {7172, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n(2-p)-1}(cW(ax^n))^p\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{x^{n(2-p)}(cW(ax^n))^p}{n(2-p)}-\frac{p\int \frac{x^{n(2-p)-1}(cW(ax^n))^p}{W(ax^n)+1}dx}{2-p}$$

$$\Big\downarrow \text{7205}$$

$$\frac{x^{n(2-p)}(cW(ax^n))^p}{n(2-p)}-\frac{p\left(\frac{cx^{n(2-p)}(cW(ax^n))^{p-1}}{n(2-p)}-\frac{c\int \frac{x^{n(2-p)-1}(cW(ax^n))^{p-1}}{W(ax^n)+1}dx}{2-p}\right)}{2-p}$$

$$\Big\downarrow \text{7201}$$

$$\frac{x^{n(2-p)}(cW(ax^n))^p}{n(2-p)}-\frac{p\left(\frac{cx^{n(2-p)}(cW(ax^n))^{p-1}}{n(2-p)}-\frac{c^2x^{n(2-p)}(cW(ax^n))^{p-2}}{n(2-p)^2}\right)}{2-p}$$

input
```
Int[x^(-1 + n*(2 - p))*(c*ProductLog[a*x^n])^p,x]
```

output
```
(x^(n*(2 - p))*(c*ProductLog[a*x^n])^p)/(n*(2 - p)) - (p*(-((c^2*x^(n*(2 -
 p))*(c*ProductLog[a*x^n])^(-2 + p))/(n*(2 - p)^2)) + (c*x^(n*(2 - p))*(c*
ProductLog[a*x^n])^(-1 + p))/(n*(2 - p))))/(2 - p)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^{-1+n(2-p)}(c\,\mathrm{LambertW}\,(a\,x^n))^p dx$$

input
```
int(x^(-1+n*(2-p))*(c*LambertW(a*x^n))^p,x)
```

output
```
int(x^(-1+n*(2-p))*(c*LambertW(a*x^n))^p,x)
```

Fricas [F]

$$\int x^{-1+n(2-p)}(cW(ax^n))^p\,dx = \int (c\,\mathrm{W}(ax^n))^p\,x^{-n(p-2)-1}\,dx$$

input
```
integrate(x^(-1+n*(2-p))*(c*lambert_w(a*x^n))^p,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^n))^p*x^(-n*p + 2*n - 1), x)
```

Sympy [F]

$$\int x^{-1+n(2-p)}(cW(ax^n))^p\,dx = \int x^{n(2-p)-1}(cW(ax^n))^p\,dx$$

input
```
integrate(x**(-1+n*(2-p))*(c*LambertW(a*x**n))**p,x)
```

output
```
Integral(x**(n*(2 - p) - 1)*(c*LambertW(a*x**n))**p, x)
```

Maxima [F]

$$\int x^{-1+n(2-p)}(cW(ax^n))^p\,dx = \int (c\,\mathrm{W}(ax^n))^p\,x^{-n(p-2)-1}\,dx$$

input
```
integrate(x^(-1+n*(2-p))*(c*lambert_w(a*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^p*x^(-n*(p - 2) - 1), x)
```

Giac [F]

$$\int x^{-1+n(2-p)}(cW(ax^n))^p\,dx = \int (c\,\mathrm{W}(ax^n))^p\,x^{-n(p-2)-1}\,dx$$

input
```
integrate(x^(-1+n*(2-p))*(c*lambert_w(a*x^n))^p,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^p*x^(-n*(p - 2) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+n(2-p)}(cW(ax^n))^p\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x^n))^p}{x^{n\,(p-2)+1}}\,dx$$

input
```
int((c*LambertW(a*x^n))^p/x^(n*(p - 2) + 1),x)
```

output
```
int((c*LambertW(a*x^n))^p/x^(n*(p - 2) + 1), x)
```

Reduce [F]

$$\int x^{-1+n(2-p)}(cW(ax^n))^p\,dx = c^p\left(\int \frac{x^{2n}lambert\_w(x^n a)^p}{x^{np}x}dx\right)$$

input
```
int(x^(-1+n*(2-p))*(c*Lambert_W(a*x^n))^p,x)
```

output
```
c**p*int((x**(2*n)*lambert_w(x**n*a)**p)/(x**(n*p)*x),x)
```

3.297 $\int x^{-1+n(3-p)}(cW(ax^n))^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 140

$$\int x^{-1+n(3-p)}(cW(ax^n))^p \, dx = -\frac{2c^3px^{n(3-p)}(cW(ax^n))^{-3+p}}{n(3-p)^4} + \frac{2c^2px^{n(3-p)}(cW(ax^n))^{-2+p}}{n(3-p)^3} - \frac{cpx^{n(3-p)}(cW(ax^n))^{-1+p}}{n(3-p)^2} + \frac{x^{n(3-p)}(cW(ax^n))^p}{n(3-p)}$$

output
```
-2*c^3*p*x^(n*(3-p))*(c*LambertW(a*x^n))^(-3+p)/n/(3-p)^4+2*c^2*p*x^(n*(3-
p))*(c*LambertW(a*x^n))^(-2+p)/n/(3-p)^3-c*p*x^(n*(3-p))*(c*LambertW(a*x^n
))^(-1+p)/n/(3-p)^2+x^(n*(3-p))*(c*LambertW(a*x^n))^p/n/(3-p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.58

$$\int x^{-1+n(3-p)}(cW(ax^n))^p \, dx = -\frac{x^{-n(-3+p)}(cW(ax^n))^p \left(2p + 2(-3+p)pW(ax^n) + (-3+p)^2pW(ax^n)^2 + (-3+p)^3W(ax^n)^3\right)}{n(-3+p)^4W(ax^n)^3}$$

input
```
Integrate[x^(-1 + n*(3 - p))*(c*ProductLog[a*x^n])^p,x]
```

output
```
-(((c*ProductLog[a*x^n])^p*(2*p + 2*(-3 + p)*p*ProductLog[a*x^n] + (-3 + p
)^2*p*ProductLog[a*x^n]^2 + (-3 + p)^3*ProductLog[a*x^n]^3))/(n*(-3 + p)^4
*x^(n*(-3 + p))*ProductLog[a*x^n]^3))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {7172, 7205, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n(3-p)-1}(cW(ax^n))^p\,dx$$

$$\Big\downarrow \text{7172}$$

$$\frac{x^{n(3-p)}(cW(ax^n))^p}{n(3-p)}-\frac{p\int \frac{x^{n(3-p)-1}(cW(ax^n))^p}{W(ax^n)+1}dx}{3-p}$$

$$\Big\downarrow \text{7205}$$

$$\frac{x^{n(3-p)}(cW(ax^n))^p}{n(3-p)}-\frac{p\left(\frac{cx^{n(3-p)}(cW(ax^n))^{p-1}}{n(3-p)}-\frac{2c\int \frac{x^{n(3-p)-1}(cW(ax^n))^{p-1}}{W(ax^n)+1}dx}{3-p}\right)}{3-p}$$

$$\Big\downarrow \text{7205}$$

$$\frac{x^{n(3-p)}(cW(ax^n))^p}{n(3-p)}-\frac{p\left(\frac{cx^{n(3-p)}(cW(ax^n))^{p-1}}{n(3-p)}-\frac{2c\left(\frac{cx^{n(3-p)}(cW(ax^n))^{p-2}}{n(3-p)}-\frac{c\int \frac{x^{n(3-p)-1}(cW(ax^n))^{p-2}}{W(ax^n)+1}dx}{3-p}\right)}{3-p}\right)}{3-p}$$

$$\Big\downarrow \text{7201}$$

$$\frac{x^{n(3-p)}(cW(ax^n))^p}{n(3-p)}-\frac{p\left(\frac{cx^{n(3-p)}(cW(ax^n))^{p-1}}{n(3-p)}-\frac{2c\left(\frac{cx^{n(3-p)}(cW(ax^n))^{p-2}}{n(3-p)}-\frac{c^2x^{n(3-p)}(cW(ax^n))^{p-3}}{n(3-p)^2}\right)}{3-p}\right)}{3-p}$$

input
```
Int[x^(-1 + n*(3 - p))*(c*ProductLog[a*x^n])^p,x]
```

output
```
(x^(n*(3 - p))*(c*ProductLog[a*x^n])^p)/(n*(3 - p)) - (p*((c*x^(n*(3 - p))
*(c*ProductLog[a*x^n])^(-1 + p))/(n*(3 - p)) - (2*c*(-((c^2*x^(n*(3 - p))*
(c*ProductLog[a*x^n])^(-3 + p))/(n*(3 - p)^2)) + (c*x^(n*(3 - p))*(c*Produ
ctLog[a*x^n])^(-2 + p))/(n*(3 - p))))/(3 - p)))/(3 - p)
```

Defintions of rubi rules used

rule 7172
```
Int[(x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Sim
p[x^(m + 1)*((c*ProductLog[a*x^n])^p/(m + 1)), x] - Simp[n*(p/(m + 1))   In
t[x^m*((c*ProductLog[a*x^n])^p/(1 + ProductLog[a*x^n])), x], x] /; FreeQ[{a
, c, m, n, p}, x] && NeQ[m, -1] && ((IntegerQ[p - 1/2] && IGtQ[2*Simplify[p
 + (m + 1)/n], 0]) || ( !IntegerQ[p - 1/2] && IGtQ[Simplify[p + (m + 1)/n]
+ 1, 0]))
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [F]

$$\int x^{-1+n(3-p)}(c\,\text{LambertW}\,(a\,x^n))^p dx$$

input
```
int(x^(-1+n*(3-p))*(c*LambertW(a*x^n))^p,x)
```

output
```
int(x^(-1+n*(3-p))*(c*LambertW(a*x^n))^p,x)
```

Fricas [F]

$$\int x^{-1+n(3-p)}(cW(ax^n))^p\,dx = \int (c\,\mathrm{W}(ax^n))^p\,x^{-n(p-3)-1}\,dx$$

input
```
integrate(x^(-1+n*(3-p))*(c*lambert_w(a*x^n))^p,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(a*x^n))^p*x^(-n*p + 3*n - 1), x)
```

Sympy [F]

$$\int x^{-1+n(3-p)}(cW(ax^n))^p\,dx = \int x^{n(3-p)-1}(cW(ax^n))^p\,dx$$

input
```
integrate(x**(-1+n*(3-p))*(c*LambertW(a*x**n))**p,x)
```

output
```
Integral(x**(n*(3 - p) - 1)*(c*LambertW(a*x**n))**p, x)
```

Maxima [F]

$$\int x^{-1+n(3-p)}(cW(ax^n))^p\,dx = \int (c\,\mathrm{W}(ax^n))^p\,x^{-n(p-3)-1}\,dx$$

input
```
integrate(x^(-1+n*(3-p))*(c*lambert_w(a*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(a*x^n))^p*x^(-n*(p - 3) - 1), x)
```

Giac [F]

$$\int x^{-1+n(3-p)}(cW(ax^n))^p\,dx = \int (c\,\mathrm{W}(ax^n))^p\,x^{-n(p-3)-1}\,dx$$

input
```
integrate(x^(-1+n*(3-p))*(c*lambert_w(a*x^n))^p,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(a*x^n))^p*x^(-n*(p - 3) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+n(3-p)}(cW(ax^n))^p\,dx = \int \frac{(c\,\mathrm{LambertW}(a\,x^n))^p}{x^{n\,(p-3)+1}}\,dx$$

input
```
int((c*LambertW(a*x^n))^p/x^(n*(p - 3) + 1),x)
```

output
```
int((c*LambertW(a*x^n))^p/x^(n*(p - 3) + 1), x)
```

Reduce [F]

$$\int x^{-1+n(3-p)}(cW(ax^n))^p\,dx = c^p\left(\int \frac{x^{3n}lambert\_w(x^n a)^p}{x^{np}x}dx\right)$$

input
```
int(x^(-1+n*(3-p))*(c*Lambert_W(a*x^n))^p,x)
```

output
```
c**p*int((x**(3*n)*lambert_w(x**n*a)**p)/(x**(n*p)*x),x)
```

3.298 $\int \frac{x^3}{1+W(ax)} dx$

Optimal result

Integrand size = 12, antiderivative size = 53

$$\int \frac{x^3}{1+W(ax)} dx = -\frac{3x^4}{128W(ax)^4} + \frac{3x^4}{32W(ax)^3} - \frac{3x^4}{16W(ax)^2} + \frac{x^4}{4W(ax)}$$

output
```
-3/128*x^4/LambertW(a*x)^4+3/32*x^4/LambertW(a*x)^3-3/16*x^4/LambertW(a*x)
^2+1/4*x^4/LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+W(ax)} dx = -\frac{3x^4}{128W(ax)^4} + \frac{3x^4}{32W(ax)^3} - \frac{3x^4}{16W(ax)^2} + \frac{x^4}{4W(ax)}$$

input
```
Integrate[x^3/(1 + ProductLog[a*x]),x]
```

output
```
(-3*x^4)/(128*ProductLog[a*x]^4) + (3*x^4)/(32*ProductLog[a*x]^3) - (3*x^4
)/(16*ProductLog[a*x]^2) + x^4/(4*ProductLog[a*x])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7194, 7205, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{W(ax)+1}dx$$

$$\Big\downarrow \text{7194}$$

$$\frac{x^4}{4W(ax)}-\frac{3}{4}\int \frac{x^3}{W(ax)(W(ax)+1)}dx$$

$$\Big\downarrow \text{7205}$$

$$\frac{x^4}{4W(ax)}-\frac{3}{4}\left(\frac{x^4}{4W(ax)^2}-\frac{1}{2}\int \frac{x^3}{W(ax)^2(W(ax)+1)}dx\right)$$

$$\Big\downarrow \text{7205}$$

$$\frac{x^4}{4W(ax)}-\frac{3}{4}\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{x^3}{W(ax)^3(W(ax)+1)}dx-\frac{x^4}{4W(ax)^3}\right)+\frac{x^4}{4W(ax)^2}\right)$$

$$\Big\downarrow \text{7201}$$

$$\frac{x^4}{4W(ax)}-\frac{3}{4}\left(\frac{x^4}{4W(ax)^2}+\frac{1}{2}\left(\frac{x^4}{16W(ax)^4}-\frac{x^4}{4W(ax)^3}\right)\right)$$

input
```
Int[x^3/(1 + ProductLog[a*x]),x]
```

output
```
(-3*((x^4/(16*ProductLog[a*x]^4) - x^4/(4*ProductLog[a*x]^3))/2 + x^4/(4*P
roductLog[a*x]^2)))/4 + x^4/(4*ProductLog[a*x])
```

Defintions of rubi rules used

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{\frac{x^4a^4}{4\,\mathrm{LambertW}(xa)}-\frac{3x^4a^4}{16\,\mathrm{LambertW}(xa)^2}+\frac{3x^4a^4}{32\,\mathrm{LambertW}(xa)^3}-\frac{3x^4a^4}{128\,\mathrm{LambertW}(xa)^4}}{a^4}$	62
default	$\frac{\frac{x^4a^4}{4\,\mathrm{LambertW}(xa)}-\frac{3x^4a^4}{16\,\mathrm{LambertW}(xa)^2}+\frac{3x^4a^4}{32\,\mathrm{LambertW}(xa)^3}-\frac{3x^4a^4}{128\,\mathrm{LambertW}(xa)^4}}{a^4}$	62

input
```
int(x^3/(1+LambertW(x*a)),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/4*x^4*a^4/LambertW(x*a)-3/16*x^4*a^4/LambertW(x*a)^2+3/32/Lambert
W(x*a)^3*x^4*a^4-3/128*x^4*a^4/LambertW(x*a)^4)
```

Fricas [F]

$$\int \frac{x^3}{1+W(ax)}\,dx = \int \frac{x^3}{\mathrm{W}(ax)+1}\,dx$$

input
```
integrate(x^3/(1+lambert_w(a*x)),x, algorithm="fricas")
```

output
```
integral(x^3/(lambert_w(a*x) + 1), x)
```

Sympy [F]

$$\int \frac{x^3}{1+W(ax)}\,dx = \int \frac{x^3}{W\left(ax\right)+1}\,dx$$

input
```
integrate(x**3/(1+LambertW(a*x)),x)
```

output
```
Integral(x**3/(LambertW(a*x) + 1), x)
```

Maxima [F]

$$\int \frac{x^3}{1+W(ax)}\,dx = \int \frac{x^3}{\mathrm{W}(ax)+1}\,dx$$

input
```
integrate(x^3/(1+lambert_w(a*x)),x, algorithm="maxima")
```

output
```
integrate(x^3/(lambert_w(a*x) + 1), x)
```

Giac [F]

$$\int \frac{x^3}{1+W(ax)}\,dx=\int \frac{x^3}{\mathrm{W}(ax)+1}\,dx$$

input
```
integrate(x^3/(1+lambert_w(a*x)),x, algorithm="giac")
```

output
```
integrate(x^3/(lambert_w(a*x) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{1+W(ax)}\,dx=\int \frac{x^3}{\mathrm{LambertW}\,(a\,x)+1}\,dx$$

input
```
int(x^3/(LambertW(a*x) + 1),x)
```

output
```
int(x^3/(LambertW(a*x) + 1), x)
```

Reduce [F]

$$\int \frac{x^3}{1+W(ax)}\,dx=\int \frac{x^3}{lambert\_w\,(ax)+1}dx$$

input
```
int(x^3/(1+Lambert_W(a*x)),x)
```

output
```
int(x**3/(lambert_w(a*x) + 1),x)
```

3.299 $\int \frac{x^2}{1+W(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{x^2}{1+W(ax)}\,dx = \frac{2x^3}{27W(ax)^3} - \frac{2x^3}{9W(ax)^2} + \frac{x^3}{3W(ax)}$$

output
```
2/27*x^3/LambertW(a*x)^3-2/9*x^3/LambertW(a*x)^2+1/3*x^3/LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{1+W(ax)}\,dx = \frac{2x^3}{27W(ax)^3} - \frac{2x^3}{9W(ax)^2} + \frac{x^3}{3W(ax)}$$

input
```
Integrate[x^2/(1 + ProductLog[a*x]),x]
```

output
```
(2*x^3)/(27*ProductLog[a*x]^3) - (2*x^3)/(9*ProductLog[a*x]^2) + x^3/(3*Pr
oductLog[a*x])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7194, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{W(ax)+1} \, dx$$

$$\Big\downarrow \text{ 7194}$$

$$\frac{x^3}{3W(ax)} - \frac{2}{3}\int \frac{x^2}{W(ax)(W(ax)+1)}dx$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{x^3}{3W(ax)} - \frac{2}{3}\left(\frac{x^3}{3W(ax)^2} - \frac{1}{3}\int \frac{x^2}{W(ax)^2(W(ax)+1)}dx\right)$$

$$\Big\downarrow \text{ 7201}$$

$$\frac{x^3}{3W(ax)} - \frac{2}{3}\left(\frac{x^3}{3W(ax)^2} - \frac{x^3}{9W(ax)^3}\right)$$

input
```
Int[x^2/(1 + ProductLog[a*x]),x]
```

output
```
(-2*(-1/9*x^3/ProductLog[a*x]^3 + x^3/(3*ProductLog[a*x]^2)))/3 + x^3/(3*P
roductLog[a*x])
```

Defintions of rubi rules used

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{\frac{x^3 a^3}{3\,\mathrm{LambertW}(xa)}-\frac{2x^3 a^3}{9\,\mathrm{LambertW}(xa)^2}+\frac{2x^3 a^3}{27\,\mathrm{LambertW}(xa)^3}}{a^3}$	48
default	$\frac{\frac{x^3 a^3}{3\,\mathrm{LambertW}(xa)}-\frac{2x^3 a^3}{9\,\mathrm{LambertW}(xa)^2}+\frac{2x^3 a^3}{27\,\mathrm{LambertW}(xa)^3}}{a^3}$	48

input
```
int(x^2/(1+LambertW(x*a)),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/3/LambertW(x*a)*x^3*a^3-2/9/LambertW(x*a)^2*x^3*a^3+2/27*x^3*a^3/
LambertW(x*a)^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{1+W(ax)}\,dx=\frac{9\,x^3\,\mathrm{W}(ax)^2-6\,x^3\,\mathrm{W}(ax)+2\,x^3}{27\,\mathrm{W}(ax)^3}$$

input
```
integrate(x^2/(1+lambert_w(a*x)),x, algorithm="fricas")
```

output
```
1/27*(9*x^3*lambert_w(a*x)^2 - 6*x^3*lambert_w(a*x) + 2*x^3)/lambert_w(a*x
)^3
```

Sympy [F]

$$\int \frac{x^2}{1+W(ax)}\,dx = \int \frac{x^2}{W(ax)+1}\,dx$$

input
```
integrate(x**2/(1+LambertW(a*x)),x)
```

output
```
Integral(x**2/(LambertW(a*x) + 1), x)
```

Maxima [F]

$$\int \frac{x^2}{1+W(ax)}\,dx = \int \frac{x^2}{\mathrm{W}(ax)+1}\,dx$$

input
```
integrate(x^2/(1+lambert_w(a*x)),x, algorithm="maxima")
```

output
```
integrate(x^2/(lambert_w(a*x) + 1), x)
```

Giac [F]

$$\int \frac{x^2}{1+W(ax)}\,dx = \int \frac{x^2}{\mathrm{W}(ax)+1}\,dx$$

input
```
integrate(x^2/(1+lambert_w(a*x)),x, algorithm="giac")
```

output
```
integrate(x^2/(lambert_w(a*x) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{1+W(ax)}\,dx = \int \frac{x^2}{\mathrm{LambertW}\left(a\,x\right)+1}\,dx$$

input
```
int(x^2/(LambertW(a*x) + 1),x)
```

output
```
int(x^2/(LambertW(a*x) + 1), x)
```

Reduce [F]

$$\int \frac{x^2}{1+W(ax)}\,dx = \int \frac{x^2}{lambert\_w(ax)+1}dx$$

input
```
int(x^2/(1+Lambert_W(a*x)),x)
```

output
```
int(x**2/(lambert_w(a*x) + 1),x)
```

3.300 $\int \frac{x}{1+W(ax)} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \frac{x}{1+W(ax)} \, dx = -\frac{x^2}{4W(ax)^2} + \frac{x^2}{2W(ax)}$$

output
```
-1/4*x^2/LambertW(a*x)^2+1/2*x^2/LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+W(ax)} \, dx = -\frac{x^2}{4W(ax)^2} + \frac{x^2}{2W(ax)}$$

input
```
Integrate[x/(1 + ProductLog[a*x]),x]
```

output
```
-1/4*x^2/ProductLog[a*x]^2 + x^2/(2*ProductLog[a*x])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7194, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{W(ax)+1}dx$$
$$\Big\downarrow \text{ 7194}$$
$$\frac{x^2}{2W(ax)}-\frac{1}{2}\int \frac{x}{W(ax)(W(ax)+1)}dx$$
$$\Big\downarrow \text{ 7201}$$
$$\frac{x^2}{2W(ax)}-\frac{x^2}{4W(ax)^2}$$

input
```
Int[x/(1 + ProductLog[a*x]),x]
```

output
```
-1/4*x^2/ProductLog[a*x]^2 + x^2/(2*ProductLog[a*x])
```

Defintions of rubi rules used

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$\frac{\frac{x^2 a^2}{2\,\mathrm{LambertW}(xa)}-\frac{x^2 a^2}{4\,\mathrm{LambertW}(xa)^2}}{a^2}$	34
default	$\frac{\frac{x^2 a^2}{2\,\mathrm{LambertW}(xa)}-\frac{x^2 a^2}{4\,\mathrm{LambertW}(xa)^2}}{a^2}$	34

input
```
int(x/(1+LambertW(x*a)),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/2/LambertW(x*a)*x^2*a^2-1/4*x^2*a^2/LambertW(x*a)^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x}{1+W(ax)}\,dx=\frac{2\,x^2\,\mathrm{W}(ax)-x^2}{4\,\mathrm{W}(ax)^2}$$

input
```
integrate(x/(1+lambert_w(a*x)),x, algorithm="fricas")
```

output
```
1/4*(2*x^2*lambert_w(a*x) - x^2)/lambert_w(a*x)^2
```

Sympy [F]

$$\int \frac{x}{1+W(ax)}\,dx=\int \frac{x}{W\,(ax)+1}\,dx$$

input
```
integrate(x/(1+LambertW(a*x)),x)
```

output
```
Integral(x/(LambertW(a*x) + 1), x)
```

Maxima [F]

$$\int \frac{x}{1 + W(ax)}\, dx = \int \frac{x}{\mathrm{W}(ax) + 1}\, dx$$

input
```
integrate(x/(1+lambert_w(a*x)),x, algorithm="maxima")
```

output
```
integrate(x/(lambert_w(a*x) + 1), x)
```

Giac [F]

$$\int \frac{x}{1 + W(ax)}\, dx = \int \frac{x}{\mathrm{W}(ax) + 1}\, dx$$

input
```
integrate(x/(1+lambert_w(a*x)),x, algorithm="giac")
```

output
```
integrate(x/(lambert_w(a*x) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{1 + W(ax)}\, dx = \int \frac{x}{\mathrm{LambertW}\left(a\,x\right) + 1}\, dx$$

input
```
int(x/(LambertW(a*x) + 1),x)
```

output
```
int(x/(LambertW(a*x) + 1), x)
```

Reduce [F]

$$\int \frac{x}{1+W(ax)}\,dx = \int \frac{x}{lambert\_w(ax)+1}dx$$

input
```
int(x/(1+Lambert_W(a*x)),x)
```

output
```
int(x/(lambert_w(a*x) + 1),x)
```

3.301 $\int \frac{1}{1+W(ax)} dx$

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{1}{1+W(ax)} dx = \frac{x}{W(ax)}$$

output
```
x/LambertW(a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+W(ax)} dx = \frac{x}{W(ax)}$$

input
```
Integrate[(1 + ProductLog[a*x])^(-1),x]
```

output
```
x/ProductLog[a*x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W(ax)+1}dx$$

$$\Big\downarrow \text{7176}$$

$$\frac{x}{W(ax)}$$

input
```
Int[(1 + ProductLog[a*x])^(-1),x]
```

output
```
x/ProductLog[a*x]
```

Defintions of rubi rules used

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\frac{x}{\mathrm{LambertW}(xa)}$	9
default	$\frac{x}{\mathrm{LambertW}(xa)}$	9
parallelrisch	$\frac{x}{\mathrm{LambertW}(xa)}$	9

input
```
int(1/(1+LambertW(x*a)),x,method=_RETURNVERBOSE)
```

output
```
x/LambertW(x*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \frac{1}{1+W(ax)}\,dx = \frac{x}{\mathrm{W}(ax)}$$

input
```
integrate(1/(1+lambert_w(a*x)),x, algorithm="fricas")
```

output
```
x/lambert_w(a*x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+W(ax)}\,dx = \begin{cases} \frac{x}{W(ax)} & \text{for } a \neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1+LambertW(a*x)),x)
```

output
```
Piecewise((x/LambertW(a*x), Ne(a, 0)), (x, True))
```

Maxima [F]

$$\int \frac{1}{1+W(ax)}\,dx = \int \frac{1}{\mathrm{W}(ax)+1}\,dx$$

input
```
integrate(1/(1+lambert_w(a*x)),x, algorithm="maxima")
```

output
```
integrate(1/(lambert_w(a*x) + 1), x)
```

Giac [F]

$$\int \frac{1}{1+W(ax)}\,dx = \int \frac{1}{\mathrm{W}(ax)+1}\,dx$$

input
```
integrate(1/(1+lambert_w(a*x)),x, algorithm="giac")
```

output
```
integrate(1/(lambert_w(a*x) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+W(ax)}\,dx = \int \frac{1}{\mathrm{LambertW}\,(a\,x)+1}\,dx$$

input
```
int(1/(LambertW(a*x) + 1),x)
```

output
```
int(1/(LambertW(a*x) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{1+W(ax)}\,dx = \frac{e^{lambert\_w(ax)}}{a}$$

input
```
int(1/(1+Lambert_W(a*x)),x)
```

output
```
e**lambert_w(a*x)/a
```

3.302 $\int \frac{1}{x(1+W(ax))}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 5

$$\int \frac{1}{x(1+W(ax))}\,dx = \log(W(ax))$$

output
```
ln(LambertW(a*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+W(ax))}\,dx = \log(W(ax))$$

input
```
Integrate[1/(x*(1 + ProductLog[a*x])),x]
```

output
```
Log[ProductLog[a*x]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {7195}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(W(ax)+1)} \, dx$$
$$\Big\downarrow \; 7195$$
$$\log(W(ax))$$

input
```
Int[1/(x*(1 + ProductLog[a*x])),x]
```

output
```
Log[ProductLog[a*x]]
```

Defintions of rubi rules used

rule 7195
```
Int[1/((x_)*((d_) + (d_.)*ProductLog[(a_.)*(x_)])), x_Symbol] :> Simp[Log[P
roductLog[a*x]]/d, x] /; FreeQ[{a, d}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\ln(\text{LambertW}(xa))$	6
default	$\ln(\text{LambertW}(xa))$	6
parallelrisch	$\ln(x) - \text{LambertW}(xa)$	10

input
```
int(1/x/(1+LambertW(x*a)),x,method=_RETURNVERBOSE)
```

output
```
ln(LambertW(x*a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(1+W(ax))}\,dx = \log\left(\mathrm{W}(ax)\right)$$

input
```
integrate(1/x/(1+lambert_w(a*x)),x, algorithm="fricas")
```

output
```
log(lambert_w(a*x))
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \frac{1}{x(1+W(ax))}\,dx = \log\left(x\right) - W(ax)$$

input
```
integrate(1/x/(1+LambertW(a*x)),x)
```

output
```
log(x) - LambertW(a*x)
```

Maxima [F]

$$\int \frac{1}{x(1+W(ax))}\,dx = \int \frac{1}{x(\mathrm{W}(ax)+1)}\,dx$$

input
```
integrate(1/x/(1+lambert_w(a*x)),x, algorithm="maxima")
```

output
```
integrate(1/(x*(lambert_w(a*x) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x(1+W(ax))}\,dx = \int \frac{1}{x(\mathrm{W}(ax)+1)}\,dx$$

input
```
integrate(1/x/(1+lambert_w(a*x)),x, algorithm="giac")
```

output
```
integrate(1/(x*(lambert_w(a*x) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(1+W(ax))}\,dx = \int \frac{1}{x\,(\mathrm{LambertW}\,(a\,x)+1)}\,dx$$

input
```
int(1/(x*(LambertW(a*x) + 1)),x)
```

output
```
int(1/(x*(LambertW(a*x) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x(1+W(ax))}\,dx = \int \frac{1}{lambert\_w(ax)\,x+x}dx$$

input
```
int(1/x/(1+Lambert_W(a*x)),x)
```

output
```
int(1/(lambert_w(a*x)*x + x),x)
```

3.303 $\int \frac{1}{x^2(1+W(ax))} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 16

$$\int \frac{1}{x^2(1+W(ax))} \, dx = -\frac{1}{x} - a \,\text{ExpIntegralEi}(-W(ax))$$

output
```
-1/x-a*Ei(-LambertW(a*x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(1+W(ax))} \, dx = -\frac{1}{x} - a \,\text{ExpIntegralEi}(-W(ax))$$

input
```
Integrate[1/(x^2*(1 + ProductLog[a*x])),x]
```

output
```
-x^(-1) - a*ExpIntegralEi[-ProductLog[a*x]]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7196, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(W(ax)+1)}dx$$
$$\Big\downarrow \text{ 7196}$$
$$-\int \frac{W(ax)}{x^2(W(ax)+1)}dx-\frac{1}{x}$$
$$\Big\downarrow \text{ 7202}$$
$$-a\,\text{ExpIntegralEi}(-W(ax))-\frac{1}{x}$$

input
```
Int[1/(x^2*(1 + ProductLog[a*x])),x]
```

output
```
-x^(-1) - a*ExpIntegralEi[-ProductLog[a*x]]
```

Defintions of rubi rules used

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$a\left(-\frac{1}{ax}+\text{expIntegral}_1\left(\text{LambertW}\left(xa\right)\right)\right)$	18
default	$a\left(-\frac{1}{ax}+\text{expIntegral}_1\left(\text{LambertW}\left(xa\right)\right)\right)$	18

input
```
int(1/x^2/(1+LambertW(x*a)),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/a/x+Ei(1,LambertW(x*a)))
```

Fricas [F]

$$\int \frac{1}{x^2(1+W(ax))}\,dx=\int \frac{1}{x^2(\mathrm{W}(ax)+1)}\,dx$$

input
```
integrate(1/x^2/(1+lambert_w(a*x)),x, algorithm="fricas")
```

output
```
integral(1/(x^2*lambert_w(a*x) + x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2(1+W(ax))}\,dx=\int \frac{1}{x^2\left(W\left(ax\right)+1\right)}\,dx$$

input
```
integrate(1/x**2/(1+LambertW(a*x)),x)
```

output
```
Integral(1/(x**2*(LambertW(a*x) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(1+W(ax))}\,dx = \int \frac{1}{x^2(\mathrm{W}(ax)+1)}\,dx$$

input
```
integrate(1/x^2/(1+lambert_w(a*x)),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*(lambert_w(a*x) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^2(1+W(ax))}\,dx = \int \frac{1}{x^2(\mathrm{W}(ax)+1)}\,dx$$

input
```
integrate(1/x^2/(1+lambert_w(a*x)),x, algorithm="giac")
```

output
```
integrate(1/(x^2*(lambert_w(a*x) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(1+W(ax))}\,dx = \int \frac{1}{x^2\,(\mathrm{LambertW}\,(a\,x)+1)}\,dx$$

input
```
int(1/(x^2*(LambertW(a*x) + 1)),x)
```

output
```
int(1/(x^2*(LambertW(a*x) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^2(1+W(ax))}\,dx = \frac{-\left(\int \frac{lambert\_w(ax)}{lambert\_w(ax)x^2+x^2}dx\right)x+3\left(\int \frac{1}{lambert\_w(ax)x^2+x^2}dx\right)x-1}{4x}$$

input
```
int(1/x^2/(1+Lambert_W(a*x)),x)
```

output
```
( - int(lambert_w(a*x)/(lambert_w(a*x)*x**2 + x**2),x)*x + 3*int(1/(lamber
t_w(a*x)*x**2 + x**2),x)*x - 1)/(4*x)
```

3.304 $\int \frac{1}{x^3(1+W(ax))}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 28

$$\int \frac{1}{x^3(1+W(ax))}\,dx = -\frac{1}{2x^2} + 2a^2\,\text{ExpIntegralEi}(-2W(ax)) + \frac{W(ax)}{x^2}$$

output
```
-1/2/x^2+2*a^2*Ei(-2*LambertW(a*x))+LambertW(a*x)/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(1+W(ax))}\,dx = -\frac{1}{2x^2} + 2a^2\,\text{ExpIntegralEi}(-2W(ax)) + \frac{W(ax)}{x^2}$$

input
```
Integrate[1/(x^3*(1 + ProductLog[a*x])),x]
```

output
```
-1/2*1/x^2 + 2*a^2*ExpIntegralEi[-2*ProductLog[a*x]] + ProductLog[a*x]/x^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7196, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(W(ax)+1)}dx$$

$$\Big\downarrow \text{ 7196}$$

$$-\int \frac{W(ax)}{x^3(W(ax)+1)}dx-\frac{1}{2x^2}$$

$$\Big\downarrow \text{ 7206}$$

$$2\int \frac{W(ax)^2}{x^3(W(ax)+1)}dx+\frac{W(ax)}{x^2}-\frac{1}{2x^2}$$

$$\Big\downarrow \text{ 7202}$$

$$2a^2 \operatorname{ExpIntegralEi}(-2W(ax))+\frac{W(ax)}{x^2}-\frac{1}{2x^2}$$

input
```
Int[1/(x^3*(1 + ProductLog[a*x])),x]
```

output
```
-1/2*1/x^2 + 2*a^2*ExpIntegralEi[-2*ProductLog[a*x]] + ProductLog[a*x]/x^2
```

Defintions of rubi rules used

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$a^2\left(-\frac{1}{2x^2a^2}+\frac{\mathrm{LambertW}(xa)}{x^2a^2}-2\ \mathrm{expIntegral}_1\left(2\,\mathrm{LambertW}\left(xa\right)\right)\right)$	35
default	$a^2\left(-\frac{1}{2x^2a^2}+\frac{\mathrm{LambertW}(xa)}{x^2a^2}-2\ \mathrm{expIntegral}_1\left(2\,\mathrm{LambertW}\left(xa\right)\right)\right)$	35

input
```
int(1/x^3/(1+LambertW(x*a)),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/x^2/a^2+LambertW(x*a)/x^2/a^2-2*Ei(1,2*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{1}{x^3(1+W(ax))}\,dx=\int \frac{1}{x^3(\mathrm{W}(ax)+1)}\,dx$$

input
```
integrate(1/x^3/(1+lambert_w(a*x)),x, algorithm="fricas")
```

output
```
integral(1/(x^3*lambert_w(a*x) + x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3(1+W(ax))}\,dx = \int \frac{1}{x^3\,(W\,(ax)+1)}\,dx$$

input
```
integrate(1/x**3/(1+LambertW(a*x)),x)
```

output
```
Integral(1/(x**3*(LambertW(a*x) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(1+W(ax))}\,dx = \int \frac{1}{x^3(\mathrm{W}(ax)+1)}\,dx$$

input
```
integrate(1/x^3/(1+lambert_w(a*x)),x, algorithm="maxima")
```

output
```
integrate(1/(x^3*(lambert_w(a*x) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^3(1+W(ax))}\,dx = \int \frac{1}{x^3(\mathrm{W}(ax)+1)}\,dx$$

input
```
integrate(1/x^3/(1+lambert_w(a*x)),x, algorithm="giac")
```

output
```
integrate(1/(x^3*(lambert_w(a*x) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(1+W(ax))}\,dx = \int \frac{1}{x^3\,\left(\mathrm{LambertW}\left(a\,x\right)+1\right)}\,dx$$

input
```
int(1/(x^3*(LambertW(a*x) + 1)),x)
```

output
```
int(1/(x^3*(LambertW(a*x) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^3(1+W(ax))}\,dx = \frac{-2\left(\int \frac{lambert\_w(ax)}{lambert\_w(ax)x^3+x^3}dx\right)x^2+6\left(\int \frac{1}{lambert\_w(ax)x^3+x^3}dx\right)x^2-1}{8x^2}$$

input
```
int(1/x^3/(1+Lambert_W(a*x)),x)
```

output
```
( - 2*int(lambert_w(a*x)/(lambert_w(a*x)*x**3 + x**3),x)*x**2 + 6*int(1/(l
ambert_w(a*x)*x**3 + x**3),x)*x**2 - 1)/(8*x**2)
```

3.305 $\int \frac{1}{x^4(1+W(ax))}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{1}{x^4(1+W(ax))}\,dx = -\frac{1}{3x^3} - \frac{9}{2}a^3\,\text{ExpIntegralEi}(-3W(ax)) + \frac{W(ax)}{2x^3} - \frac{3W(ax)^2}{2x^3}$$

output
```
-1/3/x^3-9/2*a^3*Ei(-3*LambertW(a*x))+1/2*LambertW(a*x)/x^3-3/2*LambertW(a
*x)^2/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(1+W(ax))}\,dx = -\frac{1}{3x^3} - \frac{9}{2}a^3\,\text{ExpIntegralEi}(-3W(ax)) + \frac{W(ax)}{2x^3} - \frac{3W(ax)^2}{2x^3}$$

input
```
Integrate[1/(x^4*(1 + ProductLog[a*x])),x]
```

output
```
-1/3*1/x^3 - (9*a^3*ExpIntegralEi[-3*ProductLog[a*x]])/2 + ProductLog[a*x]
/(2*x^3) - (3*ProductLog[a*x]^2)/(2*x^3)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7196, 7206, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(W(ax)+1)}dx$$
$$\Big\downarrow \text{7196}$$
$$-\int \frac{W(ax)}{x^4(W(ax)+1)}dx-\frac{1}{3x^3}$$
$$\Big\downarrow \text{7206}$$
$$\frac{3}{2}\int \frac{W(ax)^2}{x^4(W(ax)+1)}dx+\frac{W(ax)}{2x^3}-\frac{1}{3x^3}$$
$$\Big\downarrow \text{7206}$$
$$\frac{3}{2}\left(-3\int \frac{W(ax)^3}{x^4(W(ax)+1)}dx-\frac{W(ax)^2}{x^3}\right)+\frac{W(ax)}{2x^3}-\frac{1}{3x^3}$$
$$\Big\downarrow \text{7202}$$
$$\frac{3}{2}\left(-3a^3\,\text{ExpIntegralEi}(-3W(ax))-\frac{W(ax)^2}{x^3}\right)+\frac{W(ax)}{2x^3}-\frac{1}{3x^3}$$

input
```
Int[1/(x^4*(1 + ProductLog[a*x])),x]
```

output
```
-1/3*1/x^3 + ProductLog[a*x]/(2*x^3) + (3*(-3*a^3*ExpIntegralEi[-3*Product
Log[a*x]] - ProductLog[a*x]^2/x^3))/2
```

Defintions of rubi rules used

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$a^3\left(-\frac{1}{3x^3a^3}+\frac{\mathrm{LambertW}(xa)}{2x^3a^3}-\frac{3\,\mathrm{LambertW}(xa)^2}{2x^3a^3}+\frac{9\ \mathrm{expIntegral}_1(3\,\mathrm{LambertW}(xa))}{2}\right)$	50
default	$a^3\left(-\frac{1}{3x^3a^3}+\frac{\mathrm{LambertW}(xa)}{2x^3a^3}-\frac{3\,\mathrm{LambertW}(xa)^2}{2x^3a^3}+\frac{9\ \mathrm{expIntegral}_1(3\,\mathrm{LambertW}(xa))}{2}\right)$	50

input
```
int(1/x^4/(1+LambertW(x*a)),x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3/x^3/a^3+1/2*LambertW(x*a)/x^3/a^3-3/2*LambertW(x*a)^2/x^3/a^3+9/
2*Ei(1,3*LambertW(x*a)))
```

Fricas [F]

$$\int \frac{1}{x^4(1+W(ax))}\,dx = \int \frac{1}{x^4(\mathrm{W}(ax)+1)}\,dx$$

input
```
integrate(1/x^4/(1+lambert_w(a*x)),x, algorithm="fricas")
```

output
```
integral(1/(x^4*lambert_w(a*x) + x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4(1+W(ax))}\,dx = \int \frac{1}{x^4\,(W\,(ax)+1)}\,dx$$

input
```
integrate(1/x**4/(1+LambertW(a*x)),x)
```

output
```
Integral(1/(x**4*(LambertW(a*x) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^4(1+W(ax))}\,dx = \int \frac{1}{x^4(\mathrm{W}(ax)+1)}\,dx$$

input
```
integrate(1/x^4/(1+lambert_w(a*x)),x, algorithm="maxima")
```

output
```
integrate(1/(x^4*(lambert_w(a*x) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^4(1+W(ax))} dx = \int \frac{1}{x^4(\mathrm{W}(ax)+1)} dx$$

input
```
integrate(1/x^4/(1+lambert_w(a*x)),x, algorithm="giac")
```

output
```
integrate(1/(x^4*(lambert_w(a*x) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4(1+W(ax))} dx = \int \frac{1}{x^4 (\mathrm{LambertW}(a x)+1)} dx$$

input
```
int(1/(x^4*(LambertW(a*x) + 1)),x)
```

output
```
int(1/(x^4*(LambertW(a*x) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^4(1+W(ax))} dx = \frac{-3\left(\int \frac{lambert\_w(ax)}{lambert\_w(ax)x^4+x^4} dx\right)x^3+9\left(\int \frac{1}{lambert\_w(ax)x^4+x^4} dx\right)x^3-1}{12x^3}$$

input
```
int(1/x^4/(1+Lambert_W(a*x)),x)
```

output
```
( - 3*int(lambert_w(a*x)/(lambert_w(a*x)*x**4 + x**4),x)*x**3 + 9*int(1/(l
ambert_w(a*x)*x**4 + x**4),x)*x**3 - 1)/(12*x**3)
```

3.306 $\int \frac{x^5}{1+W(ax^2)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{x^5}{1+W(ax^2)}\,dx = \frac{x^6}{27W(ax^2)^3} - \frac{x^6}{9W(ax^2)^2} + \frac{x^6}{6W(ax^2)}$$

output
```
1/27*x^6/LambertW(a*x^2)^3-1/9*x^6/LambertW(a*x^2)^2+1/6*x^6/LambertW(a*x^
2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{1+W(ax^2)}\,dx = \frac{x^6}{27W(ax^2)^3} - \frac{x^6}{9W(ax^2)^2} + \frac{x^6}{6W(ax^2)}$$

input
```
Integrate[x^5/(1 + ProductLog[a*x^2]),x]
```

output
```
x^6/(27*ProductLog[a*x^2]^3) - x^6/(9*ProductLog[a*x^2]^2) + x^6/(6*Produc
tLog[a*x^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7283, 7194, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{W(ax^2)+1}dx$$
$$\Big\downarrow \text{ 7283}$$
$$\frac{1}{2}\int \frac{x^4}{W(ax^2)+1}dx^2$$
$$\Big\downarrow \text{ 7194}$$
$$\frac{1}{2}\left(\frac{x^6}{3W(ax^2)}-\frac{2}{3}\int \frac{x^4}{W(ax^2)(W(ax^2)+1)}dx^2\right)$$
$$\Big\downarrow \text{ 7205}$$
$$\frac{1}{2}\left(\frac{x^6}{3W(ax^2)}-\frac{2}{3}\left(\frac{x^6}{3W(ax^2)^2}-\frac{1}{3}\int \frac{x^4}{W(ax^2)^2(W(ax^2)+1)}dx^2\right)\right)$$
$$\Big\downarrow \text{ 7201}$$
$$\frac{1}{2}\left(\frac{x^6}{3W(ax^2)}-\frac{2}{3}\left(\frac{x^6}{3W(ax^2)^2}-\frac{x^6}{9W(ax^2)^3}\right)\right)$$

input
```
Int[x^5/(1 + ProductLog[a*x^2]),x]
```

output
```
((-2*(-1/9*x^6/ProductLog[a*x^2]^3 + x^6/(3*ProductLog[a*x^2]^2)))/3 + x^6
/(3*ProductLog[a*x^2]))/2
```

Defintions of rubi rules used

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{x^5}{1 + \mathrm{LambertW}(a\,x^2)}dx$$

input
```
int(x^5/(1+LambertW(a*x^2)),x)
```

output
```
int(x^5/(1+LambertW(a*x^2)),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{1 + W(ax^2)} \, dx = \frac{9\,x^6\,\mathrm{W}(ax^2)^2 - 6\,x^6\,\mathrm{W}(ax^2) + 2\,x^6}{54\,\mathrm{W}(ax^2)^3}$$

input
```
integrate(x^5/(1+lambert_w(a*x^2)),x, algorithm="fricas")
```

output
```
1/54*(9*x^6*lambert_w(a*x^2)^2 - 6*x^6*lambert_w(a*x^2) + 2*x^6)/lambert_w
(a*x^2)^3
```

Sympy [F]

$$\int \frac{x^5}{1 + W(ax^2)} \, dx = \int \frac{x^5}{W(ax^2) + 1} \, dx$$

input
```
integrate(x**5/(1+LambertW(a*x**2)),x)
```

output
```
Integral(x**5/(LambertW(a*x**2) + 1), x)
```

Maxima [F]

$$\int \frac{x^5}{1 + W(ax^2)} \, dx = \int \frac{x^5}{\mathrm{W}(ax^2) + 1} \, dx$$

input
```
integrate(x^5/(1+lambert_w(a*x^2)),x, algorithm="maxima")
```

output
```
integrate(x^5/(lambert_w(a*x^2) + 1), x)
```

Giac [F]

$$\int \frac{x^5}{1+W(ax^2)}\,dx = \int \frac{x^5}{\mathrm{W}(ax^2)+1}\,dx$$

input
```
integrate(x^5/(1+lambert_w(a*x^2)),x, algorithm="giac")
```

output
```
integrate(x^5/(lambert_w(a*x^2) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{1+W(ax^2)}\,dx = \int \frac{x^5}{\mathrm{LambertW}(a\,x^2)+1}\,dx$$

input
```
int(x^5/(LambertW(a*x^2) + 1),x)
```

output
```
int(x^5/(LambertW(a*x^2) + 1), x)
```

Reduce [F]

$$\int \frac{x^5}{1+W(ax^2)}\,dx = \int \frac{x^5}{lambert\_w(a\,x^2)+1}dx$$

input
```
int(x^5/(1+Lambert_W(a*x^2)),x)
```

output
```
int(x**5/(lambert_w(a*x**2) + 1),x)
```

3.307 $\int \frac{x^3}{1+W(ax^2)} dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{x^3}{1+W(ax^2)} dx = -\frac{x^4}{8W(ax^2)^2} + \frac{x^4}{4W(ax^2)}$$

output
```
-1/8*x^4/LambertW(a*x^2)^2+1/4*x^4/LambertW(a*x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+W(ax^2)} dx = -\frac{x^4}{8W(ax^2)^2} + \frac{x^4}{4W(ax^2)}$$

input
```
Integrate[x^3/(1 + ProductLog[a*x^2]),x]
```

output
```
-1/8*x^4/ProductLog[a*x^2]^2 + x^4/(4*ProductLog[a*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {7283, 7194, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{W(ax^2)+1}dx$$
$$\Big\downarrow \text{ 7283}$$
$$\frac{1}{2}\int \frac{x^2}{W(ax^2)+1}dx^2$$
$$\Big\downarrow \text{ 7194}$$
$$\frac{1}{2}\left(\frac{x^4}{2W(ax^2)}-\frac{1}{2}\int \frac{x^2}{W(ax^2)(W(ax^2)+1)}dx^2\right)$$
$$\Big\downarrow \text{ 7201}$$
$$\frac{1}{2}\left(\frac{x^4}{2W(ax^2)}-\frac{x^4}{4W(ax^2)^2}\right)$$

input
```
Int[x^3/(1 + ProductLog[a*x^2]),x]
```

output
```
(-1/4*x^4/ProductLog[a*x^2]^2 + x^4/(2*ProductLog[a*x^2]))/2
```

Defintions of rubi rules used

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{x^3}{1 + \mathrm{LambertW}\left(a\,x^2\right)}dx$$

input
```
int(x^3/(1+LambertW(a*x^2)),x)
```

output
```
int(x^3/(1+LambertW(a*x^2)),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{1 + W\left(ax^2\right)}\,dx = \frac{2\,x^4\,\mathrm{W}(ax^2) - x^4}{8\,\mathrm{W}(ax^2)^2}$$

input
```
integrate(x^3/(1+lambert_w(a*x^2)),x, algorithm="fricas")
```

output
```
1/8*(2*x^4*lambert_w(a*x^2) - x^4)/lambert_w(a*x^2)^2
```

Sympy [F]

$$\int \frac{x^3}{1 + W\left(ax^2\right)}\,dx = \int \frac{x^3}{W\left(ax^2\right) + 1}\,dx$$

input
```
integrate(x**3/(1+LambertW(a*x**2)),x)
```

output
```
Integral(x**3/(LambertW(a*x**2) + 1), x)
```

Maxima [F]

$$\int \frac{x^3}{1 + W\left(ax^2\right)}\,dx = \int \frac{x^3}{\mathrm{W}(ax^2) + 1}\,dx$$

input
```
integrate(x^3/(1+lambert_w(a*x^2)),x, algorithm="maxima")
```

output
```
integrate(x^3/(lambert_w(a*x^2) + 1), x)
```

Giac [F]

$$\int \frac{x^3}{1 + W\left(ax^2\right)}\,dx = \int \frac{x^3}{\mathrm{W}(ax^2) + 1}\,dx$$

input
```
integrate(x^3/(1+lambert_w(a*x^2)),x, algorithm="giac")
```

output
```
integrate(x^3/(lambert_w(a*x^2) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{1+W(ax^2)}\,dx = \int \frac{x^3}{\mathrm{LambertW}(a\,x^2)+1}\,dx$$

input
```
int(x^3/(LambertW(a*x^2) + 1),x)
```

output
```
int(x^3/(LambertW(a*x^2) + 1), x)
```

Reduce [F]

$$\int \frac{x^3}{1+W(ax^2)}\,dx = \int \frac{x^3}{lambert\_w(a\,x^2)+1}dx$$

input
```
int(x^3/(1+Lambert_W(a*x^2)),x)
```

output
```
int(x**3/(lambert_w(a*x**2) + 1),x)
```

3.308 $\int \frac{x}{1+W(ax^2)} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \frac{x}{1+W(ax^2)} \, dx = \frac{x^2}{2W(ax^2)}$$

output
```
1/2*x^2/LambertW(a*x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+W(ax^2)} \, dx = \frac{x^2}{2W(ax^2)}$$

input
```
Integrate[x/(1 + ProductLog[a*x^2]),x]
```

output
```
x^2/(2*ProductLog[a*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7266, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{W(ax^2)+1}dx$$
$$\Big\downarrow \text{7266}$$
$$\frac{1}{2}\int \frac{1}{W(ax^2)+1}dx^2$$
$$\Big\downarrow \text{7176}$$
$$\frac{x^2}{2W(ax^2)}$$

input
```
Int[x/(1 + ProductLog[a*x^2]),x]
```

output
```
x^2/(2*ProductLog[a*x^2])
```

Defintions of rubi rules used

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{x^{2}}{2 \operatorname{LambertW}\left(a \,x^{2}\right)}$	14
default	$\frac{x^{2}}{2 \operatorname{LambertW}\left(a \,x^{2}\right)}$	14
parallelrisch	$\frac{x^{2}}{2 \operatorname{LambertW}\left(a \,x^{2}\right)}$	14

input
```
int(x/(1+LambertW(a*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/LambertW(a*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x}{1+W\left(ax^2\right)}\,dx=\frac{x^{2}}{2 \, \mathrm{W}\left(a x^{2}\right)}$$

input
```
integrate(x/(1+lambert_w(a*x^2)),x, algorithm="fricas")
```

output
```
1/2*x^2/lambert_w(a*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+W\left(ax^2\right)}\,dx=\begin{cases} \frac{x^{2}}{2 W\left(a x^{2}\right)} & \text{for}\: a \neq 0 \\ \frac{x^{2}}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(1+LambertW(a*x**2)),x)
```

output
```
Piecewise((x**2/(2*LambertW(a*x**2)), Ne(a, 0)), (x**2/2, True))
```

Maxima [F]

$$\int \frac{x}{1+W(ax^2)}\,dx = \int \frac{x}{\mathrm{W}(ax^2)+1}\,dx$$

input
```
integrate(x/(1+lambert_w(a*x^2)),x, algorithm="maxima")
```

output
```
integrate(x/(lambert_w(a*x^2) + 1), x)
```

Giac [F]

$$\int \frac{x}{1+W(ax^2)}\,dx = \int \frac{x}{\mathrm{W}(ax^2)+1}\,dx$$

input
```
integrate(x/(1+lambert_w(a*x^2)),x, algorithm="giac")
```

output
```
integrate(x/(lambert_w(a*x^2) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{1+W(ax^2)}\,dx = \int \frac{x}{\mathrm{LambertW}\left(a\,x^2\right)+1}\,dx$$

input
```
int(x/(LambertW(a*x^2) + 1),x)
```

output
```
int(x/(LambertW(a*x^2) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{1+W(ax^2)}\,dx = \frac{e^{lambert\_w(a\,x^2)}}{2a}$$

input

```
int(x/(1+Lambert_W(a*x^2)),x)
```

output

```
e**lambert_w(a*x**2)/(2*a)
```

3.309 $\int \frac{1}{x(1+W(ax^2))}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 11

$$\int \frac{1}{x(1+W(ax^2))}\,dx = \frac{1}{2}\log\left(W(ax^2)\right)$$

output
```
1/2*ln(LambertW(a*x^2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+W(ax^2))}\,dx = \frac{1}{2}\log\left(W(ax^2)\right)$$

input
```
Integrate[1/(x*(1 + ProductLog[a*x^2])),x]
```

output
```
Log[ProductLog[a*x^2]]/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {7198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(W\left(ax^2\right)+1\right)}dx$$
$$\Big\downarrow \text{7198}$$
$$\frac{1}{2}\log\left(W\left(ax^2\right)\right)$$

input
```
Int[1/(x*(1 + ProductLog[a*x^2])),x]
```

output
```
Log[ProductLog[a*x^2]]/2
```

Defintions of rubi rules used

rule 7198
```
Int[1/((x_)*((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp
[Log[ProductLog[a*x^n]]/(d*n), x] /; FreeQ[{a, d, n}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\ln\left(\operatorname{LambertW}\left(a\,x^{2}\right)\right)}{2}$	10
default	$\frac{\ln\left(\operatorname{LambertW}\left(a\,x^{2}\right)\right)}{2}$	10
parallelrisch	$-\frac{\operatorname{LambertW}\left(a\,x^{2}\right)}{2}+\ln\left(x\right)$	12

input
```
int(1/x/(1+LambertW(a*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(LambertW(a*x^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\left(1+W\left(ax^2\right)\right)}\,dx = \frac{1}{2}\log\left(\mathrm{W}(ax^2)\right)$$

input
```
integrate(1/x/(1+lambert_w(a*x^2)),x, algorithm="fricas")
```

output
```
1/2*log(lambert_w(a*x^2))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\left(1+W\left(ax^2\right)\right)}\,dx = \log\left(x\right) - \frac{W(ax^2)}{2}$$

input
```
integrate(1/x/(1+LambertW(a*x**2)),x)
```

output
```
log(x) - LambertW(a*x**2)/2
```

Maxima [F]

$$\int \frac{1}{x\left(1+W\left(ax^2\right)\right)}\,dx=\int \frac{1}{x(\mathrm{W}(ax^2)+1)}\,dx$$

input
```
integrate(1/x/(1+lambert_w(a*x^2)),x, algorithm="maxima")
```

output
```
integrate(1/(x*(lambert_w(a*x^2) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x\left(1+W\left(ax^2\right)\right)}\,dx=\int \frac{1}{x(\mathrm{W}(ax^2)+1)}\,dx$$

input
```
integrate(1/x/(1+lambert_w(a*x^2)),x, algorithm="giac")
```

output
```
integrate(1/(x*(lambert_w(a*x^2) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(1+W\left(ax^2\right)\right)}\,dx=\int \frac{1}{x\,\left(\mathrm{LambertW}\left(a\,x^2\right)+1\right)}\,dx$$

input
```
int(1/(x*(LambertW(a*x^2) + 1)),x)
```

output
```
int(1/(x*(LambertW(a*x^2) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(1+W\left(ax^2\right)\right)}\,dx=\int \frac{1}{lambert\_w\left(a\,x^2\right)x+x}dx$$

input
```
int(1/x/(1+Lambert_W(a*x^2)),x)
```

output
```
int(1/(lambert_w(a*x**2)*x + x),x)
```

3.310 $\int \frac{1}{x^3(1+W(ax^2))} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 22

$$\int \frac{1}{x^3 (1+W(ax^2))} \, dx = -\frac{1}{2x^2} - \frac{1}{2} a \, \text{ExpIntegralEi}\left(-W\left(ax^2\right)\right)$$

output
```
-1/2/x^2-1/2*a*Ei(-LambertW(a*x^2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (1+W(ax^2))} \, dx = -\frac{1}{2x^2} - \frac{1}{2} a \, \text{ExpIntegralEi}\left(-W\left(ax^2\right)\right)$$

input
```
Integrate[1/(x^3*(1 + ProductLog[a*x^2])),x]
```

output
```
-1/2*1/x^2 - (a*ExpIntegralEi[-ProductLog[a*x^2]])/2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {7283, 7196, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (W(ax^2)+1)} \, dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int \frac{1}{x^4 (W(ax^2)+1)}dx^2$$

$$\Big\downarrow \text{7196}$$

$$\frac{1}{2}\left(-\int \frac{W(ax^2)}{x^4 (W(ax^2)+1)}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{7202}$$

$$\frac{1}{2}\left(-a\,\text{ExpIntegralEi}\left(-W(ax^2)\right)-\frac{1}{x^2}\right)$$

input
```
Int[1/(x^3*(1 + ProductLog[a*x^2])),x]
```

output
```
(-x^(-2) - a*ExpIntegralEi[-ProductLog[a*x^2]])/2
```

Defintions of rubi rules used

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{1}{x^3 \left(1+\mathrm{LambertW}\left(a\,x^2\right)\right)}dx$$

input
```
int(1/x^3/(1+LambertW(a*x^2)),x)
```

output
```
int(1/x^3/(1+LambertW(a*x^2)),x)
```

Fricas [F]

$$\int \frac{1}{x^3 \left(1+W\left(ax^2\right)\right)}\,dx = \int \frac{1}{x^3(\mathrm{W}(ax^2)+1)}\,dx$$

input
```
integrate(1/x^3/(1+lambert_w(a*x^2)),x, algorithm="fricas")
```

output
```
integral(1/(x^3*lambert_w(a*x^2) + x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3 \left(1 + W\left(ax^2\right)\right)} \, dx = \int \frac{1}{x^3 \left(W\left(ax^2\right) + 1\right)} \, dx$$

input
```
integrate(1/x**3/(1+LambertW(a*x**2)),x)
```

output
```
Integral(1/(x**3*(LambertW(a*x**2) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \left(1 + W\left(ax^2\right)\right)} \, dx = \int \frac{1}{x^3 (\mathrm{W}(ax^2) + 1)} \, dx$$

input
```
integrate(1/x^3/(1+lambert_w(a*x^2)),x, algorithm="maxima")
```

output
```
integrate(1/(x^3*(lambert_w(a*x^2) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^3 \left(1 + W\left(ax^2\right)\right)} \, dx = \int \frac{1}{x^3 (\mathrm{W}(ax^2) + 1)} \, dx$$

input
```
integrate(1/x^3/(1+lambert_w(a*x^2)),x, algorithm="giac")
```

output
```
integrate(1/(x^3*(lambert_w(a*x^2) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\,(1+W(ax^2))}\,dx = \int \frac{1}{x^3\,\left(\mathrm{LambertW}\left(a\,x^2\right)+1\right)}\,dx$$

input
```
int(1/(x^3*(LambertW(a*x^2) + 1)),x)
```

output
```
int(1/(x^3*(LambertW(a*x^2) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\,(1+W(ax^2))}\,dx = \frac{-2\left(\int \frac{lambert\_w(a\,x^2)}{lambert\_w(a\,x^2)x^3+x^3}dx\right)x^2+6\left(\int \frac{1}{lambert\_w(a\,x^2)x^3+x^3}dx\right)x^2-1}{8x^2}$$

input
```
int(1/x^3/(1+Lambert_W(a*x^2)),x)
```

output
```
( - 2*int(lambert_w(a*x**2)/(lambert_w(a*x**2)*x**3 + x**3),x)*x**2 + 6*in
t(1/(lambert_w(a*x**2)*x**3 + x**3),x)*x**2 - 1)/(8*x**2)
```

3.311 $\int \frac{1}{x^5(1+W(ax^2))}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \frac{1}{x^5(1+W(ax^2))}\,dx = -\frac{1}{4x^4} + a^2\,\text{ExpIntegralEi}\left(-2W\left(ax^2\right)\right) + \frac{W(ax^2)}{2x^4}$$

output
```
-1/4/x^4+a^2*Ei(-2*LambertW(a*x^2))+1/2*LambertW(a*x^2)/x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5(1+W(ax^2))}\,dx = -\frac{1}{4x^4} + a^2\,\text{ExpIntegralEi}\left(-2W\left(ax^2\right)\right) + \frac{W(ax^2)}{2x^4}$$

input
```
Integrate[1/(x^5*(1 + ProductLog[a*x^2])),x]
```

output
```
-1/4*1/x^4 + a^2*ExpIntegralEi[-2*ProductLog[a*x^2]] + ProductLog[a*x^2]/(
2*x^4)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7283, 7196, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (W(ax^2)+1)} \, dx$$

$$\Big\downarrow \text{ 7283}$$

$$\frac{1}{2} \int \frac{1}{x^6 (W(ax^2)+1)} dx^2$$

$$\Big\downarrow \text{ 7196}$$

$$\frac{1}{2} \left(-\int \frac{W(ax^2)}{x^6 (W(ax^2)+1)} dx^2 - \frac{1}{2x^4} \right)$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{1}{2} \left(2 \int \frac{W(ax^2)^2}{x^6 (W(ax^2)+1)} dx^2 + \frac{W(ax^2)}{x^4} - \frac{1}{2x^4} \right)$$

$$\Big\downarrow \text{ 7202}$$

$$\frac{1}{2} \left(2a^2 \,\text{ExpIntegralEi}\left(-2W(ax^2)\right) + \frac{W(ax^2)}{x^4} - \frac{1}{2x^4} \right)$$

input
```
Int[1/(x^5*(1 + ProductLog[a*x^2])),x]
```

output
```
(-1/2*1/x^4 + 2*a^2*ExpIntegralEi[-2*ProductLog[a*x^2]] + ProductLog[a*x^2
]/x^4)/2
```

Defintions of rubi rules used

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{1}{x^5 \left(1+\mathrm{LambertW}\left(a\, x^2\right)\right)}dx$$

input
```
int(1/x^5/(1+LambertW(a*x^2)),x)
```

output
```
int(1/x^5/(1+LambertW(a*x^2)),x)
```

Fricas [F]

$$\int \frac{1}{x^5\left(1+W\left(ax^2\right)\right)}\,dx = \int \frac{1}{x^5(\mathrm{W}(ax^2)+1)}\,dx$$

input
```
integrate(1/x^5/(1+lambert_w(a*x^2)),x, algorithm="fricas")
```

output
```
integral(1/(x^5*lambert_w(a*x^2) + x^5), x)
```

Sympy [F]

$$\int \frac{1}{x^5\left(1+W\left(ax^2\right)\right)}\,dx = \int \frac{1}{x^5\left(W\left(ax^2\right)+1\right)}\,dx$$

input
```
integrate(1/x**5/(1+LambertW(a*x**2)),x)
```

output
```
Integral(1/(x**5*(LambertW(a*x**2) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^5\left(1+W\left(ax^2\right)\right)}\,dx = \int \frac{1}{x^5(\mathrm{W}(ax^2)+1)}\,dx$$

input
```
integrate(1/x^5/(1+lambert_w(a*x^2)),x, algorithm="maxima")
```

output
```
integrate(1/(x^5*(lambert_w(a*x^2) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^5\left(1+W\left(ax^2\right)\right)}\,dx=\int \frac{1}{x^5(\mathrm{W}(ax^2)+1)}\,dx$$

input
```
integrate(1/x^5/(1+lambert_w(a*x^2)),x, algorithm="giac")
```

output
```
integrate(1/(x^5*(lambert_w(a*x^2) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(1+W\left(ax^2\right)\right)}\,dx=\int \frac{1}{x^5\left(\mathrm{LambertW}\left(a\,x^2\right)+1\right)}\,dx$$

input
```
int(1/(x^5*(LambertW(a*x^2) + 1)),x)
```

output
```
int(1/(x^5*(LambertW(a*x^2) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^5\left(1+W\left(ax^2\right)\right)}\,dx=\frac{-4\left(\int \frac{lambert\_w(a\,x^2)}{lambert\_w(a\,x^2)x^5+x^5}dx\right)x^4+12\left(\int \frac{1}{lambert\_w(a\,x^2)x^5+x^5}dx\right)x^4-1}{16x^4}$$

input
```
int(1/x^5/(1+Lambert_W(a*x^2)),x)
```

output
```
( - 4*int(lambert_w(a*x**2)/(lambert_w(a*x**2)*x**5 + x**5),x)*x**4 + 12*i
nt(1/(lambert_w(a*x**2)*x**5 + x**5),x)*x**4 - 1)/(16*x**4)
```

3.312 $\int \frac{x^4}{1+W(ax^2)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{x^4}{1+W(ax^2)}\,dx = -\frac{3x^5}{25W(ax^2)^2} + \frac{3e^{-\frac{3}{2}W(ax^2)}x^3\Gamma\left(\frac{1}{2},-\frac{5}{2}W(ax^2)\right)\sqrt{-W(ax^2)}}{25\sqrt{10}aW(ax^2)^2} + \frac{x^5}{5W(ax^2)}$$

output
```
-3/25*x^5/LambertW(a*x^2)^2+3/250*10^(1/2)*x^3*Pi^(1/2)*erfc(1/2*(-10*Lamb
ertW(a*x^2))^(1/2))*(-LambertW(a*x^2))^(1/2)/a/exp(3/2*LambertW(a*x^2))/La
mbertW(a*x^2)^2+1/5*x^5/LambertW(a*x^2)
```

Mathematica [F]

$$\int \frac{x^4}{1+W(ax^2)}\,dx = \int \frac{x^4}{1+W(ax^2)}\,dx$$

input
```
Integrate[x^4/(1 + ProductLog[a*x^2]),x]
```

output
```
Integrate[x^4/(1 + ProductLog[a*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{W(ax^2)+1} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{x^4}{W(ax^2)+1} dx$$

input
```
Int[x^4/(1 + ProductLog[a*x^2]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^4}{1+\mathrm{LambertW}\left(a\,x^2\right)} dx$$

input
```
int(x^4/(1+LambertW(a*x^2)),x)
```

output
```
int(x^4/(1+LambertW(a*x^2)),x)
```

Fricas [F]

$$\int \frac{x^4}{1+W(ax^2)} \, dx = \int \frac{x^4}{\mathrm{W}(ax^2)+1} \, dx$$

input
```
integrate(x^4/(1+lambert_w(a*x^2)),x, algorithm="fricas")
```

output
```
integral(x^4/(lambert_w(a*x^2) + 1), x)
```

Sympy [F]

$$\int \frac{x^4}{1+W(ax^2)}\,dx = \int \frac{x^4}{W(ax^2)+1}\,dx$$

input
```
integrate(x**4/(1+LambertW(a*x**2)),x)
```

output
```
Integral(x**4/(LambertW(a*x**2) + 1), x)
```

Maxima [F]

$$\int \frac{x^4}{1+W(ax^2)}\,dx = \int \frac{x^4}{\mathrm{W}(ax^2)+1}\,dx$$

input
```
integrate(x^4/(1+lambert_w(a*x^2)),x, algorithm="maxima")
```

output
```
integrate(x^4/(lambert_w(a*x^2) + 1), x)
```

Giac [F]

$$\int \frac{x^4}{1+W(ax^2)}\,dx = \int \frac{x^4}{\mathrm{W}(ax^2)+1}\,dx$$

input
```
integrate(x^4/(1+lambert_w(a*x^2)),x, algorithm="giac")
```

output
```
integrate(x^4/(lambert_w(a*x^2) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{1+W(ax^2)}\,dx = \int \frac{x^4}{\mathrm{LambertW}\left(a\,x^2\right)+1}\,dx$$

input
```
int(x^4/(LambertW(a*x^2) + 1),x)
```

output
```
int(x^4/(LambertW(a*x^2) + 1), x)
```

Reduce [F]

$$\int \frac{x^4}{1+W(ax^2)}\,dx = \int \frac{x^4}{lambert\_w\left(a\,x^2\right)+1}dx$$

input
```
int(x^4/(1+Lambert_W(a*x^2)),x)
```

output
```
int(x**4/(lambert_w(a*x**2) + 1),x)
```

3.313 $\int \frac{x^2}{1+W(ax^2)} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \frac{x^2}{1+W(ax^2)} \, dx = \frac{x^3}{3W(ax^2)} - \frac{e^{-\frac{1}{2}W(ax^2)}x\Gamma\left(\frac{1}{2},-\frac{3}{2}W(ax^2)\right)\sqrt{-W(ax^2)}}{3\sqrt{6}aW(ax^2)}$$

output
```
1/3*x^3/LambertW(a*x^2)-1/18*6^(1/2)*x*Pi^(1/2)*erfc(1/2*(-6*LambertW(a*x^
2))^(1/2))*(-LambertW(a*x^2))^(1/2)/a/exp(1/2*LambertW(a*x^2))/LambertW(a*
x^2)
```

Mathematica [F]

$$\int \frac{x^2}{1+W(ax^2)} \, dx = \int \frac{x^2}{1+W(ax^2)} \, dx$$

input
```
Integrate[x^2/(1 + ProductLog[a*x^2]),x]
```

output
```
Integrate[x^2/(1 + ProductLog[a*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{W(ax^2)+1} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{x^2}{W(ax^2)+1} dx$$

input
```
Int[x^2/(1 + ProductLog[a*x^2]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^2}{1+\operatorname{LambertW}(a\,x^2)} dx$$

input
```
int(x^2/(1+LambertW(a*x^2)),x)
```

output
```
int(x^2/(1+LambertW(a*x^2)),x)
```

Fricas [F]

$$\int \frac{x^2}{1+W(ax^2)} \, dx = \int \frac{x^2}{\mathrm{W}(ax^2)+1} \, dx$$

input
```
integrate(x^2/(1+lambert_w(a*x^2)),x, algorithm="fricas")
```

output
```
integral(x^2/(lambert_w(a*x^2) + 1), x)
```

Sympy [F]

$$\int \frac{x^2}{1+W\left(ax^2\right)}\,dx = \int \frac{x^2}{W\left(ax^2\right)+1}\,dx$$

input
```
integrate(x**2/(1+LambertW(a*x**2)),x)
```

output
```
Integral(x**2/(LambertW(a*x**2) + 1), x)
```

Maxima [F]

$$\int \frac{x^2}{1+W\left(ax^2\right)}\,dx = \int \frac{x^2}{\mathrm{W}(ax^2)+1}\,dx$$

input
```
integrate(x^2/(1+lambert_w(a*x^2)),x, algorithm="maxima")
```

output
```
integrate(x^2/(lambert_w(a*x^2) + 1), x)
```

Giac [F]

$$\int \frac{x^2}{1+W\left(ax^2\right)}\,dx = \int \frac{x^2}{\mathrm{W}(ax^2)+1}\,dx$$

input
```
integrate(x^2/(1+lambert_w(a*x^2)),x, algorithm="giac")
```

output
```
integrate(x^2/(lambert_w(a*x^2) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{1+W(ax^2)}\,dx = \int \frac{x^2}{\mathrm{LambertW}\left(a\,x^2\right)+1}\,dx$$

input
```
int(x^2/(LambertW(a*x^2) + 1),x)
```

output
```
int(x^2/(LambertW(a*x^2) + 1), x)
```

Reduce [F]

$$\int \frac{x^2}{1+W(ax^2)}\,dx = \int \frac{x^2}{lambert\_w\left(a\,x^2\right)+1}dx$$

input
```
int(x^2/(1+Lambert_W(a*x^2)),x)
```

output
```
int(x**2/(lambert_w(a*x**2) + 1),x)
```

3.314 $\int \frac{1}{1+W(ax^2)} dx$

Optimal result

Integrand size = 10, antiderivative size = 50

$$\int \frac{1}{1+W(ax^2)} dx = \frac{e^{\frac{1}{2}W(ax^2)}\Gamma\left(\frac{1}{2},-\frac{1}{2}W(ax^2)\right)\sqrt{-W(ax^2)}}{\sqrt{2}ax}$$

output
```
1/2*2^(1/2)*exp(1/2*LambertW(a*x^2))*Pi^(1/2)*erfc(1/2*(-2*LambertW(a*x^2)
)^(1/2))*(-LambertW(a*x^2))^(1/2)/a/x
```

Mathematica [F]

$$\int \frac{1}{1+W(ax^2)} dx = \int \frac{1}{1+W(ax^2)} dx$$

input
```
Integrate[(1 + ProductLog[a*x^2])^(-1),x]
```

output
```
Integrate[(1 + ProductLog[a*x^2])^(-1), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W\left(ax^2\right)+1}\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{1}{W\left(ax^2\right)+1}dx$$

input
```
Int[(1 + ProductLog[a*x^2])^(-1),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{1}{1+\operatorname{LambertW}\left(a\,x^{2}\right)}dx$$

input
```
int(1/(1+LambertW(a*x^2)),x)
```

output
```
int(1/(1+LambertW(a*x^2)),x)
```

Fricas [F]

$$\int \frac{1}{1+W\left(ax^2\right)}\,dx=\int \frac{1}{\mathrm{W}(ax^2)+1}\,dx$$

input
```
integrate(1/(1+lambert_w(a*x^2)),x, algorithm="fricas")
```

output
```
integral(1/(lambert_w(a*x^2) + 1), x)
```

Sympy [F]

$$\int \frac{1}{1 + W(ax^2)}\,dx = \int \frac{1}{W(ax^2) + 1}\,dx$$

input
```
integrate(1/(1+LambertW(a*x**2)),x)
```

output
```
Integral(1/(LambertW(a*x**2) + 1), x)
```

Maxima [F]

$$\int \frac{1}{1 + W(ax^2)}\,dx = \int \frac{1}{\mathrm{W}(ax^2) + 1}\,dx$$

input
```
integrate(1/(1+lambert_w(a*x^2)),x, algorithm="maxima")
```

output
```
integrate(1/(lambert_w(a*x^2) + 1), x)
```

Giac [F]

$$\int \frac{1}{1 + W(ax^2)}\,dx = \int \frac{1}{\mathrm{W}(ax^2) + 1}\,dx$$

input
```
integrate(1/(1+lambert_w(a*x^2)),x, algorithm="giac")
```

output
```
integrate(1/(lambert_w(a*x^2) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+W(ax^2)}\,dx = \int \frac{1}{\mathrm{LambertW}\left(a\,x^2\right)+1}\,dx$$

input
```
int(1/(LambertW(a*x^2) + 1),x)
```

output
```
int(1/(LambertW(a*x^2) + 1), x)
```

Reduce [F]

$$\int \frac{1}{1+W(ax^2)}\,dx = \int \frac{1}{\mathit{lambert\_w}\left(a\,x^2\right)+1}dx$$

input
```
int(1/(1+Lambert_W(a*x^2)),x)
```

output
```
int(1/(lambert_w(a*x**2) + 1),x)
```

3.315 $\int \frac{1}{x^2(1+W(ax^2))}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \frac{1}{x^2 (1+W(ax^2))}\,dx = -\frac{1}{x} + \frac{e^{\frac{3}{2}W(ax^2)}\Gamma\left(\frac{1}{2},\frac{1}{2}W(ax^2)\right)W(ax^2)^{3/2}}{\sqrt{2}ax^3}$$

output
```
-1/x+1/2*2^(1/2)*exp(3/2*LambertW(a*x^2))*Pi^(1/2)*erfc(1/2*2^(1/2)*Lamber
tW(a*x^2)^(1/2))*LambertW(a*x^2)^(3/2)/a/x^3
```

Mathematica [F]

$$\int \frac{1}{x^2 (1+W(ax^2))}\,dx = \int \frac{1}{x^2 (1+W(ax^2))}\,dx$$

input
```
Integrate[1/(x^2*(1 + ProductLog[a*x^2])),x]
```

output
```
Integrate[1/(x^2*(1 + ProductLog[a*x^2])), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (W(ax^2)+1)} dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{1}{x^2 (W(ax^2)+1)} dx$$

input
```
Int[1/(x^2*(1 + ProductLog[a*x^2])),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{1}{x^2 (1+\mathrm{LambertW}(a\,x^2))} dx$$

input
```
int(1/x^2/(1+LambertW(a*x^2)),x)
```

output
```
int(1/x^2/(1+LambertW(a*x^2)),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (1+W(ax^2))} dx = \int \frac{1}{x^2(\mathrm{W}(ax^2)+1)} dx$$

input
```
integrate(1/x^2/(1+lambert_w(a*x^2)),x, algorithm="fricas")
```

output
```
integral(1/(x^2*lambert_w(a*x^2) + x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2(1+W(ax^2))}\,dx = \int \frac{1}{x^2(W(ax^2)+1)}\,dx$$

input
```
integrate(1/x**2/(1+LambertW(a*x**2)),x)
```

output
```
Integral(1/(x**2*(LambertW(a*x**2) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(1+W(ax^2))}\,dx = \int \frac{1}{x^2(\mathrm{W}(ax^2)+1)}\,dx$$

input
```
integrate(1/x^2/(1+lambert_w(a*x^2)),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*(lambert_w(a*x^2) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^2(1+W(ax^2))}\,dx = \int \frac{1}{x^2(\mathrm{W}(ax^2)+1)}\,dx$$

input
```
integrate(1/x^2/(1+lambert_w(a*x^2)),x, algorithm="giac")
```

output
```
integrate(1/(x^2*(lambert_w(a*x^2) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (1 + W (ax^2))} \, dx = \int \frac{1}{x^2 \, (\mathrm{LambertW}\left(a\,x^2\right) + 1)} \, dx$$

input
```
int(1/(x^2*(LambertW(a*x^2) + 1)),x)
```

output
```
int(1/(x^2*(LambertW(a*x^2) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (1 + W (ax^2))} \, dx = \frac{-\left(\int \frac{lambert\_w(a\,x^2)}{lambert\_w(a\,x^2)x^2+x^2}dx\right) x + 3\left(\int \frac{1}{lambert\_w(a\,x^2)x^2+x^2}dx\right) x - 1}{4x}$$

input
```
int(1/x^2/(1+Lambert_W(a*x^2)),x)
```

output
```
( - int(lambert_w(a*x**2)/(lambert_w(a*x**2)*x**2 + x**2),x)*x + 3*int(1/(
lambert_w(a*x**2)*x**2 + x**2),x)*x - 1)/(4*x)
```

3.316 $\int \frac{1}{x^4(1+W(ax^2))} dx$

Optimal result

Integrand size = 14, antiderivative size = 69

$$\int \frac{1}{x^4 (1 + W(ax^2))} dx = -\frac{1}{3x^3} + \frac{W(ax^2)}{x^3} - \frac{\sqrt{\frac{3}{2}} e^{\frac{5}{2}W(ax^2)} \Gamma\left(\frac{1}{2}, \frac{3}{2}W(ax^2)\right) W(ax^2)^{5/2}}{ax^5}$$

output
```
-1/3/x^3+LambertW(a*x^2)/x^3-1/2*6^(1/2)*exp(5/2*LambertW(a*x^2))*Pi^(1/2)
*erfc(1/2*6^(1/2)*LambertW(a*x^2)^(1/2))*LambertW(a*x^2)^(5/2)/a/x^5
```

Mathematica [F]

$$\int \frac{1}{x^4 (1 + W(ax^2))} dx = \int \frac{1}{x^4 (1 + W(ax^2))} dx$$

input
```
Integrate[1/(x^4*(1 + ProductLog[a*x^2])),x]
```

output
```
Integrate[1/(x^4*(1 + ProductLog[a*x^2])), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(W\left(ax^2\right)+1\right)} \, dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{1}{x^4 \left(W\left(ax^2\right)+1\right)}dx$$

input
```
Int[1/(x^4*(1 + ProductLog[a*x^2])),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{1}{x^{4} \left(1+\operatorname{LambertW}\left(a \,x^{2}\right)\right)}d x$$

input
```
int(1/x^4/(1+LambertW(a*x^2)),x)
```

output
```
int(1/x^4/(1+LambertW(a*x^2)),x)
```

Fricas [F]

$$\int \frac{1}{x^4 \left(1+W\left(ax^2\right)\right)} \, dx=\int { \frac{1}{x^{4} {\left(\mathrm{W}\left(a x^{2}\right) + 1\right)}} \,d x }$$

input
```
integrate(1/x^4/(1+lambert_w(a*x^2)),x, algorithm="fricas")
```

output
```
integral(1/(x^4*lambert_w(a*x^2) + x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4 \left(1+W\left(a x^2\right)\right)} \, dx=\int \frac{1}{x^4 \left(W\left(a x^2\right)+1\right)}\, dx$$

input
```
integrate(1/x**4/(1+LambertW(a*x**2)),x)
```

output
```
Integral(1/(x**4*(LambertW(a*x**2) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(1+W\left(a x^2\right)\right)} \, dx=\int \frac{1}{x^4 (\mathrm{W}(a x^2)+1)}\, dx$$

input
```
integrate(1/x^4/(1+lambert_w(a*x^2)),x, algorithm="maxima")
```

output
```
integrate(1/(x^4*(lambert_w(a*x^2) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^4 \left(1+W\left(a x^2\right)\right)} \, dx=\int \frac{1}{x^4 (\mathrm{W}(a x^2)+1)}\, dx$$

input
```
integrate(1/x^4/(1+lambert_w(a*x^2)),x, algorithm="giac")
```

output
```
integrate(1/(x^4*(lambert_w(a*x^2) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(1+W\left(ax^2\right)\right)}\,dx=\int \frac{1}{x^4\,\left(\mathrm{LambertW}\left(a\,x^2\right)+1\right)}\,dx$$

input
```
int(1/(x^4*(LambertW(a*x^2) + 1)),x)
```

output
```
int(1/(x^4*(LambertW(a*x^2) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\left(1+W\left(ax^2\right)\right)}\,dx = \frac{-3\left(\int \frac{lambert\_w(a\,x^2)}{lambert\_w(a\,x^2)x^4+x^4}dx\right)x^3+9\left(\int \frac{1}{lambert\_w(a\,x^2)x^4+x^4}dx\right)x^3-1}{12x^3}$$

input
```
int(1/x^4/(1+Lambert_W(a*x^2)),x)
```

output
```
( - 3*int(lambert_w(a*x**2)/(lambert_w(a*x**2)*x**4 + x**4),x)*x**3 + 9*in
t(1/(lambert_w(a*x**2)*x**4 + x**4),x)*x**3 - 1)/(12*x**3)
```

