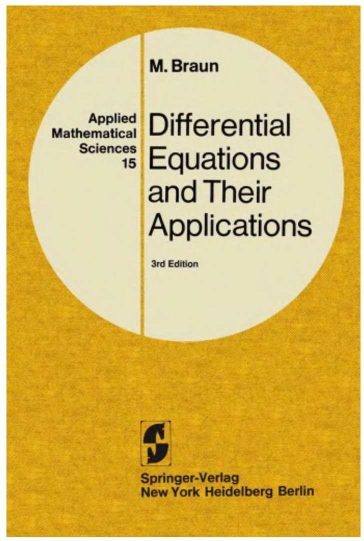
A Solution Manual For

# Differential equations and their applications, 3rd ed., M. Braun

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# 1 Section 1.2. Page 6

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## 1.1 problem Example 3

Internal problem ID [1644]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 6
**Problem number**: Example 3.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \sin(t)\, y = 0}$$

With initial conditions

$$\left[y(0) = \frac{3}{2}\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve([diff(y(t),t)+sin(t)*y(t)=0,y(0) = 3/2],y(t), singsol=all)
```

$$y(t) = \frac{3\,\mathrm{e}^{\cos(t)-1}}{2}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 15

```
DSolve[{y'[t]+Sin[t]*y[t]==0,y[0]==3/2},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{3}{2}e^{\cos(t)-1}$$

## 1.2 problem Example 4

Internal problem ID [1645]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 6
**Problem number**: Example 4.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + e^{t^2} y = 0}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 19

```
dsolve([diff(y(t),t)+exp(t^2)*y(t)=0,y(1) = 2],y(t), singsol=all)
```

$$y(t) = 2\,e^{-\frac{(-\operatorname{erfi}(1)+\operatorname{erfi}(t))\sqrt{\pi}}{2}}$$

✓ Solution by Mathematica

Time used: 0.063 (sec). Leaf size: 23

```
DSolve[{y'[t]+Exp[t^2]*y[t]==0,y[1]==2},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2e^{e\,\text{DawsonF}(1) - e^{t^2}\,\text{DawsonF}(t)}$$

## 1.3 problem Example 5

Internal problem ID [1646]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 6
**Problem number**: Example 5.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2yt - t = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(t),t)-2*t*y(t)=t,y(t), singsol=all)
```

$$y(t) = -\frac{1}{2} + e^{t^2} c_1$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 24

```
DSolve[y'[t]-2*t*y[t]==t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{1}{2} + c_1 e^{t^2}$$
$$y(t) \to -\frac{1}{2}$$

## 1.4 problem Example 6

Internal problem ID [1647]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 6
**Problem number**: Example 6.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + 2yt - t = 0}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(t),t)+2*t*y(t)=t,y(1) = 2],y(t), singsol=all)
```

$$y(t) = \frac{1}{2} + \frac{3\,\mathrm{e}^{-(t-1)(t+1)}}{2}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 22

```
DSolve[{y'[t]+2*t*y[t]==t,y[1]==2},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{3e^{1-t^2}}{2} + \frac{1}{2}$$

## 1.5 problem Example 7

Internal problem ID [1648]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 6
**Problem number**: Example 7.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y - \frac{1}{t^2 + 1} = 0}$$

With initial conditions

$$[y(2) = 3]$$

✓ Solution by Maple

Time used: 0.828 (sec). Leaf size: 65

```
dsolve([diff(y(t),t)+y(t)=1/(1+t^2),y(2) = 3],y(t), singsol=all)
```

$$y(t) = \frac{\left(i\mathrm{e}^{i} \mathrm{Ei}_1\left(-t+i\right) - i\mathrm{e}^{-i} \mathrm{Ei}_1\left(-t-i\right) - i\mathrm{e}^{i} \mathrm{Ei}_1\left(-2+i\right) + i\mathrm{e}^{-i} \mathrm{Ei}_1\left(-2-i\right) + 6\,\mathrm{e}^{2}\right)\mathrm{e}^{-t}}{2}$$

✓ Solution by Mathematica

Time used: 0.12 (sec). Leaf size: 65

```
DSolve[{y'[t]+y[t]==1/(1+t^2),y[1]==2},y[t],t,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(t) \to \frac{1}{2}e^{-t-i}(ie^{2i}(\text{ExpIntegralEi}(1-i) - \text{ExpIntegralEi}(t-i)) \\ - i(\text{ExpIntegralEi}(1+i) - \text{ExpIntegralEi}(t+i)) + 4e^{1+i}) \end{aligned}$$

# 2 Section 1.2. Page 9

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## 2.1 problem 1

Internal problem ID [1649]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 1.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(t)\, y + y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(cos(t)*y(t)+diff(y(t),t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{-\sin(t)}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 19

```
DSolve[Cos[t]*y[t]+y'[t] == 0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{-\sin(t)}$$
$$y(t) \to 0$$

## 2.2 problem 2

Internal problem ID [1650]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 2.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sqrt{t}\,\sin(t)\,y + y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(t^(1/2)*sin(t)*y(t)+diff(y(t),t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{\sqrt{t}\,\cos(t) - \frac{\mathrm{FresnelC}\left(\sqrt{2}\,\sqrt{\frac{t}{\pi}}\right)\sqrt{\pi}\,\sqrt{2}}{2}}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 47

```
DSolve[t^(1/2)*Sin[t]*y[t]+y'[t] == 0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \exp\left(-\frac{1}{2} i t^{3/2}\left(\text{ExpIntegralE}\left(-\frac{1}{2}, -it\right) - \text{ExpIntegralE}\left(-\frac{1}{2}, it\right)\right)\right)$$

$$y(t) \to 0$$

## 2.3 problem 3

Internal problem ID [1651]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 3.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\frac{2ty}{t^2+1}+y'-\frac{1}{t^2+1}=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(2*t*y(t)/(t^2+1)+diff(y(t),t) = 1/(t^2+1),y(t), singsol=all)
```

$$y(t)=\frac{t+c_1}{t^2+1}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 17

```
DSolve[2*t*y[t]/(t^2+1)+y'[t] == 1/(t^2+1),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to \frac{t+c_1}{t^2+1}$$

## 2.4 problem 4

Internal problem ID [1652]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 4.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y + y' - t\,\mathrm{e}^{t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(y(t)+diff(y(t),t) = exp(t)*t,y(t), singsol=all)
```

$$y(t) = \left(\frac{(2t-1)\,\mathrm{e}^{2t}}{4} + c_1\right)\mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 26

```
DSolve[y[t]+y'[t] == Exp[t]*t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4}e^{t}(2t-1) + c_1 e^{-t}$$

## 2.5 problem 5

Internal problem ID [1653]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 5.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{yt^2 + y' - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(t^2*y(t)+diff(y(t),t) = 1,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-\frac{t^3}{3}} c_1 + \frac{3^{\frac{1}{3}} t \left(2\sqrt{3}\,\pi - 3\Gamma\left(\frac{1}{3}, -\frac{t^3}{3}\right) \Gamma\left(\frac{2}{3}\right)\right) \mathrm{e}^{-\frac{t^3}{3}}}{9\Gamma\left(\frac{2}{3}\right) (-t^3)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 37

```
DSolve[t^2*y[t]+y'[t] == 1,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{3} e^{-\frac{t^3}{3}} \left(-t\,\text{ExpIntegralE}\left(\frac{2}{3}, -\frac{t^3}{3}\right) + 3c_1\right)$$

## 2.6 problem 6

Internal problem ID [1654]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 6.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yt^2 + y' - t^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(t^2*y(t)+diff(y(t),t) = t^2,y(t), singsol=all)
```

$$y(t) = 1 + \mathrm{e}^{-\frac{t^3}{3}} c_1$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 24

```
DSolve[t^2*y[t]+y'[t]== t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 1 + c_1 e^{-\frac{t^3}{3}}$$
$$y(t) \to 1$$

## 2.7 problem 7

Internal problem ID [1655]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 7.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\frac{ty}{t^2+1}+y'-1+\frac{t^3y}{t^4+1}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(t*y(t)/(t^2+1)+diff(y(t),t) = 1-t^3*y(t)/(t^4+1),y(t), singsol=all)
```

$$y(t)=\frac{\int (t^4+1)^{\frac{1}{4}}\sqrt{t^2+1}dt+c_1}{(t^4+1)^{\frac{1}{4}}\sqrt{t^2+1}}$$

✓ Solution by Mathematica

Time used: 22.28 (sec). Leaf size: 55

```
DSolve[t*y[t]/(t^2+1)+y'[t] == 1-t^3*y[t]/(t^4+1),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\frac{\int_1^t \sqrt{K[1]^2+1}\sqrt[4]{K[1]^4+1}dK[1]+c_1}{\sqrt{t^2+1}\sqrt[4]{t^4+1}}$$

## 2.8 problem 8

Internal problem ID [1656]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 8.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sqrt{t^2+1}\, y + y' = 0}$$

With initial conditions

$$\left[y(0) = \sqrt{5}\right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 24

```
dsolve([(t^2+1)^(1/2)*y(t)+diff(y(t),t) = 0,y(0) = 5^(1/2)],y(t), singsol=all)
```

$$y(t) = \sqrt{5}\, \mathrm{e}^{-\frac{t\sqrt{t^2+1}}{2}-\frac{\operatorname{arcsinh}(t)}{2}}$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 44

```
DSolve[{(t^2+1)^(1/2)*y[t]+y'[t] == 0,y[0]==Sqrt[5]},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sqrt{5} e^{-\frac{1}{2}t\sqrt{t^2+1}} \sqrt{\sqrt{t^2+1}-t}$$

## 2.9 problem 9

Internal problem ID [1657]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 9.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sqrt{t^2+1}\, y\, \mathrm{e}^{-t} + y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve((t^2+1)^(1/2)*y(t)/exp(t)+diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{\int -\sqrt{t^2+1}\, \mathrm{e}^{-t} dt}$$

✓ Solution by Mathematica

Time used: 0.266 (sec). Leaf size: 40

```
DSolve[(t^2+1)^(1/2)*y[t]/Exp[t]+y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \exp\left(\int_1^t -e^{-K[1]}\sqrt{K[1]^2+1} dK[1]\right)$$

$$y(t) \to 0$$

## 2.10 problem 11

Internal problem ID [1658]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 11.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2yt - t = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([-2*t*y(t)+diff(y(t),t) = t,y(0) = 1],y(t), singsol=all)
```

$$y(t) = -\frac{1}{2} + \frac{3\,e^{t^2}}{2}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 18

```
DSolve[{-2*t*y[t]+y'[t] == t,y[0]==1},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}\left(3e^{t^2} - 1\right)$$

## 2.11 problem 12

Internal problem ID [1659]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 12.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{yt + y' - t - 1 = 0}$$

With initial conditions

$$\left[y\left(\frac{3}{2}\right) = 0\right]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 50

```
dsolve([t*y(t)+diff(y(t),t) = 1+t,y(3/2) = 0],y(t), singsol=all)
```

$$y(t) = 1 - e^{\frac{9}{8}-\frac{t^2}{2}} + \frac{\sqrt{\pi}\,\sqrt{2}\left(-i\,\mathrm{erf}\left(\frac{i\sqrt{2}\,t}{2}\right) - \mathrm{erfi}\left(\frac{3\sqrt{2}}{4}\right)\right) e^{-\frac{t^2}{2}}}{2}$$

✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 54

```
DSolve[{t*y[t]+y'[t] == 1+t,y[3/2]==0},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\left(\sqrt{2}\,\mathrm{DawsonF}\left(\frac{3}{2\sqrt{2}}\right) + 1\right) e^{\frac{9}{8}-\frac{t^2}{2}} + \sqrt{2}\,\mathrm{DawsonF}\left(\frac{t}{\sqrt{2}}\right) + 1$$

## 2.12 problem 13

Internal problem ID [1660]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 13.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y - \frac{1}{t^2+1} = 0}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.703 (sec). Leaf size: 65

```
dsolve([y(t)+diff(y(t),t) = 1/(t^2+1),y(1) = 2],y(t), singsol=all)
```

$$y(t) = \frac{(-i\,\mathrm{Ei}_1(-1+i)\,\mathrm{e}^{i} + i\,\mathrm{Ei}_1(-1-i)\,\mathrm{e}^{-i} + i\mathrm{e}^{i}\,\mathrm{Ei}_1(-t+i) - i\mathrm{e}^{-i}\,\mathrm{Ei}_1(-t-i) + 4\,\mathrm{e})\,\mathrm{e}^{-t}}{2}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 65

```
DSolve[{y[t]+y'[t] == 1/(t^2+1),y[1]==2},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-t-i}(ie^{2i}(\text{ExpIntegralEi}(1-i) - \text{ExpIntegralEi}(t-i)) - i(\text{ExpIntegralEi}(1+i) - \text{ExpIntegralEi}(t+i)) + 4e^{1+i})$$

## 2.13 problem 14

Internal problem ID [1661]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 14.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-2yt + y' - 1 = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve([-2*t*y(t)+diff(y(t),t) = 1,y(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{\left(\sqrt{\pi}\,\operatorname{erf}(t) + 2\right) e^{t^2}}{2}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 24

```
DSolve[{-2*t*y[t]+y'[t] == 1,y[0]==1},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} e^{t^2} \left(\sqrt{\pi}\operatorname{erf}(t) + 2\right)$$

## 2.14 problem 15

Internal problem ID [1662]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 15.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{yt + (t^2+1)\, y' - (t^2+1)^{\frac{5}{2}} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(t*y(t)+(t^2+1)*diff(y(t),t) = (t^2+1)^(5/2),y(t), singsol=all)
```

$$y(t) = \frac{\frac{1}{5}t^5 + \frac{2}{3}t^3 + t + c_1}{\sqrt{t^2+1}}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 36

```
DSolve[t*y[t]+(t^2+1)*y'[t] == (t^2+1)^(5/2),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{3t^5 + 10t^3 + 15t + 15c_1}{15\sqrt{t^2+1}}$$

## 2.15 problem 16

Internal problem ID [1663]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 16.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{4yt + (t^2+1)\, y' - t = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([4*t*y(t)+(t^2+1)*diff(y(t),t) = t,y(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{1}{4} - \frac{1}{4\,(t^2+1)^2}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 20

```
DSolve[{4*t*y[t]+(t^2+1)*y'[t]== t,y[0]==0},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4} - \frac{1}{4\,(t^2+1)^2}$$

## 2.16 problem 20

Internal problem ID [1664]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 20.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{t} - \frac{1}{t^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(t),t)+1/t*y(t)=1/t^2,y(t), singsol=all)
```

$$y(t) = \frac{\ln(t) + c_1}{t}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 14

```
DSolve[y'[t]+1/t*y[t]==1/t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\log(t) + c_1}{t}$$

## 2.17 problem 21

Internal problem ID [1665]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 21.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{\sqrt{t}} - e^{\frac{\sqrt{t}}{2}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(t),t)+1/sqrt(t)*y(t)=exp(sqrt(t)/2),y(t), singsol=all)
```

$$y(t) = \left(\frac{4\,e^{\frac{5\sqrt{t}}{2}}\sqrt{t}}{5} - \frac{8\,e^{\frac{5\sqrt{t}}{2}}}{25} + c_1\right) e^{-2\sqrt{t}}$$

✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 42

```
DSolve[y'[t]+1/Sqrt[t]*y[t]==Exp[Sqrt[t]/2],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{4}{25} e^{\frac{\sqrt{t}}{2}} \left(5\sqrt{t} - 2\right) + c_1 e^{-2\sqrt{t}}$$

## 2.18 problem 22

Internal problem ID [1666]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 22.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{t} - \cos(t) - \frac{\sin(t)}{t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(t),t)+1/t*y(t)=cos(t)+sin(t)/t,y(t), singsol=all)
```

$$y(t) = \sin(t) + \frac{c_1}{t}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 14

```
DSolve[y'[t]+1/t*y[t]==Cos[t]+Sin[t]/t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sin(t) + \frac{c_1}{t}$$

## 2.19 problem 23

Internal problem ID [1667]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.2. Page 9
**Problem number**: 23.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \tan(t)\, y - \sin(t)\cos(t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(t),t)+tan(t)*y(t)=cos(t)*sin(t),y(t), singsol=all)
```

$$y(t) = (-\cos(t) + c_1)\cos(t)$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 15

```
DSolve[y'[t]+Tan[t]*y[t]==Cos[t]*Sin[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \cos(t)(-\cos(t) + c_1)$$

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## 3.1 problem 1

Internal problem ID [1668]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 1.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(t^2+1)\,y'-1-y^2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve((t^2+1)*diff(y(t),t) = 1+y(t)^2,y(t), singsol=all)
```

$$y(t)=\tan\left(\arctan\left(t\right)+c_1\right)$$

✓ Solution by Mathematica

Time used: 0.241 (sec). Leaf size: 25

```
DSolve[(t^2+1)*y'[t] == 1+y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\tan(\arctan(t)+c_1)$$
$$y(t)\to -i$$
$$y(t)\to i$$

## 3.2 problem 2

Internal problem ID [1669]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 2.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (t+1)(1+y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(t),t) = (1+t)*(1+y(t)),y(t), singsol=all)
```

$$y(t) = -1 + e^{\frac{t(2+t)}{2}} c_1$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 25

```
DSolve[y'[t] == (1+t)*(1+y[t]),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -1 + c_1 e^{\frac{1}{2}t(t+2)}$$
$$y(t) \to -1$$

## 3.3 problem 3

Internal problem ID [1670]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 3.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 1 + t - y^2 + ty^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(t),t) = 1-t+y(t)^2-t*y(t)^2,y(t), singsol=all)
```

$$y(t) = -\tan\left(\frac{1}{2}t^2 + c_1 - t\right)$$

✓ Solution by Mathematica

Time used: 0.18 (sec). Leaf size: 17

```
DSolve[y'[t] == 1-t+y[t]^2-t*y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \tan\left(-\frac{t^2}{2} + t + c_1\right)$$

## 3.4 problem 4

Internal problem ID [1671]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 4.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{3+t+y} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(t),t) = exp(3+t+y(t)),y(t), singsol=all)
```

$$y(t) = -3 - \ln\left(-e^{t} - c_1\right)$$

✓ Solution by Mathematica

Time used: 0.816 (sec). Leaf size: 20

```
DSolve[y'[t] == Exp[3+t+y[t]],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -\log\left(-e^{t+3} - c_1\right)$$

## 3.5 problem 5

Internal problem ID [1672]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 5.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(y)\sin(t)\, y' - \cos(t)\sin(y) = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 9

```
dsolve(cos(y(t))*sin(t)*diff(y(t),t) = cos(t)*sin(y(t)),y(t), singsol=all)
```

$$y(t) = \arcsin(c_1 \sin(t))$$

✓ Solution by Mathematica

Time used: 3.089 (sec). Leaf size: 19

