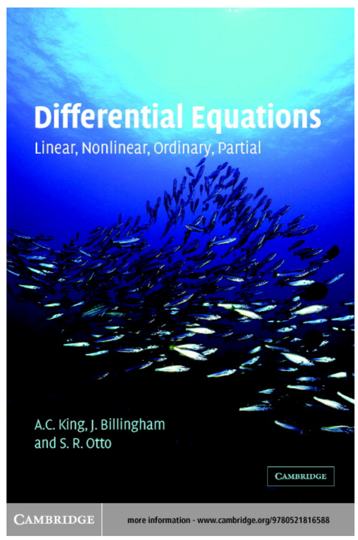
A Solution Manual For

# Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003

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# Contents

# 1 Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28

## 1.1 problem Problem 1.1(a)

Internal problem ID [11046]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
**Problem number**: Problem 1.1(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-y'x+y=0}$$

Given that one solution of the ode is

$$y_1=\mathrm{e}^x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([(x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,exp(x)],y(x), singsol=all)
```

$$y(x)=c_1x+c_2\mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^x-c_2x$$

## 1.2 problem Problem 1.1(b)

Internal problem ID [11047]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
**Problem number**: Problem 1.1(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + xy = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{\sin(x)}{x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve([x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,sin(x)/x],y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

## 1.3 problem Problem 1.3(a)

Internal problem ID [11048]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
**Problem number**: Problem 1.3(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y - x^{\frac{3}{2}} \mathrm{e}^{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=x^(3/2)*exp(x),y(x), singsol=all)
```

$$y(x) = \frac{4x^{\frac{7}{2}} \mathrm{e}^{x}}{35} + c_1 x \,\mathrm{e}^{x} + c_2 \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 29

```
DSolve[y''[x]-2*y'[x]+y[x]==x^(3/2)*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{35} e^{x} \left(4x^{7/2} + 35c_2 x + 35c_1\right)$$

## 1.4 problem Problem 1.3(b)

Internal problem ID [11049]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
**Problem number**: Problem 1.3(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y - 2\sec(2x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)+4*y(x)=2*sec(2*x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(2x) + c_1 \cos(2x) + x \sin(2x) - \frac{\ln(\sec(2x))\cos(2x)}{2}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 32

```
DSolve[y''[x]+4*y[x]==2*Sec[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_2)\sin(2x) + \cos(2x)\left(\frac{1}{2}\log(\cos(2x)) + c_1\right)$$

## 1.5 problem Problem 1.3(c)

Internal problem ID [11050]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
**Problem number**: Problem 1.3(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + \frac{y'}{x} + \left(1 - \frac{1}{4x^2}\right) y - x = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 71

```
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)+(1-1/(4*x^2))*y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{c_2 \sin(x)}{\sqrt{x}} + \frac{\cos(x) c_1}{\sqrt{x}} - \frac{3\left(\sin(x)\sqrt{2}\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right) - \cos(x)\sqrt{2}\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right) - \frac{4x^{\frac{3}{2}}}{3}\right)}{4\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.134 (sec). Leaf size: 74

```
DSolve[y''[x]+1/x*y'[x]+(1-1/(4*x^2))*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{ie^{-ix}\left(x^{5/2}\left(-\mathrm{ExpIntegralE}\left(-\frac{3}{2}, -ix\right)\right) + e^{2ix}\left(x^{5/2}\,\mathrm{ExpIntegralE}\left(-\frac{3}{2}, ix\right) - c_2\right) - 2ic_1\right)}{2\sqrt{x}}$$

