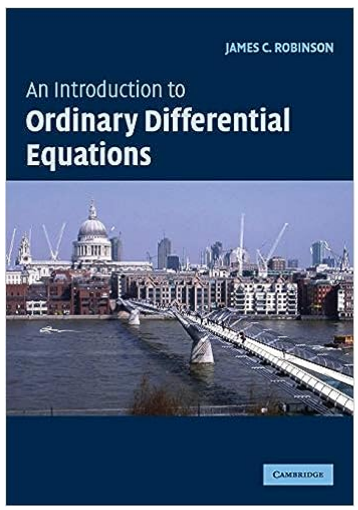
A Solution Manual For

AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004



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1 Chapter 5, Trivial differential equations. Exercises page 33

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1.1 problem 5.1 (i)

Internal problem ID [10620]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.1 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - \cos(t) - \sin(t) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(diff(x(t),t)=sin(t)+cos(t),x(t), singsol=all)
```

$$x(t) = -\cos(t) + \sin(t) + c_1$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 14

```
DSolve[x'[t]==Sin[t]+Cos[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \sin(t) - \cos(t) + c_1$$

1.2 problem 5.1 (ii)

Internal problem ID [10621]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.1 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \frac{1}{x^2 - 1} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=1/(x^2-1),y(x), singsol=all)
```

$$y(x) = -\operatorname{arctanh}(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 12

```
DSolve[y'[x]==1/(x^2-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\operatorname{arctanh}(x) + c_1$$

1.3 problem 5.1 (iii)

Internal problem ID [10622]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.1 (iii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{u' - 4t\ln(t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(u(t),t)=4*t*ln(t),u(t), singsol=all)
```

$$u(t) = 2t^2\ln(t) - t^2 + c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 20

```
DSolve[u'[t]==4*t*Log[t],u[t],t,IncludeSingularSolutions -> True]
```

$$u(t) \to -t^2 + 2t^2\log(t) + c_1$$

1.4 problem 5.1 (iv)

Internal problem ID [10623]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.1 (iv).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{z' - x\,\mathrm{e}^{-2x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(z(x),x)=x*exp(-2*x),z(x), singsol=all)
```

$$z(x) = -\frac{(2x+1)\,\mathrm{e}^{-2x}}{4} + c_1$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 22

```
DSolve[z'[x]==x*Exp[-2*x],z[x],x,IncludeSingularSolutions -> True]
```

$$z(x) \to -\frac{1}{4}e^{-2x}(2x+1) + c_1$$

1.5 problem 5.1 (v)

Internal problem ID [10624]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.1 (v).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{T' - e^{-t} \sin(2t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(T(t),t)=exp(-t)*sin(2*t),T(t), singsol=all)
```

$$T(t) = -\frac{2 e^{-t} \cos(2t)}{5} - \frac{e^{-t} \sin(2t)}{5} + c_1$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 28

```
DSolve[T'[t]==Exp[-t]*Sin[2*t],T[t],t,IncludeSingularSolutions -> True]
```

$$T(t) \to -\frac{1}{5} e^{-t} (\sin(2t) + 2\cos(2t)) + c_1$$

1.6 problem 5.4 (i)

Internal problem ID [10625]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.4 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - \sec(t)^2 = 0}$$

With initial conditions

$$\left[x\left(\frac{\pi}{4}\right) = 0\right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 8

```
dsolve([diff(x(t),t)=sec(t)^2,x(1/4*Pi) = 0],x(t), singsol=all)
```

$$x(t) = \tan(t) - 1$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 9

```
DSolve[{x'[t]==Sec[t]^2,{x[Pi/4]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \tan(t) - 1$$

1.7 problem 5.4 (ii)

Internal problem ID [10626]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.4 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - x + \frac{x^3}{3} = 0}$$

With initial conditions

$$[y(-1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)=x-1/3*x^3,y(-1) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{(x^2-3)^2}{12} + \frac{4}{3}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 21

```
DSolve[{y'[x]==x-1/3*x^3,{y[-1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}(-x^4 + 6x^2 + 7)$$

1.8 problem 5.4 (iii)

Internal problem ID [10627]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.4 (iii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - 2\sin(t)^2 = 0}$$

With initial conditions

$$\left[x\left(\frac{\pi}{4}\right) = \frac{\pi}{4}\right]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(x(t),t)=2*sin(t)^2,x(1/4*Pi) = 1/4*Pi],x(t), singsol=all)
```

$$x(t) = t + \frac{1}{2} - \frac{\sin(2t)}{2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 16

```
DSolve[{x'[t]==2*Sin[t]^2,{x[Pi/4]==Pi/4}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to t - \sin(t)\cos(t) + \frac{1}{2}$$

1.9 problem 5.4 (iv)

Internal problem ID [10628]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.4 (iv).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xV' - x^2 - 1 = 0}$$

With initial conditions

$$[V(1) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x*diff(V(x),x)=1+x^2,V(1) = 1],V(x), singsol=all)
```

$$V(x) = \frac{x^2}{2} + \ln(x) + \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 17

```
DSolve[{x*V'[x]==1+x^2,{V[1]==1}},V[x],x,IncludeSingularSolutions -> True]
```

$$V(x) \to \frac{1}{2}(x^2 + 1) + \log(x)$$

1.10 problem 5.4 (v)

Internal problem ID [10629]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 5, Trivial differential equations. Exercises page 33
Problem number: 5.4 (v).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x' e^{3t} + 3x\, e^{3t} - e^{-t} = 0}$$

With initial conditions

$$[x(0) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(x(t)*exp(3*t),t)=exp(-t),x(0) = 3],x(t), singsol=all)
```

$$x(t) = -\left(e^{-t} - 4\right) e^{-3t}$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 18

```
DSolve[{D[x[t]*Exp[3*t],t]==Exp[-t],{x[0]==3}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-4t}\left(4e^{t} - 1\right)$$

2 Chapter 7, Scalar autonomous ODEs. Exercises page 56

2.1 problem 7.1 (i) 14
2.2 problem 7.1 (ii) 15
2.3 problem 7.1 (iii) 16
2.4 problem 7.1 (iv) 17
2.5 problem 7.1 (v) 18

2.1 problem 7.1 (i)

Internal problem ID [10630]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 7, Scalar autonomous ODEs. Exercises page 56
Problem number: 7.1 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' + x - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(x(t),t)=-x(t)+1,x(t), singsol=all)
```

$$x(t) = 1 + \mathrm{e}^{-t} c_1$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 20

```
DSolve[x'[t]==-x[t]+1,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow 1 + c_1 e^{-t}$$
$$x(t) \rightarrow 1$$

2.2 problem 7.1 (ii)

Internal problem ID [10631]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 7, Scalar autonomous ODEs. Exercises page 56
Problem number: 7.1 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - x(2 - x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(x(t),t)=x(t)*(2-x(t)),x(t), singsol=all)
```

$$x(t) = \frac{2}{1 + 2\,e^{-2t}c_1}$$

✓ Solution by Mathematica

Time used: 0.32 (sec). Leaf size: 24

```
DSolve[x'[t]==x[t]*(2-x[t]),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 1 + \tanh(t - c_1)$$
$$x(t) \to 0$$
$$x(t) \to 2$$

2.3 problem 7.1 (iii)

Internal problem ID [10632]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 7, Scalar autonomous ODEs. Exercises page 56
Problem number: 7.1 (iii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - (1 + x)(2 - x)\sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(diff(x(t),t)=(1+x(t))*(2-x(t))*sin(x(t)),x(t), singsol=all)
```

$$t + \int^{x(t)} \frac{1}{(\_a+1)(\_a-2)\sin(\_a)} d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 9.49 (sec). Leaf size: 52

```
DSolve[x'[t]==(1+x[t])*(2-x[t])*Sin[x[t]],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\csc(K[1])}{(K[1]-2)(K[1]+1)} dK[1]\&\right][-t + c_1]$$

$$x(t) \to -1$$

$$x(t) \to 0$$

$$x(t) \to 2$$

2.4 problem 7.1 (iv)

Internal problem ID [10633]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 7, Scalar autonomous ODEs. Exercises page 56
Problem number: 7.1 (iv).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' + x(-x+1)(2-x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve(diff(x(t),t)=-x(t)*(1-x(t))*(2-x(t)),x(t), singsol=all)
```

$$x(t) = \frac{e^t c_1}{\sqrt{-1 + e^{2t} c_1^2}} + 1$$

✓ Solution by Mathematica

Time used: 29.045 (sec). Leaf size: 120

```
DSolve[x'[t]==-x[t]*(1-x[t])*(2-x[t]),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 1 - \frac{e^{2t}}{\sqrt{e^{4t} + e^{2(t+c_1)}}}$$
$$x(t) \to 1 + \frac{e^{2t}}{\sqrt{e^{4t} + e^{2(t+c_1)}}}$$
$$x(t) \to 0$$
$$x(t) \to 1$$
$$x(t) \to 2$$
$$x(t) \to 1 - e^{-2t}\sqrt{e^{4t}}$$
$$x(t) \to e^{-2t}\sqrt{e^{4t}} + 1$$

2.5 problem 7.1 (v)

Internal problem ID [10634]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 7, Scalar autonomous ODEs. Exercises page 56
Problem number: 7.1 (v).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - x^2 + x^4 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
dsolve(diff(x(t),t)=x(t)^2-x(t)^4,x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{\mathrm{RootOf}\left(\ln\left(\mathrm{e}^{\_Z}-2\right)\mathrm{e}^{\_Z}+2c_1\mathrm{e}^{\_Z}-\_Z\mathrm{e}^{\_Z}+2t\,\mathrm{e}^{\_Z}-\ln\left(\mathrm{e}^{\_Z}-2\right)-2c_1+\_Z-2t+2\right)} - 1$$

✓ Solution by Mathematica

Time used: 0.262 (sec). Leaf size: 53

```
DSolve[x'[t]==x[t]^2-x[t]^4,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \text{InverseFunction}\left[\frac{1}{\#1} + \frac{1}{2}\log(1-\#1) - \frac{1}{2}\log(\#1+1)\&\right][-t+c_1]$$

$$x(t) \to -1$$

$$x(t) \to 0$$

$$x(t) \to 1$$

3 Chapter 8, Separable equations. Exercises page 72

3.1 problem 8.1 (i) 20
3.2 problem 8.1 (ii) 21
3.3 problem 8.1 (iii) 22
3.4 problem 8.1 (iv) 23
3.5 problem 8.1 (v) 24
3.6 problem 8.2 25
3.7 problem 8.3 26
3.8 problem 8.4 27
3.9 problem 8.5 28
3.10 problem 8.6 29
3.11 problem 8.7 30
3.12 problem 8.8 31

3.1 problem 8.1 (i)

Internal problem ID [10635]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.1 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - t^3(-x+1) = 0}$$

With initial conditions

$$[x(0) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(x(t),t)=t^3*(1-x(t)),x(0) = 3],x(t), singsol=all)
```

$$x(t) = 1 + 2\,\mathrm{e}^{-\frac{t^4}{4}}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 18

```
DSolve[{x'[t]==t^3*(1-x[t]),{x[0]==3}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 2e^{-\frac{t^4}{4}} + 1$$

3.2 problem 8.1 (ii)

Internal problem ID [10636]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.1 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (y^2 + 1)\tan(x) = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=(1+y(x)^2)*tan(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \cot\left(\frac{\pi}{4} + \ln(\cos(x))\right)$$

✓ Solution by Mathematica

Time used: 0.291 (sec). Leaf size: 15

```
DSolve[{y'[x]==(1+y[x]^2)*Tan[x],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cot\left(\log(\cos(x)) + \frac{\pi}{4}\right)$$

3.3 problem 8.1 (iii)

Internal problem ID [10637]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.1 (iii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - t^2 x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 12

```
dsolve(diff(x(t),t)=t^2*x(t),x(t), singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{\frac{t^3}{3}}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 22

```
DSolve[x'[t]==t^2*x[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{\frac{t^3}{3}}$$
$$x(t) \to 0$$

3.4 problem 8.1 (iv)

Internal problem ID [10638]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.1 (iv).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' + x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(x(t),t)=-x(t)^2,x(t), singsol=all)
```

$$x(t) = \frac{1}{t + c_1}$$

✓ Solution by Mathematica

Time used: 0.107 (sec). Leaf size: 18

```
DSolve[x'[t]==-x[t]^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{t - c_1}$$
$$x(t) \to 0$$

3.5 problem 8.1 (v)

Internal problem ID [10639]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.1 (v).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y^2 e^{-t^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(t),t)=exp(-t^2)*y(t)^2,y(t), singsol=all)
```

$$y(t) = -\frac{2}{\sqrt{\pi}\,\mathrm{erf}\,(t) - 2c_1}$$

✓ Solution by Mathematica

Time used: 0.223 (sec). Leaf size: 27

