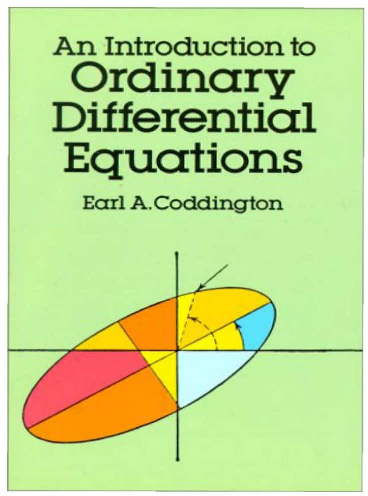
A Solution Manual For

An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961



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1 Chapter 1.3 Introduction– Linear equations of First Order. Page 38

1.1 problem 1 (a)

Internal problem ID [5158]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 1 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - e^{3x} - \sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=exp(3*x)+sin(x),y(x), singsol=all)
```

$$y(x) = \frac{e^{3x}}{3} - \cos(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 21

```
DSolve[y'[x]==Exp[3*x]+Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{3x}}{3} - \cos(x) + c_1$$

1.2 problem 1 (b)

Internal problem ID [5159]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 1 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' - 2 - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)=2+x,y(x), singsol=all)
```

$$y(x) = \frac{1}{6}x^3 + x^2 + c_1 x + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''[x]==2+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{6} + x^2 + c_2 x + c_1$$

1.3 problem 1 (d)

Internal problem ID [5160]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 1 (d).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _quadrature]]`

$$\boxed{y''' - x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$3)=x^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{60}x^5 + \frac{1}{2}c_1x^2 + xc_2 + c_3$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 25

```
DSolve[y'''[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^5}{60} + c_3x^2 + c_2x + c_1$$

1.4 problem 2 (a)

Internal problem ID [5161]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 2 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + y\cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)+cos(x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 19

```
DSolve[y'[x]+Cos[x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\sin(x)}$$
$$y(x) \to 0$$

1.5 problem 2 (b)

Internal problem ID [5162]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 2 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cos(x) - \cos(x)\sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+cos(x)*y(x)=sin(x)*cos(x),y(x), singsol=all)
```

$$y(x) = \sin(x) - 1 + c_1 \mathrm{e}^{-\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 18

```
DSolve[y'[x]+Cos[x]*y[x]==Sin[x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1 e^{-\sin(x)} - 1$$

1.6 problem 2 (c)

Internal problem ID [5163]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 2 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + e^{x}c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1e^{x} + c_2e^{-x}$$

1.7 problem 2 (f)

Internal problem ID [5164]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 2 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(2x) + c_2 \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[y''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2x) + c_2 \sin(2x)$$

1.8 problem 2 (h)

Internal problem ID [5165]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 2 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + k^2 y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+k^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(kx) + c_2 \cos(kx)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y''[x]+k^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(kx) + c_2 \sin(kx)$$

1.9 problem 3(a)

Internal problem ID [5166]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 3(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + 5y - 2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+5*y(x)=2,y(x), singsol=all)
```

$$y(x) = \frac{2}{5} + e^{-5x}c_1$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 24

```
DSolve[y'[x]+5*y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{5} + c_1e^{-5x}$$
$$y(x) \to \frac{2}{5}$$

1.10 problem 4(a)

Internal problem ID [5167]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 4(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' - 1 - 3x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)=3*x+1,y(x), singsol=all)
```

$$y(x) = \frac{1}{2}x^3 + \frac{1}{2}x^2 + c_1x + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 25

```
DSolve[y''[x]==3*x+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x^3 + x^2 + 2c_2x + 2c_1\right)$$

1.11 problem 5(a)

Internal problem ID [5168]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.3 Introduction– Linear equations of First Order. Page 38
Problem number: 5(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - ky = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=k*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{kx}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 18

```
DSolve[y'[x]==k*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{kx}$$
$$y(x) \to 0$$

2 Chapter 1.6 Introduction– Linear equations of First Order. Page 41

2.1 problem 1(a)

Internal problem ID [5169]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 2y - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)-2*y(x)=1,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2} + e^{2x}c_1$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 24

```
DSolve[y'[x]-2*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2} + c_1 e^{2x}$$

$$y(x) \to -\frac{1}{2}$$

2.2 problem 1(b)

Internal problem ID [5170]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 1(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y - e^x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \frac{e^x}{2} + e^{-x} c_1$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 21

```
DSolve[y'[x]+y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x}{2} + c_1 e^{-x}$$

2.3 problem 1(c)

Internal problem ID [5171]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 1(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - 2y - x^2 - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)-2*y(x)=x^2+x,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{2} - x - \frac{1}{2} + e^{2x}c_1$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 23

```
DSolve[y'[x]-2*y[x]==x^2+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}(x+1)^2 + c_1 e^{2x}$$

2.4 problem 1(d)

Internal problem ID [5172]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 1(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y + 3y' - 2\,e^{-x} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(3*diff(y(x),x)+y(x)=2*exp(-x),y(x), singsol=all)
```

$$y(x) = -e^{-x} + e^{-\frac{x}{3}} c_1$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 23

```
DSolve[3*y'[x]+y[x]==2*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(-1 + c_1 e^{2x/3}\right)$$

2.5 problem 1(e)

Internal problem ID [5173]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 1(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 3y - e^{ix} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)+3*y(x)=exp(I*x),y(x), singsol=all)
```

$$y(x) = \left(\left(\frac{3}{10} - \frac{i}{10}\right) e^{(3+i)x} + c_1\right) e^{-3x}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 29

```
DSolve[y'[x]+3*y[x]==Exp[I*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{3}{10} - \frac{i}{10}\right) e^{ix} + c_1 e^{-3x}$$

2.6 problem 2

Internal problem ID [5174]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + iy - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+I*y(x)=x,y(x), singsol=all)
```

$$y(x) = -ix + 1 + \mathrm{e}^{-ix} c_1$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 22

```
DSolve[y'[x]+I*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -ix + c_1 e^{-ix} + 1$$

2.7 problem 3

Internal problem ID [5175]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{Ly' + Ry - E = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(L*diff(y(x),x)+R*y(x)=E,y(x), singsol=all)
```

$$y(x) = \frac{E}{R} + \mathrm{e}^{-\frac{Rx}{L}} c_1$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 23

```
DSolve[L*y'[x]+R*y[x]==E0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\mathrm{E0} - \mathrm{E0}e^{-\frac{Rx}{L}}}{R}$$

2.8 problem 4

Internal problem ID [5176]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{Ly' + Ry - E\sin(\omega x) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
dsolve([L*diff(y(x),x)+R*y(x)=E*sin(omega*x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{E\left(L\cos(\omega x)\,\omega - \mathrm{e}^{-\frac{Rx}{L}}L\omega - \sin(\omega x)\,R\right)}{\omega^2L^2 + R^2}$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 47

```
DSolve[{L*y'[x]+R*y[x]==E0*Sin[\[Omega]*x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{\mathrm{E0}\left(L\omega e^{-\frac{Rx}{L}} - L\omega\cos(x\omega) + R\sin(x\omega)\right)}{L^2\omega^2 + R^2}$$

2.9 problem 5

Internal problem ID [5177]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{Ly' + Ry - E\,\mathrm{e}^{i\omega x} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 38

```
dsolve([L*diff(y(x),x)+R*y(x)=E*exp(I*omega*x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{E\left(\mathrm{e}^{\frac{x(iL\omega+R)}{L}} - 1\right)\mathrm{e}^{-\frac{Rx}{L}}}{iL\omega + R}$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 43

```
DSolve[{L*y'[x]+R*y[x]==E0*Exp[I*\[Omega]*x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{\mathrm{E0}e^{-\frac{Rx}{L}}\left(-1 + e^{\frac{x(R+iL\omega)}{L}}\right)}{R + iL\omega}$$

2.10 problem 7

Internal problem ID [5178]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1.6 Introduction– Linear equations of First Order. Page 41
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + ya - b(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)+a*y(x)=b(x),y(x), singsol=all)
```

$$y(x) = \left(\int b(x)\,\mathrm{e}^{ax}dx + c_1\right)\mathrm{e}^{-ax}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 32

```
DSolve[y'[x]+a*y[x]==b[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-ax}\left(\int_1^x e^{aK[1]}b(K[1])dK[1] + c_1\right)$$

3 Chapter 1. Introduction– Linear equations of First Order. Page 45

3.1 problem 1(a)

Internal problem ID [5179]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + 2xy - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+2*x*y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{1}{2} + c_1 \mathrm{e}^{-x^2}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 26

```
DSolve[y'[x]+2*x*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} + c_1 e^{-x^2}$$
$$y(x) \to \frac{1}{2}$$

3.2 problem 1(b)

Internal problem ID [5180]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 1(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y - 3x^3 + 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*diff(y(x),x)+y(x)=3*x^3-1,y(x), singsol=all)
```

$$y(x) = \frac{\frac{3}{4}x^4 - x + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 20

```
DSolve[x*y'[x]+y[x]==3*x^3-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x^3}{4} + \frac{c_1}{x} - 1$$

3.3 problem 1(c)

Internal problem ID [5181]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 1(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y' + e^x y - 3\,e^x = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)+exp(x)*y(x)=3*exp(x),y(x), singsol=all)
```

$$y(x) = 3 + e^{-e^x} c_1$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 22

```
DSolve[y'[x]+Exp[x]*y[x]==3*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3 + c_1 e^{-e^x}$$
$$y(x) \to 3$$

3.4 problem 1(d)

Internal problem ID [5182]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 1(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - y\tan(x) - e^{\sin(x)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)-tan(x)*y(x)=exp(sin(x)),y(x), singsol=all)
```

$$y(x) = \frac{e^{\sin(x)} + c_1}{\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.155 (sec). Leaf size: 15

```
DSolve[y'[x]-Tan[x]*y[x]==Exp[Sin[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sec(x)\left(e^{\sin(x)} + c_1\right)$$

3.5 problem 1(e)

Internal problem ID [5183]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 1(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + 2xy - x\,\mathrm{e}^{-x^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)+2*x*y(x)=x*exp(-x^2),y(x), singsol=all)
```

$$y(x) = \left(\frac{x^2}{2} + c_1\right)\mathrm{e}^{-x^2}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 24

```
DSolve[y'[x]+2*x*y[x]==x*Exp[-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x^2}\left(x^2 + 2c_1\right)$$

3.6 problem 2

Internal problem ID [5184]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cos(x) - \mathrm{e}^{-\sin(x)} = 0}$$

With initial conditions

$$[y(\pi) = \pi]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)+cos(x)*y(x)=exp(-sin(x)),y(Pi) = Pi],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\sin(x)}x$$

✓ Solution by Mathematica

Time used: 0.133 (sec). Leaf size: 13

```
DSolve[{y'[x]+Cos[x]*y[x]==Exp[-Sin[x]],{y[Pi]==Pi}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to xe^{-\sin(x)}$$

3.7 problem 3

Internal problem ID [5185]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x^2y' + 2xy - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x^2*diff(y(x),x)+2*x*y(x)=1,y(x), singsol=all)
```

$$y(x) = \frac{x + c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 13

```
DSolve[x^2*y'[x]+2*x*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x + c_1}{x^2}$$

3.8 problem 8

Internal problem ID [5186]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 2y - b(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)+2*y(x)=b(x),y(x), singsol=all)
```

$$y(x) = \left(\int b(x)\,\mathrm{e}^{2x}dx + c_1\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 31

```
DSolve[y'[x]+2*y[x]==b[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(\int_1^x e^{2K[1]}b(K[1])dK[1] + c_1\right)$$

3.9 problem 14(a)

Internal problem ID [5187]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 14(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 1 - y = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 8

```
dsolve([diff(y(x),x)=1+y(x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = e^x - 1$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 10

```
DSolve[{y'[x]==1+y[x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x - 1$$

3.10 problem 14(b)

Internal problem ID [5188]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 14(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 1 - y^2 = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 6

```
dsolve([diff(y(x),x)=1+y(x)^2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \tan(x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 7

```
DSolve[{y'[x]==1+y[x]^2,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(x)$$

3.11 problem 14(b)

Internal problem ID [5189]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 1. Introduction– Linear equations of First Order. Page 45
Problem number: 14(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 1 - y^2 = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 6

```
dsolve([diff(y(x),x)=1+y(x)^2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \tan(x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 7

```
DSolve[{y'[x]==1+y[x]^2,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(x)$$

4 Chapter 2. Linear equations with constant coefficients. Page 52

4.1 problem 1(a)

Internal problem ID [5190]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{2x}c_1 + c_2e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(c_1e^{4x} + c_2)$$

4.2 problem 1(b)

Internal problem ID [5191]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{3y'' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(3*diff(y(x),x$2)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\frac{\sqrt{6}\,x}{3}\right) + c_2 \cos\left(\frac{\sqrt{6}\,x}{3}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 32

```
DSolve[3*y''[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\sqrt{\frac{2}{3}}x\right) + c_2 \sin\left(\sqrt{\frac{2}{3}}x\right)$$

4.3 problem 1(c)

Internal problem ID [5192]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(4x) + c_2 \cos(4x)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y''[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(4x) + c_2 \sin(4x)$$

4.4 problem 1(d)

Internal problem ID [5193]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve(diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 x + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x + c_1$$

4.5 problem 1(e)

Internal problem ID [5194]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2iy' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+2*I*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-ix} \sin\left(\sqrt{2}\, x\right) + c_2 \mathrm{e}^{-ix} \cos\left(\sqrt{2}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 38

```
DSolve[y''[x]+2*I*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-i\left(1+\sqrt{2}\right)x}\left(c_2 e^{2i\sqrt{2}x} + c_1\right)$$

4.6 problem 1(f)

Internal problem ID [5195]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 5y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) e^{2x} + c_2 \cos(x) e^{2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[y''[x]-4*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(c_2 \cos(x) + c_1 \sin(x))$$

4.7 problem 1(g)

Internal problem ID [5196]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 1(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + (-1+3i)\,y' - 3iy = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)+(3*I-1)*diff(y(x),x)-3*I*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-3ix} + \mathrm{e}^{x} c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[y''[x]+(3*I-1)*y'[x]-3*I*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-3ix} + c_2 e^{x}$$

4.8 problem 2(a)

Internal problem ID [5197]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 2(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 6y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)+diff(y(x),x)-6*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(3\,e^{5x} + 2)\,e^{-3x}}{5}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 23

```
DSolve[{y''[x]+y'[x]-6*y[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{5}e^{-3x}\left(3e^{5x} + 2\right)$$

4.9 problem 2(b)

Internal problem ID [5198]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 2(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 6y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)+diff(y(x),x)-6*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{(e^{5x} - 1)\, e^{-3x}}{5}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 21

```
DSolve[{y''[x]+y'[x]-6*y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{5} e^{-3x} \left(e^{5x} - 1\right)$$

4.10 problem 3(a)

Internal problem ID [5199]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 3(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With initial conditions

$$\left[y(0) = 1, y\left(\frac{\pi}{2}\right) = 2\right]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)+y(x)=0,y(0) = 1, y(1/2*Pi) = 2],y(x), singsol=all)
```

$$y(x) = 2\sin(x) + \cos(x)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[{y''[x]+y[x]==0,{y[0]==1,y[Pi/2]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\sin(x) + \cos(x)$$

4.11 problem 3(b)

Internal problem ID [5200]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 3(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With initial conditions

$$[y(0) = 0, y(\pi) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 8