3.317 $\int \frac{x^2}{1+W\left(\frac{a}{x}\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int \frac{x^2}{1+W\left(\frac{a}{x}\right)}\,dx=\frac{x^3}{3}+\frac{9}{2}a^3\,\text{ExpIntegralEi}\left(-3W\left(\frac{a}{x}\right)\right)-\frac{1}{2}x^3W\left(\frac{a}{x}\right)+\frac{3}{2}x^3W\left(\frac{a}{x}\right)^2$$

output
```
1/3*x^3+9/2*a^3*Ei(-3*LambertW(a/x))-1/2*x^3*LambertW(a/x)+3/2*x^3*Lambert
W(a/x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{1+W\left(\frac{a}{x}\right)}\,dx=\frac{x^3}{3}+\frac{9}{2}a^3\,\text{ExpIntegralEi}\left(-3W\left(\frac{a}{x}\right)\right)-\frac{1}{2}x^3W\left(\frac{a}{x}\right)+\frac{3}{2}x^3W\left(\frac{a}{x}\right)^2$$

input
```
Integrate[x^2/(1 + ProductLog[a/x]),x]
```

output
```
x^3/3 + (9*a^3*ExpIntegralEi[-3*ProductLog[a/x]])/2 - (x^3*ProductLog[a/x]
)/2 + (3*x^3*ProductLog[a/x]^2)/2
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {7199, 7196, 7206, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{W\left(\frac{a}{x}\right)+1} \, dx$$

$$\Big\downarrow \text{7199}$$

$$-\int \frac{x^4}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}$$

$$\Big\downarrow \text{7196}$$

$$\int \frac{x^4 W\left(\frac{a}{x}\right)}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}+\frac{x^3}{3}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{3}{2}\int \frac{x^4 W\left(\frac{a}{x}\right)^2}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}-\frac{1}{2}x^3 W\left(\frac{a}{x}\right)+\frac{x^3}{3}$$

$$\Big\downarrow \text{7206}$$

$$-\frac{3}{2}\left(x^3\left(-W\left(\frac{a}{x}\right)^2\right)-3\int \frac{x^4 W\left(\frac{a}{x}\right)^3}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}\right)-\frac{1}{2}x^3 W\left(\frac{a}{x}\right)+\frac{x^3}{3}$$

$$\Big\downarrow \text{7202}$$

$$-\frac{3}{2}\left(-3a^3 \text{ExpIntegralEi}\left(-3W\left(\frac{a}{x}\right)\right)-x^3 W\left(\frac{a}{x}\right)^2\right)-\frac{1}{2}x^3 W\left(\frac{a}{x}\right)+\frac{x^3}{3}$$

input
```
Int[x^2/(1 + ProductLog[a/x]),x]
```

output
```
x^3/3 - (x^3*ProductLog[a/x])/2 - (3*(-3*a^3*ExpIntegralEi[-3*ProductLog[a
/x]] - x^3*ProductLog[a/x]^2))/2
```

Defintions of rubi rules used

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$-a^3\left(-\frac{x^3}{3a^3}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)x^3}{2a^3}-\frac{3\,\mathrm{LambertW}\left(\frac{a}{x}\right)^2x^3}{2a^3}+\frac{9\ \mathrm{expIntegral}_1\left(3\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)}{2}\right)$	57
default	$-a^3\left(-\frac{x^3}{3a^3}+\frac{\mathrm{LambertW}\left(\frac{a}{x}\right)x^3}{2a^3}-\frac{3\,\mathrm{LambertW}\left(\frac{a}{x}\right)^2x^3}{2a^3}+\frac{9\ \mathrm{expIntegral}_1\left(3\,\mathrm{LambertW}\left(\frac{a}{x}\right)\right)}{2}\right)$	57

input
```
int(x^2/(1+LambertW(a/x)),x,method=_RETURNVERBOSE)
```

output
```
-a^3*(-1/3*x^3/a^3+1/2*LambertW(a/x)*x^3/a^3-3/2*LambertW(a/x)^2*x^3/a^3+9
/2*Ei(1,3*LambertW(a/x)))
```

Fricas [F]

$$\int \frac{x^2}{1+W\left(\frac{a}{x}\right)}\,dx = \int \frac{x^2}{\mathrm{W}(\frac{a}{x})+1}\,dx$$

input
```
integrate(x^2/(1+lambert_w(a/x)),x, algorithm="fricas")
```

output
```
integral(x^2/(lambert_w(a/x) + 1), x)
```

Sympy [F]

$$\int \frac{x^2}{1+W\left(\frac{a}{x}\right)}\,dx = \int \frac{x^2}{W\left(\frac{a}{x}\right)+1}\,dx$$

input
```
integrate(x**2/(1+LambertW(a/x)),x)
```

output
```
Integral(x**2/(LambertW(a/x) + 1), x)
```

Maxima [F]

$$\int \frac{x^2}{1+W\left(\frac{a}{x}\right)}\,dx = \int \frac{x^2}{\mathrm{W}(\frac{a}{x})+1}\,dx$$

input
```
integrate(x^2/(1+lambert_w(a/x)),x, algorithm="maxima")
```

output
```
integrate(x^2/(lambert_w(a/x) + 1), x)
```

Giac [F]

$$\int \frac{x^2}{1+W\left(\frac{a}{x}\right)}\,dx = \int \frac{x^2}{W(\frac{a}{x})+1}\,dx$$

input
```
integrate(x^2/(1+lambert_w(a/x)),x, algorithm="giac")
```

output
```
integrate(x^2/(lambert_w(a/x) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{1+W\left(\frac{a}{x}\right)}\,dx = \int \frac{x^2}{\mathrm{LambertW}\left(\frac{a}{x}\right)+1}\,dx$$

input
```
int(x^2/(LambertW(a/x) + 1),x)
```

output
```
int(x^2/(LambertW(a/x) + 1), x)
```

Reduce [F]

$$\int \frac{x^2}{1+W\left(\frac{a}{x}\right)}\,dx = \int \frac{x^2}{lambert\_w\left(\frac{a}{x}\right)+1}dx$$

input
```
int(x^2/(1+Lambert_W(a/x)),x)
```

output
```
int(x**2/(lambert_w(a/x) + 1),x)
```

3.318 $\int \frac{x}{1+W\left(\frac{a}{x}\right)} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \frac{x}{1+W\left(\frac{a}{x}\right)} \, dx = \frac{x^2}{2} - 2a^2 \operatorname{ExpIntegralEi}\left(-2W\left(\frac{a}{x}\right)\right) - x^2 W\left(\frac{a}{x}\right)$$

output
```
1/2*x^2-2*a^2*Ei(-2*LambertW(a/x))-x^2*LambertW(a/x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+W\left(\frac{a}{x}\right)} \, dx = \frac{x^2}{2} - 2a^2 \operatorname{ExpIntegralEi}\left(-2W\left(\frac{a}{x}\right)\right) - x^2 W\left(\frac{a}{x}\right)$$

input
```
Integrate[x/(1 + ProductLog[a/x]),x]
```

output
```
x^2/2 - 2*a^2*ExpIntegralEi[-2*ProductLog[a/x]] - x^2*ProductLog[a/x]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7199, 7196, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{W\left(\frac{a}{x}\right)+1}dx$$

$$\Big\downarrow \text{ 7199}$$

$$-\int \frac{x^3}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}$$

$$\Big\downarrow \text{ 7196}$$

$$\int \frac{x^3 W\left(\frac{a}{x}\right)}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}+\frac{x^2}{2}$$

$$\Big\downarrow \text{ 7206}$$

$$-2\int \frac{x^3 W\left(\frac{a}{x}\right)^2}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}+x^2\left(-W\left(\frac{a}{x}\right)\right)+\frac{x^2}{2}$$

$$\Big\downarrow \text{ 7202}$$

$$-2a^2\,\text{ExpIntegralEi}\left(-2W\left(\frac{a}{x}\right)\right)-x^2W\left(\frac{a}{x}\right)+\frac{x^2}{2}$$

input
```
Int[x/(1 + ProductLog[a/x]),x]
```

output
```
x^2/2 - 2*a^2*ExpIntegralEi[-2*ProductLog[a/x]] - x^2*ProductLog[a/x]
```

Defintions of rubi rules used

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

method	result	size
derivativedivides	$-a^2\left(-\frac{x^2}{2a^2}+\frac{\text{LambertW}\left(\frac{a}{x}\right)x^2}{a^2}-2\,\text{expIntegral}_1\left(2\,\text{LambertW}\left(\frac{a}{x}\right)\right)\right)$	40
default	$-a^2\left(-\frac{x^2}{2a^2}+\frac{\text{LambertW}\left(\frac{a}{x}\right)x^2}{a^2}-2\,\text{expIntegral}_1\left(2\,\text{LambertW}\left(\frac{a}{x}\right)\right)\right)$	40

input
```
int(x/(1+LambertW(a/x)),x,method=_RETURNVERBOSE)
```

output
```
-a^2*(-1/2*x^2/a^2+LambertW(a/x)*x^2/a^2-2*Ei(1,2*LambertW(a/x)))
```

Fricas [F]

$$\int \frac{x}{1+W\left(\frac{a}{x}\right)}\,dx = \int \frac{x}{\mathrm{W}(\frac{a}{x})+1}\,dx$$

input
```
integrate(x/(1+lambert_w(a/x)),x, algorithm="fricas")
```

output
```
integral(x/(lambert_w(a/x) + 1), x)
```

Sympy [F]

$$\int \frac{x}{1+W\left(\frac{a}{x}\right)}\,dx = \int \frac{x}{W\left(\frac{a}{x}\right)+1}\,dx$$

input
```
integrate(x/(1+LambertW(a/x)),x)
```

output
```
Integral(x/(LambertW(a/x) + 1), x)
```

Maxima [F]

$$\int \frac{x}{1+W\left(\frac{a}{x}\right)}\,dx = \int \frac{x}{\mathrm{W}(\frac{a}{x})+1}\,dx$$

input
```
integrate(x/(1+lambert_w(a/x)),x, algorithm="maxima")
```

output
```
integrate(x/(lambert_w(a/x) + 1), x)
```

Giac [F]

$$\int \frac{x}{1+W\left(\frac{a}{x}\right)}\,dx=\int \frac{x}{\mathrm{W}(\frac{a}{x})+1}\,dx$$

input
```
integrate(x/(1+lambert_w(a/x)),x, algorithm="giac")
```

output
```
integrate(x/(lambert_w(a/x) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{1+W\left(\frac{a}{x}\right)}\,dx=\int \frac{x}{\mathrm{LambertW}\left(\frac{a}{x}\right)+1}\,dx$$

input
```
int(x/(LambertW(a/x) + 1),x)
```

output
```
int(x/(LambertW(a/x) + 1), x)
```

Reduce [F]

$$\int \frac{x}{1+W\left(\frac{a}{x}\right)}\,dx=\int \frac{x}{lambert\_w\left(\frac{a}{x}\right)+1}dx$$

input
```
int(x/(1+Lambert_W(a/x)),x)
```

output
```
int(x/(lambert_w(a/x) + 1),x)
```

3.319 $\int \frac{1}{1+W\left(\frac{a}{x}\right)} dx$

Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \frac{1}{1+W\left(\frac{a}{x}\right)} dx = x + a\,\text{ExpIntegralEi}\left(-W\left(\frac{a}{x}\right)\right)$$

output
```
x+a*Ei(-LambertW(a/x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+W\left(\frac{a}{x}\right)} dx = x + a\,\text{ExpIntegralEi}\left(-W\left(\frac{a}{x}\right)\right)$$

input
```
Integrate[(1 + ProductLog[a/x])^(-1),x]
```

output
```
x + a*ExpIntegralEi[-ProductLog[a/x]]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7186, 7196, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W\left(\frac{a}{x}\right)+1}dx$$

$$\Big\downarrow \text{7186}$$

$$-\int \frac{x^2}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}$$

$$\Big\downarrow \text{7196}$$

$$\int \frac{x^2W\left(\frac{a}{x}\right)}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}+x$$

$$\Big\downarrow \text{7202}$$

$$a\,\text{ExpIntegralEi}\left(-W\left(\frac{a}{x}\right)\right)+x$$

input
```
Int[(1 + ProductLog[a/x])^(-1),x]
```

output
```
x + a*ExpIntegralEi[-ProductLog[a/x]]
```

Defintions of rubi rules used

rule 7186
```
Int[((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)])^(-1), x_Symbol] :> -Subst[In
t[1/(x^2*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d}, x] && ILt
Q[n, 0]
```

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

method	result	size
derivativedivides	$-a\left(-\frac{x}{a}+\text{expIntegral}_1\left(\text{LambertW}\left(\frac{a}{x}\right)\right)\right)$	19
default	$-a\left(-\frac{x}{a}+\text{expIntegral}_1\left(\text{LambertW}\left(\frac{a}{x}\right)\right)\right)$	19

input
```
int(1/(1+LambertW(a/x)),x,method=_RETURNVERBOSE)
```

output
```
-a*(-x/a+Ei(1,LambertW(a/x)))
```

Fricas [F]

$$\int \frac{1}{1+W\left(\frac{a}{x}\right)}\,dx=\int \frac{1}{\mathrm{W}(\frac{a}{x})+1}\,dx$$

input
```
integrate(1/(1+lambert_w(a/x)),x, algorithm="fricas")
```

output
```
integral(1/(lambert_w(a/x) + 1), x)
```

Sympy [F]

$$\int \frac{1}{1 + W\left(\frac{a}{x}\right)}\,dx = \int \frac{1}{W\left(\frac{a}{x}\right) + 1}\,dx$$

input
```
integrate(1/(1+LambertW(a/x)),x)
```

output
```
Integral(1/(LambertW(a/x) + 1), x)
```

Maxima [F]

$$\int \frac{1}{1 + W\left(\frac{a}{x}\right)}\,dx = \int \frac{1}{\mathrm{W}\left(\frac{a}{x}\right) + 1}\,dx$$

input
```
integrate(1/(1+lambert_w(a/x)),x, algorithm="maxima")
```

output
```
integrate(1/(lambert_w(a/x) + 1), x)
```

Giac [F]

$$\int \frac{1}{1 + W\left(\frac{a}{x}\right)}\,dx = \int \frac{1}{\mathrm{W}\left(\frac{a}{x}\right) + 1}\,dx$$

input
```
integrate(1/(1+lambert_w(a/x)),x, algorithm="giac")
```

output
```
integrate(1/(lambert_w(a/x) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+W\left(\frac{a}{x}\right)} \, dx = \int \frac{1}{\mathrm{LambertW}\left(\frac{a}{x}\right)+1} \, dx$$

input
```
int(1/(LambertW(a/x) + 1),x)
```

output
```
int(1/(LambertW(a/x) + 1), x)
```

Reduce [F]

$$\int \frac{1}{1+W\left(\frac{a}{x}\right)} \, dx = \int \frac{1}{lambert\_w\left(\frac{a}{x}\right)+1} dx$$

input
```
int(1/(1+Lambert_W(a/x)),x)
```

output
```
int(1/(lambert_w(a/x) + 1),x)
```

3.320 $\int \frac{1}{x\left(1+W\left(\frac{a}{x}\right)\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 9

$$\int \frac{1}{x\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=-\log\left(W\left(\frac{a}{x}\right)\right)$$

output
```
-ln(LambertW(a/x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=-\log\left(W\left(\frac{a}{x}\right)\right)$$

input
```
Integrate[1/(x*(1 + ProductLog[a/x])),x]
```

output
```
-Log[ProductLog[a/x]]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {7198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \left(W\left(\frac{a}{x}\right)+1\right)} \, dx$$

$$\Big\downarrow \text{ 7198}$$

$$-\log \left(W\left(\frac{a}{x}\right)\right)$$

input
```
Int[1/(x*(1 + ProductLog[a/x])),x]
```

output
```
-Log[ProductLog[a/x]]
```

Defintions of rubi rules used

rule 7198
```
Int[1/((x_)*((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp
[Log[ProductLog[a*x^n]]/(d*n), x] /; FreeQ[{a, d, n}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$-\ln\left(\operatorname{LambertW}\left(\frac{a}{x}\right)\right)$	10
default	$-\ln\left(\operatorname{LambertW}\left(\frac{a}{x}\right)\right)$	10
parallelrisch	$\ln\left(x\right)+\operatorname{LambertW}\left(\frac{a}{x}\right)$	10

input
```
int(1/x/(1+LambertW(a/x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(LambertW(a/x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\left(1+W\left(\frac{a}{x}\right)\right)}\,dx = -\log\left(-\operatorname{W}(\frac{a}{x})\right)$$

input
```
integrate(1/x/(1+lambert_w(a/x)),x, algorithm="fricas")
```

output
```
-log(-lambert_w(a/x))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\left(1+W\left(\frac{a}{x}\right)\right)}\,dx = \log\left(x\right)+W\left(\frac{a}{x}\right)$$

input
```
integrate(1/x/(1+LambertW(a/x)),x)
```

output
```
log(x) + LambertW(a/x)
```

Maxima [F]

$$\int \frac{1}{x\left(1+W\left(\frac{a}{x}\right)\right)}\,dx = \int \frac{1}{x\left(\operatorname{W}(\frac{a}{x})+1\right)}\,dx$$

input
```
integrate(1/x/(1+lambert_w(a/x)),x, algorithm="maxima")
```

output
```
integrate(1/(x*(lambert_w(a/x) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=\int \frac{1}{x\left(\mathrm{W}\left(\frac{a}{x}\right)+1\right)}\,dx$$

input
```
integrate(1/x/(1+lambert_w(a/x)),x, algorithm="giac")
```

output
```
integrate(1/(x*(lambert_w(a/x) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=\int \frac{1}{x\,\left(\mathrm{LambertW}\left(\frac{a}{x}\right)+1\right)}\,dx$$

input
```
int(1/(x*(LambertW(a/x) + 1)),x)
```

output
```
int(1/(x*(LambertW(a/x) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=\int \frac{1}{\mathit{lambert\_w}\left(\frac{a}{x}\right)x+x}dx$$

input
```
int(1/x/(1+Lambert_W(a/x)),x)
```

output
```
int(1/(lambert_w(a/x)*x + x),x)
```

3.321 $\int \frac{1}{x^2\left(1+W\left(\frac{a}{x}\right)\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{1}{x^2\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=-\frac{1}{xW\left(\frac{a}{x}\right)}$$

output
```
-1/x/LambertW(a/x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=-\frac{1}{xW\left(\frac{a}{x}\right)}$$

input
```
Integrate[1/(x^2*(1 + ProductLog[a/x])),x]
```

output
```
-(1/(x*ProductLog[a/x]))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {7199, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(W\left(\frac{a}{x}\right)+1\right)}dx$$

$$\Big\downarrow \; 7199$$

$$-\int \frac{1}{W\left(\frac{a}{x}\right)+1}d\frac{1}{x}$$

$$\Big\downarrow \; 7176$$

$$-\frac{1}{xW\left(\frac{a}{x}\right)}$$

input
```
Int[1/(x^2*(1 + ProductLog[a/x])),x]
```

output
```
-(1/(x*ProductLog[a/x]))
```

Defintions of rubi rules used

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{1}{x \operatorname{LambertW}\left(\frac{a}{x}\right)}$	14
default	$-\frac{1}{x \operatorname{LambertW}\left(\frac{a}{x}\right)}$	14
parallelrisch	$-\frac{1}{x \operatorname{LambertW}\left(\frac{a}{x}\right)}$	14

input
```
int(1/x^2/(1+LambertW(a/x)),x,method=_RETURNVERBOSE)
```

output
```
-1/x/LambertW(a/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=-\frac{1}{x\,\mathrm{W}\left(\frac{a}{x}\right)}$$

input
```
integrate(1/x^2/(1+lambert_w(a/x)),x, algorithm="fricas")
```

output
```
-1/(x*lambert_w(a/x))
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=\begin{cases}-\frac{1}{xW\left(\frac{a}{x}\right)} & \text{for}\ a\neq 0\\ -\frac{1}{x} & \text{otherwise}\end{cases}$$

input
```
integrate(1/x**2/(1+LambertW(a/x)),x)
```

output
```
Piecewise((-1/(x*LambertW(a/x)), Ne(a, 0)), (-1/x, True))
```

Maxima [F]

$$\int \frac{1}{x^2\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=\int \frac{1}{x^2\left(\mathrm{W}\left(\frac{a}{x}\right)+1\right)}\,dx$$

input
```
integrate(1/x^2/(1+lambert_w(a/x)),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*(lambert_w(a/x) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=\int \frac{1}{x^2\left(\mathrm{W}\left(\frac{a}{x}\right)+1\right)}\,dx$$

input
```
integrate(1/x^2/(1+lambert_w(a/x)),x, algorithm="giac")
```

output
```
integrate(1/(x^2*(lambert_w(a/x) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=\int \frac{1}{x^2\,\left(\mathrm{LambertW}\left(\frac{a}{x}\right)+1\right)}\,dx$$

input
```
int(1/(x^2*(LambertW(a/x) + 1)),x)
```

output
```
int(1/(x^2*(LambertW(a/x) + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(1+W\left(\frac{a}{x}\right)\right)}\,dx = -\frac{e^{lambert\_w\left(\frac{a}{x}\right)}}{a}$$

input

```
int(1/x^2/(1+Lambert_W(a/x)),x)
```

output

```
( - e**lambert_w(a/x))/a
```

3.322 $\int \frac{1}{x^3\left(1+W\left(\frac{a}{x}\right)\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{x^3\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=\frac{1}{4x^2W\left(\frac{a}{x}\right)^2}-\frac{1}{2x^2W\left(\frac{a}{x}\right)}$$

output
```
1/4/x^2/LambertW(a/x)^2-1/2/x^2/LambertW(a/x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=\frac{1}{4x^2W\left(\frac{a}{x}\right)^2}-\frac{1}{2x^2W\left(\frac{a}{x}\right)}$$

input
```
Integrate[1/(x^3*(1 + ProductLog[a/x])),x]
```

output
```
1/(4*x^2*ProductLog[a/x]^2) - 1/(2*x^2*ProductLog[a/x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {7199, 7194, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(W\left(\frac{a}{x}\right)+1\right)} \, dx$$

$$\Big\downarrow \text{ 7199}$$

$$-\int \frac{1}{x \left(W\left(\frac{a}{x}\right)+1\right)} d\frac{1}{x}$$

$$\Big\downarrow \text{ 7194}$$

$$\frac{1}{2} \int \frac{1}{x W\left(\frac{a}{x}\right) \left(W\left(\frac{a}{x}\right)+1\right)} d\frac{1}{x} - \frac{1}{2 x^2 W\left(\frac{a}{x}\right)}$$

$$\Big\downarrow \text{ 7201}$$

$$\frac{1}{4 x^2 W\left(\frac{a}{x}\right)^2} - \frac{1}{2 x^2 W\left(\frac{a}{x}\right)}$$

input
```
Int[1/(x^3*(1 + ProductLog[a/x])),x]
```

output
```
1/(4*x^2*ProductLog[a/x]^2) - 1/(2*x^2*ProductLog[a/x])
```

Defintions of rubi rules used

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$-\frac{\frac{a^2}{2x^2 \operatorname{LambertW}\left(\frac{a}{x}\right)}-\frac{a^2}{4x^2 \operatorname{LambertW}\left(\frac{a}{x}\right)^2}}{a^2}$	39
default	$-\frac{\frac{a^2}{2x^2 \operatorname{LambertW}\left(\frac{a}{x}\right)}-\frac{a^2}{4x^2 \operatorname{LambertW}\left(\frac{a}{x}\right)^2}}{a^2}$	39

input
```
int(1/x^3/(1+LambertW(a/x)),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(1/2/x^2*a^2/LambertW(a/x)-1/4/x^2*a^2/LambertW(a/x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3 \left(1+W\left(\frac{a}{x}\right)\right)}\,dx=-\frac{2\,\mathrm{W}(\frac{a}{x})-1}{4\,x^2\,\mathrm{W}(\frac{a}{x})^2}$$

input
```
integrate(1/x^3/(1+lambert_w(a/x)),x, algorithm="fricas")
```

output
```
-1/4*(2*lambert_w(a/x) - 1)/(x^2*lambert_w(a/x)^2)
```

Sympy [F]

$$\int \frac{1}{x^3\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=\int \frac{1}{x^3\left(W\left(\frac{a}{x}\right)+1\right)}\,dx$$

input
```
integrate(1/x**3/(1+LambertW(a/x)),x)
```

output
```
Integral(1/(x**3*(LambertW(a/x) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=\int \frac{1}{x^3\left(\mathrm{W}\left(\frac{a}{x}\right)+1\right)}\,dx$$

input
```
integrate(1/x^3/(1+lambert_w(a/x)),x, algorithm="maxima")
```

output
```
integrate(1/(x^3*(lambert_w(a/x) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^3\left(1+W\left(\frac{a}{x}\right)\right)}\,dx=\int \frac{1}{x^3\left(\mathrm{W}\left(\frac{a}{x}\right)+1\right)}\,dx$$

input
```
integrate(1/x^3/(1+lambert_w(a/x)),x, algorithm="giac")
```

output
```
integrate(1/(x^3*(lambert_w(a/x) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(1+W\left(\frac{a}{x}\right)\right)}\,dx=\int \frac{1}{x^3\,\left(\mathrm{LambertW}\left(\frac{a}{x}\right)+1\right)}\,dx$$

input
```
int(1/(x^3*(LambertW(a/x) + 1)),x)
```

output
```
int(1/(x^3*(LambertW(a/x) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 \left(1+W\left(\frac{a}{x}\right)\right)}\,dx=\int \frac{1}{lambert\_w\left(\frac{a}{x}\right)x^3+x^3}dx$$

input
```
int(1/x^3/(1+Lambert_W(a/x)),x)
```

output
```
int(1/(lambert_w(a/x)*x**3 + x**3),x)
```

3.323 $\int \frac{1}{x^4\left(1+W\left(\frac{a}{x}\right)\right)} dx$

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{x^4\left(1+W\left(\frac{a}{x}\right)\right)} \, dx=-\frac{2}{27x^3W\left(\frac{a}{x}\right)^3}+\frac{2}{9x^3W\left(\frac{a}{x}\right)^2}-\frac{1}{3x^3W\left(\frac{a}{x}\right)}$$

output
```
-2/27/x^3/LambertW(a/x)^3+2/9/x^3/LambertW(a/x)^2-1/3/x^3/LambertW(a/x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(1+W\left(\frac{a}{x}\right)\right)} \, dx=-\frac{2}{27x^3W\left(\frac{a}{x}\right)^3}+\frac{2}{9x^3W\left(\frac{a}{x}\right)^2}-\frac{1}{3x^3W\left(\frac{a}{x}\right)}$$

input
```
Integrate[1/(x^4*(1 + ProductLog[a/x])),x]
```

output
```
-2/(27*x^3*ProductLog[a/x]^3) + 2/(9*x^3*ProductLog[a/x]^2) - 1/(3*x^3*Pro
ductLog[a/x])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7199, 7194, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(W\left(\frac{a}{x}\right)+1\right)} \, dx$$

$$\Big\downarrow \text{ 7199}$$

$$-\int \frac{1}{x^2 \left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 7194}$$

$$\frac{2}{3} \int \frac{1}{x^2 W\left(\frac{a}{x}\right) \left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}-\frac{1}{3 x^3 W\left(\frac{a}{x}\right)}$$

$$\Big\downarrow \text{ 7205}$$

$$\frac{2}{3} \left(\frac{1}{3 x^3 W\left(\frac{a}{x}\right)^2}-\frac{1}{3} \int \frac{1}{x^2 W\left(\frac{a}{x}\right)^2 \left(W\left(\frac{a}{x}\right)+1\right)}d\frac{1}{x}\right)-\frac{1}{3 x^3 W\left(\frac{a}{x}\right)}$$

$$\Big\downarrow \text{ 7201}$$

$$\frac{2}{3} \left(\frac{1}{3 x^3 W\left(\frac{a}{x}\right)^2}-\frac{1}{9 x^3 W\left(\frac{a}{x}\right)^3}\right)-\frac{1}{3 x^3 W\left(\frac{a}{x}\right)}$$

input
```
Int[1/(x^4*(1 + ProductLog[a/x])),x]
```

output
```
(2*(-1/9*1/(x^3*ProductLog[a/x]^3) + 1/(3*x^3*ProductLog[a/x]^2)))/3 - 1/(
3*x^3*ProductLog[a/x])
```

Defintions of rubi rules used

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$-\frac{\frac{a^3}{3\,\mathrm{LambertW}\left(\frac{a}{x}\right)x^3}-\frac{2a^3}{9\,\mathrm{LambertW}\left(\frac{a}{x}\right)^2x^3}+\frac{2a^3}{27x^3\,\mathrm{LambertW}\left(\frac{a}{x}\right)^3}}{a^3}$	55
default	$-\frac{\frac{a^3}{3\,\mathrm{LambertW}\left(\frac{a}{x}\right)x^3}-\frac{2a^3}{9\,\mathrm{LambertW}\left(\frac{a}{x}\right)^2x^3}+\frac{2a^3}{27x^3\,\mathrm{LambertW}\left(\frac{a}{x}\right)^3}}{a^3}$	55

input
```
int(1/x^4/(1+LambertW(a/x)),x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*(1/3/LambertW(a/x)/x^3*a^3-2/9/LambertW(a/x)^2/x^3*a^3+2/27/x^3*a^3
/LambertW(a/x)^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4 \left(1+W\left(\frac{a}{x}\right)\right)} \, dx = -\frac{9 \mathrm{W}(\frac{a}{x})^2 - 6 \mathrm{W}(\frac{a}{x}) + 2}{27 \, x^3 \mathrm{W}(\frac{a}{x})^3}$$

input
```
integrate(1/x^4/(1+lambert_w(a/x)),x, algorithm="fricas")
```

output
```
-1/27*(9*lambert_w(a/x)^2 - 6*lambert_w(a/x) + 2)/(x^3*lambert_w(a/x)^3)
```

Sympy [F]

$$\int \frac{1}{x^4 \left(1+W\left(\frac{a}{x}\right)\right)} \, dx = \int \frac{1}{x^4 \left(W\left(\frac{a}{x}\right)+1\right)} \, dx$$

input
```
integrate(1/x**4/(1+LambertW(a/x)),x)
```

output
```
Integral(1/(x**4*(LambertW(a/x) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(1+W\left(\frac{a}{x}\right)\right)} \, dx = \int \frac{1}{x^4 (\mathrm{W}(\frac{a}{x}) + 1)} \, dx$$

input
```
integrate(1/x^4/(1+lambert_w(a/x)),x, algorithm="maxima")
```

output
```
integrate(1/(x^4*(lambert_w(a/x) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^4 \left(1+W\left(\frac{a}{x}\right)\right)} \, dx = \int \frac{1}{x^4 \left(\mathrm{W}\left(\frac{a}{x}\right)+1\right)} \, dx$$

input
```
integrate(1/x^4/(1+lambert_w(a/x)),x, algorithm="giac")
```

output
```
integrate(1/(x^4*(lambert_w(a/x) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(1+W\left(\frac{a}{x}\right)\right)} \, dx = \int \frac{1}{x^4 \left(\mathrm{LambertW}\left(\frac{a}{x}\right)+1\right)} \, dx$$

input
```
int(1/(x^4*(LambertW(a/x) + 1)),x)
```

output
```
int(1/(x^4*(LambertW(a/x) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 \left(1+W\left(\frac{a}{x}\right)\right)} \, dx = \int \frac{1}{lambert\_w\left(\frac{a}{x}\right) x^4 + x^4} dx$$

input
```
int(1/x^4/(1+Lambert_W(a/x)),x)
```

output
```
int(1/(lambert_w(a/x)*x**4 + x**4),x)
```

3.324 $\int \frac{x^5}{1+W\left(\frac{a}{x^2}\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int \frac{x^5}{1+W\left(\frac{a}{x^2}\right)}\,dx=\frac{x^6}{6}+\frac{9}{4}a^3\,\text{ExpIntegralEi}\left(-3W\left(\frac{a}{x^2}\right)\right)-\frac{1}{4}x^6W\left(\frac{a}{x^2}\right)+\frac{3}{4}x^6W\left(\frac{a}{x^2}\right)^2$$

output
```
1/6*x^6+9/4*a^3*Ei(-3*LambertW(a/x^2))-1/4*x^6*LambertW(a/x^2)+3/4*x^6*Lam
bertW(a/x^2)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{1+W\left(\frac{a}{x^2}\right)}\,dx=\frac{x^6}{6}+\frac{9}{4}a^3\,\text{ExpIntegralEi}\left(-3W\left(\frac{a}{x^2}\right)\right)-\frac{1}{4}x^6W\left(\frac{a}{x^2}\right)+\frac{3}{4}x^6W\left(\frac{a}{x^2}\right)^2$$

input
```
Integrate[x^5/(1 + ProductLog[a/x^2]),x]
```

output
```
x^6/6 + (9*a^3*ExpIntegralEi[-3*ProductLog[a/x^2]])/4 - (x^6*ProductLog[a/
x^2])/4 + (3*x^6*ProductLog[a/x^2]^2)/4
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {7199, 7283, 7196, 7206, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{W\left(\frac{a}{x^2}\right)+1}dx$$

$$\Big\downarrow \text{ 7199}$$

$$-\int \frac{x^7}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x}$$

$$\Big\downarrow \text{ 7283}$$

$$-\frac{1}{2}\int \frac{x^4}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 7196}$$

$$\frac{1}{2}\left(\int \frac{x^4 W\left(\frac{a}{x^2}\right)}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x^2}+\frac{x^3}{3}\right)$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{1}{2}\left(-\frac{3}{2}\int \frac{x^4 W\left(\frac{a}{x^2}\right)^2}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x^2}-\frac{1}{2}x^3 W\left(\frac{a}{x^2}\right)+\frac{x^3}{3}\right)$$

$$\Big\downarrow \text{ 7206}$$

$$\frac{1}{2}\left(-\frac{3}{2}\left(x^3\left(-W\left(\frac{a}{x^2}\right)^2\right)-3\int \frac{x^4 W\left(\frac{a}{x^2}\right)^3}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x^2}\right)-\frac{1}{2}x^3 W\left(\frac{a}{x^2}\right)+\frac{x^3}{3}\right)$$

$$\Big\downarrow \text{ 7202}$$

$$\frac{1}{2}\left(-\frac{3}{2}\left(-3a^3\,\text{ExpIntegralEi}\left(-3W\left(\frac{a}{x^2}\right)\right)-x^3 W\left(\frac{a}{x^2}\right)^2\right)-\frac{1}{2}x^3 W\left(\frac{a}{x^2}\right)+\frac{x^3}{3}\right)$$

input
```
Int[x^5/(1 + ProductLog[a/x^2]),x]
```

output
```
(x^3/3 - (x^3*ProductLog[a/x^2])/2 - (3*(-3*a^3*ExpIntegralEi[-3*ProductLo
g[a/x^2]] - x^3*ProductLog[a/x^2]^2))/2)/2
```

Defintions of rubi rules used

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{x^5}{1 + \text{LambertW}\left(\frac{a}{x^2}\right)} dx$$

input
```
int(x^5/(1+LambertW(a/x^2)),x)
```

output
```
int(x^5/(1+LambertW(a/x^2)),x)
```

Fricas [F]

$$\int \frac{x^5}{1 + W\left(\frac{a}{x^2}\right)} \, dx = \int \frac{x^5}{\mathrm{W}(\frac{a}{x^2}) + 1} \, dx$$

input
```
integrate(x^5/(1+lambert_w(a/x^2)),x, algorithm="fricas")
```

output
```
integral(x^5/(lambert_w(a/x^2) + 1), x)
```

Sympy [F]

$$\int \frac{x^5}{1 + W\left(\frac{a}{x^2}\right)} \, dx = \int \frac{x^5}{W\left(\frac{a}{x^2}\right) + 1} \, dx$$

input
```
integrate(x**5/(1+LambertW(a/x**2)),x)
```

output
```
Integral(x**5/(LambertW(a/x**2) + 1), x)
```

Maxima [F]

$$\int \frac{x^5}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x^5}{\mathrm{W}(\frac{a}{x^2})+1}\,dx$$

input
```
integrate(x^5/(1+lambert_w(a/x^2)),x, algorithm="maxima")
```

output
```
integrate(x^5/(lambert_w(a/x^2) + 1), x)
```

Giac [F]

$$\int \frac{x^5}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x^5}{\mathrm{W}(\frac{a}{x^2})+1}\,dx$$

input
```
integrate(x^5/(1+lambert_w(a/x^2)),x, algorithm="giac")
```

output
```
integrate(x^5/(lambert_w(a/x^2) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x^5}{\mathrm{LambertW}\left(\frac{a}{x^2}\right)+1}\,dx$$

input
```
int(x^5/(LambertW(a/x^2) + 1),x)
```

output
```
int(x^5/(LambertW(a/x^2) + 1), x)
```

Reduce [F]

$$\int \frac{x^5}{1+W\left(\frac{a}{x^2}\right)}\,dx = \int \frac{x^5}{lambert\_w\left(\frac{a}{x^2}\right)+1}dx$$

input
```
int(x^5/(1+Lambert_W(a/x^2)),x)
```

output
```
int(x**5/(lambert_w(a/x**2) + 1),x)
```

3.325 $\int \frac{x^3}{1+W\left(\frac{a}{x^2}\right)} dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{x^3}{1+W\left(\frac{a}{x^2}\right)} dx = \frac{x^4}{4} - a^2 \text{ExpIntegralEi}\left(-2W\left(\frac{a}{x^2}\right)\right) - \frac{1}{2}x^4 W\left(\frac{a}{x^2}\right)$$

output
```
1/4*x^4-a^2*Ei(-2*LambertW(a/x^2))-1/2*x^4*LambertW(a/x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+W\left(\frac{a}{x^2}\right)} dx = \frac{x^4}{4} - a^2 \text{ExpIntegralEi}\left(-2W\left(\frac{a}{x^2}\right)\right) - \frac{1}{2}x^4 W\left(\frac{a}{x^2}\right)$$

input
```
Integrate[x^3/(1 + ProductLog[a/x^2]),x]
```

output
```
x^4/4 - a^2*ExpIntegralEi[-2*ProductLog[a/x^2]] - (x^4*ProductLog[a/x^2])/
2
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {7199, 7283, 7196, 7206, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{W\left(\frac{a}{x^2}\right)+1}dx$$

$$\Big\downarrow \text{7199}$$

$$-\int \frac{x^5}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x}$$

$$\Big\downarrow \text{7283}$$

$$-\frac{1}{2}\int \frac{x^3}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x^2}$$

$$\Big\downarrow \text{7196}$$

$$\frac{1}{2}\left(\int \frac{x^3W\left(\frac{a}{x^2}\right)}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x^2}+\frac{x^2}{2}\right)$$

$$\Big\downarrow \text{7206}$$

$$\frac{1}{2}\left(-2\int \frac{x^3W\left(\frac{a}{x^2}\right)^2}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x^2}+x^2\left(-W\left(\frac{a}{x^2}\right)\right)+\frac{x^2}{2}\right)$$

$$\Big\downarrow \text{7202}$$

$$\frac{1}{2}\left(-2a^2\,\text{ExpIntegralEi}\left(-2W\left(\frac{a}{x^2}\right)\right)-x^2W\left(\frac{a}{x^2}\right)+\frac{x^2}{2}\right)$$

input
```
Int[x^3/(1 + ProductLog[a/x^2]),x]
```

output
```
(x^2/2 - 2*a^2*ExpIntegralEi[-2*ProductLog[a/x^2]] - x^2*ProductLog[a/x^2]
)/2
```

Defintions of rubi rules used

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7206
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[x^(m + 1)*((c*ProductLog[a*
x^n])^p/(d*(m + n*p + 1))), x] - Simp[(m + 1)/(c*(m + n*p + 1))   Int[x^m*(
(c*ProductLog[a*x^n])^(p + 1)/(d + d*ProductLog[a*x^n])), x], x] /; FreeQ[{
a, c, d, m, n, p}, x] && NeQ[m, -1] && LtQ[Simplify[p + (m + 1)/n], 0]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{x^3}{1+\operatorname{LambertW}\left(\frac{a}{x^2}\right)}dx$$

input
```
int(x^3/(1+LambertW(a/x^2)),x)
```

output
```
int(x^3/(1+LambertW(a/x^2)),x)
```

Fricas [F]

$$\int \frac{x^3}{1+W\left(\frac{a}{x^2}\right)}\,dx = \int \frac{x^3}{\mathrm{W}(\frac{a}{x^2})+1}\,dx$$

input
```
integrate(x^3/(1+lambert_w(a/x^2)),x, algorithm="fricas")
```

output
```
integral(x^3/(lambert_w(a/x^2) + 1), x)
```

Sympy [F]

$$\int \frac{x^3}{1+W\left(\frac{a}{x^2}\right)}\,dx = \int \frac{x^3}{W\left(\frac{a}{x^2}\right)+1}\,dx$$

input
```
integrate(x**3/(1+LambertW(a/x**2)),x)
```

output
```
Integral(x**3/(LambertW(a/x**2) + 1), x)
```

Maxima [F]

$$\int \frac{x^3}{1+W\left(\frac{a}{x^2}\right)}\,dx = \int \frac{x^3}{\mathrm{W}(\frac{a}{x^2})+1}\,dx$$

input
```
integrate(x^3/(1+lambert_w(a/x^2)),x, algorithm="maxima")
```

output
```
integrate(x^3/(lambert_w(a/x^2) + 1), x)
```

Giac [F]

$$\int \frac{x^3}{1+W\left(\frac{a}{x^2}\right)}\,dx = \int \frac{x^3}{\mathrm{W}(\frac{a}{x^2})+1}\,dx$$

input
```
integrate(x^3/(1+lambert_w(a/x^2)),x, algorithm="giac")
```

output
```
integrate(x^3/(lambert_w(a/x^2) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{1+W\left(\frac{a}{x^2}\right)}\,dx = \int \frac{x^3}{\mathrm{LambertW}\left(\frac{a}{x^2}\right)+1}\,dx$$

input
```
int(x^3/(LambertW(a/x^2) + 1),x)
```

output
```
int(x^3/(LambertW(a/x^2) + 1), x)
```

Reduce [F]

$$\int \frac{x^3}{1+W\left(\frac{a}{x^2}\right)}\,dx = \int \frac{x^3}{lambert\_w\left(\frac{a}{x^2}\right)+1}dx$$

input
```
int(x^3/(1+Lambert_W(a/x^2)),x)
```

output
```
int(x**3/(lambert_w(a/x**2) + 1),x)
```

3.326 $\int \frac{x}{1+W\left(\frac{a}{x^2}\right)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{x}{1+W\left(\frac{a}{x^2}\right)}\,dx=\frac{x^2}{2}+\frac{1}{2}a\,\text{ExpIntegralEi}\left(-W\left(\frac{a}{x^2}\right)\right)$$

output
```
1/2*x^2+1/2*a*Ei(-LambertW(a/x^2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+W\left(\frac{a}{x^2}\right)}\,dx=\frac{x^2}{2}+\frac{1}{2}a\,\text{ExpIntegralEi}\left(-W\left(\frac{a}{x^2}\right)\right)$$

input
```
Integrate[x/(1 + ProductLog[a/x^2]),x]
```

output
```
x^2/2 + (a*ExpIntegralEi[-ProductLog[a/x^2]])/2
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7199, 7283, 7196, 7202}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{W\left(\frac{a}{x^2}\right)+1}dx$$

$$\Big\downarrow \text{ 7199}$$

$$-\int \frac{x^3}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x}$$

$$\Big\downarrow \text{ 7283}$$

$$-\frac{1}{2}\int \frac{x^2}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 7196}$$

$$\frac{1}{2}\left(\int \frac{x^2W\left(\frac{a}{x^2}\right)}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x^2}+x\right)$$

$$\Big\downarrow \text{ 7202}$$

$$\frac{1}{2}\left(a\,\text{ExpIntegralEi}\left(-W\left(\frac{a}{x^2}\right)\right)+x\right)$$

input
```
Int[x/(1 + ProductLog[a/x^2]),x]
```

output
```
(x + a*ExpIntegralEi[-ProductLog[a/x^2]])/2
```

Defintions of rubi rules used

rule 7196
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)), x] - Int[x^m*(ProductLog[a*x]/(d + d*ProductLog[a*x])),
 x] /; FreeQ[{a, d}, x] && LtQ[m, -1]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7202
```
Int[((x_)^(m_.)*ProductLog[(a_.)*(x_)^(n_.)]^(p_.))/((d_) + (d_.)*ProductLo
g[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[
a*x^n]]/(d*n)), x] /; FreeQ[{a, d, m, n}, x] && IntegerQ[p] && EqQ[m + n*p,
 -1]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{x}{1+\text{LambertW}\left(\frac{a}{x^2}\right)}dx$$

input
```
int(x/(1+LambertW(a/x^2)),x)
```

output
```
int(x/(1+LambertW(a/x^2)),x)
```

Fricas [F]

$$\int \frac{x}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x}{\mathrm{W}(\frac{a}{x^2})+1}\,dx$$

input
```
integrate(x/(1+lambert_w(a/x^2)),x, algorithm="fricas")
```

output
```
integral(x/(lambert_w(a/x^2) + 1), x)
```

Sympy [F]

$$\int \frac{x}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x}{W\left(\frac{a}{x^2}\right)+1}\,dx$$

input
```
integrate(x/(1+LambertW(a/x**2)),x)
```

output
```
Integral(x/(LambertW(a/x**2) + 1), x)
```

Maxima [F]

$$\int \frac{x}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x}{\mathrm{W}(\frac{a}{x^2})+1}\,dx$$

input
```
integrate(x/(1+lambert_w(a/x^2)),x, algorithm="maxima")
```

output
```
integrate(x/(lambert_w(a/x^2) + 1), x)
```

Giac [F]

$$\int \frac{x}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x}{\mathrm{W}(\frac{a}{x^2})+1}\,dx$$

input
```
integrate(x/(1+lambert_w(a/x^2)),x, algorithm="giac")
```

output
```
integrate(x/(lambert_w(a/x^2) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x}{\mathrm{LambertW}\left(\frac{a}{x^2}\right)+1}\,dx$$

input
```
int(x/(LambertW(a/x^2) + 1),x)
```

output
```
int(x/(LambertW(a/x^2) + 1), x)
```

Reduce [F]

$$\int \frac{x}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x}{lambert\_w\left(\frac{a}{x^2}\right)+1}dx$$

input
```
int(x/(1+Lambert_W(a/x^2)),x)
```

output
```
int(x/(lambert_w(a/x**2) + 1),x)
```

3.327 $\int \frac{1}{x\left(1+W\left(\frac{a}{x^2}\right)\right)} dx$

Optimal result

Integrand size = 14, antiderivative size = 11

$$\int \frac{1}{x\left(1+W\left(\frac{a}{x^2}\right)\right)} dx = -\frac{1}{2}\log\left(W\left(\frac{a}{x^2}\right)\right)$$

output
```
-1/2*ln(LambertW(a/x^2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+W\left(\frac{a}{x^2}\right)\right)} dx = -\frac{1}{2}\log\left(W\left(\frac{a}{x^2}\right)\right)$$

input
```
Integrate[1/(x*(1 + ProductLog[a/x^2])),x]
```

output
```
-1/2*Log[ProductLog[a/x^2]]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {7198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(W\left(\frac{a}{x^2}\right)+1\right)}dx$$

$$\Big\downarrow \; 7198$$

$$-\frac{1}{2}\log\left(W\left(\frac{a}{x^2}\right)\right)$$

input
```
Int[1/(x*(1 + ProductLog[a/x^2])),x]
```

output
```
-1/2*Log[ProductLog[a/x^2]]
```

Defintions of rubi rules used

rule 7198
```
Int[1/((x_)*((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_.)])), x_Symbol] :> Simp
[Log[ProductLog[a*x^n]]/(d*n), x] /; FreeQ[{a, d, n}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$-\frac{\ln\left(\mathrm{LambertW}\left(\frac{a}{x^2}\right)\right)}{2}$	10
default	$-\frac{\ln\left(\mathrm{LambertW}\left(\frac{a}{x^2}\right)\right)}{2}$	10
parallelrisch	$\frac{\mathrm{LambertW}\left(\frac{a}{x^2}\right)}{2}+\ln\left(x\right)$	12

input
```
int(1/x/(1+LambertW(a/x^2)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(LambertW(a/x^2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx = -\frac{1}{2}\log\left(-\mathrm{W}\left(\frac{a}{x^2}\right)\right)$$

input
```
integrate(1/x/(1+lambert_w(a/x^2)),x, algorithm="fricas")
```

output
```
-1/2*log(-lambert_w(a/x^2))
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx = \log(x) + \frac{W\left(\frac{a}{x^2}\right)}{2}$$

input
```
integrate(1/x/(1+LambertW(a/x**2)),x)
```

output
```
log(x) + LambertW(a/x**2)/2
```

Maxima [F]

$$\int \frac{1}{x\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=\int \frac{1}{x\left(\mathrm{W}\left(\frac{a}{x^2}\right)+1\right)}\,dx$$

input
```
integrate(1/x/(1+lambert_w(a/x^2)),x, algorithm="maxima")
```

output
```
integrate(1/(x*(lambert_w(a/x^2) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=\int \frac{1}{x\left(\mathrm{W}\left(\frac{a}{x^2}\right)+1\right)}\,dx$$

input
```
integrate(1/x/(1+lambert_w(a/x^2)),x, algorithm="giac")
```

output
```
integrate(1/(x*(lambert_w(a/x^2) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=\int \frac{1}{x\,\left(\mathrm{LambertW}\left(\frac{a}{x^2}\right)+1\right)}\,dx$$

input
```
int(1/(x*(LambertW(a/x^2) + 1)),x)
```

output
```
int(1/(x*(LambertW(a/x^2) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx = \int \frac{1}{lambert\_w\left(\frac{a}{x^2}\right)x+x}dx$$

input
```
int(1/x/(1+Lambert_W(a/x^2)),x)
```

output
```
int(1/(lambert_w(a/x**2)*x + x),x)
```

3.328 $\int \frac{1}{x^3\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 15

$$\int \frac{1}{x^3\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=-\frac{1}{2x^2W\left(\frac{a}{x^2}\right)}$$

output
```
-1/2/x^2/LambertW(a/x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=-\frac{1}{2x^2W\left(\frac{a}{x^2}\right)}$$

input
```
Integrate[1/(x^3*(1 + ProductLog[a/x^2])),x]
```

output
```
-1/2*1/(x^2*ProductLog[a/x^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {7199, 7266, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(W\left(\frac{a}{x^2}\right)+1\right)} dx$$

$$\Big\downarrow \text{7199}$$

$$-\int \frac{1}{x \left(W\left(\frac{a}{x^2}\right)+1\right)} d\frac{1}{x}$$

$$\Big\downarrow \text{7266}$$

$$-\frac{1}{2}\int \frac{1}{W\left(\frac{a}{x^2}\right)+1} d\frac{1}{x^2}$$

$$\Big\downarrow \text{7176}$$

$$-\frac{1}{2x^2 W\left(\frac{a}{x^2}\right)}$$

input
```
Int[1/(x^3*(1 + ProductLog[a/x^2])),x]
```

output
```
-1/2*1/(x^2*ProductLog[a/x^2])
```

Defintions of rubi rules used

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{1}{2x^2 \operatorname{LambertW}\left(\frac{a}{x^2}\right)}$	14
default	$-\frac{1}{2x^2 \operatorname{LambertW}\left(\frac{a}{x^2}\right)}$	14
parallelrisch	$-\frac{1}{2x^2 \operatorname{LambertW}\left(\frac{a}{x^2}\right)}$	14

input
```
int(1/x^3/(1+LambertW(a/x^2)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2/LambertW(a/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3 \left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=-\frac{1}{2\,x^2\,\mathrm{W}\left(\frac{a}{x^2}\right)}$$

input
```
integrate(1/x^3/(1+lambert_w(a/x^2)),x, algorithm="fricas")
```

output
```
-1/2/(x^2*lambert_w(a/x^2))
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^3\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=\begin{cases}-\frac{1}{2x^2W\left(\frac{a}{x^2}\right)} & \text{for } a\neq 0\\ -\frac{1}{2x^2} & \text{otherwise}\end{cases}$$

input
```
integrate(1/x**3/(1+LambertW(a/x**2)),x)
```

output
```
Piecewise((-1/(2*x**2*LambertW(a/x**2)), Ne(a, 0)), (-1/(2*x**2), True))
```

Maxima [F]

$$\int \frac{1}{x^3\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=\int \frac{1}{x^3\left(\mathrm{W}\left(\frac{a}{x^2}\right)+1\right)}\,dx$$

input
```
integrate(1/x^3/(1+lambert_w(a/x^2)),x, algorithm="maxima")
```

output
```
integrate(1/(x^3*(lambert_w(a/x^2) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^3\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=\int \frac{1}{x^3\left(\mathrm{W}\left(\frac{a}{x^2}\right)+1\right)}\,dx$$

input
```
integrate(1/x^3/(1+lambert_w(a/x^2)),x, algorithm="giac")
```

output
```
integrate(1/(x^3*(lambert_w(a/x^2) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=\int \frac{1}{x^3\left(\mathrm{LambertW}\left(\frac{a}{x^2}\right)+1\right)}\,dx$$

input
```
int(1/(x^3*(LambertW(a/x^2) + 1)),x)
```

output
```
int(1/(x^3*(LambertW(a/x^2) + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=-\frac{e^{lambert\_w\left(\frac{a}{x^2}\right)}}{2a}$$

input
```
int(1/x^3/(1+Lambert_W(a/x^2)),x)
```

output
```
( - e**lambert_w(a/x**2))/(2*a)
```

3.329 $\int \frac{1}{x^5\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{x^5\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=\frac{1}{8x^4W\left(\frac{a}{x^2}\right)^2}-\frac{1}{4x^4W\left(\frac{a}{x^2}\right)}$$

output
```
1/8/x^4/LambertW(a/x^2)^2-1/4/x^4/LambertW(a/x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=\frac{1}{8x^4W\left(\frac{a}{x^2}\right)^2}-\frac{1}{4x^4W\left(\frac{a}{x^2}\right)}$$

input
```
Integrate[1/(x^5*(1 + ProductLog[a/x^2])),x]
```

output
```
1/(8*x^4*ProductLog[a/x^2]^2) - 1/(4*x^4*ProductLog[a/x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {7199, 7283, 7194, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(W\left(\frac{a}{x^2}\right)+1\right)} dx$$

$$\Big\downarrow \text{7199}$$

$$-\int \frac{1}{x^3 \left(W\left(\frac{a}{x^2}\right)+1\right)} d\frac{1}{x}$$

$$\Big\downarrow \text{7283}$$

$$-\frac{1}{2}\int \frac{1}{x^2 \left(W\left(\frac{a}{x^2}\right)+1\right)} d\frac{1}{x^2}$$

$$\Big\downarrow \text{7194}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2 W\left(\frac{a}{x^2}\right)\left(W\left(\frac{a}{x^2}\right)+1\right)} d\frac{1}{x^2} - \frac{1}{2x^2 W\left(\frac{a}{x^2}\right)}\right)$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{2}\left(\frac{1}{4x^2 W\left(\frac{a}{x^2}\right)^2} - \frac{1}{2x^2 W\left(\frac{a}{x^2}\right)}\right)$$

input
```
Int[1/(x^5*(1 + ProductLog[a/x^2])),x]
```

output
```
(1/(4*x^2*ProductLog[a/x^2]^2) - 1/(2*x^2*ProductLog[a/x^2]))/2
```

Defintions of rubi rules used

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{1}{x^5 \left(1+\text{LambertW}\left(\frac{a}{x^2}\right)\right)}dx$$

input
```
int(1/x^5/(1+LambertW(a/x^2)),x)
```

output
```
int(1/x^5/(1+LambertW(a/x^2)),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^5 \left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx = -\frac{2\,\mathrm{W}\left(\frac{a}{x^2}\right)-1}{8\,x^4\,\mathrm{W}\left(\frac{a}{x^2}\right)^2}$$

input
```
integrate(1/x^5/(1+lambert_w(a/x^2)),x, algorithm="fricas")
```

output
```
-1/8*(2*lambert_w(a/x^2) - 1)/(x^4*lambert_w(a/x^2)^2)
```

Sympy [F]

$$\int \frac{1}{x^5 \left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx = \int \frac{1}{x^5 \left(W\left(\frac{a}{x^2}\right)+1\right)}\,dx$$

input
```
integrate(1/x**5/(1+LambertW(a/x**2)),x)
```

output
```
Integral(1/(x**5*(LambertW(a/x**2) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 \left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx = \int \frac{1}{x^5 \left(\mathrm{W}\left(\frac{a}{x^2}\right)+1\right)}\,dx$$

input
```
integrate(1/x^5/(1+lambert_w(a/x^2)),x, algorithm="maxima")
```

output
```
integrate(1/(x^5*(lambert_w(a/x^2) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^5 \left(1+W\left(\frac{a}{x^2}\right)\right)} \, dx = \int \frac{1}{x^5 \left(W\left(\frac{a}{x^2}\right)+1\right)} \, dx$$

input
```
integrate(1/x^5/(1+lambert_w(a/x^2)),x, algorithm="giac")
```

output
```
integrate(1/(x^5*(lambert_w(a/x^2) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(1+W\left(\frac{a}{x^2}\right)\right)} \, dx = \int \frac{1}{x^5 \left(\mathrm{LambertW}\left(\frac{a}{x^2}\right)+1\right)} \, dx$$

input
```
int(1/(x^5*(LambertW(a/x^2) + 1)),x)
```

output
```
int(1/(x^5*(LambertW(a/x^2) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^5 \left(1+W\left(\frac{a}{x^2}\right)\right)} \, dx = \int \frac{1}{lambert\_w\left(\frac{a}{x^2}\right) x^5 + x^5} dx$$

input
```
int(1/x^5/(1+Lambert_W(a/x^2)),x)
```

output
```
int(1/(lambert_w(a/x**2)*x**5 + x**5),x)
```

3.330 $\int \frac{1}{x^7\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{x^7\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=-\frac{1}{27x^6W\left(\frac{a}{x^2}\right)^3}+\frac{1}{9x^6W\left(\frac{a}{x^2}\right)^2}-\frac{1}{6x^6W\left(\frac{a}{x^2}\right)}$$

output
```
-1/27/x^6/LambertW(a/x^2)^3+1/9/x^6/LambertW(a/x^2)^2-1/6/x^6/LambertW(a/x
^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^7\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=-\frac{1}{27x^6W\left(\frac{a}{x^2}\right)^3}+\frac{1}{9x^6W\left(\frac{a}{x^2}\right)^2}-\frac{1}{6x^6W\left(\frac{a}{x^2}\right)}$$

input
```
Integrate[1/(x^7*(1 + ProductLog[a/x^2])),x]
```

output
```
-1/27*1/(x^6*ProductLog[a/x^2]^3) + 1/(9*x^6*ProductLog[a/x^2]^2) - 1/(6*x
^6*ProductLog[a/x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {7199, 7283, 7194, 7205, 7201}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(W\left(\frac{a}{x^2}\right)+1\right)}dx$$

$$\Big\downarrow \text{7199}$$

$$-\int \frac{1}{x^5 \left(W\left(\frac{a}{x^2}\right)+1\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{7283}$$

$$-\frac{1}{2}\int \frac{1}{x^2 \left(W\left(\frac{a}{x^2}\right)+1\right)}d\frac{1}{x^2}$$

$$\Big\downarrow \text{7194}$$

$$\frac{1}{2}\left(\frac{2}{3}\int \frac{1}{x^2 W\left(\frac{a}{x^2}\right)\left(W\left(\frac{a}{x^2}\right)+1\right)}d\frac{1}{x^2}-\frac{1}{3x^3 W\left(\frac{a}{x^2}\right)}\right)$$

$$\Big\downarrow \text{7205}$$

$$\frac{1}{2}\left(\frac{2}{3}\left(\frac{1}{3x^3 W\left(\frac{a}{x^2}\right)^2}-\frac{1}{3}\int \frac{1}{x^2 W\left(\frac{a}{x^2}\right)^2\left(W\left(\frac{a}{x^2}\right)+1\right)}d\frac{1}{x^2}\right)-\frac{1}{3x^3 W\left(\frac{a}{x^2}\right)}\right)$$

$$\Big\downarrow \text{7201}$$

$$\frac{1}{2}\left(\frac{2}{3}\left(\frac{1}{3x^3 W\left(\frac{a}{x^2}\right)^2}-\frac{1}{9x^3 W\left(\frac{a}{x^2}\right)^3}\right)-\frac{1}{3x^3 W\left(\frac{a}{x^2}\right)}\right)$$

input
```
Int[1/(x^7*(1 + ProductLog[a/x^2])),x]
```

output
```
((2*(-1/9*1/(x^3*ProductLog[a/x^2]^3) + 1/(3*x^3*ProductLog[a/x^2]^2)))/3
- 1/(3*x^3*ProductLog[a/x^2]))/2
```

Defintions of rubi rules used

rule 7194
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^(
m + 1)/(d*(m + 1)*ProductLog[a*x]), x] - Simp[m/(m + 1)   Int[x^m/(ProductL
og[a*x]*(d + d*ProductLog[a*x])), x], x] /; FreeQ[{a, d}, x] && GtQ[m, 0]
```

rule 7199
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^(n_)]), x_Symbol] :> -Su
bst[Int[1/(x^(m + 2)*(d + d*ProductLog[a/x^n])), x], x, 1/x] /; FreeQ[{a, d
}, x] && IntegerQ[m] && ILtQ[n, 0] && NeQ[m, -1]
```

rule 7201
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] /; FreeQ[{a, c, d, m, n, p}, x] && NeQ[m,
-1] && EqQ[m + n*(p - 1), -1]
```

rule 7205
```
Int[((x_)^(m_.)*((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.))/((d_) + (d_.)*P
roductLog[(a_.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x^(m + 1)*((c*ProductLog[
a*x^n])^(p - 1)/(d*(m + 1))), x] - Simp[c*((m + n*(p - 1) + 1)/(m + 1))   I
nt[x^m*((c*ProductLog[a*x^n])^(p - 1)/(d + d*ProductLog[a*x^n])), x], x] /;
 FreeQ[{a, c, d, m, n, p}, x] && NeQ[m, -1] && GtQ[Simplify[p + (m + 1)/n],
 1]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{1}{x^7 \left(1+\text{LambertW}\left(\frac{a}{x^2}\right)\right)} dx$$

input
```
int(1/x^7/(1+LambertW(a/x^2)),x)
```

output
```
int(1/x^7/(1+LambertW(a/x^2)),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7 \left(1+W\left(\frac{a}{x^2}\right)\right)} dx = -\frac{9 \text{ W}(\frac{a}{x^2})^2 - 6 \text{ W}(\frac{a}{x^2}) + 2}{54 x^6 \text{ W}(\frac{a}{x^2})^3}$$

input
```
integrate(1/x^7/(1+lambert_w(a/x^2)),x, algorithm="fricas")
```

output
```
-1/54*(9*lambert_w(a/x^2)^2 - 6*lambert_w(a/x^2) + 2)/(x^6*lambert_w(a/x^2
)^3)
```

Sympy [F]

$$\int \frac{1}{x^7 \left(1+W\left(\frac{a}{x^2}\right)\right)} dx = \int \frac{1}{x^7 \left(W\left(\frac{a}{x^2}\right)+1\right)} dx$$

input
```
integrate(1/x**7/(1+LambertW(a/x**2)),x)
```

output
```
Integral(1/(x**7*(LambertW(a/x**2) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 \left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx = \int \frac{1}{x^7\left(\mathrm{W}\left(\frac{a}{x^2}\right)+1\right)}\,dx$$

input
```
integrate(1/x^7/(1+lambert_w(a/x^2)),x, algorithm="maxima")
```

output
```
integrate(1/(x^7*(lambert_w(a/x^2) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^7 \left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx = \int \frac{1}{x^7\left(\mathrm{W}\left(\frac{a}{x^2}\right)+1\right)}\,dx$$

input
```
integrate(1/x^7/(1+lambert_w(a/x^2)),x, algorithm="giac")
```

output
```
integrate(1/(x^7*(lambert_w(a/x^2) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 \left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx = \int \frac{1}{x^7 \left(\mathrm{LambertW}\left(\frac{a}{x^2}\right)+1\right)}\,dx$$

input
```
int(1/(x^7*(LambertW(a/x^2) + 1)),x)
```

output
```
int(1/(x^7*(LambertW(a/x^2) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^7 \left(1 + W\left(\frac{a}{x^2}\right)\right)} \, dx = \int \frac{1}{lambert\_w\left(\frac{a}{x^2}\right) x^7 + x^7} dx$$

input
```
int(1/x^7/(1+Lambert_W(a/x^2)),x)
```

output
```
int(1/(lambert_w(a/x**2)*x**7 + x**7),x)
```

3.331 $\int \frac{x^4}{1+W\left(\frac{a}{x^2}\right)} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \frac{x^4}{1+W\left(\frac{a}{x^2}\right)} \, dx = \frac{x^5}{5} - \frac{1}{3}x^5 W\left(\frac{a}{x^2}\right) + \frac{5}{3}x^5 W\left(\frac{a}{x^2}\right)^2 - \frac{5\sqrt{\frac{5}{2}}e^{\frac{7}{2}W\left(\frac{a}{x^2}\right)}x^7\Gamma\left(\frac{1}{2},\frac{5}{2}W\left(\frac{a}{x^2}\right)\right)W\left(\frac{a}{x^2}\right)^{7/2}}{3a}$$

output
```
1/5*x^5-1/3*x^5*LambertW(a/x^2)+5/3*x^5*LambertW(a/x^2)^2-5/6*10^(1/2)*exp
(7/2*LambertW(a/x^2))*x^7*Pi^(1/2)*erfc(1/2*10^(1/2)*LambertW(a/x^2)^(1/2)
)*LambertW(a/x^2)^(7/2)/a
```

Mathematica [F]

$$\int \frac{x^4}{1+W\left(\frac{a}{x^2}\right)} \, dx = \int \frac{x^4}{1+W\left(\frac{a}{x^2}\right)} \, dx$$

input
```
Integrate[x^4/(1 + ProductLog[a/x^2]),x]
```

output
```
Integrate[x^4/(1 + ProductLog[a/x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{W\left(\frac{a}{x^2}\right)+1}dx$$

$$\Big\downarrow \text{ 7199}$$

$$-\int \frac{x^6}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x}$$

$$\Big\downarrow \text{ 7299}$$

$$-\int \frac{x^6}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x}$$

input
```
Int[x^4/(1 + ProductLog[a/x^2]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^4}{1+\text{LambertW}\left(\frac{a}{x^2}\right)}dx$$

input
```
int(x^4/(1+LambertW(a/x^2)),x)
```

output
```
int(x^4/(1+LambertW(a/x^2)),x)
```

Fricas [F]

$$\int \frac{x^4}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x^4}{\mathrm{W}\left(\frac{a}{x^2}\right)+1}\,dx$$

input
```
integrate(x^4/(1+lambert_w(a/x^2)),x, algorithm="fricas")
```

output
```
integral(x^4/(lambert_w(a/x^2) + 1), x)
```

Sympy [F]

$$\int \frac{x^4}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x^4}{W\left(\frac{a}{x^2}\right)+1}\,dx$$

input
```
integrate(x**4/(1+LambertW(a/x**2)),x)
```

output
```
Integral(x**4/(LambertW(a/x**2) + 1), x)
```

Maxima [F]

$$\int \frac{x^4}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x^4}{\mathrm{W}\left(\frac{a}{x^2}\right)+1}\,dx$$

input
```
integrate(x^4/(1+lambert_w(a/x^2)),x, algorithm="maxima")
```

output
```
integrate(x^4/(lambert_w(a/x^2) + 1), x)
```

Giac [F]

$$\int \frac{x^4}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x^4}{W(\frac{a}{x^2})+1}\,dx$$

input
```
integrate(x^4/(1+lambert_w(a/x^2)),x, algorithm="giac")
```

output
```
integrate(x^4/(lambert_w(a/x^2) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x^4}{\mathrm{LambertW}\left(\frac{a}{x^2}\right)+1}\,dx$$

input
```
int(x^4/(LambertW(a/x^2) + 1),x)
```

output
```
int(x^4/(LambertW(a/x^2) + 1), x)
```

Reduce [F]

$$\int \frac{x^4}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x^4}{lambert\_w\left(\frac{a}{x^2}\right)+1}dx$$

input
```
int(x^4/(1+Lambert_W(a/x^2)),x)
```

output
```
int(x**4/(lambert_w(a/x**2) + 1),x)
```

3.332 $\int \frac{x^2}{1+W\left(\frac{a}{x^2}\right)} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 69

$$\int \frac{x^2}{1+W\left(\frac{a}{x^2}\right)} \, dx = \frac{x^3}{3} - x^3 W\left(\frac{a}{x^2}\right) + \frac{\sqrt{\frac{3}{2}} e^{\frac{5}{2} W\left(\frac{a}{x^2}\right)} x^5 \Gamma\left(\frac{1}{2}, \frac{3}{2} W\left(\frac{a}{x^2}\right)\right) W\left(\frac{a}{x^2}\right)^{5/2}}{a}$$

output
```
1/3*x^3-x^3*LambertW(a/x^2)+1/2*6^(1/2)*exp(5/2*LambertW(a/x^2))*x^5*Pi^(1
/2)*erfc(1/2*6^(1/2)*LambertW(a/x^2)^(1/2))*LambertW(a/x^2)^(5/2)/a
```

Mathematica [F]

$$\int \frac{x^2}{1+W\left(\frac{a}{x^2}\right)} \, dx = \int \frac{x^2}{1+W\left(\frac{a}{x^2}\right)} \, dx$$

input
```
Integrate[x^2/(1 + ProductLog[a/x^2]),x]
```

output
```
Integrate[x^2/(1 + ProductLog[a/x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{W\left(\frac{a}{x^2}\right)+1}dx$$
$$\Big\downarrow \text{ 7199}$$
$$-\int \frac{x^4}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x}$$
$$\Big\downarrow \text{ 7299}$$
$$-\int \frac{x^4}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x}$$

input
```
Int[x^2/(1 + ProductLog[a/x^2]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^2}{1+\text{LambertW}\left(\frac{a}{x^2}\right)}dx$$

input
```
int(x^2/(1+LambertW(a/x^2)),x)
```

output
```
int(x^2/(1+LambertW(a/x^2)),x)
```

Fricas [F]

$$\int \frac{x^2}{1+W\left(\frac{a}{x^2}\right)} dx = \int \frac{x^2}{\mathrm{W}(\frac{a}{x^2})+1} dx$$

input
```
integrate(x^2/(1+lambert_w(a/x^2)),x, algorithm="fricas")
```

output
```
integral(x^2/(lambert_w(a/x^2) + 1), x)
```

Sympy [F]

$$\int \frac{x^2}{1+W\left(\frac{a}{x^2}\right)} dx = \int \frac{x^2}{W\left(\frac{a}{x^2}\right)+1} dx$$

input
```
integrate(x**2/(1+LambertW(a/x**2)),x)
```

output
```
Integral(x**2/(LambertW(a/x**2) + 1), x)
```

Maxima [F]

$$\int \frac{x^2}{1+W\left(\frac{a}{x^2}\right)} dx = \int \frac{x^2}{\mathrm{W}(\frac{a}{x^2})+1} dx$$

input
```
integrate(x^2/(1+lambert_w(a/x^2)),x, algorithm="maxima")
```

output
```
integrate(x^2/(lambert_w(a/x^2) + 1), x)
```

Giac [F]

$$\int \frac{x^2}{1 + W\left(\frac{a}{x^2}\right)}\,dx = \int \frac{x^2}{\mathrm{W}(\frac{a}{x^2}) + 1}\,dx$$

input
```
integrate(x^2/(1+lambert_w(a/x^2)),x, algorithm="giac")
```

output
```
integrate(x^2/(lambert_w(a/x^2) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{1 + W\left(\frac{a}{x^2}\right)}\,dx = \int \frac{x^2}{\mathrm{LambertW}\left(\frac{a}{x^2}\right) + 1}\,dx$$

input
```
int(x^2/(LambertW(a/x^2) + 1),x)
```

output
```
int(x^2/(LambertW(a/x^2) + 1), x)
```

Reduce [F]

$$\int \frac{x^2}{1 + W\left(\frac{a}{x^2}\right)}\,dx = \int \frac{x^2}{lambert\_w\left(\frac{a}{x^2}\right) + 1}dx$$

input
```
int(x^2/(1+Lambert_W(a/x^2)),x)
```

output
```
int(x**2/(lambert_w(a/x**2) + 1),x)
```

3.333 $\int \frac{1}{1+W\left(\frac{a}{x^2}\right)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 51

$$\int \frac{1}{1+W\left(\frac{a}{x^2}\right)}\,dx = x-\frac{e^{\frac{3}{2}W\left(\frac{a}{x^2}\right)}x^3\Gamma\left(\frac{1}{2},\frac{1}{2}W\left(\frac{a}{x^2}\right)\right)W\left(\frac{a}{x^2}\right)^{3/2}}{\sqrt{2}a}$$

output
```
x-1/2*exp(3/2*LambertW(a/x^2))*x^3*Pi^(1/2)*erfc(1/2*2^(1/2)*LambertW(a/x^
2)^(1/2))*LambertW(a/x^2)^(3/2)*2^(1/2)/a
```

Mathematica [F]

$$\int \frac{1}{1+W\left(\frac{a}{x^2}\right)}\,dx = \int \frac{1}{1+W\left(\frac{a}{x^2}\right)}\,dx$$

input
```
Integrate[(1 + ProductLog[a/x^2])^(-1),x]
```

output
```
Integrate[(1 + ProductLog[a/x^2])^(-1), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W\left(\frac{a}{x^2}\right)+1}dx$$
$$\Big\downarrow \text{ 7186}$$
$$-\int \frac{x^2}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x}$$
$$\Big\downarrow \text{ 7299}$$
$$-\int \frac{x^2}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x}$$

input
```
Int[(1 + ProductLog[a/x^2])^(-1),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{1}{1+\mathrm{LambertW}\left(\frac{a}{x^2}\right)}dx$$

input
```
int(1/(1+LambertW(a/x^2)),x)
```

output
```
int(1/(1+LambertW(a/x^2)),x)
```

Fricas [F]

$$\int \frac{1}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{1}{\mathrm{W}(\frac{a}{x^2})+1}\,dx$$

input
```
integrate(1/(1+lambert_w(a/x^2)),x, algorithm="fricas")
```

output
```
integral(1/(lambert_w(a/x^2) + 1), x)
```

Sympy [F]

$$\int \frac{1}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{1}{W\left(\frac{a}{x^2}\right)+1}\,dx$$

input
```
integrate(1/(1+LambertW(a/x**2)),x)
```

output
```
Integral(1/(LambertW(a/x**2) + 1), x)
```

Maxima [F]

$$\int \frac{1}{1+W\left(\frac{a}{x^2}\right)}\,dx=\int \frac{1}{\mathrm{W}(\frac{a}{x^2})+1}\,dx$$

input
```
integrate(1/(1+lambert_w(a/x^2)),x, algorithm="maxima")
```

output
```
integrate(1/(lambert_w(a/x^2) + 1), x)
```

Giac [F]

$$\int \frac{1}{1+W\left(\frac{a}{x^2}\right)}\,dx = \int \frac{1}{\mathrm{W}(\frac{a}{x^2})+1}\,dx$$

input
```
integrate(1/(1+lambert_w(a/x^2)),x, algorithm="giac")
```

output
```
integrate(1/(lambert_w(a/x^2) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+W\left(\frac{a}{x^2}\right)}\,dx = \int \frac{1}{\mathrm{LambertW}\left(\frac{a}{x^2}\right)+1}\,dx$$

input
```
int(1/(LambertW(a/x^2) + 1),x)
```

output
```
int(1/(LambertW(a/x^2) + 1), x)
```

Reduce [F]

$$\int \frac{1}{1+W\left(\frac{a}{x^2}\right)}\,dx = \int \frac{1}{lambert\_w\left(\frac{a}{x^2}\right)+1}dx$$

input
```
int(1/(1+Lambert_W(a/x^2)),x)
```

output
```
int(1/(lambert_w(a/x**2) + 1),x)
```

3.334 $\int \frac{1}{x^2\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int \frac{1}{x^2\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx = -\frac{e^{\frac{1}{2}W\left(\frac{a}{x^2}\right)}x\Gamma\left(\frac{1}{2},-\frac{1}{2}W\left(\frac{a}{x^2}\right)\right)\sqrt{-W\left(\frac{a}{x^2}\right)}}{\sqrt{2}a}$$

output
```
-1/2*exp(1/2*LambertW(a/x^2))*x*Pi^(1/2)*erfc(1/2*(-2*LambertW(a/x^2))^(1/
2))*(-LambertW(a/x^2))^(1/2)*2^(1/2)/a
```