```
DSolve[y'[t]==Exp[-t^2]*y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{2}{\sqrt{\pi}\mathrm{erf}(t) + 2c_1}$$

$$y(t) \to 0$$

3.6 problem 8.2

Internal problem ID [10640]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.2 .
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' + px - q = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(x(t),t)+p*x(t)=q,x(t), singsol=all)
```

$$x(t) = \frac{q}{p} + \mathrm{e}^{-pt}c_1$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 29

```
DSolve[x'[t]+p*x[t]==q,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{q}{p} + c_1 e^{-pt}$$

$$x(t) \to \frac{q}{p}$$

3.7 problem 8.3

Internal problem ID [10641]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - ky = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*diff(y(x),x)=k*y(x),y(x), singsol=all)
```

$$y(x) = c_1 x^k$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 16

```
DSolve[x*y'[x]==k*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^k$$
$$y(x) \to 0$$

3.8 problem 8.4

Internal problem ID [10642]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{i' - p(t)\, i = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(i(t),t)=p(t)*i(t),i(t), singsol=all)
```

$$i(t) = c_1 e^{\int p(t)dt}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 25

```
DSolve[i'[t]==p[t]*i[t],i[t],t,IncludeSingularSolutions -> True]
```

$$i(t) \to c_1 \exp\left(\int_1^t p(K[1])dK[1]\right)$$
$$i(t) \to 0$$

3.9 problem 8.5

Internal problem ID [10643]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - \lambda x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(x(t),t)=lambda*x(t),x(t), singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{\lambda t}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 18

```
DSolve[x'[t]==\[Lambda]*x[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{\lambda t}$$
$$x(t) \to 0$$

3.10 problem 8.6

Internal problem ID [10644]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{mv' + mg - kv^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(m*diff(v(t),t)=-m*g+k*v(t)^2,v(t), singsol=all)
```

$$v(t) = -\frac{\tanh\left(\frac{\sqrt{mgk}\,(t+c_1)}{m}\right)\sqrt{mgk}}{k}$$

✓ Solution by Mathematica

Time used: 8.753 (sec). Leaf size: 87

```
DSolve[m*v'[t]==-m*g+k*v[t]^2,v[t],t,IncludeSingularSolutions -> True]
```

$$v(t) \to \frac{\sqrt{g}\sqrt{m}\tanh\left(\frac{\sqrt{g}\sqrt{k}(-t+c_1m)}{\sqrt{m}}\right)}{\sqrt{k}}$$
$$v(t) \to -\frac{\sqrt{g}\sqrt{m}}{\sqrt{k}}$$
$$v(t) \to \frac{\sqrt{g}\sqrt{m}}{\sqrt{k}}$$

3.11 problem 8.7

Internal problem ID [10645]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - kx + x^2 = 0}$$

With initial conditions

$$[x(0) = x_0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 22

```
dsolve([diff(x(t),t)=k*x(t)-x(t)^2,x(0) = x__0],x(t), singsol=all)
```

$$x(t) = \frac{kx_0}{(-x_0 + k)\,\mathrm{e}^{-kt} + x_0}$$

✓ Solution by Mathematica

Time used: 0.658 (sec). Leaf size: 26

```
DSolve[{x'[t]==k*x[t]-x[t]^2,{x[0]==x0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{k\mathrm{x0}e^{kt}}{\mathrm{x0}\,(e^{kt} - 1) + k}$$

3.12 problem 8.8

Internal problem ID [10646]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 8, Separable equations. Exercises page 72
Problem number: 8.8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' + x(k^2 + x^2) = 0}$$

With initial conditions

$$[x(0) = x_0]$$

✗ Solution by Maple

```
dsolve([diff(x(t),t)=-x(t)*(k^2+x(t)^2),x(0) = x__0],x(t), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 1.105 (sec). Leaf size: 62

```
DSolve[{x'[t]==-x[t]*(k^2+x[t]^2),{x[0]==x0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{k}{\sqrt{e^{2k^2t}\left(\frac{k^2}{\text{x0}^2}+1\right)-1}}$$

$$x(t) \to \frac{k}{\sqrt{e^{2k^2t}\left(\frac{k^2}{\text{x0}^2}+1\right)-1}}$$

4 Chapter 9, First order linear equations and the integrating factor. Exercises page 86

4.1 problem 9.1 (i) 33
4.2 problem 9.1 (ii) 34
4.3 problem 9.1 (iii) 35
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4.7 problem 9.1 (vii) 39
4.8 problem 9.1 (viii) 40
4.9 problem 9.4 41

4.1 problem 9.1 (i)

Internal problem ID [10647]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.1 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{x} - x^2 = 0}$$

With initial conditions

$$[y(0) = y_0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 9

```
dsolve([diff(y(x),x)+y(x)/x=x^2,y(0) = y__0],y(x), singsol=all)
```

$$y(x) = \frac{x^3}{4}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]+y[x]/x==x^2,{y[0]==y0}},y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.2 problem 9.1 (ii)

Internal problem ID [10648]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.1 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xt + x' - 4t = 0}$$

With initial conditions

$$[x(0) = 2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(x(t),t)+t*x(t)=4*t,x(0) = 2],x(t), singsol=all)
```

$$x(t) = 4 - 2\,\mathrm{e}^{-\frac{t^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 18

```
DSolve[{x'[t]+t*x[t]==4*t,{x[0]==2}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 4 - 2e^{-\frac{t^2}{2}}$$

4.3 problem 9.1 (iii)

Internal problem ID [10649]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.1 (iii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{z' - z\tan(y) - \sin(y) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(z(y),y)=z(y)*tan(y)+sin(y),z(y), singsol=all)
```

$$z(y) = \frac{-\frac{\cos(2y)}{4} + c_1}{\cos(y)}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 17

```
DSolve[z'[y]==z[y]*Tan[y]+Sin[y],z[y],y,IncludeSingularSolutions -> True]
```

$$z(y) \to -\frac{\cos(y)}{2} + c_1 \sec(y)$$

4.4 problem 9.1 (iv)

Internal problem ID [10650]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.1 (iv).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + e^{-x} y - 1 = 0}$$

With initial conditions

$$[y(0) = e]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 23

```
dsolve([diff(y(x),x)+exp(-x)*y(x)=1,y(0) = exp(1)],y(x), singsol=all)
```

$$y(x) = -\left(-\operatorname{Ei}_1\left(e^{-x}\right) - 1 + \operatorname{Ei}_1(1)\right) e^{e^{-x}}$$

✓ Solution by Mathematica

Time used: 0.063 (sec). Leaf size: 27

```
DSolve[{y'[x]+Exp[-x]*y[x]==1,{y[0]==Exp[1]}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{e^{-x}}(-\text{ExpIntegralEi}(\sinh(x) - \cosh(x)) + \text{ExpIntegralEi}(-1) + 1)$$

4.5 problem 9.1 (v)

Internal problem ID [10651]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.1 (v).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x' + x\tanh(t) - 3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(x(t),t)+x(t)*tanh(t)=3,x(t), singsol=all)
```

$$x(t) = \frac{3\sinh(t) + c_1}{\cosh(t)}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 15

```
DSolve[x'[t]+x[t]*Tanh[t]==3,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \operatorname{sech}(t)(3\sinh(t) + c_1)$$

4.6 problem 9.1 (vi)

Internal problem ID [10652]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.1 (vi).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + 2\cot(x)\, y - 5 = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 1\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve([diff(y(x),x)+2*y(x)*cot(x)=5,y(1/2*Pi) = 1],y(x), singsol=all)
```

$$y(x) = \frac{-10x + 5\sin(2x) - 4 + 5\pi}{-2 + 2\cos(2x)}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 27

```
DSolve[{y'[x]+2*y[x]*Cot[x]==5,{y[Pi/2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(10x - 5\sin(2x) - 5\pi + 4)\csc^2(x)$$

4.7 problem 9.1 (vii)

Internal problem ID [10653]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.1 (vii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x' + 5x - t = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(x(t),t)+5*x(t)=t,x(t), singsol=all)
```

$$x(t) = \frac{t}{5} - \frac{1}{25} + e^{-5t} c_1$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 22

```
DSolve[x'[t]+5*x[t]==t,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{t}{5} + c_1 e^{-5t} - \frac{1}{25}$$

4.8 problem 9.1 (viii)

Internal problem ID [10654]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.1 (viii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x' + \left(a + \frac{1}{t}\right)x - b = 0}$$

With initial conditions

$$[x(1) = x_0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 38

```
dsolve([diff(x(t),t)+(a+1/t)*x(t)=b,x(1) = x__0],x(t), singsol=all)
```

$$x(t) = \frac{(x_0 a^2 - ba + b)\,\mathrm{e}^{-a(t-1)} + b(at - 1)}{a^2 t}$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 46

```
DSolve[{x'[t]+(a+1/t)*x[t]==b,{x[1]==x0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{e^{-at}(e^a(a^2\mathrm{x0} - ab + b) + be^{at}(at - 1))}{a^2 t}$$

4.9 problem 9.4

Internal problem ID [10655]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 9, First order linear equations and the integrating factor. Exercises page 86
Problem number: 9.4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{T' + k(T - \mu - a\cos(\omega(t - \phi))) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 62

```
dsolve(diff(T(t),t)=-k*(T(t)- (mu+a*cos( omega*(t-phi)))),T(t), singsol=all)
```

$$T(t) = e^{-kt}c_1 - \frac{\sin(\omega(-t+\phi))\,ak\omega - \cos(\omega(-t+\phi))\,a\,k^2 - k^2\mu - \mu\,\omega^2}{k^2+\omega^2}$$

✓ Solution by Mathematica

Time used: 0.326 (sec). Leaf size: 50

```
DSolve[T'[t]==-k*(T[t]- (mu+a*Cos[ omega*(t-phi)])),T[t],t,IncludeSingularSolutions -> True]
```

$$T(t) \to \frac{ak(k\cos(\omega(\phi-t)) - \omega\sin(\omega(\phi-t)))}{k^2+\omega^2} + c_1e^{-kt} + \mu$$

5 Chapter 10, Two tricks for nonlinear equations. Exercises page 97

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5.2 problem 10.1 (ii) 44
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5.1 problem 10.1 (i)

Internal problem ID [10656]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.1 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel, '2`

$$2xy - \sec(x)^2 + (x^2 + 2y)\, y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
dsolve((2*x*y(x)- sec(x)^2)+(x^2+2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{2} - \frac{\sqrt{x^4 + 4\tan(x) - 4c_1}}{2}$$
$$y(x) = -\frac{x^2}{2} + \frac{\sqrt{x^4 + 4\tan(x) - 4c_1}}{2}$$

✓ Solution by Mathematica

Time used: 22.22 (sec). Leaf size: 90

```
DSolve[(2*x*y[x]- Sec[x]^2)+(x^2+2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-x^2 - \sqrt{\sec^2(x)}\sqrt{\cos^2(x)\left(x^4 + 4\tan(x) + 4c_1\right)}\right)$$
$$y(x) \to \frac{1}{2}\left(-x^2 + \sqrt{\sec^2(x)}\sqrt{\cos^2(x)\left(x^4 + 4\tan(x) + 4c_1\right)}\right)$$

5.2 problem 10.1 (ii)

Internal problem ID [10657]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.1 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{1 + e^{x} y + x\,e^{x} y + (x\,e^{x} + 2)\,y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((1+exp(x)*y(x)+x*exp(x)*y(x))+(x*exp(x)+2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x + c_1}{x\,e^{x} + 2}$$