```
dsolve([diff(y(x),x$2)+y(x)=0,y(0) = 0, y(Pi) = 0],y(x), singsol=all)
```

$$y(x) = c_1 \sin(x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 10

```
DSolve[{y''[x]+y[x]==0,{y[0]==0,y[Pi]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \sin(x)$$

4.12 problem 3(c)

Internal problem ID [5201]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 3(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With initial conditions

$$\left[y(0) = 0, y'\left(\frac{\pi}{2}\right) = 0\right]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 8

```
dsolve([diff(y(x),x$2)+y(x)=0,y(0) = 0, D(y)(1/2*Pi) = 0],y(x), singsol=all)
```

$$y(x) = c_1 \sin(x)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 10

```
DSolve[{y''[x]+y[x]==0,{y[0]==0,y'[Pi/2]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \sin(x)$$

4.13 problem 3(d)

Internal problem ID [5202]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 52
Problem number: 3(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With initial conditions

$$\left[y(0) = 0, y\left(\frac{\pi}{2}\right) = 0\right]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$2)+y(x)=0,y(0) = 0, y(1/2*Pi) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 6

```
DSolve[{y''[x]+y[x]==0,{y[0]==0,y[Pi/2]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

5 Chapter 2. Linear equations with constant coefficients. Page 59

5.1 problem 1(a)

Internal problem ID [5203]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 59
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' - 3y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{e^{3x}}{4} - \frac{e^{-x}}{4}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 13

```
DSolve[{y''[x]-2*y'[x]-3*y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x \sinh(x) \cosh(x)$$

5.2 problem 1(b)

Internal problem ID [5204]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 59
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + (1 + 4i)\, y' + y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$2)+(4*I+1)*diff(y(x),x)+y(x)=0,y(0) = 0, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 6

```
DSolve[{y''[x]+(4*I+1)*y'[x]+y[x]==0,{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to 0$$

5.3 problem 1(c)

Internal problem ID [5205]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 59
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + (-1+3i)\,y' - 3iy = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(y(x),x$2)+(3*I-1)*diff(y(x),x)-3*I*y(x)=0,y(0) = 2, D(y)(0) = 0],y(x), singsol=a
```

$$y(x) = \left(\frac{1}{5} - \frac{3i}{5}\right) \mathrm{e}^{-3ix} + \left(\frac{9}{5} + \frac{3i}{5}\right) \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 31

```
DSolve[{y''[x]+(3*I-1)*y'[x]-3*I*y[x]==0,{y[0]==2,y'[0]==0}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{5} e^{-3ix} \left((9+3i)e^{(1+3i)x} + (1-3i)\right)$$

5.4 problem 1(d)

Internal problem ID [5206]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 59
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 10y = 0}$$

With initial conditions

$$[y(0) = \pi, y'(0) = \pi^2]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 27

```
dsolve([diff(y(x),x$2)+10*y(x)=0,y(0) = Pi, D(y)(0) = Pi^2],y(x), singsol=all)
```

$$y(x) = \frac{\pi\left(\pi\sqrt{10}\,\sin\left(\sqrt{10}\,x\right) + 10\cos\left(\sqrt{10}\,x\right)\right)}{10}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 33

```
DSolve[{y''[x]+10*y[x]==0,{y[0]==Pi,y'[0]==Pi^2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\pi^2 \sin\left(\sqrt{10}x\right)}{\sqrt{10}} + \pi\cos\left(\sqrt{10}x\right)$$

6 Chapter 2. Linear equations with constant coefficients. Page 69

6.1 problem 1(a)

Internal problem ID [5207]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y - \cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+4*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \sin(2x)\, c_2 + \cos(2x)\, c_1 + \frac{\cos(x)}{3}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 26

```
DSolve[y''[x]+4*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cos(x)}{3} + c_1 \cos(2x) + c_2 \sin(2x)$$

6.2 problem 1(b)

Internal problem ID [5208]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y - \sin(3x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+9*y(x)=sin(3*x),y(x), singsol=all)
```

$$y(x) = \sin(3x)\, c_2 + \cos(3x)\, c_1 - \frac{\cos(3x)\, x}{6}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 33

```
DSolve[y''[x]+9*y[x]==Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{x}{6} + c_1\right)\cos(3x) + \frac{1}{36}(1 + 36c_2)\sin(3x)$$

6.3 problem 1(c)

Internal problem ID [5209]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \tan(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=tan(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 - \cos(x) \ln(\sec(x) + \tan(x))$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)(-\text{arctanh}(\sin(x)) + c_1) + c_2 \sin(x)$$

6.4 problem 1(d)

Internal problem ID [5210]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2iy' + y - x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)+2*I*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-ix} \sin\left(\sqrt{2}\,x\right) c_2 + \mathrm{e}^{-ix} \cos\left(\sqrt{2}\,x\right) c_1 - 2i + x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 43

```
DSolve[y''[x]+2*I*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + e^{-i\left(1+\sqrt{2}\right)x}\left(c_2 e^{2i\sqrt{2}x} + c_1\right) - 2i$$

6.5 problem 1(e)

Internal problem ID [5211]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 5y - 3\,\mathrm{e}^{-x} - 2x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=3*exp(-x)+2*x^2,y(x), singsol=all)
```

$$y(x) = \sin(x)\,\mathrm{e}^{2x} c_2 + \cos(x)\,\mathrm{e}^{2x} c_1 + \frac{3\,\mathrm{e}^{-x}}{10} + \frac{2x^2}{5} + \frac{16x}{25} + \frac{44}{125}$$

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 46

```
DSolve[y''[x]-4*y'[x]+5*y[x]==3*Exp[-x]+2*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{125}(5x(5x+8)+22) + \frac{3e^{-x}}{10} + e^{2x}(c_2\cos(x) + c_1\sin(x))$$

6.6 problem 1(f)

Internal problem ID [5212]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 7y' + 6y - \sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-7*diff(y(x),x)+6*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{6x} + \mathrm{e}^{x} c_1 + \frac{7\cos(x)}{74} + \frac{5\sin(x)}{74}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 32

```
DSolve[y''[x]-7*y'[x]+6*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{5\sin(x)}{74} + \frac{7\cos(x)}{74} + c_1 e^{x} + c_2 e^{6x}$$

6.7 problem 1(g)

Internal problem ID [5213]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - 2\sin(2x)\sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+y(x)=2*sin(x)*sin(2*x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + \frac{\sin(x)(-\cos(x)\sin(x) + x)}{2}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 33

```
DSolve[y''[x]+y[x]==2*Sin[x]*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}(\cos(3x) + (-1 + 8c_1)\cos(x) + 4(x + 2c_2)\sin(x))$$

6.8 problem 1(h)

Internal problem ID [5214]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \sec(x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+y(x)=sec(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 - \ln(\sec(x)) \cos(x) + \sin(x) x$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_2) \sin(x) + \cos(x)(\log(\cos(x)) + c_1)$$

6.9 problem 1(i)

Internal problem ID [5215]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y'' - y - e^x = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(4*diff(y(x),x$2)-y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = e^{-\frac{x}{2}}c_2 + e^{\frac{x}{2}}c_1 + \frac{e^x}{3}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 33

```
DSolve[4*y''[x]-y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x}{3} + c_1 e^{x/2} + c_2 e^{-x/2}$$

6.10 problem 1(j)

Internal problem ID [5216]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 1(j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{6y'' + 5y' - 6y - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(6*diff(y(x),x$2)+5*diff(y(x),x)-6*y(x)=x,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{3x}{2}} c_2 + \mathrm{e}^{\frac{2x}{3}} c_1 - \frac{x}{6} - \frac{5}{36}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 34

```
DSolve[6*y''[x]+5*y'[x]-6*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{6} + c_1 e^{2x/3} + c_2 e^{-3x/2} - \frac{5}{36}$$

6.11 problem 4(c)

Internal problem ID [5217]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 69
Problem number: 4(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + \omega^2 y - A\cos(\omega x) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)+omega^2*y(x)=A*cos(omega*x),y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\sin(\omega x)(Ax + 2)}{2\omega}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 21

```
DSolve[{y''[x]+\[Omega]^2*y[x]==A*Cos[\[Omega]*x],{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularS
```

$$y(x) \to \frac{(Ax + 2)\sin(x\omega)}{2\omega}$$

7 Chapter 2. Linear equations with constant coefficients. Page 74

7.1 problem 4(a)

Internal problem ID [5218]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 74
Problem number: 4(a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$3)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x} c_1 + c_2 \mathrm{e}^{-x} \sin\left(\sqrt{3}\, x\right) + c_3 \mathrm{e}^{-x} \cos\left(\sqrt{3}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[y'''[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_1 e^{3x} + c_2 \cos\left(\sqrt{3}x\right) + c_3 \sin\left(\sqrt{3}x\right)\right)$$

7.2 problem 4(b)

Internal problem ID [5219]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 74
Problem number: 4(b).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 65

```
dsolve(diff(y(x),x$4)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = -c_1 e^{-\sqrt{2}\,x} \sin\left(\sqrt{2}\,x\right) - c_2 e^{\sqrt{2}\,x} \sin\left(\sqrt{2}\,x\right) + c_3 e^{-\sqrt{2}\,x} \cos\left(\sqrt{2}\,x\right) + c_4 e^{\sqrt{2}\,x} \cos\left(\sqrt{2}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 67

```
DSolve[y''''[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\sqrt{2}x}\left(\left(c_1 e^{2\sqrt{2}x} + c_2\right)\cos\left(\sqrt{2}x\right) + \left(c_4 e^{2\sqrt{2}x} + c_3\right)\sin\left(\sqrt{2}x\right)\right)$$

7.3 problem 4(c)

Internal problem ID [5220]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 74
Problem number: 4(c).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 5y'' + 6y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$3)-5*diff(y(x),x$2)+6*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + e^{2x}c_2 + c_3e^{3x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 29

```
DSolve[y'''[x]-5*y''[x]+6*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}e^{2x}(2c_2e^x + 3c_1) + c_3$$

7.4 problem 4(d)

Internal problem ID [5221]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 74
Problem number: 4(d).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - iy'' + 4y' - 4iy = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$3)-I*diff(y(x),x$2)+4*diff(y(x),x)-4*I*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2ix} + c_2 e^{ix} + c_3 e^{-2ix}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 36

```
DSolve[y'''[x]-I*y''[x]+4*y'[x]-4*I*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2ix}\left(c_2 e^{4ix} + c_3 e^{3ix} + c_1\right)$$

7.5 problem 4(f)

Internal problem ID [5222]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 74
Problem number: 4(f).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 5y'' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$4)+5*diff(y(x),x$2)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(2x) + c_2 \cos(2x) + c_3 \sin(x) + c_4 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y''''[x]+5*y''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2x) + c_4 \sin(x) + \cos(x)(2c_2 \sin(x) + c_3)$$

7.6 problem 4(g)

Internal problem ID [5223]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 74
Problem number: 4(g).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)-16*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{2x}c_1 + c_2 e^{-2x} + c_3 \sin(2x) + c_4 \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 36

```
DSolve[y''''[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2x} + c_3 e^{-2x} + c_2 \cos(2x) + c_4 \sin(2x)$$

7.7 problem 4(h)

Internal problem ID [5224]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 74
Problem number: 4(h).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x}c_1 + c_2\mathrm{e}^{-x} + c_3\mathrm{e}^{-x}x$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 26

```
DSolve[y'''[x]-3*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_2x + c_3e^{3x} + c_1\right)$$

7.8 problem 4(i)

Internal problem ID [5225]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 74
Problem number: 4(i).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3iy'' - 3y' + iy = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$3)-3*I*diff(y(x),x$2)-3*diff(y(x),x)+I*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{ix} + c_2 e^{ix} x + c_3 e^{ix} x^2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[y'''[x]-3*I*y''[x]-3*y'[x]+I*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{ix}(x(c_3 x + c_2) + c_1)$$

8 Chapter 2. Linear equations with constant coefficients. Page 79

8.1 problem 1(c)

Internal problem ID [5226]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 79
Problem number: 1(c).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 4y' = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$3)-4*diff(y(x),x)=0,y(0) = 0, D(y)(0) = 1, (D@@2)(y)(0) = 0],y(x), singso
```

$$y(x) = \frac{e^{2x}}{4} - \frac{e^{-2x}}{4}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 69

```
DSolve[{y'''[x]-4*y[x]==0,{y[0]==0,y'[0]==1,y''[0]==0}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{e^{-\frac{x}{\sqrt[3]{2}}}\left(e^{\frac{3x}{\sqrt[3]{2}}} + \sqrt{3}\sin\left(\frac{\sqrt{3}x}{\sqrt[3]{2}}\right) - \cos\left(\frac{\sqrt{3}x}{\sqrt[3]{2}}\right)\right)}{3\ 2^{2/3}}$$

8.2 problem 2(c)

Internal problem ID [5227]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 79
Problem number: 2(c).
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} - y'''' - y' + y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0, y''(0) = 0, y'''(0) = 0, y''''(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 28

```
dsolve([diff(y(x),x$5)-diff(y(x),x$4)-diff(y(x),x)+y(x)=0,y(0) = 1, D(y)(0) = 0, (D@@2)(y)(0)
```

$$y(x) = \frac{e^{-x}}{8} + \frac{(-2x+5)\,e^{x}}{8} + \frac{\cos(x)}{4} - \frac{\sin(x)}{4}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 32

```
DSolve[{y'''''[x]-y''''[x]-y'[x]+y[x]==0,{y[0]==1,y'[0]==0,y''[0]==0,y'''[0]==0,y''''[0]==0}}
```

$$y(x) \to \frac{1}{8}\left(e^{x}(5-2x) + e^{-x} - 2\sin(x) + 2\cos(x)\right)$$

9 Chapter 2. Linear equations with constant coefficients. Page 83

9.1 problem 1(a)

Internal problem ID [5228]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + c_2 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 16

```
DSolve[y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + c_2 \sin(x)$$

9.2 problem 1(b)

Internal problem ID [5229]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x} c_1 + e^{x} c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_2 e^{-x}$$

9.3 problem 1(c)

Internal problem ID [5230]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 1(c).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$4)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + \mathrm{e}^{x} c_2 + c_3 \sin(x) + c_4 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 30

```
DSolve[y''''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_3 e^{-x} + c_2 \cos(x) + c_4 \sin(x)$$

9.4 problem 1(d)

Internal problem ID [5231]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 1(d).
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 369

