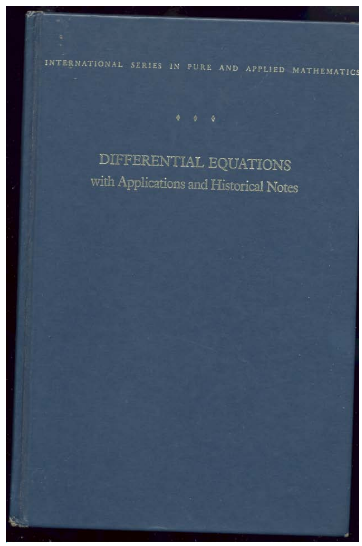
A Solution Manual For

Differential equations with applications and historial notes, George F. Simmons, 1971



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1 Chapter 2, section 7, page 37

1.1 problem 1.a

Internal problem ID [2571]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 1.a.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{x^2 - y^2 + xyy' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve((x^2-y(x)^2)+x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-2\ln(x) + c_1}\, x$$
$$y(x) = -\sqrt{-2\ln(x) + c_1}\, x$$

✓ Solution by Mathematica

Time used: 0.164 (sec). Leaf size: 36

```
DSolve[(x^2-y[x]^2)+x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{-2\log(x) + c_1}$$
$$y(x) \to x\sqrt{-2\log(x) + c_1}$$

1.2 problem 1.b

Internal problem ID [2572]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 1.b.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y'x^2 - 2yx - 2y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x)-2*x*y(x)-2*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{-2x + c_1}$$

✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 22

```
DSolve[x^2*y'[x]-2*x*y[x]-2*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{-2x + c_1}$$
$$y(x) \to 0$$

1.3 problem 1.c

Internal problem ID [2573]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 1.c.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$y'x^2 - 3(x^2 + y^2) \arctan\left(\frac{y}{x}\right) - yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x^2*diff(y(x),x)=3*(x^2+y(x)^2)*arctan(y(x)/x)+x*y(x),y(x), singsol=all)
```

$$y(x) = \tan\left(c_1 x^3\right) x$$

✓ Solution by Mathematica

Time used: 0.172 (sec). Leaf size: 37

```
DSolve[x^2*y'[x]==3*(x^2+y[x]^2)*Arctan[y[x]/x]+x*y[x],y[x],x,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{1}{\text{Arctan}(K[1])\left(K[1]^2+1\right)} dK[1] = 3\log(x) + c_1, y(x)\right]$$

1.4 problem 1.d

Internal problem ID [2574]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 1.d.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{x\sin\left(\frac{y}{x}\right)y'-\sin\left(\frac{y}{x}\right)y-x=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*sin(y(x)/x)*diff(y(x),x)=y(x)*sin(y(x)/x)+x,y(x), singsol=all)
```

$$y(x)=(\pi-\arccos(\ln(x)+c_1))\,x$$

✓ Solution by Mathematica

Time used: 0.385 (sec). Leaf size: 33

```
DSolve[x*Sin[y[x]/x]*y'[x]==y[x]*Sin[y[x]/x]+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x(-\pi+\arccos(\log(x)+c_1))$$
$$y(x)\to x(\pi-\arccos(\log(x)+c_1))$$

1.5 problem 1.

Internal problem ID [2575]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 1..
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘]]`

$$\boxed{y'x - y - 2\,\mathrm{e}^{-\frac{y}{x}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)=y(x)+2*exp(- y(x)/x),y(x), singsol=all)
```

$$y(x) = \ln\left(\frac{2c_1x - 2}{x}\right)x$$

✓ Solution by Mathematica

Time used: 0.555 (sec). Leaf size: 16

```
DSolve[x*y'[x]==y[x]+2*Exp[- y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x\log\left(-\frac{2}{x} + c_1\right)$$

1.6 problem 3.a

Internal problem ID [2576]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 3.a.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (x + y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=(x+y(x))^2,y(x), singsol=all)
```

$$y(x) = -x - \tan(c_1 - x)$$

✓ Solution by Mathematica

Time used: 0.498 (sec). Leaf size: 14

```
DSolve[y'[x]==(x+y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + \tan(x + c_1)$$

1.7 problem 3.b

Internal problem ID [2577]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 3.b.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \sin(1 + x - y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)=sin(x-y(x)+1)^2,y(x), singsol=all)
```

$$y(x) = x + 1 + \arctan(c_1 - x)$$

✓ Solution by Mathematica

Time used: 0.338 (sec). Leaf size: 33

```
DSolve[y'[x]==Sin[x-y[x]+1]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[2y(x) - 2(\tan(-y(x) + x + 1) - \arctan(\tan(-y(x) + x + 1))) = c_1, y(x)]$$

1.8 problem 5.a

Internal problem ID [2578]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 5.a.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$y' - \frac{x+y+4}{x-y-6} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)=(x+y(x)+4)/(x-y(x)-6),y(x), singsol=all)
```

$$y(x) = -5 - \tan\left(\operatorname{RootOf}\left(2\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(x-1) + 2c_1\right)\right)(x-1)$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 58

```
DSolve[y'[x]==(x+y[x]+4)/(x-y[x]-6),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{y(x)+x+4}{y(x)-x+6}\right) + \log\left(\frac{x^2+y(x)^2+10y(x)-2x+26}{2(x-1)^2}\right) + 2\log(x-1) + c_1 = 0, y(x)\right]$$

