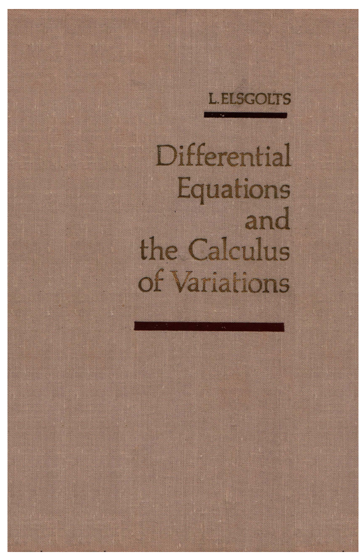
A Solution Manual For

Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.



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1.1 problem Problem 1

Internal problem ID [10764]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\tan(y) - \cot(x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 11

```
dsolve(tan(y(x))-cot(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{c_1}{\cos(x)}\right)$$

✓ Solution by Mathematica

Time used: 3.601 (sec). Leaf size: 19

```
DSolve[Tan[y[x]]-Cot[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{1}{2}c_1 \sec(x)\right)$$
$$y(x) \to 0$$

1.2 problem Problem 2

Internal problem ID [10765]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{12x + 6y - 9 + (5x + 2y - 3)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 44

```
dsolve((12*x+6*y(x)-9)+(5*x+2*y(x)-3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{3}{2} - x \operatorname{RootOf}\left(128\_Z^{25} c_1 x^5 + 640\_Z^{20} c_1 x^5 + 800\_Z^{15} c_1 x^5 - 1\right)^5 - 4x$$

✓ Solution by Mathematica

Time used: 60.071 (sec). Leaf size: 1121

```
DSolve[(12*x+6*y[x]-9)+(5*x+2*y[x]-3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\mathrm{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1^4\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\mathrm{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1^4\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\mathrm{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1^4\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\mathrm{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1^4\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\mathrm{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1^4\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\mathrm{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1^4\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\mathrm{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1^4\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\mathrm{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1^4\right.}$$

$$y(x) \to \frac{1}{2}(3-5x) + \frac{1}{2\mathrm{Root}\left[\#1^{10}\left(11664x^{10}+11664e^{60c_1}\right)-9720\#1^8x^8-1080\#1^7x^7+3105\#1^6x^6+666\#1^5x^5-425\#1^4\right.}$$

$$y(x) \to \frac{1}{2}(3-5x)$$

1.3 problem Problem 3

Internal problem ID [10766]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y'x - y - \sqrt{y^2 + x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)=y(x)+sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{y(x)}{x^2} + \frac{\sqrt{x^2 + y(x)^2}}{x^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.378 (sec). Leaf size: 27

```
DSolve[x*y'[x]==y[x]+Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-c_1}\left(-1 + e^{2c_1}x^2\right)$$

1.4 problem Problem 4

Internal problem ID [10767]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y - x^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)+y(x)=x^3,y(x), singsol=all)
```

$$y(x) = \frac{\frac{x^4}{4} + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 19

```
DSolve[x*y'[x]+y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{4} + \frac{c_1}{x}$$

1.5 problem Problem 5

Internal problem ID [10768]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$-y'x + y - yy'x^2 = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 47

```
dsolve(y(x)-x*diff(y(x),x)=x^2*y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -\frac{c_1 - \sqrt{c_1^2 + x^2}}{x c_1}$$
$$y(x) = -\frac{c_1 + \sqrt{c_1^2 + x^2}}{x c_1}$$

✓ Solution by Mathematica

Time used: 0.418 (sec). Leaf size: 68

```
DSolve[y[x]-x*y'[x]==x^2*y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1 + \sqrt{\frac{1}{x^2}} x \sqrt{1 + c_1 x^2}}{x}$$
$$y(x) \to -\frac{1}{x} + \sqrt{\frac{1}{x^2}} \sqrt{1 + c_1 x^2}$$
$$y(x) \to 0$$

1.6 problem Problem 6

Internal problem ID [10769]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x' + 3x - e^{2t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(x(t),t)+3*x(t)=exp(2*t),x(t), singsol=all)
```

$$x(t) = \left(\frac{e^{5t}}{5} + c_1\right) e^{-3t}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 23

```
DSolve[x'[t]+3*x[t]==Exp[2*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow \frac{e^{2t}}{5} + c_1 e^{-3t}$$

1.7 problem Problem 7

Internal problem ID [10770]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\sin(x)\, y + \cos(x)\, y' - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(y(x)*sin(x)+diff(y(x),x)*cos(x)=1,y(x), singsol=all)
```

$$y(x) = (\tan(x) + c_1)\cos(x)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 13

```
DSolve[y[x]*Sin[x]+y'[x]*Cos[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1 \cos(x)$$

1.8 problem Problem 8

Internal problem ID [10771]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{x-y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=exp(x-y(x)),y(x), singsol=all)
```

$$y(x) = \ln\left(e^x + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.814 (sec). Leaf size: 12

```
DSolve[y'[x]==Exp[x-y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(e^x + c_1\right)$$

1.9 problem Problem 9

Internal problem ID [10772]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x' - x - \sin(t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(x(t),t)=x(t)+sin(t),x(t), singsol=all)
```

$$x(t) = c_1 e^t - \frac{\cos(t)}{2} - \frac{\sin(t)}{2}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 24

```
DSolve[x'[t]==x[t]+Sin[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{\sin(t)}{2} - \frac{\cos(t)}{2} + c_1 e^t$$

1.10 problem Problem 10

Internal problem ID [10773]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{x(\ln(x) - \ln(y))\, y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*(ln(x)-ln(y(x)))*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^{-\operatorname{LambertW}\left(c_1 x\,\mathrm{e}^{-1}\right)-1}$$

✓ Solution by Mathematica

Time used: 5.038 (sec). Leaf size: 37

```
DSolve[x*(Log[x]-Log[y[x]])*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{c_1} W\left(-e^{-1-c_1} x\right)$$
$$y(x) \to 0$$
$$y(x) \to \frac{x}{e}$$

1.11 problem Problem 11

Internal problem ID [10774]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 11.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{xyy'^2 - (y^2 + x^2)\, y' + xy = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x*y(x)*diff(y(x),x)^2-(x^2+y(x)^2)*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x$$
$$y(x) = \sqrt{x^2 + c_1}$$
$$y(x) = -\sqrt{x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 55

```
DSolve[x*y[x]*y'[x]^2-(x^2+y[x]^2)*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x$$
$$y(x) \to -\sqrt{x^2 + 2c_1}$$
$$y(x) \to \sqrt{x^2 + 2c_1}$$
$$y(x) \to -x$$
$$y(x) \to x$$

1.12 problem Problem 12

Internal problem ID [10775]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 12.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 9y^4 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^2=9*y(x)^4,y(x), singsol=all)
```

$$y(x) = \frac{1}{-3x + c_1}$$
$$y(x) = \frac{1}{3x + c_1}$$

✓ Solution by Mathematica

Time used: 0.17 (sec). Leaf size: 34

```
DSolve[y'[x]^2==9*y[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{3x + c_1}$$
$$y(x) \to \frac{1}{3x - c_1}$$
$$y(x) \to 0$$

1.13 problem Problem 13

Internal problem ID [10776]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{x' - e^{\frac{x}{t}} - \frac{x}{t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(x(t),t)=exp(x(t)/t)+x(t)/t,x(t), singsol=all)
```

$$x(t) = t \ln \left(-\frac{1}{\ln(t) + c_1}\right)$$

✓ Solution by Mathematica

Time used: 0.349 (sec). Leaf size: 18

```
DSolve[x'[t]==Exp[x[t]/t]+x[t]/t,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -t \log(-\log(t) - c_1)$$

1.14 problem Problem 14

Internal problem ID [10777]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 14.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x^2 + {y'}^2 - 1 = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 43

```
dsolve(x^2+diff(y(x),x)^2=1,y(x), singsol=all)
```

$$y(x) = \frac{x\sqrt{-x^2+1}}{2} + \frac{\arcsin(x)}{2} + c_1$$
$$y(x) = -\frac{x\sqrt{-x^2+1}}{2} - \frac{\arcsin(x)}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 81

```
DSolve[x^2+y'[x]^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{1-x^2}x - \cot^{-1}\left(\frac{x+1}{\sqrt{1-x^2}}\right) + c_1$$
$$y(x) \to -\frac{1}{2}\sqrt{1-x^2}x + \cot^{-1}\left(\frac{x+1}{\sqrt{1-x^2}}\right) + c_1$$

1.15 problem Problem 15

Internal problem ID [10778]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - y'x - \frac{1}{y} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(y(x)=x*diff(y(x),x)+1/y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x^2 + 1}$$
$$y(x) = -\sqrt{c_1 x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.294 (sec). Leaf size: 53

```
DSolve[y[x]==x*y'[x]+1/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{1 + e^{2c_1} x^2}$$
$$y(x) \to \sqrt{1 + e^{2c_1} x^2}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

1.16 problem Problem 16

Internal problem ID [10779]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 16.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x - {y'}^3 + y' - 2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 231

```
dsolve(x=diff(y(x),x)^3-diff(y(x),x)+2,y(x), singsol=all)
```

$$y(x) = \int \frac{i\left(i\left(-216+108x+12\sqrt{81x^2-324x+312}\right)^{\frac{2}{3}} - \left(-216+108x+12\sqrt{81x^2-324x+312}\right)^{\frac{2}{3}}\sqrt{3}+12i}{12\left(-216+108x+12\sqrt{81x^2-324x+312}\right)^{\frac{1}{3}}} + c_1$$

$$y(x) = \int \frac{i\left(\left(-216+108x+12\sqrt{81x^2-324x+312}\right)^{\frac{2}{3}}\sqrt{3}-12\sqrt{3}+i\left(-216+108x+12\sqrt{81x^2-324x+312}\right)}{12\left(-216+108x+12\sqrt{81x^2-324x+312}\right)^{\frac{1}{3}}} + c_1$$

$$y(x) = \int \frac{\left(-216+108x+12\sqrt{81x^2-324x+312}\right)^{\frac{2}{3}}+12}{6\left(-216+108x+12\sqrt{81x^2-324x+312}\right)^{\frac{1}{3}}}dx + c_1$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x==y'[x]^3-y'[x]+2,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.17 problem Problem 17

Internal problem ID [10780]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y' - \frac{y}{x + y^3} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 260

