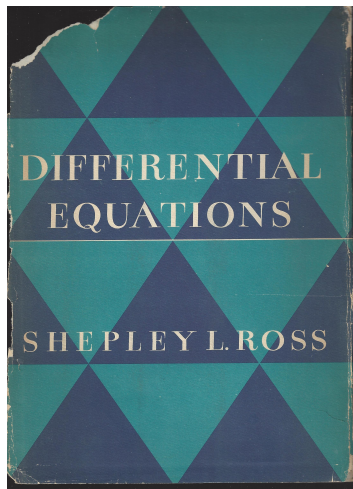


A Solution Manual For

**Differential equations, Shepley L.
Ross, 1964**



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1.1 problem 1

Internal problem ID [2479]

Book: Differential equations, Shepley L. Ross, 1964

Section: 2.4, page 55

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_rational, [_Abel, '2nd type', 'class B']]

$$5yx + 4y^2 + 1 + (x^2 + 2yx)y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 59

```
dsolve((5*x*y(x)+4*y(x)^2+1)+(x^2+2*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x^3 - \sqrt{x^6 - x^4 - 4c_1}}{2x^2}$$

$$y(x) = \frac{-x^3 + \sqrt{x^6 - x^4 - 4c_1}}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.624 (sec). Leaf size: 84

```
DSolve[(5*x*y[x]+4*y[x]^2+1)+(x^2+2*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x^5 + \sqrt{x^3}\sqrt{x^7 - x^5 + 4c_1x}}{2x^4}$$

$$y(x) \rightarrow -\frac{x}{2} + \frac{\sqrt{x^3}\sqrt{x^7 - x^5 + 4c_1x}}{2x^4}$$

1.2 problem 2

Internal problem ID [2480]

Book: Differential equations, Shepley L. Ross, 1964

Section: 2.4, page 55

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_exponential_symmetries]]`

$$2x \tan(y) + (x - x^2 \tan(y)) y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve((2*x*tan(y(x)))+(x-x^2*tan(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 2 \text{RootOf} \left(e^{-z} \left(\int^{2-z} -\frac{e^{-\frac{a}{2}}}{2 \tan(-a)} d_b \right) + e^{-z} c_1 - x \right)$$

✓ Solution by Mathematica

Time used: 0.406 (sec). Leaf size: 78

```
DSolve[(2*x*Tan[y[x]])+(x-x^2*Tan[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[x = \frac{1}{34} \left((8 - 2i) e^{2iy(x)} \text{Hypergeometric2F1} \left(1, 1 + \frac{i}{4}, 2 + \frac{i}{4}, e^{2iy(x)} \right) \right. \right. \\ \left. \left. - 34i \text{Hypergeometric2F1} \left(\frac{i}{4}, 1, 1 + \frac{i}{4}, e^{2iy(x)} \right) \right) + c_1 e^{\frac{y(x)}{2}}, y(x) \right]$$

1.3 problem 3

Internal problem ID [2481]

Book: Differential equations, Shepley L. Ross, 1964

Section: 2.4, page 55

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_rational, [_Abel, '2nd type', 'class B']]

$$y^2(x^2 + 1) + y + (2yx + 1)y' = 0$$

✗ Solution by Maple

```
dsolve((y(x)^2*(x^2+1)+y(x))+(2*x*y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y[x]^2*(x^2+1)+y[x])+(2*x*y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.4 problem 4

Internal problem ID [2482]

Book: Differential equations, Shepley L. Ross, 1964

Section: 2.4, page 55

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, [_Abel, '2nd type', 'cla`

$$4xy^2 + 6y + (5x^2y + 8x)y' = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve((4*x*y(x)^2+6*y(x))+(5*x^2*y(x)+8*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\text{RootOf}(-\ln(x) + c_1 + 4\ln(_Z) + \ln(2 + _Z))}{x}$$

✓ Solution by Mathematica

Time used: 1.879 (sec). Leaf size: 156

```
DSolve[(4*x*y[x]^2+6*y[x])+(5*x^2*y[x]+8*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \text{Root}\left[-\#1^5 - \frac{2\#1^4}{x} + \frac{e^{c_1}}{x^4} \&, 1\right]$$

$$y(x) \rightarrow \text{Root}\left[-\#1^5 - \frac{2\#1^4}{x} + \frac{e^{c_1}}{x^4} \&, 2\right]$$

$$y(x) \rightarrow \text{Root}\left[-\#1^5 - \frac{2\#1^4}{x} + \frac{e^{c_1}}{x^4} \&, 3\right]$$

$$y(x) \rightarrow \text{Root}\left[-\#1^5 - \frac{2\#1^4}{x} + \frac{e^{c_1}}{x^4} \&, 4\right]$$

$$y(x) \rightarrow \text{Root}\left[-\#1^5 - \frac{2\#1^4}{x} + \frac{e^{c_1}}{x^4} \&, 5\right]$$

1.5 problem 5

Internal problem ID [2483]

Book: Differential equations, Shepley L. Ross, 1964

Section: 2.4, page 55

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _exact, _rational, [_Abel, '2nd typ`

$$5x + 2y + 1 + (2x + y + 1)y' = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 32