✓ Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 21

```
DSolve[(1+Exp[x]*y[x]+x*Exp[x]*y[x])+(x*Exp[x]+2)*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{-x + c_1}{e^{x} x + 2}$$

5.3 problem 10.1 (iii)

Internal problem ID [10658]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.1 (iii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{(x\cos(y)+\cos(x))\,y'+\sin(y)-\sin(x)\,y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve((x*cos(y(x))+cos(x))*diff(y(x),x)+sin(y(x))-y(x)*sin(x)=0,y(x), singsol=all)
```

$$\cos(x)\,y(x)+x\sin(y(x))+c_1=0$$

✓ Solution by Mathematica

Time used: 0.164 (sec). Leaf size: 17

```
DSolve[(x*Cos[y[x]]+Cos[x])*y'[x]+Sin[y[x]]-y[x]*Sin[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}[x\sin(y(x))+y(x)\cos(x)=c_1,y(x)]$$

5.4 problem 10.1 (iv)

Internal problem ID [10659]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.1 (iv).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{e^{x} \sin \left(y \right)+y+\left(e^{x} \cos \left(y \right)+x+e^{y}\right) y^{\prime}=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(exp(x)*sin(y(x))+y(x)+ (exp(x)*cos(y(x))+x+exp(y(x)))*diff(y(x),x)=0,y(x), singsol=all
```

$$y(x)\, x+e^{x} \sin \left(y(x)\right)+e^{y(x)}+c_{1}=0$$

✓ Solution by Mathematica

Time used: 0.427 (sec). Leaf size: 22

```
DSolve[Exp[x]*Sin[y[x]]+y[x]+ (Exp[x]*Cos[y[x]]+x+Exp[y[x]])*y'[x]==0,y[x],x,IncludeSingularS
```

$$\text{Solve}\left[e^{y(x)}+xy(x)+e^{x}\sin(y(x))=c_1,y(x)\right]$$

5.5 problem 10.2

Internal problem ID [10660]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{e^{-y} \sec(x) + 2\cos(x) - e^{-y} y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 88

```
dsolve(exp(-y(x))*sec(x)+2*cos(x)-exp(-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \ln\left(\frac{\tan\left(\frac{x}{2}\right)^3 + \tan\left(\frac{x}{2}\right)^2 + \tan\left(\frac{x}{2}\right) + 1}{\tan\left(\frac{x}{2}\right)^3 c_1 + 2\tan\left(\frac{x}{2}\right)^3 x - \tan\left(\frac{x}{2}\right)^2 c_1 - 2\tan\left(\frac{x}{2}\right)^2 x + \tan\left(\frac{x}{2}\right) c_1 + 2\tan\left(\frac{x}{2}\right) x - c_1 - 2x - 4\tan\left(\right.}\right.$$

✓ Solution by Mathematica

Time used: 1.612 (sec). Leaf size: 33

```
DSolve[Exp[-y[x]]*Sec[x]+2*Cos[x]-Exp[-y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \log\left(\frac{e^{2\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)}}{2(-x + \cos(x) - 2c_1)}\right)$$

5.6 problem 10.3 (i)

Internal problem ID [10661]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.3 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{V'(x) + 2y'y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(V(x),x)+2*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-V(x) + c_1}$$
$$y(x) = -\sqrt{-V(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 37

```
DSolve[V'[x]+2*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-V(x) + 2c_1}$$
$$y(x) \to \sqrt{-V(x) + 2c_1}$$

5.7 problem 10.3 (ii)

Internal problem ID [10662]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.3 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(\frac{1}{y}-a\right)y'+\frac{2}{x}-b=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve((1/y(x)-a)*diff(y(x),x)+2/x-b=0,y(x), singsol=all)
```

$$y(x)=-\frac{\text{LambertW}\left(-\frac{a\,e^{bx}c_1}{x^2}\right)}{a}$$

✓ Solution by Mathematica

Time used: 4.223 (sec). Leaf size: 32

```
DSolve[(1/y[x]-a)*y'[x]+2/x-b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{W\left(-\frac{ae^{bx-c_1}}{x^2}\right)}{a}$$
$$y(x)\to 0$$

5.8 problem 10.4 (i)

Internal problem ID [10663]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.4 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{xy + y^2 + x^2 - x^2y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*y(x)+y(x)^2+x^2-x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \tan(\ln(x) + c_1) x$$

✓ Solution by Mathematica

Time used: 0.208 (sec). Leaf size: 13

```
DSolve[x*y[x]+y[x]^2+x^2-x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \tan(\log(x) + c_1)$$

5.9 problem 10.4 (ii)

Internal problem ID [10664]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.4 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{x' - \frac{x^2 + t\sqrt{x^2 + t^2}}{xt} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(x(t),t)=(x(t)^2+t*sqrt(t^2+x(t)^2))/(t*x(t)),x(t), singsol=all)
```

$$-\frac{\sqrt{t^2 + x(t)^2}}{t} + \ln(t) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.321 (sec). Leaf size: 48

```
DSolve[x'[t]==(x[t]^2+t*Sqrt[t^2+x[t]^2])/(t*x[t]),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -t\sqrt{(\log(t) - 1 + c_1)(\log(t) + 1 + c_1)}$$
$$x(t) \to t\sqrt{(\log(t) - 1 + c_1)(\log(t) + 1 + c_1)}$$

5.10 problem 10.5

Internal problem ID [10665]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 10, Two tricks for nonlinear equations. Exercises page 97
Problem number: 10.5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - kx + x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(x(t),t)=k*x(t)-x(t)^2,x(t), singsol=all)
```

$$x(t) = \frac{k}{1 + \mathrm{e}^{-kt} c_1 k}$$

✓ Solution by Mathematica

Time used: 0.598 (sec). Leaf size: 31

```
DSolve[x'[t]==k*x[t]-x[t]^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to k\left(1 + \frac{1}{-1 + e^{k(t+c_1)}}\right)$$
$$x(t) \to 0$$
$$x(t) \to k$$

6 Chapter 12, Homogeneous second order linear equations. Exercises page 118

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6.1 problem 12.1 (i)

Internal problem ID [10666]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - 3x' + 2x = 0}$$

With initial conditions

$$[x(0) = 2, x'(0) = 6]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(x(t),t$2)-3*diff(x(t),t)+2*x(t)=0,x(0) = 2, D(x)(0) = 6],x(t), singsol=all)
```

$$x(t) = -2\,e^{t} + 4\,e^{2t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 17

```
DSolve[{x''[t]-3*x'[t]+2*x[t]==0,{x[0]==2,x'[0]==6}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 2e^{t}\left(2e^{t} - 1\right)$$

6.2 problem 12.1 (ii)

Internal problem ID [10667]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 4y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 3]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=0,y(0) = 0, D(y)(0) = 3],y(x), singsol=all)
```

$$y(x) = 3\,\mathrm{e}^{2x}x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 13

```
DSolve[{y''[x]-4*y'[x]+4*y[x]==0,{y[0]==0,y'[0]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3e^{2x}x$$

6.3 problem 12.1 (iii)

Internal problem ID [10668]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{z'' - 4z' + 13z = 0}$$

With initial conditions

$$[z(0) = 7, z'(0) = 42]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(z(t),t$2)-4*diff(z(t),t)+13*z(t)=0,z(0) = 7, D(z)(0) = 42],z(t), singsol=all)
```

$$z(t) = \frac{7\,\mathrm{e}^{2t}(4\sin(3t) + 3\cos(3t))}{3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 27

```
DSolve[{z''[t]-4*z'[t]+13*z[t]==0,{z[0]==7,z'[0]==42}},z[t],t,IncludeSingularSolutions -> Tru
```

$$z(t) \to \frac{7}{3}e^{2t}(4\sin(3t) + 3\cos(3t))$$

6.4 problem 12.1 (iv)

Internal problem ID [10669]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (iv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 6y = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 8]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-6*y(t)=0,y(0) = -1, D(y)(0) = 8],y(t), singsol=all)
```

$$y(t) = \left(\mathrm{e}^{5t} - 2\right)\mathrm{e}^{-3t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{y''[t]+y'[t]-6*y[t]==0,{y[0]==-1,y'[0]==8}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-3t}\left(e^{5t} - 2\right)$$

6.5 problem 12.1 (v)

Internal problem ID [10670]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (v).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' = 0}$$

With initial conditions

$$[y(0) = 13, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 5

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)=0,y(0) = 13, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = 13$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 6

```
DSolve[{y''[t]-4*y'[t]==0,{y[0]==13,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 13$$

6.6 problem 12.1 (vi)

Internal problem ID [10671]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (vi).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{\theta'' + 4\theta = 0}$$

With initial conditions

$$[\theta(0) = 0, \theta'(0) = 10]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(theta(t),t$2)+4*theta(t)=0,theta(0) = 0, D(theta)(0) = 10],theta(t), singsol=all
```

$$\theta(t) = 5\sin(2t)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 11

```
DSolve[{\[Theta]''[t]+4*\[Theta][t]==0,{\[Theta][0]==0,\[Theta]'[0]==10}},\[Theta][t],t,Inclu
```

$$\theta(t) \to 5\sin(2t)$$

6.7 problem 12.1 (vii)

Internal problem ID [10672]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (vii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 10y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+10*y(t)=0,y(0) = 3, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-t}(3\cos(3t) + \sin(3t))$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 22

```
DSolve[{y''[t]+2*y'[t]+10*y[t]==0,{y[0]==3,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to e^{-t}(\sin(3t) + 3\cos(3t))$$

6.8 problem 12.1 (viii)

Internal problem ID [10673]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (viii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2z'' + 7z' - 4z = 0}$$

With initial conditions

$$[z(0) = 0, z'(0) = 9]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([2*diff(z(t),t$2)+7*diff(z(t),t)-4*z(t)=0,z(0) = 0, D(z)(0) = 9],z(t), singsol=all)
```

$$z(t) = 2\left(e^{\frac{9t}{2}} - 1\right) e^{-4t}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 49

```
DSolve[{z''[t]+7*z'[t]-4*z[t]==0,{z[0]==3,z'[0]==9}},z[t],t,IncludeSingularSolutions -> True]
```

$$z(t) \to \frac{3}{10} e^{-\frac{1}{2}\left(7+\sqrt{65}\right)t}\left(\left(5+\sqrt{65}\right) e^{\sqrt{65}t} + 5 - \sqrt{65}\right)$$

6.9 problem 12.1 (ix)

Internal problem ID [10674]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (ix).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+y(t)=0,y(0) = 0, D(y)(0) = -1],y(t), singsol=all)
```

$$y(t) = -t\,\mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 13

```
DSolve[{y''[t]+2*y'[t]+y[t]==0,{y[0]==0,y'[0]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -e^{-t}t$$

6.10 problem 12.1 (x)

Internal problem ID [10675]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (x).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 6x' + 10x = 0}$$

With initial conditions

$$[x(0) = 3, x'(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(x(t),t$2)+6*diff(x(t),t)+10*x(t)=0,x(0) = 3, D(x)(0) = 1],x(t), singsol=all)
```

$$x(t) = e^{-3t}(3\cos(t) + 10\sin(t))$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[{x''[t]+6*x'[t]+10*x[t]==0,{x[0]==3,x'[0]==1}},x[t],t,IncludeSingularSolutions -> True
```

$$x(t) \to e^{-3t}(10\sin(t) + 3\cos(t))$$

6.11 problem 12.1 (xi)

Internal problem ID [10676]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (xi).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4x'' - 20x' + 21x = 0}$$

With initial conditions

$$[x(0) = -4, x'(0) = -12]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([4*diff(x(t),t$2)-20*diff(x(t),t)+21*x(t)=0,x(0) = -4, D(x)(0) = -12],x(t), singsol=al
```

$$x(t) = -3\,\mathrm{e}^{\frac{7t}{2}} - \mathrm{e}^{\frac{3t}{2}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 23

```
DSolve[{4*x''[t]-20*x'[t]+21*x[t]==0,{x[0]==-4,x'[0]==-12}},x[t],t,IncludeSingularSolutions -
```

$$x(t) \to -e^{3t/2}\left(3e^{2t} + 1\right)$$

6.12 problem 12.1 (xii)

Internal problem ID [10677]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (xii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 2y = 0}$$

With initial conditions

$$[y(0) = 4, y'(0) = -4]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-2*y(t)=0,y(0) = 4, D(y)(0) = -4],y(t), singsol=all)
```

$$y(t) = \frac{4(\mathrm{e}^{3t} + 2)\,\mathrm{e}^{-2t}}{3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 21

```
DSolve[{y''[t]+y'[t]-2*y[t]==0,{y[0]==4,y'[0]==-4}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{4}{3}e^{-2t}\left(e^{3t} + 2\right)$$

6.13 problem 12.1 (xiii)

Internal problem ID [10678]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (xiii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y = 0}$$

With initial conditions

$$[y(0) = 10, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)-4*y(t)=0,y(0) = 10, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = 5\,e^{-2t} + 5\,e^{2t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 11

```
DSolve[{y''[t]-4*y[t]==0,{y[0]==10,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 10\cosh(2t)$$

6.14 problem 12.1 (xiv)

Internal problem ID [10679]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (xiv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 4y = 0}$$

With initial conditions

$$[y(0) = 27, y'(0) = -54]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 10

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+4*y(t)=0,y(0) = 27, D(y)(0) = -54],y(t), singsol=all)
```

$$y(t) = 27\,\mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 12

```
DSolve[{y''[t]+4*y'[t]+4*y[t]==0,{y[0]==27,y'[0]==-54}},y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to 27e^{-2t}$$

6.15 problem 12.1 (xv)

