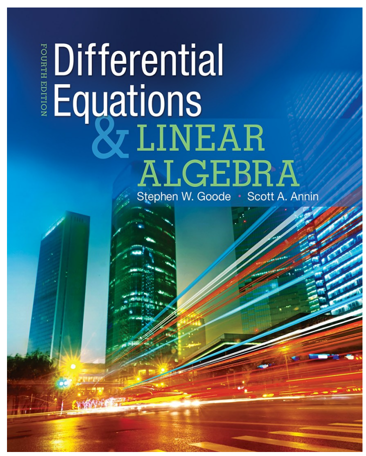
A Solution Manual For

Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015



Nasser M. Abbasi

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1 Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21

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1.1 problem Problem 7

Internal problem ID [2078]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 25y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-25*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{5x} + c_2 e^{-5x}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''[x]-25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{5x} + c_2 e^{-5x}$$

1.2 problem Problem 8

Internal problem ID [2079]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(2x) + c_2 \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2x) + c_2 \sin(2x)$$

1.3 problem Problem 9

Internal problem ID [2080]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[y''[x]+y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-2x} + c_2 e^x$$

1.4 problem Problem 10

Internal problem ID [2081]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{c_1 + x}$$

✓ Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 18

```
DSolve[y'[x]==-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x - c_1}$$
$$y(x) \to 0$$

1.5 problem Problem 11

Internal problem ID [2082]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{2x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=y(x)/(2*x),y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 18

```
DSolve[y'[x]==y[x]/(2*x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\sqrt{x}$$
$$y(x) \to 0$$

1.6 problem Problem 12

Internal problem ID [2083]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 5y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} \sin(2x) + c_2 \cos(2x) e^{-x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y''[x]+2*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2 \cos(2x) + c_1 \sin(2x))$$

1.7 problem Problem 13

Internal problem ID [2084]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 9y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} + c_2 e^{-3x}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''[x]-9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(c_1 e^{6x} + c_2\right)$$

1.8 problem Problem 14

Internal problem ID [2085]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + 5y'x + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2}{x^3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+5*x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^2 + c_1}{x^3}$$

1.9 problem Problem 15

Internal problem ID [2086]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$\boxed{x^2y'' - 3y'x + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2 + c_2x^2 \ln(x)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(2c_2 \log(x) + c_1)$$

1.10 problem Problem 16

Internal problem ID [2087]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 3y'x + 13y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+13*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2 \sin(3\ln(x)) + c_2x^2 \cos(3\ln(x))$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]-3*x*y'[x]+13*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(c_2 \cos(3\log(x)) + c_1 \sin(3\log(x)))$$

1.11 problem Problem 17

Internal problem ID [2088]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - y'x + y - 9x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=9*x^2,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\, c_2 + c_1 x + 3x^2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 23

```
DSolve[2*x^2*y''[x]-x*y'[x]+y[x]==9*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x^2 + c_2 x + c_1\sqrt{x}$$

1.12 problem Problem 18

Internal problem ID [2089]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 4y'x + 6y - \sin(x)\, x^4 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=x^4*sin(x),y(x), singsol=all)
```

$$y(x) = x^2c_2 + c_1x^3 - \sin(x)\, x^2$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 20

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==x^4*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(-\sin(x) + c_2x + c_1)$$

1.13 problem Problem 19

Internal problem ID [2090]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - (a + b)\, y' + aby = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-(a+b)*diff(y(x),x)+a*b*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{ax} + c_2 e^{bx}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[y''[x]-(a+b)*y'[x]+a*b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^{ax} + c_1 e^{bx}$$

1.14 problem Problem 20

Internal problem ID [2091]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2ay' + ya^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-2*a*diff(y(x),x)+a^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{ax} + c_2 e^{ax} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[y''[x]-2*a*y'[x]+a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{ax}(c_2 x + c_1)$$

1.15 problem Problem 21

Internal problem ID [2092]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2ay' + (a^2 + b^2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-2*a*diff(y(x),x)+(a^2+b^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{ax} \sin(bx) + c_2 e^{ax} \cos(bx)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 31

```
DSolve[y''[x]-2*a*y'[x]+(a^2+b^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x(a-ib)} \left(c_2 e^{2ibx} + c_1\right)$$

1.16 problem Problem 22

Internal problem ID [2093]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} + c_2 e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''[x]-y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(c_2 e^{5x} + c_1)$$

1.17 problem Problem 23

Internal problem ID [2094]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-3x} + c_2 \mathrm{e}^{-3x} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[y''[x]+6*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}(c_2 x + c_1)$$

1.18 problem Problem 24

Internal problem ID [2095]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x} + c_2x$$

1.19 problem Problem 25

Internal problem ID [2096]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 5y'x + 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2} + \frac{c_2 \ln(x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+5*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_2 \log(x) + c_1}{x^2}$$

1.20 problem Problem 28

Internal problem ID [2097]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \frac{e^x - \sin(y)}{x\cos(y)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=(exp(x)-sin(y(x)))/(x*cos(y(x))),y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{-c_1 + e^x}{x}\right)$$

✓ Solution by Mathematica

Time used: 11.343 (sec). Leaf size: 16

```
DSolve[y'[x]==(Exp[x]-Sin[y[x]])/(x*Cos[y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{e^x + c_1}{x}\right)$$

1.21 problem Problem 29

Internal problem ID [2098]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Abe`

$$\boxed{y' - \frac{1-y^2}{2+2yx} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=(1-y(x)^2)/(2*(1+x*y(x))),y(x), singsol=all)
```

$$c_1 + \frac{1}{(y(x)-1)(xy(x)+x+2)} = 0$$

✓ Solution by Mathematica

Time used: 0.375 (sec). Leaf size: 56

```
DSolve[y'[x]==(1-y[x]^2)/(2*(1+x*y[x])),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1+\sqrt{1+x(x+c_1)}}{x}$$
$$y(x) \to \frac{-1+\sqrt{1+x(x+c_1)}}{x}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

1.22 problem Problem 30

Internal problem ID [2099]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y' - \frac{(1 - e^{yx} y) e^{-yx}}{x} = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)=(1-y(x)*exp(x*y(x)))/(x*exp(x*y(x))),y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\ln(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.377 (sec). Leaf size: 11

```
DSolve[{y'[x]==(1-y[x]*Exp[x*y[x]])/(x*Exp[x*y[x]]),{y[1]==0}},y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{\log(x)}{x}$$

1.23 problem Problem 31

Internal problem ID [2100]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' - \frac{x^2(1-y^2) + e^{\frac{y}{x}} y}{x\left(e^{\frac{y}{x}} + 2x^2 y\right)} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=(x^2*(1-y(x)^2)+y(x)*exp(y(x)/x))/(x*(exp(y(x)/x)+2*x^2*y(x))),y(x), sing
```

$$y(x) = \operatorname{RootOf}\left(e^{\_Z} + x^3\_Z^2 + c_1 - x\right) x$$

✓ Solution by Mathematica

Time used: 0.293 (sec). Leaf size: 24

```
DSolve[y'[x]==(x^2*(1-y[x]^2)+y[x]*Exp[y[x]/x])/(x*(Exp[y[x]/x]+2*x^2*y[x])),y[x],x,IncludeSi
```

$$\text{Solve}\left[xy(x)^2 + e^{\frac{y(x)}{x}} - x = c_1, y(x)\right]$$

1.24 problem Problem 32

Internal problem ID [2101]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - \frac{\cos(x) - 2xy^2}{2x^2y} = 0}$$

With initial conditions

$$\left[y(\pi) = \frac{1}{\pi}\right]$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=(cos(x)-2*x*y(x)^2)/(2*x^2*y(x)),y(Pi) = 1/Pi],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\sin(x) + 1}}{x}$$

✓ Solution by Mathematica

Time used: 0.301 (sec). Leaf size: 17

```
DSolve[{y'[x]==(Cos[x]-2*x*y[x]^2)/(2*x^2*y[x]),{y[Pi]==1/Pi}},y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{\sqrt{\sin(x) + 1}}{x}$$

1.25 problem Problem 33

Internal problem ID [2102]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=sin(x),y(x), singsol=all)
```

$$y(x) = -\cos(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 12

```
DSolve[y'[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\cos(x) + c_1$$

1.26 problem Problem 34

Internal problem ID [2103]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \frac{1}{x^{\frac{2}{3}}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=x^(-2/3),y(x), singsol=all)
```

$$y(x) = 3x^{\frac{1}{3}} + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 15

```
DSolve[y'[x]==x^(-2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3\sqrt[3]{x} + c_1$$

1.27 problem Problem 35

Internal problem ID [2104]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' - e^{x}x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=x*exp(x),y(x), singsol=all)
```

$$y(x) = (-2 + x)\,e^{x} + c_1 x + c_2$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 19

```
DSolve[y''[x]==x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(x - 2) + c_2 x + c_1$$

1.28 problem Problem 36

Internal problem ID [2105]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' - x^n = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)=x^n,y(x), singsol=all)
```

$$y(x) = \frac{x^{2+n}}{(2+n)(n+1)} + c_1 x + c_2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 28

```
DSolve[y''[x]==x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{n+2}}{n^2 + 3n + 2} + c_2 x + c_1$$

1.29 problem Problem 37

Internal problem ID [2106]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \ln(x)\, x^2 = 0}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(x),x)=x^2*ln(x),y(1) = 2],y(x), singsol=all)
```

$$y(x) = \frac{x^3 \ln(x)}{3} - \frac{x^3}{9} + \frac{19}{9}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 23

```
DSolve[{y'[x]==x^2*Log[x],{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9}\left(-x^3 + 3x^3 \log(x) + 19\right)$$

1.30 problem Problem 38

Internal problem ID [2107]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' - \cos(x) = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)=cos(x),y(0) = 2, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = -\cos(x) + x + 3$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 12

```
DSolve[{y''[x]==Cos[x],{y[0]==2,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \cos(x) + 3$$

1.31 problem Problem 39

Internal problem ID [2108]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 39.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _quadrature]]`

$$\boxed{y''' - 6x = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1, y''(0) = -4]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$3)=6*x,y(0) = 1, D(y)(0) = -1, (D@@2)(y)(0) = -4],y(x), singsol=all)
```

$$y(x) = \frac{1}{4}x^4 - 2x^2 + 1 - x$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[{y'''[x]==6*x,{y[0]==2,y'[0]==-1,y''[0]==-4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}x\left(x^3 - 8x - 4\right) + 2$$

1.32 problem Problem 40

Internal problem ID [2109]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' - \mathrm{e}^x x = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 4]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)=x*exp(x),y(0) = 3, D(y)(0) = 4],y(x), singsol=all)
```

$$y(x) = (-2 + x)\,\mathrm{e}^x + 5x + 5$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 18

```
DSolve[{y''[x]==x*Exp[x],{y[0]==3,y'[0]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(x - 2) + 5(x + 1)$$

1.33 problem Problem 45

Internal problem ID [2110]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{-3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 19

```
DSolve[y''[x]==x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(x - 2) + c_2 x + c_1$$

1.34 problem Problem 46

Internal problem ID [2111]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x - 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^4 c_1 + \frac{c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-x*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x^6 + c_1}{x^2}$$

1.35 problem Problem 47

Internal problem ID [2112]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.2, Basic Ideas and Terminology. page 21
Problem number: Problem 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2 y'' - 3y'x + 4y - \ln(x)\, x^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=x^2*ln(x),y(x), singsol=all)
```

$$y(x) = x^2 c_2 + \ln(x)\, c_1 x^2 + \frac{\ln(x)^3 x^2}{6}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==x^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6} x^2 \left(\log^3(x) + 12 c_2 \log(x) + 6 c_1\right)$$

2 Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43

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2.1 problem Problem 1

Internal problem ID [2113]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=2*x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{x^2}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 18

```
DSolve[y'[x]==2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x^2}$$
$$y(x) \to 0$$

2.2 problem Problem 2

Internal problem ID [2114]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y^2}{x^2+1} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=y(x)^2/(x^2+1),y(x), singsol=all)
```

$$y(x) = -\frac{1}{\arctan(x) - c_1}$$

✓ Solution by Mathematica

Time used: 0.153 (sec). Leaf size: 19

```
DSolve[y'[x]==y[x]^2/(x^2+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\arctan(x) + c_1}$$
$$y(x) \to 0$$

2.3 problem Problem 3

Internal problem ID [2115]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{e^{x+y}y' - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(exp(x+y(x))*diff(y(x),x)-1=0,y(x), singsol=all)
```

$$y(x) = \ln\left(c_1 e^x - 1\right) - x$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 15

```
DSolve[Exp[x+y[x]]*y'[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(\sinh(x) - \cosh(x) + c_1)$$

2.4 problem Problem 4

Internal problem ID [2116]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{x \ln(x)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=y(x)/(x*ln(x)),y(x), singsol=all)
```

$$y(x) = \ln(x) c_1$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 15

```
DSolve[y'[x]==y[x]/(x*Log[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \log(x)$$
$$y(x) \to 0$$

2.5 problem Problem 5

Internal problem ID [2117]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - (x - 1)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve(y(x)-(x-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x - 1)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 16

```
DSolve[y[x]-(x-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x - 1)$$
$$y(x) \to 0$$

2.6 problem Problem 6

Internal problem ID [2118]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2x(y-1)}{x^2+3} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)=(2*x*(y(x)-1))/(x^2+3),y(x), singsol=all)
```

$$y(x) = 1 + \left(x^2+3\right) c_1$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 20

```
DSolve[y'[x]==(2*x*(y[x]-1))/(x^2+3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + c_1\left(x^2+3\right)$$
$$y(x) \to 1$$

2.7 problem Problem 7

Internal problem ID [2119]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - y'x - 3 + 2y'x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(y(x)-x*diff(y(x),x)=3-2*x^2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-\frac{3}{x} + c_1\right) x}{2x - 1}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 24

```
DSolve[y[x]-x*y'[x]==3-2*x^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3 + c_1 x}{1 - 2x}$$
$$y(x) \to 3$$

2.8 problem Problem 8

Internal problem ID [2120]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\cos(x-y)}{\sin(x)\sin(y)} + 1 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)=cos(x-y(x))/(sin(x)*sin(y(x)))-1,y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{1}{\sin(x)\, c_1}\right)$$

✓ Solution by Mathematica

Time used: 5.69 (sec). Leaf size: 47

```
DSolve[y'[x]==Cos[x-y[x]]/(Sin[x]*Sin[y[x]])-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(-\frac{1}{2}c_1\csc(x)\right)$$
$$y(x) \to \arccos\left(-\frac{1}{2}c_1\csc(x)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

2.9 problem Problem 9

Internal problem ID [2121]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x(-1+y^2)}{2\left(-2+x\right)\left(x-1\right)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=x*(y(x)^2-1)/(2*(x-2)*(x-1)),y(x), singsol=all)
```

$$y(x) = -\tanh\left(\ln\left(-2+x\right) - \frac{\ln\left(x-1\right)}{2} + \frac{c_1}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.742 (sec). Leaf size: 51

```
DSolve[y'[x]==x*(y[x]^2-1)/(2*(x-2)*(x-1)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x + e^{2c_1}(x-2)^2 - 1}{-x + e^{2c_1}(x-2)^2 + 1}$$

$$y(x) \to -1$$

$$y(x) \to 1$$

2.10 problem Problem 10

Internal problem ID [2122]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x^2 y - 32}{-x^2 + 16} - 2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)=(x^2*y(x)-32)/(16-x^2)+2,y(x), singsol=all)
```

$$y(x) = \frac{e^{-x}(x^2 + 8x + 16)\, c_1}{(x-4)^2} + 2\, e^{-x} e^{x}$$

✓ Solution by Mathematica

Time used: 0.143 (sec). Leaf size: 30

```
DSolve[y'[x]==(x^2*y[x]-32)/(16-x^2)+2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2 + \frac{c_1 e^{-x}(x+4)^2}{(x-4)^2}$$

$$y(x) \to 2$$

2.11 problem Problem 11

Internal problem ID [2123]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x-a)(x-b)\,y'-y+c=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve((x-a)*(x-b)*diff(y(x),x)-(y(x)-c)=0,y(x), singsol=all)
```

$$y(x)=c+(x-b)^{-\frac{1}{a-b}}(x-a)^{\frac{1}{a-b}}c_1$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 41

```
DSolve[(x-a)*(x-b)*y'[x]-(y[x]-c)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c+c_1(x-b)^{\frac{1}{b-a}}(x-a)^{\frac{1}{a-b}}$$
$$y(x)\to c$$

2.12 problem Problem 12

Internal problem ID [2124]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,y'+y^2+1=0}$$

With initial conditions

$$[y(0)=1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([(x^2+1)*diff(y(x),x)+y(x)^2=-1,y(0) = 1],y(x), singsol=all)
```

$$y(x)=\cot\left(\arctan(x)+\frac{\pi}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.226 (sec). Leaf size: 14

```
DSolve[{(x^2+1)*y'[x]+y[x]^2==-1,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\cot\left(\arctan(x)+\frac{\pi}{4}\right)$$

2.13 problem Problem 13

Internal problem ID [2125]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1-x^2)\,y'+yx-ax=0}$$

With initial conditions

$$[y(0)=2a]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve([(1-x^2)*diff(y(x),x)+x*y(x)=a*x,y(0) = 2*a],y(x), singsol=all)
```

$$y(x)=a\left(1-i\sqrt{x-1}\,\sqrt{x+1}\right)$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 21

```
DSolve[{(1-x^2)*y'[x]+x*y[x]==a*x,{y[0]==2*a}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to a-ia\sqrt{x^2-1}$$

2.14 problem Problem 14

Internal problem ID [2126]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 1 + \frac{\sin(x+y)}{\cos(x)\sin(y)} = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}\right]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=1-(sin(x+y(x)))/(sin(y(x))*cos(x)),y(1/4*Pi) = 1/4*Pi],y(x), singsol=all
```

$$y(x) = \arccos\left(\frac{\sec(x)}{2}\right)$$

✓ Solution by Mathematica

Time used: 6.063 (sec). Leaf size: 10

```
DSolve[{y'[x]==1-(Sin[x+y[x]])/(Sin[y[x]]*Cos[x]),{y[Pi/4]==Pi/4}},y[x],x,IncludeSingularSolu
```

$$y(x) \to \sec^{-1}(2\cos(x))$$

2.15 problem Problem 15

Internal problem ID [2127]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y^3 \sin(x) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=y(x)^3*sin(x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x]==y[x]^3*Sin[x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

2.16 problem Problem 16

Internal problem ID [2128]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \frac{2\sqrt{y-1}}{3} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=2/3*(y(x)-1)^(1/2),y(1) = 1],y(x), singsol=all)
```

$$y(x) = 1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 16

```
DSolve[{y'[x]==1/3*(y[x]-1)^(1/2),{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{36}((x-2)x+37)$$

2.17 problem Problem 17

Internal problem ID [2129]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.4, Separable Differential Equations. page 43
Problem number: Problem 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{mv' - mg + kv^2 = 0}$$

With initial conditions

$$[v(0) = 0]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 26

```
dsolve([m*diff(v(t),t)=m*g-k*v(t)^2,v(0) = 0],v(t), singsol=all)
```

$$v(t) = \frac{\tanh\left(\frac{t\sqrt{mgk}}{m}\right)\sqrt{mgk}}{k}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 39

```
DSolve[{m*v'[t]==m*g-k*v[t]^2,{v[0]==0}},v[t],t,IncludeSingularSolutions -> True]
```

$$v(t) \rightarrow \frac{\sqrt{g}\sqrt{m}\tanh\left(\frac{\sqrt{g}\sqrt{k}t}{\sqrt{m}}\right)}{\sqrt{k}}$$

3 Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59

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3.1 problem Problem 1

Internal problem ID [2130]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y - 4\,\mathrm{e}^{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)=4*exp(x),y(x), singsol=all)
```

$$y(x) = 2\,\mathrm{e}^{x} + \mathrm{e}^{-x} c_1$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 19

```
DSolve[y'[x]+y[x]==4*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2e^{x} + c_1 e^{-x}$$

3.2 problem Problem 2

Internal problem ID [2131]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{2y}{x} - 5x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)+2/x*y(x)=5*x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^5 + c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 15

```
DSolve[y'[x]+2/x*y[x]==5*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^5 + c_1}{x^2}$$

3.3 problem Problem 3

Internal problem ID [2132]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x^2 - 4yx - x^7 \sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)-4*x*y(x)=x^7*sin(x),y(x), singsol=all)
```

$$y(x) = (\sin(x) - \cos(x)\, x + c_1)\, x^4$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 19

```
DSolve[x^2*y'[x]-4*x*y[x]==x^7*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^4(\sin(x) - x\cos(x) + c_1)$$

3.4 problem Problem 4

Internal problem ID [2133]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + 2yx - 2x^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+2*x*y(x)=2*x^3,y(x), singsol=all)
```

$$y(x) = x^2 - 1 + e^{-x^2} c_1$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 20

```
DSolve[y'[x]+2*x*y[x]==2*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + c_1 e^{-x^2} - 1$$

3.5 problem Problem 5

Internal problem ID [2134]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{2xy}{1 - x^2} - 4x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)+2*x/(1-x^2)*y(x)=4*x,y(x), singsol=all)
```

$$y(x) = (2\ln(x-1) + 2\ln(x+1) + c_1)\left(x^2 - 1\right)$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 22

```
DSolve[y'[x]+2*x/(1-x^2)*y[x]==4*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(x^2 - 1\right)\left(2\log\left(x^2 - 1\right) + c_1\right)$$

3.6 problem Problem 6

Internal problem ID [2135]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' + \frac{2xy}{x^2+1} - \frac{4}{(x^2+1)^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)+2*x/(1+x^2)*y(x)=4/(1+x^2)^2,y(x), singsol=all)
```

$$y(x) = \frac{4\arctan(x) + c_1}{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 20

```
DSolve[y'[x]+2*x/(1+x^2)*y[x]==4/(1+x^2)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4\arctan(x) + c_1}{x^2+1}$$

3.7 problem Problem 7

Internal problem ID [2136]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2\cos(x)^2 y' + y\sin(2x) - 4\cos(x)^4 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(2*(cos(x)^2)*diff(y(x),x)+y(x)*sin(2*x)=4*cos(x)^4,y(x), singsol=all)
```

$$y(x) = (2\sin(x) + c_1)\cos(x)$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 15

```
DSolve[2*(Cos[x]^2)*y'[x]+y[x]*Sin[2*x]==4*Cos[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)(2\sin(x) + c_1)$$

3.8 problem Problem 8

Internal problem ID [2137]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{x\ln(x)} - 9x^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)+1/(x*ln(x))*y(x)=9*x^2,y(x), singsol=all)
```

$$y(x) = \frac{3x^3\ln(x) - x^3 + c_1}{\ln(x)}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 24

```
DSolve[y'[x]+1/(x*Log[x])*y[x]==9*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x^3 + \frac{-x^3 + c_1}{\log(x)}$$

3.9 problem Problem 9

Internal problem ID [2138]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - y\tan(x) - 8\sin(x)^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)-y(x)*tan(x)=8*sin(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{-\cos(2x) + \frac{\cos(4x)}{4} + c_1}{\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 19

```
DSolve[y'[x]-y[x]*Tan[x]==8*Sin[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\sin^3(x)\tan(x) + c_1\sec(x)$$

3.10 problem Problem 10

Internal problem ID [2139]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{tx' + 2x - 4\,\mathrm{e}^{t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(t*diff(x(t),t)+2*x(t)=4*exp(t),x(t), singsol=all)
```

$$x(t) = \frac{4(t-1)\,\mathrm{e}^{t} + c_1}{t^2}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 20

```
DSolve[t*x'[t]+2*x[t]==4*Exp[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{4e^t(t-1) + c_1}{t^2}$$

3.11 problem Problem 11

Internal problem ID [2140]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \sin(x)\,(y\sec(x) - 2) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=sin(x)*(y(x)*sec(x)-2),y(x), singsol=all)
```

$$y(x) = \frac{\frac{\cos(2x)}{2} + c_1}{\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 20

```
DSolve[y'[x]==Sin[x]*(y[x]*Sec[x]-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sec(x)(\cos(2x) + 2c_1)$$

3.12 problem Problem 12

Internal problem ID [2141]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{1 - \sin(x) y - y' \cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve((1-y(x)*sin(x))-cos(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (\tan(x) + c_1) \cos(x)$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 13

```
DSolve[(1-y[x]*Sin[x])-Cos[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1 \cos(x)$$

3.13 problem Problem 13

Internal problem ID [2142]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{x} - 2\ln(x)\, x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)-1/x*y(x)=2*x^2*ln(x),y(x), singsol=all)
```

$$y(x) = \left(\ln(x)\, x^2 - \frac{x^2}{2} + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 23

```
DSolve[y'[x]-1/x*y[x]==2*x^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^3}{2} + x^3 \log(x) + c_1 x$$

3.14 problem Problem 14

Internal problem ID [2143]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + \alpha y - e^{\beta x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)+alpha*y(x)=exp(beta*x),y(x), singsol=all)
```

$$y(x) = \left(\frac{e^{x(\alpha+\beta)}}{\alpha+\beta} + c_1\right) e^{-\alpha x}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 31

```
DSolve[y'[x]+\[Alpha]*y[x]==Exp[\[Beta]*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\alpha(-x)}\left(e^{x(\alpha+\beta)} + c_1(\alpha+\beta)\right)}{\alpha+\beta}$$

3.15 problem Problem 15

Internal problem ID [2144]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{my}{x} - \ln(x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(diff(y(x),x)+m/x*y(x)=ln(x),y(x), singsol=all)
```

$$y(x) = \frac{\ln(x)\,x}{m+1} - \frac{x}{m^2+2m+1} + x^{-m}c_1$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 29

```
DSolve[y'[x]+m/x*y[x]==Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x((m+1)\log(x)-1)}{(m+1)^2} + c_1x^{-m}$$

3.16 problem Problem 16

Internal problem ID [2145]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{2y}{x} - 4x = 0}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)+2/x*y(x)=4*x,y(1) = 2],y(x), singsol=all)
```

$$y(x) = \frac{x^4 + 1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 12

```
DSolve[{y'[x]+2/x*y[x]==4*x,{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + \frac{1}{x^2}$$

3.17 problem Problem 17

Internal problem ID [2146]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\sin(x) y' - \cos(x) y - \sin(2x) = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 2\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([sin(x)*diff(y(x),x)-y(x)*cos(x)=sin(2*x),y(1/2*Pi) = 2],y(x), singsol=all)
```

$$y(x) = (2\ln(\sin(x)) + 2)\sin(x)$$

✓ Solution by Mathematica

Time used: 0.503 (sec). Leaf size: 22

```
DSolve[{Sin[x]*y'[x]-y[x]*Cos[x]==Sin[2*x],{y[Pi/2]==2}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to 2\sin(x)(\log(\tan(x)) + \log(\cos(x)) - 2i\pi + 1)$$

3.18 problem Problem 18

Internal problem ID [2147]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x' + \frac{2x}{4-t} - 5 = 0}$$

With initial conditions

$$[x(0) = 4]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(x(t),t)+2/(4-t)*x(t)=5,x(0) = 4],x(t), singsol=all)
```

$$x(t) = -t^2 + 3t + 4$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 13

```
DSolve[{x'[t]+2/(4-t)*x[t]==5,{x[0]==4}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -((t-4)(t+1))$$

3.19 problem Problem 19

Internal problem ID [2148]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y + y' - e^{x} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([y(x)-exp(x)+diff(y(x),x)=0,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{e^{x}}{2} + \frac{e^{-x}}{2}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 7

```
DSolve[{y[x]-Exp[x]+y'[x]==0,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cosh(x)$$

3.20 problem Problem 20

Internal problem ID [2149]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - 2y - \left(\begin{cases} 1 & x \leq 1 \\ 0 & 1 < x \end{cases}\right) = 0}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 27

```
dsolve([diff(y(x),x)-2*y(x)=piecewise(x<=1,1,x>1,0),y(0) = 3],y(x), singsol=all)
```

$$y(x) = \frac{7\,\mathrm{e}^{2x}}{2} - \frac{\left(\begin{cases} 1 & x < 1 \\ \mathrm{e}^{2x-2} & 1 \leq x \end{cases}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 42

```
DSolve[{ode = y'[x] - 2*y[x] == Piecewise[{{1, x <= 1}, {0, x > 1}}],{y[0]==3}},y[x],x,Includ
```

$$y(x) \to \begin{cases} \frac{1}{2}(-1 + 7e^{2x}) & x \leq 1 \\ \frac{1}{2}e^{2x-2}(-1 + 7e^2) & \text{True} \end{cases}$$

3.21 problem Problem 21

Internal problem ID [2150]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - 2y - \left(\begin{cases} 1-x & x<1 \\ 0 & 1\leq x \end{cases}\right) = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 31

```
dsolve([diff(y(x),x)-2*y(x)=piecewise(x<1,1-x,x>=1,0),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{5\,\mathrm{e}^{2x}}{4} + \frac{\left(\begin{cases} 2x-1 & x<1 \\ \mathrm{e}^{2x-2} & 1\leq x \end{cases}\right)}{4}$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 45

```
DSolve[{y'[x] - 2*y[x] == Piecewise[{{1-x, x < 1}, {0, x >= 1}}],{y[0]==1}},y[x],x,IncludeSin
```

$$y(x) \to \begin{cases} \frac{1}{4}(2x + 5e^{2x} - 1) & x \leq 1 \\ \frac{1}{4}e^{2x-2}(1+5e^2) & \text{True} \end{cases}$$

3.22 problem Problem 22

Internal problem ID [2151]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + \frac{y'}{x} - 9x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)=9*x,y(x), singsol=all)
```

$$y(x) = x^3 + \ln(x)\, c_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 16

```
DSolve[y''[x]+1/x*y'[x]==9*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3 + c_1 \log(x) + c_2$$

3.23 problem Problem 30

Internal problem ID [2152]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{x} - \cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+1/x*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)\,x + \cos(x) + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 17

```
DSolve[y'[x]+1/x*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + \frac{\cos(x) + c_1}{x}$$

3.24 problem Problem 31

Internal problem ID [2153]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y - e^{-2x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+y(x)=exp(-2*x),y(x), singsol=all)
```

$$y(x) = \left(-e^{-x} + c_1\right) e^{-x}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 19

```
DSolve[y'[x]+y[x]==Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(-1 + c_1 e^x)$$

3.25 problem Problem 32

Internal problem ID [2154]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \cot(x) y - 2\cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+y(x)*cot(x)=2*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{-\frac{\cos(2x)}{2} + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 17

```
DSolve[y'[x]+y[x]*Cot[x]==2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + \left(-\frac{1}{2} + c_1\right)\csc(x)$$

3.26 problem Problem 33

Internal problem ID [2155]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.6, First-Order Linear Differential Equations. page 59
Problem number: Problem 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - y - \ln(x)\, x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)-y(x)=x^2*ln(x),y(x), singsol=all)
```

$$y(x) = (x\ln(x) - x + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 17

```
DSolve[x*y'[x]-y[x]==x^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(-x + x\log(x) + c_1)$$

4 Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79

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4.1 problem Problem 9

Internal problem ID [2156]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y' - \frac{x^2 + yx + y^2}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=(y(x)^2+x*y(x)+x^2)/x^2,y(x), singsol=all)
```

$$y(x) = \tan(\ln(x) + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.184 (sec). Leaf size: 13

```
DSolve[y'[x]==(y[x]^2+x*y[x]+x^2)/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \tan(\log(x) + c_1)$$

4.2 problem Problem 10

Internal problem ID [2157]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(-y+3x)\, y'-3y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve((3*x-y(x))*diff(y(x),x)=3*y(x),y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\mathrm{LambertW}\left(-3x\,\mathrm{e}^{-3c_1}\right)+3c_1}$$

✓ Solution by Mathematica

Time used: 5.805 (sec). Leaf size: 25

```
DSolve[(3*x-y[x])*y'[x]==3*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{W\left(-3e^{-c_1}x\right)+c_1}$$
$$y(x)\to 0$$

4.3 problem Problem 11

Internal problem ID [2158]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Riccati]`

$$\boxed{y' - \frac{(x+y)^2}{2x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=(x+y(x))^2/(2*x^2),y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{\ln(x)}{2} + \frac{c_1}{2}\right) x$$

✓ Solution by Mathematica

Time used: 0.203 (sec). Leaf size: 17

```
DSolve[y'[x]==(x+y[x])^2/(2*x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \tan\left(\frac{\log(x)}{2} + c_1\right)$$

4.4 problem Problem 12

Internal problem ID [2159]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{\sin\left(\frac{y}{x}\right)(y'x-y)-x\cos\left(\frac{y}{x}\right)=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(sin(y(x)/x)*(x*diff(y(x),x)-y(x))=x*cos(y(x)/x),y(x), singsol=all)
```

$$y(x)=x\arccos\left(\frac{1}{c_1x}\right)$$

✓ Solution by Mathematica

Time used: 24.469 (sec). Leaf size: 48

```
DSolve[Sin[y[x]/x]*(x*y'[x]-y[x])==x*Cos[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x\sec^{-1}\left(e^{c_1}x\right)$$
$$y(x)\to x\sec^{-1}\left(e^{c_1}x\right)$$
$$y(x)\to -\frac{\pi x}{2}$$
$$y(x)\to \frac{\pi x}{2}$$

4.5 problem Problem 13

Internal problem ID [2160]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y'x - \sqrt{16x^2 - y^2} - y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x)=sqrt(16*x^2-y(x)^2)+y(x),y(x), singsol=all)
```

$$-\arctan\left(\frac{y(x)}{\sqrt{16x^2 - y(x)^2}}\right) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.349 (sec). Leaf size: 18

```
DSolve[x*y'[x]==Sqrt[16*x^2-y[x]^2]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4x \cosh(i \log(x) + c_1)$$

4.6 problem Problem 14

Internal problem ID [2161]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$y'x - y - \sqrt{9x^2 + y^2} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(9*x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{y(x)}{x^2} + \frac{\sqrt{9x^2 + y(x)^2}}{x^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.325 (sec). Leaf size: 27

