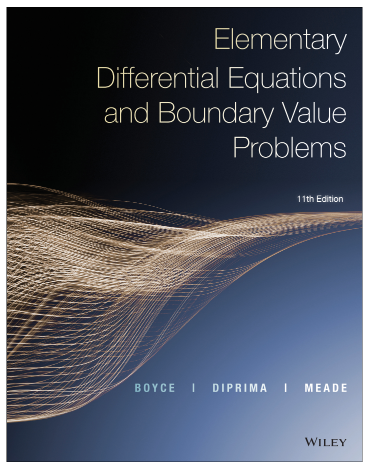
A Solution Manual For

Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade



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Contents

1 Chapter 4.1, Higher order linear differential equations. General theory. page 173

1.1 problem 1

Internal problem ID [812]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173
Problem number: 1.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{y'''' + 4y''' + 3y - t = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 227

```
dsolve(diff(y(t),t$4)+4*diff(y(t),t$3)+3*y(t)=t,y(t), singsol=all)
```

$$y(t) = \frac{t}{3} + \mathrm{e}^{-t}c_1 + c_2\mathrm{e}^{\frac{\left(\sqrt{2}\left(4+2\sqrt{2}\right)^{\frac{2}{3}}-2\left(4+2\sqrt{2}\right)^{\frac{2}{3}}-2\left(4+2\sqrt{2}\right)^{\frac{1}{3}}-2\right)t}{2}}$$
$$+c_3\mathrm{e}^{-\frac{\left(\sqrt{2}\left(4+2\sqrt{2}\right)^{\frac{2}{3}}-2\left(4+2\sqrt{2}\right)^{\frac{2}{3}}-2\left(4+2\sqrt{2}\right)^{\frac{1}{3}}+4\right)t}{4}}\cos\left(\frac{\sqrt{3}\left(4+2\sqrt{2}\right)^{\frac{1}{3}}\left(\left(4+2\sqrt{2}\right)^{\frac{1}{3}}\sqrt{2}-2\left(4+2\sqrt{2}\right)^{\frac{1}{3}}+2\right)t}{4}\right)$$
$$+c_4\mathrm{e}^{-\frac{\left(\sqrt{2}\left(4+2\sqrt{2}\right)^{\frac{2}{3}}-2\left(4+2\sqrt{2}\right)^{\frac{2}{3}}-2\left(4+2\sqrt{2}\right)^{\frac{1}{3}}+4\right)t}{4}}\sin\left(\frac{\sqrt{3}\left(4+2\sqrt{2}\right)^{\frac{1}{3}}\left(\left(4+2\sqrt{2}\right)^{\frac{1}{3}}\sqrt{2}-2\left(4+2\sqrt{2}\right)^{\frac{1}{3}}+2\right)t}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 100

```
DSolve[y''''[t]+4*y'''[t]+3*y[t]==t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_2 \exp\left(t\text{Root}\left[\#1^3+3\#1^2-3\#1+3\&,2\right]\right)+c_3 \exp\left(t\text{Root}\left[\#1^3+3\#1^2-3\#1+3\&,3\right]\right)$$
$$+ c_1 \exp\left(t\text{Root}\left[\#1^3+3\#1^2-3\#1+3\&,1\right]\right) + \frac{t}{3} + c_4e^{-t}$$

1.2 problem 2

Internal problem ID [813]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173
Problem number: 2.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{t(t-1)\, y'''' + e^{t} y'' + 4y t^{2} = 0}$$

✗ Solution by Maple

```
dsolve(t*(t-1)*diff(y(t),t$4)+exp(t)*diff(y(t),t$2)+4*t^2*y(t)=0,y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[t*(t-1)*y''''[t]+Exp[t]*y''[t]+4*t^2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

Not solved

1.3 problem 8

Internal problem ID [814]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173
Problem number: 8.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + y'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$4)+diff(y(t),t$2)=0,y(t), singsol=all)
```

$$y(t) = c_1 + c_2 t + c_3 \sin(t) + c_4 \cos(t)$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 24

```
DSolve[y''''[t]+y''[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_4 t - c_1 \cos(t) - c_2 \sin(t) + c_3$$

1.4 problem 9

Internal problem ID [815]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173
Problem number: 9.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 2y'' - y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(t),t$3)+2*diff(y(t),t$2)-diff(y(t),t)-2*y(t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-t}c_1 + \mathrm{e}^{-2t}c_2 + c_3\mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y'''[t]+2*y''[t]-y'[t]-2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-2t}\left(c_2e^{t} + c_3e^{3t} + c_1\right)$$

1.5 problem 10

Internal problem ID [816]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173
Problem number: 10.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{xy''' - y'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x$3)-diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_2x^3 + c_3x + c_1$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 21

```
DSolve[x*y'''[x]-y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1x^3}{6} + c_3x + c_2$$

1.6 problem 11

Internal problem ID [817]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173
Problem number: 11.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^3y''' + x^2y'' - 2y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)+x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + x^2c_2 + c_3x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+x^2*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3x^2 + c_2x + \frac{c_1}{x}$$

1.7 problem 16

Internal problem ID [818]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173
Problem number: 16.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' + 2y'' - y' - 3y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 181

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)-diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{\left(\left(188+12\sqrt{93}\right)^{\frac{2}{3}}-4\left(188+12\sqrt{93}\right)^{\frac{1}{3}}+28\right)x}{6\left(188+12\sqrt{93}\right)^{\frac{1}{3}}}} - c_2 e^{-\frac{\left(28+\left(188+12\sqrt{93}\right)^{\frac{2}{3}}+8\left(188+12\sqrt{93}\right)^{\frac{1}{3}}\right)x}{12\left(188+12\sqrt{93}\right)^{\frac{1}{3}}}} \sin\left(\frac{\left(\sqrt{3}\left(188+12\sqrt{93}\right)^{\frac{2}{3}}-28\sqrt{3}\right)x}{12\left(188+12\sqrt{93}\right)^{\frac{1}{3}}}\right) + c_3 e^{-\frac{\left(28+\left(188+12\sqrt{93}\right)^{\frac{2}{3}}+8\left(188+12\sqrt{93}\right)^{\frac{1}{3}}\right)x}{12\left(188+12\sqrt{93}\right)^{\frac{1}{3}}}} \cos\left(\frac{\left(\sqrt{3}\left(188+12\sqrt{93}\right)^{\frac{2}{3}}-28\sqrt{3}\right)x}{12\left(188+12\sqrt{93}\right)^{\frac{1}{3}}}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 87

```
DSolve[y'''[x]+2*y''[x]-y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \exp\left(x\text{Root}\left[\#1^3+2\#1^2-\#1-3\&,2\right]\right)+c_3 \exp\left(x\text{Root}\left[\#1^3+2\#1^2-\#1-3\&,3\right]\right) + c_1 \exp\left(x\text{Root}\left[\#1^3+2\#1^2-\#1-3\&,1\right]\right)$$