```
dsolve(diff(y(x),x)=y(x)/(x+y(x)^3),y(x), singsol=all)
```

$$y(x) = \frac{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}{3} - \frac{2c_1}{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}{6} + \frac{c_1}{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(27x+3\sqrt{24c_1^3+81x^2}\right)^{\frac{1}{3}}}{3} + \frac{2c_1}{\left(27x+3\sqrt{24c_1^3+81x^2}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}{6} + \frac{c_1}{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(27x+3\sqrt{24c_1^3+81x^2}\right)^{\frac{1}{3}}}{3} + \frac{2c_1}{\left(27x+3\sqrt{24c_1^3+81x^2}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 1.733 (sec). Leaf size: 227

```
DSolve[y'[x]==y[x]/(x+y[x]^3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2\ 3^{2/3}c_1 - \sqrt[3]{3}\left(-9x + \sqrt{81x^2 + 24c_1{}^3}\right)^{2/3}}{3\sqrt[3]{-9x + \sqrt{81x^2 + 24c_1{}^3}}}$$

$$y(x) \to \frac{-(-1)^{2/3}\left(-9x + \sqrt{81x^2 + 24c_1{}^3}\right)^{2/3} - 2\sqrt[3]{-3}c_1}{3^{2/3}\sqrt[3]{-9x + \sqrt{81x^2 + 24c_1{}^3}}}$$

$$y(x) \to \frac{2\sqrt[3]{-3}\left(-9x + \sqrt{81x^2 + 24c_1{}^3}\right)^{2/3} + 4(-3)^{2/3}c_1}{6\sqrt[3]{-9x + \sqrt{81x^2 + 24c_1{}^3}}}$$

$$y(x) \to 0$$

1.18 problem Problem 18

Internal problem ID [10781]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 18.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y - {y'}^{4} + {y'}^{3} + 2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 327

```
dsolve(y(x)=diff(y(x),x)^4-diff(y(x),x)^3-2,y(x), singsol=all)
```

$$y(x) = -2$$

$$y(x) = \frac{\left(27 - 192c_1 + 192x + 24\sqrt{64c_1^2 - 128c_1x + 64x^2 - 18c_1 + 18x}\right)^{\frac{8}{3}} + 4\left(27 - 192c_1 + 192x + 24\sqrt{64c_1^2 - \right.}{}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]==y'[x]^4-y'[x]^3-2,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.19 problem Problem 26

Internal problem ID [10782]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 26.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + y^2 - 4 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)^2+y(x)^2=4,y(x), singsol=all)
```

$$y(x) = -2$$
$$y(x) = 2$$
$$y(x) = -2\sin(c_1 - x)$$
$$y(x) = 2\sin(c_1 - x)$$

✓ Solution by Mathematica

Time used: 0.197 (sec). Leaf size: 43

```
DSolve[y'[x]^2+y[x]^2==4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\cos(x + c_1)$$
$$y(x) \to 2\cos(x - c_1)$$
$$y(x) \to -2$$
$$y(x) \to 2$$
$$y(x) \to \text{Interval}[\{-2, 2\}]$$

1.20 problem Problem 28

Internal problem ID [10783]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$
y' - \frac{2y - x - 4}{2x - y + 5} = 0
$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 184

```
dsolve(diff(y(x),x)=(2*y(x)-x-4)/(2*x-y(x)+5),y(x), singsol=all)
```

$$
y(x) = 1 + \frac{(x+2)\left(-c_1^2 - c_1^2\left(-\frac{\left(27(x+2)c_1+3\sqrt{3}\sqrt{27c_1^2(x+2)^2-1}\right)^{\frac{1}{3}}}{6c_1(x+2)} - \frac{1}{2c_1(x+2)\left(27(x+2)c_1+3\sqrt{3}\sqrt{27c_1^2(x+2)^2-1}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(27(x\right.}{}\right.}{}\right.\right.}{c_1^2}
$$

✓ Solution by Mathematica

Time used: 60.185 (sec). Leaf size: 628

```
DSolve[y'[x]==(2*y[x]-x-4)/(2*x-y[x]+5),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x + \frac{3(x+2)}{\frac{1}{\sqrt[3]{-e^{\frac{3c_1}{4}}(x+2)^4 + 2e^{\frac{3c_1}{8}}(x+2)^2 + \sqrt{e^{\frac{3c_1}{8}}(x+2)^2\left(-1+e^{\frac{3c_1}{8}}(x+2)^2\right)^3} - 1}} - \frac{\sqrt[3]{-e^{\frac{3c_1}{4}}(x+2)^4 + 2e}}{}} + 5$$

$$y(x) \to 2x + \frac{6(x+2)}{\frac{-1-i\sqrt{3}}{\sqrt[3]{-e^{\frac{3c_1}{4}}(x+2)^4 + 2e^{\frac{3c_1}{8}}(x+2)^2 + \sqrt{e^{\frac{3c_1}{8}}(x+2)^2\left(-1+e^{\frac{3c_1}{8}}(x+2)^2\right)^3} - 1}} + \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-e^{\frac{3c_1}{4}}(x+2}}{}} + 5$$

$$y(x) \to 2x + \frac{6(x+2)}{\frac{i\left(\sqrt{3}+i\right)}{\sqrt[3]{-e^{\frac{3c_1}{4}}(x+2)^4 + 2e^{\frac{3c_1}{8}}(x+2)^2 + \sqrt{e^{\frac{3c_1}{8}}(x+2)^2\left(-1+e^{\frac{3c_1}{8}}(x+2)^2\right)^3} - 1}} + \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-e^{\frac{3c_1}{4}}(x+2}}{}} + 5$$

1.21 problem Problem 29

Internal problem ID [10784]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Bernoulli]`

$$\boxed{y' - \frac{y}{x+1} + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)-y(x)/(1+x)+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{2+2x}{x^2+2c_1+2x}$$

✓ Solution by Mathematica

Time used: 0.192 (sec). Leaf size: 27

```
DSolve[y'[x]-y[x]/(1+x)+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2(x+1)}{x(x+2)+2c_1}$$

$$y(x) \to 0$$

1.22 problem Problem 30

Internal problem ID [10785]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - x - y^2 = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve([diff(y(x),x)=x+y(x)^2,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{3}\ \mathrm{AiryAi}(1, -x) + \mathrm{AiryBi}(1, -x)}{\sqrt{3}\ \mathrm{AiryAi}(-x) + \mathrm{AiryBi}(-x)}$$

✓ Solution by Mathematica

Time used: 1.248 (sec). Leaf size: 36

```
DSolve[{y'[x]==x+y[x]^2,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2\, {}_0\tilde{F}_1\left(;\frac{5}{3};-\frac{x^3}{9}\right)}{3\, {}_0\tilde{F}_1\left(;\frac{2}{3};-\frac{x^3}{9}\right)}$$

1.23 problem Problem 31

Internal problem ID [10786]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$\boxed{y' - y^3x - x^2 = 0}$$

With initial conditions

$$[y(0) = 0]$$

✗ Solution by Maple

```
dsolve([diff(y(x),x)=x*y(x)^3+x^2,y(0) = 0],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]==x*y[x]^3+x^2,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.24 problem Problem 35

Internal problem ID [10787]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - x^2 + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(diff(y(x),x)=x^2-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x\left(\operatorname{BesselI}\left(-\frac{3}{4}, \frac{x^2}{2}\right) c_1 - \operatorname{BesselK}\left(\frac{3}{4}, \frac{x^2}{2}\right)\right)}{\operatorname{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_1 + \operatorname{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.123 (sec). Leaf size: 103

```
DSolve[y'[x]==x^2-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{ix\left(\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{ix^2}{2}\right) - c_1 \operatorname{BesselJ}\left(\frac{3}{4}, \frac{ix^2}{2}\right)\right)}{\operatorname{BesselJ}\left(\frac{1}{4}, \frac{ix^2}{2}\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}$$

$$y(x) \to \frac{x \operatorname{BesselI}\left(\frac{3}{4}, \frac{x^2}{2}\right)}{\operatorname{BesselI}\left(-\frac{1}{4}, \frac{x^2}{2}\right)}$$

1.25 problem Problem 36

Internal problem ID [10788]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{2x + 2y - 1 + (x + y - 2) y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve((2*x+2*y(x)-1)+(x+y(x)-2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -x - 3 \operatorname{LambertW}\left(-\frac{e^{\frac{x}{3}} c_1 e^{-\frac{1}{3}}}{3}\right) - 1$$

✓ Solution by Mathematica

Time used: 3.875 (sec). Leaf size: 35

```
DSolve[(2*x+2*y[x]-1)+(x+y[x]-2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3W\left(-e^{\frac{x}{3}-1+c_1}\right) - x - 1$$
$$y(x) \to -x - 1$$

1.26 problem Problem 37

Internal problem ID [10789]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 37.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 - y' \mathrm{e}^{2x} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^3-diff(y(x),x)*exp(2*x)=0,y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{x} + c_1$$
$$y(x) = \mathrm{e}^{x} + c_1$$
$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 29

```
DSolve[y'[x]^3-y'[x]*Exp[2*x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$
$$y(x) \to -e^{x} + c_1$$
$$y(x) \to e^{x} + c_1$$

1.27 problem Problem 39

Internal problem ID [10790]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 39.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y - 5y'x + y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 87

```
dsolve(y(x)=5*x*diff(y(x),x)-diff(y(x),x)^2,y(x), singsol=all)
```

$$-\frac{c_1}{\left(\frac{5x}{2} - \frac{\sqrt{25x^2-4y(x)}}{2}\right)^{\frac{5}{4}}} + \frac{4x}{9} + \frac{\sqrt{25x^2 - 4y(x)}}{9} = 0$$
$$-\frac{c_1}{\left(\frac{5x}{2} + \frac{\sqrt{25x^2-4y(x)}}{2}\right)^{\frac{5}{4}}} + \frac{4x}{9} - \frac{\sqrt{25x^2 - 4y(x)}}{9} = 0$$

✓ Solution by Mathematica

Time used: 41.997 (sec). Leaf size: 2238

```
DSolve[y[x]==5*x*y'[x]-y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

1.28 problem Problem 40

Internal problem ID [10791]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - x + y^2 = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 37

```
dsolve([diff(y(x),x)=x-y(x)^2,y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{AiryBi}(1,1)\,\mathrm{AiryAi}(1,x) - \mathrm{AiryBi}(1,x)\,\mathrm{AiryAi}(1,1)}{\mathrm{AiryBi}(1,1)\,\mathrm{AiryAi}(x) - \mathrm{AiryBi}(x)\,\mathrm{AiryAi}(1,1)}$$

✓ Solution by Mathematica

Time used: 0.14 (sec). Leaf size: 119