```
dsolve(diff(y(x),x$5)+2*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) = {} & c_1 \mathrm{e}^{\left(\frac{2^{\frac{1}{5}}\cos\left(\frac{\pi}{5}\right)\sqrt{5}}{4}-\frac{\cos\left(\frac{\pi}{5}\right)2^{\frac{1}{5}}}{4}+\frac{2^{\frac{7}{10}}\sqrt{5+\sqrt{5}}\,\sin\left(\frac{\pi}{5}\right)}{4}-\frac{i2^{\frac{7}{10}}\sqrt{5+\sqrt{5}}\,\cos\left(\frac{\pi}{5}\right)}{4}+\frac{i2^{\frac{1}{5}}\sin\left(\frac{\pi}{5}\right)\sqrt{5}}{4}-\frac{i\sin\left(\frac{\pi}{5}\right)2^{\frac{1}{5}}}{4}\right)x} \\ & + c_2 \mathrm{e}^{\left(-\frac{2^{\frac{1}{5}}\cos\left(\frac{\pi}{5}\right)\sqrt{5}}{4}-\frac{\cos\left(\frac{\pi}{5}\right)2^{\frac{1}{5}}}{4}+\frac{2^{\frac{7}{10}}\sqrt{5-\sqrt{5}}\,\sin\left(\frac{\pi}{5}\right)}{4}-\frac{i2^{\frac{7}{10}}\sqrt{5-\sqrt{5}}\,\cos\left(\frac{\pi}{5}\right)}{4}-\frac{i2^{\frac{1}{5}}\sin\left(\frac{\pi}{5}\right)\sqrt{5}}{4}-\frac{i\sin\left(\frac{\pi}{5}\right)2^{\frac{1}{5}}}{4}\right)x} \\ & + c_3 \mathrm{e}^{\left(-\frac{2^{\frac{1}{5}}\cos\left(\frac{\pi}{5}\right)\sqrt{5}}{4}-\frac{\cos\left(\frac{\pi}{5}\right)2^{\frac{1}{5}}}{4}-\frac{2^{\frac{7}{10}}\sqrt{5-\sqrt{5}}\,\sin\left(\frac{\pi}{5}\right)}{4}+\frac{i2^{\frac{7}{10}}\sqrt{5-\sqrt{5}}\,\cos\left(\frac{\pi}{5}\right)}{4}-\frac{i2^{\frac{1}{5}}\sin\left(\frac{\pi}{5}\right)\sqrt{5}}{4}-\frac{i\sin\left(\frac{\pi}{5}\right)2^{\frac{1}{5}}}{4}\right)x} \\ & + c_4 \mathrm{e}^{\left(\frac{2^{\frac{1}{5}}\cos\left(\frac{\pi}{5}\right)\sqrt{5}}{4}-\frac{\cos\left(\frac{\pi}{5}\right)2^{\frac{1}{5}}}{4}-\frac{2^{\frac{7}{10}}\sqrt{5+\sqrt{5}}\,\sin\left(\frac{\pi}{5}\right)}{4}+\frac{i2^{\frac{7}{10}}\sqrt{5+\sqrt{5}}\,\cos\left(\frac{\pi}{5}\right)}{4}+\frac{i2^{\frac{1}{5}}\sin\left(\frac{\pi}{5}\right)\sqrt{5}}{4}-\frac{i\sin\left(\frac{\pi}{5}\right)2^{\frac{1}{5}}}{4}\right)x} \\ & + c_5 \mathrm{e}^{\left(\cos\left(\frac{\pi}{5}\right)2^{\frac{1}{5}}+i\sin\left(\frac{\pi}{5}\right)2^{\frac{1}{5}}\right)x} \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 168

```
DSolve[y'''''[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to {} & e^{-\frac{\left(\sqrt{5}-1\right)x}{2\ 2^{4/5}}}\left(c_5 e^{\frac{\left(\sqrt{5}-5\right)x}{2\ 2^{4/5}}}\right. \\ & \left. +c_4\cos\left(\frac{\sqrt{5+\sqrt{5}}x}{2\ 2^{3/10}}\right)+c_1\sin\left(\frac{\sqrt{5+\sqrt{5}}x}{2\ 2^{3/10}}\right)+e^{\frac{\sqrt{5}x}{2^{4/5}}}\left(c_3\cos\left(\frac{\sqrt{5-\sqrt{5}}x}{2\ 2^{3/10}}\right)+c_2\sin\left(\frac{\sqrt{5-\sqrt{5}}x}{2\ 2^{3/10}}\right)\right)\right) \end{aligned}$$

9.5 problem 1(e)

Internal problem ID [5232]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 1(e).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 5y'' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)-5*diff(y(x),x$2)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x}c_1 + c_2\mathrm{e}^{-2x} + c_3\mathrm{e}^{-x} + c_4\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 35

```
DSolve[y''''[x]-5*y''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_2e^x + e^{3x}(c_4e^x + c_3) + c_1\right)$$

9.6 problem 2

Internal problem ID [5233]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = 0]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 39

```
dsolve([diff(y(x),x$3)+y(x)=0,y(0) = 0, D(y)(0) = 1, (D@@2)(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\left(\sqrt{3}\,\mathrm{e}^{\frac{3x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) + \mathrm{e}^{\frac{3x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) - 1\right) \mathrm{e}^{-x}}{3}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 53

```
DSolve[{y'''[x]+y[x]==0,{y[0]==0,y'[0]==1,y''[0]==0}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{3} e^{-x} \left(e^{3x/2} \left(\sqrt{3} \sin\left(\frac{\sqrt{3}x}{2}\right) + \cos\left(\frac{\sqrt{3}x}{2}\right)\right) - 1\right)$$

9.7 problem 3(a)

Internal problem ID [5234]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 3(a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - iy'' + y' - iy = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$3)-I*diff(y(x),x$2)+diff(y(x),x)-I*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-ix}c_1 + c_2e^{ix} + c_3e^{ix}x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 31

```
DSolve[y'''[x]-I*y''[x]+y'[x]-I*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-ix}\left(e^{2ix}(c_3x + c_2) + c_1\right)$$

9.8 problem 3(b)

Internal problem ID [5235]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 3(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2iy' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)-2*I*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{ix} + c_2 e^{ix} x$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y''[x]-2*I*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{ix}(c_2 x + c_1)$$

9.9 problem 5(b)

Internal problem ID [5236]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 83
Problem number: 5(b).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - k^4 y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y(1) = 0, y'(1) = 0]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$4)-k^4*y(x)=0,y(0) = 0, D(y)(0) = 0, y(1) = 0, D(y)(1) = 0],y(x), singsol
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 6

```
DSolve[{y''''[x]-k^4*y[x]==0,{y[0]==0,y[1]==0,y'[0]==0,y'[1]==0}},y[x],x,IncludeSingularSolut
```

$$y(x) \to 0$$

10 Chapter 2. Linear equations with constant coefficients. Page 89

10.1 problem 1(a)

Internal problem ID [5237]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 89
Problem number: 1(a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - y - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$3)-y(x)=x,y(x), singsol=all)
```

$$y(x) = -x + e^{x} c_1 + c_2 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) + c_3 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 52

```
DSolve[y'''[x]-y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + c_1 e^{x} + e^{-x/2}\left(c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_3 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

10.2 problem 1(b)

Internal problem ID [5238]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 89
Problem number: 1(b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - 8y - e^{ix} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$3)-8*y(x)=exp(I*x),y(x), singsol=all)
```

$$y(x) = \left(-\frac{8}{65} + \frac{i}{65}\right) e^{ix} + e^{2x} c_1 + c_2 e^{-x} \cos\left(\sqrt{3}\, x\right) + c_3 e^{-x} \sin\left(\sqrt{3}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.359 (sec). Leaf size: 58

```
DSolve[y'''[x]-8*y[x]==Exp[I*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\left(\frac{8}{65} - \frac{i}{65}\right) e^{ix} + c_1 e^{2x} + e^{-x}\left(c_2 \cos\left(\sqrt{3}x\right) + c_3 \sin\left(\sqrt{3}x\right)\right)$$

10.3 problem 1(c)

Internal problem ID [5239]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 89
Problem number: 1(c).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 16y - \cos(x) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 85

```
dsolve(diff(y(x),x$4)+16*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)}{(5+2\sqrt{2})(-5+2\sqrt{2})} + c_1 e^{\sqrt{2}\,x}\cos\left(\sqrt{2}\,x\right) + c_2 e^{\sqrt{2}\,x}\sin\left(\sqrt{2}\,x\right) + c_3 e^{-\sqrt{2}\,x}\cos\left(\sqrt{2}\,x\right) + c_4 e^{-\sqrt{2}\,x}\sin\left(\sqrt{2}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.748 (sec). Leaf size: 74

```
DSolve[y''''[x]+16*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cos(x)}{17} + e^{-\sqrt{2}x}\left(\left(c_1 e^{2\sqrt{2}x} + c_2\right)\cos\left(\sqrt{2}x\right) + \left(c_4 e^{2\sqrt{2}x} + c_3\right)\sin\left(\sqrt{2}x\right)\right)$$

10.4 problem 1(d)

Internal problem ID [5240]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 89
Problem number: 1(d).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{y'''' - 4y''' + 6y'' - 4y' + y - e^x = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+6*diff(y(x),x$2)-4*diff(y(x),x)+y(x)=exp(x),y(x), sing
```

$$y(x) = \frac{e^x x^4}{24} + e^x c_1 + c_2 e^x x + c_3 e^x x^2 + c_4 e^x x^3$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 39

```
DSolve[y''''[x]-4*y'''[x]+6*y''[x]-4*y'[x]+y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{24} e^x \left(x^4 + 24c_4 x^3 + 24c_3 x^2 + 24c_2 x + 24c_1\right)$$

10.5 problem 1(e)

Internal problem ID [5241]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 89
Problem number: 1(e).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - y - \cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$4)-y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)}{4} - \frac{\sin(x)\,x}{4} + \cos(x)\,c_1 + \mathrm{e}^{x}c_2 + c_3\sin(x) + c_4\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 40

```
DSolve[y''''[x]-y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1e^{x} + c_3e^{-x} + \left(-\frac{1}{2} + c_2\right)\cos(x) + \left(-\frac{x}{4} + c_4\right)\sin(x)$$

10.6 problem 1(f)

Internal problem ID [5242]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 89
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2iy' - y - e^{ix} + 2\,e^{-ix} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$2)-2*I*diff(y(x),x)-y(x)=exp(I*x)-2*exp(-I*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{ix} + e^{ix} c_1 x + \frac{(x^2 + 2ix + 2)\cos(x)}{2} + \frac{x(ix - 2)\sin(x)}{2}$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 39

```
DSolve[y''[x]-2*I*y'[x]-y[x]==Exp[I*x]-2*Exp[-I*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-ix} \left(1 + e^{2ix} \left(x^2 + 2c_2 x + 2c_1\right)\right)$$

11 Chapter 2. Linear equations with constant coefficients. Page 93

11.1 problem 1(a)

Internal problem ID [5243]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y - \cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+4*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \sin(2x)\, c_2 + \cos(2x)\, c_1 + \frac{\cos(x)}{3}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 26

```
DSolve[y''[x]+4*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cos(x)}{3} + c_1 \cos(2x) + c_2 \sin(2x)$$

11.2 problem 1(b)

Internal problem ID [5244]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y - \sin(2x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+4*y(x)=sin(2*x),y(x), singsol=all)
```

$$y(x) = \sin(2x)\, c_2 + \cos(2x)\, c_1 - \frac{x\cos(2x)}{4}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 33

```
DSolve[y''[x]+4*y[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{x}{4} + c_1\right)\cos(2x) + \frac{1}{8}(1 + 16c_2)\sin(x)\cos(x)$$

11.3 problem 1(c)

Internal problem ID [5245]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y - 3\,\mathrm{e}^{2x} - 4\,\mathrm{e}^{-x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-4*y(x)=3*exp(2*x)+4*exp(-x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x} c_2 + \mathrm{e}^{-2x} c_1 + \frac{3(-1+4x)\,\mathrm{e}^{2x}}{16} - \frac{4\,\mathrm{e}^{-x}}{3}$$

✓ Solution by Mathematica

Time used: 0.164 (sec). Leaf size: 81

```
DSolve[y''[x]-4*y[x]==3*exp[2*x]+4*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(\int_1^x -\frac{1}{4}e^{K[2]}\left(3e^{K[2]}\exp(2K[2])+4\right)dK[2] + e^{4x}\left(\int_1^x \frac{1}{4}e^{-3K[1]}\left(3e^{K[1]}\exp(2K[1])+4\right)dK[1]+c_1\right)+c_2\right)$$

11.4 problem 1(d)

Internal problem ID [5246]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - 2y - x^2 - \cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=x^2+cos(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x} c_2 + \mathrm{e}^{-x} c_1 - \frac{x^2}{2} - \frac{3\cos(x)}{10} - \frac{\sin(x)}{10} + \frac{x}{2} - \frac{3}{4}$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 45

```
DSolve[y''[x]-y'[x]-2*y[x]==x^2+Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} + c_2 e^{2x} + \frac{1}{20}(-5(2(x-1)x+3) - 2\sin(x) - 6\cos(x))$$

11.5 problem 1(e)

Internal problem ID [5247]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y - x^2 e^{3x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+9*y(x)=x^2*exp(3*x),y(x), singsol=all)
```

$$y(x) = \sin(3x)\, c_2 + \cos(3x)\, c_1 + \frac{(3x-1)^2 e^{3x}}{162}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 36

```
DSolve[y''[x]+9*y[x]==x^2*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{162} e^{3x}(1-3x)^2 + c_1 \cos(3x) + c_2 \sin(3x)$$

11.6 problem 1(f)

Internal problem ID [5248]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - x\,\mathrm{e}^{x} \cos(2x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+y(x)=x*exp(x)*cos(2*x),y(x), singsol=all)
```

$$y(x) = \sin(x)\,c_2 + \cos(x)\,c_1 + \frac{(-5x+11)\,\mathrm{e}^{x}\cos(2x)}{50} + \frac{\mathrm{e}^{x}\left(x-\frac{1}{5}\right)\sin(2x)}{5}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 45

```
DSolve[y''[x]+y[x]==x*Exp[x]*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{50}e^{x}(2(5x-1)\sin(2x) + (11-5x)\cos(2x)) + c_1\cos(x) + c_2\sin(x)$$

11.7 problem 1(g)

Internal problem ID [5249]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + iy' + 2y - 2\cosh(2x) - e^{-2x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+I*diff(y(x),x)+2*y(x)=2*cosh(2*x)+exp(-2*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{ix} + e^{-2ix} c_1 + \left(\frac{3}{10} + \frac{i}{10}\right) e^{-2x} + \left(\frac{3}{20} - \frac{i}{20}\right) e^{2x}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 47

```
DSolve[y''[x]+I*y'[x]+2*y[x]==2*Cosh[2*x]+Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-2ix} + c_2 e^{ix} + \frac{1}{20}((9+i)\cosh(2x) - (3+3i)\sinh(2x))$$

11.8 problem 1(h)

Internal problem ID [5250]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(h).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _quadrature]]`

$$\boxed{y''' - x^2 - e^{-x} \sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$3)=x^2+exp(-x)*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{x^5}{60} + \frac{c_1 x^2}{2} - \frac{\cos(x)\, e^{-x}}{4} + \frac{\sin(x)\, e^{-x}}{4} + x c_2 + c_3$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 41

```
DSolve[y'''[x]==x^2+Exp[-x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^5}{60} + c_3 x^2 + c_2 x + \frac{1}{4} e^{-x} (\sin(x) - \cos(x)) + c_1$$

11.9 problem 1(i)

Internal problem ID [5251]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 2. Linear equations with constant coefficients. Page 93
Problem number: 1(i).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + 3y'' + 3y' + y - x^2 e^{-x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=x^2*exp(-x),y(x), singsol=all)
```

$$y(x) = \frac{x^5 e^{-x}}{60} + e^{-x} c_1 + c_2 e^{-x} x + c_3 e^{-x} x^2$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 34

```
DSolve[y'''[x]+3*y''[x]+3*y'[x]+y[x]==x^2*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{60} e^{-x} \left(x^5 + 60c_3 x^2 + 60c_2 x + 60c_1\right)$$

12 Chapter 3. Linear equations with variable coefficients. Page 108

12.1 problem 1(c.1)

