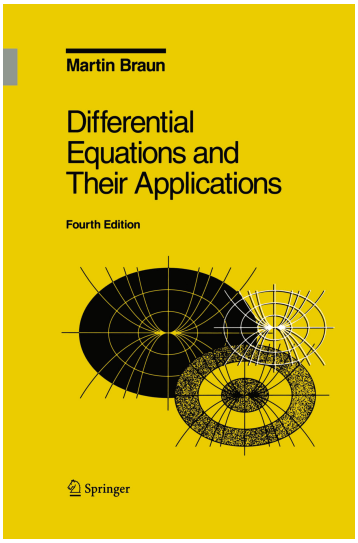
A Solution Manual For

Differential equations and their applications, 4th ed., M. Braun



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1 Section 3.8, Systems of differential equations. The eigenva1ue-eigenvector method. Page 339

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1.1 problem 1

Internal problem ID [1824]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.8, Systems of differential equations. The eigenva1ue-eigenvector method. Page 339
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 6x_1(t) - 3x_2(t)$$
$$x_2'(t) = 2x_1(t) + x_2(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve([diff(x__1(t),t)=6*x__1(t)-3*x__2(t),diff(x__2(t),t)=2*x__1(t)+1*x__2(t)],[x__1(t), x_
```

$$x_1(t) = \frac{3c_1 e^{4t}}{2} + c_2 e^{3t}$$

$$x_2(t) = c_1 e^{4t} + c_2 e^{3t}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 60

```
DSolve[{x1'[t]==6*x1[t]-3*x2[t],x2'[t]==2*x1[t]+1*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolut
```

$$\text{x1}(t) \to e^{3t}\left(c_1\left(3e^t - 2\right) - 3c_2\left(e^t - 1\right)\right)$$
$$\text{x2}(t) \to e^{3t}\left(2c_1\left(e^t - 1\right) + c_2\left(3 - 2e^t\right)\right)$$

1.2 problem 2

Internal problem ID [1825]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.8, Systems of differential equations. The eigenva1ue-eigenvector method. Page 339
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -2x_1(t) + x_2(t)$$
$$x_2'(t) = -4x_1(t) + 3x_2(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 35

```
dsolve([diff(x__1(t),t)=-2*x__1(t)+1*x__2(t),diff(x__2(t),t)=-4*x__1(t)+3*x__2(t)],[x__1(t),
```

$$x_1(t) = \mathrm{e}^{-t}c_1 + \frac{c_2\mathrm{e}^{2t}}{4}$$

$$x_2(t) = \mathrm{e}^{-t}c_1 + c_2\mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 71

```
DSolve[{x1'[t]==-2*x1[t]+1*x2[t],x2'[t]==-4*x1[t]+3*x2[t]},{x1[t],x2[t]},t,IncludeSingularSol
```

$$\mathrm{x1}(t) \to \frac{1}{3}e^{-t}\left((c_2 - c_1)e^{3t} + 4c_1 - c_2\right)$$
$$\mathrm{x2}(t) \to \frac{1}{3}e^{-t}\left(-4(c_1 - c_2)e^{3t} + 4c_1 - c_2\right)$$

1.3 problem 3

Internal problem ID [1826]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.8, Systems of differential equations. The eigenva1ue-eigenvector method. Page 339
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) + 2x_2(t) + 4x_3(t)$$
$$x_2'(t) = 2x_1(t) + 2x_3(t)$$
$$x_3'(t) = 4x_1(t) + 2x_2(t) + 3x_3(t)$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 66

```
dsolve([diff(x__1(t),t)=3*x__1(t)+2*x__2(t)+4*x__3(t),diff(x__2(t),t)=2*x__1(t)+0*x__2(t)+2*x
```

$$x_1(t) = c_2 \mathrm{e}^{8t} - \frac{5c_3 \mathrm{e}^{-t}}{4} - \frac{\mathrm{e}^{-t} c_1}{2}$$

$$x_2(t) = \frac{c_2 \mathrm{e}^{8t}}{2} + \frac{c_3 \mathrm{e}^{-t}}{2} + \mathrm{e}^{-t} c_1$$

$$x_3(t) = c_2 \mathrm{e}^{8t} + c_3 \mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 127

```
DSolve[{x1'[t]==3*x1[t]+2*x2[t]+4*x3[t],x2'[t]==2*x1[t]+0*x2[t]+2*x3[t],x3'[t]==4*x1[t]+2*x2[
```

$$\text{x1}(t) \to \frac{1}{9}e^{-t}\left(c_1\left(4e^{9t}+5\right)+2(c_2+2c_3)\left(e^{9t}-1\right)\right)$$

$$\text{x2}(t) \to \frac{1}{9}e^{-t}\left((2c_1+c_2+2c_3)e^{9t}-2(c_1-4c_2+c_3)\right)$$

$$\text{x3}(t) \to \frac{1}{9}e^{-t}\left(2(2c_1+c_2+2c_3)e^{9t}-4c_1-2c_2+5c_3\right)$$

1.4 problem 4

Internal problem ID [1827]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.8, Systems of differential equations. The eigenva1ue-eigenvector method. Page 339
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 7x_1(t) - x_2(t) + 6x_3(t)$$
$$x_2'(t) = -10x_1(t) + 4x_2(t) - 12x_3(t)$$
$$x_3'(t) = -2x_1(t) + x_2(t) - x_3(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 73

```
dsolve([diff(x__1(t),t)=7*x__1(t)-1*x__2(t)+6*x__3(t),diff(x__2(t),t)=-10*x__1(t)+4*x__2(t)-1
```

$$x_1(t) = -c_1 e^{2t} - \frac{3c_2 e^{5t}}{2} - c_3 e^{3t}$$

$$x_2(t) = c_1 e^{2t} + 3c_2 e^{5t} + 2c_3 e^{3t}$$

$$x_3(t) = c_1 e^{2t} + c_2 e^{5t} + c_3 e^{3t}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 137

```
DSolve[{x1'[t]==7*x1[t]-1*x2[t]+6*x3[t],x2'[t]==-10*x1[t]+4*x2[t]-12*x3[t],x3'[t]==-2*x1[t]+1
```

$$\text{x1}(t) \to e^{2t}\left(3(c_1 + c_3)e^{3t} - (4c_1 + c_2 + 3c_3)e^{t} + 2c_1 + c_2\right)$$
$$\text{x2}(t) \to e^{2t}\left(-6(c_1 + c_3)e^{3t} + 2(4c_1 + c_2 + 3c_3)e^{t} - 2c_1 - c_2\right)$$
$$\text{x3}(t) \to e^{2t}\left(-2(c_1 + c_3)e^{3t} + (4c_1 + c_2 + 3c_3)e^{t} - 2c_1 - c_2\right)$$

1.5 problem 5

Internal problem ID [1828]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.8, Systems of differential equations. The eigenva1ue-eigenvector method. Page 339
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -7x_1(t) + 6x_3(t)$$
$$x_2'(t) = 5x_2(t)$$
$$x_3'(t) = 6x_1(t) + 2x_3(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 45

```
dsolve([diff(x__1(t),t)=-7*x__1(t)+0*x__2(t)+6*x__3(t),diff(x__2(t),t)=0*x__1(t)+5*x__2(t)+0*
```

$$x_1(t) = -2c_2 e^{-10t} + \frac{c_3 e^{5t}}{2}$$

$$x_2(t) = c_1 e^{5t}$$

$$x_3(t) = c_2 e^{-10t} + c_3 e^{5t}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 152

```
DSolve[{x1'[t]==-7*x1[t]+0*x2[t]+6*x3[t],x2'[t]==0*x1[t]+5*x2[t]+0*x3[t],x3'[t]==6*x1[t]+0*x2
```

$$\text{x1}(t) \to \frac{1}{5}e^{-10t}\left((c_1+2c_2)e^{15t}+4c_1-2c_2\right)$$
$$\text{x3}(t) \to \frac{1}{5}e^{-10t}\left(2(c_1+2c_2)e^{15t}-2c_1+c_2\right)$$
$$\text{x2}(t) \to c_3 e^{5t}$$
$$\text{x1}(t) \to \frac{1}{5}e^{-10t}\left((c_1+2c_2)e^{15t}+4c_1-2c_2\right)$$
$$\text{x3}(t) \to \frac{1}{5}e^{-10t}\left(2(c_1+2c_2)e^{15t}-2c_1+c_2\right)$$
$$\text{x2}(t) \to 0$$

1.6 problem 6

Internal problem ID [1829]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.8, Systems of differential equations. The eigenva1ue-eigenvector method. Page 339
Problem number: 6.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_1(t) + 2x_2(t) + 3x_3(t) + 6x_4(t) \\
x_2'(t) &= 3x_1(t) + 6x_2(t) + 9x_3(t) + 18x_4(t) \\
x_3'(t) &= 5x_1(t) + 10x_2(t) + 15x_3(t) + 30x_4(t) \\
x_4'(t) &= 7x_1(t) + 14x_2(t) + 21x_3(t) + 42x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 63