1.8 problem 17

Internal problem ID [819]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173
Problem number: 17.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{ty''' + 2y'' - y' + yt = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 183

```
dsolve(t*diff(y(t),t$3)+2*diff(y(t),t$2)-diff(y(t),t)+t*y(t)=0,y(t), singsol=all)
```

$$
\begin{aligned}
y(t) = {} & c_1 \,\mathrm{KummerM}\left(\frac{1}{2}-\frac{i\sqrt{3}}{6}, 1, i\sqrt{3}\,t\right) \mathrm{e}^{-\frac{t\left(i\sqrt{3}-1\right)}{2}} \\
& + c_2 \,\mathrm{KummerU}\left(\frac{1}{2}-\frac{i\sqrt{3}}{6}, 1, i\sqrt{3}\,t\right) \mathrm{e}^{-\frac{t\left(i\sqrt{3}-1\right)}{2}} \\
& + c_3 \mathrm{e}^{-\frac{t\left(i\sqrt{3}-1\right)}{2}} \left(\left(\int \mathrm{KummerU}\left(\frac{1}{2}-\frac{i\sqrt{3}}{6}, 1, i\sqrt{3}\,t\right) \mathrm{e}^{-\frac{t\left(i\sqrt{3}+3\right)}{2}} dt\right) \mathrm{KummerM}\left(\frac{1}{2}\right.\right. \\
& \left. - \frac{i\sqrt{3}}{6}, 1, i\sqrt{3}\,t\right) - \mathrm{KummerU}\left(\frac{1}{2}\right. \\
& \left. \left. - \frac{i\sqrt{3}}{6}, 1, i\sqrt{3}\,t\right) \left(\int \mathrm{KummerM}\left(\frac{1}{2}-\frac{i\sqrt{3}}{6}, 1, i\sqrt{3}\,t\right) \mathrm{e}^{-\frac{t\left(i\sqrt{3}+3\right)}{2}} dt\right)\right)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.631 (sec). Leaf size: 452

```
DSolve[t*y'''[t]+2*y''[t]-y'[t]+t*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{\frac{1}{2}\left(t-i\sqrt{3}t\right)}\left(\text{HypergeometricU}\left(\frac{1}{6}\left(3-i\sqrt{3}\right),1,i\sqrt{3}t\right)\left(c_3\int_1^t \frac{}{K[1]\left(\text{Hypergeometric1F1}\left(\frac{1}{6}\left(9-i\sqrt{3}\right),\right.\right.}\right.\right.$$

$$+\text{LaguerreL}\left(\frac{1}{6}i\left(3i+\sqrt{3}\right),i\sqrt{3}t\right)\left(c_3\int_1^t -\frac{}{K[2]\left(\text{Hypergeometric1F1}\left(\frac{1}{6}\left(9-i\sqrt{3}\right),2,i\sqrt{3}K[2]\right)\text{Hype}\right.}\right.$$

1.9 problem 20

Internal problem ID [820]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173
Problem number: 20.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

Solve

$$\boxed{(2-t)\,y''' + (2t-3)\,y'' - y't + y = 0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^t$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve([(2-t)*diff(y(t),t$3)+(2*t-3)*diff(y(t),t$2)-t*diff(y(t),t)+y(t)=0,exp(t)],y(t), sings
```

$$y(t) = c_1 t + c_2 \mathrm{e}^t + c_3 \mathrm{e}^t t$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 23

```
DSolve[(2-t)*y'''[t]+(2*t-3)*y''[t]-t*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 t + e^t (c_2 (t-4) + c_3)$$

1.10 problem 21

Internal problem ID [821]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.1, Higher order linear differential equations. General theory. page 173
Problem number: 21.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

Solve

$$\boxed{t^2(t+3)\,y''' - 3t(t+2)\,y'' + 6(1+t)\,y' - 6y = 0}$$

Given that one solution of the ode is

$$y_1 = [t^2, t^3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([t^2*(t+3)*diff(y(t),t$3)-3*t*(t+2)*diff(y(t),t$2)+6*(1+t)*diff(y(t),t)-6*y(t)=0,[t^2,
```

$$y(t) = c_1t^2 + t^3c_2 + c_3(t+1)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 53

```
DSolve[t^2*(t+3)*y'''[t]-3*t*(t+2)*y''[t]+6*(1+t)*y'[t]-6*y[t]==0,y[t],t,IncludeSingularSolut
```

$$y(t) \to \frac{1}{8}\left(-4c_2\left(t^3 - 3t^2 + t + 1\right) + c_3(3t+1)(t-1)^2 + 2c_1(t((t-3)t+3)+3)\right)$$

2 Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180

2.1 problem 8

Internal problem ID [822]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180
Problem number: 8.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y'' - y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)-diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + e^{x}c_2 + c_3e^{x}x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[y'''[x]-y''[x]-y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1e^{-x} + e^{x}(c_3x + c_2)$$

2.2 problem 9

Internal problem ID [823]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180
Problem number: 9.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y'' + 3y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\left(-2^{\frac{1}{3}}+1\right)x} + c_2 e^{\left(\frac{2^{\frac{1}{3}}}{2}+1\right)x} \sin\left(\frac{\sqrt{3}\, 2^{\frac{1}{3}} x}{2}\right) + c_3 e^{\left(\frac{2^{\frac{1}{3}}}{2}+1\right)x} \cos\left(\frac{\sqrt{3}\, 2^{\frac{1}{3}} x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 87

```
DSolve[y'''[x]-3*y''[x]+3*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to\ & c_1 \exp\left(x\text{Root}\left[\#1^3 - 3\#1^2 + 3\#1 + 1\&, 1\right]\right) \\ & + c_2 \exp\left(x\text{Root}\left[\#1^3 - 3\#1^2 + 3\#1 + 1\&, 2\right]\right) \\ & + c_3 \exp\left(x\text{Root}\left[\#1^3 - 3\#1^2 + 3\#1 + 1\&, 3\right]\right) \end{aligned}$$

2.3 problem 10

Internal problem ID [824]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180
Problem number: 10.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 4y''' + 4y'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+4*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2x + c_3e^{2x} + c_4e^{2x}x$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''''[x]-4*y'''[x]+4*y'''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x(c_4x + c_3) + c_2) + c_1$$

2.4 problem 11

Internal problem ID [825]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180
Problem number: 11.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(6)} + y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 66

```
dsolve(diff(y(x),x$6)+y(x)=0,y(x), singsol=all)
```

$$\begin{aligned}y(x) = {} & c_1 \sin(x) + \cos(x) c_2 + c_3 \mathrm{e}^{\frac{\sqrt{3}\,x}{2}} \sin\left(\frac{x}{2}\right) \\ & - c_4 \mathrm{e}^{-\frac{\sqrt{3}\,x}{2}} \sin\left(\frac{x}{2}\right) + c_5 \mathrm{e}^{\frac{\sqrt{3}\,x}{2}} \cos\left(\frac{x}{2}\right) + c_6 \mathrm{e}^{-\frac{\sqrt{3}\,x}{2}} \cos\left(\frac{x}{2}\right)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 74

```
DSolve[y''''''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \cos(x) + c_5 \sin(x) + e^{-\frac{\sqrt{3}x}{2}} \left(\left(c_1 e^{\sqrt{3}x} + c_3 \right) \cos\left(\frac{x}{2}\right) + \left(c_6 e^{\sqrt{3}x} + c_4 \right) \sin\left(\frac{x}{2}\right) \right)$$

