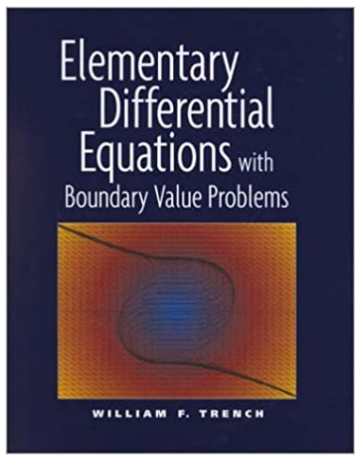
A Solution Manual For

Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001



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1 Chapter 1, Introduction. Section 1.2 Page 14

1.1 problem 2(a)

Internal problem ID [869]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x) = 2*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 16

```
DSolve[y'[x]== y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to 0$$

1.2 problem 2(b)

Internal problem ID [870]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 2(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y + y'x - x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x) +y(x)= x^2,y(x), singsol=all)
```

$$y(x) = \frac{\frac{x^3}{3} + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 19

```
DSolve[x*y'[x] +y[x]== x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{3} + \frac{c_1}{x}$$

1.3 problem 2(c)

Internal problem ID [871]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 2(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + 2yx - x = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x) +2*x*y(x)= x,y(x), singsol=all)
```

$$y(x) = \frac{1}{2} + \mathrm{e}^{-x^2} c_1$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 26

```
DSolve[y'[x] +2*x*y[x]== x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} + c_1 e^{-x^2}$$
$$y(x) \to \frac{1}{2}$$

1.4 problem 2(d)

Internal problem ID [872]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 2(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2y' + x(-1 + y^2) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(2*diff(y(x),x) +x*(y(x)^2-1)= 0,y(x), singsol=all)
```

$$y(x) = \tanh\left(\frac{x^2}{4} + \frac{c_1}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.233 (sec). Leaf size: 28

```
DSolve[2*y'[x] +x*(y[x]^2-1)== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tanh\left(\frac{1}{4}(x^2 - 4c_1)\right)$$
$$y(x) \to -1$$
$$y(x) \to 1$$

1.5 problem 2(e)

Internal problem ID [873]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 2(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (1 + y^2)\, x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) = x^2*(1+y(x)^2),y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{x^3}{3} + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.166 (sec). Leaf size: 30

```
DSolve[y'[x] == x^2*(1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(\frac{x^3}{3} + c_1\right)$$
$$y(x) \to -i$$
$$y(x) \to i$$

1.6 problem 3(a)

Internal problem ID [874]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 3(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = -x,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 15

```
DSolve[y'[x] == -x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{2} + c_1$$

1.7 problem 3(b)

Internal problem ID [875]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 3(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + \sin(x)\, x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x) = -x*sin(x),y(x), singsol=all)
```

$$y(x) = -\sin(x) + \cos(x)\, x + c_1$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 16

```
DSolve[y'[x] == -x*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sin(x) + x\cos(x) + c_1$$

1.8 problem 3(c)

Internal problem ID [876]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 3(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - x\ln(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x) = x*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{\ln(x)\, x^2}{2} - \frac{x^2}{4} + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 24

```
DSolve[y'[x] == x*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{4} + \frac{1}{2}x^2\log(x) + c_1$$

1.9 problem 4(a)

Internal problem ID [877]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 4(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + e^x x = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x) = -x*exp(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = -(x-1)\,e^x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 13

```
DSolve[{y'[x] == -x*Exp[x],y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^x(x-1)$$

1.10 problem 4(b)

Internal problem ID [878]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 4(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - x\sin\left(x^2\right) = 0}$$

With initial conditions

$$\left[y\left(\frac{\sqrt{2}\,\sqrt{\pi}}{2}\right) = 1\right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 12

```
dsolve([diff(y(x),x) = x*sin(x^2),y(1/2*2^(1/2)*Pi^(1/2)) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{\cos\left(x^2\right)}{2} + 1$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 15

```
DSolve[{y'[x] == x*Sin[x^2],y[Sqrt[Pi/2]]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 - \frac{\cos\left(x^2\right)}{2}$$

1.11 problem 4(c)

Internal problem ID [879]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 4(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \tan(x) = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = 3\right]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(x),x) = tan(x),y(1/4*Pi) = 3],y(x), singsol=all)
```

$$y(x) = -\ln(\cos(x)) + 3 - \frac{\ln(2)}{2}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 18

```
DSolve[{y'[x] == Tan[x],y[Pi/4]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log(\cos(x)) + 3 - \frac{\log(2)}{2}$$

1.12 problem 5(a)

Internal problem ID [880]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 5(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \cos(x) + y\tan(x) = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}\,\pi}{8}\right]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve([diff(y(x),x) = cos(x)-y(x)*tan(x),y(1/4*Pi) = 1/8*Pi*2^(1/2)],y(x), singsol=all)
```

$$y(x) = \cos(x)\, x$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 9

```
DSolve[{y'[x] ==Cos[x]-y[x]*Tan[x],y[Pi/4]==Pi/(4*Sqrt[2])},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to x\cos(x)$$

1.13 problem 5(b)

Internal problem ID [881]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 5(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{x^2 - 2x^2 y + 2}{x^3} = 0}$$

With initial conditions

$$\left[y(1) = \frac{3}{2}\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(y(x),x) = (x^2-2*x^2*y(x)+2)/x^3,y(1) = 3/2],y(x), singsol=all)
```

$$y(x) = \frac{\frac{x^2}{2} + 2\ln(x) + 1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 19

```
DSolve[{y'[x] ==(x^2-2*x^2*y[x]+2)/x^3,y[1]==3/2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x^2} + \frac{2\log(x)}{x^2} + \frac{1}{2}$$

1.14 problem 5(c)

Internal problem ID [882]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 5(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x(1 + y^2) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 10

```
dsolve([diff(y(x),x) = x*(1+y(x)^2),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.167 (sec). Leaf size: 13

```
DSolve[{y'[x] ==x*(1+y[x]^2),y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(\frac{x^2}{2}\right)$$

1.15 problem 5(d)

Internal problem ID [883]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 5(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{y(1+y)}{x} = 0}$$

With initial conditions

$$[y(1) = -2]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 11

```
dsolve([diff(y(x),x) = (- y(x)*(y(x)+1))/x,y(1) = -2],y(x), singsol=all)
```

$$y(x) = \frac{2}{-2+x}$$

✓ Solution by Mathematica

Time used: 0.229 (sec). Leaf size: 12

```
DSolve[{y'[x] ==(- y[x]*(y[x]+1))/x,y[1]==-2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{x-2}$$

1.16 problem 8(a)

Internal problem ID [884]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 8(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - a y^{\frac{a-1}{a}} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve(diff(y(x),x) = a*y(x)^( (a-1)/a),y(x), singsol=all)
```

$$y(x) = (c_1 + x)^a$$

✓ Solution by Mathematica

Time used: 0.771 (sec). Leaf size: 28

```
DSolve[y'[x] ==a*y[x]^( (a-1)/a),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(x + \frac{c_1}{a}\right)^a$$
$$y(x) \to 0^{\frac{a}{a-1}}$$

