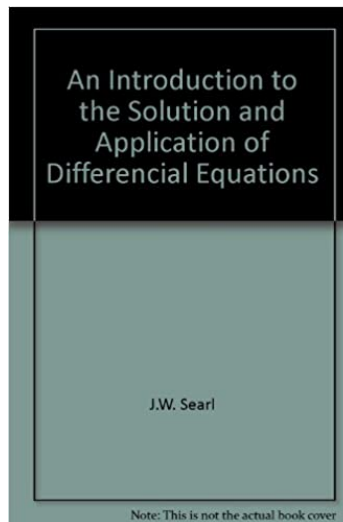


A Solution Manual For

**An introduction to the solution
and applications of differential
equations, J.W. Searl, 1966**



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1 Chapter 4, Ex. 4.1

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1.1 problem 1

Internal problem ID [2625]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966

Section: Chapter 4, Ex. 4.1

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$2yx + y'x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(2*x*y(x)+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 16

```
DSolve[2*x*y[x]+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1}{x^2}$$

$$y(x) \rightarrow 0$$

1.2 problem 2

Internal problem ID [2626]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966

Section: Chapter 4, Ex. 4.1

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _exact, _rational, [_Abel, '2nd typ`

$$x + y + (x - y)y' = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 22

```
dsolve([(x+y(x))+(x-y(x))*diff(y(x),x)=0,y(0) = 0],y(x), singsol=all)
```

$$y(x) = (1 + \sqrt{2})x$$

$$y(x) = -(\sqrt{2} - 1)x$$

✓ Solution by Mathematica

Time used: 0.451 (sec). Leaf size: 40

```
DSolve[{(x+y[x])+(x-y[x])*y'[x]==0,y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x - \sqrt{2}\sqrt{x^2}$$

$$y(x) \rightarrow \sqrt{2}\sqrt{x^2} + x$$

1.3 problem 3

Internal problem ID [2627]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966

Section: Chapter 4, Ex. 4.1

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$y' \ln(x) + \frac{x+y}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(ln(x)*diff(y(x),x)+(x+y(x))/x=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 - x}{\ln(x)}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 16

```
DSolve[Log[x]*y'[x]+(x+y[x])/x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{-x + c_1}{\log(x)}$$

1.4 problem 4

Internal problem ID [2628]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966

Section: Chapter 4, Ex. 4.1

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [exact]

$$\cos(y) - x \sin(y) y' - \sec(x)^2 = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.5 (sec). Leaf size: 17

```
dsolve([cos(y(x))-x*sin(y(x))*diff(y(x),x)=sec(x)^2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{\tan(x)}{x}\right) (1 - 2_B2)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{Cos[y[x]]-x*Sin[y[x]]*y'[x]==Sec[x]^2,y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

{}

1.5 problem 5

Internal problem ID [2629]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966

Section: Chapter 4, Ex. 4.1

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_exact]

$$y \sin\left(\frac{x}{y}\right) + x \cos\left(\frac{x}{y}\right) - 1 + \left(x \sin\left(\frac{x}{y}\right) - \frac{x^2 \cos\left(\frac{x}{y}\right)}{y}\right) y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve((y(x)*sin(x/y(x))+x*cos(x/y(x))-1)+(x*sin(x/y(x))-x^2/y(x)*cos(x/y(x)))*diff(y(x),x)=0
```

$$y(x) = \frac{x}{\text{RootOf}(x^2 \sin(_Z) + _Z c_1 - _Z x)}$$

✓ Solution by Mathematica

Time used: 0.424 (sec). Leaf size: 20

```
DSolve[(y[x]*Sin[x/y[x]]+x*Cos[x/y[x]]-1)+(x*Sin[x/y[x]]-x^2/y[x]*Cos[x/y[x]])*y'[x]==0,y[x],
```

$$\text{Solve}\left[x - xy(x) \sin\left(\frac{x}{y(x)}\right) = c_1, y(x)\right]$$

1.6 problem 6

Internal problem ID [2630]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966

Section: Chapter 4, Ex. 4.1

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _exact, _rational, _dAlembert]`

$$\frac{x}{x^2 + y^2} + \frac{y}{x^2} + \left(\frac{y}{x^2 + y^2} - \frac{1}{x} \right) y' = 0$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 33

```
dsolve([(x/(x^2+y(x)^2)+y(x)/x^2)+(y(x)/(x^2+y(x)^2)-1/x)*diff(y(x),x)=0,y(1) = 0],y(x), sing
```

$$y(x) = \frac{x(\text{RootOf}(4 + 4 \ln(x)^2 + 4 \ln(x) _Z + _Z^2 - 4 e^{-Z}) + 2 \ln(x))}{2}$$

✓ Solution by Mathematica

Time used: 0.159 (sec). Leaf size: 28

```
DSolve[{(x/(x^2+y[x]^2)+y[x]/x^2)+(y[x]/(x^2+y[x]^2)-1/x)*y'[x]==0,y[1]==0},y[x],x,IncludeSin
```

$$\text{Solve} \left[\frac{y(x)}{x} - \frac{1}{2} \log \left(\frac{y(x)^2}{x^2} + 1 \right) = \log(x), y(x) \right]$$

2 Chapter 4, Ex. 4.2

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2.1 problem 1

Internal problem ID [2631]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966

Section: Chapter 4, Ex. 4.2

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$x^2(1+y^2)y' + y^2(x^2+1) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 94

