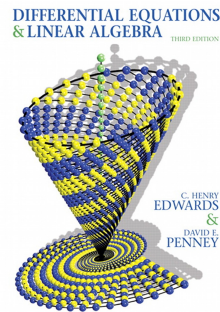


A Solution Manual For

Differential equations and linear algebra, 3rd ed., Edwards and Penney



Nasser M. Abbasi

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1 Section 1.2. Integrals as general and particular solutions. Page 16

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1.1 problem 1

Internal problem ID [1]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.2. Integrals as general and particular solutions. Page 16

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - 1 - 2x = 0$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 10

```
dsolve([diff(y(x),x) = 1+2*x,y(0) = 3],y(x), singsol=all)
```

$$y(x) = x^2 + x + 3$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 11

```
DSolve[{y'[x]==1+2*x,y[0]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2 + x + 3$$

1.2 problem 2

Internal problem ID [2]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.2. Integrals as general and particular solutions. Page 16

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - (-2 + x)^2 = 0$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve([diff(y(x),x) = (-2+x)^2,y(2) = 1],y(x), singsol=all)
```

$$y(x) = \frac{(-2 + x)^3}{3} + 1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[{y'[x]==(-2+x)^2,y[2]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{3}(x((x - 6)x + 12) - 5)$$

1.3 problem 3

Internal problem ID [3]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.2. Integrals as general and particular solutions. Page 16

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - \sqrt{x} = 0$$

With initial conditions

$$[y(4) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x) = x^(1/2),y(4) = 0],y(x), singsol=all)
```

$$y(x) = \frac{2x^{\frac{3}{2}}}{3} - \frac{16}{3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{y'[x] == x^(1/2),y[4]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2}{3}(x^{3/2} - 8)$$

1.4 problem 4

Internal problem ID [4]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.2. Integrals as general and particular solutions. Page 16

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - \frac{1}{x^2} = 0$$

With initial conditions

$$[y(1) = 5]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x) = 1/x^2,y(1) = 5],y(x), singsol=all)
```

$$y(x) = -\frac{1}{x} + 6$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[{y'[x] == 1/x^2,y[1]==5},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 6 - \frac{1}{x}$$

1.5 problem 5

Internal problem ID [5]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.2. Integrals as general and particular solutions. Page 16

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - \frac{1}{\sqrt{2+x}} = 0$$

With initial conditions

$$[y(2) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(y(x),x) = 1/(2+x)^(1/2),y(2) = -1],y(x), singular=all)
```

$$y(x) = 2\sqrt{2+x} - 5$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 16

```
DSolve[{y'[x] == 1/(2+x)^(1/2),y[2]==-1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2\sqrt{x+2} - 5$$

1.6 problem 6

Internal problem ID [6]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.2. Integrals as general and particular solutions. Page 16

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - x\sqrt{x^2 + 9} = 0$$

With initial conditions

$$[y(-4) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x) = x*(x^2+9)^(1/2),y(-4) = 0],y(x), singsol=all)
```

$$y(x) = \frac{x^2\sqrt{x^2+9}}{3} + 3\sqrt{x^2+9} - \frac{125}{3}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 20

```
DSolve[{y'[x] == x*(x^2+9)^(1/2),y[-4]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{3} \left((x^2 + 9)^{3/2} - 125 \right)$$

1.7 problem 7

Internal problem ID [7]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.2. Integrals as general and particular solutions. Page 16

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - \frac{10}{x^2 + 1} = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 8

```
dsolve([diff(y(x),x) = 10/(x^2+1),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 10 \arctan(x)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 9

```
DSolve[{y'[x]==10/(x^2+1),y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 10 \arctan(x)$$

1.8 problem 8

Internal problem ID [8]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.2. Integrals as general and particular solutions. Page 16

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - \cos(2x) = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x) = cos(2*x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\sin(2x)}{2} + 1$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 12

```
DSolve[{y'[x] == Cos[2*x],y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sin(x) \cos(x) + 1$$

1.9 problem 9

Internal problem ID [9]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.2. Integrals as general and particular solutions. Page 16

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - \frac{1}{\sqrt{-x^2 + 1}} = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 6

```
dsolve([diff(y(x),x) = 1/(-x^2+1)^(1/2),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \arcsin(x)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 29

```
DSolve[{y'[x] == 1/(-x^2+1)^(1/2),y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} \left(\pi - 4 \cot^{-1} \left(\frac{x+1}{\sqrt{1-x^2}} \right) \right)$$

1.10 problem 10

Internal problem ID [10]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.2. Integrals as general and particular solutions. Page 16

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - x e^{-x} = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([diff(y(x),x) = x/exp(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = 2 + (-x - 1) e^{-x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 17

```
DSolve[{y'[x]== x/Exp[x],y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2 - e^{-x}(x + 1)$$

2 Section 1.3. Slope fields and solution curves. Page 26

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2.1 problem 1

Internal problem ID [11]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.3. Slope fields and solution curves. Page 26

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + \sin(x) + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x) = -sin(x)-y(x),y(x), singsol=all)
```

$$y(x) = \frac{\cos(x)}{2} - \frac{\sin(x)}{2} + e^{-x}c_1$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 25

```
DSolve[y'[x]== -Sin[x]-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}(-\sin(x) + \cos(x) + 2c_1e^{-x})$$

2.2 problem 2

Internal problem ID [12]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.3. Slope fields and solution curves. Page 26

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y' - x - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) = x+y(x),y(x), singsol=all)
```

$$y(x) = -x - 1 + e^x c_1$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 16

```
DSolve[y'[x] == x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x + c_1 e^x - 1$$

2.3 problem 3

Internal problem ID [13]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.3. Slope fields and solution curves. Page 26

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + \sin(x) - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x) = -sin(x)+y(x),y(x), singsol=all)
```

$$y(x) = \frac{\cos(x)}{2} + \frac{\sin(x)}{2} + e^x c_1$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 21

```
DSolve[y'[x] == -Sin[x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}(\sin(x) + \cos(x) + 2c_1 e^x)$$

2.4 problem 4

Internal problem ID [14]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.3. Slope fields and solution curves. Page 26

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y' - x + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) = x-y(x),y(x), singsol=all)
```

$$y(x) = x - 1 + e^{-x}c_1$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 16

```
DSolve[y'[x] == x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x + c_1 e^{-x} - 1$$

2.5 problem 5

Internal problem ID [15]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.3. Slope fields and solution curves. Page 26

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y' - 1 + x - y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve(diff(y(x),x) = 1-x+y(x),y(x), singsol=all)
```

$$y(x) = x + e^x c_1$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 13

```
DSolve[y'[x] == 1-x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x + c_1 e^x$$

2.6 problem 6

Internal problem ID [16]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.3. Slope fields and solution curves. Page 26

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y' - 1 - x + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) = 1+x-y(x),y(x), singsol=all)
```

$$y(x) = x + e^{-x}c_1$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 15

```
DSolve[y'[x] == 1+x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x + c_1 e^{-x}$$

2.7 problem 8

Internal problem ID [17]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.3. Slope fields and solution curves. Page 26

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - x^2 + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x) = x^2-y(x),y(x), singsol=all)
```

$$y(x) = x^2 - 2x + 2 + e^{-x}c_1$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 20

```
DSolve[y'[x] == x^2-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (x - 2)x + c_1e^{-x} + 2$$

2.8 problem 9

Internal problem ID [18]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.3. Slope fields and solution curves. Page 26

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + 2 - x^2 + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x) = -2+x^2-y(x),y(x), singsol=all)
```

$$y(x) = x^2 - 2x + e^{-x}c_1$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 19

```
DSolve[y'[x]== -2+x^2-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (x - 2)x + c_1 e^{-x}$$

2.9 problem 11

Internal problem ID [19]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.3. Slope fields and solution curves. Page 26

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - 2x^2y^2 = 0$$

With initial conditions

$$[y(1) = -1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve([diff(y(x),x) = 2*x^2*y(x)^2,y(1) = -1],y(x), singsol=all)
```

$$y(x) = -\frac{3}{2x^3 + 1}$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 16

```
DSolve[{y'[x] == 2*x^2*y[x]^2,y[1]==-1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{3}{2x^3 + 1}$$

2.10 problem 12

Internal problem ID [20]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.3. Slope fields and solution curves. Page 26

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - x \ln(y) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) = x*ln(y(x)),y(x), singsol=all)
```

$$y(x) = e^{\text{RootOf}(x^2 + 2 \text{Ei}_1(-x) + 2c_1)}$$

✓ Solution by Mathematica

Time used: 0.255 (sec). Leaf size: 22

```
DSolve[y'[x] == x*Log[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \text{LogIntegral}^{(-1)}\left(\frac{x^2}{2} + c_1\right)$$

$$y(x) \rightarrow 1$$

2.11 problem 13

Internal problem ID [21]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.3. Slope fields and solution curves. Page 26

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - y^{\frac{1}{3}} = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 13

```
dsolve([diff(y(x),x) = y(x)^(1/3),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{(3 + 2x) \sqrt{6x + 9}}{9}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 23

```
DSolve[{y'[x] == y[x]^(1/3),y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{(2x + 3)^{3/2}}{3\sqrt{3}}$$

2.12 problem 14

Internal problem ID [22]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.3. Slope fields and solution curves. Page 26

Problem number: 14.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - y^{\frac{1}{3}} = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x) = y(x)^(1/3),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 21

```
DSolve[{y'[x] == y[x]^(1/3),y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2}{3} \sqrt{\frac{2}{3}} x^{3/2}$$

2.13 problem 17

Internal problem ID [23]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.3. Slope fields and solution curves. Page 26

Problem number: 17.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$yy' - x + 1 = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 9

```
dsolve([y(x)*diff(y(x),x) = -1+x,y(0) = 1],y(x), singsol=all)
```

$$y(x) = 1 - x$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 14

```
DSolve[{y[x]*y'[x] == -1+x,y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt{(x-1)^2}$$

2.14 problem 18

Internal problem ID [24]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.3. Slope fields and solution curves. Page 26

Problem number: 18.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$yy' - x + 1 = 0$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve([y(x)*diff(y(x),x) = -1+x,y(1) = 0],y(x), singsol=all)
```

$$y(x) = 1 - x$$

$$y(x) = x - 1$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 29

```
DSolve[{y[x]*y'[x] == -1+x,y[1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt{(x-1)^2}$$

$$y(x) \rightarrow \sqrt{(x-1)^2}$$

2.15 problem 19

Internal problem ID [25]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.3. Slope fields and solution curves. Page 26

Problem number: 19.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - \ln(1 + y^2) = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x) = ln(1+y(x)^2),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x] == Log[1+y[x]^2],y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0$$

2.16 problem 20

Internal problem ID [26]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.3. Slope fields and solution curves. Page 26

Problem number: 20.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [Riccati]

$$y' - x^2 + y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = x^2-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x \left(\text{BesselI} \left(-\frac{3}{4}, \frac{x^2}{2} \right) c_1 - \text{BesselK} \left(\frac{3}{4}, \frac{x^2}{2} \right) \right)}{c_1 \text{BesselI} \left(\frac{1}{4}, \frac{x^2}{2} \right) + \text{BesselK} \left(\frac{1}{4}, \frac{x^2}{2} \right)}$$

✓ Solution by Mathematica

Time used: 0.105 (sec). Leaf size: 103

```
DSolve[y'[x]== x^2-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{ix \left(\text{BesselJ} \left(-\frac{3}{4}, \frac{ix^2}{2} \right) - c_1 \text{BesselJ} \left(\frac{3}{4}, \frac{ix^2}{2} \right) \right)}{\text{BesselJ} \left(\frac{1}{4}, \frac{ix^2}{2} \right) + c_1 \text{BesselJ} \left(-\frac{1}{4}, \frac{ix^2}{2} \right)}$$

$$y(x) \rightarrow \frac{x \text{BesselI} \left(\frac{3}{4}, \frac{x^2}{2} \right)}{\text{BesselI} \left(-\frac{1}{4}, \frac{x^2}{2} \right)}$$

3 Section 1.4. Separable equations. Page 43

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3.1 problem 1

Internal problem ID [27]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$2yx + y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(2*x*y(x)+diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x^2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 20

```
DSolve[2*x*y[x]+y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-x^2}$$

$$y(x) \rightarrow 0$$

3.2 problem 2

Internal problem ID [28]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$2xy^2 + y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(2*x*y(x)^2+diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.092 (sec). Leaf size: 20

```
DSolve[2*x*y[x]^2+y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{x^2 - c_1}$$

$$y(x) \rightarrow 0$$

3.3 problem 3

Internal problem ID [29]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - \sin(x)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = sin(x)*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{-\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 19

```
DSolve[y'[x] == Sin[x]*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-\cos(x)}$$

$$y(x) \rightarrow 0$$

3.4 problem 4

Internal problem ID [30]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$(x + 1)y' - 4y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve((1+x)*diff(y(x),x) = 4*y(x),y(x), singsol=all)
```

$$y(x) = c_1(x + 1)^4$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 18

```
DSolve[(1+x)*y'[x] == 4*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1(x + 1)^4$$

$$y(x) \rightarrow 0$$

3.5 problem 5

Internal problem ID [31]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$2\sqrt{x}y' - \sqrt{1-y^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(2*x^(1/2)*diff(y(x),x) = (1-y(x)^2)^(1/2),y(x), singsol=all)
```

$$y(x) = \sin\left(\sqrt{x} + \frac{c_1}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.218 (sec). Leaf size: 32

```
DSolve[2*x^(1/2)*y'[x] == (1-y[x]^2)^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \cos(\sqrt{x} + c_1)$$

$$y(x) \rightarrow -1$$

$$y(x) \rightarrow 1$$

$$y(x) \rightarrow \text{Interval}[\{-1, 1\}]$$

3.6 problem 6

Internal problem ID [32]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$y' - 3\sqrt{yx} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 66

```
dsolve(diff(y(x),x) = 3*(x*y(x))^(1/2),y(x), singsol=all)
```

$$\frac{x^2}{(-x^3 + y(x)) \left(-x^2 + \sqrt{xy(x)}\right)} + \frac{\sqrt{xy(x)}}{(-x^3 + y(x)) \left(-x^2 + \sqrt{xy(x)}\right)} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.123 (sec). Leaf size: 26

```
DSolve[y'[x] == 3*(x*y[x])^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}(2x^{3/2} + c_1)^2$$

$$y(x) \rightarrow 0$$

3.7 problem 7

Internal problem ID [33]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$y' - 4(yx)^{\frac{1}{3}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 108

```
dsolve(diff(y(x),x) = 4*(x*y(x))^(1/3),y(x), singsol=all)
```

$$\frac{(xy(x))^{\frac{4}{3}}}{(-8x^4 + y(x)^2) \left((xy(x))^{\frac{2}{3}} - 2x^2 \right)^2} + \frac{2x^2(xy(x))^{\frac{2}{3}}}{(-8x^4 + y(x)^2) \left((xy(x))^{\frac{2}{3}} - 2x^2 \right)^2} + \frac{4x^4}{(-8x^4 + y(x)^2) \left((xy(x))^{\frac{2}{3}} - 2x^2 \right)^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 4.796 (sec). Leaf size: 35

```
DSolve[y'[x] == 4*(x*y[x])^(1/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2}{3} \sqrt{\frac{2}{3}} (3x^{4/3} + c_1)^{3/2}$$

$$y(x) \rightarrow 0$$

3.8 problem 8

Internal problem ID [34]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - 2x \sec(y) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) = 2*x*sec(y(x)),y(x), singsol=all)
```

$$y(x) = \arcsin(x^2 + 2c_1)$$

✓ Solution by Mathematica

Time used: 0.592 (sec). Leaf size: 12

```
DSolve[y'[x]==2*x*Sec[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \arcsin(x^2 + c_1)$$

3.9 problem 9

Internal problem ID [35]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$(-x^2 + 1)y' - 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve((-x^2+1)*diff(y(x),x) = 2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1(x+1)^2}{-x^2+1}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 22

```
DSolve[(-x^2+1)*y'[x] == 2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{c_1(x+1)}{x-1}$$

$$y(x) \rightarrow 0$$

3.10 problem 10

Internal problem ID [36]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$(x^2 + 1)y' - (1 + y)^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((x^2+1)*diff(y(x),x) = (1+y(x))^2,y(x), singsol=all)
```

$$y(x) = -\frac{\arctan(x) + c_1 + 1}{\arctan(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.181 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y'[x]== (1+y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -1 - \frac{1}{\arctan(x) + c_1}$$

$$y(x) \rightarrow -1$$

3.11 problem 11

Internal problem ID [37]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - xy^3 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = x*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-x^2 + c_1}}$$

$$y(x) = -\frac{1}{\sqrt{-x^2 + c_1}}$$

✓ Solution by Mathematica

Time used: 0.134 (sec). Leaf size: 44

```
DSolve[y'[x] == x*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{\sqrt{-x^2 - 2c_1}}$$

$$y(x) \rightarrow \frac{1}{\sqrt{-x^2 - 2c_1}}$$

$$y(x) \rightarrow 0$$

3.12 problem 12

Internal problem ID [38]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$yy' - x(1 + y^2) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(y(x)*diff(y(x),x) = x*(1+y(x)^2),y(x), singsol=all)
```

$$y(x) = \sqrt{e^{x^2}c_1 - 1}$$

$$y(x) = -\sqrt{e^{x^2}c_1 - 1}$$

✓ Solution by Mathematica

Time used: 6.907 (sec). Leaf size: 57

```
DSolve[y[x]*y'[x] == x*(1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt{-1 + e^{x^2+2c_1}}$$

$$y(x) \rightarrow \sqrt{-1 + e^{x^2+2c_1}}$$

$$y(x) \rightarrow -i$$

$$y(x) \rightarrow i$$

3.13 problem 14

Internal problem ID [39]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 14.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - \frac{1 + \sqrt{x}}{1 + \sqrt{y}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = (1+x^(1/2))/(1+y(x)^(1/2)),y(x), singsol=all)
```

$$x + \frac{2x^{\frac{3}{2}}}{3} - y(x) - \frac{2y(x)^{\frac{3}{2}}}{3} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 3.85 (sec). Leaf size: 789

`DSolve[y'[x]== (1+x^(1/2))/(1+y[x]^(1/2)),y[x],x,IncludeSingularSolutions -> True]`

$y(x)$

$$\rightarrow \frac{\left(4(2\sqrt{x}+3)x(4x^{3/2}+6x-9)+8\sqrt{(2x^{3/2}+3x-1+3c_1)(2x^{3/2}+3x+3c_1)^3+48c_1(2\sqrt{x}+3)x}\right)}{4\sqrt[3]{4(2\sqrt{x}+3)x(4x^{3/2}+6x-9)+8\sqrt{(2x^{3/2}+3x-1+3c_1)(2x^{3/2}+3x+3c_1)^3+48c_1(2\sqrt{x}+3)x}}}$$

$y(x)$

$$\rightarrow \frac{1}{16} \left(\frac{(2+2i\sqrt{3})(16x^{3/2}+24x-9+24c_1)}{\sqrt[3]{4(2\sqrt{x}+3)x(4x^{3/2}+6x-9)+8\sqrt{(2x^{3/2}+3x-1+3c_1)(2x^{3/2}+3x+3c_1)^3+48c_1(2\sqrt{x}+3)x}}}\right. \\ \left. +2i(\sqrt{3}+i)\sqrt[3]{4(2\sqrt{x}+3)x(4x^{3/2}+6x-9)+8\sqrt{(2x^{3/2}+3x-1+3c_1)(2x^{3/2}+3x+3c_1)^3+48c_1(2\sqrt{x}+3)x}}}\right) + 12$$

$y(x)$

$$\rightarrow \frac{1}{16} \left(\frac{(2-2i\sqrt{3})(16x^{3/2}+24x-9+24c_1)}{\sqrt[3]{4(2\sqrt{x}+3)x(4x^{3/2}+6x-9)+8\sqrt{(2x^{3/2}+3x-1+3c_1)(2x^{3/2}+3x+3c_1)^3+48c_1(2\sqrt{x}+3)x}}}\right) \\ -2(1+i\sqrt{3})\sqrt[3]{4(2\sqrt{x}+3)x(4x^{3/2}+6x-9)+8\sqrt{(2x^{3/2}+3x-1+3c_1)(2x^{3/2}+3x+3c_1)^3+48c_1(2\sqrt{x}+3)x}}\right) + 12$$

3.14 problem 15

Internal problem ID [40]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 15.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_separable`]

$$y' - \frac{(x-1)y^5}{x^2(-y+2y^3)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 2094

```
dsolve(diff(y(x),x) = (-1+x)*y(x)^5/x^2/(-y(x)+2*y(x)^3),y(x), singsol=all)
```

$y(x)$

$$= \frac{4^{\frac{1}{3}} \left(x \left(9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 3 \ln(x) \sqrt{9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 18 x \ln(x) + 1} \right) \right)}{3 (x \ln(x) + c_1 x + 1)}$$

+

$$\frac{3 (x \ln(x) + c_1 x + 1) \left(x \left(9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 3 \ln(x) \sqrt{9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 18 x \ln(x) + 1} \right) \right) - 2x}{3 (x \ln(x) + c_1 x + 1)}$$

$y(x) =$

$$\frac{4^{\frac{1}{3}} \left(x \left(9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 3 \ln(x) \sqrt{9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 18 x \ln(x) + 1} \right) \right)}{3 (x \ln(x) + c_1 x + 1)}$$

$$\frac{3 (x \ln(x) + c_1 x + 1) \left(x \left(9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 3 \ln(x) \sqrt{9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 18 x \ln(x) + 1} \right) \right) - 2x}{3 (x \ln(x) + c_1 x + 1)}$$

$$+ i\sqrt{3} \left(\frac{4^{\frac{1}{3}} \left(x \left(9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 3 \ln(x) \sqrt{9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 18 x \ln(x) + 18 c_1 x - 32 x^2 + 9 x + 3} \sqrt{9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 18 x \ln(x) + 1} \right) \right)}{3 (x \ln(x) + c_1 x + 1)} \right)$$

$y(x) =$

$$\frac{4^{\frac{1}{3}} \left(x \left(9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 3 \ln(x) \sqrt{9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 18 x \ln(x) + 1} \right) \right)}{3 (x \ln(x) + c_1 x + 1)}$$

$$\frac{3 (x \ln(x) + c_1 x + 1) \left(x \left(9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 3 \ln(x) \sqrt{9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 18 x \ln(x) + 1} \right) \right) - 2x}{3 (x \ln(x) + c_1 x + 1)}$$

$$+ i\sqrt{3} \left(\frac{4^{\frac{1}{3}} \left(x \left(9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 3 \ln(x) \sqrt{9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 18 x \ln(x) + 18 c_1 x - 32 x^2 + 9 x + 3} \sqrt{9 \ln(x)^2 x^2 + 18 \ln(x) c_1 x^2 + 9 c_1^2 x^2 + 18 x \ln(x) + 1} \right) \right)}{3 (x \ln(x) + c_1 x + 1)} \right)$$

