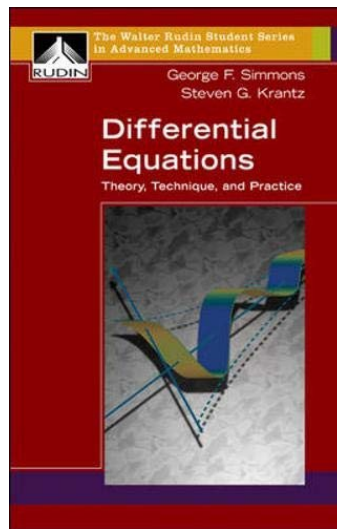


A Solution Manual For

**Differential Equations: Theory,  
Technique, and Practice by  
George Simmons, Steven Krantz.  
McGraw-Hill NY. 2007. 1st  
Edition.**



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## 1.1 problem 1(a)

Internal problem ID [5351]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS.  
Page 9

**Problem number:** 1(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - 2x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=2*x,y(x), singsol=all)
```

$$y(x) = x^2 + c_1$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 11

```
DSolve[y'[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2 + c_1$$

## 1.2 problem 1(b)

Internal problem ID [5352]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 1(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y'x - 2y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*diff(y(x),x)=2*y(x),y(x), singsol=all)
```

$$y(x) = c_1x^2$$

### ✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 16

```
DSolve[x*y'[x]==2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1x^2$$

$$y(x) \rightarrow 0$$

### 1.3 problem 1(c)

Internal problem ID [5353]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS.

Page 9

**Problem number:** 1(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$yy' - e^{2x} = 0$$

#### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(y(x)*diff(y(x),x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = \sqrt{e^{2x} + c_1}$$

$$y(x) = -\sqrt{e^{2x} + c_1}$$

#### ✓ Solution by Mathematica

Time used: 0.58 (sec). Leaf size: 39

```
DSolve[y[x]*y'[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt{e^{2x} + 2c_1}$$

$$y(x) \rightarrow \sqrt{e^{2x} + 2c_1}$$



## 1.4 problem 1(d)

Internal problem ID [5354]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 1(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - ky = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=k*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{kx}$$

### ✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 18

```
DSolve[y'[x]==k*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{kx}$$

$$y(x) \rightarrow 0$$

## 1.5 problem 1(e)

Internal problem ID [5355]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 1(e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 4y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(2x) + c_2 \cos(2x)$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos(2x) + c_2 \sin(2x)$$

## 1.6 problem 1(f)

Internal problem ID [5356]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 1(f).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x]`

$$y'' - 4y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-2x}c_1 + e^{2x}c_2$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x}(c_1 e^{4x} + c_2)$$

## 1.7 problem 1(h)

Internal problem ID [5357]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 1(h).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_rational]

$$y'x + y - y'\sqrt{1-x^2y} = 0$$

**X** Solution by Maple

```
dsolve(x*diff(y(x),x)+y(x)=diff(y(x),x)*sqrt(1-x^2*y(x)),y(x), singsol=all)
```

No solution found

**X** Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]+y[x]==y'[x]*Sqrt[1-x^2*y[x]],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 1.8 problem 1(i)

Internal problem ID [5358]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 1(i).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class D'], _rational, _Riccati]`

$$-y + y'x - y^2 - x^2 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x)=y(x)+x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \tan(x + c_1)x$$

### ✓ Solution by Mathematica

Time used: 0.174 (sec). Leaf size: 12

```
DSolve[x*y'[x]==y[x]+x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x \tan(x + c_1)$$

## 1.9 problem 1(j)

Internal problem ID [5359]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 1(j).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _dAlembert]`

$$y' - \frac{xy}{y^2 + x^2} = 0$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=(x*y(x))/(x^2+y(x)^2),y(x), singsol=all)
```

$$y(x) = \sqrt{\frac{1}{\text{LambertW}(c_1 x^2)}} x$$

### ✓ Solution by Mathematica

Time used: 7.147 (sec). Leaf size: 49

```
DSolve[y'[x]==(x*y[x])/(x^2+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x}{\sqrt{W(e^{-2c_1 x^2})}}$$

$$y(x) \rightarrow \frac{x}{\sqrt{W(e^{-2c_1 x^2})}}$$

$$y(x) \rightarrow 0$$

## 1.10 problem 1(k)

Internal problem ID [5360]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 1(k).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$2xyy' - y^2 - x^2 = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(2*x*y(x)*diff(y(x),x)=x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1x + x^2}$$

$$y(x) = -\sqrt{c_1x + x^2}$$

### ✓ Solution by Mathematica

Time used: 0.172 (sec). Leaf size: 38

```
DSolve[2*x*y[x]*y'[x]==x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt{x}\sqrt{x + c_1}$$

$$y(x) \rightarrow \sqrt{x}\sqrt{x + c_1}$$

## 1.11 problem 1(L)

Internal problem ID [5361]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS.

Page 9

**Problem number:** 1(L).

**ODE order:** 1.

**ODE degree:** 2.

CAS Maple gives this as type `[_homogeneous, 'class G'], _rational]`

$$y'x + y - x^4y'^2 = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 135

```
dsolve(y(x)+x*diff(y(x),x)=x^4*(diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = -\frac{1}{4x^2}$$

$$y(x) = \frac{-c_1^2 - c_1(2ix - c_1) - 2x^2}{2x^2c_1^2}$$

$$y(x) = \frac{-c_1^2 - c_1(-2ix - c_1) - 2x^2}{2x^2c_1^2}$$

$$y(x) = \frac{c_1(2ix + c_1) - 2x^2 - c_1^2}{2c_1^2x^2}$$

$$y(x) = \frac{c_1(-2ix + c_1) - 2x^2 - c_1^2}{2c_1^2x^2}$$



✓ Solution by Mathematica

Time used: 0.496 (sec). Leaf size: 123

```
DSolve[y[x]+x*y'[x]==x^4*(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[ -\frac{x\sqrt{4x^2y(x)+1}\operatorname{arctanh}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}} - \frac{1}{2}\log(y(x)) = c_1, y(x) \right]$$

$$\text{Solve} \left[ \frac{x\sqrt{4x^2y(x)+1}\operatorname{arctanh}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}} - \frac{1}{2}\log(y(x)) = c_1, y(x) \right]$$

$$y(x) \rightarrow 0$$

## 1.12 problem 1(m)

Internal problem ID [5362]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS.

Page 9

**Problem number:** 1(m).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cla`

$$y' - \frac{y^2}{xy - x^2} = 0$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=y(x)^2/(x*y(x)-x^2),y(x), singsol=all)
```

$$y(x) = e^{-\text{LambertW}\left(-\frac{e^{-c_1}}{x}\right) - c_1}$$

### ✓ Solution by Mathematica

Time used: 2.06 (sec). Leaf size: 25

```
DSolve[y'[x]==y[x]^2/(x*y[x]-x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -xW\left(-\frac{e^{-c_1}}{x}\right)$$

$$y(x) \rightarrow 0$$

### 1.13 problem 1(n)

Internal problem ID [5363]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 1(n).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_1st_order, _with_linear_symmetries]`

$$(y \cos(y) - \sin(y) + x)y' - y = 0$$

#### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve((y(x)*cos(y(x))-sin(y(x))+x)*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$x - c_1 y(x) - \sin(y(x)) = 0$$

#### ✓ Solution by Mathematica

Time used: 0.233 (sec). Leaf size: 14

```
DSolve[(y[x]*Cos[y[x]]-Sin[y[x]]+x)*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[x = \sin(y(x)) + c_1 y(x), y(x)]$$

## 1.14 problem 1(o)

Internal problem ID [5364]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 1(o).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$1 + y^2 + y^2 y' = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(1+y(x)^2+y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \tan(\text{RootOf}(-\_Z + x + c_1 + \tan(\_Z)))$$

### ✓ Solution by Mathematica

Time used: 0.175 (sec). Leaf size: 35

```
DSolve[1+y[x]^2+y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \text{InverseFunction}[\#1 - \arctan(\#1)\&][-x + c_1]$$

$$y(x) \rightarrow -i$$

$$y(x) \rightarrow i$$

## 1.15 problem 2(a)

Internal problem ID [5365]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 2(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - e^{3x} + x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=exp(3*x)-x,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{2} + \frac{e^{3x}}{3} + c_1$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 24

```
DSolve[y'[x]==Exp[3*x]-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x^2}{2} + \frac{e^{3x}}{3} + c_1$$

## 1.16 problem 2(b)

Internal problem ID [5366]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS.

Page 9

**Problem number:** 2(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - e^{x^2} x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=x*exp(x^2),y(x), singsol=all)
```

$$y(x) = \frac{e^{x^2}}{2} + c_1$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 17

```
DSolve[y'[x]==x*Exp[x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{x^2}}{2} + c_1$$

## 1.17 problem 2(c)

Internal problem ID [5367]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 2(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y'(1+x) - x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((1+x)*diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = x - \ln(x + 1) + c_1$$

### ✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 15

```
DSolve[(1+x)*y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x - \log(x + 1) + c_1$$

## 1.18 problem 2(d)

Internal problem ID [5368]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 2(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$(x^2 + 1) y' - x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((1+x^2)*diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = \frac{\ln(x^2 + 1)}{2} + c_1$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[(1+x^2)*y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} \log(x^2 + 1) + c_1$$



## 1.19 problem 2(e)

Internal problem ID [5369]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 2(e).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$(x^2 + 1) y' - \arctan(x) = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((1+x^2)*diff(y(x),x)=arctan(x),y(x), singsol=all)
```

$$y(x) = \frac{\arctan(x)^2}{2} + c_1$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 16

```
DSolve[(1+x^2)*y'[x]==ArcTan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\arctan(x)^2}{2} + c_1$$

## 1.20 problem 2(f)

Internal problem ID [5370]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 2(f).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y'x - 1 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(x*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = \ln(x) + c_1$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 10

```
DSolve[x*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \log(x) + c_1$$

## 1.21 problem 2(g)

Internal problem ID [5371]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 2(g).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - \arcsin(x) = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=arcsin(x),y(x), singsol=all)
```

$$y(x) = x \arcsin(x) + \sqrt{-x^2 + 1} + c_1$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 23

```
DSolve[y'[x]==ArcSin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x \arcsin(x) + \sqrt{1 - x^2} + c_1$$

## 1.22 problem 2(h)

Internal problem ID [5372]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 2(h).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' \sin(x) - 1 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(sin(x)*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = -\ln(\csc(x) + \cot(x)) + c_1$$

### ✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 24

```
DSolve[Sin[x]*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + c_1$$

## 1.23 problem 2(i)

Internal problem ID [5373]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 2(i).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$(x^3 + 1)y' - x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve((1+x^3)*diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = -\frac{\ln(x+1)}{3} + \frac{\ln(x^2-x+1)}{6} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3} + c_1$$

### ✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 47

```
DSolve[(1+x^3)*y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{6} \left( 2\sqrt{3} \arctan\left(\frac{2x-1}{\sqrt{3}}\right) - 2\log(x+1) + \log((x-1)x+1) + 6c_1 \right)$$

## 1.24 problem 2(j)

Internal problem ID [5374]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 2(j).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$(x^2 - 3x + 2)y' - x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((x^2-3*x+2)*diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = -\ln(x - 1) + 2\ln(x - 2) + c_1$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 24

```
DSolve[(x^2-3*x+2)*y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\log(1 - x) + 2\log(2 - x) + c_1$$

## 1.25 problem 3(a)

Internal problem ID [5375]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 3(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - x e^x = 0$$

With initial conditions

$$[y(1) = 3]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=x*exp(x),y(1) = 3],y(x), singsol=all)
```

$$y(x) = (x - 1)e^x + 3$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 14

```
DSolve[{y'[x]==x*Exp[x],{y[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x(x - 1) + 3$$

## 1.26 problem 3(b)

Internal problem ID [5376]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 3(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - 2 \cos(x) \sin(x) = 0$$

With initial conditions

$$[y(0) = 1]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=2*sin(x)*cos(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{\cos(2x)}{2} + \frac{3}{2}$$

### ✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 17

```
DSolve[{y'[x]==2*Sin[x]*Cos[x],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}(3 - \cos(2x))$$



## 1.27 problem 3(c)

Internal problem ID [5377]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 3(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - \ln(x) = 0$$

With initial conditions

$$[y(e) = 0]$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)=ln(x),y(exp(1)) = 0],y(x), singsol=all)
```

$$y(x) = x(\ln(x) - 1)$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 11

```
DSolve[{y'[x]==Log[x],{y[Exp[1]]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(\log(x) - 1)$$

## 1.28 problem 3(d)

Internal problem ID [5378]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 3(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$(x^2 - 1)y' - 1 = 0$$

With initial conditions

$$[y(2) = 0]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([(x^2-1)*diff(y(x),x)=1,y(2) = 0],y(x), singsol=all)
```

$$y(x) = -\operatorname{arctanh}(x) + \operatorname{arctanh}\left(\frac{1}{2}\right) - \frac{i\pi}{2}$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 27

```
DSolve[{(x^2-1)*y'[x]==1,{y[2]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}(\log(3 - 3x) - \log(x + 1) - i\pi)$$

## 1.29 problem 3(e)

Internal problem ID [5379]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 3(e).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$x(x^2 - 4)y' - 1 = 0$$

With initial conditions

$$[y(1) = 0]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve([x*(x^2-4)*diff(y(x),x)=1,y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\ln(x+2)}{8} + \frac{\ln(x-2)}{8} - \frac{\ln(x)}{4} - \frac{\ln(3)}{8} - \frac{i\pi}{8}$$

### ✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 26

```
DSolve[{x*(x^2-4)*y'[x]==1,{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{8} \left( \log \left( \frac{1}{3} (4 - x^2) \right) - 2 \log(x) \right)$$

### 1.30 problem 3(f)

Internal problem ID [5380]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 3(f).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$(1+x)(x^2+1)y' - 2x^2 - x = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve([(x+1)*(x^2+1)*diff(y(x),x)=2*x^2+x,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\ln(x+1)}{2} + \frac{3 \ln(x^2+1)}{4} - \frac{\arctan(x)}{2} + 1$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 29

```
DSolve[{(x+1)*(x^2+1)*y'[x]==2*x^2+x,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}(-2 \arctan(x) + 3 \log(x^2+1) + 2 \log(x+1) + 4)$$

### 1.31 problem 4

Internal problem ID [5381]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 4.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' - 2xy - 1 = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)=2*x*y(x)+1,y(x), singsol=all)
```

$$y(x) = \left( \frac{\sqrt{\pi} \operatorname{erf}(x)}{2} + c_1 \right) e^{x^2}$$

#### ✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 27

```
DSolve[y'[x]==2*x*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} e^{x^2} (\sqrt{\pi} \operatorname{erf}(x) + 2c_1)$$

## 1.32 problem 5

Internal problem ID [5382]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 5.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 5y' + 4y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^x c_1 + c_2 e^{4x}$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y''[x]-5*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x (c_2 e^{3x} + c_1)$$

### 1.33 problem 6

Internal problem ID [5383]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS.

Page 9

**Problem number:** 6.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, [_Abel, '2nd type', 'cla`

$$y' - \frac{2xy^2}{1 - x^2y} = 0$$

#### ✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=2*x*y(x)^2/(1-x^2*y(x)),y(x), singsol=all)
```

$$y(x) = e^{-\text{LambertW}(-x^2e^{-2c_1})-2c_1}$$

#### ✓ Solution by Mathematica

Time used: 3.458 (sec). Leaf size: 27

```
DSolve[y'[x]==2*x*y[x]^2/(1-x^2*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{W(-e^{-1+c_1}x^2)}{x^2}$$

$$y(x) \rightarrow 0$$

### 1.34 problem 7

Internal problem ID [5384]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.2 THE NATURE OF SOLUTIONS. Page 9

**Problem number:** 7.

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$2y''' + y'' - 5y' + 2y = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(2*diff(y(x),x$3)+diff(y(x),x$2)-5*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x}{2}} + c_2 e^{-2x} + c_3 e^x$$

#### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[2*y'''[x]+y''[x]-5*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{x/2} + c_2 e^{-2x} + c_3 e^x$$



## 2 Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12

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## 2.1 problem 1(a)

Internal problem ID [5385]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12

**Problem number:** 1(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y'x^5 + y^5 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 67

```
dsolve(x^5*diff(y(x),x)+y(x)^5=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{(c_1x^4 - 1)^{\frac{1}{4}}}$$

$$y(x) = -\frac{x}{(c_1x^4 - 1)^{\frac{1}{4}}}$$

$$y(x) = \frac{x}{\sqrt{-\sqrt{c_1x^4 - 1}}}$$

$$y(x) = -\frac{x}{\sqrt{-\sqrt{c_1x^4 - 1}}}$$

✓ Solution by Mathematica

Time used: 0.432 (sec). Leaf size: 135

```
DSolve[x^5*y'[x]+y[x]^5==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x}{\sqrt[4]{-1-4c_1x^4}}$$

$$y(x) \rightarrow -\frac{ix}{\sqrt[4]{-1-4c_1x^4}}$$

$$y(x) \rightarrow \frac{ix}{\sqrt[4]{-1-4c_1x^4}}$$

$$y(x) \rightarrow \frac{x}{\sqrt[4]{-1-4c_1x^4}}$$

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow -\sqrt[4]{-1}x$$

$$y(x) \rightarrow (-1)^{3/4}x$$

$$y(x) \rightarrow -(-1)^{3/4}x$$

$$y(x) \rightarrow \sqrt[4]{-1}x$$

## 2.2 problem 1(b)

Internal problem ID [5386]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12

**Problem number:** 1(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y' - 4xy = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=4*x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{2x^2}$$

### ✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 20

```
DSolve[y'[x]==4*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{2x^2}$$

$$y(x) \rightarrow 0$$

## 2.3 problem 1(c)

Internal problem ID [5387]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12

**Problem number:** 1(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_separable]`

$$y' + y \tan(x) = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)+y(x)*tan(x)=0,y(x), singsol=all)
```

$$y(x) = \cos(x) c_1$$

### ✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 15

```
DSolve[y'[x]+y[x]*Tan[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos(x)$$

$$y(x) \rightarrow 0$$

## 2.4 problem 1(d)

Internal problem ID [5388]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12

**Problem number:** 1(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$(x^2 + 1)y' + 1 + y^2 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve((1+x^2)*diff(y(x),x)+1+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = -\tan(\arctan(x) + c_1)$$

### ✓ Solution by Mathematica

Time used: 0.214 (sec). Leaf size: 29

```
DSolve[(1+x^2)*y'[x]+1+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\tan(\arctan(x) - c_1)$$

$$y(x) \rightarrow -i$$

$$y(x) \rightarrow i$$

## 2.5 problem 1(e)

Internal problem ID [5389]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12

**Problem number:** 1(e).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y \ln(y) - y'x = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 8

```
dsolve(y(x)*ln(y(x))-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^{c_1 x}$$

### ✓ Solution by Mathematica

Time used: 0.172 (sec). Leaf size: 18

```
DSolve[y[x]*Log[y[x]]-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{e^{c_1 x}}$$

$$y(x) \rightarrow 1$$

## 2.6 problem 1(f)

Internal problem ID [5390]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12

**Problem number:** 1(f).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y'x - (-4x^2 + 1) \tan(y) = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)=(1-4*x^2)*tan(y(x)),y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{x e^{-2x^2}}{c_1}\right)$$

### ✓ Solution by Mathematica

Time used: 42.771 (sec). Leaf size: 23

```
DSolve[x*y'[x]==(1-4*x^2)*Tan[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \arcsin\left(xe^{-2x^2+c_1}\right)$$

$$y(x) \rightarrow 0$$



## 2.7 problem 1(g)

Internal problem ID [5391]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS.

Page 12

**Problem number:** 1(g).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y' \sin(y) - x^2 = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)*sin(y(x))=x^2,y(x), singsol=all)
```

$$y(x) = \pi - \arccos\left(\frac{x^3}{3} + c_1\right)$$

### ✓ Solution by Mathematica

Time used: 0.425 (sec). Leaf size: 37

```
DSolve[y'[x]*Sin[y[x]]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\arccos\left(-\frac{x^3}{3} - c_1\right)$$

$$y(x) \rightarrow \arccos\left(-\frac{x^3}{3} - c_1\right)$$

## 2.8 problem 1(h)

Internal problem ID [5392]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12

**Problem number:** 1(h).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y' - y \tan(x) = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)-y(x)*tan(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\cos(x)}$$

### ✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 15

```
DSolve[y'[x]-y[x]*Tan[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \sec(x)$$

$$y(x) \rightarrow 0$$

## 2.9 problem 1(i)

Internal problem ID [5393]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12

**Problem number:** 1(i).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$xyy' - y + 1 = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*y(x)*diff(y(x),x)=y(x)-1,y(x), singsol=all)
```

$$y(x) = \text{LambertW}(xc_1e^{-1}) + 1$$

### ✓ Solution by Mathematica

Time used: 1.605 (sec). Leaf size: 21

```
DSolve[x*y[x]*y'[x]==y[x]-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 1 + W(e^{-1+c_1x})$$

$$y(x) \rightarrow 1$$

## 2.10 problem 1(j)

Internal problem ID [5394]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS.

Page 12

**Problem number:** 1(j).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$xy^2 - x^2y' = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*y(x)^2-diff(y(x),x)*x^2=0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{\ln(x) - c_1}$$

### ✓ Solution by Mathematica

Time used: 0.1 (sec). Leaf size: 19

```
DSolve[x*y[x]^2-y'[x]*x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{\log(x) + c_1}$$

$$y(x) \rightarrow 0$$

## 2.11 problem 2(a)

Internal problem ID [5395]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12

**Problem number:** 2(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$yy' - 1 - x = 0$$

With initial conditions

$$[y(1) = 3]$$

### ✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)*y(x)=x+1,y(1) = 3],y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 + 2x + 6}$$

### ✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 16

```
DSolve[{y'[x]*y[x]==x+1,{y[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt{x(x+2)+6}$$

## 2.12 problem 2(b)

Internal problem ID [5396]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12

**Problem number:** 2(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$x^2 y' - y = 0$$

With initial conditions

$$[y(1) = 0]$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)*x^2=y(x),y(1) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

### ✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x]*x^2==y[x],{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0$$

## 2.13 problem 2(c)

Internal problem ID [5397]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12

**Problem number:** 2(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$\frac{y'}{x^2 + 1} - \frac{x}{y} = 0$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 20

```
dsolve([diff(y(x),x)/(1+x^2)=x/y(x),y(1) = 3],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{2x^4 + 4x^2 + 30}}{2}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 25

```
DSolve[{y'[x]/(1+x^2)==x/y[x],{y[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\sqrt{x^4 + 2x^2 + 15}}{\sqrt{2}}$$

## 2.14 problem 2(d)

Internal problem ID [5398]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12

**Problem number:** 2(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y^2 y' - 2 - x = 0$$

With initial conditions

$$[y(0) = 4]$$

### ✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 18

```
dsolve([y(x)^2*diff(y(x),x)=x+2,y(0) = 4],y(x), singsol=all)
```

$$y(x) = \frac{(12x^2 + 48x + 512)^{\frac{1}{3}}}{2}$$

### ✓ Solution by Mathematica

Time used: 0.192 (sec). Leaf size: 19

```
DSolve[{y[x]^2*y'[x]==x+2,{y[0]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt[3]{\frac{3}{2}x(x+4) + 64}$$



## 2.15 problem 2(e)

Internal problem ID [5399]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS.

Page 12

**Problem number:** 2(e).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y' - x^2y^2 = 0$$

With initial conditions

$$[y(-1) = 2]$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)=x^2*y(x)^2,y(-1) = 2],y(x), singsol=all)
```

$$y(x) = -\frac{6}{2x^3 - 1}$$

### ✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 16

```
DSolve[{y'[x]==x^2*y[x]^2,{y[-1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{6}{1 - 2x^3}$$

## 2.16 problem 2(e)

Internal problem ID [5400]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12

**Problem number:** 2(e).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y'(1 + y) + x^2 - 1 = 0$$

With initial conditions

$$[y(-1) = -2]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 20

```
dsolve([diff(y(x),x)*(1+y(x))=1-x^2,y(-1) = -2],y(x), singsol=all)
```

$$y(x) = -1 - \frac{\sqrt{-6x^3 + 18x + 21}}{3}$$

✓ Solution by Mathematica

Time used: 0.134 (sec). Leaf size: 28

```
DSolve[{y'[x]*(1+y[x])==1-x^2,{y[-1]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt{-2x^3 + 6x + 7}}{\sqrt{3}} - 1$$

## 2.17 problem 3

Internal problem ID [5401]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS. Page 12

**Problem number:** 3.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\frac{y''}{y'} - x^2 = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)/diff(y(x),x)=x^2,y(x), singsol=all)
```

$$y(x) = c_1 + \left( 2\sqrt{3}\pi - 3\Gamma\left(\frac{1}{3}, -\frac{x^3}{3}\right) \Gamma\left(\frac{2}{3}\right) \right) c_2$$

### ✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 26

```
DSolve[y'[x]/y'[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 - \frac{1}{3}c_1 x \operatorname{ExpIntegralE}\left(\frac{2}{3}, -\frac{x^3}{3}\right)$$

## 2.18 problem 4

Internal problem ID [5402]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.3 SEPARABLE EQUATIONS.

Page 12

**Problem number:** 4.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_y], [_2nd_order, _exact, _nonlinear], [`

$$y''y' - x(1+x) = 0$$

### ✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$2)*diff(y(x),x)=x*(1+x),y(x), singsol=all)
```

$$y(x) = \int -\frac{\sqrt{6x^3 + 9x^2 + 9c_1}}{3} dx + c_2$$

$$y(x) = \int \frac{\sqrt{6x^3 + 9x^2 + 9c_1}}{3} dx + c_2$$

### ✓ Solution by Mathematica

Time used: 61.887 (sec). Leaf size: 12885

```
DSolve[y' '[x]*y' [x]==x*(1+x),y[x],x,IncludeSingularSolutions -> True]
```

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### 3 Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

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### 3.1 problem 1(a)

Internal problem ID [5403]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 1(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_separable]`

$$y' - xy = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x^2}{2}} c_1$$

#### ✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 22

```
DSolve[y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{\frac{x^2}{2}}$$

$$y(x) \rightarrow 0$$

### 3.2 problem 1(b)

Internal problem ID [5404]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 1(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y' + xy - x = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+x*y(x)=x,y(x), singsol=all)
```

$$y(x) = 1 + e^{-\frac{x^2}{2}} c_1$$

#### ✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 24

```
DSolve[y'[x]+x*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 1 + c_1 e^{-\frac{x^2}{2}}$$

$$y(x) \rightarrow 1$$

### 3.3 problem 1(c)

Internal problem ID [5405]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 1(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$y' + y - \frac{1}{e^{2x} + 1} = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+y(x)=1/(1+exp(2*x)),y(x), singsol=all)
```

$$y(x) = (\arctan(e^x) + c_1) e^{-x}$$

#### ✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]==1/(1+Exp[2*x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(\arctan(e^x) + c_1)$$



### 3.4 problem 1(d)

Internal problem ID [5406]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 1(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y' + y - 2e^{-x}x - x^2 = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)+y(x)=2*x*exp(-x)+x^2,y(x), singsol=all)
```

$$y(x) = x^2 - 2x + e^{-x}x^2 + 2 + e^{-x}c_1$$

#### ✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 24

```
DSolve[y'[x]+y[x]==2*x*Exp[-x]+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(x^2 + c_1) + (x - 2)x + 2$$

### 3.5 problem 1(e)

Internal problem ID [5407]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 1(e).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$2y - x^3 - y'x = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(2*y(x)-x^3=x*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = (-x + c_1)x^2$$

#### ✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 15

```
DSolve[2*y[x]-x^3==x*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2(-x + c_1)$$

### 3.6 problem 1(f)

Internal problem ID [5408]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 1(f).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y' + 2xy = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+2*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x^2}$$

#### ✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 20

```
DSolve[y'[x]+2*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-x^2}$$

$$y(x) \rightarrow 0$$

### 3.7 problem 1(g)

Internal problem ID [5409]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 1(g).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$y'x - 3y - x^4 = 0$$

#### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x)-3*y(x)=x^4,y(x), singsol=all)
```

$$y(x) = (x + c_1) x^3$$

#### ✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 13

```
DSolve[x*y'[x]-3*y[x]==x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^3(x + c_1)$$

### 3.8 problem 1(h)

Internal problem ID [5410]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 1(h).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$(x^2 + 1)y' + 2xy - \cot(x) = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((1+x^2)*diff(y(x),x)+2*x*y(x)=cot(x),y(x), singsol=all)
```

$$y(x) = \frac{\ln(\sin(x)) + c_1}{x^2 + 1}$$

#### ✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 22

```
DSolve[(1+x^2)*y'[x]+2*x*y[x]==Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\log(\tan(x)) + \log(\cos(x)) + c_1}{x^2 + 1}$$

### 3.9 problem 1(i)

Internal problem ID [5411]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 1(i).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' + y \cot(x) - 2x \csc(x) = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+y(x)*cot(x)=2*x*csc(x),y(x), singsol=all)
```

$$y(x) = \frac{x^2 + c_1}{\sin(x)}$$

#### ✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 14

```
DSolve[y'[x]+y[x]*Cot[x]==2*x*Csc[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (x^2 + c_1) \csc(x)$$

### 3.10 problem 1(j)

Internal problem ID [5412]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 1(j).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y - x + xy \cot(x) + y'x = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(y(x)-x+x*y(x)*cot(x)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sin(x) - \cos(x)x + c_1}{\sin(x)x}$$

#### ✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 21

```
DSolve[y[x]-x+x*y[x]*Cot[x]+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{-x \cot(x) + c_1 \csc(x) + 1}{x}$$

### 3.11 problem 2(a)

Internal problem ID [5413]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 2(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y' - xy = 0$$

With initial conditions

$$[y(1) = 3]$$

#### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)-x*y(x)=0,y(1) = 3],y(x), singsol=all)
```

$$y(x) = 3e^{\frac{(x-1)(x+1)}{2}}$$

#### ✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 18

```
DSolve[{y'[x]-x*y[x]==0,{y[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3e^{\frac{1}{2}(x^2-1)}$$



### 3.12 problem 2(b)

Internal problem ID [5414]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 2(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$y' - 2xy - 6e^{x^2}x = 0$$

With initial conditions

$$[y(1) = 1]$$

#### ✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 18

```
dsolve([diff(y(x),x)-2*x*y(x)=6*x*exp(x^2),y(1) = 1],y(x), singsol=all)
```

$$y(x) = (3x^2 - 3 + e^{-1}) e^{x^2}$$

#### ✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 23

```
DSolve[{y'[x]-2*x*y[x]==6*x*Exp[x^2],{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{x^2-1}(3e(x^2 - 1) + 1)$$

### 3.13 problem 2(c)

Internal problem ID [5415]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 2(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$x \ln(x) y' + y - 3x^3 = 0$$

With initial conditions

$$[y(1) = 0]$$

**X** Solution by Maple

```
dsolve([(x*ln(x))*diff(y(x),x)+y(x)=3*x^3,y(1) = 0],y(x), singsol=all)
```

No solution found

**X** Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(x*Log[x])*y'[x]+y[x]==3*x^3,{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

Not solved

### 3.14 problem 2(d)

Internal problem ID [5416]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 2(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' - \frac{y}{x} - x^2 = 0$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)-y(x)/x=x^2,y(1) = 3],y(x), singsol=all)
```

$$y(x) = \frac{(x^2 + 5)x}{2}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 15

```
DSolve[{y'[x]-y[x]/x==x^2,{y[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}x(x^2 + 5)$$

### 3.15 problem 2(e)

Internal problem ID [5417]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 2(e).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$4y + y' - e^{-x} = 0$$

With initial conditions

$$[y(0) = 0]$$

#### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(x),x)+4*y(x)=exp(-x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(e^{3x} - 1)e^{-4x}}{3}$$

#### ✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 21

```
DSolve[{y'[x]+4*y[x]==Exp[-x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{3}e^{-4x}(e^{3x} - 1)$$

### 3.16 problem 2(f)

Internal problem ID [5418]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 2(f).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$x^2y' + xy - 2x = 0$$

With initial conditions

$$[y(1) = 1]$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x)+x*y(x)=2*x,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{2x - 1}{x}$$

#### ✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 12

```
DSolve[{x^2*y'[x]+x*y[x]==2*x,{y[1]==1}],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2 - \frac{1}{x}$$

### 3.17 problem 3(a)

Internal problem ID [5419]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 3(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$y'x + y - x^4y^3 = 0$$

#### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x)+y(x)=x^4*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-x^2 + c_1} x}$$

$$y(x) = -\frac{1}{\sqrt{-x^2 + c_1} x}$$

#### ✓ Solution by Mathematica

Time used: 0.351 (sec). Leaf size: 48

```
DSolve[x*y'[x]+y[x]==x^4*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{\sqrt{-x^4 + c_1 x^2}}$$

$$y(x) \rightarrow \frac{1}{\sqrt{-x^4 + c_1 x^2}}$$

$$y(x) \rightarrow 0$$

### 3.18 problem 3(b)

Internal problem ID [5420]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 3(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_Bernoulli]

$$xy^2y' + y^3 - x \cos(x) = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 175

```
dsolve(x*y(x)^2*diff(y(x),x)+y(x)^3=x*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{(3 \sin(x) x^3 + 9x^2 \cos(x) - 18 \cos(x) - 18 \sin(x) x + c_1)^{\frac{1}{3}}}{x}$$

$$y(x) = \frac{-\frac{(3 \sin(x) x^3 + 9x^2 \cos(x) - 18 \cos(x) - 18 \sin(x) x + c_1)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3} (3 \sin(x) x^3 + 9x^2 \cos(x) - 18 \cos(x) - 18 \sin(x) x + c_1)^{\frac{1}{3}}}{2}}{x}$$

$$y(x) = \frac{-\frac{(3 \sin(x) x^3 + 9x^2 \cos(x) - 18 \cos(x) - 18 \sin(x) x + c_1)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3} (3 \sin(x) x^3 + 9x^2 \cos(x) - 18 \cos(x) - 18 \sin(x) x + c_1)^{\frac{1}{3}}}{2}}{x}$$

✓ Solution by Mathematica

Time used: 0.462 (sec). Leaf size: 114

`DSolve[x*y[x]^2*y'[x]+y[x]^3==x*Cos[x],y[x],x,IncludeSingularSolutions -> True]`

$$y(x) \rightarrow \frac{\sqrt[3]{3x(x^2 - 6)\sin(x) + 9(x^2 - 2)\cos(x) + c_1}}{x}$$

$$y(x) \rightarrow -\frac{\sqrt[3]{-1}\sqrt[3]{3x(x^2 - 6)\sin(x) + 9(x^2 - 2)\cos(x) + c_1}}{x}$$

$$y(x) \rightarrow \frac{(-1)^{2/3}\sqrt[3]{3x(x^2 - 6)\sin(x) + 9(x^2 - 2)\cos(x) + c_1}}{x}$$



### 3.19 problem 3(c)

Internal problem ID [5421]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 3(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$y'x + y - xy^2 = 0$$

#### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)+y(x)=x*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{1}{(\ln(x) - c_1)x}$$

#### ✓ Solution by Mathematica

Time used: 0.122 (sec). Leaf size: 22

```
DSolve[x*y'[x]+y[x]==x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{-x \log(x) + c_1 x}$$

$$y(x) \rightarrow 0$$

### 3.20 problem 3(d)

Internal problem ID [5422]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 3(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y' + xy - y^4x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 88

```
dsolve(diff(y(x),x)+x*y(x)=x*y(x)^4,y(x), singsol=all)
```

$$y(x) = \frac{1}{\left(e^{\frac{3x^2}{2}}c_1 + 1\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{1}{2\left(e^{\frac{3x^2}{2}}c_1 + 1\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}}{2\left(e^{\frac{3x^2}{2}}c_1 + 1\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{1}{2\left(e^{\frac{3x^2}{2}}c_1 + 1\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}}{2\left(e^{\frac{3x^2}{2}}c_1 + 1\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 1.926 (sec). Leaf size: 116

```
DSolve[y'[x]+x*y[x]==x*y[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{\sqrt[3]{1 + e^{\frac{3x^2}{2} + 3c_1}}}$$

$$y(x) \rightarrow -\frac{\sqrt[3]{-1}}{\sqrt[3]{1 + e^{\frac{3x^2}{2} + 3c_1}}}$$

$$y(x) \rightarrow \frac{(-1)^{2/3}}{\sqrt[3]{1 + e^{\frac{3x^2}{2} + 3c_1}}}$$

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow 1$$

$$y(x) \rightarrow -\sqrt[3]{-1}$$

$$y(x) \rightarrow (-1)^{2/3}$$

### 3.21 problem 4(a)

Internal problem ID [5423]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 4(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_exact, [\_1st\_order, ‘\_with\_symmetry\_[F(x)\*G(y),0]’]]

$$(e^y - 2xy)y' - y^2 = 0$$

#### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve((exp(y(x))-2*x*y(x))*diff(y(x),x)=y(x)^2,y(x), singsol=all)
```

$$x - \frac{e^{y(x)} + c_1}{y(x)^2} = 0$$

#### ✓ Solution by Mathematica

Time used: 0.223 (sec). Leaf size: 22

```
DSolve[(Exp[y[x]]-2*x*y[x])*y'[x]==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[ x = \frac{e^{y(x)}}{y(x)^2} + \frac{c_1}{y(x)^2}, y(x) \right]$$

### 3.22 problem 4(b)

Internal problem ID [5424]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 4(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_1st_order, _with_linear_symmetries]`

$$y - y'x - y'y^2e^y = 0$$

#### ✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 14

```
dsolve(y(x)-x*diff(y(x),x)=diff(y(x),x)*y(x)^2*exp(y(x)),y(x), singsol=all)
```

$$x - (e^{y(x)} + c_1) y(x) = 0$$

#### ✓ Solution by Mathematica

Time used: 0.197 (sec). Leaf size: 18

```
DSolve[y[x]-x*y'[x]==y'[x]*y[x]^2*Exp[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[x = e^{y(x)}y(x) + c_1y(x), y(x)]$$

### 3.23 problem 4(c)

Internal problem ID [5425]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 4(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_1st_order, '_with_symmetry_[F(x)*G(y),0]']`, `[_Abel, '2nd typ`

$$y'x + 2 - x^3(y - 1)y' = 0$$

#### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve(x*diff(y(x),x)+2=x^3*(y(x)-1)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -\frac{\text{LambertW}\left(c_1 e^{\frac{1}{x^2}}\right) x^2 - 1}{x^2}$$

#### ✓ Solution by Mathematica

Time used: 0.366 (sec). Leaf size: 33

```
DSolve[x*y'[x]+2==x^3*(y[x]-1)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{x^2} - W\left(e^{\frac{1}{x^2} + \frac{1}{2}(-2 - 9\sqrt[3]{-2}c_1)}\right)$$

### 3.24 problem 6

Internal problem ID [5426]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 6.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y'x - 2x^2y - \ln(x)y = 0$$

#### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)=2*x^2*y(x)+y(x)*ln(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{\ln(x)^2}{2} + x^2}$$

#### ✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 27

```
DSolve[x*y'[x]==2*x^2*y[x]+y[x]*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{x^2 + \frac{\log^2(x)}{2}}$$

$$y(x) \rightarrow 0$$

### 3.25 problem 7

Internal problem ID [5427]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.4 First Order Linear Equations. Page 15

**Problem number:** 7.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' \sin(2x) - 2y - 2 \cos(x) = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)*sin(2*x)=2*y(x)+2*cos(x),y(x), singsol=all)
```

$$y(x) = -\left(-\frac{1}{\sin(x)} + c_1\right) (-\csc(2x) + \cot(2x))$$

#### ✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 15

```
DSolve[y'[x]*Sin[2*x]==2*y[x]+2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sec(x)(-1 + c_1 \sin(x))$$



## 4 Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

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## 4.1 problem 1

Internal problem ID [5428]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 1.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _exact, _rational, [_Abel, '2nd typ`

$$\left(x + \frac{2}{y}\right) y' + y = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve((x+2/y(x))*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-\text{LambertW}\left(\frac{x e^{\frac{c_1}{2}}}{2}\right) + \frac{c_1}{2}}$$

### ✓ Solution by Mathematica

Time used: 9.507 (sec). Leaf size: 58

```
DSolve[(x+2/y[x])*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2W\left(-\frac{1}{2}\sqrt{e^{c_1}x^2}\right)}{x}$$

$$y(x) \rightarrow \frac{2W\left(\frac{1}{2}\sqrt{e^{c_1}x^2}\right)}{x}$$

$$y(x) \rightarrow 0$$

## 4.2 problem 2

Internal problem ID [5429]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 2.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type ['x=G(y,y)']

$$\sin(x) \tan(y) + 1 + \cos(x) \sec(x)^2 y y' = 0$$

**X** Solution by Maple

```
dsolve((sin(x)*tan(y(x))+1)+(cos(x)*sec(x)^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

**X** Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(Sin[x]*Tan[y[x]]+1)+(Cos[x]*Sec[x]^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -
```

Not solved

### 4.3 problem 3

Internal problem ID [5430]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 3.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [exact, rational]

$$y - x^3 + (x + y^3) y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve((y(x)-x^3)+(x+y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$-\frac{x^4}{4} + y(x)x + \frac{y(x)^4}{4} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.161 (sec). Leaf size: 1126

`DSolve[(y[x]-x^3)+(x+y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]`

$y(x) \rightarrow$

$$\sqrt{\sqrt[3]{9x^2 + \sqrt{3}\sqrt{27x^4 + (x^4 + 4c_1)^3}} - \frac{x^4 + 4c_1}{\sqrt[3]{3x^2 + \sqrt{9x^4 + \frac{1}{3}(x^4 + 4c_1)^3}}}} + \sqrt{\sqrt[3]{9x^2 + \sqrt{3}\sqrt{27x^4 + (x^4 + 4c_1)^3}}}$$

$y(x)$

$$\rightarrow \sqrt{\sqrt[3]{9x^2 + \sqrt{3}\sqrt{27x^4 + (x^4 + 4c_1)^3}} - \frac{6\sqrt{2}x}{\sqrt[3]{3x^2 + \sqrt{9x^4 + \frac{1}{3}(x^4 + 4c_1)^3}}}} - \sqrt[3]{9x^2 + \sqrt{3}\sqrt{27x^4 + (x^4 + 4c_1)^3}}$$

