A Solution Manual For

Collection of Kovacic problems

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Contents

1 section 1

1.1 problem 1

Internal problem ID [6734]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(x^2-1\right)y''-2y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2-1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2\left(x^2+1\right)$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 39

```
DSolve[(x^2-1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{\sqrt{x^2-1}(c_1(x-1)^2+c_2x)}{\sqrt{1-x^2}}$$

1.2 problem 2

Internal problem ID [6735]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(x^2-1\right)y''-6y'x+12y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x^2-1)*diff(y(x),x$2)-6*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^3+x\right)+c_2\left(x^4+6x^2+1\right)$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 45

```
DSolve[(x^2-1)*y''[x]-6*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\sqrt{x^2-1}(c_2x(x^2+1)+c_1(x-1)^4)}{\sqrt{1-x^2}}$$

1.3 problem 3

Internal problem ID [6736]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+3\right) y'' - 7y'x + 16y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 75

```
dsolve((x^2+3)*diff(y(x),x$2)-7*x*diff(y(x),x)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(x^4 - 9x^2 + \frac{27}{8}\right) + c_2\left(\frac{225}{2048} + \frac{\left(8x^4 - 72x^2 + 27\right)\ln\left(\sqrt{x^2+3} - x\right)}{512} + \frac{5\left(10x^3 - 33x\right)\sqrt{x^2+3}}{1536} + \frac{25x^4}{768} - \frac{75x^2}{256}\right)$$

✓ Solution by Mathematica

Time used: 0.239 (sec). Leaf size: 492

```
DSolve[(x^2+3)*y''[x]-7*x*y'[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to \frac{1}{24}c_2\Big(&12960x^2\text{RootSum}\Big[7838208000\#1^4-188281584000\#1^2-241544908800\#1 \\
&+18453344881\&,\#1\log\Big(-411757211968704000\#1^3-16606327460698080\#1^2+101387038251671139\text{6} \\
&+5248800x^2\text{RootSum}\Big[2108807205724800000\#1^4-3088288681560000\#1^2 \\
&+97825688064000\#1 \\
&+18453344881\&,\#1\log\Big(27353083060732502808000000\#1^3-272385286174100257200\#1^2-4106175\text{0} \\
&-4860\text{RootSum}\Big[7838208000\#1^4-188281584000\#1^2-241544908800\#1 \\
&+18453344881\&,\#1\log\Big(-411757211968704000\#1^3-16606327460698080\#1^2+101387038251671139\text{6} \\
&-1968300\text{RootSum}\Big[2108807205724800000\#1^4-3088288681560000\#1^2 \\
&+97825688064000\#1 \\
&+18453344881\&,\#1\log\Big(27353083060732502808000000\#1^3-272385286174100257200\#1^2-4106175\text{0} \\
&-1440x^4\text{RootSum}\Big[7838208000\#1^4-188281584000\#1^2-241544908800\#1 \\
&+18453344881\&,\#1\log\Big(-411757211968704000\#1^3-16606327460698080\#1^2+101387038251671139\text{6} \\
&-583200x^4\text{RootSum}\Big[2108807205724800000\#1^4-3088288681560000\#1^2 \\
&+97825688064000\#1 \\
&+18453344881\&,\#1\log\Big(27353083060732502808000000\#1^3-272385286174100257200\#1^2-4106175\text{0} \\
&+165\sqrt{x^2+3}x+216x^2\log\left(\sqrt{x^2+3}-x\right)-81\log\left(\sqrt{x^2+3}-x\right) \\
&-24x^4\log\left(\sqrt{x^2+3}-x\right)-50\sqrt{x^2+3}x^3\Big)+c_1\left(x^4-9x^2+\frac{27}{8}\right)
\end{aligned}
$$

1.4 problem 4

Internal problem ID [6737]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(x^2-1\right)y''+8y'x+12y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve((x^2-1)*diff(y(x),x$2)+8*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(3x^2+1)}{\left(x^2-1\right)^3}+\frac{c_2(x^3+3x)}{\left(x^2-1\right)^3}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 37

```
DSolve[(x^2-1)*y''[x]+8*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{3c_1(x-1)^3-c_2(3x^2+1)}{3\left(x^2-1\right)^3}$$

1.5 problem 5

Internal problem ID [6738]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3y'' + y'x - 4y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 57

```
dsolve(3*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(\mathrm{e}^{-\frac{x^2}{6}} \sqrt{6} \left(x^2 + 15\right) x + \left(x^4 + 18x^2 + 27\right) \operatorname{erf}\left(\frac{x\sqrt{6}}{6} \right) \sqrt{\pi} \right) + c_2 \left(x^4 + 18x^2 + 27\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 43

```
DSolve[3*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{6}} \operatorname{HermiteH}\left(-5, \frac{x}{\sqrt{6}}\right) + \frac{1}{27} c_2 \left(x^4 + 18x^2 + 27\right)$$

1.6 problem 6

Internal problem ID [6739]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{5y'' - 2y'x + 10y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(5*diff(y(x),x$2)-2*x*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\frac{4}{375}x^5 - \frac{4}{15}x^3 + x\right) + c_2 \,\text{hypergeom}\left(\left[-\frac{5}{2}\right], \left[\frac{1}{2}\right], \frac{x^2}{5}\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 77

```
DSolve[5*y''[x]-2*x*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\sqrt{5}x(4(x-5)(x+5)x^2+375)\left(64c_1 - \sqrt{\pi}c_2 \text{erfi}\left(\frac{x}{\sqrt{5}}\right)\right) + 10c_2 e^{\frac{x^2}{5}}(x^2-20)(2x^2-5)}{1000}$$

1.7 problem 7

Internal problem ID [6740]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x^2 - 3yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-3*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^3}{3}} x + \frac{c_2 \left(x^6 \operatorname{WhittakerM}\left(\frac{1}{3}, \frac{5}{6}, \frac{x^3}{3}\right) + (5x^3 + 10) \operatorname{WhittakerM}\left(\frac{4}{3}, \frac{5}{6}, \frac{x^3}{3}\right) \right) \mathrm{e}^{\frac{x^3}{6}}}{x^4}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 39

```
DSolve[y''[x]-x^2*y'[x]-3*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{\frac{x^3}{3}} \left(3c_1 x - c_2 \operatorname{ExpIntegralE}\left(\frac{4}{3}, \frac{x^3}{3}\right) \right)$$

1.8 problem 8

Internal problem ID [6741]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right) y'' + 2y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2(\arctan(x)\, x + 1)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 23

```
DSolve[(1+x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to ic_1 x - c_2(x \arctan(x) + 1)$$

1.9 problem 9

Internal problem ID [6742]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\left(x^2+1\right)\operatorname{erf}\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{\pi} + \sqrt{2}\,\mathrm{e}^{-\frac{x^2}{2}}x\right) + c_2\left(x^2+1\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 35

```
DSolve[y''[x]+x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{2}}\,\text{HermiteH}\left(-3, \frac{x}{\sqrt{2}}\right) + c_2\left(x^2+1\right)$$

1.10 problem 10

Internal problem ID [6743]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-6x+10\right)y''-4(x-3)y'+6y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve((x^2-6*x+10)*diff(y(x),x$2)-4*(x-3)*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^3-30x+60\right)+c_2\left(\frac{26}{3}+x^2-6x\right)$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 34

```
DSolve[(x^2-6*x+10)*y''[x]-4*(x-3)*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{3}i\left(3c_1(x-(3+i))^3+c_2(3(x-6)x+26)\right)$$

1.11 problem 11

Internal problem ID [6744]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+6x\right)y''+\left(3x+9\right)y'-3y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve((x^2+6*x)*diff(y(x),x$2)+(3*x+9)*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(3+x)+\frac{c_2(2x^2+12x+9)}{\sqrt{x}\sqrt{x+6}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 66

```
DSolve[(x^2+6*x)*y''[x]+(3*x+9)*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sqrt{\frac{2}{3\pi}}\left(c_1(2x(x+6)+9)-\pi c_2(x+3)\sqrt{-x(x+6)}\right)}{3\sqrt[4]{-x^2}\sqrt{x+6}}$$

1.12 problem 12

Internal problem ID [6745]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' + \left(t^2 - 1\right) y' + t^2 y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 57

```
dsolve(t*diff(y(t),t$2)+ (t^2-1)*diff(y(t),t)+t^2*y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{-\frac{(t-2)t}{2}} (t-1) + c_2 \left(\text{hypergeom}\left(\left[\frac{1}{2}\right], \left[\frac{3}{2}\right], \frac{(t-2)^2}{2} \right) (t-2) - \text{hypergeom}\left(\left[-\frac{1}{2}\right], \left[\frac{1}{2}\right], \frac{(t-2)^2}{2} \right) \right) \mathrm{e}^{-\frac{(t-2)t}{2}}$$

✓ Solution by Mathematica

Time used: 0.161 (sec). Leaf size: 69

```
DSolve[t*y''[t]+(t^2-1)*y'[t]+t^2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} e^{-\frac{t^2}{2}+t-2} \left((t-1) \left(\sqrt{2\pi} c_2 \operatorname{erfi}\left(\frac{t-2}{\sqrt{2}} \right) + 2e^2 c_1 \right) - 2c_2 e^{\frac{1}{2}(t-2)^2} \right)$$

1.13 problem 13

Internal problem ID [6746]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$t^2 y'' - t(t+2)\,y' + (t+2)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(t^2*diff(y(t),t$2)-t*(t+2)*diff(y(t),t)+(t+2)*y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2 t\,\mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]-t*(t+2)*y'[t]+(t+2)*y[t] == 0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t\left(c_2 e^t + c_1\right)$$

1.14 problem 14

Internal problem ID [6747]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{ty'' - y'(t+1) + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(t*diff(y(t),t$2)-(1+t)*diff(y(t),t)+y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1(t+1) + c_2 \mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 19

```
DSolve[t*y''[t]-(1+t)*y'[t]+y[t] == 0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^t - c_2(t+1)$$

1.15 problem 15

Internal problem ID [6748]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-t+1)\,y'' + ty' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((1-t)*diff(y(t),t$2)+t*diff(y(t),t)-y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2 \mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 17

```
DSolve[(1-t)*y''[t]+t*y'[t]-y[t] == 0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^t - c_2 t$$

1.16 problem 16

Internal problem ID [6749]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-25/100)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-25/100)*y[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2 e^{2ix})}{2\sqrt{x}}$$

1.17 problem 17

Internal problem ID [6750]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{ty'' - y'(t+1) + y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve(t*diff(y(t),t$2)-(1+t)*diff(y(t),t)+y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1(t+1) + c_2 \mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 19

```
DSolve[t*y''[t]-(1+t)*y'[t]+y[t] ==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^t - c_2(t+1)$$

1.18 problem 18

Internal problem ID [6751]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-t+1)\,y'' + ty' - y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 12

```
dsolve((1-t)*diff(y(t),t$2)+t*diff(y(t),t)-y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2 \mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 17

```
DSolve[(1-t)*y''[t]+t*y'[t]-y[t] ==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^t - c_2 t$$

1.19 problem 19

Internal problem ID [6752]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \,\mathrm{e}^{-\frac{x^2}{2}} + c_2 \left(i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-\frac{x^2}{2}}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,x}{2}\right) x + 2 \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 44

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}c_2 x\,\mathrm{DawsonF}\left(\frac{x}{\sqrt{2}}\right) + \sqrt{2}c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.20 problem 20

Internal problem ID [6753]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right)y''-4y'x+6y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve((1+x^2)*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(-3x^2+1\right)+c_2\left(x^3-3x\right)$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 33

```
DSolve[(1+x^2)*y''[x]-4*x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{3}i\left(c_2\left(3x^2-1\right)+3c_1(x-i)^3\right)$$

1.21 problem 21

Internal problem ID [6754]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\,y'' + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 17

```
DSolve[(1-x)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2 x$$

1.22 problem 22

Internal problem ID [6755]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y'' + y'x + 3y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(2*diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\sqrt{\pi}\,\mathrm{e}^{-\frac{x^2}{4}}\,\mathrm{erfi}\left(\frac{x}{2}\right)\left(x^2-2\right) - 2x\right) + c_2\mathrm{e}^{-\frac{x^2}{4}}\left(x^2-2\right)$$

✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 47

```
DSolve[2*y''[x]+x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}c_2\left(x^2-2\right)\mathrm{DawsonF}\left(\frac{x}{2}\right) + c_1 e^{-\frac{x^2}{4}}\left(x^2-2\right) - \frac{c_2 x}{4}$$

1.23 problem 23

Internal problem ID [6756]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \,\mathrm{e}^{-\frac{x^2}{2}} + c_2 \left(i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-\frac{x^2}{2}}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,x}{2}\right) x + 2 \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 44

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}c_2 x\,\mathrm{DawsonF}\left(\frac{x}{\sqrt{2}}\right) + \sqrt{2}c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.24 problem 24

Internal problem ID [6757]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\,y'' + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 17

```
DSolve[(1-x)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2 x$$

1.25 problem 25

Internal problem ID [6758]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \,\mathrm{e}^{-\frac{x^2}{2}} + c_2 \left(i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-\frac{x^2}{2}}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,x}{2}\right) x + 2 \right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 44

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}c_2 x\,\mathrm{DawsonF}\left(\frac{x}{\sqrt{2}}\right) + \sqrt{2}c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.26 problem 26

Internal problem ID [6759]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(-x^2+4\right)y''+y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve((4-x^2)*diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^2-4\right)^{\frac{3}{4}}\operatorname{LegendreP}\left(\sqrt{3}-\frac{1}{2},\frac{3}{2},\frac{x}{2}\right)+c_2\left(x^2-4\right)^{\frac{3}{4}}\operatorname{LegendreQ}\left(\sqrt{3}-\frac{1}{2},\frac{3}{2},\frac{x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 58

```
DSolve[(4-x^2)*y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\left(x^2-4\right)^{3/4}\left(c_1P^{\frac{3}{2}}_{-\frac{1}{2}+\sqrt{3}}\left(\frac{x}{2}\right)+c_2Q^{\frac{3}{2}}_{-\frac{1}{2}+\sqrt{3}}\left(\frac{x}{2}\right)\right)$$

1.27 problem 27

Internal problem ID [6760]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y''x^2 - 4y'x + \left(-16x^2 + 3\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(3-16*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sinh(2x) + c_2\sqrt{x}\,\cosh(2x)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 32

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(3-16*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}\sqrt{x}\left(c_2e^{4x} + 4c_1\right)$$

1.28 problem 28

Internal problem ID [6761]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-1+x)\,y'' - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2 x$$

1.29 problem 29

Internal problem ID [6762]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 - 2y'x + \left(x^2 + 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

1.30 problem 31

Internal problem ID [6763]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(x^2-2x\right)y''+\left(-x^2+2\right)y'+(2x-2)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((x^2-2*x)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)+(2*x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=x^2c_1+c_2\mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 18

```
DSolve[(x^2-2*x)*y''[x]+(2-x^2)*y'[x]+(2*x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to c_2x^2+c_1e^x$$

1.31 problem 32

Internal problem ID [6764]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y''x^2 + \left(-8x^2 + 4x\right)y' + \left(4x^2 - 4x - 1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)+(4*x-8*x^2)*diff(y(x),x)+(4*x^2-4*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^x}{\sqrt{x}} + c_2 \sqrt{x}\, \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 21

```
DSolve[4*x^2*y''[x]+(4*x-8*x^2)*y'[x]+(4*x^2-4*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{e^x(c_2 x + c_1)}{\sqrt{x}}$$

1.32 problem 33

Internal problem ID [6765]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y'x + \left(4x^2 + 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} x$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 20

```
DSolve[y''[x]+4*x*y'[x]+(4*x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2} (c_2 x + c_1)$$

1.33 problem 34

Internal problem ID [6766]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x+1)\,y'' - 2y' - (3+2x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((2*x+1)*diff(y(x),x$2)-2*diff(y(x),x)-(2*x+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 x\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 29

```
DSolve[(2*x+1)*y''[x]-2*y'[x]-(2*x+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x-\frac{1}{2}} \left(c_2 e^{2x+1} x + c_1\right)$$

1.34 problem 35

Internal problem ID [6767]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2x+2)\,y' + (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)-(2*x+2)*diff(y(x),x)+(x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x + c_2 \mathrm{e}^x x^3$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 23

```
DSolve[x*y''[x]-(2*x+2)*y'[x]+(x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^x \left(c_2 x^3 + 3c_1\right)$$

1.35 problem 36

Internal problem ID [6768]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 - 2y'x + \left(x^2 + 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

1.36 problem 38

Internal problem ID [6769]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y''x^2 - 4y'x + \left(-16x^2 + 3\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(3-16*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sinh(2x) + c_2\sqrt{x}\,\cosh(2x)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 32

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(3-16*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}\sqrt{x}\left(c_2e^{4x} + 4c_1\right)$$

1.37 problem 39

Internal problem ID [6770]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y''x^2 - 4y'x + \left(4x^2 + 3\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sin(x) + c_2\sqrt{x}\,\cos(x)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 39

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(4*x^2+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-ix}\sqrt{x}\left(2c_1 - ic_2e^{2ix}\right)$$

1.38 problem 40

Internal problem ID [6771]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' x^2 - 2y' x - \left(x^2 - 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)-(x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sinh(x) + c_2 x \cosh(x)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]-2*x*y'[x]-(x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} x + \frac{1}{2} c_2 e^{x} x$$

1.39 problem 41

Internal problem ID [6772]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' x^2 - 2x(1+x)\,y' + \left(x^2+2x+2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-2*x*(x+1)*diff(y(x),x)+(x^2+2*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{x} + c_2 \mathrm{e}^{x} x^2$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 17

```
DSolve[x^2*y''[x]-2*x*(x+1)*y'[x]+(x^2+2*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{x} x(c_2 x + c_1)$$

1.40 problem 42

Internal problem ID [6773]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' x^2 - 2x(x+2)\,y' + \left(x^2 + 4x + 6\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)-2*x*(x+2)*diff(y(x),x)+(x^2+4*x+6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x x^2 + c_2 \mathrm{e}^x x^3$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 19

```
DSolve[x^2*y''[x]-2*x*(x+2)*y'[x]+(x^2+4*x+6)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^x x^2 (c_2 x + c_1)$$

1.41 problem 43

Internal problem ID [6774]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 - 4y'x + \left(x^2 + 6\right) y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(x^2+6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^2 \sin(x) + c_2 \cos(x) x^2$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]-4*x*y'[x]+(x^2+6)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-ix} x^2 \left(2c_1 - ic_2 e^{2ix}\right)$$

1.42 problem 44

Internal problem ID [6775]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-1+x)\,y'' - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2 x$$

1.43 problem 45

Internal problem ID [6776]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y''x^2 - 4x(1+x)\,y' + (3+2x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*(x+1)*diff(y(x),x)+(2*x+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x}\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 20

```
DSolve[4*x^2*y''[x]-4*x*(x+1)*y'[x]+(2*x+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_2 e^{x} + c_1)$$

1.44 problem 46

Internal problem ID [6777]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x-1)\,y''-(3x+2)\,y'-(6x-8)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((3*x-1)*diff(y(x),x$2)-(3*x+2)*diff(y(x),x)-(6*x-8)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{2x}+\mathrm{e}^{-x}c_2x$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 35

```
DSolve[(3*x-1)*y''[x]-(3*x+2)*y'[x]-(6*x-8)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{e^{-x-\frac{1}{2}}\left(c_1e^{3x}+2ec_2x\right)}{\sqrt{2}}$$

1.45 problem 47

Internal problem ID [6778]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+2)\,y'' + y'x + 3y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 71

```
dsolve((2+x)*diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) = {} & c_1 \mathrm{e}^{-x}\left(x^2-6x+4\right)(x+2)^3 \\ & + c_2\left(\mathrm{e}^{-2-x}\left(x^2-6x+4\right)(x+2)^3\,\mathrm{Ei}_1\left(-2-x\right) + x^4 - x^3 - 18x^2 - 22x + 8\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.297 (sec). Leaf size: 65

```
DSolve[(2+x)*y''[x]+x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} & y(x) \\ & \to \frac{e^{-x}((x-6)x+4)(x+2)^3(c_2\,\mathrm{ExpIntegralEi}(x+2)+3840c_1)+e^2c_2(x(x(-x^2+x+18)+22)-8)}{960e} \end{aligned}$$

1.46 problem 48

Internal problem ID [6779]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 48.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x)\,y'' + x(x+4)\,y' + (2-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
dsolve(x^2*(1-x)*diff(y(x),x$2)+x*(4+x)*diff(y(x),x)+(2-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2+6x+3)}{x} + \frac{c_2(1+3(x^3+6x^2+3x)\ln(x)+51x^2+48x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 51

```
DSolve[x^2*(1-x)*y''[x]+x*(4+x)*y'[x]+(2-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3c_1x(x(x+6)+3) - c_2(51x^2+48x+3(x(x+6)+3)x\log(x)+1)}{3x^2}$$

1.47 problem 49

Internal problem ID [6780]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y'' + x(2x+1)\,y' - (4+6x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 46

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+x*(1+2*x)*diff(y(x),x)-(4+6*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2 c_1 + \frac{c_2(12\ln(x)\,x^4 - 12\ln(x+1)\,x^4 + 12x^3 - 6x^2 + 4x - 3)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 50

```
DSolve[x^2*(1+x)*y''[x]+x*(1+2*x)*y'[x]-(4+6*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to c_1 x^2 + \frac{c_2(12x^4(\log(x) - \log(x+1)) + 2(6x^2 - 3x + 2)\,x - 3)}{12x^2}$$

1.48 problem 50

Internal problem ID [6781]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(2x^2+1\right)y''+x\left(2x^2+4\right)y'+2\left(1-x^2\right)y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
dsolve(x^2*(1+2*x^2)*diff(y(x),x$2)+x*(4+2*x^2)*diff(y(x),x)+2*(1-x^2)*y(x)=0,y(x), singsol=a
```

$$y(x)=\frac{c_1}{x}+\frac{c_2\left(3\,\operatorname{arcsinh}\left(\sqrt{2}\,x\right)x+\sqrt{2}\,\sqrt{2x^2+1}\,\left(x^2-1\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 67

```
DSolve[x^2*(1+2*x^2)*y''[x]+x*(4+2*x^2)*y'[x]+2*(1-x^2)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to\frac{3\sqrt{2}c_2x\operatorname{arcsinh}\left(\sqrt{2}x\right)-2c_2\sqrt{2x^2+1}+2x\left(c_2x\sqrt{2x^2+1}+c_1\right)}{2x^2}$$

1.49 problem 51

Internal problem ID [6782]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 51.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+2\right)y''+2x\left(x^2+5\right)y'+2\left(-x^2+3\right)y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 61

```
dsolve(x^2*(2+x^2)*diff(y(x),x$2)+2*x*(x^2+5)*diff(y(x),x)+2*(3-x^2)*y(x)=0,y(x), singsol=all
```

$$y(x)=\frac{c_1(x^2+8)}{x}+\frac{c_2\left(\left(x^4+8x^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{x^2+2}}\right)+\left(-x^2+4\right)\sqrt{2}\sqrt{x^2+2}\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 85

```
DSolve[x^2*(2+x^2)*y''[x]+2*x*(x^2+5)*y'[x]+2*(3-x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to -\frac{\sqrt{2}c_2(x^2+8)\,x^2\operatorname{arctanh}\left(\frac{\sqrt{x^2+2}}{\sqrt{2}}\right)-2x^2\left(32c_1(x^2+8)+c_2\sqrt{x^2+2}\right)+8c_2\sqrt{x^2+2}}{64x^3}$$

1.50 problem 52

Internal problem ID [6783]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 52.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right)y''+6y'x+6y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve((1+x^2)*diff(y(x),x$2)+6*x*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1 x}{\left(x^2+1\right)^2}+\frac{c_2\left(x^2-1\right)}{\left(x^2+1\right)^2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 29

```
DSolve[(1+x^2)*y''[x]+6*x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_2 x-c_1(x-i)^2}{\left(x^2+1\right)^2}$$

1.51 problem 53

Internal problem ID [6784]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 53.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right) y'' + 2y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2(\arctan(x)\, x + 1)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 23

```
DSolve[(1+x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to ic_1 x - c_2(x \arctan(x) + 1)$$

1.52 problem 54

Internal problem ID [6785]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 54.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right) y'' - 8y'x + 20y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve((1+x^2)*diff(y(x),x$2)-8*x*diff(y(x),x)+20*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(5x^4 - 10x^2 + 1\right) + c_2\left(x^5 - 10x^3 + 5x\right)$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 38

```
DSolve[(1+x^2)*y''[x]-8*x*y'[x]+20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{5} i c_2 \left(5\left(x^2-2\right)x^2+1\right) + c_1(1+ix)^5$$

1.53 problem 55

Internal problem ID [6786]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 55.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(1-x^2\right)y''-8y'x-12y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve((1-x^2)*diff(y(x),x$2)-8*x*diff(y(x),x)-12*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(3x^2+1)}{\left(x^2-1\right)^3}+\frac{c_2(x^3+3x)}{\left(x^2-1\right)^3}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 37

```
DSolve[(1-x^2)*y''[x]-8*x*y'[x]-12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{3c_1(x-1)^3-c_2(3x^2+1)}{3\left(x^2-1\right)^3}$$

1.54 problem 56

Internal problem ID [6787]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 56.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(2x^2+1\right)y''+7y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 47

```
dsolve((1+2*x^2)*diff(y(x),x$2)+7*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\,\text{LegendreP}\left(\frac{1}{4},\frac{3}{4},i\sqrt{2}\,x\right)}{\left(2x^2+1\right)^{\frac{3}{8}}}+\frac{c_2\,\text{LegendreQ}\left(\frac{1}{4},\frac{3}{4},i\sqrt{2}\,x\right)}{\left(2x^2+1\right)^{\frac{3}{8}}}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 66

```
DSolve[(1+2*x^2)*y''[x]+7*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_2 Q_{\frac{1}{4}}^{\frac{3}{4}}\left(i\sqrt{2}x\right)}{\left(2x^2+1\right)^{3/8}}+\frac{2i\sqrt[4]{2}c_1 x}{\left(2x^2+1\right)^{3/4}\,\text{Gamma}\left(\frac{1}{4}\right)}$$

1.55 problem 57

Internal problem ID [6788]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 57.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(1-x^2\right)y''-5y'x-4y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
dsolve((1-x^2)*diff(y(x),x$2)-5*x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1 x}{\left(x^2-1\right)^{\frac{3}{2}}}+\frac{c_2\left(-\ln\left(x+\sqrt{x^2-1}\right)x+\sqrt{x^2-1}\right)}{\left(x^2-1\right)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 52

```
DSolve[(1-x^2)*y''[x]-5*x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-c_2\sqrt{x^2-1}-c_2 x\log\left(\sqrt{x^2-1}-x\right)+c_1 x}{\left(x^2-1\right)^{3/2}}$$

1.56 problem 58

Internal problem ID [6789]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 58.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right)y''-10y'x+28y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve((1+x^2)*diff(y(x),x$2)-10*x*diff(y(x),x)+28*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(1+\frac{35}{3}x^4-14x^2\right)+c_2\left(x^7+21x^5-105x^3+35x\right)$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 40

```
DSolve[(1+x^2)*y''[x]-10*x*y'[x]+28*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{1}{105}c_2\left(35x^4-42x^2+3\right)-c_1(x-i)^6(x+6i)$$

1.57 problem 59

Internal problem ID [6790]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 59.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \,\mathrm{e}^{-\frac{x^2}{2}} + c_2 \left(i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-\frac{x^2}{2}}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,x}{2}\right) x + 2 \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 44

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}c_2 x\,\mathrm{DawsonF}\left(\frac{x}{\sqrt{2}}\right) + \sqrt{2}c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.58 problem 60

Internal problem ID [6791]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 60.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(2x^2+1\right)y''-9y'x-6y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve((1+2*x^2)*diff(y(x),x$2)-9*x*diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(2x^2+1\right)^{\frac{13}{8}}\operatorname{LegendreP}\left(\frac{11}{4},\frac{13}{4},i\sqrt{2}\,x\right)+c_2\left(2x^2+1\right)^{\frac{13}{8}}\operatorname{LegendreQ}\left(\frac{11}{4},\frac{13}{4},i\sqrt{2}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 69

```
DSolve[(1+2*x^2)*y''[x]-9*x*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2\left(2x^2+1\right)^{13/8}Q_{\frac{11}{4}}^{\frac{13}{4}}\left(i\sqrt{2}x\right)-\frac{48\sqrt[4]{2}c_1\left(3x^6+5x^4+3x^2+1\right)}{\operatorname{Gamma}\left(-\frac{5}{4}\right)}$$

1.59 problem 61

Internal problem ID [6792]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 61.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(2x^2-8x+11\right)y''-16(x-2)y'+36y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
dsolve((11-8*x+2*x^2)*diff(y(x),x$2)-16*(x-2)*diff(y(x),x)+36*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(-\frac{31}{5}+x^3-6x^2+\frac{111}{10}x\right)+c_2\left(x^6-12x^5+\frac{165}{2}x^4-\frac{16577}{8}-\frac{5445}{4}x^2+3267x\right)$$

✓ Solution by Mathematica

Time used: 0.453 (sec). Leaf size: 87

```
DSolve[(11-8*x+2*x^2)*y''[x]-16*(x-2)*y'[x]+36*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to \frac{c_1\left(2x+5i\sqrt{6}-4\right)\left(2(x-4)x+11\right)^2\left(2ix+\sqrt{6}-4i\right)^3}{2\left(-2ix+\sqrt{6}+4i\right)^2}+\frac{1}{15}ic_2(x-2)(10(x-4)x+31)$$

1.60 problem 62

Internal problem ID [6793]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 62.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (x-3)\,y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+(x-3)*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\text{KummerM}\left(\frac{3}{2}, \frac{1}{2}, -\frac{(x-3)^2}{2}\right) + c_2 \,\text{KummerU}\left(\frac{3}{2}, \frac{1}{2}, -\frac{(x-3)^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.217 (sec). Leaf size: 58

```
DSolve[y''[x]+(x-3)*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2(x-4)(x-2)\,\text{DawsonF}\left(\frac{x-3}{\sqrt{2}}\right)}{\sqrt{2}} + c_1 e^{-\frac{1}{2}(x-6)x}(x-4)(x-2) - \frac{1}{2}c_2(x-3)$$

1.61 problem 63

Internal problem ID [6794]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 63.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-8x+14\right)y''-8(x-4)y'+20y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve((x^2-8*x+14)*diff(y(x),x$2)-8*(x-4)*diff(y(x),x)+20*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^5-140x^3+1120x^2-3500x+4032\right)+c_2\left(\frac{1604}{5}+x^4-16x^3+100x^2-288x\right)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 76

```
DSolve[(x^2-8*x+14)*y''[x]+8*(x-4)*y'[x]+20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{c_1 P^3_{\frac{1}{2}i\left(i+\sqrt{31}\right)}\left(\frac{x-4}{\sqrt{2}}\right)+c_2 Q^3_{\frac{1}{2}i\left(i+\sqrt{31}\right)}\left(\frac{x-4}{\sqrt{2}}\right)}{((x-8)x+14)^{3/2}}$$

1.62 problem 64

Internal problem ID [6795]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 64.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(2x^2+4x+5\right)y''-20(1+x)y'+60y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve((2*x^2+4*x+5)*diff(y(x),x$2)-20*(x+1)*diff(y(x),x)+60*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(-\frac{7}{4}+x^5+5x^4+5x^3-5x^2-\frac{31}{4}x\right)+c_2\left(x^6+\frac{155}{8}-\frac{75}{2}x^4-100x^3-\frac{225}{4}x^2+30x\right)$$

✓ Solution by Mathematica

Time used: 0.39 (sec). Leaf size: 73

```
DSolve[(2*x^2+4*x+5)*y''[x]-20*(x+1)*y'[x]+60*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\rightarrow\frac{(2x(x+2)+5)^{5/2}\left(c_1\left(2ix+\sqrt{6}+2i\right)^6+4c_2(x+1)(2x(x+2)-7)(2x(x+2)+1)\right)}{(4x(x+2)+10)^{5/2}}$$

1.63 problem 65

Internal problem ID [6796]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 65.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^3+1\right)y''+7y'x^2+9yx=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 28

```
dsolve((1+x^3)*diff(y(x),x$2)+7*x^2*diff(y(x),x)+9*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\text{hypergeom}\left(\left[1,1\right],\left[\frac{2}{3}\right],-x^3\right)+\frac{c_2x}{\left(x^3+1\right)^{\frac{4}{3}}}$$

✓ Solution by Mathematica

Time used: 0.6 (sec). Leaf size: 113

```
DSolve[(1+x^3)*y''[x]+7*x^2*y'[x]+9*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to \frac{c_2x\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)-2\log\left(\sqrt[3]{x^3+1}-x\right)+\log\left(\sqrt[3]{x^3+1}x+\left(x^3+1\right)^{2/3}+x^2\right)\right)-6c_2$$
$$6\left(x^3+1\right)^{4/3}$$

1.64 problem 66

Internal problem ID [6797]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 66.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(2x^5+1\right)y''+14y'x^4+10yx^3=0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 30

```
dsolve((1+2*x^5)*diff(y(x),x$2)+14*x^4*diff(y(x),x)+10*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1 x}{\left(2x^5+1\right)^{\frac{2}{5}}}+c_2\,\mathrm{hypergeom}\left(\left[\frac{1}{5},1\right],\left[\frac{4}{5}\right],-2x^5\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1+2*x^5)*y''[x]+14*x^4*y'[x]+10*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.65 problem 67

Internal problem ID [6798]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 67.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + x^6 y' + 7yx^5 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)+x^6*diff(y(x),x)+7*x^5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^7}{7}} x + \frac{c_2\left(-(-x^7)^{\frac{6}{7}} 7^{\frac{1}{7}} + x^7 \mathrm{e}^{-\frac{x^7}{7}}\left(\Gamma\left(\frac{6}{7}\right) - \Gamma\left(\frac{6}{7}, -\frac{x^7}{7}\right)\right)\right)}{x^6}$$

✓ Solution by Mathematica

Time used: 0.093 (sec). Leaf size: 39

```
DSolve[y''[x]+x^6*y'[x]+7*x^5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{7} e^{-\frac{x^7}{7}} \left(7c_1 x - c_2 \operatorname{ExpIntegralE}\left(\frac{8}{7}, -\frac{x^7}{7}\right)\right)$$

1.66 problem 68

Internal problem ID [6799]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 68.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^8+1\right)y''-16x^7y'+72yx^6=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve((1+x^8)*diff(y(x),x$2)-16*x^7*diff(y(x),x)+72*x^6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(-\frac{7}{9}+x^8\right)+c_2\left(x^9-\frac{9}{7}x\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1+x^8)*y''[x]-16*x^7*y'[x]+72*x^6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.67 problem 69

Internal problem ID [6800]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 69.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' x^5 + 6x^4 y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)+x^5*diff(y(x),x)+6*x^4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^6}{6}} x + \frac{c_2 \left(-(-x^6)^{\frac{5}{6}} 6^{\frac{1}{6}} + x^6 \mathrm{e}^{-\frac{x^6}{6}} \left(\Gamma\left(\frac{5}{6}\right) - \Gamma\left(\frac{5}{6}, -\frac{x^6}{6}\right) \right) \right)}{x^5}$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 39

```
DSolve[y''[x]+x^5*y'[x]+6*x^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6} e^{-\frac{x^6}{6}} \left(6c_1 x - c_2 \operatorname{ExpIntegralE}\left(\frac{7}{6}, -\frac{x^6}{6}\right) \right)$$

1.68 problem 70

Internal problem ID [6801]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 70.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1+3x)\,y''+y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve((1+3*x)*diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{-\frac{x}{3}}\,\mathrm{KummerM}\left(-\frac{19}{9},-\frac{1}{9},\frac{x}{3}+\frac{1}{9}\right)+c_2\left(3x^2-17x-6\right)\mathrm{e}^{-\frac{x}{3}}\left(\frac{x}{3}+\frac{1}{9}\right)^{\frac{1}{9}}$$

✓ Solution by Mathematica

Time used: 1.533 (sec). Leaf size: 106

```
DSolve[(1+3*x)*y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to \frac{e^{-\frac{x}{3}-\frac{1}{9}}\left(-\frac{1}{3}2^{8/9}c_2(x-6)(3x+1)^2\,\mathrm{ExpIntegralE}\left(\frac{1}{9},\frac{1}{9}(-3x-1)\right)-2^{8/9}c_2e^{\frac{x}{3}+\frac{1}{9}}(9x^2-48x-26)+1520c_{\text{}}\right.}{380\ 2^{17/18}}$$

1.69 problem 71

Internal problem ID [6802]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 71.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(3x^2+x+1\right)y''+\left(2+15x\right)y'+12y=0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 307

```
dsolve((1+x+3*x^2)*diff(y(x),x$2)+(2+15*x)*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)$$
$$= \frac{c_1\left(i\sqrt{11}-6x-1\right)^{\frac{3}{2}}\left(-36x^2-12x-12\right)^{-\frac{1}{4}+\frac{i\sqrt{11}}{44}}\mathrm{e}^{\frac{\sqrt{11}\arctan\left(\frac{(6x+1)\sqrt{11}}{11}\right)}{22}}\,\mathrm{hypergeom}\left(\left[\frac{\sqrt{1078+66i\sqrt{11}}}{44}+\frac{1}{2}-\frac{\sqrt{1}}{}\right.\right.}{\left(3x^2+x+1\right)^{\frac{5}{4}}}$$
$$+\frac{c_2\left(i\sqrt{11}+6x+1\right)^{\frac{5}{4}-\frac{i\sqrt{11}}{44}}\left(i\sqrt{11}-6x-1\right)^{\frac{5}{4}+\frac{i\sqrt{11}}{44}}\mathrm{e}^{\frac{\sqrt{11}\arctan\left(\frac{(6x+1)\sqrt{11}}{11}\right)}{22}}\,\mathrm{hypergeom}\left(\left[\frac{\sqrt{1078+66i\sqrt{11}}}{44}+\frac{1}{2}+\right.\right.}{\left(3x^2+x+1\right)^{\frac{5}{4}}}$$

✓ Solution by Mathematica

Time used: 1.776 (sec). Leaf size: 93

```
DSolve[(1+x+3*x^2)*y''[x]+(2+15*x)*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{xe^{\frac{\arctan\left(\frac{6x+1}{\sqrt{11}}\right)}{\sqrt{11}}}\left(c_2\int_1^x e^{-\frac{\arctan\left(\frac{6K[1]+1}{\sqrt{11}}\right)}{\sqrt{11}}}\frac{\sqrt{3K[1]^2+K[1]+1}}{K[1]^2}dK[1]+c_1\right)}{\left(3x^2+x+1\right)^{3/2}}$$

1.70 problem 72

Internal problem ID [6803]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 72.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x+2)\,y'' + (1+x)\,y' + 3y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 58

```
dsolve((2+x)*diff(y(x),x$2)+(1+x)*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} x(x-4)\,(x+2)^2 + c_2\left(\mathrm{e}^{-2-x} x(x-4)\,(x+2)^2\,\mathrm{Ei}_1\left(-2-x\right) + x^3 - x^2 - 10x - 6\right)$$

✓ Solution by Mathematica

Time used: 0.159 (sec). Leaf size: 64

```
DSolve[(2+x)*y''[x]+(1+x)*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \\ \to \frac{e^{-x-1}((x-4)x(x+2)^2(c_2\,\mathrm{ExpIntegralEi}(x+2)+384c_1) - c_2 e^{x+2}(x((x-1)x-10)-6))}{96\sqrt{2}}$$

1.71 problem 73

Internal problem ID [6804]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 73.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+4)\,y'' + (x+2)\,y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
dsolve((4+x)*diff(y(x),x$2)+(2+x)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} x (x+4)^3 + c_2 \left(\mathrm{e}^{-x-4} x (x+4)^3 \operatorname{Ei}_1(-x-4) + x^3 + 9x^2 + 22x + 6\right)$$

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 54

```
DSolve[(4+x)*y''[x]+(2+x)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{24}\left(e^{-x-4} x (x+4)^3 \left(c_2 \operatorname{ExpIntegralEi}(x+4) + 24 e^4 c_1\right) - c_2 (x(x(x+9)+22)+6)\right)$$

1.72 problem 74

Internal problem ID [6805]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 74.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(2x^2+3x\right)y''+10(1+x)y'+8y=0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 31

```
dsolve((3*x+2*x^2)*diff(y(x),x$2)+10*(1+x)*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x+2)}{\left(1+\frac{2x}{3}\right)^{\frac{2}{3}}x^{\frac{7}{3}}}+c_2\,\text{hypergeom}\left(\left[2,2\right],\left[\frac{10}{3}\right],-\frac{2x}{3}\right)$$

✓ Solution by Mathematica

Time used: 0.439 (sec). Leaf size: 167

```
DSolve[(3*x+2*x^2)*y''[x]+10*(1+x)*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to \frac{2^{2/3}c_2(x+2)\left(\log\left(2x^{2/3}+2^{2/3}\sqrt[3]{2x+3}\sqrt[3]{x}+\sqrt[3]{2}(2x+3)^{2/3}\right)-2\log\left(2^{2/3}\sqrt[3]{2x+3}-2\sqrt[3]{x}\right)+2\sqrt{3}\cot$$
$$4x^{7/3}(2x+3)^{2/3}$$

1.73 problem 75

Internal problem ID [6806]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 75.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 - (6-7x)\,y' + 8y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(x^2*diff(y(x),x$2)-(6-7*x)*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{6}{x}}(x-2)}{x^5} + \frac{c_2\left(108\,\mathrm{e}^{-\frac{6}{x}}(x-2)\,\mathrm{Ei}_1\left(-\frac{6}{x}\right) + x^3 + 12x^2 - 36x\right)}{x^5}$$

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 49

```
DSolve[x^2*y''[x]-(6-7*x)*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2e^{-6/x}(x-2)\left(c_1 - 54c_2\,\mathrm{ExpIntegralEi}\left(\frac{6}{x}\right)\right) + c_2 x(x(x+12)-36)}{2x^5}$$

1.74 problem 76

Internal problem ID [6807]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 76.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(2x^2+x+1\right)y''+\left(1+7x\right)y'+2y=0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 78

```
dsolve((1+x+2*x^2)*diff(y(x),x$2)+(1+7*x)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\text{hypergeom}\left(\left[\frac{1}{2},2\right],\left[\frac{7}{4}-\frac{3i\sqrt{7}}{28}\right],\frac{1}{2}+\frac{i(-1-4x)\sqrt{7}}{14}\right)+c_2\left(i\sqrt{7}+4x+1\right)^{-\frac{3}{4}+\frac{3i\sqrt{7}}{28}}\left(i\sqrt{7}-4x-1\right)^{-\frac{3}{4}-\frac{3i\sqrt{7}}{28}}(x+1)$$

✓ Solution by Mathematica

Time used: 1.205 (sec). Leaf size: 102

```
DSolve[(1+x+2*x^2)*y''[x]+(1+7*x)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{(x+1)e^{\frac{3\arctan\left(\frac{4x+1}{\sqrt{7}}\right)}{2\sqrt{7}}}\left(c_2\int_1^x\frac{e^{-\frac{3\arctan\left(\frac{4K[1]+1}{\sqrt{7}}\right)}{2\sqrt{7}}}}{(K[1]+1)^2\sqrt[4]{2K[1]^2+K[1]+1}}dK[1]+c_1\right)}{\left(2x^2+x+1\right)^{3/4}}$$

1.75 problem 77

Internal problem ID [6808]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 77.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+3)\,y'' + (2x+1)\,y' - (2-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve((3+x)*diff(y(x),x$2)+(1+2*x)*diff(y(x),x)-(2-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 \mathrm{e}^{-x} x (x+6) \left(x^2+9x+27\right) \left(x^2+3x+9\right)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 29

```
DSolve[(3+x)*y''[x]+(1+2*x)*y'[x]-(2-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6} e^{-x-3} \left(c_2 (x+3)^6 + 6 c_1\right)$$

1.76 problem 78

Internal problem ID [6809]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 78.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y'x + \left(2x^2 + 4\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 56

```
dsolve(diff(y(x),x$2)+3*x*diff(y(x),x)+(4+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(-2x\, \mathrm{e}^{-\frac{x^2}{2}} + \operatorname{erfi}\left(\frac{\sqrt{2}\, x}{2} \right) \sqrt{2}\, \sqrt{\pi}\, \mathrm{e}^{-x^2} (x-1)\,(x+1) \right) + c_2 \mathrm{e}^{-x^2} \left(x^2 - 1\right)$$

✓ Solution by Mathematica

Time used: 0.187 (sec). Leaf size: 60

```
DSolve[y''[x]+3*x*y'[x]+(4+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-x^2} \left(\left(x^2 - 1\right) \left(\sqrt{2\pi} c_2 \operatorname{erfi}\left(\frac{x}{\sqrt{2}} \right) + 4c_1 \right) - 2c_2 e^{\frac{x^2}{2}} x \right)$$

1.77 problem 79

Internal problem ID [6810]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 79.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(4x+2)\,y'' - 4y' - (6+4x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((2+4*x)*diff(y(x),x$2)-4*diff(y(x),x)-(6+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 x\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 29

```
DSolve[(2+4*x)*y''[x]-4*y'[x]-(6+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x-\frac{1}{2}} \left(c_2 e^{2x+1} x + c_1\right)$$

1.78 problem 80

Internal problem ID [6811]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 80.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y'x + (2x^2 + 5)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 83

```
dsolve(diff(y(x),x$2)-3*x*diff(y(x),x)+(5+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(\operatorname{erfi}\left(\frac{\sqrt{2}\, x}{2} \right) \sqrt{2}\, \sqrt{\pi}\, \left(x^6 - 15x^4 + 45x^2 - 15\right) \mathrm{e}^{\frac{x^2}{2}} - 2x\, \mathrm{e}^{x^2} \left(x^2 - 11\right) \left(x^2 - 3\right) \right) + c_2 \mathrm{e}^{\frac{x^2}{2}} \left(x^6 - 15x^4 + 45x^2 - 15\right)$$

✓ Solution by Mathematica

Time used: 0.389 (sec). Leaf size: 78

```
DSolve[y''[x]-3*x*y'[x]+(5+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\frac{x^2}{2}} \left(x^6 - 15x^4 + 45x^2 - 15\right) \left(\sqrt{2\pi} c_2 \operatorname{erfi}\left(\frac{x}{\sqrt{2}} \right) + 1440 c_1 \right) - 2c_2 e^{x^2} x (x^4 - 14x^2 + 33)}{1440}$$

1.79 problem 81

Internal problem ID [6812]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 81.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y'' + 5y'x + \left(2x^2 + 4\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(2*diff(y(x),x$2)+5*x*diff(y(x),x)+(4+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} \operatorname{erf}\left(\frac{i\sqrt{3}\, x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 42

```
DSolve[2*y''[x]+5*x*y'[x]+(4+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{-x^2} \left(\sqrt{3\pi} c_2 \operatorname{erfi}\left(\frac{\sqrt{3}x}{2}\right) + 3c_1\right)$$

1.80 problem 82

Internal problem ID [6813]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 82.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y'x + \left(4x^2 + 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(2+4*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} x$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 20

```
DSolve[y''[x]+4*x*y'[x]+(2+4*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2} (c_2 x + c_1)$$

1.81 problem 83

Internal problem ID [6814]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 83.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y'x + \left(4x^2 + 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(2+4*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} x$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 20

```
DSolve[y''[x]+4*x*y'[x]+(2+4*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2} (c_2 x + c_1)$$

1.82 problem 84

Internal problem ID [6815]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 84.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2\left(x^2+x+1\right)y''+x\left(11x^2+11x+9\right)y'+\left(7x^2+10x+6\right)y=0}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 362

```
dsolve(2*x^2*(1+x+x^2)*diff(y(x),x$2)+x*(9+11*x+11*x^2)*diff(y(x),x)+(6+10*x+7*x^2)*y(x)=0,y(
```

$$y(x)$$
$$= \frac{c_1 \mathrm{e}^{-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}} \operatorname{HeunG}\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}},0,0,\frac{5}{2},\frac{1}{2},\frac{5i\sqrt{3}-3}{3i\sqrt{3}-3},-\frac{2x}{1+i\sqrt{3}}\right)\left(i\sqrt{3}+2x+1\right)^{\frac{5i\sqrt{3}-3}{-6+6i\sqrt{3}}}\left(i\sqrt{3}-2x-1\right)^{\overline{(1+i}}}{\left(x^2+x+1\right)^{\frac{1}{4}}x^2}$$
$$+\frac{c_2 \mathrm{e}^{-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}} \operatorname{HeunG}\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}},\frac{16}{\left(1+i\sqrt{3}\right)^3\left(i\sqrt{3}-1\right)^2},\frac{1}{2},3,\frac{3}{2},\frac{5i\sqrt{3}-3}{3i\sqrt{3}-3},-\frac{2x}{1+i\sqrt{3}}\right)\left(i\sqrt{3}+2x+1\right)^{\frac{5i\sqrt{3}-3}{-6+6i\sqrt{3}}}\left(}{\left(x^2+x+1\right)^{\frac{1}{4}}x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.677 (sec). Leaf size: 93

```
DSolve[2*x^2*(1+x+x^2)*y''[x]+x*(9+11*x+11*x^2)*y'[x]+(6+10*x+7*x^2)*y[x]==0,y[x],x,IncludeSi
```

$$y(x)\to \frac{\sqrt{x^2+x+1}e^{-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}}\left(c_2\int_1^x \frac{e^{\frac{\arctan\left(\frac{2K[1]+1}{\sqrt{3}}\right)}{\sqrt{3}}}}{\sqrt{K[1]}\left(K[1]^2+K[1]+1\right)^{3/2}}dK[1]+c_1\right)}{x^2}$$

1.83 problem 85

Internal problem ID [6816]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 85.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$\boxed{3y''x^2+2x\left(-2x^2+x+1\right)y'+\left(-8x^2+2x\right)y=0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 38

```
dsolve(3*x^2*diff(y(x),x$2)+2*x*(1+x-2*x^2)*diff(y(x),x)+(2*x-8*x^2)*y(x)=0,y(x), singsol=all
```

$$y(x)=c_1x^{\frac{1}{3}}\mathrm{e}^{\frac{2x(x-1)}{3}}+c_2\,\mathrm{HeunB}\left(-\frac{1}{3},\frac{\sqrt{6}}{3},-\frac{7}{3},\frac{4\sqrt{6}}{9},-\frac{x\sqrt{6}}{3}\right)$$

✓ Solution by Mathematica

Time used: 4.536 (sec). Leaf size: 53

```
DSolve[3*x^2*y''[x]+2*x*(1+x-2*x^2)*y'[x]+(2*x-8*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to e^{\frac{2}{3}(x-1)x}\sqrt[3]{x}\left(c_2\int_1^x\frac{e^{-\frac{2}{3}(K[1]-1)K[1]}}{K[1]^{4/3}}dK[1]+c_1\right)$$

1.84 problem 86

Internal problem ID [6817]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 86.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{12x^2(1+x)\,y'' + x\left(3x^2+35x+11\right)y' - \left(-5x^2-10x+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 51

```
dsolve(12*x^2*(1+x)*diff(y(x),x$2)+x*(11+35*x+3*x^2)*diff(y(x),x)-(1-10*x-5*x^2)*y(x)=0,y(x),
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{x}{4}}\,\mathrm{HeunC}\left(\frac{1}{4},-\frac{7}{12},-\frac{3}{4},-\frac{1}{12},\frac{1}{2},-x\right)}{(x+1)^{\frac{3}{4}}\,x^{\frac{1}{4}}} + \frac{c_2 \mathrm{e}^{-\frac{x}{4}}\,\mathrm{HeunC}\left(\frac{1}{4},\frac{7}{12},-\frac{3}{4},-\frac{1}{12},\frac{1}{2},-x\right)x^{\frac{1}{3}}}{(x+1)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 10.321 (sec). Leaf size: 61

```
DSolve[12*x^2*(1+x)*y''[x]+x*(11+35*x+3*x^2)*y'[x]-(1-10*x-5*x^2)*y[x]==0,y[x],x,IncludeSingu
```

$$y(x) \to \frac{e^{-x/4}\left(c_2 \int_1^x \frac{e^{\frac{K[1]}{4}}}{K[1]^{5/12}\sqrt[4]{K[1]+1}}dK[1] + c_1\right)}{\sqrt[4]{x}(x+1)^{3/4}}$$

1.85 problem 87

Internal problem ID [6818]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 87.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$\boxed{x^2\left(10x^2+x+5\right)y''+x\left(48x^2+3x+4\right)y'+\left(36x^2+x\right)y=0}$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 242

```
dsolve(x^2*(5+x+10*x^2)*diff(y(x),x$2)+x*(4+3*x+48*x^2)*diff(y(x),x)+(x+36*x^2)*y(x)=0,y(x),
```

$$y(x)$$
$$=\frac{c_1 \mathrm{e}^{-\frac{\sqrt{199}\,\arctan\left(\frac{(20x+1)\sqrt{199}}{199}\right)}{995}}\,\mathrm{HeunG}\left(\frac{-i\sqrt{199}+1}{1+i\sqrt{199}},\frac{15721-179i\sqrt{199}}{194275i\sqrt{199}+641775},-\frac{1}{5},0,\frac{4}{5},-\frac{i\sqrt{199}}{995},-\frac{20x}{1+i\sqrt{199}}\right)\left(i\sqrt{199}+20x+\right.}{10x^2+x+5}$$
$$+\frac{c_2 \mathrm{e}^{-\frac{\sqrt{199}\,\arctan\left(\frac{(20x+1)\sqrt{199}}{199}\right)}{995}}\,\mathrm{HeunG}\left(\frac{-i\sqrt{199}+1}{1+i\sqrt{199}},0,0,\frac{1}{5},\frac{6}{5},-\frac{i\sqrt{199}}{995},-\frac{20x}{1+i\sqrt{199}}\right)x^{\frac{1}{5}}\left(i\sqrt{199}+20x+1\right)^{-\frac{i\sqrt{199}}{1990}}\left(i\sqrt{\right.}}{10x^2+x+5}$$

✓ Solution by Mathematica

Time used: 1.105 (sec). Leaf size: 88

```
DSolve[x^2*(5+x+10*x^2)*y''[x]+x*(4+3*x+48*x^2)*y'[x]+(x+36*x^2)*y[x]==0,y[x],x,IncludeSingul
```

$$y(x)\to\frac{\sqrt[5]{x}e^{-\frac{2\arctan\left(\frac{20x+1}{\sqrt{199}}\right)}{5\sqrt{199}}}\left(c_2\int_1^x\frac{e^{\frac{2\arctan\left(\frac{20K[1]+1}{\sqrt{199}}\right)}{5\sqrt{199}}}}{K[1]^{6/5}}dK[1]+c_1\right)}{10x^2+x+5}$$

1.86 problem 88

Internal problem ID [6819]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 88.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{18x^2(1+x)\,y'' + 3x\left(x^2+11x+5\right)y' - \left(-5x^2-2x+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 41

```
dsolve(18*x^2*(1+x)*diff(y(x),x$2)+3*x*(5+11*x+x^2)*diff(y(x),x)-(1-2*x-5*x^2)*y(x)=0,y(x), s
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{x}{6}}\,\mathrm{HeunC}\left(\frac{1}{6},-\frac{1}{2},-\frac{1}{6},-\frac{5}{36},\frac{1}{4},-x\right)}{x^{\frac{1}{6}}} + c_2 \mathrm{e}^{-\frac{x}{6}}\,\mathrm{HeunC}\left(\frac{1}{6},\frac{1}{2},-\frac{1}{6},-\frac{5}{36},\frac{1}{4},-x\right)x^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 1.733 (sec). Leaf size: 60

```
DSolve[18*x^2*(1+x)*y''[x]+3*x*(5+11*x+x^2)*y'[x]-(1-2*x-5*x^2)*y[x]==0,y[x],x,IncludeSingula
```

$$y(x) \to \frac{e^{-x/6}\left(c_2 \int_1^x \frac{e^{\frac{K[1]}{6}}}{\sqrt{K[1]}(K[1]+1)^{7/6}}dK[1] + c_1\right)}{\sqrt[6]{\frac{x}{x+1}}}$$

1.87 problem 89

Internal problem ID [6820]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 89.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y''x^2 + x(3+2x)\,y' - (1-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 27

```
dsolve(2*x^2*diff(y(x),x$2)+x*(3+2*x)*diff(y(x),x)-(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \,\text{WhittakerM}\left(-\frac{1}{4}, \frac{3}{4}, x\right) \mathrm{e}^{-\frac{x}{2}}}{x^{\frac{3}{4}}} + \frac{c_2 \mathrm{e}^{-x}}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 33

```
DSolve[2*x^2*y''[x]+x*(3+2*x)*y'[x]-(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}\left(c_2 x^{3/2} L^{\frac{3}{2}}_{-\frac{3}{2}}(x) + c_1\right)}{x}$$

1.88 problem 90

Internal problem ID [6821]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 90.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y''x^2 + x(x+5)\,y' - (2-3x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 55

```
dsolve(2*x^2*diff(y(x),x$2)+x*(5+x)*diff(y(x),x)-(2-3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\mathrm{e}^{-\frac{x}{2}} + \frac{c_2\left(i\mathrm{e}^{-\frac{x}{2}}x^{\frac{5}{2}}\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\frac{i\sqrt{2}\sqrt{x}}{2}\right) + 2x^2 + 2x + 6\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 59

```
DSolve[2*x^2*y''[x]+x*(5+x)*y'[x]-(2-3*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{15}c_2 e^{-x/2}x\,\text{ExpIntegralE}\left(\frac{1}{2},-\frac{x}{2}\right) - \frac{2c_2(x^2+x+3)}{15x^2} + c_1 e^{-x/2}\sqrt{x}$$

1.89 problem 91

Internal problem ID [6822]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 91.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3y''x^2 + x(1+x)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 29

```
dsolve(3*x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \,\text{WhittakerM}\left(-\frac{1}{6}, \frac{2}{3}, \frac{x}{3}\right) \mathrm{e}^{-\frac{x}{6}}}{x^{\frac{1}{6}}} + \frac{c_2 \mathrm{e}^{-\frac{x}{3}}}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 39

```
DSolve[3*x^2*y''[x]+x*(1+x)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x/3}\left(c_2 - c_1 x^{4/3}\,\text{ExpIntegralE}\left(-\frac{1}{3}, -\frac{x}{3}\right)\right)}{\sqrt[3]{x}}$$

1.90 problem 92

Internal problem ID [6823]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 92.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y''x^2 - y'x + (1-2x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+(1-2*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sinh\left(2\sqrt{x}\right) + c_2\sqrt{x}\,\cosh\left(2\sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 41

```
DSolve[2*x^2*y''[x]-x*y'[x]+(1-2*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-2\sqrt{x}}\sqrt{x}\left(2c_1e^{4\sqrt{x}} - c_2\right)$$

1.91 problem 93

Internal problem ID [6824]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 93.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3y''x^2 + x(1+x)\,y' - (1+3x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 33

```
dsolve(3*x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-(1+3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\left(x^2 + 20x + 70\right) + \frac{c_2 \mathrm{e}^{-\frac{x}{3}}\,\mathrm{hypergeom}\left([3], \left[-\frac{1}{3}\right], \frac{x}{3}\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.71 (sec). Leaf size: 70

```
DSolve[3*x^2*y''[x]+x*(1+x)*y'[x]-(1+3*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 e^{-x/3}(x(x(x+19)+54)-18)}{1680\sqrt[3]{x}} + \frac{x(x(x+20)+70)\left(5040c_1 - 3^{2/3}c_2\Gamma\left(\frac{2}{3}, \frac{x}{3}\right)\right)}{5040}$$

1.92 problem 94

Internal problem ID [6825]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 94.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x+3)\,y'' + x(5x+1)\,y' + (1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve(2*x^2*(3+x)*diff(y(x),x$2)+x*(1+5*x)*diff(y(x),x)+(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\ \text{hypergeom}\left(\left[1, \frac{3}{2}\right], \left[\frac{7}{6}\right], -\frac{x}{3}\right) + \frac{c_2 x^{\frac{1}{3}}}{(3+x)\left(\frac{x}{3}+1\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 10.025 (sec). Leaf size: 50

```
DSolve[2*x^2*(3+x)*y''[x]+x*(1+5*x)*y'[x]+(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{\sqrt[3]{x}\left(6\sqrt[3]{3}c_2\sqrt[6]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{6}, \frac{7}{6}, -\frac{x}{3}\right) + c_1\right)}{(x+3)^{4/3}}$$

1.93 problem 95

Internal problem ID [6826]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 95.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x+4)\,y'' - x(-3x+1)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 27

```
dsolve(x^2*(4+x)*diff(y(x),x$2)-x*(1-3*x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^{\frac{1}{4}}}{(x+4)^{\frac{9}{4}}} + c_2\,\text{hypergeom}\left([1,3],\left[\frac{7}{4}\right],-\frac{x}{4}\right)x$$

✓ Solution by Mathematica

Time used: 0.204 (sec). Leaf size: 87

```
DSolve[x^2*(4+x)*y''[x]-x*(1-3*x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to \frac{10c_2\sqrt[4]{x}\left(\text{arctanh}\left(\sqrt[4]{\frac{x}{x+4}}\right) - \arctan\left(\sqrt[4]{\frac{x}{x+4}}\right)\right) + c_2\sqrt[4]{x+4}x^2 + 9c_2\sqrt[4]{x+4}x + 2c_1\sqrt[4]{x}}{2(x+4)^{9/4}}$$

1.94 problem 96

Internal problem ID [6827]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 96.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2y''x^2 + 5y'x + (1+x)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\sqrt{x}\,\sqrt{2}\right)}{x} + \frac{c_2 \cos\left(\sqrt{x}\,\sqrt{2}\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 60

```
DSolve[2*x^2*y''[x]+5*x*y'[x]+(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{i\sqrt{2}\sqrt{x}} + i\sqrt{2}c_2 e^{-i\sqrt{2}\sqrt{x}}}{2x}$$

1.95 problem 97

Internal problem ID [6828]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 97.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{6y''x^2 + x(10-x)\,y' - (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 25

```
dsolve(6*x^2*diff(y(x),x$2)+x*(10-x)*diff(y(x),x)-(2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \,\text{WhittakerM}\left(-\frac{1}{6}, \frac{2}{3}, \frac{x}{6}\right) \mathrm{e}^{\frac{x}{12}}}{x^{\frac{5}{6}}} + \frac{c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 38

```
DSolve[6*x^2*y''[x]+x*(10-x)*y'[x]-(2+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sqrt[3]{x} L_{-\frac{4}{3}}^{\frac{4}{3}}\left(\frac{x}{6}\right) + \frac{6\sqrt[3]{6}c_1}{x}$$

1.96 problem 98

Internal problem ID [6829]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 98.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(4x+3)\,y'' + x(11+4x)\,y' - (4x+3)\,y = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 41

```
dsolve(x^2*(3+4*x)*diff(y(x),x$2)+x*(11+4*x)*diff(y(x),x)-(3+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(48x^2+32x+7)}{x^3} + c_2\,\text{hypergeom}\left([3,5],\left[\frac{13}{3}\right],-\frac{4x}{3}\right)(4x+3)^{\frac{11}{3}}\,x^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 0.616 (sec). Leaf size: 171

```
DSolve[x^2*(3+4*x)*y''[x]+x*(11+4*x)*y'[x]-(3+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{-6\sqrt[3]{2}c_2(16x(3x+2)+7)\left(\log\left(4x^{2/3}+2\sqrt[3]{2}\sqrt[3]{4x+3}\sqrt[3]{x}+(8x+6)^{2/3}\right)-2\log\left(\sqrt[3]{8x+6}-2\sqrt[3]{x}\right)+2\right.}{48x^3}$$

1.97 problem 99

Internal problem ID [6830]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 99.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(3x+2)\,y'' + x(4+11x)\,y' - (1-x)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(2*x^2*(2+3*x)*diff(y(x),x$2)+x*(4+11*x)*diff(y(x),x)-(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2(3x+2)^{\frac{1}{6}}}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 32

```
DSolve[2*x^2*(2+3*x)*y''[x]+x*(4+11*x)*y'[x]-(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{c_2\sqrt[6]{6x+4} + 2^{5/6}c_1}{\sqrt{x}}$$

1.98 problem 100

Internal problem ID [6831]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 100.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+2)\,y'' + 5x(1-x)\,y' - (-8x+2)\,y = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 116

```
dsolve(x^2*(2+x)*diff(y(x),x$2)+5*x*(1-x)*diff(y(x),x)-(2-8*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(40x^4 - 160x^3 + 60x^2 + 8x + 1)}{x^2} + \frac{c_2\left(x^{\frac{3}{2}}(40x^4 - 160x^3 + 60x^2 + 8x + 1)\operatorname{arcsinh}\left(\frac{\sqrt{x}\sqrt{2}}{2}\right) + \frac{\sqrt{x+2}\,x^2(8x^5+328x^4-13974x^3+26734x^2-805x-105)}{210}\right)(-}{(x+2)^{\frac{3}{4}}\,x^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 21.922 (sec). Leaf size: 1347

