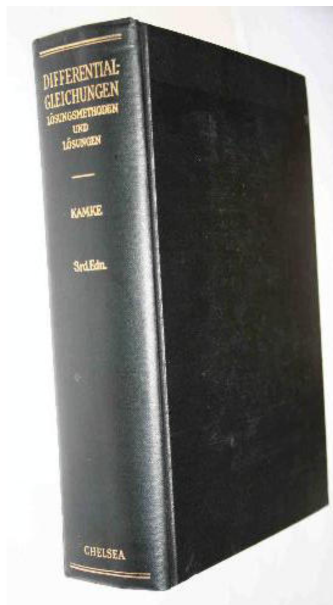
A Solution Manual For

# Differential Gleichungen, Kamke, 3rd ed, Abel ODEs

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# Contents

# 1 Abel ODE's with constant invariant

## 1.1 problem problem 38

Internal problem ID [4166]

**Book**: Differential Gleichungen, Kamke, 3rd ed, Abel ODEs
**Section**: Abel ODE's with constant invariant
**Problem number**: problem 38.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Abel]`

$$\boxed{-y^3a-\frac{b}{x^{\frac{3}{2}}}+y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(-a*y(x)^3-b/(x^(3/2))+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\operatorname{RootOf}\left(-\ln\left(x\right)+c_1+2\left(\int^{\_Z}\frac{1}{2\_a^3a+\_a+2b}d\_a\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.306 (sec). Leaf size: 320

```
DSolve[-a*y[x]^3-b/(x^(3/2))+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2}{3}ab^2\text{RootSum}\left[8\#1^9ab^2+24\#1^6ab^2+24\#1^3ab^2+\#1^3\right.\right.$$

$$\left.+8ab^2\&,\frac{4\#1^6\log\left(y(x)\sqrt[3]{\frac{ax^{3/2}}{b}}-\#1\right)+2\#1^4\sqrt[3]{-\frac{1}{ab^2}}\log\left(y(x)\sqrt[3]{\frac{ax^{3/2}}{b}}-\#1\right)+8\#1^3\log\left(y(x)\sqrt[3]{\frac{ax^3}{b}}\right.}{24\#1^8a}\right.$$

$$\left.+c_1,y(x)\right]$$

## 1.2 problem problem 41

Internal problem ID [4167]

**Book**: Differential Gleichungen, Kamke, 3rd ed, Abel ODEs
**Section**: Abel ODE's with constant invariant
**Problem number**: problem 41.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _Abel]`

$$\boxed{axy^3 + by^2 + y' = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 103

```
dsolve(a*x*y(x)^3+b*y(x)^2+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{\operatorname{RootOf}\left(2\sqrt{b^2+4a}\, b \operatorname{arctanh}\left(\frac{2a\,e^{-\_Z}+b}{\sqrt{b^2+4a}}\right) - \ln\left(x^2\left(e^{2\_Z}a+b\,e^{\_Z}-1\right)\right)b^2+2c_1b^2+2\_Zb^2-4\ln\left(x^2\left(e^{2\_Z}a+b\,e^{\_Z}-1\right)\right)a+8c_1a+8a\_Z\right)}}{x}$$

✓ Solution by Mathematica

Time used: 0.19 (sec). Leaf size: 103

```
DSolve[a*x*y[x]^3+b*y[x]^2+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{b^2\left(\frac{2\arctan\left(\frac{-2axy(x)-b}{b\sqrt{-\frac{4a}{b^2}-1}}\right)}{\sqrt{-\frac{4a}{b^2}-1}} - \log\left(\frac{a(-x)y(x)(-axy(x)-b)-a}{a^2x^2y(x)^2}\right)\right)}{2a} = -\frac{b^2\log(x)}{a} + c_1, y(x)\right]$$

## 1.3 problem problem 46

Internal problem ID [4168]

**Book**: Differential Gleichungen, Kamke, 3rd ed, Abel ODEs
**Section**: Abel ODE's with constant invariant
**Problem number**: problem 46.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_Abel]`

$$y' - x^a y^3 + 3y^2 - x^{-a} y - x^{-2a} + a\, x^{-a-1} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 1008