```
DSolve[x*y'[x]-y[x]==Sqrt[9*x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{9e^{c_1}x^2}{2} - \frac{e^{-c_1}}{2}$$

4.7 problem Problem 15

Internal problem ID [2162]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y(x^2-y^2)-x(x^2-y^2)\,y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(y(x)*(x^2-y(x)^2)-x*(x^2-y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -x$$
$$y(x) = x$$
$$y(x) = c_1x$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 33

```
DSolve[y[x]*(x^2-y[x]^2)-x*(x^2-y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x$$
$$y(x) \to x$$
$$y(x) \to c_1x$$
$$y(x) \to -x$$
$$y(x) \to x$$

4.8 problem Problem 16

Internal problem ID [2163]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'x + \ln(x)\, y - y\ln(y) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+y(x)*ln(x)=y(x)*ln(y(x)),y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^{-c_1 x}\mathrm{e}$$

✓ Solution by Mathematica

Time used: 0.222 (sec). Leaf size: 24

```
DSolve[x*y'[x]+y[x]*Log[x]==y[x]*Log[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to xe^{1+e^{c_1 x}}$$
$$y(x) \to ex$$

4.9 problem Problem 17

Internal problem ID [2164]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y' - \frac{y^2 + 2yx - 2x^2}{x^2 - yx + y^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 79

```
dsolve(diff(y(x),x)=(y(x)^2+2*x*y(x)-2*x^2)/(x^2-x*y(x)+y(x)^2),y(x), singsol=all)
```

$$y(x) = -\frac{x\left(\operatorname{RootOf}\left(2\_Z^6 + (9c_1x^2 - 1)\_Z^4 - 6x^2c_1\_Z^2 + c_1x^2\right)^2 - 1\right)}{\operatorname{RootOf}\left(2\_Z^6 + (9c_1x^2 - 1)\_Z^4 - 6x^2c_1\_Z^2 + c_1x^2\right)^2}$$

✓ Solution by Mathematica

Time used: 60.191 (sec). Leaf size: 372

```
DSolve[y'[x]==(y[x]^2+2*x*y[x]-2*x^2)/(x^2-x*y[x]+y[x]^2),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{\sqrt[3]{-54x^3 + 2\sqrt{729x^6 + (-9x^2 + 3e^{2c_1})^3}}}{3\sqrt[3]{2}} - \frac{\sqrt[3]{2}(-3x^2 + e^{2c_1})}{\sqrt[3]{-54x^3 + 2\sqrt{729x^6 + (-9x^2 + 3e^{2c_1})^3}}} + x$$

$$y(x) \to \frac{(1 + i\sqrt{3})(-3x^2 + e^{2c_1})}{2^{2/3}\sqrt[3]{-54x^3 + 2\sqrt{729x^6 + (-9x^2 + 3e^{2c_1})^3}}} + \left(-\frac{1}{3}\right)^{2/3}\sqrt[3]{-9x^3 + \sqrt{3}\sqrt{27e^{2c_1}x^4 - 9e^{4c_1}x^2 + e^{6c_1}}} + x$$

$$y(x) \to -\frac{(1 + i\sqrt{3})\sqrt[3]{-54x^3 + 2\sqrt{729x^6 + (-9x^2 + 3e^{2c_1})^3}}}{6\sqrt[3]{2}} + \frac{(1 - i\sqrt{3})(-3x^2 + e^{2c_1})}{2^{2/3}\sqrt[3]{-54x^3 + 2\sqrt{729x^6 + (-9x^2 + 3e^{2c_1})^3}}} + x$$

4.10 problem Problem 18

Internal problem ID [2165]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘]]`

$$\boxed{2xyy' - 2y^2 - x^2 e^{-\frac{y^2}{x^2}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(2*x*y(x)*diff(y(x),x)-(x^2*exp(-y(x)^2/x^2)+2*y(x)^2)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{\ln(\ln(x) + c_1)}\, x$$
$$y(x) = -\sqrt{\ln(\ln(x) + c_1)}\, x$$

✓ Solution by Mathematica

Time used: 2.141 (sec). Leaf size: 38

```
DSolve[2*x*y[x]*y'[x]-(x^2*Exp[-y[x]^2/x^2]+2*y[x]^2)==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -x\sqrt{\log(\log(x) + 2c_1)}$$
$$y(x) \to x\sqrt{\log(\log(x) + 2c_1)}$$

4.11 problem Problem 19

Internal problem ID [2166]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Riccati]`

$$\boxed{y'x^2 - y^2 - 3yx - x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x)=y(x)^2+3*x*y(x)+x^2,y(x), singsol=all)
```

$$y(x) = -\frac{x(\ln(x) + c_1 + 1)}{\ln(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.147 (sec). Leaf size: 25

```
DSolve[x^2*y'[x]==y[x]^2+3*x*y[x]+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(-1 - \frac{1}{\log(x) + c_1}\right)$$
$$y(x) \to -x$$

4.12 problem Problem 20

Internal problem ID [2167]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{yy' + x - \sqrt{x^2 + y^2} = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 28

```
dsolve(y(x)*diff(y(x),x)=sqrt(x^2+y(x)^2)-x,y(x), singsol=all)
```

$$-c_1 + \frac{\sqrt{x^2 + y(x)^2}}{y(x)^2} + \frac{x}{y(x)^2} = 0$$

✓ Solution by Mathematica

Time used: 0.375 (sec). Leaf size: 57

```
DSolve[y[x]*y'[x]==Sqrt[x^2+y[x]^2]-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -e^{\frac{c_1}{2}}\sqrt{2x + e^{c_1}}$$
$$y(x) \rightarrow e^{\frac{c_1}{2}}\sqrt{2x + e^{c_1}}$$
$$y(x) \rightarrow 0$$

4.13 problem Problem 21

Internal problem ID [2168]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{2x(2x+y)\,y'-y(4x-y)=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(2*x*(y(x)+2*x)*diff(y(x),x)=y(x)*(4*x-y(x)),y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\mathrm{LambertW}\left(2\,\mathrm{e}^{\frac{3c_1}{2}}x^{\frac{3}{2}}\right)-\frac{3c_1}{2}-\frac{3\ln(x)}{2}}x$$

✓ Solution by Mathematica

Time used: 5.204 (sec). Leaf size: 29

```
DSolve[2*x*(y[x]+2*x)*y'[x]==y[x]*(4*x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2x}{W\left(2e^{-c_1}x^{3/2}\right)}$$
$$y(x)\to 0$$

4.14 problem Problem 22

Internal problem ID [2169]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'x - \tan\left(\frac{y}{x}\right)x - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x)=x*tan(y(x)/x)+y(x),y(x), singsol=all)
```

$$y(x) = \arcsin(c_1 x)\, x$$

✓ Solution by Mathematica

Time used: 8.362 (sec). Leaf size: 19

```
DSolve[x*y'[x]==x*Tan[y[x]/x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin\left(e^{c_1} x\right)$$
$$y(x) \to 0$$

4.15 problem Problem 23

Internal problem ID [2170]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y' - \frac{\sqrt{x^2+y^2}\,x + y^2}{yx} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=(x*sqrt(y(x)^2+x^2)+y(x)^2)/(x*y(x)),y(x), singsol=all)
```

$$-\frac{\sqrt{x^2+y(x)^2}}{x} + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.283 (sec). Leaf size: 48

```
DSolve[y'[x]==(x*Sqrt[y[x]^2+x^2]+y[x]^2)/(x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{(\log(x) - 1 + c_1)(\log(x) + 1 + c_1)}$$
$$y(x) \to x\sqrt{(\log(x) - 1 + c_1)(\log(x) + 1 + c_1)}$$

4.16 problem Problem 25

Internal problem ID [2171]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{2(-x+2y)}{x+y} = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.859 (sec). Leaf size: 273

```
dsolve([diff(y(x),x)=2*(2*y(x)-x)/(x+y(x)),y(0) = 2],y(x), singsol=all)
```

$$y(x) = \frac{\left(3\sqrt{3}\,x\sqrt{x\,(27x+8)}+27x^2+36x+8\right)^{\frac{1}{3}}}{3} + \frac{4x+\frac{4}{3}}{\left(3\sqrt{3}\,x\sqrt{x\,(27x+8)}+27x^2+36x+8\right)^{\frac{1}{3}}} + 2x + \frac{2}{3}$$

✓ Solution by Mathematica

Time used: 60.261 (sec). Leaf size: 122

```
DSolve[{y'[x]==2*(2*y[x]-x)/(x+y[x]),{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(6x\left(\frac{2}{\sqrt[3]{3\sqrt{3}\sqrt{x^3(27x+8)}+9x(3x+4)+8}}+1\right)+\sqrt[3]{3\sqrt{3}\sqrt{x^3(27x+8)}+9x(3x+4)+8}+\frac{4}{\sqrt[3]{3\sqrt{3}\sqrt{x^3(27x+8)}+9x(3x+4)+8}}+2\right)$$

4.17 problem Problem 26

Internal problem ID [2172]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{2x - y}{x + 4y} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 19

```
dsolve([diff(y(x),x)=(2*x-y(x))/(x+4*y(x)),y(1) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{x}{4} + \frac{\sqrt{9x^2 + 16}}{4}$$

✓ Solution by Mathematica

Time used: 0.422 (sec). Leaf size: 24

```
DSolve[{y'[x]==(2*x-y[x])/(x+4*y[x]),{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(\sqrt{9x^2 + 16} - x\right)$$

4.18 problem Problem 27

Internal problem ID [2173]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y' - \frac{y - \sqrt{x^2 + y^2}}{x} = 0}$$

With initial conditions

$$[y(3) = 4]$$

✓ Solution by Maple

Time used: 0.422 (sec). Leaf size: 21

```
dsolve([diff(y(x),x)=(y(x)-sqrt(x^2+y(x)^2))/x,y(3) = 4],y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} - \frac{1}{2}$$
$$y(x) = -\frac{x^2}{18} + \frac{9}{2}$$

✓ Solution by Mathematica

Time used: 0.234 (sec). Leaf size: 28

```
DSolve[{y'[x]==(y[x]-Sqrt[x^2+y[x]^2])/x,{y[3]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{18}(x-9)(x+9)$$
$$y(x) \to \frac{1}{2}(x^2-1)$$

4.19 problem Problem 28

Internal problem ID [2174]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y'x - y - \sqrt{4x^2 - y^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(4*x^2-y(x)^2),y(x), singsol=all)
```

$$-\arctan\left(\frac{y(x)}{\sqrt{4x^2 - y(x)^2}}\right) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.376 (sec). Leaf size: 18

```
DSolve[x*y'[x]-y[x]==Sqrt[4*x^2-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x \cosh(i \log(x) + c_1)$$

4.20 problem Problem 29(a)

Internal problem ID [2175]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 29(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{x + ay}{ax - y} = 0}$$

✓ Solution by Maple

Time used: 0.296 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=(x+a*y(x))/(a*x-y(x)),y(x), singsol=all)
```

$$y(x) = \tan\left(\operatorname{RootOf}\left(-2a\_Z + \ln\left(\frac{x^2}{\cos(\_Z)^2}\right) + 2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 34

```
DSolve[y'[x]==(x+a*y[x])/(a*x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[a\arctan\left(\frac{y(x)}{x}\right) - \frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + 1\right) = \log(x) + c_1, y(x)\right]$$

4.21 problem Problem 29(b)

Internal problem ID [2176]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 29(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{x + \frac{y}{2}}{\frac{x}{2} - y} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 30

```
dsolve([diff(y(x),x)=(x+1/2*y(x))/(1/2*x-y(x)),y(1) = 1],y(x), singsol=all)
```

$$y(x) = \tan\left(\mathrm{RootOf}\left(4\_Z - 4\ln\left(\sec\left(\_Z\right)^2\right) - 8\ln\left(x\right) + 4\ln\left(2\right) - \pi\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 42

```
DSolve[{y'[x]==(x+1/2*y[x])/(1/2*x-y[x]),{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\log\left(\frac{y(x)^2}{x^2} + 1\right) - \arctan\left(\frac{y(x)}{x}\right) = \frac{1}{4}(4\log(2) - \pi) - 2\log(x), y(x)\right]$$

4.22 problem Problem 38

Internal problem ID [2177]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Bernoulli]`

$$\boxed{y' - \frac{y}{x} - \frac{4x^2 \cos(x)}{y} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)-1/x*y(x)=4*x^2/y(x)*cos(x),y(x), singsol=all)
```

$$y(x) = \sqrt{8\sin(x) + c_1}\, x$$
$$y(x) = -\sqrt{8\sin(x) + c_1}\, x$$

✓ Solution by Mathematica

Time used: 0.27 (sec). Leaf size: 36

```
DSolve[y'[x]-1/x*y[x]==4*x^2/y[x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{8\sin(x) + c_1}$$
$$y(x) \to x\sqrt{8\sin(x) + c_1}$$

4.23 problem Problem 39

Internal problem ID [2178]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' + \frac{y\tan(x)}{2} - 2y^3\sin(x) = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 66

```
dsolve(diff(y(x),x)+1/2*tan(x)*y(x)=2*y(x)^3*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-\left(2\sin(x)^2 - c_1\right)\cos(x)}}{2\sin(x)^2 - c_1}$$

$$y(x) = -\frac{\sqrt{-\left(2\sin(x)^2 - c_1\right)\cos(x)}}{2\sin(x)^2 - c_1}$$

✓ Solution by Mathematica

Time used: 5.026 (sec). Leaf size: 215

```
DSolve[y'[x]+1/2*Tan(x)*y[x]==2*y[x]^3*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{\frac{1}{4}/\text{Tan}}\sqrt[4]{\text{Tan}}}{\sqrt{e^{\frac{\text{Tan}x^2}{2}}\left(\sqrt{2\pi}\left(\text{erfi}\left(\frac{1+i\text{Tan}x}{\sqrt{2}\sqrt{\text{Tan}}}\right)-i\text{erf}\left(\frac{\text{Tan}x+i}{\sqrt{2}\sqrt{\text{Tan}}}\right)\right)+c_1e^{\frac{1}{2}/\text{Tan}}\sqrt{\text{Tan}}\right)}}$$

$$y(x) \to \frac{e^{\frac{1}{4}/\text{Tan}}\sqrt[4]{\text{Tan}}}{\sqrt{e^{\frac{\text{Tan}x^2}{2}}\left(\sqrt{2\pi}\left(\text{erfi}\left(\frac{1+i\text{Tan}x}{\sqrt{2}\sqrt{\text{Tan}}}\right)-i\text{erf}\left(\frac{\text{Tan}x+i}{\sqrt{2}\sqrt{\text{Tan}}}\right)\right)+c_1e^{\frac{1}{2}/\text{Tan}}\sqrt{\text{Tan}}\right)}}$$

$$y(x) \to 0$$

4.24 problem Problem 40

Internal problem ID [2179]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - \frac{3y}{2x} - 6y^{\frac{1}{3}}x^2 \ln(x) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)-3/(2*x)*y(x)=6*y(x)^(1/3)*x^2*ln(x),y(x), singsol=all)
```

$$-2x^3 \ln(x) + x^3 + y(x)^{\frac{2}{3}} - c_1 x = 0$$

✓ Solution by Mathematica

Time used: 0.727 (sec). Leaf size: 26

```
DSolve[y'[x]-3/(2*x)*y[x]==6*y[x]^(1/3)*x^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(x\left(-x^2 + 2x^2 \log(x) + c_1\right)\right)^{3/2}$$

4.25 problem Problem 41

Internal problem ID [2180]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$
\boxed{y' + \frac{2y}{x} - 6\sqrt{x^2+1}\,\sqrt{y} = 0}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)+2/x*y(x)=6*sqrt(1+x^2)*sqrt(y(x)),y(x), singsol=all)
```

$$
\sqrt{y(x)} - \frac{(x^2+1)^{\frac{3}{2}} + c_1}{x} = 0
$$

✓ Solution by Mathematica

Time used: 0.217 (sec). Leaf size: 55

```
DSolve[y'[x]+2/x*y[x]==6*Sqrt[1+x^2]*Sqrt[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to \frac{x^6 + 3x^4 + x^2\left(3 + 2c_1\sqrt{x^2+1}\right) + 2c_1\sqrt{x^2+1} + 1 + {c_1}^2}{x^2}
$$

4.26 problem Problem 42

Internal problem ID [2181]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{y' + \frac{2y}{x} - 6y^2x^4 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+2/x*y(x)=6*y(x)^2*x^4,y(x), singsol=all)
```

$$y(x) = \frac{1}{(-2x^3 + c_1)\,x^2}$$

✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 24

```
DSolve[y'[x]+2/x*y[x]==6*y[x]^2*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{-2x^5 + c_1x^2}$$

$$y(x) \to 0$$

4.27 problem Problem 43

Internal problem ID [2182]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{2x(y' + x^2y^3) + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(2*x*(diff(y(x),x)+y(x)^3*x^2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{x^3 + c_1 x}}$$
$$y(x) = -\frac{1}{\sqrt{x^3 + c_1 x}}$$

✓ Solution by Mathematica

Time used: 0.277 (sec). Leaf size: 40

```
DSolve[2*x*(y'[x]+y[x]^3*x^2)+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x(x^2 + c_1)}}$$
$$y(x) \to \frac{1}{\sqrt{x(x^2 + c_1)}}$$
$$y(x) \to 0$$

4.28 problem Problem 44

Internal problem ID [2183]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(x-a)(x-b)\left(y'-\sqrt{y}\right)-2(-a+b)\,y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 80

```
dsolve((x-a)*(x-b)*(diff(y(x),x)-sqrt(y(x)))=2*(b-a)*y(x),y(x), singsol=all)
```

$$\sqrt{y(x)}-\frac{x(x-b)}{2(x-a)}+\frac{a\ln(x-b)(x-b)}{2x-2a}-\frac{b\ln(x-b)(x-b)}{2(x-a)}-\frac{c_1(x-b)}{x-a}=0$$

✓ Solution by Mathematica

Time used: 0.457 (sec). Leaf size: 43

```
DSolve[(x-a)*(x-b)*(y'[x]-Sqrt[y[x]])==2*(b-a)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{(b-x)^2((b-a)\log(x-b)+x+2c_1)^2}{4(a-x)^2}$$

4.29 problem Problem 45

Internal problem ID [2184]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + \frac{6y}{x} - \frac{3y^{\frac{2}{3}} \cos (x)}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)+6/x*y(x)=3/x*y(x)^(2/3)*cos(x),y(x), singsol=all)
```

$$y(x)^{\frac{1}{3}} - \frac{\sin (x) x + \cos (x) + c_1}{x^2} = 0$$

✓ Solution by Mathematica

Time used: 0.193 (sec). Leaf size: 20

```
DSolve[y'[x]+6/x*y[x]==3/x*y[x]^(2/3)*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x \sin(x) + \cos(x) + c_1)^3}{x^6}$$

4.30 problem Problem 46

Internal problem ID [2185]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + 4yx - 4x^3\sqrt{y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)+4*x*y(x)=4*x^3*sqrt(y(x)),y(x), singsol=all)
```

$$-x^2 + 1 - \mathrm{e}^{-x^2} c_1 + \sqrt{y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.152 (sec). Leaf size: 29

```
DSolve[y'[x]+4*x*y[x]==4*x^3*Sqrt[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x^2}\left(e^{x^2}(x^2 - 1) + c_1\right)^2$$

4.31 problem Problem 47

Internal problem ID [2186]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - \frac{y}{2x \ln(x)} - 2x y^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 90

```
dsolve(diff(y(x),x)-1/(2*x*ln(x))*y(x)=2*x*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-(2\ln(x)\, x^2 - x^2 - c_1)\ln(x)}}{2\ln(x)\, x^2 - x^2 - c_1}$$

$$y(x) = -\frac{\sqrt{-(2\ln(x)\, x^2 - x^2 - c_1)\ln(x)}}{2\ln(x)\, x^2 - x^2 - c_1}$$

✓ Solution by Mathematica

Time used: 0.249 (sec). Leaf size: 63

```
DSolve[y'[x]-1/(2*x*Log[x])*y[x]==2*x*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\log(x)}}{\sqrt{x^2 - 2x^2\log(x) + c_1}}$$

$$y(x) \to \frac{\sqrt{\log(x)}}{\sqrt{x^2 - 2x^2\log(x) + c_1}}$$

$$y(x) \to 0$$

4.32 problem Problem 48

Internal problem ID [2187]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{y}{(\pi - 1)\,x} - \frac{3xy^{\pi}}{1 - \pi} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)-1/( (Pi-1)*x)*y(x)=3/(1-Pi)*x*y(x)^Pi,y(x), singsol=all)
```

$$y(x) = \left(\frac{x^3 + c_1}{x}\right)^{-\frac{1}{\pi - 1}}$$

✓ Solution by Mathematica

Time used: 0.913 (sec). Leaf size: 28

```
DSolve[y'[x]-1/( (Pi-1)*x)*y[x]==3/(1-Pi)*x*y[x]^Pi,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{x^3 + c_1}{x}\right)^{\frac{1}{1 - \pi}}$$
$$y(x) \to 0$$

4.33 problem Problem 49

Internal problem ID [2188]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{2y' + \cot(x)\, y - \frac{8\cos(x)^3}{y} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 64

```
dsolve(2*diff(y(x),x)+y(x)*cot(x)=8/y(x)*cos(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-\sin(x)\left(2\sin(x)^4 - 4\sin(x)^2 - c_1 + 2\right)}}{\sin(x)}$$
$$y(x) = -\frac{\sqrt{-\sin(x)\left(2\sin(x)^4 - 4\sin(x)^2 - c_1 + 2\right)}}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 3.926 (sec). Leaf size: 47

```
DSolve[2*y'[x]+y[x]*Cot[x]==8/y[x]*Cos[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-2\cos^3(x)\cot(x) + c_1\csc(x)}$$
$$y(x) \to \sqrt{-2\cos^3(x)\cot(x) + c_1\csc(x)}$$

4.34 problem Problem 50

Internal problem ID [2189]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\left(1-\sqrt{3}\right) y' + y \sec(x) - y^{\sqrt{3}} \sec(x) = 0$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 54

```
dsolve((1-sqrt(3))*diff(y(x),x)+y(x)*sec(x)=y(x)^sqrt(3)*sec(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{c_1 \cos(x)+\sin(x)+1}{\sin(x)+1}\right)^{-\frac{\sqrt{3}}{2}}}{\sqrt{\frac{\cos(x)c_1}{\sin(x)+1}+\frac{\sin(x)}{\sin(x)+1}+\frac{1}{\sin(x)+1}}}$$

✓ Solution by Mathematica

Time used: 0.573 (sec). Leaf size: 74

```
DSolve[(1-Sqrt[3])*y'[x]+y[x]*Sec[x]==y[x]^Sqrt[3]*Sec[x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \text{InverseFunction}\left[\frac{\log\left(1-\#1^{\sqrt{3}-1}\right)-\left(\sqrt{3}-1\right)\log(\#1)}{\sqrt{3}-1}\&\right]\left[-\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)+c_1\right]$$

$$y(x) \to 0$$

$$y(x) \to 1$$

4.35 problem Problem 51

Internal problem ID [2190]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{y' + \frac{2xy}{x^2+1} - xy^2 = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 23

```
dsolve([diff(y(x),x)+2*x/(1+x^2)*y(x)=x*y(x)^2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{2}{(x^2+1)(\ln(x^2+1)-2)}$$

✓ Solution by Mathematica

Time used: 0.198 (sec). Leaf size: 24

```
DSolve[{y'[x]+2*x/(1+x^2)*y[x]==x*y[x]^2,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{(x^2+1)(\log(x^2+1)-2)}$$

4.36 problem Problem 52

Internal problem ID [2191]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + \cot(x)\, y - y^3 \sin(x)^3 = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 1\right]$$

✓ Solution by Maple

Time used: 1.89 (sec). Leaf size: 34

```
dsolve([diff(y(x),x)+y(x)*cot(x)=y(x)^3*sin(x)^3,y(1/2*Pi) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{\csc(x)\sqrt{(2\cos(x)-1)^2(1+2\cos(x))}}{4\cos(x)^2-1}$$

✓ Solution by Mathematica

Time used: 0.853 (sec). Leaf size: 20

```
DSolve[{y'[x]+y[x]*Cot[x]==y[x]^3*Sin[x]^3,{y[Pi/2]==1}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{\sqrt{\sin^2(x)(2\cos(x)+1)}}$$

4.37 problem Problem 54

Internal problem ID [2192]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (9x - y)^2 = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 28

```
dsolve([diff(y(x),x)=(9*x-y(x))^2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(9x - 3)\,\mathrm{e}^{6x} + 9x + 3}{1 + \mathrm{e}^{6x}}$$

✓ Solution by Mathematica

Time used: 0.142 (sec). Leaf size: 15

```
DSolve[{y'[x]==(9*x-y[x])^2,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 9x - 3\tanh(3x)$$

4.38 problem Problem 55

Internal problem ID [2193]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _Riccati]`

$$\boxed{y' - (4x + y + 2)^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=(4*x+y(x)+2)^2,y(x), singsol=all)
```

$$y(x) = -4x - 2 - 2\tan(-2x + 2c_1)$$

✓ Solution by Mathematica

Time used: 0.156 (sec). Leaf size: 41

```
DSolve[y'[x]==(4*x+y[x]+2)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4x + \frac{1}{c_1 e^{4ix} - \frac{i}{4}} - (2 + 2i)$$
$$y(x) \to -4x - (2 + 2i)$$

4.39 problem Problem 56

Internal problem ID [2194]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{y' - \sin(3x - 3y + 1)^2 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=(sin(3*x-3*y(x)+1))^2,y(x), singsol=all)
```

$$y(x) = x + \frac{1}{3} + \frac{\arctan(-3x + 3c_1)}{3}$$

✓ Solution by Mathematica

Time used: 0.58 (sec). Leaf size: 43

```
DSolve[y'[x]==(Sin[3*x-3*y[x]+1])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2y(x) - 2\left(\frac{1}{3}\tan(-3y(x) + 3x + 1) - \frac{1}{3}\arctan(\tan(-3y(x) + 3x + 1))\right) = c_1, y(x)\right]$$

4.40 problem Problem 58

Internal problem ID [2195]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y' - \frac{y(\ln(yx) - 1)}{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=y(x)/x*(ln(x*y(x))-1),y(x), singsol=all)
```

$$y(x) = \frac{e^{\frac{x}{c_1}}}{x}$$

✓ Solution by Mathematica

Time used: 0.207 (sec). Leaf size: 24

```
DSolve[y'[x]==y[x]/x*(Log[x*y[x]]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{e^{c_1} x}}{x}$$
$$y(x) \to \frac{1}{x}$$

4.41 problem Problem 59

Internal problem ID [2196]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$\boxed{y' - 2x(x+y)^2 + 1 = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 20

```
dsolve([diff(y(x),x)=2*x*(x+y(x))^2-1,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{-x^3 + x - 1}{x^2 - 1}$$

✓ Solution by Mathematica

Time used: 0.134 (sec). Leaf size: 21

```
DSolve[{y'[x]==2*x*(x+y[x])^2-1,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-x^3 + x - 1}{x^2 - 1}$$

4.42 problem Problem 60

Internal problem ID [2197]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$y' - \frac{x + 2y - 1}{2x - y + 3} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)=(x+2*y(x)-1)/(2*x-y(x)+3),y(x), singsol=all)
```

$$y(x) = 1 - \tan\left(\operatorname{RootOf}\left(4\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(x+1) + 2c_1\right)\right)(x+1)$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 68

```
DSolve[y'[x]==(x+2*y[x]-1)/(2*x-y[x]+3),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[32\arctan\left(\frac{-2y(x) - x + 1}{-y(x) + 2x + 3}\right) + 8\log\left(\frac{x^2 + y(x)^2 - 2y(x) + 2x + 2}{5(x+1)^2}\right) + 16\log(x+1) + 5c_1 = 0, y(x)\right]$$

4.43 problem Problem 61

Internal problem ID [2198]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + p(x)\, y + q(x)\, y^2 - r(x) = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)+p(x)*y(x)+q(x)*y(x)^2=r(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]+p[x]*y[x]+q[x]*y[x]^2==r[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.44 problem Problem 62

Internal problem ID [2199]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{y' + \frac{2y}{x} - y^2 + \frac{2}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.469 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)+2/x*y(x)-y(x)^2=-2/x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^3 + 2c_1}{(-x^3 + c_1)\,x}$$

✓ Solution by Mathematica

Time used: 0.173 (sec). Leaf size: 35

```
DSolve[y'[x]+2/x*y[x]-y[x]^2==-2/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2 + 3c_1x^3}{x - 3c_1x^4}$$
$$y(x) \to -\frac{1}{x}$$

4.45 problem Problem 63

Internal problem ID [2200]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{y' + \frac{7y}{x} - 3y^2 - \frac{3}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)+7/x*y(x)-3*y(x)^2=3/x^2,y(x), singsol=all)
```

$$y(x) = \frac{3\ln(x) - 3c_1 - 1}{3x(\ln(x) - c_1)}$$

✓ Solution by Mathematica

Time used: 0.146 (sec). Leaf size: 15

```
DSolve[y'[x]+7/x*y[x]-3*y[x]^2==3/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x}$$
$$y(x) \to \frac{1}{x}$$

4.46 problem Problem 64

Internal problem ID [2201]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{\frac{y'}{y} + p(x)\ln(y) - q(x) = 0}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 36

```
dsolve(diff(y(x),x)/y(x)+p(x)*ln(y(x))=q(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{e}^{\int -p(x)dx}\left(\int q(x)\mathrm{e}^{\int p(x)dx}dx\right)}\mathrm{e}^{-\mathrm{e}^{\int -p(x)dx}c_1}$$

✓ Solution by Mathematica

Time used: 0.189 (sec). Leaf size: 109

```
DSolve[y'[x]/y[x]+p[x]*Log[y[x]]==q[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x \exp\left(-\int_1^{K[2]} -p(K[1])dK[1]\right)(\log(y(x))p(K[2]) - q(K[2]))dK[2] + \int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x -p(K[1])dK[1]\right)}{K[3]} - \int_1^x \frac{\exp\left(-\int_1^{K[2]} -p(K[1])dK[1]\right)p(K[2])}{K[3]}dK[2]\right)dK[3] = c_1, y(x)\right]$$

4.47 problem Problem 65

Internal problem ID [2202]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{\frac{y'}{y}-\frac{2\ln\left(y\right)}{x}-\frac{1-2\ln\left(x\right)}{x}=0}$$

With initial conditions

$$[y(1)=\mathrm{e}]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)/y(x)-2/x*ln(y(x))=1/x*(1-2*ln(x)),y(1) = exp(1)],y(x), singsol=all)
```

$$y(x)=x\,\mathrm{e}^{x^2}$$

✓ Solution by Mathematica

Time used: 0.205 (sec). Leaf size: 12

```
DSolve[{y'[x]/y[x]-2/x*Log[y[x]]==1/x*(1-2*Log[x]),{y[1]==Exp[1]}},y[x],x,IncludeSingularSolu
```

$$y(x)\to e^{x^2}x$$

4.48 problem Problem 67

Internal problem ID [2203]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.8, Change of Variables. page 79
Problem number: Problem 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\sec(y)^2 y' + \frac{\tan(y)}{2\sqrt{x+1}} - \frac{1}{2\sqrt{x+1}} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(sec(y(x))^2*diff(y(x),x)+1/(2*sqrt(1+x))*tan(y(x))=1/(2*sqrt(1+x)),y(x), singsol=all)
```

$$y(x) = \arctan\left(e^{-\sqrt{x+1}} c_1 + 1\right)$$

✓ Solution by Mathematica

Time used: 60.276 (sec). Leaf size: 239

```
DSolve[Sec[y[x]]^2*y'[x]+1/(2*Sqrt[1+x])*Tan[y[x]]==1/(2*Sqrt[1+x]),y[x],x,IncludeSingularSol
```

$$y(x) \to -\arccos\left(-\frac{e^{\sqrt{x+1}+2c_1}}{\sqrt{1+2e^{\sqrt{x+1}+2c_1}\left(-1+e^{\sqrt{x+1}+2c_1}\right)}}\right)$$

$$y(x) \to \arccos\left(-\frac{e^{\sqrt{x+1}+2c_1}}{\sqrt{1+2e^{\sqrt{x+1}+2c_1}\left(-1+e^{\sqrt{x+1}+2c_1}\right)}}\right)$$

$$y(x) \to -\arccos\left(\frac{e^{\sqrt{x+1}+2c_1}}{\sqrt{1+2e^{\sqrt{x+1}+2c_1}\left(-1+e^{\sqrt{x+1}+2c_1}\right)}}\right)$$

$$y(x) \to \arccos\left(\frac{e^{\sqrt{x+1}+2c_1}}{\sqrt{1+2e^{\sqrt{x+1}+2c_1}\left(-1+e^{\sqrt{x+1}+2c_1}\right)}}\right)$$

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5.1 problem Problem 1

Internal problem ID [2204]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘x=_G(y,y’)‘]`

$$e^{yx}y + (2y - e^{yx}x)\,y' = 0$$

✗ Solution by Maple

