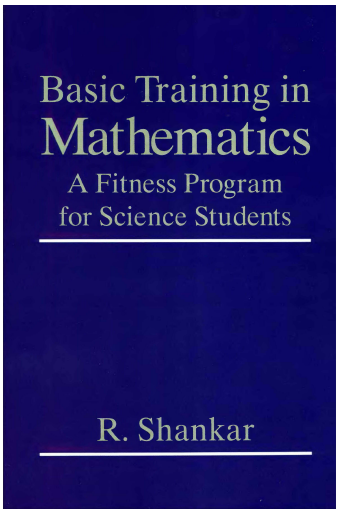
A Solution Manual For

Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995



Nasser M. Abbasi

October 12, 2023

Contents

1 Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients. page 307

1.1 problem 10.2.4

Internal problem ID [4536]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients. page 307
Problem number: 10.2.4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - \omega^2 x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(diff(x(t),t$2)-omega^2*x(t)=0,x(t), singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{-\omega t} + c_2 \mathrm{e}^{\omega t}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 23

```
DSolve[x''[t]-\[Omega]^2*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{t\omega} + c_2 e^{-t\omega}$$

1.2 problem 10.2.5

Internal problem ID [4537]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients. page 307
Problem number: 10.2.5.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{x''' - x'' + x' - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(x(t),t$3)-diff(x(t),t$2)+diff(x(t),t)-x(t)=0,x(t), singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{t} + c_2 \sin(t) + c_3 \cos(t)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[x'''[t]-x''[t]+x'[t]-x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_3 e^{t} + c_1 \cos(t) + c_2 \sin(t)$$

1.3 problem 10.2.8 part(1)

Internal problem ID [4538]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients. page 307
Problem number: 10.2.8 part(1).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 42x' + x = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 43

```
dsolve([diff(x(t),t$2)+42*diff(x(t),t)+x(t)=0,x(0) = 1, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{\left(220 + 21\sqrt{110}\right) \mathrm{e}^{\left(-21+2\sqrt{110}\right)t}}{440} + \frac{\left(220 - 21\sqrt{110}\right) \mathrm{e}^{\left(-21-2\sqrt{110}\right)t}}{440}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 53

```
DSolve[{x''[t]+42*x'[t]+x[t]==0,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{e^{-\left(\left(21+2\sqrt{110}\right)t\right)} \left(\left(881 + 84\sqrt{110}\right) e^{4\sqrt{110}t} - 1\right)}{880 + 84\sqrt{110}}$$

1.4 problem 10.2.8 part(2)

Internal problem ID [4539]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients. page 307
Problem number: 10.2.8 part(2).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{x'''' + x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 71

```
dsolve(diff(x(t),t$4)+x(t)=0,x(t), singsol=all)
```

$$x(t) = -c_1 \mathrm{e}^{-\frac{\sqrt{2}t}{2}} \sin\left(\frac{\sqrt{2}t}{2}\right) - c_2 \mathrm{e}^{\frac{\sqrt{2}t}{2}} \sin\left(\frac{\sqrt{2}t}{2}\right) + c_3 \mathrm{e}^{-\frac{\sqrt{2}t}{2}} \cos\left(\frac{\sqrt{2}t}{2}\right) + c_4 \mathrm{e}^{\frac{\sqrt{2}t}{2}} \cos\left(\frac{\sqrt{2}t}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 65

```
DSolve[x''''[t]+x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-\frac{t}{\sqrt{2}}} \left(\left(c_1 e^{\sqrt{2}t} + c_2 \right) \cos\left(\frac{t}{\sqrt{2}}\right) + \left(c_4 e^{\sqrt{2}t} + c_3 \right) \sin\left(\frac{t}{\sqrt{2}}\right) \right)$$

1.5 problem 10.2.8 part(3)

Internal problem ID [4540]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients. page 307
Problem number: 10.2.8 part(3).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{x''' - 3x'' - 9x' - 5x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(x(t),t$3)-3*diff(x(t),t$2)-9*diff(x(t),t)-5*x(t)=0,x(t), singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{5t} + c_2 \mathrm{e}^{-t} + c_3 \mathrm{e}^{-t} t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[x'''[t]-3*x''[t]-9*x'[t]-5*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-t}\left(c_2 t + c_3 e^{6t} + c_1\right)$$