```
dsolve(x^2*(1+y(x)^2)*diff(y(x),x)+y(x)^2*(x^2+1)=0,y(x), singsol=all)
```

$$y(x) = \frac{-c_1x - x^2 + 1 + \sqrt{c_1^2x^2 + 2c_1x^3 + x^4 - 2c_1x + 2x^2 + 1}}{2x}$$

$$y(x) = -\frac{c_1x + x^2 + \sqrt{c_1^2x^2 + 2c_1x^3 + x^4 - 2c_1x + 2x^2 + 1} - 1}{2x}$$

✓ Solution by Mathematica

Time used: 1.044 (sec). Leaf size: 95

```
DSolve[x^2*(1+y[x]^2)*y'[x]+y[x]^2*(x^2+1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x^2 + \sqrt{4x^2 + (-x^2 + c_1x + 1)^2} - c_1x - 1}{2x}$$

$$y(x) \rightarrow \frac{-x^2 + \sqrt{4x^2 + (-x^2 + c_1x + 1)^2} + c_1x + 1}{2x}$$

$$y(x) \rightarrow 0$$

2.2 problem 2

Internal problem ID [2632]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966

Section: Chapter 4, Ex. 4.2

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$x(x-1)y' - \cot(y) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*(x-1)*diff(y(x),x)=cot(y(x)),y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{x}{c_1(x-1)}\right)$$

✓ Solution by Mathematica

Time used: 53.226 (sec). Leaf size: 55

```
DSolve[x*(x-1)*y'[x]==Cot[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sec^{-1}\left(-\frac{e^{c_1}(x-1)}{x}\right)$$

$$y(x) \rightarrow \sec^{-1}\left(-\frac{e^{c_1}(x-1)}{x}\right)$$

$$y(x) \rightarrow -\frac{\pi}{2}$$

$$y(x) \rightarrow \frac{\pi}{2}$$

2.3 problem 3

Internal problem ID [2633]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966

Section: Chapter 4, Ex. 4.2

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$ry' - \frac{(a^2 - r^2) \tan(y)}{a^2 + r^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(r*diff(y(r),r)= (a^2-r^2)/(a^2+r^2)*tan(y(r)),y(r), singsol=all)
```

$$y(r) = \arcsin\left(\frac{rc_1}{a^2 + r^2}\right)$$

✓ Solution by Mathematica

Time used: 23.044 (sec). Leaf size: 26

```
DSolve[r*y'[r]== (a^2-r^2)/(a^2+r^2)*Tan[y[r]],y[r],r,IncludeSingularSolutions -> True]
```

$$y(r) \rightarrow \arcsin\left(\frac{e^{c_1}r}{a^2 + r^2}\right)$$

$$y(r) \rightarrow 0$$

2.4 problem 4

Internal problem ID [2634]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966

Section: Chapter 4, Ex. 4.2

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$\sqrt{x^2 + 1} y' + \sqrt{1 + y^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(sqrt(1+x^2)*diff(y(x),x)+sqrt(1+y(x)^2)=0,y(x), singsol=all)
```

$$y(x) = -\sinh(\operatorname{arcsinh}(x) + c_1)$$

✓ Solution by Mathematica

Time used: 0.317 (sec). Leaf size: 39

```
DSolve[Sqrt[1+x^2]*y'[x]+Sqrt[1+y[x]^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt{x^2 + 1} \sinh(c_1) - x \cosh(c_1)$$

$$y(x) \rightarrow -i$$

$$y(x) \rightarrow i$$

2.5 problem 5

Internal problem ID [2635]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966

Section: Chapter 4, Ex. 4.2

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - \frac{x(1+y^2)}{y(x^2+1)} = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=(x*(1+y(x)^2))/(y(x)*(1+x^2)),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \sqrt{2x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.517 (sec). Leaf size: 16

```
DSolve[{y'[x]==(x*(1+y[x]^2))/(y[x]*(1+x^2)),y[0]==1},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow \sqrt{2x^2 + 1}$$

2.6 problem 6

Internal problem ID [2636]

Book: An introduction to the solution and applications of differential equations, J.W. Searl, 1966

Section: Chapter 4, Ex. 4.2

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y^2 y' - 2 - 3y^6 = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 77

```
dsolve([y(x)^2*diff(y(x),x)=2+3*y(x)^6,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{3^{\frac{5}{6}} 2^{\frac{1}{6}} \tan(3\sqrt{6}x)^{\frac{1}{3}}}{3}$$

$$y(x) = \frac{2^{\frac{1}{6}} \tan(3\sqrt{6}x)^{\frac{1}{3}} \left(-3^{\frac{5}{6}} + 3i3^{\frac{1}{3}}\right)}{6}$$

$$y(x) = -\frac{2^{\frac{1}{6}} \tan(3\sqrt{6}x)^{\frac{1}{3}} \left(3^{\frac{5}{6}} + 3i3^{\frac{1}{3}}\right)}{6}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 87

```
DSolve[{y[x]^2*y'[x]==2+3*y[x]^6,y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt[6]{\frac{2}{3}} \sqrt[3]{\tan(3\sqrt{6}x)}$$

$$y(x) \rightarrow -\sqrt[3]{-1} \sqrt[6]{\frac{2}{3}} \sqrt[3]{\tan(3\sqrt{6}x)}$$

$$y(x) \rightarrow (-1)^{2/3} \sqrt[6]{\frac{2}{3}} \sqrt[3]{\tan(3\sqrt{6}x)}$$