Internal problem ID [5252]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 108
Problem number: 1(c.1).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + \frac{y'}{x} - \frac{y}{x^2} = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)+1/x*diff(y(x),x)-1/x^2*y(x)=0,y(1) = 1, D(y)(1) = 0],y(x), singsol=all
```

$$y(x) = \frac{1}{2x} + \frac{x}{2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 17

```
DSolve[{y''[x]+1/x*y'[x]-1/x^2*y[x]==0,{y[1]==1,y'[1]==0}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{x^2 + 1}{2x}$$

12.2 problem 1(c.2)

Internal problem ID [5253]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 108
Problem number: 1(c.2).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + \frac{y'}{x} - \frac{y}{x^2} = 0}$$

With initial conditions

$$[y(1) = 0, y'(1) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)+1/x*diff(y(x),x)-1/x^2*y(x)=0,y(1) = 0, D(y)(1) = 1],y(x), singsol=all
```

$$y(x) = -\frac{1}{2x} + \frac{x}{2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 17

```
DSolve[{y''[x]+1/x*y'[x]-1/x^2*y[x]==0,{y[1]==0,y'[1]==1}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{x^2 - 1}{2x}$$

12.3 problem 2

Internal problem ID [5254]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 108
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(3x-1)^2 y'' + (9x-3) y' - 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((3*x-1)^2*diff(y(x),x$2)+(9*x-3)*diff(y(x),x)-9*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x-\frac{1}{3}} + \left(x - \frac{1}{3}\right) c_2$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 39

```
DSolve[(3*x-1)^2*y''[x]+(9*x-3)*y'[x]-9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1(-9x^2+6x-2) - 3ic_2x(3x-2)}{6x-2}$$

13 Chapter 3. Linear equations with variable coefficients. Page 121

13.1 problem 1(a)

Internal problem ID [5255]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 121
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 7y'x + 15y = 0}$$

Given that one solution of the ode is

$$y_1 = x^3$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-7*x*diff(y(x),x)+15*y(x)=0,x^3],y(x), singsol=all)
```

$$y(x) = c_2x^5 + c_1x^3$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-7*x*y'[x]+15*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3(c_2x^2 + c_1)$$

13.2 problem 1(b)

Internal problem ID [5256]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 121
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x + y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,x],y(x), singsol=all)
```

$$y(x) = c_1x + c_2x\ln(x)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 15

```
DSolve[x^2*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2\log(x) + c_1)$$

13.3 problem 1(c)

Internal problem ID [5257]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 121
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y'x + (4x^2 - 2)\,y = 0}$$

Given that one solution of the ode is

$$y_1 = e^{x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,exp(x^2)],y(x), singsol=all)
```

$$y(x) = c_1 e^{x^2} + c_2 x\, e^{x^2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(c_2 x + c_1)$$

13.4 problem 1(d)

Internal problem ID [5258]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 121
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - y'(1+x) + y = 0}$$

Given that one solution of the ode is

$$y_1 = e^x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([x*diff(y(x),x$2)-(x+1)*diff(y(x),x)+y(x)=0,exp(x)],y(x), singsol=all)
```

$$y(x) = c_1(x+1) + e^x c_2$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 19

```
DSolve[x*y''[x]-(x+1)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2(x+1)$$

13.5 problem 1(e)

Internal problem ID [5259]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 121
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\,y''-2y'x+2y=0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve([(1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,x],y(x), singsol=all)
```

$$y(x) = c_1 x + c_2\left(\frac{\ln(x-1)\,x}{2}-\frac{\ln(x+1)\,x}{2}+1\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 19

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2(x\operatorname{arctanh}(x)-1)+c_1 x$$

13.6 problem 1(f)

Internal problem ID [5260]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 121
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 2y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve([diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,x],y(x), singsol=all)
```

$$y(x) = c_1 x + c_2 \left(\sqrt{\pi}\, \operatorname{erfi}(x)\, x - e^{x^2}\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 31

```
DSolve[y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\pi} c_2 x \operatorname{erfi}(x) + c_2 e^{x^2} + 2c_1 x$$

13.7 problem 2

Internal problem ID [5261]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 121
Problem number: 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

Solve

$$\boxed{x^3 y''' - 3x^2 y'' + 6y'x - 6y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([x^3*diff(y(x),x$3)-3*x^2*diff(y(x),x$2)+6*x*diff(y(x),x)-6*y(x)=0,x],y(x), singsol=al
```

$$y(x) = c_2 x^3 + c_1 x^2 + c_3 x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 19

```
DSolve[x^3*y'''[x]-3*x^2*y''[x]+6*x*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x(c_3 x + c_2) + c_1)$$

14 Chapter 3. Linear equations with variable coefficients. Page 124

14.1 problem 1

Internal problem ID [5262]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 124
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' - 2y = 0}$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-2*y(x)=0,x^2],y(x), singsol=all)
```

$$y(x) = c_1x^2 + \frac{c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^3 + c_1}{x}$$

14.2 problem 2

Internal problem ID [5263]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 124
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x + y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,x],y(x), singsol=all)
```

$$y(x) = c_1x + c_2x\ln(x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 15

```
DSolve[x^2*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2\log(x) + c_1)$$

14.3 problem 3

Internal problem ID [5264]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 124
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 4y'x + (x^2 + 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(2+x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x^2} + \frac{c_2 \cos(x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+4*x*y'[x]+(2+x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x^2}$$

15 Chapter 3. Linear equations with variable coefficients. Page 130

15.1 problem 1(a)

Internal problem ID [5265]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{24}x^4\right) y(0) + D(y)(0)\, x + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 27

```
AsymptoticDSolveValue[y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(-\frac{x^4}{24} - \frac{x^2}{2} + 1\right) + c_2 x$$

15.2 problem 1(b)

Internal problem ID [5266]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3x^2y' - xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+3*x^2*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^3}{6}\right) y(0) + \left(x - \frac{1}{6}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+3*x^2*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^4}{6}\right) + c_1\left(\frac{x^3}{6} + 1\right)$$

15.3 problem 1(c)

Internal problem ID [5267]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - x^2 y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^4}{12}\right) y(0) + \left(x + \frac{1}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]-x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(\frac{x^5}{20} + x\right) + c_1 \left(\frac{x^4}{12} + 1\right)$$

15.4 problem 1(d)

Internal problem ID [5268]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + x^3 y' + x^2 y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x^3*diff(y(x),x)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^4}{12}\right) y(0) + \left(x - \frac{1}{10}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x^3*y'[x]+x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^5}{10}\right) + c_1\left(1 - \frac{x^4}{12}\right)$$

15.5 problem 1(e)

Internal problem ID [5269]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{120} - \frac{x^3}{6} + x\right) + c_1\left(\frac{x^4}{24} - \frac{x^2}{2} + 1\right)$$

15.6 problem 2

Internal problem ID [5270]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (x-1)^2 y' - (x-1)\, y = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
Order:=6;
dsolve([diff(y(x),x$2)+(x-1)^2*diff(y(x),x)-(x-1)*y(x)=0,y(1) = 1, D(y)(1) = 0],y(x),type='se
```

$$y(x) = 1 + \frac{1}{6}(x-1)^3 + \mathrm{O}\left((x-1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 14

```
AsymptoticDSolveValue[{y''[x]+(x-1)^2*y'[x]-(x-1)*y[x]==0,{y[1]==1,y'[1]==0}},y[x],{x,1,5}]
```

$$y(x) \to \frac{1}{6}(x-1)^3 + 1$$

15.7 problem 3

Internal problem ID [5271]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(x^2+1)\,y''+y=0}$$

With initial conditions

$$[y(0)=0, y'(0)=1]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([(1+x^2)*diff(y(x),x$2)+y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',x=0);
```

$$y(x)=x-\frac{1}{6}x^3+\frac{7}{120}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{(1+x^2)*y''[x]+y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x)\to\frac{7x^5}{120}-\frac{x^3}{6}+x$$

15.8 problem 4

Internal problem ID [5272]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + e^x y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
Order:=6;
dsolve([diff(y(x),x$2)+exp(x)*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y(x) = 1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{40}x^5 + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[{y''[x]+Exp[x]*y[x]==0,{}},y[x],{x,0,5}]
```

$$y(x) \to c_2\left(-\frac{x^5}{60} - \frac{x^4}{12} - \frac{x^3}{6} + x\right) + c_1\left(\frac{x^5}{40} - \frac{x^3}{6} - \frac{x^2}{2} + 1\right)$$

15.9 problem 5

Internal problem ID [5273]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 5.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - xy = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0, y''(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$3)-x*y(x)=0,y(0) = 1, D(y)(0) = 0, (D@@2)(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \text{hypergeom}\left([], \left[\frac{1}{2}, \frac{3}{4}\right], \frac{x^4}{64}\right)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 21

```
DSolve[{y'''[x]-x*y[x]==0,{y[0]==1,y'[0]==0,y''[0]==0}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to {}_0F_2\left(; \frac{1}{2}, \frac{3}{4}; \frac{x^4}{64}\right)$$

15.10 problem 6

Internal problem ID [5274]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$(-x^2+1)\,y'' - 2y'x + \alpha(\alpha+1)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 101

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+alpha*(alpha+1)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = \left(1 - \frac{\alpha(\alpha+1)\,x^2}{2} + \frac{\alpha(\alpha^3+2\alpha^2-5\alpha-6)\,x^4}{24}\right) y(0) + \left(x - \frac{(\alpha^2+\alpha-2)\,x^3}{6} + \frac{(\alpha^4+2\alpha^3-13\alpha^2-14\alpha+24)\,x^5}{120}\right) D(y)\,(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 127

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-2*x*y'[x]+\[Alpha]*(\[Alpha]+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{1}{60}(-\alpha^2-\alpha)\,x^5 - \frac{1}{120}(-\alpha^2-\alpha)\,(\alpha^2+\alpha)\,x^5 - \frac{1}{10}(\alpha^2+\alpha)\,x^5 + \frac{x^5}{5} - \frac{1}{6}(\alpha^2+\alpha)\,x^3 + \frac{x^3}{3} + x\right) + c_1\left(\frac{1}{24}(\alpha^2+\alpha)^2 x^4 - \frac{1}{4}(\alpha^2+\alpha)\,x^4 - \frac{1}{2}(\alpha^2+\alpha)\,x^2 + 1\right)$$

15.11 problem 7

Internal problem ID [5275]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{(-x^2+1)\,y''-y'x+\alpha^2 y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+alpha^2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x+\sqrt{x^2-1}\right)^{-\alpha}+c_2\left(x+\sqrt{x^2-1}\right)^{\alpha}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 45

```
DSolve[(1-x^2)*y''[x]-x*y'[x]+\[Alpha]^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cosh\left(\alpha\operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)\right)+ic_2\sinh\left(\alpha\operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)\right)$$

15.12 problem 8

Internal problem ID [5276]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 3. Linear equations with variable coefficients. Page 130
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 2\alpha y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+2*alpha*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \operatorname{KummerM}\left(\frac{1}{2} - \frac{\alpha}{2}, \frac{3}{2}, x^2\right) + c_2 x \operatorname{KummerU}\left(\frac{1}{2} - \frac{\alpha}{2}, \frac{3}{2}, x^2\right)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 45

```
DSolve[(1-x^2)*y''[x]-x*y'[x]+\[Alpha]^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cosh\left(\alpha \operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)\right) + ic_2 \sinh\left(\alpha \operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)\right)$$

16 Chapter 4. Linear equations with Regular Singular Points. Page 149

16.1 problem 1(a)

Internal problem ID [5277]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$x^2y'' + 2y'x - 6y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2 + \frac{c_2}{x^3}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+2*x*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^5 + c_1}{x^3}$$

16.2 problem 1(b)

Internal problem ID [5278]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2x^2y'' + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve(2*x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + xc_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[2*x^2*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{\sqrt{x}} + c_2x$$

16.3 problem 1(c)

Internal problem ID [5279]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$x^2y'' + y'x - 4y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2 + \frac{c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^4 + c_1}{x^2}$$

16.4 problem 1(d)

Internal problem ID [5280]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 5y'x + 9y - x^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+9*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = c_2x^3 + x^3 \ln(x) c_1 + x^2$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-5*x*y'[x]+9*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(c_1x + 3c_2x \log(x) + 1)$$

16.5 problem 1(e)

Internal problem ID [5281]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 1(e).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^3y''' + 2x^2y'' - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + xc_2 + c_3x\ln(x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+2*x^2*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x} + c_2x + c_3x\log(x)$$

16.6 problem 2(a)

Internal problem ID [5282]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 2(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + 4y - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=1,y(x), singsol=all)
```

$$y(x) = \sin(2\ln(x))\, c_2 + \cos(2\ln(x))\, c_1 + \frac{1}{4}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]+x*y'[x]+4*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2\log(x)) + c_2 \sin(2\log(x)) + \frac{1}{4}$$

16.7 problem 2(b)

Internal problem ID [5283]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 2(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 3y'x + 5y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(\ln(x))\, x^2 + c_2 \cos(\ln(x))\, x^2$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-3*x*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(c_2 \cos(\log(x)) + c_1 \sin(\log(x)))$$

16.8 problem 2(c)

Internal problem ID [5284]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 2(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + (-2 - i)\, xy' + 3iy = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x$2)-(2+I)*x*diff(y(x),x)+3*I*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^3 + c_2x^i$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 20

```
DSolve[x^2*y''[x]-(2+I)*x*y'[x]+3*I*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1x^i + c_2x^3$$

16.9 problem 2(d)

Internal problem ID [5285]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 149
Problem number: 2(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x - 4\pi y - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-4*Pi*y(x)=x,y(x), singsol=all)
```

$$y(x) = x^{-2\sqrt{\pi}}c_2 + x^{2\sqrt{\pi}}c_1 - \frac{x}{4\pi - 1}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]-4*Pi*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^{2\sqrt{\pi}} + c_1x^{-2\sqrt{\pi}} + \frac{x}{1 - 4\pi}$$

17 Chapter 4. Linear equations with Regular Singular Points. Page 154

17.1 problem 1(a)

Internal problem ID [5286]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (x^2 + x)\, y' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+(x+x^2)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x\left(1 - \frac{1}{3}x + \frac{1}{12}x^2 - \frac{1}{60}x^3 + \frac{1}{360}x^4 - \frac{1}{2520}x^5 + \frac{1}{20160}x^6 - \frac{1}{181440}x^7 + \mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(-2 + 2x - x^2 + \frac{1}{3}x^3 - \frac{1}{12}x^4 + \frac{1}{60}x^5 - \frac{1}{360}x^6 + \frac{1}{2520}x^7 + \mathrm{O}\left(x^8\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 92

```
AsymptoticDSolveValue[x^2*y''[x]+(x+x^2)*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^5}{720} - \frac{x^4}{120} + \frac{x^3}{24} - \frac{x^2}{6} + \frac{x}{2} + \frac{1}{x} - 1\right) + c_2\left(\frac{x^7}{20160} - \frac{x^6}{2520} + \frac{x^5}{360} - \frac{x^4}{60} + \frac{x^3}{12} - \frac{x^2}{3} + x\right)$$

17.2 problem 1(b)