1.9 problem 5.b

Internal problem ID [2579]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 5.b.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$y' - \frac{x+y+4}{x+y-6} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=(x+y(x)+4)/(x+y(x)-6),y(x), singsol=all)
```

$$y(x) = -x - 5\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{-\frac{2x}{5}} c_1 \mathrm{e}^{\frac{1}{5}}}{5}\right) + 1$$

✓ Solution by Mathematica

Time used: 3.764 (sec). Leaf size: 35

```
DSolve[y'[x]==(x+y[x]+4)/(x+y[x]-6),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -5W\left(-e^{-\frac{2x}{5}-1+c_1}\right) - x + 1$$

$$y(x) \to 1 - x$$

2 Chapter 2, section 8, page 41

2.1 problem 1

Internal problem ID [2580]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, [_Abel, ‘2nd typ`

$$\left(x+\frac{2}{y}\right)y'+y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x+2/y(x))*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\mathrm{LambertW}\left(\frac{x\,\mathrm{e}^{\frac{c_1}{2}}}{2}\right)+\frac{c_1}{2}}$$

✓ Solution by Mathematica

Time used: 10.94 (sec). Leaf size: 58

```
DSolve[(x+2/y[x])*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2W\left(-\frac{1}{2}\sqrt{e^{c_1}x^2}\right)}{x}$$
$$y(x)\to\frac{2W\left(\frac{1}{2}\sqrt{e^{c_1}x^2}\right)}{x}$$
$$y(x)\to 0$$

2.2 problem 2

Internal problem ID [2581]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\sin(x)\tan(y) + 1 + \cos(x)\sec(y)^2 y' = 0$$

✗ Solution by Maple

```
dsolve((sin(x)*tan(y(x))+1)+(cos(x)*sec(y(x))^2)*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 2.075 (sec). Leaf size: 54

```
DSolve[(Sin[x]*Tan[y[x]]+1)+(Cos[x]*Sec[y[x]]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\arctan(\sin(x) + c_1\cos(x))$$

$$y(x) \to -\frac{1}{2}\pi\sqrt{\cos^2(x)}\sec(x)$$

$$y(x) \to \frac{1}{2}\pi\sqrt{\cos^2(x)}\sec(x)$$

2.3 problem 3

Internal problem ID [2582]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{y - x^3 + (y^3 + x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve((y(x)-x^3)+(x+y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$-\frac{x^4}{4} + y(x)\, x + \frac{y(x)^4}{4} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.165 (sec). Leaf size: 1126

```
DSolve[(y[x]-x^3)+(x+y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x^4+4c_1)^3}}-\frac{x^4+4c_1}{\sqrt[3]{3x^2+\sqrt{9x^4+\frac{1}{3}(x^4+4c_1)^3}}}}+\sqrt{\frac{\cdots}{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x\cdots}}}}}}{\cdots}$$

$$y(x) \to \frac{\sqrt{\frac{6\sqrt{2}x}{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x^4+4c_1)^3}}-\frac{x^4+4c_1}{\sqrt[3]{3x^2+\sqrt{9x^4+\frac{1}{3}(x^4+4c_1)^3}}}}}-\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x^4+4\cdots}}}}{\cdots}$$

$$y(x) \to \frac{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x^4+4c_1)^3}}-\frac{x^4+4c_1}{\sqrt[3]{3x^2+\sqrt{9x^4+\frac{1}{3}(x^4+4c_1)^3}}}}-\sqrt{-\frac{\cdots}{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x\cdots}}}}}}{\cdots}$$

$$y(x) \to \frac{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x^4+4c_1)^3}}-\frac{x^4+4c_1}{\sqrt[3]{3x^2+\sqrt{9x^4+\frac{1}{3}(x^4+4c_1)^3}}}}+\sqrt{-\frac{\cdots}{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+(x\cdots}}}}}}{\cdots}$$

2.4 problem 4

Internal problem ID [2583]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$2y^2 - 4x + 5 - (4 - 2y + 4yx)\, y' = 0$$

✗ Solution by Maple

```
dsolve((2*y(x)^2-4*x+5)=(4-2*y(x)+4*x*y(x))*diff(y(x),x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(2*y[x]^2-4*x+5)==(4-2*y[x]+4*x*y[x])*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.5 problem 5

Internal problem ID [2584]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + y\cos(yx) + (x + x\cos(yx))\,y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve((y(x)+y(x)*cos(x*y(x)))+(x+x*cos(x*y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\pi}{x}$$
$$y(x) = \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 49

```
DSolve[(y[x]+y[x]*Cos[x*y[x]])+(x+x*Cos[x*y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{\pi}{x}$$
$$y(x) \to \frac{\pi}{x}$$
$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to -\frac{\pi}{x}$$
$$y(x) \to \frac{\pi}{x}$$

2.6 problem 6

Internal problem ID [2585]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(x)\cos(y)^2 + 2\sin(x)\sin(y)\cos(y)\,y' = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 31

```
dsolve(cos(x)*cos(y(x))^2+(2*sin(x)*sin(y(x))*cos(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\pi}{2}$$
$$y(x) = \arccos\left(\sqrt{\sin(x)\,c_1}\right)$$
$$y(x) = \pi - \arccos\left(\sqrt{\sin(x)\,c_1}\right)$$

✓ Solution by Mathematica

Time used: 6.216 (sec). Leaf size: 85