```
dsolve(y(x)*exp(x*y(x))+(2*y(x)-x*exp(x*y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*Exp[x*y[x]]+(2*y[x]-x*Exp[x*y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

5.2 problem Problem 2

Internal problem ID [2205]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact]`

$$\cos(yx) - xy\sin(yx) - x^2\sin(yx)\,y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve((cos(x*y(x))-x*y(x)*sin(x*y(x)))-x^2*sin(x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\arccos\left(\frac{c_1}{x}\right)}{x}$$

✓ Solution by Mathematica

Time used: 5.515 (sec). Leaf size: 34

```
DSolve[(Cos[x*y[x]]-x*y[x]*Sin[x*y[x]])-x^2*Sin[x*y[x]]*y'[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to -\frac{\arccos\left(-\frac{c_1}{x}\right)}{x}$$
$$y(x) \to \frac{\arccos\left(-\frac{c_1}{x}\right)}{x}$$

5.3 problem Problem 3

Internal problem ID [2206]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y + 3x^2 + y'x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((y(x)+3*x^2)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x^3 + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 17

```
DSolve[(y[x]+3*x^2)+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-x^3 + c_1}{x}$$

5.4 problem Problem 4

Internal problem ID [2207]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{2\,e^{y}x+\left(3y^{2}+x^{2}e^{y}\right)y'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(2*x*exp(y(x))+(3*y(x)^2+x^2*exp(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$x^{2}e^{y(x)}+y(x)^{3}+c_{1}=0$$

✓ Solution by Mathematica

Time used: 0.241 (sec). Leaf size: 19

```
DSolve[2*x*Exp[y[x]]+(3*y[x]^2+x^2*Exp[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[x^{2}e^{y(x)}+y(x)^{3}=c_{1},y(x)\right]$$

5.5 problem Problem 5

Internal problem ID [2208]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2yx + (x^2 + 1)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(2*x*y(x)+(x^2+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 20

```
DSolve[2*x*y[x]+(x^2+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^2 + 1}$$
$$y(x) \to 0$$

5.6 problem Problem 6

Internal problem ID [2209]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact, _rational, _Bernoulli]`

$$\boxed{y^2 - 2x + 2xyy' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve((y(x)^2-2*x)+2*x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x\,(x^2+c_1)}}{x}$$
$$y(x) = -\frac{\sqrt{x\,(x^2+c_1)}}{x}$$

✓ Solution by Mathematica

Time used: 0.198 (sec). Leaf size: 42

```
DSolve[(y[x]^2-2*x)+2*x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x^2+c_1}}{\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{x^2+c_1}}{\sqrt{x}}$$

5.7 problem Problem 7

Internal problem ID [2210]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{4\,\mathrm{e}^{2x}+2yx-y^2+(x-y)^2\,y'=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 117

```
dsolve((4*exp(2*x)+2*x*y(x)-y(x)^2)+(x-y(x))^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\left(-x^3-6\,\mathrm{e}^{2x}-3c_1\right)^{\frac{1}{3}}+x$$
$$y(x)=-\frac{\left(-x^3-6\,\mathrm{e}^{2x}-3c_1\right)^{\frac{1}{3}}}{2}-\frac{i\sqrt{3}\,\left(-x^3-6\,\mathrm{e}^{2x}-3c_1\right)^{\frac{1}{3}}}{2}+x$$
$$y(x)=-\frac{\left(-x^3-6\,\mathrm{e}^{2x}-3c_1\right)^{\frac{1}{3}}}{2}+\frac{i\sqrt{3}\,\left(-x^3-6\,\mathrm{e}^{2x}-3c_1\right)^{\frac{1}{3}}}{2}+x$$

✓ Solution by Mathematica

Time used: 1.43 (sec). Leaf size: 112

```
DSolve[(4*Exp[2*x]+2*x*y[x]-y[x]^2)+(x-y[x])^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to x+\sqrt[3]{-x^3-6e^{2x}+3c_1}$$
$$y(x)\to x+\frac{1}{2}i\left(\sqrt{3}+i\right)\sqrt[3]{-x^3-6e^{2x}+3c_1}$$
$$y(x)\to x-\frac{1}{2}\left(1+i\sqrt{3}\right)\sqrt[3]{-x^3-6e^{2x}+3c_1}$$

5.8 problem Problem 8

Internal problem ID [2211]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _Riccati]`

$$\boxed{\frac{1}{x} - \frac{y}{x^2 + y^2} + \frac{xy'}{x^2 + y^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 12

```
dsolve((1/x-y(x)/(x^2+y(x)^2))+x/(x^2+y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\tan\left(\ln\left(x\right) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.2 (sec). Leaf size: 15

```
DSolve[(1/x-y[x]/(x^2+y[x]^2))+x/(x^2+y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to x \tan(-\log(x) + c_1)$$

5.9 problem Problem 9

Internal problem ID [2212]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y\cos(yx)-\sin(x)+x\cos(yx)\,y'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve((y(x)*cos(x*y(x))-sin(x))+x*cos(x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{\arcsin(\cos(x)+c_1)}{x}$$

✓ Solution by Mathematica

Time used: 0.576 (sec). Leaf size: 17

```
DSolve[(y[x]*Cos[x*y[x]]-Sin[x])+x*Cos[x*y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to\frac{\arcsin(-\cos(x)+c_1)}{x}$$

5.10 problem Problem 10

Internal problem ID [2213]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _Bernoulli]`

$$\boxed{2y^2 e^{2x} + 3x^2 + 2y\,e^{2x}y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 50

```
dsolve((2*y(x)^2*exp(2*x)+3*x^2)+2*y(x)*exp(2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^{-2x}\sqrt{e^{2x}\left(-x^3 + c_1\right)}$$
$$y(x) = -e^{-2x}\sqrt{e^{2x}\left(-x^3 + c_1\right)}$$

✓ Solution by Mathematica

Time used: 7.475 (sec). Leaf size: 47

```
DSolve[(2*y[x]^2*Exp[2*x]+3*x^2)+2*y[x]*Exp[2*x]*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\sqrt{e^{-2x}\left(-x^3 + c_1\right)}$$
$$y(x) \to \sqrt{e^{-2x}\left(-x^3 + c_1\right)}$$

5.11 problem Problem 11

Internal problem ID [2214]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y^2 + \cos(x) + (2yx + \sin(y))\, y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve((y(x)^2+cos(x))+(2*x*y(x)+sin(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$xy(x)^2 + \sin(x) - \cos(y(x)) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.214 (sec). Leaf size: 20

```
DSolve[(y[x]^2+Cos[x])+(2*x*y[x]+Sin[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[xy(x)^2 - \cos(y(x)) + \sin(x) = c_1, y(x)\right]$$

5.12 problem Problem 12

Internal problem ID [2215]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 1, First-Order Differential Equations. Section 1.9, Exact Differential Equations. page 91
Problem number: Problem 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\sin(y) + \cos(x) y + (x\cos(y) + \sin(x)) y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve((sin(y(x))+y(x)*cos(x))+(x*cos(y(x))+sin(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)\sin(x) + x\sin(y(x)) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 17

```
DSolve[(Sin[y[x]]+y[x]*Cos[x])+(x*Cos[y[x]]+Sin[x])*y'[x]==0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}[x\sin(y(x)) + y(x)\sin(x) = c_1, y(x)]$$

6 Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502

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6.1 problem Problem 23

Internal problem ID [2216]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' - 3y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + c_2e^{3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[y''[x]-2*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2e^{4x} + c_1)$$

6.2 problem Problem 24

Internal problem ID [2217]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 7y' + 10y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+7*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-5x} + c_2 e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''[x]+7*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-5x} \left(c_2 e^{3x} + c_1\right)$$

6.3 problem Problem 25

Internal problem ID [2218]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 36y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-36*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-6x} + c_2 e^{6x}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''[x]-36*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{6x} + c_2 e^{-6x}$$

6.4 problem Problem 26

Internal problem ID [2219]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \mathrm{e}^{-4x}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 19

```
DSolve[y''[x]+4*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{4} c_1 e^{-4x}$$

6.5 problem Problem 27

Internal problem ID [2220]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 27.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y'' - y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)-diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 \mathrm{e}^{3x} + c_3 \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y'''[x]-3*y''[x]-y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} + c_2 e^{x} + c_3 e^{3x}$$

6.6 problem Problem 28

Internal problem ID [2221]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 28.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 3y'' - 4y' - 12y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)-4*diff(y(x),x)-12*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{-3x} + c_3 e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y'''[x]+3*y''[x]-4*y'[x]-12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(c_2 e^x + c_3 e^{5x} + c_1\right)$$

6.7 problem Problem 29

Internal problem ID [2222]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 29.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 3y'' - 18y' - 40y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)-18*diff(y(x),x)-40*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{4x} + c_2 e^{-5x} + c_3 e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y'''[x]+3*y''[x]-18*y'[x]-40*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-5x}\left(c_2 e^{3x} + c_3 e^{9x} + c_1\right)$$

6.8 problem Problem 30

Internal problem ID [2223]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 30.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y'' - 2y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)-2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \mathrm{e}^{2x} + c_3 \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 28

```
DSolve[y'''[x]-y''[x]-2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \left(-e^{-x}\right) + \frac{1}{2} c_2 e^{2x} + c_3$$

6.9 problem Problem 31

Internal problem ID [2224]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 31.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y'' - 10y' + 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-10*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{-4x} + c_3 e^{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y'''[x]+y''[x]-10*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-4x} + c_2 e^{x} + c_3 e^{2x}$$

6.10 problem Problem 32

Internal problem ID [2225]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 32.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 2y''' - y'' + 2y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$3)-diff(y(x),x$2)+2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{2x} + c_3 e^{-x} + c_4 e^{x}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 34

```
DSolve[y''''[x]-2*y'''[x]-y''[x]+2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(-e^{-x}\right) + c_2 e^{x} + \frac{1}{2} c_3 e^{2x} + c_4$$

6.11 problem Problem 33

Internal problem ID [2226]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 33.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 13y'' + 36y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)-13*diff(y(x),x$2)+36*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{3x} + c_3 e^{-3x} + c_4 e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 35

```
DSolve[y''''[x]-13*y''[x]+36*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(c_2 e^x + e^{5x}(c_4 e^x + c_3) + c_1\right)$$

6.12 problem Problem 34

Internal problem ID [2227]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$\boxed{x^2y'' + 3y'x - 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2 + \frac{c_2}{x^4}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+3*x*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^6 + c_1}{x^4}$$

6.13 problem Problem 35

Internal problem ID [2228]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{2x^2y'' + 5y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{c_2}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[2*x^2*y''[x]+5*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2\sqrt{x} + c_1}{x}$$

6.14 problem Problem 36

Internal problem ID [2229]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 36.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^3y''' + x^2y'' - 2y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)+x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2 + \frac{c_2}{x} + c_3x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+x^2*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3x^2 + c_2x + \frac{c_1}{x}$$

6.15 problem Problem 37

Internal problem ID [2230]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 37.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{x^3y''' + 3x^2y'' - 6y'x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^3*diff(y(x),x$3)+3*x^2*diff(y(x),x$2)-6*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2x^{\sqrt{7}} + c_3x^{-\sqrt{7}}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 40

```
DSolve[x^3*y'''[x]+3*x^2*y''[x]-6*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-\sqrt{7}}\left(c_2x^{2\sqrt{7}} - c_1\right)}{\sqrt{7}} + c_3$$

6.16 problem Problem 38

Internal problem ID [2231]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' - 6y - 18\,\mathrm{e}^{5x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-6*y(x)=18*exp(5*x),y(x), singsol=all)
```

$$y(x) = c_2\mathrm{e}^{2x} + c_1\mathrm{e}^{-3x} + \frac{3\,\mathrm{e}^{5x}}{4}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 31

```
DSolve[y''[x]+y'[x]-6*y[x]==18*Exp[5*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3e^{5x}}{4} + c_1e^{-3x} + c_2e^{2x}$$

6.17 problem Problem 39

Internal problem ID [2232]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' - 2y - 4x^2 - 5 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=4*x^2+5,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} c_2 + \mathrm{e}^{-2x} c_1 - 2x^2 - 2x - \frac{11}{2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 29

```
DSolve[y''[x]+y'[x]-2*y[x]==4*x^2+5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x(x+1) + c_1 e^{-2x} + c_2 e^{x} - \frac{11}{2}$$

6.18 problem Problem 40

Internal problem ID [2233]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 40.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + 2y'' - y' - 2y - 4\,e^{2x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)-diff(y(x),x)-2*y(x)=4*exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{e^{2x}}{3} + c_1 e^{x} + c_2 e^{-2x} + c_3 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 37

```
DSolve[y'''[x]+2*y''[x]-y'[x]-2*y[x]==4*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2x}}{3} + c_1 e^{-2x} + c_2 e^{-x} + c_3 e^{x}$$

6.19 problem Problem 41

Internal problem ID [2234]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 41.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' + y'' - 10y' + 8y - 24\,\mathrm{e}^{-3x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-10*diff(y(x),x)+8*y(x)=24*exp(-3*x),y(x), singsol=all)
```

$$y(x) = \frac{6\,\mathrm{e}^{-3x}}{5} + c_1\mathrm{e}^{x} + c_2\mathrm{e}^{-4x} + c_3\mathrm{e}^{2x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 37

```
DSolve[y'''[x]+y''[x]-10*y'[x]+8*y[x]==24*Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{6e^{-3x}}{5} + c_1e^{-4x} + c_2e^{x} + c_3e^{2x}$$

6.20 problem Problem 42

Internal problem ID [2235]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.1, General Theory for Linear Differential Equations. page 502
Problem number: Problem 42.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 5y'' + 6y' - 6\,\mathrm{e}^{-x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$3)+5*diff(y(x),x$2)+6*diff(y(x),x)=6*exp(-x),y(x), singsol=all)
```

$$y(x) = -\frac{c_1 \mathrm{e}^{-3x}}{3} - \frac{c_2 \mathrm{e}^{-2x}}{2} - 3\,\mathrm{e}^{-x} + c_3$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 35

```
DSolve[y'''[x]+5*y''[x]+6*y'[x]==6*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6} e^{-3x}(-3e^x(6e^x + c_2) - 2c_1) + c_3$$

7 Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525

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7.1 problem Problem 25

Internal problem ID [2236]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y - 6\,\mathrm{e}^{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+y(x)=6*exp(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\,c_2 + c_1\cos(x) + 3\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 21

```
DSolve[y''[x]+y[x]==6*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3e^{x} + c_1\cos(x) + c_2\sin(x)$$

7.2 problem Problem 26

Internal problem ID [2237]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y - 5\,\mathrm{e}^{-2x}x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=5*x*exp(-2*x),y(x), singsol=all)
```

$$y(x) = c_2\mathrm{e}^{-2x} + \mathrm{e}^{-2x}xc_1 + \frac{5\,\mathrm{e}^{-2x}x^3}{6}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 29

```
DSolve[y''[x]+4*y'[x]+4*y[x]==5*x*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}e^{-2x}(5x^3 + 6c_2x + 6c_1)$$

7.3 problem Problem 27

Internal problem ID [2238]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y - 8\sin(2x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+4*y(x)=8*sin(2*x),y(x), singsol=all)
```

$$y(x) = \sin(2x)\, c_2 + \cos(2x)\, c_1 - 2x\cos(2x)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 29

```
DSolve[y''[x]+4*y[x]==8*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x)\cos(x) + (-2x + c_1)\cos(2x) + c_2\sin(2x)$$

7.4 problem Problem 28

Internal problem ID [2239]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 2y - 5\,\mathrm{e}^{2x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=5*exp(2*x),y(x), singsol=all)
```

$$y(x) = c_2\mathrm{e}^{2x} + \mathrm{e}^{-x}c_1 + \frac{5\,\mathrm{e}^{2x}x}{3}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 31

```
DSolve[y''[x]-y'[x]-2*y[x]==5*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1e^{-x} + e^{2x}\left(\frac{5x}{3} - \frac{5}{9} + c_2\right)$$

7.5 problem Problem 29

Internal problem ID [2240]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 5y - 3\sin(2x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=3*sin(2*x),y(x), singsol=all)
```

$$y(x) = e^{-x}\sin(2x)\, c_2 + \cos(2x)\, e^{-x} c_1 + \frac{3\sin(2x)}{17} - \frac{12\cos(2x)}{17}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 44

```
DSolve[y''[x]+2*y'[x]+5*y[x]==3*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3}{17}(4\cos(2x) - \sin(2x)) + e^{-x}(c_2\cos(2x) + c_1\sin(2x))$$

7.6 problem Problem 30

Internal problem ID [2241]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 30.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + 2y'' - 5y' - 6y - 4x^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)-5*diff(y(x),x)-6*y(x)=4*x^2,y(x), singsol=all)
```

$$y(x) = -\frac{2x^2}{3} + \frac{10x}{9} - \frac{37}{27} + c_1 e^{-3x} + e^{-x} c_2 + c_3 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 43

```
DSolve[y'''[x]+2*y''[x]-5*y'[x]-6*y[x]==4*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{9}(5 - 3x)x + c_1 e^{-3x} + c_2 e^{-x} + c_3 e^{2x} - \frac{37}{27}$$

7.7 problem Problem 31

Internal problem ID [2242]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 31.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - y'' + y' - y - 9\,\mathrm{e}^{-x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+diff(y(x),x)-y(x)=9*exp(-x),y(x), singsol=all)
```

$$y(x) = -\frac{9\,\mathrm{e}^{-x}}{4} + c_1 \cos(x) + \mathrm{e}^{x} c_2 + c_3 \sin(x)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 31

```
DSolve[y'''[x]-y''[x]+y'[x]-y[x]==9*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{9e^{-x}}{4} + c_3 e^{x} + c_1 \cos(x) + c_2 \sin(x)$$

7.8 problem Problem 32

Internal problem ID [2243]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 32.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + 3y'' + 3y' + y - 2\,e^{-x} - 3\,e^{2x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=2*exp(-x)+3*exp(2*x),y(x), singsol
```

$$y(x) = \frac{e^{-x}x^3}{3} + \frac{e^{2x}}{9} + e^{-x}c_1 + c_2e^{-x}x + c_3x^2e^{-x}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 41

```
DSolve[y'''[x]+3*y''[x]+3*y'[x]+y[x]==2*Exp[-x]+3*Exp[2*x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{9}e^{-x}\left(3x^3 + 9c_3x^2 + e^{3x} + 9c_2x + 9c_1\right)$$

7.9 problem Problem 33

Internal problem ID [2244]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y - 5\cos(2x) = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 3]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+9*y(x)=5*cos(2*x),y(0) = 2, D(y)(0) = 3],y(x), singsol=all)
```

$$y(x) = \sin(3x) + \cos(3x) + \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[{y''[x]+9*y[x]==5*Cos[2*x],{y[0]==2,y'[0]==3}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \sin(3x) + \cos(2x) + \cos(3x)$$

7.10 problem Problem 34

Internal problem ID [2245]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y - 9x\,\mathrm{e}^{2x} = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 7]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$2)-y(x)=9*x*exp(2*x),y(0) = 0, D(y)(0) = 7],y(x), singsol=all)
```

$$y(x) = -4\,\mathrm{e}^{-x} + 8\,\mathrm{e}^{x} + (3x - 4)\,\mathrm{e}^{2x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 29

```
DSolve[{y''[x]-y[x]==9*x*Exp[2*x],{y[0]==0,y'[0]==7}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{2x}(3x - 4) - 4e^{-x} + 8e^{x}$$

7.11 problem Problem 35

Internal problem ID [2246]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 2y + 10\sin(x) = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)+diff(y(x),x)-2*y(x)=-10*sin(x),y(0) = 2, D(y)(0) = 1],y(x), singsol=al
```

$$y(x) = e^{-2x} + \cos(x) + 3\sin(x)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 17

```
DSolve[{y''[x]+y'[x]-2*y[x]==-10*Sin[x],{y[0]==2,y'[0]==1}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to e^{-2x} + 3\sin(x) + \cos(x)$$

7.12 problem Problem 36

Internal problem ID [2247]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 2y - 4\cos(x) + 2\sin(x) = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 4]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)+diff(y(x),x)-2*y(x)=4*cos(x)-2*sin(x),y(0) = -1, D(y)(0) = 4],y(x), si
```

$$y(x) = -\left((\cos(x) - \sin(x))\,\mathrm{e}^{2x} - \mathrm{e}^{3x} + 1\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 22

```
DSolve[{y''[x]+y'[x]-2*y[x]==4*Cos[x]-2*Sin[x],{y[0]==-1,y'[0]==4}},y[x],x,IncludeSingularSol
```

$$y(x) \to -e^{-2x} + e^x + \sin(x) - \cos(x)$$

7.13 problem Problem 38

Internal problem ID [2248]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + \omega^2 y - \frac{F_0 \cos(\omega t)}{m} = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)+omega^2*y(t)=F__0/m*cos(omega*t),y(0) = 1, D(y)(0) = 0],y(t), singsol=
```

$$y(t) = \cos(\omega t) + \frac{F_0 \sin(\omega t)\, t}{2\omega m}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 26

```
DSolve[{y''[t]+\[Omega]^2*y[t]==F0/m*Cos[\[Omega]*t],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingul
```

$$y(t) \to \frac{\text{F0} t \sin(t\omega)}{2m\omega} + \cos(t\omega)$$

7.14 problem Problem 39

Internal problem ID [2249]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' + 6y - 7\,\mathrm{e}^{2x} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+6*y(x)=7*exp(2*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x} \sin\left(\sqrt{2}\,x\right) c_2 + \mathrm{e}^{2x} \cos\left(\sqrt{2}\,x\right) c_1 + \frac{7\,\mathrm{e}^{2x}}{2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 40

```
DSolve[y''[x]-4*y'[x]+6*y[x]==7*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{2x} \left(2c_2 \cos\left(\sqrt{2}x\right) + 2c_1 \sin\left(\sqrt{2}x\right) + 7\right)$$

7.15 problem Problem 40

Internal problem ID [2250]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 40.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + y'' + y' + y - 4\,\mathrm{e}^{x} x = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)+diff(y(x),x)+y(x)=4*x*exp(x),y(x), singsol=all)
```

$$y(x) = \frac{(2x-3)\,\mathrm{e}^{x}}{2} + c_1 \cos(x) + \sin(x)\, c_2 + c_3 \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 33

```
DSolve[y'''[x]+y''[x]+y'[x]+y[x]==4*x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}\left(x - \frac{3}{2}\right) + c_3 e^{-x} + c_1 \cos(x) + c_2 \sin(x)$$

7.16 problem Problem 41

Internal problem ID [2251]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 41.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + 104y''' + 2740y'' - 5\,\mathrm{e}^{-2x}\cos(3x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 73

```
dsolve(diff(y(x),x$4)+104*diff(y(x),x$3)+2740*diff(y(x),x$2)=5*exp(-2*x)*cos(3*x),y(x), sings
```

$$y(x) = \frac{667\,\mathrm{e}^{-52x}\cos(6x)\,c_1}{1876900} - \frac{39c_1\mathrm{e}^{-52x}\sin(6x)}{469225} + \frac{39c_2\mathrm{e}^{-52x}\cos(6x)}{469225} + \frac{667\,\mathrm{e}^{-52x}\sin(6x)\,c_2}{1876900} - \frac{3475\,\mathrm{e}^{-2x}\cos(3x)}{84184477} - \frac{12240\,\mathrm{e}^{-2x}\sin(3x)}{84184477} + c_3x + c_4$$

✓ Solution by Mathematica

Time used: 2.322 (sec). Leaf size: 72

```
DSolve[y''''[x]+104*y'''[x]+2740*y''[x]==5*Exp[-2*x]*Cos[3*x],y[x],x,IncludeSingularSolutions
```

$$y(x) \to c_4x - \frac{5e^{-2x}(2448\sin(3x) + 695\cos(3x))}{84184477} + \frac{e^{-52x}((156c_1 + 667c_2)\cos(6x) + (667c_1 - 156c_2)\sin(6x))}{1876900} + c_3$$

7.17 problem Problem 46

Internal problem ID [2252]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' - 3y - \sin(x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-3*y(x)=sin(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} c_2 + c_1 \mathrm{e}^{-3x} - \frac{1}{6} - \frac{2\sin(2x)}{65} + \frac{7\cos(2x)}{130}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 39

```
DSolve[y''[x]+2*y'[x]-3*y[x]==Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{65}\sin(2x) + \frac{7}{130}\cos(2x) + c_1 e^{-3x} + c_2 e^{x} - \frac{1}{6}$$

7.18 problem Problem 47

Internal problem ID [2253]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.3, The Method of Undetermined Coefficients. page 525
Problem number: Problem 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 6y - \cos(x)^2 \sin(x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+6*y(x)=sin(x)^2*cos(x)^2,y(x), singsol=all)
```

$$y(x) = \sin\left(\sqrt{6}\,x\right) c_2 + \cos\left(\sqrt{6}\,x\right) c_1 + \frac{\cos(4x)}{80} + \frac{1}{48}$$

✓ Solution by Mathematica

Time used: 0.375 (sec). Leaf size: 39

```
DSolve[y''[x]+6*y[x]==Sin[x]^2*Cos[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{80}\cos(4x) + c_1 \cos\left(\sqrt{6}x\right) + c_2 \sin\left(\sqrt{6}x\right) + \frac{1}{48}$$

8 Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529

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8.1 problem Problem 1

Internal problem ID [2254]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 16y - 20\cos(4x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-16*y(x)=20*cos(4*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{4x}c_2 + c_1\mathrm{e}^{-4x} - \frac{5\cos(4x)}{8}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 30

```
DSolve[y''[x]-16*y[x]==20*Cos[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{5}{8}\cos(4x) + c_1e^{4x} + c_2e^{-4x}$$

8.2 problem Problem 2

Internal problem ID [2255]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y - 50\sin(3x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=50*sin(3*x),y(x), singsol=all)
```

$$y(x) = e^{-x}c_2 + x\,e^{-x}c_1 - 3\cos(3x) - 4\sin(3x)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 31

```
DSolve[y''[x]+2*y'[x]+y[x]==50*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4\sin(3x) - 3\cos(3x) + e^{-x}(c_2x + c_1)$$

8.3 problem Problem 3

Internal problem ID [2256]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y - 10\,\mathrm{e}^{2x}\cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-y(x)=10*exp(2*x)*cos(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}c_2 + c_1\mathrm{e}^{x} + \mathrm{e}^{2x}(2\sin(x) + \cos(x))$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 33

```
DSolve[y''[x]-y[x]==10*Exp[2*x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1e^{x} + c_2e^{-x} + e^{2x}(2\sin(x) + \cos(x))$$

8.4 problem Problem 4

Internal problem ID [2257]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y - 169\sin(3x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=169*sin(3*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-2x} + e^{-2x} x c_1 - 12\cos(3x) - 5\sin(3x)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 31

```
DSolve[y''[x]+4*y'[x]+4*y[x]==169*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -5\sin(3x) - 12\cos(3x) + e^{-2x}(c_2 x + c_1)$$

8.5 problem Problem 5

Internal problem ID [2258]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - 2y - 40\sin(x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=40*sin(x)^2,y(x), singsol=all)
```

$$y(x) = c_2 e^{2x} + e^{-x} c_1 - 10 + \sin(2x) + 3\cos(2x)$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 33

```
DSolve[y''[x]-y'[x]-2*y[x]==40*Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(2x) + 3\cos(2x) + c_1 e^{-x} + c_2 e^{2x} - 10$$

8.6 problem Problem 6

Internal problem ID [2259]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - 3\,\mathrm{e}^{x} \cos (2x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+y(x)=3*exp(x)*cos(2*x),y(x), singsol=all)
```

$$y(x) = \sin (x)\, c_2 + c_1 \cos (x) - \frac{3\,\mathrm{e}^{x}(\cos (2x) - 2\sin (2x))}{10}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 34

```
DSolve[y''[x]+y[x]==3*Exp[x]*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3}{10}e^{x}(\cos(2x) - 2\sin(2x)) + c_1\cos(x) + c_2\sin(x)$$

8.7 problem Problem 7

Internal problem ID [2260]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 2y - 2\,\mathrm{e}^{-x} \sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+2*y(x)=2*exp(-x)*sin(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\,\mathrm{e}^{-x} c_2 + \mathrm{e}^{-x} \cos(x)\, c_1 - \mathrm{e}^{-x}(\cos(x)\, x - \sin(x))$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 34

```
DSolve[y''[x]+2*y'[x]+2*y[x]==2*Exp[-x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x}(2(-x + c_2)\cos(x) + (1 + 2c_1)\sin(x))$$

8.8 problem Problem 8

Internal problem ID [2261]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y - 100\,\mathrm{e}^{x} \sin(x)\, x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)-4*y(x)=100*x*exp(x)*sin(x),y(x), singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{2x} + \mathrm{e}^{-2x} c_1 - 2\,\mathrm{e}^{x}(5\cos(x)\,x + 10\sin(x)\,x + 7\cos(x) - \sin(x))$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 44

```
DSolve[y''[x]-4*y[x]==100*x*Exp[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2x} + c_2 e^{-2x} - 2e^{x}((10x - 1)\sin(x) + (5x + 7)\cos(x))$$

8.9 problem Problem 9

Internal problem ID [2262]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 5y - 4\cos(2x)\,\mathrm{e}^{-x} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=4*exp(-x)*cos(2*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}\sin(2x)\,c_2 + \cos(2x)\,\mathrm{e}^{-x}c_1 + \frac{\mathrm{e}^{-x}(2\sin(2x)\,x + \cos(2x))}{2}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 36

```
DSolve[y''[x]+2*y'[x]+5*y[x]==4*Exp[-x]*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-x}((1+4c_2)\cos(2x) + 4(x+c_1)\sin(2x))$$

8.10 problem Problem 10

Internal problem ID [2263]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + 10y - 24\,\mathrm{e}^{x} \cos(3x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+10*y(x)=24*exp(x)*cos(3*x),y(x), singsol=all)
```

$$y(x) = \sin(3x)\,\mathrm{e}^{x} c_2 + \cos(3x)\,\mathrm{e}^{x} c_1 + \frac{4\,\mathrm{e}^{x}(3\sin(3x)\,x + \cos(3x))}{3}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 36

```
DSolve[y''[x]-2*y'[x]+10*y[x]==24*Exp[x]*Cos[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{x}((2 + 3c_2)\cos(3x) + 3(4x + c_1)\sin(3x))$$

8.11 problem Problem 11

Internal problem ID [2264]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.4, Complex-Valued Trial Solutions. page 529
Problem number: Problem 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 16y - 34\,\mathrm{e}^{x} - 16\cos(4x) + 8\sin(4x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)+16*y(x)=34*exp(x)+16*cos(4*x)-8*sin(4*x),y(x), singsol=all)
```

$$y(x) = \sin(4x)\,c_2 + \cos(4x)\,c_1 - \frac{\sin(4x)}{4} + \cos(4x)\,x + 2\sin(4x)\,x + 2\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.308 (sec). Leaf size: 37

```
DSolve[y''[x]+16*y[x]==34*Exp[x]+16*Cos[4*x]-8*Sin[4*x],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to 2e^{x} + \left(x + \frac{1}{4} + c_1\right)\cos(4x) + \left(2x - \frac{1}{8} + c_2\right)\sin(4x)$$

9 Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556

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9.1 problem Problem 1

Internal problem ID [2265]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 6y' + 9y - 4\,\mathrm{e}^{3x} \ln(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=4*exp(3*x)*ln(x),y(x), singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{3x} + x\,\mathrm{e}^{3x} c_1 + x^2 \mathrm{e}^{3x}(2\ln(x) - 3)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 29

```
DSolve[y''[x]-6*y'[x]+9*y[x]==4*Exp[3*x]*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}\left(2x^2 \log(x) + x(-3x + c_2) + c_1\right)$$

9.2 problem Problem 2

Internal problem ID [2266]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y' + 4y - \frac{e^{-2x}}{x^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=x^(-2)*exp(-2*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-2x} + e^{-2x} x c_1 - (\ln(x) + 1) e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 23

```
DSolve[y''[x]+4*y'[x]+4*y[x]==x^(-2)*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(-\log(x) + c_2 x - 1 + c_1)$$

9.3 problem Problem 3

Internal problem ID [2267]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y - 18\sec(3x)^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+9*y(x)=18*sec(3*x)^3,y(x), singsol=all)
```

$$y(x) = \sin(3x)\, c_2 + \cos(3x)\, c_1 - 2\cos(3x) + \sec(3x)$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 32

```
DSolve[y''[x]+9*y[x]==18*Sec[3*x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sec(3x)((-2 + c_1)\cos(6x) + c_2\sin(6x) + c_1)$$

9.4 problem Problem 4

Internal problem ID [2268]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 6y' + 9y - \frac{2\,\mathrm{e}^{-3x}}{x^2+1} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=2*exp(-3*x)/(x^2+1),y(x), singsol=all)
```

$$y(x) = c_2\mathrm{e}^{-3x} + x\,\mathrm{e}^{-3x}c_1 + \left(2x\arctan\left(x\right) - \ln\left(x^2+1\right)\right)\mathrm{e}^{-3x}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 31

```
DSolve[y''[x]+6*y'[x]+9*y[x]==2*Exp[-3*x]/(x^2+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(2x\arctan(x) - \log\left(x^2+1\right) + c_2x + c_1\right)$$

9.5 problem Problem 5

Internal problem ID [2269]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y - \frac{8}{e^{2x} + 1} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 46