+

✓ Solution by Mathematica

Time used: 19.262 (sec). Leaf size: 842

```
DSolve[y'[x] == (-1+x)*y[x]^5/x^2/(-y[x]+2*y[x]^3),y[x],x,IncludeSingularSolutions -> True]
```

$y(x) \rightarrow$

$$\frac{8\sqrt[3]{2}x^2}{\sqrt[3]{16x^3 - 9x^3 \log^2(x) - 9c_1^2x^3 - 18c_1x^2 + 3\sqrt{x^2(x \log(x) + c_1x + 1)^2 (9x^2 \log^2(x) + (-32 + 9c_1^2)x^2 + 1)}}}$$

$y(x)$

$$\rightarrow \frac{8\sqrt[3]{2}(1+i\sqrt{3})x^2}{\sqrt[3]{16x^3 - 9x^3 \log^2(x) - 9c_1^2x^3 - 18c_1x^2 + 3\sqrt{x^2(x \log(x) + c_1x + 1)^2 (9x^2 \log^2(x) + (-32 + 9c_1^2)x^2 + 1)}}}$$

$y(x)$

$$\rightarrow \frac{8\sqrt[3]{2}(1-i\sqrt{3})x^2}{\sqrt[3]{16x^3 - 9x^3 \log^2(x) - 9c_1^2x^3 - 18c_1x^2 + 3\sqrt{x^2(x \log(x) + c_1x + 1)^2 (9x^2 \log^2(x) + (-32 + 9c_1^2)x^2 + 1)}}}$$

$y(x) \rightarrow 0$

3.15 problem 16

Internal problem ID [41]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 16.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$(x^2 + 1) \tan(y) y' - x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((x^2+1)*tan(y(x))*diff(y(x),x) = x,y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{1}{\sqrt{x^2 + 1} c_1}\right)$$

✓ Solution by Mathematica

Time used: 15.24 (sec). Leaf size: 59

```
DSolve[(x^2+1)*Tan[y[x]]*y'[x] == x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sec^{-1}\left(e^{c_1}\sqrt{x^2 + 1}\right)$$

$$y(x) \rightarrow \sec^{-1}\left(e^{c_1}\sqrt{x^2 + 1}\right)$$

$$y(x) \rightarrow -\frac{\pi}{2}$$

$$y(x) \rightarrow \frac{\pi}{2}$$

3.16 problem 17

Internal problem ID [42]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 17.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - 1 - x - y - yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) = 1+x+y(x)+x*y(x),y(x), singsol=all)
```

$$y(x) = -1 + e^{\frac{x(2+x)}{2}} c_1$$

✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 25

```
DSolve[y'[x] == 1+x+y[x]+x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -1 + c_1 e^{\frac{1}{2}x(x+2)}$$

$$y(x) \rightarrow -1$$

3.17 problem 18

Internal problem ID [43]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 18.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y'x^2 - 1 + x^2 - y^2 + x^2y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x) = 1-x^2+y(x)^2-x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\tan\left(\frac{c_1x + x^2 + 1}{x}\right)$$

✓ Solution by Mathematica

Time used: 0.256 (sec). Leaf size: 17

```
DSolve[x^2*y'[x] == 1-x^2+y[x]^2-x^2*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\tan\left(x + \frac{1}{x} - c_1\right)$$

3.18 problem 19

Internal problem ID [44]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 19.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - e^x y = 0$$

With initial conditions

$$[y(0) = 2e]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve([diff(y(x),x) = exp(x)*y(x),y(0) = 2*exp(1)],y(x), singsol=all)
```

$$y(x) = 2e^{e^x}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 12

```
DSolve[{y'[x] == Exp[x]*y[x],y[0]==2*Exp[1]},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2e^{e^x}$$

3.19 problem 20

Internal problem ID [45]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 20.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - 3(1 + y^2)x^2 = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x) = 3*x^2*(1+y(x)^2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \tan\left(x^3 + \frac{\pi}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.165 (sec). Leaf size: 15

```
DSolve[{y'[x]== 3*x^2*(1+y[x]^2),y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \tan\left(x^3 + \frac{\pi}{4}\right)$$

3.20 problem 21

Internal problem ID [46]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 21.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$2yy' - \frac{x}{\sqrt{x^2 - 16}} = 0$$

With initial conditions

$$[y(5) = 2]$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 34

```
dsolve([2*y(x)*diff(y(x),x) = x/(x^2-16)^(1/2),y(5) = 2],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\sqrt{x^2 - 16}} (x^2 + \sqrt{x^2 - 16} - 16)}{\sqrt{x^2 - 16}}$$

✓ Solution by Mathematica

Time used: 1.937 (sec). Leaf size: 20

```
DSolve[{2*y[x]*y'[x] == x/(x^2-16)^(1/2),y[5]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt{\sqrt{x^2 - 16} + 1}$$

3.21 problem 22

Internal problem ID [47]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 22.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' + y - 4x^3y = 0$$

With initial conditions

$$[y(1) = -3]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(y(x),x) = -y(x)+4*x^3*y(x),y(1) = -3],y(x), singsol=all)
```

$$y(x) = -3e^{x(x-1)(x^2+x+1)}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 16

```
DSolve[{y'[x]== -y[x]+4*x^3*y[x],y[1]==-3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -3e^{x^4-x}$$

3.22 problem 23

Internal problem ID [48]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 23.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$1 + y' - 2y = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([1+diff(y(x),x) = 2*y(x),y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{2} + \frac{e^{2x-2}}{2}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 18

```
DSolve[{1+y'[x] == 2*y[x],y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}(e^{2x-2} + 1)$$

3.23 problem 24

Internal problem ID [49]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 24.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$\tan(x)y' - y = 0$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = \frac{\pi}{2} \right]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve([tan(x)*diff(y(x),x) = y(x),y(1/2*Pi) = 1/2*Pi],y(x), singsol=all)
```

$$y(x) = \frac{\pi \sin(x)}{2}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 12

```
DSolve[{Tan[x]*y'[x] == y[x],y[Pi/2]==Pi/2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}\pi \sin(x)$$

3.24 problem 25

Internal problem ID [50]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 25.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$-y + y'x - 2x^2y = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([-y(x)+x*diff(y(x),x) = 2*x^2*y(x),y(1) = 1],y(x), singsol=all)
```

$$y(x) = x e^{(x-1)(x+1)}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 14

```
DSolve[{-y[x]+x*y'[x] == 2*x^2*y[x],y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{x^2-1}x$$

3.25 problem 26

Internal problem ID [51]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 26.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - 2xy^2 - 3x^2y^2 = 0$$

With initial conditions

$$[y(1) = -1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(x),x) = 2*x*y(x)^2+3*x^2*y(x)^2,y(1) = -1],y(x), singsol=all)
```

$$y(x) = -\frac{1}{x^3 + x^2 - 1}$$

✓ Solution by Mathematica

Time used: 0.122 (sec). Leaf size: 17

```
DSolve[{y'[x] == 2*x*y[x]^2+3*x^2*y[x]^2,y[1]==-1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{1 - x^2(x + 1)}$$

3.26 problem 27

Internal problem ID [52]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 27.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - 6e^{2x-y} = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 13

```
dsolve([diff(y(x),x) = 6*exp(2*x-y(x)),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \ln(-2 + 3e^{2x})$$

✓ Solution by Mathematica

Time used: 0.717 (sec). Leaf size: 15

```
DSolve[{y'[x] == 6*Exp[2*x-y[x]],y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \log(3e^{2x} - 2)$$

3.27 problem 28

Internal problem ID [53]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.4. Separable equations. Page 43

Problem number: 28.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$2\sqrt{x}y' - \cos(y)^2 = 0$$

With initial conditions

$$\left[y(4) = \frac{\pi}{4} \right]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 10

```
dsolve([2*x^(1/2)*diff(y(x),x) = cos(y(x))^2,y(4) = 1/4*Pi],y(x), singsol=all)
```

$$y(x) = \arctan(-1 + \sqrt{x})$$

✓ Solution by Mathematica

Time used: 0.451 (sec). Leaf size: 17

```
DSolve[{2*x^(1/2)*y'[x] == Cos[y[x]]^2,y[4]==Pi/4},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\arctan(1 - \sqrt{x})$$

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4.1 problem 1

Internal problem ID [54]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y + y' - 2 = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([y(x)+diff(y(x),x) = 2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = 2 - 2e^{-x}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 14

```
DSolve[{y[x]+y'[x] == 2,y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2 - 2e^{-x}$$

4.2 problem 2

Internal problem ID [55]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$-2y + y' - 3e^{2x} = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([-2*y(x)+diff(y(x),x) = 3*exp(2*x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 3e^{2x}x$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 13

```
DSolve[{-2*y[x]+y'[x] == 3*Exp[2*x],y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3e^{2x}x$$

4.3 problem 3

Internal problem ID [56]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$3y + y' - 2xe^{-3x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(3*y(x)+diff(y(x),x) = 2*x/exp(3*x),y(x), singsol=all)
```

$$y(x) = (x^2 + c_1) e^{-3x}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 17

```
DSolve[3*y[x]+y'[x] == 2*x/Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-3x}(x^2 + c_1)$$

4.4 problem 4

Internal problem ID [57]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$-2yx + y' - e^{x^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(-2*x*y(x)+diff(y(x),x) = exp(x^2),y(x), singsol=all)
```

$$y(x) = (c_1 + x) e^{x^2}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 15

```
DSolve[-2*x*y[x]+y'[x] == Exp[x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{x^2}(x + c_1)$$

4.5 problem 5

Internal problem ID [58]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$2y + y'x - 3x = 0$$

With initial conditions

$$[y(1) = 5]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([2*y(x)+x*diff(y(x),x) = 3*x,y(1) = 5],y(x), singsol=all)
```

$$y(x) = x + \frac{4}{x^2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 12

```
DSolve[{2*y[x]+x*y'[x] == 3*x,y[1]==5},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{4}{x^2} + x$$

4.6 problem 6

Internal problem ID [59]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y + 2y'x - 10\sqrt{x} = 0$$

With initial conditions

$$[y(2) = 5]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([y(x)+2*x*diff(y(x),x) = 10*x^(1/2),y(2) = 5],y(x), singsol=all)
```

$$y(x) = \frac{-10 + 5\sqrt{2} + 5x}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 20

```
DSolve[{y[x]+2*x*y'[x]== 10*x^(1/2),y[2]==5},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{5(x + \sqrt{2} - 2)}{\sqrt{x}}$$

4.7 problem 7

Internal problem ID [60]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$y + 2y'x - 10\sqrt{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(y(x)+2*x*diff(y(x),x) = 10*x^(1/2),y(x), singsol=all)
```

$$y(x) = \frac{5x + c_1}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 17

```
DSolve[y[x]+2*x*y'[x] == 10*x^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{5x + c_1}{\sqrt{x}}$$

4.8 problem 8

Internal problem ID [61]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y + 3y'x - 12x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(y(x)+3*x*diff(y(x),x) = 12*x,y(x), singsol=all)
```

$$y(x) = 3x + \frac{c_1}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 17

```
DSolve[y[x]+3*x*y'[x] == 12*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3x + \frac{c_1}{\sqrt[3]{x}}$$

4.9 problem 9

Internal problem ID [62]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$-y + y'x - x = 0$$

With initial conditions

$$[y(1) = 7]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 10

```
dsolve([-y(x)+x*diff(y(x),x) = x,y(1) = 7],y(x), singsol=all)
```

$$y(x) = (\ln(x) + 7)x$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 11

```
DSolve[{-y[x]+x*y'[x]== x,y[1]==7},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(\log(x) + 7)$$

4.10 problem 10

Internal problem ID [63]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$-3y + 2y'x - 9x^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(-3*y(x)+2*x*diff(y(x),x) = 9*x^3,y(x), singsol=all)
```

$$y(x) = 3x^3 + x^{\frac{3}{2}}c_1$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 19

```
DSolve[-3*y[x]+2*x*y'[x] == 9*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3x^3 + c_1x^{3/2}$$

4.11 problem 11

Internal problem ID [64]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y + y'x - 3yx = 0$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([y(x)+x*diff(y(x),x) = 3*x*y(x),y(1) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y[x]+x*y'[x] == 3*x*y[x],y[1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0$$

4.12 problem 12

Internal problem ID [65]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$3y + y'x - 2x^5 = 0$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([3*y(x)+x*diff(y(x),x) = 2*x^5,y(2) = 1],y(x), singsol=all)
```

$$y(x) = \frac{x^8 - 224}{4x^3}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 17

```
DSolve[{3*y[x]+x*y'[x] == 2*x^5,y[2]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^8 - 224}{4x^3}$$

4.13 problem 13

Internal problem ID [66]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y + y' - e^x = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([y(x)+diff(y(x),x) = exp(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{e^x}{2} + \frac{e^{-x}}{2}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 7

```
DSolve[{y[x]+y'[x] == Exp[x],y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \cosh(x)$$

4.14 problem 14

Internal problem ID [67]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 14.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$-3y + y'x - x^3 = 0$$

With initial conditions

$$[y(1) = 10]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([-3*y(x)+x*diff(y(x),x) = x^3,y(1) = 10],y(x), singsol=all)
```

$$y(x) = (\ln(x) + 10)x^3$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 13

```
DSolve[{-3*y[x]+x*y'[x] == x^3,y[1]==10},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^3(\log(x) + 10)$$

4.15 problem 15

Internal problem ID [68]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 15.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$2yx + y' - x = 0$$

With initial conditions

$$[y(0) = -2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([2*x*y(x)+diff(y(x),x) = x,y(0) = -2],y(x), singsol=all)
```

$$y(x) = \frac{1}{2} - \frac{5e^{-x^2}}{2}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 20

```
DSolve[{2*x*y[x]+y'[x] == x,y[0]==-2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} - \frac{5e^{-x^2}}{2}$$

4.16 problem 16

Internal problem ID [69]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 16.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - \cos(x)(1 - y) = 0$$

With initial conditions

$$[y(\pi) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x) = cos(x)*(1-y(x)),y(Pi) = 2],y(x), singsol=all)
```

$$y(x) = 1 + e^{-\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 13

```
DSolve[{y'[x] == Cos[x]*(1-y[x]),y[Pi]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-\sin(x)} + 1$$

4.17 problem 17

Internal problem ID [70]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 17.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y + (x + 1)y' - \cos(x) = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([y(x)+(1+x)*diff(y(x),x) = cos(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\sin(x) + 1}{x + 1}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 15

```
DSolve[{y[x]+(1+x)*y'[x] == Cos[x],y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\sin(x) + 1}{x + 1}$$

4.18 problem 18

Internal problem ID [71]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 18.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y'x - x^3 \cos(x) - 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x) = x^3*cos(x)+2*y(x),y(x), singsol=all)
```

$$y(x) = (\sin(x) + c_1) x^2$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 14

```
DSolve[x*y'[x]== x^3*Cos[x]+2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2(\sin(x) + c_1)$$

4.19 problem 19

Internal problem ID [72]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 19.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\cot(x)y + y' - \cos(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(cot(x)*y(x)+diff(y(x),x) = cos(x),y(x), singsol=all)
```

$$y(x) = \frac{-\frac{\cos(2x)}{4} + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 19

```
DSolve[Cot[x]*y[x]+y'[x] == Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{2} \cos(x) \cot(x) + c_1 \csc(x)$$

4.20 problem 20

Internal problem ID [73]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 20.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - 1 - x - y - yx = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(y(x),x) = 1+x+y(x)+x*y(x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = -1 + e^{\frac{x(2+x)}{2}}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 17

```
DSolve[{y'[x]== 1+x+y[x]+x*y[x],y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{\frac{1}{2}x(x+2)} - 1$$

4.21 problem 21

Internal problem ID [74]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 21.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y'x - x^4 \cos(x) - 3y = 0$$

With initial conditions

$$[y(2\pi) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve([x*diff(y(x),x) = x^4*cos(x)+3*y(x),y(2*Pi) = 0],y(x), singsol=all)
```

$$y(x) = \sin(x) x^3$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 11

```
DSolve[{x*y'[x] == x^4*Cos[x]+3*y[x],y[2*Pi]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^3 \sin(x)$$

4.22 problem 22

Internal problem ID [75]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 22.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$y' - 3x^2e^{x^2} - 2yx = 0$$

With initial conditions

$$[y(0) = 5]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x) = 3*exp(x^2)*x^2+2*x*y(x),y(0) = 5],y(x), singsol=all)
```

$$y(x) = (x^3 + 5) e^{x^2}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 16

```
DSolve[{y'[x] == 3*Exp[x^2]*x^2+2*x*y[x],y[0]==5},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{x^2} (x^3 + 5)$$

4.23 problem 23

Internal problem ID [76]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 23.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$(-3 + 2x)y + y'x - 4x^4 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((-3+2*x)*y(x)+x*diff(y(x),x) = 4*x^4,y(x), singsol=all)
```

$$y(x) = 2x^3 + e^{-2x}c_1x^3$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 19

```
DSolve[(-3+2*x)*y[x]+x*y'[x] == 4*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^3(2 + c_1e^{-2x})$$

4.24 problem 24

Internal problem ID [77]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 24.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$3yx + (x^2 + 4)y' - x = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([3*x*y(x)+(x^2+4)*diff(y(x),x) = x,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{3} + \frac{16}{3(x^2 + 4)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 22

```
DSolve[{3*x*y[x]+(x^2+4)*y'[x] == x,y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{16}{3(x^2 + 4)^{3/2}} + \frac{1}{3}$$

4.25 problem 25

Internal problem ID [78]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.5. Linear first order equations. Page 56

Problem number: 25.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$3x^3y + (x^2 + 1)y' - 6xe^{-\frac{3x^2}{2}} = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve([3*x^3*y(x)+(x^2+1)*diff(y(x),x) = 6*x/exp(3/2*x^2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \left(3x^2\sqrt{x^2+1} + 3\sqrt{x^2+1} - 2\right)e^{-\frac{3x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 28

```
DSolve[{3*x^3*y[x]+(x^2+1)*y'[x] == 6*x/Exp[3/2*x^2],y[0]==1},y[x],x,IncludeSingularSolutions
```

$$y(x) \rightarrow e^{-\frac{3x^2}{2}} \left(3(x^2+1)^{3/2} - 2\right)$$

5 Section 1.6, Substitution methods and exact equations. Page 74

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5.1 problem 1

Internal problem ID [79]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _exact, _rational, [_Abel, '2nd typ`

$$(x + y)y' - x + y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
dsolve((x+y(x))*diff(y(x),x) = x-y(x),y(x), singsol=all)
```

$$y(x) = \frac{-c_1x - \sqrt{2c_1^2x^2 + 1}}{c_1}$$

$$y(x) = \frac{-c_1x + \sqrt{2c_1^2x^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.418 (sec). Leaf size: 94

```
DSolve[(x+y[x])*y'[x]== x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x - \sqrt{2x^2 + e^{2c_1}}$$

$$y(x) \rightarrow -x + \sqrt{2x^2 + e^{2c_1}}$$

$$y(x) \rightarrow -\sqrt{2}\sqrt{x^2} - x$$

$$y(x) \rightarrow \sqrt{2}\sqrt{x^2} - x$$

5.2 problem 2

Internal problem ID [80]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$2xyy' - x^2 - y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(2*x*y(x)*diff(y(x),x) = x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1x + x^2}$$

$$y(x) = -\sqrt{c_1x + x^2}$$

✓ Solution by Mathematica

Time used: 0.169 (sec). Leaf size: 38

```
DSolve[2*x*y[x]*y'[x] == x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt{x}\sqrt{x + c_1}$$

$$y(x) \rightarrow \sqrt{x}\sqrt{x + c_1}$$

5.3 problem 3

Internal problem ID [81]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A'], _dAlembert]`

$$y'x - y - 2\sqrt{yx} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x) = y(x)+2*(x*y(x))^(1/2),y(x), singsol=all)
```

$$-\frac{y(x)}{\sqrt{xy(x)}} + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.162 (sec). Leaf size: 19

```
DSolve[x*y'[x] == y[x]+2*(x*y[x])^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}x(2\log(x) + c_1)^2$$

5.4 problem 4

Internal problem ID [82]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cla`

$$(x - y)y' - x - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve((x-y(x))*diff(y(x),x) = x+y(x),y(x), singsol=all)
```

$$y(x) = \tan \left(\text{RootOf} \left(-2_Z + \ln \left(\frac{1}{\cos(_Z)^2} \right) + 2 \ln(x) + 2c_1 \right) \right) x$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 36

```
DSolve[(x-y[x])*y'[x] == x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[\frac{1}{2} \log \left(\frac{y(x)^2}{x^2} + 1 \right) - \arctan \left(\frac{y(x)}{x} \right) = -\log(x) + c_1, y(x) \right]$$

5.5 problem 5

Internal problem ID [83]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cla`

$$x(x+y)y' - y(x-y) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*(x+y(x))*diff(y(x),x) = y(x)*(x-y(x)),y(x), singsol=all)
```

$$y(x) = \frac{x}{\text{LambertW}(c_1 x^2)}$$

✓ Solution by Mathematica

Time used: 4.131 (sec). Leaf size: 25

```
DSolve[x*(x+y[x])*y'[x] == y[x]*(x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x}{W(e^{-c_1} x^2)}$$

$$y(x) \rightarrow 0$$

5.6 problem 6

Internal problem ID [84]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cla`

$$(x + 2y)y' - y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve((x+2*y(x))*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x) = e^{\text{LambertW}\left(\frac{x e^{\frac{c_1}{2}}}{2}\right) - \frac{c_1}{2}}$$

✓ Solution by Mathematica

Time used: 4.567 (sec). Leaf size: 31