```
dsolve([diff(x__1(t),t)=1*x__1(t)+2*x__2(t)+3*x__3(t)+6*x__4(t),diff(x__2(t),t)=3*x__1(t)+6*x
```

$$x_1(t) = \frac{c_4 \mathrm{e}^{64t}}{7} - 9c_3 - 2c_1 - 3c_2$$

$$x_2(t) = \frac{3c_3}{7} + \frac{3c_4 \mathrm{e}^{64t}}{7} + c_1$$

$$x_3(t) = \frac{5c_3}{7} + \frac{5c_4 \mathrm{e}^{64t}}{7} + c_2$$

$$x_4(t) = c_3 + c_4 \mathrm{e}^{64t}$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 516

```
DSolve[{x1'[t]==1*x1[t]+2*x2[t]+3*x3[t]+6*x4[t],x2'[t]==3*x1[t]+6*x2[t]+9*x3[t]+19*x4[t],x3'[
```

x1(t)

$$\to \frac{e^{-\sqrt{1038}t}\left(2076(7c_1-c_4)e^{\sqrt{1038}t}-\left(7\sqrt{1038}c_1+14\sqrt{1038}c_2+21\sqrt{1038}c_3+2\left(5\sqrt{1038}-519\right)c_4\right)e^{32t}+(\right.}{14532}$$

x2(t)

$$\to \frac{\left(\left(3633-91\sqrt{1038}\right)c_1+\left(7266-182\sqrt{1038}\right)c_2+21\left(519-13\sqrt{1038}\right)c_3+\left(389\sqrt{1038}-8304\right)c_4\right)e^{\left(32-\right.}}{}$$

x3(t)

$$\to \frac{e^{-\sqrt{1038}t}\left(2076(7c_3-5c_4)e^{\sqrt{1038}t}-5\left(7\sqrt{1038}c_1+14\sqrt{1038}c_2+21\sqrt{1038}c_3+2\left(5\sqrt{1038}-519\right)c_4\right)e^{32t}-\right.}{14532}$$

x4(t)

$$\to \frac{e^{-\left(\left(\sqrt{1038}-32\right)t\right)}\left(\left(7(c_1+2c_2+3c_3)+\left(10+\sqrt{1038}\right)c_4\right)e^{2\sqrt{1038}t}-7c_1-14c_2-21c_3+\left(\sqrt{1038}-10\right)c_4\right)}{2\sqrt{1038}}$$

1.7 problem 7

Internal problem ID [1830]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.8, Systems of differential equations. The eigenva1ue-eigenvector method. Page 339
Problem number: 7.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + x_2(t)$$
$$x_2'(t) = 4x_1(t) + x_2(t)$$

With initial conditions

$$[x_1(0) = 2, x_2(0) = 3]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve([diff(x__1(t),t) = x__1(t)+x__2(t), diff(x__2(t),t) = 4*x__1(t)+x__2(t), x__1(0) = 2,
```

$$x_1(t) = \frac{\mathrm{e}^{-t}}{4} + \frac{7\,\mathrm{e}^{3t}}{4}$$

$$x_2(t) = -\frac{\mathrm{e}^{-t}}{2} + \frac{7\,\mathrm{e}^{3t}}{2}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 43

```
DSolve[{x1'[t]==1*x1[t]+1*x2[t],x2'[t]==4*x1[t]+1*x2[t]},{x1[0]==2,x2[0]==3},{x1[t],x2[t]},t,
```

$$\text{x1}(t) \to \frac{1}{4}e^{-t}\left(7e^{4t} + 1\right)$$
$$\text{x2}(t) \to e^{t}(4\sinh(2t) + 3\cosh(2t))$$

1.8 problem 8

Internal problem ID [1831]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.8, Systems of differential equations. The eigenva1ue-eigenvector method. Page 339
Problem number: 8.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - 3x_2(t)$$
$$x_2'(t) = -2x_1(t) + 2x_2(t)$$

With initial conditions

$$[x_1(0) = 0, x_2(0) = 5]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve([diff(x__1(t),t) = x__1(t)-3*x__2(t), diff(x__2(t),t) = -2*x__1(t)+2*x__2(t), x__1(0)
```

$$x_1(t) = 3\,\mathrm{e}^{-t} - 3\,\mathrm{e}^{4t}$$

$$x_2(t) = 2\,\mathrm{e}^{-t} + 3\,\mathrm{e}^{4t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 37

```
DSolve[{x1'[t]==1*x1[t]-3*x2[t],x2'[t]==-2*x1[t]+2*x2[t]},{x1[0]==0,x2[0]==5},{x1[t],x2[t]},t
```

$$\mathrm{x1}(t) \to -3e^{-t}\left(e^{5t} - 1\right)$$
$$\mathrm{x2}(t) \to e^{-t}\left(3e^{5t} + 2\right)$$

1.9 problem 9

Internal problem ID [1832]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.8, Systems of differential equations. The eigenva1ue-eigenvector method. Page 339
Problem number: 9.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) + x_2(t) - x_3(t)$$
$$x_2'(t) = x_1(t) + 3x_2(t) - x_3(t)$$
$$x_3'(t) = 3x_1(t) + 3x_2(t) - x_3(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = -2, x_3(0) = -1]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 27

```
dsolve([diff(x__1(t),t) = 3*x__1(t)+x__2(t)-x__3(t), diff(x__2(t),t) = x__1(t)+3*x__2(t)-x__3
```

$$x_1(t) = \mathrm{e}^{2t}$$

$$x_2(t) = -2\,\mathrm{e}^{2t}$$

$$x_3(t) = -\mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 30

```
DSolve[{x1'[t]==3*x1[t]+1*x2[t]-1*x3[t],x2'[t]==1*x1[t]+3*x2[t]-1*x3[t],x3'[t]==3*x1[t]+3*x2[
```

$$\text{x1}(t) \to e^{2t}$$
$$\text{x2}(t) \to -2e^{2t}$$
$$\text{x3}(t) \to -e^{2t}$$

1.10 problem 10

Internal problem ID [1833]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.8, Systems of differential equations. The eigenva1ue-eigenvector method. Page 339
Problem number: 10.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - x_2(t)$$
$$x_2'(t) = x_1(t) + 2x_2(t) + x_3(t)$$
$$x_3'(t) = x_1(t) + 10x_2(t) + 2x_3(t)$$

With initial conditions

$$[x_1(0) = -1, x_2(0) = -4, x_3(0) = 13]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 37

```
dsolve([diff(x__1(t),t) = x__1(t)-x__2(t), diff(x__2(t),t) = x__1(t)+2*x__2(t)+x__3(t), diff(
```

$$x_1(t) = -2\,\mathrm{e}^{-t} + \mathrm{e}^{t}$$

$$x_2(t) = -4\,\mathrm{e}^{-t}$$

$$x_3(t) = 14\,\mathrm{e}^{-t} - \mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 36

```
DSolve[{x1'[t]==1*x1[t]-1*x2[t]-0*x3[t],x2'[t]==1*x1[t]+2*x2[t]+1*x3[t],x3'[t]==1*x1[t]+10*x2
```

$$\mathrm{x1}(t) \to 3\sinh(t) - \cosh(t)$$
$$\mathrm{x2}(t) \to -4e^{-t}$$
$$\mathrm{x3}(t) \to 13\cosh(t) - 15\sinh(t)$$

1.11 problem 11

Internal problem ID [1834]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.8, Systems of differential equations. The eigenva1ue-eigenvector method. Page 339
Problem number: 11.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - 3x_2(t) + 2x_3(t)$$
$$x_2'(t) = -x_2(t)$$
$$x_3'(t) = -x_2(t) - 2x_3(t)$$

With initial conditions

$$[x_1(0) = -2, x_2(0) = 0, x_3(0) = 3]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 24

```
dsolve([diff(x__1(t),t) = x__1(t)-3*x__2(t)+2*x__3(t), diff(x__2(t),t) = -x__2(t), diff(x__3(
```

$$x_1(t) = -2\,\mathrm{e}^{-2t}$$

$$x_2(t) = 0$$

$$x_3(t) = 3\,\mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 26

```
DSolve[{x1'[t]==1*x1[t]-3*x2[t]+2*x3[t],x2'[t]==0*x1[t]-1*x2[t]+0*x3[t],x3'[t]==0*x1[t]-1*x2[
```

$$\mathrm{x1}(t) \to -2e^{-2t}$$
$$\mathrm{x2}(t) \to 0$$
$$\mathrm{x3}(t) \to 3e^{-2t}$$

1.12 problem 12

Internal problem ID [1835]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.8, Systems of differential equations. The eigenva1ue-eigenvector method. Page 339
Problem number: 12.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) + x_2(t) - 2x_3(t)$$
$$x_2'(t) = -x_1(t) + 2x_2(t) + x_3(t)$$
$$x_3'(t) = 4x_1(t) + x_2(t) - 3x_3(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 4, x_3(0) = -7]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 58

```
dsolve([diff(x__1(t),t) = 3*x__1(t)+x__2(t)-2*x__3(t), diff(x__2(t),t) = -x__1(t)+2*x__2(t)+x
```

$$x_1(t) = \frac{4\,\mathrm{e}^{2t}}{3} - \frac{28\,\mathrm{e}^{-t}}{3} + 9\,\mathrm{e}^{t}$$

$$x_2(t) = \frac{4\,\mathrm{e}^{2t}}{3} + \frac{8\,\mathrm{e}^{-t}}{3}$$

$$x_3(t) = \frac{4\,\mathrm{e}^{2t}}{3} - \frac{52\,\mathrm{e}^{-t}}{3} + 9\,\mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 75