Internal problem ID [10680]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 12, Homogeneous second order linear equations. Exercises page 118
Problem number: 12.1 (xv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + \omega^2 y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(t),t$2)+omega^2*y(t)=0,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{\sin(t\omega)}{\omega}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 13

```
DSolve[{y''[t]+w^2*y[t]==0,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\sin(tw)}{w}$$

7 Chapter 14, Inhomogeneous second order linear equations. Exercises page 140

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7.1 problem 14.1 (i)

Internal problem ID [10681]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' - 4x - t^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(x(t),t$2)-4*x(t)=t^2,x(t), singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{2t} + c_2 \mathrm{e}^{-2t} - \frac{t^2}{4} - \frac{1}{8}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 32

```
DSolve[x''[t]-4*x[t]==t^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{t^2}{4} + c_1 e^{2t} + c_2 e^{-2t} - \frac{1}{8}$$

7.2 problem 14.1 (ii)

Internal problem ID [10682]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x'' - 4x' - t^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(x(t),t$2)-4*diff(x(t),t)=t^2,x(t), singsol=all)
```

$$x(t) = -\frac{t^2}{16} - \frac{t^3}{12} + \frac{c_1 e^{4t}}{4} - \frac{t}{32} + c_2$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 34

```
DSolve[x''[t]-4*x'[t]==t^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{1}{96}t(8t^2 + 6t + 3) + \frac{1}{4}c_1 e^{4t} + c_2$$

7.3 problem 14.1 (iii)

Internal problem ID [10683]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + x' - 2x - 3\,\mathrm{e}^{-t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(x(t),t$2)+diff(x(t),t)-2*x(t)=3*exp(-t),x(t), singsol=all)
```

$$x(t) = c_1\mathrm{e}^{t} + c_2\mathrm{e}^{-2t} - \frac{3\,\mathrm{e}^{-t}}{2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 29

```
DSolve[x''[t]+x'[t]-2*x[t]==3*Exp[-t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{3e^{-t}}{2} + c_1e^{-2t} + c_2e^{t}$$

7.4 problem 14.1 (iv)

Internal problem ID [10684]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (iv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + x' - 2x - e^{t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(x(t),t$2)+diff(x(t),t)-2*x(t)=exp(t),x(t), singsol=all)
```

$$x(t) = c_1 e^{t} + c_2 e^{-2t} + \frac{t\,e^{t}}{3}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 29

```
DSolve[x''[t]+x'[t]-2*x[t]==Exp[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{-2t} + e^{t}\left(\frac{t}{3} - \frac{1}{9} + c_2\right)$$

7.5 problem 14.1 (v)

Internal problem ID [10685]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (v).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + 2x' + x - e^{-t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(x(t),t$2)+2*diff(x(t),t)+x(t)=exp(-t),x(t), singsol=all)
```

$$x(t) = c_1 t\, e^{-t} + \frac{e^{-t} t^2}{2} + c_2 e^{-t}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 27

```
DSolve[x''[t]+2*x'[t]+x[t]==Exp[-t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2} e^{-t} \left(t^2 + 2c_2 t + 2c_1\right)$$

7.6 problem 14.1 (vi)

Internal problem ID [10686]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (vi).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + \omega^2 x - \sin(\alpha t) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve(diff(x(t),t$2)+omega^2*x(t)=sin(alpha*t),x(t), singsol=all)
```

$$x(t) = \sin(t\omega)\, c_2 + \cos(t\omega)\, c_1 - \frac{\sin(\alpha t)}{\alpha^2 - \omega^2}$$

✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 37

```
DSolve[x''[t]+w^2*x[t]==Sin[a*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{\sin(at)}{a^2 - w^2} + c_1 \cos(tw) + c_2 \sin(tw)$$

7.7 problem 14.1 (vii)

Internal problem ID [10687]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (vii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + \omega^2 x - \sin(\omega t) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve(diff(x(t),t$2)+omega^2*x(t)=sin(omega*t),x(t), singsol=all)
```

$$x(t) = \sin(t\omega)\, c_2 + \cos(t\omega)\, c_1 + \frac{\sin(t\omega) - \cos(t\omega)\,\omega t}{2\omega^2}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 29

```
DSolve[x''[t]+w^2*x[t]==Sin[w*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \left(-\frac{t}{2w} + c_1\right)\cos(tw) + c_2\sin(tw)$$

7.8 problem 14.1 (viii)

Internal problem ID [10688]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (viii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + 2x' + 10x - e^{-t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(x(t),t$2)+2*diff(x(t),t)+10*x(t)=exp(-t),x(t), singsol=all)
```

$$x(t) = e^{-t} \sin(3t) c_2 + e^{-t} \cos(3t) c_1 + \frac{e^{-t}}{9}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 32

```
DSolve[x''[t]+2*x'[t]+10*x[t]==Exp[-t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{9} e^{-t} (9c_2 \cos(3t) + 9c_1 \sin(3t) + 1)$$

7.9 problem 14.1 (ix)

Internal problem ID [10689]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (ix).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 2x' + 10x - e^{-t} \cos(3t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(x(t),t$2)+2*diff(x(t),t)+10*x(t)=exp(-t)*cos(3*t),x(t), singsol=all)
```

$$x(t) = e^{-t} \sin(3t)\, c_2 + e^{-t} \cos(3t)\, c_1 + \frac{e^{-t}(\cos(3t) + 3t \sin(3t))}{18}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 38

```
DSolve[x''[t]+2*x'[t]+10*x[t]==Exp[-t]*Cos[3*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{36} e^{-t}((1 + 36c_2)\cos(3t) + 6(t + 6c_1)\sin(3t))$$

7.10 problem 14.1 (x)

Internal problem ID [10690]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (x).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 6x' + 10x - e^{-2t}\cos(t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(x(t),t$2)+6*diff(x(t),t)+10*x(t)=exp(-2*t)*cos(t),x(t), singsol=all)
```

$$x(t) = \sin(t)\,e^{-3t}c_2 + \cos(t)\,e^{-3t}c_1 + \frac{e^{-2t}(\cos(t) + 2\sin(t))}{5}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 33

```
DSolve[x''[t]+6*x'[t]+10*x[t]==Exp[-3*t]*Cos[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2}e^{-3t}((1 + 2c_2)\cos(t) + (t + 2c_1)\sin(t))$$

7.11 problem 14.1 (xi)

Internal problem ID [10691]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.1 (xi).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + 4x' + 4x - e^{2t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(x(t),t$2)+4*diff(x(t),t)+4*x(t)=exp(2*t),x(t), singsol=all)
```

$$x(t) = \frac{e^{2t}}{16} + c_1 t\, e^{-2t} + c_2 e^{-2t}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 28

```
DSolve[x''[t]+4*x'[t]+4*x[t]==Exp[2*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{e^{2t}}{16} + e^{-2t}(c_2 t + c_1)$$

7.12 problem 14.2

Internal problem ID [10692]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x' - 2x - 12\,\mathrm{e}^{-t} + 6\,\mathrm{e}^{t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(x(t),t$2)+diff(x(t),t)-2*x(t)=12*exp(-t)-6*exp(t),x(t), singsol=all)
```

$$x(t) = c_2\mathrm{e}^{-2t} + c_1\mathrm{e}^{t} - 6\,\mathrm{e}^{-t} - 2t\,\mathrm{e}^{t} + \frac{2\,\mathrm{e}^{t}}{3}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 34

```
DSolve[x''[t]+x'[t]-2*x[t]==12*Exp[-t]-6*Exp[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-2t}\left(-6e^{t} + e^{3t}\left(-2t + \frac{2}{3} + c_2\right) + c_1\right)$$

7.13 problem 14.3

Internal problem ID [10693]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 14, Inhomogeneous second order linear equations. Exercises page 140
Problem number: 14.3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 4x - 289t\,\mathrm{e}^{t} \sin(2t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(x(t),t$2)+4*x(t)=289*t*exp(t)*sin(2*t),x(t), singsol=all)
```

$$x(t) = c_2 \sin(2t) + c_1 \cos(2t) - \mathrm{e}^{t}(68 \cos(2t)\, t - 17t \sin(2t) - 76 \cos(2t) + 2 \sin(2t))$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 40

```
DSolve[x''[t]+4*x[t]==289*t*Exp[t]*Sin[2*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \left(e^{t}(76 - 68t) + c_1\right) \cos(2t) + \left(e^{t}(17t - 2) + c_2\right) \sin(2t)$$

8 Chapter 15, Resonance. Exercises page 148

8.1 problem 15.1 84
8.2 problem 15.3 85

8.1 problem 15.1

Internal problem ID [10694]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 15, Resonance. Exercises page 148
Problem number: 15.1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + \omega^2 x - \cos(\alpha t) = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve([diff(x(t),t$2)+omega^2*x(t)=cos(alpha*t),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{\cos(t\omega) - \cos(\alpha t)}{\alpha^2 - \omega^2}$$

✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 28

```
DSolve[{x''[t]+w^2*x[t]==Cos[a*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True
```

$$x(t) \to \frac{\cos(tw) - \cos(at)}{a^2 - w^2}$$

8.2 problem 15.3

Internal problem ID [10695]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 15, Resonance. Exercises page 148
Problem number: 15.3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + \omega^2 x - \cos(\omega t) = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(x(t),t$2)+omega^2*x(t)=cos(omega*t),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{\sin(t\omega)\, t}{2\omega}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 17

```
DSolve[{x''[t]+w^2*x[t]==Cos[w*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True
```

$$x(t) \to \frac{t\sin(tw)}{2w}$$

9 Chapter 16, Higher order linear equations with constant coefficients. Exercises page 153

9.1 problem 16.1 (i) 87
9.2 problem 16.1 (ii) 88
9.3 problem 16.1 (iii) 89
9.4 problem 16.1 (iv) 90

9.1 problem 16.1 (i)

Internal problem ID [10696]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 16, Higher order linear equations with constant coefficients. Exercises page 153
Problem number: 16.1 (i).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x''' - 6x'' + 11x' - 6x - e^{-t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(x(t),t$3)-6*diff(x(t),t$2)+11*diff(x(t),t)-6*x(t)=exp(-t),x(t), singsol=all)
```

$$x(t) = c_1 e^t + e^{2t} c_2 + c_3 e^{3t} - \frac{e^{-t}}{24}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 37

```
DSolve[x'''[t]-6*x''[t]+11*x'[t]-6*x[t]==Exp[-t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{e^{-t}}{24} + c_1 e^t + c_2 e^{2t} + c_3 e^{3t}$$

9.2 problem 16.1 (ii)

Internal problem ID [10697]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 16, Higher order linear equations with constant coefficients. Exercises page 153
Problem number: 16.1 (ii).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 3y'' + 2y - \sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+2*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = \frac{-15\sin(x) - 3\cos(x)}{6\left(5+2\sqrt{3}\right)\left(-5+2\sqrt{3}\right)} + e^{x} c_1 + e^{\left(1+\sqrt{3}\right)x} c_2 + e^{-\left(\sqrt{3}-1\right)x} c_3$$

✓ Solution by Mathematica

Time used: 0.129 (sec). Leaf size: 49

```
DSolve[y'''[x]-3*y''[x]+2*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{26}\left(5\sin(x) + \cos(x) + 26e^{x}\left(c_1 e^{-\sqrt{3}x} + c_2 e^{\sqrt{3}x} + c_3\right)\right)$$

9.3 problem 16.1 (iii)

Internal problem ID [10698]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 16, Higher order linear equations with constant coefficients. Exercises page 153
Problem number: 16.1 (iii).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{x'''' - 4x''' + 8x'' - 8x' + 4x - \sin(t) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(x(t),t$4)-4*diff(x(t),t$3)+8*diff(x(t),t$2)-8*diff(x(t),t)+4*x(t)=sin(t),x(t), si
```

$$x(t) = -\frac{3\sin(t)}{25} + \frac{4\cos(t)}{25} + c_1 e^t \cos(t) + c_2 e^t \sin(t) + c_3 e^t \cos(t)\, t + c_4 e^t \sin(t)\, t$$