## 1.6 problem Problem 1.3(d)

Internal problem ID [11051]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
**Problem number**: Problem 1.3(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - f(x) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve([diff(y(x),x$2)+y(x)=f(x),y(0) = 0, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \left(\int_0^x \cos(\_z1)\, f(\_z1)\, d\_z1\right)\sin(x) - \left(\int_0^x \sin(\_z1)\, f(\_z1)\, d\_z1\right)\cos(x)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 76

```
DSolve[{y''[x]+y[x]==f[x],{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x)\left(\int_1^x \cos(K[2])f(K[2])dK[2] - \int_1^0 \cos(K[2])f(K[2])dK[2]\right) + \cos(x)\left(\int_1^x -f(K[1])\sin(K[1])dK[1] - \int_1^0 -f(K[1])\sin(K[1])dK[1]\right)$$

## 1.7 problem Problem 1.6(a)

Internal problem ID [11052]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
**Problem number**: Problem 1.6(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x\left(x - \frac{1}{2}\right)y' + \frac{y}{2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+x*(x-1/2)*diff(y(x),x)+1/2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{WhittakerM}\left(\frac{1}{4}, \frac{1}{4}, x\right) x^{\frac{1}{4}} \mathrm{e}^{-\frac{x}{2}} + c_2\sqrt{x}\,\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]+x*(x-1/2)*y'[x]+1/2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_2\sqrt{x} - c_1 x \operatorname{ExpIntegralE}\left(\frac{1}{2}, -x\right)\right)$$

## 1.8 problem Problem 1.6(b)

Internal problem ID [11053]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
**Problem number**: Problem 1.6(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x+1)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x-1)}{x} + \frac{c_2 \mathrm{e}^{-x}}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+x*(1+x)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(c_1 e^x (x-1) + c_2)}{x}$$

## 1.9 problem Problem 1.7

Internal problem ID [11054]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
**Problem number**: Problem 1.7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{x(1-x)\,y''+(1-5x)\,y'-4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*(1-x)*diff(y(x),x$2)+(1-5*x)*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x)=&\,(c_2\ln(x)+c_1)\left(1+4x+9x^2+16x^3+25x^4+36x^5+\mathrm{O}\left(x^6\right)\right)\\&+\left((-4)\,x-12x^2-24x^3-40x^4-60x^5+\mathrm{O}\left(x^6\right)\right)c_2\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x*(1-x)*y''[x]+(1-5*x)*y'[x]-4*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned}y(x)\to&\ c_1\left(36x^5+25x^4+16x^3+9x^2+4x+1\right)\\&+c_2\left(-60x^5-40x^4-24x^3-12x^2+\left(36x^5+25x^4+16x^3+9x^2+4x+1\right)\log(x)-4x\right)\end{aligned}$$

## 1.10 problem Problem 1.8(a)

Internal problem ID [11055]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
**Problem number**: Problem 1.8(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-1)^2 y'' + (x+1) y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((x^2-1)^2*diff(y(x),x$2)+(x+1)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1+\frac{1}{2}x^2-\frac{1}{6}x^3+\frac{1}{6}x^4-\frac{7}{60}x^5\right) y(0)+\left(x-\frac{1}{2}x^2+\frac{1}{6}x^3-\frac{1}{6}x^4+\frac{7}{60}x^5\right) D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 70

```
AsymptoticDSolveValue[(x^2-1)^2*y''[x]+(x+1)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{7x^5}{60}+\frac{x^4}{6}-\frac{x^3}{6}+\frac{x^2}{2}+1\right)+c_2\left(\frac{7x^5}{60}-\frac{x^4}{6}+\frac{x^3}{6}-\frac{x^2}{2}+x\right)$$

## 1.11 problem Problem 1.8(b)

Internal problem ID [11056]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
**Problem number**: Problem 1.8(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 4y' - xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
Order:=6;
dsolve(x*diff(y(x),x$2)+4*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 + \frac{1}{10}x^2 + \frac{1}{280}x^4 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(12 - 6x^2 - \frac{3}{2}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 42

```
AsymptoticDSolveValue[x*y''[x]+4*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{x^3} - \frac{x}{8} - \frac{1}{2x}\right) + c_2\left(\frac{x^4}{280} + \frac{x^2}{10} + 1\right)$$

## 1.12 problem Problem 1.9

Internal problem ID [11057]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
**Problem number**: Problem 1.9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + (x+1)\,y' - ky = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 132

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)+(1+x)*diff(y(x),x)-k*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 + \left(\frac{k}{3} - \frac{1}{6}\right)x + \left(\frac{1}{30}k^2 - \frac{1}{15}k + \frac{1}{40}\right)x^2 + \frac{1}{5040}(2k-5)(2k-3)(2k-1)x^3 + \frac{1}{362880}(2k-7)(2k-5)(2k-3)(2k-1)x^4 + \frac{1}{39916800}(2k-9)(2k-7)(2k-5)(2k-3)(2k-1)x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 + kx + \frac{1}{6}(k-1)kx^2 + \frac{1}{90}(k-2)(k-1)kx^3 + \frac{1}{2520}(k-3)(k-2)(k-1)kx^4 + \frac{1}{113400}(k-4)(k-3)(k-2)(k-1)kx^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 304