```
DSolve[{y'[x]==x-y[x]^2,{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{i\sqrt{x}\left(\mathrm{BesselJ}\left(\frac{2}{3},\frac{2i}{3}\right)\mathrm{BesselJ}\left(-\frac{2}{3},\frac{2}{3}ix^{3/2}\right) - \mathrm{BesselJ}\left(-\frac{2}{3},\frac{2i}{3}\right)\mathrm{BesselJ}\left(\frac{2}{3},\frac{2}{3}ix^{3/2}\right)\right)}{\mathrm{BesselJ}\left(-\frac{2}{3},\frac{2i}{3}\right)\mathrm{BesselJ}\left(-\frac{1}{3},\frac{2}{3}ix^{3/2}\right) + \mathrm{BesselJ}\left(\frac{2}{3},\frac{2i}{3}\right)\mathrm{BesselJ}\left(\frac{1}{3},\frac{2}{3}ix^{3/2}\right)}$$

1.29 problem Problem 42

Internal problem ID [10792]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - (x - 5y)^{\frac{1}{3}} - 2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 80

```
dsolve(diff(y(x),x)=(x-5*y(x))^(1/3)+2,y(x), singsol=all)
```

$$x + \frac{81 \ln \left(729 + 125x - 625y(x)\right)}{125} - \frac{27(x - 5y(x))^{\frac{1}{3}}}{25} + \frac{162 \ln \left(5(x - 5y(x))^{\frac{1}{3}} + 9\right)}{125} - \frac{81 \ln \left(25(x - 5y(x))^{\frac{2}{3}} - 45(x - 5y(x))^{\frac{1}{3}} + 81\right)}{125} + \frac{3(x - 5y(x))^{\frac{2}{3}}}{10} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.218 (sec). Leaf size: 70

```
DSolve[y'[x]==(x-5*y[x])^(1/3)+2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[5y(x) + 5\left(-y(x) + \frac{3}{50}(x - 5y(x))^{2/3} - \frac{27}{125}\sqrt[3]{x - 5y(x)} + \frac{243}{625}\log\left(5\sqrt[3]{x - 5y(x)} + 9\right) + \frac{x}{5}\right) = c_1, y(x)\right]$$

1.30 problem Problem 43

Internal problem ID [10793]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{(x-y)\,y-x^2y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-y(x))*y(x)-x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{x}{\ln(x)+c_1}$$

✓ Solution by Mathematica

Time used: 0.152 (sec). Leaf size: 19

```
DSolve[(x-y[x])*y[x]-x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{x}{\log(x)+c_1}$$
$$y(x)\to 0$$

1.31 problem Problem 45

Internal problem ID [10794]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x' + 5x - 10t - 2 = 0}$$

With initial conditions

$$[x(1) = 2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve([diff(x(t),t)+5*x(t)=10*t+2,x(1) = 2],x(t), singsol=all)
```

$$x(t) = 2t$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 8

```
DSolve[{x'[t]+5*x[t]==10*t+2,{x[1]==2}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 2t$$

1.32 problem Problem 46

Internal problem ID [10795]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{x' - \frac{x}{t} - \frac{x^2}{t^3} = 0}$$

With initial conditions

$$[x(2) = 4]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 7

```
dsolve([diff(x(t),t)=x(t)/t+x(t)^2/t^3,x(2) = 4],x(t), singsol=all)
```

$$x(t) = t^2$$

✓ Solution by Mathematica

Time used: 0.171 (sec). Leaf size: 8

```
DSolve[{x'[t]==x[t]/t+x[t]^2/t^3,{x[2]==4}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to t^2$$

1.33 problem Problem 47

Internal problem ID [10796]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 47.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - y'x - y'^2 = 0}$$

With initial conditions

$$[y(2) = -1]$$

✓ Solution by Maple

Time used: 0.375 (sec). Leaf size: 17

```
dsolve([y(x)=x*diff(y(x),x)+diff(y(x),x)^2,y(2) = -1],y(x), singsol=all)
```

$$y(x) = 1 - x$$
$$y(x) = -\frac{x^2}{4}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 21

```
DSolve[{y[x]==x*y'[x]+y'[x]^2,{y[2]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 - x$$
$$y(x) \to -\frac{x^2}{4}$$

1.34 problem Problem 48

Internal problem ID [10797]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 48.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - y'x - y'^2 = 0}$$

With initial conditions

$$[y(1) = -1]$$

✓ Solution by Maple

Time used: 0.343 (sec). Leaf size: 66

```
dsolve([y(x)=x*diff(y(x),x)+diff(y(x),x)^2,y(1) = -1],y(x), singsol=all)
```

$$y(x) = -\frac{1}{2} + \frac{i(x-1)\sqrt{3}}{2} - \frac{x}{2}$$
$$y(x) = \frac{(1 + i\sqrt{3})(i\sqrt{3} - 2x + 1)}{4}$$
$$y(x) = \frac{(i\sqrt{3} - 1)(i\sqrt{3} + 2x - 1)}{4}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 38

```
DSolve[{y[x]==x*y'[x]+y'[x]^2,{y[1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (-1)^{2/3} - \sqrt[3]{-1}x$$
$$y(x) \to \sqrt[3]{-1}(\sqrt[3]{-1}x - 1)$$

1.35 problem Problem 49

Internal problem ID [10798]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{3x - 4y - 2}{3x - 4y - 3} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=(3*x-4*y(x)-2)/(3*x-4*y(x)-3),y(x), singsol=all)
```

$$y(x) = \frac{3x}{4} + \text{LambertW}\left(\frac{\mathrm{e}^{-\frac{1}{4}}\mathrm{e}^{\frac{x}{4}}c_1}{4}\right) + \frac{1}{4}$$

✓ Solution by Mathematica

Time used: 3.882 (sec). Leaf size: 41

```
DSolve[y'[x]==(3*x-4*y[x]-2)/(3*x-4*y[x]-3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W\left(-e^{\frac{x}{4}-1+c_1}\right) + \frac{3x}{4} + \frac{1}{4}$$

$$y(x) \to \frac{1}{4}(3x + 1)$$

1.36 problem Problem 50

Internal problem ID [10799]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x' - x\cot(t) - 4\sin(t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(x(t),t)-x(t)*cot(t)=4*sin(t),x(t), singsol=all)
```

$$x(t) = (4t + c_1)\sin(t)$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 14

```
DSolve[x'[t]-x[t]*Cot[t]==4*Sin[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to (4t + c_1)\sin(t)$$

1.37 problem Problem 51

Internal problem ID [10800]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 51.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y - x^2 - 2y'x - \frac{y'^2}{2} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 95

```
dsolve(y(x)=x^2+2*diff(y(x),x)*x+(diff(y(x),x)^2)/2,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= -x^2 \\
y(x) &= -\frac{3x^2}{2} - x(-x - c_1) + \frac{c_1^2}{2} \\
y(x) &= -\frac{3x^2}{2} - x(-x + c_1) + \frac{c_1^2}{2} \\
y(x) &= -\frac{3x^2}{2} + x(x - c_1) + \frac{c_1^2}{2} \\
y(x) &= -\frac{3x^2}{2} + x(x + c_1) + \frac{c_1^2}{2}
\end{aligned}
$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]==x^2+2*y'[x]*x+(y'[x]^2)/2,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.38 problem Problem 52

Internal problem ID [10801]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{y' - \frac{3y}{x} + x^3 y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)-3*y(x)/x+x^3*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{7x^3}{x^7 + 7c_1}$$

✓ Solution by Mathematica

Time used: 0.173 (sec). Leaf size: 25

```
DSolve[y'[x]-3*y[x]/x+x^3*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{7x^3}{x^7 + 7c_1}$$
$$y(x) \to 0$$

1.39 problem Problem 53

Internal problem ID [10802]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 53.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y\left(y'^2+1\right)-a=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 1583

```
dsolve(y(x)*(1+diff(y(x),x)^2)=a,y(x), singsol=all)
```

$$y(x) = a$$

$$y(x) = \frac{\tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2\_Z^2 + 4\tan(\_Z)^2 c_1 a\_Z - 4\tan(\_Z)^2 ax\_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c\right.\right.}{} + \tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2\_Z^2 + 4\tan(\_Z)^2 c_1 a\_Z - 4\tan(\_Z)^2 ax\_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c_1 x + 4\tan(\_Z)^2 x^2 + \_Z^2 a^2 + 4c_1\_Za - 4\_Zax + 4c_1^2 - 8c_1 x - a^2 + 4x^2\right)\right) c_1 - \tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2\_Z^2 + 4\tan(\_Z)^2 c_1 a\_Z - 4\tan(\_Z)^2 ax\_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c_1 x + 4\tan(\_Z)^2 x^2 + \_Z^2 a^2 + 4c_1\_Za - 4\_Zax + 4c_1^2 - 8c_1 x - a^2 + 4x^2\right)\right) x + \frac{a}{2}$$

$$y(x) = \frac{\tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2\_Z^2 + 4\tan(\_Z)^2 c_1 a\_Z - 4\tan(\_Z)^2 ax\_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c\right.\right.}{} + \tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2\_Z^2 + 4\tan(\_Z)^2 c_1 a\_Z - 4\tan(\_Z)^2 ax\_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c_1 x + 4\tan(\_Z)^2 x^2 + \_Z^2 a^2 + 4c_1\_Za - 4\_Zax + 4c_1^2 - 8c_1 x - a^2 + 4x^2\right)\right) c_1 - \tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2\_Z^2 + 4\tan(\_Z)^2 c_1 a\_Z - 4\tan(\_Z)^2 ax\_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c_1 x + 4\tan(\_Z)^2 x^2 + \_Z^2 a^2 + 4c_1\_Za - 4\_Zax + 4c_1^2 - 8c_1 x - a^2 + 4x^2\right)\right) x + \frac{a}{2}$$

$$y(x) = \frac{\tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2\_Z^2 - 4\tan(\_Z)^2 c_1 a\_Z + 4\tan(\_Z)^2 ax\_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c\right.\right.}{} - \tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2\_Z^2 - 4\tan(\_Z)^2 c_1 a\_Z + 4\tan(\_Z)^2 ax\_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c_1 x + 4\tan(\_Z)^2 x^2 + \_Z^2 a^2 - 4c_1\_Za + 4\_Zax + 4c_1^2 - 8c_1 x - a^2 + 4x^2\right)\right) c_1 + \tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2\_Z^2 - 4\tan(\_Z)^2 c_1 a\_Z + 4\tan(\_Z)^2 ax\_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c_1 x + 4\tan(\_Z)^2 x^2 + \_Z^2 a^2 - 4c_1\_Za + 4\_Zax + 4c_1^2 - 8c_1 x - a^2 + 4x^2\right)\right) x + \frac{a}{2}$$

$$y(x) = \frac{\tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2\_Z^2 - 4\tan(\_Z)^2 c_1 a\_Z + 4\tan(\_Z)^2 ax\_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c\right.\right.}{} - \tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2\_Z^2 - 4\tan(\_Z)^2 c_1 a\_Z + 4\tan(\_Z)^2 ax\_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c_1 x + 4\tan(\_Z)^2 x^2 + \_Z^2 a^2 - 4c_1\_Za + 4\_Zax + 4c_1^2 - 8c_1 x - a^2 + 4x^2\right)\right) c_1 + \tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2\_Z^2 - 4\tan(\_Z)^2 c_1 a\_Z + 4\tan(\_Z)^2 ax\_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c_1 x + 4\tan(\_Z)^2 x^2 + \_Z^2 a^2 - 4c_1\_Za + 4\_Zax + 4c_1^2 - 8c_1 x - a^2 + 4x^2\right)\right) x + \frac{a}{2}$$

✓ Solution by Mathematica

Time used: 0.476 (sec). Leaf size: 106

