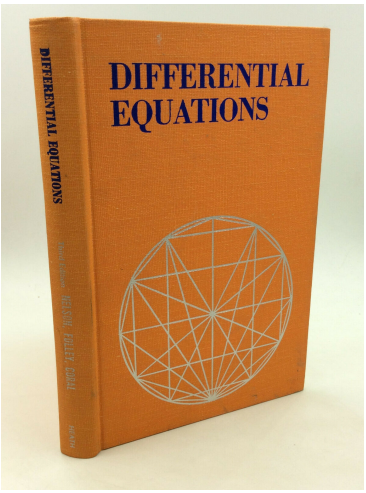
A Solution Manual For

Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964



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Contents

1 Exercis 5, page 21

1.1 problem 1

Internal problem ID [1870]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yx + \left(x^2+1\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*y(x)+(x^2+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 22

```
DSolve[x*y[x]+(x^2+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{\sqrt{x^2+1}}$$
$$y(x) \to 0$$

1.2 problem 2

Internal problem ID [1871]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy^2 + x + \left(y - x^2 y\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve((x*y(x)^2+x)+(y(x)-x^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x^2 - c_1 - 1}$$
$$y(x) = -\sqrt{c_1 x^2 - c_1 - 1}$$

✓ Solution by Mathematica

Time used: 1.226 (sec). Leaf size: 61

```
DSolve[(x*y[x]^2+x)+(y[x]-x^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-1 + e^{2c_1}\left(x^2 - 1\right)}$$
$$y(x) \to \sqrt{-1 + e^{2c_1}\left(x^2 - 1\right)}$$
$$y(x) \to -i$$
$$y(x) \to i$$

1.3 problem 3

Internal problem ID [1872]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{1+y^2+\left(x^2+1\right)y'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve((1+y(x)^2)+(1+x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\tan\left(\arctan\left(x\right)+c_1\right)$$

✓ Solution by Mathematica

Time used: 0.234 (sec). Leaf size: 29

```
DSolve[(1+y[x]^2)+(1+x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\tan(\arctan(x)-c_1)$$
$$y(x)\to -i$$
$$y(x)\to i$$

1.4 problem 4

Internal problem ID [1873]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + y'x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve(y(x)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 16

```
DSolve[y[x]+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to 0$$

1.5 problem 5

Internal problem ID [1874]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=2*x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x^2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 18

```
DSolve[y'[x]==2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x^2}$$
$$y(x) \to 0$$

1.6 problem 6

Internal problem ID [1875]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy^2 + x + \left(x^2 y - y\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve((x*y(x)^2+x)+(x^2*y(x)-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\left(x^2-1\right)\left(-x^2+c_1\right)}}{x^2-1}$$

$$y(x) = -\frac{\sqrt{\left(x^2-1\right)\left(-x^2+c_1\right)}}{x^2-1}$$

✓ Solution by Mathematica

Time used: 0.35 (sec). Leaf size: 133

```
DSolve[(x*y[x]^2+x)+(x^2*y[x]-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^2+1-e^{2c_1}}}{\sqrt{x^2-1}}$$

$$y(x) \to \frac{\sqrt{-x^2+1-e^{2c_1}}}{\sqrt{x^2-1}}$$

$$y(x) \to -i$$

$$y(x) \to i$$

$$y(x) \to -\frac{\sqrt{1-x^2}}{\sqrt{x^2-1}}$$

$$y(x) \to \frac{\sqrt{1-x^2}}{\sqrt{x^2-1}}$$

1.7 problem 7

Internal problem ID [1876]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sqrt{1-x^2}+\sqrt{1-y^2}\,y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(sqrt(1-x^2)+sqrt(1-y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$c_1+x\sqrt{-x^2+1}+\arcsin(x)+y(x)\sqrt{1-y(x)^2}+\arcsin(y(x))=0$$