```
AsymptoticDSolveValue[2*x*y''[x]+(1+x)*y'[x]-k*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{4\left(\frac{3}{4}-\frac{k}{2}\right)\left(\frac{5}{4}-\frac{k}{2}\right)\left(\frac{7}{4}-\frac{k}{2}\right)\left(\frac{9}{4}-\frac{k}{2}\right)\left(\frac{k}{2}-\frac{1}{4}\right)x^5}{155925} - \frac{2\left(\frac{3}{4}-\frac{k}{2}\right)\left(\frac{5}{4}-\frac{k}{2}\right)\left(\frac{7}{4}-\frac{k}{2}\right)\left(\frac{k}{2}-\frac{1}{4}\right)x^4}{2835} + \frac{4}{315}\left(\frac{3}{4}-\frac{k}{2}\right)\left(\frac{5}{4}-\frac{k}{2}\right)\left(\frac{k}{2}-\frac{1}{4}\right)x^3 - \frac{2}{15}\left(\frac{3}{4}-\frac{k}{2}\right)\left(\frac{k}{2}-\frac{1}{4}\right)x^2 + \frac{2}{3}\left(\frac{k}{2}-\frac{1}{4}\right)x+1\right) + c_2\left(\frac{2\left(\frac{1}{2}-\frac{k}{2}\right)\left(1-\frac{k}{2}\right)\left(\frac{3}{2}-\frac{k}{2}\right)\left(2-\frac{k}{2}\right)kx^5}{14175} - \frac{1}{315}\left(\frac{1}{2}-\frac{k}{2}\right)\left(1-\frac{k}{2}\right)\left(\frac{3}{2}-\frac{k}{2}\right)kx^4 + \frac{2}{45}\left(\frac{1}{2}-\frac{k}{2}\right)\left(1-\frac{k}{2}\right)kx^3 - \frac{1}{3}\left(\frac{1}{2}-\frac{k}{2}\right)kx^2 + kx + 1\right)$$

## 1.13 problem Problem 1.11(a)

Internal problem ID [11058]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
**Problem number**: Problem 1.11(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^3y'' + x^2y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^3*diff(y(x),x$2)+x^2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 222

```
AsymptoticDSolveValue[x^3*y''[x]+x^2*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 e^{-\frac{2i}{\sqrt{x}}} \sqrt[4]{x}\left(\frac{418854310875ix^{9/2}}{8796093022208} - \frac{57972915ix^{7/2}}{4294967296} + \frac{59535ix^{5/2}}{8388608} - \frac{75ix^{3/2}}{8192} - \frac{30241281245175x^5}{281474976710656} + \frac{13043905875x^4}{549755813888} - \frac{2401245x^3}{268435456} + \frac{3675x^2}{524288} - \frac{9x}{512} + \frac{i\sqrt{x}}{16} + 1\right) + c_2 e^{\frac{2i}{\sqrt{x}}} \sqrt[4]{x}\left(-\frac{418854310875ix^{9/2}}{8796093022208} + \frac{57972915ix^{7/2}}{4294967296} - \frac{59535ix^{5/2}}{8388608} + \frac{75ix^{3/2}}{8192} - \frac{30241281245175x^5}{281474976710656} + \frac{13}{5}\right.$$

## 1.14 problem Problem 1.11(b)

Internal problem ID [11059]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
**Problem number**: Problem 1.11(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + y' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 28