```
DSolve[{x1'[t]==3*x1[t]+1*x2[t]-2*x3[t],x2'[t]==-1*x1[t]+2*x2[t]+1*x3[t],x3'[t]==4*x1[t]+1*x2
```

$$\text{x1}(t) \to -\frac{28e^{-t}}{3} + 9e^{t} + \frac{4e^{2t}}{3}$$

$$\text{x2}(t) \to \frac{4}{3}e^{-t}\left(e^{3t} + 2\right)$$

$$\text{x3}(t) \to -\frac{52e^{-t}}{3} + 9e^{t} + \frac{4e^{2t}}{3}$$

2 Section 3.9, Systems of differential equations. Complex roots. Page 344

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2.1 problem 1

Internal problem ID [1836]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.9, Systems of differential equations. Complex roots. Page 344
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -3x_1(t) + 2x_2(t)$$
$$x_2'(t) = -x_1(t) - x_2(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
dsolve([diff(x__1(t),t)=-3*x__1(t)+2*x__2(t),diff(x__2(t),t)=-1*x__1(t)-1*x__2(t)],[x__1(t),
```

$$x_1(t) = \mathrm{e}^{-2t}(c_1 \sin(t) + c_2 \sin(t) - \cos(t) c_1 + c_2 \cos(t))$$

$$x_2(t) = \mathrm{e}^{-2t}(c_1 \sin(t) + c_2 \cos(t))$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 52

```
DSolve[{x1'[t]==-3*x1[t]+2*x2[t],x2'[t]==-1*x1[t]-1*x2[t]},{x1[t],x2[t]},t,IncludeSingularSol
```

$$\text{x1}(t) \to e^{-2t}(c_1 \cos(t) - (c_1 - 2c_2)\sin(t))$$
$$\text{x2}(t) \to e^{-2t}(c_2(\sin(t) + \cos(t)) - c_1 \sin(t))$$

2.2 problem 2

Internal problem ID [1837]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.9, Systems of differential equations. Complex roots. Page 344
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - 5x_2(t)$$
$$x_2'(t) = x_1(t) - 3x_2(t)$$
$$x_3'(t) = x_3(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 54

```
dsolve([diff(x__1(t),t)=1*x__1(t)-5*x__2(t)+0*x__3(t),diff(x__2(t),t)=1*x__1(t)-3*x__2(t)+0*x
```

$$x_1(t) = \mathrm{e}^{-t}(\cos(t)\,c_1 - c_2\sin(t) + 2c_1\sin(t) + 2c_2\cos(t))$$

$$x_2(t) = \mathrm{e}^{-t}(c_1\sin(t) + c_2\cos(t))$$

$$x_3(t) = c_3\mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 120

```
DSolve[{x1'[t]==1*x1[t]-5*x2[t]+0*x3[t],x2'[t]==1*x1[t]-3*x2[t]+0*x3[t],x3'[t]==0*x1[t]-0*x2[
```

$$\begin{aligned}
\text{x1}(t) &\to e^{-t}(c_1 \cos(t) + (2c_1 - 5c_2)\sin(t)) \\
\text{x2}(t) &\to e^{-t}(c_2 \cos(t) + (c_1 - 2c_2)\sin(t)) \\
\text{x3}(t) &\to c_3 e^{t} \\
\text{x1}(t) &\to e^{-t}(c_1 \cos(t) + (2c_1 - 5c_2)\sin(t)) \\
\text{x2}(t) &\to e^{-t}(c_2 \cos(t) + (c_1 - 2c_2)\sin(t)) \\
\text{x3}(t) &\to 0
\end{aligned}$$

2.3 problem 3

Internal problem ID [1838]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.9, Systems of differential equations. Complex roots. Page 344
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_1(t) \\
x_2'(t) &= 3x_1(t) + x_2(t) - 2x_3(t) \\
x_3'(t) &= 2x_1(t) + 2x_2(t) + x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 55

```
dsolve([diff(x__1(t),t)=1*x__1(t)-0*x__2(t)+0*x__3(t),diff(x__2(t),t)=3*x__1(t)+1*x__2(t)-2*x
```

$$x_1(t) = \frac{2c_1 \mathrm{e}^t}{3}$$

$$x_2(t) = \frac{\mathrm{e}^t(3c_2 \cos(2t) - 3c_3 \sin(2t) - 2c_1)}{3}$$

$$x_3(t) = \mathrm{e}^t(c_2 \sin(2t) + c_3 \cos(2t) + c_1)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 91

```
DSolve[{x1'[t]==1*x1[t]-0*x2[t]+0*x3[t],x2'[t]==3*x1[t]+1*x2[t]-2*x3[t],x3'[t]==2*x1[t]+2*x2[
```

$$\mathrm{x1}(t) \to c_1 e^t$$

$$\mathrm{x2}(t) \to \frac{1}{2}e^t(2(c_1 + c_2)\cos(2t) + (3c_1 - 2c_3)\sin(2t) - 2c_1)$$

$$\mathrm{x3}(t) \to \frac{1}{2}e^t((2c_3 - 3c_1)\cos(2t) + 2(c_1 + c_2)\sin(2t) + 3c_1)$$

2.4 problem 4

Internal problem ID [1839]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.9, Systems of differential equations. Complex roots. Page 344
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + x_3(t)$$
$$x_2'(t) = x_2(t) - x_3(t)$$
$$x_3'(t) = -2x_1(t) - x_3(t)$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 66

```
dsolve([diff(x__1(t),t)=1*x__1(t)-0*x__2(t)+1*x__3(t),diff(x__2(t),t)=0*x__1(t)+1*x__2(t)-1*x
```

$$x_1(t) = \frac{c_3 \sin(t)}{2} - \frac{c_2 \cos(t)}{2} - \frac{c_3 \cos(t)}{2} - \frac{c_2 \sin(t)}{2}$$

$$x_2(t) = \frac{c_2 \sin(t)}{2} - \frac{c_3 \sin(t)}{2} + \frac{c_2 \cos(t)}{2} + \frac{c_3 \cos(t)}{2} + c_1 e^t$$

$$x_3(t) = c_3 \cos(t) + c_2 \sin(t)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 67

```
DSolve[{x1'[t]==1*x1[t]-0*x2[t]+1*x3[t],x2'[t]==0*x1[t]+1*x2[t]-1*x3[t],x3'[t]==-2*x1[t]-0*x2
```

$$\text{x1}(t) \to c_1 \cos(t) + (c_1 + c_3) \sin(t)$$
$$\text{x2}(t) \to (c_1 + c_2) e^t - c_1 \cos(t) - (c_1 + c_3) \sin(t)$$
$$\text{x3}(t) \to c_3 \cos(t) - (2c_1 + c_3) \sin(t)$$

2.5 problem 5

Internal problem ID [1840]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.9, Systems of differential equations. Complex roots. Page 344
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - x_2(t)$$
$$x_2'(t) = 5x_1(t) - 3x_2(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve([diff(x__1(t),t) = x__1(t)-x__2(t), diff(x__2(t),t) = 5*x__1(t)-3*x__2(t), x__1(0) = 1
```

$$x_1(t) = \mathrm{e}^{-t}\cos(t)$$

$$x_2(t) = \mathrm{e}^{-t}(\sin(t) + 2\cos(t))$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 29

```
DSolve[{x1'[t]==1*x1[t]-1*x2[t],x2'[t]==5*x1[t]-3*x2[t]},{x1[0]==1,x2[0]==2},{x1[t],x2[t]},t,
```

$$\mathrm{x1}(t) \to e^{-t}\cos(t)$$
$$\mathrm{x2}(t) \to e^{-t}(\sin(t) + 2\cos(t))$$

2.6 problem 6

Internal problem ID [1841]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.9, Systems of differential equations. Complex roots. Page 344
Problem number: 6.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) - 2x_2(t)$$
$$x_2'(t) = 4x_1(t) - x_2(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 5]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 41

```
dsolve([diff(x__1(t),t) = 3*x__1(t)-2*x__2(t), diff(x__2(t),t) = 4*x__1(t)-x__2(t), x__1(0) =
```

$$x_1(t) = \frac{e^t(-8\sin(2t) + 2\cos(2t))}{2}$$

$$x_2(t) = e^t(-3\sin(2t) + 5\cos(2t))$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 40

```
DSolve[{x1'[t]==3*x1[t]-2*x2[t],x2'[t]==4*x1[t]-1*x2[t]},{x1[0]==1,x2[0]==5},{x1[t],x2[t]},t,
```

$$\text{x1}(t) \to e^t(\cos(2t) - 4\sin(2t))$$
$$\text{x2}(t) \to e^t(5\cos(2t) - 3\sin(2t))$$

2.7 problem 7

Internal problem ID [1842]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.9, Systems of differential equations. Complex roots. Page 344
Problem number: 7.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -3x_1(t) + 2x_3(t)$$
$$x_2'(t) = x_1(t) - x_2(t)$$
$$x_3'(t) = -2x_1(t) - x_2(t)$$

With initial conditions

$$[x_1(0) = 0, x_2(0) = -1, x_3(0) = -2]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 95