```
dsolve(diff(y(x),x)-x^a*y(x)^3+3*y(x)^2-x^(-a)*y(x)-x^(-2*a)+a*x^(-a-1) = 0,y(x), singsol=all
```

$$y(x) = -\frac{}{\sqrt{c_1 - \frac{2\,2^{-\frac{2a}{1-a}-\frac{2}{1-a}} \left(\frac{1}{1-a}\right)^{\frac{a+1}{a-1}} \left(-\frac{2^{-3+\frac{2a}{1-a}+\frac{2}{1-a}+\frac{2}{a-1}} (a-1) x^{-\frac{a^2}{1-a}+\frac{1}{1-a}-1+a} \left(\frac{1}{1-a}\right)^{-\frac{a+1}{a-1}} \left(-\frac{4x^{1-a}a^2}{1-a}+\frac{8a\,x^{1-a}}{1-a}-\frac{4x^{1-a}}{1-a}+2a-2\right)(1-}{(a+1)(-3+a)}\right.}{}}} + x^{-a}$$

$$y(x) = \frac{}{\sqrt{c_1 - \frac{2\,2^{-\frac{2a}{1-a}-\frac{2}{1-a}} \left(\frac{1}{1-a}\right)^{\frac{a+1}{a-1}} \left(-\frac{2^{-3+\frac{2a}{1-a}+\frac{2}{1-a}+\frac{2}{a-1}} (a-1) x^{-\frac{a^2}{1-a}+\frac{1}{1-a}-1+a} \left(\frac{1}{1-a}\right)^{-\frac{a+1}{a-1}} \left(-\frac{4x^{1-a}a^2}{1-a}+\frac{8a\,x^{1-a}}{1-a}-\frac{4x^{1-a}}{1-a}+2a-2\right)(1-a)\left(}{(a+1)(-3+a)}\right.}{}}} + x^{-a}$$

✓ Solution by Mathematica

Time used: 14.951 (sec). Leaf size: 149

```
DSolve[y'[x]-x^a*y[x]^3+3*y[x]^2-x^(-a)*y[x]-x^(-2*a)+a*x^(-a-1) == 0,y[x],x,IncludeSingularS
```

$$y(x)\to x^{-a}-\frac{e^{\frac{2x^{1-a}}{a-1}}}{\sqrt{-\frac{2x^{a+1}\,\text{ExpIntegralE}\left(\frac{2a}{a-1},-\frac{4x^{1-a}}{a-1}\right)}{a-1}+c_1}}$$

$$y(x)\to x^{-a}+\frac{e^{\frac{2x^{1-a}}{a-1}}}{\sqrt{-\frac{2x^{a+1}\,\text{ExpIntegralE}\left(\frac{2a}{a-1},-\frac{4x^{1-a}}{a-1}\right)}{a-1}+c_1}}$$

$$y(x)\to x^{-a}$$

## 1.4 problem problem 51

Internal problem ID [4169]

**Book**: Differential Gleichungen, Kamke, 3rd ed, Abel ODEs
**Section**: Abel ODE's with constant invariant
**Problem number**: problem 51.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_Abel]`

$$y' - (y - f(x))\,(y - g(x))\left(y - \frac{f(x)\,a + bg(x)}{a + b}\right)h(x) - \frac{f'(x)\,(y - g(x))}{f\,(x) - g\,(x)} - \frac{g'(x)\,(y - f(x))}{g\,(x) - f\,(x)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 2428

```
dsolve(diff(y(x),x)-(y(x)-f(x))*(y(x)-g(x))*(y(x)-(a*f(x)+b*g(x))/(a+b))*h(x)-diff(f(x),x)*(y
```

Expression too large to display

✓ Solution by Mathematica

Time used: 1.136 (sec). Leaf size: 355