```
DSolve[Cos[y[t]]*Sin[t]*y'[t] == Cos[t]*Sin[y[t]],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \arcsin\left(\frac{1}{2}c_1 \sin(t)\right)$$
$$y(t) \to 0$$

## 3.6 problem 6

Internal problem ID [1673]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 6.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{t^2(1+y^2)+2y'y=0}$$

With initial conditions

$$[y(0)=1]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 16

```
dsolve([t^2*(1+y(t)^2)+2*y(t)*diff(y(t),t) = 0,y(0) = 1],y(t), singsol=all)
```

$$y(t)=\sqrt{2\,\mathrm{e}^{-\frac{t^3}{3}}-1}$$

✓ Solution by Mathematica

Time used: 3.68 (sec). Leaf size: 43

```
DSolve[{t^2*(1+y[t]^2)+2*y[t]*y'[t] == 0,y[0]==1},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\sqrt{2e^{-\frac{t^3}{3}}-1}$$
$$y(t)\to\sqrt{2e^{-\frac{t^3}{3}}-1}$$

## 3.7 problem 7

Internal problem ID [1674]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 7.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2t}{y + yt^2} = 0}$$

With initial conditions

$$[y(2) = 3]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 20

```
dsolve([diff(y(t),t) = 2*t/(y(t)+t^2*y(t)),y(2) = 3],y(t), singsol=all)
```

$$y(t) = \sqrt{2\ln(t^2+1) - 2\ln(5) + 9}$$

✓ Solution by Mathematica

Time used: 0.093 (sec). Leaf size: 23

```
DSolve[{y'[t] == 2*t/(y[t]+t^2*y[t]),y[2]==3},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sqrt{2\log(t^2+1) + 9 - 2\log(5)}$$

## 3.8 problem 8

Internal problem ID [1675]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 8.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sqrt{t^2+1}\,y' - \frac{t y^3}{\sqrt{t^2+1}} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 16

```
dsolve([(t^2+1)^(1/2)*diff(y(t),t) = t*y(t)^3/(t^2+1)^(1/2),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{1}{\sqrt{1-\ln(t^2+1)}}$$

✓ Solution by Mathematica

Time used: 0.207 (sec). Leaf size: 19

```
DSolve[{(t^2+1)^(1/2)*y'[t] == t*y[t]^3/(t^2+1)^(1/2),y[0]==1},y[t],t,IncludeSingularSolution
```

$$y(t) \to \frac{1}{\sqrt{1-\log(t^2+1)}}$$

## 3.9 problem 9

Internal problem ID [1676]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 9.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{3t^2 + 4t + 2}{-2 + 2y} = 0}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 19

```
dsolve([diff(y(t),t) = (3*t^2+4*t+2)/(-2+2*y(t)),y(0) = -1],y(t), singsol=all)
```

$$y(t) = -\sqrt{(2+t)(t^2+2)} + 1$$

✓ Solution by Mathematica

Time used: 0.126 (sec). Leaf size: 22

```
DSolve[{y'[t] == (3*t^2+4*t+2)/(-2+2*y[t]),y[0]==-1},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 1 - \sqrt{(t+2)(t^2+2)}$$

## 3.10 problem 10

Internal problem ID [1677]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 10.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{\cos(y)\,y' + \frac{t\sin(y)}{t^2+1} = 0}$$

With initial conditions

$$\left[y(1) = \frac{\pi}{2}\right]$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 32

```
dsolve([cos(y(t))*diff(y(t),t) = -t*sin(y(t))/(t^2+1),y(1) = 1/2*Pi],y(t), singsol=all)
```

$$y(t) = \arcsin\left(\frac{\sqrt{2}}{\sqrt{t^2+1}}\right) + 2\arccos\left(\frac{\sqrt{2}}{\sqrt{t^2+1}}\right)\_B3$$

✓ Solution by Mathematica

Time used: 17.138 (sec). Leaf size: 21

```
DSolve[{Cos[y[t]]*y'[t] == -t*Sin[y[t]]/(t^2+1),y[1]==Pi/2},y[t],t,IncludeSingularSolutions -
```

$$y(t) \to \arcsin\left(\frac{\sqrt{2}}{\sqrt{t^2+1}}\right)$$

## 3.11 problem 11

Internal problem ID [1678]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 11.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - k(a-y)(b-y) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.437 (sec). Leaf size: 35

```
dsolve([diff(y(t),t) = k*(a-y(t))*(b-y(t)),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{ab\left(e^{tk(a-b)} - 1\right)}{e^{tk(a-b)}a - b}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 40

```
DSolve[{y'[t] == k*(a-y[t])*(b-y[t]),y[0]==0},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{a(a-b)e^{akt}}{be^{bkt} - ae^{akt}} + a$$

## 3.12 problem 12

Internal problem ID [1679]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 12.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{3ty' - \cos(t)\, y = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 5

```
dsolve([3*t*diff(y(t),t) = cos(t)*y(t),y(1) = 0],y(t), singsol=all)
```

$$y(t) = 0$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{3*t*y'[t] == Cos[t]*y[t],y[1]==0},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 0$$

## 3.13 problem 15

Internal problem ID [1680]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 15.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{ty' - y - \sqrt{t^2 + y^2} = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.359 (sec). Leaf size: 21

```
dsolve([t*diff(y(t),t)=y(t)+sqrt(t^2+y(t)^2),y(1) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{t^2}{2} + \frac{1}{2}$$
$$y(t) = \frac{t^2}{2} - \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.343 (sec). Leaf size: 14

```
DSolve[{t*y'[t]==y[t]+Sqrt[t^2+y[t]^2],y[1]==0},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}(t^2 - 1)$$

## 3.14 problem 16

Internal problem ID [1681]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 16.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{2tyy' - 3y^2 + t^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(2*t*y(t)*diff(y(t),t)=3*y(t)^2-t^2,y(t), singsol=all)
```

$$y(t) = \sqrt{c_1 t + 1}\, t$$
$$y(t) = -\sqrt{c_1 t + 1}\, t$$

✓ Solution by Mathematica

Time used: 0.212 (sec). Leaf size: 34

```
DSolve[2*t*y[t]*y'[t]==3*y[t]^2-t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -t\sqrt{1 + c_1 t}$$
$$y(t) \to t\sqrt{1 + c_1 t}$$

## 3.15 problem 17

Internal problem ID [1682]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 17.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{\left(t-\sqrt{yt}\right)y'-y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((t-sqrt(t*y(t)))*diff(y(t),t)=y(t),y(t), singsol=all)
```

$$\ln\left(y(t)\right)+\frac{2t}{\sqrt{ty\left(t\right)}}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.21 (sec). Leaf size: 31

```
DSolve[(t-Sqrt[t*y[t]])*y'[t]==y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2}{\sqrt{\frac{y(t)}{t}}}+\log\left(\frac{y(t)}{t}\right)=-\log(t)+c_1,y(t)\right]$$

## 3.16 problem 18

Internal problem ID [1683]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 18.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{t+y}{t-y} = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 24

```
dsolve(diff(y(t),t)=(t+y(t))/(t-y(t)),y(t), singsol=all)
```

$$y(t) = \tan\left(\mathrm{RootOf}\left(-2\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(t) + 2c_1\right)\right) t$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 36

```
DSolve[y'[t]==(t+y[t])/(t-y[t]),y[t],t,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{1}{2}\log\left(\frac{y(t)^2}{t^2}+1\right) - \arctan\left(\frac{y(t)}{t}\right) = -\log(t) + c_1, y(t)\right]$$

## 3.17 problem 19

Internal problem ID [1684]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 19.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\mathrm{e}^{\frac{t}{y}}(y-t)\,y'+y\left(1+\mathrm{e}^{\frac{t}{y}}\right)=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 20

```
dsolve(exp(t/y(t))*(y(t)-t)*diff(y(t),t)+y(t)*(1+exp(t/y(t)))=0,y(t), singsol=all)
```

$$y(t)=-\frac{t}{\mathrm{LambertW}\left(\frac{c_1 t}{c_1 t-1}\right)}$$

✓ Solution by Mathematica

Time used: 1.234 (sec). Leaf size: 34

```
DSolve[Exp[t/y[t]]*(y[t]-t)*y'[t]+y[t]*(1+Exp[t/y[t]])==0,y[t],t,IncludeSingularSolutions ->
```

$$y(t)\to-\frac{t}{W\left(\frac{t}{t-e^{c_1}}\right)}$$

$$y(t)\to t\left(-e^{W(1)}\right)$$

## 3.18 problem 20

Internal problem ID [1685]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 20.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{t+y+1}{t-y+3} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve(diff(y(t),t)=(t+y(t)+1)/(t-y(t)+3),y(t), singsol=all)
```

$$y(t) = 1 - \tan\left(\operatorname{RootOf}\left(2\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(2+t) + 2c_1\right)\right)(2+t)$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 57

```
DSolve[y'[t]==(t+y[t]+1)/(t-y[t]+3),y[t],t,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{y(t)+t+1}{-y(t)+t+3}\right) = \log\left(\frac{t^2+y(t)^2-2y(t)+4t+5}{2(t+2)^2}\right) + 2\log(t+2) + c_1, y(t)\right]$$

## 3.19 problem 22

Internal problem ID [1686]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 22.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{1 + t - 2y + (4t - 3y - 6)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.422 (sec). Leaf size: 39

```
dsolve((1+t-2*y(t))+(4*t-3*y(t)-6)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = 2 - \frac{(t-3)\left(c_1 \operatorname{RootOf}\left(3(t-3)^4 c_1\_Z^{20} - \_Z^4 - 4\right)^4 + c_1\right)}{3c_1}$$

✓ Solution by Mathematica

Time used: 60.067 (sec). Leaf size: 1511

```
DSolve[(1+t-2*y[t])+(4*t-3*y[t]-6)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{2}{3}(2t-3)$$

$$- \overline{3\mathrm{Root}\left[\#1^5\left(3125e^{\frac{5c_1}{9}}t^5 - 46875e^{\frac{5c_1}{9}}t^4 + 281250e^{\frac{5c_1}{9}}t^3 - 843750e^{\frac{5c_1}{9}}t^2 + 3125t + 1265625e^{\frac{5c_1}{9}}t - 9375\right.\right.}$$

$$y(t) \to \frac{2}{3}(2t-3)$$

$$- \overline{3\mathrm{Root}\left[\#1^5\left(3125e^{\frac{5c_1}{9}}t^5 - 46875e^{\frac{5c_1}{9}}t^4 + 281250e^{\frac{5c_1}{9}}t^3 - 843750e^{\frac{5c_1}{9}}t^2 + 3125t + 1265625e^{\frac{5c_1}{9}}t - 9375\right.\right.}$$

$$y(t) \to \frac{2}{3}(2t-3)$$

$$- \overline{3\mathrm{Root}\left[\#1^5\left(3125e^{\frac{5c_1}{9}}t^5 - 46875e^{\frac{5c_1}{9}}t^4 + 281250e^{\frac{5c_1}{9}}t^3 - 843750e^{\frac{5c_1}{9}}t^2 + 3125t + 1265625e^{\frac{5c_1}{9}}t - 9375\right.\right.}$$

$$y(t) \to \frac{2}{3}(2t-3)$$

$$- \overline{3\mathrm{Root}\left[\#1^5\left(3125e^{\frac{5c_1}{9}}t^5 - 46875e^{\frac{5c_1}{9}}t^4 + 281250e^{\frac{5c_1}{9}}t^3 - 843750e^{\frac{5c_1}{9}}t^2 + 3125t + 1265625e^{\frac{5c_1}{9}}t - 9375\right.\right.}$$

$$y(t) \to \frac{2}{3}(2t-3)$$

$$- \overline{3\mathrm{Root}\left[\#1^5\left(3125e^{\frac{5c_1}{9}}t^5 - 46875e^{\frac{5c_1}{9}}t^4 + 281250e^{\frac{5c_1}{9}}t^3 - 843750e^{\frac{5c_1}{9}}t^2 + 3125t + 1265625e^{\frac{5c_1}{9}}t - 9375\right.\right.}$$

## 3.20 problem 23

Internal problem ID [1687]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.4. Page 24
**Problem number**: 23.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd typ`

$$t + 2y + 3 + (2t + 4y - 1)\, y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
dsolve((t+2*y(t)+3)+(2*t+4*y(t)-1)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = -\frac{t}{2} + \frac{1}{4} - \frac{\sqrt{28c_1 - 28t + 1}}{4}$$
$$y(t) = -\frac{t}{2} + \frac{1}{4} + \frac{\sqrt{28c_1 - 28t + 1}}{4}$$

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 55

```
DSolve[(t+2*y[t]+3)+(2*t+4*y[t]-1)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4}\left(-2t - \sqrt{-28t + 1 + 16c_1} + 1\right)$$
$$y(t) \to \frac{1}{4}\left(-2t + \sqrt{-28t + 1 + 16c_1} + 1\right)$$

# 4 Section 1.9. Page 66

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## 4.1 problem 3

Internal problem ID [1688]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.9. Page 66
**Problem number**: 3.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact]`

$$2t\sin(y) + e^t y^3 + \left(t^2\cos(y) + 3e^t y^2\right) y' = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 19

```
dsolve(2*t*sin(y(t))+exp(t)*y(t)^3+(t^2*cos(y(t))+3*exp(t)*y(t)^2)*diff(y(t),t) = 0,y(t), sin
```

$$e^t y(t)^3 + t^2 \sin(y(t)) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.406 (sec). Leaf size: 22

```
DSolve[2*t*Sin[y[t]]+Exp[t]*y[t]^3+(t^2*Cos[y[t]]+3*Exp[t]*y[t]^2)*y'[t]== 0,y[t],t,IncludeSi
```

$$\text{Solve}\left[t^2\sin(y(t)) + e^t y(t)^3 = c_1, y(t)\right]$$

## 4.2 problem 4

Internal problem ID [1689]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.9. Page 66
**Problem number**: 4.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact]`

$$1 + e^{yt}(1 + yt) + \left(1 + e^{yt}t^2\right) y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(1+exp(t*y(t))*(1+t*y(t))+(1+exp(t*y(t))*t^2)*diff(y(t),t) = 0,y(t), singsol=all)
```

$$y(t) = -\frac{tc_1 + t^2 + \text{LambertW}\left(t^2 e^{-tc_1} e^{-t^2}\right)}{t}$$

✓ Solution by Mathematica

Time used: 2.836 (sec). Leaf size: 31

```
DSolve[1+Exp[t*y[t]]*(1+t*y[t])+(1+Exp[t*y[t]]*t^2)*y'[t] == 0,y[t],t,IncludeSingularSolution
```

$$y(t) \to -\frac{W\left(t^2 e^{t(-t+c_1)}\right)}{t} - t + c_1$$

## 4.3 problem 5

Internal problem ID [1690]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.9. Page 66
**Problem number**: 5.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, [_Abel, '2nd type', 'class A']]`

$$\sec(t)\tan(t) + \sec(t)^2 y + (\tan(t) + 2y)\, y' = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 63

```
dsolve(sec(t)*tan(t)+sec(t)^2*y(t)+(tan(t)+2*y(t))*diff(y(t),t) = 0,y(t), singsol=all)
```

$$y(t) = -\frac{\sin(t) + \sqrt{-4\cos(t)^2 c_1 + \sin(t)^2 - 4\cos(t)}}{2\cos(t)}$$

$$y(t) = \frac{-\sin(t) + \sqrt{-4\cos(t)^2 c_1 + \sin(t)^2 - 4\cos(t)}}{2\cos(t)}$$

✓ Solution by Mathematica

Time used: 1.067 (sec). Leaf size: 96

```
DSolve[Sec[t]*Tan[t]+Sec[t]^2*y[t]+(Tan[t]+2*y[t])*y'[t]== 0,y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{1}{4}\left(-2\tan(t) - \sqrt{\sec^2(t)}\sqrt{-16\cos(t) + (-2 + 8c_1)\cos(2t) + 2 + 8c_1}\right)$$

$$y(t) \to \frac{1}{4}\left(-2\tan(t) + \sqrt{\sec^2(t)}\sqrt{-16\cos(t) + (-2 + 8c_1)\cos(2t) + 2 + 8c_1}\right)$$

## 4.4 problem 6

Internal problem ID [1691]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.9. Page 66
**Problem number**: 6.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], [_Abel, '2nd type', 'cl`

$$\boxed{\frac{y^2}{2} - 2\,\mathrm{e}^t y + \left(-\mathrm{e}^t + y\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 39

```
dsolve(1/2*y(t)^2-2*exp(t)*y(t)+(-exp(t)+y(t))*diff(y(t),t) = 0,y(t), singsol=all)
```

$$y(t) = \left(1 - \sqrt{1 + c_1 \mathrm{e}^{-3t}}\right) \mathrm{e}^t$$
$$y(t) = \left(1 + \sqrt{1 + c_1 \mathrm{e}^{-3t}}\right) \mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 1.187 (sec). Leaf size: 70

```
DSolve[1/2*y[t]^2-2*Exp[t]*y[t]+(-Exp[t]+y[t])*y'[t] == 0,y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to e^t - \frac{\sqrt{-e^{3t} - c_1}}{\sqrt{-e^t}}$$
$$y(t) \to e^t + \frac{\sqrt{-e^{3t} - c_1}}{\sqrt{-e^t}}$$

## 4.5 problem 7

Internal problem ID [1692]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.9. Page 66
**Problem number**: 7.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2ty^3 + 3t^2y^2y' = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 7

```
dsolve([2*t*y(t)^3+3*t^2*y(t)^2*diff(y(t),t) = 0,y(1) = 1],y(t), singsol=all)
```

$$y(t) = \frac{1}{t^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 10

```
DSolve[{2*t*y[t]^3+3*t^2*y[t]^2*y'[t] == 0,y[1]==1},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{t^{2/3}}$$

## 4.6 problem 8

Internal problem ID [1693]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.9. Page 66
**Problem number**: 8.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact]`

$$2t\cos(y) + 3yt^2 + \left(t^3 - t^2\sin(y) - y\right)y' = 0$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 23

```
dsolve([2*t*cos(y(t))+3*t^2*y(t)+(t^3-t^2*sin(y(t))-y(t))*diff(y(t),t) = 0,y(0) = 2],y(t), si
```

$$y(t) = \text{RootOf}\left(-2\_Zt^3 - 2\cos(\_Z)\,t^2 + \_Z^2 - 4\right)$$

✓ Solution by Mathematica

Time used: 0.252 (sec). Leaf size: 27

```
DSolve[{2*t*Cos[y[t]]+3*t^2*y[t]+(t^3-t^2*Sin[y[t]]-y[t])*y'[t] == 0,y[0]==2},y[t],t,IncludeS
```

$$\text{Solve}\left[t^3y(t) + t^2\cos(y(t)) - \frac{y(t)^2}{2} = -2, y(t)\right]$$