```
dsolve(diff(y(x),x$2)-4*y(x)=8/(exp(2*x)+1),y(x), singsol=all)
```

$$y(x) = c_2 e^{2x} + e^{-2x} c_1 + \left(-e^{-2x} + e^{2x}\right) \ln\left(e^{2x} + 1\right) - 2\ln\left(e^{x}\right) e^{2x} - 1$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 47

```
DSolve[y''[x]-4*y[x]==8/(Exp[2*x]+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}\left(2\operatorname{arctanh}\left(2e^{2x} + 1\right) + c_1\right) + e^{-2x}\left(-\log\left(e^{2x} + 1\right) + c_2\right) - 1$$

9.6 problem Problem 6

Internal problem ID [2270]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 5y - e^{2x} \tan(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=exp(2*x)*tan(x),y(x), singsol=all)
```

$$y(x) = e^{2x} \sin(x) c_2 + e^{2x} \cos(x) c_1 - e^{2x} \cos(x) \ln(\sec(x) + \tan(x))$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 28

```
DSolve[y''[x]-4*y'[x]+5*y[x]==Exp[2*x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(\cos(x)(-\text{arctanh}(\sin(x)) + c_2) + c_1 \sin(x))$$

9.7 problem Problem 7

Internal problem ID [2271]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y - \frac{36}{4 - \cos(3x)^2} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 59

```
dsolve(diff(y(x),x$2)+9*y(x)=36/(4-cos(3*x)^2),y(x), singsol=all)
```

$$y(x) = \sin(3x) c_2 + \cos(3x) c_1 + \frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sin(3x)}{3}\right)\sin(3x)}{3} - (-\ln(\cos(3x)+2) + \ln(\cos(3x)-2))\cos(3x)$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 52

```
DSolve[y''[x]+9*y[x]==36/(4-Cos[3*x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sin(3x) + \frac{4\sin(3x)\cot^{-1}\left(\sqrt{3}\csc(3x)\right)}{\sqrt{3}} + \cos(3x)\left(2\coth^{-1}(2\sec(3x)) + c_1\right)$$

9.8 problem Problem 8

Internal problem ID [2272]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 10y' + 25y - \frac{2\,\mathrm{e}^{5x}}{x^2+4} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)-10*diff(y(x),x)+25*y(x)=2*exp(5*x)/(4+x^2),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{5x}c_2 + \mathrm{e}^{5x}xc_1 + \mathrm{e}^{5x}\left(-\ln\left(x^2+4\right) + x\arctan\left(\frac{x}{2}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 33

```
DSolve[y''[x]-10*y'[x]+25*y[x]==2*Exp[5*x]/(4+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{5x}\left(x\left(\arctan\left(\frac{x}{2}\right) + c_2\right) - \log\left(x^2+4\right) + c_1\right)$$

9.9 problem Problem 9

Internal problem ID [2273]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 6y' + 13y - 4\,e^{3x}\sec(2x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+13*y(x)=4*exp(3*x)*sec(2*x)^2,y(x), singsol=all)
```

$$y(x) = e^{3x}\sin(2x)\,c_2 + e^{3x}\cos(2x)\,c_1 + e^{3x}(\sin(2x)\ln(\sec(2x)+\tan(2x)) - 1)$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 33

```
DSolve[y''[x]-6*y'[x]+13*y[x]==4*Exp[3*x]*Sec[2*x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}(\sin(2x)(\operatorname{arctanh}(\sin(2x)) + c_1) + c_2\cos(2x) - 1)$$

9.10 problem Problem 10

Internal problem ID [2274]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \sec(x) - 4\,e^{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+y(x)=sec(x)+4*exp(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\,c_2 + c_1\cos(x) + \cos(x)\ln(\cos(x)) + \sin(x)\,x + 2\,e^{x}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 90

```
DSolve[y''[x]+y[x]==4*Exp[x]*Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to & -4ie^{x}\,\text{Hypergeometric2F1}\left(-\frac{i}{2}, 1, 1-\frac{i}{2}, -e^{2ix}\right)\cos(x) \\ & + \left(\frac{8}{5}+\frac{4i}{5}\right)e^{(1+2i)x}\,\text{Hypergeometric2F1}\left(1, 1-\frac{i}{2}, 2-\frac{i}{2}, -e^{2ix}\right)\cos(x) \\ & + c_1\cos(x) + (4e^{x}+c_2)\sin(x) \end{aligned}$$

9.11 problem Problem 11

Internal problem ID [2275]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \csc(x) - 2x^2 - 5x - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+y(x)=csc(x)+2*x^2+5*x+1,y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + c_1 \cos(x) - \cos(x)\, x + \sin(x) \ln(\sin(x)) + 2x^2 + 5x - 3$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 36

```
DSolve[y''[x]+y[x]==Csc[x]+2*x^2+5*x+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x+3)(2x-1) + (-x + c_1)\cos(x) + \sin(x)(\log(\tan(x)) + \log(\cos(x)) + c_2)$$

9.12 problem Problem 12

Internal problem ID [2276]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y - 2\tanh(x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-y(x)=2*tanh(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}c_2 + c_1\mathrm{e}^{x} + 2\arctan(\mathrm{e}^{x})\left(\mathrm{e}^{x} + \mathrm{e}^{-x}\right)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 28

```
DSolve[y''[x]-y[x]==2*Tanh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 4\arctan(e^{x})\cosh(x) + c_1e^{x} + c_2e^{-x}$$

9.13 problem Problem 13

Internal problem ID [2277]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2my' + m^2 y - \frac{e^{mx}}{x^2 + 1} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)-2*m*diff(y(x),x)+m^2*y(x)=exp(m*x)/(1+x^2),y(x), singsol=all)
```

$$y(x) = e^{mx} c_2 + e^{mx} x c_1 + e^{mx} \left(-\frac{\ln(x^2 + 1)}{2} + x \arctan(x)\right)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 35

```
DSolve[y''[x]-2*m*y'[x]+m^2*y[x]==Exp[m*x]/(1+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{mx} \left(-\log(x^2 + 1) + 2(x(\arctan(x) + c_2) + c_1)\right)$$

9.14 problem Problem 13

Internal problem ID [2278]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + y - \frac{4\,\mathrm{e}^{x} \ln(x)}{x^{3}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=4*exp(x)*x^(-3)*ln(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} c_2 + x\,\mathrm{e}^{x} c_1 + \frac{2\,\mathrm{e}^{x} \ln(x) + 3\,\mathrm{e}^{x}}{x}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 27

```
DSolve[y''[x]-2*y'[x]+y[x]==4*Exp[x]*x^(-3)*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{x}(2\log(x) + x(c_2 x + c_1) + 3)}{x}$$

9.15 problem Problem 15

Internal problem ID [2279]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + y - \frac{e^{-x}}{\sqrt{-x^2+4}} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=exp(-x)/sqrt(4-x^2),y(x), singsol=all)
```

$$y(x) = e^{-x}c_2 + x\,e^{-x}c_1 - \frac{e^{-x}\left(-\arcsin\left(\frac{x}{2}\right)x\sqrt{-x^2+4} + x^2 - 4\right)}{\sqrt{-x^2+4}}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 48

```
DSolve[y''[x]+2*y'[x]+y[x]==Exp[-x]/Sqrt[4-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(\sqrt{4-x^2} - 2x\cot^{-1}\left(\frac{x+2}{\sqrt{4-x^2}}\right) + c_2x + c_1\right)$$

9.16 problem Problem 16

Internal problem ID [2280]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 17y - \frac{64\,\mathrm{e}^{-x}}{3 + \sin(4x)^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 73

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+17*y(x)=64*exp(-x)/(3+sin(4*x)^2),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}\sin(4x)\,c_2 + \mathrm{e}^{-x}\cos(4x)\,c_1 + \frac{4\left(\sin(4x)\sqrt{3}\arctan\left(\frac{\sqrt{3}\sin(4x)}{3}\right) - \frac{3\cos(4x)(-\ln(\cos(4x)+2)+\ln(\cos(4x)-2))}{4}\right)\mathrm{e}^{-x}}{3}$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 61

```
DSolve[y''[x]+2*y'[x]+17*y[x]==64*Exp[-x]/(3+Sin[4*x]^2),y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{3}e^{-x}\left(3\cos(4x)\left(2\coth^{-1}(2\sec(4x))+c_2\right)+\sin(4x)\left(4\sqrt{3}\cot^{-1}\left(\sqrt{3}\csc(4x)\right)+3c_1\right)\right)$$

9.17 problem Problem 17

Internal problem ID [2281]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y - \frac{4\,e^{-2x}}{x^2+1} - 2x^2 + 1 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 46

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=4*exp(-2*x)/(1+x^2)+2*x^2-1,y(x), singsol=all)
```

$$y(x) = c_2\mathrm{e}^{-2x} + \mathrm{e}^{-2x}xc_1 - 2\,\mathrm{e}^{-2x}\ln\left(x^2+1\right) + 4\arctan\left(x\right)\mathrm{e}^{-2x}x + \frac{\left(x-1\right)^2}{2}$$

✓ Solution by Mathematica

Time used: 0.287 (sec). Leaf size: 41

```
DSolve[y''[x]+4*y'[x]+4*y[x]==4*Exp[-2*x]/(1+x^2)+2*x^2-1,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{2}(x-1)^2 + e^{-2x}\left(4x\arctan(x) - 2\log\left(x^2+1\right) + c_2x + c_1\right)$$

9.18 problem Problem 18

Internal problem ID [2282]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y - 15\,\mathrm{e}^{-2x} \ln(x) - 25\cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=15*exp(-2*x)*ln(x)+25*cos(x),y(x), singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{-2x} + \mathrm{e}^{-2x} x c_1 + \frac{15x^2 \left(\ln(x) - \frac{3}{2}\right) \mathrm{e}^{-2x}}{2} + 3\cos(x) + 4\sin(x)$$

✓ Solution by Mathematica

Time used: 0.099 (sec). Leaf size: 45

```
DSolve[y''[x]+4*y'[x]+4*y[x]==15*Exp[-2*x]*Log[x]+25*Cos[x],y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{4} e^{-2x} \left(-45x^2 + 30x^2 \log(x) + 4c_2 x + 4c_1\right) + 4\sin(x) + 3\cos(x)$$

9.19 problem Problem 19

Internal problem ID [2283]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 19.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 3y'' + 3y' - y - \frac{2\,\mathrm{e}^{x}}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+3*diff(y(x),x)-y(x)=2*x^(-2)*exp(x),y(x), singsol=all)
```

$$y(x) = -2\,\mathrm{e}^{x} \ln(x)\, x + c_1 \mathrm{e}^{x} + c_2 x\, \mathrm{e}^{x} + c_3 x^2 \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.379 (sec). Leaf size: 627

```
DSolve[y'''[x]-6*y''[x]+3*y'[x]-y[x]==2*x^(-2)*Exp[x],y[x],x,IncludeSingularSolutions -> True
```

$y(x)$

$$\to \frac{2i(\text{Root}[\#1^3-6\#1^2+3\#1-1\&,1]-\text{Root}[\#1^3-6\#1^2+3\#1-1\&,2])\exp(x\text{Root}[\#1^3-6\#1^2+}{}$$

$$+\frac{2i(\text{Root}[\#1^3-6\#1^2+3\#1-1\&,2]-\text{Root}[\#1^3-6\#1^2+3\#1-1\&,3])\exp(x\text{Root}[\#1^3-6\#1^2}{}$$

$$-\frac{2i(\text{Root}[\#1^3-6\#1^2+3\#1-1\&,1]-\text{Root}[\#1^3-6\#1^2+3\#1-1\&,3])\exp(x\text{Root}[\#1^3-6\#1^2}{}$$

$$+c_2\exp(x\text{Root}[\#1^3-6\#1^2+3\#1-1\&,2])+c_3\exp(x\text{Root}[\#1^3-6\#1^2+3\#1-1\&,3])$$
$$+c_1\exp(x\text{Root}[\#1^3-6\#1^2+3\#1-1\&,1])$$

9.20 problem Problem 20

Internal problem ID [2284]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 20.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 6y'' + 12y' - 8y - 36\,\mathrm{e}^{2x}\ln(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+12*diff(y(x),x)-8*y(x)=36*exp(2*x)*ln(x),y(x), singsol
```

$$y(x) = 6\ln(x)\,\mathrm{e}^{2x}x^3 - 11\,\mathrm{e}^{2x}x^3 + c_1\mathrm{e}^{2x} + c_2\mathrm{e}^{2x}x + c_3\mathrm{e}^{2x}x^2$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 34

```
DSolve[y'''[x]-6*y''[x]+12*y'[x]-8*y[x]==36*Exp[2*x]*Log[x],y[x],x,IncludeSingularSolutions -
```

$$y(x) \to e^{2x}\left(6x^3\log(x) + x(x(-11x + c_3) + c_2) + c_1\right)$$

9.21 problem Problem 21

Internal problem ID [2285]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 21.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + 3y'' + 3y' + y - \frac{2\,e^{-x}}{x^2+1} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 64

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=2*exp(-x)/(1+x^2),y(x), singsol=al
```

$$y(x) = \arctan(x)\,x^2 e^{-x} - \ln(x^2+1)\,x\,e^{-x} - e^{-x}\arctan(x) + x\,e^{-x} + e^{-x}c_1 + c_2 e^{-x}x + c_3 x^2 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 40

```
DSolve[y'''[x]+3*y''[x]+3*y'[x]+y[x]==2*Exp[-x]/(1+x^2),y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to e^{-x}\left((x^2-1)\arctan(x) + x\left(-\log(x^2+1) + c_3 x + c_2\right) + x + c_1\right)$$

9.22 problem Problem 22

Internal problem ID [2286]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 22.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 6y'' + 9y' - 12\,e^{3x} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+9*diff(y(x),x)=12*exp(3*x),y(x), singsol=all)
```

$$y(x) = \frac{(3c_1x + 18x^2 - c_1 + 3c_2 - 12x + 4)\,e^{3x}}{9} + c_3$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 37

```
DSolve[y'''[x]-6*y''[x]+9*y'[x]==12*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9}e^{3x}(3x(6x - 4 + c_2) + 4 + 3c_1 - c_2) + c_3$$

9.23 problem Problem 23

Internal problem ID [2287]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 9y - F(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)-9*y(x)=F(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{3x} + c_1 e^{-3x} + \frac{\left(\int e^{-3x} F(x)\, dx\right) e^{3x}}{6} - \frac{\left(\int e^{3x} F(x)\, dx\right) e^{-3x}}{6}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 60

```
DSolve[y''[x]-y[x]==F[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x \left(\int_1^x \frac{1}{2} e^{-K[1]} F(K[1]) dK[1] + c_1 \right) + e^{-x} \left(\int_1^x -\frac{1}{2} e^{K[2]} F(K[2]) dK[2] + c_2 \right)$$

9.24 problem Problem 24

Internal problem ID [2288]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 5y' + 4y - F(x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)+5*diff(y(x),x)+4*y(x)=F(x),y(x), singsol=all)
```

$$y(x) = e^{-x}c_2 + c_1e^{-4x} + \frac{\left(\left(\int e^{x}F(x)\,dx\right)e^{3x} - \left(\int F(x)\,e^{4x}dx\right)\right)e^{-4x}}{3}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 61

```
DSolve[y''[x]+5*y'[x]+4*y[x]==F[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-4x}\left(\int_1^x -\frac{1}{3}e^{4K[1]}F(K[1])dK[1] + e^{3x}\left(\int_1^x \frac{1}{3}e^{K[2]}F(K[2])dK[2] + c_2\right) + c_1\right)$$

9.25 problem Problem 25

Internal problem ID [2289]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 2y - F(x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=F(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} c_2 + \mathrm{e}^{-2x} c_1 + \frac{\left(\left(\int \mathrm{e}^{-x} F(x)\, dx\right) \mathrm{e}^{3x} - \left(\int F(x)\, \mathrm{e}^{2x} dx\right)\right) \mathrm{e}^{-2x}}{3}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 62

```
DSolve[y''[x]+y'[x]-2*y[x]==F[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x} \left(\int_1^x -\frac{1}{3} e^{2K[1]} F(K[1]) dK[1] + c_1 \right) + e^{x} \left(\int_1^x \frac{1}{3} e^{-K[2]} F(K[2]) dK[2] + c_2 \right)$$

9.26 problem Problem 26

Internal problem ID [2290]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' - 12y - F(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)-12*y(x)=F(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{2x} + c_1 e^{-6x} + \frac{\left(\left(\int F(x)\, e^{-2x} dx\right) e^{8x} - \left(\int F(x)\, e^{6x} dx\right)\right) e^{-6x}}{8}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 63

```
DSolve[y''[x]+4*y'[x]-12*y[x]==F[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-6x}\left(\int_1^x -\frac{1}{8}e^{6K[1]}F(K[1])dK[1] + e^{8x}\left(\int_1^x \frac{1}{8}e^{-2K[2]}F(K[2])dK[2] + c_2\right) + c_1\right)$$

9.27 problem Problem 27

Internal problem ID [2291]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 4y - 5x\,\mathrm{e}^{2x} = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=5*x*exp(2*x),y(0) = 1, D(y)(0) = 0],y(x), singso
```

$$y(x) = \frac{\mathrm{e}^{2x}(5x^3 - 12x + 6)}{6}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 24

```
DSolve[{y''[x]-4*y'[x]+4*y[x]==5*x*Exp[2*x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{1}{6}e^{2x}(5x^3 - 12x + 6)$$

9.28 problem Problem 28

Internal problem ID [2292]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.7, The Variation of Parameters Method. page 556
Problem number: Problem 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \sec(x) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)+y(x)=sec(x),y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \sin(x) + \sin(x)\,x - \cos(x)\ln(\sec(x))$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 24

```
DSolve[{y''[x]-4*y'[x]+4*y[x]==5*x*Exp[2*x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{1}{6}e^{2x}(5x^3 - 12x + 6)$$

10 Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567

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10.1 problem Problem 14

Internal problem ID [2293]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 4y'x + 2y - 4\ln(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=4*ln(x),y(x), singsol=all)
```

$$y(x) = 2\ln(x) + \frac{c_1}{x} - 3 + \frac{c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+4*x*y'[x]+2*y[x]==4*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x + c_1}{x^2} + 2\log(x) - 3$$

10.2 problem Problem 15

Internal problem ID [2294]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 4y'x + 2y - \cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} - \frac{\cos(x)}{x^2} + \frac{c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 20

```
DSolve[x^2*y''[x]+4*x*y'[x]+2*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\cos(x) + c_2x + c_1}{x^2}$$

10.3 problem Problem 16

Internal problem ID [2295]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + 9y - 9\ln(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+9*y(x)=9*ln(x),y(x), singsol=all)
```

$$y(x) = \sin(3\ln(x))\,c_2 + \cos(3\ln(x))\,c_1 + \ln(x)$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 24

```
DSolve[x^2*y''[x]+x*y'[x]+9*y[x]==9*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x) + c_1\cos(3\log(x)) + c_2\sin(3\log(x))$$

10.4 problem Problem 17

Internal problem ID [2296]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - y'x + 5y - 8x\ln(x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+5*y(x)=8*x*(ln(x))^2,y(x), singsol=all)
```

$$y(x) = x\sin(2\ln(x))\,c_2 + x\cos(2\ln(x))\,c_1 + 2\ln(x)^2\,x - x$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 31

```
DSolve[x^2*y''[x]-x*y'[x]+5*y[x]==8*x*(Log[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(2\log^2(x) + c_2\cos(2\log(x)) + c_1\sin(2\log(x)) - 1\right)$$

10.5 problem Problem 18

Internal problem ID [2297]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 4y'x + 6y - \sin(x)\, x^4 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=x^4*sin(x),y(x), singsol=all)
```

$$y(x) = x^2c_2 + c_1x^3 - \sin(x)\, x^2$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 20

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==x^4*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(-\sin(x) + c_2x + c_1)$$

10.6 problem Problem 19

Internal problem ID [2298]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 6y'x + 6y - 4\,e^{2x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x),x$2)+6*x*diff(y(x),x)+6*y(x)=4*exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{-\frac{c_1}{x} - \frac{e^{2x}}{x} + e^{2x} + c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]+6*x*y'[x]+6*y[x]==4*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2x}(x-1) + c_2x + c_1}{x^3}$$

10.7 problem Problem 20

Internal problem ID [2299]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 3y'x + 4y - \frac{x^2}{\ln(x)} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=x^2/ln(x),y(x), singsol=all)
```

$$y(x) = x^2c_2 + \ln(x)\,c_1x^2 + \ln(x)\,x^2(-1 + \ln(\ln(x)))$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 24

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==x^2/Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(\log(x)(\log(\log(x)) - 1 + 2c_2) + c_1)$$

10.8 problem Problem 21

Internal problem ID [2300]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - (2m-1)\,xy' + m^2y - x^m \ln(x)^k = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(x^2*diff(y(x),x$2)-(2*m-1)*x*diff(y(x),x)+m^2*y(x)=x^m*(ln(x))^k,y(x), singsol=all)
```

$$y(x) = x^m c_2 + \ln(x)\,x^m c_1 + \frac{x^m \ln(x)^{k+2}}{k^2 + 3k + 2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]-(2*m-1)*x*y'[x]+m^2*y[x]==x^m*(Log[x])^k,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to x^m \left(\frac{\log^{k+2}(x)}{k^2 + 3k + 2} + c_2 m \log(x) + c_1\right)$$

10.9 problem Problem 22

Internal problem ID [2301]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x + 5y = 0}$$

With initial conditions

$$\left[y(1) = \sqrt{2}, y'(1) = 3\sqrt{2}\right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 20

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)+5*y(x)=0,y(1) = 2^(1/2), D(y)(1) = 3*2^(1/2)],y(x),
```

$$y(x) = \sqrt{2}\, x(\sin(2\ln(x)) + \cos(2\ln(x)))$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 23

```
DSolve[{x^2*y''[x]-x*y'[x]+5*y[x]==0,{y[1]==Sqrt[2],y'[1]==3*Sqrt[2]}},y[x],x,IncludeSingular
```

$$y(x) \to \sqrt{2}x(\sin(2\log(x)) + \cos(2\log(x)))$$

10.10 problem Problem 23

Internal problem ID [2302]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.8, A Differential Equation with Nonconstant Coefficients. page 567
Problem number: Problem 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$\boxed{t^2y'' + ty' + 25y = 0}$$

With initial conditions

$$\left[y(1) = \frac{3\sqrt{3}}{2}, y'(1) = \frac{15}{2}\right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 22

```
dsolve([t^2*diff(y(t),t$2)+t*diff(y(t),t)+25*y(t)=0,y(1) = 3/2*3^(1/2), D(y)(1) = 15/2],y(t),
```

$$y(t) = \frac{3\sin(5\ln(t))}{2} + \frac{3\sqrt{3}\cos(5\ln(t))}{2}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 26

```
DSolve[{t^2*y''[t]+t*y'[t]+25*y[t]==0,{y[1]==3*Sqrt[3]/2,y'[1]==15/2}},y[t],t,IncludeSingular
```

$$y(t) \to \frac{3}{2}\left(\sin(5\log(t)) + \sqrt{3}\cos(5\log(t))\right)$$

11 Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572

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11.1 problem Problem 1

Internal problem ID [2303]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$\boxed{x^2y'' - 3y'x + 4y = 0}$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=0,x^2],y(x), singsol=all)
```

$$y(x) = c_1x^2 + c_2x^2 \ln(x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(2c_2 \log(x) + c_1)$$

11.2 problem Problem 2

Internal problem ID [2304]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1-2x)\,y' + (x-1)\,y = 0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^x$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([x*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)+(x-1)*y(x)=0,exp(x)],y(x), singsol=all)
```

$$y(x) = c_1\mathrm{e}^x + c_2\mathrm{e}^x \ln(x)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 17

```
DSolve[x*y''[x]+(1-2*x)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(c_2\log(x) + c_1)$$

11.3 problem Problem 3

Internal problem ID [2305]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + (x^2 + 2)\, y = 0}$$

Given that one solution of the ode is

$$y_1 = \sin(x)\, x$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,x*sin(x)],y(x), singsol=all)
```

$$y(x) = c_1 \sin(x)\, x + c_2 \cos(x)\, x$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

11.4 problem Problem 4

Internal problem ID [2306]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(1-x^2)\, y'' - 2y'x + 2y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve([(1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,x],y(x), singsol=all)
```

$$y(x) = c_1 x + c_2 \left(\frac{\ln(x-1)\, x}{2} - \frac{\ln(x+1)\, x}{2} + 1\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 19

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2(x\text{arctanh}(x) - 1) + c_1 x$$

11.5 problem Problem 5

Internal problem ID [2307]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$y'' - \frac{y'}{x} + 4x^2y = 0$$

Given that one solution of the ode is

$$y_1 = \sin\left(x^2\right)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-1/x*diff(y(x),x)+4*x^2*y(x)=0,sin(x^2)],y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(x^2\right) + c_2 \cos\left(x^2\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 20

```
DSolve[y''[x]-1/x*y'[x]+4*x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(x^2\right) + c_2 \sin\left(x^2\right)$$

11.6 problem Problem 6

Internal problem ID [2308]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4y'x + (4x^2 - 1)\, y = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{\sin(x)}{\sqrt{x}}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 19

```
dsolve([4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2-1)*y(x)=0,sin(x)/x^(1/2)],y(x), singsol=
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 39

```
DSolve[4*x^2*y''[x]+4*x*y'[x]+(4*x^2-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

11.7 problem Problem 10

Internal problem ID [2309]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \csc(x) = 0}$$

Given that one solution of the ode is

$$y_1 = \sin(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$2)+y(x)=csc(x),sin(x)],y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + c_1 \cos(x) - \ln(\csc(x))\sin(x) - \cos(x)\, x$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 27

```
DSolve[y''[x]+y[x]==Csc[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (-x + c_1)\cos(x) + \sin(x)(\log(\tan(x)) + \log(\cos(x)) + c_2)$$

11.8 problem Problem 11

Internal problem ID [2310]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (1+2x)\,y' + 2y - 8x^2\mathrm{e}^{2x} = 0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{2x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve([x*diff(y(x),x$2)-(2*x+1)*diff(y(x),x)+2*y(x)=8*x^2*exp(2*x),exp(2*x)],y(x), singsol=a
```

$$y(x) = (1+2x)\,c_2 + c_1\mathrm{e}^{2x} + 2\,\mathrm{e}^{2x}x^2$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 32

```
DSolve[x*y''[x]-(2*x+1)*y'[x]+2*y[x]==8*x^2*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}\left(2x^2 - 1 + c_1\right) - \frac{1}{4}c_2(2x+1)$$

11.9 problem Problem 12

Internal problem ID [2311]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 3y'x + 4y - 8x^4 = 0}$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=8*x^4,x^2],y(x), singsol=all)
```

$$y(x) = x^2c_2 + \ln(x)\, c_1x^2 + 2x^4$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==8*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2\left(2x^2 + 2c_2\log(x) + c_1\right)$$

11.10 problem Problem 13

Internal problem ID [2312]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 6y' + 9y - 15\,\mathrm{e}^{3x}\sqrt{x} = 0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{3x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve([diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=15*exp(3*x)*sqrt(x),exp(3*x)],y(x), singsol=all)
```

$$y(x) = c_2\mathrm{e}^{3x} + x\,\mathrm{e}^{3x}c_1 + 4x^{\frac{5}{2}}\mathrm{e}^{3x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 25

```
DSolve[y''[x]-6*y'[x]+9*y[x]==15*Exp[3*x]*Sqrt[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}\left(4x^{5/2} + c_2x + c_1\right)$$

11.11 problem Problem 14

Internal problem ID [2313]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 4y - 4\,\mathrm{e}^{2x}\ln(x) = 0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{2x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=4*exp(2*x)*ln(x),exp(2*x)],y(x), singsol=all)
```

$$y(x) = c_2\mathrm{e}^{2x} + \mathrm{e}^{2x}xc_1 + \mathrm{e}^{2x}x^2(2\ln(x) - 3)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 29

```
DSolve[y''[x]-4*y'[x]+4*y[x]==4*Exp[2*x]*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}\left(2x^2\log(x) + x(-3x + c_2) + c_1\right)$$

11.12 problem Problem 15

Internal problem ID [2314]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.9, Reduction of Order. page 572
Problem number: Problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{4x^2y'' + y - \sqrt{x}\,\ln(x) = 0}$$

Given that one solution of the ode is

$$y_1 = \sqrt{x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve([4*x^2*diff(y(x),x$2)+y(x)=sqrt(x)*ln(x),sqrt(x)],y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,c_2 + \sqrt{x}\,\ln(x)\,c_1 + \frac{\ln(x)^3\sqrt{x}}{24}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 29

```
DSolve[4*x^2*y''[x]+y[x]==Sqrt[x]*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{24}\sqrt{x}\left(\log^3(x) + 12c_2\log(x) + 24c_1\right)$$

12 Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575

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12.1 problem Problem 7

Internal problem ID [2315]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 7.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 3y'' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2 \mathrm{e}^{-2x} + c_3 \mathrm{e}^{-2x} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[y'''[x]+3*y''[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(c_2 x + c_1) + c_3 e^{x}$$

12.2 problem Problem 8

Internal problem ID [2316]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 8.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 11y'' + 36y' + 26y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$3)+11*diff(y(x),x$2)+36*diff(y(x),x)+26*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 \mathrm{e}^{-5x} \sin(x) + c_3 \mathrm{e}^{-5x} \cos(x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y'''[x]+11*y''[x]+36*y'[x]+26*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-5x}\left(c_3 e^{4x} + c_2 \cos(x) + c_1 \sin(x)\right)$$

12.3 problem Problem 18

Internal problem ID [2317]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 6y' + 9y - 4\,\mathrm{e}^{-3x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=4*exp(-3*x),y(x), singsol=all)
```

$$y(x) = c_2\mathrm{e}^{-3x} + x\,\mathrm{e}^{-3x}c_1 + 2\,\mathrm{e}^{-3x}x^2$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 22

```
DSolve[y''[x]+6*y'[x]+9*y[x]==4*Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}(x(2x + c_2) + c_1)$$

12.4 problem Problem 19

Internal problem ID [2318]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 6y' + 9y - 4\,\mathrm{e}^{-2x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=4*exp(-2*x),y(x), singsol=all)
```

$$y(x) = c_2\mathrm{e}^{-3x} + x\,\mathrm{e}^{-3x}c_1 + 4\,\mathrm{e}^{-2x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 23

```
DSolve[y''[x]+6*y'[x]+9*y[x]==4*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}(4e^x + c_2x + c_1)$$

12.5 problem Problem 20

Internal problem ID [2319]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 20.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 6y'' + 25y' - x^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+25*diff(y(x),x)=x^2,y(x), singsol=all)
```

$$\begin{aligned}y(x) = {} & \frac{6x^2}{625} + \frac{x^3}{75} + \frac{3\,\mathrm{e}^{3x}\cos(4x)\,c_1}{25} + \frac{4c_1\mathrm{e}^{3x}\sin(4x)}{25} \\ & - \frac{4c_2\mathrm{e}^{3x}\cos(4x)}{25} + \frac{3\,\mathrm{e}^{3x}\sin(4x)\,c_2}{25} + \frac{22x}{15625} + c_3\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.129 (sec). Leaf size: 61

```
DSolve[y'''[x]-6*y''[x]+25*y'[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(25x(25x+18)+66)}{46875} + \frac{1}{25}e^{3x}((3c_2 - 4c_1)\cos(4x) + (3c_1 + 4c_2)\sin(4x)) + c_3$$

12.6 problem Problem 21

Internal problem ID [2320]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 21.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 6y'' + 25y' - \sin(4x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 62

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+25*diff(y(x),x)=sin(4*x),y(x), singsol=all)
```

$$y(x) = \frac{3\,e^{3x}\cos(4x)\,c_1}{25} + \frac{4c_1 e^{3x}\sin(4x)}{25} - \frac{4c_2 e^{3x}\cos(4x)}{25} + \frac{3\,e^{3x}\sin(4x)\,c_2}{25} + \frac{2\sin(4x)}{219} - \frac{\cos(4x)}{292} + c_3$$

✓ Solution by Mathematica

Time used: 0.337 (sec). Leaf size: 62

```
DSolve[y'''[x]-6*y''[x]+25*y'[x]==Sin[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{219}\sin(4x) - \frac{1}{292}\cos(4x) + \frac{1}{25}e^{3x}((3c_2 - 4c_1)\cos(4x) + (3c_1 + 4c_2)\sin(4x)) + c_3$$

12.7 problem Problem 22

Internal problem ID [2321]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 22.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' + 9y'' + 24y' + 16y - 8\,\mathrm{e}^{-x} - 1 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$3)+9*diff(y(x),x$2)+24*diff(y(x),x)+16*y(x)=8*exp(-x)+1,y(x), singsol=all)
```

$$y(x) = \frac{1}{16} - \frac{16\,\mathrm{e}^{-x}}{27} + \frac{8x\,\mathrm{e}^{-x}}{9} + c_1\mathrm{e}^{-4x} + \mathrm{e}^{-x}c_2 + c_3x\,\mathrm{e}^{-4x}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 39

```
DSolve[y'''[x]+9*y''[x]+24*y'[x]+16*y[x]==8*Exp[-x]+1,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{16} + e^{-4x}\left(c_2x + e^{3x}\left(\frac{8x}{9} - \frac{16}{27} + c_3\right) + c_1\right)$$

12.8 problem Problem 27

Internal problem ID [2322]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y - 5\,\mathrm{e}^{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-4*y(x)=5*exp(x),y(x), singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{2x} + \mathrm{e}^{-2x} c_1 - \frac{5\,\mathrm{e}^{x}}{3}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 29