```
DSolve[(x+2*y[x])*y'[x] == y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x}{2W\left(\frac{1}{2}e^{-\frac{c_1}{2}}x\right)}$$

$$y(x) \rightarrow 0$$

5.7 problem 7

Internal problem ID [85]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$y^2 y' x - x^3 - y^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 74

```
dsolve(x*y(x)^2*diff(y(x),x) = x^3+y(x)^3,y(x), singsol=all)
```

$$y(x) = (3 \ln(x) + c_1)^{\frac{1}{3}} x$$

$$y(x) = \left(-\frac{(3 \ln(x) + c_1)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}(3 \ln(x) + c_1)^{\frac{1}{3}}}{2} \right) x$$

$$y(x) = \left(-\frac{(3 \ln(x) + c_1)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}(3 \ln(x) + c_1)^{\frac{1}{3}}}{2} \right) x$$

✓ Solution by Mathematica

Time used: 0.177 (sec). Leaf size: 63

```
DSolve[x*y[x]^2*y'[x] == x^3+y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x \sqrt[3]{3 \log(x) + c_1}$$

$$y(x) \rightarrow -\sqrt[3]{-1} x \sqrt[3]{3 \log(x) + c_1}$$

$$y(x) \rightarrow (-1)^{2/3} x \sqrt[3]{3 \log(x) + c_1}$$

5.8 problem 8

Internal problem ID [86]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A'], _dAlembert]`

$$y'x^2 - e^{\frac{y}{x}}x^2 - yx = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x) = exp(y(x)/x)*x^2+x*y(x),y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{1}{\ln(x) + c_1}\right)x$$

✓ Solution by Mathematica

Time used: 0.302 (sec). Leaf size: 18

```
DSolve[x^2*y'[x] == Exp[y[x]/x]*x^2+x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x \log(-\log(x) - c_1)$$

5.9 problem 9

Internal problem ID [87]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$y'x^2 - yx - y^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x) = x*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{x}{\ln(x) - c_1}$$

✓ Solution by Mathematica

Time used: 0.123 (sec). Leaf size: 21

```
DSolve[x^2*y'[x] == x*y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x}{-\log(x) + c_1}$$

$$y(x) \rightarrow 0$$

5.10 problem 10

Internal problem ID [88]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$xyy' - x^2 - 3y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x*y(x)*diff(y(x),x) = x^2+3*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{4c_1x^4 - 2}x}{2}$$

$$y(x) = \frac{\sqrt{4c_1x^4 - 2}x}{2}$$

✓ Solution by Mathematica

Time used: 0.487 (sec). Leaf size: 42

```
DSolve[x*y[x]*y'[x] == x^2+3*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x\sqrt{-\frac{1}{2} + c_1x^4}$$

$$y(x) \rightarrow x\sqrt{-\frac{1}{2} + c_1x^4}$$

5.11 problem 11

Internal problem ID [89]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$(x^2 - y^2) y' - 2yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve((x^2-y(x)^2)*diff(y(x),x) = 2*x*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{-1 + \sqrt{-4c_1^2 x^2 + 1}}{2c_1}$$

$$y(x) = \frac{1 + \sqrt{-4c_1^2 x^2 + 1}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.954 (sec). Leaf size: 66

```
DSolve[(x^2-y[x]^2)*y'[x]== 2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} \left(e^{c_1} - \sqrt{-4x^2 + e^{2c_1}} \right)$$

$$y(x) \rightarrow \frac{1}{2} \left(\sqrt{-4x^2 + e^{2c_1}} + e^{c_1} \right)$$

$$y(x) \rightarrow 0$$

5.12 problem 12

Internal problem ID [90]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$xyy' - y^2 - x\sqrt{4x^2 + y^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(x*y(x)*diff(y(x),x) = y(x)^2+x*(4*x^2+y(x)^2)^(1/2),y(x), singsol=all)
```

$$-\frac{\sqrt{4x^2 + y(x)^2}}{x} + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.265 (sec). Leaf size: 48

```
DSolve[x*y[x]*y'[x] == y[x]^2+x*(4*x^2+y[x]^2)^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x\sqrt{(\log(x) - 2 + c_1)(\log(x) + 2 + c_1)}$$

$$y(x) \rightarrow x\sqrt{(\log(x) - 2 + c_1)(\log(x) + 2 + c_1)}$$

5.13 problem 13

Internal problem ID [91]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _dAlembert]`

$$y'x - y - \sqrt{x^2 + y^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x) = y(x)+(x^2+y(x)^2)^(1/2),y(x), singsol=all)
```

$$\frac{y(x)}{x^2} + \frac{\sqrt{x^2 + y(x)^2}}{x^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.332 (sec). Leaf size: 27

```
DSolve[x*y'[x] == y[x]+(x^2+y[x]^2)^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}e^{-c_1}(-1 + e^{2c_1}x^2)$$

5.14 problem 14

Internal problem ID [92]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 14.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$x + yy' - \sqrt{x^2 + y^2} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 28

```
dsolve(x+y(x)*diff(y(x),x) = (x^2+y(x)^2)^(1/2),y(x), singsol=all)
```

$$-c_1 + \frac{\sqrt{x^2 + y(x)^2}}{y(x)^2} + \frac{x}{y(x)^2} = 0$$

✓ Solution by Mathematica

Time used: 0.377 (sec). Leaf size: 57

```
DSolve[x+y[x]*y'[x] == (x^2+y[x]^2)^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -e^{\frac{c_1}{2}} \sqrt{2x + e^{c_1}}$$

$$y(x) \rightarrow e^{\frac{c_1}{2}} \sqrt{2x + e^{c_1}}$$

$$y(x) \rightarrow 0$$

5.15 problem 15

Internal problem ID [93]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 15.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cla`

$$y(3x + y) + x(x + y)y' = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 59

```
dsolve(y(x)*(3*x+y(x))+x*(x+y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-c_1x^2 - \sqrt{c_1^2x^4 + 1}}{c_1x}$$

$$y(x) = \frac{-c_1x^2 + \sqrt{c_1^2x^4 + 1}}{c_1x}$$

✓ Solution by Mathematica

Time used: 0.591 (sec). Leaf size: 93

```
DSolve[y[x]*(3*x+y[x])+x*(x+y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x^2 + \sqrt{x^4 + e^{2c_1}}}{x}$$

$$y(x) \rightarrow -x + \frac{\sqrt{x^4 + e^{2c_1}}}{x}$$

$$y(x) \rightarrow -\frac{\sqrt{x^4 + x^2}}{x}$$

$$y(x) \rightarrow \frac{\sqrt{x^4}}{x} - x$$

5.16 problem 16

Internal problem ID [94]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 16.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$y' - \sqrt{1+x+y} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = (1+x+y(x))^(1/2),y(x), singsol=all)
```

$$x - 2\sqrt{1+x+y(x)} - \ln\left(-1 + \sqrt{1+x+y(x)}\right) + \ln\left(1 + \sqrt{1+x+y(x)}\right) + \ln(x+y(x)) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 9.01 (sec). Leaf size: 55

```
DSolve[y'[x] == (1+x+y[x])^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x + W\left(-e^{\frac{1}{2}(-x-3-c_1)}\right) \left(2 + W\left(-e^{\frac{1}{2}(-x-3-c_1)}\right)\right)$$

$$y(x) \rightarrow -x$$

5.17 problem 17

Internal problem ID [95]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 17.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _Riccati]`

$$y' - (4x + y)^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x) = (4*x+y(x))^2,y(x), singsol=all)
```

$$y(x) = -4x - 2 \tan(-2x + 2c_1)$$

✓ Solution by Mathematica

Time used: 0.126 (sec). Leaf size: 41

```
DSolve[y'[x] == (4*x+y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -4x + \frac{1}{c_1 e^{4ix} - \frac{i}{4}} - 2i$$

$$y(x) \rightarrow -4x - 2i$$

5.18 problem 18

Internal problem ID [96]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 18.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$(x + y)y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve((x+y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = -x$$

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 14

```
DSolve[(x+y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x$$

$$y(x) \rightarrow c_1$$

5.19 problem 19

Internal problem ID [97]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 19.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$2yx + y'x^2 - 5y^3 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(2*x*y(x)+x^2*diff(y(x),x) = 5*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(c_1x^5 + 2)}x}{c_1x^5 + 2}$$

$$y(x) = -\frac{\sqrt{(c_1x^5 + 2)}x}{c_1x^5 + 2}$$

✓ Solution by Mathematica

Time used: 0.377 (sec). Leaf size: 51

```
DSolve[2*x*y[x]+x^2*y'[x] == 5*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt{x}}{\sqrt{2 + c_1x^5}}$$

$$y(x) \rightarrow \frac{\sqrt{x}}{\sqrt{2 + c_1x^5}}$$

$$y(x) \rightarrow 0$$

5.20 problem 20

Internal problem ID [98]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 20.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$2xy^3 + y^2y' - 6x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 88

```
dsolve(2*x*y(x)^3+y(x)^2*diff(y(x),x) = 6*x,y(x), singsol=all)
```

$$y(x) = \left(e^{-3x^2}c_1 + 3\right)^{\frac{1}{3}}$$

$$y(x) = -\frac{\left(e^{-3x^2}c_1 + 3\right)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\left(e^{-3x^2}c_1 + 3\right)^{\frac{1}{3}}}{2}$$

$$y(x) = -\frac{\left(e^{-3x^2}c_1 + 3\right)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\left(e^{-3x^2}c_1 + 3\right)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 1.91 (sec). Leaf size: 115

```
DSolve[2*x*y[x]^3+y[x]^2*y'[x] == 6*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt[3]{3 + e^{-3x^2+3c_1}}$$

$$y(x) \rightarrow -\sqrt[3]{-1} \sqrt[3]{3 + e^{-3x^2+3c_1}}$$

$$y(x) \rightarrow (-1)^{2/3} \sqrt[3]{3 + e^{-3x^2+3c_1}}$$

$$y(x) \rightarrow -\sqrt[3]{-3}$$

$$y(x) \rightarrow \sqrt[3]{3}$$

$$y(x) \rightarrow (-1)^{2/3} \sqrt[3]{3}$$

5.21 problem 21

Internal problem ID [99]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 21.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - y - y^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = y(x)+y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{e^{-2x}c_1 - 1}}$$

$$y(x) = -\frac{1}{\sqrt{e^{-2x}c_1 - 1}}$$

✓ Solution by Mathematica

Time used: 60.062 (sec). Leaf size: 57

```
DSolve[y'[x] == y[x]+y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{ie^{x+c_1}}{\sqrt{-1 + e^{2(x+c_1)}}}$$

$$y(x) \rightarrow \frac{ie^{x+c_1}}{\sqrt{-1 + e^{2(x+c_1)}}}$$

5.22 problem 22

Internal problem ID [100]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 22.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$2yx + y'x^2 - 5y^4 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 164

```
dsolve(2*x*y(x)+x^2*diff(y(x),x) = 5*y(x)^4,y(x), singsol=all)
```

$$y(x) = \frac{7^{\frac{1}{3}} \left(x(7c_1x^7 + 15)^2 \right)^{\frac{1}{3}}}{7c_1x^7 + 15}$$

$$y(x) = -\frac{7^{\frac{1}{3}} \left(x(7c_1x^7 + 15)^2 \right)^{\frac{1}{3}}}{2(7c_1x^7 + 15)} - \frac{i\sqrt{3} 7^{\frac{1}{3}} \left(x(7c_1x^7 + 15)^2 \right)^{\frac{1}{3}}}{2(7c_1x^7 + 15)}$$

$$y(x) = -\frac{7^{\frac{1}{3}} \left(x(7c_1x^7 + 15)^2 \right)^{\frac{1}{3}}}{2(7c_1x^7 + 15)} + \frac{i\sqrt{3} 7^{\frac{1}{3}} \left(x(7c_1x^7 + 15)^2 \right)^{\frac{1}{3}}}{14c_1x^7 + 30}$$

✓ Solution by Mathematica

Time used: 0.422 (sec). Leaf size: 96

```
DSolve[2*x*y[x]+x^2*y'[x] == 5*y[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt[3]{-7}\sqrt[3]{x}}{\sqrt[3]{15+7c_1x^7}}$$

$$y(x) \rightarrow \frac{\sqrt[3]{7}\sqrt[3]{x}}{\sqrt[3]{15+7c_1x^7}}$$

$$y(x) \rightarrow \frac{(-1)^{2/3}\sqrt[3]{7}\sqrt[3]{x}}{\sqrt[3]{15+7c_1x^7}}$$

$$y(x) \rightarrow 0$$

5.23 problem 23

Internal problem ID [101]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 23.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$6y + y'x - 3xy^{\frac{4}{3}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(6*y(x)+x*diff(y(x),x) = 3*x*y(x)^(4/3),y(x), singsol=all)
```

$$\frac{1}{y(x)^{\frac{1}{3}}} - x - c_1x^2 = 0$$

✓ Solution by Mathematica

Time used: 0.147 (sec). Leaf size: 22

```
DSolve[6*y[x]+x*y'[x] == 3*x*y[x]^(4/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{x^3(1 + c_1x)^3}$$

$$y(x) \rightarrow 0$$

5.24 problem 24

Internal problem ID [102]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 24.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_Bernoulli]

$$y^3 e^{-2x} + 2y'x - 2yx = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
dsolve(y(x)^3/exp(2*x)+2*x*diff(y(x),x) = 2*x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(\ln(x) + c_1) e^{2x}}}{\ln(x) + c_1}$$

$$y(x) = -\frac{\sqrt{(\ln(x) + c_1) e^{2x}}}{\ln(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.334 (sec). Leaf size: 41

```
DSolve[y[x]^3/Exp[2*x]+2*x*y'[x] == 2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{e^x}{\sqrt{\log(x) + c_1}}$$

$$y(x) \rightarrow \frac{e^x}{\sqrt{\log(x) + c_1}}$$

$$y(x) \rightarrow 0$$

5.25 problem 25

Internal problem ID [103]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 25.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_Bernoulli]

$$\sqrt{x^4 + 1} y^2 (y + y'x) - x = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 135

```
dsolve((x^4+1)^(1/2)*y(x)^2*(y(x)+x*diff(y(x),x)) = x,y(x), singsol=all)
```

$$y(x) = \frac{\left(3 \left(\int \frac{x^3}{\sqrt{x^4+1}} dx\right) + c_1\right)^{\frac{1}{3}}}{x}$$

$$y(x) = -\frac{\left(3 \left(\int \frac{x^3}{\sqrt{x^4+1}} dx\right) + c_1\right)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3} \left(3 \left(\int \frac{x^3}{\sqrt{x^4+1}} dx\right) + c_1\right)^{\frac{1}{3}}}{2x}$$

$$y(x) = -\frac{\left(3 \left(\int \frac{x^3}{\sqrt{x^4+1}} dx\right) + c_1\right)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3} \left(3 \left(\int \frac{x^3}{\sqrt{x^4+1}} dx\right) + c_1\right)^{\frac{1}{3}}}{2x}$$

✓ Solution by Mathematica

Time used: 3.913 (sec). Leaf size: 106

`DSolve[(x^4+1)^(1/2)*y[x]^2*(y[x]+x*y'[x]) ==x,y[x],x,IncludeSingularSolutions -> True]`

$$y(x) \rightarrow \sqrt[3]{\frac{3\sqrt{x^4+1}}{2x^3} + \frac{c_1}{x^3}}$$

$$y(x) \rightarrow -\sqrt[3]{-\frac{1}{2}} \sqrt[3]{\frac{3\sqrt{x^4+1} + 2c_1}{x^3}}$$

$$y(x) \rightarrow (-1)^{2/3} \sqrt[3]{\frac{3\sqrt{x^4+1}}{2x^3} + \frac{c_1}{x^3}}$$

5.26 problem 26

Internal problem ID [104]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 26.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_1st_order, _with_linear_symmetries], _Bernoulli]`

$$y^3 + 3y^2y' - e^{-x} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 99

```
dsolve(y(x)^3+3*y(x)^2*diff(y(x),x) = exp(-x),y(x), singsol=all)
```

$$y(x) = e^{-x}((c_1 + x)e^{2x})^{\frac{1}{3}}$$

$$y(x) = -\frac{e^{-x}((c_1 + x)e^{2x})^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}e^{-x}((c_1 + x)e^{2x})^{\frac{1}{3}}}{2}$$

$$y(x) = -\frac{e^{-x}((c_1 + x)e^{2x})^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}e^{-x}((c_1 + x)e^{2x})^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 0.305 (sec). Leaf size: 72

```
DSolve[y[x]^3+3*y[x]^2*y'[x] == Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x/3} \sqrt[3]{x + c_1}$$

$$y(x) \rightarrow -\sqrt[3]{-1} e^{-x/3} \sqrt[3]{x + c_1}$$

$$y(x) \rightarrow (-1)^{2/3} e^{-x/3} \sqrt[3]{x + c_1}$$

5.27 problem 27

Internal problem ID [105]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 27.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$3y^2y'x - 3x^4 - y^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 73

```
dsolve(3*x*y(x)^2*diff(y(x),x) = 3*x^4+y(x)^3,y(x), singsol=all)
```

$$y(x) = (x^4 + c_1x)^{\frac{1}{3}}$$

$$y(x) = -\frac{(x^4 + c_1x)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}(x^4 + c_1x)^{\frac{1}{3}}}{2}$$

$$y(x) = -\frac{(x^4 + c_1x)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}(x^4 + c_1x)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 0.198 (sec). Leaf size: 72

```
DSolve[3*x*y[x]^2*y'[x] == 3*x^4+y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt[3]{x} \sqrt[3]{x^3 + c_1}$$

$$y(x) \rightarrow -\sqrt[3]{-1} \sqrt[3]{x} \sqrt[3]{x^3 + c_1}$$

$$y(x) \rightarrow (-1)^{2/3} \sqrt[3]{x} \sqrt[3]{x^3 + c_1}$$

5.28 problem 28

Internal problem ID [106]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 28.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$e^y x y' - 2 e^y - 2 e^{2x} x^3 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(exp(y(x))*x*diff(y(x),x) = 2*exp(y(x))+2*exp(2*x)*x^3,y(x), singsol=all)
```

$$y(x) = \ln(e^{2x} x^2 - c_1 x^2)$$

✓ Solution by Mathematica

Time used: 4.23 (sec). Leaf size: 18

```
DSolve[Exp[y[x]]*x*y'[x] == 2*Exp[y[x]]+2*Exp[2*x]*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \log(x^2(e^{2x} + c_1))$$

5.29 problem 29

Internal problem ID [107]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 29.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [$y = G(x, y')$]

$$2x \cos(y) \sin(y) y' - 4x^2 - \sin(y)^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve(2*x*cos(y(x))*sin(y(x))*diff(y(x),x) = 4*x^2+sin(y(x))^2,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\sqrt{-c_1x + 4x^2}\right)$$

$$y(x) = -\arcsin\left(\sqrt{-c_1x + 4x^2}\right)$$

✓ Solution by Mathematica

Time used: 6.095 (sec). Leaf size: 41

```
DSolve[2*x*Cos[y[x]]*Sin[y[x]]*y'[x] == 4*x^2+Sin[y[x]]^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow -\arcsin\left(2\sqrt{x(x + 2c_1)}\right)$$

$$y(x) \rightarrow \arcsin\left(2\sqrt{x(x + 2c_1)}\right)$$

5.30 problem 30

Internal problem ID [108]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 30.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$(e^y + x)y' + 1 - xe^{-y} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve((exp(y(x))+x)*diff(y(x),x) = -1+x/exp(y(x)),y(x), singsol=all)
```

$$y(x) = \ln\left(-x - \sqrt{2x^2 + c_1}\right)$$

$$y(x) = \ln\left(-x + \sqrt{2x^2 + c_1}\right)$$

✓ Solution by Mathematica

Time used: 2.653 (sec). Leaf size: 52

```
DSolve[(Exp[y[x]]+x)*y'[x]== -1+x/Exp[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \log\left(-x - \sqrt{2}\sqrt{x^2 + c_1}\right)$$

$$y(x) \rightarrow \log\left(-x + \sqrt{2}\sqrt{x^2 + c_1}\right)$$

5.31 problem 31

Internal problem ID [109]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 31.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, [_Abel, '2nd typ`

$$2x + 3y + (3x + 2y)y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 53

```
dsolve(2*x+3*y(x)+(3*x+2*y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-\frac{3c_1x}{2} - \frac{\sqrt{5c_1^2x^2+4}}{2}}{c_1}$$

$$y(x) = \frac{-\frac{3c_1x}{2} + \frac{\sqrt{5c_1^2x^2+4}}{2}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.411 (sec). Leaf size: 110

```
DSolve[2*x+3*y[x]+(3*x+2*y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} \left(-3x - \sqrt{5x^2 + 4e^{c_1}} \right)$$

$$y(x) \rightarrow \frac{1}{2} \left(-3x + \sqrt{5x^2 + 4e^{c_1}} \right)$$

$$y(x) \rightarrow \frac{1}{2} \left(-\sqrt{5}\sqrt{x^2} - 3x \right)$$

$$y(x) \rightarrow \frac{1}{2} \left(\sqrt{5}\sqrt{x^2} - 3x \right)$$

5.32 problem 32

Internal problem ID [110]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 32.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _exact, _rational, [_Abel, '2nd typ`

$$4x - y + (-x + 6y)y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve(4*x-y(x)+(-x+6*y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\frac{c_1 x}{6} - \frac{\sqrt{-23c_1^2 x^2 + 12}}{6}}{c_1}$$

$$y(x) = \frac{\frac{c_1 x}{6} + \frac{\sqrt{-23c_1^2 x^2 + 12}}{6}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.41 (sec). Leaf size: 106