1.17 problem 9

Internal problem ID [885]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - |y| - 1 = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 19

```
dsolve([diff(y(x),x) = abs(y(x))+1,y(0) = 0],y(x), singsol=all)
```

$$y(x) = e^{x} - 1$$
$$y(x) = 1 - e^{-x}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x] ==Abs[y[x]]+1,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

{}

1.18 problem 10(a)

Internal problem ID [886]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 1, Introduction. Section 1.2 Page 14
Problem number: 10(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y' + \frac{x}{2} + 1 - \frac{\sqrt{x^2 + 4x + 4y}}{2} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = 1/2*(-(x+2)+sqrt(x^2+4*x+4*y(x))),y(x), singsol=all)
```

$$x - 2\sqrt{y(x) + \frac{x^2}{4} + x} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.788 (sec). Leaf size: 46

```
DSolve[y'[x] ==1/2*(-(x+2)+Sqrt[x^2+4*x+4*y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(-2x + e^{c_1}(2x + 2 + e^{c_1}) + 1)$$
$$y(x) \to 1$$
$$y(x) \to \frac{1}{4}(1 - 2x)$$

2 Chapter 2, First order equations. Linear first order. Section 2.1 Page 41

2.1 problem 1

Internal problem ID [887]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + ay = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) + a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-ax}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 19

```
DSolve[y'[x] + a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ax}$$
$$y(x) \to 0$$

2.2 problem 2

Internal problem ID [888]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + 3x^2 y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) + 3*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x^3}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 20

```
DSolve[y'[x] +3*x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x^3}$$
$$y(x) \to 0$$

2.3 problem 3

Internal problem ID [889]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x + \ln(x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x) + ln(x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{\ln(x)^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 23

```
DSolve[x*y'[x] +Log[x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{1}{2}\log^2(x)}$$
$$y(x) \to 0$$

2.4 problem 4

Internal problem ID [890]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*diff(y(x),x) + 3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^3}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 16

```
DSolve[x*y'[x] +3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^3}$$
$$y(x) \to 0$$

2.5 problem 5

Internal problem ID [891]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x^2 + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x^2*diff(y(x),x) + y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{1}{x}}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 18

```
DSolve[x^2*y'[x] +y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{1}{x}}$$
$$y(x) \to 0$$

2.6 problem 6

Internal problem ID [892]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{(x+1)\,y}{x} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x) + ((1+x)/x)*y(x)=0,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{e^{1-x}}{x}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 16

```
DSolve[{y'[x] +((1+x)/x)*y[x]==0,y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{1-x}}{x}$$

2.7 problem 7

Internal problem ID [893]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x + \left(1 + \frac{1}{\ln(x)}\right)y = 0}$$

With initial conditions

$$[y(\mathrm{e}) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([x*diff(y(x),x) + (1+1/ln(x))*y(x)=0,y(exp(1)) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}}{x\ln(x)}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 18

```
DSolve[{y'[x] +(1+1/Log[x])*y[x]==0,y[Exp[1]]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\text{LogIntegral}(x)+\text{LogIntegral}(e)-x+e}$$

2.8 problem 8

Internal problem ID [894]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x + (1 + x\cot(x))\, y = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 2\right]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([x*diff(y(x),x) + (1+x*cot(x))*y(x)=0,y(1/2*Pi) = 2],y(x), singsol=all)
```

$$y(x) = \frac{\csc(x)\,\pi}{x}$$

✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 65

```
DSolve[{y'[x] +(1+x*Cot[x])*y[x]==0,y[Pi/2]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2^{1+\frac{\pi}{2}} \left(1 - e^{2ix}\right)^{-x} \exp\left(-\frac{1}{12} i \left(-6\,\text{PolyLog}\left(2, e^{2ix}\right) - 6x(x + 2i) + \pi(\pi + 6i)\right)\right)$$

2.9 problem 9

Internal problem ID [895]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2xy}{x^2+1} = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x) - (2*x)/(1+x^2)*y(x)=0,y(0) = 2],y(x), singsol=all)
```

$$y(x) = 2x^2 + 2$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 12

```
DSolve[{y'[x] -(2*x)/(1+x^2)*y[x]==0,y[0]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2(x^2 + 1)$$

2.10 problem 10

Internal problem ID [896]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{ky}{x} = 0}$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x) +k/x*y(x)=0,y(1) = 3],y(x), singsol=all)
```

$$y(x) = 3x^{-k}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 12

```
DSolve[{y'[x] +k/x*y[x]==0,y[1]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x^{-k}$$

2.11 problem 11

Internal problem ID [897]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \tan(kx)\, y = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve([diff(y(x),x) +tan(k*x)*y(x)=0,y(0) = 2],y(x), singsol=all)
```

$$y(x) = 2\left(\sec(kx)^2\right)^{-\frac{1}{2k}}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 15

```
DSolve[{y'[x] +Tan[k*x]*y[x]==0,y[0]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\sqrt[k]{\cos(kx)}$$

2.12 problem 12

Internal problem ID [898]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + 3y - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) +3*y(x)=1,y(x), singsol=all)
```

$$y(x) = \frac{1}{3} + \mathrm{e}^{-3x} c_1$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 24

```
DSolve[y'[x] +3*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} + c_1 e^{-3x}$$
$$y(x) \to \frac{1}{3}$$

2.13 problem 13

Internal problem ID [899]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' + \left(\frac{1}{x} - 1\right) y + \frac{2}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x) +(1/x-1)*y(x)=-2/x,y(x), singsol=all)
```

$$y(x) = \frac{2}{x} + \frac{e^x c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 17

```
DSolve[y'[x] +(1/x-1)*y[x]==-2/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2 + c_1 e^x}{x}$$

2.14 problem 14

Internal problem ID [900]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + 2yx - e^{-x^2}x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x) +2*x*y(x)=x*exp(-x^2),y(x), singsol=all)
```

$$y(x) = \left(\frac{x^2}{2} + c_1\right) e^{-x^2}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 24

```
DSolve[y'[x] +2*x*y[x]==x*Exp[-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x^2} (x^2 + 2c_1)$$

2.15 problem 15

Internal problem ID [901]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{2xy}{x^2+1} - \frac{e^{-x^2}}{x^2+1} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) +(2*x)/(1+x^2)*y(x)=exp(-x^2)/(1+x^2),y(x), singsol=all)
```

$$y(x) = \frac{\frac{\sqrt{\pi}\,\mathrm{erf}(x)}{2} + c_1}{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 28

```
DSolve[y'[x] +(2*x)/(1+x^2)*y[x]==Exp[-x^2]/(1+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\pi}\mathrm{erf}(x) + 2c_1}{2x^2+2}$$

2.16 problem 16

Internal problem ID [902]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{x} - \frac{7}{x^2} - 3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x) +1/x*y(x)=7/x^2+3,y(x), singsol=all)
```

$$y(x) = \frac{\frac{3x^2}{2} + 7\ln(x) + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 24

```
DSolve[y'[x] +1/x*y[x]==7/x^2+3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x}{2} + \frac{7\log(x)}{x} + \frac{c_1}{x}$$

2.17 problem 17

Internal problem ID [903]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' + \frac{4y}{x-1} - \frac{1}{(x-1)^5} - \frac{\sin(x)}{(x-1)^4} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) +4/(x-1)*y(x)=1/(x-1)^5+sin(x)/(x-1)^4,y(x), singsol=all)
```

$$y(x) = \frac{-\cos(x) + \ln(x-1) + c_1}{(x-1)^4}$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 22

```
DSolve[y'[x] +4/(x-1)*y[x]==1/(x-1)^5+Sin[x]/(x-1)^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log(x-1) - \cos(x) + c_1}{(x-1)^4}$$

2.18 problem 18

Internal problem ID [904]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y(2x^2 + 1) - x^3 \mathrm{e}^{-x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x) +(1+2*x^2)*y(x)=x^3*exp(-x^2),y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{x^4}{4} + c_1\right) \mathrm{e}^{-x^2}}{x}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 27