```
DSolve[y[x]*(1+y'[x]^2)==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[a \arctan\left(\frac{\sqrt{\#1}}{\sqrt{a-\#1}}\right) - \sqrt{\#1}\sqrt{a-\#1}\&\right][-x+c_1]$$

$$y(x) \to \text{InverseFunction}\left[a \arctan\left(\frac{\sqrt{\#1}}{\sqrt{a-\#1}}\right) - \sqrt{\#1}\sqrt{a-\#1}\&\right][x+c_1]$$

$$y(x) \to a$$

1.40 problem Problem 54

Internal problem ID [10803]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{x^2 - y + \left(y^2x^2 + x\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 508

```
dsolve((x^2-y(x))+(x^2*y(x)^2+x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(-12c_1x - 12x^2 + 4\sqrt{\frac{9c_1^2x^3+18c_1x^4+9x^5+4}{x}}\right)x^2\right)^{\frac{1}{3}}}{2x} - \frac{2}{\left(\left(-12c_1x - 12x^2 + 4\sqrt{\frac{9c_1^2x^3+18c_1x^4+9x^5+4}{x}}\right)x^2\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(\left(-12c_1x - 12x^2 + 4\sqrt{\frac{9c_1^2x^3+18c_1x^4+9x^5+4}{x}}\right)x^2\right)^{\frac{1}{3}}}{4x} + \frac{1}{\left(\left(-12c_1x - 12x^2 + 4\sqrt{\frac{9c_1^2x^3+18c_1x^4+9x^5+4}{x}}\right)x^2\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(\left(-12c_1x-12x^2+4\sqrt{\frac{9c_1^2x^3+18c_1x^4+9x^5+4}{x}}\right)x^2\right)^{\frac{1}{3}}}{2x} + \frac{2}{\left(\left(-12c_1x-12x^2+4\sqrt{\frac{9c_1^2x^3+18c_1x^4+9x^5+4}{x}}\right)x^2\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(\left(-12c_1x - 12x^2 + 4\sqrt{\frac{9c_1^2x^3+18c_1x^4+9x^5+4}{x}}\right)x^2\right)^{\frac{1}{3}}}{4x} + \frac{1}{\left(\left(-12c_1x - 12x^2 + 4\sqrt{\frac{9c_1^2x^3+18c_1x^4+9x^5+4}{x}}\right)x^2\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(\left(-12c_1x-12x^2+4\sqrt{\frac{9c_1^2x^3+18c_1x^4+9x^5+4}{x}}\right)x^2\right)^{\frac{1}{3}}}{2x} + \frac{2}{\left(\left(-12c_1x-12x^2+4\sqrt{\frac{9c_1^2x^3+18c_1x^4+9x^5+4}{x}}\right)x^2\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 48.634 (sec). Leaf size: 319

```
DSolve[(x^2-y[x])+(x^2*y[x]^2+x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2\sqrt[3]{2}x + 2^{2/3}\left(3x^3(-x+c_1) + \sqrt{x^3\left(4+9x^3(x-c_1)^2\right)}\right)^{2/3}}{2x\sqrt[3]{3x^3(-x+c_1) + \sqrt{x^3\left(4+9x^3(x-c_1)^2\right)}}}$$

$$y(x) \to \frac{2\sqrt[3]{-2}x + (-2)^{2/3}\left(3x^3(-x+c_1) + \sqrt{x^3\left(4+9x^3(x-c_1)^2\right)}\right)^{2/3}}{2x\sqrt[3]{3x^3(-x+c_1) + \sqrt{x^3\left(4+9x^3(x-c_1)^2\right)}}}$$

$$y(x) \to \frac{-\sqrt[3]{-2}\left(3x^3(-x+c_1) + \sqrt{x^3\left(4+9x^3(x-c_1)^2\right)}\right)^{2/3} - i\sqrt{3}x + x}{2^{2/3}x\sqrt[3]{3x^3(-x+c_1) + \sqrt{x^3\left(4+9x^3(x-c_1)^2\right)}}}$$

1.41 problem Problem 55

Internal problem ID [10804]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$3y^2 - x + 2y\left(y^2 - 3x\right) y' = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 105

```
dsolve((3*y(x)^2-x)+(2*y(x))*(y(x)^2-3*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2c_1 - 2\sqrt{c_1^2 - 8c_1 x} - 4x}}{2}$$
$$y(x) = \frac{\sqrt{2c_1 - 2\sqrt{c_1^2 - 8c_1 x} - 4x}}{2}$$
$$y(x) = -\frac{\sqrt{2c_1 + 2\sqrt{c_1^2 - 8c_1 x} - 4x}}{2}$$
$$y(x) = \frac{\sqrt{2c_1 + 2\sqrt{c_1^2 - 8c_1 x} - 4x}}{2}$$

✓ Solution by Mathematica

Time used: 11.815 (sec). Leaf size: 185

```
DSolve[(3*y[x]^2-x)+(2*y[x])*(y[x]^2-3*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-2x - e^{\frac{c_1}{2}}\sqrt{8x + e^{c_1}} - e^{c_1}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-2x - e^{\frac{c_1}{2}}\sqrt{8x + e^{c_1}} - e^{c_1}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{-2x + e^{\frac{c_1}{2}}\sqrt{8x + e^{c_1}} - e^{c_1}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-2x + e^{\frac{c_1}{2}}\sqrt{8x + e^{c_1}} - e^{c_1}}}{\sqrt{2}}$$

1.42 problem Problem 56

Internal problem ID [10805]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{(x-y)\,y-x^2y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-y(x))*y(x)- x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{x}{\ln(x)+c_1}$$

✓ Solution by Mathematica

Time used: 0.16 (sec). Leaf size: 19

```
DSolve[(x-y[x])*y[x]- x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x}{\log(x)+c_1}$$
$$y(x)\to 0$$

1.43 problem Problem 57

Internal problem ID [10806]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$y' - \frac{x+y-3}{1-x+y} = 0$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)= (x+y(x)-3)/(1-x+y(x)),y(x), singsol=all)
```

$$y(x) = 1 - \frac{-(x-2)\,c_1 + \sqrt{2\,(x-2)^2\,c_1^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.134 (sec). Leaf size: 55

```
DSolve[y'[x]== (x+y[x]-3)/(1-x+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - i\sqrt{-2(x-4)x - 1 - c_1} - 1$$
$$y(x) \to x + i\sqrt{-2(x-4)x - 1 - c_1} - 1$$

1.44 problem Problem 58

Internal problem ID [10807]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y'x - y^2 \ln(x) + y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)-y(x)^2*ln(x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{1 + c_1 x + \ln(x)}$$

✓ Solution by Mathematica

Time used: 0.148 (sec). Leaf size: 20

```
DSolve[x*y'[x]-y[x]^2*Log[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\log(x) + c_1 x + 1}$$
$$y(x) \to 0$$

1.45 problem Problem 59

Internal problem ID [10808]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2-1)\,y'+2xy-\cos(x)=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x^2-1)*diff(y(x),x)+2*x*y(x)-cos(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\sin(x)+c_1}{(x-1)(x+1)}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 18

```
DSolve[(x^2-1)*y'[x]+2*x*y[x]-Cos[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sin(x)+c_1}{x^2-1}$$

1.46 problem Problem 60

Internal problem ID [10809]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(4y + 2x + 3) y' - 2y - x - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve((4*y(x)+2*x+3)*diff(y(x),x)-2*y(x)-x-1=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + \frac{\mathrm{LambertW}\left(\mathrm{e}^{5}\mathrm{e}^{8x}c_1\right)}{8} - \frac{5}{8}$$

✓ Solution by Mathematica

Time used: 4.975 (sec). Leaf size: 39

```
DSolve[(4*y[x]+2*x+3)*y'[x]-2*y[x]-x-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(W\left(-e^{8x-1+c_1}\right) - 4x - 5\right)$$

$$y(x) \to \frac{1}{8}(-4x - 5)$$

1.47 problem Problem 61

Internal problem ID [10810]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(-x+y^2)\,y'-y+x^2=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 402

```
dsolve((y(x)^2-x)*diff(y(x),x)-y(x)+x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + 6c_1x^3 - 4x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{2x}{\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + 6c_1x^3 - 4x^3 + 9c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + 6c_1x^3 - 4x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x}{\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + 6c_1x^3 - 4x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(-4x^3-12c_1+4\sqrt{x^6+6c_1x^3-4x^3+9c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{2x}{\left(-4x^3-12c_1+4\sqrt{x^6+6c_1x^3-4x^3+9c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + 6c_1x^3 - 4x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x}{\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + 6c_1x^3 - 4x^3 + 9c_1^2}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(-4x^3-12c_1+4\sqrt{x^6+6c_1x^3-4x^3+9c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{2x}{\left(-4x^3-12c_1+4\sqrt{x^6+6c_1x^3-4x^3+9c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 3.915 (sec). Leaf size: 326