2.5 problem 12

Internal problem ID [826]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180
Problem number: 12.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(6)} - 3y'''' + 3y'' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$6)-3*diff(y(x),x$4)+3*diff(y(x),x$2)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}c_1 + c_2\mathrm{e}^{-x}x + c_3\mathrm{e}^{-x}x^2 + c_4\mathrm{e}^{x} + c_5\mathrm{e}^{x}x + c_6\mathrm{e}^{x}x^2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 41

```
DSolve[y''''''[x]-3*y''''[x]+3*y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(x(c_3x + c_2) + e^{2x}(x(c_6x + c_5) + c_4) + c_1\right)$$

2.6 problem 13

Internal problem ID [827]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180
Problem number: 13.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(6)} - y'' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$6)-diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2x + c_3\mathrm{e}^{-x} + c_4\mathrm{e}^{x} + c_5\sin(x) + c_6\cos(x)$$

✓ Solution by Mathematica

Time used: 0.112 (sec). Leaf size: 38

```
DSolve[y''''''[x]-y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1e^x + c_3e^{-x} + c_6x - c_2\cos(x) - c_4\sin(x) + c_5$$

2.7 problem 14

Internal problem ID [828]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180
Problem number: 14.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} - 3y'''' + 3y''' - 3y'' + 2y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$5)-3*diff(y(x),x$4)+3*diff(y(x),x$3)-3*diff(y(x),x$2)+2*diff(y(x),x)=0,y(x
```

$$y(x) = c_1 + c_2 e^{2x} + c_3 e^x + c_4 \sin(x) + c_5 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 36

```
DSolve[y'''''[x]-3*y''''[x]+3*y'''[x]-3*y''[x]+2*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to c_3 e^x + \frac{1}{2} c_4 e^{2x} - c_2 \cos(x) + c_1 \sin(x) + c_5$$

2.8 problem 15

Internal problem ID [829]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180
Problem number: 15.
ODE order: 8.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(8)} + 8y'''' + 16y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 65

```
dsolve(diff(y(x),x$8)+8*diff(y(x),x$4)+16*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned}y(x) = {} & c_1 \mathrm{e}^{-x} \sin(x) + c_2 \mathrm{e}^{-x} \cos(x) + c_3 \mathrm{e}^{-x} \sin(x)\, x + c_4 \mathrm{e}^{-x} \cos(x)\, x \\ & + c_5 \sin(x)\, \mathrm{e}^{x} + c_6 \cos(x)\, \mathrm{e}^{x} + c_7 \sin(x)\, \mathrm{e}^{x} x + c_8 \cos(x)\, \mathrm{e}^{x} x\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 238

```
DSolve[D[y[x],{x,8}]+8*y''''[x]+3*y'''[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}y(x) \to {} & c_1 \exp\left(x\mathrm{Root}\left[\#1^8 + 8\#1^4 + 3\#1^3 + 16\&, 1\right]\right) \\ & + c_2 \exp\left(x\mathrm{Root}\left[\#1^8 + 8\#1^4 + 3\#1^3 + 16\&, 2\right]\right) \\ & + c_5 \exp\left(x\mathrm{Root}\left[\#1^8 + 8\#1^4 + 3\#1^3 + 16\&, 5\right]\right) \\ & + c_6 \exp\left(x\mathrm{Root}\left[\#1^8 + 8\#1^4 + 3\#1^3 + 16\&, 6\right]\right) \\ & + c_3 \exp\left(x\mathrm{Root}\left[\#1^8 + 8\#1^4 + 3\#1^3 + 16\&, 3\right]\right) \\ & + c_4 \exp\left(x\mathrm{Root}\left[\#1^8 + 8\#1^4 + 3\#1^3 + 16\&, 4\right]\right) \\ & + c_7 \exp\left(x\mathrm{Root}\left[\#1^8 + 8\#1^4 + 3\#1^3 + 16\&, 7\right]\right) \\ & + c_8 \exp\left(x\mathrm{Root}\left[\#1^8 + 8\#1^4 + 3\#1^3 + 16\&, 8\right]\right)\end{aligned}$$

2.9 problem 16

Internal problem ID [830]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180
Problem number: 16.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2y'' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + \cos(x) c_2 + c_3 \sin(x) x + c_4 \cos(x) x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y''''[x]+2*y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (c_2 x + c_1) \cos(x) + (c_4 x + c_3) \sin(x)$$

2.10 problem 17

Internal problem ID [831]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180
Problem number: 17.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 5y'' + 6y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$3)+5*diff(y(x),x$2)+6*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 \mathrm{e}^{\left(\sqrt{2}-2\right)x} + c_3 \mathrm{e}^{-\left(2+\sqrt{2}\right)x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 43

```
DSolve[y'''[x]+5*y''[x]+6*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \left(e^{-\left(\left(1+\sqrt{2}\right)x\right)} \left(c_2 e^{2\sqrt{2}x} + c_1 \right) + c_3 \right)$$

2.11 problem 18

Internal problem ID [832]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 4.2, Higher order linear differential equations. Constant coefficients. page 180
Problem number: 18.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - 7y''' + 6y'' + 30y' - 36y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$4)-7*diff(y(x),x$3)+6*diff(y(x),x$2)+30*diff(y(x),x)-36*y(x)=0,y(x), sings
```

$$y(x) = c_1 e^{3x} + c_2 e^{-2x} + c_3 e^{\left(3+\sqrt{3}\right)x} + c_4 e^{-\left(-3+\sqrt{3}\right)x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 51

```
DSolve[y''''[x]-7*y'''[x]+6*y''[x]+30*y'[x]-36*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to c_1 e^{-\left(\left(\sqrt{3}-3\right)x\right)} + c_2 e^{\left(3+\sqrt{3}\right)x} + c_3 e^{-2x} + c_4 e^{3x}$$

3 Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255

3.1 problem 8

Internal problem ID [833]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 6y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)-diff(y(t),t)-6*y(t)=0,y(0) = 1, D(y)(0) = -1],y(t), singsol=all)
```

$$y(t) = \frac{(\mathrm{e}^{5t} + 4)\,\mathrm{e}^{-2t}}{5}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 21

```
DSolve[{y''[t]-y'[t]-6*y[t]==0,{y[0]==1,y'[0]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{5}e^{-2t}(e^{5t} + 4)$$

3.2 problem 9

Internal problem ID [834]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y' + 2y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=0,y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = 2\,\mathrm{e}^{-t} - \mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{y''[t]+3*y'[t]+2*y[t]==0,{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-2t}(2e^t - 1)$$

3.3 problem 10

Internal problem ID [835]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 2y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)+2*y(t)=0,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \sin(t) e^t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 11

```
DSolve[{y''[t]-2*y'[t]+2*y[t]==0,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^t \sin(t)$$

3.4 problem 11

Internal problem ID [836]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 4y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 27

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)+4*y(t)=0,y(0) = 2, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{2\,\mathrm{e}^{t}\left(\sqrt{3}\,\sin\left(\sqrt{3}\,t\right) - 3\cos\left(\sqrt{3}\,t\right)\right)}{3}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 37

```
DSolve[{y''[t]-2*y'[t]+4*y[t]==0,{y[0]==2,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{2}{3}e^{t}\left(\sqrt{3}\sin\left(\sqrt{3}t\right) - 3\cos\left(\sqrt{3}t\right)\right)$$

3.5 problem 12

Internal problem ID [837]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 5y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+5*y(t)=0,y(0) = 2, D(y)(0) = -1],y(t), singsol=all)
```

$$y(t) = \frac{e^{-t}(\sin(2t) + 4\cos(2t))}{2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[{y''[t]+2*y'[t]+5*y[t]==0,{y[0]==2,y'[0]==-1}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to \frac{1}{2}e^{-t}(\sin(2t) + 4\cos(2t))$$