✓ Solution by Mathematica

Time used: 0.597 (sec). Leaf size: 83

```
DSolve[Sqrt[1-x^2]+Sqrt[1-y[x]^2]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{InverseFunction}\left[\frac{1}{2}\#1\sqrt{1-\#1^2}-\arctan\left(\frac{\sqrt{1-\#1^2}}{\#1+1}\right)\&\right]\left[-\frac{1}{2}\sqrt{1-x^2}x+\cot^{-1}\left(\frac{x+1}{\sqrt{1-x^2}}\right)+c_1\right]$$

1.8 problem 8

Internal problem ID [1877]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+1)\,y' - 1 + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((1+x)*diff(y(x),x)-(1-y(x))=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 + x}{x+1}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 20

```
DSolve[(1+x)*y'[x]-(1-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x + c_1}{x+1}$$
$$y(x) \to 1$$

1.9 problem 9

Internal problem ID [1878]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\tan(x)\,y' - y - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)*tan(x)-y(x)=1,y(x), singsol=all)
```

$$y(x) = (-\csc(x) + c_1)\sin(x)$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 17

```
DSolve[y'[x]*Tan[x]-y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + c_1 \sin(x)$$
$$y(x) \to -1$$

1.10 problem 10

Internal problem ID [1879]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + 3 + \cot(x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((y(x)+3)+cot(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (-3\sec(x) + c_1)\cos(x)$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 17

```
DSolve[(y[x]+3)+Cot[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3 + c_1 \cos(x)$$
$$y(x) \to -3$$

1.11 problem 11

Internal problem ID [1880]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x}{y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)=x/y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 + c_1}$$
$$y(x) = -\sqrt{x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 35

```
DSolve[y'[x]==x/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^2 + 2c_1}$$
$$y(x) \to \sqrt{x^2 + 2c_1}$$

1.12 problem 12

Internal problem ID [1881]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - 1 + \sin(2t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(x(t),t)=1-sin(2*t),x(t), singsol=all)
```

$$x(t) = \frac{\cos(2t)}{2} + t + c_1$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 17

```
DSolve[x'[t]==1-Sin[2*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to t + \frac{1}{2}\cos(2t) + c_1$$

1.13 problem 13

Internal problem ID [1882]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + y'x - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x)+y(x)=y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{c_1 x + 1}$$

✓ Solution by Mathematica

Time used: 0.243 (sec). Leaf size: 25

```
DSolve[x*y'[x]+y[x]==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{1 + e^{c_1} x}$$
$$y(x) \to 0$$
$$y(x) \to 1$$

1.14 problem 14

Internal problem ID [1883]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sin(x)\cos(y)^2+\cos(x)^2y'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve(sin(x)*cos(y(x))^2+cos(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\arctan(\sec(x)+c_1)$$

✓ Solution by Mathematica

Time used: 1.557 (sec). Leaf size: 31

```
DSolve[Sin[x]*Cos[y[x]]^2+Cos[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \arctan(-\sec(x)+c_1)$$
$$y(x)\to -\frac{\pi}{2}$$
$$y(x)\to \frac{\pi}{2}$$

1.15 problem 15

Internal problem ID [1884]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sec(x)\cos(y)^2 - \cos(x)\sin(y)\,y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve(sec(x)*cos(y(x))^2=cos(x)*sin(y(x))*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{1}{\tan(x) + c_1}\right)$$

✓ Solution by Mathematica

Time used: 0.788 (sec). Leaf size: 45

```
DSolve[Sec[x]*Cos[y[x]]^2==Cos[x]*Sin[y[x]]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sec^{-1}(\tan(x) + 2c_1)$$
$$y(x) \to \sec^{-1}(\tan(x) + 2c_1)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

1.16 problem 16

Internal problem ID [1885]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + y'x - xy(y' - 1) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(y(x)+x*diff(y(x),x)=x*y(x)*(diff(y(x),x)-1),y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-x}}{c_1 x}\right)$$

✓ Solution by Mathematica

Time used: 3.033 (sec). Leaf size: 28

```
DSolve[y[x]+x*y'[x]==x*y[x]*(y'[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -W\left(-\frac{e^{-x-c_1}}{x}\right)$$
$$y(x) \to 0$$