```
DSolve[y''[x]-4*y[x]==5*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{5e^{x}}{3} + c_1 e^{2x} + c_2 e^{-2x}$$

12.9 problem Problem 28

Internal problem ID [2323]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y - 2x\,\mathrm{e}^{-x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=2*x*exp(-x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_2 + x\,\mathrm{e}^{-x} c_1 + \frac{\mathrm{e}^{-x} x^3}{3}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 27

```
DSolve[y''[x]+2*y'[x]+y[x]==2*x*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{-x} \left(x^3 + 3c_2 x + 3c_1\right)$$

12.10 problem Problem 29

Internal problem ID [2324]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y - 4\,e^{x} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)-y(x)=4*exp(x),y(x), singsol=all)
```

$$y(x) = e^{-x}c_2 + c_1 e^{x} + 2x\,e^{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 25

```
DSolve[y''[x]-y[x]==4*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(2x - 1 + c_1) + c_2 e^{-x}$$

12.11 problem Problem 30

Internal problem ID [2325]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + yx - \sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$2)+x*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = \operatorname{AiryAi}(-x)\, c_2 + \operatorname{AiryBi}(-x)\, c_1 + \pi\left(\operatorname{AiryAi}(-x)\left(\int \operatorname{AiryBi}(-x)\sin(x)\, dx\right) - \operatorname{AiryBi}(-x)\left(\int \operatorname{AiryAi}(-x)\sin(x)\, dx\right)\right)$$

✓ Solution by Mathematica

Time used: 51.516 (sec). Leaf size: 99

```
DSolve[y''[x]+x*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \operatorname{AiryAi}\left(\sqrt[3]{-1}x\right)\int_1^x (-1)^{2/3}\pi \operatorname{AiryBi}\left(\sqrt[3]{-1}K[1]\right)\sin(K[1])dK[1] + \operatorname{AiryBi}\left(\sqrt[3]{-1}x\right)\int_1^x -(-1)^{2/3}\pi \operatorname{AiryAi}\left(\sqrt[3]{-1}K[2]\right)\sin(K[2])dK[2] + c_1 \operatorname{AiryAi}\left(\sqrt[3]{-1}x\right) + c_2 \operatorname{AiryBi}\left(\sqrt[3]{-1}x\right)$$

12.12 problem Problem 31

Internal problem ID [2326]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y - \ln(x) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 65

```
dsolve(diff(y(x),x$2)+4*y(x)=ln(x),y(x), singsol=all)
```

$$y(x) = \sin(2x) c_2 + \cos(2x) c_1 + \frac{i\pi \cos(2x) (\operatorname{csgn}(x) - 1) \operatorname{csgn}(ix)}{8} - \frac{\cos(2x) \operatorname{Ci}(2x)}{4} + \frac{(\pi \operatorname{csgn}(x) - 2 \operatorname{Si}(2x)) \sin(2x)}{8} + \frac{\ln(x)}{4}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 44

```
DSolve[y''[x]+4*y[x]==Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(\cos(2x)(-\operatorname{CosIntegral}(2x) + 4c_1) + \sin(2x)(-\operatorname{Si}(2x) + 4c_2) + \log(x))$$

12.13 problem Problem 32

Internal problem ID [2327]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' - 3y - 5\,e^{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-3*y(x)=5*exp(x),y(x), singsol=all)
```

$$y(x) = e^{x} c_2 + c_1 e^{-3x} + \frac{5x\,e^{x}}{4}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 29

```
DSolve[y''[x]+2*y'[x]-3*y[x]==5*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-3x} + e^{x}\left(\frac{5x}{4} - \frac{5}{16} + c_2\right)$$

12.14 problem Problem 33

Internal problem ID [2328]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \tan(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=tan(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + c_1 \cos(x) - \ln(\sec(x) + \tan(x)) \cos(x)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)(-\text{arctanh}(\sin(x)) + c_1) + c_2 \sin(x)$$

12.15 problem Problem 34

Internal problem ID [2329]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 8, Linear differential equations of order n. Section 8.10, Chapter review. page 575
Problem number: Problem 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - 4\cos(2x) - 3\,\mathrm{e}^{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=4*cos(2*x)+3*exp(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + c_1 \cos(x) - \frac{4\cos(2x)}{3} + \frac{3\,\mathrm{e}^{x}}{2}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 30

```
DSolve[y''[x]+y[x]==4*Cos[x]*3*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{12}{5} e^{x}(2\sin(x) + \cos(x)) + c_1 \cos(x) + c_2 \sin(x)$$

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13.1 problem Problem 1

Internal problem ID [2330]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - 2y - 6\,\mathrm{e}^{5t} = 0}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(t),t)-2*y(t)=6*exp(5*t),y(0) = 3],y(t), singsol=all)
```

$$y(t) = \left(2\,\mathrm{e}^{3t} + 1\right)\mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 18

```
DSolve[{y'[t]-2*y[t]==6*Exp[5*t],{y[0]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{2t} + 2e^{5t}$$

13.2 problem Problem 2

Internal problem ID [2331]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y - 8\,e^{3t} = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve([diff(y(t),t)+y(t)=8*exp(3*t),y(0) = 2],y(t), singsol=all)
```

$$y(t) = 2\,e^{3t}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 12

```
DSolve[{y'[t]+y[t]==8*Exp[3*t],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2e^{3t}$$

13.3 problem Problem 3

Internal problem ID [2332]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 3y - 2\,\mathrm{e}^{-t} = 0}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)+3*y(t)=2*exp(-t),y(0) = 3],y(t), singsol=all)
```

$$y(t) = \left(\mathrm{e}^{2t} + 2\right)\mathrm{e}^{-3t}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 18

```
DSolve[{y'[t]+3*y[t]==2*Exp[-t],{y[0]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-3t}\left(e^{2t} + 2\right)$$

13.4 problem Problem 4

Internal problem ID [2333]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{2y + y' - 4t = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)+2*y(t)=4*t,y(0) = 1],y(t), singsol=all)
```

$$y(t) = 2t - 1 + 2\,\mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 17

```
DSolve[{y'[t]+2*y[t]==4*t,{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2t + 2e^{-2t} - 1$$

13.5 problem Problem 5

Internal problem ID [2334]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y - 6\cos(t) = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(t),t)-y(t)=6*cos(t),y(0) = 2],y(t), singsol=all)
```

$$y(t) = 3\sin(t) - 3\cos(t) + 5\,\mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 19

```
DSolve[{y'[t]-y[t]==6*Cos[t],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 5e^{t} + 3\sin(t) - 3\cos(t)$$

13.6 problem Problem 6

Internal problem ID [2335]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y - 5\sin(2t) = 0}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve([diff(y(t),t)-y(t)=5*sin(2*t),y(0) = -1],y(t), singsol=all)
```

$$y(t) = -2\cos(2t) - \sin(2t) + e^t$$

✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 21

```
DSolve[{y'[t]-y[t]==5*Sin[2*t],{y[0]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^t - \sin(2t) - 2\cos(2t)$$

13.7 problem Problem 7

Internal problem ID [2336]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y - 5\,\mathrm{e}^{t} \sin\left(t\right) = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(t),t)+y(t)=5*exp(t)*sin(t),y(0) = 1],y(t), singsol=all)
```

$$y(t) = 2\,\mathrm{e}^{-t} + \mathrm{e}^{t}(-\cos\left(t\right) + 2\sin\left(t\right))$$

✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 25

```
DSolve[{y'[t]+y[t]==5*Exp[t]*Sin[t],{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2e^{-t} - e^{t}(\cos(t) - 2\sin(t))$$

13.8 problem Problem 8

Internal problem ID [2337]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 2y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 4]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-2*y(t)=0,y(0) = 1, D(y)(0) = 4],y(t), singsol=all)
```

$$y(t) = (2\,e^{3t} - 1)\,e^{-2t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{y''[t]+y'[t]-2*y[t]==0,{y[0]==1,y'[0]==4}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2e^t - e^{-2t}$$

13.9 problem Problem 9

Internal problem ID [2338]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y = 0}$$

With initial conditions

$$[y(0) = 5, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+4*y(t)=0,y(0) = 5, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{\sin(2t)}{2} + 5\cos(2t)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 17

```
DSolve[{y''[t]+4*y[t]==0,{y[0]==5,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 5\cos(2t) + \sin(t)\cos(t)$$

13.10 problem Problem 10

Internal problem ID [2339]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' + 2y - 4 = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)-3*diff(y(t),t)+2*y(t)=4,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = 3\,e^{2t} - 5\,e^{t} + 2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{y''[t]-3*y'[t]+2*y[t]==4,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to (e^t - 1)\,(3e^t - 2)$$

13.11 problem Problem 11

Internal problem ID [2340]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 12y - 36 = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 12]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 12

```
dsolve([diff(y(t),t$2)-diff(y(t),t)-12*y(t)=36,y(0) = 0, D(y)(0) = 12],y(t), singsol=all)
```

$$y(t) = 3\,\mathrm{e}^{4t} - 3$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 14

```
DSolve[{y''[t]-y'[t]-12*y[t]==36,{y[0]==0,y'[0]==12}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to 3\left(e^{4t} - 1\right)$$

13.12 problem Problem 12

Internal problem ID [2341]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' - 2y - 10\,\mathrm{e}^{-t} = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-2*y(t)=10*exp(-t),y(0) = 0, D(y)(0) = 1],y(t), singsol=al
```

$$y(t) = \left(2\,\mathrm{e}^{3t} - 5\,\mathrm{e}^{t} + 3\right)\mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 25

```
DSolve[{y''[t]+y'[t]-2*y[t]==10*Exp[-t],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -
```

$$y(t) \rightarrow e^{-2t}\left(-5e^{t} + 2e^{3t} + 3\right)$$

13.13 problem Problem 13

Internal problem ID [2342]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y' + 2y - 4\,\mathrm{e}^{3t} = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve([diff(y(t),t$2)-3*diff(y(t),t)+2*y(t)=4*exp(3*t),y(0) = 0, D(y)(0) = 0],y(t), singsol=
```

$$y(t) = -4\,\mathrm{e}^{2t} + 2\,\mathrm{e}^{2t}\mathrm{e}^{t} + 2\,\mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 17

```
DSolve[{y''[t]-3*y'[t]+2*y[t]==4*Exp[3*t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to 2e^t\left(e^t - 1\right)^2$$

13.14 problem Problem 14

Internal problem ID [2343]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 2y' - 30\,\mathrm{e}^{-3t} = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)=30*exp(-3*t),y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \left(3\,\mathrm{e}^{5t} - 4\,\mathrm{e}^{3t} + 2\right)\mathrm{e}^{-3t}$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 21

```
DSolve[{y''[t]-2*y'[t]==30*Exp[-3*t],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to 2e^{-3t} + 3e^{2t} - 4$$

13.15 problem Problem 15

Internal problem ID [2344]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y - 12\,\mathrm{e}^{2t} = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)-y(t)=12*exp(2*t),y(0) = 1, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = 2\,\mathrm{e}^{-t} - 5\,\mathrm{e}^{t} + 4\,\mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 25

```
DSolve[{y''[t]-y[t]==12*Exp[2*t],{y[0]==1,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2e^{-t} - 5e^{t} + 4e^{2t}$$

13.16 problem Problem 16

Internal problem ID [2345]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y - 10\,\mathrm{e}^{-t} = 0}$$

With initial conditions

$$[y(0) = 4, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+4*y(t)=10*exp(-t),y(0) = 4, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \sin(2t) + 2\cos(2t) + 2\,\mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 23

```
DSolve[{y''[t]+4*y[t]==10*Exp[-t],{y[0]==4,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to 2e^{-t} + \sin(2t) + 2\cos(2t)$$

13.17 problem Problem 17

Internal problem ID [2346]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 6y - 12 + 6\,\mathrm{e}^{t} = 0}$$

With initial conditions

$$[y(0) = 5, y'(0) = -3]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)-diff(y(t),t)-6*y(t)=6*(2-exp(t)),y(0) = 5, D(y)(0) = -3],y(t), singsol
```

$$y(t) = \frac{(8\,\mathrm{e}^{5t} + 5\,\mathrm{e}^{3t} - 10\,\mathrm{e}^{2t} + 22)\,\mathrm{e}^{-2t}}{5}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 28

```
DSolve[{y''[t]-y'[t]-6*y[t]==6*(2-Exp[t]),{y[0]==5,y'[0]==-3}},y[t],t,IncludeSingularSolution
```

$$y(t) \to \frac{22e^{-2t}}{5} + e^{t} + \frac{8e^{3t}}{5} - 2$$

13.18 problem Problem 18

Internal problem ID [2347]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y - 6\cos(t) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 4]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve([diff(y(t),t$2)-y(t)=6*cos(t),y(0) = 0, D(y)(0) = 4],y(t), singsol=all)
```

$$y(t) = -\frac{e^{-t}}{2} + \frac{7\,e^{t}}{2} - 3\cos(t)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 18

```
DSolve[{y''[t]-y[t]==6*Cos[t],{y[0]==0,y'[0]==4}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -3\cos(t) + 4\sinh(t) + 3\cosh(t)$$

13.19 problem Problem 19

Internal problem ID [2348]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 9y - 13\sin(2t) = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)-9*y(t)=13*sin(2*t),y(0) = 3, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = 2\,\mathrm{e}^{3t} + \mathrm{e}^{-3t} - \sin(2t)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 22

```
DSolve[{y''[t]-9*y[t]==13*Sin[2*t],{y[0]==3,y'[0]==1}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to -\sin(2t) + \sinh(3t) + 3\cosh(3t)$$

13.20 problem Problem 20

Internal problem ID [2349]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y - 8\sin(t) + 6\cos(t) = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)-y(t)=8*sin(t)-6*cos(t),y(0) = 2, D(y)(0) = -1],y(t), singsol=all)
```

$$y(t) = -2\,e^{-t} + e^{t} - 4\sin(t) + 3\cos(t)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 22

```
DSolve[{y''[t]-y[t]==8*Sin[t]-6*Cos[t],{y[0]==2,y'[0]==-1}},y[t],t,IncludeSingularSolutions -
```

$$y(t) \to -4\sin(t) + 3\cos(t) + 3\sinh(t) - \cosh(t)$$

13.21 problem Problem 21

Internal problem ID [2350]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - 2y - 10\cos(t) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)-diff(y(t),t)-2*y(t)=10*cos(t),y(0) = 0, D(y)(0) = -1],y(t), singsol=al
```

$$y(t) = e^{2t} + 2\,e^{-t} - 3\cos(t) - \sin(t)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 26

```
DSolve[{y''[t]-y'[t]-2*y[t]==10*Cos[t],{y[0]==0,y'[0]==-1}},y[t],t,IncludeSingularSolutions -
```

$$y(t) \to 2e^{-t} + e^{2t} - \sin(t) - 3\cos(t)$$

13.22 problem Problem 22

Internal problem ID [2351]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 5y' + 4y - 20\sin(2t) = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)+5*diff(y(t),t)+4*y(t)=20*sin(2*t),y(0) = -1, D(y)(0) = 2],y(t), singso
```

$$y(t) = 2\,\mathrm{e}^{-t} - \mathrm{e}^{-4t} - 2\cos(2t)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 27

```
DSolve[{y''[t]+5*y'[t]+4*y[t]==20*Sin[2*t],{y[0]==-1,y'[0]==2}},y[t],t,IncludeSingularSolutio
```

$$y(t) \to e^{-4t}\left(2e^{3t} - 1\right) - 2\cos(2t)$$

13.23 problem Problem 23

Internal problem ID [2352]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 5y' + 4y - 20\sin(2t) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -2]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)+5*diff(y(t),t)+4*y(t)=20*sin(2*t),y(0) = 1, D(y)(0) = -2],y(t), singso
```

$$y(t) = \frac{10\,e^{-t}}{3} - \frac{e^{-4t}}{3} - 2\cos(2t)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 30

```
DSolve[{y''[t]+5*y'[t]+4*y[t]==20*Sin[2*t],{y[0]==1,y'[0]==-2}},y[t],t,IncludeSingularSolutio
```

$$y(t) \to \frac{1}{3}e^{-4t}\left(10e^{3t} - 1\right) - 2\cos(2t)$$

13.24 problem Problem 24

Internal problem ID [2353]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 3y' + 2y - 3\cos(t) - \sin(t) = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)-3*diff(y(t),t)+2*y(t)=3*cos(t)+sin(t),y(0) = 1, D(y)(0) = 1],y(t), sin
```

$$y(t) = \frac{7\,e^{2t}}{5} + \frac{3\cos(t)}{5} - \frac{4\sin(t)}{5} - e^{t}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 29

```
DSolve[{y''[t]-3*y'[t]+2*y[t]==3*Cos[t]+Sin[t],{y[0]==1,y'[0]==1}},y[t],t,IncludeSingularSolu
```

$$y(t) \to \frac{1}{5}\left(e^t\left(7e^t - 5\right) - 4\sin(t) + 3\cos(t)\right)$$

13.25 problem Problem 25

Internal problem ID [2354]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y - 9\sin(t) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 19

```
dsolve([diff(y(t),t$2)+4*y(t)=9*sin(t),y(0) = 1, D(y)(0) = -1],y(t), singsol=all)
```

$$y(t) = -2\sin(2t) + \cos(2t) + 3\sin(t)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 20

```
DSolve[{y''[t]+4*y[t]==9*Sin[t],{y[0]==1,y'[0]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 3\sin(t) - 2\sin(2t) + \cos(2t)$$

13.26 problem Problem 26

Internal problem ID [2355]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - 6\cos(2t) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 19

```
dsolve([diff(y(t),t$2)+y(t)=6*cos(2*t),y(0) = 0, D(y)(0) = 2],y(t), singsol=all)
```

$$y(t) = 2\sin(t) + 2\cos(t) - 2\cos(2t)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 18

```
DSolve[{y''[t]+y[t]==6*Cos[2*t],{y[0]==0,y'[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2(\sin(t) + \cos(t) - \cos(2t))$$

13.27 problem Problem 27

Internal problem ID [2356]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y - 7\sin(4t) - 14\cos(4t) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 29

```
dsolve([diff(y(t),t$2)+9*y(t)=7*sin(4*t)+14*cos(4*t),y(0) = 1, D(y)(0) = 2],y(t), singsol=all
```

$$y(t) = 2\sin(3t) + 3\cos(3t) - \sin(4t) - 2\cos(4t)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 50

```
DSolve[{y''[t]+8*y[t]==7*Sin[4*t]+14*Cos[4*t],{y[0]==1,y'[0]==2}},y[t],t,IncludeSingularSolut
```

$$y(t) \to \frac{1}{8}\left(11\sqrt{2}\sin\left(2\sqrt{2}t\right) + 22\cos\left(2\sqrt{2}t\right) - 7(\sin(4t) + 2\cos(4t))\right)$$

13.28 problem Problem 28

Internal problem ID [2357]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.4. page 689
Problem number: Problem 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

With initial conditions

$$[y(0) = A, y'(0) = B]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)-y(t)=0,y(0) = A, D(y)(0) = B],y(t), singsol=all)
```

$$y(t) = \frac{(A-B)\,\mathrm{e}^{-t}}{2} + \frac{\mathrm{e}^{t}(B+A)}{2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 14

```
DSolve[{y''[t]-y[t]==0,{y[0]==a,y'[0]==b}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to a\cosh(t) + b\sinh(t)$$

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14.1 problem Problem 27

Internal problem ID [2358]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{2y + y' - 2\,\text{Heaviside}\,(t-1) = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 25

```
dsolve([diff(y(t),t)+2*y(t)=2*Heaviside(t-1),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \text{Heaviside}\,(t-1) - \text{Heaviside}\,(t-1)\,\mathrm{e}^{-2t+2} + \mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 25

```
DSolve[{y'[t]-y[t]==2*UnitStep[t-1],{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \left\{ \begin{array}{cc} e^t & t \leq 1 \\ -2 + e^{t-1}(2+e) & \text{True} \end{array} \right.$$

14.2 problem Problem 28

Internal problem ID [2359]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - 2y - \text{Heaviside}\,(t-2)\,\mathrm{e}^{t-2} = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve([diff(y(t),t)-2*y(t)=Heaviside(t-2)*exp(t-2),y(0) = 2],y(t), singsol=all)
```

$$y(t) = \left(-\text{Heaviside}\,(t-2)\,\mathrm{e}^{-t-2} + \text{Heaviside}\,(t-2)\,\mathrm{e}^{-4} + 2\right)\mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 40

```
DSolve[{y'[t]-2*y[t]==UnitStep[t-2]*Exp[t-2],{y[0]==2}},y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to \begin{cases} 2e^{2t} & t \leq 2 \\ e^{t-4}(-e^2 + e^t + 2e^{t+4}) & \text{True} \end{cases}$$

14.3 problem Problem 29

Internal problem ID [2360]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - y - 4\,\mathrm{Heaviside}\left(t - \frac{\pi}{4}\right)\sin\left(t + \frac{\pi}{4}\right) = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 39

```
dsolve([diff(y(t),t)-y(t)=4*Heaviside(t-Pi/4)*cos(t-Pi/4),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \left(-2\cos\left(t + \frac{\pi}{4}\right) + 2\,\mathrm{e}^{t-\frac{\pi}{4}} - 2\sin\left(t + \frac{\pi}{4}\right)\right)\mathrm{Heaviside}\left(t - \frac{\pi}{4}\right) + \mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 40

```
DSolve[{y'[t]-y[t]==4*UnitStep[t-Pi/4]*Cos[t-Pi/4],{y[0]==1}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to \left\{ \begin{array}{cc} e^{t} & 4t \leq \pi \\ -2\sqrt{2}\cos(t) + e^{t} + 2e^{t-\frac{\pi}{4}} & \text{True} \end{array} \right.$$

14.4 problem Problem 30

Internal problem ID [2361]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{2y + y' - \text{Heaviside}\left(-\pi + t\right)\sin\left(2t\right) = 0}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 46

```
dsolve([diff(y(t),t)+2*y(t)=Heaviside(t-Pi)*sin(2*t),y(0) = 3],y(t), singsol=all)
```

$$y(t) = \frac{\text{Heaviside}\left(-\pi + t\right)\mathrm{e}^{-2t+2\pi}}{4} + \frac{\text{Heaviside}\left(-\pi + t\right)\left(-\cos\left(2t\right) + \sin\left(2t\right)\right)}{4} + 3\,\mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.117 (sec). Leaf size: 46

```
DSolve[{y'[t]+2*y[t]==UnitStep[t-Pi]*Sin[2*t],{y[0]==3}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to \begin{cases} 3e^{-2t} & t \leq \pi \\ \frac{1}{4}(-\cos(2t) + \sin(2t) + e^{-2t}(12 + e^{2\pi})) & \text{True} \end{cases}$$

14.5 problem Problem 31

Internal problem ID [2362]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + 3y - \left(\begin{cases} 1 & 0 \leq t < 1 \\ 0 & 1 \leq t \end{cases}\right) = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 41

```
dsolve([diff(y(t),t)+3*y(t)=piecewise(0<=t and t<1,1,t>=1,0),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \begin{cases} e^{-3t} & t < 0 \\ \frac{2\,e^{-3t}}{3} + \frac{1}{3} & t < 1 \\ \frac{2\,e^{-3t}}{3} + \frac{e^{3-3t}}{3} & 1 \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 47

```
DSolve[{y'[t]+3*y[t]==Piecewise[{{1,0<=t<1},{0,t >= 1}}],{y[0]==1}},y[t],t,IncludeSingularSol
```

$$y(t) \rightarrow \{ \begin{array}{cc} e^{-3t} & t \leq 0 \\ \frac{1}{3}e^{-3t}(2 + e^3) & t > 1 \\ \frac{1}{3} + \frac{2e^{-3t}}{3} & \text{True} \end{array}$$

14.6 problem Problem 32

Internal problem ID [2363]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - 3y - \left(\begin{cases} \sin(t) & 0 \leq t < \frac{\pi}{2} \\ 1 & \frac{\pi}{2} \leq t \end{cases}\right) = 0$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 57

```
dsolve([diff(y(t),t)-3*y(t)=piecewise(0<=t and t<Pi/2,sin(t),t>=Pi/2,1),y(0) = 2],y(t), sings
```

$$y(t) = \begin{cases} 2\,e^{3t} & t < 0 \\ \frac{21\,e^{3t}}{10} - \frac{\cos(t)}{10} - \frac{3\sin(t)}{10} & t < \frac{\pi}{2} \\ \frac{21\,e^{3t}}{10} + \frac{e^{3t-\frac{3\pi}{2}}}{30} - \frac{1}{3} & \frac{\pi}{2} \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 65

```
DSolve[{y'[t]-3*y[t]==Piecewise[{{Sin[t],0<=t<Pi/2},{1,t >= Pi/2}}],{y[0]==2}},y[t],t,Include
```

$$y(t) \to \begin{cases} 2e^{3t} & t \leq 0 \\ \frac{1}{30}\left(-10 + e^{3t}\left(63 + e^{-3\pi/2}\right)\right) & 2t > \pi \\ \frac{1}{10}\left(-\cos(t) + 21e^{3t} - 3\sin(t)\right) & \text{True} \end{cases}$$

14.7 problem Problem 33

Internal problem ID [2364]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - 3y + 10\,\mathrm{e}^{-t+a}\sin(-2t+2a)\,\mathrm{Heaviside}(t-a) = 0$$

With initial conditions

$$[y(0) = 5]$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 100

```
dsolve([diff(y(t),t)-3*y(t)=10*exp(-(t-a))*sin(2*(t-a))*Heaviside(t-a),y(0) = 5],y(t), singso
```

$$y(t) = -\left(\left((\cos(2t) + 2\sin(2t))\cos(2a) - 2\sin(2a)\left(\cos(2t) - \frac{\sin(2t)}{2}\right)\right)\mathrm{Heaviside}(t - a)\,\mathrm{e}^{4a-4t} - \mathrm{Heaviside}(t-a) + (\mathrm{Heaviside}(a) - 1)\,\mathrm{e}^{4a}\cos(2a) + (-2\,\mathrm{Heaviside}(a) + 2)\sin(2a)\,\mathrm{e}^{4a} - 5\,\mathrm{e}^{3a} - \mathrm{Heaviside}(a) + 1\right)\mathrm{e}^{3t-3a}$$

✓ Solution by Mathematica

Time used: 0.444 (sec). Leaf size: 88

```
DSolve[{y'[t]-3*y[t]==10*Exp[-(t-a)]*Sin[2*(t-a)]*UnitStep[t-a],{y[0]==5}},y[t],t,IncludeSing
```

$$y(t) \to e^{3t-3a}\left(\theta(t-a) + \theta(-a)\left(e^{4a}(\cos(2a) - 2\sin(2a)) - 1\right) + 5e^{3a}\right) - e^{a-t}\theta(t-a)(\cos(2(a-t)) - 2\sin(2(a-t)))$$

14.8 problem Problem 34

Internal problem ID [2365]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y - \text{Heaviside}\,(t-1) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 33

```
dsolve([diff(y(t),t$2)-y(t)=Heaviside(t-1),y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\text{Heaviside}\,(t-1)\,\mathrm{e}^{-t+1}}{2} + \frac{(\mathrm{e}^{t-1}-2)\,\text{Heaviside}\,(t-1)}{2} + \frac{\mathrm{e}^{-t}}{2} + \frac{\mathrm{e}^{t}}{2}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 23

```
DSolve[{y''[t]-y[t]==UnitStep[t-1],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to \begin{cases} \cosh(t) & t \leq 1 \\ \cosh(1-t) + \cosh(t) - 1 & \text{True} \end{cases}$$

14.9 problem Problem 35

Internal problem ID [2366]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - 2y - 1 + 3\,\text{Heaviside}\,(t-2) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 48

```
dsolve([diff(y(t),t$2)-diff(y(t),t)-2*y(t)=1-3*Heaviside(t-2),y(0) = 1, D(y)(0) = -2],y(t), s
```

$$y(t) = -\frac{e^{2t}}{6} + \frac{5\,e^{-t}}{3} + \frac{3\,\text{Heaviside}\,(t-2)}{2} - \frac{\text{Heaviside}\,(t-2)\,e^{2t-4}}{2} - \frac{1}{2} - \text{Heaviside}\,(t-2)\,e^{2-t}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 65

```
DSolve[{y''[t]-y'[t]-2*y[t]==1-3*UnitStep[t-2],{y[0]==1,y'[0]==-2}},y[t],t,IncludeSingularSol
```

$$y(t) \to \begin{cases} \frac{1}{6}(-3 + 10e^{-t} - e^{2t}) & t \leq 2 \\ 1 + \frac{1}{3}e^{-t}(5 - 3e^2) - \frac{1}{6}e^{2t-4}(3 + e^4) & \text{True} \end{cases}$$

14.10 problem Problem 36

Internal problem ID [2367]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y - \text{Heaviside}(t-1) + \text{Heaviside}(t-2) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 4]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 75

```
dsolve([diff(y(t),t$2)-4*y(t)=Heaviside(t-1)-Heaviside(t-2),y(0) = 0, D(y)(0) = 4],y(t), sing
```

$$y(t) = e^{2t} - e^{-2t} - \frac{\text{Heaviside}(t-1)}{4} + \frac{\text{Heaviside}(t-1)\,e^{2t-2}}{8} + \frac{\text{Heaviside}(t-2)}{4} - \frac{\text{Heaviside}(t-2)\,e^{2t-4}}{8} + \frac{\text{Heaviside}(t-1)\,e^{-2t+2}}{8} - \frac{\text{Heaviside}(t-2)\,e^{-2t+4}}{8}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 61

```
DSolve[{y''[t]-4*y[t]==UnitStep[t-1]-UnitStep[t-2],{y[0]==0,y'[0]==4}},y[t],t,IncludeSingular
```

$$y(t) \to \frac{1}{4}(-\theta(1-t)(\cosh(2-2t)-1) + \theta(2-t)(\cosh(4-2t)-1) + 8\sinh(2t) + \cosh(2-2t) - \cosh(4-2t))$$

14.11 problem Problem 37

Internal problem ID [2368]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - t + \text{Heaviside}\,(t-1)\,(t-1) = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve([diff(y(t),t$2)+y(t)=t-Heaviside(t-1)*(t-1),y(0) = 2, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = 2\cos(t) + (-t + \sin(t-1) + 1)\,\text{Heaviside}\,(t-1) + t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 31

```
DSolve[{y''[t]+y[t]==t-UnitStep[t-1]*(t-1),{y[0]==2,y'[0]==1}},y[t],t,IncludeSingularSolution
```

$$y(t) \to \begin{cases} t + 2\cos(t) & t \leq 1 \\ 2\cos(t) - \sin(1-t) + 1 & \text{True} \end{cases}$$

14.12 problem Problem 38

Internal problem ID [2369]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 3y' + 2y + 10\,\text{Heaviside}\left(t - \frac{\pi}{4}\right)\cos\left(t + \frac{\pi}{4}\right) = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 67

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=10*Heaviside(t-Pi/4)*sin(t-Pi/4),y(0) = 1, D(y)(
```

$$y(t) = -2\,\text{Heaviside}\left(t - \frac{\pi}{4}\right)e^{-2t+\frac{\pi}{2}} + 5\,\text{Heaviside}\left(t - \frac{\pi}{4}\right)e^{-t+\frac{\pi}{4}} - 2\left(\cos(t) + \frac{\sin(t)}{2}\right)\sqrt{2}\,\text{Heaviside}\left(t - \frac{\pi}{4}\right) - e^{-2t} + 2\,e^{-t}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 76

```
DSolve[{y''[t]+3*y'[t]+2*y[t]==10*UnitStep[t-Pi/4]*Sin[t-Pi/4],{y[0]==1,y'[0]==0}},y[t],t,Inc
```

$$y(t) \to \begin{cases} e^{-2t}(-1 + 2e^t) & 4t \leq \pi \\ -\sqrt{2}(2\cos(t) + \sin(t)) - e^{-2t}\left(1 + 2e^{\pi/2}\right) + e^{-t}\left(2 + 5e^{\pi/4}\right) & \text{True} \end{cases}$$

14.13 problem Problem 39

Internal problem ID [2370]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 6y - 30\,\mathrm{Heaviside}\,(t-1)\,\mathrm{e}^{-t+1} = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = -4]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 45

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-6*y(t)=30*Heaviside(t-1)*exp(-(t-1)),y(0) = 3, D(y)(0) =
```

$$y(t) = \left(\mathrm{e}^{5t} + 3\,\mathrm{Heaviside}\,(t-1)\,\mathrm{e}^{3} + 2\,\mathrm{Heaviside}\,(t-1)\,\mathrm{e}^{-2+5t} - 5\,\mathrm{Heaviside}\,(t-1)\,\mathrm{e}^{1+2t} + 2\right)\mathrm{e}^{-3t}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 59

```
DSolve[{y''[t]+y'[t]-6*y[t]==30*UnitStep[t-1]*Exp[-(t-1)],{y[0]==3,y'[0]==-4}},y[t],t,Include
```

$$y(t) \to \begin{cases} e^{-3t}(2+e^{5t}) & t \leq 1 \\ e^{-3t}(2+3e^{3}-5e^{2t+1}+e^{5t-2}(2+e^{2})) & \text{True} \end{cases}$$

14.14 problem Problem 40

Internal problem ID [2371]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 5y - 5\,\text{Heaviside}\,(t-3) = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 46