3.6 problem 13

Internal problem ID [838]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255
Problem number: 13.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 4y''' + 6y'' - 4y' + y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = 0, y'''(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve([diff(y(t),t$4)-4*diff(y(t),t$3)+6*diff(y(t),t$2)-4*diff(y(t),t)+y(t)=0,y(0) = 0, D(y)
```

$$y(t) = \frac{e^t t(2t^2 - 3t + 3)}{3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[{y''''[t]-4*y'''[t]+6*y''[t]-4*y'[t]+y[t]==0,{y[0]==0,y'[0]==1,y''[0]==0,y'''[0]==1}},
```

$$y(t) \to \frac{1}{3}e^t t(t(2t - 3) + 3)$$

3.7 problem 14

Internal problem ID [839]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255
Problem number: 14.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 4y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0, y''(0) = 1, y'''(0) = 0]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 30

```
dsolve([diff(y(t),t$4)-4*y(t)=0,y(0) = 1, D(y)(0) = 0, (D@@2)(y)(0) = 1, (D@@3)(y)(0) = 0],y(
```

$$y(t) = \frac{3\,e^{t\sqrt{2}}}{8} + \frac{3\,e^{-t\sqrt{2}}}{8} + \frac{\cos\left(t\sqrt{2}\right)}{4}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 28

```
DSolve[{y''''[t]-4*y[t]==0,{y[0]==1,y'[0]==0,y''[0]==1,y'''[0]==0}},y[t],t,IncludeSingularSol
```

$$y(t) \to \frac{1}{4}\left(\cos\left(\sqrt{2}t\right) + 3\cosh\left(\sqrt{2}t\right)\right)$$

3.8 problem 15

Internal problem ID [840]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + \omega^2 y - \cos(2t) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve([diff(y(t),t$2)+omega^2*y(t)=cos(2*t),y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\cos(\omega t)\,\omega^2 - 5\cos(\omega t) + \cos(2t)}{\omega^2 - 4}$$

✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 28

```
DSolve[{y''[t]+w^2*y[t]==Cos[2*t],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to \frac{(w^2 - 5)\cos(tw) + \cos(2t)}{w^2 - 4}$$

3.9 problem 16

Internal problem ID [841]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' + 2y - e^{-t} = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)+2*y(t)=exp(-t),y(0) = 0, D(y)(0) = 1],y(t), singsol=all
```

$$y(t) = \frac{e^{-t}}{5} + \frac{(-\cos(t) + 7\sin(t))\, e^{t}}{5}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 27

```
DSolve[{y''[t]-2*y'[t]+2*y[t]==Exp[-t],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to \frac{1}{5}\left(e^{-t} - e^{t}(\cos(t) - 7\sin(t))\right)$$

3.10 problem 17

Internal problem ID [842]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y - \left(\begin{cases} 1 & 0 \leq t < \pi \\ 0 & \pi \leq t < \infty \end{cases} \right) = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 30

```
dsolve([diff(y(t),t$2)+4*y(t)=piecewise(0<=t and t<Pi,1,Pi<=t and t<infinity,0),y(0) = 1, D(y
```

$$y(t) = \begin{cases} \cos(2t) & t < 0 \\ \frac{1}{4} + \frac{3\cos(2t)}{4} & t < \pi \\ \cos(2t) & \pi \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 31

```
DSolve[{y''[t]+4*y[t]==Piecewise[{{1,0<t<Pi},{0,Pi<=t<Infinity}}],{y[0]==1,y'[0]==0}},y[t],t,
```

$$y(t) \to \begin{cases} \cos(2t) & t > \pi \lor t \leq 0 \\ \frac{1}{4}(3\cos(2t) + 1) & \text{True} \end{cases}$$

3.11 problem 18

Internal problem ID [843]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y - \left(\begin{cases} 1 & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases} \right) = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 38

```
dsolve([diff(y(t),t$2)+4*y(t)=piecewise(0<=t and t<1,1,1<=t and t<infinity,0),y(0) = 0, D(y)(
```

$$y(t) = \frac{\left(\begin{cases} 0 & t < 0 \\ 1 - \cos(2t) & t < 1 \\ \cos(2t-2) - \cos(2t) & 1 \leq t \end{cases} \right)}{4}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 39

```
DSolve[{y''[t]+4*y[t]==Piecewise[{{1,0<t<1},{0,1<=t<Infinity}}],{y[0]==0,y'[0]==0}},y[t],t,In
```

$$y(t) \to \begin{cases} 0 & t \leq 0 \\ \frac{\sin^2(t)}{2} & 0 < t \leq 1 \\ -\frac{1}{2}\sin(1)\sin(1-2t) & \text{True} \end{cases}$$

3.12 problem 19

Internal problem ID [844]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.2, The Laplace Transform. Solution of Initial Value Problems. page 255
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - \left(\begin{cases} t & 0 \leq t < 1 \\ -t + 2 & 1 \leq t < 2 \\ 0 & 2 \leq t < \infty \end{cases} \right) = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 59

```
dsolve([diff(y(t),t$2)+y(t)=piecewise(0<=t and t<1,t,1<=t and t<2,2-t,2<=t and t<infinity,0),
```

$$y(t) = \cos(t) + \left(\begin{cases} 0 & t < 0 \\ -\sin(t) + t & t < 1 \\ 2\sin(t-1) - t - \sin(t) + 2 & t < 2 \\ 2\sin(t-1) - \sin(t) - \sin(t-2) & 2 \leq t \end{cases} \right)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 68

```
DSolve[{y''[t]+y[t]==Piecewise[{{t,0<t<1},{2-t,1<=t<2},{0,2<=t<Infinity}}],{y[0]==1,y'[0]==0}
```

$$y(t) \to \begin{cases} \cos(t) & t \leq 0 \\ \cos(t) - 4\sin^2\left(\frac{1}{2}\right)\sin(1-t) & t > 2 \\ t + \cos(t) - \sin(t) & 0 < t \leq 1 \\ -t + \cos(t) - 2\sin(1-t) - \sin(t) + 2 & \text{True} \end{cases}$$

4 Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268

4.1 problem 1

Internal problem ID [845]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - \left(\begin{cases} 1 & 0 \leq t < 3\pi \\ 0 & 3\pi \leq t < \infty \end{cases}\right) = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 32

```
dsolve([diff(y(t),t$2)+y(t)=piecewise(0<=t and t<3*Pi,1,3*Pi<=t and t<infinity,0),y(0) = 0, D
```

$$y(t) = \sin(t) - \left(\begin{cases} 0 & t < 0 \\ \cos(t) - 1 & t < 3\pi \\ 2\cos(t) & 3\pi \leq t \end{cases}\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 34