Mathematica [F]

$$\int \frac{1}{x^2\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx = \int \frac{1}{x^2\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx$$

input
```
Integrate[1/(x^2*(1 + ProductLog[a/x^2])),x]
```

output
```
Integrate[1/(x^2*(1 + ProductLog[a/x^2])), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(W\left(\frac{a}{x^2}\right)+1\right)} \, dx$$

$$\Big\downarrow \text{ 7199}$$

$$-\int \frac{1}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x}$$

$$\Big\downarrow \text{ 7299}$$

$$-\int \frac{1}{W\left(\frac{a}{x^2}\right)+1}d\frac{1}{x}$$

input
```
Int[1/(x^2*(1 + ProductLog[a/x^2])),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{1}{x^2 \left(1+\operatorname{LambertW}\left(\frac{a}{x^2}\right)\right)}dx$$

input
```
int(1/x^2/(1+LambertW(a/x^2)),x)
```

output
```
int(1/x^2/(1+LambertW(a/x^2)),x)
```

Fricas [F]

$$\int \frac{1}{x^2\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=\int \frac{1}{x^2\left(\mathrm{W}\left(\frac{a}{x^2}\right)+1\right)}\,dx$$

input
```
integrate(1/x^2/(1+lambert_w(a/x^2)),x, algorithm="fricas")
```

output
```
integral(1/(x^2*lambert_w(a/x^2) + x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=\int \frac{1}{x^2\left(W\left(\frac{a}{x^2}\right)+1\right)}\,dx$$

input
```
integrate(1/x**2/(1+LambertW(a/x**2)),x)
```

output
```
Integral(1/(x**2*(LambertW(a/x**2) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=\int \frac{1}{x^2\left(\mathrm{W}\left(\frac{a}{x^2}\right)+1\right)}\,dx$$

input
```
integrate(1/x^2/(1+lambert_w(a/x^2)),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*(lambert_w(a/x^2) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx = \int \frac{1}{x^2\left(\mathrm{W}\left(\frac{a}{x^2}\right)+1\right)}\,dx$$

input
```
integrate(1/x^2/(1+lambert_w(a/x^2)),x, algorithm="giac")
```

output
```
integrate(1/(x^2*(lambert_w(a/x^2) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx = \int \frac{1}{x^2\,\left(\mathrm{LambertW}\left(\frac{a}{x^2}\right)+1\right)}\,dx$$

input
```
int(1/(x^2*(LambertW(a/x^2) + 1)),x)
```

output
```
int(1/(x^2*(LambertW(a/x^2) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx = \int \frac{1}{lambert\_w\left(\frac{a}{x^2}\right)x^2+x^2}dx$$

input
```
int(1/x^2/(1+Lambert_W(a/x^2)),x)
```

output
```
int(1/(lambert_w(a/x**2)*x**2 + x**2),x)
```

3.335 $\int \frac{1}{x^4\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 77

$$\int \frac{1}{x^4\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=-\frac{1}{3x^3W\left(\frac{a}{x^2}\right)}+\frac{e^{-\frac{1}{2}W\left(\frac{a}{x^2}\right)}\Gamma\left(\frac{1}{2},-\frac{3}{2}W\left(\frac{a}{x^2}\right)\right)\sqrt{-W\left(\frac{a}{x^2}\right)}}{3\sqrt{6}axW\left(\frac{a}{x^2}\right)}$$

output
```
-1/3/x^3/LambertW(a/x^2)+1/18*Pi^(1/2)*erfc(1/2*(-6*LambertW(a/x^2))^(1/2)
)*(-LambertW(a/x^2))^(1/2)*6^(1/2)/a/exp(1/2*LambertW(a/x^2))/x/LambertW(a
/x^2)
```

Mathematica [F]

$$\int \frac{1}{x^4\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx=\int \frac{1}{x^4\left(1+W\left(\frac{a}{x^2}\right)\right)}\,dx$$

input
```
Integrate[1/(x^4*(1 + ProductLog[a/x^2])),x]
```

output
```
Integrate[1/(x^4*(1 + ProductLog[a/x^2])), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(W\left(\frac{a}{x^2}\right)+1\right)} \, dx$$
$$\Big\downarrow \; 7199$$
$$-\int \frac{1}{x^2 \left(W\left(\frac{a}{x^2}\right)+1\right)} d\frac{1}{x}$$
$$\Big\downarrow \; 7299$$
$$-\int \frac{1}{x^2 \left(W\left(\frac{a}{x^2}\right)+1\right)} d\frac{1}{x}$$

input
```
Int[1/(x^4*(1 + ProductLog[a/x^2])),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{1}{x^4 \left(1+\operatorname{LambertW}\left(\frac{a}{x^2}\right)\right)} dx$$

input
```
int(1/x^4/(1+LambertW(a/x^2)),x)
```

output
```
int(1/x^4/(1+LambertW(a/x^2)),x)
```

Fricas [F]

$$\int \frac{1}{x^4 \left(1+W\left(\frac{a}{x^2}\right)\right)} \, dx = \int \frac{1}{x^4 \left(\mathrm{W}\left(\frac{a}{x^2}\right)+1\right)} \, dx$$

input
```
integrate(1/x^4/(1+lambert_w(a/x^2)),x, algorithm="fricas")
```

output
```
integral(1/(x^4*lambert_w(a/x^2) + x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4 \left(1+W\left(\frac{a}{x^2}\right)\right)} \, dx = \int \frac{1}{x^4 \left(W\left(\frac{a}{x^2}\right)+1\right)} \, dx$$

input
```
integrate(1/x**4/(1+LambertW(a/x**2)),x)
```

output
```
Integral(1/(x**4*(LambertW(a/x**2) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(1+W\left(\frac{a}{x^2}\right)\right)} \, dx = \int \frac{1}{x^4 \left(\mathrm{W}\left(\frac{a}{x^2}\right)+1\right)} \, dx$$

input
```
integrate(1/x^4/(1+lambert_w(a/x^2)),x, algorithm="maxima")
```

output
```
integrate(1/(x^4*(lambert_w(a/x^2) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^4 \left(1+W\left(\frac{a}{x^2}\right)\right)} \, dx = \int \frac{1}{x^4 \left(\mathrm{W}\left(\frac{a}{x^2}\right)+1\right)} \, dx$$

input
```
integrate(1/x^4/(1+lambert_w(a/x^2)),x, algorithm="giac")
```

output
```
integrate(1/(x^4*(lambert_w(a/x^2) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(1+W\left(\frac{a}{x^2}\right)\right)} \, dx = \int \frac{1}{x^4 \left(\mathrm{LambertW}\left(\frac{a}{x^2}\right)+1\right)} \, dx$$

input
```
int(1/(x^4*(LambertW(a/x^2) + 1)),x)
```

output
```
int(1/(x^4*(LambertW(a/x^2) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 \left(1+W\left(\frac{a}{x^2}\right)\right)} \, dx = \int \frac{1}{lambert\_w\left(\frac{a}{x^2}\right) x^4 + x^4} dx$$

input
```
int(1/x^4/(1+Lambert_W(a/x^2)),x)
```

output
```
int(1/(lambert_w(a/x**2)*x**4 + x**4),x)
```

3.336 $\int \frac{x^m}{d+dW(ax^3)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{x^m}{d+dW(ax^3)} \, dx = \frac{3^{-\frac{2}{3}+\frac{m}{3}} e^{\frac{1}{3}(2-m)W(ax^3)} x^{-2+m} \Gamma\left(\frac{1+m}{3}, -\frac{1}{3}(1+m)W(ax^3)\right) \left(-((1+m)W(ax^3))\right)^{\frac{2-m}{3}}}{ad(1+m)}$$

output
```
3^(-2/3+1/3*m)*exp(1/3*(2-m)*LambertW(a*x^3))*x^(-2+m)*GAMMA(1/3+1/3*m,-1/
3*(1+m)*LambertW(a*x^3))*(-(1+m)*LambertW(a*x^3))^(2/3-1/3*m)/a/d/(1+m)
```

Mathematica [F]

$$\int \frac{x^m}{d+dW(ax^3)} \, dx = \int \frac{x^m}{d+dW(ax^3)} \, dx$$

input
```
Integrate[x^m/(d + d*ProductLog[a*x^3]),x]
```

output
```
Integrate[x^m/(d + d*ProductLog[a*x^3]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{dW(ax^3)+d}dx$$

$$\big\downarrow \text{ 7292}$$

$$\int \frac{x^m}{d(W(ax^3)+1)}dx$$

$$\big\downarrow \text{ 27}$$

$$\frac{\int \frac{x^m}{W(ax^3)+1}dx}{d}$$

$$\big\downarrow \text{ 7299}$$

$$\frac{\int \frac{x^m}{W(ax^3)+1}dx}{d}$$

input
```
Int[x^m/(d + d*ProductLog[a*x^3]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^m}{d+d\,\mathrm{LambertW}\left(a\,x^3\right)}dx$$

input
```
int(x^m/(d+d*LambertW(a*x^3)),x)
```

output
```
int(x^m/(d+d*LambertW(a*x^3)),x)
```

Fricas [F]

$$\int \frac{x^m}{d+dW\left(ax^3\right)}\,dx=\int \frac{x^m}{d\,\mathrm{W}(ax^3)+d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a*x^3)),x, algorithm="fricas")
```

output
```
integral(x^m/(d*lambert_w(a*x^3) + d), x)
```

Sympy [F]

$$\int \frac{x^m}{d+dW\left(ax^3\right)}\,dx=\frac{\int \frac{x^m}{W(ax^3)+1}\,dx}{d}$$

input
```
integrate(x**m/(d+d*LambertW(a*x**3)),x)
```

output
```
Integral(x**m/(LambertW(a*x**3) + 1), x)/d
```

Maxima [F]

$$\int \frac{x^m}{d+dW\left(ax^3\right)}\,dx=\int \frac{x^m}{d\,\mathrm{W}(ax^3)+d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a*x^3)),x, algorithm="maxima")
```

output
```
integrate(x^m/(d*lambert_w(a*x^3) + d), x)
```

Giac [F]

$$\int \frac{x^m}{d+dW(ax^3)}\,dx = \int \frac{x^m}{d\,\mathrm{W}(ax^3)+d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a*x^3)),x, algorithm="giac")
```

output
```
integrate(x^m/(d*lambert_w(a*x^3) + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{d+dW(ax^3)}\,dx = \int \frac{x^m}{d+d\,\mathrm{LambertW}(a\,x^3)}\,dx$$

input
```
int(x^m/(d + d*LambertW(a*x^3)),x)
```

output
```
int(x^m/(d + d*LambertW(a*x^3)), x)
```

Reduce [F]

$$\int \frac{x^m}{d+dW(ax^3)}\,dx = \frac{x^m x+3\left(\int \frac{x^m}{lambert\_w(a\,x^3)+1}dx\right)m+3\left(\int \frac{x^m}{lambert\_w(a\,x^3)+1}dx\right)-\left(\int \frac{x^m lambert\_w(a\,x^3)}{lambert\_w(a\,x^3)+1}dx\right)m-\left(\int \right.}{4d(m+1)}$$

input
```
int(x^m/(d+d*Lambert_W(a*x^3)),x)
```

output
```
(x**m*x + 3*int(x**m/(lambert_w(a*x**3) + 1),x)*m + 3*int(x**m/(lambert_w(
a*x**3) + 1),x) - int((x**m*lambert_w(a*x**3))/(lambert_w(a*x**3) + 1),x)*
m - int((x**m*lambert_w(a*x**3))/(lambert_w(a*x**3) + 1),x))/(4*d*(m + 1))
```

3.337 $\int \frac{x^m}{d+dW(ax^2)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{x^m}{d+dW(ax^2)}\,dx = \frac{2^{-\frac{1}{2}+\frac{m}{2}}e^{\frac{1}{2}(1-m)W(ax^2)}x^{-1+m}\Gamma\left(\frac{1+m}{2},-\frac{1}{2}(1+m)W(ax^2)\right)\left(-((1+m)W(ax^2))\right)^{\frac{1-m}{2}}}{ad(1+m)}$$

output
```
2^(-1/2+1/2*m)*exp(1/2*(1-m)*LambertW(a*x^2))*x^(-1+m)*GAMMA(1/2+1/2*m,-1/
2*(1+m)*LambertW(a*x^2))*(-(1+m)*LambertW(a*x^2))^(1/2-1/2*m)/a/d/(1+m)
```

Mathematica [F]

$$\int \frac{x^m}{d+dW(ax^2)}\,dx = \int \frac{x^m}{d+dW(ax^2)}\,dx$$

input
```
Integrate[x^m/(d + d*ProductLog[a*x^2]),x]
```

output
```
Integrate[x^m/(d + d*ProductLog[a*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{dW(ax^2)+d}\,dx$$

$$\Big\downarrow \text{ 7292}$$

$$\int \frac{x^m}{d(W(ax^2)+1)}\,dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{x^m}{W(ax^2)+1}dx}{d}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\int \frac{x^m}{W(ax^2)+1}dx}{d}$$

input
```
Int[x^m/(d + d*ProductLog[a*x^2]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^m}{d+d\,\mathrm{LambertW}(a\,x^2)}dx$$

input
```
int(x^m/(d+d*LambertW(a*x^2)),x)
```

output
```
int(x^m/(d+d*LambertW(a*x^2)),x)
```

Fricas [F]

$$\int \frac{x^m}{d+dW\left(ax^2\right)}\,dx=\int \frac{x^m}{d\,\mathrm{W}(ax^2)+d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a*x^2)),x, algorithm="fricas")
```

output
```
integral(x^m/(d*lambert_w(a*x^2) + d), x)
```

Sympy [F]

$$\int \frac{x^m}{d+dW\left(ax^2\right)}\,dx=\frac{\int \frac{x^m}{W(ax^2)+1}\,dx}{d}$$

input
```
integrate(x**m/(d+d*LambertW(a*x**2)),x)
```

output
```
Integral(x**m/(LambertW(a*x**2) + 1), x)/d
```

Maxima [F]

$$\int \frac{x^m}{d+dW\left(ax^2\right)}\,dx=\int \frac{x^m}{d\,\mathrm{W}(ax^2)+d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a*x^2)),x, algorithm="maxima")
```

output
```
integrate(x^m/(d*lambert_w(a*x^2) + d), x)
```

Giac [F]

$$\int \frac{x^m}{d+dW(ax^2)}\,dx=\int \frac{x^m}{d\,\mathrm{W}(ax^2)+d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a*x^2)),x, algorithm="giac")
```

output
```
integrate(x^m/(d*lambert_w(a*x^2) + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{d+dW(ax^2)}\,dx=\int \frac{x^m}{d+d\,\mathrm{LambertW}\left(a\,x^2\right)}\,dx$$

input
```
int(x^m/(d + d*LambertW(a*x^2)),x)
```

output
```
int(x^m/(d + d*LambertW(a*x^2)), x)
```

Reduce [F]

$$\int \frac{x^m}{d+dW(ax^2)}\,dx$$
$$=\frac{x^m x+3\left(\int \frac{x^m}{lambert\_w(a\,x^2)+1}dx\right)m+3\left(\int \frac{x^m}{lambert\_w(a\,x^2)+1}dx\right)-\left(\int \frac{x^m\,lambert\_w(a\,x^2)}{lambert\_w(a\,x^2)+1}dx\right)m-\left(\int \cdots\right.}{4d\,(m+1)}$$

input
```
int(x^m/(d+d*Lambert_W(a*x^2)),x)
```

output
```
(x**m*x + 3*int(x**m/(lambert_w(a*x**2) + 1),x)*m + 3*int(x**m/(lambert_w(
a*x**2) + 1),x) - int((x**m*lambert_w(a*x**2))/(lambert_w(a*x**2) + 1),x)*
m - int((x**m*lambert_w(a*x**2))/(lambert_w(a*x**2) + 1),x))/(4*d*(m + 1))
```

3.338 $\int \frac{x^m}{d+dW(ax)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int \frac{x^m}{d+dW(ax)}\,dx = \frac{e^{-mW(ax)}x^m\Gamma(1+m,(-1-m)W(ax))((-1-m)W(ax))^{-m}}{ad(1+m)}$$

output
```
x^m*GAMMA(1+m,(-1-m)*LambertW(a*x))/a/d/exp(m*LambertW(a*x))/(1+m)/(((-1-m
)*LambertW(a*x))^m)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{d+dW(ax)}\,dx = \frac{e^{-mW(ax)}x^m\Gamma(1+m,-((1+m)W(ax)))(-((1+m)W(ax)))^{-m}}{ad(1+m)}$$

input
```
Integrate[x^m/(d + d*ProductLog[a*x]),x]
```

output
```
(x^m*Gamma[1 + m, -((1 + m)*ProductLog[a*x])])/(a*d*E^(m*ProductLog[a*x])*
(1 + m)*(-((1 + m)*ProductLog[a*x]))^m)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {7197}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{dW(ax)+d}\,dx$$

$$\Big\downarrow \text{7197}$$

$$\frac{x^m e^{-mW(ax)}(-((m+1)W(ax)))^{-m}\Gamma(m+1,-((m+1)W(ax)))}{ad(m+1)}$$

input
```
Int[x^m/(d + d*ProductLog[a*x]),x]
```

output
```
(x^m*Gamma[1 + m, -((1 + m)*ProductLog[a*x])])/(a*d*E^(m*ProductLog[a*x])*
(1 + m)*(-((1 + m)*ProductLog[a*x]))^m)
```

Defintions of rubi rules used

rule 7197
```
Int[(x_)^(m_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)]), x_Symbol] :> Simp[x^m
*(Gamma[m + 1, (-(m + 1))*ProductLog[a*x]]/(a*d*(m + 1)*E^(m*ProductLog[a*x
])*((-(m + 1))*ProductLog[a*x])^m)), x] /; FreeQ[{a, d, m}, x] &&  !Integer
Q[m]
```

Maple [F]

$$\int \frac{x^m}{d+d\,\text{LambertW}\left(xa\right)}dx$$

input
```
int(x^m/(d+d*LambertW(x*a)),x)
```

output
```
int(x^m/(d+d*LambertW(x*a)),x)
```

Fricas [F]

$$\int \frac{x^m}{d+dW(ax)}\,dx = \int \frac{x^m}{d\,\mathrm{W}(ax)+d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a*x)),x, algorithm="fricas")
```

output
```
integral(x^m/(d*lambert_w(a*x) + d), x)
```

Sympy [F]

$$\int \frac{x^m}{d+dW(ax)}\,dx = \frac{\int \frac{x^m}{W(ax)+1}\,dx}{d}$$

input
```
integrate(x**m/(d+d*LambertW(a*x)),x)
```

output
```
Integral(x**m/(LambertW(a*x) + 1), x)/d
```

Maxima [F]

$$\int \frac{x^m}{d+dW(ax)}\,dx = \int \frac{x^m}{d\,\mathrm{W}(ax)+d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a*x)),x, algorithm="maxima")
```

output
```
integrate(x^m/(d*lambert_w(a*x) + d), x)
```

Giac [F]

$$\int \frac{x^m}{d+dW(ax)}\,dx=\int \frac{x^m}{d\,\mathrm{W}(ax)+d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a*x)),x, algorithm="giac")
```

output
```
integrate(x^m/(d*lambert_w(a*x) + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{d+dW(ax)}\,dx=\int \frac{x^m}{d+d\,\mathrm{LambertW}\left(a\,x\right)}\,dx$$

input
```
int(x^m/(d + d*LambertW(a*x)),x)
```

output
```
int(x^m/(d + d*LambertW(a*x)), x)
```

Reduce [F]

$$\int \frac{x^m}{d+dW(ax)}\,dx =\frac{3x^mx+\left(\int \frac{x^m}{lambert\_w(ax)+1}dx\right)m+\int \frac{x^m}{lambert\_w(ax)+1}dx-3\left(\int \frac{x^m lambert\_w(ax)}{lambert\_w(ax)+1}dx\right)m-3\left(\int \frac{x^m lam}{lambe}\right.}{4d\,(m+1)}$$

input
```
int(x^m/(d+d*Lambert_W(a*x)),x)
```

output
```
(3*x**m*x + int(x**m/(lambert_w(a*x) + 1),x)*m + int(x**m/(lambert_w(a*x)
+ 1),x) - 3*int((x**m*lambert_w(a*x))/(lambert_w(a*x) + 1),x)*m - 3*int((x
**m*lambert_w(a*x))/(lambert_w(a*x) + 1),x))/(4*d*(m + 1))
```

3.339 $\int \frac{x^m}{d+dW\left(\frac{a}{x}\right)} dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \frac{x^m}{d+dW\left(\frac{a}{x}\right)} dx = \frac{e^{(2+m)W\left(\frac{a}{x}\right)}x^{2+m}\Gamma\left(-1-m,(1+m)W\left(\frac{a}{x}\right)\right)\left((1+m)W\left(\frac{a}{x}\right)\right)^{2+m}}{ad(1+m)}$$

output
```
exp((2+m)*LambertW(a/x))*x^(2+m)*GAMMA(-1-m,(1+m)*LambertW(a/x))*((1+m)*La
mbertW(a/x))^(2+m)/a/d/(1+m)
```

Mathematica [F]

$$\int \frac{x^m}{d+dW\left(\frac{a}{x}\right)} dx = \int \frac{x^m}{d+dW\left(\frac{a}{x}\right)} dx$$

input
```
Integrate[x^m/(d + d*ProductLog[a/x]),x]
```

output
```
Integrate[x^m/(d + d*ProductLog[a/x]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{dW\left(\frac{a}{x}\right)+d}dx$$
$$\Big\downarrow \text{ 7292}$$
$$\int \frac{x^m}{d\left(W\left(\frac{a}{x}\right)+1\right)}dx$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{x^m}{W\left(\frac{a}{x}\right)+1}dx}{d}$$
$$\Big\downarrow \text{ 7299}$$
$$\frac{\int \frac{x^m}{W\left(\frac{a}{x}\right)+1}dx}{d}$$

input
```
Int[x^m/(d + d*ProductLog[a/x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^m}{d+d\,\mathrm{LambertW}\left(\frac{a}{x}\right)}dx$$

input
```
int(x^m/(d+d*LambertW(a/x)),x)
```

output
```
int(x^m/(d+d*LambertW(a/x)),x)
```

Fricas [F]

$$\int \frac{x^m}{d + dW\left(\frac{a}{x}\right)}\,dx = \int \frac{x^m}{d\,\mathrm{W}\left(\frac{a}{x}\right) + d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a/x)),x, algorithm="fricas")
```

output
```
integral(x^m/(d*lambert_w(a/x) + d), x)
```

Sympy [F]

$$\int \frac{x^m}{d + dW\left(\frac{a}{x}\right)}\,dx = \frac{\int \frac{x^m}{W\left(\frac{a}{x}\right)+1}\,dx}{d}$$

input
```
integrate(x**m/(d+d*LambertW(a/x)),x)
```

output
```
Integral(x**m/(LambertW(a/x) + 1), x)/d
```

Maxima [F]

$$\int \frac{x^m}{d + dW\left(\frac{a}{x}\right)}\,dx = \int \frac{x^m}{d\,\mathrm{W}\left(\frac{a}{x}\right) + d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a/x)),x, algorithm="maxima")
```

output
```
integrate(x^m/(d*lambert_w(a/x) + d), x)
```

Giac [F]

$$\int \frac{x^m}{d+dW\left(\frac{a}{x}\right)}\,dx=\int \frac{x^m}{d\,\mathrm{W}(\frac{a}{x})+d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a/x)),x, algorithm="giac")
```

output
```
integrate(x^m/(d*lambert_w(a/x) + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{d+dW\left(\frac{a}{x}\right)}\,dx=\int \frac{x^m}{d+d\,\mathrm{LambertW}\left(\frac{a}{x}\right)}\,dx$$

input
```
int(x^m/(d + d*LambertW(a/x)),x)
```

output
```
int(x^m/(d + d*LambertW(a/x)), x)
```

Reduce [F]

$$\int \frac{x^m}{d+dW\left(\frac{a}{x}\right)}\,dx=\frac{3x^mx+\left(\int \frac{x^m}{lambert\_w(\frac{a}{x})+1}dx\right)m+\int \frac{x^m}{lambert\_w(\frac{a}{x})+1}dx-3\left(\int \frac{x^m lambert\_w(\frac{a}{x})}{lambert\_w(\frac{a}{x})+1}dx\right)m-3\left(\int \frac{x^m la}{lamb}\right.}{4d\,(m+1)}$$

input
```
int(x^m/(d+d*Lambert_W(a/x)),x)
```

output
```
(3*x**m*x + int(x**m/(lambert_w(a/x) + 1),x)*m + int(x**m/(lambert_w(a/x)
+ 1),x) - 3*int((x**m*lambert_w(a/x))/(lambert_w(a/x) + 1),x)*m - 3*int((x
**m*lambert_w(a/x))/(lambert_w(a/x) + 1),x))/(4*d*(m + 1))
```

3.340 $\int \frac{x^m}{d+dW\left(\frac{a}{x^2}\right)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int \frac{x^m}{d+dW\left(\frac{a}{x^2}\right)}\,dx = \frac{2^{-\frac{3}{2}-\frac{m}{2}}e^{\frac{1}{2}(3+m)W\left(\frac{a}{x^2}\right)}x^{3+m}\Gamma\left(\frac{1}{2}(-1-m),\frac{1}{2}(1+m)W\left(\frac{a}{x^2}\right)\right)\left((1+m)W\left(\frac{a}{x^2}\right)\right)^{\frac{3+m}{2}}}{ad(1+m)}$$

output
```
2^(-3/2-1/2*m)*exp(1/2*(3+m)*LambertW(a/x^2))*x^(3+m)*GAMMA(-1/2-1/2*m,1/2
*(1+m)*LambertW(a/x^2))*((1+m)*LambertW(a/x^2))^(3/2+1/2*m)/a/d/(1+m)
```

Mathematica [F]

$$\int \frac{x^m}{d+dW\left(\frac{a}{x^2}\right)}\,dx = \int \frac{x^m}{d+dW\left(\frac{a}{x^2}\right)}\,dx$$

input
```
Integrate[x^m/(d + d*ProductLog[a/x^2]),x]
```

output
```
Integrate[x^m/(d + d*ProductLog[a/x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{dW\left(\frac{a}{x^2}\right)+d}dx$$
$$\Big\downarrow \text{ 7292}$$
$$\int \frac{x^m}{d\left(W\left(\frac{a}{x^2}\right)+1\right)}dx$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{x^m}{W\left(\frac{a}{x^2}\right)+1}dx}{d}$$
$$\Big\downarrow \text{ 7299}$$
$$\frac{\int \frac{x^m}{W\left(\frac{a}{x^2}\right)+1}dx}{d}$$

input
```
Int[x^m/(d + d*ProductLog[a/x^2]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^m}{d+d\,\text{LambertW}\left(\frac{a}{x^2}\right)}dx$$

input
```
int(x^m/(d+d*LambertW(a/x^2)),x)
```

output
```
int(x^m/(d+d*LambertW(a/x^2)),x)
```

Fricas [F]

$$\int \frac{x^m}{d+dW\left(\frac{a}{x^2}\right)}\,dx = \int \frac{x^m}{d\,\mathrm{W}(\frac{a}{x^2})+d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a/x^2)),x, algorithm="fricas")
```

output
```
integral(x^m/(d*lambert_w(a/x^2) + d), x)
```

Sympy [F]

$$\int \frac{x^m}{d+dW\left(\frac{a}{x^2}\right)}\,dx = \frac{\int \frac{x^m}{W\left(\frac{a}{x^2}\right)+1}\,dx}{d}$$

input
```
integrate(x**m/(d+d*LambertW(a/x**2)),x)
```

output
```
Integral(x**m/(LambertW(a/x**2) + 1), x)/d
```

Maxima [F]

$$\int \frac{x^m}{d+dW\left(\frac{a}{x^2}\right)}\,dx = \int \frac{x^m}{d\,\mathrm{W}(\frac{a}{x^2})+d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a/x^2)),x, algorithm="maxima")
```

output
```
integrate(x^m/(d*lambert_w(a/x^2) + d), x)
```

Giac [F]

$$\int \frac{x^m}{d+dW\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x^m}{d\,\mathrm{W}(\frac{a}{x^2})+d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a/x^2)),x, algorithm="giac")
```

output
```
integrate(x^m/(d*lambert_w(a/x^2) + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{d+dW\left(\frac{a}{x^2}\right)}\,dx=\int \frac{x^m}{d+d\,\mathrm{LambertW}\left(\frac{a}{x^2}\right)}\,dx$$

input
```
int(x^m/(d + d*LambertW(a/x^2)),x)
```

output
```
int(x^m/(d + d*LambertW(a/x^2)), x)
```

Reduce [F]

$$\int \frac{x^m}{d+dW\left(\frac{a}{x^2}\right)}\,dx$$
$$=\frac{3x^m x+\left(\int \frac{x^m}{lambert\_w\left(\frac{a}{x^2}\right)+1}dx\right)m+\int \frac{x^m}{lambert\_w\left(\frac{a}{x^2}\right)+1}dx-3\left(\int \frac{x^m lambert\_w\left(\frac{a}{x^2}\right)}{lambert\_w\left(\frac{a}{x^2}\right)+1}dx\right)m-3\left(\int \frac{x}{l}\right.}{4d\,(m+1)}$$

input
```
int(x^m/(d+d*Lambert_W(a/x^2)),x)
```

output
```
(3*x**m*x + int(x**m/(lambert_w(a/x**2) + 1),x)*m + int(x**m/(lambert_w(a/
x**2) + 1),x) - 3*int((x**m*lambert_w(a/x**2))/(lambert_w(a/x**2) + 1),x)*
m - 3*int((x**m*lambert_w(a/x**2))/(lambert_w(a/x**2) + 1),x))/(4*d*(m + 1
))
```

3.341 $\int \frac{x^m}{d+dW(ax^n)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 86

$$\int \frac{x^m}{d+dW(ax^n)}\,dx = \frac{e^{\left(1-\frac{1+m}{n}\right)W(ax^n)}x^{1+m-n}\Gamma\left(\frac{1+m}{n},-\frac{(1+m)W(ax^n)}{n}\right)\left(-\frac{(1+m)W(ax^n)}{n}\right)^{1-\frac{1+m}{n}}}{ad(1+m)}$$

output
```
exp((1-(1+m)/n)*LambertW(a*x^n))*x^(1+m-n)*GAMMA((1+m)/n,-(1+m)*LambertW(a
*x^n)/n)*(-(1+m)*LambertW(a*x^n)/n)^(1-(1+m)/n)/a/d/(1+m)
```

Mathematica [F]

$$\int \frac{x^m}{d+dW(ax^n)}\,dx = \int \frac{x^m}{d+dW(ax^n)}\,dx$$

input
```
Integrate[x^m/(d + d*ProductLog[a*x^n]),x]
```

output
```
Integrate[x^m/(d + d*ProductLog[a*x^n]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{dW(ax^n)+d}dx$$
$$\Big\downarrow \text{ 7292}$$
$$\int \frac{x^m}{d(W(ax^n)+1)}dx$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{x^m}{W(ax^n)+1}dx}{d}$$
$$\Big\downarrow \text{ 7299}$$
$$\frac{\int \frac{x^m}{W(ax^n)+1}dx}{d}$$

input
```
Int[x^m/(d + d*ProductLog[a*x^n]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^m}{d+d\,\mathrm{LambertW}(a\,x^n)}dx$$

input
```
int(x^m/(d+d*LambertW(a*x^n)),x)
```

output
```
int(x^m/(d+d*LambertW(a*x^n)),x)
```

Fricas [F]

$$\int \frac{x^m}{d + dW(ax^n)}\,dx = \int \frac{x^m}{d\,\mathrm{W}(ax^n) + d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a*x^n)),x, algorithm="fricas")
```

output
```
integral(x^m/(d*lambert_w(a*x^n) + d), x)
```

Sympy [F]

$$\int \frac{x^m}{d + dW(ax^n)}\,dx = \frac{\int \frac{x^m}{W(ax^n)+1}\,dx}{d}$$

input
```
integrate(x**m/(d+d*LambertW(a*x**n)),x)
```

output
```
Integral(x**m/(LambertW(a*x**n) + 1), x)/d
```

Maxima [F]

$$\int \frac{x^m}{d + dW(ax^n)}\,dx = \int \frac{x^m}{d\,\mathrm{W}(ax^n) + d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a*x^n)),x, algorithm="maxima")
```

output
```
integrate(x^m/(d*lambert_w(a*x^n) + d), x)
```

Giac [F]

$$\int \frac{x^m}{d+dW(ax^n)}\,dx=\int \frac{x^m}{d\,\mathrm{W}(ax^n)+d}\,dx$$

input
```
integrate(x^m/(d+d*lambert_w(a*x^n)),x, algorithm="giac")
```

output
```
integrate(x^m/(d*lambert_w(a*x^n) + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{d+dW(ax^n)}\,dx=\int \frac{x^m}{d+d\,\mathrm{LambertW}\left(a\,x^n\right)}\,dx$$

input
```
int(x^m/(d + d*LambertW(a*x^n)),x)
```

output
```
int(x^m/(d + d*LambertW(a*x^n)), x)
```

Reduce [F]

$$\int \frac{x^m}{d+dW(ax^n)}\,dx = \frac{x^m x+3\left(\int \frac{x^m}{lambert\_w(x^n a)+1}dx\right)m+3\left(\int \frac{x^m}{lambert\_w(x^n a)+1}dx\right)-\left(\int \frac{x^m lambert\_w(x^n a)}{lambert\_w(x^n a)+1}dx\right)m-\left(\int \frac{x}{l}\right.}{4d(m+1)}$$

input
```
int(x^m/(d+d*Lambert_W(a*x^n)),x)
```

output
```
(x**m*x + 3*int(x**m/(lambert_w(x**n*a) + 1),x)*m + 3*int(x**m/(lambert_w(
x**n*a) + 1),x) - int((x**m*lambert_w(x**n*a))/(lambert_w(x**n*a) + 1),x)*
m - int((x**m*lambert_w(x**n*a))/(lambert_w(x**n*a) + 1),x))/(4*d*(m + 1))
```

3.342 $\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^5}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)} dx$

Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^5}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)} dx = xW\left(\frac{a}{\sqrt[4]{x}}\right)^4$$

output
```
x*LambertW(a/x^(1/4))^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^5}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)} dx = xW\left(\frac{a}{\sqrt[4]{x}}\right)^4$$

input
```
Integrate[ProductLog[a/x^(1/4)]^5/(1 + ProductLog[a/x^(1/4)]),x]
```

output
```
x*ProductLog[a/x^(1/4)]^4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{7187\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^5}{W\left(\frac{a}{\sqrt[4]{x}}\right)+1}dx$$

$$\Big\downarrow \text{ 7187}$$

$$xW\left(\frac{a}{\sqrt[4]{x}}\right)^4$$

input
```
Int[ProductLog[a/x^(1/4)]^5/(1 + ProductLog[a/x^(1/4)]),x]
```

output
```
x*ProductLog[a/x^(1/4)]^4
```

Defintions of rubi rules used

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$x \operatorname{LambertW}\left(\frac{a}{x^{\frac{1}{4}}}\right)^{4}$	11
default	$x \operatorname{LambertW}\left(\frac{a}{x^{\frac{1}{4}}}\right)^{4}$	11

input
```
int(LambertW(a/x^(1/4))^5/(1+LambertW(a/x^(1/4))),x,method=_RETURNVERBOSE)
```

output
```
x*LambertW(a/x^(1/4))^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^{5}}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)}\,dx = x\,\mathrm{W}\left(\frac{a}{x^{\frac{1}{4}}}\right)^{4}$$

input
```
integrate(lambert_w(a/x^(1/4))^5/(1+lambert_w(a/x^(1/4))),x, algorithm="fr
icas")
```

output
```
x*lambert_w(a/x^(1/4))^4
```

Sympy [F]

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^5}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)}\,dx=\int \frac{W^5\left(\frac{a}{\sqrt[4]{x}}\right)}{W\left(\frac{a}{\sqrt[4]{x}}\right)+1}\,dx$$

input
```
integrate(LambertW(a/x**(1/4))**5/(1+LambertW(a/x**(1/4))),x)
```

output
```
Integral(LambertW(a/x**(1/4))**5/(LambertW(a/x**(1/4)) + 1), x)
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^5}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)}\,dx=\int \frac{\mathrm{W}\left(\frac{a}{x^{\frac{1}{4}}}\right)^5}{\mathrm{W}\left(\frac{a}{x^{\frac{1}{4}}}\right)+1}\,dx$$

input
```
integrate(lambert_w(a/x^(1/4))^5/(1+lambert_w(a/x^(1/4))),x, algorithm="ma
xima")
```

output
```
integrate(lambert_w(a/x^(1/4))^5/(lambert_w(a/x^(1/4)) + 1), x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^5}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)}\,dx=\int \frac{\mathrm{W}\left(\frac{a}{x^{\frac{1}{4}}}\right)^5}{\mathrm{W}\left(\frac{a}{x^{\frac{1}{4}}}\right)+1}\,dx$$

input
```
integrate(lambert_w(a/x^(1/4))^5/(1+lambert_w(a/x^(1/4))),x, algorithm="gi
ac")
```

output
```
integrate(lambert_w(a/x^(1/4))^5/(lambert_w(a/x^(1/4)) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^5}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)}\,dx=\int \frac{\mathrm{LambertW}\left(\frac{a}{x^{1/4}}\right)^5}{\mathrm{LambertW}\left(\frac{a}{x^{1/4}}\right)+1}\,dx$$

input
```
int(LambertW(a/x^(1/4))^5/(LambertW(a/x^(1/4)) + 1),x)
```

output
```
int(LambertW(a/x^(1/4))^5/(LambertW(a/x^(1/4)) + 1), x)
```

Reduce [F]

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^5}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)}\,dx=\int \frac{\mathit{lambert\_w}\left(\frac{a}{x^{\frac{1}{4}}}\right)^5}{\mathit{lambert\_w}\left(\frac{a}{x^{\frac{1}{4}}}\right)+1}dx$$

input
```
int(Lambert_W(a/x^(1/4))^5/(1+Lambert_W(a/x^(1/4))),x)
```

output
```
int(lambert_w(a/x**(1/4))**5/(lambert_w(a/x**(1/4)) + 1),x)
```

3.343 $\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)}\,dx=xW\left(\frac{a}{\sqrt[3]{x}}\right)^3$$

output
```
x*LambertW(a/x^(1/3))^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)}\,dx=xW\left(\frac{a}{\sqrt[3]{x}}\right)^3$$

input
```
Integrate[ProductLog[a/x^(1/3)]^4/(1 + ProductLog[a/x^(1/3)]),x]
```

output
```
x*ProductLog[a/x^(1/3)]^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^4}{W\left(\frac{a}{\sqrt[3]{x}}\right)+1}dx$$

$$\Big\downarrow \text{7187}$$

$$xW\left(\frac{a}{\sqrt[3]{x}}\right)^3$$

input
```
Int[ProductLog[a/x^(1/3)]^4/(1 + ProductLog[a/x^(1/3)]),x]
```

output
```
x*ProductLog[a/x^(1/3)]^3
```

Defintions of rubi rules used

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$x \operatorname{LambertW}\left(\frac{a}{x^{\frac{1}{3}}}\right)^{3}$	11
default	$x \operatorname{LambertW}\left(\frac{a}{x^{\frac{1}{3}}}\right)^{3}$	11

input
```
int(LambertW(a/x^(1/3))^4/(1+LambertW(a/x^(1/3))),x,method=_RETURNVERBOSE)
```

output
```
x*LambertW(a/x^(1/3))^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^{4}}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)}\,dx = x\,\mathrm{W}\left(\frac{a}{x^{\frac{1}{3}}}\right)^{3}$$

input
```
integrate(lambert_w(a/x^(1/3))^4/(1+lambert_w(a/x^(1/3))),x, algorithm="fr
icas")
```

output
```
x*lambert_w(a/x^(1/3))^3
```

Sympy [F]

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)}\,dx=\int \frac{W^4\left(\frac{a}{\sqrt[3]{x}}\right)}{W\left(\frac{a}{\sqrt[3]{x}}\right)+1}\,dx$$

input
```
integrate(LambertW(a/x**(1/3))**4/(1+LambertW(a/x**(1/3))),x)
```

output
```
Integral(LambertW(a/x**(1/3))**4/(LambertW(a/x**(1/3)) + 1), x)
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)}\,dx=\int \frac{\mathrm{W}\left(\frac{a}{x^{\frac{1}{3}}}\right)^4}{\mathrm{W}\left(\frac{a}{x^{\frac{1}{3}}}\right)+1}\,dx$$

input
```
integrate(lambert_w(a/x^(1/3))^4/(1+lambert_w(a/x^(1/3))),x, algorithm="ma
xima")
```

output
```
integrate(lambert_w(a/x^(1/3))^4/(lambert_w(a/x^(1/3)) + 1), x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)}\,dx=\int \frac{\mathrm{W}\left(\frac{a}{x^{\frac{1}{3}}}\right)^4}{\mathrm{W}\left(\frac{a}{x^{\frac{1}{3}}}\right)+1}\,dx$$

input
```
integrate(lambert_w(a/x^(1/3))^4/(1+lambert_w(a/x^(1/3))),x, algorithm="gi
ac")
```

output
```
integrate(lambert_w(a/x^(1/3))^4/(lambert_w(a/x^(1/3)) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)}\,dx=\int \frac{\mathrm{LambertW}\left(\frac{a}{x^{1/3}}\right)^4}{\mathrm{LambertW}\left(\frac{a}{x^{1/3}}\right)+1}\,dx$$

input
```
int(LambertW(a/x^(1/3))^4/(LambertW(a/x^(1/3)) + 1),x)
```

output
```
int(LambertW(a/x^(1/3))^4/(LambertW(a/x^(1/3)) + 1), x)
```

Reduce [F]

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)}\,dx=\int \frac{lambert\_w\left(\frac{a}{x^{\frac{1}{3}}}\right)^4}{lambert\_w\left(\frac{a}{x^{\frac{1}{3}}}\right)+1}dx$$

input
```
int(Lambert_W(a/x^(1/3))^4/(1+Lambert_W(a/x^(1/3))),x)
```

output
```
int(lambert_w(a/x**(1/3))**4/(lambert_w(a/x**(1/3)) + 1),x)
```

3.344 $\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^3}{1+W\left(\frac{a}{\sqrt{x}}\right)} dx$

Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^3}{1+W\left(\frac{a}{\sqrt{x}}\right)} dx = xW\left(\frac{a}{\sqrt{x}}\right)^2$$

output
```
x*LambertW(a/x^(1/2))^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^3}{1+W\left(\frac{a}{\sqrt{x}}\right)} dx = xW\left(\frac{a}{\sqrt{x}}\right)^2$$

input
```
Integrate[ProductLog[a/Sqrt[x]]^3/(1 + ProductLog[a/Sqrt[x]]),x]
```

output
```
x*ProductLog[a/Sqrt[x]]^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^3}{W\left(\frac{a}{\sqrt{x}}\right)+1} \, dx$$
$$\Big\downarrow \text{ 7187}$$
$$xW\left(\frac{a}{\sqrt{x}}\right)^2$$

input
```
Int[ProductLog[a/Sqrt[x]]^3/(1 + ProductLog[a/Sqrt[x]]),x]
```

output
```
x*ProductLog[a/Sqrt[x]]^2
```

Defintions of rubi rules used

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$x \operatorname{LambertW}\left(\frac{a}{\sqrt{x}}\right)^2$	11
default	$x \operatorname{LambertW}\left(\frac{a}{\sqrt{x}}\right)^2$	11

input
```
int(LambertW(a/x^(1/2))^3/(1+LambertW(a/x^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
x*LambertW(a/x^(1/2))^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^3}{1+W\left(\frac{a}{\sqrt{x}}\right)}\,dx = x\,\mathrm{W}(\frac{a}{\sqrt{x}})^2$$

input
```
integrate(lambert_w(a/x^(1/2))^3/(1+lambert_w(a/x^(1/2))),x, algorithm="fr
icas")
```

output
```
x*lambert_w(a/sqrt(x))^2
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^3}{1+W\left(\frac{a}{\sqrt{x}}\right)}\,dx = xW^2\left(\frac{a}{\sqrt{x}}\right)$$

input
```
integrate(LambertW(a/x**(1/2))**3/(1+LambertW(a/x**(1/2))),x)
```

output
```
x*LambertW(a/sqrt(x))**2
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^3}{1+W\left(\frac{a}{\sqrt{x}}\right)}\,dx=\int \frac{W(\frac{a}{\sqrt{x}})^3}{W(\frac{a}{\sqrt{x}})+1}\,dx$$

input
```
integrate(lambert_w(a/x^(1/2))^3/(1+lambert_w(a/x^(1/2))),x, algorithm="ma
xima")
```

output
```
integrate(lambert_w(a/sqrt(x))^3/(lambert_w(a/sqrt(x)) + 1), x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^3}{1+W\left(\frac{a}{\sqrt{x}}\right)}\,dx=\int \frac{W(\frac{a}{\sqrt{x}})^3}{W(\frac{a}{\sqrt{x}})+1}\,dx$$

input
```
integrate(lambert_w(a/x^(1/2))^3/(1+lambert_w(a/x^(1/2))),x, algorithm="gi
ac")
```

output
```
integrate(lambert_w(a/sqrt(x))^3/(lambert_w(a/sqrt(x)) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^3}{1+W\left(\frac{a}{\sqrt{x}}\right)}\,dx=\int \frac{\mathrm{LambertW}\left(\frac{a}{\sqrt{x}}\right)^3}{\mathrm{LambertW}\left(\frac{a}{\sqrt{x}}\right)+1}\,dx$$

input
```
int(LambertW(a/x^(1/2))^3/(LambertW(a/x^(1/2)) + 1),x)
```

output
```
int(LambertW(a/x^(1/2))^3/(LambertW(a/x^(1/2)) + 1), x)
```

Reduce [F]

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^3}{1+W\left(\frac{a}{\sqrt{x}}\right)}\,dx = \int \frac{lambert\_w\left(\frac{a}{\sqrt{x}}\right)^3}{lambert\_w\left(\frac{a}{\sqrt{x}}\right)+1}dx$$

input
```
int(Lambert_W(a/x^(1/2))^3/(1+Lambert_W(a/x^(1/2))),x)
```

output
```
int(lambert_w(a/sqrt(x))**3/(lambert_w(a/sqrt(x)) + 1),x)
```

3.345 $\int \frac{W\left(\frac{a}{x}\right)^2}{1+W\left(\frac{a}{x}\right)} dx$

Optimal result

Integrand size = 19, antiderivative size = 8

$$\int \frac{W\left(\frac{a}{x}\right)^2}{1+W\left(\frac{a}{x}\right)} dx = xW\left(\frac{a}{x}\right)$$

output
```
x*LambertW(a/x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{W\left(\frac{a}{x}\right)^2}{1+W\left(\frac{a}{x}\right)} dx = xW\left(\frac{a}{x}\right)$$

input
```
Integrate[ProductLog[a/x]^2/(1 + ProductLog[a/x]),x]
```

output
```
x*ProductLog[a/x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{x}\right)^2}{W\left(\frac{a}{x}\right)+1}dx$$

$$\Big\downarrow \; 7187$$

$$xW\left(\frac{a}{x}\right)$$

input
```
Int[ProductLog[a/x]^2/(1 + ProductLog[a/x]),x]
```

output
```
x*ProductLog[a/x]
```

Defintions of rubi rules used

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$x\,\mathrm{LambertW}\left(\frac{a}{x}\right)$	9
default	$x\,\mathrm{LambertW}\left(\frac{a}{x}\right)$	9
parallelrisch	$x\,\mathrm{LambertW}\left(\frac{a}{x}\right)$	9

input
```
int(LambertW(a/x)^2/(1+LambertW(a/x)),x,method=_RETURNVERBOSE)
```

output
```
x*LambertW(a/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \frac{W\left(\frac{a}{x}\right)^2}{1+W\left(\frac{a}{x}\right)}\,dx = x\,\mathrm{W}(\frac{a}{x})$$

input
```
integrate(lambert_w(a/x)^2/(1+lambert_w(a/x)),x, algorithm="fricas")
```

output
```
x*lambert_w(a/x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{W\left(\frac{a}{x}\right)^2}{1+W\left(\frac{a}{x}\right)}\,dx = xW\left(\frac{a}{x}\right)$$

input
```
integrate(LambertW(a/x)**2/(1+LambertW(a/x)),x)
```

output
```
x*LambertW(a/x)
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{1+W\left(\frac{a}{x}\right)}\,dx=\int \frac{\mathrm{W}(\frac{a}{x})^2}{\mathrm{W}(\frac{a}{x})+1}\,dx$$

input
```
integrate(lambert_w(a/x)^2/(1+lambert_w(a/x)),x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x)^2/(lambert_w(a/x) + 1), x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{1+W\left(\frac{a}{x}\right)}\,dx=\int \frac{\mathrm{W}(\frac{a}{x})^2}{\mathrm{W}(\frac{a}{x})+1}\,dx$$

input
```
integrate(lambert_w(a/x)^2/(1+lambert_w(a/x)),x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x)^2/(lambert_w(a/x) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{x}\right)^2}{1+W\left(\frac{a}{x}\right)}\,dx=\int \frac{\mathrm{LambertW}\left(\frac{a}{x}\right)^2}{\mathrm{LambertW}\left(\frac{a}{x}\right)+1}\,dx$$

input
```
int(LambertW(a/x)^2/(LambertW(a/x) + 1),x)
```

output
```
int(LambertW(a/x)^2/(LambertW(a/x) + 1), x)
```

Reduce [F]

$$\int \frac{W\left(\frac{a}{x}\right)^2}{1+W\left(\frac{a}{x}\right)}\,dx = \int \frac{lambert\_w\left(\frac{a}{x}\right)^2}{lambert\_w\left(\frac{a}{x}\right)+1}dx$$

input
```
int(Lambert_W(a/x)^2/(1+Lambert_W(a/x)),x)
```

output
```
int(lambert_w(a/x)**2/(lambert_w(a/x) + 1),x)
```

3.346 $\int \frac{1}{W(a\sqrt{x})(1+W(a\sqrt{x}))} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{1}{W\left(a\sqrt{x}\right)\left(1+W\left(a\sqrt{x}\right)\right)} \, dx = \frac{x}{W\left(a\sqrt{x}\right)^2}$$

output
```
x/LambertW(a*x^(1/2))^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{W\left(a\sqrt{x}\right)\left(1+W\left(a\sqrt{x}\right)\right)} \, dx = \frac{x}{W\left(a\sqrt{x}\right)^2}$$

input
```
Integrate[1/(ProductLog[a*Sqrt[x]]*(1 + ProductLog[a*Sqrt[x]])),x]
```

output
```
x/ProductLog[a*Sqrt[x]]^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W\left(a\sqrt{x}\right)\left(W\left(a\sqrt{x}\right)+1\right)}dx$$

$$\Big\downarrow \text{ 7187}$$

$$\frac{x}{W\left(a\sqrt{x}\right)^2}$$

input
```
Int[1/(ProductLog[a*Sqrt[x]]*(1 + ProductLog[a*Sqrt[x]])),x]
```

output
```
x/ProductLog[a*Sqrt[x]]^2
```

Defintions of rubi rules used

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{x}{\operatorname{LambertW}\left(a\sqrt{x}\right)^{2}}$	11
default	$\frac{x}{\operatorname{LambertW}\left(a\sqrt{x}\right)^{2}}$	11

input
```
int(1/LambertW(a*x^(1/2))/(1+LambertW(a*x^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
x/LambertW(a*x^(1/2))^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{W\left(a\sqrt{x}\right)\left(1+W\left(a\sqrt{x}\right)\right)}\,dx = \frac{x}{\mathrm{W}(a\sqrt{x})^2}$$

input
```
integrate(1/lambert_w(a*x^(1/2))/(1+lambert_w(a*x^(1/2))),x, algorithm="fr
icas")
```

output
```
x/lambert_w(a*sqrt(x))^2
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{W\left(a\sqrt{x}\right)\left(1+W\left(a\sqrt{x}\right)\right)}\,dx = \frac{x}{W^2\left(a\sqrt{x}\right)}$$

input
```
integrate(1/LambertW(a*x**(1/2))/(1+LambertW(a*x**(1/2))),x)
```

output
```
x/LambertW(a*sqrt(x))**2
```

Maxima [F]

$$\int \frac{1}{W\left(a\sqrt{x}\right)\left(1+W\left(a\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(\mathrm{W}(a\sqrt{x})+1\right)\mathrm{W}(a\sqrt{x})}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/2))/(1+lambert_w(a*x^(1/2))),x, algorithm="ma
xima")
```

output
```
integrate(1/((lambert_w(a*sqrt(x)) + 1)*lambert_w(a*sqrt(x))), x)
```

Giac [F]

$$\int \frac{1}{W\left(a\sqrt{x}\right)\left(1+W\left(a\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(\mathrm{W}(a\sqrt{x})+1\right)\mathrm{W}(a\sqrt{x})}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/2))/(1+lambert_w(a*x^(1/2))),x, algorithm="gi
ac")
```

output
```
integrate(1/((lambert_w(a*sqrt(x)) + 1)*lambert_w(a*sqrt(x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W\left(a\sqrt{x}\right)\left(1+W\left(a\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\mathrm{LambertW}\left(a\,\sqrt{x}\right)\,\left(\mathrm{LambertW}\left(a\,\sqrt{x}\right)+1\right)}\,dx$$

input
```
int(1/(LambertW(a*x^(1/2))*(LambertW(a*x^(1/2)) + 1)),x)
```

output
```
int(1/(LambertW(a*x^(1/2))*(LambertW(a*x^(1/2)) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{W\left(a\sqrt{x}\right)\left(1+W\left(a\sqrt{x}\right)\right)}\,dx=\int \frac{1}{lambert\_w\left(\sqrt{x}\,a\right)^{2}+lambert\_w\left(\sqrt{x}\,a\right)}dx$$

input
```
int(1/Lambert_W(a*x^(1/2))/(1+Lambert_W(a*x^(1/2))),x)
```

output
```
int(1/(lambert_w(sqrt(x)*a)**2 + lambert_w(sqrt(x)*a)),x)
```

3.347 $\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx=\frac{x}{W\left(a\sqrt[3]{x}\right)^3}$$

output
```
x/LambertW(a*x^(1/3))^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx=\frac{x}{W\left(a\sqrt[3]{x}\right)^3}$$

input
```
Integrate[1/(ProductLog[a*x^(1/3)]^2*(1 + ProductLog[a*x^(1/3)])),x]
```

output
```
x/ProductLog[a*x^(1/3)]^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2\left(W\left(a\sqrt[3]{x}\right)+1\right)}dx$$

$$\Big\downarrow \text{7187}$$

$$\frac{x}{W\left(a\sqrt[3]{x}\right)^3}$$

input
```
Int[1/(ProductLog[a*x^(1/3)]^2*(1 + ProductLog[a*x^(1/3)])),x]
```

output
```
x/ProductLog[a*x^(1/3)]^3
```

Defintions of rubi rules used

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{x}{\text{LambertW}\left(a\,x^{\frac{1}{3}}\right)^{3}}$	11
default	$\frac{x}{\text{LambertW}\left(a\,x^{\frac{1}{3}}\right)^{3}}$	11

input
```
int(1/LambertW(a*x^(1/3))^2/(1+LambertW(a*x^(1/3))),x,method=_RETURNVERBOS
E)
```

output
```
x/LambertW(a*x^(1/3))^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx=\frac{x}{\mathrm{W}(ax^{\frac{1}{3}})^3}$$

input
```
integrate(1/lambert_w(a*x^(1/3))^2/(1+lambert_w(a*x^(1/3))),x, algorithm="
fricas")
```

output
```
x/lambert_w(a*x^(1/3))^3
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx=\frac{x}{W^3\left(a\sqrt[3]{x}\right)}$$

input
```
integrate(1/LambertW(a*x**(1/3))**2/(1+LambertW(a*x**(1/3))),x)
```

output
```
x/LambertW(a*x**(1/3))**3
```

Maxima [F]

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^{2}\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx=\int \frac{1}{\left(\mathrm{W}(ax^{\frac{1}{3}})+1\right)\mathrm{W}(ax^{\frac{1}{3}})^{2}}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/3))^2/(1+lambert_w(a*x^(1/3))),x, algorithm="
maxima")
```

output
```
integrate(1/((lambert_w(a*x^(1/3)) + 1)*lambert_w(a*x^(1/3))^2), x)
```

Giac [F]

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^{2}\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx=\int \frac{1}{\left(\mathrm{W}(ax^{\frac{1}{3}})+1\right)\mathrm{W}(ax^{\frac{1}{3}})^{2}}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/3))^2/(1+lambert_w(a*x^(1/3))),x, algorithm="
giac")
```

output
```
integrate(1/((lambert_w(a*x^(1/3)) + 1)*lambert_w(a*x^(1/3))^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^{2}\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx=\int \frac{1}{\mathrm{LambertW}\left(a\,x^{1/3}\right)^{2}\,\left(\mathrm{LambertW}\left(a\,x^{1/3}\right)+1\right)}\,dx$$

input
```
int(1/(LambertW(a*x^(1/3))^2*(LambertW(a*x^(1/3)) + 1)),x)
```

output
```
int(1/(LambertW(a*x^(1/3))^2*(LambertW(a*x^(1/3)) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^2\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx = \int \frac{1}{lambert\_w\left(x^{\frac{1}{3}}a\right)^3 + lambert\_w\left(x^{\frac{1}{3}}a\right)^2}dx$$

input
```
int(1/Lambert_W(a*x^(1/3))^2/(1+Lambert_W(a*x^(1/3))),x)
```

output
```
int(1/(lambert_w(x**(1/3)*a)**3 + lambert_w(x**(1/3)*a)**2),x)
```

3.348 $\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx=\frac{x}{W\left(a\sqrt[4]{x}\right)^4}$$

output
```
x/LambertW(a*x^(1/4))^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx=\frac{x}{W\left(a\sqrt[4]{x}\right)^4}$$

input
```
Integrate[1/(ProductLog[a*x^(1/4)]^3*(1 + ProductLog[a*x^(1/4)])),x]
```

output
```
x/ProductLog[a*x^(1/4)]^4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3\left(W\left(a\sqrt[4]{x}\right)+1\right)}dx$$

$$\Big\downarrow \ 7187$$

$$\frac{x}{W\left(a\sqrt[4]{x}\right)^4}$$

input
```
Int[1/(ProductLog[a*x^(1/4)]^3*(1 + ProductLog[a*x^(1/4)])),x]
```

output
```
x/ProductLog[a*x^(1/4)]^4
```

Defintions of rubi rules used

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{x}{\operatorname{LambertW}\left(a\,x^{\frac{1}{4}}\right)^{4}}$	11
default	$\frac{x}{\operatorname{LambertW}\left(a\,x^{\frac{1}{4}}\right)^{4}}$	11

input
```
int(1/LambertW(a*x^(1/4))^3/(1+LambertW(a*x^(1/4))),x,method=_RETURNVERBOS
E)
```

output
```
x/LambertW(a*x^(1/4))^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx=\frac{x}{\mathrm{W}(ax^{\frac{1}{4}})^4}$$

input
```
integrate(1/lambert_w(a*x^(1/4))^3/(1+lambert_w(a*x^(1/4))),x, algorithm="
fricas")
```

output
```
x/lambert_w(a*x^(1/4))^4
```

Sympy [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx=\frac{x}{W^4\left(a\sqrt[4]{x}\right)}$$

input
```
integrate(1/LambertW(a*x**(1/4))**3/(1+LambertW(a*x**(1/4))),x)
```

output
```
x/LambertW(a*x**(1/4))**4
```

Maxima [F]

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx=\int \frac{1}{\left(\mathrm{W}(ax^{\frac{1}{4}})+1\right)\mathrm{W}(ax^{\frac{1}{4}})^3}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/4))^3/(1+lambert_w(a*x^(1/4))),x, algorithm="
maxima")
```

output
```
integrate(1/((lambert_w(a*x^(1/4)) + 1)*lambert_w(a*x^(1/4))^3), x)
```

Giac [F]

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx=\int \frac{1}{\left(\mathrm{W}(ax^{\frac{1}{4}})+1\right)\mathrm{W}(ax^{\frac{1}{4}})^3}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/4))^3/(1+lambert_w(a*x^(1/4))),x, algorithm="
giac")
```

output
```
integrate(1/((lambert_w(a*x^(1/4)) + 1)*lambert_w(a*x^(1/4))^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx=\int \frac{1}{\mathrm{LambertW}\left(a\,x^{1/4}\right)^3\,\left(\mathrm{LambertW}\left(a\,x^{1/4}\right)+1\right)}\,dx$$

input
```
int(1/(LambertW(a*x^(1/4))^3*(LambertW(a*x^(1/4)) + 1)),x)
```

output
```
int(1/(LambertW(a*x^(1/4))^3*(LambertW(a*x^(1/4)) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^3\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx = \int \frac{1}{lambert\_w\left(x^{\frac{1}{4}}a\right)^4 + lambert\_w\left(x^{\frac{1}{4}}a\right)^3}dx$$

input
```
int(1/Lambert_W(a*x^(1/4))^3/(1+Lambert_W(a*x^(1/4))),x)
```

output
```
int(1/(lambert_w(x**(1/4)*a)**4 + lambert_w(x**(1/4)*a)**3),x)
```

3.349 $\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)} dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)} dx = -4a^4 \operatorname{ExpIntegralEi}\left(-4W\left(\frac{a}{\sqrt[4]{x}}\right)\right)$$

output
```
-4*a^4*Ei(-4*LambertW(a/x^(1/4)))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)} dx = -4a^4 \operatorname{ExpIntegralEi}\left(-4W\left(\frac{a}{\sqrt[4]{x}}\right)\right)$$

input
```
Integrate[ProductLog[a/x^(1/4)]^4/(1 + ProductLog[a/x^(1/4)]),x]
```

output
```
-4*a^4*ExpIntegralEi[-4*ProductLog[a/x^(1/4)]]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {7188}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^4}{W\left(\frac{a}{\sqrt[4]{x}}\right)+1}dx$$

$$\Big\downarrow \text{7188}$$

$$-4a^4 \operatorname{ExpIntegralEi}\left(-4W\left(\frac{a}{\sqrt[4]{x}}\right)\right)$$

input
```
Int[ProductLog[a/x^(1/4)]^4/(1 + ProductLog[a/x^(1/4)]),x]
```

output
```
-4*a^4*ExpIntegralEi[-4*ProductLog[a/x^(1/4)]]
```

Defintions of rubi rules used

rule 7188
```
Int[ProductLog[(a_.)*(x_)^(n_.)]^(p_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^
(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[a*x^n]]/(d*n)
), x] /; FreeQ[{a, d}, x] && IntegerQ[p] && EqQ[n*p, -1]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$4a^{4} \operatorname{expIntegral}_{1}\left(4 \operatorname{LambertW}\left(\frac{a}{x^{\frac{1}{4}}}\right)\right)$	16
default	$4a^{4} \operatorname{expIntegral}_{1}\left(4 \operatorname{LambertW}\left(\frac{a}{x^{\frac{1}{4}}}\right)\right)$	16

input
```
int(LambertW(a/x^(1/4))^4/(1+LambertW(a/x^(1/4))),x,method=_RETURNVERBOSE)
```

output
```
4*a^4*Ei(1,4*LambertW(a/x^(1/4)))
```

Fricas [F]

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x^{\frac{1}{4}}}\right)^{4}}{\mathrm{W}\left(\frac{a}{x^{\frac{1}{4}}}\right)+1}\,dx$$

input
```
integrate(lambert_w(a/x^(1/4))^4/(1+lambert_w(a/x^(1/4))),x, algorithm="fr
icas")
```

output
```
integral(lambert_w(a/x^(1/4))^4/(lambert_w(a/x^(1/4)) + 1), x)
```

Sympy [F]

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)}\,dx = \int \frac{W^{4}\left(\frac{a}{\sqrt[4]{x}}\right)}{W\left(\frac{a}{\sqrt[4]{x}}\right)+1}\,dx$$

input
```
integrate(LambertW(a/x**(1/4))**4/(1+LambertW(a/x**(1/4))),x)
```

output
```
Integral(LambertW(a/x**(1/4))**4/(LambertW(a/x**(1/4)) + 1), x)
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)}\,dx=\int \frac{\mathrm{W}(\frac{a}{x^{\frac{1}{4}}})^4}{\mathrm{W}(\frac{a}{x^{\frac{1}{4}}})+1}\,dx$$

input
```
integrate(lambert_w(a/x^(1/4))^4/(1+lambert_w(a/x^(1/4))),x, algorithm="ma
xima")
```

output
```
integrate(lambert_w(a/x^(1/4))^4/(lambert_w(a/x^(1/4)) + 1), x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)}\,dx=\int \frac{\mathrm{W}(\frac{a}{x^{\frac{1}{4}}})^4}{\mathrm{W}(\frac{a}{x^{\frac{1}{4}}})+1}\,dx$$

input
```
integrate(lambert_w(a/x^(1/4))^4/(1+lambert_w(a/x^(1/4))),x, algorithm="gi
ac")
```

output
```
integrate(lambert_w(a/x^(1/4))^4/(lambert_w(a/x^(1/4)) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)}\,dx=\int \frac{\mathrm{LambertW}\left(\frac{a}{x^{1/4}}\right)^4}{\mathrm{LambertW}\left(\frac{a}{x^{1/4}}\right)+1}\,dx$$

input
```
int(LambertW(a/x^(1/4))^4/(LambertW(a/x^(1/4)) + 1),x)
```

output
```
int(LambertW(a/x^(1/4))^4/(LambertW(a/x^(1/4)) + 1), x)
```

Reduce [F]

$$\int \frac{W\left(\frac{a}{\sqrt[4]{x}}\right)^4}{1+W\left(\frac{a}{\sqrt[4]{x}}\right)}\,dx=\int \frac{lambert\_w\left(\frac{a}{x^{\frac{1}{4}}}\right)^4}{lambert\_w\left(\frac{a}{x^{\frac{1}{4}}}\right)+1}dx$$

input
```
int(Lambert_W(a/x^(1/4))^4/(1+Lambert_W(a/x^(1/4))),x)
```

output
```
int(lambert_w(a/x**(1/4))**4/(lambert_w(a/x**(1/4)) + 1),x)
```

3.350 $\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^3}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)} dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^3}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)} dx = -3a^3 \text{ExpIntegralEi}\left(-3W\left(\frac{a}{\sqrt[3]{x}}\right)\right)$$

output
```
-3*a^3*Ei(-3*LambertW(a/x^(1/3)))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^3}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)} dx = -3a^3 \text{ExpIntegralEi}\left(-3W\left(\frac{a}{\sqrt[3]{x}}\right)\right)$$

input
```
Integrate[ProductLog[a/x^(1/3)]^3/(1 + ProductLog[a/x^(1/3)]),x]
```

output
```
-3*a^3*ExpIntegralEi[-3*ProductLog[a/x^(1/3)]]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {7188}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^3}{W\left(\frac{a}{\sqrt[3]{x}}\right)+1}dx$$
$$\Big\downarrow \; 7188$$
$$-3a^3 \operatorname{ExpIntegralEi}\left(-3W\left(\frac{a}{\sqrt[3]{x}}\right)\right)$$

input
```
Int[ProductLog[a/x^(1/3)]^3/(1 + ProductLog[a/x^(1/3)]),x]
```

output
```
-3*a^3*ExpIntegralEi[-3*ProductLog[a/x^(1/3)]]
```

Defintions of rubi rules used

rule 7188
```
Int[ProductLog[(a_.)*(x_)^(n_.)]^(p_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^
(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[a*x^n]]/(d*n)
), x] /; FreeQ[{a, d}, x] && IntegerQ[p] && EqQ[n*p, -1]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$3a^{3} \operatorname{expIntegral}_{1}\left(3 \operatorname{LambertW}\left(\frac{a}{x^{\frac{1}{3}}}\right)\right)$	16
default	$3a^{3} \operatorname{expIntegral}_{1}\left(3 \operatorname{LambertW}\left(\frac{a}{x^{\frac{1}{3}}}\right)\right)$	16

input
```
int(LambertW(a/x^(1/3))^3/(1+LambertW(a/x^(1/3))),x,method=_RETURNVERBOSE)
```

output
```
3*a^3*Ei(1,3*LambertW(a/x^(1/3)))
```

Fricas [F]

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^{3}}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x^{\frac{1}{3}}}\right)^{3}}{\mathrm{W}\left(\frac{a}{x^{\frac{1}{3}}}\right)+1}\,dx$$

input
```
integrate(lambert_w(a/x^(1/3))^3/(1+lambert_w(a/x^(1/3))),x, algorithm="fr
icas")
```

output
```
integral(lambert_w(a/x^(1/3))^3/(lambert_w(a/x^(1/3)) + 1), x)
```

Sympy [F]

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^{3}}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)}\,dx = \int \frac{W^{3}\left(\frac{a}{\sqrt[3]{x}}\right)}{W\left(\frac{a}{\sqrt[3]{x}}\right)+1}\,dx$$

input
```
integrate(LambertW(a/x**(1/3))**3/(1+LambertW(a/x**(1/3))),x)
```

output
```
Integral(LambertW(a/x**(1/3))**3/(LambertW(a/x**(1/3)) + 1), x)
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^3}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x^{\frac{1}{3}}}\right)^3}{\mathrm{W}\left(\frac{a}{x^{\frac{1}{3}}}\right)+1}\,dx$$

input
```
integrate(lambert_w(a/x^(1/3))^3/(1+lambert_w(a/x^(1/3))),x, algorithm="ma
xima")
```

output
```
integrate(lambert_w(a/x^(1/3))^3/(lambert_w(a/x^(1/3)) + 1), x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^3}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x^{\frac{1}{3}}}\right)^3}{\mathrm{W}\left(\frac{a}{x^{\frac{1}{3}}}\right)+1}\,dx$$

input
```
integrate(lambert_w(a/x^(1/3))^3/(1+lambert_w(a/x^(1/3))),x, algorithm="gi
ac")
```

output
```
integrate(lambert_w(a/x^(1/3))^3/(lambert_w(a/x^(1/3)) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^3}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)}\,dx=\int \frac{\mathrm{LambertW}\left(\frac{a}{x^{1/3}}\right)^3}{\mathrm{LambertW}\left(\frac{a}{x^{1/3}}\right)+1}\,dx$$

input
```
int(LambertW(a/x^(1/3))^3/(LambertW(a/x^(1/3)) + 1),x)
```

output
```
int(LambertW(a/x^(1/3))^3/(LambertW(a/x^(1/3)) + 1), x)
```

Reduce [F]

$$\int \frac{W\left(\frac{a}{\sqrt[3]{x}}\right)^3}{1+W\left(\frac{a}{\sqrt[3]{x}}\right)}\,dx=\int \frac{lambert\_w\left(\frac{a}{x^{\frac{1}{3}}}\right)^3}{lambert\_w\left(\frac{a}{x^{\frac{1}{3}}}\right)+1}dx$$

input
```
int(Lambert_W(a/x^(1/3))^3/(1+Lambert_W(a/x^(1/3))),x)
```

output
```
int(lambert_w(a/x**(1/3))**3/(lambert_w(a/x**(1/3)) + 1),x)
```

3.351 $\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^2}{1+W\left(\frac{a}{\sqrt{x}}\right)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^2}{1+W\left(\frac{a}{\sqrt{x}}\right)}\,dx = -2a^2\,\text{ExpIntegralEi}\left(-2W\left(\frac{a}{\sqrt{x}}\right)\right)$$

output
```
-2*a^2*Ei(-2*LambertW(a/x^(1/2)))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^2}{1+W\left(\frac{a}{\sqrt{x}}\right)}\,dx = -2a^2\,\text{ExpIntegralEi}\left(-2W\left(\frac{a}{\sqrt{x}}\right)\right)$$

input
```
Integrate[ProductLog[a/Sqrt[x]]^2/(1 + ProductLog[a/Sqrt[x]]),x]
```

output
```
-2*a^2*ExpIntegralEi[-2*ProductLog[a/Sqrt[x]]]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {7188}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^2}{W\left(\frac{a}{\sqrt{x}}\right)+1}\,dx$$

$$\Big\downarrow \text{ 7188}$$

$$-2a^2\,\text{ExpIntegralEi}\left(-2W\left(\frac{a}{\sqrt{x}}\right)\right)$$

input
```
Int[ProductLog[a/Sqrt[x]]^2/(1 + ProductLog[a/Sqrt[x]]),x]
```

output
```
-2*a^2*ExpIntegralEi[-2*ProductLog[a/Sqrt[x]]]
```

Defintions of rubi rules used

rule 7188
```
Int[ProductLog[(a_.)*(x_)^(n_.)]^(p_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^
(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[a*x^n]]/(d*n)
), x] /; FreeQ[{a, d}, x] && IntegerQ[p] && EqQ[n*p, -1]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$2a^2\,\text{expIntegral}_1\left(2\,\text{LambertW}\left(\frac{a}{\sqrt{x}}\right)\right)$	16
default	$2a^2\,\text{expIntegral}_1\left(2\,\text{LambertW}\left(\frac{a}{\sqrt{x}}\right)\right)$	16

input
```
int(LambertW(a/x^(1/2))^2/(1+LambertW(a/x^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
2*a^2*Ei(1,2*LambertW(a/x^(1/2)))
```

Fricas [F]

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^2}{1+W\left(\frac{a}{\sqrt{x}}\right)}\,dx = \int \frac{\mathrm{W}(\frac{a}{\sqrt{x}})^2}{\mathrm{W}(\frac{a}{\sqrt{x}})+1}\,dx$$

input
```
integrate(lambert_w(a/x^(1/2))^2/(1+lambert_w(a/x^(1/2))),x, algorithm="fr
icas")
```

output
```
integral(lambert_w(a/sqrt(x))^2/(lambert_w(a/sqrt(x)) + 1), x)
```

Sympy [F]

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^2}{1+W\left(\frac{a}{\sqrt{x}}\right)}\,dx = \int \frac{W^2\left(\frac{a}{\sqrt{x}}\right)}{W\left(\frac{a}{\sqrt{x}}\right)+1}\,dx$$

input
```
integrate(LambertW(a/x**(1/2))**2/(1+LambertW(a/x**(1/2))),x)
```

output
```
Integral(LambertW(a/sqrt(x))**2/(LambertW(a/sqrt(x)) + 1), x)
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^2}{1+W\left(\frac{a}{\sqrt{x}}\right)}\,dx = \int \frac{W(\frac{a}{\sqrt{x}})^2}{W(\frac{a}{\sqrt{x}})+1}\,dx$$

input
```
integrate(lambert_w(a/x^(1/2))^2/(1+lambert_w(a/x^(1/2))),x, algorithm="ma
xima")
```

output
```
integrate(lambert_w(a/sqrt(x))^2/(lambert_w(a/sqrt(x)) + 1), x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^2}{1+W\left(\frac{a}{\sqrt{x}}\right)}\,dx = \int \frac{W(\frac{a}{\sqrt{x}})^2}{W(\frac{a}{\sqrt{x}})+1}\,dx$$

input
```
integrate(lambert_w(a/x^(1/2))^2/(1+lambert_w(a/x^(1/2))),x, algorithm="gi
ac")
```

output
```
integrate(lambert_w(a/sqrt(x))^2/(lambert_w(a/sqrt(x)) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^2}{1+W\left(\frac{a}{\sqrt{x}}\right)}\,dx = \int \frac{\mathrm{LambertW}\left(\frac{a}{\sqrt{x}}\right)^2}{\mathrm{LambertW}\left(\frac{a}{\sqrt{x}}\right)+1}\,dx$$

input
```
int(LambertW(a/x^(1/2))^2/(LambertW(a/x^(1/2)) + 1),x)
```

output
```
int(LambertW(a/x^(1/2))^2/(LambertW(a/x^(1/2)) + 1), x)
```

Reduce [F]

$$\int \frac{W\left(\frac{a}{\sqrt{x}}\right)^2}{1+W\left(\frac{a}{\sqrt{x}}\right)}\,dx = \int \frac{lambert\_w\left(\frac{a}{\sqrt{x}}\right)^2}{lambert\_w\left(\frac{a}{\sqrt{x}}\right)+1}dx$$

input
```
int(Lambert_W(a/x^(1/2))^2/(1+Lambert_W(a/x^(1/2))),x)
```

output
```
int(lambert_w(a/sqrt(x))**2/(lambert_w(a/sqrt(x)) + 1),x)
```

3.352 $\int \frac{W\left(\frac{a}{x}\right)}{1+W\left(\frac{a}{x}\right)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{W\left(\frac{a}{x}\right)}{1+W\left(\frac{a}{x}\right)}\,dx = -a\,\text{ExpIntegralEi}\left(-W\left(\frac{a}{x}\right)\right)$$

output
```
-a*Ei(-LambertW(a/x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{W\left(\frac{a}{x}\right)}{1+W\left(\frac{a}{x}\right)}\,dx = -a\,\text{ExpIntegralEi}\left(-W\left(\frac{a}{x}\right)\right)$$

input
```
Integrate[ProductLog[a/x]/(1 + ProductLog[a/x]),x]
```

output
```
-(a*ExpIntegralEi[-ProductLog[a/x]])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {7188}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(\frac{a}{x}\right)}{W\left(\frac{a}{x}\right)+1}\,dx$$
$$\Big\downarrow \text{ 7188}$$
$$-a\,\text{ExpIntegralEi}\left(-W\left(\frac{a}{x}\right)\right)$$

input
```
Int[ProductLog[a/x]/(1 + ProductLog[a/x]),x]
```

output
```
-(a*ExpIntegralEi[-ProductLog[a/x]])
```

Defintions of rubi rules used

rule 7188
```
Int[ProductLog[(a_.)*(x_)^(n_.)]^(p_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^
(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[a*x^n]]/(d*n)
), x] /; FreeQ[{a, d}, x] && IntegerQ[p] && EqQ[n*p, -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$a\ \text{expIntegral}_1\left(\text{LambertW}\left(\frac{a}{x}\right)\right)$	11
default	$a\ \text{expIntegral}_1\left(\text{LambertW}\left(\frac{a}{x}\right)\right)$	11

input
```
int(LambertW(a/x)/(1+LambertW(a/x)),x,method=_RETURNVERBOSE)
```

output
```
a*Ei(1,LambertW(a/x))
```

Fricas [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{1+W\left(\frac{a}{x}\right)}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)}{\mathrm{W}\left(\frac{a}{x}\right)+1}\,dx$$

input
```
integrate(lambert_w(a/x)/(1+lambert_w(a/x)),x, algorithm="fricas")
```

output
```
integral(lambert_w(a/x)/(lambert_w(a/x) + 1), x)
```

Sympy [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{1+W\left(\frac{a}{x}\right)}\,dx = \int \frac{W\left(\frac{a}{x}\right)}{W\left(\frac{a}{x}\right)+1}\,dx$$

input
```
integrate(LambertW(a/x)/(1+LambertW(a/x)),x)
```

output
```
Integral(LambertW(a/x)/(LambertW(a/x) + 1), x)
```

Maxima [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{1+W\left(\frac{a}{x}\right)}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)}{\mathrm{W}\left(\frac{a}{x}\right)+1}\,dx$$

input
```
integrate(lambert_w(a/x)/(1+lambert_w(a/x)),x, algorithm="maxima")
```

output
```
integrate(lambert_w(a/x)/(lambert_w(a/x) + 1), x)
```

Giac [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{1+W\left(\frac{a}{x}\right)}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x}\right)}{\mathrm{W}\left(\frac{a}{x}\right)+1}\,dx$$

input
```
integrate(lambert_w(a/x)/(1+lambert_w(a/x)),x, algorithm="giac")
```

output
```
integrate(lambert_w(a/x)/(lambert_w(a/x) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(\frac{a}{x}\right)}{1+W\left(\frac{a}{x}\right)}\,dx = \int \frac{\mathrm{LambertW}\left(\frac{a}{x}\right)}{\mathrm{LambertW}\left(\frac{a}{x}\right)+1}\,dx$$

input
```
int(LambertW(a/x)/(LambertW(a/x) + 1),x)
```

output
```
int(LambertW(a/x)/(LambertW(a/x) + 1), x)
```

Reduce [F]

$$\int \frac{W\left(\frac{a}{x}\right)}{1+W\left(\frac{a}{x}\right)}\,dx = \int \frac{lambert\_w\left(\frac{a}{x}\right)}{lambert\_w\left(\frac{a}{x}\right)+1}dx$$

input
```
int(Lambert_W(a/x)/(1+Lambert_W(a/x)),x)
```

output
```
int(lambert_w(a/x)/(lambert_w(a/x) + 1),x)
```

3.353 $\int \frac{1}{W(ax)(1+W(ax))} dx$

Optimal result

Integrand size = 15, antiderivative size = 9

$$\int \frac{1}{W(ax)(1+W(ax))} dx = \frac{\text{ExpIntegralEi}(W(ax))}{a}$$

output
```
Ei(LambertW(a*x))/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{W(ax)(1+W(ax))} dx = \frac{\text{ExpIntegralEi}(W(ax))}{a}$$

input
```
Integrate[1/(ProductLog[a*x]*(1 + ProductLog[a*x])),x]
```

output
```
ExpIntegralEi[ProductLog[a*x]]/a
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {7179}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W(ax)(W(ax)+1)}dx$$
$$\Big\downarrow \text{7179}$$
$$\frac{\text{ExpIntegralEi}(W(ax))}{a}$$

input
```
Int[1/(ProductLog[a*x]*(1 + ProductLog[a*x])),x]
```

output
```
ExpIntegralEi[ProductLog[a*x]]/a
```

Defintions of rubi rules used

rule 7179
```
Int[1/(ProductLog[(a_.) + (b_.)*(x_)]*((d_) + (d_.)*ProductLog[(a_.) + (b_.
)*(x_)])), x_Symbol] :> Simp[ExpIntegralEi[ProductLog[a + b*x]]/(b*d), x] /
; FreeQ[{a, b, d}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.56

method	result	size
derivativedivides	$-\frac{\text{expIntegral}_1(-\text{LambertW}(xa))}{a}$	14
default	$-\frac{\text{expIntegral}_1(-\text{LambertW}(xa))}{a}$	14

input
```
int(1/LambertW(x*a)/(1+LambertW(x*a)),x,method=_RETURNVERBOSE)
```

output
```
-1/a*Ei(1,-LambertW(x*a))
```

Fricas [F]

$$\int \frac{1}{W(ax)(1+W(ax))}\,dx = \int \frac{1}{(\mathrm{W}(ax)+1)\,\mathrm{W}(ax)}\,dx$$

input
```
integrate(1/lambert_w(a*x)/(1+lambert_w(a*x)),x, algorithm="fricas")
```

output
```
integral(1/(lambert_w(a*x)^2 + lambert_w(a*x)), x)
```

Sympy [F]

$$\int \frac{1}{W(ax)(1+W(ax))}\,dx = \int \frac{1}{(W(ax)+1)\,W(ax)}\,dx$$

input
```
integrate(1/LambertW(a*x)/(1+LambertW(a*x)),x)
```

output
```
Integral(1/((LambertW(a*x) + 1)*LambertW(a*x)), x)
```

Maxima [F]

$$\int \frac{1}{W(ax)(1+W(ax))}\,dx = \int \frac{1}{(\mathrm{W}(ax)+1)\,\mathrm{W}(ax)}\,dx$$

input
```
integrate(1/lambert_w(a*x)/(1+lambert_w(a*x)),x, algorithm="maxima")
```

output
```
integrate(1/((lambert_w(a*x) + 1)*lambert_w(a*x)), x)
```

Giac [F]

$$\int \frac{1}{W(ax)(1+W(ax))}\,dx = \int \frac{1}{(\mathrm{W}(ax)+1)\,\mathrm{W}(ax)}\,dx$$

input
```
integrate(1/lambert_w(a*x)/(1+lambert_w(a*x)),x, algorithm="giac")
```

output
```
integrate(1/((lambert_w(a*x) + 1)*lambert_w(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W(ax)(1+W(ax))}\,dx = \int \frac{1}{\mathrm{LambertW}\,(a\,x)\ (\mathrm{LambertW}\,(a\,x)+1)}\,dx$$

input
```
int(1/(LambertW(a*x)*(LambertW(a*x) + 1)),x)
```

output
```
int(1/(LambertW(a*x)*(LambertW(a*x) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{W(ax)(1+W(ax))}\,dx = \int \frac{1}{lambert\_w(ax)^2 + lambert\_w(ax)}\,dx$$

input
```
int(1/Lambert_W(a*x)/(1+Lambert_W(a*x)),x)
```

output
```
int(1/(lambert_w(a*x)**2 + lambert_w(a*x)),x)
```

3.354 $\int \frac{1}{W(a\sqrt{x})^2(1+W(a\sqrt{x}))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{1}{W\left(a\sqrt{x}\right)^2\left(1+W\left(a\sqrt{x}\right)\right)}\,dx=\frac{2\,\text{ExpIntegralEi}\left(2W\left(a\sqrt{x}\right)\right)}{a^2}$$

output
```
2*Ei(2*LambertW(a*x^(1/2)))/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{W\left(a\sqrt{x}\right)^2\left(1+W\left(a\sqrt{x}\right)\right)}\,dx=\frac{2\,\text{ExpIntegralEi}\left(2W\left(a\sqrt{x}\right)\right)}{a^2}$$

input
```
Integrate[1/(ProductLog[a*Sqrt[x]]^2*(1 + ProductLog[a*Sqrt[x]])),x]
```

output
```
(2*ExpIntegralEi[2*ProductLog[a*Sqrt[x]]])/a^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {7188}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W\left(a\sqrt{x}\right)^2\left(W\left(a\sqrt{x}\right)+1\right)}dx$$

$$\Big\downarrow \; 7188$$

$$\frac{2\,\text{ExpIntegralEi}\left(2W\left(a\sqrt{x}\right)\right)}{a^2}$$

input
```
Int[1/(ProductLog[a*Sqrt[x]]^2*(1 + ProductLog[a*Sqrt[x]])),x]
```

output
```
(2*ExpIntegralEi[2*ProductLog[a*Sqrt[x]]])/a^2
```

Defintions of rubi rules used

rule 7188
```
Int[ProductLog[(a_.)*(x_)^(n_.)]^(p_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^
(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[a*x^n]]/(d*n)
), x] /; FreeQ[{a, d}, x] && IntegerQ[p] && EqQ[n*p, -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{2\,\text{expIntegral}_1\left(-2\,\text{LambertW}\left(a\sqrt{x}\right)\right)}{a^2}$	16
default	$-\frac{2\,\text{expIntegral}_1\left(-2\,\text{LambertW}\left(a\sqrt{x}\right)\right)}{a^2}$	16

input
```
int(1/LambertW(a*x^(1/2))^2/(1+LambertW(a*x^(1/2))),x,method=_RETURNVERBOS
E)
```

output
```
-2/a^2*Ei(1,-2*LambertW(a*x^(1/2)))
```

Fricas [F]

$$\int \frac{1}{W\left(a\sqrt{x}\right)^2\left(1+W\left(a\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(\mathrm{W}(a\sqrt{x})+1\right)\mathrm{W}(a\sqrt{x})^2}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/2))^2/(1+lambert_w(a*x^(1/2))),x, algorithm="
fricas")
```

output
```
integral(1/(lambert_w(a*sqrt(x))^3 + lambert_w(a*sqrt(x))^2), x)
```

Sympy [F]

$$\int \frac{1}{W\left(a\sqrt{x}\right)^2\left(1+W\left(a\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(W\left(a\sqrt{x}\right)+1\right)W^2\left(a\sqrt{x}\right)}\,dx$$

input
```
integrate(1/LambertW(a*x**(1/2))**2/(1+LambertW(a*x**(1/2))),x)
```

output
```
Integral(1/((LambertW(a*sqrt(x)) + 1)*LambertW(a*sqrt(x))**2), x)
```

Maxima [F]

$$\int \frac{1}{W\left(a\sqrt{x}\right)^2 \left(1+W\left(a\sqrt{x}\right)\right)}\,dx = \int \frac{1}{\left(\mathrm{W}(a\sqrt{x})+1\right)\mathrm{W}(a\sqrt{x})^2}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/2))^2/(1+lambert_w(a*x^(1/2))),x, algorithm="
maxima")
```

output
```
integrate(1/((lambert_w(a*sqrt(x)) + 1)*lambert_w(a*sqrt(x))^2), x)
```