✓ Solution by Mathematica

Time used: 0.181 (sec). Leaf size: 42

```
DSolve[x''''[t]-4*x'''[t]+8*x''[t]-8*x'[t]+4*x[t]==Sin[t],x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to \left(\frac{4}{25} + e^t(c_4 t + c_3)\right)\cos(t) + \left(-\frac{3}{25} + e^t(c_2 t + c_1)\right)\sin(t)$$

9.4 problem 16.1 (iv)

Internal problem ID [10699]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 16, Higher order linear equations with constant coefficients. Exercises page 153
Problem number: 16.1 (iv).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{x'''' - 5x'' + 4x - e^t = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(diff(x(t),t$4)-5*diff(x(t),t$2)+4*x(t)=exp(t),x(t), singsol=all)
```

$$x(t) = -\frac{t\,e^t}{6} + c_1 e^t + c_2 e^{-2t} + c_3 e^{-t} + e^{2t} c_4$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 45

```
DSolve[x''''[t]-5*x''[t]+4*x[t]==Exp[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-2t}\left(c_2 e^t + e^{3t}\left(-\frac{t}{6} - \frac{1}{36} + c_3\right) + c_4 e^{4t} + c_1\right)$$

10 Chapter 17, Reduction of order. Exercises page 162

10.1 problem 17.1 92
10.2 problem 17.2 93
10.3 problem 17.3 94
10.4 problem 17.4 95
10.5 problem 17.5 96
10.6 problem 17.6 97

10.1 problem 17.1

Internal problem ID [10700]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 17, Reduction of order. Exercises page 162
Problem number: 17.1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' - (t^2 + 2t)\, y' + (t + 2)\, y = 0}$$

Given that one solution of the ode is

$$y_1 = t$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve([t^2*diff(y(t),t$2)-(t^2+2*t)*diff(y(t),t)+(t+2)*y(t)=0,t],y(t), singsol=all)
```

$$y(t) = c_1 t + c_2 t\, \mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]-(t^2+2*t)*y'[t]+(t+2)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t\left(c_2 e^t + c_1\right)$$

10.2 problem 17.2

Internal problem ID [10701]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 17, Reduction of order. Exercises page 162
Problem number: 17.2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-y'x+y=0}$$

Given that one solution of the ode is

$$y_1=\mathrm{e}^x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([(x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,exp(x)],y(x), singsol=all)
```

$$y(x)=c_1x+c_2\mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^x-c_2x$$

10.3 problem 17.3

Internal problem ID [10702]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 17, Reduction of order. Exercises page 162
Problem number: 17.3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(\cos(t)\, t-\sin(t))\, x''-x' t\sin(t)-x\sin(t)=0}$$

Given that one solution of the ode is

$$x_1 = t$$

✗ Solution by Maple

```
dsolve([(t*cos(t)-sin(t))*diff(x(t),t$2)-diff(x(t),t)*t*sin(t)-x(t)*sin(t)=0,t],x(t), singsol
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(t*Cos[t]-Sin[t])*x''[t]-x'[t]*t*Sin[t]-x[t]*Sin[t]==0,x[t],t,IncludeSingularSolutions
```

Not solved

10.4 problem 17.4

Internal problem ID [10703]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 17, Reduction of order. Exercises page 162
Problem number: 17.4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-t^2+t)\, x'' + (-t^2+2)\, x' + (-t+2)\, x = 0}$$

Given that one solution of the ode is

$$x_1 = \mathrm{e}^{-t}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([(t-t^2)*diff(x(t),t$2)+(2-t^2)*diff(x(t),t)+(2-t)*x(t)=0,exp(-t)],x(t), singsol=all)
```

$$x(t) = \frac{c_1}{t} + c_2\mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 42

```
DSolve[(t-t^2)*x''[t]+(2-t^2)*x'[t]+(2-t)*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{e^{-t}\sqrt{1-t}(c_1e^t - c_2t)}{\sqrt{t-1}t}$$

10.5 problem 17.5

Internal problem ID [10704]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 17, Reduction of order. Exercises page 162
Problem number: 17.5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - y'x + y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve([diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,x],y(x), singsol=all)
```

$$y(x) = c_1 x + c_2 \left(i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{x^2}{2}} - \pi\ \mathrm{erf}\left(\frac{i\sqrt{2}\,x}{2} \right) x \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 47

```
DSolve[y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\left(2c_1 - \sqrt{\pi} c_2 \mathrm{erfi}\left(\frac{x}{\sqrt{2}}\right)\right)}{\sqrt{2}} + c_2 e^{\frac{x^2}{2}}$$

10.6 problem 17.6

Internal problem ID [10705]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 17, Reduction of order. Exercises page 162
Problem number: 17.6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\tan(t)\, x'' - 3x' + (\tan(t) + 3\cot(t))\, x = 0}$$

Given that one solution of the ode is

$$x_1 = \sin(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 15

```
dsolve([tan(t)*diff(x(t),t$2)-3*diff(x(t),t)+(tan(t)+3*cot(t))*x(t)=0,sin(t)],x(t), singsol=a
```

$$x(t) = c_1 \sin(t) + c_2 \sin(t)\cos(t)$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 24

```
DSolve[Tan[t]*x''[t]-3*x'[t]+(Tan[t]+3*Cot[t])*x[t]==0,x[t],t,IncludeSingularSolutions -> Tru
```

$$x(t) \to \sqrt{-\sin^2(t)}(c_2\cos(t) + c_1)$$

11 Chapter 18, The variation of constants formula. Exercises page 168

11.1 problem 18.1 (i) 99
11.2 problem 18.1 (ii) 100
11.3 problem 18.1 (iii) 101
11.4 problem 18.1 (iv) 102
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11.6 problem 18.1 (vi) 104

11.1 problem 18.1 (i)

Internal problem ID [10706]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 18, The variation of constants formula. Exercises page 168
Problem number: 18.1 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 6y - \mathrm{e}^x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-6*y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = c_1\mathrm{e}^{3x} + c_2\mathrm{e}^{-2x} - \frac{\mathrm{e}^x}{6}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 29

```
DSolve[y''[x]-y'[x]-6*y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^x}{6} + c_1e^{-2x} + c_2e^{3x}$$

11.2 problem 18.1 (ii)

Internal problem ID [10707]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 18, The variation of constants formula. Exercises page 168
Problem number: 18.1 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' - x - \frac{1}{t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(x(t),t$2)-x(t)=1/t,x(t), singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{t} + c_2 \mathrm{e}^{-t} - \frac{\mathrm{Ei}_1(t)\, \mathrm{e}^{t}}{2} + \frac{\mathrm{Ei}_1(-t)\, \mathrm{e}^{-t}}{2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 38

```
DSolve[x''[t]-x[t]==1/t,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2} e^{-t} \left(-\operatorname{ExpIntegralEi}(t) + e^{2t} (\operatorname{ExpIntegralEi}(-t) + 2c_1) + 2c_2 \right)$$

11.3 problem 18.1 (iii)

Internal problem ID [10708]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 18, The variation of constants formula. Exercises page 168
Problem number: 18.1 (iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y - \cot(2x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+4*y(x)=cot(2*x),y(x), singsol=all)
```

$$y(x) = c_1 \cos(2x) + c_2 \sin(2x) + \frac{\sin(2x) \ln(\csc(2x) - \cot(2x))}{4}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 34

```
DSolve[y''[x]+4*y[x]==Cot[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2x) + \frac{1}{4} \sin(2x)(\log(\sin(x)) - \log(\cos(x)) + 4c_2)$$

11.4 problem 18.1 (iv)

Internal problem ID [10709]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 18, The variation of constants formula. Exercises page 168
Problem number: 18.1 (iv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{t^2x'' - 2x - t^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(t^2*diff(x(t),t$2)-2*x(t)=t^3,x(t), singsol=all)
```

$$x(t) = \frac{t^3}{4} + \frac{c_1}{t} + c_2t^2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 25

```
DSolve[t^2*x''[t]-2*x[t]==t^3,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{t^3}{4} + c_2t^2 + \frac{c_1}{t}$$

11.5 problem 18.1 (v)

Internal problem ID [10710]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 18, The variation of constants formula. Exercises page 168
Problem number: 18.1 (v).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x'' - 4x' - \tan(t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(x(t),t$2)-4*diff(x(t),t)=tan(t),x(t), singsol=all)
```

$$x(t) = \int \left(\int \tan(t)\,\mathrm{e}^{-4t}dt + c_1\right)\mathrm{e}^{4t}dt + c_2$$

✓ Solution by Mathematica

Time used: 60.141 (sec). Leaf size: 82

```
DSolve[x''[t]-4*x'[t]==Tan[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \int_1^t \left(e^{4K[1]}c_1 + \frac{1}{20}\left(-5i\,\text{Hypergeometric2F1}\left(2i, 1, 1+2i, -e^{2iK[1]}\right)\right.\right.$$
$$\left.\left. - (2-4i)e^{2iK[1]}\,\text{Hypergeometric2F1}\left(1, 1+2i, 2+2i, -e^{2iK[1]}\right)\right)\right) dK[1] + c_2$$

11.6 problem 18.1 (vi)

Internal problem ID [10711]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 18, The variation of constants formula. Exercises page 168
Problem number: 18.1 (vi).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\left(\tan(x)^2-1\right)y''-4\tan(x)^3y'+2y\sec(x)^4-\left(\tan(x)^2-1\right)\left(1-2\sin(x)^2\right)=0$$

Given that one solution of the ode is

$$y_1=\sec(x)^2$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 29

```
dsolve([(tan(x)^2-1)*diff(y(x),x$2)-4*tan(x)^3*diff(y(x),x)+2*y(x)*sec(x)^4=(tan(x)^2-1)*(1-2
```

$$y(x)=\sec(x)^2c_2+\sec(x)\sin(x)c_1-\frac{\cos(x)^2}{4}+\frac{x\tan(x)}{2}+\frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.228 (sec). Leaf size: 54

```
DSolve[(Tan[x]^2-1)*y''[x]-4*Tan[x]^3*y'[x]+2*y[x]*Sec[x]^4==(Tan[x]^2-1)*(1-2*Sin[x]^2),y[x]
```

$$y(x)\to-\frac{1}{4}\cos^2(x)+c_1\sec^2(x)+\sqrt{\sin^2(x)}\sec(x)\left(\cot^{-1}\left(\sec(x)-\sqrt{\sin^2(x)}\sec(x)\right)+c_2\right)+\frac{1}{2}$$

12 Chapter 19, CauchyEuler equations. Exercises page 174

12.1 problem 19.1 (i) 106
12.2 problem 19.1 (ii) 107
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12.1 problem 19.1 (i)

Internal problem ID [10712]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, ‘_with_symmetry_[0,F(`

$$\boxed{x^2y'' - 4y'x + 6y = 0}$$

With initial conditions

$$[y(1) = 0, y'(1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve([x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(1) = 0, D(y)(1) = 1],y(x), singsol=all
```

$$y(x) = x^2(x - 1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 12

```
DSolve[{x^2*y''[x]-4*x*y'[x]+6*y[x]==0,{y[1]==0,y'[1]==1}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to (x - 1)x^2$$

12.2 problem 19.1 (ii)

Internal problem ID [10713]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4x^2y'' + y = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 13

```
dsolve([4*x^2*diff(y(x),x$2)+y(x)=0,y(1) = 1, D(y)(1) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{x}\,(-2 + \ln(x))}{2}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 47

```
DSolve[{x^2*y''[x]+y[x]==0,{y[1]==1,y'[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{3}\sqrt{x}\left(\sqrt{3}\sin\left(\frac{1}{2}\sqrt{3}\log(x)\right) - 3\cos\left(\frac{1}{2}\sqrt{3}\log(x)\right)\right)$$

12.3 problem 19.1 (iii)

Internal problem ID [10714]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2x'' - 5x't + 10x = 0}$$

With initial conditions

$$[x(1) = 2, x'(1) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve([t^2*diff(x(t),t$2)-5*t*diff(x(t),t)+10*x(t)=0,x(1) = 2, D(x)(1) = 1],x(t), singsol=al
```

$$x(t) = t^3(-5\sin(\ln(t)) + 2\cos(\ln(t)))$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 194

```
DSolve[{t^2*x''[t]-5*t*x[t]+10*x[t]==0,{x[1]==2,x'[1]==1}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{t^{\frac{1}{2}-\frac{i\sqrt{39}}{2}}\left(\left(2\,{}_0F_1(;-i\sqrt{39};5) - {}_0F_1(;1-i\sqrt{39};5)\right)t^{i\sqrt{39}}\,{}_0F_1(;1+i\sqrt{39};5t) + \left(2\,{}_0F_1(;i\sqrt{39};5) - {}_0F_1(;1\right.}{{}_0F_1\left(;1-i\sqrt{39};5\right)\left({}_0F_1\left(;i\sqrt{39};5\right) - {}_0F_1\left(;1+i\sqrt{39};5\right)\right) + {}_0F_1\left(;-i\sqrt{39};5\right)\,{}_0F_1\left(\right.}$$

12.4 problem 19.1 (iv)

Internal problem ID [10715]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (iv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{t^2x'' + x't - x = 0}$$

With initial conditions

$$[x(1) = 1, x'(1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 5

```
dsolve([t^2*diff(x(t),t$2)+t*diff(x(t),t)-x(t)=0,x(1) = 1, D(x)(1) = 1],x(t), singsol=all)
```

$$x(t) = t$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 151