$y(x)$

$$\rightarrow \sqrt{\sqrt[3]{9x^2 + \sqrt{3}\sqrt{27x^4 + (x^4 + 4c_1)^3}} - \frac{x^4 + 4c_1}{\sqrt[3]{3x^2 + \sqrt{9x^4 + \frac{1}{3}(x^4 + 4c_1)^3}}}} - \sqrt{\sqrt[3]{9x^2 + \sqrt{3}\sqrt{27x^4 + (x^4 + 4c_1)^3}}}$$

$y(x)$

$$\rightarrow \sqrt{\sqrt[3]{9x^2 + \sqrt{3}\sqrt{27x^4 + (x^4 + 4c_1)^3}} - \frac{x^4 + 4c_1}{\sqrt[3]{3x^2 + \sqrt{9x^4 + \frac{1}{3}(x^4 + 4c_1)^3}}}} + \sqrt{\sqrt[3]{9x^2 + \sqrt{3}\sqrt{27x^4 + (x^4 + 4c_1)^3}}}$$

## 4.4 problem 4

Internal problem ID [5431]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 4.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$2y^2 - 4x + 5 - (4 - 2y + 4xy)y' = 0$$

### ✗ Solution by Maple

```
dsolve((2*y(x)^2-4*x+5)=(4-2*y(x)+4*x*y(x))*diff(y(x),x),y(x), singsol=all)
```

No solution found

### ✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(2*y[x]^2-4*x+5)==(4-2*y[x]+4*x*y[x])*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 4.5 problem 5

Internal problem ID [5432]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 5.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_separable]`

$$y + y \cos(xy) + (x + x \cos(xy)) y' = 0$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 17

```
dsolve((y(x)+y(x)*cos(x*y(x)))+(x+x*cos(x*y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\pi}{x}$$

$$y(x) = \frac{C_1}{x}$$

### ✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 49

```
DSolve[(y[x]+y[x]*Cos[x*y[x]])+(x+x*Cos[x*y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow -\frac{\pi}{x}$$

$$y(x) \rightarrow \frac{\pi}{x}$$

$$y(x) \rightarrow \frac{C_1}{x}$$

$$y(x) \rightarrow -\frac{\pi}{x}$$

$$y(x) \rightarrow \frac{\pi}{x}$$

## 4.6 problem 6

Internal problem ID [5433]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 6.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$\cos(x) \cos(y)^2 + 2 \sin(x) \sin(y) \cos(y) y' = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 31

```
dsolve((cos(x)*cos(y(x))^2)+(2*sin(x)*sin(y(x))*cos(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\pi}{2}$$

$$y(x) = \arccos\left(\sqrt{c_1 \sin(x)}\right)$$

$$y(x) = \pi - \arccos\left(\sqrt{c_1 \sin(x)}\right)$$



✓ Solution by Mathematica

Time used: 6.156 (sec). Leaf size: 85

```
DSolve[(Cos[x]*Cos[y[x]]^2)+(2*Sin[x]*Sin[y[x]]*Cos[y[x]])*y'[x]==0,y[x],x,IncludeSingularSol
```

$$y(x) \rightarrow -\frac{\pi}{2}$$

$$y(x) \rightarrow \frac{\pi}{2}$$

$$y(x) \rightarrow -\arccos\left(-\frac{1}{4}c_1\sqrt{\cos(x)}\sqrt{\tan(x)}\right)$$

$$y(x) \rightarrow \arccos\left(-\frac{1}{4}c_1\sqrt{\cos(x)}\sqrt{\tan(x)}\right)$$

$$y(x) \rightarrow -\frac{\pi}{2}$$

$$y(x) \rightarrow \frac{\pi}{2}$$

## 4.7 problem 7

Internal problem ID [5434]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 7.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_exact]`

$$(\sin(x) \sin(y) - x e^y) y' - e^y - \cos(x) \cos(y) = 0$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 16

```
dsolve((sin(x)*sin(y(x))-x*exp(y(x)))*diff(y(x),x)=exp(y(x))+cos(x)*cos(y(x)),y(x), singsol=a
```

$$c_1 + \sin(x) \cos(y(x)) + x e^{y(x)} = 0$$

### ✓ Solution by Mathematica

Time used: 0.598 (sec). Leaf size: 21

```
DSolve[(Sin[x]*Sin[y[x]]-x*Exp[y[x]])*y'[x]==Exp[y[x]]+Cos[x]*Cos[y[x]],y[x],x,IncludeSingular
```

$$\text{Solve}[2(xe^{y(x)} + \sin(x) \cos(y(x))) = c_1, y(x)]$$

## 4.8 problem 8

Internal problem ID [5435]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 8.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$-\frac{\sin\left(\frac{x}{y}\right)}{y} + \frac{x \sin\left(\frac{x}{y}\right) y'}{y^2} = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve(-1/y(x)*sin(x/y(x))+(x/y(x)^2*sin(x/y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\pi - c_1}$$

### ✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 19

```
DSolve[-1/y[x]*Sin[x/y[x]]+(x/y[x]^2*Sin[x/y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow c_1 x$$

$$y(x) \rightarrow \text{ComplexInfinity}$$

$$y(x) \rightarrow \text{ComplexInfinity}$$

## 4.9 problem 9

Internal problem ID [5436]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 9.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$1 + y + (1 - x)y' = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve((1+y(x))+(1-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -1 + c_1(x - 1)$$

### ✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 18

```
DSolve[(1+y[x])+(1-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -1 + c_1(x - 1)$$

$$y(x) \rightarrow -1$$

## 4.10 problem 10

Internal problem ID [5437]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 10.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_exact, [\_1st\_order, ‘\_with\_symmetry\_[F(x),G(x)\*y+H(x)]’]]

$$2y^3x + y \cos(x) + (3x^2y^2 + \sin(x))y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 375

```
dsolve((2*x*y(x)^3+y(x)*cos(x))+(3*x^2*y(x)^2+sin(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(12\sqrt{3}\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}{6x} - \frac{2\sin(x)}{x\left(12\sqrt{3}\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(12\sqrt{3}\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}{12x\sin(x)} + \frac{1}{x\left(12\sqrt{3}\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}{6x} + \frac{2\sin(x)}{x\left(12\sqrt{3}\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(12\sqrt{3}\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}{12x\sin(x)} + \frac{1}{x\left(12\sqrt{3}\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}{6x} + \frac{2\sin(x)}{x\left(12\sqrt{3}\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 26.727 (sec). Leaf size: 339

`DSolve[(2*x*y[x]^3+y[x]*Cos[x])+(3*x^2*y[x]^2+Sin[x])*y'[x]==0,y[x],x,IncludeSingularSolution`

$$y(x) \rightarrow \frac{\sqrt[3]{9c_1x^4 + \sqrt{12x^6 \sin^3(x) + 81c_1^2x^8}}}{\sqrt[3]{2}3^{2/3}x^2} - \frac{\sqrt[3]{\frac{2}{3}} \sin(x)}{\sqrt[3]{9c_1x^4 + \sqrt{12x^6 \sin^3(x) + 81c_1^2x^8}}}$$

$$y(x) \rightarrow \frac{(1 + i\sqrt{3}) \sin(x)}{2^{2/3} \sqrt[3]{27c_1x^4 + 3\sqrt{12x^6 \sin^3(x) + 81c_1^2x^8}}} - \frac{(1 - i\sqrt{3}) \sqrt[3]{27c_1x^4 + \sqrt{108x^6 \sin^3(x) + 729c_1^2x^8}}}{6\sqrt[3]{2}x^2}$$

$$y(x) \rightarrow \frac{(1 - i\sqrt{3}) \sin(x)}{2^{2/3} \sqrt[3]{27c_1x^4 + 3\sqrt{12x^6 \sin^3(x) + 81c_1^2x^8}}} - \frac{(1 + i\sqrt{3}) \sqrt[3]{27c_1x^4 + \sqrt{108x^6 \sin^3(x) + 729c_1^2x^8}}}{6\sqrt[3]{2}x^2}$$

## 4.11 problem 11

Internal problem ID [5438]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 11.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_exact, _rational, _Riccati]`

$$\frac{y}{1-x^2y^2} + \frac{xy'}{1-x^2y^2} - 1 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(y(x)/(1-x^2*y(x)^2)+x/(1-x^2*y(x)^2)*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = -\frac{e^{-2x}c_1 + 1}{x(e^{-2x}c_1 - 1)}$$

### ✓ Solution by Mathematica

Time used: 0.146 (sec). Leaf size: 18

```
DSolve[y[x]/(1-x^2*y[x]^2)+x/(1-x^2*y[x]^2)*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\tanh(x + ic_1)}{x}$$



## 4.12 problem 12

Internal problem ID [5439]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 12.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_exact]`

$$2y^4x + \sin(y) + (4y^3x^2 + x \cos(y)) y' = 0$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve((2*x*y(x)^4+sin(y(x)))+(4*x^2*y(x)^3+x*cos(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$x^2y(x)^4 + x \sin(y(x)) + c_1 = 0$$

### ✓ Solution by Mathematica

Time used: 0.221 (sec). Leaf size: 20

```
DSolve[(2*x*y[x]^4+Sin[y[x]])+(4*x^2*y[x]^3+x*Cos[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$\text{Solve}[x^2y(x)^4 + x \sin(y(x)) = c_1, y(x)]$$

### 4.13 problem 13

Internal problem ID [5440]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 13.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [`_exact`, `_rational`, `_Riccati`]

$$\frac{y'x + y}{1 - x^2y^2} + x = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((y(x)+x*diff(y(x),x))/(1-x^2*y(x)^2)+x=0,y(x), singsol=all)
```

$$y(x) = \frac{i \tan\left(\frac{ix^2}{2} + c_1\right)}{x}$$

#### ✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 25

```
DSolve[(y[x]+x*y'[x])/(1-x^2*y[x]^2)+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\tanh\left(\frac{1}{2}(x^2 - 2ic_1)\right)}{x}$$

## 4.14 problem 14

Internal problem ID [5441]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 14.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_exact, [\_1st\_order, ‘\_with\_symmetry\_[F(x),G(y)]’]]

$$2x(1 + \sqrt{x^2 - y}) - \sqrt{x^2 - y}y' = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve((2*x*(1+sqrt(x^2-y(x))))=sqrt(x^2-y(x))*diff(y(x),x),y(x), singsol=all)
```

$$x^2 + \frac{2(x^2 - y(x))^{\frac{3}{2}}}{3} + c_1 = 0$$

### ✓ Solution by Mathematica

Time used: 0.846 (sec). Leaf size: 121

```
DSolve[2*x*(1+Sqrt[x^2-y[x]])==Sqrt[x^2-y[x]]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2 + \left(\frac{3}{2}\right)^{2/3} \sqrt[3]{-(x^2 + c_1)^2}$$

$$y(x) \rightarrow x^2 - \frac{\sqrt[6]{3}(\sqrt{3} - 3i) \sqrt[3]{-(x^2 + c_1)^2}}{2 \cdot 2^{2/3}}$$

$$y(x) \rightarrow x^2 - \frac{\sqrt[6]{3}(\sqrt{3} + 3i) \sqrt[3]{-(x^2 + c_1)^2}}{2 \cdot 2^{2/3}}$$

## 4.15 problem 15

Internal problem ID [5442]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 15.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$x \ln(y) + xy + (\ln(x)y + xy)y' = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve((x*ln(y(x))+x*y(x))+(y(x)*ln(x)+x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\int \frac{x}{\ln(x) + x} dx + \int^{y(x)} \frac{-a}{-a + \ln(-a)} d_{-a} + c_1 = 0$$

### ✓ Solution by Mathematica

Time used: 34.588 (sec). Leaf size: 54

```
DSolve[(x*Log[y[x]]+x*y[x])+(y[x]*Log[x]+x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow \text{InverseFunction} \left[ \int_1^{\#1} \frac{K[1]}{K[1] + \log(K[1])} dK[1] \& \right] \left[ \int_1^x -\frac{K[2]}{K[2] + \log(K[2])} dK[2] + c_1 \right]$$

$$y(x) \rightarrow W(1)$$

## 4.16 problem 16

Internal problem ID [5443]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 16.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_exact]`

$$e^{y^2} - \csc(y) \csc(x)^2 + (2xy e^{y^2} - \csc(y) \cot(y) \cot(x)) y' = 0$$

### ✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 18

```
dsolve((exp(y(x)^2)-csc(y(x))*csc(x)^2)+(2*x*y(x)*exp(y(x)^2)-csc(y(x))*cot(y(x))*cot(x))*dif
```

$$\csc(y(x)) \cot(x) + x e^{y(x)^2} + c_1 = 0$$

### ✓ Solution by Mathematica

Time used: 174.413 (sec). Leaf size: 23

```
DSolve[(Exp[y[x]^2]-Csc[y[x]]*Csc[x]^2)+(2*x*y[x]*Exp[y[x]^2]-Csc[y[x]]*Cot[y[x]]*Cot[x])*y'
```

$$\text{Solve}\left[-2x e^{y(x)^2} - 2 \cot(x) \csc(y(x)) = c_1, y(x)\right]$$

## 4.17 problem 17

Internal problem ID [5444]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 17.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_exact, \_Bernoulli]

$$1 + y^2 \sin(2x) - 2y \cos(x)^2 y' = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve((1+y(x)^2*sin(2*x))-(2*y(x)*cos(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x + c_1}}{\cos(x)}$$

$$y(x) = -\frac{\sqrt{x + c_1}}{\cos(x)}$$

### ✓ Solution by Mathematica

Time used: 0.267 (sec). Leaf size: 32

```
DSolve[(1+y[x]^2*Sin[2*x])-(2*y[x]*Cos[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow -\sqrt{x + c_1} \sec(x)$$

$$y(x) \rightarrow \sqrt{x + c_1} \sec(x)$$

## 4.18 problem 18

Internal problem ID [5445]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 18.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$\frac{x}{(y^2 + x^2)^{\frac{3}{2}}} + \frac{yy'}{(y^2 + x^2)^{\frac{3}{2}}} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve((x/(x^2+y(x)^2)^(3/2))+y(x)/(x^2+y(x)^2)^(3/2))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + c_1}$$

$$y(x) = -\sqrt{-x^2 + c_1}$$

### ✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 39

```
DSolve[(x/(x^2+y[x]^2)^(3/2))+y[x]/(x^2+y[x]^2)^(3/2))*y'[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \rightarrow -\sqrt{-x^2 + 2c_1}$$

$$y(x) \rightarrow \sqrt{-x^2 + 2c_1}$$

## 4.19 problem 19

Internal problem ID [5446]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 19.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_exact, [\_1st\_order, ‘\_with\_symmetry\_[F(x)\*G(y),0]’]]

$$3x^2(1 + \ln(y)) + \left(\frac{x^3}{y} - 2y\right)y' = 0$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 39

```
dsolve((3*x^2*(1+ln(y(x))))+(x^3/y(x)-2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^{-\frac{x^3 \operatorname{LambertW}\left(-\frac{2e^{-2+\frac{2c_1}{x^3}}}{x^3}\right) + 2x^3 + 2c_1}{2x^3}}$$

### ✓ Solution by Mathematica

Time used: 60.155 (sec). Leaf size: 79

```
DSolve[(3*x^2*(1+Log[y[x]]))+(x^3/y[x]-2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \rightarrow -\frac{ix^{3/2} \sqrt{W\left(-\frac{2e^{-2+\frac{2c_1}{x^3}}}{x^3}\right)}}{\sqrt{2}}$$

$$y(x) \rightarrow \frac{ix^{3/2} \sqrt{W\left(-\frac{2e^{-2+\frac{2c_1}{x^3}}}{x^3}\right)}}{\sqrt{2}}$$



## 4.20 problem 20

Internal problem ID [5447]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 20.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _exact, _rational]`

$$\frac{y - y'x}{(x + y)^2} + y' - 1 = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 65

```
dsolve((y(x)-x*diff(y(x),x))/(x+y(x))^2+diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = \frac{1}{4} + \frac{c_1}{4} - \frac{\sqrt{c_1^2 + 8c_1x + 16x^2 + 2c_1 - 8x + 1}}{4}$$

$$y(x) = \frac{1}{4} + \frac{c_1}{4} + \frac{\sqrt{c_1^2 + 8c_1x + 16x^2 + 2c_1 - 8x + 1}}{4}$$

### ✓ Solution by Mathematica

Time used: 0.417 (sec). Leaf size: 70

```
DSolve[(y[x]-x*y'[x])/(x+y[x])^2+y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} \left( -\sqrt{(2x + c_1)^2 + 1 + 2c_1} + 1 + c_1 \right)$$

$$y(x) \rightarrow \frac{1}{2} \left( \sqrt{(2x + c_1)^2 + 1 + 2c_1} + 1 + c_1 \right)$$

$$y(x) \rightarrow -x$$

## 4.21 problem 21

Internal problem ID [5448]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.5. Exact Equations. Page 20

**Problem number:** 21.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _exact, _rational, _dAlembert]`

$$\frac{4y^2 - 2x^2}{4xy^2 - x^3} + \frac{(8y^2 - x^2)y'}{4y^3 - x^2y} = 0$$

✓ Solution by Maple

Time used: 0.438 (sec). Leaf size: 225

```
dsolve(( (4*y(x)^2-2*x^2)/(4*x*y(x)^2-x^3))+( (8*y(x)^2-x^2)/(4*y(x)^3-x^2*y(x)) )*diff(y(x),
```

$$y(x) = \frac{-c_1x - \frac{-2c_1^2x^2 + \sqrt{2x^4c_1^4 - 2\sqrt{c_1^6x^6 + 16c_1x}}}{2c_1x}}{2c_1}$$

$$y(x) = \frac{-c_1x - \frac{-2c_1^2x^2 + \sqrt{2x^4c_1^4 + 2\sqrt{c_1^6x^6 + 16c_1x}}}{2c_1x}}{2c_1}$$

$$y(x) = \frac{-c_1x + \frac{2c_1^2x^2 + \sqrt{2x^4c_1^4 - 2\sqrt{c_1^6x^6 + 16c_1x}}}{2c_1x}}{2c_1}$$

$$y(x) = \frac{-c_1x + \frac{2c_1^2x^2 + \sqrt{2x^4c_1^4 + 2\sqrt{c_1^6x^6 + 16c_1x}}}{2c_1x}}{2c_1}$$

✓ Solution by Mathematica

Time used: 11.856 (sec). Leaf size: 297

`DSolve[( (4*y[x]^2-2*x^2)/(4*x*y[x]^2-x^3))+( (8*y[x]^2-x^2)/(4*y[x]^3-x^2*y[x]) ) *y' [x]==0,y`

$$y(x) \rightarrow -\frac{\sqrt{x^2 - \frac{\sqrt{x^6 - 16e^{2c_1}}}{x}}}{2\sqrt{2}}$$

$$y(x) \rightarrow \frac{\sqrt{x^2 - \frac{\sqrt{x^6 - 16e^{2c_1}}}{x}}}{2\sqrt{2}}$$

$$y(x) \rightarrow -\frac{\sqrt{\frac{x^3 + \sqrt{x^6 - 16e^{2c_1}}}{x}}}{2\sqrt{2}}$$

$$y(x) \rightarrow \frac{\sqrt{\frac{x^3 + \sqrt{x^6 - 16e^{2c_1}}}{x}}}{2\sqrt{2}}$$

$$y(x) \rightarrow -\frac{\sqrt{x^2 - \frac{\sqrt{x^6}}{x}}}{2\sqrt{2}}$$

$$y(x) \rightarrow \frac{\sqrt{x^2 - \frac{\sqrt{x^6}}{x}}}{2\sqrt{2}}$$

$$y(x) \rightarrow -\frac{\sqrt{\frac{\sqrt{x^6} + x^3}{x}}}{2\sqrt{2}}$$

$$y(x) \rightarrow \frac{\sqrt{\frac{\sqrt{x^6} + x^3}{x}}}{2\sqrt{2}}$$

## 5 Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

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## 5.1 problem 1(a)

Internal problem ID [5449]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 1(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$x^2 - 2y^2 + xy y' = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve((x^2-2*y(x)^2)+(x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x^2 + 1} x$$

$$y(x) = -\sqrt{c_1 x^2 + 1} x$$

### ✓ Solution by Mathematica

Time used: 0.43 (sec). Leaf size: 39

```
DSolve[(x^2-2*y[x]^2)+(x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt{x^2 + c_1 x^4}$$

$$y(x) \rightarrow \sqrt{x^2 + c_1 x^4}$$

## 5.2 problem 1(b)

Internal problem ID [5450]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 1(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$x^2y' - 3xy - 2y^2 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)-3*x*y(x)-2*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{-x^2 + c_1}$$

### ✓ Solution by Mathematica

Time used: 0.131 (sec). Leaf size: 24

```
DSolve[x^2*y'[x]-3*x*y[x]-2*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^3}{-x^2 + c_1}$$

$$y(x) \rightarrow 0$$

### 5.3 problem 1(c)

Internal problem ID [5451]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 1(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A'], _dAlembert]`

$$x^2 y' - 3(y^2 + x^2) \arctan\left(\frac{y}{x}\right) - xy = 0$$

#### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 12

```
dsolve(x^2*diff(y(x),x)=3*(x^2+y(x)^2)*arctan(y(x)/x)+x*y(x),y(x), singsol=all)
```

$$y(x) = \tan(c_1 x^3) x$$

#### ✓ Solution by Mathematica

Time used: 5.471 (sec). Leaf size: 23

```
DSolve[x^2*y'[x]==3*(x^2+y[x]^2)*ArcTan[x,y[x]]+x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x \tan(e^{-3c_1} x^3)$$

$$y(x) \rightarrow 0$$

## 5.4 problem 1(d)

Internal problem ID [5452]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 1(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A'], _dAlembert]`

$$x \sin\left(\frac{y}{x}\right) y' - y \sin\left(\frac{y}{x}\right) - x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*sin(y(x)/x)*diff(y(x),x)=y(x)*sin(y(x)/x)+x,y(x), singsol=all)
```

$$y(x) = (\pi - \arccos(\ln(x) + c_1)) x$$

### ✓ Solution by Mathematica

Time used: 0.406 (sec). Leaf size: 33

```
DSolve[x*Sin[y[x]/x]*y'[x]==y[x]*Sin[y[x]/x]+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(-\pi + \arccos(\log(x) + c_1))$$

$$y(x) \rightarrow x(\pi - \arccos(\log(x) + c_1))$$



## 5.5 problem 1(e)

Internal problem ID [5453]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 1(e).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A'], _dAlembert]`

$$y'x - y - 2xe^{-\frac{y}{x}} = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)=y(x)+2*x*exp(-y(x)/x),y(x), singsol=all)
```

$$y(x) = \ln(2 \ln(x) + 2c_1) x$$

### ✓ Solution by Mathematica

Time used: 0.385 (sec). Leaf size: 15

```
DSolve[x*y'[x]==y[x]+2*x*Exp[-y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x \log(2 \log(x) + c_1)$$

## 5.6 problem 1(f)

Internal problem ID [5454]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 1(f).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _exact, _rational, [_Abel, '2nd typ`

$$x - y - (y + x)y' = 0$$

### ✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 51

```
dsolve((x-y(x))-(x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-c_1x - \sqrt{2c_1^2x^2 + 1}}{c_1}$$

$$y(x) = \frac{-c_1x + \sqrt{2c_1^2x^2 + 1}}{c_1}$$

### ✓ Solution by Mathematica

Time used: 0.446 (sec). Leaf size: 94

```
DSolve[(x-y[x])-(x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x - \sqrt{2x^2 + e^{2c_1}}$$

$$y(x) \rightarrow -x + \sqrt{2x^2 + e^{2c_1}}$$

$$y(x) \rightarrow -\sqrt{2}\sqrt{x^2} - x$$

$$y(x) \rightarrow \sqrt{2}\sqrt{x^2} - x$$

## 5.7 problem 1(g)

Internal problem ID [5455]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 1(g).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y'x - 2x + 6y = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)=2*x-6*y(x),y(x), singsol=all)
```

$$y(x) = \frac{2x}{7} + \frac{c_1}{x^6}$$

### ✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 17

```
DSolve[x*y'[x]==2*x-6*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2x}{7} + \frac{c_1}{x^6}$$

## 5.8 problem 1(h)

Internal problem ID [5456]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 1(h).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$y'x - \sqrt{y^2 + x^2} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(x*diff(y(x),x)=sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{y(x)^2}{x^2} + \frac{y(x)\sqrt{x^2 + y(x)^2}}{x^2} + \ln\left(y(x) + \sqrt{x^2 + y(x)^2}\right) - 3\ln(x) - c_1 = 0$$

### ✓ Solution by Mathematica

Time used: 0.299 (sec). Leaf size: 66

```
DSolve[x*y'[x]==Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[ \frac{1}{2} \left( \frac{y(x) \left( \sqrt{\frac{y(x)^2}{x^2} + 1} + \frac{y(x)}{x} \right)}{x} - \log \left( \sqrt{\frac{y(x)^2}{x^2} + 1} - \frac{y(x)}{x} \right) \right) = \log(x) + c_1, y(x) \right]$$

## 5.9 problem 1(i)

Internal problem ID [5457]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 1(i).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$x^2y' - 2xy - y^2 = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x)=y(x)^2+2*x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{-x + c_1}$$

### ✓ Solution by Mathematica

Time used: 0.128 (sec). Leaf size: 22

```
DSolve[x^2*y'[x]==y[x]^2+2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^2}{-x + c_1}$$

$$y(x) \rightarrow 0$$

## 5.10 problem 1(j)

Internal problem ID [5458]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 1(j).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$x^3 + y^3 - xy^2y' = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 74

```
dsolve((x^3+y(x)^3)-(x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (3 \ln(x) + c_1)^{\frac{1}{3}} x$$

$$y(x) = \left( -\frac{(3 \ln(x) + c_1)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}(3 \ln(x) + c_1)^{\frac{1}{3}}}{2} \right) x$$

$$y(x) = \left( -\frac{(3 \ln(x) + c_1)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}(3 \ln(x) + c_1)^{\frac{1}{3}}}{2} \right) x$$

### ✓ Solution by Mathematica

Time used: 0.174 (sec). Leaf size: 63

```
DSolve[(x^3+y[x]^3)-(x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x \sqrt[3]{3 \log(x) + c_1}$$

$$y(x) \rightarrow -\sqrt[3]{-1} x \sqrt[3]{3 \log(x) + c_1}$$

$$y(x) \rightarrow (-1)^{2/3} x \sqrt[3]{3 \log(x) + c_1}$$

## 5.11 problem 4(a)

Internal problem ID [5459]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 4(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _rational, [_Abel, '2nd type', 'cla`

$$y' - \frac{x + y + 4}{x - y - 6} = 0$$

### ✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)=(x+y(x)+4)/(x-y(x)-6),y(x), singsol=all)
```

$$y(x) = -5 - \tan \left( \text{RootOf} \left( 2\_Z + \ln \left( \frac{1}{\cos(\_Z)^2} \right) + 2 \ln(x - 1) + 2c_1 \right) \right) (x - 1)$$

### ✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 58

```
DSolve[y'[x]==(x+y[x]+4)/(x-y[x]-6),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[ 2 \arctan \left( \frac{y(x) + x + 4}{y(x) - x + 6} \right) + \log \left( \frac{x^2 + y(x)^2 + 10y(x) - 2x + 26}{2(x - 1)^2} \right) + 2 \log(x - 1) + c_1 = 0, y(x) \right]$$

## 5.12 problem 4(b)

Internal problem ID [5460]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 4(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _rational, [_Abel, '2nd type', 'cla`

$$y' - \frac{x + y + 4}{x + y - 6} = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=(x+y(x)+4)/(x+y(x)-6),y(x), singsol=all)
```

$$y(x) = -x - 5 \operatorname{LambertW}\left(-\frac{e^{-\frac{2x}{5}} c_1 e^{\frac{1}{5}}}{5}\right) + 1$$

### ✓ Solution by Mathematica

Time used: 3.76 (sec). Leaf size: 35

```
DSolve[y'[x]==(x+y[x]+4)/(x+y[x]-6),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -5W\left(-e^{-\frac{2x}{5}-1+c_1}\right) - x + 1$$

$$y(x) \rightarrow 1 - x$$



### 5.13 problem 4(c)

Internal problem ID [5461]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 4(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _rational, [_Abel, '2nd type', 'cla`

$$2x - 2y + (y - 1)y' = 0$$

#### ✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve((2*x-2*y(x))+(y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\tan\left(\text{RootOf}\left(-2\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(x-1) + 2c_1\right)\right)(x-1) + x$$

#### ✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 60

```
DSolve[(2*x-2*y[x])+(y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2 \arctan\left(\frac{y(x) - 2x + 1}{y(x) - 1}\right) + \log\left(\frac{2x^2 - 2xy(x) + y(x)^2 - 2x + 1}{2(x-1)^2}\right) + 2\log(x-1) + c_1 = 0, y(x)\right]$$

## 5.14 problem 4(d)

Internal problem ID [5462]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 4(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _rational, [_Abel, '2nd type', 'cla`

$$y' - \frac{y + x - 1}{x + 4y + 2} = 0$$

### ✓ Solution by Maple

Time used: 1.719 (sec). Leaf size: 65

```
dsolve(diff(y(x),x)=(x+y(x)-1)/(x+4*y(x)+2),y(x), singsol=all)
```

$$y(x) = -1 + \frac{(x-2) \left( \text{RootOf}(\_Z^{16} + 2(x-2)^4 c_1 \_Z^4 - (x-2)^4 c_1)^4 - 1 \right)}{2 \text{RootOf}(\_Z^{16} + 2(x-2)^4 c_1 \_Z^4 - (x-2)^4 c_1)^4}$$

### ✓ Solution by Mathematica

Time used: 60.302 (sec). Leaf size: 8141

```
DSolve[y'[x]==(x+y[x]-1)/(x+4*y[x]+2),y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

## 5.15 problem 4(e)

Internal problem ID [5463]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 4(e).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$2x + 3y - 1 - 4y'(1 + x) = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((2*x+3*y(x)-1)-4*(x+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (x + 1)^{\frac{3}{4}} c_1 + 2x + 3$$

### ✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 20

```
DSolve[(2*x+3*y[x]-1)-4*(x+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2x + c_1(x + 1)^{3/4} + 3$$

## 5.16 problem 5(a)

Internal problem ID [5464]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 5(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$y' - \frac{-xy^2 + 1}{2x^2y} = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)=(1-x*y(x)^2)/(2*x^2*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x(\ln(x) + c_1)}}{x}$$

$$y(x) = -\frac{\sqrt{x(\ln(x) + c_1)}}{x}$$

### ✓ Solution by Mathematica

Time used: 0.167 (sec). Leaf size: 40

```
DSolve[y'[x]==(1-x*y[x]^2)/(2*x^2*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt{\log(x) + c_1}}{\sqrt{x}}$$

$$y(x) \rightarrow \frac{\sqrt{\log(x) + c_1}}{\sqrt{x}}$$

## 5.17 problem 5(b)

Internal problem ID [5465]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 5(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$y' - \frac{2 + 3xy^2}{4x^2y} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x)=(2+3*x*y(x)^2)/(4*x^2*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{5} \sqrt{x \left(5x^{\frac{5}{2}}c_1 - 2\right)}}{5x}$$

$$y(x) = \frac{\sqrt{5} \sqrt{x \left(5x^{\frac{5}{2}}c_1 - 2\right)}}{5x}$$

### ✓ Solution by Mathematica

Time used: 3.617 (sec). Leaf size: 51

```
DSolve[y'[x]==(2+3*x*y[x]^2)/(4*x^2*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt{-\frac{2}{5x} + c_1x^{3/2}}$$

$$y(x) \rightarrow \sqrt{-\frac{2}{5x} + c_1x^{3/2}}$$

## 5.18 problem 5(c)

Internal problem ID [5466]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 5(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, [_Abel, '2nd type', 'cla`

$$y' - \frac{-xy^2 + y}{x + x^2y} = 0$$

### ✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)=(y(x)-x*y(x)^2)/(x+x^2*y(x)),y(x), singsol=all)
```

$$y(x) = x e^{-\text{LambertW}(x^2 e^{-2c_1}) - 2c_1}$$

### ✓ Solution by Mathematica

Time used: 60.433 (sec). Leaf size: 31

```
DSolve[y'[x]==(y[x]-x*y[x]^2)/(x+x^2*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{W\left(e^{\frac{1}{2}(-2-9\sqrt[3]{-2c_1})}x^2\right)}{x}$$

## 5.19 problem 7(a)

Internal problem ID [5467]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 7(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$y' - \sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right) = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=sin(y(x)/x)-cos(y(x)/x),y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(\int^{-Z} \frac{1}{-\sin(\_a) + \cos(\_a) + \_a} d\_a + \ln(x) + c_1\right) x$$

### ✓ Solution by Mathematica

Time used: 0.348 (sec). Leaf size: 36

```
DSolve[y'[x]==Sin[y[x]/x]-Cos[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{1}{\cos(K[1]) + K[1] - \sin(K[1])} dK[1] = -\log(x) + c_1, y(x)\right]$$

## 5.20 problem 7(b)

Internal problem ID [5468]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 7(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$e^{\frac{x}{y}} - \frac{y'y}{x} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(exp(x/y(x))-y(x)/x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \text{RootOf} \left( - \left( \int^{-z} \frac{-a}{-a^2 + e^{\frac{1}{-a}}} d_a \right) + \ln(x) + c_1 \right) x$$

### ✓ Solution by Mathematica

Time used: 0.176 (sec). Leaf size: 41

```
DSolve[Exp[x/y[x]]-y[x]/x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[ \int_1^{\frac{y(x)}{x}} \frac{K[1]}{K[1]^2 - e^{\frac{1}{K[1]}}} dK[1] = -\log(x) + c_1, y(x) \right]$$



## 5.21 problem 7(c)

Internal problem ID [5469]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 7(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A'], _dAlembert]`

$$y' - \frac{-xy + x^2}{y^2 \cos\left(\frac{x}{y}\right)} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x)=(x^2-x*y(x))/(y(x)^2*cos(x/y(x))),y(x), singsol=all)
```

$$y(x) = \text{RootOf} \left( \int^{-z} \frac{-a^2 \cos\left(\frac{1}{-a}\right)}{-a^3 \cos\left(\frac{1}{-a}\right) + -a - 1} d_{-a} + \ln(x) + c_1 \right) x$$

### ✓ Solution by Mathematica

Time used: 1.054 (sec). Leaf size: 49

```
DSolve[y'[x]==(x^2-x*y[x])/(y[x]^2*Cos[x/y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[ \int_1^{\frac{y(x)}{x}} \frac{\cos\left(\frac{1}{K[1]}\right) K[1]^2}{\cos\left(\frac{1}{K[1]}\right) K[1]^3 + K[1] - 1} dK[1] = -\log(x) + c_1, y(x) \right]$$

## 5.22 problem 7(d)

Internal problem ID [5470]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.7. Homogeneous Equations. Page 28

**Problem number:** 7(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A'], _dAlembert]`

$$y' - \frac{y \tan\left(\frac{y}{x}\right)}{x} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)=y(x)/x*tan(y(x)/x),y(x), singsol=all)
```

$$y(x) = \text{RootOf} \left( \ln(x) + c_1 - \left( \int \frac{1}{-a(-1 + \tan(-a))} d_{-a} \right) \right) x$$

### ✓ Solution by Mathematica

Time used: 1.675 (sec). Leaf size: 33

```
DSolve[y'[x]==y[x]/x*Tan[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[ \int_1^{\frac{y(x)}{x}} \frac{1}{K[1](\tan(K[1]) - 1)} dK[1] = \log(x) + c_1, y(x) \right]$$

## 6 Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32

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## 6.1 problem 1(a)

Internal problem ID [5471]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32

**Problem number:** 1(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _dAlembert]`

$$(3x^2 - y^2) y' - 2xy = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 402

```
dsolve((3*x^2-y(x)^2)*diff(y(x),x)-2*x*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned}
 y(x) &= \frac{\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}}{6c_1} \\
 &+ \frac{2}{3c_1\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}} + \frac{1}{3c_1} \\
 y(x) &= -\frac{\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}}{12c_1} \\
 &- \frac{1}{3c_1\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}} + \frac{1}{3c_1} \\
 &- \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}}{6c_1} - \frac{2}{3c_1\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}}\right)}{2} \\
 y(x) &= -\frac{\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}}{12c_1} \\
 &- \frac{1}{3c_1\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}} + \frac{1}{3c_1} \\
 &+ \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}}{6c_1} - \frac{2}{3c_1\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}}\right)}{2}
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 60.175 (sec). Leaf size: 458

`DSolve[(3*x^2-y[x]^2)*y'[x]-2*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]`

$$y(x) \rightarrow \frac{1}{3} \left( \frac{\sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}e^{2c_1}}{\sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}} - e^{c_1} \right)$$

$$y(x) \rightarrow \frac{i(\sqrt{3} + i) \sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}}{6\sqrt[3]{2}} - \frac{i(\sqrt{3} - i) e^{2c_1}}{3 \cdot 2^{2/3} \sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

$$y(x) \rightarrow -\frac{i(\sqrt{3} - i) \sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}}{6\sqrt[3]{2}} + \frac{i(\sqrt{3} + i) e^{2c_1}}{3 \cdot 2^{2/3} \sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

## 6.2 problem 1(b)

Internal problem ID [5472]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32

**Problem number:** 1(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_rational, [_1st_order, ' _with_symmetry_[F(x),G(x)]', [_Abel,`

$$xy - 1 + (-xy + x^2)y' = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve((x*y(x)-1)+(x^2-x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x - \sqrt{x^2 - 2 \ln(x) + 2c_1}$$

$$y(x) = x + \sqrt{x^2 - 2 \ln(x) + 2c_1}$$

### ✓ Solution by Mathematica

Time used: 0.392 (sec). Leaf size: 68

```
DSolve[(x*y[x]-1)+(x^2-x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x + \sqrt{-\frac{1}{x}} \sqrt{-x(x^2 - 2 \log(x) + c_1)}$$

$$y(x) \rightarrow x + x \left(-\frac{1}{x}\right)^{3/2} \sqrt{-x(x^2 - 2 \log(x) + c_1)}$$

### 6.3 problem 1(c)

Internal problem ID [5473]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32

**Problem number:** 1(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$y'x + y + 3y^4y'x^3 = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 129

```
dsolve(x*diff(y(x),x)+y(x)+3*x^3*y(x)^4*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-6xc_1 \left(-x + \sqrt{12c_1^2 + x^2}\right)}}{6xc_1}$$

$$y(x) = \frac{\sqrt{-6xc_1 \left(-x + \sqrt{12c_1^2 + x^2}\right)}}{6xc_1}$$

$$y(x) = -\frac{\sqrt{6} \sqrt{xc_1 \left(x + \sqrt{12c_1^2 + x^2}\right)}}{6xc_1}$$

$$y(x) = \frac{\sqrt{6} \sqrt{xc_1 \left(x + \sqrt{12c_1^2 + x^2}\right)}}{6xc_1}$$



✓ Solution by Mathematica

Time used: 9.515 (sec). Leaf size: 166

```
DSolve[x*y'[x]+y[x]+3*x^3*y[x]^4*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt{c_1 - \frac{\sqrt{x^2(3+c_1^2x^2)}}{x^2}}}{\sqrt{3}}$$

$$y(x) \rightarrow \frac{\sqrt{c_1 - \frac{\sqrt{x^2(3+c_1^2x^2)}}{x^2}}}{\sqrt{3}}$$

$$y(x) \rightarrow -\frac{\sqrt{\frac{\sqrt{x^2(3+c_1^2x^2)}}{x^2} + c_1}}{\sqrt{3}}$$

$$y(x) \rightarrow \frac{\sqrt{\frac{\sqrt{x^2(3+c_1^2x^2)}}{x^2} + c_1}}{\sqrt{3}}$$

$$y(x) \rightarrow 0$$

## 6.4 problem 1(d)

Internal problem ID [5474]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32

**Problem number:** 1(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$e^x + (e^x \cot(y) + 2y \csc(y)) y' = 0$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve(exp(x)+(exp(x)*cot(y(x))+2*y(x)*csc(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$e^x \sin(y(x)) + y(x)^2 + c_1 = 0$$

### ✓ Solution by Mathematica

Time used: 0.308 (sec). Leaf size: 18

```
DSolve[Exp[x]+(Exp[x]*Cot[y[x]]+2*y[x]*Csc[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}[y(x)^2 + e^x \sin(y(x)) = c_1, y(x)]$$

## 6.5 problem 1(e)

Internal problem ID [5475]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32

**Problem number:** 1(e).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$(2 + x) \sin(y) + x \cos(y) y' = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((x+2)*sin(y(x))+x*cos(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{e^{-x}}{c_1 x^2}\right)$$

### ✓ Solution by Mathematica

Time used: 47.884 (sec). Leaf size: 23

```
DSolve[(x+2)*Sin[y[x]]+x*Cos[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \csc^{-1}(x^2 e^{x-c_1})$$

$$y(x) \rightarrow 0$$

## 6.6 problem 1(f)

Internal problem ID [5476]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32

**Problem number:** 1(f).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class G'], _rational]`

$$y + (x - 2y^3x^2) y' = 0$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 432