```
DSolve[{y''[t]+y[t]==Piecewise[{{1,0<=t<3*Pi},{0,3*Pi<=t<Infinity}}],{y[0]==0,y'[0]==1}},y[t]
```

$$y(t) \to \begin{cases} \sin(t) & t \leq 0 \\ \sin(t) - 2\cos(t) & t > 3\pi \\ -\cos(t) + \sin(t) + 1 & \text{True} \end{cases}$$

4.2 problem 2

Internal problem ID [846]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 2y - \left(\begin{cases} 1 & \pi \le t < 2\pi \\ 0 & \text{otherwise} \end{cases}\right) = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.5 (sec). Leaf size: 76

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=piecewise(Pi<=t and t<2*Pi,1,true,0),y(0) = 0, D
```

$$y(t) = \begin{cases} e^{-t}\sin(t) & t < \pi \\ \frac{(\cos(t)+\sin(t))e^{\pi-t}}{2} + e^{-t}\sin(t) + \frac{1}{2} & t < 2\pi \\ \frac{e^{-t}((\cos(t)+\sin(t))e^{2\pi}+(\cos(t)+\sin(t))e^{\pi}+2\sin(t))}{2} & 2\pi \le t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 82

```
DSolve[{y''[t]+2*y'[t]+2*y[t]==Piecewise[{{1,Pi<=t<2*Pi},{0,True}}],{y[0]==0,y'[0]==1}},y[t],
```

$$y(t) \to \left\{ \begin{array}{cc} e^{-t}\sin(t) & t \le \pi \\ \frac{1}{2}e^{-t}(2\sin(t) + e^t + e^{\pi}(\cos(t) + \sin(t))) & \pi < t \le 2\pi \\ \frac{1}{2}e^{-t}(2\sin(t) + e^{\pi}(1 + e^{\pi})(\cos(t) + \sin(t))) & \text{True} \end{array} \right.$$

4.3 problem 3

Internal problem ID [847]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y - \sin(t) + \text{Heaviside}(-2\pi + t)\sin(t) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)+4*y(t)=sin(t)-Heaviside(t-2*Pi)*sin(t-2*Pi),y(0) = 0, D(y)(0) = 0],y(t
```

$$y(t) = \frac{\sin(t)(-1 + \text{Heaviside}(-2\pi + t))(\cos(t) - 1)}{3}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 27

```
DSolve[{y''[t]+4*y[t]==Sin[t]-UnitStep[t-2*Pi]*Sin[t-2*Pi],{y[0]==0,y'[0]==0}},y[t],t,Include
```

$$y(t) \to \frac{2}{3}\theta(2\pi - t)\sin^2\left(\frac{t}{2}\right)\sin(t)$$

4.4 problem 4

Internal problem ID [848]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y - \left(\begin{cases} 1 & 0 \le t < 10 \\ 0 & \text{otherwise} \end{cases}\right) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 56

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=piecewise(0<=t and t<10,1,true,0),y(0) = 0, D(y)
```

$$y(t) = \frac{\left(\begin{cases} 0 & t < 0 \\ 1 - 2\,\mathrm{e}^{-t} + \mathrm{e}^{-2t} & t < 10 \\ 2\,\mathrm{e}^{10-t} - \mathrm{e}^{-2t+20} - 2\,\mathrm{e}^{-t} + \mathrm{e}^{-2t} & 10 \le t \end{cases}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 61

```
DSolve[{y''[t]+3*y'[t]+2*y[t]==Piecewise[{{1,0<=t<10},{0,True}}],{y[0]==0,y'[0]==0}},y[t],t,I
```

$$y(t) \to \begin{cases} 0 & t \leq 0 \\ \frac{1}{2}e^{-2t}(-1+e^t)^2 & 0 < t \leq 10 \\ \frac{1}{2}e^{-2t}(-1+e^{10})\left(-1-e^{10}+2e^t\right) & \text{True} \end{cases}$$

4.5 problem 5

Internal problem ID [849]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y' + \frac{5y}{4} - t + \text{Heaviside}\left(t - \frac{\pi}{2}\right)\left(t - \frac{\pi}{2}\right) = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 63

```
dsolve([diff(y(t),t$2)+diff(y(t),t)+5/4*y(t)=t-Heaviside(t-Pi/2)*(t-Pi/2),y(0) = 0, D(y)(0) =
```

$$y(t) = -\frac{16}{25} - \frac{12\,\text{Heaviside}\left(t-\frac{\pi}{2}\right)\left(\cos(t) + \frac{4\sin(t)}{3}\right)\mathrm{e}^{-\frac{t}{2}+\frac{\pi}{4}}}{25} + \frac{2(8-10t+5\pi)\,\text{Heaviside}\left(t-\frac{\pi}{2}\right)}{25} + \frac{4(4\cos(t) - 3\sin(t))\,\mathrm{e}^{-\frac{t}{2}}}{25} + \frac{4t}{5}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 87

```
DSolve[{y''[t]+y'[t]+5/4*y[t]==t-UnitStep[t-Pi/2]*(t-Pi/2),{y[0]==0,y'[0]==0}},y[t],t,Include
```

$$y(t) \to \begin{cases} \frac{4}{25}\left(5t + e^{-t/2}(4\cos(t) - 3\sin(t)) - 4\right) & 2t \leq \pi \\ \frac{2}{25}e^{-t/2}\left(8\cos(t) - 6\sin(t) - 2e^{\pi/4}(3\cos(t) + 4\sin(t)) + 5e^{t/2}\pi\right) & \text{True} \end{cases}$$

4.6 problem 6

Internal problem ID [850]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y' + \frac{5y}{4} - \left(\begin{cases} \sin(t) & 0 \le t < \pi \\ 0 & \text{otherwise} \end{cases} \right) = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.453 (sec). Leaf size: 59

```
dsolve([diff(y(t),t$2)+diff(y(t),t)+5/4*y(t)=piecewise(0<=t and t<Pi,sin(t),true,0),y(0) = 0,
```

$$y(t) = \frac{4 \left(\begin{cases} 0 & t < 0 \\ (4\cos(t) + \sin(t))\, e^{-\frac{t}{2}} - 4\cos(t) + \sin(t) & t < \pi \\ (4\cos(t) + \sin(t)) \left(e^{-\frac{t}{2}} - e^{-\frac{t}{2} + \frac{\pi}{2}} \right) & \pi \le t \end{cases} \right)}{17}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 72

```
DSolve[{y''[t]+y'[t]+5/4*y[t]==Piecewise[{{Sin[t],0<=t<Pi},{0,True}}],{y[0]==0,y'[0]==0}},y[t
```

$$y(t) \to \begin{cases} 0 & t \leq 0 \\ \frac{4}{17}\left(-4\cos(t) + \sin(t) + e^{-t/2}(4\cos(t) + \sin(t))\right) & 0 < t \leq \pi \\ -\frac{4}{17}e^{-t/2}\left(-1 + e^{\pi/2}\right)(4\cos(t) + \sin(t)) & \text{True} \end{cases}$$

4.7 problem 7

Internal problem ID [851]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y - \text{Heaviside}(-\pi + t) + \text{Heaviside}(-3\pi + t) = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve([diff(y(t),t$2)+4*y(t)=Heaviside(t-Pi)-Heaviside(t-3*Pi),y(0) = 0, D(y)(0) = 0],y(t),
```

$$y(t) = \frac{(-1 + \cos(2t))(-\text{Heaviside}(-\pi + t) + \text{Heaviside}(-3\pi + t))}{4}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 25