```
dsolve([diff(x__1(t),t) = -3*x__1(t)+2*x__3(t), diff(x__2(t),t) = x__1(t)-x__2(t), diff(x__3(
```

$$x_1(t) = 2\,\mathrm{e}^{-2t} - 2\,\mathrm{e}^{-t}\cos\left(t\sqrt{2}\right) - \mathrm{e}^{-t}\sqrt{2}\,\sin\left(t\sqrt{2}\right)$$

$$x_2(t) = -2\,\mathrm{e}^{-2t} + \mathrm{e}^{-t}\cos\left(t\sqrt{2}\right) - \mathrm{e}^{-t}\sqrt{2}\,\sin\left(t\sqrt{2}\right)$$

$$x_3(t) = \mathrm{e}^{-2t} - 3\,\mathrm{e}^{-t}\cos\left(t\sqrt{2}\right)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 106

```
DSolve[{x1'[t]==-3*x1[t]-0*x2[t]+2*x3[t],x2'[t]==1*x1[t]-1*x2[t]-0*x3[t],x3'[t]==-2*x1[t]-1*x
```

$$\text{x1}(t) \to e^{-2t}\left(2 - e^{t}\left(\sqrt{2}\sin\left(\sqrt{2}t\right) + 2\cos\left(\sqrt{2}t\right)\right)\right)$$

$$\text{x2}(t) \to e^{-2t}\left(e^{t}\left(\cos\left(\sqrt{2}t\right) - \sqrt{2}\sin\left(\sqrt{2}t\right)\right) - 2\right)$$

$$\text{x3}(t) \to e^{-2t}\left(1 - 3e^{t}\cos\left(\sqrt{2}t\right)\right)$$

2.8 problem 8

Internal problem ID [1843]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.9, Systems of differential equations. Complex roots. Page 344
Problem number: 8.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 2x_2(t) \\
x_2'(t) &= -2x_1(t) \\
x_3'(t) &= -3x_4(t) \\
x_4'(t) &= 3x_3(t)
\end{aligned}
$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 1, x_3(0) = 1, x_4(0) = 0]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 42

```
dsolve([diff(x__1(t),t) = 2*x__2(t), diff(x__2(t),t) = -2*x__1(t), diff(x__3(t),t) = -3*x__4(
```

$$x_1(t) = \cos(2t) + \sin(2t)$$

$$x_2(t) = -\sin(2t) + \cos(2t)$$

$$x_3(t) = \cos(3t)$$

$$x_4(t) = \sin(3t)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[{x1'[t]==-0*x1[t]+2*x2[t]+0*x3[t]+0*x4[t],x2'[t]==-2*x1[t]-0*x2[t]-0*x3[t]+0*x4[t],x3'
```

$$\begin{aligned}
\text{x1}(t) &\to \sin(2t) + \cos(2t) \\
\text{x2}(t) &\to \cos(2t) - \sin(2t) \\
\text{x3}(t) &\to \cos(3t) \\
\text{x4}(t) &\to \sin(3t)
\end{aligned}$$

3 Section 3.10, Systems of differential equations. Equal roots. Page 352

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3.1 problem Example 1, page 348

Internal problem ID [1844]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.10, Systems of differential equations. Equal roots. Page 352
Problem number: Example 1, page 348.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + x_2(t)$$
$$x_2'(t) = x_2(t)$$
$$x_3'(t) = 2x_3(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 29

```
dsolve([diff(x__1(t),t)=1*x__1(t)+1*x__2(t)+0*x__3(t),diff(x__2(t),t)=0*x__1(t)+1*x__2(t)-0*x
```

$$x_1(t) = (c_2 t + c_1)\,\mathrm{e}^{t}$$

$$x_2(t) = c_2 \mathrm{e}^{t}$$

$$x_3(t) = c_3 \mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 64

```
DSolve[{x1'[t]==1*x1[t]+1*x2[t]+0*x3[t],x2'[t]==0*x1[t]+1*x2[t]-0*x3[t],x3'[t]==0*x1[t]-0*x2[
```

$$\text{x1}(t) \to e^t(c_2 t + c_1)$$
$$\text{x2}(t) \to c_2 e^t$$
$$\text{x3}(t) \to c_3 e^{2t}$$
$$\text{x1}(t) \to e^t(c_2 t + c_1)$$
$$\text{x2}(t) \to c_2 e^t$$
$$\text{x3}(t) \to 0$$

3.2 problem Example 2, page 349

Internal problem ID [1845]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.10, Systems of differential equations. Equal roots. Page 352
Problem number: Example 2, page 349.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) + x_2(t) + 3x_3(t)$$
$$x_2'(t) = 2x_2(t) - x_3(t)$$
$$x_3'(t) = 2x_3(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 2, x_3(0) = 1]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 41

```
dsolve([diff(x__1(t),t) = 2*x__1(t)+x__2(t)+3*x__3(t), diff(x__2(t),t) = 2*x__2(t)-x__3(t), d
```

$$x_1(t) = \frac{(-t^2 + 10t + 2)\,\mathrm{e}^{2t}}{2}$$

$$x_2(t) = (2 - t)\,\mathrm{e}^{2t}$$

$$x_3(t) = \mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 42

```
DSolve[{x1'[t]==2*x1[t]+1*x2[t]+3*x3[t],x2'[t]==0*x1[t]+2*x2[t]-1*x3[t],x3'[t]==0*x1[t]-0*x2[
```

$$\text{x1}(t) \to -\frac{1}{2}e^{2t}((t-10)t-2)$$
$$\text{x2}(t) \to -e^{2t}(t-2)$$
$$\text{x3}(t) \to e^{2t}$$

3.3 problem 1

Internal problem ID [1846]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.10, Systems of differential equations. Equal roots. Page 352
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -x_2(t) + x_3(t)$$
$$x_2'(t) = 2x_1(t) - 3x_2(t) + x_3(t)$$
$$x_3'(t) = x_1(t) - x_2(t) - x_3(t)$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 51

```
dsolve([diff(x__1(t),t)=0*x__1(t)-1*x__2(t)+1*x__3(t),diff(x__2(t),t)=2*x__1(t)-3*x__2(t)+1*x
```

$$x_1(t) = \mathrm{e}^{-t}(c_2 t + c_1)$$

$$x_2(t) = \mathrm{e}^{-t}(c_2 t + c_1) + c_3 \mathrm{e}^{-2t}$$

$$x_3(t) = c_2 \mathrm{e}^{-t} + c_3 \mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 92

```
DSolve[{x1'[t]==0*x1[t]-1*x2[t]+1*x3[t],x2'[t]==2*x1[t]-3*x2[t]+1*x3[t],x3'[t]==1*x1[t]-1*x2[
```

$$\text{x1}(t) \to e^{-t}(c_1(t+1) + (c_3 - c_2)t)$$
$$\text{x2}(t) \to e^{-2t}\left(e^t(c_1(t+1) + (c_3 - c_2)t) - c_1 + c_2\right)$$
$$\text{x3}(t) \to e^{-2t}\left((c_1 - c_2 + c_3)e^t - c_1 + c_2\right)$$

3.4 problem 2

Internal problem ID [1847]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.10, Systems of differential equations. Equal roots. Page 352
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_1(t) + x_2(t) + x_3(t) \\
x_2'(t) &= 2x_1(t) + x_2(t) - x_3(t) \\
x_3'(t) &= -3x_1(t) + 2x_2(t) + 4x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 65

```
dsolve([diff(x__1(t),t)=1*x__1(t)+1*x__2(t)+1*x__3(t),diff(x__2(t),t)=2*x__1(t)+1*x__2(t)-1*x
```

$$x_1(t) = -\mathrm{e}^{2t}(2c_3t + c_2 + 4c_3)$$

$$x_2(t) = -\mathrm{e}^{2t}\left(c_3t^2 + c_2t + 2c_3t + c_1 + c_2 + 6c_3\right)$$

$$x_3(t) = \mathrm{e}^{2t}\left(c_3t^2 + c_2t + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 96

```
DSolve[{x1'[t]==1*x1[t]+1*x2[t]+1*x3[t],x2'[t]==2*x1[t]+1*x2[t]-1*x3[t],x3'[t]==-3*x1[t]+2*x2
```

$$
\begin{aligned}
\mathrm{x1}(t) &\to e^{2t}((c_2 + c_3)t - c_1(t-1)) \\
\mathrm{x2}(t) &\to \frac{1}{2}e^{2t}(-(c_1(t-4)t) + (c_2 + c_3)(t-2)t + 2c_2) \\
\mathrm{x3}(t) &\to \frac{1}{2}e^{2t}(c_1(t-6)t - (c_2 + c_3)(t-4)t + 2c_3)
\end{aligned}
$$

3.5 problem 3

Internal problem ID [1848]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.10, Systems of differential equations. Equal roots. Page 352
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -x_1(t) - x_2(t)$$
$$x_2'(t) = -x_2(t)$$
$$x_3'(t) = -2x_3(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve([diff(x__1(t),t)=-1*x__1(t)-1*x__2(t)+0*x__3(t),diff(x__2(t),t)=0*x__1(t)-1*x__2(t)+0*
```

$$x_1(t) = (-c_2 t + c_1)\,\mathrm{e}^{-t}$$

$$x_2(t) = c_2 \mathrm{e}^{-t}$$

$$x_3(t) = c_3 \mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 74