1.17 problem 17

Internal problem ID [1886]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yx + \sqrt{x^2+1}\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*y(x)+sqrt(1+x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 26

```
DSolve[x*y[x]+Sqrt[1+x^2]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\sqrt{x^2+1}}$$
$$y(x) \to 0$$

1.18 problem 18

Internal problem ID [1887]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - yx - y'x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(y(x)=x*y(x)+x^2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{1}{x}}}{x}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 23

```
DSolve[y[x]==x*y[x]+x^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 e^{-1/x}}{x}$$
$$y(x) \to 0$$

1.19 problem 19

Internal problem ID [1888]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\tan(x)\sin(x)^2+\cos(x)^2\cot(y)y'=0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 40

```
dsolve(tan(x)*sin(x)^2+cos(x)^2*cot(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\arcsin\left(\frac{\sqrt{2}\sqrt{\frac{1}{1+\cos(2x)}}\,\mathrm{e}^{\frac{-1+\cos(2x)}{2\cos(2x)+2}}}{c_1}\right)$$

✓ Solution by Mathematica

Time used: 16.527 (sec). Leaf size: 24

```
DSolve[Tan[x]*Sin[x]^2+Cos[x]^2*Cot[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\arcsin\left(\frac{1}{8}c_1e^{-\frac{1}{2}\sec^2(x)}\sec(x)\right)$$

1.20 problem 20

Internal problem ID [1889]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 + yy' + y'x^2y - 1 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(y(x)^2+y(x)*diff(y(x),x)+x^2*y(x)*diff(y(x),x)-1=0,y(x), singsol=all)
```

$$y(x) = \sqrt{\mathrm{e}^{-2\arctan(x)}c_1 + 1}$$
$$y(x) = -\sqrt{\mathrm{e}^{-2\arctan(x)}c_1 + 1}$$

✓ Solution by Mathematica

Time used: 0.845 (sec). Leaf size: 55

```
DSolve[y[x]^2+y[x]*y'[x]+x^2*y[x]*y'[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{1 + e^{-2\arctan(x)+2c_1}}$$
$$y(x) \to \sqrt{1 + e^{-2\arctan(x)+2c_1}}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

1.21 problem 21

Internal problem ID [1890]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{x} = 0}$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 7

```
dsolve([diff(y(x),x)=y(x)/x,y(1) = 3],y(x), singsol=all)
```

$$y(x) = 3x$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 8

```
DSolve[{y'[x]==y[x]/x,y[1]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x$$

1.22 problem 22

Internal problem ID [1891]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x + 2y = 0}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve([x*diff(y(x),x)+2*y(x)=0,y(2) = 1],y(x), singsol=all)
```

$$y(x) = \frac{4}{x^2}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 10

```
DSolve[{x*y'[x]+2*y[x]==0,y[2]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4}{x^2}$$

1.23 problem 23

Internal problem ID [1892]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sin(x)\cos(y)+\cos(x)\sin(y)y'=0}$$

With initial conditions

$$[y(0)=0]$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 13

```
dsolve([sin(x)*cos(y(x))+cos(x)*sin(y(x))*diff(y(x),x)=0,y(0) = 0],y(x), singsol=all)
```

$$y(x)=(1-2\_B5)\arccos(\sec(x))$$

✓ Solution by Mathematica

Time used: 6.057 (sec). Leaf size: 17

```
DSolve[{Sin[x]*Cos[y[x]]+Cos[x]*Sin[y[x]]*y'[x]==0,y[0]==0},y[x],x,IncludeSingularSolutions -
```

$$y(x)\to -\arccos(\sec(x))$$
$$y(x)\to \arccos(\sec(x))$$

1.24 problem 24

Internal problem ID [1893]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x^2 + y^2 = 0}$$

With initial conditions

$$[y(3) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 14

```
dsolve([x^2*diff(y(x),x)+y(x)^2=0,y(3) = 1],y(x), singsol=all)
```

$$y(x) = \frac{3x}{4x - 3}$$

✓ Solution by Mathematica

Time used: 0.128 (sec). Leaf size: 15