```
DSolve[4*x-y[x]+(-x+6*y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{6} \left(x - \sqrt{-23x^2 + 12e^{c_1}} \right)$$

$$y(x) \rightarrow \frac{1}{6} \left(x + \sqrt{-23x^2 + 12e^{c_1}} \right)$$

$$y(x) \rightarrow \frac{1}{6} \left(x - \sqrt{23}\sqrt{-x^2} \right)$$

$$y(x) \rightarrow \frac{1}{6} \left(\sqrt{23}\sqrt{-x^2} + x \right)$$

5.33 problem 33

Internal problem ID [111]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 33.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _exact, _rational, _dAlembert]`

$$3x^2 + 2y^2 + (4yx + 6y^2) y' = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 441

```
dsolve(3*x^2+2*y(x)^2+(4*x*y(x)+6*y(x)^2)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{54-62x^3c_1^3+6\sqrt{105c_1^6x^6-186x^3c_1^3+81}}{6}\right)^{\frac{1}{3}} + \frac{2x^2c_1^2}{3\left(\frac{54-62x^3c_1^3+6\sqrt{105c_1^6x^6-186x^3c_1^3+81}}{6}\right)^{\frac{1}{3}}} - \frac{c_1x}{3}}{c_1}$$

$$y(x) = \frac{\left(\frac{54-62x^3c_1^3+6\sqrt{105c_1^6x^6-186x^3c_1^3+81}}{12}\right)^{\frac{1}{3}} - \frac{x^2c_1^2}{3\left(\frac{54-62x^3c_1^3+6\sqrt{105c_1^6x^6-186x^3c_1^3+81}}{6}\right)^{\frac{1}{3}}} - \frac{c_1x}{3} - i\sqrt{3}\left(\frac{54-62x^3c_1^3+6\sqrt{105c_1^6x^6-186x^3c_1^3+81}}{6}\right)^{\frac{1}{3}}}{c_1}$$

$$y(x) = \frac{\left(\frac{54-62x^3c_1^3+6\sqrt{105c_1^6x^6-186x^3c_1^3+81}}{12}\right)^{\frac{1}{3}} - \frac{x^2c_1^2}{3\left(\frac{54-62x^3c_1^3+6\sqrt{105c_1^6x^6-186x^3c_1^3+81}}{6}\right)^{\frac{1}{3}}} - \frac{c_1x}{3} + i\sqrt{3}\left(\frac{54-62x^3c_1^3+6\sqrt{105c_1^6x^6-186x^3c_1^3+81}}{6}\right)^{\frac{1}{3}}}{c_1}$$

✓ Solution by Mathematica

Time used: 39.372 (sec). Leaf size: 679

`DSolve[3*x^2+2*y[x]^2+(4*x*y[x]+6*y[x]^2)*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]`

$$y(x) \rightarrow \frac{\sqrt[3]{-124x^3 + \sqrt{-256x^6 + (-124x^3 + 108e^{2c_1})^2} + 108e^{2c_1}}}{6\sqrt[3]{2}} + \frac{2\sqrt[3]{2}x^2}{3\sqrt[3]{-124x^3 + \sqrt{-256x^6 + (-124x^3 + 108e^{2c_1})^2} + 108e^{2c_1}}} - \frac{x}{3}$$

$$y(x) \rightarrow \frac{1}{12}i(\sqrt{3} + i) \sqrt[3]{-62x^3 + 6\sqrt{3}\sqrt{35x^6 - 62e^{2c_1}x^3 + 27e^{4c_1}} + 54e^{2c_1}} - \frac{i(\sqrt{3} - i)x^2}{3\sqrt[3]{-62x^3 + 6\sqrt{3}\sqrt{35x^6 - 62e^{2c_1}x^3 + 27e^{4c_1}} + 54e^{2c_1}}} - \frac{x}{3}$$

$$y(x) \rightarrow -\frac{1}{12}i(\sqrt{3} - i) \sqrt[3]{-62x^3 + 6\sqrt{3}\sqrt{35x^6 - 62e^{2c_1}x^3 + 27e^{4c_1}} + 54e^{2c_1}} + \frac{i(\sqrt{3} + i)x^2}{3\sqrt[3]{-62x^3 + 6\sqrt{3}\sqrt{35x^6 - 62e^{2c_1}x^3 + 27e^{4c_1}} + 54e^{2c_1}}} - \frac{x}{3}$$

$$y(x) \rightarrow \frac{1}{6} \left(\sqrt[3]{6\sqrt{105}\sqrt{x^6} - 62x^3} + \frac{2 \cdot 2^{2/3}x^2}{\sqrt[3]{3\sqrt{105}\sqrt{x^6} - 31x^3}} - 2x \right)$$

$$y(x) \rightarrow \frac{1}{12} \left(i(\sqrt{3} + i) \sqrt[3]{6\sqrt{105}\sqrt{x^6} - 62x^3} - \frac{2i2^{2/3}(\sqrt{3} - i)x^2}{\sqrt[3]{3\sqrt{105}\sqrt{x^6} - 31x^3}} - 4x \right)$$

$$y(x) \rightarrow \frac{1}{12} \left((-1 - i\sqrt{3}) \sqrt[3]{6\sqrt{105}\sqrt{x^6} - 62x^3} + \frac{2i2^{2/3}(\sqrt{3} + i)x^2}{\sqrt[3]{3\sqrt{105}\sqrt{x^6} - 31x^3}} - 4x \right)$$

5.34 problem 34

Internal problem ID [112]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 34.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_exact`, `_rational`]

$$3x^2 + 2xy^2 + (2x^2y + 4y^3) y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 117

```
dsolve(3*x^2+2*x*y(x)^2+(2*x^2*y(x)+4*y(x)^3)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2x^2 - 2\sqrt{x^4 - 4x^3 - 4c_1}}}{2}$$

$$y(x) = \frac{\sqrt{-2x^2 - 2\sqrt{x^4 - 4x^3 - 4c_1}}}{2}$$

$$y(x) = -\frac{\sqrt{-2x^2 + 2\sqrt{x^4 - 4x^3 - 4c_1}}}{2}$$

$$y(x) = \frac{\sqrt{-2x^2 + 2\sqrt{x^4 - 4x^3 - 4c_1}}}{2}$$

✓ Solution by Mathematica

Time used: 5.861 (sec). Leaf size: 151

`DSolve[3*x^2+2*x*y[x]^2+(2*x^2*y[x]+4*y[x]^3)*y'[x]== 0,y[x],x,IncludeSingularSolutions -> Tr`

$$y(x) \rightarrow -\frac{\sqrt{-x^2 - \sqrt{(x-4)x^3 + 4c_1}}}{\sqrt{2}}$$

$$y(x) \rightarrow \frac{\sqrt{-x^2 - \sqrt{(x-4)x^3 + 4c_1}}}{\sqrt{2}}$$

$$y(x) \rightarrow -\frac{\sqrt{-x^2 + \sqrt{(x-4)x^3 + 4c_1}}}{\sqrt{2}}$$

$$y(x) \rightarrow \frac{\sqrt{-x^2 + \sqrt{(x-4)x^3 + 4c_1}}}{\sqrt{2}}$$

5.35 problem 35

Internal problem ID [113]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 35.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_exact`]

$$x^3 + \frac{y}{x} + (\ln(x) + y^2) y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 415

```
dsolve(x^3+y(x)/x+(ln(x)+y(x)^2)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$\begin{aligned}
 y(x) &= \frac{\left(-3x^4 - 12c_1 + \sqrt{64 \ln(x)^3 + 9x^8 + 72x^4c_1 + 144c_1^2}\right)^{\frac{1}{3}}}{2 \ln(x)} \\
 &\quad - \frac{\left(-3x^4 - 12c_1 + \sqrt{64 \ln(x)^3 + 9x^8 + 72x^4c_1 + 144c_1^2}\right)^{\frac{1}{3}}}{2 \ln(x)} \\
 y(x) &= - \frac{\left(-3x^4 - 12c_1 + \sqrt{64 \ln(x)^3 + 9x^8 + 72x^4c_1 + 144c_1^2}\right)^{\frac{1}{3}}}{4 \ln(x)} \\
 &\quad + \frac{\left(-3x^4 - 12c_1 + \sqrt{64 \ln(x)^3 + 9x^8 + 72x^4c_1 + 144c_1^2}\right)^{\frac{1}{3}}}{4 \ln(x)} \\
 &\quad + i\sqrt{3} \left(\frac{\left(-3x^4 - 12c_1 + \sqrt{64 \ln(x)^3 + 9x^8 + 72x^4c_1 + 144c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{2 \ln(x)}{\left(-3x^4 - 12c_1 + \sqrt{64 \ln(x)^3 + 9x^8 + 72x^4c_1 + 144c_1^2}\right)^{\frac{1}{3}}} \right) \\
 &\quad - \frac{\left(-3x^4 - 12c_1 + \sqrt{64 \ln(x)^3 + 9x^8 + 72x^4c_1 + 144c_1^2}\right)^{\frac{1}{3}}}{2} \\
 y(x) &= - \frac{\left(-3x^4 - 12c_1 + \sqrt{64 \ln(x)^3 + 9x^8 + 72x^4c_1 + 144c_1^2}\right)^{\frac{1}{3}}}{4 \ln(x)} \\
 &\quad + \frac{\left(-3x^4 - 12c_1 + \sqrt{64 \ln(x)^3 + 9x^8 + 72x^4c_1 + 144c_1^2}\right)^{\frac{1}{3}}}{4 \ln(x)} \\
 &\quad + i\sqrt{3} \left(\frac{\left(-3x^4 - 12c_1 + \sqrt{64 \ln(x)^3 + 9x^8 + 72x^4c_1 + 144c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{2 \ln(x)}{\left(-3x^4 - 12c_1 + \sqrt{64 \ln(x)^3 + 9x^8 + 72x^4c_1 + 144c_1^2}\right)^{\frac{1}{3}}} \right) \\
 &\quad + \frac{\left(-3x^4 - 12c_1 + \sqrt{64 \ln(x)^3 + 9x^8 + 72x^4c_1 + 144c_1^2}\right)^{\frac{1}{3}}}{2}
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 1.675 (sec). Leaf size: 307

`DSolve[x^3+y[x]/x+(Log[x]+y[x]^2)*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]`

$$y(x) \rightarrow \frac{-4 \log(x) + \left(-3x^4 + \sqrt{64 \log^3(x) + 9(x^4 - 4c_1)^2 + 12c_1}\right)^{2/3}}{2 \sqrt[3]{-3x^4 + \sqrt{64 \log^3(x) + 9(x^4 - 4c_1)^2 + 12c_1}}}$$

$$y(x) \rightarrow \frac{i(\sqrt{3} + i) \left(-3x^4 + \sqrt{64 \log^3(x) + 9(x^4 - 4c_1)^2 + 12c_1}\right)^{2/3} + (4 + 4i\sqrt{3}) \log(x)}{4 \sqrt[3]{-3x^4 + \sqrt{64 \log^3(x) + 9(x^4 - 4c_1)^2 + 12c_1}}}$$

$$y(x) \rightarrow \frac{(-1 - i\sqrt{3}) \left(-3x^4 + \sqrt{64 \log^3(x) + 9(x^4 - 4c_1)^2 + 12c_1}\right)^{2/3} + (4 - 4i\sqrt{3}) \log(x)}{4 \sqrt[3]{-3x^4 + \sqrt{64 \log^3(x) + 9(x^4 - 4c_1)^2 + 12c_1}}}$$

5.36 problem 36

Internal problem ID [114]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 36.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [exact]

$$1 + e^{yx}y + (e^{yx}x + 2y)y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(1+exp(x*y(x))*y(x)+(exp(x*y(x))*x+2*y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$e^{xy(x)} + x + y(x)^2 + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.236 (sec). Leaf size: 18

```
DSolve[1+Exp[x*y[x]]*y[x]+(Exp[x*y[x]]*x+2*y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -
```

$$\text{Solve}[y(x)^2 + e^{xy(x)} + x = c_1, y(x)]$$

5.37 problem 37

Internal problem ID [115]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 37.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [exact]

$$\cos(x) + \ln(y) + \left(e^y + \frac{x}{y}\right) y' = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 24

```
dsolve(cos(x)+ln(y(x))+(exp(y(x))+x/y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = e^{\text{RootOf}(e^{-Z} - \ln(-x_Z - c_1 - \sin(x)))}$$

✓ Solution by Mathematica

Time used: 0.365 (sec). Leaf size: 18

```
DSolve[Cos[x]+Log[y[x]]+(Exp[y[x]]+x/y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True
```

$$\text{Solve}[e^{y(x)} + x \log(y(x)) + \sin(x) = c_1, y(x)]$$

5.38 problem 38

Internal problem ID [116]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 38.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [exact]

$$x + \arctan(y) + \frac{(x+y)y'}{1+y^2} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve(x+arctan(y(x))+(x+y(x))*diff(y(x),x)/(1+y(x)^2) = 0,y(x), singsol=all)
```

$$y(x) = \tan(\text{RootOf}(2x_Z + x^2 - 2 \ln(\cos(_Z)) + 2c_1))$$

✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 30

```
DSolve[x+ArcTan[y[x]]+(x+y[x])*y'[x]/(1+y[x]^2) == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x \arctan(y(x)) + \frac{x^2}{2} + \frac{1}{2} \log(y(x)^2 + 1) = c_1, y(x)\right]$$

5.39 problem 39

Internal problem ID [117]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 39.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$3x^2y^3 + y^4 + (3x^3y^2 + 4xy^3 + y^4)y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(3*x^2*y(x)^3+y(x)^4+(3*x^3*y(x)^2+4*x*y(x)^3+y(x)^4)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$xy(x)^4 + x^3y(x)^3 + \frac{y(x)^5}{5} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 33.396 (sec). Leaf size: 171

```
DSolve[3*x^2*y[x]^3+y[x]^4+(3*x^3*y[x]^2+4*x*y[x]^3+y[x]^4)*y'[x] == 0,y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow \text{Root}[\#1^5 + 5\#1^4x + 5\#1^3x^3 - 5c_1 \&, 1]$$

$$y(x) \rightarrow \text{Root}[\#1^5 + 5\#1^4x + 5\#1^3x^3 - 5c_1 \&, 2]$$

$$y(x) \rightarrow \text{Root}[\#1^5 + 5\#1^4x + 5\#1^3x^3 - 5c_1 \&, 3]$$

$$y(x) \rightarrow \text{Root}[\#1^5 + 5\#1^4x + 5\#1^3x^3 - 5c_1 \&, 4]$$

$$y(x) \rightarrow \text{Root}[\#1^5 + 5\#1^4x + 5\#1^3x^3 - 5c_1 \&, 5]$$

$$y(x) \rightarrow 0$$

5.40 problem 40

Internal problem ID [118]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 40.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_exact]

$$e^x \sin(y) + \tan(y) + (e^x \cos(y) + x \sec(y)^2) y' = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 153

```
dsolve(exp(x)*sin(y(x))+tan(y(x))+(exp(x)*cos(y(x))+x*sec(y(x))^2)*diff(y(x),x) = 0,y(x), sin
```

$$y(x) = \arctan \left(-\frac{c_1 \operatorname{RootOf}(_Z^4 e^{2x} + 2x e^x _Z^3 + (c_1^2 + x^2 - e^{2x}) _Z^2 - 2x e^x _Z - x^2)}{\operatorname{RootOf}(_Z^4 e^{2x} + 2x e^x _Z^3 + (c_1^2 + x^2 - e^{2x}) _Z^2 - 2x e^x _Z - x^2) e^x + x}, \operatorname{RootOf}(_Z^4 e^{2x} + 2x e^x _Z^3 + (c_1^2 + x^2 - e^{2x}) _Z^2 - 2x e^x _Z - x^2) \right)$$

✓ Solution by Mathematica

Time used: 60.836 (sec). Leaf size: 5539

```
DSolve[Exp[x]*Sin[y[x]]+Tan[y[x]]+(Exp[x]*Cos[y[x]]+x*Sec[y[x]]^2)*y'[x] == 0,y[x],x,IncludesS
```

Too large to display

5.41 problem 41

Internal problem ID [119]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 41.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [exact, rational]

$$\frac{2x}{y} - \frac{3y^2}{x^4} + \left(-\frac{x^2}{y^2} + \frac{1}{\sqrt{y}} + \frac{2y}{x^3} \right) y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(2*x/y(x)-3*y(x)^2/x^4+(-x^2/y(x)^2+1/y(x)^(1/2)+2*y(x)/x^3)*diff(y(x),x) = 0,y(x), sin
```

$$\frac{y(x)^2}{x^3} + \frac{x^2}{y(x)} + 2\sqrt{y(x)} + c_1 = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*x/y[x]-3*y[x]^2/x^4+(-x^2/y[x]^2+1/y[x]^(1/2)+2*y[x]/x^3)*y'[x] == 0,y[x],x,IncludeS
```

Not solved

5.42 problem 42

Internal problem ID [120]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 1.6, Substitution methods and exact equations. Page 74

Problem number: 42.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_1st_order, _with_linear_symmetries], _exact, _rational]`

$$\frac{2x^{\frac{5}{2}} - 3y^{\frac{5}{3}}}{2x^{\frac{5}{2}}y^{\frac{2}{3}}} + \frac{(-2x^{\frac{5}{2}} + 3y^{\frac{5}{3}})y'}{3x^{\frac{3}{2}}y^{\frac{5}{3}}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 189

```
dsolve(1/2*(2*x^(5/2)-3*y(x)^(5/3))/x^(5/2)/y(x)^(2/3)+1/3*(-2*x^(5/2)+3*y(x)^(5/3))*diff(y(x)
```

$$y(x) = \frac{2^{\frac{3}{5}}3^{\frac{2}{5}}\left(x^{\frac{5}{2}}\right)^{\frac{3}{5}}}{3}$$

$$y(x) = \frac{\left(-\frac{\sqrt{5}}{4} - \frac{1}{4} - \frac{i\sqrt{2}\sqrt{5-\sqrt{5}}}{4}\right)^3 2^{\frac{3}{5}}3^{\frac{2}{5}}\left(x^{\frac{5}{2}}\right)^{\frac{3}{5}}}{3}$$

$$y(x) = \frac{\left(-\frac{\sqrt{5}}{4} - \frac{1}{4} + \frac{i\sqrt{2}\sqrt{5-\sqrt{5}}}{4}\right)^3 2^{\frac{3}{5}}3^{\frac{2}{5}}\left(x^{\frac{5}{2}}\right)^{\frac{3}{5}}}{3}$$

$$y(x) = \frac{\left(\frac{\sqrt{5}}{4} - \frac{1}{4} - \frac{i\sqrt{2}\sqrt{5+\sqrt{5}}}{4}\right)^3 2^{\frac{3}{5}}3^{\frac{2}{5}}\left(x^{\frac{5}{2}}\right)^{\frac{3}{5}}}{3}$$

$$y(x) = \frac{\left(\frac{\sqrt{5}}{4} - \frac{1}{4} + \frac{i\sqrt{2}\sqrt{5+\sqrt{5}}}{4}\right)^3 2^{\frac{3}{5}}3^{\frac{2}{5}}\left(x^{\frac{5}{2}}\right)^{\frac{3}{5}}}{3}$$

$$\frac{x}{y(x)^{\frac{2}{3}}} + \frac{y(x)}{x^{\frac{3}{2}}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 260

`DSolve[1/2*(2*x^(5/2)-3*y[x]^(5/3))/x^(5/2)/y[x]^(2/3)+1/3*(-2*x^(5/2)+3*y[x]^(5/3))*y'[x]/x^`

$$y(x) \rightarrow \left(\frac{2}{3}\right)^{3/5} (x^{5/2})^{3/5}$$

$$y(x) \rightarrow c_1 x^{3/2}$$

$$y(x) \rightarrow -\left(-\frac{2}{3}\right)^{3/5} x^{3/2}$$

$$y(x) \rightarrow \left(-\frac{2}{3}\right)^{3/5} x^{3/2}$$

$$y(x) \rightarrow -\left(\frac{2}{3}\right)^{3/5} x^{3/2}$$

$$y(x) \rightarrow \left(\frac{2}{3}\right)^{3/5} x^{3/2}$$

$$y(x) \rightarrow -\sqrt[5]{-1} \left(\frac{2}{3}\right)^{3/5} x^{3/2}$$

$$y(x) \rightarrow \sqrt[5]{-1} \left(\frac{2}{3}\right)^{3/5} x^{3/2}$$

$$y(x) \rightarrow -(-1)^{2/5} \left(\frac{2}{3}\right)^{3/5} x^{3/2}$$

$$y(x) \rightarrow (-1)^{2/5} \left(\frac{2}{3}\right)^{3/5} x^{3/2}$$

$$y(x) \rightarrow -(-1)^{4/5} \left(\frac{2}{3}\right)^{3/5} x^{3/2}$$

$$y(x) \rightarrow (-1)^{4/5} \left(\frac{2}{3}\right)^{3/5} x^{3/2}$$

$$y(x) \rightarrow \left(\frac{2}{3}\right)^{3/5} (x^{5/2})^{3/5}$$

6 Chapter 1 review problems. Page 78

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6.1 problem 1

Internal problem ID [121]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$x^3 + 3y - y'x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x^3+3*y(x)-x*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = (\ln(x) + c_1)x^3$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 14

```
DSolve[x^3+3*y[x]-x*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^3(\log(x) + c_1)$$

6.2 problem 2

Internal problem ID [122]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$3y^2 + xy^2 - y'x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(3*y(x)^2+x*y(x)^2-x^2*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{x \ln(x) - c_1x - 3}$$

✓ Solution by Mathematica

Time used: 0.131 (sec). Leaf size: 25

```
DSolve[3*y[x]^2+x*y[x]^2-x^2*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x}{x \log(x) + c_1x - 3}$$

$$y(x) \rightarrow 0$$

6.3 problem 3

Internal problem ID [123]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$yx + y^2 - y'x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*y(x)+y(x)^2-x^2*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{\ln(x) - c_1}$$

✓ Solution by Mathematica

Time used: 0.121 (sec). Leaf size: 21

```
DSolve[x*y[x]+y[x]^2-x^2*y'[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x}{-\log(x) + c_1}$$

$$y(x) \rightarrow 0$$

6.4 problem 4

Internal problem ID [124]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [exact]

$$e^x + 2xy^3 + (\sin(y) + 3x^2y^2)y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(exp(x)+2*x*y(x)^3+(sin(y(x))+3*x^2*y(x)^2)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$x^2y(x)^3 + e^x - \cos(y(x)) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.253 (sec). Leaf size: 23

```
DSolve[Exp[x]+2*x*y[x]^3+(Sin[y[x]]+3*x^2*y[x]^2)*y'[x]== 0,y[x],x,IncludeSingularSolutions -
```

$$\text{Solve}[x^2y(x)^3 - \cos(y(x)) + e^x = c_1, y(x)]$$

6.5 problem 5

Internal problem ID [125]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$3y + x^4 y' - 2yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(3*y(x)+x^4*diff(y(x),x) = 2*x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x-1}{x^3}}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 24

```
DSolve[3*y[x]+x^4*y'[x] == 2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{\frac{1-x}{x^3}}$$

$$y(x) \rightarrow 0$$

6.6 problem 6

Internal problem ID [126]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$2xy^2 + y'x^2 - y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(2*x*y(x)^2+x^2*diff(y(x),x) = y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x}{1 + 2x \ln(x) + c_1 x}$$

✓ Solution by Mathematica

Time used: 0.134 (sec). Leaf size: 26

```
DSolve[2*x*y[x]^2+x^2*y'[x] == y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x}{2x \log(x) + c_1(-x) + 1}$$

$$y(x) \rightarrow 0$$

6.7 problem 7

Internal problem ID [127]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$2x^2y + y'x^3 - 1 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(2*x^2*y(x)+x^3*diff(y(x),x) = 1,y(x), singsol=all)
```

$$y(x) = \frac{\ln(x) + c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 14

