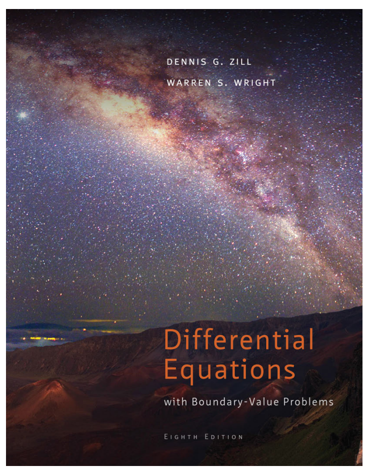
A Solution Manual For

DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.



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October 12, 2023

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1 CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246

1.1 problem 3. series method

Internal problem ID [5796]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 3. series method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{x^7}{5040} + \frac{x^5}{120} - \frac{x^3}{6} + x\right) + c_1\left(-\frac{x^6}{720} + \frac{x^4}{24} - \frac{x^2}{2} + 1\right)$$

1.2 problem 3. direct method

Internal problem ID [5797]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 3. direct method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + c_2 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 16

```
DSolve[y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + c_2 \sin(x)$$

1.3 problem 4. series method

Internal problem ID [5798]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 4. series method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{720}x^6\right) y(0) + \left(x + \frac{1}{6}x^3 + \frac{1}{120}x^5 + \frac{1}{5040}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{x^7}{5040} + \frac{x^5}{120} + \frac{x^3}{6} + x\right) + c_1\left(\frac{x^6}{720} + \frac{x^4}{24} + \frac{x^2}{2} + 1\right)$$

1.4 problem 4. direct method

Internal problem ID [5799]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 4. direct method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + e^{x}c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_2 e^{-x}$$

1.5 problem 5. series method

Internal problem ID [5800]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 5. series method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)-diff(y(x),x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + \left(x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 53

```
AsymptoticDSolveValue[y''[x]-y'[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x\right) + c_1$$

1.6 problem 5. direct method

Internal problem ID [5801]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 5. direct method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x$2)-diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + e^x c_2$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 14

```
DSolve[y''[x]-y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + c_2$$

1.7 problem 6. series method

Internal problem ID [5802]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 6. series method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)+2*diff(y(x),x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + \left(x - x^2 + \frac{2}{3}x^3 - \frac{1}{3}x^4 + \frac{2}{15}x^5 - \frac{2}{45}x^6 + \frac{4}{315}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 51

```
AsymptoticDSolveValue[y''[x]+2*y'[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{4x^7}{315} - \frac{2x^6}{45} + \frac{2x^5}{15} - \frac{x^4}{3} + \frac{2x^3}{3} - x^2 + x\right) + c_1$$

1.8 problem 6. direct method

Internal problem ID [5803]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 6. direct method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \mathrm{e}^{-2x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 19

```
DSolve[y''[x]+2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{2} c_1 e^{-2x}$$

1.9 problem 7

Internal problem ID [5804]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
Order:=8;
dsolve(diff(y(x),x$2)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{6}x^3 + \frac{1}{180}x^6\right) y(0) + \left(x + \frac{1}{12}x^4 + \frac{1}{504}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]-x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{x^7}{504} + \frac{x^4}{12} + x\right) + c_1\left(\frac{x^6}{180} + \frac{x^3}{6} + 1\right)$$

1.10 problem 8

Internal problem ID [5805]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + x^2 y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=8;
dsolve(diff(y(x),x$2)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^4}{12}\right) y(0) + \left(x - \frac{1}{20}x^5\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x^2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(x - \frac{x^5}{20}\right) + c_1\left(1 - \frac{x^4}{12}\right)$$

1.11 problem 9

Internal problem ID [5806]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{y'' - 2y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{7}{240}x^6\right) y(0) + \left(x + \frac{1}{6}x^3 + \frac{1}{24}x^5 + \frac{1}{112}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{x^7}{112} + \frac{x^5}{24} + \frac{x^3}{6} + x\right) + c_1\left(-\frac{7x^6}{240} - \frac{x^4}{8} - \frac{x^2}{2} + 1\right)$$

1.12 problem 10

Internal problem ID [5807]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - y'x + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=8;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(-x^2 + 1\right) y(0) + \left(x - \frac{1}{6}x^3 - \frac{1}{120}x^5 - \frac{1}{1680}x^7\right) D(y)\left(0\right) + O\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[y''[x]-x*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(1 - x^2\right) + c_2\left(-\frac{x^7}{1680} - \frac{x^5}{120} - \frac{x^3}{6} + x\right)$$

1.13 problem 11

Internal problem ID [5808]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + x^2 y' + xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=8;
dsolve(diff(y(x),x$2)+x^2*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 + \frac{1}{45}x^6\right) y(0) + \left(x - \frac{1}{6}x^4 + \frac{5}{252}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+x^2*y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{5x^7}{252} - \frac{x^4}{6} + x\right) + c_1\left(\frac{x^6}{45} - \frac{x^3}{6} + 1\right)$$

1.14 problem 12

Internal problem ID [5809]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + 2y'x + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 + \frac{1}{2}x^4 - \frac{1}{6}x^6\right) y(0) + \left(x - \frac{2}{3}x^3 + \frac{4}{15}x^5 - \frac{8}{105}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 54

```
AsymptoticDSolveValue[y''[x]+2*x*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{8x^7}{105} + \frac{4x^5}{15} - \frac{2x^3}{3} + x\right) + c_1\left(-\frac{x^6}{6} + \frac{x^4}{2} - x^2 + 1\right)$$

1.15 problem 13

Internal problem ID [5810]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(x-1)\,y''+y'=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve((x-1)*diff(y(x),x$2)+diff(y(x),x)=0,y(x),type='series',x=0);
```

$$y(x)=y(0)+\left(x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{4}x^4+\frac{1}{5}x^5+\frac{1}{6}x^6+\frac{1}{7}x^7\right)D(y)\,(0)+O\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 53

```
AsymptoticDSolveValue[(x-1)*y''[x]+y'[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(\frac{x^7}{7}+\frac{x^6}{6}+\frac{x^5}{5}+\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+x\right)+c_1$$

1.16 problem 14

Internal problem ID [5811]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2+x)\,y''+y'x-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
Order:=8;
dsolve((x+2)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{4}x^2-\frac{1}{24}x^3+\frac{1}{480}x^5-\frac{1}{1440}x^6+\frac{1}{6720}x^7\right)y(0)+D(y)\,(0)\,x+O\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 91

```
AsymptoticDSolveValue[(x+2)*y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(\frac{29x^7}{20160}-\frac{7x^6}{1440}+\frac{x^5}{240}+\frac{x^4}{24}-\frac{x^3}{6}+x\right)+c_1\left(\frac{x^7}{8064}+\frac{x^6}{576}-\frac{x^5}{96}+\frac{x^4}{48}+\frac{x^3}{24}-\frac{x^2}{4}+1\right)$$

1.17 problem 15

Internal problem ID [5812]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' - y'(1+x) - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 74

```
Order:=8;
dsolve(diff(y(x),x$2)-(x+1)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{6}x^4 + \frac{1}{15}x^5 + \frac{7}{180}x^6 + \frac{19}{1260}x^7\right) y(0) + \left(x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{4}x^4 + \frac{3}{20}x^5 + \frac{1}{15}x^6 + \frac{13}{420}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 98

```
AsymptoticDSolveValue[y''[x]-(x+1)*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{19x^7}{1260} + \frac{7x^6}{180} + \frac{x^5}{15} + \frac{x^4}{6} + \frac{x^3}{6} + \frac{x^2}{2} + 1\right) + c_2\left(\frac{13x^7}{420} + \frac{x^6}{15} + \frac{3x^5}{20} + \frac{x^4}{4} + \frac{x^3}{2} + \frac{x^2}{2} + x\right)$$

1.18 problem 16

Internal problem ID [5813]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(x^2+1)\,y''-6y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
Order:=8;
dsolve((x^2+1)*diff(y(x),x$2)-6*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+3x^2+x^4-\frac{1}{5}x^6\right)y(0)+\left(x^3+x\right)D(y)\,(0)+O\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[(x^2+1)*y''[x]-6*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(x^3+x\right)+c_1\left(-\frac{x^6}{5}+x^4+3x^2+1\right)$$

1.19 problem 17

Internal problem ID [5814]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+2)\,y''+3y'x-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve((x^2+2)*diff(y(x),x$2)+3*x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{4}x^2-\frac{7}{96}x^4+\frac{161}{5760}x^6\right)y(0)+\left(x-\frac{1}{6}x^3+\frac{7}{120}x^5-\frac{17}{720}x^7\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[(x^2+2)*y''[x]+3*x*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(-\frac{17x^7}{720}+\frac{7x^5}{120}-\frac{x^3}{6}+x\right)+c_1\left(\frac{161x^6}{5760}-\frac{7x^4}{96}+\frac{x^2}{4}+1\right)$$

1.20 problem 18

Internal problem ID [5815]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2-1)\,y''+y'x-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
Order:=8;
dsolve((x^2-1)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{2}x^2-\frac{1}{8}x^4-\frac{1}{16}x^6\right)y(0)+D(y)\,(0)\,x+O\big(x^8\big)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[(x^2-1)*y''[x]+x*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(-\frac{x^6}{16}-\frac{x^4}{8}-\frac{x^2}{2}+1\right)+c_2x$$

1.21 problem 19

Internal problem ID [5816]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-y'x+y=0}$$

With initial conditions

$$[y(0)=-2, y'(0)=6]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
Order:=8;
dsolve([(x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(0) = -2, D(y)(0) = 6],y(x),type='series'
```

$$y(x)=-2+6x-x^2-\frac{1}{3}x^3-\frac{1}{12}x^4-\frac{1}{60}x^5-\frac{1}{360}x^6-\frac{1}{2520}x^7+\mathrm{O}\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 48

```
AsymptoticDSolveValue[{(x-1)*y''[x]-x*y'[x]+y[x]==0,{y[0]==-2,y'[0]==6}},y[x],{x,0,7}]
```

$$y(x)\to -\frac{x^7}{2520}-\frac{x^6}{360}-\frac{x^5}{60}-\frac{x^4}{12}-\frac{x^3}{3}-x^2+6x-2$$

1.22 problem 20

Internal problem ID [5817]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(1+x)\,y'' - (-x+2)\,y' + y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
Order:=8;
dsolve([(x+1)*diff(y(x),x$2)-(2-x)*diff(y(x),x)+y(x)=0,y(0) = 2, D(y)(0) = -1],y(x),type='ser
```

$$y(x) = 2 - x - 2x^2 - \frac{1}{3}x^3 + \frac{1}{2}x^4 - \frac{1}{30}x^5 - \frac{13}{180}x^6 + \frac{1}{28}x^7 + O\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 48

```
AsymptoticDSolveValue[{(x+1)*y''[x]-(2-x)*y'[x]+y[x]==0,{y[0]==2,y'[0]==-1}},y[x],{x,0,7}]
```

$$y(x) \to \frac{x^7}{28} - \frac{13x^6}{180} - \frac{x^5}{30} + \frac{x^4}{2} - \frac{x^3}{3} - 2x^2 - x + 2$$

1.23 problem 21

Internal problem ID [5818]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 8y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
Order:=8;
dsolve([diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(0) = 3, D(y)(0) = 0],y(x),type='series',x=
```

$$y(x) = 4x^4 - 12x^2 + 3$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 15

```
AsymptoticDSolveValue[{y''[x]-2*x*y'[x]+8*y[x]==0,{y[0]==3,y'[0]==0}},y[x],{x,0,7}]
```

$$y(x) \to 4x^4 - 12x^2 + 3$$

1.24 problem 22

Internal problem ID [5819]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(x^2+1)\, y''+2y'x=0}$$

With initial conditions

$$[y(0)=0, y'(0)=1]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
Order:=8;
dsolve([(x^2+1)*diff(y(x),x$2)+2*x*diff(y(x),x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',x
```

$$y(x)=x-\frac{1}{3}x^3+\frac{1}{5}x^5-\frac{1}{7}x^7+\mathrm{O}\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 26

```
AsymptoticDSolveValue[{(x^2+1)*y''[x]+2*x*y'[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,7}]
```

$$y(x)\to -\frac{x^7}{7}+\frac{x^5}{5}-\frac{x^3}{3}+x$$

1.25 problem 23

Internal problem ID [5820]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y\sin(x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=8;
dsolve(diff(y(x),x$2)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \frac{1}{180}x^6 - \frac{1}{5040}x^7\right) y(0) + \left(x - \frac{1}{12}x^4 + \frac{1}{180}x^6 + \frac{1}{504}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]+Sin[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{x^7}{504} + \frac{x^6}{180} - \frac{x^4}{12} + x\right) + c_1\left(-\frac{x^7}{5040} + \frac{x^6}{180} + \frac{x^5}{120} - \frac{x^3}{6} + 1\right)$$

1.26 problem 24

Internal problem ID [5821]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \mathrm{e}^{x} y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
Order:=8;
dsolve(diff(y(x),x$2)+exp(x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = &\left(1+\frac{1}{2}x^{2}-\frac{1}{6}x^{3}-\frac{1}{120}x^{5}+\frac{1}{240}x^{6}+\frac{1}{840}x^{7}\right)y(0) \\ &+\left(x-\frac{1}{2}x^{2}+\frac{1}{6}x^{3}-\frac{1}{24}x^{4}+\frac{1}{120}x^{5}-\frac{1}{720}x^{6}+\frac{1}{5040}x^{7}\right)D(y)(0)+O(x^{8}) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 91

```
AsymptoticDSolveValue[y''[x]+Exp[x]*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^7}{840}+\frac{x^6}{240}-\frac{x^5}{120}-\frac{x^3}{6}+\frac{x^2}{2}+1\right)+c_2\left(\frac{x^7}{5040}-\frac{x^6}{720}+\frac{x^5}{120}-\frac{x^4}{24}+\frac{x^3}{6}-\frac{x^2}{2}+x\right)$$

1.27 problem 25 expansion at 0

Internal problem ID [5822]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 25 expansion at 0.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\cos(x)\, y'' + y' + 5y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=8;
dsolve(cos(x)*diff(y(x),x$2)+diff(y(x),x)+5*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1-\frac{5}{2}x^2+\frac{5}{6}x^3+\frac{5}{8}x^4-\frac{5}{24}x^5+\frac{1}{16}x^6-\frac{13}{336}x^7\right)y(0) + \left(x-\frac{1}{2}x^2-\frac{2}{3}x^3+\frac{1}{3}x^4+\frac{1}{80}x^6+\frac{11}{5040}x^7\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 91

```
AsymptoticDSolveValue[Cos[x]*y''[x]+y'[x]+5*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{11x^7}{5040}+\frac{x^6}{80}+\frac{x^4}{3}-\frac{2x^3}{3}-\frac{x^2}{2}+x\right)+c_1\left(-\frac{13x^7}{336}+\frac{x^6}{16}-\frac{5x^5}{24}+\frac{5x^4}{8}+\frac{5x^3}{6}-\frac{5x^2}{2}+1\right)$$

1.28 problem 25 expansion at 1

Internal problem ID [5823]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 25 expansion at 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\cos(x) y'' + y' + 5y = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 860

```
Order:=8;
dsolve(cos(x)*diff(y(x),x$2)+diff(y(x),x)+5*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 - \frac{5\sec(1)(x-1)^2}{2} - \frac{5(\sin(1)-1)\sec(1)^2(x-1)^3}{6} + \frac{5\sec(1)^3\left(\cos(1)^2+5\cos(1)+3\sin(1)-3\right)(x-1)^4}{24} + \frac{\sec(1)^4\left(12+\left(\cos(1)^2+20\cos(1)-12\right)\sin(1)-7\cos(1)^2-10\cos(1)\right)(x-1)^5}{24} - \frac{\sec(1)^5\left(\cos(1)^4+55\cos(1)^3+(15\sin(1)-20)\cos(1)^2+(75\sin(1)-105)\cos(1)-60\sin(1)+60\right)}{144} + \frac{\sec(1)^6\left(360-\left(\cos(1)^4+130\cos(1)^3+75\cos(1)^2-660\cos(1)+360\right)\sin(1)+31\cos(1)^4+365\cos\right.}{1008}\right.$$

$$+ \left(x-1-\frac{\sec(1)(x-1)^2}{2} - \frac{5\left(\cos(1)+\frac{\sin(1)}{5}-\frac{1}{5}\right)\sec(1)^2(x-1)^3}{6} + \frac{5\left(\frac{\cos(1)^2}{5}+(-2\sin(1)+2)\cos(1)+\frac{3\sin(1)}{5}-\frac{3}{5}\right)\sec(1)^3(x-1)^4}{24} + \frac{\sec(1)^4\left(\left(\cos(1)^2+45\cos(1)-12\right)\sin(1)+15\cos(1)^3+18\cos(1)^2-45\cos(1)+12\right)(x-1)^5}{120} - \frac{\sec(1)^5\left(\frac{\cos(1)^4}{5}+(-4\sin(1)+28)\cos(1)^3+(-27\sin(1)+6)\cos(1)^2+(48\sin(1)-48)\cos(1)-12\right.}{144} - \frac{\sec(1)^6\left(\left(\cos(1)^4+375\cos(1)^3+600\cos(1)^2-1500\cos(1)+360\right)\sin(1)+25\cos(1)^5+544\cos(1)^4\right.}{5040}\right.$$

$$+ O\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 1808

```
AsymptoticDSolveValue[Cos[x]*y''[x]+y'[x]+5*y[x]==0,y[x],{x,1,7}]
```

Too large to display

1.29 problem 26 (a)

Internal problem ID [5824]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 26 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - xy - 1 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
Order:=8;
dsolve(diff(y(x),x$2)-x*y(x)=1,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{6}x^3 + \frac{1}{180}x^6\right) y(0) + \left(x + \frac{1}{12}x^4 + \frac{1}{504}x^7\right) D(y)(0) + \frac{x^2}{2} + \frac{x^5}{40} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]-x*y[x]==1,y[x],{x,0,7}]
```

$$y(x) \to \frac{x^5}{40} + \frac{x^2}{2} + c_2\left(\frac{x^7}{504} + \frac{x^4}{12} + x\right) + c_1\left(\frac{x^6}{180} + \frac{x^3}{6} + 1\right)$$

1.30 problem 26 (b)

Internal problem ID [5825]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 26 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y'x - 4y - e^{x} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 56

```
Order:=8;
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)-4*y(x)=exp(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + 2x^2 + 2x^4 + \frac{4}{3}x^6\right) y(0) + \left(x + \frac{4}{3}x^3 + \frac{16}{15}x^5 + \frac{64}{105}x^7\right) D(y)(0) + \frac{x^2}{2} + \frac{x^3}{6} + \frac{13x^4}{24} + \frac{17x^5}{120} + \frac{29x^6}{80} + \frac{409x^7}{5040} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 94

```
AsymptoticDSolveValue[y''[x]-4*x*y'[x]-4*y[x]==Exp[x],y[x],{x,0,7}]
```

$$y(x) \to \frac{409x^7}{5040} + \frac{29x^6}{80} + \frac{17x^5}{120} + \frac{13x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + c_2\left(\frac{64x^7}{105} + \frac{16x^5}{15} + \frac{4x^3}{3} + x\right) + c_1\left(\frac{4x^6}{3} + 2x^4 + 2x^2 + 1\right)$$

1.31 problem 27

Internal problem ID [5826]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + y\sin(x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(x*diff(y(x),x$2)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{18}x^4 - \frac{53}{10800}x^6\right)y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{60}x^5 - \frac{19}{15120}x^7\right)D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[x*y''[x]+Sin[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{19x^7}{15120} + \frac{x^5}{60} - \frac{x^3}{6} + x\right) + c_1\left(-\frac{53x^6}{10800} + \frac{x^4}{18} - \frac{x^2}{2} + 1\right)$$

1.32 problem 28

Internal problem ID [5827]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 5y'x + \sqrt{x}\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=8;
dsolve(diff(y(x),x$2)+5*x*diff(y(x),x)+sqrt(x)*y(x)=0,y(x),type='series',x=0);
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
AsymptoticDSolveValue[y''[x]+5*x*y'[x]+Sqrt[x]*y[x]==0,y[x],{x,0,7}]
```

Not solved

1.33 problem 29 (a)

Internal problem ID [5828]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 29 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 - \frac{1}{48}x^6\right) y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{15}x^5 - \frac{1}{105}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{x^7}{105} + \frac{x^5}{15} - \frac{x^3}{3} + x\right) + c_1\left(-\frac{x^6}{48} + \frac{x^4}{8} - \frac{x^2}{2} + 1\right)$$

1.34 problem 30 (b)

Internal problem ID [5829]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. Section 6.2 SOLUTIONS ABOUT ORDINARY POINTS. EXERCISES 6.2. Page 246
Problem number: 30 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y\cos(x) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=8;
dsolve([diff(y(x),x$2)+cos(x)*y(x)=0,y(0) = 1, D(y)(0) = 1],y(x),type='series',x=0);
```

$$y(x) = 1 + x - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{12}x^4 + \frac{1}{30}x^5 - \frac{1}{80}x^6 - \frac{19}{5040}x^7 + \mathrm{O}\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 48

```
AsymptoticDSolveValue[{y''[x]+Cos[x]*y[x]==0,{y[0]==1,y'[0]==1}},y[x],{x,0,7}]
```

$$y(x) \to -\frac{19x^7}{5040} - \frac{x^6}{80} + \frac{x^5}{30} + \frac{x^4}{12} - \frac{x^3}{6} - \frac{x^2}{2} + x + 1$$

2 CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255

2.1 problem 1

Internal problem ID [5830]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^3y'' + 4x^2y' + 3y = 0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=8;
dsolve(x^3*diff(y(x),x$2)+4*x^2*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 374

```
AsymptoticDSolveValue[x^3*y''[x]+4*x^2*y'[x]+3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{c_1 e^{-\frac{2i\sqrt{3}}{\sqrt{x}}}\left(-\frac{9447234753831875i\sqrt{3}x^{13/2}}{4611686018427387904} + \frac{3806522094375i\sqrt{3}x^{11/2}}{4503599627370496} - \frac{14315125825ix^{9/2}}{8796093022208\sqrt{3}} + \frac{8083075ix^{7/2}}{4294967296\sqrt{3}} - \frac{15015i\sqrt{3}x^{5/2}}{8388608} + 385\right.}{} + \frac{c_2 e^{\frac{2i\sqrt{3}}{\sqrt{x}}}\left(\frac{9447234753831875i\sqrt{3}x^{13/2}}{4611686018427387904} - \frac{3806522094375i\sqrt{3}x^{11/2}}{4503599627370496} + \frac{14315125825ix^{9/2}}{8796093022208\sqrt{3}} - \frac{8083075ix^{7/2}}{4294967296\sqrt{3}} + \frac{15015i\sqrt{3}x^{5/2}}{8388608} - 385i\right.}{81}$$

2.2 problem 2

Internal problem ID [5831]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x+3)^2 y'' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 70

```
Order:=8;
dsolve(x*(x+3)^2*diff(y(x),x$2)-y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x \left(1 + \frac{1}{18}x - \frac{11}{972}x^2 + \frac{277}{104976}x^3 - \frac{12539}{18895680}x^4 + \frac{893821}{5101833600}x^5 \right. \\ & \left. - \frac{13183337}{275499014400}x^6 + \frac{265861081}{19835929036800}x^7 + \mathrm{O}\left(x^8\right)\right) \\ & + c_2 \left(\ln(x)\left(\frac{1}{9}x + \frac{1}{162}x^2 - \frac{11}{8748}x^3 + \frac{277}{944784}x^4 - \frac{12539}{170061120}x^5 + \frac{893821}{45916502400}x^6 \right.\right. \\ & \left. - \frac{13183337}{2479491129600}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(1 - \frac{5}{108}x^2 + \frac{167}{26244}x^3 - \frac{13583}{11337408}x^4 \right. \\ & \left.\left. + \frac{1327279}{5101833600}x^5 - \frac{21146863}{344373768000}x^6 + \frac{379766273}{24794911296000}x^7 + \mathrm{O}\left(x^8\right)\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.745 (sec). Leaf size: 121

```
AsymptoticDSolveValue[x*(x+3)^2*y''[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x(893821x^5 - 3385530x^4 + 13462200x^3 - 57736800x^2 + 283435200x + 5101833600)\log(x)}{45916502400} + \frac{24742849x^6 - 74732085x^5 + 184497750x^4 + 52488000x^3 - 10628820000x^2 + 382637520000x + 68874}{688747536000}\right. \\ \left. + c_2\left(-\frac{13183337x^7}{275499014400} + \frac{893821x^6}{5101833600} - \frac{12539x^5}{18895680} + \frac{277x^4}{104976} - \frac{11x^3}{972} + \frac{x^2}{18} + x\right)\right.$$

2.3 problem 3

Internal problem ID [5832]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-9)^2 y'' + (x+3)\, y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 74