## 4.7 problem 9

Internal problem ID [1694]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.9. Page 66
**Problem number**: 9.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'],`

$$\boxed{3t^2 + 4yt + (2t^2 + 2y)\, y' = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 22

```
dsolve([3*t^2+4*t*y(t)+(2*t^2+2*y(t))*diff(y(t),t) = 0,y(0) = 1],y(t), singsol=all)
```

$$y(t) = -t^2 + \sqrt{t^4 - t^3 + 1}$$

✓ Solution by Mathematica

Time used: 0.132 (sec). Leaf size: 24

```
DSolve[{3*t^2+4*t*y[t]+(2*t^2+2*y[t])*y'[t] == 0,y[0]==1},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to \sqrt{(t-1)t^3 + 1} - t^2$$

## 4.8 problem 10

Internal problem ID [1695]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.9. Page 66
**Problem number**: 10.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact]`

$$2t - 2\,\mathrm{e}^{yt}\sin(2t) + \mathrm{e}^{yt}\cos(2t)\,y + \left(-3 + \mathrm{e}^{yt} t\cos(2t)\right) y' = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 39.109 (sec). Leaf size: 36

```
dsolve([2*t-2*exp(t*y(t))*sin(2*t)+exp(t*y(t))*cos(2*t)*y(t)+(-3+exp(t*y(t))*t*cos(2*t))*diff
```

$$y(t) = \frac{t^3 - 3\,\mathrm{LambertW}\left(-\frac{t\cos(2t)\mathrm{e}^{\frac{t(t-1)(t+1)}{3}}}{3}\right) - t}{3t}$$

✓ Solution by Mathematica

Time used: 4.287 (sec). Leaf size: 41

```
DSolve[{2*t-2*Exp[t*y[t]]*Sin[2*t]+Exp[t*y[t]]*Cos[2*t]*y[t]+(-3+Exp[t*y[t]]*t*Cos[2*t])*y'[t
```

$$y(t) \to \frac{1}{3}\left(-\frac{3W\left(-\frac{1}{3}e^{\frac{1}{3}t(t^2-1)}t\cos(2t)\right)}{t} + t^2 - 1\right)$$

## 4.9 problem 11

Internal problem ID [1696]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.9. Page 66
**Problem number**: 11.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{3yt + y^2 + (t^2 + yt)\, y' = 0}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 21

```
dsolve([3*t*y(t)+y(t)^2+(t^2+t*y(t))*diff(y(t),t) = 0,y(2) = 1],y(t), singsol=all)
```

$$y(t) = \frac{-t^2 + \sqrt{t^4 + 20}}{t}$$

✓ Solution by Mathematica

Time used: 0.669 (sec). Leaf size: 22

```
DSolve[{3*t*y[t]+y[t]^2+(t^2+t*y[t])*y'[t] == 0,y[2]==1},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to \frac{\sqrt{t^4 + 20}}{t} - t$$

# 5 Section 1.10. Page 80

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## 5.1 problem 4

Internal problem ID [1697]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.10. Page 80
**Problem number**: 4.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \cos\left(t^2\right) = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(t),t)= y(t)^2+cos(t^2),y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[t]== y[t]^2+Cos[t^2],y[t],t,IncludeSingularSolutions -> True]
```

Not solved

## 5.2 problem 5

Internal problem ID [1698]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.10. Page 80
**Problem number**: 5.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - 1 - y - y^2 \cos(t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 1211

```
dsolve(diff(y(t),t)= 1+y(t)+y(t)^2*cos(t),y(t), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[t]== 1+y[t]+y[t]^2*Cos[t],y[t],t,IncludeSingularSolutions -> True]
```

Not solved

## 5.3 problem 6

Internal problem ID [1699]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.10. Page 80
**Problem number**: 6.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - t - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(t),t)= t+y(t)^2,y(t), singsol=all)
```

$$y(t) = \frac{c_1 \operatorname{AiryAi}(1, -t) + \operatorname{AiryBi}(1, -t)}{c_1 \operatorname{AiryAi}(-t) + \operatorname{AiryBi}(-t)}$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 110

```
DSolve[y'[t]== t+y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\sqrt{t}\left(-\operatorname{BesselJ}\left(-\frac{2}{3}, \frac{2t^{3/2}}{3}\right) + c_1 \operatorname{BesselJ}\left(\frac{2}{3}, \frac{2t^{3/2}}{3}\right)\right)}{\operatorname{BesselJ}\left(\frac{1}{3}, \frac{2t^{3/2}}{3}\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{3}, \frac{2t^{3/2}}{3}\right)}$$

$$y(t) \to \frac{t^2 \, {}_0\tilde{F}_1\left(; \frac{5}{3}; -\frac{t^3}{9}\right)}{3 \, {}_0\tilde{F}_1\left(; \frac{2}{3}; -\frac{t^3}{9}\right)}$$

## 5.4 problem 7

Internal problem ID [1700]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.10. Page 80
**Problem number**: 7.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \mathrm{e}^{-t^2} - y^2 = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(t),t)= exp(-t^2)+y(t)^2,y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[t]== Exp[-t^2]+y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

Not solved

## 5.5 problem 8

Internal problem ID [1701]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.10. Page 80
**Problem number**: 8.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - e^{-t^2} - y^2 = 0$$

✗ Solution by Maple

```
dsolve(diff(y(t),t)= exp(-t^2)+y(t)^2,y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[t]== Exp[-t^2]+y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

Not solved

## 5.6 problem 9

Internal problem ID [1702]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.10. Page 80
**Problem number**: 9.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \mathrm{e}^{-t^2} - y^2 = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(t),t)= exp(-t^2)+y(t)^2,y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[t]== Exp[-t^2]+y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

Not solved

## 5.7 problem 10

Internal problem ID [1703]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.10. Page 80
**Problem number**: 10.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - y - e^{-y} - e^{-t} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(t),t)= y(t)+exp(-y(t))+exp(-t),y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[t]== y[t]+Exp[-y[t]]+Exp[-t],y[t],t,IncludeSingularSolutions -> True]
```

Not solved

## 5.8 problem 11

Internal problem ID [1704]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.10. Page 80
**Problem number**: 11.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_Abel]`

$$y' - y^3 - e^{-5t} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(t),t)= y(t)^3+exp(-5*t),y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[t]== y[t]^3+Exp[-5*t],y[t],t,IncludeSingularSolutions -> True]
```

Not solved

## 5.9 problem 12

Internal problem ID [1705]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.10. Page 80
**Problem number**: 12.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \mathrm{e}^{(y-t)^2} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 24

```
dsolve(diff(y(t),t)= exp((y(t)-t)^2),y(t), singsol=all)
```

$$y(t) = t + \mathrm{RootOf}\left(-t + \int^{\_Z} \frac{1}{-1 + \mathrm{e}^{\_a^2}} d\_a + c_1\right)$$

✓ Solution by Mathematica

Time used: 1.017 (sec). Leaf size: 241

```
DSolve[y'[t]== Exp[(y[t]-t)^2],y[t],t,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\int_1^t -\frac{e^{(y(t)-K[1])^2}}{-1 + e^{(y(t)-K[1])^2}} dK[1] + \int_1^{y(t)}\right.$$

$$-\frac{e^{(t-K[2])^2} \int_1^t \left(\frac{2e^{2(K[2]-K[1])^2}(K[2]-K[1])}{\left(-1+e^{(K[2]-K[1])^2}\right)^2} - \frac{2e^{(K[2]-K[1])^2}(K[2]-K[1])}{-1+e^{(K[2]-K[1])^2}}\right) dK[1] - \int_1^t \left(\frac{2e^{2(K[2]-K[1])^2}(K[2]-K[1])}{\left(-1+e^{(K[2]-K[1])^2}\right)^2} - \frac{2e^{(K[2]-}{-1+}\right.}{-1 + e^{(t-K[2])^2}}$$

## 5.10 problem 13

Internal problem ID [1706]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.10. Page 80
**Problem number**: 13.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' - \left(4y + e^{-t^2}\right) e^{2y} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(t),t)=(4*y(t)+exp(-t^2))*exp(2*y(t)),y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[t]==(4*y[t]+Exp[-t^2])*Exp[2*y[t]],y[t],t,IncludeSingularSolutions -> True]
```

Not solved

## 5.11 problem 14

Internal problem ID [1707]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.10. Page 80
**Problem number**: 14.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - e^{-t} - \ln(1 + y^2) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✗ Solution by Maple

```
dsolve([diff(y(t),t)=exp(-t)+ln(1+y(t)^2),y(0) = 0],y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[t]==Exp[-t]+Log[1+y[t]^2],y[0]==0},y[t],t,IncludeSingularSolutions -> True]
```

Not solved

## 5.12 problem 15

Internal problem ID [1708]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.10. Page 80
**Problem number**: 15.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - \frac{(1+\cos(4t))\,y}{4} + \frac{(1-\cos(4t))\,y^2}{800} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(t),t)=1/4*(1+cos(4*t))*y(t)-1/800*(1-cos(4*t))*y(t)^2,y(t), singsol=all)
```

$$y(t) = \frac{e^{\frac{t}{4}+\frac{\sin(4t)}{16}}}{c_1 + \int -\frac{e^{\frac{t}{4}+\frac{\sin(4t)}{16}}(-1+\cos(4t))}{800}dt}$$

✓ Solution by Mathematica

Time used: 15.205 (sec). Leaf size: 122

```
DSolve[y'[t]==1/4*(1+Cos[4*t])*y[t]-1/800*(1-Cos[4*t])*y[t]^2,y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{e^{\frac{1}{16}(4t+\sin(4t))}}{-\int_1^t -\frac{1}{400}e^{\frac{1}{16}(4K[1]+\sin(4K[1]))}\sin^2(2K[1])dK[1] + c_1}$$

$$y(t) \to 0$$

$$y(t) \to -\frac{e^{\frac{1}{16}(4t+\sin(4t))}}{\int_1^t -\frac{1}{400}e^{\frac{1}{16}(4K[1]+\sin(4K[1]))}\sin^2(2K[1])dK[1]}$$

## 5.13 problem 16

Internal problem ID [1709]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.10. Page 80
**Problem number**: 16.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - t^2 - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(t),t)=t^2+y(t)^2,y(t), singsol=all)
```

$$y(t) = \frac{\left(-\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{t^2}{2}\right) c_1 - \operatorname{BesselY}\left(-\frac{3}{4}, \frac{t^2}{2}\right)\right) t}{c_1 \operatorname{BesselJ}\left(\frac{1}{4}, \frac{t^2}{2}\right) + \operatorname{BesselY}\left(\frac{1}{4}, \frac{t^2}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.12 (sec). Leaf size: 93

```
DSolve[y'[t]==t^2+y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t\left(-\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{t^2}{2}\right) + c_1 \operatorname{BesselJ}\left(\frac{3}{4}, \frac{t^2}{2}\right)\right)}{\operatorname{BesselJ}\left(\frac{1}{4}, \frac{t^2}{2}\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{t^2}{2}\right)}$$

$$y(t) \to \frac{t \operatorname{BesselJ}\left(\frac{3}{4}, \frac{t^2}{2}\right)}{\operatorname{BesselJ}\left(-\frac{1}{4}, \frac{t^2}{2}\right)}$$

## 5.14 problem 17

Internal problem ID [1710]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.10. Page 80
**Problem number**: 17.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - t(1 + y) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve(diff(y(t),t)=t*(1+y(t)),y(t), singsol=all)
```

$$y(t) = -1 + \mathrm{e}^{\frac{t^2}{2}} c_1$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 24

```
DSolve[y'[t]==t*(1+y[t]),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -1 + c_1 e^{\frac{t^2}{2}}$$
$$y(t) \to -1$$

## 5.15 problem 19

Internal problem ID [1711]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 1.10. Page 80
**Problem number**: 19.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - t\sqrt{1 - y^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(diff(y(t),t)=t*sqrt(1-y(t)^2),y(t), singsol=all)
```

$$y(t) = \sin\left(c_1 + \frac{t^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.201 (sec). Leaf size: 34

```
DSolve[y'[t]==t*Sqrt[1-y[t]^2],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \cos\left(\frac{t^2}{2} + c_1\right)$$
$$y(t) \to -1$$
$$y(t) \to 1$$
$$y(t) \to \text{Interval}[\{-1, 1\}]$$

# 6 Section 2.1, second order linear differential equations. Page 134

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6.2 problem 5(d) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 78
6.3 problem 6(a) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 79
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## 6.1 problem 5(a)

Internal problem ID [1712]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.1, second order linear differential equations. Page 134
**Problem number**: 5(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{2t^2y'' + 3ty' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*t^2*diff(y(t),t$2)+3*t*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1}{t} + c_2\sqrt{t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[2*t^2*y''[t]+3*t*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2t^{3/2} + c_1}{t}$$

## 6.2 problem 5(d)

Internal problem ID [1713]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.1, second order linear differential equations. Page 134
**Problem number**: 5(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{2t^2y'' + 3ty' - y = 0}$$

With initial conditions

$$[y(1) = 2, y'(1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 9

```
dsolve([2*t^2*diff(y(t),t$2)+3*t*diff(y(t),t)-y(t)=0,y(1) = 2, D(y)(1) = 1],y(t), singsol=all
```

$$y(t) = 2\sqrt{t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 12

```
DSolve[{2*t^2*y''[t]+3*t*y'[t]-y[t]==0,{y[1]==2,y'[1]==1}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \rightarrow 2\sqrt{t}$$

## 6.3 problem 6(a)

Internal problem ID [1714]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.1, second order linear differential equations. Page 134
**Problem number**: 6(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + ty' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(t),t$2)+t*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = \operatorname{erf}\left(\frac{i\sqrt{2}\,t}{2}\right) \mathrm{e}^{-\frac{t^2}{2}} c_1 + c_2 \mathrm{e}^{-\frac{t^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 34

```
DSolve[y''[t]+t*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sqrt{2} c_1 \operatorname{DawsonF}\left(\frac{t}{\sqrt{2}}\right) + c_2 e^{-\frac{t^2}{2}}$$

## 6.4 problem 6(d)

Internal problem ID [1715]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.1, second order linear differential equations. Page 134
**Problem number**: 6(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + ty' + y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve([diff(y(t),t$2)+t*diff(y(t),t)+y(t)=0,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = -\frac{i\mathrm{e}^{-\frac{t^2}{2}}\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,t}{2}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 19

```
DSolve[{y''[t]+t*y'[t]+y[t]==0,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sqrt{2}\,\mathrm{DawsonF}\left(\frac{t}{\sqrt{2}}\right)$$

# 7 Section 2.2, linear equations with constant coefficients. Page 138

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## 7.1 problem 1

Internal problem ID [1716]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2, linear equations with constant coefficients. Page 138
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(t),t$2)-y(t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-t} c_1 + c_2 \mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y''[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^t + c_2 e^{-t}$$

## 7.2 problem 2

Internal problem ID [1717]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2, linear equations with constant coefficients. Page 138
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{6y'' - 7y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(6*diff(y(t),t$2)-7*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^{\frac{t}{6}} + c_2 e^t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[6*y''[t]-7*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{t/6} + c_2 e^t$$

## 7.3 problem 3

Internal problem ID [1718]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2, linear equations with constant coefficients. Page 138
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(t),t$2)-3*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{\frac{(\sqrt{5}+3)t}{2}} + c_2 \mathrm{e}^{-\frac{(\sqrt{5}-3)t}{2}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 35

```
DSolve[y''[t]-3*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-\frac{1}{2}(\sqrt{5}-3)t}\left(c_2 e^{\sqrt{5}t} + c_1\right)$$

## 7.4 problem 4

Internal problem ID [1719]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2, linear equations with constant coefficients. Page 138
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{3y'' + 6y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(3*diff(y(t),t$2)+6*diff(y(t),t)+3*y(t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-t}c_1 + c_2\mathrm{e}^{-t}t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[3*y''[t]+6*y'[t]+3*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}(c_2t + c_1)$$

## 7.5 problem 5

Internal problem ID [1720]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2, linear equations with constant coefficients. Page 138
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' - 4y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)-3*diff(y(t),t)-4*y(t)=0,y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{4\,\mathrm{e}^{-t}}{5} + \frac{\mathrm{e}^{4t}}{5}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 21

```
DSolve[{y''[t]-3*y'[t]-4*y[t]==0,{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{5}e^{-t}\left(e^{5t} + 4\right)$$

## 7.6 problem 6

Internal problem ID [1721]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2, linear equations with constant coefficients. Page 138
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' + y' - 10y = 0}$$

With initial conditions

$$[y(1) = 5, y'(1) = 2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve([2*diff(y(t),t$2)+diff(y(t),t)-10*y(t)=0,y(1) = 5, D(y)(1) = 2],y(t), singsol=all)
```

$$y(t) = \frac{16\,\mathrm{e}^{\frac{5}{2}-\frac{5t}{2}}}{9} + \frac{29\,\mathrm{e}^{2t-2}}{9}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 30

```
DSolve[{2*y''[t]+y'[t]-10*y[t]==0,{y[1]==5,y'[1]==2}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to \frac{16}{9}e^{-\frac{5}{2}(t-1)} + \frac{29}{9}e^{2t-2}$$

## 7.7 problem 7

Internal problem ID [1722]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2, linear equations with constant coefficients. Page 138
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{5y'' + 5y' - y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 37

```
dsolve([5*diff(y(t),t$2)+5*diff(y(t),t)-y(t)=0,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{\left(\mathrm{e}^{\frac{3t\sqrt{5}}{10}-\frac{t}{2}} - \mathrm{e}^{-\frac{t}{2}-\frac{3t\sqrt{5}}{10}}\right)\sqrt{5}}{3}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 32

```
DSolve[{5*y''[t]+5*y'[t]-y[t]==0,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{2}{3}\sqrt{5}e^{-t/2}\sinh\left(\frac{3t}{2\sqrt{5}}\right)$$

## 7.8 problem 8

Internal problem ID [1723]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2, linear equations with constant coefficients. Page 138
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + y = 0}$$

With initial conditions

$$[y(2) = 1, y'(2) = 1]$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 44

```
dsolve([diff(y(t),t$2)-6*diff(y(t),t)+y(t)=0,y(2) = 1, D(y)(2) = 1],y(t), singsol=all)
```

$$y(t) = \frac{(2+\sqrt{2})\,\mathrm{e}^{-(t-2)\left(-3+2\sqrt{2}\right)}}{4} - \frac{\mathrm{e}^{(t-2)\left(3+2\sqrt{2}\right)}\left(\sqrt{2}-2\right)}{4}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 47

```
DSolve[{y''[t]-6*y'[t]+y[t]==0,{y[2]==1,y'[2]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{1}{2}e^{3t-6}\left(\sqrt{2}\sinh\left(2\sqrt{2}(t-2)\right) - 2\cosh\left(2\sqrt{2}(t-2)\right)\right)$$