```
DSolve[x*y'[x] +(1+2*x^2)*y[x]==x^3*Exp[-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x^2}(x^4 + 4c_1)}{4x}$$

2.19 problem 19

Internal problem ID [905]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + 2y - \frac{2}{x^2} - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x) +2*y(x)=2/x^2+1,y(x), singsol=all)
```

$$y(x) = \frac{\frac{x^2}{2} + 2\ln(x) + c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 22

```
DSolve[x*y'[x] +2*y[x]==2/x^2+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2\log(x)}{x^2} + \frac{c_1}{x^2} + \frac{1}{2}$$

2.20 problem 20

Internal problem ID [906]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\tan(x) - \cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x) +tan(x)*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = (c_1 + x)\cos(x)$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 12

```
DSolve[y'[x] +Tan[x]*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_1)\cos(x)$$

2.21 problem 21

Internal problem ID [907]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$(x+1)\,y'+2y-\frac{\sin(x)}{x+1}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((1+x)*diff(y(x),x) +2*y(x)=sin(x)/(1+x),y(x), singsol=all)
```

$$y(x)=\frac{-\cos(x)+c_1}{(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 18

```
DSolve[(1+x)*y'[x] +2*y[x]==Sin[x]/(1+x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-\cos(x)+c_1}{(x+1)^2}$$

2.22 problem 22

Internal problem ID [908]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(-2+x)\,(x-1)\,y' - (4x-3)\,y - (-2+x)^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve((x-2)*(x-1)*diff(y(x),x) -(4*x-3)*y(x)=(x-2)^3,y(x), singsol=all)
```

$$y(x) = \frac{(x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32)\,c_1}{x-1} - \frac{x^3 - 6x^2 + 12x - 8}{2\,(x-1)}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 30

```
DSolve[(x-2)*(x-1)*y'[x] -(4*x-3)*y[x]==(x-2)^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x-2)^3\,(-1 + 2c_1(x-2)^2)}{2(x-1)}$$

2.23 problem 23

Internal problem ID [909]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + 2\sin(x)\cos(x)\,y - \mathrm{e}^{-\sin(x)^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x) +(2*sin(x)*cos(x))*y(x)=exp(-sin(x)^2),y(x), singsol=all)
```

$$y(x) = (c_1 + x)\,\mathrm{e}^{-\frac{1}{2}+\frac{\cos(2x)}{2}}$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 24

```
DSolve[y'[x] +(2*Sin[x]*Cos[x])*y[x]==Exp[-Sin[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(x + \sqrt{e}c_1\right) e^{-\sin^2(x)}$$

2.24 problem 24

Internal problem ID [910]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x^2 + 3yx - e^x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x) +3*x*y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \frac{(x-1)\,e^x + c_1}{x^3}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 19

```
DSolve[x^2*y'[x] +3*x*y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x(x-1) + c_1}{x^3}$$

2.25 problem 25

Internal problem ID [911]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 7y - e^{3x} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(y(x),x) +7*y(x)=exp(3*x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(e^{10x} - 1) e^{-7x}}{10}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 21

```
DSolve[{y'[x] +7*y[x]==Exp[3*x],y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{10} e^{-7x} \left(e^{10x} - 1\right)$$

2.26 problem 26

Internal problem ID [912]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y'+4yx-\frac{2}{x^2+1}=0}$$

With initial conditions

$$[y(0)=1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([(1+x^2)*diff(y(x),x)+4*x*y(x)=2/(1+x^2),y(0) = 1],y(x), singsol=all)
```

$$y(x)=\frac{1+2x}{(x^2+1)^2}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 18

```
DSolve[{(1+x^2)*y'[x]+4*x*y[x]==2/(1+x^2),y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2x+1}{(x^2+1)^2}$$

2.27 problem 27

Internal problem ID [913]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + 3y - \frac{2}{(x^2+1)\,x} = 0}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([x*diff(y(x),x)+3*y(x)=2/(x*(1+x^2)),y(-1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\ln(x^2+1) - \ln(2)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 19

```
DSolve[{x*y'[x]+3*y[x]==2/(x*(1+x^2)),y[-1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log\left(\frac{1}{2}(x^2+1)\right)}{x^3}$$

2.28 problem 28

Internal problem ID [914]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \cot(x)\, y - \cos(x) = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 1\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)+cot(x)*y(x)=cos(x),y(1/2*Pi) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)\cot(x)}{2} + \csc(x)$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 16

```
DSolve[{y'[x]+Cot[x]*y[x]==Cos[x],y[Pi/2]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \csc(x) - \frac{1}{2}\cos(x)\cot(x)$$

2.29 problem 29

Internal problem ID [915]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{x} - \frac{2}{x^2} - 1 = 0}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([diff(y(x),x)+y(x)/x=2/x^2+1,y(-1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{-4i\pi + x^2 + 4\ln(x) - 1}{2x}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 26

```
DSolve[{y'[x]+y[x]/x==2/x^2+1,y[-1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2 + 4\log(x) - 4i\pi - 1}{2x}$$

2.30 problem 30

Internal problem ID [916]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x-1)\,y'+3y-\frac{1}{(x-1)^3}-\frac{\sin(x)}{(x-1)^2}=0}$$

With initial conditions

$$[y(0)=1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([(x-1)*diff(y(x),x)+3*y(x)=1/(x-1)^3+sin(x)/(x-1)^2,y(0) = 1],y(x), singsol=all)
```

$$y(x)=\frac{-\cos(x)+\ln(x-1)-i\pi}{(x-1)^3}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 25

```
DSolve[{(x-1)*y'[x]+3*y[x]==1/(x-1)^3+Sin[x]/(x-1)^2,y[0]==1},y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{\log(x-1)-\cos(x)-i\pi}{(x-1)^3}$$

2.31 problem 31

Internal problem ID [917]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + 2y - 8x^2 = 0}$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x*diff(y(x),x)+2*y(x)=8*x^2,y(1) = 3],y(x), singsol=all)
```

$$y(x) = \frac{2x^4 + 1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 14

```
DSolve[{x*y'[x]+2*y[x]==8*x^2,y[1]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x^2 + \frac{1}{x^2}$$

2.32 problem 32

Internal problem ID [918]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 2y + x^2 = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([x*diff(y(x),x)-2*y(x)=-x^2,y(1) = 1],y(x), singsol=all)
```

$$y(x) = (-\ln(x) + 1)\, x^2$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 14

```
DSolve[{x*y'[x]-2*y[x]==-x^2,y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2(\log(x) - 1)$$