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+5*y(t)=5*Heaviside(t-3),y(0) = 2, D(y)(0) = 1],y(t), si
```

$$\begin{aligned} y(t) = &-\text{Heaviside}\,(t-3)\,(\cos\,(t-3) + 2\sin\,(t-3))\,\mathrm{e}^{-2t+6} \\ &+ \text{Heaviside}\,(t-3) + (2\cos\,(t) + 5\sin\,(t))\,\mathrm{e}^{-2t} \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 63

```
DSolve[{y''[t]+4*y'[t]+5*y[t]==5*UnitStep[t-3],{y[0]==2,y'[0]==1}},y[t],t,IncludeSingularSolu
```

$$y(t) \to \begin{cases} e^{-2t}(2\cos(t) + 5\sin(t)) & t \leq 3 \\ e^{-2t}(2\cos(t) - e^6(\cos(3-t) - 2\sin(3-t)) + 5\sin(t)) + 1 & \text{True} \end{cases}$$

14.15 problem Problem 41

Internal problem ID [2372]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + 5y - 2\sin(t) - \text{Heaviside}\left(t - \frac{\pi}{2}\right)(1 + \cos(t)) = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 68

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)+5*y(t)=2*sin(t)+Heaviside(t-Pi/2)*(1-sin(t-Pi/2)),y(0)
```

$$y(t) = \frac{\left(\left(2\cos(t)^2 - 3\cos(t)\sin(t) - 1\right)e^{t-\frac{\pi}{2}} + 2\cos(t) - \sin(t) + 2\right)\text{Heaviside}\left(t - \frac{\pi}{2}\right)}{10} - \frac{2\,e^t\cos(t)^2}{5} - \frac{e^t\cos(t)\sin(t)}{5} + \frac{\cos(t)}{5} + \frac{e^t}{5} + \frac{2\sin(t)}{5}$$

✓ Solution by Mathematica

Time used: 0.241 (sec). Leaf size: 90

```
DSolve[{y''[t]-2*y'[t]+5*y[t]==2*Sin[t]+UnitStep[t-Pi/2]*(1-Sin[t-Pi/2]),{y[0]==0,y'[0]==0}},
```

$$y(t) \to \begin{cases} \frac{1}{5}(\cos(t) + 2\sin(t) - e^t(\cos(2t) + \cos(t)\sin(t))) & 2t \leq \pi \\ \frac{1}{20}\left(8\cos(t) + 2e^t\left(-2 + e^{-\pi/2}\right)\cos(2t) + 6\sin(t) + e^t\left(-2 - 3e^{-\pi/2}\right)\sin(2t) + 4\right) & \text{True} \end{cases}$$

14.16 problem Problem 46 part a

Internal problem ID [2373]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 46 part a.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$y' - y - \left(\begin{cases} 2 & 0 \leq t < 1 \\ -1 & 1 \leq t \end{cases} \right) = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 34

```
dsolve([diff(y(t),t)-y(t)=piecewise(0<=t and t<1,2,t>=1,-1),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \begin{cases} e^{t} & t < 0 \\ 3\,e^{t} - 2 & t < 1 \\ 3\,e^{t} - 3\,e^{t-1} + 1 & 1 \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 40

```
DSolve[{y'[t]-y[t]==Piecewise[{{2,0<=t<1},{-1,t>=1}}],{y[0]==1}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to \{ \begin{array}{cc} e^t & t \leq 0 \\ -2 + 3e^t & 0 < t \leq 1 \\ 1 + 3(-1 + e)e^{t-1} & \text{True} \end{array}$$

14.17 problem Problem 46 part b

Internal problem ID [2374]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.7. page 704
Problem number: Problem 46 part b.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$y' - y - \left(\begin{cases} 2 & 0 \leq t < 1 \\ -1 & 1 \leq t \end{cases}\right) = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 34

```
dsolve([diff(y(t),t)-y(t)=piecewise(0<=t and t<1,2,t>=1,-1),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \begin{cases} e^{t} & t < 0 \\ 3\,e^{t} - 2 & t < 1 \\ 3\,e^{t} - 3\,e^{t-1} + 1 & 1 \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 40

```
DSolve[{y'[t]-y[t]==Piecewise[{{2,0<=t<1},{-1,t>=1}}],{y[0]==1}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to \{ \begin{array}{cc} e^{t} & t \leq 0 \\ -2 + 3e^{t} & 0 < t \leq 1 \\ 1 + 3(-1 + e)e^{t-1} & \text{True} \end{array}$$

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15.1 problem Problem 1

Internal problem ID [2375]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y - (\delta(t-5)) = 0}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(t),t)+y(t)=Dirac(t-5),y(0) = 3],y(t), singsol=all)
```

$$y(t) = \left(\mathrm{e}^{5} \operatorname{Heaviside}(t-5) + 3\right) \mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 21

```
DSolve[{y'[t]+y[t]==DiracDelta[t-5],{y[0]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}\left(e^{5}\theta(t-5) + 3\right)$$

15.2 problem Problem 2

Internal problem ID [2376]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - 2y - (\delta(t-2)) = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve([diff(y(t),t)-2*y(t)=Dirac(t-2),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \left(\text{Heaviside}\,(t-2)\,\mathrm{e}^{-4} + 1\right)\mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 23

```
DSolve[{y'[t]-2*y[t]==DiracDelta[t-2],{y[0]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{2t-4}\left(\theta(t-2) + 3e^4\right)$$

15.3 problem Problem 3

Internal problem ID [2377]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 4y - 3(\delta(t-1)) = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(y(t),t)+4*y(t)=3*Dirac(t-1),y(0) = 2],y(t), singsol=all)
```

$$y(t) = 3\,\mathrm{e}^{-4t}\,\mathrm{Heaviside}\,(t-1)\,\mathrm{e}^{4} + 2\,\mathrm{e}^{-4t}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 22

```
DSolve[{y'[t]+4*y[t]==3*DiracDelta[t-1],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-4t}\left(3e^{4}\theta(t-1) + 2\right)$$

15.4 problem Problem 4

Internal problem ID [2378]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - 5y - 2\,\mathrm{e}^{-t} - (\delta(t-3)) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 24

```
dsolve([diff(y(t),t)-5*y(t)=2*exp(-t)+Dirac(t-3),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\mathrm{e}^{5t}}{3} + \mathrm{Heaviside}\,(t-3)\,\mathrm{e}^{5t-15} - \frac{\mathrm{e}^{-t}}{3}$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 34

```
DSolve[{y'[t]-5*y[t]==2*Exp[-t]+DiracDelta[t-3],{y[0]==0}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \rightarrow \frac{1}{3}e^{-t}\left(3e^{6t-15}\theta(t-3) + e^{6t} - 1\right)$$

15.5 problem Problem 5

Internal problem ID [2379]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y - (\delta(t-1)) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve([diff(y(t),t$2)-3*diff(y(t),t)+2*y(t)=Dirac(t-1),y(0) = 1, D(y)(0) = 0],y(t), singsol=
```

$$y(t) = -\operatorname{Heaviside}(t-1)\,\mathrm{e}^{t-1} + \operatorname{Heaviside}(t-1)\,\mathrm{e}^{2t-2} - \mathrm{e}^{2t} + 2\,\mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 31

```
DSolve[{y''[t]-3*y'[t]+2*y[t]==DiracDelta[t-1],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolu
```

$$y(t) \to e^t\left(\frac{(e^t - e)\,\theta(t-1)}{e^2} - e^t + 2\right)$$

15.6 problem Problem 6

Internal problem ID [2380]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y - (\delta(t-3)) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 38

```
dsolve([diff(y(t),t$2)-4*y(t)=Dirac(t-3),y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{e^{2t}}{4} - \frac{e^{-2t}}{4} - \frac{\text{Heaviside}(t-3)\, e^{-2t+6}}{4} + \frac{\text{Heaviside}(t-3)\, e^{2t-6}}{4}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 26

```
DSolve[{y''[t]-4*y[t]==DiracDelta[t-3],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to \frac{1}{2}(\sinh(2t) - \theta(t-3)\sinh(6-2t))$$

15.7 problem Problem 7

Internal problem ID [2381]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 5y - \left(\delta\left(t - \frac{\pi}{2}\right)\right) = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 34

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+5*y(t)=Dirac(t-Pi/2),y(0) = 0, D(y)(0) = 2],y(t), sings
```

$$y(t) = \frac{\sin(2t)\left(-\operatorname{Heaviside}\left(t - \frac{\pi}{2}\right)e^{-t+\frac{\pi}{2}} + 2\,e^{-t}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 34

```
DSolve[{y''[t]+2*y'[t]+5*y[t]==DiracDelta[t-Pi/2],{y[0]==0,y'[0]==2}},y[t],t,IncludeSingularS
```

$$y(t) \rightarrow -e^{-t}\left(e^{\pi/2}\theta(2t - \pi) - 2\right)\sin(t)\cos(t)$$

15.8 problem Problem 8

Internal problem ID [2382]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 13y - \left(\delta\left(t - \frac{\pi}{4}\right)\right) = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 53

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)+13*y(t)=Dirac(t-Pi/4),y(0) = 3, D(y)(0) = 0],y(t), sing
```

$$y(t) = -\frac{\sqrt{2}\, \mathrm{e}^{2t-\frac{\pi}{2}} \operatorname{Heaviside}\left(t - \frac{\pi}{4}\right) (\sin(3t) + \cos(3t))}{6} + 3\left(\cos(3t) - \frac{2\sin(3t)}{3}\right) \mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 61

```
DSolve[{y''[t]-4*y'[t]+13*y[t]==DiracDelta[t-Pi/4],{y[0]==3,y'[0]==0}},y[t],t,IncludeSingular
```

$$y(t) \rightarrow \frac{1}{6} e^{2t}\left(6(3\cos(3t) - 2\sin(3t)) - \sqrt{2} e^{-\pi/2} \theta(4t - \pi)(\sin(3t) + \cos(3t))\right)$$

15.9 problem Problem 9

Internal problem ID [2383]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 3y - (\delta(t-2)) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 30

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+3*y(t)=Dirac(t-2),y(0) = 1, D(y)(0) = -1],y(t), singsol
```

$$y(t) = \mathrm{e}^{-t} - \frac{\mathrm{Heaviside}\,(t-2)\,\mathrm{e}^{6-3t}}{2} + \frac{\mathrm{Heaviside}\,(t-2)\,\mathrm{e}^{2-t}}{2}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 37

```
DSolve[{y''[t]+4*y'[t]+3*y[t]==DiracDelta[t-2],{y[0]==1,y'[0]==-1}},y[t],t,IncludeSingularSol
```

$$y(t) \rightarrow \frac{1}{2}e^{2-3t}\left(e^{2t} - e^{4}\right)\theta(t-2) + e^{-t}$$

15.10 problem Problem 10

Internal problem ID [2384]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 6y' + 13y - \left(\delta\left(t - \frac{\pi}{4}\right)\right) = 0}$$

With initial conditions

$$[y(0) = 5, y'(0) = 5]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 42

```
dsolve([diff(y(t),t$2)+6*diff(y(t),t)+13*y(t)=Dirac(t-Pi/4),y(0) = 5, D(y)(0) = 5],y(t), sing
```

$$y(t) = -\frac{\text{Heaviside}\left(t - \frac{\pi}{4}\right)\cos(2t)\,\mathrm{e}^{\frac{3\pi}{4}-3t}}{2} + 5\,\mathrm{e}^{-3t}(\cos(2t) + 2\sin(2t))$$

✓ Solution by Mathematica

Time used: 0.116 (sec). Leaf size: 121

```
DSolve[{y''[t]+46*y'[t]+13*y[t]==DiracDelta[t-Pi/4],{y[0]==1,y'[0]==-1}},y[t],t,IncludeSingul
```

$$y(t) \to \frac{1}{516}e^{-2\sqrt{129}t-23t-\frac{\sqrt{129}\pi}{2}}\left(2e^{\frac{\sqrt{129}\pi}{2}}\left(\left(129+11\sqrt{129}\right)e^{4\sqrt{129}t}+129-11\sqrt{129}\right)\right.$$
$$\left.-\sqrt{129}e^{23\pi/4}\left(e^{\sqrt{129}\pi}-e^{4\sqrt{129}t}\right)\theta(4t-\pi)\right)$$

15.11 problem Problem 11

Internal problem ID [2385]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y - 15\sin(2t) - \left(\delta\left(t - \frac{\pi}{6}\right)\right) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 29

```
dsolve([diff(y(t),t$2)+9*y(t)=15*sin(2*t)+Dirac(t-Pi/6),y(0) = 0, D(y)(0) = 0],y(t), singsol=
```

$$y(t) = -2\sin(3t) + 3\sin(2t) - \frac{\cos(3t)\,\text{Heaviside}\left(t - \frac{\pi}{6}\right)}{3}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 34

```
DSolve[{y''[t]+9*y[t]==15*Sin[2*t]+DiracDelta[t-Pi/6],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingu
```

$$y(t) \to -\frac{1}{3}\theta(6t - \pi)\cos(3t) + 3\sin(2t) - 2\sin(3t)$$

15.12 problem Problem 12

Internal problem ID [2386]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 16y - 4\cos(3t) - \left(\delta\left(t - \frac{\pi}{3}\right)\right) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 40

```
dsolve([diff(y(t),t$2)+16*y(t)=4*cos(3*t)+Dirac(t-Pi/3),y(0) = 0, D(y)(0) = 0],y(t), singsol=
```

$$y(t) = -\frac{4\cos(4t)}{7} + \frac{\left(\sqrt{3}\,\cos(4t) - \sin(4t)\right)\text{Heaviside}\left(t - \frac{\pi}{3}\right)}{8} + \frac{4\cos(3t)}{7}$$

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 50

```
DSolve[{y''[t]+16*y[t]==4*Cos[3*t]+DiracDelta[t-Pi/3],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingu
```

$$y(t) \rightarrow \frac{1}{8}\theta(3t - \pi)\left(\sqrt{3}\cos(4t) - \sin(4t)\right) + \frac{4}{7}(\cos(3t) - \cos(4t))$$

15.13 problem Problem 13

Internal problem ID [2387]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 10, The Laplace Transform and Some Elementary Applications. Exercises for 10.8. page 710
Problem number: Problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 5y - 4\sin(t) - \left(\delta\left(t - \frac{\pi}{6}\right)\right) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 69

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+5*y(t)=4*sin(t)+Dirac(t-Pi/6),y(0) = 0, D(y)(0) = 1],y(
```

$$y(t) = -\frac{\left(\cos(t)^2\sqrt{3} - \cos(t)\sin(t) - \frac{\sqrt{3}}{2}\right)\text{Heaviside}\left(t - \frac{\pi}{6}\right)e^{-t+\frac{\pi}{6}}}{2} + \frac{\left(4\cos(t)^2 + 3\cos(t)\sin(t) - 2\right)e^{-t}}{5} - \frac{2\cos(t)}{5} + \frac{4\sin(t)}{5}$$

✓ Solution by Mathematica

Time used: 0.296 (sec). Leaf size: 73

```
DSolve[{y''[t]+2*y'[t]+5*y[t]==4*Sin[t]+DiracDelta[t-Pi/6],{y[0]==0,y'[0]==1}},y[t],t,Include
```

$$y(t) \to \frac{1}{20}e^{-t}\left(-5e^{\pi/6}\theta(6t-\pi)\left(\sqrt{3}\cos(2t) - \sin(2t)\right) + 6\sin(2t) + 8\cos(2t) - 8e^t(\cos(t) - 2\sin(t))\right)$$

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16.1 problem Problem 1

Internal problem ID [2388]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{24}x^4\right) y(0) + \left(x + \frac{1}{6}x^3 + \frac{1}{120}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(\frac{x^5}{120} + \frac{x^3}{6} + x\right) + c_1 \left(\frac{x^4}{24} + \frac{x^2}{2} + 1\right)$$

16.2 problem Problem 2

Internal problem ID [2389]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_erf]`

$$\boxed{y'' + 2y'x + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - 2x^2 + \frac{4}{3}x^4\right) y(0) + \left(x - x^3 + \frac{1}{2}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 38

```
AsymptoticDSolveValue[y''[x]+2*x*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{2} - x^3 + x\right) + c_1\left(\frac{4x^4}{3} - 2x^2 + 1\right)$$

16.3 problem Problem 3

Internal problem ID [2390]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' - 2y'x - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
Order:=6;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x^2 + \frac{1}{2}x^4\right) y(0) + \left(x + \frac{2}{3}x^3 + \frac{4}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 38

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{4x^5}{15} + \frac{2x^3}{3} + x\right) + c_1\left(\frac{x^4}{2} + x^2 + 1\right)$$

16.4 problem Problem 4

Internal problem ID [2391]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' - y'x^2 - 2yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^3}{3}\right) y(0) + \left(x + \frac{1}{4}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]-x^2*y'[x]-2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2\left(\frac{x^4}{4} + x\right) + c_1\left(\frac{x^3}{3} + 1\right)$$

16.5 problem Problem 5

Internal problem ID [2392]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^3}{6}\right) y(0) + \left(x - \frac{1}{12}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(x - \frac{x^4}{12}\right) + c_1 \left(1 - \frac{x^3}{6}\right)$$

16.6 problem Problem 6

Internal problem ID [2393]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{3}{2}x^2 + \frac{5}{8}x^4\right) y(0) + \left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{5} - \frac{2x^3}{3} + x\right) + c_1\left(\frac{5x^4}{8} - \frac{3x^2}{2} + 1\right)$$

16.7 problem Problem 7

Internal problem ID [2394]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x^2 - 3yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-3*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^3}{2}\right) y(0) + \left(x + \frac{1}{3}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]-x^2*y'[x]-3*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^4}{3} + x\right) + c_1\left(\frac{x^3}{2} + 1\right)$$

16.8 problem Problem 8

Internal problem ID [2395]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y'x^2 + 2yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+2*x^2*diff(y(x),x)+2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^3}{3}\right) y(0) + \left(x - \frac{1}{3}x^4\right) D(y)\,(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+2*x^2*y'[x]+2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^4}{3}\right) + c_1\left(1 - \frac{x^3}{3}\right)$$

16.9 problem Problem 9

Internal problem ID [2396]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2-3)\,y''-3y'x-5y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((x^2-3)*diff(y(x),x$2)-3*x*diff(y(x),x)-5*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{5}{6}x^2+\frac{5}{24}x^4\right)y(0)+\left(x-\frac{4}{9}x^3+\frac{8}{135}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(x^2-3)*y''[x]-3*x*y'[x]-5*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{8x^5}{135}-\frac{4x^3}{9}+x\right)+c_1\left(\frac{5x^4}{24}-\frac{5x^2}{6}+1\right)$$

16.10 problem Problem 10

Internal problem ID [2397]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2+1)\,y''+4y'x+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
Order:=6;
dsolve((1+x^2)*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(x^4-x^2+1\right)y(0)+\left(x^5-x^3+x\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 30

```
AsymptoticDSolveValue[(1+x^2)*y''[x]+4*x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(x^5-x^3+x\right)+c_1\left(x^4-x^2+1\right)$$

16.11 problem Problem 11

Internal problem ID [2398]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-4x^2+1)\,y''-20y'x-16y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((1-4*x^2)*diff(y(x),x$2)-20*x*diff(y(x),x)-16*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+8x^2+\frac{128}{3}x^4\right)y(0)+\left(30x^5+6x^3+x\right)D(y)(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[(1-4*x^2)*y''[x]-20*x*y'[x]-16*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(30x^5+6x^3+x\right)+c_1\left(\frac{128x^4}{3}+8x^2+1\right)$$

16.12 problem Problem 12

Internal problem ID [2399]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\,y''-6y'x+12y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
Order:=6;
dsolve((x^2-1)*diff(y(x),x$2)-6*x*diff(y(x),x)+12*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(x^4+6x^2+1\right)y(0)+\left(x^3+x\right)D(y)(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 25

```
AsymptoticDSolveValue[(x^2-1)*y''[x]-6*x*y'[x]+12*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(x^3+x\right)+c_1\left(x^4+6x^2+1\right)$$

16.13 problem Problem 13

Internal problem ID [2400]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' + 4yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+4*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{2}{3}x^3 + \frac{1}{3}x^4 - \frac{2}{15}x^5\right) y(0) + \left(x - x^2 + \frac{2}{3}x^3 - \frac{2}{3}x^4 + \frac{7}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 61

```
AsymptoticDSolveValue[y''[x]+2*y'[x]+4*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{2x^5}{15} + \frac{x^4}{3} - \frac{2x^3}{3} + 1\right) + c_2\left(\frac{7x^5}{15} - \frac{2x^4}{3} + \frac{2x^3}{3} - x^2 + x\right)$$

16.14 problem Problem 14

Internal problem ID [2401]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + (2 + x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+(2+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 - \frac{1}{6}x^3 + \frac{1}{3}x^4 + \frac{11}{120}x^5\right) y(0) + \left(x - \frac{1}{2}x^3 - \frac{1}{12}x^4 + \frac{1}{8}x^5\right) D(y)\,(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 61

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+(2+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{8} - \frac{x^4}{12} - \frac{x^3}{2} + x\right) + c_1\left(\frac{11x^5}{120} + \frac{x^4}{3} - \frac{x^3}{6} - x^2 + 1\right)$$

16.15 problem Problem 15

Internal problem ID [2402]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - e^x y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)-exp(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{12}x^4 + \frac{1}{24}x^5\right) y(0) + \left(x + \frac{1}{6}x^3 + \frac{1}{12}x^4 + \frac{1}{30}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]-Exp[x]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{30} + \frac{x^4}{12} + \frac{x^3}{6} + x\right) + c_1\left(\frac{x^5}{24} + \frac{x^4}{12} + \frac{x^3}{6} + \frac{x^2}{2} + 1\right)$$

16.16 problem Problem 17

Internal problem ID [2403]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (x-1)\,y' - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=6;
dsolve(x*diff(y(x),x$2)-(x-1)*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 + \frac{1}{4}x^2 + \frac{1}{18}x^3 + \frac{5}{192}x^4 + \frac{23}{3600}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(x + \frac{11}{108}x^3 + \frac{11}{1152}x^4 + \frac{883}{216000}x^5 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 96

```
AsymptoticDSolveValue[x*y''[x]-(x-1)*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{23x^5}{3600} + \frac{5x^4}{192} + \frac{x^3}{18} + \frac{x^2}{4} + 1\right) + c_2\left(\frac{883x^5}{216000} + \frac{11x^4}{1152} + \frac{11x^3}{108} + \left(\frac{23x^5}{3600} + \frac{5x^4}{192} + \frac{x^3}{18} + \frac{x^2}{4} + 1\right)\log(x) + x\right)$$

16.17 problem Problem 18

Internal problem ID [2404]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1+2x^2)\,y''+7y'x+2y=0}$$

With initial conditions

$$[y(0)=0,y'(0)=1]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([(1+2*x^2)*diff(y(x),x$2)+7*x*diff(y(x),x)+2*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='
```

$$y(x)=x-\frac{3}{2}x^3+\frac{21}{8}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{(1+2*x^2)*y''[x]+7*x*y'[x]+2*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x)\to\frac{21x^5}{8}-\frac{3x^3}{2}+x$$

16.18 problem Problem 19

Internal problem ID [2405]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{4y'' + y'x + 4y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([4*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='series',x=
```

$$y(x) = 1 - \frac{1}{2}x^2 + \frac{1}{16}x^4 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{4*y''[x]+x*y'[x]+4*y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^4}{16} - \frac{x^2}{2} + 1$$

16.19 problem Problem 20

Internal problem ID [2406]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y'x^2 + yx - 2\cos(x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
Order:=6;
dsolve(diff(y(x),x$2)+2*x^2*diff(y(x),x)+x*y(x)=2*cos(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^3}{6}\right) y(0) + \left(x - \frac{1}{4}x^4\right) D(y)(0) + x^2 - \frac{x^4}{12} - \frac{x^5}{4} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 45

```
AsymptoticDSolveValue[y''[x]+2*x^2*y'[x]+x*y[x]==2*Cos[x],y[x],{x,0,5}]
```

$$y(x) \to -\frac{x^5}{4} - \frac{x^4}{12} + c_2\left(x - \frac{x^4}{4}\right) + c_1\left(1 - \frac{x^3}{6}\right) + x^2$$

16.20 problem Problem 21

Internal problem ID [2407]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.2. page 739
Problem number: Problem 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y'x - 4y - 6\,\mathrm{e}^{x} = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
Order:=6;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=6*exp(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + 2x^2 + \frac{1}{3}x^4\right) y(0) + \left(x + \frac{1}{2}x^3 + \frac{1}{40}x^5\right) D(y)(0) + 3x^2 + x^3 + \frac{3x^4}{4} + \frac{x^5}{10} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 62

```
AsymptoticDSolveValue[y''[x]+x*y'[x]-4*y[x]==6*Exp[x],y[x],{x,0,5}]
```

$$y(x) \to \frac{x^5}{10} + \frac{3x^4}{4} + x^3 + 3x^2 + c_2\left(\frac{x^5}{40} + \frac{x^3}{2} + x\right) + c_1\left(\frac{x^4}{3} + 2x^2 + 1\right)$$

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17.1 problem 1

Internal problem ID [2408]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{y'}{1-x} + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
Order:=6;
dsolve(diff(y(x),x$2)+1/(1-x)*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{60}x^5\right) y(0) + \left(x - \frac{1}{2}x^2 - \frac{1}{12}x^4 + \frac{1}{24}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]+1/(1-x)*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{x^5}{60} + \frac{x^4}{24} - \frac{x^3}{6} + 1\right) + c_2 \left(\frac{x^5}{24} - \frac{x^4}{12} - \frac{x^2}{2} + x\right)$$

17.2 problem 3

Internal problem ID [2409]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \frac{xy'}{(1-x^2)^2} + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x/(1-x^2)^2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^{-i}\left(1 + \left(-\frac{1}{4} + \frac{i}{4}\right)x^2 + \left(-\frac{1}{80} + \frac{7i}{80}\right)x^4 + \mathrm{O}\left(x^6\right)\right) + c_2x^i\left(1 + \left(-\frac{1}{4} - \frac{i}{4}\right)x^2 + \left(-\frac{1}{80} - \frac{7i}{80}\right)x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 70

```
AsymptoticDSolveValue[x^2*y''[x]+x/(1-x^2)^2*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \left(\frac{1}{80} + \frac{3i}{80}\right)c_2x^{-i}\left((2+i)x^4 + (4+8i)x^2 + (8-24i)\right) - \left(\frac{3}{80} + \frac{i}{80}\right)c_1x^i\left((1+2i)x^4 + (8+4i)x^2 - (24-8i)\right)$$

17.3 problem 4

Internal problem ID [2410]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(-2+x)^2 y'' + (-2+x)\,\mathrm{e}^x y' + \frac{4y}{x} = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 60

```
Order:=6;
dsolve((x-2)^2*diff(y(x),x$2)+(x-2)*exp(x)*diff(y(x),x)+4/x*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x\left(1-\frac{1}{4}x-\frac{1}{24}x^2-\frac{13}{576}x^3-\frac{35}{2304}x^4-\frac{1297}{138240}x^5+\mathrm{O}\left(x^6\right)\right) \\ & + c_2\left(\ln(x)\left(-x+\frac{1}{4}x^2+\frac{1}{24}x^3+\frac{13}{576}x^4+\frac{35}{2304}x^5+\mathrm{O}\left(x^6\right)\right)\right. \\ & \left. + \left(1+\frac{1}{2}x-\frac{5}{4}x^2-\frac{41}{144}x^3-\frac{1097}{6912}x^4-\frac{397}{4320}x^5+\mathrm{O}\left(x^6\right)\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 87

```
AsymptoticDSolveValue[(x-2)^2*y''[x]+(x-2)*Exp[x]*y'[x]+4/x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{576}x(13x^3+24x^2+144x-576)\log(x) + \frac{-1097x^4-1968x^3-8640x^2+3456x+6912}{6912}\right) + c_2\left(-\frac{35x^5}{2304}-\frac{13x^4}{576}-\frac{x^3}{24}-\frac{x^2}{4}+x\right)$$

17.4 problem 5

Internal problem ID [2411]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{2y'}{x(x-3)} - \frac{y}{x^3(x+3)} = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x$2)+2/(x*(x-3))*diff(y(x),x)-1/(x^3*(x+3))*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.227 (sec). Leaf size: 258

```
AsymptoticDSolveValue[y''[x]+2/(x*(x-3))*y'[x]-1/(x^3*(x+3))*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 e^{-\frac{2}{\sqrt{3}\sqrt{x}}} \left(\frac{10879996003390494539x^{9/2}}{6059672463464202240\sqrt{3}} + \frac{647134806104 17x^{7/2}}{328758271672320\sqrt{3}} + \frac{287821451x^{5/2}}{3397386240\sqrt{3}} + \frac{19817x^{3/2}}{73728\sqrt{3}} - \frac{489456448614940132045 7x^5}{1246561192484064460800} - \frac{1166128129822977 97x^4}{378729528966512640} - \frac{22160647459x^3}{587068342272} + \frac{463507x^2}{42467328} + \frac{587x}{4608} + \frac{25\sqrt{x}}{16\sqrt{3}} + 1 \right) x^{13/12} + c_2 e^{\frac{2}{\sqrt{3}\sqrt{x}}} \left(-\frac{10879996003390494539x^{9/2}}{6059672463464202240\sqrt{3}} - \frac{64713480610417x^{7/2}}{328758271672320\sqrt{3}} - \frac{287821451x^{5/2}}{3397386240\sqrt{3}} - \frac{19817x^{3/}}{73728\sqrt{}}\right.$$

17.5 problem 6

Internal problem ID [2412]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(1-x)\,y' - 7y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 478

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(1-x)*diff(y(x),x)-7*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^{-\sqrt{7}}c_1\left(1 + \frac{\sqrt{7}}{-1+2\sqrt{7}}x + \frac{\sqrt{7}}{-4+8\sqrt{7}}x^2 + \frac{\sqrt{7}\,(\sqrt{7}-2)}{372-96\sqrt{7}}x^3 + \frac{\sqrt{7}\,(\sqrt{7}-3)}{2976-768\sqrt{7}}x^4 + \frac{\sqrt{7}\,(\sqrt{7}-3)\,(\sqrt{7}-4)}{48960\sqrt{7}-128160}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2x^{\sqrt{7}}\left(1 + \frac{\sqrt{7}}{1+2\sqrt{7}}x + \frac{\sqrt{7}}{4+8\sqrt{7}}x^2 + \frac{\sqrt{7}\,(\sqrt{7}+2)}{372+96\sqrt{7}}x^3 + \frac{(\sqrt{7}+3)\,\sqrt{7}}{2976+768\sqrt{7}}x^4 + \frac{(\sqrt{7}+4)\,(\sqrt{7}+3)\,\sqrt{7}}{48960\sqrt{7}+128160}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 1066

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1-x)*y'[x]-7*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \left(\frac{\sqrt{7}(1+\sqrt{7})\,(2+\sqrt{7})\,(3+\sqrt{7})\,(4+\sqrt{7})\,x}{(-6+\sqrt{7}+\sqrt{7}\,(1+\sqrt{7}))\,(-5+\sqrt{7}+(1+\sqrt{7})\,(2+\sqrt{7}))\,(-4+\sqrt{7}+(2+\sqrt{7})\,(3+\sqrt{7}))\,(-3+}\right.$$
$$+\frac{\sqrt{7}(1+\sqrt{7})\,(2+\sqrt{7})\,(3+\sqrt{7})\,x^4}{(-6+\sqrt{7}+\sqrt{7}\,(1+\sqrt{7}))\,(-5+\sqrt{7}+(1+\sqrt{7})\,(2+\sqrt{7}))\,(-4+\sqrt{7}+(2+\sqrt{7})\,(3+\sqrt{7}))\,(-3+}$$
$$+\frac{\sqrt{7}(1+\sqrt{7})\,(2+\sqrt{7})\,x^3}{(-6+\sqrt{7}+\sqrt{7}\,(1+\sqrt{7}))\,(-5+\sqrt{7}+(1+\sqrt{7})\,(2+\sqrt{7}))\,(-4+\sqrt{7}+(2+\sqrt{7})\,(3+\sqrt{7}))}$$
$$+\frac{\sqrt{7}(1+\sqrt{7})\,x^2}{(-6+\sqrt{7}+\sqrt{7}\,(1+\sqrt{7}))\,(-5+\sqrt{7}+(1+\sqrt{7})\,(2+\sqrt{7}))}$$
$$\left.+\frac{\sqrt{7}x}{-6+\sqrt{7}+\sqrt{7}\,(1+\sqrt{7})}+1\right)c_1x^{\sqrt{7}}$$
$$+\left(-\frac{\sqrt{7}(1-\sqrt{7})\,(2-\sqrt{7})\,(3-\sqrt{7})\,(4-\sqrt{}}{(-6-\sqrt{7}-\sqrt{7}\,(1-\sqrt{7}))\,(-5-\sqrt{7}+(1-\sqrt{7})\,(2-\sqrt{7}))\,(-4-\sqrt{7}+(2-\sqrt{7})\,(3-\sqrt{7}))\,(-}\right.$$
$$-\frac{\sqrt{7}(1-\sqrt{7})\,(2-\sqrt{7})\,(3-\sqrt{7})\,x^4}{(-6-\sqrt{7}-\sqrt{7}\,(1-\sqrt{7}))\,(-5-\sqrt{7}+(1-\sqrt{7})\,(2-\sqrt{7}))\,(-4-\sqrt{7}+(2-\sqrt{7})\,(3-\sqrt{7}))\,(-3-}$$
$$-\frac{\sqrt{7}(1-\sqrt{7})\,(2-\sqrt{7})\,x^3}{(-6-\sqrt{7}-\sqrt{7}\,(1-\sqrt{7}))\,(-5-\sqrt{7}+(1-\sqrt{7})\,(2-\sqrt{7}))\,(-4-\sqrt{7}+(2-\sqrt{7})\,(3-\sqrt{7}))}$$
$$-\frac{\sqrt{7}(1-\sqrt{7})\,x^2}{(-6-\sqrt{7}-\sqrt{7}\,(1-\sqrt{7}))\,(-5-\sqrt{7}+(1-\sqrt{7})\,(2-\sqrt{7}))}$$
$$\left.-\frac{\sqrt{7}x}{-6-\sqrt{7}-\sqrt{7}\,(1-\sqrt{7})}+1\right)c_2x^{-\sqrt{7}}$$

17.6 problem 7

Internal problem ID [2413]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4x^2y'' + y'\mathrm{e}^x x - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+x*exp(x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{5}{4}}\left(1 - \frac{1}{9}x - \frac{5}{468}x^2 - \frac{11}{23868}x^3 + \frac{79}{501228}x^4 + \frac{16043}{313267500}x^5 + \mathrm{O}\left(x^6\right)\right) + c_1\left(1 - \frac{1}{4}x + \frac{5}{96}x^2 + \frac{17}{8064}x^3 - \frac{313}{1419264}x\right.}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 86