Giac [F]

$$\int \frac{1}{W\left(a\sqrt{x}\right)^2 \left(1+W\left(a\sqrt{x}\right)\right)}\,dx = \int \frac{1}{\left(\mathrm{W}(a\sqrt{x})+1\right)\mathrm{W}(a\sqrt{x})^2}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/2))^2/(1+lambert_w(a*x^(1/2))),x, algorithm="
giac")
```

output
```
integrate(1/((lambert_w(a*sqrt(x)) + 1)*lambert_w(a*sqrt(x))^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W\left(a\sqrt{x}\right)^2 \left(1+W\left(a\sqrt{x}\right)\right)}\,dx = \int \frac{1}{\mathrm{LambertW}\left(a\sqrt{x}\right)^2\,\left(\mathrm{LambertW}\left(a\sqrt{x}\right)+1\right)}\,dx$$

input
```
int(1/(LambertW(a*x^(1/2))^2*(LambertW(a*x^(1/2)) + 1)),x)
```

output
```
int(1/(LambertW(a*x^(1/2))^2*(LambertW(a*x^(1/2)) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{W\left(a\sqrt{x}\right)^2\left(1+W\left(a\sqrt{x}\right)\right)}\,dx = \int \frac{1}{lambert\_w\left(\sqrt{x}\,a\right)^3 + lambert\_w\left(\sqrt{x}\,a\right)^2}dx$$

input
```
int(1/Lambert_W(a*x^(1/2))^2/(1+Lambert_W(a*x^(1/2))),x)
```

output
```
int(1/(lambert_w(sqrt(x)*a)**3 + lambert_w(sqrt(x)*a)**2),x)
```

3.355 $\int \frac{1}{W\left(a\sqrt[3]{x}\right)^3\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^3\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx=\frac{3\,\text{ExpIntegralEi}\left(3W\left(a\sqrt[3]{x}\right)\right)}{a^3}$$

output
```
3*Ei(3*LambertW(a*x^(1/3)))/a^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^3\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx=\frac{3\,\text{ExpIntegralEi}\left(3W\left(a\sqrt[3]{x}\right)\right)}{a^3}$$

input
```
Integrate[1/(ProductLog[a*x^(1/3)]^3*(1 + ProductLog[a*x^(1/3)])),x]
```

output
```
(3*ExpIntegralEi[3*ProductLog[a*x^(1/3)]])/a^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {7188}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^3\left(W\left(a\sqrt[3]{x}\right)+1\right)}dx$$
$$\Big\downarrow \text{7188}$$
$$\frac{3\,\text{ExpIntegralEi}\left(3W\left(a\sqrt[3]{x}\right)\right)}{a^3}$$

input
```
Int[1/(ProductLog[a*x^(1/3)]^3*(1 + ProductLog[a*x^(1/3)])),x]
```

output
```
(3*ExpIntegralEi[3*ProductLog[a*x^(1/3)]])/a^3
```

Defintions of rubi rules used

rule 7188
```
Int[ProductLog[(a_.)*(x_)^(n_.)]^(p_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^
(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[a*x^n]]/(d*n)
), x] /; FreeQ[{a, d}, x] && IntegerQ[p] && EqQ[n*p, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{3\,\text{expIntegral}_1\left(-3\,\text{LambertW}\left(a\,x^{\frac{1}{3}}\right)\right)}{a^3}$	16
default	$-\frac{3\,\text{expIntegral}_1\left(-3\,\text{LambertW}\left(a\,x^{\frac{1}{3}}\right)\right)}{a^3}$	16

input
```
int(1/LambertW(a*x^(1/3))^3/(1+LambertW(a*x^(1/3))),x,method=_RETURNVERBOS
E)
```

output
```
-3/a^3*Ei(1,-3*LambertW(a*x^(1/3)))
```

Fricas [F]

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^3\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx=\int \frac{1}{\left(\mathrm{W}(ax^{\frac{1}{3}})+1\right)\mathrm{W}(ax^{\frac{1}{3}})^3}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/3))^3/(1+lambert_w(a*x^(1/3))),x, algorithm="
fricas")
```

output
```
integral(1/(lambert_w(a*x^(1/3))^4 + lambert_w(a*x^(1/3))^3), x)
```

Sympy [F]

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^3\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx=\int \frac{1}{\left(W\left(a\sqrt[3]{x}\right)+1\right)W^{3}\left(a\sqrt[3]{x}\right)}\,dx$$

input
```
integrate(1/LambertW(a*x**(1/3))**3/(1+LambertW(a*x**(1/3))),x)
```

output
```
Integral(1/((LambertW(a*x**(1/3)) + 1)*LambertW(a*x**(1/3))**3), x)
```

Maxima [F]

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^3\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx = \int \frac{1}{\left(\mathrm{W}(ax^{\frac{1}{3}})+1\right)\mathrm{W}(ax^{\frac{1}{3}})^3}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/3))^3/(1+lambert_w(a*x^(1/3))),x, algorithm="
maxima")
```

output
```
integrate(1/((lambert_w(a*x^(1/3)) + 1)*lambert_w(a*x^(1/3))^3), x)
```

Giac [F]

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^3\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx = \int \frac{1}{\left(\mathrm{W}(ax^{\frac{1}{3}})+1\right)\mathrm{W}(ax^{\frac{1}{3}})^3}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/3))^3/(1+lambert_w(a*x^(1/3))),x, algorithm="
giac")
```

output
```
integrate(1/((lambert_w(a*x^(1/3)) + 1)*lambert_w(a*x^(1/3))^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^3\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx = \int \frac{1}{\mathrm{LambertW}\left(a\,x^{1/3}\right)^3\,\left(\mathrm{LambertW}\left(a\,x^{1/3}\right)+1\right)}\,dx$$

input
```
int(1/(LambertW(a*x^(1/3))^3*(LambertW(a*x^(1/3)) + 1)),x)
```

output
```
int(1/(LambertW(a*x^(1/3))^3*(LambertW(a*x^(1/3)) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{W\left(a\sqrt[3]{x}\right)^3\left(1+W\left(a\sqrt[3]{x}\right)\right)}\,dx = \int \frac{1}{lambert\_w\left(x^{\frac{1}{3}}a\right)^4 + lambert\_w\left(x^{\frac{1}{3}}a\right)^3}dx$$

input
```
int(1/Lambert_W(a*x^(1/3))^3/(1+Lambert_W(a*x^(1/3))),x)
```

output
```
int(1/(lambert_w(x**(1/3)*a)**4 + lambert_w(x**(1/3)*a)**3),x)
```

3.356 $\int \frac{1}{W\left(a\sqrt[4]{x}\right)^4\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^4\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx=\frac{4\,\mathrm{ExpIntegralEi}\left(4W\left(a\sqrt[4]{x}\right)\right)}{a^4}$$

output
```
4*Ei(4*LambertW(a*x^(1/4)))/a^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^4\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx=\frac{4\,\mathrm{ExpIntegralEi}\left(4W\left(a\sqrt[4]{x}\right)\right)}{a^4}$$

input
```
Integrate[1/(ProductLog[a*x^(1/4)]^4*(1 + ProductLog[a*x^(1/4)])),x]
```

output
```
(4*ExpIntegralEi[4*ProductLog[a*x^(1/4)]])/a^4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {7188}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^4\left(W\left(a\sqrt[4]{x}\right)+1\right)}\,dx$$

$$\Big\downarrow \text{7188}$$

$$\frac{4\,\text{ExpIntegralEi}\left(4W\left(a\sqrt[4]{x}\right)\right)}{a^4}$$

input
```
Int[1/(ProductLog[a*x^(1/4)]^4*(1 + ProductLog[a*x^(1/4)])),x]
```

output
```
(4*ExpIntegralEi[4*ProductLog[a*x^(1/4)]])/a^4
```

Defintions of rubi rules used

rule 7188
```
Int[ProductLog[(a_.)*(x_)^(n_.)]^(p_.)/((d_) + (d_.)*ProductLog[(a_.)*(x_)^
(n_.)]), x_Symbol] :> Simp[a^p*(ExpIntegralEi[(-p)*ProductLog[a*x^n]]/(d*n)
), x] /; FreeQ[{a, d}, x] && IntegerQ[p] && EqQ[n*p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{4\,\text{expIntegral}_1\left(-4\,\text{LambertW}\left(a\,x^{\frac{1}{4}}\right)\right)}{a^4}$	16
default	$-\frac{4\,\text{expIntegral}_1\left(-4\,\text{LambertW}\left(a\,x^{\frac{1}{4}}\right)\right)}{a^4}$	16

input
```
int(1/LambertW(a*x^(1/4))^4/(1+LambertW(a*x^(1/4))),x,method=_RETURNVERBOS
E)
```

output
```
-4/a^4*Ei(1,-4*LambertW(a*x^(1/4)))
```

Fricas [F]

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^4\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx = \int \frac{1}{\left(\mathrm{W}(ax^{\frac{1}{4}})+1\right)\mathrm{W}(ax^{\frac{1}{4}})^4}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/4))^4/(1+lambert_w(a*x^(1/4))),x, algorithm="
fricas")
```

output
```
integral(1/(lambert_w(a*x^(1/4))^5 + lambert_w(a*x^(1/4))^4), x)
```

Sympy [F]

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^4\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx = \int \frac{1}{\left(W\left(a\sqrt[4]{x}\right)+1\right)W^4\left(a\sqrt[4]{x}\right)}\,dx$$

input
```
integrate(1/LambertW(a*x**(1/4))**4/(1+LambertW(a*x**(1/4))),x)
```

output
```
Integral(1/((LambertW(a*x**(1/4)) + 1)*LambertW(a*x**(1/4))**4), x)
```

Maxima [F]

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^4\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx=\int \frac{1}{\left(\mathrm{W}(ax^{\frac{1}{4}})+1\right)\mathrm{W}(ax^{\frac{1}{4}})^4}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/4))^4/(1+lambert_w(a*x^(1/4))),x, algorithm="
maxima")
```

output
```
integrate(1/((lambert_w(a*x^(1/4)) + 1)*lambert_w(a*x^(1/4))^4), x)
```

Giac [F]

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^4\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx=\int \frac{1}{\left(\mathrm{W}(ax^{\frac{1}{4}})+1\right)\mathrm{W}(ax^{\frac{1}{4}})^4}\,dx$$

input
```
integrate(1/lambert_w(a*x^(1/4))^4/(1+lambert_w(a*x^(1/4))),x, algorithm="
giac")
```

output
```
integrate(1/((lambert_w(a*x^(1/4)) + 1)*lambert_w(a*x^(1/4))^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^4\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx=\int \frac{1}{\mathrm{LambertW}\left(a\,x^{1/4}\right)^4\,\left(\mathrm{LambertW}\left(a\,x^{1/4}\right)+1\right)}\,dx$$

input
```
int(1/(LambertW(a*x^(1/4))^4*(LambertW(a*x^(1/4)) + 1)),x)
```

output
```
int(1/(LambertW(a*x^(1/4))^4*(LambertW(a*x^(1/4)) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{W\left(a\sqrt[4]{x}\right)^4\left(1+W\left(a\sqrt[4]{x}\right)\right)}\,dx = \int \frac{1}{lambert\_w\left(x^{\frac{1}{4}}a\right)^5 + lambert\_w\left(x^{\frac{1}{4}}a\right)^4}dx$$

input
```
int(1/Lambert_W(a*x^(1/4))^4/(1+Lambert_W(a*x^(1/4))),x)
```

output
```
int(1/(lambert_w(x**(1/4)*a)**5 + lambert_w(x**(1/4)*a)**4),x)
```

3.357 $\int \frac{W(ax^n)^p}{d+dW(ax^n)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{W(ax^n)^p}{d+dW(ax^n)}\,dx=\frac{e^{-\frac{(1-n)W(ax^n)}{n}}x^{1-n}\Gamma\left(\frac{1}{n}+p,-\frac{W(ax^n)}{n}\right)W(ax^n)^p\left(-\frac{W(ax^n)}{n}\right)^{1-\frac{1}{n}-p}}{ad}$$

output
```
x^(1-n)*GAMMA(1/n+p,-LambertW(a*x^n)/n)*LambertW(a*x^n)^p*(-LambertW(a*x^n
)/n)^(1-1/n-p)/a/d/exp((1-n)*LambertW(a*x^n)/n)
```

Mathematica [F]

$$\int \frac{W(ax^n)^p}{d+dW(ax^n)}\,dx=\int \frac{W(ax^n)^p}{d+dW(ax^n)}\,dx$$

input
```
Integrate[ProductLog[a*x^n]^p/(d + d*ProductLog[a*x^n]),x]
```

output
```
Integrate[ProductLog[a*x^n]^p/(d + d*ProductLog[a*x^n]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^n)^p}{dW(ax^n)+d}dx$$
$$\Big\downarrow \text{ 7292}$$
$$\int \frac{W(ax^n)^p}{d(W(ax^n)+1)}dx$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{W(ax^n)^p}{W(ax^n)+1}dx}{d}$$
$$\Big\downarrow \text{ 7299}$$
$$\frac{\int \frac{W(ax^n)^p}{W(ax^n)+1}dx}{d}$$

input
```
Int[ProductLog[a*x^n]^p/(d + d*ProductLog[a*x^n]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{\operatorname{LambertW}(a\,x^n)^p}{d+d\operatorname{LambertW}(a\,x^n)}dx$$

input
```
int(LambertW(a*x^n)^p/(d+d*LambertW(a*x^n)),x)
```

output
```
int(LambertW(a*x^n)^p/(d+d*LambertW(a*x^n)),x)
```

Fricas [F]

$$\int \frac{W(ax^n)^p}{d+dW(ax^n)}\,dx = \int \frac{\mathrm{W}(ax^n)^p}{d\,\mathrm{W}(ax^n)+d}\,dx$$

input
```
integrate(lambert_w(a*x^n)^p/(d+d*lambert_w(a*x^n)),x, algorithm="fricas")
```

output
```
integral(lambert_w(a*x^n)^p/(d*lambert_w(a*x^n) + d), x)
```

Sympy [F]

$$\int \frac{W(ax^n)^p}{d+dW(ax^n)}\,dx = \frac{\int \frac{W^p(ax^n)}{W(ax^n)+1}\,dx}{d}$$

input
```
integrate(LambertW(a*x**n)**p/(d+d*LambertW(a*x**n)),x)
```

output
```
Integral(LambertW(a*x**n)**p/(LambertW(a*x**n) + 1), x)/d
```

Maxima [F]

$$\int \frac{W(ax^n)^p}{d+dW(ax^n)}\,dx = \int \frac{\mathrm{W}(ax^n)^p}{d\,\mathrm{W}(ax^n)+d}\,dx$$

input
```
integrate(lambert_w(a*x^n)^p/(d+d*lambert_w(a*x^n)),x, algorithm="maxima")
```

output
```
integrate(lambert_w(a*x^n)^p/(d*lambert_w(a*x^n) + d), x)
```

Giac [F]

$$\int \frac{W(ax^n)^p}{d+dW(ax^n)}\,dx = \int \frac{\mathrm{W}(ax^n)^p}{d\,\mathrm{W}(ax^n)+d}\,dx$$

input
```
integrate(lambert_w(a*x^n)^p/(d+d*lambert_w(a*x^n)),x, algorithm="giac")
```

output
```
integrate(lambert_w(a*x^n)^p/(d*lambert_w(a*x^n) + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^n)^p}{d+dW(ax^n)}\,dx = \int \frac{\mathrm{LambertW}(a\,x^n)^p}{d+d\,\mathrm{LambertW}(a\,x^n)}\,dx$$

input
```
int(LambertW(a*x^n)^p/(d + d*LambertW(a*x^n)),x)
```

output
```
int(LambertW(a*x^n)^p/(d + d*LambertW(a*x^n)), x)
```

Reduce [F]

$$\int \frac{W(ax^n)^p}{d+dW(ax^n)}\,dx = \frac{lambert\_w(x^na)^p\,x + 3\left(\int \frac{lambert\_w(x^na)^p}{lambert\_w(x^na)+1}dx\right) - \left(\int \frac{lambert\_w(x^na)^p\,lambert\_w(x^na)}{lambert\_w(x^na)+1}dx\right) - \left(\int \frac{}{e^{lambert\_}}\right.}{4d}$$

input
```
int(Lambert_W(a*x^n)^p/(d+d*Lambert_W(a*x^n)),x)
```

output
```
(lambert_w(x**n*a)**p*x + 3*int(lambert_w(x**n*a)**p/(lambert_w(x**n*a) +
1),x) - int((lambert_w(x**n*a)**p*lambert_w(x**n*a))/(lambert_w(x**n*a) +
1),x) - int((x**n*lambert_w(x**n*a)**p)/(e**lambert_w(x**n*a)*lambert_w(x*
*n*a)**2 + e**lambert_w(x**n*a)*lambert_w(x**n*a)),x)*a*n*p)/(4*d)
```

3.358 $\int \frac{W(ax^n)^{1-\frac{1}{n}}}{1+W(ax^n)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 14

$$\int \frac{W(ax^n)^{1-\frac{1}{n}}}{1+W(ax^n)}\,dx = xW(ax^n)^{-1/n}$$

output
```
x/(LambertW(a*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{W(ax^n)^{1-\frac{1}{n}}}{1+W(ax^n)}\,dx = xW(ax^n)^{-1/n}$$

input
```
Integrate[ProductLog[a*x^n]^(1 - n^(-1))/(1 + ProductLog[a*x^n]),x]
```

output
```
x/ProductLog[a*x^n]^n^(-1)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(ax^n)^{1-\frac{1}{n}}}{W(ax^n)+1}\,dx$$

$$\Big\downarrow \text{ 7187}$$

$$xW(ax^n)^{-1/n}$$

input
```
Int[ProductLog[a*x^n]^(1 - n^(-1))/(1 + ProductLog[a*x^n]),x]
```

output
```
x/ProductLog[a*x^n]^n^(-1)
```

Defintions of rubi rules used

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

method	result	size
parallelrisch	$\frac{\text{LambertW}(a\,x^n)^{\frac{-1+n}{n}}\,x}{\text{LambertW}(a\,x^n)}$	25

input
```
int(LambertW(a*x^n)^(1-1/n)/(1+LambertW(a*x^n)),x,method=_RETURNVERBOSE)
```

output
```
LambertW(a*x^n)^((-1+n)/n)*x/LambertW(a*x^n)
```

Fricas [F]

$$\int \frac{W(ax^n)^{1-\frac{1}{n}}}{1+W(ax^n)}\,dx=\int \frac{\mathrm{W}(ax^n)^{-\frac{1}{n}+1}}{\mathrm{W}(ax^n)+1}\,dx$$

input
```
integrate(lambert_w(a*x^n)^(1-1/n)/(1+lambert_w(a*x^n)),x, algorithm="fric
as")
```

output
```
integral(lambert_w(a*x^n)^((n - 1)/n)/(lambert_w(a*x^n) + 1), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.89 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{W(ax^n)^{1-\frac{1}{n}}}{1+W(ax^n)}\,dx=\begin{cases} \frac{xW^{1-\frac{1}{n}}(ax^n)}{W(ax^n)} & \text{for } a\neq 0\\ 0^{1-\frac{1}{n}}x & \text{otherwise}\end{cases}$$

input
```
integrate(LambertW(a*x**n)**(1-1/n)/(1+LambertW(a*x**n)),x)
```

output
```
Piecewise((x*LambertW(a*x**n)**(1 - 1/n)/LambertW(a*x**n), Ne(a, 0)), (0**
(1 - 1/n)*x, True))
```

Maxima [F]

$$\int \frac{W(ax^n)^{1-\frac{1}{n}}}{1+W(ax^n)}\,dx = \int \frac{W(ax^n)^{-\frac{1}{n}+1}}{W(ax^n)+1}\,dx$$

input
```
integrate(lambert_w(a*x^n)^(1-1/n)/(1+lambert_w(a*x^n)),x, algorithm="maxi
ma")
```

output
```
integrate(lambert_w(a*x^n)^(-1/n + 1)/(lambert_w(a*x^n) + 1), x)
```

Giac [F]

$$\int \frac{W(ax^n)^{1-\frac{1}{n}}}{1+W(ax^n)}\,dx = \int \frac{W(ax^n)^{-\frac{1}{n}+1}}{W(ax^n)+1}\,dx$$

input
```
integrate(lambert_w(a*x^n)^(1-1/n)/(1+lambert_w(a*x^n)),x, algorithm="giac
")
```

output
```
integrate(lambert_w(a*x^n)^(-1/n + 1)/(lambert_w(a*x^n) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W(ax^n)^{1-\frac{1}{n}}}{1+W(ax^n)}\,dx = \int \frac{\mathrm{LambertW}(a\,x^n)^{1-\frac{1}{n}}}{\mathrm{LambertW}\,(a\,x^n)+1}\,dx$$

input
```
int(LambertW(a*x^n)^(1 - 1/n)/(LambertW(a*x^n) + 1),x)
```

output
```
int(LambertW(a*x^n)^(1 - 1/n)/(LambertW(a*x^n) + 1), x)
```

Reduce [F]

$$\int \frac{W(ax^n)^{1-\frac{1}{n}}}{1+W(ax^n)}\,dx = \int \frac{lambert\_w(x^n a)}{lambert\_w(x^n a)^{\frac{1}{n}}\, lambert\_w(x^n a) + lambert\_w(x^n a)^{\frac{1}{n}}}dx$$

input
```
int(Lambert_W(a*x^n)^(1-1/n)/(1+Lambert_W(a*x^n)),x)
```

output
```
int(lambert_w(x**n*a)/(lambert_w(x**n*a)**(1/n)*lambert_w(x**n*a) + lamber
t_w(x**n*a)**(1/n)),x)
```

3.359 $\int \frac{W\left(ax^{\frac{1}{1-p}}\right)^p}{1+W\left(ax^{\frac{1}{1-p}}\right)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 18

$$\int \frac{W\left(ax^{\frac{1}{1-p}}\right)^p}{1+W\left(ax^{\frac{1}{1-p}}\right)}\,dx = xW\left(ax^{\frac{1}{1-p}}\right)^{-1+p}$$

output
```
x*LambertW(a*x^(1/(1-p)))^(-1+p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{W\left(ax^{\frac{1}{1-p}}\right)^p}{1+W\left(ax^{\frac{1}{1-p}}\right)}\,dx = xW\left(ax^{\frac{1}{1-p}}\right)^{-1+p}$$

input
```
Integrate[ProductLog[a*x^(1 - p)^(-1)]^p/(1 + ProductLog[a*x^(1 - p)^(-1)]
),x]
```

output
```
x*ProductLog[a*x^(1 - p)^(-1)]^(-1 + p)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {7187}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W\left(ax^{\frac{1}{1-p}}\right)^p}{W\left(ax^{\frac{1}{1-p}}\right)+1}dx$$

$$\Big\downarrow \text{ 7187}$$

$$xW\left(ax^{\frac{1}{1-p}}\right)^{p-1}$$

input
```
Int[ProductLog[a*x^(1 - p)^(-1)]^p/(1 + ProductLog[a*x^(1 - p)^(-1)]),x]
```

output
```
x*ProductLog[a*x^(1 - p)^(-1)]^(-1 + p)
```

Defintions of rubi rules used

rule 7187
```
Int[((c_.)*ProductLog[(a_.)*(x_)^(n_.)])^(p_.)/((d_) + (d_.)*ProductLog[(a_
.)*(x_)^(n_.)]), x_Symbol] :> Simp[c*x*((c*ProductLog[a*x^n])^(p - 1)/d), x
] /; FreeQ[{a, c, d, n, p}, x] && EqQ[n*(p - 1), -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

method	result	size
parallelrisch	$\frac{x\,\mathrm{LambertW}\left(a\,x^{\frac{1}{1-p}}\right)^{p}}{\mathrm{LambertW}\left(a\,x^{\frac{1}{1-p}}\right)}$	31

input
```
int(LambertW(a*x^(1/(1-p)))^p/(1+LambertW(a*x^(1/(1-p)))),x,method=_RETURN
VERBOSE)
```

output
```
x*LambertW(a*x^(1/(1-p)))^p/LambertW(a*x^(1/(1-p)))
```

Fricas [F]

$$\int \frac{W\left(ax^{\frac{1}{1-p}}\right)^{p}}{1+W\left(ax^{\frac{1}{1-p}}\right)}\,dx=\int \frac{\mathrm{W}\left(\frac{a}{x^{\left(\frac{1}{p-1}\right)}}\right)^{p}}{\mathrm{W}\left(\frac{a}{x^{\left(\frac{1}{p-1}\right)}}\right)+1}\,dx$$

input
```
integrate(lambert_w(a*x^(1/(1-p)))^p/(1+lambert_w(a*x^(1/(1-p)))),x, algor
ithm="fricas")
```

output
```
integral(lambert_w(a/x^(1/(p - 1)))^p/(lambert_w(a/x^(1/(p - 1))) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{W\left(ax^{\frac{1}{1-p}}\right)^p}{1+W\left(ax^{\frac{1}{1-p}}\right)}\,dx = \text{Timed out}$$

input
```
integrate(LambertW(a*x**(1/(1-p)))**p/(1+LambertW(a*x**(1/(1-p)))),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{W\left(ax^{\frac{1}{1-p}}\right)^p}{1+W\left(ax^{\frac{1}{1-p}}\right)}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x^{\left(\frac{1}{p-1}\right)}}\right)^p}{\mathrm{W}\left(\frac{a}{x^{\left(\frac{1}{p-1}\right)}}\right)+1}\,dx$$

input
```
integrate(lambert_w(a*x^(1/(1-p)))^p/(1+lambert_w(a*x^(1/(1-p)))),x, algor
ithm="maxima")
```

output
```
integrate(lambert_w(a/x^(1/(p - 1)))^p/(lambert_w(a/x^(1/(p - 1))) + 1), x
)
```

Giac [F]

$$\int \frac{W\left(ax^{\frac{1}{1-p}}\right)^p}{1+W\left(ax^{\frac{1}{1-p}}\right)}\,dx = \int \frac{\mathrm{W}\left(\frac{a}{x^{\left(\frac{1}{p-1}\right)}}\right)^p}{\mathrm{W}\left(\frac{a}{x^{\left(\frac{1}{p-1}\right)}}\right)+1}\,dx$$

input
```
integrate(lambert_w(a*x^(1/(1-p)))^p/(1+lambert_w(a*x^(1/(1-p)))),x, algor
ithm="giac")
```

output
```
integrate(lambert_w(a/x^(1/(p - 1)))^p/(lambert_w(a/x^(1/(p - 1))) + 1), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{W\left(ax^{\frac{1}{1-p}}\right)^p}{1+W\left(ax^{\frac{1}{1-p}}\right)}\,dx = \int \frac{\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{p-1}}}\right)^p}{\mathrm{LambertW}\left(\frac{a}{x^{\frac{1}{p-1}}}\right)+1}\,dx$$

input
```
int(LambertW(a/x^(1/(p - 1)))^p/(LambertW(a/x^(1/(p - 1))) + 1),x)
```

output
```
int(LambertW(a/x^(1/(p - 1)))^p/(LambertW(a/x^(1/(p - 1))) + 1), x)
```

Reduce [F]

$$\int \frac{W\left(ax^{\frac{1}{1-p}}\right)^p}{1+W\left(ax^{\frac{1}{1-p}}\right)}\,dx = \text{Too large to display}$$

input
```
int(Lambert_W(a*x^(1/(1-p)))^p/(1+Lambert_W(a*x^(1/(1-p)))),x)
```

output

```
(3*lambert_w(a/x**(1/(p - 1)))**p*x + int(lambert_w(a/x**(1/(p - 1)))**p/(
x**(1/(p - 1))*e**lambert_w(a/x**(1/(p - 1)))*lambert_w(a/x**(1/(p - 1)))*
*2*p**2 - 2*x**(1/(p - 1))*e**lambert_w(a/x**(1/(p - 1)))*lambert_w(a/x**(
1/(p - 1)))**2*p + x**(1/(p - 1))*e**lambert_w(a/x**(1/(p - 1)))*lambert_w
(a/x**(1/(p - 1)))**2 + x**(1/(p - 1))*e**lambert_w(a/x**(1/(p - 1)))*lamb
ert_w(a/x**(1/(p - 1)))*p**2 - 2*x**(1/(p - 1))*e**lambert_w(a/x**(1/(p -
1)))*lambert_w(a/x**(1/(p - 1)))*p + x**(1/(p - 1))*e**lambert_w(a/x**(1/(
p - 1)))*lambert_w(a/x**(1/(p - 1)))),x)*a*p**2 - int(lambert_w(a/x**(1/(p
 - 1)))**p/(x**(1/(p - 1))*e**lambert_w(a/x**(1/(p - 1)))*lambert_w(a/x**(
1/(p - 1)))**2*p**2 - 2*x**(1/(p - 1))*e**lambert_w(a/x**(1/(p - 1)))*lamb
ert_w(a/x**(1/(p - 1)))**2*p + x**(1/(p - 1))*e**lambert_w(a/x**(1/(p - 1)
))*lambert_w(a/x**(1/(p - 1)))**2 + x**(1/(p - 1))*e**lambert_w(a/x**(1/(p
 - 1)))*lambert_w(a/x**(1/(p - 1)))*p**2 - 2*x**(1/(p - 1))*e**lambert_w(a
/x**(1/(p - 1)))*lambert_w(a/x**(1/(p - 1)))*p + x**(1/(p - 1))*e**lambert
_w(a/x**(1/(p - 1)))*lambert_w(a/x**(1/(p - 1)))),x)*a*p + 2*int(lambert_w
(a/x**(1/(p - 1)))**p/(x**(1/(p - 1))*e**lambert_w(a/x**(1/(p - 1)))*lambe
rt_w(a/x**(1/(p - 1)))**2*p - x**(1/(p - 1))*e**lambert_w(a/x**(1/(p - 1))
)*lambert_w(a/x**(1/(p - 1)))**2 + x**(1/(p - 1))*e**lambert_w(a/x**(1/(p
- 1)))*lambert_w(a/x**(1/(p - 1)))*p - x**(1/(p - 1))*e**lambert_w(a/x**(1
/(p - 1)))*lambert_w(a/x**(1/(p - 1)))),x)*a*p + int(lambert_w(a/x**(1/...
```

3.360 $\int \frac{x^m W(ax^n)^p}{d+dW(ax^n)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{x^m W(ax^n)^p}{d+dW(ax^n)}\,dx = \frac{e^{\left(1-\frac{1+m}{n}\right)W(ax^n)}x^{1+m-n}\Gamma\left(\frac{1+m}{n}+p,-\frac{(1+m)W(ax^n)}{n}\right)W(ax^n)^p\left(-\frac{(1+m)W(ax^n)}{n}\right)^{1-\frac{1+m}{n}-p}}{ad(1+m)}$$

output
```
exp((1-(1+m)/n)*LambertW(a*x^n))*x^(1+m-n)*GAMMA((1+m)/n+p,-(1+m)*LambertW
(a*x^n)/n)*LambertW(a*x^n)^p*(-(1+m)*LambertW(a*x^n)/n)^(1-(1+m)/n-p)/a/d/
(1+m)
```

Mathematica [F]

$$\int \frac{x^m W(ax^n)^p}{d+dW(ax^n)}\,dx = \int \frac{x^m W(ax^n)^p}{d+dW(ax^n)}\,dx$$

input
```
Integrate[(x^m*ProductLog[a*x^n]^p)/(d + d*ProductLog[a*x^n]),x]
```

output
```
Integrate[(x^m*ProductLog[a*x^n]^p)/(d + d*ProductLog[a*x^n]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m W(ax^n)^p}{dW(ax^n)+d}dx$$

$$\Big\downarrow \text{ 7292}$$

$$\int \frac{x^m W(ax^n)^p}{d(W(ax^n)+1)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{x^m W(ax^n)^p}{W(ax^n)+1}dx}{d}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\int \frac{x^m W(ax^n)^p}{W(ax^n)+1}dx}{d}$$

input
```
Int[(x^m*ProductLog[a*x^n]^p)/(d + d*ProductLog[a*x^n]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{x^m \operatorname{LambertW}(a x^n)^p}{d+d \operatorname{LambertW}(a x^n)}dx$$

input
```
int(x^m*LambertW(a*x^n)^p/(d+d*LambertW(a*x^n)),x)
```

output
```
int(x^m*LambertW(a*x^n)^p/(d+d*LambertW(a*x^n)),x)
```

Fricas [F]

$$\int \frac{x^m W(ax^n)^p}{d + dW(ax^n)} \, dx = \int \frac{x^m \mathrm{W}(ax^n)^p}{d \, \mathrm{W}(ax^n) + d} \, dx$$

input
```
integrate(x^m*lambert_w(a*x^n)^p/(d+d*lambert_w(a*x^n)),x, algorithm="fric
as")
```

output
```
integral(x^m*lambert_w(a*x^n)^p/(d*lambert_w(a*x^n) + d), x)
```

Sympy [F]

$$\int \frac{x^m W(ax^n)^p}{d + dW(ax^n)} \, dx = \frac{\int \frac{x^m W^p(ax^n)}{W(ax^n)+1} \, dx}{d}$$

input
```
integrate(x**m*LambertW(a*x**n)**p/(d+d*LambertW(a*x**n)),x)
```

output
```
Integral(x**m*LambertW(a*x**n)**p/(LambertW(a*x**n) + 1), x)/d
```

Maxima [F]

$$\int \frac{x^m W(ax^n)^p}{d + dW(ax^n)} \, dx = \int \frac{x^m \mathrm{W}(ax^n)^p}{d \, \mathrm{W}(ax^n) + d} \, dx$$

input
```
integrate(x^m*lambert_w(a*x^n)^p/(d+d*lambert_w(a*x^n)),x, algorithm="maxi
ma")
```

output
```
integrate(x^m*lambert_w(a*x^n)^p/(d*lambert_w(a*x^n) + d), x)
```

Giac [F]

$$\int \frac{x^m W(ax^n)^p}{d+dW(ax^n)}\,dx = \int \frac{x^m\,\mathrm{W}(ax^n)^p}{d\,\mathrm{W}(ax^n)+d}\,dx$$

input
```
integrate(x^m*lambert_w(a*x^n)^p/(d+d*lambert_w(a*x^n)),x, algorithm="giac
")
```

output
```
integrate(x^m*lambert_w(a*x^n)^p/(d*lambert_w(a*x^n) + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m W(ax^n)^p}{d+dW(ax^n)}\,dx = \int \frac{x^m\,\mathrm{LambertW}(a\,x^n)^p}{d+d\,\mathrm{LambertW}(a\,x^n)}\,dx$$

input
```
int((x^m*LambertW(a*x^n)^p)/(d + d*LambertW(a*x^n)),x)
```

output
```
int((x^m*LambertW(a*x^n)^p)/(d + d*LambertW(a*x^n)), x)
```

Reduce [F]

$$\int \frac{x^m W(ax^n)^p}{d+dW(ax^n)}\,dx = \text{Too large to display}$$

input
```
int(x^m*Lambert_W(a*x^n)^p/(d+d*Lambert_W(a*x^n)),x)
```

output

```
(x**m*lambert_w(x**n*a)**p*x + int((x**(m + n)*lambert_w(x**n*a)**p)/(e**l
ambert_w(x**n*a)*lambert_w(x**n*a)**2*m**2 + 2*e**lambert_w(x**n*a)*lamber
t_w(x**n*a)**2*m + e**lambert_w(x**n*a)*lambert_w(x**n*a)**2 + e**lambert_
w(x**n*a)*lambert_w(x**n*a)*m**2 + 2*e**lambert_w(x**n*a)*lambert_w(x**n*a
)*m + e**lambert_w(x**n*a)*lambert_w(x**n*a)),x)*a*m**2*n*p + 2*int((x**(m
 + n)*lambert_w(x**n*a)**p)/(e**lambert_w(x**n*a)*lambert_w(x**n*a)**2*m**
2 + 2*e**lambert_w(x**n*a)*lambert_w(x**n*a)**2*m + e**lambert_w(x**n*a)*l
ambert_w(x**n*a)**2 + e**lambert_w(x**n*a)*lambert_w(x**n*a)*m**2 + 2*e**l
ambert_w(x**n*a)*lambert_w(x**n*a)*m + e**lambert_w(x**n*a)*lambert_w(x**n
*a)),x)*a*m*n*p + int((x**(m + n)*lambert_w(x**n*a)**p)/(e**lambert_w(x**n
*a)*lambert_w(x**n*a)**2*m**2 + 2*e**lambert_w(x**n*a)*lambert_w(x**n*a)**
2*m + e**lambert_w(x**n*a)*lambert_w(x**n*a)**2 + e**lambert_w(x**n*a)*lam
bert_w(x**n*a)*m**2 + 2*e**lambert_w(x**n*a)*lambert_w(x**n*a)*m + e**lamb
ert_w(x**n*a)*lambert_w(x**n*a)),x)*a*n*p - 2*int((x**(m + n)*lambert_w(x*
*n*a)**p)/(e**lambert_w(x**n*a)*lambert_w(x**n*a)**2*m + e**lambert_w(x**n
*a)*lambert_w(x**n*a)**2 + e**lambert_w(x**n*a)*lambert_w(x**n*a)*m + e**l
ambert_w(x**n*a)*lambert_w(x**n*a)),x)*a*m*n*p - 2*int((x**(m + n)*lambert
_w(x**n*a)**p)/(e**lambert_w(x**n*a)*lambert_w(x**n*a)**2*m + e**lambert_w
(x**n*a)*lambert_w(x**n*a)**2 + e**lambert_w(x**n*a)*lambert_w(x**n*a)*m +
 e**lambert_w(x**n*a)*lambert_w(x**n*a)),x)*a*n*p - int((x**m*lambert_w...
```

3.361 $\int W(a+bx)^4 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 91

$$\int W(a+bx)^4 \, dx = 96x - \frac{96(a+bx)}{bW(a+bx)} - \frac{48(a+bx)W(a+bx)}{b} + \frac{16(a+bx)W(a+bx)^2}{b} - \frac{4(a+bx)W(a+bx)^3}{b} + \frac{(a+bx)W(a+bx)^4}{b}$$

output
```
96*x-96*(b*x+a)/b/LambertW(b*x+a)-48*(b*x+a)*LambertW(b*x+a)/b+16*(b*x+a)*
LambertW(b*x+a)^2/b-4*(b*x+a)*LambertW(b*x+a)^3/b+(b*x+a)*LambertW(b*x+a)^
4/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69

$$\int W(a+bx)^4 \, dx = \frac{(a+bx)(-2+W(a+bx))\left(48-24W(a+bx)+12W(a+bx)^2-2W(a+bx)^3+W(a+bx)^4\right)}{bW(a+bx)}$$

input
```
Integrate[ProductLog[a + b*x]^4,x]
```

output
```
((a + b*x)*(-2 + ProductLog[a + b*x])*(48 - 24*ProductLog[a + b*x] + 12*Pr
oductLog[a + b*x]^2 - 2*ProductLog[a + b*x]^3 + ProductLog[a + b*x]^4))/(b
*ProductLog[a + b*x])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {7167, 7178, 7178, 7178, 7177, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W(a+bx)^4\,dx$$

$$\Big\downarrow \text{7167}$$

$$\frac{(a+bx)W(a+bx)^4}{b}-4\int \frac{W(a+bx)^4}{W(a+bx)+1}dx$$

$$\Big\downarrow \text{7178}$$

$$\frac{(a+bx)W(a+bx)^4}{b}-4\left(\frac{(a+bx)W(a+bx)^3}{b}-4\int \frac{W(a+bx)^3}{W(a+bx)+1}dx\right)$$

$$\Big\downarrow \text{7178}$$

$$\frac{(a+bx)W(a+bx)^4}{b}-4\left(\frac{(a+bx)W(a+bx)^3}{b}-4\left(\frac{(a+bx)W(a+bx)^2}{b}-3\int \frac{W(a+bx)^2}{W(a+bx)+1}dx\right)\right)$$

$$\Big\downarrow \text{7178}$$

$$\frac{(a+bx)W(a+bx)^4}{b}-4\left(\frac{(a+bx)W(a+bx)^3}{b}-4\left(\frac{(a+bx)W(a+bx)^2}{b}-3\left(\frac{(a+bx)W(a+bx)}{b}-2\int \frac{W(a+bx)}{W(a+bx)+1}dx\right)\right)\right)$$

$$\Big\downarrow \text{7177}$$

$$\frac{(a+bx)W(a+bx)^4}{b}-4\left(\frac{(a+bx)W(a+bx)^3}{b}-4\left(\frac{(a+bx)W(a+bx)^2}{b}-3\left(\frac{(a+bx)W(a+bx)}{b}-2\left(x-\int \frac{1}{W(a+bx)+1}dx\right)\right)\right)\right)$$

$$\Big\downarrow \text{7176}$$

$$\frac{(a+bx)W(a+bx)^4}{b}-4\left(\frac{(a+bx)W(a+bx)^3}{b}-4\left(\frac{(a+bx)W(a+bx)^2}{b}-3\left(\frac{(a+bx)W(a+bx)}{b}-2\left(x-\frac{a+bx}{bW(a+bx)}\right)\right)\right)\right)$$

input
```
Int[ProductLog[a + b*x]^4,x]
```

output
```
((a + b*x)*ProductLog[a + b*x]^4)/b - 4*(((a + b*x)*ProductLog[a + b*x]^3)
/b - 4*(((a + b*x)*ProductLog[a + b*x]^2)/b - 3*(-2*(x - (a + b*x)/(b*Prod
uctLog[a + b*x])) + ((a + b*x)*ProductLog[a + b*x])/b)))
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7177
```
Int[ProductLog[(a_.) + (b_.)*(x_)]/((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(
x_)]), x_Symbol] :> Simp[d*x, x] - Int[1/(d + d*ProductLog[a + b*x]), x] /;
 FreeQ[{a, b, d}, x]
```

rule 7178
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[c*(a + b*x)*((c*ProductLog[a + b*x])^
(p - 1)/(b*d)), x] - Simp[c*p   Int[(c*ProductLog[a + b*x])^(p - 1)/(d + d*
ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-4\,\mathrm{LambertW}(bx+a)^3(bx+a)+16\,\mathrm{LambertW}(bx+a)^2(bx+a)-48(bx+a)\,\mathrm{LambertW}(bx+a)+96bx+96a-\frac{96(bx+a}{\mathrm{LambertW}(b}}{b}$
default	$\frac{-4\,\mathrm{LambertW}(bx+a)^3(bx+a)+16\,\mathrm{LambertW}(bx+a)^2(bx+a)-48(bx+a)\,\mathrm{LambertW}(bx+a)+96bx+96a-\frac{96(bx+a}{\mathrm{LambertW}(b}}{b}$
parallelrisch	$-\frac{-x\,\mathrm{LambertW}(bx+a)^5b+4x\,\mathrm{LambertW}(bx+a)^4b-\mathrm{LambertW}(bx+a)^5a-16x\,\mathrm{LambertW}(bx+a)^3b+4\,\mathrm{LambertW}(}{}$

input
```
int(LambertW(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-4*LambertW(b*x+a)^3*(b*x+a)+16*LambertW(b*x+a)^2*(b*x+a)-48*(b*x+a)*
LambertW(b*x+a)+96*b*x+96*a-96*(b*x+a)/LambertW(b*x+a)+LambertW(b*x+a)^4*(
b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.38

$$\int W(a+bx)^4\,dx$$
$$= \frac{(bx+a)\,\mathrm{W}(bx+a)^5-4\,(bx+a)\,\mathrm{W}(bx+a)^4-48\,bx\,\mathrm{W}(bx+a)^2+16\,(bx+a)\,\mathrm{W}(bx+a)^3+96\,bx\,\mathrm{W}(}{b\,\mathrm{W}(bx+a)}$$

input
```
integrate(lambert_w(b*x+a)^4,x, algorithm="fricas")
```

output
```
((b*x + a)*lambert_w(b*x + a)^5 - 4*(b*x + a)*lambert_w(b*x + a)^4 - 48*b*
x*lambert_w(b*x + a)^2 + 16*(b*x + a)*lambert_w(b*x + a)^3 + 96*b*x*lamber
t_w(b*x + a) - 48*a*lambert_w(b*x + a)*log(b*x + a) + 48*a*lambert_w(b*x +
 a)*log(lambert_w(b*x + a)) - 96*b*x - 96*a)/(b*lambert_w(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.44

$$\int W(a+bx)^4\,dx$$
$$= \begin{cases} 0 \\ xW^4(a) \\ 0 \\ \frac{aW^4(a+bx)}{b} - \frac{4aW^3(a+bx)}{b} + \frac{16aW^2(a+bx)}{b} - \frac{48aW(a+bx)}{b} - \frac{96a}{bW(a+bx)} + xW^4(a+bx) - 4xW^3(a+bx) + 16xW \end{cases}$$

input
```
integrate(LambertW(b*x+a)**4,x)
```

output
```
Piecewise((0, Eq(a, 0) & Eq(b, 0)), (x*LambertW(a)**4, Eq(b, 0)), (0, Eq(a
, -b*x)), (a*LambertW(a + b*x)**4/b - 4*a*LambertW(a + b*x)**3/b + 16*a*La
mbertW(a + b*x)**2/b - 48*a*LambertW(a + b*x)/b - 96*a/(b*LambertW(a + b*x
)) + x*LambertW(a + b*x)**4 - 4*x*LambertW(a + b*x)**3 + 16*x*LambertW(a +
 b*x)**2 - 48*x*LambertW(a + b*x) + 96*x - 96*x/LambertW(a + b*x), True))
```

Maxima [F]

$$\int W(a+bx)^4\,dx = \int \mathrm{W}(bx+a)^4\,dx$$

input
```
integrate(lambert_w(b*x+a)^4,x, algorithm="maxima")
```

output
```
integrate(lambert_w(b*x + a)^4, x)
```

Giac [F]

$$\int W(a+bx)^4\,dx=\int \mathrm{W}(bx+a)^4\,dx$$

input
```
integrate(lambert_w(b*x+a)^4,x, algorithm="giac")
```

output
```
integrate(lambert_w(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int W(a+bx)^4\,dx=\int \mathrm{LambertW}(a+b\,x)^4\,dx$$

input
```
int(LambertW(a + b*x)^4,x)
```

output
```
int(LambertW(a + b*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int W(a+bx)^4\,dx = \frac{e^{lambert\_w(bx+a)}\left(lambert\_w(bx+a)^5-4lambert\_w(bx+a)^4+16lambert\_w(bx+a)^3-48lambert\_w(b\right.}{b}$$

input
```
int(Lambert_W(b*x+a)^4,x)
```

output
```
(e**lambert_w(a + b*x)*(lambert_w(a + b*x)**5 - 4*lambert_w(a + b*x)**4 +
16*lambert_w(a + b*x)**3 - 48*lambert_w(a + b*x)**2 + 96*lambert_w(a + b*x
) - 96))/b
```

3.362 $\int W(a+bx)^3\,dx$

Optimal result

Integrand size = 8, antiderivative size = 73

$$\int W(a+bx)^3\,dx=-18x+\frac{18(a+bx)}{bW(a+bx)}+\frac{9(a+bx)W(a+bx)}{b}-\frac{3(a+bx)W(a+bx)^2}{b}+\frac{(a+bx)W(a+bx)^3}{b}$$

output
```
-18*x+18*(b*x+a)/b/LambertW(b*x+a)+9*(b*x+a)*LambertW(b*x+a)/b-3*(b*x+a)*L
ambertW(b*x+a)^2/b+(b*x+a)*LambertW(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int W(a+bx)^3\,dx=\frac{(a+bx)\left(6+W(a+bx)^2\right)\left(3-3W(a+bx)+W(a+bx)^2\right)}{bW(a+bx)}$$

input
```
Integrate[ProductLog[a + b*x]^3,x]
```

output
```
((a + b*x)*(6 + ProductLog[a + b*x]^2)*(3 - 3*ProductLog[a + b*x] + Produc
tLog[a + b*x]^2))/(b*ProductLog[a + b*x])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {7167, 7178, 7178, 7177, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W(a+bx)^3\,dx$$

$$\Big\downarrow \text{ 7167}$$

$$\frac{(a+bx)W(a+bx)^3}{b}-3\int \frac{W(a+bx)^3}{W(a+bx)+1}dx$$

$$\Big\downarrow \text{ 7178}$$

$$\frac{(a+bx)W(a+bx)^3}{b}-3\left(\frac{(a+bx)W(a+bx)^2}{b}-3\int \frac{W(a+bx)^2}{W(a+bx)+1}dx\right)$$

$$\Big\downarrow \text{ 7178}$$

$$\frac{(a+bx)W(a+bx)^3}{b}-3\left(\frac{(a+bx)W(a+bx)^2}{b}-3\left(\frac{(a+bx)W(a+bx)}{b}-2\int \frac{W(a+bx)}{W(a+bx)+1}dx\right)\right)$$

$$\Big\downarrow \text{ 7177}$$

$$\frac{(a+bx)W(a+bx)^3}{b}-3\left(\frac{(a+bx)W(a+bx)^2}{b}-3\left(\frac{(a+bx)W(a+bx)}{b}-2\left(x-\int \frac{1}{W(a+bx)+1}dx\right)\right)\right)$$

$$\Big\downarrow \text{ 7176}$$

$$\frac{(a+bx)W(a+bx)^3}{b}-3\left(\frac{(a+bx)W(a+bx)^2}{b}-3\left(\frac{(a+bx)W(a+bx)}{b}-2\left(x-\frac{a+bx}{bW(a+bx)}\right)\right)\right)$$

input
```
Int[ProductLog[a + b*x]^3,x]
```

output
```
((a + b*x)*ProductLog[a + b*x]^3)/b - 3*(((a + b*x)*ProductLog[a + b*x]^2)
/b - 3*(-2*(x - (a + b*x)/(b*ProductLog[a + b*x])) + ((a + b*x)*ProductLog
[a + b*x])/b))
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7177
```
Int[ProductLog[(a_.) + (b_.)*(x_)]/((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(
x_)]), x_Symbol] :> Simp[d*x, x] - Int[1/(d + d*ProductLog[a + b*x]), x] /;
 FreeQ[{a, b, d}, x]
```

rule 7178
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[c*(a + b*x)*((c*ProductLog[a + b*x])^
(p - 1)/(b*d)), x] - Simp[c*p   Int[(c*ProductLog[a + b*x])^(p - 1)/(d + d*
ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-3\,\mathrm{LambertW}(bx+a)^2(bx+a)+9(bx+a)\,\mathrm{LambertW}(bx+a)-18bx-18a+\frac{18bx+18a}{\mathrm{LambertW}(bx+a)}+\mathrm{LambertW}(bx+a)^3(bx+a}{b}$
default	$\frac{-3\,\mathrm{LambertW}(bx+a)^2(bx+a)+9(bx+a)\,\mathrm{LambertW}(bx+a)-18bx-18a+\frac{18bx+18a}{\mathrm{LambertW}(bx+a)}+\mathrm{LambertW}(bx+a)^3(bx+a}{b}$
parallelrisch	$-\frac{-x\,\mathrm{LambertW}(bx+a)^4b+3x\,\mathrm{LambertW}(bx+a)^3b-\mathrm{LambertW}(bx+a)^4a-9\,\mathrm{LambertW}(bx+a)^2xb+3\,\mathrm{LambertW}(b}{b\,\mathrm{LambertW}(bx+a)}$

input
```
int(LambertW(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-3*LambertW(b*x+a)^2*(b*x+a)+9*(b*x+a)*LambertW(b*x+a)-18*b*x-18*a+18
*(b*x+a)/LambertW(b*x+a)+LambertW(b*x+a)^3*(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.53

$$\int W(a+bx)^3\,dx$$
$$=\frac{(bx+a)\,\mathrm{W}(bx+a)^4+9\,bx\,\mathrm{W}(bx+a)^2-3\,(bx+a)\,\mathrm{W}(bx+a)^3-18\,bx\,\mathrm{W}(bx+a)+9\,a\,\mathrm{W}(bx+a)\log}{b\,\mathrm{W}(bx+a)}$$

input
```
integrate(lambert_w(b*x+a)^3,x, algorithm="fricas")
```

output
```
((b*x + a)*lambert_w(b*x + a)^4 + 9*b*x*lambert_w(b*x + a)^2 - 3*(b*x + a)
*lambert_w(b*x + a)^3 - 18*b*x*lambert_w(b*x + a) + 9*a*lambert_w(b*x + a)
*log(b*x + a) - 9*a*lambert_w(b*x + a)*log(-lambert_w(b*x + a)) + 18*b*x +
 18*a)/(b*lambert_w(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int W(a+bx)^3\,dx$$
$$=\begin{cases}0\\ xW^3(a)\\ 0\\ \frac{aW^3(a+bx)}{b}-\frac{3aW^2(a+bx)}{b}+\frac{9aW(a+bx)}{b}+\frac{18a}{bW(a+bx)}+xW^3(a+bx)-3xW^2(a+bx)+9xW(a+bx)-18x\end{cases}$$

input
```
integrate(LambertW(b*x+a)**3,x)
```

output
```
Piecewise((0, Eq(a, 0) & Eq(b, 0)), (x*LambertW(a)**3, Eq(b, 0)), (0, Eq(a
, -b*x)), (a*LambertW(a + b*x)**3/b - 3*a*LambertW(a + b*x)**2/b + 9*a*Lam
bertW(a + b*x)/b + 18*a/(b*LambertW(a + b*x)) + x*LambertW(a + b*x)**3 - 3
*x*LambertW(a + b*x)**2 + 9*x*LambertW(a + b*x) - 18*x + 18*x/LambertW(a +
 b*x), True))
```

Maxima [F]

$$\int W(a+bx)^3\,dx = \int \mathrm{W}(bx+a)^3\,dx$$

input
```
integrate(lambert_w(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(b*x + a)^3, x)
```

Giac [F]

$$\int W(a+bx)^3\,dx = \int \mathrm{W}(bx+a)^3\,dx$$

input
```
integrate(lambert_w(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int W(a+bx)^3\,dx=\int \mathrm{LambertW}(a+b\,x)^3\,dx$$

input
```
int(LambertW(a + b*x)^3,x)
```

output
```
int(LambertW(a + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int W(a+bx)^3\,dx$$
$$=\frac{e^{lambert\_w(bx+a)}\left(lambert\_w(bx+a)^4-3lambert\_w(bx+a)^3+9lambert\_w(bx+a)^2-18lambert\_w(bx}{b}$$

input
```
int(Lambert_W(b*x+a)^3,x)
```

output
```
(e**lambert_w(a + b*x)*(lambert_w(a + b*x)**4 - 3*lambert_w(a + b*x)**3 +
9*lambert_w(a + b*x)**2 - 18*lambert_w(a + b*x) + 18))/b
```

3.363 $\int W(a+bx)^2\,dx$

Optimal result

Integrand size = 8, antiderivative size = 55

$$\int W(a+bx)^2\,dx=4x-\frac{4(a+bx)}{bW(a+bx)}-\frac{2(a+bx)W(a+bx)}{b}+\frac{(a+bx)W(a+bx)^2}{b}$$

output
```
4*x-4*(b*x+a)/b/LambertW(b*x+a)-2*(b*x+a)*LambertW(b*x+a)/b+(b*x+a)*Lamber
tW(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int W(a+bx)^2\,dx=\frac{(a+bx)\left(-4+4W(a+bx)-2W(a+bx)^2+W(a+bx)^3\right)}{bW(a+bx)}$$

input
```
Integrate[ProductLog[a + b*x]^2,x]
```

output
```
((a + b*x)*(-4 + 4*ProductLog[a + b*x] - 2*ProductLog[a + b*x]^2 + Product
Log[a + b*x]^3))/(b*ProductLog[a + b*x])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7167, 7178, 7177, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W(a+bx)^2\,dx$$

$$\Big\downarrow \text{7167}$$

$$\frac{(a+bx)W(a+bx)^2}{b}-2\int \frac{W(a+bx)^2}{W(a+bx)+1}dx$$

$$\Big\downarrow \text{7178}$$

$$\frac{(a+bx)W(a+bx)^2}{b}-2\left(\frac{(a+bx)W(a+bx)}{b}-2\int \frac{W(a+bx)}{W(a+bx)+1}dx\right)$$

$$\Big\downarrow \text{7177}$$

$$\frac{(a+bx)W(a+bx)^2}{b}-2\left(\frac{(a+bx)W(a+bx)}{b}-2\left(x-\int \frac{1}{W(a+bx)+1}dx\right)\right)$$

$$\Big\downarrow \text{7176}$$

$$\frac{(a+bx)W(a+bx)^2}{b}-2\left(\frac{(a+bx)W(a+bx)}{b}-2\left(x-\frac{a+bx}{bW(a+bx)}\right)\right)$$

input
```
Int[ProductLog[a + b*x]^2,x]
```

output
```
((a + b*x)*ProductLog[a + b*x]^2)/b - 2*(-2*(x - (a + b*x)/(b*ProductLog[a
 + b*x])) + ((a + b*x)*ProductLog[a + b*x])/b)
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7177
```
Int[ProductLog[(a_.) + (b_.)*(x_)]/((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(
x_)]), x_Symbol] :> Simp[d*x, x] - Int[1/(d + d*ProductLog[a + b*x]), x] /;
 FreeQ[{a, b, d}, x]
```

rule 7178
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[c*(a + b*x)*((c*ProductLog[a + b*x])^
(p - 1)/(b*d)), x] - Simp[c*p   Int[(c*ProductLog[a + b*x])^(p - 1)/(d + d*
ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\text{LambertW}(bx+a)^2(bx+a)-2(bx+a)\,\text{LambertW}(bx+a)+4bx+4a-\frac{4(bx+a)}{\text{LambertW}(bx+a)}}{b}$
default	$\frac{\text{LambertW}(bx+a)^2(bx+a)-2(bx+a)\,\text{LambertW}(bx+a)+4bx+4a-\frac{4(bx+a)}{\text{LambertW}(bx+a)}}{b}$
parallelrisch	$-\frac{-x\,\text{LambertW}(bx+a)^3b+2\,\text{LambertW}(bx+a)^2xb-\text{LambertW}(bx+a)^3a-4x\,\text{LambertW}(bx+a)b+2a\,\text{LambertW}(b}{b\,\text{LambertW}(bx+a)}$

input
```
int(LambertW(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(LambertW(b*x+a)^2*(b*x+a)-2*(b*x+a)*LambertW(b*x+a)+4*b*x+4*a-4*(b*x+
a)/LambertW(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.75

$$\int W(a+bx)^2\,dx = -\frac{2\,bx\,\mathrm{W}(bx+a)^2-(bx+a)\,\mathrm{W}(bx+a)^3-4\,bx\,\mathrm{W}(bx+a)+2\,a\,\mathrm{W}(bx+a)\log(bx+a)-2\,a\,\mathrm{W}(bx+}{b\,\mathrm{W}(bx+a)}$$

input
```
integrate(lambert_w(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*b*x*lambert_w(b*x + a)^2 - (b*x + a)*lambert_w(b*x + a)^3 - 4*b*x*lamb
ert_w(b*x + a) + 2*a*lambert_w(b*x + a)*log(b*x + a) - 2*a*lambert_w(b*x +
 a)*log(lambert_w(b*x + a)) + 4*b*x + 4*a)/(b*lambert_w(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int W(a+bx)^2\,dx = \begin{cases} 0 & \text{for } a=0 \wedge b=0 \\ xW^2(a) & \text{for } b=0 \\ 0 & \text{for } a=-bx \\ \frac{aW^2(a+bx)}{b}-\frac{2aW(a+bx)}{b}-\frac{4a}{bW(a+bx)}+xW^2(a+bx)-2xW(a+bx)+4x-\frac{4x}{W(a+bx)} & \text{otherwise} \end{cases}$$

input
```
integrate(LambertW(b*x+a)**2,x)
```

output
```
Piecewise((0, Eq(a, 0) & Eq(b, 0)), (x*LambertW(a)**2, Eq(b, 0)), (0, Eq(a
, -b*x)), (a*LambertW(a + b*x)**2/b - 2*a*LambertW(a + b*x)/b - 4*a/(b*Lam
bertW(a + b*x)) + x*LambertW(a + b*x)**2 - 2*x*LambertW(a + b*x) + 4*x - 4
*x/LambertW(a + b*x), True))
```

Maxima [F]

$$\int W(a+bx)^2\,dx=\int \mathrm{W}(bx+a)^2\,dx$$

input
```
integrate(lambert_w(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(b*x + a)^2, x)
```

Giac [F]

$$\int W(a+bx)^2\,dx=\int \mathrm{W}(bx+a)^2\,dx$$

input
```
integrate(lambert_w(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int W(a+bx)^2\,dx=\int \mathrm{LambertW}(a+b\,x)^2\,dx$$

input
```
int(LambertW(a + b*x)^2,x)
```

output
```
int(LambertW(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int W(a+bx)^2\,dx$$
$$=\frac{e^{lambert\_w(bx+a)}\left(lambert\_w(bx+a)^3-2lambert\_w(bx+a)^2+4lambert\_w(bx+a)-4\right)}{b}$$

input
```
int(Lambert_W(b*x+a)^2,x)
```

output
```
(e**lambert_w(a + b*x)*(lambert_w(a + b*x)**3 - 2*lambert_w(a + b*x)**2 +
4*lambert_w(a + b*x) - 4))/b
```

3.364 $\int W(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 36

$$\int W(a+bx)\,dx=-x+\frac{a+bx}{bW(a+bx)}+\frac{(a+bx)W(a+bx)}{b}$$

output
```
-x+(b*x+a)/b/LambertW(b*x+a)+(b*x+a)*LambertW(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int W(a+bx)\,dx=\frac{(a+bx)\left(1-W(a+bx)+W(a+bx)^2\right)}{bW(a+bx)}$$

input
```
Integrate[ProductLog[a + b*x],x]
```

output
```
((a + b*x)*(1 - ProductLog[a + b*x] + ProductLog[a + b*x]^2))/(b*ProductLo
g[a + b*x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {7167, 7177, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W(a+bx)\,dx$$
$$\Big\downarrow \text{7167}$$
$$\frac{(a+bx)W(a+bx)}{b}-\int \frac{W(a+bx)}{W(a+bx)+1}dx$$
$$\Big\downarrow \text{7177}$$
$$\int \frac{1}{W(a+bx)+1}dx+\frac{(a+bx)W(a+bx)}{b}-x$$
$$\Big\downarrow \text{7176}$$
$$\frac{(a+bx)W(a+bx)}{b}+\frac{a+bx}{bW(a+bx)}-x$$

input
```
Int[ProductLog[a + b*x],x]
```

output
```
-x + (a + b*x)/(b*ProductLog[a + b*x]) + ((a + b*x)*ProductLog[a + b*x])/b
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7177
```
Int[ProductLog[(a_.) + (b_.)*(x_)]/((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(
x_)]), x_Symbol] :> Simp[d*x, x] - Int[1/(d + d*ProductLog[a + b*x]), x] /;
 FreeQ[{a, b, d}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{-bx-a+\frac{bx+a}{\mathrm{LambertW}(bx+a)}+(bx+a)\,\mathrm{LambertW}(bx+a)}{b}$	39
default	$\frac{-bx-a+\frac{bx+a}{\mathrm{LambertW}(bx+a)}+(bx+a)\,\mathrm{LambertW}(bx+a)}{b}$	39
parallelrisch	$-\frac{-\mathrm{LambertW}(bx+a)^{2}xb+x\,\mathrm{LambertW}(bx+a)b-a\,\mathrm{LambertW}(bx+a)^{2}-bx-a\,\mathrm{LambertW}(bx+a)-a}{b\,\mathrm{LambertW}(bx+a)}$	63

input
```
int(LambertW(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-b*x-a+(b*x+a)/LambertW(b*x+a)+(b*x+a)*LambertW(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int W(a+bx)\,dx = \frac{bx\,\mathrm{W}(bx+a)^2-bx\,\mathrm{W}(bx+a)+a\,\mathrm{W}(bx+a)\log(bx+a)-a\,\mathrm{W}(bx+a)\log(-\mathrm{W}(bx+a))+bx+a}{b\,\mathrm{W}(bx+a)}$$

input
```
integrate(lambert_w(b*x+a),x, algorithm="fricas")
```

output
```
(b*x*lambert_w(b*x + a)^2 - b*x*lambert_w(b*x + a) + a*lambert_w(b*x + a)*
log(b*x + a) - a*lambert_w(b*x + a)*log(-lambert_w(b*x + a)) + b*x + a)/(b
*lambert_w(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int W(a+bx)\,dx = \begin{cases} 0 & \text{for } a=0 \wedge b=0 \\ xW(a) & \text{for } b=0 \\ 0 & \text{for } a=-bx \\ \frac{aW(a+bx)}{b}+\frac{a}{bW(a+bx)}+xW(a+bx)-x+\frac{x}{W(a+bx)} & \text{otherwise} \end{cases}$$

input
```
integrate(LambertW(b*x+a),x)
```

output
```
Piecewise((0, Eq(a, 0) & Eq(b, 0)), (x*LambertW(a), Eq(b, 0)), (0, Eq(a, -
b*x)), (a*LambertW(a + b*x)/b + a/(b*LambertW(a + b*x)) + x*LambertW(a + b
*x) - x + x/LambertW(a + b*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int W(a+bx)\,dx = \frac{(bx+a)(\mathrm{W}(bx+a)^2-\mathrm{W}(bx+a)+1)}{b\,\mathrm{W}(bx+a)}$$

input
```
integrate(lambert_w(b*x+a),x, algorithm="maxima")
```

output
```
(b*x + a)*(lambert_w(b*x + a)^2 - lambert_w(b*x + a) + 1)/(b*lambert_w(b*x
 + a))
```

Giac [F]

$$\int W(a+bx)\,dx=\int \mathrm{W}(bx+a)\,dx$$

input
```
integrate(lambert_w(b*x+a),x, algorithm="giac")
```

output
```
integrate(lambert_w(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int W(a+bx)\,dx=\int \mathrm{LambertW}(a+b\,x)\;dx$$

input
```
int(LambertW(a + b*x),x)
```

output
```
int(LambertW(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int W(a+bx)\,dx=\frac{e^{lambert\_w(bx+a)}\left(lambert\_w(bx+a)^2-lambert\_w(bx+a)+1\right)}{b}$$

input
```
int(Lambert_W(b*x+a),x)
```

output
```
(e**lambert_w(a + b*x)*(lambert_w(a + b*x)**2 - lambert_w(a + b*x) + 1))/b
```

3.365 $\int \frac{1}{W(a+bx)} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 29

$$\int \frac{1}{W(a+bx)} \, dx = \frac{\text{ExpIntegralEi}(W(a+bx))}{b} + \frac{a+bx}{bW(a+bx)}$$

output
```
Ei(LambertW(b*x+a))/b+(b*x+a)/b/LambertW(b*x+a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{1}{W(a+bx)} \, dx = \frac{\text{ExpIntegralEi}(W(a+bx)) + \frac{a+bx}{W(a+bx)}}{b}$$

input
```
Integrate[ProductLog[a + b*x]^(-1),x]
```

output
```
(ExpIntegralEi[ProductLog[a + b*x]] + (a + b*x)/ProductLog[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7167, 7179}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W(a+bx)}dx$$
$$\Big\downarrow \text{ 7167}$$
$$\int \frac{1}{W(a+bx)(W(a+bx)+1)}dx+\frac{a+bx}{bW(a+bx)}$$
$$\Big\downarrow \text{ 7179}$$
$$\frac{\text{ExpIntegralEi}(W(a+bx))}{b}+\frac{a+bx}{bW(a+bx)}$$

input
```
Int[ProductLog[a + b*x]^(-1),x]
```

output
```
ExpIntegralEi[ProductLog[a + b*x]]/b + (a + b*x)/(b*ProductLog[a + b*x])
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7179
```
Int[1/(ProductLog[(a_.) + (b_.)*(x_)]*((d_) + (d_.)*ProductLog[(a_.) + (b_.
)*(x_)])), x_Symbol] :> Simp[ExpIntegralEi[ProductLog[a + b*x]]/(b*d), x] /
; FreeQ[{a, b, d}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{-\operatorname{expIntegral}_1(-\operatorname{LambertW}(bx+a))+\frac{bx+a}{\operatorname{LambertW}(bx+a)}}{b}$	32
default	$\frac{-\operatorname{expIntegral}_1(-\operatorname{LambertW}(bx+a))+\frac{bx+a}{\operatorname{LambertW}(bx+a)}}{b}$	32

input
```
int(1/LambertW(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-Ei(1,-LambertW(b*x+a))+(b*x+a)/LambertW(b*x+a))
```

Fricas [F]

$$\int \frac{1}{W(a+bx)}\,dx = \int \frac{1}{\mathrm{W}(bx+a)}\,dx$$

input
```
integrate(1/lambert_w(b*x+a),x, algorithm="fricas")
```

output
```
integral(1/lambert_w(b*x + a), x)
```

Sympy [F]

$$\int \frac{1}{W(a+bx)}\,dx = \int \frac{1}{W(a+bx)}\,dx$$

input
```
integrate(1/LambertW(b*x+a),x)
```

output
```
Integral(1/LambertW(a + b*x), x)
```

Maxima [F]

$$\int \frac{1}{W(a+bx)}\,dx = \int \frac{1}{\mathrm{W}(bx+a)}\,dx$$

input
```
integrate(1/lambert_w(b*x+a),x, algorithm="maxima")
```

output
```
integrate(1/lambert_w(b*x + a), x)
```

Giac [F]

$$\int \frac{1}{W(a+bx)}\,dx = \int \frac{1}{\mathrm{W}(bx+a)}\,dx$$

input
```
integrate(1/lambert_w(b*x+a),x, algorithm="giac")
```

output
```
integrate(1/lambert_w(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W(a+bx)}\,dx = \int \frac{1}{\mathrm{LambertW}\,(a+b\,x)}\,dx$$

input
```
int(1/LambertW(a + b*x),x)
```

output
```
int(1/LambertW(a + b*x), x)
```

Reduce [F]

$$\int \frac{1}{W(a+bx)}\,dx = \frac{e^{lambert\_w(bx+a)} + \left(\int \frac{1}{lambert\_w(bx+a)^2 + lambert\_w(bx+a)}dx\right)b}{b}$$

input