Internal problem ID [5287]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2y'' + y'x^6 + 2xy = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 70

```
Order:=8;
dsolve(3*x^2*diff(y(x),x$2)+x^6*diff(y(x),x)+2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x\left(1 - \frac{1}{3}x + \frac{1}{27}x^2 - \frac{1}{486}x^3 + \frac{1}{14580}x^4 - \frac{7291}{656100}x^5 + \frac{225991}{41334300}x^6 - \frac{2522341}{3472081200}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2\left(\ln\left(x\right)\left(-\frac{2}{3}x + \frac{2}{9}x^2 - \frac{2}{81}x^3 + \frac{1}{729}x^4 - \frac{1}{21870}x^5 + \frac{7291}{984150}x^6 - \frac{225991}{62001450}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(1 - \frac{1}{3}x^2 + \frac{14}{243}x^3 - \frac{35}{8748}x^4 + \frac{101}{656100}x^5 + \frac{69199}{14762250}x^6 + \frac{19882543}{4340101500}x^7 + \mathrm{O}\left(x^8\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 121

```
AsymptoticDSolveValue[3*x^2*y''[x]+x^6*y'[x]+2*x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \left(\frac{x(7291x^5 - 45x^4 + 1350x^3 - 24300x^2 + 218700x - 656100)\log(x)}{984150} + \frac{-80332x^6 + 5895x^5 - 158625x^4 + 2430000x^3 - 16402500x^2 + 19683000x + 29524500}{29524500} \right) + c_2 \left(\frac{225991x^7}{41334300} - \frac{7291x^6}{656100} + \frac{x^5}{14580} - \frac{x^4}{486} + \frac{x^3}{27} - \frac{x^2}{3} + x \right)$$

17.3 problem 1(c)

Internal problem ID [5288]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 5y' + 3x^2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-5*diff(y(x),x)+3*x^2*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 106

```
AsymptoticDSolveValue[x^2*y''[x]-5*y'[x]+3*x^2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{339x^7}{8750} + \frac{49x^6}{1250} + \frac{18x^5}{625} + \frac{3x^4}{50} + \frac{x^3}{5} + 1\right) + c_2e^{-5/x}\left(-\frac{302083x^7}{218750} + \frac{5243x^6}{6250} - \frac{357x^5}{625} + \frac{113x^4}{250} - \frac{49x^3}{125} + \frac{6x^2}{25} - \frac{2x}{5} + 1\right)x^2$$

17.4 problem 1(d)

Internal problem ID [5289]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 70

```
Order:=8;
dsolve(x*diff(y(x),x$2)+4*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x \left(1 - 2x + \frac{4}{3}x^2 - \frac{4}{9}x^3 + \frac{4}{45}x^4 - \frac{8}{675}x^5 + \frac{16}{14175}x^6 - \frac{8}{99225}x^7 + \mathrm{O}\left(x^8\right)\right) \\ & + c_2 \left(\ln(x) \left((-4)x + 8x^2 - \frac{16}{3}x^3 + \frac{16}{9}x^4 - \frac{16}{45}x^5 + \frac{32}{675}x^6 - \frac{64}{14175}x^7 + \mathrm{O}\left(x^8\right)\right)\right. \\ & \left. + \left(1 - 12x^2 + \frac{112}{9}x^3 - \frac{140}{27}x^4 + \frac{808}{675}x^5 - \frac{1792}{10125}x^6 + \frac{9056}{496125}x^7 + \mathrm{O}\left(x^8\right)\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 119

```
AsymptoticDSolveValue[x*y''[x]+4*y[x]==0,y[x],{x,0,7}]
```

$$\begin{aligned} y(x) \to {} & c_1 \left(\frac{4}{675}x\left(8x^5 - 60x^4 + 300x^3 - 900x^2 + 1350x - 675\right)\log(x)\right. \\ & \left. + \frac{-2272x^6 + 15720x^5 - 70500x^4 + 180000x^3 - 202500x^2 + 40500x + 10125}{10125}\right) \\ & + c_2 \left(\frac{16x^7}{14175} - \frac{8x^6}{675} + \frac{4x^5}{45} - \frac{4x^4}{9} + \frac{4x^3}{3} - 2x^2 + x\right) \end{aligned}$$

17.5 problem 1(e)

Internal problem ID [5290]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\, y'' - 2y'x + 2y = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 49

```
Order:=8;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(-\frac{5}{2}(x-1) - \frac{3}{8}(x-1)^2 + \frac{1}{12}(x-1)^3 - \frac{5}{192}(x-1)^4 + \frac{3}{320}(x-1)^5 - \frac{7}{1920}(x-1)^6 + \frac{1}{672}(x-1)^7 + \mathrm{O}\left((x-1)^8\right)\right) c_2 + \left(1 + (x-1) + \mathrm{O}\left((x-1)^8\right)\right)\left(\ln(x-1)\, c_2 + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 86

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],{x,1,7}]
```

$$y(x) \to c_1 x + c_2\left(\frac{1}{672}(x-1)^7 - \frac{7(x-1)^6}{1920} + \frac{3}{320}(x-1)^5 - \frac{5}{192}(x-1)^4 + \frac{1}{12}(x-1)^3 - \frac{3}{8}(x-1)^2 - 2(x-1) + \frac{1-x}{2} + x\log(x-1)\right)$$

17.6 problem 1(f)

Internal problem ID [5291]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2+x-2)^2 y''+3(2+x) y'+(x-1) y=0$$

With the expansion point for the power series method at $x=-2$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 57

```
Order:=8;
dsolve((x^2+x-2)^2*diff(y(x),x$2)+3*(x+2)*diff(y(x),x)+(x-1)*y(x)=0,y(x),type='series',x=-2);
```

$$y(x) = \frac{c_1\left(1-\frac{5}{9}(x+2)+\frac{23}{324}(x+2)^2+\frac{271}{43740}(x+2)^3+\frac{10517}{12597120}(x+2)^4+\frac{778801}{6235574400}(x+2)^5+\frac{16965493}{942818849280}(x+2)\right.}{}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 148

```
AsymptoticDSolveValue[(x^2+x-2)^2*y''[x]+3*(x+2)*y'[x]+(x-1)*y[x]==0,y[x],{x,-2,7}]
```

$$y(x) \to c_1(x+2)\left(-\frac{52991201(x+2)^7}{11727918720000}-\frac{5797423(x+2)^6}{290405606400}-\frac{709507(x+2)^5}{8066822400}-\frac{11093(x+2)^4}{28304640}-\frac{53(x+2)^3}{29484}-\frac{11(x+2)^2}{1260}+\frac{1}{21}(-x-2)+1\right)+\frac{c_2\left(\frac{899971067(x+2)^7}{458981357990400}+\frac{16965493(x+2)^6}{942818849280}+\frac{778801(x+2)^5}{6235574400}+\frac{10517(x+2)^4}{12597120}+\frac{271(x+2)^3}{43740}+\frac{23}{324}(x+2)^2-\frac{5(x+2)}{9}+1\right)}{\sqrt[3]{x+2}}$$

17.7 problem 1(g)

Internal problem ID [5292]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 1(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'\sin(x) + y\cos(x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+sin(x)*diff(y(x),x)+cos(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^{-i}\left(1+\left(\frac{1}{12}+\frac{i}{24}\right)x^2+\left(\frac{29}{28800}+\frac{67i}{28800}\right)x^4+\left(-\frac{893}{14515200}-\frac{17i}{4838400}\right)x^6 + \mathrm{O}\left(x^8\right)\right)+c_2x^i\left(1+\left(\frac{1}{12}-\frac{i}{24}\right)x^2+\left(\frac{29}{28800}-\frac{67i}{28800}\right)x^4 + \left(-\frac{893}{14515200}+\frac{17i}{4838400}\right)x^6+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 112

```
AsymptoticDSolveValue[x^2*y''[x]+Sin[x]*y'[x]+Cos[x]*y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \to\; & c_1 x^{-i}\left(\left(-\frac{26459}{59222016000}-\frac{12449 i}{7402752000}\right) x^8-\left(\frac{893}{14515200}+\frac{17 i}{4838400}\right) x^6\right. \\
& \left.+\left(\frac{29}{28800}+\frac{67 i}{28800}\right) x^4+\left(\frac{1}{12}+\frac{i}{24}\right) x^2+1\right) \\
& + c_2 x^{i}\left(\left(-\frac{26459}{59222016000}+\frac{12449 i}{7402752000}\right) x^8-\left(\frac{893}{14515200}-\frac{17 i}{4838400}\right) x^6\right. \\
& \left.+\left(\frac{29}{28800}-\frac{67 i}{28800}\right) x^4+\left(\frac{1}{12}-\frac{i}{24}\right) x^2+1\right)
\end{aligned}
$$

17.8 problem 2(b)

Internal problem ID [5293]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 2(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 - \frac{1}{5040}x^6 + \mathrm{O}\left(x^8\right)\right) + c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \mathrm{O}\left(x^8\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 76

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^{11/2}}{720} + \frac{x^{7/2}}{24} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}}\right) + c_2\left(-\frac{x^{13/2}}{5040} + \frac{x^{9/2}}{120} - \frac{x^{5/2}}{6} + \sqrt{x}\right)$$

17.9 problem 2(c)

Internal problem ID [5294]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 2(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + (4x^4 - 5x)\, y' + (x^2 + 2)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 51

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)+(4*x^4-5*x)*diff(y(x),x)+(x^2+2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{4}} \left(1 - \frac{1}{2}x^2 - \frac{1}{15}x^3 + \frac{1}{72}x^4 + \frac{137}{1950}x^5 + \frac{307}{36720}x^6 - \frac{7169}{3439800}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2 x^2 \left(1 - \frac{1}{30}x^2 - \frac{8}{57}x^3 + \frac{1}{2760}x^4 + \frac{64}{12825}x^5 + \frac{147181}{9753840}x^6 - \frac{4037}{72268875}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 106

```
AsymptoticDSolveValue[4*x^2*y''[x]+(4*x^4-5*x)*y'[x]+(x^2+2)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \left(-\frac{4037x^7}{72268875} + \frac{147181x^6}{9753840} + \frac{64x^5}{12825} + \frac{x^4}{2760} - \frac{8x^3}{57} - \frac{x^2}{30} + 1\right) x^2 + c_2 \left(-\frac{7169x^7}{3439800} + \frac{307x^6}{36720} + \frac{137x^5}{1950} + \frac{x^4}{72} - \frac{x^3}{15} - \frac{x^2}{2} + 1\right) \sqrt[4]{x}$$

17.10 problem 2(d)

Internal problem ID [5295]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 154
Problem number: 2(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(-3x^2 + x\right) y' + \mathrm{e}^x y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 85

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+(x-3*x^2)*diff(y(x),x)+exp(x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x^{-i}\left(1 + (1-i)\,x + \left(\frac{7}{16} - \frac{13i}{16}\right)x^2 + \left(\frac{7}{39} - \frac{395i}{936}\right)x^3 + \left(\frac{2117}{29952} - \frac{5197i}{29952}\right)x^4 \right. \\ & + \left(\frac{5521}{217152} - \frac{642043i}{10857600}\right)x^5 + \left(\frac{782461}{97718400} - \frac{8813057i}{521164800}\right)x^6 \\ & \left. + \left(\frac{1238071931}{580056422400} - \frac{3271304833i}{812078991360}\right)x^7 + \mathrm{O}\left(x^8\right)\right) \\ & + c_2 x^{i}\left(1 + (1+i)\,x + \left(\frac{7}{16} + \frac{13i}{16}\right)x^2 + \left(\frac{7}{39} + \frac{395i}{936}\right)x^3 + \left(\frac{2117}{29952} + \frac{5197i}{29952}\right)x^4 \right. \\ & + \left(\frac{5521}{217152} + \frac{642043i}{10857600}\right)x^5 + \left(\frac{782461}{97718400} + \frac{8813057i}{521164800}\right)x^6 \\ & \left. + \left(\frac{1238071931}{580056422400} + \frac{3271304833i}{812078991360}\right)x^7 + \mathrm{O}\left(x^8\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 122

```
AsymptoticDSolveValue[x^2*y''[x]+(x-3*x^2)*y'[x]+Exp[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to \left(\frac{1}{97718400} + \frac{11i}{1563494400}\right) c_1 x^i \left((1302761 + 756800i)x^6 + (4384656 + 2763936i)x^5 + (12605400 + 8289000i)x^4 + (31161600 + 19814400i)x^3 + (66096000 + 33955200i)x^2 + (111974400 + 20736000i)x + (66355200 - 45619200i)\right) - \left(\frac{11}{1563494400} + \frac{i}{97718400}\right) c_2 x^{-i} \left((756800 + 1302761i)x^6 + (2763936 + 4384656i)x^5 + (8289000 + 12605400i)x^4 + (19814400 + 31161600i)x^3 + (33955200 + 66096000i)x^2 + (20736000 + 111974400i)x - (45619200 - 66355200i)\right)$$

18 Chapter 4. Linear equations with Regular Singular Points. Page 159

18.1 problem 1(a)

Internal problem ID [5296]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 159
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{3x^2y'' + 5y'x + 3xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 52

```
Order:=8;
dsolve(3*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+3*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2\left(1 - \frac{3}{5}x + \frac{9}{80}x^2 - \frac{9}{880}x^3 + \frac{27}{49280}x^4 - \frac{81}{4188800}x^5 + \frac{81}{167552000}x^6 - \frac{243}{26975872000}x^7 + \mathrm{O}\left(x^8\right)\right)x^{\frac{2}{3}} + c_1\left(1 - 3x + \right.}{x^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 111

```
AsymptoticDSolveValue[3*x^2*y''[x]+5*x*y'[x]+3*x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{243x^7}{26975872000} + \frac{81x^6}{167552000} - \frac{81x^5}{4188800} + \frac{27x^4}{49280} - \frac{9x^3}{880} + \frac{9x^2}{80} - \frac{3x}{5} + 1\right) + \frac{c_2\left(-\frac{243x^7}{619673600} + \frac{81x^6}{4659200} - \frac{81x^5}{145600} + \frac{27x^4}{2240} - \frac{9x^3}{56} + \frac{9x^2}{8} - 3x + 1\right)}{x^{2/3}}$$

18.2 problem 1(b)

Internal problem ID [5297]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 159
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{x^2y'' + y'x + x^2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x) c_2 + c_1)\left(1 - \frac{1}{4}x^2 + \frac{1}{64}x^4 - \frac{1}{2304}x^6 + \mathrm{O}(x^8)\right) + \left(\frac{1}{4}x^2 - \frac{3}{128}x^4 + \frac{11}{13824}x^6 + \mathrm{O}(x^8)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 81

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+x^2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^6}{2304} + \frac{x^4}{64} - \frac{x^2}{4} + 1\right) + c_2\left(\frac{11x^6}{13824} - \frac{3x^4}{128} + \frac{x^2}{4} + \left(-\frac{x^6}{2304} + \frac{x^4}{64} - \frac{x^2}{4} + 1\right)\log(x)\right)$$

18.3 problem 2

Internal problem ID [5298]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 159
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + y'\mathrm{e}^x x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 85

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*exp(x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-i}\left(1+\left(-\frac{2}{5}+\frac{i}{5}\right)x+\left(\frac{3}{80}+\frac{i}{80}\right)x^2+\left(\frac{67}{9360}-\frac{9i}{1040}\right)x^3 + \left(-\frac{103}{149760}-\frac{229i}{149760}\right)x^4+\left(-\frac{2831}{7238400}+\frac{607i}{4343040}\right)x^5 + \left(-\frac{59077}{1563494400}+\frac{26063i}{260582400}\right)x^6+\left(\frac{22952047}{2030197478400}+\frac{8634893i}{580056422400}\right)x^7 + \mathrm{O}\left(x^8\right)\right) + c_2 x^{i}\left(1+\left(-\frac{2}{5}-\frac{i}{5}\right)x+\left(\frac{3}{80}-\frac{i}{80}\right)x^2+\left(\frac{67}{9360}+\frac{9i}{1040}\right)x^3 + \left(-\frac{103}{149760}+\frac{229i}{149760}\right)x^4+\left(-\frac{2831}{7238400}-\frac{607i}{4343040}\right)x^5 + \left(-\frac{59077}{1563494400}-\frac{26063i}{260582400}\right)x^6+\left(\frac{22952047}{2030197478400}-\frac{8634893i}{580056422400}\right)x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 122