```
DSolve[{t^2*x''[t]+t*x[t]-x[t]==0,{x[1]==1,x'[1]==1}},x[t],t,IncludeSingularSolutions -> True
```

$$x(t) \rightarrow \frac{\sqrt{t}\left(\left(\left(1+\sqrt{5}\right) \text{BesselJ}\left(\sqrt{5},2\right) - 2\,{}_0\tilde{F}_1\left(;\sqrt{5};-1\right)\right) \text{BesselJ}\left(-\sqrt{5},2\sqrt{t}\right) + \left(2\,{}_0\tilde{F}_1\left(;-\sqrt{5};-1\right) + \left(\sqrt{5}-1\right)\right.}{2\,{}_0\tilde{F}_1\left(;-\sqrt{5};-1\right) \text{BesselJ}\left(\sqrt{5},2\right) + \text{BesselJ}\left(-\sqrt{5},2\right)\left(2\sqrt{5}\,\text{BesselJ}\left(\sqrt{5},2\right) - 2\right.}$$

12.5 problem 19.1 (v)

Internal problem ID [10716]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (v).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2z'' + 3z'x + 4z = 0$$

With initial conditions

$$[z(1) = 0, z'(1) = 5]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve([x^2*diff(z(x),x$2)+3*x*diff(z(x),x)+4*z(x)=0,z(1) = 0, D(z)(1) = 5],z(x), singsol=all
```

$$z(x) = \frac{5\sqrt{3}\,\sin\left(\sqrt{3}\,\ln(x)\right)}{3x}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 171

```
DSolve[{x^2*z''[x]+3*x*z[x]+4*z[x]==0,{z[1]==0,z'[1]==5}},z[x],x,IncludeSingularSolutions ->
```

$$z(x) \to \frac{5\sqrt{x}\left(\text{BesselJ}\left(-i\sqrt{15},2\sqrt{3}\right)\text{BesselJ}\left(i\sqrt{15},2\sqrt{3}\sqrt{x}\right) - \text{BesselJ}\left(i\sqrt{15},2\sqrt{3}\right)\text{BesselJ}\left(-i\sqrt{15},2\sqrt{3}\sqrt{\right.}\right.}{{}_0\tilde{F}_1\left(;i\sqrt{15};-3\right)\,{}_0\tilde{F}_1\left(;1-i\sqrt{15};-3\right) - \left({}_0\tilde{F}_1\left(;-i\sqrt{15};-3\right) + i\sqrt{15}\,{}_0\tilde{F}_1\left(;1-i\sqrt{15};-3\right)\right){}_0\tilde{F}_1\left(;1+i\sqrt{\right.}\right.}$$

12.6 problem 19.1 (vi)

Internal problem ID [10717]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (vi).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' - y'x - 3y = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)-3*y(x)=0,y(1) = 1, D(y)(1) = -1],y(x), singsol=all)
```

$$y(x) = \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 151

```
DSolve[{x^2*y''[x]-x*y[x]-3*y[x]==0,{y[1]==1,y'[1]==-1}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{\sqrt{x}\left(\left(\left(\sqrt{13}-3\right) \text{BesselI}\left(\sqrt{13},2\right) - 2\,{}_0\tilde{F}_1\left(;\sqrt{13};1\right)\right) \text{BesselI}\left(-\sqrt{13},2\sqrt{x}\right) + \left(2\,{}_0\tilde{F}_1\left(;-\sqrt{13};1\right) + \left(3+\sqrt{\right.\right.\right.}{2\,{}_0\tilde{F}_1\left(;-\sqrt{13};1\right) \text{BesselI}\left(\sqrt{13},2\right) + \text{BesselI}\left(-\sqrt{13},2\right)\left(2\sqrt{13}\,\text{BesselI}\left(\sqrt{13},2\right)\right.}$$

12.7 problem 19.1 (vii)

Internal problem ID [10718]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (vii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4t^2x'' + 8x't + 5x = 0}$$

With initial conditions

$$[x(1) = 2, x'(1) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve([4*t^2*diff(x(t),t$2)+8*t*diff(x(t),t)+5*x(t)=0,x(1) = 2, D(x)(1) = 0],x(t), singsol=a
```

$$x(t) = \frac{\sin(\ln(t)) + 2\cos(\ln(t))}{\sqrt{t}}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 79

```
DSolve[{4*t^2*x''[t]+8*t*x[t]+5*x[t]==0,{x[1]==2,x'[1]==0}},x[t],t,IncludeSingularSolutions -
```

$$x(t) \to \frac{1}{2} i\pi t^{\frac{1}{2}-i}\text{csch}(\pi)\text{sech}(\pi)\left((2\,{}_0\tilde{F}_1(;-2i;-2) + (1+2i)\,{}_0\tilde{F}_1(;1-2i;-2))t^{2i}\,{}_0\tilde{F}_1(;1+2i;-2t) + (-2\,{}_0\tilde{F}_1(;2i;-2) - (1-2i)\,{}_0\tilde{F}_1(;1+2i;-2))\,{}_0\tilde{F}_1(;1-2i;-2t)\right)$$

12.8 problem 19.1 (viii)

Internal problem ID [10719]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (viii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 5y'x + 5y = 0}$$

With initial conditions

$$[y(1) = -2, y'(1) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+5*y(x)=0,y(1) = -2, D(y)(1) = 1],y(x), singsol=al
```

$$y(x) = \frac{3}{4}x^5 - \frac{11}{4}x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 17

```
DSolve[{x^2*y''[x]-5*x*y'[x]+5*y[x]==0,{y[1]==-2,y'[1]==1}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{4}x(3x^4 - 11)$$

12.9 problem 19.1 (ix)

Internal problem ID [10720]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (ix).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{3x^2z'' + 5z'x - z = 0}$$

With initial conditions

$$[z(1) = 2, z'(1) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([3*x^2*diff(z(x),x$2)+5*x*diff(z(x),x)-z(x)=0,z(1) = 2, D(z)(1) = -1],z(x), singsol=al
```

$$z(x) = \frac{3x^{\frac{4}{3}} + 5}{4x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 21

```
DSolve[{3*x^2*z''[x]+5*x*z'[x]-z[x]==0,{z[1]==2,z'[1]==-1}},z[x],x,IncludeSingularSolutions -
```

$$z(x) \to \frac{3x^{4/3} + 5}{4x}$$

12.10 problem 19.1 (x)

Internal problem ID [10721]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.1 (x).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2x'' + 3x't + 13x = 0}$$

With initial conditions

$$[x(1) = -1, x'(1) = 2]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 32

```
dsolve([t^2*diff(x(t),t$2)+3*t*diff(x(t),t)+13*x(t)=0,x(1) = -1, D(x)(1) = 2],x(t), singsol=a
```

$$x(t) = \frac{\sqrt{3}\,\sin\left(2\sqrt{3}\,\ln\left(t\right)\right) - 6\cos\left(2\sqrt{3}\,\ln\left(t\right)\right)}{6t}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 41

```
DSolve[{t^2*x''[t]+3*t*x'[t]+13*x[t]==0,{x[1]==-1,x'[1]==2}},x[t],t,IncludeSingularSolutions
```

$$x(t) \to \frac{\sqrt{3}\sin\left(2\sqrt{3}\log(t)\right) - 6\cos\left(2\sqrt{3}\log(t)\right)}{6t}$$

12.11 problem 19.2

Internal problem ID [10722]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 19, CauchyEuler equations. Exercises page 174
Problem number: 19.2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{ay'' + (-a+b)\,y' + cy = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve(a*diff(y(z),z$2)+(b-a)*diff(y(z),z)+c*y(z)=0,y(z), singsol=all)
```

$$y(z) = c_1 \mathrm{e}^{\frac{\left(-b+a+\sqrt{a^2-2ba-4ca+b^2}\right)z}{2a}} + c_2 \mathrm{e}^{-\frac{\left(b-a+\sqrt{a^2-2ba-4ca+b^2}\right)z}{2a}}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 66

```
DSolve[a*y''[z]+(b-a)*y'[z]+c*y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z) \to e^{-\frac{z\left(\sqrt{(a-b)^2-4ac}-a+b\right)}{2a}}\left(c_2 e^{\frac{z\sqrt{(a-b)^2-4ac}}{a}} + c_1\right)$$

13 Chapter 20, Series solutions of second order linear equations. Exercises page 195

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13.1 problem 20.1

Internal problem ID [10723]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\, y''-2y'x+n(1+n)\, y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 101

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+n*(n+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{n(n+1)\,x^2}{2}+\frac{n(n^3+2n^2-5n-6)\,x^4}{24}\right)y(0)+\left(x-\frac{(n^2+n-2)\,x^3}{6}+\frac{(n^4+2n^3-13n^2-14n+24)\,x^5}{120}\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 120

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-2*x*y'[x]+n*(n+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{1}{120}(n^2+n)^2x^5+\frac{7}{60}(-n^2-n)\,x^5+\frac{1}{6}(-n^2-n)\,x^3+\frac{x^5}{5}+\frac{x^3}{3}+x\right)+c_1\left(\frac{1}{24}(n^2+n)^2x^4+\frac{1}{4}(-n^2-n)\,x^4+\frac{1}{2}(-n^2-n)\,x^2+1\right)$$

13.2 problem 20.2 (i)

Internal problem ID [10724]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.2 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{24}x^4\right) y(0) + xD(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 27

```
AsymptoticDSolveValue[y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^4}{24} - \frac{x^2}{2} + 1\right) + c_2 x$$

13.3 problem 20.2 (ii)

Internal problem ID [10725]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.2 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(x^2+1)\,y''+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
Order:=6;
dsolve((1+x^2)*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{2}x^2+\frac{1}{8}x^4\right)y(0)+\left(x-\frac{1}{6}x^3+\frac{7}{120}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(1+x^2)*y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{7x^5}{120}-\frac{x^3}{6}+x\right)+c_1\left(\frac{x^4}{8}-\frac{x^2}{2}+1\right)$$

13.4 problem 20.2 (iii)

Internal problem ID [10726]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.2 (iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$2xy'' + y' - 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)+diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{2}{3}x + \frac{2}{15}x^2 + \frac{4}{315}x^3 + \frac{2}{2835}x^4 + \frac{4}{155925}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 + 2x + \frac{2}{3}x^2 + \frac{4}{45}x^3 + \frac{2}{315}x^4 + \frac{4}{14175}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 83

```
AsymptoticDSolveValue[2*x*y''[x]+y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{4x^5}{155925} + \frac{2x^4}{2835} + \frac{4x^3}{315} + \frac{2x^2}{15} + \frac{2x}{3} + 1\right) + c_2\left(\frac{4x^5}{14175} + \frac{2x^4}{315} + \frac{4x^3}{45} + \frac{2x^2}{3} + 2x + 1\right)$$

13.5 problem 20.2 (iv) (k=-2)

Internal problem ID [10727]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.2 (iv) (k=-2).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x - 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
Order:=6;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + 2x^2 + \frac{4}{3}x^4\right) y(0) + \left(x + x^3 + \frac{1}{2}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]-4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{2} + x^3 + x\right) + c_1\left(\frac{4x^4}{3} + 2x^2 + 1\right)$$

13.6 problem 20.2 (iv) (k=2)

Internal problem ID [10728]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.2 (iv) (k=2).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(-2x^2 + 1\right) y(0) + \left(x - \frac{1}{3}x^3 - \frac{1}{30}x^5\right) D(y)\left(0\right) + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(1 - 2x^2\right) + c_2\left(-\frac{x^5}{30} - \frac{x^3}{3} + x\right)$$