```
DSolve[{y''[t]+4*y[t]==UnitStep[t-Pi]-UnitStep[t-3*Pi],{y[0]==0,y'[0]==0}},y[t],t,IncludeSing
```

$$y(t) \to \begin{cases} \frac{\sin^2(t)}{2} & \pi < t \leq 3\pi \\ 0 & \text{True} \end{cases}$$

4.8 problem 8

Internal problem ID [852]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268
Problem number: 8.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 5y'' + 4y - 1 + \text{Heaviside}\,(-\pi + t) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 0, y'''(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 27

```
dsolve([diff(y(t),t$4)+5*diff(y(t),t$2)+4*y(t)=1-Heaviside(t-Pi),y(0) = 0, D(y)(0) = 0, (D@@2
```

$$y(t) = -\frac{(\cos(t) + 1)^2 \text{Heaviside}\,(-\pi + t)}{6} + \frac{(\cos(t) - 1)^2}{6}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 29

```
DSolve[{y''''[t]+5*y''[t]+4*y[t]==1-UnitStep[t-Pi],{y[0]==0,y'[0]==0,y''[0]==0,y'''[0]==0}},y
```

$$y(t) \to \begin{cases} \frac{2}{3}\sin^4\left(\frac{t}{2}\right) & t \leq \pi \\ -\frac{2\cos(t)}{3} & \text{True} \end{cases}$$

4.9 problem 11(b)

Internal problem ID [853]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268
Problem number: 11(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$u'' + \frac{u'}{4} + u - k\left(\text{Heaviside}\left(t - \frac{3}{2}\right) - \text{Heaviside}\left(t - \frac{5}{2}\right)\right) = 0$$

With initial conditions

$$[u(0) = 0, u'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 117

```
dsolve([diff(u(t),t$2)+1/4*diff(u(t),t)+u(t)=k*(Heaviside(t-3/2)-Heaviside(t-5/2)),u(0) = 0,
```

$u(t) =$

$$-\frac{k\left(\text{Heaviside}\left(t-\frac{3}{2}\right)\sqrt{7}\,\mathrm{e}^{-\frac{t}{8}+\frac{3}{16}}\sin\left(\frac{3\sqrt{7}\,(2t-3)}{16}\right) - \text{Heaviside}\left(t-\frac{5}{2}\right)\sqrt{7}\,\mathrm{e}^{-\frac{t}{8}+\frac{5}{16}}\sin\left(\frac{3\sqrt{7}\,(2t-5)}{16}\right) - 21\,\text{Hea}\right.}{}$$

✓ Solution by Mathematica

Time used: 0.105 (sec). Leaf size: 182

```
DSolve[{u''[t]+1/4*u'[t]+u[t]==k*(UnitStep[t-3/2]-UnitStep[t-5/2]),{u[0]==0,u'[0]==0}},u[t],t
```

$u(t)$

$$\to \left\{ \begin{array}{c} \frac{1}{21}e^{\frac{3}{16}-\frac{t}{8}}\left(\sqrt{7}\sin\left(\frac{3}{16}\sqrt{7}(3-2t)\right) - 21\cos\left(\frac{3}{16}\sqrt{7}(3-2t)\right)\right)k + k \\ \frac{1}{21}e^{\frac{3}{16}-\frac{t}{8}}k\left(-21\cos\left(\frac{3}{16}\sqrt{7}(3-2t)\right) + 21\sqrt[8]{e}\cos\left(\frac{3}{16}\sqrt{7}(5-2t)\right) + \sqrt{7}\left(\sin\left(\frac{3}{16}\sqrt{7}(3-2t)\right) - \sqrt[8]{e}\sin\left(\right.\right.\right. \end{array} \right.$$

4.10 problem 11(c) k=1/2

Internal problem ID [854]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268
Problem number: 11(c) k=1/2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$u'' + \frac{u'}{4} + u - \frac{\text{Heaviside}\left(t-\frac{3}{2}\right)}{2} + \frac{\text{Heaviside}\left(t-\frac{5}{2}\right)}{2} = 0$$

With initial conditions

$$[u(0) = 0, u'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 115

```
dsolve([diff(u(t),t$2)+1/4*diff(u(t),t)+u(t)=1/2*(Heaviside(t-3/2)-Heaviside(t-5/2)),u(0) = 0
```

$$u(t) = \frac{\text{Heaviside}\left(t-\frac{3}{2}\right)}{2} - \frac{\text{Heaviside}\left(t-\frac{5}{2}\right)}{2} + \frac{\text{Heaviside}\left(t-\frac{5}{2}\right)\sqrt{7}\,\mathrm{e}^{-\frac{t}{8}+\frac{5}{16}}\sin\left(\frac{3\sqrt{7}\,(2t-5)}{16}\right)}{42} - \frac{\text{Heaviside}\left(t-\frac{3}{2}\right)\sqrt{7}\,\mathrm{e}^{-\frac{t}{8}+\frac{3}{16}}\sin\left(\frac{3\sqrt{7}\,(2t-3)}{16}\right)}{42} + \frac{\text{Heaviside}\left(t-\frac{5}{2}\right)\mathrm{e}^{-\frac{t}{8}+\frac{5}{16}}\cos\left(\frac{3\sqrt{7}\,(2t-5)}{16}\right)}{2} - \frac{\text{Heaviside}\left(t-\frac{3}{2}\right)\mathrm{e}^{-\frac{t}{8}+\frac{3}{16}}\cos\left(\frac{3\sqrt{7}\,(2t-3)}{16}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 182

```
DSolve[{u''[t]+1/4*u'[t]+u[t]==1/2*(UnitStep[t-3/2]-UnitStep[t-5/2]),{u[0]==0,u'[0]==0}},u[t]
```

$u(t)$

$$\rightarrow \{ \quad \begin{array}{c} \frac{1}{42}e^{\frac{3}{16}-\frac{t}{8}}\left(\sqrt{7}\sin\left(\frac{3}{16}\sqrt{7}(3-2t)\right)-21\cos\left(\frac{3}{16}\sqrt{7}(3-2t)\right)\right)+\frac{1}{2} \\ \frac{1}{42}e^{\frac{3}{16}-\frac{t}{8}}\left(-21\cos\left(\frac{3}{16}\sqrt{7}(3-2t)\right)+21\sqrt[8]{e}\cos\left(\frac{3}{16}\sqrt{7}(5-2t)\right)+\sqrt{7}\left(\sin\left(\frac{3}{16}\sqrt{7}(3-2t)\right)-\sqrt[8]{e}\sin\left(\frac{3}{16}\right.\right.\right. \end{array}$$

4.11 problem 12

Internal problem ID [855]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.4, The Laplace Transform. Differential equations with discontinuous forcing functions. page 268
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$u'' + \frac{u'}{4} + u - \frac{\text{Heaviside}\,(t-5)\,(t-5) - \text{Heaviside}\,(t-5-k)\,(t-5-k)}{k} = 0$$

With initial conditions

$$[u(0) = 0, u'(0) = 0]$$

✓ Solution by Maple

Time used: 0.453 (sec). Leaf size: 182