```
Order:=8;
dsolve((x^2-9)^2*diff(y(x),x$2)+(x+3)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{81}x^2 + \frac{1}{6561}x^3 - \frac{289}{708588}x^4 + \frac{304}{23914845}x^5 - \frac{194981}{7748409780}x^6 + \frac{1732937}{1464449448420}x^7\right) y(0) + \left(x - \frac{1}{54}x^2 - \frac{13}{2187}x^3 - \frac{131}{236196}x^4 - \frac{596}{1594323}x^5 - \frac{78469}{2582803260}x^6 - \frac{13738871}{488149816140}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 98

```
AsymptoticDSolveValue[(x^2-9)^2*y''[x]+(x+3)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{1732937x^7}{1464449448420} - \frac{194981x^6}{7748409780} + \frac{304x^5}{23914845} - \frac{289x^4}{708588} + \frac{x^3}{6561} - \frac{x^2}{81} + 1\right) + c_2\left(-\frac{13738871x^7}{488149816140} - \frac{78469x^6}{2582803260} - \frac{596x^5}{1594323} - \frac{131x^4}{236196} - \frac{13x^3}{2187} - \frac{x^2}{54} + x\right)$$

2.4 problem 4

Internal problem ID [5833]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{y'}{x} + \frac{y}{(x-1)^3} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 64

```
Order:=8;
dsolve(diff(y(x),x$2)-1/x*diff(y(x),x)+1/(x-1)^3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^2 \left(1 + \frac{1}{8}x^2 + \frac{1}{5}x^3 + \frac{49}{192}x^4 + \frac{423}{1400}x^5 + \frac{15941}{46080}x^6 + \frac{30511}{78400}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2 \left(\ln(x) \left(-x^2 - \frac{1}{8}x^4 - \frac{1}{5}x^5 - \frac{49}{192}x^6 - \frac{423}{1400}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(-2 - 2x^3 - \frac{45}{32}x^4 - \frac{34}{25}x^5 - \frac{1673}{1152}x^6 - \frac{313337}{196000}x^7 + \mathrm{O}\left(x^8\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.37 (sec). Leaf size: 107

```
AsymptoticDSolveValue[y''[x]-1/x*y'[x]+1/(x-1)^3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{(245x^4+192x^3+120x^2+960)\,x^2\log(x)}{1920} + \frac{-25025x^6-16416x^5-2250x^4+28800x^3-180000x^2+28800}{28800}\right) + c_2\left(\frac{15941x^8}{46080}+\frac{423x^7}{1400}+\frac{49x^6}{192}+\frac{x^5}{5}+\frac{x^4}{8}+x^2\right)$$

2.5 problem 5

Internal problem ID [5834]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^3+4x)\,y''-2y'x+6y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 72

```
Order:=8;
dsolve((x^3+4*x)*diff(y(x),x$2)-2*x*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x\left(1-\frac{1}{2}x+\frac{1}{24}x^2+\frac{1}{48}x^3-\frac{1}{384}x^4-\frac{5}{2304}x^5+\frac{5}{21504}x^6+\frac{15}{50176}x^7+\mathrm{O}\left(x^8\right)\right) \\ & +\left(-\frac{3}{2}x+\frac{3}{4}x^2-\frac{1}{16}x^3-\frac{1}{32}x^4+\frac{1}{256}x^5+\frac{5}{1536}x^6-\frac{5}{14336}x^7+\mathrm{O}\left(x^8\right)\right)\ln(x)\,c_2 \\ & +\left(1+\frac{1}{2}x-\frac{7}{4}x^2+\frac{31}{96}x^3+\frac{1}{24}x^4-\frac{67}{3072}x^5-\frac{43}{10240}x^6+\frac{43061}{18063360}x^7+\mathrm{O}\left(x^8\right)\right)c_2 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.234 (sec). Leaf size: 121

```
AsymptoticDSolveValue[(x^3+4*x)*y''[x]-2*x*y'[x]+6*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x(5x^5+6x^4-48x^3-96x^2+1152x-2304)\log(x)}{1536} + \frac{-229x^6-790x^5+2240x^4+11840x^3-76800x^2+61440x+30720}{30720}\right) + c_2\left(\frac{5x^7}{21504}-\frac{5x^6}{2304}-\frac{x^5}{384}+\frac{x^4}{48}+\frac{x^3}{24}-\frac{x^2}{2}+x\right)$$

2.6 problem 6

Internal problem ID [5835]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x-5)^2 y'' + 4y'x + \left(x^2-25\right) y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 2223

```
Order:=8;
dsolve(x^2*(x-5)^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(x^2-25)*y(x)=0,y(x),type='series',x=0);
```

$$
\begin{aligned}
y(x) = x^{\frac{21}{50}} \Bigg(c_1 x^{-\frac{\sqrt{2941}}{50}} \Bigg(1 &+ \frac{-1166-4\sqrt{2941}}{-3125+125\sqrt{2941}} x - \frac{9}{15625} \frac{879\sqrt{2941}-79709}{\left(-25+\sqrt{2941}\right)\left(-50+\sqrt{2941}\right)} x^2 \\
&+ \frac{\frac{15291084\sqrt{2941}}{1953125}-\frac{906742764}{1953125}}{\left(-25+\sqrt{2941}\right)\left(-50+\sqrt{2941}\right)\left(-75+\sqrt{2941}\right)} x^3 \\
&- \frac{12}{244140625} \frac{2200649681\sqrt{2941}-122814219551}{\left(-25+\sqrt{2941}\right)\left(-50+\sqrt{2941}\right)\left(-75+\sqrt{2941}\right)\left(-100+\sqrt{2941}\right)} x^4 \\
&+ \frac{\frac{181292058002304\sqrt{2941}}{152587890625}-\frac{10008934775328384}{152587890625}}{\left(-25+\sqrt{2941}\right)\left(-50+\sqrt{2941}\right)\left(-75+\sqrt{2941}\right)\left(-100+\sqrt{2941}\right)\left(-125+\sqrt{2941}\right)} x^5 \\
&+ \frac{\frac{250187169310576512\sqrt{2941}}{19073486328125}-\frac{13371141904684696752}{19073486328125}}{\left(-25+\sqrt{2941}\right)\left(-50+\sqrt{2941}\right)\left(-75+\sqrt{2941}\right)\left(-100+\sqrt{2941}\right)\left(-125+\sqrt{2941}\right)\left(-150+\sqrt{2941}\right)\cdots} \\
&- \frac{96}{16689300537109375} \frac{381820145596656632404\sqrt{2941}-206899473876390\cdots}{\left(-25+\sqrt{2941}\right)\left(-50+\sqrt{2941}\right)\left(-75+\sqrt{2941}\right)\left(-100+\sqrt{2941}\right)\left(-125+\sqrt{294\cdots}\right.} \\
&+ O\left(x^8\right) \Bigg) + c_2 x^{\frac{\sqrt{2941}}{50}} \Bigg(1 + \frac{1166-4\sqrt{2941}}{125\sqrt{2941}+3125} x + \frac{\frac{7911\sqrt{2941}}{15625}+\frac{717381}{15625}}{\left(\sqrt{2941}+25\right)\left(50+\sqrt{2941}\right)} x^2 \\
&+ \frac{\frac{15291084\sqrt{2941}}{1953125}+\frac{906742764}{1953125}}{\left(\sqrt{2941}+25\right)\left(50+\sqrt{2941}\right)\left(\sqrt{2941}+75\right)} x^3 \\
&+ \frac{\frac{26407796172\sqrt{2941}}{244140625}+\frac{1473770634612}{244140625}}{\left(\sqrt{2941}+25\right)\left(50+\sqrt{2941}\right)\left(\sqrt{2941}+75\right)\left(100+\sqrt{2941}\right)} x^4 \\
&+ \frac{\frac{181292058002304\sqrt{2941}}{152587890625}+\frac{10008934775328384}{152587890625}}{\left(\sqrt{2941}+25\right)\left(50+\sqrt{2941}\right)\left(\sqrt{2941}+75\right)\left(100+\sqrt{2941}\right)\left(125+\sqrt{2941}\right)} x^5 \\
&- \frac{48}{19073486328125} \frac{5212232693970344\sqrt{2941}+278565456347597849}{\left(\sqrt{2941}+25\right)\left(50+\sqrt{2941}\right)\left(\sqrt{2941}+75\right)\left(100+\sqrt{2941}\right)\left(125+\sqrt{2941}\right)\left(150+\sqrt{\cdots}\right.} \\
&- \frac{96}{16689300537109375} \frac{381820145596656632404\sqrt{2941}+2068994738763901566\cdots}{\left(\sqrt{2941}+25\right)\left(50+\sqrt{2941}\right)\left(\sqrt{2941}+75\right)\left(100+\sqrt{2941}\right)\left(125+\sqrt{2941}\right)\left(150+\cdots\right.} \\
&+ O\left(x^8\right) \Bigg) \Bigg)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 22488

```
AsymptoticDSolveValue[x^2*(x-5)^2*y''[x]+4*x*y'[x]+(x^2-25)*y[x]==0,y[x],{x,0,7}]
```

Too large to display

2.7 problem 7

Internal problem ID [5836]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 + x - 6)\, y'' + (x + 3)\, y' + (x - 2)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 74

```
Order:=8;
dsolve((x^2+x-6)*diff(y(x),x$2)+(x+3)*diff(y(x),x)+(x-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^2 - \frac{1}{108}x^3 - \frac{17}{2592}x^4 - \frac{7}{2160}x^5 - \frac{139}{116640}x^6 - \frac{5377}{9797760}x^7\right) y(0) + \left(x + \frac{1}{4}x^2 + \frac{1}{36}x^3 + \frac{23}{864}x^4 + \frac{13}{1440}x^5 + \frac{619}{155520}x^6 + \frac{689}{408240}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 98

```
AsymptoticDSolveValue[(x^2+x-6)*y''[x]+(x+3)*y'[x]+(x-2)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{5377x^7}{9797760} - \frac{139x^6}{116640} - \frac{7x^5}{2160} - \frac{17x^4}{2592} - \frac{x^3}{108} - \frac{x^2}{6} + 1\right) + c_2\left(\frac{689x^7}{408240} + \frac{619x^6}{155520} + \frac{13x^5}{1440} + \frac{23x^4}{864} + \frac{x^3}{36} + \frac{x^2}{4} + x\right)$$

2.8 problem 8

Internal problem ID [5837]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x^2+1)^2 y''+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 70

```
Order:=8;
dsolve(x*(x^2+1)^2*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}
y(x) = {} & c_1 x\left(1-\frac{1}{2}x+\frac{1}{12}x^2+\frac{23}{144}x^3-\frac{167}{2880}x^4-\frac{7993}{86400}x^5+\frac{23599}{518400}x^6+\frac{1860281}{29030400}x^7+\mathrm{O}\left(x^8\right)\right) \\
& + c_2\left(\ln(x)\left(-x+\frac{1}{2}x^2-\frac{1}{12}x^3-\frac{23}{144}x^4+\frac{167}{2880}x^5+\frac{7993}{86400}x^6-\frac{23599}{518400}x^7+\mathrm{O}\left(x^8\right)\right)\right. \\
& \left.+\left(1-\frac{3}{4}x^2+\frac{19}{36}x^3+\frac{85}{1728}x^4-\frac{21907}{86400}x^5+\frac{787}{81000}x^6+\frac{5987917}{36288000}x^7+\mathrm{O}\left(x^8\right)\right)\right)
\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 121

```
AsymptoticDSolveValue[x*(x^2+1)^2*y''[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x(7993x^5 + 5010x^4 - 13800x^3 - 7200x^2 + 43200x - 86400)\log(x)}{86400} + \frac{-107303x^6 - 403755x^5 + 270750x^4 + 792000x^3 - 1620000x^2 + 1296000x + 1296000}{1296000}\right) + c_2\left(\frac{23599x^7}{518400} - \frac{7993x^6}{86400} - \frac{167x^5}{2880} + \frac{23x^4}{144} + \frac{x^3}{12} - \frac{x^2}{2} + x\right)$$

2.9 problem 9

Internal problem ID [5838]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3\left(x^2-25\right)(x-2)^2 y''+3x(x-2)\,y'+7(5+x)\,y=0$$

With the expansion point for the power series method at $x=0$.

✗ Solution by Maple

```
Order:=8;
dsolve(x^3*(x^2-25)*(x-2)^2*diff(y(x),x$2)+3*x*(x-2)*diff(y(x),x)+7*(x+5)*y(x)=0,y(x),type='s
```

No solution found

✓ Solution by Mathematica

Time used: 0.146 (sec). Leaf size: 127

```
AsymptoticDSolveValue[x^3*(x^2-25)*(x-2)^2*y''[x]+3*x*(x-2)*y'[x]+7*(x+5)*y[x]==0,y[x],{x,0,7
```

$$\begin{aligned} y(x) \to c_2\Bigg(&-\frac{8443721376476278698619699192242145x^7}{24679069470425088} \\ &+\frac{2562764390339723899972072769999x^6}{228509902503936}-\frac{13376987201697821906188881x^5}{352638738432} \\ &+\frac{428403015376532645 05x^4}{3265173504}-\frac{344729362309955x^3}{7558272}+\frac{3590248795x^2}{23328}-\frac{50309x}{108}+1\Bigg)x^{35/6} \\ &+\frac{c_1 e^{\frac{3}{50}/x}\left(\frac{2767048014517770038583814974166571582382979230 1x^7}{41187640736812500000000000000000000000}+\frac{31041725168697182475839769685531080609 01x^6}{4358480501250000000000000000000000}-\frac{37907198008\ldots}{538084\ldots}\right.}{} \end{aligned}$$

2.10 problem 10

Internal problem ID [5839]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(x^{3}-2x^{2}+3x\right)^{2} y''+x(x-3)^{2} y'-(1+x)\, y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 55

```
Order:=8;
dsolve((x^3-2*x^2+3*x)^2*diff(y(x),x$2)+x*(x-3)^2*diff(y(x),x)-(x+1)*y(x)=0,y(x),type='series
```

$$y(x) = \frac{c_2 x^{\frac{2}{3}}\left(1+\frac{1}{45}x+\frac{149}{3240}x^2+\frac{2701}{192456}x^3+\frac{236933}{121247280}x^4-\frac{67092967}{92754169200}x^5-\frac{30839263691}{50087251368000}x^6-\frac{14846109458423}{72576427232232000}x^7+\mathrm{O}\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 118

```
AsymptoticDSolveValue[(x^3-2*x^2+3*x)^2*y''[x]+x*(x-3)^2*y'[x]-(x+1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\rightarrow c_1\sqrt[3]{x}\left(-\frac{14846109458423x^7}{72576427232232000}-\frac{30839263691x^6}{50087251368000}-\frac{67092967x^5}{92754169200}+\frac{236933x^4}{121247280}+\frac{2701x^3}{192456}+\frac{149x^2}{3240}+\frac{x}{45}+1\right)+\frac{c_2\left(-\frac{2917066898x^7}{2604972321315}-\frac{70024699x^6}{43525017900}+\frac{7435523x^5}{3224075400}+\frac{106583x^4}{5511240}+\frac{1591x^3}{30618}-\frac{5x^2}{162}+\frac{13x}{9}+1\right)}{\sqrt[3]{x}}$$

2.11 problem 11

Internal problem ID [5840]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-1)\, y''+5y'(1+x)+(x^2-x)\, y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
Order:=8;
dsolve((x^2-1)*diff(y(x),x$2)+5*(x+1)*diff(y(x),x)+(x^2-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{6}x^3-\frac{1}{8}x^4-\frac{3}{10}x^5-\frac{17}{45}x^6-\frac{199}{336}x^7\right)y(0)+\left(x+\frac{5}{2}x^2+5x^3+\frac{26}{3}x^4+\frac{1661}{120}x^5+\frac{4967}{240}x^6+\frac{14881}{504}x^7\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 89

```
AsymptoticDSolveValue[(x^2-1)*y''[x]+5*(x+1)*y'[x]+(x^2-x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(-\frac{199x^7}{336}-\frac{17x^6}{45}-\frac{3x^5}{10}-\frac{x^4}{8}-\frac{x^3}{6}+1\right)+c_2\left(\frac{14881x^7}{504}+\frac{4967x^6}{240}+\frac{1661x^5}{120}+\frac{26x^4}{3}+5x^3+\frac{5x^2}{2}+x\right)$$

2.12 problem 12

Internal problem ID [5841]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (x+3)\,y' + 7x^2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 68

```
Order:=8;
dsolve(x*diff(y(x),x$2)+(x+3)*diff(y(x),x)+7*x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 - \frac{7}{15}x^3 + \frac{7}{120}x^4 - \frac{1}{150}x^5 + \frac{11}{160}x^6 - \frac{197}{15120}x^7 + \mathrm{O}\left(x^8\right)\right)$$
$$+ \frac{c_2\left(\ln\left(x\right)\left(2x^2 - \frac{14}{15}x^5 + \frac{7}{60}x^6 - \frac{1}{75}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(-2 + 4x - 3x^2 + 4x^3 - 4x^4 + \frac{547}{225}x^5 - \frac{5329}{3600}x^6 + \frac{7642}{7875}\right.}{x^2}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 96

```
AsymptoticDSolveValue[x*y''[x]+(x+3)*y'[x]+7*x^2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{11x^6}{160} - \frac{x^5}{150} + \frac{7x^4}{120} - \frac{7x^3}{15} + 1\right) + c_1\left(\frac{5539x^6 - 10432x^5 + 14400x^4 - 14400x^3 + 14400x^2 - 14400x + 7200}{7200x^2} - \frac{1}{120}\left(7x^4 - 56x^3 + 120\right)\log(x)\right)$$

2.13 problem 13

Internal problem ID [5842]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(\frac{5}{3}x + x^2\right)y' - \frac{y}{3} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+(5/3*x+x^2)*diff(y(x),x)-1/3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}}\left(1 - \frac{1}{7}x + \frac{1}{35}x^2 - \frac{1}{195}x^3 + \frac{1}{1248}x^4 - \frac{1}{9120}x^5 + \frac{1}{75240}x^6 - \frac{1}{693000}x^7 + \mathrm{O}\left(x^8\right)\right) + c_1\left(1 - 3x + \mathrm{O}\left(x^8\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 72

```
AsymptoticDSolveValue[x^2*y''[x]+(5/3*x+x^2)*y'[x]-1/3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\sqrt[3]{x}\left(-\frac{x^7}{693000} + \frac{x^6}{75240} - \frac{x^5}{9120} + \frac{x^4}{1248} - \frac{x^3}{195} + \frac{x^2}{35} - \frac{x}{7} + 1\right) + \frac{c_2(1-3x)}{x}$$

2.14 problem 14

Internal problem ID [5843]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' + 10y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
Order:=8;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+10*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x) c_2 + c_1)\left(1 - 10x + 25x^2 - \frac{250}{9}x^3 + \frac{625}{36}x^4 - \frac{125}{18}x^5 + \frac{625}{324}x^6 - \frac{3125}{7938}x^7 + \mathrm{O}(x^8)\right) + \left(20x - 75x^2 + \frac{2750}{27}x^3 - \frac{15625}{216}x^4 + \frac{3425}{108}x^5 - \frac{6125}{648}x^6 + \frac{75625}{37044}x^7 + \mathrm{O}(x^8)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 147

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+10*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{3125x^7}{7938} + \frac{625x^6}{324} - \frac{125x^5}{18} + \frac{625x^4}{36} - \frac{250x^3}{9} + 25x^2 - 10x + 1\right) + c_2\left(\frac{75625x^7}{37044} - \frac{6125x^6}{648} + \frac{3425x^5}{108} - \frac{15625x^4}{216} + \frac{2750x^3}{27} - 75x^2 + \left(-\frac{3125x^7}{7938} + \frac{625x^6}{324} - \frac{125x^5}{18} + \frac{625x^4}{36} - \frac{250x^3}{9} + 25x^2 - 10x + 1\right)\log(x) + 20x\right)$$

2.15 problem 15

Internal problem ID [5844]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2xy'' - y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{3}{2}} \left(1 - \frac{2}{5}x + \frac{2}{35}x^2 - \frac{4}{945}x^3 + \frac{2}{10395}x^4 - \frac{4}{675675}x^5 + \frac{4}{30405375}x^6 - \frac{8}{3618239625}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2 \left(1 + 2x - 2x^2 + \frac{4}{9}x^3 - \frac{2}{45}x^4 + \frac{4}{1575}x^5 - \frac{4}{42525}x^6 + \frac{8}{3274425}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 109

```
AsymptoticDSolveValue[2*x*y''[x]-y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2 \left(\frac{8x^7}{3274425} - \frac{4x^6}{42525} + \frac{4x^5}{1575} - \frac{2x^4}{45} + \frac{4x^3}{9} - 2x^2 + 2x + 1\right) + c_1 \left(-\frac{8x^7}{3618239625} + \frac{4x^6}{30405375} - \frac{4x^5}{675675} + \frac{2x^4}{10395} - \frac{4x^3}{945} + \frac{2x^2}{35} - \frac{2x}{5} + 1\right) x^{3/2}$$

2.16 problem 16

Internal problem ID [5845]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + 5y' + xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+5*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2\left(1 - \frac{1}{14}x^2 + \frac{1}{616}x^4 - \frac{1}{55440}x^6 + \mathrm{O}\left(x^8\right)\right)x^{\frac{3}{2}} + c_1\left(1 - \frac{1}{2}x^2 + \frac{1}{40}x^4 - \frac{1}{2160}x^6 + \mathrm{O}\left(x^8\right)\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 61

```
AsymptoticDSolveValue[2*x*y''[x]+5*y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^6}{55440} + \frac{x^4}{616} - \frac{x^2}{14} + 1\right) + \frac{c_2\left(-\frac{x^6}{2160} + \frac{x^4}{40} - \frac{x^2}{2} + 1\right)}{x^{3/2}}$$

2.17 problem 17

Internal problem ID [5846]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4xy'' + \frac{y'}{2} + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
Order:=8;
dsolve(4*x*diff(y(x),x$2)+1/2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{7}{8}} \left(1 - \frac{2}{15}x + \frac{2}{345}x^2 - \frac{4}{32085}x^3 + \frac{2}{1251315}x^4 - \frac{4}{294059025}x^5 + \frac{4}{48519739125}x^6 - \frac{8}{21397204954125}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2 \left(1 - 2x + \frac{2}{9}x^2 - \frac{4}{459}x^3 + \frac{2}{11475}x^4 - \frac{4}{1893375}x^5 + \frac{4}{232885125}x^6 - \frac{8}{79879597875}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 111

```
AsymptoticDSolveValue[4*x*y''[x]+1/2*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \to\; & c_2\left(-\frac{8x^7}{79879597875}+\frac{4x^6}{232885125}-\frac{4x^5}{1893375}+\frac{2x^4}{11475}-\frac{4x^3}{459}+\frac{2x^2}{9}-2x+1\right)\\
&+c_1x^{7/8}\left(-\frac{8x^7}{21397204954125}+\frac{4x^6}{48519739125}-\frac{4x^5}{294059025}+\frac{2x^4}{1251315}-\frac{4x^3}{32085}\right.\\
&\left.+\frac{2x^2}{345}-\frac{2x}{15}+1\right)
\end{aligned}
$$

2.18 problem 18

Internal problem ID [5847]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - y'x + y(x^2+1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 37

```
Order:=8;
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+(x^2+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1-\frac{1}{6}x^2+\frac{1}{168}x^4-\frac{1}{11088}x^6+\mathrm{O}\left(x^8\right)\right) + c_2x\left(1-\frac{1}{10}x^2+\frac{1}{360}x^4-\frac{1}{28080}x^6+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 62

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(x^2+1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1x\left(-\frac{x^6}{28080}+\frac{x^4}{360}-\frac{x^2}{10}+1\right)+c_2\sqrt{x}\left(-\frac{x^6}{11088}+\frac{x^4}{168}-\frac{x^2}{6}+1\right)$$

2.19 problem 19

Internal problem ID [5848]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{3xy'' + (-x+2)\,y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
Order:=8;
dsolve(3*x*diff(y(x),x$2)+(2-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{3}}\left(1+\frac{1}{3}x+\frac{1}{18}x^2+\frac{1}{162}x^3+\frac{1}{1944}x^4+\frac{1}{29160}x^5+\frac{1}{524880}x^6+\frac{1}{11022480}x^7+\mathrm{O}\left(x^8\right)\right) + c_2\left(1+\frac{1}{2}x+\frac{1}{10}x^2+\frac{1}{80}x^3+\frac{1}{880}x^4+\frac{1}{12320}x^5+\frac{1}{209440}x^6+\frac{1}{4188800}x^7+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 113

```
AsymptoticDSolveValue[3*x*y''[x]+(2-x)*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\sqrt[3]{x}\left(\frac{x^7}{11022480}+\frac{x^6}{524880}+\frac{x^5}{29160}+\frac{x^4}{1944}+\frac{x^3}{162}+\frac{x^2}{18}+\frac{x}{3}+1\right) + c_2\left(\frac{x^7}{4188800}+\frac{x^6}{209440}+\frac{x^5}{12320}+\frac{x^4}{880}+\frac{x^3}{80}+\frac{x^2}{10}+\frac{x}{2}+1\right)$$

2.20 problem 20

Internal problem ID [5849]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - \left(x - \frac{2}{9}\right) y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-(x-2/9)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{3}} \left(1 + \frac{3}{2}x + \frac{9}{20}x^2 + \frac{9}{160}x^3 + \frac{27}{7040}x^4 + \frac{81}{492800}x^5 + \frac{81}{16755200}x^6 + \frac{243}{2345728000}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2 x^{\frac{2}{3}} \left(1 + \frac{3}{4}x + \frac{9}{56}x^2 + \frac{9}{560}x^3 + \frac{27}{29120}x^4 + \frac{81}{2329600}x^5 + \frac{81}{88524800}x^6 + \frac{243}{13632819200}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 118