```
AsymptoticDSolveValue[x^2*y''[x]+x*Exp[x]*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \to & \left(\frac{11}{1563494400}+\frac{i}{97718400}\right) c_2 x^{-i}\left((4913+7070 i) x^6-(8568-32328 i) x^5\right. \\
& -(132840+24120 i) x^4-(247680+869760 i) x^3+(2540160-1918080 i) x^2 \\
& \left.-(4976640-35665920 i) x+(45619200-66355200 i)\right) \\
& -\left(\frac{1}{97718400}+\frac{11 i}{1563494400}\right) c_1 x^i\left((7070+4913 i) x^6+(32328-8568 i) x^5\right. \\
& -(24120+132840 i) x^4-(869760+247680 i) x^3-(1918080-2540160 i) x^2 \\
& \left.+(35665920-4976640 i) x-(66355200-45619200 i)\right)
\end{aligned}
$$

19 Chapter 4. Linear equations with Regular Singular Points. Page 166

19.1 problem 1(i)

Internal problem ID [5299]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 1(i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' + (x^2 + 5x)\, y' + (x^2 - 2)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 55

```
Order:=8;
dsolve(2*x^2*diff(y(x),x$2)+(5*x+x^2)*diff(y(x),x)+(x^2-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{5}{2}} \left(1 - \frac{1}{14}x - \frac{25}{504}x^2 + \frac{197}{33264}x^3 + \frac{1921}{3459456}x^4 - \frac{11653}{103783680}x^5 + \frac{12923}{21171870720}x^6 + \frac{917285}{1126343522304}x^7 + \mathrm{O}\left(x^8\right)\right) + c_1 \left(}{x^2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 116

```
AsymptoticDSolveValue[2*x^2*y''[x]+(5*x+x^2)*y'[x]+(x^2-2)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{917285x^7}{1126343522304} + \frac{12923x^6}{21171870720} - \frac{11653x^5}{103783680} + \frac{1921x^4}{3459456} + \frac{197x^3}{33264} - \frac{25x^2}{504} - \frac{x}{14} + 1\right) + \frac{c_2\left(-\frac{4x^7}{35721} + \frac{101x^6}{45360} - \frac{x^5}{540} - \frac{19x^4}{216} + \frac{2x^3}{9} + \frac{5x^2}{6} - \frac{2x}{3} + 1\right)}{x^2}$$

19.2 problem 1(ii)

Internal problem ID [5300]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 1(ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4y'\mathrm{e}^x x + 3y\cos(x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 81

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)-4*x*exp(x)*diff(y(x),x)+3*cos(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left(x\left(1+\frac{3}{4}x+\frac{1}{2}x^2+\frac{103}{384}x^3+\frac{669}{5120}x^4+\frac{54731}{921600}x^5+\frac{123443}{4838400}x^6+\frac{30273113}{2890137600}x^7 + \mathrm{O}\left(x^8\right)\right)c_1 + c_2\left(\ln(x)\left(\frac{1}{2}x+\frac{3}{8}x^2+\frac{1}{4}x^3+\frac{103}{768}x^4+\frac{669}{10240}x^5+\frac{54731}{1843200}x^6+\frac{123443}{9676800}x^7+\mathrm{O}\left(x^8\right)\right) + \left(1+x+\frac{3}{4}x^2+\frac{59}{144}x^3+\frac{5701}{27648}x^4+\frac{17519}{184320}x^5+\frac{6852157}{165888000}x^6+\frac{417496453}{24385536000}x^7+\mathrm{O}\left(x^8\right)\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.139 (sec). Leaf size: 146

```
AsymptoticDSolveValue[4*x^2*y''[x]-4*x*Exp[x]*y'[x]+3*Cos[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{123443x^{15/2}}{4838400}+\frac{54731x^{13/2}}{921600}+\frac{669x^{11/2}}{5120}+\frac{103x^{9/2}}{384}+\frac{x^{7/2}}{2}+\frac{3x^{5/2}}{4} + x^{3/2}\right) + c_1\left(\frac{(54731x^5+120420x^4+247200x^3+460800x^2+691200x+921600)\,x^{3/2}\log(x)}{1843200} + \frac{(1926367

19.3 problem 1(iii)

Internal problem ID [5301]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 1(iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-x^2+1)\,x^2y''+3(x^2+x)\,y'+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 81

```
Order:=8;
dsolve((1-x^2)*x^2*diff(y(x),x$2)+3*(x+x^2)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(\ln(x)\,c_2+c_1)\left(1+3x+\frac{1}{2}x^2-\frac{1}{6}x^3+\frac{1}{16}x^4-\frac{43}{1200}x^5+\frac{161}{7200}x^6-\frac{1837}{117600}x^7+\mathrm{O}(x^8)\right)+\left((-9)\,x-\frac{7}{2}x^2+\frac{7}{9}x\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 84

```
AsymptoticDSolveValue[(1-x^2)*y''[x]+3*(x+x^2)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(\frac{53x^7}{630}+\frac{5x^6}{24}+\frac{2x^5}{15}-\frac{x^4}{4}-\frac{2x^3}{3}+x\right)+c_1\left(-\frac{19x^7}{420}-\frac{x^6}{144}+\frac{3x^5}{20}+\frac{5x^4}{24}-\frac{x^2}{2}+1\right)$$

19.4 problem 3(a)

Internal problem ID [5302]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 3(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 3y'x + (1+x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(\ln(x)\,c_2 + c_1)\left(1 - x + \frac{1}{4}x^2 - \frac{1}{36}x^3 + \frac{1}{576}x^4 - \frac{1}{14400}x^5 + \frac{1}{518400}x^6 - \frac{1}{25401600}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(2x - \frac{3}{4}x^2 + \frac{1}{10}\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 164

```
AsymptoticDSolveValue[x^2*y''[x]+3*x*y'[x]+(1+x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to \frac{c_1\left(-\frac{x^7}{25401600} + \frac{x^6}{518400} - \frac{x^5}{14400} + \frac{x^4}{576} - \frac{x^3}{36} + \frac{x^2}{4} - x + 1\right)}{x} + c_2\left(\frac{\frac{121x^7}{592704000} - \frac{49x^6}{5184000} + \frac{137x^5}{432000} - \frac{25x^4}{3456} + \frac{11x^3}{108} - \frac{3x^2}{4} + 2x}{x} + \frac{\left(-\frac{x^7}{25401600} + \frac{x^6}{518400} - \frac{x^5}{14400} + \frac{x^4}{576} - \frac{x^3}{36} + \frac{x^2}{4} - x + 1\right)\log(x)}{x}\right)$$

19.5 problem 3(b)

Internal problem ID [5303]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 3(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2x^2y' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+2*x^2*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 - x + \frac{3}{5}x^2 - \frac{4}{15}x^3 + \frac{2}{21}x^4 - \frac{1}{35}x^5 + \frac{1}{135}x^6 - \frac{8}{4725}x^7 + \mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(12 - 12x + 8x^3 - 8x^4 + \frac{24}{5}x^5 - \frac{32}{15}x^6 + \frac{16}{21}x^7 + \mathrm{O}\left(x^8\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x^2*y''[x]+2*x^2*y'[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{8x^5}{45} + \frac{2x^4}{5} - \frac{2x^3}{3} + \frac{2x^2}{3} + \frac{1}{x} - 1\right) + c_2\left(\frac{x^8}{135} - \frac{x^7}{35} + \frac{2x^6}{21} - \frac{4x^5}{15} + \frac{3x^4}{5} - x^3 + x^2\right)$$

19.6 problem 3(c)

Internal problem ID [5304]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 3(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 5y'x + (-x^3 + 3) y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+(3-x^3)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 + \frac{1}{15}x^3 + \frac{1}{720}x^6 + \mathrm{O}\left(x^8\right)\right)}{x} + \frac{c_2\left(-2 - \frac{2}{3}x^3 - \frac{1}{36}x^6 + \mathrm{O}\left(x^8\right)\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 40

```
AsymptoticDSolveValue[x^2*y''[x]+5*x*y'[x]+(3-3*x^3)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^3}{8} + \frac{1}{x^3} + 1\right) + c_2\left(\frac{x^5}{80} + \frac{x^2}{5} + \frac{1}{x}\right)$$

19.7 problem 3(d)

Internal problem ID [5305]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 3(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2x(1+x)\,y' + 2(1+x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-2*x*(x+1)*diff(y(x),x)+2*(x+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 + x + \frac{2}{3}x^2 + \frac{1}{3}x^3 + \frac{2}{15}x^4 + \frac{2}{45}x^5 + \frac{4}{315}x^6 + \frac{1}{315}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2x\left(1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{15}x^5 + \frac{4}{45}x^6 + \frac{8}{315}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 92

```
AsymptoticDSolveValue[x^2*y''[x]-2*x*(x+1)*y'[x]+2*(1+x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{4x^7}{45} + \frac{4x^6}{15} + \frac{2x^5}{3} + \frac{4x^4}{3} + 2x^3 + 2x^2 + x\right) + c_2\left(\frac{4x^8}{315} + \frac{2x^7}{45} + \frac{2x^6}{15} + \frac{x^5}{3} + \frac{2x^4}{3} + x^3 + x^2\right)$$

19.8 problem 3(e)

Internal problem ID [5306]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 3(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{x^2y'' + y'x + (x^2 - 1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1 - \frac{1}{8}x^2 + \frac{1}{192}x^4 - \frac{1}{9216}x^6 + \mathrm{O}\left(x^8\right)\right) + c_2\left(\ln\left(x\right)\left(x^2 - \frac{1}{8}x^4 + \frac{1}{192}x^6 + \mathrm{O}\left(x^8\right)\right) + \left(-2 + \frac{3}{32}x^4 - \frac{7}{1152}x^6\right.\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 75

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1)*y[x]==0,y[x],{x,0,7}]
```

$$\begin{aligned} y(x) \to\ & c_2\left(-\frac{x^7}{9216} + \frac{x^5}{192} - \frac{x^3}{8} + x\right) \\ & + c_1\left(\frac{5x^6 - 90x^4 + 288x^2 + 1152}{1152x} - \frac{1}{384}x\left(x^4 - 24x^2 + 192\right)\log(x)\right) \end{aligned}$$

19.9 problem 3(f)

Internal problem ID [5307]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 166
Problem number: 3(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$\boxed{x^2y'' - 2x^2y' + (4x-2)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 55

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-2*x^2*diff(y(x),x)+(4*x-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1+\mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(\ln(x)\left((-48)\,x^3+\mathrm{O}\left(x^8\right)\right)+\left(12+36x+72x^2+88x^3-24x^4-\frac{24}{5}x^5-\frac{16}{15}x^6-\frac{8}{35}x^7+\mathrm{O}\left(x^8\right)\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.1 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]-2*x^2*y'[x]+(4*x-2)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2x^2 + c_1\left(-4x^2\log(x) - \frac{4x^6+18x^5+90x^4-390x^3-270x^2-135x-45}{45x}\right)$$

20 Chapter 4. Linear equations with Regular Singular Points. Page 182

20.1 problem 4

Internal problem ID [5308]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 4. Linear equations with Regular Singular Points. Page 182
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\,y''-2y'x+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
Order:=8;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-x^2-\frac{1}{3}x^4-\frac{1}{5}x^6\right)y(0)+D(y)\,(0)\,x+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(-\frac{x^6}{5}-\frac{x^4}{3}-x^2+1\right)+c_2x$$

21 Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190

21.1 problem 1(a)

Internal problem ID [5309]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x^2 y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=x^2*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^3}{3}}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 22

```
DSolve[y'[x]==x^2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{x^3}{3}}$$
$$y(x) \to 0$$

21.2 problem 1(b)

Internal problem ID [5310]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 1(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(y(x)*diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 + c_1}$$
$$y(x) = -\sqrt{x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 35

```
DSolve[y[x]*y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^2 + 2c_1}$$
$$y(x) \to \sqrt{x^2 + 2c_1}$$

21.3 problem 1(c)

Internal problem ID [5311]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 1(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x^2 + x}{y - y^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 720

```
dsolve(diff(y(x),x)=(x+x^2)/(y(x)-y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\left(1-4x^3-6x^2-12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}}{2} + \frac{1}{2\left(1-4x^3-6x^2-12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}} + \frac{1}{2}$$

$$y(x) = -\frac{\left(1-4x^3-6x^2-12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}}{4} - \frac{1}{4\left(1-4x^3-6x^2-12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}} + \frac{1}{2} - \frac{i\sqrt{3}\left(\frac{\left(1-4x^3-6x^2-12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}}{2} - \frac{}{2\left(1-4x^3-6x^2-12c_1+2\sqrt{4x^6+12x^5+24c\right.}\right.}{2}$$

$$y(x) = -\frac{\left(1-4x^3-6x^2-12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}}{4} - \frac{1}{4\left(1-4x^3-6x^2-12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}} + \frac{1}{2} + \frac{i\sqrt{3}\left(\frac{\left(1-4x^3-6x^2-12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}}{2} - \frac{}{2\left(1-4x^3-6x^2-12c_1+2\sqrt{4x^6+12x^5+24c\right.}\right.}{2}$$

✓ Solution by Mathematica

Time used: 4.069 (sec). Leaf size: 346

```
DSolve[y'[x]==(x+x^2)/(y[x]-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\sqrt[3]{-4x^3-6x^2+\sqrt{-1+(-4x^3-6x^2+1+12c_1)^2}+1+12c_1} + \frac{1}{\sqrt[3]{-4x^3-6x^2+\sqrt{-1+(-4x^3-6x^2+1+12c_1)^2}+1+12c_1}}+1\right)$$

$$y(x) \to \frac{1}{8}\left(2i\left(\sqrt{3}+i\right)\sqrt[3]{-4x^3-6x^2+\sqrt{-1+(-4x^3-6x^2+1+12c_1)^2}+1+12c_1} + \frac{-2-2i\sqrt{3}}{\sqrt[3]{-4x^3-6x^2+\sqrt{-1+(-4x^3-6x^2+1+12c_1)^2}+1+12c_1}}+4\right)$$

$$y(x) \to \frac{1}{8}\left(-2\left(1+i\sqrt{3}\right)\sqrt[3]{-4x^3-6x^2+\sqrt{-1+(-4x^3-6x^2+1+12c_1)^2}+1+12c_1} + \frac{2i(\sqrt{3}+i)}{\sqrt[3]{-4x^3-6x^2+\sqrt{-1+(-4x^3-6x^2+1+12c_1)^2}+1+12c_1}}+4\right)$$

21.4 problem 1(d)

Internal problem ID [5312]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 1(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{e^{-y+x}}{e^x + 1} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=exp(x-y(x))/(1+exp(x)),y(x), singsol=all)
```

$$y(x) = \ln(\ln(e^x + 1) + c_1)$$

✓ Solution by Mathematica

Time used: 0.465 (sec). Leaf size: 15

```
DSolve[y'[x]==Exp[x-y[x]]/(1+Exp[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(\log(e^x + 1) + c_1)$$

21.5 problem 1(e)

Internal problem ID [5313]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 1(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x^2y^2 + 4x^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)=x^2*y(x)^2-4*x^2,y(x), singsol=all)
```

$$y(x) = -\frac{2\left(e^{\frac{4x^3}{3}}c_1 + 1\right)}{-1 + e^{\frac{4x^3}{3}}c_1}$$