```
DSolve[x^2*(2+x)*y''[x]+5*x*(1-x)*y'[x]-(2-8*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

Too large to display

1.99 problem 101

Internal problem ID [6832]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 101.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{8x^2\left(1-x^2\right)y''+2x\left(-13x^2+1\right)y'+\left(-9x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 41

```
dsolve(8*x^2*(1-x^2)*diff(y(x),x$2)+2*x*(1-13*x^2)*diff(y(x),x)+(1-9*x^2)*y(x)=0,y(x), singso
```

$$y(x)=\frac{c_1x^{\frac{1}{4}}}{\sqrt{x^2-1}}+\frac{c_2x^{\frac{3}{8}}\,\text{LegendreQ}\left(-\frac{1}{8},\frac{1}{8},\sqrt{-x^2+1}\right)}{\sqrt{x^2-1}}$$

✓ Solution by Mathematica

Time used: 10.036 (sec). Leaf size: 47

```
DSolve[8*x^2*(1-x^2)*y''[x]+2*x*(1-13*x^2)*y'[x]+(1-9*x^2)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x)\to\frac{\sqrt[4]{x}\left(4c_2\sqrt[4]{x}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{9}{8},x^2\right)+c_1\right)}{\sqrt{1-x^2}}$$

1.100 problem 102

Internal problem ID [6833]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 102.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''-2x\left(-x^2+2\right)y'+4y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-2*x*(2-x^2)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^4}{\left(x^2+1\right)^2}+\frac{c_2(3x^3+x)}{\left(x^2+1\right)^2}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 35

```
DSolve[x^2*(1+x^2)*y''[x]-2*x*(2-x^2)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to -\frac{-3c_1x^4+3c_2x^3+c_2x}{3\left(x^2+1\right)^2}$$

1.101 problem 103

Internal problem ID [6834]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 103.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x\left(x^2+3\right)y''+\left(-x^2+2\right)y'-8yx=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(x*(3+x^2)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)-8*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x^{\frac{1}{3}}\left(x^2+3\right)^{\frac{11}{6}}+c_2\left(x^4+\frac{11}{2}x^2+\frac{55}{8}\right)$$

✓ Solution by Mathematica

Time used: 0.704 (sec). Leaf size: 41

```
DSolve[x*(3+x^2)*y''[x]+(2-x^2)*y'[x]-8*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\sqrt[3]{x}\left(x^2+3\right)^{11/6}-\frac{1}{55}c_2\left(8x^4+44x^2+55\right)$$

1.102 problem 104

Internal problem ID [6835]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 104.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2\left(1-x^2\right)y''+x\left(-19x^2+7\right)y'-\left(14x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 53

```
dsolve(4*x^2*(1-x^2)*diff(y(x),x$2)+x*(7-19*x^2)*diff(y(x),x)-(1+14*x^2)*y(x)=0,y(x), singsol
```

$$y(x)=\frac{c_1\,\text{LegendreP}\left(-\frac{3}{8},\frac{5}{8},\sqrt{-x^2+1}\right)}{x^{\frac{3}{8}}\sqrt{x^2-1}}+\frac{c_2\,\text{LegendreQ}\left(-\frac{3}{8},\frac{5}{8},\sqrt{-x^2+1}\right)}{x^{\frac{3}{8}}\sqrt{x^2-1}}$$

✓ Solution by Mathematica

Time used: 10.043 (sec). Leaf size: 50

```
DSolve[4*x^2*(1-x^2)*y''[x]+x*(7-19*x^2)*y'[x]-(1+14*x^2)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x)\to\frac{4c_2x^{5/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{13}{8},x^2\right)+5c_1}{5x\sqrt{1-x^2}}$$

1.103 problem 105

Internal problem ID [6836]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 105.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2\left(-x^2+2\right)y''+x\left(-11x^2+1\right)y'+\left(-5x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve(3*x^2*(2-x^2)*diff(y(x),x$2)+x*(1-11*x^2)*diff(y(x),x)+(1-5*x^2)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1\sqrt{x}}{\left(-2x^2+4\right)^{\frac{3}{4}}}+c_2x^{\frac{1}{3}}\,\text{hypergeom}\left(\left[\frac{2}{3},1\right],\left[\frac{11}{12}\right],\frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 10.041 (sec). Leaf size: 57

```
DSolve[3*x^2*(2-x^2)*y''[x]+x*(1-11*x^2)*y'[x]+(1-5*x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to\frac{c_1\sqrt{x}-3\ 2^{3/4}c_2\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{12},\frac{1}{4},\frac{11}{12},\frac{x^2}{2}\right)}{(2-x^2)^{3/4}}$$

1.104 problem 106

Internal problem ID [6837]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 106.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2\left(x^2+2\right)y''-x\left(-7x^2+12\right)y'+\left(3x^2+7\right)y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 42

```
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)-x*(12-7*x^2)*diff(y(x),x)+(7+3*x^2)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1x^{\frac{7}{2}}}{\left(x^2+2\right)^2\left(2x^2+4\right)^{\frac{1}{4}}}+c_2\sqrt{x}\ \text{hypergeom}\left(\left[\frac{3}{4},1\right],\left[-\frac{1}{2}\right],-\frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 10.043 (sec). Leaf size: 57

```
DSolve[2*x^2*(2+x^2)*y''[x]-x*(12-7*x^2)*y'[x]+(7+3*x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to\frac{\sqrt{x}\left(3c_1x^3-2\sqrt[4]{2}c_2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{2},-\frac{x^2}{2}\right)\right)}{3\left(x^2+2\right)^{9/4}}$$

1.105 problem 107

Internal problem ID [6838]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 107.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2\left(x^2+2\right)y''+x\left(7x^2+4\right)y'-\left(-3x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 39

```
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)+x*(4+7*x^2)*diff(y(x),x)-(1-3*x^2)*y(x)=0,y(x), singsol=a
```

$$y(x)=\frac{c_1}{\sqrt{x}\,\left(x^2+2\right)^{\frac{1}{4}}}+\frac{c_2\,\mathrm{LegendreQ}\left(-\frac{1}{4},\frac{1}{4},\frac{i\sqrt{2}\,x}{2}\right)}{\sqrt{x}\,\left(x^2+2\right)^{\frac{1}{8}}}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 68

```
DSolve[2*x^2*(2+x^2)*y''[x]+x*(4+7*x^2)*y'[x]-(1-3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to\frac{c_2\sqrt[8]{x^2+2}\,\Gamma\left(\frac{3}{4}\right)Q^{\frac{1}{4}}_{-\frac{1}{4}}\left(\frac{ix}{\sqrt{2}}\right)+2^{3/8}c_1}{\sqrt{x}\sqrt[4]{x^2+2}\,\Gamma\left(\frac{3}{4}\right)}$$

1.106 problem 108

Internal problem ID [6839]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 108.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2\left(2x^2+1\right)y''+5x\left(6x^2+1\right)y'-\left(-40x^2+2\right)y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve(2*x^2*(1+2*x^2)*diff(y(x),x$2)+5*x*(1+6*x^2)*diff(y(x),x)-(2-40*x^2)*y(x)=0,y(x), sing
```

$$y(x)=\frac{c_1\sqrt{x}}{\left(2x^2+1\right)^{\frac{3}{2}}}+\frac{c_2\,\text{hypergeom}\left(\left[\frac{1}{4},1\right],\left[-\frac{1}{4}\right],-2x^2\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 10.049 (sec). Leaf size: 52

```
DSolve[2*x^2*(1+2*x^2)*y''[x]+5*x*(1+6*x^2)*y'[x]-(2-40*x^2)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x)\to\frac{5c_1x^{5/2}-2c_2\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-2x^2\right)}{5x^2\left(2x^2+1\right)^{3/2}}$$

1.107 problem 109

Internal problem ID [6840]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 109.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x\left(x^2+1\right)y''+\left(7x^2+4\right)y'+8yx=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 42

```
dsolve(x*(1+x^2)*diff(y(x),x$2)+(4+7*x^2)*diff(y(x),x)+8*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{\sqrt{x^2+1}\,x^3}+\frac{c_2\left(-x\sqrt{x^2+1}+\operatorname{arcsinh}\left(x\right)\right)}{\sqrt{x^2+1}\,x^3}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 45

```
DSolve[x*(1+x^2)*y''[x]+(4+7*x^2)*y'[x]+8*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-c_2\text{arcsinh}(x)+c_2x\sqrt{x^2+1}+2c_1}{2x^3\sqrt{x^2+1}}$$

1.108 problem 110

Internal problem ID [6841]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 110.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2\left(x^2+1\right)y''+x\left(8x^2+3\right)y'-\left(-4x^2+3\right)y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve(2*x^2*(1+x^2)*diff(y(x),x$2)+x*(3+8*x^2)*diff(y(x),x)-(3-4*x^2)*y(x)=0,y(x), singsol=a
```

$$y(x)=c_1 x\,\text{hypergeom}\left(\left[1,\frac{3}{2}\right],\left[\frac{9}{4}\right],-x^2\right)+\frac{c_2}{\left(x^2+1\right)^{\frac{1}{4}}x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 9.951 (sec). Leaf size: 49

```
DSolve[2*x^2*(1+x^2)*y''[x]+x*(3+8*x^2)*y'[x]-(3-4*x^2)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to -\frac{c_2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1,\frac{5}{4},-x^2\right)}{x}+\frac{c_1}{x^{3/2}\sqrt[4]{x^2+1}}+\frac{c_2}{x}$$

1.109 problem 111

Internal problem ID [6842]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 111.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9y''x^2+3x\left(x^2+3\right)y'-\left(-5x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve(9*x^2*diff(y(x),x$2)+3*x*(3+x^2)*diff(y(x),x)-(1-5*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\,\text{WhittakerM}\left(\frac{1}{3},\frac{1}{6},\frac{x^2}{6}\right)\mathrm{e}^{-\frac{x^2}{12}}}{x}+\frac{c_2\mathrm{e}^{-\frac{x^2}{6}}}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 47

```
DSolve[9*x^2*y''[x]+3*x*(3+x^2)*y'[x]-(1-5*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to -\frac{e^{-\frac{x^2}{6}}\left(c_2x^{2/3}\,\text{ExpIntegralE}\left(\frac{2}{3},-\frac{x^2}{6}\right)-2c_1\right)}{2\sqrt[3]{x}}$$

1.110 problem 112

Internal problem ID [6843]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 112.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{6y''x^2 + x\left(6x^2+1\right)y' + \left(9x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve(6*x^2*diff(y(x),x$2)+x*(1+6*x^2)*diff(y(x),x)+(1+9*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{WhittakerM}\left(\frac{11}{24}, \frac{1}{24}, \frac{x^2}{2}\right) \mathrm{e}^{-\frac{x^2}{4}}}{x^{\frac{7}{12}}} + c_2 x^{\frac{1}{3}} \mathrm{e}^{-\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.17 (sec). Leaf size: 47

```
DSolve[6*x^2*y''[x]+x*(1+6*x^2)*y'[x]+(1+9*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{1}{2} e^{-\frac{x^2}{2}} \sqrt[3]{x} \left(c_2 \sqrt[6]{x} \operatorname{ExpIntegralE}\left(\frac{11}{12}, -\frac{x^2}{2}\right) - 2c_1 \right)$$

1.111 problem 113

Internal problem ID [6844]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 113.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2\left(x^2+1\right)y''+3x\left(13x^2+3\right)y'-\left(-25x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 33

```
dsolve(9*x^2*(1+x^2)*diff(y(x),x$2)+3*x*(3+13*x^2)*diff(y(x),x)-(1-25*x^2)*y(x)=0,y(x), sings
```

$$y(x)=\frac{c_1}{\left(x^2+1\right)^{\frac{2}{3}}x^{\frac{1}{3}}}+c_2x^{\frac{1}{3}}\,\text{hypergeom}\left(\left[1,1\right],\left[\frac{4}{3}\right],-x^2\right)$$

✓ Solution by Mathematica

Time used: 0.433 (sec). Leaf size: 115

```
DSolve[9*x^2*(1+x^2)*y''[x]+3*x*(3+13*x^2)*y'[x]-(1-25*x^2)*y[x]==0,y[x],x,IncludeSingularSol
```

$$y(x)$$
$$\to \frac{c_2\left(-2\log\left(\sqrt[3]{x^2+1}-x^{2/3}\right)+\log\left(x^{4/3}+\left(x^2+1\right)^{2/3}+\sqrt[3]{x^2+1}x^{2/3}\right)+2\sqrt{3}\cot^{-1}\left(\frac{\frac{2\sqrt[3]{x^2+1}}{x^{2/3}}+1}{\sqrt{3}}\right)\right)}{4\sqrt[3]{x}\left(x^2+1\right)^{2/3}}$$

1.112 problem 114

Internal problem ID [6845]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 114.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2\left(x^2+1\right)y''+4x\left(6x^2+1\right)y'-\left(-25x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 43

```
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+4*x*(1+6*x^2)*diff(y(x),x)-(1-25*x^2)*y(x)=0,y(x), singso
```

$$y(x)=\frac{c_1\sqrt{x}}{\left(x^2+1\right)^{\frac{3}{2}}}+\frac{c_2\left(\operatorname{arcsinh}\left(x\right)x-\sqrt{x^2+1}\right)}{\sqrt{x}\,\left(x^2+1\right)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 44

```
DSolve[4*x^2*(1+x^2)*y''[x]+4*x*(1+6*x^2)*y'[x]-(1-25*x^2)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x)\to\frac{c_2x\operatorname{arcsinh}(x)-c_2\sqrt{x^2+1}+c_1x}{\sqrt{x}\left(x^2+1\right)^{3/2}}$$

1.113 problem 115

Internal problem ID [6846]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 115.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{8x^2\left(2x^2+1\right)y''+2x\left(34x^2+5\right)y'-\left(-30x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 57

```
dsolve(8*x^2*(1+2*x^2)*diff(y(x),x$2)+2*x*(5+34*x^2)*diff(y(x),x)-(1-30*x^2)*y(x)=0,y(x), sin
```

$$y(x)=\frac{c_1\,\mathrm{LegendreP}\left(\frac{3}{8},\frac{3}{8},\sqrt{2x^2+1}\right)}{x^{\frac{1}{8}}\sqrt{2x^2+1}}+\frac{c_2\,\mathrm{LegendreQ}\left(\frac{3}{8},\frac{3}{8},\sqrt{2x^2+1}\right)}{x^{\frac{1}{8}}\sqrt{2x^2+1}}$$

✓ Solution by Mathematica

Time used: 10.043 (sec). Leaf size: 54

```
DSolve[8*x^2*(1+2*x^2)*y''[x]+2*x*(5+34*x^2)*y'[x]-(1-30*x^2)*y[x]==0,y[x],x,IncludeSingularS
```

$$y(x)\to\frac{3c_1x^{3/4}-4c_2\,\mathrm{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{5}{8},-2x^2\right)}{3\sqrt{x}\sqrt{2x^2+1}}$$

1.114 problem 116

Internal problem ID [6847]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 116.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(1+x)\,y'' - x(-3x+1)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 23

```
dsolve(2*x^2*(1+x)*diff(y(x),x$2)-x*(1-3*x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{x+1} + \frac{c_2\sqrt{x}}{x+1}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 25

```
DSolve[2*x^2*(1+x)*y''[x]-x*(1-3*x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1\sqrt{x} + 2c_2 x}{x+1}$$

1.115 problem 117

Internal problem ID [6848]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 117.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{6x^2\left(2x^2+1\right)y''+x\left(50x^2+1\right)y'+\left(30x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 33

```
dsolve(6*x^2*(1+2*x^2)*diff(y(x),x$2)+x*(1+50*x^2)*diff(y(x),x)+(1+30*x^2)*y(x)=0,y(x), sings
```

$$y(x)=\frac{c_1\sqrt{x}}{2x^2+1}+\frac{c_2x^{\frac{1}{3}}}{2x^2+1}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 32

```
DSolve[6*x^2*(1+2*x^2)*y''[x]+x*(1+50*x^2)*y'[x]+(1+30*x^2)*y[x]==0,y[x],x,IncludeSingularSol
```

$$y(x)\to\frac{\sqrt[3]{x}\left(6c_2\sqrt[6]{x}+c_1\right)}{2x^2+1}$$

1.116 problem 118

Internal problem ID [6849]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 118.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{28x^2(-3x+1)\,y'' - 7x(5+9x)\,y' + 7(2+9x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(28*x^2*(1-3*x)*diff(y(x),x$2)-7*x*(5+9*x)*diff(y(x),x)+7*(2+9*x)*y(x)=0,y(x), singsol=
```

$$y(x) = \frac{c_1 x^2}{3x-1} + \frac{c_2 x^{\frac{1}{4}}}{3x-1}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 30

```
DSolve[28*x^2*(1-3*x)*y''[x]-7*x*(5+9*x)*y'[x]+7*(2+9*x)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{4c_2 x^2 + 7c_1 \sqrt[4]{x}}{7-21x}$$

1.117 problem 119

Internal problem ID [6850]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 119.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{8x^2\left(-x^2+2\right)y''+2x\left(-21x^2+10\right)y'-\left(35x^2+2\right)y=0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 29

```
dsolve(8*x^2*(2-x^2)*diff(y(x),x$2)+2*x*(10-21*x^2)*diff(y(x),x)-(2+35*x^2)*y(x)=0,y(x), sing
```

$$y(x)=\frac{c_1}{\sqrt{x}\,\left(x^2-2\right)}+\frac{c_2x^{\frac{1}{4}}}{x^2-2}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 34

```
DSolve[8*x^2*(2-x^2)*y''[x]+2*x*(10-21*x^2)*y'[x]-(2+35*x^2)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x)\to\frac{\frac{3c_1}{\sqrt{x}}+4c_2\sqrt[4]{x}}{6-3x^2}$$

1.118 problem 120

Internal problem ID [6851]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 120.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2\left(x^2+3x+1\right)y''-4x\left(-3x^2-3x+1\right)y'+3\left(x^2-x+1\right)y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(4*x^2*(1+3*x+x^2)*diff(y(x),x$2)-4*x*(1-3*x-3*x^2)*diff(y(x),x)+3*(1-x+x^2)*y(x)=0,y(x
```

$$y(x)=\frac{c_1\sqrt{x}}{x^2+3x+1}+\frac{c_2x^{\frac{3}{2}}}{x^2+3x+1}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 27

```
DSolve[4*x^2*(1+3*x+x^2)*y''[x]-4*x*(1-3*x-3*x^2)*y'[x]+3*(1-x+x^2)*y[x]==0,y[x],x,IncludeSin
```

$$y(x)\to\frac{\sqrt{x}(c_2x+c_1)}{x(x+3)+1}$$

1.119 problem 121

Internal problem ID [6852]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 121.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2(1+x)^2 y'' - x\left(-11x^2-10x+1\right) y' + \left(5x^2+1\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(3*x^2*(1+x)^2*diff(y(x),x$2)-x*(1-10*x-11*x^2)*diff(y(x),x)+(1+5*x^2)*y(x)=0,y(x), sin
```

$$y(x) = \frac{c_1 x}{(x+1)^2} + \frac{c_2 x^{\frac{1}{3}}}{(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 29

```
DSolve[3*x^2*(1+x)^2*y''[x]-x*(1-10*x-11*x^2)*y'[x]+(1+5*x^2)*y[x]==0,y[x],x,IncludeSingularS
```

$$y(x) \to \frac{2c_1 \sqrt[3]{x} + 3c_2 x}{2(x+1)^2}$$

1.120 problem 122

Internal problem ID [6853]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 122.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2\left(x^2+2x+3\right)y''-x\left(-15x^2-14x+3\right)y'+\left(7x^2+3\right)y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(4*x^2*(3+2*x+x^2)*diff(y(x),x$2)-x*(3-14*x-15*x^2)*diff(y(x),x)+(3+7*x^2)*y(x)=0,y(x),
```

$$y(x)=\frac{c_1x}{x^2+2x+3}+\frac{c_2x^{\frac{1}{4}}}{x^2+2x+3}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 31

```
DSolve[4*x^2*(3+2*x+x^2)*y''[x]-x*(3-14*x-15*x^2)*y'[x]+(3+7*x^2)*y[x]==0,y[x],x,IncludeSingu
```

$$y(x)\to\frac{3c_1\sqrt[4]{x}+4c_2x}{3x(x+2)+9}$$

1.121 problem 123

Internal problem ID [6854]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 123.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2-2x+1\right)y''-x(x+3)y'+(x+4)y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
dsolve(x^2*(1-2*x+x^2)*diff(y(x),x$2)-x*(3+x)*diff(y(x),x)+(4+x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^2\mathrm{e}^{-\frac{4}{x-1}}}{x-1}+\frac{c_2x^2\,\mathrm{Ei}_1\left(-\frac{4x}{x-1}\right)\mathrm{e}^{-\frac{4x}{x-1}}}{x-1}$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 54

```
DSolve[x^2*(1-2*x+x^2)*y''[x]-x*(3+x)*y'[x]+(4+x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{e^{-\frac{4x}{x-1}}\sqrt{1-x}x^2\left(c_2\,\mathrm{ExpIntegralEi}\left(\frac{4x}{x-1}\right)+e^4c_1\right)}{(x-1)^{3/2}}$$

1.122 problem 124

Internal problem ID [6855]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 124.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x+2)\,y'' + 5y'x^2 + (1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 48

```
dsolve(2*x^2*(2+x)*diff(y(x),x$2)+5*x^2*diff(y(x),x)+(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\sqrt{x}}{(x+2)^{\frac{3}{2}}} + \frac{c_2\sqrt{x}\left(\sqrt{2}\,\sqrt{x+2} - 2\,\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x+2}}{2}\right)\right)}{(x+2)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 55

```
DSolve[2*x^2*(2+x)*y''[x]+5*x^2*y'[x]+(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{x}\left(-2\sqrt{2}c_2\operatorname{arctanh}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right) + 2c_2\sqrt{x+2} + c_1\right)}{(x+2)^{3/2}}$$

1.123 problem 125

Internal problem ID [6856]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 125.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(-x^2+2\right)y''-2x\left(2x^2+1\right)y'+\left(-2x^2+2\right)y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 51

```
dsolve(x^2*(2-x^2)*diff(y(x),x$2)-2*x*(1+2*x^2)*diff(y(x),x)+(2-2*x^2)*y(x)=0,y(x), singsol=a
```

$$y(x)=\frac{c_1 x}{\left(x^2-2\right)^{\frac{3}{2}}}+\frac{c_2 x\left(2\arctan\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)+\sqrt{2}\sqrt{x^2-2}\right)}{\left(x^2-2\right)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 58

```
DSolve[x^2*(2-x^2)*y''[x]-2*x*(1+2*x^2)*y'[x]+(2-2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to\frac{x\left(-\sqrt{2}c_2\operatorname{arctanh}\left(\sqrt{1-\frac{x^2}{2}}\right)+c_2\sqrt{2-x^2}+c_1\right)}{\left(2-x^2\right)^{3/2}}$$

1.124 problem 126

Internal problem ID [6857]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 126.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 - x(5-x)\,y' + (9-4x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(x^2*diff(y(x),x$2)-x*(5-x)*diff(y(x),x)+(9-4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^3 (x+1) + c_2 x^3 \left((x+1)\,\mathrm{Ei}_1(x) - \mathrm{e}^{-x}\right)$$

✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 34

```
DSolve[x^2*y''[x]-x*(5-x)*y'[x]+(9-4*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} x^3 (e^x (x+1)(c_2\,\mathrm{ExpIntegralEi}(-x) + c_1) + c_2)$$

1.125 problem 127

Internal problem ID [6858]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 127.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2\left(x^2+x+1\right)y''+12x^2(1+x)y'+\left(3x^2+3x+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 475

```
dsolve(4*x^2*(1+x+x^2)*diff(y(x),x$2)+12*x^2*(1+x)*diff(y(x),x)+(1+3*x+3*x^2)*y(x)=0,y(x), si
```

$$y(x)$$

$$= \frac{c_1\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{\frac{1}{4}-\frac{i\sqrt{3}}{4}}\sqrt{i\sqrt{3}-2x-1}\,\mathrm{e}^{-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{2}}\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{\sqrt{\frac{1-i\sqrt{3}}{1+i\sqrt{3}}}}{2}-\sqrt{\frac{1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)^3}},\frac{1}{2}-\frac{\sqrt{\frac{1-i}{1+i}}}{2}\right.\right.}{(x^2+x+1)^{\frac{3}{4}}}$$

$$+\frac{c_2\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{\frac{3}{4}+\frac{i\sqrt{3}}{4}}\sqrt{i\sqrt{3}-2x-1}\,\mathrm{e}^{-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{2}}\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{\sqrt{\frac{1-i\sqrt{3}}{1+i\sqrt{3}}}}{2}+\sqrt{\frac{1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)^3}},\frac{1}{2}-\frac{\sqrt{}}{}\right.\right.}{(x^2+x+1)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 0.361 (sec). Leaf size: 93

```
DSolve[4*x^2*(1+x+x^2)*y''[x]+12*x^2*(1+x)*y'[x]+(1+3*x+3*x^2)*y[x]==0,y[x],x,IncludeSingular
```

$$y(x)\to \frac{\sqrt{x}e^{-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}\left(c_2\int_1^x\frac{e^{\sqrt{3}\arctan\left(\frac{2K[1]+1}{\sqrt{3}}\right)}}{K[1]\sqrt{K[1]^2+K[1]+1}}dK[1]+c_1\right)}{\sqrt{x^2+x+1}}$$

1.126 problem 128

Internal problem ID [6859]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 128.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2\left(x^2+x+1\right)y''-x\left(-2x^2-4x+1\right)y'+y=0$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 301

```
dsolve(x^2*(1+x+x^2)*diff(y(x),x$2)-x*(1-4*x-2*x^2)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\left(i\sqrt{3}-2x-1\right)^{\frac{1}{4}+\frac{7i\sqrt{3}}{12}}\left(i\sqrt{3}+2x+1\right)^{\frac{1}{4}-\frac{7i\sqrt{3}}{12}}x\,\mathrm{e}^{-\frac{7\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}}}{\left(x^2+x+1\right)^{\frac{3}{4}}}$$
$$+\frac{c_2\left(i\sqrt{3}-2x-1\right)^{-\frac{1}{4}-\frac{7i\sqrt{3}}{12}}\left(i\sqrt{3}+2x+1\right)^{\frac{3}{4}+\frac{7i\sqrt{3}}{12}}x\,\mathrm{e}^{-\frac{7\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}}\operatorname{hypergeom}\left(\left[\frac{1}{2}+\frac{\sqrt{\frac{-45i\sqrt{3}-3}{1+i\sqrt{3}}}}{6}+\right.\right.}{\left(x^2+x+1\right)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 0.382 (sec). Leaf size: 90

```
DSolve[x^2*(1+x+x^2)*y''[x]-x*(1-4*x-2*x^2)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{xe^{-\frac{7\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}}\left(c_2\int_1^x\frac{e^{\frac{7\arctan\left(\frac{2K[1]+1}{\sqrt{3}}\right)}{\sqrt{3}}}}{K[1]\sqrt{K[1]^2+K[1]+1}}dK[1]+c_1\right)}{\sqrt{x^2+x+1}}$$

1.127 problem 129

Internal problem ID [6860]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 129.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9y''x^2+3x\left(-2x^2+3x+5\right)y'+\left(-14x^2+12x+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 42

```
dsolve(9*x^2*diff(y(x),x$2)+3*x*(5+3*x-2*x^2)*diff(y(x),x)+(1+12*x-14*x^2)*y(x)=0,y(x), sings
```

$$y(x)=\frac{c_1\mathrm{e}^{\frac{x(x-3)}{3}}}{x^{\frac{1}{3}}}+\frac{c_2\mathrm{e}^{\frac{x(x-3)}{3}}\left(\int\frac{\mathrm{e}^{-\frac{x(x-3)}{3}}}{x}dx\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.172 (sec). Leaf size: 52

```
DSolve[9*x^2*y''[x]+3*x*(5+3*x-2*x^2)*y'[x]+(1+12*x-14*x^2)*y[x]==0,y[x],x,IncludeSingularSol
```

$$y(x)\to\frac{e^{\frac{1}{3}(x-3)x}\left(c_2\int_1^x\frac{e^{K[1]-\frac{K[1]^2}{3}}}{K[1]}dK[1]+c_1\right)}{\sqrt[3]{x}}$$

1.128 problem 130

Internal problem ID [6861]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 130.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x+1)\,y'' + x\left(3x^2+14x+5\right)y' + \left(12x^2+18x+4\right)y = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 52

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(5+14*x+3*x^2)*diff(y(x),x)+(4+18*x+12*x^2)*y(x)=0,y(x),
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{3x}{2}}\,\mathrm{HeunC}\left(-\frac{3}{4},-\frac{1}{4},0,\frac{21}{32},-\frac{5}{32},2x+1\right)}{x^2\,(2x+1)^{\frac{1}{4}}} + \frac{c_2 \mathrm{e}^{-\frac{3x}{2}}\,\mathrm{HeunC}\left(-\frac{3}{4},\frac{1}{4},0,\frac{21}{32},-\frac{5}{32},2x+1\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 7.27 (sec). Leaf size: 61

```
DSolve[x^2*(1+2*x)*y''[x]+x*(5+14*x+3*x^2)*y'[x]+(4+18*x+12*x^2)*y[x]==0,y[x],x,IncludeSingul
```

$$y(x) \to \frac{e^{-3x/2}\left(c_2 \int_1^x \frac{e^{\frac{3K[1]}{2}}}{K[1](2K[1]+1)^{3/4}}\,dK[1] + c_1\right)}{x^2\sqrt[4]{2x+1}}$$

1.129 problem 131

Internal problem ID [6862]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 131.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16y''x^2+4x\left(2x^2+x+6\right)y'+\left(18x^2+5x+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 42

```
dsolve(16*x^2*diff(y(x),x$2)+4*x*(6+x+2*x^2)*diff(y(x),x)+(1+5*x+18*x^2)*y(x)=0,y(x), singsol
```

$$y(x)=\frac{c_1\mathrm{e}^{-\frac{x(x+1)}{4}}}{x^{\frac{1}{4}}}+\frac{c_2\mathrm{e}^{-\frac{x(x+1)}{4}}\left(\int\frac{\mathrm{e}^{\frac{x(x+1)}{4}}}{x}dx\right)}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 51

```
DSolve[16*x^2*y''[x]+4*x*(6+x+2*x^2)*y'[x]+(1+5*x+18*x^2)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x)\to\frac{e^{-\frac{1}{4}x(x+1)}\left(c_2\int_1^x\frac{e^{\frac{1}{4}K[1](K[1]+1)}}{K[1]}dK[1]+c_1\right)}{\sqrt[4]{x}}$$

1.130 problem 132

Internal problem ID [6863]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 132.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2(1+x)\,y'' + 3x\left(-x^2+11x+5\right)y' + \left(-7x^2+16x+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 38

```
dsolve(9*x^2*(1+x)*diff(y(x),x$2)+3*x*(5+11*x-x^2)*diff(y(x),x)+(1+16*x-7*x^2)*y(x)=0,y(x), s
```

$$y(x) = \frac{c_1\,\mathrm{HeunC}\left(-\frac{1}{3},-\frac{4}{3},0,-\frac{1}{9},\frac{11}{18},x+1\right)}{x^{\frac{1}{3}}\,(x+1)^{\frac{4}{3}}} + \frac{c_2\,\mathrm{HeunC}\left(-\frac{1}{3},\frac{4}{3},0,-\frac{1}{9},\frac{11}{18},x+1\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 3.987 (sec). Leaf size: 50

```
DSolve[9*x^2*(1+x)*y''[x]+3*x*(5+11*x-x^2)*y'[x]+(1+16*x-7*x^2)*y[x]==0,y[x],x,IncludeSingula
```

$$y(x) \to \frac{e^{x/3}\left(c_1 - \sqrt[3]{3}ec_2\Gamma\left(\frac{1}{3},\frac{x+1}{3}\right)\right)}{\sqrt[3]{x}(x+1)^{4/3}}$$

1.131 problem 133

Internal problem ID [6864]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 133.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{36x^2(1-2x)\,y'' + 24x(1-9x)\,y' + (1-70x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 123

```
dsolve(36*x^2*(1-2*x)*diff(y(x),x$2)+24*x*(1-9*x)*diff(y(x),x)+(1-70*x)*y(x)=0,y(x), singsol=
```

$$y(x) = \frac{c_1 x^{\frac{1}{6}}}{(2x-1)^{\frac{4}{3}}} + c_2\left(-\frac{4x^{\frac{1}{6}}\ln\left(1+(2x-1)^{\frac{1}{3}}\right)}{3\,(2x-1)^{\frac{4}{3}}} + \frac{2x^{\frac{1}{6}}\ln\left(1-(2x-1)^{\frac{1}{3}}+(2x-1)^{\frac{2}{3}}\right)}{3\,(2x-1)^{\frac{4}{3}}} + \frac{4\sqrt{3}\,x^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\,(2x-1)^{\frac{1}{3}}}{-2+(2x-1)^{\frac{1}{3}}}\right)}{3\,(2x-1)^{\frac{4}{3}}} + \frac{4x^{\frac{1}{6}}}{2x-1}\right)$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 108

```
DSolve[36*x^2*(1-2*x)*y''[x]+24*x*(1-9*x)*y'[x]+(1-70*x)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{\sqrt[6]{x}\left(-c_2\left(2\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-2x}+1}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{1-2x}-1\right) + \log\left((1-2x)^{2/3}+\sqrt[3]{1-2x}+1\right)\right) + 6c_2\right.}{2(1-2x)^{4/3}}$$

1.132 problem 134

Internal problem ID [6865]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 134.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y'' - x(3-x)\,y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(3-x)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2(x-1)}{(x+1)^3} + \frac{c_2 x^2(-4+(x-1)\ln(x))}{(x+1)^3}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 33

```
DSolve[x^2*(1+x)*y''[x]-x*(3-x)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2(c_1(x-1) + c_2(x-1)\log(x) - 4c_2)}{(x+1)^3}$$

1.133 problem 135

Internal problem ID [6866]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 135.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-2x)\,y'' - x(5-4x)\,y' + (9-4x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)-x*(5-4*x)*diff(y(x),x)+(9-4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^3}{(2x-1)^2} + \frac{c_2 x^3 (2x - \ln(x))}{(2x-1)^2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 29

```
DSolve[x^2*(1-2*x)*y''[x]-x*(5-4*x)*y'[x]+(9-4*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{x^3(-2c_2 x + c_2 \log(x) + c_1)}{(1-2x)^2}$$

1.134 problem 136

Internal problem ID [6867]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 136.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x+2)\,y'' + y'x^2 + (1-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 50

```
dsolve(2*x^2*(2+x)*diff(y(x),x$2)+x^2*diff(y(x),x)+(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x\,(x+2)} + \frac{c_2\sqrt{x}\left(\sqrt{2}\,\sqrt{x+2} - (x+2)\,\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x+2}}{2}\right)\right)}{\sqrt{x+2}}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 65

```
DSolve[2*x^2*(2+x)*y''[x]+x^2*y'[x]+(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{x}\left(2\left(c_1\sqrt{x+2}+c_2\right) - \sqrt{2}c_2\sqrt{x+2}\operatorname{arctanh}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right)\right)}{2\sqrt[4]{2}}$$

1.135 problem 137

Internal problem ID [6868]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 137.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(1+x)\,y'' - x(6-x)\,y' + (8-x)\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
dsolve(2*x^2*(1+x)*diff(y(x),x$2)-x*(6-x)*diff(y(x),x)+(8-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2}{(x+1)^{\frac{5}{2}}} + \frac{c_2 x^2\left(2\sqrt{x+1}\,x + 8\sqrt{x+1} + 3\ln\left(\sqrt{x+1}-1\right) - 3\ln\left(\sqrt{x+1}+1\right)\right)}{(x+1)^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 50

```
DSolve[2*x^2*(1+x)*y''[x]-x*(6-x)*y'[x]+(8-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{x^2\left(-6c_2 \operatorname{arctanh}\left(\sqrt{x+1}\right) + 2c_2\sqrt{x+1}(x+4) + 3c_1\right)}{3(x+1)^{5/2}}$$

1.136 problem 138

Internal problem ID [6869]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 138.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x+1)\,y'' + x(5+9x)\,y' + (4+3x)\,y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 74

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(5+9*x)*diff(y(x),x)+(4+3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(2x+1)^{\frac{3}{2}}}{x^2} + \frac{c_2\left(\left(x+\frac{1}{2}\right)^2 \ln\left(\sqrt{2x+1}-1\right) - \left(x+\frac{1}{2}\right)^2 \ln\left(\sqrt{2x+1}+1\right) + \sqrt{2x+1}\left(x+\frac{2}{3}\right)\right)}{\sqrt{2x+1}\,x^2}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 56

```
DSolve[x^2*(1+2*x)*y''[x]+x*(5+9*x)*y'[x]+(4+3*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{2c_2\left(-3(2x+1)^{3/2}\operatorname{arctanh}\left(\sqrt{2x+1}\right) + 6x + 4\right) + 3c_1(2x+1)^{3/2}}{3x^2}$$

1.137 problem 139

Internal problem ID [6870]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 139.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-2x)\,y'' - x(4x+5)\,y' + (4x+9)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)-x*(5+4*x)*diff(y(x),x)+(9+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(8x^4+x^3)}{(2x-1)^6} + \frac{c_2 x^3\left(\left(-6x-\frac{3}{4}\right)\ln(x) + x^4 - 4x^3 + 9x^2 + \frac{609x}{512} - \frac{9375}{4096}\right)}{(2x-1)^6}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 61

```
DSolve[x^2*(1-2*x)*y''[x]-x*(5+4*x)*y'[x]+(9+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{x^3(-8(8x+1)(c_2x(8x(8x-33)+609)-6c_1)+3072c_2(8x+1)\log(x)+9375c_2)}{384(1-2x)^6}$$

1.138 problem 140

Internal problem ID [6871]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 140.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x)\,y'' + x(7+x)\,y' + (9-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 70

```
dsolve(x^2*(1-x)*diff(y(x),x$2)+x*(7+x)*diff(y(x),x)+(9-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^4+16x^3+36x^2+16x+1)}{x^3} + \frac{c_2(25+3(x^4+16x^3+36x^2+16x+1)\ln(x)+120x^3+450x^2+280x)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 70

```
DSolve[x^2*(1-x)*y''[x]+x*(7+x)*y'[x]+(9-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3c_1(x(x+2)(x(x+14)+8)+1)+5c_2(2x(3x(4x+15)+28)+5)+3c_2(x(x+2)(x(x+14)+8)+1)l}{3x^3}$$

1.139 problem 141

Internal problem ID [6872]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 141.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 - x\left(1-x^2\right)y' + \left(x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(x^2*diff(y(x),x$2)-x*(1-x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{-\frac{x^2}{2}} + c_2 x\,\mathrm{e}^{-\frac{x^2}{2}}\,\mathrm{Ei}_1\left(-\frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]-x*(1-x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-\frac{x^2}{2}}x\left(c_1\,\mathrm{ExpIntegralEi}\left(\frac{x^2}{2}\right) + 2c_2\right)$$

1.140 problem 142

Internal problem ID [6873]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 142.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''-3x\left(1-x^2\right)y'+4y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-3*x*(1-x^2)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^2}{\left(x^2+1\right)^2}+\frac{c_2x^2\left(\frac{x^2}{2}+\ln\left(x\right)\right)}{\left(x^2+1\right)^2}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 36

```
DSolve[x^2*(1+x^2)*y''[x]-3*x*(1-x^2)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to\frac{x^2(c_2x^2+2c_2\log(x)+2c_1)}{2\left(x^2+1\right)^2}$$

1.141 problem 143

Internal problem ID [6874]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 143.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y''x^2 + 2y'x^3 + \left(3x^2 + 1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve(4*x^2*diff(y(x),x$2)+2*x^3*diff(y(x),x)+(1+3*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\mathrm{e}^{-\frac{x^2}{4}} + c_2\sqrt{x}\,\mathrm{e}^{-\frac{x^2}{4}}\,\mathrm{Ei}_1\left(-\frac{x^2}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 39

```
DSolve[4*x^2*y''[x]+2*x^3*y'[x]+(1+3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-\frac{x^2}{4}}\sqrt{x}\left(c_2\,\mathrm{ExpIntegralEi}\left(\frac{x^2}{4}\right) + 2c_1\right)$$

1.142 problem 144

Internal problem ID [6875]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 144.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''-x\left(-2x^2+1\right)y'+y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1-2*x^2)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1 x}{\sqrt{x^2+1}}+\frac{c_2 x\ \operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 33

```
DSolve[x^2*(1+x^2)*y''[x]-x*(1-2*x^2)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x\left(c_1-c_2\operatorname{arctanh}\left(\sqrt{x^2+1}\right)\right)}{\sqrt{x^2+1}}$$

1.143 problem 145

Internal problem ID [6876]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 145.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2\left(x^2+2\right)y''+7y'x^3+\left(3x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 85

```
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)+7*x^3*diff(y(x),x)+(1+3*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\sqrt{x}}{\left(x^2+2\right)^{\frac{3}{4}}}+\frac{c_2\sqrt{x}\left(\ln\left(1-\frac{\sqrt{2}\left(2x^2+4\right)^{\frac{1}{4}}}{2}\right)-\ln\left(1+\frac{\sqrt{2}\left(2x^2+4\right)^{\frac{1}{4}}}{2}\right)+2\arctan\left(\frac{\sqrt{2}\left(2x^2+4\right)^{\frac{1}{4}}}{2}\right)\right)}{\left(2x^2+4\right)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 71

```
DSolve[2*x^2*(2+x^2)*y''[x]+7*x^3*y'[x]+(1+3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{\sqrt{x}\left(2^{3/4}c_2\left(\arctan\left(\frac{\sqrt[4]{x^2+2}}{\sqrt[4]{2}}\right)-\operatorname{arctanh}\left(\frac{\sqrt[4]{x^2+2}}{\sqrt[4]{2}}\right)\right)+2c_1\right)}{2\left(x^2+2\right)^{3/4}}$$

1.144 problem 146

Internal problem ID [6877]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 146.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''-x\left(-4x^2+1\right)y'+\left(2x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 43

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1-4*x^2)*diff(y(x),x)+(1+2*x^2)*y(x)=0,y(x), singsol=all
```

$$y(x)=\frac{c_1x}{\left(x^2+1\right)^{\frac{3}{2}}}+\frac{c_2x\left(-\sqrt{x^2+1}+\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)\right)}{\left(x^2+1\right)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 45

```
DSolve[x^2*(1+x^2)*y''[x]-x*(1-4*x^2)*y'[x]+(1+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{x\left(-c_2\operatorname{arctanh}\left(\sqrt{x^2+1}\right)+c_2\sqrt{x^2+1}+c_1\right)}{\left(x^2+1\right)^{3/2}}$$

1.145 problem 147

Internal problem ID [6878]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 147.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2\left(x^2+4\right)y''+3x\left(3x^2+8\right)y'+\left(-9x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 68

```
dsolve(4*x^2*(4+x^2)*diff(y(x),x$2)+3*x*(8+3*x^2)*diff(y(x),x)+(1-9*x^2)*y(x)=0,y(x), singsol
```

$$y(x)=\frac{c_1\left(x^2+4\right)^{\frac{5}{8}}}{x^{\frac{1}{4}}}+\frac{c_2\left(-\frac{1024}{25}+\left(x^2\,\mathrm{hypergeom}\left(\left[1,1,\frac{13}{8}\right],[2,2],-\frac{x^2}{4}\right)-\frac{32\gamma}{5}+\frac{64\ln(2)}{5}-\frac{64\ln(x)}{5}-\frac{32\Psi\left(\frac{5}{8}\right)}{5}\right)\left(x^2+4\right)^{\frac{5}{8}}2^{\frac{3}{4}}\right)}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.261 (sec). Leaf size: 185

```
DSolve[4*x^2*(4+x^2)*y''[x]+3*x*(8+3*x^2)*y'[x]+(1-9*x^2)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x)\to\frac{5\sqrt[4]{2}c_2\left(x^2+4\right)^{5/8}\left(\sqrt{2}\arctan\left(\frac{\sqrt[8]{x^2+4}}{\sqrt[4]{2}}\right)+\arctan\left(\frac{\sqrt{2}-\sqrt[4]{x^2+4}}{2^{3/4}\sqrt[8]{x^2+4}}\right)\right)+5\sqrt[4]{2}c_2\left(x^2+4\right)^{5/8}\left(\operatorname{arctanh}\left(\frac{2}{\sqrt{}}\right.\right.}{80\sqrt[4]{x}}$$

1.146 problem 148

Internal problem ID [6879]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 148.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2\left(x^2+3\right)y''+x\left(11x^2+3\right)y'+\left(5x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 107

```
dsolve(3*x^2*(3+x^2)*diff(y(x),x$2)+x*(3+11*x^2)*diff(y(x),x)+(1+5*x^2)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1x^{\frac{1}{3}}}{\left(x^2+3\right)^{\frac{2}{3}}}+\frac{c_2x^{\frac{1}{3}}\left(2\arctan\left(\frac{\left(9x^2+27\right)^{\frac{1}{3}}\sqrt{3}}{6+\left(9x^2+27\right)^{\frac{1}{3}}}\right)\sqrt{3}-\ln\left(1+\frac{\left(9x^2+27\right)^{\frac{1}{3}}}{3}+\frac{\left(9x^2+27\right)^{\frac{2}{3}}}{9}\right)+2\ln\left(1-\frac{\left(9x^2+27\right)^{\frac{1}{3}}}{3}\right)\right)}{\left(9x^2+27\right)^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 94

```
DSolve[3*x^2*(3+x^2)*y'[x]+x*(3+11*x^2)*y'[x]+(1+5*x^2)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to\frac{c_1\exp\left(\frac{1}{3}\text{RootSum}\left[3\#1^3+11\#1^2+9\#1+3\&,\frac{3\#1^2\log(x-\#1)-4\#1\log(x-\#1)+9\log(x-\#1)}{9\#1^2+22\#1+9}\&\right]\right)}{\sqrt[3]{x}}$$

$$y(x)\to 0$$

1.147 problem 149

Internal problem ID [6880]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 149.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9y''x^2 - 3x\left(-2x^2+7\right)y' + \left(2x^2+25\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(9*x^2*diff(y(x),x$2)-3*x*(7-2*x^2)*diff(y(x),x)+(25+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{5}{3}} \mathrm{e}^{-\frac{x^2}{3}} + c_2 x^{\frac{5}{3}} \mathrm{e}^{-\frac{x^2}{3}} \operatorname{Ei}_1\left(-\frac{x^2}{3}\right)$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 39

```
DSolve[9*x^2*y''[x]-3*x*(7-2*x^2)*y'[x]+(25+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{2} e^{-\frac{x^2}{3}} x^{5/3} \left(c_2 \operatorname{ExpIntegralEi}\left(\frac{x^2}{3}\right) + 2c_1\right)$$

1.148 problem 150

Internal problem ID [6881]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 150.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 - x\left(1-x^2\right)y' + \left(x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x^2*diff(y(x),x$2)-x*(1-x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\, \mathrm{e}^{-\frac{x^2}{2}} + c_2 x\, \mathrm{e}^{-\frac{x^2}{2}}\, \mathrm{Ei}_1\left(-\frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]-x*(1-x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-\frac{x^2}{2}} x \left(c_1 \operatorname{ExpIntegralEi}\left(\frac{x^2}{2}\right) + 2c_2\right)$$

1.149 problem 151

Internal problem ID [6882]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 151.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-2x)\,y'' + 3y'x + (1+4x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)+3*x*diff(y(x),x)+(1+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2(-8x^3 + 18x^2 + 3\ln(x) - 18x)}{x}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 35

```
DSolve[x^2*(1-2*x)*y''[x]+3*x*y'[x]+(1+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x} - \frac{2}{3}c_2(x(4x-9)+9) + \frac{c_2\log(x)}{x}$$

1.150 problem 152

Internal problem ID [6883]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 152.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(1+x)\,y'' + (1-x)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x*(1+x)*diff(y(x),x$2)+(1-x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x-1) + c_2(-4 + (x-1)\ln(x))$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 23

```
DSolve[x*(1+x)*y''[x]+(1-x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x-1) + c_2((x-1)\log(x) - 4)$$

1.151 problem 153

Internal problem ID [6884]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 153.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x)\,y'' - x(3-5x)\,y' + (4-5x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
dsolve(x^2*(1-x)*diff(y(x),x$2)-x*(3-5*x)*diff(y(x),x)+(4-5*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^2 (x-1)^3 + c_2 x^2 \left(-(x-1)^3 \ln(x-1) + (x-1)^3 \ln(x) - x^2 + \frac{5x}{2} - \frac{11}{6} \right)$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 53

```
DSolve[x^2*(1-x)*y''[x]-x*(3-5*x)*y'[x]+(4-5*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{6} x^2 \left(-6c_1 (x-1)^3 + c_2 (3x(2x-5)+11) + 6c_2 (x-1)^3 (\log(x-1) - \log(x)) \right)$$

1.152 problem 154

Internal problem ID [6885]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 154.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''-x\left(9x^2+1\right)y'+\left(25x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1+9*x^2)*diff(y(x),x)+(1+25*x^2)*y(x)=0,y(x), singsol=al
```

$$y(x)=c_1x\left(x^4-4x^2+1\right)+c_2\left(\left(x^4-4x^2+1\right)\ln\left(x\right)-6x^2+3\right)x$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 43

```
DSolve[x^2*(1+x^2)*y''[x]-x*(1+9*x^2)*y'[x]+(1+25*x^2)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to c_1\left(x^5-4x^3+x\right)+c_2x\left(-6x^2+\left(x^4-4x^2+1\right)\log(x)+3\right)$$

1.153 problem 155

Internal problem ID [6886]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 155.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9y''x^2+3x\left(1-x^2\right)y'+\left(7x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(9*x^2*diff(y(x),x$2)+3*x*(1-x^2)*diff(y(x),x)+(1+7*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x^{\frac{1}{3}}\,\text{hypergeom}\left(\left[-1\right],\left[1\right],\frac{x^2}{6}\right)+c_2x^{\frac{1}{3}}\,\text{KummerU}\left(-1,1,\frac{x^2}{6}\right)$$

✓ Solution by Mathematica

Time used: 1.706 (sec). Leaf size: 50

```
DSolve[9*x^2*y''[x]+3*x*(1-x^2)*y'[x]+(1+7*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to\frac{1}{72}\sqrt[3]{x}\left(\left(x^2-6\right)\left(c_2\,\text{ExpIntegralEi}\left(\frac{x^2}{6}\right)+72c_1\right)-6c_2e^{\frac{x^2}{6}}\right)$$

1.154 problem 156

Internal problem ID [6887]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 156.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x\left(x^2+1\right)y''+\left(1-x^2\right)y'-8yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(x*(1+x^2)*diff(y(x),x$2)+(1-x^2)*diff(y(x),x)-8*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^2+1\right)^2+c_2\left(-\frac{\left(x^2+1\right)^2\ln\left(x^2+1\right)}{2}+\left(x^2+1\right)^2\ln\left(x\right)+\frac{x^2}{2}+\frac{3}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 55

```
DSolve[x*(1+x^2)*y''[x]+(1-x^2)*y'[x]-8*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\left(x^2+1\right)^2+\frac{1}{4}c_2\left(2x^2+4\left(x^2+1\right)^2\log(x)-2\left(x^2+1\right)^2\log\left(x^2+1\right)+3\right)$$

1.155 problem 157

Internal problem ID [6888]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 157.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y''x^2+2x\left(-x^2+4\right)y'+\left(7x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 33

```
dsolve(4*x^2*diff(y(x),x$2)+2*x*(4-x^2)*diff(y(x),x)+(1+7*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\,\mathrm{hypergeom}\left(\left[-2\right],\left[1\right],\frac{x^2}{4}\right)}{\sqrt{x}}+\frac{c_2\,\mathrm{KummerU}\left(-2,1,\frac{x^2}{4}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.199 (sec). Leaf size: 60

```
DSolve[4*x^2*y''[x]+2*x*(4-x^2)*y'[x]+(1+7*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to\frac{\left(x^4-16x^2+32\right)\left(c_2\,\mathrm{ExpIntegralEi}\left(\frac{x^2}{4}\right)+2048c_1\right)-4c_2e^{\frac{x^2}{4}}\left(x^2-12\right)}{2048\sqrt{x}}$$

1.156 problem 158

Internal problem ID [6889]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 158.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(1+x)\,y'' + 8y'x^2 + (1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+8*x^2*diff(y(x),x)+(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\sqrt{x}}{x+1} + \frac{c_2\sqrt{x}\,\ln(x)}{x+1}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 24

```
DSolve[4*x^2*(1+x)*y''[x]+8*x^2*y'[x]+(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{x}(c_2\log(x) + c_1)}{x+1}$$

1.157 problem 159

Internal problem ID [6890]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 159.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(x+3)\,y'' + 3x(3+7x)\,y' + (4x+3)\,y = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 27

```
dsolve(9*x^2*(3+x)*diff(y(x),x$2)+3*x*(3+7*x)*diff(y(x),x)+(3+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^{\frac{1}{3}}}{3+x} + \frac{c_2 x^{\frac{1}{3}} \ln(x)}{3+x}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 24

```
DSolve[9*x^2*(3+x)*y''[x]+3*x*(3+7*x)*y'[x]+(3+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{\sqrt[3]{x}(c_2 \log(x) + c_1)}{x+3}$$

1.158 problem 160

Internal problem ID [6891]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 160.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(-x^2+2\right)y''-x\left(3x^2+2\right)y'+\left(-x^2+2\right)y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x^2*(2-x^2)*diff(y(x),x$2)-x*(2+3*x^2)*diff(y(x),x)+(2-x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{x^2-2}+\frac{c_2x\ln\left(x\right)}{x^2-2}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 23

```
DSolve[x^2*(2-x^2)*y''[x]-x*(2+3*x^2)*y'[x]+(2-x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x)\to -\frac{x(c_2\log(x)+c_1)}{x^2-2}$$

1.159 problem 161

Internal problem ID [6892]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 161.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16x^2\left(x^2+1\right)y''+8x\left(9x^2+1\right)y'+\left(49x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve(16*x^2*(1+x^2)*diff(y(x),x$2)+8*x*(1+9*x^2)*diff(y(x),x)+(1+49*x^2)*y(x)=0,y(x), sings
```

$$y(x)=\frac{c_1x^{\frac{1}{4}}}{x^2+1}+\frac{c_2x^{\frac{1}{4}}\ln\left(x\right)}{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 26

```
DSolve[16*x^2*(1+x^2)*y''[x]+8*x*(1+9*x^2)*y'[x]+(1+49*x^2)*y[x]==0,y[x],x,IncludeSingularSol
```

$$y(x)\to\frac{\sqrt[4]{x}(c_2\log(x)+c_1)}{x^2+1}$$

1.160 problem 162

Internal problem ID [6893]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 162.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(4+3x)\,y'' - x(4-3x)\,y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(x^2*(4+3*x)*diff(y(x),x$2)-x*(4-3*x)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{3x+4} + \frac{c_2 x \ln(x)}{3x+4}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 22

```
DSolve[x^2*(4+3*x)*y''[x]-x*(4-3*x)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(c_2 \log(x) + c_1)}{3x+4}$$

1.161 problem 163

Internal problem ID [6894]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 163.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2\left(x^2+3x+1\right)y''+8x^2(3+2x)\,y'+\left(9x^2+3x+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 37

```
dsolve(4*x^2*(1+3*x+x^2)*diff(y(x),x$2)+8*x^2*(3+2*x)*diff(y(x),x)+(1+3*x+9*x^2)*y(x)=0,y(x),
```

$$y(x)=\frac{c_1\sqrt{x}}{x^2+3x+1}+\frac{c_2\sqrt{x}\,\ln(x)}{x^2+3x+1}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 28

```
DSolve[4*x^2*(1+3*x+x^2)*y''[x]+8*x^2*(3+2*x)*y'[x]+(1+3*x+9*x^2)*y[x]==0,y[x],x,IncludeSingu
```

$$y(x)\to\frac{\sqrt{x}(c_2\log(x)+c_1)}{x(x+3)+1}$$

1.162 problem 164

Internal problem ID [6895]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 164.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x)^2 y'' - x\left(-3x^2+2x+1\right) y' + \left(x^2+1\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*(1-x)^2*diff(y(x),x$2)-x*(1+2*x-3*x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x), singsol=a
```

$$y(x) = \frac{c_1 x}{(x-1)^2} + \frac{c_2 x \ln(x)}{(x-1)^2}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 20

```
DSolve[x^2*(1-x)^2*y''[x]-x*(1+2*x-3*x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{x(c_2 \log(x) + c_1)}{(x-1)^2}$$

1.163 problem 165

Internal problem ID [6896]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 165.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2\left(x^2+x+1\right)y''+3x\left(13x^2+7x+1\right)y'+\left(25x^2+4x+1\right)y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(9*x^2*(1+x+x^2)*diff(y(x),x$2)+3*x*(1+7*x+13*x^2)*diff(y(x),x)+(1+4*x+25*x^2)*y(x)=0,y
```

$$y(x)=\frac{c_1x^{\frac{1}{3}}}{x^2+x+1}+\frac{c_2x^{\frac{1}{3}}\ln\left(x\right)}{x^2+x+1}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 27

```
DSolve[9*x^2*(1+x+x^2)*y''[x]+3*x*(1+7*x+13*x^2)*y'[x]+(1+4*x+25*x^2)*y[x]==0,y[x],x,IncludeS
```

$$y(x)\to\frac{\sqrt[3]{x}(c_2\log(x)+c_1)}{x^2+x+1}$$

1.164 problem 166

Internal problem ID [6897]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 166.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x+2)\,y'' - x(4-7x)\,y' - (5-3x)\,y = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 61

```
dsolve(2*x^2*(2+x)*diff(y(x),x$2)-x*(4-7*x)*diff(y(x),x)-(5-3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^{\frac{5}{2}}}{(x+2)^{\frac{7}{2}}} + \frac{c_2\left(\frac{\sqrt{2}\sqrt{x+2}\,(33x^2+52x+32)}{15} + \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x+2}}{2}\right)x^3\right)}{(x+2)^{\frac{7}{2}}\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.148 (sec). Leaf size: 92

```
DSolve[2*x^2*(2+x)*y''[x]-x*(4-7*x)*y'[x]-(5-3*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{15\sqrt{2}c_2x^3\operatorname{arctanh}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right) - 48c_1x^3 + 66c_2\sqrt{x+2}x^2 + 104c_2\sqrt{x+2}x + 64c_2\sqrt{x+2}}{48\sqrt{x}(x+2)^{7/2}}$$

1.165 problem 167

Internal problem ID [6898]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 167.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-2x)\,y'' + x(8-9x)\,y' + (6-3x)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)+x*(8-9*x)*diff(y(x),x)+(6-3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(3x+4)\,(2x-1)^{\frac{9}{2}}}{x^6} + \frac{c_2(231x^3 - 198x^2 + 66x - 8)}{x^6}$$

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 48

```
DSolve[x^2*(1-2*x)*y''[x]+x*(8-9*x)*y'[x]+(6-3*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{385c_1(3x+4)(1-2x)^{9/2} + c_2(33x(x(7x-6)+2)-8)}{1155x^6}$$

1.166 problem 168

Internal problem ID [6899]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 168.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''+x\left(10x^2+3\right)y'-\left(-14x^2+15\right)y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 88

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(3+10*x^2)*diff(y(x),x)-(15-14*x^2)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1x^3}{\left(x^2+1\right)^{\frac{5}{2}}}+\frac{c_2\left(3\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)x^8-3\sqrt{x^2+1}\,x^6+2x^4\sqrt{x^2+1}+24\sqrt{x^2+1}\,x^2+16\sqrt{x^2+1}\right)}{x^5\left(x^2+1\right)^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 75

```
DSolve[x^2*(1+x^2)*y''[x]+x*(3+10*x^2)*y'[x]-(15-14*x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to\frac{c_2\left(\sqrt{x^2+1}(x^2+2)\left(3x^4-8x^2-8\right)-3x^8\operatorname{arctanh}\left(\sqrt{x^2+1}\right)\right)+128c_1x^8}{128x^5\left(x^2+1\right)^{5/2}}$$

1.167 problem 169

Internal problem ID [6900]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 169.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(-2x^2+1\right)y''+x\left(-13x^2+7\right)y'-14yx^2=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve(x^2*(1-2*x^2)*diff(y(x),x$2)+x*(7-13*x^2)*diff(y(x),x)-14*x^2*y(x)=0,y(x), singsol=all
```

$$y(x)=\frac{c_1(2x^2-1)^{\frac{5}{4}}}{x^6}+\frac{c_2(5x^4-20x^2+8)}{x^6}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 43

```
DSolve[x^2*(1-2*x^2)*y''[x]+x*(7-13*x^2)*y'[x]-14*x^2*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{15c_1(1-2x^2)^{5/4}+c_2(-5x^4+20x^2-8)}{15x^6}$$

1.168 problem 170

Internal problem ID [6901]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 170.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(1+x)\,y'' + 4x(2x+1)\,y' - (1+3x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(1+2*x)*diff(y(x),x)-(1+3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + \frac{c_2(\ln(x)\,x - \ln(x+1)\,x + 1)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 32

```
DSolve[4*x^2*(1+x)*y''[x]+4*x*(1+2*x)*y'[x]-(1+3*x)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{c_1 x + c_2(-x\log(x) + x\log(x+1) - 1)}{\sqrt{x}}$$

1.169 problem 171

Internal problem ID [6902]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 171.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(3x+2)\,y'' + x(4+21x)\,y' - (1-9x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 57

```
dsolve(2*x^2*(2+3*x)*diff(y(x),x$2)+x*(4+21*x)*diff(y(x),x)-(1-9*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\sqrt{x}}{(3x+2)^{\frac{3}{2}}} + \frac{c_2\left(\sqrt{2}\,\sqrt{3x+2} + 3\,\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{3x+2}}{2}\right)x\right)}{\sqrt{x}\,(3x+2)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 64

```
DSolve[2*x^2*(2+3*x)*y''[x]+x*(4+21*x)*y'[x]-(1-9*x)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\frac{3\sqrt{2}c_2 x\operatorname{arctanh}\left(\sqrt{\frac{3x}{2}+1}\right) - 2c_1 x + 2c_2\sqrt{3x+2}}{2\sqrt{x}(3x+2)^{3/2}}$$

1.170 problem 172

Internal problem ID [6903]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 172.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 + x(x+2)\,y' - (2-3x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(x^2*diff(y(x),x$2)+x*(2+x)*diff(y(x),x)-(2-3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} x + \frac{c_2(\mathrm{e}^{-x}\,\mathrm{Ei}_1(-x)\,x^3 + x^2 + x + 2)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 41

```
DSolve[x^2*y''[x]+x*(2+x)*y'[x]-(2-3*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}e^{-x}x(c_2\,\mathrm{ExpIntegralEi}(x) + 6c_1) - \frac{c_2(x^2+x+2)}{6x^2}$$

1.171 problem 173

Internal problem ID [6904]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 173.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(1+x)\,y'' + 4x(3+8x)\,y' - (5-49x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(3+8*x)*diff(y(x),x)-(5-49*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\sqrt{x}}{(x+1)^4} + \frac{c_2(6x^3\ln(x) - 18x^2 - 9x - 2)}{(x+1)^4\,x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 50

```
DSolve[4*x^2*(1+x)*y''[x]+4*x*(3+8*x)*y'[x]-(5-49*x)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{6c_1x^3 + 6c_2x^3\log(x) - c_2(9x(2x+1)+2)}{6x^{5/2}(x+1)^4}$$

1.172 problem 174

Internal problem ID [6905]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 174.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y'' - x(10x+3)\,y' + 30yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 64

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(3+10*x)*diff(y(x),x)+30*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(2x^5 - 5x^4\right) + c_2\left(\left(3x^5 - \frac{15}{2}x^4\right)\ln(x) + \frac{x^6}{4} - \frac{5x^5}{8} - \frac{299x^4}{16} + 5x^3 + \frac{5x^2}{4} + \frac{x}{4} + \frac{1}{40}\right)$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 67

```
DSolve[x^2*(1+x)*y''[x]-x*(3+10*x)*y'[x]+30*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$\begin{aligned} y(x) \to{}& \frac{1}{2}c_1(2x-5)x^4 + 6c_2(2x-5)x^4\log(x) \\ &+ \frac{1}{20}c_2(5x(x(x(x(2x(2x-5)-299)+80)+20)+4)+2) \end{aligned}$$