```
dsolve(y(x)+(x-2*x^2*y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left( \left( 12\sqrt{3} \sqrt{\frac{27c_1^3 - 4x^2}{c_1}} - 108c_1 \right) c_1^2 x^2 \right)^{\frac{1}{3}}}{6c_1 x} + \frac{2x}{\left( \left( 12\sqrt{3} \sqrt{\frac{27c_1^3 - 4x^2}{c_1}} - 108c_1 \right) c_1^2 x^2 \right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left( \left( 12\sqrt{3} \sqrt{\frac{27c_1^3 - 4x^2}{c_1}} - 108c_1 \right) c_1^2 x^2 \right)^{\frac{1}{3}}}{12c_1 x} - \frac{x}{\left( \left( 12\sqrt{3} \sqrt{\frac{27c_1^3 - 4x^2}{c_1}} - 108c_1 \right) c_1^2 x^2 \right)^{\frac{1}{3}}}$$

$$- \frac{i\sqrt{3} \left( \frac{\left( \left( 12\sqrt{3} \sqrt{\frac{27c_1^3 - 4x^2}{c_1}} - 108c_1 \right) c_1^2 x^2 \right)^{\frac{1}{3}}}{6c_1 x} - \frac{2x}{\left( \left( 12\sqrt{3} \sqrt{\frac{27c_1^3 - 4x^2}{c_1}} - 108c_1 \right) c_1^2 x^2 \right)^{\frac{1}{3}}} \right)}{2}$$

$$y(x) = -\frac{\left( \left( 12\sqrt{3} \sqrt{\frac{27c_1^3 - 4x^2}{c_1}} - 108c_1 \right) c_1^2 x^2 \right)^{\frac{1}{3}}}{12c_1 x} - \frac{x}{\left( \left( 12\sqrt{3} \sqrt{\frac{27c_1^3 - 4x^2}{c_1}} - 108c_1 \right) c_1^2 x^2 \right)^{\frac{1}{3}}}$$

$$+ \frac{i\sqrt{3} \left( \frac{\left( \left( 12\sqrt{3} \sqrt{\frac{27c_1^3 - 4x^2}{c_1}} - 108c_1 \right) c_1^2 x^2 \right)^{\frac{1}{3}}}{6c_1 x} - \frac{2x}{\left( \left( 12\sqrt{3} \sqrt{\frac{27c_1^3 - 4x^2}{c_1}} - 108c_1 \right) c_1^2 x^2 \right)^{\frac{1}{3}}} \right)}{2}$$

✓ Solution by Mathematica

Time used: 29.06 (sec). Leaf size: 287

`DSolve[y[x]+(x-2*x^2*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]`

$$y(x) \rightarrow \frac{2\sqrt[3]{3}c_1x^2 + \sqrt[3]{2}(-9x^2 + \sqrt{81x^4 - 12c_1^3x^6})^{2/3}}{6^{2/3}x^3\sqrt[3]{-9x^2 + \sqrt{81x^4 - 12c_1^3x^6}}}$$

$$y(x) \rightarrow \frac{\sqrt[3]{-1}\left(\sqrt[3]{-2}(-9x^2 + \sqrt{81x^4 - 12c_1^3x^6})^{2/3} - 2\sqrt[3]{3}c_1x^2\right)}{6^{2/3}x^3\sqrt[3]{-9x^2 + \sqrt{81x^4 - 12c_1^3x^6}}}$$

$$y(x) \rightarrow \frac{\sqrt[3]{-1}\left(2\sqrt[3]{-3}c_1x^2 - \sqrt[3]{2}(-9x^2 + \sqrt{81x^4 - 12c_1^3x^6})^{2/3}\right)}{6^{2/3}x^3\sqrt[3]{-9x^2 + \sqrt{81x^4 - 12c_1^3x^6}}}$$

$$y(x) \rightarrow 0$$

## 6.7 problem 1(g)

Internal problem ID [5477]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32

**Problem number:** 1(g).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$x + 3y^2 + 2xyy' = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve((x+3*y(x)^2)+(2*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{x(-x^4 + 4c_1)}}{2x^2}$$

$$y(x) = \frac{\sqrt{x(-x^4 + 4c_1)}}{2x^2}$$

### ✓ Solution by Mathematica

Time used: 0.198 (sec). Leaf size: 55

```
DSolve[(x+3*y[x]^2)+(2*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt{-x^4 + 4c_1}}{2x^{3/2}}$$

$$y(x) \rightarrow \frac{\sqrt{-x^4 + 4c_1}}{2x^{3/2}}$$

## 6.8 problem 1(h)

Internal problem ID [5478]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32

**Problem number:** 1(h).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$y + (2x - y e^y) y' = 0$$

### ✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 27

```
dsolve(y(x)+(2*x-y(x)*exp(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$x - \frac{(y(x)^2 - 2y(x) + 2) e^{y(x)} + c_1}{y(x)^2} = 0$$

### ✓ Solution by Mathematica

Time used: 0.219 (sec). Leaf size: 32

```
DSolve[y[x]+(2*x-y[x]*Exp[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[ x = \frac{e^{y(x)}(y(x)^2 - 2y(x) + 2)}{y(x)^2} + \frac{c_1}{y(x)^2}, y(x) \right]$$



## 6.9 problem 1(i)

Internal problem ID [5479]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32

**Problem number:** 1(i).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type ['y=\_G(x,y)']

$$y \ln(y) - 2xy + (y + x)y' = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve((y(x)*ln(y(x))-2*x*y(x))+(x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^{-\frac{x \operatorname{LambertW}\left(\frac{e^x e^{-\frac{c_1}{x}}}{x}\right) - x^2 + c_1}{x}}$$

### ✓ Solution by Mathematica

Time used: 1.063 (sec). Leaf size: 22

```
DSolve[(y[x]*Log[y[x]]-2*x*y[x])+(x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow xW\left(\frac{e^{x+\frac{c_1}{x}}}{x}\right)$$

## 6.10 problem 1(j)

Internal problem ID [5480]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32

**Problem number:** 1(j).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y^2 + xy + 1 + (x^2 + xy + 1)y' = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve((y(x)^2+x*y(x)+1)+(x^2+x*y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x^2 + \text{LambertW}\left(-2x e^{x^2} c_1 e^{-1}\right)}{x}$$

### ✓ Solution by Mathematica

Time used: 6.26 (sec). Leaf size: 56

```
DSolve[(y[x]^2+x*y[x]+1)+(x^2+x*y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x + \frac{W\left(x\left(-e^{x^2-1+c_1}\right)\right)}{x}$$

$$y(x) \rightarrow -x$$

$$y(x) \rightarrow \frac{W\left(-e^{x^2-1}x\right)}{x} - x$$

## 6.11 problem 1(k)

Internal problem ID [5481]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32

**Problem number:** 1(k).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$x^3 + y^3 x + 3y^2 y' = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 113

```
dsolve((x^3+x*y(x)^3)+(3*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \left( e^{-\frac{x^2}{2}} c_1 - x^2 + 2 \right)^{\frac{1}{3}}$$

$$y(x) = -\frac{\left( e^{-\frac{x^2}{2}} c_1 - x^2 + 2 \right)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3} \left( e^{-\frac{x^2}{2}} c_1 - x^2 + 2 \right)^{\frac{1}{3}}}{2}$$

$$y(x) = -\frac{\left( e^{-\frac{x^2}{2}} c_1 - x^2 + 2 \right)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3} \left( e^{-\frac{x^2}{2}} c_1 - x^2 + 2 \right)^{\frac{1}{3}}}{2}$$

### ✓ Solution by Mathematica

Time used: 10.627 (sec). Leaf size: 95

```
DSolve[(x^3+x*y[x]^3)+(3*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt[3]{-x^2 + c_1 e^{-\frac{x^2}{2}} + 2}$$

$$y(x) \rightarrow -\sqrt[3]{-1} \sqrt[3]{-x^2 + c_1 e^{-\frac{x^2}{2}} + 2}$$

$$y(x) \rightarrow (-1)^{2/3} \sqrt[3]{-x^2 + c_1 e^{-\frac{x^2}{2}} + 2}$$

## 6.12 problem 4

Internal problem ID [5482]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.8. Integrating Factors. Page 32

**Problem number:** 4.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$y' - \frac{2y}{x} - \frac{x^3}{y} - x \tan\left(\frac{y}{x^2}\right) = 0$$

### ✓ Solution by Maple

Time used: 0.89 (sec). Leaf size: 216

```
dsolve(diff(y(x),x)=2*y(x)/x+x^3/y(x)+x*tan(y(x)/x^2),y(x), singsol=all)
```

$$y(x) = \frac{x^2(c_1 \cos(\text{RootOf}(c_1^2 Z^2 \cos(2_Z) + 4c_1 \sin(_Z) x_Z - c_1^2 Z^2 + c_1^2 \cos(2_Z) + c_1^2 - 2x^2)) - x)}{c_1 \sin(\text{RootOf}(c_1^2 Z^2 \cos(2_Z) + 4c_1 \sin(_Z) x_Z - c_1^2 Z^2 + c_1^2 \cos(2_Z) + c_1^2 - 2x^2))}$$

$$y(x) = \frac{x^2(c_1 \cos(\text{RootOf}(c_1^2 Z^2 \cos(2_Z) + 4c_1 \sin(_Z) x_Z - c_1^2 Z^2 + c_1^2 \cos(2_Z) + c_1^2 - 2x^2)) + x)}{c_1 \sin(\text{RootOf}(c_1^2 Z^2 \cos(2_Z) + 4c_1 \sin(_Z) x_Z - c_1^2 Z^2 + c_1^2 \cos(2_Z) + c_1^2 - 2x^2))}$$

### ✓ Solution by Mathematica

Time used: 1.209 (sec). Leaf size: 36

```
DSolve[y'[x]==2*y[x]/x+x^3/y[x]+x*Tan[y[x]/x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[3 \log(x) - \log\left(y(x) \sin\left(\frac{y(x)}{x^2}\right) + x^2 \cos\left(\frac{y(x)}{x^2}\right)\right) = c_1, y(x)\right]$$

## **7 Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38**

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## 7.1 problem 1(a)

Internal problem ID [5483]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38

**Problem number:** 1(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _L`

$$yy'' + y'^2 = 0$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 33

```
dsolve(y(x)*diff(y(x),x$2)+(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \sqrt{2c_1x + 2c_2}$$

$$y(x) = -\sqrt{2c_1x + 2c_2}$$

### ✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 20

```
DSolve[y[x]*y'[x]+(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2\sqrt{2x - c_1}$$

## 7.2 problem 1(b)

Internal problem ID [5484]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38

**Problem number:** 1(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [NONE]

$$xyy'' - y' - y'^3 = 0$$

### ✗ Solution by Maple

```
dsolve(x*y(x)*diff(y(x),x$2)=diff(y(x),x)+(diff(y(x),x))^3,y(x), singsol=all)
```

No solution found

### ✓ Solution by Mathematica

Time used: 1.405 (sec). Leaf size: 103

```
DSolve[x*y'[x]==y'[x]+(y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 - ie^{-c_1} \sqrt{-1 + e^{2c_1} x^2}$$

$$y(x) \rightarrow ie^{-c_1} \sqrt{-1 + e^{2c_1} x^2} + c_2$$

$$y(x) \rightarrow c_2 - i\sqrt{x^2}$$

$$y(x) \rightarrow i\sqrt{x^2} + c_2$$

### 7.3 problem 1(c)

Internal problem ID [5485]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38

**Problem number:** 1(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - k^2 y = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-k^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-kx} + c_2 e^{kx}$$

#### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 23

```
DSolve[y''[x]-k^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{kx} + c_2 e^{-kx}$$



## 7.4 problem 1(d)

Internal problem ID [5486]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38

**Problem number:** 1(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$x^2 y'' - 2y'x - y'^2 = 0$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 26

```
dsolve(x^2*diff(y(x),x$2)=2*x*diff(y(x),x)+(diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{2} - c_1 x - c_1^2 \ln(x - c_1) + c_2$$

### ✓ Solution by Mathematica

Time used: 0.428 (sec). Leaf size: 37

```
DSolve[x^2*y'[x]==2*x*y'[x]+(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{2}(x - c_1)(x + 3c_1) + c_1^2(-\log(x - c_1)) + c_2$$

## 7.5 problem 1(e)

Internal problem ID [5487]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38

**Problem number:** 1(e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$2yy'' - 1 - y'^2 = 0$$

### ✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 22

```
dsolve(2*y(x)*diff(y(x),x$2)=1+(diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = \frac{(c_1^2 + 1)x^2}{4c_2} + c_1x + c_2$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 29

```
DSolve[2*y[x]*y'[x]==1+(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{(1 + c_1^2)x^2}{4c_2} + c_1x + c_2$$

## 7.6 problem 1(f)

Internal problem ID [5488]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38

**Problem number:** 1(f).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible,`

$$yy'' - y'^2 = 0$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 14

```
dsolve(y(x)*diff(y(x),x$2)-(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = e^{c_1 x} c_2$$

### ✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 14

```
DSolve[y[x]*y'[x]-(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 e^{c_1 x}$$

## 7.7 problem 1(g)

Internal problem ID [5489]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38

**Problem number:** 1(g).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$xy'' + y' - 4x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x$2)+diff(y(x),x)=4*x,y(x), singsol=all)
```

$$y(x) = x^2 + c_1 \ln(x) + c_2$$

### ✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 16

```
DSolve[x*y''[x]+y'[x]==4*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2 + c_1 \log(x) + c_2$$

## 7.8 problem 2(a)

Internal problem ID [5490]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38

**Problem number:** 2(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_y], [_2nd_order, _exact, _nonlinear], [`

$$(x^2 + 2y') y'' + 2y'x = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

### ✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 15

```
dsolve([(x^2+2*diff(y(x),x))*diff(y(x),x$2)+2*x*diff(y(x),x)=0,y(0) = 1, D(y)(0) = 0],y(x), s
```

$$y(x) = 1$$

$$y(x) = -\frac{x^3}{3} + 1$$

### ✓ Solution by Mathematica

Time used: 0.137 (sec). Leaf size: 19

```
DSolve[{(x^2+2*y'[x])*y''[x]+2*x*y'[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions
```

$$y(x) \rightarrow \text{Indeterminate}$$

$$y(x) \rightarrow 1 - \frac{x^3}{6}$$

## 7.9 problem 2(b)

Internal problem ID [5491]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38

**Problem number:** 2(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x]`, `[_2nd_order, _with_potential_symmetr`

$$yy'' - y^2y' - y'^2 = 0$$

With initial conditions

$$\left[ y(0) = -\frac{1}{2}, y'(0) = 1 \right]$$

### ✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 16

```
dsolve([y(x)*diff(y(x),x$2)=y(x)^2*diff(y(x),x)+(diff(y(x),x))^2,y(0) = -1/2, D(y)(0) = 1],y(x)
```

$$y(x) = -\frac{3}{8e^{\frac{3x}{2}} - 2}$$

### ✓ Solution by Mathematica

Time used: 0.544 (sec). Leaf size: 20

```
DSolve[{y[x]*y'[x]==y[x]^2*y'[x]+(y'[x])^2,{y[0]==-1/2,y'[0]==1}},y[x],x,IncludeSingularSolu
```

$$y(x) \rightarrow \frac{3}{2 - 8e^{3x/2}}$$

## 7.10 problem 2(c)

Internal problem ID [5492]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38

**Problem number:** 2(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$y'' - y'e^y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.516 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)=diff(y(x),x)*exp(y(x)),y(0) = 0, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = x + \ln\left(-\frac{1}{e^x - 2}\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]==y'[x]*Exp[y[x]],{y[0]==0,y'[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

```
{}
```

## 7.11 problem 3(a)

Internal problem ID [5493]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38

**Problem number:** 3(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]`

$$y'' - 1 - y'^2 = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)=1+(diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = -\ln\left(\frac{-c_2 + \tan(x) c_1}{\sec(x)}\right)$$

### ✓ Solution by Mathematica

Time used: 1.746 (sec). Leaf size: 16

```
DSolve[y''[x]==1+(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 - \log(\cos(x + c_1))$$



## 7.12 problem 3(b)

Internal problem ID [5494]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Section 1.9. Reduction of Order. Page 38

**Problem number:** 3(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]`

$$y'' + y'^2 - 1 = 0$$

### ✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+(diff(y(x),x))^2=1,y(x), singsol=all)
```

$$y(x) = x + \ln\left(\frac{e^{-2x}c_1}{2} - \frac{c_2}{2}\right)$$

### ✓ Solution by Mathematica

Time used: 0.371 (sec). Leaf size: 46

```
DSolve[y''[x]+(y'[x])^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\log(e^x) + \log(e^{2x} + e^{2c_1}) + c_2$$

$$y(x) \rightarrow -\log(e^x) + \log(e^{2x}) + c_2$$

## 8 Chapter 1. What is a differential equation.

### Problems for Review and Discovery. Page 53

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## 8.1 problem 1(a)

Internal problem ID [5495]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 1(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y'x + y - x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{x}{2} + \frac{c_1}{x}$$

### ✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 17

```
DSolve[x*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x}{2} + \frac{c_1}{x}$$

## 8.2 problem 1(b)

Internal problem ID [5496]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 1(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$x^2y' + y - x^2 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x)+y(x)=x^2,y(x), singsol=all)
```

$$y(x) = x - \text{Ei}_1\left(\frac{1}{x}\right) e^{\frac{1}{x}} + e^{\frac{1}{x}} c_1$$

### ✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 22

```
DSolve[x^2*y'[x]+y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x + e^{\frac{1}{x}} \left( \text{ExpIntegralEi}\left(-\frac{1}{x}\right) + c_1 \right)$$

### 8.3 problem 1(c)

Internal problem ID [5497]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 1(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$x^2 y' - y = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x^2*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{1}{x}}$$

#### ✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 20

```
DSolve[x^2*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-1/x}$$

$$y(x) \rightarrow 0$$

## 8.4 problem 1(d)

Internal problem ID [5498]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 1(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_separable]`

$$y' \sec(x) - \sec(y) = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(sec(x)*diff(y(x),x)=sec(y(x)),y(x), singsol=all)
```

$$y(x) = \arcsin(\sin(x) + c_1)$$

### ✓ Solution by Mathematica

Time used: 0.323 (sec). Leaf size: 11

```
DSolve[Sec[x]*y'[x]==Sec[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \arcsin(\sin(x) + c_1)$$

## 8.5 problem 1(e)

Internal problem ID [5499]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 1(e).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _dAlembert]`

$$y' - \frac{y^2 + x^2}{x^2 - y^2} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x)=(x^2+y(x)^2)/(x^2-y(x)^2),y(x), singsol=all)
```

$$y(x) = \text{RootOf} \left( \int^{-z} \frac{-a^2 - 1}{-a^3 + -a^2 - a + 1} d_a + \ln(x) + c_1 \right) x$$

### ✓ Solution by Mathematica

Time used: 0.117 (sec). Leaf size: 67

```
DSolve[y'[x]==(x^2+y[x]^2)/(x^2-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[ \text{RootSum} \left[ \#1^3 + \#1^2 - \#1 + 1 \&, \frac{\#1 \log \left( \frac{y(x)}{x} - \#1 \right) - \log \left( \frac{y(x)}{x} - \#1 \right)}{3\#1 - 1} \& \right] = \right. \\ \left. -\log(x) + c_1, y(x) \right]$$

## 8.6 problem 1(f)

Internal problem ID [5500]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 1(f).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cla`

$$y' - \frac{x + 2y}{-y + 2x} = 0$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=(x+2*y(x))/(2*x-y(x)),y(x), singsol=all)
```

$$y(x) = \tan \left( \text{RootOf} \left( -4\_Z + \ln \left( \frac{1}{\cos(\_Z)^2} \right) + 2 \ln(x) + 2c_1 \right) \right) x$$

### ✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(x+2*y[x]^2)/(2*x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

Not solved



## 8.7 problem 1(g)

Internal problem ID [5501]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 1(g).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$2xy + x^2y' = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve(2*x*y(x)+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2}$$

### ✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 16

```
DSolve[2*x*y[x]+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1}{x^2}$$

$$y(x) \rightarrow 0$$

## 8.8 problem 1(h)

Internal problem ID [5502]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 1(h).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$-\sin(x)\sin(y) + \cos(x)\cos(y)y' = 0$$

### ✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 11

```
dsolve(-sin(x)*sin(y(x))+cos(x)*cos(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{c_1}{\cos(x)}\right)$$

### ✓ Solution by Mathematica

Time used: 3.399 (sec). Leaf size: 19

```
DSolve[-Sin[x]*Sin[y[x]]+Cos[x]*Cos[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \arcsin\left(\frac{1}{2}c_1 \sec(x)\right)$$

$$y(x) \rightarrow 0$$

## 8.9 problem 2(a)

Internal problem ID [5503]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 2(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$-y + y'x - 2x = 0$$

With initial conditions

$$[y(1) = 0]$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve([x*diff(y(x),x)-y(x)=2*x,y(1) = 0],y(x), singsol=all)
```

$$y(x) = 2 \ln(x) x$$

### ✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 10

```
DSolve[{x*y'[x]-y[x]==2*x,{y[1]==0}],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2x \log(x)$$

## 8.10 problem 2(b)

Internal problem ID [5504]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 2(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear]`

$$x^2 y' - 2y - 3x^2 = 0$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 39

```
dsolve([x^2*diff(y(x),x)-2*y(x)=3*x^2,y(1) = 2],y(x), singsol=all)
```

$$y(x) = 3x - e^{2-\frac{2}{x}} + 6e^{-\frac{2}{x}} \operatorname{Ei}_1\left(-\frac{2}{x}\right) - 6 \operatorname{Ei}_1(-2) e^{-\frac{2}{x}}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 34

```
DSolve[{x^2*y'[x]-2*y[x]==3*x^2,{y[1]==2}],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3x - e^{-2/x} \left( 6 \operatorname{ExpIntegralEi}\left(\frac{2}{x}\right) - 6 \operatorname{ExpIntegralEi}(2) + e^2 \right)$$

## 8.11 problem 2(c)

Internal problem ID [5505]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 2(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y^2 y' - x = 0$$

With initial conditions

$$[y(-1) = 3]$$

### ✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 15

```
dsolve([y(x)^2*diff(y(x),x)=x,y(-1) = 3],y(x), singsol=all)
```

$$y(x) = \frac{(12x^2 + 204)^{\frac{1}{3}}}{2}$$

### ✓ Solution by Mathematica

Time used: 0.192 (sec). Leaf size: 22

```
DSolve[{y[x]^2*y'[x]==x,{y[-1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt[3]{\frac{3}{2} \sqrt[3]{x^2 + 17}}$$

## 8.12 problem 2(d)

Internal problem ID [5506]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 2(d).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y' \csc(x) - \csc(y) = 0$$

With initial conditions

$$\left[ y\left(\frac{\pi}{2}\right) = 1 \right]$$

### ✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 10

```
dsolve([csc(x)*diff(y(x),x)=csc(y(x)),y(1/2*Pi) = 1],y(x), singsol=all)
```

$$y(x) = \arccos(\cos(x) + \cos(1))$$

### ✓ Solution by Mathematica

Time used: 0.371 (sec). Leaf size: 11

```
DSolve[{Csc[x]*y'[x]==Csc[y[x]],{y[Pi/2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \arccos(\cos(x) + \cos(1))$$

### 8.13 problem 2(e)

Internal problem ID [5507]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 2(e).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cla`

$$y' - \frac{y+x}{-y+x} = 0$$

With initial conditions

$$[y(1) = 1]$$

#### ✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 30

```
dsolve([diff(y(x),x)=(x+y(x))/(x-y(x)),y(1) = 1],y(x), singsol=all)
```

$$y(x) = \tan(\text{RootOf}(4_Z - 2 \ln(\sec(_Z)^2) - 4 \ln(x) + 2 \ln(2) - \pi)) x$$

#### ✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 46

```
DSolve[{y'[x]==(x+y[x])/(x-y[x]),{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2} \log\left(\frac{y(x)^2}{x^2} + 1\right) - \arctan\left(\frac{y(x)}{x}\right) = \frac{1}{4}(2 \log(2) - \pi) - \log(x), y(x)\right]$$

## 8.14 problem 2(f)

Internal problem ID [5508]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 2(f).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$y' - \frac{x^2 + 2y^2}{x^2 - 2y^2} = 0$$

With initial conditions

$$[y(0) = 1]$$

**X** Solution by Maple

```
dsolve([diff(y(x),x)=(x^2+2*y(x)^2)/(x^2-2*y(x)^2),y(0) = 1],y(x), singsol=all)
```

No solution found

**X** Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]==(x^2+2*y[x]^2)/(x^2-2*y[x]^2),{y[0]==1}},y[x],x,IncludeSingularSolutions -> Tr
```

{}



## 8.15 problem 2(g)

Internal problem ID [5509]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 2(g).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [separable]

$$2x \cos(y) - x^2 \sin(y) y' = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.766 (sec). Leaf size: 11

```
dsolve([2*x*cos(y(x))-x^2*sin(y(x))*diff(y(x),x)=0,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{\cos(1)}{x^2}\right)$$

✓ Solution by Mathematica

Time used: 29.176 (sec). Leaf size: 12

```
DSolve[{2*x*Cos[y[x]]-x^2*Sin[y[x]]*y'[x]==0,{y[1]==1}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \rightarrow \sec^{-1}(x^2 \sec(1))$$

## 8.16 problem 2(h)

Internal problem ID [5510]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 2(h).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$\frac{1}{y} - \frac{xy'}{y^2} = 0$$

With initial conditions

$$[y(0) = 2]$$

**X** Solution by Maple

```
dsolve([1/y(x)-x/y(x)^2*diff(y(x),x)=0,y(0) = 2],y(x), singsol=all)
```

No solution found

**X** Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{1/y[x]-x/y[x]^2*y'[x]==0,{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

{}

## 8.17 problem 4(a)

Internal problem ID [5511]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 4(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible,`

$$yy'' - y'^2 = 0$$

### ✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 14

```
dsolve(y(x)*diff(y(x),x$2)-(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = e^{c_1 x} c_2$$

### ✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 14

```
DSolve[y[x]*y'[x]-(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 e^{c_1 x}$$

## 8.18 problem 4(b)

Internal problem ID [5512]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 4(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$xy'' - y' + 2y'^3 = 0$$

### ✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 37

```
dsolve(x*diff(y(x),x$2)=diff(y(x),x)-2*(diff(y(x),x))^3,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{2x^2 - c_1}}{2} + c_2$$

$$y(x) = -\frac{\sqrt{2x^2 - c_1}}{2} + c_2$$

### ✓ Solution by Mathematica

Time used: 0.591 (sec). Leaf size: 96

```
DSolve[x*y''[x]==y'[x]-2*(y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 - \frac{1}{2}\sqrt{2x^2 + e^{2c_1}}$$

$$y(x) \rightarrow \frac{1}{2}\sqrt{2x^2 + e^{2c_1}} + c_2$$

$$y(x) \rightarrow -\frac{\sqrt{x^2}}{\sqrt{2}} + c_2$$

$$y(x) \rightarrow \frac{\sqrt{x^2}}{\sqrt{2}} + c_2$$

## 8.19 problem 4(c)

Internal problem ID [5513]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 4(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$yy'' + y' = 0$$

### ✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 24

```
dsolve(y(x)*diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = e^{\text{RootOf}(-e^{c_1} \text{Ei}_1(-_Z+c_1)+x+c_2)}$$

### ✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 26

```
DSolve[y[x]*y'[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \text{InverseFunction}[-e^{c_1} \text{ExpIntegralEi}(\log(\#1) - c_1)\&][x + c_2]$$

## 8.20 problem 4(d)

Internal problem ID [5514]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 1. What is a differential equation. Problems for Review and Discovery. Page 53

**Problem number:** 4(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$xy'' - 3y' - 5x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x$2)-3*diff(y(x),x)=5*x,y(x), singsol=all)
```

$$y(x) = \frac{(2c_1x^2 - 5)^2}{16c_1} + c_2$$

### ✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 23

```
DSolve[x*y''[x]-3*y'[x]==5*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}x^2(-5 + c_1x^2) + c_2$$

## 9 Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients.

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## 9.1 problem 1(a)

Internal problem ID [5515]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x]`

$$y'' + y' - 6y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-3x} + e^{2x} c_2$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''[x]+y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-3x} (c_2 e^{5x} + c_1)$$



## 9.2 problem 1(b)

Internal problem ID [5516]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 2y' + y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + c_2e^{-x}x$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[y''[x]+2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(c_2x + c_1)$$

### 9.3 problem 1(c)

Internal problem ID [5517]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 8y = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(2\sqrt{2}x) + c_2 \cos(2\sqrt{2}x)$$

#### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y''[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos(2\sqrt{2}x) + c_2 \sin(2\sqrt{2}x)$$

## 9.4 problem 1(d)

Internal problem ID [5518]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$2y'' - 4y' + 4y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x \sin(x) + c_2 e^x \cos(x)$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[2*y''[x]-4*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x (c_2 \cos(x) + c_1 \sin(x))$$

## 9.5 problem 1(e)

Internal problem ID [5519]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 4y' + 4y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{2x}c_1 + c_2e^{2x}x$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[y''[x]-4*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{2x}(c_2x + c_1)$$

## 9.6 problem 1(f)

Internal problem ID [5520]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(f).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 9y' + 20y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-9*diff(y(x),x)+20*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{5x} + c_2 e^{4x}$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y''[x]-9*y'[x]+20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{4x}(c_2 e^x + c_1)$$

## 9.7 problem 1(g)

Internal problem ID [5521]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(g).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$2y'' + 2y' + 3y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(2*diff(y(x),x$2)+2*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{5}x}{2}\right) + c_2 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{5}x}{2}\right)$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[2*y''[x]+2*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x/2} \left( c_2 \cos\left(\frac{\sqrt{5}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{5}x}{2}\right) \right)$$

## 9.8 problem 1(h)

Internal problem ID [5522]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(h).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$4y'' - 12y' + 9y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(4*diff(y(x),x$2)-12*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{3x}{2}} + c_2 e^{\frac{3x}{2}} x$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[4*y'[x]-12*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{3x/2}(c_2 x + c_1)$$

## 9.9 problem 1(i)

Internal problem ID [5523]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(i).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + c_2 \cos(x)$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 16

```
DSolve[y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos(x) + c_2 \sin(x)$$



## 9.10 problem 1(j)

Internal problem ID [5524]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(j).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 6y' + 25y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+25*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} \sin(4x) + c_2 e^{3x} \cos(4x)$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 26

```
DSolve[y''[x]-6*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{3x}(c_2 \cos(4x) + c_1 \sin(4x))$$

## 9.11 problem 1(k)

Internal problem ID [5525]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(k).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$4y'' + 20y' + 25y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(4*diff(y(x),x$2)+20*diff(y(x),x)+25*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{5x}{2}} + c_2 e^{-\frac{5x}{2}} x$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[4*y''[x]+20*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-5x/2}(c_2 x + c_1)$$

## 9.12 problem 1(1)

Internal problem ID [5526]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(1).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 2y' + 3y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} \sin(\sqrt{2}x) + c_2 e^{-x} \cos(\sqrt{2}x)$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 34

```
DSolve[y''[x]+2*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x} \left( c_2 \cos(\sqrt{2}x) + c_1 \sin(\sqrt{2}x) \right)$$

### 9.13 problem 1(m)

Internal problem ID [5527]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(m).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 4y = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)=4*y(x),y(x), singsol=all)
```

$$y(x) = e^{-2x}c_1 + e^{2x}c_2$$

#### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''[x]==4*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x}(c_1 e^{4x} + c_2)$$

## 9.14 problem 1(n)

Internal problem ID [5528]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(n).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$4y'' - 8y' + 7y = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(4*diff(y(x),x$2)-8*diff(y(x),x)+7*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x \sin\left(\frac{\sqrt{3}x}{2}\right) + c_2 e^x \cos\left(\frac{\sqrt{3}x}{2}\right)$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 38

```
DSolve[4*y''[x]-8*y'[x]+7*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x \left( c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right) \right)$$

## 9.15 problem 1(o)

Internal problem ID [5529]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(o).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$2y'' + y' - y = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(2*diff(y(x),x$2)+diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x}{2}} + c_2 e^{-x}$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 24

```
DSolve[2*y''[x]+y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x} (c_1 e^{3x/2} + c_2)$$

## 9.16 problem 1(p)

Internal problem ID [5530]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(p).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 4y' + 5y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-2x} \sin(x) + c_2 e^{-2x} \cos(x)$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''[x]+4*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x} (c_2 \cos(x) + c_1 \sin(x))$$

## 9.17 problem 1(q)

Internal problem ID [5531]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(q).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 4y' + 5y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-2x} \sin(x) + c_2 e^{-2x} \cos(x)$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''[x]+4*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x} (c_2 \cos(x) + c_1 \sin(x))$$



## 9.18 problem 1(r)

Internal problem ID [5532]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 1(r).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 4y' - 5y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)-5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-5x} + e^x c_2$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y''[x]+4*y'[x]-5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-5x} + c_2 e^x$$

## 9.19 problem 2(a)

Internal problem ID [5533]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 2(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 5y' + 6y = 0$$

With initial conditions

$$[y(1) = e^2, y'(1) = 3e^2]$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=0,y(1) = exp(2), D(y)(1) = 3*exp(2)],y(x), sings
```

$$y(x) = e^{3x-1}$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 12

```
DSolve[{y'[x]-5*y'[x]+6*y[x]==0,{y[1]==Exp[2],y'[1]==3*Exp[2]}],y[x],x,IncludeSingularSoluti
```

$$y(x) \rightarrow e^{3x-1}$$

## 9.20 problem 2(b)

Internal problem ID [5534]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 2(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 6y' + 5y = 0$$

With initial conditions

$$[y(0) = 3, y'(0) = 11]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)-6*diff(y(x),x)+5*y(x)=0,y(0) = 3, D(y)(0) = 11],y(x), singsol=all)
```

$$y(x) = 2e^{5x} + e^x$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{y'[x]-5*y'[x]+6*y[x]==0,{y[0]==3,y'[0]==11}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow e^{2x}(5e^x - 2)$$

## 9.21 problem 2(c)

Internal problem ID [5535]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 2(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 6y' + 9y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 5]$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=0,y(0) = 0, D(y)(0) = 5],y(x), singsol=all)
```

$$y(x) = 5e^{3x}x$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 13

```
DSolve[{y'[x]-6*y'[x]+9*y[x]==0,{y[0]==0,y'[0]==5}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 5e^{3x}x$$

## 9.22 problem 2(d)

Internal problem ID [5536]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 2(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 4y' + 5y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = e^{-2x}(2 \sin(x) + \cos(x))$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{y''[x]+4*y'[x]+5*y[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x}(2 \sin(x) + \cos(x))$$

## 9.23 problem 2(e)

Internal problem ID [5537]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 2(e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 4y' + 2y = 0$$

With initial conditions

$$[y(0) = -1, y'(0) = 2 + 3\sqrt{2}]$$

### ✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 24

```
dsolve([diff(y(x),x$2)+4*diff(y(x),x)+2*y(x)=0,y(0) = -1, D(y)(0) = 2+3*2^(1/2)],y(x), singso
```

$$y(x) = e^{(-2+\sqrt{2})x} - 2e^{-(2+\sqrt{2})x}$$

### ✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 30

```
DSolve[{y'[x]+4*y'[x]+2*y[x]==0,{y[0]==-1,y'[0]==2+3*Sqrt[2]}],y[x],x,IncludeSingularSolutio
```

$$y(x) \rightarrow e^{-((2+\sqrt{2})x)} (e^{2\sqrt{2}x} - 2)$$

## 9.24 problem 2(f)

Internal problem ID [5538]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 2(f).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 8y' - 9y = 0$$

With initial conditions

$$[y(1) = 2, y'(1) = 0]$$

### ✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)+8*diff(y(x),x)-9*y(x)=0,y(1) = 2, D(y)(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{e^{9-9x}}{5} + \frac{9e^{x-1}}{5}$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[{y'[x]+8*y'[x]-9*y[x]==0,{y[1]==2,y'[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{5}e^{9-9x} + \frac{9e^{x-1}}{5}$$

## 9.25 problem 5(a)

Internal problem ID [5539]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 5(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_Emden, _Fowler]`

$$x^2 y'' + 3y'x + 10y = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(3 \ln(x))}{x} + \frac{c_2 \cos(3 \ln(x))}{x}$$

### ✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 26

```
DSolve[x^2*y'[x]+3*x*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2 \cos(3 \log(x)) + c_1 \sin(3 \log(x))}{x}$$



## 9.26 problem 5(b)

Internal problem ID [5540]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 5(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_Emden, _Fowler]`

$$2x^2y'' + 10y'x + 8y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*x^2*diff(y(x),x$2)+10*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2} + \frac{c_2 \ln(x)}{x^2}$$

### ✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 18

```
DSolve[2*x^2*y''[x]+10*x*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2c_2 \log(x) + c_1}{x^2}$$

## 9.27 problem 5(c)

Internal problem ID [5541]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 5(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2 y'' + 2y'x - 12y = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-12*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^3 + \frac{c_2}{x^4}$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+2*x*y'[x]-12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2 x^7 + c_1}{x^4}$$

## 9.28 problem 5(d)

Internal problem ID [5542]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 5(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [[\_Emden, \_Fowler]]

$$4x^2y'' - 3y = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(4*x^2*diff(y(x),x$2)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{\frac{3}{2}}c_1 + \frac{c_2}{\sqrt{x}}$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[4*x^2*y''[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2x^2 + c_1}{\sqrt{x}}$$

## 9.29 problem 5(e)

Internal problem ID [5543]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 5(e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$x^2 y'' - 3y'x + 4y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^2 + c_2 x^2 \ln(x)$$

### ✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[x^2*y'[x]-3*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2(2c_2 \log(x) + c_1)$$

### 9.30 problem 5(f)

Internal problem ID [5544]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 5(f).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$x^2 y'' + 2y'x - 6y = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^3} + c_2 x^2$$

#### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[x^2*y'[x]+2*x*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2 x^5 + c_1}{x^3}$$

### 9.31 problem 5(g)

Internal problem ID [5545]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 5(g).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2 y'' + 2y'x + 3y = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{\sqrt{11} \ln(x)}{2}\right)}{\sqrt{x}} + \frac{c_2 \cos\left(\frac{\sqrt{11} \ln(x)}{2}\right)}{\sqrt{x}}$$

#### ✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 42

```
DSolve[x^2*y'[x]+2*x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2 \cos\left(\frac{1}{2}\sqrt{11} \log(x)\right) + c_1 \sin\left(\frac{1}{2}\sqrt{11} \log(x)\right)}{\sqrt{x}}$$

## 9.32 problem 5(h)

Internal problem ID [5546]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 5(h).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$x^2 y'' + y' x - 2y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\sqrt{2}} + c_2 x^{-\sqrt{2}}$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 28

```
DSolve[x^2*y'[x]+x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 x^{-\sqrt{2}} + c_2 x^{\sqrt{2}}$$

### 9.33 problem 5(i)

Internal problem ID [5547]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.1. Linear Equations with Constant Coefficients. Page 62

**Problem number:** 5(i).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$x^2y'' + y'x - 16y = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-16*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^4} + c_2x^4$$

#### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2x^8 + c_1}{x^4}$$



**10 Chapter 2. Second-Order Linear Equations.**  
**Section 2.2. THE METHOD OF**  
**UNDETERMINED COEFFICIENTS. Page 67**

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## 10.1 problem 1(a)

Internal problem ID [5548]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67

**Problem number:** 1(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 3y' - 10y - 6e^{4x} = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)-10*y(x)=6*exp(4*x),y(x), singsol=all)
```

$$y(x) = e^{-5x}c_2 + e^{2x}c_1 + \frac{e^{4x}}{3}$$

### ✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 31

```
DSolve[y''[x]+3*y'[x]-10*y[x]==6*Exp[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{4x}}{3} + c_1 e^{-5x} + c_2 e^{2x}$$

## 10.2 problem 1(b)

Internal problem ID [5549]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67

**Problem number:** 1(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + 4y - 3\sin(x) = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+4*y(x)=3*sin(x),y(x), singsol=all)
```

$$y(x) = \sin(2x)c_2 + \cos(2x)c_1 + \sin(x)$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 22

```
DSolve[y''[x]+4*y[x]==3*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sin(x) + c_1 \cos(2x) + c_2 \sin(2x)$$

### 10.3 problem 1(c)

Internal problem ID [5550]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67

**Problem number:** 1(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' + 10y' + 25y - 14e^{-5x} = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+10*diff(y(x),x)+25*y(x)=14*exp(-5*x),y(x), singsol=all)
```

$$y(x) = e^{-5x}c_2 + e^{-5x}xc_1 + 7x^2e^{-5x}$$

#### ✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 22

```
DSolve[y''[x]+10*y'[x]+25*y[x]==14*Exp[-5*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-5x}(x(7x + c_2) + c_1)$$

## 10.4 problem 1(d)

Internal problem ID [5551]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67

**Problem number:** 1(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y' + 5y - 25x^2 - 12 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+5*y(x)=25*x^2+12,y(x), singsol=all)
```

$$y(x) = e^x \sin(2x) c_2 + e^x \cos(2x) c_1 + 5x^2 + 4x + 2$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 33

```
DSolve[y''[x]-2*y'[x]+5*y[x]==25*x^2+12,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(5x + 4) + e^x(c_2 \cos(2x) + c_1 \sin(2x)) + 2$$

## 10.5 problem 1(e)

Internal problem ID [5552]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67

**Problem number:** 1(e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y' - 6y - 20e^{-2x} = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-6*y(x)=20*exp(-2*x),y(x), singsol=all)
```

$$y(x) = e^{3x}c_2 + e^{-2x}c_1 - 4e^{-2x}x$$

### ✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 32

```
DSolve[y''[x]-y'[x]-6*y[x]==20*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{5}e^{-2x}(-20x + 5c_2e^{5x} - 4 + 5c_1)$$

## 10.6 problem 1(f)

Internal problem ID [5553]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67

**Problem number:** 1(f).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' - 3y' + 2y - 14 \sin(2x) + 18 \cos(2x) = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=14*sin(2*x)-18*cos(2*x),y(x), singsol=all)
```

$$y(x) = e^{2x}c_1 + e^x c_2 + 2 \sin(2x) + 3 \cos(2x)$$

### ✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 31

```
DSolve[y''[x]-3*y'[x]+2*y[x]==14*Sin[2*x]-18*Cos[2*x],y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow 2 \sin(2x) + 3 \cos(2x) + e^x(c_2 e^x + c_1)$$

## 10.7 problem 1(g)

Internal problem ID [5554]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67

**Problem number:** 1(g).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + y - 2 \cos(x) = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+y(x)=2*cos(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + \sin(x) x$$

### ✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 20

```
DSolve[y''[x]+y[x]==2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (1 + c_1) \cos(x) + (x + c_2) \sin(x)$$



## 10.8 problem 1(h)

Internal problem ID [5555]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67

**Problem number:** 1(h).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' - 2y' - 12x + 10 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)=12*x-10,y(x), singsol=all)
```

$$y(x) = \frac{e^{2x}c_1}{2} - 3x^2 + 2x + c_2$$

### ✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 26

```
DSolve[y''[x]-2*y'[x]==12*x-10,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (2 - 3x)x + \frac{1}{2}c_1e^{2x} + c_2$$

## 10.9 problem 1(i)

Internal problem ID [5556]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67

**Problem number:** 1(i).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' - 2y' + y - 6e^x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=6*exp(x),y(x), singsol=all)
```

$$y(x) = e^x c_2 + e^x x c_1 + 3e^x x^2$$

### ✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 20

```
DSolve[y''[x]-2*y'[x]+y[x]==6*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x(x(3x + c_2) + c_1)$$

## 10.10 problem 1(j)

Internal problem ID [5557]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67

**Problem number:** 1(j).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + 2y - \sin(x)e^x = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=exp(x)*sin(x),y(x), singsol=all)
```

$$y(x) = e^x \sin(x) c_2 + e^x \cos(x) c_1 + \frac{e^x (\sin(x) - \cos(x) x)}{2}$$

### ✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 28

```
DSolve[y''[x]-2*y'[x]+2*y[x]==Exp[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{2}e^x((x - 2c_2) \cos(x) - 2c_1 \sin(x))$$

## 10.11 problem 1(k)

Internal problem ID [5558]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67

**Problem number:** 1(k).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' + y' - 10x^4 - 2 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=10*x^4+2,y(x), singsol=all)
```

$$y(x) = -e^{-x}c_1 - 120x^2 + 40x^3 - 10x^4 + 2x^5 + 242x + c_2$$

### ✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 35

```
DSolve[y''[x]+y'[x]==10*x^4+2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2x(x(x((x - 5)x + 20) - 60) + 121) - c_1e^{-x} + c_2$$

## 10.12 problem 3(a)

Internal problem ID [5559]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67

**Problem number:** 3(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y - 4 \cos(2x) - 6 \cos(x) - 8x^2 + 4x = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+4*y(x)=4*cos(2*x)+6*cos(x)+8*x^2-4*x,y(x), singsol=all)
```

$$y(x) = \sin(2x) c_2 + \cos(2x) c_1 + x \sin(2x) + 2x^2 - 1 - x + 2 \cos(x) + \frac{\cos(2x)}{4}$$

### ✓ Solution by Mathematica

Time used: 0.19 (sec). Leaf size: 39

```
DSolve[y''[x]+4*y[x]==4*Cos[2*x]+6*Cos[x]+8*x^2-4*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (x - 1)(2x + 1) + 2 \cos(x) + \left(\frac{1}{2} + c_1\right) \cos(2x) + (x + c_2) \sin(2x)$$

### 10.13 problem 3(b)

Internal problem ID [5560]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67

**Problem number:** 3(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y - 2 \sin(3x) - 4 \sin(x) + 26e^{-2x} - 27x^3 = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)+9*y(x)=2*sin(3*x)+4*sin(x)-26*exp(-2*x)+27*x^3,y(x), singsol=all)
```

$$y(x) = \sin(3x) c_2 + \cos(3x) c_1 - \frac{4x \cos(x)^3}{3} + 3x^3 + 2 \sin(x) \cos(x)^2 + \cos(x) x - 2x - 2e^{-2x}$$

#### ✓ Solution by Mathematica

Time used: 1.067 (sec). Leaf size: 51

```
DSolve[y''[x]+9*y[x]==2*Sin[3*x]+4*Sin[x]-26*Exp[-2*x]+27*x^3,y[x],x,IncludeSingularSolutions
```

$$y(x) \rightarrow 3x^3 - 2x - 2e^{-2x} + \frac{\sin(x)}{2} + \left(-\frac{x}{3} + c_1\right) \cos(3x) + \left(\frac{1}{18} + c_2\right) \sin(3x)$$

## 10.14 problem 4(a)

Internal problem ID [5561]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67

**Problem number:** 4(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' - 3y - e^{2x} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-3*y(x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = e^{\sqrt{3}x}c_2 + e^{-\sqrt{3}x}c_1 + e^{2x}$$

### ✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 36

```
DSolve[y''[x]-3*y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{2x} + c_1 e^{\sqrt{3}x} + c_2 e^{-\sqrt{3}x}$$

## 10.15 problem 4(b)

Internal problem ID [5562]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.2. THE METHOD OF UNDETERMINED COEFFICIENTS. Page 67

**Problem number:** 4(b).

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$y''' + y' - \sin(x) = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$3)+diff(y(x),x)=sin(x),y(x), singsol=all)
```

$$y(x) = -\cos(x) - \frac{\sin(x)x}{2} + c_1 \sin(x) - c_2 \cos(x) + c_3$$

### ✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 31

```
DSolve[y'''[x]+y'[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{2}(1 + 2c_2) \cos(x) + \left(-\frac{x}{2} + c_1\right) \sin(x) + c_3$$



**11 Chapter 2. Second-Order Linear Equations.**  
**Section 2.3. THE METHOD OF VARIATION OF**  
**PARAMETERS. Page 71**

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## 11.1 problem 1(a)

Internal problem ID [5563]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 1(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y - \tan(2x) = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+4*y(x)=tan(2*x),y(x), singsol=all)
```

$$y(x) = \sin(2x) c_2 + \cos(2x) c_1 - \frac{\cos(2x) \ln(\sec(2x) + \tan(2x))}{4}$$

### ✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 37

```
DSolve[y''[x]+4*y[x]==Tan[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}((-1 + 4c_2) \sin(2x) - \cos(2x)(\operatorname{arctanh}(\sin(2x)) - 4c_1))$$

## 11.2 problem 1(b)

Internal problem ID [5564]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 1(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + y - e^{-x} \ln(x) = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=exp(-x)*ln(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + e^{-x} x c_1 + \frac{x^2(2 \ln(x) - 3) e^{-x}}{4}$$

### ✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 36

```
DSolve[y''[x]+2*y'[x]+y[x]==Exp[-x]*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4} e^{-x} (-3x^2 + 2x^2 \log(x) + 4c_2 x + 4c_1)$$

### 11.3 problem 1(c)

Internal problem ID [5565]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 1(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' - 3y - 64e^{-x}x = 0$$

#### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=64*x*exp(-x),y(x), singsol=all)
```

$$y(x) = e^{3x}c_2 + e^{-x}c_1 + (-8x^2 - 4x)e^{-x}$$

#### ✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 31

```
DSolve[y''[x]-2*y'[x]-3*y[x]==64*x*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(-4x(2x + 1) + c_2e^{4x} - 1 + c_1)$$

## 11.4 problem 1(d)

Internal problem ID [5566]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 1(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + 2y' + 5y - e^{-x} \sec(2x) = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=exp(-x)*sec(2*x),y(x), singsol=all)
```

$$y(x) = e^{-x} \sin(2x) c_2 + e^{-x} \cos(2x) c_1 - \frac{(\cos(2x) \ln(\sec(2x)) - 2x \sin(2x)) e^{-x}}{4}$$

### ✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 42

```
DSolve[y''[x]+2*y'[x]+5*y[x]==Exp[-x]*Sec[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4} e^{-x} (2(x + 2c_1) \sin(2x) + \cos(2x) (\log(\cos(2x)) + 4c_2))$$

## 11.5 problem 1(e)

Internal problem ID [5567]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 1(e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2y'' + 3y' + y - e^{-3x} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(2*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=exp(-3*x),y(x), singsol=all)
```

$$y(x) = \frac{e^{-3x}}{10} - 2e^{-x}c_1 + c_2e^{-\frac{x}{2}}$$

### ✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 45

```
DSolve[y''[x]+3*y'[x]+y[x]==Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-3x} + c_1e^{-\frac{1}{2}(3+\sqrt{5})x} + c_2e^{\frac{1}{2}(\sqrt{5}-3)x}$$

## 11.6 problem 1(f)

Internal problem ID [5568]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 1(f).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' - 3y' + 2y - \frac{1}{e^{-x} + 1} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=1/(1+exp(-x)),y(x), singsol=all)
```

$$y(x) = (e^x c_1 - \ln(e^x) - \ln(e^x) e^x + \ln(e^x + 1)(e^x + 1) - 1 + c_2) e^x$$

### ✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 34

```
DSolve[y''[x]-3*y'[x]+2*y[x]==1/(1+Exp[-x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x(2(e^x + 1) \operatorname{arctanh}(2e^x + 1) + c_2 e^x - 1 + c_1)$$

## 11.7 problem 2(a)

Internal problem ID [5569]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 2(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + y - \sec(x) = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+y(x)=sec(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + \sin(x) x - \ln(\sec(x)) \cos(x)$$

### ✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (x + c_2) \sin(x) + \cos(x)(\log(\cos(x)) + c_1)$$



## 11.8 problem 2(b)

Internal problem ID [5570]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 2(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - \cot(x)^2 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+y(x)=cot(x)^2,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 - 2 - \cos(x) \ln(\csc(x) - \cot(x))$$

### ✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 34

```
DSolve[y''[x]+y[x]==Cot[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 \sin(x) + \cos(x) \left( -\log\left(\sin\left(\frac{x}{2}\right)\right) + \log\left(\cos\left(\frac{x}{2}\right)\right) + c_1 \right) - 2$$

## 11.9 problem 2(c)

Internal problem ID [5571]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 2(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + y - \cot(2x) = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+y(x)=cot(2*x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + \frac{\sin(x) \ln(\csc(x) - \cot(x))}{2} + \frac{\cos(x) \ln(\sec(x) + \tan(x))}{2}$$

### ✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 45

```
DSolve[y''[x]+y[x]==Cot[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} \left( \cos(x) (\operatorname{arctanh}(\sin(x)) + 2c_1) + \sin(x) \left( \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + 2c_2 \right) \right)$$

## 11.10 problem 2(d)

Internal problem ID [5572]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 2(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - x \cos(x) = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+y(x)=x*cos(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + \frac{\cos(x) x}{4} + \frac{\sin(x) x^2}{4} - \frac{\sin(x)}{4}$$

### ✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 34

```
DSolve[y''[x]+y[x]==x*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{8}((2x^2 - 1 + 8c_2) \sin(x) + 2(x + 4c_1) \cos(x))$$

## 11.11 problem 2(e)

Internal problem ID [5573]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 2(e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - \tan(x) = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=tan(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 - \cos(x) \ln(\sec(x) + \tan(x))$$

### ✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \cos(x)(-\operatorname{arctanh}(\sin(x)) + c_1) + c_2 \sin(x)$$

## 11.12 problem 2(f)

Internal problem ID [5574]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 2(f).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - \sec(x) \tan(x) = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+y(x)=sec(x)*tan(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + \ln(\sec(x)) \sin(x) - \sin(x) + \cos(x) x$$

### ✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 27

```
DSolve[y''[x]+y[x]==Sec[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \cos(x)(\arctan(\tan(x)) + c_1) + \sin(x)(-\log(\cos(x)) - 1 + c_2)$$

## 11.13 problem 2(g)

Internal problem ID [5575]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 2(g).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - \sec(x) \csc(x) = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)+y(x)=sec(x)*csc(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + \sin(x) \ln(\csc(x) - \cot(x)) - \cos(x) \ln(\sec(x) + \tan(x))$$

### ✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 39

```
DSolve[y''[x]+y[x]==Sec[x]*Csc[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \cos(x)(-\operatorname{arctanh}(\sin(x)) + c_1) + \sin(x) \left( \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + c_2 \right)$$

### 11.14 problem 3

Internal problem ID [5576]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 3.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' - 2y' + y - 2x = 0$$

#### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=2*x,y(x), singsol=all)
```

$$y(x) = e^x c_2 + e^x x c_1 + 2x + 4$$

#### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[y''[x]+2*y'[x]+y[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2(x - 2) + e^{-x}(c_2 x + c_1)$$

## 11.15 problem 4

Internal problem ID [5577]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 4.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y' - 6y - e^{-x} = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-6*y(x)=exp(-x),y(x), singsol=all)
```

$$y(x) = e^{3x}c_2 + e^{-2x}c_1 - \frac{e^{-x}}{4}$$

### ✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 31

```
DSolve[y''[x]-y'[x]-6*y[x]==Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{e^{-x}}{4} + c_1 e^{-2x} + c_2 e^{3x}$$



## 11.16 problem 5(a)

Internal problem ID [5578]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 5(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$(x^2 - 1)y'' - 2y'x + 2y - (x^2 - 1)^2 = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve((x^2-1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=(x^2-1)^2,y(x), singsol=all)
```

$$y(x) = xc_2 + (x^2 + 1)c_1 + \frac{1}{2} + \frac{x^4}{6}$$

### ✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 96

```
DSolve[(x^2-1)*y''[x]-2*x*y'[x]+2*y[x]==(x^2-1)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x \left( x^5 - 4x^3 + 2x^2 - 6c_1 \sqrt{-(x^2 - 1)^2} x + 6(2c_1 - c_2) \sqrt{-(x^2 - 1)^2} + 3x - 2 \right) - 6c_1 \sqrt{-(x^2 - 1)^2}}{6(x^2 - 1)}$$

## 11.17 problem 5(b)

Internal problem ID [5579]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 5(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(x^2 + x)y'' + (-x^2 + 2)y' - (2 + x)y - x(1 + x)^2 = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve((x^2+x)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)-(2+x)*y(x)=x*(x+1)^2,y(x), singsol=all)
```

$$y(x) = \frac{c_2}{x} + e^x c_1 - \frac{x^2}{3} - x - 1$$

### ✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 45

```
DSolve[(x^2+x)*y''[x]+(2-x^2)*y'[x]-(2+x)*y[x]==x*(x+1)^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow -\frac{x^2}{3} - x + \sqrt{2}c_2e^{x+\frac{1}{2}} + \frac{c_1}{\sqrt{2}ex} - 1$$

## 11.18 problem 5(c)

Internal problem ID [5580]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 5(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$(1-x)y'' + y'x - y - (1-x)^2 = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=(1-x)^2,y(x), singsol=all)
```

$$y(x) = xc_2 + e^x c_1 + x^2 + 1$$

### ✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 22

```
DSolve[(1-x)*y'[x]+x*y'[x]-y[x]==(1-x)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2 + x - c_2x + c_1e^x + 1$$

## 11.19 problem 5(d)

Internal problem ID [5581]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 5(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' - y'(1+x) + y - x^2e^{2x} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)-(1+x)*diff(y(x),x)+y(x)=x^2*exp(2*x),y(x), singsol=all)
```

$$y(x) = (x + 1)c_2 + e^x c_1 + \frac{(x - 1)e^{2x}}{2}$$

### ✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 31

```
DSolve[x*y''[x]-(1+x)*y'[x]+y[x]==x^2*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}e^{2x}(x - 1) + c_1e^x - c_2(x + 1)$$

## 11.20 problem 5(e)

Internal problem ID [5582]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.3. THE METHOD OF VARIATION OF PARAMETERS. Page 71

**Problem number:** 5(e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - 2y'x + 2y - e^{-x}x = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=x*exp(-x),y(x), singsol=all)
```

$$y(x) = xc_2 + c_1x^2 + (-e^{-x} + \text{Ei}_1(x)(x+1))x$$

### ✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 29

```
DSolve[x^2*y'[x]-2*x*y'[x]+2*y[x]==x*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(-(x+1)\text{ExpIntegralEi}(-x) + \sinh(x) - \cosh(x) + c_2x + c_1)$$

**12 Chapter 2. Second-Order Linear Equations.**  
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## 12.1 problem 1(a)

Internal problem ID [5583]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74

**Problem number:** 1(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y = 0$$

Given that one solution of the ode is

$$y_1 = \sin(x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)+y(x)=0,sin(x)],y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + c_2 \cos(x)$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 16

```
DSolve[y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos(x) + c_2 \sin(x)$$

## 12.2 problem 1(b)

Internal problem ID [5584]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74

**Problem number:** 1(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - y = 0$$

Given that one solution of the ode is

$$y_1 = e^x$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)-y(x)=0,exp(x)],y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + e^x c_2$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^x + c_2 e^{-x}$$



## 12.3 problem 2

Internal problem ID [5585]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74

**Problem number:** 2.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$xy'' + 3y' = 0$$

Given that one solution of the ode is

$$y_1 = 1$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([x*diff(y(x),x$2)+3*diff(y(x),x)=0,1],y(x), singsol=all)
```

$$y(x) = c_1 + \frac{c_2}{x^2}$$

### ✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 17

```
DSolve[x*y'[x]+3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 - \frac{c_1}{2x^2}$$

## 12.4 problem 3

Internal problem ID [5586]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74

**Problem number:** 3.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$x^2 y'' + y' x - 4y = 0$$

Given that one solution of the ode is

$$y_1 = x^2$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,x^2],y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2} + c_2 x^2$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2 x^4 + c_1}{x^2}$$

## 12.5 problem 4

Internal problem ID [5587]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74

**Problem number:** 4.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Gegenbauer]

$$(-x^2 + 1)y'' - 2y'x + 2y = 0$$

Given that one solution of the ode is

$$y_1 = x$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve([(1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,x],y(x), singsol=all)
```

$$y(x) = c_1x + c_2\left(\frac{\ln(x-1)x}{2} - \frac{\ln(x+1)x}{2} + 1\right)$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 19

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2(x \operatorname{arctanh}(x) - 1) + c_1x$$

## 12.6 problem 5

Internal problem ID [5588]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74

**Problem number:** 5.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$x^2 y'' + y' x + \left(x^2 - \frac{1}{4}\right) y = 0$$

Given that one solution of the ode is

$$y_1 = \frac{\sin(x)}{\sqrt{x}}$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,x^(-1/2)*sin(x)],y(x), singsol=all
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

### ✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{-ix}(2c_1 - ic_2 e^{2ix})}{2\sqrt{x}}$$

## 12.7 problem 6(a)

Internal problem ID [5589]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74

**Problem number:** 6(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{xy'}{x-1} + \frac{y}{x-1} = 0$$

Given that one solution of the ode is

$$y_1 = x$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)-x/(x-1)*diff(y(x),x)+1/(x-1)*y(x)=0,x],y(x), singsol=all)
```

$$y(x) = c_1x + e^x c_2$$

### ✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y' '[x]-x/(x-1)*x*y' [x]+1/(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 12.8 problem 6(b)

Internal problem ID [5590]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74

**Problem number:** 6(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [[\_Emden, \_Fowler]]

$$x^2 y'' + 2y'x - 2y = 0$$

Given that one solution of the ode is

$$y_1 = x$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,x],y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2} + xc_2$$

### ✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]-x/(x-1)*x*y'[x]+1/(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 12.9 problem 6(c)

Internal problem ID [5591]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74

**Problem number:** 6(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - x(2+x)y' + (2+x)y = 0$$

Given that one solution of the ode is

$$y_1 = x$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x$2)-x*(x+2)*diff(y(x),x)+(x+2)*y(x)=0,x],y(x), singsol=all)
```

$$y(x) = c_1 x + c_2 e^x x$$

### ✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]-x*(x+2)*y'[x]+(x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(c_2 e^x + c_1)$$

## 12.10 problem 7

Internal problem ID [5592]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74

**Problem number:** 7.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' - xf(x)y' + f(x)y = 0$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-x*f(x)*diff(y(x),x)+f(x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left( \int e^{\int \frac{-2+f(x)x^2}{x} dx} dx \right) x + xc_2$$

### ✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 44

```
DSolve[y''[x]-x*f[x]*y'[x]+f[x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x \left( c_2 \int_1^x \frac{\exp \left( - \int_1^{K[2]} -f(K[1])K[1]dK[1] \right)}{K[2]^2} dK[2] + c_1 \right)$$



## 12.11 problem 8

Internal problem ID [5593]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Second-Order Linear Equations. Section 2.4. THE USE OF A KNOWN SOLUTION TO FIND ANOTHER. Page 74

**Problem number:** 8.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' - (1 + 2x)y' + (1 + x)y = 0$$

Given that one solution of the ode is

$$y_1 = e^x$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([x*diff(y(x),x$2)-(2*x+1)*diff(y(x),x)+(x+1)*y(x)=0,exp(x)],y(x), singsol=all)
```

$$y(x) = e^x c_1 + c_2 x^2 e^x$$

### ✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 23

```
DSolve[x*y''[x]-(2*x+1)*y'[x]+(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}e^x(c_2x^2 + 2c_1)$$

## 13 Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

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## 13.1 problem 1

Internal problem ID [5594]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 1.

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - 3y'' + 2y' = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + e^x c_2 + c_3 e^{2x}$$

### ✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 25

```
DSolve[y'''[x]-3*y''[x]+2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^x + \frac{1}{2} c_2 e^{2x} + c_3$$

## 13.2 problem 2

Internal problem ID [5595]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 2.

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[_3rd_order, _missing_x]`

$$y''' - 3y'' + 4y' - 2y = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+4*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^x c_1 + e^x \sin(x) c_2 + c_3 e^x \cos(x)$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[y'''[x]-3*y''[x]+4*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x(c_2 \cos(x) + c_1 \sin(x) + c_3)$$

### 13.3 problem 3

Internal problem ID [5596]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 3.

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - y = 0$$

#### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$3)-y(x)=0,y(x), singsol=all)
```

$$y(x) = e^x c_1 + c_2 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}x}{2}\right) + c_3 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}x}{2}\right)$$

#### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 49

```
DSolve[y'''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^x + e^{-x/2} \left( c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_3 \sin\left(\frac{\sqrt{3}x}{2}\right) \right)$$

## 13.4 problem 4

Internal problem ID [5597]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 4.

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' + y = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$3)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + c_2e^{\frac{x}{2}} \sin\left(\frac{\sqrt{3}x}{2}\right) + c_3e^{\frac{x}{2}} \cos\left(\frac{\sqrt{3}x}{2}\right)$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 51

```
DSolve[y'''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1e^{-x} + e^{x/2}\left(c_3 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_2 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

## 13.5 problem 5

Internal problem ID [5598]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 5.

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' + 3y'' + 3y' + y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + c_2e^{-x}x + c_3e^{-x}x^2$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 23

```
DSolve[y'''[x]+3*y''[x]+3*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(x(c_3x + c_2) + c_1)$$

## 13.6 problem 6

Internal problem ID [5599]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 6.

**ODE order:** 4.

**ODE degree:** 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + 4y''' + 6y'' + 4y' + y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$4)+4*diff(y(x),x$3)+6*diff(y(x),x$2)+4*diff(y(x),x)+y(x)=0,y(x), singsol=a
```

$$y(x) = e^{-x}c_1 + c_2e^{-x}x + c_3e^{-x}x^2 + c_4e^{-x}x^3$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y''''[x]+4*y'''[x]+6*y''[x]+4*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(x(x(c_4x + c_3) + c_2) + c_1)$$



## 13.7 problem 7

Internal problem ID [5600]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 7.

**ODE order:** 4.

**ODE degree:** 1.

CAS Maple gives this as type `[_high_order, _missing_x]`

$$y'''' - y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$4)-y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + e^x c_2 + c_3 \sin(x) + c_4 \cos(x)$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 30

```
DSolve[y''''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^x + c_3 e^{-x} + c_2 \cos(x) + c_4 \sin(x)$$

## 13.8 problem 8

Internal problem ID [5601]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 8.

**ODE order:** 4.

**ODE degree:** 1.

CAS Maple gives this as type `[_high_order, _missing_x]`

$$y'''' + 5y'' + 4y = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$4)+5*diff(y(x),x$2)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(2x) + c_2 \cos(2x) + c_3 \sin(x) + c_4 \cos(x)$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y''''[x]+5*y''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos(2x) + c_4 \sin(x) + \cos(x)(2c_2 \sin(x) + c_3)$$

## 13.9 problem 9

Internal problem ID [5602]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 9.

**ODE order:** 4.

**ODE degree:** 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - 2a^2y'' + a^4y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$4)-2*a^2*diff(y(x),x$2)+a^4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1e^{ax} + c_2e^{ax}x + c_3e^{-ax} + c_4e^{-ax}x$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 33

```
DSolve[y''''[x]-2*a^2*y''[x]+a^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-ax}(c_2x + c_1) + e^{ax}(c_4x + c_3)$$

## 13.10 problem 10

Internal problem ID [5603]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 10.

**ODE order:** 4.

**ODE degree:** 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + 2a^2y'' + a^4y = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$4)+2*a^2*diff(y(x),x$2)+a^4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(ax) + c_2 \cos(ax) + c_3 \sin(ax)x + c_4 \cos(ax)x$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y''''[x]+2*a^2*y''[x]+a^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (c_2x + c_1) \cos(ax) + (c_4x + c_3) \sin(ax)$$

### 13.11 problem 11

Internal problem ID [5604]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 11.

**ODE order:** 4.

**ODE degree:** 1.

CAS Maple gives this as type `[_high_order, _missing_x]`

$$y'''' + 2y''' + 2y'' + 2y' + y = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)+2*diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,y(x), singsol=a
```

$$y(x) = e^{-x}c_1 + c_2e^{-x}x + c_3 \sin(x) + c_4 \cos(x)$$

#### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 29

```
DSolve[y''''[x]+2*y'''[x]+2*y''[x]+2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(c_4x + c_3) + c_1 \cos(x) + c_2 \sin(x)$$

## 13.12 problem 12

Internal problem ID [5605]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 12.

**ODE order:** 4.

**ODE degree:** 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + 2y''' - 2y'' - 6y' + 5y = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)-2*diff(y(x),x$2)-6*diff(y(x),x)+5*y(x)=0,y(x), singsol
```

$$y(x) = e^x c_1 + c_2 e^x x + c_3 e^{-2x} \sin(x) + c_4 e^{-2x} \cos(x)$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 34

```
DSolve[y''''[x]+2*y'''[x]-2*y''[x]-6*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x (c_4 x + c_3) + e^{-2x} (c_2 \cos(x) + c_1 \sin(x))$$

### 13.13 problem 13

Internal problem ID [5606]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 13.

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - 6y'' + 11y' - 6y = 0$$

#### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+11*diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} + e^x c_2 + c_3 e^{2x}$$

#### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[y'''[x]-6*y''[x]+11*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x (e^x (c_3 e^x + c_2) + c_1)$$

## 13.14 problem 14

Internal problem ID [5607]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 14.

**ODE order:** 4.

**ODE degree:** 1.

CAS Maple gives this as type `[_high_order, _missing_x]`

$$y'''' + y''' - 3y'' - 5y' - 2y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$4)+diff(y(x),x$3)-3*diff(y(x),x$2)-5*diff(y(x),x)-2*y(x)=0,y(x), singsol=a
```

$$y(x) = e^{2x}c_1 + c_2e^{-x} + c_3e^{-x}x + c_4e^{-x}x^2$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 31

```
DSolve[y''''[x]+y'''[x]-3*y''[x]-5*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(x(c_3x + c_2) + c_4e^{3x} + c_1)$$



## 13.15 problem 15

Internal problem ID [5608]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 15.

**ODE order:** 5.

**ODE degree:** 1.

CAS Maple gives this as type `[_high_order, _missing_x]`

$$y^{(5)} - 6y'''' - 8y''' + 48y'' + 16y' - 96y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$5)-6*diff(y(x),x$4)-8*diff(y(x),x$3)+48*diff(y(x),x$2)+16*diff(y(x),x)-96*y(x),x)-96*y(x),x)
```

$$y(x) = c_1 e^{6x} + c_2 e^{-2x} + c_3 e^{-2x} x + c_4 e^{2x} + c_5 e^{2x} x$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 39

```
DSolve[y'''''[x]-6*y''''[x]-8*y'''[x]+48*y''[x]+16*y'[x]-96*y[x]==0,y[x],x,IncludeSingularSol
```

$$y(x) \rightarrow e^{-2x} (c_2 x + e^{4x} (c_4 x + c_3) + c_5 e^{8x} + c_1)$$

### 13.16 problem 16(a)

Internal problem ID [5609]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 16(a).

**ODE order:** 4.

**ODE degree:** 1.

CAS Maple gives this as type `[_high_order, _quadrature]`

$$y'''' = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$4)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{6}c_1x^3 + \frac{1}{2}c_2x^2 + c_3x + c_4$$

#### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(x(c_4x + c_3) + c_2) + c_1$$

### 13.17 problem 16(b)

Internal problem ID [5610]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 16(b).

**ODE order:** 4.

**ODE degree:** 1.

CAS Maple gives this as type `[[_high_order, _quadrature]]`

$$y'''' - \sin(x) - 24 = 0$$

#### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$4)=sin(x)+24,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^3}{6} + x^4 + \frac{c_2 x^2}{2} + \sin(x) + c_3 x + c_4$$

#### ✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 26

```
DSolve[y''''[x]==Sin[x]+24,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sin(x) + x(x(x(x + c_4) + c_3) + c_2) + c_1$$

### 13.18 problem 17

Internal problem ID [5611]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 17.

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$y''' - 3y'' + 2y' - 10 - 42e^{3x} = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+2*diff(y(x),x)=10+42*exp(3*x),y(x), singsol=all)
```

$$y(x) = \frac{e^{2x}c_1}{2} + e^x c_2 + 7e^{3x} + 5x + c_3$$

#### ✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 35

```
DSolve[y'''[x]-3*y''[x]+2*y'[x]==10+42*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 5x + 7e^{3x} + c_1 e^x + \frac{1}{2}c_2 e^{2x} + c_3$$

### 13.19 problem 18

Internal problem ID [5612]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 18.

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' - y' - 1 = 0$$

With initial conditions

$$[y(0) = 4, y'(0) = 4, y''(0) = 4]$$

#### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$3)-diff(y(x),x)=1,y(0) = 4, D(y)(0) = 4, (D@@2)(y)(0) = 4],y(x), singsol=
```

$$y(x) = -\frac{e^{-x}}{2} + \frac{9e^x}{2} - x$$

#### ✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 17

```
DSolve[{y'''[x]-y'[x]==1,{y[0]==4,y'[0]==4,y''[0]==4}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \rightarrow -x + 5 \sinh(x) + 4 \cosh(x)$$

### 13.20 problem 19(a)

Internal problem ID [5613]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 19(a).

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$x^3 y''' + 3x^2 y'' = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^3*diff(y(x),x$3)+3*x^2*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 + \frac{c_2}{x} + c_3 x$$

#### ✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 21

```
DSolve[x^3*y'''[x]+3*x^2*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1}{2x} + c_3 x + c_2$$

### 13.21 problem 19(b)

Internal problem ID [5614]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 19(b).

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$x^3y''' + x^2y'' - 2y'x + 2y = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)+x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + xc_2 + c_3x^2$$

#### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+x^2*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_3x^2 + c_2x + \frac{c_1}{x}$$

## 13.22 problem 19(c)

Internal problem ID [5615]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 19(c).

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3 y''' + 2x^2 y'' + y' x - y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x + \sin(\ln(x)) c_2 + c_3 \cos(\ln(x))$$

### ✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+2*x^2*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_3 x + c_1 \cos(\log(x)) + c_2 \sin(\log(x))$$



### 13.23 problem 20

Internal problem ID [5616]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Section 2.7. HIGHER ORDER LINEAR EQUATIONS, COUPLED HARMONIC OSCILLATORS Page 98

**Problem number:** 20.

**ODE order:** 4.

**ODE degree:** 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$x^3 y'''' + 8x^2 y''' + 8xy'' - 8y' = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^3*diff(y(x),x$4)+8*x^2*diff(y(x),x$3)+8*x*diff(y(x),x$2)-8*diff(y(x),x)=0,y(x),sing
```

$$y(x) = c_1 + \frac{c_2}{x} + \frac{c_3}{x^3} + c_4 x^2$$

#### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 33

```
DSolve[x^3*y''''[x]+8*x^2*y'''[x]+8*x*y''[x]-8*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \rightarrow -\frac{c_1}{3x^3} + \frac{c_3 x^2}{2} - \frac{c_2}{x} + c_4$$

## 14 Chapter 2. Problems for Review and Discovery.

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## 14.1 problem 1(a)

Internal problem ID [5617]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 1(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 3y' + y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{(3+\sqrt{5})x}{2}} + c_2 e^{-\frac{(\sqrt{5}-3)x}{2}}$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 35

```
DSolve[y''[x]-3*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-\frac{1}{2}(\sqrt{5}-3)x} (c_2 e^{\sqrt{5}x} + c_1)$$

## 14.2 problem 1(b)

Internal problem ID [5618]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 1(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y' + y = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}x}{2}\right) + c_2 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}x}{2}\right)$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[y''[x]+y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x/2} \left( c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right) \right)$$

### 14.3 problem 1(c)

Internal problem ID [5619]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 1(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 6y' + 9y = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-3x} + c_2 e^{-3x} x$$

#### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[y''[x]+6*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-3x}(c_2 x + c_1)$$

## 14.4 problem 1(d)

Internal problem ID [5620]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 1(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - y' + 6y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x}{2}} \sin\left(\frac{\sqrt{23}x}{2}\right) + c_2 e^{\frac{x}{2}} \cos\left(\frac{\sqrt{23}x}{2}\right)$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[y''[x]-y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{x/2} \left( c_2 \cos\left(\frac{\sqrt{23}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{23}x}{2}\right) \right)$$

## 14.5 problem 1(e)

Internal problem ID [5621]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 1(e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y' - 5y - x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-5*y(x)=x,y(x), singsol=all)
```

$$y(x) = e^{(1+\sqrt{6})x} c_2 + e^{-(1+\sqrt{6})x} c_1 - \frac{x}{5} + \frac{2}{25}$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 43

```
DSolve[y'[x]-2*y'[x]-5*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x}{5} + c_1 e^{x-\sqrt{6}x} + c_2 e^{(1+\sqrt{6})x} + \frac{2}{25}$$

## 14.6 problem 1(f)

Internal problem ID [5622]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 1(f).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y - e^x = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + \frac{e^x}{2}$$

### ✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^x}{2} + c_1 \cos(x) + c_2 \sin(x)$$



## 14.7 problem 1(g)

Internal problem ID [5623]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 1(g).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y' + y - \sin(x) = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}x}{2}\right) + e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}x}{2}\right) c_1 - \cos(x)$$

### ✓ Solution by Mathematica

Time used: 0.487 (sec). Leaf size: 47

```
DSolve[y''[x]+y'[x]+y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\cos(x) + e^{-x/2} \left( c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right) \right)$$

## 14.8 problem 1(h)

Internal problem ID [5624]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 1(h).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y - e^{3x} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-y(x)=exp(3*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + e^x c_1 + \frac{e^{3x}}{8}$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 29

```
DSolve[y''[x]-y[x]==Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{3x}}{8} + c_1 e^x + c_2 e^{-x}$$

## 14.9 problem 2(a)

Internal problem ID [5625]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 2(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 9y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 2]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)+9*y(x)=0,y(0) = 1, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = \frac{2 \sin(3x)}{3} + \cos(3x)$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{y'[x]+9*y[x]==0,{y[0]==1,y'[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2}{3} \sin(3x) + \cos(3x)$$

## 14.10 problem 2(b)

Internal problem ID [5626]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 2(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y' + 4y - x = 0$$

With initial conditions

$$[y(1) = 2, y'(1) = 1]$$

### ✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 73

```
dsolve([diff(y(x),x$2)-diff(y(x),x)+4*y(x)=x,y(1) = 2, D(y)(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\left( \left( \sin\left(\frac{\sqrt{15}}{2}\right) \sqrt{15} + 135 \cos\left(\frac{\sqrt{15}}{2}\right) \right) \cos\left(\frac{\sqrt{15}x}{2}\right) - \sin\left(\frac{\sqrt{15}x}{2}\right) \left( \sqrt{15} \cos\left(\frac{\sqrt{15}}{2}\right) - 135 \sin\left(\frac{\sqrt{15}}{2}\right) \right) \right) e^{\frac{x}{2} - \frac{1}{2}}}{80} + \frac{x}{4} + \frac{1}{16}$$

### ✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 60

```
DSolve[{y'[x]-y'[x]+4*y[x]==x,{y[1]==2,y'[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{80} \left( 20x + e^{\frac{x-1}{2}} \left( 135 \cos\left(\frac{1}{2}\sqrt{15}(x-1)\right) - \sqrt{15} \sin\left(\frac{1}{2}\sqrt{15}(x-1)\right) \right) \right) + 5$$

## 14.11 problem 2(c)

Internal problem ID [5627]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 2(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2y' + 5y - e^x = 0$$

With initial conditions

$$[y(0) = -1, y'(0) = 1]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve([diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=exp(x),y(0) = -1, D(y)(0) = 1],y(x), singsol=all
```

$$y(x) = \frac{(-9 \cos(2x) - \sin(2x)) e^{-x}}{8} + \frac{e^x}{8}$$

### ✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 32

```
DSolve[{y'[x]+2*y'[x]+5*y[x]==Exp[x],{y[0]==-1,y'[0]==1}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow \frac{1}{8} e^{-x} (e^{2x} - \sin(2x) - 9 \cos(2x))$$

## 14.12 problem 2(d)

Internal problem ID [5628]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 2(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 3y' + 4y - \sin(x) = 0$$

With initial conditions

$$\left[ y\left(\frac{\pi}{2}\right) = 1, y'\left(\frac{\pi}{2}\right) = -1 \right]$$

### ✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 95

```
dsolve([diff(y(x),x$2)+3*diff(y(x),x)+4*y(x)=sin(x),y(1/2*Pi) = 1, D(y)(1/2*Pi) = -1],y(x), s
```

$$y(x) = \frac{\left( \left( \sqrt{7} \sin\left(\frac{\sqrt{7}x}{2}\right) + 35 \cos\left(\frac{\sqrt{7}x}{2}\right) \right) \cos\left(\frac{\sqrt{7}\pi}{4}\right) - \sin\left(\frac{\sqrt{7}\pi}{4}\right) \left( \sqrt{7} \cos\left(\frac{\sqrt{7}x}{2}\right) - 35 \sin\left(\frac{\sqrt{7}x}{2}\right) \right) \right) e^{-\frac{3x}{2} + \frac{3\pi}{4}}}{42} - \frac{\cos(x)}{6} + \frac{\sin(x)}{6}$$

### ✓ Solution by Mathematica

Time used: 0.546 (sec). Leaf size: 70

```
DSolve[{y'[x]+3*y'[x]+4*y[x]==Sin[x],{y[Pi/2]==1,y'[Pi/2]==-1}},y[x],x,IncludeSingularSoluti
```

$$y(x) \rightarrow \frac{1}{42} \left( 7 \sin(x) - 7 \cos(x) + e^{\frac{3}{4}(\pi-2x)} \left( 35 \cos\left(\frac{1}{4}\sqrt{7}(\pi-2x)\right) - \sqrt{7} \sin\left(\frac{1}{4}\sqrt{7}(\pi-2x)\right) \right) \right)$$

### 14.13 problem 2(e)

Internal problem ID [5629]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 2(e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y - e^{-x} = 0$$

With initial conditions

$$[y(2) = 0, y'(2) = -2]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 50

```
dsolve([diff(y(x),x$2)+y(x)=exp(-x),y(2) = 0, D(y)(2) = -2],y(x), singsol=all)
```

$$y(x) = \frac{e^{-x}}{2} + \frac{((- \cos(x) + \sin(x)) \cos(2) - \cos(x) \sin(2) - \sin(x) \sin(2)) e^{-2}}{2} - 2 \sin(x) \cos(2) + 2 \cos(x) \sin(2)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 40

```
DSolve[{y'[x]+y[x]==Exp[-x],{y[2]==0,y'[2]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} \left( e^{-x} + \left( 4 - \frac{1}{e^2} \right) \sin(2-x) - \frac{\cos(2-x)}{e^2} \right)$$

## 14.14 problem 2(f)

Internal problem ID [5630]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 2(f).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - y - \cos(x) = 0$$

With initial conditions

$$[y(0) = 3, y'(2) = 2]$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 54

```
dsolve([diff(y(x),x$2)-y(x)=cos(x),y(0) = 3, D(y)(2) = 2],y(x), singsol=all)
```

$$y(x) = \frac{(\sin(2) - 4)e^{-x+2} + 7e^{-x+4} + (-\sin(2) + 4)e^{x+2} + (-e^4 - 1)\cos(x) + 7e^x}{2e^4 + 2}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 45

```
DSolve[{y'[x]-y[x]==Cos[x],{y[0]==3,y'[2]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} \left( -\cos(x) + 7 \cosh(x) + \frac{(7 + e^2(8 - 7e^2 - 2\sin(2))) \sinh(x)}{1 + e^4} \right)$$



## 14.15 problem 2(g)

Internal problem ID [5631]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 2(g).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$y'' - \tan(x) = 0$$

With initial conditions

$$[y(1) = 1, y'(1) = -1]$$

### ✓ Solution by Maple

Time used: 0.579 (sec). Leaf size: 135

```
dsolve([diff(y(x),x$2)=tan(x),y(1) = 1, D(y)(1) = -1],y(x), singsol=all)
```

$$y(x) = \frac{(-ie^{2i} - i) \operatorname{polylog}(2, -e^{2ix}) + 2x(1 + e^{2i}) \ln(1 + e^{2ix}) + (ie^{2i} + i) \operatorname{polylog}(2, -e^{2i}) + (-2e^{2i} - 2) \ln(1 + e^{2i})}{2}$$

### ✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 82

```
DSolve[{y'[x]==Tan[x],{y[1]==1,y'[1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} \left( -i \operatorname{PolyLog}(2, -e^{2ix}) + i \operatorname{PolyLog}(2, -e^{2i}) + (-2 - ix)x \right. \\ \left. + 2x \log((1 + e^{2ix}) \cos(1)) - 2x \log(\cos(x)) + (4 + i) - 2 \log(1 + e^{2i}) \right)$$

## 14.16 problem 2(h)

Internal problem ID [5632]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 2(h).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' - 2y' - \ln(x) = 0$$

With initial conditions

$$[y(1) = e, y'(1) = e^{-1}]$$

### ✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 42

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)=ln(x),y(1) = exp(1), D(y)(1) = 1/exp(1)],y(x), singsol=
```

$$y(x) = \frac{\left(\int_1^x (-e^{2-z1} \text{Ei}_1(2-z1) + e^{2-z1} \text{Ei}_1(2) - \ln(z1) + 2e^{2-z1-3}) d_z1\right)}{2} + e$$

### ✓ Solution by Mathematica

Time used: 0.093 (sec). Leaf size: 66