2.33 problem 33

Internal problem ID [919]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + 2yx - x = 0}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)+2*x*y(x)=x,y(0) = 3],y(x), singsol=all)
```

$$y(x) = \frac{1}{2} + \frac{5\,\mathrm{e}^{-x^2}}{2}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 20

```
DSolve[{y'[x]+2*x*y[x]==x,y[0]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{5e^{-x^2}}{2} + \frac{1}{2}$$

2.34 problem 34

Internal problem ID [920]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$(x-1)\,y' + 3y - \frac{1 + (x-1)\sec(x)^2}{(x-1)^3} = 0$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([(x-1)*diff(y(x),x)+3*y(x)= (1+(x-1)*sec(x)^2)/(x-1)^3,y(0) = -1],y(x), singsol=all)
```

$$y(x) = \frac{\ln(x-1) + \tan(x) + 1 - i\pi}{(x-1)^3}$$

✓ Solution by Mathematica

Time used: 0.169 (sec). Leaf size: 21

```
DSolve[{(x-1)*y'[x]+3*y[x]==(1+(x-1)*Sec[x]^2)/(x-1)^3,y[0]==-1},y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{\log(1-x) + \tan(x) + 1}{(x-1)^3}$$

2.35 problem 35

Internal problem ID [921]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(2+x)\,y'+4y-\frac{2x^2+1}{x\,(2+x)^3}=0}$$

With initial conditions

$$[y(-1)=2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([(x+2)*diff(y(x),x)+4*y(x)= (1+2*x^2)/(x*(x+2)^3),y(-1) = 2],y(x), singsol=all)
```

$$y(x)=\frac{x^2+\ln(x)+1-i\pi}{(2+x)^4}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 23

```
DSolve[{(x+2)*y'[x]+4*y[x]== (1+2*x^2)/(x*(x+2)^3),y[-1]==2},y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{x^2+\log(x)-i\pi+1}{(x+2)^4}$$

2.36 problem 36

Internal problem ID [922]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2-1)\,y'-2yx-x(x^2-1)=0}$$

With initial conditions

$$[y(0)=4]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve([(x^2-1)*diff(y(x),x)-2*x*y(x)= x*(x^2-1),y(0) = 4],y(x), singsol=all)
```

$$y(x)=-\frac{(i\pi-\ln(x+1)-\ln(x-1)+8)(x^2-1)}{2}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 27

```
DSolve[{(x^2-1)*y'[x]-2*x*y[x]== x*(x^2-1),y[0]==4},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}(x^2-1)\left(\log\left(x^2-1\right)-i\pi-8\right)$$

2.37 problem 44

Internal problem ID [923]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - 2y + 1 = 0}$$

With initial conditions

$$\left[y(0) = \frac{1}{2}\right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 11

```
dsolve([x*diff(y(x),x)-2*y(x)= -1,y(0) = 1/2],y(x), singsol=all)
```

$$y(x) = \frac{1}{2} + c_1 x^2$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 8

```
DSolve[{x*y'[x]-2*y[x]== -1,y[0]==1/2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}$$

2.38 problem 48(a)

Internal problem ID [924]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 48(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\sec(y)^2 y' - 3\tan(y) + 1 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve(sec(y(x))^2*diff(y(x),x)-3*tan(y(x))= -1,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{c_1 e^{3x}}{3} + \frac{1}{3}\right)$$

✓ Solution by Mathematica

Time used: 60.224 (sec). Leaf size: 177

```
DSolve[Sec[y[x]]^2*y'[x]-3*Tan[y[x]]== -1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(-\frac{3e^{6c_1}}{\sqrt{e^{6x} - 2e^{3x+6c_1} + 10e^{12c_1}}}\right)$$
$$y(x) \to \arccos\left(-\frac{3e^{6c_1}}{\sqrt{e^{6x} - 2e^{3x+6c_1} + 10e^{12c_1}}}\right)$$
$$y(x) \to -\arccos\left(\frac{3e^{6c_1}}{\sqrt{e^{6x} - 2e^{3x+6c_1} + 10e^{12c_1}}}\right)$$
$$y(x) \to \arccos\left(\frac{3e^{6c_1}}{\sqrt{e^{6x} - 2e^{3x+6c_1} + 10e^{12c_1}}}\right)$$

2.39 problem 48(b)

Internal problem ID [925]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 48(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{e^{y^2}\left(2yy' + \frac{2}{x}\right) - \frac{1}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(exp(y(x)^2)*(2*y(x)*diff(y(x),x)+2/x)= 1/x^2,y(x), singsol=all)
```

$$y(x) = \sqrt{\ln\left(-\frac{c_1 - x}{x^2}\right)}$$
$$y(x) = -\sqrt{\ln\left(-\frac{c_1 - x}{x^2}\right)}$$

✓ Solution by Mathematica

Time used: 7.149 (sec). Leaf size: 37

```
DSolve[Exp[y[x]^2]*(2*y[x]*y'[x]+2/x)== 1/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\log\left(\frac{x + c_1}{x^2}\right)}$$
$$y(x) \to \sqrt{\log\left(\frac{x + c_1}{x^2}\right)}$$

2.40 problem 48(c)

Internal problem ID [926]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 48(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(x)*y+H(x)]‘]]`

$$\boxed{\frac{xy'}{y} + 2\ln(y) - 4x^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)/y(x)+2*ln(y(x))= 4*x^2,y(x), singsol=all)
```

$$y(x) = e^{x^2} e^{-\frac{c_1}{x^2}}$$

✓ Solution by Mathematica

Time used: 0.209 (sec). Leaf size: 17

```
DSolve[x*y'[x]/y[x]+2*Log[y[x]]== 4*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2+\frac{c_1}{x^2}}$$

2.41 problem 48(d)

Internal problem ID [927]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Linear first order. Section 2.1 Page 41
Problem number: 48(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, _Riccati]`

$$\boxed{\frac{y'}{(1+y)^2}-\frac{1}{x\,(1+y)}+\frac{3}{x^2}=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)/(1+y(x))^2-1/(x*(1+y(x)))= -3/x^2,y(x), singsol=all)
```

$$y(x)=-1+\frac{x}{3\ln(x)+3c_1}$$

✓ Solution by Mathematica

Time used: 0.223 (sec). Leaf size: 24

```
DSolve[y'[x]/(1+y[x])^2-1/(x*(1+y[x]))== -3/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -1+\frac{x}{3(\log(x)+c_1)}$$
$$y(x)\to -1$$

3 Chapter 2, First order equations. separable equations. Section 2.2 Page 52

3.1 problem 1

Internal problem ID [928]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{3x^2 + 2x + 1}{y - 2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(diff(y(x),x)= (3*x^2+2*x+1)/(y(x)-2),y(x), singsol=all)
```

$$y(x) = 2 - \sqrt{2x^3 + 2x^2 + 2c_1 + 2x + 4}$$
$$y(x) = 2 + \sqrt{2x^3 + 2x^2 + 2c_1 + 2x + 4}$$

✓ Solution by Mathematica

Time used: 0.12 (sec). Leaf size: 56

```
DSolve[y'[x]== (3*x^2+2*x+1)/(y[x]-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2 - \sqrt{2}\sqrt{x^3 + x^2 + x + 2 + c_1}$$
$$y(x) \to 2 + \sqrt{2}\sqrt{x^3 + x^2 + x + 2 + c_1}$$