13.7 problem 20.3

Internal problem ID [10729]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(1-x)\,y''-3y'x-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
Order:=6;
dsolve(x*(1-x)*diff(y(x),x$2)-3*x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x) = {} & c_1x\left(1+2x+3x^2+4x^3+5x^4+6x^5+\mathrm{O}\left(x^6\right)\right)\\ & +\left(x+2x^2+3x^3+4x^4+5x^5+\mathrm{O}\left(x^6\right)\right)\ln\left(x\right)c_2\\ & +\left(1+3x+5x^2+7x^3+9x^4+11x^5+\mathrm{O}\left(x^6\right)\right)c_2\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 63

```
AsymptoticDSolveValue[x*(1-x)*y''[x]-3*x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(x^4+x^3+x^2+\left(4x^3+3x^2+2x+1\right)x\log(x)+x+1\right)+c_2\left(5x^5+4x^4+3x^3+2x^2+x\right)$$

13.8 problem 20.4

Internal problem ID [10730]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x - x^2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 + \frac{1}{4}x^2 + \frac{1}{64}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-\frac{1}{4}x^2 - \frac{3}{128}x^4 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]-x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{64} + \frac{x^2}{4} + 1\right) + c_2\left(-\frac{3x^4}{128} - \frac{x^2}{4} + \left(\frac{x^4}{64} + \frac{x^2}{4} + 1\right)\log(x)\right)$$

13.9 problem 20.5

Internal problem ID [10731]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$x^2y'' + y'x + (x^2 - 1)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1 - \frac{1}{8}x^2 + \frac{1}{192}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(x^2 - \frac{1}{8}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-2 + \frac{3}{32}x^4 + \mathrm{O}\left(x^6\right)\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{192} - \frac{x^3}{8} + x\right) + c_1\left(\frac{1}{16}x\left(x^2 - 8\right)\log(x) - \frac{5x^4 - 16x^2 - 64}{64x}\right)$$

13.10 problem 20.7

Internal problem ID [10732]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 20, Series solutions of second order linear equations. Exercises page 195
Problem number: 20.7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{x^2y'' + y'x + (-n^2 + x^2)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 77

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-n^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-n}\left(1 + \frac{1}{4n-4}x^2 + \frac{1}{32}\frac{1}{(n-2)(n-1)}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 x^n\left(1 - \frac{1}{4n+4}x^2 + \frac{1}{32}\frac{1}{(n+2)(n+1)}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 160

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-n^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^4}{(-n^2-n+(1-n)(2-n)+2)(-n^2-n+(3-n)(4-n)+4)} - \frac{x^2}{-n^2-n+(1-n)(2-n)+2} + 1\right)x^{-n} + c_1\left(\frac{x^4}{(-n^2+n+(n+1)(n+2)+2)(-n^2+n+(n+3)(n+4)+4)} - \frac{x^2}{-n^2+n+(n+1)(n+2)+2} + 1\right)x^n$$

14 Chapter 26, Explicit solutions of coupled linear systems. Exercises page 257

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14.1 problem 26.1 (i)

Internal problem ID [10733]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 26, Explicit solutions of coupled linear systems. Exercises page 257
Problem number: 26.1 (i).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4x(t) - y(t)$$
$$y'(t) = 2x(t) + y(t) + t^2$$

With initial conditions

$$[x(0) = 0, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 52

```
dsolve([diff(x(t),t) = 4*x(t)-y(t), diff(y(t),t) = 2*x(t)+y(t)+t^2, x(0) = 0, y(0) = 1],[x(t)
```

$$x(t) = -\frac{t^2}{6} + \frac{5\,e^{2t}}{4} - \frac{29\,e^{3t}}{27} - \frac{5t}{18} - \frac{19}{108}$$

$$y(t) = \frac{5\,e^{2t}}{2} - \frac{29\,e^{3t}}{27} - \frac{2t^2}{3} - \frac{7t}{9} - \frac{23}{54}$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 62

```
DSolve[{x'[t]==4*x[t]-y[t],y'[t]==2*x[t]+y[t]+t^2},{x[0]==0,y[0]==1},{x[t],y[t]},t,IncludeSin
```

$$x(t) \to \frac{1}{108}\left(e^{2t}\left(135 - 116e^t\right) - 6t(3t+5) - 19\right)$$
$$y(t) \to \frac{1}{54}\left(e^{2t}\left(135 - 58e^t\right) - 6t(6t+7) - 23\right)$$

14.2 problem 26.1 (ii)

Internal problem ID [10734]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 26, Explicit solutions of coupled linear systems. Exercises page 257
Problem number: 26.1 (ii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - 4y(t) + 2\cos(t)^2 - 1$$
$$y'(t) = x(t) + y(t)$$

With initial conditions

$$[x(0) = 1, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 66

```
dsolve([diff(x(t),t) = x(t)-4*y(t)+cos(2*t), diff(y(t),t) = x(t)+y(t), x(0) = 1, y(0) = 1],[x
```

$$x(t) = \frac{26\,\mathrm{e}^t\cos(2t)}{17} - \frac{32\,\mathrm{e}^t\sin(2t)}{17} - \frac{9\cos(2t)}{17} + \frac{2\sin(2t)}{17}$$

$$y(t) = \frac{13\,\mathrm{e}^t\sin(2t)}{17} + \frac{16\,\mathrm{e}^t\cos(2t)}{17} - \frac{4\sin(2t)}{17} + \frac{\cos(2t)}{17}$$

✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 85

```
DSolve[{x'[t]==x[t]-4*y[t]+cos(2*t),y'[t]==x[t]+y[t]},{x[0]==1,y[0]==1},{x[t],y[t]},t,Include
```

$$x(t) \to \frac{1}{25}\left(2(3-5t)\cos + e^t((25-6\cos)\cos(2t) + 2(4\cos-25)\sin(2t))\right)$$
$$y(t) \to \frac{1}{50}\left(4(5t+2)\cos + e^t((50-8\cos)\cos(2t) + (25-6\cos)\sin(2t))\right)$$

14.3 problem 26.1 (iii)

Internal problem ID [10735]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 26, Explicit solutions of coupled linear systems. Exercises page 257
Problem number: 26.1 (iii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) + 2y(t)$$
$$y'(t) = 6x(t) + 3y(t) + e^t$$

With initial conditions

$$[x(0) = 0, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve([diff(x(t),t) = 2*x(t)+2*y(t), diff(y(t),t) = 6*x(t)+3*y(t)+exp(t), x(0) = 0, y(0) = 1
```

$$x(t) = \frac{12\,e^{6t}}{35} - \frac{e^{-t}}{7} - \frac{e^{t}}{5}$$

$$y(t) = \frac{24\,e^{6t}}{35} + \frac{3\,e^{-t}}{14} + \frac{e^{t}}{10}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 58

```
DSolve[{x'[t]==2*x[t]+2*y[t],y'[t]==6*x[t]+3*y[t]+Exp[t]},{x[0]==0,y[0]==1},{x[t],y[t]},t,Inc
```

$$x(t) \to \frac{1}{35} e^{-t} \left(-7e^{2t} + 12e^{7t} - 5\right)$$
$$y(t) \to \frac{1}{70} e^{-t} \left(7e^{2t} + 48e^{7t} + 15\right)$$

14.4 problem 26.1 (iv)

Internal problem ID [10736]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 26, Explicit solutions of coupled linear systems. Exercises page 257
Problem number: 26.1 (iv).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 5x(t) - 4y(t) + e^{3t}$$
$$y'(t) = x(t) + y(t)$$

With initial conditions

$$[x(0) = 1, y(0) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve([diff(x(t),t) = 5*x(t)-4*y(t)+exp(3*t), diff(y(t),t) = x(t)+y(t), x(0) = 1, y(0) = -1]
```

$$x(t) = e^{3t}(t^2 + 7t + 1)$$

$$y(t) = \frac{e^{3t}(t^2 + 6t - 2)}{2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 37

```
DSolve[{x'[t]==5*x[t]-4*y[t]+Exp[3*t],y'[t]==x[t]+y[t]},{x[0]==1,y[0]==-1},{x[t],y[t]},t,Incl
```

$$x(t) \to e^{3t}(t(t+7) + 1)$$
$$y(t) \to \frac{1}{2}e^{3t}(t(t+6) - 2)$$

14.5 problem 26.1 (v)

Internal problem ID [10737]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 26, Explicit solutions of coupled linear systems. Exercises page 257
Problem number: 26.1 (v).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) + 5y(t)$$
$$y'(t) = -2x(t) + 4\cos(t)^3 - 3\cos(t)$$

With initial conditions

$$[x(0) = 2, y(0) = -1]$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 66

```
dsolve([diff(x(t),t) = 2*x(t)+5*y(t), diff(y(t),t) = -2*x(t)+cos(3*t), x(0) = 2, y(0) = -1],[
```

$$x(t) = -\frac{16\,e^{t}\sin(3t)}{111} + \frac{69\,e^{t}\cos(3t)}{37} - \frac{30\sin(3t)}{37} + \frac{5\cos(3t)}{37}$$

$$y(t) = -\frac{121\,e^{t}\sin(3t)}{111} - \frac{17\,e^{t}\cos(3t)}{37} - \frac{20\cos(3t)}{37} + \frac{9\sin(3t)}{37}$$

✓ Solution by Mathematica

Time used: 0.236 (sec). Leaf size: 70

```
DSolve[{x'[t]==2*x[t]+5*y[t],y'[t]==-2*x[t]+Cos[3*t]},{x[0]==2,y[0]==-1},{x[t],y[t]},t,Includ
```

$$x(t) \to \frac{1}{111}\left(3\left(69e^t + 5\right)\cos(3t) - 2\left(8e^t + 45\right)\sin(3t)\right)$$
$$y(t) \to \frac{1}{111}\left(-\left(121e^t - 27\right)\sin(3t) - 3\left(17e^t + 20\right)\cos(3t)\right)$$

14.6 problem 26.1 (vi)

Internal problem ID [10738]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 26, Explicit solutions of coupled linear systems. Exercises page 257
Problem number: 26.1 (vi).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + y(t) + \mathrm{e}^{-t}$$
$$y'(t) = 4x(t) - 2y(t) + \mathrm{e}^{2t}$$

With initial conditions

$$[x(0) = 1, y(0) = -1]$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 60

```
dsolve([diff(x(t),t) = x(t)+y(t)+exp(-t), diff(y(t),t) = 4*x(t)-2*y(t)+exp(2*t), x(0) = 1, y(
```

$$x(t) = \frac{62\,\mathrm{e}^{2t}}{75} + \frac{\mathrm{e}^{2t}t}{5} + \frac{17\,\mathrm{e}^{-3t}}{50} - \frac{\mathrm{e}^{-t}}{6}$$

$$y(t) = \frac{77\,\mathrm{e}^{2t}}{75} - \frac{34\,\mathrm{e}^{-3t}}{25} + \frac{\mathrm{e}^{2t}t}{5} - \frac{2\,\mathrm{e}^{-t}}{3}$$

✓ Solution by Mathematica

Time used: 0.621 (sec). Leaf size: 67

```
DSolve[{x'[t]==x[t]+y[t]+Exp[-t],y'[t]==4*x[t]-2*y[t]+Exp[2*t]},{x[0]==1,y[0]==-1},{x[t],y[t]
```

$$x(t) \to \frac{1}{150}e^{-3t}\left(2e^{5t}(15t+62) - 25e^{2t} + 51\right)$$
$$y(t) \to \frac{1}{75}e^{-3t}\left(e^{5t}(15t+77) - 50e^{2t} - 102\right)$$

14.7 problem 26.1 (vii)

Internal problem ID [10739]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 26, Explicit solutions of coupled linear systems. Exercises page 257
Problem number: 26.1 (vii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 8x(t) + 14y(t)$$
$$y'(t) = 7x(t) + y(t)$$

With initial conditions

$$[x(0) = 1, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve([diff(x(t),t) = 8*x(t)+14*y(t), diff(y(t),t) = 7*x(t)+y(t), x(0) = 1, y(0) = 1],[x(t),
```

$$x(t) = \frac{4\,\mathrm{e}^{15t}}{3} - \frac{\mathrm{e}^{-6t}}{3}$$

$$y(t) = \frac{2\,\mathrm{e}^{15t}}{3} + \frac{\mathrm{e}^{-6t}}{3}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 44

```
DSolve[{x'[t]==8*x[t]+14*y[t],y'[t]==7*x[t]+y[t]},{x[0]==1,y[0]==1},{x[t],y[t]},t,IncludeSing
```

$$x(t) \to \frac{1}{3}e^{-6t}\left(4e^{21t} - 1\right)$$
$$y(t) \to \frac{1}{3}e^{-6t}\left(2e^{21t} + 1\right)$$

15 Chapter 28, Distinct real eigenvalues. Exercises page 282

15.1 problem 28.2 (i) 137
15.2 problem 28.2 (ii) 138
15.3 problem 28.2 (iii) 139
15.4 problem 28.2 (iv) 140
15.5 problem 28.6 (iii) 141

15.1 problem 28.2 (i)

Internal problem ID [10749]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 28, Distinct real eigenvalues. Exercises page 282
Problem number: 28.2 (i).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 8x(t) + 14y(t)$$
$$y'(t) = 7x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=8*x(t)+14*y(t),diff(y(t),t)=7*x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = 2c_1 e^{15t} - c_2 e^{-6t}$$

$$y(t) = c_1 e^{15t} + c_2 e^{-6t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 65

```
DSolve[{x'[t]==8*x[t]+14*y[t],y'[t]==7*x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to \frac{1}{3}e^{-6t}\left(2(c_1 + c_2)e^{21t} + c_1 - 2c_2\right)$$
$$y(t) \to \frac{1}{3}e^{-6t}\left((c_1 + c_2)e^{21t} - c_1 + 2c_2\right)$$

15.2 problem 28.2 (ii)

Internal problem ID [10750]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 28, Distinct real eigenvalues. Exercises page 282
Problem number: 28.2 (ii).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 2x(t) \\
y'(t) &= -5x(t) - 3y(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve([diff(x(t),t)=2*x(t),diff(y(t),t)=-5*x(t)-3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -c_2 e^{2t}$$

$$y(t) = c_1 e^{-3t} + c_2 e^{2t}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 36