```
DSolve[Cos[x]*Cos[y[x]]^2+(2*Sin[x]*Sin[y[x]]*Cos[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$
$$y(x) \to -\arccos\left(-\frac{1}{4}c_1\sqrt{\cos(x)}\sqrt{\tan(x)}\right)$$
$$y(x) \to \arccos\left(-\frac{1}{4}c_1\sqrt{\cos(x)}\sqrt{\tan(x)}\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

2.7 problem 7

Internal problem ID [2586]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{(\sin(x)\sin(y) - e^{y}x)\, y' - e^{y} - \cos(x)\cos(y) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((sin(x)*sin(y(x))-x*exp(y(x)))*diff(y(x),x)=exp(y(x))+cos(x)*cos(y(x)),y(x), singsol=a
```

$$c_1 + \sin(x)\cos(y(x)) + x\,e^{y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.616 (sec). Leaf size: 21

```
DSolve[(Sin[x]*Sin[y[x]]-x*Exp[y[x]])*y'[x]==Exp[y[x]]+Cos[x]*Cos[y[x]],y[x],x,IncludeSingula
```

$$\text{Solve}\left[2\left(xe^{y(x)} + \sin(x)\cos(y(x))\right) = c_1, y(x)\right]$$

2.8 problem 8

Internal problem ID [2587]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$-\frac{\sin\left(\frac{x}{y}\right)}{y}+\frac{x\sin\left(\frac{x}{y}\right)y'}{y^2}=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(-1/y(x)*sin(x/y(x))+x/y(x)^2*sin(x/y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{x}{\pi-c_1}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 19

```
DSolve[-1/y[x]*Sin[x/y[x]]+x/y[x]^2*Sin[x/y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to c_1x$$
$$y(x)\to \text{ComplexInfinity}$$
$$y(x)\to \text{ComplexInfinity}$$

2.9 problem 9

Internal problem ID [2588]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{1 + y + (1 - x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve((1+y(x))+(1-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -1 + c_1(x - 1)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 18

```
DSolve[(1+y[x])+(1-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + c_1(x - 1)$$
$$y(x) \to -1$$

2.10 problem 10

Internal problem ID [2589]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{2xy^3 + \cos(x)\, y + \left(3x^2y^2 + \sin(x)\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 375

```
dsolve((2*x*y(x)^3+y(x)*cos(x))+(3*x^2*y(x)^2+sin(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin\left(x\right)^3}-108c_1x\right)^{\frac{1}{3}}}{6x}-\frac{2\sin\left(x\right)}{x\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin\left(x\right)^3}-108c_1x\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin\left(x\right)^3}-108c_1x\right)^{\frac{1}{3}}}{12x}+\frac{\sin\left(x\right)}{x\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin\left(x\right)^3}-108c_1x\right)^{\frac{1}{3}}}-\frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}{6x}+\frac{2\sin(x)}{x\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin\left(x\right)^3}-108c_1x\right)^{\frac{1}{3}}}{12x}+\frac{\sin\left(x\right)}{x\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin\left(x\right)^3}-108c_1x\right)^{\frac{1}{3}}}+\frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}{6x}+\frac{2\sin(x)}{x\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 27.111 (sec). Leaf size: 339

```
DSolve[(2*x*y[x]^3+y[x]*Cos[x])+(3*x^2*y[x]^2+Sin[x])*y'[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{\sqrt[3]{9c_1x^4 + \sqrt{12x^6\sin^3(x) + 81c_1{}^2x^8}}}{\sqrt[3]{2}3^{2/3}x^2} - \frac{\sqrt[3]{\frac{2}{3}}\sin(x)}{\sqrt[3]{9c_1x^4 + \sqrt{12x^6\sin^3(x) + 81c_1{}^2x^8}}}$$

$$y(x) \to \frac{(1 + i\sqrt{3})\sin(x)}{2^{2/3}\sqrt[3]{27c_1x^4 + 3\sqrt{12x^6\sin^3(x) + 81c_1{}^2x^8}}} - \frac{(1 - i\sqrt{3})\sqrt[3]{27c_1x^4 + \sqrt{108x^6\sin^3(x) + 729c_1{}^2x^8}}}{6\sqrt[3]{2}x^2}$$

$$y(x) \to \frac{(1 - i\sqrt{3})\sin(x)}{2^{2/3}\sqrt[3]{27c_1x^4 + 3\sqrt{12x^6\sin^3(x) + 81c_1{}^2x^8}}} - \frac{(1 + i\sqrt{3})\sqrt[3]{27c_1x^4 + \sqrt{108x^6\sin^3(x) + 729c_1{}^2x^8}}}{6\sqrt[3]{2}x^2}$$

2.11 problem 11

Internal problem ID [2590]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, _Riccati]`

$$1 - \frac{y}{1 - x^2y^2} - \frac{xy'}{1 - x^2y^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(1=y(x)/(1-x^2*y(x)^2)+x/(1-x^2*y(x)^2)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -\frac{e^{-2x}c_1 + 1}{x\left(e^{-2x}c_1 - 1\right)}$$

✓ Solution by Mathematica

Time used: 0.147 (sec). Leaf size: 18