```
DSolve[{x1'[t]==-1*x1[t]-1*x2[t]+0*x3[t],x2'[t]==0*x1[t]-1*x2[t]+0*x3[t],x3'[t]==0*x1[t]-0*x2
```

$$\text{x1}(t) \to e^{-t}(c_1 - c_2 t)$$
$$\text{x2}(t) \to c_2 e^{-t}$$
$$\text{x3}(t) \to c_3 e^{-2t}$$
$$\text{x1}(t) \to e^{-t}(c_1 - c_2 t)$$
$$\text{x2}(t) \to c_2 e^{-t}$$
$$\text{x3}(t) \to 0$$

3.6 problem 4

Internal problem ID [1849]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.10, Systems of differential equations. Equal roots. Page 352
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 2x_1(t) - x_3(t) \\
x_2'(t) &= 2x_2(t) + x_3(t) \\
x_3'(t) &= 2x_3(t) \\
x_4'(t) &= -x_3(t) + 2x_4(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 56

```
dsolve([diff(x__1(t),t)=2*x__1(t)+0*x__2(t)-1*x__3(t)+0*x__4(t),diff(x__2(t),t)=0*x__1(t)+2*x
```

$$x_1(t) = \mathrm{e}^{2t}(c_4 t + c_1 + c_3)$$

$$x_2(t) = \mathrm{e}^{2t}(-c_4 t + c_2 - c_3)$$

$$x_3(t) = -c_4 \mathrm{e}^{2t}$$

$$x_4(t) = (c_4 t + c_3)\,\mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 63

```
DSolve[{x1'[t]==2*x1[t]+0*x2[t]-1*x3[t]+0*x4[t],x2'[t]==0*x1[t]+2*x2[t]+1*x3[t]+0*x4[t],x3'[t
```

$$\begin{aligned}
\text{x1}(t) &\to e^{2t}(c_1 - c_3 t) \\
\text{x2}(t) &\to e^{2t}(c_3 t + c_2) \\
\text{x3}(t) &\to c_3 e^{2t} \\
\text{x4}(t) &\to e^{2t}(c_4 - c_3 t)
\end{aligned}$$

3.7 problem 5

Internal problem ID [1850]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.10, Systems of differential equations. Equal roots. Page 352
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -x_1(t) + x_2(t) + 2x_3(t)$$
$$x_2'(t) = -x_1(t) + x_2(t) + x_3(t)$$
$$x_3'(t) = -2x_1(t) + x_2(t) + 3x_3(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 0, x_3(0) = 1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 16

```
dsolve([diff(x__1(t),t) = -x__1(t)+x__2(t)+2*x__3(t), diff(x__2(t),t) = -x__1(t)+x__2(t)+x__3
```

$$x_1(t) = \mathrm{e}^t$$

$$x_2(t) = 0$$

$$x_3(t) = \mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 18

```
DSolve[{x1'[t]==-1*x1[t]+1*x2[t]+2*x3[t],x2'[t]==-1*x1[t]+1*x2[t]+1*x3[t],x3'[t]==-2*x1[t]+1*
```

$$\mathrm{x1}(t) \to e^t$$
$$\mathrm{x2}(t) \to 0$$
$$\mathrm{x3}(t) \to e^t$$

3.8 problem 6

Internal problem ID [1851]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.10, Systems of differential equations. Equal roots. Page 352
Problem number: 6.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -4x_1(t) - 4x_2(t)$$
$$x_2'(t) = 10x_1(t) + 9x_2(t) + x_3(t)$$
$$x_3'(t) = -4x_1(t) - 3x_2(t) + x_3(t)$$

With initial conditions

$$[x_1(0) = 2, x_2(0) = 1, x_3(0) = -1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 57

```
dsolve([diff(x__1(t),t) = -4*x__1(t)-4*x__2(t), diff(x__2(t),t) = 10*x__1(t)+9*x__2(t)+x__3(t
```

$$x_1(t) = \mathrm{e}^{2t}\left(-4t^2 - 16t + 2\right)$$

$$x_2(t) = -\mathrm{e}^{2t}\left(-6t^2 - 26t - 1\right)$$

$$x_3(t) = \mathrm{e}^{2t}\left(-2t^2 - 10t - 1\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 57

```
DSolve[{x1'[t]==-4*x1[t]-4*x2[t]+0*x3[t],x2'[t]==10*x1[t]+9*x2[t]+1*x3[t],x3'[t]==-4*x1[t]-3*
```

$$\mathrm{x1}(t) \to -2e^{2t}(2t(t+4) - 1)$$
$$\mathrm{x2}(t) \to e^{2t}\left(6t^2 + 26t + 1\right)$$
$$\mathrm{x3}(t) \to -e^{2t}(2t(t+5) + 1)$$

3.9 problem 7

Internal problem ID [1852]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.10, Systems of differential equations. Equal roots. Page 352
Problem number: 7.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + 2x_2(t) - 3x_3(t)$$
$$x_2'(t) = x_1(t) + x_2(t) + 2x_3(t)$$
$$x_3'(t) = x_1(t) - x_2(t) + 4x_3(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 0, x_3(0) = 0]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 32

```
dsolve([diff(x__1(t),t) = x__1(t)+2*x__2(t)-3*x__3(t), diff(x__2(t),t) = x__1(t)+x__2(t)+2*x_
```

$$x_1(t) = -(t-1)\,\mathrm{e}^{2t}$$

$$x_2(t) = \mathrm{e}^{2t}t$$

$$x_3(t) = \mathrm{e}^{2t}t$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 35

```
DSolve[{x1'[t]==1*x1[t]+2*x2[t]-3*x3[t],x2'[t]==1*x1[t]+1*x2[t]+2*x3[t],x3'[t]==1*x1[t]-1*x2[
```

$$\mathrm{x1}(t) \to -e^{2t}(t-1)$$
$$\mathrm{x2}(t) \to e^{2t}t$$
$$\mathrm{x3}(t) \to e^{2t}t$$

3.10 problem 8

Internal problem ID [1853]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.10, Systems of differential equations. Equal roots. Page 352
Problem number: 8.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 3x_1(t) \\
x_2'(t) &= x_1(t) + 3x_2(t) \\
x_3'(t) &= 3x_3(t) \\
x_4'(t) &= 2x_3(t) + 3x_4(t)
\end{aligned}
$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 1, x_3(0) = 1, x_4(0) = 1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 40

```
dsolve([diff(x__1(t),t) = 3*x__1(t), diff(x__2(t),t) = x__1(t)+3*x__2(t), diff(x__3(t),t) = 3
```

$$x_1(t) = \mathrm{e}^{3t}$$

$$x_2(t) = \mathrm{e}^{3t}(t + 1)$$

$$x_3(t) = \mathrm{e}^{3t}$$

$$x_4(t) = \mathrm{e}^{3t}(1 + 2t)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 44

```
DSolve[{x1'[t]==3*x1[t]+0*x2[t]+0*x3[t]+0*x4[t],x2'[t]==1*x1[t]+3*x2[t]-0*x3[t]+0*x4[t],x3'[t
```

$$\text{x1}(t) \to e^{3t}$$
$$\text{x2}(t) \to e^{3t}(t+1)$$
$$\text{x3}(t) \to e^{3t}$$
$$\text{x4}(t) \to e^{3t}(2t+1)$$

4 Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366

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4.1 problem Example 1, page 361

Internal problem ID [1854]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366
Problem number: Example 1, page 361.
ODE order: 1.
ODE degree: 1.

Solve

$$\begin{aligned} x_1'(t) &= x_1(t) \\ x_2'(t) &= 2x_1(t) + x_2(t) - 2x_3(t) \\ x_3'(t) &= 3x_1(t) + 2x_2(t) + x_3(t) + 2\,\mathrm{e}^t \cos(t)^2 - \mathrm{e}^t \end{aligned}$$

✓ Solution by Maple

Time used: 0.641 (sec). Leaf size: 85

```
dsolve([diff(x__1(t),t)=1*x__1(t)+0*x__2(t)+0*x__3(t),diff(x__2(t),t)=2*x__1(t)+1*x__2(t)-2*x
```

$$x_1(t) = c_1 \mathrm{e}^t$$

$$x_2(t) = \frac{\mathrm{e}^t(2c_3\cos(2t) - 2c_2\sin(2t) - \sin(2t)\,t - \cos(2t) - 3c_1)}{2}$$

$$x_3(t) = \frac{\mathrm{e}^t(4c_2\cos(2t) + 2\cos(2t)\,t + 4c_3\sin(2t) - \sin(2t) + 4c_1)}{4}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 103

```
DSolve[{x1'[t]==1*x1[t]+0*x2[t]+0*x3[t],x2'[t]==2*x1[t]+1*x2[t]-2*x3[t],x3'[t]==3*x1[t]+2*x2[
```

$$\text{x1}(t) \to c_1 e^t$$

$$\text{x2}(t) \to -\frac{1}{8}e^t((1-12c_1-8c_2)\cos(2t)+4(t-2c_1+2c_3)\sin(2t)+12c_1)$$

$$\text{x3}(t) \to \frac{1}{8}e^t(4(t-2c_1+2c_3)\cos(2t)+(1+12c_1+8c_2)\sin(2t)+8c_1)$$

4.2 problem Example 2, page 364

Internal problem ID [1855]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366
Problem number: Example 2, page 364.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + \mathrm{e}^{ct}$$
$$x_2'(t) = 2x_1(t) + x_2(t) - 2x_3(t)$$
$$x_3'(t) = 3x_1(t) + 2x_2(t) + x_3(t)$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 225