```
DSolve[{x^2*y'[x]+y[x]^2==0,y[3]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x}{4x - 3}$$

1.25 problem 25

Internal problem ID [1894]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \mathrm{e}^{y} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=exp(y(x)),y(0) = 0],y(x), singsol=all)
```

$$y(x) = -\ln(1 - x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 13

```
DSolve[{y'[x]==Exp[y[x]],y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log(1 - x)$$

1.26 problem 26

Internal problem ID [1895]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{e^{y}(y'+1)-1=0}$$

With initial conditions

$$[y(0)=1]$$

✓ Solution by Maple

Time used: 0.438 (sec). Leaf size: 32

```
dsolve([exp(y(x))*(diff(y(x),x)+1)=1,y(0) = 1],y(x), singsol=all)
```

$$y(x)=-x+\ln\left(-e^{x}-e+1\right)-i\pi$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 17

```
DSolve[{Exp[y[x]]*(y'[x]+1)==1,y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \log\left((e-1)e^{-x}+1\right)$$

1.27 problem 27

Internal problem ID [1896]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{1+y^2-\frac{y'}{x^3\left(x-1\right)}=0}$$

With initial conditions

$$[y(2)=0]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 17

```
dsolve([(1+y(x)^2)=diff(y(x),x)/(x^3*(x-1)),y(2) = 0],y(x), singsol=all)
```

$$y(x)=\tan\left(\frac{1}{5}x^5-\frac{1}{4}x^4-\frac{12}{5}\right)$$

✓ Solution by Mathematica

Time used: 0.33 (sec). Leaf size: 21

```
DSolve[{(1+y[x]^2)==y'[x]/(x^3*(x-1)),y[2]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\tan\left(\frac{1}{20}\left(x^4(4x-5)-48\right)\right)$$

1.28 problem 28

Internal problem ID [1897]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$\boxed{x^2 + 3y'x - y^3 - 2y = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 7

```
dsolve([x^2+3*x*diff(y(x),x)=y(x)^3+2*y(x),y(1) = 1],y(x), singsol=all)
```

$$y(x) = x^{\frac{2}{3}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x^2+3*x*y'[x]==y[x]^3+2*y[x],y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

{}

1.29 problem 29

Internal problem ID [1898]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(x^2+x+1\right)y'-y^2-2y-5=0}$$

With initial conditions

$$[y(1)=1]$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 35

```
dsolve([(x^2+x+1)*diff(y(x),x)=y(x)^2+2*y(x)+5,y(1) = 1],y(x), singsol=all)
```

$$y(x)=-1+2\cot\left(\frac{4\sqrt{3}\,\pi}{9}-\frac{4\sqrt{3}\,\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)}{3}+\frac{\pi}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.816 (sec). Leaf size: 44

```
DSolve[{(x^2+x+1)*y'[x]==y[x]^2+2*y[x]+5,y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 2\tan\left(\frac{4\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{36}\left(9-16\sqrt{3}\right)\pi\right)-1$$

1.30 problem 30

Internal problem ID [1899]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 5, page 21
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(x^2-2x-8\right)y'-y^2-y+2=0}$$

With initial conditions

$$[y(0)=0]$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 31

```
dsolve([(x^2-2*x-8)*diff(y(x),x)=y(x)^2+y(x)-2,y(0) = 0],y(x), singsol=all)
```

$$y(x)=\frac{2x+8-2\sqrt{-2x^2+4x+16}}{4+3x}$$

✓ Solution by Mathematica

Time used: 3.81 (sec). Leaf size: 34

```
DSolve[{(x^2-2*x-8)*y'[x]==y[x]^2+y[x]-2,y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-2\sqrt{-2x^2+4x+16}+2x+8}{3x+4}$$

2 Exercis 6, page 25

2.1 problem 1

Internal problem ID [1900]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x + y - y'x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x+y(x)=x*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = (\ln(x) + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 12

```
DSolve[x+y[x]==x*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(\log(x) + c_1)$$

2.2 problem 2

Internal problem ID [1901]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(x+y)\,y'+x-y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 24