```
AsymptoticDSolveValue[4*x^2*y''[x]+x*Exp[x]*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x\left(\frac{16043x^5}{313267500} + \frac{79x^4}{501228} - \frac{11x^3}{23868} - \frac{5x^2}{468} - \frac{x}{9} + 1\right) + \frac{c_2\left(-\frac{69703x^5}{709632000} - \frac{313x^4}{1419264} + \frac{17x^3}{8064} + \frac{5x^2}{96} - \frac{x}{4} + 1\right)}{\sqrt[4]{x}}$$

17.7 problem 8

Internal problem ID [2414]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4xy'' - y'x + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 46

```
Order:=6;
dsolve(4*x*diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \ln(x)\left(-\frac{1}{2}x + \frac{1}{16}x^2 + \mathrm{O}\left(x^6\right)\right)c_2 + c_1 x\left(1 - \frac{1}{8}x + \mathrm{O}\left(x^6\right)\right) + \left(1 + \frac{1}{4}x - \frac{3}{16}x^2 + \frac{1}{384}x^3 + \frac{1}{18432}x^4 + \frac{1}{737280}x^5 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 52

```
AsymptoticDSolveValue[4*x*y''[x]-x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^2}{8}\right) + c_1\left(\frac{x^4 + 48x^3 - 4608x^2 + 13824x + 18432}{18432} + \frac{1}{16}(x-8)x\log(x)\right)$$

17.8 problem 9

Internal problem ID [2415]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y' \cos(x)\, x + 5y\, \mathrm{e}^{2x} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 71

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*cos(x)*diff(y(x),x)+5*exp(2*x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}
y(x) = c_1 x^{1-2i}\Bigg(1 &+ \left(-\frac{10}{17} - \frac{40i}{17}\right)x + \left(-\frac{365}{136} + \frac{13i}{17}\right)x^2 + \left(\frac{223}{1020} + \frac{1723i}{765}\right)x^3 \\
&+ \left(\frac{114911}{78336} + \frac{24835i}{78336}\right)x^4 + \left(\frac{4041077}{8029440} - \frac{1112267i}{1605888}\right)x^5 + \mathrm{O}\left(x^6\right)\Bigg) \\
+ c_2 x^{1+2i}\Bigg(1 &+ \left(-\frac{10}{17} + \frac{40i}{17}\right)x + \left(-\frac{365}{136} - \frac{13i}{17}\right)x^2 + \left(\frac{223}{1020} - \frac{1723i}{765}\right)x^3 \\
&+ \left(\frac{114911}{78336} - \frac{24835i}{78336}\right)x^4 + \left(\frac{4041077}{8029440} + \frac{1112267i}{1605888}\right)x^5 + \mathrm{O}\left(x^6\right)\Bigg)
\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 94

```
AsymptoticDSolveValue[x^2*y''[x]-x*Cos[x]*y'[x]+5*Exp[2*x]*y[x]==0,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to & \left(\frac{11}{391680}+\frac{7i}{391680}\right)c_1\big((32064-31693i)x^4-(30784+60608i)x^3 \\
& -(80352-23904i)x^2+(23040+69120i)x+(25344-16128i)\big)\,x^{1+2i} \\
& +\left(\frac{7}{391680}+\frac{11i}{391680}\right)c_2\big((31693-32064i)x^4+(60608+30784i)x^3 \\
& -(23904-80352i)x^2-(69120+23040i)x+(16128-25344i)\big)\,x^{1-2i}
\end{aligned}
$$

17.9 problem 10

Internal problem ID [2416]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4x^2y'' + 3y'x + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{4}}\left(1 - \frac{1}{5}x + \frac{1}{90}x^2 - \frac{1}{3510}x^3 + \frac{1}{238680}x^4 - \frac{1}{25061400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 - \frac{1}{3}x + \frac{1}{42}x^2 - \frac{1}{1386}x^3 + \frac{1}{83160}x^4 - \frac{1}{7900200}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 85

```
AsymptoticDSolveValue[4*x^2*y''[x]+3*x*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \sqrt[4]{x}\left(-\frac{x^5}{25061400} + \frac{x^4}{238680} - \frac{x^3}{3510} + \frac{x^2}{90} - \frac{x}{5} + 1\right) + c_2\left(-\frac{x^5}{7900200} + \frac{x^4}{83160} - \frac{x^3}{1386} + \frac{x^2}{42} - \frac{x}{3} + 1\right)$$

17.10 problem 11

Internal problem ID [2417]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$6x^2y'' + x(1+18x)\,y' + (1+12x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 47

```
Order:=6;
dsolve(6*x^2*diff(y(x),x$2)+x*(1+18*x)*diff(y(x),x)+(1+12*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{3}}\left(1 - \frac{18}{5}x + \frac{324}{55}x^2 - \frac{5832}{935}x^3 + \frac{104976}{21505}x^4 - \frac{1889568}{623645}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\sqrt{x}\left(1 - 3x + \frac{9}{2}x^2 - \frac{9}{2}x^3 + \frac{27}{8}x^4 - \frac{81}{40}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 88

```
AsymptoticDSolveValue[6*x^2*y''[x]+x*(1+18*x)*y'[x]+(1+12*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{81x^5}{40} + \frac{27x^4}{8} - \frac{9x^3}{2} + \frac{9x^2}{2} - 3x + 1\right) + c_2\sqrt[3]{x}\left(-\frac{1889568x^5}{623645} + \frac{104976x^4}{21505} - \frac{5832x^3}{935} + \frac{324x^2}{55} - \frac{18x}{5} + 1\right)$$

17.11 problem 12

Internal problem ID [2418]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x - (2+x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 321

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(2+x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x) = {} & x^{-\sqrt{2}}c_1\left(1 - \frac{1}{-1+2\sqrt{2}}x + \frac{1}{20-12\sqrt{2}}x^2 - \frac{1}{228\sqrt{2}-324}x^3 + \frac{1}{8832-6240\sqrt{2}}x^4\right.\\ & \left. - \frac{1}{480}\frac{1}{\left(-1+2\sqrt{2}\right)\left(\sqrt{2}-1\right)\left(-3+2\sqrt{2}\right)\left(\sqrt{2}-2\right)\left(-5+2\sqrt{2}\right)}x^5 + \mathrm{O}\left(x^6\right)\right)\\ & + c_2x^{\sqrt{2}}\left(1 + \frac{1}{1+2\sqrt{2}}x + \frac{1}{20+12\sqrt{2}}x^2 + \frac{1}{228\sqrt{2}+324}x^3 + \frac{1}{8832+6240\sqrt{2}}x^4\right.\\ & \left. + \frac{1}{244320\sqrt{2}+345600}x^5 + \mathrm{O}\left(x^6\right)\right)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 843

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]-(2+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow \left(\frac{x^5}{(-1+\sqrt{2}+\sqrt{2}\,(1+\sqrt{2}))\,(\sqrt{2}+(1+\sqrt{2})\,(2+\sqrt{2}))\,(1+\sqrt{2}+(2+\sqrt{2})\,(3+\sqrt{2}))\,(2+\sqrt{2}+(3\cdots}\right.$$
$$+\frac{x^4}{(-1+\sqrt{2}+\sqrt{2}\,(1+\sqrt{2}))\,(\sqrt{2}+(1+\sqrt{2})\,(2+\sqrt{2}))\,(1+\sqrt{2}+(2+\sqrt{2})\,(3+\sqrt{2}))\,(2+\sqrt{2}+(3\cdots}$$
$$+\frac{x^3}{(-1+\sqrt{2}+\sqrt{2}\,(1+\sqrt{2}))\,(\sqrt{2}+(1+\sqrt{2})\,(2+\sqrt{2}))\,(1+\sqrt{2}+(2+\sqrt{2})\,(3+\sqrt{2}))}$$
$$+\frac{x^2}{(-1+\sqrt{2}+\sqrt{2}\,(1+\sqrt{2}))\,(\sqrt{2}+(1+\sqrt{2})\,(2+\sqrt{2}))}+\frac{x}{-1+\sqrt{2}+\sqrt{2}\,(1+\sqrt{2})}$$
$$\left.+1\right)c_1x^{\sqrt{2}}$$
$$+\left(\frac{x^5}{(-1-\sqrt{2}-\sqrt{2}\,(1-\sqrt{2}))\,(-\sqrt{2}+(1-\sqrt{2})\,(2-\sqrt{2}))\,(1-\sqrt{2}+(2-\sqrt{2})\,(3-\sqrt{2}))\,(2-\sqrt{2}+\cdots}\right.$$
$$+\frac{x^4}{(-1-\sqrt{2}-\sqrt{2}\,(1-\sqrt{2}))\,(-\sqrt{2}+(1-\sqrt{2})\,(2-\sqrt{2}))\,(1-\sqrt{2}+(2-\sqrt{2})\,(3-\sqrt{2}))\,(2-\sqrt{2}+(\cdots}$$
$$+\frac{x^3}{(-1-\sqrt{2}-\sqrt{2}\,(1-\sqrt{2}))\,(-\sqrt{2}+(1-\sqrt{2})\,(2-\sqrt{2}))\,(1-\sqrt{2}+(2-\sqrt{2})\,(3-\sqrt{2}))}$$
$$+\frac{x^2}{(-1-\sqrt{2}-\sqrt{2}\,(1-\sqrt{2}))\,(-\sqrt{2}+(1-\sqrt{2})\,(2-\sqrt{2}))}+\frac{x}{-1-\sqrt{2}-\sqrt{2}\,(1-\sqrt{2})}$$
$$\left.+1\right)c_2x^{-\sqrt{2}}$$

17.12 problem 13

Internal problem ID [2419]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + y' - 2yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)+diff(y(x),x)-2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{5}x^2 + \frac{1}{90}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 + \frac{1}{3}x^2 + \frac{1}{42}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 47

```
AsymptoticDSolveValue[2*x*y''[x]+y'[x]-2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^4}{90} + \frac{x^2}{5} + 1\right) + c_2\left(\frac{x^4}{42} + \frac{x^2}{3} + 1\right)$$

17.13 problem 14

Internal problem ID [2420]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2y'' - x(x+8)\,y' + 6y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(3*x^2*diff(y(x),x$2)-x*(x+8)*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^{\frac{2}{3}}\left(1 - \frac{1}{6}x + \frac{5}{36}x^2 + \frac{5}{81}x^3 + \frac{11}{972}x^4 + \frac{77}{58320}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2x^3\left(1 + \frac{3}{10}x + \frac{3}{65}x^2 + \frac{1}{208}x^3 + \frac{3}{7904}x^4 + \frac{21}{869440}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 88

```
AsymptoticDSolveValue[3*x^2*y''[x]-x*(x+8)*y'[x]+6*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{21x^5}{869440} + \frac{3x^4}{7904} + \frac{x^3}{208} + \frac{3x^2}{65} + \frac{3x}{10} + 1\right)x^3 + c_2\left(\frac{77x^5}{58320} + \frac{11x^4}{972} + \frac{5x^3}{81} + \frac{5x^2}{36} - \frac{x}{6} + 1\right)x^{2/3}$$

17.14 problem 15

Internal problem ID [2421]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - x(1+2x)\,y' + 2(4x-1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 41

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)-x*(1+2*x)*diff(y(x),x)+2*(4*x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{5}{2}}\left(1 - \frac{4}{7}x + \frac{4}{63}x^2 + \mathrm{O}\left(x^6\right)\right) + c_1\left(1 + 3x + \frac{21}{2}x^2 - \frac{35}{2}x^3 + \frac{35}{8}x^4 - \frac{7}{40}x^5 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 65

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*(1+2*x)*y'[x]+2*(4*x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{4x^2}{63} - \frac{4x}{7} + 1\right)x^2 + \frac{c_2\left(-\frac{7x^5}{40} + \frac{35x^4}{8} - \frac{35x^3}{2} + \frac{21x^2}{2} + 3x + 1\right)}{\sqrt{x}}$$

17.15 problem 16

Internal problem ID [2422]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(1-x)\,y' - (x+5)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 503

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(1-x)*diff(y(x),x)-(5+x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x) = {}& c_1 x^{-\sqrt{5}}\left(1+\frac{\sqrt{5}-1}{-1+2\sqrt{5}}x+\frac{-2+\sqrt{5}}{8\sqrt{5}-4}x^2+\frac{\left(-2+\sqrt{5}\right)\left(\sqrt{5}-3\right)}{276-96\sqrt{5}}x^3\right.\\ &\left.+\frac{\left(\sqrt{5}-3\right)\left(\sqrt{5}-4\right)}{2208-768\sqrt{5}}x^4+\frac{\left(-5+\sqrt{5}\right)\left(\sqrt{5}-3\right)\left(\sqrt{5}-4\right)}{41280\sqrt{5}-93600}x^5+\mathrm{O}\left(x^6\right)\right)\\ &+c_2 x^{\sqrt{5}}\left(1+\frac{\sqrt{5}+1}{1+2\sqrt{5}}x+\frac{\sqrt{5}+2}{8\sqrt{5}+4}x^2+\frac{\left(\sqrt{5}+3\right)\left(\sqrt{5}+2\right)}{276+96\sqrt{5}}x^3\right.\\ &\left.+\frac{\left(\sqrt{5}+4\right)\left(\sqrt{5}+3\right)}{2208+768\sqrt{5}}x^4+\frac{\left(5+\sqrt{5}\right)\left(\sqrt{5}+4\right)\left(\sqrt{5}+3\right)}{41280\sqrt{5}+93600}x^5+\mathrm{O}\left(x^6\right)\right)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 1093

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1-x)*y'[x]-(5+x)*y[x]==0,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) & \\
\rightarrow \Bigg(& \frac{\left(-5-\sqrt{5}\right)\left(-4-\sqrt{5}\right)\left(-3-\sqrt{5}\right)\left(-2-\sqrt{5}\right)\left(1+\sqrt{\right.}}{\left(-4+\sqrt{5}+\sqrt{5}\left(1+\sqrt{5}\right)\right)\left(-3+\sqrt{5}+\left(1+\sqrt{5}\right)\left(2+\sqrt{5}\right)\right)\left(-2+\sqrt{5}+\left(2+\sqrt{5}\right)\left(3+\sqrt{5}\right)\right)\left(-1+\right.} \\
& - \frac{\left(-4-\sqrt{5}\right)\left(-3-\sqrt{5}\right)\left(-2-\sqrt{5}\right)\left(1+\sqrt{5}\right)x^4}{\left(-4+\sqrt{5}+\sqrt{5}\left(1+\sqrt{5}\right)\right)\left(-3+\sqrt{5}+\left(1+\sqrt{5}\right)\left(2+\sqrt{5}\right)\right)\left(-2+\sqrt{5}+\left(2+\sqrt{5}\right)\left(3+\sqrt{5}\right)\right)\left(-1+\right.} \\
& + \frac{\left(-3-\sqrt{5}\right)\left(-2-\sqrt{5}\right)\left(1+\sqrt{5}\right)x^3}{\left(-4+\sqrt{5}+\sqrt{5}\left(1+\sqrt{5}\right)\right)\left(-3+\sqrt{5}+\left(1+\sqrt{5}\right)\left(2+\sqrt{5}\right)\right)\left(-2+\sqrt{5}+\left(2+\sqrt{5}\right)\left(3+\sqrt{5}\right)\right)} \\
& - \frac{\left(-2-\sqrt{5}\right)\left(1+\sqrt{5}\right)x^2}{\left(-4+\sqrt{5}+\sqrt{5}\left(1+\sqrt{5}\right)\right)\left(-3+\sqrt{5}+\left(1+\sqrt{5}\right)\left(2+\sqrt{5}\right)\right)} \\
& + \frac{\left(1+\sqrt{5}\right)x}{-4+\sqrt{5}+\sqrt{5}\left(1+\sqrt{5}\right)}+1\Bigg) c_1 x^{\sqrt{5}} \\
+ \Bigg(& \frac{\left(1-\sqrt{5}\right)\left(-5+\sqrt{5}\right)\left(-4+\sqrt{5}\right)\left(-3+\sqrt{5}\right)\left(-2+\right.}{\left(-4-\sqrt{5}-\sqrt{5}\left(1-\sqrt{5}\right)\right)\left(-3-\sqrt{5}+\left(1-\sqrt{5}\right)\left(2-\sqrt{5}\right)\right)\left(-2-\sqrt{5}+\left(2-\sqrt{5}\right)\left(3-\sqrt{5}\right)\right)\left(-1\right.} \\
& - \frac{\left(1-\sqrt{5}\right)\left(-4+\sqrt{5}\right)\left(-3+\sqrt{5}\right)\left(-2+\sqrt{5}\right)x^4}{\left(-4-\sqrt{5}-\sqrt{5}\left(1-\sqrt{5}\right)\right)\left(-3-\sqrt{5}+\left(1-\sqrt{5}\right)\left(2-\sqrt{5}\right)\right)\left(-2-\sqrt{5}+\left(2-\sqrt{5}\right)\left(3-\sqrt{5}\right)\right)\left(-1-\right.} \\
& + \frac{\left(1-\sqrt{5}\right)\left(-3+\sqrt{5}\right)\left(-2+\sqrt{5}\right)x^3}{\left(-4-\sqrt{5}-\sqrt{5}\left(1-\sqrt{5}\right)\right)\left(-3-\sqrt{5}+\left(1-\sqrt{5}\right)\left(2-\sqrt{5}\right)\right)\left(-2-\sqrt{5}+\left(2-\sqrt{5}\right)\left(3-\sqrt{5}\right)\right)} \\
& - \frac{\left(1-\sqrt{5}\right)\left(-2+\sqrt{5}\right)x^2}{\left(-4-\sqrt{5}-\sqrt{5}\left(1-\sqrt{5}\right)\right)\left(-3-\sqrt{5}+\left(1-\sqrt{5}\right)\left(2-\sqrt{5}\right)\right)} \\
& + \frac{\left(1-\sqrt{5}\right)x}{-4-\sqrt{5}-\sqrt{5}\left(1-\sqrt{5}\right)}+1\Bigg) c_2 x^{-\sqrt{5}}
\end{aligned}
$$

17.16 problem 17

Internal problem ID [2423]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{3x^2y'' + x(7+3x)\,y' + (6x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(3*x^2*diff(y(x),x$2)+x*(7+3*x)*diff(y(x),x)+(1+6*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - 3x + \frac{9}{4}x^2 - \frac{27}{28}x^3 + \frac{81}{280}x^4 - \frac{243}{3640}x^5 + \mathrm{O}\left(x^6\right)\right)x^{\frac{1}{3}} + c_2\left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{4}{3}}}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 84

```
AsymptoticDSolveValue[3*x^2*y''[x]+x*(7+3*x)*y'[x]+(1+6*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1\left(-\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1\right)}{\sqrt[3]{x}} + \frac{c_2\left(-\frac{243x^5}{3640} + \frac{81x^4}{280} - \frac{27x^3}{28} + \frac{9x^2}{4} - 3x + 1\right)}{x}$$

17.17 problem 18

Internal problem ID [2424]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + (1-x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(1-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^{-i}\left(1+\left(\frac{1}{5}+\frac{2i}{5}\right)x+\left(-\frac{1}{40}+\frac{3i}{40}\right)x^2+\left(-\frac{3}{520}+\frac{7i}{1560}\right)x^3+\left(-\frac{1}{2496}+\frac{i}{12480}\right)x^4 + \left(-\frac{9}{603200}-\frac{i}{361920}\right)x^5+\mathrm{O}\left(x^6\right)\right)+c_2x^i\left(1+\left(\frac{1}{5}-\frac{2i}{5}\right)x+\left(-\frac{1}{40}-\frac{3i}{40}\right)x^2 + \left(-\frac{3}{520}-\frac{7i}{1560}\right)x^3+\left(-\frac{1}{2496}-\frac{i}{12480}\right)x^4+\left(-\frac{9}{603200}+\frac{i}{361920}\right)x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 90

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(1-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \left(\frac{1}{12480}+\frac{i}{2496}\right)c_2x^{-i}(ix^4+(8+16i)x^3+(168+96i)x^2+(1056-288i)x + (480-2400i)) - \left(\frac{1}{2496}+\frac{i}{12480}\right)c_1x^i(x^4+(16+8i)x^3+(96+168i)x^2 - (288-1056i)x-(2400-480i))$$

17.18 problem 19

Internal problem ID [2425]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2y'' + x(3x^2+1)\,y' - 2yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(3*x^2*diff(y(x),x$2)+x*(1+3*x^2)*diff(y(x),x)-2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{2}{3}}\left(1+\frac{2}{5}x-\frac{3}{40}x^2-\frac{43}{660}x^3+\frac{31}{3696}x^4+\frac{2259}{261800}x^5+\mathrm{O}\left(x^6\right)\right) + c_2\left(1+2x+\frac{1}{2}x^2-\frac{5}{21}x^3-\frac{73}{840}x^4+\frac{827}{27300}x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 83

```
AsymptoticDSolveValue[3*x^2*y''[x]+x*(1+3*x^2)*y'[x]-2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{827x^5}{27300}-\frac{73x^4}{840}-\frac{5x^3}{21}+\frac{x^2}{2}+2x+1\right) + c_1 x^{2/3}\left(\frac{2259x^5}{261800}+\frac{31x^4}{3696}-\frac{43x^3}{660}-\frac{3x^2}{40}+\frac{2x}{5}+1\right)$$

17.19 problem 20

Internal problem ID [2426]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - 4y'x^2 + (1 + 2x)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)-4*x^2*diff(y(x),x)+(1+2*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(\left(x + \frac{1}{4}x^2 + \frac{1}{18}x^3 + \frac{1}{96}x^4 + \frac{1}{600}x^5 + \mathrm{O}\left(x^6\right)\right) c_2 + \left(c_2 \ln\left(x\right) + c_1\right)\left(1 + \mathrm{O}\left(x^6\right)\right)\right)\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 60

```
AsymptoticDSolveValue[4*x^2*y''[x]-4*x^2*y'[x]+(1+2*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\sqrt{x}\left(\frac{x^5}{600} + \frac{x^4}{96} + \frac{x^3}{18} + \frac{x^2}{4} + x\right) + \sqrt{x}\log(x)\right) + c_1\sqrt{x}$$

17.20 problem 21

Internal problem ID [2427]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.4. page 758
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(-2x+3)\,y' + (1-2x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(3-2*x)*diff(y(x),x)+(1-2*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(c_2 \ln(x) + c_1)(1 + \mathrm{O}(x^6)) + \left(2x + x^2 + \frac{4}{9}x^3 + \frac{1}{6}x^4 + \frac{4}{75}x^5 + \mathrm{O}(x^6)\right)c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 52

```
AsymptoticDSolveValue[x^2*y''[x]+x*(3-2*x)*y'[x]+(1-2*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{\frac{4x^5}{75} + \frac{x^4}{6} + \frac{4x^3}{9} + x^2 + 2x}{x} + \frac{\log(x)}{x}\right) + \frac{c_1}{x}$$

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18.1 problem Example 11.5.2 page 763

Internal problem ID [2428]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: Example 11.5.2 page 763.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(x+3)\,y' + (-x+4)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(3+x)*diff(y(x),x)+(4-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^2\left((c_2\ln(x)+c_1)\left(1+3x+3x^2+\frac{5}{3}x^3+\frac{5}{8}x^4+\frac{7}{40}x^5+\mathrm{O}\left(x^6\right)\right) + \left((-5)\,x-\frac{29}{4}x^2-\frac{173}{36}x^3-\frac{193}{96}x^4-\frac{1459}{2400}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 118

```
AsymptoticDSolveValue[x^2*y''[x]-x*(3+x)*y'[x]+(4-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{7x^5}{40}+\frac{5x^4}{8}+\frac{5x^3}{3}+3x^2+3x+1\right)x^2 + c_2\left(\left(-\frac{1459x^5}{2400}-\frac{193x^4}{96}-\frac{173x^3}{36}-\frac{29x^2}{4}-5x\right)x^2 + \left(\frac{7x^5}{40}+\frac{5x^4}{8}+\frac{5x^3}{3}+3x^2+3x+1\right)x^2\log(x)\right)$$

18.2 problem Example 11.5.4 page 765

Internal problem ID [2429]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: Example 11.5.4 page 765.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(-x+3)\,y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 53

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(3-x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(c_2 \ln(x) + c_1)\left(1 - x + \mathrm{O}\left(x^6\right)\right) + \left(3x - \frac{1}{4}x^2 - \frac{1}{36}x^3 - \frac{1}{288}x^4 - \frac{1}{2400}x^5 + \mathrm{O}\left(x^6\right)\right)c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 66

```
AsymptoticDSolveValue[x^2*y''[x]+x*(3-x)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{-\frac{x^5}{2400} - \frac{x^4}{288} - \frac{x^3}{36} - \frac{x^2}{4} + 3x}{x} + \frac{(1-x)\log(x)}{x}\right) + \frac{c_1(1-x)}{x}$$

18.3 problem Example 11.5.5 page 768

Internal problem ID [2430]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: Example 11.5.5 page 768.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x - (x+4)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(4+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^4\left(1+\frac{1}{5}x+\frac{1}{60}x^2+\frac{1}{1260}x^3+\frac{1}{40320}x^4+\frac{1}{1814400}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(\ln\left(x\right)\left(x^4+\frac{1}{5}x^5+\mathrm{O}\left(x^6\right)\right)+\left(-144+\right.\right.}{x^2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 77

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]-(4+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4-16x^3+48x^2-192x+576}{576x^2}-\frac{1}{144}x^2\log(x)\right)+c_2\left(\frac{x^6}{40320}+\frac{x^5}{1260}+\frac{x^4}{60}+\frac{x^3}{5}+x^2\right)$$

18.4 problem (a)

Internal problem ID [2431]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - (-x^2 + x)\, y' + (x^3 + 1)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-(x-x^2)*diff(y(x),x)+(1+x^3)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x\left((c_2 \ln(x) + c_1)\left(1 - x + \frac{1}{2}x^2 - \frac{5}{18}x^3 + \frac{19}{144}x^4 - \frac{167}{3600}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(x - \frac{3}{4}x^2 + \frac{41}{108}x^3 - \frac{89}{432}x^4 + \frac{2281}{27000}x^5 + \mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 114

```
AsymptoticDSolveValue[x^2*y''[x]-(x-x^2)*y'[x]+(1+x^3)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x\left(-\frac{167x^5}{3600} + \frac{19x^4}{144} - \frac{5x^3}{18} + \frac{x^2}{2} - x + 1\right) + c_2\left(x\left(\frac{2281x^5}{27000} - \frac{89x^4}{432} + \frac{41x^3}{108} - \frac{3x^2}{4} + x\right) + x\left(-\frac{167x^5}{3600} + \frac{19x^4}{144} - \frac{5x^3}{18} + \frac{x^2}{2} - x + 1\right)\log(x)\right)$$

18.5 problem (b)

Internal problem ID [2432]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - \left(-1+2\sqrt{5}\right)xy' + \left(\frac{19}{4} - 3x^2\right)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 325

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-(2*sqrt(5)-1)*x*diff(y(x),x)+(19/4-3*x^2)*y(x)=0,y(x),type='series'
```

$$y(x) = \left(\left(1+\frac{3}{2}x^2+\frac{3}{8}x^4+\mathrm{O}\left(x^6\right)\right)c_1 + xc_2\left(\ln(x)\left(1+\frac{1}{2}x^2+\frac{3}{40}x^4+\mathrm{O}\left(x^6\right)\right)+\left(-\frac{5}{12}x^2-\frac{77}{800}x^4+\mathrm{O}\left(x^6\right)\right)\right)\right)x^{-\frac{1}{2}+\sqrt{5}}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 94

```
AsymptoticDSolveValue[x^2*y''[x]-(2*Sqrt[5]-1)*x*y'[x]+(19/4-3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{3}{8}x^{\frac{7}{2}+\sqrt{5}}+\frac{3}{2}x^{\frac{3}{2}+\sqrt{5}}+x^{\sqrt{5}-\frac{1}{2}}\right)+c_2\left(\frac{3}{40}x^{\frac{9}{2}+\sqrt{5}}+\frac{1}{2}x^{\frac{5}{2}+\sqrt{5}}+x^{\frac{1}{2}+\sqrt{5}}\right)$$

18.6 problem (c)

Internal problem ID [2433]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (-2x^5 + 9x)\, y' + (10x^4 + 5x^2 + 25)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
Order:=7;
dsolve(x^2*diff(y(x),x$2)+(9*x-2*x^5)*diff(y(x),x)+(25+5*x^2+10*x^4)*y(x)=0,y(x),type='series
```

$$y(x) = c_1 x^{-4-3i}\left(1+\left(-\frac{1}{8}-\frac{3i}{8}\right)x^2+\left(-\frac{179}{832}-\frac{483i}{832}\right)x^4+\left(-\frac{433}{3744}+\frac{3943i}{29952}\right)x^6+\mathrm{O}\left(x^7\right)\right)$$
$$+ c_2 x^{-4+3i}\left(1+\left(-\frac{1}{8}+\frac{3i}{8}\right)x^2+\left(-\frac{179}{832}+\frac{483i}{832}\right)x^4+\left(-\frac{433}{3744}-\frac{3943i}{29952}\right)x^6 + \mathrm{O}\left(x^7\right)\right)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 70

```
AsymptoticDSolveValue[x^2*y''[x]+(9*x-2*x^5)*y'[x]+(25+5*x^2+10*x^4)*y[x]==0,y[x],{x,0,6}]
```

$$y(x) \to \left(\frac{1}{832}+\frac{5i}{832}\right)c_1 x^{-4+3i}\left((86+53i)x^4+(56+32i)x^2+(32-160i)\right)$$
$$-\left(\frac{5}{832}+\frac{i}{832}\right)c_2 x^{-4-3i}\left((53+86i)x^4+(32+56i)x^2-(160-32i)\right)$$

18.7 problem (d)

Internal problem ID [2434]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + \left(4x + \frac{1}{2}x^2 - \frac{1}{3}x^3\right)y' - \frac{7y}{4} = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+(4*x+1/2*x^2-1/3*x^3)*diff(y(x),x)-7/4*y(x)=0,y(x),type='series',x=
```

$$y(x) = \frac{c_1x^4\left(1 - \frac{1}{20}x + \frac{49}{2880}x^2 - \frac{533}{241920}x^3 + \frac{277}{491520}x^4 - \frac{203759}{2388787200}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\left(\frac{8491}{768}x^4 - \frac{8491}{15360}x^5 + \mathrm{O}\left(x^6\right)\right)\ln}{x^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 93

```
AsymptoticDSolveValue[x^2*y''[x]+(4*x+1/2*x^2-1/3*x^3)*y'[x]-7/4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{277x^{9/2}}{491520} - \frac{533x^{7/2}}{241920} + \frac{49x^{5/2}}{2880} - \frac{x^{3/2}}{20} + \sqrt{x}\right) + c_1\left(\frac{65067x^4 - 124096x^3 + 209664x^2 - 258048x + 442368}{442368x^{7/2}} - \frac{8491\sqrt{x}\log(x)}{110592}\right)$$

18.8 problem (e)

Internal problem ID [2435]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$x^2y'' + y'x^2 + yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 58

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x\left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \mathrm{O}\left(x^6\right)\right) \\ & + c_2\left(\ln\left(x\right)\left(-x + x^2 - \frac{1}{2}x^3 + \frac{1}{6}x^4 - \frac{1}{24}x^5 + \mathrm{O}\left(x^6\right)\right)\right. \\ & \left. + \left(1 - x + \frac{1}{4}x^3 - \frac{5}{36}x^4 + \frac{13}{288}x^5 + \mathrm{O}\left(x^6\right)\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 80

```
AsymptoticDSolveValue[x^2*y''[x]+x^2*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \to {} & c_1\left(\frac{1}{6}x\left(x^3 - 3x^2 + 6x - 6\right)\log(x) + \frac{1}{36}\left(-11x^4 + 27x^3 - 36x^2 + 36\right)\right) \\ & + c_2\left(\frac{x^5}{24} - \frac{x^4}{6} + \frac{x^3}{2} - x^2 + x\right) \end{aligned}$$

18.9 problem 1

Internal problem ID [2436]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(x-3)\,y' + (-x+4)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(x-3)*diff(y(x),x)+(4-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^2\left((c_2\ln(x)+c_1)\left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(x - \frac{3}{4}x^2 + \frac{11}{36}x^3 - \frac{25}{288}x^4 + \frac{137}{7200}x^5 + \mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 120