```
dsolve([diff(u(t),t$2)+1/4*diff(u(t),t)+u(t)=1/k*(Heaviside(t-5)*(t-5)-Heaviside(t-(5+k))*(t-
```

$$u(t) = \frac{-31(\text{Heaviside}\,(5+k) + \text{Heaviside}\,(t-5-k) - 1)\left(\left(\left(\sin\left(\frac{15\sqrt{7}}{8}\right)\sqrt{7} + \frac{21\cos\left(\frac{15\sqrt{7}}{8}\right)}{31}\right)\cos\left(\frac{3\sqrt{7}\,k}{8}\right) + \sin\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 7.337 (sec). Leaf size: 436

```
DSolve[{u''[t]+1/4*u'[t]+u[t]==1/k*(UnitStep[t-5]*(t-5)-UnitStep[t-(5+k)]*(t-(5+k)) ),{u[0]==
```

$u(t)$

$$\rightarrow \frac{e^{-t/8}\left(-21(4k+21)\cos\left(\frac{3\sqrt{7}t}{8}\right)+e^{\frac{k+5}{8}}\left(21\cos\left(\frac{3}{8}\sqrt{7}(k-t+5)\right)+31\sqrt{7}\sin\left(\frac{3}{8}\sqrt{7}(k-t+5)\right)\right)+\sqrt{7}(11-4k)\sin\left(\frac{3\sqrt{7}t}{8}\right)+\left(21e^{t/8}(4t-21)+e^{5/8}\left(2\right.\right.\right.}{}$$

$u(t)$

$$\rightarrow \frac{e^{-t/8}\left(\left(3\sqrt{7}e^{t/8}(4t-21)+e^{5/8}\left(3\sqrt{7}\cos\left(\frac{3}{8}\sqrt{7}(t-5)\right)-31\sin\left(\frac{3}{8}\sqrt{7}(t-5)\right)\right)\right)\theta(t-5)+\left(-3\sqrt{7}e^{t/8}(-4k+4t-21)-e^{\frac{k+5}{8}}\left(3\sqrt{7}\cos\left(\frac{3}{8}\sqrt{7}(k-t+\right.\right.\right.\right.}{12\sqrt{7}k}$$

5 Chapter 6.5, The Laplace Transform. Impulse functions. page 273

5.1 problem 1

Internal problem ID [856]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 2y - (\delta(-\pi + t)) = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=Dirac(t-Pi),y(0) = 1, D(y)(0) = 0],y(t), singsol
```

$$y(t) = -\sin(t)\,\mathrm{Heaviside}(-\pi + t)\,\mathrm{e}^{\pi - t} + \mathrm{e}^{-t}(\cos(t) + \sin(t))$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 29

```
DSolve[{y''[t]+2*y'[t]+2*y[t]==DiracDelta[t-Pi],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSol
```

$$y(t) \to e^{-t}(-e^{\pi}\theta(t - \pi)\sin(t) + \sin(t) + \cos(t))$$

5.2 problem 2

Internal problem ID [857]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y - (\delta(-\pi + t)) + \delta(-2\pi + t) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve([diff(y(t),t$2)+4*y(t)=Dirac(t-Pi)-Dirac(t-2*Pi),y(0) = 0, D(y)(0) = 0],y(t), singsol=
```

$$y(t) = -\frac{(\text{Heaviside}\,(-2\pi + t) - \text{Heaviside}\,(-\pi + t))\sin\,(2t)}{2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 25

```
DSolve[{y''[t]+4*y[t]==DiracDelta[t-Pi]-DiracDelta[t-2*Pi],{y[0]==0,y'[0]==0}},y[t],t,Include
```

$$y(t) \to (\theta(t - \pi) - \theta(t - 2\pi))\sin(t)\cos(t)$$

5.3 problem 3

Internal problem ID [858]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y - (\delta(t-5)) - \text{Heaviside}(-10+t) = 0}$$

With initial conditions

$$\left[y(0) = 0, y'(0) = \frac{1}{2}\right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 70

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=Dirac(t-5)+Heaviside(t-10),y(0) = 0, D(y)(0) = 1
```

$$y(t) = -\frac{e^{-2t}}{2} - \text{Heaviside}(t-5)\,e^{-2t+10} + \text{Heaviside}(t-5)\,e^{-t+5} + \frac{\text{Heaviside}(-10+t)}{2} - \text{Heaviside}(-10+t)\,e^{10-t} + \frac{\text{Heaviside}(-10+t)\,e^{-2t+20}}{2} + \frac{e^{-t}}{2}$$

✓ Solution by Mathematica

Time used: 0.161 (sec). Leaf size: 71

```
DSolve[{y''[t]+3*y'[t]+2*y[t]==DiracDelta[t-5]+UnitStep[t-10],{y[0]==0,y'[0]==1/2}},y[t],t,In
```

$$y(t) \to \frac{1}{2}e^{-2t}\left(2e^5\left(e^t - e^5\right)\theta(t-5) + \left(e^{10} - e^t\right)^2(-\theta(10-t)) + e^t + e^{2t} - 2e^{t+10} + e^{20} - 1\right)$$

5.4 problem 4

Internal problem ID [859]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 3y - \sin(t) - (\delta(-3\pi + t)) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 54

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+3*y(t)=sin(t)+Dirac(t-3*Pi),y(0) = 0, D(y)(0) = 0],y(t)
```

$$y(t) = \frac{e^{-t} \cos(t\sqrt{2})}{4} + \frac{\sqrt{2}\, e^{3\pi - t}\, \mathrm{Heaviside}(-3\pi + t) \sin\left(\sqrt{2}\,(-3\pi + t)\right)}{2} + \frac{\sin(t)}{4} - \frac{\cos(t)}{4}$$

✓ Solution by Mathematica

Time used: 0.796 (sec). Leaf size: 80

```
DSolve[{y''[t]+2*y'[t]+3*y[t]==Sin[t]+DiracDelta[t-3*Pi],{y[0]==0,y'[0]==1/2}},y[t],t,Include
```

$$y(t) \to \frac{1}{4}e^{-t}\left(-2\sqrt{2}e^{3\pi}\theta(t-3\pi)\sin\left(\sqrt{2}(3\pi - t)\right) + \sqrt{2}\sin\left(\sqrt{2}t\right) + \cos\left(\sqrt{2}t\right) + e^{t}(\sin(t) - \cos(t))\right)$$

5.5 problem 5

Internal problem ID [860]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - (\delta(-2\pi + t)) \cos(t) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)+y(t)=Dirac(t-2*Pi)*cos(t),y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \sin(t) (\text{Heaviside}(-2\pi + t) + 1)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 16

```
DSolve[{y''[t]+y[t]==DiracDelta[t-2*Pi]*Cos[t],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolu
```

$$y(t) \to (\theta(t - 2\pi) + 1) \sin(t)$$

5.6 problem 6

Internal problem ID [861]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y - 2\left(\delta\left(t - \frac{\pi}{4}\right)\right) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)+4*y(t)=2*Dirac(t-Pi/4),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -\operatorname{Heaviside}\left(t - \frac{\pi}{4}\right)\cos(2t)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 25