```
int(1/Lambert_W(b*x+a),x)
```

output

```
(e**lambert_w(a + b*x) + int(1/(lambert_w(a + b*x)**2 + lambert_w(a + b*x)
),x)*b)/b
```

3.366 $\int \frac{1}{W(a+bx)^2} dx$

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int \frac{1}{W(a+bx)^2} dx = \frac{2\,\text{ExpIntegralEi}(W(a+bx))}{b} - \frac{a+bx}{bW(a+bx)^2}$$

output
```
2*Ei(LambertW(b*x+a))/b-(b*x+a)/b/LambertW(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{1}{W(a+bx)^2} dx = -\frac{a+bx-2\,\text{ExpIntegralEi}(W(a+bx))W(a+bx)^2}{bW(a+bx)^2}$$

input
```
Integrate[ProductLog[a + b*x]^(-2),x]
```

output
```
-((a + b*x - 2*ExpIntegralEi[ProductLog[a + b*x]]*ProductLog[a + b*x]^2)/(
b*ProductLog[a + b*x]^2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7166, 7179}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W(a+bx)^2} \, dx$$
$$\Big\downarrow \text{7166}$$
$$2 \int \frac{1}{W(a+bx)(W(a+bx)+1)}dx - \frac{a+bx}{bW(a+bx)^2}$$
$$\Big\downarrow \text{7179}$$
$$\frac{2\,\text{ExpIntegralEi}(W(a+bx))}{b} - \frac{a+bx}{bW(a+bx)^2}$$

input
```
Int[ProductLog[a + b*x]^(-2),x]
```

output
```
(2*ExpIntegralEi[ProductLog[a + b*x]])/b - (a + b*x)/(b*ProductLog[a + b*x
]^2)
```

Defintions of rubi rules used

rule 7166
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_), x_Symbol] :> Simp[(a + b*x
)*((c*ProductLog[a + b*x])^p/(b*(p + 1))), x] + Simp[p/(c*(p + 1))   Int[(c
*ProductLog[a + b*x])^(p + 1)/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a
, b, c}, x] && LtQ[p, -1]
```

rule 7179
```
Int[1/(ProductLog[(a_.) + (b_.)*(x_)]*((d_) + (d_.)*ProductLog[(a_.) + (b_.
)*(x_)])), x_Symbol] :> Simp[ExpIntegralEi[ProductLog[a + b*x]]/(b*d), x] /
; FreeQ[{a, b, d}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{-\frac{bx+a}{\text{LambertW}(bx+a)^2}-2\,\text{expIntegral}_1(-\text{LambertW}(bx+a))}{b}$	33
default	$\frac{-\frac{bx+a}{\text{LambertW}(bx+a)^2}-2\,\text{expIntegral}_1(-\text{LambertW}(bx+a))}{b}$	33

input
```
int(1/LambertW(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/LambertW(b*x+a)^2*(b*x+a)-2*Ei(1,-LambertW(b*x+a)))
```

Fricas [F]

$$\int \frac{1}{W(a+bx)^2}\,dx = \int \frac{1}{\mathrm{W}(bx+a)^2}\,dx$$

input
```
integrate(1/lambert_w(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(lambert_w(b*x + a)^(-2), x)
```

Sympy [F]

$$\int \frac{1}{W(a+bx)^2}\,dx = \int \frac{1}{W^2\,(a+bx)}\,dx$$

input
```
integrate(1/LambertW(b*x+a)**2,x)
```

output
```
Integral(LambertW(a + b*x)**(-2), x)
```

Maxima [F]

$$\int \frac{1}{W(a+bx)^2}\,dx = \int \frac{1}{\mathrm{W}(bx+a)^2}\,dx$$

input
```
integrate(1/lambert_w(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(b*x + a)^(-2), x)
```

Giac [F]

$$\int \frac{1}{W(a+bx)^2}\,dx = \int \frac{1}{\mathrm{W}(bx+a)^2}\,dx$$

input
```
integrate(1/lambert_w(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(b*x + a)^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W(a+bx)^2}\,dx = \int \frac{1}{\mathrm{LambertW}\left(a+b\,x\right)^2}\,dx$$

input
```
int(1/LambertW(a + b*x)^2,x)
```

output
```
int(1/LambertW(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{1}{W(a+bx)^2} \, dx = \int \frac{1}{lambert\_w\,(bx+a)^2} dx$$

input
```
int(1/Lambert_W(b*x+a)^2,x)
```

output
```
int(1/lambert_w(a + b*x)**2,x)
```

3.367 $\int \frac{1}{W(a+bx)^3} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 55

$$\int \frac{1}{W(a+bx)^3} \, dx = \frac{3\,\text{ExpIntegralEi}(W(a+bx))}{2b} - \frac{a+bx}{2bW(a+bx)^3} - \frac{3(a+bx)}{2bW(a+bx)^2}$$

output
```
3/2*Ei(LambertW(b*x+a))/b-1/2*(b*x+a)/b/LambertW(b*x+a)^3-3/2*(b*x+a)/b/La
mbertW(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{1}{W(a+bx)^3} \, dx = -\frac{a+bx+3(a+bx)W(a+bx) - 3\,\text{ExpIntegralEi}(W(a+bx))W(a+bx)^3}{2bW(a+bx)^3}$$

input
```
Integrate[ProductLog[a + b*x]^(-3),x]
```

output
```
-1/2*(a + b*x + 3*(a + b*x)*ProductLog[a + b*x] - 3*ExpIntegralEi[ProductL
og[a + b*x]]*ProductLog[a + b*x]^3)/(b*ProductLog[a + b*x]^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {7166, 7182, 7179}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W(a+bx)^3}dx$$
$$\Big\downarrow \text{ 7166}$$
$$\frac{3}{2}\int \frac{1}{W(a+bx)^2(W(a+bx)+1)}dx-\frac{a+bx}{2bW(a+bx)^3}$$
$$\Big\downarrow \text{ 7182}$$
$$\frac{3}{2}\left(\int \frac{1}{W(a+bx)(W(a+bx)+1)}dx-\frac{a+bx}{bW(a+bx)^2}\right)-\frac{a+bx}{2bW(a+bx)^3}$$
$$\Big\downarrow \text{ 7179}$$
$$\frac{3}{2}\left(\frac{\text{ExpIntegralEi}(W(a+bx))}{b}-\frac{a+bx}{bW(a+bx)^2}\right)-\frac{a+bx}{2bW(a+bx)^3}$$

input
```
Int[ProductLog[a + b*x]^(-3),x]
```

output
```
(3*(ExpIntegralEi[ProductLog[a + b*x]]/b - (a + b*x)/(b*ProductLog[a + b*x
]^2)))/2 - (a + b*x)/(2*b*ProductLog[a + b*x]^3)
```

Defintions of rubi rules used

rule 7166
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_), x_Symbol] :> Simp[(a + b*x
)*((c*ProductLog[a + b*x])^p/(b*(p + 1))), x] + Simp[p/(c*(p + 1))   Int[(c
*ProductLog[a + b*x])^(p + 1)/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a
, b, c}, x] && LtQ[p, -1]
```

rule 7179
```
Int[1/(ProductLog[(a_.) + (b_.)*(x_)]*((d_) + (d_.)*ProductLog[(a_.) + (b_.
)*(x_)])), x_Symbol] :> Simp[ExpIntegralEi[ProductLog[a + b*x]]/(b*d), x] /
; FreeQ[{a, b, d}, x]
```

rule 7182
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[(a + b*x)*((c*ProductLog[a + b*x])^p/
(b*d*(p + 1))), x] - Simp[1/(c*(p + 1))   Int[(c*ProductLog[a + b*x])^(p +
1)/(d + d*ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p,
-1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{-\frac{3(bx+a)}{2\,\mathrm{LambertW}(bx+a)^2}-\frac{3\,\mathrm{expIntegral}_1(-\mathrm{LambertW}(bx+a))}{2}-\frac{bx+a}{2\,\mathrm{LambertW}(bx+a)^3}}{b}$	48
default	$\frac{-\frac{3(bx+a)}{2\,\mathrm{LambertW}(bx+a)^2}-\frac{3\,\mathrm{expIntegral}_1(-\mathrm{LambertW}(bx+a))}{2}-\frac{bx+a}{2\,\mathrm{LambertW}(bx+a)^3}}{b}$	48

input
```
int(1/LambertW(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-3/2/LambertW(b*x+a)^2*(b*x+a)-3/2*Ei(1,-LambertW(b*x+a))-1/2/Lambert
W(b*x+a)^3*(b*x+a))
```

Fricas [F]

$$\int \frac{1}{W(a+bx)^3}\,dx=\int \frac{1}{\mathrm{W}(bx+a)^3}\,dx$$

input
```
integrate(1/lambert_w(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(lambert_w(b*x + a)^(-3), x)
```

Sympy [F]

$$\int \frac{1}{W(a+bx)^3}\,dx = \int \frac{1}{W^3(a+bx)}\,dx$$

input
```
integrate(1/LambertW(b*x+a)**3,x)
```

output
```
Integral(LambertW(a + b*x)**(-3), x)
```

Maxima [F]

$$\int \frac{1}{W(a+bx)^3}\,dx = \int \frac{1}{\mathrm{W}(bx+a)^3}\,dx$$

input
```
integrate(1/lambert_w(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(lambert_w(b*x + a)^(-3), x)
```

Giac [F]

$$\int \frac{1}{W(a+bx)^3}\,dx = \int \frac{1}{\mathrm{W}(bx+a)^3}\,dx$$

input
```
integrate(1/lambert_w(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(lambert_w(b*x + a)^(-3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W(a+bx)^3}\,dx = \int \frac{1}{\mathrm{LambertW}\left(a+b\,x\right)^3}\,dx$$

input
```
int(1/LambertW(a + b*x)^3,x)
```

output
```
int(1/LambertW(a + b*x)^3, x)
```

Reduce [F]

$$\int \frac{1}{W(a+bx)^3}\,dx = \int \frac{1}{lambert\_w\left(bx+a\right)^3}dx$$

input
```
int(1/Lambert_W(b*x+a)^3,x)
```

output
```
int(1/lambert_w(a + b*x)**3,x)
```

3.368 $\int \frac{1}{W(a+bx)^4} dx$

Optimal result

Integrand size = 8, antiderivative size = 75

$$\int \frac{1}{W(a+bx)^4} dx = \frac{2\,\text{ExpIntegralEi}(W(a+bx))}{3b} - \frac{a+bx}{3bW(a+bx)^4} - \frac{2(a+bx)}{3bW(a+bx)^3} - \frac{2(a+bx)}{3bW(a+bx)^2}$$

output
```
2/3*Ei(LambertW(b*x+a))/b-1/3*(b*x+a)/b/LambertW(b*x+a)^4-2/3*(b*x+a)/b/La
mbertW(b*x+a)^3-2/3*(b*x+a)/b/LambertW(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \frac{1}{W(a+bx)^4} dx = -\frac{2(a+bx) + \frac{a+bx}{W(a+bx)} + 2(a+bx)W(a+bx) - 2\,\text{ExpIntegralEi}(W(a+bx))W(a+bx)^3}{3bW(a+bx)^3}$$

input
```
Integrate[ProductLog[a + b*x]^(-4),x]
```

output
```
-1/3*(2*(a + b*x) + (a + b*x)/ProductLog[a + b*x] + 2*(a + b*x)*ProductLog
[a + b*x] - 2*ExpIntegralEi[ProductLog[a + b*x]]*ProductLog[a + b*x]^3)/(b
*ProductLog[a + b*x]^3)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {7166, 7182, 7182, 7179}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W(a+bx)^4}\,dx$$

$$\Big\downarrow \text{7166}$$

$$\frac{4}{3}\int \frac{1}{W(a+bx)^3(W(a+bx)+1)}dx-\frac{a+bx}{3bW(a+bx)^4}$$

$$\Big\downarrow \text{7182}$$

$$\frac{4}{3}\left(\frac{1}{2}\int \frac{1}{W(a+bx)^2(W(a+bx)+1)}dx-\frac{a+bx}{2bW(a+bx)^3}\right)-\frac{a+bx}{3bW(a+bx)^4}$$

$$\Big\downarrow \text{7182}$$

$$\frac{4}{3}\left(\frac{1}{2}\left(\int \frac{1}{W(a+bx)(W(a+bx)+1)}dx-\frac{a+bx}{bW(a+bx)^2}\right)-\frac{a+bx}{2bW(a+bx)^3}\right)-\frac{a+bx}{3bW(a+bx)^4}$$

$$\Big\downarrow \text{7179}$$

$$\frac{4}{3}\left(\frac{1}{2}\left(\frac{\text{ExpIntegralEi}(W(a+bx))}{b}-\frac{a+bx}{bW(a+bx)^2}\right)-\frac{a+bx}{2bW(a+bx)^3}\right)-\frac{a+bx}{3bW(a+bx)^4}$$

input
```
Int[ProductLog[a + b*x]^(-4),x]
```

output
```
(4*((ExpIntegralEi[ProductLog[a + b*x]]/b - (a + b*x)/(b*ProductLog[a + b*
x]^2))/2 - (a + b*x)/(2*b*ProductLog[a + b*x]^3)))/3 - (a + b*x)/(3*b*Prod
uctLog[a + b*x]^4)
```

Defintions of rubi rules used

rule 7166
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_), x_Symbol] :> Simp[(a + b*x
)*((c*ProductLog[a + b*x])^p/(b*(p + 1))), x] + Simp[p/(c*(p + 1))   Int[(c
*ProductLog[a + b*x])^(p + 1)/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a
, b, c}, x] && LtQ[p, -1]
```

rule 7179
```
Int[1/(ProductLog[(a_.) + (b_.)*(x_)]*((d_) + (d_.)*ProductLog[(a_.) + (b_.
)*(x_)])), x_Symbol] :> Simp[ExpIntegralEi[ProductLog[a + b*x]]/(b*d), x] /
; FreeQ[{a, b, d}, x]
```

rule 7182
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[(a + b*x)*((c*ProductLog[a + b*x])^p/
(b*d*(p + 1))), x] - Simp[1/(c*(p + 1))   Int[(c*ProductLog[a + b*x])^(p +
1)/(d + d*ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p,
-1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{-\frac{bx+a}{3\,\mathrm{LambertW}(bx+a)^4}-\frac{2(bx+a)}{3\,\mathrm{LambertW}(bx+a)^3}-\frac{2(bx+a)}{3\,\mathrm{LambertW}(bx+a)^2}-\frac{2\,\mathrm{expIntegral}_1(-\,\mathrm{LambertW}(bx+a))}{3}}{b}$	63
default	$\frac{-\frac{bx+a}{3\,\mathrm{LambertW}(bx+a)^4}-\frac{2(bx+a)}{3\,\mathrm{LambertW}(bx+a)^3}-\frac{2(bx+a)}{3\,\mathrm{LambertW}(bx+a)^2}-\frac{2\,\mathrm{expIntegral}_1(-\,\mathrm{LambertW}(bx+a))}{3}}{b}$	63

input
```
int(1/LambertW(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3*(b*x+a)/LambertW(b*x+a)^4-2/3/LambertW(b*x+a)^3*(b*x+a)-2/3/Lamb
ertW(b*x+a)^2*(b*x+a)-2/3*Ei(1,-LambertW(b*x+a)))
```

Fricas [F]

$$\int \frac{1}{W(a+bx)^4}\,dx = \int \frac{1}{\mathrm{W}(bx+a)^4}\,dx$$

input
```
integrate(1/lambert_w(b*x+a)^4,x, algorithm="fricas")
```

output
```
integral(lambert_w(b*x + a)^(-4), x)
```

Sympy [F]

$$\int \frac{1}{W(a+bx)^4}\,dx = \int \frac{1}{W^4(a+bx)}\,dx$$

input
```
integrate(1/LambertW(b*x+a)**4,x)
```

output
```
Integral(LambertW(a + b*x)**(-4), x)
```

Maxima [F]

$$\int \frac{1}{W(a+bx)^4}\,dx = \int \frac{1}{\mathrm{W}(bx+a)^4}\,dx$$

input
```
integrate(1/lambert_w(b*x+a)^4,x, algorithm="maxima")
```

output
```
integrate(lambert_w(b*x + a)^(-4), x)
```

Giac [F]

$$\int \frac{1}{W(a+bx)^4}\,dx = \int \frac{1}{\mathrm{W}(bx+a)^4}\,dx$$

input
```
integrate(1/lambert_w(b*x+a)^4,x, algorithm="giac")
```

output
```
integrate(lambert_w(b*x + a)^(-4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W(a+bx)^4}\,dx = \int \frac{1}{\mathrm{LambertW}\,(a+b\,x)^4}\,dx$$

input
```
int(1/LambertW(a + b*x)^4,x)
```

output
```
int(1/LambertW(a + b*x)^4, x)
```

Reduce [F]

$$\int \frac{1}{W(a+bx)^4}\,dx = \int \frac{1}{lambert\_w\,(bx+a)^4}dx$$

input
```
int(1/Lambert_W(b*x+a)^4,x)
```

output
```
int(1/lambert_w(a + b*x)**4,x)
```

3.369 $\int \frac{1}{W(a+bx)^5} dx$

Optimal result

Integrand size = 8, antiderivative size = 95

$$\int \frac{1}{W(a+bx)^5} dx = \frac{5\,\text{ExpIntegralEi}(W(a+bx))}{24b} - \frac{a+bx}{4bW(a+bx)^5} - \frac{5(a+bx)}{12bW(a+bx)^4} - \frac{5(a+bx)}{24bW(a+bx)^3} - \frac{5(a+bx)}{24bW(a+bx)^2}$$

output
```
5/24*Ei(LambertW(b*x+a))/b-1/4*(b*x+a)/b/LambertW(b*x+a)^5-5/12*(b*x+a)/b/
LambertW(b*x+a)^4-5/24*(b*x+a)/b/LambertW(b*x+a)^3-5/24*(b*x+a)/b/LambertW
(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \frac{1}{W(a+bx)^5} dx = -\frac{10(a+bx) + \frac{6(a+bx)}{W(a+bx)} + 5(a+bx)W(a+bx) + 5(a+bx)W(a+bx)^2 - 5\,\text{ExpIntegralEi}(W(a+bx))W}{24bW(a+bx)^4}$$

input
```
Integrate[ProductLog[a + b*x]^(-5),x]
```

output
```
-1/24*(10*(a + b*x) + (6*(a + b*x))/ProductLog[a + b*x] + 5*(a + b*x)*Prod
uctLog[a + b*x] + 5*(a + b*x)*ProductLog[a + b*x]^2 - 5*ExpIntegralEi[Prod
uctLog[a + b*x]]*ProductLog[a + b*x]^4)/(b*ProductLog[a + b*x]^4)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {7166, 7182, 7182, 7182, 7179}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{W(a+bx)^5}\,dx$$

$$\Big\downarrow \text{7166}$$

$$\frac{5}{4}\int \frac{1}{W(a+bx)^4(W(a+bx)+1)}dx-\frac{a+bx}{4bW(a+bx)^5}$$

$$\Big\downarrow \text{7182}$$

$$\frac{5}{4}\left(\frac{1}{3}\int \frac{1}{W(a+bx)^3(W(a+bx)+1)}dx-\frac{a+bx}{3bW(a+bx)^4}\right)-\frac{a+bx}{4bW(a+bx)^5}$$

$$\Big\downarrow \text{7182}$$

$$\frac{5}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{W(a+bx)^2(W(a+bx)+1)}dx-\frac{a+bx}{2bW(a+bx)^3}\right)-\frac{a+bx}{3bW(a+bx)^4}\right)-\frac{a+bx}{4bW(a+bx)^5}$$

$$\Big\downarrow \text{7182}$$

$$\frac{5}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\int \frac{1}{W(a+bx)(W(a+bx)+1)}dx-\frac{a+bx}{bW(a+bx)^2}\right)-\frac{a+bx}{2bW(a+bx)^3}\right)-\frac{a+bx}{3bW(a+bx)^4}\right)-\frac{a+bx}{4bW(a+bx)^5}$$

$$\Big\downarrow \text{7179}$$

$$\frac{5}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{\text{ExpIntegralEi}(W(a+bx))}{b}-\frac{a+bx}{bW(a+bx)^2}\right)-\frac{a+bx}{2bW(a+bx)^3}\right)-\frac{a+bx}{3bW(a+bx)^4}\right)-\frac{a+bx}{4bW(a+bx)^5}$$

input
```
Int[ProductLog[a + b*x]^(-5),x]
```

output
```
(5*(((ExpIntegralEi[ProductLog[a + b*x]]/b - (a + b*x)/(b*ProductLog[a + b
*x]^2))/2 - (a + b*x)/(2*b*ProductLog[a + b*x]^3))/3 - (a + b*x)/(3*b*Prod
uctLog[a + b*x]^4)))/4 - (a + b*x)/(4*b*ProductLog[a + b*x]^5)
```

Defintions of rubi rules used

rule 7166
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_), x_Symbol] :> Simp[(a + b*x
)*((c*ProductLog[a + b*x])^p/(b*(p + 1))), x] + Simp[p/(c*(p + 1))   Int[(c
*ProductLog[a + b*x])^(p + 1)/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a
, b, c}, x] && LtQ[p, -1]
```

rule 7179
```
Int[1/(ProductLog[(a_.) + (b_.)*(x_)]*((d_) + (d_.)*ProductLog[(a_.) + (b_.
)*(x_)])), x_Symbol] :> Simp[ExpIntegralEi[ProductLog[a + b*x]]/(b*d), x] /
; FreeQ[{a, b, d}, x]
```

rule 7182
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[(a + b*x)*((c*ProductLog[a + b*x])^p/
(b*d*(p + 1))), x] - Simp[1/(c*(p + 1))   Int[(c*ProductLog[a + b*x])^(p +
1)/(d + d*ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p,
-1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-\frac{bx+a}{4\,\mathrm{LambertW}(bx+a)^5}-\frac{5(bx+a)}{12\,\mathrm{LambertW}(bx+a)^4}-\frac{5(bx+a)}{24\,\mathrm{LambertW}(bx+a)^3}-\frac{5(bx+a)}{24\,\mathrm{LambertW}(bx+a)^2}-\frac{5\,\mathrm{expIntegral}_1(-\mathrm{LambertW}(bx}{24}}{b}$
default	$\frac{-\frac{bx+a}{4\,\mathrm{LambertW}(bx+a)^5}-\frac{5(bx+a)}{12\,\mathrm{LambertW}(bx+a)^4}-\frac{5(bx+a)}{24\,\mathrm{LambertW}(bx+a)^3}-\frac{5(bx+a)}{24\,\mathrm{LambertW}(bx+a)^2}-\frac{5\,\mathrm{expIntegral}_1(-\mathrm{LambertW}(bx}{24}}{b}$

input
```
int(1/LambertW(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/LambertW(b*x+a)^5*(b*x+a)-5/12*(b*x+a)/LambertW(b*x+a)^4-5/24/La
mbertW(b*x+a)^3*(b*x+a)-5/24/LambertW(b*x+a)^2*(b*x+a)-5/24*Ei(1,-LambertW
(b*x+a)))
```

Fricas [F]

$$\int \frac{1}{W(a+bx)^5}\,dx=\int \frac{1}{\mathrm{W}(bx+a)^5}\,dx$$

input
```
integrate(1/lambert_w(b*x+a)^5,x, algorithm="fricas")
```

output
```
integral(lambert_w(b*x + a)^(-5), x)
```

Sympy [F]

$$\int \frac{1}{W(a+bx)^5}\,dx=\int \frac{1}{W^5\,(a+bx)}\,dx$$

input
```
integrate(1/LambertW(b*x+a)**5,x)
```

output
```
Integral(LambertW(a + b*x)**(-5), x)
```

Maxima [F]

$$\int \frac{1}{W(a+bx)^5}\,dx = \int \frac{1}{\mathrm{W}(bx+a)^5}\,dx$$

input
```
integrate(1/lambert_w(b*x+a)^5,x, algorithm="maxima")
```

output
```
integrate(lambert_w(b*x + a)^(-5), x)
```

Giac [F]

$$\int \frac{1}{W(a+bx)^5}\,dx = \int \frac{1}{\mathrm{W}(bx+a)^5}\,dx$$

input
```
integrate(1/lambert_w(b*x+a)^5,x, algorithm="giac")
```

output
```
integrate(lambert_w(b*x + a)^(-5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{W(a+bx)^5}\,dx = \int \frac{1}{\mathrm{LambertW}\left(a+b\,x\right)^5}\,dx$$

input
```
int(1/LambertW(a + b*x)^5,x)
```

output
```
int(1/LambertW(a + b*x)^5, x)
```

Reduce [F]

$$\int \frac{1}{W(a+bx)^5}\,dx=\int \frac{1}{lambert\_w\,(bx+a)^5}dx$$

input
```
int(1/Lambert_W(b*x+a)^5,x)
```

output
```
int(1/lambert_w(a + b*x)**5,x)
```

3.370 $\int (cW(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 137

$$\int (cW(a+bx))^{5/2}\,dx = \frac{75c^{5/2}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{16b} - \frac{75c^3(a+bx)}{8b\sqrt{cW(a+bx)}} + \frac{25c^2(a+bx)\sqrt{cW(a+bx)}}{4b} - \frac{5c(a+bx)(cW(a+bx))^{3/2}}{2b} + \frac{(a+bx)(cW(a+bx))^{5/2}}{b}$$

output
```
75/16*c^(5/2)*Pi^(1/2)*erfi((c*LambertW(b*x+a))^(1/2)/c^(1/2))/b-75/8*c^3*
(b*x+a)/b/(c*LambertW(b*x+a))^(1/2)+25/4*c^2*(b*x+a)*(c*LambertW(b*x+a))^(
1/2)/b-5/2*c*(b*x+a)*(c*LambertW(b*x+a))^(3/2)/b+(b*x+a)*(c*LambertW(b*x+a
))^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.74

$$\int (cW(a+bx))^{5/2}\,dx=\frac{c^3\left(-150(a+bx)+75\sqrt{\pi}\text{erfi}\left(\sqrt{W(a+bx)}\right)\sqrt{W(a+bx)}+100(a+bx)W(a+bx)-40(a\right.}{16b\sqrt{cW(a+bx)}}$$

input
```
Integrate[(c*ProductLog[a + b*x])^(5/2),x]
```

output
```
(c^3*(-150*(a + b*x) + 75*Sqrt[Pi]*Erfi[Sqrt[ProductLog[a + b*x]]]*Sqrt[Pr
oductLog[a + b*x]] + 100*(a + b*x)*ProductLog[a + b*x] - 40*(a + b*x)*Prod
uctLog[a + b*x]^2 + 16*(a + b*x)*ProductLog[a + b*x]^3))/(16*b*Sqrt[c*Prod
uctLog[a + b*x]])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {7167, 7178, 7178, 7178, 7180}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cW(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{7167}$$

$$\frac{(a+bx)(cW(a+bx))^{5/2}}{b}-\frac{5}{2}\int \frac{(cW(a+bx))^{5/2}}{W(a+bx)+1}dx$$

$$\Big\downarrow \text{7178}$$

$$\frac{(a+bx)(cW(a+bx))^{5/2}}{b}-\frac{5}{2}\left(\frac{c(a+bx)(cW(a+bx))^{3/2}}{b}-\frac{5}{2}c\int \frac{(cW(a+bx))^{3/2}}{W(a+bx)+1}dx\right)$$

$$\Big\downarrow \text{7178}$$

$$\frac{(a+bx)(cW(a+bx))^{5/2}}{b}-\frac{5}{2}\left(\frac{c(a+bx)(cW(a+bx))^{3/2}}{b}-\frac{5}{2}c\left(\frac{c(a+bx)\sqrt{cW(a+bx)}}{b}-\frac{3}{2}c\int\frac{\sqrt{cW(a+bx)}}{W(a+bx)+1}dx\right)\right)$$

$\Big\downarrow$ 7178

$$\frac{(a+bx)(cW(a+bx))^{5/2}}{b}-\frac{5}{2}\left(\frac{c(a+bx)(cW(a+bx))^{3/2}}{b}-\frac{5}{2}c\left(\frac{c(a+bx)\sqrt{cW(a+bx)}}{b}-\frac{3}{2}c\left(\frac{c(a+bx)}{b\sqrt{cW(a+bx)}}-\frac{1}{2}c\int\frac{1}{\sqrt{cW(a+bx)}(W(\cdots$$

$\Big\downarrow$ 7180

$$\frac{(a+bx)(cW(a+bx))^{5/2}}{b}-\frac{5}{2}\left(\frac{c(a+bx)(cW(a+bx))^{3/2}}{b}-\frac{5}{2}c\left(\frac{c(a+bx)\sqrt{cW(a+bx)}}{b}-\frac{3}{2}c\left(\frac{c(a+bx)}{b\sqrt{cW(a+bx)}}-\frac{\sqrt{\pi}\sqrt{c}\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{2b}\cdots$$

input
```
Int[(c*ProductLog[a + b*x])^(5/2),x]
```

output
```
((a + b*x)*(c*ProductLog[a + b*x])^(5/2))/b - (5*((c*(a + b*x)*(c*ProductL
og[a + b*x])^(3/2))/b - (5*c*((c*(a + b*x)*Sqrt[c*ProductLog[a + b*x]])/b
- (3*c*(-1/2*(Sqrt[c]*Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a + b*x]]/Sqrt[c]])/
b + (c*(a + b*x))/(b*Sqrt[c*ProductLog[a + b*x]])))/2))/2))/2
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7178
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[c*(a + b*x)*((c*ProductLog[a + b*x])^
(p - 1)/(b*d)), x] - Simp[c*p   Int[(c*ProductLog[a + b*x])^(p - 1)/(d + d*
ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0]
```

rule 7180
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[Pi*c, 2]*(Erfi[Sqrt[c*ProductLo
g[a + b*x]]/Rt[c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && PosQ[c]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.15

method	result
default	$\frac{\frac{(c\,\mathrm{LambertW}(bx+a))^{\frac{7}{2}}(bx+a)c}{\mathrm{LambertW}(bx+a)}-5c\left(\frac{c(c\,\mathrm{LambertW}(bx+a))^{\frac{5}{2}}(bx+a)}{2\,\mathrm{LambertW}(bx+a)}-\frac{5c\left(\frac{(c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}(bx+a)c}{2\,\mathrm{LambertW}(bx+a)}-\frac{3c\left(\frac{c\sqrt{c\,\mathrm{LambertW}(bx+a)}\,(bx+a)}{2\,\mathrm{LambertW}(bx+a)}\right.}{}\right.}{2}\right.}{b\,c^{2}}$

input
```
int((c*LambertW(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(1/2*(c*LambertW(b*x+a))^(7/2)*(b*x+a)/LambertW(b*x+a)*c-5/2*c*(1/
2*c*(c*LambertW(b*x+a))^(5/2)*(b*x+a)/LambertW(b*x+a)-5/2*c*(1/2*(c*Lamber
tW(b*x+a))^(3/2)*(b*x+a)/LambertW(b*x+a)*c-3/2*c*(1/2*c*(c*LambertW(b*x+a)
)^(1/2)*(b*x+a)/LambertW(b*x+a)-1/4*c*Pi^(1/2)/(-1/c)^(1/2)*erf((-1/c)^(1/
2)*(c*LambertW(b*x+a))^(1/2))))))
```

Fricas [F]

$$\int (cW(a+bx))^{5/2}\,dx=\int (c\,\mathrm{W}(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))*c^2*lambert_w(b*x + a)^2, x)
```

Sympy [F]

$$\int (cW(a+bx))^{5/2}\,dx = \int (cW(a+bx))^{\frac{5}{2}}\,dx$$

input
```
integrate((c*LambertW(b*x+a))**(5/2),x)
```

output
```
Integral((c*LambertW(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int (cW(a+bx))^{5/2}\,dx = \int (c\,\mathrm{W}(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(b*x + a))^(5/2), x)
```

Giac [F]

$$\int (cW(a+bx))^{5/2}\,dx = \int (c\,\mathrm{W}(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cW(a+bx))^{5/2}\,dx = \int (c\,\mathrm{LambertW}(a+b\,x))^{5/2}\,dx$$

input
```
int((c*LambertW(a + b*x))^(5/2),x)
```

output
```
int((c*LambertW(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int (cW(a+bx))^{5/2}\,dx = \sqrt{c}\left(\int \sqrt{lambert\_w(bx+a)}\,lambert\_w(bx+a)^2\,dx\right)c^2$$

input
```
int((c*Lambert_W(b*x+a))^(5/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a + b*x))*lambert_w(a + b*x)**2,x)*c**2
```

3.371 $\int (cW(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 110

$$\int (cW(a+bx))^{3/2}\,dx = -\frac{9c^{3/2}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{8b} + \frac{9c^2(a+bx)}{4b\sqrt{cW(a+bx)}} - \frac{3c(a+bx)\sqrt{cW(a+bx)}}{2b} + \frac{(a+bx)(cW(a+bx))^{3/2}}{b}$$

output
```
-9/8*c^(3/2)*Pi^(1/2)*erfi((c*LambertW(b*x+a))^(1/2)/c^(1/2))/b+9/4*c^2*(b
*x+a)/b/(c*LambertW(b*x+a))^(1/2)-3/2*c*(b*x+a)*(c*LambertW(b*x+a))^(1/2)/
b+(b*x+a)*(c*LambertW(b*x+a))^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int (cW(a+bx))^{3/2}\,dx = \frac{c^2\left(18(a+bx) - 9\sqrt{\pi}\,\text{erfi}\left(\sqrt{W(a+bx)}\right)\sqrt{W(a+bx)} - 12(a+bx)W(a+bx) + 8(a+bx)\right.}{8b\sqrt{cW(a+bx)}}$$

input
```
Integrate[(c*ProductLog[a + b*x])^(3/2),x]
```

output
```
(c^2*(18*(a + b*x) - 9*Sqrt[Pi]*Erfi[Sqrt[ProductLog[a + b*x]]]*Sqrt[Produ
ctLog[a + b*x]] - 12*(a + b*x)*ProductLog[a + b*x] + 8*(a + b*x)*ProductLo
g[a + b*x]^2))/(8*b*Sqrt[c*ProductLog[a + b*x]])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7167, 7178, 7178, 7180}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cW(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{7167}$$

$$\frac{(a+bx)(cW(a+bx))^{3/2}}{b}-\frac{3}{2}\int \frac{(cW(a+bx))^{3/2}}{W(a+bx)+1}dx$$

$$\Big\downarrow \text{7178}$$

$$\frac{(a+bx)(cW(a+bx))^{3/2}}{b}-\frac{3}{2}\left(\frac{c(a+bx)\sqrt{cW(a+bx)}}{b}-\frac{3}{2}c\int \frac{\sqrt{cW(a+bx)}}{W(a+bx)+1}dx\right)$$

$$\Big\downarrow \text{7178}$$

$$\frac{(a+bx)(cW(a+bx))^{3/2}}{b}-\frac{3}{2}\left(\frac{c(a+bx)\sqrt{cW(a+bx)}}{b}-\frac{3}{2}c\left(\frac{c(a+bx)}{b\sqrt{cW(a+bx)}}-\frac{1}{2}c\int \frac{1}{\sqrt{cW(a+bx)}(W(a+bx)+1)}dx\right)\right)$$

$$\Big\downarrow \text{7180}$$

$$\frac{(a+bx)(cW(a+bx))^{3/2}}{b}-\frac{3}{2}\left(\frac{c(a+bx)\sqrt{cW(a+bx)}}{b}-\frac{3}{2}c\left(\frac{c(a+bx)}{b\sqrt{cW(a+bx)}}-\frac{\sqrt{\pi}\sqrt{c}\,\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{2b}\right)\right)$$

input
```
Int[(c*ProductLog[a + b*x])^(3/2),x]
```

output
```
((a + b*x)*(c*ProductLog[a + b*x])^(3/2))/b - (3*((c*(a + b*x)*Sqrt[c*Prod
uctLog[a + b*x]])/b - (3*c*(-1/2*(Sqrt[c]*Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[
a + b*x]]/Sqrt[c]])/b + (c*(a + b*x))/(b*Sqrt[c*ProductLog[a + b*x]])))/2)
)/2
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7178
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[c*(a + b*x)*((c*ProductLog[a + b*x])^
(p - 1)/(b*d)), x] - Simp[c*p   Int[(c*ProductLog[a + b*x])^(p - 1)/(d + d*
ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0]
```

rule 7180
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[Pi*c, 2]*(Erfi[Sqrt[c*ProductLo
g[a + b*x]]/Rt[c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && PosQ[c]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16

method	result
default	$\frac{\frac{c(c\,\mathrm{LambertW}(bx+a))^{\frac{5}{2}}(bx+a)}{\mathrm{LambertW}(bx+a)}-3c\left(\frac{(c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}(bx+a)c}{2\,\mathrm{LambertW}(bx+a)}-\frac{3c\left(\frac{c\sqrt{c\,\mathrm{LambertW}(bx+a)}\,(bx+a)}{2\,\mathrm{LambertW}(bx+a)}-\frac{c\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{c}}\,\sqrt{c\,\mathrm{LambertW}(bx+a}\right.}{4\sqrt{-\frac{1}{c}}}\right.}{2}\right.}{b\,c^{2}}$

input
```
int((c*LambertW(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(1/2*c*(c*LambertW(b*x+a))^(5/2)*(b*x+a)/LambertW(b*x+a)-3/2*c*(1/
2*(c*LambertW(b*x+a))^(3/2)*(b*x+a)/LambertW(b*x+a)*c-3/2*c*(1/2*c*(c*Lamb
ertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)-1/4*c*Pi^(1/2)/(-1/c)^(1/2)*erf
((-1/c)^(1/2)*(c*LambertW(b*x+a))^(1/2)))))
```

Fricas [F]

$$\int (cW(a+bx))^{3/2}\,dx = \int (c\,\mathrm{W}(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))*c*lambert_w(b*x + a), x)
```

Sympy [F]

$$\int (cW(a+bx))^{3/2}\,dx = \int (cW(a+bx))^{\frac{3}{2}}\,dx$$

input
```
integrate((c*LambertW(b*x+a))**(3/2),x)
```

output
```
Integral((c*LambertW(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int (cW(a+bx))^{3/2}\,dx = \int (c\,\mathrm{W}(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(b*x + a))^(3/2), x)
```

Giac [F]

$$\int (cW(a+bx))^{3/2}\,dx = \int (c\,\mathrm{W}(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cW(a+bx))^{3/2}\,dx = \int (c\,\mathrm{LambertW}(a+b\,x))^{3/2}\,dx$$

input
```
int((c*LambertW(a + b*x))^(3/2),x)
```

output
```
int((c*LambertW(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int (cW(a+bx))^{3/2}\,dx = \sqrt{c}\left(\int \sqrt{lambert\_w(bx+a)}\,lambert\_w(bx+a)\,dx\right)c$$

input
```
int((c*Lambert_W(b*x+a))^(3/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a + b*x))*lambert_w(a + b*x),x)*c
```

3.372 $\int \sqrt{cW(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 83

$$\int \sqrt{cW(a+bx)}\,dx = \frac{\sqrt{c}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{4b} - \frac{c(a+bx)}{2b\sqrt{cW(a+bx)}} + \frac{(a+bx)\sqrt{cW(a+bx)}}{b}$$

output
```
1/4*c^(1/2)*Pi^(1/2)*erfi((c*LambertW(b*x+a))^(1/2)/c^(1/2))/b-1/2*c*(b*x+
a)/b/(c*LambertW(b*x+a))^(1/2)+(b*x+a)*(c*LambertW(b*x+a))^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \sqrt{cW(a+bx)}\,dx = \frac{c\left(-2(a+bx)+\sqrt{\pi}\,\text{erfi}\left(\sqrt{W(a+bx)}\right)\sqrt{W(a+bx)}+4(a+bx)W(a+bx)\right)}{4b\sqrt{cW(a+bx)}}$$

input
```
Integrate[Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
(c*(-2*(a + b*x) + Sqrt[Pi]*Erfi[Sqrt[ProductLog[a + b*x]]]*Sqrt[ProductLo
g[a + b*x]] + 4*(a + b*x)*ProductLog[a + b*x]))/(4*b*Sqrt[c*ProductLog[a +
 b*x]])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7167, 7178, 7180}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cW(a+bx)}dx$$

$$\Big\downarrow \text{7167}$$

$$\frac{(a+bx)\sqrt{cW(a+bx)}}{b}-\frac{1}{2}\int \frac{\sqrt{cW(a+bx)}}{W(a+bx)+1}dx$$

$$\Big\downarrow \text{7178}$$

$$\frac{1}{2}\left(\frac{1}{2}c\int \frac{1}{\sqrt{cW(a+bx)}(W(a+bx)+1)}dx-\frac{c(a+bx)}{b\sqrt{cW(a+bx)}}\right)+\frac{(a+bx)\sqrt{cW(a+bx)}}{b}$$

$$\Big\downarrow \text{7180}$$

$$\frac{1}{2}\left(\frac{\sqrt{\pi}\sqrt{c}\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{2b}-\frac{c(a+bx)}{b\sqrt{cW(a+bx)}}\right)+\frac{(a+bx)\sqrt{cW(a+bx)}}{b}$$

input
```
Int[Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
((a + b*x)*Sqrt[c*ProductLog[a + b*x]])/b + ((Sqrt[c]*Sqrt[Pi]*Erfi[Sqrt[c
*ProductLog[a + b*x]]/Sqrt[c]])/(2*b) - (c*(a + b*x))/(b*Sqrt[c*ProductLog
[a + b*x]]))/2
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7178
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[c*(a + b*x)*((c*ProductLog[a + b*x])^
(p - 1)/(b*d)), x] - Simp[c*p   Int[(c*ProductLog[a + b*x])^(p - 1)/(d + d*
ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0]
```

rule 7180
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[Pi*c, 2]*(Erfi[Sqrt[c*ProductLo
g[a + b*x]]/Rt[c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && PosQ[c]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{\frac{(c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}(bx+a)c}{\mathrm{LambertW}(bx+a)}-c\left(\frac{c\sqrt{c\,\mathrm{LambertW}(bx+a)}\,(bx+a)}{2\,\mathrm{LambertW}(bx+a)}-\frac{c\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{c}}\,\sqrt{c\,\mathrm{LambertW}(bx+a)}\right)}{4\sqrt{-\frac{1}{c}}}\right)}{b\,c^{2}}$	98

input
```
int((c*LambertW(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(1/2*(c*LambertW(b*x+a))^(3/2)*(b*x+a)/LambertW(b*x+a)*c-1/2*c*(1/
2*c*(c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)-1/4*c*Pi^(1/2)/(-1/c
)^(1/2)*erf((-1/c)^(1/2)*(c*LambertW(b*x+a))^(1/2))))
```

Fricas [F]

$$\int \sqrt{cW(a+bx)}\,dx = \int \sqrt{c\,\mathrm{W}(bx+a)}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a)), x)
```

Sympy [F]

$$\int \sqrt{cW(a+bx)}\,dx = \int \sqrt{cW\left(a+bx\right)}\,dx$$

input
```
integrate((c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(c*LambertW(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{cW(a+bx)}\,dx = \int \sqrt{c\,\mathrm{W}(bx+a)}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{cW(a+bx)}\,dx = \int \sqrt{c\,\mathrm{W}(bx+a)}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cW(a+bx)}\,dx = \int \sqrt{c\,\mathrm{LambertW}\,(a+b\,x)}\,dx$$

input
```
int((c*LambertW(a + b*x))^(1/2),x)
```

output
```
int((c*LambertW(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{cW(a+bx)}\,dx = \sqrt{c}\left(\int \sqrt{lambert\_w(bx+a)}dx\right)$$

input
```
int((c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a + b*x)),x)
```

3.373 $\int \frac{1}{\sqrt{cW(a+bx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{\sqrt{cW(a+bx)}}\,dx=\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{2b\sqrt{c}}+\frac{a+bx}{b\sqrt{cW(a+bx)}}$$

output
```
1/2*Pi^(1/2)*erfi((c*LambertW(b*x+a))^(1/2)/c^(1/2))/b/c^(1/2)+(b*x+a)/b/(
c*LambertW(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{cW(a+bx)}}\,dx=\frac{2(a+bx)+\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{W(a+bx)}\right)\sqrt{W(a+bx)}}{2b\sqrt{cW(a+bx)}}$$

input
```
Integrate[1/Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
(2*(a + b*x) + Sqrt[Pi]*Erfi[Sqrt[ProductLog[a + b*x]]]*Sqrt[ProductLog[a
+ b*x]])/(2*b*Sqrt[c*ProductLog[a + b*x]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7167, 7180}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cW(a+bx)}}dx$$
$$\Big\downarrow \text{7167}$$
$$\frac{1}{2}\int \frac{1}{\sqrt{cW(a+bx)}(W(a+bx)+1)}dx+\frac{a+bx}{b\sqrt{cW(a+bx)}}$$
$$\Big\downarrow \text{7180}$$
$$\frac{\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{2b\sqrt{c}}+\frac{a+bx}{b\sqrt{cW(a+bx)}}$$

input
```
Int[1/Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
(Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a + b*x]]/Sqrt[c]])/(2*b*Sqrt[c]) + (a +
b*x)/(b*Sqrt[c*ProductLog[a + b*x]])
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7180
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[Pi*c, 2]*(Erfi[Sqrt[c*ProductLo
g[a + b*x]]/Rt[c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && PosQ[c]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{\frac{c\sqrt{c\,\mathrm{LambertW}(bx+a)}\,(bx+a)}{\mathrm{LambertW}(bx+a)}+\frac{c\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{c}}\,\sqrt{c\,\mathrm{LambertW}(bx+a)}\right)}{2\sqrt{-\frac{1}{c}}}}{b\,c^{2}}$	68

input
```
int(1/(c*LambertW(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(1/2*c*(c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)+1/4*c*Pi^
(1/2)/(-1/c)^(1/2)*erf((-1/c)^(1/2)*(c*LambertW(b*x+a))^(1/2)))
```

Fricas [F]

$$\int \frac{1}{\sqrt{cW(a+bx)}}\,dx=\int \frac{1}{\sqrt{c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(1/(c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))/(c*lambert_w(b*x + a)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{cW(a+bx)}}\,dx=\int \frac{1}{\sqrt{cW\left(a+bx\right)}}\,dx$$

input
```
integrate(1/(c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(1/sqrt(c*LambertW(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(1/(c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*lambert_w(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(1/(c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*lambert_w(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{LambertW}\,(a+b\,x)}}\,dx$$

input
```
int(1/(c*LambertW(a + b*x))^(1/2),x)
```

output
```
int(1/(c*LambertW(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{cW(a+bx)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}}{lambert\_w(bx+a)}dx\right)}{c}$$

input
```
int(1/(c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a + b*x))/lambert_w(a + b*x),x))/c
```

3.374 $\int \frac{1}{(cW(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{1}{(cW(a+bx))^{3/2}}\,dx = \frac{3\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{bc^{3/2}} - \frac{2(a+bx)}{b(cW(a+bx))^{3/2}}$$

output
```
3*Pi^(1/2)*erfi((c*LambertW(b*x+a))^(1/2)/c^(1/2))/b/c^(3/2)-2*(b*x+a)/b/(
c*LambertW(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{1}{(cW(a+bx))^{3/2}}\,dx = \frac{-2(a+bx)+3\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{W(a+bx)}\right)W(a+bx)^{3/2}}{b(cW(a+bx))^{3/2}}$$

input
```
Integrate[(c*ProductLog[a + b*x])^(-3/2),x]
```

output
```
(-2*(a + b*x) + 3*Sqrt[Pi]*Erfi[Sqrt[ProductLog[a + b*x]]]*ProductLog[a +
b*x]^(3/2))/(b*(c*ProductLog[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7166, 7180}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cW(a+bx))^{3/2}}dx$$
$$\Big\downarrow \text{7166}$$
$$\frac{3\int \frac{1}{\sqrt{cW(a+bx)}(W(a+bx)+1)}dx}{c}-\frac{2(a+bx)}{b(cW(a+bx))^{3/2}}$$
$$\Big\downarrow \text{7180}$$
$$\frac{3\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{bc^{3/2}}-\frac{2(a+bx)}{b(cW(a+bx))^{3/2}}$$

input
```
Int[(c*ProductLog[a + b*x])^(-3/2),x]
```

output
```
(3*Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a + b*x]]/Sqrt[c]])/(b*c^(3/2)) - (2*(a
 + b*x))/(b*(c*ProductLog[a + b*x])^(3/2))
```

Defintions of rubi rules used

rule 7166
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_), x_Symbol] :> Simp[(a + b*x
)*((c*ProductLog[a + b*x])^p/(b*(p + 1))), x] + Simp[p/(c*(p + 1))   Int[(c
*ProductLog[a + b*x])^(p + 1)/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a
, b, c}, x] && LtQ[p, -1]
```

rule 7180
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[Pi*c, 2]*(Erfi[Sqrt[c*ProductLo
g[a + b*x]]/Rt[c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && PosQ[c]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(47) = 94$.

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.79

method	result	size
default	$\frac{2c\left(-\frac{bx+a}{\sqrt{c\,\mathrm{LambertW}(bx+a)}\,\mathrm{LambertW}(bx+a)}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{c}}\,\sqrt{c\,\mathrm{LambertW}(bx+a)}\right)}{c\sqrt{-\frac{1}{c}}}\right)+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{c}}\,\sqrt{c\,\mathrm{LambertW}(bx+a)}\right)}{\sqrt{-\frac{1}{c}}}}{b\,c^2}$	102

input
```
int(1/(c*LambertW(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(c*(-1/(c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)+1/c*Pi^(1
/2)/(-1/c)^(1/2)*erf((-1/c)^(1/2)*(c*LambertW(b*x+a))^(1/2)))+1/2*Pi^(1/2)
/(-1/c)^(1/2)*erf((-1/c)^(1/2)*(c*LambertW(b*x+a))^(1/2)))
```

Fricas [F]

$$\int \frac{1}{(cW(a+bx))^{3/2}}\,dx = \int \frac{1}{\left(c\,\mathrm{W}(bx+a)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*lambert_w(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))/(c^2*lambert_w(b*x + a)^2), x)
```

Sympy [F]

$$\int \frac{1}{(cW(a+bx))^{3/2}}\,dx = \int \frac{1}{\left(cW(a+bx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*LambertW(b*x+a))**(3/2),x)
```

output
```
Integral((c*LambertW(a + b*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(cW(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\,\mathrm{W}(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*lambert_w(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(b*x + a))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(cW(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\,\mathrm{W}(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*lambert_w(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(b*x + a))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cW(a+bx))^{3/2}}\,dx = \int \frac{1}{\left(c\,\mathrm{LambertW}\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(c*LambertW(a + b*x))^(3/2),x)
```

output
```
int(1/(c*LambertW(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(cW(a+bx))^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}}{lambert\_w(bx+a)^2}dx\right)}{c^2}$$

input
```
int(1/(c*Lambert_W(b*x+a))^(3/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a + b*x))/lambert_w(a + b*x)**2,x))/c**2
```

3.375 $\int \frac{1}{(cW(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{1}{(cW(a+bx))^{5/2}}\,dx=\frac{10\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{3bc^{5/2}}-\frac{2(a+bx)}{3b(cW(a+bx))^{5/2}}-\frac{10(a+bx)}{3bc(cW(a+bx))^{3/2}}$$

output
```
10/3*Pi^(1/2)*erfi((c*LambertW(b*x+a))^(1/2)/c^(1/2))/b/c^(5/2)-2/3*(b*x+a
)/b/(c*LambertW(b*x+a))^(5/2)-10/3*(b*x+a)/b/c/(c*LambertW(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \frac{1}{(cW(a+bx))^{5/2}}\,dx=\frac{2\left(-a-bx-5(a+bx)W(a+bx)+5\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{W(a+bx)}\right)W(a+bx)^{5/2}\right)}{3b(cW(a+bx))^{5/2}}$$

input
```
Integrate[(c*ProductLog[a + b*x])^(-5/2),x]
```

output
```
(2*(-a - b*x - 5*(a + b*x)*ProductLog[a + b*x] + 5*Sqrt[Pi]*Erfi[Sqrt[Prod
uctLog[a + b*x]]]*ProductLog[a + b*x]^(5/2)))/(3*b*(c*ProductLog[a + b*x])
^(5/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7166, 7182, 7180}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cW(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{7166}$$

$$\frac{5\int \frac{1}{(cW(a+bx))^{3/2}(W(a+bx)+1)}dx}{3c}-\frac{2(a+bx)}{3b(cW(a+bx))^{5/2}}$$

$$\Big\downarrow \text{7182}$$

$$\frac{5\left(\frac{2\int \frac{1}{\sqrt{cW(a+bx)}(W(a+bx)+1)}dx}{c}-\frac{2(a+bx)}{b(cW(a+bx))^{3/2}}\right)}{3c}-\frac{2(a+bx)}{3b(cW(a+bx))^{5/2}}$$

$$\Big\downarrow \text{7180}$$

$$\frac{5\left(\frac{2\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{bc^{3/2}}-\frac{2(a+bx)}{b(cW(a+bx))^{3/2}}\right)}{3c}-\frac{2(a+bx)}{3b(cW(a+bx))^{5/2}}$$

input
```
Int[(c*ProductLog[a + b*x])^(-5/2),x]
```

output
```
(-2*(a + b*x))/(3*b*(c*ProductLog[a + b*x])^(5/2)) + (5*((2*Sqrt[Pi]*Erfi[
Sqrt[c*ProductLog[a + b*x]]/Sqrt[c]])/(b*c^(3/2)) - (2*(a + b*x))/(b*(c*Pr
oductLog[a + b*x])^(3/2))))/(3*c)
```

Defintions of rubi rules used

rule 7166
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_), x_Symbol] :> Simp[(a + b*x
)*((c*ProductLog[a + b*x])^p/(b*(p + 1))), x] + Simp[p/(c*(p + 1))   Int[(c
*ProductLog[a + b*x])^(p + 1)/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a
, b, c}, x] && LtQ[p, -1]
```

rule 7180
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[Pi*c, 2]*(Erfi[Sqrt[c*ProductLo
g[a + b*x]]/Rt[c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && PosQ[c]
```

rule 7182
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[(a + b*x)*((c*ProductLog[a + b*x])^p/
(b*d*(p + 1))), x] - Simp[1/(c*(p + 1))   Int[(c*ProductLog[a + b*x])^(p +
1)/(d + d*ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(70) = 140$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.82

method	result
default	$\frac{-\frac{2(bx+a)}{\sqrt{c\,\mathrm{LambertW}(bx+a)}\,\mathrm{LambertW}(bx+a)}+\frac{2\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{c}}\,\sqrt{c\,\mathrm{LambertW}(bx+a)}\right)}{c\sqrt{-\frac{1}{c}}}+2c\Bigg(-\frac{bx+a}{3(c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}\,\mathrm{LambertW}(bx+a)}+\frac{-\frac{\cdots}{3\sqrt{c}\cdots}}{\cdots}}{b\,c^2}$

input
```
int(1/(c*LambertW(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(-1/(c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)+1/c*Pi^(1/2)
/(-1/c)^(1/2)*erf((-1/c)^(1/2)*(c*LambertW(b*x+a))^(1/2))+c*(-1/3/(c*Lambe
rtW(b*x+a))^(3/2)*(b*x+a)/LambertW(b*x+a)+2/3/c*(-1/(c*LambertW(b*x+a))^(1
/2)*(b*x+a)/LambertW(b*x+a)+1/c*Pi^(1/2)/(-1/c)^(1/2)*erf((-1/c)^(1/2)*(c*
LambertW(b*x+a))^(1/2)))))
```

Fricas [F]

$$\int \frac{1}{(cW(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\,\mathrm{W}(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*lambert_w(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))/(c^3*lambert_w(b*x + a)^3), x)
```

Sympy [F]

$$\int \frac{1}{(cW(a+bx))^{5/2}}\,dx = \int \frac{1}{(cW(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*LambertW(b*x+a))**(5/2),x)
```

output
```
Integral((c*LambertW(a + b*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(cW(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\,\mathrm{W}(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*lambert_w(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(b*x + a))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(cW(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\,\mathrm{W}(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*lambert_w(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(b*x + a))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cW(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\,\mathrm{LambertW}\,(a+b\,x))^{5/2}}\,dx$$

input
```
int(1/(c*LambertW(a + b*x))^(5/2),x)
```

output
```
int(1/(c*LambertW(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(cW(a+bx))^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}}{lambert\_w(bx+a)^{3}}dx\right)}{c^{3}}$$

input
```
int(1/(c*Lambert_W(b*x+a))^(5/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a + b*x))/lambert_w(a + b*x)**3,x))/c**3
```

3.376 $\int \frac{1}{(cW(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 115

$$\int \frac{1}{(cW(a+bx))^{7/2}}\,dx = \frac{28\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{15bc^{7/2}} - \frac{2(a+bx)}{5b(cW(a+bx))^{7/2}} - \frac{14(a+bx)}{15bc(cW(a+bx))^{5/2}} - \frac{28(a+bx)}{15bc^2(cW(a+bx))^{3/2}}$$

output
```
28/15*Pi^(1/2)*erfi((c*LambertW(b*x+a))^(1/2)/c^(1/2))/b/c^(7/2)-2/5*(b*x+
a)/b/(c*LambertW(b*x+a))^(7/2)-14/15*(b*x+a)/b/c/(c*LambertW(b*x+a))^(5/2)
-28/15*(b*x+a)/b/c^2/(c*LambertW(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int \frac{1}{(cW(a+bx))^{7/2}}\,dx = \frac{2\sqrt{cW(a+bx)}\left(-3(a+bx) - 7(a+bx)W(a+bx) - 14(a+bx)W(a+bx)^2 + \right.}{15bc^4W(a+bx)^4}$$

input
```
Integrate[(c*ProductLog[a + b*x])^(-7/2),x]
```

output
```
(2*Sqrt[c*ProductLog[a + b*x]]*(-3*(a + b*x) - 7*(a + b*x)*ProductLog[a +
b*x] - 14*(a + b*x)*ProductLog[a + b*x]^2 + 14*Sqrt[Pi]*Erfi[Sqrt[ProductL
og[a + b*x]]]*ProductLog[a + b*x]^(7/2)))/(15*b*c^4*ProductLog[a + b*x]^4)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7166, 7182, 7182, 7180}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cW(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{7166}$$

$$\frac{7\int \frac{1}{(cW(a+bx))^{5/2}(W(a+bx)+1)}dx}{5c}-\frac{2(a+bx)}{5b(cW(a+bx))^{7/2}}$$

$$\Big\downarrow \text{7182}$$

$$\frac{7\left(\frac{2\int \frac{1}{(cW(a+bx))^{3/2}(W(a+bx)+1)}dx}{3c}-\frac{2(a+bx)}{3b(cW(a+bx))^{5/2}}\right)}{5c}-\frac{2(a+bx)}{5b(cW(a+bx))^{7/2}}$$

$$\Big\downarrow \text{7182}$$

$$\frac{7\left(\frac{2\left(\frac{2\int \frac{1}{\sqrt{cW(a+bx)}(W(a+bx)+1)}dx}{c}-\frac{2(a+bx)}{b(cW(a+bx))^{3/2}}\right)}{3c}-\frac{2(a+bx)}{3b(cW(a+bx))^{5/2}}\right)}{5c}-\frac{2(a+bx)}{5b(cW(a+bx))^{7/2}}$$

$$\Big\downarrow \text{7180}$$

$$\frac{7\left(\frac{2\left(\frac{2\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{bc^{3/2}}-\frac{2(a+bx)}{b(cW(a+bx))^{3/2}}\right)}{3c}-\frac{2(a+bx)}{3b(cW(a+bx))^{5/2}}\right)}{5c}-\frac{2(a+bx)}{5b(cW(a+bx))^{7/2}}$$

input
```
Int[(c*ProductLog[a + b*x])^(-7/2),x]
```

output
```
(-2*(a + b*x))/(5*b*(c*ProductLog[a + b*x])^(7/2)) + (7*((-2*(a + b*x))/(3
*b*(c*ProductLog[a + b*x])^(5/2)) + (2*((2*Sqrt[Pi]*Erfi[Sqrt[c*ProductLog
[a + b*x]]/Sqrt[c]])/(b*c^(3/2)) - (2*(a + b*x))/(b*(c*ProductLog[a + b*x]
)^(3/2))))/(3*c)))/(5*c)
```

Defintions of rubi rules used

rule 7166
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_), x_Symbol] :> Simp[(a + b*x
)*((c*ProductLog[a + b*x])^p/(b*(p + 1))), x] + Simp[p/(c*(p + 1))   Int[(c
*ProductLog[a + b*x])^(p + 1)/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a
, b, c}, x] && LtQ[p, -1]
```

rule 7180
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[Pi*c, 2]*(Erfi[Sqrt[c*ProductLo
g[a + b*x]]/Rt[c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && PosQ[c]
```

rule 7182
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[(a + b*x)*((c*ProductLog[a + b*x])^p/
(b*d*(p + 1))), x] - Simp[1/(c*(p + 1))   Int[(c*ProductLog[a + b*x])^(p +
1)/(d + d*ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(93) = 186.

Time = 0.04 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.93

method	result
default	$\frac{-\frac{2(bx+a)}{3(c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}\,\mathrm{LambertW}(bx+a)}+\frac{2\left(-\frac{2(bx+a)}{3\sqrt{c\,\mathrm{LambertW}(bx+a)}\,\mathrm{LambertW}(bx+a)}+\frac{2\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{c}}\,\sqrt{c\,\mathrm{LambertW}(bx+a)}\right)}{3c\sqrt{-\frac{1}{c}}}\right)}{c}+2c\left(\cdots\right.}{}$

input
```
int(1/(c*LambertW(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(-1/3/(c*LambertW(b*x+a))^(3/2)*(b*x+a)/LambertW(b*x+a)+2/3/c*(-1/
(c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)+1/c*Pi^(1/2)/(-1/c)^(1/2
)*erf((-1/c)^(1/2)*(c*LambertW(b*x+a))^(1/2)))+c*(-1/5/(c*LambertW(b*x+a))
^(5/2)*(b*x+a)/LambertW(b*x+a)+2/5/c*(-1/3/(c*LambertW(b*x+a))^(3/2)*(b*x+
a)/LambertW(b*x+a)+2/3/c*(-1/(c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*
x+a)+1/c*Pi^(1/2)/(-1/c)^(1/2)*erf((-1/c)^(1/2)*(c*LambertW(b*x+a))^(1/2))
))))
```

Fricas [F]

$$\int \frac{1}{(cW(a+bx))^{7/2}}\,dx=\int \frac{1}{(c\,\mathrm{W}(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*lambert_w(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))/(c^4*lambert_w(b*x + a)^4), x)
```

Sympy [F]

$$\int \frac{1}{(cW(a+bx))^{7/2}}\,dx = \int \frac{1}{(cW(a+bx))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*LambertW(b*x+a))**(7/2),x)
```

output
```
Integral((c*LambertW(a + b*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(cW(a+bx))^{7/2}}\,dx = \int \frac{1}{(c\,\mathrm{W}(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*lambert_w(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(b*x + a))^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(cW(a+bx))^{7/2}}\,dx = \int \frac{1}{(c\,\mathrm{W}(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*lambert_w(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate((c*lambert_w(b*x + a))^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cW(a+bx))^{7/2}} \, dx = \int \frac{1}{(c \operatorname{LambertW}(a+b\,x))^{7/2}} \, dx$$

input
```
int(1/(c*LambertW(a + b*x))^(7/2),x)
```

output
```
int(1/(c*LambertW(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(cW(a+bx))^{7/2}} \, dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}}{lambert\_w(bx+a)^4} dx\right)}{c^4}$$

input
```
int(1/(c*Lambert_W(b*x+a))^(7/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a + b*x))/lambert_w(a + b*x)**4,x))/c**4
```

3.377 $\int (-cW(a+bx))^{5/2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 142

$$\int (-cW(a+bx))^{5/2} \, dx = \frac{75c^{5/2}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{16b} + \frac{75c^3(a+bx)}{8b\sqrt{-cW(a+bx)}} + \frac{25c^2(a+bx)\sqrt{-cW(a+bx)}}{4b} + \frac{5c(a+bx)(-cW(a+bx))^{3/2}}{2b} + \frac{(a+bx)(-cW(a+bx))^{5/2}}{b}$$

output
```
75/16*c^(5/2)*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2))/b+75/8*c^3*
(b*x+a)/b/(-c*LambertW(b*x+a))^(1/2)+25/4*c^2*(b*x+a)*(-c*LambertW(b*x+a))
^(1/2)/b+5/2*c*(b*x+a)*(-c*LambertW(b*x+a))^(3/2)/b+(b*x+a)*(-c*LambertW(b
*x+a))^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

$$\int (-cW(a+bx))^{5/2}\,dx = -\frac{c^3\left(-150(a+bx)+75\sqrt{\pi}\text{erfi}\left(\sqrt{W(a+bx)}\right)\sqrt{W(a+bx)}+100(a+bx)W(a+bx)-40(a+bx)W(a+\cdots\right.}{16b\sqrt{-cW(a+bx)}}$$

input
```
Integrate[(-(c*ProductLog[a + b*x]))^(5/2),x]
```

output
```
-1/16*(c^3*(-150*(a + b*x) + 75*Sqrt[Pi]*Erfi[Sqrt[ProductLog[a + b*x]]]*S
qrt[ProductLog[a + b*x]] + 100*(a + b*x)*ProductLog[a + b*x] - 40*(a + b*x
)*ProductLog[a + b*x]^2 + 16*(a + b*x)*ProductLog[a + b*x]^3))/(b*Sqrt[-(c
*ProductLog[a + b*x])])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {7167, 7178, 7178, 7178, 7181}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-cW(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{7167}$$

$$\frac{(a+bx)(-cW(a+bx))^{5/2}}{b}-\frac{5}{2}\int \frac{(-cW(a+bx))^{5/2}}{W(a+bx)+1}dx$$

$$\Big\downarrow \text{7178}$$

$$\frac{(a+bx)(-cW(a+bx))^{5/2}}{b}-\frac{5}{2}\left(\frac{5}{2}c\int \frac{(-cW(a+bx))^{3/2}}{W(a+bx)+1}dx-\frac{c(a+bx)(-cW(a+bx))^{3/2}}{b}\right)$$

$$\Big\downarrow \text{7178}$$

$$\frac{(a+bx)(-cW(a+bx))^{5/2}}{b}-\frac{5}{2}\left(\frac{5}{2}c\left(\frac{3}{2}c\int\frac{\sqrt{-cW(a+bx)}}{W(a+bx)+1}dx-\frac{c(a+bx)\sqrt{-cW(a+bx)}}{b}\right)-\frac{c(a+bx)(-cW(a+bx))^{3/2}}{b}\right)$$

$$\Big\downarrow \text{ 7178}$$

$$\frac{(a+bx)(-cW(a+bx))^{5/2}}{b}-\frac{5}{2}\left(\frac{5}{2}c\left(\frac{3}{2}c\left(\frac{1}{2}c\int\frac{1}{\sqrt{-cW(a+bx)}(W(a+bx)+1)}dx-\frac{c(a+bx)}{b\sqrt{-cW(a+bx)}}\right)-\frac{c(a+bx)\sqrt{-cW(a+bx)}}{b}\right)-c(\cdots$$

$$\Big\downarrow \text{ 7181}$$

$$\frac{(a+bx)(-cW(a+bx))^{5/2}}{b}-\frac{5}{2}\left(\frac{5}{2}c\left(\frac{3}{2}c\left(-\frac{\sqrt{\pi}\sqrt{c}\,\mathrm{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{2b}-\frac{c(a+bx)}{b\sqrt{-cW(a+bx)}}\right)-\frac{c(a+bx)\sqrt{-cW(a+bx)}}{b}\right)-\frac{c(a+bx)(-cW(\cdots}{b}$$

input
```
Int[(-(c*ProductLog[a + b*x]))^(5/2),x]
```

output
```
((a + b*x)*(-(c*ProductLog[a + b*x]))^(5/2))/b - (5*(-(((c*(a + b*x)*(-(c*P
roductLog[a + b*x]))^(3/2))/b) + (5*c*(-(((c*(a + b*x)*Sqrt[-(c*ProductLog[
a + b*x])])/b) + (3*c*(-1/2*(Sqrt[c]*Sqrt[Pi]*Erf[Sqrt[-(c*ProductLog[a +
b*x])]/Sqrt[c]])/b - (c*(a + b*x))/(b*Sqrt[-(c*ProductLog[a + b*x])])))/2)
)/2))/2
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7178
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[c*(a + b*x)*((c*ProductLog[a + b*x])^
(p - 1)/(b*d)), x] - Simp[c*p   Int[(c*ProductLog[a + b*x])^(p - 1)/(d + d*
ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0]
```

rule 7181
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[(-Pi)*c, 2]*(Erf[Sqrt[c*Product
Log[a + b*x]]/Rt[-c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && NegQ[c]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.08

method	result
default	$\frac{-\frac{c(-c\,\mathrm{LambertW}(bx+a))^{\frac{7}{2}}(bx+a)}{\mathrm{LambertW}(bx+a)}+5c\left(-\frac{c(-c\,\mathrm{LambertW}(bx+a))^{\frac{5}{2}}(bx+a)}{2\,\mathrm{LambertW}(bx+a)}+\frac{5c\left(-\frac{c(-c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}(bx+a)}{2\,\mathrm{LambertW}(bx+a)}+\frac{3c\left(-\frac{c\sqrt{-c\,\mathrm{LambertW}}}{2\,\mathrm{Lambert}}\right.}{}\right.}{2}\right.}{b\,c^2}$

input
```
int((-c*LambertW(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(-1/2*c*(-c*LambertW(b*x+a))^(7/2)*(b*x+a)/LambertW(b*x+a)+5/2*c*(
-1/2*c*(-c*LambertW(b*x+a))^(5/2)*(b*x+a)/LambertW(b*x+a)+5/2*c*(-1/2*c*(-
c*LambertW(b*x+a))^(3/2)*(b*x+a)/LambertW(b*x+a)+3/2*c*(-1/2*c*(-c*Lambert
W(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)+1/4*c^(3/2)*Pi^(1/2)*erf((-c*Lambe
rtW(b*x+a))^(1/2)/c^(1/2)))))
```

Fricas [F]

$$\int (-cW(a+bx))^{5/2}\,dx=\int (-c\,\mathrm{W}(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((-c*lambert_w(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*lambert_w(b*x + a))*c^2*lambert_w(b*x + a)^2, x)
```

Sympy [F]

$$\int (-cW(a+bx))^{5/2} \, dx = \int (-cW(a+bx))^{\frac{5}{2}} \, dx$$

input
```
integrate((-c*LambertW(b*x+a))**(5/2),x)
```

output
```
Integral((-c*LambertW(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int (-cW(a+bx))^{5/2} \, dx = \int (-c\,\mathrm{W}(bx+a))^{\frac{5}{2}} \, dx$$

input
```
integrate((-c*lambert_w(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((-c*lambert_w(b*x + a))^(5/2), x)
```

Giac [F]

$$\int (-cW(a+bx))^{5/2} \, dx = \int (-c\,\mathrm{W}(bx+a))^{\frac{5}{2}} \, dx$$

input
```
integrate((-c*lambert_w(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((-c*lambert_w(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (-cW(a+bx))^{5/2}\,dx = \int (-c\,\mathrm{LambertW}(a+b\,x))^{5/2}\,dx$$

input
```
int((-c*LambertW(a + b*x))^(5/2),x)
```

output
```
int((-c*LambertW(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int (-cW(a+bx))^{5/2}\,dx = \sqrt{c}\left(\int \sqrt{lambert\_w(bx+a)}\,lambert\_w(bx+a)^2\,dx\right)c^2 i$$

input
```
int((-c*Lambert_W(b*x+a))^(5/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a + b*x))*lambert_w(a + b*x)**2,x)*c**2*i
```

3.378 $\int (-cW(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 114

$$\int (-cW(a+bx))^{3/2}\,dx=\frac{9c^{3/2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{8b}+\frac{9c^2(a+bx)}{4b\sqrt{-cW(a+bx)}}+\frac{3c(a+bx)\sqrt{-cW(a+bx)}}{2b}+\frac{(a+bx)(-cW(a+bx))^{3/2}}{b}$$

output
```
9/8*c^(3/2)*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2))/b+9/4*c^2*(b*
x+a)/b/(-c*LambertW(b*x+a))^(1/2)+3/2*c*(b*x+a)*(-c*LambertW(b*x+a))^(1/2)
/b+(b*x+a)*(-c*LambertW(b*x+a))^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

$$\int (-cW(a+bx))^{3/2}\,dx=\frac{c^2\left(18(a+bx)-9\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{W(a+bx)}\right)\sqrt{W(a+bx)}-12(a+bx)W(a+bx)+8(a+bx)}{8b\sqrt{-cW(a+bx)}}$$

input
```
Integrate[(-(c*ProductLog[a + b*x]))^(3/2),x]
```

output
```
(c^2*(18*(a + b*x) - 9*Sqrt[Pi]*Erfi[Sqrt[ProductLog[a + b*x]]]*Sqrt[Produ
ctLog[a + b*x]] - 12*(a + b*x)*ProductLog[a + b*x] + 8*(a + b*x)*ProductLo
g[a + b*x]^2))/(8*b*Sqrt[-(c*ProductLog[a + b*x])])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {7167, 7178, 7178, 7181}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-cW(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{7167}$$

$$\frac{(a+bx)(-cW(a+bx))^{3/2}}{b}-\frac{3}{2}\int \frac{(-cW(a+bx))^{3/2}}{W(a+bx)+1}dx$$

$$\Big\downarrow \text{7178}$$

$$\frac{(a+bx)(-cW(a+bx))^{3/2}}{b}-\frac{3}{2}\left(\frac{3}{2}c\int \frac{\sqrt{-cW(a+bx)}}{W(a+bx)+1}dx-\frac{c(a+bx)\sqrt{-cW(a+bx)}}{b}\right)$$

$$\Big\downarrow \text{7178}$$

$$\frac{(a+bx)(-cW(a+bx))^{3/2}}{b}-\frac{3}{2}\left(\frac{3}{2}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{-cW(a+bx)}(W(a+bx)+1)}dx-\frac{c(a+bx)}{b\sqrt{-cW(a+bx)}}\right)-\frac{c(a+bx)\sqrt{-cW(a+bx)}}{b}\right)$$

$$\Big\downarrow \text{7181}$$

$$\frac{(a+bx)(-cW(a+bx))^{3/2}}{b}-\frac{3}{2}\left(\frac{3}{2}c\left(-\frac{\sqrt{\pi}\sqrt{c}\,\text{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{2b}-\frac{c(a+bx)}{b\sqrt{-cW(a+bx)}}\right)-\frac{c(a+bx)\sqrt{-cW(a+bx)}}{b}\right)$$

input
```
Int[(-(c*ProductLog[a + b*x]))^(3/2),x]
```

output
```
((a + b*x)*(-(c*ProductLog[a + b*x]))^(3/2))/b - (3*(-((c*(a + b*x)*Sqrt[-
(c*ProductLog[a + b*x])])/b) + (3*c*(-1/2*(Sqrt[c]*Sqrt[Pi]*Erf[Sqrt[-(c*P
roductLog[a + b*x])]/Sqrt[c]])/b - (c*(a + b*x))/(b*Sqrt[-(c*ProductLog[a
+ b*x])])))/2))/2
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7178
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[c*(a + b*x)*((c*ProductLog[a + b*x])^
(p - 1)/(b*d)), x] - Simp[c*p   Int[(c*ProductLog[a + b*x])^(p - 1)/(d + d*
ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0]
```

rule 7181
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[(-Pi)*c, 2]*(Erf[Sqrt[c*Product
Log[a + b*x]]/Rt[-c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && NegQ[c]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

method	result
default	$\frac{-\frac{c(-c\,\mathrm{LambertW}(bx+a))^{\frac{5}{2}}(bx+a)}{\mathrm{LambertW}(bx+a)}+3c\left(-\frac{c(-c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}(bx+a)}{2\,\mathrm{LambertW}(bx+a)}+\frac{3c\left(-\frac{c\sqrt{-c\,\mathrm{LambertW}(bx+a)}\,(bx+a)}{2\,\mathrm{LambertW}(bx+a)}+\frac{c^{\frac{3}{2}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-c\,\mathrm{Lambe}}}{\sqrt{}}\right.}{4}\right.}{2}\right.}{b\,c^{2}}$

input
```
int((-c*LambertW(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(-1/2*c*(-c*LambertW(b*x+a))^(5/2)*(b*x+a)/LambertW(b*x+a)+3/2*c*(
-1/2*c*(-c*LambertW(b*x+a))^(3/2)*(b*x+a)/LambertW(b*x+a)+3/2*c*(-1/2*c*(-
c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)+1/4*c^(3/2)*Pi^(1/2)*erf(
(-c*LambertW(b*x+a))^(1/2)/c^(1/2)))))
```

Fricas [F]

$$\int (-cW(a+bx))^{3/2}\,dx=\int (-c\,\mathrm{W}(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((-c*lambert_w(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*lambert_w(b*x + a))*c*lambert_w(b*x + a), x)
```

Sympy [F]

$$\int (-cW(a+bx))^{3/2}\,dx=\int (-cW(a+bx))^{\frac{3}{2}}\,dx$$

input
```
integrate((-c*LambertW(b*x+a))**(3/2),x)
```

output
```
Integral((-c*LambertW(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int (-cW(a+bx))^{3/2}\,dx=\int (-c\,\mathrm{W}(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((-c*lambert_w(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-c*lambert_w(b*x + a))^(3/2), x)
```

Giac [F]

$$\int (-cW(a+bx))^{3/2}\,dx=\int (-c\,\mathrm{W}(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((-c*lambert_w(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*lambert_w(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (-cW(a+bx))^{3/2}\,dx=\int (-c\,\mathrm{LambertW}(a+b\,x))^{3/2}\,dx$$

input
```
int((-c*LambertW(a + b*x))^(3/2),x)
```

output
```
int((-c*LambertW(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int (-cW(a+bx))^{3/2}\,dx=-\sqrt{c}\left(\int \sqrt{lambert\_w(bx+a)}\,lambert\_w(bx+a)\,dx\right)ci$$

input
```
int((-c*Lambert_W(b*x+a))^(3/2),x)
```

output
```
- sqrt(c)*int(sqrt(lambert_w(a + b*x))*lambert_w(a + b*x),x)*c*i
```

3.379 $\int \sqrt{-cW(a+bx)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 86

$$\int \sqrt{-cW(a+bx)}\,dx = \frac{\sqrt{c}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{4b} + \frac{c(a+bx)}{2b\sqrt{-cW(a+bx)}} + \frac{(a+bx)\sqrt{-cW(a+bx)}}{b}$$

output
```
1/4*c^(1/2)*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2))/b+1/2*c*(b*x+
a)/b/(-c*LambertW(b*x+a))^(1/2)+(b*x+a)*(-c*LambertW(b*x+a))^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\int \sqrt{-cW(a+bx)}\,dx = -\frac{c\left(-2(a+bx)+\sqrt{\pi}\operatorname{erfi}\left(\sqrt{W(a+bx)}\right)\sqrt{W(a+bx)}+4(a+bx)W(a+bx)\right)}{4b\sqrt{-cW(a+bx)}}$$

input
```
Integrate[Sqrt[-(c*ProductLog[a + b*x])],x]
```

output
```
-1/4*(c*(-2*(a + b*x) + Sqrt[Pi]*Erfi[Sqrt[ProductLog[a + b*x]]]*Sqrt[Prod
uctLog[a + b*x]] + 4*(a + b*x)*ProductLog[a + b*x]))/(b*Sqrt[-(c*ProductLo
g[a + b*x])])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7167, 7178, 7181}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-cW(a+bx)}\,dx$$

$$\Big\downarrow \text{7167}$$

$$\frac{(a+bx)\sqrt{-cW(a+bx)}}{b}-\frac{1}{2}\int \frac{\sqrt{-cW(a+bx)}}{W(a+bx)+1}dx$$

$$\Big\downarrow \text{7178}$$

$$\frac{1}{2}\left(\frac{c(a+bx)}{b\sqrt{-cW(a+bx)}}-\frac{1}{2}c\int \frac{1}{\sqrt{-cW(a+bx)}(W(a+bx)+1)}dx\right)+\frac{(a+bx)\sqrt{-cW(a+bx)}}{b}$$

$$\Big\downarrow \text{7181}$$

$$\frac{1}{2}\left(\frac{\sqrt{\pi}\sqrt{c}\,\text{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{2b}+\frac{c(a+bx)}{b\sqrt{-cW(a+bx)}}\right)+\frac{(a+bx)\sqrt{-cW(a+bx)}}{b}$$

input
```
Int[Sqrt[-(c*ProductLog[a + b*x])],x]
```

output
```
((a + b*x)*Sqrt[-(c*ProductLog[a + b*x])])/b + ((Sqrt[c]*Sqrt[Pi]*Erf[Sqrt
[-(c*ProductLog[a + b*x])]/Sqrt[c]])/(2*b) + (c*(a + b*x))/(b*Sqrt[-(c*Pro
ductLog[a + b*x])]))/2
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7178
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[c*(a + b*x)*((c*ProductLog[a + b*x])^
(p - 1)/(b*d)), x] - Simp[c*p   Int[(c*ProductLog[a + b*x])^(p - 1)/(d + d*
ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0]
```

rule 7181
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[(-Pi)*c, 2]*(Erf[Sqrt[c*Product
Log[a + b*x]]/Rt[-c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && NegQ[c]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{-\frac{c(-c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}(bx+a)}{\mathrm{LambertW}(bx+a)}+c\left(-\frac{c\sqrt{-c\,\mathrm{LambertW}(bx+a)}\,(bx+a)}{2\,\mathrm{LambertW}(bx+a)}+\frac{c^{\frac{3}{2}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-c\,\mathrm{LambertW}(bx+a)}}{\sqrt{c}}\right)}{4}\right)}{b\,c^{2}}$	92

input
```
int((-c*LambertW(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(-1/2*c*(-c*LambertW(b*x+a))^(3/2)*(b*x+a)/LambertW(b*x+a)+1/2*c*(
-1/2*c*(-c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)+1/4*c^(3/2)*Pi^(
1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2))))
```

Fricas [F]

$$\int \sqrt{-cW(a+bx)}\,dx = \int \sqrt{-c\,\mathrm{W}(bx+a)}\,dx$$

input
```
integrate((-c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*lambert_w(b*x + a)), x)
```

Sympy [F]

$$\int \sqrt{-cW(a+bx)}\,dx = \int \sqrt{-cW\left(a+bx\right)}\,dx$$

input
```
integrate((-c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(-c*LambertW(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{-cW(a+bx)}\,dx = \int \sqrt{-c\,\mathrm{W}(bx+a)}\,dx$$

input
```
integrate((-c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*lambert_w(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{-cW(a+bx)}\,dx = \int \sqrt{-c\,\mathrm{W}(bx+a)}\,dx$$

input
```
integrate((-c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*lambert_w(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{-cW(a+bx)}\,dx = \int \sqrt{-c\,\mathrm{LambertW}\left(a+b\,x\right)}\,dx$$

input
```
int((-c*LambertW(a + b*x))^(1/2),x)
```

output
```
int((-c*LambertW(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{-cW(a+bx)}\,dx = \sqrt{c}\left(\int \sqrt{lambert\_w(bx+a)}dx\right)i$$

input
```
int((-c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a + b*x)),x)*i
```

3.380 $\int \frac{1}{\sqrt{-cW(a+bx)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{1}{\sqrt{-cW(a+bx)}}\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{2b\sqrt{c}} + \frac{a+bx}{b\sqrt{-cW(a+bx)}}$$

output
```
-1/2*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2))/b/c^(1/2)+(b*x+a)/b/
(-c*LambertW(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-cW(a+bx)}}\,dx = \frac{2(a+bx)+\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{W(a+bx)}\right)\sqrt{W(a+bx)}}{2b\sqrt{-cW(a+bx)}}$$

input
```
Integrate[1/Sqrt[-(c*ProductLog[a + b*x])],x]
```

output
```
(2*(a + b*x) + Sqrt[Pi]*Erfi[Sqrt[ProductLog[a + b*x]]]*Sqrt[ProductLog[a
+ b*x]])/(2*b*Sqrt[-(c*ProductLog[a + b*x])])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {7167, 7181}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-cW(a+bx)}}dx$$
$$\Big\downarrow \text{ 7167}$$
$$\frac{1}{2}\int \frac{1}{\sqrt{-cW(a+bx)}(W(a+bx)+1)}dx+\frac{a+bx}{b\sqrt{-cW(a+bx)}}$$
$$\Big\downarrow \text{ 7181}$$
$$\frac{a+bx}{b\sqrt{-cW(a+bx)}}-\frac{\sqrt{\pi}\text{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{2b\sqrt{c}}$$

input
```
Int[1/Sqrt[-(c*ProductLog[a + b*x])],x]
```

output
```
-1/2*(Sqrt[Pi]*Erf[Sqrt[-(c*ProductLog[a + b*x])]/Sqrt[c]])/(b*Sqrt[c]) +
(a + b*x)/(b*Sqrt[-(c*ProductLog[a + b*x])])
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7181
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[(-Pi)*c, 2]*(Erf[Sqrt[c*Product
Log[a + b*x]]/Rt[-c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && NegQ[c]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{-\frac{c\sqrt{-c\,\mathrm{LambertW}(bx+a)}\,(bx+a)}{\mathrm{LambertW}(bx+a)}-\frac{c^{\frac{3}{2}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-c\,\mathrm{LambertW}(bx+a)}}{\sqrt{c}}\right)}{2}}{b\,c^{2}}$	61

input
```
int(1/(-c*LambertW(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(-1/2*c*(-c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)-1/4*c^(
3/2)*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2)))
```