```
AsymptoticDSolveValue[x^2*y''[x]-(x-2/9)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2 \sqrt[3]{x} \left(\frac{243x^7}{2345728000} + \frac{81x^6}{16755200} + \frac{81x^5}{492800} + \frac{27x^4}{7040} + \frac{9x^3}{160} + \frac{9x^2}{20} + \frac{3x}{2} + 1\right) + c_1 x^{2/3} \left(\frac{243x^7}{13632819200} + \frac{81x^6}{88524800} + \frac{81x^5}{2329600} + \frac{27x^4}{29120} + \frac{9x^3}{560} + \frac{9x^2}{56} + \frac{3x}{4} + 1\right)$$

2.21 problem 21

Internal problem ID [5850]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{2xy'' - (3+2x)\,y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)-(3+2*x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{5}{2}} \left(1+\frac{4}{7}x+\frac{4}{21}x^2+\frac{32}{693}x^3+\frac{80}{9009}x^4+\frac{64}{45045}x^5+\frac{64}{328185}x^6+\frac{1024}{43648605}x^7+\mathrm{O}\left(x^8\right)\right) + c_2\left(1+\frac{1}{3}x-\frac{1}{6}x^2-\frac{1}{6}x^3-\frac{5}{72}x^4-\frac{7}{360}x^5-\frac{1}{240}x^6-\frac{11}{15120}x^7+\mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 113

```
AsymptoticDSolveValue[2*x*y''[x]-(3+2*x)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(-\frac{11x^7}{15120}-\frac{x^6}{240}-\frac{7x^5}{360}-\frac{5x^4}{72}-\frac{x^3}{6}-\frac{x^2}{6}+\frac{x}{3}+1\right) + c_1\left(\frac{1024x^7}{43648605}+\frac{64x^6}{328185}+\frac{64x^5}{45045}+\frac{80x^4}{9009}+\frac{32x^3}{693}+\frac{4x^2}{21}+\frac{4x}{7}+1\right)x^{5/2}$$

2.22 problem 22

Internal problem ID [5851]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{4}{9}\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-4/9)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}}\left(1 - \frac{3}{20}x^2 + \frac{9}{1280}x^4 - \frac{9}{56320}x^6 + \mathrm{O}\left(x^8\right)\right) + c_1\left(1 - \frac{3}{4}x^2 + \frac{9}{128}x^4 - \frac{9}{3584}x^6 + \mathrm{O}\left(x^8\right)\right)}{x^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 66

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-4/9)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 x^{2/3}\left(-\frac{9x^6}{56320} + \frac{9x^4}{1280} - \frac{3x^2}{20} + 1\right) + \frac{c_2\left(-\frac{9x^6}{3584} + \frac{9x^4}{128} - \frac{3x^2}{4} + 1\right)}{x^{2/3}}$$

2.23 problem 23

Internal problem ID [5852]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' + 9x^2y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
Order:=8;
dsolve(9*x^2*diff(y(x),x$2)+9*x^2*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{3}} \left(1 - \frac{1}{2}x + \frac{1}{5}x^2 - \frac{7}{120}x^3 + \frac{7}{528}x^4 - \frac{13}{5280}x^5 + \frac{13}{33660}x^6 - \frac{247}{4712400}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2 x^{\frac{2}{3}} \left(1 - \frac{1}{2}x + \frac{5}{28}x^2 - \frac{1}{21}x^3 + \frac{11}{1092}x^4 - \frac{11}{6240}x^5 + \frac{187}{711360}x^6 - \frac{17}{497952}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 118

```
AsymptoticDSolveValue[9*x^2*y''[x]+9*x^2*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2 \sqrt[3]{x} \left(-\frac{247x^7}{4712400} + \frac{13x^6}{33660} - \frac{13x^5}{5280} + \frac{7x^4}{528} - \frac{7x^3}{120} + \frac{x^2}{5} - \frac{x}{2} + 1\right) + c_1 x^{2/3} \left(-\frac{17x^7}{497952} + \frac{187x^6}{711360} - \frac{11x^5}{6240} + \frac{11x^4}{1092} - \frac{x^3}{21} + \frac{5x^2}{28} - \frac{x}{2} + 1\right)$$

2.24 problem 24

Internal problem ID [5853]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + 3y'x + (-1 + 2x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 55

```
Order:=8;
dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+(2*x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{3}{2}}\left(1 - \frac{2}{5}x + \frac{2}{35}x^2 - \frac{4}{945}x^3 + \frac{2}{10395}x^4 - \frac{4}{675675}x^5 + \frac{4}{30405375}x^6 - \frac{8}{3618239625}x^7 + \mathrm{O}\left(x^8\right)\right) + c_1\left(1 + 2x - 2x^2 \cdots\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 112

```
AsymptoticDSolveValue[2*x^2*y''[x]+3*x*y'[x]+(2*x-1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{8x^7}{3618239625} + \frac{4x^6}{30405375} - \frac{4x^5}{675675} + \frac{2x^4}{10395} - \frac{4x^3}{945} + \frac{2x^2}{35} - \frac{2x}{5} + 1\right) + \frac{c_2\left(\frac{8x^7}{3274425} - \frac{4x^6}{42525} + \frac{4x^5}{1575} - \frac{2x^4}{45} + \frac{4x^3}{9} - 2x^2 + 2x + 1\right)}{x}$$

2.25 problem 25

Internal problem ID [5854]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 2y' - xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 36

```
Order:=8;
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 \left(1 + \frac{1}{6}x^2 + \frac{1}{120}x^4 + \frac{1}{5040}x^6 + \mathrm{O}\left(x^8\right)\right) + \frac{c_2\left(1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{720}x^6 + \mathrm{O}\left(x^8\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 56

```
AsymptoticDSolveValue[x*y''[x]+2*y'[x]-x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \left(\frac{x^5}{720} + \frac{x^3}{24} + \frac{x}{2} + \frac{1}{x}\right) + c_2 \left(\frac{x^6}{5040} + \frac{x^4}{120} + \frac{x^2}{6} + 1\right)$$

2.26 problem 26

Internal problem ID [5855]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 - \frac{1}{5040}x^6 + \mathrm{O}\left(x^8\right)\right)x + c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \mathrm{O}\left(x^8\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 76

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^{11/2}}{720} + \frac{x^{7/2}}{24} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}}\right) + c_2\left(-\frac{x^{13/2}}{5040} + \frac{x^{9/2}}{120} - \frac{x^{5/2}}{6} + \sqrt{x}\right)$$

2.27 problem 27

Internal problem ID [5856]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{xy'' - y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 46

```
Order:=8;
dsolve(x*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \ln(x)\left(-x + \mathrm{O}\left(x^8\right)\right)c_2 + c_1 x\left(1 + \mathrm{O}\left(x^8\right)\right) + \left(1 + x - \frac{1}{2}x^2 - \frac{1}{12}x^3 - \frac{1}{72}x^4 - \frac{1}{480}x^5 - \frac{1}{3600}x^6 - \frac{1}{30240}x^7 + \mathrm{O}\left(x^8\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 51

```
AsymptoticDSolveValue[x*y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{-2x^6 - 15x^5 - 100x^4 - 600x^3 - 3600x^2 + 14400x + 7200}{7200} - x\log(x)\right) + c_2 x$$

2.28 problem 28

Internal problem ID [5857]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{3y'}{x} - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
Order:=8;
dsolve(diff(y(x),x$2)+3/x*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 + \frac{1}{4}x^2 + \frac{1}{48}x^4 + \frac{1}{1152}x^6 + \mathrm{O}\left(x^8\right)\right)x^2 + c_2\left(\ln\left(x\right)\left(\left(-2\right)x^2 - \frac{1}{2}x^4 - \frac{1}{24}x^6 + \mathrm{O}\left(x^8\right)\right) + \left(-2 + \frac{3}{8}x^4 + \frac{7}{144}\right.\right.}{x^2}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 74

```
AsymptoticDSolveValue[y''[x]+3/x*y'[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{x^6}{1152} + \frac{x^4}{48} + \frac{x^2}{4} + 1\right) + c_1\left(\frac{1}{48}\left(x^4 + 12x^2 + 48\right)\log(x) - \frac{5x^6 + 45x^4 + 72x^2 - 144}{144x^2}\right)$$

2.29 problem 29

Internal problem ID [5858]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy'' + (1-x)\,y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
Order:=8;
dsolve(x*diff(y(x),x$2)+(1-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)\,c_2 + c_1)\left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(-x - \frac{3}{4}x^2 - \frac{11}{36}x^3 - \frac{25}{288}x^4 - \frac{137}{7200}x^5 - \frac{49}{14400}x^6 - \frac{121}{235200}x^7 + \mathrm{O}\left(x^8\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 149

```
AsymptoticDSolveValue[x*y''[x]+(1-x)*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right) + c_2\left(-\frac{121x^7}{235200} - \frac{49x^6}{14400} - \frac{137x^5}{7200} - \frac{25x^4}{288} - \frac{11x^3}{36} - \frac{3x^2}{4} + \left(\frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right)\log(x) - x\right)$$

2.30 problem 30

Internal problem ID [5859]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 71

```
Order:=8;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x) c_2 + c_1)\left(1 - x + \frac{1}{4}x^2 - \frac{1}{36}x^3 + \frac{1}{576}x^4 - \frac{1}{14400}x^5 + \frac{1}{518400}x^6 - \frac{1}{25401600}x^7 + \mathrm{O}(x^8)\right) + \left(2x - \frac{3}{4}x^2 + \frac{11}{108}x^3 - \frac{25}{3456}x^4 + \frac{137}{432000}x^5 - \frac{49}{5184000}x^6 + \frac{121}{592704000}x^7 + \mathrm{O}(x^8)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 153

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^7}{25401600}+\frac{x^6}{518400}-\frac{x^5}{14400}+\frac{x^4}{576}-\frac{x^3}{36}+\frac{x^2}{4}-x+1\right) + c_2\left(\frac{121x^7}{592704000}-\frac{49x^6}{5184000}+\frac{137x^5}{432000}-\frac{25x^4}{3456}+\frac{11x^3}{108}-\frac{3x^2}{4} + \left(-\frac{x^7}{25401600}+\frac{x^6}{518400}-\frac{x^5}{14400}+\frac{x^4}{576}-\frac{x^3}{36}+\frac{x^2}{4}-x+1\right)\log(x)+2x\right)$$

2.31 problem 31

Internal problem ID [5860]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (x-6)\,y' - 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 46

```
Order:=8;
dsolve(x*diff(y(x),x$2)+(x-6)*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^7 \left(1 - \frac{1}{2}x + \frac{5}{36}x^2 - \frac{1}{36}x^3 + \frac{7}{1584}x^4 - \frac{7}{11880}x^5 + \frac{7}{102960}x^6 - \frac{1}{144144}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2\left(3628800 - 1814400x + 362880x^2 - 30240x^3 + 36x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 77

```
AsymptoticDSolveValue[x*y''[x]+(x-6)*y'[x]-3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{x^3}{120} + \frac{x^2}{10} - \frac{x}{2} + 1\right) + c_2\left(\frac{7x^{13}}{102960} - \frac{7x^{12}}{11880} + \frac{7x^{11}}{1584} - \frac{x^{10}}{36} + \frac{5x^9}{36} - \frac{x^8}{2} + x^7\right)$$

2.32 problem 32

Internal problem ID [5861]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(x-1)\, y'' + 3y' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 50

```
Order:=8;
dsolve(x*(x-1)*diff(y(x),x$2)+3*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x^4 \left(1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + 7x^6 + 8x^7 + \mathrm{O}\left(x^8\right)\right) \\ & + c_2 \left(-144 - 96x - 48x^2 + 48x^4 + 96x^5 + 144x^6 + 192x^7 + \mathrm{O}\left(x^8\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.366 (sec). Leaf size: 77

```
AsymptoticDSolveValue[x*(x-1)*y''[x]+3*y'[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \left(-x^6 - \frac{2x^5}{3} - \frac{x^4}{3} + \frac{x^2}{3} + \frac{2x}{3} + 1\right) + c_2 \left(7x^{10} + 6x^9 + 5x^8 + 4x^7 + 3x^6 + 2x^5 + x^4\right)$$

2.33 problem 33(b)

Internal problem ID [5862]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 33(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{2y'}{t} + \lambda y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 56

```
Order:=8;
dsolve(diff(y(t),t$2)+2/t*diff(y(t),t)+lambda*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = c_1\left(1 - \frac{1}{6}\lambda t^2 + \frac{1}{120}\lambda^2 t^4 - \frac{1}{5040}\lambda^3 t^6 + \mathrm{O}\left(t^8\right)\right) + \frac{c_2\left(1 - \frac{1}{2}\lambda t^2 + \frac{1}{24}\lambda^2 t^4 - \frac{1}{720}\lambda^3 t^6 + \mathrm{O}\left(t^8\right)\right)}{t}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 70

```
AsymptoticDSolveValue[y''[t]+2/t*y'[t]+\[Lambda]*y[t]==0,y[t],{t,0,7}]
```

$$y(t) \to c_1\left(-\frac{1}{720}\lambda^3 t^5 + \frac{\lambda^2 t^3}{24} - \frac{\lambda t}{2} + \frac{1}{t}\right) + c_2\left(-\frac{\lambda^3 t^6}{5040} + \frac{\lambda^2 t^4}{120} - \frac{\lambda t^2}{6} + 1\right)$$

2.34 problem 36 (a)

Internal problem ID [5863]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 36 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^3y'' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=8;
dsolve(x^3*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 294

```
AsymptoticDSolveValue[x^3*y''[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 e^{-\frac{2i}{\sqrt{x}}} x^{3/4} \left(-\frac{111004588013375 30625 i x^{13/2}}{4611686018427387904} + \frac{1327867167401775 i x^{11/2}}{4503599627370496} - \frac{468131288625 i x^{9/2}}{8796093022208} + \frac{66891825 i x^{7/2}}{4294967296} - \frac{72765 i x^{5/2}}{8388608} + \frac{105 i x^{3/2}}{8192} + \frac{1149690375852815671875 x^7}{147573952589676412928} - \frac{2323767542953106 25 x^6}{288230376151711744} + \frac{33424574007825 x^5}{281474976710656} - \frac{14783093325 x^4}{549755813888} + \frac{2837835 x^3}{268435456} - \frac{4725 x^2}{524288} + \frac{15x}{512} - \frac{3i\sqrt{x}}{16} + 1 \right) + c_2 e^{\frac{2i}{\sqrt{x}}} x^{3/4} \left(\frac{11100458801337530625 i x^{13/2}}{4611686018427387904} - \frac{1327867167401775 i x^{11/2}}{4503599627370496} + \frac{468131288625 i x^{9/2}}{8796093022208} - \frac{6689\ldots}{429\ldots}\right.$$

2.35 problem 36(b)

Internal problem ID [5864]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.3 SOLUTIONS ABOUT SINGULAR POINTS. EXERCISES 6.3. Page 255
Problem number: 36(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + (3x-1)\,y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+(3*x-1)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 53

```
AsymptoticDSolveValue[x^2*y''[x]+(3*x-1)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(5040x^7 + 720x^6 + 120x^5 + 24x^4 + 6x^3 + 2x^2 + x + 1\right) + \frac{c_2e^{-1/x}}{x}$$

3 CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267

3.1 problem 1

Internal problem ID [5865]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{9}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/9)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(\frac{1}{3}, x\right) + c_2 \operatorname{BesselY}\left(\frac{1}{3}, x\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/9)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(\frac{1}{3}, x\right) + c_2 Y_{\frac{1}{3}}(x)$$

3.2 problem 2

Internal problem ID [5866]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{x^2 y'' + y'x + (x^2 - 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}(1, x) + c_2 \operatorname{BesselY}(1, x)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}(1, x) + c_2 Y_1(x)$$

3.3 problem 3

Internal problem ID [5867]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4y'x + (4x^2 - 25)\, y = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 45

```
dsolve(4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2-25)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{ix}(x^2 + 3ix - 3)}{x^{\frac{5}{2}}} + \frac{c_2 \mathrm{e}^{-ix}(-x^2 + 3ix + 3)}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 57

```
DSolve[4*x^2*y''[x]+4*x*y'[x]+(4*x^2-25)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((3c_1x - c_2(x^2 - 3))\cos(x) + (c_1(x^2 - 3) + 3c_2x)\sin(x))}{x^{5/2}}$$

3.4 problem 4

Internal problem ID [5868]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16x^2y'' + 16y'x + (16x^2 - 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(16*x^2*diff(y(x),x$2)+16*x*diff(y(x),x)+(16*x^2-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(\frac{1}{4}, x\right) + c_2 \operatorname{BesselY}\left(\frac{1}{4}, x\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 22

```
DSolve[16*x^2*y''[x]+16*x*y'[x]+(16*x^2-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(\frac{1}{4}, x\right) + c_2 Y_{\frac{1}{4}}(x)$$

3.5 problem 5

Internal problem ID [5869]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + y' + xy = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}(0, x) + c_2 \operatorname{BesselY}(0, x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[x*y''[x]+y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}(0, x) + c_2 Y_0(x)$$

3.6 problem 6

Internal problem ID [5870]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{xy'' + y' + \left(x - \frac{4}{x}\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(x*diff(y(x),x),x)+(x-4/x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}(2, x) + c_2 \operatorname{BesselY}(2, x)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 18

```
DSolve[D[x*y'[x],x]+(x-4/x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}(2, x) + c_2 Y_2(x)$$

3.7 problem 7

Internal problem ID [5871]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + (9x^2 - 4)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(9*x^2-4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}(2, 3x) + c_2 \operatorname{BesselY}(2, 3x)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+x*y'[x]+(9*x^2-4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}(2, 3x) + c_2 Y_2(3x)$$

3.8 problem 8

Internal problem ID [5872]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y'x + \left(36x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(36*x^2-1/4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(6x)}{\sqrt{x}} + \frac{c_2 \cos(6x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(36*x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-6ix}(12c_1 - ic_2e^{12ix})}{12\sqrt{x}}$$

3.9 problem 9

Internal problem ID [5873]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(25x^2 - \frac{4}{9}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(25*x^2-4/9)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(\frac{2}{3}, 5x\right) + c_2 \operatorname{BesselY}\left(\frac{2}{3}, 5x\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+x*y'[x]+(25*x^2-4/9)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(\frac{2}{3}, 5x\right) + c_2 Y_{\frac{2}{3}}(5x)$$

3.10 problem 10

Internal problem ID [5874]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + (2x^2 - 64)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(2*x^2-64)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(8, \sqrt{2}\, x\right) + c_2 \operatorname{BesselY}\left(8, \sqrt{2}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 30

```
DSolve[x^2*y''[x]+x*y'[x]+(2*x^2-64)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(8, \sqrt{2}x\right) + c_2 Y_8\left(\sqrt{2}x\right)$$

3.11 problem 13

Internal problem ID [5875]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + 2y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{BesselJ}\left(1, 4\sqrt{x}\right)}{\sqrt{x}} + \frac{c_2 \operatorname{BesselY}\left(1, 4\sqrt{x}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 34

```
DSolve[x*y''[x]+2*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\, {}_0\tilde{F}_1(;2;-4x) - \frac{ic_2 Y_1\left(4\sqrt{x}\right)}{\sqrt{x}}$$

3.12 problem 14

Internal problem ID [5876]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 3y' + xy = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{BesselJ}(1, x)}{x} + \frac{c_2 \operatorname{BesselY}(1, x)}{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x*y''[x]+3*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 \operatorname{BesselJ}(1, x) + c_2 Y_1(x)}{x}$$

3.13 problem 15

Internal problem ID [5877]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' - y' + xy = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \operatorname{BesselJ}(1, x) + c_2 x \operatorname{BesselY}(1, x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[x*y''[x]-y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x \operatorname{BesselJ}(1, x) + c_2 x Y_1(x)$$

3.14 problem 16

Internal problem ID [5878]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' - 5y' + xy = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x$2)-5*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^3 \operatorname{BesselJ}(3, x) + c_2 x^3 \operatorname{BesselY}(3, x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x*y''[x]-5*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3 (c_1 \operatorname{BesselJ}(3, x) + c_2 Y_3(x))$$

3.15 problem 17

Internal problem ID [5879]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (x^2 - 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(x^2*diff(y(x),x$2)+(x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(\cos(x)\, x - \sin(x))}{x} + \frac{c_2(\cos(x) + \sin(x)\, x)}{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]+(x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{2}{\pi}}((c_1x + c_2)\cos(x) + (c_2x - c_1)\sin(x))}{x}$$

3.16 problem 18

Internal problem ID [5880]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + (16x^2 + 1) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(4*x^2*diff(y(x),x$2)+(16*x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\ \mathrm{BesselJ}(0, 2x) + c_2\sqrt{x}\ \mathrm{BesselY}(0, 2x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 28

```
DSolve[4*x^2*y''[x]+(16*x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_1 \mathrm{BesselJ}(0, 2x) + c_2 Y_0(2x))$$

3.17 problem 19

Internal problem ID [5881]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + 3y' + yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{x^2}{2}\right)}{x^2} + \frac{c_2 \cos\left(\frac{x^2}{2}\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 43

```
DSolve[x*y''[x]+3*y'[x]+x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{ix^2}{2}}\left(2c_1 - ic_2 e^{ix^2}\right)}{2x^2}$$

3.18 problem 20

Internal problem ID [5882]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' + 9y'x + (x^6 - 36)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(9*x^2*diff(y(x),x$2)+9*x*diff(y(x),x)+(x^6-36)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(\frac{2}{3}, \frac{x^3}{9}\right) + c_2 \operatorname{BesselY}\left(\frac{2}{3}, \frac{x^3}{9}\right)$$

✓ Solution by Mathematica

Time used: 0.063 (sec). Leaf size: 42

```
DSolve[9*x^2*y''[x]+9*x*y'[x]+(x^6-36)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{Gamma}\left(\frac{1}{3}\right) \operatorname{BesselJ}\left(-\frac{2}{3}, \frac{x^3}{9}\right) + c_2 \operatorname{Gamma}\left(\frac{5}{3}\right) \operatorname{BesselJ}\left(\frac{2}{3}, \frac{x^3}{9}\right)$$

3.19 problem 22(a)

Internal problem ID [5883]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 22(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - x^2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\text{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) + c_2\sqrt{x}\,\text{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 37

```
DSolve[y''[x]-x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\,\text{ParabolicCylinderD}\left(-\frac{1}{2}, i\sqrt{2}x\right) + c_1\,\text{ParabolicCylinderD}\left(-\frac{1}{2}, \sqrt{2}x\right)$$

3.20 problem 22 (b)

Internal problem ID [5884]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 22 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' - 7yx^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x$2)+diff(y(x),x)-7*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselI}\left(0, \frac{\sqrt{7}\, x^2}{2}\right) + c_2 \operatorname{BesselK}\left(0, \frac{\sqrt{7}\, x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 36

```
DSolve[x*y''[x]+y'[x]-7*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\, {}_0\tilde{F}_1\left(;1;\frac{7x^4}{16}\right) + 2c_2 K_0\left(\frac{\sqrt{7}x^2}{2}\right)$$

3.21 problem 23

Internal problem ID [5885]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + c_2 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 16

```
DSolve[y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + c_2 \sin(x)$$

3.22 problem 24

Internal problem ID [5886]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 4y'x + \left(x^2 + 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x^2} + \frac{c_2 \cos(x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+4*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x^2}$$

3.23 problem 25

Internal problem ID [5887]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16x^2y'' + 32y'x + (x^4 - 12)\, y = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 27

```
dsolve(16*x^2*diff(y(x),x$2)+32*x*diff(y(x),x)+(x^4-12)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{x^2}{8}\right)}{x^{\frac{3}{2}}} + \frac{c_2 \cos\left(\frac{x^2}{8}\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 42

```
DSolve[16*x^2*y''[x]+32*x*y'[x]+(x^4-12)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{ix^2}{8}}\left(c_1 - 2ic_2 e^{\frac{ix^2}{4}}\right)}{x^{3/2}}$$

3.24 problem 26

Internal problem ID [5888]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. 6.4 SPECIAL FUNCTIONS. EXERCISES 6.4. Page 267
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4y'x + (16x^2 + 3)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(16*x^2+3)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sin(2x) + c_2\sqrt{x}\,\cos(2x)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 39

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(16*x^2+3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2ix}\sqrt{x}\left(4c_1 - ic_2e^{4ix}\right)$$