## 7.9 problem 9

Internal problem ID [1724]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2, linear equations with constant coefficients. Page 138
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 5y' + 6y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = v]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)+5*diff(y(t),t)+6*y(t)=0,y(0) = 1, D(y)(0) = v],y(t), singsol=all)
```

$$y(t) = (-2 - v)\,\mathrm{e}^{-3t} + (v + 3)\,\mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 23

```
DSolve[{y''[t]+5*y'[t]+6*y[t]==0,{y[0]==1,y'[0]==v}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-3t}\left(e^t(v + 3) - v - 2\right)$$

## 7.10 problem 10

Internal problem ID [1725]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2, linear equations with constant coefficients. Page 138
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$t^2y'' + \alpha t y' + \beta y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve(t^2*diff(y(t),t$2)+alpha*t*diff(y(t),t)+beta*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t^{-\frac{\alpha}{2}+\frac{1}{2}+\frac{\sqrt{\alpha^2-2\alpha-4\beta+1}}{2}} + c_2 t^{-\frac{\alpha}{2}+\frac{1}{2}-\frac{\sqrt{\alpha^2-2\alpha-4\beta+1}}{2}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 53

```
DSolve[t^2*y''[t]+\[Alpha]*t*y'[t]+\[Beta]*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t^{\frac{1}{2}\left(-\sqrt{(\alpha-1)^2-4\beta}-\alpha+1\right)}\left(c_2 t^{\sqrt{(\alpha-1)^2-4\beta}} + c_1\right)$$

## 7.11 problem 11

Internal problem ID [1726]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2, linear equations with constant coefficients. Page 138
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$t^2y'' + 5ty' - 5y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(t^2*diff(y(t),t$2)+5*t*diff(y(t),t)-5*y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1}{t^5} + c_2t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]+5*t*y'[t]-5*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_1}{t^5} + c_2t$$

## 7.12 problem 12

Internal problem ID [1727]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2, linear equations with constant coefficients. Page 138
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' - ty' - 2y = 0}$$

With initial conditions

$$[y(1) = 0, y'(1) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 25

```
dsolve([t^2*diff(y(t),t$2)-t*diff(y(t),t)-2*y(t)=0,y(1) = 0, D(y)(1) = 1],y(t), singsol=all)
```

$$y(t) = \frac{\sqrt{3}\, t \left(t^{\sqrt{3}} - t^{-\sqrt{3}}\right)}{6}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 36

```
DSolve[{t^2*y''[t]-t*y'[t]-2*y[t]==0,{y[1]==0,y'[1]==1}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to \frac{t^{1-\sqrt{3}}\left(t^{2\sqrt{3}} - 1\right)}{2\sqrt{3}}$$

# 8 Section 2.2.1, Complex roots. Page 141


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## 8.1 problem Example 2

Internal problem ID [1728]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.1, Complex roots. Page 141
**Problem number**: Example 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 4y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+4*y(t)=0,y(0) = 1, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{e^{-t}\left(2\sqrt{3}\,\sin\left(\sqrt{3}\,t\right) + 3\cos\left(\sqrt{3}\,t\right)\right)}{3}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 40

```
DSolve[{y''[t]+2*y'[t]+4*y[t]==0,{y[0]==1,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{3}e^{-t}\left(2\sqrt{3}\sin\left(\sqrt{3}t\right) + 3\cos\left(\sqrt{3}t\right)\right)$$

## 8.2 problem 1

Internal problem ID [1729]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.1, Complex roots. Page 141
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(t),t$2)+diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\, t}{2}\right) + c_2 e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\, t}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[y''[t]+y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t/2}\left(c_2 \cos\left(\frac{\sqrt{3}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$

## 8.3 problem 2

Internal problem ID [1730]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.1, Complex roots. Page 141
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' + 3y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(2*diff(y(t),t$2)+3*diff(y(t),t)+4*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{-\frac{3t}{4}} \sin\left(\frac{\sqrt{23}\,t}{4}\right) + c_2 \mathrm{e}^{-\frac{3t}{4}} \cos\left(\frac{\sqrt{23}\,t}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 42

```
DSolve[2*y''[t]+3*y'[t]+4*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-3t/4}\left(c_2 \cos\left(\frac{\sqrt{23}t}{4}\right) + c_1 \sin\left(\frac{\sqrt{23}t}{4}\right)\right)$$

## 8.4 problem 3

Internal problem ID [1731]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.1, Complex roots. Page 141
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(t),t$2)+2*diff(y(t),t)+3*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^{-t} \sin\left(t\sqrt{2}\right) + c_2 e^{-t} \cos\left(t\sqrt{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 34

```
DSolve[y''[t]+2*y'[t]+3*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}\left(c_2 \cos\left(\sqrt{2}t\right) + c_1 \sin\left(\sqrt{2}t\right)\right)$$

## 8.5 problem 4

Internal problem ID [1732]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.1, Complex roots. Page 141
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(4*diff(y(t),t$2)-diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^{\frac{t}{8}} \sin\left(\frac{\sqrt{15}\, t}{8}\right) + c_2 e^{\frac{t}{8}} \cos\left(\frac{\sqrt{15}\, t}{8}\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 42

```
DSolve[4*y''[t]-y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{t/8}\left(c_2 \cos\left(\frac{\sqrt{15}t}{8}\right) + c_1 \sin\left(\frac{\sqrt{15}t}{8}\right)\right)$$

## 8.6 problem 5

Internal problem ID [1733]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.1, Complex roots. Page 141
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' + 2y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 32

```
dsolve([diff(y(t),t$2)+diff(y(t),t)+2*y(t)=0,y(0) = 1, D(y)(0) = 2],y(t), singsol=all)
```

$$y(t) = \frac{e^{-\frac{t}{2}} \left(5\sqrt{7}\, \sin\left(\frac{\sqrt{7}\, t}{2}\right) + 7 \cos\left(\frac{\sqrt{7}\, t}{2}\right)\right)}{7}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 48

```
DSolve[{2*y''[t]+3*y'[t]+4*y[t]==0,{y[0]==1,y'[0]==2}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to \frac{1}{23} e^{-3t/4} \left(11\sqrt{23} \sin\left(\frac{\sqrt{23}t}{4}\right) + 23 \cos\left(\frac{\sqrt{23}t}{4}\right)\right)$$

## 8.7 problem 6

Internal problem ID [1734]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.1, Complex roots. Page 141
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 5y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+5*y(t)=0,y(0) = 0, D(y)(0) = 2],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-t} \sin(2t)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 15

```
DSolve[{y''[t]+2*y'[t]+5*y[t]==0,{y[0]==0,y'[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t} \sin(2t)$$

## 8.8 problem 8

Internal problem ID [1735]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.1, Complex roots. Page 141
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' - y' + 3y = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = 1]$$

✓ Solution by Maple

Time used: 0.171 (sec). Leaf size: 79

```
dsolve([2*diff(y(t),t$2)-diff(y(t),t)+3*y(t)=0,y(1) = 1, D(y)(1) = 1],y(t), singsol=all)
```

$$y(t) = \frac{e^{-\frac{1}{4}+\frac{t}{4}} \left(3 \sin\left(\frac{\sqrt{23}\, t}{4}\right) \sqrt{23}\, \cos\left(\frac{\sqrt{23}}{4}\right) - 3 \cos\left(\frac{\sqrt{23}\, t}{4}\right) \sqrt{23}\, \sin\left(\frac{\sqrt{23}}{4}\right) + 23 \sin\left(\frac{\sqrt{23}\, t}{4}\right) \sin\left(\frac{\sqrt{23}}{4}\right) + 23 \cos\left(\right.\right.}{23}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 54

```
DSolve[{2*y''[t]-y'[t]+3*y[t]==0,{y[1]==1,y'[1]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{23} e^{\frac{t-1}{4}} \left(3\sqrt{23} \sin\left(\frac{1}{4}\sqrt{23}(t-1)\right) + 23 \cos\left(\frac{1}{4}\sqrt{23}(t-1)\right)\right)$$

## 8.9 problem 9

Internal problem ID [1736]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.1, Complex roots. Page 141
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{3y'' - 2y' + 4y = 0}$$

With initial conditions

$$[y(2) = 1, y'(2) = -1]$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 79

```
dsolve([3*diff(y(t),t$2)-2*diff(y(t),t)+4*y(t)=0,y(2) = 1, D(y)(2) = -1],y(t), singsol=all)
```

$$y(t) = \frac{e^{-\frac{2}{3}+\frac{t}{3}} \left(-4 \sin\left(\frac{\sqrt{11}\, t}{3}\right) \cos\left(\frac{2\sqrt{11}}{3}\right) \sqrt{11} + 4 \cos\left(\frac{\sqrt{11}\, t}{3}\right) \sin\left(\frac{2\sqrt{11}}{3}\right) \sqrt{11} + 11 \sin\left(\frac{\sqrt{11}\, t}{3}\right) \sin\left(\frac{2\sqrt{11}}{3}\right) + 11 \cos\right.}{11}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 54

```
DSolve[{3*y''[t]-2*y'[t]+4*y[t]==0,{y[2]==1,y'[2]==-1}},y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to \frac{1}{11} e^{\frac{t-2}{3}} \left(11 \cos\left(\frac{1}{3}\sqrt{11}(t-2)\right) - 4\sqrt{11} \sin\left(\frac{1}{3}\sqrt{11}(t-2)\right)\right)$$

## 8.10 problem 18

Internal problem ID [1737]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.1, Complex roots. Page 141
**Problem number**: 18.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$\boxed{t^2y'' + ty' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(t^2*diff(y(t),t$2)+t*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = \sin(\ln(t))\, c_1 + c_2 \cos(\ln(t))$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 18

```
DSolve[t^2*y''[t]+t*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \cos(\log(t)) + c_2 \sin(\log(t))$$

## 8.11 problem 19

Internal problem ID [1738]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.1, Complex roots. Page 141
**Problem number**: 19.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' + 2ty' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(t^2*diff(y(t),t$2)+2*t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1 \sin\left(\frac{\sqrt{7}\ln(t)}{2}\right)}{\sqrt{t}} + \frac{c_2 \cos\left(\frac{\sqrt{7}\ln(t)}{2}\right)}{\sqrt{t}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 42

```
DSolve[t^2*y''[t]+2*t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 \cos\left(\frac{1}{2}\sqrt{7}\log(t)\right) + c_1 \sin\left(\frac{1}{2}\sqrt{7}\log(t)\right)}{\sqrt{t}}$$

# 9 Section 2.2.2, Equal roots, reduction of order. Page 147


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## 9.1 problem 1

Internal problem ID [1739]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.2, Equal roots, reduction of order. Page 147
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(t),t$2)-6*diff(y(t),t)+9*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^{3t} + c_2 e^{3t} t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[y''[t]-6*y'[t]+9*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{3t}(c_2 t + c_1)$$

## 9.2 problem 2

Internal problem ID [1740]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.2, Equal roots, reduction of order. Page 147
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - 12y' + 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(4*diff(y(t),t$2)-12*diff(y(t),t)+9*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^{\frac{3t}{2}} + c_2 e^{\frac{3t}{2}} t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[4*y''[t]-12*y'[t]+9*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow e^{3t/2}(c_2 t + c_1)$$

## 9.3 problem 3

Internal problem ID [1741]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.2, Equal roots, reduction of order. Page 147
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{9y'' + 6y' + y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([9*diff(y(t),t$2)+6*diff(y(t),t)+y(t)=0,y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{e^{-\frac{t}{3}}(3 + t)}{3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 19

```
DSolve[{9*y''[t]+6*y'[t]+y[t]==0,{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{3}e^{-t/3}(t + 3)$$

## 9.4 problem 4

Internal problem ID [1742]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.2, Equal roots, reduction of order. Page 147
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - 4y' + y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve([4*diff(y(t),t$2)-4*diff(y(t),t)+y(t)=0,y(0) = 0, D(y)(0) = 3],y(t), singsol=all)
```

$$y(t) = 3t\,\mathrm{e}^{\frac{t}{2}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 15

```
DSolve[{4*y''[t]-4*y'[t]+y[t]==0,{y[0]==0,y'[0]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 3e^{t/2}t$$

## 9.5 problem 6

Internal problem ID [1743]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.2, Equal roots, reduction of order. Page 147
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + y = 0}$$

With initial conditions

$$[y(2) = 1, y'(2) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+y(t)=0,y(2) = 1, D(y)(2) = -1],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{2-t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 12

```
DSolve[{y''[t]+2*y'[t]+y[t]==0,{y[2]==1,y'[2]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{2-t}$$

## 9.6 problem 7

Internal problem ID [1744]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.2, Equal roots, reduction of order. Page 147
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{9y'' - 12y' + 4y = 0}$$

With initial conditions

$$[y(\pi) = 0, y'(\pi) = 2]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 19

```
dsolve([9*diff(y(t),t$2)-12*diff(y(t),t)+4*y(t)=0,y(Pi) = 0, D(y)(Pi) = 2],y(t), singsol=all)
```

$$y(t) = -2\,\mathrm{e}^{-\frac{2\pi}{3}+\frac{2t}{3}}(\pi - t)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 24

```
DSolve[{9*y''[t]-12*y'[t]+4*y[t]==0,{y[Pi]==0,y'[Pi]==2}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to e^{-\frac{2}{3}(\pi - t)}(2t - 2\pi)$$

## 9.7 problem 10

Internal problem ID [1745]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.2, Equal roots, reduction of order. Page 147
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{2(t+1)\,y'}{t^2+2t-1} + \frac{2y}{t^2+2t-1} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)-2*(t+1)/(t^2+2*t-1)*diff(y(t),t)+2/(t^2+2*t-1)*y(t)=0,y(t), singsol=all
```

$$y(t) = c_1(t+1) + c_2(t^2+1)$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 60

```
DSolve[y''[t]-2*(t+1)/(t^2+2*t-1)*y'[t]+2/(t^2+2*t-1)*y[t]==0,y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{\sqrt{t(t+2)-1}\left(c_1\left(t\left(t-2\sqrt{2}+2\right)-2\sqrt{2}+3\right)+c_2(t+1)\right)}{\sqrt{1-t(t+2)}}$$

## 9.8 problem 11

Internal problem ID [1746]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.2, Equal roots, reduction of order. Page 147
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4ty' + (4t^2 - 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(t),t$2)-4*t*diff(y(t),t)+(4*t^2-2)*y(t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{t^2} c_1 + c_2 t\, \mathrm{e}^{t^2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[y''[t]-4*t*y'[t]+(4*t^2-2)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{t^2} (c_2 t + c_1)$$

## 9.9 problem 12

Internal problem ID [1747]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.2, Equal roots, reduction of order. Page 147
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-t^2+1)\,y''-2ty'+2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve((1-t^2)*diff(y(t),t$2)-2*t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t)=tc_1+c_2\left(-\frac{\ln(t+1)\,t}{2}+\frac{\ln(t-1)\,t}{2}+1\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 19

```
DSolve[(1-t^2)*y''[t]-2*t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to c_2(t\operatorname{arctanh}(t)-1)+c_1t$$

## 9.10 problem 13

Internal problem ID [1748]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.2, Equal roots, reduction of order. Page 147
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(t^2+1)\,y''-2ty'+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+t^2)*diff(y(t),t$2)-2*t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t)=tc_1+c_2(t^2-1)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 21

```
DSolve[(1+t^2)*y''[t]-2*t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to c_2t-c_1(t-i)^2$$

## 9.11 problem 14

Internal problem ID [1749]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.2, Equal roots, reduction of order. Page 147
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-t^2+1)\,y''-2ty'+6y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve((1-t^2)*diff(y(t),t$2)-2*t*diff(y(t),t)+6*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1(-3t^2+1)+c_2\left(\left(\frac{3t^2}{8}-\frac{1}{8}\right)\ln(t-1)+\left(-\frac{3t^2}{8}+\frac{1}{8}\right)\ln(t+1)+\frac{3t}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 37

```
DSolve[(1-t^2)*y''[t]-2*t*y'[t]+6*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\frac{1}{2}\left(c_2(3t^2-1)\operatorname{arctanh}(t)+c_1(3t^2-1)-3c_2t\right)$$

## 9.12 problem 15

Internal problem ID [1750]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.2, Equal roots, reduction of order. Page 147
**Problem number**: 15.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1+2t)\,y''-4(t+1)\,y'+4y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((2*t+1)*diff(y(t),t$2)-4*(t+1)*diff(y(t),t)+4*y(t)=0,y(t), singsol=all)
```

$$y(t)=c_1(t+1)+c_2\mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 23

```
DSolve[(2*t+1)*y''[t]-4*(t+1)*y'[t]+4*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to c_1e^{2t+1}-c_2(t+1)$$

## 9.13 problem 16

Internal problem ID [1751]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.2, Equal roots, reduction of order. Page 147
**Problem number**: 16.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' + ty' + \left(t^2 - \frac{1}{4}\right) y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(t^2*diff(y(t),t$2)+t*diff(y(t),t)+(t^2-1/4)*y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1 \sin(t)}{\sqrt{t}} + \frac{c_2 \cos(t)}{\sqrt{t}}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 39

```
DSolve[t^2*y''[t]+t*y'[t]+(t^2-1/4)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{-it}(2c_1 - ic_2e^{2it})}{2\sqrt{t}}$$

## 9.14 problem 19

Internal problem ID [1752]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.2, Equal roots, reduction of order. Page 147
**Problem number**: 19.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{t^2y'' + 3ty' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(t^2*diff(y(t),t$2)+3*t*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1}{t} + \frac{c_2 \ln(t)}{t}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 17

```
DSolve[t^2*y''[t]+3*t*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 \log(t) + c_1}{t}$$