✓ Solution by Mathematica

Time used: 0.247 (sec). Leaf size: 30

```
DSolve[y'[x]==x^2*y[x]^2-4*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\tanh\left(\frac{2}{3}(x^3 + 3c_1)\right)$$
$$y(x) \to -2$$
$$y(x) \to 2$$

21.6 problem 2(a)

Internal problem ID [5314]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 = 0}$$

With initial conditions

$$[y(x_0) = y_0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve([diff(y(x),x)=y(x)^2,y(x__0) = y__0],y(x), singsol=all)
```

$$y(x) = -\frac{y_0}{-1 + (x - x_0)\, y_0}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 16

```
DSolve[{y'[x]==x2*y[x],{y[x0]==y0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \mathrm{y0}e^{\mathrm{x2}(x-\mathrm{x0})}$$

21.7 problem 3(a)

Internal problem ID [5315]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 3(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 2\sqrt{y} = 0}$$

With initial conditions

$$[y(x_0) = y_0]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 28

```
dsolve([diff(y(x),x)=2*sqrt(y(x)),y(x__0) = y__0],y(x), singsol=all)
```

$$y(x) = (2x - 2x_0)\sqrt{y_0} + x^2 - 2xx_0 + x_0^2 + y_0$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 33

```
DSolve[{y'[x]==2*Sqrt[y[x]],{y[x0]==y0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(x - \mathrm{x0} + \sqrt{\mathrm{y0}}\right)^2$$
$$y(x) \to \left(-x + \mathrm{x0} + \sqrt{\mathrm{y0}}\right)^2$$

21.8 problem 3(b)

Internal problem ID [5316]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 3(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 2\sqrt{y} = 0}$$

With initial conditions

$$[y(x_0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=2*sqrt(y(x)),y(x__0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x]==2*Sqrt[y[x]],{y[x0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

21.9 problem 4(a)

Internal problem ID [5317]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 4(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cla`

$$\boxed{y' - \frac{y + x}{-y + x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=(x+y(x))/(x-y(x)),y(x), singsol=all)
```

$$y(x) = \tan\left(\operatorname{RootOf}\left(-2\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(x) + 2c_1\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 36

```
DSolve[y'[x]==(x+y[x])/(x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + 1\right) - \arctan\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

21.10 problem 4(b)

Internal problem ID [5318]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 4(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cla`

$$y' - \frac{y^2}{xy + x^2} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)=y(x)^2/(x*y(x)+x^2),y(x), singsol=all)
```

$$y(x) = e^{-\operatorname{LambertW}\left(\frac{e^{-c_1}}{x}\right)-c_1}$$

✓ Solution by Mathematica

Time used: 2.271 (sec). Leaf size: 21

```
DSolve[y'[x]==y[x]^2/(x*y[x]+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to xW\left(\frac{e^{c_1}}{x}\right)$$
$$y(x) \to 0$$

21.11 problem 4(c)

Internal problem ID [5319]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 4(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y' - \frac{x^2 + xy + y^2}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=(x^2+x*y(x)+y(x)^2)/x^2,y(x), singsol=all)
```

$$y(x) = \tan(\ln(x) + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.184 (sec). Leaf size: 13

```
DSolve[y'[x]==(x^2+x*y[x]+y[x]^2)/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \tan(\log(x) + c_1)$$

21.12 problem 4(d)

Internal problem ID [5320]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 4(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y' - \frac{y + x\,\mathrm{e}^{-\frac{2y}{x}}}{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=(y(x)+x*exp(-2*y(x)/x))/x,y(x), singsol=all)
```

$$y(x) = \frac{\ln\left(2\ln\left(x\right) + 2c_1\right)x}{2}$$

✓ Solution by Mathematica

Time used: 0.404 (sec). Leaf size: 18

```
DSolve[y'[x]==(y[x]+x*Exp[-2*y[x]/x])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x\log(2(\log(x) + c_1))$$

21.13 problem 5(a)

Internal problem ID [5321]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 5(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cla`

$$y' - \frac{x-y+2}{y+x-1} = 0$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)=(x-y(x)+2)/(x+y(x)-1),y(x), singsol=all)
```

$$y(x) = \frac{3}{2} - \frac{(2x+1)\,c_1 + \sqrt{2\,(2x+1)^2\,c_1^2 + 1}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.146 (sec). Leaf size: 49

```
DSolve[y'[x]==(x-y[x]+2)/(x+y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \sqrt{2x(x+1)+1+c_1} + 1$$
$$y(x) \to -x + \sqrt{2x(x+1)+1+c_1} + 1$$

21.14 problem 5(b)

Internal problem ID [5322]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 5(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cla`

$$\boxed{y' - \frac{2x+3y+1}{x-2y-1} = 0}$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 59

```
dsolve(diff(y(x),x)=(2*x+3*y(x)+1)/(x-2*y(x)-1),y(x), singsol=all)
```

$$y(x) = -\frac{5}{14} - \frac{x}{2} + \frac{\sqrt{3}\,(7x-1)\tan\left(\operatorname{RootOf}\left(\sqrt{3}\,\ln\left(\frac{3(7x-1)^2}{4} + \frac{3(7x-1)^2\tan(\_Z)^2}{4}\right) + 2\sqrt{3}\,c_1 - 4\_Z\right)\right)}{14}$$

✓ Solution by Mathematica

Time used: 0.116 (sec). Leaf size: 85

```
DSolve[y'[x]==(2*x+3*y[x]+1)/(x-2*y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[32\sqrt{3}\arctan\left(\frac{4y(x)+5x+1}{\sqrt{3}(-2y(x)+x-1)}\right) = 3\left(8\log\left(\frac{4(7x^2+7y(x)^2+(7x+5)y(x)+x+1)}{(1-7x)^2}\right) + 16\log(7x-1)+7c_1\right), y(x)\right]$$

21.15 problem 5(c)

Internal problem ID [5323]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 5(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$y' - \frac{y + x + 1}{2x + 2y - 1} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)=(x+y(x)+1)/(2*x+2*y(x)-1),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(-2\,\mathrm{e}^{-3x}\mathrm{e}^{3c_1}\right)-3x+3c_1} - x$$

✓ Solution by Mathematica

Time used: 4.024 (sec). Leaf size: 32

```
DSolve[y'[x]==(x+y[x]+1)/(2*x+2*y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \frac{1}{2}W\left(-e^{-3x-1+c_1}\right)$$
$$y(x) \to -x$$

21.16 problem 6(b)

Internal problem ID [5324]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 190
Problem number: 6(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, _Riccati]`

$$y' - \frac{(y + x - 1)^2}{2(2 + x)^2} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=1/2*((x+y(x)-1)/(x+2))^2,y(x), singsol=all)
```

$$y(x) = 3 + \tan\left(\frac{\ln(x+2)}{2} + \frac{c_1}{2}\right)(x+2)$$

✓ Solution by Mathematica

Time used: 0.353 (sec). Leaf size: 57

```
DSolve[y'[x]==1/2*((x+y[x]-1)/(x+2))^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i(x + (2+3i)) - \frac{4c_1(x+2)}{2^i(x+2)^i + 2ic_1}$$

$$y(x) \to ix + (3+2i)$$

22 Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198

22.1 problem 1(a)

Internal problem ID [5325]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$\boxed{2xy + (x^2 + 3y^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 257

```
dsolve(2*x*y(x)+(x^2+3*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\frac{\left(108+12\sqrt{12c_1^3x^6+81}\right)^{\frac{1}{3}}}{6} - \frac{2x^2c_1}{\left(108+12\sqrt{12c_1^3x^6+81}\right)^{\frac{1}{3}}}}{\sqrt{c_1}}$$

$$y(x) = \frac{-\frac{\left(108+12\sqrt{12c_1^3x^6+81}\right)^{\frac{1}{3}}}{12} + \frac{x^2c_1}{\left(108+12\sqrt{12c_1^3x^6+81}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(108+12\sqrt{12c_1^3x^6+81}\right)^{\frac{1}{3}}}{6} + \frac{2x^2c_1}{\left(108+12\sqrt{12c_1^3x^6+81}\right)^{\frac{1}{3}}}\right)}{2}}{\sqrt{c_1}}$$

$$y(x) = \frac{-\frac{\left(108+12\sqrt{12c_1^3x^6+81}\right)^{\frac{1}{3}}}{12} + \frac{x^2c_1}{\left(108+12\sqrt{12c_1^3x^6+81}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(108+12\sqrt{12c_1^3x^6+81}\right)^{\frac{1}{3}}}{6} + \frac{2x^2c_1}{\left(108+12\sqrt{12c_1^3x^6+81}\right)^{\frac{1}{3}}}\right)}{2}}{\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 27.301 (sec). Leaf size: 396

```
DSolve[2*x*y[x]+(x^2+3*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2\sqrt[3]{3}x^2 + \sqrt[3]{2}\left(\sqrt{12x^6 + 81e^{2c_1}} + 9e^{c_1}\right)^{2/3}}{6^{2/3}\sqrt[3]{\sqrt{12x^6 + 81e^{2c_1}} + 9e^{c_1}}}$$

$$y(x) \to \frac{\sqrt[3]{-1}\left(2\sqrt[3]{3}x^2 + \sqrt[3]{-2}\left(\sqrt{12x^6 + 81e^{2c_1}} + 9e^{c_1}\right)^{2/3}\right)}{6^{2/3}\sqrt[3]{\sqrt{12x^6 + 81e^{2c_1}} + 9e^{c_1}}}$$

$$y(x) \to -\frac{\sqrt[3]{-1}\left(2\sqrt[3]{-3}x^2 + \sqrt[3]{2}\left(\sqrt{12x^6 + 81e^{2c_1}} + 9e^{c_1}\right)^{2/3}\right)}{6^{2/3}\sqrt[3]{\sqrt{12x^6 + 81e^{2c_1}} + 9e^{c_1}}}$$

$$y(x) \to 0$$

$$y(x) \to \frac{\sqrt[3]{x^6} - x^2}{\sqrt{3}\sqrt[6]{x^6}}$$

$$y(x) \to \frac{\left(\sqrt{3} - 3i\right)x^2 - \left(\sqrt{3} + 3i\right)\sqrt[3]{x^6}}{6\sqrt[6]{x^6}}$$

$$y(x) \to \frac{\left(\sqrt{3} + 3i\right)x^2 - \left(\sqrt{3} - 3i\right)\sqrt[3]{x^6}}{6\sqrt[6]{x^6}}$$

22.2 problem 1(b)

Internal problem ID [5326]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 1(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x^2 + xy + (y + x)\,y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((x^2+x*y(x))+(x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -x$$
$$y(x) = -\frac{x^2}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.127 (sec). Leaf size: 53

```
DSolve[(x^2+y[x])+(x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \sqrt{-\frac{2x^3}{3} + x^2 + c_1}$$
$$y(x) \to -x + \sqrt{-\frac{2x^3}{3} + x^2 + c_1}$$

22.3 problem 1(c)

Internal problem ID [5327]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 1(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{e^{x}+e^{y}(1+y)\,y'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(exp(x)+(exp(y(x))*(y(x)+1))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\operatorname{LambertW}\left(-c_1-e^{x}\right)$$

✓ Solution by Mathematica

Time used: 60.16 (sec). Leaf size: 14

```
DSolve[Exp[x]+(Exp[y[x]]*(y[x]+1))*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to W\left(-e^{x}+c_1\right)$$

22.4 problem 1(d)

Internal problem ID [5328]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 1(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(x)\cos(y)^2 - \sin(x)\sin(2y)\,y' = 0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 25

```
dsolve(cos(x)*cos(y(x))^2-sin(x)*sin(2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{1}{\sqrt{c_1 \sin(x)}}\right)$$
$$y(x) = \pi - \arccos\left(\frac{1}{\sqrt{c_1 \sin(x)}}\right)$$

✓ Solution by Mathematica

Time used: 8.954 (sec). Leaf size: 89

```
DSolve[Cos[x]*Cos[y[x]]^2-Sin[x]*Sin[2*y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$
$$y(x) \to -\arccos\left(-\frac{1}{4}c_1\sqrt{\cos(x)}\sqrt{\tan(x)}\csc(x)\right)$$
$$y(x) \to \arccos\left(-\frac{1}{4}c_1\sqrt{\cos(x)}\sqrt{\tan(x)}\csc(x)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

22.5 problem 1(e)

Internal problem ID [5329]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 1(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^3x^2 - x^3y^2y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x^2*y(x)^3-x^3*y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1x$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 19

```
DSolve[x^2*y[x]^3-x^3*y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to c_1x$$
$$y(x) \to 0$$

22.6 problem 1(f)

Internal problem ID [5330]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 1(f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd typ`

$$\boxed{x + y + (-y + x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 49

```
dsolve((x+y(x))+(x-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x - \sqrt{2c_1^2 x^2 + 1}}{c_1}$$
$$y(x) = \frac{c_1 x + \sqrt{2c_1^2 x^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.429 (sec). Leaf size: 86

```
DSolve[(x+y[x])+(x-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to x + \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to x - \sqrt{2}\sqrt{x^2}$$
$$y(x) \to \sqrt{2}\sqrt{x^2} + x$$

22.7 problem 1(g)

Internal problem ID [5331]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 1(g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{2\,\mathrm{e}^{2x}y+2x\cos\left(y\right)+\left(\mathrm{e}^{2x}-x^{2}\sin\left(y\right)\right)y'=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve((2*y(x)*exp(2*x)+2*x*cos(y(x)))+(exp(2*x)-x^2*sin(y(x)))*diff(y(x),x)=0,y(x), singsol=
```

$$\cos\left(y(x)\right)x^{2}+y(x)\,\mathrm{e}^{2x}+c_{1}=0$$

✓ Solution by Mathematica

Time used: 0.416 (sec). Leaf size: 30

```
DSolve[(2*y[x]*Exp[2*x]+2*x*Cos[y[x]])+(Exp[2*x]-x^2*Sin[y[x]])*y'[x]==0,y[x],x,IncludeSingul
```

$$\mathrm{Solve}\left[2\left(\frac{1}{2}x^{2}\cos(y(x))+\frac{1}{2}e^{2x}y(x)\right)=c_{1},y(x)\right]$$

22.8 problem 1(h)

Internal problem ID [5332]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 1(h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{3\ln(x)\,x^2 + x^2 + y + y'x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve((3*x^2*ln(x)+x^2+y(x))+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x^3\ln(x) + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 19

```
DSolve[(3*x^2*Log[x]+x^2+y[x])+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-x^3\log(x) + c_1}{x}$$

22.9 problem 2(a)

Internal problem ID [5333]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2y^3 + 2 + 3xy^2y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 99

```
dsolve((2*y(x)^3+2)+(3*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(-x^2 + c_1\right) x\right)^{\frac{1}{3}}}{x}$$
$$y(x) = -\frac{\left(\left(-x^2 + c_1\right) x\right)^{\frac{1}{3}}}{2x} - \frac{i\sqrt{3}\left(\left(-x^2 + c_1\right) x\right)^{\frac{1}{3}}}{2x}$$
$$y(x) = -\frac{\left(\left(-x^2 + c_1\right) x\right)^{\frac{1}{3}}}{2x} + \frac{i\sqrt{3}\left(\left(-x^2 + c_1\right) x\right)^{\frac{1}{3}}}{2x}$$