```
AsymptoticDSolveValue[x^2*y''[x]+x*(x-3)*y'[x]+(4-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1\right)x^2 + c_2\left(\left(\frac{137x^5}{7200} - \frac{25x^4}{288} + \frac{11x^3}{36} - \frac{3x^2}{4} + x\right)x^2 + \left(-\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1\right)x^2\log(x)\right)$$

18.10 problem 2

Internal problem ID [2437]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 2y'x^2 + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 67

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+2*x^2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_2 \ln(x) + c_1)\left(1 - \frac{1}{4}x + \frac{3}{64}x^2 - \frac{5}{768}x^3 + \frac{35}{49152}x^4 - \frac{21}{327680}x^5 + \mathrm{O}(x^6)\right) + \left(-\frac{1}{64}x^2 + \frac{1}{256}x^3 - \frac{19}{32768}x^4 + \frac{25}{393216}x^5 + \mathrm{O}(x^6)\right)c_2\right)\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 129

```
AsymptoticDSolveValue[4*x^2*y''[x]+2*x^2*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{21x^5}{327680} + \frac{35x^4}{49152} - \frac{5x^3}{768} + \frac{3x^2}{64} - \frac{x}{4} + 1\right) + c_2\left(\sqrt{x}\left(\frac{25x^5}{393216} - \frac{19x^4}{32768} + \frac{x^3}{256} - \frac{x^2}{64}\right) + \sqrt{x}\left(-\frac{21x^5}{327680} + \frac{35x^4}{49152} - \frac{5x^3}{768} + \frac{3x^2}{64} - \frac{x}{4} + 1\right)\log(x)\right)$$

18.11 problem 3

Internal problem ID [2438]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + y' \cos(x)\, x - 2\,\mathrm{e}^x y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 389

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*cos(x)*diff(y(x),x)-2*exp(x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x^{-\sqrt{2}} \left(1 - 2\frac{1}{-1+2\sqrt{2}}x + \frac{-5\sqrt{2}+14}{40-24\sqrt{2}}x^2 + \frac{-122+75\sqrt{2}}{684\sqrt{2}-972}x^3 + \frac{-1626\sqrt{2}+2375}{52992-37440\sqrt{2}}x^4 \right. \\ & \left. + \frac{1}{7200}\frac{-75763+52810\sqrt{2}}{(-1+2\sqrt{2})(\sqrt{2}-1)(-3+2\sqrt{2})(-2+\sqrt{2})(-5+2\sqrt{2})}x^5 + \mathrm{O}(x^6)\right) \\ & + c_2 x^{\sqrt{2}} \left(1 + 2\frac{1}{1+2\sqrt{2}}x + \frac{5\sqrt{2}+14}{40+24\sqrt{2}}x^2 + \frac{122+75\sqrt{2}}{684\sqrt{2}+972}x^3 + \frac{1626\sqrt{2}+2375}{52992+37440\sqrt{2}}x^4 \right. \\ & \left. + \frac{1}{7200}\frac{75763+52810\sqrt{2}}{(1+2\sqrt{2})(1+\sqrt{2})(3+2\sqrt{2})(2+\sqrt{2})(5+2\sqrt{2})}x^5 + \mathrm{O}(x^6)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 2210

```
AsymptoticDSolveValue[x^2*y''[x]+x*Cos[x]*y'[x]-2*Exp[x]*y[x]==0,y[x],{x,0,5}]
```

Too large to display

18.12 problem 4

Internal problem ID [2439]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x^2 - (2+x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)-(2+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 - \frac{1}{4}x + \frac{1}{20}x^2 - \frac{1}{120}x^3 + \frac{1}{840}x^4 - \frac{1}{6720}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(12 - 12x + 6x^2 - 2x^3 + \frac{1}{2}x^4 - \frac{1}{10}x^5 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 66

```
AsymptoticDSolveValue[x^2*y''[x]+x^2*y'[x]-(2+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^3}{24} - \frac{x^2}{6} + \frac{x}{2} + \frac{1}{x} - 1\right) + c_2\left(\frac{x^6}{840} - \frac{x^5}{120} + \frac{x^4}{20} - \frac{x^3}{4} + x^2\right)$$

18.13 problem 5

Internal problem ID [2440]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 2y'x^2 + \left(x - \frac{3}{4}\right)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+2*x^2*diff(y(x),x)+(x-3/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1 - \frac{4}{3}x + x^2 - \frac{8}{15}x^3 + \frac{2}{9}x^4 - \frac{8}{105}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(-2 + 4x^2 - \frac{16}{3}x^3 + 4x^4 - \frac{32}{15}x^5 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 77

```
AsymptoticDSolveValue[x^2*y''[x]+2*x^2*y'[x]+(x-3/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-2x^{7/2} + \frac{8x^{5/2}}{3} - 2x^{3/2} + \frac{1}{\sqrt{x}}\right) + c_2\left(\frac{2x^{11/2}}{9} - \frac{8x^{9/2}}{15} + x^{7/2} - \frac{4x^{5/2}}{3} + x^{3/2}\right)$$

18.14 problem 6

Internal problem ID [2441]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + (2x - 1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(2*x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^2 \left(1 - \frac{2}{3}x + \frac{1}{6}x^2 - \frac{1}{45}x^3 + \frac{1}{540}x^4 - \frac{1}{9450}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln\left(x\right)\left(4x^2 - \frac{8}{3}x^3 + \frac{2}{3}x^4 - \frac{4}{45}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(\right.\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 83

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(2*x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{31x^4 - 88x^3 + 36x^2 + 72x + 36}{36x} - \frac{1}{3}x\left(x^2 - 4x + 6\right)\log(x)\right) + c_2\left(\frac{x^5}{540} - \frac{x^4}{45} + \frac{x^3}{6} - \frac{2x^2}{3} + x\right)$$

18.15 problem 7

Internal problem ID [2442]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x^3y' - (2+x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 65

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x^3*diff(y(x),x)-(2+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^3\left(1+\frac{1}{4}x-\frac{7}{40}x^2-\frac{37}{720}x^3+\frac{467}{20160}x^4+\frac{5647}{806400}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(\ln\left(x\right)\left(-x^3-\frac{1}{4}x^4+\frac{7}{40}x^5+\mathrm{O}\left(x^6\right)\right)+(\cdots\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 82

```
AsymptoticDSolveValue[x^2*y''[x]+x^3*y'[x]-(2+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{91x^4+160x^3-144x^2-288x+576}{576x}-\frac{1}{48}x^2(x+4)\log(x)\right) + c_2\left(\frac{467x^6}{20160}-\frac{37x^5}{720}-\frac{7x^4}{40}+\frac{x^3}{4}+x^2\right)$$

18.16 problem 8

Internal problem ID [2443]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''+7y'\mathrm{e}^x x+9(1+\tan(x))\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 75

```
Order:=7;
dsolve(x^2*(x^2+1)*diff(y(x),x$2)+7*x*exp(x)*diff(y(x),x)+9*(1+tan(x))*y(x)=0,y(x),type='seri
```

$$y(x) = \frac{(c_2\ln(x)+c_1)\left(1+12x+\frac{117}{8}x^2-\frac{67}{36}x^3+\frac{505}{256}x^4-\frac{262}{125}x^5+\frac{2443637}{2304000}x^6+\mathrm{O}(x^7)\right)+\left((-31)\,x-\frac{147}{2}x^2+\frac{37}{8}x\right.}{x^3}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 143

```
AsymptoticDSolveValue[x^2*(x^2+1)*y''[x]+7*x*Exp[x]*y'[x]+9*(1+Tan[x])*y[x]==0,y[x],{x,0,6}]
```

$$y(x)\to \frac{c_1\left(\frac{2443637x^6}{2304000}-\frac{262x^5}{125}+\frac{505x^4}{256}-\frac{67x^3}{36}+\frac{117x^2}{8}+12x+1\right)}{x^3}+c_2\left(\frac{-\frac{3797765581x^6}{622080000}+\frac{5057587x^5}{480000}-\frac{44803x^4}{4608}+\frac{37x^3}{8}-\frac{147x^2}{2}-31x}{x^3}+\frac{\left(\frac{2443637x^6}{2304000}-\frac{262x^5}{125}+\frac{505x^4}{256}-\frac{67x^3}{36}+\frac{117x^2}{8}+12x+1\right)\log(x)}{x^3}\right)$$

18.17 problem 11

Internal problem ID [2444]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x+1)\,y''+y'x^2-2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
Order:=6;
dsolve(x^2*(1+x)*diff(y(x),x$2)+x^2*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^2\left(1-x+\frac{9}{10}x^2-\frac{4}{5}x^3+\frac{5}{7}x^4-\frac{9}{14}x^5+\mathrm{O}\left(x^6\right)\right)+\frac{c_2(12+6x+\mathrm{O}\left(x^6\right))}{x}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 47

```
AsymptoticDSolveValue[x^2*(1+x)*y''[x]+x^2*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{5x^6}{7}-\frac{4x^5}{5}+\frac{9x^4}{10}-x^3+x^2\right)+c_1\left(\frac{1}{x}+\frac{1}{2}\right)$$

18.18 problem 12

Internal problem ID [2445]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 3y'x + (1-x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+(1-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(c_2 \ln(x) + c_1)\left(1 + x + \frac{1}{4}x^2 + \frac{1}{36}x^3 + \frac{1}{576}x^4 + \frac{1}{14400}x^5 + \mathrm{O}(x^6)\right) + \left((-2)\,x - \frac{3}{4}x^2 - \frac{11}{108}x^3 - \frac{25}{3456}x^4 - \frac{13}{4320}\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 118

```
AsymptoticDSolveValue[x^2*y''[x]+3*x*y'[x]+(1-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1\left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right)}{x} + c_2\left(\frac{-\frac{137x^5}{432000} - \frac{25x^4}{3456} - \frac{11x^3}{108} - \frac{3x^2}{4} - 2x}{x} + \frac{\left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right)\log(x)}{x}\right)$$

18.19 problem 13

Internal problem ID [2446]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
Order:=6;
dsolve(x*diff(y(x),x$2)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x \left(1 + \frac{1}{2}x + \frac{1}{12}x^2 + \frac{1}{144}x^3 + \frac{1}{2880}x^4 + \frac{1}{86400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln(x)\left(x + \frac{1}{2}x^2 + \frac{1}{12}x^3 + \frac{1}{144}x^4 + \frac{1}{2880}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(1 - \frac{3}{4}x^2 - \frac{7}{36}x^3 - \frac{35}{1728}x^4 - \frac{101}{86400}x^5 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x*y''[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{144}x\left(x^3+12x^2+72x+144\right)\log(x) + \frac{-47x^4 - 480x^3 - 2160x^2 - 1728x + 1728}{1728}\right) + c_2\left(\frac{x^5}{2880} + \frac{x^4}{144} + \frac{x^3}{12} + \frac{x^2}{2} + x\right)$$

18.20 problem 14

Internal problem ID [2447]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(x^2 + 6)\, y' + 6y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(6+x^2)*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 + \frac{1}{3}x^2 + \mathrm{O}\left(x^6\right)\right)x + c_2\left(1 + \frac{3}{2}x^2 + \frac{1}{8}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 33

```
AsymptoticDSolveValue[x^2*y''[x]+x*(6+x^2)*y'[x]+6*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{x^3} + \frac{x}{8} + \frac{3}{2x}\right) + c_2\left(\frac{1}{x^2} + \frac{1}{3}\right)$$

18.21 problem 15

Internal problem ID [2448]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(1-x)\,y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(1-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x\left(1+\frac{1}{3}x+\frac{1}{12}x^2+\frac{1}{60}x^3+\frac{1}{360}x^4+\frac{1}{2520}x^5+\mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(-2-2x-x^2-\frac{1}{3}x^3-\frac{1}{12}x^4-\frac{1}{60}x^5+\mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 64

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1-x)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^3}{24}+\frac{x^2}{6}+\frac{x}{2}+\frac{1}{x}+1\right)+c_2\left(\frac{x^5}{360}+\frac{x^4}{60}+\frac{x^3}{12}+\frac{x^2}{3}+x\right)$$

18.22 problem 16

Internal problem ID [2449]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + (1-4x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+(1-4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_2\ln(x)+c_1)\left(1+x+\frac{1}{4}x^2+\frac{1}{36}x^3+\frac{1}{576}x^4+\frac{1}{14400}x^5+\mathrm{O}\left(x^6\right)\right) + \left((-2)\,x-\frac{3}{4}x^2-\frac{11}{108}x^3-\frac{25}{3456}x^4-\frac{137}{432000}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 124

```
AsymptoticDSolveValue[4*x^2*y''[x]+(1-4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^5}{14400}+\frac{x^4}{576}+\frac{x^3}{36}+\frac{x^2}{4}+x+1\right) + c_2\left(\sqrt{x}\left(-\frac{137x^5}{432000}-\frac{25x^4}{3456}-\frac{11x^3}{108}-\frac{3x^2}{4}-2x\right) + \sqrt{x}\left(\frac{x^5}{14400}+\frac{x^4}{576}+\frac{x^3}{36}+\frac{x^2}{4}+x+1\right)\log(x)\right)$$

18.23 problem 17

Internal problem ID [2450]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 + 2x + x^2 + \frac{2}{9}x^3 + \frac{1}{36}x^4 + \frac{1}{450}x^5 + \mathrm{O}\left(x^6\right)\right) + \left((-4)\,x - 3x^2 - \frac{22}{27}x^3 - \frac{25}{216}x^4 - \frac{137}{13500}x^5 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 101

```
AsymptoticDSolveValue[x*y''[x]+y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{450} + \frac{x^4}{36} + \frac{2x^3}{9} + x^2 + 2x + 1\right) + c_2\left(-\frac{137x^5}{13500} - \frac{25x^4}{216} - \frac{22x^3}{27} - 3x^2 + \left(\frac{x^5}{450} + \frac{x^4}{36} + \frac{2x^3}{9} + x^2 + 2x + 1\right)\log(x) - 4x\right)$$

18.24 problem 18

Internal problem ID [2451]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x - (x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1+\frac{1}{3}x+\frac{1}{24}x^2+\frac{1}{360}x^3+\frac{1}{8640}x^4+\frac{1}{302400}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(\ln\left(x\right)\left(x^2+\frac{1}{3}x^3+\frac{1}{24}x^4+\frac{1}{360}x^5+\mathrm{O}\left(x^6\right)\right.\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 83

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]-(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{31x^4+176x^3+144x^2-576x+576}{576x}-\frac{1}{48}x\left(x^2+8x+24\right)\log(x)\right) + c_2\left(\frac{x^5}{8640}+\frac{x^4}{360}+\frac{x^3}{24}+\frac{x^2}{3}+x\right)$$

18.25 problem 19

Internal problem ID [2452]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(x+3)\,y' + 4y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(x+3)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^2\left((c_2\ln(x)+c_1)\left(1+2x+\frac{3}{2}x^2+\frac{2}{3}x^3+\frac{5}{24}x^4+\frac{1}{20}x^5+\mathrm{O}\left(x^6\right)\right)\right.$$
$$\left.+\left((-3)\,x-\frac{13}{4}x^2-\frac{31}{18}x^3-\frac{173}{288}x^4-\frac{187}{1200}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 122

```
AsymptoticDSolveValue[x^2*y''[x]-x*(x+3)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{20}+\frac{5x^4}{24}+\frac{2x^3}{3}+\frac{3x^2}{2}+2x+1\right)x^2+c_2\left(\left(-\frac{187x^5}{1200}-\frac{173x^4}{288}-\frac{31x^3}{18}-\frac{13x^2}{4}-3x\right)x^2\right.$$
$$\left.+\left(\frac{x^5}{20}+\frac{5x^4}{24}+\frac{2x^3}{3}+\frac{3x^2}{2}+2x+1\right)x^2\log(x)\right)$$

18.26 problem 20

Internal problem ID [2453]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x^2 - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 + \frac{1}{2}x + \frac{3}{20}x^2 + \frac{1}{30}x^3 + \frac{1}{168}x^4 + \frac{1}{1120}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(12 + 6x - x^3 - \frac{1}{2}x^4 - \frac{3}{20}x^5 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 63

```
AsymptoticDSolveValue[x^2*y''[x]-x^2*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^3}{24} - \frac{x^2}{12} + \frac{1}{x} + \frac{1}{2}\right) + c_2\left(\frac{x^6}{168} + \frac{x^5}{30} + \frac{3x^4}{20} + \frac{x^3}{2} + x^2\right)$$

18.27 problem 21

Internal problem ID [2454]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x^2 - (3x+2)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 65

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x^2*diff(y(x),x)-(3*x+2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^3\left(1 + \frac{5}{4}x + \frac{3}{4}x^2 + \frac{7}{24}x^3 + \frac{1}{12}x^4 + \frac{3}{160}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2(\ln\left(x\right)\left(24x^3 + 30x^4 + 18x^5 + \mathrm{O}\left(x^6\right)\right) + (12 - 1}{x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 84

```
AsymptoticDSolveValue[x^2*y''[x]-x^2*y'[x]-(3*x+2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{2}x^2(5x+4)\log(x) - \frac{3x^4 - 6x^3 - 6x^2 + 4x - 4}{4x}\right) + c_2\left(\frac{x^6}{12} + \frac{7x^5}{24} + \frac{3x^4}{4} + \frac{5x^3}{4} + x^2\right)$$

18.28 problem 22

Internal problem ID [2455]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(5-x)\,y' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 57

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(5-x)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(c_2\ln(x) + c_1)\left(1 - 2x + \frac{1}{2}x^2 + \mathrm{O}(x^6)\right) + \left(5x - \frac{9}{4}x^2 + \frac{1}{18}x^3 + \frac{1}{288}x^4 + \frac{1}{3600}x^5 + \mathrm{O}(x^6)\right)c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 80

```
AsymptoticDSolveValue[x^2*y''[x]+x*(5-x)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1\left(\frac{x^2}{2} - 2x + 1\right)}{x^2} + c_2\left(\frac{\left(\frac{x^2}{2} - 2x + 1\right)\log(x)}{x^2} + \frac{\frac{x^5}{3600} + \frac{x^4}{288} + \frac{x^3}{18} - \frac{9x^2}{4} + 5x}{x^2}\right)$$

18.29 problem 23

Internal problem ID [2456]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 4x(1-x)\,y' + (2x-9)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+4*x*(1-x)*diff(y(x),x)+(2*x-9)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^3\left(1+\frac{1}{4}x+\frac{1}{20}x^2+\frac{1}{120}x^3+\frac{1}{840}x^4+\frac{1}{6720}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(12+12x+6x^2+2x^3+\frac{1}{2}x^4+\frac{1}{10}x^5+\mathrm{O}\left(x^6\right)\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 90

```
AsymptoticDSolveValue[4*x^2*y''[x]+4*x*(1-x)*y'[x]+(2*x-9)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^{5/2}}{24}+\frac{x^{3/2}}{6}+\frac{1}{x^{3/2}}+\frac{\sqrt{x}}{2}+\frac{1}{\sqrt{x}}\right)+c_2\left(\frac{x^{11/2}}{840}+\frac{x^{9/2}}{120}+\frac{x^{7/2}}{20}+\frac{x^{5/2}}{4}+x^{3/2}\right)$$

18.30 problem 24

Internal problem ID [2457]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2x(2 + x)\, y' + 2(x + 1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+2*x*(2+x)*diff(y(x),x)+2*(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1(1 + \mathrm{O}(x^6))\, x + (2x + \mathrm{O}(x^6)) \ln(x)\, c_2 + \left(1 - 2x - 2x^2 + \frac{2}{3}x^3 - \frac{2}{9}x^4 + \frac{1}{15}x^5 + \mathrm{O}(x^6)\right) c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 48

```
AsymptoticDSolveValue[x^2*y''[x]+2*x*(2+x)*y'[x]+2*(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{2\log(x)}{x} - \frac{2x^4 - 6x^3 + 18x^2 + 36x - 9}{9x^2}\right) + \frac{c_2}{x}$$

18.31 problem 25

Internal problem ID [2458]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(1-x)\,y' + (1-x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(1-x)*diff(y(x),x)+(1-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x\left((c_2 \ln(x) + c_1)\left(1 + \mathrm{O}\left(x^6\right)\right) + \left(-x + \frac{1}{4}x^2 - \frac{1}{18}x^3 + \frac{1}{96}x^4 - \frac{1}{600}x^5 + \mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 50

```
AsymptoticDSolveValue[x^2*y''[x]-x*(1-x)*y'[x]+(1-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x\left(-\frac{x^5}{600} + \frac{x^4}{96} - \frac{x^3}{18} + \frac{x^2}{4} - x\right) + x\log(x)\right) + c_1 x$$

18.32 problem 26

Internal problem ID [2459]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 4x(1+2x)\,y' + (4x-1)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+4*x*(1+2*x)*diff(y(x),x)+(4*x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x\left(1 - x + \frac{2}{3}x^2 - \frac{1}{3}x^3 + \frac{2}{15}x^4 - \frac{2}{45}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 - 2x + 2x^2 - \frac{4}{3}x^3 + \frac{2}{3}x^4 - \frac{4}{15}x^5 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 88

```
AsymptoticDSolveValue[4*x^2*y''[x]+4*x*(1+2*x)*y'[x]+(4*x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{2x^{7/2}}{3} - \frac{4x^{5/2}}{3} + 2x^{3/2} - 2\sqrt{x} + \frac{1}{\sqrt{x}}\right) + c_2\left(\frac{2x^{9/2}}{15} - \frac{x^{7/2}}{3} + \frac{2x^{5/2}}{3} - x^{3/2} + \sqrt{x}\right)$$

18.33 problem 27

Internal problem ID [2460]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' - (4x+3)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 65

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)-(3+4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1+\frac{1}{3}x+\frac{1}{24}x^2+\frac{1}{360}x^3+\frac{1}{8640}x^4+\frac{1}{302400}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(\ln\left(x\right)\left(x^2+\frac{1}{3}x^3+\frac{1}{24}x^4+\frac{1}{360}x^5+\mathrm{O}\left(x^6\right)\right.\right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 101

```
AsymptoticDSolveValue[4*x^2*y''[x]-(3+4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^{11/2}}{8640}+\frac{x^{9/2}}{360}+\frac{x^{7/2}}{24}+\frac{x^{5/2}}{3}+x^{3/2}\right)+c_1\left(\frac{31x^4+176x^3+144x^2-576x+576}{576\sqrt{x}}-\frac{1}{48}x^{3/2}\left(x^2+8x+24\right)\log(x)\right)$$

18.34 problem 28

Internal problem ID [2461]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{xy'' - y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
Order:=6;
dsolve(x*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (-x + O(x^6)) \ln(x) c_2 + c_1(1 + O(x^6)) x + \left(1 + x - \frac{1}{2}x^2 - \frac{1}{12}x^3 - \frac{1}{72}x^4 - \frac{1}{480}x^5 + O(x^6)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 41

```
AsymptoticDSolveValue[x*y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{72}(-x^4 - 6x^3 - 36x^2 + 144x + 72) - x\log(x)\right) + c_2x$$

18.35 problem 29

Internal problem ID [2462]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.5. page 771
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + x(x+4)\, y' + (2+x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(4+x)*diff(y(x),x)+(2+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(x + \mathrm{O}(x^6)) \ln(x)\, c_2 + c_1(1 + \mathrm{O}(x^6))\, x + \left(1 - x - \frac{1}{2}x^2 + \frac{1}{12}x^3 - \frac{1}{72}x^4 + \frac{1}{480}x^5 + \mathrm{O}(x^6)\right) c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 45

```
AsymptoticDSolveValue[x^2*y''[x]+x*(4+x)*y'[x]+(2+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{\log(x)}{x} - \frac{x^4 - 6x^3 + 36x^2 + 144x - 72}{72x^2} \right) + \frac{c_2}{x}$$

19 Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.6. page 783

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19.1 problem 2

Internal problem ID [2463]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.6. page 783
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{9}{4}\right) y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-9/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^3 \left(1 - \frac{1}{10}x^2 + \frac{1}{280}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(12 + 6x^2 - \frac{3}{2}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-9/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(-\frac{x^{5/2}}{8} + \frac{1}{x^{3/2}} + \frac{\sqrt{x}}{2}\right) + c_2 \left(\frac{x^{11/2}}{280} - \frac{x^{7/2}}{10} + x^{3/2}\right)$$

19.2 problem 3

Internal problem ID [2464]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Exercises for 11.6. page 783
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' - y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
Order:=6;
dsolve(x*diff(y(x),x$2)-diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^2 \left(1 - \frac{1}{8}x^2 + \frac{1}{192}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln\left(x\right)\left(x^2 - \frac{1}{8}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-2 + \frac{3}{32}x^4 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 59

```
AsymptoticDSolveValue[x*y''[x]-y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{1}{16}\left(x^2 - 8\right)x^2 \log(x) + \frac{1}{64}\left(-5x^4 + 16x^2 + 64\right)\right) + c_2 \left(\frac{x^6}{192} - \frac{x^4}{8} + x^2\right)$$

20 Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788

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20.1 problem 1

Internal problem ID [2465]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^3}{6}\right) y(0) + \left(x - \frac{1}{12}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^4}{12}\right) + c_1\left(1 - \frac{x^3}{6}\right)$$

20.2 problem 2

Internal problem ID [2466]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - x^2 y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^4}{12}\right) y(0) + \left(x + \frac{1}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]-x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{20} + x\right) + c_1\left(\frac{x^4}{12} + 1\right)$$

20.3 problem 3

Internal problem ID [2467]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(1-x^2)\,y''-6y'x-4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-6*x*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(3x^4+2x^2+1)\,y(0)+\left(x+\frac{5}{3}x^3+\frac{7}{3}x^5\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 38

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-6*x*y'[x]-4*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{7x^5}{3}+\frac{5x^3}{3}+x\right)+c_1(3x^4+2x^2+1)$$

20.4 problem 4

Internal problem ID [2468]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln (x) + c_1) \left(1 - 2x + x^2 - \frac{2}{9}x^3 + \frac{1}{36}x^4 - \frac{1}{450}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(4x - 3x^2 + \frac{22}{27}x^3 - \frac{25}{216}x^4 + \frac{137}{13500}x^5 + \mathrm{O}\left(x^6\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 101

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(-\frac{x^5}{450} + \frac{x^4}{36} - \frac{2x^3}{9} + x^2 - 2x + 1\right) + c_2 \left(\frac{137x^5}{13500} - \frac{25x^4}{216} + \frac{22x^3}{27} - 3x^2 + \left(-\frac{x^5}{450} + \frac{x^4}{36} - \frac{2x^3}{9} + x^2 - 2x + 1\right) \log(x) + 4x\right)$$

20.5 problem 5

Internal problem ID [2469]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
Order:=6;
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 42

```
AsymptoticDSolveValue[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^3}{24} - \frac{x}{2} + \frac{1}{x}\right) + c_2\left(\frac{x^4}{120} - \frac{x^2}{6} + 1\right)$$

20.6 problem 6

Internal problem ID [2470]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + 5(1-2x)\,y' - 5y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)+5*(1-2*x)*diff(y(x),x)-5*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2\left(1 + x + \frac{15}{14}x^2 + \frac{125}{126}x^3 + \frac{625}{792}x^4 + \frac{625}{1144}x^5 + \mathrm{O}\left(x^6\right)\right)x^{\frac{3}{2}} + c_1\left(1 + 10x + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 51

```
AsymptoticDSolveValue[2*x*y''[x]+5*(1-2*x)*y'[x]-5*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_2(10x+1)}{x^{3/2}} + c_1\left(\frac{625x^5}{1144} + \frac{625x^4}{792} + \frac{125x^3}{126} + \frac{15x^2}{14} + x + 1\right)$$

20.7 problem 7

Internal problem ID [2471]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 - \frac{1}{4}x^2 + \frac{1}{64}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(\frac{1}{4}x^2 - \frac{3}{128}x^4 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right) + c_2\left(-\frac{3x^4}{128} + \frac{x^2}{4} + \left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right)\log(x)\right)$$

20.8 problem 8

Internal problem ID [2472]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(4x^2+1)\,y''-8y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve((1+4*x^2)*diff(y(x),x$2)-8*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(4x^2+1)\,y(0)+\left(x+\frac{4}{3}x^3-\frac{16}{15}x^5\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[(1+4*x^2)*y''[x]-8*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1(4x^2+1)+c_2\left(-\frac{16x^5}{15}+\frac{4x^3}{3}+x\right)$$

20.9 problem 9

Internal problem ID [2473]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)\right)x + c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^{7/2}}{24} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}}\right) + c_2\left(\frac{x^{9/2}}{120} - \frac{x^{5/2}}{6} + \sqrt{x}\right)$$

20.10 problem 10

Internal problem ID [2474]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4xy'' + 3y' + 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=6;
dsolve(4*x*diff(y(x),x$2)+3*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{4}} \left(1 - \frac{3}{5}x + \frac{1}{10}x^2 - \frac{1}{130}x^3 + \frac{3}{8840}x^4 - \frac{3}{309400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(1 - x + \frac{3}{14}x^2 - \frac{3}{154}x^3 + \frac{3}{3080}x^4 - \frac{9}{292600}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 83

```
AsymptoticDSolveValue[4*x*y''[x]+3*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \sqrt[4]{x} \left(-\frac{3x^5}{309400} + \frac{3x^4}{8840} - \frac{x^3}{130} + \frac{x^2}{10} - \frac{3x}{5} + 1\right) + c_2 \left(-\frac{9x^5}{292600} + \frac{3x^4}{3080} - \frac{3x^3}{154} + \frac{3x^2}{14} - x + 1\right)$$

20.11 problem 11

Internal problem ID [2475]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + \frac{3y'x}{2} - \frac{(x+1)\,y}{2} = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+3/2*x*diff(y(x),x)-1/2*(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{3}{2}}\left(1+\frac{1}{5}x+\frac{1}{70}x^2+\frac{1}{1890}x^3+\frac{1}{83160}x^4+\frac{1}{5405400}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1-x-\frac{1}{2}x^2-\frac{1}{18}x^3-\frac{1}{360}x^4-\frac{1}{12600}x^5}{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 86

```
AsymptoticDSolveValue[x^2*y''[x]+3/2*x*y'[x]-1/2*(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^5}{5405400}+\frac{x^4}{83160}+\frac{x^3}{1890}+\frac{x^2}{70}+\frac{x}{5}+1\right)+\frac{c_2\left(-\frac{x^5}{12600}-\frac{x^4}{360}-\frac{x^3}{18}-\frac{x^2}{2}-x+1\right)}{x}$$

20.12 problem 12

Internal problem ID [2476]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(2-x)\,y' + \left(x^2+2\right)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 65

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(2-x)*diff(y(x),x)+(2+x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x\left(c_1x\left(1 - x + \frac{1}{3}x^2 - \frac{1}{36}x^3 - \frac{7}{720}x^4 + \frac{31}{10800}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(-x + x^2 - \frac{1}{3}x^3 + \frac{1}{36}x^4 + \frac{7}{720}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(1 - x - \frac{1}{2}x^2 + \frac{19}{36}x^3 - \frac{53}{432}x^4 - \frac{1}{675}x^5 + \mathrm{O}\left(x^6\right)\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x^2*y''[x]-x*(2-x)*y'[x]+(2+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{36}x^2\left(x^3 - 12x^2 + 36x - 36\right)\log(x) - \frac{1}{432}x\left(65x^4 - 372x^3 + 648x^2 - 432\right)\right) + c_2\left(-\frac{7x^6}{720} - \frac{x^5}{36} + \frac{x^4}{3} - x^3 + x^2\right)$$

20.13 problem 13

Internal problem ID [2477]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 3y'x + 4(x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*(x+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^2\left((c_2\ln(x) + c_1)\left(1 - 4x + 4x^2 - \frac{16}{9}x^3 + \frac{4}{9}x^4 - \frac{16}{225}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(8x - 12x^2 + \frac{176}{27}x^3 - \frac{50}{27}x^4 + \frac{1096}{3375}x^5 + \mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 116

```
AsymptoticDSolveValue[x^2*y''[x]-3*x*y'[x]+4*(x+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{16x^5}{225} + \frac{4x^4}{9} - \frac{16x^3}{9} + 4x^2 - 4x + 1\right)x^2 + c_2\left(\left(\frac{1096x^5}{3375} - \frac{50x^4}{27} + \frac{176x^3}{27} - 12x^2 + 8x\right)x^2 + \left(-\frac{16x^5}{225} + \frac{4x^4}{9} - \frac{16x^3}{9} + 4x^2 - 4x + 1\right)x^2\log(x)\right)$$

20.14 problem 20

Internal problem ID [2478]

Book: Differential equations and linear algebra, Stephen W. Goode and Scott A Annin. Fourth edition, 2015
Section: Chapter 11, Series Solutions to Linear Differential Equations. Additional problems. Section 11.7. page 788
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(1 - \frac{3}{4x^2}\right) y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)+(1-3/(4*x^2))*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^2 \left(1 - \frac{1}{8}x^2 + \frac{1}{192}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln\left(x\right) \left(x^2 - \frac{1}{8}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-2 + \frac{3}{32}x^4 + \mathrm{O}\left(x^6\right)\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 72

```
AsymptoticDSolveValue[y''[x]+(1-3/(4*x^2))*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(\frac{x^{11/2}}{192} - \frac{x^{7/2}}{8} + x^{3/2}\right) + c_1 \left(\frac{1}{16}x^{3/2}\left(x^2 - 8\right)\log(x) - \frac{5x^4 - 16x^2 - 64}{64\sqrt{x}}\right)$$