```
dsolve([diff(x__1(t),t)=1*x__1(t)+0*x__2(t)+0*x__3(t)+exp(c*t),diff(x__2(t),t)=2*x__1(t)+1*x_
```

$$x_1(t) = \frac{\mathrm{e}^t c_1 c - c_1 \mathrm{e}^t + \mathrm{e}^{ct}}{c-1}$$

$$x_2(t) = \frac{2\cos(2t)\,\mathrm{e}^t c_3 c^3 - 2\sin(2t)\,\mathrm{e}^t c_2 c^3 - 6\cos(2t)\,\mathrm{e}^t c_3 c^2 + 6\sin(2t)\,\mathrm{e}^t c_2 c^2 - 3\,\mathrm{e}^t c_1 c^3 + 14\cos(2t)\,\mathrm{e}^t c_3 c - 14\sin}{2(c-1)(c^2-2c+}$$

$$x_3(t) = \frac{3c\,\mathrm{e}^{ct} + \mathrm{e}^{ct}}{c^3 - 3c^2 + 7c - 5} + c_1 \mathrm{e}^t + c_2 \mathrm{e}^t \cos(2t) + c_3 \mathrm{e}^t \sin(2t)$$

✓ Solution by Mathematica

Time used: 0.494 (sec). Leaf size: 162

```
DSolve[{x1'[t]==1*x1[t]+0*x2[t]+0*x3[t]+Exp[c*t],x2'[t]==2*x1[t]+1*x2[t]-2*x3[t],x3'[t]==3*x1
```

$$\text{x1}(t) \to e^t \left(\frac{e^{(c-1)t}}{c-1} + c_1 \right)$$

$$\text{x2}(t) \to \frac{1}{2} e^t \left(\frac{4(c-4)e^{(c-1)t}}{(c-1)((c-2)c+5)} + (3c_1 + 2c_2)\cos(2t) + 2(c_1 - c_3)\sin(2t) - 3c_1 \right)$$

$$\text{x3}(t) \to \frac{1}{2} e^t \left(\frac{(6c+2)e^{(c-1)t}}{(c-1)((c-2)c+5)} + 2(c_3 - c_1)\cos(2t) + (3c_1 + 2c_2)\sin(2t) + 2c_1 \right)$$

4.3 problem 1

Internal problem ID [1856]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 4x_1(t) + 5x_2(t) + 4\,\mathrm{e}^t \cos(t)$$
$$x_2'(t) = -2x_1(t) - 2x_2(t)$$

With initial conditions

$$[x_1(0) = 0, x_2(0) = 0]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 34

```
dsolve([diff(x__1(t),t) = 4*x__1(t)+5*x__2(t)+4*exp(t)*cos(t), diff(x__2(t),t) = -2*x__1(t)-2
```

$$x_1(t) = \frac{\mathrm{e}^t(12\sin(t)\,t + 4\cos(t)\,t + 4\sin(t))}{2}$$

$$x_2(t) = -4\sin(t)\,\mathrm{e}^t t$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 33

```
DSolve[{x1'[t]==4*x1[t]+5*x2[t]+4*Exp[t]*Cos[t],x2'[t]==-2*x1[t]-2*x2[t]},{x1[0]==0,x2[0]==0}
```

$$\mathrm{x1}(t) \to 2e^t(3t\sin(t) + \sin(t) + t\cos(t))$$
$$\mathrm{x2}(t) \to -4e^t t\sin(t)$$

4.4 problem 2

Internal problem ID [1857]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) - 4x_2(t) + \mathrm{e}^t$$
$$x_2'(t) = x_1(t) - x_2(t) + \mathrm{e}^t$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 32

```
dsolve([diff(x__1(t),t) = 3*x__1(t)-4*x__2(t)+exp(t), diff(x__2(t),t) = x__1(t)-x__2(t)+exp(t
```

$$x_1(t) = \mathrm{e}^t\left(-t^2 - t + 1\right)$$

$$x_2(t) = \frac{\mathrm{e}^t(-t^2 + 2)}{2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 31

```
DSolve[{x1'[t]==3*x1[t]-4*x2[t]+Exp[t],x2'[t]==1*x1[t]-1*x2[t]+Exp[t]},{x1[0]==1,x2[0]==1},{x
```

$$\mathrm{x1}(t) \to -e^t\left(t^2 + t - 1\right)$$
$$\mathrm{x2}(t) \to -\frac{1}{2}e^t\left(t^2 - 2\right)$$

4.5 problem 3

Internal problem ID [1858]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) - 5x_2(t) + \sin(t)$$
$$x_2'(t) = x_1(t) - 2x_2(t) + \tan(t)$$

With initial conditions

$$[x_1(0) = 0, x_2(0) = 0]$$

✓ Solution by Maple

Time used: 0.469 (sec). Leaf size: 172

```
dsolve([diff(x__1(t),t) = 2*x__1(t)-5*x__2(t)+sin(t), diff(x__2(t),t) = x__1(t)-2*x__2(t)+tan
```

$x_1(t)$

$$= \frac{-2\sec(t)\tan(t) + 2\sin(t) + 4\cos(t) - 4\sec(t) - 4\tan(t) - 2\tan(t)^2 - 8\sin(t)\sec(t) + 10\ln(\sec(t) +}{}$$

$$x_2(t) = -\frac{3\sin(t)}{2} + \cos(t) - 1 + \ln(\sec(t) + \tan(t))\sin(t)$$
$$+ 2\ln(\sec(t) + \tan(t))\cos(t) - \frac{\cos(t)\,t}{2}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 58

```
DSolve[{x1'[t]==2*x1[t]-5*x2[t]+Sin[t],x2'[t]==1*x1[t]-2*x2[t]+Tan[t]},{x1[0]==0,x2[0]==0},{x
```

$$\text{x1}(t) \to 5\cos(t)\operatorname{arctanh}(\sin(t)) + \frac{1}{2}(t-8)\sin(t) - t\cos(t)$$

$$\text{x2}(t) \to \operatorname{arctanh}(\sin(t))(\sin(t) + 2\cos(t)) - \frac{3\sin(t)}{2} - \frac{1}{2}t\cos(t) + \cos(t) - 1$$

4.6 problem 4

Internal problem ID [1859]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_2(t) + f_1(t)$$
$$x_2'(t) = -x_1(t) + f_2(t)$$

With initial conditions

$$[x_1(0) = 0, x_2(0) = 0]$$

✓ Solution by Maple

Time used: 0.516 (sec). Leaf size: 110

```
dsolve([diff(x__1(t),t) = x__2(t)+f__1(t), diff(x__2(t),t) = -x__1(t)+f__2(t), x__1(0) = 0, x
```

$$x_1(t) = -f_2(0)\cos(t) - \left(\int_0^t \cos(\_z1)\left(\frac{d}{d\_z1}f_2(\_z1) - f_1(\_z1)\right)d\_z1\right)\cos(t) - \left(\int_0^t \sin(\_z1)\left(\frac{d}{d\_z1}f_2(\_z1) - f_1(\_z1)\right)d\_z1\right)\sin(t) + f_2(t)$$

$$x_2(t) = f_2(0)\sin(t) + \left(\int_0^t \cos(\_z1)\left(\frac{d}{d\_z1}f_2(\_z1) - f_1(\_z1)\right)d\_z1\right)\sin(t) - \left(\int_0^t \sin(\_z1)\left(\frac{d}{d\_z1}f_2(\_z1) - f_1(\_z1)\right)d\_z1\right)\cos(t)$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 214

```
DSolve[{x1'[t]==0*x1[t]+1*x2[t]+f1[t],x2'[t]==-1*x1[t]-0*x2[t]+f2[t]},{x1[0]==0,x2[0]==0},{x1
```

$$\begin{aligned}
\text{x1}(t) \to \cos(t) &\left(\int_1^t (\cos(K[1])\text{f1}(K[1]) - \text{f2}(K[1])\sin(K[1]))dK[1] \right. \\
&\left. - \int_1^0 (\cos(K[1])\text{f1}(K[1]) - \text{f2}(K[1])\sin(K[1]))dK[1] \right) \\
+ \sin(t) &\left(\int_1^t (\cos(K[2])\text{f2}(K[2]) + \text{f1}(K[2])\sin(K[2]))dK[2] \right. \\
&\left. - \int_1^0 (\cos(K[2])\text{f2}(K[2]) + \text{f1}(K[2])\sin(K[2]))dK[2] \right) \\
\text{x2}(t) \to \sin(t) &\left(\int_1^0 (\cos(K[1])\text{f1}(K[1]) - \text{f2}(K[1])\sin(K[1]))dK[1] \right. \\
&\left. - \int_1^t (\cos(K[1])\text{f1}(K[1]) - \text{f2}(K[1])\sin(K[1]))dK[1] \right) \\
+ \cos(t) &\left(\int_1^t (\cos(K[2])\text{f2}(K[2]) + \text{f1}(K[2])\sin(K[2]))dK[2] \right. \\
&\left. - \int_1^0 (\cos(K[2])\text{f2}(K[2]) + \text{f1}(K[2])\sin(K[2]))dK[2] \right)
\end{aligned}$$

4.7 problem 5

Internal problem ID [1860]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) + x_3(t) + \mathrm{e}^{2t}$$
$$x_2'(t) = 2x_2(t)$$
$$x_3'(t) = x_2(t) + 3x_3(t) + \mathrm{e}^{2t}$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 1, x_3(0) = 1]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 45