1.6 problem 10.2.10

Internal problem ID [4541]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients. page 307
Problem number: 10.2.10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 2\gamma x' + \omega_0 x - F\cos(\omega t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 89

```
dsolve(diff(x(t),t$2)+2*gamma*diff(x(t),t)+omega__0*x(t)=F*cos(omega*t),x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{\left(-\gamma+\sqrt{\gamma^2-\omega_0}\right)t}c_2 + \mathrm{e}^{\left(-\gamma-\sqrt{\gamma^2-\omega_0}\right)t}c_1 + \frac{F((-\omega^2+\omega_0)\cos(\omega t) + 2\sin(\omega t)\gamma\omega)}{\omega^4 + 2(2\gamma^2-\omega_0)\omega^2 + \omega_0^2}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 106

```
DSolve[x''[t]+2*\[Gamma]*x'[t]+Subscript[\[Omega],0]*x[t]==F*Cos[\[Omega]*t],x[t],t,IncludeSi
```

$$x(t) \to \frac{2\gamma F\omega\sin(t\omega) + F(\omega_0-\omega^2)\cos(t\omega)}{4\gamma^2\omega^2+\omega^4-2\omega_0\omega^2+\omega_0^2} + c_1 e^{-t\left(\sqrt{\gamma^2-\omega_0}+\gamma\right)} + c_2 e^{t\left(\sqrt{\gamma^2-\omega_0}-\gamma\right)}$$

1.7 problem 10.2.11 (i)

Internal problem ID [4542]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients. page 307
Problem number: 10.2.11 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 2y - \mathrm{e}^{2x} = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-2*y(x)=exp(2*x),y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(3x+2)\,\mathrm{e}^{2x}}{9} + \frac{7\,\mathrm{e}^{-x}}{9}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 27

```
DSolve[{y''[x]-y'[x]-2*y[x]==Exp[2*x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{9}e^{-x}\left(e^{3x}(3x+2)+7\right)$$

1.8 problem 10.2.11 (ii)

Internal problem ID [4543]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients. page 307
Problem number: 10.2.11 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y - 2\cos(x) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+y(x)=2*cos(x),y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} - \sin(x)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 13

```
DSolve[{y''[x]-2*y'[x]+y[x]==2*Cos[x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{x} - \sin(x)$$

1.9 problem 10.2.11 (iii)

Internal problem ID [4544]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients. page 307
Problem number: 10.2.11 (iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 16y - 16\cos(4x) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)+16*y(x)=16*cos(4*x),y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \cos(4x) + 2\sin(4x)\, x$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 17

```
DSolve[{y''[x]+16*y[x]==16*Cos[4*x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to 2x\sin(4x) + \cos(4x)$$

1.10 problem 10.2.11 (iv)

Internal problem ID [4545]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.2, ODEs with constant Coefficients. page 307
Problem number: 10.2.11 (iv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y - \cosh(x) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)-y(x)=cosh(x),y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(-x+2)\,\mathrm{e}^{-x}}{4} + \frac{\mathrm{e}^{x}(x+2)}{4}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 15

```
DSolve[{y''[x]-y[x]==Cosh[x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x\sinh(x) + \cosh(x)$$

2 Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients. First order. page 315

2.1 problem 10.3.2

Internal problem ID [4546]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients. First order. page 315
Problem number: 10.3.2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y - \mathrm{e}^{2x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)-y(x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = (\mathrm{e}^{x} + c_1)\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 15

```
DSolve[y'[x]-y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(e^{x} + c_1)$$

2.2 problem 10.3.3

Internal problem ID [4547]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients. First order. page 315
Problem number: 10.3.3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x^2y' + 2xy - x + 1 = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([x^2*diff(y(x),x)+2*x*y(x)-x+1=0,y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(x-1)^2}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 17

```
DSolve[{x^2*y'[x]+2*x*y[x]-x+1==0,{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x-1)^2}{2x^2}$$

2.3 problem 10.3.4

Internal problem ID [4548]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients. First order. page 315
Problem number: 10.3.4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y - (x+1)^2 = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)+y(x)=(x+1)^2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = x^2 + 1 - \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.103 (sec). Leaf size: 16

```
DSolve[{y'[x]+y[x]==(x+1)^2,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + \sinh(x) - \cosh(x) + 1$$

2.4 problem 10.3.5

Internal problem ID [4549]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients. First order. page 315
Problem number: 10.3.5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x^2 y' + 2xy - \sinh(x) = 0}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([x^2*diff(y(x),x)+2*x*y(x)=sinh(x),y(1) = 2],y(x), singsol=all)
```

$$y(x) = \frac{\cosh(x) + 2 - \cosh(1)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 17