```
dsolve((x+y(x))*diff(y(x),x)+x=y(x),y(x), singsol=all)
```

$$y(x)=\tan\left(\text{RootOf}\left(2\_Z+\ln\left(\frac{1}{\cos\left(\_Z\right)^2}\right)+2\ln\left(x\right)+2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 34

```
DSolve[(x+y[x])*y'[x]+x==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{y(x)}{x}\right)+\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)=-\log(x)+c_1,y(x)\right]$$

2.3 problem 3

Internal problem ID [1902]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y'x - y - \sqrt{yx} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(x*y(x)),y(x), singsol=all)
```

$$-\frac{y(x)}{\sqrt{xy\left(x\right)}} + \frac{\ln\left(x\right)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.158 (sec). Leaf size: 17

```
DSolve[x*y'[x]-y[x]==Sqrt[x*y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}x(\log(x) + c_1)^2$$

2.4 problem 4

Internal problem ID [1903]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cla`

$$\boxed{y' - \frac{2x - y}{x + 4y} = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 53

```
dsolve(diff(y(x),x)=(2*x-y(x))/(x+4*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{-\frac{c_1 x}{4} - \frac{\sqrt{9c_1^2 x^2 + 8}}{4}}{c_1}$$

$$y(x) = \frac{-\frac{c_1 x}{4} + \frac{\sqrt{9c_1^2 x^2 + 8}}{4}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.418 (sec). Leaf size: 101

```
DSolve[y'[x]==(2*x-y[x])/(x+4*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-x - \sqrt{9x^2 + 8e^{c_1}}\right)$$

$$y(x) \to \frac{1}{4}\left(-x + \sqrt{9x^2 + 8e^{c_1}}\right)$$

$$y(x) \to \frac{1}{4}\left(-3\sqrt{x^2} - x\right)$$

$$y(x) \to \frac{1}{4}\left(3\sqrt{x^2} - x\right)$$

2.5 problem 5

Internal problem ID [1904]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y'x - y - \sqrt{x^2 - y^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(x^2-y(x)^2),y(x), singsol=all)
```

$$-\arctan\left(\frac{y(x)}{\sqrt{x^2 - y(x)^2}}\right) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.261 (sec). Leaf size: 18

```
DSolve[x*y'[x]-y[x]==Sqrt[x^2-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \cosh(i \log(x) + c_1)$$

2.6 problem 6

Internal problem ID [1905]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{x + yy' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve(x+y(x)*diff(y(x),x)=2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x(\text{LambertW}\left(c_1 x\right) + 1)}{\text{LambertW}\left(c_1 x\right)}$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 33

```
DSolve[x+y[x]*y'[x]==2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\log\left(\frac{y(x)}{x} - 1\right) - \frac{1}{\frac{y(x)}{x} - 1} = -\log(x) + c_1, y(x)\right]$$

2.7 problem 7

Internal problem ID [1906]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y'x - y + \sqrt{y^2 - x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x)-y(x)+sqrt(y(x)^2-x^2)=0,y(x), singsol=all)
```

$$y(x) + \sqrt{y(x)^2 - x^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.345 (sec). Leaf size: 16

```
DSolve[x*y'[x]-y[x]+Sqrt[y[x]^2-x^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \cosh(-\log(x) + c_1)$$

2.8 problem 8

Internal problem ID [1907]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{x^2 + y^2 - xyy' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve((x^2+y(x)^2)=x*y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \sqrt{2\ln(x) + c_1}\, x$$
$$y(x) = -\sqrt{2\ln(x) + c_1}\, x$$

✓ Solution by Mathematica

Time used: 0.162 (sec). Leaf size: 36

```
DSolve[(x^2+y[x]^2)==x*y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{2\log(x) + c_1}$$
$$y(x) \to x\sqrt{2\log(x) + c_1}$$

2.9 problem 9

Internal problem ID [1908]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{\left(yx - x^2\right) y' - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 21

```
dsolve((x*y(x)-x^2)*diff(y(x),x)-y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-c_1}}{x}\right)-c_1}$$

✓ Solution by Mathematica

Time used: 2.153 (sec). Leaf size: 25