3.2 problem 2

Internal problem ID [929]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sin(x)\sin(y) + y'\cos(y) = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 12

```
dsolve(sin(x)*sin(y(x))+cos(y(x))*diff(y(x),x)= 0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{e^{\cos(x)}}{c_1}\right)$$

✓ Solution by Mathematica

Time used: 28.472 (sec). Leaf size: 22

```
DSolve[Sin[x]*Sin[y[x]]+Cos[y[x]]*y'[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(e^{\cos(x)+\frac{c_1}{2}}\right)$$
$$y(x) \to 0$$

3.3 problem 3

Internal problem ID [930]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x + y^2 + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x)+y(x)^2+y(x)= 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{c_1 x - 1}$$

✓ Solution by Mathematica

Time used: 0.2 (sec). Leaf size: 27

```
DSolve[x*y'[x]+y[x]^2+y[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{-1 + e^{-c_1} x}$$
$$y(x) \to -1$$
$$y(x) \to 0$$

3.4 problem 5

Internal problem ID [931]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$(3y^3 + 3y\cos(y) + 1)\, y' + \frac{(1+2x)\, y}{x^2+1} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve((3*y(x)^3+3*y(x)*cos(y(x))+1)*diff(y(x),x)+((2*x+1)*y(x))/(1+x^2)= 0,y(x), singsol=all
```

$$\ln(x^2+1) + \arctan(x) + y(x)^3 + 3\sin(y(x)) + \ln(y(x)) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.351 (sec). Leaf size: 40

```
DSolve[(3*y[x]^3+3*y[x]*Cos[y[x]]+1)*y'[x]+((2*x+1)*y[x])/(1+x^2)== 0,y[x],x,IncludeSingularS
```

$$y(x) \to \text{InverseFunction}\left[\#1^3 + \log(\#1) + 3\sin(\#1)\&\right]\left[-\arctan(x) - \log\left(x^2+1\right) + c_1\right]$$
$$y(x) \to 0$$

3.5 problem 6

Internal problem ID [932]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x^2y - (-1 + y^2)^{\frac{3}{2}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*y(x)*diff(y(x),x)= (y(x)^2-1)^(3/2),y(x), singsol=all)
```

$$-\frac{1}{x} + \frac{(y(x) - 1)(y(x) + 1)}{(y(x)^2 - 1)^{\frac{3}{2}}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.663 (sec). Leaf size: 105

```
DSolve[x^2*y[x]*y'[x]== (y[x]^2-1)^(3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{1 + x(x + c_1{}^2x - 2c_1)}}{1 - c_1x}$$

$$y(x) \to \frac{\sqrt{1 + x(x + c_1{}^2x - 2c_1)}}{-1 + c_1x}$$

$$y(x) \to -1$$

$$y(x) \to 1$$

$$y(x) \to -\frac{x}{\sqrt{x^2}}$$

$$y(x) \to \frac{x}{\sqrt{x^2}}$$

3.6 problem 7

Internal problem ID [933]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (1 + y^2)\, x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)= x^2*(1+y(x)^2),y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{x^3}{3} + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.163 (sec). Leaf size: 30

```
DSolve[y'[x]== x^2*(1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(\frac{x^3}{3} + c_1\right)$$
$$y(x) \to -i$$
$$y(x) \to i$$

3.7 problem 8

Internal problem ID [934]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,y'+yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)*(1+x^2)+x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 22

```
DSolve[y'[x]*(1+x^2)+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1}{\sqrt{x^2+1}}$$
$$y(x)\to 0$$

3.8 problem 9

Internal problem ID [935]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (x-1)(y-1)(y-2) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)=(x-1)*(y(x)-1)*(y(x)-2),y(x), singsol=all)
```

$$y(x) = \frac{-2 + c_1 e^{\frac{1}{2}x^2 - x}}{c_1 e^{\frac{1}{2}x^2 - x} - 1}$$

✓ Solution by Mathematica

Time used: 0.288 (sec). Leaf size: 36

```
DSolve[y'[x]==(x-1)*(y[x]-1)*(y[x]-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + \frac{1}{1 - e^{\frac{1}{2}(x-2)x + c_1}}$$
$$y(x) \to 1$$
$$y(x) \to 2$$

3.9 problem 10

Internal problem ID [936]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(y-1)^2 y' - 2x - 3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 102

```
dsolve((y(x)-1)^2*diff(y(x),x)=2*x+3,y(x), singsol=all)
```

$$y(x) = \left(3x^2 + 3c_1 + 9x\right)^{\frac{1}{3}} + 1$$
$$y(x) = -\frac{\left(3x^2 + 3c_1 + 9x\right)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\left(3x^2 + 3c_1 + 9x\right)^{\frac{1}{3}}}{2} + 1$$
$$y(x) = -\frac{\left(3x^2 + 3c_1 + 9x\right)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\left(3x^2 + 3c_1 + 9x\right)^{\frac{1}{3}}}{2} + 1$$

✓ Solution by Mathematica

Time used: 0.423 (sec). Leaf size: 97

```
DSolve[(y[x]-1)^2*y'[x]==2*x+3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + \sqrt[3]{3x(x+3) - 1 + 3c_1}$$
$$y(x) \to 1 + \frac{1}{2}i\left(\sqrt{3} + i\right)\sqrt[3]{3x(x+3) - 1 + 3c_1}$$
$$y(x) \to 1 + \frac{1}{2}\left(-1 - i\sqrt{3}\right)\sqrt[3]{3x(x+3) - 1 + 3c_1}$$

3.10 problem 11

Internal problem ID [937]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x^2 + 3x + 2}{y - 2} = 0}$$

With initial conditions

$$[y(1) = 4]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 25

```
dsolve([diff(y(x),x)=(x^2+3*x+2)/(y(x)-2),y(1) = 4],y(x), singsol=all)
```

$$y(x) = 2 + \frac{\sqrt{6x^3 + 27x^2 + 36x - 33}}{3}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 30

```
DSolve[{y'[x]==(x^2+3*x+2)/(y[x]-2),y[1]==4},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{\frac{2x^3}{3} + 3x^2 + 4x - \frac{11}{3}} + 2$$

3.11 problem 12

Internal problem ID [938]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + x(y^2 + y) = 0}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve([diff(y(x),x)+x*(y(x)^2+y(x))=0,y(2) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{-1 + 2\,\mathrm{e}^{\frac{(2+x)(-2+x)}{2}}}$$

✓ Solution by Mathematica

Time used: 0.212 (sec). Leaf size: 22

```
DSolve[{y'[x]+x*(y[x]^2+y[x])==0,y[2]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2e^{\frac{x^2}{2}-2} - 1}$$

3.12 problem 13

Internal problem ID [939]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(3y^2+4y)\,y'+2x+\cos(x)=0}$$

With initial conditions

$$[y(0)=1]$$

✓ Solution by Maple

Time used: 0.5 (sec). Leaf size: 102

```
dsolve([(3*y(x)^2+4*y(x))*diff(y(x),x)+2*x+cos(x)=0,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\left(260-108x^2-108\sin(x)+12\sqrt{441-390x^2-390\sin(x)+81x^4+162\sin(x)\,x^2+81\sin(x)^2}\right)^{\frac{1}{3}}}{6} + \frac{8}{3\left(260-108x^2-108\sin(x)+12\sqrt{441-390x^2-390\sin(x)+81x^4+162\sin(x)\,x^2+81\sin(x)^2}\right)^{\frac{1}{3}}} - \frac{2}{3}$$