```
DSolve[{y'[x]-2*y'[x]==Log[x],{y[1]==Exp[1],y'[1]==1/Exp[1]}},y[x],x,IncludeSingularSolution
```

$$y(x) \rightarrow \frac{1}{4} \left( e^{2x} \text{ExpIntegralEi}(-2x) - \text{ExpIntegralEi}(-2)e^{2x} + 2x + 2e^{2x-3} - 2x \log(x) - \log(-x) + i\pi + 4e - \frac{2}{e} - 2 \right)$$

## 14.17 problem 3(a)

Internal problem ID [5633]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 3(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 3y' + 2y + 1 - 2x = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=2*x-1,y(x), singsol=all)
```

$$y(x) = -e^{-2x}c_1 + c_2e^{-x} + x - 2$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 23

```
DSolve[y''[x]+3*y'[x]+2*y[x]==2*x-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x + e^{-2x}(c_2e^x + c_1) - 2$$

## 14.18 problem 3(b)

Internal problem ID [5634]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 3(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 3y' + 2y - e^{-x} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=exp(-x),y(x), singsol=all)
```

$$y(x) = \left( e^x c_1 + \frac{e^{-2x}}{6} + c_2 \right) e^x$$

### ✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 29

```
DSolve[y''[x]-3*y'[x]+2*y[x]==Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{-x}}{6} + c_1 e^x + c_2 e^{2x}$$

### 14.19 problem 3(c)

Internal problem ID [5635]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 3(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - y' - 2y - \cos(x) = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + e^{2x} c_1 - \frac{3 \cos(x)}{10} - \frac{\sin(x)}{10}$$

#### ✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 34

```
DSolve[y''[x]-y'[x]-2*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sin(x)}{10} - \frac{3 \cos(x)}{10} + c_1 e^{-x} + c_2 e^{2x}$$

## 14.20 problem 3(d)

Internal problem ID [5636]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 3(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' - y - x e^x \sin(x) = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-y(x)=x*exp(x)*sin(x),y(x), singsol=all)
```

$$y(x) = e^{(\sqrt{2}-1)x} c_2 + e^{-(1+\sqrt{2})x} c_1 + \frac{e^x(17 \sin(x) x - 68 \cos(x) x + 44 \sin(x) + 62 \cos(x))}{289}$$

### ✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 59

```
DSolve[y''[x]+2*y'[x]-y[x]==x*Exp[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{289} e^x ((17x + 44) \sin(x) + (62 - 68x) \cos(x)) + e^{-((1+\sqrt{2})x)} (c_2 e^{2\sqrt{2}x} + c_1)$$

## 14.21 problem 3(e)

Internal problem ID [5637]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 3(e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y - \sec(2x) = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
dsolve(diff(y(x),x$2)+9*y(x)=sec(2*x),y(x), singsol=all)
```

$$y(x) = \sin(3x)c_2 + \cos(3x)c_1 - \frac{2}{3} + \frac{\sin(x)(-4\cos(x)^2 + 1)\sqrt{2}\operatorname{arctanh}(\sqrt{2}\sin(x))}{6} \\ + \frac{(-4\cos(x)^3 + 3\cos(x))\sqrt{2}\operatorname{arctanh}(\cos(x)\sqrt{2})}{6} + \frac{4\cos(x)^2}{3}$$

### ✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 93

```
DSolve[y''[x]+9*y[x]==Sec[2*x],y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow \frac{1}{6} \left( \sin(3x) \left( -\sqrt{2}\operatorname{arctanh}(\sqrt{2}\sin(x)) + 6c_2 \right) \right. \\ \left. - \cos(3x) \left( \sqrt{2}\operatorname{arctanh}(\sqrt{2} - \tan\left(\frac{x}{2}\right)) + \sqrt{2}\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right) + \sqrt{2}\right) - 6c_1 \right) \right. \\ \left. + 4\cos(2x) \right)$$

## 14.22 problem 3(f)

Internal problem ID [5638]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 3(f).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y' + 4y - \ln(x)x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=x*ln(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-2x} + e^{-2x} x c_1 - \frac{e^{-2x}(x+1) \operatorname{Ei}_1(-2x)}{4} - \frac{3}{8} + \frac{(2x-2) \ln(x)}{8}$$

### ✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 48

```
DSolve[y''[x]+4*y'[x]+4*y[x]==x*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{8} e^{-2x} (2(x+1) \operatorname{ExpIntegralEi}(2x) + e^{2x} (2(x-1) \log(x) - 3) + 8(c_2 x + c_1))$$



### 14.23 problem 3(g)

Internal problem ID [5639]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 3(g).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$x^2 y'' + 3y'x + y - \frac{2}{x} = 0$$

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=2/x,y(x), singsol=all)
```

$$y(x) = \frac{\ln(x) c_1}{x} + \frac{\ln(x)^2}{x} + \frac{c_2}{x}$$

#### ✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 20

```
DSolve[x^2*y''[x]+3*x*y'[x]+y[x]==2/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\log(x)(\log(x) + c_2) + c_1}{x}$$

## 14.24 problem 3(h)

Internal problem ID [5640]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 3(h).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y - \tan(x)^2 = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+4*y(x)=tan(x)^2,y(x), singsol=all)
```

$$y(x) = \sin(2x) c_2 + \cos(2x) c_1 + (2 \cos(x)^2 - 1) \ln(\cos(x)) + 2 \cos(x) \sin(x) x - \frac{3 \sin(x)^2}{2}$$

### ✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 32

```
DSolve[y''[x]+4*y[x]==Tan[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (x + c_2) \sin(2x) + \cos(2x) \left( \log(\cos(x)) + \frac{1}{4} + c_1 \right) - \frac{3}{4}$$

## 14.25 problem 4(a)

Internal problem ID [5641]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 4(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y - 3e^{2x} = 0$$

Given that one solution of the ode is

$$y_1 = e^{2x}$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)-y(x)=3*exp(2*x),exp(2*x)],y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + e^x c_1 + e^{2x}$$

### ✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 25

```
DSolve[y''[x]-y[x]==3*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{2x} + c_1 e^x + c_2 e^{-x}$$

## 14.26 problem 4(b)

Internal problem ID [5642]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 4(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y + 8 \sin(3x) = 0$$

Given that one solution of the ode is

$$y_1 = \sin(3x)$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+y(x)=-8*sin(3*x),sin(3*x)],y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + \sin(3x)$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 20

```
DSolve[y''[x]+y[x]==-8*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sin(3x) + c_1 \cos(x) + c_2 \sin(x)$$

## 14.27 problem 4(c)

Internal problem ID [5643]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 4(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y' + y - x^2 - 2x - 2 = 0$$

Given that one solution of the ode is

$$y_1 = x^2$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve([diff(y(x),x$2)+diff(y(x),x)+y(x)=x^2+2*x+2,x^2],y(x), singsol=all)
```

$$y(x) = c_2 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}x}{2}\right) + e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}x}{2}\right) c_1 + x^2$$

### ✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 46

```
DSolve[y''[x]+y'[x]+y[x]==x^2+2*x+2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2 + e^{-x/2} \left( c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right) \right)$$

## 14.28 problem 4(d)

Internal problem ID [5644]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Drill exercises. Page 105

**Problem number:** 4(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' + y' - \frac{x-1}{x} = 0$$

Given that one solution of the ode is

$$y_1 = \ln(x)$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve([diff(y(x),x$2)+diff(y(x),x)=(x-1)/x,ln(x)],y(x), singsol=all)
```

$$y(x) = \int (1 + e^{-x} \text{Ei}_1(-x) + e^{-x} c_1) dx + c_2$$

### ✓ Solution by Mathematica

Time used: 0.107 (sec). Leaf size: 26

```
DSolve[y''[x]+y'[x]==(x-1)/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(\text{ExpIntegralEi}(x) - c_1) + x - \log(x) + c_2$$

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Challenge excercises. Page 105**

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## 15.1 problem 3

Internal problem ID [5645]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Challenge exercises. Page 105

**Problem number:** 3.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$x^2 y'' - 2y'x + 2y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 9

```
dsolve([x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(0) = 0, D(y)(0) = 0],y(x), singsol=all
```

$$y(x) = c_2 x^2$$

### ✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 11

```
DSolve[{x^2*y'[x]-2*x*y'[x]+2*y[x]==0,{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow c_2 x^2$$



## 15.2 problem 4

Internal problem ID [5646]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Challenge exercises. Page 105

**Problem number:** 4.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y + 3 \cos(2x) = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+9*y(x)=-3*cos(2*x),y(x), singsol=all)
```

$$y(x) = \sin(3x) c_2 + \cos(3x) c_1 - \frac{3 \cos(2x)}{5}$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 28

```
DSolve[y''[x]+9*y[x]==-3*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{3}{5} \cos(2x) + c_1 \cos(3x) + c_2 \sin(3x)$$

**16 Chapter 2. Problems for Review and Discovery.  
Problems for Discussion and Exploration. Page  
105**

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## 16.1 problem 1

Internal problem ID [5647]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Problems for Discussion and Exploration. Page 105

**Problem number:** 1.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y' + y - \cos(x) = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)+y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \frac{\cos(x)}{2} + \frac{\sin(x)}{2} + e^{-x}c_1$$

### ✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 23

```
DSolve[y'[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}(\sin(x) + \cos(x) + 2c_1e^{-x})$$

## 16.2 problem 2

Internal problem ID [5648]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Problems for Discussion and Exploration. Page 105

**Problem number:** 2.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 3y = 0$$

With initial conditions

$$[y(0) = -1]$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)=-3*y(x),y(0) = -1],y(x), singsol=all)
```

$$y(x) = c_1 \sin(\sqrt{3}x) - \cos(\sqrt{3}x)$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 27

```
DSolve[{y'[x]==-3*y[x],{y[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\cos(\sqrt{3}x) + c_2 \sin(\sqrt{3}x)$$

### 16.3 problem 4

Internal problem ID [5649]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 2. Problems for Review and Discovery. Problems for Discussion and Exploration. Page 105

**Problem number:** 4.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$y'' + \sin(y) = 0$$

#### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)+sin(y(x))=0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{2 \cos(\_a) + c_1}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} -\frac{1}{\sqrt{2 \cos(\_a) + c_1}} d\_a - x - c_2 = 0$$

#### ✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 69

```
DSolve[y''[x]+Sin[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -2 \operatorname{JacobiAmplitude} \left( \frac{1}{2} \sqrt{(c_1 + 2)(x + c_2)^2}, \frac{4}{c_1 + 2} \right)$$

$$y(x) \rightarrow 2 \operatorname{JacobiAmplitude} \left( \frac{1}{2} \sqrt{(c_1 + 2)(x + c_2)^2}, \frac{4}{c_1 + 2} \right)$$

## 17 Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

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## 17.1 problem 1(a) solving using series

Internal problem ID [5650]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 1(a) solving using series.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y' - 2xy = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
Order:=8;
dsolve(diff(y(x),x)=2*x*y(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + x^2 + \frac{1}{2}x^4 + \frac{1}{6}x^6\right) y(0) + O(x^8)$$

### ✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 25

```
AsymptoticDSolveValue[y'[x]==2*x*y[x],y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( \frac{x^6}{6} + \frac{x^4}{2} + x^2 + 1 \right)$$

## 17.2 problem 1(a) solving directly

Internal problem ID [5651]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 1(a) solving directly.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y' - 2xy = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=2*x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{x^2}$$

### ✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 18

```
DSolve[y'[x]==2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{x^2}$$

$$y(x) \rightarrow 0$$



### 17.3 problem 1(b) solving using series

Internal problem ID [5652]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 1(b) solving using series.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' + y - 1 = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
Order:=8;
dsolve(diff(y(x),x)+y(x)=1,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \frac{1}{720}x^6 - \frac{1}{5040}x^7\right) y(0) \\ + x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{24} + \frac{x^5}{120} - \frac{x^6}{720} + \frac{x^7}{5040} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 97

```
AsymptoticDSolveValue[y'[x]+y[x]==1,y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{x^7}{5040} - \frac{x^6}{720} + \frac{x^5}{120} - \frac{x^4}{24} + \frac{x^3}{6} - \frac{x^2}{2} + c_1 \left( -\frac{x^7}{5040} + \frac{x^6}{720} - \frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1 \right) + x$$

## 17.4 problem 1(b) solving directly

Internal problem ID [5653]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 1(b) solving directly.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' + y - 1 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+y(x)=1,y(x), singsol=all)
```

$$y(x) = 1 + e^{-x}c_1$$

### ✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 20

```
DSolve[y'[x]+y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 1 + c_1 e^{-x}$$

$$y(x) \rightarrow 1$$

## 17.5 problem 1(c) solving using series

Internal problem ID [5654]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 1(c) solving using series.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - y - 2 = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
Order:=8;
dsolve(diff(y(x),x)-y(x)=2,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7\right) y(0) \\ + 2x + x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{60} + \frac{x^6}{360} + \frac{x^7}{2520} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 93

```
AsymptoticDSolveValue[y'[x]-y[x]==2,y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{x^7}{2520} + \frac{x^6}{360} + \frac{x^5}{60} + \frac{x^4}{12} + \frac{x^3}{3} + x^2 + c_1 \left( \frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right) + 2x$$

## 17.6 problem 1(c) solving directly

Internal problem ID [5655]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 1(c) solving directly.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - y - 2 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)-y(x)=2,y(x), singsol=all)
```

$$y(x) = -2 + e^x c_1$$

### ✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 18

```
DSolve[y'[x]-y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -2 + c_1 e^x$$

$$y(x) \rightarrow -2$$

## 17.7 problem 1(d) solving using series

Internal problem ID [5656]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 1(d) solving using series.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
Order:=8;
dsolve(diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \frac{1}{720}x^6 - \frac{1}{5040}x^7\right) y(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 53

```
AsymptoticDSolveValue[y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( -\frac{x^7}{5040} + \frac{x^6}{720} - \frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1 \right)$$

## 17.8 problem 1(d) solving directly

Internal problem ID [5657]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 1(d) solving directly.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' + y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1$$

### ✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-x}$$

$$y(x) \rightarrow 0$$

## 17.9 problem 1(e) solving using series

Internal problem ID [5658]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 1(e) solving using series.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - y = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7\right) y(0) + O(x^8)$$

### ✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 51

```
AsymptoticDSolveValue[y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( \frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right)$$

## 17.10 problem 1(e) solving directly

Internal problem ID [5659]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 1(e) solving directly.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = e^x c_1$$

### ✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 16

```
DSolve[y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^x$$

$$y(x) \rightarrow 0$$



## 17.11 problem 1(f) solving using series

Internal problem ID [5660]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 1(f) solving using series.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - y - x^2 = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=8;
dsolve(diff(y(x),x)-y(x)=x^2,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7\right) y(0) + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{60} + \frac{x^6}{360} + \frac{x^7}{2520} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 87

```
AsymptoticDSolveValue[y'[x]-y[x]==x^2,y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{x^7}{2520} + \frac{x^6}{360} + \frac{x^5}{60} + \frac{x^4}{12} + \frac{x^3}{3} + c_1 \left( \frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right)$$

## 17.12 problem 1(f) solving directly

Internal problem ID [5661]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 1(f) solving directly.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y' - y - x^2 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)-y(x)=x^2,y(x), singsol=all)
```

$$y(x) = -x^2 - 2x - 2 + e^x c_1$$

### ✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 19

```
DSolve[y'[x]-y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x(x + 2) + c_1 e^x - 2$$

## 17.13 problem 2(a) solving using series

Internal problem ID [5662]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 2(a) solving using series.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$-y + y'x = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
Order:=8;
dsolve(x*diff(y(x),x)=y(x),y(x),type='series',x=0);
```

$$y(x) = c_1x + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 7

```
AsymptoticDSolveValue[x*y'[x]==y[x],y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1x$$

## 17.14 problem 2(a) solving directly

Internal problem ID [5663]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 2(a) solving directly.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$-y + y'x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve(x*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = c_1x$$

### ✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 14

```
DSolve[x*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1x$$

$$y(x) \rightarrow 0$$

## 17.15 problem 2(b) solving using series

Internal problem ID [5664]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 2(b) solving using series.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$x^2 y' - y = 0$$

With the expansion point for the power series method at  $x = 0$ .

 Solution by Maple

```
Order:=8;
dsolve(x^2*diff(y(x),x)=y(x),y(x),type='series',x=0);
```

No solution found

 Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 13

```
AsymptoticDSolveValue[x^2*y'[x]==y[x],y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 e^{-1/x}$$

## 17.16 problem 2(b) solving directly

Internal problem ID [5665]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 2(b) solving directly.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$x^2 y' - y = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x^2*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{1}{x}}$$

### ✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 20

```
DSolve[x^2*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-1/x}$$

$$y(x) \rightarrow 0$$

## 17.17 problem 2(c) solving using series

Internal problem ID [5666]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 2(c) solving using series.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' - \frac{y}{x} - x^2 = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
Order:=8;
dsolve(diff(y(x),x)-(1/x)*y(x)=x^2,y(x),type='series',x=0);
```

$$y(x) = c_1x(1 + O(x^8)) + x^3\left(\frac{1}{2} + O(x^5)\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 15

```
AsymptoticDSolveValue[y'[x]-1/x*y[x]==x^2,y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{x^3}{2} + c_1x$$

## 17.18 problem 2(c) solving directly

Internal problem ID [5667]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 2(c) solving directly.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' - \frac{y}{x} - x^2 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)-(1/x)*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = \left( \frac{x^2}{2} + c_1 \right) x$$

### ✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 17

```
DSolve[y'[x]-1/x*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^3}{2} + c_1 x$$



## 17.19 problem 2(d) solving using series

Internal problem ID [5668]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 2(d) solving using series.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' + \frac{y}{x} - x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+(1/x)*y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{\frac{x^3}{3} + c_1}{x}$$

### ✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 19

```
DSolve[y'[x]+1/x*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^2}{3} + \frac{c_1}{x}$$

## 17.20 problem 3

Internal problem ID [5669]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 3.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - \frac{1}{\sqrt{-x^2 + 1}} = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
Order:=8;
dsolve(diff(y(x),x)=(1-x^2)^(-1/2),y(x),type='series',x=0);
```

$$y(x) = y(0) + x + \frac{x^3}{6} + \frac{3x^5}{40} + \frac{5x^7}{112} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y'[x]==(1-x^2)^(-1/2),y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{5x^7}{112} + \frac{3x^5}{40} + \frac{x^3}{6} + x + c_1$$

## 17.21 problem 4

Internal problem ID [5670]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 4.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - 1 - y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
Order:=8;
dsolve(diff(y(x),x)=1+y(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7\right) y(0) \\ + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 95

```
AsymptoticDSolveValue[y'[x]==1+y[x],y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + c_1 \left( \frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right) + x$$

## 17.22 problem 5 solved using series

Internal problem ID [5671]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 5 solved using series.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y' + y - x = 0$$

With initial conditions

$$[y(0) = 0]$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
Order:=8;
dsolve([diff(y(x),x)=x-y(x),y(0) = 0],y(x),type='series',x=0);
```

$$y(x) = \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \frac{1}{720}x^6 - \frac{1}{5040}x^7 + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{y'[x]==x-y[x],{y[0]==0}},y[x],{x,0,7}]
```

$$y(x) \rightarrow -\frac{x^7}{5040} + \frac{x^6}{720} - \frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2}$$

## 17.23 problem 5 solved directly

Internal problem ID [5672]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.2. Series Solutions of First-Order Differential Equations Page 162

**Problem number:** 5 solved directly.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + y - x = 0$$

With initial conditions

$$[y(0) = 0]$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=x-y(x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = x - 1 + e^{-x}$$

### ✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 13

```
DSolve[{y'[x]==x-y[x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x + e^{-x} - 1$$

**18 Chapter 4. Power Series Solutions and Special  
Functions. Section 4.3. Second-Order Linear  
Equations: Ordinary Points. Page 169**

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## 18.1 problem 1(a)

Internal problem ID [5673]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169

**Problem number:** 1(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$y'' + y'x + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 - \frac{1}{48}x^6\right) y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{15}x^5 - \frac{1}{105}x^7\right) D(y)(0) + O(x^8)$$

### ✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left( -\frac{x^7}{105} + \frac{x^5}{15} - \frac{x^3}{3} + x \right) + c_1 \left( -\frac{x^6}{48} + \frac{x^4}{8} - \frac{x^2}{2} + 1 \right)$$

## 18.2 problem 1(b)

Internal problem ID [5674]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169

**Problem number:** 1(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' - y' + xy = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
Order:=8;
dsolve(diff(y(x),x$2)-diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 - \frac{1}{24}x^4 - \frac{1}{120}x^5 + \frac{1}{240}x^6 + \frac{1}{630}x^7\right) y(0) \\ + \left(x + \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{24}x^4 - \frac{1}{30}x^5 - \frac{1}{90}x^6 - \frac{1}{1680}x^7\right) D(y)(0) + O(x^8)$$

### ✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 91

```
AsymptoticDSolveValue[y''[x]-y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( \frac{x^7}{630} + \frac{x^6}{240} - \frac{x^5}{120} - \frac{x^4}{24} - \frac{x^3}{6} + 1 \right) + c_2 \left( -\frac{x^7}{1680} - \frac{x^6}{90} - \frac{x^5}{30} - \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x \right)$$



### 18.3 problem 1(c)

Internal problem ID [5675]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169

**Problem number:** 1(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2y'x - y - x = 0$$

With the expansion point for the power series method at  $x = 0$ .

#### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
Order:=8;
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)-y(x)=x,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \frac{7}{240}x^6\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{24}x^5 - \frac{1}{112}x^7\right) D(y)(0) + \frac{x^3}{6} - \frac{x^5}{24} + \frac{x^7}{112} + O(x^8)$$

#### ✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 77

```
AsymptoticDSolveValue[y''[x]+2*x*y'[x]-y[x]==x,y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{x^7}{112} - \frac{x^5}{24} + \frac{x^3}{6} + c_2 \left( -\frac{x^7}{112} + \frac{x^5}{24} - \frac{x^3}{6} + x \right) + c_1 \left( \frac{7x^6}{240} - \frac{x^4}{8} + \frac{x^2}{2} + 1 \right)$$

## 18.4 problem 1(d)

Internal problem ID [5676]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169

**Problem number:** 1(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y' - x^2y - 1 = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 72

```
Order:=8;
dsolve(diff(y(x),x$2)+diff(y(x),x)-x^2*y(x)=1,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = & \left(1 + \frac{1}{12}x^4 - \frac{1}{60}x^5 + \frac{1}{360}x^6 - \frac{1}{2520}x^7\right) y(0) \\ & + \left(x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{24}x^4 + \frac{7}{120}x^5 - \frac{19}{720}x^6 + \frac{13}{1680}x^7\right) D(y)(0) \\ & + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \frac{x^5}{120} + \frac{13x^6}{720} - \frac{11x^7}{1680} + O(x^8) \end{aligned}$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 126

```
AsymptoticDSolveValue[y''[x]+y'[x]+x^2*y[x]==1,y[x],{x,0,7}]
```

$$\begin{aligned} y(x) \rightarrow & \frac{31x^7}{5040} - \frac{11x^6}{720} - \frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} + c_1 \left( \frac{x^7}{2520} - \frac{x^6}{360} + \frac{x^5}{60} - \frac{x^4}{12} + 1 \right) \\ & + c_2 \left( -\frac{37x^7}{5040} + \frac{17x^6}{720} - \frac{x^5}{24} - \frac{x^4}{24} + \frac{x^3}{6} - \frac{x^2}{2} + x \right) \end{aligned}$$

## 18.5 problem 1(e)

Internal problem ID [5677]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169

**Problem number:** 1(e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`, `[_2nd_order, _linear]`, ‘

$$(x^2 + 1)y'' + y'x + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=8;
dsolve((1+x^2)*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{5}{24}x^4 - \frac{17}{144}x^6\right)y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{6}x^5 - \frac{13}{126}x^7\right)D(y)(0) + O(x^8)$$

### ✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[(1+x^2)*y'[x]+x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left( -\frac{13x^7}{126} + \frac{x^5}{6} - \frac{x^3}{3} + x \right) + c_1 \left( -\frac{17x^6}{144} + \frac{5x^4}{24} - \frac{x^2}{2} + 1 \right)$$

## 18.6 problem 1(f)

Internal problem ID [5678]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169

**Problem number:** 1(f).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y'(1+x) - y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 64

```
Order:=8;
dsolve(diff(y(x),x$2)+(1+x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{60}x^5 - \frac{1}{360}x^6 - \frac{1}{840}x^7\right) y(0) \\ + \left(x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{60}x^5 + \frac{1}{360}x^6 + \frac{1}{840}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 84

```
AsymptoticDSolveValue[y'[x]+(1+x)*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( -\frac{x^7}{840} - \frac{x^6}{360} + \frac{x^5}{60} - \frac{x^3}{6} + \frac{x^2}{2} + 1 \right) + c_2 \left( \frac{x^7}{840} + \frac{x^6}{360} - \frac{x^5}{60} + \frac{x^3}{6} - \frac{x^2}{2} + x \right)$$

## 18.7 problem 2

Internal problem ID [5679]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169

**Problem number:** 2.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 1)y'' + 2y'x - 2y = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
Order:=8;
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x^2 - \frac{1}{3}x^4 + \frac{1}{5}x^6\right)y(0) + D(y)(0)x + O(x^8)$$

### ✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 30

```
AsymptoticDSolveValue[(1+x^2)*y'[x]+2*x*y'[x]-2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( \frac{x^6}{5} - \frac{x^4}{3} + x^2 + 1 \right) + c_2 x$$

## 18.8 problem 3

Internal problem ID [5680]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169

**Problem number:** 3.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$y'' + y'x + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 - \frac{1}{48}x^6\right) y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{15}x^5 - \frac{1}{105}x^7\right) D(y)(0) + O(x^8)$$

### ✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left( -\frac{x^7}{105} + \frac{x^5}{15} - \frac{x^3}{3} + x \right) + c_1 \left( -\frac{x^6}{48} + \frac{x^4}{8} - \frac{x^2}{2} + 1 \right)$$

## 18.9 problem 4(a)

Internal problem ID [5681]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169

**Problem number:** 4(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' + y' - xy = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
Order:=8;
dsolve([diff(y(x),x$2)+diff(y(x),x)-x*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y(x) = 1 + \frac{1}{6}x^3 - \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{240}x^6 - \frac{1}{630}x^7 + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[{y'[x]+y[x]-x*y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,7}]
```

$$y(x) \rightarrow -\frac{x^7}{630} + \frac{x^6}{240} + \frac{x^5}{120} - \frac{x^4}{24} + \frac{x^3}{6} + 1$$

## 18.10 problem 4(b)

Internal problem ID [5682]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169

**Problem number:** 4(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y' - xy = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
Order:=8;
dsolve([diff(y(x),x$2)+diff(y(x),x)-x*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',x=0);
```

$$y(x) = x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{30}x^5 + \frac{1}{90}x^6 - \frac{1}{1680}x^7 + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 47

```
AsymptoticDSolveValue[{y'[x]+y[x]-x*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,7}]
```

$$y(x) \rightarrow -\frac{x^7}{1680} + \frac{x^6}{90} - \frac{x^5}{30} + \frac{x^4}{24} + \frac{x^3}{6} - \frac{x^2}{2} + x$$



## 18.11 problem 5

Internal problem ID [5683]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169

**Problem number:** 5.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left( p + \frac{1}{2} - \frac{x^2}{4} \right) y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 120

```
Order:=8;
dsolve(diff(y(x),x$2)+(p+1/2-x^2/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left( 1 - \frac{(2p+1)x^2}{4} + \frac{(4p^2+4p+3)x^4}{96} - \frac{(8p^3+12p^2+34p+15)x^6}{5760} \right) y(0) \\ + \left( x - \frac{(2p+1)x^3}{12} + \frac{(4p^2+4p+7)x^5}{480} - \frac{(8p^3+12p^2+58p+27)x^7}{40320} \right) D(y)(0) \\ + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 142

AsymptoticDSolveValue[y' [x]+(p+1/2-x^2/4)\*y[x]==0,y[x],{x,0,7}]

$$\begin{aligned}
 y(x) \rightarrow & c_2 \left( \frac{(-4p-2)(4p+2)^2 x^7}{322560} + \frac{13(-4p-2)x^7}{40320} + \frac{(4p+2)^2 x^5}{1920} + \frac{1}{24}(-4p-2)x^3 + \frac{x^5}{80} + x \right) \\
 & + c_1 \left( \frac{(-4p-2)(4p+2)^2 x^6}{46080} + \frac{7(-4p-2)x^6}{5760} + \frac{1}{384}(4p+2)^2 x^4 + \frac{1}{8}(-4p-2)x^2 \right. \\
 & \left. + \frac{x^4}{48} + 1 \right)
 \end{aligned}$$

## 18.12 problem 6

Internal problem ID [5684]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169

**Problem number:** 6.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_Emden, _Fowler]`

$$y'' + xy = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
Order:=8;
dsolve(diff(y(x),x$2)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 + \frac{1}{180}x^6\right) y(0) + \left(x - \frac{1}{12}x^4 + \frac{1}{504}x^7\right) D(y)(0) + O(x^8)$$

### ✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left( \frac{x^7}{504} - \frac{x^4}{12} + x \right) + c_1 \left( \frac{x^6}{180} - \frac{x^3}{6} + 1 \right)$$

## 18.13 problem 7

Internal problem ID [5685]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169

**Problem number:** 7.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Gegenbauer, [\_2nd\_order, \_linear, ‘\_with\_symmetry\_[0,F(x)]’]]

$$(-x^2 + 1)y'' - y'x + p^2y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 121

```
Order:=8;
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+p^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{p^2 x^2}{2} + \frac{p^2(p^2 - 4)x^4}{24} - \frac{p^2(p^4 - 20p^2 + 64)x^6}{720}\right) y(0) \\ + \left(x - \frac{(p^2 - 1)x^3}{6} + \frac{(p^4 - 10p^2 + 9)x^5}{120} - \frac{(p^6 - 35p^4 + 259p^2 - 225)x^7}{5040}\right) D(y)(0) \\ + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 155

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-x*y'[x]+p^2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left( -\frac{p^6 x^7}{5040} + \frac{p^4 x^7}{144} + \frac{p^4 x^5}{120} - \frac{37p^2 x^7}{720} - \frac{p^2 x^5}{12} - \frac{p^2 x^3}{6} + \frac{5x^7}{112} + \frac{3x^5}{40} + \frac{x^3}{6} + x \right) \\ + c_1 \left( -\frac{1}{720} p^6 x^6 + \frac{p^4 x^6}{36} + \frac{p^4 x^4}{24} - \frac{4p^2 x^6}{45} - \frac{p^2 x^4}{6} - \frac{p^2 x^2}{2} + 1 \right)$$

## 18.14 problem 8

Internal problem ID [5686]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.3. Second-Order Linear Equations: Ordinary Points. Page 169

**Problem number:** 8.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y'x + 2yp = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 109

```
Order:=8;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+2*p*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - px^2 + \frac{p(p-2)x^4}{6} - \frac{p(p-2)(p-4)x^6}{90}\right) y(0) \\ + \left(x - \frac{(p-1)x^3}{3} + \frac{(p^2-4p+3)x^5}{30} - \frac{(p^3-9p^2+23p-15)x^7}{630}\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 141

```
AsymptoticDSolveValue[y'[x]-2*x*y'[x]+2*p*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left( -\frac{1}{630}p^3x^7 + \frac{p^2x^7}{70} + \frac{p^2x^5}{30} - \frac{23px^7}{630} - \frac{2px^5}{15} - \frac{px^3}{3} + \frac{x^7}{42} + \frac{x^5}{10} + \frac{x^3}{3} + x \right) \\ + c_1 \left( -\frac{1}{90}p^3x^6 + \frac{p^2x^6}{15} + \frac{p^2x^4}{6} - \frac{4px^6}{45} - \frac{px^4}{3} - px^2 + 1 \right)$$

**19 Chapter 4. Power Series Solutions and Special  
Functions. Section 4.4. REGULAR SINGULAR  
POINTS. Page 175**

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## 19.1 problem 1(a)

Internal problem ID [5687]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 1(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3(x-1)y'' - 2(x-1)y' + 3xy = 0$$

With the expansion point for the power series method at  $x = 0$ .

**X** Solution by Maple

```
Order:=8;
dsolve(x^3*(x-1)*diff(y(x),x$2)-2*(x-1)*diff(y(x),x)+3*x*y(x)=0,y(x),type='series',x=0);
```

No solution found

**✓** Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 108

```
AsymptoticDSolveValue[x^3*(x-1)*y'[x]-2*(x-1)*y'[x]+3*x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 e^{-\frac{1}{x^2}} \left( \frac{1731x^7}{320} - \frac{795x^6}{128} - \frac{51x^5}{40} + \frac{63x^4}{32} + \frac{x^3}{2} - \frac{3x^2}{4} + 1 \right) x^3 \\ + c_1 \left( -\frac{51x^7}{320} - \frac{19x^6}{128} - \frac{9x^5}{40} - \frac{9x^4}{32} - \frac{x^3}{2} - \frac{3x^2}{4} + 1 \right)$$

## 19.2 problem 1(b)

Internal problem ID [5688]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 1(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2 - 1)y'' - x(1 - x)y' + 2y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 654

Order:=8;

`dsolve(x^2*(x^2-1)*diff(y(x),x$2)-x*(1-x)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);`

$$\begin{aligned}
 y(x) = & c_1 x^{-\sqrt{2}} \left( 1 + \frac{\sqrt{2}}{-1 + 2\sqrt{2}}x + \frac{\sqrt{2}}{-5 + 3\sqrt{2}}x^2 + \frac{6\sqrt{2} - 8}{57\sqrt{2} - 81}x^3 + \frac{-49\sqrt{2} + 69}{1104 - 780\sqrt{2}}x^4 \right. \\
 & \left. + \frac{293\sqrt{2} - 414}{6108\sqrt{2} - 8640}x^5 + \frac{-2757\sqrt{2} + 3898}{114408 - 80892\sqrt{2}}x^6 \right. \\
 & \left. + \frac{1}{126} \frac{77567\sqrt{2} - 109686}{(-1 + 2\sqrt{2})(\sqrt{2} - 1)(-3 + 2\sqrt{2})(-2 + \sqrt{2})(-5 + 2\sqrt{2})(-3 + \sqrt{2})(-7 + 2\sqrt{2})} x^7 \right. \\
 & \left. + O(x^8) \right) + c_2 x^{\sqrt{2}} \left( 1 + \frac{\sqrt{2}}{1 + 2\sqrt{2}}x + \frac{\sqrt{2}}{5 + 3\sqrt{2}}x^2 + \frac{6\sqrt{2} + 8}{57\sqrt{2} + 81}x^3 + \frac{49\sqrt{2} + 69}{1104 + 780\sqrt{2}}x^4 \right. \\
 & \left. + \frac{293\sqrt{2} + 414}{6108\sqrt{2} + 8640}x^5 + \frac{2757\sqrt{2} + 3898}{114408 + 80892\sqrt{2}}x^6 \right. \\
 & \left. + \frac{1}{126} \frac{77567\sqrt{2} + 109686}{(1 + 2\sqrt{2})(1 + \sqrt{2})(3 + 2\sqrt{2})(2 + \sqrt{2})(5 + 2\sqrt{2})(3 + \sqrt{2})(7 + 2\sqrt{2})} x^7 \right. \\
 & \left. + O(x^8) \right)
 \end{aligned}$$



✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 10352

```
AsymptoticDSolveValue[x^2*(x^2-1)*y'[x]-x*(1-x)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

Too large to display

### 19.3 problem 1(c)

Internal problem ID [5689]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 1(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$x^2 y'' + (-x + 2) y' = 0$$

With the expansion point for the power series method at  $x = 0$ .

 Solution by Maple

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+(2-x)*diff(y(x),x)=0,y(x),type='series',x=0);
```

No solution found

 Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 64

```
AsymptoticDSolveValue[x^2*y'[x]+(2-x)*y'[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 e^{2/x} \left( \frac{2835x^7}{2} + 315x^6 + \frac{315x^5}{4} + \frac{45x^4}{2} + \frac{15x^3}{2} + 3x^2 + \frac{3x}{2} + 1 \right) x^3 + c_1$$

## 19.4 problem 1(d)

Internal problem ID [5690]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 1(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(1 + 3x)xy'' - y'(1 + x) + 2y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 72

**Order:=8;**

**dsolve((3\*x+1)\*x\*diff(y(x),x\$2)-(x+1)\*diff(y(x),x)+2\*y(x)=0,y(x),type='series',x=0);**

$$y(x) = c_1 x^2 \left( 1 - 2x + \frac{17}{4}x^2 - \frac{289}{30}x^3 + \frac{5491}{240}x^4 - \frac{236113}{4200}x^5 + \frac{28569673}{201600}x^6 - \frac{28569673}{78400}x^7 + O(x^8) \right) + c_2 \left( \ln(x) \left( 2x^2 - 4x^3 + \frac{17}{2}x^4 - \frac{289}{15}x^5 + \frac{5491}{120}x^6 - \frac{236113}{2100}x^7 + O(x^8) \right) + \left( -2 - 4x + 6x^2 - 12x^3 + \frac{209}{8}x^4 - \frac{54247}{900}x^5 + \frac{521849}{3600}x^6 - \frac{158526173}{441000}x^7 + O(x^8) \right) \right)$$

✓ Solution by Mathematica

Time used: 0.416 (sec). Leaf size: 118

AsymptoticDSolveValue[(3\*x+1)\*x\*y'[x]-(x+1)\*y'[x]+2\*y[x]==0,y[x],{x,0,7}]

$$\begin{aligned}
 y(x) \rightarrow & c_1 \left( \frac{27353x^6 - 12886x^5 + 6525x^4 - 3600x^3 + 1800x^2 + 7200x + 3600}{3600} \right. \\
 & \left. - \frac{1}{240}x^2(5491x^4 - 2312x^3 + 1020x^2 - 480x + 240) \log(x) \right) \\
 & + c_2 \left( \frac{28569673x^8}{201600} - \frac{236113x^7}{4200} + \frac{5491x^6}{240} - \frac{289x^5}{30} + \frac{17x^4}{4} - 2x^3 + x^2 \right)
 \end{aligned}$$

## 19.5 problem 2(a)

Internal problem ID [5691]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 2(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y \sin(x) = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
Order:=8;
dsolve(diff(y(x),x$2)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \frac{1}{180}x^6 - \frac{1}{5040}x^7\right) y(0) \\ + \left(x - \frac{1}{12}x^4 + \frac{1}{180}x^6 + \frac{1}{504}x^7\right) D(y)(0) + O(x^8)$$

### ✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]+Sin[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left( \frac{x^7}{504} + \frac{x^6}{180} - \frac{x^4}{12} + x \right) + c_1 \left( -\frac{x^7}{5040} + \frac{x^6}{180} + \frac{x^5}{120} - \frac{x^3}{6} + 1 \right)$$

## 19.6 problem 2(b)

Internal problem ID [5692]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 2(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + y \sin(x) = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(x*diff(y(x),x$2)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{18}x^4 - \frac{53}{10800}x^6\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{60}x^5 - \frac{19}{15120}x^7\right) D(y)(0) + O(x^8)$$

### ✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[x*y''[x]+Sin[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left( -\frac{19x^7}{15120} + \frac{x^5}{60} - \frac{x^3}{6} + x \right) + c_1 \left( -\frac{53x^6}{10800} + \frac{x^4}{18} - \frac{x^2}{2} + 1 \right)$$

## 19.7 problem 2(c)

Internal problem ID [5693]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 2(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + y \sin(x) = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 70

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}
 y(x) = c_1 x & \left( 1 - \frac{1}{2}x + \frac{1}{12}x^2 + \frac{1}{144}x^3 - \frac{13}{2880}x^4 + \frac{29}{86400}x^5 + \frac{431}{3628800}x^6 - \frac{4961}{203212800}x^7 \right. \\
 & \left. + O(x^8) \right) \\
 & + c_2 \left( \ln(x) \left( -x + \frac{1}{2}x^2 - \frac{1}{12}x^3 - \frac{1}{144}x^4 + \frac{13}{2880}x^5 - \frac{29}{86400}x^6 - \frac{431}{3628800}x^7 + O(x^8) \right) \right. \\
 & \left. + \left( 1 - \frac{3}{4}x^2 + \frac{2}{9}x^3 - \frac{25}{1728}x^4 - \frac{689}{86400}x^5 + \frac{263}{162000}x^6 + \frac{71809}{762048000}x^7 + O(x^8) \right) \right)
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 121

```
AsymptoticDSolveValue[x^2*y''[x]+Sin[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( \frac{2539x^6 - 16185x^5 - 9750x^4 + 396000x^3 - 1620000x^2 + 1296000x + 1296000}{1296000} - \frac{x(29x^5 - 390x^4 + 600x^3 + 7200x^2 - 43200x + 86400) \log(x)}{86400} \right) + c_2 \left( \frac{431x^7}{3628800} + \frac{29x^6}{86400} - \frac{13x^5}{2880} + \frac{x^4}{144} + \frac{x^3}{12} - \frac{x^2}{2} + x \right)$$



## 19.8 problem 2(d)

Internal problem ID [5694]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 2(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3 y'' + \sin(x) y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 427

```
Order:=8;
dsolve(x^3*diff(y(x),x$2)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}
 y(x) = & \sqrt{x} \left( c_2 x^{\frac{i\sqrt{3}}{2}} \left( 1 + \frac{1}{12i\sqrt{3} + 24} x^2 - \frac{1}{1440} \frac{3i\sqrt{3} + 1}{(i\sqrt{3} + 4)(i\sqrt{3} + 2)} x^4 \right. \right. \\
 & \left. \left. + \frac{1}{362880} \frac{9i\sqrt{3} - 115}{(i\sqrt{3} + 6)(i\sqrt{3} + 4)(i\sqrt{3} + 2)} x^6 + O(x^8) \right) \right. \\
 & \left. + c_1 x^{-\frac{i\sqrt{3}}{2}} \left( 1 - \frac{1}{12i\sqrt{3} - 24} x^2 + \frac{1}{1440} \frac{3i\sqrt{3} - 1}{(i\sqrt{3} - 4)(-2 + i\sqrt{3})} x^4 \right. \right. \\
 & \left. \left. + \frac{1}{362880} \frac{9i\sqrt{3} + 115}{(i\sqrt{3} - 6)(i\sqrt{3} - 4)(-2 + i\sqrt{3})} x^6 + O(x^8) \right) \right)
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 410

AsymptoticDSolveValue[x^3\*y''[x]+Sin[x]\*y[x]==0,y[x],{x,0,7}]

$$\begin{aligned}
 y(x) \rightarrow c_1 & \left( \frac{\left( \frac{1}{5040} - \frac{1}{720(1+(-1)^{2/3})(2-(-1)^{2/3})} + \frac{36(1+(1-(-1)^{2/3})(2-(-1)^{2/3}))^{-\frac{1}{120}}}{6(1+(3-(-1)^{2/3})(4-(-1)^{2/3}))} \right) x^6}{1+(5-(-1)^{2/3})(6-(-1)^{2/3})} \right. \\
 & + \frac{\left( \frac{1}{36(1+(1-(-1)^{2/3})(2-(-1)^{2/3}))} - \frac{1}{120} \right) x^4}{1+(3-(-1)^{2/3})(4-(-1)^{2/3})} + \frac{x^2}{6(1+(1-(-1)^{2/3})(2-(-1)^{2/3}))} \\
 & \left. + 1 \right) x^{-(-1)^{2/3}} + c_2 \left( \frac{\left( \frac{1}{5040} - \frac{1}{720(1+(1+\sqrt[3]{-1})(2+\sqrt[3]{-1}))} + \frac{36(1+(1+\sqrt[3]{-1})(2+\sqrt[3]{-1}))^{-\frac{1}{120}}}{6(1+(3+\sqrt[3]{-1})(4+\sqrt[3]{-1}))} \right) x^6}{1+(5+\sqrt[3]{-1})(6+\sqrt[3]{-1})} + \frac{\left( \frac{1}{36(1+} \right)}{1}
 \end{aligned}$$

## 19.9 problem 2(e)

Internal problem ID [5695]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 2(e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$x^4 y'' + y \sin(x) = 0$$

With the expansion point for the power series method at  $x = 0$ .