1.173 problem 175

Internal problem ID [6906]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 175.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 + x(1+x)\,y' - 3(x+3)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-3*(3+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^3 + \frac{c_2(\mathrm{Ei}_1(x)\,x^6 + (-x^5 + x^4 - 2x^3 + 6x^2 - 24x + 120)\,\mathrm{e}^{-x})}{x^3}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 56

```
DSolve[x^2*y''[x]+x*(1+x)*y'[x]-3*(3+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 e^{-x}(e^x x^6\,\mathrm{ExpIntegralEi}(-x) + (x(x((x-1)x+2)-6)+24)x - 120)}{720x^3} + c_1 x^3$$

1.174 problem 176

Internal problem ID [6907]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 176.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(2x+1)\,y'' + x(9+13x)\,y' + (7+5x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(9+13*x)*diff(y(x),x)+(7+5*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(143x^2+104x+20)}{x^7} + \frac{c_2(35x^3-45x^2+36x-20)\,(2x+1)^{\frac{7}{2}}}{x^7}$$

✓ Solution by Mathematica

Time used: 0.937 (sec). Leaf size: 57

```
DSolve[x^2*(1+2*x)*y''[x]+x*(9+13*x)*y'[x]+(7+5*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{c_2(x(5x(7x-9)+36)-20)(2x+1)^{7/2}}{315x^7} + \frac{c_1(13x(11x+8)+20)}{143x^7}$$

1.175 problem 177

Internal problem ID [6908]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 177.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(2x+1)\,y'' - 2x(4-x)\,y' - (7+5x)\,y = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 37

```
dsolve(4*x^2*(1+2*x)*diff(y(x),x$2)-2*x*(4-x)*diff(y(x),x)-(7+5*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2(5x^3 - 10x^2 - 40x - 16)}{(2x+1)^{\frac{5}{4}}\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 43

```
DSolve[4*x^2*(1+2*x)*y''[x]-2*x*(4-x)*y'[x]-(7+5*x)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{\frac{2c_2(5(x-4)x(x+2)-16)}{(2x+1)^{5/4}} + 35c_1}{35\sqrt{x}}$$

1.176 problem 178

Internal problem ID [6909]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 178.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2(x+3)\,y'' - x(15+x)\,y' - 20y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 33

```
dsolve(3*x^2*(3+x)*diff(y(x),x$2)-x*(15+x)*diff(y(x),x)-20*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2 - 36x - 243)}{x^{\frac{2}{3}}} + \frac{c_2(7x+27)}{x^{\frac{2}{3}}\,(3+x)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 42

```
DSolve[3*x^2*(3+x)*y''[x]-x*(15+x)*y'[x]-20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\frac{4c_1(7x+27)}{\sqrt[3]{x+3}} + 21c_2((x-36)x - 243)}{28x^{2/3}}$$

1.177 problem 179

Internal problem ID [6910]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 179.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y'' + x(1-10x)\,y' - (9-10x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 73

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+x*(1-10*x)*diff(y(x),x)-(9-10*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(715x^4 + 572x^3 + 234x^2 + 52x + 5)}{x^3} + c_2\left(8x^{10} + 91x^9 + 468x^8 + 1430x^7 + 2860x^6 + 3861x^5 + 3432x^4 + 1716x^3\right)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 50

```
DSolve[x^2*(1+x)*y''[x]+x*(1-10*x)*y'[x]-(9-10*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{6435c_1(x+1)^{12}(8x-5) - 8c_2(13x(x(11x(5x+4)+18)+4)+5)}{51480x^3}$$

1.178 problem 180

Internal problem ID [6911]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 180.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y'' + 3y'x^2 - (6-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+3*x^2*diff(y(x),x)-(6-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x+4)}{x^2} + \frac{c_2\left(6(x+4)\,(x+1)^2\ln(x+1) + 60x^2 + 129x + 68\right)}{x^2\,(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 49

```
DSolve[x^2*(1+x)*y''[x]+3*x^2*y'[x]-(6-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{6c_1(x+4) + \frac{c_2(3x(20x+43)+68)}{(x+1)^2} + 6c_2(x+4)\log(x+1)}{6x^2}$$

1.179 problem 181

Internal problem ID [6912]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 181.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(2x+1)\,y'' - 2x(3+14x)\,y' + (6+100x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 57

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)-2*x*(3+14*x)*diff(y(x),x)+(6+100*x)*y(x)=0,y(x), singsol=al
```

$$y(x) = c_1\left(2016x^5 + 672x^4 + 144x^3 + 18x^2 + x\right) + c_2\left(8x^{10} + 36x^9 + 72x^8 + 84x^7 + 63x^6\right)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 44

```
DSolve[x^2*(1+2*x)*y''[x]-2*x*(3+14*x)*y'[x]+(6+100*x)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to c_1 x(2x+1)^9 - \frac{c_2 x(6x(8x(14x(3x+1)+3)+3)+1)}{20160}$$

1.180 problem 182

Internal problem ID [6913]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 182.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y'' - x(6+11x)\,y' + (6+32x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(6+11*x)*diff(y(x),x)+(6+32*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(3x^8 + 14x^7 + 21x^6\right) + c_2\left(35x^4 + 42x^3 + 21x^2 + 4x\right)$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 44

```
DSolve[x^2*(1+x)*y''[x]-x*(6+11*x)*y'[x]+(6+32*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{3}c_1 x(x+1)^6(3x-4) - \frac{1}{140}c_2 x(7x(x(5x+6)+3)+4)$$

1.181 problem 183

Internal problem ID [6914]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 183.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(1+x)\,y'' + 4x(1+4x)\,y' - (49+27x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 30

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(1+4*x)*diff(y(x),x)-(49+27*x)*y(x)=0,y(x), singsol=all
```

$$y(x) = \frac{c_1 x^{\frac{7}{2}}}{(x+1)^2} + \frac{c_2(7x+6)}{(x+1)^2 x^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 36

```
DSolve[4*x^2*(1+x)*y''[x]+4*x*(1+4*x)*y'[x]-(49+27*x)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{42c_1x^7 - c_2(7x+6)}{42x^{7/2}(x+1)^2}$$

1.182 problem 184

Internal problem ID [6915]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 184.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''-x\left(-2x^2+7\right)y'+12y=0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 62

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(7-2*x^2)*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^6}{\left(x^2+1\right)^{\frac{7}{2}}}+c_2\left(\frac{15x^6\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)}{\left(x^2+1\right)^{\frac{7}{2}}}-\frac{8x^2\left(x^4-\frac{9}{8}x^2-\frac{1}{4}\right)}{\left(x^2+1\right)^3}\right)$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 88

```
DSolve[x^2*(1+x^2)*y''[x]-x*(7-2*x^2)*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{-15c_2x^6\operatorname{arctanh}\left(\sqrt{x^2+1}\right)-2c_2\sqrt{x^2+1}x^2+8x^6\left(c_2\sqrt{x^2+1}+c_1\right)-9c_2\sqrt{x^2+1}x^4}{8\left(x^2+1\right)^{7/2}}$$

1.183 problem 185

Internal problem ID [6916]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 185.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 - x\left(-x^2+7\right)y' + 12y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve(x^2*diff(y(x),x$2)-x*(7-x^2)*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^6 \mathrm{e}^{-\frac{x^2}{2}} + c_2 x^2 \left(\mathrm{e}^{-\frac{x^2}{2}}\,\mathrm{Ei}_1\left(-\frac{x^2}{2}\right)x^4 + 2x^2 + 4\right)$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 51

```
DSolve[x^2*y''[x]-x*(7-x^2)*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16}x^2\left(e^{-\frac{x^2}{2}}x^4\left(c_2\,\mathrm{ExpIntegralEi}\left(\frac{x^2}{2}\right) + 16c_1\right) - 2c_2\left(x^2+2\right)\right)$$

1.184 problem 186

Internal problem ID [6917]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 186.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2+x\left(2x^2+1\right)y'-\left(-10x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 37

```
dsolve(x^2*diff(y(x),x$2)+x*(1+2*x^2)*diff(y(x),x)-(1-10*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{-x^2}x\,\mathrm{hypergeom}\left(\left[-1\right],\left[2\right],x^2\right)+c_2\mathrm{e}^{-x^2}x\,\mathrm{KummerU}\left(-1,2,x^2\right)$$

✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 49

```
DSolve[x^2*y''[x]+x*(1+2*x^2)*y'[x]-(1-10*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to \frac{1}{4}e^{-x^2}x\left(x^2-2\right)\left(c_2\,\mathrm{ExpIntegralEi}\left(x^2\right)+4c_1\right)-\frac{c_2(x^2-1)}{4x}$$

1.185 problem 187

Internal problem ID [6918]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 187.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 + x\left(-2x^2+1\right)y' - 4\left(2x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve(x^2*diff(y(x),x$2)+x*(1-2*x^2)*diff(y(x),x)-4*(1+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^2 \mathrm{e}^{x^2} + \frac{c_2\left(-x^4 \mathrm{e}^{x^2}\,\mathrm{Ei}_1\left(x^2\right) + x^2 - 1\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 44

```
DSolve[x^2*y''[x]+x*(1-2*x^2)*y'[x]-4*(1+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{e^{x^2} x^4 (c_2\,\mathrm{ExpIntegralEi}\left(-x^2\right) + 4c_1) + c_2(x^2-1)}{4x^2}$$

1.186 problem 188

Internal problem ID [6919]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 188.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 + x\left(-3x^2+1\right)y' - 4\left(-3x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x),x$2)+x*(1-3*x^2)*diff(y(x),x)-4*(1-3*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(-\frac{x^2}{2}+1\right)x^2 + c_2\left(\frac{3x^2}{2}-3\right)x^2$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 64

```
DSolve[x^2*y''[x]+x*(1-3*x^2)*y'[x]-4*(1-3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{x^4(x^2-2)\left(27c_2\,\text{ExpIntegralEi}\left(\frac{3x^2}{2}\right)+64c_1\right) - 2c_2e^{\frac{3x^2}{2}}\left(9x^4-12x^2-4\right)}{64x^2}$$

1.187 problem 189

Internal problem ID [6920]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 189.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''+x\left(11x^2+5\right)y'+24yx^2=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(5+11*x^2)*diff(y(x),x)+24*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{\left(x^2+1\right)^2}+\frac{c_2(2x^2+1)}{x^4\left(x^2+1\right)^2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 36

```
DSolve[x^2*(1+x^2)*y''[x]+x*(5+11*x^2)*y'[x]+24*x^2*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x)\to -\frac{-4c_1x^4+2c_2x^2+c_2}{4x^4\left(x^2+1\right)^2}$$

1.188 problem 190

Internal problem ID [6921]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 190.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2\left(x^2+1\right)y''+8y'x-\left(-x^2+35\right)y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+8*x*diff(y(x),x)-(35-x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x^2+1)^2}{x^{\frac{7}{2}}}+\frac{c_2\left(\left(x^2+1\right)^2\ln\left(x^2+1\right)+2x^2+\frac{3}{2}\right)}{x^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 53

```
DSolve[4*x^2*(1+x^2)*y''[x]+8*x*y'[x]-(35-x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{4c_1\left(x^2+1\right)^2+c_2(4x^2+3)+2c_2\left(x^2+1\right)^2\log\left(x^2+1\right)}{4x^{7/2}}$$

1.189 problem 191

Internal problem ID [6922]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 191.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''-x\left(-x^2+5\right)y'-\left(25x^2+7\right)y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(5-x^2)*diff(y(x),x)-(7+25*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{x\left(x^2+1\right)^2}+\frac{c_2x^7(4x^2+5)}{\left(x^2+1\right)^2}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 37

```
DSolve[x^2*(1+x^2)*y''[x]-x*(5-x^2)*y'[x]-(7+25*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{c_2(4x^2+5)\,x^8+40c_1}{40x\left(x^2+1\right)^2}$$

1.190 problem 192

Internal problem ID [6923]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 192.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''+x\left(2x^2+5\right)y'-21y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(5+2*x^2)*diff(y(x),x)-21*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x^2+1)^{\frac{5}{2}}\left(x^2+8\right)}{x^7}+\frac{c_2(35x^6+140x^4+168x^2+64)}{x^7}$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 54

```
DSolve[x^2*(1+x^2)*y''[x]+x*(5+2*x^2)*y'[x]-21*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{35c_1(x^2+1)^{5/2}\left(x^2+8\right)-c_2(7(5(x^2+4)\,x^2+24)\,x^2+64)}{35x^7}$$

1.191 problem 193

Internal problem ID [6924]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 193.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2\left(x^2+1\right)y''+4x\left(x^2+2\right)y'-\left(x^2+15\right)y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+4*x*(2+x^2)*diff(y(x),x)-(15+x^2)*y(x)=0,y(x), singsol=al
```

$$y(x)=\frac{c_1(3x^2+2)}{x^{\frac{5}{2}}}+\frac{c_2(x^2+1)^{\frac{3}{2}}}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 39

```
DSolve[4*x^2*(1+x^2)*y''[x]+4*x*(2+x^2)*y'[x]-(15+x^2)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to\frac{3c_1(x^2+1)^{3/2}-c_2(3x^2+2)}{3x^{5/2}}$$

1.192 problem 194

Internal problem ID [6925]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 194.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{2(t+1)\,y'}{t^2+2t-1} + \frac{2y}{t^2+2t-1} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)-2*(t+1)/(t^2+2*t-1)*diff(y(t),t)+2/(t^2+2*t-1)*y(t)=0,y(t), singsol=all
```

$$y(t) = c_1(t+1) + c_2\left(t^2+1\right)$$

✓ Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 60

```
DSolve[y''[t]-2*(t+1)/(t^2+2*t-1)*y'[t]+2/(t^2+2*t-1)*y[t]==0,y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{\sqrt{t(t+2)-1}\left(c_1\left(t\left(t-2\sqrt{2}+2\right)-2\sqrt{2}+3\right)+c_2(t+1)\right)}{\sqrt{1-t(t+2)}}$$

1.193 problem 195

Internal problem ID [6926]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 195.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4ty' + \left(4t^2 - 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(t),t$2)-4*t*diff(y(t),t)+(4*t^2-2)*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{t^2} + c_2 \mathrm{e}^{t^2} t$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[y''[t]-4*t*y'[t]+(4*t^2-2)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{t^2} (c_2 t + c_1)$$

1.194 problem 196

Internal problem ID [6927]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 196.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(-t^2+1\right) y''-2ty'+2y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve((1-t^2)*diff(y(t),t$2)-2*t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1 t+c_2\left(\frac{\ln\left(t-1\right)t}{2}-\frac{\ln\left(t+1\right)t}{2}+1\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 19

```
DSolve[(1-t^2)*y''[t]-2*t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to c_2(t\operatorname{arctanh}(t)-1)+c_1 t$$

1.195 problem 197

Internal problem ID [6928]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 197.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(t^2+1\right)y''-2ty'+2y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve((1+t^2)*diff(y(t),t$2)-2*t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1t+c_2\left(t^2-1\right)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 21

```
DSolve[(1+t^2)*y''[t]-2*t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to c_2t-c_1(t-i)^2$$

1.196 problem 198

Internal problem ID [6929]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 198.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(-t^2+1\right) y''-2ty'+6y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve((1-t^2)*diff(y(t),t$2)-2*t*diff(y(t),t)+6*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1\left(-3t^2+1\right)+c_2\left(\left(\frac{3t^2}{8}-\frac{1}{8}\right)\ln\left(t-1\right)+\left(-\frac{3t^2}{8}+\frac{1}{8}\right)\ln\left(t+1\right)+\frac{3t}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 37

```
DSolve[(1-t^2)*y''[t]-2*t*y'[t]+6*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\frac{1}{2}\left(c_2\left(3t^2-1\right)\operatorname{arctanh}(t)+c_1\left(3t^2-1\right)-3c_2t\right)$$

1.197 problem 199

Internal problem ID [6930]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 199.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2t+1)\,y'' - 4y'(t+1) + 4y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve((2*t+1)*diff(y(t),t$2)-4*(t+1)*diff(y(t),t)+4*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1(t+1) + c_2 \mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 23

```
DSolve[(2*t+1)*y''[t]-4*(t+1)*y'[t]+4*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{2t+1} - c_2(t+1)$$

1.198 problem 200

Internal problem ID [6931]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 200.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$t^2y'' + ty' + \left(t^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(t^2*diff(y(t),t$2)+t*diff(y(t),t)+(t^2-1/4)*y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1 \sin(t)}{\sqrt{t}} + \frac{c_2 \cos(t)}{\sqrt{t}}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 39

```
DSolve[t^2*y''[t]+t*y'[t]+(t^2-1/4)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{-it}(2c_1 - ic_2 e^{2it})}{2\sqrt{t}}$$

1.199 problem 201

Internal problem ID [6932]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 201.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{2ty'}{t^2+1} + \frac{2y}{t^2+1} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(t),t$2)-2*t/(1+t^2)*diff(y(t),t)+2/(1+t^2)*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2\left(t^2 - 1\right)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 21

```
DSolve[y''[t]-2*t/(1+t^2)*y'[t]+2/(1+t^2)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_2 t - c_1 (t-i)^2$$

1.200 problem 202

Internal problem ID [6933]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 202.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(t^2 + 2t + 1\right) y' - (4t + 4)\, y = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 78

```
dsolve(diff(y(t),t$2)+(t^2+2*t+1)*diff(y(t),t)-(4+4*t)*y(t)=0,y(t), singsol=all)
```

$$\begin{aligned} y(t) = {} & c_1\left(t^4 + 4t^3 + 6t^2 + 8t + 5\right) \\ & + c_2(t+1)\left(t^3 + 3t^2 + 3t + 5\right)\left(\int \frac{\mathrm{e}^{-\frac{t\left(t^2+3t+3\right)}{3}}}{(t+1)^2\left(t^3 + 3t^2 + 3t + 5\right)^2} dt\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 1.418 (sec). Leaf size: 114

```
DSolve[y''[t]+(t^2+2*t+1)*y'[t]-(4+4*t)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(t) \to \frac{1}{36} e^{-\frac{1}{3}t(t(t+3)+3)} \Bigg(& c_2 e^{\frac{1}{3}(t+1)^3} (t(t(t+3)+3)+5)(t+1)^3 \operatorname{ExpIntegralE}\left(\frac{1}{3}, \frac{1}{3}(t+1)^3\right) \\ & + 36 c_1 e^{\frac{t^3}{3}+t^2+t} (t(t(t+3)+3)+5)(t+1) - 3c_2(t(t(t+3)+3)+4) \Bigg) \end{aligned}$$

1.201 problem 204

Internal problem ID [6934]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 204.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{2ty'' + (1-2t)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 17

```
dsolve(2*t*diff(y(t),t$2)+(1-2*t)*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^t \operatorname{erf}\left(\sqrt{t}\right) + c_2 \mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 21

```
DSolve[2*t*y''[t]+(1-2*t)*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^t \left(c_1 - c_2 \Gamma\left(\frac{1}{2}, t\right)\right)$$

1.202 problem 205

Internal problem ID [6935]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 205.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2ty'' + y'(t+1) - 2y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 56

```
dsolve(2*t*diff(y(t),t$2)+(1+t)*diff(y(t),t)-2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1\left(\sqrt{\pi}\left(t^2+6t+3\right)\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{t}}{2}\right) + \sqrt{2}\left(t^{\frac{3}{2}}+5\sqrt{t}\right)\mathrm{e}^{-\frac{t}{2}}\right) + c_2\left(t^2+6t+3\right)$$

✓ Solution by Mathematica

Time used: 5.271 (sec). Leaf size: 64

```
DSolve[2*t*y''[t]+(1+t)*y'[t]-2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{24}\left((t(t+6)+3)\left(\sqrt{2\pi}c_2\operatorname{erf}\left(\frac{\sqrt{t}}{\sqrt{2}}\right)+24c_1\right) + 2c_2e^{-t/2}\sqrt{t}(t+5)\right)$$

1.203 problem 206

Internal problem ID [6936]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 206.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2t^2y'' - ty' + (t+1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(2*t^2*diff(y(t),t$2)-t*diff(y(t),t)+(1+t)*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \sin\left(\sqrt{2}\,\sqrt{t}\right)\sqrt{t} + c_2\sqrt{t}\,\cos\left(\sqrt{2}\,\sqrt{t}\right)$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 62

```
DSolve[2*t^2*y''[t]-t*y'[t]+(1+t)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-i\sqrt{2}\sqrt{t}}\sqrt{t}\left(2c_1e^{2i\sqrt{2}\sqrt{t}} + i\sqrt{2}c_2\right)$$

1.204 problem 207

Internal problem ID [6937]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 207.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2t^2y'' + \left(t^2 - t\right)y' + y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve(2*t^2*diff(y(t),t$2)+(t^2-t)*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \,\text{WhittakerM}\left(\frac{1}{4}, \frac{1}{4}, \frac{t}{2}\right) t^{\frac{1}{4}} \mathrm{e}^{-\frac{t}{4}} + c_2\sqrt{t}\,\mathrm{e}^{-\frac{t}{2}}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 36

```
DSolve[2*t^2*y''[t]+(t^2-t)*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t/2}\left(c_2\sqrt{t} - c_1 t\,\text{ExpIntegralE}\left(\frac{1}{2}, -\frac{t}{2}\right)\right)$$

1.205 problem 208

Internal problem ID [6938]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 208.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' + \left(-t^2 + t\right)y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(t^2*diff(y(t),t$2)+(t-t^2)*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1(t+1)}{t} + \frac{c_2 e^t}{t}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 23

```
DSolve[t^2*y''[t]+(t-t^2)*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 e^t - c_1(t+1)}{t}$$

1.206 problem 209

Internal problem ID [6939]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 209.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{ty'' - \left(t^2+2\right)y' + yt = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(t*diff(y(t),t$2)-(t^2+2)*diff(y(t),t)+t*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{\frac{t^2}{2}} + c_2 \left(\sqrt{\pi}\, t\sqrt{2} - \operatorname{erf}\left(\frac{\sqrt{2}\, t}{2} \right) \pi\, \mathrm{e}^{\frac{t^2}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 47

```
DSolve[t*y''[t]-(t^2+2)*y'[t]+t*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} e^{\frac{t^2}{2}} \left(\sqrt{2\pi} c_2 \operatorname{erf}\left(\frac{t}{\sqrt{2}} \right) + 2c_1 \right) - c_2 t$$

1.207 problem 210

Internal problem ID [6940]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 210.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' + t(t+1)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(t^2*diff(y(t),t$2)+t*(t+1)*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1(t-1)}{t} + \frac{c_2 \mathrm{e}^{-t}}{t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 26

```
DSolve[t^2*y''[t]+t*(t+1)*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{-t}(c_1 e^t(t-1) + c_2)}{t}$$

1.208 problem 211

Internal problem ID [6941]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 211.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{ty'' - (t+4)\,y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(t*diff(y(t),t$2)-(4+t)*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1\left(t^2 + 6t + 12\right) + c_2 \mathrm{e}^t\left(t^2 - 6t + 12\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 63

```
DSolve[t*y''[t]-(4+t)*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\sqrt{t}((c_2 - ic_1)(t(t+6)+12) + (ic_1 + c_2)e^t((t-6)t+12))}{\sqrt{\pi}\sqrt{-it}}$$

1.209 problem 212

Internal problem ID [6942]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 212.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2 y'' + \left(t^2 - 3t\right) y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(t^2*diff(y(t),t$2)+(t^2-3*t)*diff(y(t),t)+3*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t^3 \mathrm{e}^{-t} + c_2 t\left(\mathrm{e}^{-t}\,\mathrm{Ei}_1\left(-t\right) t^2 + t + 1\right)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 37

```
DSolve[t^2*y''[t]+(t^2-3*t)*y'[t]+3*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} t\left(e^{-t} t^2 (c_1 \operatorname{ExpIntegralEi}(t) + 2c_2) - c_1 (t+1)\right)$$

1.210 problem 213

Internal problem ID [6943]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 213.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' + ty' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(t*diff(y(t),t$2)+t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{-t}(t-2)\,t + c_2\left(\mathrm{e}^{-t}(t-2)\,t\ \mathrm{Ei}_1\left(-t\right) + t - 1\right)$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 37

```
DSolve[t*y''[t]+t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}\left(e^{-t}(t-2)t(c_2\,\text{ExpIntegralEi}(t) + 2c_1) - c_2(t-1)\right)$$

1.211 problem 214

Internal problem ID [6944]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 214.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' + \left(-t^2 + 1\right) y' + 4yt = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(t*diff(y(t),t$2)+(1-t^2)*diff(y(t),t)+4*t*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1\left(1 - t^2 + \frac{1}{8}t^4\right) + c_2\left(2 - 2t^2 + \frac{1}{4}t^4\right)$$

✓ Solution by Mathematica

Time used: 0.202 (sec). Leaf size: 55

```
DSolve[t*y''[t]+(1-t^2)*y'[t]+4*t*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{128}\left(\left(t^4 - 8t^2 + 8\right)\left(c_2\,\text{ExpIntegralEi}\left(\frac{t^2}{2}\right) + 128c_1\right) - 2c_2 e^{\frac{t^2}{2}}\left(t^2 - 6\right)\right)$$

1.212 problem 215

Internal problem ID [6945]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 215.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2 y'' - t(t+1)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(t^2*diff(y(t),t$2)-t*(1+t)*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^t c_1 t + c_2 \mathrm{e}^t t \operatorname{Ei}_1(t)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 20

```
DSolve[t^2*y''[t]-t*(1+t)*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^t t(c_1 \operatorname{ExpIntegralEi}(-t) + c_2)$$

1.213 problem 216

Internal problem ID [6946]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 216.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y'x + \left(4x^2 + 6\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} \cos\left(2x\right) + c_2 \mathrm{e}^{-x^2} \sin\left(2x\right)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 37

```
DSolve[y''[x]+4*x*y'[x]+(4*x^2+6)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-x(x+2i)} \left(4c_1 - ic_2 e^{4ix}\right)$$

1.214 problem 217

Internal problem ID [6947]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 217.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(-z^2+1\right) y'' - 3zy' + \lambda y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 55

```
dsolve((1-z^2)*diff(y(z),z$2)-3*z*diff(y(z),z)+lambda*y(z)=0,y(z), singsol=all)
```

$$y(z) = \frac{c_1\left(z+\sqrt{z^2-1}\right)^{\sqrt{\lambda+1}}}{\sqrt{z^2-1}} + \frac{c_2\left(z+\sqrt{z^2-1}\right)^{-\sqrt{\lambda+1}}}{\sqrt{z^2-1}}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 54

```
DSolve[(1-z^2)*y''[z]-3*z*y'[z]+\[Lambda]*y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z) \to \frac{c_1 P^{\frac{1}{2}}_{\sqrt{\lambda+1}-\frac{1}{2}}(z) + c_2 Q^{\frac{1}{2}}_{\sqrt{\lambda+1}-\frac{1}{2}}(z)}{\sqrt[4]{z^2-1}}$$

1.215 problem 218

Internal problem ID [6948]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 218.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4zy'' + 2(1-z)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(4*z*diff(y(z),z$2)+2*(1-z)*diff(y(z),z)-y(z)=0,y(z), singsol=all)
```

$$y(z) = c_1 \mathrm{e}^{\frac{z}{2}} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{z}}{2}\right) + c_2 \mathrm{e}^{\frac{z}{2}}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 34

```
DSolve[4*z*y''[z]+2*(1-z)*y'[z]-y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z) \to e^{z/2}\left(c_1 - \sqrt{2}c_2\Gamma\left(\frac{1}{2}, \frac{z}{2}\right)\right)$$

1.216 problem 219

Internal problem ID [6949]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 219.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{f'' + 2(z-1)\,f' + 4f = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve(diff(f(z),z$2)+2*(z-1)*diff(f(z),z)+4*f(z)=0,f(z), singsol=all)
```

$$f(z) = c_1 \mathrm{e}^{-z(z-2)}(z-1) + c_2\left(i(z-1)\sqrt{\pi}\ \mathrm{erf}\left(i(z-1)\right)\mathrm{e}^{-(z-1)^2} + 1\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 56

```
DSolve[f''[z]+2*(z-a)*f'[z]+4*f[z]==0,f[z],z,IncludeSingularSolutions -> True]
```

$$f(z) \to e^{z(2a-z)}\left(c_2 e^{(a-z)^2} - (a-z)\left(\sqrt{\pi}c_2 \mathrm{erfi}(a-z) + 2c_1\right)\right)$$

1.217 problem 220

Internal problem ID [6950]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{zy'' - 2y' + yz = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(z*diff(y(z),z$2)-2*diff(y(z),z)+z*y(z)=0,y(z), singsol=all)
```

$$y(z) = c_1(\cos(z)\, z - \sin(z)) + c_2(\cos(z) + \sin(z)\, z)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 39

```
DSolve[z*y''[z]-2*y'[z]+z*y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z) \to -\sqrt{\frac{2}{\pi}}((c_1 z + c_2)\cos(z) + (c_2 z - c_1)\sin(z))$$

1.218 problem 221

Internal problem ID [6951]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 221.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{zy'' + (2z-3)\,y' + \frac{4y}{z} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(z*diff(y(z),z$2)+(2*z-3)*diff(y(z),z)+4/z*y(z)=0,y(z), singsol=all)
```

$$y(z) = c_1 z^2 \mathrm{e}^{-2z}(2z-1) + c_2 z^2 \left(\frac{1}{2} + \mathrm{e}^{-2z}\left(z - \frac{1}{2}\right) \mathrm{Ei}_1\left(-2z\right)\right)$$

✓ Solution by Mathematica

Time used: 0.29 (sec). Leaf size: 39

```
DSolve[z*y''[z]+(2*z-3)*y'[z]+4/z*y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z) \to \frac{1}{2} z^2 \left(e^{-2z}(2z-1)(4c_2\,\text{ExpIntegralEi}(2z) + c_1) - 4c_2\right)$$

1.219 problem 222

Internal problem ID [6952]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 222.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_erf]`

$$\boxed{y'' + 2y'x + 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} x + c_2 \left(-\sqrt{\pi}\ \mathrm{erfi}\left(x\right) \mathrm{e}^{-x^2} x + 1 \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 27

```
DSolve[y''[x]+2*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2c_2 x\, \mathrm{DawsonF}(x) + 2c_1 e^{-x^2} x + c_2$$

1.220 problem 223

Internal problem ID [6953]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 223.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 3y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\sqrt{2}\sqrt{\pi}\ \operatorname{erfi}\left(\frac{\sqrt{2}\,x}{2}\right)(x-1)(x+1)\,\mathrm{e}^{-\frac{x^2}{2}} - 2x\right) + c_2\mathrm{e}^{-\frac{x^2}{2}}\left(x^2-1\right)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 53

```
DSolve[y''[x]+x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(e^{-\frac{x^2}{2}}\left(x^2-1\right)\left(\sqrt{2\pi}c_2\operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) + 4c_1\right) - 2c_2x\right)$$

1.221 problem 224

Internal problem ID [6954]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 224.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x^2 - 3yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-3*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^3}{3}} x + \frac{c_2 \left(x^6 \operatorname{WhittakerM}\left(\frac{1}{3}, \frac{5}{6}, \frac{x^3}{3}\right) + (5x^3 + 10) \operatorname{WhittakerM}\left(\frac{4}{3}, \frac{5}{6}, \frac{x^3}{3}\right) \right) \mathrm{e}^{\frac{x^3}{6}}}{x^4}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 39

```
DSolve[y''[x]-x^2*y'[x]-3*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{\frac{x^3}{3}} \left(3c_1 x - c_2 \operatorname{ExpIntegralE}\left(\frac{4}{3}, \frac{x^3}{3}\right) \right)$$

1.222 problem 225

Internal problem ID [6955]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 225.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(-4x^2+1\right)y''-20y'x-16y=0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 57

```
dsolve((1-4*x^2)*diff(y(x),x$2)-20*x*diff(y(x),x)-16*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1 x}{\left(4x^2-1\right)^{\frac{3}{2}}}+\frac{c_2\left(2\ln\left(2x+\sqrt{4x^2-1}\right)x-\sqrt{4x^2-1}\right)}{\left(4x^2-1\right)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 71

```
DSolve[(1-4*x^2)*y''[x]-20*x*y'[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-c_2\sqrt{1-4x^2}+4c_2 x\cot^{-1}\left(\frac{2x+1}{\sqrt{1-4x^2}}\right)+c_1 x}{\sqrt[4]{1-4x^2}\left(4x^2-1\right)^{5/4}}$$

1.223 problem 226

Internal problem ID [6956]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 226.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(x^2-1\right)y''-6y'x+12y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve((x^2-1)*diff(y(x),x$2)-6*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^3+x\right)+c_2\left(x^4+6x^2+1\right)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 45

```
DSolve[(x^2-1)*y''[x]-6*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\sqrt{x^2-1}\left(c_2x\left(x^2+1\right)+c_1(x-1)^4\right)}{\sqrt{1-x^2}}$$

1.224 problem 227

Internal problem ID [6957]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 227.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+(2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}\,\mathrm{KummerM}\left(\frac{3}{2}, \frac{1}{2}, -\frac{(x-2)^2}{2}\right) + c_2 \mathrm{e}^{-x}\,\mathrm{KummerU}\left(\frac{3}{2}, \frac{1}{2}, -\frac{(x-2)^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.205 (sec). Leaf size: 84

```
DSolve[y''[x]+x*y'[x]+(2+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-\frac{x^2}{2}+x-\frac{9}{2}} \left(e^{5/2}(x-3)(x-1)\left(\sqrt{2\pi}c_2 \operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right) + 4e^2 c_1\right) - 2c_2 e^{\frac{1}{2}(x-3)^2+x}(x-2)\right)$$

1.225 problem 228

Internal problem ID [6958]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 228.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(2x^2+1\right)y''+7y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
dsolve((1+2*x^2)*diff(y(x),x$2)+7*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\,\text{LegendreP}\left(\frac{1}{4},\frac{3}{4},i\sqrt{2}\,x\right)}{\left(2x^2+1\right)^{\frac{3}{8}}}+\frac{c_2\,\text{LegendreQ}\left(\frac{1}{4},\frac{3}{4},i\sqrt{2}\,x\right)}{\left(2x^2+1\right)^{\frac{3}{8}}}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 66

```
DSolve[(1+2*x^2)*y''[x]+7*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_2 Q_{\frac{1}{4}}^{\frac{3}{4}}\left(i\sqrt{2}x\right)}{\left(2x^2+1\right)^{3/8}}+\frac{2i\sqrt[4]{2}c_1 x}{\left(2x^2+1\right)^{3/4}\,\text{Gamma}\left(\frac{1}{4}\right)}$$

1.226 problem 229

Internal problem ID [6959]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 229.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{4y'' + y'x + 4y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(4*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^2}{8}} \operatorname{hypergeom}\left(\left[-1\right], \left[\frac{3}{2}\right], \frac{x^2}{8}\right) x + c_2 \mathrm{e}^{-\frac{x^2}{8}} \operatorname{hypergeom}\left(\left[-\frac{3}{2}\right], \left[\frac{1}{2}\right], \frac{x^2}{8}\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 64

```
DSolve[4*y''[x]+x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{32}\left(\sqrt{2}e^{-\frac{x^2}{8}} x\left(x^2-12\right)\left(\sqrt{\pi}c_2 \operatorname{erfi}\left(\frac{x}{2\sqrt{2}}\right) + 8c_1\right) - 4c_2\left(x^2-8\right)\right)$$

1.227 problem 230

Internal problem ID [6960]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 230.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x - 4y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 57

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\sqrt{2}\,\mathrm{e}^{-\frac{x^2}{2}}\left(x^2+5\right)x + \operatorname{erf}\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{\pi}\left(x^4+6x^2+3\right)\right) + c_2\left(x^4+6x^2+3\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 43

```
DSolve[y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{2}}\,\text{HermiteH}\left(-5, \frac{x}{\sqrt{2}}\right) + \frac{1}{3}c_2\left(x^4+6x^2+3\right)$$

1.228 problem 231

Internal problem ID [6961]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4xy'' - y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(4*x*diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^2 - 8x) + c_2\left(\frac{(x^2 - 8x)\,\mathrm{Ei}_1\left(-\frac{x}{4}\right)}{128} + \frac{\mathrm{e}^{\frac{x}{4}}(x-4)}{32}\right)$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 43

```
DSolve[4*x*y''[x]-x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{128}c_2\left((x-8)x\,\mathrm{ExpIntegralEi}\left(\frac{x}{4}\right) - 4e^{x/4}(x-4)\right) + c_1(x-8)x$$

1.229 problem 232

Internal problem ID [6962]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 232.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{6x^2y'' + x(1+18x)\,y' + (1+12x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 46

```
dsolve(6*x^2*diff(y(x),x$2)+x*(1+18*x)*diff(y(x),x)+(1+12*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\mathrm{e}^{-3x} + \frac{c_2\left(-\frac{(-x)^{\frac{5}{6}}3^{\frac{5}{6}}}{3} + x\,\mathrm{e}^{-3x}\left(\Gamma\left(\frac{5}{6}\right) - \Gamma\left(\frac{5}{6}, -3x\right)\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 36

```
DSolve[6*x^2*y''[x]+x*(1+18*x)*y'[x]+(1+12*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{-3x}\sqrt[3]{x}\left(c_1\sqrt[6]{x} - c_2\,\text{ExpIntegralE}\left(\frac{7}{6}, -3x\right)\right)$$

1.230 problem 233

Internal problem ID [6963]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 233.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2y'' - x(x+8)\,y' + 6y = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 31

```
dsolve(3*x^2*diff(y(x),x$2)-x*(x+8)*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\text{hypergeom}\left([3], \left[\frac{10}{3}\right], \frac{x}{3}\right) x^3 + c_2 \,\text{hypergeom}\left(\left[\frac{2}{3}\right], \left[-\frac{4}{3}\right], \frac{x}{3}\right) x^{\frac{2}{3}}$$

✓ Solution by Mathematica

Time used: 0.555 (sec). Leaf size: 62

```
DSolve[3*x^2*y''[x]-x*(x+8)*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{18} e^{x/3} x^{2/3} ((x-2)x+4) \left(18c_1 - \sqrt[3]{3} c_2 \Gamma\left(\frac{1}{3}, \frac{x}{3}\right)\right) + \frac{1}{6} c_2 (x-4)x$$

1.231 problem 234

Internal problem ID [6964]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 234.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - x(2x+1)\,y' + 2(4x-1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 32

```
dsolve(2*x^2*diff(y(x),x$2)-x*(1+2*x)*diff(y(x),x)+2*(4*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\frac{4}{63}x^4 - \frac{4}{7}x^3 + x^2\right) + \frac{c_2\,\text{hypergeom}\left(\left[-\frac{9}{2}\right],\left[-\frac{3}{2}\right],x\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.855 (sec). Leaf size: 71

```
DSolve[2*x^2*y''[x]-x*(1+2*x)*y'[x]+2*(4*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \\ \to \frac{x^{5/2}(4(x-9)x+63)\left(32c_2e^x\,\text{DawsonF}\left(\sqrt{x}\right)+945c_1\right) - 32c_2e^x(x(x(x(2x-17)+24)+6)+3)}{3780\sqrt{x}}$$

1.232 problem 235

Internal problem ID [6965]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 235.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4y'x^2 + (2x+1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(4*x^2*diff(y(x),x$2)-4*x^2*diff(y(x),x)+(1+2*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x}\ \mathrm{Ei}_1\left(-x\right)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 19

```
DSolve[4*x^2*y''[x]-4*x^2*y'[x]+(1+2*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_2\,\mathrm{ExpIntegralEi}(x) + c_1)$$

1.233 problem 236

Internal problem ID [6966]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 236.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(3-2x)\,y' + (1-2x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*(3-2*x)*diff(y(x),x)+(1-2*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2\,\mathrm{Ei}_1\left(-2x\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 19

```
DSolve[x^2*y''[x]+x*(3-2*x)*y'[x]+(1-2*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2\,\mathrm{ExpIntegralEi}(2x) + c_1}{x}$$

1.234 problem 237

Internal problem ID [6967]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 237.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(x+3)\,y' + (4-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x$2)-x*(3+x)*diff(y(x),x)+(4-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x x^2 \left(x^2+4x+2\right) + c_2 x^2 \left(\mathrm{e}^x \left(x^2+4x+2\right) \mathrm{Ei}_1\left(x\right) - x - 3\right)$$

✓ Solution by Mathematica

Time used: 0.1 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]-x*(3+x)*y'[x]+(4-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}x^2(e^x(x(x+4)+2)(c_2\,\text{ExpIntegralEi}(-x)+4c_1)+c_2(x+3))$$

1.235 problem 238

Internal problem ID [6968]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 238.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(3-x)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x^2*diff(y(x),x$2)+x*(3-x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x-1)}{x} + \frac{c_2((x-1)\,\mathrm{Ei}_1(-x) + \mathrm{e}^x)}{x}$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 29

```
DSolve[x^2*y''[x]+x*(3-x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x-1)(c_2\,\mathrm{ExpIntegralEi}(x) + c_1) - c_2 e^x}{x}$$

1.236 problem 239

Internal problem ID [6969]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 239.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - \left(2\sqrt{5}-1\right)xy' + \left(\frac{19}{4} - 3x^2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x$2)-(2*sqrt(5)-1)*x*diff(y(x),x)+(19/4-3*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{-\frac{1}{2}+\sqrt{5}} \sinh\left(\sqrt{3}\,x\right) + c_2 x^{-\frac{1}{2}+\sqrt{5}} \cosh\left(\sqrt{3}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 53

```
DSolve[x^2*y''[x]-(2*Sqrt[5]-1)*x*y'[x]+(19/4-3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{6} e^{-\sqrt{3}x} x^{\sqrt{5}-\frac{1}{2}} \left(\sqrt{3}c_2 e^{2\sqrt{3}x} + 6c_1\right)$$

1.237 problem 240

Internal problem ID [6970]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x-3)\,y' + (4-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(x^2*diff(y(x),x$2)+x*(x-3)*diff(y(x),x)+(4-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}c_1x^2 + c_2x^2\mathrm{e}^{-x}\,\mathrm{Ei}_1\left(-x\right)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+x*(x-3)*y'[x]+(4-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}x^2(c_2\,\text{ExpIntegralEi}(x) + c_1)$$

1.238 problem 241

Internal problem ID [6971]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 241.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x^2 - (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)-(2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{-x}}{x} + \frac{c_2(x^2 - 2x + 2)}{x}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 30

```
DSolve[x^2*y''[x]+x^2*y'[x]-(2+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(c_2 e^x((x-2)x+2) + c_1)}{x}$$

1.239 problem 242

Internal problem ID [6972]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 242.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2y'x^2 + \left(x - \frac{3}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x$2)+2*x^2*diff(y(x),x)+(x-3/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2 \mathrm{e}^{-2x}(2x+1)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]+2*x^2*y'[x]+(x-3/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 - c_2 e^{-2x}(2x+1)}{4\sqrt{x}}$$

1.240 problem 243

Internal problem ID [6973]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 243.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y'' + y'x^2 - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+x^2*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x+2)}{x} + \frac{c_2(4+(x+2)\ln(x+1))}{x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 30

```
DSolve[x^2*(1+x)*y''[x]+x^2*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1(x+2) + c_2(x+2)\log(x+1) + 4c_2}{x}$$

1.241 problem 244

Internal problem ID [6974]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 244.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x\left(x^2+6\right)y' + 6y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve(x^2*diff(y(x),x$2)+x*(6+x^2)*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2+3)}{x^2} + \frac{c_2 \mathrm{e}^{-\frac{x^2}{2}}\,\mathrm{hypergeom}\left([2],\left[\frac{1}{2}\right],\frac{x^2}{2}\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.246 (sec). Leaf size: 63

```
DSolve[x^2*y''[x]+x*(6+x^2)*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2c_2 e^{-\frac{x^2}{2}}(x^2+2) - x(x^2+3)\left(12c_1 - \sqrt{2\pi}c_2 \mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)\right)}{12x^3}$$

1.242 problem 245

Internal problem ID [6975]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 245.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(1-x)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*(1-x)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x+1)}{x} + \frac{c_2 \mathrm{e}^x}{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]+x*(1-x)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 e^x - c_1(x+1)}{x}$$

1.243 problem 246

Internal problem ID [6976]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 246.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(x+3)y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(x^2*diff(y(x),x$2)-x*(x+3)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x x^2(x+1) + c_2 x^2(-1 + \mathrm{e}^x(x+1)\,\mathrm{Ei}_1(x))$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 29

```
DSolve[x^2*y''[x]-x*(x+3)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(e^x(x+1)(c_2\,\mathrm{ExpIntegralEi}(-x) + c_1) + c_2)$$

1.244 problem 247

Internal problem ID [6977]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 247.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x^2 - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x+2)}{x} + \frac{c_2 e^x (x-2)}{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 57

```
DSolve[x^2*y''[x]-x^2*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(c_1 - ic_2)\left(-e^x\right)(x-2) - (c_1 + ic_2)(x+2)}{\sqrt{\pi}\sqrt{-ix}\sqrt{x}}$$

1.245 problem 248

Internal problem ID [6978]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 248.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x^2 - (3x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)-(3*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2\mathrm{e}^x(x+4) + \frac{c_2(x^3\mathrm{e}^x(x+4)\,\mathrm{Ei}_1(x) - x^3 - 3x^2 + 2x - 2)}{x}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 51

```
DSolve[x^2*y''[x]-x^2*y'[x]-(3*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x x^3(x+4)(24c_1 - c_2\,\mathrm{ExpIntegralEi}(-x)) - c_2(x(x(x+3)-2)+2)}{24x}$$

1.246 problem 249

Internal problem ID [6979]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 249.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(5-x)\,y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x$2)+x*(5-x)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2-4x+2)}{x^2} + \frac{c_2((x^2-4x+2)\,\mathrm{Ei}_1(-x) + \mathrm{e}^x(x-3))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 41

```
DSolve[x^2*y''[x]+x*(5-x)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{((x-4)x+2)(c_2\,\mathrm{ExpIntegralEi}(x) + 4c_1) - c_2e^x(x-3)}{4x^2}$$

1.247 problem 250

Internal problem ID [6980]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 250.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4x(1-x)\,y' + (2x-9)\,y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 25

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*(1-x)*diff(y(x),x)+(2*x-9)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^x}{x^{\frac{3}{2}}} + \frac{c_2(x^2+2x+2)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 29

```
DSolve[4*x^2*y''[x]+4*x*(1-x)*y'[x]+(2*x-9)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 e^x - c_2(x(x+2)+2)}{x^{3/2}}$$

1.248 problem 251

Internal problem ID [6981]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 251.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2x(x+2)\,y' + 2(1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(x^2*diff(y(x),x$2)+2*x*(2+x)*diff(y(x),x)+2*(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2(2\ \mathrm{Ei}_1\left(2x\right)x - \mathrm{e}^{-2x})}{x^2}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]+2*x*(2+x)*y'[x]+2*(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2c_2x\,\mathrm{ExpIntegralEi}(-2x) + c_1x - c_2e^{-2x}}{x^2}$$

1.249 problem 252

Internal problem ID [6982]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 252.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(1-x)\,y' + (1-x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)-x*(1-x)*diff(y(x),x)+(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2\,\mathrm{Ei}_1(x)\,x$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 17

```
DSolve[x^2*y''[x]-x*(1-x)*y'[x]+(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2\,\mathrm{ExpIntegralEi}(-x) + c_1)$$

1.250 problem 253

Internal problem ID [6983]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 253.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 4x(2x+1)\,y' + (4x-1)\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*(1+2*x)*diff(y(x),x)+(4*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2 \mathrm{e}^{-2x}}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 26

```
DSolve[4*x^2*y''[x]+4*x*(1+2*x)*y'[x]+(4*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{2c_1 e^{-2x} + c_2}{2\sqrt{x}}$$

1.251 problem 254

Internal problem ID [6984]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 254.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x+4)\,y' + (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+x*(4+x)*diff(y(x),x)+(2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2(\mathrm{Ei}_1(x)\,x - \mathrm{e}^{-x})}{x^2}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]+x*(4+x)*y'[x]+(2+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-c_2 x\,\mathrm{ExpIntegralEi}(-x) + c_1 x - c_2 e^{-x}}{x^2}$$

1.252 problem 255

Internal problem ID [6985]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 255.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{9}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-9/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{ix}(x+i)}{x^{\frac{3}{2}}} + \frac{c_2 e^{-ix}(-x+i)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 44

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-9/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((c_1 x + c_2)\cos(x) + (c_2 x - c_1)\sin(x))}{x^{3/2}}$$

1.253 problem 256

Internal problem ID [6986]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 256.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.254 problem 257

Internal problem ID [6987]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 257.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + 5(1-2x)\,y' - 5y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 50

```
dsolve(2*x*diff(y(x),x$2)+5*(1-2*x)*diff(y(x),x)-5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\frac{\operatorname{erfi}\left(\sqrt{5}\,\sqrt{x}\right)(1+10x)\sqrt{5}\,\sqrt{\pi}}{x^{\frac{3}{2}}} - \frac{10\,\mathrm{e}^{5x}}{x}\right) + \frac{c_2(1+10x)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 40

```
DSolve[2*x*y''[x]+5*(1-2*x)*y'[x]-5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 L^{\frac{3}{2}}_{-\frac{1}{2}}(5x) + \frac{c_1(10x+1)}{10\sqrt{5}x^{3/2}}$$

1.255 problem 258

Internal problem ID [6988]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 258.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2 e^{2ix})}{2\sqrt{x}}$$

1.256 problem 259

Internal problem ID [6989]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 259.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (x+n)\,y' + (n+1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(x*diff(y(x),x$2)+(x+n)*diff(y(x),x)+(n+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}\,\mathrm{hypergeom}\left([-1],[n],x\right) + c_2 x^{-n+1}\mathrm{e}^{-x}\,\mathrm{hypergeom}\left([-n],[-n+2],x\right)$$

✓ Solution by Mathematica

Time used: 0.421 (sec). Leaf size: 48

```
DSolve[x*y''[x]+(x+n)*y'[x]+(n+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(n-x)\left(c_2 \int_1^x \frac{e^{K[1]} K[1]^{-n}}{(n-K[1])^2} dK[1] + c_1\right)$$

1.257 problem 260

Internal problem ID [6990]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 260.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^4 y'' + y' x + y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 50

```
dsolve(x^4*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(\frac{\operatorname{erfi}\left(\frac{\sqrt{2}}{2x}\right) (x^2-1)\sqrt{2}\sqrt{\pi}}{x} + 2\,\mathrm{e}^{\frac{1}{2x^2}} \right) + \frac{c_2 (x^2-1)}{x}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 59

```
DSolve[x^4*y''[x]+x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x^2-1)\left(4c_1 - \sqrt{2\pi} c_2 \operatorname{erfi}\left(\frac{1}{\sqrt{2}x}\right)\right) - 2c_2 e^{\frac{1}{2x^2}} x}{4x}$$

1.258 problem 261

Internal problem ID [6991]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 261.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(2x^2 + x\right)y' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve(x^2*diff(y(x),x$2)+(x+2*x^2)*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(2x^2 - 4x + 3)}{x^2} + \frac{c_2 \mathrm{e}^{-2x}(2x + 3)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.176 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]+(x+2*x^2)*y'[x]-4*y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(\frac{c_1 e^{-2x}(4x + 6) + c_2(3 - 4x)}{x^2} + 2(-1 + c_2)\right)$$

1.259 problem 262

Internal problem ID [6992]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 262.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(4x^3-14x^2-2x\right)y''-\left(6x^2-7x+1\right)y'+(6x-1)y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve((4*x^3-14*x^2-2*x)*diff(y(x),x$2)-(6*x^2-7*x+1)*diff(y(x),x)+(6*x-1)*y(x)=0,y(x), sing
```

$$y(x)=c_1(x-1)+c_2\sqrt{x}\,(2x+1)$$

✓ Solution by Mathematica

Time used: 3.654 (sec). Leaf size: 26

```
DSolve[(4*x^3-14*x^2-2*x)*y''[x]-(6*x^2-7*x+1)*y'[x]+(6*x-1)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x)\to c_1(x-1)-2c_2\sqrt{x}(2x+1)$$

1.260 problem 263

Internal problem ID [6993]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 263.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x^2 + (x-2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)+(x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2 \mathrm{e}^{-x}(x^2+2x+2)}{x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 28

```
DSolve[x^2*y''[x]+x^2*y'[x]+(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 - c_2 e^{-x}(x(x+2)+2)}{x}$$

1.261 problem 264

Internal problem ID [6994]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 264.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x^2 + (x-2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)+(x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^x}{x} + \frac{c_2(x^2+2x+2)}{x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]-x^2*y'[x]+(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 e^x - c_2(x(x+2)+2)}{x}$$

1.262 problem 265

Internal problem ID [6995]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 265.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-4x)\,y'' - \frac{y'x}{2} - \frac{3yx}{4} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 30

```
dsolve(x^2*(1-4*x)*diff(y(x),x$2)+((1-(3/2))*x-(6-4*(3/2))*x^2)*diff(y(x),x)+(3/2)*(1-(3/2))*
```

$$y(x) = c_1 \,\text{hypergeom}\left(\left[-\frac{3}{4}, -\frac{1}{4}\right], \left[-\frac{1}{2}\right], 4x\right) + c_2 x^{\frac{3}{2}} \,\text{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{4}\right], \left[\frac{5}{2}\right], 4x\right)$$

✓ Solution by Mathematica

Time used: 0.171 (sec). Leaf size: 111

```
DSolve[x^2*(1-4*x)*y''[x]+((1-(3/2))*x-(6-4*(3/2))*x^2)*y'[x]+(3/2)*(1-(3/2))*x*y[x]==0,y[x],
```

$$y(x) \to \frac{\sqrt[4]{x}\sqrt[4]{4x-1}\left(6c_1\left(\sqrt{4x-1}-i\right)^{3/2} + ic_2\left(\sqrt{4x-1}+i\right)^{3/2}\right)}{6\sqrt[4]{1-4x}\sqrt[4]{\sqrt{4x-1}-i}\sqrt[4]{\sqrt{4x-1}+i}}$$

1.263 problem 266

Internal problem ID [6996]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 266.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(x^2 + x\right)y' + (-9 + x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(x^2*diff(y(x),x$2)+(x+x^2)*diff(y(x),x)+(x-9)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2 - 8x + 20)}{x^3} + \frac{c_2 \mathrm{e}^{-x}(x^3 + 9x^2 + 36x + 60)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 40

```
DSolve[x^2*y''[x]+(x+x^2)*y'[x]+(x-9)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1((x-8)x + 20) - c_2 e^{-x}(x(x(x+9)+36)+60)}{x^3}$$

1.264 problem 267

Internal problem ID [6997]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 267.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(1+x)\,y' + (3x-1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x$2)+x*(x+1)*diff(y(x),x)+(3*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{-x}(x-3) + \frac{c_2(x^2\mathrm{e}^{-x}(x-3)\,\mathrm{Ei}_1(-x) + x^2 - 2x - 1)}{x}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 45

```
DSolve[x^2*y''[x]+x*(x+1)*y'[x]+(3*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}e^{-x}(x-3)x(c_2\,\mathrm{ExpIntegralEi}(x) + 6c_1) - \frac{c_2((x-2)x-1)}{6x}$$

1.265 problem 268

Internal problem ID [6998]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 268.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - \left(x^2 + 4x\right)y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve(x^2*diff(y(x),x$2)-(x^2+4*x)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x x^4 + c_2 x\left(\mathrm{e}^x x^3 \operatorname{Ei}_1(x) - x^2 + x - 2\right)$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 40

```
DSolve[x^2*y''[x]-(x^2+4*x)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^x x^4 - \frac{1}{6}c_1 x\left(e^x x^3 \operatorname{ExpIntegralEi}(-x) + (x-1)x + 2\right)$$

1.266 problem 269

Internal problem ID [6999]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 269.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - (3x+2)\,y' + \frac{(2x-1)\,y}{x} = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 34

```
dsolve(2*x^2*diff(y(x),x$2)-(3*x+2)*diff(y(x),x)+(2*x-1)/x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(5x+2)}{\sqrt{x}} + c_2 \mathrm{e}^{-\frac{1}{x}}\,\mathrm{hypergeom}\left([2], \left[-\frac{1}{2}\right], \frac{1}{x}\right) x^2$$

✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 60

```
DSolve[2*x^2*y''[x]-(3*x+2)*y'[x]+(2*x-1)/x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(5x+2)\left(3c_1 - 10\sqrt{\pi}c_2 \mathrm{erf}\left(\frac{1}{\sqrt{x}}\right)\right)}{15\sqrt{x}} + \frac{2}{3}c_2 e^{-1/x}((x-4)x-2)$$

1.267 problem 270

Internal problem ID [7000]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 270.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{x(1-x)\,y'' + \left(-2x + \frac{3}{2}\right) y' - \frac{y}{4} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(x*(1-x)*diff(y(x),x$2)+(3/2-2*x)*diff(y(x),x)-1/4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2 \ln\left(x - \frac{1}{2} + \sqrt{x\,(x-1)}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 50

```
DSolve[x*(1-x)*y''[x]+(3/2-2*x)*y'[x]-1/4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\frac{2c_2\sqrt{1-x}\log\left(\sqrt{x-1}-\sqrt{x}\right)}{\sqrt{x-1}} + c_1}{\sqrt{x}}$$

1.268 problem 271

Internal problem ID [7001]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 271.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x(1-x)\,y'' + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(2*x*(1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2\left(\arctan\left(\sqrt{x-1}\right)x - \sqrt{x-1}\right)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 43

```
DSolve[2*x*(1-x)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[4]{2}\left(c_2 x \operatorname{arctanh}\left(\sqrt{1-x}\right) + c_1 x - c_2\sqrt{1-x}\right)$$

1.269 problem 272

Internal problem ID [7002]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 272.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{2x(1-x)\,y'' + (1-11x)\,y' - 10y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(2*x*(1-x)*diff(y(x),x$2)+(1-11*x)*diff(y(x),x)-10*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2+6x+1)}{(x-1)^4} + \frac{c_2\sqrt{x}\,(x+1)}{(x-1)^4}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 34

```
DSolve[2*x*(1-x)*y''[x]+(1-11*x)*y'[x]-10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1\sqrt{x}(x+1) - 2c_2(x(x+6)+1)}{(x-1)^4}$$

1.270 problem 273

Internal problem ID [7003]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 273.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{x(1-x)\,y'' + \frac{(1-2x)\,y'}{3} + \frac{20y}{9} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*(1-x)*diff(y(x),x$2)+1/3*(1-2*x)*diff(y(x),x)+20/9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(6x-5)\,x^{\frac{2}{3}} + c_2(6x-1)\,(x-1)^{\frac{2}{3}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 48

```
DSolve[x*(1-x)*y''[x]+1/3*(1-2*x)*y'[x]+20/9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sqrt[3]{-((x-1)x)}\,Q_1^{\frac{2}{3}}(2x-1) + \frac{c_1 x^{2/3}(6x-5)}{\text{Gamma}\left(\frac{1}{3}\right)}$$

1.271 problem 274

Internal problem ID [7004]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 274.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y'' + \frac{3(-x^2+2)\,y}{(1-x^2)^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(4*diff(y(x),x$2)+3*(2-x^2)/(1-x^2)^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(x^2-1\right)^{\frac{3}{4}} + c_2 \left(x^2-1\right)^{\frac{1}{4}} x$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 51

```
DSolve[4*y''[x]+3*(2-x^2)/(1-x^2)^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x^2-1}\left(c_2 Q^{\frac{1}{2}}_{\frac{1}{2}}(x) + \frac{\sqrt{\frac{2}{\pi}}c_1 x}{\sqrt[4]{1-x^2}}\right)$$

1.272 problem 275

Internal problem ID [7005]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 275.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' - \frac{2u'}{x} - a^2 u = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(u(x),x$2)-2/x*diff(u(x),x)-a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = c_1 \mathrm{e}^{ax}(ax - 1) + c_2 \mathrm{e}^{-ax}(ax + 1)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 68

```
DSolve[u''[x]-2/x*u'[x]-a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{\sqrt{\frac{2}{\pi}}\sqrt{x}((iac_2x + c_1)\sinh(ax) - (ac_1x + ic_2)\cosh(ax))}{a\sqrt{-iax}}$$

1.273 problem 276

Internal problem ID [7006]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 276.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{2u'}{x} - a^2 u = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(u(x),x$2)+2/x*diff(u(x),x)-a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1 \sinh(ax)}{x} + \frac{c_2 \cosh(ax)}{x}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 35

```
DSolve[u''[x]+2/x*u'[x]-a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{2ac_1 e^{-ax} + c_2 e^{ax}}{2ax}$$

1.274 problem 277

Internal problem ID [7007]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 277.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{2u'}{x} + a^2 u = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(u(x),x$2)+2/x*diff(u(x),x)+a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1 \sin(ax)}{x} + \frac{c_2 \cos(ax)}{x}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 42

```
DSolve[u''[x]+2/x*u'[x]+a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{e^{-iax}\left(2c_1 - \frac{ic_2 e^{2iax}}{a}\right)}{2x}$$

1.275 problem 278

Internal problem ID [7008]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 278.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{4u'}{x} - a^2 u = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(u(x),x$2)+4/x*diff(u(x),x)-a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1 \mathrm{e}^{ax}(ax-1)}{x^3} + \frac{c_2 \mathrm{e}^{-ax}(ax+1)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 68

```
DSolve[u''[x]+4/x*u'[x]-a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{\sqrt{\frac{2}{\pi}}((iac_2x + c_1)\sinh(ax) - (ac_1x + ic_2)\cosh(ax))}{ax^{5/2}\sqrt{-iax}}$$

1.276 problem 279

Internal problem ID [7009]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 279.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{4u'}{x} + a^2 u = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 41

```
dsolve(diff(u(x),x$2)+4/x*diff(u(x),x)+a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1(\cos(ax)\,ax - \sin(ax))}{x^3} + \frac{c_2(\cos(ax) + \sin(ax)\,ax)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 57

```
DSolve[u''[x]+4/x*u'[x]+a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to -\frac{\sqrt{\frac{2}{\pi}}((ac_1x + c_2)\cos(ax) + (ac_2x - c_1)\sin(ax))}{x^{3/2}(ax)^{3/2}}$$

1.277 problem 280

Internal problem ID [7010]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 280.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - a^2 y - \frac{6y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)-a^2*y(x)=6*y(x)/x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{ax}(a^2x^2 - 3ax + 3)}{x^2} + \frac{c_2 \mathrm{e}^{-ax}(a^2x^2 + 3ax + 3)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 88

```
DSolve[y''[x]-a^2*y[x]==6*y[x]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\frac{2}{\pi}}(i(c_1(a^2x^2+3)+3iac_2x)\sinh(ax)+(ax(ac_2x-3ic_1)+3c_2)\cosh(ax))}{a^2x^{3/2}\sqrt{-iax}}$$

1.278 problem 281

Internal problem ID [7011]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 281.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + n^2 y - \frac{6y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$2)+n^2*y(x)=6*y(x)/x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1((n^2x^2-3)\cos(nx) - 3\sin(nx)\,nx)}{x^2} + \frac{c_2(3\cos(nx)\,nx + (n^2x^2-3)\sin(nx))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 77

```
DSolve[y''[x]+n^2*y[x]==6*y[x]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}\sqrt{x}((c_1(n^2x^2-3)+3c_2nx)\sin(nx) + (nx(3c_1 - c_2nx)+3c_2)\cos(nx))}{(nx)^{5/2}}$$

1.279 problem 282

Internal problem ID [7012]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 282.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x - \left(x^2 + \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(x^2+1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh(x)}{\sqrt{x}} + \frac{c_2 \cosh(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]+x*y'[x]-(x^2+1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(c_2 e^{2x} + 2c_1)}{2\sqrt{x}}$$

1.280 problem 283

Internal problem ID [7013]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 283.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \frac{(-9a^2 + 4x^2)\,y}{4a^2} = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 45

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(4*x^2-9*a^2)/(4*a^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{\frac{ix}{a}}(ix - a)}{x^{\frac{3}{2}}} + \frac{c_2 \mathrm{e}^{-\frac{ix}{a}}(ix + a)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 62

```
DSolve[x^2*y''[x]+x*y'[x]+(4*x^2-9*a^2)/(4*a^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}\left((ac_2 + c_1x)\cos\left(\frac{x}{a}\right) + (c_2x - ac_1)\sin\left(\frac{x}{a}\right)\right)}{x\sqrt{\frac{x}{a}}}$$

1.281 problem 284

Internal problem ID [7014]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 284.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{25}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-25/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{ix}(x^2 + 3ix - 3)}{x^{\frac{5}{2}}} + \frac{c_2 e^{-ix}(-x^2 + 3ix + 3)}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 57

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-25/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((3c_1x - c_2(x^2-3))\cos(x) + (c_1(x^2-3) + 3c_2x)\sin(x))}{x^{5/2}}$$

1.282 problem 285

Internal problem ID [7015]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 285.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + qy' - \frac{2y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+q*diff(y(x),x)=2*y(x)/x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1(qx-2)}{x} + \frac{c_2 \mathrm{e}^{-qx}(qx+2)}{x}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 80