```
DSolve[1==y[x]/(1-x^2*y[x]^2)+x/(1-x^2*y[x]^2)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\tanh(x + ic_1)}{x}$$

3 Chapter 2, section 10, page 47

3.1 problem 2(a)

Internal problem ID [2591]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 10, page 47
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(3x^2 - y^2)\, y' - 2yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 402

```
dsolve((3*x^2-y(x)^2)*diff(y(x),x)-2*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1 - 108c_1^2x^2+8\right)^{\frac{1}{3}}}{6c_1} + \frac{2}{3c_1\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1 - 108c_1^2x^2+8\right)^{\frac{1}{3}}} + \frac{1}{3c_1}$$

$$y(x) = -\frac{\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1 - 108c_1^2x^2+8\right)^{\frac{1}{3}}}{12c_1} - \frac{1}{3c_1\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1 - 108c_1^2x^2+8\right)^{\frac{1}{3}}} + \frac{1}{3c_1} - \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{6c_1} - \frac{2}{3c_1\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1 - 108c_1^2x^2+8\right)^{\frac{1}{3}}}{12c_1} - \frac{1}{3c_1\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1 - 108c_1^2x^2+8\right)^{\frac{1}{3}}} + \frac{1}{3c_1} + \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{6c_1} - \frac{2}{3c_1\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 60.192 (sec). Leaf size: 458

```
DSolve[(3*x^2-y[x]^2)*y'[x]-2*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(\frac{\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}e^{2c_1}}{\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - e^{c_1}\right)$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{6\sqrt[3]{2}} - \frac{i(\sqrt{3}-i)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

$$y(x) \to -\frac{i(\sqrt{3}-i)\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{6\sqrt[3]{2}} + \frac{i(\sqrt{3}+i)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

3.2 problem 2(b)

Internal problem ID [2592]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 10, page 47
Problem number: 2(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel,`

$$yx - 1 + (x^2 - yx)\, y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve((x*y(x)-1)+(x^2-x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x - \sqrt{x^2 - 2\ln(x) + 2c_1}$$
$$y(x) = x + \sqrt{x^2 - 2\ln(x) + 2c_1}$$

✓ Solution by Mathematica

Time used: 0.398 (sec). Leaf size: 68

```
DSolve[(x*y[x]-1)+(x^2-x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \sqrt{-\frac{1}{x}}\sqrt{-x\left(x^2 - 2\log(x) + c_1\right)}$$
$$y(x) \to x + x\left(-\frac{1}{x}\right)^{3/2}\sqrt{-x\left(x^2 - 2\log(x) + c_1\right)}$$

3.3 problem 2(c)

Internal problem ID [2593]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 10, page 47
Problem number: 2(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{(x + 3x^3y^4)\, y' + y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 129

```
dsolve((x+3*x^3*y(x)^4)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-6xc_1\left(-x + \sqrt{12c_1^2 + x^2}\right)}}{6xc_1}$$

$$y(x) = \frac{\sqrt{-6xc_1\left(-x + \sqrt{12c_1^2 + x^2}\right)}}{6xc_1}$$

$$y(x) = -\frac{\sqrt{6}\,\sqrt{xc_1\left(x + \sqrt{12c_1^2 + x^2}\right)}}{6xc_1}$$

$$y(x) = \frac{\sqrt{6}\,\sqrt{xc_1\left(x + \sqrt{12c_1^2 + x^2}\right)}}{6xc_1}$$

✓ Solution by Mathematica

Time used: 9.742 (sec). Leaf size: 166

```
DSolve[(x+3*x^3*y[x]^4)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{c_1 - \frac{\sqrt{x^2(3+c_1{}^2x^2)}}{x^2}}}{\sqrt{3}}$$
$$y(x) \to \frac{\sqrt{c_1 - \frac{\sqrt{x^2(3+c_1{}^2x^2)}}{x^2}}}{\sqrt{3}}$$
$$y(x) \to -\frac{\sqrt{\frac{\sqrt{x^2(3+c_1{}^2x^2)}}{x^2} + c_1}}{\sqrt{3}}$$
$$y(x) \to \frac{\sqrt{\frac{\sqrt{x^2(3+c_1{}^2x^2)}}{x^2} + c_1}}{\sqrt{3}}$$
$$y(x) \to 0$$

3.4 problem 4(a)

Internal problem ID [2594]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 10, page 47
Problem number: 4(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{(x-1-y^2)\,y'-y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve((x-1-y(x)^2)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{2}-\frac{\sqrt{c_1^2-4x+4}}{2}$$
$$y(x)=\frac{c_1}{2}+\frac{\sqrt{c_1^2-4x+4}}{2}$$

✓ Solution by Mathematica

Time used: 0.259 (sec). Leaf size: 56

```
DSolve[(x-1-y[x]^2)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(c_1-\sqrt{-4x+4+c_1{}^2}\right)$$
$$y(x)\to\frac{1}{2}\left(\sqrt{-4x+4+c_1{}^2}+c_1\right)$$
$$y(x)\to 0$$

3.5 problem 4(b)