```
DSolve[(y[x]^2-x)*y'[x]-y[x]+x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x + \sqrt[3]{2}\left(x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9c_1{}^2} + 3c_1\right)^{2/3}}{2^{2/3}\sqrt[3]{x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9c_1{}^2} + 3c_1}}$$

$$y(x) \to \frac{2^{2/3}(1 - i\sqrt{3})\left(x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9c_1{}^2} + 3c_1\right)^{2/3} + \sqrt[3]{2}(2 + 2i\sqrt{3})\,x}{4\sqrt[3]{x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9c_1{}^2} + 3c_1}}$$

$$y(x) \to \frac{2^{2/3}(1 + i\sqrt{3})\left(x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9c_1{}^2} + 3c_1\right)^{2/3} + \sqrt[3]{2}(2 - 2i\sqrt{3})\,x}{4\sqrt[3]{x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9c_1{}^2} + 3c_1}}$$

1.48 problem Problem 62

Internal problem ID [10811]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{(y^2 - x^2)\, y' + 2xy = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve((y(x)^2-x^2)*diff(y(x),x)+2*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{-1 + \sqrt{-4c_1^2x^2 + 1}}{2c_1}$$
$$y(x) = \frac{1 + \sqrt{-4c_1^2x^2 + 1}}{2c_1}$$

✓ Solution by Mathematica

Time used: 1.118 (sec). Leaf size: 66

```
DSolve[(y[x]^2-x^2)*y'[x]+2*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(e^{c_1} - \sqrt{-4x^2 + e^{2c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{-4x^2 + e^{2c_1}} + e^{c_1}\right)$$
$$y(x) \to 0$$

1.49 problem Problem 63

Internal problem ID [10812]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, _Bernoulli]`

$$\boxed{3y'y^2x + y^3 - 2x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 99

```
dsolve(3*x*y(x)^2*diff(y(x),x)+y(x)^3-2*x=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(x^2+c_1\right)x^2\right)^{\frac{1}{3}}}{x}$$

$$y(x) = -\frac{\left(\left(x^2+c_1\right)x^2\right)^{\frac{1}{3}}}{2x} - \frac{i\sqrt{3}\left(\left(x^2+c_1\right)x^2\right)^{\frac{1}{3}}}{2x}$$

$$y(x) = -\frac{\left(\left(x^2+c_1\right)x^2\right)^{\frac{1}{3}}}{2x} + \frac{i\sqrt{3}\left(\left(x^2+c_1\right)x^2\right)^{\frac{1}{3}}}{2x}$$

✓ Solution by Mathematica

Time used: 0.242 (sec). Leaf size: 72

```
DSolve[3*x*y[x]^2*y'[x]+y[x]^3-2*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{x^2+c_1}}{\sqrt[3]{x}}$$

$$y(x) \to -\frac{\sqrt[3]{-1}\sqrt[3]{x^2+c_1}}{\sqrt[3]{x}}$$

$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{x^2+c_1}}{\sqrt[3]{x}}$$

1.50 problem Problem 64

Internal problem ID [10813]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 64.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y'^2 + (a + x) y' - y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)^2+(x+a)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{4}x^2 - \frac{1}{2}xa - \frac{1}{4}a^2$$
$$y(x) = ac_1 + c_1^2 + c_1 x$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 26

```
DSolve[y'[x]^2+(x+a)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(a + x + c_1)$$
$$y(x) \to -\frac{1}{4}(a + x)^2$$

1.51 problem Problem 65

Internal problem ID [10814]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 65.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^2 - 2y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 656

```
dsolve(diff(y(x),x)^2-2*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = -\left(\frac{\left(-6c_1+x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{2}+\frac{x^2}{2\left(-6c_1+x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}+\frac{x}{2}\right)^2 + 2x\left(\frac{\left(-6c_1+x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{2}+\frac{x^2}{2\left(-6c_1+x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}+\frac{x}{2}\right)$$

$$y(x) = -\left(-\frac{\left(-6c_1+x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{4}-\frac{x^2}{4\left(-6c_1+x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}+\frac{x}{2}-\frac{i\sqrt{3}\left(\frac{\left(-6c_1+x^3+2\right.}{}\right.}{}\right.$$

$$+2x\left(-\frac{\left(-6c_1+x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{4}-\frac{x^2}{4\left(-6c_1+x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}+\frac{x}{2}-\frac{i\sqrt{3}\left(\frac{\left(-6c_1+x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{2}-\frac{x^2}{2\left(-6c_1+x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}\right)}{2}\right)$$

$$y(x) = -\left(-\frac{\left(-6c_1+x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{4}-\frac{x^2}{4\left(-6c_1+x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}+\frac{x}{2}+\frac{i\sqrt{3}\left(\frac{\left(-6c_1+x^3+2\right.}{}\right.}{}\right.$$

✓ Solution by Mathematica

Time used: 60.107 (sec). Leaf size: 950

```
DSolve[y'[x]^2-2*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{1}{4}\left(x^2+\frac{x(x^3+8e^{3c_1})}{\sqrt[3]{x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}-8e^{6c_1}}}+\sqrt[3]{x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}-8e^{6c_1}}\right)$$

$$y(x)\to \frac{1}{72}\left(18x^2+\frac{\left(-9-9i\sqrt{3}\right)x(x^3+8e^{3c_1})}{\sqrt[3]{x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}-8e^{6c_1}}}+9i\left(\sqrt{3}+i\right)\sqrt[3]{x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}-8e^{6c_1}}\right)$$

$$y(x)\to \frac{1}{72}\left(18x^2+\frac{9i\left(\sqrt{3}+i\right)x(x^3+8e^{3c_1})}{\sqrt[3]{x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}-8e^{6c_1}}}-9\left(1+i\sqrt{3}\right)\sqrt[3]{x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}-8e^{6c_1}}\right)$$

$$y(x)\to \frac{x^4+\left(x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}\right)^{2/3}+x^2\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}}{4\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}}$$

$$y(x)\to \frac{1}{72}\left(18x^2+\frac{\left(-9-9i\sqrt{3}\right)x(x^3-8e^{3c_1})}{\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}}+9i\left(\sqrt{3}+i\right)\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}\right)$$

$$y(x)\to \frac{1}{72}\left(18x^2+\frac{9i\left(\sqrt{3}+i\right)x(x^3-8e^{3c_1})}{\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}}-9\left(1+i\sqrt{3}\right)\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}\right)$$

1.52 problem Problem 66

Internal problem ID [10815]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 1, First-Order Differential Equations. Problems page 88
Problem number: Problem 66.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'^2 + 2yy' \cot(x) - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 61

```
dsolve(diff(y(x),x)^2+2*y(x)*diff(y(x),x)*cot(x)-y(x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{c_1 (\tan(x)^2 + 1) \sqrt{\frac{\tan(x)^2}{\tan(x)^2+1}}}{\left(1 + \sqrt{\tan(x)^2 + 1}\right) \tan(x)}$$

$$y(x) = \frac{c_1 e^{\operatorname{arctanh}\left(\frac{1}{\sqrt{\tan(x)^2+1}}\right)} \sqrt{\tan(x)^2 + 1}}{\tan(x)}$$

✓ Solution by Mathematica

Time used: 0.177 (sec). Leaf size: 36

```
DSolve[y'[x]^2+2*y[x]*y'[x]*Cot[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \csc^2\left(\frac{x}{2}\right)$$

$$y(x) \to c_1 \sec^2\left(\frac{x}{2}\right)$$

$$y(x) \to 0$$

2 Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172

2.1 problem Problem 1

Internal problem ID [10816]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 10y - 100 = 0}$$

With initial conditions

$$[y(0) = 10, y'(0) = 5]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)-6*diff(y(x),x)+10*y(x)=100,y(0) = 10, D(y)(0) = 5],y(x), singsol=all)
```

$$y(x) = 10 + 5\,e^{3x} \sin(x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 17

```
DSolve[{y''[x]-6*y'[x]+10*y[x]==100,{y[0]==10,y'[0]==5}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to 5\left(e^{3x} \sin(x) + 2\right)$$

2.2 problem Problem 2

Internal problem ID [10817]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x - \sin(t) + \cos(2t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(x(t),t$2)+x(t)=sin(t)-cos(2*t),x(t), singsol=all)
```

$$x(t) = c_2 \sin(t) + c_1 \cos(t) + \frac{\cos(2t)}{3} + \frac{\sin(t)}{4} - \frac{t\cos(t)}{2}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 30

```
DSolve[x''[t]+x[t]==Sin[t]-Cos[2*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{3}\cos(2t) + \left(-\frac{t}{2} + c_1\right)\cos(t) + c_2 \sin(t)$$

2.3 problem Problem 3

Internal problem ID [10818]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 3.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y' + y''' - 3y'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)+diff(y(x),x$3)-3*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \mathrm{e}^{\frac{(3+\sqrt{5})x}{2}} + c_3 \mathrm{e}^{-\frac{(\sqrt{5}-3)x}{2}}$$

✓ Solution by Mathematica

Time used: 0.112 (sec). Leaf size: 57

```
DSolve[y'[x]+y'''[x]-3*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-\frac{1}{2}(\sqrt{5}-3)x} \left(\left(3+\sqrt{5}\right) c_1 - \left(\sqrt{5}-3\right) c_2 e^{\sqrt{5}x} \right) + c_3$$

2.4 problem Problem 4

Internal problem ID [10819]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \frac{1}{\sin(x)^3} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+y(x)=1/sin(x)^3,y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + c_1 \cos(x) + \cot(x) \cos(x) - \frac{\csc(x)}{2}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 24

```
DSolve[y''[x]+y[x]==1/Sin[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\csc(x)}{2} + c_1 \cos(x) + (-1 + c_2) \sin(x)$$

2.5 problem Problem 5

Internal problem ID [10820]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 4y'x + 6y - 2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=2,y(x), singsol=all)
```

$$y(x) = x^3c_2 + c_1x^2 + \frac{1}{3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} + x^2(c_2x + c_1)$$

2.6 problem Problem 6

Internal problem ID [10821]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \cosh(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=cosh(x),y(x), singsol=all)
```

$$y(x) = c_1 \cos(x) + c_2 \sin(x) + \frac{e^x}{4} + \frac{e^{-x}}{4}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Cosh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cosh(x)}{2} + c_1 \cos(x) + c_2 \sin(x)$$

2.7 problem Problem 7

Internal problem ID [10822]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible,`

$$\boxed{y'' + \frac{2y'^2}{1-y} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+2/(1-y(x))*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x + c_2 - 1}{c_1 x + c_2}$$