```
DSolve[{x'[t]==2*x[t],y'[t]==-5*x[t]-3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to c_1 e^{2t} \\
y(t) &\to e^{-3t}\left(c_1\left(-e^{5t}\right) + c_1 + c_2\right)
\end{aligned}
$$

15.3 problem 28.2 (iii)

Internal problem ID [10751]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 28, Distinct real eigenvalues. Exercises page 282
Problem number: 28.2 (iii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 11x(t) - 2y(t)$$
$$y'(t) = 3x(t) + 4y(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=11*x(t)-2*y(t),diff(y(t),t)=3*x(t)+4*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = 2c_1 e^{10t} + \frac{c_2 e^{5t}}{3}$$

$$y(t) = c_1 e^{10t} + c_2 e^{5t}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 95

```
DSolve[{x'[t]==2*x[t]-2*y[t],y'[t]==3*x[t]+4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{5}e^{3t}\left(5c_1 \cos\left(\sqrt{5}t\right) - \sqrt{5}(c_1 + 2c_2)\sin\left(\sqrt{5}t\right)\right)$$
$$y(t) \to \frac{1}{5}e^{3t}\left(5c_2 \cos\left(\sqrt{5}t\right) + \sqrt{5}(3c_1 + c_2)\sin\left(\sqrt{5}t\right)\right)$$

15.4 problem 28.2 (iv)

Internal problem ID [10752]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 28, Distinct real eigenvalues. Exercises page 282
Problem number: 28.2 (iv).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + 20y(t)$$
$$y'(t) = 40x(t) - 19y(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=x(t)+20*y(t),diff(y(t),t)=40*x(t)-19*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = c_1 e^{21t} - \frac{c_2 e^{-39t}}{2}$$

$$y(t) = c_1 e^{21t} + c_2 e^{-39t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 68

```
DSolve[{x'[t]==x[t]+20*y[t],y'[t]==40*x[t]-19*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{3} e^{-39t} \left((2c_1 + c_2) e^{60t} + c_1 - c_2 \right)$$
$$y(t) \to \frac{1}{3} e^{-39t} \left((2c_1 + c_2) e^{60t} - 2c_1 + 2c_2 \right)$$

15.5 problem 28.6 (iii)

Internal problem ID [10753]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 28, Distinct real eigenvalues. Exercises page 282
Problem number: 28.6 (iii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -2x(t) + 2y(t)$$
$$y'(t) = x(t) - y(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve([diff(x(t),t)=-2*x(t)+2*y(t),diff(y(t),t)=x(t)-y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -2c_2 e^{-3t} + c_1$$

$$y(t) = c_1 + c_2 e^{-3t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 62

```
DSolve[{x'[t]==-2*x[t]+2*y[t],y'[t]==x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tru
```

$$x(t) \to \frac{1}{3}\left(2(c_1 - c_2)e^{-3t} + c_1 + 2c_2\right)$$
$$y(t) \to \frac{1}{3}e^{-3t}\left((c_1 + 2c_2)e^{3t} - c_1 + c_2\right)$$

16 Chapter 29, Complex eigenvalues. Exercises page 292

16.1 problem 29.3 (i) 143
16.2 problem 29.3 (ii) 144
16.3 problem 29.3 (iii) 145
16.4 problem 29.3 (iv) 146

16.1 problem 29.3 (i)

Internal problem ID [10754]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 29, Complex eigenvalues. Exercises page 292
Problem number: 29.3 (i).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -y(t)$$
$$y'(t) = x(t) - y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 84

```
dsolve([diff(x(t),t)=-y(t),diff(y(t),t)=x(t)-y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^{-\frac{t}{2}} \left(\sin\left(\frac{\sqrt{3}\,t}{2}\right)\sqrt{3}\,c_2 - \cos\left(\frac{\sqrt{3}\,t}{2}\right)\sqrt{3}\,c_1 - \sin\left(\frac{\sqrt{3}\,t}{2}\right)c_1 - \cos\left(\frac{\sqrt{3}\,t}{2}\right)c_2\right)}{2}$$

$$y(t) = e^{-\frac{t}{2}} \left(\sin\left(\frac{\sqrt{3}\,t}{2}\right)c_1 + \cos\left(\frac{\sqrt{3}\,t}{2}\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 112

```
DSolve[{x'[t]==-y[t],y'[t]==x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{3}e^{-t/2}\left(3c_1\cos\left(\frac{\sqrt{3}t}{2}\right) + \sqrt{3}(c_1 - 2c_2)\sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$
$$y(t) \to \frac{1}{3}e^{-t/2}\left(3c_2\cos\left(\frac{\sqrt{3}t}{2}\right) + \sqrt{3}(2c_1 - c_2)\sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$

16.2 problem 29.3 (ii)

Internal problem ID [10755]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 29, Complex eigenvalues. Exercises page 292
Problem number: 29.3 (ii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -2x(t) + 3y(t)$$
$$y'(t) = -6x(t) + 4y(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
dsolve([diff(x(t),t)=-2*x(t)+3*y(t),diff(y(t),t)=-6*x(t)+4*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{e^{t}(\sin(3t) c_1 + \sin(3t) c_2 - \cos(3t) c_1 + \cos(3t) c_2)}{2}$$

$$y(t) = e^{t}(\sin(3t) c_1 + \cos(3t) c_2)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 56

```
DSolve[{x'[t]==-2*x[t]+3*y[t],y'[t]==-6*x[t]+4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -
```

$$x(t) \to e^t(c_1 \cos(3t) + (c_2 - c_1)\sin(3t))$$
$$y(t) \to e^t(c_2 \cos(3t) + (c_2 - 2c_1)\sin(3t))$$

16.3 problem 29.3 (iii)

Internal problem ID [10756]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 29, Complex eigenvalues. Exercises page 292
Problem number: 29.3 (iii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -11x(t) - 2y(t)$$
$$y'(t) = 13x(t) - 9y(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 59

```
dsolve([diff(x(t),t)=-11*x(t)-2*y(t),diff(y(t),t)=13*x(t)-9*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{\mathrm{e}^{-10t}(\sin(5t)\, c_1 + 5\sin(5t)\, c_2 - 5\cos(5t)\, c_1 + \cos(5t)\, c_2)}{13}$$

$$y(t) = \mathrm{e}^{-10t}(\sin(5t)\, c_1 + \cos(5t)\, c_2)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 69

```
DSolve[{x'[t]==-11*x[t]-2*y[t],y'[t]==13*x[t]-9*y[t]},{x[t],y[t]},t,IncludeSingularSolutions
```

$$x(t) \to \frac{1}{5}e^{-10t}(5c_1\cos(5t) - (c_1 + 2c_2)\sin(5t))$$
$$y(t) \to \frac{1}{5}e^{-10t}(5c_2\cos(5t) + (13c_1 + c_2)\sin(5t))$$

16.4 problem 29.3 (iv)

Internal problem ID [10757]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 29, Complex eigenvalues. Exercises page 292
Problem number: 29.3 (iv).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 7x(t) - 5y(t)$$
$$y'(t) = 10x(t) - 3y(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 58

```
dsolve([diff(x(t),t)=7*x(t)-5*y(t),diff(y(t),t)=10*x(t)-3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{e^{2t}(\sin(5t)\,c_1 - \sin(5t)\,c_2 + \cos(5t)\,c_1 + \cos(5t)\,c_2)}{2}$$

$$y(t) = e^{2t}(\sin(5t)\,c_1 + \cos(5t)\,c_2)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 62

```
DSolve[{x'[t]==7*x[t]-5*y[t],y'[t]==10*x[t]-3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to e^{2t}(c_1\cos(5t) + (c_1 - c_2)\sin(5t))$$
$$y(t) \to e^{2t}(c_2\cos(5t) + (2c_1 - c_2)\sin(5t))$$

17 Chapter 30, A repeated real eigenvalue. Exercises page 299

17.1 problem 30.1 (i) 148
17.2 problem 30.1 (ii) 149
17.3 problem 30.1 (iii) 150
17.4 problem 30.1 (iv) 151
17.5 problem 30.1 (v) 153
17.6 problem 30.5 (iii) 154

17.1 problem 30.1 (i)

Internal problem ID [10758]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 30, A repeated real eigenvalue. Exercises page 299
Problem number: 30.1 (i).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 5x(t) - 4y(t)$$
$$y'(t) = x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)=5*x(t)-4*y(t),diff(y(t),t)=x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = e^{3t}(2c_2t + 2c_1 + c_2)$$

$$y(t) = e^{3t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 45

```
DSolve[{x'[t]==5*x[t]-4*y[t],y'[t]==x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True
```

$$x(t) \to e^{3t}(2c_1t - 4c_2t + c_1)$$
$$y(t) \to e^{3t}((c_1 - 2c_2)t + c_2)$$

17.2 problem 30.1 (ii)

Internal problem ID [10759]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 30, A repeated real eigenvalue. Exercises page 299
Problem number: 30.1 (ii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -6x(t) + 2y(t)$$
$$y'(t) = -2x(t) - 2y(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=-6*x(t)+2*y(t),diff(y(t),t)=-2*x(t)-2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{e^{-4t}(2c_2t + 2c_1 - c_2)}{2}$$

$$y(t) = e^{-4t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 46

```
DSolve[{x'[t]==-6*x[t]+2*y[t],y'[t]==-2*x[t]-2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -
```

$$x(t) \to e^{-4t}(-2c_1t + 2c_2t + c_1)$$
$$y(t) \to e^{-4t}(2(c_2 - c_1)t + c_2)$$

17.3 problem 30.1 (iii)

Internal problem ID [10760]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 30, A repeated real eigenvalue. Exercises page 299
Problem number: 30.1 (iii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -3x(t) - y(t)$$
$$y'(t) = x(t) - 5y(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve([diff(x(t),t)=-3*x(t)-y(t),diff(y(t),t)=x(t)-5*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = e^{-4t}(c_2 t + c_1 + c_2)$$

$$y(t) = e^{-4t}(c_2 t + c_1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 44

```
DSolve[{x'[t]==-3*x[t]-y[t],y'[t]==x[t]-5*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tru
```

$$x(t) \to e^{-4t}(c_1(t+1) - c_2 t)$$
$$y(t) \to e^{-4t}((c_1 - c_2)t + c_2)$$

17.4 problem 30.1 (iv)

Internal problem ID [10761]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 30, A repeated real eigenvalue. Exercises page 299
Problem number: 30.1 (iv).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 13x(t)$$
$$y'(t) = 13y(t)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve([diff(x(t),t)=13*x(t),diff(y(t),t)=13*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = c_1 e^{13t}$$

$$y(t) = c_2 e^{13t}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 65

```
DSolve[{x'[t]==13*x[t],y'[t]==13*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{13t}$$
$$y(t) \to c_2 e^{13t}$$
$$x(t) \to c_1 e^{13t}$$
$$y(t) \to 0$$
$$x(t) \to 0$$
$$y(t) \to c_2 e^{13t}$$
$$x(t) \to 0$$
$$y(t) \to 0$$

17.5 problem 30.1 (v)

Internal problem ID [10762]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 30, A repeated real eigenvalue. Exercises page 299
Problem number: 30.1 (v).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 7x(t) - 4y(t)$$
$$y'(t) = x(t) + 3y(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)=7*x(t)-4*y(t),diff(y(t),t)=x(t)+3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = e^{5t}(2c_2t + 2c_1 + c_2)$$

$$y(t) = e^{5t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 45

```
DSolve[{x'[t]==7*x[t]-4*y[t],y'[t]==x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$x(t) \to e^{5t}(2c_1t - 4c_2t + c_1)$$
$$y(t) \to e^{5t}((c_1 - 2c_2)t + c_2)$$

17.6 problem 30.5 (iii)

Internal problem ID [10763]

Book: AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS by JAMES C. ROBINSON. Cambridge University Press 2004
Section: Chapter 30, A repeated real eigenvalue. Exercises page 299
Problem number: 30.5 (iii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t) + y(t)$$
$$y'(t) = -x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve([diff(x(t),t)=-x(t)+y(t),diff(y(t),t)=-x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = c_1 t - c_1 + c_2$$

$$y(t) = c_1 t + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 32

```
DSolve[{x'[t]==-x[t]+y[t],y'[t]==-x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1(-t) + c_2 t + c_1$$
$$y(t) \to (c_2 - c_1)t + c_2$$