```
DSolve[y''[x]+q*y'[x]==2*y[x]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{qx^{3/2}e^{-\frac{qx}{2}}\left(2(ic_2qx+2c_1)\sinh\left(\frac{qx}{2}\right)-2(c_1qx+2ic_2)\cosh\left(\frac{qx}{2}\right)\right)}{\sqrt{\pi}(-iqx)^{5/2}}$$

1.283 problem 286

Internal problem ID [7016]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 286.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + 3y' + 4yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+4*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x^2)}{x^2} + \frac{c_2 \cos(x^2)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 41

```
DSolve[x*y''[x]+3*y'[x]+4*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-ix^2} - ic_2 e^{ix^2}}{4x^2}$$

1.284 problem 287

Internal problem ID [7017]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 287.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+2x\right)y''-2(1+x)y'+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+2*x)*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=x^2c_1+c_2(x+1)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 19

```
DSolve[(x^2+2*x)*y''[x]-2*(x+1)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x^2-c_2(x+1)$$

1.285 problem 288

Internal problem ID [7018]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 288.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+2x\right)y''-2(1+x)y'+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+2*x)*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=x^2c_1+c_2(x+1)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 19

```
DSolve[(x^2+2*x)*y''[x]-2*(x+1)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x^2-c_2(x+1)$$

1.286 problem 289

Internal problem ID [7019]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 289.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right) y'' - 2y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2\left(x^2 - 1\right)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x - c_1 (x - i)^2$$

1.287 problem 290

Internal problem ID [7020]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 290.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right) y'' - 2y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2\left(x^2 - 1\right)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x - c_1 (x - i)^2$$

1.288 problem 291

Internal problem ID [7021]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 291.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y'x + \left(4x^2 - 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x^2} + c_2 x \,\mathrm{e}^{x^2}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 18

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(c_2 x + c_1)$$

1.289 problem 292

Internal problem ID [7022]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 292.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y'x + \left(4x^2 - 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x^2} + c_2 x \,\mathrm{e}^{x^2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 18

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(c_2 x + c_1)$$

1.290 problem 293

Internal problem ID [7023]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 293.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x-3)\,y'' - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve((2*x-3)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(-3+2x)^{\frac{7}{4}}\,\text{KummerM}\left(\frac{3}{4},\frac{11}{4},-\frac{3}{4}+\frac{x}{2}\right) + c_2 x$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 63

```
DSolve[(2*x-3)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\ 2^{3/4}(2x-3)\left(c_2(2x-3)^{3/4}L^{\frac{7}{4}}_{-\frac{3}{4}}\left(\frac{x}{2}-\frac{3}{4}\right) + \frac{4\sqrt{2}c_1 x}{2x-3}\right)$$

1.291 problem 294

Internal problem ID [7024]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 294.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - y'x - 3y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 48

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(\sqrt{\pi}\ \operatorname{erf}\left(\frac{\sqrt{2}\, x}{2} \right) \mathrm{e}^{\frac{x^2}{2}} \left(x^2+1\right) + \sqrt{2}\, x \right) + c_2 \mathrm{e}^{\frac{x^2}{2}} \left(x^2+1\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 35

```
DSolve[y''[x]-x*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{HermiteH}\left(-3, \frac{x}{\sqrt{2}} \right) + c_2 e^{\frac{x^2}{2}} \left(x^2+1\right)$$

1.292 problem 295

Internal problem ID [7025]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 295.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right)y''-y'x+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve((1+x^2)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2\left(\operatorname{arcsinh}\left(x\right)x-\sqrt{x^2+1}\right)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 29

```
DSolve[(1+x^2)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x\operatorname{arcsinh}(x)-c_2\sqrt{x^2+1}+c_1x$$

1.293 problem 296

Internal problem ID [7026]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 296.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(-2x\,\mathrm{e}^{\frac{x^2}{2}} + \sqrt{2}\,\sqrt{\pi}\,\operatorname{erfi}\left(\frac{\sqrt{2}\,x}{2}\right)(x-1)\,(x+1)\right) + c_2\left(x^2-1\right)$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 53

```
DSolve[y''[x]-x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(\left(x^2-1\right)\left(\sqrt{2\pi}c_2\operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right)+4c_1\right) - 2c_2 e^{\frac{x^2}{2}}x\right)$$

1.294 problem 297

Internal problem ID [7027]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 297.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(1-x^2\right)y''-y'+y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 62

```
dsolve((1-x^2)*diff(y(x),x$2)-diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\text{hypergeom}\left(\left[-\frac{1}{2}-\frac{\sqrt{5}}{2},\frac{\sqrt{5}}{2}-\frac{1}{2}\right],\left[-\frac{1}{2}\right],\frac{x}{2}+\frac{1}{2}\right)$$
$$+c_2(2x+2)^{\frac{3}{2}}\,\text{hypergeom}\left(\left[\frac{\sqrt{5}}{2}+1,-\frac{\sqrt{5}}{2}+1\right],\left[\frac{5}{2}\right],\frac{x}{2}+\frac{1}{2}\right)$$

✓ Solution by Mathematica

Time used: 14.811 (sec). Leaf size: 198

```
DSolve[(1-x^2)*y''[x]-y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to\frac{\sqrt[4]{x+1}\left(\sqrt{x+1}-\sqrt{x-1}\right)^{-1-\sqrt{5}}\left(-2x+2\sqrt{x-1}\sqrt{x+1}+\sqrt{5}-3\right)e^{-\text{arctanh}\left(x-\sqrt{x-1}\sqrt{x+1}\right)}\left(c_2\int_1^x\frac{e^{2\text{arcta}}}{}\right.}{\sqrt[4]{1-x}}$$

1.295 problem 298

Internal problem ID [7028]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 298.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(1+x)^2 y'' + \left(1-x^2\right) y' + y(-1+x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*(x+1)^2*diff(y(x),x$2)+(1-x^2)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x+1) + c_2(x+1)\ln(x)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 17

```
DSolve[x*(x+1)^2*y''[x]+(1-x^2)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x+1)(c_2 \log(x) + c_1)$$

1.296 problem 299

Internal problem ID [7029]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 299.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2xy'' - y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
dsolve(2*x*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(2\cos\left(2\sqrt{x}\right)\sqrt{x} - \sin\left(2\sqrt{x}\right)\right) + c_2\left(2\sin\left(2\sqrt{x}\right)\sqrt{x} + \cos\left(2\sqrt{x}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 59

```
DSolve[2*x*y''[x]-y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2i\sqrt{x}}\left(2\sqrt{x} + i\right) + \frac{1}{8}c_2 e^{-2i\sqrt{x}}\left(1 + 2i\sqrt{x}\right)$$

1.297 problem 300

Internal problem ID [7030]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 300.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(x*diff(y(x),x$2)+x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(x^2 + 2x\right) + c_2\left(\frac{(-x-1)\,\mathrm{e}^{-x}}{2} + \frac{x\ \mathrm{Ei}_1\left(x\right)(x+2)}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 39

```
DSolve[x*y''[x]+x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x(x+2) - \frac{1}{2}c_2 e^{-x}(e^x(x+2)x\,\mathrm{ExpIntegralEi}(-x) + x + 1)$$

1.298 problem 301

Internal problem ID [7031]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 301.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(-1+x)^2 y'' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x*(x-1)^2*diff(y(x),x$2)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{x-1} + \frac{c_2(2\ln(x)\,x - x^2 + 1)}{x-1}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 31

```
DSolve[x*(x-1)^2*y''[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-x(c_2 x + c_1) + 2c_2 x \log(x) + c_2}{x-1}$$

1.299 problem 302

Internal problem ID [7032]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 302.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + yx^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^2}{2}} \cos(x) + c_2 \mathrm{e}^{\frac{x^2}{2}} \sin(x)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 39

```
DSolve[y''[x]-2*x*y'[x]+x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{\frac{1}{2}x(x-2i)} \left(2c_1 - ic_2 e^{2ix}\right)$$

1.300 problem 303

Internal problem ID [7033]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 303.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x\left(-x^2+2\right)y''-\left(x^2+4x+2\right)\left(\left(1-x\right)y'+y\right)=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x*(2-x^2)*diff(y(x),x$2)-(x^2+4*x+2)*((1-x)*diff(y(x),x)+y(x))=0,y(x), singsol=all)
```

$$y(x)=c_1(x-1)+c_2\mathrm{e}^x x^2$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 21

```
DSolve[x*(2-x^2)*y''[x]-(x^2+4*x+2)*((1-x)*y'[x]+y[x])==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to c_1e^x x^2+c_2(x-1)$$

1.301 problem 304

Internal problem ID [7034]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 304.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y'' - (2x+1)\,(y'x - y) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-(1+2*x)*(x*diff(y(x),x)-y(x))=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 x(x + \ln(x))$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 132

```
DSolve[x^2*(1+x)*y''[x]-(1+2*x)*(x*y'[x]+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x^{1+\sqrt{2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+\sqrt{2}-\frac{\sqrt{17}}{2}, \frac{1}{2}\left(-1+2\sqrt{2}+\sqrt{17}\right), 1+2\sqrt{2}, -x\right) + c_1 x^{1-\sqrt{2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-1-2\sqrt{2}-\sqrt{17}\right), \frac{1}{2}\left(-1-2\sqrt{2}+\sqrt{17}\right), 1-2\sqrt{2}, -x\right)$$

1.302 problem 305

Internal problem ID [7035]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 305.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2(2-x)\,x^2y'' - (4-x)\,xy' + (3-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(2*(2-x)*x^2*diff(y(x),x$2)-(4-x)*x*diff(y(x),x)+(3-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x\,(x-2)}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 41

```
DSolve[2*(2-x)*x^2*y''[x]-(4-x)*x*y'[x]+(3-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{\sqrt[4]{x-2}\sqrt{x}\left(2c_2\sqrt{x-2}+c_1\right)}{\sqrt[4]{2-x}}$$

1.303 problem 306

Internal problem ID [7036]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 306.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\,x^2y'' + (5x-4)\,xy' + (6-9x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((1-x)*x^2*diff(y(x),x$2)+(5*x-4)*x*diff(y(x),x)+(6-9*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^3 + c_2x^2(\ln(x)\,x + 1)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 24

```
DSolve[(1-x)*x^2*y''[x]+(5*x-4)*x*y'[x]+(6-9*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to x^2(c_1x - c_2(x\log(x) + 1))$$

1.304 problem 307

Internal problem ID [7037]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 307.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + \left(4x^2+1\right)y' + 4x\left(x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+(4*x^2+1)*diff(y(x),x)+4*x*(x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} \ln\left(x\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 21

```
DSolve[x*y''[x]+(4*x^2+1)*y'[x]+4*x*(x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}\left(c_2 \log(x) + c_1\right)$$

1.305 problem 308

Internal problem ID [7038]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 308.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 8y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\left(-4x^3 + 10x\right)\mathrm{e}^{x^2} + 4\,\mathrm{erfi}\left(x\right)\sqrt{\pi}\left(x^4 - 3x^2 + \frac{3}{4}\right)\right) + c_2\left(4x^4 - 12x^2 + 3\right)$$

✓ Solution by Mathematica

Time used: 0.57 (sec). Leaf size: 55

```
DSolve[y''[x]-2*x*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(\left(4\left(x^2-3\right)x^2+3\right)\left(\sqrt{\pi}c_2\mathrm{erfi}(x)+3c_1\right) - 2c_2e^{x^2}x\left(2x^2-5\right)\right)$$

1.306 problem 309

Internal problem ID [7039]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 309.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 8y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\left(-4x^3 + 10x\right)\mathrm{e}^{x^2} + 4\,\mathrm{erfi}\left(x\right)\sqrt{\pi}\left(x^4 - 3x^2 + \frac{3}{4}\right)\right) + c_2\left(4x^4 - 12x^2 + 3\right)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 55

```
DSolve[y''[x]-2*x*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(\left(4\left(x^2-3\right)x^2+3\right)\left(\sqrt{\pi}c_2\mathrm{erfi}(x)+3c_1\right) - 2c_2e^{x^2}x\left(2x^2-5\right)\right)$$

1.307 problem 310

Internal problem ID [7040]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 310.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(1-x^2\right)y''-2y'x+12y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(-\frac{5}{3}x^3+x\right)+c_2\left(-\frac{1}{9}+\frac{(5x^3-3x)\ln(x-1)}{24}+\frac{(-5x^3+3x)\ln(x+1)}{24}+\frac{5x^2}{12}\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 44

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{6}\left(c_2\left(3\left(5x^2-3\right)x\operatorname{arctanh}(x)-15x^2+4\right)+3c_1x\left(5x^2-3\right)\right)$$

1.308 problem 311

Internal problem ID [7041]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 311.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x+2)\,y'' + 2(1+x)\,y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(x*(x+2)*diff(y(x),x$2)+2*(x+1)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x+1) + c_2\left(\frac{(-x-1)\ln(x+2)}{2} + 1 + \frac{(x+1)\ln(x)}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 26

```
DSolve[x*(x+2)*y''[x]+2*(x+1)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2(x+1)\text{arctanh}(x+1) + c_1(x+1) - c_2$$

1.309 problem 313

Internal problem ID [7042]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 313.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$\boxed{x(x+2)y'' + (1+x)y' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x*(x+2)*diff(y(x),x$2)+(x+1)*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(2x^2+4x+1\right) + c_2(x+1)\sqrt{x\,(x+2)}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 53

```
DSolve[x*(x+2)*y''[x]+(x+1)*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cosh\left(4\log\left(\sqrt{x+2}-\sqrt{x}\right)\right) - ic_2 \sinh\left(4\log\left(\sqrt{x+2}-\sqrt{x}\right)\right)$$

1.310 problem 314

Internal problem ID [7043]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 314.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-1+x)\,y'' - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2 x$$

1.311 problem 315

Internal problem ID [7044]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 315.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right)y''-2y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve((1+x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2\left(x^2-1\right)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 21

```
DSolve[(1+x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x-c_1(x-i)^2$$

1.312 problem 316

Internal problem ID [7045]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 316.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-2x+10\right)y''+y'x-4y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve((x^2-2*x+10)*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^2-\frac{4}{3}x+5\right)+c_2(3x-4)\left(x-1+3i\right)^{\frac{1}{2}-\frac{i}{6}}\left(x-1-3i\right)^{\frac{1}{2}+\frac{i}{6}}$$

✓ Solution by Mathematica

Time used: 0.279 (sec). Leaf size: 90

```
DSolve[(x^2-2*x+10)*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}&y(x)\\&\to\frac{1}{3}(3x\\&\quad-4)\sqrt{(x-2)x+10}e^{-\frac{1}{3}\arctan\left(\frac{x-1}{3}\right)}\left(c_2\int_1^x\frac{9e^{\frac{1}{3}\arctan\left(\frac{1}{3}(K[1]-1)\right)}}{(4-3K[1])^2((K[1]-2)K[1]+10)^{3/2}}dK[1]\right.\\&\qquad\left.+c_1\right)\end{aligned}$$

1.313 problem 317

Internal problem ID [7046]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 317.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-2x+10\right)y''+y'x-4y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 40

```
dsolve((x^2-2*x+10)*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^2-\frac{4}{3}x+5\right)+c_2(3x-4)\left(x-1+3i\right)^{\frac{1}{2}-\frac{i}{6}}\left(x-1-3i\right)^{\frac{1}{2}+\frac{i}{6}}$$

✓ Solution by Mathematica

Time used: 0.103 (sec). Leaf size: 90

```
DSolve[(x^2-2*x+10)*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}&y(x)\\&\to\frac{1}{3}(3x\\&\quad-4)\sqrt{(x-2)x+10}e^{-\frac{1}{3}\arctan\left(\frac{x-1}{3}\right)}\left(c_2\int_1^x\frac{9e^{\frac{1}{3}\arctan\left(\frac{1}{3}(K[1]-1)\right)}}{(4-3K[1])^2((K[1]-2)K[1]+10)^{3/2}}dK[1]\right.\\&\qquad\left.+c_1\right)\end{aligned}$$

1.314 problem 318

Internal problem ID [7047]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 318.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(-2x\,\mathrm{e}^{\frac{x^2}{2}} + \sqrt{2}\,\sqrt{\pi}\,\operatorname{erfi}\left(\frac{\sqrt{2}\,x}{2}\right)(x-1)\,(x+1)\right) + c_2\left(x^2-1\right)$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 53

```
DSolve[y''[x]-x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(\left(x^2-1\right)\left(\sqrt{2\pi}c_2\operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right)+4c_1\right) - 2c_2 e^{\frac{x^2}{2}}x\right)$$

1.315 problem 319

Internal problem ID [7048]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 319.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+2)\,y'' + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((x+2)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 \mathrm{e}^{-x}(x+4)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 71

```
DSolve[(x+2)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\sqrt{\frac{2}{\pi}}e^{-x-2}\sqrt{x+2}((c_1 - ic_2)e^{x+2}x + (c_1 + ic_2)(x+4))}{\sqrt{-i(x+2)}}$$

1.316 problem 320

Internal problem ID [7049]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 320.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{\left(x^2+1\right)y''-6y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve((x^2+1)*diff(y(x),x$2)-6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^3+x\right)+c_2\left(\frac{\left(3x^3+3x\right)\arctan\left(x\right)}{2}+\frac{3x^2}{2}+1\right)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 36

```
DSolve[(x^2+1)*y''[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\left(x^3+x\right)-\frac{1}{2}c_2\left(3\left(x^3+x\right)\arctan(x)+3x^2+2\right)$$

1.317 problem 321

Internal problem ID [7050]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 321.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+2\right)y''+3y'x-y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 51

```
dsolve((x^2+2)*diff(y(x),x$2)+3*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\left(\sqrt{x^2+2}+x\right)^{\sqrt{2}}}{\sqrt{x^2+2}}+\frac{c_2\left(\sqrt{x^2+2}+x\right)^{-\sqrt{2}}}{\sqrt{x^2+2}}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 80

```
DSolve[(x^2+2)*y''[x]+3*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2c_1\cos\left(\sqrt{2}\arccos\left(\frac{ix}{\sqrt{2}}\right)\right)-\pi c_2\sin\left(2\sqrt{2}\csc^{-1}\left(\frac{2}{\sqrt{2-i\sqrt{2}x}}\right)\right)}{\sqrt[4]{2}\sqrt{\pi}\sqrt{x^2+2}}$$

1.318 problem 322

Internal problem ID [7051]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 322.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-1+x)\,y'' - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2 x$$

1.319 problem 323

Internal problem ID [7052]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 323.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 8y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\left(-4x^3 + 10x\right)\mathrm{e}^{x^2} + 4\,\mathrm{erfi}\left(x\right)\sqrt{\pi}\left(x^4 - 3x^2 + \frac{3}{4}\right)\right) + c_2\left(4x^4 - 12x^2 + 3\right)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 49

```
DSolve[y''[x]-2*x*y''[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{4x-2}\left(c_1\,\mathrm{BesselI}\left(1, 4\sqrt{x-\frac{1}{2}}\right) - c_2 K_1\left(4\sqrt{x-\frac{1}{2}}\right)\right)$$

1.320 problem 325

Internal problem ID [7053]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 325.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(\frac{5}{3}x + x^2\right)y' - \frac{y}{3} = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x$2)+(5/3*x+x^2)*diff(y(x),x)-1/3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} x^{\frac{1}{3}} \operatorname{hypergeom}\left([2], \left[\frac{7}{3}\right], x\right) + \frac{c_2 \mathrm{e}^{-x} \operatorname{hypergeom}\left(\left[\frac{2}{3}\right], \left[-\frac{1}{3}\right], x\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.169 (sec). Leaf size: 43

```
DSolve[x^2*y''[x]+(5/3*x+x^2)*y'[x]-1/3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(3x-1)\left(c_2\Gamma\left(\frac{1}{3},x\right)+c_1\right)}{3x} - \frac{c_2 e^{-x}}{x^{2/3}}$$

1.321 problem 326

Internal problem ID [7054]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 326.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2xy'' - y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
dsolve(2*x*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(2\cos\left(2\sqrt{x}\right)\sqrt{x} - \sin\left(2\sqrt{x}\right)\right) + c_2\left(2\sin\left(2\sqrt{x}\right)\sqrt{x} + \cos\left(2\sqrt{x}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 59

```
DSolve[2*x*y''[x]-y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2i\sqrt{x}}\left(2\sqrt{x} + i\right) + \frac{1}{8}c_2 e^{-2i\sqrt{x}}\left(1 + 2i\sqrt{x}\right)$$

1.322 problem 327

Internal problem ID [7055]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 327.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{2xy'' - (3+2x)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 25

```
dsolve(2*x*diff(y(x),x$2)-(3+2*x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\text{hypergeom}\left([2], \left[\frac{7}{2}\right], x\right) x^{\frac{5}{2}} + c_2\left(-\frac{2x}{3} + 1\right) \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.277 (sec). Leaf size: 48

```
DSolve[2*x*y''[x]-(3+2*x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(e^{x}(2x-3)\left(2c_1 - \sqrt{\pi}c_2 \text{erf}\left(\sqrt{x}\right)\right) - 6c_2\sqrt{x}\right)$$

1.323 problem 328

Internal problem ID [7056]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 328.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + 3y'x + (2x-1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 85

```
dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+(2*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{2i\sqrt{x}}\sqrt{1+4x}\,\sqrt{\frac{2\sqrt{x}+i}{-2\sqrt{x}+i}}}{x} + \frac{c_2 \mathrm{e}^{-2i\sqrt{x}}\sqrt{1+4x}\,\sqrt{\frac{-2\sqrt{x}+i}{2\sqrt{x}+i}}}{x}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 63

```
DSolve[2*x^2*y''[x]+3*x*y'[x]+(2*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-2i\sqrt{x}}\left(8c_1 e^{4i\sqrt{x}}\left(2\sqrt{x}+i\right)+2ic_2\sqrt{x}+c_2\right)}{8x}$$

1.324 problem 329

Internal problem ID [7057]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 329.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 2y' - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh(x)}{x} + \frac{c_2 \cosh(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 28

```
DSolve[x*y''[x]+2*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-x} + c_2 e^{x}}{2x}$$

1.325 problem 330

Internal problem ID [7058]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 330.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2 e^{2ix})}{2\sqrt{x}}$$

1.326 problem 331

Internal problem ID [7059]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 331.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (x-6)\,y' - 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x$2)+(x-6)*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(x^3 - 12x^2 + 60x - 120\right) + c_2 \mathrm{e}^{-x}\left(x^3 + 12x^2 + 60x + 120\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 92

```
DSolve[x*y''[x]+(x-6)*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to \frac{2e^{-x/2}\sqrt{x}\left(\left(c_1 x(x^2+60) + 12ic_2(x^2+10)\right)\cosh\left(\frac{x}{2}\right) - \left(12c_1(x^2+10) + ic_2 x(x^2+60)\right)\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{\pi}\sqrt{-ix}}$$

1.327 problem 332

Internal problem ID [7060]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 332.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^4 y'' + \lambda y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(x^4*diff(y(x),x$2)+lambda*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sinh\left(\frac{\sqrt{-\lambda}}{x}\right) + c_2 x \cosh\left(\frac{\sqrt{-\lambda}}{x}\right)$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 52

```
DSolve[x^4*y''[x]+\[Lambda]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x e^{\frac{i\sqrt{\lambda}}{x}} - \frac{i c_2 x e^{-\frac{i\sqrt{\lambda}}{x}}}{2\sqrt{\lambda}}$$

1.328 problem 333

Internal problem ID [7061]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 333.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4y'x + \left(4x^2 - 25\right)y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 45

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2-25)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{ix}(x^2 + 3ix - 3)}{x^{\frac{5}{2}}} + \frac{c_2 \mathrm{e}^{-ix}(-x^2 + 3ix + 3)}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 57

```
DSolve[4*x^2*y''[x]+4*x*y'[x]+(4*x^2-25)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((3c_1x - c_2(x^2 - 3))\cos(x) + (c_1(x^2 - 3) + 3c_2x)\sin(x))}{x^{5/2}}$$

1.329 problem 334

Internal problem ID [7062]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 334.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y'x + \left(36x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(36*x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(6x)}{\sqrt{x}} + \frac{c_2 \cos(6x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(36*x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-6ix}(12c_1 - ic_2 e^{12ix})}{12\sqrt{x}}$$

1.330 problem 335

Internal problem ID [7063]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 335.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(x^2 - 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(x^2*diff(y(x),x$2)+(x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(-\sin(x) + \cos(x)\, x)}{x} + \frac{c_2(\cos(x) + x\sin(x))}{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]+(x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((c_1 x + c_2)\cos(x) + (c_2 x - c_1)\sin(x))}{x}$$

1.331 problem 336

Internal problem ID [7064]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 336.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + 3y' + yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{x^2}{2}\right)}{x^2} + \frac{c_2 \cos\left(\frac{x^2}{2}\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 43

```
DSolve[x*y''[x]+3*y'[x]+x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{ix^2}{2}}\left(2c_1 - ic_2 e^{ix^2}\right)}{2x^2}$$

1.332 problem 337

Internal problem ID [7065]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 337.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 4y'x + \left(x^2 + 2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x^2} + \frac{c_2 \cos(x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+4*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x^2}$$

1.333 problem 338

Internal problem ID [7066]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 338.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16x^2y'' + 32y'x + \left(x^4 - 12\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(16*x^2*diff(y(x),x$2)+32*x*diff(y(x),x)+(x^4-12)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{x^2}{8}\right)}{x^{\frac{3}{2}}} + \frac{c_2 \cos\left(\frac{x^2}{8}\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 42

```
DSolve[16*x^2*y''[x]+32*x*y'[x]+(x^4-12)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{ix^2}{8}}\left(c_1 - 2ic_2 e^{\frac{ix^2}{4}}\right)}{x^{3/2}}$$

1.334 problem 339

Internal problem ID [7067]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 339.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x^2 + yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2\left(6\left(-x^3\right)^{\frac{1}{3}} 3^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right) - 6\left(-x^3\right)^{\frac{1}{3}} 3^{\frac{2}{3}}\Gamma\left(\frac{2}{3}, -\frac{x^3}{3}\right) + 18\,\mathrm{e}^{\frac{x^3}{3}}\right)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 27

```
DSolve[y''[x]-x^2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{1}{3}c_2\,\text{ExpIntegralE}\left(\frac{4}{3}, -\frac{x^3}{3}\right)$$

1.335 problem 340

Internal problem ID [7068]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 340.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - (x+2)\,y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)-(x+2)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2\left(x^2 + 2x + 2\right)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 23

```
DSolve[x*y''[x]-(x+2)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} - c_2(x(x+2)+2)$$

1.336 problem 341

Internal problem ID [7069]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 341.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \, \mathrm{e}^{-\frac{x^2}{2}} + c_2 \left(-i\sqrt{\pi}\,\sqrt{2} + \operatorname{erf}\left(\frac{i\sqrt{2}\,x}{2} \right) \mathrm{e}^{-\frac{x^2}{2}} \pi x \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 44

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}c_2 x \operatorname{DawsonF}\left(\frac{x}{\sqrt{2}} \right) + \sqrt{2}c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.337 problem 342

Internal problem ID [7070]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 342.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(1-x^2\right)y''-2y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2\left(\frac{\ln\left(x-1\right)x}{2}-\frac{\ln\left(x+1\right)x}{2}+1\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 19

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2(x\operatorname{arctanh}(x)-1)+c_1x$$

1.338 problem 343

Internal problem ID [7071]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 343.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y'x + \left(4x^2 - 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x^2} + c_2 x \,\mathrm{e}^{x^2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(c_2 x + c_1)$$

1.339 problem 344

Internal problem ID [7072]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 344.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(1-x^2\right)y''-2y'x+30y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 73

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+30*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(\frac{21}{5}x^5-\frac{14}{3}x^3+x\right)+c_2\left(\frac{1}{225}+\frac{\left(63x^5-70x^3+15x\right)\ln\left(x-1\right)}{1920}+\frac{\left(-63x^5+70x^3-15x\right)\ln\left(x+1\right)}{1920}+\frac{21x^4}{320}-\frac{49x^2}{960}\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 66

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+30*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{8}c_2x\left(63x^4-70x^2+15\right)\operatorname{arctanh}(x)+\frac{1}{8}x\left(7c_2x\left(7-9x^2\right)+c_1\left(63x^4-70x^2+15\right)\right)-\frac{8c_2}{15}$$

1.340 problem 345

Internal problem ID [7073]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 345.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.341 problem 346

Internal problem ID [7074]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 346.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (2x+1)\,y' + (1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+(2*x+1)*diff(y(x),x)+(x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 \mathrm{e}^{-x} \ln(x)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 19

```
DSolve[x*y''[x]+(2*x+1)*y'[x]+(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2 \log(x) + c_1)$$

1.342 problem 347

Internal problem ID [7075]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 347.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{2x(-1+x)\,y'' - (1+x)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(2*x*(x-1)*diff(y(x),x$2)-(x+1)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x+1) + c_2\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 21

```
DSolve[2*x*(x-1)*y''[x]-(x+1)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\sqrt{x} - 2c_2(x+1)$$

1.343 problem 348

Internal problem ID [7076]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 348.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 2y' + 4yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+4*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(2x)}{x} + \frac{c_2 \cos(2x)}{x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+4*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-2ix} - ic_2 e^{2ix}}{4x}$$

1.344 problem 349

Internal problem ID [7077]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 349.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (-2x+2)\,y' + (x-2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)+(2-2*x)*diff(y(x),x)+(x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x + \frac{c_2 \mathrm{e}^x}{x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 19

```
DSolve[x*y''[x]+(2-2*x)*y'[x]+(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x(c_2 x + c_1)}{x}$$

1.345 problem 350

Internal problem ID [7078]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 350.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 6y'x + \left(4x^2 + 6\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+6*x*diff(y(x),x)+(4*x^2+6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(2x)}{x^3} + \frac{c_2 \cos(2x)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+6*x*y'[x]+(4*x^2+6)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-2ix} - ic_2 e^{2ix}}{4x^3}$$

1.346 problem 351

Internal problem ID [7079]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 351.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1-2x)\,y' + y(-1+x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2 \mathrm{e}^{x} \ln(x)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 17

```
DSolve[x*y''[x]+(1-2*x)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(c_2 \log(x) + c_1)$$

1.347 problem 352

Internal problem ID [7080]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 352.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{x(1-x)\,y'' + \left(\frac{1}{2}+2x\right)y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
dsolve(x*(1-x)*diff(y(x),x$2)+(1/2+2*x)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(1+4x) + c_2\left((-12x-3)\ln\left(x-\frac{1}{2}+\sqrt{x\,(x-1)}\right) + (4x+26)\sqrt{x\,(x-1)}\right)$$

✓ Solution by Mathematica

Time used: 0.198 (sec). Leaf size: 62

```
DSolve[x*(1-x)*y''[x]+(1/2+2*x)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(x+\frac{1}{4}\right) + \frac{1}{2}c_2\left(\sqrt{-((x-1)x)}(2x+13) - 6(4x+1)\cot^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{1-x}}\right)\right)$$

1.348 problem 353

Internal problem ID [7081]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 353.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4\left(t^2-3t+2\right)y''-2y'+y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 48

```
dsolve(4*(t^2-3*t+2)*diff(y(t),t$2)-2*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1\sqrt{t-1}+\frac{c_2\left(-\frac{\sqrt{t^2-3t+2}\,\ln\left(t-\frac{3}{2}+\sqrt{t^2-3t+2}\right)}{2}+t-2\right)}{\sqrt{t-2}}$$

✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 49

```
DSolve[4*(t^2-3*t+2)*y''[t]-2*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\sqrt{1-t}\left(\frac{2c_2}{\sqrt{\frac{1}{t-2}+1}}-2c_2\coth^{-1}\left(\sqrt{\frac{1}{t-2}+1}\right)+c_1\right)$$

1.349 problem 354

Internal problem ID [7082]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 354.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2\left(t^2-5t+6\right)y''+(2t-3)y'-8y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 32

```
dsolve(2*(t^2-5*t+6)*diff(y(t),t$2)+(2*t-3)*diff(y(t),t)-8*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1\left(t^2-\frac{13}{3}t+\frac{37}{8}\right)+\frac{c_2(6t-17)\,(t-2)^{\frac{3}{2}}}{\sqrt{t-3}}$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 78

```
DSolve[2*(t^2-5*t+6)*y''[t]+(2*t-3)*y'[t]-8*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\frac{\sqrt[4]{2-t}\sqrt[4]{\frac{t-3}{t-2}}\left(5c_1(6t-17)(t-2)^{3/2}+24c_2\sqrt{t-3}(8t(3t-13)+111)\right)}{30(3-t)^{3/4}}$$

1.350 problem 355

Internal problem ID [7083]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 355.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3t(t+1)\,y'' + ty' - y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 66

```
dsolve(3*t*(1+t)*diff(y(t),t$2)+t*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2 \left(2\sqrt{3}\,\arctan\left(\frac{\left(2(t+1)^{\frac{1}{3}}+1\right)\sqrt{3}}{3} \right) t + 6(t+1)^{\frac{2}{3}} + 2\ln\left((t+1)^{\frac{1}{3}}-1\right) t - \ln\left((t+1)^{\frac{2}{3}} + (t+1)^{\frac{1}{3}} + 1\right) t \right)$$

✓ Solution by Mathematica

Time used: 0.099 (sec). Leaf size: 90

```
DSolve[3*t*(1+t)*y''[t]+t*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 t\left(-2\sqrt{3}\arctan\left(\frac{2\sqrt[3]{t+1}+1}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{t+1}-1\right) + \log\left((t+1)^{2/3} + \sqrt[3]{t+1} + 1\right)\right) + 6c_1 t - 6c_2(t}{6\sqrt[6]{3}}$$

1.351 problem 356

Internal problem ID [7084]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 356.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + \frac{\left(x + \frac{3}{4}\right) y}{4} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+1/4*(x+3/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\sqrt{x}\right) x^{\frac{1}{4}} + c_2 x^{\frac{1}{4}} \cos\left(\sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 43

```
DSolve[x^2*y''[x]+1/4*(x+3/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-i\sqrt{x}} \sqrt[4]{x} \left(c_1 e^{2i\sqrt{x}} + i c_2\right)$$

1.352 problem 357

Internal problem ID [7085]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 357.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \frac{(x^2-1)\,y}{4} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+1/4*(x^2-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{x}{2}\right)}{\sqrt{x}} + \frac{c_2 \cos\left(\frac{x}{2}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]+x*y'[x]+1/4*(x^2-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{ix}{2}}\left(c_2(\sin(x) - i\cos(x)) + c_1\right)}{\sqrt{x}}$$

1.353 problem 358

Internal problem ID [7086]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 358.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1-2x)\,y' + y(-1+x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2 \mathrm{e}^{x} \ln(x)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 17

```
DSolve[x*y''[x]+(1-2*x)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(c_2 \log(x) + c_1)$$

1.354 problem 359

Internal problem ID [7087]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 359.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - (1+x)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x$2)-(x+1)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x+1) + c_2 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 19

```
DSolve[x*y''[x]-(x+1)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2(x+1)$$

1.355 problem 360

Internal problem ID [7088]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 360.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + 3y' + 4yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+4*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x^2)}{x^2} + \frac{c_2 \cos(x^2)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 41

```
DSolve[x*y''[x]+3*y'[x]+4*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-ix^2} - ic_2 e^{ix^2}}{4x^2}$$

1.356 problem 361

Internal problem ID [7089]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 361.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x^2)\,y'' + 2x(1-x^2)\,y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
dsolve(x^2*(1-x^2)*diff(y(x),x$2)+2*x*(1-x^2)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2-1)}{x^2} + \frac{c_2(\ln(x-1)\,x^2 - \ln(x+1)\,x^2 - \ln(x-1) + \ln(x+1) - 2x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 35

```
DSolve[x^2*(1-x^2)*y''[x]+2*x*(1-x^2)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{c_2((x^2-1)\operatorname{arctanh}(x) + x) - 2c_1(x^2-1)}{2x^2}$$

1.357 problem 362

Internal problem ID [7090]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 362.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + (x-2)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(2*x*diff(y(x),x$2)+(x-2)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x-2) + c_2 \mathrm{e}^{-\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 23

```
DSolve[2*x*y''[x]+(x-2)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x/2} + 2c_2(x-2)$$

1.358 problem 363

Internal problem ID [7091]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 363.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.359 problem 364

Internal problem ID [7092]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 364.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y'x^2 + \left(x^4 + 2x - 1\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+2*x^2*diff(y(x),x)+(x^4+2*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x\left(x^2-3\right)}{3}} + c_2 \mathrm{e}^{-\frac{x\left(x^2+3\right)}{3}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 34

```
DSolve[y''[x]+2*x^2*y'[x]+(x^4+2*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-\frac{1}{3}x\left(x^2+3\right)} \left(c_2 e^{2x} + 2c_1\right)$$

1.360 problem 365

Internal problem ID [7093]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 365.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{u'' + \frac{u}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(u(x),x$2)+1/x^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = c_1\sqrt{x}\,\sin\left(\frac{\sqrt{3}\,\ln(x)}{2}\right) + c_2\sqrt{x}\,\cos\left(\frac{\sqrt{3}\,\ln(x)}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 42

```
DSolve[u''[x]+1/x^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \sqrt{x}\left(c_1 \cos\left(\frac{1}{2}\sqrt{3}\log(x)\right) + c_2 \sin\left(\frac{1}{2}\sqrt{3}\log(x)\right)\right)$$

1.361 problem 366

Internal problem ID [7094]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 366.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' - (2x+1)\,u' + \left(x^2 + x - 1\right) u = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(u(x),x$2)-(2*x+1)*diff(u(x),x)+(x^2+x-1)*u(x)=0,u(x), singsol=all)
```

$$u(x) = c_1 \mathrm{e}^{\frac{x^2}{2}} + c_2 \mathrm{e}^{\frac{x(x+2)}{2}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 24

```
DSolve[u''[x]-(2*x+1)*u'[x]+(x^2+x-1)*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to e^{\frac{x^2}{2}} \left(c_2 e^x + c_1\right)$$

1.362 problem 367

Internal problem ID [7095]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 367.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' + \left(1 + \frac{2}{(1+3x)^2}\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+(1+2/(1+3*x)^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 (3x+1)^{\frac{1}{3}} \mathrm{e}^{-x} + c_2 (3x+1)^{\frac{2}{3}} \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 35

```
DSolve[y''[x]+2*y'[x]+(1+2/(1+3*x)^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \sqrt[3]{3x+1} \left(c_2 \sqrt[3]{3x+1} + c_1\right)$$

1.363 problem 368

Internal problem ID [7096]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 368.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + \left(x^2 + 2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

1.364 problem 369

Internal problem ID [7097]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 369.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{2y'}{x} - \frac{2y}{(1+x)^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+2/x*diff(y(x),x)-2/(1+x)^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x\,(x+1)} + \frac{c_2(x^2+3x+3)}{x+1}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 33

```
DSolve[y''[x]+2/x*y'[x]-2/(1+x)^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x(x(x+3)+3)+3c_1}{3x(x+1)}$$

1.365 problem 370

Internal problem ID [7098]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 370.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + \frac{y}{2x^4} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+1/(2*x^4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin\left(\frac{\sqrt{2}}{2x}\right) + c_2 x \cos\left(\frac{\sqrt{2}}{2x}\right)$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 50

```
DSolve[y''[x]+1/(2*x^4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{i}{\sqrt{2}x}} x - \frac{i c_2 e^{-\frac{i}{\sqrt{2}x}} x}{\sqrt{2}}$$

1.366 problem 371

Internal problem ID [7099]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 371.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2} \right) \pi (x+2)\,\mathrm{e}^{-2-x} - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x} \left(\sqrt{2}(x+2) \left(2c_1 - \sqrt{\pi} c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}} \right) \right) + 2c_2 e^{\frac{1}{2}(x+2)^2} \right)$$

1.367 problem 372

Internal problem ID [7100]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 372.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2} \right) \pi (x+2)\,\mathrm{e}^{-2-x} - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x} \left(\sqrt{2}(x+2) \left(2c_1 - \sqrt{\pi} c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}} \right) \right) + 2c_2 e^{\frac{1}{2}(x+2)^2} \right)$$

1.368 problem 373

Internal problem ID [7101]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 373.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2} \right) \pi (x+2)\,\mathrm{e}^{-2-x} - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x} \left(\sqrt{2}(x+2) \left(2c_1 - \sqrt{\pi} c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}} \right) \right) + 2c_2 e^{\frac{1}{2}(x+2)^2} \right)$$

1.369 problem 374

Internal problem ID [7102]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 374.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2} \right) \pi (x+2)\,\mathrm{e}^{-2-x} - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x} \left(\sqrt{2}(x+2) \left(2c_1 - \sqrt{\pi} c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}} \right) \right) + 2c_2 e^{\frac{1}{2}(x+2)^2} \right)$$

1.370 problem 375

Internal problem ID [7103]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 375.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2} \right) \pi (x+2)\,\mathrm{e}^{-2-x} - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x} \left(\sqrt{2}(x+2) \left(2c_1 - \sqrt{\pi} c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}} \right) \right) + 2c_2 e^{\frac{1}{2}(x+2)^2} \right)$$

1.371 problem 376

Internal problem ID [7104]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 376.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2} \right) \pi (x+2)\,\mathrm{e}^{-2-x} - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x} \left(\sqrt{2}(x+2) \left(2c_1 - \sqrt{\pi} c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}} \right) \right) + 2c_2 e^{\frac{1}{2}(x+2)^2} \right)$$

1.372 problem 377

Internal problem ID [7105]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 377.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2} \right) \pi (x+2)\,\mathrm{e}^{-2-x} - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x} \left(\sqrt{2}(x+2) \left(2c_1 - \sqrt{\pi} c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}} \right) \right) + 2c_2 e^{\frac{1}{2}(x+2)^2} \right)$$

1.373 problem 378

Internal problem ID [7106]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 378.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2} \right) \pi (x+2)\,\mathrm{e}^{-2-x} - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x} \left(\sqrt{2}(x+2) \left(2c_1 - \sqrt{\pi} c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}} \right) \right) + 2c_2 e^{\frac{1}{2}(x+2)^2} \right)$$

1.374 problem 379

Internal problem ID [7107]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 379.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2} \right) \pi (x+2)\,\mathrm{e}^{-2-x} - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x} \left(\sqrt{2}(x+2) \left(2c_1 - \sqrt{\pi} c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}} \right) \right) + 2c_2 e^{\frac{1}{2}(x+2)^2} \right)$$

1.375 problem 380

Internal problem ID [7108]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 380.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2} \right) \pi (x+2)\,\mathrm{e}^{-2-x} - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x} \left(\sqrt{2}(x+2) \left(2c_1 - \sqrt{\pi} c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}} \right) \right) + 2c_2 e^{\frac{1}{2}(x+2)^2} \right)$$

1.376 problem 381

Internal problem ID [7109]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 381.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2} \right) \pi (x+2)\,\mathrm{e}^{-2-x} - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x} \left(\sqrt{2}(x+2) \left(2c_1 - \sqrt{\pi} c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}} \right) \right) + 2c_2 e^{\frac{1}{2}(x+2)^2} \right)$$

1.377 problem 382

Internal problem ID [7110]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 382.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.378 problem 383

Internal problem ID [7111]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 383.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2x^2y'' + 3y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh\left(\sqrt{x}\,\sqrt{2}\right)}{\sqrt{x}} + \frac{c_2 \cosh\left(\sqrt{x}\,\sqrt{2}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 56

```
DSolve[2*x^2*y''[x]+3*x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\sqrt{2}\sqrt{x}}\left(2c_1 e^{2\sqrt{2}\sqrt{x}} - \sqrt{2}c_2\right)}{2\sqrt{x}}$$

1.379 problem 384

Internal problem ID [7112]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 384.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(3x^2 + 2x\right)y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x), x, x) + (2*x+3*x^2)*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{-3x}}{x^2} + \frac{c_2(9x^2 - 6x + 2)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]+(2*x+3*x^2)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1(9x^2 - 6x + 2) + 27c_2e^{-3x}}{27x^2}$$

1.380 problem 385

Internal problem ID [7113]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 385.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2\left(x^2+x+1\right)y''+x\left(11x^2+11x+9\right)y'+\left(7x^2+10x+6\right)y=0}$$

✓ Solution by Maple

Time used: 0.282 (sec). Leaf size: 362

```
dsolve(2*x^2*(1+x+x^2)*diff(y(x), x$2) + x*(9+11*x+11*x^2)*diff(y(x), x) + (6+10*x+7*x^2)*y(x
```

$$y(x)$$
$$= \frac{c_1 \mathrm{e}^{-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}} \operatorname{HeunG}\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}},0,0,\frac{5}{2},\frac{1}{2},\frac{5i\sqrt{3}-3}{3i\sqrt{3}-3},-\frac{2x}{1+i\sqrt{3}}\right)\left(i\sqrt{3}+2x+1\right)^{\frac{5i\sqrt{3}-3}{-6+6i\sqrt{3}}}\left(i\sqrt{3}-2x-1\right)^{\overline{(1+i}}}{\left(x^2+x+1\right)^{\frac{1}{4}}x^2}$$
$$+\frac{c_2 \mathrm{e}^{-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}} \operatorname{HeunG}\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}},\frac{16}{\left(1+i\sqrt{3}\right)^3\left(i\sqrt{3}-1\right)^2},\frac{1}{2},3,\frac{3}{2},\frac{5i\sqrt{3}-3}{3i\sqrt{3}-3},-\frac{2x}{1+i\sqrt{3}}\right)\left(i\sqrt{3}+2x+1\right)^{\frac{5i\sqrt{3}-3}{-6+6i\sqrt{3}}}\left(}{\left(x^2+x+1\right)^{\frac{1}{4}}x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.199 (sec). Leaf size: 93

```
DSolve[2*x^2*(1+x+x^2)*y''[x] + x*(9+11*x+11*x^2)*y'[x] + (6+10*x+7*x^2)*y[x] == 0,y[x],x,Inc
```

$$y(x)\to \frac{\sqrt{x^2+x+1}e^{-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}}\left(c_2\int_1^x \frac{e^{\frac{\arctan\left(\frac{2K[1]+1}{\sqrt{3}}\right)}{\sqrt{3}}}}{\sqrt{K[1]}(K[1]^2+K[1]+1)^{3/2}}dK[1]+c_1\right)}{x^2}$$

1.381 problem 388

Internal problem ID [7114]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 388.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1+x)\,y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(x*diff(y(x), x$2) +(1+x)*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x-1) + c_2\left(\mathrm{e}^{-x}(x-1)\,\mathrm{Ei}_1\left(-x\right) + 1\right)$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 27

```
DSolve[x*y''[x] +(1+x)*y'[x]+2*y[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(x-1)(c_2\,\mathrm{ExpIntegralEi}(x) + c_1) - c_2$$

1.382 problem 389

Internal problem ID [7115]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 389.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2-2x+1\right)y''-x(x+3)y'+(x+4)y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
dsolve(x^2*(1-2*x+x^2)*diff(y(x), x$2) -x*(3+x)*diff(y(x),x)+(4+x)*y(x) = 0,y(x), singsol=all
```

$$y(x)=\frac{c_1x^2\mathrm{e}^{-\frac{4}{x-1}}}{x-1}+\frac{c_2x^2\,\mathrm{Ei}_1\left(-\frac{4x}{x-1}\right)\mathrm{e}^{-\frac{4x}{x-1}}}{x-1}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 54

```
DSolve[x^2*(1-2*x+x^2)*y''[x] -x*(3+x)*y'[x]+(4+x)*y[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{e^{-\frac{4x}{x-1}}\sqrt{1-x}x^2\left(c_2\,\mathrm{ExpIntegralEi}\left(\frac{4x}{x-1}\right)+e^4c_1\right)}{(x-1)^{3/2}}$$

1.383 problem 390

Internal problem ID [7116]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 390.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x+2)\,y'' + 5y'x^2 + (1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 48

```
dsolve(2*x^2*(2+x)*diff(y(x), x$2) +5*x^2*diff(y(x),x)+(1+x)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\sqrt{x}}{(x+2)^{\frac{3}{2}}} + \frac{c_2\sqrt{x}\left(\sqrt{2}\,\sqrt{x+2} - 2\,\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x+2}}{2}\right)\right)}{(x+2)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 55

```
DSolve[2*x^2*(2+x)*y''[x] +5*x^2*y'[x]+(1+x)*y[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{\sqrt{x}\left(-2\sqrt{2}c_2\operatorname{arctanh}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right) + 2c_2\sqrt{x+2} + c_1\right)}{(x+2)^{3/2}}$$

1.384 problem 391

Internal problem ID [7117]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 391.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 4y'x + \left(x^2 + 2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x), x, x) + 4*x*diff(y(x), x) + (x^2+2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x^2} + \frac{c_2 \cos(x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+4*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x^2}$$

1.385 problem 392

Internal problem ID [7118]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 392.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2 e^{2ix})}{2\sqrt{x}}$$

1.386 problem 394

Internal problem ID [7119]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 394.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x - \left(x^2 + \frac{5}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)-(x^2+5/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{x}(x-1)}{\sqrt{x}} + \frac{c_2 \mathrm{e}^{-x}(x+1)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 53

```
DSolve[x^2*y''[x]-x*y'[x]-(x^2+5/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\frac{2}{\pi}}((ic_2x + c_1)\sinh(x) - (c_1x + ic_2)\cosh(x))}{\sqrt{-ix}}$$

1.387 problem 395

Internal problem ID [7120]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 395.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2 e^{2ix})}{2\sqrt{x}}$$

1.388 problem 396

Internal problem ID [7121]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 396.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 3y'x + 4x^4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+4*x^4*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x^2)}{x^2} + \frac{c_2 \cos(x^2)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 41

```
DSolve[x^2*y''[x]+3*x*y'[x]+4*x^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-ix^2} - ic_2 e^{ix^2}}{4x^2}$$

1.389 problem 398

Internal problem ID [7122]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 398.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(x^2 + 3\right) y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)=(x^2+3)*y(x),y(x), singsol=all)
```

$$y(x) = c_1 x \, \mathrm{e}^{\frac{x^2}{2}} + c_2 \left(\mathrm{e}^{\frac{x^2}{2}} \sqrt{\pi} \, \mathrm{erf}\left(x\right) x + \mathrm{e}^{-\frac{x^2}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 41

```
DSolve[y''[x]==(x^2+3)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} \left(e^{x^2} x \left(c_1 - \sqrt{\pi} c_2 \mathrm{erf}(x) \right) - c_2 \right)$$

1.390 problem 399

Internal problem ID [7123]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 399.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y'x + \left(x^2 + 1\right) y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+(x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^2}{2}} + c_2 x \,\mathrm{e}^{-\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 22

```
DSolve[y''[x]+2*x*y'[x]+(x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} (c_2 x + c_1)$$

1.391 problem 400

Internal problem ID [7124]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 400.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3 y'' + y' - \frac{y}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^3*diff(y(x),x$2)+ diff(y(x),x)-1/x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = x c_1 + c_2 x \operatorname{erf}\left(\frac{i\sqrt{2}}{2x}\right)$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 34

```
DSolve[x^3*y''[x]+ y'[x]-1/x*y[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \sqrt{\frac{\pi}{2}} c_2 x \operatorname{erfi}\left(\frac{1}{\sqrt{2}x}\right)$$

1.392 problem 401

Internal problem ID [7125]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 401.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x^2*diff(diff(y(x),x),x)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2 e^{2ix})}{2\sqrt{x}}$$

1.393 problem 402

Internal problem ID [7126]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 402.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + \left(-8x^2 + 4x\right)y' + \left(4x^2 - 4x - 1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(diff(y(x),x),x)+(-8*x^2+4*x)*diff(y(x),x)+(4*x^2-4*x-1)*y(x) = 0,y(x), sing
```

$$y(x) = \frac{c_1 \mathrm{e}^x}{\sqrt{x}} + c_2 \sqrt{x}\, \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 21

```
DSolve[4*x^2*y''[x]+(-8*x^2+4*x)*y'[x]+(4*x^2-4*x-1)*y[x] == 0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{e^x(c_2 x + c_1)}{\sqrt{x}}$$

1.394 problem 404

Internal problem ID [7127]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 404.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' + y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) + c_2 \mathrm{e}^{\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[y''[x]-y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/2}\left(c_1 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_2 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

1.395 problem 405

Internal problem ID [7128]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 405.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(x^2-1\right)y''-2y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2-1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2\left(x^2+1\right)$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 39

```
DSolve[(x^2-1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{\sqrt{x^2-1}(c_1(x-1)^2+c_2x)}{\sqrt{1-x^2}}$$

1.396 problem 406

Internal problem ID [7129]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 406.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(x+2)\,y' + (x+2)\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)-x*(x+2)*diff(y(x),x)+(x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 x\,\mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]-x*(x+2)*y'[x]+(x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2 e^x + c_1)$$

1.397 problem 407

Internal problem ID [7130]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 407.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1+x)\,y'' - (x+2)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve((x+1)*diff(y(x),x$2)-(x+2)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x+2) + c_2 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 29

```
DSolve[(x+1)*y''[x]-(x+2)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 e^{x+1} - 2c_2(x+2)}{\sqrt{2e}}$$

1.398 problem 408

Internal problem ID [7131]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 408.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(1-x^2\right)y''+2y'x-2y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve((1-x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2\left(x^2+1\right)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 39

```
DSolve[(1-x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{\sqrt{x^2-1}(c_1(x-1)^2+c_2x)}{\sqrt{1-x^2}}$$

1.399 problem 409

Internal problem ID [7132]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 409.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(1-x^2\right)y''-2y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2\left(\frac{\ln\left(x-1\right)x}{2}-\frac{\ln\left(x+1\right)x}{2}+1\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 19

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2(x\operatorname{arctanh}(x)-1)+c_1x$$

1.400 problem 410

Internal problem ID [7133]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 410.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 19

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2(x\text{arctanh}(x) - 1) + c_1 x$$

1.401 problem 411

Internal problem ID [7134]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 411.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(x^2-1\right)y''-6y'x+12y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x^2-1)*diff(y(x),x$2)-6*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^3+x\right)+c_2\left(x^4+6x^2+1\right)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 45

```
DSolve[(x^2-1)*y''[x]-6*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\sqrt{x^2-1}\left(c_2x\left(x^2+1\right)+c_1(x-1)^4\right)}{\sqrt{1-x^2}}$$

1.402 problem 412

Internal problem ID [7135]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 412.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+3\right) y'' - 7y'x + 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 75

```
dsolve((x^2+3)*diff(y(x),x$2)-7*x*diff(y(x),x)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(x^4 - 9x^2 + \frac{27}{8}\right) + c_2\left(\frac{225}{2048} + \frac{\left(8x^4 - 72x^2 + 27\right)\ln\left(\sqrt{x^2+3} - x\right)}{512} + \frac{5(10x^3 - 33x)\sqrt{x^2+3}}{1536} + \frac{25x^4}{768} - \frac{75x^2}{256}\right)$$

✓ Solution by Mathematica

Time used: 0.112 (sec). Leaf size: 492

```
DSolve[(x^2+3)*y''[x]-7*x*y'[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to \frac{1}{24}c_2\Big(&12960x^2\text{RootSum}\Big[7838208000\#1^4-188281584000\#1^2-241544908800\#1 \\
&+18453344881\&,\#1\log\Big(-411757211968704000\#1^3-16606327460698080\#1^2+1013870382516711396 \\
&+5248800x^2\text{RootSum}\Big[2108807205724800000\#1^4-3088288681560000\#1^2 \\
&+97825688064000\#1 \\
&+18453344881\&,\#1\log\Big(27353083060732502808000000\#1^3-272385286174100257200\#1^2-4106175 \\
&-4860\text{RootSum}\Big[7838208000\#1^4-188281584000\#1^2-241544908800\#1 \\
&+18453344881\&,\#1\log\Big(-411757211968704000\#1^3-16606327460698080\#1^2+1013870382516711396 \\
&-1968300\text{RootSum}\Big[2108807205724800000\#1^4-3088288681560000\#1^2 \\
&+97825688064000\#1 \\
&+18453344881\&,\#1\log\Big(27353083060732502808000000\#1^3-272385286174100257200\#1^2-4106175 \\
&-1440x^4\text{RootSum}\Big[7838208000\#1^4-188281584000\#1^2-241544908800\#1 \\
&+18453344881\&,\#1\log\Big(-411757211968704000\#1^3-16606327460698080\#1^2+1013870382516711396 \\
&-583200x^4\text{RootSum}\Big[2108807205724800000\#1^4-3088288681560000\#1^2 \\
&+97825688064000\#1 \\
&+18453344881\&,\#1\log\Big(27353083060732502808000000\#1^3-272385286174100257200\#1^2-4106175 \\
&+165\sqrt{x^2+3}x+216x^2\log\left(\sqrt{x^2+3}-x\right)-81\log\left(\sqrt{x^2+3}-x\right) \\
&-24x^4\log\left(\sqrt{x^2+3}-x\right)-50\sqrt{x^2+3}x^3\Big)+c_1\left(x^4-9x^2+\frac{27}{8}\right)
\end{aligned}
$$

1.403 problem 413

Internal problem ID [7136]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 413.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(x^2-1\right)y''+8y'x+12y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve((x^2-1)*diff(y(x),x$2)+8*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(3x^2+1)}{\left(x^2-1\right)^3}+\frac{c_2(x^3+3x)}{\left(x^2-1\right)^3}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 37

```
DSolve[(x^2-1)*y''[x]+8*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{3c_1(x-1)^3-c_2(3x^2+1)}{3\left(x^2-1\right)^3}$$

1.404 problem 414

Internal problem ID [7137]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 414.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3y'' + y'x - 4y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 57

```
dsolve(3*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\left(x^4 + 18x^2 + 27\right)\operatorname{erf}\left(\frac{x\sqrt{6}}{6}\right)\sqrt{\pi} + \mathrm{e}^{-\frac{x^2}{6}}\sqrt{6}\left(x^2 + 15\right)x\right) + c_2\left(x^4 + 18x^2 + 27\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 43

```
DSolve[3*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{6}}\operatorname{HermiteH}\left(-5, \frac{x}{\sqrt{6}}\right) + \frac{1}{27}c_2\left(x^4 + 18x^2 + 27\right)$$

1.405 problem 415

Internal problem ID [7138]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 415.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{5y'' - 2y'x + 10y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(5*diff(y(x),x$2)-2*x*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\frac{4}{375}x^5 - \frac{4}{15}x^3 + x\right) + c_2 \,\text{hypergeom}\left(\left[-\frac{5}{2}\right], \left[\frac{1}{2}\right], \frac{x^2}{5}\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 77

```
DSolve[5*y''[x]-2*x*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\sqrt{5}x(4(x-5)(x+5)x^2+375)\left(64c_1 - \sqrt{\pi}c_2 \text{erfi}\left(\frac{x}{\sqrt{5}}\right)\right) + 10c_2 e^{\frac{x^2}{5}}(x^2-20)(2x^2-5)}{1000}$$

1.406 problem 416

Internal problem ID [7139]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 416.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x^2 - 3yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-3*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^3}{3}} x + \frac{c_2 \left(x^6 \operatorname{WhittakerM}\left(\frac{1}{3}, \frac{5}{6}, \frac{x^3}{3}\right) + (5x^3 + 10) \operatorname{WhittakerM}\left(\frac{4}{3}, \frac{5}{6}, \frac{x^3}{3}\right) \right) \mathrm{e}^{\frac{x^3}{6}}}{x^4}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 39

```
DSolve[y''[x]-x^2*y'[x]-3*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{\frac{x^3}{3}} \left(3c_1 x - c_2 \operatorname{ExpIntegralE}\left(\frac{4}{3}, \frac{x^3}{3}\right) \right)$$

1.407 problem 417

Internal problem ID [7140]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 417.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right) y'' + 2y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2(\arctan(x)\, x + 1)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 23

```
DSolve[(1+x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to ic_1 x - c_2(x \arctan(x) + 1)$$

1.408 problem 418

Internal problem ID [7141]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 418.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\left(x^2+1\right)\operatorname{erf}\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{\pi} + \sqrt{2}\,\mathrm{e}^{-\frac{x^2}{2}}x\right) + c_2\left(x^2+1\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 35

```
DSolve[y''[x]+x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{2}}\,\text{HermiteH}\left(-3, \frac{x}{\sqrt{2}}\right) + c_2\left(x^2+1\right)$$

1.409 problem 419

Internal problem ID [7142]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 419.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-6x+10\right)y''-4(x-3)y'+6y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve((x^2-6*x+10)*diff(y(x),x$2)-4*(x-3)*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^3-30x+60\right)+c_2\left(\frac{26}{3}+x^2-6x\right)$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 34

```
DSolve[(x^2-6*x+10)*y''[x]-4*(x-3)*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{3}i\left(3c_1(x-(3+i))^3+c_2(3(x-6)x+26)\right)$$

1.410 problem 420

Internal problem ID [7143]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 420.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+6x\right)y''+\left(3x+9\right)y'-3y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve((x^2+6*x)*diff(y(x),x$2)+(3*x+9)*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(3+x)+\frac{c_2(2x^2+12x+9)}{\sqrt{x}\sqrt{x+6}}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 66

```
DSolve[(x^2+6*x)*y''[x]+(3*x+9)*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{\sqrt{\frac{2}{3\pi}}\left(c_1(2x(x+6)+9)-\pi c_2(x+3)\sqrt{-x(x+6)}\right)}{3\sqrt[4]{-x^2}\sqrt{x+6}}$$

1.411 problem 421

Internal problem ID [7144]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 421.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' + \left(t^2 - 1\right) y' + t^3 y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
dsolve(t*diff(y(t),t$2)+ (t^2-1)*diff(y(t),t)+t^3*y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{-\frac{t^2}{4}} \cos\left(\frac{t^2\sqrt{3}}{4}\right) + c_2 \mathrm{e}^{-\frac{t^2}{4}} \sin\left(\frac{t^2\sqrt{3}}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 48

```
DSolve[t*y''[t]+(t^2-1)*y'[t]+t^3*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-\frac{t^2}{4}} \left(c_2 \cos\left(\frac{\sqrt{3}t^2}{4}\right) + c_1 \sin\left(\frac{\sqrt{3}t^2}{4}\right)\right)$$

1.412 problem 422

Internal problem ID [7145]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 422.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$t^2 y'' - t(t+2)\,y' + (t+2)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(t^2*diff(y(t),t$2)-t*(t+2)*diff(y(t),t)+(t+2)*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t + \mathrm{e}^t c_2 t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]-t*(t+2)*y'[t]+(t+2)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t\left(c_2 e^t + c_1\right)$$

1.413 problem 423

Internal problem ID [7146]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 423.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-1+x)\,y'' - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2 x$$

1.414 problem 424

Internal problem ID [7147]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 424.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - \left(x - \frac{3}{16}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)-(x-1875/10000)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{1}{4}} \sinh\left(2\sqrt{x}\right) + c_2 x^{\frac{1}{4}} \cosh\left(2\sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 41

```
DSolve[x^2*y''[x]-(x-1875/10000)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-2\sqrt{x}} \sqrt[4]{x} \left(2c_1 e^{4\sqrt{x}} - c_2\right)$$

1.415 problem 425

Internal problem ID [7148]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 425.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-25/100)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-25/100)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

1.416 problem 426

Internal problem ID [7149]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 426.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2 y'' - t(t+2)\,y' + (t+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(t^2*diff(y(t),t$2)-t*(t+2)*diff(y(t),t)+(t+2)*y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 t + \mathrm{e}^t c_2 t$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]-t*(t+2)*y'[t]+(t+2)*y[t] == 0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t\left(c_2 e^t + c_1\right)$$

1.417 problem 427

Internal problem ID [7150]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 427.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{ty'' - y'(t+1) + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(t*diff(y(t),t$2)-(1+t)*diff(y(t),t)+y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1(t+1) + c_2 \mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 19

```
DSolve[t*y''[t]-(1+t)*y'[t]+y[t] == 0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^t - c_2(t+1)$$

1.418 problem 428

Internal problem ID [7151]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 428.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-t+1)\,y''+ty'-y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((1-t)*diff(y(t),t$2)+t*diff(y(t),t)-y(t) = 0,y(t), singsol=all)
```

$$y(t)=c_1t+c_2\mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 17

```
DSolve[(1-t)*y''[t]+t*y'[t]-y[t] == 0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to c_1e^t-c_2t$$

1.419 problem 429

Internal problem ID [7152]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 429.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-25/100)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-25/100)*y[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2 e^{2ix})}{2\sqrt{x}}$$

1.420 problem 430

Internal problem ID [7153]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 430.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{ty'' - y'(t+1) + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(t*diff(y(t),t$2)-(1+t)*diff(y(t),t)+y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1(t+1) + c_2 \mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 19

```
DSolve[t*y''[t]-(1+t)*y'[t]+y[t] ==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^t - c_2(t+1)$$

1.421 problem 431

Internal problem ID [7154]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 431.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-t+1)\,y'' + ty' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((1-t)*diff(y(t),t$2)+t*diff(y(t),t)-y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2 \mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 17

```
DSolve[(1-t)*y''[t]+t*y'[t]-y[t] ==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^t - c_2 t$$