## 9.15 problem 20

Internal problem ID [1753]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.2.2, Equal roots, reduction of order. Page 147
**Problem number**: 20.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' - ty' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(t^2*diff(y(t),t$2)-t*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = tc_1 + c_2t\ln(t)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 15

```
DSolve[t^2*y''[t]-t*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t(c_2\log(t) + c_1)$$

# 10 Section 2.4, The method of variation of parameters. Page 154

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## 10.1 problem 1

Internal problem ID [1754]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.4, The method of variation of parameters. Page 154
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \sec(t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(t),t$2)+y(t)=sec(t),y(t), singsol=all)
```

$$y(t) = c_2 \sin(t) + \cos(t) c_1 + \sin(t) t - \ln(\sec(t)) \cos(t)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 22

```
DSolve[y''[t]+y[t]==Sec[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to (t + c_2) \sin(t) + \cos(t)(\log(\cos(t)) + c_1)$$

## 10.2 problem 2

Internal problem ID [1755]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.4, The method of variation of parameters. Page 154
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type **[[_2nd_order, _linear, _nonhomogeneous]]**

$$\boxed{y'' - 4y' + 4y - e^{2t}t = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(t),t$2)-4*diff(y(t),t)+4*y(t)=t*exp(2*t),y(t), singsol=all)
```

$$y(t) = c_2 e^{2t} + e^{2t} t c_1 + \frac{e^{2t} t^3}{6}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 27

```
DSolve[y''[t]-4*y'[t]+4*y[t]==t*Exp[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{6} e^{2t} (t^3 + 6c_2 t + 6c_1)$$

## 10.3 problem 3

Internal problem ID [1756]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.4, The method of variation of parameters. Page 154
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2y'' - 3y' + y - (t^2 + 1)\,\mathrm{e}^t = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
dsolve(2*diff(y(t),t$2)-3*diff(y(t),t)+y(t)=(t^2+1)*exp(t),y(t), singsol=all)
```

$$y(t) = \frac{\mathrm{e}^t t^3}{3} - 2\,\mathrm{e}^t t^2 + 9t\,\mathrm{e}^t - 18\,\mathrm{e}^t + 2c_1\mathrm{e}^t + c_2\mathrm{e}^{\frac{t}{2}}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 36

```
DSolve[2*y''[t]-3*y'[t]+y[t]==(t^2+1)*Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{t/2} + e^t\left(\frac{1}{3}t((t-6)t+27) - 18 + c_2\right)$$

## 10.4 problem 4

Internal problem ID [1757]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.4, The method of variation of parameters. Page 154
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y - t\,e^{3t} - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(t),t$2)-3*diff(y(t),t)+2*y(t)=t*exp(3*t)+1,y(t), singsol=all)
```

$$y(t) = c_1 e^{2t} + \frac{1}{2} - \frac{3\,e^{3t}}{4} + \frac{e^{3t}t}{2} + c_2 e^{t}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 37

```
DSolve[y''[t]-3*y'[t]+2*y[t]==t*Exp[3*t]+1,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4}e^{3t}(2t-3) + c_1 e^{t} + c_2 e^{2t} + \frac{1}{2}$$

## 10.5 problem 5

Internal problem ID [1758]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.4, The method of variation of parameters. Page 154
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{3y'' + 4y' + y - e^{-t} \sin(t) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve([3*diff(y(t),t$2)+4*diff(y(t),t)+y(t)=sin(t)*exp(-t),y(0) = 1, D(y)(0) = 0],y(t), sing
```

$$y(t) = \frac{24\, e^{-\frac{t}{3}}}{13} + \frac{(2\cos(t) - 3\sin(t) - 13)\, e^{-t}}{13}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 33

```
DSolve[{3*y''[t]+4*y'[t]+y[t]==Sin[t]*Exp[-t],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolut
```

$$y(t) \to \frac{1}{13} e^{-t} \left(24e^{2t/3} - 3\sin(t) + 2\cos(t) - 13\right)$$

## 10.6 problem 6

Internal problem ID [1759]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.4, The method of variation of parameters. Page 154
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y' + 4y - t^{\frac{5}{2}} e^{-2t} = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+4*y(t)=t^(5/2)*exp(-2*t),y(0) = 0, D(y)(0) = 0],y(t), s
```

$$y(t) = \frac{4t^{\frac{9}{2}} e^{-2t}}{63}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 19

```
DSolve[{y''[t]+4*y'[t]+4*y[t]==t^(5/2)*Exp[-2*t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSo
```

$$y(t) \to \frac{4}{63} e^{-2t} t^{9/2}$$

## 10.7 problem 7

Internal problem ID [1760]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.4, The method of variation of parameters. Page 154
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y - \sqrt{t+1} = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 84

```
dsolve([diff(y(t),t$2)-3*diff(y(t),t)+2*y(t)=sqrt(1+t),y(0) = 0, D(y)(0) = 0],y(t), singsol=a
```

$$y(t) = -\frac{\sqrt{2}\, e^{2t+2}\sqrt{\pi}\, \operatorname{erf}\left(\sqrt{2}\right)}{8} + \frac{e^{2t}}{2} + \frac{\sqrt{2}\,\sqrt{\pi}\, \operatorname{erf}\left(\sqrt{2}\,\sqrt{t+1}\right) e^{2t+2}}{8} - \frac{\operatorname{erf}\left(\sqrt{t+1}\right)\sqrt{\pi}\, e^{t+1}}{2} + \frac{\sqrt{t+1}}{2} + \frac{\operatorname{erf}(1)\, e^{t+1}\sqrt{\pi}}{2} - e^{t}$$

✓ Solution by Mathematica

Time used: 0.246 (sec). Leaf size: 102

```
DSolve[{y''[t]-3*y'[t]+2*y[t]==Sqrt[1+t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{1}{8}\left(4e^t\left(-e\sqrt{\pi}\operatorname{erf}\left(\sqrt{t+1}\right) + e\sqrt{\pi}\operatorname{erf}(1) - 2\right) + e^{2t}\left(e^2\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{t+1}\right) - e^2\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\right) + 4\right) + 4\sqrt{t+1}\right)$$

## 10.8 problem 8

Internal problem ID [1761]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.4, The method of variation of parameters. Page 154
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y - f(t) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 39

```
dsolve([diff(y(t),t$2)-y(t)=f(t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\left(\int_0^t \mathrm{e}^{-\_z1} f(\_z1)\, d\_z1\right) \mathrm{e}^{t}}{2} - \frac{\left(\int_0^t \mathrm{e}^{\_z1} f(\_z1)\, d\_z1\right) \mathrm{e}^{-t}}{2}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 99

```
DSolve[{y''[t]-y[t]==f[t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}\left(e^{2t}\left(\int_1^t \frac{1}{2}e^{-K[1]}f(K[1])dK[1] - \int_1^0 \frac{1}{2}e^{-K[1]}f(K[1])dK[1]\right) + \int_1^t -\frac{1}{2}e^{K[2]}f(K[2])dK[2] - \int_1^0 -\frac{1}{2}e^{K[2]}f(K[2])dK[2]\right)$$

## 10.9 problem 11

Internal problem ID [1762]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.4, The method of variation of parameters. Page 154
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + \frac{y t^2}{4} - f\cos(t) = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 84

```
dsolve(diff(y(t),t$2)+(1/4*t^2)*y(t)=f*cos(t),y(t), singsol=all)
```

$$y(t) = \sqrt{t}\,\mathrm{BesselJ}\left(\frac{1}{4}, \frac{t^2}{4}\right) c_2 + \sqrt{t}\,\mathrm{BesselY}\left(\frac{1}{4}, \frac{t^2}{4}\right) c_1 - \frac{f\pi\sqrt{t}\left(\left(\int \sqrt{t}\,\mathrm{BesselY}\left(\frac{1}{4}, \frac{t^2}{4}\right)\cos(t)\,dt\right)\mathrm{BesselJ}\left(\frac{1}{4}, \frac{t^2}{4}\right) - \left(\int \sqrt{t}\,\mathrm{BesselJ}\left(\frac{1}{4}, \frac{t^2}{4}\right)\cos(t)\,dt\right)\mathrm{BesselY}\left(\right.}{4}$$

✓ Solution by Mathematica

Time used: 14.116 (sec). Leaf size: 208

```
DSolve[y''[t]+(1/4*t^2)*y[t]==f*Cos[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \mathrm{ParabolicCylinderD}\left(-\frac{1}{2}, \sqrt[4]{-1}t\right)\left(\int_1^t \frac{f\cos(}{\mathrm{ParabolicCylinderD}\left(-\frac{1}{2}, \sqrt[4]{-1}K[1]\right)\left(iK[1] + \frac{(-1)^{3/4}\,\mathrm{ParabolicCyl}}{\mathrm{ParabolicCylinder}}\right.}\right.$$

## 10.10 problem 12

Internal problem ID [1763]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.4, The method of variation of parameters. Page 154
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{2ty'}{t^2+1} + \frac{2y}{t^2+1} - t^2 - 1 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(t),t$2)-2*t/(1+t^2)*diff(y(t),t)+2/(1+t^2)*y(t)=1+t^2,y(t), singsol=all)
```

$$y(t) = c_2 t + \left(t^2 - 1\right) c_1 + \frac{1}{2} + \frac{t^4}{6}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 33

```
DSolve[y''[t]-2*t/(1+t^2)*y'[t]+2/(1+t^2)*y[t]==1+t^2,y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to \frac{1}{6}\left(t^2 + 3\right) t^2 + c_2 t - c_1 (t - i)^2$$

# 11 Section 2.6, Mechanical Vibrations. Page 171

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## 11.1 problem 13

Internal problem ID [1764]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.6, Mechanical Vibrations. Page 171
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{my'' + cy' + yk - F_0 \cos(\omega t) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 97

```
dsolve(m*diff(y(t),t$2)+c*diff(y(t),t)+k*y(t)=F__0*cos(omega*t),y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{\frac{\left(-c+\sqrt{c^2-4km}\right)t}{2m}} c_2 + \mathrm{e}^{-\frac{\left(c+\sqrt{c^2-4km}\right)t}{2m}} c_1 + \frac{F_0\left(\left(-m\,\omega^2+k\right)\cos(\omega t)+\sin(\omega t)\,c\omega\right)}{m^2\omega^4+\left(c^2-2km\right)\omega^2+k^2}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 100

```
DSolve[m*y''[t]+c*y'[t]+k*y[t]==F0*Cos[\[Omega]*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c\mathrm{F0}\omega\sin(t\omega) + \mathrm{F0}(k-m\omega^2)\cos(t\omega)}{c^2\omega^2+(k-m\omega^2)^2} + e^{-\frac{t\left(\sqrt{c^2-4km}+c\right)}{2m}}\left(c_2 e^{\frac{t\sqrt{c^2-4km}}{m}} + c_1\right)$$

# 12 Section 2.8, Series solutions. Page 195

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## 12.1 problem 1

Internal problem ID [1765]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + ty' + y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(t),t$2)+t*diff(y(t),t)+y(t)=0,y(t),type='series',t=0);
```

$$y(t) = \left(1 - \frac{1}{2}t^2 + \frac{1}{8}t^4\right) y(0) + \left(t - \frac{1}{3}t^3 + \frac{1}{15}t^5\right) D(y)(0) + O\left(t^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[t]+t*y'[t]+y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_2\left(\frac{t^5}{15} - \frac{t^3}{3} + t\right) + c_1\left(\frac{t^4}{8} - \frac{t^2}{2} + 1\right)$$

## 12.2 problem 2

Internal problem ID [1766]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - yt = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(t),t$2)-t*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = \left(1 + \frac{t^3}{6}\right) y(0) + \left(t + \frac{1}{12}t^4\right) D(y)(0) + O(t^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[t]-t*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_2\left(\frac{t^4}{12} + t\right) + c_1\left(\frac{t^3}{6} + 1\right)$$

## 12.3 problem 3

Internal problem ID [1767]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(t^2+2)\,y''-ty'-3y=0}$$

With the expansion point for the power series method at $t=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve((2+t^2)*diff(y(t),t$2)-t*diff(y(t),t)-3*y(t)=0,y(t),type='series',t=0);
```

$$y(t)=\left(1+\frac{3}{4}t^2+\frac{3}{32}t^4\right)y(0)+\left(\frac{1}{3}t^3+t\right)D(y)\,(0)+O\left(t^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 35

```
AsymptoticDSolveValue[(2+t^2)*y''[t]-t*y'[t]-3*y[t]==0,y[t],{t,0,5}]
```

$$y(t)\to c_2\left(\frac{t^3}{3}+t\right)+c_1\left(\frac{3t^4}{32}+\frac{3t^2}{4}+1\right)$$

## 12.4 problem 4

Internal problem ID [1768]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - y t^3 = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
Order:=6;
dsolve(diff(y(t),t$2)-t^3*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = \left(1 + \frac{t^5}{20}\right) y(0) + tD(y)\left(0\right) + O\left(t^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 20

```
AsymptoticDSolveValue[y''[t]-t^3*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1\left(\frac{t^5}{20} + 1\right) + c_2 t$$

## 12.5 problem 5

Internal problem ID [1769]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{t(-t+2)\,y''-6(t-1)\,y'-4y=0}$$

With initial conditions

$$[y(1)=1,y'(1)=0]$$

With the expansion point for the power series method at $t=1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([t*(2-t)*diff(y(t),t$2)-6*(t-1)*diff(y(t),t)-4*y(t)=0,y(1) = 1, D(y)(1) = 0],y(t),type
```

$$y(t)=1+2(t-1)^2+3(t-1)^4+\mathrm{O}\left((t-1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{t*(2-t)*y''[t]-6*(t-1)*y'[t]-4*y[t]==0,{y[1]==1,y'[1]==0}},y[t],{t,1,5
```

$$y(t)\to 3(t-1)^4+2(t-1)^2+1$$

## 12.6 problem 6

Internal problem ID [1770]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + yt^2 = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
Order:=6;
dsolve([diff(y(t),t$2)+t^2*y(t)=0,y(0) = 2, D(y)(0) = -1],y(t),type='series',t=0);
```

$$y(t) = 2 - t - \frac{1}{6}t^4 + \frac{1}{20}t^5 + \mathrm{O}\left(t^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 22

```
AsymptoticDSolveValue[{y''[t]+t^2*y[t]==0,{y[0]==2,y'[0]==-1}},y[t],{t,0,5}]
```

$$y(t) \to \frac{t^5}{20} - \frac{t^4}{6} - t + 2$$

## 12.7 problem 7

Internal problem ID [1771]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - y t^3 = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = -2]$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
Order:=6;
dsolve([diff(y(t),t$2)-t^3*y(t)=0,y(0) = 0, D(y)(0) = -2],y(t),type='series',t=0);
```

$$y(t) = (-2)\,t + \mathrm{O}\left(t^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 6

```
AsymptoticDSolveValue[{y''[t]-t^3*y[t]==0,{y[0]==0,y'[0]==-2}},y[t],{t,0,5}]
```

$$y(t) \to -2t$$

## 12.8 problem 8

Internal problem ID [1772]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (t^2 + 2t + 1)\, y' - (4t + 4)\, y = 0}$$

With initial conditions

$$[y(-1) = 0, y'(-1) = 1]$$

With the expansion point for the power series method at $t = -1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
Order:=6;
dsolve([diff(y(t),t$2)+(t^2+2*t+1)*diff(y(t),t)-(4+4*t)*y(t)=0,y(-1) = 0, D(y)(-1) = 1],y(t),
```

$$y(t) = (t + 1) + \frac{1}{4}(t + 1)^4 + O\left((t + 1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 15

```
AsymptoticDSolveValue[{y''[t]+(t^2+2*t+1)*y'[t]-(4+4*t)*y[t]==0,{y[-1]==0,y'[-1]==1}},y[t],{t
```

$$y(t) \to \frac{1}{4}(t + 1)^4 + t + 1$$

## 12.9 problem 9

Internal problem ID [1773]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2ty' + \lambda y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
Order:=6;
dsolve(diff(y(t),t$2)-2*t*diff(y(t),t)+lambda*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = \left(1 - \frac{\lambda t^2}{2} + \frac{\lambda(\lambda-4)\,t^4}{24}\right) y(0) + \left(t - \frac{(\lambda-2)\,t^3}{6} + \frac{(\lambda-2)\,(-6+\lambda)\,t^5}{120}\right) D(y)\,(0) + O\left(t^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 80

```
AsymptoticDSolveValue[y''[t]-2*t*y'[t]+\[Lambda]*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_2\left(\frac{\lambda^2 t^5}{120} - \frac{\lambda t^5}{15} + \frac{t^5}{10} - \frac{\lambda t^3}{6} + \frac{t^3}{3} + t\right) + c_1\left(\frac{\lambda^2 t^4}{24} - \frac{\lambda t^4}{6} - \frac{\lambda t^2}{2} + 1\right)$$

## 12.10 problem 10

Internal problem ID [1774]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$(-t^2+1)\,y''-2ty'+\alpha(\alpha+1)\,y=0$$

With the expansion point for the power series method at $t=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 101

```
Order:=6;
dsolve((1-t^2)*diff(y(t),t$2)-2*t*diff(y(t),t)+alpha*(alpha+1)*y(t)=0,y(t),type='series',t=0)
```

$$y(t)=\left(1-\frac{\alpha(1+\alpha)\,t^2}{2}+\frac{\alpha(\alpha^3+2\alpha^2-5\alpha-6)\,t^4}{24}\right)y(0)+\left(t-\frac{(\alpha^2+\alpha-2)\,t^3}{6}+\frac{(\alpha^4+2\alpha^3-13\alpha^2-14\alpha+24)\,t^5}{120}\right)D(y)\,(0)+O(t^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 127

```
AsymptoticDSolveValue[(1-t^2)*y''[t]-2*t*y'[t]+\[Alpha]*(\[Alpha]+1)*y[t]==0,y[t],{t,0,5}]
```

$$y(t)\to c_2\left(\frac{1}{60}(-\alpha^2-\alpha)\,t^5-\frac{1}{120}(-\alpha^2-\alpha)\,(\alpha^2+\alpha)\,t^5-\frac{1}{10}(\alpha^2+\alpha)\,t^5+\frac{t^5}{5}-\frac{1}{6}(\alpha^2+\alpha)\,t^3+\frac{t^3}{3}+t\right)+c_1\left(\frac{1}{24}(\alpha^2+\alpha)^2\,t^4-\frac{1}{4}(\alpha^2+\alpha)\,t^4-\frac{1}{2}(\alpha^2+\alpha)\,t^2+1\right)$$

## 12.11 problem 11

Internal problem ID [1775]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{(-t^2+1)\,y''-ty'+\alpha^2 y=0}$$

With the expansion point for the power series method at $t=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 71

```
Order:=6;
dsolve((1-t^2)*diff(y(t),t$2)-t*diff(y(t),t)+alpha^2*y(t)=0,y(t),type='series',t=0);
```

$$y(t)=\left(1-\frac{\alpha^2 t^2}{2}+\frac{\alpha^2(\alpha^2-4)\,t^4}{24}\right)y(0)+\left(t-\frac{(\alpha^2-1)\,t^3}{6}+\frac{(\alpha^4-10\alpha^2+9)\,t^5}{120}\right)D(y)\,(0)+O(t^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 88

```
AsymptoticDSolveValue[(1-t^2)*y''[t]-t*y'[t]+\[Alpha]^2*y[t]==0,y[t],{t,0,5}]
```

$$y(t)\to c_2\left(\frac{\alpha^4t^5}{120}-\frac{\alpha^2t^5}{12}+\frac{3t^5}{40}-\frac{\alpha^2t^3}{6}+\frac{t^3}{6}+t\right)+c_1\left(\frac{\alpha^4t^4}{24}-\frac{\alpha^2t^4}{6}-\frac{\alpha^2t^2}{2}+1\right)$$