Fricas [F]

$$\int \frac{1}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{-c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(1/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*lambert_w(b*x + a))/(c*lambert_w(b*x + a)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{-cW\left(a+bx\right)}}\,dx$$

input
```
integrate(1/(-c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(1/sqrt(-c*LambertW(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-cW(a+bx)}}\,dx=\int \frac{1}{\sqrt{-c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(1/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-c*lambert_w(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-cW(a+bx)}}\,dx=\int \frac{1}{\sqrt{-c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(1/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-c*lambert_w(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-cW(a+bx)}}\,dx=\int \frac{1}{\sqrt{-c\,\mathrm{LambertW}\left(a+b\,x\right)}}\,dx$$

input
```
int(1/(-c*LambertW(a + b*x))^(1/2),x)
```

output
```
int(1/(-c*LambertW(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-cW(a+bx)}}\,dx = -\frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}}{lambert\_w(bx+a)}dx\right)i}{c}$$

input
```
int(1/(-c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
( - sqrt(c)*int(sqrt(lambert_w(a + b*x))/lambert_w(a + b*x),x)*i)/c
```

3.381 $\int \frac{1}{(-cW(a+bx))^{3/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{1}{(-cW(a+bx))^{3/2}} dx = \frac{3\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{bc^{3/2}} - \frac{2(a+bx)}{b(-cW(a+bx))^{3/2}}$$

output
```
3*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2))/b/c^(3/2)-2*(b*x+a)/b/(
-c*LambertW(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-cW(a+bx))^{3/2}} dx = -\frac{2(a+bx) - 3\sqrt{\pi}\operatorname{erfi}\left(\sqrt{W(a+bx)}\right) W(a+bx)^{3/2}}{b(-cW(a+bx))^{3/2}}$$

input
```
Integrate[(-(c*ProductLog[a + b*x]))^(-3/2),x]
```

output
```
-((2*(a + b*x) - 3*Sqrt[Pi]*Erfi[Sqrt[ProductLog[a + b*x]]]*ProductLog[a +
 b*x]^(3/2))/(b*(-(c*ProductLog[a + b*x]))^(3/2)))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {7166, 7181}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-cW(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \; 7166$$

$$-\frac{3\int \frac{1}{\sqrt{-cW(a+bx)}(W(a+bx)+1)}dx}{c}-\frac{2(a+bx)}{b(-cW(a+bx))^{3/2}}$$

$$\Big\downarrow \; 7181$$

$$\frac{3\sqrt{\pi}\text{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{bc^{3/2}}-\frac{2(a+bx)}{b(-cW(a+bx))^{3/2}}$$

input
```
Int[(-(c*ProductLog[a + b*x]))^(-3/2),x]
```

output
```
(3*Sqrt[Pi]*Erf[Sqrt[-(c*ProductLog[a + b*x])]/Sqrt[c]])/(b*c^(3/2)) - (2*
(a + b*x))/(b*(-(c*ProductLog[a + b*x]))^(3/2))
```

Defintions of rubi rules used

rule 7166
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_), x_Symbol] :> Simp[(a + b*x
)*((c*ProductLog[a + b*x])^p/(b*(p + 1))), x] + Simp[p/(c*(p + 1))   Int[(c
*ProductLog[a + b*x])^(p + 1)/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a
, b, c}, x] && LtQ[p, -1]
```

rule 7181
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[(-Pi)*c, 2]*(Erf[Sqrt[c*Product
Log[a + b*x]]/Rt[-c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && NegQ[c]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{\sqrt{\pi}\,\sqrt{c}\,\operatorname{erf}\left(\frac{\sqrt{-c\,\mathrm{LambertW}(bx+a)}}{\sqrt{c}}\right)-2c\left(-\frac{bx+a}{\sqrt{-c\,\mathrm{LambertW}(bx+a)}\,\mathrm{LambertW}(bx+a)}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-c\,\mathrm{LambertW}(bx+a)}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{b\,c^{2}}$	88

input
```
int(1/(-c*LambertW(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(1/2*Pi^(1/2)*c^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2))-c*(-
1/(-c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)-1/c^(1/2)*Pi^(1/2)*er
f((-c*LambertW(b*x+a))^(1/2)/c^(1/2))))
```

Fricas [F]

$$\int \frac{1}{(-cW(a+bx))^{3/2}}\,dx = \int \frac{1}{(-c\,\mathrm{W}(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*lambert_w(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*lambert_w(b*x + a))/(c^2*lambert_w(b*x + a)^2), x)
```

Sympy [F]

$$\int \frac{1}{(-cW(a+bx))^{3/2}}\,dx = \int \frac{1}{(-cW(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*LambertW(b*x+a))**(3/2),x)
```

output
```
Integral((-c*LambertW(a + b*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-cW(a+bx))^{3/2}}\,dx = \int \frac{1}{(-c\,\mathrm{W}(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*lambert_w(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-c*lambert_w(b*x + a))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-cW(a+bx))^{3/2}}\,dx = \int \frac{1}{(-c\,\mathrm{W}(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*lambert_w(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*lambert_w(b*x + a))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-cW(a+bx))^{3/2}}\,dx = \int \frac{1}{(-c\,\mathrm{LambertW}\,(a+b\,x))^{3/2}}\,dx$$

input
```
int(1/(-c*LambertW(a + b*x))^(3/2),x)
```

output
```
int(1/(-c*LambertW(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-cW(a+bx))^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}}{lambert\_w(bx+a)^2}dx\right)i}{c^2}$$

input
```
int(1/(-c*Lambert_W(b*x+a))^(3/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a + b*x))/lambert_w(a + b*x)**2,x)*i)/c**2
```

3.382 $\int \frac{1}{(-cW(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 91

$$\int \frac{1}{(-cW(a+bx))^{5/2}}\,dx = -\frac{10\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{3bc^{5/2}} - \frac{2(a+bx)}{3b(-cW(a+bx))^{5/2}} + \frac{10(a+bx)}{3bc(-cW(a+bx))^{3/2}}$$

output
```
-10/3*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2))/b/c^(5/2)-2/3*(b*x+
a)/b/(-c*LambertW(b*x+a))^(5/2)+10/3*(b*x+a)/b/c/(-c*LambertW(b*x+a))^(3/2
)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.76

$$\int \frac{1}{(-cW(a+bx))^{5/2}}\,dx = \frac{2\left(-a-bx-5(a+bx)W(a+bx)+5\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{W(a+bx)}\right)W(a+bx)^{5/2}\right)}{3b(-cW(a+bx))^{5/2}}$$

input
```
Integrate[(-(c*ProductLog[a + b*x]))^(-5/2),x]
```

output
```
(2*(-a - b*x - 5*(a + b*x)*ProductLog[a + b*x] + 5*Sqrt[Pi]*Erfi[Sqrt[Prod
uctLog[a + b*x]]]*ProductLog[a + b*x]^(5/2)))/(3*b*(-(c*ProductLog[a + b*x
]))^(5/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7166, 7182, 7181}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-cW(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{7166}$$

$$-\frac{5\int \frac{1}{(-cW(a+bx))^{3/2}(W(a+bx)+1)}dx}{3c}-\frac{2(a+bx)}{3b(-cW(a+bx))^{5/2}}$$

$$\Big\downarrow \text{7182}$$

$$-\frac{5\left(-\frac{2\int \frac{1}{\sqrt{-cW(a+bx)}(W(a+bx)+1)}dx}{c}-\frac{2(a+bx)}{b(-cW(a+bx))^{3/2}}\right)}{3c}-\frac{2(a+bx)}{3b(-cW(a+bx))^{5/2}}$$

$$\Big\downarrow \text{7181}$$

$$-\frac{5\left(\frac{2\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{bc^{3/2}}-\frac{2(a+bx)}{b(-cW(a+bx))^{3/2}}\right)}{3c}-\frac{2(a+bx)}{3b(-cW(a+bx))^{5/2}}$$

input
```
Int[(-(c*ProductLog[a + b*x]))^(-5/2),x]
```

output
```
(-2*(a + b*x))/(3*b*(-(c*ProductLog[a + b*x]))^(5/2)) - (5*((2*Sqrt[Pi]*Er
f[Sqrt[-(c*ProductLog[a + b*x])]/Sqrt[c]])/(b*c^(3/2)) - (2*(a + b*x))/(b*
(-(c*ProductLog[a + b*x]))^(3/2))))/(3*c)
```

Defintions of rubi rules used

rule 7166
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_), x_Symbol] :> Simp[(a + b*x
)*((c*ProductLog[a + b*x])^p/(b*(p + 1))), x] + Simp[p/(c*(p + 1))   Int[(c
*ProductLog[a + b*x])^(p + 1)/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a
, b, c}, x] && LtQ[p, -1]
```

rule 7181
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[(-Pi)*c, 2]*(Erf[Sqrt[c*Product
Log[a + b*x]]/Rt[-c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && NegQ[c]
```

rule 7182
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[(a + b*x)*((c*ProductLog[a + b*x])^p/
(b*d*(p + 1))), x] - Simp[1/(c*(p + 1))   Int[(c*ProductLog[a + b*x])^(p +
1)/(d + d*ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p,
-1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.60

method	result
default	$\frac{-\frac{2(bx+a)}{\sqrt{-c\,\mathrm{LambertW}(bx+a)}\,\mathrm{LambertW}(bx+a)}-\frac{2\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-c\,\mathrm{LambertW}(bx+a)}}{\sqrt{c}}\right)}{\sqrt{c}}-2c\left(-\frac{bx+a}{3(-c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}\,\mathrm{LambertW}(bx+a)}-\frac{2\left(-\cdots\right.}{\cdots}\right.}{b\,c^{2}}$

input
```
int(1/(-c*LambertW(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(-1/(-c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)-1/c^(1/2)*P
i^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2))-c*(-1/3/(-c*LambertW(b*x+a
))^(3/2)*(b*x+a)/LambertW(b*x+a)-2/3/c*(-1/(-c*LambertW(b*x+a))^(1/2)*(b*x
+a)/LambertW(b*x+a)-1/c^(1/2)*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1
/2))))
```

Fricas [F]

$$\int \frac{1}{(-cW(a+bx))^{5/2}}\,dx = \int \frac{1}{(-c\,\mathrm{W}(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-c*lambert_w(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*lambert_w(b*x + a))/(c^3*lambert_w(b*x + a)^3), x)
```

Sympy [F]

$$\int \frac{1}{(-cW(a+bx))^{5/2}}\,dx = \int \frac{1}{(-cW\,(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-c*LambertW(b*x+a))**(5/2),x)
```

output
```
Integral((-c*LambertW(a + b*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(-cW(a+bx))^{5/2}}\,dx = \int \frac{1}{(-c\,\mathrm{W}(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-c*lambert_w(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((-c*lambert_w(b*x + a))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(-cW(a+bx))^{5/2}}\,dx = \int \frac{1}{(-c\,\mathrm{W}(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-c*lambert_w(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((-c*lambert_w(b*x + a))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-cW(a+bx))^{5/2}}\,dx = \int \frac{1}{(-c\,\mathrm{LambertW}\,(a+b\,x))^{5/2}}\,dx$$

input
```
int(1/(-c*LambertW(a + b*x))^(5/2),x)
```

output
```
int(1/(-c*LambertW(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(-cW(a+bx))^{5/2}}\,dx = -\frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}}{lambert\_w(bx+a)^3}dx\right)i}{c^3}$$

input
```
int(1/(-c*Lambert_W(b*x+a))^(5/2),x)
```

output
```
( - sqrt(c)*int(sqrt(lambert_w(a + b*x))/lambert_w(a + b*x)**3,x)*i)/c**3
```

3.383 $\int \frac{1}{(-cW(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 119

$$\int \frac{1}{(-cW(a+bx))^{7/2}}\,dx = \frac{28\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{15bc^{7/2}} - \frac{2(a+bx)}{5b(-cW(a+bx))^{7/2}} + \frac{14(a+bx)}{15bc(-cW(a+bx))^{5/2}} - \frac{28(a+bx)}{15bc^2(-cW(a+bx))^{3/2}}$$

output
```
28/15*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2))/b/c^(7/2)-2/5*(b*x+
a)/b/(-c*LambertW(b*x+a))^(7/2)+14/15*(b*x+a)/b/c/(-c*LambertW(b*x+a))^(5/
2)-28/15*(b*x+a)/b/c^2/(-c*LambertW(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.80

$$\int \frac{1}{(-cW(a+bx))^{7/2}}\,dx = \frac{6(a+bx) + 14(a+bx)W(a+bx) + 28(a+bx)W(a+bx)^2 - 28\sqrt{\pi}\operatorname{erfi}\left(\sqrt{W(\right.}{15bc^3W(a+bx)^3\sqrt{-cW(a+bx)}}$$

input
```
Integrate[(-(c*ProductLog[a + b*x]))^(-7/2),x]
```

output
```
(6*(a + b*x) + 14*(a + b*x)*ProductLog[a + b*x] + 28*(a + b*x)*ProductLog[
a + b*x]^2 - 28*Sqrt[Pi]*Erfi[Sqrt[ProductLog[a + b*x]]]*ProductLog[a + b*
x]^(7/2))/(15*b*c^3*ProductLog[a + b*x]^3*Sqrt[-(c*ProductLog[a + b*x])])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {7166, 7182, 7182, 7181}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-cW(a+bx))^{7/2}}\,dx$$

$$\Big\downarrow \text{7166}$$

$$-\frac{7\int \frac{1}{(-cW(a+bx))^{5/2}(W(a+bx)+1)}dx}{5c}-\frac{2(a+bx)}{5b(-cW(a+bx))^{7/2}}$$

$$\Big\downarrow \text{7182}$$

$$-\frac{7\left(-\frac{2\int \frac{1}{(-cW(a+bx))^{3/2}(W(a+bx)+1)}dx}{3c}-\frac{2(a+bx)}{3b(-cW(a+bx))^{5/2}}\right)}{5c}-\frac{2(a+bx)}{5b(-cW(a+bx))^{7/2}}$$

$$\Big\downarrow \text{7182}$$

$$-\frac{7\left(-\frac{2\left(-\frac{2\int \frac{1}{\sqrt{-cW(a+bx)}(W(a+bx)+1)}dx}{c}-\frac{2(a+bx)}{b(-cW(a+bx))^{3/2}}\right)}{3c}-\frac{2(a+bx)}{3b(-cW(a+bx))^{5/2}}\right)}{5c}-\frac{2(a+bx)}{5b(-cW(a+bx))^{7/2}}$$

$$\Big\downarrow \text{7181}$$

$$-\frac{7\left(-\frac{2\left(\frac{2\sqrt{\pi}\text{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{bc^{3/2}}-\frac{2(a+bx)}{b(-cW(a+bx))^{3/2}}\right)}{3c}-\frac{2(a+bx)}{3b(-cW(a+bx))^{5/2}}\right)}{5c}-\frac{2(a+bx)}{5b(-cW(a+bx))^{7/2}}$$

input
```
Int[(-(c*ProductLog[a + b*x]))^(-7/2),x]
```

output
```
(-2*(a + b*x))/(5*b*(-(c*ProductLog[a + b*x]))^(7/2)) - (7*((-2*(a + b*x))
/(3*b*(-(c*ProductLog[a + b*x]))^(5/2)) - (2*((2*Sqrt[Pi]*Erf[Sqrt[-(c*Pro
ductLog[a + b*x])]/Sqrt[c]])/(b*c^(3/2)) - (2*(a + b*x))/(b*(-(c*ProductLo
g[a + b*x]))^(3/2))))/(3*c)))/(5*c)
```

Defintions of rubi rules used

rule 7166
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_), x_Symbol] :> Simp[(a + b*x
)*((c*ProductLog[a + b*x])^p/(b*(p + 1))), x] + Simp[p/(c*(p + 1))   Int[(c
*ProductLog[a + b*x])^(p + 1)/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a
, b, c}, x] && LtQ[p, -1]
```

rule 7181
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[(-Pi)*c, 2]*(Erf[Sqrt[c*Product
Log[a + b*x]]/Rt[-c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && NegQ[c]
```

rule 7182
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[(a + b*x)*((c*ProductLog[a + b*x])^p/
(b*d*(p + 1))), x] - Simp[1/(c*(p + 1))   Int[(c*ProductLog[a + b*x])^(p +
1)/(d + d*ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(97) = 194$.

Time = 0.05 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.76

method	result
default	$\frac{-\frac{2(bx+a)}{3(-c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}\,\mathrm{LambertW}(bx+a)}-\frac{4\left(-\frac{bx+a}{\sqrt{-c\,\mathrm{LambertW}(bx+a)}\,\mathrm{LambertW}(bx+a)}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-c\,\mathrm{LambertW}(bx+a)}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{3c}-2c\left(-\frac{\cdots}{5\cdots}\right.}{\cdots}$

input
```
int(1/(-c*LambertW(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(-1/3/(-c*LambertW(b*x+a))^(3/2)*(b*x+a)/LambertW(b*x+a)-2/3/c*(-1
/(-c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)-1/c^(1/2)*Pi^(1/2)*erf
((-c*LambertW(b*x+a))^(1/2)/c^(1/2)))-c*(-1/5/(-c*LambertW(b*x+a))^(5/2)*(
b*x+a)/LambertW(b*x+a)-2/5/c*(-1/3/(-c*LambertW(b*x+a))^(3/2)*(b*x+a)/Lamb
ertW(b*x+a)-2/3/c*(-1/(-c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)-1
/c^(1/2)*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2))))))
```

Fricas [F]

$$\int \frac{1}{(-cW(a+bx))^{7/2}}\,dx = \int \frac{1}{(-c\,\mathrm{W}(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(-c*lambert_w(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*lambert_w(b*x + a))/(c^4*lambert_w(b*x + a)^4), x)
```

Sympy [F]

$$\int \frac{1}{(-cW(a+bx))^{7/2}}\,dx = \int \frac{1}{(-cW(a+bx))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(-c*LambertW(b*x+a))**(7/2),x)
```

output
```
Integral((-c*LambertW(a + b*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(-cW(a+bx))^{7/2}}\,dx = \int \frac{1}{(-c\,\mathrm{W}(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(-c*lambert_w(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
integrate((-c*lambert_w(b*x + a))^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(-cW(a+bx))^{7/2}}\,dx = \int \frac{1}{(-c\,\mathrm{W}(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(-c*lambert_w(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate((-c*lambert_w(b*x + a))^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-cW(a+bx))^{7/2}}\,dx = \int \frac{1}{\left(-c\,\mathrm{LambertW}\left(a+b\,x\right)\right)^{7/2}}\,dx$$

input
```
int(1/(-c*LambertW(a + b*x))^(7/2),x)
```

output
```
int(1/(-c*LambertW(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(-cW(a+bx))^{7/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}}{lambert\_w(bx+a)^4}dx\right)i}{c^4}$$

input
```
int(1/(-c*Lambert_W(b*x+a))^(7/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a + b*x))/lambert_w(a + b*x)**4,x)*i)/c**4
```

3.384 $\int (cW(a+bx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int (cW(a+bx))^n\,dx = \frac{(a+bx)(cW(a+bx))^n}{b} - \frac{n\Gamma(1+n,-W(a+bx))(-W(a+bx))^{-n}(cW(a+bx))^n}{b}$$

output
```
(b*x+a)*(c*LambertW(b*x+a))^n/b-n*GAMMA(1+n,-LambertW(b*x+a))*(c*LambertW(
b*x+a))^n/b/((-LambertW(b*x+a))^n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int (cW(a+bx))^n\,dx = \frac{(\Gamma(1+n,-W(a+bx)) - \Gamma(2+n,-W(a+bx)))(-W(a+bx))^{-n}(cW(a+bx))^n}{b}$$

input
```
Integrate[(c*ProductLog[a + b*x])^n,x]
```

output
```
((Gamma[1 + n, -ProductLog[a + b*x]] - Gamma[2 + n, -ProductLog[a + b*x]])
*(c*ProductLog[a + b*x])^n)/(b*(-ProductLog[a + b*x])^n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7167, 7183}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cW(a+bx))^n \, dx$$

$$\Big\downarrow \text{ 7167}$$

$$\frac{(a+bx)(cW(a+bx))^n}{b} - n\int \frac{(cW(a+bx))^n}{W(a+bx)+1}dx$$

$$\Big\downarrow \text{ 7183}$$

$$\frac{(a+bx)(cW(a+bx))^n}{b} - \frac{n(-W(a+bx))^{-n}(cW(a+bx))^n\Gamma(n+1,-W(a+bx))}{b}$$

input
```
Int[(c*ProductLog[a + b*x])^n,x]
```

output
```
((a + b*x)*(c*ProductLog[a + b*x])^n)/b - (n*Gamma[1 + n, -ProductLog[a +
b*x]]*(c*ProductLog[a + b*x])^n)/(b*(-ProductLog[a + b*x])^n)
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] && !LtQ[p, -1]
```

rule 7183
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.)/((d_) + (d_.)*ProductLog[(
a_.) + (b_.)*(x_)]), x_Symbol] :> Simp[Gamma[p + 1, -ProductLog[a + b*x]]*(
(c*ProductLog[a + b*x])^p/(b*d*(-ProductLog[a + b*x])^p)), x] /; FreeQ[{a,
b, c, d, p}, x]
```

Maple [F]

$$\int (c\,\mathrm{LambertW}\left(bx+a\right))^{n}\,dx$$

input
```
int((c*LambertW(b*x+a))^n,x)
```

output
```
int((c*LambertW(b*x+a))^n,x)
```

Fricas [F]

$$\int (cW(a+bx))^n\,dx = \int \left(c\,\mathrm{W}(bx+a)\right)^n\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((c*lambert_w(b*x + a))^n, x)
```

Sympy [F]

$$\int (cW(a+bx))^n\,dx = \int \left(cW(a+bx)\right)^n\,dx$$

input
```
integrate((c*LambertW(b*x+a))**n,x)
```

output
```
Integral((c*LambertW(a + b*x))**n, x)
```

Maxima [F]

$$\int (cW(a+bx))^n\,dx = \int (c\,\mathrm{W}(bx+a))^n\;dx$$

input
```
integrate((c*lambert_w(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((c*lambert_w(b*x + a))^n, x)
```

Giac [F]

$$\int (cW(a+bx))^n\,dx = \int (c\,\mathrm{W}(bx+a))^n\;dx$$

input
```
integrate((c*lambert_w(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((c*lambert_w(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cW(a+bx))^n\,dx = \int (c\,\mathrm{LambertW}(a+b\,x))^n\,dx$$

input
```
int((c*LambertW(a + b*x))^n,x)
```

output
```
int((c*LambertW(a + b*x))^n, x)
```

Reduce [F]

$$\int (cW(a+bx))^n\,dx = c^n\left(\int lambert\_w(bx+a)^n\,dx\right)$$

input
```
int((c*Lambert_W(b*x+a))^n,x)
```

output
```
c**n*int(lambert_w(a + b*x)**n,x)
```

3.385 $\int x^3 W(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 364

$$\int x^3W(a+bx)\,dx=\frac{a^3x}{b^3}-\frac{3a^2(a+bx)^2}{4b^4}+\frac{a(a+bx)^3}{3b^4}-\frac{(a+bx)^4}{16b^4}-\frac{3(a+bx)^4}{512b^4W(a+bx)^4}-\frac{2a(a+bx)^3}{27b^4W(a+bx)^3}+\frac{3(a+bx)^4}{128b^4W(a+bx)^3}-\frac{3a^2(a+bx)^2}{8b^4W(a+bx)^2}+\frac{2a(a+bx)^3}{9b^4W(a+bx)^2}-\frac{3(a+bx)^4}{64b^4W(a+bx)^2}-\frac{a^3(a+bx)}{b^4W(a+bx)}+\frac{3a^2(a+bx)^2}{4b^4W(a+bx)}-\frac{a(a+bx)^3}{3b^4W(a+bx)}+\frac{(a+bx)^4}{16b^4W(a+bx)}-\frac{a^3(a+bx)W(a+bx)}{b^4}+\frac{3a^2(a+bx)^2W(a+bx)}{2b^4}-\frac{a(a+bx)^3W(a+bx)}{b^4}+\frac{(a+bx)^4W(a+bx)}{4b^4}$$

output
```
a^3*x/b^3-3/4*a^2*(b*x+a)^2/b^4+1/3*a*(b*x+a)^3/b^4-1/16*(b*x+a)^4/b^4-3/5
12*(b*x+a)^4/b^4/LambertW(b*x+a)^4-2/27*a*(b*x+a)^3/b^4/LambertW(b*x+a)^3+
3/128*(b*x+a)^4/b^4/LambertW(b*x+a)^3-3/8*a^2*(b*x+a)^2/b^4/LambertW(b*x+a
)^2+2/9*a*(b*x+a)^3/b^4/LambertW(b*x+a)^2-3/64*(b*x+a)^4/b^4/LambertW(b*x+
a)^2-a^3*(b*x+a)/b^4/LambertW(b*x+a)+3/4*a^2*(b*x+a)^2/b^4/LambertW(b*x+a)
-1/3*a*(b*x+a)^3/b^4/LambertW(b*x+a)+1/16*(b*x+a)^4/b^4/LambertW(b*x+a)-a^
3*(b*x+a)*LambertW(b*x+a)/b^4+3/2*a^2*(b*x+a)^2*LambertW(b*x+a)/b^4-a*(b*x
+a)^3*LambertW(b*x+a)/b^4+1/4*(b*x+a)^4*LambertW(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.58

$$\int x^3W(a+bx)\,dx =$$
$$-\frac{(a+bx)\,(81(a+bx)^3+4(175a-81bx)(a+bx)^2W(a+bx)+24(115a^3+41a^2bx-47ab^2x^2+27b^3x^3$$

input
```
Integrate[x^3*ProductLog[a + b*x],x]
```

output
```
-1/13824*((a + b*x)*(81*(a + b*x)^3 + 4*(175*a - 81*b*x)*(a + b*x)^2*Produ
ctLog[a + b*x] + 24*(115*a^3 + 41*a^2*b*x - 47*a*b^2*x^2 + 27*b^3*x^3)*Pro
ductLog[a + b*x]^2 + 288*(25*a^3 - 13*a^2*b*x + 7*a*b^2*x^2 - 3*b^3*x^3)*P
roductLog[a + b*x]^3 - 288*(25*a^3 - 13*a^2*b*x + 7*a*b^2*x^2 - 3*b^3*x^3)
*ProductLog[a + b*x]^4 + 3456*(a^3 - a^2*b*x + a*b^2*x^2 - b^3*x^3)*Produc
tLog[a + b*x]^5))/(b^4*ProductLog[a + b*x]^4)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.87, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3W(a+bx)\,dx$$
$$\Big\downarrow \text{ 7168}$$
$$\frac{\int \left(-W(a+bx)a^3+3(a+bx)W(a+bx)a^2-3(a+bx)^2W(a+bx)a+(a+bx)^3W(a+bx)\right)d(a+bx)}{b^4}$$
$$\Big\downarrow \text{ 2009}$$
$$-a^3(a+bx)W(a+bx)-\frac{a^3(a+bx)}{W(a+bx)}+a^3(a+bx)+\frac{3}{2}a^2(a+bx)^2W(a+bx)+\frac{3a^2(a+bx)^2}{4W(a+bx)}-\frac{3a^2(a+bx)^2}{8W(a+bx)^2}-\frac{3}{4}a^2(a+$$

input
```
Int[x^3*ProductLog[a + b*x],x]
```

output
```
(a^3*(a + b*x) - (3*a^2*(a + b*x)^2)/4 + (a*(a + b*x)^3)/3 - (a + b*x)^4/1
6 - (3*(a + b*x)^4)/(512*ProductLog[a + b*x]^4) - (2*a*(a + b*x)^3)/(27*Pr
oductLog[a + b*x]^3) + (3*(a + b*x)^4)/(128*ProductLog[a + b*x]^3) - (3*a^
2*(a + b*x)^2)/(8*ProductLog[a + b*x]^2) + (2*a*(a + b*x)^3)/(9*ProductLog
[a + b*x]^2) - (3*(a + b*x)^4)/(64*ProductLog[a + b*x]^2) - (a^3*(a + b*x)
)/ProductLog[a + b*x] + (3*a^2*(a + b*x)^2)/(4*ProductLog[a + b*x]) - (a*(
a + b*x)^3)/(3*ProductLog[a + b*x]) + (a + b*x)^4/(16*ProductLog[a + b*x])
 - a^3*(a + b*x)*ProductLog[a + b*x] + (3*a^2*(a + b*x)^2*ProductLog[a + b
*x])/2 - a*(a + b*x)^3*ProductLog[a + b*x] + ((a + b*x)^4*ProductLog[a + b
*x])/4)/b^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7168
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((c_.)*ProductLog[(a_) + (b_.)*(x_)])^(p_.),
 x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[(c*ProductLog[x]
)^p, (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, e, f,
p}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{-\frac{(bx+a)^4}{16}+\frac{(bx+a)^4}{16\,\mathrm{LambertW}(bx+a)}-\frac{3(bx+a)^4}{64\,\mathrm{LambertW}(bx+a)^2}+\frac{3(bx+a)^4}{128\,\mathrm{LambertW}(bx+a)^3}-\frac{3(bx+a)^4}{512\,\mathrm{LambertW}(bx+a)^4}+\frac{\mathrm{LambertW}(bx+a)}{4}}{}$
default	$\frac{-\frac{(bx+a)^4}{16}+\frac{(bx+a)^4}{16\,\mathrm{LambertW}(bx+a)}-\frac{3(bx+a)^4}{64\,\mathrm{LambertW}(bx+a)^2}+\frac{3(bx+a)^4}{128\,\mathrm{LambertW}(bx+a)^3}-\frac{3(bx+a)^4}{512\,\mathrm{LambertW}(bx+a)^4}+\frac{\mathrm{LambertW}(bx+a)}{4}}{}$

input
```
int(x^3*LambertW(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(-1/16*(b*x+a)^4+1/16/LambertW(b*x+a)*(b*x+a)^4-3/64*(b*x+a)^4/Lambe
rtW(b*x+a)^2+3/128/LambertW(b*x+a)^3*(b*x+a)^4-3/512*(b*x+a)^4/LambertW(b*
x+a)^4+1/4*LambertW(b*x+a)*(b*x+a)^4-a^3*(b*x+a-(b*x+a)/LambertW(b*x+a))-a
^3*((b*x+a)*LambertW(b*x+a)-2*b*x-2*a+2*(b*x+a)/LambertW(b*x+a))+3*a^2*(1/
2*(b*x+a)^2-1/2/LambertW(b*x+a)*(b*x+a)^2+1/4*(b*x+a)^2/LambertW(b*x+a)^2)
+3*a^2*(1/2*LambertW(b*x+a)*(b*x+a)^2-3/4*(b*x+a)^2+3/4/LambertW(b*x+a)*(b
*x+a)^2-3/8*(b*x+a)^2/LambertW(b*x+a)^2)-3*a*(1/3*(b*x+a)^3-1/3/LambertW(b
*x+a)*(b*x+a)^3+2/9/LambertW(b*x+a)^2*(b*x+a)^3-2/27*(b*x+a)^3/LambertW(b*
x+a)^3)-3*a*(1/3*LambertW(b*x+a)*(b*x+a)^3-4/9*(b*x+a)^3+4/9/LambertW(b*x+
a)*(b*x+a)^3-8/27/LambertW(b*x+a)^2*(b*x+a)^3+8/81*(b*x+a)^3/LambertW(b*x+
a)^3))
```

Fricas [F]

$$\int x^3 W(a+bx)\,dx = \int x^3 \mathrm{W}(bx+a)\,dx$$

input
```
integrate(x^3*lambert_w(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^3*lambert_w(b*x + a), x)
```

Sympy [F]

$$\int x^3 W(a+bx)\,dx = \int x^3 W(a+bx)\;dx$$

input
```
integrate(x**3*LambertW(b*x+a),x)
```

output
```
Integral(x**3*LambertW(a + b*x), x)
```

Maxima [F]

$$\int x^3 W(a+bx)\,dx = \int x^3\,\mathrm{W}(bx+a)\,dx$$

input
```
integrate(x^3*lambert_w(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^3*lambert_w(b*x + a), x)
```

Giac [F]

$$\int x^3 W(a+bx)\,dx = \int x^3\,\mathrm{W}(bx+a)\,dx$$

input
```
integrate(x^3*lambert_w(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*lambert_w(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 W(a+bx)\,dx = \int x^3\,\mathrm{LambertW}(a+b\,x)\,dx$$

input
```
int(x^3*LambertW(a + b*x),x)
```

output
```
int(x^3*LambertW(a + b*x), x)
```

Reduce [F]

$$\int x^3 W(a+bx)\,dx = \int lambert\_w(bx+a)\,x^3 dx$$

input
```
int(x^3*Lambert_W(b*x+a),x)
```

output
```
int(lambert_w(a + b*x)*x**3,x)
```

3.386 $\int x^2W(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 228

$$\begin{aligned}\int x^2W(a+bx)\,dx = &-\frac{a^2x}{b^2}+\frac{a(a+bx)^2}{2b^3}-\frac{(a+bx)^3}{9b^3}+\frac{2(a+bx)^3}{81b^3W(a+bx)^3}\\&+\frac{a(a+bx)^2}{4b^3W(a+bx)^2}-\frac{2(a+bx)^3}{27b^3W(a+bx)^2}+\frac{a^2(a+bx)}{b^3W(a+bx)}\\&-\frac{a(a+bx)^2}{2b^3W(a+bx)}+\frac{(a+bx)^3}{9b^3W(a+bx)}+\frac{a^2(a+bx)W(a+bx)}{b^3}\\&-\frac{a(a+bx)^2W(a+bx)}{b^3}+\frac{(a+bx)^3W(a+bx)}{3b^3}\end{aligned}$$

output

```
-a^2*x/b^2+1/2*a*(b*x+a)^2/b^3-1/9*(b*x+a)^3/b^3+2/81*(b*x+a)^3/b^3/Lamber
tW(b*x+a)^3+1/4*a*(b*x+a)^2/b^3/LambertW(b*x+a)^2-2/27*(b*x+a)^3/b^3/Lambe
rtW(b*x+a)^2+a^2*(b*x+a)/b^3/LambertW(b*x+a)-1/2*a*(b*x+a)^2/b^3/LambertW(
b*x+a)+1/9*(b*x+a)^3/b^3/LambertW(b*x+a)+a^2*(b*x+a)*LambertW(b*x+a)/b^3-a
*(b*x+a)^2*LambertW(b*x+a)/b^3+1/3*(b*x+a)^3*LambertW(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.62

$$\int x^2 W(a+bx)\,dx$$
$$= \frac{(a+bx)\left(8(a+bx)^2+3(19a^2+11abx-8b^2x^2)W(a+bx)+18(11a^2-5abx+2b^2x^2)W(a+bx)^2-18\right.}{324b^3W(a+bx)^3}$$

input
```
Integrate[x^2*ProductLog[a + b*x],x]
```

output
```
((a + b*x)*(8*(a + b*x)^2 + 3*(19*a^2 + 11*a*b*x - 8*b^2*x^2)*ProductLog[a
 + b*x] + 18*(11*a^2 - 5*a*b*x + 2*b^2*x^2)*ProductLog[a + b*x]^2 - 18*(11
*a^2 - 5*a*b*x + 2*b^2*x^2)*ProductLog[a + b*x]^3 + 108*(a^2 - a*b*x + b^2
*x^2)*ProductLog[a + b*x]^4))/(324*b^3*ProductLog[a + b*x]^3)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.88, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 W(a+bx)\,dx$$
$$\Big\downarrow \; 7168$$
$$\frac{\int \left(W(a+bx)a^2-2(a+bx)W(a+bx)a+(a+bx)^2W(a+bx)\right)d(a+bx)}{b^3}$$
$$\Big\downarrow \; 2009$$
$$\frac{a^2(a+bx)W(a+bx)+\frac{a^2(a+bx)}{W(a+bx)}-a^2(a+bx)+\frac{1}{3}(a+bx)^3W(a+bx)+\frac{(a+bx)^3}{9W(a+bx)}-\frac{2(a+bx)^3}{27W(a+bx)^2}+\frac{2(a+bx)^3}{81W(a+bx)^3}-}{b^3}$$

input
```
Int[x^2*ProductLog[a + b*x],x]
```

output
```
(-(a^2*(a + b*x)) + (a*(a + b*x)^2)/2 - (a + b*x)^3/9 + (2*(a + b*x)^3)/(8
1*ProductLog[a + b*x]^3) + (a*(a + b*x)^2)/(4*ProductLog[a + b*x]^2) - (2*
(a + b*x)^3)/(27*ProductLog[a + b*x]^2) + (a^2*(a + b*x))/ProductLog[a + b
*x] - (a*(a + b*x)^2)/(2*ProductLog[a + b*x]) + (a + b*x)^3/(9*ProductLog[
a + b*x]) + a^2*(a + b*x)*ProductLog[a + b*x] - a*(a + b*x)^2*ProductLog[a
 + b*x] + ((a + b*x)^3*ProductLog[a + b*x])/3)/b^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7168
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((c_.)*ProductLog[(a_) + (b_.)*(x_)])^(p_.),
 x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[(c*ProductLog[x]
)^p, (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, e, f,
p}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.11

method	result
derivativedivides	$-\frac{(bx+a)^3}{9}+\frac{(bx+a)^3}{9\,\mathrm{LambertW}(bx+a)}-\frac{2(bx+a)^3}{27\,\mathrm{LambertW}(bx+a)^2}+\frac{2(bx+a)^3}{81\,\mathrm{LambertW}(bx+a)^3}+\frac{\mathrm{LambertW}(bx+a)(bx+a)^3}{3}+a^2\Big(bx+a-\frac{}{\mathrm{Lam}}$
default	$-\frac{(bx+a)^3}{9}+\frac{(bx+a)^3}{9\,\mathrm{LambertW}(bx+a)}-\frac{2(bx+a)^3}{27\,\mathrm{LambertW}(bx+a)^2}+\frac{2(bx+a)^3}{81\,\mathrm{LambertW}(bx+a)^3}+\frac{\mathrm{LambertW}(bx+a)(bx+a)^3}{3}+a^2\Big(bx+a-\frac{}{\mathrm{Lam}}$

input
```
int(x^2*LambertW(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(-1/9*(b*x+a)^3+1/9/LambertW(b*x+a)*(b*x+a)^3-2/27/LambertW(b*x+a)^2
*(b*x+a)^3+2/81*(b*x+a)^3/LambertW(b*x+a)^3+1/3*LambertW(b*x+a)*(b*x+a)^3+
a^2*(b*x+a-(b*x+a)/LambertW(b*x+a))+a^2*((b*x+a)*LambertW(b*x+a)-2*b*x-2*a
+2*(b*x+a)/LambertW(b*x+a))-2*a*(1/2*(b*x+a)^2-1/2/LambertW(b*x+a)*(b*x+a)
^2+1/4*(b*x+a)^2/LambertW(b*x+a)^2)-2*a*(1/2*LambertW(b*x+a)*(b*x+a)^2-3/4
*(b*x+a)^2+3/4/LambertW(b*x+a)*(b*x+a)^2-3/8*(b*x+a)^2/LambertW(b*x+a)^2))
```

Fricas [F]

$$\int x^2W(a+bx)\,dx = \int x^2\,\mathrm{W}(bx+a)\,dx$$

input
```
integrate(x^2*lambert_w(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^2*lambert_w(b*x + a), x)
```

Sympy [F]

$$\int x^2W(a+bx)\,dx = \int x^2W(a+bx)\,\,dx$$

input
```
integrate(x**2*LambertW(b*x+a),x)
```

output
```
Integral(x**2*LambertW(a + b*x), x)
```

Maxima [F]

$$\int x^2W(a+bx)\,dx = \int x^2\,\mathrm{W}(bx+a)\,dx$$

input
```
integrate(x^2*lambert_w(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^2*lambert_w(b*x + a), x)
```

Giac [F]

$$\int x^2 W(a+bx)\,dx = \int x^2\,\mathrm{W}(bx+a)\,dx$$

input
```
integrate(x^2*lambert_w(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*lambert_w(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 W(a+bx)\,dx = \int x^2\,\mathrm{LambertW}(a+b\,x)\,dx$$

input
```
int(x^2*LambertW(a + b*x),x)
```

output
```
int(x^2*LambertW(a + b*x), x)
```

Reduce [F]

$$\int x^2 W(a+bx)\,dx = \int lambert\_w(bx+a)\,x^2dx$$

input
```
int(x^2*Lambert_W(b*x+a),x)
```

output
```
int(lambert_w(a + b*x)*x**2,x)
```

3.387 $\int xW(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 121

$$\int xW(a+bx)\,dx = \frac{ax}{b} - \frac{(a+bx)^2}{4b^2} - \frac{(a+bx)^2}{8b^2W(a+bx)^2} - \frac{a(a+bx)}{b^2W(a+bx)} + \frac{(a+bx)^2}{4b^2W(a+bx)} - \frac{a(a+bx)W(a+bx)}{b^2} + \frac{(a+bx)^2W(a+bx)}{2b^2}$$

output
```
a*x/b-1/4*(b*x+a)^2/b^2-1/8*(b*x+a)^2/b^2/LambertW(b*x+a)^2-a*(b*x+a)/b^2/
LambertW(b*x+a)+1/4*(b*x+a)^2/b^2/LambertW(b*x+a)-a*(b*x+a)*LambertW(b*x+a
)/b^2+1/2*(b*x+a)^2*LambertW(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.60

$$\int xW(a+bx)\,dx = -\frac{(a+bx)\left(a+bx+(6a-2bx)W(a+bx)+(-6a+2bx)W(a+bx)^2+4(a-bx)W(a+bx)^3\right)}{8b^2W(a+bx)^2}$$

input
```
Integrate[x*ProductLog[a + b*x],x]
```

output
```
-1/8*((a + b*x)*(a + b*x + (6*a - 2*b*x)*ProductLog[a + b*x] + (-6*a + 2*b
*x)*ProductLog[a + b*x]^2 + 4*(a - b*x)*ProductLog[a + b*x]^3))/(b^2*Produ
ctLog[a + b*x]^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xW(a+bx)\,dx$$

$$\Big\downarrow \text{ 7168}$$

$$\frac{\int ((a+bx)W(a+bx)-aW(a+bx))d(a+bx)}{b^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2}(a+bx)^2W(a+bx)+\frac{(a+bx)^2}{4W(a+bx)}-\frac{(a+bx)^2}{8W(a+bx)^2}-a(a+bx)W(a+bx)-\frac{a(a+bx)}{W(a+bx)}-\frac{1}{4}(a+bx)^2+a(a+bx)}{b^2}$$

input
```
Int[x*ProductLog[a + b*x],x]
```

output
```
(a*(a + b*x) - (a + b*x)^2/4 - (a + b*x)^2/(8*ProductLog[a + b*x]^2) - (a*
(a + b*x))/ProductLog[a + b*x] + (a + b*x)^2/(4*ProductLog[a + b*x]) - a*(
a + b*x)*ProductLog[a + b*x] + ((a + b*x)^2*ProductLog[a + b*x])/2)/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7168
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((c_.)*ProductLog[(a_) + (b_.)*(x_)])^(p_.),
 x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[(c*ProductLog[x]
)^p, (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, e, f,
p}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{-\frac{(bx+a)^2}{4}+\frac{(bx+a)^2}{4\,\mathrm{LambertW}(bx+a)}-\frac{(bx+a)^2}{8\,\mathrm{LambertW}(bx+a)^2}+\frac{\mathrm{LambertW}(bx+a)(bx+a)^2}{2}-a\left(bx+a-\frac{bx+a}{\mathrm{LambertW}(bx+a)}\right)-a\left((bx+a\right.}{b^2}$
default	$\frac{-\frac{(bx+a)^2}{4}+\frac{(bx+a)^2}{4\,\mathrm{LambertW}(bx+a)}-\frac{(bx+a)^2}{8\,\mathrm{LambertW}(bx+a)^2}+\frac{\mathrm{LambertW}(bx+a)(bx+a)^2}{2}-a\left(bx+a-\frac{bx+a}{\mathrm{LambertW}(bx+a)}\right)-a\left((bx+a\right.}{b^2}$

input
```
int(x*LambertW(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(-1/4*(b*x+a)^2+1/4/LambertW(b*x+a)*(b*x+a)^2-1/8*(b*x+a)^2/LambertW
(b*x+a)^2+1/2*LambertW(b*x+a)*(b*x+a)^2-a*(b*x+a-(b*x+a)/LambertW(b*x+a))-
a*((b*x+a)*LambertW(b*x+a)-2*b*x-2*a+2*(b*x+a)/LambertW(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18

$$\int xW(a+bx)\,dx$$
$$=\frac{4\,b^2x^2\,\mathrm{W}(bx+a)^3-4\,a^2\,\mathrm{W}(bx+a)^2\log{(bx+a)}+4\,a^2\,\mathrm{W}(bx+a)^2\log{(\mathrm{W}(bx+a))}-b^2x^2-2\,abx-2}{8\,b^2\,\mathrm{W}(bx+a)^2}$$

input
```
integrate(x*lambert_w(b*x+a),x, algorithm="fricas")
```

output
```
1/8*(4*b^2*x^2*lambert_w(b*x + a)^3 - 4*a^2*lambert_w(b*x + a)^2*log(b*x +
 a) + 4*a^2*lambert_w(b*x + a)^2*log(lambert_w(b*x + a)) - b^2*x^2 - 2*a*b
*x - 2*(b^2*x^2 - 2*a*b*x)*lambert_w(b*x + a)^2 - a^2 + 2*(b^2*x^2 - 2*a*b
*x - 3*a^2)*lambert_w(b*x + a))/(b^2*lambert_w(b*x + a)^2)
```

Sympy [F]

$$\int xW(a+bx)\,dx = \int xW(a+bx)\;dx$$

input
```
integrate(x*LambertW(b*x+a),x)
```

output
```
Integral(x*LambertW(a + b*x), x)
```

Maxima [F]

$$\int xW(a+bx)\,dx = \int x\,\mathrm{W}(bx+a)\,dx$$

input
```
integrate(x*lambert_w(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*lambert_w(b*x + a), x)
```

Giac [F]

$$\int xW(a+bx)\,dx = \int x\,\mathrm{W}(bx+a)\,dx$$

input
```
integrate(x*lambert_w(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*lambert_w(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int xW(a+bx)\,dx=\int x\,\mathrm{LambertW}(a+b\,x)\;dx$$

input
```
int(x*LambertW(a + b*x),x)
```

output
```
int(x*LambertW(a + b*x), x)
```

Reduce [F]

$$\int xW(a+bx)\,dx=\int lambert\_w(bx+a)\,xdx$$

input
```
int(x*Lambert_W(b*x+a),x)
```

output
```
int(lambert_w(a + b*x)*x,x)
```

3.388 $\int W(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 36

$$\int W(a+bx)\,dx = -x + \frac{a+bx}{bW(a+bx)} + \frac{(a+bx)W(a+bx)}{b}$$

output
```
-x+(b*x+a)/b/LambertW(b*x+a)+(b*x+a)*LambertW(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int W(a+bx)\,dx = \frac{(a+bx)\left(1-W(a+bx)+W(a+bx)^2\right)}{bW(a+bx)}$$

input
```
Integrate[ProductLog[a + b*x],x]
```

output
```
((a + b*x)*(1 - ProductLog[a + b*x] + ProductLog[a + b*x]^2))/(b*ProductLo
g[a + b*x])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7167, 7177, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W(a+bx)\,dx$$

$$\Big\downarrow \text{7167}$$

$$\frac{(a+bx)W(a+bx)}{b}-\int \frac{W(a+bx)}{W(a+bx)+1}dx$$

$$\Big\downarrow \text{7177}$$

$$\int \frac{1}{W(a+bx)+1}dx+\frac{(a+bx)W(a+bx)}{b}-x$$

$$\Big\downarrow \text{7176}$$

$$\frac{(a+bx)W(a+bx)}{b}+\frac{a+bx}{bW(a+bx)}-x$$

input
```
Int[ProductLog[a + b*x],x]
```

output
```
-x + (a + b*x)/(b*ProductLog[a + b*x]) + ((a + b*x)*ProductLog[a + b*x])/b
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7177
```
Int[ProductLog[(a_.) + (b_.)*(x_)]/((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(
x_)]), x_Symbol] :> Simp[d*x, x] - Int[1/(d + d*ProductLog[a + b*x]), x] /;
 FreeQ[{a, b, d}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{-bx-a+\frac{bx+a}{\mathrm{LambertW}(bx+a)}+(bx+a)\,\mathrm{LambertW}(bx+a)}{b}$	39
default	$\frac{-bx-a+\frac{bx+a}{\mathrm{LambertW}(bx+a)}+(bx+a)\,\mathrm{LambertW}(bx+a)}{b}$	39
parallelrisch	$-\frac{-\mathrm{LambertW}(bx+a)^2xb+x\,\mathrm{LambertW}(bx+a)b-a\,\mathrm{LambertW}(bx+a)^2-bx-a\,\mathrm{LambertW}(bx+a)-a}{b\,\mathrm{LambertW}(bx+a)}$	63

input
```
int(LambertW(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-b*x-a+(b*x+a)/LambertW(b*x+a)+(b*x+a)*LambertW(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int W(a+bx)\,dx = \frac{bx\,\mathrm{W}(bx+a)^2-bx\,\mathrm{W}(bx+a)+a\,\mathrm{W}(bx+a)\log(bx+a)-a\,\mathrm{W}(bx+a)\log(-\mathrm{W}(bx+a))+bx+a}{b\,\mathrm{W}(bx+a)}$$

input
```
integrate(lambert_w(b*x+a),x, algorithm="fricas")
```

output
```
(b*x*lambert_w(b*x + a)^2 - b*x*lambert_w(b*x + a) + a*lambert_w(b*x + a)*
log(b*x + a) - a*lambert_w(b*x + a)*log(-lambert_w(b*x + a)) + b*x + a)/(b
*lambert_w(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int W(a+bx)\,dx = \begin{cases} 0 & \text{for } a=0 \wedge b=0 \\ xW(a) & \text{for } b=0 \\ 0 & \text{for } a=-bx \\ \frac{aW(a+bx)}{b} + \frac{a}{bW(a+bx)} + xW(a+bx) - x + \frac{x}{W(a+bx)} & \text{otherwise} \end{cases}$$

input
```
integrate(LambertW(b*x+a),x)
```

output
```
Piecewise((0, Eq(a, 0) & Eq(b, 0)), (x*LambertW(a), Eq(b, 0)), (0, Eq(a, -
b*x)), (a*LambertW(a + b*x)/b + a/(b*LambertW(a + b*x)) + x*LambertW(a + b
*x) - x + x/LambertW(a + b*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int W(a+bx)\,dx = \frac{(bx+a)(\mathrm{W}(bx+a)^2 - \mathrm{W}(bx+a)+1)}{b\,\mathrm{W}(bx+a)}$$

input
```
integrate(lambert_w(b*x+a),x, algorithm="maxima")
```

output
```
(b*x + a)*(lambert_w(b*x + a)^2 - lambert_w(b*x + a) + 1)/(b*lambert_w(b*x
 + a))
```

Giac [F]

$$\int W(a+bx)\,dx = \int \mathrm{W}(bx+a)\,dx$$

input
```
integrate(lambert_w(b*x+a),x, algorithm="giac")
```

output
```
integrate(lambert_w(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int W(a+bx)\,dx = \int \mathrm{LambertW}(a+b\,x)\,dx$$

input
```
int(LambertW(a + b*x),x)
```

output
```
int(LambertW(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int W(a+bx)\,dx = \frac{e^{lambert\_w(bx+a)}\left(lambert\_w(bx+a)^2 - lambert\_w(bx+a) + 1\right)}{b}$$

input
```
int(Lambert_W(b*x+a),x)
```

output
```
(e**lambert_w(a + b*x)*(lambert_w(a + b*x)**2 - lambert_w(a + b*x) + 1))/b
```

3.389 $\int \frac{W(a+bx)}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{W(a+bx)}{x} dx = \text{Int}\left(\frac{W(a+bx)}{x}, x\right)$$

output
```
Defer(Int)(LambertW(b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{W(a+bx)}{x} dx = \int \frac{W(a+bx)}{x} dx$$

input
```
Integrate[ProductLog[a + b*x]/x,x]
```

output
```
Integrate[ProductLog[a + b*x]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(a+bx)}{x} \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{W(a+bx)}{x}dx$$

input
```
Int[ProductLog[a + b*x]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{LambertW}\,(bx+a)}{x}dx$$

input
```
int(LambertW(b*x+a)/x,x)
```

output
```
int(LambertW(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{W(a+bx)}{x} \, dx = \int \frac{\mathrm{W}(bx+a)}{x} \, dx$$

input
```
integrate(lambert_w(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(lambert_w(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{W(a+bx)}{x} \, dx = \int \frac{W(a+bx)}{x} \, dx$$

input
```
integrate(LambertW(b*x+a)/x,x)
```

output
```
Integral(LambertW(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{W(a+bx)}{x} \, dx = \int \frac{\mathrm{W}(bx+a)}{x} \, dx$$

input
```
integrate(lambert_w(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(lambert_w(b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{W(a+bx)}{x}\,dx = \int \frac{\mathrm{W}(bx+a)}{x}\,dx$$

input
```
integrate(lambert_w(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(lambert_w(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{W(a+bx)}{x}\,dx = \int \frac{\mathrm{LambertW}(a+b\,x)}{x}\,dx$$

input
```
int(LambertW(a + b*x)/x,x)
```

output
```
int(LambertW(a + b*x)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.40

$$\int \frac{W(a+bx)}{x}\,dx = \int \frac{lambert\_w(bx+a)}{x}dx - 2\left(\int \frac{1}{x}dx\right) + 2\log(x)$$

input

```
int(Lambert_W(b*x+a)/x,x)
```

output

```
int(lambert_w(a + b*x)/x,x) - 2*int(1/x,x) + 2*log(x)
```

3.390 $\int \frac{W(a+bx)}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{W(a+bx)}{x^2} dx = \text{Int}\left(\frac{W(a+bx)}{x^2}, x\right)$$

output
```
Defer(Int)(LambertW(b*x+a)/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{W(a+bx)}{x^2} dx = \int \frac{W(a+bx)}{x^2} dx$$

input
```
Integrate[ProductLog[a + b*x]/x^2,x]
```

output
```
Integrate[ProductLog[a + b*x]/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(a+bx)}{x^2} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{W(a+bx)}{x^2}dx$$

input
```
Int[ProductLog[a + b*x]/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{LambertW}\,(bx+a)}{x^2}dx$$

input
```
int(LambertW(b*x+a)/x^2,x)
```

output
```
int(LambertW(b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{W(a+bx)}{x^2} dx = \int \frac{\mathrm{W}(bx+a)}{x^2} dx$$

input
```
integrate(lambert_w(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(lambert_w(b*x + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{W(a+bx)}{x^2} dx = \int \frac{W(a+bx)}{x^2} dx$$

input
```
integrate(LambertW(b*x+a)/x**2,x)
```

output
```
Integral(LambertW(a + b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{W(a+bx)}{x^2} dx = \int \frac{\mathrm{W}(bx+a)}{x^2} dx$$

input
```
integrate(lambert_w(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(b*x + a)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{W(a+bx)}{x^2}\,dx = \int \frac{\mathrm{W}(bx+a)}{x^2}\,dx$$

input
```
integrate(lambert_w(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(b*x + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{W(a+bx)}{x^2}\,dx = \int \frac{\mathrm{LambertW}(a+b\,x)}{x^2}\,dx$$

input
```
int(LambertW(a + b*x)/x^2,x)
```

output
```
int(LambertW(a + b*x)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 4.70

$$\int \frac{W(a+bx)}{x^2}\,dx = \frac{\left(\int \frac{1}{e^{lambert\_w(bx+a)}lambert\_w(bx+a)x+e^{lambert\_w(bx+a)}x}dx\right)bx - lambert\_w(bx+a)}{x}$$

input
```
int(Lambert_W(b*x+a)/x^2,x)
```

output
```
(int(1/(e**lambert_w(a + b*x)*lambert_w(a + b*x)*x + e**lambert_w(a + b*x)
*x),x)*b*x - lambert_w(a + b*x))/x
```

3.391 $\int x^3 W(a+bx)^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 456

$$\begin{aligned}\int x^3W(a+bx)^2\,dx = {} & -\frac{4a^3x}{b^3}+\frac{9a^2(a+bx)^2}{4b^4}-\frac{8a(a+bx)^3}{9b^4}+\frac{5(a+bx)^4}{32b^4} \\ & +\frac{15(a+bx)^4}{1024b^4W(a+bx)^4}+\frac{16a(a+bx)^3}{81b^4W(a+bx)^3} \\ & -\frac{15(a+bx)^4}{256b^4W(a+bx)^3}+\frac{9a^2(a+bx)^2}{8b^4W(a+bx)^2} \\ & -\frac{16a(a+bx)^3}{27b^4W(a+bx)^2}+\frac{15(a+bx)^4}{128b^4W(a+bx)^2}+\frac{4a^3(a+bx)}{b^4W(a+bx)} \\ & -\frac{9a^2(a+bx)^2}{4b^4W(a+bx)}+\frac{8a(a+bx)^3}{9b^4W(a+bx)}-\frac{5(a+bx)^4}{32b^4W(a+bx)} \\ & +\frac{2a^3(a+bx)W(a+bx)}{b^4}-\frac{3a^2(a+bx)^2W(a+bx)}{2b^4} \\ & +\frac{2a(a+bx)^3W(a+bx)}{3b^4}-\frac{(a+bx)^4W(a+bx)}{8b^4} \\ & -\frac{a^3(a+bx)W(a+bx)^2}{b^4}+\frac{3a^2(a+bx)^2W(a+bx)^2}{2b^4} \\ & -\frac{a(a+bx)^3W(a+bx)^2}{b^4}+\frac{(a+bx)^4W(a+bx)^2}{4b^4}\end{aligned}$$

output
```
-4*a^3*x/b^3+9/4*a^2*(b*x+a)^2/b^4-8/9*a*(b*x+a)^3/b^4+5/32*(b*x+a)^4/b^4+
15/1024*(b*x+a)^4/b^4/LambertW(b*x+a)^4+16/81*a*(b*x+a)^3/b^4/LambertW(b*x
+a)^3-15/256*(b*x+a)^4/b^4/LambertW(b*x+a)^3+9/8*a^2*(b*x+a)^2/b^4/Lambert
W(b*x+a)^2-16/27*a*(b*x+a)^3/b^4/LambertW(b*x+a)^2+15/128*(b*x+a)^4/b^4/La
mbertW(b*x+a)^2+4*a^3*(b*x+a)/b^4/LambertW(b*x+a)-9/4*a^2*(b*x+a)^2/b^4/La
mbertW(b*x+a)+8/9*a*(b*x+a)^3/b^4/LambertW(b*x+a)-5/32*(b*x+a)^4/b^4/Lambe
rtW(b*x+a)+2*a^3*(b*x+a)*LambertW(b*x+a)/b^4-3/2*a^2*(b*x+a)^2*LambertW(b*
x+a)/b^4+2/3*a*(b*x+a)^3*LambertW(b*x+a)/b^4-1/8*(b*x+a)^4*LambertW(b*x+a)
/b^4-a^3*(b*x+a)*LambertW(b*x+a)^2/b^4+3/2*a^2*(b*x+a)^2*LambertW(b*x+a)^2
/b^4-a*(b*x+a)^3*LambertW(b*x+a)^2/b^4+1/4*(b*x+a)^4*LambertW(b*x+a)^2/b^4
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.55

$$\int x^3 W(a+bx)^2 \, dx$$
$$= \frac{(a+bx)\left(1215(a+bx)^3+4(2881a-1215bx)(a+bx)^2W(a+bx)+24(2245a^3+1007a^2bx-833ab^2x^2\right.}{}$$

input
```
Integrate[x^3*ProductLog[a + b*x]^2,x]
```

output
```
((a + b*x)*(1215*(a + b*x)^3 + 4*(2881*a - 1215*b*x)*(a + b*x)^2*ProductLo
g[a + b*x] + 24*(2245*a^3 + 1007*a^2*b*x - 833*a*b^2*x^2 + 405*b^3*x^3)*Pr
oductLog[a + b*x]^2 + 288*(715*a^3 - 271*a^2*b*x + 121*a*b^2*x^2 - 45*b^3*
x^3)*ProductLog[a + b*x]^3 - 288*(715*a^3 - 271*a^2*b*x + 121*a*b^2*x^2 -
45*b^3*x^3)*ProductLog[a + b*x]^4 + 3456*(25*a^3 - 13*a^2*b*x + 7*a*b^2*x^
2 - 3*b^3*x^3)*ProductLog[a + b*x]^5 - 20736*(a^3 - a^2*b*x + a*b^2*x^2 -
b^3*x^3)*ProductLog[a + b*x]^6))/(82944*b^4*ProductLog[a + b*x]^4)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.87, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 W(a+bx)^2 \, dx$$

$$\Big\downarrow \text{7168}$$

$$\frac{\int \left(-W(a+bx)^2a^3+3(a+bx)W(a+bx)^2a^2-3(a+bx)^2W(a+bx)^2a+(a+bx)^3W(a+bx)^2\right)d(a+bx)}{b^4}$$

$$\Big\downarrow \text{2009}$$

$$-a^3(a+bx)W(a+bx)^2+2a^3(a+bx)W(a+bx)+\frac{4a^3(a+bx)}{W(a+bx)}-4a^3(a+bx)+\frac{3}{2}a^2(a+bx)^2W(a+bx)^2-\frac{3}{2}a^2($$

input
```
Int[x^3*ProductLog[a + b*x]^2,x]
```

output
```
(-4*a^3*(a + b*x) + (9*a^2*(a + b*x)^2)/4 - (8*a*(a + b*x)^3)/9 + (5*(a +
b*x)^4)/32 + (15*(a + b*x)^4)/(1024*ProductLog[a + b*x]^4) + (16*a*(a + b*
x)^3)/(81*ProductLog[a + b*x]^3) - (15*(a + b*x)^4)/(256*ProductLog[a + b*
x]^3) + (9*a^2*(a + b*x)^2)/(8*ProductLog[a + b*x]^2) - (16*a*(a + b*x)^3)
/(27*ProductLog[a + b*x]^2) + (15*(a + b*x)^4)/(128*ProductLog[a + b*x]^2)
 + (4*a^3*(a + b*x))/ProductLog[a + b*x] - (9*a^2*(a + b*x)^2)/(4*ProductL
og[a + b*x]) + (8*a*(a + b*x)^3)/(9*ProductLog[a + b*x]) - (5*(a + b*x)^4)
/(32*ProductLog[a + b*x]) + 2*a^3*(a + b*x)*ProductLog[a + b*x] - (3*a^2*(
a + b*x)^2*ProductLog[a + b*x])/2 + (2*a*(a + b*x)^3*ProductLog[a + b*x])/
3 - ((a + b*x)^4*ProductLog[a + b*x])/8 - a^3*(a + b*x)*ProductLog[a + b*x
]^2 + (3*a^2*(a + b*x)^2*ProductLog[a + b*x]^2)/2 - a*(a + b*x)^3*ProductL
og[a + b*x]^2 + ((a + b*x)^4*ProductLog[a + b*x]^2)/4)/b^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7168
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((c_.)*ProductLog[(a_) + (b_.)*(x_)])^(p_.),
 x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[(c*ProductLog[x]
)^p, (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, e, f,
p}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{-\frac{\mathrm{LambertW}(bx+a)(bx+a)^4}{8}+\frac{5(bx+a)^4}{32}-\frac{5(bx+a)^4}{32\,\mathrm{LambertW}(bx+a)}+\frac{15(bx+a)^4}{128\,\mathrm{LambertW}(bx+a)^2}-\frac{15(bx+a)^4}{256\,\mathrm{LambertW}(bx+a)^3}+\frac{15(bx\ldots}{1024\,\mathrm{Lamber}\ldots}\ldots}{\ldots}$
default	$\frac{-\frac{\mathrm{LambertW}(bx+a)(bx+a)^4}{8}+\frac{5(bx+a)^4}{32}-\frac{5(bx+a)^4}{32\,\mathrm{LambertW}(bx+a)}+\frac{15(bx+a)^4}{128\,\mathrm{LambertW}(bx+a)^2}-\frac{15(bx+a)^4}{256\,\mathrm{LambertW}(bx+a)^3}+\frac{15(bx\ldots}{1024\,\mathrm{Lamber}\ldots}\ldots}{\ldots}$

input
```
int(x^3*LambertW(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(-1/8*LambertW(b*x+a)*(b*x+a)^4+5/32*(b*x+a)^4-5/32/LambertW(b*x+a)*
(b*x+a)^4+15/128*(b*x+a)^4/LambertW(b*x+a)^2-15/256/LambertW(b*x+a)^3*(b*x
+a)^4+15/1024*(b*x+a)^4/LambertW(b*x+a)^4+1/4*LambertW(b*x+a)^2*(b*x+a)^4-
a^3*((b*x+a)*LambertW(b*x+a)-2*b*x-2*a+2*(b*x+a)/LambertW(b*x+a))-a^3*(Lam
bertW(b*x+a)^2*(b*x+a)-3*(b*x+a)*LambertW(b*x+a)+6*b*x+6*a-6*(b*x+a)/Lambe
rtW(b*x+a))+3*a^2*(1/2*LambertW(b*x+a)*(b*x+a)^2-3/4*(b*x+a)^2+3/4/Lambert
W(b*x+a)*(b*x+a)^2-3/8*(b*x+a)^2/LambertW(b*x+a)^2)+3*a^2*(1/2*LambertW(b*
x+a)^2*(b*x+a)^2-LambertW(b*x+a)*(b*x+a)^2+3/2*(b*x+a)^2-3/2/LambertW(b*x+
a)*(b*x+a)^2+3/4*(b*x+a)^2/LambertW(b*x+a)^2)-3*a*(1/3*LambertW(b*x+a)*(b*
x+a)^3-4/9*(b*x+a)^3+4/9/LambertW(b*x+a)*(b*x+a)^3-8/27/LambertW(b*x+a)^2*
(b*x+a)^3+8/81*(b*x+a)^3/LambertW(b*x+a)^3)-3*a*(1/3*LambertW(b*x+a)^2*(b*
x+a)^3-5/9*LambertW(b*x+a)*(b*x+a)^3+20/27*(b*x+a)^3-20/27/LambertW(b*x+a)
*(b*x+a)^3+40/81/LambertW(b*x+a)^2*(b*x+a)^3-40/243*(b*x+a)^3/LambertW(b*x
+a)^3))
```

Fricas [F]

$$\int x^3W(a+bx)^2\,dx=\int x^3\,\mathrm{W}(bx+a)^2\,dx$$

input
```
integrate(x^3*lambert_w(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^3*lambert_w(b*x + a)^2, x)
```

Sympy [F]

$$\int x^3W(a+bx)^2\,dx=\int x^3W^2(a+bx)\,\,dx$$

input
```
integrate(x**3*LambertW(b*x+a)**2,x)
```

output
```
Integral(x**3*LambertW(a + b*x)**2, x)
```

Maxima [F]

$$\int x^3W(a+bx)^2\,dx=\int x^3\,\mathrm{W}(bx+a)^2\,dx$$

input
```
integrate(x^3*lambert_w(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*lambert_w(b*x + a)^2, x)
```

Giac [F]

$$\int x^3W(a+bx)^2\,dx=\int x^3\,\mathrm{W}(bx+a)^2\,dx$$

input
```
integrate(x^3*lambert_w(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*lambert_w(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3W(a+bx)^2\,dx=\int x^3\,\mathrm{LambertW}(a+b\,x)^2\,dx$$

input
```
int(x^3*LambertW(a + b*x)^2,x)
```

output
```
int(x^3*LambertW(a + b*x)^2, x)
```

Reduce [F]

$$\int x^3W(a+bx)^2\,dx=\int lambert\_w(bx+a)^2\,x^3dx$$

input
```
int(x^3*Lambert_W(b*x+a)^2,x)
```

output
```
int(lambert_w(a + b*x)**2*x**3,x)
```

3.392 $\int x^2 W(a+bx)^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 292

$$\begin{aligned}\int x^2W(a+bx)^2\,dx = {} & \frac{4a^2x}{b^2}-\frac{3a(a+bx)^2}{2b^3}+\frac{8(a+bx)^3}{27b^3}-\frac{16(a+bx)^3}{243b^3W(a+bx)^3}\\ & -\frac{3a(a+bx)^2}{4b^3W(a+bx)^2}+\frac{16(a+bx)^3}{81b^3W(a+bx)^2}\\ & -\frac{4a^2(a+bx)}{b^3W(a+bx)}+\frac{3a(a+bx)^2}{2b^3W(a+bx)}-\frac{8(a+bx)^3}{27b^3W(a+bx)}\\ & -\frac{2a^2(a+bx)W(a+bx)}{b^3}+\frac{a(a+bx)^2W(a+bx)}{b^3}\\ & -\frac{2(a+bx)^3W(a+bx)}{9b^3}+\frac{a^2(a+bx)W(a+bx)^2}{b^3}\\ & -\frac{a(a+bx)^2W(a+bx)^2}{b^3}+\frac{(a+bx)^3W(a+bx)^2}{3b^3}\end{aligned}$$

output

```
4*a^2*x/b^2-3/2*a*(b*x+a)^2/b^3+8/27*(b*x+a)^3/b^3-16/243*(b*x+a)^3/b^3/La
mbertW(b*x+a)^3-3/4*a*(b*x+a)^2/b^3/LambertW(b*x+a)^2+16/81*(b*x+a)^3/b^3/
LambertW(b*x+a)^2-4*a^2*(b*x+a)/b^3/LambertW(b*x+a)+3/2*a*(b*x+a)^2/b^3/La
mbertW(b*x+a)-8/27*(b*x+a)^3/b^3/LambertW(b*x+a)-2*a^2*(b*x+a)*LambertW(b*
x+a)/b^3+a*(b*x+a)^2*LambertW(b*x+a)/b^3-2/9*(b*x+a)^3*LambertW(b*x+a)/b^3
+a^2*(b*x+a)*LambertW(b*x+a)^2/b^3-a*(b*x+a)^2*LambertW(b*x+a)^2/b^3+1/3*(
b*x+a)^3*LambertW(b*x+a)^2/b^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.58

$$\int x^2W(a+bx)^2\,dx$$
$$= \frac{(a+bx)\left(-64(a+bx)^2-3(179a^2+115abx-64b^2x^2)\,W(a+bx)-18(151a^2-49abx+16b^2x^2)\,W(a-$$

input
```
Integrate[x^2*ProductLog[a + b*x]^2,x]
```

output
```
((a + b*x)*(-64*(a + b*x)^2 - 3*(179*a^2 + 115*a*b*x - 64*b^2*x^2)*Product
Log[a + b*x] - 18*(151*a^2 - 49*a*b*x + 16*b^2*x^2)*ProductLog[a + b*x]^2
+ 18*(151*a^2 - 49*a*b*x + 16*b^2*x^2)*ProductLog[a + b*x]^3 - 108*(11*a^2
 - 5*a*b*x + 2*b^2*x^2)*ProductLog[a + b*x]^4 + 324*(a^2 - a*b*x + b^2*x^2
)*ProductLog[a + b*x]^5))/(972*b^3*ProductLog[a + b*x]^3)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.87, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2W(a+bx)^2\,dx$$
$$\Big\downarrow \text{7168}$$
$$\frac{\int \left(a^2W(a+bx)^2+(a+bx)^2W(a+bx)^2-2a(a+bx)W(a+bx)^2\right)d(a+bx)}{b^3}$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^2(a+bx)W(a+bx)^2-2a^2(a+bx)W(a+bx)-\frac{4a^2(a+bx)}{W(a+bx)}+4a^2(a+bx)+\frac{1}{3}(a+bx)^3W(a+bx)^2-\frac{2}{9}(a+bx)$$

input
```
Int[x^2*ProductLog[a + b*x]^2,x]
```

output
```
(4*a^2*(a + b*x) - (3*a*(a + b*x)^2)/2 + (8*(a + b*x)^3)/27 - (16*(a + b*x
)^3)/(243*ProductLog[a + b*x]^3) - (3*a*(a + b*x)^2)/(4*ProductLog[a + b*x
]^2) + (16*(a + b*x)^3)/(81*ProductLog[a + b*x]^2) - (4*a^2*(a + b*x))/Pro
ductLog[a + b*x] + (3*a*(a + b*x)^2)/(2*ProductLog[a + b*x]) - (8*(a + b*x
)^3)/(27*ProductLog[a + b*x]) - 2*a^2*(a + b*x)*ProductLog[a + b*x] + a*(a
 + b*x)^2*ProductLog[a + b*x] - (2*(a + b*x)^3*ProductLog[a + b*x])/9 + a^
2*(a + b*x)*ProductLog[a + b*x]^2 - a*(a + b*x)^2*ProductLog[a + b*x]^2 +
((a + b*x)^3*ProductLog[a + b*x]^2)/3)/b^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7168
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((c_.)*ProductLog[(a_) + (b_.)*(x_)])^(p_.),
 x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[(c*ProductLog[x]
)^p, (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, e, f,
p}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.14

method	result
derivativedivides	$-\frac{2 \operatorname{LambertW}(bx+a)(bx+a)^3}{9}+\frac{8(bx+a)^3}{27}-\frac{8(bx+a)^3}{27 \operatorname{LambertW}(bx+a)}+\frac{16(bx+a)^3}{81 \operatorname{LambertW}(bx+a)^2}-\frac{16(bx+a)^3}{243 \operatorname{LambertW}(bx+a)^3}+\operatorname{LambertW}($
default	$-\frac{2 \operatorname{LambertW}(bx+a)(bx+a)^3}{9}+\frac{8(bx+a)^3}{27}-\frac{8(bx+a)^3}{27 \operatorname{LambertW}(bx+a)}+\frac{16(bx+a)^3}{81 \operatorname{LambertW}(bx+a)^2}-\frac{16(bx+a)^3}{243 \operatorname{LambertW}(bx+a)^3}+\operatorname{LambertW}($

input
```
int(x^2*LambertW(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(-2/9*LambertW(b*x+a)*(b*x+a)^3+8/27*(b*x+a)^3-8/27/LambertW(b*x+a)*
(b*x+a)^3+16/81/LambertW(b*x+a)^2*(b*x+a)^3-16/243*(b*x+a)^3/LambertW(b*x+
a)^3+1/3*LambertW(b*x+a)^2*(b*x+a)^3+a^2*((b*x+a)*LambertW(b*x+a)-2*b*x-2*
a+2*(b*x+a)/LambertW(b*x+a))+a^2*(LambertW(b*x+a)^2*(b*x+a)-3*(b*x+a)*Lamb
ertW(b*x+a)+6*b*x+6*a-6*(b*x+a)/LambertW(b*x+a))-2*a*(1/2*LambertW(b*x+a)*
(b*x+a)^2-3/4*(b*x+a)^2+3/4/LambertW(b*x+a)*(b*x+a)^2-3/8*(b*x+a)^2/Lamber
tW(b*x+a)^2)-2*a*(1/2*LambertW(b*x+a)^2*(b*x+a)^2-LambertW(b*x+a)*(b*x+a)^
2+3/2*(b*x+a)^2-3/2/LambertW(b*x+a)*(b*x+a)^2+3/4*(b*x+a)^2/LambertW(b*x+a
)^2))
```

Fricas [F]

$$\int x^2 W(a+bx)^2 \, dx = \int x^2 \operatorname{W}(bx+a)^2 \, dx$$

input
```
integrate(x^2*lambert_w(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^2*lambert_w(b*x + a)^2, x)
```

Sympy [F]

$$\int x^2 W(a+bx)^2 \, dx = \int x^2 W^2(a+bx) \, dx$$

input
```
integrate(x**2*LambertW(b*x+a)**2,x)
```

output
```
Integral(x**2*LambertW(a + b*x)**2, x)
```

Maxima [F]

$$\int x^2 W(a+bx)^2 \, dx = \int x^2 \operatorname{W}(bx+a)^2 \, dx$$

input
```
integrate(x^2*lambert_w(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^2*lambert_w(b*x + a)^2, x)
```

Giac [F]

$$\int x^2 W(a+bx)^2 \, dx = \int x^2 \operatorname{W}(bx+a)^2 \, dx$$

input
```
integrate(x^2*lambert_w(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*lambert_w(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 W(a+bx)^2 \, dx = \int x^2 \operatorname{LambertW}(a+b\,x)^2 \, dx$$

input
```
int(x^2*LambertW(a + b*x)^2,x)
```

output
```
int(x^2*LambertW(a + b*x)^2, x)
```

Reduce [F]

$$\int x^2W(a+bx)^2\,dx=\int lambert\_w(bx+a)^2\,x^2dx$$

input
```
int(x^2*Lambert_W(b*x+a)^2,x)
```

output
```
int(lambert_w(a + b*x)**2*x**2,x)
```

3.393 $\int xW(a+bx)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 163

$$\int xW(a+bx)^2\,dx=-\frac{4ax}{b}+\frac{3(a+bx)^2}{4b^2}+\frac{3(a+bx)^2}{8b^2W(a+bx)^2}+\frac{4a(a+bx)}{b^2W(a+bx)}-\frac{3(a+bx)^2}{4b^2W(a+bx)}+\frac{2a(a+bx)W(a+bx)}{b^2}-\frac{(a+bx)^2W(a+bx)}{2b^2}-\frac{a(a+bx)W(a+bx)^2}{b^2}+\frac{(a+bx)^2W(a+bx)^2}{2b^2}$$

output
```
-4*a*x/b+3/4*(b*x+a)^2/b^2+3/8*(b*x+a)^2/b^2/LambertW(b*x+a)^2+4*a*(b*x+a)
/b^2/LambertW(b*x+a)-3/4*(b*x+a)^2/b^2/LambertW(b*x+a)+2*a*(b*x+a)*Lambert
W(b*x+a)/b^2-1/2*(b*x+a)^2*LambertW(b*x+a)/b^2-a*(b*x+a)*LambertW(b*x+a)^2
/b^2+1/2*(b*x+a)^2*LambertW(b*x+a)^2/b^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.58

$$\int xW(a+bx)^2\,dx=\frac{(a+bx)\left(3(a+bx)+(26a-6bx)W(a+bx)+(-26a+6bx)W(a+bx)^2+4(3a-bx)W(a+bx)^3-4(\cdots\right.}{8b^2W(a+bx)^2}$$

input
```
Integrate[x*ProductLog[a + b*x]^2,x]
```

output
```
((a + b*x)*(3*(a + b*x) + (26*a - 6*b*x)*ProductLog[a + b*x] + (-26*a + 6*
b*x)*ProductLog[a + b*x]^2 + 4*(3*a - b*x)*ProductLog[a + b*x]^3 - 4*(a -
b*x)*ProductLog[a + b*x]^4))/(8*b^2*ProductLog[a + b*x]^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.88, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xW(a+bx)^2\,dx$$

$$\Big\downarrow \text{7168}$$

$$\frac{\int \left((a+bx)W(a+bx)^2-aW(a+bx)^2\right)d(a+bx)}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}(a+bx)^2W(a+bx)^2-\frac{1}{2}(a+bx)^2W(a+bx)-\frac{3(a+bx)^2}{4W(a+bx)}+\frac{3(a+bx)^2}{8W(a+bx)^2}-a(a+bx)W(a+bx)^2+2a(a+bx)W}{b^2}$$

input
```
Int[x*ProductLog[a + b*x]^2,x]
```

output
```
(-4*a*(a + b*x) + (3*(a + b*x)^2)/4 + (3*(a + b*x)^2)/(8*ProductLog[a + b*
x]^2) + (4*a*(a + b*x))/ProductLog[a + b*x] - (3*(a + b*x)^2)/(4*ProductLo
g[a + b*x]) + 2*a*(a + b*x)*ProductLog[a + b*x] - ((a + b*x)^2*ProductLog[
a + b*x])/2 - a*(a + b*x)*ProductLog[a + b*x]^2 + ((a + b*x)^2*ProductLog[
a + b*x]^2)/2)/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7168
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((c_.)*ProductLog[(a_) + (b_.)*(x_)])^(p_.),
 x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[(c*ProductLog[x]
)^p, (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, e, f,
p}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{-\frac{\mathrm{LambertW}(bx+a)(bx+a)^2}{2}+\frac{3(bx+a)^2}{4}-\frac{3(bx+a)^2}{4\,\mathrm{LambertW}(bx+a)}+\frac{3(bx+a)^2}{8\,\mathrm{LambertW}(bx+a)^2}+\frac{\mathrm{LambertW}(bx+a)^2(bx+a)^2}{2}-a\Big((bx+a)}{}$
default	$\frac{-\frac{\mathrm{LambertW}(bx+a)(bx+a)^2}{2}+\frac{3(bx+a)^2}{4}-\frac{3(bx+a)^2}{4\,\mathrm{LambertW}(bx+a)}+\frac{3(bx+a)^2}{8\,\mathrm{LambertW}(bx+a)^2}+\frac{\mathrm{LambertW}(bx+a)^2(bx+a)^2}{2}-a\Big((bx+a)}{}$

input
```
int(x*LambertW(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(-1/2*LambertW(b*x+a)*(b*x+a)^2+3/4*(b*x+a)^2-3/4/LambertW(b*x+a)*(b
*x+a)^2+3/8*(b*x+a)^2/LambertW(b*x+a)^2+1/2*LambertW(b*x+a)^2*(b*x+a)^2-a*
((b*x+a)*LambertW(b*x+a)-2*b*x-2*a+2*(b*x+a)/LambertW(b*x+a))-a*(LambertW(
b*x+a)^2*(b*x+a)-3*(b*x+a)*LambertW(b*x+a)+6*b*x+6*a-6*(b*x+a)/LambertW(b*
x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.09

$$\int xW(a+bx)^2\,dx$$
$$=\frac{4\,(b^2x^2-a^2)\,\mathrm{W}(bx+a)^4+12\,a^2\,\mathrm{W}(bx+a)^2\log{(bx+a)}-12\,a^2\,\mathrm{W}(bx+a)^2\log{(-\,\mathrm{W}(bx+a))}+3\,b^2x}{}$$

input
```
integrate(x*lambert_w(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*(b^2*x^2 - a^2)*lambert_w(b*x + a)^4 + 12*a^2*lambert_w(b*x + a)^2*
log(b*x + a) - 12*a^2*lambert_w(b*x + a)^2*log(-lambert_w(b*x + a)) + 3*b^
2*x^2 - 4*(b^2*x^2 - 2*a*b*x)*lambert_w(b*x + a)^3 + 6*a*b*x + 2*(3*b^2*x^
2 - 10*a*b*x)*lambert_w(b*x + a)^2 + 3*a^2 - 2*(3*b^2*x^2 - 10*a*b*x - 13*
a^2)*lambert_w(b*x + a))/(b^2*lambert_w(b*x + a)^2)
```