✓ Solution by Mathematica

Time used: 2.384 (sec). Leaf size: 114

```
DSolve[{(3*y[x]^2+4*y[x])*y'[x]+2*x+Cos[x]==0,y[0]==1},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{6}\left(2^{2/3}\sqrt[3]{-27x^2+3\sqrt{3}\sqrt{(x^2+\sin(x)-3)\left(27x^2+27\sin(x)-49\right)}-27\sin(x)+65} + \frac{8\sqrt[3]{2}}{\sqrt[3]{-27x^2+3\sqrt{3}\sqrt{(x^2+\sin(x)-3)\left(27x^2+27\sin(x)-49\right)}-27\sin(x)+65}}-4\right)$$

3.13 problem 14

Internal problem ID [940]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' + \frac{(1+y)(y-1)(y-2)}{x+1} = 0$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 2.625 (sec). Leaf size: 111

```
dsolve([diff(y(x),x)+((y(x)+1)*(y(x)-1)*(y(x)-2))/(x+1)=0,y(1) = 0],y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(-2048 + \left(x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 257\right)\_Z^{18} + \left(-6x^6 - 36x^5 - 90x^4 - 120x^3 -\right.\right.$$
$$-1$$

✓ Solution by Mathematica

Time used: 60.886 (sec). Leaf size: 1128

```
DSolve[{y'[x]+((y[x]+1)*(y[x]-1)*(y[x]-2))/(x+1)==0,y[1]==0},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{\left(-1 - i\sqrt{3}\right)x^{12} - 12i\left(-i + \sqrt{3}\right)x^{11} - 66i\left(-i + \sqrt{3}\right)x^{10} - 220i\left(-i + \sqrt{3}\right)x^9 - 495i\left(-i + \sqrt{3}\right)x^8 - 79}{}$$

3.14 problem 15

Internal problem ID [941]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + 2x(1 + y) = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)+2*x*(y(x)+1)=0,y(0) = 2],y(x), singsol=all)
```

$$y(x) = -1 + 3\,\mathrm{e}^{-x^2}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 16

```
DSolve[{y'[x]+2*x*(y[x]+1)==0,y[0]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3e^{-x^2} - 1$$

3.15 problem 16

Internal problem ID [942]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2xy(1 + y^2) = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 16

```
dsolve([diff(y(x),x)=2*x*y(x)*(1+y(x)^2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{2\,\mathrm{e}^{-2x^2} - 1}}$$

✓ Solution by Mathematica

Time used: 60.091 (sec). Leaf size: 27

```
DSolve[{y'[x]==2*x*y[x]*(1+y[x]^2),y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{ie^{x^2}}{\sqrt{e^{2x^2} - 2}}$$

3.16 problem 17

Internal problem ID [943]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+2)\,y' - 4x(y^2+2y+1) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)*(x^2+2)=4*x*(y(x)^2+2*y(x)+1),y(x), singsol=all)
```

$$y(x) = -\frac{2\ln(x^2+2)+4c_1+1}{2(\ln(x^2+2)+2c_1)}$$

✓ Solution by Mathematica

Time used: 0.197 (sec). Leaf size: 27

```
DSolve[y'[x]*(x^2+2)==4*x*(y[x]^2+2*y[x]+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 - \frac{1}{2\log(x^2+2)+c_1}$$

$$y(x) \to -1$$

3.17 problem 18

Internal problem ID [944]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + 2x(y^3 - 3y + 2) = 0}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 0.421 (sec). Leaf size: 70

```
dsolve([diff(y(x),x)=-2*x*(y(x)^3-3*y(x)+2),y(0) = 3],y(x), singsol=all)
```

$$y(x) = e^{\operatorname{RootOf}\left(18x^2 e^{\_Z} - 2\ln(e^{\_Z} - 3) e^{\_Z} + 2 e^{\_Z} \ln(2) - 2 e^{\_Z} \ln(5) + 2\_Z e^{\_Z} - 54x^2 + 6\ln(e^{\_Z} - 3) + 3 e^{\_Z} - 6\ln(2) + 6\ln(5) - 6\_Z - 15\right)} - 2$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]==-2*x*(y[x]^3-3*y[x]+2),y[1]==-1},y[x],x,IncludeSingularSolutions -> True]
```

{}

3.18 problem 19

Internal problem ID [945]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2x}{1+2y} = 0}$$

With initial conditions

$$[y(2) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)=2*x/(1+2*y(x)),y(2) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{1}{2} + \frac{\sqrt{4x^2 - 15}}{2}$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 22

```
DSolve[{y'[x]==2*x/(1+2*y[x]),y[2]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\sqrt{4x^2 - 15} - 1\right)$$

3.19 problem 20

Internal problem ID [946]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 2y + y^2 = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=2*y(x)-y(x)^2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{2}{1 + e^{-2x}}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 9

```
DSolve[{y'[x]==2*y[x]-y[x]^2,y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tanh(x) + 1$$

3.20 problem 21

Internal problem ID [947]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x + yy' = 0}$$

With initial conditions

$$[y(3) = -4]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 15

```
dsolve([x+y(x)*diff(y(x),x)=0,y(3) = -4],y(x), singsol=all)
```

$$y(x) = -\sqrt{-x^2 + 25}$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 18

```
DSolve[{x+y[x]*y'[x]==0,y[3]==-4},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{25 - x^2}$$

3.21 problem 22

Internal problem ID [948]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + x^2(1+y)(y-2)^2 = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)+x^2*(y(x)+1)*(y(x)-2)^2=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(3x^3\mathrm{e}^{\_Z}+\ln\left(\mathrm{e}^{\_Z}+3\right)\mathrm{e}^{\_Z}+9c_1\mathrm{e}^{\_Z}-\_Z\mathrm{e}^{\_Z}-3\right)} + 2$$

✓ Solution by Mathematica

Time used: 0.476 (sec). Leaf size: 52

```
DSolve[y'[x]+x^2*(y[x]+1)*(y[x]-2)^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{1}{9}\left(-\frac{3}{\#1-2} - \log(\#1-2) + \log(\#1+1)\right)\&\right]\left[-\frac{x^3}{3} + c_1\right]$$
$$y(x) \to -1$$
$$y(x) \to 2$$

3.22 problem 23

Internal problem ID [949]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+1)(-2+x)\,y'+y=0}$$