```
DSolve[2*x^2*y[x]+x^3*y'[x] == 1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\log(x) + c_1}{x^2}$$

6.8 problem 8

Internal problem ID [128]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$2yx + y'x^2 - y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*x*y(x)+x^2*diff(y(x),x) = y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{3x}{3c_1x^3 + 1}$$

✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 24

```
DSolve[2*x*y[x]+x^2*y'[x] == y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{3x}{1 + 3c_1x^3}$$

$$y(x) \rightarrow 0$$

6.9 problem 9

Internal problem ID [129]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$2y + y'x - 6x^2\sqrt{y} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(2*y(x)+x*diff(y(x),x) = 6*x^2*y(x)^(1/2),y(x), singsol=all)
```

$$-x^2 - \frac{c_1}{x} + \sqrt{y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.149 (sec). Leaf size: 17

```
DSolve[2*y[x]+x*y'[x] == 6*x^2*y[x]^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{(x^3 + c_1)^2}{x^2}$$

6.10 problem 10

Internal problem ID [130]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - 1 - x^2 - y^2 - x^2y^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) = 1+x^2+y(x)^2+x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{1}{3}x^3 + c_1 + x\right)$$

✓ Solution by Mathematica

Time used: 0.173 (sec). Leaf size: 17

```
DSolve[y'[x] == 1+x^2+y[x]^2+x^2*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \tan\left(\frac{x^3}{3} + x + c_1\right)$$

6.11 problem 11

Internal problem ID [131]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$y'x^2 - yx - 3y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x) = x*y(x)+3*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{x}{3 \ln(x) - c_1}$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 21

```
DSolve[x^2*y'[x] == x*y[x]+3*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x}{-3 \log(x) + c_1}$$

$$y(x) \rightarrow 0$$

6.12 problem 12

Internal problem ID [132]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A'], _exact, _rational, [_Abel, '2nd typ`

$$6xy^3 + 2y^4 + (9x^2y^2 + 8xy^3)y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(6*x*y(x)^3+2*y(x)^4+(9*x^2*y(x)^2+8*x*y(x)^3)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$3x^2y(x)^3 + 2xy(x)^4 + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.141 (sec). Leaf size: 1714

`DSolve[6*x*y[x]^3+2*y[x]^4+(9*x^2*y[x]^2+8*x*y[x]^3)*y'[x] == 0,y[x],x,IncludeSingularSolutions->True]`

$y(x) \rightarrow 0$

$y(x)$

$$\rightarrow \frac{1}{2} \sqrt{\frac{9x^2}{16} - \frac{4\sqrt[3]{\frac{2}{3}}e^{c_1}}{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3(2187x^5+2048e^{c_1})}-81e^{c_1}x^4}}} + \frac{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3(2187x^5+2048e^{c_1})}-81e^{c_1}x^4}}{2\sqrt[3]{23^{2/3}x}}}$$

$$-\frac{1}{2} \sqrt{\frac{9x^2}{8} + \frac{4\sqrt[3]{\frac{2}{3}}e^{c_1}}{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3(2187x^5+2048e^{c_1})}-81e^{c_1}x^4}}} - \frac{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3(2187x^5+2048e^{c_1})}-81e^{c_1}x^4}}{2\sqrt[3]{23^{2/3}x}}}$$

$$\sqrt{-\frac{3x}{8}}$$

$y(x)$

$$\rightarrow \frac{1}{2} \sqrt{\frac{9x^2}{16} - \frac{4\sqrt[3]{\frac{2}{3}}e^{c_1}}{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3(2187x^5+2048e^{c_1})}-81e^{c_1}x^4}}} + \frac{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3(2187x^5+2048e^{c_1})}-81e^{c_1}x^4}}{2\sqrt[3]{23^{2/3}x}}}$$

$$+\frac{1}{2} \sqrt{\frac{9x^2}{8} + \frac{4\sqrt[3]{\frac{2}{3}}e^{c_1}}{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3(2187x^5+2048e^{c_1})}-81e^{c_1}x^4}}} - \frac{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3(2187x^5+2048e^{c_1})}-81e^{c_1}x^4}}{2\sqrt[3]{23^{2/3}x}}}$$

$$\sqrt{-\frac{3x}{8}}$$

$y(x) \rightarrow$

$$-\frac{1}{2} \sqrt{\frac{9x^2}{16} - \frac{4\sqrt[3]{\frac{2}{3}}e^{c_1}}{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3(2187x^5+2048e^{c_1})}-81e^{c_1}x^4}}} + \frac{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3(2187x^5+2048e^{c_1})}-81e^{c_1}x^4}}{2\sqrt[3]{23^{2/3}x}}}$$

$$-\frac{1}{2} \sqrt{\frac{9x^2}{8} + \frac{4\sqrt[3]{\frac{2}{3}}e^{c_1}}{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3(2187x^5+2048e^{c_1})}-81e^{c_1}x^4}}} - \frac{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3(2187x^5+2048e^{c_1})}-81e^{c_1}x^4}}{2\sqrt[3]{23^{2/3}x}}}$$

$$\sqrt{-\frac{3x}{8}}$$

6.13 problem 13

Internal problem ID [133]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [$y = G(x, y')$]

$$y' - 1 - x^2 - y^2 - x^2 y^4 = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = 1+x^2+y(x)^2+x^2*y(x)^4,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] == 1+x^2+y[x]^2+x^2*y[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

6.14 problem 14

Internal problem ID [134]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 14.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$y'x^3 - x^2y + y^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x^3*diff(y(x),x) = x^2*y(x)-y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{x}{\sqrt{2 \ln(x) + c_1}}$$

$$y(x) = -\frac{x}{\sqrt{2 \ln(x) + c_1}}$$

✓ Solution by Mathematica

Time used: 0.165 (sec). Leaf size: 41

```
DSolve[x^3*y'[x] == x^2*y[x]-y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x}{\sqrt{2 \log(x) + c_1}}$$

$$y(x) \rightarrow \frac{x}{\sqrt{2 \log(x) + c_1}}$$

$$y(x) \rightarrow 0$$

6.15 problem 15

Internal problem ID [135]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 15.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$3y + y' - 3x^2e^{-3x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(3*y(x)+diff(y(x),x) = 3*x^2/exp(3*x),y(x), singsol=all)
```

$$y(x) = (x^3 + c_1) e^{-3x}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 17

```
DSolve[3*y[x]+y'[x] == 3*x^2/Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-3x}(x^3 + c_1)$$

6.16 problem 16

Internal problem ID [136]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 16.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _Riccati]`

$$y' - x^2 + 2yx - y^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x) = x^2-2*x*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x e^{-2x} c_1 + e^{-2x} c_1 - x + 1}{e^{-2x} c_1 - 1}$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 29

```
DSolve[y'[x] == x^2-2*x*y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x + \frac{1}{\frac{1}{2} + c_1 e^{2x}} - 1$$

$$y(x) \rightarrow x - 1$$

6.17 problem 17

Internal problem ID [137]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 17.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [exact]

$$e^x + e^{yx}y + (e^y + e^{yx}x)y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(exp(x)+exp(x*y(x))*y(x)+(exp(y(x))+exp(x*y(x))*x)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$e^{xy(x)} + e^x + e^{y(x)} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.216 (sec). Leaf size: 20

```
DSolve[Exp[x]+Exp[x*y[x]]*y[x]+(Exp[y[x]]+Exp[x*y[x]]*x)*y'[x] == 0,y[x],x,IncludeSingularSol
```

$$\text{Solve}[e^{y(x)} + e^{xy(x)} + e^x = c_1, y(x)]$$

6.18 problem 18

Internal problem ID [138]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 18.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$2x^2y - y'x^3 - y^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(2*x^2*y(x)-x^3*diff(y(x),x) = y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{\sqrt{x^2 + c_1}}$$

$$y(x) = -\frac{x^2}{\sqrt{x^2 + c_1}}$$

✓ Solution by Mathematica

Time used: 0.172 (sec). Leaf size: 43

```
DSolve[2*x^2*y[x]-x^3*y'[x] == y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x^2}{\sqrt{x^2 + c_1}}$$

$$y(x) \rightarrow \frac{x^2}{\sqrt{x^2 + c_1}}$$

$$y(x) \rightarrow 0$$

6.19 problem 19

Internal problem ID [139]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 19.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$3y^2x^5 + y'x^3 - 2y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(3*x^5*y(x)^2+x^3*diff(y(x),x) = 2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{x^5 + c_1x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.142 (sec). Leaf size: 28

```
DSolve[3*x^5*y[x]^2+x^3*y'[x] == 2*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^2}{x^5 - c_1x^2 + 1}$$

$$y(x) \rightarrow 0$$

6.20 problem 20

Internal problem ID [140]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 20.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$3y + y'x - \frac{3}{x^{\frac{3}{2}}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(3*y(x)+x*diff(y(x),x) = 3/x^(3/2),y(x), singsol=all)
```

$$y(x) = \frac{2x^{\frac{3}{2}} + c_1}{x^3}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 19

```
DSolve[3*y[x]+x*y'[x]== 3/x^(3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2x^{3/2} + c_1}{x^3}$$

6.21 problem 21

Internal problem ID [141]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 21.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$(x - 1)y + (x^2 - 1)y' - 1 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((-1+x)*y(x)+(x^2-1)*diff(y(x),x) = 1,y(x), singsol=all)
```

$$y(x) = \frac{\ln(x - 1) + c_1}{x + 1}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 18

```
DSolve[(-1+x)*y[x]+(x^2-1)*y'[x] == 1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\log(x - 1) + c_1}{x + 1}$$

6.22 problem 22

Internal problem ID [142]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 22.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$y'x - 12x^4y^{\frac{2}{3}} - 6y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x) = 12*x^4*y(x)^(2/3)+6*y(x),y(x), singsol=all)
```

$$-2x^4 - c_1x^2 + y(x)^{\frac{1}{3}} = 0$$

✓ Solution by Mathematica

Time used: 0.16 (sec). Leaf size: 19

```
DSolve[x*y'[x]== 12*x^4*y[x]^(2/3)+6*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^6(2x^2 + c_1)^3$$

6.23 problem 23

Internal problem ID [143]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 23.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$e^y + \cos(x)y + (e^y x + \sin(x))y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(exp(y(x))+cos(x)*y(x)+(exp(y(x))*x+sin(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = -\text{LambertW}\left(\frac{x e^{-\frac{c_1}{\sin(x)}}}{\sin(x)}\right) - \frac{c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 3.99 (sec). Leaf size: 25

```
DSolve[Exp[y[x]]+Cos[x]*y[x]+(Exp[y[x]]*x+Sin[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions
```

$$y(x) \rightarrow c_1 \csc(x) - W(x \csc(x) e^{c_1 \csc(x)})$$

6.24 problem 24

Internal problem ID [144]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 24.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$9x^2y^2 + x^{\frac{3}{2}}y' - y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(9*x^2*y(x)^2+x^(3/2)*diff(y(x),x) = y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x}}{2 + 6x^2 + c_1\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.169 (sec). Leaf size: 30

```
DSolve[9*x^2*y[x]^2+x^(3/2)*y'[x] == y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{\frac{6x^2+2}{\sqrt{x}} - c_1}$$

$$y(x) \rightarrow 0$$

6.25 problem 25

Internal problem ID [145]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 25.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$2y + (x + 1)y' - 3 - 3x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(2*y(x)+(1+x)*diff(y(x),x) = 3+3*x,y(x), singsol=all)
```

$$y(x) = x + 1 + \frac{c_1}{(x + 1)^2}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 23

```
DSolve[2*y[x]+(1+x)*y'[x] == 3+3*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x(x(x + 3) + 3) + c_1}{(x + 1)^2}$$

6.26 problem 26

Internal problem ID [146]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 26.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact, _rational]`

$$9\sqrt{x}y^{\frac{4}{3}} - 12x^{\frac{1}{5}}y^{\frac{3}{2}} + \left(8x^{\frac{3}{2}}y^{\frac{1}{3}} - 15x^{\frac{6}{5}}\sqrt{y}\right)y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(9*x^(1/2)*y(x)^(4/3)-12*x^(1/5)*y(x)^(3/2)+(8*x^(3/2)*y(x)^(1/3)-15*x^(6/5)*y(x)^(1/2)
```

$$125y(x)^{\frac{9}{2}}x^{\frac{18}{5}} - 225y(x)^{\frac{13}{3}}x^{\frac{39}{10}} + 135y(x)^{\frac{25}{6}}x^{\frac{21}{5}} - 27y(x)^4x^{\frac{9}{2}} - c_1 = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[9*x^(1/2)*y[x]^(4/3)-12*x^(1/5)*y[x]^(3/2)+(8*x^(3/2)*y[x]^(1/3)-15*x^(6/5)*y[x]^(1/2)
```

Timed out

6.27 problem 27

Internal problem ID [147]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 27.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$3y + x^3y^4 + 3y'x = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 70

```
dsolve(3*y(x)+x^3*y(x)^4+3*x*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{(\ln(x) + c_1)^{\frac{1}{3}} x}$$

$$y(x) = \frac{-\frac{1}{2(\ln(x)+c_1)^{\frac{1}{3}}} - \frac{i\sqrt{3}}{2(\ln(x)+c_1)^{\frac{1}{3}}}}{x}$$

$$y(x) = \frac{-\frac{1}{2(\ln(x)+c_1)^{\frac{1}{3}}} + \frac{i\sqrt{3}}{2(\ln(x)+c_1)^{\frac{1}{3}}}}{x}$$

✓ Solution by Mathematica

Time used: 0.41 (sec). Leaf size: 70

```
DSolve[3*y[x]+x^3*y[x]^4+3*x*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{\sqrt[3]{x^3(\log(x) + c_1)}}$$

$$y(x) \rightarrow -\frac{\sqrt[3]{-1}}{\sqrt[3]{x^3(\log(x) + c_1)}}$$

$$y(x) \rightarrow \frac{(-1)^{2/3}}{\sqrt[3]{x^3(\log(x) + c_1)}}$$

$$y(x) \rightarrow 0$$

6.28 problem 28

Internal problem ID [148]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 28.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y + y'x - 2e^{2x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(y(x)+x*diff(y(x),x) = 2*exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{e^{2x} + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 17

```
DSolve[y[x]+x*y'[x] == 2*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{2x} + c_1}{x}$$

6.29 problem 29

Internal problem ID [149]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 29.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y + (1 + 2x)y' - (1 + 2x)^{\frac{3}{2}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(y(x)+(1+2*x)*diff(y(x),x) = (1+2*x)^(3/2),y(x), singsol=all)
```

$$y(x) = \frac{x^2 + c_1 + x}{\sqrt{1 + 2x}}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 32

```
DSolve[y[x]+(1+2*x)*y'[x] == (1+2*x)^(3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x(x+1)}{\sqrt{2x+1}} + \frac{c_1}{\sqrt{-2x-1}}$$

6.30 problem 31(a)

Internal problem ID [150]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 31(a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - 3x^2(7 + y) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) = 3*x^2*(7+y(x)),y(x), singsol=all)
```

$$y(x) = -7 + e^{x^3} c_1$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 20

```
DSolve[y'[x] == 3*x^2*(7+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -7 + c_1 e^{x^3}$$

$$y(x) \rightarrow -7$$

6.31 problem 31 (b)

Internal problem ID [151]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 31 (b).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - 3x^2(7 + y) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) = 3*x^2*(7+y(x)),y(x), singsol=all)
```

$$y(x) = -7 + e^{x^3} c_1$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 20

```
DSolve[y'[x] == 3*x^2*(7+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -7 + c_1 e^{x^3}$$

$$y(x) \rightarrow -7$$

6.32 problem 32 (b)

Internal problem ID [152]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 32 (b).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' + yx - xy^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = -x*y(x)+x*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{e^{x^2}c_1 + 1}}$$

$$y(x) = -\frac{1}{\sqrt{e^{x^2}c_1 + 1}}$$

✓ Solution by Mathematica

Time used: 1.87 (sec). Leaf size: 58

```
DSolve[y'[x] == -x*y[x]+x*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{\sqrt{1 + e^{x^2+2c_1}}}$$

$$y(x) \rightarrow \frac{1}{\sqrt{1 + e^{x^2+2c_1}}}$$

$$y(x) \rightarrow -1$$

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow 1$$

6.33 problem 33 (a)

Internal problem ID [153]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 33 (a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$y' - \frac{-3x^2 - 2y^2}{4yx} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(diff(y(x),x) = 1/4*(-3*x^2-2*y(x)^2)/(x*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2} \sqrt{x(-x^3 + 2c_1)}}{2x}$$

$$y(x) = \frac{\sqrt{2} \sqrt{x(-x^3 + 2c_1)}}{2x}$$

✓ Solution by Mathematica

Time used: 0.207 (sec). Leaf size: 60

```
DSolve[y'[x] == 1/4*(-3*x^2-2*y[x]^2)/(x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt{-x^3 + 2c_1}}{\sqrt{2}\sqrt{x}}$$

$$y(x) \rightarrow \frac{\sqrt{-x^3 + 2c_1}}{\sqrt{2}\sqrt{x}}$$

6.34 problem 34 (a)

Internal problem ID [154]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 34 (a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cla`

$$y' - \frac{x + 3y}{-3x + y} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 51

```
dsolve(diff(y(x),x) = (x+3*y(x))/(-3*x+y(x)),y(x), singsol=all)
```

$$y(x) = \frac{3c_1x - \sqrt{10c_1^2x^2 + 1}}{c_1}$$

$$y(x) = \frac{3c_1x + \sqrt{10c_1^2x^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.415 (sec). Leaf size: 94

```
DSolve[y'[x] == (x+3*y[x])/(-3*x+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3x - \sqrt{10x^2 + e^{2c_1}}$$

$$y(x) \rightarrow 3x + \sqrt{10x^2 + e^{2c_1}}$$

$$y(x) \rightarrow 3x - \sqrt{10}\sqrt{x^2}$$

$$y(x) \rightarrow \sqrt{10}\sqrt{x^2} + 3x$$

6.35 problem 35 (a)

Internal problem ID [155]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 35 (a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - \frac{2x + 2yx}{x^2 + 1} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) = (2*x+2*x*y(x))/(x^2+1),y(x), singsol=all)
```

$$y(x) = -1 + (x^2 + 1) c_1$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 20

```
DSolve[y'[x] == (2*x+2*x*y[x])/(x^2+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -1 + c_1(x^2 + 1)$$

$$y(x) \rightarrow -1$$

6.36 problem 36 (a)

Internal problem ID [156]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Chapter 1 review problems. Page 78

Problem number: 36 (a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - \cot(x)(\sqrt{y} - y) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x) = cot(x)*(y(x)^(1/2)-y(x)),y(x), singsol=all)
```

$$\sqrt{y(x)} - \frac{\int \frac{\sqrt{\sin(x)} \cot(x)}{2} dx + c_1}{\sqrt{\sin(x)}} = 0$$

✓ Solution by Mathematica

Time used: 0.274 (sec). Leaf size: 47

```
DSolve[y'[x] == Cot[x]*(y[x]^(1/2)-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \csc(x) \left(\sin(x) + 2e^{\frac{c_1}{2}} \sqrt{\cos(x)} \sqrt{\tan(x)} + e^{c_1} \right)$$

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow 1$$

7 Section 5.1, second order linear equations. Page 299

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7.1 problem 1

Internal problem ID [157]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 5]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)-y(x)=0,y(0) = 0, D(y)(0) = 5],y(x), singsol=all)
```

$$y(x) = \frac{5e^x}{2} - \frac{5e^{-x}}{2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 9

```
DSolve[{y'[x]-y[x]==0,{y[0]==0,y'[0]==5}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 5 \sinh(x)$$

7.2 problem 2

Internal problem ID [158]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 9y = 0$$

With initial conditions

$$[y(0) = -1, y'(0) = 15]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-9*y(x)=0,y(0) = -1, D(y)(0) = 15],y(x), singsol=all)
```

$$y(x) = 2e^{3x} - 3e^{-3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{y'[x]-9*y[x]==0,{y[0]==-1,y'[0]==15}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 5 \sinh(3x) - \cosh(3x)$$

7.3 problem 3

Internal problem ID [159]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 4y = 0$$

With initial conditions

$$[y(0) = 3, y'(0) = 8]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+4*y(x)=0,y(0) = 3, D(y)(0) = 8],y(x), singsol=all)
```

$$y(x) = 4 \sin(2x) + 3 \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[{y'[x]+4*y[x]==0,{y[0]==3,y'[0]==8}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 4 \sin(2x) + 3 \cos(2x)$$

7.4 problem 4

Internal problem ID [160]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 25y = 0$$

With initial conditions

$$[y(0) = 10, y'(0) = -10]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+25*y(x)=0,y(0) = 10, D(y)(0) = -10],y(x), singsol=all)
```

$$y(x) = -2 \sin(5x) + 10 \cos(5x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{y'[x]+25*y[x]==0,{y[0]==10,y'[0]==-10}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 10 \cos(5x) - 2 \sin(5x)$$

7.5 problem 5

Internal problem ID [161]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 3y' + 2y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = 2e^x - e^{2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 15

```
DSolve[{y'[x]-3*y'[x]+2*y[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -e^x(e^x - 2)$$

7.6 problem 6

Internal problem ID [162]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y' - 6y = 0$$

With initial conditions

$$[y(0) = 7, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+diff(y(x),x)-6*y(x)=0,y(0) = 7, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = (4e^{5x} + 3)e^{-3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[{y'[x]+y'[x]-6*y[x]==0,{y[0]==7,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-3x}(4e^{5x} + 3)$$

7.7 problem 7

Internal problem ID [163]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y' = 0$$

With initial conditions

$$[y(0) = -2, y'(0) = 8]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)+diff(y(x),x)=0,y(0) = -2, D(y)(0) = 8],y(x), singsol=all)
```

$$y(x) = 6 - 8e^{-x}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 14

```
DSolve[{y'[x]+y'[x]==0,{y[0]==-2,y'[0]==8}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 6 - 8e^{-x}$$

7.8 problem 8

Internal problem ID [164]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 3y' = 0$$

With initial conditions

$$[y(0) = 4, y'(0) = -2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)-3*diff(y(x),x)=0,y(0) = 4, D(y)(0) = -2],y(x), singsol=all)
```

$$y(x) = \frac{14}{3} - \frac{2e^{3x}}{3}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 16