```
AsymptoticDSolveValue[x^2*y''[x]+y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 e^{\frac{1}{x}} x^2 + c_1 (2x^2 + 2x + 1)$$

## 1.15 problem Problem 1.12

Internal problem ID [11060]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
**Problem number**: Problem 1.12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + x(1-x)\,y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+x*(1-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{3}{2}}\left(1+\frac{1}{5}x+\frac{1}{35}x^2+\frac{1}{315}x^3+\frac{1}{3465}x^4+\frac{1}{45045}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1+\frac{1}{2}x+\frac{1}{8}x^2+\frac{1}{48}x^3+\frac{1}{384}x^4+\frac{1}{3840}x^5+ \right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 86

```
AsymptoticDSolveValue[2*x^2*y''[x]+x*(1-x)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x\left(\frac{x^5}{45045}+\frac{x^4}{3465}+\frac{x^3}{315}+\frac{x^2}{35}+\frac{x}{5}+1\right)+\frac{c_2\left(\frac{x^5}{3840}+\frac{x^4}{384}+\frac{x^3}{48}+\frac{x^2}{8}+\frac{x}{2}+1\right)}{\sqrt{x}}$$

## 1.16 problem Problem 1.13

Internal problem ID [11061]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 1 VARIABLE COEFFICIENT, SECOND ORDER DIFFERENTIAL EQUATIONS. Problems page 28
**Problem number**: Problem 1.13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(x-1)\,y''+3y'x+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
Order:=6;
dsolve(x*(x-1)*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & \ln\left(x\right)\left(x+2x^2+3x^3+4x^4+5x^5+\mathrm{O}\left(x^6\right)\right)c_2 \\ & + c_1 x\left(1+2x+3x^2+4x^3+5x^4+6x^5+\mathrm{O}\left(x^6\right)\right) \\ & + \left(1+3x+5x^2+7x^3+9x^4+11x^5+\mathrm{O}\left(x^6\right)\right)c_2 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 63

```
AsymptoticDSolveValue[x*(x-1)*y''[x]+3*x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(x^4+x^3+x^2+\left(4x^3+3x^2+2x+1\right)x\log(x)+x+1\right)+c_2\left(5x^5+4x^4+3x^3+2x^2+x\right)$$

# 2 Chapter 3 Bessel functions. Problems page 89

## 2.1 problem Problem 3.7(a)

Internal problem ID [11062]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 3 Bessel functions. Problems page 89
**Problem number**: Problem 3.7(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - x^2 y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sqrt{x}\, \mathrm{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) + c_2 \sqrt{x}\, \mathrm{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 37

```
DSolve[y''[x]-x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\, \mathrm{ParabolicCylinderD}\left(-\frac{1}{2}, i\sqrt{2}x\right) + c_1\, \mathrm{ParabolicCylinderD}\left(-\frac{1}{2}, \sqrt{2}x\right)$$

## 2.2 problem Problem 3.7(b)

Internal problem ID [11063]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 3 Bessel functions. Problems page 89
**Problem number**: Problem 3.7(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(0, 2\sqrt{x}\right) + c_2 \operatorname{BesselY}\left(0, 2\sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 27

```
DSolve[x*y''[x]+y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\, {}_0\tilde{F}_1(;1;-x) + 2c_2 Y_0\left(2\sqrt{x}\right)$$

## 2.3 problem Problem 3.7(c)

Internal problem ID [11064]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 3 Bessel functions. Problems page 89
**Problem number**: Problem 3.7(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (x+1)^2 y = 0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+(x+1)^2*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(x+1)^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 2.4 problem Problem 3.7(d)

Internal problem ID [11065]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 3 Bessel functions. Problems page 89
**Problem number**: Problem 3.7(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + \alpha^2 y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+alpha^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(\alpha x) + c_2 \cos(\alpha x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[y''[x]+a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(ax) + c_2 \sin(ax)$$