Sympy [F]

$$\int xW(a+bx)^2\,dx=\int xW^2(a+bx)\,dx$$

input
```
integrate(x*LambertW(b*x+a)**2,x)
```

output
```
Integral(x*LambertW(a + b*x)**2, x)
```

Maxima [F]

$$\int xW(a+bx)^2\,dx=\int x\,\mathrm{W}(bx+a)^2\,dx$$

input
```
integrate(x*lambert_w(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x*lambert_w(b*x + a)^2, x)
```

Giac [F]

$$\int xW(a+bx)^2\,dx = \int x\,\mathrm{W}(bx+a)^2\,dx$$

input
```
integrate(x*lambert_w(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x*lambert_w(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int xW(a+bx)^2\,dx = \int x\,\mathrm{LambertW}(a+b\,x)^2\,dx$$

input
```
int(x*LambertW(a + b*x)^2,x)
```

output
```
int(x*LambertW(a + b*x)^2, x)
```

Reduce [F]

$$\int xW(a+bx)^2\,dx = \int lambert\_w(bx+a)^2\,xdx$$

input
```
int(x*Lambert_W(b*x+a)^2,x)
```

output
```
int(lambert_w(a + b*x)**2*x,x)
```

3.394 $\int W(a+bx)^2\,dx$

Optimal result

Integrand size = 8, antiderivative size = 55

$$\int W(a+bx)^2\,dx=4x-\frac{4(a+bx)}{bW(a+bx)}-\frac{2(a+bx)W(a+bx)}{b}+\frac{(a+bx)W(a+bx)^2}{b}$$

output
```
4*x-4*(b*x+a)/b/LambertW(b*x+a)-2*(b*x+a)*LambertW(b*x+a)/b+(b*x+a)*Lamber
tW(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int W(a+bx)^2\,dx=\frac{(a+bx)\left(-4+4W(a+bx)-2W(a+bx)^2+W(a+bx)^3\right)}{bW(a+bx)}$$

input
```
Integrate[ProductLog[a + b*x]^2,x]
```

output
```
((a + b*x)*(-4 + 4*ProductLog[a + b*x] - 2*ProductLog[a + b*x]^2 + Product
Log[a + b*x]^3))/(b*ProductLog[a + b*x])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7167, 7178, 7177, 7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int W(a+bx)^2\,dx$$

$$\Big\downarrow \text{7167}$$

$$\frac{(a+bx)W(a+bx)^2}{b}-2\int \frac{W(a+bx)^2}{W(a+bx)+1}dx$$

$$\Big\downarrow \text{7178}$$

$$\frac{(a+bx)W(a+bx)^2}{b}-2\left(\frac{(a+bx)W(a+bx)}{b}-2\int \frac{W(a+bx)}{W(a+bx)+1}dx\right)$$

$$\Big\downarrow \text{7177}$$

$$\frac{(a+bx)W(a+bx)^2}{b}-2\left(\frac{(a+bx)W(a+bx)}{b}-2\left(x-\int \frac{1}{W(a+bx)+1}dx\right)\right)$$

$$\Big\downarrow \text{7176}$$

$$\frac{(a+bx)W(a+bx)^2}{b}-2\left(\frac{(a+bx)W(a+bx)}{b}-2\left(x-\frac{a+bx}{bW(a+bx)}\right)\right)$$

input
```
Int[ProductLog[a + b*x]^2,x]
```

output
```
((a + b*x)*ProductLog[a + b*x]^2)/b - 2*(-2*(x - (a + b*x)/(b*ProductLog[a
 + b*x])) + ((a + b*x)*ProductLog[a + b*x])/b)
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

rule 7177
```
Int[ProductLog[(a_.) + (b_.)*(x_)]/((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(
x_)]), x_Symbol] :> Simp[d*x, x] - Int[1/(d + d*ProductLog[a + b*x]), x] /;
 FreeQ[{a, b, d}, x]
```

rule 7178
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[c*(a + b*x)*((c*ProductLog[a + b*x])^
(p - 1)/(b*d)), x] - Simp[c*p   Int[(c*ProductLog[a + b*x])^(p - 1)/(d + d*
ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\mathrm{LambertW}(bx+a)^2(bx+a)-2(bx+a)\,\mathrm{LambertW}(bx+a)+4bx+4a-\frac{4(bx+a)}{\mathrm{LambertW}(bx+a)}}{b}$
default	$\frac{\mathrm{LambertW}(bx+a)^2(bx+a)-2(bx+a)\,\mathrm{LambertW}(bx+a)+4bx+4a-\frac{4(bx+a)}{\mathrm{LambertW}(bx+a)}}{b}$
parallelrisch	$-\frac{-x\,\mathrm{LambertW}(bx+a)^3b+2\,\mathrm{LambertW}(bx+a)^2xb-\mathrm{LambertW}(bx+a)^3a-4x\,\mathrm{LambertW}(bx+a)b+2a\,\mathrm{LambertW}(b}{b\,\mathrm{LambertW}(bx+a)}$

input
```
int(LambertW(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(LambertW(b*x+a)^2*(b*x+a)-2*(b*x+a)*LambertW(b*x+a)+4*b*x+4*a-4*(b*x+
a)/LambertW(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.75

$$\int W(a+bx)^2\,dx =$$
$$-\frac{2\,bx\,\mathrm{W}(bx+a)^2-(bx+a)\,\mathrm{W}(bx+a)^3-4\,bx\,\mathrm{W}(bx+a)+2\,a\,\mathrm{W}(bx+a)\log{(bx+a)}-2\,a\,\mathrm{W}(bx+}{b\,\mathrm{W}(bx+a)}$$

input
```
integrate(lambert_w(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*b*x*lambert_w(b*x + a)^2 - (b*x + a)*lambert_w(b*x + a)^3 - 4*b*x*lamb
ert_w(b*x + a) + 2*a*lambert_w(b*x + a)*log(b*x + a) - 2*a*lambert_w(b*x +
 a)*log(lambert_w(b*x + a)) + 4*b*x + 4*a)/(b*lambert_w(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int W(a+bx)^2\,dx$$
$$=\begin{cases} 0 & \text{for } a=0\wedge b=0 \\ xW^2(a) & \text{for } b=0 \\ 0 & \text{for } a=-bx \\ \frac{aW^2(a+bx)}{b}-\frac{2aW(a+bx)}{b}-\frac{4a}{bW(a+bx)}+xW^2(a+bx)-2xW(a+bx)+4x-\frac{4x}{W(a+bx)} & \text{otherwise} \end{cases}$$

input
```
integrate(LambertW(b*x+a)**2,x)
```

output
```
Piecewise((0, Eq(a, 0) & Eq(b, 0)), (x*LambertW(a)**2, Eq(b, 0)), (0, Eq(a
, -b*x)), (a*LambertW(a + b*x)**2/b - 2*a*LambertW(a + b*x)/b - 4*a/(b*Lam
bertW(a + b*x)) + x*LambertW(a + b*x)**2 - 2*x*LambertW(a + b*x) + 4*x - 4
*x/LambertW(a + b*x), True))
```

Maxima [F]

$$\int W(a+bx)^2\,dx = \int \mathrm{W}(bx+a)^2\,dx$$

input
```
integrate(lambert_w(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(b*x + a)^2, x)
```

Giac [F]

$$\int W(a+bx)^2\,dx = \int \mathrm{W}(bx+a)^2\,dx$$

input
```
integrate(lambert_w(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int W(a+bx)^2\,dx = \int \mathrm{LambertW}(a+b\,x)^2\,dx$$

input
```
int(LambertW(a + b*x)^2,x)
```

output
```
int(LambertW(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int W(a+bx)^2\,dx$$
$$= \frac{e^{lambert\_w(bx+a)}\left(lambert\_w(bx+a)^3 - 2lambert\_w(bx+a)^2 + 4lambert\_w(bx+a) - 4\right)}{b}$$

input
```
int(Lambert_W(b*x+a)^2,x)
```

output
```
(e**lambert_w(a + b*x)*(lambert_w(a + b*x)**3 - 2*lambert_w(a + b*x)**2 +
4*lambert_w(a + b*x) - 4))/b
```

3.395 $\int \frac{W(a+bx)^2}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{W(a+bx)^2}{x} \, dx = \text{Int}\left(\frac{W(a+bx)^2}{x}, x\right)$$

output
```
Defer(Int)(LambertW(b*x+a)^2/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{W(a+bx)^2}{x} \, dx = \int \frac{W(a+bx)^2}{x} \, dx$$

input
```
Integrate[ProductLog[a + b*x]^2/x,x]
```

output
```
Integrate[ProductLog[a + b*x]^2/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(a+bx)^2}{x} \, dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{W(a+bx)^2}{x}dx$$

input
```
Int[ProductLog[a + b*x]^2/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{LambertW}(bx+a)^2}{x}dx$$

input
```
int(LambertW(b*x+a)^2/x,x)
```

output
```
int(LambertW(b*x+a)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{W(a+bx)^2}{x} \, dx = \int \frac{\mathrm{W}(bx+a)^2}{x} \, dx$$

input
```
integrate(lambert_w(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(lambert_w(b*x + a)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{W(a+bx)^2}{x} \, dx = \int \frac{W^2(a+bx)}{x} \, dx$$

input
```
integrate(LambertW(b*x+a)**2/x,x)
```

output
```
Integral(LambertW(a + b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{W(a+bx)^2}{x} \, dx = \int \frac{\mathrm{W}(bx+a)^2}{x} \, dx$$

input
```
integrate(lambert_w(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
integrate(lambert_w(b*x + a)^2/x, x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{W(a+bx)^2}{x}\,dx = \int \frac{\mathrm{W}(bx+a)^2}{x}\,dx$$

input
```
integrate(lambert_w(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(lambert_w(b*x + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{W(a+bx)^2}{x}\,dx = \int \frac{\mathrm{LambertW}(a+b\,x)^2}{x}\,dx$$

input
```
int(LambertW(a + b*x)^2/x,x)
```

output
```
int(LambertW(a + b*x)^2/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \frac{W(a+bx)^2}{x} \, dx = \int \frac{lambert\_w(bx+a)^2}{x} dx - \left(\int \frac{1}{x} dx \right) + \log(x)$$

input
```
int(Lambert_W(b*x+a)^2/x,x)
```

output
```
int(lambert_w(a + b*x)**2/x,x) - int(1/x,x) + log(x)
```

3.396 $\int \frac{W(a+bx)^2}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{W(a+bx)^2}{x^2}\,dx = \operatorname{Int}\left(\frac{W(a+bx)^2}{x^2},x\right)$$

output
```
Defer(Int)(LambertW(b*x+a)^2/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{W(a+bx)^2}{x^2}\,dx = \int \frac{W(a+bx)^2}{x^2}\,dx$$

input
```
Integrate[ProductLog[a + b*x]^2/x^2,x]
```

output
```
Integrate[ProductLog[a + b*x]^2/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{W(a+bx)^2}{x^2} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{W(a+bx)^2}{x^2}dx$$

input
```
Int[ProductLog[a + b*x]^2/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\text{LambertW}\,(bx+a)^2}{x^2}dx$$

input
```
int(LambertW(b*x+a)^2/x^2,x)
```

output
```
int(LambertW(b*x+a)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{W(a+bx)^2}{x^2} \, dx = \int \frac{\mathrm{W}(bx+a)^2}{x^2} \, dx$$

input
```
integrate(lambert_w(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(lambert_w(b*x + a)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{W(a+bx)^2}{x^2} \, dx = \int \frac{W^2(a+bx)}{x^2} \, dx$$

input
```
integrate(LambertW(b*x+a)**2/x**2,x)
```

output
```
Integral(LambertW(a + b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{W(a+bx)^2}{x^2} \, dx = \int \frac{\mathrm{W}(bx+a)^2}{x^2} \, dx$$

input
```
integrate(lambert_w(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
integrate(lambert_w(b*x + a)^2/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{W(a+bx)^2}{x^2}\,dx = \int \frac{\mathrm{W}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(lambert_w(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(lambert_w(b*x + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{W(a+bx)^2}{x^2}\,dx = \int \frac{\mathrm{LambertW}(a+b\,x)^2}{x^2}\,dx$$

input
```
int(LambertW(a + b*x)^2/x^2,x)
```

output
```
int(LambertW(a + b*x)^2/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 8.92

$$\int \frac{W(a+bx)^2}{x^2}\,dx$$
$$= \frac{2\left(\int \frac{lambert\_w(bx+a)}{e^{lambert\_w(bx+a)}lambert\_w(bx+a)x+e^{lambert\_w(bx+a)}x}dx\right)bx+2\left(\int \frac{lambert\_w(bx+a)}{lambert\_w(bx+a)x^2+x^2}dx\right)x+2\left(\int \frac{1}{lambert\_w(bx+a)x^2+x^2}dx\right)x-lambert\_w(bx+a)^2+2}{x}$$

input
```
int(Lambert_W(b*x+a)^2/x^2,x)
```

output
```
(2*int(lambert_w(a + b*x)/(e**lambert_w(a + b*x)*lambert_w(a + b*x)*x + e*
*lambert_w(a + b*x)*x),x)*b*x + 2*int(lambert_w(a + b*x)/(lambert_w(a + b*
x)*x**2 + x**2),x)*x + 2*int(1/(lambert_w(a + b*x)*x**2 + x**2),x)*x - lam
bert_w(a + b*x)**2 + 2)/x
```

3.397 $\int x^3 \sqrt{cW(a+bx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 568

$$\begin{aligned}\int x^3\sqrt{cW(a+bx)}\,dx &= -\frac{a^3\sqrt{c}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{4b^4}-\frac{105\sqrt{c}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{65536b^4}\\
&\quad-\frac{9a^2\sqrt{c}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{64b^4}-\frac{5a\sqrt{c}\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{144b^4}\\
&\quad+\frac{105c^4(a+bx)^4}{16384b^4(cW(a+bx))^{7/2}}+\frac{5ac^3(a+bx)^3}{72b^4(cW(a+bx))^{5/2}}\\
&\quad-\frac{35c^3(a+bx)^4}{2048b^4(cW(a+bx))^{5/2}}+\frac{9a^2c^2(a+bx)^2}{32b^4(cW(a+bx))^{3/2}}\\
&\quad-\frac{5ac^2(a+bx)^3}{36b^4(cW(a+bx))^{3/2}}+\frac{7c^2(a+bx)^4}{256b^4(cW(a+bx))^{3/2}}\\
&\quad+\frac{a^3c(a+bx)}{2b^4\sqrt{cW(a+bx)}}-\frac{3a^2c(a+bx)^2}{8b^4\sqrt{cW(a+bx)}}\\
&\quad+\frac{ac(a+bx)^3}{6b^4\sqrt{cW(a+bx)}}-\frac{c(a+bx)^4}{32b^4\sqrt{cW(a+bx)}}\\
&\quad-\frac{a^3(a+bx)\sqrt{cW(a+bx)}}{b^4}+\frac{3a^2(a+bx)^2\sqrt{cW(a+bx)}}{2b^4}\\
&\quad-\frac{a(a+bx)^3\sqrt{cW(a+bx)}}{b^4}+\frac{(a+bx)^4\sqrt{cW(a+bx)}}{4b^4}\end{aligned}$$

output
```
-1/4*a^3*c^(1/2)*Pi^(1/2)*erfi((c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^4-105/
65536*c^(1/2)*Pi^(1/2)*erfi(2*(c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^4-9/128
*a^2*c^(1/2)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(b*x+a))^(1/2)/c^(1/
2))/b^4-5/432*a*c^(1/2)*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(c*LambertW(b*x+a))^
(1/2)/c^(1/2))/b^4+105/16384*c^4*(b*x+a)^4/b^4/(c*LambertW(b*x+a))^(7/2)+5
/72*a*c^3*(b*x+a)^3/b^4/(c*LambertW(b*x+a))^(5/2)-35/2048*c^3*(b*x+a)^4/b^
4/(c*LambertW(b*x+a))^(5/2)+9/32*a^2*c^2*(b*x+a)^2/b^4/(c*LambertW(b*x+a))
^(3/2)-5/36*a*c^2*(b*x+a)^3/b^4/(c*LambertW(b*x+a))^(3/2)+7/256*c^2*(b*x+a
)^4/b^4/(c*LambertW(b*x+a))^(3/2)+1/2*a^3*c*(b*x+a)/b^4/(c*LambertW(b*x+a)
)^(1/2)-3/8*a^2*c*(b*x+a)^2/b^4/(c*LambertW(b*x+a))^(1/2)+1/6*a*c*(b*x+a)^
3/b^4/(c*LambertW(b*x+a))^(1/2)-1/32*c*(b*x+a)^4/b^4/(c*LambertW(b*x+a))^(
1/2)-a^3*(b*x+a)*(c*LambertW(b*x+a))^(1/2)/b^4+3/2*a^2*(b*x+a)^2*(c*Lamber
tW(b*x+a))^(1/2)/b^4-a*(b*x+a)^3*(c*LambertW(b*x+a))^(1/2)/b^4+1/4*(b*x+a)
^4*(c*LambertW(b*x+a))^(1/2)/b^4
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.48

$$\int x^3\sqrt{cW(a+bx)}\,dx$$
$$= \frac{\sqrt{cW(a+bx)}\Big(11340(a+bx)^4+480(193a-63bx)(a+bx)^3W(a+bx)+768(a+bx)^2\left(391a^2-194ab\right.}{}$$

input
```
Integrate[x^3*Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
(Sqrt[c*ProductLog[a + b*x]]*(11340*(a + b*x)^4 + 480*(193*a - 63*b*x)*(a
+ b*x)^3*ProductLog[a + b*x] + 768*(a + b*x)^2*(391*a^2 - 194*a*b*x + 63*b
^2*x^2)*ProductLog[a + b*x]^2 + 18432*(25*a^4 + 12*a^3*b*x - 6*a^2*b^2*x^2
 + 4*a*b^3*x^3 - 3*b^4*x^4)*ProductLog[a + b*x]^3 - Sqrt[Pi]*(442368*a^3*E
rfi[Sqrt[ProductLog[a + b*x]]] + 2835*Erfi[2*Sqrt[ProductLog[a + b*x]]] +
512*a*(243*Sqrt[2]*a*Erfi[Sqrt[2]*Sqrt[ProductLog[a + b*x]]] + 40*Sqrt[3]*
Erfi[Sqrt[3]*Sqrt[ProductLog[a + b*x]]]))*ProductLog[a + b*x]^(7/2) - 4423
68*(a^4 - b^4*x^4)*ProductLog[a + b*x]^4))/(1769472*b^4*ProductLog[a + b*x
]^4)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 518, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {7168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{cW(a+bx)}\,dx$$

$$\Big\downarrow \; 7168$$

$$\frac{\int \left(-\sqrt{cW(a+bx)}a^3+3(a+bx)\sqrt{cW(a+bx)}a^2-3(a+bx)^2\sqrt{cW(a+bx)}a+(a+bx)^3\sqrt{cW(a+bx)}\right)d(a+}{b^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{1}{4}\sqrt{\pi}a^3\sqrt{c}\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)-a^3(a+bx)\sqrt{cW(a+bx)}+\frac{a^3c(a+bx)}{2\sqrt{cW(a+bx)}}+\frac{9a^2c^2(a+bx)^2}{32(cW(a+bx))^{3/2}}-\frac{9}{64}\sqrt{\frac{\pi}{2}}a^2\sqrt{c}\text{erfi}\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{}$$

input
```
Int[x^3*Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
(-1/4*(a^3*Sqrt[c]*Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a + b*x]]/Sqrt[c]]) - (
105*Sqrt[c]*Sqrt[Pi]*Erfi[(2*Sqrt[c*ProductLog[a + b*x]])/Sqrt[c]])/65536
- (9*a^2*Sqrt[c]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[c*ProductLog[a + b*x]])/Sqr
t[c]])/64 - (5*a*Sqrt[c]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[c*ProductLog[a + b*
x]])/Sqrt[c]])/144 + (105*c^4*(a + b*x)^4)/(16384*(c*ProductLog[a + b*x])^
(7/2)) + (5*a*c^3*(a + b*x)^3)/(72*(c*ProductLog[a + b*x])^(5/2)) - (35*c^
3*(a + b*x)^4)/(2048*(c*ProductLog[a + b*x])^(5/2)) + (9*a^2*c^2*(a + b*x)
^2)/(32*(c*ProductLog[a + b*x])^(3/2)) - (5*a*c^2*(a + b*x)^3)/(36*(c*Prod
uctLog[a + b*x])^(3/2)) + (7*c^2*(a + b*x)^4)/(256*(c*ProductLog[a + b*x])
^(3/2)) + (a^3*c*(a + b*x))/(2*Sqrt[c*ProductLog[a + b*x]]) - (3*a^2*c*(a
+ b*x)^2)/(8*Sqrt[c*ProductLog[a + b*x]]) + (a*c*(a + b*x)^3)/(6*Sqrt[c*Pr
oductLog[a + b*x]]) - (c*(a + b*x)^4)/(32*Sqrt[c*ProductLog[a + b*x]]) - a
^3*(a + b*x)*Sqrt[c*ProductLog[a + b*x]] + (3*a^2*(a + b*x)^2*Sqrt[c*Produ
ctLog[a + b*x]])/2 - a*(a + b*x)^3*Sqrt[c*ProductLog[a + b*x]] + ((a + b*x
)^4*Sqrt[c*ProductLog[a + b*x]])/4)/b^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7168
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((c_.)*ProductLog[(a_) + (b_.)*(x_)])^(p_.),
 x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[(c*ProductLog[x]
)^p, (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, e, f,
p}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.39

method	result	size
default	Expression too large to display	787

input
```
int(x^3*(c*LambertW(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^4/c^5*(1/8*c*(c*LambertW(b*x+a))^(9/2)*exp(4*LambertW(b*x+a))-1/8*c*(1
/8*c*(c*LambertW(b*x+a))^(7/2)*exp(4*LambertW(b*x+a))-7/8*c*(1/8*c*(c*Lamb
ertW(b*x+a))^(5/2)*exp(4*LambertW(b*x+a))-5/8*c*(1/8*c*(c*LambertW(b*x+a))
^(3/2)*exp(4*LambertW(b*x+a))-3/8*c*(1/8*c*(c*LambertW(b*x+a))^(1/2)*exp(4
*LambertW(b*x+a))-1/32*c*Pi^(1/2)/(-1/c)^(1/2)*erf(2*(-1/c)^(1/2)*(c*Lambe
rtW(b*x+a))^(1/2))))))-a^3*c^4*(1/2*c*(c*LambertW(b*x+a))^(1/2)*(b*x+a)/La
mbertW(b*x+a)-1/4*c*Pi^(1/2)/(-1/c)^(1/2)*erf((-1/c)^(1/2)*(c*LambertW(b*x
+a))^(1/2)))-a^3*c^3*(1/2*(c*LambertW(b*x+a))^(3/2)*(b*x+a)/LambertW(b*x+a
)*c-3/2*c*(1/2*c*(c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)-1/4*c*P
i^(1/2)/(-1/c)^(1/2)*erf((-1/c)^(1/2)*(c*LambertW(b*x+a))^(1/2))))+3*a^2*c
^3*(1/4*c*(c*LambertW(b*x+a))^(3/2)*exp(2*LambertW(b*x+a))-3/4*c*(1/4*c*(c
*LambertW(b*x+a))^(1/2)*exp(2*LambertW(b*x+a))-1/8*c*Pi^(1/2)/(-2/c)^(1/2)
*erf((-2/c)^(1/2)*(c*LambertW(b*x+a))^(1/2))))+3*a^2*c^2*(1/4*c*(c*Lambert
W(b*x+a))^(5/2)*exp(2*LambertW(b*x+a))-5/4*c*(1/4*c*(c*LambertW(b*x+a))^(3
/2)*exp(2*LambertW(b*x+a))-3/4*c*(1/4*c*(c*LambertW(b*x+a))^(1/2)*exp(2*La
mbertW(b*x+a))-1/8*c*Pi^(1/2)/(-2/c)^(1/2)*erf((-2/c)^(1/2)*(c*LambertW(b*
x+a))^(1/2)))))-3*a*c^2*(1/6*c*(c*LambertW(b*x+a))^(5/2)*exp(3*LambertW(b*
x+a))-5/6*c*(1/6*c*(c*LambertW(b*x+a))^(3/2)*exp(3*LambertW(b*x+a))-1/2*c*
(1/6*c*(c*LambertW(b*x+a))^(1/2)*exp(3*LambertW(b*x+a))-1/12*c*Pi^(1/2)/(-
3/c)^(1/2)*erf((-3/c)^(1/2)*(c*LambertW(b*x+a))^(1/2)))))-3*a*c*(1/6*c*...
```

Fricas [F]

$$\int x^3 \sqrt{cW(a+bx)}\,dx = \int \sqrt{c\,\mathrm{W}(bx+a)}x^3\,dx$$

input
```
integrate(x^3*(c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))*x^3, x)
```

Sympy [F]

$$\int x^3\sqrt{cW(a+bx)}\,dx=\int x^3\sqrt{cW\left(a+bx\right)}\,dx$$

input
```
integrate(x**3*(c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(x**3*sqrt(c*LambertW(a + b*x)), x)
```

Maxima [F]

$$\int x^3\sqrt{cW(a+bx)}\,dx=\int \sqrt{c\,\mathrm{W}(bx+a)}x^3\,dx$$

input
```
integrate(x^3*(c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(b*x + a))*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{cW(a+bx)}\,dx=\int \sqrt{c\,\mathrm{W}(bx+a)}x^3\,dx$$

input
```
integrate(x^3*(c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(b*x + a))*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{cW(a+bx)}\,dx = \int x^3\,\sqrt{c\,\mathrm{LambertW}\left(a+b\,x\right)}\,dx$$

input
```
int(x^3*(c*LambertW(a + b*x))^(1/2),x)
```

output
```
int(x^3*(c*LambertW(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{cW(a+bx)}\,dx = \sqrt{c}\left(\int \sqrt{lambert\_w\left(bx+a\right)}\,x^3dx\right)$$

input
```
int(x^3*(c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a + b*x))*x**3,x)
```

3.398 $\int x^2 \sqrt{cW(a+bx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 377

$$\int x^2 \sqrt{cW(a+bx)}\,dx = \frac{a^2\sqrt{c}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{4b^3} + \frac{3a\sqrt{c}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{32b^3} + \frac{5\sqrt{c}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{432b^3} - \frac{5c^3(a+bx)^3}{216b^3(cW(a+bx))^{5/2}} - \frac{3ac^2(a+bx)^2}{16b^3(cW(a+bx))^{3/2}} + \frac{5c^2(a+bx)^3}{108b^3(cW(a+bx))^{3/2}} - \frac{a^2c(a+bx)}{2b^3\sqrt{cW(a+bx)}} + \frac{ac(a+bx)^2}{4b^3\sqrt{cW(a+bx)}} - \frac{c(a+bx)^3}{18b^3\sqrt{cW(a+bx)}} + \frac{a^2(a+bx)\sqrt{cW(a+bx)}}{b^3} - \frac{a(a+bx)^2\sqrt{cW(a+bx)}}{b^3} + \frac{(a+bx)^3\sqrt{cW(a+bx)}}{3b^3}$$

output
```
1/4*a^2*c^(1/2)*Pi^(1/2)*erfi((c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^3+3/64*
a*c^(1/2)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(b*x+a))^(1/2)/c^(1/2))
/b^3+5/1296*c^(1/2)*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(c*LambertW(b*x+a))^(1/2
)/c^(1/2))/b^3-5/216*c^3*(b*x+a)^3/b^3/(c*LambertW(b*x+a))^(5/2)-3/16*a*c^
2*(b*x+a)^2/b^3/(c*LambertW(b*x+a))^(3/2)+5/108*c^2*(b*x+a)^3/b^3/(c*Lambe
rtW(b*x+a))^(3/2)-1/2*a^2*c*(b*x+a)/b^3/(c*LambertW(b*x+a))^(1/2)+1/4*a*c*
(b*x+a)^2/b^3/(c*LambertW(b*x+a))^(1/2)-1/18*c*(b*x+a)^3/b^3/(c*LambertW(b
*x+a))^(1/2)+a^2*(b*x+a)*(c*LambertW(b*x+a))^(1/2)/b^3-a*(b*x+a)^2*(c*Lamb
ertW(b*x+a))^(1/2)/b^3+1/3*(b*x+a)^3*(c*LambertW(b*x+a))^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.54

$$\int x^2\sqrt{cW(a+bx)}\,dx$$
$$= \frac{\sqrt{cW(a+bx)}\Big(-120(a+bx)^3-12(61a-20bx)(a+bx)^2W(a+bx)-144(11a^3+6a^2bx-3ab^2x^2+2\ldots}{}$$

input
```
Integrate[x^2*Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
(Sqrt[c*ProductLog[a + b*x]]*(-120*(a + b*x)^3 - 12*(61*a - 20*b*x)*(a + b
*x)^2*ProductLog[a + b*x] - 144*(11*a^3 + 6*a^2*b*x - 3*a*b^2*x^2 + 2*b^3*
x^3)*ProductLog[a + b*x]^2 + Sqrt[Pi]*(1296*a^2*Erfi[Sqrt[ProductLog[a + b
*x]]] + 243*Sqrt[2]*a*Erfi[Sqrt[2]*Sqrt[ProductLog[a + b*x]]] + 20*Sqrt[3]
*Erfi[Sqrt[3]*Sqrt[ProductLog[a + b*x]]])*ProductLog[a + b*x]^(5/2) + 1728
*(a^3 + b^3*x^3)*ProductLog[a + b*x]^3))/(5184*b^3*ProductLog[a + b*x]^3)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {7168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{cW(a+bx)}\,dx$$

$$\Big\downarrow \text{7168}$$

$$\frac{\int \left(\sqrt{cW(a+bx)}a^2-2(a+bx)\sqrt{cW(a+bx)}a+(a+bx)^2\sqrt{cW(a+bx)}\right)d(a+bx)}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}\sqrt{\pi}a^2\sqrt{c}\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)+a^2(a+bx)\sqrt{cW(a+bx)}-\frac{a^2c(a+bx)}{2\sqrt{cW(a+bx)}}-\frac{5c^3(a+bx)^3}{216(cW(a+bx))^{5/2}}+\frac{5c^2(a+bx)^3}{108(cW(a+bx))^{3/2}}-\frac{3}{16(}}{}$$

input
```
Int[x^2*Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
((a^2*Sqrt[c]*Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a + b*x]]/Sqrt[c]])/4 + (3*a
*Sqrt[c]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[c*ProductLog[a + b*x]])/Sqrt[c]])/3
2 + (5*Sqrt[c]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[c*ProductLog[a + b*x]])/Sqrt[
c]])/432 - (5*c^3*(a + b*x)^3)/(216*(c*ProductLog[a + b*x])^(5/2)) - (3*a*
c^2*(a + b*x)^2)/(16*(c*ProductLog[a + b*x])^(3/2)) + (5*c^2*(a + b*x)^3)/
(108*(c*ProductLog[a + b*x])^(3/2)) - (a^2*c*(a + b*x))/(2*Sqrt[c*ProductL
og[a + b*x]]) + (a*c*(a + b*x)^2)/(4*Sqrt[c*ProductLog[a + b*x]]) - (c*(a
+ b*x)^3)/(18*Sqrt[c*ProductLog[a + b*x]]) + a^2*(a + b*x)*Sqrt[c*ProductL
og[a + b*x]] - a*(a + b*x)^2*Sqrt[c*ProductLog[a + b*x]] + ((a + b*x)^3*Sq
rt[c*ProductLog[a + b*x]])/3)/b^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7168
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((c_.)*ProductLog[(a_) + (b_.)*(x_)])^(p_.),
 x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[(c*ProductLog[x]
)^p, (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, e, f,
p}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.33

method	result
default	$\frac{\frac{c(c\,\mathrm{LambertW}(bx+a))^{\frac{7}{2}}\,\mathrm{e}^{3\,\mathrm{LambertW}(bx+a)}}{3}-\frac{c\left(\frac{c(c\,\mathrm{LambertW}(bx+a))^{\frac{5}{2}}\,\mathrm{e}^{3\,\mathrm{LambertW}(bx+a)}}{6}-\frac{5c\left(\frac{c(c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}\,\mathrm{e}^{3\,\mathrm{LambertW}(bx+}}{6}\right.}{}\right.}{}}{}$

input
```
int(x^2*(c*LambertW(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^3/c^4*(1/6*c*(c*LambertW(b*x+a))^(7/2)*exp(3*LambertW(b*x+a))-1/6*c*(1
/6*c*(c*LambertW(b*x+a))^(5/2)*exp(3*LambertW(b*x+a))-5/6*c*(1/6*c*(c*Lamb
ertW(b*x+a))^(3/2)*exp(3*LambertW(b*x+a))-1/2*c*(1/6*c*(c*LambertW(b*x+a))
^(1/2)*exp(3*LambertW(b*x+a))-1/12*c*Pi^(1/2)/(-3/c)^(1/2)*erf((-3/c)^(1/2
)*(c*LambertW(b*x+a))^(1/2)))))+a^2*c^3*(1/2*c*(c*LambertW(b*x+a))^(1/2)*(
b*x+a)/LambertW(b*x+a)-1/4*c*Pi^(1/2)/(-1/c)^(1/2)*erf((-1/c)^(1/2)*(c*Lam
bertW(b*x+a))^(1/2)))+a^2*c^2*(1/2*(c*LambertW(b*x+a))^(3/2)*(b*x+a)/Lambe
rtW(b*x+a)*c-3/2*c*(1/2*c*(c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a
)-1/4*c*Pi^(1/2)/(-1/c)^(1/2)*erf((-1/c)^(1/2)*(c*LambertW(b*x+a))^(1/2)))
)-2*a*c^2*(1/4*c*(c*LambertW(b*x+a))^(3/2)*exp(2*LambertW(b*x+a))-3/4*c*(1
/4*c*(c*LambertW(b*x+a))^(1/2)*exp(2*LambertW(b*x+a))-1/8*c*Pi^(1/2)/(-2/c
)^(1/2)*erf((-2/c)^(1/2)*(c*LambertW(b*x+a))^(1/2))))-2*a*c*(1/4*c*(c*Lamb
ertW(b*x+a))^(5/2)*exp(2*LambertW(b*x+a))-5/4*c*(1/4*c*(c*LambertW(b*x+a))
^(3/2)*exp(2*LambertW(b*x+a))-3/4*c*(1/4*c*(c*LambertW(b*x+a))^(1/2)*exp(2
*LambertW(b*x+a))-1/8*c*Pi^(1/2)/(-2/c)^(1/2)*erf((-2/c)^(1/2)*(c*LambertW
(b*x+a))^(1/2))))))
```

Fricas [F]

$$\int x^2 \sqrt{cW(a+bx)}\,dx = \int \sqrt{c\,\mathrm{W}(bx+a)}x^2\,dx$$

input
```
integrate(x^2*(c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))*x^2, x)
```

Sympy [F]

$$\int x^2 \sqrt{cW(a+bx)}\,dx = \int x^2 \sqrt{cW\,(a+bx)}\,dx$$

input
```
integrate(x**2*(c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(x**2*sqrt(c*LambertW(a + b*x)), x)
```

Maxima [F]

$$\int x^2 \sqrt{cW(a+bx)}\,dx = \int \sqrt{c\,\mathrm{W}(bx+a)}x^2\,dx$$

input
```
integrate(x^2*(c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(b*x + a))*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{cW(a+bx)}\,dx=\int\sqrt{c\,\mathrm{W}(bx+a)}x^2\,dx$$

input
```
integrate(x^2*(c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(b*x + a))*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{cW(a+bx)}\,dx=\int x^2\,\sqrt{c\,\mathrm{LambertW}\left(a+b\,x\right)}\,dx$$

input
```
int(x^2*(c*LambertW(a + b*x))^(1/2),x)
```

output
```
int(x^2*(c*LambertW(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{cW(a+bx)}\,dx=\sqrt{c}\left(\int\sqrt{lambert\_w(bx+a)}\,x^2dx\right)$$

input
```
int(x^2*(c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a + b*x))*x**2,x)
```

3.399 $\int x\sqrt{cW(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 214

$$\int x\sqrt{cW(a+bx)}\,dx = -\frac{a\sqrt{c}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{4b^2} - \frac{3\sqrt{c}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{64b^2} + \frac{3c^2(a+bx)^2}{32b^2(cW(a+bx))^{3/2}} + \frac{ac(a+bx)}{2b^2\sqrt{cW(a+bx)}} - \frac{c(a+bx)^2}{8b^2\sqrt{cW(a+bx)}} - \frac{a(a+bx)\sqrt{cW(a+bx)}}{b^2} + \frac{(a+bx)^2\sqrt{cW(a+bx)}}{2b^2}$$

output
```
-1/4*a*c^(1/2)*Pi^(1/2)*erfi((c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^2-3/128*
c^(1/2)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(b*x+a))^(1/2)/c^(1/2))/b
^2+3/32*c^2*(b*x+a)^2/b^2/(c*LambertW(b*x+a))^(3/2)+1/2*a*c*(b*x+a)/b^2/(c
*LambertW(b*x+a))^(1/2)-1/8*c*(b*x+a)^2/b^2/(c*LambertW(b*x+a))^(1/2)-a*(b
*x+a)*(c*LambertW(b*x+a))^(1/2)/b^2+1/2*(b*x+a)^2*(c*LambertW(b*x+a))^(1/2
)/b^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.64

$$\int x\sqrt{cW(a+bx)}\,dx = \frac{c^2\Big(12(a+bx)^2+16(3a^2+2abx-b^2x^2)W(a+bx)-\sqrt{\pi}\Big(32a\text{erfi}\Big(\sqrt{W(a+bx)}\Big)+3\sqrt{2}\text{erfi}\Big(\sqrt{2}\sqrt{W(}}{128b^2(cW(a+bx))^{3/2}}$$

input
```
Integrate[x*Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
(c^2*(12*(a + b*x)^2 + 16*(3*a^2 + 2*a*b*x - b^2*x^2)*ProductLog[a + b*x]
- Sqrt[Pi]*(32*a*Erfi[Sqrt[ProductLog[a + b*x]]] + 3*Sqrt[2]*Erfi[Sqrt[2]*
Sqrt[ProductLog[a + b*x]]])*ProductLog[a + b*x]^(3/2) - 64*(a^2 - b^2*x^2)
*ProductLog[a + b*x]^2))/(128*b^2*(c*ProductLog[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{cW(a+bx)}\,dx$$

$$\Big\downarrow \text{7168}$$

$$\frac{\int \left((a+bx)\sqrt{cW(a+bx)}-a\sqrt{cW(a+bx)}\right)d(a+bx)}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3c^2(a+bx)^2}{32(cW(a+bx))^{3/2}}-\frac{1}{4}\sqrt{\pi}a\sqrt{c}\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)-\frac{3}{64}\sqrt{\frac{\pi}{2}}\sqrt{c}\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(a+bx)}}{\sqrt{c}}\right)+\frac{1}{2}(a+bx)^2\sqrt{cW(a+bx)}-\frac{c(a+}{8\sqrt{cW(}}}{b^2}$$

input
```
Int[x*Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
(-1/4*(a*Sqrt[c]*Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a + b*x]]/Sqrt[c]]) - (3*
Sqrt[c]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[c*ProductLog[a + b*x]])/Sqrt[c]])/64
 + (3*c^2*(a + b*x)^2)/(32*(c*ProductLog[a + b*x])^(3/2)) + (a*c*(a + b*x)
)/(2*Sqrt[c*ProductLog[a + b*x]]) - (c*(a + b*x)^2)/(8*Sqrt[c*ProductLog[a
 + b*x]]) - a*(a + b*x)*Sqrt[c*ProductLog[a + b*x]] + ((a + b*x)^2*Sqrt[c*
ProductLog[a + b*x]])/2)/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7168
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((c_.)*ProductLog[(a_) + (b_.)*(x_)])^(p_.),
 x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[(c*ProductLog[x]
)^p, (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, e, f,
p}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.28

method	result
default	$\frac{\frac{c(c\,\mathrm{LambertW}(bx+a))^{\frac{5}{2}}\mathrm{e}^{2\,\mathrm{LambertW}(bx+a)}}{2}-\frac{c\left(\frac{c(c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}\mathrm{e}^{2\,\mathrm{LambertW}(bx+a)}}{4}-\frac{3c\left(\frac{c\sqrt{c\,\mathrm{LambertW}(bx+a)}\,\mathrm{e}^{2\,\mathrm{LambertW}(bx+a)}}{4}\right.}{4}\right.}{2}}{}$

input
```
int(x*(c*LambertW(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^2/c^3*(1/4*c*(c*LambertW(b*x+a))^(5/2)*exp(2*LambertW(b*x+a))-1/4*c*(1
/4*c*(c*LambertW(b*x+a))^(3/2)*exp(2*LambertW(b*x+a))-3/4*c*(1/4*c*(c*Lamb
ertW(b*x+a))^(1/2)*exp(2*LambertW(b*x+a))-1/8*c*Pi^(1/2)/(-2/c)^(1/2)*erf(
(-2/c)^(1/2)*(c*LambertW(b*x+a))^(1/2))))-a*c^2*(1/2*c*(c*LambertW(b*x+a))
^(1/2)*(b*x+a)/LambertW(b*x+a)-1/4*c*Pi^(1/2)/(-1/c)^(1/2)*erf((-1/c)^(1/2
)*(c*LambertW(b*x+a))^(1/2)))-a*c*(1/2*(c*LambertW(b*x+a))^(3/2)*(b*x+a)/L
ambertW(b*x+a)*c-3/2*c*(1/2*c*(c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b
*x+a)-1/4*c*Pi^(1/2)/(-1/c)^(1/2)*erf((-1/c)^(1/2)*(c*LambertW(b*x+a))^(1/
2)))))
```

Fricas [F]

$$\int x\sqrt{cW(a+bx)}\,dx=\int\sqrt{c\,\mathrm{W}(bx+a)}x\,dx$$

input
```
integrate(x*(c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))*x, x)
```

Sympy [F]

$$\int x\sqrt{cW(a+bx)}\,dx=\int x\sqrt{cW\left(a+bx\right)}\,dx$$

input
```
integrate(x*(c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(x*sqrt(c*LambertW(a + b*x)), x)
```

Maxima [F]

$$\int x\sqrt{cW(a+bx)}\,dx = \int \sqrt{c\,\mathrm{W}(bx+a)}x\,dx$$

input
```
integrate(x*(c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(b*x + a))*x, x)
```

Giac [F]

$$\int x\sqrt{cW(a+bx)}\,dx = \int \sqrt{c\,\mathrm{W}(bx+a)}x\,dx$$

input
```
integrate(x*(c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(b*x + a))*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{cW(a+bx)}\,dx = \int x\,\sqrt{c\,\mathrm{LambertW}\left(a+b\,x\right)}\,dx$$

input
```
int(x*(c*LambertW(a + b*x))^(1/2),x)
```

output
```
int(x*(c*LambertW(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{cW(a+bx)}\,dx = \sqrt{c}\left(\int \sqrt{lambert\_w(bx+a)}\,xdx\right)$$

input
```
int(x*(c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a + b*x))*x,x)
```

3.400 $\int \sqrt{cW(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 83

$$\int \sqrt{cW(a+bx)}\,dx = \frac{\sqrt{c}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{4b} - \frac{c(a+bx)}{2b\sqrt{cW(a+bx)}} + \frac{(a+bx)\sqrt{cW(a+bx)}}{b}$$

output
```
1/4*c^(1/2)*Pi^(1/2)*erfi((c*LambertW(b*x+a))^(1/2)/c^(1/2))/b-1/2*c*(b*x+
a)/b/(c*LambertW(b*x+a))^(1/2)+(b*x+a)*(c*LambertW(b*x+a))^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \sqrt{cW(a+bx)}\,dx = \frac{c\left(-2(a+bx)+\sqrt{\pi}\,\text{erfi}\left(\sqrt{W(a+bx)}\right)\sqrt{W(a+bx)}+4(a+bx)W(a+bx)\right)}{4b\sqrt{cW(a+bx)}}$$

input
```
Integrate[Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
(c*(-2*(a + b*x) + Sqrt[Pi]*Erfi[Sqrt[ProductLog[a + b*x]]]*Sqrt[ProductLo
g[a + b*x]] + 4*(a + b*x)*ProductLog[a + b*x]))/(4*b*Sqrt[c*ProductLog[a +
 b*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7167, 7178, 7180}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cW(a+bx)}dx$$

$$\Big\downarrow \text{7167}$$

$$\frac{(a+bx)\sqrt{cW(a+bx)}}{b}-\frac{1}{2}\int \frac{\sqrt{cW(a+bx)}}{W(a+bx)+1}dx$$

$$\Big\downarrow \text{7178}$$

$$\frac{1}{2}\left(\frac{1}{2}c\int \frac{1}{\sqrt{cW(a+bx)}(W(a+bx)+1)}dx-\frac{c(a+bx)}{b\sqrt{cW(a+bx)}}\right)+\frac{(a+bx)\sqrt{cW(a+bx)}}{b}$$

$$\Big\downarrow \text{7180}$$

$$\frac{1}{2}\left(\frac{\sqrt{\pi}\sqrt{c}\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{2b}-\frac{c(a+bx)}{b\sqrt{cW(a+bx)}}\right)+\frac{(a+bx)\sqrt{cW(a+bx)}}{b}$$

input
```
Int[Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
((a + b*x)*Sqrt[c*ProductLog[a + b*x]])/b + ((Sqrt[c]*Sqrt[Pi]*Erfi[Sqrt[c
*ProductLog[a + b*x]]/Sqrt[c]])/(2*b) - (c*(a + b*x))/(b*Sqrt[c*ProductLog
[a + b*x]]))/2
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7178
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_)/((d_) + (d_.)*ProductLog[(a
_.) + (b_.)*(x_)]), x_Symbol] :> Simp[c*(a + b*x)*((c*ProductLog[a + b*x])^
(p - 1)/(b*d)), x] - Simp[c*p   Int[(c*ProductLog[a + b*x])^(p - 1)/(d + d*
ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0]
```

rule 7180
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[Pi*c, 2]*(Erfi[Sqrt[c*ProductLo
g[a + b*x]]/Rt[c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && PosQ[c]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{\frac{(c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}(bx+a)c}{\mathrm{LambertW}(bx+a)}-c\left(\frac{c\sqrt{c\,\mathrm{LambertW}(bx+a)}\,(bx+a)}{2\,\mathrm{LambertW}(bx+a)}-\frac{c\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{c}}\,\sqrt{c\,\mathrm{LambertW}(bx+a)}\right)}{4\sqrt{-\frac{1}{c}}}\right)}{b\,c^{2}}$	98

input
```
int((c*LambertW(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(1/2*(c*LambertW(b*x+a))^(3/2)*(b*x+a)/LambertW(b*x+a)*c-1/2*c*(1/
2*c*(c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)-1/4*c*Pi^(1/2)/(-1/c
)^(1/2)*erf((-1/c)^(1/2)*(c*LambertW(b*x+a))^(1/2))))
```

Fricas [F]

$$\int \sqrt{cW(a+bx)}\,dx=\int \sqrt{c\,\mathrm{W}(bx+a)}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a)), x)
```

Sympy [F]

$$\int \sqrt{cW(a+bx)}\,dx=\int \sqrt{cW\left(a+bx\right)}\,dx$$

input
```
integrate((c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(c*LambertW(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{cW(a+bx)}\,dx=\int \sqrt{c\,\mathrm{W}(bx+a)}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{cW(a+bx)}\,dx = \int \sqrt{c\,\mathrm{W}(bx+a)}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cW(a+bx)}\,dx = \int \sqrt{c\,\mathrm{LambertW}\left(a+b\,x\right)}\,dx$$

input
```
int((c*LambertW(a + b*x))^(1/2),x)
```

output
```
int((c*LambertW(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{cW(a+bx)}\,dx = \sqrt{c}\left(\int \sqrt{lambert\_w(bx+a)}dx\right)$$

input
```
int((c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a + b*x)),x)
```

3.401 $\int \frac{\sqrt{cW(a+bx)}}{x} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sqrt{cW(a+bx)}}{x} dx = \frac{\sqrt{cW(a+bx)}\text{Int}\left(\frac{\sqrt{W(a+bx)}}{x}, x\right)}{\sqrt{W(a+bx)}}$$

output
```
(c*LambertW(b*x+a))^(1/2)*Defer(Int)(LambertW(b*x+a)^(1/2)/x,x)/LambertW(b
*x+a)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{cW(a+bx)}}{x} dx = \int \frac{\sqrt{cW(a+bx)}}{x} dx$$

input
```
Integrate[Sqrt[c*ProductLog[a + b*x]]/x,x]
```

output
```
Integrate[Sqrt[c*ProductLog[a + b*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cW(a+bx)}}{x}dx$$
$$\Big\downarrow \text{7271}$$
$$\frac{\sqrt{cW(a+bx)}\int \frac{\sqrt{W(a+bx)}}{x}dx}{\sqrt{W(a+bx)}}$$
$$\Big\downarrow \text{7299}$$
$$\frac{\sqrt{cW(a+bx)}\int \frac{\sqrt{W(a+bx)}}{x}dx}{\sqrt{W(a+bx)}}$$

input
```
Int[Sqrt[c*ProductLog[a + b*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{c\,\mathrm{LambertW}\left(bx+a\right)}}{x}dx$$

input
```
int((c*LambertW(b*x+a))^(1/2)/x,x)
```

output
```
int((c*LambertW(b*x+a))^(1/2)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{cW(a+bx)}}{x}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(bx+a)}}{x}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{cW(a+bx)}}{x}\,dx = \int \frac{\sqrt{cW\left(a+bx\right)}}{x}\,dx$$

input
```
integrate((c*LambertW(b*x+a))**(1/2)/x,x)
```

output
```
Integral(sqrt(c*LambertW(a + b*x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{cW(a+bx)}}{x}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(bx+a)}}{x}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(b*x + a))/x, x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{cW(a+bx)}}{x}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(bx+a)}}{x}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(b*x + a))/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{cW(a+bx)}}{x}\,dx = \int \frac{\sqrt{c\,\mathrm{LambertW}\left(a+b\,x\right)}}{x}\,dx$$

input
```
int((c*LambertW(a + b*x))^(1/2)/x,x)
```

output
```
int((c*LambertW(a + b*x))^(1/2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{cW(a+bx)}}{x}\,dx = \sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}}{x}dx\right)$$

input
```
int((c*Lambert_W(b*x+a))^(1/2)/x,x)
```

output
```
sqrt(c)*int(sqrt(lambert_w(a + b*x))/x,x)
```

3.402 $\int \frac{\sqrt{cW(a+bx)}}{x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sqrt{cW(a+bx)}}{x^2} dx = \frac{\sqrt{cW(a+bx)}\mathrm{Int}\left(\frac{\sqrt{W(a+bx)}}{x^2}, x\right)}{\sqrt{W(a+bx)}}$$

output
```
(c*LambertW(b*x+a))^(1/2)*Defer(Int)(LambertW(b*x+a)^(1/2)/x^2,x)/LambertW
(b*x+a)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{cW(a+bx)}}{x^2} dx = \int \frac{\sqrt{cW(a+bx)}}{x^2} dx$$

input
```
Integrate[Sqrt[c*ProductLog[a + b*x]]/x^2,x]
```

output
```
Integrate[Sqrt[c*ProductLog[a + b*x]]/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cW(a+bx)}}{x^2} \, dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sqrt{cW(a+bx)} \int \frac{\sqrt{W(a+bx)}}{x^2}dx}{\sqrt{W(a+bx)}}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\sqrt{cW(a+bx)} \int \frac{\sqrt{W(a+bx)}}{x^2}dx}{\sqrt{W(a+bx)}}$$

input
```
Int[Sqrt[c*ProductLog[a + b*x]]/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{c \,\mathrm{LambertW}\left(bx+a\right)}}{x^2}dx$$

input
```
int((c*LambertW(b*x+a))^(1/2)/x^2,x)
```

output
```
int((c*LambertW(b*x+a))^(1/2)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{cW(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(bx+a)}}{x^2}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{cW(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{cW\left(a+bx\right)}}{x^2}\,dx$$

input
```
integrate((c*LambertW(b*x+a))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(c*LambertW(a + b*x))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{cW(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(bx+a)}}{x^2}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*lambert_w(b*x + a))/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{cW(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{c\,\mathrm{W}(bx+a)}}{x^2}\,dx$$

input
```
integrate((c*lambert_w(b*x+a))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c*lambert_w(b*x + a))/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{cW(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{c\,\mathrm{LambertW}\left(a+b\,x\right)}}{x^2}\,dx$$

input
```
int((c*LambertW(a + b*x))^(1/2)/x^2,x)
```

output
```
int((c*LambertW(a + b*x))^(1/2)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 4.19

$$\int \frac{\sqrt{cW(a+bx)}}{x^2}\,dx$$
$$= \frac{\sqrt{c}\left(-2\sqrt{lambert\_w(bx+a)} + \left(\int \frac{\sqrt{lambert\_w(bx+a)}}{e^{lambert\_w(bx+a)}\,lambert\_w(bx+a)^2 x + e^{lambert\_w(bx+a)}\,lambert\_w(bx+a)x}dx\right.\right.}{2x}$$

input
```
int((c*Lambert_W(b*x+a))^(1/2)/x^2,x)
```

output
```
(sqrt(c)*( - 2*sqrt(lambert_w(a + b*x)) + int(sqrt(lambert_w(a + b*x))/(e*
*lambert_w(a + b*x)*lambert_w(a + b*x)**2*x + e**lambert_w(a + b*x)*lamber
t_w(a + b*x)*x),x)*b*x))/(2*x)
```

3.403 $\int \frac{x^3}{\sqrt{cW(a+bx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 449

$$\begin{aligned}\int \frac{x^3}{\sqrt{cW(a+bx)}}\,dx = &-\frac{a^3\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{2b^4\sqrt{c}}-\frac{15\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{8192b^4\sqrt{c}}\\&-\frac{3a^2\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{16b^4\sqrt{c}}-\frac{a\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{24b^4\sqrt{c}}\\&+\frac{15c^3(a+bx)^4}{2048b^4(cW(a+bx))^{7/2}}+\frac{ac^2(a+bx)^3}{12b^4(cW(a+bx))^{5/2}}\\&-\frac{5c^2(a+bx)^4}{256b^4(cW(a+bx))^{5/2}}+\frac{3a^2c(a+bx)^2}{8b^4(cW(a+bx))^{3/2}}\\&-\frac{ac(a+bx)^3}{6b^4(cW(a+bx))^{3/2}}+\frac{c(a+bx)^4}{32b^4(cW(a+bx))^{3/2}}-\frac{a^3(a+bx)}{b^4\sqrt{cW(a+bx)}}\\&+\frac{3a^2(a+bx)^2}{2b^4\sqrt{cW(a+bx)}}-\frac{a(a+bx)^3}{b^4\sqrt{cW(a+bx)}}+\frac{(a+bx)^4}{4b^4\sqrt{cW(a+bx)}}\end{aligned}$$

output
```
-1/2*a^3*Pi^(1/2)*erfi((c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^4/c^(1/2)-15/8
192*Pi^(1/2)*erfi(2*(c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^4/c^(1/2)-3/32*a^
2*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^4/c^(
1/2)-1/72*a*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(c*LambertW(b*x+a))^(1/2)/c^(1/2
))/b^4/c^(1/2)+15/2048*c^3*(b*x+a)^4/b^4/(c*LambertW(b*x+a))^(7/2)+1/12*a*
c^2*(b*x+a)^3/b^4/(c*LambertW(b*x+a))^(5/2)-5/256*c^2*(b*x+a)^4/b^4/(c*Lam
bertW(b*x+a))^(5/2)+3/8*a^2*c*(b*x+a)^2/b^4/(c*LambertW(b*x+a))^(3/2)-1/6*
a*c*(b*x+a)^3/b^4/(c*LambertW(b*x+a))^(3/2)+1/32*c*(b*x+a)^4/b^4/(c*Lamber
tW(b*x+a))^(3/2)-a^3*(b*x+a)/b^4/(c*LambertW(b*x+a))^(1/2)+3/2*a^2*(b*x+a)
^2/b^4/(c*LambertW(b*x+a))^(1/2)-a*(b*x+a)^3/b^4/(c*LambertW(b*x+a))^(1/2)
+1/4*(b*x+a)^4/b^4/(c*LambertW(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.49

$$\int \frac{x^3}{\sqrt{cW(a+bx)}}\,dx$$
$$= \frac{540(a+bx)^4+96(49a-15bx)(a+bx)^3W(a+bx)+768(a+bx)^2\left(23a^2-10abx+3b^2x^2\right)W(a+bx)^2}{}$$

input
```
Integrate[x^3/Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
(540*(a + b*x)^4 + 96*(49*a - 15*b*x)*(a + b*x)^3*ProductLog[a + b*x] + 76
8*(a + b*x)^2*(23*a^2 - 10*a*b*x + 3*b^2*x^2)*ProductLog[a + b*x]^2 - 1843
2*(a^4 - b^4*x^4)*ProductLog[a + b*x]^3 - Sqrt[Pi]*(36864*a^3*Erfi[Sqrt[Pr
oductLog[a + b*x]]] + 135*Erfi[2*Sqrt[ProductLog[a + b*x]]] + 256*a*(27*Sq
rt[2]*a*Erfi[Sqrt[2]*Sqrt[ProductLog[a + b*x]]] + 4*Sqrt[3]*Erfi[Sqrt[3]*S
qrt[ProductLog[a + b*x]]]))*ProductLog[a + b*x]^(7/2))/(73728*b^4*ProductL
og[a + b*x]^3*Sqrt[c*ProductLog[a + b*x]])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{cW(a+bx)}}dx$$

$$\Big\downarrow \text{7168}$$

$$\frac{\int \left(-\frac{a^3}{\sqrt{cW(a+bx)}}+\frac{3(a+bx)a^2}{\sqrt{cW(a+bx)}}-\frac{3(a+bx)^2a}{\sqrt{cW(a+bx)}}+\frac{(a+bx)^3}{\sqrt{cW(a+bx)}}\right)d(a+bx)}{b^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\sqrt{\pi}a^3\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{2\sqrt{c}}-\frac{a^3(a+bx)}{\sqrt{cW(a+bx)}}-\frac{3\sqrt{\frac{\pi}{2}}a^2\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{16\sqrt{c}}+\frac{3a^2(a+bx)^2}{2\sqrt{cW(a+bx)}}+\frac{3a^2c(a+bx)^2}{8(cW(a+bx))^{3/2}}+\frac{15c^3(a+bx)^4}{2048(cW(a+bx))^{7/}}}{}$$

input
```
Int[x^3/Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
(-1/2*(a^3*Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a + b*x]]/Sqrt[c]])/Sqrt[c] - (
15*Sqrt[Pi]*Erfi[(2*Sqrt[c*ProductLog[a + b*x]])/Sqrt[c]])/(8192*Sqrt[c])
- (3*a^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[c*ProductLog[a + b*x]])/Sqrt[c]])/(
16*Sqrt[c]) - (a*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[c*ProductLog[a + b*x]])/Sqr
t[c]])/(24*Sqrt[c]) + (15*c^3*(a + b*x)^4)/(2048*(c*ProductLog[a + b*x])^(
7/2)) + (a*c^2*(a + b*x)^3)/(12*(c*ProductLog[a + b*x])^(5/2)) - (5*c^2*(a
 + b*x)^4)/(256*(c*ProductLog[a + b*x])^(5/2)) + (3*a^2*c*(a + b*x)^2)/(8*
(c*ProductLog[a + b*x])^(3/2)) - (a*c*(a + b*x)^3)/(6*(c*ProductLog[a + b*
x])^(3/2)) + (c*(a + b*x)^4)/(32*(c*ProductLog[a + b*x])^(3/2)) - (a^3*(a
+ b*x))/Sqrt[c*ProductLog[a + b*x]] + (3*a^2*(a + b*x)^2)/(2*Sqrt[c*Produc
tLog[a + b*x]]) - (a*(a + b*x)^3)/Sqrt[c*ProductLog[a + b*x]] + (a + b*x)^
4/(4*Sqrt[c*ProductLog[a + b*x]]))/b^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7168
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((c_.)*ProductLog[(a_) + (b_.)*(x_)])^(p_.),
 x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[(c*ProductLog[x]
)^p, (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, e, f,
p}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.33

method	result
default	$\frac{\frac{c(c\,\mathrm{LambertW}(bx+a))^{\frac{7}{2}}\mathrm{e}^{4\,\mathrm{LambertW}(bx+a)}}{4}+\frac{c\left(\frac{c(c\,\mathrm{LambertW}(bx+a))^{\frac{5}{2}}\mathrm{e}^{4\,\mathrm{LambertW}(bx+a)}}{8}-\frac{5c\left(\frac{c(c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}\mathrm{e}^{4\,\mathrm{LambertW}(bx+}}{8}\right.}{}\right.}{}}{}$

input
```
int(x^3/(c*LambertW(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^4/c^5*(1/8*c*(c*LambertW(b*x+a))^(7/2)*exp(4*LambertW(b*x+a))+1/8*c*(1
/8*c*(c*LambertW(b*x+a))^(5/2)*exp(4*LambertW(b*x+a))-5/8*c*(1/8*c*(c*Lamb
ertW(b*x+a))^(3/2)*exp(4*LambertW(b*x+a))-3/8*c*(1/8*c*(c*LambertW(b*x+a))
^(1/2)*exp(4*LambertW(b*x+a))-1/32*c*Pi^(1/2)/(-1/c)^(1/2)*erf(2*(-1/c)^(1
/2)*(c*LambertW(b*x+a))^(1/2)))))-1/2*a^3*c^4*Pi^(1/2)/(-1/c)^(1/2)*erf((-
1/c)^(1/2)*(c*LambertW(b*x+a))^(1/2))-a^3*c^3*(1/2*c*(c*LambertW(b*x+a))^(
1/2)*(b*x+a)/LambertW(b*x+a)-1/4*c*Pi^(1/2)/(-1/c)^(1/2)*erf((-1/c)^(1/2)*
(c*LambertW(b*x+a))^(1/2)))+3*a^2*c^3*(1/4*c*(c*LambertW(b*x+a))^(1/2)*exp
(2*LambertW(b*x+a))-1/8*c*Pi^(1/2)/(-2/c)^(1/2)*erf((-2/c)^(1/2)*(c*Lamber
tW(b*x+a))^(1/2)))+3*a^2*c^2*(1/4*c*(c*LambertW(b*x+a))^(3/2)*exp(2*Lamber
tW(b*x+a))-3/4*c*(1/4*c*(c*LambertW(b*x+a))^(1/2)*exp(2*LambertW(b*x+a))-1
/8*c*Pi^(1/2)/(-2/c)^(1/2)*erf((-2/c)^(1/2)*(c*LambertW(b*x+a))^(1/2))))-3
*a*c^2*(1/6*c*(c*LambertW(b*x+a))^(3/2)*exp(3*LambertW(b*x+a))-1/2*c*(1/6*
c*(c*LambertW(b*x+a))^(1/2)*exp(3*LambertW(b*x+a))-1/12*c*Pi^(1/2)/(-3/c)^
(1/2)*erf((-3/c)^(1/2)*(c*LambertW(b*x+a))^(1/2))))-3*a*c*(1/6*c*(c*Lamber
tW(b*x+a))^(5/2)*exp(3*LambertW(b*x+a))-5/6*c*(1/6*c*(c*LambertW(b*x+a))^(
3/2)*exp(3*LambertW(b*x+a))-1/2*c*(1/6*c*(c*LambertW(b*x+a))^(1/2)*exp(3*L
ambertW(b*x+a))-1/12*c*Pi^(1/2)/(-3/c)^(1/2)*erf((-3/c)^(1/2)*(c*LambertW(
b*x+a))^(1/2)))))
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{cW(a+bx)}}\,dx = \int \frac{x^3}{\sqrt{c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x^3/(c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))*x^3/(c*lambert_w(b*x + a)), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{cW(a+bx)}}\,dx = \int \frac{x^3}{\sqrt{cW\left(a+bx\right)}}\,dx$$

input
```
integrate(x**3/(c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(x**3/sqrt(c*LambertW(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{cW(a+bx)}}\,dx = \int \frac{x^3}{\sqrt{c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x^3/(c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(c*lambert_w(b*x + a)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{cW(a+bx)}}\,dx = \int \frac{x^3}{\sqrt{c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x^3/(c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(c*lambert_w(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{cW(a+bx)}}\,dx = \int \frac{x^3}{\sqrt{c\,\mathrm{LambertW}\left(a+b\,x\right)}}\,dx$$

input
```
int(x^3/(c*LambertW(a + b*x))^(1/2),x)
```

output
```
int(x^3/(c*LambertW(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{cW(a+bx)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}\,x^3}{lambert\_w(bx+a)}dx\right)}{c}$$

input
```
int(x^3/(c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(lambert_w(a + b*x))*x**3)/lambert_w(a + b*x),x))/c
```

3.404 $\int \frac{x^2}{\sqrt{cW(a+bx)}} dx$

Optimal result

Integrand size = 16, antiderivative size = 290

$$\begin{aligned}\int \frac{x^2}{\sqrt{cW(a+bx)}} dx &= \frac{a^2\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{2b^3\sqrt{c}} + \frac{a\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{8b^3\sqrt{c}} \\ &+ \frac{\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{72b^3\sqrt{c}} - \frac{c^2(a+bx)^3}{36b^3(cW(a+bx))^{5/2}} \\ &- \frac{ac(a+bx)^2}{4b^3(cW(a+bx))^{3/2}} + \frac{c(a+bx)^3}{18b^3(cW(a+bx))^{3/2}} \\ &+ \frac{a^2(a+bx)}{b^3\sqrt{cW(a+bx)}} - \frac{a(a+bx)^2}{b^3\sqrt{cW(a+bx)}} + \frac{(a+bx)^3}{3b^3\sqrt{cW(a+bx)}}\end{aligned}$$

output
```
1/2*a^2*Pi^(1/2)*erfi((c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^3/c^(1/2)+1/16*
a*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^3/c^(
1/2)+1/216*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(c*LambertW(b*x+a))^(1/2)/c^(1/2)
)/b^3/c^(1/2)-1/36*c^2*(b*x+a)^3/b^3/(c*LambertW(b*x+a))^(5/2)-1/4*a*c*(b*
x+a)^2/b^3/(c*LambertW(b*x+a))^(3/2)+1/18*c*(b*x+a)^3/b^3/(c*LambertW(b*x+
a))^(3/2)+a^2*(b*x+a)/b^3/(c*LambertW(b*x+a))^(1/2)-a*(b*x+a)^2/b^3/(c*Lam
bertW(b*x+a))^(1/2)+1/3*(b*x+a)^3/b^3/(c*LambertW(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.56

$$\int \frac{x^2}{\sqrt{cW(a+bx)}}\,dx = \frac{-12(a+bx)^3-12(7a-2bx)(a+bx)^2W(a+bx)+144(a^3+b^3x^3)\,W(a+bx)^2+\sqrt{\pi}\Big(216a^2\text{erfi}\Big(\sqrt{W(}}{432b^3W(a+bx)^2\sqrt{cW(}}$$

input
```
Integrate[x^2/Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
(-12*(a + b*x)^3 - 12*(7*a - 2*b*x)*(a + b*x)^2*ProductLog[a + b*x] + 144*
(a^3 + b^3*x^3)*ProductLog[a + b*x]^2 + Sqrt[Pi]*(216*a^2*Erfi[Sqrt[Produc
tLog[a + b*x]]] + 27*Sqrt[2]*a*Erfi[Sqrt[2]*Sqrt[ProductLog[a + b*x]]] + 2
*Sqrt[3]*Erfi[Sqrt[3]*Sqrt[ProductLog[a + b*x]]])*ProductLog[a + b*x]^(5/2
))/(432*b^3*ProductLog[a + b*x]^2*Sqrt[c*ProductLog[a + b*x]])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {7168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{cW(a+bx)}}\,dx$$

$$\Big\downarrow \; 7168$$

$$\frac{\int \left(\frac{a^2}{\sqrt{cW(a+bx)}}-\frac{2(a+bx)a}{\sqrt{cW(a+bx)}}+\frac{(a+bx)^2}{\sqrt{cW(a+bx)}}\right)d(a+bx)}{b^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\sqrt{\pi}a^2\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{a^2(a+bx)}{\sqrt{cW(a+bx)}}-\frac{c^2(a+bx)^3}{36(cW(a+bx))^{5/2}}+\frac{\sqrt{\frac{\pi}{2}}a\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{8\sqrt{c}}+\frac{\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{72\sqrt{c}}+\frac{(a+bx)^3}{3\sqrt{cW(a+}}}{b^3}$$

input
```
Int[x^2/Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
((a^2*Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a + b*x]]/Sqrt[c]])/(2*Sqrt[c]) + (a
*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[c*ProductLog[a + b*x]])/Sqrt[c]])/(8*Sqrt[c
]) + (Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[c*ProductLog[a + b*x]])/Sqrt[c]])/(72*
Sqrt[c]) - (c^2*(a + b*x)^3)/(36*(c*ProductLog[a + b*x])^(5/2)) - (a*c*(a
+ b*x)^2)/(4*(c*ProductLog[a + b*x])^(3/2)) + (c*(a + b*x)^3)/(18*(c*Produ
ctLog[a + b*x])^(3/2)) + (a^2*(a + b*x))/Sqrt[c*ProductLog[a + b*x]] - (a*
(a + b*x)^2)/Sqrt[c*ProductLog[a + b*x]] + (a + b*x)^3/(3*Sqrt[c*ProductLo
g[a + b*x]]))/b^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7168
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((c_.)*ProductLog[(a_) + (b_.)*(x_)])^(p_.),
 x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[(c*ProductLog[x]
)^p, (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, e, f,
p}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.26

method	result
default	$\frac{\frac{c(c\,\mathrm{LambertW}(bx+a))^{\frac{5}{2}}\mathrm{e}^{3\,\mathrm{LambertW}(bx+a)}}{3}+\frac{c\left(\frac{c(c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}\mathrm{e}^{3\,\mathrm{LambertW}(bx+a)}}{6}-\frac{c\left(\frac{c\sqrt{c\,\mathrm{LambertW}(bx+a)}\,\mathrm{e}^{3\,\mathrm{LambertW}(bx+a)}}{6}\right.}{2}\right.}{3}}{}$

input
```
int(x^2/(c*LambertW(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^3/c^4*(1/6*c*(c*LambertW(b*x+a))^(5/2)*exp(3*LambertW(b*x+a))+1/6*c*(1
/6*c*(c*LambertW(b*x+a))^(3/2)*exp(3*LambertW(b*x+a))-1/2*c*(1/6*c*(c*Lamb
ertW(b*x+a))^(1/2)*exp(3*LambertW(b*x+a))-1/12*c*Pi^(1/2)/(-3/c)^(1/2)*erf
((-3/c)^(1/2)*(c*LambertW(b*x+a))^(1/2))))+1/2*a^2*c^3*Pi^(1/2)/(-1/c)^(1/
2)*erf((-1/c)^(1/2)*(c*LambertW(b*x+a))^(1/2))+a^2*c^2*(1/2*c*(c*LambertW(
b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)-1/4*c*Pi^(1/2)/(-1/c)^(1/2)*erf((-1/
c)^(1/2)*(c*LambertW(b*x+a))^(1/2)))-2*a*c^2*(1/4*c*(c*LambertW(b*x+a))^(1
/2)*exp(2*LambertW(b*x+a))-1/8*c*Pi^(1/2)/(-2/c)^(1/2)*erf((-2/c)^(1/2)*(c
*LambertW(b*x+a))^(1/2)))-2*a*c*(1/4*c*(c*LambertW(b*x+a))^(3/2)*exp(2*Lam
bertW(b*x+a))-3/4*c*(1/4*c*(c*LambertW(b*x+a))^(1/2)*exp(2*LambertW(b*x+a)
)-1/8*c*Pi^(1/2)/(-2/c)^(1/2)*erf((-2/c)^(1/2)*(c*LambertW(b*x+a))^(1/2)))
))
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{cW(a+bx)}}\,dx = \int \frac{x^2}{\sqrt{c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x^2/(c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))*x^2/(c*lambert_w(b*x + a)), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{cW(a+bx)}}\,dx = \int \frac{x^2}{\sqrt{cW\,(a+bx)}}\,dx$$

input
```
integrate(x**2/(c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(x**2/sqrt(c*LambertW(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{cW(a+bx)}}\,dx = \int \frac{x^2}{\sqrt{c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x^2/(c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(c*lambert_w(b*x + a)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{cW(a+bx)}}\,dx = \int \frac{x^2}{\sqrt{c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x^2/(c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(c*lambert_w(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{cW(a+bx)}}\,dx = \int \frac{x^2}{\sqrt{c\,\mathrm{LambertW}\left(a+b\,x\right)}}\,dx$$

input
```
int(x^2/(c*LambertW(a + b*x))^(1/2),x)
```

output
```
int(x^2/(c*LambertW(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{cW(a+bx)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}\,x^2}{lambert\_w(bx+a)}dx\right)}{c}$$

input
```
int(x^2/(c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(lambert_w(a + b*x))*x**2)/lambert_w(a + b*x),x))/c
```