4 CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271

4.1 problem 9

Internal problem ID [5889]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$\boxed{2xy'' + y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 - \frac{1}{3}x + \frac{1}{30}x^2 - \frac{1}{630}x^3 + \frac{1}{22680}x^4 - \frac{1}{1247400}x^5 + \frac{1}{97297200}x^6 - \frac{1}{10216206000}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2\left(1 - x + \frac{1}{6}x^2 - \frac{1}{90}x^3 + \frac{1}{2520}x^4 - \frac{1}{113400}x^5 + \frac{1}{7484400}x^6 - \frac{1}{681080400}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 111

```
AsymptoticDSolveValue[2*x*y''[x]+y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{x^7}{10216206000} + \frac{x^6}{97297200} - \frac{x^5}{1247400} + \frac{x^4}{22680} - \frac{x^3}{630} + \frac{x^2}{30} - \frac{x}{3} + 1\right) + c_2\left(-\frac{x^7}{681080400} + \frac{x^6}{7484400} - \frac{x^5}{113400} + \frac{x^4}{2520} - \frac{x^3}{90} + \frac{x^2}{6} - x + 1\right)$$

4.2 problem 10

Internal problem ID [5890]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' - y'x - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{8}x^4 + \frac{1}{48}x^6\right) y(0) + \left(x + \frac{1}{3}x^3 + \frac{1}{15}x^5 + \frac{1}{105}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]-x*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{x^7}{105} + \frac{x^5}{15} + \frac{x^3}{3} + x\right) + c_1\left(\frac{x^6}{48} + \frac{x^4}{8} + \frac{x^2}{2} + 1\right)$$

4.3 problem 11

Internal problem ID [5891]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(x-1)\,y''+3y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
Order:=8;
dsolve((x-1)*diff(y(x),x$2)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{3}{2}x^2+\frac{1}{2}x^3+\frac{5}{8}x^4+\frac{9}{20}x^5+\frac{29}{80}x^6+\frac{163}{560}x^7\right)y(0)+\left(x+\frac{1}{2}x^3+\frac{1}{4}x^4+\frac{9}{40}x^5+\frac{7}{40}x^6+\frac{79}{560}x^7\right)D(y)(0)+O\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 91

```
AsymptoticDSolveValue[(x-1)*y''[x]+3*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(\frac{79x^7}{560}+\frac{7x^6}{40}+\frac{9x^5}{40}+\frac{x^4}{4}+\frac{x^3}{2}+x\right)+c_1\left(\frac{163x^7}{560}+\frac{29x^6}{80}+\frac{9x^5}{20}+\frac{5x^4}{8}+\frac{x^3}{2}+\frac{3x^2}{2}+1\right)$$

4.4 problem 12

Internal problem ID [5892]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - x^2 y' + xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=8;
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 - \frac{1}{90}x^6\right) y(0) + D(y)(0)\, x + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 27

```
AsymptoticDSolveValue[y''[x]-x^2*y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \left(-\frac{x^6}{90} - \frac{x^3}{6} + 1\right) + c_2 x$$

4.5 problem 13

Internal problem ID [5893]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - (2+x)\,y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 52

```
Order:=8;
dsolve(x*diff(y(x),x$2)-(x+2)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^3 \left(1 + \frac{1}{4}x + \frac{1}{20}x^2 + \frac{1}{120}x^3 + \frac{1}{840}x^4 + \frac{1}{6720}x^5 + \frac{1}{60480}x^6 + \frac{1}{604800}x^7 + \mathrm{O}\left(x^8\right)\right) + c_2 \left(12 + 12x + 6x^2 + 2x^3 + \frac{1}{2}x^4 + \frac{1}{10}x^5 + \frac{1}{60}x^6 + \frac{1}{420}x^7 + \mathrm{O}\left(x^8\right)\right)$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 94

```
AsymptoticDSolveValue[x*y''[x]-(x+2)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \left(\frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right) + c_2 \left(\frac{x^9}{60480} + \frac{x^8}{6720} + \frac{x^7}{840} + \frac{x^6}{120} + \frac{x^5}{20} + \frac{x^4}{4} + x^3\right)$$

4.6 problem 14

Internal problem ID [5894]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\cos(x)\, y'' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
Order:=8;
dsolve(cos(x)*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{720}x^6\right) y(0) + \left(x - \frac{1}{6}x^3 - \frac{1}{60}x^5 - \frac{13}{5040}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 49

```
AsymptoticDSolveValue[Cos[x]*y''[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(\frac{x^6}{720} - \frac{x^2}{2} + 1\right) + c_2\left(-\frac{13x^7}{5040} - \frac{x^5}{60} - \frac{x^3}{6} + x\right)$$

4.7 problem 15

Internal problem ID [5895]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 2y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = -2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
Order:=8;
dsolve([diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(0) = 3, D(y)(0) = -2],y(x),type='series',x=0
```

$$y(x) = 3 - 2x - 3x^2 + x^3 + x^4 - \frac{1}{4}x^5 - \frac{1}{5}x^6 + \frac{1}{24}x^7 + \mathrm{O}\left(x^8\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[{y''[x]+x*y'[x]+2*y[x]==0,{y[0]==3,y'[0]==-2}},y[x],{x,0,7}]
```

$$y(x) \to \frac{x^7}{24} - \frac{x^6}{5} - \frac{x^5}{4} + x^4 + x^3 - 3x^2 - 2x + 3$$

4.8 problem 16

Internal problem ID [5896]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(2+x)\, y''+3y=0}$$

With initial conditions

$$[y(0)=0, y'(0)=1]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=8;
dsolve([(x+2)*diff(y(x),x$2)+3*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',x=0);
```

$$y(x)=x-\frac{1}{4}x^{3}+\frac{1}{16}x^{4}-\frac{1}{320}x^{6}+\frac{1}{896}x^{7}+\mathrm{O}\left(x^{8}\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[{(x+2)*y''[x]+3*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,7}]
```

$$y(x)\to \frac{x^7}{896}-\frac{x^6}{320}+\frac{x^4}{16}-\frac{x^3}{4}+x$$

4.9 problem 17

Internal problem ID [5897]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-2\sin(x)+1)\,y''+xy=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
Order:=8;
dsolve((1-2*sin(x))*diff(y(x),x$2)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{6}x^3-\frac{1}{6}x^4-\frac{1}{5}x^5-\frac{1}{4}x^6-\frac{85}{252}x^7\right)y(0)+\left(x-\frac{1}{12}x^4-\frac{1}{10}x^5-\frac{2}{15}x^6-\frac{13}{72}x^7\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 77

```
AsymptoticDSolveValue[(1-2*Sin[x])*y''[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_2\left(-\frac{13x^7}{72}-\frac{2x^6}{15}-\frac{x^5}{10}-\frac{x^4}{12}+x\right)+c_1\left(-\frac{85x^7}{252}-\frac{x^6}{4}-\frac{x^5}{5}-\frac{x^4}{6}-\frac{x^3}{6}+1\right)$$

4.10 problem 18

Internal problem ID [5898]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y'x + y = 0}$$

With initial conditions

$$[y(1) = -6, y'(1) = 3]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
Order:=8;
dsolve([diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(1) = -6, D(y)(1) = 3],y(x),type='series',x=1);
```

$$y(x) = -6 + 3(x-1) + \frac{3}{2}(x-1)^2 - \frac{3}{2}(x-1)^3 + \frac{3}{10}(x-1)^5 - \frac{1}{20}(x-1)^6 - \frac{1}{28}(x-1)^7 + \mathrm{O}\left((x-1)^8\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 55

```
AsymptoticDSolveValue[{y''[x]+x*y'[x]+y[x]==0,{y[1]==-6,y'[1]==3}},y[x],{x,1,7}]
```

$$y(x) \to -\frac{1}{28}(x-1)^7 - \frac{1}{20}(x-1)^6 + \frac{3}{10}(x-1)^5 - \frac{3}{2}(x-1)^3 + \frac{3}{2}(x-1)^2 + 3(x-1) - 6$$

4.11 problem 19

Internal problem ID [5899]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + (1-\cos(x))\,y' + x^2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
Order:=8;
dsolve(x*diff(y(x),x$2)+(1-cos(x))*diff(y(x),x)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1-\frac{1}{6}x^3+\frac{1}{80}x^5+\frac{1}{180}x^6-\frac{5}{4032}x^7\right)y(0) + \left(x-\frac{1}{12}x^3-\frac{1}{12}x^4+\frac{1}{120}x^5+\frac{1}{120}x^6+\frac{73}{60480}x^7\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 36

```
AsymptoticDSolveValue[x*y''[x]+(1-Cos[x])*y'[x]+x^2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to -\frac{53x^7}{8640}+\frac{x^5}{48}+\frac{x^4}{6}-\frac{x^3}{3}-2x+3$$

4.12 problem 20

Internal problem ID [5900]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(e^x - 1 - x)\, y'' + xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 70

```
Order:=8;
dsolve((exp(x)-1-x)*diff(y(x),x$2)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x \left(1 - x + \frac{4}{9}x^2 - \frac{29}{216}x^3 + \frac{37}{1200}x^4 - \frac{58}{10125}x^5 + \frac{14209}{15876000}x^6 - \frac{107329}{889056000}x^7 + O\left(x^8\right)\right) + c_2 \left(\ln(x) \left((-2)\, x + 2x^2 - \frac{8}{9}x^3 + \frac{29}{108}x^4 - \frac{37}{600}x^5 + \frac{116}{10125}x^6 - \frac{14209}{7938000}x^7 + O\left(x^8\right)\right) + \left(1 - \frac{8}{3}x^2 + \frac{175}{108}x^3 - \frac{3727}{6480}x^4 + \frac{47531}{324000}x^5 - \frac{3003737}{102060000}x^6 + \frac{48833381}{10001880000}x^7 + O\left(x^8\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.359 (sec). Leaf size: 133

```
AsymptoticDSolveValue[(Exp[x]-1-x)*y''[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \to\ & c_1\left(x^6\left(\frac{116\log(x)}{10125}-\frac{3003737}{102060000}\right)+x^5\left(\frac{47531}{324000}-\frac{37\log(x)}{600}\right)\right. \\
& \left.+x^4\left(\frac{29\log(x)}{108}-\frac{3727}{6480}\right)+x^3\left(\frac{175}{108}-\frac{8\log(x)}{9}\right)+x^2\left(2\log(x)-\frac{8}{3}\right)-2x\log(x)+1\right) \\
& +c_2x\left(-\frac{107329x^7}{889056000}+\frac{14209x^6}{15876000}-\frac{58x^5}{10125}+\frac{37x^4}{1200}-\frac{29x^3}{216}+\frac{4x^2}{9}-x+1\right)
\end{aligned}
$$

4.13 problem 21

Internal problem ID [5901]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 6 SERIES SOLUTIONS OF LINEAR EQUATIONS. CHAPTER 6 IN REVIEW. Page 271
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{y'' + x^2y' + 2xy - 10x^3 + 2x - 5 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
Order:=8;
dsolve(diff(y(x),x$2)+x^2*diff(y(x),x)+2*x*y(x)=5-2*x+10*x^3,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{3}x^3 + \frac{1}{18}x^6\right)y(0) + \left(x - \frac{1}{4}x^4 + \frac{1}{28}x^7\right)D(y)(0) + \frac{5x^2}{2} - \frac{x^3}{3} + \frac{x^6}{18} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]+x^2*y'[x]+2*x*y[x]==5-2*x+10*x^3,y[x],{x,0,7}]
```

$$y(x) \to \frac{x^6}{18} - \frac{x^3}{3} + \frac{5x^2}{2} + c_2\left(\frac{x^7}{28} - \frac{x^4}{4} + x\right) + c_1\left(\frac{x^6}{18} - \frac{x^3}{3} + 1\right)$$

5 CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289

5.1 problem 31

Internal problem ID [5902]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y - 1 = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 8

```
dsolve([diff(y(t),t)-y(t)=1,y(0) = 0],y(t), singsol=all)
```

$$y(t) = -1 + e^{t}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 10

```
DSolve[{y'[t]-y[t]==1,{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{t} - 1$$

5.2 problem 32

Internal problem ID [5903]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{2y' + y = 0}$$

With initial conditions

$$[y(0) = -3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve([2*diff(y(t),t)+y(t)=0,y(0) = -3],y(t), singsol=all)
```

$$y(t) = -3\,\mathrm{e}^{-\frac{t}{2}}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 14

```
DSolve[{2*y'[t]+y[t]==0,{y[0]==-3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -3e^{-t/2}$$

5.3 problem 33

Internal problem ID [5904]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 6y - e^{4t} = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(y(t),t)+6*y(t)=exp(4*t),y(0) = 2],y(t), singsol=all)
```

$$y(t) = \frac{(e^{10t} + 19)\, e^{-6t}}{10}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 21

```
DSolve[{y'[t]+6*y[t]==Exp[4*t],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{10} e^{-6t} \left(e^{10t} + 19\right)$$

5.4 problem 34

Internal problem ID [5905]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y - 2\cos(5t) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve([diff(y(t),t)-y(t)=2*cos(5*t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{\cos(5t)}{13} + \frac{5\sin(5t)}{13} + \frac{e^t}{13}$$

✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 25

```
DSolve[{y'[t]-y[t]==2*Cos[5*t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{13}\left(e^t + 5\sin(5t) - \cos(5t)\right)$$

5.5 problem 35

Internal problem ID [5906]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 5y' + 4y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+5*diff(y(t),t)+4*y(t)=0,y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{4\,\mathrm{e}^{-t}}{3} - \frac{\mathrm{e}^{-4t}}{3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 23

```
DSolve[{y''[t]+5*y'[t]+4*y[t]==0,{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{3}e^{-4t}(4e^{3t} - 1)$$

5.6 problem 36

Internal problem ID [5907]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 4y' - 6\,\mathrm{e}^{3t} + 3\,\mathrm{e}^{-t} = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)=6*exp(3*t)-3*exp(-t),y(0) = 1, D(y)(0) = -1],y(t), sing
```

$$y(t) = \frac{11\,\mathrm{e}^{4t}}{10} - \frac{3\,\mathrm{e}^{-t}}{5} - 2\,\mathrm{e}^{3t} + \frac{5}{2}$$

✓ Solution by Mathematica

Time used: 0.131 (sec). Leaf size: 34

```
DSolve[{y''[t]-4*y'[t]==6*Exp[3*t]-3*Exp[-t],{y[0]==1,y'[0]==-1}},y[t],t,IncludeSingularSolut
```

$$y(t) \to -\frac{3e^{-t}}{5} - 2e^{3t} + \frac{11e^{4t}}{10} + \frac{5}{2}$$

5.7 problem 37

Internal problem ID [5908]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \sqrt{2}\,\sin\left(t\sqrt{2}\right) = 0}$$

With initial conditions

$$[y(0) = 10, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 24

```
dsolve([diff(y(t),t$2)+y(t)=sqrt(2)*sin(sqrt(2)*t),y(0) = 10, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = 2\sin(t) + 10\cos(t) - \sqrt{2}\,\sin\left(\sqrt{2}\,t\right)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 29

```
DSolve[{y''[t]+y[t]==Sqrt[2]*Sin[Sqrt[2]*t],{y[0]==10,y'[0]==0}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to 2\sin(t) - \sqrt{2}\sin\left(\sqrt{2}t\right) + 10\cos(t)$$

5.8 problem 38

Internal problem ID [5909]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 9y - e^t = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+9*y(t)=exp(t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{\sin(3t)}{30} - \frac{\cos(3t)}{10} + \frac{e^t}{10}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 27

```
DSolve[{y''[t]+9*y[t]==Exp[t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{30}(3e^t - \sin(3t) - 3\cos(3t))$$

5.9 problem 39

Internal problem ID [5910]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289
Problem number: 39.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{2y''' + 3y'' - 3y' - 2y - \mathrm{e}^{-t} = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 28

```
dsolve([2*diff(y(t),t$3)+3*diff(y(t),t$2)-3*diff(y(t),t)-2*y(t)=exp(-t),y(0) = 0, D(y)(0) = 0
```

$$y(t) = -\frac{\left(-5\,\mathrm{e}^{3t} + 16\,\mathrm{e}^{\frac{3t}{2}} - 9\,\mathrm{e}^{t} - 2\right)\mathrm{e}^{-2t}}{18}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 37

```
DSolve[{2*y'''[t]+3*y''[t]-3*y'[t]-2*y[t]==Exp[-t],{y[0]==0,y'[0]==0,y''[0]==1}},y[t],t,Inclu
```

$$y(t) \to \frac{1}{18}e^{-2t}\left(9e^{t} - 16e^{3t/2} + 5e^{3t} + 2\right)$$

5.10 problem 40

Internal problem ID [5911]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289
Problem number: 40.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + 2y'' - y' - 2y - \sin(3t) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 33

```
dsolve([diff(y(t),t$3)+2*diff(y(t),t$2)-diff(y(t),t)-2*y(t)=sin(3*t),y(0) = 0, D(y)(0) = 0, (
```

$$y(t) = -\frac{\left(12\sin(3t)\,e^{2t} - 18\cos(3t)\,e^{2t} - 169\,e^{3t} + 507\,e^{t} - 320\right)e^{-2t}}{780}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 42

```
DSolve[{y'''[t]+2*y''[t]-y'[t]-2*y[t]==Sin[3*t],{y[0]==0,y'[0]==0,y''[0]==1}},y[t],t,IncludeS
```

$$y(t) \to \frac{1}{780}\left(e^{-2t}\left(169e^t\left(e^{2t} - 3\right) + 320\right) - 12\sin(3t) + 18\cos(3t)\right)$$

5.11 problem 41

Internal problem ID [5912]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y - e^{-3t} \cos(2t) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 28

```
dsolve([diff(y(t),t)+y(t)=exp(-3*t)*cos(2*t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{(-1 + (\cos(2t) - \sin(2t)) e^{-2t}) e^{-t}}{4}$$

✓ Solution by Mathematica

Time used: 0.12 (sec). Leaf size: 30

```
DSolve[{y'[t]+y[t]==Exp[-3*t]*Cos[2*t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4} e^{-3t} \left(e^{2t} + \sin(2t) - \cos(2t)\right)$$

5.12 problem 42

Internal problem ID [5913]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.2.2 TRANSFORMS OF DERIVATIVES Page 289
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 5y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 3]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)+5*y(t)=0,y(0) = 1, D(y)(0) = 3],y(t), singsol=all)
```

$$y(t) = e^t(\sin(2t) + \cos(2t))$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{y''[t]-2*y'[t]+5*y[t]==0,{y[0]==1,y'[0]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^t(\sin(2t) + \cos(2t))$$

6 CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297

6.1 problem 21

Internal problem ID [5914]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 4y - e^{-4t} = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 12

```
dsolve([diff(y(t),t)+4*y(t)=exp(-4*t),y(0) = 2],y(t), singsol=all)
```

$$y(t) = (t + 2)\,e^{-4t}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 14

```
DSolve[{y'[t]+4*y[t]==Exp[-4*t],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-4t}(t + 2)$$

6.2 problem 22

Internal problem ID [5915]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y - 1 - t\,\mathrm{e}^{t} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)-y(t)=1+t*exp(t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\mathrm{e}^{t} t^{2}}{2} - 1 + \mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 19

```
DSolve[{y'[t]-y[t]==1+t*Exp[t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{t}\left(t^{2} + 2\right) - 1$$

6.3 problem 23

Internal problem ID [5916]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+y(t)=0,y(0) = 1, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-t}(2t + 1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{y''[t]+2*y'[t]+y[t]==0,{y[0]==1,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}(2t + 1)$$

6.4 problem 24

Internal problem ID [5917]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 4y - t^3 e^{2t} = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)+4*y(t)=t^3*exp(2*t),y(0) = 0, D(y)(0) = 0],y(t), singso
```

$$y(t) = \frac{t^5 e^{2t}}{20}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 17

```
DSolve[{y''[t]-4*y'[t]+4*y[t]==t^3*Exp[2*t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutio
```

$$y(t) \to \frac{1}{20} e^{2t} t^5$$

6.5 problem 25

Internal problem ID [5918]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 6y' + 9y - t = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)-6*diff(y(t),t)+9*y(t)=t,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{(30t - 2)\,e^{3t}}{27} + \frac{2}{27} + \frac{t}{9}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[{y''[t]-6*y'[t]+9*y[t]==t,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{27}\left(3t + e^{3t}(30t - 2) + 2\right)$$

6.6 problem 26

Internal problem ID [5919]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 4y - t^3 = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 30

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)+4*y(t)=t^3,y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{(-13t + 2)\,e^{2t}}{8} + \frac{t^3}{4} + \frac{3t^2}{4} + \frac{9t}{8} + \frac{3}{4}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 32

```
DSolve[{y''[t]-4*y'[t]+4*y[t]==t^3,{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to \frac{1}{8}\left(e^{2t}(2 - 13t) + t(2t(t + 3) + 9) + 6\right)$$

6.7 problem 27

Internal problem ID [5920]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 13y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = -3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(t),t$2)-6*diff(y(t),t)+13*y(t)=0,y(0) = 0, D(y)(0) = -3],y(t), singsol=all)
```

$$y(t) = -\frac{3\,\mathrm{e}^{3t}\sin(2t)}{2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{y''[t]-6*y'[t]+13*y[t]==0,{y[0]==0,y'[0]==-3}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to -3e^{3t}\sin(t)\cos(t)$$

6.8 problem 28

Internal problem ID [5921]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' + 20y' + 51y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 30

```
dsolve([2*diff(y(t),t$2)+20*diff(y(t),t)+51*y(t)=0,y(0) = 2, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = 2\,\mathrm{e}^{-5t}\left(5\sqrt{2}\,\sin\left(\frac{\sqrt{2}\,t}{2}\right) + \cos\left(\frac{\sqrt{2}\,t}{2}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 36

```
DSolve[{2*y''[t]+20*y'[t]+51*y[t]==0,{y[0]==2,y'[0]==0}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to 2e^{-5t}\left(5\sqrt{2}\sin\left(\frac{t}{\sqrt{2}}\right) + \cos\left(\frac{t}{\sqrt{2}}\right)\right)$$

6.9 problem 29

Internal problem ID [5922]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y - e^{t} \cos(t) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve([diff(y(t),t$2)-y(t)=exp(t)*cos(t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{e^{-t}}{5} + \frac{e^{t}(-\cos(t) + 2\sin(t))}{5}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 27

```
DSolve[{y''[t]-y[t]==Exp[t]*Cos[t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to \frac{1}{5}\left(e^{-t} - e^{t}(\cos(t) - 2\sin(t))\right)$$

6.10 problem 30

Internal problem ID [5923]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' + 5y - t - 1 = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 4]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)+5*y(t)=1+t,y(0) = 0, D(y)(0) = 4],y(t), singsol=all)
```

$$y(t) = \frac{51\,\mathrm{e}^{t} \sin(2t)}{25} - \frac{7\,\mathrm{e}^{t} \cos(2t)}{25} + \frac{t}{5} + \frac{7}{25}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 31

```
DSolve[{y''[t]-2*y'[t]+5*y[t]==1+t,{y[0]==0,y'[0]==4}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to \frac{1}{25}\left(5t + e^t(51\sin(2t) - 7\cos(2t)) + 7\right)$$

6.11 problem 31

Internal problem ID [5924]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

Solve

$$\boxed{y'' + 2y' + y = 0}$$

With initial conditions

$$[y(1) = 2, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+y(t)=0,y(1) = 2, D(y)(0) = 2],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-t}(t\,\mathrm{e} + \mathrm{e} + t - 1)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 18

```
DSolve[{y''[t]+2*y'[t]+y[t]==0,{y[1]==2,y'[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow e^{-t}(et + t + e - 1)$$

6.12 problem 32

Internal problem ID [5925]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

Solve

$$\boxed{y'' + 8y' + 20y = 0}$$

With initial conditions

$$[y(0) = 0, y'(\pi) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 5

```
dsolve([diff(y(t),t$2)+8*diff(y(t),t)+20*y(t)=0,y(0) = 0, D(y)(Pi) = 0],y(t), singsol=all)
```

$$y(t) = 0$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 6

```
DSolve[{y''[t]+8*y'[t]+20*y[t]==0,{y[0]==0,y'[Pi]==0}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to 0$$

6.13 problem 63

Internal problem ID [5926]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$y' + y - \left(\begin{cases} 0 & 0 \leq t < 1 \\ 5 & 1 \leq t \end{cases}\right) = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 22

```
dsolve([diff(y(t),t)+y(t)=piecewise(0<=t and t<1,0,t>=1,5),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \begin{cases} 0 & t < 1 \\ -5\,\mathrm{e}^{-t+1} + 5 & 1 \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 23

```
DSolve[{y'[t]+y[t]==Piecewise[{{0,0<=t<1},{5,t>=1}}],{y[0]==0}},y[t],t,IncludeSingularSolutio
```

$$y(t) \to \left\{ \begin{array}{cc} 0 & t \leq 1 \\ 5 - 5e^{1-t} & \text{True} \end{array} \right.$$

6.14 problem 64

Internal problem ID [5927]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$y' + y - \left(\begin{cases} 1 & 0 \leq t < 1 \\ -1 & 1 \leq t \end{cases}\right) = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 39

```
dsolve([diff(y(t),t)+y(t)=piecewise(0<=t and t<1,1,t>=1,-1),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \begin{cases} 0 & t < 0 \\ 1 - e^{-t} & t < 1 \\ 2\,e^{-t+1} - 1 - e^{-t} & 1 \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 40

```
DSolve[{y'[t]+y[t]==Piecewise[{{1,0<=t<1},{-1,t>=1}}],{y[0]==0}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to \{ \begin{array}{cc} 0 & t \leq 0 \\ -\cosh(t) + \sinh(t) + 1 & 0 < t \leq 1 \\ -1 + e^{-t}(-1 + 2e) & \text{True} \end{array}$$

6.15 problem 65

Internal problem ID [5928]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + y - \left(\begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t \end{cases}\right) = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 26