```
DSolve[{y'[x]-3*y'[x]==0,{y[0]==4,y'[0]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{2}{3}(e^{3x} - 7)$$

7.9 problem 9

Internal problem ID [165]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 2y' + y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,y(0) = 2, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = e^{-x}(2 + x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 14

```
DSolve[{y'[x]+2*y'[x]+y[x]==0,{y[0]==2,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(x + 2)$$

7.10 problem 10

Internal problem ID [166]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 10y' + 25y = 0$$

With initial conditions

$$[y(0) = 3, y'(0) = 13]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)-10*diff(y(x),x)+25*y(x)=0,y(0) = 3, D(y)(0) = 13],y(x), singsol=all)
```

$$y(x) = e^{5x}(3 - 2x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{y''[x]-10*y'[x]+25*y[x]==0,{y[0]==3,y'[0]==13}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \rightarrow e^{5x}(3 - 2x)$$

7.11 problem 11

Internal problem ID [167]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 2y' + 2y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 5]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=0,y(0) = 0, D(y)(0) = 5],y(x), singsol=all)
```

$$y(x) = 5 \sin(x) e^x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 12

```
DSolve[{y'[x]-2*y'[x]+2*y[x]==0,{y[0]==0,y'[0]==5}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 5e^x \sin(x)$$

7.12 problem 12

Internal problem ID [168]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 6y' + 13y = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve([diff(y(x),x$2)+6*diff(y(x),x)+13*y(x)=0,y(0) = 2, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = e^{-3x}(3 \sin(2x) + 2 \cos(2x))$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[{y'[x]+6*y'[x]+13*y[x]==0,{y[0]==2,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow e^{-3x}(3 \sin(2x) + 2 \cos(2x))$$

7.13 problem 13

Internal problem ID [169]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_Emden, _Fowler], [_2nd_order, _linear, ' _with_symmetry_[0,F(`

$$x^2 y'' - 2y'x + 2y = 0$$

With initial conditions

$$[y(1) = 3, y'(1) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(1) = 3, D(y)(1) = 1],y(x), singsol=all
```

$$y(x) = -2x^2 + 5x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 12

```
DSolve[{x^2*y'[x]-2*x*y'[x]+2*y[x]==0,{y[1]==3,y'[1]==1}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow (5 - 2x)x$$

7.14 problem 14

Internal problem ID [170]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$x^2 y'' + 2y'x - 6y = 0$$

With initial conditions

$$[y(2) = 10, y'(2) = 15]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-6*y(x)=0,y(2) = 10, D(y)(2) = 15],y(x), singsol=a
```

$$y(x) = 3x^2 - \frac{16}{x^3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{x^2*y''[x]+2*x*y'[x]-6*y[x]==0,{y[2]==10,y'[2]==15}},y[x],x,IncludeSingularSolutions
```

$$y(x) \rightarrow \frac{3x^5 - 16}{x^3}$$

7.15 problem 15

Internal problem ID [171]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_Emden, _Fowler]`

$$x^2 y'' - y' x + y = 0$$

With initial conditions

$$[y(1) = 7, y'(1) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 12

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(1) = 7, D(y)(1) = 2],y(x), singsol=all)
```

$$y(x) = x(7 - 5 \ln(x))$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 13

```
DSolve[{x^2*y''[x]-x*y'[x]+y[x]==0,{y[1]==7,y'[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(7 - 5 \log(x))$$

7.16 problem 16

Internal problem ID [172]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_Emden, _Fowler], [_2nd_order, _linear, ' _with_symmetry_[0,F(`

$$x^2 y'' + y' x + y = 0$$

With initial conditions

$$[y(1) = 2, y'(1) = 3]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(1) = 2, D(y)(1) = 3],y(x), singsol=all)
```

$$y(x) = 3 \sin(\ln(x)) + 2 \cos(\ln(x))$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 16

```
DSolve[{x^2*y''[x]+x*y'[x]+y[x]==0,{y[1]==2,y'[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3 \sin(\log(x)) + 2 \cos(\log(x))$$

7.17 problem 33

Internal problem ID [173]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 33.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 3y' + 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^x c_1 + c_2 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[y''[x]-3*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x (c_2 e^x + c_1)$$

7.18 problem 34

Internal problem ID [174]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 34.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 2y' - 15y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-15*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} + c_2 e^{-5x}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''[x]+2*y'[x]-15*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-5x}(c_2 e^{8x} + c_1)$$

7.19 problem 35

Internal problem ID [175]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 35.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 5y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)+5*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{-5x}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 19

```
DSolve[y''[x]+5*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 - \frac{1}{5}c_1 e^{-5x}$$

7.20 problem 36

Internal problem ID [176]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 36.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$2y'' + 3y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(2*diff(y(x),x$2)+3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{-\frac{3x}{2}}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 21

```
DSolve[2*y''[x]+3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 - \frac{2}{3}c_1 e^{-3x/2}$$

7.21 problem 37

Internal problem ID [177]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 37.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$2y'' - y' - y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(2*diff(y(x),x$2)-diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = e^x c_1 + c_2 e^{-\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[2*y''[x]-y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-x/2} + c_2 e^x$$

7.22 problem 38

Internal problem ID [178]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 38.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$4y'' + 8y' + 3y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*diff(y(x),x$2)+8*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x}{2}} + c_2 e^{-\frac{3x}{2}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[4*y''[x]+8*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-3x/2}(c_2 e^x + c_1)$$

7.23 problem 39

Internal problem ID [179]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 39.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$4y'' + 4y' + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(4*diff(y(x),x$2)+4*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x}{2}} + c_2 e^{-\frac{x}{2}} x$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[4*y''[x]+4*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x/2}(c_2 x + c_1)$$

7.24 problem 40

Internal problem ID [180]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 40.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$9y'' - 12y' + 4y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(9*diff(y(x),x$2)-12*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{2x}{3}} + c_2 e^{\frac{2x}{3}} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[9*y''[x]-12*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{2x/3}(c_2 x + c_1)$$

7.25 problem 41

Internal problem ID [181]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 41.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$6y'' - 7y' - 20y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(6*diff(y(x),x$2)-7*diff(y(x),x)-20*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{5x}{2}} + c_2 e^{-\frac{4x}{3}}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 26

```
DSolve[6*y''[x]-7*y'[x]-20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-4x/3} + c_2 e^{5x/2}$$

7.26 problem 42

Internal problem ID [182]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 42.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$35y'' - y' - 12y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(35*dif(y(x),x$2)-dif(y(x),x)-12*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{4x}{7}} + c_2 e^{\frac{3x}{5}}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 26

```
DSolve[35*y''[x]-y'[x]-12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-4x/7} + c_2 e^{3x/5}$$

7.27 problem 52

Internal problem ID [183]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 52.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2 y'' + y'x - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x + \frac{c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 16

```
DSolve[x^2*y'[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1}{x} + c_2 x$$

7.28 problem 53

Internal problem ID [184]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 53.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2 y'' + 2y'x - 12y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-12*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^4} + c_2 x^3$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[x^2*y'[x]+2*x*y'[x]-12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2 x^7 + c_1}{x^4}$$

7.29 problem 54

Internal problem ID [185]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 54.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$4x^2y'' + 8y'x - 3y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(4*x^2*diff(y(x),x$2)+8*x*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + \frac{c_2}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[4*x^2*y''[x]+8*x*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2x^2 + c_1}{x^{3/2}}$$

7.30 problem 55

Internal problem ID [186]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 55.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$x^2 y'' + y' x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_2 \ln(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 13

```
DSolve[x^2*y''[x]+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \log(x) + c_2$$

7.31 problem 56

Internal problem ID [187]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.1, second order linear equations. Page 299

Problem number: 56.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$x^2 y'' - 3y'x + 4y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^2 + c_2 x^2 \ln(x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2(2c_2 \log(x) + c_1)$$

8 Section 5.2, second order linear equations. Page 311

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8.1 problem 21

Internal problem ID [188]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.2, second order linear equations. Page 311

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y - 3x = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = -2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)+y(x)=3*x,y(0) = 2, D(y)(0) = -2],y(x), singsol=all)
```

$$y(x) = -5 \sin(x) + 2 \cos(x) + 3x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 17

```
DSolve[{y'[x]+y[x]==3*x,{y[0]==2,y'[0]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3x - 5 \sin(x) + 2 \cos(x)$$

8.2 problem 22

Internal problem ID [189]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.2, second order linear equations. Page 311

Problem number: 22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 4y - 12 = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 10]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)-4*y(x)=12,y(0) = 0, D(y)(0) = 10],y(x), singsol=all)
```

$$y(x) = 4e^{2x} - e^{-2x} - 3$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 19

```
DSolve[{y'[x]-4*y[x]==12,{y[0]==0,y'[0]==10}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 5 \sinh(2x) + 3 \cosh(2x) - 3$$

8.3 problem 23

Internal problem ID [190]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.2, second order linear equations. Page 311

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 2y' - 3y - 6 = 0$$

With initial conditions

$$[y(0) = 3, y'(0) = 11]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=6,y(0) = 3, D(y)(0) = 11],y(x), singsol=all)
```

$$y(x) = e^{-x} + 4e^{3x} - 2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 19

```
DSolve[{y'[x]-2*y'[x]-3*y[x]==6,{y[0]==3,y'[0]==11}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow e^{-x} + 4e^{3x} - 2$$

8.4 problem 24

Internal problem ID [191]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.2, second order linear equations. Page 311

Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y' + 2y - 2x = 0$$

With initial conditions

$$[y(0) = 4, y'(0) = 8]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=2*x,y(0) = 4, D(y)(0) = 8],y(x), singsol=all)
```

$$y(x) = (3 \cos(x) + 4 \sin(x)) e^x + x + 1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 21

```
DSolve[{y'[x]-2*y'[x]+2*y[x]==2*x,{y[0]==4,y'[0]==8}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x + e^x(4 \sin(x) + 3 \cos(x)) + 1$$

8.5 problem 26(a.1)

Internal problem ID [192]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.2, second order linear equations. Page 311

Problem number: 26(a.1).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 2y - 4 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+2*y(x)=4,y(x), singsol=all)
```

$$y(x) = \sin(\sqrt{2}x) c_2 + \cos(\sqrt{2}x) c_1 + 2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 29

```
DSolve[y''[x]+2*y[x]==4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos(\sqrt{2}x) + c_2 \sin(\sqrt{2}x) + 2$$

8.6 problem 26(a.2)

Internal problem ID [193]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.2, second order linear equations. Page 311

Problem number: 26(a.2).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2y - 6x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+2*y(x)=6*x,y(x), singsol=all)
```

$$y(x) = \sin(\sqrt{2}x) c_2 + \cos(\sqrt{2}x) c_1 + 3x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 31

```
DSolve[y''[x]+2*y[x]==6*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3x + c_1 \cos(\sqrt{2}x) + c_2 \sin(\sqrt{2}x)$$

8.7 problem 26(b)

Internal problem ID [194]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.2, second order linear equations. Page 311

Problem number: 26(b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2y - 6x - 4 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+2*y(x)=6*x+4,y(x), singsol=all)
```

$$y(x) = \sin(\sqrt{2}x) c_2 + \cos(\sqrt{2}x) c_1 + 3x + 2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 32

```
DSolve[y''[x]+2*y[x]==6*x+4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3x + c_1 \cos(\sqrt{2}x) + c_2 \sin(\sqrt{2}x) + 2$$

9 Section 5.3, second order linear equations. Page 323

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9.1 problem 1

Internal problem ID [195]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 4y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x}(c_1 e^{4x} + c_2)$$

9.2 problem 2

Internal problem ID [196]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$2y'' - 3y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(2*diff(y(x),x$2)-3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{\frac{3x}{2}}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 21

```
DSolve[2*y''[x]-3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2}{3}c_1 e^{3x/2} + c_2$$

9.3 problem 3

Internal problem ID [197]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 3y' - 10y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)-10*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{-5x}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''[x]+3*y'[x]-10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-5x} (c_2 e^{7x} + c_1)$$

9.4 problem 4

Internal problem ID [198]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$2y'' - 7y' + 3y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*diff(y(x),x$2)-7*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} + c_2 e^{\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 24

```
DSolve[2*y''[x]-7*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{x/2} + c_2 e^{3x}$$

9.5 problem 5

Internal problem ID [199]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 6y' + 9y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-3x} + c_2 e^{-3x} x$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[y''[x]+6*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-3x}(c_2 x + c_1)$$

9.6 problem 6

Internal problem ID [200]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 5y' + 5y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+5*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{(-5+\sqrt{5})x}{2}} + c_2 e^{-\frac{(5+\sqrt{5})x}{2}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 35

```
DSolve[y''[x]+5*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-\frac{1}{2}(5+\sqrt{5})x} (c_2 e^{\sqrt{5}x} + c_1)$$

9.7 problem 7

Internal problem ID [201]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$4y'' - 12y' + 9y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(4*diff(y(x),x$2)-12*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{3x}{2}} + c_2 e^{\frac{3x}{2}} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[4*y''[x]-12*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{3x/2}(c_2 x + c_1)$$

9.8 problem 8

Internal problem ID [202]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 6y' + 13y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+13*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} \sin(2x) + c_2 e^{3x} \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y''[x]-6*y'[x]+13*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{3x}(c_2 \cos(2x) + c_1 \sin(2x))$$

9.9 problem 9

Internal problem ID [203]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 8y' + 25y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+8*diff(y(x),x)+25*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-4x} \sin(3x) + c_2 e^{-4x} \cos(3x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y''[x]+8*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-4x}(c_2 \cos(3x) + c_1 \sin(3x))$$

9.10 problem 21

Internal problem ID [204]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 4y' + 3y = 0$$

With initial conditions

$$[y(0) = 7, y'(0) = 11]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+3*y(x)=0,y(0) = 7, D(y)(0) = 11],y(x), singsol=all)
```

$$y(x) = 5e^x + 2e^{3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{y'[x]-4*y'[x]+3*y[x]==0,{y[0]==7,y'[0]==11}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow e^x(2e^{2x} + 5)$$

9.11 problem 22

Internal problem ID [205]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$9y'' + 6y' + 4y = 0$$

With initial conditions

$$[y(0) = 3, y'(0) = 4]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve([9*dif(y(x),x$2)+6*dif(y(x),x)+4*y(x)=0,y(0) = 3, D(y)(0) = 4],y(x), singsol=all)
```

$$y(x) = e^{-\frac{x}{3}} \left(5\sqrt{3} \sin\left(\frac{\sqrt{3}x}{3}\right) + 3 \cos\left(\frac{\sqrt{3}x}{3}\right) \right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 39

```
DSolve[{9*y'[x]+6*y'[x]+4*y[x]==0,{y[0]==3,y'[0]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x/3} \left(5\sqrt{3} \sin\left(\frac{x}{\sqrt{3}}\right) + 3 \cos\left(\frac{x}{\sqrt{3}}\right) \right)$$

9.12 problem 23

Internal problem ID [206]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 6y' + 25y = 0$$

With initial conditions

$$[y(0) = 4, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)-6*diff(y(x),x)+25*y(x)=0,y(0) = 4, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{e^{3x}(11 \sin(4x) - 16 \cos(4x))}{4}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 27

```
DSolve[{y'[x]-6*y'[x]+25*y[x]==0,{y[0]==4,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow \frac{1}{4}e^{3x}(16 \cos(4x) - 11 \sin(4x))$$

9.13 problem 45

Internal problem ID [207]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 45.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 2iy' + 3y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-2*I*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3ix} + c_2 e^{-ix}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y''[x]-2*I*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-ix} (c_1 e^{4ix} + c_2)$$

9.14 problem 46

Internal problem ID [208]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 46.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - iy' + 6y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-I*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-2ix} + c_2 e^{3ix}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y''[x]-I*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2ix} (c_1 e^{5ix} + c_2)$$

9.15 problem 47

Internal problem ID [209]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 47.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - (-2 + 2i\sqrt{3})y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)=(-2+2*I*sqrt(3))*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{(-i\sqrt{3}-1)x} + c_2 e^{(1+i\sqrt{3})x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 41

```
DSolve[y''[x]==(-2+2*I*Sqrt[3])*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{x+i\sqrt{3}x} + c_2 e^{(-1-i\sqrt{3})x}$$

9.16 problem 52

Internal problem ID [210]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 52.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$x^2 y'' + y' x + 9y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(3 \ln(x)) + c_2 \cos(3 \ln(x))$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+x*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos(3 \log(x)) + c_2 \sin(3 \log(x))$$

9.17 problem 53

Internal problem ID [211]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.3, second order linear equations. Page 323

Problem number: 53.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2 y'' + 7y'x + 25y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)+7*x*diff(y(x),x)+25*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(4 \ln(x))}{x^3} + \frac{c_2 \cos(4 \ln(x))}{x^3}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+7*x*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2 \cos(4 \log(x)) + c_1 \sin(4 \log(x))}{x^3}$$

10 Section 5.4, Mechanical Vibrations. Page 337

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10.1 problem 15

Internal problem ID [212]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.4, Mechanical Vibrations. Page 337

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\frac{x''}{2} + 3x' + 4x = 0$$

With initial conditions

$$[x(0) = 2, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([1/2*diff(x(t),t$2)+3*diff(x(t),t)+4*x(t)=0,x(0) = 2, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = 4e^{-2t} - 2e^{-4t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[{1/2*x''[t]+3*x'[t]+4*x[t]==0,{x[0]==2,x'[0]==0}},x[t],t,IncludeSingularSolutions -> T
```

$$x(t) \rightarrow e^{-4t}(4e^{2t} - 2)$$

10.2 problem 16

Internal problem ID [213]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.4, Mechanical Vibrations. Page 337

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$3x'' + 30x' + 63x = 0$$

With initial conditions

$$[x(0) = 2, x'(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([3*diff(x(t),t$2)+30*diff(x(t),t)+63*x(t)=0,x(0) = 2, D(x)(0) = 2],x(t), singsol=all)
```

$$x(t) = -2e^{-7t} + 4e^{-3t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[{3*x'[t]+30*x'[t]+63*x[t]==0,{x[0]==2,x'[0]==2}},x[t],t,IncludeSingularSolutions ->T
```

$$x(t) \rightarrow e^{-7t}(4e^{4t} - 2)$$

10.3 problem 17

Internal problem ID [214]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.4, Mechanical Vibrations. Page 337

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$x'' + 8x' + 16x = 0$$

With initial conditions

$$[x(0) = 5, x'(0) = -10]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(x(t),t$2)+8*diff(x(t),t)+16*x(t)=0,x(0) = 5, D(x)(0) = -10],x(t), singsol=all)
```

$$x(t) = (5 + 10t)e^{-4t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 17

```
DSolve[{x'[t]+8*x'[t]+16*x[t]==0,{x[0]==5,x'[0]==-10}},x[t],t,IncludeSingularSolutions -> Tr
```

$$x(t) \rightarrow 5e^{-4t}(2t + 1)$$

10.4 problem 18

Internal problem ID [215]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.4, Mechanical Vibrations. Page 337

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$2x'' + 12x' + 50x = 0$$

With initial conditions

$$[x(0) = 0, x'(0) = -8]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([2*diff(x(t),t$2)+12*diff(x(t),t)+50*x(t)=0,x(0) = 0, D(x)(0) = -8],x(t), singsol=all)
```

$$x(t) = -2e^{-3t} \sin(4t)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{2*x'[t]+12*x'[t]+50*x[t]==0,{x[0]==0,x'[0]==-8}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \rightarrow -2e^{-3t} \sin(4t)$$

10.5 problem 19

Internal problem ID [216]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.4, Mechanical Vibrations. Page 337

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$4x'' + 20x' + 169x = 0$$

With initial conditions

$$[x(0) = 4, x'(0) = 16]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([4*diff(x(t),t$2)+20*diff(x(t),t)+169*x(t)=0,x(0) = 4, D(x)(0) = 16],x(t), singsol=all
```

$$x(t) = \frac{e^{-\frac{5t}{2}}(13 \sin(6t) + 12 \cos(6t))}{3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 29

```
DSolve[{4*x''[t]+20*x'[t]+169*x[t]==0,{x[0]==4,x'[0]==16}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \rightarrow \frac{1}{3}e^{-5t/2}(13 \sin(6t) + 12 \cos(6t))$$

10.6 problem 20

Internal problem ID [217]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.4, Mechanical Vibrations. Page 337

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$2x'' + 16x' + 40x = 0$$

With initial conditions

$$[x(0) = 5, x'(0) = 4]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve([2*diff(x(t),t$2)+16*diff(x(t),t)+40*x(t)=0,x(0) = 5, D(x)(0) = 4],x(t), singsol=all)
```

$$x(t) = e^{-4t}(12 \sin(2t) + 5 \cos(2t))$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[{2*x'[t]+16*x'[t]+40*x[t]==0,{x[0]==5,x'[0]==4}},x[t],t,IncludeSingularSolutions ->T
```

$$x(t) \rightarrow e^{-4t}(12 \sin(2t) + 5 \cos(2t))$$

10.7 problem 21

Internal problem ID [218]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.4, Mechanical Vibrations. Page 337

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$x'' + 10x' + 125x = 0$$

With initial conditions

$$[x(0) = 6, x'(0) = 50]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(x(t),t$2)+10*diff(x(t),t)+125*x(t)=0,x(0) = 6, D(x)(0) = 50],x(t), singsol=all)
```

$$x(t) = 2e^{-5t}(4\sin(10t) + 3\cos(10t))$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[{x'[t]+10*x'[t]+125*x[t]==0,{x[0]==6,x'[0]==50}},x[t],t,IncludeSingularSolutions ->T
```

$$x(t) \rightarrow e^{-5t}(8\sin(10t) + 6\cos(10t))$$

11 Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

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11.1 problem 1

Internal problem ID [219]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 16y - e^{3x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+16*y(x)=exp(3*x),y(x), singsol=all)
```

$$y(x) = \sin(4x) c_2 + \cos(4x) c_1 + \frac{e^{3x}}{25}$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 29

```
DSolve[y''[x]+16*y[x]==Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{3x}}{25} + c_1 \cos(4x) + c_2 \sin(4x)$$

11.2 problem 2

Internal problem ID [220]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y' - 2y - 3x - 4 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=3*x+4,y(x), singsol=all)
```

$$y(x) = c_2 e^{2x} + e^{-x} c_1 - \frac{3x}{2} - \frac{5}{4}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y''[x]-y'[x]-2*y[x]==3*x+4,y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow -\frac{3x}{2} + c_1 e^{-x} + c_2 e^{2x} - \frac{5}{4}$$