Internal problem ID [2595]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 10, page 47
Problem number: 4(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - (x + xy^3)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(y(x)-(x+x*y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\left(\frac{1}{\operatorname{LambertW}(c_1 x^3)}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 4.156 (sec). Leaf size: 76

```
DSolve[y[x]-(x+x*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{W\left(e^{3c_1}x^3\right)}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{W\left(e^{3c_1}x^3\right)}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{W\left(e^{3c_1}x^3\right)}$$
$$y(x) \to 0$$

3.6 problem 4(c)

Internal problem ID [2596]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 10, page 47
Problem number: 4(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{y'x - x^5 - x^3y^2 - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x)=x^5+x^3*y(x)^2+y(x),y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{x^4}{4} + c_1\right)x$$

✓ Solution by Mathematica

Time used: 0.191 (sec). Leaf size: 18

```
DSolve[x*y'[x]==x^5+x^3*y[x]^2+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\tan\left(\frac{x^4}{4} + c_1\right)$$

3.7 problem 4(d)

Internal problem ID [2597]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 10, page 47
Problem number: 4(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(x+y)\,y'-y+x=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve((y(x)+x)*diff(y(x),x)=(y(x)-x),y(x), singsol=all)
```

$$y(x)=\tan\left(\operatorname{RootOf}\left(2\_Z+\ln\left(\frac{1}{\cos\left(\_Z\right)^2}\right)+2\ln\left(x\right)+2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 34

```
DSolve[(y[x]+x)*y'[x]==(y[x]-x),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{y(x)}{x}\right)+\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)=-\log(x)+c_1,y(x)\right]$$

3.8 problem 4(e)

Internal problem ID [2598]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 10, page 47
Problem number: 4(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{y'x - y - x^2 - 9y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)=y(x)+x^2+9*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\tan(3x + 3c_1)\, x}{3}$$

✓ Solution by Mathematica

Time used: 0.248 (sec). Leaf size: 17

```
DSolve[x*y'[x]==y[x]+x^2+9*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} x \tan(3(x + c_1))$$

4 Chapter 2, section 11, page 49

4.1 problem 2(a)

Internal problem ID [2599]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 11, page 49
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 3y - x^4 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x)-3*y(x)=x^4,y(x), singsol=all)
```

$$y(x) = (x + c_1)\, x^3$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 13

```
DSolve[x*y'[x]-3*y[x]==x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3(x + c_1)$$

4.2 problem 2(b)

Internal problem ID [2600]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 11, page 49
Problem number: 2(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y + y' - \frac{1}{e^{2x} + 1} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+y(x)=1/(1+exp(2*x)),y(x), singsol=all)
```

$$y(x) = (\arctan(e^{x}) + c_1) e^{-x}$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]==1/(1+Exp[2*x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(\arctan(e^{x}) + c_1)$$

4.3 problem 2(c)

Internal problem ID [2601]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 11, page 49
Problem number: 2(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\, y' + 2yx - \cot(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((1+x^2)*diff(y(x),x)+2*x*y(x)=cot(x),y(x), singsol=all)
```

$$y(x) = \frac{\ln(\sin(x)) + c_1}{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 22

```
DSolve[(1+x^2)*y'[x]+2*x*y[x]==Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log(\tan(x)) + \log(\cos(x)) + c_1}{x^2+1}$$

4.4 problem 2(d)

Internal problem ID [2602]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 11, page 49
Problem number: 2(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y + y' - 2x\,\mathrm{e}^{-x} - x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)+y(x)=2*x*exp(-x)+x^2,y(x), singsol=all)
```

$$y(x) = x^2 - 2x + \mathrm{e}^{-x}x^2 + 2 + c_1\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 24

```
DSolve[y'[x]+y[x]==2*x*Exp[-x]+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(x^2 + c_1\right) + (x - 2)x + 2$$

4.5 problem 2(e)

Internal problem ID [2603]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 11, page 49
Problem number: 2(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \cot(x)\, y - 2\csc(x)\, x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+y(x)*cot(x)=2*x*csc(x),y(x), singsol=all)
```

$$y(x) = \frac{x^2 + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 14

```
DSolve[y'[x]+y[x]*Cot[x]==2*x*Csc[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(x^2 + c_1\right) \csc(x)$$

4.6 problem 2(f)

Internal problem ID [2604]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 11, page 49
Problem number: 2(f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y - x^3 - y'x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((2*y(x)-x^3)=x*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = (c_1 - x)\, x^2$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 15

```
DSolve[(2*y[x]-x^3)==x*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(-x + c_1)$$

5 Chapter 2, End of chapter, page 61

5.1 problem 2

Internal problem ID [2605]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(-yx+1)\,y' - y^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve((1-x*y(x))*diff(y(x),x)=y(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(-x\,\mathrm{e}^{-c_1}\right)-c_1}$$

✓ Solution by Mathematica

Time used: 2.074 (sec). Leaf size: 25

```
DSolve[(1-x*y[x])*y'[x]==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{W(-e^{-c_1}x)}{x}$$
$$y(x) \to 0$$