```
DSolve[(x*y[x]-x^2)*y'[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -xW\left(-\frac{e^{-c_1}}{x}\right)$$
$$y(x) \to 0$$

2.10 problem 10

Internal problem ID [1909]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y + y'x - 2\sqrt{yx} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
dsolve(x*diff(y(x),x)+y(x)=2*sqrt(x*y(x)),y(x), singsol=all)
```

$$\frac{\sqrt{xy\left(x\right)}}{\left(-x+y\left(x\right)\right)\left(-x+\sqrt{xy\left(x\right)}\right)x}+\frac{1}{\left(-x+y\left(x\right)\right)\left(-x+\sqrt{xy\left(x\right)}\right)}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.187 (sec). Leaf size: 26

```
DSolve[x*y'[x]+y[x]==2*Sqrt[x*y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\left(x+e^{\frac{c_1}{2}}\right)^2}{x}$$
$$y(x)\to x$$

2.11 problem 11

Internal problem ID [1910]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, [_Abel, '2nd typ`

$$\boxed{x + y + (x - y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 49

```
dsolve((x+y(x))+(x-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x - \sqrt{2c_1^2 x^2 + 1}}{c_1}$$
$$y(x) = \frac{c_1 x + \sqrt{2c_1^2 x^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.435 (sec). Leaf size: 86

```
DSolve[(x+y[x])+(x-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to x + \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to x - \sqrt{2}\sqrt{x^2}$$
$$y(x) \to \sqrt{2}\sqrt{x^2} + x$$

2.12 problem 12

Internal problem ID [1911]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y(x^2 - yx + y^2) + xy'(x^2 + yx + y^2) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(y(x)*(x^2-x*y(x)+y(x)^2)+x*diff(y(x),x)*(x^2+x*y(x)+y(x)^2)=0,y(x), singsol=all)
```

$$y(x) = \tan\left(\text{RootOf}\left(\ln\left(\tan\left(\_Z\right)\right) + \_Z + 2\ln\left(x\right) + 2c_1\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.115 (sec). Leaf size: 26

```
DSolve[y[x]*(x^2-x*y[x]+y[x]^2)+x*y'[x]*(x^2+x*y[x]+y[x]^2)==0,y[x],x,IncludeSingularSolution
```

$$\text{Solve}\left[\arctan\left(\frac{y(x)}{x}\right) + \log\left(\frac{y(x)}{x}\right) = -2\log(x) + c_1, y(x)\right]$$

2.13 problem 13

Internal problem ID [1912]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y'x - y - x\sin\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 44

```
dsolve(diff(y(x),x)*x-y(x)-x*sin(y(x)/x)=0,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{2c_1 x}{c_1^2 x^2 + 1}, -\frac{c_1^2 x^2 - 1}{c_1^2 x^2 + 1}\right) x$$

✓ Solution by Mathematica

Time used: 2.781 (sec). Leaf size: 33

```
DSolve[y'[x]*x-y[x]-x*Sin[y[x]/x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x \arctan\left(e^{c_1} x\right)$$
$$y(x) \to 0$$
$$y(x) \to \pi\sqrt{x^2}$$

2.14 problem 14

Internal problem ID [1913]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y' - \frac{y}{x} - \cosh\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=y(x)/x+cosh(y(x)/x),y(x), singsol=all)
```

$$y(x) = \ln\left(\tan\left(\frac{\ln(x)}{2} + \frac{c_1}{2}\right)\right) x$$

✓ Solution by Mathematica

Time used: 2.085 (sec). Leaf size: 14

```
DSolve[y'[x]==y[x]/x+Cosh[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \operatorname{arcsinh}(\tan(\log(x) + c_1))$$

2.15 problem 15

Internal problem ID [1914]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{x^2 + y^2 - 2xyy' = 0}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 23

```
dsolve([(x^2+y(x)^2)=2*x*y(x)*diff(y(x),x),y(-1) = 0],y(x), singsol=all)
```

$$y(x) = \sqrt{(x+1)\,x}$$
$$y(x) = -\sqrt{(x+1)\,x}$$

✓ Solution by Mathematica

Time used: 0.186 (sec). Leaf size: 36