✓ Solution by Mathematica

Time used: 0.1 (sec). Leaf size: 19

```
DSolve[y''[x]+2/(1-y[x])*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 - \frac{1}{c_1(x + c_2)}$$

2.8 problem Problem 8

Internal problem ID [10823]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' - 4x' + 4x - e^{t} - e^{2t} - 1 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(diff(x(t),t$2)-4*diff(x(t),t)+4*x(t)=exp(t)+exp(2*t)+1,x(t), singsol=all)
```

$$x(t) = c_1 t\, e^{2t} + \frac{t^2 e^{2t}}{2} + c_2 e^{2t} + e^{t} + \frac{1}{4}$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 32

```
DSolve[x''[t]-4*x'[t]+4*x[t]==Exp[t]+Exp[2*t]+1,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{2t}\left(\frac{t^2}{2} + c_2 t + c_1\right) + e^{t} + \frac{1}{4}$$

2.9 problem Problem 9

Internal problem ID [10824]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{(x^2+1)\,y''+y'^2+1=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve((1+x^2)*diff(y(x),x$2)+diff(y(x),x)^2+1=0,y(x), singsol=all)
```

$$y(x)=\frac{x}{c_1}-\frac{(-c_1^2-1)\ln(c_1x-1)}{c_1^2}+c_2$$

✓ Solution by Mathematica

Time used: 7.919 (sec). Leaf size: 33

```
DSolve[(1+x^2)*y''[x]+y'[x]^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x\cot(c_1)+\csc^2(c_1)\log(-x\sin(c_1)-\cos(c_1))+c_2$$

2.10 problem Problem 10

Internal problem ID [10825]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{x^3x''+1=0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 70

```
dsolve(x(t)^3*diff(x(t),t$2)+1=0,x(t), singsol=all)
```

$$x(t)=\frac{\sqrt{c_1\left(c_1^2c_2^2+2c_1^2c_2t+c_1^2t^2-1\right)}}{c_1}$$
$$x(t)=-\frac{\sqrt{c_1\left(c_1^2c_2^2+2c_1^2c_2t+c_1^2t^2-1\right)}}{c_1}$$

✓ Solution by Mathematica

Time used: 1.036 (sec). Leaf size: 58

```
DSolve[x[t]^3*x''[t]+1==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t)\to-\frac{\sqrt{-1+c_1{}^2(t+c_2)^2}}{\sqrt{c_1}}$$
$$x(t)\to\frac{\sqrt{-1+c_1{}^2(t+c_2)^2}}{\sqrt{c_1}}$$

2.11 problem Problem 11

Internal problem ID [10826]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 11.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - 16y - x^2 + e^x = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 65

```
dsolve(diff(y(x),x$4)-16*y(x)=x^2-exp(x),y(x), singsol=all)
```

$$y(x) = -\frac{e^{-2x} e^{2x} \cos(2x)}{64} + \frac{e^{-2x}(-15x^2 e^{2x} + 16\, e^{3x})}{240} + c_1 \cos(2x) + c_2 e^{-2x} + c_3 e^{2x} + c_4 \sin(2x)$$

✓ Solution by Mathematica

Time used: 0.199 (sec). Leaf size: 50

```
DSolve[y''''[x]-16*y[x]==x^2-Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{16} + \frac{e^x}{15} + c_1 e^{2x} + c_3 e^{-2x} + c_2 \cos(2x) + c_4 \sin(2x)$$

2.12 problem Problem 12

Internal problem ID [10827]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 12.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _missing_y], [_3rd_orde`

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$3)^2+diff(y(x),x$2)^2=1,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2}x^2 + c_1x + c_2$$
$$y(x) = \frac{1}{2}x^2 + c_1x + c_2$$
$$y(x) = c_1 + xc_2 + \sqrt{-c_3^2 + 1}\, \sin(x) + c_3 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.244 (sec). Leaf size: 54

```
DSolve[y'''[x]^2+y''[x]^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3x - \cos(x - c_1) + c_2$$
$$y(x) \to c_3x - \cos(x + c_1) + c_2$$
$$y(x) \to \text{Interval}[\{-1, 1\}] + c_3x + c_2$$

2.13 problem Problem 13

Internal problem ID [10828]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 13.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{x^{(6)} - x'''' - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(x(t),t$6)-diff(x(t),t$4)=1,x(t), singsol=all)
```

$$x(t) = -\frac{t^4}{24} + e^t c_1 + \frac{c_3 t^3}{6} + \frac{t^2 c_4}{2} + e^{-t} c_2 + c_5 t + c_6$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 45

```
DSolve[x''''''[t]-x''''[t]==1,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{t^4}{24} + c_6 t^3 + c_5 t^2 + c_4 t + c_1 e^t + c_2 e^{-t} + c_3$$

2.14 problem Problem 14

Internal problem ID [10829]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 14.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{x'''' - 2x'' + x - t^2 + 3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(x(t),t$4)-2*diff(x(t),t$2)+x(t)=t^2-3,x(t), singsol=all)
```

$$x(t) = t^2 + 1 + \mathrm{e}^t c_1 + \mathrm{e}^{-t} c_2 + c_3 \mathrm{e}^t t + c_4 t\,\mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 34

```
DSolve[x''''[t]-2*x''[t]+x[t]==t^2-3,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow t^2 + e^{-t}(c_2 t + c_1) + e^t(c_4 t + c_3) + 1$$

2.15 problem Problem 15

Internal problem ID [10830]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + 4xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+4*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{2x^3}{3}\right) y(0) + \left(x - \frac{1}{3}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+4*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^4}{3}\right) + c_1\left(1 - \frac{2x^3}{3}\right)$$

2.16 problem Problem 16

Internal problem ID [10831]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(9x^2 - \frac{1}{25}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(9*x^2-1/25)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(\frac{1}{5}, 3x\right) + c_2 \operatorname{BesselY}\left(\frac{1}{5}, 3x\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+x*y'[x]+(9*x^2-1/25)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \operatorname{BesselJ}\left(\frac{1}{5}, 3x\right) + c_2 Y_{\frac{1}{5}}(3x)$$

2.17 problem Problem 17

Internal problem ID [10832]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' + y'^2 - 1 = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$2)+diff(y(x),x)^2=1,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 6

```
DSolve[{y''[x]+y'[x]^2==1,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x$$

2.18 problem Problem 18

Internal problem ID [10833]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - 3\sqrt{y} = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)=3*sqrt(y(x)),y(0) = 1, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = \frac{(x+2)^4}{16}$$

✓ Solution by Mathematica

Time used: 0.063 (sec). Leaf size: 14

```
DSolve[{y''[x]==3*Sqrt[y[x]],{y[0]==1,y'[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16}(x+2)^4$$

2.19 problem Problem 19

Internal problem ID [10834]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - 1 + \frac{1}{\sin(x)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+y(x)=1-1/sin(x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + c_1 \cos(x) - \sin(x) \ln(\sin(x)) + 1 + \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 30

```
DSolve[y''[x]+y[x]==1-1/Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_1)\cos(x) + \sin(x)(-\log(\tan(x)) - \log(\cos(x)) + c_2) + 1$$

2.20 problem Problem 20

Internal problem ID [10835]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{u'' + \frac{2u'}{r} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(u(r),r$2)+2/r*diff(u(r),r)=0,u(r), singsol=all)
```

$$u(r) = c_1 + \frac{c_2}{r}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 15

```
DSolve[u''[r]+2/r*u'[r]==0,u[r],r,IncludeSingularSolutions -> True]
```

$$u(r) \to c_2 - \frac{c_1}{r}$$

2.21 problem Problem 30

Internal problem ID [10836]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _with_linear_symmetries], [_2nd_order`

$$yy'' + y'^2 - \frac{yy'}{\sqrt{x^2+1}} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
dsolve(y(x)*diff(y(x),x$2)+diff(y(x),x)^2= y(x)*diff(y(x),x)/sqrt(1+x^2),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \sqrt{c_1 x\sqrt{x^2+1} + c_1 x^2 + c_1 \operatorname{arcsinh}(x) + 2c_2}$$
$$y(x) = -\sqrt{c_1 x\sqrt{x^2+1} + c_1 x^2 + c_1 \operatorname{arcsinh}(x) + 2c_2}$$

✓ Solution by Mathematica

Time used: 0.588 (sec). Leaf size: 44

```
DSolve[y[x]*y''[x]+y'[x]^2== y[x]*y'[x]/Sqrt[1+x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \exp\left(\int_1^x \frac{1}{K[1] + (\operatorname{arcsinh}(K[1]) + c_1)\left(\sqrt{K[1]^2+1} - K[1]\right)} dK[1]\right)$$

2.22 problem Problem 31

Internal problem ID [10837]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 31.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{yy'y'' - y'^3 - y''^2 = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 53

```
dsolve(y(x)*diff(y(x),x)*diff(y(x),x$2)=diff(y(x),x)^3+diff(y(x),x$2)^2,y(x), singsol=all)
```

$$y(x) = \frac{4}{4c_1 - x}$$
$$y(x) = c_1$$
$$y(x) = e^{-c_1c_2}e^{-c_1x} - c_1$$
$$y(x) = e^{c_1c_2}e^{c_1x} + c_1$$

✓ Solution by Mathematica

Time used: 0.912 (sec). Leaf size: 67

```
DSolve[y[x]*y'[x]*y''[x]==y'[x]^3+y''[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(e^{-\frac{1}{2}(1+e^{c_1})(x+c_2)} - 1 - e^{c_1}\right)$$
$$y(x) \to \frac{1 + e^{-\frac{1}{2}(1+e^{c_1})(x+c_2)}}{-1 + \tanh\left(\frac{c_1}{2}\right)}$$

2.23 problem Problem 32

Internal problem ID [10838]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 9x - t\sin(3t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(x(t),t$2)+9*x(t)=t*sin(3*t),x(t), singsol=all)
```

$$x(t) = c_2 \sin(3t) + \cos(3t)\, c_1 + \frac{t\sin(3t)}{36} - \frac{\cos(3t)\, t^2}{12}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 38

```
DSolve[x''[t]+9*x[t]==t*Sin[3*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \left(-\frac{t^2}{12} + \frac{1}{216} + c_1\right)\cos(3t) + \frac{1}{36}(t + 36c_2)\sin(3t)$$

2.24 problem Problem 33

Internal problem ID [10839]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y - \sinh(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=sinh(x),y(x), singsol=all)
```

$$y(x) = e^{-x} c_2 + x\,e^{-x} c_1 + \frac{(-2x^2 + 2x + 1)\,e^{-x}}{8} + \frac{e^x}{8}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 34

```
DSolve[y''[x]+2*y'[x]+y[x]==Sinh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8} e^{-x} \left(-2x^2 + e^{2x} + 8c_2 x + 8c_1\right)$$