5.2 problem 3

Internal problem ID [2606]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$2x+3y+1+(2y-3x+5)\,y'=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve((2*x+3*y(x)+1)+(2*y(x)-3*x+5)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-1-\tan\left(\operatorname{RootOf}\left(3\_Z+\ln\left(\frac{1}{\cos(\_Z)^2}\right)+2\ln(x-1)+2c_1\right)\right)(x-1)$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 68

```
DSolve[(2*x+3*y[x]+1)+(2*y[x]-3*x+5)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[54\arctan\left(\frac{3y(x)+2x+1}{2y(x)-3x+5}\right)+18\log\left(\frac{4(x^2+y(x)^2+2y(x)-2x+2)}{13(x-1)^2}\right)+36\log(x-1)+13c_1=0,y(x)\right]$$

5.3 problem 4

Internal problem ID [2607]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y'x - \sqrt{x^2 + y^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(x*diff(y(x),x)=sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{y(x)^2}{x^2} + \frac{y(x)\sqrt{x^2 + y(x)^2}}{x^2} + \ln\left(y(x) + \sqrt{x^2 + y(x)^2}\right) - 3\ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.307 (sec). Leaf size: 66

```
DSolve[x*y'[x]==Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\left(\frac{y(x)\left(\sqrt{\frac{y(x)^2}{x^2}+1}+\frac{y(x)}{x}\right)}{x} - \log\left(\sqrt{\frac{y(x)^2}{x^2}+1} - \frac{y(x)}{x}\right)\right) = \log(x) + c_1, y(x)\right]$$

5.4 problem 5

Internal problem ID [2608]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y^2 - (x^3 - yx)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 285

```
dsolve(y(x)^2=(x^3-x*y(x))*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = c_1 \left(\frac{\left(-x^3 + \sqrt{x^6 - c_1^3}\right)^{\frac{1}{3}}}{x^3} + \frac{c_1}{x^3 \left(-x^3 + \sqrt{x^6 - c_1^3}\right)^{\frac{1}{3}}} \right) x^2$$

$$y(x) = \frac{c_1 \left(-\frac{2\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}}{x^3} - \frac{2c_1}{x^3\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}} - 2i\sqrt{3}\left(\frac{\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}}{x^3} - \frac{c_1}{x^3\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}} \right) \right) x^2}{4}$$

$$y(x) = \frac{c_1 \left(-\frac{2\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}}{x^3} - \frac{2c_1}{x^3\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}} + 2i\sqrt{3}\left(\frac{\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}}{x^3} - \frac{c_1}{x^3\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}} \right) \right) x^2}{4}$$

✓ Solution by Mathematica

Time used: 60.119 (sec). Leaf size: 534

```
DSolve[y[x]^2==(x^3-x*y[x])*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 \left(1 \right.$$

$$- \frac{9}{\frac{9\sqrt[3]{-e^{\frac{3c_1}{4}}x^{12} + 2e^{\frac{3c_1}{8}}x^6 + \sqrt{e^{\frac{3c_1}{8}}x^6\left(-1 + e^{\frac{3c_1}{8}}x^6\right)^3} - 1}}{-1+e^{\frac{3c_1}{8}}x^6} - \frac{9}{\sqrt[3]{-e^{\frac{3c_1}{4}}x^{12} + 2e^{\frac{3c_1}{8}}x^6 + \sqrt{e^{\frac{3c_1}{8}}x^6\left(-1 + e^{\frac{3 \cdots}{}} \right.}}}}$$

$$y(x) \to x^2 \left(1 \right.$$

$$- \frac{18}{\frac{9i\left(\sqrt{3}+i\right)\sqrt[3]{-e^{\frac{3c_1}{4}}x^{12} + 2e^{\frac{3c_1}{8}}x^6 + \sqrt{e^{\frac{3c_1}{8}}x^6\left(-1 + e^{\frac{3c_1}{8}}x^6\right)^3} - 1}}{-1+e^{\frac{3c_1}{8}}x^6} + \frac{9+9i\sqrt{3}}{\sqrt[3]{-e^{\frac{3c_1}{4}}x^{12} + 2e^{\frac{3c_1}{8}}x^6 + \sqrt{e^{\frac{3c_1}{8}}x^6\left(- \cdots \right.}}}}$$

5.5 problem 6

Internal problem ID [2609]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$x^2 + y + y^3 - (x^3y^2 - x)\, y' = 0$$

✗ Solution by Maple

```
dsolve((x^2+y(x)^3+y(x))=(x^3*y(x)^2-x)*diff(y(x),x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x^2+y[x]^3+y[x])==(x^3*y[x]^2-x)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

5.6 problem 8

Internal problem ID [2610]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y - x\cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+y(x)=x*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{\cos(x) + x\sin(x) + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 17

```
DSolve[x*y'[x]+y[x]==x*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + \frac{\cos(x) + c_1}{x}$$

5.7 problem 9

Internal problem ID [2611]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(yx - x^2)\, y' - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve((x*y(x)-x^2)*diff(y(x),x)=y(x)^2,y(x), singsol=all)
```

$$y(x) = e^{-\operatorname{LambertW}\left(-\frac{e^{-c_1}}{x}\right)-c_1}$$

✓ Solution by Mathematica

Time used: 2.225 (sec). Leaf size: 25

```
DSolve[(x*y[x]-x^2)*y'[x]==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -xW\left(-\frac{e^{-c_1}}{x}\right)$$
$$y(x) \to 0$$