11.3 problem 3

Internal problem ID [221]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - y' - 6y - 2 \sin(3x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-6*y(x)=2*sin(3*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-2x} + c_1 e^{3x} + \frac{\cos(3x)}{39} - \frac{5 \sin(3x)}{39}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 37

```
DSolve[y''[x]-y'[x]-6*y[x]==2*Sin[3*x],y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow c_1 e^{-2x} + c_2 e^{3x} + \frac{1}{39}(\cos(3x) - 5 \sin(3x))$$

11.4 problem 4

Internal problem ID [222]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$4y'' + 4y' + y - 3e^x x = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(4*diff(y(x),x$2)+4*diff(y(x),x)+y(x)=3*x*exp(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-\frac{x}{2}} + e^{-\frac{x}{2}} x c_1 + \frac{(3x - 4)e^x}{9}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 33

```
DSolve[4*y''[x]+4*y'[x]+y[x]==3*x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{9}e^x(3x - 4) + e^{-x/2}(c_2x + c_1)$$

11.5 problem 5

Internal problem ID [223]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y' + y - \sin(x)^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=sin(x)^2,y(x), singsol=all)
```

$$y(x) = e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}x}{2}\right) c_2 + e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}x}{2}\right) c_1 + \frac{3 \cos(2x)}{26} - \frac{\sin(2x)}{13} + \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.754 (sec). Leaf size: 61

```
DSolve[y''[x]+y'[x]+y[x]==Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{26}(-2 \sin(2x) + 3 \cos(2x) + 13) + e^{-x/2} \left(c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right) \right)$$

11.6 problem 6

Internal problem ID [224]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2y'' + 4y' + 7y - x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(2*diff(y(x),x$2)+4*diff(y(x),x)+7*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = e^{-x} \sin\left(\frac{\sqrt{10}x}{2}\right) c_2 + e^{-x} \cos\left(\frac{\sqrt{10}x}{2}\right) c_1 + \frac{x^2}{7} - \frac{8x}{49} + \frac{4}{343}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 53

```
DSolve[2*y''[x]+4*y'[x]+7*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{343}(7x(7x - 8) + 4) + e^{-x} \left(c_2 \cos\left(\sqrt{\frac{5}{2}}x\right) + c_1 \sin\left(\sqrt{\frac{5}{2}}x\right) \right)$$

11.7 problem 7

Internal problem ID [225]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y - \sinh(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 64

```
dsolve(diff(y(x),x$2)-4*y(x)=sinh(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{2x} + e^{-2x} c_1 + \frac{(-2 \sinh(x)^2 \cosh(x) - 2 \sinh(x)^3 + \cosh(x)) e^{-2x}}{12} + \frac{e^{2x} \left(\sinh(x)^2 \cosh(x) - \sinh(x)^3 - \frac{\cosh(x)}{2} \right)}{6}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 38

```
DSolve[y''[x]-4*y[x]==Sinh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{6} e^{-2x} (e^x - e^{3x} + 6c_1 e^{4x} + 6c_2)$$

11.8 problem 8

Internal problem ID [226]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y - \cosh(2x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)-4*y(x)=cosh(2*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{2x} + e^{-2x} c_1 + \frac{(-4x - 2) e^{-2x}}{32} + \frac{e^{2x}(4x - 1)}{32}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 38

```
DSolve[y''[x]-4*y[x]==Cosh[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{32} e^{-2x} (-4x + e^{4x}(4x - 1 + 32c_1) - 1 + 32c_2)$$

11.9 problem 9

Internal problem ID [227]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' - 3y - 1 - e^x x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-3*y(x)=1+x*exp(x),y(x), singsol=all)
```

$$y(x) = e^x c_2 + c_1 e^{-3x} - \frac{1}{3} + \frac{(8x^2 - 4x + 1) e^x}{64}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 38

```
DSolve[y''[x]+2*y'[x]-3*y[x]==1+x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{64} e^x (8x^2 - 4x + 1 + 64c_2) + c_1 e^{-3x} - \frac{1}{3}$$

11.10 problem 10

Internal problem ID [228]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y - 2 \cos(3x) - 3 \sin(3x) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+9*y(x)=2*cos(3*x)+3*sin(3*x),y(x), singsol=all)
```

$$y(x) = \sin(3x) c_2 + \cos(3x) c_1 + \frac{(-9x + 2) \cos(3x)}{18} + \frac{x \sin(3x)}{3}$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 39

```
DSolve[y''[x]+9*y[x]==2*Cos[3*x]+3*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \left(-\frac{x}{2} + \frac{1}{9} + c_1\right) \cos(3x) + \frac{1}{12}(4x + 1 + 12c_2) \sin(3x)$$

11.11 problem 16

Internal problem ID [229]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y - 2x^2 e^{3x} - 5 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+9*y(x)=2*x^2*exp(3*x)+5,y(x), singsol=all)
```

$$y(x) = \sin(3x) c_2 + \cos(3x) c_1 + \frac{5}{9} + \frac{\left(x - \frac{1}{3}\right)^2 e^{3x}}{9}$$

✓ Solution by Mathematica

Time used: 0.123 (sec). Leaf size: 40

```
DSolve[y''[x]+9*y[x]==2*x^2*Exp[3*x]+5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{81} (e^{3x}(1 - 3x)^2 + 81c_1 \cos(3x) + 81c_2 \sin(3x) + 45)$$

11.12 problem 21

Internal problem ID [230]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + 2y - e^x \sin(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=exp(x)*sin(x),y(x), singsol=all)
```

$$y(x) = \sin(x) e^x c_2 + \cos(x) e^x c_1 + \frac{e^x(\sin(x) - \cos(x)x)}{2}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 28

```
DSolve[y''[x]-2*y'[x]+2*y[x]==Exp[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{2}e^x((x - 2c_2) \cos(x) - 2c_1 \sin(x))$$

11.13 problem 23

Internal problem ID [231]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y - 3x \cos(2x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)+4*y(x)=3*x*cos(2*x),y(x), singsol=all)
```

$$y(x) = \sin(2x) c_2 + \cos(2x) c_1 + \frac{3 \sin(2x) x^2}{8} - \frac{3 \sin(2x)}{64} + \frac{3x \cos(2x)}{16}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 38

```
DSolve[y''[x]+4*y[x]==3*x*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{64} (24x^2 - 3 + 64c_2) \sin(2x) + \left(\frac{3x}{16} + c_1 \right) \cos(2x)$$

11.14 problem 25

Internal problem ID [232]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 25.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 3y' + 2y - x(e^{-x} - e^{-2x}) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=x*(exp(-x)-exp(-2*x)),y(x), singsol=all)
```

$$y(x) = \left(-e^{-x}c_1 + \frac{x^2}{2} - x + \frac{e^{-x}x^2}{2} + xe^{-x} + e^{-x} + c_2 \right) e^{-x}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 41

```
DSolve[y''[x]+3*y'[x]+2*y[x]==x*(Exp[-x]-Exp[-2*x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}e^{-2x}(x^2 + e^x(x^2 - 2x + 2 + 2c_2)) + 2(x + 1 + c_1)$$

11.15 problem 26

Internal problem ID [233]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 26.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 6y' + 13y - x e^{3x} \sin(2x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+13*y(x)=x*exp(3*x)*sin(2*x),y(x), singsol=all)
```

$$y(x) = e^{3x} \sin(2x) c_2 + e^{3x} \cos(2x) c_1 - \frac{e^{3x} x \left(x \cos(2x) - \frac{\sin(2x)}{2} \right)}{8}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 43

```
DSolve[y''[x]-6*y'[x]+13*y[x]==x*Exp[3*x]*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{64} e^{3x} ((-8x^2 + 1 + 64c_2) \cos(2x) + 4(x + 16c_1) \sin(2x))$$

11.16 problem 31

Internal problem ID [234]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 31.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 4y - 2x = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)+4*y(x)=2*x,y(0) = 1, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = \frac{3 \sin(2x)}{4} + \cos(2x) + \frac{x}{2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[{y'[x]+4*y[x]==2*x,{y[0]==1,y'[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \cos(2x) + \frac{1}{2}(x + 3 \sin(x) \cos(x))$$

11.17 problem 32

Internal problem ID [235]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 32.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 3y' + 2y - e^x = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=exp(x),y(0) = 0, D(y)(0) = 3],y(x), singsol=all)
```

$$y(x) = \frac{(e^{3x} + 15e^x - 16)e^{-2x}}{6}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 26

```
DSolve[{y''[x]+3*y'[x]+2*y[x]==Exp[x],{y[0]==0,y'[0]==3}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow \frac{1}{6}e^{-2x}(15e^x + e^{3x} - 16)$$

11.18 problem 33

Internal problem ID [236]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 33.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y - \sin(2x) = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)+9*y(x)=sin(2*x),y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{2 \sin(3x)}{15} + \cos(3x) + \frac{\sin(2x)}{5}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 26

```
DSolve[{y''[x]+9*y[x]==Sin[2*x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{5} \sin(2x) - \frac{2}{15} \sin(3x) + \cos(3x)$$

11.19 problem 34

Internal problem ID [237]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 34.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - \cos(x) = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)+y(x)=cos(x),y(0) = 1, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = \frac{(-2 + x) \sin(x)}{2} + \cos(x)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 17

```
DSolve[{y'[x]+y[x]==Cos[x],{y[0]==1,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}(x - 2) \sin(x) + \cos(x)$$

11.20 problem 35

Internal problem ID [238]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 35.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y' + 2y - x - 1 = 0$$

With initial conditions

$$[y(0) = 3, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=x+1,y(0) = 3, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(4 \cos(x) - 5 \sin(x)) e^x}{2} + 1 + \frac{x}{2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 25

```
DSolve[{y'[x]-2*y'[x]+2*y[x]==x+1,{y[0]==3,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}(x + e^x(4 \cos(x) - 5 \sin(x)) + 2)$$

11.21 problem 44

Internal problem ID [239]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 44.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y' + y - \sin(x) \sin(3x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=sin(x)*sin(3*x),y(x), singsol=all)
```

$$y(x) = e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}x}{2}\right) c_2 + e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}x}{2}\right) c_1 \\ + \frac{\sin(2x)}{13} - \frac{3 \cos(2x)}{26} + \frac{15 \cos(4x)}{482} - \frac{2 \sin(4x)}{241}$$

✓ Solution by Mathematica

Time used: 2.482 (sec). Leaf size: 75

```
DSolve[y''[x]+y'[x]+y[x]==Sin[x]*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{13} \sin(2x) - \frac{2}{241} \sin(4x) - \frac{3}{26} \cos(2x) + \frac{15}{482} \cos(4x) \\ + e^{-x/2} \left(c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right) \right)$$

11.22 problem 45

Internal problem ID [240]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 45.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y - \sin(x)^4 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+9*y(x)=sin(x)^4,y(x), singsol=all)
```

$$y(x) = \sin(3x) c_2 + \cos(3x) c_1 - \frac{\cos(2x)}{10} - \frac{\cos(2x)^2}{28} + \frac{5}{84}$$

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 39

```
DSolve[y''[x]+9*y[x]==Sin[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{10} \cos(2x) - \frac{1}{56} \cos(4x) + c_1 \cos(3x) + c_2 \sin(3x) + \frac{1}{24}$$

11.23 problem 46

Internal problem ID [241]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 46.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - \cos(x)^3 x = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+y(x)=x*cos(x)^3,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + \frac{(12x^2 + 6 \cos(x)^2 + 9) \sin(x)}{64} - \frac{x \cos(x)^3}{8} + \frac{9 \cos(x) x}{32}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 50

```
DSolve[y''[x]+y[x]==x*Cos[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{128} (4(6x^2 - 3 + 32c_2) \sin(x) + 3 \sin(3x) - 4x \cos(3x) + 8(3x + 16c_1) \cos(x))$$

11.24 problem 47

Internal problem ID [242]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 47.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 3y' + 2y - 4e^x = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=4*exp(x),y(x), singsol=all)
```

$$y(x) = \frac{2e^x}{3} - e^{-2x}c_1 + e^{-x}c_2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 29

```
DSolve[y''[x]+3*y'[x]+2*y[x]==4*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2e^x}{3} + c_1e^{-2x} + c_2e^{-x}$$

11.25 problem 48

Internal problem ID [243]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 48.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y' - 8y - 3e^{-2x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-8*y(x)=3*exp(-2*x),y(x), singsol=all)
```

$$y(x) = e^{4x}c_2 + e^{-2x}c_1 - \frac{e^{-2x}x}{2}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 30

```
DSolve[y''[x]-2*y'[x]-8*y[x]==3*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x} \left(-\frac{x}{2} + c_2 e^{6x} - \frac{1}{12} + c_1 \right)$$

11.26 problem 49

Internal problem ID [244]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 49.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 4y' + 4y - 2e^{2x} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=2*exp(2*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{2x} + e^{2x} x c_1 + e^{2x} x^2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 20

```
DSolve[y''[x]-4*y'[x]+4*y[x]==2*Exp[2*x],y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow e^{2x}(x(x + c_2) + c_1)$$

11.27 problem 50

Internal problem ID [245]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 50.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y - \sinh(2x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)-4*y(x)=sinh(2*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{2x} + e^{-2x} c_1 + \frac{e^{2x}(4x - 1)}{32} + \frac{e^{-2x}x}{8}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 38

```
DSolve[y''[x]-4*y[x]==Sinh[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{32} e^{-2x} (4x + e^{4x} (4x - 1 + 32c_1) + 1 + 32c_2)$$

11.28 problem 51

Internal problem ID [246]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 51.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y - \cos(3x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+4*y(x)=cos(3*x),y(x), singsol=all)
```

$$y(x) = \sin(2x)c_2 + \cos(2x)c_1 - \frac{\cos(3x)}{5}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 28

```
DSolve[y''[x]+4*y[x]==Cos[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{5}\cos(3x) + c_1\cos(2x) + c_2\sin(2x)$$

11.29 problem 52

Internal problem ID [247]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 52.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y - \sin(3x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+9*y(x)=sin(3*x),y(x), singsol=all)
```

$$y(x) = \sin(3x) c_2 + \cos(3x) c_1 - \frac{x \cos(3x)}{6}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 33

```
DSolve[y''[x]+9*y[x]==Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \left(-\frac{x}{6} + c_1\right) \cos(3x) + \frac{1}{36}(1 + 36c_2) \sin(3x)$$

11.30 problem 53

Internal problem ID [248]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 53.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y - 2 \sec(3x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+9*y(x)=2*sec(3*x),y(x), singsol=all)
```

$$y(x) = \sin(3x) c_2 + \cos(3x) c_1 + \frac{2x \sin(3x)}{3} - \frac{2 \ln(\sec(3x)) \cos(3x)}{9}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 39

```
DSolve[y''[x]+9*y[x]==2*Sec[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{3}(2x + 3c_2) \sin(3x) + \cos(3x) \left(\frac{2}{9} \log(\cos(3x)) + c_1 \right)$$

11.31 problem 54

Internal problem ID [249]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 54.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - \csc(x)^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+y(x)=csc(x)^2,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 - 1 - \ln(\csc(x) - \cot(x)) \cos(x)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 34

```
DSolve[y''[x]+y[x]==Csc[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 \sin(x) + \cos(x) \left(-\log\left(\sin\left(\frac{x}{2}\right)\right) + \log\left(\cos\left(\frac{x}{2}\right)\right) + c_1 \right) - 1$$

11.32 problem 55

Internal problem ID [250]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 55.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y - \sin(x)^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+4*y(x)=sin(x)^2,y(x), singsol=all)
```

$$y(x) = \sin(2x) c_2 + \cos(2x) c_1 - \frac{\sin(2x)x}{8} + \frac{1}{8} - \frac{\cos(2x)}{8}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 63

```
DSolve[y''[x]+4*y[x]==sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sin(2x) \left(\int_1^x \frac{1}{2} \cos(2K[2]) \sin(K[2])^2 dK[2] + c_2 \right) \\ + \cos(2x) \left(\int_1^x -\cos(K[1]) \sin(K[1])^2 \sin(K[1]) dK[1] + c_1 \right)$$

11.33 problem 56

Internal problem ID [251]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 56.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y - e^x x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-4*y(x)=x*exp(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{2x} + e^{-2x} c_1 - \frac{(3x + 2) e^x}{9}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 34

```
DSolve[y''[x]-4*y[x]==x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{9} e^x (3x + 2) + c_1 e^{2x} + c_2 e^{-2x}$$

11.34 problem 57

Internal problem ID [252]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 57.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$x^2 y'' + y'x - y - 72x^5 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=72*x^5,y(x), singsol=all)
```

$$y(x) = xc_2 + 3x^5 + \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 21

```
DSolve[x^2*y'[x]+x*y'[x]-y[x]==72*x^5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3x^5 + c_2x + \frac{c_1}{x}$$

11.35 problem 58

Internal problem ID [253]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 58.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - 4y'x + 6y - x^3 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=x^3,y(x), singsol=all)
```

$$y(x) = x^2 c_2 + c_1 x^3 + x^3(-1 + \ln(x))$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2(x \log(x) + (-1 + c_2)x + c_1)$$

11.36 problem 59

Internal problem ID [254]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 59.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - 3y'x + 4y - x^4 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=x^4,y(x), singsol=all)
```

$$y(x) = x^2 c_2 + \ln(x) c_1 x^2 + \frac{x^4}{4}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}x^2(x^2 + 8c_2 \log(x) + 4c_1)$$

11.37 problem 60

Internal problem ID [255]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 60.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - 4y'x + 3y - 8x^{\frac{4}{3}} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+3*y(x)=8*x^(4/3),y(x), singsol=all)
```

$$y(x) = x^{\frac{3}{2}}c_2 + c_1\sqrt{x} - \frac{72x^{\frac{4}{3}}}{5}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 31

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+3*y[x]==8*x^(4/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{5}\sqrt{x}(-72x^{5/6} + 5c_2x + 5c_1)$$

11.38 problem 61

Internal problem ID [256]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 61.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + y' x + y - \ln(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=ln(x),y(x), singsol=all)
```

$$y(x) = \sin(\ln(x)) c_2 + \cos(\ln(x)) c_1 + \ln(x)$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 20

```
DSolve[x^2*y''[x]+x*y'[x]+y[x]==Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \log(x) + c_1 \cos(\log(x)) + c_2 \sin(\log(x))$$

11.39 problem 62

Internal problem ID [257]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.5, Nonhomogeneous equations and undetermined coefficients Page 351

Problem number: 62.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 - 1)y'' - 2y'x + 2y - x^2 + 1 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve((x^2-1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=x^2-1,y(x), singsol=all)
```

$$y(x) = xc_2 + (x^2 + 1)c_1 + \frac{(x-1)^2 \ln(x-1)}{2} + \frac{(x+1)^2 \ln(x+1)}{2} - x^2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2(x \log(x) + (-1 + c_2)x + c_1)$$

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12.1 problem 1

Internal problem ID [258]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.6, Forced Oscillations and Resonance. Page 362

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + 9x - 10 \cos(2t) = 0$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(x(t),t$2)+9*x(t)=10*cos(2*t),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = -8 \cos(t)^3 + 6 \cos(t) + 4 \cos(t)^2 - 2$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[{x'[t]+9*x[t]==10*Cos[2*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow 2(\cos(2t) - \cos(3t))$$

12.2 problem 2

Internal problem ID [259]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.6, Forced Oscillations and Resonance. Page 362

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + 4x - 5 \sin(3t) = 0$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(x(t),t$2)+4*x(t)=5*sin(3*t),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{3 \sin(2t)}{2} - \sin(3t)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[{x'[t]+4*x[t]==5*Sin[3*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True
```

$$x(t) \rightarrow 3 \sin(t) \cos(t) - \sin(3t)$$

12.3 problem 3

Internal problem ID [260]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.6, Forced Oscillations and Resonance. Page 362

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + 100x - 225 \cos(5t) - 300 \sin(5t) = 0$$

With initial conditions

$$[x(0) = 375, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve([diff(x(t),t$2)+100*x(t)=225*cos(5*t)+300*sin(5*t),x(0) = 375, D(x)(0) = 0],x(t),sing
```

$$x(t) = -2 \sin(10t) + 372 \cos(10t) + 3 \cos(5t) + 4 \sin(5t)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 30

```
DSolve[{x''[t]+100*x[t]==225*Cos[5*t]+300*Sin[5*t]},{x[0]==375,x'[0]==0},x[t],t,IncludeSingul
```

$$x(t) \rightarrow 4 \sin(5t) - 2 \sin(10t) + 3 \cos(5t) + 372 \cos(10t)$$

12.4 problem 4

Internal problem ID [261]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.6, Forced Oscillations and Resonance. Page 362

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + 25x - 90 \cos(4t) = 0$$

With initial conditions

$$[x(0) = 0, x'(0) = 90]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve([diff(x(t),t$2)+25*x(t)=90*cos(4*t),x(0) = 0, D(x)(0) = 90],x(t), singsol=all)
```

$$x(t) = 18 \sin(5t) - 10 \cos(5t) + 10 \cos(4t)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 26