```
DSolve[{(x^2+y[x]^2)==2*x*y[x]*y'[x],y[-1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{x+1}$$
$$y(x) \to \sqrt{x}\sqrt{x+1}$$

2.16 problem 16

Internal problem ID [1915]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{\left(\frac{x}{y}+\frac{y}{x}\right)y'+1=0}$$

✓ Solution by Maple

Time used: 0.703 (sec). Leaf size: 223

```
dsolve((x/y(x)+y(x)/x)*diff(y(x),x)+1=0,y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{x^2c_1\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)c_1}$$

$$y(x)=\frac{\sqrt{-x^2c_1\left(-c_1x^2+\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2-\sqrt{c_1^2x^4+1}\right)c_1}$$

$$y(x)=-\frac{\sqrt{x^2c_1\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)c_1}$$

$$y(x)=-\frac{\sqrt{-x^2c_1\left(-c_1x^2+\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2-\sqrt{c_1^2x^4+1}\right)c_1}$$

✓ Solution by Mathematica

Time used: 0.093 (sec). Leaf size: 121

```
DSolve[(x/y[x]+y[x]/x)*y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 - \sqrt{x^4 + e^{4c_1}}}$$

$$y(x) \to \sqrt{-x^2 - \sqrt{x^4 + e^{4c_1}}}$$

$$y(x) \to -\sqrt{-x^2 + \sqrt{x^4 + e^{4c_1}}}$$

$$y(x) \to \sqrt{-x^2 + \sqrt{x^4 + e^{4c_1}}}$$

2.17 problem 17

Internal problem ID [1916]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{e^{\frac{y}{x}} x + y - y' x = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 15

```
dsolve([x*exp(y(x)/x)+y(x)=x*diff(y(x),x),y(1) = 0],y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{1}{-1 + \ln(x)}\right) x$$

✓ Solution by Mathematica

Time used: 0.311 (sec). Leaf size: 15

```
DSolve[{x*Exp[y[x]/x]+y[x]==x*y'[x],y[1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \log(1 - \log(x))$$

2.18 problem 18

Internal problem ID [1917]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{x+y}{x-y} = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 23

```
dsolve([diff(y(x),x)=(x+y(x))/(x-y(x)),y(1) = 0],y(x), singsol=all)
```

$$y(x) = \tan\left(\text{RootOf}\left(2\_Z - \ln\left(\sec\left(\_Z\right)^2\right) - 2\ln\left(x\right)\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 33

```
DSolve[{y'[x]==(x+y[x])/(x-y[x]),y[1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right) - \arctan\left(\frac{y(x)}{x}\right) = -\log(x), y(x)\right]$$

2.19 problem 19

Internal problem ID [1918]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y' - \frac{y}{x} - \tan\left(\frac{y}{x}\right) = 0}$$

With initial conditions

$$[y(6) = \pi]$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)=y(x)/x+tan(y(x)/x),y(6) = Pi],y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{x}{12}\right) x$$

✓ Solution by Mathematica

Time used: 4.616 (sec). Leaf size: 13

```
DSolve[{y'[x]==y[x]/x+Tan[y[x]/x],y[6]==Pi},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin\left(\frac{x}{12}\right)$$

2.20 problem 20

Internal problem ID [1919]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cla`

$$\boxed{\left(3yx-2x^2\right)y'-2y^2+yx=0}$$

With initial conditions

$$[y(1)=-1]$$

✓ Solution by Maple

Time used: 0.672 (sec). Leaf size: 114

```
dsolve([(3*x*y(x)-2*x^2)*diff(y(x),x)=2*y(x)^2-x*y(x),y(1) = -1],y(x), singsol=all)
```