**X** Solution by Maple

```
Order:=8;
dsolve(x^4*diff(y(x),x$2)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

No solution found

**✓** Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 294

```
AsymptoticDSolveValue[x^4*y''[x]+Sin[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 e^{-\frac{2i}{\sqrt{x}} x^{3/4}} \left( \frac{16487484152477478659746223ix^{13/2}}{2773583263632691770163200} - \frac{4594934148364735183693ix^{11/2}}{6320013947079701299200} \right. \\ + \frac{12579783586699513ix^{9/2}}{96185277197844480} - \frac{21896783401ix^{7/2}}{579820584960} + \frac{856783ix^{5/2}}{41943040} - \frac{3151ix^{3/2}}{73728} \\ - \frac{3986263268940827572255963529x^7}{21730712888356628741772337x^6} \\ \left. - \frac{207094217017907652172185600}{10920984100553723845017600} + \frac{1500040357444099007x^5}{5129881450551705600} + \frac{4885269094757x^4}{74217034874880} - \frac{2835642457x^3}{108716359680} + \frac{11659x^2}{524288} + \frac{15x}{512} - \frac{3i\sqrt{x}}{16} \right) \\ + 1) + c_2 e^{\frac{2i}{\sqrt{x}} x^{3/4}} \left( -\frac{16487484152477478659746223ix^{13/2}}{2773583263632691770163200} + \frac{4594934148364735183693ix^{11/2}}{6320013947079701299200} - \frac{125797835}{9618527}$$

## 19.10 problem 3(a)

Internal problem ID [5696]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 3(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3 y'' + (-1 + \cos(2x)) y' + 2xy = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 37

```
Order:=8;
dsolve(x^3*diff(y(x),x$2)+(cos(2*x)-1)*diff(y(x),x)+2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^2 \left( 1 - \frac{2}{9} x^2 + \frac{26}{675} x^4 - \frac{1742}{297675} x^6 + O(x^8) \right) \\ + c_2 x \left( 1 - \frac{1}{3} x^2 + \frac{17}{270} x^4 - \frac{173}{17010} x^6 + O(x^8) \right)$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 74

```
AsymptoticDSolveValue[x^3*y''[x]+(Cos[2*x]-1)*y'[x]+2*x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left( \frac{32351x^8}{40186125} - \frac{1742x^6}{297675} + \frac{26x^4}{675} - \frac{2x^2}{9} + 1 \right) x^2 \\ + c_1 \left( \frac{10471x^8}{7144200} - \frac{173x^6}{17010} + \frac{17x^4}{270} - \frac{x^2}{3} + 1 \right) x$$

## 19.11 problem 3(b)

Internal problem ID [5697]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 3(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + (2x^4 - 5x)y' + (3x^2 + 2)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

Order:=8;

`dsolve(4*x^2*diff(y(x),x$2)+(2*x^4-5*x)*diff(y(x),x)+(3*x^2+2)*y(x)=0,y(x),type='series',x=0)`

$$y(x) = c_1 x^{\frac{1}{4}} \left( 1 - \frac{3}{2}x^2 - \frac{1}{30}x^3 + \frac{1}{8}x^4 + \frac{137}{1300}x^5 - \frac{19}{12240}x^6 - \frac{7169}{764400}x^7 + O(x^8) \right) \\ + c_2 x^2 \left( 1 - \frac{1}{10}x^2 - \frac{4}{57}x^3 + \frac{3}{920}x^4 + \frac{32}{4275}x^5 + \frac{36287}{9753840}x^6 - \frac{4037}{16059750}x^7 + O(x^8) \right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 106

`AsymptoticDSolveValue[4*x^2*y'[x]+(2*x^4-5*x)*y'[x]+(3*x^2+2)*y[x]==0,y[x],{x,0,7}]`

$$y(x) \rightarrow c_1 \left( -\frac{4037x^7}{16059750} + \frac{36287x^6}{9753840} + \frac{32x^5}{4275} + \frac{3x^4}{920} - \frac{4x^3}{57} - \frac{x^2}{10} + 1 \right) x^2 \\ + c_2 \left( -\frac{7169x^7}{764400} - \frac{19x^6}{12240} + \frac{137x^5}{1300} + \frac{x^4}{8} - \frac{x^3}{30} - \frac{3x^2}{2} + 1 \right) \sqrt[4]{x}$$

## 19.12 problem 3(c)

Internal problem ID [5698]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 3(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2 y'' + 3y'x + 4xy = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 74

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+4*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 \left(1 - \frac{4}{3}x + \frac{2}{3}x^2 - \frac{8}{45}x^3 + \frac{4}{135}x^4 - \frac{16}{4725}x^5 + \frac{4}{14175}x^6 - \frac{16}{893025}x^7 + O(x^8)\right) x^2 + c_2 (\ln(x) (16x^2 - \frac{64}{3}x^3 + \frac{32}{3}x^4))}{1}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 116

```
AsymptoticDSolveValue[x^2*y''[x]+3*x*y'[x]+4*x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left( \frac{4x^6}{14175} - \frac{16x^5}{4725} + \frac{4x^4}{135} - \frac{8x^3}{45} + \frac{2x^2}{3} - \frac{4x}{3} + 1 \right) + c_1 \left( \frac{1696x^6 - 8976x^5 + 27900x^4 - 39600x^3 + 8100x^2 + 8100x + 2025}{2025x^2} - \frac{8}{135} (4x^4 - 24x^3 + 90x^2 - 180x + 135) \log(x) \right)$$

### 19.13 problem 3(d)

Internal problem ID [5699]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 3(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_Emden, _Fowler]`

$$x^3 y'' - 4x^2 y' + 3xy = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
Order:=8;
dsolve(x^3*diff(y(x),x$2)-4*x^2*diff(y(x),x)+3*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^{\frac{5}{2}} \left( x^{-\frac{\sqrt{13}}{2}} c_1 + x^{\frac{\sqrt{13}}{2}} c_2 \right) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 38

```
AsymptoticDSolveValue[x^3*y''[x]-4*x^2*y'[x]+3*x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 x^{\frac{1}{2}(5+\sqrt{13})} + c_2 x^{\frac{1}{2}(5-\sqrt{13})}$$

## 19.14 problem 4(a)

Internal problem ID [5700]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 4(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_Emden, _Fowler]`

$$4xy'' + 3y' + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
Order:=8;
dsolve(4*x*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{4}} \left( 1 - \frac{1}{5}x + \frac{1}{90}x^2 - \frac{1}{3510}x^3 + \frac{1}{238680}x^4 - \frac{1}{25061400}x^5 + \frac{1}{3759210000}x^6 - \frac{1}{763119630000}x^7 + O(x^8) \right) + c_2 \left( 1 - \frac{1}{3}x + \frac{1}{42}x^2 - \frac{1}{1386}x^3 + \frac{1}{83160}x^4 - \frac{1}{7900200}x^5 + \frac{1}{1090227600}x^6 - \frac{1}{206053016400}x^7 + O(x^8) \right)$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 113

```
AsymptoticDSolveValue[4*x*y''[x]+3*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \sqrt[4]{x} \left( -\frac{x^7}{763119630000} + \frac{x^6}{3759210000} - \frac{x^5}{25061400} + \frac{x^4}{238680} - \frac{x^3}{3510} + \frac{x^2}{90} - \frac{x}{5} + 1 \right) + c_2 \left( -\frac{x^7}{206053016400} + \frac{x^6}{1090227600} - \frac{x^5}{7900200} + \frac{x^4}{83160} - \frac{x^3}{1386} + \frac{x^2}{42} - \frac{x}{3} + 1 \right)$$



## 19.15 problem 4(b)

Internal problem ID [5701]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 4(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$2xy'' + (-x + 3)y' - y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+(3-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 \left( 1 + \frac{1}{3}x + \frac{1}{15}x^2 + \frac{1}{105}x^3 + \frac{1}{945}x^4 + \frac{1}{10395}x^5 + \frac{1}{135135}x^6 + \frac{1}{2027025}x^7 + O(x^8) \right) \sqrt{x} + c_1 \left( 1 + \frac{1}{2}x + \frac{1}{8}x^2 + \frac{1}{48}x^3 + \dots \right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 113

```
AsymptoticDSolveValue[2*x*y''[x]+(3-x)*y'[x]-y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( \frac{x^7}{2027025} + \frac{x^6}{135135} + \frac{x^5}{10395} + \frac{x^4}{945} + \frac{x^3}{105} + \frac{x^2}{15} + \frac{x}{3} + 1 \right) + \frac{c_2 \left( \frac{x^7}{645120} + \frac{x^6}{46080} + \frac{x^5}{3840} + \frac{x^4}{384} + \frac{x^3}{48} + \frac{x^2}{8} + \frac{x}{2} + 1 \right)}{\sqrt{x}}$$

## 19.16 problem 4(c)

Internal problem ID [5702]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 4(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2xy'' + y'(1+x) + 3y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 52

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+(x+1)*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 \sqrt{x} \left( 1 - \frac{7}{6}x + \frac{21}{40}x^2 - \frac{11}{80}x^3 + \frac{143}{5760}x^4 - \frac{13}{3840}x^5 + \frac{17}{46080}x^6 - \frac{323}{9676800}x^7 + O(x^8) \right) \\ + c_2 \left( 1 - 3x + 2x^2 - \frac{2}{3}x^3 + \frac{1}{7}x^4 - \frac{1}{45}x^5 + \frac{4}{1485}x^6 - \frac{4}{15015}x^7 + O(x^8) \right)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 106

```
AsymptoticDSolveValue[2*x*x*y'[x]+(x+1)*y'[x]+3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( -\frac{1386072x^7}{35} + \frac{20088x^6}{5} - \frac{2511x^5}{5} + 81x^4 - 18x^3 + 6x^2 - 3x + 1 \right) \\ + c_2 e^{\frac{1}{2}/x} \left( \frac{257243688x^7}{35} + \frac{2381886x^6}{5} + \frac{176436x^5}{5} + 3042x^4 + 312x^3 + 39x^2 + 6x \right. \\ \left. + 1 \right) x^{3/2}$$

## 19.17 problem 4(d)

Internal problem ID [5703]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 4(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' + y'x - (1+x)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=8;
dsolve(2*x^2*diff(y(x),x$2)+x*diff(y(x),x)-(x+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{3}{2}} \left( 1 + \frac{1}{5}x + \frac{1}{70}x^2 + \frac{1}{1890}x^3 + \frac{1}{83160}x^4 + \frac{1}{5405400}x^5 + \frac{1}{486486000}x^6 + \frac{1}{57891834000}x^7 + O(x^8) \right) + c_1 \left( 1 - x - \frac{1}{2} \right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 112

```
AsymptoticDSolveValue[2*x^2*y'[x]+x*y'[x]-(x+1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 x \left( \frac{x^7}{57891834000} + \frac{x^6}{486486000} + \frac{x^5}{5405400} + \frac{x^4}{83160} + \frac{x^3}{1890} + \frac{x^2}{70} + \frac{x}{5} + 1 \right) + \frac{c_2 \left( -\frac{x^7}{52390800} - \frac{x^6}{680400} - \frac{x^5}{12600} - \frac{x^4}{360} - \frac{x^3}{18} - \frac{x^2}{2} - x + 1 \right)}{\sqrt{x}}$$

## 19.18 problem 5

Internal problem ID [5704]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 5.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [Lienard]

$$x^2 y'' + y' x + x^2 y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x) c_2 + c_1) \left( 1 - \frac{1}{4}x^2 + \frac{1}{64}x^4 - \frac{1}{2304}x^6 + O(x^8) \right) \\ + \left( \frac{1}{4}x^2 - \frac{3}{128}x^4 + \frac{11}{13824}x^6 + O(x^8) \right) c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 81

```
AsymptoticDSolveValue[x^2*y'[x]+x*y'[x]+x^2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( -\frac{x^6}{2304} + \frac{x^4}{64} - \frac{x^2}{4} + 1 \right) + c_2 \left( \frac{11x^6}{13824} - \frac{3x^4}{128} + \frac{x^2}{4} + \left( -\frac{x^6}{2304} + \frac{x^4}{64} - \frac{x^2}{4} + 1 \right) \log(x) \right)$$

## 19.19 problem 6

Internal problem ID [5705]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 6.


**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`, `[_2nd_order, _linear]`, ‘

$$y'' + \frac{y'}{x^2} - \frac{y}{x^3} = 0$$

With the expansion point for the power series method at  $x = 0$ .

 Solution by Maple

```
Order:=8;
dsolve(diff(y(x),x$2)+1/x^2*diff(y(x),x)-1/x^3*y(x)=0,y(x),type='series',x=0);
```

No solution found

 Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 17

```
AsymptoticDSolveValue[y''[x]+1/x^2*y'[x]-1/x^3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 x + c_2 e^{\frac{1}{x}} x$$

## 19.20 problem 8

Internal problem ID [5706]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.4. REGULAR SINGULAR POINTS. Page 175

**Problem number:** 8.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2 y'' + (3x - 1) y' + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

**X** Solution by Maple

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+(3*x-1)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

No solution found

**✓** Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 53

```
AsymptoticDSolveValue[x^2*y'[x]+(3*x-1)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1(5040x^7 + 720x^6 + 120x^5 + 24x^4 + 6x^3 + 2x^2 + x + 1) + \frac{c_2 e^{-1/x}}{x}$$

**20 Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183**

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## 20.1 problem 1

Internal problem ID [5707]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183

**Problem number:** 1.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - 3y'x + (4 + 4x)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+(4*x+4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left( (\ln(x) c_2 + c_1) \left( 1 - 4x + 4x^2 - \frac{16}{9}x^3 + \frac{4}{9}x^4 - \frac{16}{225}x^5 + \frac{16}{2025}x^6 - \frac{64}{99225}x^7 + O(x^8) \right) \right. \\ \left. + \left( 8x - 12x^2 + \frac{176}{27}x^3 - \frac{50}{27}x^4 + \frac{1096}{3375}x^5 - \frac{392}{10125}x^6 + \frac{3872}{1157625}x^7 + O(x^8) \right) c_2 \right) x^2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 158

```
AsymptoticDSolveValue[x^2*y''[x]-3*x*y'[x]+(4*x+4)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( -\frac{64x^7}{99225} + \frac{16x^6}{2025} - \frac{16x^5}{225} + \frac{4x^4}{9} - \frac{16x^3}{9} + 4x^2 - 4x + 1 \right) x^2 \\ + c_2 \left( \left( \frac{3872x^7}{1157625} - \frac{392x^6}{10125} + \frac{1096x^5}{3375} - \frac{50x^4}{27} + \frac{176x^3}{27} - 12x^2 + 8x \right) x^2 \right. \\ \left. + \left( -\frac{64x^7}{99225} + \frac{16x^6}{2025} - \frac{16x^5}{225} + \frac{4x^4}{9} - \frac{16x^3}{9} + 4x^2 - 4x + 1 \right) x^2 \log(x) \right)$$



## 20.2 problem 2

Internal problem ID [5708]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183

**Problem number:** 2.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - 8x^2y' + (4x^2 + 1)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 57

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)-8*x^2*diff(y(x),x)+(4*x^2+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x} \left( 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 \right) (\ln(x) c_2 + c_1) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 112

```
AsymptoticDSolveValue[4*x^2*y''[x]-8*x^2*y'[x]+(4*x^2+1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left( \frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right) + c_2 \sqrt{x} \left( \frac{x^7}{5040} + \frac{x^6}{720} + \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right) \log(x)$$

## 20.3 problem 3(a)

Internal problem ID [5709]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183

**Problem number:** 3(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Lienard]

$$xy'' + 2y' + xy = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
Order:=8;
dsolve(x*dif(y(x),x$2)+2*dif(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 \left( 1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 - \frac{1}{5040}x^6 + O(x^8) \right) + \frac{c_2 \left( 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + O(x^8) \right)}{x}$$

### ✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 56

```
AsymptoticDSolveValue[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( -\frac{x^5}{720} + \frac{x^3}{24} - \frac{x}{2} + \frac{1}{x} \right) + c_2 \left( -\frac{x^6}{5040} + \frac{x^4}{120} - \frac{x^2}{6} + 1 \right)$$

## 20.4 problem 3(b)

Internal problem ID [5710]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183

**Problem number:** 3(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - x^2 y' + (x^2 - 2)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 55

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)+(x^2-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^2 \left( 1 + \frac{1}{2}x + \frac{1}{20}x^2 - \frac{1}{60}x^3 - \frac{1}{210}x^4 - \frac{1}{3360}x^5 + \frac{1}{20160}x^6 + \frac{1}{100800}x^7 + O(x^8) \right) \\ + \frac{c_2 (12 + 6x + 6x^2 + 5x^3 + x^4 - \frac{1}{5}x^5 - \frac{1}{10}x^6 - \frac{3}{280}x^7 + O(x^8))}{x}$$

✓ Solution by Mathematica

Time used: 0.063 (sec). Leaf size: 96

```
AsymptoticDSolveValue[x^2*y'[x]-x^2*y'[x]+(x^2-2)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( -\frac{x^5}{120} - \frac{x^4}{60} + \frac{x^3}{12} + \frac{5x^2}{12} + \frac{x}{2} + \frac{1}{x} + \frac{1}{2} \right) \\ + c_2 \left( \frac{x^8}{20160} - \frac{x^7}{3360} - \frac{x^6}{210} - \frac{x^5}{60} + \frac{x^4}{20} + \frac{x^3}{2} + x^2 \right)$$

## 20.5 problem 3(c)

Internal problem ID [5711]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183

**Problem number:** 3(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$xy'' - y' + 4yx^3 = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 28

```
Order:=8;
dsolve(x*dif(y(x),x$2)-dif(y(x),x)+4*x^3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^2 \left( 1 - \frac{1}{6} x^4 + O(x^8) \right) + c_2 (-2 + x^4 + O(x^8))$$

### ✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 30

```
AsymptoticDSolveValue[x*y''[x]-y'[x]+4*x^3*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( 1 - \frac{x^4}{2} \right) + c_2 \left( x^2 - \frac{x^6}{6} \right)$$

## 20.6 problem 4

Internal problem ID [5712]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183

**Problem number:** 4.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$(x - 1)^2 y'' - 3(x - 1) y' + 2y = 0$$

With the expansion point for the power series method at  $x = 1$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
Order:=8;
dsolve((x-1)^2*diff(y(x),x$2)-3*(x-1)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = (x - 1)^2 \left( c_1 (x - 1)^{-\sqrt{2}} + c_2 (x - 1)^{\sqrt{2}} \right) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 34

```
AsymptoticDSolveValue[(x-1)^2*y'[x]-3*(x-1)*y'[x]+2*y[x]==0,y[x],{x,1,7}]
```

$$y(x) \rightarrow c_1 (x - 1)^{2+\sqrt{2}} + c_2 (x - 1)^{2-\sqrt{2}}$$

## 20.7 problem 5

Internal problem ID [5713]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183

**Problem number:** 5.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3(1+x)^2 y'' - y'(1+x) - y = 0$$

With the expansion point for the power series method at  $x = -1$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
Order:=8;
dsolve(3*(x+1)^2*diff(y(x),x$2)-(x+1)*diff(y(x),x)-y(x)=0,y(x),type='series',x=-1);
```

$$y(x) = (x+1)^{\frac{2}{3}} \left( (x+1)^{-\frac{\sqrt{7}}{3}} c_1 + (x+1)^{\frac{\sqrt{7}}{3}} c_2 \right) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 42

```
AsymptoticDSolveValue[3*(x+1)^2*y'[x]-(x+1)*y'[x]-y[x]==0,y[x],{x,-1,7}]
```

$$y(x) \rightarrow c_1(x+1)^{\frac{1}{3}(2+\sqrt{7})} + c_2(x+1)^{\frac{1}{3}(2-\sqrt{7})}$$

## 20.8 problem 6

Internal problem ID [5714]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183

**Problem number:** 6.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Bessel]

$$x^2 y'' + y' x + (x^2 - 1) y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^2 \left(1 - \frac{1}{8} x^2 + \frac{1}{192} x^4 - \frac{1}{9216} x^6 + O(x^8)\right) + c_2 (\ln(x) (x^2 - \frac{1}{8} x^4 + \frac{1}{192} x^6 + O(x^8)) + (-2 + \frac{3}{32} x^4 - \frac{7}{1152} x^6))}{x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 75

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left( -\frac{x^7}{9216} + \frac{x^5}{192} - \frac{x^3}{8} + x \right) + c_1 \left( \frac{5x^6 - 90x^4 + 288x^2 + 1152}{1152x} - \frac{1}{384} x (x^4 - 24x^2 + 192) \log(x) \right)$$

## 20.9 problem 7

Internal problem ID [5715]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.5. More on Regular Singular Points. Page 183

**Problem number:** 7.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + y' x + \left(x^2 - \frac{1}{4}\right) y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 \left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 - \frac{1}{5040}x^6 + O(x^8)\right) x + c_2 \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + O(x^8)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 76

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( -\frac{x^{11/2}}{720} + \frac{x^{7/2}}{24} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}} \right) + c_2 \left( -\frac{x^{13/2}}{5040} + \frac{x^{9/2}}{120} - \frac{x^{5/2}}{6} + \sqrt{x} \right)$$



**21 Chapter 4. Power Series Solutions and Special Functions. Section 4.6. Gauss's Hypergeometric Equation. Page 187**

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## 21.1 problem 2(a)

Internal problem ID [5716]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.6. Gauss's Hypergeometric Equation. Page 187

**Problem number:** 2(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Jacobi]

$$x(1-x)y'' + \left(\frac{3}{2} - 2x\right)y' + 2y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
Order:=8;
dsolve(x*(1-x)*diff(y(x),x$2)+(3/2-2*x)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 \left(1 - \frac{4}{3}x + O(x^8)\right) \sqrt{x} + c_1 \left(1 - \frac{9}{2}x + \frac{15}{8}x^2 + \frac{7}{16}x^3 + \frac{27}{128}x^4 + \frac{33}{256}x^5 + \frac{91}{1024}x^6 + \frac{135}{2048}x^7 + O(x^8)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 71

```
AsymptoticDSolveValue[x*(1-x)*y'[x]+(3/2-2*x)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{c_2 \left(\frac{135x^7}{2048} + \frac{91x^6}{1024} + \frac{33x^5}{256} + \frac{27x^4}{128} + \frac{7x^3}{16} + \frac{15x^2}{8} - \frac{9x}{2} + 1\right)}{\sqrt{x}} + c_1 \left(1 - \frac{4x}{3}\right)$$

## 21.2 problem 2(b)

Internal problem ID [5717]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.6. Gauss's Hypergeometric Equation. Page 187

**Problem number:** 2(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(2x^2 + 2x)y'' + (5x + 1)y' + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 52

```
Order:=8;
dsolve((2*x^2+2*x)*diff(y(x),x$2)+(1+5*x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (-x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x + 1)(c_1\sqrt{x} + c_2) + O(x^8)$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 73

```
AsymptoticDSolveValue[(2*x^2+2*x)*y'[x]+(1+5*x)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1\sqrt{x}(-x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x + 1) + c_2(-x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x + 1)$$

## 21.3 problem 2(x)

Internal problem ID [5718]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.6. Gauss's Hypergeometric Equation. Page 187

**Problem number:** 2(x).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 - 1)y'' + (5x + 4)y' + 4y = 0$$

With the expansion point for the power series method at  $x = -1$ .

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 54

```
Order:=8;
dsolve((x^2-1)*diff(y(x),x$2)+(5*x+4)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=-1);
```

$$y(x) = c_1 \sqrt{x+1} \left( 1 + \frac{25}{12}(x+1) + \frac{245}{96}(x+1)^2 + \frac{315}{128}(x+1)^3 + \frac{4235}{2048}(x+1)^4 + \frac{13013}{8192}(x+1)^5 \right. \\ \left. + \frac{75075}{65536}(x+1)^6 + \frac{206635}{262144}(x+1)^7 + O((x+1)^8) \right) + c_2 \left( 1 + 4(x+1) + 6(x+1)^2 \right. \\ \left. + \frac{32}{5}(x+1)^3 + \frac{40}{7}(x+1)^4 + \frac{32}{7}(x+1)^5 + \frac{112}{33}(x+1)^6 + \frac{1024}{429}(x+1)^7 + O((x+1)^8) \right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 139

AsymptoticDSolveValue[(x^2-1)\*y'[x]+(5\*x+4)\*y'[x]+4\*y[x]==0,y[x],{x,-1,7}]

$$\begin{aligned}
 y(x) \rightarrow & c_1 \sqrt{x+1} \left( \frac{206635(x+1)^7}{262144} + \frac{75075(x+1)^6}{65536} + \frac{13013(x+1)^5}{8192} + \frac{4235(x+1)^4}{2048} \right. \\
 & + \frac{315}{128}(x+1)^3 + \frac{245}{96}(x+1)^2 + \frac{25(x+1)}{12} + 1 \left. \right) + c_2 \left( \frac{1024}{429}(x+1)^7 + \frac{112}{33}(x+1)^6 \right. \\
 & \left. + \frac{32}{7}(x+1)^5 + \frac{40}{7}(x+1)^4 + \frac{32}{5}(x+1)^3 + 6(x+1)^2 + 4(x+1) + 1 \right)
 \end{aligned}$$

## 21.4 problem 2(d)

Internal problem ID [5719]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.6. Gauss's Hypergeometric Equation. Page 187

**Problem number:** 2(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(x^2 - x - 6)y'' + (5 + 3x)y' + y = 0$$

With the expansion point for the power series method at  $x = 3$ .

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 54

```
Order:=8;
dsolve((x^2-x-6)*diff(y(x),x$2)+(5+3*x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=3);
```

$$y(x) = \frac{c_2 \left( 1 - \frac{1}{14}(x-3) + \frac{1}{133}(x-3)^2 - \frac{1}{1064}(x-3)^3 + \frac{1}{7714}(x-3)^4 - \frac{5}{262276}(x-3)^5 + \frac{5}{1704794}(x-3)^6 - \frac{5}{10715848}(x-3)^7 \right)}{1}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 145

```
AsymptoticDSolveValue[(x^2-x-6)*y'[x]+(5+3*x)*y'[x]+y[x]==0,y[x],{x,3,7}]
```

$$y(x) \rightarrow c_1 \left( -\frac{5(x-3)^7}{10715848} + \frac{5(x-3)^6}{1704794} - \frac{5(x-3)^5}{262276} + \frac{(x-3)^4}{7714} - \frac{(x-3)^3}{1064} + \frac{1}{133}(x-3)^2 + \frac{3-x}{14} + 1 \right) + \frac{c_2 \left( \frac{2288(x-3)^7}{30517578125} - \frac{616(x-3)^6}{1220703125} + \frac{176(x-3)^5}{48828125} - \frac{11(x-3)^4}{390625} + \frac{4(x-3)^3}{15625} - \frac{2}{625}(x-3)^2 + \frac{4(x-3)}{25} + 1 \right)}{(x-3)^{9/5}}$$

## 21.5 problem 3

Internal problem ID [5720]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.6. Gauss's Hypergeometric Equation. Page 187

**Problem number:** 3.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Gegenbauer, [\_2nd\_order, \_linear, ‘\_with\_symmetry\_[0,F(x)]’]]

$$(-x^2 + 1)y'' - y'x + p^2y = 0$$

With the expansion point for the power series method at  $x = 1$ .

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 338

Order:=8;

dsolve((1-x^2)\*diff(y(x),x\$2)-x\*diff(y(x),x)+p^2\*y(x)=0,y(x),type='series',x=1);

$$\begin{aligned}
 y(x) = & c_1 \sqrt{x-1} \left( 1 + \left( \frac{p^2}{3} - \frac{1}{12} \right) (x-1) + \left( \frac{1}{30} p^4 - \frac{1}{12} p^2 + \frac{3}{160} \right) (x-1)^2 \right. \\
 & + \left( \frac{1}{630} p^6 - \frac{1}{72} p^4 + \frac{37}{1440} p^2 - \frac{5}{896} \right) (x-1)^3 \\
 & + \left( \frac{1}{22680} p^8 - \frac{1}{1080} p^6 + \frac{47}{8640} p^4 - \frac{3229}{362880} p^2 + \frac{35}{18432} \right) (x-1)^4 \\
 & + \left( \frac{1}{1247400} p^{10} - \frac{1}{30240} p^8 + \frac{19}{43200} p^6 - \frac{1571}{725760} p^4 + \frac{10679}{3225600} p^2 - \frac{63}{90112} \right) (x-1)^5 \\
 & + \left( \frac{1}{97297200} p^{12} - \frac{1}{1360800} p^{10} + \frac{67}{3628800} p^8 - \frac{2159}{10886400} p^6 + \frac{153617}{174182400} p^4 \right. \\
 & \quad \left. - \frac{550499}{425779200} p^2 + \frac{231}{851968} \right) (x-1)^6 \\
 & + \left( \frac{1}{10216206000} p^{14} - \frac{1}{89812800} p^{12} + \frac{11}{23328000} p^{10} - \frac{8521}{914457600} p^8 + \frac{230443}{2612736000} p^6 \right. \\
 & \quad \left. - \frac{1206053}{3284582400} p^4 + \frac{2430898831}{4649508864000} p^2 - \frac{143}{1310720} \right) (x-1)^7 + O((x-1)^8) \\
 & + c_2 \left( 1 + p^2(x-1) + \left( \frac{1}{6} p^4 - \frac{1}{6} p^2 \right) (x-1)^2 + \left( \frac{1}{90} p^6 - \frac{1}{18} p^4 + \frac{2}{45} p^2 \right) (x-1)^3 \right. \\
 & \quad + \left( \frac{1}{2520} p^8 - \frac{1}{180} p^6 + \frac{7}{360} p^4 - \frac{1}{70} p^2 \right) (x-1)^4 \\
 & \quad + \left( \frac{1}{113400} p^{10} - \frac{1}{3780} p^8 + \frac{13}{5400} p^6 - \frac{41}{5670} p^4 + \frac{8}{1575} p^2 \right) (x-1)^5 \\
 & + \left( \frac{1}{7484400} p^{12} - \frac{1}{136080} p^{10} + \frac{31}{226800} p^8 - \frac{139}{136080} p^6 + \frac{479}{170100} p^4 - \frac{4}{2079} p^2 \right) (x-1)^6 \\
 & + \left( \frac{1}{681080400} p^{14} - \frac{1}{7484400} p^{12} + \frac{1}{226800} p^{10} - \frac{311}{4762800} p^8 + \frac{37}{85050} p^6 - \frac{59}{51975} p^4 \right. \\
 & \quad \left. + \frac{16}{21021} p^2 \right) (x-1)^7 + O((x-1)^8)
 \end{aligned}$$



✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 5699

```
AsymptoticDSolveValue[(1-x^2)*y'[x]-x*y'[x]+p^2*y[x]==0,y[x],{x,1,7}]
```

Too large to display

## 21.6 problem 5

Internal problem ID [5721]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Section 4.6. Gauss's Hypergeometric Equation. Page 187

**Problem number:** 5.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(1 - e^x) y'' + \frac{y'}{2} + e^x y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 52

```
Order:=8;
dsolve((1-exp(x))*diff(y(x),x$2)+1/2*diff(y(x),x)+exp(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{3}{2}} \left( 1 + \frac{1}{4}x + \frac{3}{32}x^2 + \frac{7}{384}x^3 + \frac{109}{30720}x^4 + \frac{13}{24576}x^5 + \frac{4439}{61931520}x^6 + \frac{2069}{247726080}x^7 + O(x^8) \right) + c_2 \left( 1 - 2x - x^2 - \frac{1}{3}x^3 - \frac{1}{12}x^4 - \frac{1}{60}x^5 - \frac{1}{360}x^6 - \frac{1}{2520}x^7 + O(x^8) \right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 109

```
AsymptoticDSolveValue[(1-Exp[x])*y'[x]+1/2*y'[x]+Exp[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left( -\frac{x^7}{2520} - \frac{x^6}{360} - \frac{x^5}{60} - \frac{x^4}{12} - \frac{x^3}{3} - x^2 - 2x + 1 \right) + c_1 \left( \frac{2069x^7}{247726080} + \frac{4439x^6}{61931520} + \frac{13x^5}{24576} + \frac{109x^4}{30720} + \frac{7x^3}{384} + \frac{3x^2}{32} + \frac{x}{4} + 1 \right) x^{3/2}$$

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Functions. Problems for review and discover. (A)  
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## 22.1 problem 1(a)

Internal problem ID [5722]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 1(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2xy - x^2 = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
Order:=8;
dsolve(diff(y(x),x$2)+2*x*y(x)=x^2,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{3}x^3 + \frac{1}{45}x^6\right) y(0) + \left(x - \frac{1}{6}x^4 + \frac{1}{126}x^7\right) D(y)(0) + \frac{x^4}{12} - \frac{x^7}{252} + O(x^8)$$

### ✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+2*x*y[x]==x^2,y[x],{x,0,7}]
```

$$y(x) \rightarrow -\frac{x^7}{252} + \frac{x^4}{12} + c_2 \left( \frac{x^7}{126} - \frac{x^4}{6} + x \right) + c_1 \left( \frac{x^6}{45} - \frac{x^3}{3} + 1 \right)$$

## 22.2 problem 1(b)

Internal problem ID [5723]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 1(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y'x + y - x = 0$$

With the expansion point for the power series method at  $x = 0$ .

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=8;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+y(x)=x,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{24}x^4 - \frac{1}{240}x^6\right)y(0) + D(y)(0)x + \frac{x^3}{6} + \frac{x^5}{60} + \frac{x^7}{630} + O(x^8)$$

### ✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 55

```
AsymptoticDSolveValue[y''[x]-x*y'[x]+y[x]==x,y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{x^7}{630} + \frac{x^5}{60} + \frac{x^3}{6} + c_1 \left( -\frac{x^6}{240} - \frac{x^4}{24} - \frac{x^2}{2} + 1 \right) + c_2 x$$

## 22.3 problem 1(c)

Internal problem ID [5724]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 1(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y' + y - x^3 + x = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
Order:=8;
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=x^3-x,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{120}x^5 + \frac{1}{720}x^6\right) y(0) \\ + \left(x - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \frac{1}{5040}x^7\right) D(y)(0) - \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{20} - \frac{7x^6}{720} + \frac{x^7}{5040} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 105

```
AsymptoticDSolveValue[y''[x]+y'[x]+y[x]==x^3-x,y[x],{x,0,7}]
```

$$y(x) \rightarrow \frac{x^7}{5040} - \frac{7x^6}{720} + \frac{x^5}{20} + \frac{x^4}{24} - \frac{x^3}{6} + c_2 \left( \frac{x^7}{5040} - \frac{x^5}{120} + \frac{x^4}{24} - \frac{x^2}{2} + x \right) \\ + c_1 \left( \frac{x^6}{720} - \frac{x^5}{120} + \frac{x^3}{6} - \frac{x^2}{2} + 1 \right)$$

## 22.4 problem 1(d)

Internal problem ID [5725]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 1(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$2y'' + y'x + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=8;
dsolve(2*dif(y(x),x$2)+x*dif(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{4}x^2 + \frac{1}{32}x^4 - \frac{1}{384}x^6\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{60}x^5 - \frac{1}{840}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[2*y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_2 \left( -\frac{x^7}{840} + \frac{x^5}{60} - \frac{x^3}{6} + x \right) + c_1 \left( -\frac{x^6}{384} + \frac{x^4}{32} - \frac{x^2}{4} + 1 \right)$$

## 22.5 problem 1(e)

Internal problem ID [5726]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 1(e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 4)y'' - y' + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 74

```
Order:=8;
dsolve((4+x^2)*diff(y(x),x$2)-diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{8}x^2 - \frac{1}{96}x^3 + \frac{11}{1536}x^4 + \frac{13}{10240}x^5 - \frac{533}{737280}x^6 - \frac{3809}{20643840}x^7\right) y(0) \\ + \left(x + \frac{1}{8}x^2 - \frac{1}{32}x^3 - \frac{5}{512}x^4 + \frac{23}{10240}x^5 + \frac{283}{245760}x^6 - \frac{1649}{6881280}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 98

```
AsymptoticDSolveValue[(4+x^2)*y''[x]-y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( -\frac{3809x^7}{20643840} - \frac{533x^6}{737280} + \frac{13x^5}{10240} + \frac{11x^4}{1536} - \frac{x^3}{96} - \frac{x^2}{8} + 1 \right) \\ + c_2 \left( -\frac{1649x^7}{6881280} + \frac{283x^6}{245760} + \frac{23x^5}{10240} - \frac{5x^4}{512} - \frac{x^3}{32} + \frac{x^2}{8} + x \right)$$



## 22.6 problem 1(f)

Internal problem ID [5727]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 1(f).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 1)y'' - y'x + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
Order:=8;
dsolve((x^2+1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{80}x^6\right)y(0) + D(y)(0)x + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[(x^2+1)*y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( -\frac{x^6}{80} + \frac{x^4}{24} - \frac{x^2}{2} + 1 \right) + c_2 x$$

## 22.7 problem 1(g)

Internal problem ID [5728]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 1(g).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y'(1+x) - xy = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
Order:=8;
dsolve(diff(y(x),x$2)-(x+1)*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{30}x^5 + \frac{1}{60}x^6 + \frac{37}{5040}x^7\right) y(0) \\ + \left(x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{8}x^5 + \frac{47}{720}x^6 + \frac{19}{630}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 91

```
AsymptoticDSolveValue[y'[x]-(x+1)*y'[x]-x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( \frac{37x^7}{5040} + \frac{x^6}{60} + \frac{x^5}{30} + \frac{x^4}{24} + \frac{x^3}{6} + 1 \right) + c_2 \left( \frac{19x^7}{630} + \frac{47x^6}{720} + \frac{x^5}{8} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x \right)$$

## 22.8 problem 1(h)

Internal problem ID [5729]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 1(h).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(x - 1)y'' + y'(1 + x) + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 74

```
Order:=8;
dsolve((x-1)*diff(y(x),x$2)+(x+1)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{3}{8}x^4 + \frac{11}{30}x^5 + \frac{53}{144}x^6 + \frac{103}{280}x^7\right) y(0) \\ + \left(x + \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{5}{8}x^4 + \frac{19}{30}x^5 + \frac{91}{144}x^6 + \frac{177}{280}x^7\right) D(y)(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 98

```
AsymptoticDSolveValue[(x-1)*y''[x]+(x+1)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( \frac{103x^7}{280} + \frac{53x^6}{144} + \frac{11x^5}{30} + \frac{3x^4}{8} + \frac{x^3}{3} + \frac{x^2}{2} + 1 \right) \\ + c_2 \left( \frac{177x^7}{280} + \frac{91x^6}{144} + \frac{19x^5}{30} + \frac{5x^4}{8} + \frac{2x^3}{3} + \frac{x^2}{2} + x \right)$$

## 22.9 problem 2(a)

Internal problem ID [5730]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 2(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 1)x^2y'' - y'x + (2 + x)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 87