1.422 problem 432

Internal problem ID [7155]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 432.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \, \mathrm{e}^{-\frac{x^2}{2}} + c_2 \left(-i\sqrt{\pi}\,\sqrt{2} + \operatorname{erf}\left(\frac{i\sqrt{2}\,x}{2} \right) \mathrm{e}^{-\frac{x^2}{2}} \pi x \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 44

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2} c_2 x \operatorname{DawsonF}\left(\frac{x}{\sqrt{2}} \right) + \sqrt{2} c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.423 problem 433

Internal problem ID [7156]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 433.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right)y''-4y'x+6y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve((1+x^2)*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(-3x^2+1\right)+c_2\left(x^3-3x\right)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 33

```
DSolve[(1+x^2)*y''[x]-4*x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{3}i\left(c_2\left(3x^2-1\right)+3c_1(x-i)^3\right)$$

1.424 problem 434

Internal problem ID [7157]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 434.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\,y'' + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve((1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 17

```
DSolve[(1-x)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2 x$$

1.425 problem 435

Internal problem ID [7158]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 435.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y'' + y'x + 3y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
dsolve(2*diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\sqrt{\pi}\,\mathrm{e}^{-\frac{x^2}{4}}\,\mathrm{erfi}\left(\frac{x}{2}\right)\left(x^2-2\right) - 2x\right) + c_2\mathrm{e}^{-\frac{x^2}{4}}\left(x^2-2\right)$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 47

```
DSolve[2*y''[x]+x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}c_2\left(x^2-2\right)\mathrm{DawsonF}\left(\frac{x}{2}\right) + c_1 e^{-\frac{x^2}{4}}\left(x^2-2\right) - \frac{c_2 x}{4}$$

1.426 problem 436

Internal problem ID [7159]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 436.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \, \mathrm{e}^{-\frac{x^2}{2}} + c_2 \left(-i\sqrt{\pi}\,\sqrt{2} + \operatorname{erf}\left(\frac{i\sqrt{2}\,x}{2} \right) \mathrm{e}^{-\frac{x^2}{2}} \pi x \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 44

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2} c_2 x \operatorname{DawsonF}\left(\frac{x}{\sqrt{2}} \right) + \sqrt{2} c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.427 problem 437

Internal problem ID [7160]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 437.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\,y'' + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 17

```
DSolve[(1-x)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2 x$$

1.428 problem 438

Internal problem ID [7161]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 438.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \, \mathrm{e}^{-\frac{x^2}{2}} + c_2 \left(-i\sqrt{\pi}\,\sqrt{2} + \operatorname{erf}\left(\frac{i\sqrt{2}\,x}{2} \right) \mathrm{e}^{-\frac{x^2}{2}} \pi x \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 44

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}c_2 x \operatorname{DawsonF}\left(\frac{x}{\sqrt{2}} \right) + \sqrt{2}c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.429 problem 439

Internal problem ID [7162]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 439.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(-x^2+4\right)y''+y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve((4-x^2)*diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^2-4\right)^{\frac{3}{4}}\operatorname{LegendreP}\left(\sqrt{3}-\frac{1}{2},\frac{3}{2},\frac{x}{2}\right)+c_2\left(x^2-4\right)^{\frac{3}{4}}\operatorname{LegendreQ}\left(\sqrt{3}-\frac{1}{2},\frac{3}{2},\frac{x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 58

```
DSolve[(4-x^2)*y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\left(x^2-4\right)^{3/4}\left(c_1P^{\frac{3}{2}}_{-\frac{1}{2}+\sqrt{3}}\left(\frac{x}{2}\right)+c_2Q^{\frac{3}{2}}_{-\frac{1}{2}+\sqrt{3}}\left(\frac{x}{2}\right)\right)$$

1.430 problem 440

Internal problem ID [7163]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 440.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4y'x + \left(-16x^2 + 3\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(3-16*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sinh(2x) + c_2\sqrt{x}\,\cosh(2x)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 32

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(3-16*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}\sqrt{x}\left(c_2e^{4x} + 4c_1\right)$$

1.431 problem 441

Internal problem ID [7164]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 441.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-1+x)\,y'' - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2 x$$

1.432 problem 442

Internal problem ID [7165]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 442.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + \left(x^2 + 2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

1.433 problem 444

Internal problem ID [7166]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 444.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-2x\right)y''+\left(-x^2+2\right)y'+\left(2x-2\right)y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve((x^2-2*x)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)+(2*x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=x^2c_1+c_2\mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 18

```
DSolve[(x^2-2*x)*y''[x]+(2-x^2)*y'[x]+(2*x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to c_2x^2+c_1e^x$$

1.434 problem 445

Internal problem ID [7167]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 445.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x+1)\,y'' - 2y' - (3+2x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((2*x+1)*diff(y(x),x$2)-2*diff(y(x),x)-(2*x+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 x\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 29

```
DSolve[(2*x+1)*y''[x]-2*y'[x]-(2*x+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x-\frac{1}{2}} \left(c_2 e^{2x+1} x + c_1\right)$$

1.435 problem 446

Internal problem ID [7168]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 446.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + \left(-8x^2 + 4x\right)y' + \left(4x^2 - 4x - 1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)+(4*x-8*x^2)*diff(y(x),x)+(4*x^2-4*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^x}{\sqrt{x}} + c_2 \sqrt{x}\, \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 21

```
DSolve[4*x^2*y''[x]+(4*x-8*x^2)*y'[x]+(4*x^2-4*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{e^x(c_2 x + c_1)}{\sqrt{x}}$$

1.436 problem 447

Internal problem ID [7169]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 447.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y'x + \left(4x^2 + 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} x$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 20

```
DSolve[y''[x]+4*x*y'[x]+(4*x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2} (c_2 x + c_1)$$

1.437 problem 448

Internal problem ID [7170]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 448.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2x(-1+x)\,y' + \left(x^2 - 2x + 2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x$2)+2*x*(x-1)*diff(y(x),x)+(x^2-2*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} x + \mathrm{e}^{-x} c_2 x^2$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 19

```
DSolve[x^2*y''[x]+2*x*(x-1)*y'[x]+(x^2-2*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{-x} x (c_2 x + c_1)$$

1.438 problem 449

Internal problem ID [7171]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 449.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(2x-1)\,y' + \left(x^2 - x - 1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-x*(2*x-1)*diff(y(x),x)+(x^2-x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^x}{x} + c_2 x\,\mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]-x*(2*x-1)*y'[x]+(x^2-x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x\left(\frac{c_1}{x} + \frac{c_2 x}{2}\right)$$

1.439 problem 450

Internal problem ID [7172]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 450.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-2x)\,y''+2y'+(2x-3)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((1-2*x)*diff(y(x),x$2)+2*diff(y(x),x)+(2*x-3)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{x}+\mathrm{e}^{-x}c_2x$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 48

```
DSolve[(1-2*x)*y''[x]+2*y'[x]+(2*x-3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{e^{-x-\frac{1}{2}}\sqrt{1-2x}(c_1e^{2x}-ec_2x)}{\sqrt{2x-1}}$$

1.440 problem 451

Internal problem ID [7173]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 451.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + (1+4x)\,y' + (2x+1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(2*x*diff(y(x),x$2)+(4*x+1)*diff(y(x),x)+(2*x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 \sqrt{x}\, \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 23

```
DSolve[2*x*y''[x]+(4*x+1)*y'[x]+(2*x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(2c_2\sqrt{x} + c_1\right)$$

1.441 problem 452

Internal problem ID [7174]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 452.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2x+1)\,y' + (1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)-(2*x+1)*diff(y(x),x)+(x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2 \mathrm{e}^{x} x^2$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 23

```
DSolve[x*y''[x]-(2*x+1)*y'[x]+(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{x} \left(c_2 x^2 + 2c_1\right)$$

1.442 problem 453

Internal problem ID [7175]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 453.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - 4x(1+x)\,y' + (3+2x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*(x+1)*diff(y(x),x)+(2*x+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x}\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 20

```
DSolve[4*x^2*y''[x]-4*x*(x+1)*y'[x]+(2*x+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_2e^x + c_1)$$

1.443 problem 454

Internal problem ID [7176]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 454.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (-2x+2)\,y' + (x-2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)+(2-2*x)*diff(y(x),x)+(x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x + \frac{c_2 \mathrm{e}^x}{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 19

```
DSolve[x*y''[x]+(2-2*x)*y'[x]+(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x(c_2 x + c_1)}{x}$$

1.444 problem 455

Internal problem ID [7177]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 455.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$\boxed{x^2y'' - 2y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2c_1 + c_2x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 14

```
DSolve[x^2*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2x + c_1)$$

1.445 problem 456

Internal problem ID [7178]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 456.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2x+2)\,y' + (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)-(2*x+2)*diff(y(x),x)+(x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x + c_2 \mathrm{e}^x x^3$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 23

```
DSolve[x*y''[x]-(2*x+2)*y'[x]+(x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^x \left(c_2 x^3 + 3c_1\right)$$

1.446 problem 457

Internal problem ID [7179]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 457.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + \left(x^2 + 2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

1.447 problem 458

Internal problem ID [7180]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 458.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (1+4x)\,y' + (4x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x*diff(y(x),x$2)-(4*x+1)*diff(y(x),x)+(4*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{2x} + \mathrm{e}^{2x} c_2 x^2$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 25

```
DSolve[x*y''[x]-(4*x+1)*y'[x]+(4*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{2x} \left(c_2 x^2 + 2c_1\right)$$

1.448 problem 460

Internal problem ID [7181]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 460.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4y'x + \left(-16x^2 + 3\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(3-16*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sinh(2x) + c_2\sqrt{x}\,\cosh(2x)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 32

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(3-16*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}\sqrt{x}\left(c_2e^{4x} + 4c_1\right)$$

1.449 problem 461

Internal problem ID [7182]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 461.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x+1)\,xy'' - 2\left(2x^2-1\right)y' - 4(1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((2*x+1)*x*diff(y(x),x$2)-2*(2*x^2-1)*diff(y(x),x)-4*(x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2 \mathrm{e}^{2x}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 28

```
DSolve[(2*x+1)*x*y''[x]-2*(2*x^2-1)*y'[x]-4*(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{c_2 e^{2x+1} x + c_1}{\sqrt{e} x}$$

1.450 problem 462

Internal problem ID [7183]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 462.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-2x\right)y''+\left(-x^2+2\right)y'+\left(2x-2\right)y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve((x^2-2*x)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)+(2*x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=x^2c_1+c_2\mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 18

```
DSolve[(x^2-2*x)*y''[x]+(2-x^2)*y'[x]+(2*x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to c_2x^2+c_1e^x$$

1.451 problem 463

Internal problem ID [7184]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 463.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (1+4x)\,y' + (4x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x*diff(y(x),x$2)-(4*x+1)*diff(y(x),x)+(4*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{2x} + \mathrm{e}^{2x} c_2 x^2$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 25

```
DSolve[x*y''[x]-(4*x+1)*y'[x]+(4*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{2x} \left(c_2 x^2 + 2c_1\right)$$

1.452 problem 464

Internal problem ID [7185]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 464.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x-1)\,y''-(3x+2)\,y'-(6x-8)\,y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve((3*x-1)*diff(y(x),x$2)-(3*x+2)*diff(y(x),x)-(6*x-8)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{2x}+\mathrm{e}^{-x}c_2x$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(3*x-1)*y''[x]-(3*x+2)*x*y'[x]-(6*x-8)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

Not solved

1.453 problem 465

Internal problem ID [7186]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 465.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1+x)^2 y'' - 2(1+x) y' - \left(x^2+2x-1\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve((x+1)^2*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)-(x^2+2*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sinh(x)(x+1) + c_2 \cosh(x)(x+1)$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 147

```
DSolve[(x+1)^2*y''[x]-2*(x+1)*x*y'[x]-(x^2+2*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to 2^{\frac{1}{2}i\left(\sqrt{7}+i\right)} e^{-\left(\left(\sqrt{2}-1\right)(x+1)\right)} (x + 1)^{\frac{1}{2}i\left(\sqrt{7}+i\right)} \left(c_1 \text{HypergeometricU}\left(\frac{1}{2}\left(1-\sqrt{2}+i\sqrt{7}\right), 1+i\sqrt{7}, 2\sqrt{2}(x+1)\right) + c_2 L^{i\sqrt{7}}_{\frac{1}{2}\left(-1+\sqrt{2}-i\sqrt{7}\right)}\left(2\sqrt{2}(x+1)\right)\right)$$

1.454 problem 466

Internal problem ID [7187]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 466.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + \left(-8x^2 + 4x\right)y' + \left(4x^2 - 4x - 1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)+(4*x-8*x^2)*diff(y(x),x)+(4*x^2-4*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^x}{\sqrt{x}} + c_2 \sqrt{x}\, \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 21

```
DSolve[4*x^2*y''[x]+(4*x-8*x^2)*y'[x]+(4*x^2-4*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{e^x(c_2 x + c_1)}{\sqrt{x}}$$

1.455 problem 467

Internal problem ID [7188]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 467.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y'x + \left(4x^2 + 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 21

```
DSolve[4*x^2*y''[x]+(4*x-8*x^2)*y'[x]+(4*x^2-4*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{e^x (c_2 x + c_1)}{\sqrt{x}}$$

1.456 problem 468

Internal problem ID [7189]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 468.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x+1)\,y'' - 2y' - (3+2x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((2*x+1)*diff(y(x),x$2)-2*diff(y(x),x)-(2*x+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 x\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 29

```
DSolve[(2*x+1)*y''[x]-2*y'[x]-(2*x+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x-\frac{1}{2}} \left(c_2 e^{2x+1} x + c_1\right)$$

1.457 problem 469

Internal problem ID [7190]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 469.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2x+2)\,y' + (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)-(2*x+2)*diff(y(x),x)+(x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2 \mathrm{e}^{x} x^3$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 29

```
DSolve[x*y''[x]-(2*x+2)*y'[x]+(x+2)*y[x]==6*x^3*Exp[x],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{6} e^{x} \left(9x^4 + 2c_2 x^3 + 6c_1\right)$$

1.458 problem 470

Internal problem ID [7191]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 470.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + \left(x^2 + 2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

1.459 problem 472

Internal problem ID [7192]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 472.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4y'x + \left(-16x^2 + 3\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(3-16*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sinh(2x) + c_2\sqrt{x}\,\cosh(2x)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 32

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(3-16*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}\sqrt{x}\left(c_2e^{4x} + 4c_1\right)$$

1.460 problem 473

Internal problem ID [7193]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 473.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4y'x + \left(4x^2 + 3\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sin(x) + c_2\sqrt{x}\,\cos(x)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 39

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(4*x^2+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-ix}\sqrt{x}\left(2c_1 - ic_2e^{2ix}\right)$$

1.461 problem 474

Internal problem ID [7194]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 474.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x - \left(x^2 - 2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)-(x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sinh(x) + c_2 x \cosh(x)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]-2*x*y'[x]-(x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} x + \frac{1}{2} c_2 e^{x} x$$

1.462 problem 475

Internal problem ID [7195]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 475.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2x(1+x)\,y' + \left(x^2+2x+2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-2*x*(x+1)*diff(y(x),x)+(x^2+2*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} c_1 x + c_2 \mathrm{e}^{x} x^2$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 41

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{ix} x(c_1 \operatorname{HypergeometricU}(-i, 0, -2ix) + c_2 L_i^{-1}(-2ix))$$

1.463 problem 476

Internal problem ID [7196]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 476.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2x(x+2)y' + \left(x^2+4x+6\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)-2*x*(x+2)*diff(y(x),x)+(x^2+4*x+6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x x^2 + c_2 \mathrm{e}^x x^3$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 19

```
DSolve[x^2*y''[x]-2*x*(x+2)*y'[x]+(x^2+4*x+6)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^x x^2 (c_2 x + c_1)$$

1.464 problem 477

Internal problem ID [7197]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 477.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 4y'x + \left(x^2 + 6\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(x^2+6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^2 \sin(x) + c_2 \cos(x) x^2$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]-4*x*y'[x]+(x^2+6)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-ix}x^2\left(2c_1 - ic_2e^{2ix}\right)$$

1.465 problem 478

Internal problem ID [7198]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 478.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-1+x)\,y'' - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2 x$$

1.466 problem 479

Internal problem ID [7199]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 479.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - 4x(1+x)\,y' + (3+2x)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*(x+1)*diff(y(x),x)+(2*x+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x}\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 20

```
DSolve[4*x^2*y''[x]-4*x*(x+1)*y'[x]+(2*x+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_2e^x + c_1)$$

1.467 problem 480

Internal problem ID [7200]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 480.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x-1)\,y''-(3x+2)\,y'-(6x-8)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((3*x-1)*diff(y(x),x$2)-(3*x+2)*diff(y(x),x)-(6*x-8)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{2x}+\mathrm{e}^{-x}c_2x$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 35

```
DSolve[(3*x-1)*y''[x]-(3*x+2)*y'[x]-(6*x-8)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{e^{-x-\frac{1}{2}}\left(c_1e^{3x}+2ec_2x\right)}{\sqrt{2}}$$

1.468 problem 481

Internal problem ID [7201]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 481.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+2)\,y'' + y'x + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 71

```
dsolve((2+x)*diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) = {} & c_1 \mathrm{e}^{-x}\left(x^2-6x+4\right)(x+2)^3 \\ & + c_2\left(\mathrm{e}^{-2-x}\left(x^2-6x+4\right)(x+2)^3\,\mathrm{Ei}_1\left(-2-x\right) + x^4 - x^3 - 18x^2 - 22x + 8\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 65

```
DSolve[(2+x)*y''[x]+x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} & y(x) \\ & \to \frac{e^{-x}((x-6)x+4)(x+2)^3(c_2\,\mathrm{ExpIntegralEi}(x+2)+3840c_1)+e^2c_2(x(x(-x^2+x+18)+22)-8)}{960e} \end{aligned}$$

1.469 problem 482

Internal problem ID [7202]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 482.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x)\,y'' + x(x+4)\,y' + (2-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
dsolve(x^2*(1-x)*diff(y(x),x$2)+x*(4+x)*diff(y(x),x)+(2-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2+6x+3)}{x} + \frac{c_2(1+3(x^3+6x^2+3x)\ln(x)+51x^2+48x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 51

```
DSolve[x^2*(1-x)*y''[x]+x*(4+x)*y'[x]+(2-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3c_1x(x(x+6)+3) - c_2(51x^2+48x+3(x(x+6)+3)x\log(x)+1)}{3x^2}$$

1.470 problem 483

Internal problem ID [7203]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 483.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y'' + x(2x+1)\,y' - (4+6x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+x*(1+2*x)*diff(y(x),x)-(4+6*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2 c_1 + \frac{c_2(12\ln(x+1)\,x^4 - 12x^4\ln(x) - 12x^3 + 6x^2 - 4x + 3)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 50

```
DSolve[x^2*(1+x)*y''[x]+x*(1+2*x)*y'[x]-(4+6*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to c_1 x^2 + \frac{c_2(12x^4(\log(x) - \log(x+1)) + 2(6x^2 - 3x + 2)\,x - 3)}{12x^2}$$

1.471 problem 484

Internal problem ID [7204]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 484.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(2x^2+1\right)y''+x\left(2x^2+4\right)y'+2\left(1-x^2\right)y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(x^2*(1+2*x^2)*diff(y(x),x$2)+x*(4+2*x^2)*diff(y(x),x)+2*(1-x^2)*y(x)=0,y(x), singsol=a
```

$$y(x)=\frac{c_1}{x}+\frac{c_2\left(3\,\operatorname{arcsinh}\left(\sqrt{2}\,x\right)x+\sqrt{2}\,\sqrt{2x^2+1}\,\left(x^2-1\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 67

```
DSolve[x^2*(1+2*x^2)*y''[x]+x*(4+2*x^2)*y'[x]+2*(1-x^2)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to\frac{3\sqrt{2}c_2x\operatorname{arcsinh}\left(\sqrt{2}x\right)-2c_2\sqrt{2x^2+1}+2x\left(c_2x\sqrt{2x^2+1}+c_1\right)}{2x^2}$$

1.472 problem 485

Internal problem ID [7205]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 485.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+2\right)y''+2x\left(x^2+5\right)y'+2\left(-x^2+3\right)y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
dsolve(x^2*(2+x^2)*diff(y(x),x$2)+2*x*(x^2+5)*diff(y(x),x)+2*(3-x^2)*y(x)=0,y(x), singsol=all
```

$$y(x)=\frac{c_1(x^2+8)}{x}+\frac{c_2\left(\left(x^4+8x^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{x^2+2}}\right)+\left(-x^2+4\right)\sqrt{2}\sqrt{x^2+2}\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 85

```
DSolve[x^2*(2+x^2)*y''[x]+2*x*(x^2+5)*y'[x]+2*(3-x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to -\frac{\sqrt{2}c_2(x^2+8)\,x^2\operatorname{arctanh}\left(\frac{\sqrt{x^2+2}}{\sqrt{2}}\right)-2x^2\left(32c_1(x^2+8)+c_2\sqrt{x^2+2}\right)+8c_2\sqrt{x^2+2}}{64x^3}$$

1.473 problem 486

Internal problem ID [7206]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 486.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right)y''+6y'x+6y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve((1+x^2)*diff(y(x),x$2)+6*x*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1 x}{\left(x^2+1\right)^2}+\frac{c_2\left(x^2-1\right)}{\left(x^2+1\right)^2}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 29

```
DSolve[(1+x^2)*y''[x]+6*x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_2 x-c_1(x-i)^2}{\left(x^2+1\right)^2}$$

1.474 problem 487

Internal problem ID [7207]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 487.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right) y'' + 2y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2(\arctan(x)\, x + 1)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 23

```
DSolve[(1+x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to ic_1 x - c_2(x \arctan(x) + 1)$$

1.475 problem 488

Internal problem ID [7208]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 488.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right) y'' - 8y'x + 20y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve((1+x^2)*diff(y(x),x$2)-8*x*diff(y(x),x)+20*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(5x^4 - 10x^2 + 1\right) + c_2\left(x^5 - 10x^3 + 5x\right)$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 38

```
DSolve[(1+x^2)*y''[x]-8*x*y'[x]+20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{5} i c_2 \left(5\left(x^2-2\right)x^2+1\right) + c_1 (1+ix)^5$$

1.476 problem 489

Internal problem ID [7209]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 489.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(1-x^2\right)y''-8y'x-12y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve((1-x^2)*diff(y(x),x$2)-8*x*diff(y(x),x)-12*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(3x^2+1)}{\left(x^2-1\right)^3}+\frac{c_2(x^3+3x)}{\left(x^2-1\right)^3}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 37

```
DSolve[(1-x^2)*y''[x]-8*x*y'[x]-12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{3c_1(x-1)^3-c_2(3x^2+1)}{3\left(x^2-1\right)^3}$$

1.477 problem 490

Internal problem ID [7210]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 490.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(2x^2+1\right)y''+7y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
dsolve((1+2*x^2)*diff(y(x),x$2)+7*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\,\text{LegendreP}\left(\frac{1}{4},\frac{3}{4},i\sqrt{2}\,x\right)}{\left(2x^2+1\right)^{\frac{3}{8}}}+\frac{c_2\,\text{LegendreQ}\left(\frac{1}{4},\frac{3}{4},i\sqrt{2}\,x\right)}{\left(2x^2+1\right)^{\frac{3}{8}}}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 66

```
DSolve[(1+2*x^2)*y''[x]+7*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_2 Q_{\frac{1}{4}}^{\frac{3}{4}}\left(i\sqrt{2}x\right)}{\left(2x^2+1\right)^{3/8}}+\frac{2i\sqrt[4]{2}c_1 x}{\left(2x^2+1\right)^{3/4}\,\text{Gamma}\left(\frac{1}{4}\right)}$$

1.478 problem 491

Internal problem ID [7211]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 491.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(1-x^2\right)y''-5y'x-4y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 46

```
dsolve((1-x^2)*diff(y(x),x$2)-5*x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1 x}{\left(x^2-1\right)^{\frac{3}{2}}}+\frac{c_2\left(x\ln\left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}\right)}{\left(x^2-1\right)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 52

```
DSolve[(1-x^2)*y''[x]-5*x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-c_2\sqrt{x^2-1}-c_2 x\log\left(\sqrt{x^2-1}-x\right)+c_1 x}{\left(x^2-1\right)^{3/2}}$$

1.479 problem 492

Internal problem ID [7212]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 492.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right) y'' - 10y'x + 28y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve((1+x^2)*diff(y(x),x$2)-10*x*diff(y(x),x)+28*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(1 + \frac{35}{3}x^4 - 14x^2\right) + c_2\left(x^7 + 21x^5 - 105x^3 + 35x\right)$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 40

```
DSolve[(1+x^2)*y''[x]-10*x*y'[x]+28*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{105}c_2\left(35x^4 - 42x^2 + 3\right) - c_1(x-i)^6(x+6i)$$

1.480 problem 493

Internal problem ID [7213]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 493.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \, \mathrm{e}^{-\frac{x^2}{2}} + c_2 \left(-i\sqrt{\pi}\,\sqrt{2} + \operatorname{erf}\left(\frac{i\sqrt{2}\,x}{2} \right) \mathrm{e}^{-\frac{x^2}{2}} \pi x \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 44

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}c_2 x \operatorname{DawsonF}\left(\frac{x}{\sqrt{2}} \right) + \sqrt{2}c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.481 problem 495

Internal problem ID [7214]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 495.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(2x^2-8x+11\right)y''-16(x-2)y'+36y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve((11-8*x+2*x^2)*diff(y(x),x$2)-16*(x-2)*diff(y(x),x)+36*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(-\frac{31}{5}+x^3-6x^2+\frac{111}{10}x\right)+c_2\left(x^6-12x^5+\frac{165}{2}x^4-\frac{16577}{8}-\frac{5445}{4}x^2+3267x\right)$$

✓ Solution by Mathematica

Time used: 0.451 (sec). Leaf size: 87

```
DSolve[(11-8*x+2*x^2)*y''[x]-16*(x-2)*y'[x]+36*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$\begin{aligned}y(x)\to&\frac{c_1\left(2x+5i\sqrt{6}-4\right)\left(2(x-4)x+11\right)^2\left(2ix+\sqrt{6}-4i\right)^3}{2\left(-2ix+\sqrt{6}+4i\right)^2}\\&+\frac{1}{15}ic_2(x-2)(10(x-4)x+31)\end{aligned}$$

1.482 problem 496

Internal problem ID [7215]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 496.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (x-3)\,y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+(x-3)*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\mathrm{KummerM}\left(\frac{3}{2}, \frac{1}{2}, -\frac{(x-3)^2}{2}\right) + c_2 \,\mathrm{KummerU}\left(\frac{3}{2}, \frac{1}{2}, -\frac{(x-3)^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 58

```
DSolve[y''[x]+(x-3)*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2(x-4)(x-2)\,\mathrm{DawsonF}\left(\frac{x-3}{\sqrt{2}}\right)}{\sqrt{2}} + c_1 e^{-\frac{1}{2}(x-6)x}(x-4)(x-2) - \frac{1}{2}c_2(x-3)$$

1.483 problem 497

Internal problem ID [7216]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 497.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-8x+14\right)y''-8(x-4)y'+20y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve((x^2-8*x+14)*diff(y(x),x$2)-8*(x-4)*diff(y(x),x)+20*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^5-140x^3+1120x^2-3500x+4032\right)+c_2\left(\frac{1604}{5}+x^4-16x^3+100x^2-288x\right)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 76

```
DSolve[(x^2-8*x+14)*y''[x]+8*(x-4)*y'[x]+20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{c_1 P^3_{\frac{1}{2}i\left(i+\sqrt{31}\right)}\left(\frac{x-4}{\sqrt{2}}\right)+c_2 Q^3_{\frac{1}{2}i\left(i+\sqrt{31}\right)}\left(\frac{x-4}{\sqrt{2}}\right)}{((x-8)x+14)^{3/2}}$$

1.484 problem 498

Internal problem ID [7217]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 498.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(2x^2+4x+5\right)y''-20(1+x)y'+60y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve((2*x^2+4*x+5)*diff(y(x),x$2)-20*(x+1)*diff(y(x),x)+60*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(-\frac{7}{4}+x^5+5x^4+5x^3-5x^2-\frac{31}{4}x\right)+c_2\left(x^6+\frac{155}{8}-\frac{75}{2}x^4-100x^3-\frac{225}{4}x^2+30x\right)$$

✓ Solution by Mathematica

Time used: 0.326 (sec). Leaf size: 73

```
DSolve[(2*x^2+4*x+5)*y''[x]-20*(x+1)*y'[x]+60*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\rightarrow\frac{(2x(x+2)+5)^{5/2}\left(c_1\left(2ix+\sqrt{6}+2i\right)^6+4c_2(x+1)(2x(x+2)-7)(2x(x+2)+1)\right)}{(4x(x+2)+10)^{5/2}}$$

1.485 problem 499

Internal problem ID [7218]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 499.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^3+1\right)y''+7y'x^2+9yx=0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 28

```
dsolve((1+x^3)*diff(y(x),x$2)+7*x^2*diff(y(x),x)+9*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\text{hypergeom}\left(\left[1,1\right],\left[\frac{2}{3}\right],-x^3\right)+\frac{c_2x}{\left(x^3+1\right)^{\frac{4}{3}}}$$

✓ Solution by Mathematica

Time used: 0.481 (sec). Leaf size: 113

```
DSolve[(1+x^3)*y''[x]+7*x^2*y'[x]+9*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to\frac{c_2x\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)-2\log\left(\sqrt[3]{x^3+1}-x\right)+\log\left(\sqrt[3]{x^3+1}x+\left(x^3+1\right)^{2/3}+x^2\right)\right)-6c_2$$

$$6\left(x^3+1\right)^{4/3}$$

1.486 problem 500

Internal problem ID [7219]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 500.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(2x^5+1\right)y''+14y'x^4+10yx^3=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 30

```
dsolve((1+2*x^5)*diff(y(x),x$2)+14*x^4*diff(y(x),x)+10*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1 x}{\left(2x^5+1\right)^{\frac{2}{5}}}+c_2\,\mathrm{hypergeom}\left(\left[\frac{1}{5},1\right],\left[\frac{4}{5}\right],-2x^5\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1+2*x^5)*y''[x]+14*x^4*y'[x]+10*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.487 problem 501

Internal problem ID [7220]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 501.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + x^6 y' + 7yx^5 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)+x^6*diff(y(x),x)+7*x^5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^7}{7}} x + \frac{c_2 \left((-x^7)^{\frac{6}{7}} 7^{\frac{1}{7}} - x^7 \mathrm{e}^{-\frac{x^7}{7}} \left(\Gamma\left(\frac{6}{7}\right) - \Gamma\left(\frac{6}{7}, -\frac{x^7}{7}\right) \right) \right)}{x^6}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 39

```
DSolve[y''[x]+x^6*y'[x]+7*x^5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{7} e^{-\frac{x^7}{7}} \left(7c_1 x - c_2 \operatorname{ExpIntegralE}\left(\frac{8}{7}, -\frac{x^7}{7}\right) \right)$$

1.488 problem 502

Internal problem ID [7221]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 502.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^8+1\right)y''-16x^7y'+72yx^6=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((1+x^8)*diff(y(x),x$2)-16*x^7*diff(y(x),x)+72*x^6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(-\frac{7}{9}+x^8\right)+c_2\left(x^9-\frac{9}{7}x\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1+x^8)*y''[x]-16*x^7*y'[x]+72*x^6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.489 problem 503

Internal problem ID [7222]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 503.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' x^5 + 6x^4 y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)+x^5*diff(y(x),x)+6*x^4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^6}{6}} x + \frac{c_2 \left((-x^6)^{\frac{5}{6}} 6^{\frac{1}{6}} - x^6 \mathrm{e}^{-\frac{x^6}{6}} \left(\Gamma\left(\frac{5}{6}\right) - \Gamma\left(\frac{5}{6}, -\frac{x^6}{6}\right) \right) \right)}{x^5}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 39

```
DSolve[y''[x]+x^5*y'[x]+6*x^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6} e^{-\frac{x^6}{6}} \left(6c_1 x - c_2 \operatorname{ExpIntegralE}\left(\frac{7}{6}, -\frac{x^6}{6}\right) \right)$$

1.490 problem 504

Internal problem ID [7223]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1+3x)\,y''+y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 42

```
dsolve((1+3*x)*diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{-\frac{x}{3}}\,\mathrm{KummerM}\left(-\frac{19}{9},-\frac{1}{9},\frac{x}{3}+\frac{1}{9}\right)+c_2\left(3x^2-17x-6\right)\mathrm{e}^{-\frac{x}{3}}\left(\frac{x}{3}+\frac{1}{9}\right)^{\frac{1}{9}}$$

✓ Solution by Mathematica

Time used: 0.394 (sec). Leaf size: 106

```
DSolve[(1+3*x)*y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to \frac{e^{-\frac{x}{3}-\frac{1}{9}}\left(-\frac{1}{3}2^{8/9}c_2(x-6)(3x+1)^2\,\mathrm{ExpIntegralE}\left(\frac{1}{9},\frac{1}{9}(-3x-1)\right)-2^{8/9}c_2e^{\frac{x}{3}+\frac{1}{9}}(9x^2-48x-26)+1520c_{\text{}}\right.}{380\ 2^{17/18}}$$

1.491 problem 505

Internal problem ID [7224]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 505.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(3x^2+x+1\right)y''+\left(2+15x\right)y'+12y=0}$$

✓ Solution by Maple

Time used: 0.171 (sec). Leaf size: 307

```
dsolve((1+x+3*x^2)*diff(y(x),x$2)+(2+15*x)*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)$$
$$=\frac{c_1\left(i\sqrt{11}-6x-1\right)^{\frac{3}{2}}\left(-36x^2-12x-12\right)^{-\frac{1}{4}+\frac{i\sqrt{11}}{44}}\mathrm{e}^{\frac{\sqrt{11}\arctan\left(\frac{(6x+1)\sqrt{11}}{11}\right)}{22}}\operatorname{hypergeom}\left(\left[\frac{\sqrt{1078+66i\sqrt{11}}}{44}+\frac{1}{2}-\frac{\sqrt{1}}{\left(3x^2+x+1\right)^{\frac{5}{4}}}\right.\right.$$
$$+\frac{c_2\left(i\sqrt{11}+6x+1\right)^{\frac{5}{4}-\frac{i\sqrt{11}}{44}}\left(i\sqrt{11}-6x-1\right)^{\frac{5}{4}+\frac{i\sqrt{11}}{44}}\mathrm{e}^{\frac{\sqrt{11}\arctan\left(\frac{(6x+1)\sqrt{11}}{11}\right)}{22}}\operatorname{hypergeom}\left(\left[\frac{\sqrt{1078+66i\sqrt{11}}}{44}+\frac{1}{2}+\right.\right.}{\left(3x^2+x+1\right)^{\frac{5}{4}}}$$

✓ Solution by Mathematica

Time used: 1.617 (sec). Leaf size: 93

```
DSolve[(1+x+3*x^2)*y''[x]+(2+15*x)*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{xe^{\frac{\arctan\left(\frac{6x+1}{\sqrt{11}}\right)}{\sqrt{11}}}\left(c_2\int_1^x e^{-\frac{\arctan\left(\frac{6K[1]+1}{\sqrt{11}}\right)}{\sqrt{11}}}\frac{\sqrt{3K[1]^2+K[1]+1}}{K[1]^2}dK[1]+c_1\right)}{\left(3x^2+x+1\right)^{3/2}}$$

1.492 problem 506

Internal problem ID [7225]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 506.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+2)\,y'' + (1+x)\,y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 58

```
dsolve((2+x)*diff(y(x),x$2)+(1+x)*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} x(x-4)\,(x+2)^2 + c_2\left(\mathrm{e}^{-2-x} x(x-4)\,(x+2)^2\,\mathrm{Ei}_1\left(-2-x\right) + x^3 - x^2 - 10x - 6\right)$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 64

```
DSolve[(2+x)*y''[x]+(1+x)*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \\ \to \frac{e^{-x-1}((x-4)x(x+2)^2(c_2\,\mathrm{ExpIntegralEi}(x+2)+384c_1) - c_2 e^{x+2}(x((x-1)x-10)-6))}{96\sqrt{2}}$$

1.493 problem 507

Internal problem ID [7226]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 507.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+4)\,y'' + (x+2)\,y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
dsolve((4+x)*diff(y(x),x$2)+(2+x)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} x (x+4)^3 + c_2 \left(\mathrm{e}^{-x-4} x (x+4)^3 \operatorname{Ei}_1(-x-4) + x^3 + 9x^2 + 22x + 6\right)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 54

```
DSolve[(4+x)*y''[x]+(2+x)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{24}\left(e^{-x-4} x (x+4)^3 \left(c_2 \operatorname{ExpIntegralEi}(x+4) + 24 e^4 c_1\right) - c_2 (x(x(x+9)+22)+6)\right)$$

1.494 problem 508

Internal problem ID [7227]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 508.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(2x^2+3x\right)y''+10(1+x)y'+8y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 31

```
dsolve((3*x+2*x^2)*diff(y(x),x$2)+10*(1+x)*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x+2)}{\left(1+\frac{2x}{3}\right)^{\frac{2}{3}}x^{\frac{7}{3}}}+c_2\,\text{hypergeom}\left(\left[2,2\right],\left[\frac{10}{3}\right],-\frac{2x}{3}\right)$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 167

```
DSolve[(3*x+2*x^2)*y''[x]+10*(1+x)*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to \frac{2^{2/3}c_2(x+2)\left(\log\left(2x^{2/3}+2^{2/3}\sqrt[3]{2x+3}\sqrt[3]{x}+\sqrt[3]{2}(2x+3)^{2/3}\right)-2\log\left(2^{2/3}\sqrt[3]{2x+3}-2\sqrt[3]{x}\right)+2\sqrt{3}\cot$$
$$4x^{7/3}(2x+3)^{2/3}$$

1.495 problem 509

Internal problem ID [7228]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 509.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - (6-7x)\,y' + 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(x^2*diff(y(x),x$2)-(6-7*x)*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{6}{x}}(x-2)}{x^5} + \frac{c_2\left(108\,\mathrm{e}^{-\frac{6}{x}}(x-2)\,\mathrm{Ei}_1\left(-\frac{6}{x}\right) + x^3 + 12x^2 - 36x\right)}{x^5}$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 49

```
DSolve[x^2*y''[x]-(6-7*x)*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2e^{-6/x}(x-2)\left(c_1 - 54c_2\,\mathrm{ExpIntegralEi}\left(\frac{6}{x}\right)\right) + c_2 x(x(x+12)-36)}{2x^5}$$

1.496 problem 510

Internal problem ID [7229]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 510.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(2x^2+x+1\right)y''+\left(1+7x\right)y'+2y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 78

```
dsolve((1+x+2*x^2)*diff(y(x),x$2)+(1+7*x)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\text{hypergeom}\left(\left[\frac{1}{2},2\right],\left[\frac{7}{4}-\frac{3i\sqrt{7}}{28}\right],\frac{1}{2}+\frac{i(-1-4x)\sqrt{7}}{14}\right)+c_2\left(i\sqrt{7}+4x+1\right)^{-\frac{3}{4}+\frac{3i\sqrt{7}}{28}}\left(i\sqrt{7}-4x-1\right)^{-\frac{3}{4}-\frac{3i\sqrt{7}}{28}}(x+1)$$

✓ Solution by Mathematica

Time used: 1.038 (sec). Leaf size: 102

```
DSolve[(1+x+2*x^2)*y''[x]+(1+7*x)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{(x+1)e^{\frac{3\arctan\left(\frac{4x+1}{\sqrt{7}}\right)}{2\sqrt{7}}}\left(c_2\int_1^x\frac{e^{-\frac{3\arctan\left(\frac{4K[1]+1}{\sqrt{7}}\right)}{2\sqrt{7}}}}{(K[1]+1)^2\sqrt[4]{2K[1]^2+K[1]+1}}dK[1]+c_1\right)}{\left(2x^2+x+1\right)^{3/4}}$$

1.497 problem 511

Internal problem ID [7230]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 511.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+3)\,y'' + (2x+1)\,y' - (2-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve((3+x)*diff(y(x),x$2)+(1+2*x)*diff(y(x),x)-(2-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 \mathrm{e}^{-x} x (x+6) \left(x^2+9x+27\right) \left(x^2+3x+9\right)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 29

```
DSolve[(3+x)*y''[x]+(1+2*x)*y'[x]-(2-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6} e^{-x-3} \left(c_2 (x+3)^6 + 6 c_1\right)$$

1.498 problem 512

Internal problem ID [7231]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 512.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y'x + \left(2x^2 + 4\right) y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 56

```
dsolve(diff(y(x),x$2)+3*x*diff(y(x),x)+(4+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(-2x\, \mathrm{e}^{-\frac{x^2}{2}} + \operatorname{erfi}\left(\frac{\sqrt{2}\, x}{2} \right) \sqrt{2}\, \sqrt{\pi}\, \mathrm{e}^{-x^2} (x-1)\,(x+1) \right) + c_2 \mathrm{e}^{-x^2} \left(x^2 - 1\right)$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 60

```
DSolve[y''[x]+3*x*y'[x]+(4+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-x^2} \left(\left(x^2 - 1\right) \left(\sqrt{2\pi} c_2 \operatorname{erfi}\left(\frac{x}{\sqrt{2}} \right) + 4c_1 \right) - 2c_2 e^{\frac{x^2}{2}} x \right)$$

1.499 problem 513

Internal problem ID [7232]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 513.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(4x+2)\,y'' - 4y' - (4x+6)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((2+4*x)*diff(y(x),x$2)-4*diff(y(x),x)-(6+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 x\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 29

```
DSolve[(2+4*x)*y''[x]-4*y'[x]-(6+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x-\frac{1}{2}} \left(c_2 e^{2x+1} x + c_1\right)$$

1.500 problem 514

Internal problem ID [7233]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 514.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y'x + (2x^2 + 5)\, y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 83

```
dsolve(diff(y(x),x$2)-3*x*diff(y(x),x)+(5+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(\operatorname{erfi}\left(\frac{\sqrt{2}\, x}{2} \right) \sqrt{\pi}\, \sqrt{2}\, \left(x^6 - 15x^4 + 45x^2 - 15 \right) \mathrm{e}^{\frac{x^2}{2}} - 2x\, \mathrm{e}^{x^2} \left(x^2 - 11 \right) \left(x^2 - 3 \right) \right) + c_2 \mathrm{e}^{\frac{x^2}{2}} \left(x^6 - 15x^4 + 45x^2 - 15 \right)$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 78

```
DSolve[y''[x]-3*x*y'[x]+(5+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\frac{x^2}{2}} \left(x^6 - 15x^4 + 45x^2 - 15 \right) \left(\sqrt{2\pi} c_2 \operatorname{erfi}\left(\frac{x}{\sqrt{2}} \right) + 1440 c_1 \right) - 2c_2 e^{x^2} x (x^4 - 14x^2 + 33)}{1440}$$

1.501 problem 515

Internal problem ID [7234]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 515.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y'' + 5y'x + \left(2x^2 + 4\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(2*diff(y(x),x$2)+5*x*diff(y(x),x)+(4+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} \operatorname{erf}\left(\frac{i\sqrt{3}\, x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 42

```
DSolve[2*y''[x]+5*x*y'[x]+(4+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{-x^2} \left(\sqrt{3\pi} c_2 \operatorname{erfi}\left(\frac{\sqrt{3}x}{2}\right) + 3c_1\right)$$

1.502 problem 516

Internal problem ID [7235]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 516.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y'x + \left(4x^2 + 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(2+4*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 20

```
DSolve[y''[x]+4*x*y'[x]+(2+4*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2} (c_2 x + c_1)$$

1.503 problem 517

Internal problem ID [7236]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 517.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y'x + \left(4x^2 + 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(2+4*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} x$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 20

```
DSolve[y''[x]+4*x*y'[x]+(2+4*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2} (c_2 x + c_1)$$

1.504 problem 518

Internal problem ID [7237]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 518.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2\left(x^2+x+1\right)y''+x\left(11x^2+11x+9\right)y'+\left(7x^2+10x+6\right)y=0}$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 362

```
dsolve(2*x^2*(1+x+x^2)*diff(y(x),x$2)+x*(9+11*x+11*x^2)*diff(y(x),x)+(6+10*x+7*x^2)*y(x)=0,y(
```

$$y(x)$$
$$= \frac{c_1 \mathrm{e}^{-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}} \operatorname{HeunG}\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}},0,0,\frac{5}{2},\frac{1}{2},\frac{5i\sqrt{3}-3}{3i\sqrt{3}-3},-\frac{2x}{1+i\sqrt{3}}\right)\left(i\sqrt{3}+2x+1\right)^{\frac{5i\sqrt{3}-3}{-6+6i\sqrt{3}}}\left(i\sqrt{3}-2x-1\right)^{\overline{(1+i}}}{\left(x^2+x+1\right)^{\frac{1}{4}}x^2}$$
$$+\frac{c_2 \mathrm{e}^{-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}} \operatorname{HeunG}\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}},\frac{16}{\left(1+i\sqrt{3}\right)^3\left(i\sqrt{3}-1\right)^2},\frac{1}{2},3,\frac{3}{2},\frac{5i\sqrt{3}-3}{3i\sqrt{3}-3},-\frac{2x}{1+i\sqrt{3}}\right)\left(i\sqrt{3}+2x+1\right)^{\frac{5i\sqrt{3}-3}{-6+6i\sqrt{3}}}\left(}{\left(x^2+x+1\right)^{\frac{1}{4}}x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.204 (sec). Leaf size: 93

```
DSolve[2*x^2*(1+x+x^2)*y''[x]+x*(9+11*x+11*x^2)*y'[x]+(6+10*x+7*x^2)*y[x]==0,y[x],x,IncludeSi
```

$$y(x)\to \frac{\sqrt{x^2+x+1}e^{-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}}\left(c_2\int_1^x \frac{e^{\frac{\arctan\left(\frac{2K[1]+1}{\sqrt{3}}\right)}{\sqrt{3}}}}{\sqrt{K[1]}\left(K[1]^2+K[1]+1\right)^{3/2}}dK[1]+c_1\right)}{x^2}$$

1.505 problem 519

Internal problem ID [7238]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 519.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$\boxed{3x^2y'' + 2x\left(-2x^2 + x + 1\right)y' + \left(-8x^2 + 2x\right)y = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 38

```
dsolve(3*x^2*diff(y(x),x$2)+2*x*(1+x-2*x^2)*diff(y(x),x)+(2*x-8*x^2)*y(x)=0,y(x), singsol=all
```

$$y(x) = c_1 x^{\frac{1}{3}} \mathrm{e}^{\frac{2x(x-1)}{3}} + c_2 \,\mathrm{HeunB}\left(-\frac{1}{3}, \frac{\sqrt{6}}{3}, -\frac{7}{3}, \frac{4\sqrt{6}}{9}, -\frac{x\sqrt{6}}{3}\right)$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 53

```
DSolve[3*x^2*y''[x]+2*x*(1+x-2*x^2)*y'[x]+(2*x-8*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to e^{\frac{2}{3}(x-1)x} \sqrt[3]{x}\left(c_2 \int_1^x \frac{e^{-\frac{2}{3}(K[1]-1)K[1]}}{K[1]^{4/3}} dK[1] + c_1\right)$$

1.506 problem 520

Internal problem ID [7239]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 520.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{12x^2(1+x)\,y'' + x\left(3x^2+35x+11\right)y' - \left(-5x^2-10x+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 51

```
dsolve(12*x^2*(1+x)*diff(y(x),x$2)+x*(11+35*x+3*x^2)*diff(y(x),x)-(1-10*x-5*x^2)*y(x)=0,y(x),
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{x}{4}}\,\mathrm{HeunC}\left(\frac{1}{4},-\frac{7}{12},-\frac{3}{4},-\frac{1}{12},\frac{1}{2},-x\right)}{(x+1)^{\frac{3}{4}}\,x^{\frac{1}{4}}} + \frac{c_2 \mathrm{e}^{-\frac{x}{4}}\,\mathrm{HeunC}\left(\frac{1}{4},\frac{7}{12},-\frac{3}{4},-\frac{1}{12},\frac{1}{2},-x\right)x^{\frac{1}{3}}}{(x+1)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 61

```
DSolve[12*x^2*(1+x)*y''[x]+x*(11+35*x+3*x^2)*y'[x]-(1-10*x-5*x^2)*y[x]==0,y[x],x,IncludeSingu
```

$$y(x) \to \frac{e^{-x/4}\left(c_2 \int_1^x \frac{e^{\frac{K[1]}{4}}}{K[1]^{5/12}\sqrt[4]{K[1]+1}}dK[1] + c_1\right)}{\sqrt[4]{x}(x+1)^{3/4}}$$

1.507 problem 521

Internal problem ID [7240]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 521.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{3x}{2}} \sin\left(\frac{\sqrt{7}\,x}{2}\right) + c_2 \mathrm{e}^{-\frac{3x}{2}} \cos\left(\frac{\sqrt{7}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 42

```
DSolve[y''[x]+3*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x/2}\left(c_2 \cos\left(\frac{\sqrt{7}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{7}x}{2}\right)\right)$$

1.508 problem 522

Internal problem ID [7241]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 522.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{18x^2(1+x)\,y'' + 3x\left(x^2+11x+5\right)y' - \left(-5x^2-2x+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 41

```
dsolve(18*x^2*(1+x)*diff(y(x),x$2)+3*x*(5+11*x+x^2)*diff(y(x),x)-(1-2*x-5*x^2)*y(x)=0,y(x), s
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{x}{6}}\,\mathrm{HeunC}\left(\frac{1}{6},-\frac{1}{2},-\frac{1}{6},-\frac{5}{36},\frac{1}{4},-x\right)}{x^{\frac{1}{6}}} + c_2 \mathrm{e}^{-\frac{x}{6}}\,\mathrm{HeunC}\left(\frac{1}{6},\frac{1}{2},-\frac{1}{6},-\frac{5}{36},\frac{1}{4},-x\right)x^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 60

```
DSolve[18*x^2*(1+x)*y''[x]+3*x*(5+11*x+x^2)*y'[x]-(1-2*x-5*x^2)*y[x]==0,y[x],x,IncludeSingula
```

$$y(x) \to \frac{e^{-x/6}\left(c_2 \int_1^x \frac{e^{\frac{K[1]}{6}}}{\sqrt{K[1]}(K[1]+1)^{7/6}}dK[1] + c_1\right)}{\sqrt[6]{\frac{x}{x+1}}}$$

1.509 problem 523

Internal problem ID [7242]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 523.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + x(3+2x)\,y' - (1-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve(2*x^2*diff(y(x),x$2)+x*(3+2*x)*diff(y(x),x)-(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\,\text{WhittakerM}\left(-\frac{1}{4},\frac{3}{4},x\right)\mathrm{e}^{-\frac{x}{2}}}{x^{\frac{3}{4}}} + \frac{c_2\mathrm{e}^{-x}}{x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 33

```
DSolve[2*x^2*y''[x]+x*(3+2*x)*y'[x]-(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}\left(c_2x^{3/2}L^{\frac{3}{2}}_{-\frac{3}{2}}(x) + c_1\right)}{x}$$

1.510 problem 524

Internal problem ID [7243]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 524.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + x(x+5)\,y' - (2-3x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(2*x^2*diff(y(x),x$2)+x*(5+x)*diff(y(x),x)-(2-3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\mathrm{e}^{-\frac{x}{2}} + \frac{c_2\left(i\mathrm{e}^{-\frac{x}{2}}x^{\frac{5}{2}}\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\frac{i\sqrt{2}\sqrt{x}}{2}\right) + 2x^2 + 2x + 6\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 59

```
DSolve[2*x^2*y''[x]+x*(5+x)*y'[x]-(2-3*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{15}c_2 e^{-x/2}x\,\text{ExpIntegralE}\left(\frac{1}{2},-\frac{x}{2}\right) - \frac{2c_2(x^2+x+3)}{15x^2} + c_1 e^{-x/2}\sqrt{x}$$

1.511 problem 525

Internal problem ID [7244]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 525.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2y'' + x(1+x)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 29

```
dsolve(3*x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \,\text{WhittakerM}\left(-\frac{1}{6}, \frac{2}{3}, \frac{x}{3}\right) \mathrm{e}^{-\frac{x}{6}}}{x^{\frac{1}{6}}} + \frac{c_2 \mathrm{e}^{-\frac{x}{3}}}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 39

```
DSolve[3*x^2*y''[x]+x*(1+x)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x/3}\left(c_2 - c_1 x^{4/3}\,\text{ExpIntegralE}\left(-\frac{1}{3}, -\frac{x}{3}\right)\right)}{\sqrt[3]{x}}$$

1.512 problem 526

Internal problem ID [7245]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 526.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - y'x + (1-2x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+(1-2*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sinh\left(2\sqrt{x}\right) + c_2\sqrt{x}\,\cosh\left(2\sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 41

```
DSolve[2*x^2*y''[x]-x*y'[x]+(1-2*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-2\sqrt{x}}\sqrt{x}\left(2c_1e^{4\sqrt{x}} - c_2\right)$$

1.513 problem 527

Internal problem ID [7246]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 527.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2y'' + x(1+x)\,y' - (1+3x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(3*x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-(1+3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\left(x^2 + 20x + 70\right) + \frac{c_2 \mathrm{e}^{-\frac{x}{3}}\,\mathrm{hypergeom}\left([3], \left[-\frac{1}{3}\right], \frac{x}{3}\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 70

```
DSolve[3*x^2*y''[x]+x*(1+x)*y'[x]-(1+3*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 e^{-x/3}(x(x(x+19)+54)-18)}{1680\sqrt[3]{x}} + \frac{x(x(x+20)+70)\left(5040c_1 - 3^{2/3}c_2\Gamma\left(\frac{2}{3}, \frac{x}{3}\right)\right)}{5040}$$

1.514 problem 528

Internal problem ID [7247]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 528.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x+3)\,y'' + x(5x+1)\,y' + (1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve(2*x^2*(3+x)*diff(y(x),x$2)+x*(1+5*x)*diff(y(x),x)+(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\ \text{hypergeom}\left(\left[1, \frac{3}{2}\right], \left[\frac{7}{6}\right], -\frac{x}{3}\right) + \frac{c_2 x^{\frac{1}{3}}}{(3+x)\left(\frac{x}{3}+1\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 50

```
DSolve[2*x^2*(3+x)*y''[x]+x*(1+5*x)*y'[x]+(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{\sqrt[3]{x}\left(6\sqrt[3]{3}c_2\sqrt[6]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{6}, \frac{7}{6}, -\frac{x}{3}\right) + c_1\right)}{(x+3)^{4/3}}$$

1.515 problem 529

Internal problem ID [7248]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 529.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+4)\,y'' - x(-3x+1)\,y' + y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 27

```
dsolve(x^2*(4+x)*diff(y(x),x$2)-x*(1-3*x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^{\frac{1}{4}}}{(x+4)^{\frac{9}{4}}} + c_2\,\text{hypergeom}\left([1,3],\left[\frac{7}{4}\right],-\frac{x}{4}\right)x$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 87

```
DSolve[x^2*(4+x)*y''[x]-x*(1-3*x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{10c_2\sqrt[4]{x}\left(\text{arctanh}\left(\sqrt[4]{\frac{x}{x+4}}\right) - \arctan\left(\sqrt[4]{\frac{x}{x+4}}\right)\right) + c_2\sqrt[4]{x+4}x^2 + 9c_2\sqrt[4]{x+4}x + 2c_1\sqrt[4]{x}}{2(x+4)^{9/4}}$$

1.516 problem 530

Internal problem ID [7249]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 530.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + 5y'x + (1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\sqrt{x}\,\sqrt{2}\right)}{x} + \frac{c_2 \cos\left(\sqrt{x}\,\sqrt{2}\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 60

```
DSolve[2*x^2*y''[x]+5*x*y'[x]+(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{i\sqrt{2}\sqrt{x}} + i\sqrt{2}c_2 e^{-i\sqrt{2}\sqrt{x}}}{2x}$$

1.517 problem 531

Internal problem ID [7250]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 531.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{6x^2y'' + x(10-x)\,y' - (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 25

```
dsolve(6*x^2*diff(y(x),x$2)+x*(10-x)*diff(y(x),x)-(2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \,\text{WhittakerM}\left(-\frac{1}{6}, \frac{2}{3}, \frac{x}{6}\right) \mathrm{e}^{\frac{x}{12}}}{x^{\frac{5}{6}}} + \frac{c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 38

```
DSolve[6*x^2*y''[x]+x*(10-x)*y'[x]-(2+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sqrt[3]{x} L_{-\frac{4}{3}}^{\frac{4}{3}}\left(\frac{x}{6}\right) + \frac{6\sqrt[3]{6}c_1}{x}$$

1.518 problem 532

Internal problem ID [7251]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 532.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(4x+3)\,y'' + x(11+4x)\,y' - (4x+3)\,y = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 41

```
dsolve(x^2*(3+4*x)*diff(y(x),x$2)+x*(11+4*x)*diff(y(x),x)-(3+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(48x^2+32x+7)}{x^3} + c_2\,\text{hypergeom}\left([3,5],\left[\frac{13}{3}\right],-\frac{4x}{3}\right)(4x+3)^{\frac{11}{3}}\,x^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 171

```
DSolve[x^2*(3+4*x)*y''[x]+x*(11+4*x)*y'[x]-(3+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{-6\sqrt[3]{2}c_2(16x(3x+2)+7)\left(\log\left(4x^{2/3}+2\sqrt[3]{2}\sqrt[3]{4x+3}\sqrt[3]{x}+(8x+6)^{2/3}\right)-2\log\left(\sqrt[3]{8x+6}-2\sqrt[3]{x}\right)+2\right.}{48x^3}$$

1.519 problem 533

Internal problem ID [7252]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 533.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(3x+2)\,y'' + x(4+11x)\,y' - (1-x)\,y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 22

```
dsolve(2*x^2*(2+3*x)*diff(y(x),x$2)+x*(4+11*x)*diff(y(x),x)-(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2(3x+2)^{\frac{1}{6}}}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 32

```
DSolve[2*x^2*(2+3*x)*y''[x]+x*(4+11*x)*y'[x]-(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{c_2\sqrt[6]{6x+4} + 2^{5/6}c_1}{\sqrt{x}}$$

1.520 problem 534

Internal problem ID [7253]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 534.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+2)\,y'' + 5x(1-x)\,y' - (-8x+2)\,y = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 116

```
dsolve(x^2*(2+x)*diff(y(x),x$2)+5*x*(1-x)*diff(y(x),x)-(2-8*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(40x^4 - 160x^3 + 60x^2 + 8x + 1)}{x^2} + \frac{c_2\left(x^{\frac{3}{2}}(40x^4 - 160x^3 + 60x^2 + 8x + 1)\operatorname{arcsinh}\left(\frac{\sqrt{x}\sqrt{2}}{2}\right) + \frac{\sqrt{x+2}\,x^2(8x^5+328x^4-13974x^3+26734x^2-805x-105)}{210}\right)(-}{(x+2)^{\frac{3}{4}}\,x^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 20.789 (sec). Leaf size: 1347

```
DSolve[x^2*(2+x)*y''[x]+5*x*(1-x)*y'[x]-(2-8*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

Too large to display

1.521 problem 535

Internal problem ID [7254]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 535.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{8x^2\left(1-x^2\right)y''+2x\left(-13x^2+1\right)y'+\left(-9x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 41

```
dsolve(8*x^2*(1-x^2)*diff(y(x),x$2)+2*x*(1-13*x^2)*diff(y(x),x)+(1-9*x^2)*y(x)=0,y(x), singso
```

$$y(x)=\frac{c_1x^{\frac{1}{4}}}{\sqrt{x^2-1}}+\frac{c_2x^{\frac{3}{8}}\,\text{LegendreQ}\left(-\frac{1}{8},\frac{1}{8},\sqrt{-x^2+1}\right)}{\sqrt{x^2-1}}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 47

```
DSolve[8*x^2*(1-x^2)*y''[x]+2*x*(1-13*x^2)*y'[x]+(1-9*x^2)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x)\to\frac{\sqrt[4]{x}\left(4c_2\sqrt[4]{x}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{9}{8},x^2\right)+c_1\right)}{\sqrt{1-x^2}}$$

1.522 problem 536

Internal problem ID [7255]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 536.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''-2x\left(-x^2+2\right)y'+4y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-2*x*(2-x^2)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^4}{\left(x^2+1\right)^2}+\frac{c_2(3x^3+x)}{\left(x^2+1\right)^2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 35

```
DSolve[x^2*(1+x^2)*y''[x]-2*x*(2-x^2)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to -\frac{-3c_1x^4+3c_2x^3+c_2x}{3\left(x^2+1\right)^2}$$

1.523 problem 537

Internal problem ID [7256]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 537.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x\left(x^2+3\right)y''+\left(-x^2+2\right)y'-8yx=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(x*(3+x^2)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)-8*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x^{\frac{1}{3}}\left(x^2+3\right)^{\frac{11}{6}}+c_2\left(x^4+\frac{11}{2}x^2+\frac{55}{8}\right)$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 41

```
DSolve[x*(3+x^2)*y''[x]+(2-x^2)*y'[x]-8*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\sqrt[3]{x}\left(x^2+3\right)^{11/6}-\frac{1}{55}c_2\left(8x^4+44x^2+55\right)$$

1.524 problem 538

Internal problem ID [7257]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 538.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2\left(1-x^2\right)y''+x\left(-19x^2+7\right)y'-\left(14x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 53

```
dsolve(4*x^2*(1-x^2)*diff(y(x),x$2)+x*(7-19*x^2)*diff(y(x),x)-(1+14*x^2)*y(x)=0,y(x), singsol
```

$$y(x)=\frac{c_1\,\text{LegendreP}\left(-\frac{3}{8},\frac{5}{8},\sqrt{-x^2+1}\right)}{x^{\frac{3}{8}}\sqrt{x^2-1}}+\frac{c_2\,\text{LegendreQ}\left(-\frac{3}{8},\frac{5}{8},\sqrt{-x^2+1}\right)}{x^{\frac{3}{8}}\sqrt{x^2-1}}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 50

```
DSolve[4*x^2*(1-x^2)*y''[x]+x*(7-19*x^2)*y'[x]-(1+14*x^2)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x)\to\frac{4c_2x^{5/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{13}{8},x^2\right)+5c_1}{5x\sqrt{1-x^2}}$$

1.525 problem 539

Internal problem ID [7258]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 539.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2\left(-x^2+2\right)y''+x\left(-11x^2+1\right)y'+\left(-5x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 35

```
dsolve(3*x^2*(2-x^2)*diff(y(x),x$2)+x*(1-11*x^2)*diff(y(x),x)+(1-5*x^2)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1\sqrt{x}}{\left(-2x^2+4\right)^{\frac{3}{4}}}+c_2x^{\frac{1}{3}}\,\text{hypergeom}\left(\left[\frac{2}{3},1\right],\left[\frac{11}{12}\right],\frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 57

```
DSolve[3*x^2*(2-x^2)*y''[x]+x*(1-11*x^2)*y'[x]+(1-5*x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to\frac{c_1\sqrt{x}-3\ 2^{3/4}c_2\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{12},\frac{1}{4},\frac{11}{12},\frac{x^2}{2}\right)}{\left(2-x^2\right)^{3/4}}$$

1.526 problem 540

Internal problem ID [7259]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 540.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2\left(x^2+2\right)y''-x\left(-7x^2+12\right)y'+\left(3x^2+7\right)y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 42

```
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)-x*(12-7*x^2)*diff(y(x),x)+(7+3*x^2)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1x^{\frac{7}{2}}}{\left(x^2+2\right)^2\left(2x^2+4\right)^{\frac{1}{4}}}+c_2\sqrt{x}\ \text{hypergeom}\left(\left[\frac{3}{4},1\right],\left[-\frac{1}{2}\right],-\frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 57

```
DSolve[2*x^2*(2+x^2)*y''[x]-x*(12-7*x^2)*y'[x]+(7+3*x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to\frac{\sqrt{x}\left(3c_1x^3-2\sqrt[4]{2}c_2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{2},-\frac{x^2}{2}\right)\right)}{3\left(x^2+2\right)^{9/4}}$$

1.527 problem 541

Internal problem ID [7260]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 541.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2\left(x^2+2\right)y''+x\left(7x^2+4\right)y'-\left(-3x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 39

```
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)+x*(4+7*x^2)*diff(y(x),x)-(1-3*x^2)*y(x)=0,y(x), singsol=a
```

$$y(x)=\frac{c_1}{\sqrt{x}\,\left(x^2+2\right)^{\frac{1}{4}}}+\frac{c_2\,\mathrm{LegendreQ}\left(-\frac{1}{4},\frac{1}{4},\frac{i\sqrt{2}\,x}{2}\right)}{\sqrt{x}\,\left(x^2+2\right)^{\frac{1}{8}}}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 68