$$y(x)$$
$$=\frac{i\left(\left(-27x^2+x^3+3\sqrt{3}\sqrt{-2x^5+27x^4}\right)^{\frac{2}{3}}-x^2\right)\sqrt{3}-\left(\left(-27x^2+x^3+3\sqrt{3}\sqrt{-2x^5+27x^4}\right)^{\frac{1}{3}}-x\right)^2}{6\left(-27x^2+x^3+3\sqrt{3}\sqrt{-2x^5+27x^4}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 60.351 (sec). Leaf size: 134

```
DSolve[{(3*x*y[x]-2*x^2)*y'[x]==2*y[x]^2-x*y[x],y[1]==-1},y[x],x,IncludeSingularSolutions ->
```

$$y(x)$$
$$\to\frac{\left(\sqrt[3]{3\sqrt{3}\sqrt{(27-2x)x^4}+(x-27)x^2}-x\right)\left(i\left(\sqrt{3}+i\right)\sqrt[3]{3\sqrt{3}\sqrt{(27-2x)x^4}+(x-27)x^2}+i\sqrt{3}x+x\right)}{6\sqrt[3]{3\sqrt{3}\sqrt{(27-2x)x^4}+(x-27)x^2}}$$

2.21 problem 21

Internal problem ID [1920]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y' - \frac{y}{x - k\sqrt{x^2 + y^2}} = 0}$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)=y(x)/(x-k*sqrt(x^2+y(x)^2)),y(x), singsol=all)
```

$$-c_1 + y(x)^{k-1}\sqrt{x^2 + y\left(x\right)^2} + y(x)^{k-1}\, x = 0$$

✓ Solution by Mathematica

Time used: 0.235 (sec). Leaf size: 59

```
DSolve[y'[x]==y[x]/(x-k*Sqrt[x^2+y[x]^2]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\left((k-1)\log\left(\sqrt{\frac{y(x)^2}{x^2}+1}-1\right)+(k+1)\log\left(\sqrt{\frac{y(x)^2}{x^2}+1}+1\right)\right)=\right.$$
$$\left.-k\log(x)+c_1, y(x)\right]$$

2.22 problem 22

Internal problem ID [1921]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y^2(yy' - x) + x^3 = 0}$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 50

```
dsolve(y(x)^2*(y(x)*diff(y(x),x)-x)+x^3=0,y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(2\_Z^2 + \sqrt{3}\,\tan\left(\text{RootOf}\left(\sqrt{3}\,\ln\left(\frac{3\tan(\_Z)^2 x^4}{4} + \frac{3x^4}{4}\right) + 4\sqrt{3}\,c_1 - 2\_Z\right)\right) - 1\right) x$$

✓ Solution by Mathematica

Time used: 0.117 (sec). Leaf size: 63

```
DSolve[y[x]^2*(y[x]*y'[x]-x)+x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\arctan\left(\frac{\frac{2y(x)^2}{x^2}-1}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{4}\log\left(\frac{y(x)^4}{x^4} - \frac{y(x)^2}{x^2} + 1\right) = -\log(x) + c_1, y(x)\right]$$

2.23 problem 23

Internal problem ID [1922]

Book: Differential Equations, Nelson, Folley, Coral, 3rd ed, 1964
Section: Exercis 6, page 25
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y' - \frac{y}{x} - \tanh\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple

Time used: 0.578 (sec). Leaf size: 113

```
dsolve(diff(y(x),x)=y(x)/x+tanh(y(x)/x),y(x), singsol=all)
```

$$y(x) = \operatorname{arctanh}\left(\frac{c_1 x^2 - \sqrt{c_1^2 x^4 - c_1 x^2}}{c_1 x^2 - 1 - \sqrt{c_1^2 x^4 - c_1 x^2}}\right) x$$

$$y(x) = \operatorname{arctanh}\left(\frac{c_1 x^2 + \sqrt{c_1^2 x^4 - c_1 x^2}}{c_1 x^2 - 1 + \sqrt{c_1^2 x^4 - c_1 x^2}}\right) x$$

✓ Solution by Mathematica

Time used: 2.01 (sec). Leaf size: 19

```
DSolve[y'[x]==y[x]/x+Tanh[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \operatorname{arcsinh}(e^{c_1} x)$$

$$y(x) \to 0$$