```
DSolve[{y''[t]+4*y[t]==2*DiracDelta[t-Pi/4],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutio
```

$$y(t) \to \sin(t)\cos(t) - \theta(4t - \pi)\cos(2t)$$

5.7 problem 7

Internal problem ID [862]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 2y - \cos(t) - \left(\delta\left(t - \frac{\pi}{2}\right)\right) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 47

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=cos(t)+Dirac(t-Pi/2),y(0) = 0, D(y)(0) = 0],y(t)
```

$$y(t) = -\cos(t)\,\mathrm{Heaviside}\left(t - \frac{\pi}{2}\right) \mathrm{e}^{-t+\frac{\pi}{2}} + \frac{(-\cos(t) - 3\sin(t))\,\mathrm{e}^{-t}}{5} + \frac{\cos(t)}{5} + \frac{2\sin(t)}{5}$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 50

```
DSolve[{y''[t]+2*y'[t]+2*y[t]==Cos[t]+DiracDelta[t-Pi/2],{y[0]==0,y'[0]==0}},y[t],t,IncludeSi
```

$$y(t) \to \frac{1}{5}e^{-t}\left(\left(-5e^{\pi/2}\theta(2t-\pi) + e^t - 1\right)\cos(t) + \left(2e^t - 3\right)\sin(t)\right)$$

5.8 problem 8

Internal problem ID [863]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273
Problem number: 8.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - y - (\delta(t-1)) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 0, y'''(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 29

```
dsolve([diff(y(t),t$4)-y(t)=Dirac(t-1),y(0) = 0, D(y)(0) = 0, (D@@2)(y)(0) = 0, (D@@3)(y)(0)
```

$$y(t) = \frac{\text{Heaviside}(t-1)\left(e^{t-1} - e^{-t+1} - 2\sin(t-1)\right)}{4}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 28

```
DSolve[{y''''[t]-y[t]==DiracDelta[t-1],{y[0]==0,y'[0]==0,y''[0]==0,y'''[0]==0}},y[t],t,Includ
```

$$y(t) \to \frac{1}{2}\theta(t-1)(\sin(1-t) - \sinh(1-t))$$

5.9 problem 10(a)

Internal problem ID [864]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273
Problem number: 10(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + \frac{y'}{2} + y - (\delta(t-1)) = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 28

```
dsolve([diff(y(t),t$2)+1/2*diff(y(t),t)+y(t)=Dirac(t-1),y(0) = 0, D(y)(0) = 0],y(t), singsol=
```

$$y(t) = \frac{4\,\text{Heaviside}\,(t-1)\,\mathrm{e}^{\frac{1}{4}-\frac{t}{4}}\sqrt{15}\,\sin\left(\frac{\sqrt{15}\,(t-1)}{4}\right)}{15}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 40

```
DSolve[{y''[t]+1/2*y'[t]+y[t]==DiracDelta[t-1],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolu
```

$$y(t) \to \frac{4e^{\frac{1}{4}-\frac{t}{4}}\theta(t-1)\sin\left(\frac{1}{4}\sqrt{15}(t-1)\right)}{\sqrt{15}}$$

5.10 problem 10(c)

Internal problem ID [865]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273
Problem number: 10(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + \frac{y'}{4} + y - (\delta(t-1)) = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve([diff(y(t),t$2)+1/4*diff(y(t),t)+y(t)=Dirac(t-1),y(0) = 0, D(y)(0) = 0],y(t), singsol=
```

$$y(t) = \frac{8 \operatorname{Heaviside}(t-1) \, \mathrm{e}^{\frac{1}{8}-\frac{t}{8}} \sqrt{7}\, \sin\left(\frac{3\sqrt{7}\,(t-1)}{8}\right)}{21}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 42

```
DSolve[{y''[t]+1/4*y'[t]+y[t]==DiracDelta[t-1],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolu
```

$$y(t) \to \frac{8e^{\frac{1}{8}-\frac{t}{8}}\theta(t-1)\sin\left(\frac{3}{8}\sqrt{7}(t-1)\right)}{3\sqrt{7}}$$

5.11 problem 12

Internal problem ID [866]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - \frac{\text{Heaviside}\,(t-4+k) - \text{Heaviside}\,(t-4-k)}{2k} = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 83

```
dsolve([diff(y(t),t$2)+y(t)=1/(2*k)*(Heaviside(t-(4-k)) -  Heaviside(t-(4+k)) ),y(0) = 0, D(y
```

$$y(t) = \frac{(\text{Heaviside}\,(4+k) + \text{Heaviside}\,(t-4-k) - 1)\cos(-t+4+k) - \text{Heaviside}\,(t-4-k) + (-\cos(t-4}{}$$

✓ Solution by Mathematica

Time used: 0.676 (sec). Leaf size: 181

```
DSolve[{y''[t]+y[t]==1/(2*k)*(UnitStep[t-(4-k)] -  UnitStep[t-(4+k)] ),{y[0]==0,y'[0]==0}},y[
```

$$y(t) \to \boxed{\frac{(\cos(k-t+4)-1)\theta(-k+t-4)-(\cos(-k-t+4)-1)\theta(k+t-4)}{2k} \text{ if } -4 < k < 4}$$

$$y(t) \to \boxed{\frac{\cos(-k-t+4)-\cos(t)+(\cos(k-t+4)-1)\theta(-k+t-4)-(\cos(-k-t+4)-1)\theta(k+t-4)}{2k} \text{ if } k > 4}$$

$$y(t) \to \boxed{\frac{-\cos(k-t+4)+\cos(t)+(\cos(k-t+4)-1)\theta(-k+t-4)-(\cos(-k-t+4)-1)\theta(k+t-4)}{2k} \text{ if } k < -4}$$

5.12 problem 19(a)

Internal problem ID [867]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273
Problem number: 19(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 2y - f(t) = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 43

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=f(t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \left(\sin(t)\left(\int_0^t f(\_z1)\cos(\_z1)\,\mathrm{e}^{\_z1}d\_z1\right) - \cos(t)\left(\int_0^t f(\_z1)\sin(\_z1)\,\mathrm{e}^{\_z1}d\_z1\right)\right)\mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 98

```
DSolve[{y''[t]+2*y'[t]+2*y[t]==f[t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to e^{-t}\left(\sin(t)\left(\int_1^t e^{K[1]}\cos(K[1])f(K[1])dK[1] - \int_1^0 e^{K[1]}\cos(K[1])f(K[1])dK[1]\right) + \cos(t)\left(\int_1^t -e^{K[2]}f(K[2])\sin(K[2])dK[2] - \int_1^0 -e^{K[2]}f(K[2])\sin(K[2])dK[2]\right)\right)$$

5.13 problem 19(b)

Internal problem ID [868]

Book: Elementary differential equations and boundary value problems, 11th ed., Boyce, DiPrima, Meade
Section: Chapter 6.5, The Laplace Transform. Impulse functions. page 273
Problem number: 19(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 2y - (\delta(-\pi + t)) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=Dirac(t-Pi),y(0) = 0, D(y)(0) = 0],y(t), singsol
```

$$y(t) = -\sin(t)\,\mathrm{Heaviside}(-\pi + t)\,\mathrm{e}^{\pi - t}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 22

```
DSolve[{y''[t]+2*y'[t]+2*y[t]==DiracDelta[t-Pi],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSol
```

$$y(t) \to -e^{\pi - t}\theta(t - \pi)\sin(t)$$