```
DSolve[y'[x]-(y[x]-f[x])*(y[x]-g[x])*(y[x]-(a*f[x]+b*g[x])/(a+b))*h[x]-f'[x]*(y[x]-g[x])/(f[x
```

$$\text{Solve}\left[-\frac{1}{3}(a\right.$$

$$-b)^{2/3}(2a+b)^{2/3}(a+2b)^{2/3}\text{RootSum}\left[\#1^3(a-b)^{2/3}(2a+b)^{2/3}(a+2b)^{2/3}-3\#1a^2-3\#1ab-3\#1b^2+(a-b)^{2/3}\right.$$

## 1.5 problem problem 146

Internal problem ID [4170]

**Book**: Differential Gleichungen, Kamke, 3rd ed, Abel ODEs
**Section**: Abel ODE's with constant invariant
**Problem number**: problem 146.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$\boxed{x^2y' + y^3x + y^2a = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 82

```
dsolve(x^2*diff(y(x),x)+x*y(x)^3+a*y(x)^2 = 0,y(x), singsol=all)
```

$$c_1 + \left(x + \frac{a\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\frac{\sqrt{2}\,(ay(x)+x)}{2y(x)x}\right)\mathrm{e}^{\frac{(ay(x)+x)^2}{2y(x)^2x^2}}}{2}\right)\mathrm{e}^{-\frac{((x+a)y(x)+x)((-x+a)y(x)+x)}{2x^2y(x)^2}} = 0$$

✓ Solution by Mathematica

Time used: 0.615 (sec). Leaf size: 78

```
DSolve[x^2*y'[x]+x*y[x]^3+a*y[x]^2 == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{ia}{x} = \frac{2e^{\frac{1}{2}\left(-\frac{ia}{x}-\frac{i}{y(x)}\right)^2}}{\sqrt{2\pi}\operatorname{erfi}\left(\frac{-\frac{ia}{x}-\frac{i}{y(x)}}{\sqrt{2}}\right)+2c_1}, y(x)\right]$$

## 1.6 problem problem 169

Internal problem ID [4171]

**Book**: Differential Gleichungen, Kamke, 3rd ed, Abel ODEs
**Section**: Abel ODE's with constant invariant
**Problem number**: problem 169.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$(ax+b)^2 y' + (ax+b) y^3 + c y^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 153

```
dsolve((a*x+b)^2*diff(y(x),x)+(a*x+b)*y(x)^3+c*y(x)^2 = 0,y(x), singsol=all)
```

$$c_1 + \left(x + \frac{b}{a} + \frac{c\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\frac{\sqrt{2}\,(a^2x+ab+cy(x))}{2\sqrt{a}\,y(x)(ax+b)}\right) \mathrm{e}^{\frac{\left(a^2x+ab+cy(x)\right)^2}{2y(x)^2(ax+b)^2 a}}}{2a^{\frac{3}{2}}}\right) \mathrm{e}^{-\frac{\left(a^2x+axy(x)+ab+by(x)+cy(x)\right)\left(a^2x-axy(x)+ab-by(x)+cy(x)\right)}{2y(x)^2(ax+b)^2 a}} = 0$$

✓ Solution by Mathematica

Time used: 1.43 (sec). Leaf size: 149

```
DSolve[(a*x+b)^2*y'[x]+(a*x+b)*y[x]^3+c*y[x]^2 == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{c}{\sqrt{-a(ax+b)^2}}=\frac{2\exp\left(\frac{1}{2}\left(-\frac{c}{\sqrt{-a(ax+b)^2}}-\frac{\left(-a(ax+b)^2\right)^{3/2}}{ay(x)(ax+b)^3}\right)^2\right)}{\sqrt{2\pi}\,\text{erfi}\left(\frac{-\frac{c}{\sqrt{-a(ax+b)^2}}-\frac{\left(-a(ax+b)^2\right)^{3/2}}{ay(x)(ax+b)^3}}{\sqrt{2}}\right)+2c_1},y(x)\right]$$