With initial conditions

$$[y(1)=-3]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 22

```
dsolve([(x+1)*(x-2)*diff(y(x),x)+y(x)=0,y(1) = -3],y(x), singsol=all)
```

$$y(x)=-\frac{3\left(1+i\sqrt{3}\right)2^{\frac{2}{3}}(x+1)^{\frac{1}{3}}}{4\,(-2+x)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 23

```
DSolve[{(x+1)*(x-2)*y'[x]+y[x]==0,y[1]==-3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{3\sqrt[3]{x+1}}{\sqrt[3]{4-2x}}$$

3.23 problem 24

Internal problem ID [950]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{1+y^2}{x^2+1} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=(1+y(x)^2)/(1+x^2),y(x), singsol=all)
```

$$y(x) = \tan(\arctan(x) + c_1)$$

✓ Solution by Mathematica

Time used: 0.216 (sec). Leaf size: 25

```
DSolve[y'[x]==(1+y[x]^2)/(1+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(\arctan(x) + c_1)$$
$$y(x) \to -i$$
$$y(x) \to i$$

3.24 problem 25

Internal problem ID [951]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'\sqrt{-x^2+1}+\sqrt{1-y^2}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)*sqrt(1-x^2)+sqrt(1-y(x)^2)=0,y(x), singsol=all)
```

$$y(x) = -\sin(\arcsin(x)+c_1)$$

✓ Solution by Mathematica

Time used: 0.307 (sec). Leaf size: 45

```
DSolve[y'[x]*Sqrt[1-x^2]+Sqrt[1-y[x]^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos\left(2\cot^{-1}\left(\frac{x+1}{\sqrt{1-x^2}}\right)+c_1\right)$$
$$y(x) \to -1$$
$$y(x) \to 1$$
$$y(x) \to \text{Interval}[\{-1,1\}]$$

3.25 problem 26

Internal problem ID [952]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\cos(x)}{\sin(y)} = 0}$$

With initial conditions

$$\left[y(\pi) = \frac{\pi}{2}\right]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=cos(x)/sin(y(x)),y(Pi) = 1/2*Pi],y(x), singsol=all)
```

$$y(x) = \frac{\pi}{2} + \arcsin(\sin(x))$$

✓ Solution by Mathematica

Time used: 0.407 (sec). Leaf size: 10

```
DSolve[{y'[x]==Cos[x]/Sin[y[x]],y[Pi]==Pi/2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arccos(-\sin(x))$$

3.26 problem 27

Internal problem ID [953]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - ay + by^2 = 0}$$

With initial conditions

$$[y(0) = \mathrm{y0}]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 25

```
dsolve([diff(y(x),x)=a*y(x)-b*y(x)^2,y(0) = y0],y(x), singsol=all)
```

$$y(x) = \frac{a\,\mathrm{y0}}{(-\,\mathrm{y0}\,b + a)\,\mathrm{e}^{-ax} + \mathrm{y0}\,b}$$

✓ Solution by Mathematica

Time used: 0.743 (sec). Leaf size: 27

```
DSolve[{y'[x]==a*y[x]-b*y[x]^2,y[0]==y0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a\mathrm{y0}e^{ax}}{b\mathrm{y0}\left(e^{ax} - 1\right) + a}$$

3.27 problem 35

Internal problem ID [954]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\boxed{y' + y - \frac{2x\,\mathrm{e}^{-x}}{1 + \mathrm{e}^{x} y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x)+y(x)=(2*x*exp(-x))/(1+y(x)*exp(x)),y(x), singsol=all)
```

$$y(x) = \left(-1 - \sqrt{2x^2 - 2c_1 + 1}\right)\mathrm{e}^{-x}$$
$$y(x) = \left(-1 + \sqrt{2x^2 - 2c_1 + 1}\right)\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 32.175 (sec). Leaf size: 70

```
DSolve[y'[x]+y[x]==(2*x*Exp[-x])/(1+y[x]*Exp[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{-2x}\left(e^{x} + \sqrt{e^{2x}\left(2x^2 + 1 + c_1\right)}\right)$$
$$y(x) \to e^{-2x}\left(-e^{x} + \sqrt{e^{2x}\left(2x^2 + 1 + c_1\right)}\right)$$

3.28 problem 36

Internal problem ID [955]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'x - 2y - \frac{x^6}{y + x^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(x*diff(y(x),x)-2*y(x)=x^6/(y(x)+x^2),y(x), singsol=all)
```

$$y(x) = \left(-1 - \sqrt{x^2 - 2c_1 + 1}\right) x^2$$
$$y(x) = \left(-1 + \sqrt{x^2 - 2c_1 + 1}\right) x^2$$

✓ Solution by Mathematica

Time used: 0.554 (sec). Leaf size: 70

```
DSolve[x*y'[x]-2*y[x]==x^6/(y[x]+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2\left(1 + \sqrt{\frac{1}{x^5}}x^2\sqrt{x\left(x^2 + 1 + c_1\right)}\right)$$
$$y(x) \to -x^2 + \sqrt{\frac{1}{x^5}}x^4\sqrt{x\left(x^2 + 1 + c_1\right)}$$

3.29 problem 37

Internal problem ID [956]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - y - \frac{(x+1)\,\mathrm{e}^{4x}}{(y+\mathrm{e}^{x})^{2}} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 112

```
dsolve(diff(y(x),x)-y(x)=((x+1)*exp(4*x))/(y(x)+exp(x))^2,y(x), singsol=all)
```

$$y(x) = (3x\,\mathrm{e}^{x} - 3c_1 + 1)^{\frac{1}{3}}\,\mathrm{e}^{x} - \mathrm{e}^{x}$$

$$y(x) = \left(-\frac{(3x\,\mathrm{e}^{x} - 3c_1 + 1)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\,(3x\,\mathrm{e}^{x} - 3c_1 + 1)^{\frac{1}{3}}}{2}\right)\mathrm{e}^{x} - \mathrm{e}^{x}$$

$$y(x) = \left(-\frac{(3x\,\mathrm{e}^{x} - 3c_1 + 1)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\,(3x\,\mathrm{e}^{x} - 3c_1 + 1)^{\frac{1}{3}}}{2}\right)\mathrm{e}^{x} - \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 18.719 (sec). Leaf size: 143

```
DSolve[y'[x]-y[x]==((x+1)*Exp[4*x])/(y[x]+Exp[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{x} + e^{3x}\sqrt[3]{e^{-6x}\left(3e^{x}x + 1 + 3c_1\right)}$$

$$y(x) \to -e^{x} + \frac{1}{2}i\left(\sqrt{3} + i\right)e^{3x}\sqrt[3]{e^{-6x}\left(3e^{x}x + 1 + 3c_1\right)}$$

$$y(x) \to -e^{x} - \frac{1}{2}\left(1 + i\sqrt{3}\right)e^{3x}\sqrt[3]{e^{-6x}\left(3e^{x}x + 1 + 3c_1\right)}$$

3.30 problem 38

Internal problem ID [957]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. separable equations. Section 2.2 Page 52
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{y' - 2y - \frac{x\,\mathrm{e}^{2x}}{1 - y\,\mathrm{e}^{-2x}} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 84

```
dsolve(diff(y(x),x)-2*y(x)=x*exp(2*x)/(1-y(x)*exp(-2*x)),y(x), singsol=all)
```

$$y(x) = \left(\mathrm{e}^{-2x} + \sqrt{-\mathrm{e}^{-4x}x^2 + \mathrm{e}^{-4x} - 2c_1\mathrm{e}^{-4x}}\right)\mathrm{e}^{4x}$$
$$y(x) = -\left(-\mathrm{e}^{-2x} + \sqrt{-\mathrm{e}^{-4x}x^2 + \mathrm{e}^{-4x} - 2c_1\mathrm{e}^{-4x}}\right)\mathrm{e}^{4x}$$

✓ Solution by Mathematica

Time used: 0.688 (sec). Leaf size: 72