```
DSolve[2*x^2*(2+x^2)*y''[x]+x*(4+7*x^2)*y'[x]-(1-3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to\frac{c_2\sqrt[8]{x^2+2}\,\Gamma\left(\frac{3}{4}\right)Q^{\frac{1}{4}}_{-\frac{1}{4}}\left(\frac{ix}{\sqrt{2}}\right)+2^{3/8}c_1}{\sqrt{x}\sqrt[4]{x^2+2}\,\Gamma\left(\frac{3}{4}\right)}$$

1.528 problem 542

Internal problem ID [7261]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 542.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2\left(2x^2+1\right)y''+5x\left(6x^2+1\right)y'-\left(-40x^2+2\right)y=0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 35

```
dsolve(2*x^2*(1+2*x^2)*diff(y(x),x$2)+5*x*(1+6*x^2)*diff(y(x),x)-(2-40*x^2)*y(x)=0,y(x), sing
```

$$y(x)=\frac{c_1\sqrt{x}}{\left(2x^2+1\right)^{\frac{3}{2}}}+\frac{c_2\,\text{hypergeom}\left(\left[\frac{1}{4},1\right],\left[-\frac{1}{4}\right],-2x^2\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 52

```
DSolve[2*x^2*(1+2*x^2)*y''[x]+5*x*(1+6*x^2)*y'[x]-(2-40*x^2)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x)\to\frac{5c_1x^{5/2}-2c_2\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-2x^2\right)}{5x^2\left(2x^2+1\right)^{3/2}}$$

1.529 problem 543

Internal problem ID [7262]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 543.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x\left(x^2+1\right)y''+\left(7x^2+4\right)y'+8yx=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 42

```
dsolve(x*(1+x^2)*diff(y(x),x$2)+(4+7*x^2)*diff(y(x),x)+8*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{\sqrt{x^2+1}\,x^3}+\frac{c_2\left(-x\sqrt{x^2+1}+\operatorname{arcsinh}\left(x\right)\right)}{\sqrt{x^2+1}\,x^3}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 45

```
DSolve[x*(1+x^2)*y''[x]+(4+7*x^2)*y'[x]+8*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-c_2\text{arcsinh}(x)+c_2x\sqrt{x^2+1}+2c_1}{2x^3\sqrt{x^2+1}}$$

1.530 problem 544

Internal problem ID [7263]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 544.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2\left(x^2+1\right)y''+x\left(8x^2+3\right)y'-\left(-4x^2+3\right)y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve(2*x^2*(1+x^2)*diff(y(x),x$2)+x*(3+8*x^2)*diff(y(x),x)-(3-4*x^2)*y(x)=0,y(x), singsol=a
```

$$y(x)=c_1 x\,\text{hypergeom}\left(\left[1,\frac{3}{2}\right],\left[\frac{9}{4}\right],-x^2\right)+\frac{c_2}{\left(x^2+1\right)^{\frac{1}{4}}x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 49

```
DSolve[2*x^2*(1+x^2)*y''[x]+x*(3+8*x^2)*y'[x]-(3-4*x^2)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to -\frac{c_2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1,\frac{5}{4},-x^2\right)}{x}+\frac{c_1}{x^{3/2}\sqrt[4]{x^2+1}}+\frac{c_2}{x}$$

1.531 problem 545

Internal problem ID [7264]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 545.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' + 3x\left(x^2+3\right)y' - \left(-5x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve(9*x^2*diff(y(x),x$2)+3*x*(3+x^2)*diff(y(x),x)-(1-5*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{WhittakerM}\left(\frac{1}{3}, \frac{1}{6}, \frac{x^2}{6}\right) \mathrm{e}^{-\frac{x^2}{12}}}{x} + \frac{c_2 \mathrm{e}^{-\frac{x^2}{6}}}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 47

```
DSolve[9*x^2*y''[x]+3*x*(3+x^2)*y'[x]-(1-5*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{e^{-\frac{x^2}{6}}\left(c_2 x^{2/3} \operatorname{ExpIntegralE}\left(\frac{2}{3}, -\frac{x^2}{6}\right) - 2c_1\right)}{2\sqrt[3]{x}}$$

1.532 problem 546

Internal problem ID [7265]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 546.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{6x^2y'' + x\left(6x^2+1\right)y' + \left(9x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve(6*x^2*diff(y(x),x$2)+x*(1+6*x^2)*diff(y(x),x)+(1+9*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{WhittakerM}\left(\frac{11}{24}, \frac{1}{24}, \frac{x^2}{2}\right) \mathrm{e}^{-\frac{x^2}{4}}}{x^{\frac{7}{12}}} + c_2 x^{\frac{1}{3}} \mathrm{e}^{-\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 47

```
DSolve[6*x^2*y''[x]+x*(1+6*x^2)*y'[x]+(1+9*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{1}{2} e^{-\frac{x^2}{2}} \sqrt[3]{x} \left(c_2 \sqrt[6]{x} \operatorname{ExpIntegralE}\left(\frac{11}{12}, -\frac{x^2}{2}\right) - 2c_1 \right)$$

1.533 problem 547

Internal problem ID [7266]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 547.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2\left(x^2+1\right)y''+3x\left(13x^2+3\right)y'-\left(-25x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 33

```
dsolve(9*x^2*(1+x^2)*diff(y(x),x$2)+3*x*(3+13*x^2)*diff(y(x),x)-(1-25*x^2)*y(x)=0,y(x), sings
```

$$y(x)=\frac{c_1}{\left(x^2+1\right)^{\frac{2}{3}}x^{\frac{1}{3}}}+c_2x^{\frac{1}{3}}\,\text{hypergeom}\left(\left[1,1\right],\left[\frac{4}{3}\right],-x^2\right)$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 115

```
DSolve[9*x^2*(1+x^2)*y''[x]+3*x*(3+13*x^2)*y'[x]-(1-25*x^2)*y[x]==0,y[x],x,IncludeSingularSol
```

$$y(x)$$

$$\to \frac{c_2\left(-2\log\left(\sqrt[3]{x^2+1}-x^{2/3}\right)+\log\left(x^{4/3}+\left(x^2+1\right)^{2/3}+\sqrt[3]{x^2+1}x^{2/3}\right)+2\sqrt{3}\cot^{-1}\left(\frac{\frac{2\sqrt[3]{x^2+1}}{x^{2/3}}+1}{\sqrt{3}}\right)\right)}{4\sqrt[3]{x}\left(x^2+1\right)^{2/3}}$$

1.534 problem 548

Internal problem ID [7267]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 548.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2\left(x^2+1\right)y''+4x\left(6x^2+1\right)y'-\left(-25x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 43

```
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+4*x*(1+6*x^2)*diff(y(x),x)-(1-25*x^2)*y(x)=0,y(x), singso
```

$$y(x)=\frac{c_1\sqrt{x}}{\left(x^2+1\right)^{\frac{3}{2}}}+\frac{c_2\left(\operatorname{arcsinh}\left(x\right)x-\sqrt{x^2+1}\right)}{\sqrt{x}\,\left(x^2+1\right)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 44

```
DSolve[4*x^2*(1+x^2)*y''[x]+4*x*(1+6*x^2)*y'[x]-(1-25*x^2)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x)\to\frac{c_2x\operatorname{arcsinh}(x)-c_2\sqrt{x^2+1}+c_1x}{\sqrt{x}\left(x^2+1\right)^{3/2}}$$

1.535 problem 549

Internal problem ID [7268]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 549.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{8x^2\left(2x^2+1\right)y''+2x\left(34x^2+5\right)y'-\left(-30x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 57

```
dsolve(8*x^2*(1+2*x^2)*diff(y(x),x$2)+2*x*(5+34*x^2)*diff(y(x),x)-(1-30*x^2)*y(x)=0,y(x), sin
```

$$y(x)=\frac{c_1\,\mathrm{LegendreP}\left(\frac{3}{8},\frac{3}{8},\sqrt{2x^2+1}\right)}{x^{\frac{1}{8}}\sqrt{2x^2+1}}+\frac{c_2\,\mathrm{LegendreQ}\left(\frac{3}{8},\frac{3}{8},\sqrt{2x^2+1}\right)}{x^{\frac{1}{8}}\sqrt{2x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 54

```
DSolve[8*x^2*(1+2*x^2)*y''[x]+2*x*(5+34*x^2)*y'[x]-(1-30*x^2)*y[x]==0,y[x],x,IncludeSingularS
```

$$y(x)\to\frac{3c_1x^{3/4}-4c_2\,\mathrm{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{5}{8},-2x^2\right)}{3\sqrt{x}\sqrt{2x^2+1}}$$

1.536 problem 550

Internal problem ID [7269]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 550.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(1+x)\,y'' - x(-3x+1)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(2*x^2*(1+x)*diff(y(x),x$2)-x*(1-3*x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{x+1} + \frac{c_2\sqrt{x}}{x+1}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 25

```
DSolve[2*x^2*(1+x)*y''[x]-x*(1-3*x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1\sqrt{x} + 2c_2 x}{x+1}$$

1.537 problem 551

Internal problem ID [7270]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 551.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{6x^2\left(2x^2+1\right)y''+x\left(50x^2+1\right)y'+\left(30x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 33

```
dsolve(6*x^2*(1+2*x^2)*diff(y(x),x$2)+x*(1+50*x^2)*diff(y(x),x)+(1+30*x^2)*y(x)=0,y(x), sings
```

$$y(x)=\frac{c_1\sqrt{x}}{2x^2+1}+\frac{c_2x^{\frac{1}{3}}}{2x^2+1}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 32

```
DSolve[6*x^2*(1+2*x^2)*y''[x]+x*(1+50*x^2)*y'[x]+(1+30*x^2)*y[x]==0,y[x],x,IncludeSingularSol
```

$$y(x)\to\frac{\sqrt[3]{x}\left(6c_2\sqrt[6]{x}+c_1\right)}{2x^2+1}$$

1.538 problem 552

Internal problem ID [7271]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 552.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{28x^2(-3x+1)\,y'' - 7x(5+9x)\,y' + 7(2+9x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(28*x^2*(1-3*x)*diff(y(x),x$2)-7*x*(5+9*x)*diff(y(x),x)+7*(2+9*x)*y(x)=0,y(x), singsol=
```

$$y(x) = \frac{c_1 x^2}{3x-1} + \frac{c_2 x^{\frac{1}{4}}}{3x-1}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 30

```
DSolve[28*x^2*(1-3*x)*y''[x]-7*x*(5+9*x)*y'[x]+7*(2+9*x)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{4c_2 x^2 + 7c_1 \sqrt[4]{x}}{7-21x}$$

1.539 problem 553

Internal problem ID [7272]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 553.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{8x^2\left(-x^2+2\right)y''+2x\left(-21x^2+10\right)y'-\left(35x^2+2\right)y=0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 29

```
dsolve(8*x^2*(2-x^2)*diff(y(x),x$2)+2*x*(10-21*x^2)*diff(y(x),x)-(2+35*x^2)*y(x)=0,y(x), sing
```

$$y(x)=\frac{c_1}{\sqrt{x}\,\left(x^2-2\right)}+\frac{c_2x^{\frac{1}{4}}}{x^2-2}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 34

```
DSolve[8*x^2*(2-x^2)*y''[x]+2*x*(10-21*x^2)*y'[x]-(2+35*x^2)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x)\to\frac{\frac{3c_1}{\sqrt{x}}+4c_2\sqrt[4]{x}}{6-3x^2}$$

1.540 problem 554

Internal problem ID [7273]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 554.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2\left(x^2+3x+1\right)y''-4x\left(-3x^2-3x+1\right)y'+3\left(x^2-x+1\right)y=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve(4*x^2*(1+3*x+x^2)*diff(y(x),x$2)-4*x*(1-3*x-3*x^2)*diff(y(x),x)+3*(1-x+x^2)*y(x)=0,y(x
```

$$y(x)=\frac{c_1\sqrt{x}}{x^2+3x+1}+\frac{c_2x^{\frac{3}{2}}}{x^2+3x+1}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 27

```
DSolve[4*x^2*(1+3*x+x^2)*y''[x]-4*x*(1-3*x-3*x^2)*y'[x]+3*(1-x+x^2)*y[x]==0,y[x],x,IncludeSin
```

$$y(x)\to\frac{\sqrt{x}(c_2x+c_1)}{x(x+3)+1}$$

1.541 problem 555

Internal problem ID [7274]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 555.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2(1+x)^2 y'' - x\left(-11x^2-10x+1\right) y' + \left(5x^2+1\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(3*x^2*(1+x)^2*diff(y(x),x$2)-x*(1-10*x-11*x^2)*diff(y(x),x)+(1+5*x^2)*y(x)=0,y(x), sin
```

$$y(x) = \frac{c_1 x}{(x+1)^2} + \frac{c_2 x^{\frac{1}{3}}}{(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 29

```
DSolve[3*x^2*(1+x)^2*y''[x]-x*(1-10*x-11*x^2)*y'[x]+(1+5*x^2)*y[x]==0,y[x],x,IncludeSingularS
```

$$y(x) \to \frac{2c_1 \sqrt[3]{x} + 3c_2 x}{2(x+1)^2}$$

1.542 problem 556

Internal problem ID [7275]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 556.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2\left(x^2+2x+3\right)y''-x\left(-15x^2-14x+3\right)y'+\left(7x^2+3\right)y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(4*x^2*(3+2*x+x^2)*diff(y(x),x$2)-x*(3-14*x-15*x^2)*diff(y(x),x)+(3+7*x^2)*y(x)=0,y(x),
```

$$y(x)=\frac{c_1x}{x^2+2x+3}+\frac{c_2x^{\frac{1}{4}}}{x^2+2x+3}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 31

```
DSolve[4*x^2*(3+2*x+x^2)*y''[x]-x*(3-14*x-15*x^2)*y'[x]+(3+7*x^2)*y[x]==0,y[x],x,IncludeSingu
```

$$y(x)\to\frac{3c_1\sqrt[4]{x}+4c_2x}{3x(x+2)+9}$$

1.543 problem 557

Internal problem ID [7276]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 557.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2-2x+1\right)y''-x(x+3)y'+(x+4)y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(x^2*(1-2*x+x^2)*diff(y(x),x$2)-x*(3+x)*diff(y(x),x)+(4+x)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^2\mathrm{e}^{-\frac{4}{x-1}}}{x-1}+\frac{c_2x^2\,\mathrm{Ei}_1\left(-\frac{4x}{x-1}\right)\mathrm{e}^{-\frac{4x}{x-1}}}{x-1}$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 54

```
DSolve[x^2*(1-2*x+x^2)*y''[x]-x*(3+x)*y'[x]+(4+x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{e^{-\frac{4x}{x-1}}\sqrt{1-x}x^2\left(c_2\,\mathrm{ExpIntegralEi}\left(\frac{4x}{x-1}\right)+e^4c_1\right)}{(x-1)^{3/2}}$$

1.544 problem 558

Internal problem ID [7277]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 558.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x+2)\,y'' + 5y'x^2 + (1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 48

```
dsolve(2*x^2*(2+x)*diff(y(x),x$2)+5*x^2*diff(y(x),x)+(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\sqrt{x}}{(x+2)^{\frac{3}{2}}} + \frac{c_2\sqrt{x}\left(\sqrt{2}\,\sqrt{x+2} - 2\,\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x+2}}{2}\right)\right)}{(x+2)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 55

```
DSolve[2*x^2*(2+x)*y''[x]+5*x^2*y'[x]+(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{x}\left(-2\sqrt{2}c_2\operatorname{arctanh}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right) + 2c_2\sqrt{x+2} + c_1\right)}{(x+2)^{3/2}}$$

1.545 problem 559

Internal problem ID [7278]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 559.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(-x^2+2\right)y''-2x\left(2x^2+1\right)y'+\left(-2x^2+2\right)y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 51

```
dsolve(x^2*(2-x^2)*diff(y(x),x$2)-2*x*(1+2*x^2)*diff(y(x),x)+(2-2*x^2)*y(x)=0,y(x), singsol=a
```

$$y(x)=\frac{c_1 x}{\left(x^2-2\right)^{\frac{3}{2}}}+\frac{c_2 x\left(2\arctan\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)+\sqrt{2}\sqrt{x^2-2}\right)}{\left(x^2-2\right)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 58

```
DSolve[x^2*(2-x^2)*y''[x]-2*x*(1+2*x^2)*y'[x]+(2-2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to\frac{x\left(-\sqrt{2}c_2\operatorname{arctanh}\left(\sqrt{1-\frac{x^2}{2}}\right)+c_2\sqrt{2-x^2}+c_1\right)}{\left(2-x^2\right)^{3/2}}$$

1.546 problem 560

Internal problem ID [7279]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 560.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(5-x)\,y' + (9-4x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve(x^2*diff(y(x),x$2)-x*(5-x)*diff(y(x),x)+(9-4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^3(x+1) + c_2x^3\left((x+1)\,\mathrm{Ei}_1\left(x\right) - \mathrm{e}^{-x}\right)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 34

```
DSolve[x^2*y''[x]-x*(5-x)*y'[x]+(9-4*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}x^3(e^x(x+1)(c_2\,\mathrm{ExpIntegralEi}(-x) + c_1) + c_2)$$

1.547 problem 561

Internal problem ID [7280]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 561.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2\left(x^2+x+1\right)y''+12x^2(1+x)y'+\left(3x^2+3x+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 475

```
dsolve(4*x^2*(1+x+x^2)*diff(y(x),x$2)+12*x^2*(1+x)*diff(y(x),x)+(1+3*x+3*x^2)*y(x)=0,y(x), si
```

$$y(x)$$
$$= \frac{c_1\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{\frac{1}{4}-\frac{i\sqrt{3}}{4}}\sqrt{i\sqrt{3}-2x-1}\,\mathrm{e}^{-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{2}}\operatorname{hypergeom}\left(\left[\frac{1}{2}+\frac{\sqrt{\frac{1-i\sqrt{3}}{1+i\sqrt{3}}}}{2}-\sqrt{\frac{1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)^3}},\frac{1}{2}-\frac{\sqrt{\frac{1-i}{1+i}}}{2}\right.\right.}{(x^2+x+1)^{\frac{3}{4}}}$$
$$+\frac{c_2\left(\frac{i\sqrt{3}+2x+1}{i\sqrt{3}-2x-1}\right)^{\frac{3}{4}+\frac{i\sqrt{3}}{4}}\sqrt{i\sqrt{3}-2x-1}\,\mathrm{e}^{-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{2}}\operatorname{hypergeom}\left(\left[\frac{1}{2}+\frac{\sqrt{\frac{1-i\sqrt{3}}{1+i\sqrt{3}}}}{2}+\sqrt{\frac{1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)^3}},\frac{1}{2}-\frac{\sqrt{}}{}\right.\right.}{(x^2+x+1)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 0.281 (sec). Leaf size: 93

```
DSolve[4*x^2*(1+x+x^2)*y''[x]+12*x^2*(1+x)*y'[x]+(1+3*x+3*x^2)*y[x]==0,y[x],x,IncludeSingular
```

$$y(x)\to \frac{\sqrt{x}e^{-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}\left(c_2\int_1^x\frac{e^{\sqrt{3}\arctan\left(\frac{2K[1]+1}{\sqrt{3}}\right)}}{K[1]\sqrt{K[1]^2+K[1]+1}}dK[1]+c_1\right)}{\sqrt{x^2+x+1}}$$

1.548 problem 562

Internal problem ID [7281]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 562.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+x+1\right)y''-x\left(-2x^2-4x+1\right)y'+y=0}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 301

```
dsolve(x^2*(1+x+x^2)*diff(y(x),x$2)-x*(1-4*x-2*x^2)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\left(i\sqrt{3}-2x-1\right)^{\frac{1}{4}+\frac{7i\sqrt{3}}{12}}\left(i\sqrt{3}+2x+1\right)^{\frac{1}{4}-\frac{7i\sqrt{3}}{12}}x\,\mathrm{e}^{-\frac{7\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}}}{\left(x^2+x+1\right)^{\frac{3}{4}}}$$
$$+\frac{c_2\left(i\sqrt{3}-2x-1\right)^{-\frac{1}{4}-\frac{7i\sqrt{3}}{12}}\left(i\sqrt{3}+2x+1\right)^{\frac{3}{4}+\frac{7i\sqrt{3}}{12}}x\,\mathrm{e}^{-\frac{7\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}}\text{hypergeom}\left(\left[\frac{1}{2}+\frac{\sqrt{\frac{-45i\sqrt{3}-3}{1+i\sqrt{3}}}}{6}+\right.\right.}{\left(x^2+x+1\right)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 0.3 (sec). Leaf size: 90

```
DSolve[x^2*(1+x+x^2)*y''[x]-x*(1-4*x-2*x^2)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{xe^{-\frac{7\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}}\left(c_2\int_1^x\frac{e^{\frac{7\arctan\left(\frac{2K[1]+1}{\sqrt{3}}\right)}{\sqrt{3}}}}{K[1]\sqrt{K[1]^2+K[1]+1}}dK[1]+c_1\right)}{\sqrt{x^2+x+1}}$$

1.549 problem 563

Internal problem ID [7282]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 563.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' + 3x\left(-2x^2+3x+5\right)y' + \left(-14x^2+12x+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 42

```
dsolve(9*x^2*diff(y(x),x$2)+3*x*(5+3*x-2*x^2)*diff(y(x),x)+(1+12*x-14*x^2)*y(x)=0,y(x), sings
```

$$y(x) = \frac{c_1 \mathrm{e}^{\frac{x(x-3)}{3}}}{x^{\frac{1}{3}}} + \frac{c_2 \mathrm{e}^{\frac{x(x-3)}{3}} \left(\int \frac{\mathrm{e}^{-\frac{x(x-3)}{3}}}{x} dx \right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 52

```
DSolve[9*x^2*y''[x]+3*x*(5+3*x-2*x^2)*y'[x]+(1+12*x-14*x^2)*y[x]==0,y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{e^{\frac{1}{3}(x-3)x} \left(c_2 \int_1^x \frac{e^{K[1]-\frac{K[1]^2}{3}}}{K[1]} dK[1] + c_1 \right)}{\sqrt[3]{x}}$$

1.550 problem 564

Internal problem ID [7283]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 564.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(2x+1)\,y'' + x\left(3x^2+14x+5\right)y' + \left(12x^2+18x+4\right)y = 0}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 52

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(5+14*x+3*x^2)*diff(y(x),x)+(4+18*x+12*x^2)*y(x)=0,y(x),
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{3x}{2}}\,\mathrm{HeunC}\left(-\frac{3}{4},-\frac{1}{4},0,\frac{21}{32},-\frac{5}{32},2x+1\right)}{x^2\,(2x+1)^{\frac{1}{4}}} + \frac{c_2 \mathrm{e}^{-\frac{3x}{2}}\,\mathrm{HeunC}\left(-\frac{3}{4},\frac{1}{4},0,\frac{21}{32},-\frac{5}{32},2x+1\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 61

```
DSolve[x^2*(1+2*x)*y''[x]+x*(5+14*x+3*x^2)*y'[x]+(4+18*x+12*x^2)*y[x]==0,y[x],x,IncludeSingul
```

$$y(x) \to \frac{e^{-3x/2}\left(c_2 \int_1^x \frac{e^{\frac{3K[1]}{2}}}{K[1](2K[1]+1)^{3/4}}\,dK[1] + c_1\right)}{x^2\sqrt[4]{2x+1}}$$

1.551 problem 565

Internal problem ID [7284]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 565.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16x^2y'' + 4x\left(2x^2 + x + 6\right)y' + \left(18x^2 + 5x + 1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 42

```
dsolve(16*x^2*diff(y(x),x$2)+4*x*(6+x+2*x^2)*diff(y(x),x)+(1+5*x+18*x^2)*y(x)=0,y(x), singsol
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{x(x+1)}{4}}}{x^{\frac{1}{4}}} + \frac{c_2 \mathrm{e}^{-\frac{x(x+1)}{4}} \left(\int \frac{\mathrm{e}^{\frac{x(x+1)}{4}}}{x} dx \right)}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 51

```
DSolve[16*x^2*y''[x]+4*x*(6+x+2*x^2)*y'[x]+(1+5*x+18*x^2)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{e^{-\frac{1}{4}x(x+1)} \left(c_2 \int_1^x \frac{e^{\frac{1}{4}K[1](K[1]+1)}}{K[1]} dK[1] + c_1 \right)}{\sqrt[4]{x}}$$

1.552 problem 566

Internal problem ID [7285]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 566.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2(1+x)\,y'' + 3x\left(-x^2+11x+5\right)y' + \left(-7x^2+16x+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 38

```
dsolve(9*x^2*(1+x)*diff(y(x),x$2)+3*x*(5+11*x-x^2)*diff(y(x),x)+(1+16*x-7*x^2)*y(x)=0,y(x), s
```

$$y(x) = \frac{c_1 \operatorname{HeunC}\left(-\frac{1}{3}, -\frac{4}{3}, 0, -\frac{1}{9}, \frac{11}{18}, x+1\right)}{x^{\frac{1}{3}}\,(x+1)^{\frac{4}{3}}} + \frac{c_2 \operatorname{HeunC}\left(-\frac{1}{3}, \frac{4}{3}, 0, -\frac{1}{9}, \frac{11}{18}, x+1\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 50

```
DSolve[9*x^2*(1+x)*y''[x]+3*x*(5+11*x-x^2)*y'[x]+(1+16*x-7*x^2)*y[x]==0,y[x],x,IncludeSingula
```

$$y(x) \to \frac{e^{x/3}\left(c_1 - \sqrt[3]{3}ec_2\Gamma\left(\frac{1}{3}, \frac{x+1}{3}\right)\right)}{\sqrt[3]{x}(x+1)^{4/3}}$$

1.553 problem 567

Internal problem ID [7286]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 567.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{36x^2(1-2x)\,y'' + 24x(1-9x)\,y' + (1-70x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 123

```
dsolve(36*x^2*(1-2*x)*diff(y(x),x$2)+24*x*(1-9*x)*diff(y(x),x)+(1-70*x)*y(x)=0,y(x), singsol=
```

$$y(x) = \frac{c_1 x^{\frac{1}{6}}}{(2x-1)^{\frac{4}{3}}} + c_2\left(-\frac{4x^{\frac{1}{6}}\ln\left(1+(2x-1)^{\frac{1}{3}}\right)}{3\,(2x-1)^{\frac{4}{3}}} + \frac{2x^{\frac{1}{6}}\ln\left(1-(2x-1)^{\frac{1}{3}}+(2x-1)^{\frac{2}{3}}\right)}{3\,(2x-1)^{\frac{4}{3}}} + \frac{4\sqrt{3}\,x^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\,(2x-1)^{\frac{1}{3}}}{-2+(2x-1)^{\frac{1}{3}}}\right)}{3\,(2x-1)^{\frac{4}{3}}} + \frac{4x^{\frac{1}{6}}}{2x-1}\right)$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 108

```
DSolve[36*x^2*(1-2*x)*y''[x]+24*x*(1-9*x)*y'[x]+(1-70*x)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{\sqrt[6]{x}\left(-c_2\left(2\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-2x}+1}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{1-2x}-1\right) + \log\left((1-2x)^{2/3}+\sqrt[3]{1-2x}+1\right)\right) + 6c_2\right.}{2(1-2x)^{4/3}}$$

1.554 problem 568

Internal problem ID [7287]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 568.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y'' - x(3-x)\,y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(3-x)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2(x-1)}{(x+1)^3} + \frac{c_2 x^2(-4+(x-1)\ln(x))}{(x+1)^3}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 33

```
DSolve[x^2*(1+x)*y''[x]-x*(3-x)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2(c_1(x-1) + c_2(x-1)\log(x) - 4c_2)}{(x+1)^3}$$

1.555 problem 569

Internal problem ID [7288]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 569.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-2x)\,y'' - x(5-4x)\,y' + (9-4x)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)-x*(5-4*x)*diff(y(x),x)+(9-4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^3}{(2x-1)^2} + \frac{c_2 x^3 (2x - \ln(x))}{(2x-1)^2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 29

```
DSolve[x^2*(1-2*x)*y''[x]-x*(5-4*x)*y'[x]+(9-4*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{x^3(-2c_2 x + c_2 \log(x) + c_1)}{(1-2x)^2}$$

1.556 problem 570

Internal problem ID [7289]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 570.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x+2)\,y'' + y'x^2 + (1-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(2*x^2*(2+x)*diff(y(x),x$2)+x^2*diff(y(x),x)+(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x\,(x+2)} + \frac{c_2\sqrt{x}\left(\sqrt{2}\,\sqrt{x+2} - (x+2)\,\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x+2}}{2}\right)\right)}{\sqrt{x+2}}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 65

```
DSolve[2*x^2*(2+x)*y''[x]+x^2*y'[x]+(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{x}\left(2\left(c_1\sqrt{x+2}+c_2\right) - \sqrt{2}c_2\sqrt{x+2}\operatorname{arctanh}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right)\right)}{2\sqrt[4]{2}}$$

1.557 problem 571

Internal problem ID [7290]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 571.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(1+x)\,y'' - x(6-x)\,y' + (8-x)\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
dsolve(2*x^2*(1+x)*diff(y(x),x$2)-x*(6-x)*diff(y(x),x)+(8-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2}{(x+1)^{\frac{5}{2}}} + \frac{c_2 x^2\left(2\sqrt{x+1}\,x + 8\sqrt{x+1} + 3\ln\left(\sqrt{x+1}-1\right) - 3\ln\left(\sqrt{x+1}+1\right)\right)}{(x+1)^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 50

```
DSolve[2*x^2*(1+x)*y''[x]-x*(6-x)*y'[x]+(8-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{x^2\left(-6c_2 \operatorname{arctanh}\left(\sqrt{x+1}\right) + 2c_2\sqrt{x+1}(x+4) + 3c_1\right)}{3(x+1)^{5/2}}$$

1.558 problem 572

Internal problem ID [7291]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 572.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x+1)\,y'' + x(5+9x)\,y' + (4+3x)\,y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 74

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(5+9*x)*diff(y(x),x)+(4+3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(2x+1)^{\frac{3}{2}}}{x^2} + \frac{c_2\left(\left(x+\frac{1}{2}\right)^2 \ln\left(\sqrt{2x+1}-1\right) - \left(x+\frac{1}{2}\right)^2 \ln\left(\sqrt{2x+1}+1\right) + \sqrt{2x+1}\left(x+\frac{2}{3}\right)\right)}{\sqrt{2x+1}\,x^2}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 56

```
DSolve[x^2*(1+2*x)*y''[x]+x*(5+9*x)*y'[x]+(4+3*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{2c_2\left(-3(2x+1)^{3/2}\operatorname{arctanh}\left(\sqrt{2x+1}\right) + 6x + 4\right) + 3c_1(2x+1)^{3/2}}{3x^2}$$

1.559 problem 573

Internal problem ID [7292]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 573.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-2x)\,y'' - x(4x+5)\,y' + (4x+9)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 61

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)-x*(5+4*x)*diff(y(x),x)+(9+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(8x^4+x^3)}{(2x-1)^6} + \frac{c_2 x^3\left(\left(-6x-\frac{3}{4}\right)\ln(x) + x^4 - 4x^3 + 9x^2 + \frac{609x}{512} - \frac{9375}{4096}\right)}{(2x-1)^6}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 61

```
DSolve[x^2*(1-2*x)*y''[x]-x*(5+4*x)*y'[x]+(9+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{x^3(-8(8x+1)(c_2 x(8x(8x-33)+609) - 6c_1) + 3072c_2(8x+1)\log(x) + 9375c_2)}{384(1-2x)^6}$$

1.560 problem 574

Internal problem ID [7293]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 574.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-x)\,y'' + x(7+x)\,y' + (9-x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 70

```
dsolve(x^2*(1-x)*diff(y(x),x$2)+x*(7+x)*diff(y(x),x)+(9-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^4+16x^3+36x^2+16x+1)}{x^3} + \frac{c_2(25+3(x^4+16x^3+36x^2+16x+1)\ln(x)+120x^3+450x^2+280x)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 70

```
DSolve[x^2*(1-x)*y''[x]+x*(7+x)*y'[x]+(9-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3c_1(x(x+2)(x(x+14)+8)+1)+5c_2(2x(3x(4x+15)+28)+5)+3c_2(x(x+2)(x(x+14)+8)+1)l}{3x^3}$$

1.561 problem 575

Internal problem ID [7294]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 575.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x\left(1-x^2\right)y' + \left(x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x^2*diff(y(x),x$2)-x*(1-x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{-\frac{x^2}{2}} + c_2 x\,\mathrm{e}^{-\frac{x^2}{2}}\,\mathrm{Ei}_1\left(-\frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]-x*(1-x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-\frac{x^2}{2}}x\left(c_1\,\mathrm{ExpIntegralEi}\left(\frac{x^2}{2}\right) + 2c_2\right)$$

1.562 problem 576

Internal problem ID [7295]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 576.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''-3x\left(1-x^2\right)y'+4y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-3*x*(1-x^2)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^2}{\left(x^2+1\right)^2}+\frac{c_2x^2\left(\frac{x^2}{2}+\ln\left(x\right)\right)}{\left(x^2+1\right)^2}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 36

```
DSolve[x^2*(1+x^2)*y''[x]-3*x*(1-x^2)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to\frac{x^2(c_2x^2+2c_2\log(x)+2c_1)}{2\left(x^2+1\right)^2}$$

1.563 problem 577

Internal problem ID [7296]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 577.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 2y'x^3 + \left(3x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(4*x^2*diff(y(x),x$2)+2*x^3*diff(y(x),x)+(1+3*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\mathrm{e}^{-\frac{x^2}{4}} + c_2\sqrt{x}\,\mathrm{e}^{-\frac{x^2}{4}}\,\mathrm{Ei}_1\left(-\frac{x^2}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 39

```
DSolve[4*x^2*y''[x]+2*x^3*y'[x]+(1+3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-\frac{x^2}{4}}\sqrt{x}\left(c_2\,\mathrm{ExpIntegralEi}\left(\frac{x^2}{4}\right) + 2c_1\right)$$

1.564 problem 578

Internal problem ID [7297]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 578.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''-x\left(-2x^2+1\right)y'+y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1-2*x^2)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1 x}{\sqrt{x^2+1}}+\frac{c_2 x\ \operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 33

```
DSolve[x^2*(1+x^2)*y''[x]-x*(1-2*x^2)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x\left(c_1-c_2\operatorname{arctanh}\left(\sqrt{x^2+1}\right)\right)}{\sqrt{x^2+1}}$$

1.565 problem 579

Internal problem ID [7298]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 579.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2\left(x^2+2\right)y''+7y'x^3+\left(3x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 85

```
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)+7*x^3*diff(y(x),x)+(1+3*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\sqrt{x}}{\left(x^2+2\right)^{\frac{3}{4}}}+\frac{c_2\sqrt{x}\left(-\ln\left(1-\frac{\sqrt{2}\left(2x^2+4\right)^{\frac{1}{4}}}{2}\right)+\ln\left(1+\frac{\sqrt{2}\left(2x^2+4\right)^{\frac{1}{4}}}{2}\right)-2\arctan\left(\frac{\sqrt{2}\left(2x^2+4\right)^{\frac{1}{4}}}{2}\right)\right)}{\left(2x^2+4\right)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 71

```
DSolve[2*x^2*(2+x^2)*y''[x]+7*x^3*y'[x]+(1+3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{\sqrt{x}\left(2^{3/4}c_2\left(\arctan\left(\frac{\sqrt[4]{x^2+2}}{\sqrt[4]{2}}\right)-\operatorname{arctanh}\left(\frac{\sqrt[4]{x^2+2}}{\sqrt[4]{2}}\right)\right)+2c_1\right)}{2\left(x^2+2\right)^{3/4}}$$

1.566 problem 580

Internal problem ID [7299]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 580.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''-x\left(-4x^2+1\right)y'+\left(2x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 43

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1-4*x^2)*diff(y(x),x)+(1+2*x^2)*y(x)=0,y(x), singsol=all
```

$$y(x)=\frac{c_1x}{\left(x^2+1\right)^{\frac{3}{2}}}+\frac{c_2x\left(-\sqrt{x^2+1}+\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)\right)}{\left(x^2+1\right)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 45

```
DSolve[x^2*(1+x^2)*y''[x]-x*(1-4*x^2)*y'[x]+(1+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{x\left(-c_2\operatorname{arctanh}\left(\sqrt{x^2+1}\right)+c_2\sqrt{x^2+1}+c_1\right)}{\left(x^2+1\right)^{3/2}}$$

1.567 problem 581

Internal problem ID [7300]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 581.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2\left(x^2+4\right)y''+3x\left(3x^2+8\right)y'+\left(-9x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 68

```
dsolve(4*x^2*(4+x^2)*diff(y(x),x$2)+3*x*(8+3*x^2)*diff(y(x),x)+(1-9*x^2)*y(x)=0,y(x), singsol
```

$$y(x)=\frac{c_1\left(x^2+4\right)^{\frac{5}{8}}}{x^{\frac{1}{4}}}+\frac{c_2\left(-\frac{1024}{25}+\left(x^2\,\mathrm{hypergeom}\left(\left[1,1,\frac{13}{8}\right],[2,2],-\frac{x^2}{4}\right)-\frac{32\gamma}{5}+\frac{64\ln(2)}{5}-\frac{64\ln(x)}{5}-\frac{32\Psi\left(\frac{5}{8}\right)}{5}\right)\left(x^2+4\right)^{\frac{5}{8}}2^{\frac{3}{4}}\right)}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 185

```
DSolve[4*x^2*(4+x^2)*y''[x]+3*x*(8+3*x^2)*y'[x]+(1-9*x^2)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x)\to\frac{5\sqrt[4]{2}c_2\left(x^2+4\right)^{5/8}\left(\sqrt{2}\arctan\left(\frac{\sqrt[8]{x^2+4}}{\sqrt[4]{2}}\right)+\arctan\left(\frac{\sqrt{2}-\sqrt[4]{x^2+4}}{2^{3/4}\sqrt[8]{x^2+4}}\right)\right)+5\sqrt[4]{2}c_2\left(x^2+4\right)^{5/8}\left(\operatorname{arctanh}\left(\frac{2}{\sqrt{}}\right.\right.}{80\sqrt[4]{x}}$$

1.568 problem 582

Internal problem ID [7301]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 582.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2\left(x^2+3\right)y''+x\left(11x^2+3\right)y'+\left(5x^2+1\right)y=0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 107

```
dsolve(3*x^2*(3+x^2)*diff(y(x),x$2)+x*(3+11*x^2)*diff(y(x),x)+(1+5*x^2)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1x^{\frac{1}{3}}}{\left(x^2+3\right)^{\frac{2}{3}}}+\frac{c_2x^{\frac{1}{3}}\left(2\sqrt{3}\arctan\left(\frac{\left(9x^2+27\right)^{\frac{1}{3}}\sqrt{3}}{6+\left(9x^2+27\right)^{\frac{1}{3}}}\right)-\ln\left(1+\frac{\left(9x^2+27\right)^{\frac{1}{3}}}{3}+\frac{\left(9x^2+27\right)^{\frac{2}{3}}}{9}\right)+2\ln\left(1-\frac{\left(9x^2+27\right)^{\frac{1}{3}}}{3}\right)\right)}{\left(9x^2+27\right)^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 94

```
DSolve[3*x^2*(3+x^2)*y'[x]+x*(3+11*x^2)*y'[x]+(1+5*x^2)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to\frac{c_1\exp\left(\frac{1}{3}\text{RootSum}\left[3\#1^3+11\#1^2+9\#1+3\&,\frac{3\#1^2\log(x-\#1)-4\#1\log(x-\#1)+9\log(x-\#1)}{9\#1^2+22\#1+9}\&\right]\right)}{\sqrt[3]{x}}$$

$$y(x)\to 0$$

1.569 problem 583

Internal problem ID [7302]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 583.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' - 3x\left(-2x^2+7\right)y' + \left(2x^2+25\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(9*x^2*diff(y(x),x$2)-3*x*(7-2*x^2)*diff(y(x),x)+(25+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{5}{3}} \mathrm{e}^{-\frac{x^2}{3}} + c_2 x^{\frac{5}{3}} \mathrm{e}^{-\frac{x^2}{3}} \operatorname{Ei}_1\left(-\frac{x^2}{3}\right)$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 39

```
DSolve[9*x^2*y''[x]-3*x*(7-2*x^2)*y'[x]+(25+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{2} e^{-\frac{x^2}{3}} x^{5/3} \left(c_2 \operatorname{ExpIntegralEi}\left(\frac{x^2}{3}\right) + 2c_1\right)$$

1.570 problem 584

Internal problem ID [7303]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 584.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x\left(1-x^2\right)y' + \left(x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(x^2*diff(y(x),x$2)-x*(1-x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{-\frac{x^2}{2}} + c_2 x\,\mathrm{e}^{-\frac{x^2}{2}}\,\mathrm{Ei}_1\left(-\frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]-x*(1-x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-\frac{x^2}{2}}x\left(c_1\,\mathrm{ExpIntegralEi}\left(\frac{x^2}{2}\right) + 2c_2\right)$$

1.571 problem 585

Internal problem ID [7304]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 585.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-2x)\,y'' + 3y'x + (1+4x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)+3*x*diff(y(x),x)+(1+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2(-8x^3 + 18x^2 + 3\ln(x) - 18x)}{x}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 35

```
DSolve[x^2*(1-2*x)*y''[x]+3*x*y'[x]+(1+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x} - \frac{2}{3}c_2(x(4x-9)+9) + \frac{c_2\log(x)}{x}$$

1.572 problem 586

Internal problem ID [7305]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 586.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(1+x)\,y'' + (1-x)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x*(1+x)*diff(y(x),x$2)+(1-x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x-1) + c_2(-4 + (x-1)\ln(x))$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 23

```
DSolve[x*(1+x)*y''[x]+(1-x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x-1) + c_2((x-1)\log(x) - 4)$$

1.573 problem 587

Internal problem ID [7306]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 587.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x)\,y'' - x(3-5x)\,y' + (4-5x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(x^2*(1-x)*diff(y(x),x$2)-x*(3-5*x)*diff(y(x),x)+(4-5*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^2 (x-1)^3 + c_2 x^2 \left(-(x-1)^3 \ln(x-1) + (x-1)^3 \ln(x) - x^2 + \frac{5x}{2} - \frac{11}{6} \right)$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 53

```
DSolve[x^2*(1-x)*y''[x]-x*(3-5*x)*y'[x]+(4-5*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{6} x^2 \left(-6c_1 (x-1)^3 + c_2 (3x(2x-5)+11) + 6c_2 (x-1)^3 (\log(x-1) - \log(x)) \right)$$

1.574 problem 588

Internal problem ID [7307]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 588.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''-x\left(9x^2+1\right)y'+\left(25x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1+9*x^2)*diff(y(x),x)+(1+25*x^2)*y(x)=0,y(x), singsol=al
```

$$y(x)=c_1x\left(x^4-4x^2+1\right)+c_2\left(\left(x^4-4x^2+1\right)\ln\left(x\right)-6x^2+3\right)x$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 43

```
DSolve[x^2*(1+x^2)*y''[x]-x*(1+9*x^2)*y'[x]+(1+25*x^2)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to c_1\left(x^5-4x^3+x\right)+c_2x\left(-6x^2+\left(x^4-4x^2+1\right)\log(x)+3\right)$$

1.575 problem 589

Internal problem ID [7308]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 589.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' + 3x\left(1-x^2\right)y' + \left(7x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(9*x^2*diff(y(x),x$2)+3*x*(1-x^2)*diff(y(x),x)+(1+7*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{1}{3}} \operatorname{hypergeom}\left(\left[-1\right], \left[1\right], \frac{x^2}{6}\right) + c_2 x^{\frac{1}{3}} \operatorname{KummerU}\left(-1, 1, \frac{x^2}{6}\right)$$

✓ Solution by Mathematica

Time used: 0.156 (sec). Leaf size: 50

```
DSolve[9*x^2*y''[x]+3*x*(1-x^2)*y'[x]+(1+7*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{72}\sqrt[3]{x}\left(\left(x^2-6\right)\left(c_2 \operatorname{ExpIntegralEi}\left(\frac{x^2}{6}\right) + 72c_1\right) - 6c_2 e^{\frac{x^2}{6}}\right)$$

1.576 problem 590

Internal problem ID [7309]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 590.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x\left(x^2+1\right)y''+\left(1-x^2\right)y'-8yx=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 48

```
dsolve(x*(1+x^2)*diff(y(x),x$2)+(1-x^2)*diff(y(x),x)-8*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^2+1\right)^2+c_2\left(-\frac{\left(x^2+1\right)^2\ln\left(x^2+1\right)}{2}+\left(x^2+1\right)^2\ln\left(x\right)+\frac{x^2}{2}+\frac{3}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 55

```
DSolve[x*(1+x^2)*y''[x]+(1-x^2)*y'[x]-8*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\left(x^2+1\right)^2+\frac{1}{4}c_2\left(2x^2+4\left(x^2+1\right)^2\log(x)-2\left(x^2+1\right)^2\log\left(x^2+1\right)+3\right)$$

1.577 problem 591

Internal problem ID [7310]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 591.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 2x\left(-x^2+4\right)y' + \left(7x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 33

```
dsolve(4*x^2*diff(y(x),x$2)+2*x*(4-x^2)*diff(y(x),x)+(1+7*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{hypergeom}\left([-2],[1],\frac{x^2}{4}\right)}{\sqrt{x}} + \frac{c_2 \operatorname{KummerU}\left(-2,1,\frac{x^2}{4}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 60

```
DSolve[4*x^2*y''[x]+2*x*(4-x^2)*y'[x]+(1+7*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{\left(x^4-16x^2+32\right)\left(c_2 \operatorname{ExpIntegralEi}\left(\frac{x^2}{4}\right)+2048c_1\right) - 4c_2 e^{\frac{x^2}{4}}\left(x^2-12\right)}{2048\sqrt{x}}$$

1.578 problem 592

Internal problem ID [7311]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 592.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(1+x)\,y'' + 8y'x^2 + (1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+8*x^2*diff(y(x),x)+(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\sqrt{x}}{x+1} + \frac{c_2\sqrt{x}\,\ln(x)}{x+1}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 24

```
DSolve[4*x^2*(1+x)*y''[x]+8*x^2*y'[x]+(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{x}(c_2\log(x) + c_1)}{x+1}$$

1.579 problem 593

Internal problem ID [7312]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 593.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(x+3)\,y'' + 3x(3+7x)\,y' + (4x+3)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(9*x^2*(3+x)*diff(y(x),x$2)+3*x*(3+7*x)*diff(y(x),x)+(3+4*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^{\frac{1}{3}}}{3+x} + \frac{c_2 x^{\frac{1}{3}} \ln(x)}{3+x}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 24

```
DSolve[9*x^2*(3+x)*y''[x]+3*x*(3+7*x)*y'[x]+(3+4*x)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{\sqrt[3]{x}(c_2 \log(x) + c_1)}{x+3}$$

1.580 problem 594

Internal problem ID [7313]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 594.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2\left(-x^2+2\right)y''-x\left(3x^2+2\right)y'+\left(-x^2+2\right)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*(2-x^2)*diff(y(x),x$2)-x*(2+3*x^2)*diff(y(x),x)+(2-x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{x^2-2}+\frac{c_2x\ln\left(x\right)}{x^2-2}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 23

```
DSolve[x^2*(2-x^2)*y''[x]-x*(2+3*x^2)*y'[x]+(2-x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x)\to -\frac{x(c_2\log(x)+c_1)}{x^2-2}$$

1.581 problem 595

Internal problem ID [7314]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 595.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16x^2\left(x^2+1\right)y''+8x\left(9x^2+1\right)y'+\left(49x^2+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve(16*x^2*(1+x^2)*diff(y(x),x$2)+8*x*(1+9*x^2)*diff(y(x),x)+(1+49*x^2)*y(x)=0,y(x), sings
```

$$y(x)=\frac{c_1x^{\frac{1}{4}}}{x^2+1}+\frac{c_2x^{\frac{1}{4}}\ln\left(x\right)}{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 26

```
DSolve[16*x^2*(1+x^2)*y''[x]+8*x*(1+9*x^2)*y'[x]+(1+49*x^2)*y[x]==0,y[x],x,IncludeSingularSol
```

$$y(x)\to\frac{\sqrt[4]{x}(c_2\log(x)+c_1)}{x^2+1}$$

1.582 problem 596

Internal problem ID [7315]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 596.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(4+3x)\,y'' - x(4-3x)\,y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x^2*(4+3*x)*diff(y(x),x$2)-x*(4-3*x)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{3x+4} + \frac{c_2 x \ln(x)}{3x+4}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 22

```
DSolve[x^2*(4+3*x)*y''[x]-x*(4-3*x)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(c_2 \log(x) + c_1)}{3x+4}$$

1.583 problem 597

Internal problem ID [7316]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 597.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2\left(x^2+3x+1\right)y''+8x^2(3+2x)\,y'+\left(9x^2+3x+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 37

```
dsolve(4*x^2*(1+3*x+x^2)*diff(y(x),x$2)+8*x^2*(3+2*x)*diff(y(x),x)+(1+3*x+9*x^2)*y(x)=0,y(x),
```

$$y(x)=\frac{c_1\sqrt{x}}{x^2+3x+1}+\frac{c_2\sqrt{x}\,\ln(x)}{x^2+3x+1}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 28

```
DSolve[4*x^2*(1+3*x+x^2)*y''[x]+8*x^2*(3+2*x)*y'[x]+(1+3*x+9*x^2)*y[x]==0,y[x],x,IncludeSingu
```

$$y(x)\to\frac{\sqrt{x}(c_2\log(x)+c_1)}{x(x+3)+1}$$

1.584 problem 598

Internal problem ID [7317]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 598.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x)^2 y'' - x\left(-3x^2+2x+1\right) y' + \left(x^2+1\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^2*(1-x)^2*diff(y(x),x$2)-x*(1+2*x-3*x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x), singsol=a
```

$$y(x) = \frac{c_1 x}{(x-1)^2} + \frac{c_2 x \ln(x)}{(x-1)^2}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 20

```
DSolve[x^2*(1-x)^2*y''[x]-x*(1+2*x-3*x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{x(c_2 \log(x) + c_1)}{(x-1)^2}$$

1.585 problem 599

Internal problem ID [7318]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 599.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2\left(x^2+x+1\right)y''+3x\left(13x^2+7x+1\right)y'+\left(25x^2+4x+1\right)y=0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 33

```
dsolve(9*x^2*(1+x+x^2)*diff(y(x),x$2)+3*x*(1+7*x+13*x^2)*diff(y(x),x)+(1+4*x+25*x^2)*y(x)=0,y
```

$$y(x)=\frac{c_1x^{\frac{1}{3}}}{x^2+x+1}+\frac{c_2x^{\frac{1}{3}}\ln\left(x\right)}{x^2+x+1}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 27

```
DSolve[9*x^2*(1+x+x^2)*y''[x]+3*x*(1+7*x+13*x^2)*y'[x]+(1+4*x+25*x^2)*y[x]==0,y[x],x,IncludeS
```

$$y(x)\to\frac{\sqrt[3]{x}(c_2\log(x)+c_1)}{x^2+x+1}$$

1.586 problem 600

Internal problem ID [7319]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 600.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x+2)\,y'' - x(4-7x)\,y' - (5-3x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 62

```
dsolve(2*x^2*(2+x)*diff(y(x),x$2)-x*(4-7*x)*diff(y(x),x)-(5-3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^{\frac{5}{2}}}{(x+2)^{\frac{7}{2}}} + \frac{c_2\left(\frac{\sqrt{2}\sqrt{x+2}\,(33x^2+52x+32)}{3} + 5\,\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x+2}}{2}\right)x^3\right)}{(x+2)^{\frac{7}{2}}\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 92

```
DSolve[2*x^2*(2+x)*y''[x]-x*(4-7*x)*y'[x]-(5-3*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{15\sqrt{2}c_2x^3\operatorname{arctanh}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right) - 48c_1x^3 + 66c_2\sqrt{x+2}x^2 + 104c_2\sqrt{x+2}x + 64c_2\sqrt{x+2}}{48\sqrt{x}(x+2)^{7/2}}$$

1.587 problem 601

Internal problem ID [7320]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 601.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-2x)\,y'' + x(8-9x)\,y' + (6-3x)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(x^2*(1-2*x)*diff(y(x),x$2)+x*(8-9*x)*diff(y(x),x)+(6-3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(3x+4)\,(2x-1)^{\frac{9}{2}}}{x^6} + \frac{c_2(231x^3 - 198x^2 + 66x - 8)}{x^6}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 48

```
DSolve[x^2*(1-2*x)*y''[x]+x*(8-9*x)*y'[x]+(6-3*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{385c_1(3x+4)(1-2x)^{9/2} + c_2(33x(x(7x-6)+2)-8)}{1155x^6}$$

1.588 problem 602

Internal problem ID [7321]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 602.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''+x\left(10x^2+3\right)y'-\left(-14x^2+15\right)y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 88

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(3+10*x^2)*diff(y(x),x)-(15-14*x^2)*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1x^3}{\left(x^2+1\right)^{\frac{5}{2}}}+\frac{c_2\left(3\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)x^8-3\sqrt{x^2+1}\,x^6+2\sqrt{x^2+1}\,x^4+24\sqrt{x^2+1}\,x^2+16\sqrt{x^2+1}\right)}{x^5\left(x^2+1\right)^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 75

```
DSolve[x^2*(1+x^2)*y''[x]+x*(3+10*x^2)*y'[x]-(15-14*x^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to\frac{c_2\left(\sqrt{x^2+1}(x^2+2)\left(3x^4-8x^2-8\right)-3x^8\operatorname{arctanh}\left(\sqrt{x^2+1}\right)\right)+128c_1x^8}{128x^5\left(x^2+1\right)^{5/2}}$$

1.589 problem 603

Internal problem ID [7322]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 603.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(-2x^2+1\right)y''+x\left(-13x^2+7\right)y'-14yx^2=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(x^2*(1-2*x^2)*diff(y(x),x$2)+x*(7-13*x^2)*diff(y(x),x)-14*x^2*y(x)=0,y(x), singsol=all
```

$$y(x)=\frac{c_1(2x^2-1)^{\frac{5}{4}}}{x^6}+\frac{c_2(5x^4-20x^2+8)}{x^6}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 43

```
DSolve[x^2*(1-2*x^2)*y''[x]+x*(7-13*x^2)*y'[x]-14*x^2*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{15c_1(1-2x^2)^{5/4}+c_2(-5x^4+20x^2-8)}{15x^6}$$

1.590 problem 604

Internal problem ID [7323]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 604.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(1+x)\,y'' + 4x(2x+1)\,y' - (1+3x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(1+2*x)*diff(y(x),x)-(1+3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + \frac{c_2(\ln(x)\,x - \ln(x+1)\,x + 1)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 32

```
DSolve[4*x^2*(1+x)*y''[x]+4*x*(1+2*x)*y'[x]-(1+3*x)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{c_1 x + c_2(-x\log(x) + x\log(x+1) - 1)}{\sqrt{x}}$$

1.591 problem 605

Internal problem ID [7324]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 605.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(3x+2)\,y'' + x(4+21x)\,y' - (1-9x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 57

```
dsolve(2*x^2*(2+3*x)*diff(y(x),x$2)+x*(4+21*x)*diff(y(x),x)-(1-9*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\sqrt{x}}{(3x+2)^{\frac{3}{2}}} + \frac{c_2\left(\sqrt{2}\,\sqrt{3x+2} + 3\,\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{3x+2}}{2}\right)x\right)}{\sqrt{x}\,(3x+2)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 64

```
DSolve[2*x^2*(2+3*x)*y''[x]+x*(4+21*x)*y'[x]-(1-9*x)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\frac{3\sqrt{2}c_2 x\operatorname{arctanh}\left(\sqrt{\frac{3x}{2}+1}\right) - 2c_1 x + 2c_2\sqrt{3x+2}}{2\sqrt{x}(3x+2)^{3/2}}$$

1.592 problem 606

Internal problem ID [7325]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 606.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x+2)\,y' - (2-3x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve(x^2*diff(y(x),x$2)+x*(2+x)*diff(y(x),x)-(2-3*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} x + \frac{c_2(\mathrm{e}^{-x}\,\mathrm{Ei}_1\left(-x\right)x^3 + x^2 + x + 2)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 41

```
DSolve[x^2*y''[x]+x*(2+x)*y'[x]-(2-3*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}e^{-x}x(c_2\,\mathrm{ExpIntegralEi}(x) + 6c_1) - \frac{c_2(x^2+x+2)}{6x^2}$$

1.593 problem 607

Internal problem ID [7326]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 607.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(1+x)\,y'' + 4x(3+8x)\,y' - (5-49x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 42

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(3+8*x)*diff(y(x),x)-(5-49*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\sqrt{x}}{(x+1)^4} + \frac{c_2(6x^3\ln(x) - 18x^2 - 9x - 2)}{(x+1)^4\,x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 50

```
DSolve[4*x^2*(1+x)*y''[x]+4*x*(3+8*x)*y'[x]-(5-49*x)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{6c_1x^3 + 6c_2x^3\log(x) - c_2(9x(2x+1)+2)}{6x^{5/2}(x+1)^4}$$

1.594 problem 608

Internal problem ID [7327]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 608.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y'' - x(10x+3)\,y' + 30yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 64

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(3+10*x)*diff(y(x),x)+30*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(2x^5 - 5x^4\right) + c_2\left(\left(3x^5 - \frac{15}{2}x^4\right)\ln(x) + \frac{x^6}{4} - \frac{5x^5}{8} - \frac{299x^4}{16} + 5x^3 + \frac{5x^2}{4} + \frac{x}{4} + \frac{1}{40}\right)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 67

```
DSolve[x^2*(1+x)*y''[x]-x*(3+10*x)*y'[x]+30*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$\begin{aligned} y(x) \to{}& \frac{1}{2}c_1(2x-5)x^4 + 6c_2(2x-5)x^4\log(x) \\ &+ \frac{1}{20}c_2(5x(x(x(x(2x(2x-5)-299)+80)+20)+4)+2) \end{aligned}$$

1.595 problem 609

Internal problem ID [7328]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 609.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(1+x)\,y' - 3(x+3)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-3*(3+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^3 + \frac{c_2(-\operatorname{Ei}_1(x)\,x^6 + (x^5 - x^4 + 2x^3 - 6x^2 + 24x - 120)\,\mathrm{e}^{-x})}{x^3}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 56

```
DSolve[x^2*y''[x]+x*(1+x)*y'[x]-3*(3+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2e^{-x}(e^xx^6\,\text{ExpIntegralEi}(-x) + (x(x((x-1)x+2)-6)+24)x - 120)}{720x^3} + c_1x^3$$

1.596 problem 610

Internal problem ID [7329]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 610.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(2x+1)\,y'' + x(9+13x)\,y' + (7+5x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(9+13*x)*diff(y(x),x)+(7+5*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(143x^2+104x+20)}{x^7} + \frac{c_2(35x^3-45x^2+36x-20)\,(2x+1)^{\frac{7}{2}}}{x^7}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 57

```
DSolve[x^2*(1+2*x)*y''[x]+x*(9+13*x)*y'[x]+(7+5*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{c_2(x(5x(7x-9)+36)-20)(2x+1)^{7/2}}{315x^7} + \frac{c_1(13x(11x+8)+20)}{143x^7}$$

1.597 problem 611

Internal problem ID [7330]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 611.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(2x+1)\,y'' - 2(4-x)\,xy' - (7+5x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(4*x^2*(1+2*x)*diff(y(x),x$2)-2*x*(4-x)*diff(y(x),x)-(7+5*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2(5x^3 - 10x^2 - 40x - 16)}{(2x+1)^{\frac{5}{4}}\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 43

```
DSolve[4*x^2*(1+2*x)*y''[x]-2*x*(4-x)*y'[x]-(7+5*x)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{\frac{2c_2(5(x-4)x(x+2)-16)}{(2x+1)^{5/4}} + 35c_1}{35\sqrt{x}}$$

1.598 problem 612

Internal problem ID [7331]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 612.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2(x+3)\,y'' - x(15+x)\,y' - 20y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(3*x^2*(3+x)*diff(y(x),x$2)-x*(15+x)*diff(y(x),x)-20*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2 - 36x - 243)}{x^{\frac{2}{3}}} + \frac{c_2(7x+27)}{x^{\frac{2}{3}}\,(3+x)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 42

```
DSolve[3*x^2*(3+x)*y''[x]-x*(15+x)*y'[x]-20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\frac{4c_1(7x+27)}{\sqrt[3]{x+3}} + 21c_2((x-36)x - 243)}{28x^{2/3}}$$

1.599 problem 613

Internal problem ID [7332]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 613.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y'' + x(1-10x)\,y' - (9-10x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 73

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+x*(1-10*x)*diff(y(x),x)-(9-10*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(715x^4 + 572x^3 + 234x^2 + 52x + 5)}{x^3} + c_2\left(8x^{10} + 91x^9 + 468x^8 + 1430x^7 + 2860x^6 + 3861x^5 + 3432x^4 + 1716x^3\right)$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 50

```
DSolve[x^2*(1+x)*y''[x]+x*(1-10*x)*y'[x]-(9-10*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{6435c_1(x+1)^{12}(8x-5) - 8c_2(13x(x(11x(5x+4)+18)+4)+5)}{51480x^3}$$

1.600 problem 614

Internal problem ID [7333]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 614.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y'' + 3y'x^2 - (6-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+3*x^2*diff(y(x),x)-(6-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x+4)}{x^2} + \frac{c_2\left(6(x+4)\,(x+1)^2\ln(x+1) + 60x^2 + 129x + 68\right)}{x^2\,(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 49

```
DSolve[x^2*(1+x)*y''[x]+3*x^2*y'[x]-(6-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{6c_1(x+4) + \frac{c_2(3x(20x+43)+68)}{(x+1)^2} + 6c_2(x+4)\log(x+1)}{6x^2}$$

1.601 problem 615

Internal problem ID [7334]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 615.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(2x+1)\,y'' - 2x(3+14x)\,y' + (6+100x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 57

```
dsolve(x^2*(1+2*x)*diff(y(x),x$2)-2*x*(3+14*x)*diff(y(x),x)+(6+100*x)*y(x)=0,y(x), singsol=al
```

$$y(x) = c_1\left(2016x^5 + 672x^4 + 144x^3 + 18x^2 + x\right) + c_2\left(8x^{10} + 36x^9 + 72x^8 + 84x^7 + 63x^6\right)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 44

```
DSolve[x^2*(1+2*x)*y''[x]-2*x*(3+14*x)*y'[x]+(6+100*x)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to c_1 x(2x+1)^9 - \frac{c_2 x(6x(8x(14x(3x+1)+3)+3)+1)}{20160}$$

1.602 problem 616

Internal problem ID [7335]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 616.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1+x)\,y'' - x(6+11x)\,y' + (6+32x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(6+11*x)*diff(y(x),x)+(6+32*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(3x^8 + 14x^7 + 21x^6\right) + c_2\left(35x^4 + 42x^3 + 21x^2 + 4x\right)$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 44

```
DSolve[x^2*(1+x)*y''[x]-x*(6+11*x)*y'[x]+(6+32*x)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{3}c_1 x(x+1)^6(3x-4) - \frac{1}{140}c_2 x(7x(x(5x+6)+3)+4)$$

1.603 problem 617

Internal problem ID [7336]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 617.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(1+x)\,y'' + 4x(1+4x)\,y' - (49+27x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 30

```
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(1+4*x)*diff(y(x),x)-(49+27*x)*y(x)=0,y(x), singsol=all
```

$$y(x) = \frac{c_1 x^{\frac{7}{2}}}{(x+1)^2} + \frac{c_2(7x+6)}{(x+1)^2 x^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 36

```
DSolve[4*x^2*(1+x)*y''[x]+4*x*(1+4*x)*y'[x]-(49+27*x)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{42c_1x^7 - c_2(7x+6)}{42x^{7/2}(x+1)^2}$$

1.604 problem 618

Internal problem ID [7337]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 618.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''-x\left(-2x^2+7\right)y'+12y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 62

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(7-2*x^2)*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^6}{\left(x^2+1\right)^{\frac{7}{2}}}+c_2\left(\frac{15x^6\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)}{\left(x^2+1\right)^{\frac{7}{2}}}-\frac{8x^2\left(x^4-\frac{9}{8}x^2-\frac{1}{4}\right)}{\left(x^2+1\right)^3}\right)$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 88

```
DSolve[x^2*(1+x^2)*y''[x]-x*(7-2*x^2)*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{-15c_2x^6\operatorname{arctanh}\left(\sqrt{x^2+1}\right)-2c_2\sqrt{x^2+1}x^2+8x^6\left(c_2\sqrt{x^2+1}+c_1\right)-9c_2\sqrt{x^2+1}x^4}{8\left(x^2+1\right)^{7/2}}$$