```
dsolve((5*x+2*y(x)+1)+(2*x+y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -3 - \frac{2c_1(x-1) + \sqrt{-(x-1)^2 c_1^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 49

```
DSolve[(5*x+2*y[x]+1)+(2*x+y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -2x - \sqrt{-((x-2)x) + 1 + c_1} - 1$$

$$y(x) \rightarrow -2x + \sqrt{-((x-2)x) + 1 + c_1} - 1$$

1.6 problem 6

Internal problem ID [2484]

Book: Differential equations, Shepley L. Ross, 1964

Section: 2.4, page 55

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _rational, [_Abel, '2nd type', 'cla`

$$3x - y + 1 - (6x - 2y - 3)y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve((3*x-y(x)+1)-(6*x-2*y(x)-3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^{-\text{LambertW}(-2e^{-4}e^{5x}e^{-5c_1})-4+5x-5c_1} - 2 + 3x$$

✓ Solution by Mathematica

Time used: 3.223 (sec). Leaf size: 35

```
DSolve[(3*x-y[x]+1)-(6*x-2*y[x]-3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{2}W(-e^{5x-1+c_1}) + 3x - 2$$

$$y(x) \rightarrow 3x - 2$$

1.7 problem 7

Internal problem ID [2485]

Book: Differential equations, Shepley L. Ross, 1964

Section: 2.4, page 55

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _rational, [_Abel, '2nd type', 'cla`

$$x - 2y - 3 + (2x + y - 1)y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve((x-2*y(x)-3)+(2*x+y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -1 - \tan \left(\text{RootOf} \left(-4_Z + \ln \left(\frac{1}{\cos(_Z)^2} \right) + 2 \ln(x - 1) + 2c_1 \right) \right) (x - 1)$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 66

```
DSolve[(x-2*y[x]-3)+(2*x+y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[32 \arctan \left(\frac{2y(x) - x + 3}{y(x) + 2x - 1} \right) + 8 \log \left(\frac{x^2 + y(x)^2 + 2y(x) - 2x + 2}{5(x - 1)^2} \right) + 16 \log(x - 1) + 5c_1 = 0, y(x) \right]$$

1.8 problem 8

Internal problem ID [2486]

Book: Differential equations, Shepley L. Ross, 1964

Section: 2.4, page 55

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _exact, _rational, [_Abel, '2nd typ`

$$6x + 4y + 1 + (4x + 2y + 2)y' = 0$$

With initial conditions

$$\left[y\left(\frac{1}{2}\right) = 3 \right]$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 23

```
dsolve([(6*x+4*y(x)+1)+(4*x+2*y(x)+2)*diff(y(x),x)=0,y(1/2) = 3],y(x), singsol=all)
```

$$y(x) = -2x - 1 + \frac{\sqrt{4x^2 + 12x + 93}}{2}$$

✓ Solution by Mathematica

Time used: 0.121 (sec). Leaf size: 26

```
DSolve[{(6*x+4*y[x]+1)+(4*x+2*y[x]+2)*y'[x]==0,y[1/2]==3},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow \frac{1}{2} \left(-4x + \sqrt{4x(x+3) + 93} - 2 \right)$$

1.9 problem 9

Internal problem ID [2487]

Book: Differential equations, Shepley L. Ross, 1964

Section: 2.4, page 55

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _rational, [_Abel, '2nd type', 'cla`

$$3x - y - 6 + (x + y + 2)y' = 0$$

With initial conditions

$$[y(2) = -2]$$

✓ Solution by Maple

Time used: 0.828 (sec). Leaf size: 120

```
dsolve([(3*x-y(x)-6)+(x+y(x)+2)*diff(y(x),x)=0,y(2) = -2],y(x), singsol=all)
```

$$y(x) = -3 - \tan \left(\text{RootOf} \left(-3\sqrt{3} \ln((x-1)^2 \sec(_Z)^2) + 6\sqrt{3} \ln(2) - 3\sqrt{3} \ln(3) + \pi + 6_Z \right) \right) \sqrt{3}(x-1)$$

✓ Solution by Mathematica

Time used: 0.144 (sec). Leaf size: 90

```
DSolve[{(3*x-y[x]-6)+(x+y[x]+2)*y'[x]==0,y[2]==-2},y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[\frac{\arctan \left(\frac{-y(x)+3x-6}{\sqrt{3}(y(x)+x+2)} \right)}{\sqrt{3}} + \log(2) = \frac{1}{2} \log \left(\frac{3x^2 + y(x)^2 + 6y(x) - 6x + 12}{(x-1)^2} \right) + \log(x-1) + \frac{1}{18} \left(\sqrt{3}\pi + 18 \log(2) - 9 \log(4) \right), y(x) \right]$$

1.10 problem 10

Internal problem ID [2488]

Book: Differential equations, Shepley L. Ross, 1964

Section: 2.4, page 55

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _rational, [_Abel, '2nd type', 'cla`

$$2x + 3y + 1 + (4x + 6y + 1)y' = 0$$

With initial conditions

$$[y(-2) = 2]$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 20

```
dsolve([(2*x+3*y(x)+1)+(4*x+6*y(x)+1)*diff(y(x),x)=0,y(-2) = 2],y(x), singsol=all)
```

$$y(x) = \frac{1}{3} - \frac{2x}{3} + \frac{\text{LambertW}\left(\frac{2e^{\frac{4}{3} + \frac{x}{3}}}{3}\right)}{2}$$

✓ Solution by Mathematica

Time used: 5.325 (sec). Leaf size: 30

```
DSolve[{(2*x+3*y[x]+1)+(4*x+6*y[x]+1)*y'[x]==0,y[-2]==2},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \rightarrow \frac{1}{6} \left(3W\left(\frac{2}{3}e^{\frac{x+4}{3}}\right) - 4x + 2 \right)$$