```
dsolve([diff(y(t),t)+y(t)=piecewise(0<=t and t<1,t,t>=1,0),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \begin{cases} 0 & t < 0 \\ t - 1 + e^{-t} & t < 1 \\ e^{-t} & 1 \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 32

```
DSolve[{y'[t]+y[t]==Piecewise[{{t,0<=t<1},{0,t>=1}}],{y[0]==0}},y[t],t,IncludeSingularSolutio
```

$$y(t) \rightarrow \{ \begin{array}{cc} 0 & t \leq 0 \\ t + e^{-t} - 1 & 0 < t \leq 1 \\ e^{-t} & \text{True} \end{array}$$

6.16 problem 66

Internal problem ID [5929]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 66.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y - \left(\begin{cases} 1 & 0 \leq t < 1 \\ 0 & 1 \leq t \end{cases}\right) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 45

```
dsolve([diff(y(t),t$2)+4*y(t)=piecewise(0<=t and t<1,1,t>=1,0),y(0) = 0, D(y)(0) = -1],y(t),
```

$$y(t) = -\frac{\sin(2t)}{2} + \frac{\left(\begin{cases} 0 & t < 0 \\ 1 - \cos(2t) & t < 1 \\ \cos(2t - 2) - \cos(2t) & 1 \leq t \end{cases}\right)}{4}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 65

```
DSolve[{y''[t]+4*y[t]==Piecewise[{{1,0<=t<1},{0,t>=1}}],{y[0]==0,y'[0]==-1}},y[t],t,IncludeSi
```

$$y(t) \to \begin{cases} -\cos(t)\sin(t) & t \leq 0 \\ \frac{1}{4}(-\cos(2t) - 2\sin(2t) + 1) & 0 < t \leq 1 \\ \frac{1}{4}(\cos(2-2t) - \cos(2t) - 2\sin(2t)) & \text{True} \end{cases}$$

6.17 problem 67

Internal problem ID [5930]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 67.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y - \text{Heaviside}\,(-2\pi + t)\sin(t) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)+4*y(t)=sin(t)*Heaviside(t-2*Pi),y(0) = 1, D(y)(0) = 0],y(t), singsol=a
```

$$y(t) = -\frac{(\cos(t) - 1)\sin(t)\,\text{Heaviside}\,(-2\pi + t)}{3} + 2\cos(t)^2 - 1$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 32

```
DSolve[{y''[t]+4*y[t]==Sin[t]*UnitStep[t-2*Pi],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolu
```

$$y(t) \to \begin{cases} \cos(2t) & t \leq 2\pi \\ \cos(2t) - \frac{1}{3}(\cos(t) - 1)\sin(t) & \text{True} \end{cases}$$

6.18 problem 68

Internal problem ID [5931]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 68.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 5y' + 6y - \text{Heaviside}\,(t-1) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve([diff(y(t),t$2)-5*diff(y(t),t)+6*y(t)=Heaviside(t-1),y(0) = 0, D(y)(0) = 1],y(t), sing
```

$$y(t) = e^{3t} - e^{2t} + \frac{\text{Heaviside}\,(t-1)\,e^{3t-3}}{3} - \frac{\text{Heaviside}\,(t-1)\,e^{2t-2}}{2} + \frac{\text{Heaviside}\,(t-1)}{6}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 57

```
DSolve[{y''[t]-5*y'[t]+6*y[t]==UnitStep[t-1],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to \begin{cases} e^{2t}(-1+e^t) & t \leq 1 \\ \frac{1}{6}(e^{2t-3}(-3e - 6e^3 + 2e^t + 6e^{t+3}) + 1) & \text{True} \end{cases}$$

6.19 problem 69

Internal problem ID [5932]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 69.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \left(\begin{cases} 0 & 0 \leq t < \pi \\ 1 & \pi \leq t < 2\pi \\ 0 & 2\pi \leq t \end{cases}\right) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 30

```
dsolve([diff(y(t),t$2)+y(t)=piecewise(0<=t and t<Pi,0,Pi<=t and t<2*Pi,1,t>=2*Pi,0),y(0) = 0,
```

$$y(t) = \sin(t) + \left(\begin{cases} 0 & t < \pi \\ \cos(t) + 1 & t < 2\pi \\ 2\cos(t) & 2\pi \leq t \end{cases}\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 35

```
DSolve[{y''[t]+y[t]==Piecewise[{{0,0<=t<Pi},{1,Pi<=t<2*Pi},{0,t>=2*Pi}}],{y[0]==0,y'[0]==1}},
```

$$y(t) \to \begin{cases} \sin(t) & t \leq \pi \\ \cos(t) + \sin(t) + 1 & \pi < t \leq 2\pi \\ 2\cos(t) + \sin(t) & \text{True} \end{cases}$$

6.20 problem 70

Internal problem ID [5933]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.3.1 TRANSLATION ON THE s-AXIS. Page 297
Problem number: 70.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y' + 3y - 1 + \text{Heaviside}\,(t-2) + \text{Heaviside}\,(t-4) - \text{Heaviside}\,(t-6) = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 108

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+3*y(t)=1-Heaviside(t-2)-Heaviside(t-4)+Heaviside(t-6),y
```

$$\begin{aligned} y(t) = &-\frac{e^{-t}}{2} + \frac{e^{-3t}}{6} - \frac{\text{Heaviside}\,(t-2)}{3} + \frac{\text{Heaviside}\,(t-2)\,e^{-t+2}}{2} - \frac{\text{Heaviside}\,(t-4)}{3} \\ &+ \frac{\text{Heaviside}\,(t-4)\,e^{-t+4}}{2} + \frac{\text{Heaviside}\,(t-6)}{3} - \frac{\text{Heaviside}\,(t-6)\,e^{-t+6}}{2} + \frac{1}{3} \\ &- \frac{\text{Heaviside}\,(t-2)\,e^{-3t+6}}{6} - \frac{\text{Heaviside}\,(t-4)\,e^{-3t+12}}{6} + \frac{\text{Heaviside}\,(t-6)\,e^{-3t+18}}{6} \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 129

```
DSolve[{y''[t]+4*y'[t]+3*y[t]==1-UnitStep[t-2]-UnitStep[t-4]+UnitStep[t-6],{y[0]==0,y'[0]==0}
```

$$\begin{aligned} y(t) \to \frac{1}{6}e^{-3t}\Big(&(2e^t + e^2)\left(e^2 - e^t\right)^2 \theta(2-t) + \left(e^4 - e^t\right)^2 (2e^t + e^4)\,\theta(4-t) \\ &- \left(e^6 - e^t\right)^2 (2e^t + e^6)\,\theta(6-t) - 3\left(e^2 - 1\right)^2 (1 + e^2)\,e^{2t} + \left(e^6 - 1\right)^2 (1 + e^6)\Big) \end{aligned}$$

7 CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309

7.1 problem 9

Internal problem ID [5934]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y - t\sin(t) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve([diff(y(t),t)+y(t)=t*sin(t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{e^{-t}}{2} + \frac{(-t+1)\cos(t)}{2} + \frac{t\sin(t)}{2}$$

✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 27

```
DSolve[{y'[t]+y[t]==t*Sin[t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}(t\sin(t) - t\cos(t) + \cos(t) + \sinh(t) - \cosh(t))$$

7.2 problem 10

Internal problem ID [5935]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y - t\,\mathrm{e}^{t} \sin(t) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)-y(t)=t*exp(t)*sin(t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = -\mathrm{e}^{t}(\cos(t)\,t - \sin(t))$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 17

```
DSolve[{y'[t]-y[t]==t*Exp[t]*Sin[t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{t}(\sin(t) - t\cos(t))$$

7.3 problem 11

Internal problem ID [5936]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y - \cos(3t) = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 5]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)+9*y(t)=cos(3*t),y(0) = 2, D(y)(0) = 5],y(t), singsol=all)
```

$$y(t) = \frac{(t+10)\sin(3t)}{6} + 2\cos(3t)$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 23

```
DSolve[{y''[t]+9*y[t]==Cos[3*t],{y[0]==2,y'[0]==5}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{6}(t+10)\sin(3t) + 2\cos(3t)$$

7.4 problem 12

Internal problem ID [5937]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \sin(t) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)+y(t)=sin(t),y(0) = 1, D(y)(0) = -1],y(t), singsol=all)
```

$$y(t) = -\frac{\sin(t)}{2} + \cos(t) - \frac{\cos(t)\, t}{2}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 21

```
DSolve[{y''[t]+y[t]==Sin[t],{y[0]==1,y'[0]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{\sin(t)}{2} - \frac{1}{2}t\cos(t) + \cos(t)$$

7.5 problem 13

Internal problem ID [5938]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 16y - \left(\left\{\begin{array}{cc} \cos(4t) & 0 \leq t < \pi \\ 0 & \pi \leq t \end{array}\right.\right) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.329 (sec). Leaf size: 25

```
dsolve([diff(y(t),t$2)+16*y(t)=piecewise(0<=t and t<Pi,cos(4*t),t>= Pi,0),y(0) = 0, D(y)(0) =
```

$$y(t) = \frac{\sin(4t)\left(2 + \left(\left\{\begin{array}{cc} 0 & t < 0 \\ t & t < \pi \\ \pi & \pi \leq t \end{array}\right.\right)\right)}{8}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 60

```
DSolve[{y''[t]+16*y[t]==Piecewise[{{Cos[4*t],0<=t<Pi},{0,t>=Pi}}],{y[0]==1,y'[0]==1}},y[t],t,
```

$$y(t) \rightarrow \begin{cases} \cos(4t) + \frac{1}{4}\sin(4t) & t \leq 0 \\ \cos(4t) + \frac{1}{8}(2+\pi)\sin(4t) & t > \pi \\ \cos(4t) + \frac{1}{8}(t+2)\sin(4t) & \text{True} \end{cases}$$

7.6 problem 14

Internal problem ID [5939]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - \left(\begin{cases} 1 & 0 \leq t < \frac{\pi}{2} \\ \sin(t) & \frac{\pi}{2} \leq t \end{cases} \right) = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 7.547 (sec). Leaf size: 33

```
dsolve([diff(y(t),t$2)+y(t)=piecewise(0<=t and t<Pi/2,1,t>= Pi/2,sin(t)),y(0) = 1, D(y)(0) =
```

$$y(t) = \begin{cases} \cos(t) & t < 0 \\ 1 & t < \frac{\pi}{2} \\ \frac{(-2t+\pi)\cos(t)}{4} + \sin(t) & \frac{\pi}{2} \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 38

```
DSolve[{y''[t]+y[t]==Piecewise[{{1,0<=t<Pi/2},{Sin[t],t>=Pi/2}}],{y[0]==1,y'[0]==0}},y[t],t,I
```

$$y(t) \to \{ \begin{array}{cc} \cos(t) & t \leq 0 \\ 1 & t > 0 \land 2t \leq \pi \\ \frac{1}{4}(\pi - 2t)\cos(t) + \sin(t) & \text{True} \end{array}$$

7.7 problem 17

Internal problem ID [5940]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{ty'' - y' - 2t^2 = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 16

```
dsolve([t*diff(y(t),t$2)-diff(y(t),t)=2*t^2,y(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{t^2(4t + 3c_1)}{6}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 26

```
DSolve[{y''[t]-y'[t]==2*t^2,{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{2}{3}t(t(t+3)+6) + c_1(e^t - 1)$$

7.8 problem 18

Internal problem ID [5941]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y'' + ty' - 2y - 10 = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 9

```
dsolve([2*diff(y(t),t$2)+t*diff(y(t),t)-2*y(t)=10,y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{5t^2}{2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 10

```
DSolve[{y''[t]+t*y'[t]-2*y[t]==10,{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to 5t^2$$

7.9 problem 36

Internal problem ID [5942]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. 7.4.1 DERIVATIVES OF A TRANSFORM. Page 309
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \sin(t) - t\sin(t) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve([diff(y(t),t$2)+y(t)=sin(t)+t*sin(t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{(t+2)(\cos(t)\,t - \sin(t))}{4}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 21

```
DSolve[{y''[t]+y[t]==Sin[t]+t*Sin[t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to -\frac{1}{4}(t+2)(t\cos(t) - \sin(t))$$

8 CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315

8.1 problem 1

Internal problem ID [5943]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - 3y - (\delta(t-2)) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)-3*y(t)=Dirac(t-2),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \text{Heaviside}\,(t-2)\,\mathrm{e}^{3t-6}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 17

```
DSolve[{y'[t]-3*y[t]==DiracDelta[t-2],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{3t-6}\theta(t-2)$$

8.2 problem 2

Internal problem ID [5944]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y - (\delta(t-1)) = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(t),t)+y(t)=Dirac(t-1),y(0) = 2],y(t), singsol=all)
```

$$y(t) = (\text{Heaviside}\,(t-1)\,\mathrm{e} + 2)\,\mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 19

```
DSolve[{y'[t]+y[t]==DiracDelta[t-1],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}(e\theta(t-1) + 2)$$

8.3 problem 3

Internal problem ID [5945]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - (\delta(-2\pi + t)) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)+y(t)=Dirac(t-2*Pi),y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \sin(t)\,(\text{Heaviside}\,(-2\pi + t) + 1)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 16

```
DSolve[{y''[t]+y[t]==DiracDelta[t-2*Pi],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -
```

$$y(t) \to (\theta(t - 2\pi) + 1)\sin(t)$$

8.4 problem 4

Internal problem ID [5946]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 16y - (\delta(-2\pi + t)) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)+16*y(t)=Dirac(t-2*Pi),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\text{Heaviside}\,(-2\pi + t) \sin{(4t)}}{4}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 19

```
DSolve[{y''[t]+16*y[t]==DiracDelta[t-2*Pi],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolution
```

$$y(t) \rightarrow \frac{1}{4}\theta(t - 2\pi)\sin(4t)$$

8.5 problem 5

Internal problem ID [5947]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - \left(\delta\left(t - \frac{\pi}{2}\right)\right) - \left(\delta\left(t - \frac{3\pi}{2}\right)\right) = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([diff(y(t),t$2)+y(t)=Dirac(t-1/2*Pi)+Dirac(t-3/2*Pi),y(0) = 0, D(y)(0) = 0],y(t), sing
```

$$y(t) = \left(\text{Heaviside}\left(t - \frac{3\pi}{2}\right) - \text{Heaviside}\left(t - \frac{\pi}{2}\right)\right)\cos(t)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 27

```
DSolve[{y''[t]+y[t]==DiracDelta[t-1/2*Pi]+DiracDelta[t-3/2*Pi],{y[0]==0,y'[0]==0}},y[t],t,Inc
```

$$y(t) \to (\theta(2t - 3\pi) - \theta(2t - \pi))\cos(t)$$

8.6 problem 6

Internal problem ID [5948]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - (\delta(-2\pi + t)) - (\delta(t - 4\pi)) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)+y(t)=Dirac(t-2*Pi)+Dirac(t-4*Pi),y(0) = 1, D(y)(0) = 0],y(t), singsol=
```

$$y(t) = \sin(t) \operatorname{Heaviside}(t - 4\pi) + \sin(t) \operatorname{Heaviside}(-2\pi + t) + \cos(t)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 24

```
DSolve[{y''[t]+y[t]==DiracDelta[t-2*Pi]+DiracDelta[t-4*Pi],{y[0]==1,y'[0]==0}},y[t],t,Include
```

$$y(t) \to (\theta(t - 4\pi) + \theta(t - 2\pi)) \sin(t) + \cos(t)$$

8.7 problem 7

Internal problem ID [5949]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 2y' - (\delta(t-1)) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 30

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)=Dirac(t-1),y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = -\frac{e^{-2t}}{2} - \frac{\text{Heaviside}(t-1)\, e^{-2t+2}}{2} + \frac{\text{Heaviside}(t-1)}{2} + \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 32

```
DSolve[{y''[t]+2*y'[t]==DiracDelta[t-1],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -
```

$$y(t) \to \frac{1}{2}\left(\theta(t-1) - e^{-2t}\left(e^2\theta(t-1) + 1\right) + 1\right)$$

8.8 problem 8

Internal problem ID [5950]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 2y' - 1 - (\delta(t - 2)) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 33

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)=1+Dirac(t-2),y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{3\,\mathrm{e}^{2t}}{4} + \frac{\mathrm{Heaviside}\,(t-2)\,\mathrm{e}^{2t-4}}{2} - \frac{\mathrm{Heaviside}\,(t-2)}{2} - \frac{t}{2} - \frac{3}{4}$$

✓ Solution by Mathematica

Time used: 0.102 (sec). Leaf size: 37

```
DSolve[{y''[t]-2*y'[t]==1+DiracDelta[t-2],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{1}{4}\left(\left(2e^{2t-4} - 2\right)\theta(t-2) - 2t + 3e^{2t} - 3\right)$$

8.9 problem 9

Internal problem ID [5951]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 5y - (\delta(-2\pi + t)) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+5*y(t)=Dirac(t-2*Pi),y(0) = 0, D(y)(0) = 0],y(t), sings
```

$$y(t) = \sin(t)\,\text{Heaviside}(-2\pi + t)\,e^{4\pi - 2t}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 23

```
DSolve[{y''[t]+4*y'[t]+5*y[t]==DiracDelta[t-2*Pi],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularS
```

$$y(t) \to e^{4\pi - 2t}\theta(t - 2\pi)\sin(t)$$

8.10 problem 10

Internal problem ID [5952]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y - (\delta(t-1)) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+y(t)=Dirac(t-1),y(0) = 0, D(y)(0) = 0],y(t), singsol=al
```

$$y(t) = (t-1)\,\mathrm{Heaviside}\,(t-1)\,\mathrm{e}^{-t+1}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 20

```
DSolve[{y''[t]+2*y'[t]+y[t]==DiracDelta[t-1],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to e^{1-t}(t-1)\theta(t-1)$$

8.11 problem 11

Internal problem ID [5953]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 13y - (\delta(-\pi + t)) - (\delta(t - 3\pi)) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 56

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+13*y(t)=Dirac(t-Pi)+Dirac(t-3*Pi),y(0) = 1, D(y)(0) = 0
```

$$y(t) = -\frac{\sin(3t)\,\text{Heaviside}(t - 3\pi)\,\mathrm{e}^{6\pi - 2t}}{3} - \frac{\sin(3t)\,\text{Heaviside}(-\pi + t)\,\mathrm{e}^{-2t + 2\pi}}{3} + \mathrm{e}^{-2t}\left(\cos(3t) + \frac{2\sin(3t)}{3}\right)$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 53

```
DSolve[{y''[t]+4*y'[t]+13*y[t]==DiracDelta[t-Pi]+DiracDelta[t-3*Pi],{y[0]==1,y'[0]==0}},y[t],
```

$$y(t) \to \frac{1}{3}e^{-2t}\left(3\cos(3t) - \left(e^{6\pi}\theta(t - 3\pi) + e^{2\pi}\theta(t - \pi) - 2\right)\sin(3t)\right)$$

8.12 problem 12

Internal problem ID [5954]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 7y' + 6y - e^t - (\delta(t-2)) - (\delta(t-4)) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 64

```
dsolve([diff(y(t),t$2)-7*diff(y(t),t)+6*y(t)=exp(t)+Dirac(t-2)+Dirac(t-4),y(0) = 0, D(y)(0) =
```

$$y(t) = \frac{e^{6t}}{25} + \frac{e^{-24+6t}\,\text{Heaviside}(t-4)}{5} + \frac{e^{-12+6t}\,\text{Heaviside}(t-2)}{5} - \frac{e^{t-4}\,\text{Heaviside}(t-4)}{5} - \frac{e^{t-2}\,\text{Heaviside}(t-2)}{5} + \frac{(-5t-1)\,e^t}{25}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 67

```
DSolve[{y''[t]-7*y'[t]+6*y[t]==Exp[t]+DiracDelta[t-2]+DiracDelta[t-4],{y[0]==9,y'[0]==0}},y[t
```

$$y(t) \to \frac{1}{25}e^{t-24}\left(5\left(e^{5t}-e^{20}\right)\theta(t-4) + 5\left(e^{5t+12}-e^{22}\right)\theta(t-2) + e^{24}\left(-5t-44e^{5t}+269\right)\right)$$

8.13 problem 15(a)

Internal problem ID [5955]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315
Problem number: 15(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 10y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+10*y(t)=0,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{e^{-t} \sin(3t)}{3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{y''[t]+2*y'[t]+10*y[t]==0,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to \frac{1}{3} e^{-t} \sin(3t)$$

8.14 problem 15(b)

Internal problem ID [5956]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 7 THE LAPLACE TRANSFORM. EXERCISES 7.5. Page 315
Problem number: 15(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 10y - (\delta(t)) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 18

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+10*y(t)=Dirac(t),y(0) = 0, D(y)(0) = 0],y(t), singsol=a
```

$$y(t) = \frac{e^{-t} \sin(3t) (2 \operatorname{Heaviside}(t) - 1)}{6}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 25

```
DSolve[{y''[t]+2*y'[t]+10*y[t]==DiracDelta[t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolut
```

$$y(t) \to \frac{1}{3} e^{-t} (\theta(t) - \theta(0)) \sin(3t)$$

9 CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332

9.1 problem 1

Internal problem ID [5957]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 3x(t) - 5y(t)$$
$$y'(t) = 4x(t) + 8y(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 84

```
dsolve([diff(x(t),t)=3*x(t)-5*y(t),diff(y(t),t)=4*x(t)+8*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^{\frac{11t}{2}} \left(\sin\left(\frac{\sqrt{55}\,t}{2}\right)\sqrt{55}\,c_2 - \cos\left(\frac{\sqrt{55}\,t}{2}\right)\sqrt{55}\,c_1 + 5\sin\left(\frac{\sqrt{55}\,t}{2}\right)c_1 + 5\cos\left(\frac{\sqrt{55}\,t}{2}\right)c_2\right)}{8}$$

$$y(t) = e^{\frac{11t}{2}} \left(\sin\left(\frac{\sqrt{55}\,t}{2}\right)c_1 + \cos\left(\frac{\sqrt{55}\,t}{2}\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 113

```
DSolve[{x'[t]==3*x[t]-5*y[t],y'[t]==4*x[t]+8*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{11}e^{11t/2}\left(11c_1\cos\left(\frac{\sqrt{55}t}{2}\right) - \sqrt{55}(c_1 + 2c_2)\sin\left(\frac{\sqrt{55}t}{2}\right)\right)$$
$$y(t) \to \frac{1}{55}e^{11t/2}\left(55c_2\cos\left(\frac{\sqrt{55}t}{2}\right) + \sqrt{55}(8c_1 + 5c_2)\sin\left(\frac{\sqrt{55}t}{2}\right)\right)$$

9.2 problem 2

Internal problem ID [5958]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4x(t) - 7y(t)$$
$$y'(t) = 5x(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 78

```
dsolve([diff(x(t),t)=4*x(t)-7*y(t),diff(y(t),t)=5*x(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^{2t}\left(\sin\left(\sqrt{31}\,t\right)\sqrt{31}\,c_2 - \cos\left(\sqrt{31}\,t\right)\sqrt{31}\,c_1 - 2\sin\left(\sqrt{31}\,t\right)c_1 - 2\cos\left(\sqrt{31}\,t\right)c_2\right)}{5}$$

$$y(t) = e^{2t}\left(\sin\left(\sqrt{31}\,t\right)c_1 + \cos\left(\sqrt{31}\,t\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 98

```
DSolve[{x'[t]==4*x[t]-7*y[t],y'[t]==5*x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{2t}\cos\left(\sqrt{31}t\right) + \frac{(2c_1 - 7c_2)e^{2t}\sin\left(\sqrt{31}t\right)}{\sqrt{31}}$$
$$y(t) \to c_2 e^{2t}\cos\left(\sqrt{31}t\right) + \frac{(5c_1 - 2c_2)e^{2t}\sin\left(\sqrt{31}t\right)}{\sqrt{31}}$$

9.3 problem 3

Internal problem ID [5959]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$\begin{aligned} x'(t) &= -3x(t) + 4y(t) - 9z(t) \\ y'(t) &= 6x(t) - y(t) \\ z'(t) &= 10x(t) + 4y(t) + 3z(t) \end{aligned}$$

✓ Solution by Maple

Time used: 0.61 (sec). Leaf size: 3117

```
dsolve([diff(x(t),t)=-3*x(t)+4*y(t)-9*z(t),diff(y(t),t)=6*x(t)-y(t),diff(z(t),t)=10*x(t)+4*y(
```