5.8 problem 10

Internal problem ID [2612]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\left(e^{x}-3x^{2}y^{2}\right)y'+e^{x}y-2xy^{3}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 347

```
dsolve((exp(x)-3*x^2*y(x)^2)*diff(y(x),x)+y(x)*exp(x)=2*x*y(x)^3,y(x), singsol=all)
```

$$y(x)=\frac{\left(108c_1x+12\sqrt{81c_1^2x^2-12\,e^{3x}}\right)^{\frac{1}{3}}}{6x}+\frac{2\,e^{x}}{x\left(108c_1x+12\sqrt{81c_1^2x^2-12\,e^{3x}}\right)^{\frac{1}{3}}}$$

$$y(x)=-\frac{\left(108c_1x+12\sqrt{81c_1^2x^2-12\,e^{3x}}\right)^{\frac{1}{3}}}{12x}-\frac{e^{x}}{x\left(108c_1x+12\sqrt{81c_1^2x^2-12\,e^{3x}}\right)^{\frac{1}{3}}}-\frac{i\sqrt{3}\left(\frac{\left(108c_1x+12\sqrt{81c_1^2x^2-12\,e^{3x}}\right)^{\frac{1}{3}}}{6x}-\frac{2\,e^{x}}{x\left(108c_1x+12\sqrt{81c_1^2x^2-12\,e^{3x}}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x)=-\frac{\left(108c_1x+12\sqrt{81c_1^2x^2-12\,e^{3x}}\right)^{\frac{1}{3}}}{12x}-\frac{e^{x}}{x\left(108c_1x+12\sqrt{81c_1^2x^2-12\,e^{3x}}\right)^{\frac{1}{3}}}+\frac{i\sqrt{3}\left(\frac{\left(108c_1x+12\sqrt{81c_1^2x^2-12\,e^{3x}}\right)^{\frac{1}{3}}}{6x}-\frac{2\,e^{x}}{x\left(108c_1x+12\sqrt{81c_1^2x^2-12\,e^{3x}}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 54.59 (sec). Leaf size: 364

```
DSolve[(Exp[x]-3*x^2*y[x]^2)*y'[x]+y[x]*Exp[x]==2*x*y[x]^3,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{2\sqrt[3]{3}e^x x^2 + \sqrt[3]{2}\left(9c_1x^4 + \sqrt{-12e^{3x}x^6 + 81{c_1}^2x^8}\right)^{2/3}}{6^{2/3}x^2\sqrt[3]{9c_1x^4 + \sqrt{-12e^{3x}x^6 + 81{c_1}^2x^8}}}$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{9c_1x^4 + \sqrt{-12e^{3x}x^6 + 81{c_1}^2x^8}}}{2\sqrt[3]{2}3^{2/3}x^2} - \frac{(\sqrt{3}+3i)\,e^x}{2^{2/3}3^{5/6}\sqrt[3]{9c_1x^4 + \sqrt{-12e^{3x}x^6 + 81{c_1}^2x^8}}}$$

$$y(x) \to \frac{(-1-i\sqrt{3})\sqrt[3]{9c_1x^4 + \sqrt{-12e^{3x}x^6 + 81{c_1}^2x^8}}}{2\sqrt[3]{2}3^{2/3}x^2} - \frac{(\sqrt{3}-3i)\,e^x}{2^{2/3}3^{5/6}\sqrt[3]{9c_1x^4 + \sqrt{-12e^{3x}x^6 + 81{c_1}^2x^8}}}$$

5.9 problem 12

Internal problem ID [2613]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y + x^2 - y'x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve((x^2+y(x))=x*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = (x + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 11

```
DSolve[(x^2+y[x])==x*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x + c_1)$$

5.10 problem 13

Internal problem ID [2614]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y - x^2 \cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x*diff(y(x),x)+y(x)=x^2*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)\, x^2 - 2\sin(x) + 2x\cos(x) + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 25

```
DSolve[x*y'[x]+y[x]==x^2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x^2 - 2)\sin(x) + 2x\cos(x) + c_1}{x}$$

5.11 problem 14

Internal problem ID [2615]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cla`

$$\boxed{6x + 4y + 3 + (3x + 2y + 2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve((6*x+4*y(x)+3)+(3*x+2*y(x)+2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{3x}{2} + \operatorname{LambertW}\left(\mathrm{e}^{-\frac{x}{2}} c_1\right)$$

✓ Solution by Mathematica

Time used: 4.049 (sec). Leaf size: 34

```
DSolve[(6*x+4*y[x]+3)+(3*x+2*y[x]+2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3x}{2} + W\left(-e^{-\frac{x}{2}-1+c_1}\right)$$

$$y(x) \to -\frac{3x}{2}$$

5.12 problem 15

Internal problem ID [2616]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _exact]`

$$\boxed{\cos(x+y) - x\sin(x+y) - x\sin(x+y)\,y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(cos(x+y(x))-x*sin(x+y(x))=x*sin(x+y(x))*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -x + \arccos\left(\frac{c_1}{x}\right)$$