## 12.12 problem 12(a)

Internal problem ID [1776]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 12(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y't^3 + 3yt^2 = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(t),t$2)+t^3*diff(y(t),t)+3*t^2*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = \left(1 - \frac{t^4}{4}\right) y(0) + \left(t - \frac{1}{5}t^5\right) D(y)(0) + O(t^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[t]+t^3*y'[t]+3*t^2*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_2\left(t - \frac{t^5}{5}\right) + c_1\left(1 - \frac{t^4}{4}\right)$$

## 12.13 problem 12(b)

Internal problem ID [1777]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 12(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y't^3 + 3yt^2 = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
Order:=6;
dsolve([diff(y(t),t$2)+t^3*diff(y(t),t)+3*t^2*y(t)=0,y(0) = 0, D(y)(0) = 0],y(t),type='series
```

$$y(t) = 0$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 4

```
AsymptoticDSolveValue[{y''[t]+t^3*y'[t]+3*t^2*y[t]==0,{y[0]==0,y'[0]==0}},y[t],{t,0,5}]
```

$$y(t) \to 0$$

## 12.14 problem 13

Internal problem ID [1778]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(-t+1)\, y'' + ty' + y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve([(1-t)*diff(y(t),t$2)+t*diff(y(t),t)+y(t)=0,y(0) = 1, D(y)(0) = 0],y(t),type='series',
```

$$y(t) = 1 - \frac{1}{2}t^2 - \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{7}{120}t^5 + \mathrm{O}\left(t^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[{(1-t)*y''[t]+t*y'[t]+y[t]==0,{y[0]==1,y'[0]==0}},y[t],{t,0,5}]
```

$$y(t) \to \frac{7t^5}{120} + \frac{t^4}{24} - \frac{t^3}{6} - \frac{t^2}{2} + 1$$

## 12.15 problem 14

Internal problem ID [1779]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' + yt = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 2]$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(t),t$2)+diff(y(t),t)+t*y(t)=0,y(0) = -1, D(y)(0) = 2],y(t),type='series',t=0);
```

$$y(t) = -1 + 2t - t^2 + \frac{1}{2}t^3 - \frac{7}{24}t^4 + \frac{13}{120}t^5 + \mathrm{O}\left(t^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{y''[t]+y'[t]+t*y[t]==0,{y[0]==-1,y'[0]==2}},y[t],{t,0,5}]
```

$$y(t) \to \frac{13t^5}{120} - \frac{7t^4}{24} + \frac{t^3}{2} - t^2 + 2t - 1$$

## 12.16 problem 15

Internal problem ID [1780]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 15.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + ty' + e^t y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve([diff(y(t),t$2)+t*diff(y(t),t)+exp(t)*y(t)=0,y(0) = 1, D(y)(0) = 0],y(t),type='series'
```

$$y(t) = 1 - \frac{1}{2}t^2 - \frac{1}{6}t^3 + \frac{1}{12}t^4 + \frac{1}{20}t^5 + O\left(t^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[{y''[t]+t*y'[t]+Exp[t]*y[t]==0,{y[0]==1,y'[0]==0}},y[t],{t,0,5}]
```

$$y(t) \to \frac{t^5}{20} + \frac{t^4}{12} - \frac{t^3}{6} - \frac{t^2}{2} + 1$$

## 12.17 problem 16

Internal problem ID [1781]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 16.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' + e^t y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = -1]$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
Order:=6;
dsolve([diff(y(t),t$2)+diff(y(t),t)+exp(t)*y(t)=0,y(0) = 0, D(y)(0) = -1],y(t),type='series',
```

$$y(t) = -t + \frac{1}{2}t^2 + \frac{1}{24}t^4 - \frac{1}{120}t^5 + O\left(t^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[{y''[t]+y'[t]+Exp[t]*y[t]==0,{y[0]==0,y'[0]==-1}},y[t],{t,0,5}]
```

$$y(t) \rightarrow -\frac{t^5}{120} + \frac{t^4}{24} + \frac{t^2}{2} - t$$

## 12.18 problem 17

Internal problem ID [1782]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8, Series solutions. Page 195
**Problem number**: 17.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' + y\,\mathrm{e}^{-t} = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 5]$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(t),t$2)+diff(y(t),t)+exp(-t)*y(t)=0,y(0) = 3, D(y)(0) = 5],y(t),type='series',
```

$$y(t) = 3 + 5t - 4t^2 + t^3 + \frac{3}{8}t^4 - \frac{17}{40}t^5 + \mathrm{O}\left(t^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 30

```
AsymptoticDSolveValue[{y''[t]+y'[t]+Exp[-t]*y[t]==0,{y[0]==3,y'[0]==5}},y[t],{t,0,5}]
```

$$y(t) \to -\frac{17t^5}{40} + \frac{3t^4}{8} + t^3 - 4t^2 + 5t + 3$$

# 13 Section 2.8.1, Singular points, Euler equations. Page 201

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## 13.1 problem Example 2

Internal problem ID [1783]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.1, Singular points, Euler equations. Page 201
**Problem number**: Example 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' - 5ty' + 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(t^2*diff(y(t),t$2)-5*t*diff(y(t),t)+9*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1t^3 + c_2t^3 \ln(t)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 18

```
DSolve[t^2*y''[t]-5*t*y'[t]+9*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t^3(3c_2 \log(t) + c_1)$$

## 13.2 problem 1

Internal problem ID [1784]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.1, Singular points, Euler equations. Page 201
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' + 5ty' - 5y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(t^2*diff(y(t),t$2)+5*t*diff(y(t),t)-5*y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1}{t^5} + c_2t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]+5*t*y'[t]-5*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_1}{t^5} + c_2t$$

## 13.3 problem 2

Internal problem ID [1785]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.1, Singular points, Euler equations. Page 201
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{2t^2y'' + 3ty' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*t^2*diff(y(t),t$2)+3*t*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1}{t} + c_2\sqrt{t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[2*t^2*y''[t]+3*t*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2t^{3/2} + c_1}{t}$$

## 13.4 problem 3

Internal problem ID [1786]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.1, Singular points, Euler equations. Page 201
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$\boxed{(t-1)^2 y'' - 2(t-1)\, y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve((t-1)^2*diff(y(t),t$2)-2*(t-1)*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1(t-1)^2 + c_2(t-1)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 18

```
DSolve[(t-1)^2*y''[t]-2*(t-1)*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to (t-1)(c_2(t-1) + c_1)$$

## 13.5 problem 4

Internal problem ID [1787]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.1, Singular points, Euler equations. Page 201
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{t^2y'' + 3ty' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(t^2*diff(y(t),t$2)+3*t*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1}{t} + \frac{c_2 \ln(t)}{t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 17

```
DSolve[t^2*y''[t]+3*t*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 \log(t) + c_1}{t}$$

## 13.6 problem 5

Internal problem ID [1788]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.1, Singular points, Euler equations. Page 201
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' - ty' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(t^2*diff(y(t),t$2)-t*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = tc_1 + c_2t\ln(t)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 15

```
DSolve[t^2*y''[t]-t*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t(c_2\log(t) + c_1)$$

## 13.7 problem 6

Internal problem ID [1789]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.1, Singular points, Euler equations. Page 201
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(t-2)^2 y'' + 5(t-2)\, y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve((t-2)^2*diff(y(t),t$2)+5*(t-2)*diff(y(t),t)+4*y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1}{(t-2)^2} + \frac{c_2 \ln (t-2)}{(t-2)^2}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 22

```
DSolve[(t-2)^2*y''[t]+5*(t-2)*y'[t]+4*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{2c_2 \log(t-2) + c_1}{(t-2)^2}$$

## 13.8 problem 7

Internal problem ID [1790]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.1, Singular points, Euler equations. Page 201
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, ‘_with_symmetry_[0,F(`

$$\boxed{t^2y'' + ty' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(t^2*diff(y(t),t$2)+t*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = \sin(\ln(t)) c_1 + c_2 \cos(\ln(t))$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 18

```
DSolve[t^2*y''[t]+t*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \cos(\log(t)) + c_2 \sin(\log(t))$$

## 13.9 problem 9

Internal problem ID [1791]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.1, Singular points, Euler equations. Page 201
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' - ty' + 2y = 0}$$

With initial conditions

$$[y(1) = 0, y'(1) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve([t^2*diff(y(t),t$2)-t*diff(y(t),t)+2*y(t)=0,y(1) = 0, D(y)(1) = 1],y(t), singsol=all)
```

$$y(t) = \sin(\ln(t))\, t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 10

```
DSolve[{t^2*y''[t]-t*y'[t]+2*y[t]==0,{y[1]==0,y'[1]==1}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to t \sin(\log(t))$$

## 13.10 problem 10

Internal problem ID [1792]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.1, Singular points, Euler equations. Page 201
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$\boxed{t^2y'' - 3ty' + 4y = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([t^2*diff(y(t),t$2)-3*t*diff(y(t),t)+4*y(t)=0,y(1) = 1, D(y)(1) = 0],y(t), singsol=all
```

$$y(t) = t^2(1 - 2\ln(t))$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 15

```
DSolve[{t^2*y''[t]-3*t*y'[t]+4*y[t]==0,{y[1]==1,y'[1]==0}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to t^2(1 - 2\log(t))$$

# 14 Section 2.8.2, Regular singular points, the method of Frobenius. Page 214


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## 14.1 problem 1

Internal problem ID [1793]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t(t-2)^2 y'' + ty' + y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 60

```
Order:=6;
dsolve(t*(t-2)^2*diff(y(t),t$2)+t*diff(y(t),t)+y(t)=0,y(t),type='series',t=0);
```

$$\begin{aligned} y(t) = {} & c_1 t\left(1 - \frac{1}{4}t - \frac{5}{96}t^2 - \frac{13}{1152}t^3 - \frac{199}{92160}t^4 - \frac{1123}{5529600}t^5 + \mathrm{O}\left(t^6\right)\right) \\ & + c_2\left(\ln(t)\left(-\frac{1}{4}t + \frac{1}{16}t^2 + \frac{5}{384}t^3 + \frac{13}{4608}t^4 + \frac{199}{368640}t^5 + \mathrm{O}\left(t^6\right)\right)\right. \\ & \left. + \left(1 - \frac{1}{4}t - \frac{1}{8}t^2 + \frac{5}{2304}t^3 + \frac{79}{13824}t^4 + \frac{62027}{22118400}t^5 + \mathrm{O}\left(t^6\right)\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 87

```
AsymptoticDSolveValue[t*(t-2)^2*y''[t]+t*y'[t]+y[t]==0,y[t],{t,0,5}]
```

$$\begin{aligned} y(t) \to {} & c_1\left(\frac{t(13t^3 + 60t^2 + 288t - 1152)\log(t)}{4608} + \frac{98t^4 + 285t^3 + 432t^2 - 6912t + 6912}{6912}\right) \\ & + c_2\left(-\frac{199t^5}{92160} - \frac{13t^4}{1152} - \frac{5t^3}{96} - \frac{t^2}{4} + t\right) \end{aligned}$$

## 14.2 problem 2

Internal problem ID [1794]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t(t-2)^2 y'' + ty' + y = 0}$$

With the expansion point for the power series method at $t = 2$.

✗ Solution by Maple

```
Order:=6;
dsolve(t*(t-2)^2*diff(y(t),t$2)+t*diff(y(t),t)+y(t)=0,y(t),type='series',t=2);
```

No solution found

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 112

```
AsymptoticDSolveValue[t*(t-2)^2*y''[t]+t*y'[t]+y[t]==0,y[t],{t,2,5}]
```

$$y(t) \to c_2 e^{\frac{1}{t-2}} \left(\frac{247853}{240}(t-2)^5 + \frac{4069}{24}(t-2)^4 + \frac{199}{6}(t-2)^3 + 8(t-2)^2 + \frac{5(t-2)}{2} + 1\right)(t-2)^2 + c_1\left(-\frac{641}{480}(t-2)^5 + \frac{25}{48}(t-2)^4 - \frac{7}{24}(t-2)^3 + \frac{1}{4}(t-2)^2 + \frac{2-t}{2} + 1\right)$$

## 14.3 problem 3

Internal problem ID [1795]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\sin(t)\, y'' + \cos(t)\, y' + \frac{y}{t} = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 45

```
Order:=6;
dsolve(sin(t)*diff(y(t),t$2)+cos(t)*diff(y(t),t)+1/t*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = c_1 t^{-i}\left(1+\left(\frac{1}{48}-\frac{i}{16}\right)t^2+\left(\frac{1}{57600}-\frac{217i}{57600}\right)t^4+\mathrm{O}\left(t^6\right)\right) + c_2 t^{i}\left(1+\left(\frac{1}{48}+\frac{i}{16}\right)t^2+\left(\frac{1}{57600}+\frac{217i}{57600}\right)t^4+\mathrm{O}\left(t^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 70

```
AsymptoticDSolveValue[Sin[t]*y''[t]+Cos[t]*y'[t]+1/t*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to \left(\frac{1}{19200}+\frac{i}{57600}\right)c_1 t^{i}\left((22+65i)t^4+(720+960i)t^2+(17280-5760i)\right) - \left(\frac{1}{57600}+\frac{i}{19200}\right)c_2 t^{-i}\left((65+22i)t^4+(960+720i)t^2-(5760-17280i)\right)$$

## 14.4 problem 4

Internal problem ID [1796]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(e^t - 1)\, y'' + e^t y' + y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.218 (sec). Leaf size: 59

```
Order:=6;
dsolve((exp(t)-1)*diff(y(t),t$2)+exp(t)*diff(y(t),t)+y(t)=0,y(t),type='series',t=0);
```

$$y(t) = (c_2 \ln(t) + c_1)\left(1 - t + \frac{1}{2}t^2 - \frac{1}{6}t^3 + \frac{1}{24}t^4 - \frac{1}{120}t^5 + O(t^6)\right) + \left(\frac{3}{2}t - \frac{23}{24}t^2 + \frac{3}{8}t^3 - \frac{301}{2880}t^4 + \frac{13}{576}t^5 + O(t^6)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 113

```
AsymptoticDSolveValue[(Exp[t]-1)*y''[t]+Exp[t]*y'[t]+y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1\left(-\frac{t^5}{120} + \frac{t^4}{24} - \frac{t^3}{6} + \frac{t^2}{2} - t + 1\right) + c_2\left(\frac{13t^5}{576} - \frac{301t^4}{2880} + \frac{3t^3}{8} - \frac{23t^2}{24} + \left(-\frac{t^5}{120} + \frac{t^4}{24} - \frac{t^3}{6} + \frac{t^2}{2} - t + 1\right)\log(t) + \frac{3t}{2}\right)$$

## 14.5 problem 5

Internal problem ID [1797]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-t^2+1)\,y''+\frac{y'}{\sin(t+1)}+y=0}$$

With the expansion point for the power series method at $t=-1$.

✗ Solution by Maple

```
Order:=6;
dsolve((1-t^2)*diff(y(t),t$2)+1/sin(t+1)*diff(y(t),t)+y(t)=0,y(t),type='series',t=-1);
```

No solution found

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 111

```
AsymptoticDSolveValue[(1-t^2)*y''[t]+1/Sin[t+1]*y'[t]+y[t]==0,y[t],{t,-1,5}]
```

$$y(t)\to c_2e^{\frac{1}{2(t+1)}}\left(\frac{516353141702117(t+1)^5}{33443020800}+\frac{53349163853(t+1)^4}{39813120}+\frac{58276991(t+1)^3}{414720}+\frac{21397(t+1)^2}{1152}+\frac{79(t+1)}{24}+1\right)(t+1)^{7/4}+c_1\left(\frac{53}{5}(t+1)^5-\frac{25}{12}(t+1)^4+\frac{2}{3}(t+1)^3-\frac{1}{2}(t+1)^2+1\right)$$

## 14.6 problem 6

Internal problem ID [1798]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$t^3y'' + \sin\left(t^3\right) y' + ty = 0$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 907

```
Order:=6;
dsolve(t^3*diff(y(t),t$2)+sin(t^3)*diff(y(t),t)+t*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = \sqrt{t}\left(c_2 t^{\frac{i\sqrt{3}}{2}}\left(1 - \frac{1}{2}t + \frac{i\sqrt{3}+3}{8i\sqrt{3}+16}t^2 + \frac{-i\sqrt{3}-5}{48i\sqrt{3}+96}t^3 + \frac{1}{384}\frac{\left(i\sqrt{3}+5\right)\left(i\sqrt{3}+7\right)}{\left(i\sqrt{3}+4\right)\left(i\sqrt{3}+2\right)}t^4 - \frac{1}{3840}\frac{\left(i\sqrt{3}+7\right)\left(i\sqrt{3}+9\right)}{\left(i\sqrt{3}+4\right)\left(i\sqrt{3}+2\right)}t^5 + \mathrm{O}\left(t^6\right)\right) + c_1 t^{-\frac{i\sqrt{3}}{2}}\left(1 - \frac{1}{2}t + \frac{i\sqrt{3}-3}{8i\sqrt{3}-16}t^2 + \frac{-i\sqrt{3}+5}{48i\sqrt{3}-96}t^3 + \frac{1}{384}\frac{\left(i\sqrt{3}-5\right)\left(i\sqrt{3}-7\right)}{\left(i\sqrt{3}-4\right)\left(i\sqrt{3}-2\right)}t^4 - \frac{1}{3840}\frac{\left(i\sqrt{3}-7\right)\left(i\sqrt{3}-9\right)}{\left(i\sqrt{3}-4\right)\left(i\sqrt{3}-2\right)}t^5 + \mathrm{O}\left(t^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 886

```
AsymptoticDSolveValue[t^3*y''[t]+Sin[t^3]*y'[t]+t*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to \left(\frac{(-1)^{2/3}\left(1-(-1)^{2/3}\right)\left(2-(-1)^{2/3}\right)\left(3-(-1)^{2/3}\right)\left(4-\right.}{\left(1-(-1)^{2/3}\left(1-(-1)^{2/3}\right)\right)\left(1+\left(1-(-1)^{2/3}\right)\left(2-(-1)^{2/3}\right)\right)\left(1+\left(2-(-1)^{2/3}\right)\left(3-(-1)^{2/3}\right)\right)\left(1+(3\right.}\right.$$
$$-\frac{(-1)^{2/3}\left(1-(-1)^{2/3}\right)\left(2-(-1)^{2/3}\right)\left(3-(-1)^{2/3}\right)t^4}{\left(1-(-1)^{2/3}\left(1-(-1)^{2/3}\right)\right)\left(1+\left(1-(-1)^{2/3}\right)\left(2-(-1)^{2/3}\right)\right)\left(1+\left(2-(-1)^{2/3}\right)\left(3-(-1)^{2/3}\right)\right)\left(1+(3\right.}$$
$$+\frac{(-1)^{2/3}\left(1-(-1)^{2/3}\right)\left(2-(-1)^{2/3}\right)t^3}{\left(1-(-1)^{2/3}\left(1-(-1)^{2/3}\right)\right)\left(1+\left(1-(-1)^{2/3}\right)\left(2-(-1)^{2/3}\right)\right)\left(1+\left(2-(-1)^{2/3}\right)\left(3-(-1)^{2/3}\right)\right)}$$
$$-\frac{(-1)^{2/3}\left(1-(-1)^{2/3}\right)t^2}{\left(1-(-1)^{2/3}\left(1-(-1)^{2/3}\right)\right)\left(1+\left(1-(-1)^{2/3}\right)\left(2-(-1)^{2/3}\right)\right)}$$
$$+\frac{(-1)^{2/3}t}{1-(-1)^{2/3}\left(1-(-1)^{2/3}\right)}$$
$$\left.+1\right)c_1t^{-(-1)^{2/3}}+\left(-\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(2+\sqrt[3]{-1}\right)\left(3+\sqrt[3]{-1}\right.}{\left(1+\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\right)\left(1+\left(1+\sqrt[3]{-1}\right)\left(2+\sqrt[3]{-1}\right)\right)\left(1+\left(2+\sqrt[3]{-1}\right)\left(3+\sqrt[3]{-1}\right)\right)}\right.$$

## 14.7 problem 7

Internal problem ID [1799]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2t^2y'' + 3ty' - (t+1)\,y = 0$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(2*t^2*diff(y(t),t$2)+3*t*diff(y(t),t)-(1+t)*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = \frac{c_2 t^{\frac{3}{2}}\left(1 + \frac{1}{5}t + \frac{1}{70}t^2 + \frac{1}{1890}t^3 + \frac{1}{83160}t^4 + \frac{1}{5405400}t^5 + \mathrm{O}\left(t^6\right)\right) + c_1\left(1 - t - \frac{1}{2}t^2 - \frac{1}{18}t^3 - \frac{1}{360}t^4 - \frac{1}{12600}t^5 + \mathrm{O}\left(\right.\right.}{t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 86