Expression too large to display

Expression too large to display

$$z(t) = c_2 \mathrm{e}^{\frac{\left(-170+\left(4726+306\sqrt{291}\right)^{\frac{2}{3}}-2\left(4726+306\sqrt{291}\right)^{\frac{1}{3}}\right)t}{6\left(4726+306\sqrt{291}\right)^{\frac{1}{3}}}} \sin\left(\frac{\left(\left(4726+306\sqrt{291}\right)^{\frac{2}{3}}+170\right)t\sqrt{3}\,1156^{\frac{1}{3}}}{204\left(139+9\sqrt{291}\right)^{\frac{1}{3}}}\right)$$
$$+ c_3 \mathrm{e}^{\frac{\left(-170+\left(4726+306\sqrt{291}\right)^{\frac{2}{3}}-2\left(4726+306\sqrt{291}\right)^{\frac{1}{3}}\right)t}{6\left(4726+306\sqrt{291}\right)^{\frac{1}{3}}}} \cos\left(\frac{\left(\left(4726+306\sqrt{291}\right)^{\frac{2}{3}}+170\right)t\sqrt{3}\,1156^{\frac{1}{3}}}{204\left(139+9\sqrt{291}\right)^{\frac{1}{3}}}\right)$$
$$+ c_1 \mathrm{e}^{-\frac{\left(\left(4726+306\sqrt{291}\right)^{\frac{2}{3}}+\left(4726+306\sqrt{291}\right)^{\frac{1}{3}}-170\right)t}{3\left(4726+306\sqrt{291}\right)^{\frac{1}{3}}}}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 510

```
DSolve[{x'[t]==-3*x[t]+4*y[t]-9*z[t],y'[t]==6*x[t]-y[t],z'[t]==10*x[t]+4*y[t]+3*z[t]},{x[t],y
```

$$
\begin{aligned}
x(t) \to\ & 4c_2\text{RootSum}\left[\#1^3+\#1^2+57\#1+369\&,\frac{\#1e^{\#1t}-12e^{\#1t}}{3\#1^2+2\#1+57}\&\right] \\
& -9c_3\text{RootSum}\left[\#1^3+\#1^2+57\#1+369\&,\frac{\#1e^{\#1t}+e^{\#1t}}{3\#1^2+2\#1+57}\&\right] \\
& +c_1\text{RootSum}\left[\#1^3+\#1^2+57\#1+369\&,\frac{\#1^2e^{\#1t}-2\#1e^{\#1t}-3e^{\#1t}}{3\#1^2+2\#1+57}\&\right] \\
y(t) \to\ & -54c_3\text{RootSum}\left[\#1^3+\#1^2+57\#1+369\&,\frac{e^{\#1t}}{3\#1^2+2\#1+57}\&\right] \\
& +6c_1\text{RootSum}\left[\#1^3+\#1^2+57\#1+369\&,\frac{\#1e^{\#1t}-3e^{\#1t}}{3\#1^2+2\#1+57}\&\right] \\
& +c_2\text{RootSum}\left[\#1^3+\#1^2+57\#1+369\&,\frac{\#1^2e^{\#1t}+81e^{\#1t}}{3\#1^2+2\#1+57}\&\right] \\
z(t) \to\ & 4c_2\text{RootSum}\left[\#1^3+\#1^2+57\#1+369\&,\frac{\#1e^{\#1t}+13e^{\#1t}}{3\#1^2+2\#1+57}\&\right] \\
& +2c_1\text{RootSum}\left[\#1^3+\#1^2+57\#1+369\&,\frac{5\#1e^{\#1t}+17e^{\#1t}}{3\#1^2+2\#1+57}\&\right] \\
& +c_3\text{RootSum}\left[\#1^3+\#1^2+57\#1+369\&,\frac{\#1^2e^{\#1t}+4\#1e^{\#1t}-21e^{\#1t}}{3\#1^2+2\#1+57}\&\right]
\end{aligned}
$$

9.4 problem 4

Internal problem ID [5960]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= x(t) - y(t) \\
y'(t) &= x(t) + 2z(t) \\
z'(t) &= -x(t) + z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.687 (sec). Leaf size: 2271

```
dsolve([diff(x(t),t)=x(t)-y(t),diff(y(t),t)=x(t)+2*z(t),diff(z(t),t)=-x(t)+z(t)],[x(t), y(t),
```

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Expression too large to display

$$
\begin{aligned}
z(t) = &-c_2 \mathrm{e}^{-\frac{\left(-8+\left(244+12\sqrt{417}\right)^{\frac{2}{3}}-8\left(244+12\sqrt{417}\right)^{\frac{1}{3}}\right)t}{12\left(244+12\sqrt{417}\right)^{\frac{1}{3}}}} \sin\left(\frac{\left(\left(244+12\sqrt{417}\right)^{\frac{2}{3}}+8\right)t\sqrt{3}\,2^{\frac{1}{3}}}{24\left(61+3\sqrt{417}\right)^{\frac{1}{3}}}\right) \\
&+ c_3 \mathrm{e}^{-\frac{\left(-8+\left(244+12\sqrt{417}\right)^{\frac{2}{3}}-8\left(244+12\sqrt{417}\right)^{\frac{1}{3}}\right)t}{12\left(244+12\sqrt{417}\right)^{\frac{1}{3}}}} \cos\left(\frac{\left(\left(244+12\sqrt{417}\right)^{\frac{2}{3}}+8\right)t\sqrt{3}\,2^{\frac{1}{3}}}{24\left(61+3\sqrt{417}\right)^{\frac{1}{3}}}\right) \\
&+ c_1 \mathrm{e}^{\frac{\left(\left(244+12\sqrt{417}\right)^{\frac{2}{3}}+4\left(244+12\sqrt{417}\right)^{\frac{1}{3}}-8\right)t}{6\left(244+12\sqrt{417}\right)^{\frac{1}{3}}}}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 503

```
DSolve[{x'[t]==x[t]-y[t],y'[t]==x[t]+2*z[t],z'[t]==-x[t]+z[t]},{x[t],y[t],z[t]},t,IncludeSing
```

$$
\begin{aligned}
x(t) \to & -2c_3 \text{RootSum}\left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{e^{\#1t}}{3\#1^2 - 4\#1 + 2}\&\right] \\
& - c_2 \text{RootSum}\left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{\#1e^{\#1t} - e^{\#1t}}{3\#1^2 - 4\#1 + 2}\&\right] \\
& + c_1 \text{RootSum}\left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{\#1^2e^{\#1t} - \#1e^{\#1t}}{3\#1^2 - 4\#1 + 2}\&\right] \\
y(t) \to & \, c_1 \text{RootSum}\left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{\#1e^{\#1t} - 3e^{\#1t}}{3\#1^2 - 4\#1 + 2}\&\right] \\
& + 2c_3 \text{RootSum}\left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{\#1e^{\#1t} - e^{\#1t}}{3\#1^2 - 4\#1 + 2}\&\right] \\
& + c_2 \text{RootSum}\left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{\#1^2e^{\#1t} - 2\#1e^{\#1t} + e^{\#1t}}{3\#1^2 - 4\#1 + 2}\&\right] \\
z(t) \to & \, c_2 \text{RootSum}\left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{e^{\#1t}}{3\#1^2 - 4\#1 + 2}\&\right] \\
& - c_1 \text{RootSum}\left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{\#1e^{\#1t}}{3\#1^2 - 4\#1 + 2}\&\right] \\
& + c_3 \text{RootSum}\left[\#1^3 - 2\#1^2 + 2\#1 - 3\&, \frac{\#1^2e^{\#1t} - \#1e^{\#1t} + e^{\#1t}}{3\#1^2 - 4\#1 + 2}\&\right]
\end{aligned}
$$

9.5 problem 5

Internal problem ID [5961]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - y(t) + z(t) + t - 1$$
$$y'(t) = 2x(t) + y(t) - z(t) - 3t^2$$
$$z'(t) = x(t) + y(t) + z(t) + t^2 - t + 2$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 172

```
dsolve([diff(x(t),t)=x(t)-y(t)+z(t)+t-1,diff(y(t),t)=2*x(t)+y(t)-z(t)-3*t^2,diff(z(t),t)=x(t)
```

$$x(t) = t^2 - \frac{1}{6} + \frac{2c_1 e^{2t}}{3} - c_2 e^{\frac{t}{2}} \cos\left(\frac{\sqrt{11}\, t}{2}\right) - c_3 e^{\frac{t}{2}} \sin\left(\frac{\sqrt{11}\, t}{2}\right)$$

$$y(t) = -\frac{t^2}{2} - \frac{3t}{2} - \frac{7}{4} + \frac{c_1 e^{2t}}{3} + \frac{c_2 e^{\frac{t}{2}} \cos\left(\frac{\sqrt{11}\, t}{2}\right)}{2} + \frac{c_3 e^{\frac{t}{2}} \sin\left(\frac{\sqrt{11}\, t}{2}\right)}{2} - \frac{c_2 e^{\frac{t}{2}} \sqrt{11}\, \sin\left(\frac{\sqrt{11}\, t}{2}\right)}{2} + \frac{c_3 e^{\frac{t}{2}} \sqrt{11}\, \cos\left(\frac{\sqrt{11}\, t}{2}\right)}{2}$$

$$z(t) = -\frac{3t^2}{2} - \frac{t}{2} - \frac{7}{12} + c_1 e^{2t} + c_2 e^{\frac{t}{2}} \cos\left(\frac{\sqrt{11}\, t}{2}\right) + c_3 e^{\frac{t}{2}} \sin\left(\frac{\sqrt{11}\, t}{2}\right)$$

✓ Solution by Mathematica

Time used: 15.858 (sec). Leaf size: 273

```
DSolve[{x'[t]==x[t]-y[t]+z[t]+t-1,y'[t]==2*x[t]+y[t]-z[t]-3*t^2,z'[t]==x[t]+y[t]+z[t]+t^2-t+2
```

$$x(t) \to t^2 + \frac{2}{5}(c_1 + c_3)e^{2t} + \frac{1}{55}e^{t/2}\left(11(3c_1 - 2c_3)\cos\left(\frac{\sqrt{11}t}{2}\right) - \sqrt{11}(c_1 + 10c_2 - 4c_3)\sin\left(\frac{\sqrt{11}t}{2}\right)\right) - \frac{1}{6}$$

$$y(t) \to \frac{1}{220}\left(-55(2t(t+3)+7) + 44(c_1 + c_3)e^{2t} + 4e^{t/2}\left(\sqrt{11}(17c_1 + 5c_2 - 13c_3)\sin\left(\frac{\sqrt{11}t}{2}\right) - 11(c_1 - 5c_2 + c_3)\cos\left(\frac{\sqrt{11}t}{2}\right)\right)\right)$$

$$z(t) \to -\frac{1}{2}t(3t+1) + \frac{3}{5}(c_1 + c_3)e^{2t} + \frac{1}{55}e^{t/2}\left((22c_3 - 33c_1)\cos\left(\frac{\sqrt{11}t}{2}\right) + \sqrt{11}(c_1 + 10c_2 - 4c_3)\sin\left(\frac{\sqrt{11}t}{2}\right)\right) - \frac{7}{12}$$

9.6 problem 6

Internal problem ID [5962]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332
Problem number: 6.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -3x(t) + 4y(t) + 2\,\mathrm{e}^{-t}\cos(t)\sin(t) \\
y'(t) &= 5x(t) + 9z(t) + 8\,\mathrm{e}^{-t}\cos(t)^2 - 4\,\mathrm{e}^{-t} \\
z'(t) &= y(t) + 6z(t) - \mathrm{e}^{-t}
\end{aligned}
$$

✓ Solution by Maple

Time used: 46.781 (sec). Leaf size: 12874

```
dsolve([diff(x(t),t)=-3*x(t)+4*y(t)+exp(-t)*sin(2*t),diff(y(t),t)=5*x(t)+9*z(t)+4*exp(-t)*cos
```

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Expression too large to display

Expression too large to display

✓ Solution by Mathematica

Time used: 0.621 (sec). Leaf size: 2949

```
DSolve[{x'[t]==-3*x[t]+4*y[t]+Exp[-t]*Sin[2*t],y'[t]==5*x[t]+9*z[t]+4*Exp[-t]*Cos[2*t],z'[t]=
```

Too large to display

9.7 problem 7

Internal problem ID [5963]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332
Problem number: 7.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4x(t) + 2y(t) + e^t$$
$$y'(t) = -x(t) + 3y(t) - e^t$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 106

```
dsolve([diff(x(t),t)=4*x(t)+2*y(t)+exp(t),diff(y(t),t)=-x(t)+3*y(t)-exp(t)],[x(t), y(t)], sin
```

$$x(t) = -\frac{e^{\frac{7t}{2}} \sin\left(\frac{\sqrt{7}\,t}{2}\right) c_2}{2} - \frac{e^{\frac{7t}{2}} \sqrt{7} \cos\left(\frac{\sqrt{7}\,t}{2}\right) c_2}{2} - \frac{e^{\frac{7t}{2}} \cos\left(\frac{\sqrt{7}\,t}{2}\right) c_1}{2} + \frac{e^{\frac{7t}{2}} \sqrt{7} \sin\left(\frac{\sqrt{7}\,t}{2}\right) c_1}{2} - \frac{e^t}{2}$$

$$y(t) = e^{\frac{7t}{2}} \sin\left(\frac{\sqrt{7}\,t}{2}\right) c_2 + e^{\frac{7t}{2}} \cos\left(\frac{\sqrt{7}\,t}{2}\right) c_1 + \frac{e^t}{4}$$

✓ Solution by Mathematica

Time used: 0.304 (sec). Leaf size: 127

```
DSolve[{x'[t]==4*x[t]+2*y[t]+Exp[t],y'[t]==-x[t]+3*y[t]-Exp[t]},{x[t],y[t]},t,IncludeSingular
```

$$x(t) \to -\frac{e^t}{2} + \frac{1}{7} e^{7t/2} \left(7c_1 \cos\left(\frac{\sqrt{7}t}{2}\right) + \sqrt{7}(c_1 + 4c_2) \sin\left(\frac{\sqrt{7}t}{2}\right)\right)$$
$$y(t) \to \frac{e^t}{4} + \frac{1}{7} e^{7t/2} \left(7c_2 \cos\left(\frac{\sqrt{7}t}{2}\right) - \sqrt{7}(2c_1 + c_2) \sin\left(\frac{\sqrt{7}t}{2}\right)\right)$$

9.8 problem 8

Internal problem ID [5964]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332
Problem number: 8.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 7x(t) + 5y(t) - 9z(t) - 8\,\mathrm{e}^{-2t} \\
y'(t) &= 4x(t) + y(t) + z(t) + 2\,\mathrm{e}^{5t} \\
z'(t) &= -2y(t) + 3z(t) + \mathrm{e}^{5t} - 3\,\mathrm{e}^{-2t}
\end{aligned}
$$

✓ Solution by Maple

Time used: 7.375 (sec). Leaf size: 8847

```
dsolve([diff(x(t),t)=7*x(t)+5*y(t)-9*z(t)-8*exp(-2*t),diff(y(t),t)=4*x(t)+y(t)+z(t)+2*exp(5*t
```

Expression too large to display

Expression too large to display

Expression too large to display

✓ Solution by Mathematica

Time used: 0.195 (sec). Leaf size: 3002

```
DSolve[{x'[t]==7*x[t]+5*y[t]-9*z[t]-8*Exp[-2*t],y'[t]==4*x[t]+y[t]+z[t]+2*Exp[5*t],z'[t]==-2*
```

Too large to display

9.9 problem 9

Internal problem ID [5965]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332
Problem number: 9.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= x(t) - y(t) + 2z(t) + e^{-t} - 3t \\
y'(t) &= 3x(t) - 4y(t) + z(t) + 2\,e^{-t} + t \\
z'(t) &= -2x(t) + 5y(t) + 6z(t) + 2\,e^{-t} - t
\end{aligned}
$$

✗ Solution by Maple

```
dsolve([diff(x(t),t)=x(t)-y(t)+2*z(t)+exp(-t)-3*t,diff(y(t),t)=3*x(t)-4*y(t)+z(t)+2*exp(-t)+t
```

No solution found

✓ Solution by Mathematica

Time used: 0.186 (sec). Leaf size: 3251

```
DSolve[{x'[t]==x[t]-y[t]+2*z[t]+Exp[-t]-3*t,y'[t]==3*x[t]-4*y[t]+z[t]+2*Exp[-t]+t,z'[t]==-2*x
```

Too large to display

9.10 problem 10

Internal problem ID [5966]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332
Problem number: 10.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = e^{4t}t + 3x(t) - 7y(t) + 4\sin(t) - 4\,e^{4t}$$
$$y'(t) = 2\,e^{4t}t + e^{4t} + x(t) + y(t) + 8\sin(t)$$

✓ Solution by Maple

Time used: 0.953 (sec). Leaf size: 131

```
dsolve([diff(x(t),t)=3*x(t)-7*y(t)+4*sin(t)+(t-4)*exp(4*t),diff(y(t),t)=x(t)+y(t)+8*sin(t)+(2
```

$$x(t) = -\frac{11\,e^{4t}t}{10} - \frac{34\,e^{4t}}{25} - e^{2t}\sqrt{6}\,\sin\left(\sqrt{6}\,t\right)c_1 + e^{2t}\sqrt{6}\,\cos\left(\sqrt{6}\,t\right)c_2 + e^{2t}\sin\left(\sqrt{6}\,t\right)c_2 + e^{2t}\cos\left(\sqrt{6}\,t\right)c_1 - \frac{204\cos(t)}{97} - \frac{556\sin(t)}{97}$$

$$y(t) = e^{2t}\sin\left(\sqrt{6}\,t\right)c_2 + e^{2t}\cos\left(\sqrt{6}\,t\right)c_1 + \frac{3\,e^{4t}t}{10} - \frac{11\,e^{4t}}{50} - \frac{8\cos(t)}{97} - \frac{212\sin(t)}{97}$$

✓ Solution by Mathematica

Time used: 11.24 (sec). Leaf size: 150

```
DSolve[{x'[t]==3*x[t]-7*y[t]+4*Sin[t]+(t-4)*Exp[4*t],y'[t]==x[t]+y[t]+8*Sin[t]+(2*t+1)*Exp[4*
```

$$
\begin{aligned}
x(t) \to & -\frac{1}{50}e^{4t}(55t+68) - \frac{4}{97}(139\sin(t) + 51\cos(t)) \\
& + \frac{1}{6}e^{2t}\left(6c_1\cos\left(\sqrt{6}t\right) + \sqrt{6}(c_1 - 7c_2)\sin\left(\sqrt{6}t\right)\right) \\
y(t) \to & \frac{1}{50}e^{4t}(15t-11) - \frac{4}{97}(53\sin(t) + 2\cos(t)) \\
& + \frac{1}{6}e^{2t}\left(6c_2\cos\left(\sqrt{6}t\right) + \sqrt{6}(c_1 - c_2)\sin\left(\sqrt{6}t\right)\right)
\end{aligned}
$$

9.11 problem 11

Internal problem ID [5967]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332
Problem number: 11.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 3x(t) - 4y(t)$$
$$y'(t) = 4x(t) - 7y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)=3*x(t)-4*y(t),diff(y(t),t)=4*x(t)-7*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = 2c_1 e^t + \frac{c_2 e^{-5t}}{2}$$

$$y(t) = c_1 e^t + c_2 e^{-5t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 73

```
DSolve[{x'[t]==3*x[t]-4*y[t],y'[t]==4*x[t]-7*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{3}e^{-5t}\left(c_1\left(4e^{6t} - 1\right) - 2c_2\left(e^{6t} - 1\right)\right)$$
$$y(t) \to \frac{1}{3}e^{-5t}\left(2c_1\left(e^{6t} - 1\right) - c_2\left(e^{6t} - 4\right)\right)$$

9.12 problem 12

Internal problem ID [5968]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332
Problem number: 12.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -2x(t) + 5y(t)$$
$$y'(t) = -2x(t) + 4y(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 44

```
dsolve([diff(x(t),t)=-2*x(t)+5*y(t),diff(y(t),t)=-2*x(t)+4*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^t(\cos(t)\,c_1 - 3c_2\cos(t) - 3c_1\sin(t) - \sin(t)\,c_2)}{2}$$

$$y(t) = e^t(c_2\cos(t) + c_1\sin(t))$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 51

```
DSolve[{x'[t]==-2*x[t]+5*y[t],y'[t]==-2*x[t]+4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -
```

$$x(t) \to e^t(c_1\cos(t) + (5c_2 - 3c_1)\sin(t))$$
$$y(t) \to e^t(c_2(3\sin(t) + \cos(t)) - 2c_1\sin(t))$$

9.13 problem 13

Internal problem ID [5969]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332
Problem number: 13.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t) + \frac{y(t)}{4}$$
$$y'(t) = x(t) - y(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=-x(t)+1/4*y(t),diff(y(t),t)=x(t)-y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{c_1 e^{-\frac{t}{2}}}{2} - \frac{c_2 e^{-\frac{3t}{2}}}{2}$$

$$y(t) = c_1 e^{-\frac{t}{2}} + c_2 e^{-\frac{3t}{2}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 63

```
DSolve[{x'[t]==-x[t]+1/4*y[t],y'[t]==x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tru
```

$$x(t) \to \frac{1}{4} e^{-3t/2} (2c_1 (e^t + 1) + c_2 (e^t - 1))$$
$$y(t) \to e^{-t} \left(c_2 \cosh\left(\frac{t}{2}\right) + 2c_1 \sinh\left(\frac{t}{2}\right)\right)$$

9.14 problem 14

Internal problem ID [5970]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332
Problem number: 14.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) + y(t)$$
$$y'(t) = -x(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 26

```
dsolve([diff(x(t),t)=2*x(t)+y(t),diff(y(t),t)=-x(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\mathrm{e}^{t}(c_2 t + c_1 + c_2)$$

$$y(t) = \mathrm{e}^{t}(c_2 t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 38

```
DSolve[{x'[t]==2*x[t]+y[t],y'[t]==-x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{t}(c_1(t+1) + c_2 t)$$
$$y(t) \to e^{t}(c_2 - (c_1 + c_2)t)$$

9.15 problem 15

Internal problem ID [5971]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332
Problem number: 15.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + 2y(t) + z(t)$$
$$y'(t) = 6x(t) - y(t)$$
$$z'(t) = -x(t) - 2y(t) - z(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 61

```
dsolve([diff(x(t),t)=x(t)+2*y(t)+z(t),diff(y(t),t)=6*x(t)-y(t),diff(z(t),t)=-x(t)-2*y(t)-z(t)
```

$$x(t) = -c_2 e^{-4t} - c_3 e^{3t} - \frac{c_1}{13}$$

$$y(t) = 2c_2 e^{-4t} - \frac{3c_3 e^{3t}}{2} - \frac{6c_1}{13}$$

$$z(t) = c_1 + c_2 e^{-4t} + c_3 e^{3t}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 176

```
DSolve[{x'[t]==x[t]+2*y[t]+z[t],y'[t]==6*x[t]-y[t],z'[t]==-x[t]-2*y[t]-z[t]},{x[t],y[t],z[t]}
```

$$x(t) \to \frac{1}{84}e^{-4t}\left(-7(c_1+c_3)e^{4t}+8(8c_1+3c_2+2c_3)e^{7t}+3(9c_1-8c_2-3c_3)\right)$$
$$y(t) \to \frac{1}{14}e^{-4t}\left(-7(c_1+c_3)e^{4t}+2(8c_1+3c_2+2c_3)e^{7t}-9c_1+8c_2+3c_3\right)$$
$$z(t) \to \frac{1}{84}e^{-4t}\left(91(c_1+c_3)e^{4t}-8(8c_1+3c_2+2c_3)e^{7t}-27c_1+24c_2+9c_3\right)$$

9.16 problem 16

Internal problem ID [5972]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.1. Page 332
Problem number: 16.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= x(t) + z(t)\\
y'(t) &= x(t) + y(t)\\
z'(t) &= -2x(t) - z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 56

```
dsolve([diff(x(t),t)=x(t)+z(t),diff(y(t),t)=x(t)+y(t),diff(z(t),t)=-2*x(t)-z(t)],[x(t), y(t),
```

$$x(t) = -\frac{c_2 \cos(t)}{2} + \frac{c_3 \sin(t)}{2} - \frac{\sin(t) c_2}{2} - \frac{c_3 \cos(t)}{2}$$

$$y(t) = \frac{c_2 \cos(t)}{2} - \frac{c_3 \sin(t)}{2} + c_1 e^t$$

$$z(t) = \sin(t) c_2 + c_3 \cos(t)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 76

```
DSolve[{x'[t]==x[t]+z[t],y'[t]==x[t]+y[t],z'[t]==-2*x[t]-z[t]},{x[t],y[t],z[t]},t,IncludeSing
```

$$x(t) \to c_1 \cos(t) + (c_1 + c_3) \sin(t)$$
$$y(t) \to c_2 e^t + c_1 \left(e^t - \cos(t)\right) - \frac{1}{2} c_3 \left(-e^t + \sin(t) + \cos(t)\right)$$
$$z(t) \to c_3 \cos(t) - (2c_1 + c_3) \sin(t)$$

10 CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346

10.1 problem 1

Internal problem ID [5973]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + 2y(t)$$
$$y'(t) = 4x(t) + 3y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=x(t)+2*y(t),diff(y(t),t)=4*x(t)+3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -c_1 \mathrm{e}^{-t} + \frac{c_2 \mathrm{e}^{5t}}{2}$$

$$y(t) = c_1 \mathrm{e}^{-t} + c_2 \mathrm{e}^{5t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 65

```
DSolve[{x'[t]==x[t]+2*y[t],y'[t]==4*x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$x(t) \to \frac{1}{3} e^{-t} \left((c_1 + c_2) e^{6t} + 2c_1 - c_2 \right)$$
$$y(t) \to \frac{1}{3} e^{-t} \left(2(c_1 + c_2) e^{6t} - 2c_1 + c_2 \right)$$

10.2 problem 2

Internal problem ID [5974]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) + 2y(t)$$
$$y'(t) = x(t) + 3y(t)$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 31