2.25 problem Problem 34

Internal problem ID [10840]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 34.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - y - e^{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$3)-y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \frac{x\,e^{x}}{3} + c_1 e^{x} + c_2 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) + c_3 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.332 (sec). Leaf size: 57

```
DSolve[y'''[x]-y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{x}(x - 1 + 3c_1) + e^{-x/2}\left(c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_3 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

2.26 problem Problem 35

Internal problem ID [10841]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + 2y - x\,\mathrm{e}^{x}\cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x) +2*y(x)=x*exp(x)*cos(x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(x)\,\mathrm{e}^{x} + \mathrm{e}^{x}\cos(x)\,c_1 + \frac{\mathrm{e}^{x}(x^2\sin(x) + \cos(x)\,x - \sin(x))}{4}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 37

```
DSolve[y''[x]-2*y'[x] +2*y[x]==x*Exp[x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}e^{x}\left((2x^2 - 1 + 8c_1)\sin(x) + 2(x + 4c_2)\cos(x)\right)$$

2.27 problem Problem 36

Internal problem ID [10842]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-1)\,y''-6y-1=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve((x^2-1)*diff(y(x),x$2)-6*y(x)=1,y(x), singsol=all)
```

$$y(x)=(x^3-x)\,c_2+\left(\frac{(3x^3-3x)\ln(x-1)}{4}+\frac{(-3x^3+3x)\ln(x+1)}{4}+\frac{3x^2}{2}-1\right)c_1-\frac{1}{6}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 43

```
DSolve[(x^2-1)*y''[x]-6*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{3}{2}c_2(x^2-1)\,x\operatorname{arctanh}(x)-\frac{3c_2x^2}{2}+c_1(x^2-1)\,x-\frac{1}{6}+c_2$$

2.28 problem Problem 40(a)

Internal problem ID [10843]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 40(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{mx'' - f(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 62

```
dsolve(m*diff(x(t),t$2)=f(x(t)),x(t), singsol=all)
```

$$\int^{x(t)} \frac{m}{\sqrt{m\left(c_1 m + 2\left(\int f(\_b)\, d\_b\right)\right)}} d\_b - t - c_2 = 0$$

$$\int^{x(t)} -\frac{m}{\sqrt{m\left(c_1 m + 2\left(\int f(\_b)\, d\_b\right)\right)}} d\_b - t - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 44

```
DSolve[m*x''[t]==f[x[t]],x[t],t,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{x(t)} \frac{1}{\sqrt{c_1 + 2\int_1^{K[2]} \frac{f(K[1])}{m} dK[1]}} dK[2]^2 = (t + c_2)^2, x(t)\right]$$

2.29 problem Problem 40(b)

Internal problem ID [10844]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 40(b).
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{mx'' - f(x') = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(m*diff(x(t),t$2)=f(diff(x(t),t)),x(t), singsol=all)
```

$$x(t) = \int \operatorname{RootOf}\left(t - m\left(\int^{\_Z} \frac{1}{f(\_f)} d\_f\right) + c_1\right) dt + c_2$$

✓ Solution by Mathematica

Time used: 1.546 (sec). Leaf size: 39

```
DSolve[m*x''[t]==f[x'[t]],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \int_1^t \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{f(K[1])} dK[1]\&\right]\left[c_1 + \frac{K[2]}{m}\right] dK[2] + c_2$$

2.30 problem Problem 41

Internal problem ID [10845]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 41.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y^{(6)} - 3y^{(5)} + 3y'''' - y''' - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
dsolve(diff(y(x),x$6)-3*diff(y(x),x$5)+3*diff(y(x),x$4)-diff(y(x),x$3)=x,y(x), singsol=all)
```

$$y(x) = -\frac{x^3}{2} - \frac{x^4}{24} + c_1 e^x + c_2(x e^x - 3 e^x) + c_3(e^x x^2 - 6x e^x + 12 e^x) + \frac{c_4 x^2}{2} + c_5 x + c_6$$

✓ Solution by Mathematica

Time used: 0.168 (sec). Leaf size: 60

```
DSolve[y''''''[x]-3*y'''''[x]+3*y''''[x]-y'''[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^4}{24} - \frac{x^3}{2} + c_6 x^2 + c_5 x + c_1 e^x + c_2 e^x (x-3) + c_3 e^x ((x-6)x + 12) + c_4$$

2.31 problem Problem 42

Internal problem ID [10846]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 42.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{x'''' + 2x'' + x - \cos(t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(x(t),t$4)+2*diff(x(t),t$2)+x(t)=cos(t),x(t), singsol=all)
```

$$x(t) = \left(-\frac{t^2}{8} + \frac{1}{4}\right)\cos(t) + \frac{3t\sin(t)}{8} + c_1\cos(t) + c_2\sin(t) + \cos(t)\,c_3 t + c_4 t\sin(t)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 43

```
DSolve[x''''[t]+2*x''[t]+x[t]==Cos[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \left(-\frac{t^2}{8} + c_2 t + \frac{5}{16} + c_1\right)\cos(t) + \frac{1}{4}(t + 4c_4 t + 4c_3)\sin(t)$$

2.32 problem Problem 43

Internal problem ID [10847]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(x+1)^2 y'' + (x+1) y' + y - 2\cos(\ln(x+1)) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve((1+x)^2*diff(y(x),x$2)+(1+x)*diff(y(x),x)+y(x)=2*cos(ln(1+x)),y(x), singsol=all)
```

$$y(x) = c_2 \sin(\ln(x+1)) + c_1 \cos(\ln(x+1)) + \ln(x+1)\sin(\ln(x+1))$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 31

```
DSolve[(1+x)^2*y''[x]+(1+x)*y'[x]+y[x]==2*Cos[Log[1+x]],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \left(\frac{1}{2} + c_1\right)\cos(\log(x+1)) + (\log(x+1) + c_2)\sin(\log(x+1))$$

2.33 problem Problem 47

Internal problem ID [10848]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$\boxed{x^3y'' - y'x + y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([x^3*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,x],y(x), singsol=all)
```

$$y(x) = \left(\mathrm{e}^{-\frac{1}{x}} c_1 + c_2\right) x$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 20

```
DSolve[x^3*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(c_2 e^{-1/x} + c_1\right)$$

2.34 problem Problem 49

Internal problem ID [10849]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 49.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{x'''' + x - t^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 72

```
dsolve(diff(x(t),t$4)+x(t)=t^3,x(t), singsol=all)
```

$$x(t) = t^3 + c_1 \mathrm{e}^{-\frac{t\sqrt{2}}{2}} \cos\left(\frac{t\sqrt{2}}{2}\right) + c_2 \mathrm{e}^{-\frac{t\sqrt{2}}{2}} \sin\left(\frac{t\sqrt{2}}{2}\right) + c_3 \mathrm{e}^{\frac{t\sqrt{2}}{2}} \cos\left(\frac{t\sqrt{2}}{2}\right) + c_4 \mathrm{e}^{\frac{t\sqrt{2}}{2}} \sin\left(\frac{t\sqrt{2}}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 78

```
DSolve[x''''[t]+x[t]==t^3,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-\frac{t}{\sqrt{2}}} \left(e^{\frac{t}{\sqrt{2}}} t^3 + \left(c_1 e^{\sqrt{2}t} + c_2\right) \cos\left(\frac{t}{\sqrt{2}}\right) + \left(c_4 e^{\sqrt{2}t} + c_3\right) \sin\left(\frac{t}{\sqrt{2}}\right) \right)$$

2.35 problem Problem 50

Internal problem ID [10850]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 50.
ODE order: 2.
ODE degree: 3.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$y''^3 + y'' + 1 - x = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 243

```
dsolve(diff(y(x),x$2)^3+diff(y(x),x$2)+1=x,y(x), singsol=all)
```

$$y(x) = \int \left(\int \frac{\left(-108+108x+12\sqrt{81x^2-162x+93}\right)^{\frac{2}{3}}-12}{6\left(-108+108x+12\sqrt{81x^2-162x+93}\right)^{\frac{1}{3}}}dx\right)dx + c_1x + c_2$$

$$y(x) = \int \left(\int -\frac{i\left(-108+108x+12\sqrt{81x^2-162x+93}\right)^{\frac{2}{3}}\sqrt{3}+12i\sqrt{3}+\left(-108+108x+12\sqrt{81x^2-162x+93}\right)^{\frac{2}{3}} -}{12\left(-108+108x+12\sqrt{81x^2-162x+93}\right)^{\frac{1}{3}}} \right. + c_1x + c_2$$

$$y(x) = \int \left(\int \frac{i\left(-108+108x+12\sqrt{81x^2-162x+93}\right)^{\frac{2}{3}}\sqrt{3}+12i\sqrt{3}-\left(-108+108x+12\sqrt{81x^2-162x+93\right.}}{12\left(-108+108x+12\sqrt{81x^2-162x+93}\right)^{\frac{1}{3}}} \right. + c_1x + c_2$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]^3+y''[x]+1==x,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

2.36 problem Problem 51

Internal problem ID [10851]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 51.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 10x' + 25x - 2^t - t\,\mathrm{e}^{-5t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve(diff(x(t),t$2)+10*diff(x(t),t)+25*x(t)=2^t+t*exp(-5*t),x(t), singsol=all)
```

$$x(t) = c_2\mathrm{e}^{-5t} + t\,\mathrm{e}^{-5t}c_1 + \frac{t^3(\ln(2)+5)^2\,\mathrm{e}^{-5t} + 6\,2^t}{6\,(\ln(2)+5)^2}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 36

```
DSolve[x''[t]+10*x'[t]+25*x[t]==2^t+t*Exp[-5*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{2^t}{(5+\log(2))^2} + e^{-5t}\left(\frac{t^3}{6} + c_2 t + c_1\right)$$

2.37 problem Problem 52

Internal problem ID [10852]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 52.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _with_linear_symmetries], [_2nd_order`

$$\boxed{xyy'' - xy'^2 - y'y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(x*y(x)*diff(y(x),x$2)-x*diff(y(x),x)^2-y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \mathrm{e}^{\frac{c_1 x^2}{2}} c_2$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 19

```
DSolve[x*y[x]*y''[x]-x*y'[x]^2-y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^{\frac{c_1 x^2}{2}}$$

2.38 problem Problem 53

Internal problem ID [10853]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 53.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{y^{(6)} - y - e^{2x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 73