Order:=8;

```
dsolve((x^2+1)*x^2*diff(y(x),x$2)-x*diff(y(x),x)+(2+x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}
 y(x) = & c_1 x^{1-i} \left( 1 + \left( -\frac{1}{5} - \frac{2i}{5} \right) x + \left( -\frac{1}{40} + \frac{13i}{40} \right) x^2 + \left( \frac{71}{520} + \frac{17i}{520} \right) x^3 + \left( -\frac{31}{832} - \frac{541i}{4160} \right) x^4 \right. \\
 & + \left( -\frac{1423}{20800} + \frac{7i}{4160} \right) x^5 + \left( \frac{12849}{416000} + \frac{10853i}{156000} \right) x^6 + \left( \frac{209609}{5088000} - \frac{106907i}{17808000} \right) x^7 \\
 & \left. + O(x^8) \right) + c_2 x^{1+i} \left( 1 + \left( -\frac{1}{5} + \frac{2i}{5} \right) x + \left( -\frac{1}{40} - \frac{13i}{40} \right) x^2 + \left( \frac{71}{520} - \frac{17i}{520} \right) x^3 \right. \\
 & + \left( -\frac{31}{832} + \frac{541i}{4160} \right) x^4 + \left( -\frac{1423}{20800} - \frac{7i}{4160} \right) x^5 + \left( \frac{12849}{416000} - \frac{10853i}{156000} \right) x^6 \\
 & \left. + \left( \frac{209609}{5088000} + \frac{106907i}{17808000} \right) x^7 + O(x^8) \right)
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 122

AsymptoticDSolveValue[(x^2+1)\*x^2\*y'[x]-x\*y'[x]+(2+x)\*y[x]==0,y[x],{x,0,7}]

$$\begin{aligned}
 y(x) \rightarrow & \left( \frac{1}{156000} + \frac{i}{1248000} \right) c_2 x^{1-i} \left( (6080 + 10093i)x^6 - (10476 - 1572i)x^5 \right. \\
 & - (8220 + 19260i)x^4 + (21600 + 2400i)x^3 + (2400 + 50400i)x^2 - (38400 + 57600i)x \\
 & \left. + (153600 - 19200i) \right) - \left( \frac{1}{1248000} + \frac{i}{156000} \right) c_1 x^{1+i} \left( (10093 + 6080i)x^6 \right. \\
 & + (1572 - 10476i)x^5 - (19260 + 8220i)x^4 + (2400 + 21600i)x^3 + (50400 + 2400i)x^2 \\
 & \left. - (57600 + 38400i)x - (19200 - 153600i) \right)
 \end{aligned}$$

## 22.10 problem 2(b)

Internal problem ID [5731]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 2(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + y' x + (1 + x) y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 85

`Order:=8;`

`dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);`

$$\begin{aligned}
 y(x) = & c_1 x^{-i} \left( 1 + \left( -\frac{1}{5} - \frac{2i}{5} \right) x + \left( -\frac{1}{40} + \frac{3i}{40} \right) x^2 + \left( \frac{3}{520} - \frac{7i}{1560} \right) x^3 \right. \\
 & \left. + \left( -\frac{1}{2496} + \frac{i}{12480} \right) x^4 + \left( \frac{9}{603200} + \frac{i}{361920} \right) x^5 \right. \\
 & \left. + \left( -\frac{19}{54288000} - \frac{7i}{36192000} \right) x^6 + \left( \frac{1}{179829000} + \frac{223i}{40281696000} \right) x^7 + O(x^8) \right) \\
 & + c_2 x^i \left( 1 + \left( -\frac{1}{5} + \frac{2i}{5} \right) x + \left( -\frac{1}{40} - \frac{3i}{40} \right) x^2 + \left( \frac{3}{520} + \frac{7i}{1560} \right) x^3 \right. \\
 & \left. + \left( -\frac{1}{2496} - \frac{i}{12480} \right) x^4 + \left( \frac{9}{603200} - \frac{i}{361920} \right) x^5 \right. \\
 & \left. + \left( -\frac{19}{54288000} + \frac{7i}{36192000} \right) x^6 + \left( \frac{1}{179829000} - \frac{223i}{40281696000} \right) x^7 + O(x^8) \right)
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 118

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(1+x)*y[x]==0,y[x],{x,0,7}]
```

$$\begin{aligned}
 y(x) \rightarrow & \left( \frac{7}{36192000} + \frac{19i}{54288000} \right) c_1 x^i (ix^6 + (12 - 36i)x^5 - (660 - 780i)x^4 + (16800 - 7200i)x^3 \\
 & - (194400 + 36000i)x^2 + (633600 + 921600i)x + (1209600 - 2188800i)) - \left( \frac{19}{54288000} \right. \\
 & \left. + \frac{7i}{36192000} \right) c_2 x^{-i} (x^6 - (36 - 12i)x^5 + (780 - 660i)x^4 - (7200 - 16800i)x^3 \\
 & - (36000 + 194400i)x^2 + (921600 + 633600i)x - (2188800 - 1209600i))
 \end{aligned}$$

## 22.11 problem 2(c)

Internal problem ID [5732]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 2(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Lienard]

$$xy'' - 4y' + xy = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
Order:=8;
dsolve(x*diff(y(x),x$2)-4*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^5 \left( 1 - \frac{1}{14} x^2 + \frac{1}{504} x^4 - \frac{1}{33264} x^6 + O(x^8) \right) \\ + c_2 (2880 + 480x^2 + 120x^4 - 20x^6 + O(x^8))$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x*y''[x]-4*y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( -\frac{x^6}{144} + \frac{x^4}{24} + \frac{x^2}{6} + 1 \right) + c_2 \left( -\frac{x^{11}}{33264} + \frac{x^9}{504} - \frac{x^7}{14} + x^5 \right)$$



## 22.12 problem 2(d)

Internal problem ID [5733]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 2(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 4x^2y' + 2y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 83

Order:=8;

```
dsolve(4*x^2*diff(y(x),x$2)+4*x^2*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}
 y(x) = & c_1 x^{\frac{1}{2}-\frac{i}{2}} \left( 1 - \frac{1}{2}x + \left( \frac{7}{40} + \frac{i}{40} \right) x^2 + \left( -\frac{11}{240} - \frac{i}{80} \right) x^3 + \left( \frac{31}{3264} + \frac{i}{272} \right) x^4 \right. \\
 & \left. + \left( -\frac{53}{32640} - \frac{13i}{16320} \right) x^5 + \left( \frac{3421}{14492160} + \frac{223i}{1610240} \right) x^6 \right. \\
 & \left. + \left( -\frac{30269}{1014451200} - \frac{977i}{48307200} \right) x^7 + O(x^8) \right) + c_2 x^{\frac{1}{2}+\frac{i}{2}} \left( 1 - \frac{1}{2}x + \left( \frac{7}{40} - \frac{i}{40} \right) x^2 \right. \\
 & \left. + \left( -\frac{11}{240} + \frac{i}{80} \right) x^3 + \left( \frac{31}{3264} - \frac{i}{272} \right) x^4 + \left( -\frac{53}{32640} + \frac{13i}{16320} \right) x^5 \right. \\
 & \left. + \left( \frac{3421}{14492160} - \frac{223i}{1610240} \right) x^6 + \left( -\frac{30269}{1014451200} + \frac{977i}{48307200} \right) x^7 + O(x^8) \right)
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 226

AsymptoticDSolveValue[4\*x^2\*y'[x]+4\*x^2\*y'[x]+2\*y[x]==0,y[x],{x,0,7}]

$$\begin{aligned}
 y(x) \rightarrow & c_1 \left( \left( \frac{3421}{14492160} - \frac{223i}{1610240} \right) x^{\frac{13}{2} + \frac{i}{2}} - \left( \frac{53}{32640} - \frac{13i}{16320} \right) x^{\frac{11}{2} + \frac{i}{2}} + \left( \frac{31}{3264} - \frac{i}{272} \right) x^{\frac{9}{2} + \frac{i}{2}} \right. \\
 & \left. - \left( \frac{11}{240} - \frac{i}{80} \right) x^{\frac{7}{2} + \frac{i}{2}} + \left( \frac{7}{40} - \frac{i}{40} \right) x^{\frac{5}{2} + \frac{i}{2}} - \frac{1}{2} x^{\frac{3}{2} + \frac{i}{2}} + x^{\frac{1}{2} + \frac{i}{2}} \right) \\
 & + c_2 \left( \left( \frac{3421}{14492160} + \frac{223i}{1610240} \right) x^{\frac{13}{2} - \frac{i}{2}} - \left( \frac{53}{32640} + \frac{13i}{16320} \right) x^{\frac{11}{2} - \frac{i}{2}} \right. \\
 & \left. + \left( \frac{31}{3264} + \frac{i}{272} \right) x^{\frac{9}{2} - \frac{i}{2}} - \left( \frac{11}{240} + \frac{i}{80} \right) x^{\frac{7}{2} - \frac{i}{2}} + \left( \frac{7}{40} + \frac{i}{40} \right) x^{\frac{5}{2} - \frac{i}{2}} - \frac{1}{2} x^{\frac{3}{2} - \frac{i}{2}} + x^{\frac{1}{2} - \frac{i}{2}} \right)
 \end{aligned}$$

## 22.13 problem 2(e)

Internal problem ID [5734]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 2(e).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2xy'' + (1 - x)y' + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 40

```
Order:=8;
dsolve(2*x*diff(y(x),x$2)+(1-x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 \sqrt{x} \left( 1 - \frac{1}{6}x - \frac{1}{120}x^2 - \frac{1}{1680}x^3 - \frac{1}{24192}x^4 - \frac{1}{380160}x^5 - \frac{1}{6589440}x^6 - \frac{1}{125798400}x^7 + O(x^8) \right) + c_2(1 - x + O(x^8))$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 111

```
AsymptoticDSolveValue[2*x*y''[x]+(1-x)*y'[x]+(2+x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left( \frac{17333x^7}{48432384000} - \frac{34817x^6}{691891200} - \frac{1171x^5}{4435200} + \frac{121x^4}{40320} + \frac{37x^3}{1680} - \frac{3x^2}{40} - \frac{x}{2} + 1 \right) + c_2 \left( \frac{4x^7}{143325} - \frac{x^6}{8400} - \frac{19x^5}{6300} - \frac{x^4}{840} + \frac{2x^3}{15} + \frac{x^2}{6} - 2x + 1 \right)$$

## 22.14 problem 2(f)

Internal problem ID [5735]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 2(f).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Laguerre]

$$xy'' - (x - 1)y' + 2y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=8;
dsolve(x*dif(y(x),x$2)-(x-1)*dif(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x) c_2 + c_1) \left( 1 - 2x + \frac{1}{2}x^2 + O(x^8) \right) + \left( 5x - \frac{9}{4}x^2 + \frac{1}{18}x^3 + \frac{1}{288}x^4 + \frac{1}{3600}x^5 + \frac{1}{43200}x^6 + \frac{1}{529200}x^7 + O(x^8) \right) c_2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 83

```
AsymptoticDSolveValue[x*y''[x]-(x-1)*y'[x]+2*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( \frac{x^2}{2} - 2x + 1 \right) + c_2 \left( \frac{x^7}{529200} + \frac{x^6}{43200} + \frac{x^5}{3600} + \frac{x^4}{288} + \frac{x^3}{18} - \frac{9x^2}{4} + \left( \frac{x^2}{2} - 2x + 1 \right) \log(x) + 5x \right)$$

## 22.15 problem 2(g)

Internal problem ID [5736]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 2(g).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + x(1-x)y' + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 85

`Order:=8;`

`dsolve(x^2*diff(y(x),x$2)+x*(1-x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);`

$$\begin{aligned}
 y(x) = & c_1 x^{-i} \left( 1 + \left( \frac{2}{5} - \frac{i}{5} \right) x + \left( \frac{1}{10} - \frac{i}{20} \right) x^2 + \left( \frac{17}{780} - \frac{i}{130} \right) x^3 + \left( \frac{5}{1248} - \frac{i}{1248} \right) x^4 \right. \\
 & + \left( \frac{113}{180960} - \frac{7i}{180960} \right) x^5 + \left( \frac{911}{10857600} + \frac{19i}{3619200} \right) x^6 \\
 & \left. + \left( \frac{39799}{4028169600} + \frac{1009i}{575452800} \right) x^7 + O(x^8) \right) + c_2 x^i \left( 1 + \left( \frac{2}{5} + \frac{i}{5} \right) x \right. \\
 & + \left( \frac{1}{10} + \frac{i}{20} \right) x^2 + \left( \frac{17}{780} + \frac{i}{130} \right) x^3 + \left( \frac{5}{1248} + \frac{i}{1248} \right) x^4 + \left( \frac{113}{180960} + \frac{7i}{180960} \right) x^5 \\
 & \left. + \left( \frac{911}{10857600} - \frac{19i}{3619200} \right) x^6 + \left( \frac{39799}{4028169600} - \frac{1009i}{575452800} \right) x^7 + O(x^8) \right)
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 122

```
AsymptoticDSolveValue[x^2*y'[x]+x*(1-x)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$\begin{aligned}
 y(x) \rightarrow & \left( \frac{59}{10857600} - \frac{17i}{10857600} \right) c_2 x^{-i} \left( (14 + 5i)x^6 + (108 + 24i)x^5 + (720 + 60i)x^4 \right. \\
 & \left. + (4080 - 240i)x^3 + (19440 - 3600i)x^2 + (77760 - 14400i)x + (169920 + 48960i) \right) \\
 & + \left( \frac{59}{10857600} + \frac{17i}{10857600} \right) c_1 x^i \left( (14 - 5i)x^6 + (108 - 24i)x^5 + (720 - 60i)x^4 \right. \\
 & \left. + (4080 + 240i)x^3 + (19440 + 3600i)x^2 + (77760 + 14400i)x + (169920 - 48960i) \right)
 \end{aligned}$$

## 22.16 problem 2(h)

Internal problem ID [5737]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 2(h).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$xy'' + y'(1+x) + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
Order:=8;
dsolve(x*dif(y(x),x$2)+(x+1)*dif(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x) c_2 + c_1) \left( 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \frac{1}{720}x^6 - \frac{1}{5040}x^7 + O(x^8) \right) \\ + \left( x - \frac{3}{4}x^2 + \frac{11}{36}x^3 - \frac{25}{288}x^4 + \frac{137}{7200}x^5 - \frac{49}{14400}x^6 + \frac{121}{235200}x^7 + O(x^8) \right) c_2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 151

```
AsymptoticDSolveValue[x*y'[x]+(x+1)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \rightarrow c_1 \left( -\frac{x^7}{5040} + \frac{x^6}{720} - \frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1 \right) + c_2 \left( \frac{121x^7}{235200} - \frac{49x^6}{14400} + \frac{137x^5}{7200} \right. \\ \left. - \frac{25x^4}{288} + \frac{11x^3}{36} - \frac{3x^2}{4} + \left( -\frac{x^7}{5040} + \frac{x^6}{720} - \frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1 \right) \log(x) + x \right)$$

**22.17 problem 3(a)**

Internal problem ID [5738]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 3(a).

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

Solve

$$x^3 y''' + 2x^2 y'' + (x^2 + x) y' + yx = 0$$

With the expansion point for the power series method at  $x = 0$ .



✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 1219

Order:=8;

dsolve(x^3\*diff(y(x),x\$3)+2\*x^2\*diff(y(x),x\$2)+(x+x^2)\*diff(y(x),x)+x\*y(x)=0,y(x),type='series

$$\begin{aligned}
 y(x) = & c_1 x^{\frac{1}{2} - \frac{i\sqrt{3}}{2}} \left( 1 + \frac{1}{-1 + i\sqrt{3}} x + \frac{1}{2} \frac{1}{(-2 + i\sqrt{3})(-1 + i\sqrt{3})} x^2 \right. \\
 & + \frac{1}{6} \frac{1}{(-3 + i\sqrt{3})(-2 + i\sqrt{3})(-1 + i\sqrt{3})} x^3 \\
 & + \frac{1}{24} \frac{1}{(i\sqrt{3} - 4)(-3 + i\sqrt{3})(-2 + i\sqrt{3})(-1 + i\sqrt{3})} x^4 \\
 & + \frac{1}{120} \frac{1}{(i\sqrt{3} - 5)(i\sqrt{3} - 4)(-3 + i\sqrt{3})(-2 + i\sqrt{3})(-1 + i\sqrt{3})} x^5 \\
 & + \frac{1}{720} \frac{1}{(i\sqrt{3} - 6)(i\sqrt{3} - 5)(i\sqrt{3} - 4)(-3 + i\sqrt{3})(-2 + i\sqrt{3})(-1 + i\sqrt{3})} x^6 \\
 & + \frac{1}{5040} \frac{1}{(i\sqrt{3} - 7)(i\sqrt{3} - 6)(i\sqrt{3} - 5)(i\sqrt{3} - 4)(-3 + i\sqrt{3})(-2 + i\sqrt{3})(-1 + i\sqrt{3})} x^7 \\
 & \left. + O(x^8) \right) + c_2 x^{\frac{1}{2} + \frac{i\sqrt{3}}{2}} \left( 1 - \frac{1}{1 + i\sqrt{3}} x + \frac{1}{2} \frac{1}{(i\sqrt{3} + 2)(1 + i\sqrt{3})} x^2 \right. \\
 & - \frac{1}{6} \frac{1}{(i\sqrt{3} + 3)(i\sqrt{3} + 2)(1 + i\sqrt{3})} x^3 + \frac{1}{24} \frac{1}{(i\sqrt{3} + 4)(i\sqrt{3} + 3)(i\sqrt{3} + 2)(1 + i\sqrt{3})} x^4 \\
 & - \frac{1}{120} \frac{1}{(i\sqrt{3} + 5)(i\sqrt{3} + 4)(i\sqrt{3} + 3)(i\sqrt{3} + 2)(1 + i\sqrt{3})} x^5 \\
 & + \frac{1}{720} \frac{1}{(i\sqrt{3} + 6)(i\sqrt{3} + 5)(i\sqrt{3} + 4)(i\sqrt{3} + 3)(i\sqrt{3} + 2)(1 + i\sqrt{3})} x^6 \\
 & - \frac{1}{5040} \frac{1}{(i\sqrt{3} + 7)(i\sqrt{3} + 6)(i\sqrt{3} + 5)(i\sqrt{3} + 4)(i\sqrt{3} + 3)(i\sqrt{3} + 2)(1 + i\sqrt{3})} x^7 \\
 & \left. + O(x^8) \right) + c_3 \left( 1 - x + \frac{1}{3} x^2 - \frac{1}{21} x^3 + \frac{1}{273} x^4 - \frac{1}{5733} x^5 + \frac{1}{177723} x^6 - \frac{1}{7642089} x^7 + O(x^8) \right)
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 3447

AsymptoticDSolveValue[x^3\*y'''[x]+2\*x^2\*y''[x]+(x+x^2)\*y'[x]+x\*y[x]==0,y[x],{x,0,7}]

Too large to display

## 22.18 problem 3(b)

Internal problem ID [5739]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 3(b).

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

Solve

$$x^3 y''' + x^2 y'' - 3xy' + y(x-1) = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 12916

```
Order:=8;
dsolve(x^3*diff(y(x),x$3)+x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+(x-1)*y(x)=0,y(x),type='series'
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 11815

```
AsymptoticDSolveValue[x^3*y'''[x]+x^2*y''[x]-3*x*y'[x]+(x-1)*y[x]==0,y[x],{x,0,7}]
```

Too large to display

## 22.19 problem 3(c)

Internal problem ID [5740]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 3(c).

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

Solve

$$x^3 y''' - 2x^2 y'' + (x^2 + 2x) y' - yx = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 120

`Order:=8;`

`dsolve(x^3*diff(y(x),x$3)-2*x^2*diff(y(x),x$2)+(x^2+2*x)*diff(y(x),x)-x*y(x)=0,y(x),type='ser`

$$\begin{aligned}
 y(x) = & c_1 x^3 \left( 1 - \frac{1}{4}x + \frac{1}{40}x^2 - \frac{1}{720}x^3 + \frac{1}{20160}x^4 - \frac{1}{806400}x^5 + \frac{1}{43545600}x^6 \right. \\
 & \left. - \frac{1}{3048192000}x^7 + O(x^8) \right) \\
 & + c_2 x^2 \left( \ln(x) \left( (-240)x + 60x^2 - 6x^3 + \frac{1}{3}x^4 - \frac{1}{84}x^5 + \frac{1}{3360}x^6 - \frac{1}{181440}x^7 + O(x^8) \right) \right. \\
 & \left. + \left( 720 - 908x + 152x^2 - 11x^3 + \frac{4}{9}x^4 - \frac{79}{7056}x^5 + \frac{517}{2822400}x^6 - \frac{851}{457228800}x^7 \right. \right. \\
 & \left. \left. + O(x^8) \right) \right) + c_3 \left( 2 \ln(x) \left( x^3 - \frac{1}{4}x^4 + \frac{1}{40}x^5 - \frac{1}{720}x^6 + \frac{1}{20160}x^7 + O(x^8) \right) \right. \\
 & \left. + \left( -24 - 12x - 6x^2 + \frac{5}{8}x^4 - \frac{39}{400}x^5 + \frac{49}{7200}x^6 - \frac{199}{705600}x^7 + O(x^8) \right) \right)
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.623 (sec). Leaf size: 186

AsymptoticDSolveValue[x^3\*y'''[x]-2\*x^2\*y''[x]+(x^2+2\*x)\*y'[x]-x\*y[x]==0,y[x],{x,0,7}]

$$\begin{aligned}
 y(x) \rightarrow & c_1 \left( \frac{(x^3 - 18x^2 + 180x - 720)x^3 \log(x)}{4320} \right. \\
 & \left. + \frac{-167x^6 + 2466x^5 - 17100x^4 + 14400x^3 + 129600x^2 + 259200x + 518400}{259200} \right) \\
 & + c_2 \left( \frac{x^3(x^5 - 40x^4 + 1120x^3 - 20160x^2 + 201600x - 806400) \log(x)}{2419200} \right. \\
 & \left. - \frac{x^2(2941x^6 - 106720x^5 + 2618560x^4 - 38666880x^3 + 268128000x^2 - 225792000x - 2032128000)}{2032128000} \right) \\
 & + c_3 \left( \frac{x^9}{43545600} - \frac{x^8}{806400} + \frac{x^7}{20160} - \frac{x^6}{720} + \frac{x^5}{40} - \frac{x^4}{4} + x^3 \right)
 \end{aligned}$$

## 22.20 problem 3(d)

Internal problem ID [5741]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (A) Drill Exercises . Page 194

**Problem number:** 3(d).

**ODE order:** 3.

**ODE degree:** 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

Solve

$$x^3 y''' + (2x^3 - x^2) y'' - y' x + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 1506

Order:=8;

`dsolve(x^3*diff(y(x),x$3)+(2*x^3-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series'`

$$\begin{aligned}
 y(x) = & c_3 x (1 + O(x^8)) \\
 & + c_2 x^{\frac{3}{2} - \frac{\sqrt{13}}{2}} \left( 1 - x + \frac{-3 + \sqrt{13}}{2\sqrt{13} - 4} x^2 + \frac{5 - \sqrt{13}}{6\sqrt{13} - 12} x^3 + \frac{1}{24} \frac{(-5 + \sqrt{13})(-7 + \sqrt{13})}{(-2 + \sqrt{13})(-4 + \sqrt{13})} x^4 \right. \\
 & + \frac{1}{30} \frac{-19 + 4\sqrt{13}}{(-2 + \sqrt{13})(-4 + \sqrt{13})} x^5 + \frac{1}{20} \frac{-29 + 7\sqrt{13}}{(-2 + \sqrt{13})(-4 + \sqrt{13})(-6 + \sqrt{13})} x^6 \\
 & \left. + \frac{-\frac{117}{35} + \frac{6\sqrt{13}}{7}}{(-2 + \sqrt{13})(-4 + \sqrt{13})(-6 + \sqrt{13})(-7 + \sqrt{13})} x^7 + O(x^8) \right) \\
 & + c_1 x^{\frac{3}{2} + \frac{\sqrt{13}}{2}} \left( 1 - x + \frac{3 + \sqrt{13}}{4 + 2\sqrt{13}} x^2 + \frac{-5 - \sqrt{13}}{6\sqrt{13} + 12} x^3 + \frac{1}{24} \frac{(5 + \sqrt{13})(7 + \sqrt{13})}{(2 + \sqrt{13})(4 + \sqrt{13})} x^4 \right. \\
 & - \frac{1}{30} \frac{19 + 4\sqrt{13}}{(2 + \sqrt{13})(4 + \sqrt{13})} x^5 + \frac{1}{20} \frac{29 + 7\sqrt{13}}{(2 + \sqrt{13})(4 + \sqrt{13})(6 + \sqrt{13})} x^6 \\
 & \left. + \frac{-\frac{117}{35} - \frac{6\sqrt{13}}{7}}{(2 + \sqrt{13})(4 + \sqrt{13})(6 + \sqrt{13})(7 + \sqrt{13})} x^7 + O(x^8) \right)
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.216 (sec). Leaf size: 310

AsymptoticDSolveValue[x^3\*y'''[x]+(2\*x^3-x^2)\*y''[x]-y'[x]+y[x]==0,y[x],{x,0,7}]

$$\begin{aligned}
 y(x) \rightarrow & c_1 \left( \frac{99473x^7}{1008} + \frac{1043x^6}{144} + \frac{19x^5}{24} + \frac{11x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right) \\
 & + c_2 e^{-\frac{2}{\sqrt{x}}} \left( -\frac{279112936065458899252220570230691x^{13/2}}{160251477454333302276096} \right. \\
 & \quad - \frac{2430057902534044595693470483x^{11/2}}{100317681699677798400} - \frac{1545013796231079344731x^{9/2}}{3562417673994240} \\
 & \quad - \frac{2005991558758787x^{7/2}}{193273528320} - \frac{43999069453x^{5/2}}{125829120} - \frac{438565x^{3/2}}{24576} \\
 & \quad + \frac{14436319972596450047835320516938615783x^7}{897408273744266492746137600} \\
 & \quad + \frac{3840864007433053956366665361751x^6}{19260994886338137292800} + \frac{1786308115320202497636167x^5}{569986827839078400} \\
 & \quad + \frac{319234145332261451x^4}{4947802324992} + \frac{21959100963217x^3}{12079595520} + \frac{117706529x^2}{1572864} + \frac{2353x}{512} - \frac{29\sqrt{x}}{16} \\
 & \left. + 1 \right) x^{11/4} + c_3 e^{\frac{2}{\sqrt{x}}} \left( \frac{279112936065458899252220570230691x^{13/2}}{160251477454333302276096} + \frac{2430057902534044595693470483x^{11/2}}{100317681699677798400} \right)
 \end{aligned}$$

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Challenge Problems . Page 194**

23.1 problem 1(a)	447
23.2 problem 1(b)	449
23.3 problem 1(c)	450

## 23.1 problem 1(a)

Internal problem ID [5742]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (B) Challenge Problems . Page 194

**Problem number:** 1(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [[\_Emden, \_Fowler]]

$$x^3 y'' + x^2 y' + y = 0$$

With the expansion point for the power series method at  $x = \infty$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 207

Order:=8;

```
dsolve(x^3*diff(y(x),x$2)+x^2*diff(y(x),x)+y(x)=0,y(x),type='series',x=Infinity);
```

$$\begin{aligned}
 y(x) = & \left( 1 - \frac{(x - \text{Infinity})^2}{2 \text{Infinity}^3} + \frac{2(x - \text{Infinity})^3}{3 \text{Infinity}^4} + \frac{(-18 \text{Infinity} + 1)(x - \text{Infinity})^4}{24 \text{Infinity}^6} \right. \\
 & + \frac{(96 \text{Infinity} - 14)(x - \text{Infinity})^5}{120 \text{Infinity}^7} + \frac{(-600 \text{Infinity}^2 + 156 \text{Infinity} - 1)(x - \text{Infinity})^6}{720 \text{Infinity}^9} \\
 & \left. + \frac{(4320 \text{Infinity}^2 - 1692 \text{Infinity} + 30)(x - \text{Infinity})^7}{5040 \text{Infinity}^{10}} \right) y(\text{Infinity}) \\
 & + \left( x - \text{Infinity} - \frac{(x - \text{Infinity})^2}{2 \text{Infinity}} + \frac{(2 \text{Infinity}^2 - \text{Infinity})(x - \text{Infinity})^3}{6 \text{Infinity}^4} \right. \\
 & - \frac{(\text{Infinity} - \frac{4}{3})(x - \text{Infinity})^4}{4 \text{Infinity}^4} + \frac{(24 \text{Infinity}^3 - 58 \text{Infinity}^2 + \text{Infinity})(x - \text{Infinity})^5}{120 \text{Infinity}^7} \\
 & + \frac{(-120 \text{Infinity}^4 + 444 \text{Infinity}^3 - 21 \text{Infinity}^2)(x - \text{Infinity})^6}{720 \text{Infinity}^9} \\
 & \left. + \frac{(720 \text{Infinity}^4 - 3708 \text{Infinity}^3 + 324 \text{Infinity}^2 - \text{Infinity})(x - \text{Infinity})^7}{5040 \text{Infinity}^{10}} \right) D(y)(\text{Infinity}) \\
 & + O(x^8)
 \end{aligned}$$



✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 171

```
AsymptoticDSolveValue[x^3*y'[x]+x^2*y'[x]+y[x]==0,y[x],{x,Infinity,7}]
```

$$\begin{aligned}
 y(x) \rightarrow & c_1 \left( -\frac{1}{25401600x^7} + \frac{1}{518400x^6} - \frac{1}{14400x^5} + \frac{1}{576x^4} - \frac{1}{36x^3} + \frac{1}{4x^2} - \frac{1}{x} + 1 \right) \\
 & + c_2 \left( \frac{121}{592704000x^7} + \frac{\log(x)}{25401600x^7} - \frac{49}{5184000x^6} - \frac{\log(x)}{518400x^6} + \frac{137}{432000x^5} + \frac{\log(x)}{14400x^5} \right. \\
 & \left. - \frac{25}{3456x^4} - \frac{\log(x)}{576x^4} + \frac{11}{108x^3} + \frac{\log(x)}{36x^3} - \frac{3}{4x^2} - \frac{\log(x)}{4x^2} + \frac{2}{x} + \frac{\log(x)}{x} - \log(x) \right)
 \end{aligned}$$

## 23.2 problem 1(b)

Internal problem ID [5743]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (B) Challenge Problems . Page 194

**Problem number:** 1(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9(x-2)^2(x-3)y'' + 6x(x-2)y' + 16y = 0$$

With the expansion point for the power series method at  $x = \infty$ .

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 414

Order:=8;

`dsolve(9*(x-2)^2*(x-3)*diff(y(x),x$2)+6*x*(x-2)*diff(y(x),x)+16*y(x)=0,y(x),type='series',x=I`

Expression too large to display

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 130

`AsymptoticDSolveValue[9*(x-2)^2*(x-3)*y'[x]+6*x*(x-2)*y'[x]+16*y[x]==0,y[x],{x,Infinity,7}]`

$$y(x) \rightarrow c_2 \left( -\frac{13}{3x^{2/3}} - \frac{251}{45x^{5/3}} - \frac{7781}{810x^{8/3}} - \frac{22151}{1215x^{11/3}} - \frac{669229}{18225x^{14/3}} - \frac{216463313}{2788425x^{17/3}} - \frac{7179886604}{41826375x^{20/3}} + \sqrt[3]{x} \right) + c_1 \left( -\frac{401483448544}{1336967775x^7} - \frac{4666732192}{40514175x^6} - \frac{822592}{18225x^5} - \frac{285704}{15795x^4} - \frac{3004}{405x^3} - \frac{28}{9x^2} - \frac{4}{3x} + 1 \right)$$

### 23.3 problem 1(c)

Internal problem ID [5744]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 4. Power Series Solutions and Special Functions. Problems for review and discover. (B) Challenge Problems . Page 194

**Problem number:** 1(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Gegenbauer]

$$(-x^2 + 1)y'' - 2y'x + p(p + 1)y = 0$$

With the expansion point for the power series method at  $x = \infty$ .

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 1124

```
Order:=8;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+p*(p+1)*y(x)=0,y(x),type='series',x=Infinity);
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 2707

AsymptoticDSolveValue[(1-x^2)\*y'[x]-2\*x\*y'[x]+p\*(p+1)\*y[x]==0,y[x],{x,Infinity,7}]

$$\begin{aligned}
 y(x) \rightarrow & \left( \frac{p^2 x^{-p-7}}{-p^2 - p + (p+6)(p+7)} + \frac{3px^{-p-7}}{-p^2 - p + (p+6)(p+7)} \right. \\
 & + \frac{p^4 x^{-p-7}}{(-p^2 - p + (p+2)(p+3))(-p^2 - p + (p+6)(p+7))} \\
 & + \frac{6p^3 x^{-p-7}}{(-p^2 - p + (p+2)(p+3))(-p^2 - p + (p+6)(p+7))} \\
 & + \frac{17p^2 x^{-p-7}}{(-p^2 - p + (p+2)(p+3))(-p^2 - p + (p+6)(p+7))} \\
 & + \frac{24px^{-p-7}}{(-p^2 - p + (p+2)(p+3))(-p^2 - p + (p+6)(p+7))} \\
 & + \frac{12x^{-p-7}}{(-p^2 - p + (p+2)(p+3))(-p^2 - p + (p+6)(p+7))} \\
 & + \frac{p^4 x^{-p-7}}{(-p^2 - p + (p+4)(p+5))(-p^2 - p + (p+6)(p+7))} \\
 & + \frac{6p^3 x^{-p-7}}{(-p^2 - p + (p+4)(p+5))(-p^2 - p + (p+6)(p+7))} \\
 & + \frac{21p^2 x^{-p-7}}{(-p^2 - p + (p+4)(p+5))(-p^2 - p + (p+6)(p+7))} \\
 & + \frac{36px^{-p-7}}{(-p^2 - p + (p+4)(p+5))(-p^2 - p + (p+6)(p+7))} \\
 & + \frac{p^6 x^{-p-7}}{(-p^2 - p + (p+2)(p+3))(-p^2 - p + (p+4)(p+5))(-p^2 - p + (p+6)(p+7))} \\
 & + \frac{9p^5 x^{-p-7}}{(-p^2 - p + (p+2)(p+3))(-p^2 - p + (p+4)(p+5))(-p^2 - p + (p+6)(p+7))} \\
 & + \frac{45p^4 x^{-p-7}}{(-p^2 - p + (p+2)(p+3))(-p^2 - p + (p+4)(p+5))(-p^2 - p + (p+6)(p+7))} \\
 & + \frac{135p^3 x^{-p-7}}{(-p^2 - p + (p+2)(p+3))(-p^2 - p + (p+4)(p+5))(-p^2 - p + (p+6)(p+7))} \\
 & + \frac{254p^2 x^{-p-7}}{(-p^2 - p + (p+2)(p+3))(-p^2 - p + (p+4)(p+5))(-p^2 - p + (p+6)(p+7))} \\
 & + \frac{276px^{-p-7}}{(-p^2 - p + (p+2)(p+3))(-p^2 - p + (p+4)(p+5))(-p^2 - p + (p+6)(p+7))} \\
 & + \frac{120x^{-p-7}}{(-p^2 - p + (p+2)(p+3))(-p^2 - p + (p+4)(p+5))(-p^2 - p + (p+6)(p+7))} \\
 & + \frac{20x^{-p-7}}{(-p^2 - p + (p+4)(p+5))(-p^2 - p + (p+6)(p+7))} + \frac{2x^{-p-7}}{-p^2 - p + (p+6)(p+7)} \\
 & + \frac{p^2 x^{-p-5}}{-p^2 - p + (p+4)(p+5)} + \frac{3px^{-p-5}}{-p^2 - p + (p+4)(p+5)} \\
 & + \frac{p^4 x^{-p-5}}{-p^2 - p + (p+4)(p+5)}
 \end{aligned}$$

## 24 Chapter 7. Laplace Transforms. Section 7.5 The Unit Step and Impulse Functions. Page 303

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## 24.1 problem 1(a)

Internal problem ID [5745]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 7. Laplace Transforms. Section 7.5 The Unit Step and Impulse Functions. Page 303

**Problem number:** 1(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' + 5y' + 6y - 5e^{3t} = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)+5*diff(y(t),t)+6*y(t)=5*exp(3*t),y(0) = 0, D(y)(0) = 0],y(t), singsol=
```

$$y(t) = \frac{(e^{6t} - 6e^t + 5)e^{-3t}}{6}$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 26

```
DSolve[{y''[t]+5*y'[t]+6*y[t]==5*Exp[3*t]},{y[0]==0,y'[0]==0}],y[t],t,IncludeSingularSolutions
```

$$y(t) \rightarrow \frac{1}{6}e^{-3t}(-6e^t + e^{6t} + 5)$$

## 24.2 problem 1(b)

Internal problem ID [5746]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 7. Laplace Transforms. Section 7.5 The Unit Step and Impulse Functions. Page 303

**Problem number:** 1(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' + y' - 6y - t = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-6*y(t)=t,y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{(9e^{5t} - 30te^{3t} - 5e^{3t} - 4)e^{-3t}}{180}$$

### ✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 28

```
DSolve[{y'[t]+y'[t]-6*y[t]==t,{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{180}(-30t - 4e^{-3t} + 9e^{2t} - 5)$$

## 24.3 problem 1(c)

Internal problem ID [5747]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 7. Laplace Transforms. Section 7.5 The Unit Step and Impulse Functions. Page 303

**Problem number:** 1(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' - y - t^2 = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)-y(t)=t^2,y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = e^{-t} + e^t - t^2 - 2$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{y'[t]-y[t]==t^2,{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -t^2 + 2 \cosh(t) - 2$$



## 24.4 problem 7(a)

Internal problem ID [5748]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 7. Laplace Transforms. Section 7.5 The Unit Step and Impulse Functions. Page 303

**Problem number:** 7(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$Li' + Ri - E_0 \text{Heaviside}(t) = 0$$

With initial conditions

$$[i(0) = 0]$$

### ✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 22

```
dsolve([L*diff(i(t),t)+R*i(t)=E__0*Heaviside(t),i(0) = 0],i(t), singsol=all)
```

$$i(t) = -\frac{E_0 \text{Heaviside}(t) \left( e^{-\frac{Rt}{L}} - 1 \right)}{R}$$

### ✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 25

```
DSolve[{L*i'[t]+R*i[t]==E0*UnitStep[t],{i[0]==0}},i[t],t,IncludeSingularSolutions -> True]
```

$$i(t) \rightarrow \frac{E0\theta(t) \left( 1 - e^{-\frac{Rt}{L}} \right)}{R}$$

## 24.5 problem 7(b)

Internal problem ID [5749]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 7. Laplace Transforms. Section 7.5 The Unit Step and Impulse Functions. Page 303

**Problem number:** 7(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$Li' + Ri - E_0(\delta(t)) = 0$$

With initial conditions

$$[i(0) = 0]$$

### ✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 21

```
dsolve([L*diff(i(t),t)+R*i(t)=E__0*Dirac(t),i(0) = 0],i(t), singsol=all)
```

$$i(t) = \frac{E_0 e^{-\frac{Rt}{L}} (2 \text{Heaviside}(t) - 1)}{2L}$$

### ✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 26

```
DSolve[{L*i'[t]+R*i[t]==E0*DiracDelta[t],{i[0]==0}},i[t],t,IncludeSingularSolutions -> True]
```

$$i(t) \rightarrow \frac{E_0(\theta(t) - \theta(0))e^{-\frac{Rt}{L}}}{L}$$

## 24.6 problem 7(c)

Internal problem ID [5750]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 7. Laplace Transforms. Section 7.5 The Unit Step and Impulse Functions. Page 303

**Problem number:** 7(c).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$Li' + Ri - E_0 \sin(\omega t) = 0$$

With initial conditions

$$[i(0) = 0]$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
dsolve([L*diff(i(t),t)+R*i(t)=E__0*sin(omega*t),i(0) = 0],i(t), singsol=all)
```

$$i(t) = \frac{E_0 \left( e^{-\frac{Rt}{L}} L\omega - L \cos(\omega t) \omega + \sin(\omega t) R \right)}{\omega^2 L^2 + R^2}$$

### ✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 47

```
DSolve[{L*i'[t]+R*i[t]==E0*Sin[\[Omega]*t],{i[0]==0}},i[t],t,IncludeSingularSolutions -> True
```

$$i(t) \rightarrow \frac{E_0 \left( L\omega e^{-\frac{Rt}{L}} - L\omega \cos(t\omega) + R \sin(t\omega) \right)}{L^2\omega^2 + R^2}$$

**25 Chapter 7. Laplace Transforms. Section 7.5**  
**Problem for review and discovery. Section A,**  
**Drill exercises. Page 309**

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## 25.1 problem 3(a)

Internal problem ID [5751]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 7. Laplace Transforms. Section 7.5 Problem for review and discovery. Section A, Drill exercises. Page 309

**Problem number:** 3(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 3y' - 5y - 1 = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

### ✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 40

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)-5*y(t)=1,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{(29 + 13\sqrt{29}) e^{\frac{(-3+\sqrt{29})t}{2}}}{290} - \frac{1}{5} + \frac{(29 - 13\sqrt{29}) e^{-\frac{(3+\sqrt{29})t}{2}}}{290}$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 52

```
DSolve[{y''[t]+3*y'[t]-5*y[t]==1,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{145} e^{-3t/2} \left( 13\sqrt{29} \sinh\left(\frac{\sqrt{29}t}{2}\right) + 29 \cosh\left(\frac{\sqrt{29}t}{2}\right) \right) - \frac{1}{5}$$

## 25.2 problem 3(b)

Internal problem ID [5752]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 7. Laplace Transforms. Section 7.5 Problem for review and discovery. Section A, Drill exercises. Page 309

**Problem number:** 3(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' + 3y' - 2y + 6e^{\pi-t} = 0$$

With initial conditions

$$[y(\pi) = 1, y'(\pi) = 4]$$

### ✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 57

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)-2*y(t)=-6*exp(Pi-t),y(Pi) = 1, D(y)(Pi) = 4],y(t), sing
```

$$y(t) = \frac{(19\sqrt{17} - 17) e^{-\frac{(-3+\sqrt{17})(\pi-t)}{2}}}{68} + \frac{(-19\sqrt{17} - 17) e^{\frac{(3+\sqrt{17})(\pi-t)}{2}}}{68} + \frac{3e^{\pi-t}}{2}$$

### ✓ Solution by Mathematica

Time used: 0.175 (sec). Leaf size: 103

```
DSolve[{y'[t]+3*y'[t]-2*y[t]==-6*Exp[Pi-t],{y[Pi]==1,y'[Pi]==4}},y[t],t,IncludeSingularSolut
```

$$y(t) \rightarrow \frac{1}{68} e^{-\frac{1}{2}(3+\sqrt{17})t - \frac{1}{2}(\sqrt{17}-3)\pi} \left( (19\sqrt{17} - 17) e^{\sqrt{17}t} + 102e^{\frac{1}{2}((1+\sqrt{17})t + (\sqrt{17}-1)\pi)} - \left( (17 + 19\sqrt{17}) e^{\sqrt{17}\pi} \right) \right)$$

## 25.3 problem 3(c)

Internal problem ID [5753]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 7. Laplace Transforms. Section 7.5 Problem for review and discovery. Section A, Drill exercises. Page 309

**Problem number:** 3(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' - y - te^{-t} = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

### ✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 39

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)-y(t)=t*exp(-t),y(0) = 0, D(y)(0) = 1],y(t), singsol=all
```

$$y(t) = \frac{3e^{(\sqrt{2}-1)t}\sqrt{2}}{8} - \frac{3e^{-(1+\sqrt{2})t}\sqrt{2}}{8} - \frac{e^{-t}t}{2}$$

### ✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 33

```
DSolve[{y'[t]+2*y'[t]-y[t]==t*Exp[-t]},{y[0]==0,y'[0]==1},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \rightarrow \frac{1}{4}e^{-t}\left(3\sqrt{2}\sinh(\sqrt{2}t) - 2t\right)$$

## 25.4 problem 3(d)

Internal problem ID [5754]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 7. Laplace Transforms. Section 7.5 Problem for review and discovery. Section A, Drill exercises. Page 309

**Problem number:** 3(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y' + y - 3e^{-t} = 0$$

With initial conditions

$$[y(0) = 3, y'(0) = 2]$$

### ✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 38

```
dsolve([diff(y(t),t$2)-diff(y(t),t)+y(t)=3*exp(-t),y(0) = 3, D(y)(0) = 2],y(t), singsol=all)
```

$$y(t) = \frac{\left(4 e^{\frac{3t}{2}} \sin\left(\frac{\sqrt{3}t}{2}\right) \sqrt{3} + 6 e^{\frac{3t}{2}} \cos\left(\frac{\sqrt{3}t}{2}\right) + 3\right) e^{-t}}{3}$$

### ✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 54

```
DSolve[{y'[t]-y[t]+y[t]==3*Exp[-t],{y[0]==3,y'[0]==2}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \rightarrow e^{-t} + \frac{2}{3}e^{t/2} \left( 2\sqrt{3} \sin\left(\frac{\sqrt{3}t}{2}\right) + 3 \cos\left(\frac{\sqrt{3}t}{2}\right) \right)$$



## 25.5 problem 4(a)

Internal problem ID [5755]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 7. Laplace Transforms. Section 7.5 Problem for review and discovery. Section A, Drill exercises. Page 309

**Problem number:** 4(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 5y' + 4y = 0$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(t),t$2)-5*diff(y(t),t)+4*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^t + c_2 e^{4t}$$

### ✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 20

```
DSolve[y''[t]-5*y'[t]+4*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow e^t (c_2 e^{3t} + c_1)$$

## 25.6 problem 4(b)

Internal problem ID [5756]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 7. Laplace Transforms. Section 7.5 Problem for review and discovery. Section A, Drill exercises. Page 309

**Problem number:** 4(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 3y' + 3y - 2 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(t),t$2)+3*diff(y(t),t)+3*y(t)=2,y(t), singsol=all)
```

$$y(t) = e^{-\frac{3t}{2}} \sin\left(\frac{\sqrt{3}t}{2}\right) c_2 + e^{-\frac{3t}{2}} \cos\left(\frac{\sqrt{3}t}{2}\right) c_1 + \frac{2}{3}$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 46

```
DSolve[y''[t]+3*y'[t]+3*y[t]==2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{2}{3} + e^{-3t/2} \left( c_2 \cos\left(\frac{\sqrt{3}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}t}{2}\right) \right)$$

## 25.7 problem 4(c)

Internal problem ID [5757]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 7. Laplace Transforms. Section 7.5 Problem for review and discovery. Section A, Drill exercises. Page 309

**Problem number:** 4(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y' + 2y - t = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(t),t$2)+diff(y(t),t)+2*y(t)=t,y(t), singsol=all)
```

$$y(t) = e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{7}t}{2}\right) c_2 + e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{7}t}{2}\right) c_1 - \frac{1}{4} + \frac{t}{2}$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 52

```
DSolve[y''[t]+y'[t]+2*y[t]==t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{4}(2t - 1) + e^{-t/2} \left( c_2 \cos\left(\frac{\sqrt{7}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{7}t}{2}\right) \right)$$

## 25.8 problem 4(d)

Internal problem ID [5758]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 7. Laplace Transforms. Section 7.5 Problem for review and discovery. Section A, Drill exercises. Page 309

**Problem number:** 4(d).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 7y' + 12y - te^{2t} = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(t),t$2)-7*diff(y(t),t)+12*y(t)=t*exp(2*t),y(t), singsol=all)
```

$$y(t) = c_2 e^{3t} + e^{4t} c_1 + \frac{(2t + 3)e^{2t}}{4}$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 33

```
DSolve[y''[t]-7*y'[t]+12*y[t]==t*Exp[2*t],y[t],t,IncludeSingularSolutions->True]
```

$$y(t) \rightarrow \frac{1}{4}e^{2t}(2t + 4e^t(c_2e^t + c_1) + 3)$$

**26 Chapter 7. Laplace Transforms. Section 7.5**  
**Problem for review and discovery. Section B,**  
**Challenge Problems. Page 310**

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## 26.1 problem 3

Internal problem ID [5759]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 7. Laplace Transforms. Section 7.5 Problem for review and discovery. Section B, Challenge Problems. Page 310

**Problem number:** 3.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$i'' + 2i' + 3i - \begin{cases} 30 & 0 < t < 2\pi \\ 0 & 2\pi \leq t \leq 5\pi \\ 10 & 5\pi < t < \infty \end{cases} = 0$$

With initial conditions

$$[i(0) = 8, i'(0) = 0]$$

✗ Solution by Maple

```
dsolve([diff(i(t),t$2)+2*diff(i(t),t)+3*i(t)=piecewise(0<t and t<2*Pi,30,2*Pi<= t and t<= 5*P
```

No solution found

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 289

```
DSolve[{i''[t]+2*i'[t]+3*i[t]==Piecewise[{{30,0<t<2*Pi},{0,2*Pi<= t <= 5*Pi},{10,5*Pi<t<Infin
```

$i(t)$

$$\rightarrow \begin{cases} 10 - e^{-t}(2 \cos(\sqrt{2}t) + \sqrt{2} \sin(\sqrt{2}t)) \\ 4e^{-t}(2 \cos(\sqrt{2}t) + \sqrt{2} \sin(\sqrt{2}t)) \\ e^{-t}(-2 \cos(\sqrt{2}t) - \sqrt{2} \sin(\sqrt{2}t) + 5e^{2\pi}(2 \cos(\sqrt{2}(t-2\pi)) + \sqrt{2} \sin(\sqrt{2}(t-2\pi))) \\ \frac{1}{3}e^{-t}(-6 \cos(\sqrt{2}t) + 10e^t - 3\sqrt{2} \sin(\sqrt{2}t) - 5e^{5\pi}(2 \cos(\sqrt{2}(t-5\pi)) + \sqrt{2} \sin(\sqrt{2}(t-5\pi))) + 15 \end{cases}$$

**27 Chapter 10. Systems of First-Order Equations.****Section 10.2 Linear Systems. Page 380**

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27.6 problem 6(a)	476

## 27.1 problem 2(a)

Internal problem ID [5760]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section 10.2 Linear Systems. Page 380

**Problem number:** 2(a).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = x(t) + 3y(t)$$

$$y'(t) = 3x(t) + y(t)$$

### ✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=x(t)+3*y(t),diff(y(t),t)=3*x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -c_1 e^{-2t} + c_2 e^{4t}$$

$$y(t) = c_1 e^{-2t} + c_2 e^{4t}$$

### ✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 46

```
DSolve[{x'[t]==x[t]+3*y[t],y'[t]==3*x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True
```

$$x(t) \rightarrow e^t (c_1 \cosh(3t) + c_2 \sinh(3t))$$

$$y(t) \rightarrow e^t (c_2 \cosh(3t) + c_1 \sinh(3t))$$



## 27.2 problem 2(c)

Internal problem ID [5761]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section 10.2 Linear Systems. Page 380

**Problem number:** 2(c).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = x(t) + 3y(t)$$

$$y'(t) = 3x(t) + y(t)$$

With initial conditions

$$[x(0) = 5, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve([diff(x(t),t) = x(t)+3*y(t), diff(y(t),t) = 3*x(t)+y(t), x(0) = 5, y(0) = 1],[x(t), y(t)
```

$$x(t) = 2e^{-2t} + 3e^{4t}$$

$$y(t) = -2e^{-2t} + 3e^{4t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 38

```
DSolve[{x'[t]==x[t]+3*y[t],y'[t]==3*x[t]+y[t]},{x[0]==5,y[0]==1},{x[t],y[t]},t,IncludeSingular
```

$$x(t) \rightarrow e^{-2t}(3e^{6t} + 2)$$

$$y(t) \rightarrow e^{-2t}(3e^{6t} - 2)$$

### 27.3 problem 3(a)

Internal problem ID [5762]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section 10.2 Linear Systems. Page 380

**Problem number:** 3(a).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = x(t) + 2y(t)$$

$$y'(t) = 3x(t) + 2y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=x(t)+2*y(t),diff(y(t),t)=3*x(t)+2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -c_1 e^{-t} + \frac{2c_2 e^{4t}}{3}$$

$$y(t) = c_1 e^{-t} + c_2 e^{4t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 68

```
DSolve[{x'[t]==x[t]+2*y[t],y'[t]==3*x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$x(t) \rightarrow \frac{1}{5} e^{-t} (2(c_1 + c_2) e^{5t} + 3c_1 - 2c_2)$$

$$y(t) \rightarrow \frac{1}{5} e^{-t} (3(c_1 + c_2) e^{5t} - 3c_1 + 2c_2)$$

## 27.4 problem 3(c)

Internal problem ID [5763]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section 10.2 Linear Systems. Page 380

**Problem number:** 3(c).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = x(t) + 2y(t) + t - 1$$

$$y'(t) = 3x(t) + 2y(t) - 5t - 2$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 44

```
dsolve([diff(x(t),t)=x(t)+2*y(t)+t-1,diff(y(t),t)=3*x(t)+2*y(t)-5*t-2],[x(t), y(t)], singsol=
```

$$x(t) = -e^{-t}c_2 + \frac{2e^{4t}c_1}{3} - 2 + 3t$$

$$y(t) = e^{-t}c_2 + e^{4t}c_1 - 2t + 3$$

✓ Solution by Mathematica

Time used: 0.172 (sec). Leaf size: 78

```
DSolve[{x'[t]==x[t]+2*y[t]+t-1,y'[t]==3*x[t]+2*y[t]-5*t-2},{x[t],y[t]},t,IncludeSingularSolut
```

$$x(t) \rightarrow 3t + \frac{1}{5}e^{-t}(2(c_1 + c_2)e^{5t} + 3c_1 - 2c_2) - 2$$

$$y(t) \rightarrow -2t + \frac{1}{5}e^{-t}(3(c_1 + c_2)e^{5t} - 3c_1 + 2c_2) + 3$$

## 27.5 problem 5

Internal problem ID [5764]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section 10.2 Linear Systems. Page 380

**Problem number:** 5.

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = x(t) + y(t)$$

$$y'(t) = y(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 20

```
dsolve([diff(x(t),t)=x(t)+y(t),diff(y(t),t)=y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = (c_2 t + c_1) e^t$$

$$y(t) = c_2 e^t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 25

```
DSolve[{x'[t]==x[t]+y[t],y'[t]==y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow e^t(c_2 t + c_1)$$

$$y(t) \rightarrow c_2 e^t$$

## 27.6 problem 6(a)

Internal problem ID [5765]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section 10.2 Linear Systems. Page 380

**Problem number:** 6(a).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = x(t)$$

$$y'(t) = y(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 16

```
dsolve([diff(x(t),t)=x(t),diff(y(t),t)=y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = c_1 e^t$$

$$y(t) = c_2 e^t$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 57

```
DSolve[{x'[t]==x[t],y'[t]==y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow c_1 e^t$$

$$y(t) \rightarrow c_2 e^t$$

$$x(t) \rightarrow c_1 e^t$$

$$y(t) \rightarrow 0$$

$$x(t) \rightarrow 0$$

$$y(t) \rightarrow c_2 e^t$$

$$x(t) \rightarrow 0$$

$$y(t) \rightarrow 0$$

**28 Chapter 10. Systems of First-Order Equations.  
Section 10.3 Homogeneous Linear Systems with  
Constant Coefficients. Page 387**

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## 28.1 problem 1(a)

Internal problem ID [5766]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387

**Problem number:** 1(a).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = -3x(t) + 4y(t)$$

$$y'(t) = -2x(t) + 3y(t)$$

### ✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 31

```
dsolve([diff(x(t),t)=-3*x(t)+4*y(t),diff(y(t),t)=-2*x(t)+3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = 2c_1e^{-t} + c_2e^t$$

$$y(t) = c_1e^{-t} + c_2e^t$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 43

```
DSolve[{x'[t]==-3*x[t]+4*y[t],y'[t]==-2*x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -
```

$$x(t) \rightarrow c_1 \cosh(t) + (4c_2 - 3c_1) \sinh(t)$$

$$y(t) \rightarrow c_2(3 \sinh(t) + \cosh(t)) - 2c_1 \sinh(t)$$

## 28.2 problem 1(b)

Internal problem ID [5767]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387

**Problem number:** 1(b).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = 4x(t) - 2y(t)$$

$$y'(t) = 5x(t) + 2y(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 59

```
dsolve([diff(x(t),t)=4*x(t)-2*y(t),diff(y(t),t)=5*x(t)+2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{e^{3t}(\sin(3t)c_1 - 3\sin(3t)c_2 + 3\cos(3t)c_1 + \cos(3t)c_2)}{5}$$

$$y(t) = e^{3t}(\sin(3t)c_1 + \cos(3t)c_2)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 70

```
DSolve[{x'[t]==4*x[t]-2*y[t],y'[t]==5*x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \rightarrow \frac{1}{3}e^{3t}(3c_1 \cos(3t) + (c_1 - 2c_2) \sin(3t))$$

$$y(t) \rightarrow \frac{1}{3}e^{3t}(3c_2 \cos(3t) + (5c_1 - c_2) \sin(3t))$$



## 28.3 problem 1(c)

Internal problem ID [5768]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387

**Problem number:** 1(c).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = 5x(t) + 4y(t)$$

$$y'(t) = -x(t) + y(t)$$

### ✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 33

```
dsolve([diff(x(t),t)=5*x(t)+4*y(t),diff(y(t),t)=-x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -e^{3t}(2c_2t + 2c_1 + c_2)$$

$$y(t) = e^{3t}(c_2t + c_1)$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{x'[t]==5*x[t]+4*y[t],y'[t]==-x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow e^{3t}(2c_1t + 4c_2t + c_1)$$

$$y(t) \rightarrow e^{3t}(c_2 - (c_1 + 2c_2)t)$$

## 28.4 problem 1(d)

Internal problem ID [5769]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387

**Problem number:** 1(d).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = 4x(t) - 3y(t)$$

$$y'(t) = 8x(t) - 6y(t)$$

### ✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 27

```
dsolve([diff(x(t),t)=4*x(t)-3*y(t),diff(y(t),t)=8*x(t)-6*y(t)],[x(t),y(t)],singsol=all)
```

$$x(t) = \frac{e^{-2t}c_2}{2} + \frac{3c_1}{4}$$

$$y(t) = c_1 + e^{-2t}c_2$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 55

```
DSolve[{x'[t]==4*x[t]-3*y[t],y'[t]==8*x[t]-6*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \rightarrow e^{-t}(c_1 \cosh(t) + (5c_1 - 3c_2) \sinh(t))$$

$$y(t) \rightarrow e^{-t}(8c_1 \sinh(t) + c_2(\cosh(t) - 5 \sinh(t)))$$

## 28.5 problem 1(e)

Internal problem ID [5770]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387

**Problem number:** 1(e).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = 2x(t)$$

$$y'(t) = 3y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 20

```
dsolve([diff(x(t),t)=2*x(t),diff(y(t),t)=3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = c_1 e^{2t}$$

$$y(t) = c_2 e^{3t}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 65

```
DSolve[{x'[t]==2*x[t],y'[t]==3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow c_1 e^{2t}$$

$$y(t) \rightarrow c_2 e^{3t}$$

$$x(t) \rightarrow c_1 e^{2t}$$

$$y(t) \rightarrow 0$$

$$x(t) \rightarrow 0$$

$$y(t) \rightarrow c_2 e^{3t}$$

$$x(t) \rightarrow 0$$

$$y(t) \rightarrow 0$$

## 28.6 problem 1(f)

Internal problem ID [5771]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387

**Problem number:** 1(f).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = -4x(t) - y(t)$$

$$y'(t) = x(t) - 2y(t)$$

### ✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)=-4*x(t)-y(t),diff(y(t),t)=x(t)-2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -e^{-3t}(c_2t + c_1 - c_2)$$

$$y(t) = e^{-3t}(c_2t + c_1)$$

### ✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 43

```
DSolve[{x'[t]==-4*x[t]-y[t],y'[t]==x[t]-2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions->True]
```

$$x(t) \rightarrow e^{-3t}(c_1(-t) - c_2t + c_1)$$

$$y(t) \rightarrow e^{-3t}((c_1 + c_2)t + c_2)$$

## 28.7 problem 1(g)

Internal problem ID [5772]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387

**Problem number:** 1(g).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = 7x(t) + 6y(t)$$

$$y'(t) = 2x(t) + 6y(t)$$

### ✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=7*x(t)+6*y(t),diff(y(t),t)=2*x(t)+6*y(t)],[x(t),y(t)],singsol=all)
```

$$x(t) = -\frac{3c_1e^{3t}}{2} + 2c_2e^{10t}$$

$$y(t) = c_1e^{3t} + c_2e^{10t}$$

### ✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 74

```
DSolve[{x'[t]==7*x[t]+6*y[t],y'[t]==2*x[t]+6*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \rightarrow \frac{1}{7}e^{3t}(c_1(4e^{7t} + 3) + 6c_2(e^{7t} - 1))$$

$$y(t) \rightarrow \frac{1}{7}e^{3t}(2c_1(e^{7t} - 1) + c_2(3e^{7t} + 4))$$

## 28.8 problem 1(h)

Internal problem ID [5773]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387

**Problem number:** 1(h).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$\begin{aligned}x'(t) &= x(t) - 2y(t) \\y'(t) &= 4x(t) + 5y(t)\end{aligned}$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 58

```
dsolve([diff(x(t),t)=x(t)-2*y(t),diff(y(t),t)=4*x(t)+5*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^{3t}(\sin(2t)c_1 + \sin(2t)c_2 - \cos(2t)c_1 + \cos(2t)c_2)}{2}$$

$$y(t) = e^{3t}(\sin(2t)c_1 + \cos(2t)c_2)$$

### ✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 59

```
DSolve[{x'[t]==x[t]-2*y[t],y'[t]==4*x[t]+5*y[t]},{x[t],y[t]},t,IncludeSingularSolutions->True]
```

$$x(t) \rightarrow e^{3t}(c_1 \cos(2t) - (c_1 + c_2) \sin(2t))$$

$$y(t) \rightarrow e^{3t}(c_2 \cos(2t) + (2c_1 + c_2) \sin(2t))$$

## 28.9 problem 5(b)

Internal problem ID [5774]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section 10.3 Homogeneous Linear Systems with Constant Coefficients. Page 387

**Problem number:** 5(b).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$\begin{aligned}x'(t) &= x(t) + y(t) - 5t + 2 \\y'(t) &= 4x(t) - 2y(t) - 8t - 8\end{aligned}$$

### ✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 43

```
dsolve([diff(x(t),t)=x(t)+y(t)-5*t+2,diff(y(t),t)=4*x(t)-2*y(t)-8*t-8],[x(t), y(t)], singsol=
```

$$x(t) = -\frac{c_2 e^{-3t}}{4} + c_1 e^{2t} + 2 + 3t$$

$$y(t) = c_2 e^{-3t} + c_1 e^{2t} + 2t - 1$$

### ✓ Solution by Mathematica

Time used: 0.134 (sec). Leaf size: 81

```
DSolve[{x'[t]==x[t]+y[t]-5*t+2,y'[t]==4*x[t]-2*y[t]-8*t-8},{x[t],y[t]},t,IncludeSingularSolut
```

$$x(t) \rightarrow 3t + \frac{1}{5}((c_1 - c_2)e^{-3t} + (4c_1 + c_2)e^{2t} + 10)$$

$$y(t) \rightarrow 2t + \frac{1}{5}(-4(c_1 - c_2)e^{-3t} + (4c_1 + c_2)e^{2t} - 5)$$



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**Section A. Drill exercises. Page 400**

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## 29.1 problem 2(a)

Internal problem ID [5775]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400

**Problem number:** 2(a).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = 3x(t) - 4y(t)$$

$$y'(t) = 4x(t) - 7y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)=3*x(t)-4*y(t),diff(y(t),t)=4*x(t)-7*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = 2c_1e^t + \frac{c_2e^{-5t}}{2}$$

$$y(t) = c_1e^t + c_2e^{-5t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 73

```
DSolve[{x'[t]==3*x[t]-4*y[t],y'[t]==4*x[t]-7*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \rightarrow \frac{1}{3}e^{-5t}(c_1(4e^{6t} - 1) - 2c_2(e^{6t} - 1))$$

$$y(t) \rightarrow \frac{1}{3}e^{-5t}(2c_1(e^{6t} - 1) - c_2(e^{6t} - 4))$$

## 29.2 problem 2(b)

Internal problem ID [5776]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400

**Problem number:** 2(b).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = x(t) + y(t)$$

$$y'(t) = 4x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=x(t)+y(t),diff(y(t),t)=4*x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{c_1 e^{3t}}{2} - \frac{e^{-t} c_2}{2}$$

$$y(t) = c_1 e^{3t} + e^{-t} c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 51

```
DSolve[{x'[t]==x[t]+y[t],y'[t]==4*x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow \frac{1}{2} e^t (2c_1 \cosh(2t) + c_2 \sinh(2t))$$

$$y(t) \rightarrow e^t (c_2 \cosh(2t) + 2c_1 \sinh(2t))$$

### 29.3 problem 2(c)

Internal problem ID [5777]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400

**Problem number:** 2(c).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = -3x(t) + \sqrt{2}y(t)$$

$$y'(t) = \sqrt{2}x(t) - 2y(t)$$

#### ✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 41

```
dsolve([diff(x(t),t)=-3*x(t)+sqrt(2)*y(t),diff(y(t),t)=sqrt(2)*x(t)-2*y(t)],[x(t), y(t)], sin
```

$$x(t) = -\frac{(2c_1e^{-4t} - e^{-t}c_2)\sqrt{2}}{2}$$

$$y(t) = c_1e^{-4t} + e^{-t}c_2$$

#### ✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 79

```
DSolve[{x'[t]==-3*x[t]+Sqrt[2]*y[t],y'[t]==Sqrt[2]*x[t]-2*y[t]},{x[t],y[t]},t,IncludeSingular
```

$$x(t) \rightarrow \frac{1}{3}e^{-4t} \left( c_1(e^{3t} + 2) + \sqrt{2}c_2(e^{3t} - 1) \right)$$

$$y(t) \rightarrow \frac{1}{3}e^{-4t} \left( \sqrt{2}c_1(e^{3t} - 1) + 2c_2e^{3t} + c_2 \right)$$

## 29.4 problem 2(d)

Internal problem ID [5778]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400

**Problem number:** 2(d).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = 5x(t) + 3y(t)$$

$$y'(t) = -6x(t) - 4y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=5*x(t)+3*y(t),diff(y(t),t)=-6*x(t)-4*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{c_1 e^{-t}}{2} - c_2 e^{2t}$$

$$y(t) = c_1 e^{-t} + c_2 e^{2t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 61

```
DSolve[{x'[t]==5*x[t]+3*y[t],y'[t]==-6*x[t]-4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \rightarrow (2c_1 + c_2)e^{2t} - (c_1 + c_2)e^{-t}$$

$$y(t) \rightarrow 2(c_1 + c_2)e^{-t} - (2c_1 + c_2)e^{2t}$$

## 29.5 problem 3(a)

Internal problem ID [5779]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400

**Problem number:** 3(a).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = 3x(t) + 2y(t)$$

$$y'(t) = -2x(t) - y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 29

```
dsolve([diff(x(t),t)=3*x(t)+2*y(t),diff(y(t),t)=-2*x(t)-y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^t(2c_2t + 2c_1 + c_2)}{2}$$

$$y(t) = (c_2t + c_1)e^t$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 40

```
DSolve[{x'[t]==3*x[t]+2*y[t],y'[t]==-2*x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions->T
```

$$x(t) \rightarrow e^t(2c_1t + 2c_2t + c_1)$$

$$y(t) \rightarrow e^t(c_2 - 2(c_1 + c_2)t)$$

## 29.6 problem 3(b)

Internal problem ID [5780]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400

**Problem number:** 3(b).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$\begin{aligned}x'(t) &= x(t) + y(t) \\y'(t) &= -x(t) + y(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve([diff(x(t),t)=x(t)+y(t),diff(y(t),t)=-x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -e^t(\cos(t) c_1 - \sin(t) c_2)$$

$$y(t) = e^t(c_2 \cos(t) + c_1 \sin(t))$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 39

```
DSolve[{x'[t]==x[t]+y[t],y'[t]==-x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow e^t(c_1 \cos(t) + c_2 \sin(t))$$

$$y(t) \rightarrow e^t(c_2 \cos(t) - c_1 \sin(t))$$

## 29.7 problem 3(c)

Internal problem ID [5781]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400

**Problem number:** 3(c).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = 3x(t) - 5y(t)$$

$$y'(t) = -x(t) + 2y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 86

```
dsolve([diff(x(t),t)=3*x(t)-5*y(t),diff(y(t),t)=-x(t)+2*y(t)],[x(t),y(t)],singsol=all)
```

$$x(t) = -\frac{c_1 e^{\frac{(5+\sqrt{21})t}{2}} \sqrt{21}}{2} + \frac{c_2 e^{-\frac{(-5+\sqrt{21})t}{2}} \sqrt{21}}{2} - \frac{c_1 e^{\frac{(5+\sqrt{21})t}{2}}}{2} - \frac{c_2 e^{-\frac{(-5+\sqrt{21})t}{2}}}{2}$$

$$y(t) = c_1 e^{\frac{(5+\sqrt{21})t}{2}} + c_2 e^{-\frac{(-5+\sqrt{21})t}{2}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 126

```
DSolve[{x'[t]==3*x[t]-5*y[t],y'[t]==-x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions->T
```

$$x(t) \rightarrow \frac{1}{21} e^{5t/2} \left( 21c_1 \cosh\left(\frac{\sqrt{21}t}{2}\right) + \sqrt{21}(c_1 - 10c_2) \sinh\left(\frac{\sqrt{21}t}{2}\right) \right)$$

$$y(t) \rightarrow \frac{1}{42} e^{-\frac{1}{2}(\sqrt{21}-5)t} \left( c_2 \left( -(\sqrt{21}-21) e^{\sqrt{21}t} + 21 + \sqrt{21} \right) - 2\sqrt{21}c_1 (e^{\sqrt{21}t} - 1) \right)$$



## 29.8 problem 3(d)

Internal problem ID [5782]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400

**Problem number:** 3(d).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$\begin{aligned}x'(t) &= x(t) + 2y(t) \\y'(t) &= -4x(t) + y(t)\end{aligned}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 57

```
dsolve([diff(x(t),t)=x(t)+2*y(t),diff(y(t),t)=-4*x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{\sqrt{2}e^t(\cos(2\sqrt{2}t)c_1 - \sin(2\sqrt{2}t)c_2)}{2}$$

$$y(t) = e^t(c_2 \cos(2\sqrt{2}t) + c_1 \sin(2\sqrt{2}t))$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 79

```
DSolve[{x'[t]==x[t]+2*y[t],y'[t]==-4*x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions->True]
```

$$x(t) \rightarrow c_1 e^t \cos(2\sqrt{2}t) + \frac{c_2 e^t \sin(2\sqrt{2}t)}{\sqrt{2}}$$

$$y(t) \rightarrow e^t(c_2 \cos(2\sqrt{2}t) - \sqrt{2}c_1 \sin(2\sqrt{2}t))$$

## 29.9 problem 3(e)

Internal problem ID [5783]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400

**Problem number:** 3(e).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = 3x(t) + 2y(t) + z(t)$$

$$y'(t) = -2x(t) - y(t) + 3z(t)$$

$$z'(t) = x(t) + y(t) + z(t)$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 63

```
dsolve([diff(x(t),t)=3*x(t)+2*y(t)+z(t),diff(y(t),t)=-2*x(t)-y(t)+3*z(t),diff(z(t),t)=x(t)+y(t)+z(t)],t)
```

$$x(t) = \frac{5c_2e^{3t}}{2} - c_1e^t + \frac{3c_3e^{-t}}{2}$$

$$y(t) = -\frac{c_2e^{3t}}{2} + c_1e^t - \frac{7c_3e^{-t}}{2}$$

$$z(t) = c_2e^{3t} + c_3e^{-t}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 143

```
DSolve[{x'[t]==3*x[t]+2*y[t]+z[t], y'[t]==-2*x[t]-y[t]+3*z[t], z'[t]==x[t]+y[t]+z[t]}, {x[t], y[t], z[t]}
```

$$x(t) \rightarrow \frac{1}{4}e^t(2 \sinh(t)(2(2c_2 + c_3) \cosh(t) + (c_2 + 8c_3) \sinh(t)) + c_1(4 \sinh(2t) + \cosh(2t) + 3))$$

$$y(t) \rightarrow \frac{1}{8}e^{-t}(-2(3c_1 - c_2 - 8c_3)e^{2t} - (c_1 + c_2 + 2c_3)e^{4t} + 7(c_1 + c_2 - 2c_3))$$

$$z(t) \rightarrow \frac{1}{2}e^t(2c_3 \cosh(2t) + (c_1 + c_2) \sinh(2t))$$

## 29.10 problem 3(f)

Internal problem ID [5784]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400

**Problem number:** 3(f).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = -x(t) + y(t) - z(t)$$

$$y'(t) = 2x(t) - y(t) - 4z(t)$$

$$z'(t) = 3x(t) - y(t) + z(t)$$

✓ Solution by Maple

Time used: 0.265 (sec). Leaf size: 2535

```
dsolve([diff(x(t),t)=-x(t)+y(t)-z(t),diff(y(t),t)=2*x(t)-y(t)-4*z(t),diff(z(t),t)=3*x(t)-y(t)
```

Expression too large to display

Expression too large to display

$$z(t) = c_2 e^{\frac{\left(13 + (154 + 3\sqrt{2391})^{\frac{2}{3}} - 2(154 + 3\sqrt{2391})^{\frac{1}{3}}\right)t}{6(154 + 3\sqrt{2391})^{\frac{1}{3}}}} \sin\left(\frac{\sqrt{3}\left((154 + 3\sqrt{2391})^{\frac{2}{3}} - 13\right)t}{6(154 + 3\sqrt{2391})^{\frac{1}{3}}}\right) \\ + c_3 e^{\frac{\left(13 + (154 + 3\sqrt{2391})^{\frac{2}{3}} - 2(154 + 3\sqrt{2391})^{\frac{1}{3}}\right)t}{6(154 + 3\sqrt{2391})^{\frac{1}{3}}}} \cos\left(\frac{\sqrt{3}\left((154 + 3\sqrt{2391})^{\frac{2}{3}} - 13\right)t}{6(154 + 3\sqrt{2391})^{\frac{1}{3}}}\right) \\ + c_1 e^{-\frac{\left((154 + 3\sqrt{2391})^{\frac{2}{3}} + (154 + 3\sqrt{2391})^{\frac{1}{3}} + 13\right)t}{3(154 + 3\sqrt{2391})^{\frac{1}{3}}}}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 501

`DSolve[{x'[t]==-x[t]+y[t]-z[t],y'[t]==2*x[t]-y[t]-4*z[t],z'[t]==3*x[t]-y[t]+z[t]},{x[t],y[t],z[t]}`

$$\begin{aligned}
 x(t) &\rightarrow c_2 \text{RootSum} \left[ \#1^3 + \#1^2 - 4\#1 + 10\&, \frac{\#1 e^{\#1 t}}{3\#1^2 + 2\#1 - 4} \& \right] \\
 &\quad - c_3 \text{RootSum} \left[ \#1^3 + \#1^2 - 4\#1 + 10\&, \frac{\#1 e^{\#1 t} + 5e^{\#1 t}}{3\#1^2 + 2\#1 - 4} \& \right] \\
 &\quad + c_1 \text{RootSum} \left[ \#1^3 + \#1^2 - 4\#1 + 10\&, \frac{\#1^2 e^{\#1 t} - 5e^{\#1 t}}{3\#1^2 + 2\#1 - 4} \& \right] \\
 y(t) &\rightarrow 2c_1 \text{RootSum} \left[ \#1^3 + \#1^2 - 4\#1 + 10\&, \frac{\#1 e^{\#1 t} - 7e^{\#1 t}}{3\#1^2 + 2\#1 - 4} \& \right] \\
 &\quad - 2c_3 \text{RootSum} \left[ \#1^3 + \#1^2 - 4\#1 + 10\&, \frac{2\#1 e^{\#1 t} + 3e^{\#1 t}}{3\#1^2 + 2\#1 - 4} \& \right] \\
 &\quad + c_2 \text{RootSum} \left[ \#1^3 + \#1^2 - 4\#1 + 10\&, \frac{\#1^2 e^{\#1 t} + 2e^{\#1 t}}{3\#1^2 + 2\#1 - 4} \& \right] \\
 z(t) &\rightarrow -c_2 \text{RootSum} \left[ \#1^3 + \#1^2 - 4\#1 + 10\&, \frac{\#1 e^{\#1 t} - 2e^{\#1 t}}{3\#1^2 + 2\#1 - 4} \& \right] \\
 &\quad + c_1 \text{RootSum} \left[ \#1^3 + \#1^2 - 4\#1 + 10\&, \frac{3\#1 e^{\#1 t} + e^{\#1 t}}{3\#1^2 + 2\#1 - 4} \& \right] \\
 &\quad + c_3 \text{RootSum} \left[ \#1^3 + \#1^2 - 4\#1 + 10\&, \frac{\#1^2 e^{\#1 t} + 2\#1 e^{\#1 t} - e^{\#1 t}}{3\#1^2 + 2\#1 - 4} \& \right]
 \end{aligned}$$

## 29.11 problem 4(a)

Internal problem ID [5785]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400

**Problem number:** 4(a).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$\begin{aligned}x'(t) &= x(t) + 2y(t) - 4t + 1 \\y'(t) &= -x(t) + 2y(t) + 3t + 4\end{aligned}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 106

```
dsolve([diff(x(t),t)=x(t)+2*y(t)-4*t+1,diff(y(t),t)=-x(t)+2*y(t)+3*t+4],[x(t), y(t)], singsol
```

$$\begin{aligned}x(t) &= \frac{e^{\frac{3t}{2}} \sin\left(\frac{\sqrt{7}t}{2}\right) c_2}{2} - \frac{e^{\frac{3t}{2}} \sqrt{7} \cos\left(\frac{\sqrt{7}t}{2}\right) c_2}{2} \\ &+ \frac{e^{\frac{3t}{2}} \cos\left(\frac{\sqrt{7}t}{2}\right) c_1}{2} + \frac{e^{\frac{3t}{2}} \sqrt{7} \sin\left(\frac{\sqrt{7}t}{2}\right) c_1}{2} + \frac{25}{8} + \frac{7t}{2}\end{aligned}$$

$$y(t) = e^{\frac{3t}{2}} \sin\left(\frac{\sqrt{7}t}{2}\right) c_2 + e^{\frac{3t}{2}} \cos\left(\frac{\sqrt{7}t}{2}\right) c_1 + \frac{t}{4} - \frac{5}{16}$$

✓ Solution by Mathematica

Time used: 1.911 (sec). Leaf size: 123

`DSolve[{x'[t]==x[t]+2*y[t]-4+t+1,y'[t]==-x[t]+2*y[t]+3*t+4},{x[t],y[t]},t,IncludeSingularSolu`

$$x(t) \rightarrow t + \frac{1}{7}e^{3t/2} \left( 7c_1 \cos\left(\frac{\sqrt{7}t}{2}\right) - \sqrt{7}(c_1 - 4c_2) \sin\left(\frac{\sqrt{7}t}{2}\right) \right) + \frac{9}{2}$$

$$y(t) \rightarrow -t + \frac{1}{7}e^{3t/2} \left( 7c_2 \cos\left(\frac{\sqrt{7}t}{2}\right) + \sqrt{7}(c_2 - 2c_1) \sin\left(\frac{\sqrt{7}t}{2}\right) \right) - \frac{1}{4}$$

## 29.12 problem 4(b)

Internal problem ID [5786]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400

**Problem number:** 4(b).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = -2x(t) + y(t) - t + 3$$

$$y'(t) = x(t) + 4y(t) + t - 2$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 90

```
dsolve([diff(x(t),t)=-2*x(t)+y(t)-t+3,diff(y(t),t)=x(t)+4*y(t)+t-2],[x(t), y(t)], singsol=all
```

$$x(t) = e^{(1+\sqrt{10})t} c_2 \sqrt{10} - e^{-(1+\sqrt{10})t} c_1 \sqrt{10} - 3e^{(1+\sqrt{10})t} c_2 - 3e^{-(1+\sqrt{10})t} c_1 - \frac{5t}{9} + \frac{145}{81}$$

$$y(t) = e^{(1+\sqrt{10})t} c_2 + e^{-(1+\sqrt{10})t} c_1 - \frac{t}{9} + \frac{2}{81}$$

✓ Solution by Mathematica

Time used: 10.584 (sec). Leaf size: 188

```
DSolve[{x'[t]==-2*x[t]+y[t]-t+3,y'[t]==x[t]+4*y[t]+t-2},{x[t],y[t]},t,IncludeSingularSolution
```

$$x(t) \rightarrow \frac{e^{t-\sqrt{10}t} \left( -100e^{(\sqrt{10}-1)t} (9t-29) + 81 \left( c_1 \left( (10-3\sqrt{10}) e^{2\sqrt{10}t} + 10 + 3\sqrt{10} \right) + \sqrt{10} c_2 \left( e^{2\sqrt{10}t} - 1 \right) \right) \right)}{1620}$$

$$y(t) \rightarrow \frac{e^{t-\sqrt{10}t} \left( -20e^{(\sqrt{10}-1)t} (9t-2) + 81 \left( \sqrt{10} c_1 \left( e^{2\sqrt{10}t} - 1 \right) + c_2 \left( (10+3\sqrt{10}) e^{2\sqrt{10}t} + 10 - 3\sqrt{10} \right) \right) \right)}{1620}$$



## 29.13 problem 4(c)

Internal problem ID [5787]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section A. Drill exercises. Page 400

**Problem number:** 4(c).

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = -4x(t) + y(t) - t + 3$$

$$y'(t) = -x(t) - 5y(t) + t + 1$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 106

```
dsolve([diff(x(t),t)=-4*x(t)+y(t)-t+3,diff(y(t),t)=-x(t)-5*y(t)+t+1],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^{-\frac{9t}{2}} \sin\left(\frac{\sqrt{3}t}{2}\right) c_2}{2} - \frac{e^{-\frac{9t}{2}} \sqrt{3} \cos\left(\frac{\sqrt{3}t}{2}\right) c_2}{2} \\ - \frac{e^{-\frac{9t}{2}} \cos\left(\frac{\sqrt{3}t}{2}\right) c_1}{2} + \frac{e^{-\frac{9t}{2}} \sqrt{3} \sin\left(\frac{\sqrt{3}t}{2}\right) c_1}{2} + \frac{39}{49} - \frac{4t}{21}$$

$$y(t) = e^{-\frac{9t}{2}} \sin\left(\frac{\sqrt{3}t}{2}\right) c_2 + e^{-\frac{9t}{2}} \cos\left(\frac{\sqrt{3}t}{2}\right) c_1 + \frac{5t}{21} - \frac{1}{147}$$

✓ Solution by Mathematica

Time used: 1.488 (sec). Leaf size: 127

```
DSolve[{x'[t]==-4*x[t]+y[t]-t+3,y'[t]==-x[t]-5*y[t]+t+1},{x[t],y[t]},t,IncludeSingularSolutio
```

$$x(t) \rightarrow -\frac{4t}{21} + \frac{1}{3}e^{-9t/2} \left( 3c_1 \cos\left(\frac{\sqrt{3}t}{2}\right) + \sqrt{3}(c_1 + 2c_2) \sin\left(\frac{\sqrt{3}t}{2}\right) \right) + \frac{39}{49}$$

$$y(t) \rightarrow \frac{1}{147} \left( 35t + 49e^{-9t/2} \left( 3c_2 \cos\left(\frac{\sqrt{3}t}{2}\right) - \sqrt{3}(2c_1 + c_2) \sin\left(\frac{\sqrt{3}t}{2}\right) \right) - 1 \right)$$

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**Section B. Challenge Problems. Page 401**

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### 30.1 problem 1

Internal problem ID [5788]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section B. Challenge Problems. Page 401

**Problem number:** 1.

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = x(t)y(t) + 1$$

$$y'(t) = -x(t) + y(t)$$

With initial conditions

$$[x(0) = 2, y(0) = -1]$$

**X** Solution by Maple

```
dsolve([diff(x(t),t) = x(t)*y(t)+1, diff(y(t),t) = -x(t)+y(t), x(0) = 2, y(0) = -1],[x(t), y(t)
```

No solution found

**X** Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==x[t]*y[t]+1,y'[t]==-x[t]+y[t]},{x[0]==2,y[0]==-1},{x[t],y[t]},t,IncludeSingular
```

Not solved

## 30.2 problem 2

Internal problem ID [5789]

**Book:** Differential Equations: Theory, Technique, and Practice by George Simmons, Steven Krantz. McGraw-Hill NY. 2007. 1st Edition.

**Section:** Chapter 10. Systems of First-Order Equations. Section B. Challenge Problems. Page 401

**Problem number:** 2.

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$\begin{aligned}x'(t) &= ty(t) + 1 \\y'(t) &= -x(t)t + y(t)\end{aligned}$$

With initial conditions

$$[x(0) = 0, y(0) = -1]$$

**X** Solution by Maple

```
dsolve([diff(x(t),t) = t*y(t)+1, diff(y(t),t) = -x(t)*t+y(t), x(0) = 0, y(0) = -1],[x(t), y(t)
```

No solution found

**X** Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==x[t]*y[t]+1,y'[t]==-x[t]+y[t]},{x[0]==2,y[0]==-1},{x[t],y[t]},t,IncludeSingular
```

Not solved