```
dsolve([diff(x(t),t)=2*x(t)+2*y(t),diff(y(t),t)=x(t)+3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -2c_1 e^t + c_2 e^{4t}$$

$$y(t) = c_1 e^t + c_2 e^{4t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 64

```
DSolve[{x'[t]==2*x[t]+2*y[t],y'[t]==x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$x(t) \to \frac{1}{3}e^t\left(c_1\left(e^{3t}+2\right)+2c_2\left(e^{3t}-1\right)\right)$$
$$y(t) \to \frac{1}{3}e^t\left((c_1+2c_2)e^{3t}-c_1+c_2\right)$$

10.3 problem 3

Internal problem ID [5975]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -4x(t) + 2y(t)$$
$$y'(t) = -\frac{5x(t)}{2} + 2y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)=-4*x(t)+2*y(t),diff(y(t),t)=-5/2*x(t)+2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = 2c_1 e^{-3t} + \frac{2c_2 e^t}{5}$$

$$y(t) = c_1 e^{-3t} + c_2 e^t$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 98

```
DSolve[{x'[t]==-4*x[t]+2*y[t],y'[t]==5/2*x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions
```

$$x(t) \to \frac{1}{14} e^{-t} \left(14c_1 \cosh\left(\sqrt{14}t\right) + \sqrt{14}(2c_2 - 3c_1) \sinh\left(\sqrt{14}t\right)\right)$$
$$y(t) \to \frac{1}{28} e^{-t} \left(28c_2 \cosh\left(\sqrt{14}t\right) + \sqrt{14}(5c_1 + 6c_2) \sinh\left(\sqrt{14}t\right)\right)$$

10.4 problem 4

Internal problem ID [5976]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -\frac{5x(t)}{2} + 2y(t)$$
$$y'(t) = \frac{3x(t)}{4} - 2y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=-5/2*x(t)+2*y(t),diff(y(t),t)=3/4*x(t)-2*y(t)],[x(t), y(t)], singsol=all
```

$$x(t) = \frac{4c_1 e^{-t}}{3} - 2c_2 e^{-\frac{7t}{2}}$$

$$y(t) = c_1 e^{-t} + c_2 e^{-\frac{7t}{2}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 114

```
DSolve[{x'[t]==5/2*x[t]+2*y[t],y'[t]==3/4*x[t]-2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions
```

$$x(t) \to c_1 e^{t/4} \cosh\left(\frac{\sqrt{105}t}{4}\right) + \frac{(9c_1 + 8c_2)e^{t/4} \sinh\left(\frac{\sqrt{105}t}{4}\right)}{\sqrt{105}}$$
$$y(t) \to \frac{1}{35} e^{t/4}\left(35c_2 \cosh\left(\frac{\sqrt{105}t}{4}\right) + \sqrt{105}(c_1 - 3c_2) \sinh\left(\frac{\sqrt{105}t}{4}\right)\right)$$

10.5 problem 5

Internal problem ID [5977]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 10x(t) - 5y(t)$$
$$y'(t) = 8x(t) - 12y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=10*x(t)-5*y(t),diff(y(t),t)=8*x(t)-12*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{5c_1 e^{8t}}{2} + \frac{c_2 e^{-10t}}{4}$$

$$y(t) = c_1 e^{8t} + c_2 e^{-10t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 73

```
DSolve[{x'[t]==10*x[t]-5*y[t],y'[t]==8*x[t]-12*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -
```

$$x(t) \to \frac{1}{9} e^{-t}(9c_1 \cosh(9t) + (11c_1 - 5c_2)\sinh(9t))$$
$$y(t) \to \frac{1}{9} e^{-10t}\left((4c_1 - c_2)e^{18t} - 4c_1 + 10c_2\right)$$

10.6 problem 6

Internal problem ID [5978]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 6.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -6x(t) + 2y(t)$$
$$y'(t) = -3x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve([diff(x(t),t)=-6*x(t)+2*y(t),diff(y(t),t)=-3*x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = 2c_2 e^{-5t} + \frac{c_1}{3}$$

$$y(t) = c_1 + c_2 e^{-5t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 64

```
DSolve[{x'[t]==-6*x[t]+2*y[t],y'[t]==-3*x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{5}\left((6c_1 - 2c_2)e^{-5t} - c_1 + 2c_2\right)$$
$$y(t) \to \frac{1}{5}\left((3c_1 - c_2)e^{-5t} - 3c_1 + 6c_2\right)$$

10.7 problem 7

Internal problem ID [5979]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 7.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= x(t) + y(t) - z(t) \\
y'(t) &= 2y(t) \\
z'(t) &= y(t) - z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 50

```
dsolve([diff(x(t),t)=x(t)+y(t)-z(t),diff(y(t),t)=2*y(t),diff(z(t),t)=y(t)-z(t)],[x(t), y(t),
```

$$
x(t) = 2c_2 e^{2t} + c_1 e^{t} + \frac{c_3 e^{-t}}{2}
$$

$$
y(t) = 3c_2 e^{2t}
$$

$$
z(t) = c_2 e^{2t} + c_3 e^{-t}
$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 86

```
DSolve[{x'[t]==x[t]+y[t]-z[t],y'[t]==2*y[t],z'[t]==y[t]-z[t]},{x[t],y[t],z[t]},t,IncludeSingu
```

$$x(t) \to \frac{1}{6}e^{-t}\left(e^{2t}\left(4c_2e^t + 6c_1 - 3(c_2 + c_3)\right) - c_2 + 3c_3\right)$$
$$y(t) \to c_2e^{2t}$$
$$z(t) \to \frac{1}{3}e^{-t}\left(c_2\left(e^{3t} - 1\right) + 3c_3\right)$$

10.8 problem 8

Internal problem ID [5980]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 8.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) - 7y(t)$$
$$y'(t) = 5x(t) + 10y(t) + 4z(t)$$
$$z'(t) = 5y(t) + 2z(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 66

```
dsolve([diff(x(t),t)=2*x(t)-7*y(t),diff(y(t),t)=5*x(t)+10*y(t)+4*z(t),diff(z(t),t)=5*y(t)+2*z
```

$$x(t) = -\frac{7c_1 e^{7t}}{5} - \frac{4c_2 e^{2t}}{5} - \frac{7c_3 e^{5t}}{5}$$

$$y(t) = c_1 e^{7t} + \frac{3c_3 e^{5t}}{5}$$

$$z(t) = c_1 e^{7t} + c_2 e^{2t} + c_3 e^{5t}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 158

```
DSolve[{x'[t]==2*x[t]-7*y[t],y'[t]==5*x[t]+10*y[t]+4*z[t],z'[t]==5*y[t]+2*z[t]},{x[t],y[t],z[
```

$$x(t) \to \frac{1}{30}\left(35(5c_1+3c_2+4c_3)e^{5t}-21(5(c_1+c_2)+4c_3)e^{7t}-8(5c_1+7c_3)e^{2t}\right)$$
$$y(t) \to e^{6t}(c_2\cosh(t)+(5c_1+4(c_2+c_3))\sinh(t))$$
$$z(t) \to \frac{1}{6}\left(-5(5c_1+3c_2+4c_3)e^{5t}+3(5(c_1+c_2)+4c_3)e^{7t}+2(5c_1+7c_3)e^{2t}\right)$$

10.9 problem 9

Internal problem ID [5981]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 9.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t) + y(t)$$
$$y'(t) = x(t) + 2y(t) + z(t)$$
$$z'(t) = 3y(t) - z(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 67

```
dsolve([diff(x(t),t)=-x(t)+y(t),diff(y(t),t)=x(t)+2*y(t)+z(t),diff(z(t),t)=3*y(t)-z(t)],[x(t)
```

$$x(t) = -c_1 e^{-t} + \frac{e^{-2t} c_2}{3} + \frac{c_3 e^{3t}}{3}$$

$$y(t) = -\frac{e^{-2t} c_2}{3} + \frac{4 c_3 e^{3t}}{3}$$

$$z(t) = c_1 e^{-t} + e^{-2t} c_2 + c_3 e^{3t}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 147

```
DSolve[{x'[t]==-x[t]+y[t],y'[t]==x[t]+2*y[t]+z[t],z'[t]==3*y[t]-z[t]},{x[t],y[t],z[t]},t,Incl
```

$$x(t) \rightarrow \frac{1}{20}e^{-2t}\left(5(3c_1 - c_3)e^t + (c_1 + 4c_2 + c_3)e^{5t} + 4(c_1 - c_2 + c_3)\right)$$
$$y(t) \rightarrow \frac{1}{5}e^{-2t}\left((c_1 + 4c_2 + c_3)e^{5t} - c_1 + c_2 - c_3\right)$$
$$z(t) \rightarrow \frac{1}{20}e^{-2t}\left(-5(3c_1 - c_3)e^t + 3(c_1 + 4c_2 + c_3)e^{5t} + 12(c_1 - c_2 + c_3)\right)$$

10.10 problem 10

Internal problem ID [5982]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 10.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + z(t)$$
$$y'(t) = y(t)$$
$$z'(t) = x(t) + z(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 33

```
dsolve([diff(x(t),t)=x(t)+z(t),diff(y(t),t)=y(t),diff(z(t),t)=x(t)+z(t)],[x(t), y(t), z(t)],
```

$$x(t) = c_3 e^{2t} - c_2$$

$$y(t) = c_1 e^{t}$$

$$z(t) = c_2 + c_3 e^{2t}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 88

```
DSolve[{x'[t]==x[t]+z[t],y'[t]==y[t],z'[t]==x[t]+z[t]},{x[t],y[t],z[t]},t,IncludeSingularSolu
```

$$x(t) \to e^t(c_1 \cosh(t) + c_2 \sinh(t))$$
$$z(t) \to e^t(c_2 \cosh(t) + c_1 \sinh(t))$$
$$y(t) \to c_3 e^t$$
$$x(t) \to e^t(c_1 \cosh(t) + c_2 \sinh(t))$$
$$z(t) \to e^t(c_2 \cosh(t) + c_1 \sinh(t))$$
$$y(t) \to 0$$

10.11 problem 11

Internal problem ID [5983]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 11.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -x(t) - y(t) \\
y'(t) &= \frac{3x(t)}{4} - \frac{3y(t)}{2} + 3z(t) \\
z'(t) &= \frac{x(t)}{8} + \frac{y(t)}{4} - \frac{z(t)}{2}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 67

```
dsolve([diff(x(t),t)=-x(t)-y(t),diff(y(t),t)=3/4*x(t)-3/2*y(t)+3*z(t),diff(z(t),t)=1/8*x(t)+1
```

$$x(t) = -\frac{12c_1 e^{-\frac{t}{2}}}{5} - 4\,e^{-t}c_2 - 4c_3 e^{-\frac{3t}{2}}$$

$$y(t) = \frac{6c_1 e^{-\frac{t}{2}}}{5} - 2c_3 e^{-\frac{3t}{2}}$$

$$z(t) = c_1 e^{-\frac{t}{2}} + e^{-t}c_2 + c_3 e^{-\frac{3t}{2}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 162

```
DSolve[{x'[t]==-x[t]-y[t],y'[t]==3/4*x[t]-3/2*y[t]+3*z[t],z'[t]==1/8x[t]+1/4*y[t]-1/2*z[t]},{
```

$$x(t) \to \frac{1}{2}e^{-3t/2}\left(c_1\left(8e^{t/2} - 3e^t - 3\right) - 4\left(e^{t/2} - 1\right)\left(3c_3\left(e^{t/2} - 1\right) + c_2\right)\right)$$

$$y(t) \to \frac{1}{4}e^{-3t/2}\left(3c_1\left(e^t - 1\right) + 4\left(3c_3\left(e^t - 1\right) + c_2\right)\right)$$

$$z(t) \to \frac{1}{8}e^{-3t/2}\left(5(c_1 + 4c_3)e^t - 4(2c_1 - c_2 + 6c_3)e^{t/2} + 3c_1 - 4c_2 + 12c_3\right)$$

10.12 problem 11

Internal problem ID [5984]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 11.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t) - y(t)$$
$$y'(t) = \frac{3x(t)}{4} - \frac{3y(t)}{2} + 3z(t)$$
$$z'(t) = \frac{x(t)}{8} + \frac{y(t)}{4} - \frac{z(t)}{2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 67

```
dsolve([diff(x(t),t)=-x(t)-y(t),diff(y(t),t)=3/4*x(t)-3/2*y(t)+3*z(t),diff(z(t),t)=1/8*x(t)+1
```

$$x(t) = -\frac{12c_1 e^{-\frac{t}{2}}}{5} - 4\,e^{-t}c_2 - 4c_3 e^{-\frac{3t}{2}}$$

$$y(t) = \frac{6c_1 e^{-\frac{t}{2}}}{5} - 2c_3 e^{-\frac{3t}{2}}$$

$$z(t) = c_1 e^{-\frac{t}{2}} + e^{-t}c_2 + c_3 e^{-\frac{3t}{2}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 162

```
DSolve[{x'[t]==-x[t]-y[t],y'[t]==3/4*x[t]-3/2*y[t]+3*z[t],z'[t]==1/8x[t]+1/4*y[t]-1/2*z[t]},{
```

$$x(t) \to \frac{1}{2}e^{-3t/2}\left(c_1\left(8e^{t/2} - 3e^t - 3\right) - 4\left(e^{t/2} - 1\right)\left(3c_3\left(e^{t/2} - 1\right) + c_2\right)\right)$$

$$y(t) \to \frac{1}{4}e^{-3t/2}\left(3c_1\left(e^t - 1\right) + 4\left(3c_3\left(e^t - 1\right) + c_2\right)\right)$$

$$z(t) \to \frac{1}{8}e^{-3t/2}\left(5(c_1 + 4c_3)e^t - 4(2c_1 - c_2 + 6c_3)e^{t/2} + 3c_1 - 4c_2 + 12c_3\right)$$

10.13 problem 12

Internal problem ID [5985]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 12.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -x(t) + 4y(t) + 2z(t) \\
y'(t) &= 4x(t) - y(t) - 2z(t) \\
z'(t) &= 6z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 58

```
dsolve([diff(x(t),t)=-x(t)+4*y(t)+2*z(t),diff(y(t),t)=4*x(t)-y(t)-2*z(t),diff(z(t),t)=6*z(t)]
```

$$x(t) = \frac{2c_3 e^{6t}}{11} + c_1 e^{3t} - c_2 e^{-5t}$$

$$y(t) = c_2 e^{-5t} + c_1 e^{3t} - \frac{2c_3 e^{6t}}{11}$$

$$z(t) = c_3 e^{6t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 105

```
DSolve[{x'[t]==-x[t]+4*y[t]+2*z[t],y'[t]==4*x[t]-y[t]-2*z[t],z'[t]==6*z[t]},{x[t],y[t],z[t]},
```

$$x(t) \to \frac{1}{22}e^{-5t}\left(11c_1\left(e^{8t}+1\right)+11c_2\left(e^{8t}-1\right)+4c_3\left(e^{11t}-1\right)\right)$$

$$y(t) \to \frac{1}{22}e^{-5t}\left(11c_1\left(e^{8t}-1\right)+11c_2\left(e^{8t}+1\right)-4c_3\left(e^{11t}-1\right)\right)$$

$$z(t) \to c_3e^{6t}$$

10.14 problem 13

Internal problem ID [5986]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 13.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{x(t)}{2}$$
$$y'(t) = x(t) - \frac{y(t)}{2}$$

With initial conditions

$$[x(0) = 4, y(0) = 5]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 25

```
dsolve([diff(x(t),t) = 1/2*x(t), diff(y(t),t) = x(t)-1/2*y(t), x(0) = 4, y(0) = 5],[x(t), y(t
```

$$x(t) = 4\,\mathrm{e}^{\frac{t}{2}}$$

$$y(t) = \mathrm{e}^{-\frac{t}{2}} + 4\,\mathrm{e}^{\frac{t}{2}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 32

```
DSolve[{x'[t]==1/2*x[t],y'[t]==x[t]-1/2*y[t]},{x[0]==4,y[0]==5},{x[t],y[t]},t,IncludeSingular
```

$$x(t) \to 4e^{t/2}$$
$$y(t) \to e^{-t/2}(4e^t + 1)$$

10.15 problem 14

Internal problem ID [5987]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 14.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= x(t) + y(t) + 4z(t) \\
y'(t) &= 2y(t) \\
z'(t) &= x(t) + y(t) + z(t)
\end{aligned}
$$

With initial conditions

$$[x(0) = 1, y(0) = 3, z(0) = 0]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 53

```
dsolve([diff(x(t),t) = x(t)+y(t)+4*z(t), diff(y(t),t) = 2*y(t), diff(z(t),t) = x(t)+y(t)+z(t)
```

$$x(t) = -5\,\mathrm{e}^{2t} + \mathrm{e}^{-t} + 5\,\mathrm{e}^{3t}$$

$$y(t) = 3\,\mathrm{e}^{2t}$$

$$z(t) = -2\,\mathrm{e}^{2t} - \frac{\mathrm{e}^{-t}}{2} + \frac{5\,\mathrm{e}^{3t}}{2}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 62

```
DSolve[{x'[t]==x[t]+y[t]+4*z[t],y'[t]==2*y[t],z'[t]==x[t]+y[t]+z[t]},{x[0]==1,y[0]==3,z[0]==0
```

$$x(t) \to e^{-t} - 5e^{2t} + 5e^{3t}$$
$$y(t) \to 3e^{2t}$$
$$z(t) \to \frac{1}{2}e^{-t}\left(e^{3t}\left(5e^{t} - 4\right) - 1\right)$$

10.16 problem 15

Internal problem ID [5988]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 15.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{9x(t)}{10} + \frac{21y(t)}{10} + \frac{16z(t)}{5}$$
$$y'(t) = \frac{7x(t)}{10} + \frac{13y(t)}{2} + \frac{21z(t)}{5}$$
$$z'(t) = \frac{11x(t)}{10} + \frac{17y(t)}{10} + \frac{17z(t)}{5}$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 1014

```
dsolve([diff(x(t),t)=9/10*x(t)+21/10*y(t)+32/10*z(t),diff(y(t),t)=7/10*x(t)+65/10*y(t)+42/10*
```

$x(t)$

$$= \frac{\left(-17i\left(329940 + 60i\sqrt{29760999}\right)^{\frac{4}{3}}\sqrt{3} + 17\left(329940 + 60i\sqrt{29760999}\right)^{\frac{4}{3}} + 12420000i\left(329940 + 60i\sqrt{2976\right.\right.}{}$$

$$- \frac{\left(-17i\left(329940 + 60i\sqrt{29760999}\right)^{\frac{4}{3}}\sqrt{3} - 17\left(329940 + 60i\sqrt{29760999}\right)^{\frac{4}{3}} + 12420000i\left(329940 + 60i\sqrt{29\right.\right.}{}$$

$$- \frac{\left(17\left(329940 + 60i\sqrt{29760999}\right)^{\frac{4}{3}} + 27870\left(329940 + 60i\sqrt{29760999}\right)^{\frac{2}{3}} - 124200i\sqrt{29760999} - 1242000\right.}{98910\left(329940 + 60i\sqrt{\right.}}$$

$y(t)$

$$= \frac{\left(11i\left(329940 + 60i\sqrt{29760999}\right)^{\frac{4}{3}}\sqrt{3} - 11\left(329940 + 60i\sqrt{29760999}\right)^{\frac{4}{3}} + 3600000i\left(329940 + 60i\sqrt{297609\right.\right.}{}$$

$$- \frac{\left(11i\left(329940 + 60i\sqrt{29760999}\right)^{\frac{4}{3}}\sqrt{3} + 11\left(329940 + 60i\sqrt{29760999}\right)^{\frac{4}{3}} + 3600000i\left(329940 + 60i\sqrt{2976\right.\right.}{}$$

$$+ \frac{\left(11\left(329940 + 60i\sqrt{29760999}\right)^{\frac{4}{3}} + 29670\left(329940 + 60i\sqrt{29760999}\right)^{\frac{2}{3}} + 36000i\sqrt{29760999} + 3600000\left(3\right.\right.}{98910\left(329940 + 60i\sqrt{2\right.}}$$

$$z(t) = c_1 e^{-\frac{\left(i\left(329940+60i\sqrt{29760999}\right)^{\frac{2}{3}}\sqrt{3}+\left(329940+60i\sqrt{29760999}\right)^{\frac{2}{3}}-6000i\sqrt{3}-216\left(329940+60i\sqrt{29760999}\right)^{\frac{1}{3}}+6000\right)t}{60\left(329940+60i\sqrt{29760999}\right)^{\frac{1}{3}}}}$$
$$+ c_2 e^{\frac{\left(i\left(329940+60i\sqrt{29760999}\right)^{\frac{2}{3}}\sqrt{3}-\left(329940+60i\sqrt{29760999}\right)^{\frac{2}{3}}-6000i\sqrt{3}+216\left(329940+60i\sqrt{29760999}\right)^{\frac{1}{3}}-6000\right)t}{60\left(329940+60i\sqrt{29760999}\right)^{\frac{1}{3}}}}$$
$$+ c_3 e^{\frac{\left(\left(329940+60i\sqrt{29760999}\right)^{\frac{2}{3}}+108\left(329940+60i\sqrt{29760999}\right)^{\frac{1}{3}}+6000\right)t}{30\left(329940+60i\sqrt{29760999}\right)^{\frac{1}{3}}}}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 616

```
DSolve[{x'[t]==9/10*x[t]+21/10*y[t]+32/10*z[t],y'[t]==7/10*x[t]+65/10*y[t]+42/10*z[t],z'[t]==
```

$$x(t) \to 2c_3 \text{RootSum}\left[\#1^3 - 108\#1^2 + 1888\#1 + 904\&, \frac{16\#1e^{\frac{\#1t}{10}} - 599e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\&\right] + c_2 \text{RootSum}\left[\#1^3 - 108\#1^2 + 1888\#1 + 904\&, \frac{21\#1e^{\frac{\#1t}{10}} - 170e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\&\right] + c_1 \text{RootSum}\left[\#1^3 - 108\#1^2 + 1888\#1 + 904\&, \frac{\#1^2e^{\frac{\#1t}{10}} - 99\#1e^{\frac{\#1t}{10}} + 1496e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\&\right]$$

$$y(t) \to 7c_1 \text{RootSum}\left[\#1^3 - 108\#1^2 + 1888\#1 + 904\&, \frac{\#1e^{\frac{\#1t}{10}} + 32e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\&\right] + 14c_3 \text{RootSum}\left[\#1^3 - 108\#1^2 + 1888\#1 + 904\&, \frac{3\#1e^{\frac{\#1t}{10}} - 11e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\&\right] + c_2 \text{RootSum}\left[\#1^3 - 108\#1^2 + 1888\#1 + 904\&, \frac{\#1^2e^{\frac{\#1t}{10}} - 43\#1e^{\frac{\#1t}{10}} - 46e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\&\right]$$

$$z(t) \to c_1 \text{RootSum}\left[\#1^3 - 108\#1^2 + 1888\#1 + 904\&, \frac{11\#1e^{\frac{\#1t}{10}} - 596e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\&\right] + c_2 \text{RootSum}\left[\#1^3 - 108\#1^2 + 1888\#1 + 904\&, \frac{17\#1e^{\frac{\#1t}{10}} + 78e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\&\right] + c_3 \text{RootSum}\left[\#1^3 - 108\#1^2 + 1888\#1 + 904\&, \frac{\#1^2e^{\frac{\#1t}{10}} - 74\#1e^{\frac{\#1t}{10}} + 438e^{\frac{\#1t}{10}}}{3\#1^2 - 216\#1 + 1888}\&\right]$$

10.17 problem 16

Internal problem ID [5989]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 16.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_1(t) + 2x_3(t) - \frac{9x_4(t)}{5} \\
x_2'(t) &= \frac{51x_2(t)}{10} - x_4(t) + 3x_5(t) \\
x_3'(t) &= x_1(t) + 2x_2(t) - 3x_3(t) \\
x_4'(t) &= x_2(t) - \frac{31x_3(t)}{10} + 4x_4(t) \\
x_5'(t) &= -\frac{14x_1(t)}{5} + \frac{3x_4(t)}{2} - x_5(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 1389