```
dsolve(diff(y(x),x$6)-y(x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{e^{2x}}{63} + c_1 e^{x} + e^{-x} c_2 + c_3 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) + c_4 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) + c_5 e^{\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) + c_6 e^{\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.429 (sec). Leaf size: 80

```
DSolve[y''''''[x]-y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2x}}{63} + c_1 e^{x} + c_4 e^{-x} + e^{-x/2}\left((c_2 e^{x} + c_3) \cos\left(\frac{\sqrt{3}x}{2}\right) + (c_6 e^{x} + c_5) \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

2.39 problem Problem 54

Internal problem ID [10854]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 54.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y^{(6)} + 2y'''' + y'' - x - e^x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(diff(y(x),x$6)+2*diff(y(x),x$4)+diff(y(x),x$2)=x+exp(x),y(x), singsol=all)
```

$$y(x) = \frac{x^3}{6} - c_1 \cos(x) - c_2 \sin(x) + c_3(2\sin(x) - \cos(x)\, x) + c_4(-\sin(x)\, x - 2\cos(x)) + \frac{e^x}{4} + c_5 x + c_6$$

✓ Solution by Mathematica

Time used: 0.404 (sec). Leaf size: 56

```
DSolve[y''''''[x]+2*y''''[x]+y''[x]==x+Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{6} + \frac{e^x}{4} + c_6 x - (c_2 x + c_1 + 2c_4)\cos(x) - (c_4 x - 2c_2 + c_3)\sin(x) + c_5$$

2.40 problem Problem 55

Internal problem ID [10855]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 55.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_high_order, _missing_x], [_high_order, _missing_y], [_high_o`

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(6*diff(y(x),x$2)*diff(y(x),x$4)-5*diff(y(x),x$3)^2=0,y(x), singsol=all)
```

$$y(x) = c_1 x + c_2$$
$$y(x) = \frac{(c_2 + x)^8 c_1}{2612736} + c_3 x + c_4$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 26

```
DSolve[6*y''[x]*y''''[x]-5*y'''[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{56} c_2 (x - 6c_1)^8 + c_4 x + c_3$$

2.41 problem Problem 56

Internal problem ID [10856]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 56.
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - y' \ln\left(\frac{y'}{x}\right) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x$2)=diff(y(x),x)*ln(diff(y(x),x)/x),y(x), singsol=all)
```

$$y(x) = \frac{(c_1 x - 1)\, \mathrm{e}^{c_1 x} \mathrm{e}}{c_1^2} + c_2$$

✓ Solution by Mathematica

Time used: 0.583 (sec). Leaf size: 31

```
DSolve[x*y''[x]==y'[x]*Log[y'[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{e^{c_1} x+1-2c_1}\left(-1 + e^{c_1} x\right) + c_2$$

2.42 problem Problem 57

Internal problem ID [10857]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 57.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \sin(3x)\cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+y(x)=sin(3*x)*cos(x),y(x), singsol=all)
```

$$y(x) = c_1 \cos(x) + c_2 \sin(x) - \frac{\sin(2x)}{6} - \frac{\sin(4x)}{30}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 30

```
DSolve[y''[x]+y[x]==Sin[3*x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) - \frac{1}{15} \sin(x)(6\cos(x) + \cos(3x) - 15c_2)$$

2.43 problem Problem 58

Internal problem ID [10858]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 58.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - 2y^3 = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)=2*y(x)^3,y(1) = 1, D(y)(1) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{1}{x-2}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 12

```
DSolve[{y''[x]==2*y[x]^3,{y[1]==1,y'[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2-x}$$

2.44 problem Problem 59

Internal problem ID [10859]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 2, DIFFERENTIAL EQUATIONS OF THE SECOND ORDER AND HIGHER. Problems page 172
Problem number: Problem 59.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{yy'' - y'^2 - y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve(y(x)*diff(y(x),x$2)-diff(y(x),x)^2=diff(y(x),x),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{e^{c_1 c_2} e^{c_1 x} + 1}{c_1}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 21

```
DSolve[y[x]*y''[x]-y'[x]^2==y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1 + e^{c_1(x+c_2)}}{c_1}$$

3 Chapter 3, SYSTEMS OF DIFFERENTIAL EQUATIONS. Problems page 209

3.1 problem Problem 1

Internal problem ID [10860]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 3, SYSTEMS OF DIFFERENTIAL EQUATIONS. Problems page 209
Problem number: Problem 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = y(t)$$
$$y'(t) = -x(t)$$

With initial conditions

$$[x(0) = 0, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 12

```
dsolve([diff(x(t),t) = y(t), diff(y(t),t) = -x(t), x(0) = 0, y(0) = 1],[x(t), y(t)], singsol=
```

$$x(t) = \sin(t)$$

$$y(t) = \cos(t)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 31

```
DSolve[{x'[t]==y[t],y'[t]==-x[t]},{},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 \cos(t) + c_2 \sin(t)$$
$$y(t) \to c_2 \cos(t) - c_1 \sin(t)$$

3.2 problem Problem 3

Internal problem ID [10861]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 3, SYSTEMS OF DIFFERENTIAL EQUATIONS. Problems page 209
Problem number: Problem 3.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -5x(t) - y(t) + \mathrm{e}^t$$
$$y'(t) = x(t) + 3y(t) + \mathrm{e}^{2t}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 102

```
dsolve([diff(x(t),t)+5*x(t)+y(t)=exp(t),diff(y(t),t)-x(t)-3*y(t)=exp(2*t)],[x(t), y(t)], sing
```

$$x(t) = \mathrm{e}^{\left(-1+\sqrt{15}\right)t} c_2 \sqrt{15} - \mathrm{e}^{-\left(1+\sqrt{15}\right)t} c_1 \sqrt{15} + \frac{\mathrm{e}^{2t}}{6} - 4\,\mathrm{e}^{\left(-1+\sqrt{15}\right)t} c_2 - 4\,\mathrm{e}^{-\left(1+\sqrt{15}\right)t} c_1 + \frac{2\,\mathrm{e}^t}{11}$$

$$y(t) = \mathrm{e}^{\left(-1+\sqrt{15}\right)t} c_2 + \mathrm{e}^{-\left(1+\sqrt{15}\right)t} c_1 - \frac{7\,\mathrm{e}^{2t}}{6} - \frac{\mathrm{e}^t}{11}$$

✓ Solution by Mathematica

Time used: 2.793 (sec). Leaf size: 194

```
DSolve[{x'[t]+5*x[t]+y[t]==Exp[t],y'[t]-x[t]-3*y[t]==Exp[2*t]},{x[t],y[t]},t,IncludeSingularS
```

$$\begin{aligned} x(t) \to \frac{1}{330} e^{-\left(\left(1+\sqrt{15}\right)t\right)} \Big(& 5e^{\left(2+\sqrt{15}\right)t}(11e^t + 12) - 11\left(\left(4\sqrt{15} - 15\right)c_1 + \sqrt{15}c_2\right)e^{2\sqrt{15}t} \\ & + 11\left(\left(15 + 4\sqrt{15}\right)c_1 + \sqrt{15}c_2\right)\Big) \end{aligned}$$

$$\begin{aligned} y(t) \to \frac{1}{330} e^{-\left(\left(1+\sqrt{15}\right)t\right)} \Big(& -5e^{\left(2+\sqrt{15}\right)t}(77e^t + 6) + 11\left(\sqrt{15}c_1 + \left(15 + 4\sqrt{15}\right)c_2\right)e^{2\sqrt{15}t} \\ & - 11\left(\sqrt{15}c_1 + \left(4\sqrt{15} - 15\right)c_2\right)\Big) \end{aligned}$$

3.3 problem Problem 4

Internal problem ID [10862]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 3, SYSTEMS OF DIFFERENTIAL EQUATIONS. Problems page 209
Problem number: Problem 4.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = y(t)$$
$$y'(t) = z(t)$$
$$z'(t) = x(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 176

```
dsolve([diff(x(t),t)=y(t),diff(y(t),t)=z(t),diff(z(t),t)=x(t)],[x(t), y(t), z(t)], singsol=al
```

$$x(t) = e^{t}c_1 - \frac{c_2 e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right)}{2} + \frac{c_2\sqrt{3}\,e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right)}{2} - \frac{c_3 e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right)}{2} - \frac{c_3 e^{-\frac{t}{2}}\sqrt{3} \sin\left(\frac{\sqrt{3}\,t}{2}\right)}{2}$$

$$y(t) = e^{t}c_1 - \frac{c_2 e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right)}{2} - \frac{c_2\sqrt{3}\,e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right)}{2} - \frac{c_3 e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right)}{2} + \frac{c_3 e^{-\frac{t}{2}}\sqrt{3} \sin\left(\frac{\sqrt{3}\,t}{2}\right)}{2}$$

$$z(t) = e^{t}c_1 + c_2 e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right) + c_3 e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 233

```
DSolve[{x'[t]==y[t],y'[t]==z[t],z'[t]==x[t]},{x[t],y[t],z[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to \frac{1}{3}e^{-t/2}\left((c_1+c_2+c_3)e^{3t/2}+(2c_1-c_2-c_3)\cos\left(\frac{\sqrt{3}t}{2}\right)+\sqrt{3}(c_2-c_3)\sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$

$$y(t) \to \frac{1}{3}e^{-t/2}\left((c_1+c_2+c_3)e^{3t/2}-(c_1-2c_2+c_3)\cos\left(\frac{\sqrt{3}t}{2}\right)+\sqrt{3}(c_3-c_1)\sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$

$$z(t) \to \frac{1}{3}e^{-t/2}\left((c_1+c_2+c_3)e^{3t/2}-(c_1+c_2-2c_3)\cos\left(\frac{\sqrt{3}t}{2}\right)+\sqrt{3}(c_1-c_2)\sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$

3.4 problem Problem 5

Internal problem ID [10863]

Book: Differential equations and the calculus of variations by L. ElSGOLTS. MIR PUBLISHERS, MOSCOW, Third printing 1977.
Section: Chapter 3, SYSTEMS OF DIFFERENTIAL EQUATIONS. Problems page 209
Problem number: Problem 5.
ODE order: 1.
ODE degree: 2.

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 38

```
dsolve([diff(x(t),t)=y(t),diff(y(t),t)=y(t)^2/x(t)],[x(t), y(t)], singsol=all)
```

$$\{y(t) = 0\}$$
$$\{x(t) = c_1\}$$

$$\{y(t) = \mathrm{e}^{c_1 t} c_2\}$$
$$\left\{x(t) = \frac{y(t)^2}{\frac{d}{dt} y(t)}\right\}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 28

```
DSolve[{x'[t]==y[t],y'[t]==y[t]^2/x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 c_2 e^{c_1 t}$$
$$x(t) \to c_2 e^{c_1 t}$$