```
dsolve([diff(x__1(t),t) = 2*x__1(t)+x__3(t)+exp(2*t), diff(x__2(t),t) = 2*x__2(t), diff(x__3(
```

$$x_1(t) = (-t-2)\,\mathrm{e}^{2t} + 3\,\mathrm{e}^{3t}$$

$$x_2(t) = \mathrm{e}^{2t}$$

$$x_3(t) = -2\,\mathrm{e}^{2t} + 3\,\mathrm{e}^{3t}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 45

```
DSolve[{x1'[t]==2*x1[t]+0*x2[t]+1*x3[t]+Exp[2*t],x2'[t]==0*x1[t]+2*x2[t]+0*x3[t],x3'[t]==0*x1
```

$$\mathrm{x1}(t) \to e^{2t}\left(-t + 3e^{t} - 2\right)$$
$$\mathrm{x2}(t) \to e^{2t}$$
$$\mathrm{x3}(t) \to e^{2t}\left(3e^{t} - 2\right)$$

4.8 problem 6

Internal problem ID [1861]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366
Problem number: 6.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -x_1(t) - x_2(t) - 2x_3(t) + \mathrm{e}^t$$
$$x_2'(t) = x_1(t) + x_2(t) + x_3(t)$$
$$x_3'(t) = 2x_1(t) + x_2(t) + 3x_3(t)$$

With initial conditions

$$[x_1(0) = 0, x_2(0) = 0, x_3(0) = 0]$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 47

```
dsolve([diff(x__1(t),t) = -x__1(t)-x__2(t)-2*x__3(t)+exp(t), diff(x__2(t),t) = x__1(t)+x__2(t
```

$$x_1(t) = -\frac{\mathrm{e}^t(t^3 + 6t^2 - 6t)}{6}$$

$$x_2(t) = \frac{\mathrm{e}^t t^2}{2}$$

$$x_3(t) = \frac{\mathrm{e}^t(t^3 + 6t^2)}{6}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 49

```
DSolve[{x1'[t]==-1*x1[t]-1*x2[t]-2*x3[t]+Exp[t],x2'[t]==1*x1[t]+1*x2[t]+1*x3[t],x3'[t]==2*x1[
```

$$\text{x1}(t) \to -\frac{1}{6}e^t t(t(t+6)-6)$$
$$\text{x2}(t) \to \frac{e^t t^2}{2}$$
$$\text{x3}(t) \to \frac{1}{6}e^t t^2(t+6)$$

4.9 problem 10

Internal problem ID [1862]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366
Problem number: 10.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) + x_2(t) + \mathrm{e}^{3t}$$
$$x_2'(t) = 3x_1(t) - 2x_2(t) + \mathrm{e}^{3t}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 83

```
dsolve([diff(x__1(t),t)=2*x__1(t)+1*x__2(t)+1*exp(3*t),diff(x__2(t),t)=3*x__1(t)-2*x__2(t)+ex
```

$$x_1(t) = \frac{\sqrt{7}\,\mathrm{e}^{\sqrt{7}\,t}c_2}{3} - \frac{\sqrt{7}\,\mathrm{e}^{-\sqrt{7}\,t}c_1}{3} + 3\,\mathrm{e}^{3t} + \frac{2\,\mathrm{e}^{\sqrt{7}\,t}c_2}{3} + \frac{2\,\mathrm{e}^{-\sqrt{7}\,t}c_1}{3}$$

$$x_2(t) = \mathrm{e}^{\sqrt{7}\,t}c_2 + \mathrm{e}^{-\sqrt{7}\,t}c_1 + 2\,\mathrm{e}^{3t}$$

✓ Solution by Mathematica

Time used: 0.378 (sec). Leaf size: 100

```
DSolve[{x1'[t]==2*x1[t]+1*x2[t]+Exp[3*t],x2'[t]==3*x1[t]-2*x2[t]+Exp[3*t]},{x1[t],x2[t]},t,In
```

$$\mathrm{x1}(t) \to 3\sinh(3t) + 3\cosh(3t) + c_1\cosh\left(\sqrt{7}t\right) + \frac{(2c_1 + c_2)\sinh\left(\sqrt{7}t\right)}{\sqrt{7}}$$
$$\mathrm{x2}(t) \to 2\sinh(3t) + 2\cosh(3t) + c_2\cosh\left(\sqrt{7}t\right) + \frac{(3c_1 - 2c_2)\sinh\left(\sqrt{7}t\right)}{\sqrt{7}}$$

4.10 problem 11

Internal problem ID [1863]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366
Problem number: 11.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - x_2(t) - t^2$$
$$x_2'(t) = x_1(t) + 3x_2(t) + 2t$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 62

```
dsolve([diff(x__1(t),t)=1*x__1(t)-1*x__2(t)-t^2,diff(x__2(t),t)=1*x__1(t)+3*x__2(t)+2*t],[x__
```

$$x_1(t) = -c_2 e^{2t} - e^{2t} t c_1 + c_1 e^{2t} + \frac{t}{2} + \frac{1}{8} + \frac{3t^2}{4}$$

$$x_2(t) = c_2 e^{2t} + e^{2t} t c_1 - \frac{t^2}{4} - t - \frac{3}{8}$$

✓ Solution by Mathematica

Time used: 0.25 (sec). Leaf size: 90

```
DSolve[{x1'[t]==1*x1[t]+3*x2[t]-t^2,x2'[t]==1*x1[t]+3*x2[t]+2*t},{x1[t],x2[t]},t,IncludeSingu
```

$$\text{x1}(t) \to \frac{1}{128}\left(-4t\left(8t^2+22t+11\right)+32c_1\left(e^{4t}+3\right)+96c_2\left(e^{4t}-1\right)-11\right)$$
$$\text{x2}(t) \to \frac{1}{384}\left(4t\left(8t^2+30t-33\right)+96(c_1+3c_2)e^{4t}-33-96c_1+96c_2\right)$$

4.11 problem 12

Internal problem ID [1864]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366
Problem number: 12.
ODE order: 1.
ODE degree: 1.

Solve

$$\begin{aligned} x_1'(t) &= x_1(t) + 3x_2(t) + 2x_3(t) + \sin(t) \\ x_2'(t) &= -x_1(t) + 2x_2(t) + x_3(t) \\ x_3'(t) &= 4x_1(t) - x_2(t) - x_3(t) \end{aligned}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 84

```
dsolve([diff(x__1(t),t)=1*x__1(t)+3*x__2(t)+2*x__3(t)+sin(t),diff(x__2(t),t)=-1*x__1(t)+2*x__
```

$$x_1(t) = -\frac{\sin(t)}{10} - \frac{\cos(t)}{5} + \frac{c_1 e^t}{3} - \frac{e^{-2t} c_2}{3} + c_3 e^{3t}$$

$$x_2(t) = \frac{\cos(t)}{10} + \frac{3\sin(t)}{10} - \frac{2c_1 e^t}{3} - \frac{e^{-2t} c_2}{3}$$

$$x_3(t) = -\frac{\cos(t)}{10} - \frac{4\sin(t)}{5} + c_1 e^t + e^{-2t} c_2 + c_3 e^{3t}$$

✓ Solution by Mathematica

Time used: 0.312 (sec). Leaf size: 187

```
DSolve[{x1'[t]==1*x1[t]+3*x2[t]+2*x3[t]+Sin[t],x2'[t]==-1*x1[t]+2*x2[t]+1*x3[t],x3'[t]==4*x1[
```

$$\text{x1}(t) \to \frac{1}{30}\left(-3\sin(t)-6\cos(t)+5e^{-2t}\left((c_1-4c_2-c_3)e^{3t}+3(c_1+2c_2+c_3)e^{5t}+2c_1-2(c_2+c_3)\right)\right)$$

$$\text{x2}(t) \to \frac{1}{3}(c_1-c_2-c_3)e^{-2t}+\frac{1}{3}(-c_1+4c_2+c_3)e^{t}+\frac{1}{10}(3\sin(t)+\cos(t))$$

$$\text{x3}(t) \to \frac{1}{10}\left(-8\sin(t)-\cos(t)+5(c_1-4c_2-c_3)e^{t}+10(-c_1+c_2+c_3)e^{-2t}+5(c_1+2c_2+c_3)e^{3t}\right)$$

4.12 problem 13

Internal problem ID [1865]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366
Problem number: 13.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + 2x_2(t) - 3x_3(t) + \mathrm{e}^t$$
$$x_2'(t) = x_1(t) + x_2(t) + 2x_3(t)$$
$$x_3'(t) = x_1(t) - x_2(t) + 4x_3(t) - \mathrm{e}^t$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 123

```
dsolve([diff(x__1(t),t)=1*x__1(t)+2*x__2(t)-3*x__3(t)+exp(t),diff(x__2(t),t)=1*x__1(t)+1*x__2
```

$$x_1(t) = 2\,\mathrm{e}^t - c_1\mathrm{e}^{2t} - c_2\mathrm{e}^{2t}t + c_2\mathrm{e}^{2t} - \mathrm{e}^{2t}c_3t^2 + 2\,\mathrm{e}^{2t}c_3t + 2c_3\mathrm{e}^{2t}$$

$$x_2(t) = -2\,\mathrm{e}^t + c_1\mathrm{e}^{2t} + c_2\mathrm{e}^{2t}t + \mathrm{e}^{2t}c_3t^2 + 2c_3\mathrm{e}^{2t}$$

$$x_3(t) = -\mathrm{e}^t + c_1\mathrm{e}^{2t} + c_2\mathrm{e}^{2t}t + \mathrm{e}^{2t}c_3t^2$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 122