```
DSolve[{x'[t]+25*x[t]==90*Cos[4*t],{x[0]==0,x'[0]==90}},x[t],t,IncludeSingularSolutions -> T
```

$$x(t) \rightarrow 2(9 \sin(5t) + 5 \cos(4t) - 5 \cos(5t))$$

12.5 problem 5

Internal problem ID [262]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.6, Forced Oscillations and Resonance. Page 362

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$mx'' + kx - F_0 \cos(\omega t) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
dsolve(m*diff(x(t),t$2)+k*x(t)=F_0*cos(omega*t),x(t), singsol=all)
```

$$x(t) = \sin\left(\frac{\sqrt{k}t}{\sqrt{m}}\right) c_2 + \cos\left(\frac{\sqrt{k}t}{\sqrt{m}}\right) c_1 + \frac{F_0 \cos(\omega t)}{-m\omega^2 + k}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 54

```
DSolve[m*x''[t]+k*x[t]==F0*Cos[omega*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow \frac{F_0 \cos(\omega t)}{k - m\omega^2} + c_1 \cos\left(\frac{\sqrt{kt}}{\sqrt{m}}\right) + c_2 \sin\left(\frac{\sqrt{kt}}{\sqrt{m}}\right)$$

12.6 problem 7

Internal problem ID [263]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.6, Forced Oscillations and Resonance. Page 362

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + 4x' + 4x - 10 \cos(3t) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(x(t),t$2)+4*diff(x(t),t)+4*x(t)=10*cos(3*t),x(t), singsol=all)
```

$$x(t) = e^{-2t}c_2 + e^{-2t}tc_1 - \frac{50 \cos(3t)}{169} + \frac{120 \sin(3t)}{169}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 36

```
DSolve[x''[t]+4*x'[t]+4*x[t]==10*Cos[3*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow -\frac{10}{169}(5 \cos(3t) - 12 \sin(3t)) + e^{-2t}(c_2t + c_1)$$

12.7 problem 8

Internal problem ID [264]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.6, Forced Oscillations and Resonance. Page 362

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + 3x' + 5x + 4 \cos(5t) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(diff(x(t),t$2)+3*diff(x(t),t)+5*x(t)=-4*cos(5*t),x(t), singsol=all)
```

$$x(t) = e^{-\frac{3t}{2}} \sin\left(\frac{\sqrt{11}t}{2}\right) c_2 + e^{-\frac{3t}{2}} \cos\left(\frac{\sqrt{11}t}{2}\right) c_1 - \frac{12 \sin(5t)}{125} + \frac{16 \cos(5t)}{125}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 60

```
DSolve[x''[t]+3*x'[t]+5*x[t]==-4*Cos[5*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow \frac{4}{125}(4 \cos(5t) - 3 \sin(5t)) + e^{-3t/2} \left(c_2 \cos\left(\frac{\sqrt{11}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{11}t}{2}\right) \right)$$

12.8 problem 9

Internal problem ID [265]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.6, Forced Oscillations and Resonance. Page 362

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x'' + 2x' + x - 3\sin(10t) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(2*dif(x(t),t$2)+2*dif(x(t),t)+x(t)=3*sin(10*t),x(t), singsol=all)
```

$$x(t) = e^{-\frac{t}{2}} \sin\left(\frac{t}{2}\right) c_2 + e^{-\frac{t}{2}} \cos\left(\frac{t}{2}\right) c_1 - \frac{597 \sin(10t)}{40001} - \frac{60 \cos(10t)}{40001}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 50

```
DSolve[2*x''[t]+2*x'[t]+x[t]==3*Sin[10*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow -\frac{3(199 \sin(10t) + 20 \cos(10t))}{40001} + e^{-t/2} \left(c_2 \cos\left(\frac{t}{2}\right) + c_1 \sin\left(\frac{t}{2}\right) \right)$$

12.9 problem 10

Internal problem ID [266]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.6, Forced Oscillations and Resonance. Page 362

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + 3x' + 3x - 8 \cos(10t) - 6 \sin(10t) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(x(t),t$2)+3*diff(x(t),t)+3*x(t)=8*cos(10*t)+6*sin(10*t),x(t), singsol=all)
```

$$x(t) = e^{-\frac{3t}{2}} \sin\left(\frac{\sqrt{3}t}{2}\right) c_2 + e^{-\frac{3t}{2}} \cos\left(\frac{\sqrt{3}t}{2}\right) c_1 - \frac{342 \sin(10t)}{10309} - \frac{956 \cos(10t)}{10309}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 60

```
DSolve[x''[t]+3*x'[t]+3*x[t]==8*Cos[10*t]+6*Sin[10*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow -\frac{2(171 \sin(10t) + 478 \cos(10t))}{10309} + e^{-3t/2} \left(c_2 \cos\left(\frac{\sqrt{3}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}t}{2}\right) \right)$$

12.10 problem 11

Internal problem ID [267]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.6, Forced Oscillations and Resonance. Page 362

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + 4x' + 5x - 10 \cos(3t) = 0$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 30

```
dsolve([diff(x(t),t$2)+4*diff(x(t),t)+5*x(t)=10*cos(3*t),x(0) = 0, D(x)(0) = 0],x(t), singsol
```

$$x(t) = \frac{(\cos(t) - 7 \sin(t)) e^{-2t}}{4} - \frac{\cos(3t)}{4} + \frac{3 \sin(3t)}{4}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 35

```
DSolve[{x'[t]+4*x'[t]+5*x[t]==10*Cos[3*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolution
```

$$x(t) \rightarrow \frac{1}{4} (3 \sin(3t) - \cos(3t) + e^{-2t} (\cos(t) - 7 \sin(t)))$$

12.11 problem 12

Internal problem ID [268]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.6, Forced Oscillations and Resonance. Page 362

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + 6x' + 13x - 10 \sin(5t) = 0$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 37

```
dsolve([diff(x(t),t$2)+6*diff(x(t),t)+13*x(t)=10*sin(5*t),x(0) = 0, D(x)(0) = 0],x(t), singso
```

$$x(t) = \frac{25(2 \cos(2t) + 5 \sin(2t)) e^{-3t}}{174} - \frac{25 \cos(5t)}{87} - \frac{10 \sin(5t)}{87}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 45

```
DSolve[{x''[t]+6*x'[t]+13*x[t]==10*Sin[5*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutio
```

$$x(t) \rightarrow \frac{5}{174} (5e^{-3t}(5 \sin(2t) + 2 \cos(2t)) - 2(2 \sin(5t) + 5 \cos(5t)))$$

12.12 problem 12

Internal problem ID [269]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.6, Forced Oscillations and Resonance. Page 362

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + 6x' + 13x - 10 \sin(5t) = 0$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve([diff(x(t),t$2)+6*diff(x(t),t)+13*x(t)=10*sin(5*t),x(0) = 0, D(x)(0) = 0],x(t), singso
```

$$x(t) = \frac{25(2 \cos(2t) + 5 \sin(2t)) e^{-3t}}{174} - \frac{25 \cos(5t)}{87} - \frac{10 \sin(5t)}{87}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 45

```
DSolve[{x''[t]+6*x'[t]+13*x[t]==10*Sin[5*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutio
```

$$x(t) \rightarrow \frac{5}{174} (5e^{-3t}(5 \sin(2t) + 2 \cos(2t)) - 2(2 \sin(5t) + 5 \cos(5t)))$$

12.13 problem 13

Internal problem ID [270]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.6, Forced Oscillations and Resonance. Page 362

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + 2x' + 26x - 600 \cos(10t) = 0$$

With initial conditions

$$[x(0) = 10, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 37

```
dsolve([diff(x(t),t$2)+2*diff(x(t),t)+26*x(t)=600*cos(10*t),x(0) = 10, D(x)(0) = 0],x(t), sin
```

$$x(t) = \frac{(25790 \cos(5t) - 842 \sin(5t)) e^{-t}}{1469} - \frac{11100 \cos(10t)}{1469} + \frac{3000 \sin(10t)}{1469}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 45

```
DSolve[{x''[t]+2*x'[t]+26*x[t]==600*Cos[10*t],{x[0]==10,x'[0]==0}},x[t],t,IncludeSingularSolu
```

$$x(t) \rightarrow \frac{e^{-t}(-842 \sin(5t) + 25790 \cos(5t) - 300e^t(37 \cos(10t) - 10 \sin(10t)))}{1469}$$

12.14 problem 14

Internal problem ID [271]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 5.6, Forced Oscillations and Resonance. Page 362

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + 8x' + 25x - 200 \cos(t) - 520 \sin(t) = 0$$

With initial conditions

$$[x(0) = -30, x'(0) = -10]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve([diff(x(t),t$2)+8*diff(x(t),t)+25*x(t)=200*cos(t)+520*sin(t),x(0) = -30, D(x)(0) = -10
```

$$x(t) = (-31 \cos(3t) - 52 \sin(3t)) e^{-4t} + 22 \sin(t) + \cos(t)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 32

```
DSolve[{x'[t]+8*x'[t]+25*x[t]==200*Cos[t]+520*Sin[t],{x[0]==-30,x'[0]==-10}},x[t],t,IncludeS
```

$$x(t) \rightarrow 22 \sin(t) + \cos(t) - e^{-4t}(52 \sin(3t) + 31 \cos(3t))$$

13 Section 7.2, Matrices and Linear systems. Page 417

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13.1 problem problem 3

Internal problem ID [272]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 7.2, Matrices and Linear systems. Page 417

Problem number: problem 3.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = -3y(t)$$

$$y'(t) = 3x(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=-3*y(t),diff(y(t),t)=3*x(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = c_1 \cos(3t) - c_2 \sin(3t)$$

$$y(t) = c_1 \sin(3t) + c_2 \cos(3t)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 38

```
DSolve[{x'[t]==3*y[t],y'[t]==3*x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow c_1 \cosh(3t) + c_2 \sinh(3t)$$

$$y(t) \rightarrow c_2 \cosh(3t) + c_1 \sinh(3t)$$

13.2 problem problem 4

Internal problem ID [273]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 7.2, Matrices and Linear systems. Page 417

Problem number: problem 4.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = 3x(t) - 2y(t)$$

$$y'(t) = 2x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 78

```
dsolve([diff(x(t),t)=3*x(t)-2*y(t),diff(y(t),t)=2*x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^{2t}(\sin(\sqrt{3}t)\sqrt{3}c_2 - \cos(\sqrt{3}t)\sqrt{3}c_1 - \sin(\sqrt{3}t)c_1 - \cos(\sqrt{3}t)c_2)}{2}$$

$$y(t) = e^{2t}(\sin(\sqrt{3}t)c_1 + \cos(\sqrt{3}t)c_2)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 96

```
DSolve[{x'[t]==3*x[t]-2*y[t],y'[t]==2*x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions->True]
```

$$x(t) \rightarrow \frac{1}{3}e^{2t}\left(3c_1 \cos(\sqrt{3}t) + \sqrt{3}(c_1 - 2c_2) \sin(\sqrt{3}t)\right)$$

$$y(t) \rightarrow \frac{1}{3}e^{2t}\left(3c_2 \cos(\sqrt{3}t) + \sqrt{3}(2c_1 - c_2) \sin(\sqrt{3}t)\right)$$

13.3 problem problem 5

Internal problem ID [274]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 7.2, Matrices and Linear systems. Page 417

Problem number: problem 5.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = 2x(t) + 4y(t) + 3e^t$$

$$y'(t) = 5x(t) - y(t) - t^2$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 112

```
dsolve([diff(x(t),t)=2*x(t)+4*y(t)+3*exp(t),diff(y(t),t)=5*x(t)-y(t)-t^2],[x(t), y(t)], sings
```

$$x(t) = \frac{e^{\frac{(1+\sqrt{89})t}{2}} c_2 \sqrt{89}}{10} - \frac{e^{-\frac{(-1+\sqrt{89})t}{2}} c_1 \sqrt{89}}{10} + \frac{3e^{\frac{(1+\sqrt{89})t}{2}} c_2}{10} \\ + \frac{3e^{-\frac{(-1+\sqrt{89})t}{2}} c_1}{10} + \frac{2t^2}{11} - \frac{3e^t}{11} - \frac{2t}{121} + \frac{23}{1331}$$

$$y(t) = e^{\frac{(1+\sqrt{89})t}{2}} c_2 + e^{-\frac{(-1+\sqrt{89})t}{2}} c_1 - \frac{t^2}{11} - \frac{15e^t}{22} + \frac{12t}{121} - \frac{17}{1331}$$

✓ Solution by Mathematica

Time used: 0.712 (sec). Leaf size: 212

```
DSolve[{x'[t]==2*x[t]+4*y[t]+3*Exp[t],y'[t]==5*x[t]-y[t]-t^2},{x[t],y[t]},t,IncludeSingularSo
```

$$\begin{aligned}
 x(t) &\rightarrow \frac{22t(11t-1)+23}{1331} - \frac{3e^t}{11} + \frac{1}{178} \left((89-3\sqrt{89})c_1 - 8\sqrt{89}c_2 \right) e^{-\frac{1}{2}(\sqrt{89}-1)t} \\
 &\quad + \frac{1}{178} \left((89+3\sqrt{89})c_1 + 8\sqrt{89}c_2 \right) e^{\frac{1}{2}(1+\sqrt{89})t} \\
 y(t) &\rightarrow \frac{11(12-11t)t-17}{1331} - \frac{15e^t}{22} + \left(\frac{5c_1}{\sqrt{89}} + \frac{1}{178} (89-3\sqrt{89})c_2 \right) e^{\frac{1}{2}(1+\sqrt{89})t} \\
 &\quad + \left(\frac{1}{178} (89+3\sqrt{89})c_2 - \frac{5c_1}{\sqrt{89}} \right) e^{-\frac{1}{2}(\sqrt{89}-1)t}
 \end{aligned}$$

13.4 problem problem 7

Internal problem ID [275]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 7.2, Matrices and Linear systems. Page 417

Problem number: problem 7.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) = y(t) + z(t)$$

$$y'(t) = z(t) + x(t)$$

$$z'(t) = x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 64

```
dsolve([diff(x(t),t)=y(t)+z(t),diff(y(t),t)=z(t)+x(t),diff(z(t),t)=x(t)+y(t)],[x(t), y(t), z(t)
```

$$x(t) = c_2 e^{2t} - 2c_3 e^{-t} - e^{-t} c_1$$

$$y(t) = c_2 e^{2t} + c_3 e^{-t} + e^{-t} c_1$$

$$z(t) = c_2 e^{2t} + c_3 e^{-t}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 114

```
DSolve[{x'[t]==y[t]+z[t],y'[t]==z[t]+x[t],z'[t]==x[t]+y[t]},{x[t],y[t],z[t]},t,IncludeSingular
```

$$x(t) \rightarrow \frac{1}{3} e^{-t} (c_1 (e^{3t} + 2) + (c_2 + c_3) (e^{3t} - 1))$$

$$y(t) \rightarrow \frac{1}{3} e^{-t} ((c_1 + c_2 + c_3) e^{3t} - c_1 + 2c_2 - c_3)$$

$$z(t) \rightarrow \frac{1}{3} e^{-t} ((c_1 + c_2 + c_3) e^{3t} - c_1 - c_2 + 2c_3)$$

13.5 problem problem 11

Internal problem ID [276]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 7.2, Matrices and Linear systems. Page 417

Problem number: problem 11.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = x_2(t)$$

$$x_2'(t) = 2x_3(t)$$

$$x_3'(t) = 3x_4(t)$$

$$x_4'(t) = 4x_1(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 171

```
dsolve([diff(x__1(t),t)=x__2(t),diff(x__2(t),t)=2*x__3(t),diff(x__3(t),t)=3*x__4(t),diff(x__4
```

$$x_1(t) = -\frac{24^{\frac{1}{4}} \left(c_1 e^{-24^{\frac{1}{4}} t} - c_2 e^{24^{\frac{1}{4}} t} + \cos \left(24^{\frac{1}{4}} t \right) c_3 + \sin \left(24^{\frac{1}{4}} t \right) c_4 \right)}{4}$$

$$x_2(t) = \frac{\sqrt{6} \left(c_1 e^{-24^{\frac{1}{4}} t} + c_2 e^{24^{\frac{1}{4}} t} - c_4 \cos \left(24^{\frac{1}{4}} t \right) + c_3 \sin \left(24^{\frac{1}{4}} t \right) \right)}{2}$$

$$x_3(t) = -\frac{24^{\frac{3}{4}} \left(c_1 e^{-24^{\frac{1}{4}} t} - c_2 e^{24^{\frac{1}{4}} t} - \cos \left(24^{\frac{1}{4}} t \right) c_3 - \sin \left(24^{\frac{1}{4}} t \right) c_4 \right)}{8}$$

$$x_4(t) = c_1 e^{-24^{\frac{1}{4}} t} + c_2 e^{24^{\frac{1}{4}} t} - c_3 \sin \left(24^{\frac{1}{4}} t \right) + c_4 \cos \left(24^{\frac{1}{4}} t \right)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 400

```
DSolve[{x1'[t]==x2[t],x2'[t]==2*x3[t],x3'[t]==3*x4[t],x4'[t]==4*x1[t]},{x1[t],x2[t],x3[t],x4[t]}
```

$$\begin{aligned}
 x1(t) &\rightarrow \frac{1}{4}c_1\text{RootSum}\left[\#1^4 - 24\&, \frac{e^{\#1t}}{\#1}\&\right] + \frac{1}{4}c_2\text{RootSum}\left[\#1^4 - 24\&, \frac{e^{\#1t}}{\#1}\&\right] \\
 &\quad + \frac{3}{2}c_4\text{RootSum}\left[\#1^4 - 24\&, \frac{e^{\#1t}}{\#1^3}\&\right] + \frac{1}{2}c_3\text{RootSum}\left[\#1^4 - 24\&, \frac{e^{\#1t}}{\#1^2}\&\right] \\
 x2(t) &\rightarrow \frac{1}{4}c_2\text{RootSum}\left[\#1^4 - 24\&, \frac{e^{\#1t}}{\#1}\&\right] + \frac{1}{2}c_3\text{RootSum}\left[\#1^4 - 24\&, \frac{e^{\#1t}}{\#1}\&\right] \\
 &\quad + 6c_1\text{RootSum}\left[\#1^4 - 24\&, \frac{e^{\#1t}}{\#1^3}\&\right] + \frac{3}{2}c_4\text{RootSum}\left[\#1^4 - 24\&, \frac{e^{\#1t}}{\#1^2}\&\right] \\
 x3(t) &\rightarrow \frac{1}{4}c_3\text{RootSum}\left[\#1^4 - 24\&, \frac{e^{\#1t}}{\#1}\&\right] + \frac{3}{4}c_4\text{RootSum}\left[\#1^4 - 24\&, \frac{e^{\#1t}}{\#1}\&\right] \\
 &\quad + 3c_2\text{RootSum}\left[\#1^4 - 24\&, \frac{e^{\#1t}}{\#1^3}\&\right] + 3c_1\text{RootSum}\left[\#1^4 - 24\&, \frac{e^{\#1t}}{\#1^2}\&\right] \\
 x4(t) &\rightarrow \frac{1}{4}c_4\text{RootSum}\left[\#1^4 - 24\&, \frac{e^{\#1t}}{\#1}\&\right] + c_1\text{RootSum}\left[\#1^4 - 24\&, \frac{e^{\#1t}}{\#1}\&\right] \\
 &\quad + 2c_3\text{RootSum}\left[\#1^4 - 24\&, \frac{e^{\#1t}}{\#1^3}\&\right] + c_2\text{RootSum}\left[\#1^4 - 24\&, \frac{e^{\#1t}}{\#1^2}\&\right]
 \end{aligned}$$

13.6 problem problem 12

Internal problem ID [277]

Book: Differential equations and linear algebra, 3rd ed., Edwards and Penney

Section: Section 7.2, Matrices and Linear systems. Page 417

Problem number: problem 12.

ODE order: 1.

ODE degree: 1.

Solve

$$x_1'(t) = x_2(t) + x_3(t) + 1$$

$$x_2'(t) = x_3(t) + x_4(t) + t$$

$$x_3'(t) = x_1(t) + x_4(t) + t^2$$

$$x_4'(t) = x_1(t) + x_2(t) + t^3$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 273

```
dsolve([diff(x__1(t),t)=x__2(t)+x__3(t)+1,diff(x__2(t),t)=x__3(t)+x__4(t)+t,diff(x__3(t),t)=x__4(t)+t,diff(x__4(t),t)=x__3(t)+x__4(t)+t])
```

$$x_1(t) = -\frac{11t}{16} + \frac{t^2}{16} - \frac{t^4}{16} - c_4 + \frac{c_1 e^{2t}}{2} + \frac{c_2 e^{-t} \cos(t)}{2} + \frac{c_2 \sin(t) e^{-t}}{2} - \frac{c_3 e^{-t} \cos(t)}{2} + \frac{c_3 e^{-t} \sin(t)}{2} - \frac{7t^3}{24} + \frac{5}{16}$$

$$x_2(t) = -\frac{3t}{16} - \frac{3}{2} + \frac{t^2}{16} + \frac{t^4}{16} + c_4 + \frac{c_1 e^{2t}}{2} + \frac{c_2 e^{-t} \cos(t)}{2} - \frac{c_2 \sin(t) e^{-t}}{2} + \frac{c_3 e^{-t} \cos(t)}{2} + \frac{c_3 e^{-t} \sin(t)}{2} - \frac{11t^3}{24}$$

$$x_3(t) = \frac{5t}{16} - \frac{15t^2}{16} - \frac{t^4}{16} - c_4 + \frac{c_1 e^{2t}}{2} - \frac{c_2 e^{-t} \cos(t)}{2} - \frac{c_2 \sin(t) e^{-t}}{2} + \frac{c_3 e^{-t} \cos(t)}{2} - \frac{c_3 e^{-t} \sin(t)}{2} + \frac{5t^3}{24} - \frac{3}{16}$$

$$x_4(t) = \frac{t^3}{24} - \frac{7t^2}{16} + \frac{t^4}{16} + \frac{c_1 e^{2t}}{2} - \frac{c_2 e^{-t} \cos(t)}{2} + \frac{c_2 \sin(t) e^{-t}}{2} - \frac{c_3 e^{-t} \cos(t)}{2} - \frac{c_3 e^{-t} \sin(t)}{2} - \frac{19t}{16} + c_4$$

✓ Solution by Mathematica

Time used: 1.47 (sec). Leaf size: 369

```
DSolve[{x1'[t]==x2[t]+x3[t]+1,x2'[t]==x3[t]+x4[t]+t,x3'[t]==x1[t]+x4[t]+t^2,x4'[t]==x1[t]+x2[t]}
```

$$x1(t) \rightarrow \frac{1}{96}e^{-t}(e^t(-2t(t(t(3t+14)-3)+33) + 3(8(c_1+c_2+c_3+c_4)e^{2t}-3+8c_1-8c_2+8c_3-8c_4)) + 48(c_1-c_3)\cos(t) + 48(c_2-c_4)\sin(t))$$

$$x2(t) \rightarrow \frac{1}{96}e^{-t}(e^t(2t(t(t(3t-22)+3)-9)+3(8(c_1+c_2+c_3+c_4)e^{2t}-35-8c_1+8c_2-8c_3+8c_4)) + 48(c_2-c_4)\cos(t) + 48(c_3-c_1)\sin(t))$$

$$x3(t) \rightarrow \frac{1}{96}e^{-t}(e^t(2t(t((10-3t)t-45)+15) + 3(8(c_1+c_2+c_3+c_4)e^{2t}-19+8c_1-8c_2+8c_3-8c_4)) + 48(c_3-c_1)\cos(t) + 48(c_4-c_2)\sin(t))$$

$$x4(t) \rightarrow \frac{1}{96}e^{-t}(e^t(2t(t(t+3)(3t-7)-57)+3(8(c_1+c_2+c_3+c_4)e^{2t}+13-8c_1+8c_2-8c_3+8c_4)) + 48(c_4-c_2)\cos(t) + 48(c_1-c_3)\sin(t))$$