```
DSolve[{x^2*y'[x]+2*x*y[x]==Sinh[x],{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cosh(x) + 2 - \cosh(1)}{x^2}$$

2.5 problem 10.3.6

Internal problem ID [4550]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients. First order. page 315
Problem number: 10.3.6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{1-x} + 2x - x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)+y(x)/(1-x)+2*x-x^2=0,y(x), singsol=all)
```

$$y(x) = \left(\frac{x^2}{2} - x - \ln(x-1) + c_1\right)(x-1)$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 27

```
DSolve[y'[x]+y[x]/(1-x)+2*x-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x-1)\left(\frac{1}{2}(x-1)^2 - \log(x-1) + c_1\right)$$

2.6 problem 10.3.7

Internal problem ID [4551]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients. First order. page 315
Problem number: 10.3.7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{1-x} + x - x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)/(1-x)+x-x^2=0,y(x), singsol=all)
```

$$y(x) = \left(\frac{x^2}{2} + c_1\right)(x-1)$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 20

```
DSolve[y'[x]+y[x]/(1-x)+x-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(x-1)\left(x^2 + 2c_1\right)$$

2.7 problem 10.3.8

Internal problem ID [4552]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients. First order. page 315
Problem number: 10.3.8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\left(x^2+1\right)y' - xy - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+x^2)*diff(y(x),x)=1+x*y(x),y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x^2+1} + x$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 19

```
DSolve[(1+x^2)*y'[x]==1+x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1\sqrt{x^2+1}$$

2.8 problem 10.3.9 (a)

Internal problem ID [4553]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients. First order. page 315
Problem number: 10.3.9 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy + y' - xy^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+x*y(x)=x*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{1 + \mathrm{e}^{\frac{x^2}{2}} c_1}$$

✓ Solution by Mathematica

Time used: 0.259 (sec). Leaf size: 31

```
DSolve[y'[x]+x*y[x]==x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{1 + e^{\frac{x^2}{2} + c_1}}$$
$$y(x) \to 0$$
$$y(x) \to 1$$

2.9 problem 10.3.9 (b)

Internal problem ID [4554]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.3, ODEs with variable Coefficients. First order. page 315
Problem number: 10.3.9 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{3y'x + y + x^2y^4 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 124

```
dsolve(3*x*diff(y(x),x)+y(x)+x^2*y(x)^4=0,y(x), singsol=all)
```

$$y(x) = \frac{\left((x+c_1)^2 x^2\right)^{\frac{1}{3}}}{(x+c_1)\,x}$$

$$y(x) = -\frac{\left((x+c_1)^2 x^2\right)^{\frac{1}{3}}}{2\,(x+c_1)\,x} - \frac{i\sqrt{3}\left((x+c_1)^2 x^2\right)^{\frac{1}{3}}}{2\,(x+c_1)\,x}$$

$$y(x) = -\frac{\left((x+c_1)^2 x^2\right)^{\frac{1}{3}}}{2\,(x+c_1)\,x} + \frac{i\sqrt{3}\left((x+c_1)^2 x^2\right)^{\frac{1}{3}}}{2\,(x+c_1)\,x}$$

✓ Solution by Mathematica

Time used: 0.307 (sec). Leaf size: 61

```
DSolve[3*x*y'[x]+y[x]+x^2*y[x]^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\sqrt[3]{x(x+c_1)}}$$

$$y(x) \to -\frac{\sqrt[3]{-1}}{\sqrt[3]{x(x+c_1)}}$$

$$y(x) \to \frac{(-1)^{2/3}}{\sqrt[3]{x(x+c_1)}}$$

$$y(x) \to 0$$

3 Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients. Second order and Homogeneous. page 318

3.1 problem 10.4.8 (a)

Internal problem ID [4555]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients. Second order and Homogeneous. page 318
Problem number: 10.4.8 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x+1)^2 y'' + \left(-x^2+1\right) y' + (x-1)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*(x+1)^2*diff(y(x),x$2)+(1-x^2)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x+1) + c_2(x+1)\ln(x)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 17

```
DSolve[x*(x+1)^2*y''[x]+(1-x^2)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x+1)(c_2 \log(x) + c_1)$$

3.2 problem 10.4.8 (b)