```
DSolve[{x1'[t]==1*x1[t]+2*x2[t]-3*x3[t]+Exp[t],x2'[t]==1*x1[t]+1*x2[t]+2*x3[t],x3'[t]==1*x1[t
```

$$\mathrm{x1}(t) \to \frac{1}{2}e^t\left(4 + e^t(-2c_1(t-1) - c_2(t-4)t + c_3(t-6)t)\right)$$
$$\mathrm{x2}(t) \to \frac{1}{2}e^t\left(-4 + e^t(t(c_2(t-2) - c_3(t-4) + 2c_1) + 2c_2)\right)$$
$$\mathrm{x3}(t) \to \frac{1}{2}e^t\left(-2 + e^t(2c_1t + c_2(t-2)t + c_3(2 - (t-4)t))\right)$$

4.13 problem 14

Internal problem ID [1866]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366
Problem number: 14.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -x_1(t) - x_2(t) + 1 \\
x_2'(t) &= -4x_2(t) - x_3(t) + t \\
x_3'(t) &= 5x_2(t) + \mathrm{e}^{t}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 124

```
dsolve([diff(x__1(t),t)=-1*x__1(t)-1*x__2(t)+0*x__3(t)+1,diff(x__2(t),t)=0*x__1(t)-4*x__2(t)-
```

$$x_1(t) = \frac{c_2 \mathrm{e}^{-2t} \sin(t)}{10} - \frac{3\,\mathrm{e}^{-2t} \sin(t)\, c_3}{10} - \frac{3 c_2 \mathrm{e}^{-2t} \cos(t)}{10} - \frac{\mathrm{e}^{-2t} \cos(t)\, c_3}{10} + \frac{\mathrm{e}^{t}}{20} + \frac{4}{5} + \mathrm{e}^{-t} c_1$$

$$x_2(t) = -\frac{2\,\mathrm{e}^{-2t} \sin(t)\, c_3}{5} + \frac{\mathrm{e}^{-2t} \cos(t)\, c_3}{5} - \frac{2 c_2 \mathrm{e}^{-2t} \cos(t)}{5} - \frac{c_2 \mathrm{e}^{-2t} \sin(t)}{5} + \frac{1}{5} - \frac{\mathrm{e}^{t}}{10}$$

$$x_3(t) = \mathrm{e}^{-2t} \sin(t)\, c_3 + c_2 \mathrm{e}^{-2t} \cos(t) + t - \frac{4}{5} + \frac{\mathrm{e}^{t}}{2}$$

✓ Solution by Mathematica

Time used: 1.725 (sec). Leaf size: 135

```
DSolve[{x1'[t]==-1*x1[t]-1*x2[t]+0*x3[t]+1,x2'[t]==0*x1[t]-4*x2[t]-1*x3[t]+t,x3'[t]==0*x1[t]+
```

$$\text{x1}(t) \to \frac{1}{20}e^{-2t}\left(e^t\left(e^t\left(e^t+16\right)+10(2c_1+c_2+c_3)\right)-10(c_2+c_3)\cos(t)-10(3c_2+c_3)\sin(t)\right)$$

$$\text{x2}(t) \to -\frac{e^t}{10}+e^{-2t}(c_2\cos(t)-(2c_2+c_3)\sin(t))+\frac{1}{5}$$

$$\text{x3}(t) \to t+\frac{e^t}{2}+e^{-2t}(5c_2\sin(t)+c_3(2\sin(t)+\cos(t)))-\frac{4}{5}$$

4.14 problem 16

Internal problem ID [1867]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366
Problem number: 16.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + x_2(t) - x_3(t) + \mathrm{e}^{2t}$$
$$x_2'(t) = 2x_1(t) + 3x_2(t) - 4x_3(t) + 2\,\mathrm{e}^{2t}$$
$$x_3'(t) = 4x_1(t) + x_2(t) - 4x_3(t) + \mathrm{e}^{2t}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 84

```
dsolve([diff(x__1(t),t)=1*x__1(t)+1*x__2(t)-1*x__3(t)+exp(2*t),diff(x__2(t),t)=2*x__1(t)+3*x_
```

$$x_1(t) = \mathrm{e}^{2t}t + c_1\mathrm{e}^{t} + \frac{c_2\mathrm{e}^{-3t}}{11} + c_3\mathrm{e}^{2t}$$

$$x_2(t) = 2\,\mathrm{e}^{2t}t + c_1\mathrm{e}^{t} + \frac{7c_2\mathrm{e}^{-3t}}{11} + 2c_3\mathrm{e}^{2t}$$

$$x_3(t) = \mathrm{e}^{2t}t + c_1\mathrm{e}^{t} + c_2\mathrm{e}^{-3t} + c_3\mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 2491

```
DSolve[{x1'[t]==1*x1[t]+1*x2[t]-1*x3[t]+Exp[2*t],x2'[t]==2*x1[t]+3*x2[t]-4*x3[t]+2*Exp[2*t],x
```

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4.15 problem 17

Internal problem ID [1868]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366
Problem number: 17.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - x_2(t) - x_3(t) + \mathrm{e}^{3t}$$
$$x_2'(t) = x_1(t) + 3x_2(t) + x_3(t) - \mathrm{e}^{3t}$$
$$x_3'(t) = -3x_1(t) + x_2(t) - x_3(t) - \mathrm{e}^{3t}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 86

```
dsolve([diff(x__1(t),t)=1*x__1(t)-1*x__2(t)-1*x__3(t)+exp(3*t),diff(x__2(t),t)=1*x__1(t)+3*x_
```

$$x_1(t) = \mathrm{e}^{3t}t + \frac{c_1\mathrm{e}^{-2t}}{4} - c_2\mathrm{e}^{2t} - c_3\mathrm{e}^{3t}$$

$$x_2(t) = -\mathrm{e}^{3t}t - \frac{c_1\mathrm{e}^{-2t}}{4} + c_3\mathrm{e}^{3t}$$

$$x_3(t) = -\mathrm{e}^{3t}t + c_1\mathrm{e}^{-2t} + c_2\mathrm{e}^{2t} + c_3\mathrm{e}^{3t}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 139

```
DSolve[{x1'[t]==1*x1[t]-1*x2[t]-1*x3[t]+Exp[3*t],x2'[t]==1*x1[t]+3*x2[t]+1*x3[t]-Exp[3*t],x3'
```

$$\text{x1}(t) \to \frac{1}{5}e^{-2t}\left(5(c_1+c_2)e^{4t} - e^{5t}(-5t+c_1+5c_2+c_3) + c_1 + c_3\right)$$

$$\text{x2}(t) \to \frac{1}{5}e^{-2t}\left(e^{5t}(-5t+c_1+5c_2+c_3) - c_1 - c_3\right)$$

$$\text{x3}(t) \to \frac{1}{5}e^{-2t}\left(-5(c_1+c_2)e^{4t} + e^{5t}(-5t+c_1+5c_2+c_3) + 4(c_1+c_3)\right)$$

4.16 problem 18

Internal problem ID [1869]

Book: Differential equations and their applications, 4th ed., M. Braun
Section: Section 3.12, Systems of differential equations. The nonhomogeneous equation. variation of parameters. Page 366
Problem number: 18.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 3x_1(t) + 2x_2(t) + 4x_3(t) + 2\,\mathrm{e}^{8t} \\
x_2'(t) &= 2x_1(t) + 2x_3(t) + \mathrm{e}^{8t} \\
x_3'(t) &= 4x_1(t) + 2x_2(t) + 3x_3(t) + 2\,\mathrm{e}^{8t}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 86

```
dsolve([diff(x__1(t),t)=3*x__1(t)+2*x__2(t)+4*x__3(t)+2*exp(8*t),diff(x__2(t),t)=2*x__1(t)+0*
```

$$x_1(t) = c_3\mathrm{e}^{8t} - \frac{5c_2\mathrm{e}^{-t}}{4} + 2t\,\mathrm{e}^{8t} - \frac{\mathrm{e}^{-t}c_1}{2}$$

$$x_2(t) = \frac{c_3\mathrm{e}^{8t}}{2} + \frac{c_2\mathrm{e}^{-t}}{2} + t\,\mathrm{e}^{8t} + \mathrm{e}^{-t}c_1$$

$$x_3(t) = c_3\mathrm{e}^{8t} + c_2\mathrm{e}^{-t} + 2t\,\mathrm{e}^{8t}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 139

```
DSolve[{x1'[t]==3*x1[t]+2*x2[t]+4*x3[t]+2*Exp[8*t],x2'[t]==2*x1[t]+0*x2[t]+2*x3[t]+Exp[8*t],x
```

$$\text{x1}(t) \to \frac{1}{9}e^{-t}\left(2e^{9t}(9t+2c_1+c_2+2c_3)+5c_1-2(c_2+2c_3)\right)$$

$$\text{x2}(t) \to \frac{1}{9}e^{-t}\left(e^{9t}(9t+2c_1+c_2+2c_3)-2(c_1-4c_2+c_3)\right)$$

$$\text{x3}(t) \to \frac{1}{9}e^{-t}\left(2e^{9t}(9t+2c_1+c_2+2c_3)-4c_1-2c_2+5c_3\right)$$