✓ Solution by Mathematica

Time used: 10.063 (sec). Leaf size: 35

```
DSolve[Cos[x+y[x]]-x*Sin[x+y[x]]==x*Sin[x+y[x]]*y'[x],y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -x - \arccos\left(-\frac{c_1}{x}\right)$$
$$y(x) \to -x + \arccos\left(-\frac{c_1}{x}\right)$$

5.13 problem 17

Internal problem ID [2617]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y^2 e^{yx} + \cos(x) + (e^{yx} + xy\, e^{yx})\, y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve((y(x)^2*exp(x*y(x))+cos(x))+(exp(x*y(x))+x*y(x)*exp(x*y(x)))*diff(y(x),x)=0,y(x), sing
```

$$y(x) = \frac{\text{LambertW}(-x(c_1 + \sin(x)))}{x}$$

✓ Solution by Mathematica

Time used: 60.254 (sec). Leaf size: 19

```
DSolve[(y[x]^2*Exp[x*y[x]]+Cos[x])+(Exp[x*y[x]]+x*y[x]*Exp[x*y[x]])*y'[x]==0,y[x],x,IncludeSi
```

$$y(x) \to \frac{W(x(-\sin(x) + c_1))}{x}$$

5.14 problem 18

Internal problem ID [2618]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _dAlembert]`

$$\boxed{y' \ln (x-y)-1-\ln (x-y)=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)*ln(x-y(x))=1+ln(x-y(x)),y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{\mathrm{LambertW}\left((c_1-x)\mathrm{e}^{-1}\right)+1}+x$$

✓ Solution by Mathematica

Time used: 0.112 (sec). Leaf size: 26

```
DSolve[y'[x]*Log[x-y[x]]==1+Log[x-y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}[(x-y(x))(-\log(x-y(x)))-y(x)=c_1, y(x)]$$

5.15 problem 19

Internal problem ID [2619]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + 2yx - e^{-x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+2*x*y(x)=exp(-x^2),y(x), singsol=all)
```

$$y(x) = (x + c_1)\, e^{-x^2}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 17

```
DSolve[y'[x]+2*x*y[x]==Exp[-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}(x + c_1)$$

5.16 problem 20

Internal problem ID [2620]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$y^2 - 3yx - 2x^2 - (x^2 - yx)\, y' = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 59

```
dsolve((y(x)^2-3*x*y(x)-2*x^2)=(x^2-x*y(x))*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2 - \sqrt{2c_1^2 x^4 + 1}}{c_1 x}$$

$$y(x) = \frac{c_1 x^2 + \sqrt{2c_1^2 x^4 + 1}}{c_1 x}$$

✓ Solution by Mathematica

Time used: 0.659 (sec). Leaf size: 99

```
DSolve[(y[x]^2-3*x*y[x]-2*x^2)==(x^2-x*y[x])*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \frac{\sqrt{2x^4 + e^{2c_1}}}{x}$$

$$y(x) \to x + \frac{\sqrt{2x^4 + e^{2c_1}}}{x}$$

$$y(x) \to x - \frac{\sqrt{2}\sqrt{x^4}}{x}$$

$$y(x) \to \frac{\sqrt{2}\sqrt{x^4}}{x} + x$$

5.17 problem 21

Internal problem ID [2621]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y'+2yx-4x^3=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((1+x^2)*diff(y(x),x)+2*x*y(x)=4*x^3,y(x), singsol=all)
```

$$y(x)=\frac{x^4+c_1}{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 19

```
DSolve[(1+x^2)*y'[x]+2*x*y[x]==4*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x^4+c_1}{x^2+1}$$

5.18 problem 22

Internal problem ID [2622]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{e^x \sin(y) - y\sin(yx) + (e^x \cos(y) - x\sin(yx))\, y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve((exp(x)*sin(y(x))-y(x)*sin(x*y(x)))+(exp(x)*cos(y(x))-x*sin(x*y(x)))*diff(y(x),x)=0,y(
```

$$e^x \sin(y(x)) + \cos(y(x)\, x) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.547 (sec). Leaf size: 19

```
DSolve[(Exp[x]*Sin[y[x]]-y[x]*Sin[x*y[x]])+(Exp[x]*Cos[y[x]]-x*Sin[x*y[x]])*y'[x]==0,y[x],x,I
```

$$\text{Solve}[e^x \sin(y(x)) + \cos(xy(x)) = c_1, y(x)]$$

5.19 problem 24

Internal problem ID [2623]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{(e^y x + y - x^2)\, y' - 2yx + e^y + x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve((x*exp(y(x))+y(x)-x^2)*diff(y(x),x)=(2*x*y(x) -exp(y(x))-x),y(x), singsol=all)
```

$$-y(x)\, x^2 + x\, e^{y(x)} + \frac{x^2}{2} + \frac{y(x)^2}{2} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.31 (sec). Leaf size: 35

```
DSolve[(x*Exp[y[x]]+y[x]-x^2)*y'[x]==(2*x*y[x] -Exp[y[x]]-x),y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[x^2(-y(x)) + \frac{x^2}{2} + xe^{y(x)} + \frac{y(x)^2}{2} = c_1, y(x)\right]$$

5.20 problem 25

Internal problem ID [2624]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{e^{x}(x+1)-(e^{x}x-e^{y}y)\,y'=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(exp(x)*(1+x)=(x*exp(x)-y(x)*exp(y(x)))*diff(y(x),x),y(x), singsol=all)
```

$$x\,e^{-y(x)+x}+\frac{y(x)^{2}}{2}+c_1=0$$

✓ Solution by Mathematica

Time used: 0.307 (sec). Leaf size: 26

```
DSolve[Exp[x]*(1+x)==(x*Exp[x]-y[x]*Exp[y[x]])*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{1}{2}y(x)^{2}-xe^{x-y(x)}=c_1,y(x)\right]$$