## 2.5 problem Problem 3.7(e)

Internal problem ID [11066]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 3 Bessel functions. Problems page 89
**Problem number**: Problem 3.7(e).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - \alpha^2 y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-alpha^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\alpha x} + c_2 \mathrm{e}^{-\alpha x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 23

```
DSolve[y''[x]-a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{ax} + c_2 e^{-ax}$$

## 2.6 problem Problem 3.7(f)

Internal problem ID [11067]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 3 Bessel functions. Problems page 89
**Problem number**: Problem 3.7(f).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + \beta y' + \gamma y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$2)+beta*diff(y(x),x)+gamma*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\left(-\frac{\beta}{2}+\frac{\sqrt{\beta^2-4\gamma}}{2}\right)x} + c_2 \mathrm{e}^{\left(-\frac{\beta}{2}-\frac{\sqrt{\beta^2-4\gamma}}{2}\right)x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 47

```
DSolve[y''[x]+\[Beta]*y'[x]+\[Gamma]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}x\left(\sqrt{\beta^2-4\gamma}+\beta\right)}\left(c_2 e^{x\sqrt{\beta^2-4\gamma}} + c_1\right)$$

## 2.7 problem Problem 3.7(g)

Internal problem ID [11068]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 3 Bessel functions. Problems page 89
**Problem number**: Problem 3.7(g).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\,y''-2y'x+n(1+n)\,y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+n*(n+1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\mathrm{LegendreP}\,(n,x)+c_2\,\mathrm{LegendreQ}\,(n,x)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 18

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+n*(n+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\,\mathrm{LegendreP}(n,x)+c_2\,\mathrm{LegendreQ}(n,x)$$

## 2.8 problem Problem 3.12

Internal problem ID [11069]

**Book**: Differential Equations, Linear, Nonlinear, Ordinary, Partial. A.C. King, J.Billingham, S.R.Otto. Cambridge Univ. Press 2003
**Section**: Chapter 3 Bessel functions. Problems page 89
**Problem number**: Problem 3.12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2y'' + y'x + (-\nu^2 + x^2)\,y - \sin(x) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 161

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-nu^2)*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = \operatorname{BesselJ}(\nu, x)\, c_2 + \operatorname{BesselY}(\nu, x)\, c_1 - \frac{x\left(\operatorname{hypergeom}\left(\left[\frac{1}{2} - \frac{\nu}{2}, \frac{5}{4} - \frac{\nu}{2}, \frac{3}{4} - \frac{\nu}{2}\right], \left[\frac{3}{2}, 1 - \nu, \frac{3}{2} - \nu, \frac{3}{2} - \frac{\nu}{2}\right], -x^2\right) \operatorname{BesselJ}(\nu, x)\, \Gamma(\nu + 2)^2\, 2^\nu x^{-\nu} + \pi\right.}{2}$$

✓ Solution by Mathematica

Time used: 0.404 (sec). Leaf size: 200

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-\[Nu]^2)*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-2^\nu \operatorname{Gamma}(\nu - 1)x^{1-\nu} \operatorname{BesselJ}(\nu, x)\, {}_3F_4\left(\frac{1}{2} - \frac{\nu}{2}, \frac{3}{4} - \frac{\nu}{2}, \frac{5}{4} - \frac{\nu}{2}; \frac{3}{2}, 1 - \nu, \frac{3}{2} - \nu, \frac{3}{2} - \frac{\nu}{2}; -x^2\right) - \frac{2^{-\nu} \csc(\pi\nu) \operatorname{BesselJ}(-\nu, x) \left(\pi x^{\nu+1}\, {}_3F_4\left(\frac{\nu}{2} + \frac{1}{2}, \frac{\nu}{2} + \frac{3}{4}, \frac{\nu}{2} + \frac{5}{4}; \frac{3}{2}, \frac{\nu}{2} + \frac{3}{2}, \nu + 1, \nu + \frac{3}{2}; -x^2\right) + c_2 2^{\nu+1} \operatorname{Gamm}\right.}{\operatorname{Gamma}(\nu + 2)} + 2(c_2 \cot(\pi\nu) + c_1) \operatorname{BesselJ}(\nu, x)\right)$$