```
AsymptoticDSolveValue[2*t^2*y''[t]+3*t*y'[t]-(1+t)*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1\sqrt{t}\left(\frac{t^5}{5405400} + \frac{t^4}{83160} + \frac{t^3}{1890} + \frac{t^2}{70} + \frac{t}{5} + 1\right) + \frac{c_2\left(-\frac{t^5}{12600} - \frac{t^4}{360} - \frac{t^3}{18} - \frac{t^2}{2} - t + 1\right)}{t}$$

## 14.8 problem 8

Internal problem ID [1800]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{2ty'' + (1-2t)\,y' - y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(2*t*diff(y(t),t$2)+(1-2*t)*diff(y(t),t)-y(t)=0,y(t),type='series',t=0);
```

$$y(t) = c_1\sqrt{t}\left(1 + \frac{2}{3}t + \frac{4}{15}t^2 + \frac{8}{105}t^3 + \frac{16}{945}t^4 + \frac{32}{10395}t^5 + \mathrm{O}\left(t^6\right)\right) + c_2\left(1 + t + \frac{1}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \mathrm{O}\left(t^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 81

```
AsymptoticDSolveValue[2*t*y''[t]+(1-2*t)*y'[t]-y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1\sqrt{t}\left(\frac{32t^5}{10395} + \frac{16t^4}{945} + \frac{8t^3}{105} + \frac{4t^2}{15} + \frac{2t}{3} + 1\right) + c_2\left(\frac{t^5}{120} + \frac{t^4}{24} + \frac{t^3}{6} + \frac{t^2}{2} + t + 1\right)$$

## 14.9 problem 9

Internal problem ID [1801]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2ty'' + (t+1)\,y' - 2y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 38

```
Order:=6;
dsolve(2*t*diff(y(t),t$2)+(1+t)*diff(y(t),t)-2*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = c_1\sqrt{t}\left(1+\frac{1}{2}t+\frac{1}{40}t^2-\frac{1}{1680}t^3+\frac{1}{40320}t^4-\frac{1}{887040}t^5+\mathrm{O}\left(t^6\right)\right) + c_2\left(1+2t+\frac{1}{3}t^2+\mathrm{O}\left(t^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 62

```
AsymptoticDSolveValue[2*t*y''[t]+(1+t)*y'[t]-2*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_2\left(\frac{t^2}{3}+2t+1\right)+c_1\sqrt{t}\left(-\frac{t^5}{887040}+\frac{t^4}{40320}-\frac{t^3}{1680}+\frac{t^2}{40}+\frac{t}{2}+1\right)$$

## 14.10 problem 10

Internal problem ID [1802]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2t^2y'' - ty' + (t+1)\,y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*t^2*diff(y(t),t$2)-t*diff(y(t),t)+(1+t)*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = c_1\sqrt{t}\left(1 - t + \frac{1}{6}t^2 - \frac{1}{90}t^3 + \frac{1}{2520}t^4 - \frac{1}{113400}t^5 + \mathrm{O}\left(t^6\right)\right) + c_2 t\left(1 - \frac{1}{3}t + \frac{1}{30}t^2 - \frac{1}{630}t^3 + \frac{1}{22680}t^4 - \frac{1}{1247400}t^5 + \mathrm{O}\left(t^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 84

```
AsymptoticDSolveValue[2*t^2*y''[t]-t*y'[t]+(1+t)*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1 t\left(-\frac{t^5}{1247400} + \frac{t^4}{22680} - \frac{t^3}{630} + \frac{t^2}{30} - \frac{t}{3} + 1\right) + c_2\sqrt{t}\left(-\frac{t^5}{113400} + \frac{t^4}{2520} - \frac{t^3}{90} + \frac{t^2}{6} - t + 1\right)$$

## 14.11 problem 11

Internal problem ID [1803]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4ty'' + 3y' - 3y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 44

```
Order:=6;
dsolve(4*t*diff(y(t),t$2)+3*diff(y(t),t)-3*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = c_1 t^{\frac{1}{4}} \left(1 + \frac{3}{5}t + \frac{1}{10}t^2 + \frac{1}{130}t^3 + \frac{3}{8840}t^4 + \frac{3}{309400}t^5 + \mathrm{O}\left(t^6\right)\right) + c_2 \left(1 + t + \frac{3}{14}t^2 + \frac{3}{154}t^3 + \frac{3}{3080}t^4 + \frac{9}{292600}t^5 + \mathrm{O}\left(t^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 81

```
AsymptoticDSolveValue[4*t*y''[t]+3*y'[t]-3*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1 \sqrt[4]{t} \left(\frac{3t^5}{309400} + \frac{3t^4}{8840} + \frac{t^3}{130} + \frac{t^2}{10} + \frac{3t}{5} + 1\right) + c_2 \left(\frac{9t^5}{292600} + \frac{3t^4}{3080} + \frac{3t^3}{154} + \frac{3t^2}{14} + t + 1\right)$$

## 14.12 problem 12

Internal problem ID [1804]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2t^2y'' + (t^2 - t)\, y' + y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*t^2*diff(y(t),t$2)+(t^2-t)*diff(y(t),t)+y(t)=0,y(t),type='series',t=0);
```

$$\begin{aligned} y(t) = c_1\sqrt{t}\left(1 - \frac{1}{2}t + \frac{1}{8}t^2 - \frac{1}{48}t^3 + \frac{1}{384}t^4 - \frac{1}{3840}t^5 + \mathrm{O}\left(t^6\right)\right) \\ + c_2 t\left(1 - \frac{1}{3}t + \frac{1}{15}t^2 - \frac{1}{105}t^3 + \frac{1}{945}t^4 - \frac{1}{10395}t^5 + \mathrm{O}\left(t^6\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 86

```
AsymptoticDSolveValue[2*t^2*y''[t]+(t^2-t)*y'[t]+y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1 t\left(-\frac{t^5}{10395} + \frac{t^4}{945} - \frac{t^3}{105} + \frac{t^2}{15} - \frac{t}{3} + 1\right) + c_2\sqrt{t}\left(-\frac{t^5}{3840} + \frac{t^4}{384} - \frac{t^3}{48} + \frac{t^2}{8} - \frac{t}{2} + 1\right)$$

## 14.13 problem 13

Internal problem ID [1805]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$t^3y'' - ty' - \left(t^2 + \frac{5}{4}\right)y = 0$$

With the expansion point for the power series method at $t = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(t^3*diff(y(t),t$2)-t*diff(y(t),t)-(t^2+5/4)*y(t)=0,y(t),type='series',t=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 97

```
AsymptoticDSolveValue[t^3*y''[t]-t*y'[t]-(t^2+5/4)*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_2 e^{-1/t}\left(-\frac{239684276027t^5}{8388608} + \frac{1648577803t^4}{524288} - \frac{3127415t^3}{8192} + \frac{26113t^2}{512} - \frac{117t}{16} + 1\right)t^{13/4} + \frac{c_1\left(-\frac{784957t^5}{8388608} - \frac{152693t^4}{524288} - \frac{7649t^3}{8192} - \frac{31t^2}{512} + \frac{45t}{16} + 1\right)}{t^{5/4}}$$

## 14.14 problem 14

Internal problem ID [1806]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' + \left(-t^2 + t\right) y' - y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(t^2*diff(y(t),t$2)+(t-t^2)*diff(y(t),t)-y(t)=0,y(t),type='series',t=0);
```

$$y(t) = c_1 t\left(1 + \frac{1}{3}t + \frac{1}{12}t^2 + \frac{1}{60}t^3 + \frac{1}{360}t^4 + \frac{1}{2520}t^5 + \mathrm{O}\left(t^6\right)\right) + \frac{c_2\left(-2 - 2t - t^2 - \frac{1}{3}t^3 - \frac{1}{12}t^4 - \frac{1}{60}t^5 + \mathrm{O}\left(t^6\right)\right)}{t}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 64

```
AsymptoticDSolveValue[t^2*y''[t]+(t-t^2)*y'[t]-y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1\left(\frac{t^3}{24} + \frac{t^2}{6} + \frac{t}{2} + \frac{1}{t} + 1\right) + c_2\left(\frac{t^5}{360} + \frac{t^4}{60} + \frac{t^3}{12} + \frac{t^2}{3} + t\right)$$

## 14.15 problem 15

Internal problem ID [1807]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 15.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{ty'' - (t^2 + 2)\, y' + yt = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
Order:=6;
dsolve(t*diff(y(t),t$2)-(t^2+2)*diff(y(t),t)+t*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = c_1 t^3 \left(1 + \frac{1}{5}t^2 + \frac{1}{35}t^4 + \mathrm{O}\left(t^6\right)\right) + c_2 \left(12 + 6t^2 + \frac{3}{2}t^4 + \mathrm{O}\left(t^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 44

```
AsymptoticDSolveValue[t*y''[t]-(t^2+2)*y'[t]+t*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1 \left(\frac{t^4}{8} + \frac{t^2}{2} + 1\right) + c_2 \left(\frac{t^7}{35} + \frac{t^5}{5} + t^3\right)$$

## 14.16 problem 16

Internal problem ID [1808]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 16.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Laguerre, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{t^2y'' + (-t^2 + 3t)\,y' - yt = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(t^2*diff(y(t),t$2)+(3*t-t^2)*diff(y(t),t)-t*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = c_1\left(1 + \frac{1}{3}t + \frac{1}{12}t^2 + \frac{1}{60}t^3 + \frac{1}{360}t^4 + \frac{1}{2520}t^5 + \mathrm{O}\left(t^6\right)\right) + \frac{c_2\left(-2 - 2t - t^2 - \frac{1}{3}t^3 - \frac{1}{12}t^4 - \frac{1}{60}t^5 + \mathrm{O}\left(t^6\right)\right)}{t^2}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 60

```
AsymptoticDSolveValue[t^2*y''[t]+(3*t-t^2)*y'[t]-t*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1\left(\frac{t^2}{24} + \frac{1}{t^2} + \frac{t}{6} + \frac{1}{t} + \frac{1}{2}\right) + c_2\left(\frac{t^4}{360} + \frac{t^3}{60} + \frac{t^2}{12} + \frac{t}{3} + 1\right)$$

## 14.17 problem 17

Internal problem ID [1809]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 17.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' + t(t+1)\,y' - y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(t^2*diff(y(t),t$2)+t*(t+1)*diff(y(t),t)-y(t)=0,y(t),type='series',t=0);
```

$$y(t) = c_1 t\left(1 - \frac{1}{3}t + \frac{1}{12}t^2 - \frac{1}{60}t^3 + \frac{1}{360}t^4 - \frac{1}{2520}t^5 + \mathrm{O}\left(t^6\right)\right) + \frac{c_2\left(-2 + 2t - t^2 + \frac{1}{3}t^3 - \frac{1}{12}t^4 + \frac{1}{60}t^5 + \mathrm{O}\left(t^6\right)\right)}{t}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 64

```
AsymptoticDSolveValue[t^2*y''[t]+t*(t+1)*y'[t]-y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1\left(\frac{t^3}{24} - \frac{t^2}{6} + \frac{t}{2} + \frac{1}{t} - 1\right) + c_2\left(\frac{t^5}{360} - \frac{t^4}{60} + \frac{t^3}{12} - \frac{t^2}{3} + t\right)$$

## 14.18 problem 18

Internal problem ID [1810]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 18.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{ty'' - y'(t+4) + 2y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 40

```
Order:=6;
dsolve(t*diff(y(t),t$2)-(4+t)*diff(y(t),t)+2*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = c_1 t^5 \left(1 + \frac{1}{2}t + \frac{1}{7}t^2 + \frac{5}{168}t^3 + \frac{5}{1008}t^4 + \frac{1}{1440}t^5 + \mathrm{O}\left(t^6\right)\right) + c_2\left(2880 + 1440t + 240t^2 + 4t^5 + \mathrm{O}\left(t^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 56

```
AsymptoticDSolveValue[t*y''[t]-(4+t)*y'[t]+2*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1\left(\frac{t^2}{12} + \frac{t}{2} + 1\right) + c_2\left(\frac{5t^9}{1008} + \frac{5t^8}{168} + \frac{t^7}{7} + \frac{t^6}{2} + t^5\right)$$

## 14.19 problem 19

Internal problem ID [1811]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 19.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' + (t^2 - 3t)\, y' + 3y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
Order:=6;
dsolve(t^2*diff(y(t),t$2)+(t^2-3*t)*diff(y(t),t)+3*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = t\left(c_1 t^2\left(1 - t + \frac{1}{2}t^2 - \frac{1}{6}t^3 + \frac{1}{24}t^4 - \frac{1}{120}t^5 + \mathrm{O}\left(t^6\right)\right) + c_2\left(\ln(t)\left(2t^2 - 2t^3 + t^4 - \frac{1}{3}t^5 + \mathrm{O}\left(t^6\right)\right) + \left(-2 - 2t + 3t^2 - t^3 + \frac{1}{9}t^5 + \mathrm{O}\left(t^6\right)\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 76

```
AsymptoticDSolveValue[t^2*y''[t]+(t^2-3*t)*y'[t]+3*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1\left(\frac{1}{4}t(t^4 - 4t^2 + 4t + 4) - \frac{1}{2}t^3(t^2 - 2t + 2)\log(t)\right) + c_2\left(\frac{t^7}{24} - \frac{t^6}{6} + \frac{t^5}{2} - t^4 + t^3\right)$$

## 14.20 problem 20

Internal problem ID [1812]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 20.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' + ty' - (t+1)\,y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
Order:=6;
dsolve(t^2*diff(y(t),t$2)+t*diff(y(t),t)-(1+t)*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = \frac{c_1 t^2\left(1 + \frac{1}{3}t + \frac{1}{24}t^2 + \frac{1}{360}t^3 + \frac{1}{8640}t^4 + \frac{1}{302400}t^5 + \mathrm{O}\left(t^6\right)\right) + c_2\left(\ln(t)\left(t^2 + \frac{1}{3}t^3 + \frac{1}{24}t^4 + \frac{1}{360}t^5 + \mathrm{O}\left(t^6\right)\right) + (-\right.}{t}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 83

```
AsymptoticDSolveValue[t^2*y''[t]+t*y'[t]-(1+t)*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1\left(\frac{31t^4 + 176t^3 + 144t^2 - 576t + 576}{576t} - \frac{1}{48}t\left(t^2 + 8t + 24\right)\log(t)\right) + c_2\left(\frac{t^5}{8640} + \frac{t^4}{360} + \frac{t^3}{24} + \frac{t^2}{3} + t\right)$$

## 14.21 problem 21

Internal problem ID [1813]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 21.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' + ty' + 2y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 60

```
Order:=6;
dsolve(t*diff(y(t),t$2)+t*diff(y(t),t)+2*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = c_1 t\left(1 - \frac{3}{2}t + t^2 - \frac{5}{12}t^3 + \frac{1}{8}t^4 - \frac{7}{240}t^5 + \mathrm{O}\left(t^6\right)\right) + c_2\left(\ln(t)\left((-2)\,t + 3t^2 - 2t^3 + \frac{5}{6}t^4 - \frac{1}{4}t^5 + \mathrm{O}\left(t^6\right)\right) + \left(1 - t - 2t^2 + \frac{5}{2}t^3 - \frac{49}{36}t^4 + \frac{23}{48}t^5 + \mathrm{O}\left(t^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 83

```
AsymptoticDSolveValue[t*y''[t]+t*y'[t]+2*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1\left(\frac{1}{6}t\left(5t^3 - 12t^2 + 18t - 12\right)\log(t) + \frac{1}{36}\left(-79t^4 + 162t^3 - 180t^2 + 36t + 36\right)\right) + c_2\left(\frac{t^5}{8} - \frac{5t^4}{12} + t^3 - \frac{3t^2}{2} + t\right)$$

## 14.22 problem 22

Internal problem ID [1814]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 22.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$ty'' + (-t^2 + 1)\, y' + 4yt = 0$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
Order:=6;
dsolve(t*diff(y(t),t$2)+(1-t^2)*diff(y(t),t)+4*t*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = (c_2 \ln(t) + c_1)\left(1 - t^2 + \frac{1}{8}t^4 + \mathrm{O}(t^6)\right) + \left(\frac{5}{4}t^2 - \frac{9}{32}t^4 + \mathrm{O}(t^6)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 56

```
AsymptoticDSolveValue[t*y''[t]+(1-t^2)*y'[t]+4*t*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1\left(\frac{t^4}{8} - t^2 + 1\right) + c_2\left(-\frac{9t^4}{32} + \frac{5t^2}{4} + \left(\frac{t^4}{8} - t^2 + 1\right)\log(t)\right)$$

## 14.23 problem 23

Internal problem ID [1815]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 23.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{t^2y'' + ty' + yt^2 = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
Order:=6;
dsolve(t^2*diff(y(t),t$2)+t*diff(y(t),t)+t^2*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = (c_2 \ln(t) + c_1)\left(1 - \frac{1}{4}t^2 + \frac{1}{64}t^4 + \mathrm{O}\left(t^6\right)\right) + \left(\frac{1}{4}t^2 - \frac{3}{128}t^4 + \mathrm{O}\left(t^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 60