```
dsolve([diff(x__1(t),t)=x__1(t)+2*x__3(t)-18/10*x__4(t),diff(x__2(t),t)=51/10*x__2(t)-x__4(t)
```

$x_1(t) =$

$$-\frac{346378788000\left(\sum_{\_a=1}^{5} \text{RootOf}\left(500\_Z^5-3050\_Z^4-4450\_Z^3+35110\_Z^2+20779\_Z-81879, \text{index}\right.\right.}{2512446718921}$$

$$+\frac{15248812500\left(\sum_{\_a=1}^{5} \text{RootOf}\left(500\_Z^5-3050\_Z^4-4450\_Z^3+35110\_Z^2+20779\_Z-81879, \text{index} =\right.\right.}{2512446718921}$$

$$-\frac{1155099105820\left(\sum_{\_a=1}^{5} \text{RootOf}\left(500\_Z^5-3050\_Z^4-4450\_Z^3+35110\_Z^2+20779\_Z-81879, \text{index}\right.\right.}{2512446718921}$$

$$-\frac{538124307820\left(\sum_{\_a=1}^{5} e^{\text{RootOf}(500\_Z^5-3050\_Z^4-4450\_Z^3+35110\_Z^2+20779\_Z-81879, \text{index}=\_a)t}\_C\_a\right)}{2512446718921}$$

$$+\frac{24122625000\left(\sum_{\_a=1}^{5} \text{RootOf}\left(500\_Z^5-3050\_Z^4-4450\_Z^3+35110\_Z^2+20779\_Z-81879, \text{index} =\right.\right.}{2512446718921}$$

$x_2(t) =$

$$-\frac{1216113967980\left(\sum_{\_a=1}^{5} \text{RootOf}\left(500\_Z^5-3050\_Z^4-4450\_Z^3+35110\_Z^2+20779\_Z-81879, \text{index}\right.\right.}{2512446718921}$$

$$+\frac{13519594578350\left(\sum_{\_a=1}^{5} e^{\text{RootOf}(500\_Z^5-3050\_Z^4-4450\_Z^3+35110\_Z^2+20779\_Z-81879, \text{index}=\_a)t}\_C\_a\right)}{2512446718921}$$

$$-\frac{6739842774000\left(\sum_{\_a=1}^{5} \text{RootOf}\left(500\_Z^5-3050\_Z^4-4450\_Z^3+35110\_Z^2+20779\_Z-81879, \text{index}\right.\right.}{2512446718921}$$

$$-\frac{508009681000\left(\sum_{\_a=1}^{5} \text{RootOf}\left(500\_Z^5-3050\_Z^4-4450\_Z^3+35110\_Z^2+20779\_Z-81879, \text{index}\right.\right.}{2512446718921}$$

$$+\frac{462980781000\left(\sum_{\_a=1}^{5} \text{RootOf}\left(500\_Z^5-3050\_Z^4-4450\_Z^3+35110\_Z^2+20779\_Z-81879, \text{index}\right.\right.}{2512446718921}$$

$x_3(t) =$

$$-\frac{625855092300\left(\sum_{\_a=1}^{5} \text{RootOf}\left(500\_Z^5-3050\_Z^4-4450\_Z^3+35110\_Z^2+20779\_Z-81879, \text{index}\right.\right.}{2512446718921}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 2839

```
DSolve[{x1'[t]==x1[t]+2*x3[t]-18/10*x4[t],x2'[t]==51/10*x2[t]-x4[t]+3*x5[t],x3'[t]==x1[t]+2*x
```

Too large to display

10.18 problem 19

Internal problem ID [5990]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 19.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 3x(t) - y(t)$$
$$y'(t) = 9x(t) - 3y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 24

```
dsolve([diff(x(t),t)=3*x(t)-y(t),diff(y(t),t)=9*x(t)-3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{1}{9}c_1 + \frac{1}{3}c_1 t + \frac{1}{3}c_2$$

$$y(t) = c_1 t + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 34

```
DSolve[{x'[t]==3*x[t]-y[t],y'[t]==9*x[t]-3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$x(t) \to 3c_1 t - c_2 t + c_1$$
$$y(t) \to 9c_1 t - 3c_2 t + c_2$$

10.19 problem 20

Internal problem ID [5991]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 20.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -6x(t) + 5y(t)$$
$$y'(t) = -5x(t) + 4y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=-6*x(t)+5*y(t),diff(y(t),t)=-5*x(t)+4*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{e^{-t}(5c_2t + 5c_1 - c_2)}{5}$$

$$y(t) = e^{-t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{x'[t]==-6*x[t]+5*y[t],y'[t]==-5*x[t]+4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -
```

$$x(t) \to e^{-t}(-5c_1t + 5c_2t + c_1)$$
$$y(t) \to e^{-t}(5(c_2 - c_1)t + c_2)$$

10.20 problem 21

Internal problem ID [5992]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 21.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t) + 3y(t)$$
$$y'(t) = -3x(t) + 5y(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=-x(t)+3*y(t),diff(y(t),t)=-3*x(t)+5*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{e^{2t}(3c_2t + 3c_1 - c_2)}{3}$$

$$y(t) = e^{2t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{x'[t]==-x[t]+3*y[t],y'[t]==-3*x[t]+5*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to e^{2t}(-3c_1t + 3c_2t + c_1)$$
$$y(t) \to e^{2t}(3(c_2 - c_1)t + c_2)$$

10.21 problem 22

Internal problem ID [5993]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 22.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 12x(t) - 9y(t)$$
$$y'(t) = 4x(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 33

```
dsolve([diff(x(t),t)=12*x(t)-9*y(t),diff(y(t),t)=4*x(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{e^{6t}(6c_2t + 6c_1 + c_2)}{4}$$

$$y(t) = e^{6t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{x'[t]==12*x[t]-9*y[t],y'[t]==4*x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{6t}(6c_1t - 9c_2t + c_1)$$
$$y(t) \to e^{6t}(4c_1t - 6c_2t + c_2)$$

10.22 problem 23

Internal problem ID [5994]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 23.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 3x(t) - y(t) - z(t) \\
y'(t) &= x(t) + y(t) - z(t) \\
z'(t) &= x(t) - y(t) + z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 57

```
dsolve([diff(x(t),t)=3*x(t)-y(t)-z(t),diff(y(t),t)=x(t)+y(t)-z(t),diff(z(t),t)=x(t)-y(t)+z(t)
```

$$x(t) = 2c_2 e^{2t} + c_3 e^{t} + c_1 e^{2t}$$

$$y(t) = c_2 e^{2t} + c_3 e^{t} + c_1 e^{2t}$$

$$z(t) = c_2 e^{2t} + c_3 e^{t}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 86

```
DSolve[{x'[t]==3*x[t]-y[t]-z[t],y'[t]==x[t]+y[t]-z[t],z'[t]==x[t]-y[t]+z[t]},{x[t],y[t],z[t]}
```

$$
\begin{aligned}
x(t) &\to e^{t}\left(c_1\left(2e^{t} - 1\right) - (c_2 + c_3)\left(e^{t} - 1\right)\right) \\
y(t) &\to e^{t}\left(c_1\left(e^{t} - 1\right) - c_3 e^{t} + c_2 + c_3\right) \\
z(t) &\to e^{t}\left(c_1\left(e^{t} - 1\right) - c_2 e^{t} + c_2 + c_3\right)
\end{aligned}
$$

10.23 problem 24

Internal problem ID [5995]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 24.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 3x(t) + 2y(t) + 4z(t)$$
$$y'(t) = 2x(t) + 2z(t)$$
$$z'(t) = 4x(t) + 2y(t) + 3z(t)$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 66

```
dsolve([diff(x(t),t)=3*x(t)+2*y(t)+4*z(t),diff(y(t),t)=2*x(t)+2*z(t),diff(z(t),t)=4*x(t)+2*y(
```

$$x(t) = c_2 e^{8t} - \frac{5c_3 e^{-t}}{4} - \frac{c_1 e^{-t}}{2}$$

$$y(t) = \frac{c_2 e^{8t}}{2} + \frac{c_3 e^{-t}}{2} + c_1 e^{-t}$$

$$z(t) = c_2 e^{8t} + c_3 e^{-t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 127

```
DSolve[{x'[t]==3*x[t]+2*y[t]+4*z[t],y'[t]==2*x[t]+2*z[t],z'[t]==4*x[t]+2*y[t]+3*z[t]},{x[t],y
```

$$x(t) \to \frac{1}{9}e^{-t}\left(c_1\left(4e^{9t}+5\right)+2(c_2+2c_3)\left(e^{9t}-1\right)\right)$$

$$y(t) \to \frac{1}{9}e^{-t}\left((2c_1+c_2+2c_3)e^{9t}-2(c_1-4c_2+c_3)\right)$$

$$z(t) \to \frac{1}{9}e^{-t}\left(2(2c_1+c_2+2c_3)e^{9t}-4c_1-2c_2+5c_3\right)$$

10.24 problem 25

Internal problem ID [5996]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 25.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 5x(t) - 4y(t) \\
y'(t) &= x(t) + 2z(t) \\
z'(t) &= 2y(t) + 5z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 63

```
dsolve([diff(x(t),t)=5*x(t)-4*y(t),diff(y(t),t)=x(t)+2*z(t),diff(z(t),t)=2*y(t)+5*z(t)],[x(t)
```

$$x(t) = -2c_2 e^{5t} - 2c_3 e^{5t} t + \frac{5c_3 e^{5t}}{2} - 2c_1$$

$$y(t) = \frac{c_3 e^{5t}}{2} - \frac{5c_1}{2}$$

$$z(t) = c_1 + c_2 e^{5t} + c_3 e^{5t} t$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 132

```
DSolve[{x'[t]==5*x[t]-4*y[t],y'[t]==x[t]+2*z[t],z'[t]==2*y[t]+5*z[t]},{x[t],y[t],z[t]},t,Incl
```

$$x(t) \to \frac{1}{25}\left(e^{5t}(c_1(29-20t)+8c_3(1-5t)-20c_2)-4(c_1-5c_2+2c_3)\right)$$
$$y(t) \to \frac{1}{5}c_1\left(e^{5t}-1\right)+\frac{2}{5}c_3\left(e^{5t}-1\right)+c_2$$
$$z(t) \to \frac{1}{25}\left(e^{5t}(2c_1(5t-1)+c_3(20t+21)+10c_2)+2(c_1-5c_2+2c_3)\right)$$

10.25 problem 26

Internal problem ID [5997]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 26.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t)$$
$$y'(t) = 3y(t) + z(t)$$
$$z'(t) = -y(t) + z(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 37

```
dsolve([diff(x(t),t)=x(t),diff(y(t),t)=3*y(t)+z(t),diff(z(t),t)=-y(t)+z(t)],[x(t), y(t), z(t)
```

$$x(t) = c_1 e^t$$

$$y(t) = -e^{2t}(c_3 t + c_2 + c_3)$$

$$z(t) = e^{2t}(c_3 t + c_2)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 96

```
DSolve[{x'[t]==x[t],y'[t]==3*y[t]+z[t],z'[t]==-y[t]+z[t]},{x[t],y[t],z[t]},t,IncludeSingularS
```

$$x(t) \to c_1 e^t$$
$$y(t) \to e^{2t}(c_2(t+1) + c_3 t)$$
$$z(t) \to e^{2t}(c_3 - (c_2 + c_3)t)$$
$$x(t) \to 0$$
$$y(t) \to e^{2t}(c_2(t+1) + c_3 t)$$
$$z(t) \to e^{2t}(c_3 - (c_2 + c_3)t)$$

10.26 problem 27

Internal problem ID [5998]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 27.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t)$$
$$y'(t) = 2x(t) + 2y(t) - z(t)$$
$$z'(t) = y(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 46

```
dsolve([diff(x(t),t)=x(t),diff(y(t),t)=2*x(t)+2*y(t)-z(t),diff(z(t),t)=y(t)],[x(t), y(t), z(t
```

$$x(t) = c_3 e^t$$

$$y(t) = e^t (c_3 t^2 + c_2 t + 2c_3 t + c_1 + c_2)$$

$$z(t) = e^t (c_3 t^2 + c_2 t + c_1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 59

```
DSolve[{x'[t]==x[t],y'[t]==2*x[t]+2*y[t]-z[t],z'[t]==y[t]},{x[t],y[t],z[t]},t,IncludeSingular
```

$$x(t) \to c_1 e^t$$
$$y(t) \to e^t (t(c_1(t+2) + c_2 - c_3) + c_2)$$
$$z(t) \to e^t (t(c_1 t + c_2 - c_3) + c_3)$$

10.27 problem 28

Internal problem ID [5999]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 28.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4x(t) + y(t)$$
$$y'(t) = 4y(t) + z(t)$$
$$z'(t) = 4z(t)$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 46

```
dsolve([diff(x(t),t)=4*x(t)+y(t),diff(y(t),t)=4*y(t)+z(t),diff(z(t),t)=4*z(t)],[x(t), y(t), z
```

$$x(t) = \frac{(c_3 t^2 + 2c_2 t + 2c_1) e^{4t}}{2}$$

$$y(t) = (c_3 t + c_2) e^{4t}$$

$$z(t) = c_3 e^{4t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 57

```
DSolve[{x'[t]==4*x[t]+y[t],y'[t]==4*y[t]+z[t],z'[t]==4*z[t]},{x[t],y[t],z[t]},t,IncludeSingul
```

$$x(t) \to \frac{1}{2}e^{4t}(t(c_3t + 2c_2) + 2c_1)$$
$$y(t) \to e^{4t}(c_3t + c_2)$$
$$z(t) \to c_3e^{4t}$$

10.28 problem 29

Internal problem ID [6000]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 29.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) + 4y(t)$$
$$y'(t) = -x(t) + 6y(t)$$

With initial conditions

$$[x(0) = -1, y(0) = 6]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 28

```
dsolve([diff(x(t),t) = 2*x(t)+4*y(t), diff(y(t),t) = -x(t)+6*y(t), x(0) = -1, y(0) = 6],[x(t)
```

$$x(t) = e^{4t}(26t - 1)$$

$$y(t) = e^{4t}(13t + 6)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[{x'[t]==2*x[t]+4*y[t],y'[t]==-x[t]+6*y[t]},{x[0]==-1,y[0]==6},{x[t],y[t]},t,IncludeSin
```

$$x(t) \to e^{4t}(26t - 1)$$
$$y(t) \to e^{4t}(13t + 6)$$

10.29 problem 30

Internal problem ID [6001]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 30.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = z(t)$$
$$y'(t) = y(t)$$
$$z'(t) = x(t)$$

With initial conditions

$$[x(0) = 1, y(0) = 2, z(0) = 5]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 37

```
dsolve([diff(x(t),t) = z(t), diff(y(t),t) = y(t), diff(z(t),t) = x(t), x(0) = 1, y(0) = 2, z(
```

$$x(t) = -2\,e^{-t} + 3\,e^{t}$$

$$y(t) = 2\,e^{t}$$

$$z(t) = 2\,e^{-t} + 3\,e^{t}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 30

```
DSolve[{x'[t]==z[t],y'[t]==y[t],z'[t]==x[t]},{x[0]==1,y[0]==2,z[0]==5},{x[t],y[t],z[t]},t,Inc
```

$$x(t) \to 5\sinh(t) + \cosh(t)$$
$$z(t) \to \sinh(t) + 5\cosh(t)$$
$$y(t) \to 2e^{t}$$

10.30 problem 33

Internal problem ID [6002]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 33.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 6x(t) - y(t)$$
$$y'(t) = 5x(t) + 2y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 48

```
dsolve([diff(x(t),t)=6*x(t)-y(t),diff(y(t),t)=5*x(t)+2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{e^{4t}(\cos(t)\, c_1 + 2c_2 \cos(t) + 2c_1 \sin(t) - \sin(t)\, c_2)}{5}$$

$$y(t) = e^{4t}(c_2 \cos(t) + c_1 \sin(t))$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 54

```
DSolve[{x'[t]==6*x[t]-y[t],y'[t]==5*x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$x(t) \to e^{4t}(c_1(2\sin(t) + \cos(t)) - c_2 \sin(t))$$
$$y(t) \to e^{4t}(5c_1 \sin(t) + c_2(\cos(t) - 2\sin(t)))$$

10.31 problem 34

Internal problem ID [6003]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 34.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + y(t)$$
$$y'(t) = -2x(t) - y(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 38

```
dsolve([diff(x(t),t)=x(t)+y(t),diff(y(t),t)=-2*x(t)-y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{\sin(t)\, c_2}{2} - \frac{\cos(t)\, c_1}{2} - \frac{c_2 \cos(t)}{2} - \frac{c_1 \sin(t)}{2}$$

$$y(t) = c_2 \cos(t) + c_1 \sin(t)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 39

```
DSolve[{x'[t]==x[t]+y[t],y'[t]==-2*x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 \cos(t) + (c_1 + c_2) \sin(t)$$
$$y(t) \to c_2 \cos(t) - (2c_1 + c_2) \sin(t)$$

10.32 problem 35

Internal problem ID [6004]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 35.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 5x(t) + y(t)$$
$$y'(t) = -2x(t) + 3y(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 46

```
dsolve([diff(x(t),t)=5*x(t)+y(t),diff(y(t),t)=-2*x(t)+3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^{4t}(\cos(t)\, c_1 + c_2 \cos(t) + c_1 \sin(t) - \sin(t)\, c_2)}{2}$$

$$y(t) = e^{4t}(c_2 \cos(t) + c_1 \sin(t))$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 51

```
DSolve[{x'[t]==5*x[t]+y[t],y'[t]==-2*x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to e^{4t}(c_1 \cos(t) + (c_1 + c_2) \sin(t))$$
$$y(t) \to e^{4t}(c_2 \cos(t) - (2c_1 + c_2) \sin(t))$$

10.33 problem 36

Internal problem ID [6005]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 36.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4x(t) + 5y(t)$$
$$y'(t) = -2x(t) + 6y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 59

```
dsolve([diff(x(t),t)=4*x(t)+5*y(t),diff(y(t),t)=-2*x(t)+6*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{e^{5t}(\sin(3t)\,c_1 + 3\sin(3t)\,c_2 - 3\cos(3t)\,c_1 + \cos(3t)\,c_2)}{2}$$

$$y(t) = e^{5t}(\sin(3t)\,c_1 + \cos(3t)\,c_2)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 69

```
DSolve[{x'[t]==4*x[t]+5*y[t],y'[t]==-2*x[t]+6*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{3}e^{5t}(3c_1\cos(3t) - (c_1 - 5c_2)\sin(3t))$$
$$y(t) \to \frac{1}{3}e^{5t}(3c_2\cos(3t) + (c_2 - 2c_1)\sin(3t))$$

10.34 problem 37

Internal problem ID [6006]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 37.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4x(t) - 5y(t)$$
$$y'(t) = 5x(t) - 4y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 50

```
dsolve([diff(x(t),t)=4*x(t)-5*y(t),diff(y(t),t)=5*x(t)-4*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{3\cos(3t)\,c_1}{5} - \frac{3\sin(3t)\,c_2}{5} + \frac{4\sin(3t)\,c_1}{5} + \frac{4\cos(3t)\,c_2}{5}$$

$$y(t) = \sin(3t)\,c_1 + \cos(3t)\,c_2$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 58

```
DSolve[{x'[t]==4*x[t]-5*y[t],y'[t]==5*x[t]-4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to c_1\cos(3t) + \frac{1}{3}(4c_1 - 5c_2)\sin(3t)$$
$$y(t) \to c_2\cos(3t) + \frac{1}{3}(5c_1 - 4c_2)\sin(3t)$$

10.35 problem 38

Internal problem ID [6007]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 38.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - 8y(t)$$
$$y'(t) = x(t) - 3y(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 58

```
dsolve([diff(x(t),t)=x(t)-8*y(t),diff(y(t),t)=x(t)-3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = 2\,\mathrm{e}^{-t}(\sin(2t)\,c_1 - \sin(2t)\,c_2 + \cos(2t)\,c_1 + \cos(2t)\,c_2)$$

$$y(t) = \mathrm{e}^{-t}(\sin(2t)\,c_1 + \cos(2t)\,c_2)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 64

```
DSolve[{x'[t]==x[t]-8*y[t],y'[t]==x[t]-3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True
```

$$x(t) \to e^{-t}(c_1\cos(2t) + (c_1 - 4c_2)\sin(2t))$$
$$y(t) \to \frac{1}{2}e^{-t}(2c_2\cos(2t) + (c_1 - 2c_2)\sin(2t))$$

10.36 problem 39

Internal problem ID [6008]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 39.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= z(t) \\
y'(t) &= -z(t) \\
z'(t) &= y(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 41

```
dsolve([diff(x(t),t)=z(t),diff(y(t),t)=-z(t),diff(z(t),t)=y(t)],[x(t), y(t), z(t)], singsol=a
```

$$x(t) = -c_2 \cos(t) + c_3 \sin(t) + c_1$$

$$y(t) = c_2 \cos(t) - c_3 \sin(t)$$

$$z(t) = \sin(t) c_2 + c_3 \cos(t)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 50

```
DSolve[{x'[t]==z[t],y'[t]==-z[t],z'[t]==y[t]},{x[t],y[t],z[t]},t,IncludeSingularSolutions ->
```

$$
\begin{aligned}
x(t) &\to -c_2 \cos(t) + c_3 \sin(t) + c_1 + c_2 \\
y(t) &\to c_2 \cos(t) - c_3 \sin(t) \\
z(t) &\to c_3 \cos(t) + c_2 \sin(t)
\end{aligned}
$$

10.37 problem 40

Internal problem ID [6009]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 40.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) + y(t) + 2z(t)$$
$$y'(t) = 3x(t) + 6z(t)$$
$$z'(t) = -4x(t) - 3z(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 92

```
dsolve([diff(x(t),t)=2*x(t)+y(t)+2*z(t),diff(y(t),t)=3*x(t)+6*z(t),diff(z(t),t)=-4*x(t)-3*z(t
```

$$x(t) = -\frac{e^{t}(2\sin(2t)c_2 - \sin(2t)c_3 + \cos(2t)c_2 + 2\cos(2t)c_3)}{2}$$

$$y(t) = -2c_1e^{-3t} - \frac{3c_2e^{t}\cos(2t)}{2} + \frac{3c_3e^{t}\sin(2t)}{2}$$

$$z(t) = c_1e^{-3t} + c_2e^{t}\sin(2t) + c_3e^{t}\cos(2t)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 168

```
DSolve[{x'[t]==2*x[t]+y[t]+2*z[t],y'[t]==3*x[t]+6*z[t],z'[t]==-4*x[t]-3*z[t]},{x[t],y[t],z[t]
```

$$x(t) \to \frac{1}{2}e^{t}(2c_1\cos(2t) + (c_1 + c_2 + 2c_3)\sin(2t))$$

$$y(t) \to \frac{1}{5}e^{-3t}\left(\frac{3}{2}e^{4t}(2(2c_1 + c_2 + 2c_3)\cos(2t) + (-3c_1 + c_2 + 2c_3)\sin(2t)) - 6c_1 + 2c_2 - 6c_3\right)$$

$$z(t) \to \frac{1}{5}e^{-3t}\left(e^{4t}((-3c_1 + c_2 + 2c_3)\cos(2t) - 2(2c_1 + c_2 + 2c_3)\sin(2t)) + 3c_1 - c_2 + 3c_3\right)$$

10.38 problem 45

Internal problem ID [6010]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.2. Page 346
Problem number: 45.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= x(t) - 12y(t) - 14z(t) \\
y'(t) &= x(t) + 2y(t) - 3z(t) \\
z'(t) &= x(t) + y(t) - 2z(t)
\end{aligned}
$$

With initial conditions

$$[x(0) = 4, y(0) = 6, z(0) = -7]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 62

```
dsolve([diff(x(t),t) = x(t)-12*y(t)-14*z(t), diff(y(t),t) = x(t)+2*y(t)-3*z(t), diff(z(t),t)
```

$$x(t) = -25\,\mathrm{e}^{t} + 29\cos(5t) + 11\sin(5t)$$

$$y(t) = 7\,\mathrm{e}^{t} + 6\sin(5t) - \cos(5t)$$

$$z(t) = -6\,\mathrm{e}^{t} + 6\sin(5t) - \cos(5t)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 65

```
DSolve[{x'[t]==x[t]-12*y[t]-14*z[t],y'[t]==x[t]+2*y[t]-3*z[t],z'[t]==x[t]+y[t]-2*z[t]},{x[0]=
```

$$x(t) \to -25e^t + 11\sin(5t) + 29\cos(5t)$$
$$y(t) \to 7e^t + 6\sin(5t) - \cos(5t)$$
$$z(t) \to -6e^t + 6\sin(5t) - \cos(5t)$$

11 CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.3. Page 354

11.1 problem 1

Internal problem ID [6011]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.3. Page 354
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) + 3y(t) - 7$$
$$y'(t) = -x(t) - 2y(t) + 5$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve([diff(x(t),t)=2*x(t)+3*y(t)-7,diff(y(t),t)=-x(t)-2*y(t)+5],[x(t), y(t)], singsol=all)
```

$$x(t) = -e^{-t}c_2 - 3c_1e^t - 1$$

$$y(t) = e^{-t}c_2 + c_1e^t + 3$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 45

```
DSolve[{x'[t]==2*x[t]+3*y[t]-7,y'[t]==-x[t]-2*y[t]+5},{x[t],y[t]},t,IncludeSingularSolutions
```

$$x(t) \to c_1 \cosh(t) + (2c_1 + 3c_2)\sinh(t) - 1$$
$$y(t) \to c_2 \cosh(t) - (c_1 + 2c_2)\sinh(t) + 3$$

11.2 problem 2

Internal problem ID [6012]

Book: DIFFERENTIAL EQUATIONS with Boundary Value Problems. DENNIS G. ZILL, WARREN S. WRIGHT, MICHAEL R. CULLEN. Brooks/Cole. Boston, MA. 2013. 8th edition.
Section: CHAPTER 8 SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS. EXERCISES 8.3. Page 354
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 5x(t) + 9y(t) + 2$$
$$y'(t) = -x(t) + 11y(t) + 6$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 38

```
dsolve([diff(x(t),t)=5*x(t)+9*y(t)+2,diff(y(t),t)=-x(t)+11*y(t)+6],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{1}{2} + \mathrm{e}^{8t}(3c_1t - c_1 + 3c_2)$$

$$y(t) = -\frac{1}{2} + \mathrm{e}^{8t}(c_1t + c_2)$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 54

```
DSolve[{x'[t]==5*x[t]+9*y[t]+2,y'[t]==-x[t]+11*y[t]+6},{x[t],y[t]},t,IncludeSingularSolutions
```

$$x(t) \to \frac{1}{2} + e^{8t}(-3c_1t + 9c_2t + c_1)$$
$$y(t) \to -\frac{1}{2} + e^{8t}(c_1(-t) + 3c_2t + c_2)$$