3.405 $\int \frac{x}{\sqrt{cW(a+bx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 159

$$\int \frac{x}{\sqrt{cW(a+bx)}}\,dx = -\frac{a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{2b^2\sqrt{c}} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{16b^2\sqrt{c}} + \frac{c(a+bx)^2}{8b^2(cW(a+bx))^{3/2}} - \frac{a(a+bx)}{b^2\sqrt{cW(a+bx)}} + \frac{(a+bx)^2}{2b^2\sqrt{cW(a+bx)}}$$

output
```
-1/2*a*Pi^(1/2)*erfi((c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^2/c^(1/2)-1/32*2
^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^2/c^(1/2
)+1/8*c*(b*x+a)^2/b^2/(c*LambertW(b*x+a))^(3/2)-a*(b*x+a)/b^2/(c*LambertW(
b*x+a))^(1/2)+1/2*(b*x+a)^2/b^2/(c*LambertW(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.66

$$\int \frac{x}{\sqrt{cW(a+bx)}}\,dx = \frac{c\left(4(a+bx)^2 - 16(a^2-b^2x^2)\,W(a+bx) - \sqrt{\pi}\left(16a\operatorname{erfi}\left(\sqrt{W(a+bx)}\right) + \sqrt{2}\operatorname{erfi}\left(\sqrt{2}\sqrt{W(a+bx)}\right)\right) W}{32b^2(cW(a+bx))^{3/2}}$$

input
```
Integrate[x/Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
(c*(4*(a + b*x)^2 - 16*(a^2 - b^2*x^2)*ProductLog[a + b*x] - Sqrt[Pi]*(16*
a*Erfi[Sqrt[ProductLog[a + b*x]]] + Sqrt[2]*Erfi[Sqrt[2]*Sqrt[ProductLog[a
 + b*x]]])*ProductLog[a + b*x]^(3/2)))/(32*b^2*(c*ProductLog[a + b*x])^(3/
2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {7168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{cW(a+bx)}}dx$$

$$\Big\downarrow \text{7168}$$

$$\frac{\int \left(\frac{a+bx}{\sqrt{cW(a+bx)}}-\frac{a}{\sqrt{cW(a+bx)}}\right)d(a+bx)}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\sqrt{\pi}a\,\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{16\sqrt{c}}+\frac{(a+bx)^2}{2\sqrt{cW(a+bx)}}+\frac{c(a+bx)^2}{8(cW(a+bx))^{3/2}}-\frac{a(a+bx)}{\sqrt{cW(a+bx)}}}{b^2}$$

input
```
Int[x/Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
(-1/2*(a*Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a + b*x]]/Sqrt[c]])/Sqrt[c] - (Sq
rt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[c*ProductLog[a + b*x]])/Sqrt[c]])/(16*Sqrt[c])
 + (c*(a + b*x)^2)/(8*(c*ProductLog[a + b*x])^(3/2)) - (a*(a + b*x))/Sqrt[
c*ProductLog[a + b*x]] + (a + b*x)^2/(2*Sqrt[c*ProductLog[a + b*x]]))/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7168
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((c_.)*ProductLog[(a_) + (b_.)*(x_)])^(p_.),
 x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[(c*ProductLog[x]
)^p, (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, e, f,
p}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.18

method	result
default	$\frac{\frac{c(c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}\,\mathrm{e}^{2\,\mathrm{LambertW}(bx+a)}}{2}+\frac{c\left(\frac{c\sqrt{c\,\mathrm{LambertW}(bx+a)}\,\mathrm{e}^{2\,\mathrm{LambertW}(bx+a)}}{4}-\frac{c\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{2}{c}}\,\sqrt{c\,\mathrm{LambertW}(bx+a)}\right)}{8\sqrt{-\frac{2}{c}}}\right)}{2}-\frac{a\,c^{2}\sqrt{\ }}{}}{b^{2}c^{3}}$

input
```
int(x/(c*LambertW(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^2/c^3*(1/4*c*(c*LambertW(b*x+a))^(3/2)*exp(2*LambertW(b*x+a))+1/4*c*(1
/4*c*(c*LambertW(b*x+a))^(1/2)*exp(2*LambertW(b*x+a))-1/8*c*Pi^(1/2)/(-2/c
)^(1/2)*erf((-2/c)^(1/2)*(c*LambertW(b*x+a))^(1/2)))-1/2*a*c^2*Pi^(1/2)/(-
1/c)^(1/2)*erf((-1/c)^(1/2)*(c*LambertW(b*x+a))^(1/2))-a*c*(1/2*c*(c*Lambe
rtW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)-1/4*c*Pi^(1/2)/(-1/c)^(1/2)*erf(
(-1/c)^(1/2)*(c*LambertW(b*x+a))^(1/2))))
```

Fricas [F]

$$\int \frac{x}{\sqrt{cW(a+bx)}}\,dx = \int \frac{x}{\sqrt{c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x/(c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))*x/(c*lambert_w(b*x + a)), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{cW(a+bx)}}\,dx = \int \frac{x}{\sqrt{cW\left(a+bx\right)}}\,dx$$

input
```
integrate(x/(c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(x/sqrt(c*LambertW(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{cW(a+bx)}}\,dx = \int \frac{x}{\sqrt{c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x/(c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(c*lambert_w(b*x + a)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{cW(a+bx)}}\,dx = \int \frac{x}{\sqrt{c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x/(c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(c*lambert_w(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{cW(a+bx)}}\,dx = \int \frac{x}{\sqrt{c\,\mathrm{LambertW}\,(a+b\,x)}}\,dx$$

input
```
int(x/(c*LambertW(a + b*x))^(1/2),x)
```

output
```
int(x/(c*LambertW(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{cW(a+bx)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}\,x}{lambert\_w(bx+a)}dx\right)}{c}$$

input
```
int(x/(c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(lambert_w(a + b*x))*x)/lambert_w(a + b*x),x))/c
```

3.406 $\int \frac{1}{\sqrt{cW(a+bx)}} dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{\sqrt{cW(a+bx)}} dx = \frac{\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{2b\sqrt{c}} + \frac{a+bx}{b\sqrt{cW(a+bx)}}$$

output
```
1/2*Pi^(1/2)*erfi((c*LambertW(b*x+a))^(1/2)/c^(1/2))/b/c^(1/2)+(b*x+a)/b/(
c*LambertW(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{cW(a+bx)}} dx = \frac{2(a+bx) + \sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{W(a+bx)}\right)\sqrt{W(a+bx)}}{2b\sqrt{cW(a+bx)}}$$

input
```
Integrate[1/Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
(2*(a + b*x) + Sqrt[Pi]*Erfi[Sqrt[ProductLog[a + b*x]]]*Sqrt[ProductLog[a
+ b*x]])/(2*b*Sqrt[c*ProductLog[a + b*x]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7167, 7180}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cW(a+bx)}}dx$$
$$\Big\downarrow \text{7167}$$
$$\frac{1}{2}\int \frac{1}{\sqrt{cW(a+bx)}(W(a+bx)+1)}dx+\frac{a+bx}{b\sqrt{cW(a+bx)}}$$
$$\Big\downarrow \text{7180}$$
$$\frac{\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{cW(a+bx)}}{\sqrt{c}}\right)}{2b\sqrt{c}}+\frac{a+bx}{b\sqrt{cW(a+bx)}}$$

input
```
Int[1/Sqrt[c*ProductLog[a + b*x]],x]
```

output
```
(Sqrt[Pi]*Erfi[Sqrt[c*ProductLog[a + b*x]]/Sqrt[c]])/(2*b*Sqrt[c]) + (a +
b*x)/(b*Sqrt[c*ProductLog[a + b*x]])
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7180
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[Pi*c, 2]*(Erfi[Sqrt[c*ProductLo
g[a + b*x]]/Rt[c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && PosQ[c]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{\frac{c\sqrt{c\,\mathrm{LambertW}(bx+a)}\,(bx+a)}{\mathrm{LambertW}(bx+a)}+\frac{c\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{c}}\,\sqrt{c\,\mathrm{LambertW}(bx+a)}\right)}{2\sqrt{-\frac{1}{c}}}}{b\,c^{2}}$	68

input
```
int(1/(c*LambertW(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(1/2*c*(c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)+1/4*c*Pi^
(1/2)/(-1/c)^(1/2)*erf((-1/c)^(1/2)*(c*LambertW(b*x+a))^(1/2)))
```

Fricas [F]

$$\int \frac{1}{\sqrt{cW(a+bx)}}\,dx=\int \frac{1}{\sqrt{c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(1/(c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))/(c*lambert_w(b*x + a)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{cW(a+bx)}}\,dx=\int \frac{1}{\sqrt{cW\,(a+bx)}}\,dx$$

input
```
integrate(1/(c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(1/sqrt(c*LambertW(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(1/(c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*lambert_w(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(1/(c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*lambert_w(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{LambertW}\,(a+b\,x)}}\,dx$$

input
```
int(1/(c*LambertW(a + b*x))^(1/2),x)
```

output
```
int(1/(c*LambertW(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{cW(a+bx)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}}{lambert\_w(bx+a)}dx\right)}{c}$$

input
```
int(1/(c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a + b*x))/lambert_w(a + b*x),x))/c
```

3.407 $\int \frac{1}{x\sqrt{cW(a+bx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x\sqrt{cW(a+bx)}}\,dx = \frac{\sqrt{W(a+bx)}\text{Int}\left(\frac{1}{x\sqrt{W(a+bx)}},x\right)}{\sqrt{cW(a+bx)}}$$

output
```
LambertW(b*x+a)^(1/2)*Defer(Int)(1/x/LambertW(b*x+a)^(1/2),x)/(c*LambertW(
b*x+a))^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{x\sqrt{cW(a+bx)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[c*ProductLog[a + b*x]]),x]
```

output
```
Integrate[1/(x*Sqrt[c*ProductLog[a + b*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{cW(a+bx)}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\sqrt{W(a+bx)}\int \frac{1}{x\sqrt{W(a+bx)}}dx}{\sqrt{cW(a+bx)}}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\sqrt{W(a+bx)}\int \frac{1}{x\sqrt{W(a+bx)}}dx}{\sqrt{cW(a+bx)}}$$

input
```
Int[1/(x*Sqrt[c*ProductLog[a + b*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{c\,\mathrm{LambertW}\left(bx+a\right)}}dx$$

input
```
int(1/x/(c*LambertW(b*x+a))^(1/2),x)
```

output
```
int(1/x/(c*LambertW(b*x+a))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{1}{x\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(bx+a)}x}\,dx$$

input
```
integrate(1/x/(c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))/(c*x*lambert_w(b*x + a)), x)
```

Sympy [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{x\sqrt{cW\left(a+bx\right)}}\,dx$$

input
```
integrate(1/x/(c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(c*LambertW(a + b*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(bx+a)}x}\,dx$$

input
```
integrate(1/x/(c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*lambert_w(b*x + a))*x), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(bx+a)}x}\,dx$$

input
```
integrate(1/x/(c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*lambert_w(b*x + a))*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{x\,\sqrt{c\,\mathrm{LambertW}\left(a+b\,x\right)}}\,dx$$

input
```
int(1/(x*(c*LambertW(a + b*x))^(1/2)),x)
```

output
```
int(1/(x*(c*LambertW(a + b*x))^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{1}{x\sqrt{cW(a+bx)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}}{lambert\_w(bx+a)x}dx\right)}{c}$$

input
```
int(1/x/(c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a + b*x))/(lambert_w(a + b*x)*x),x))/c
```

3.408 $\int \frac{1}{x^2\sqrt{cW(a+bx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x^2\sqrt{cW(a+bx)}}\,dx = \frac{\sqrt{W(a+bx)}\text{Int}\left(\frac{1}{x^2\sqrt{W(a+bx)}},x\right)}{\sqrt{cW(a+bx)}}$$

output
```
LambertW(b*x+a)^(1/2)*Defer(Int)(1/x^2/LambertW(b*x+a)^(1/2),x)/(c*Lambert
W(b*x+a))^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{x^2\sqrt{cW(a+bx)}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[c*ProductLog[a + b*x]]),x]
```

output
```
Integrate[1/(x^2*Sqrt[c*ProductLog[a + b*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{cW(a+bx)}}dx$$
$$\Big\downarrow \text{ 7271}$$
$$\frac{\sqrt{W(a+bx)}\int \frac{1}{x^2\sqrt{W(a+bx)}}dx}{\sqrt{cW(a+bx)}}$$
$$\Big\downarrow \text{ 7299}$$
$$\frac{\sqrt{W(a+bx)}\int \frac{1}{x^2\sqrt{W(a+bx)}}dx}{\sqrt{cW(a+bx)}}$$

input
```
Int[1/(x^2*Sqrt[c*ProductLog[a + b*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\sqrt{c\,\mathrm{LambertW}\,(bx+a)}}dx$$

input
```
int(1/x^2/(c*LambertW(b*x+a))^(1/2),x)
```

output
```
int(1/x^2/(c*LambertW(b*x+a))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{1}{x^2\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(bx+a)}x^2}\,dx$$

input
```
integrate(1/x^2/(c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*lambert_w(b*x + a))/(c*x^2*lambert_w(b*x + a)), x)
```

Sympy [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{x^2\sqrt{cW\left(a+bx\right)}}\,dx$$

input
```
integrate(1/x**2/(c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(c*LambertW(a + b*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(bx+a)}x^2}\,dx$$

input
```
integrate(1/x^2/(c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*lambert_w(b*x + a))*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\,\mathrm{W}(bx+a)}x^2}\,dx$$

input
```
integrate(1/x^2/(c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*lambert_w(b*x + a))*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{cW(a+bx)}}\,dx = \int \frac{1}{x^2\,\sqrt{c\,\mathrm{LambertW}\left(a+b\,x\right)}}\,dx$$

input
```
int(1/(x^2*(c*LambertW(a + b*x))^(1/2)),x)
```

output
```
int(1/(x^2*(c*LambertW(a + b*x))^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^2\sqrt{cW(a+bx)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}}{lambert\_w(bx+a)x^2}dx\right)}{c}$$

input
```
int(1/x^2/(c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(lambert_w(a + b*x))/(lambert_w(a + b*x)*x**2),x))/c
```

3.409 $\int \frac{x^3}{\sqrt{-cW(a+bx)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 463

$$\begin{aligned}\int \frac{x^3}{\sqrt{-cW(a+bx)}}\,dx &= \frac{a^3\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{2b^4\sqrt{c}}+\frac{15\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{8192b^4\sqrt{c}}\\
&\quad+\frac{3a^2\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{16b^4\sqrt{c}}+\frac{a\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{24b^4\sqrt{c}}\\
&\quad-\frac{15c^3(a+bx)^4}{2048b^4(-cW(a+bx))^{7/2}}+\frac{ac^2(a+bx)^3}{12b^4(-cW(a+bx))^{5/2}}\\
&\quad-\frac{5c^2(a+bx)^4}{256b^4(-cW(a+bx))^{5/2}}-\frac{3a^2c(a+bx)^2}{8b^4(-cW(a+bx))^{3/2}}\\
&\quad+\frac{ac(a+bx)^3}{6b^4(-cW(a+bx))^{3/2}}-\frac{c(a+bx)^4}{32b^4(-cW(a+bx))^{3/2}}\\
&\quad-\frac{a^3(a+bx)}{b^4\sqrt{-cW(a+bx)}}+\frac{3a^2(a+bx)^2}{2b^4\sqrt{-cW(a+bx)}}\\
&\quad-\frac{a(a+bx)^3}{b^4\sqrt{-cW(a+bx)}}+\frac{(a+bx)^4}{4b^4\sqrt{-cW(a+bx)}}\end{aligned}$$

output
```
1/2*a^3*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^4/c^(1/2)+15/81
92*Pi^(1/2)*erf(2*(-c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^4/c^(1/2)+3/32*a^2
*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(-c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^4/c^(1
/2)+1/72*a*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(-c*LambertW(b*x+a))^(1/2)/c^(1/2)
)/b^4/c^(1/2)-15/2048*c^3*(b*x+a)^4/b^4/(-c*LambertW(b*x+a))^(7/2)+1/12*a*
c^2*(b*x+a)^3/b^4/(-c*LambertW(b*x+a))^(5/2)-5/256*c^2*(b*x+a)^4/b^4/(-c*L
ambertW(b*x+a))^(5/2)-3/8*a^2*c*(b*x+a)^2/b^4/(-c*LambertW(b*x+a))^(3/2)+1
/6*a*c*(b*x+a)^3/b^4/(-c*LambertW(b*x+a))^(3/2)-1/32*c*(b*x+a)^4/b^4/(-c*L
ambertW(b*x+a))^(3/2)-a^3*(b*x+a)/b^4/(-c*LambertW(b*x+a))^(1/2)+3/2*a^2*(
b*x+a)^2/b^4/(-c*LambertW(b*x+a))^(1/2)-a*(b*x+a)^3/b^4/(-c*LambertW(b*x+a
))^(1/2)+1/4*(b*x+a)^4/b^4/(-c*LambertW(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.48

$$\int \frac{x^3}{\sqrt{-cW(a+bx)}}\,dx = \frac{540(a+bx)^4+96(49a-15bx)(a+bx)^3W(a+bx)+768(a+bx)^2\left(23a^2-10abx+3b^2x^2\right)W(a+bx)^2}{}$$

input
```
Integrate[x^3/Sqrt[-(c*ProductLog[a + b*x])],x]
```

output
```
(540*(a + b*x)^4 + 96*(49*a - 15*b*x)*(a + b*x)^3*ProductLog[a + b*x] + 76
8*(a + b*x)^2*(23*a^2 - 10*a*b*x + 3*b^2*x^2)*ProductLog[a + b*x]^2 - 1843
2*(a^4 - b^4*x^4)*ProductLog[a + b*x]^3 - Sqrt[Pi]*(36864*a^3*Erfi[Sqrt[Pr
oductLog[a + b*x]]] + 135*Erfi[2*Sqrt[ProductLog[a + b*x]]] + 256*a*(27*Sq
rt[2]*a*Erfi[Sqrt[2]*Sqrt[ProductLog[a + b*x]]] + 4*Sqrt[3]*Erfi[Sqrt[3]*S
qrt[ProductLog[a + b*x]]]))*ProductLog[a + b*x]^(7/2))/(73728*b^4*ProductL
og[a + b*x]^3*Sqrt[-(c*ProductLog[a + b*x])])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{-cW(a+bx)}}dx$$

$$\Big\downarrow \text{7168}$$

$$\frac{\int \left(-\frac{a^3}{\sqrt{-cW(a+bx)}}+\frac{3(a+bx)a^2}{\sqrt{-cW(a+bx)}}-\frac{3(a+bx)^2a}{\sqrt{-cW(a+bx)}}+\frac{(a+bx)^3}{\sqrt{-cW(a+bx)}}\right)d(a+bx)}{b^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{\pi}a^3\text{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{2\sqrt{c}}-\frac{a^3(a+bx)}{\sqrt{-cW(a+bx)}}+\frac{3\sqrt{\frac{\pi}{2}}a^2\text{erf}\left(\frac{\sqrt{2}\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{16\sqrt{c}}+\frac{3a^2(a+bx)^2}{2\sqrt{-cW(a+bx)}}-\frac{3a^2c(a+bx)^2}{8(-cW(a+bx))^{3/2}}-\frac{15c^3(a+b}{2048(-cW(a+}$$

input
```
Int[x^3/Sqrt[-(c*ProductLog[a + b*x])],x]
```

output
```
((a^3*Sqrt[Pi]*Erf[Sqrt[-(c*ProductLog[a + b*x])]/Sqrt[c]])/(2*Sqrt[c]) +
(15*Sqrt[Pi]*Erf[(2*Sqrt[-(c*ProductLog[a + b*x])])/Sqrt[c]])/(8192*Sqrt[c
]) + (3*a^2*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[-(c*ProductLog[a + b*x])])/Sqrt[c
]])/(16*Sqrt[c]) + (a*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[-(c*ProductLog[a + b*x]
)])/Sqrt[c]])/(24*Sqrt[c]) - (15*c^3*(a + b*x)^4)/(2048*(-(c*ProductLog[a
+ b*x]))^(7/2)) + (a*c^2*(a + b*x)^3)/(12*(-(c*ProductLog[a + b*x]))^(5/2)
) - (5*c^2*(a + b*x)^4)/(256*(-(c*ProductLog[a + b*x]))^(5/2)) - (3*a^2*c*
(a + b*x)^2)/(8*(-(c*ProductLog[a + b*x]))^(3/2)) + (a*c*(a + b*x)^3)/(6*(
-(c*ProductLog[a + b*x]))^(3/2)) - (c*(a + b*x)^4)/(32*(-(c*ProductLog[a +
 b*x]))^(3/2)) - (a^3*(a + b*x))/Sqrt[-(c*ProductLog[a + b*x])] + (3*a^2*(
a + b*x)^2)/(2*Sqrt[-(c*ProductLog[a + b*x])]) - (a*(a + b*x)^3)/Sqrt[-(c*
ProductLog[a + b*x])] + (a + b*x)^4/(4*Sqrt[-(c*ProductLog[a + b*x])]))/b^
4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7168
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((c_.)*ProductLog[(a_) + (b_.)*(x_)])^(p_.),
 x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[(c*ProductLog[x]
)^p, (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, e, f,
p}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.24

method	result
default	$-\frac{2\left(-\frac{c(-c\,\mathrm{LambertW}(bx+a))^{\frac{7}{2}}\mathrm{e}^{4\,\mathrm{LambertW}(bx+a)}}{8}-\frac{c\left(-\frac{c(-c\,\mathrm{LambertW}(bx+a))^{\frac{5}{2}}\mathrm{e}^{4\,\mathrm{LambertW}(bx+a)}}{8}+\frac{5c\left(-\frac{c(-c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}\mathrm{e}\cdots}{8}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int(x^3/(-c*LambertW(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b^4/c^5*(-1/8*c*(-c*LambertW(b*x+a))^(7/2)*exp(4*LambertW(b*x+a))-1/8*c
*(-1/8*c*(-c*LambertW(b*x+a))^(5/2)*exp(4*LambertW(b*x+a))+5/8*c*(-1/8*c*(
-c*LambertW(b*x+a))^(3/2)*exp(4*LambertW(b*x+a))+3/8*c*(-1/8*c*(-c*Lambert
W(b*x+a))^(1/2)*exp(4*LambertW(b*x+a))+1/32*c^(3/2)*Pi^(1/2)*erf(2*(-c*Lam
bertW(b*x+a))^(1/2)/c^(1/2)))))+a^3*c^3*(-1/2*c*(-c*LambertW(b*x+a))^(1/2)
*(b*x+a)/LambertW(b*x+a)+1/4*c^(3/2)*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/
2)/c^(1/2)))-1/2*a^3*c^(9/2)*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/
2))-3*a*c^2*(-1/6*c*(-c*LambertW(b*x+a))^(3/2)*exp(3*LambertW(b*x+a))+1/2*
c*(-1/6*c*(-c*LambertW(b*x+a))^(1/2)*exp(3*LambertW(b*x+a))+1/36*c^(3/2)*P
i^(1/2)*3^(1/2)*erf(3^(1/2)*(-c*LambertW(b*x+a))^(1/2)/c^(1/2))))+3*a*c*(-
1/6*c*(-c*LambertW(b*x+a))^(5/2)*exp(3*LambertW(b*x+a))+5/6*c*(-1/6*c*(-c*
LambertW(b*x+a))^(3/2)*exp(3*LambertW(b*x+a))+1/2*c*(-1/6*c*(-c*LambertW(b
*x+a))^(1/2)*exp(3*LambertW(b*x+a))+1/36*c^(3/2)*Pi^(1/2)*3^(1/2)*erf(3^(1
/2)*(-c*LambertW(b*x+a))^(1/2)/c^(1/2)))))-3*a^2*c^3*(-1/4*c*(-c*LambertW(
b*x+a))^(1/2)*exp(2*LambertW(b*x+a))+1/16*c^(3/2)*Pi^(1/2)*2^(1/2)*erf(2^(
1/2)*(-c*LambertW(b*x+a))^(1/2)/c^(1/2)))+3*a^2*c^2*(-1/4*c*(-c*LambertW(b
*x+a))^(3/2)*exp(2*LambertW(b*x+a))+3/4*c*(-1/4*c*(-c*LambertW(b*x+a))^(1/
2)*exp(2*LambertW(b*x+a))+1/16*c^(3/2)*Pi^(1/2)*2^(1/2)*erf(2^(1/2)*(-c*La
mbertW(b*x+a))^(1/2)/c^(1/2))))
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{-cW(a+bx)}}\,dx=\int \frac{x^3}{\sqrt{-c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x^3/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*lambert_w(b*x + a))*x^3/(c*lambert_w(b*x + a)), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{x^3}{\sqrt{-cW\left(a+bx\right)}}\,dx$$

input
```
integrate(x**3/(-c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(x**3/sqrt(-c*LambertW(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{x^3}{\sqrt{-c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x^3/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(-c*lambert_w(b*x + a)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{x^3}{\sqrt{-c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x^3/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(-c*lambert_w(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{-cW(a+bx)}} \, dx = \int \frac{x^3}{\sqrt{-c\,\mathrm{LambertW}\left(a+b\,x\right)}} \,dx$$

input
```
int(x^3/(-c*LambertW(a + b*x))^(1/2),x)
```

output
```
int(x^3/(-c*LambertW(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{-cW(a+bx)}} \, dx = -\frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}\,x^3}{lambert\_w(bx+a)}dx\right)i}{c}$$

input
```
int(x^3/(-c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
( - sqrt(c)*int((sqrt(lambert_w(a + b*x))*x**3)/lambert_w(a + b*x),x)*i)/c
```

3.410 $\int \frac{x^2}{\sqrt{-cW(a+bx)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 299

$$\int \frac{x^2}{\sqrt{-cW(a+bx)}}\,dx = -\frac{a^2\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{2b^3\sqrt{c}} - \frac{a\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{8b^3\sqrt{c}} - \frac{\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{72b^3\sqrt{c}} - \frac{c^2(a+bx)^3}{36b^3(-cW(a+bx))^{5/2}} + \frac{ac(a+bx)^2}{4b^3(-cW(a+bx))^{3/2}} - \frac{c(a+bx)^3}{18b^3(-cW(a+bx))^{3/2}} + \frac{a^2(a+bx)}{b^3\sqrt{-cW(a+bx)}} - \frac{a(a+bx)^2}{b^3\sqrt{-cW(a+bx)}} + \frac{(a+bx)^3}{3b^3\sqrt{-cW(a+bx)}}$$

output

```
-1/2*a^2*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^3/c^(1/2)-1/16
*a*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(-c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^3/c^
(1/2)-1/216*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(-c*LambertW(b*x+a))^(1/2)/c^(1/2
))/b^3/c^(1/2)-1/36*c^2*(b*x+a)^3/b^3/(-c*LambertW(b*x+a))^(5/2)+1/4*a*c*(
b*x+a)^2/b^3/(-c*LambertW(b*x+a))^(3/2)-1/18*c*(b*x+a)^3/b^3/(-c*LambertW(
b*x+a))^(3/2)+a^2*(b*x+a)/b^3/(-c*LambertW(b*x+a))^(1/2)-a*(b*x+a)^2/b^3/(
-c*LambertW(b*x+a))^(1/2)+1/3*(b*x+a)^3/b^3/(-c*LambertW(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{\sqrt{-cW(a+bx)}}\,dx = \frac{-12(a+bx)^3-12(7a-2bx)(a+bx)^2W(a+bx)+144(a^3+b^3x^3)\,W(a+bx)^2+\sqrt{\pi}\Big(216a^2\text{erfi}\Big(\sqrt{W(}}{432b^3W(a+bx)^2\sqrt{-cW}}$$

input
```
Integrate[x^2/Sqrt[-(c*ProductLog[a + b*x])],x]
```

output
```
(-12*(a + b*x)^3 - 12*(7*a - 2*b*x)*(a + b*x)^2*ProductLog[a + b*x] + 144*
(a^3 + b^3*x^3)*ProductLog[a + b*x]^2 + Sqrt[Pi]*(216*a^2*Erfi[Sqrt[Produc
tLog[a + b*x]]] + 27*Sqrt[2]*a*Erfi[Sqrt[2]*Sqrt[ProductLog[a + b*x]]] + 2
*Sqrt[3]*Erfi[Sqrt[3]*Sqrt[ProductLog[a + b*x]]])*ProductLog[a + b*x]^(5/2
))/(432*b^3*ProductLog[a + b*x]^2*Sqrt[-(c*ProductLog[a + b*x])])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{-cW(a+bx)}}\,dx$$

$$\Big\downarrow \text{7168}$$

$$\frac{\int \left(\frac{a^2}{\sqrt{-cW(a+bx)}}-\frac{2(a+bx)a}{\sqrt{-cW(a+bx)}}+\frac{(a+bx)^2}{\sqrt{-cW(a+bx)}}\right)d(a+bx)}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\sqrt{\pi}a^2\text{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{a^2(a+bx)}{\sqrt{-cW(a+bx)}}-\frac{c^2(a+bx)^3}{36(-cW(a+bx))^{5/2}}-\frac{\sqrt{\frac{\pi}{2}}a\text{erf}\left(\frac{\sqrt{2}\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{8\sqrt{c}}-\frac{\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{72\sqrt{c}}+\frac{\ }{3\sqrt{}}}{b^3}$$

input
```
Int[x^2/Sqrt[-(c*ProductLog[a + b*x])],x]
```

output
```
(-1/2*(a^2*Sqrt[Pi]*Erf[Sqrt[-(c*ProductLog[a + b*x])]/Sqrt[c]])/Sqrt[c] -
 (a*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[-(c*ProductLog[a + b*x])])/Sqrt[c]])/(8*S
qrt[c]) - (Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[-(c*ProductLog[a + b*x])])/Sqrt[c]
])/(72*Sqrt[c]) - (c^2*(a + b*x)^3)/(36*(-(c*ProductLog[a + b*x]))^(5/2))
+ (a*c*(a + b*x)^2)/(4*(-(c*ProductLog[a + b*x]))^(3/2)) - (c*(a + b*x)^3)
/(18*(-(c*ProductLog[a + b*x]))^(3/2)) + (a^2*(a + b*x))/Sqrt[-(c*ProductL
og[a + b*x])] - (a*(a + b*x)^2)/Sqrt[-(c*ProductLog[a + b*x])] + (a + b*x)
^3/(3*Sqrt[-(c*ProductLog[a + b*x])]))/b^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7168
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((c_.)*ProductLog[(a_) + (b_.)*(x_)])^(p_.),
 x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[(c*ProductLog[x]
)^p, (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, e, f,
p}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.16

method	result
default	$-\frac{c(-c\,\mathrm{LambertW}(bx+a))^{\frac{5}{2}}\,\mathrm{e}^{3\,\mathrm{LambertW}(bx+a)}}{3}-\frac{c\left(-\frac{c(-c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}\,\mathrm{e}^{3\,\mathrm{LambertW}(bx+a)}}{6}+\frac{c\left(-\frac{c\sqrt{-c\,\mathrm{LambertW}(bx+a)}\,\mathrm{e}^{3\,\mathrm{Lamber}}}{6}\right.}{}\right.}{3}$

input
```
int(x^2/(-c*LambertW(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^3/c^4*(-1/6*c*(-c*LambertW(b*x+a))^(5/2)*exp(3*LambertW(b*x+a))-1/6*c*
(-1/6*c*(-c*LambertW(b*x+a))^(3/2)*exp(3*LambertW(b*x+a))+1/2*c*(-1/6*c*(-
c*LambertW(b*x+a))^(1/2)*exp(3*LambertW(b*x+a))+1/36*c^(3/2)*Pi^(1/2)*3^(1
/2)*erf(3^(1/2)*(-c*LambertW(b*x+a))^(1/2)/c^(1/2))))+a^2*c^2*(-1/2*c*(-c*
LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)+1/4*c^(3/2)*Pi^(1/2)*erf((-
c*LambertW(b*x+a))^(1/2)/c^(1/2)))-1/2*a^2*c^(7/2)*Pi^(1/2)*erf((-c*Lamber
tW(b*x+a))^(1/2)/c^(1/2))-2*a*c^2*(-1/4*c*(-c*LambertW(b*x+a))^(1/2)*exp(2
*LambertW(b*x+a))+1/16*c^(3/2)*Pi^(1/2)*2^(1/2)*erf(2^(1/2)*(-c*LambertW(b
*x+a))^(1/2)/c^(1/2)))+2*a*c*(-1/4*c*(-c*LambertW(b*x+a))^(3/2)*exp(2*Lamb
ertW(b*x+a))+3/4*c*(-1/4*c*(-c*LambertW(b*x+a))^(1/2)*exp(2*LambertW(b*x+a
))+1/16*c^(3/2)*Pi^(1/2)*2^(1/2)*erf(2^(1/2)*(-c*LambertW(b*x+a))^(1/2)/c^
(1/2)))))
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{-cW(a+bx)}}\,dx=\int \frac{x^2}{\sqrt{-c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x^2/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*lambert_w(b*x + a))*x^2/(c*lambert_w(b*x + a)), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{x^2}{\sqrt{-cW\left(a+bx\right)}}\,dx$$

input
```
integrate(x**2/(-c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(x**2/sqrt(-c*LambertW(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{x^2}{\sqrt{-c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x^2/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(-c*lambert_w(b*x + a)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{x^2}{\sqrt{-c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x^2/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(-c*lambert_w(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{x^2}{\sqrt{-c\,\mathrm{LambertW}\left(a+b\,x\right)}}\,dx$$

input
```
int(x^2/(-c*LambertW(a + b*x))^(1/2),x)
```

output
```
int(x^2/(-c*LambertW(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{-cW(a+bx)}}\,dx = -\frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}\,x^2}{lambert\_w(bx+a)}dx\right)i}{c}$$

input
```
int(x^2/(-c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
( - sqrt(c)*int((sqrt(lambert_w(a + b*x))*x**2)/lambert_w(a + b*x),x)*i)/c
```

3.411 $\int \frac{x}{\sqrt{-cW(a+bx)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 164

$$\int \frac{x}{\sqrt{-cW(a+bx)}}\,dx = \frac{a\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{2b^2\sqrt{c}} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{16b^2\sqrt{c}} - \frac{c(a+bx)^2}{8b^2(-cW(a+bx))^{3/2}} - \frac{a(a+bx)}{b^2\sqrt{-cW(a+bx)}} + \frac{(a+bx)^2}{2b^2\sqrt{-cW(a+bx)}}$$

output

```
1/2*a*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^2/c^(1/2)+1/32*2^
(1/2)*Pi^(1/2)*erf(2^(1/2)*(-c*LambertW(b*x+a))^(1/2)/c^(1/2))/b^2/c^(1/2)
-1/8*c*(b*x+a)^2/b^2/(-c*LambertW(b*x+a))^(3/2)-a*(b*x+a)/b^2/(-c*LambertW
(b*x+a))^(1/2)+1/2*(b*x+a)^2/b^2/(-c*LambertW(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.65

$$\int \frac{x}{\sqrt{-cW(a+bx)}}\,dx = -\frac{c\left(4(a+bx)^2-16(a^2-b^2x^2)\,W(a+bx)-\sqrt{\pi}\left(16a\,\text{erfi}\left(\sqrt{W(a+bx)}\right)+\sqrt{2}\,\text{erfi}\left(\sqrt{2}\sqrt{W(a+bx)}\right)\right.\right.}{32b^2(-cW(a+bx))^{3/2}}$$

input
```
Integrate[x/Sqrt[-(c*ProductLog[a + b*x])],x]
```

output
```
-1/32*(c*(4*(a + b*x)^2 - 16*(a^2 - b^2*x^2)*ProductLog[a + b*x] - Sqrt[Pi
]*(16*a*Erfi[Sqrt[ProductLog[a + b*x]]] + Sqrt[2]*Erfi[Sqrt[2]*Sqrt[Produc
tLog[a + b*x]]])*ProductLog[a + b*x]^(3/2)))/(b^2*(-(c*ProductLog[a + b*x]
))^(3/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {7168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{-cW(a+bx)}}dx$$

$$\Big\downarrow \ 7168$$

$$\frac{\int \left(\frac{a+bx}{\sqrt{-cW(a+bx)}}-\frac{a}{\sqrt{-cW(a+bx)}}\right)d(a+bx)}{b^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\sqrt{\pi}a\,\text{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{16\sqrt{c}}+\frac{(a+bx)^2}{2\sqrt{-cW(a+bx)}}-\frac{c(a+bx)^2}{8(-cW(a+bx))^{3/2}}-\frac{a(a+bx)}{\sqrt{-cW(a+bx)}}}{b^2}$$

input
```
Int[x/Sqrt[-(c*ProductLog[a + b*x])],x]
```

output
```
((a*Sqrt[Pi]*Erf[Sqrt[-(c*ProductLog[a + b*x])]/Sqrt[c]])/(2*Sqrt[c]) + (S
qrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[-(c*ProductLog[a + b*x])])/Sqrt[c]])/(16*Sqrt[
c]) - (c*(a + b*x)^2)/(8*(-(c*ProductLog[a + b*x]))^(3/2)) - (a*(a + b*x))
/Sqrt[-(c*ProductLog[a + b*x])] + (a + b*x)^2/(2*Sqrt[-(c*ProductLog[a + b
*x])]))/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7168
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((c_.)*ProductLog[(a_) + (b_.)*(x_)])^(p_.),
 x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[(c*ProductLog[x]
)^p, (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, e, f,
p}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.04

method	result
default	$-\frac{2\left(-\frac{c(-c\,\mathrm{LambertW}(bx+a))^{\frac{3}{2}}\,\mathrm{e}^{2\,\mathrm{LambertW}(bx+a)}}{4}-\frac{c\left(-\frac{c\sqrt{-c\,\mathrm{LambertW}(bx+a)}\,\mathrm{e}^{2\,\mathrm{LambertW}(bx+a)}}{4}+\frac{c^{\frac{3}{2}}\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\frac{\sqrt{2}\,\sqrt{-c\,\mathrm{LambertW}}}{\sqrt{c}}\right.}{16}}{4}\right.}{b^2c^3}$

input
```
int(x/(-c*LambertW(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b^2/c^3*(-1/4*c*(-c*LambertW(b*x+a))^(3/2)*exp(2*LambertW(b*x+a))-1/4*c
*(-1/4*c*(-c*LambertW(b*x+a))^(1/2)*exp(2*LambertW(b*x+a))+1/16*c^(3/2)*Pi
^(1/2)*2^(1/2)*erf(2^(1/2)*(-c*LambertW(b*x+a))^(1/2)/c^(1/2)))+a*c*(-1/2*
c*(-c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)+1/4*c^(3/2)*Pi^(1/2)*
erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2)))-1/2*a*c^(5/2)*Pi^(1/2)*erf((-c*La
mbertW(b*x+a))^(1/2)/c^(1/2)))
```

Fricas [F]

$$\int \frac{x}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{x}{\sqrt{-c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*lambert_w(b*x + a))*x/(c*lambert_w(b*x + a)), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{x}{\sqrt{-cW\left(a+bx\right)}}\,dx$$

input
```
integrate(x/(-c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(x/sqrt(-c*LambertW(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{x}{\sqrt{-c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(-c*lambert_w(b*x + a)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{x}{\sqrt{-c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(x/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(-c*lambert_w(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{x}{\sqrt{-c\,\mathrm{LambertW}\left(a+b\,x\right)}}\,dx$$

input
```
int(x/(-c*LambertW(a + b*x))^(1/2),x)
```

output
```
int(x/(-c*LambertW(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{-cW(a+bx)}}\,dx = -\frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}\,x}{lambert\_w(bx+a)}dx\right)i}{c}$$

input
```
int(x/(-c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
( - sqrt(c)*int((sqrt(lambert_w(a + b*x))*x)/lambert_w(a + b*x),x)*i)/c
```

3.412 $\int \frac{1}{\sqrt{-cW(a+bx)}} dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{1}{\sqrt{-cW(a+bx)}} dx = -\frac{\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{2b\sqrt{c}} + \frac{a+bx}{b\sqrt{-cW(a+bx)}}$$

output
```
-1/2*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2))/b/c^(1/2)+(b*x+a)/b/
(-c*LambertW(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-cW(a+bx)}} dx = \frac{2(a+bx) + \sqrt{\pi}\operatorname{erfi}\left(\sqrt{W(a+bx)}\right)\sqrt{W(a+bx)}}{2b\sqrt{-cW(a+bx)}}$$

input
```
Integrate[1/Sqrt[-(c*ProductLog[a + b*x])],x]
```

output
```
(2*(a + b*x) + Sqrt[Pi]*Erfi[Sqrt[ProductLog[a + b*x]]]*Sqrt[ProductLog[a
+ b*x]])/(2*b*Sqrt[-(c*ProductLog[a + b*x])])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {7167, 7181}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-cW(a+bx)}}dx$$

$$\Big\downarrow \text{ 7167}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{-cW(a+bx)}(W(a+bx)+1)}dx+\frac{a+bx}{b\sqrt{-cW(a+bx)}}$$

$$\Big\downarrow \text{ 7181}$$

$$\frac{a+bx}{b\sqrt{-cW(a+bx)}}-\frac{\sqrt{\pi}\text{erf}\left(\frac{\sqrt{-cW(a+bx)}}{\sqrt{c}}\right)}{2b\sqrt{c}}$$

input
```
Int[1/Sqrt[-(c*ProductLog[a + b*x])],x]
```

output
```
-1/2*(Sqrt[Pi]*Erf[Sqrt[-(c*ProductLog[a + b*x])]/Sqrt[c]])/(b*Sqrt[c]) +
(a + b*x)/(b*Sqrt[-(c*ProductLog[a + b*x])])
```

Defintions of rubi rules used

rule 7167
```
Int[((c_.)*ProductLog[(a_.) + (b_.)*(x_)])^(p_.), x_Symbol] :> Simp[(a + b*
x)*((c*ProductLog[a + b*x])^p/b), x] - Simp[p   Int[(c*ProductLog[a + b*x])
^p/(1 + ProductLog[a + b*x]), x], x] /; FreeQ[{a, b, c}, x] &&  !LtQ[p, -1]
```

rule 7181
```
Int[1/(Sqrt[(c_.)*ProductLog[(a_.) + (b_.)*(x_)]]*((d_) + (d_.)*ProductLog[
(a_.) + (b_.)*(x_)])), x_Symbol] :> Simp[Rt[(-Pi)*c, 2]*(Erf[Sqrt[c*Product
Log[a + b*x]]/Rt[-c, 2]]/(b*c*d)), x] /; FreeQ[{a, b, c, d}, x] && NegQ[c]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{-\frac{c\sqrt{-c\,\mathrm{LambertW}(bx+a)}\,(bx+a)}{\mathrm{LambertW}(bx+a)}-\frac{c^{\frac{3}{2}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-c\,\mathrm{LambertW}(bx+a)}}{\sqrt{c}}\right)}{2}}{b\,c^{2}}$	61

input
```
int(1/(-c*LambertW(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/c^2*(-1/2*c*(-c*LambertW(b*x+a))^(1/2)*(b*x+a)/LambertW(b*x+a)-1/4*c^(
3/2)*Pi^(1/2)*erf((-c*LambertW(b*x+a))^(1/2)/c^(1/2)))
```

Fricas [F]

$$\int \frac{1}{\sqrt{-cW(a+bx)}}\,dx=\int \frac{1}{\sqrt{-c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(1/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*lambert_w(b*x + a))/(c*lambert_w(b*x + a)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-cW(a+bx)}}\,dx=\int \frac{1}{\sqrt{-cW\,(a+bx)}}\,dx$$

input
```
integrate(1/(-c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(1/sqrt(-c*LambertW(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{-c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(1/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-c*lambert_w(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{-c\,\mathrm{W}(bx+a)}}\,dx$$

input
```
integrate(1/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-c*lambert_w(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{-c\,\mathrm{LambertW}\left(a+b\,x\right)}}\,dx$$

input
```
int(1/(-c*LambertW(a + b*x))^(1/2),x)
```

output
```
int(1/(-c*LambertW(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-cW(a+bx)}}\,dx = -\frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}}{lambert\_w(bx+a)}dx\right)i}{c}$$

input
```
int(1/(-c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
( - sqrt(c)*int(sqrt(lambert_w(a + b*x))/lambert_w(a + b*x),x)*i)/c
```

3.413 $\int \frac{1}{x\sqrt{-cW(a+bx)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1}{x\sqrt{-cW(a+bx)}}\,dx = \frac{\sqrt{W(a+bx)}\text{Int}\left(\frac{1}{x\sqrt{W(a+bx)}},x\right)}{\sqrt{-cW(a+bx)}}$$

output
```
LambertW(b*x+a)^(1/2)*Defer(Int)(1/x/LambertW(b*x+a)^(1/2),x)/(-c*LambertW
(b*x+a))^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{x\sqrt{-cW(a+bx)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[-(c*ProductLog[a + b*x])]),x]
```

output
```
Integrate[1/(x*Sqrt[-(c*ProductLog[a + b*x])]), x]
```

Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{-cW(a+bx)}}dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sqrt{W(a+bx)}\int \frac{1}{x\sqrt{W(a+bx)}}dx}{\sqrt{-cW(a+bx)}}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\sqrt{W(a+bx)}\int \frac{1}{x\sqrt{W(a+bx)}}dx}{\sqrt{-cW(a+bx)}}$$

input
```
Int[1/(x*Sqrt[-(c*ProductLog[a + b*x])]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{-c\,\mathrm{LambertW}(bx+a)}}dx$$

input
```
int(1/x/(-c*LambertW(b*x+a))^(1/2),x)
```

output
```
int(1/x/(-c*LambertW(b*x+a))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \frac{1}{x\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{-c\,\mathrm{W}(bx+a)}x}\,dx$$

input
```
integrate(1/x/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*lambert_w(b*x + a))/(c*x*lambert_w(b*x + a)), x)
```

Sympy [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{x\sqrt{-cW\left(a+bx\right)}}\,dx$$

input
```
integrate(1/x/(-c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-c*LambertW(a + b*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{-c\,\mathrm{W}(bx+a)}x}\,dx$$

input
```
integrate(1/x/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*lambert_w(b*x + a))*x), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{-c\,\mathrm{W}(bx+a)}x}\,dx$$

input
```
integrate(1/x/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*lambert_w(b*x + a))*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{x\,\sqrt{-c\,\mathrm{LambertW}\left(a+b\,x\right)}}\,dx$$

input
```
int(1/(x*(-c*LambertW(a + b*x))^(1/2)),x)
```

output
```
int(1/(x*(-c*LambertW(a + b*x))^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{1}{x\sqrt{-cW(a+bx)}}\,dx = -\frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}}{lambert\_w(bx+a)x}dx\right)i}{c}$$

input
```
int(1/x/(-c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
( - sqrt(c)*int(sqrt(lambert_w(a + b*x))/(lambert_w(a + b*x)*x),x)*i)/c
```

3.414 $\int \frac{1}{x^2\sqrt{-cW(a+bx)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1}{x^2\sqrt{-cW(a+bx)}}\,dx = \frac{\sqrt{W(a+bx)}\,\mathrm{Int}\left(\frac{1}{x^2\sqrt{W(a+bx)}},x\right)}{\sqrt{-cW(a+bx)}}$$

output
```
LambertW(b*x+a)^(1/2)*Defer(Int)(1/x^2/LambertW(b*x+a)^(1/2),x)/(-c*Lamber
tW(b*x+a))^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{x^2\sqrt{-cW(a+bx)}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[-(c*ProductLog[a + b*x])]),x]
```

output
```
Integrate[1/(x^2*Sqrt[-(c*ProductLog[a + b*x])]), x]
```

Rubi [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{-cW(a+bx)}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\sqrt{W(a+bx)}\int \frac{1}{x^2\sqrt{W(a+bx)}}dx}{\sqrt{-cW(a+bx)}}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\sqrt{W(a+bx)}\int \frac{1}{x^2\sqrt{W(a+bx)}}dx}{\sqrt{-cW(a+bx)}}$$

input
```
Int[1/(x^2*Sqrt[-(c*ProductLog[a + b*x])]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\sqrt{-c\,\mathrm{LambertW}\left(bx+a\right)}}dx$$

input
```
int(1/x^2/(-c*LambertW(b*x+a))^(1/2),x)
```

output
```
int(1/x^2/(-c*LambertW(b*x+a))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{-c\,\mathrm{W}(bx+a)}x^2}\,dx$$

input
```
integrate(1/x^2/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*lambert_w(b*x + a))/(c*x^2*lambert_w(b*x + a)), x)
```

Sympy [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{x^2\sqrt{-cW\left(a+bx\right)}}\,dx$$

input
```
integrate(1/x**2/(-c*LambertW(b*x+a))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(-c*LambertW(a + b*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{-c\,\mathrm{W}(bx+a)}x^2}\,dx$$

input
```
integrate(1/x^2/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*lambert_w(b*x + a))*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{\sqrt{-c\,\mathrm{W}(bx+a)}x^2}\,dx$$

input
```
integrate(1/x^2/(-c*lambert_w(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*lambert_w(b*x + a))*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{-cW(a+bx)}}\,dx = \int \frac{1}{x^2\,\sqrt{-c\,\mathrm{LambertW}\left(a+b\,x\right)}}\,dx$$

input
```
int(1/(x^2*(-c*LambertW(a + b*x))^(1/2)),x)
```

output
```
int(1/(x^2*(-c*LambertW(a + b*x))^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{1}{x^2\sqrt{-cW(a+bx)}}\,dx = -\frac{\sqrt{c}\left(\int \frac{\sqrt{lambert\_w(bx+a)}}{lambert\_w(bx+a)x^2}dx\right)i}{c}$$

input
```
int(1/x^2/(-c*Lambert_W(b*x+a))^(1/2),x)
```

output
```
( - sqrt(c)*int(sqrt(lambert_w(a + b*x))/(lambert_w(a + b*x)*x**2),x)*i)/c
```

3.415 $\int \frac{x^3}{d+dW(a+bx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 257

$$\begin{aligned}\int \frac{x^3}{d+dW(a+bx)}\,dx = & -\frac{3(a+bx)^4}{128b^4dW(a+bx)^4}-\frac{2a(a+bx)^3}{9b^4dW(a+bx)^3} \\ & +\frac{3(a+bx)^4}{32b^4dW(a+bx)^3}-\frac{3a^2(a+bx)^2}{4b^4dW(a+bx)^2} \\ & +\frac{2a(a+bx)^3}{3b^4dW(a+bx)^2}-\frac{3(a+bx)^4}{16b^4dW(a+bx)^2}-\frac{a^3(a+bx)}{b^4dW(a+bx)} \\ & +\frac{3a^2(a+bx)^2}{2b^4dW(a+bx)}-\frac{a(a+bx)^3}{b^4dW(a+bx)}+\frac{(a+bx)^4}{4b^4dW(a+bx)}\end{aligned}$$

output
```
-3/128*(b*x+a)^4/b^4/d/LambertW(b*x+a)^4-2/9*a*(b*x+a)^3/b^4/d/LambertW(b*
x+a)^3+3/32*(b*x+a)^4/b^4/d/LambertW(b*x+a)^3-3/4*a^2*(b*x+a)^2/b^4/d/Lamb
ertW(b*x+a)^2+2/3*a*(b*x+a)^3/b^4/d/LambertW(b*x+a)^2-3/16*(b*x+a)^4/b^4/d
/LambertW(b*x+a)^2-a^3*(b*x+a)/b^4/d/LambertW(b*x+a)+3/2*a^2*(b*x+a)^2/b^4
/d/LambertW(b*x+a)-a*(b*x+a)^3/b^4/d/LambertW(b*x+a)+1/4*(b*x+a)^4/b^4/d/L
ambertW(b*x+a)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.52

$$\int \frac{x^3}{d+dW(a+bx)}\,dx = -\frac{(a+bx)\left(27(a+bx)^3+4(37a-27bx)(a+bx)^2W(a+bx)+24(13a^3-a^2bx-5ab^2x^2+9b^3x^3)\,W(a\right.}{1152b^4dW(a+bx)^4}$$

input
```
Integrate[x^3/(d + d*ProductLog[a + b*x]),x]
```

output
```
-1/1152*((a + b*x)*(27*(a + b*x)^3 + 4*(37*a - 27*b*x)*(a + b*x)^2*Product
Log[a + b*x] + 24*(13*a^3 - a^2*b*x - 5*a*b^2*x^2 + 9*b^3*x^3)*ProductLog[
a + b*x]^2 + 288*(a^3 - a^2*b*x + a*b^2*x^2 - b^3*x^3)*ProductLog[a + b*x]
^3))/(b^4*d*ProductLog[a + b*x]^4)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {7184, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{dW(a+bx)+d}dx$$

$$\Big\downarrow \text{ 7184}$$

$$\frac{\int \left(-\frac{a^3}{W(a+bx)d+d}+\frac{3(a+bx)a^2}{W(a+bx)d+d}-\frac{3(a+bx)^2a}{W(a+bx)d+d}+\frac{(a+bx)^3}{W(a+bx)d+d}\right)d(a+bx)}{b^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{a^3(a+bx)}{dW(a+bx)}+\frac{3a^2(a+bx)^2}{2dW(a+bx)}-\frac{3a^2(a+bx)^2}{4dW(a+bx)^2}+\frac{(a+bx)^4}{4dW(a+bx)}-\frac{3(a+bx)^4}{16dW(a+bx)^2}+\frac{3(a+bx)^4}{32dW(a+bx)^3}-\frac{3(a+bx)^4}{128dW(a+bx)^4}-\frac{a(a+bx)^3}{dW(a+bx)}+\frac{2}{3a}}{b^4}$$

input
```
Int[x^3/(d + d*ProductLog[a + b*x]),x]
```

output
```
((-3*(a + b*x)^4)/(128*d*ProductLog[a + b*x]^4) - (2*a*(a + b*x)^3)/(9*d*P
roductLog[a + b*x]^3) + (3*(a + b*x)^4)/(32*d*ProductLog[a + b*x]^3) - (3*
a^2*(a + b*x)^2)/(4*d*ProductLog[a + b*x]^2) + (2*a*(a + b*x)^3)/(3*d*Prod
uctLog[a + b*x]^2) - (3*(a + b*x)^4)/(16*d*ProductLog[a + b*x]^2) - (a^3*(
a + b*x))/(d*ProductLog[a + b*x]) + (3*a^2*(a + b*x)^2)/(2*d*ProductLog[a
+ b*x]) - (a*(a + b*x)^3)/(d*ProductLog[a + b*x]) + (a + b*x)^4/(4*d*Produ
ctLog[a + b*x]))/b^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7184
```
Int[((e_.) + (f_.)*(x_))^(m_.)/((d_) + (d_.)*ProductLog[(a_) + (b_.)*(x_)])
, x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[1/(d + d*Produc
tLog[x]), (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, d, e
, f}, x] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(241) = 482$.

Time = 0.14 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.90

method	result
derivativedivides	$\frac{\frac{(bx+a)^4}{4\,\mathrm{LambertW}(bx+a)}-\frac{3(bx+a)^4}{16\,\mathrm{LambertW}(bx+a)^2}+\frac{3(bx+a)^4}{32\,\mathrm{LambertW}(bx+a)^3}-\frac{3(bx+a)^4}{128\,\mathrm{LambertW}(bx+a)^4}}{d}+\frac{a^3\mathrm{e}^{-1}\,\mathrm{expIntegral}_1(-\,\mathrm{LambertW}}{d}$
default	$\frac{\frac{(bx+a)^4}{4\,\mathrm{LambertW}(bx+a)}-\frac{3(bx+a)^4}{16\,\mathrm{LambertW}(bx+a)^2}+\frac{3(bx+a)^4}{32\,\mathrm{LambertW}(bx+a)^3}-\frac{3(bx+a)^4}{128\,\mathrm{LambertW}(bx+a)^4}}{d}+\frac{a^3\mathrm{e}^{-1}\,\mathrm{expIntegral}_1(-\,\mathrm{LambertW}}{d}$

input
```
int(x^3/(d+d*LambertW(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/d*(1/4/LambertW(b*x+a)*(b*x+a)^4-3/16*(b*x+a)^4/LambertW(b*x+a)^2
+3/32/LambertW(b*x+a)^3*(b*x+a)^4-3/128*(b*x+a)^4/LambertW(b*x+a)^4)+a^3/d
*exp(-1)*Ei(1,-LambertW(b*x+a)-1)-a^3*(1/d*(b*x+a)/LambertW(b*x+a)+1/d*exp
(-1)*Ei(1,-LambertW(b*x+a)-1))+3*a^2*(1/2/d*(b*x+a)^2/LambertW(b*x+a)^2+1/
d*exp(-2)*Ei(1,-2*LambertW(b*x+a)-2))+3*a^2*(1/d*(1/2/LambertW(b*x+a)*(b*x
+a)^2-1/4*(b*x+a)^2/LambertW(b*x+a)^2)-1/2/d*(b*x+a)^2/LambertW(b*x+a)^2-1
/d*exp(-2)*Ei(1,-2*LambertW(b*x+a)-2))-3*a*(1/d*(1/3/LambertW(b*x+a)^2*(b*
x+a)^3-1/9*(b*x+a)^3/LambertW(b*x+a)^3)-1/3/d*(b*x+a)^3/LambertW(b*x+a)^3-
1/d*exp(-3)*Ei(1,-3*LambertW(b*x+a)-3))-3*a*(1/d*(1/3/LambertW(b*x+a)*(b*x
+a)^3-2/9/LambertW(b*x+a)^2*(b*x+a)^3+2/27*(b*x+a)^3/LambertW(b*x+a)^3)-1/
d*(1/3/LambertW(b*x+a)^2*(b*x+a)^3-1/9*(b*x+a)^3/LambertW(b*x+a)^3)+1/3/d*
(b*x+a)^3/LambertW(b*x+a)^3+1/d*exp(-3)*Ei(1,-3*LambertW(b*x+a)-3)))
```

Fricas [F]

$$\int \frac{x^3}{d+dW(a+bx)}\,dx=\int \frac{x^3}{d\,\mathrm{W}(bx+a)+d}\,dx$$

input
```
integrate(x^3/(d+d*lambert_w(b*x+a)),x, algorithm="fricas")
```

output
```
integral(x^3/(d*lambert_w(b*x + a) + d), x)
```

Sympy [F]

$$\int \frac{x^3}{d+dW(a+bx)}\,dx=\frac{\int \frac{x^3}{W(a+bx)+1}\,dx}{d}$$

input
```
integrate(x**3/(d+d*LambertW(b*x+a)),x)
```

output
```
Integral(x**3/(LambertW(a + b*x) + 1), x)/d
```

Maxima [F]

$$\int \frac{x^3}{d+dW(a+bx)}\,dx=\int \frac{x^3}{d\,\mathrm{W}(bx+a)+d}\,dx$$

input
```
integrate(x^3/(d+d*lambert_w(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(x^3/(d*lambert_w(b*x + a) + d), x)
```

Giac [F]

$$\int \frac{x^3}{d+dW(a+bx)}\,dx=\int \frac{x^3}{d\,\mathrm{W}(bx+a)+d}\,dx$$

input
```
integrate(x^3/(d+d*lambert_w(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^3/(d*lambert_w(b*x + a) + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{d+dW(a+bx)}\,dx=\int \frac{x^3}{d+d\,\mathrm{LambertW}\left(a+b\,x\right)}\,dx$$

input
```
int(x^3/(d + d*LambertW(a + b*x)),x)
```

output
```
int(x^3/(d + d*LambertW(a + b*x)), x)
```

Reduce [F]

$$\int \frac{x^3}{d + dW(a + bx)} dx = \frac{\int \frac{x^3}{lambert\_w(bx+a)+1} dx}{d}$$

input
```
int(x^3/(d+d*Lambert_W(b*x+a)),x)
```

output
```
int(x**3/(lambert_w(a + b*x) + 1),x)/d
```

3.416 $\int \frac{x^2}{d+dW(a+bx)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 149

$$\int \frac{x^2}{d+dW(a+bx)} \, dx = \frac{2(a+bx)^3}{27b^3dW(a+bx)^3} + \frac{a(a+bx)^2}{2b^3dW(a+bx)^2} - \frac{2(a+bx)^3}{9b^3dW(a+bx)^2} + \frac{a^2(a+bx)}{b^3dW(a+bx)} - \frac{a(a+bx)^2}{b^3dW(a+bx)} + \frac{(a+bx)^3}{3b^3dW(a+bx)}$$

output
```
2/27*(b*x+a)^3/b^3/d/LambertW(b*x+a)^3+1/2*a*(b*x+a)^2/b^3/d/LambertW(b*x+
a)^2-2/9*(b*x+a)^3/b^3/d/LambertW(b*x+a)^2+a^2*(b*x+a)/b^3/d/LambertW(b*x+
a)-a*(b*x+a)^2/b^3/d/LambertW(b*x+a)+1/3*(b*x+a)^3/b^3/d/LambertW(b*x+a)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.57

$$\int \frac{x^2}{d+dW(a+bx)} \, dx = \frac{(a+bx)\left(4(a+bx)^2+3(5a^2+abx-4b^2x^2)W(a+bx)+18(a^2-abx+b^2x^2)W(a+bx)^2\right)}{54b^3dW(a+bx)^3}$$

input
```
Integrate[x^2/(d + d*ProductLog[a + b*x]),x]
```

output
```
((a + b*x)*(4*(a + b*x)^2 + 3*(5*a^2 + a*b*x - 4*b^2*x^2)*ProductLog[a + b
*x] + 18*(a^2 - a*b*x + b^2*x^2)*ProductLog[a + b*x]^2))/(54*b^3*d*Product
Log[a + b*x]^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {7184, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{dW(a+bx)+d}\,dx$$
$$\Big\downarrow \text{7184}$$
$$\frac{\int \left(\frac{a^2}{W(a+bx)d+d}-\frac{2(a+bx)a}{W(a+bx)d+d}+\frac{(a+bx)^2}{W(a+bx)d+d}\right)d(a+bx)}{b^3}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{a^2(a+bx)}{dW(a+bx)}+\frac{(a+bx)^3}{3dW(a+bx)}-\frac{2(a+bx)^3}{9dW(a+bx)^2}+\frac{2(a+bx)^3}{27dW(a+bx)^3}-\frac{a(a+bx)^2}{dW(a+bx)}+\frac{a(a+bx)^2}{2dW(a+bx)^2}}{b^3}$$

input
```
Int[x^2/(d + d*ProductLog[a + b*x]),x]
```

output
```
((2*(a + b*x)^3)/(27*d*ProductLog[a + b*x]^3) + (a*(a + b*x)^2)/(2*d*Produ
ctLog[a + b*x]^2) - (2*(a + b*x)^3)/(9*d*ProductLog[a + b*x]^2) + (a^2*(a
+ b*x))/(d*ProductLog[a + b*x]) - (a*(a + b*x)^2)/(d*ProductLog[a + b*x])
+ (a + b*x)^3/(3*d*ProductLog[a + b*x]))/b^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7184
```
Int[((e_.) + (f_.)*(x_))^(m_.)/((d_) + (d_.)*ProductLog[(a_) + (b_.)*(x_)])
, x_Symbol] :> Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[1/(d + d*Produc
tLog[x]), (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, d, e
, f}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.66

method	result
derivativedivides	$-\frac{a^2 e^{-1}\,\mathrm{expIntegral}_1(-\mathrm{LambertW}(bx+a)-1)}{d}+\frac{\frac{(bx+a)^3}{3\,\mathrm{LambertW}(bx+a)}-\frac{2(bx+a)^3}{9\,\mathrm{LambertW}(bx+a)^2}+\frac{2(bx+a)^3}{27\,\mathrm{LambertW}(bx+a)^3}}{d}+a^2\Big(\frac{}{d\,\mathrm{Lamb}\cdots}$
default	$-\frac{a^2 e^{-1}\,\mathrm{expIntegral}_1(-\mathrm{LambertW}(bx+a)-1)}{d}+\frac{\frac{(bx+a)^3}{3\,\mathrm{LambertW}(bx+a)}-\frac{2(bx+a)^3}{9\,\mathrm{LambertW}(bx+a)^2}+\frac{2(bx+a)^3}{27\,\mathrm{LambertW}(bx+a)^3}}{d}+a^2\Big(\frac{}{d\,\mathrm{Lamb}\cdots}$

input
```
int(x^2/(d+d*LambertW(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(-a^2/d*exp(-1)*Ei(1,-LambertW(b*x+a)-1)+1/d*(1/3/LambertW(b*x+a)*(b
*x+a)^3-2/9/LambertW(b*x+a)^2*(b*x+a)^3+2/27*(b*x+a)^3/LambertW(b*x+a)^3)+
a^2*(1/d*(b*x+a)/LambertW(b*x+a)+1/d*exp(-1)*Ei(1,-LambertW(b*x+a)-1))-2*a
*(1/2/d*(b*x+a)^2/LambertW(b*x+a)^2+1/d*exp(-2)*Ei(1,-2*LambertW(b*x+a)-2)
)-2*a*(1/d*(1/2/LambertW(b*x+a)*(b*x+a)^2-1/4*(b*x+a)^2/LambertW(b*x+a)^2)
-1/2/d*(b*x+a)^2/LambertW(b*x+a)^2-1/d*exp(-2)*Ei(1,-2*LambertW(b*x+a)-2))
)
```

Fricas [F]

$$\int \frac{x^2}{d+dW(a+bx)}\,dx = \int \frac{x^2}{d\,\mathrm{W}(bx+a)+d}\,dx$$

input
```
integrate(x^2/(d+d*lambert_w(b*x+a)),x, algorithm="fricas")
```

output
```
integral(x^2/(d*lambert_w(b*x + a) + d), x)
```

Sympy [F]

$$\int \frac{x^2}{d+dW(a+bx)}\,dx = \frac{\int \frac{x^2}{W(a+bx)+1}\,dx}{d}$$

input
```
integrate(x**2/(d+d*LambertW(b*x+a)),x)
```

output
```
Integral(x**2/(LambertW(a + b*x) + 1), x)/d
```

Maxima [F]

$$\int \frac{x^2}{d+dW(a+bx)}\,dx = \int \frac{x^2}{d\,\mathrm{W}(bx+a)+d}\,dx$$

input
```
integrate(x^2/(d+d*lambert_w(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(x^2/(d*lambert_w(b*x + a) + d), x)
```

Giac [F]

$$\int \frac{x^2}{d+dW(a+bx)}\,dx = \int \frac{x^2}{d\,\mathrm{W}(bx+a)+d}\,dx$$

input
```
integrate(x^2/(d+d*lambert_w(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2/(d*lambert_w(b*x + a) + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{d+dW(a+bx)}\,dx = \int \frac{x^2}{d+d\,\mathrm{LambertW}\left(a+b\,x\right)}\,dx$$

input
```
int(x^2/(d + d*LambertW(a + b*x)),x)
```

output
```
int(x^2/(d + d*LambertW(a + b*x)), x)
```

Reduce [F]

$$\int \frac{x^2}{d+dW(a+bx)}\,dx = \frac{\int \frac{x^2}{lambert\_w(bx+a)+1}dx}{d}$$

input
```
int(x^2/(d+d*Lambert_W(b*x+a)),x)
```

output
```
int(x**2/(lambert_w(a + b*x) + 1),x)/d
```

3.417 $\int \frac{x}{d+dW(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 73

$$\int \frac{x}{d+dW(a+bx)}\,dx=-\frac{(a+bx)^2}{4b^2dW(a+bx)^2}-\frac{a(a+bx)}{b^2dW(a+bx)}+\frac{(a+bx)^2}{2b^2dW(a+bx)}$$

output
```
-1/4*(b*x+a)^2/b^2/d/LambertW(b*x+a)^2-a*(b*x+a)/b^2/d/LambertW(b*x+a)+1/2
*(b*x+a)^2/b^2/d/LambertW(b*x+a)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58

$$\int \frac{x}{d+dW(a+bx)}\,dx=-\frac{(a+bx)(a+bx+2(a-bx)W(a+bx))}{4b^2dW(a+bx)^2}$$

input
```
Integrate[x/(d + d*ProductLog[a + b*x]),x]
```

output
```
-1/4*((a + b*x)*(a + b*x + 2*(a - b*x)*ProductLog[a + b*x]))/(b^2*d*Produc
tLog[a + b*x]^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7184, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{dW(a+bx)+d}dx$$

$$\Big\downarrow \text{7184}$$

$$\frac{\int \left(\frac{a+bx}{W(a+bx)d+d}-\frac{a}{W(a+bx)d+d}\right)d(a+bx)}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a+bx)^2}{2dW(a+bx)}-\frac{(a+bx)^2}{4dW(a+bx)^2}-\frac{a(a+bx)}{dW(a+bx)}}{b^2}$$

input
```
Int[x/(d + d*ProductLog[a + b*x]),x]
```

output
```
(-1/4*(a + b*x)^2/(d*ProductLog[a + b*x]^2) - (a*(a + b*x))/(d*ProductLog[
a + b*x]) + (a + b*x)^2/(2*d*ProductLog[a + b*x]))/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7184
```
Int[((e_.) + (f_.)*(x_))^(m_.)/((d_) + (d_.)*ProductLog[(a_) + (b_.)*(x_)])
, x_Symbol] :> Simp[1/b^(m + 1)    Subst[Int[ExpandIntegrand[1/(d + d*Produc
tLog[x]), (b*e - a*f + f*x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, d, e
, f}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\dfrac{\frac{\frac{(bx+a)^2}{2\,\mathrm{LambertW}(bx+a)}-\frac{(bx+a)^2}{4\,\mathrm{LambertW}(bx+a)^2}}{d}+\frac{a\,e^{-1}\,\mathrm{expIntegral}_1(-\mathrm{LambertW}(bx+a)-1)}{d}-a\left(\frac{bx+a}{d\,\mathrm{LambertW}(bx+a)}+\frac{e^{-1}\,\mathrm{expInteg}}{}\right.}{b^2}$
default	$\dfrac{\frac{\frac{(bx+a)^2}{2\,\mathrm{LambertW}(bx+a)}-\frac{(bx+a)^2}{4\,\mathrm{LambertW}(bx+a)^2}}{d}+\frac{a\,e^{-1}\,\mathrm{expIntegral}_1(-\mathrm{LambertW}(bx+a)-1)}{d}-a\left(\frac{bx+a}{d\,\mathrm{LambertW}(bx+a)}+\frac{e^{-1}\,\mathrm{expInteg}}{}\right.}{b^2}$

input
```
int(x/(d+d*LambertW(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/d*(1/2/LambertW(b*x+a)*(b*x+a)^2-1/4*(b*x+a)^2/LambertW(b*x+a)^2)
+a/d*exp(-1)*Ei(1,-LambertW(b*x+a)-1)-a*(1/d*(b*x+a)/LambertW(b*x+a)+1/d*e
xp(-1)*Ei(1,-LambertW(b*x+a)-1)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \frac{x}{d+dW(a+bx)}\,dx = -\frac{b^2x^2+2\,abx+a^2-2\,(b^2x^2-a^2)\,\mathrm{W}(bx+a)}{4\,b^2d\,\mathrm{W}(bx+a)^2}$$

input
```
integrate(x/(d+d*lambert_w(b*x+a)),x, algorithm="fricas")
```

output
```
-1/4*(b^2*x^2 + 2*a*b*x + a^2 - 2*(b^2*x^2 - a^2)*lambert_w(b*x + a))/(b^2
*d*lambert_w(b*x + a)^2)
```

Sympy [F]

$$\int \frac{x}{d + dW(a + bx)}\,dx = \frac{\int \frac{x}{W(a+bx)+1}\,dx}{d}$$

input
```
integrate(x/(d+d*LambertW(b*x+a)),x)
```

output
```
Integral(x/(LambertW(a + b*x) + 1), x)/d
```

Maxima [F]

$$\int \frac{x}{d + dW(a + bx)}\,dx = \int \frac{x}{d\,\mathrm{W}(bx + a) + d}\,dx$$

input
```
integrate(x/(d+d*lambert_w(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(x/(d*lambert_w(b*x + a) + d), x)
```

Giac [F]

$$\int \frac{x}{d + dW(a + bx)}\,dx = \int \frac{x}{d\,\mathrm{W}(bx + a) + d}\,dx$$

input
```
integrate(x/(d+d*lambert_w(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x/(d*lambert_w(b*x + a) + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{d+dW(a+bx)}\,dx=\int \frac{x}{d+d\,\mathrm{LambertW}\left(a+b\,x\right)}\,dx$$

input
```
int(x/(d + d*LambertW(a + b*x)),x)
```

output
```
int(x/(d + d*LambertW(a + b*x)), x)
```

Reduce [F]

$$\int \frac{x}{d+dW(a+bx)}\,dx=\frac{\int \frac{x}{lambert\_w(bx+a)+1}dx}{d}$$

input
```
int(x/(d+d*Lambert_W(b*x+a)),x)
```

output
```
int(x/(lambert_w(a + b*x) + 1),x)/d
```

3.418 $\int \frac{1}{d+dW(a+bx)} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{1}{d+dW(a+bx)} \, dx = \frac{a+bx}{bdW(a+bx)}$$

output
```
(b*x+a)/b/d/LambertW(b*x+a)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{d+dW(a+bx)} \, dx = \frac{a+bx}{bdW(a+bx)}$$

input
```
Integrate[(d + d*ProductLog[a + b*x])^(-1),x]
```

output
```
(a + b*x)/(b*d*ProductLog[a + b*x])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {7176}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{dW(a+bx)+d}\,dx$$
$$\Big\downarrow \text{ 7176}$$
$$\frac{a+bx}{bdW(a+bx)}$$

input
```
Int[(d + d*ProductLog[a + b*x])^(-1),x]
```

output
```
(a + b*x)/(b*d*ProductLog[a + b*x])
```

Defintions of rubi rules used

rule 7176
```
Int[((d_) + (d_.)*ProductLog[(a_.) + (b_.)*(x_)])^(-1), x_Symbol] :> Simp[(
a + b*x)/(b*d*ProductLog[a + b*x]), x] /; FreeQ[{a, b, d}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{bx+a}{bd\,\mathrm{LambertW}(bx+a)}$	21
default	$\frac{bx+a}{bd\,\mathrm{LambertW}(bx+a)}$	21
parallelrisch	$-\frac{-bx-a}{db\,\mathrm{LambertW}(bx+a)}$	25

input
```
int(1/(d+d*LambertW(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)/b/d/LambertW(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{1}{d + dW(a + bx)}\,dx = \frac{bx + a}{bd\,\mathrm{W}(bx + a)}$$

input
```
integrate(1/(d+d*lambert_w(b*x+a)),x, algorithm="fricas")
```

output
```
(b*x + a)/(b*d*lambert_w(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.

Time = 0.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{1}{d + dW(a + bx)}\,dx = \begin{cases} \frac{x}{d} & \text{for } a = 0 \wedge b = 0 \\ \frac{x}{dW(a)+d} & \text{for } b = 0 \\ \frac{x}{d} & \text{for } a = -bx \\ \frac{a}{bdW(a+bx)} + \frac{x}{dW(a+bx)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(d+d*LambertW(b*x+a)),x)
```

output
```
Piecewise((x/d, Eq(a, 0) & Eq(b, 0)), (x/(d*LambertW(a) + d), Eq(b, 0)), (
x/d, Eq(a, -b*x)), (a/(b*d*LambertW(a + b*x)) + x/(d*LambertW(a + b*x)), T
rue))
```

Maxima [F]

$$\int \frac{1}{d+dW(a+bx)}\,dx = \int \frac{1}{d\,\mathrm{W}(bx+a)+d}\,dx$$

input
```
integrate(1/(d+d*lambert_w(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(1/(d*lambert_w(b*x + a) + d), x)
```

Giac [F]

$$\int \frac{1}{d+dW(a+bx)}\,dx = \int \frac{1}{d\,\mathrm{W}(bx+a)+d}\,dx$$

input
```
integrate(1/(d+d*lambert_w(b*x+a)),x, algorithm="giac")
```

output
```
integrate(1/(d*lambert_w(b*x + a) + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{d+dW(a+bx)}\,dx = \int \frac{1}{d\,(\mathrm{LambertW}\,(a+b\,x)+1)}\,dx$$

input
```
int(1/(d + d*LambertW(a + b*x)),x)
```

output
```
int(1/(d*(LambertW(a + b*x) + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{1}{d+dW(a+bx)}\,dx=\frac{e^{lambert\_w(bx+a)}}{bd}$$

input
```
int(1/(d+d*Lambert_W(b*x+a)),x)
```

output
```
e**lambert_w(a + b*x)/(b*d)
```

3.419 $\int \frac{1}{x(d+dW(a+bx))}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x(d+dW(a+bx))}\,dx = \frac{\text{Int}\left(\frac{1}{x(1+W(a+bx))},x\right)}{d}$$

output
```
Defer(Int)(1/x/(1+LambertW(b*x+a)),x)/d
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x(d+dW(a+bx))}\,dx = \int \frac{1}{x(d+dW(a+bx))}\,dx$$

input
```
Integrate[1/(x*(d + d*ProductLog[a + b*x])),x]
```

output
```
Integrate[1/(x*(d + d*ProductLog[a + b*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(dW(a+bx)+d)}dx$$
$$\Big\downarrow \text{7292}$$
$$\int \frac{1}{dx(W(a+bx)+1)}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{1}{x(W(a+bx)+1)}dx}{d}$$
$$\Big\downarrow \text{7299}$$
$$\frac{\int \frac{1}{x(W(a+bx)+1)}dx}{d}$$

input
```
Int[1/(x*(d + d*ProductLog[a + b*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(d+d\,\mathrm{LambertW}\left(bx+a\right)\right)}dx$$

input
```
int(1/x/(d+d*LambertW(b*x+a)),x)
```

output
```
int(1/x/(d+d*LambertW(b*x+a)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x(d+dW(a+bx))} dx = \int \frac{1}{(d\,\mathrm{W}(bx+a)+d)x} dx$$

input
```
integrate(1/x/(d+d*lambert_w(b*x+a)),x, algorithm="fricas")
```

output
```
integral(1/(d*x*lambert_w(b*x + a) + d*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x(d+dW(a+bx))} dx = \frac{\int \frac{1}{xW(a+bx)+x} dx}{d}$$

input
```
integrate(1/x/(d+d*LambertW(b*x+a)),x)
```

output
```
Integral(1/(x*LambertW(a + b*x) + x), x)/d
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{x(d+dW(a+bx))}\,dx = \int \frac{1}{(d\,\mathrm{W}(bx+a)+d)x}\,dx$$

input
```
integrate(1/x/(d+d*lambert_w(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(1/((d*lambert_w(b*x + a) + d)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{x(d+dW(a+bx))}\,dx = \int \frac{1}{(d\,\mathrm{W}(bx+a)+d)x}\,dx$$

input
```
integrate(1/x/(d+d*lambert_w(b*x+a)),x, algorithm="giac")
```

output
```
integrate(1/((d*lambert_w(b*x + a) + d)*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x(d+dW(a+bx))}\,dx = \int \frac{1}{x\,\left(d+d\,\mathrm{LambertW}\left(a+b\,x\right)\right)}\,dx$$

input
```
int(1/(x*(d + d*LambertW(a + b*x))),x)
```

output
```
int(1/(x*(d + d*LambertW(a + b*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x(d + dW(a + bx))} \, dx = \frac{\int \frac{1}{lambert\_w(bx+a)x+x} dx}{d}$$

input
```
int(1/x/(d+d*Lambert_W(b*x+a)),x)
```

output
```
int(1/(lambert_w(a + b*x)*x + x),x)/d
```

3.420 $\int \frac{1}{x^2(d+dW(a+bx))}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x^2(d+dW(a+bx))}\,dx=\frac{\text{Int}\left(\frac{1}{x^2(1+W(a+bx))},x\right)}{d}$$

output
```
Defer(Int)(1/x^2/(1+LambertW(b*x+a)),x)/d
```

Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2(d+dW(a+bx))}\,dx=\int \frac{1}{x^2(d+dW(a+bx))}\,dx$$

input
```
Integrate[1/(x^2*(d + d*ProductLog[a + b*x])),x]
```

output
```
Integrate[1/(x^2*(d + d*ProductLog[a + b*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(dW(a+bx)+d)}dx$$

$$\Big\downarrow \text{ 7292}$$

$$\int \frac{1}{dx^2(W(a+bx)+1)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{x^2(W(a+bx)+1)}dx}{d}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\int \frac{1}{x^2(W(a+bx)+1)}dx}{d}$$

input
```
Int[1/(x^2*(d + d*ProductLog[a + b*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(d+d\,\mathrm{LambertW}\left(bx+a\right)\right)}dx$$

input
```
int(1/x^2/(d+d*LambertW(b*x+a)),x)
```

output
```
int(1/x^2/(d+d*LambertW(b*x+a)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{1}{x^2(d+dW(a+bx))}\,dx = \int \frac{1}{(d\,\mathrm{W}(bx+a)+d)x^2}\,dx$$

input
```
integrate(1/x^2/(d+d*lambert_w(b*x+a)),x, algorithm="fricas")
```

output
```
integral(1/(d*x^2*lambert_w(b*x + a) + d*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2(d+dW(a+bx))}\,dx = \frac{\int \frac{1}{x^2W(a+bx)+x^2}\,dx}{d}$$

input
```
integrate(1/x**2/(d+d*LambertW(b*x+a)),x)
```

output
```
Integral(1/(x**2*LambertW(a + b*x) + x**2), x)/d
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^2(d + dW(a + bx))} \, dx = \int \frac{1}{(d \, \mathrm{W}(bx + a) + d)x^2} \, dx$$

input
```
integrate(1/x^2/(d+d*lambert_w(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(1/((d*lambert_w(b*x + a) + d)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^2(d + dW(a + bx))} \, dx = \int \frac{1}{(d \, \mathrm{W}(bx + a) + d)x^2} \, dx$$

input
```
integrate(1/x^2/(d+d*lambert_w(b*x+a)),x, algorithm="giac")
```

output
```
integrate(1/((d*lambert_w(b*x + a) + d)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2(d + dW(a + bx))} \, dx = \int \frac{1}{x^2 \, (d + d \, \mathrm{LambertW}\,(a + b\,x))} \, dx$$

input
```
int(1/(x^2*(d + d*LambertW(a + b*x))),x)
```

output
```
int(1/(x^2*(d + d*LambertW(a + b*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.69

$$\int \frac{1}{x^2(d+dW(a+bx))}\,dx = \frac{-\left(\int \frac{lambert\_w(bx+a)}{lambert\_w(bx+a)x^2+x^2}dx\right)x+3\left(\int \frac{1}{lambert\_w(bx+a)x^2+x^2}dx\right)x-1}{4dx}$$

input
```
int(1/x^2/(d+d*Lambert_W(b*x+a)),x)
```

output
```
( - int(lambert_w(a + b*x)/(lambert_w(a + b*x)*x**2 + x**2),x)*x + 3*int(1
/(lambert_w(a + b*x)*x**2 + x**2),x)*x - 1)/(4*d*x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
    if m:
      print ("func ", func , " is elementary_function")
    else:
      print ("func ", func , " is NOT elementary_function")


  return m

def is_special_function(func):
  #debug=False
  if debug:
    print ("type(func)=", type(func))

  m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
       'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
       'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
       'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
       'elliptic_pi','exp_integral_e','log_integral',
       'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
       'weierstrassPPrime','weierstrassSigma']

  if debug:
    print ("m=",m)
    if m:
      print ("func ", func ," is special_function")
    else:
      print ("func ", func ," is NOT special_function")


  return m


def is_hypergeometric_function(func):
  return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
  return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

  #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file