```
AsymptoticDSolveValue[t^2*y''[t]+t*y'[t]+t^2*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1\left(\frac{t^4}{64} - \frac{t^2}{4} + 1\right) + c_2\left(-\frac{3t^4}{128} + \frac{t^2}{4} + \left(\frac{t^4}{64} - \frac{t^2}{4} + 1\right)\log(t)\right)$$

## 14.24 problem 24

Internal problem ID [1816]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 24.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Bessel]`

$$t^2y'' + y't + \left(t^2 - v^2\right) y = 0$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
Order:=6;
dsolve(t^2*diff(y(t),t$2)+t*diff(y(t),t)+(t^2-v^2)*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = c_1 t^{-v} \left(1 + \frac{1}{4v-4} t^2 + \frac{1}{32} \frac{1}{(v-2)(v-1)} t^4 + \mathrm{O}\left(t^6\right)\right) + c_2 t^v \left(1 - \frac{1}{4v+4} t^2 + \frac{1}{32} \frac{1}{(v+2)(v+1)} t^4 + \mathrm{O}\left(t^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 160

```
AsymptoticDSolveValue[t^2*y''[t]+t*y'[t]+(t^2-v^2)*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_2 \left(\frac{t^4}{(-v^2 - v + (1-v)(2-v) + 2)(-v^2 - v + (3-v)(4-v) + 4)} - \frac{t^2}{-v^2 - v + (1-v)(2-v) + 2} + 1\right) t^{-v} + c_1 \left(\frac{t^4}{(-v^2 + v + (v+1)(v+2) + 2)(-v^2 + v + (v+3)(v+4) + 4)} - \frac{t^2}{-v^2 + v + (v+1)(v+2) + 2} + 1\right) t^v$$

## 14.25 problem 25

Internal problem ID [1817]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 25.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Laguerre]`

$$ty'' + (-t+1)\,y' + \lambda y = 0$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 309

```
Order:=6;
dsolve(t*diff(y(t),t$2)+(1-t)*diff(y(t),t)+lambda*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = \left((2\lambda+1)\,t + \left(\frac{1}{4}\lambda + \frac{1}{4} - \frac{3}{4}\lambda^2\right)t^2 + \left(-\frac{2}{9}\lambda^2 + \frac{1}{27}\lambda + \frac{1}{18} + \frac{11}{108}\lambda^3\right)t^3 + \left(\frac{7}{192}\lambda^3 - \frac{167}{3456}\lambda^2 + \frac{1}{192}\lambda + \frac{1}{96} - \frac{25}{3456}\lambda^4\right)t^4 + \left(\frac{1}{1500}\lambda - \frac{37}{4320}\lambda^2 + \frac{719}{86400}\lambda^3 - \frac{61}{21600}\lambda^4 + \frac{137}{432000}\lambda^5 + \frac{1}{600}\right)t^5 + \mathrm{O}\left(t^6\right)\right)c_2 + \left(1 - \lambda t + \frac{1}{4}(-1+\lambda)\,\lambda t^2 - \frac{1}{36}(\lambda-2)\,(-1+\lambda)\,\lambda t^3 + \frac{1}{576}(\lambda-3)\,(\lambda-2)\,(-1+\lambda)\,\lambda t^4 - \frac{1}{14400}(\lambda-4)\,(\lambda-3)\,(\lambda-2)\,(-1+\lambda)\,\lambda t^5 + \mathrm{O}\left(t^6\right)\right)(c_2\ln(t) + c_1)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 415

```
AsymptoticDSolveValue[t*y''[t]+(1-t)*y'[t]+\[Lambda]*y[t]==0,y[t],{t,0,5}]
```

$$
\begin{aligned}
y(t) \rightarrow\ & c_1\left(-\frac{(\lambda-4)(\lambda-3)(\lambda-2)(\lambda-1)\lambda t^5}{14400}+\frac{1}{576}(\lambda-3)(\lambda-2)(\lambda-1)\lambda t^4\right. \\
& \left.-\frac{1}{36}(\lambda-2)(\lambda-1)\lambda t^3+\frac{1}{4}(\lambda-1)\lambda t^2-\lambda t+1\right)+c_2\left(\frac{(\lambda-4)(\lambda-3)(\lambda-2)(\lambda-1)t^5}{14400}\right. \\
& +\frac{(\lambda-4)(\lambda-3)(\lambda-2)\lambda t^5}{14400}+\frac{(\lambda-4)(\lambda-3)(\lambda-1)\lambda t^5}{14400}+\frac{(\lambda-4)(\lambda-2)(\lambda-1)\lambda t^5}{14400} \\
& +\frac{137(\lambda-4)(\lambda-3)(\lambda-2)(\lambda-1)\lambda t^5}{432000}+\frac{(\lambda-3)(\lambda-2)(\lambda-1)\lambda t^5}{14400} \\
& -\frac{1}{576}(\lambda-3)(\lambda-2)(\lambda-1)t^4-\frac{1}{576}(\lambda-3)(\lambda-2)\lambda t^4-\frac{1}{576}(\lambda-3)(\lambda-1)\lambda t^4 \\
& -\frac{25(\lambda-3)(\lambda-2)(\lambda-1)\lambda t^4}{3456}-\frac{1}{576}(\lambda-2)(\lambda-1)\lambda t^4+\frac{1}{36}(\lambda-2)(\lambda-1)t^3 \\
& +\frac{1}{36}(\lambda-2)\lambda t^3+\frac{11}{108}(\lambda-2)(\lambda-1)\lambda t^3+\frac{1}{36}(\lambda-1)\lambda t^3-\frac{1}{4}(\lambda-1)t^2-\frac{3}{4}(\lambda-1)\lambda t^2 \\
& -\frac{\lambda t^2}{4}+\left(-\frac{(\lambda-4)(\lambda-3)(\lambda-2)(\lambda-1)\lambda t^5}{14400}+\frac{1}{576}(\lambda-3)(\lambda-2)(\lambda-1)\lambda t^4\right. \\
& \left.\left.-\frac{1}{36}(\lambda-2)(\lambda-1)\lambda t^3+\frac{1}{4}(\lambda-1)\lambda t^2-\lambda t+1\right)\log(t)+2\lambda t+t\right)
\end{aligned}
$$

## 14.26 problem 27

Internal problem ID [1818]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 27.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2\sin(t)\,y'' + (-t+1)\,y' - 2y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.375 (sec). Leaf size: 44

```
Order:=6;
dsolve(2*sin(t)*diff(y(t),t$2)+(1-t)*diff(y(t),t)-2*y(t)=0,y(t),type='series',t=0);
```

$$\begin{aligned} y(t) = {} & c_1\sqrt{t}\left(1+\frac{5}{6}t+\frac{17}{60}t^2+\frac{89}{1260}t^3+\frac{941}{45360}t^4+\frac{14989}{2494800}t^5+\mathrm{O}\left(t^6\right)\right) \\ & + c_2\left(1+2t+t^2+\frac{4}{15}t^3+\frac{1}{14}t^4+\frac{101}{4725}t^5+\mathrm{O}\left(t^6\right)\right) \end{aligned}$$

## ✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 1303

```
AsymptoticDSolveValue[2*sin(t)*y''[t]+(1-t)*y'[t]-2*y[t]==0,y[t],{t,0,5}]
```

$$
\begin{aligned}
y(t) \to & \left( \frac{\left(\frac{2\sin-1}{4\sin^2}+\frac{1}{\sin}\right)\left(-\frac{\frac{2\sin-1}{2\sin}+1}{2\sin}-\frac{1}{\sin}\right)\left(-\frac{\frac{2\sin-1}{2\sin}+2}{2\sin}-\frac{1}{\sin}\right)\left(\cdots\right.}{\left(\frac{(2\sin-1)\left(\frac{2\sin-1}{2\sin}+1\right)}{2\sin}+\frac{\frac{2\sin-1}{2\sin}+1}{2\sin}\right)\left(\left(\frac{2\sin-1}{2\sin}+1\right)\left(\frac{2\sin-1}{2\sin}+2\right)+\frac{\frac{2\sin-1}{2\sin}+2}{2\sin}\right)\left(\left(\frac{2\sin-1}{2\sin}+2\right)\left(\frac{2\sin-1}{2\sin}+3\right)+\cdots\right.} \right. \\
& - \frac{\left(\frac{2\sin-1}{4\sin^2}+\frac{1}{\sin}\right)\left(-\frac{\frac{2\sin-1}{2\sin}+1}{2\sin}-\frac{1}{\sin}\right)\left(-\frac{\frac{2\sin-1}{2\sin}+2}{2\sin}-\frac{1}{\sin}\right)\left(-\frac{\frac{2\sin-1}{2\sin}+3}{2\sin}\cdots\right.}{\left(\frac{(2\sin-1)\left(\frac{2\sin-1}{2\sin}+1\right)}{2\sin}+\frac{\frac{2\sin-1}{2\sin}+1}{2\sin}\right)\left(\left(\frac{2\sin-1}{2\sin}+1\right)\left(\frac{2\sin-1}{2\sin}+2\right)+\frac{\frac{2\sin-1}{2\sin}+2}{2\sin}\right)\left(\left(\frac{2\sin-1}{2\sin}+2\right)\left(\frac{2\sin-1}{2\sin}+3\right)+\cdots\right.} \\
& + \frac{\left(\frac{2\sin-1}{4\sin^2}+\frac{1}{\sin}\right)\left(-\frac{\frac{2\sin-1}{2\sin}+1}{2\sin}-\frac{1}{\sin}\right)\left(-\frac{\frac{2\sin-1}{2\sin}+2}{2\sin}-\frac{1}{\sin}\right)t^3}{\left(\frac{(2\sin-1)\left(\frac{2\sin-1}{2\sin}+1\right)}{2\sin}+\frac{\frac{2\sin-1}{2\sin}+1}{2\sin}\right)\left(\left(\frac{2\sin-1}{2\sin}+1\right)\left(\frac{2\sin-1}{2\sin}+2\right)+\frac{\frac{2\sin-1}{2\sin}+2}{2\sin}\right)\left(\left(\frac{2\sin-1}{2\sin}+2\right)\left(\frac{2\sin-1}{2\sin}+3\right)+\cdots\right.} \\
& - \frac{\left(\frac{2\sin-1}{4\sin^2}+\frac{1}{\sin}\right)\left(-\frac{\frac{2\sin-1}{2\sin}+1}{2\sin}-\frac{1}{\sin}\right)t^2}{\left(\frac{(2\sin-1)\left(\frac{2\sin-1}{2\sin}+1\right)}{2\sin}+\frac{\frac{2\sin-1}{2\sin}+1}{2\sin}\right)\left(\left(\frac{2\sin-1}{2\sin}+1\right)\left(\frac{2\sin-1}{2\sin}+2\right)+\frac{\frac{2\sin-1}{2\sin}+2}{2\sin}\right)} \\
& \left. + \frac{\left(\frac{2\sin-1}{4\sin^2}+\frac{1}{\sin}\right)t}{\frac{(2\sin-1)\left(\frac{2\sin-1}{2\sin}+1\right)}{2\sin}+\frac{\frac{2\sin-1}{2\sin}+1}{2\sin}}+1\right)c_1 t^{\frac{2\sin-1}{2\sin}} \\
& + \left(\frac{45t^5}{\left(2+\frac{1}{\sin}\right)\left(6+\frac{3}{2\sin}\right)\left(12+\frac{2}{\sin}\right)\left(20+\frac{5}{2\sin}\right)\sin^4}+\frac{15t^4}{\left(2+\frac{1}{\sin}\right)\left(6+\frac{3}{2\sin}\right)\left(12+\frac{2}{\sin}\right)\sin^3}\right. \\
& \left. + \frac{6t^3}{\left(2+\frac{1}{\sin}\right)\left(6+\frac{3}{2\sin}\right)\sin^2}+\frac{3t^2}{\left(2+\frac{1}{\sin}\right)\sin}+2t+1\right)c_2
\end{aligned}
$$

## 14.27 problem 29

Internal problem ID [1819]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.2, Regular singular points, the method of Frobenius. Page 214
**Problem number**: 29.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$t^2y'' + ty' + (t+1)\,y = 0$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
Order:=6;
dsolve(t^2*diff(y(t),t$2)+t*diff(y(t),t)+(1+t)*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = c_1 t^{-i}\left(1+\left(-\frac{1}{5}-\frac{2i}{5}\right)t+\left(-\frac{1}{40}+\frac{3i}{40}\right)t^2+\left(\frac{3}{520}-\frac{7i}{1560}\right)t^3+\left(-\frac{1}{2496}+\frac{i}{12480}\right)t^4 + \left(\frac{9}{603200}+\frac{i}{361920}\right)t^5+\mathrm{O}\left(t^6\right)\right)+c_2 t^{i}\left(1+\left(-\frac{1}{5}+\frac{2i}{5}\right)t+\left(-\frac{1}{40}-\frac{3i}{40}\right)t^2 + \left(\frac{3}{520}+\frac{7i}{1560}\right)t^3+\left(-\frac{1}{2496}-\frac{i}{12480}\right)t^4+\left(\frac{9}{603200}-\frac{i}{361920}\right)t^5+\mathrm{O}\left(t^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 90

```
AsymptoticDSolveValue[t^2*y''[t]+t*y'[t]+(1+t)*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to \left(\frac{1}{12480}+\frac{i}{2496}\right)c_2 t^{-i}\left(it^4-(8+16i)t^3+(168+96i)t^2-(1056-288i)t+(480-2400i)\right) - \left(\frac{1}{2496}+\frac{i}{12480}\right)c_1 t^{i}\left(t^4-(16+8i)t^3+(96+168i)t^2+(288-1056i)t-(2400-480i)\right)$$

# 15 Section 2.8.3, The method of Frobenius. Equal roots, and roots differering by an integer. Page 223

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## 15.1 problem 1

Internal problem ID [1820]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.3, The method of Frobenius. Equal roots, and roots differering by an integer. Page 223
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{ty'' + y' - 4y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 59

```
Order:=6;
dsolve(t*diff(y(t),t$2)+diff(y(t),t)-4*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = (c_2 \ln(t) + c_1)\left(1 + 4t + 4t^2 + \frac{16}{9}t^3 + \frac{4}{9}t^4 + \frac{16}{225}t^5 + O(t^6)\right) + \left((-8)t - 12t^2 - \frac{176}{27}t^3 - \frac{50}{27}t^4 - \frac{1096}{3375}t^5 + O(t^6)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 105

```
AsymptoticDSolveValue[t*y''[t]+y'[t]-4*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1\left(\frac{16t^5}{225} + \frac{4t^4}{9} + \frac{16t^3}{9} + 4t^2 + 4t + 1\right) + c_2\left(-\frac{1096t^5}{3375} - \frac{50t^4}{27} - \frac{176t^3}{27} - 12t^2 + \left(\frac{16t^5}{225} + \frac{4t^4}{9} + \frac{16t^3}{9} + 4t^2 + 4t + 1\right)\log(t) - 8t\right)$$

## 15.2 problem 2

Internal problem ID [1821]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.3, The method of Frobenius. Equal roots, and roots differering by an integer. Page 223
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' - t(t+1)\,y' + y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
Order:=6;
dsolve(t^2*diff(y(t),t$2)-t*(1+t)*diff(y(t),t)+y(t)=0,y(t),type='series',t=0);
```

$$y(t) = t\left((c_2 \ln(t) + c_1)\left(1 + t + \frac{1}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \mathrm{O}\left(t^6\right)\right) + \left(-t - \frac{3}{4}t^2 - \frac{11}{36}t^3 - \frac{25}{288}t^4 - \frac{137}{7200}t^5 + \mathrm{O}\left(t^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 112

```
AsymptoticDSolveValue[t^2*y''[t]-t*(1+t)*y'[t]+y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_1 t\left(\frac{t^5}{120} + \frac{t^4}{24} + \frac{t^3}{6} + \frac{t^2}{2} + t + 1\right) + c_2\left(t\left(-\frac{137t^5}{7200} - \frac{25t^4}{288} - \frac{11t^3}{36} - \frac{3t^2}{4} - t\right) + t\left(\frac{t^5}{120} + \frac{t^4}{24} + \frac{t^3}{6} + \frac{t^2}{2} + t + 1\right)\log(t)\right)$$

## 15.3 problem 3

Internal problem ID [1822]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.3, The method of Frobenius. Equal roots, and roots differering by an integer. Page 223
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Bessel]`

$$t^2y'' + ty' + (t^2 - 1)\,y = 0$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(t^2*diff(y(t),t$2)+t*diff(y(t),t)+(t^2-1)*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = \frac{c_1 t^2\left(1 - \frac{1}{8}t^2 + \frac{1}{192}t^4 + \mathrm{O}\left(t^6\right)\right) + c_2\left(\ln(t)\left(t^2 - \frac{1}{8}t^4 + \mathrm{O}\left(t^6\right)\right) + \left(-2 + \frac{3}{32}t^4 + \mathrm{O}\left(t^6\right)\right)\right)}{t}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 58

```
AsymptoticDSolveValue[t^2*y''[t]+t*y'[t]+(t^2-1)*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_2\left(\frac{t^5}{192} - \frac{t^3}{8} + t\right) + c_1\left(\frac{1}{16}t\left(t^2 - 8\right)\log(t) - \frac{5t^4 - 16t^2 - 64}{64t}\right)$$

## 15.4 problem 4

Internal problem ID [1823]

**Book**: Differential equations and their applications, 3rd ed., M. Braun
**Section**: Section 2.8.3, The method of Frobenius. Equal roots, and roots differering by an integer. Page 223
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{ty'' + 3y' - 3y = 0}$$

With the expansion point for the power series method at $t = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 62

```
Order:=6;
dsolve(t*diff(y(t),t$2)+3*diff(y(t),t)-3*y(t)=0,y(t),type='series',t=0);
```

$$y(t) = \frac{c_1\left(1 + t + \frac{3}{8}t^2 + \frac{3}{40}t^3 + \frac{3}{320}t^4 + \frac{9}{11200}t^5 + \mathrm{O}\left(t^6\right)\right)t^2 + c_2\left(\ln\left(t\right)\left(9t^2 + 9t^3 + \frac{27}{8}t^4 + \frac{27}{40}t^5 + \mathrm{O}\left(t^6\right)\right) + \left(-2 + \right.\right.}{t^2}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 78

```
AsymptoticDSolveValue[t*y''[t]+3*y'[t]-3*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \to c_2\left(\frac{3t^4}{320} + \frac{3t^3}{40} + \frac{3t^2}{8} + t + 1\right) + c_1\left(\frac{279t^4 + 528t^3 + 144t^2 - 192t + 64}{64t^2} - \frac{9}{16}\left(3t^2 + 8t + 8\right)\log(t)\right)$$