1.605 problem 619

Internal problem ID [7338]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 619.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x\left(-x^2+7\right)y' + 12y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
dsolve(x^2*diff(y(x),x$2)-x*(7-x^2)*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^2}{2}} x^6 + c_2 x^2 \left(\mathrm{e}^{-\frac{x^2}{2}} \operatorname{Ei}_1\left(-\frac{x^2}{2}\right) x^4 + 2x^2 + 4\right)$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 51

```
DSolve[x^2*y''[x]-x*(7-x^2)*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16} x^2 \left(e^{-\frac{x^2}{2}} x^4 \left(c_2 \operatorname{ExpIntegralEi}\left(\frac{x^2}{2}\right) + 16 c_1\right) - 2c_2\left(x^2+2\right)\right)$$

1.606 problem 620

Internal problem ID [7339]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 620.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x\left(2x^2+1\right)y' - \left(-10x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(x^2*diff(y(x),x$2)+x*(1+2*x^2)*diff(y(x),x)-(1-10*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} x \,\mathrm{hypergeom}\left([-1],[2],x^2\right) + c_2 \mathrm{e}^{-x^2} x \,\mathrm{KummerU}\left(-1,2,x^2\right)$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 49

```
DSolve[x^2*y''[x]+x*(1+2*x^2)*y'[x]-(1-10*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{4} e^{-x^2} x\left(x^2-2\right)\left(c_2 \,\mathrm{ExpIntegralEi}\left(x^2\right) + 4c_1\right) - \frac{c_2\left(x^2-1\right)}{4x}$$

1.607 problem 621

Internal problem ID [7340]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 621.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x\left(-2x^2+1\right)y' - 4\left(2x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(x^2*diff(y(x),x$2)+x*(1-2*x^2)*diff(y(x),x)-4*(1+2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2\mathrm{e}^{x^2} + \frac{c_2\left(-x^4\mathrm{e}^{x^2}\operatorname{Ei}_1\left(x^2\right) + x^2 - 1\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 44

```
DSolve[x^2*y''[x]+x*(1-2*x^2)*y'[x]-4*(1+2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{e^{x^2}x^4(c_2\,\text{ExpIntegralEi}\left(-x^2\right) + 4c_1) + c_2(x^2-1)}{4x^2}$$

1.608 problem 622

Internal problem ID [7341]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 622.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x\left(-3x^2+1\right)y' - 4\left(-3x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x),x$2)+x*(1-3*x^2)*diff(y(x),x)-4*(1-3*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(-\frac{x^2}{2}+1\right)x^2 + c_2\left(\frac{3x^2}{2}-3\right)x^2$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 64

```
DSolve[x^2*y''[x]+x*(1-3*x^2)*y'[x]-4*(1-3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{x^4(x^2-2)\left(27c_2\,\text{ExpIntegralEi}\left(\frac{3x^2}{2}\right)+64c_1\right) - 2c_2e^{\frac{3x^2}{2}}\left(9x^4-12x^2-4\right)}{64x^2}$$

1.609 problem 623

Internal problem ID [7342]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 623.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''+x\left(11x^2+5\right)y'+24yx^2=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(5+11*x^2)*diff(y(x),x)+24*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{\left(x^2+1\right)^2}+\frac{c_2(2x^2+1)}{\left(x^2+1\right)^2x^4}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 36

```
DSolve[x^2*(1+x^2)*y''[x]+x*(5+11*x^2)*y'[x]+24*x^2*y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x)\to -\frac{-4c_1x^4+2c_2x^2+c_2}{4x^4\left(x^2+1\right)^2}$$

1.610 problem 624

Internal problem ID [7343]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 624.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2\left(x^2+1\right)y''+8y'x-\left(-x^2+35\right)y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 43

```
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+8*x*diff(y(x),x)-(35-x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x^2+1)^2}{x^{\frac{7}{2}}}+\frac{c_2\left(\left(x^2+1\right)^2\ln\left(x^2+1\right)+2x^2+\frac{3}{2}\right)}{x^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 53

```
DSolve[4*x^2*(1+x^2)*y''[x]+8*x*y'[x]-(35-x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{4c_1\left(x^2+1\right)^2+c_2(4x^2+3)+2c_2\left(x^2+1\right)^2\log\left(x^2+1\right)}{4x^{7/2}}$$

1.611 problem 625

Internal problem ID [7344]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 625.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''-x\left(-x^2+5\right)y'-\left(25x^2+7\right)y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(5-x^2)*diff(y(x),x)-(7+25*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{\left(x^2+1\right)^2x}+\frac{c_2x^7(4x^2+5)}{\left(x^2+1\right)^2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 37

```
DSolve[x^2*(1+x^2)*y''[x]-x*(5-x^2)*y'[x]-(7+25*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{c_2(4x^2+5)\,x^8+40c_1}{40x\left(x^2+1\right)^2}$$

1.612 problem 626

Internal problem ID [7345]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 626.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+1\right)y''+x\left(2x^2+5\right)y'-21y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 44

```
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(5+2*x^2)*diff(y(x),x)-21*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x^2+1)^{\frac{5}{2}}\left(x^2+8\right)}{x^7}+\frac{c_2(35x^6+140x^4+168x^2+64)}{x^7}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 54

```
DSolve[x^2*(1+x^2)*y''[x]+x*(5+2*x^2)*y'[x]-21*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{35c_1(x^2+1)^{5/2}\left(x^2+8\right)-c_2(7(5(x^2+4)\,x^2+24)\,x^2+64)}{35x^7}$$

1.613 problem 627

Internal problem ID [7346]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 627.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2\left(x^2+1\right)y''+4x\left(x^2+2\right)y'-\left(x^2+15\right)y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+4*x*(2+x^2)*diff(y(x),x)-(15+x^2)*y(x)=0,y(x), singsol=al
```

$$y(x)=\frac{c_1(3x^2+2)}{x^{\frac{5}{2}}}+\frac{c_2(x^2+1)^{\frac{3}{2}}}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 39

```
DSolve[4*x^2*(1+x^2)*y''[x]+4*x*(2+x^2)*y'[x]-(15+x^2)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to\frac{3c_1(x^2+1)^{3/2}-c_2(3x^2+2)}{3x^{5/2}}$$

1.614 problem 628

Internal problem ID [7347]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 628.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{2(t+1)\,y'}{t^2+2t-1} + \frac{2y}{t^2+2t-1} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)-2*(t+1)/(t^2+2*t-1)*diff(y(t),t)+2/(t^2+2*t-1)*y(t)=0,y(t), singsol=all
```

$$y(t) = c_1(t+1) + c_2\left(t^2+1\right)$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 60

```
DSolve[y''[t]-2*(t+1)/(t^2+2*t-1)*y'[t]+2/(t^2+2*t-1)*y[t]==0,y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{\sqrt{t(t+2)-1}\left(c_1\left(t\left(t-2\sqrt{2}+2\right)-2\sqrt{2}+3\right)+c_2(t+1)\right)}{\sqrt{1-t(t+2)}}$$

1.615 problem 629

Internal problem ID [7348]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 629.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4ty' + \left(4t^2 - 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(t),t$2)-4*t*diff(y(t),t)+(4*t^2-2)*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{t^2} + c_2 \mathrm{e}^{t^2} t$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[y''[t]-4*t*y'[t]+(4*t^2-2)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{t^2} (c_2 t + c_1)$$

1.616 problem 630

Internal problem ID [7349]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 630.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(-t^2+1\right) y''-2ty'+2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve((1-t^2)*diff(y(t),t$2)-2*t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1t+c_2\left(\frac{\ln\left(t-1\right)t}{2}-\frac{\ln\left(t+1\right)t}{2}+1\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 19

```
DSolve[(1-t^2)*y''[t]-2*t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to c_2(t\operatorname{arctanh}(t)-1)+c_1t$$

1.617 problem 631

Internal problem ID [7350]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 631.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(t^2+1\right) y'' - 2ty' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+t^2)*diff(y(t),t$2)-2*t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2\left(t^2 - 1\right)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 21

```
DSolve[(1+t^2)*y''[t]-2*t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_2 t - c_1 (t-i)^2$$

1.618 problem 632

Internal problem ID [7351]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 632.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(-t^2+1\right)y''-2ty'+6y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve((1-t^2)*diff(y(t),t$2)-2*t*diff(y(t),t)+6*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1\left(-3t^2+1\right)+c_2\left(\left(\frac{3t^2}{8}-\frac{1}{8}\right)\ln\left(t-1\right)+\left(-\frac{3t^2}{8}+\frac{1}{8}\right)\ln\left(t+1\right)+\frac{3t}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 37

```
DSolve[(1-t^2)*y''[t]-2*t*y'[t]+6*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\frac{1}{2}\left(c_2\left(3t^2-1\right)\operatorname{arctanh}(t)+c_1\left(3t^2-1\right)-3c_2t\right)$$

1.619 problem 633

Internal problem ID [7352]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 633.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2t+1)\,y'' - 4y'(t+1) + 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((2*t+1)*diff(y(t),t$2)-4*(t+1)*diff(y(t),t)+4*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1(t+1) + c_2 \mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 23

```
DSolve[(2*t+1)*y''[t]-4*(t+1)*y'[t]+4*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{2t+1} - c_2(t+1)$$

1.620 problem 634

Internal problem ID [7353]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 634.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$t^2y'' + ty' + \left(t^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(t^2*diff(y(t),t$2)+t*diff(y(t),t)+(t^2-1/4)*y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1 \sin(t)}{\sqrt{t}} + \frac{c_2 \cos(t)}{\sqrt{t}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 39

```
DSolve[t^2*y''[t]+t*y'[t]+(t^2-1/4)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{-it}(2c_1 - ic_2 e^{2it})}{2\sqrt{t}}$$

1.621 problem 635

Internal problem ID [7354]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 635.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{2ty'}{t^2+1} + \frac{2y}{t^2+1} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(t),t$2)-2*t/(1+t^2)*diff(y(t),t)+2/(1+t^2)*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2\left(t^2 - 1\right)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 21

```
DSolve[y''[t]-2*t/(1+t^2)*y'[t]+2/(1+t^2)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_2 t - c_1 (t-i)^2$$

1.622 problem 636

Internal problem ID [7355]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 636.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(t^2 + 2t + 1\right) y' - (4t + 4)\, y = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 78

```
dsolve(diff(y(t),t$2)+(t^2+2*t+1)*diff(y(t),t)-(4+4*t)*y(t)=0,y(t), singsol=all)
```

$$\begin{aligned} y(t) = {} & c_1\left(t^4 + 4t^3 + 6t^2 + 8t + 5\right) \\ & + c_2(t+1)\left(t^3 + 3t^2 + 3t + 5\right)\left(\int \frac{\mathrm{e}^{-\frac{t\left(t^2+3t+3\right)}{3}}}{(t+1)^2\left(t^3 + 3t^2 + 3t + 5\right)^2} dt\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.117 (sec). Leaf size: 114

```
DSolve[y''[t]+(t^2+2*t+1)*y'[t]-(4+4*t)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(t) \to \frac{1}{36} e^{-\frac{1}{3}t(t(t+3)+3)} \Bigg(& c_2 e^{\frac{1}{3}(t+1)^3} (t(t(t+3)+3)+5)(t+1)^3 \operatorname{ExpIntegralE}\left(\frac{1}{3}, \frac{1}{3}(t+1)^3\right) \\ & + 36 c_1 e^{\frac{t^3}{3}+t^2+t} (t(t(t+3)+3)+5)(t+1) - 3c_2(t(t(t+3)+3)+4) \Bigg) \end{aligned}$$

1.623 problem 638

Internal problem ID [7356]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 638.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{2ty'' + (1-2t)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(2*t*diff(y(t),t$2)+(1-2*t)*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^t \operatorname{erf}\left(\sqrt{t}\right) + c_2 \mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 21

```
DSolve[2*t*y''[t]+(1-2*t)*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^t \left(c_1 - c_2 \Gamma\left(\frac{1}{2}, t\right)\right)$$

1.624 problem 639

Internal problem ID [7357]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 639.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2ty'' + y'(t+1) - 2y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 56

```
dsolve(2*t*diff(y(t),t$2)+(1+t)*diff(y(t),t)-2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1\left(\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{t}}{2}\right)\sqrt{\pi}\left(t^2+6t+3\right) + \sqrt{2}\,\mathrm{e}^{-\frac{t}{2}}\left(t^{\frac{3}{2}}+5\sqrt{t}\right)\right) + c_2\left(t^2+6t+3\right)$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 64

```
DSolve[2*t*y''[t]+(1+t)*y'[t]-2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{24}\left((t(t+6)+3)\left(\sqrt{2\pi}c_2\operatorname{erf}\left(\frac{\sqrt{t}}{\sqrt{2}}\right)+24c_1\right) + 2c_2e^{-t/2}\sqrt{t}(t+5)\right)$$

1.625 problem 640

Internal problem ID [7358]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 640.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2t^2y'' - ty' + (t+1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(2*t^2*diff(y(t),t$2)-t*diff(y(t),t)+(1+t)*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \sin\left(\sqrt{2}\,\sqrt{t}\right)\sqrt{t} + c_2\sqrt{t}\,\cos\left(\sqrt{2}\,\sqrt{t}\right)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 62

```
DSolve[2*t^2*y''[t]-t*y'[t]+(1+t)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-i\sqrt{2}\sqrt{t}}\sqrt{t}\left(2c_1e^{2i\sqrt{2}\sqrt{t}} + i\sqrt{2}c_2\right)$$

1.626 problem 641

Internal problem ID [7359]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 641.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2t^2y'' + \left(t^2 - t\right)y' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(2*t^2*diff(y(t),t$2)+(t^2-t)*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \,\text{WhittakerM}\left(\frac{1}{4}, \frac{1}{4}, \frac{t}{2}\right) t^{\frac{1}{4}} \mathrm{e}^{-\frac{t}{4}} + c_2\sqrt{t}\,\mathrm{e}^{-\frac{t}{2}}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 36

```
DSolve[2*t^2*y''[t]+(t^2-t)*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t/2}\left(c_2\sqrt{t} - c_1 t\,\text{ExpIntegralE}\left(\frac{1}{2}, -\frac{t}{2}\right)\right)$$

1.627 problem 642

Internal problem ID [7360]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 642.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' + \left(-t^2 + t\right)y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(t^2*diff(y(t),t$2)+(t-t^2)*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1(t+1)}{t} + \frac{c_2 e^t}{t}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 23

```
DSolve[t^2*y''[t]+(t-t^2)*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 e^t - c_1(t+1)}{t}$$

1.628 problem 643

Internal problem ID [7361]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 643.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{ty'' - \left(t^2 + 2\right) y' + yt = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 40

```
dsolve(t*diff(y(t),t$2)-(t^2+2)*diff(y(t),t)+t*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{\frac{t^2}{2}} + c_2 \left(\sqrt{\pi}\, t\sqrt{2} - \pi \ \mathrm{erf}\left(\frac{\sqrt{2}\, t}{2} \right) \mathrm{e}^{\frac{t^2}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 47

```
DSolve[t*y''[t]-(t^2+2)*y'[t]+t*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} e^{\frac{t^2}{2}} \left(\sqrt{2\pi} c_2 \mathrm{erf}\left(\frac{t}{\sqrt{2}} \right) + 2c_1 \right) - c_2 t$$

1.629 problem 644

Internal problem ID [7362]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 644.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' + t(t+1)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(t^2*diff(y(t),t$2)+t*(t+1)*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1(t-1)}{t} + \frac{c_2 \mathrm{e}^{-t}}{t}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 26

```
DSolve[t^2*y''[t]+t*(t+1)*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{-t}(c_1 e^t(t-1) + c_2)}{t}$$

1.630 problem 645

Internal problem ID [7363]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 645.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{ty'' - (t+4)\,y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(t*diff(y(t),t$2)-(4+t)*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1\left(t^2 + 6t + 12\right) + c_2 \mathrm{e}^t\left(t^2 - 6t + 12\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 63

```
DSolve[t*y''[t]-(4+t)*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\sqrt{t}((c_2 - ic_1)(t(t+6)+12) + (ic_1 + c_2)e^t((t-6)t+12))}{\sqrt{\pi}\sqrt{-it}}$$

1.631 problem 646

Internal problem ID [7364]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 646.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2 y'' + \left(t^2 - 3t\right) y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(t^2*diff(y(t),t$2)+(t^2-3*t)*diff(y(t),t)+3*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t^3 \mathrm{e}^{-t} + c_2 t\left(\mathrm{e}^{-t}\,\mathrm{Ei}_1\left(-t\right) t^2 + t + 1\right)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 37

```
DSolve[t^2*y''[t]+(t^2-3*t)*y'[t]+3*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} t\left(e^{-t} t^2 (c_1 \operatorname{ExpIntegralEi}(t) + 2c_2) - c_1 (t+1)\right)$$

1.632 problem 647

Internal problem ID [7365]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 647.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' + ty' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(t*diff(y(t),t$2)+t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{-t}(t-2)\,t + c_2\left(\mathrm{e}^{-t}(t-2)\,t\ \mathrm{Ei}_1\left(-t\right) + t - 1\right)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 37

```
DSolve[t*y''[t]+t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}\left(e^{-t}(t-2)t(c_2\,\text{ExpIntegralEi}(t) + 2c_1) - c_2(t-1)\right)$$

1.633 problem 648

Internal problem ID [7366]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 648.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' + \left(-t^2 + 1\right) y' + 4yt = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(t*diff(y(t),t$2)+(1-t^2)*diff(y(t),t)+4*t*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1\left(1 - t^2 + \frac{1}{8}t^4\right) + c_2\left(2 - 2t^2 + \frac{1}{4}t^4\right)$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 55

```
DSolve[t*y''[t]+(1-t^2)*y'[t]+4*t*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{128}\left(\left(t^4 - 8t^2 + 8\right)\left(c_2\,\text{ExpIntegralEi}\left(\frac{t^2}{2}\right) + 128c_1\right) - 2c_2 e^{\frac{t^2}{2}}\left(t^2 - 6\right)\right)$$

1.634 problem 649

Internal problem ID [7367]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 649.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2 y'' - t(t+1)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(t^2*diff(y(t),t$2)-t*(1+t)*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^t c_1 t + c_2 \mathrm{e}^t t \ \mathrm{Ei}_1\left(t\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 20

```
DSolve[t^2*y''[t]-t*(1+t)*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^t t(c_1 \operatorname{ExpIntegralEi}(-t) + c_2)$$

1.635 problem 650

Internal problem ID [7368]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 650.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y'x + \left(4x^2 + 6\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} \cos\left(2x\right) + c_2 \mathrm{e}^{-x^2} \sin\left(2x\right)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 37

```
DSolve[y''[x]+4*x*y'[x]+(4*x^2+6)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-x(x+2i)} \left(4c_1 - ic_2 e^{4ix}\right)$$

1.636 problem 651

Internal problem ID [7369]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 651.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(-z^2+1\right)y''-3zy'+y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 51

```
dsolve((1-z^2)*diff(y(z),z$2)-3*z*diff(y(z),z)+y(z)=0,y(z), singsol=all)
```

$$y(z)=\frac{c_1\left(z+\sqrt{z^2-1}\right)^{\sqrt{2}}}{\sqrt{z^2-1}}+\frac{c_2\left(z+\sqrt{z^2-1}\right)^{-\sqrt{2}}}{\sqrt{z^2-1}}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 69

```
DSolve[(1-z^2)*y''[z]-3*z*y'[z]+y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z)\to\frac{\sqrt{\frac{2}{\pi}}c_1\cos\left(\sqrt{2}\arccos(z)\right)+c_2\sqrt[4]{1-z^2}Q^{\frac{1}{2}}_{-\frac{1}{2}+\sqrt{2}}(z)}{\sqrt[4]{-\left(z^2-1\right)^2}}$$

1.637 problem 652

Internal problem ID [7370]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 652.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4zy'' + 2(1-z)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(4*z*diff(y(z),z$2)+2*(1-z)*diff(y(z),z)-y(z)=0,y(z), singsol=all)
```

$$y(z) = c_1 \mathrm{e}^{\frac{z}{2}} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{z}}{2}\right) + c_2 \mathrm{e}^{\frac{z}{2}}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 34

```
DSolve[4*z*y''[z]+2*(1-z)*y'[z]-y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z) \to e^{z/2}\left(c_1 - \sqrt{2}c_2\Gamma\left(\frac{1}{2}, \frac{z}{2}\right)\right)$$

1.638 problem 653

Internal problem ID [7371]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 653.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{f'' + 2(z-1)\,f' + 4f = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
dsolve(diff(f(z),z$2)+2*(z-1)*diff(f(z),z)+4*f(z)=0,f(z), singsol=all)
```

$$f(z) = c_1 \mathrm{e}^{-z(z-2)}(z-1) + c_2\left((z-1)\sqrt{\pi}\ \mathrm{erf}\left(i(z-1)\right)\mathrm{e}^{-(z-1)^2} - i\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 56

```
DSolve[f''[z]+2*(z-a)*f'[z]+4*f[z]==0,f[z],z,IncludeSingularSolutions -> True]
```

$$f(z) \to e^{z(2a-z)}\left(c_2 e^{(a-z)^2} - (a-z)\left(\sqrt{\pi}c_2 \mathrm{erfi}(a-z) + 2c_1\right)\right)$$

1.639 problem 654

Internal problem ID [7372]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 654.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{zy'' - 2y' + yz = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve(z*diff(y(z),z$2)-2*diff(y(z),z)+z*y(z)=0,y(z), singsol=all)
```

$$y(z) = c_1(\cos(z)\, z - \sin(z)) + c_2(\cos(z) + \sin(z)\, z)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 39

```
DSolve[z*y''[z]-2*y'[z]+z*y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z) \to -\sqrt{\frac{2}{\pi}}((c_1 z + c_2)\cos(z) + (c_2 z - c_1)\sin(z))$$

1.640 problem 655

Internal problem ID [7373]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 655.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{zy'' + (2z-3)\,y' + \frac{4y}{z} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(z*diff(y(z),z$2)+(2*z-3)*diff(y(z),z)+4/z*y(z)=0,y(z), singsol=all)
```

$$y(z) = c_1 z^2 \mathrm{e}^{-2z}(2z-1) + c_2 z^2 \left(\frac{1}{2} + \mathrm{e}^{-2z}\left(z - \frac{1}{2}\right) \mathrm{Ei}_1\left(-2z\right)\right)$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 39

```
DSolve[z*y''[z]+(2*z-3)*y'[z]+4/z*y[z]==0,y[z],z,IncludeSingularSolutions -> True]
```

$$y(z) \to \frac{1}{2} z^2 \left(e^{-2z}(2z-1)(4c_2\,\text{ExpIntegralEi}(2z) + c_1) - 4c_2\right)$$

1.641 problem 656

Internal problem ID [7374]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 656.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1-2x)\,y' + y(-1+x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2 \mathrm{e}^{x} \ln(x)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 17

```
DSolve[x*y''[x]+(1-2*x)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(c_2 \log(x) + c_1)$$

1.642 problem 657

Internal problem ID [7375]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 657.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + \left(x^2 + 2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

1.643 problem 658

Internal problem ID [7376]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 658.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(1-x^2\right)y''-2y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2\left(\frac{\ln\left(x-1\right)x}{2}-\frac{\ln\left(x+1\right)x}{2}+1\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 19

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2(x\operatorname{arctanh}(x)-1)+c_1x$$

1.644 problem 659

Internal problem ID [7377]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 659.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4y'x + \left(4x^2 - 1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 39

```
DSolve[4*x^2*y''[x]+4*x*y'[x]+(4*x^2-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2 e^{2ix})}{2\sqrt{x}}$$

1.645 problem 660

Internal problem ID [7378]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 660.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2x+1)\,y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(x*diff(y(x),x$2)-(2*x+1)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(2x+1) + c_2 \mathrm{e}^{2x}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 25

```
DSolve[x*y''[x]-(2*x+1)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2x} - \frac{1}{4} c_2 (2x+1)$$

1.646 problem 661

Internal problem ID [7379]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 661.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_erf]`

$$\boxed{y'' + 2y'x + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} x + c_2 \left(\sqrt{\pi}\ \mathrm{erfi}\left(x\right) \mathrm{e}^{-x^2} x - 1 \right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 27

```
DSolve[y''[x]+2*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2c_2 x \operatorname{DawsonF}(x) + 2c_1 e^{-x^2} x + c_2$$

1.647 problem 662

Internal problem ID [7380]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 662.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 3y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\sqrt{2}\sqrt{\pi}\ \operatorname{erfi}\left(\frac{\sqrt{2}\,x}{2}\right)(x-1)(x+1)\,\mathrm{e}^{-\frac{x^2}{2}} - 2x\right) + c_2\mathrm{e}^{-\frac{x^2}{2}}\left(x^2-1\right)$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 53

```
DSolve[y''[x]+x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(e^{-\frac{x^2}{2}}\left(x^2-1\right)\left(\sqrt{2\pi}c_2\operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right)+4c_1\right) - 2c_2x\right)$$

1.648 problem 663

Internal problem ID [7381]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 663.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x^2 - 3yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-3*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^3}{3}} x + \frac{c_2 \left(x^6 \operatorname{WhittakerM}\left(\frac{1}{3}, \frac{5}{6}, \frac{x^3}{3}\right) + (5x^3 + 10) \operatorname{WhittakerM}\left(\frac{4}{3}, \frac{5}{6}, \frac{x^3}{3}\right) \right) \mathrm{e}^{\frac{x^3}{6}}}{x^4}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 39

```
DSolve[y''[x]-x^2*y'[x]-3*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{\frac{x^3}{3}} \left(3c_1 x - c_2 \operatorname{ExpIntegralE}\left(\frac{4}{3}, \frac{x^3}{3}\right) \right)$$

1.649 problem 664

Internal problem ID [7382]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 664.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(-4x^2+1\right)y''-20y'x-16y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 57

```
dsolve((1-4*x^2)*diff(y(x),x$2)-20*x*diff(y(x),x)-16*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1 x}{\left(4x^2-1\right)^{\frac{3}{2}}}+\frac{c_2\left(2\ln\left(2x+\sqrt{4x^2-1}\right)x-\sqrt{4x^2-1}\right)}{\left(4x^2-1\right)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 71

```
DSolve[(1-4*x^2)*y''[x]-20*x*y'[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-c_2\sqrt{1-4x^2}+4c_2 x\cot^{-1}\left(\frac{2x+1}{\sqrt{1-4x^2}}\right)+c_1 x}{\sqrt[4]{1-4x^2}\left(4x^2-1\right)^{5/4}}$$

1.650 problem 665

Internal problem ID [7383]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 665.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(x^2-1\right)y''-6y'x+12y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x^2-1)*diff(y(x),x$2)-6*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^3+x\right)+c_2\left(x^4+6x^2+1\right)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 45

```
DSolve[(x^2-1)*y''[x]-6*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\sqrt{x^2-1}\left(c_2x\left(x^2+1\right)+c_1(x-1)^4\right)}{\sqrt{1-x^2}}$$

1.651 problem 666

Internal problem ID [7384]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 666.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+(2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}\,\mathrm{KummerM}\left(\frac{3}{2}, \frac{1}{2}, -\frac{(x-2)^2}{2}\right) + c_2 \mathrm{e}^{-x}\,\mathrm{KummerU}\left(\frac{3}{2}, \frac{1}{2}, -\frac{(x-2)^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 84

```
DSolve[y''[x]+x*y'[x]+(2+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-\frac{x^2}{2}+x-\frac{9}{2}} \left(e^{5/2}(x-3)(x-1)\left(\sqrt{2\pi}c_2 \operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right) + 4e^2 c_1\right) - 2c_2 e^{\frac{1}{2}(x-3)^2+x}(x-2)\right)$$

1.652 problem 667

Internal problem ID [7385]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 667.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(2x^2+1\right)y''+7y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
dsolve((1+2*x^2)*diff(y(x),x$2)+7*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\,\text{LegendreP}\left(\frac{1}{4},\frac{3}{4},i\sqrt{2}\,x\right)}{\left(2x^2+1\right)^{\frac{3}{8}}}+\frac{c_2\,\text{LegendreQ}\left(\frac{1}{4},\frac{3}{4},i\sqrt{2}\,x\right)}{\left(2x^2+1\right)^{\frac{3}{8}}}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 66

```
DSolve[(1+2*x^2)*y''[x]+7*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_2 Q_{\frac{1}{4}}^{\frac{3}{4}}\left(i\sqrt{2}x\right)}{\left(2x^2+1\right)^{3/8}}+\frac{2i\sqrt[4]{2}c_1 x}{\left(2x^2+1\right)^{3/4}\,\text{Gamma}\left(\frac{1}{4}\right)}$$

1.653 problem 668

Internal problem ID [7386]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 668.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{4y'' + y'x + 4y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(4*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^2}{8}} \operatorname{hypergeom}\left(\left[-1\right], \left[\frac{3}{2}\right], \frac{x^2}{8}\right) x + c_2 \mathrm{e}^{-\frac{x^2}{8}} \operatorname{hypergeom}\left(\left[-\frac{3}{2}\right], \left[\frac{1}{2}\right], \frac{x^2}{8}\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 64

```
DSolve[4*y''[x]+x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{32}\left(\sqrt{2}e^{-\frac{x^2}{8}} x\left(x^2-12\right)\left(\sqrt{\pi}c_2 \operatorname{erfi}\left(\frac{x}{2\sqrt{2}}\right) + 8c_1\right) - 4c_2\left(x^2-8\right)\right)$$

1.654 problem 669

Internal problem ID [7387]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 669.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x - 4y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 57

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\sqrt{2}\,\mathrm{e}^{-\frac{x^2}{2}}\left(x^2+5\right)x + \operatorname{erf}\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{\pi}\left(x^4+6x^2+3\right)\right) + c_2\left(x^4+6x^2+3\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 43

```
DSolve[y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{2}}\,\text{HermiteH}\left(-5, \frac{x}{\sqrt{2}}\right) + \frac{1}{3}c_2\left(x^4+6x^2+3\right)$$

1.655 problem 670

Internal problem ID [7388]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 670.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4xy'' - y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(4*x*diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^2 - 8x) + c_2\left(\frac{(x^2 - 8x)\,\mathrm{Ei}_1\left(-\frac{x}{4}\right)}{128} + \frac{\mathrm{e}^{\frac{x}{4}}(x-4)}{32}\right)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 43

```
DSolve[4*x*y''[x]-x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{128}c_2\left((x-8)x\,\mathrm{ExpIntegralEi}\left(\frac{x}{4}\right) - 4e^{x/4}(x-4)\right) + c_1(x-8)x$$

1.656 problem 671

Internal problem ID [7389]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 671.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{6x^2y'' + x(1+18x)\,y' + (1+12x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 46

```
dsolve(6*x^2*diff(y(x),x$2)+x*(1+18*x)*diff(y(x),x)+(1+12*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\mathrm{e}^{-3x} + \frac{c_2\left(-\frac{(-x)^{\frac{5}{6}}3^{\frac{5}{6}}}{3} + x\,\mathrm{e}^{-3x}\left(\Gamma\left(\frac{5}{6}\right) - \Gamma\left(\frac{5}{6}, -3x\right)\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 36

```
DSolve[6*x^2*y''[x]+x*(1+18*x)*y'[x]+(1+12*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{-3x}\sqrt[3]{x}\left(c_1\sqrt[6]{x} - c_2\,\text{ExpIntegralE}\left(\frac{7}{6}, -3x\right)\right)$$

1.657 problem 672

Internal problem ID [7390]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 672.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2y'' - x(x+8)\,y' + 6y = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 31

```
dsolve(3*x^2*diff(y(x),x$2)-x*(x+8)*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\text{hypergeom}\left([3], \left[\frac{10}{3}\right], \frac{x}{3}\right) x^3 + c_2 \,\text{hypergeom}\left(\left[\frac{2}{3}\right], \left[-\frac{4}{3}\right], \frac{x}{3}\right) x^{\frac{2}{3}}$$

✓ Solution by Mathematica

Time used: 0.063 (sec). Leaf size: 62

```
DSolve[3*x^2*y''[x]-x*(x+8)*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{18} e^{x/3} x^{2/3} ((x-2)x+4) \left(18c_1 - \sqrt[3]{3} c_2 \Gamma\left(\frac{1}{3}, \frac{x}{3}\right)\right) + \frac{1}{6} c_2 (x-4)x$$

1.658 problem 673

Internal problem ID [7391]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 673.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - x(2x+1)\,y' + 2(4x-1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 32

```
dsolve(2*x^2*diff(y(x),x$2)-x*(1+2*x)*diff(y(x),x)+2*(4*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\frac{4}{63}x^4 - \frac{4}{7}x^3 + x^2\right) + \frac{c_2\,\text{hypergeom}\left(\left[-\frac{9}{2}\right],\left[-\frac{3}{2}\right],x\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 71

```
DSolve[2*x^2*y''[x]-x*(1+2*x)*y'[x]+2*(4*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \\ \to \frac{x^{5/2}(4(x-9)x+63)\left(32c_2e^x\,\text{DawsonF}\left(\sqrt{x}\right)+945c_1\right) - 32c_2e^x(x(x(x(2x-17)+24)+6)+3)}{3780\sqrt{x}}$$

1.659 problem 674

Internal problem ID [7392]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 674.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4y'x^2 + (2x+1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(4*x^2*diff(y(x),x$2)-4*x^2*diff(y(x),x)+(1+2*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x}\ \mathrm{Ei}_1\left(-x\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 19

```
DSolve[4*x^2*y''[x]-4*x^2*y'[x]+(1+2*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_2\,\mathrm{ExpIntegralEi}(x) + c_1)$$

1.660 problem 675

Internal problem ID [7393]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 675.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(-2x+3)\,y' + (1-2x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*(3-2*x)*diff(y(x),x)+(1-2*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2\,\mathrm{Ei}_1\left(-2x\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 19

```
DSolve[x^2*y''[x]+x*(3-2*x)*y'[x]+(1-2*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2\,\mathrm{ExpIntegralEi}(2x) + c_1}{x}$$

1.661 problem 676

Internal problem ID [7394]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 676.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(x+3)\,y' + (4-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x$2)-x*(3+x)*diff(y(x),x)+(4-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x x^2 \left(x^2+4x+2\right) + c_2 \left(\mathrm{e}^x \left(x^2+4x+2\right) \mathrm{Ei}_1\left(x\right) - x - 3\right) x^2$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]-x*(3+x)*y'[x]+(4-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}x^2(e^x(x(x+4)+2)(c_2\,\text{ExpIntegralEi}(-x)+4c_1)+c_2(x+3))$$

1.662 problem 677

Internal problem ID [7395]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 677.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(3-x)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x^2*diff(y(x),x$2)+x*(3-x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x-1)}{x} + \frac{c_2((x-1)\,\mathrm{Ei}_1(-x) + \mathrm{e}^x)}{x}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 29

```
DSolve[x^2*y''[x]+x*(3-x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x-1)(c_2\,\mathrm{ExpIntegralEi}(x) + c_1) - c_2e^x}{x}$$

1.663 problem 678

Internal problem ID [7396]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 678.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - \left(2\sqrt{5}-1\right)xy' + \left(\frac{19}{4} - 3x^2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x$2)-(2*sqrt(5)-1)*x*diff(y(x),x)+(19/4-3*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{-\frac{1}{2}+\sqrt{5}} \sinh\left(\sqrt{3}\,x\right) + c_2 x^{-\frac{1}{2}+\sqrt{5}} \cosh\left(\sqrt{3}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 53

```
DSolve[x^2*y''[x]-(2*Sqrt[5]-1)*x*y'[x]+(19/4-3*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{6} e^{-\sqrt{3}x} x^{\sqrt{5}-\frac{1}{2}} \left(\sqrt{3}c_2 e^{2\sqrt{3}x} + 6c_1\right)$$

1.664 problem 679

Internal problem ID [7397]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 679.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x-3)\,y' + (4-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x^2*diff(y(x),x$2)+x*(x-3)*diff(y(x),x)+(4-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}c_1x^2 + c_2x^2\mathrm{e}^{-x}\,\mathrm{Ei}_1\left(-x\right)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+x*(x-3)*y'[x]+(4-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}x^2(c_2\,\text{ExpIntegralEi}(x) + c_1)$$

1.665 problem 680

Internal problem ID [7398]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 680.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x^2 - (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)-(2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{-x}}{x} + \frac{c_2(x^2 - 2x + 2)}{x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 30

```
DSolve[x^2*y''[x]+x^2*y'[x]-(2+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(c_2 e^x((x-2)x+2) + c_1)}{x}$$

1.666 problem 681

Internal problem ID [7399]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 681.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2y'x^2 + \left(x - \frac{3}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x$2)+2*x^2*diff(y(x),x)+(x-3/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2 \mathrm{e}^{-2x}(2x+1)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]+2*x^2*y'[x]+(x-3/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 - c_2 e^{-2x}(2x+1)}{4\sqrt{x}}$$

1.667 problem 682

Internal problem ID [7400]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 682.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y'' + y'x^2 - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x^2*(1+x)*diff(y(x),x$2)+x^2*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x+2)}{x} + \frac{c_2(4+(x+2)\ln(x+1))}{x}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 30

```
DSolve[x^2*(1+x)*y''[x]+x^2*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1(x+2) + c_2(x+2)\log(x+1) + 4c_2}{x}$$

1.668 problem 683

Internal problem ID [7401]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 683.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x\left(x^2+6\right)y' + 6y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(x^2*diff(y(x),x$2)+x*(6+x^2)*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2+3)}{x^2} + \frac{c_2 \mathrm{e}^{-\frac{x^2}{2}}\,\mathrm{hypergeom}\left([2],\left[\frac{1}{2}\right],\frac{x^2}{2}\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.063 (sec). Leaf size: 63

```
DSolve[x^2*y''[x]+x*(6+x^2)*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2c_2 e^{-\frac{x^2}{2}}(x^2+2) - x(x^2+3)\left(12c_1 - \sqrt{2\pi}c_2 \mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)\right)}{12x^3}$$

1.669 problem 684

Internal problem ID [7402]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 684.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(1-x)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*(1-x)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x+1)}{x} + \frac{c_2 \mathrm{e}^x}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]+x*(1-x)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 e^x - c_1(x+1)}{x}$$

1.670 problem 685

Internal problem ID [7403]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 685.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(x+3)y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(x^2*diff(y(x),x$2)-x*(x+3)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x x^2(x+1) + c_2 x^2(-1 + \mathrm{e}^x(x+1)\,\mathrm{Ei}_1(x))$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 29

```
DSolve[x^2*y''[x]-x*(x+3)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(e^x(x+1)(c_2\,\mathrm{ExpIntegralEi}(-x) + c_1) + c_2)$$

1.671 problem 686

Internal problem ID [7404]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 686.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x^2 - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x+2)}{x} + \frac{c_2 e^x (x-2)}{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 57

```
DSolve[x^2*y''[x]-x^2*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(c_1 - ic_2)\left(-e^x\right)(x-2) - (c_1 + ic_2)(x+2)}{\sqrt{\pi}\sqrt{-ix}\sqrt{x}}$$

1.672 problem 687

Internal problem ID [7405]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 687.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x^2 - (3x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)-(3*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2\mathrm{e}^x(x+4) + \frac{c_2(-x^3\mathrm{e}^x(x+4)\,\mathrm{Ei}_1(x) + x^3 + 3x^2 - 2x + 2)}{x}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 51

```
DSolve[x^2*y''[x]-x^2*y'[x]-(3*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x x^3(x+4)(24c_1 - c_2\,\mathrm{ExpIntegralEi}(-x)) - c_2(x(x(x+3)-2)+2)}{24x}$$

1.673 problem 688

Internal problem ID [7406]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 688.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(5-x)\,y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x$2)+x*(5-x)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2-4x+2)}{x^2} + \frac{c_2((x^2-4x+2)\,\mathrm{Ei}_1(-x) + \mathrm{e}^x(x-3))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 41

```
DSolve[x^2*y''[x]+x*(5-x)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{((x-4)x+2)(c_2\,\mathrm{ExpIntegralEi}(x) + 4c_1) - c_2e^x(x-3)}{4x^2}$$

1.674 problem 689

Internal problem ID [7407]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 689.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4x(1-x)\,y' + (2x-9)\,y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*(1-x)*diff(y(x),x)+(2*x-9)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^x}{x^{\frac{3}{2}}} + \frac{c_2(x^2+2x+2)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 29

```
DSolve[4*x^2*y''[x]+4*x*(1-x)*y'[x]+(2*x-9)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 e^x - c_2(x(x+2)+2)}{x^{3/2}}$$

1.675 problem 690

Internal problem ID [7408]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 690.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2x(x+2)\,y' + 2(1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(x^2*diff(y(x),x$2)+2*x*(2+x)*diff(y(x),x)+2*(1+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2(-2\ \mathrm{Ei}_1\left(2x\right)x + \mathrm{e}^{-2x})}{x^2}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]+2*x*(2+x)*y'[x]+2*(1+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2c_2x\,\mathrm{ExpIntegralEi}(-2x) + c_1x - c_2e^{-2x}}{x^2}$$

1.676 problem 691

Internal problem ID [7409]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 691.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(1-x)\,y' + (1-x)\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)-x*(1-x)*diff(y(x),x)+(1-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2\,\mathrm{Ei}_1(x)\,x$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 17

```
DSolve[x^2*y''[x]-x*(1-x)*y'[x]+(1-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2\,\mathrm{ExpIntegralEi}(-x) + c_1)$$

1.677 problem 692

Internal problem ID [7410]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 692.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4x(2x+1)\,y' + (4x-1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*(1+2*x)*diff(y(x),x)+(4*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2 \mathrm{e}^{-2x}}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 26

```
DSolve[4*x^2*y''[x]+4*x*(1+2*x)*y'[x]+(4*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{2c_1 e^{-2x} + c_2}{2\sqrt{x}}$$

1.678 problem 693

Internal problem ID [7411]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 693.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x+4)\,y' + (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x^2*diff(y(x),x$2)+x*(4+x)*diff(y(x),x)+(2+x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2(-\operatorname{Ei}_1(x)\,x + \mathrm{e}^{-x})}{x^2}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]+x*(4+x)*y'[x]+(2+x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-c_2 x\,\text{ExpIntegralEi}(-x) + c_1 x - c_2 e^{-x}}{x^2}$$

1.679 problem 694

Internal problem ID [7412]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 694.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{9}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-9/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{ix}(x+i)}{x^{\frac{3}{2}}} + \frac{c_2 e^{-ix}(-x+i)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 44

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-9/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((c_1 x + c_2)\cos(x) + (c_2 x - c_1)\sin(x))}{x^{3/2}}$$

1.680 problem 695

Internal problem ID [7413]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 695.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.681 problem 696

Internal problem ID [7414]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 696.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + 5(1-2x)\,y' - 5y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 50

```
dsolve(2*x*diff(y(x),x$2)+5*(1-2*x)*diff(y(x),x)-5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\frac{\operatorname{erfi}\left(\sqrt{5}\,\sqrt{x}\right)(1+10x)\sqrt{5}\,\sqrt{\pi}}{x^{\frac{3}{2}}} - \frac{10\,\mathrm{e}^{5x}}{x}\right) + \frac{c_2(1+10x)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 40

```
DSolve[2*x*y''[x]+5*(1-2*x)*y'[x]-5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 L_{-\frac{1}{2}}^{\frac{3}{2}}(5x) + \frac{c_1(10x+1)}{10\sqrt{5}x^{3/2}}$$

1.682 problem 697

Internal problem ID [7415]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 697.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2 e^{2ix})}{2\sqrt{x}}$$

1.683 problem 698

Internal problem ID [7416]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 698.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (x+n)\,y' + (n+1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(x*diff(y(x),x$2)+(x+n)*diff(y(x),x)+(n+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}\,\mathrm{hypergeom}\left([-1],[n],x\right) + c_2 x^{-n+1}\mathrm{e}^{-x}\,\mathrm{hypergeom}\left([-n],[-n+2],x\right)$$

✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 48

```
DSolve[x*y''[x]+(x+n)*y'[x]+(n+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(n-x)\left(c_2 \int_1^x \frac{e^{K[1]} K[1]^{-n}}{(n-K[1])^2} dK[1] + c_1\right)$$

1.684 problem 699

Internal problem ID [7417]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 699.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^4 y'' + y' x + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(x^4*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(\frac{\operatorname{erfi}\left(\frac{\sqrt{2}}{2x}\right) (x^2-1)\sqrt{2}\sqrt{\pi}}{x} + 2\,\mathrm{e}^{\frac{1}{2x^2}} \right) + \frac{c_2 (x^2-1)}{x}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 59

```
DSolve[x^4*y''[x]+x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x^2-1)\left(4c_1 - \sqrt{2\pi} c_2 \operatorname{erfi}\left(\frac{1}{\sqrt{2}x}\right)\right) - 2c_2 e^{\frac{1}{2x^2}} x}{4x}$$

1.685 problem 700

Internal problem ID [7418]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 700.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(2x^2 + x\right)y' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(x^2*diff(y(x),x$2)+(x+2*x^2)*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(2x^2 - 4x + 3)}{x^2} + \frac{c_2 \mathrm{e}^{-2x}(2x + 3)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]+(x+2*x^2)*y'[x]-4*y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(\frac{c_1 e^{-2x}(4x + 6) + c_2(3 - 4x)}{x^2} + 2(-1 + c_2)\right)$$

1.686 problem 701

Internal problem ID [7419]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 701.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(4x^3-14x^2-2x\right)y''-\left(6x^2-7x+1\right)y'+(6x-1)y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve((4*x^3-14*x^2-2*x)*diff(y(x),x$2)-(6*x^2-7*x+1)*diff(y(x),x)+(6*x-1)*y(x)=0,y(x), sing
```

$$y(x)=c_1(x-1)+c_2\sqrt{x}\,(2x+1)$$

✓ Solution by Mathematica

Time used: 3.88 (sec). Leaf size: 26

```
DSolve[(4*x^3-14*x^2-2*x)*y''[x]-(6*x^2-7*x+1)*y'[x]+(6*x-1)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x)\to c_1(x-1)-2c_2\sqrt{x}(2x+1)$$

1.687 problem 702

Internal problem ID [7420]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 702.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x^2 + (x-2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)+(x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2 \mathrm{e}^{-x}(x^2+2x+2)}{x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 28

```
DSolve[x^2*y''[x]+x^2*y'[x]+(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 - c_2 e^{-x}(x(x+2)+2)}{x}$$

1.688 problem 703

Internal problem ID [7421]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 703.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x^2 + (x-2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)+(x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^x}{x} + \frac{c_2(x^2+2x+2)}{x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]-x^2*y'[x]+(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 e^x - c_2(x(x+2)+2)}{x}$$

1.689 problem 704

Internal problem ID [7422]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 704.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-4x)\,y'' + \left(-\frac{1}{4}x - x^2\right)y' - \frac{5yx}{16} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(x^2*(1-4*x)*diff(y(x),x$2)+((1-(5/4))*x-(6-4*(5/4))*x^2)*diff(y(x),x)+(5/4)*(1-(5/4))*
```

$$y(x) = c_1\,\text{hypergeom}\left(\left[-\frac{5}{8},-\frac{1}{8}\right],\left[-\frac{1}{4}\right],4x\right) + c_2 x^{\frac{5}{4}}\,\text{hypergeom}\left(\left[\frac{5}{8},\frac{9}{8}\right],\left[\frac{9}{4}\right],4x\right)$$

✓ Solution by Mathematica

Time used: 0.157 (sec). Leaf size: 111

```
DSolve[x^2*(1-4*x)*y''[x]+((1-(5/4))*x-(6-4*(5/4))*x^2)*y'[x]+(5/4)*(1-(5/4))*x*y[x]==0,y[x],
```

$$y(x) \to \frac{\sqrt[8]{x}\sqrt[4]{4x-1}\left(5c_1\left(\sqrt{4x-1}-i\right)^{5/4} + ic_2\left(\sqrt{4x-1}+i\right)^{5/4}\right)}{5\sqrt[4]{1-4x}\sqrt[8]{\sqrt{4x-1}-i}\sqrt[8]{\sqrt{4x-1}+i}}$$

1.690 problem 705

Internal problem ID [7423]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 705.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(x^2 + x\right)y' + (-9 + x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(x^2*diff(y(x),x$2)+(x+x^2)*diff(y(x),x)+(x-9)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2 - 8x + 20)}{x^3} + \frac{c_2 \mathrm{e}^{-x}(x^3 + 9x^2 + 36x + 60)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 40

```
DSolve[x^2*y''[x]+(x+x^2)*y'[x]+(x-9)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1((x-8)x + 20) - c_2 e^{-x}(x(x(x+9)+36)+60)}{x^3}$$

1.691 problem 706

Internal problem ID [7424]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 706.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(1+x)\,y' + (3x-1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x$2)+x*(x+1)*diff(y(x),x)+(3*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{-x}(x-3) + \frac{c_2(x^2\mathrm{e}^{-x}(x-3)\,\mathrm{Ei}_1(-x) + x^2 - 2x - 1)}{x}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 45

```
DSolve[x^2*y''[x]+x*(x+1)*y'[x]+(3*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}e^{-x}(x-3)x(c_2\,\mathrm{ExpIntegralEi}(x) + 6c_1) - \frac{c_2((x-2)x-1)}{6x}$$

1.692 problem 707

Internal problem ID [7425]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 707.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - \left(x^2 + 4x\right)y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x^2*diff(y(x),x$2)-(x^2+4*x)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x x^4 + c_2 x\left(\mathrm{e}^x x^3 \operatorname{Ei}_1(x) - x^2 + x - 2\right)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 40

```
DSolve[x^2*y''[x]-(x^2+4*x)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^x x^4 - \frac{1}{6}c_1 x\left(e^x x^3 \operatorname{ExpIntegralEi}(-x) + (x-1)x + 2\right)$$

1.693 problem 708

Internal problem ID [7426]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 708.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - (3x+2)\,y' + \frac{(2x-1)\,y}{x} = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 34

```
dsolve(2*x^2*diff(y(x),x$2)-(3*x+2)*diff(y(x),x)+(2*x-1)/x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(5x+2)}{\sqrt{x}} + c_2 \mathrm{e}^{-\frac{1}{x}}\,\mathrm{hypergeom}\left([2], \left[-\frac{1}{2}\right], \frac{1}{x}\right) x^2$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 60

```
DSolve[2*x^2*y''[x]-(3*x+2)*y'[x]+(2*x-1)/x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(5x+2)\left(3c_1 - 10\sqrt{\pi}c_2 \mathrm{erf}\left(\frac{1}{\sqrt{x}}\right)\right)}{15\sqrt{x}} + \frac{2}{3}c_2 e^{-1/x}((x-4)x-2)$$

1.694 problem 709

Internal problem ID [7427]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 709.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{x(1-x)\,y'' + \left(-2x + \frac{3}{2}\right) y' - \frac{y}{4} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(x*(1-x)*diff(y(x),x$2)+(3/2-2*x)*diff(y(x),x)-1/4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2 \ln\left(x - \frac{1}{2} + \sqrt{x\,(x-1)}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 50

```
DSolve[x*(1-x)*y''[x]+(3/2-2*x)*y'[x]-1/4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\frac{2c_2\sqrt{1-x}\log\left(\sqrt{x-1}-\sqrt{x}\right)}{\sqrt{x-1}} + c_1}{\sqrt{x}}$$

1.695 problem 710

Internal problem ID [7428]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 710.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x(1-x)\,y'' + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(2*x*(1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2\left(\arctan\left(\sqrt{x-1}\right)x - \sqrt{x-1}\right)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 43

```
DSolve[2*x*(1-x)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[4]{2}\left(c_2 x \operatorname{arctanh}\left(\sqrt{1-x}\right) + c_1 x - c_2\sqrt{1-x}\right)$$

1.696 problem 711

Internal problem ID [7429]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 711.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{2x(1-x)\,y'' + (1-11x)\,y' - 10y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(2*x*(1-x)*diff(y(x),x$2)+(1-11*x)*diff(y(x),x)-10*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2+6x+1)}{(x-1)^4} + \frac{c_2\sqrt{x}\,(x+1)}{(x-1)^4}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 34

```
DSolve[2*x*(1-x)*y''[x]+(1-11*x)*y'[x]-10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1\sqrt{x}(x+1) - 2c_2(x(x+6)+1)}{(x-1)^4}$$

1.697 problem 712

Internal problem ID [7430]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 712.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{x(1-x)\,y'' + \frac{(1-2x)\,y'}{3} + \frac{20y}{9} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(x*(1-x)*diff(y(x),x$2)+1/3*(1-2*x)*diff(y(x),x)+20/9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(6x-5)\,x^{\frac{2}{3}} + c_2(6x-1)\,(x-1)^{\frac{2}{3}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 48

```
DSolve[x*(1-x)*y''[x]+1/3*(1-2*x)*y'[x]+20/9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sqrt[3]{-((x-1)x)}\,Q_1^{\frac{2}{3}}(2x-1) + \frac{c_1 x^{2/3}(6x-5)}{\text{Gamma}\left(\frac{1}{3}\right)}$$

1.698 problem 713

Internal problem ID [7431]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 713.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y'' + \frac{3(-x^2+2)\,y}{(1-x^2)^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(4*diff(y(x),x$2)+3*(2-x^2)/(1-x^2)^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(x^2-1\right)^{\frac{3}{4}} + c_2 \left(x^2-1\right)^{\frac{1}{4}} x$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 51

```
DSolve[4*y''[x]+3*(2-x^2)/(1-x^2)^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x^2-1}\left(c_2 Q^{\frac{1}{2}}_{\frac{1}{2}}(x) + \frac{\sqrt{\frac{2}{\pi}}c_1 x}{\sqrt[4]{1-x^2}}\right)$$

1.699 problem 714

Internal problem ID [7432]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 714.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' - \frac{2u'}{x} - a^2 u = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(u(x),x$2)-2/x*diff(u(x),x)-a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = c_1 \mathrm{e}^{ax}(ax - 1) + c_2 \mathrm{e}^{-ax}(ax + 1)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 68

```
DSolve[u''[x]-2/x*u'[x]-a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{\sqrt{\frac{2}{\pi}}\sqrt{x}((iac_2x + c_1)\sinh(ax) - (ac_1x + ic_2)\cosh(ax))}{a\sqrt{-iax}}$$

1.700 problem 715

Internal problem ID [7433]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 715.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{2u'}{x} - a^2 u = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(diff(u(x),x$2)+2/x*diff(u(x),x)-a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1 \sinh(ax)}{x} + \frac{c_2 \cosh(ax)}{x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 35

```
DSolve[u''[x]+2/x*u'[x]-a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{2ac_1 e^{-ax} + c_2 e^{ax}}{2ax}$$

1.701 problem 716

Internal problem ID [7434]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 716.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{2u'}{x} + a^2 u = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(u(x),x$2)+2/x*diff(u(x),x)+a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1 \sin(ax)}{x} + \frac{c_2 \cos(ax)}{x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 42

```
DSolve[u''[x]+2/x*u'[x]+a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{e^{-iax}\left(2c_1 - \frac{ic_2 e^{2iax}}{a}\right)}{2x}$$

1.702 problem 717

Internal problem ID [7435]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 717.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{4u'}{x} - a^2 u = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve(diff(u(x),x$2)+4/x*diff(u(x),x)-a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1 \mathrm{e}^{ax}(ax-1)}{x^3} + \frac{c_2 \mathrm{e}^{-ax}(ax+1)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 68

```
DSolve[u''[x]+4/x*u'[x]-a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{\sqrt{\frac{2}{\pi}}((iac_2x + c_1)\sinh(ax) - (ac_1x + ic_2)\cosh(ax))}{ax^{5/2}\sqrt{-iax}}$$

1.703 problem 718

Internal problem ID [7436]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 718.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' + \frac{4u'}{x} + a^2 u = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 41

```
dsolve(diff(u(x),x$2)+4/x*diff(u(x),x)+a^2*u(x)=0,u(x), singsol=all)
```

$$u(x) = \frac{c_1(\cos(ax)\,ax - \sin(ax))}{x^3} + \frac{c_2(\cos(ax) + \sin(ax)\,ax)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 57

```
DSolve[u''[x]+4/x*u'[x]+a^2*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to -\frac{\sqrt{\frac{2}{\pi}}((ac_1x + c_2)\cos(ax) + (ac_2x - c_1)\sin(ax))}{x^{3/2}(ax)^{3/2}}$$

1.704 problem 719

Internal problem ID [7437]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 719.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - a^2 y - \frac{6y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)-a^2*y(x)=6*y(x)/x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{ax}(a^2x^2 - 3ax + 3)}{x^2} + \frac{c_2 \mathrm{e}^{-ax}(a^2x^2 + 3ax + 3)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 88

```
DSolve[y''[x]-a^2*y[x]==6*y[x]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\frac{2}{\pi}}(i(c_1(a^2x^2+3)+3iac_2x)\sinh(ax)+(ax(ac_2x-3ic_1)+3c_2)\cosh(ax))}{a^2x^{3/2}\sqrt{-iax}}$$

1.705 problem 720

Internal problem ID [7438]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 720.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + n^2 y - \frac{6y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$2)+n^2*y(x)=6*y(x)/x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1((n^2x^2-3)\cos(nx) - 3\sin(nx)\,nx)}{x^2} + \frac{c_2(3\cos(nx)\,nx + (n^2x^2-3)\sin(nx))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 77

```
DSolve[y''[x]+n^2*y[x]==6*y[x]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}\sqrt{x}((c_1(n^2x^2-3)+3c_2nx)\sin(nx) + (nx(3c_1 - c_2nx)+3c_2)\cos(nx))}{(nx)^{5/2}}$$

1.706 problem 721

Internal problem ID [7439]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 721.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x - \left(x^2 + \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(x^2+1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh(x)}{\sqrt{x}} + \frac{c_2 \cosh(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]+x*y'[x]-(x^2+1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(c_2 e^{2x} + 2c_1)}{2\sqrt{x}}$$

1.707 problem 722

Internal problem ID [7440]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 722.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \frac{(-9a^2 + 4x^2)\,y}{4a^2} = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 45

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(4*x^2-9*a^2)/(4*a^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{\frac{ix}{a}}(ix - a)}{x^{\frac{3}{2}}} + \frac{c_2 \mathrm{e}^{-\frac{ix}{a}}(ix + a)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 62

```
DSolve[x^2*y''[x]+x*y'[x]+(4*x^2-9*a^2)/(4*a^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}\left((ac_2 + c_1x)\cos\left(\frac{x}{a}\right) + (c_2x - ac_1)\sin\left(\frac{x}{a}\right)\right)}{x\sqrt{\frac{x}{a}}}$$

1.708 problem 723

Internal problem ID [7441]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 723.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{25}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 45

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-25/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{ix}(x^2 + 3ix - 3)}{x^{\frac{5}{2}}} + \frac{c_2 e^{-ix}(-x^2 + 3ix + 3)}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 57

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-25/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((3c_1x - c_2(x^2-3))\cos(x) + (c_1(x^2-3) + 3c_2x)\sin(x))}{x^{5/2}}$$

1.709 problem 724

Internal problem ID [7442]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 724.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + qy' - \frac{2y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+q*diff(y(x),x)=2*y(x)/x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1(qx-2)}{x} + \frac{c_2 \mathrm{e}^{-qx}(qx+2)}{x}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 80

```
DSolve[y''[x]+q*y'[x]==2*y[x]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{qx^{3/2}e^{-\frac{qx}{2}}\left(2(ic_2qx+2c_1)\sinh\left(\frac{qx}{2}\right)-2(c_1qx+2ic_2)\cosh\left(\frac{qx}{2}\right)\right)}{\sqrt{\pi}(-iqx)^{5/2}}$$

1.710 problem 725

Internal problem ID [7443]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 725.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + 3y' + 4yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+4*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x^2)}{x^2} + \frac{c_2 \cos(x^2)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 41

```
DSolve[x*y''[x]+3*y'[x]+4*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-ix^2} - ic_2 e^{ix^2}}{4x^2}$$

1.711 problem 726

Internal problem ID [7444]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 726.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(2-x)\,y'' + 2y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x^2*(2-x)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + \frac{c_2(x-1)}{x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 24

```
DSolve[x^2*(2-x)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1(x-2)^2 + c_2(x-1)}{x}$$

1.712 problem 727

Internal problem ID [7445]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 727.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right)y''-2y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2\left(x^2-1\right)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x-c_1(x-i)^2$$

1.713 problem 728

Internal problem ID [7446]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 728.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - 2(1+x)y' + (x+2)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+(x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x + c_2 \mathrm{e}^x x^3$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 23

```
DSolve[x*y''[x]-2*(x+1)*y'[x]+(x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^x \left(c_2 x^3 + 3c_1\right)$$

1.714 problem 729

Internal problem ID [7447]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 729.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3xy'' - 2(3x-1)\,y' + (3x-2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(3*x*diff(y(x),x$2)-2*(3*x-1)*diff(y(x),x)+(3*x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2 x^{\frac{1}{3}} \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 21

```
DSolve[3*x*y''[x]-2*(3*x-1)*y'[x]+(3*x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}\left(3c_2 \sqrt[3]{x} + c_1\right)$$

1.715 problem 730

Internal problem ID [7448]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 730.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(1+x)\,y'' - (-1+x)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x*(x+1)*diff(y(x),x$2)-(x-1)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x-1) + c_2(-4 + (x-1)\ln(x))$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 23

```
DSolve[x*(x+1)*y''[x]-(x-1)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x-1) + c_2((x-1)\log(x) - 4)$$

1.716 problem 731

Internal problem ID [7449]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 731.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+2x\right)y''-2(1+x)y'+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+2*x)*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=x^2c_1+c_2(x+1)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 19

```
DSolve[(x^2+2*x)*y''[x]-2*(x+1)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x^2-c_2(x+1)$$

1.717 problem 732

Internal problem ID [7450]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 732.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+2x\right)y''-2(1+x)y'+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+2*x)*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=x^2c_1+c_2(x+1)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 19

```
DSolve[(x^2+2*x)*y''[x]-2*(x+1)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x^2-c_2(x+1)$$

1.718 problem 733

Internal problem ID [7451]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 733.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right)y''-2y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2\left(x^2-1\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x-c_1(x-i)^2$$

1.719 problem 734

Internal problem ID [7452]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 734.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right)y''-2y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2\left(x^2-1\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x-c_1(x-i)^2$$

1.720 problem 735

Internal problem ID [7453]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 735.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y'x + \left(4x^2 - 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x^2} + c_2 x \,\mathrm{e}^{x^2}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 18

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(c_2 x + c_1)$$

1.721 problem 736

Internal problem ID [7454]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 736.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y'x + \left(4x^2 - 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x^2} + c_2 x \,\mathrm{e}^{x^2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 18

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(c_2 x + c_1)$$

1.722 problem 737

Internal problem ID [7455]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 737.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x-3)\,y'' - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve((2*x-3)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(-3+2x)^{\frac{7}{4}}\,\text{KummerM}\left(\frac{3}{4},\frac{11}{4},-\frac{3}{4}+\frac{x}{2}\right) + c_2 x$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 63

```
DSolve[(2*x-3)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\ 2^{3/4}(2x-3)\left(c_2(2x-3)^{3/4}L^{\frac{7}{4}}_{-\frac{3}{4}}\left(\frac{x}{2}-\frac{3}{4}\right) + \frac{4\sqrt{2}c_1 x}{2x-3}\right)$$

1.723 problem 738

Internal problem ID [7456]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 738.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - y'x - 3y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 48

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(\sqrt{\pi} \left(x^2+1\right) \mathrm{e}^{\frac{x^2}{2}} \operatorname{erf}\left(\frac{\sqrt{2}\, x}{2} \right) + \sqrt{2}\, x \right) + c_2 \mathrm{e}^{\frac{x^2}{2}} \left(x^2+1\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 35

```
DSolve[y''[x]-x*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{HermiteH}\left(-3, \frac{x}{\sqrt{2}}\right) + c_2 e^{\frac{x^2}{2}} \left(x^2+1\right)$$

1.724 problem 739

Internal problem ID [7457]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 739.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right)y''-y'x+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve((1+x^2)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2\left(\operatorname{arcsinh}\left(x\right)x-\sqrt{x^2+1}\right)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 29

```
DSolve[(1+x^2)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x\operatorname{arcsinh}(x)-c_2\sqrt{x^2+1}+c_1x$$

1.725 problem 740

Internal problem ID [7458]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 740.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(-2x\,\mathrm{e}^{\frac{x^2}{2}} + \sqrt{2}\,\sqrt{\pi}\,\operatorname{erfi}\left(\frac{\sqrt{2}\,x}{2}\right)(x-1)\,(x+1)\right) + c_2\left(x^2-1\right)$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 53

```
DSolve[y''[x]-x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(\left(x^2-1\right)\left(\sqrt{2\pi}c_2\operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right)+4c_1\right) - 2c_2 e^{\frac{x^2}{2}}x\right)$$

1.726 problem 741

Internal problem ID [7459]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 741.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(1-x^2\right)y''-y'+y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 62