Internal problem ID [4556]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients. Second order and Homogeneous. page 318
Problem number: 10.4.8 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x(1-x)\,y'' + 2(-2x+1)\,y' - 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*(1-x)*diff(y(x),x$2)+2*(1-2*x)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x + c_2}{x\,(x-1)}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 22

```
DSolve[x*(1-x)*y''[x]+2*(1-2*x)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x + c_1}{x - x^2}$$

3.3 problem 10.4.8 (c)

Internal problem ID [4557]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients. Second order and Homogeneous. page 318
Problem number: 10.4.8 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$\boxed{x^2y'' + y'x - 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^3 + \frac{c_2}{x^3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]-9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^6 + c_1}{x^3}$$

3.4 problem 10.4.8 (d)

Internal problem ID [4558]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients. Second order and Homogeneous. page 318
Problem number: 10.4.8 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$\boxed{xy'' + \frac{y'}{2} + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x$2)+1/2*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(2\sqrt{x}\,\sqrt{2}\right) + c_2 \cos\left(2\sqrt{x}\,\sqrt{2}\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 38

```
DSolve[x*y''[x]+1/2*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(2\sqrt{2}\sqrt{x}\right) + c_2 \sin\left(2\sqrt{2}\sqrt{x}\right)$$

3.5 problem 10.4.8 (e)

Internal problem ID [4559]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients. Second order and Homogeneous. page 318
Problem number: 10.4.8 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x + c_2x\ln(x)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 15

```
DSolve[x^2*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2\log(x) + c_1)$$

3.6 problem 10.4.8 (f)

Internal problem ID [4560]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients. Second order and Homogeneous. page 318
Problem number: 10.4.8 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2xy'' - y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve(2*x*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(2\cos\left(2\sqrt{x}\right)\sqrt{x} - \sin\left(2\sqrt{x}\right)\right) + c_2\left(2\sin\left(2\sqrt{x}\right)\sqrt{x} + \cos\left(2\sqrt{x}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.092 (sec). Leaf size: 59

```
DSolve[2*x*y''[x]-y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2i\sqrt{x}}\left(2\sqrt{x} + i\right) + \frac{1}{8}c_2 e^{-2i\sqrt{x}}\left(1 + 2i\sqrt{x}\right)$$

3.7 problem 10.4.8 (g)

Internal problem ID [4561]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients. Second order and Homogeneous. page 318
Problem number: 10.4.8 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(x*diff(y(x),x$2)+x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(x^2 + 2x\right) + c_2\left(\frac{(-x-1)\,\mathrm{e}^{-x}}{2} + \frac{\mathrm{Ei}_1\left(x\right)x(x+2)}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 39

```
DSolve[x*y''[x]+x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x(x+2) - \frac{1}{2}c_2 e^{-x}(e^x(x+2)x\,\mathrm{ExpIntegralEi}(-x) + x + 1)$$

3.8 problem 10.4.8 (h)

Internal problem ID [4562]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients. Second order and Homogeneous. page 318
Problem number: 10.4.8 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x-1)^2 y'' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x*(x-1)^2*diff(y(x),x$2)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{x-1} + \frac{c_2(2\ln(x)\,x - x^2 + 1)}{x-1}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 31

```
DSolve[x*(x-1)^2*y''[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-x(c_2 x + c_1) + 2c_2 x \log(x) + c_2}{x-1}$$

3.9 problem 10.4.9 (i)

Internal problem ID [4563]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients. Second order and Homogeneous. page 318
Problem number: 10.4.9 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{2y}{x} - x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)-2*y(x)/x-x^2=0,y(x), singsol=all)
```

$$y(x) = (x + c_1)\, x^2$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 13

```
DSolve[y'[x]-2*y[x]/x-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(x + c_1)$$

3.10 problem 10.4.9 (ii)

Internal problem ID [4564]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients. Second order and Homogeneous. page 318
Problem number: 10.4.9 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{2y}{x} - x^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+2*y(x)/x-x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{\frac{x^6}{6} + c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 13

```
DSolve[y'[x]-2*y[x]/x-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(x + c_1)$$

3.11 problem 10.4.10

Internal problem ID [4565]

Book: Basic Training in Mathematics. By R. Shankar. Plenum Press. NY. 1995
Section: Chapter 10, Differential equations. Section 10.4, ODEs with variable Coefficients. Second order and Homogeneous. page 318
Problem number: 10.4.10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' + (1-x)\,y' + my = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x$2)+(1-x)*diff(y(x),x)+m*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\,\text{KummerM}\left(-m, 1, x\right) + c_2\,\text{KummerU}\left(-m, 1, x\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 21

```
DSolve[x*y''[x]+(1-x)*y'[x]+m*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\,\text{HypergeometricU}(-m, 1, x) + c_2\,\text{LaguerreL}(m, x)$$