✓ Solution by Mathematica

Time used: 0.279 (sec). Leaf size: 215

```
DSolve[(3*y[x]^3+2)+(3*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{-\frac{1}{3}}\sqrt[3]{-2x^3+e^{9c_1}}}{x}$$
$$y(x) \to \frac{\sqrt[3]{-2x^3+e^{9c_1}}}{\sqrt[3]{3}x}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{-2x^3+e^{9c_1}}}{\sqrt[3]{3}x}$$
$$y(x) \to \sqrt[3]{-\frac{2}{3}}$$
$$y(x) \to -\sqrt[3]{\frac{2}{3}}$$
$$y(x) \to -(-1)^{2/3}\sqrt[3]{\frac{2}{3}}$$
$$y(x) \to \frac{\sqrt[3]{-\frac{2}{3}}x^2}{(-x^3)^{2/3}}$$
$$y(x) \to \frac{\sqrt[3]{\frac{2}{3}}\sqrt[3]{-x^3}}{x}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{\frac{2}{3}}\sqrt[3]{-x^3}}{x}$$

22.10 problem 2(b)

Internal problem ID [5334]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 2(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-2y' \sin(y) \sin(x) + \cos(x) \cos(y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(cos(x)*cos(y(x))-2*sin(x)*sin(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{1}{\sqrt{c_1 \sin(x)}}\right)$$
$$y(x) = \pi - \arccos\left(\frac{1}{\sqrt{c_1 \sin(x)}}\right)$$

✓ Solution by Mathematica

Time used: 0.505 (sec). Leaf size: 47

```
DSolve[Cos[x]*cos[y[x]]-(2*Sin[x]*Sin[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sin(K[1])}{\cos(K[1])} dK[1]\&\right]\left[\frac{1}{2}(\log(\tan(x)) + \log(\cos(x))) + c_1\right]$$
$$y(x) \to \cos^{(-1)}(0)$$

22.11 problem 2(c)

Internal problem ID [5335]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 2(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$5y^2x^3 + 2y + (3yx^4 + 2x)\, y' = 0$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 347

```
dsolve((5*x^3*y(x)^2+2*y(x))+(3*x^4*y(x)+2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\frac{\left(\frac{6\left(\left(108x^2+12\sqrt{-12c_1^4+81x^4}\right)c_1\right)^{\frac{1}{3}}}{c_1} + \frac{72c_1}{\left(\left(108x^2+12\sqrt{-12c_1^4+81x^4}\right)c_1\right)^{\frac{1}{3}}}\right)^2}{1296} - 1}{x^3}$$

$$y(x) = \frac{\frac{\left(-\frac{3\left(\left(108x^2+12\sqrt{-12c_1^4+81x^4}\right)c_1\right)^{\frac{1}{3}}}{c_1} - \frac{36c_1}{\left(\left(108x^2+12\sqrt{-12c_1^4+81x^4}\right)c_1\right)^{\frac{1}{3}}} - 18i\sqrt{3}\left(\frac{\left(\left(108x^2+12\sqrt{-12c_1^4+81x^4}\right)c_1\right)^{\frac{1}{3}}}{6c_1} - \frac{2c_1}{\left(\left(108x^2+12\sqrt{-12c_1^4+81x^4}\right)c\right.}\right.}{1296}}{x^3}$$

$$y(x) = \frac{\frac{\left(-\frac{3\left(\left(108x^2+12\sqrt{-12c_1^4+81x^4}\right)c_1\right)^{\frac{1}{3}}}{c_1} - \frac{36c_1}{\left(\left(108x^2+12\sqrt{-12c_1^4+81x^4}\right)c_1\right)^{\frac{1}{3}}} + 18i\sqrt{3}\left(\frac{\left(\left(108x^2+12\sqrt{-12c_1^4+81x^4}\right)c_1\right)^{\frac{1}{3}}}{6c_1} - \frac{2c_1}{\left(\left(108x^2+12\sqrt{-12c_1^4+81x^4}\right)c\right.}\right.}{1296}}{x^3}$$

✓ Solution by Mathematica

Time used: 36.242 (sec). Leaf size: 400

```
DSolve[(5*x^3*y[x]^2+2*y[x])+(3*x^4*y[x]+2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{-2x^2 + \frac{2x^4}{\sqrt[3]{\frac{27c_1x^{10}}{2} - x^6 + \frac{3}{2}\sqrt{3}\sqrt{c_1x^{16}\left(-4+27c_1x^4\right)}}} + 2^{2/3}\sqrt[3]{27c_1x^{10} - 2x^6 + 3\sqrt{3}\sqrt{c_1x^{16}\left(-4+27c_1\right.}}}{6x^5}$$

$$y(x) \to \frac{-4x^2 - \frac{2\left(1+i\sqrt{3}\right)x^4}{\sqrt[3]{\frac{27c_1x^{10}}{2} - x^6 + \frac{3}{2}\sqrt{3}\sqrt{c_1x^{16}\left(-4+27c_1x^4\right)}}} + i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{27c_1x^{10} - 2x^6 + 3\sqrt{3}\sqrt{c_1x^{16}\left(\right.}}}{12x^5}$$

$$y(x) \to -\frac{4x^2 - \frac{2i\left(\sqrt{3}+i\right)x^4}{\sqrt[3]{\frac{27c_1x^{10}}{2} - x^6 + \frac{3}{2}\sqrt{3}\sqrt{c_1x^{16}\left(-4+27c_1x^4\right)}}} + 2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{27c_1x^{10} - 2x^6 + 3\sqrt{3}\sqrt{c_1x^{16}\left(\right.}}}{12x^5}$$

22.12 problem 2(d)

Internal problem ID [5336]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 5. Existence and uniqueness of solutions to first order equations. Page 198
Problem number: 2(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{e^{y}+x\,e^{y}+x\,e^{y}y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((exp(y(x))+x*exp(y(x)))+(x*exp(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-x-\ln(x)+c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 15

```
DSolve[(Exp[y[x]]+x*Exp[y[x]])+(x*Exp[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to -x-\log(x)+c_1$$

23 Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238

23.1 problem 1(a)

Internal problem ID [5337]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-x} c_1 + x + c_2$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 18

```
DSolve[y''[x]+y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - c_1 e^{-x} + c_2$$

23.2 problem 1(b)

Internal problem ID [5338]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + e^x y' - e^x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+exp(x)*diff(y(x),x)=exp(x),y(x), singsol=all)
```

$$y(x) = -c_1 \operatorname{Ei}_1(e^x) + x + c_2$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 18

```
DSolve[y''[x]+Exp[x]*y'[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{ExpIntegralEi}(-e^x) + x + c_2$$

23.3 problem 1(c)

Internal problem ID [5339]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible,`

$$\boxed{yy'' + 4y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 158

```
dsolve(y(x)*diff(y(x),x$2)+4*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = (5c_1x + 5c_2)^{\frac{1}{5}}$$
$$y(x) = \left(-\frac{\sqrt{5}}{4} - \frac{1}{4} - \frac{i\sqrt{2}\,\sqrt{5-\sqrt{5}}}{4}\right)(5c_1x + 5c_2)^{\frac{1}{5}}$$
$$y(x) = \left(-\frac{\sqrt{5}}{4} - \frac{1}{4} + \frac{i\sqrt{2}\,\sqrt{5-\sqrt{5}}}{4}\right)(5c_1x + 5c_2)^{\frac{1}{5}}$$
$$y(x) = \left(\frac{\sqrt{5}}{4} - \frac{1}{4} - \frac{i\sqrt{2}\,\sqrt{5+\sqrt{5}}}{4}\right)(5c_1x + 5c_2)^{\frac{1}{5}}$$
$$y(x) = \left(\frac{\sqrt{5}}{4} - \frac{1}{4} + \frac{i\sqrt{2}\,\sqrt{5+\sqrt{5}}}{4}\right)(5c_1x + 5c_2)^{\frac{1}{5}}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 20

```
DSolve[y[x]*y''[x]+4*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sqrt[5]{5x - c_1}$$

23.4 problem 1(d)

Internal problem ID [5340]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + k^2 y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+k^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(kx) + c_2 \cos(kx)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y''[x]+k^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(kx) + c_2 \sin(kx)$$

23.5 problem 1(e)

Internal problem ID [5341]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _L`

$$\boxed{y'' - yy' = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)=y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\frac{(c_2+x)\sqrt{2}}{2c_1}\right)\sqrt{2}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 34

```
DSolve[y''[x]==y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{2}\sqrt{c_1}\tan\left(\frac{\sqrt{c_1}(x+c_2)}{\sqrt{2}}\right)$$

23.6 problem 1(f)

Internal problem ID [5342]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - 2y' - x^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)-2*diff(y(x),x)=x^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{4}x^4 + \frac{1}{3}c_1x^3 + c_2$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 24

```
DSolve[x*y''[x]-2*y'[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4}{4} + \frac{c_1x^3}{3} + c_2$$

23.7 problem 2

Internal problem ID [5343]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' - 1 - {y'}^2 = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 7

```
dsolve([diff(y(x),x$2)=1+diff(y(x),x)^2,y(0) = 0, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \ln(\sec(x))$$

✓ Solution by Mathematica

Time used: 1.809 (sec). Leaf size: 27

```
DSolve[{y''[x]==1+(y'[x])^2,{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log(-\cos(x)) + i\pi$$
$$y(x) \to -\log(\cos(x))$$

23.8 problem 3

Internal problem ID [5344]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_poly_yn`

$$\boxed{y'' + \frac{1}{2y'^2} = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.438 (sec). Leaf size: 26

```
dsolve([diff(y(x),x$2)=-1/(2*diff(y(x),x)^2),y(0) = 1, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = \frac{3\left(x + \frac{2}{3}\right)(-12x - 8)^{\frac{1}{3}}\left(-1 + i\sqrt{3}\right)}{16} + \frac{3}{2}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 27

```
DSolve[{y''[x]==-1/(2*(y'[x])^2),{y[0]==1,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{8}\left(12 - (-2)^{2/3}(-3x - 2)^{4/3}\right)$$

23.9 problem 5(b)

Internal problem ID [5345]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 5(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' + \sin(y) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = \beta]$$

✓ Solution by Maple

Time used: 0.812 (sec). Leaf size: 53

```
dsolve([diff(y(x),x$2)+sin(y(x))=0,y(0) = 0, D(y)(0) = beta],y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(-\left(\int_0^{\_Z} \frac{1}{\sqrt{2\cos(\_a) + \beta^2 - 2}} d\_a\right) + x\right)$$
$$y(x) = \text{RootOf}\left(\int_0^{\_Z} \frac{1}{\sqrt{2\cos(\_a) + \beta^2 - 2}} d\_a + x\right)$$

✓ Solution by Mathematica

Time used: 0.315 (sec). Leaf size: 19

```
DSolve[{y''[x]+Sin[y[x]]==0,{y[0]==0,y'[0]==\[Beta]}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to 2\,\text{JacobiAmplitude}\left(\frac{x\beta}{2}, \frac{4}{\beta^2}\right)$$

23.10 problem 5(c)

Internal problem ID [5346]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 238
Problem number: 5(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' + \sin(y) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)+sin(y(x))=0,y(0) = 0, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(-\left(\int_0^{\_Z} \frac{1}{\sqrt{2\cos(\_a) + 2}} d\_a\right) + x\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]+Sin[y[x]]==0,{y[0]==0,y'[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

{}

24 Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 250

24.1 problem 3

Internal problem ID [5347]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 250
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(x) = y_1(x)$$
$$y_2'(x) = y_1(x) + y_2(x)$$

With initial conditions

$$[y_1(0) = 1, y_2(0) = 2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 16

```
dsolve([diff(y__1(x),x) = y__1(x), diff(y__2(x),x) = y__1(x)+y__2(x), y__1(0) = 1, y__2(0) =
```

$$y_1(x) = e^x$$

$$y_2(x) = e^x(x + 2)$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 18

```
DSolve[{y1'[x]==y1[x],y2'[x]==y1[x]+y2[x]},{y1[0]==1,y2[0]==2},{y1[x],y2[x]},x,IncludeSingula
```

$$\text{y1}(x) \to e^x$$
$$\text{y2}(x) \to e^x(x + 2)$$

24.2 problem 4

Internal problem ID [5348]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 250
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(x) = y_2(x)$$
$$y_2'(x) = 6y_1(x) + y_2(x)$$

With initial conditions

$$[y_1(0) = 1, y_2(0) = -1]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 34

```
dsolve([diff(y__1(x),x) = y__2(x), diff(y__2(x),x) = 6*y__1(x)+y__2(x), y__1(0) = 1, y__2(0)
```

$$y_1(x) = \frac{e^{3x}}{5} + \frac{4\,e^{-2x}}{5}$$

$$y_2(x) = \frac{3\,e^{3x}}{5} - \frac{8\,e^{-2x}}{5}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 42

```
DSolve[{y1'[x]==y2[x],y2'[x]==6*y1[x]+y2[x]},{y1[0]==1,y2[0]==-1},{y1[x],y2[x]},x,IncludeSing
```

$$\text{y1}(x) \to \frac{1}{5}e^{-2x}\left(e^{5x} + 4\right)$$
$$\text{y2}(x) \to \frac{1}{5}e^{-2x}\left(3e^{5x} - 8\right)$$

24.3 problem 5

Internal problem ID [5349]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 250
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(x) = y_1(x) + y_2(x)$$
$$y_2'(x) = y_1(x) + y_2(x) + e^{3x}$$

With initial conditions

$$[y_1(0) = 0, y_2(0) = 0]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 36

```
dsolve([diff(y__1(x),x) = y__1(x)+y__2(x), diff(y__2(x),x) = y__1(x)+y__2(x)+exp(3*x), y__1(0
```

$$y_1(x) = -\frac{e^{2x}}{2} + \frac{e^{3x}}{3} + \frac{1}{6}$$

$$y_2(x) = -\frac{e^{2x}}{2} + \frac{2\,e^{3x}}{3} - \frac{1}{6}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 45

```
DSolve[{y1'[x]==y1[x]+y2[x],y2'[x]==y1[x]+y2[x]+Exp[3*x]},{y1[0]==0,y2[0]==0},{y1[x],y2[x]},x
```

$$\text{y1}(x) \to \frac{1}{6}(e^x - 1)^2 (2e^x + 1)$$
$$\text{y2}(x) \to \frac{1}{6}\left(e^{2x}(4e^x - 3) - 1\right)$$

25 Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 254

25.1 problem 2

Internal problem ID [5350]

Book: An introduction to Ordinary Differential Equations. Earl A. Coddington. Dover. NY 1961
Section: Chapter 6. Existence and uniqueness of solutions to systems and nth order equations. Page 254
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(x) &= 3y_1(x) + xy_3(x) \\
y_2'(x) &= y_2(x) + x^3 y_3(x) \\
y_3'(x) &= 2xy_2(x) - y_2(x) + e^x y_3(x)
\end{aligned}
$$

✗ Solution by Maple

```
dsolve([diff(y__1(x),x)=3*y__1(x)+x*y__3(x),diff(y__2(x),x)=y__2(x)+x^3*y__3(x),diff(y__3(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y1'[x]==3*y1[x]+x*y3[x],y2'[x]==y2[x]+x^3*y3[x],y3'[x]==2*x*y1[x]-y2[x]+Exp[x]*y3[x]}
```

Not solved