```
dsolve((1-x^2)*diff(y(x),x$2)-diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\text{hypergeom}\left(\left[-\frac{1}{2}-\frac{\sqrt{5}}{2},\frac{\sqrt{5}}{2}-\frac{1}{2}\right],\left[-\frac{1}{2}\right],\frac{x}{2}+\frac{1}{2}\right)$$
$$+c_2(2x+2)^{\frac{3}{2}}\,\text{hypergeom}\left(\left[\frac{\sqrt{5}}{2}+1,-\frac{\sqrt{5}}{2}+1\right],\left[\frac{5}{2}\right],\frac{x}{2}+\frac{1}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.808 (sec). Leaf size: 198

```
DSolve[(1-x^2)*y''[x]-y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to\frac{\sqrt[4]{x+1}\left(\sqrt{x+1}-\sqrt{x-1}\right)^{-1-\sqrt{5}}\left(-2x+2\sqrt{x-1}\sqrt{x+1}+\sqrt{5}-3\right)e^{-\text{arctanh}\left(x-\sqrt{x-1}\sqrt{x+1}\right)}\left(c_2\int_1^x\frac{e^{2\text{arcta}}}{}\right.}{\sqrt[4]{1-x}}$$

1.727 problem 742

Internal problem ID [7460]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 742.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(1+x)^2 y'' + \left(1-x^2\right) y' + y(-1+x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(x*(x+1)^2*diff(y(x),x$2)+(1-x^2)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x+1) + c_2(x+1)\ln(x)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 17

```
DSolve[x*(x+1)^2*y''[x]+(1-x^2)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x+1)(c_2 \log(x) + c_1)$$

1.728 problem 743

Internal problem ID [7461]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 743.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2xy'' - y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
dsolve(2*x*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(2\cos\left(2\sqrt{x}\right)\sqrt{x} - \sin\left(2\sqrt{x}\right)\right) + c_2\left(2\sin\left(2\sqrt{x}\right)\sqrt{x} + \cos\left(2\sqrt{x}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 59

```
DSolve[2*x*y''[x]-y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2i\sqrt{x}}\left(2\sqrt{x} + i\right) + \frac{1}{8}c_2 e^{-2i\sqrt{x}}\left(1 + 2i\sqrt{x}\right)$$

1.729 problem 744

Internal problem ID [7462]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 744.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(x*diff(y(x),x$2)+x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(x^2 + 2x\right) + c_2\left(\frac{(-x-1)\,\mathrm{e}^{-x}}{2} + \frac{x\,\mathrm{Ei}_1\left(x\right)(x+2)}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 39

```
DSolve[x*y''[x]+x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x(x+2) - \frac{1}{2}c_2 e^{-x}(e^x(x+2)x\,\mathrm{ExpIntegralEi}(-x) + x + 1)$$

1.730 problem 745

Internal problem ID [7463]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 745.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(-1+x)^2 y'' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve(x*(x-1)^2*diff(y(x),x$2)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{x-1} + \frac{c_2(2\ln(x)\,x - x^2 + 1)}{x-1}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 31

```
DSolve[x*(x-1)^2*y''[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-x(c_2 x + c_1) + 2c_2 x \log(x) + c_2}{x-1}$$

1.731 problem 746

Internal problem ID [7464]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 746.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + yx^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x^2}{2}} \cos(x) + c_2 \mathrm{e}^{\frac{x^2}{2}} \sin(x)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 39

```
DSolve[y''[x]-2*x*y'[x]+x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{\frac{1}{2}x(x-2i)} \left(2c_1 - ic_2 e^{2ix}\right)$$

1.732 problem 747

Internal problem ID [7465]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 747.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x\left(-x^2+2\right)y''-\left(x^2+4x+2\right)\left(\left(1-x\right)y'+y\right)=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x*(2-x^2)*diff(y(x),x$2)-(x^2+4*x+2)*((1-x)*diff(y(x),x)+y(x))=0,y(x), singsol=all)
```

$$y(x)=c_1(x-1)+c_2\mathrm{e}^x x^2$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 21

```
DSolve[x*(2-x^2)*y''[x]-(x^2+4*x+2)*((1-x)*y'[x]+y[x])==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to c_1e^x x^2+c_2(x-1)$$

1.733 problem 748

Internal problem ID [7466]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 748.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+x)\,y'' - (2x+1)\,(y'x - y) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*(1+x)*diff(y(x),x$2)-(1+2*x)*(x*diff(y(x),x)-y(x))=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 x(x + \ln(x))$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 132

```
DSolve[x^2*(1+x)*y''[x]-(1+2*x)*(x*y'[x]+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x^{1+\sqrt{2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+\sqrt{2}-\frac{\sqrt{17}}{2}, \frac{1}{2}\left(-1+2\sqrt{2}+\sqrt{17}\right), 1+2\sqrt{2}, -x\right) + c_1 x^{1-\sqrt{2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-1-2\sqrt{2}-\sqrt{17}\right), \frac{1}{2}\left(-1-2\sqrt{2}+\sqrt{17}\right), 1-2\sqrt{2}, -x\right)$$

1.734 problem 749

Internal problem ID [7467]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 749.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(2-x)\,y'' - (4-x)\,xy' + (3-x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(2*(2-x)*x^2*diff(y(x),x$2)-(4-x)*x*diff(y(x),x)+(3-x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x\,(x-2)}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 41

```
DSolve[2*(2-x)*x^2*y''[x]-(4-x)*x*y'[x]+(3-x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{\sqrt[4]{x-2}\sqrt{x}\left(2c_2\sqrt{x-2}+c_1\right)}{\sqrt[4]{2-x}}$$

1.735 problem 750

Internal problem ID [7468]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 750.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\,x^2y'' + (5x-4)\,xy' + (6-9x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((1-x)*x^2*diff(y(x),x$2)+(5*x-4)*x*diff(y(x),x)+(6-9*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^3 + c_2x^2(\ln(x)\,x + 1)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 24

```
DSolve[(1-x)*x^2*y''[x]+(5*x-4)*x*y'[x]+(6-9*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to x^2(c_1x - c_2(x\log(x) + 1))$$

1.736 problem 751

Internal problem ID [7469]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 751.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + \left(4x^2+1\right)y' + 4x\left(x^2+1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+(4*x^2+1)*diff(y(x),x)+4*x*(x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} \ln\left(x\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 21

```
DSolve[x*y''[x]+(4*x^2+1)*y'[x]+4*x*(x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}\left(c_2 \log(x) + c_1\right)$$

1.737 problem 752

Internal problem ID [7470]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 752.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 8y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\left(-4x^3 + 10x\right)\mathrm{e}^{x^2} + 4\,\mathrm{erfi}\left(x\right)\sqrt{\pi}\left(x^4 - 3x^2 + \frac{3}{4}\right)\right) + c_2\left(4x^4 - 12x^2 + 3\right)$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 55

```
DSolve[y''[x]-2*x*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(\left(4\left(x^2-3\right)x^2+3\right)\left(\sqrt{\pi}c_2\mathrm{erfi}(x)+3c_1\right) - 2c_2e^{x^2}x\left(2x^2-5\right)\right)$$

1.738 problem 753

Internal problem ID [7471]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 753.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 8y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\left(-4x^3 + 10x\right)\mathrm{e}^{x^2} + 4\,\mathrm{erfi}\left(x\right)\sqrt{\pi}\left(x^4 - 3x^2 + \frac{3}{4}\right)\right) + c_2\left(4x^4 - 12x^2 + 3\right)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 55

```
DSolve[y''[x]-2*x*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(\left(4\left(x^2-3\right)x^2+3\right)\left(\sqrt{\pi}c_2\mathrm{erfi}(x)+3c_1\right) - 2c_2e^{x^2}x\left(2x^2-5\right)\right)$$

1.739 problem 754

Internal problem ID [7472]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 754.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(1-x^2\right)y''-2y'x+12y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(-\frac{5}{3}x^3+x\right)+c_2\left(-\frac{1}{9}+\frac{(5x^3-3x)\ln(x-1)}{24}+\frac{(-5x^3+3x)\ln(x+1)}{24}+\frac{5x^2}{12}\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 44

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{6}\left(c_2\left(3\left(5x^2-3\right)x\operatorname{arctanh}(x)-15x^2+4\right)+3c_1x\left(5x^2-3\right)\right)$$

1.740 problem 755

Internal problem ID [7473]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 755.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x+2)\,y'' + 2(1+x)\,y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(x*(x+2)*diff(y(x),x$2)+2*(x+1)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x+1) + c_2\left(\frac{(-x-1)\ln(x+2)}{2} + 1 + \frac{(x+1)\ln(x)}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 26

```
DSolve[x*(x+2)*y''[x]+2*(x+1)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2(x+1)\text{arctanh}(x+1) + c_1(x+1) - c_2$$

1.741 problem 757

Internal problem ID [7474]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 757.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$\boxed{x(x+2)y'' + (1+x)y' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x*(x+2)*diff(y(x),x$2)+(x+1)*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(2x^2+4x+1\right) + c_2(x+1)\sqrt{x\,(x+2)}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 53

```
DSolve[x*(x+2)*y''[x]+(x+1)*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cosh\left(4\log\left(\sqrt{x+2}-\sqrt{x}\right)\right) - ic_2 \sinh\left(4\log\left(\sqrt{x+2}-\sqrt{x}\right)\right)$$

1.742 problem 758

Internal problem ID [7475]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 758.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-1+x)\,y'' - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2 x$$

1.743 problem 759

Internal problem ID [7476]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 759.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right)y''-2y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve((1+x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2\left(x^2-1\right)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 21

```
DSolve[(1+x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x-c_1(x-i)^2$$

1.744 problem 760

Internal problem ID [7477]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 760.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-2x+10\right)y''+y'x-4y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 40

```
dsolve((x^2-2*x+10)*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^2-\frac{4}{3}x+5\right)+c_2(3x-4)\left(x-1+3i\right)^{\frac{1}{2}-\frac{i}{6}}\left(x-1-3i\right)^{\frac{1}{2}+\frac{i}{6}}$$

✓ Solution by Mathematica

Time used: 0.198 (sec). Leaf size: 90

```
DSolve[(x^2-2*x+10)*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}&y(x)\\&\to\frac{1}{3}(3x\\&\quad-4)\sqrt{(x-2)x+10}e^{-\frac{1}{3}\arctan\left(\frac{x-1}{3}\right)}\left(c_2\int_1^x\frac{9e^{\frac{1}{3}\arctan\left(\frac{1}{3}(K[1]-1)\right)}}{(4-3K[1])^2((K[1]-2)K[1]+10)^{3/2}}dK[1]\right.\\&\qquad\left.+c_1\right)\end{aligned}$$

1.745 problem 761

Internal problem ID [7478]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 761.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-2x+10\right)y''+y'x-4y=0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 40

```
dsolve((x^2-2*x+10)*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^2-\frac{4}{3}x+5\right)+c_2(3x-4)\left(x-1+3i\right)^{\frac{1}{2}-\frac{i}{6}}\left(x-1-3i\right)^{\frac{1}{2}+\frac{i}{6}}$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 90

```
DSolve[(x^2-2*x+10)*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}&y(x)\\&\to\frac{1}{3}(3x\\&\quad-4)\sqrt{(x-2)x+10}e^{-\frac{1}{3}\arctan\left(\frac{x-1}{3}\right)}\left(c_2\int_1^x\frac{9e^{\frac{1}{3}\arctan\left(\frac{1}{3}(K[1]-1)\right)}}{(4-3K[1])^2((K[1]-2)K[1]+10)^{3/2}}dK[1]\right.\\&\qquad\left.+c_1\right)\end{aligned}$$

1.746 problem 762

Internal problem ID [7479]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 762.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(-2x\,\mathrm{e}^{\frac{x^2}{2}} + \sqrt{2}\,\sqrt{\pi}\,\operatorname{erfi}\left(\frac{\sqrt{2}\,x}{2}\right)(x-1)\,(x+1)\right) + c_2\left(x^2-1\right)$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 53

```
DSolve[y''[x]-x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(\left(x^2-1\right)\left(\sqrt{2\pi}c_2\operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right)+4c_1\right) - 2c_2 e^{\frac{x^2}{2}}x\right)$$

1.747 problem 763

Internal problem ID [7480]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 763.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+2)\,y'' + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve((x+2)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 \mathrm{e}^{-x}(x+4)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 71

```
DSolve[(x+2)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\sqrt{\frac{2}{\pi}}e^{-x-2}\sqrt{x+2}((c_1 - ic_2)e^{x+2}x + (c_1 + ic_2)(x+4))}{\sqrt{-i(x+2)}}$$

1.748 problem 764

Internal problem ID [7481]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 764.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{\left(x^2+1\right)y''-6y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve((x^2+1)*diff(y(x),x$2)-6*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x^3+x\right)+c_2\left(\frac{\left(3x^3+3x\right)\arctan\left(x\right)}{2}+\frac{3x^2}{2}+1\right)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 36

```
DSolve[(x^2+1)*y''[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\left(x^3+x\right)-\frac{1}{2}c_2\left(3\left(x^3+x\right)\arctan(x)+3x^2+2\right)$$

1.749 problem 765

Internal problem ID [7482]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 765.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+2\right)y''+3y'x-y=0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 51

```
dsolve((x^2+2)*diff(y(x),x$2)+3*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\left(\sqrt{x^2+2}+x\right)^{\sqrt{2}}}{\sqrt{x^2+2}}+\frac{c_2\left(\sqrt{x^2+2}+x\right)^{-\sqrt{2}}}{\sqrt{x^2+2}}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 80

```
DSolve[(x^2+2)*y''[x]+3*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2c_1\cos\left(\sqrt{2}\arccos\left(\frac{ix}{\sqrt{2}}\right)\right)-\pi c_2\sin\left(2\sqrt{2}\csc^{-1}\left(\frac{2}{\sqrt{2-i\sqrt{2}x}}\right)\right)}{\sqrt[4]{2}\sqrt{\pi}\sqrt{x^2+2}}$$

1.750 problem 766

Internal problem ID [7483]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 766.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-1+x)\,y'' - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2 x$$

1.751 problem 767

Internal problem ID [7484]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 767.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 8y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(\left(-4x^3 + 10x\right)\mathrm{e}^{x^2} + 4\,\mathrm{erfi}\left(x\right)\sqrt{\pi}\left(x^4 - 3x^2 + \frac{3}{4}\right)\right) + c_2\left(4x^4 - 12x^2 + 3\right)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 49

```
DSolve[y''[x]-2*x*y''[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{4x-2}\left(c_1\,\mathrm{BesselI}\left(1, 4\sqrt{x-\frac{1}{2}}\right) - c_2 K_1\left(4\sqrt{x-\frac{1}{2}}\right)\right)$$

1.752 problem 769

Internal problem ID [7485]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 769.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(\frac{5}{3}x + x^2\right)y' - \frac{y}{3} = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x$2)+(5/3*x+x^2)*diff(y(x),x)-1/3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} x^{\frac{1}{3}} \operatorname{hypergeom}\left([2], \left[\frac{7}{3}\right], x\right) + \frac{c_2 \mathrm{e}^{-x} \operatorname{hypergeom}\left(\left[\frac{2}{3}\right], \left[-\frac{1}{3}\right], x\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 43

```
DSolve[x^2*y''[x]+(5/3*x+x^2)*y'[x]-1/3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(3x-1)\left(c_2\Gamma\left(\frac{1}{3},x\right)+c_1\right)}{3x} - \frac{c_2 e^{-x}}{x^{2/3}}$$

1.753 problem 770

Internal problem ID [7486]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 770.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2xy'' - y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
dsolve(2*x*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(2\cos\left(2\sqrt{x}\right)\sqrt{x} - \sin\left(2\sqrt{x}\right)\right) + c_2\left(2\sin\left(2\sqrt{x}\right)\sqrt{x} + \cos\left(2\sqrt{x}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 59

```
DSolve[2*x*y''[x]-y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2i\sqrt{x}}\left(2\sqrt{x} + i\right) + \frac{1}{8}c_2 e^{-2i\sqrt{x}}\left(1 + 2i\sqrt{x}\right)$$

1.754 problem 771

Internal problem ID [7487]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 771.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{2xy'' - (3+2x)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 25

```
dsolve(2*x*diff(y(x),x$2)-(3+2*x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\mathrm{hypergeom}\left([2], \left[\frac{7}{2}\right], x\right) x^{\frac{5}{2}} + c_2\left(-\frac{2x}{3}+1\right)\mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 48

```
DSolve[2*x*y''[x]-(3+2*x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(e^x(2x-3)\left(2c_1 - \sqrt{\pi}c_2 \mathrm{erf}\left(\sqrt{x}\right)\right) - 6c_2\sqrt{x}\right)$$

1.755 problem 772

Internal problem ID [7488]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 772.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + 3y'x + (2x-1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 85

```
dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+(2*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{2i\sqrt{x}}\sqrt{1+4x}\,\sqrt{\frac{2\sqrt{x}+i}{-2\sqrt{x}+i}}}{x} + \frac{c_2 \mathrm{e}^{-2i\sqrt{x}}\sqrt{1+4x}\,\sqrt{\frac{-2\sqrt{x}+i}{2\sqrt{x}+i}}}{x}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 63

```
DSolve[2*x^2*y''[x]+3*x*y'[x]+(2*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-2i\sqrt{x}}\left(8c_1 e^{4i\sqrt{x}}\left(2\sqrt{x}+i\right)+2ic_2\sqrt{x}+c_2\right)}{8x}$$

1.756 problem 773

Internal problem ID [7489]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 773.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 2y' - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh(x)}{x} + \frac{c_2 \cosh(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 28

```
DSolve[x*y''[x]+2*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-x} + c_2 e^{x}}{2x}$$

1.757 problem 774

Internal problem ID [7490]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 774.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2 e^{2ix})}{2\sqrt{x}}$$

1.758 problem 775

Internal problem ID [7491]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 775.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (x-6)\,y' - 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x$2)+(x-6)*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\left(x^3 - 12x^2 + 60x - 120\right) + c_2 \mathrm{e}^{-x}\left(x^3 + 12x^2 + 60x + 120\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 92

```
DSolve[x*y''[x]+(x-6)*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to \frac{2e^{-x/2}\sqrt{x}\left(\left(c_1 x(x^2+60) + 12ic_2(x^2+10)\right)\cosh\left(\frac{x}{2}\right) - \left(12c_1(x^2+10) + ic_2 x(x^2+60)\right)\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{\pi}\sqrt{-ix}}$$

1.759 problem 776

Internal problem ID [7492]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 776.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^4 y'' + \lambda y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(x^4*diff(y(x),x$2)+lambda*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sinh\left(\frac{\sqrt{-\lambda}}{x}\right) + c_2 x \cosh\left(\frac{\sqrt{-\lambda}}{x}\right)$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 52

```
DSolve[x^4*y''[x]+\[Lambda]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x e^{\frac{i\sqrt{\lambda}}{x}} - \frac{i c_2 x e^{-\frac{i\sqrt{\lambda}}{x}}}{2\sqrt{\lambda}}$$

1.760 problem 777

Internal problem ID [7493]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 777.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4y'x + \left(4x^2 - 25\right)y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2-25)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{ix}(x^2 + 3ix - 3)}{x^{\frac{5}{2}}} + \frac{c_2 \mathrm{e}^{-ix}(-x^2 + 3ix + 3)}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 57

```
DSolve[4*x^2*y''[x]+4*x*y'[x]+(4*x^2-25)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((3c_1x - c_2(x^2 - 3))\cos(x) + (c_1(x^2 - 3) + 3c_2x)\sin(x))}{x^{5/2}}$$

1.761 problem 778

Internal problem ID [7494]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 778.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(36x^2 - \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(36*x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(6x)}{\sqrt{x}} + \frac{c_2 \cos(6x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(36*x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-6ix}(12c_1 - ic_2 e^{12ix})}{12\sqrt{x}}$$

1.762 problem 779

Internal problem ID [7495]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 779.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(x^2 - 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve(x^2*diff(y(x),x$2)+(x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(-\sin(x) + \cos(x)\, x)}{x} + \frac{c_2(\cos(x) + x\sin(x))}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]+(x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((c_1 x + c_2)\cos(x) + (c_2 x - c_1)\sin(x))}{x}$$

1.763 problem 780

Internal problem ID [7496]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 780.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + 3y' + yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{x^2}{2}\right)}{x^2} + \frac{c_2 \cos\left(\frac{x^2}{2}\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 43

```
DSolve[x*y''[x]+3*y'[x]+x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{ix^2}{2}}\left(2c_1 - ic_2 e^{ix^2}\right)}{2x^2}$$

1.764 problem 781

Internal problem ID [7497]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 781.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 4y'x + \left(x^2 + 2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x^2} + \frac{c_2 \cos(x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+4*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x^2}$$

1.765 problem 782

Internal problem ID [7498]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 782.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16x^2y'' + 32y'x + \left(x^4 - 12\right)y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve(16*x^2*diff(y(x),x$2)+32*x*diff(y(x),x)+(x^4-12)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{x^2}{8}\right)}{x^{\frac{3}{2}}} + \frac{c_2 \cos\left(\frac{x^2}{8}\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 42

```
DSolve[16*x^2*y''[x]+32*x*y'[x]+(x^4-12)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{ix^2}{8}}\left(c_1 - 2ic_2 e^{\frac{ix^2}{4}}\right)}{x^{3/2}}$$

1.766 problem 783

Internal problem ID [7499]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 783.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x^2 + yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + \frac{c_2\left(3^{\frac{1}{3}}(-x^3)^{\frac{2}{3}}\,\mathrm{e}^{\frac{x^3}{3}} - x^3\left(\Gamma\left(\frac{2}{3}\right) - \Gamma\left(\frac{2}{3}, -\frac{x^3}{3}\right)\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 27

```
DSolve[y''[x]-x^2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{1}{3}c_2 \operatorname{ExpIntegralE}\left(\frac{4}{3}, -\frac{x^3}{3}\right)$$

1.767 problem 784

Internal problem ID [7500]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 784.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - (x+2)\,y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)-(x+2)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2\left(x^2 + 2x + 2\right)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 23

```
DSolve[x*y''[x]-(x+2)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} - c_2(x(x+2)+2)$$

1.768 problem 785

Internal problem ID [7501]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 785.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \,\mathrm{e}^{-\frac{x^2}{2}} + c_2 \left(i\sqrt{\pi}\,\sqrt{2} - \operatorname{erf}\left(\frac{i\sqrt{2}\,x}{2} \right) \mathrm{e}^{-\frac{x^2}{2}} \pi x \right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 44

```
DSolve[y''[x]+x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}c_2 x \,\mathrm{DawsonF}\left(\frac{x}{\sqrt{2}} \right) + \sqrt{2}c_1 e^{-\frac{x^2}{2}} x + c_2$$

1.769 problem 786

Internal problem ID [7502]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 786.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(1-x^2\right)y''-2y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2\left(\frac{\ln\left(x-1\right)x}{2}-\frac{\ln\left(x+1\right)x}{2}+1\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 19

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2(x\operatorname{arctanh}(x)-1)+c_1x$$

1.770 problem 787

Internal problem ID [7503]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 787.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y'x + \left(4x^2 - 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x^2} + c_2 x \,\mathrm{e}^{x^2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(c_2 x + c_1)$$

1.771 problem 788

Internal problem ID [7504]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 788.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(1-x^2\right)y''-2y'x+30y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 73

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+30*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(\frac{21}{5}x^5-\frac{14}{3}x^3+x\right)+c_2\left(\frac{1}{225}+\frac{\left(63x^5-70x^3+15x\right)\ln\left(x-1\right)}{1920}+\frac{\left(-63x^5+70x^3-15x\right)\ln\left(x+1\right)}{1920}+\frac{21x^4}{320}-\frac{49x^2}{960}\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 66

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+30*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{8}c_2x\left(63x^4-70x^2+15\right)\operatorname{arctanh}(x)+\frac{1}{8}x\left(7c_2x\left(7-9x^2\right)+c_1\left(63x^4-70x^2+15\right)\right)-\frac{8c_2}{15}$$

1.772 problem 789

Internal problem ID [7505]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 789.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.773 problem 790

Internal problem ID [7506]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 790.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (2x+1)\,y' + (1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+(2*x+1)*diff(y(x),x)+(x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 \mathrm{e}^{-x} \ln(x)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 19

```
DSolve[x*y''[x]+(2*x+1)*y'[x]+(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2 \log(x) + c_1)$$

1.774 problem 791

Internal problem ID [7507]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 791.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{2x(-1+x)\,y'' - (1+x)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*x*(x-1)*diff(y(x),x$2)-(x+1)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x+1) + c_2\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 21

```
DSolve[2*x*(x-1)*y''[x]-(x+1)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\sqrt{x} - 2c_2(x+1)$$

1.775 problem 792

Internal problem ID [7508]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 792.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 2y' + 4yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+4*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(2x)}{x} + \frac{c_2 \cos(2x)}{x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+4*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-2ix} - ic_2 e^{2ix}}{4x}$$

1.776 problem 793

Internal problem ID [7509]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 793.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (-2x+2)\,y' + (x-2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)+(2-2*x)*diff(y(x),x)+(x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x + \frac{c_2 \mathrm{e}^x}{x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 19

```
DSolve[x*y''[x]+(2-2*x)*y'[x]+(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x(c_2 x + c_1)}{x}$$

1.777 problem 794

Internal problem ID [7510]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 794.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 6y'x + \left(4x^2 + 6\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+6*x*diff(y(x),x)+(4*x^2+6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(2x)}{x^3} + \frac{c_2 \cos(2x)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+6*x*y'[x]+(4*x^2+6)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-2ix} - ic_2 e^{2ix}}{4x^3}$$

1.778 problem 795

Internal problem ID [7511]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 795.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1-2x)\,y' + y(-1+x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2 \mathrm{e}^{x} \ln(x)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 17

```
DSolve[x*y''[x]+(1-2*x)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(c_2 \log(x) + c_1)$$

1.779 problem 796

Internal problem ID [7512]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 796.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{x(1-x)\,y'' + \left(\frac{1}{2}+2x\right)y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(x*(1-x)*diff(y(x),x$2)+(1/2+2*x)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(1+4x) + c_2\left((-12x-3)\ln\left(x-\frac{1}{2}+\sqrt{x\,(x-1)}\right) + (4x+26)\sqrt{x\,(x-1)}\right)$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 62

```
DSolve[x*(1-x)*y''[x]+(1/2+2*x)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(x+\frac{1}{4}\right) + \frac{1}{2}c_2\left(\sqrt{-((x-1)x)}(2x+13) - 6(4x+1)\cot^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{1-x}}\right)\right)$$

1.780 problem 797

Internal problem ID [7513]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 797.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4\left(t^2-3t+2\right)y''-2y'+y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 48

```
dsolve(4*(t^2-3*t+2)*diff(y(t),t$2)-2*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1\sqrt{t-1}+\frac{c_2\left(-\frac{\sqrt{t^2-3t+2}\,\ln\left(t-\frac{3}{2}+\sqrt{t^2-3t+2}\right)}{2}+t-2\right)}{\sqrt{t-2}}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 49

```
DSolve[4*(t^2-3*t+2)*y''[t]-2*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\sqrt{1-t}\left(\frac{2c_2}{\sqrt{\frac{1}{t-2}+1}}-2c_2\coth^{-1}\left(\sqrt{\frac{1}{t-2}+1}\right)+c_1\right)$$

1.781 problem 798

Internal problem ID [7514]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 798.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2\left(t^2-5t+6\right)y''+(2t-3)\,y'-8y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve(2*(t^2-5*t+6)*diff(y(t),t$2)+(2*t-3)*diff(y(t),t)-8*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1\left(t^2-\frac{13}{3}t+\frac{37}{8}\right)+\frac{c_2(6t-17)\,(t-2)^{\frac{3}{2}}}{\sqrt{t-3}}$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 78

```
DSolve[2*(t^2-5*t+6)*y''[t]+(2*t-3)*y'[t]-8*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\frac{\sqrt[4]{2-t}\sqrt[4]{\frac{t-3}{t-2}}\left(5c_1(6t-17)(t-2)^{3/2}+24c_2\sqrt{t-3}(8t(3t-13)+111)\right)}{30(3-t)^{3/4}}$$

1.782 problem 799

Internal problem ID [7515]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 799.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3t(t+1)\,y'' + ty' - y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 66

```
dsolve(3*t*(1+t)*diff(y(t),t$2)+t*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2 \left(2\sqrt{3}\,\arctan\left(\frac{\left(2(t+1)^{\frac{1}{3}}+1\right)\sqrt{3}}{3} \right) t + 6(t+1)^{\frac{2}{3}} + 2\ln\left((t+1)^{\frac{1}{3}}-1\right) t - \ln\left((t+1)^{\frac{2}{3}} + (t+1)^{\frac{1}{3}} + 1\right) t \right)$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 90

```
DSolve[3*t*(1+t)*y''[t]+t*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 t\left(-2\sqrt{3}\arctan\left(\frac{2\sqrt[3]{t+1}+1}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{t+1}-1\right) + \log\left((t+1)^{2/3} + \sqrt[3]{t+1} + 1\right)\right) + 6c_1 t - 6c_2(t}{6\sqrt[6]{3}}$$

1.783 problem 800

Internal problem ID [7516]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 800.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + \frac{\left(x + \frac{3}{4}\right) y}{4} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+1/4*(x+3/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\sqrt{x}\right) x^{\frac{1}{4}} + c_2 x^{\frac{1}{4}} \cos\left(\sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 43

```
DSolve[x^2*y''[x]+1/4*(x+3/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-i\sqrt{x}} \sqrt[4]{x} \left(c_1 e^{2i\sqrt{x}} + i c_2\right)$$

1.784 problem 801

Internal problem ID [7517]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 801.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \frac{(x^2-1)\,y}{4} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+1/4*(x^2-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{x}{2}\right)}{\sqrt{x}} + \frac{c_2 \cos\left(\frac{x}{2}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]+x*y'[x]+1/4*(x^2-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{ix}{2}}\left(c_2(\sin(x) - i\cos(x)) + c_1\right)}{\sqrt{x}}$$

1.785 problem 802

Internal problem ID [7518]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 802.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1-2x)\,y' + y(-1+x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2 \mathrm{e}^{x} \ln(x)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 17

```
DSolve[x*y''[x]+(1-2*x)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(c_2 \log(x) + c_1)$$

1.786 problem 803

Internal problem ID [7519]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 803.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - (1+x)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x$2)-(x+1)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x+1) + c_2 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 19

```
DSolve[x*y''[x]-(x+1)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2(x+1)$$

1.787 problem 804

Internal problem ID [7520]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 804.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + 3y' + 4yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+4*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x^2)}{x^2} + \frac{c_2 \cos(x^2)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 41

```
DSolve[x*y''[x]+3*y'[x]+4*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-ix^2} - ic_2 e^{ix^2}}{4x^2}$$

1.788 problem 805

Internal problem ID [7521]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 805.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x^2)\,y'' + 2x(1-x^2)\,y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
dsolve(x^2*(1-x^2)*diff(y(x),x$2)+2*x*(1-x^2)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2-1)}{x^2} + \frac{c_2(\ln(x-1)\,x^2 - \ln(x+1)\,x^2 - \ln(x-1) + \ln(x+1) - 2x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 35

```
DSolve[x^2*(1-x^2)*y''[x]+2*x*(1-x^2)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{c_2((x^2-1)\operatorname{arctanh}(x) + x) - 2c_1(x^2-1)}{2x^2}$$

1.789 problem 806

Internal problem ID [7522]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 806.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + (x-2)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve(2*x*diff(y(x),x$2)+(x-2)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x-2) + c_2 \mathrm{e}^{-\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 23

```
DSolve[2*x*y''[x]+(x-2)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x/2} + 2c_2(x-2)$$

1.790 problem 807

Internal problem ID [7523]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 807.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.791 problem 808

Internal problem ID [7524]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 808.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y'x^2 + \left(x^4 + 2x - 1\right) y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+2*x^2*diff(y(x),x)+(x^4+2*x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x\left(x^2+3\right)}{3}} + c_2 \mathrm{e}^{-\frac{x\left(x^2-3\right)}{3}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 34

```
DSolve[y''[x]+2*x^2*y'[x]+(x^4+2*x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-\frac{1}{3}x\left(x^2+3\right)} \left(c_2 e^{2x} + 2c_1\right)$$

1.792 problem 809

Internal problem ID [7525]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 809.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{u'' + 2u' + u = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(diff(u(x),x$2)+2*diff(u(x),x)+u(x)=0,u(x), singsol=all)
```

$$u(x) = \mathrm{e}^{-x} c_1 + \mathrm{e}^{-x} c_2 x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[u''[x]+2*u'[x]+u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to e^{-x}(c_2 x + c_1)$$

1.793 problem 810

Internal problem ID [7526]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 810.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{u'' - (2x+1)\,u' + \left(x^2 + x - 1\right) u = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve(diff(u(x),x$2)-(2*x+1)*diff(u(x),x)+(x^2+x-1)*u(x)=0,u(x), singsol=all)
```

$$u(x) = c_1 \mathrm{e}^{\frac{x^2}{2}} + c_2 \mathrm{e}^{\frac{x(x+2)}{2}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 24

```
DSolve[u''[x]-(2*x+1)*u'[x]+(x^2+x-1)*u[x]==0,u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to e^{\frac{x^2}{2}} \left(c_2 e^x + c_1\right)$$

1.794 problem 811

Internal problem ID [7527]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 811.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' + \left(1 + \frac{2}{(1+3x)^2}\right) y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+(1+2/(1+3*x)^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 (3x+1)^{\frac{1}{3}} \mathrm{e}^{-x} + c_2 (3x+1)^{\frac{2}{3}} \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 35

```
DSolve[y''[x]+2*y'[x]+(1+2/(1+3*x)^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \sqrt[3]{3x+1} \left(c_2 \sqrt[3]{3x+1} + c_1\right)$$

1.795 problem 812

Internal problem ID [7528]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 812.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + \left(x^2 + 2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

1.796 problem 813

Internal problem ID [7529]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 813.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{2y'}{x} - \frac{2y}{(1+x)^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+2/x*diff(y(x),x)-2/(1+x)^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x\,(x+1)} + \frac{c_2(x^2+3x+3)}{x+1}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 33

```
DSolve[y''[x]+2/x*y'[x]-2/(1+x)^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x(x(x+3)+3)+3c_1}{3x(x+1)}$$

1.797 problem 815

Internal problem ID [7530]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 815.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(i(x+2)\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-2-x} + 2\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(\sqrt{2}(x+2)\left(2c_1 - \sqrt{\pi}c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}}\right)\right) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

1.798 problem 816

Internal problem ID [7531]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 816.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(i(x+2)\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-2-x} + 2\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(\sqrt{2}(x+2)\left(2c_1 - \sqrt{\pi}c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}}\right)\right) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

1.799 problem 817

Internal problem ID [7532]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 817.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(i(x+2)\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-2-x} + 2\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(\sqrt{2}(x+2)\left(2c_1 - \sqrt{\pi}c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}}\right)\right) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

1.800 problem 818

Internal problem ID [7533]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 818.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(i(x+2)\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-2-x} + 2\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(\sqrt{2}(x+2)\left(2c_1 - \sqrt{\pi}c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}}\right)\right) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

1.801 problem 819

Internal problem ID [7534]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 819.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(i(x+2)\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-2-x} + 2\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(\sqrt{2}(x+2)\left(2c_1 - \sqrt{\pi}c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}}\right)\right) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

1.802 problem 820

Internal problem ID [7535]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 820.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(i(x+2)\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-2-x} + 2\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(\sqrt{2}(x+2)\left(2c_1 - \sqrt{\pi}c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}}\right)\right) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

1.803 problem 821

Internal problem ID [7536]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 821.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(i(x+2)\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-2-x} + 2\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(\sqrt{2}(x+2)\left(2c_1 - \sqrt{\pi}c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}}\right)\right) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

1.804 problem 822

Internal problem ID [7537]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 822.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(i(x+2)\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-2-x} + 2\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(\sqrt{2}(x+2)\left(2c_1 - \sqrt{\pi}c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}}\right)\right) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

1.805 problem 823

Internal problem ID [7538]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 823.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(i(x+2)\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-2-x} + 2\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(\sqrt{2}(x+2)\left(2c_1 - \sqrt{\pi}c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}}\right)\right) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

1.806 problem 824

Internal problem ID [7539]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 824.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(i(x+2)\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-2-x} + 2\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(\sqrt{2}(x+2)\left(2c_1 - \sqrt{\pi}c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}}\right)\right) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

1.807 problem 825

Internal problem ID [7540]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 825.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x+2) + c_2 \left(i(x+2)\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-2-x} + 2\,\mathrm{e}^{\frac{x(x+2)}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 63

```
DSolve[y''[x]-x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(\sqrt{2}(x+2)\left(2c_1 - \sqrt{\pi}c_2 \operatorname{erfi}\left(\frac{x+2}{\sqrt{2}}\right)\right) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

1.808 problem 826

Internal problem ID [7541]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 826.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

1.809 problem 827

Internal problem ID [7542]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 827.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2x^2y'' + 3y'x - yx = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh\left(\sqrt{x}\,\sqrt{2}\right)}{\sqrt{x}} + \frac{c_2 \cosh\left(\sqrt{x}\,\sqrt{2}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 56

```
DSolve[2*x^2*y''[x]+3*x*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\sqrt{2}\sqrt{x}}\left(2c_1 e^{2\sqrt{2}\sqrt{x}} - \sqrt{2}c_2\right)}{2\sqrt{x}}$$

1.810 problem 828

Internal problem ID [7543]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 828.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(3x^2 + 2x\right)y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x), x, x) + (2*x+3*x^2)*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{-3x}}{x^2} + \frac{c_2(9x^2 - 6x + 2)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]+(2*x+3*x^2)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1(9x^2 - 6x + 2) + 27c_2e^{-3x}}{27x^2}$$

1.811 problem 829

Internal problem ID [7544]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 829.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2\left(x^2+x+1\right)y''+x\left(11x^2+11x+9\right)y'+\left(7x^2+10x+6\right)y=0}$$

✓ Solution by Maple

Time used: 0.312 (sec). Leaf size: 362

```
dsolve(2*x^2*(1+x+x^2)*diff(y(x), x$2) + x*(9+11*x+11*x^2)*diff(y(x), x) + (6+10*x+7*x^2)*y(x
```

$$y(x)$$
$$= \frac{c_1 \mathrm{e}^{-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}} \operatorname{HeunG}\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}, 0, 0, \frac{5}{2}, \frac{1}{2}, \frac{5i\sqrt{3}-3}{3i\sqrt{3}-3}, -\frac{2x}{1+i\sqrt{3}}\right)\left(i\sqrt{3}+2x+1\right)^{\frac{5i\sqrt{3}-3}{-6+6i\sqrt{3}}}\left(i\sqrt{3}-2x-1\right)^{\overline{(1+i}}}{\left(x^2+x+1\right)^{\frac{1}{4}}x^2}$$
$$+ \frac{c_2 \mathrm{e}^{-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}} \operatorname{HeunG}\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}, \frac{16}{\left(1+i\sqrt{3}\right)^3\left(i\sqrt{3}-1\right)^2}, \frac{1}{2}, 3, \frac{3}{2}, \frac{5i\sqrt{3}-3}{3i\sqrt{3}-3}, -\frac{2x}{1+i\sqrt{3}}\right)\left(i\sqrt{3}+2x+1\right)^{\frac{5i\sqrt{3}-3}{-6+6i\sqrt{3}}}\left(}{\left(x^2+x+1\right)^{\frac{1}{4}}x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.192 (sec). Leaf size: 93

```
DSolve[2*x^2*(1+x+x^2)*y''[x] + x*(9+11*x+11*x^2)*y'[x] + (6+10*x+7*x^2)*y[x] == 0,y[x],x,Inc
```

$$y(x) \to \frac{\sqrt{x^2+x+1}e^{-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}}\left(c_2\int_1^x \frac{e^{\frac{\arctan\left(\frac{2K[1]+1}{\sqrt{3}}\right)}{\sqrt{3}}}}{\sqrt{K[1]}(K[1]^2+K[1]+1)^{3/2}}dK[1]+c_1\right)}{x^2}$$

1.812 problem 830

Internal problem ID [7545]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 830.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1+x)\,y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x*diff(y(x), x$2) +(1+x)*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x}(x-1) + c_2\left(\mathrm{e}^{-x}(x-1)\,\mathrm{Ei}_1\left(-x\right) + 1\right)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 27

```
DSolve[x*y''[x] +(1+x)*y'[x]+2*y[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(x-1)(c_2\,\mathrm{ExpIntegralEi}(x) + c_1) - c_2$$

1.813 problem 831

Internal problem ID [7546]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 831.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2-2x+1\right)y''-x(x+3)y'+(x+4)y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
dsolve(x^2*(1-2*x+x^2)*diff(y(x), x$2) -x*(3+x)*diff(y(x),x)+(4+x)*y(x) = 0,y(x), singsol=all
```

$$y(x)=\frac{c_1x^2\mathrm{e}^{-\frac{4}{x-1}}}{x-1}+\frac{c_2x^2\,\mathrm{Ei}_1\left(-\frac{4x}{x-1}\right)\mathrm{e}^{-\frac{4x}{x-1}}}{x-1}$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 54

```
DSolve[x^2*(1-2*x+x^2)*y''[x] -x*(3+x)*y'[x]+(4+x)*y[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{e^{-\frac{4x}{x-1}}\sqrt{1-x}x^2\left(c_2\,\mathrm{ExpIntegralEi}\left(\frac{4x}{x-1}\right)+e^4c_1\right)}{(x-1)^{3/2}}$$

1.814 problem 832

Internal problem ID [7547]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 832.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x+2)\,y'' + 5y'x^2 + (1+x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 48

```
dsolve(2*x^2*(2+x)*diff(y(x), x$2) +5*x^2*diff(y(x),x)+(1+x)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\sqrt{x}}{(x+2)^{\frac{3}{2}}} + \frac{c_2\sqrt{x}\left(\sqrt{2}\,\sqrt{x+2} - 2\,\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x+2}}{2}\right)\right)}{(x+2)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 55

```
DSolve[2*x^2*(2+x)*y''[x] +5*x^2*y'[x]+(1+x)*y[x] == 0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{\sqrt{x}\left(-2\sqrt{2}c_2\operatorname{arctanh}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right) + 2c_2\sqrt{x+2} + c_1\right)}{(x+2)^{3/2}}$$

1.815 problem 833

Internal problem ID [7548]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 833.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 4y'x + \left(x^2 + 2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x), x, x) + 4*x*diff(y(x), x) + (x^2+2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x^2} + \frac{c_2 \cos(x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+4*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x^2}$$

1.816 problem 834

Internal problem ID [7549]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 834.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2 e^{2ix})}{2\sqrt{x}}$$

1.817 problem 835

Internal problem ID [7550]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 835.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x - \left(x^2 + \frac{5}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)-(x^2+5/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{x}(x-1)}{\sqrt{x}} + \frac{c_2 \mathrm{e}^{-x}(x+1)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 53

```
DSolve[x^2*y''[x]-x*y'[x]-(x^2+5/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\frac{2}{\pi}}((ic_2x + c_1)\sinh(x) - (c_1x + ic_2)\cosh(x))}{\sqrt{-ix}}$$

1.818 problem 836

Internal problem ID [7551]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 836.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2 e^{2ix})}{2\sqrt{x}}$$

1.819 problem 837

Internal problem ID [7552]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 837.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 3y'x + 4x^4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+4*x^4*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x^2)}{x^2} + \frac{c_2 \cos(x^2)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 41

```
DSolve[x^2*y''[x]+3*x*y'[x]+4*x^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4c_1 e^{-ix^2} - ic_2 e^{ix^2}}{4x^2}$$

1.820 problem 838

Internal problem ID [7553]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 838.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(x^2 + 3\right) y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)=(x^2+3)*y(x),y(x), singsol=all)
```

$$y(x) = c_1 x \, \mathrm{e}^{\frac{x^2}{2}} + c_2 \left(\mathrm{e}^{\frac{x^2}{2}} \sqrt{\pi} \, \operatorname{erf}(x) \, x + \mathrm{e}^{-\frac{x^2}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 41

```
DSolve[y''[x]==(x^2+3)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} \left(e^{x^2} x \left(c_1 - \sqrt{\pi} c_2 \operatorname{erf}(x) \right) - c_2 \right)$$

1.821 problem 839

Internal problem ID [7554]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 839.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y'x + \left(x^2 + 1\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+(x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^2}{2}} + c_2 x \,\mathrm{e}^{-\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 22

```
DSolve[y''[x]+2*x*y'[x]+(x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} (c_2 x + c_1)$$

1.822 problem 840

Internal problem ID [7555]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 840.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(x^2*diff(diff(y(x),x),x)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2 e^{2ix})}{2\sqrt{x}}$$

1.823 problem 841

Internal problem ID [7556]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 841.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + \left(-8x^2 + 4x\right)y' + \left(4x^2 - 4x - 1\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(diff(y(x),x),x)+(-8*x^2+4*x)*diff(y(x),x)+(4*x^2-4*x-1)*y(x) = 0,y(x), sing
```

$$y(x) = \frac{c_1 \mathrm{e}^x}{\sqrt{x}} + c_2 \sqrt{x}\, \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 21

```
DSolve[4*x^2*y''[x]+(-8*x^2+4*x)*y'[x]+(4*x^2-4*x-1)*y[x] == 0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{e^x(c_2 x + c_1)}{\sqrt{x}}$$

1.824 problem 843

Internal problem ID [7557]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 843.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x$2)=((4*(1/2)^2-1)/(4*x^2))*y(x),y(x), singsol=all)
```

$$y(x) = xc_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[y''[x]==((4*(1/2)^2-1)/(4*x^2))*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x + c_1$$

1.825 problem 844

Internal problem ID [7558]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 844.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$\boxed{y'' - \frac{2y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=((4*(3/2)^2-1)/(4*x^2))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2 x^2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[y''[x]==((4*(3/2)^2-1)/(4*x^2))*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x^3 + c_1}{x}$$

1.826 problem 845

Internal problem ID [7559]

Book: Collection of Kovacic problems
Section: section 1
Problem number: 845.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - \frac{6y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=((4*(5/2)^2-1)/(4*x^2))*y(x),y(x), singsol=all)
```

$$y(x) = c_1 x^3 + \frac{c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[y''[x]==((4*(5/2)^2-1)/(4*x^2))*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x^5 + c_1}{x^2}$$

2 section 2. Solution found using all possible Kovacic cases

2.1 problem 1

Internal problem ID [7560]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \left(-\frac{3}{16x^2} - \frac{2}{9(-1+x)^2} + \frac{3}{16x(-1+x)} \right) y = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)= ( -3/(16*x^2)- 2/(9*(x-1)^2) + 3/(16*x*(x-1))) *y(x),y(x), singsol=all
```

$$y(x) = c_1 \sqrt{x-1}\, x^{\frac{1}{4}} \operatorname{LegendreP}\left(-\frac{1}{6}, \frac{1}{3}, \sqrt{x} \right) + c_2 \sqrt{x-1}\, x^{\frac{1}{4}} \operatorname{LegendreQ}\left(-\frac{1}{6}, \frac{1}{3}, \sqrt{x} \right)$$

✓ Solution by Mathematica

Time used: 0.173 (sec). Leaf size: 550

```
DSolve[y''[x]== ( -3/(16*x^2)- 2/(9*(x-1)^2) + 3/(16*x*(x-1))) *y[x],y[x],x,IncludeSingularSo
```

$$
\begin{aligned}
y(x) \to c_1 \exp\Bigg(\int_1^x \text{Root}\big[&2048K[1]^4-3484K[1]^3+2313K[1]^2-702K[1]\\
&+\left(20736K[1]^8-82944K[1]^7+124416K[1]^6-82944K[1]^5+20736K[1]^4\right)\#1^4\\
&+\left(-48384K[1]^7+165888K[1]^6-207360K[1]^5+110592K[1]^4-20736K[1]^3\right)\#1^3\\
&+\left(41472K[1]^6-118368K[1]^5+120096K[1]^4-50976K[1]^3+7776K[1]^2\right)\#1^2\\
&+\left(-15360K[1]^5+34992K[1]^4-28272K[1]^3+9936K[1]^2-1296K[1]\right)\#1\\
&+81\&,1\big]\,dK[1]\Bigg)+c_2\exp\Bigg(\int_1^x \text{Root}\big[2048K[1]^4-3484K[1]^3+2313K[1]^2-702K[1]\\
&+\left(20736K[1]^8-82944K[1]^7+124416K[1]^6-82944K[1]^5+20736K[1]^4\right)\#1^4\\
&+\left(-48384K[1]^7+165888K[1]^6-207360K[1]^5+110592K[1]^4-20736K[1]^3\right)\#1^3\\
&+\left(41472K[1]^6-118368K[1]^5+120096K[1]^4-50976K[1]^3+7776K[1]^2\right)\#1^2\\
&+\left(-15360K[1]^5+34992K[1]^4-28272K[1]^3+9936K[1]^2-1296K[1]\right)\#1\\
&+81\&,1\big]\,dK[1]\Bigg)\int_1^x \exp\Bigg(-2\int_1^{K[2]} \text{Root}\big[2048K[1]^4-3484K[1]^3+2313K[1]^2-702K[1]\\
&+\left(20736K[1]^8-82944K[1]^7+124416K[1]^6-82944K[1]^5+20736K[1]^4\right)\#1^4+\left(-48384K[1]^7+165888K[1]\right.\\
&+\left(41472K[1]^6-118368K[1]^5+120096K[1]^4-50976K[1]^3+7776K[1]^2\right)\#1^2+\left(-15360K[1]^5+34992K[1]\right.\\
&+81\&,1\big]\,dK[1]\Bigg)\,dK[2]
\end{aligned}
$$

2.2 problem 2

Internal problem ID [7561]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - \frac{20y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=((4*(9/2)^2-1)/(4*x^2))*y(x),y(x), singsol=all)
```

$$y(x) = c_1 x^5 + \frac{c_2}{x^4}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[y''[x]==((4*(9/2)^2-1)/(4*x^2))*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x^9 + c_1}{x^4}$$

2.3 problem 3

Internal problem ID [7562]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - \frac{12y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=((4*(7/2)^2-1)/(4*x^2))*y(x),y(x), singsol=all)
```

$$y(x) = c_1 x^4 + \frac{c_2}{x^3}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[y''[x]==((4*(7/2)^2-1)/(4*x^2))*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x^7 + c_1}{x^3}$$

2.4 problem 4

Internal problem ID [7563]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - \frac{y}{4x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-1/(4*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{\sqrt{2}}{2}+\frac{1}{2}} + c_2 x^{-\frac{\sqrt{2}}{2}+\frac{1}{2}}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 32

```
DSolve[y''[x]-1/(4*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{\frac{1}{2}-\frac{1}{\sqrt{2}}}\left(c_2 x^{\sqrt{2}} + c_1\right)$$

2.5 problem 5

Internal problem ID [7564]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2x+2)\,y' + (x+2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(diff(y(x),x),x)-(2*x+2)*diff(y(x),x)+(2+x)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2 \mathrm{e}^{x} x^3$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 23

```
DSolve[x*y''[x]-(2*x+2)*y'[x]+(2+x)*y[x] ==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{x} \left(c_2 x^3 + 3c_1\right)$$

2.6 problem 6

Internal problem ID [7565]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + \frac{y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+1/x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sin\left(\frac{\sqrt{3}\,\ln(x)}{2}\right) + c_2\sqrt{x}\,\cos\left(\frac{\sqrt{3}\,\ln(x)}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 42

```
DSolve[y''[x]+1/x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}\left(c_1 \cos\left(\frac{1}{2}\sqrt{3}\log(x)\right) + c_2 \sin\left(\frac{1}{2}\sqrt{3}\log(x)\right)\right)$$

2.7 problem 7

Internal problem ID [7566]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(1-x^2\right)y''+y'+y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 58

```
dsolve((1-x^2)*diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\text{hypergeom}\left(\left[-\frac{1}{2}-\frac{\sqrt{5}}{2},\frac{\sqrt{5}}{2}-\frac{1}{2}\right],\left[\frac{1}{2}\right],\frac{x}{2}+\frac{1}{2}\right)+c_2\sqrt{2x+2}\,\text{hypergeom}\left(\left[\frac{\sqrt{5}}{2},-\frac{\sqrt{5}}{2}\right],\left[\frac{3}{2}\right],\frac{x}{2}+\frac{1}{2}\right)$$

✓ Solution by Mathematica

Time used: 20.474 (sec). Leaf size: 170

```
DSolve[(1-x^2)*y''[x]+y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\sqrt[4]{1-x}\left(\sqrt{5}\sqrt{x-1}-\sqrt{x+1}\right)e^{2\sqrt{5}\text{arctanh}\left(\frac{\sqrt{x+1}+\sqrt{2}}{\sqrt{x-1}}\right)}\left(c_2\int_1^x\frac{2e^{-4\sqrt{5}\text{arctanh}\left(\frac{\sqrt{K[1]+1}+\sqrt{2}}{\sqrt{K[1]-1}}\right)}\sqrt{1-\frac{2}{K[1]+1}}}{\left(\sqrt{K[1]+1}-\sqrt{5}\sqrt{K[1]-1}\right)^2}dK[1]+c_1\right.}{\sqrt{2}\sqrt[4]{x-1}}$$

2.8 problem 8

Internal problem ID [7567]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-x\right)y''-y'x+y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((x^2-x)*diff(y(x), x$2)-x*diff(y(x), x)+y(x) = 0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2(\ln(x)x+1)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 20

```
DSolve[(x^2-x)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x-c_2(x\log(x)+1)$$

2.9 problem 9

Internal problem ID [7568]

Book: Collection of Kovacic problems
Section: section 2. Solution found using all possible Kovacic cases
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(-x^2+2\right)y''-x\left(4x^2+3\right)y'+\left(-2x^2+2\right)y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 47

```
dsolve(x^2*(2-x^2)*diff(y(x), x$2) - x*(3+4*x^2)*diff(y(x), x) + (2-2*x^2)*y(x) = 0,y(x), sin
```

$$y(x)=c_1x^2\,\text{hypergeom}\left(\left[\frac{3}{2},2\right],\left[\frac{7}{4}\right],\frac{x^2}{2}\right)+\frac{c_2\sqrt{x}\left(x^2+1\right)}{\left(x^2-2\right)\left(-2x^2+4\right)^{\frac{3}{4}}}$$

✓ Solution by Mathematica

Time used: 10.08 (sec). Leaf size: 86

```
DSolve[x^2*(2-x^2)*y''[x] - x*(3+4*x^2)*y'[x] + (2-2*x^2)*y[x] == 0,y[x],x,IncludeSingularSol
```

$$y(x)\rightarrow\frac{2^{3/4}c_2(x^2+1)\,x^2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{x^2}{2}\right)+3c_2(2-x^2)^{3/4}\,x^2+6c_1(x^2+1)\,\sqrt{x}}{6\,(2-x^2)^{7/4}}$$

3 section 3. Problems from Kovacic related papers

3.1 problem Kovacic 1985 paper. page 13. section 3.2, example 1

Internal problem ID [7569]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Kovacic 1985 paper. page 13. section 3.2, example 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{\left(4x^6 - 8x^5 + 12x^4 + 4x^3 + 7x^2 - 20x + 4\right) y}{4x^4} = 0}$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 90

```
dsolve(diff(y(x),x$2)= (4*x^6-8*x^5+12*x^4+4*x^3+7*x^2-20*x+4)/(4*x^4)*y(x),y(x), singsol=all
```

$$y(x) = \frac{c_1 \mathrm{e}^{\frac{x^3-2x^2-2}{2x}} (x^2-1)}{x^{\frac{3}{2}}} + \frac{c_2 \mathrm{e}^{\frac{x^3-2x^2-2}{2x}} (x^2-1) \left(\int \frac{x^3 \mathrm{e}^{\frac{-x^3+2x^2+2}{x}}}{(x-1)^2 (x+1)^2} dx \right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.184 (sec). Leaf size: 79

```
DSolve[y''[x]== (4*x^6-8*x^5+12*x^4+4*x^3+7*x^2-20*x+4)/(4*x^4)*y[x],y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{e^{\frac{x^2}{2}-x-\frac{1}{x}} (x^2-1) \left(c_2 \int_1^x \frac{e^{-K[1]^2+2K[1]+\frac{2}{K[1]}} K[1]^3}{(K[1]^2-1)^2} dK[1] + c_1 \right)}{x^{3/2}}$$

3.2 problem Kovacic 1985 paper. page 14. section 3.2, example 2

Internal problem ID [7570]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Kovacic 1985 paper. page 14. section 3.2, example 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(\frac{6}{x^2} - 1\right) y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)= ( (4*(5/2)^2-1)/(4*x^2)-1)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1(\cos(x)\, x^2 - 3x\sin(x) - 3\cos(x))}{x^2} + \frac{c_2(x^2\sin(x) + 3\cos(x)\, x - 3\sin(x))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 55

```
DSolve[y''[x]== ( (4*(5/2)^2-1)/(4*x^2)-1)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((3c_1x - c_2(x^2-3))\cos(x) + (c_1(x^2-3) + 3c_2x)\sin(x))}{x^2}$$

3.3 problem Kovacic 1985 paper. page 15. Weber equation

Internal problem ID [7571]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Kovacic 1985 paper. page 15. Weber equation.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(\frac{x^2}{4} - \frac{11}{2}\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)= (1/4*x^2-1/2-5)*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^2}{4}} \operatorname{hypergeom}\left(\left[-2\right], \left[\frac{3}{2}\right], \frac{x^2}{2}\right) x + c_2 \mathrm{e}^{-\frac{x^2}{4}} \operatorname{hypergeom}\left(\left[-\frac{5}{2}\right], \left[\frac{1}{2}\right], \frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 22

```
DSolve[y''[x]== (1/4*x^2-1/2-5)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \operatorname{ParabolicCylinderD}(-6, ix) + c_1 \operatorname{ParabolicCylinderD}(5, x)$$

3.4 problem Kovacic 1985 paper. page 19. section 4.2. Example 1

Internal problem ID [7572]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Kovacic 1985 paper. page 19. section 4.2. Example 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(\frac{1}{x} - \frac{3}{16x^2}\right) y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)= (1/x-3/(16*x^2))*y(x),y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{1}{4}} \sinh\left(2\sqrt{x}\right) + c_2 x^{\frac{1}{4}} \cosh\left(2\sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 41

```
DSolve[y''[x]== (1/x-3/(16*x^2))*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-2\sqrt{x}} \sqrt[4]{x} \left(2c_1 e^{4\sqrt{x}} - c_2\right)$$

3.5 problem Kovacic 1985 paper. page 23. section 5.2. Example 1

Internal problem ID [7573]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Kovacic 1985 paper. page 23. section 5.2. Example 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \left(-\frac{3}{16x^2} - \frac{2}{9(-1+x)^2} + \frac{3}{16x(-1+x)} \right) y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)= ( -3/(16*x^2) - 2/(9*(x-1)^2) + 3/(16*x*(x-1))  )*y(x),y(x), singsol=a
```

$$y(x) = c_1 \sqrt{x-1}\, x^{\frac{1}{4}} \operatorname{LegendreP}\left(-\frac{1}{6}, \frac{1}{3}, \sqrt{x} \right) + c_2 \sqrt{x-1}\, x^{\frac{1}{4}} \operatorname{LegendreQ}\left(-\frac{1}{6}, \frac{1}{3}, \sqrt{x} \right)$$

✓ Solution by Mathematica

Time used: 0.165 (sec). Leaf size: 550

```
DSolve[y''[x]== ( -3/(16*x^2) - 2/(9*(x-1)^2) + 3/(16*x*(x-1))  )*y[x],y[x],x,IncludeSingular
```

$$
\begin{aligned}
y(x) \to c_1 \exp\Bigg(\int_1^x \text{Root}\big[&2048K[1]^4-3484K[1]^3+2313K[1]^2-702K[1]\\
&+\left(20736K[1]^8-82944K[1]^7+124416K[1]^6-82944K[1]^5+20736K[1]^4\right)\#1^4\\
&+\left(-48384K[1]^7+165888K[1]^6-207360K[1]^5+110592K[1]^4-20736K[1]^3\right)\#1^3\\
&+\left(41472K[1]^6-118368K[1]^5+120096K[1]^4-50976K[1]^3+7776K[1]^2\right)\#1^2\\
&+\left(-15360K[1]^5+34992K[1]^4-28272K[1]^3+9936K[1]^2-1296K[1]\right)\#1\\
&+81\&,1\big]\,dK[1]\Bigg)+c_2\exp\Bigg(\int_1^x \text{Root}\big[2048K[1]^4-3484K[1]^3+2313K[1]^2-702K[1]\\
&+\left(20736K[1]^8-82944K[1]^7+124416K[1]^6-82944K[1]^5+20736K[1]^4\right)\#1^4\\
&+\left(-48384K[1]^7+165888K[1]^6-207360K[1]^5+110592K[1]^4-20736K[1]^3\right)\#1^3\\
&+\left(41472K[1]^6-118368K[1]^5+120096K[1]^4-50976K[1]^3+7776K[1]^2\right)\#1^2\\
&+\left(-15360K[1]^5+34992K[1]^4-28272K[1]^3+9936K[1]^2-1296K[1]\right)\#1\\
&+81\&,1\big]\,dK[1]\Bigg)\int_1^x\exp\Bigg(-2\int_1^{K[2]}\text{Root}\big[2048K[1]^4-3484K[1]^3+2313K[1]^2-702K[1]\\
&+\left(20736K[1]^8-82944K[1]^7+124416K[1]^6-82944K[1]^5+20736K[1]^4\right)\#1^4+\left(-48384K[1]^7+165888K[1]\right.\\
&+\left(41472K[1]^6-118368K[1]^5+120096K[1]^4-50976K[1]^3+7776K[1]^2\right)\#1^2+\left(-15360K[1]^5+34992K[1]\right.\\
&+81\&,1\big]\,dK[1]\Bigg)\,dK[2]
\end{aligned}
$$

3.6 problem Kovacic 1985 paper. page 25. section 5.2. Example 2

Internal problem ID [7574]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Kovacic 1985 paper. page 25. section 5.2. Example 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{(5x^2+27)\,y}{36\,(x^2-1)^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)= -(5*x^2+27)/(36*(x^2-1)^2)*y(x),y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x^2-1}\ \text{LegendreP}\left(-\frac{1}{6},\frac{1}{3},x\right) + c_2\sqrt{x^2-1}\ \text{LegendreQ}\left(-\frac{1}{6},\frac{1}{3},x\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 38

```
DSolve[y''[x]== -(5*x^2+27)/(36*(x^2-1)^2)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x^2-1}\left(c_1 P^{\frac{1}{3}}_{-\frac{1}{6}}(x) + c_2 Q^{\frac{1}{3}}_{-\frac{1}{6}}(x)\right)$$

3.7 problem Kovacic 2005 paper. Example 2

Internal problem ID [7575]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Kovacic 2005 paper. Example 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + \frac{y}{4x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)= -1/(4*x^2)*y(x),y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x}\,\ln(x)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 24

```
DSolve[y''[x]== -1/(4*x^2)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{x}(c_2 \log(x) + 2c_1)$$

3.8 problem David Saunders 1981 paper. Example 1

Internal problem ID [7576]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: David Saunders 1981 paper. Example 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(x^2 + 3\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)= (x^2+3)*y(x),y(x), singsol=all)
```

$$y(x) = c_1 x \,\mathrm{e}^{\frac{x^2}{2}} + c_2 \left(\mathrm{e}^{\frac{x^2}{2}} \sqrt{\pi}\, \operatorname{erf}(x)\, x + \mathrm{e}^{-\frac{x^2}{2}}\right)$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 41

```
DSolve[y''[x]== (x^2+3)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} \left(e^{x^2} x \left(c_1 - \sqrt{\pi} c_2 \operatorname{erf}(x)\right) - c_2\right)$$

3.9 problem David Saunders 1981 paper. Example 3

Internal problem ID [7577]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: David Saunders 1981 paper. Example 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)= 2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2x^2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]== 2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^3 + c_1}{x}$$

3.10 problem Carolyn J. Smith 1984 paper. Appendix B examples and tests. Example 1

Internal problem ID [7578]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Carolyn J. Smith 1984 paper. Appendix B examples and tests. Example 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y'x + \left(4x^2 + 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^2} + c_2 \mathrm{e}^{-x^2} x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 20

```
DSolve[y''[x]+4*x*y'[x]+(4*x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}(c_2 x + c_1)$$

3.11 problem Carolyn J. Smith 1984 paper. Appendix B examples and tests. Example 2

Internal problem ID [7579]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Carolyn J. Smith 1984 paper. Appendix B examples and tests. Example 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + \left(x^2+2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(x) + c_2 \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

3.12 problem Carolyn J. Smith 1984 paper. Appendix B examples and tests. Example 3

Internal problem ID [7580]

Book: Collection of Kovacic problems
Section: section 3. Problems from Kovacic related papers
Problem number: Carolyn J. Smith 1984 paper. Appendix B examples and tests. Example 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x-2)^2 y'' - (x-2) y' - 3y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve((x-2)^2*diff(y(x),x$2)-(x-2)*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x-2} + c_2 (x-2)^3$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 22

```
DSolve[(x-2)^2*y''[x]-(x-2)*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 (x-2)^3 + \frac{c_2}{x-2}$$