```
DSolve[y'[x]-2*y[x]==x*Exp[2*x]/(1-y[x]*Exp[-2*x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x} - \frac{\sqrt{x^2 - 1 - c_1}}{\sqrt{-e^{-4x}}}$$
$$y(x) \to e^{2x} + \frac{\sqrt{x^2 - 1 - c_1}}{\sqrt{-e^{-4x}}}$$

4 Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60

4.1 problem 1

Internal problem ID [958]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \frac{x^2 + y^2}{\sin(x)} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=(x^2+y(x)^2)/sin(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(x^2+y[x]^2)/Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.2 problem 2

Internal problem ID [959]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' - \frac{y + e^x}{x^2 + y^2} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=(exp(x)+y(x))/(x^2+y(x)^2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(Exp[x]+y[x])/(x^2+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.3 problem 3

Internal problem ID [960]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \tan (yx) = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 44

```
dsolve(diff(y(x),x)=tan(x*y(x)),y(x), singsol=all)
```

$$y(x) = -i \operatorname{RootOf}\left(\sqrt{2}\, c_1 - \operatorname{erf}\left(\frac{(-x + \_Z)\sqrt{2}}{2}\right)\sqrt{\pi} - \operatorname{erf}\left(\frac{\sqrt{2}\,(x + \_Z)}{2}\right)\sqrt{\pi}\right)$$

✓ Solution by Mathematica

Time used: 0.308 (sec). Leaf size: 69

```
DSolve[y'[x]==Tan[x*y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\sqrt{\frac{\pi}{2}}e^{\frac{x^2}{2}}\left(\operatorname{erfi}\left(\frac{y(x) - ix}{\sqrt{2}}\right) + \operatorname{erfi}\left(\frac{y(x) + ix}{\sqrt{2}}\right)\right) = c_1 e^{\frac{x^2}{2}}, y(x)\right]$$

4.4 problem 4

Internal problem ID [961]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \frac{x^2 + y^2}{\ln(yx)} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=(x^2+y(x)^2)/ln(x*y(x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(x^2+y[x]^2)/Log[x*y[x]],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.5 problem 5

Internal problem ID [962]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' - (x^2 + y^2)\, y^{\frac{1}{3}} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=(x^2+y(x)^2)*y(x)^(1/3),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(x^2+y[x]^2)*y[x]^(1/3),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.6 problem 6

Internal problem ID [963]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-2yx + y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=2*x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{x^2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 18

```
DSolve[y'[x]==2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x^2}$$
$$y(x) \to 0$$

4.7 problem 7

Internal problem ID [964]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \ln\left(1 + x^2 + y^2\right) = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=ln(1+x^2+y(x)^2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==Log[1+x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.8 problem 8

Internal problem ID [965]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{2x+3y}{x-4y} = 0}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 46

```
dsolve(diff(y(x),x)=(2*x+3*y(x))/(x-4*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{x}{4} + \frac{\sqrt{7}\,x\tan\left(\operatorname{RootOf}\left(\sqrt{7}\ln\left(\frac{7x^2}{8} + \frac{7x^2\tan(\_Z)^2}{8}\right) + 2\sqrt{7}\,c_1 - 4\_Z\right)\right)}{4}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 53

```
DSolve[y'[x]==(2*x+3*y[x])/(x-4*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\log\left(\frac{2y(x)^2}{x^2} + \frac{y(x)}{x} + 1\right) - \frac{4\arctan\left(\frac{\frac{4y(x)}{x}+1}{\sqrt{7}}\right)}{\sqrt{7}} = -2\log(x) + c_1, y(x)\right]$$

4.9 problem 9

Internal problem ID [966]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \sqrt{x^2 + y^2} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=(x^2+y(x)^2)^(1/2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(x^2+y[x]^2)^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.10 problem 10

Internal problem ID [967]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x(-1+y^2)^{\frac{2}{3}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x)=x*(y(x)^2-1)^(2/3),y(x), singsol=all)
```

$$\frac{x^2}{2} - \frac{\left(-\operatorname{signum}\left(y(x)^2-1\right)\right)^{\frac{2}{3}} y(x) \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{2}{3}\right], \left[\frac{3}{2}\right], y(x)^2\right)}{\operatorname{signum}\left(y(x)^2-1\right)^{\frac{2}{3}}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 6.664 (sec). Leaf size: 66

```
DSolve[y'[x]==x*(y[x]^2-1)^(2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{\#1(1-\#1^2)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \#1^2\right)}{(\#1^2-1)^{2/3}} \&\right]\left[\frac{x^2}{2} + c_1\right]$$

$$y(x) \to -1$$

$$y(x) \to 1$$

4.11 problem 11

Internal problem ID [968]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - (x^2 + y^2)^2 = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=(x^2+y(x)^2)^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(x^2+y[x]^2)^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.12 problem 12

Internal problem ID [969]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \sqrt{x+y} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(diff(y(x),x)=(x+y(x))^(1/2),y(x), singsol=all)
```

$$x - 2\sqrt{x+y(x)} - \ln\left(-1+\sqrt{x+y(x)}\right) + \ln\left(1+\sqrt{x+y(x)}\right) + \ln\left(x+y(x)-1\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 8.698 (sec). Leaf size: 58

```
DSolve[y'[x]==(x+y[x])^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W\left(-e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right)\left(2+W\left(-e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right)\right) - x + 1$$
$$y(x) \to 1 - x$$

4.13 problem 13

Internal problem ID [970]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\tan(y)}{x-1} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=tan(y(x))/(x-1),y(x), singsol=all)
```

$$y(x) = \arcsin(c_1(x-1))$$

✓ Solution by Mathematica

Time used: 7.493 (sec). Leaf size: 19

```
DSolve[y'[x]==Tan[y[x]]/(x-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(e^{c_1}(x-1)\right)$$
$$y(x) \to 0$$

4.14 problem 16

Internal problem ID [971]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^{\frac{2}{5}} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=y(x)^(2/5),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{(3x+5)\left(\frac{3x}{5}+1\right)^{\frac{2}{3}}}{5}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 23

```
DSolve[{y'[x]==y[x]^(2/5),y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(3x+5)^{5/3}}{5\,5^{2/3}}$$

4.15 problem 18

Internal problem ID [972]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3x(y-1)^{\frac{1}{3}} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=3*x*(y(x)-1)^(1/3),y(0) = 1],y(x), singsol=all)
```

$$y(x) = 1$$

✓ Solution by Mathematica

Time used: 0.272 (sec). Leaf size: 19

```
DSolve[{y'[x]==3*x*(y[x]-1)^(1/3),y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1$$
$$y(x) \to (x^2)^{3/2} + 1$$

4.16 problem 19

Internal problem ID [973]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3x(y-1)^{\frac{1}{3}} = 0}$$

With initial conditions

$$[y(0) = 9]$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=3*x*(y(x)-1)^(1/3),y(0) = 9],y(x), singsol=all)
```

$$y(x) = x^2\sqrt{x^2+4} + 4\sqrt{x^2+4} + 1$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 16

```
DSolve[{y'[x]==3*x*(y[x]-1)^(1/3),y[0]==9},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x^2+4)^{3/2} + 1$$

4.17 problem 20(a)

Internal problem ID [974]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Existence and Uniqueness of Solutions of Nonlinear Equations. Section 2.3 Page 60
Problem number: 20(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3x(y-1)^{\frac{1}{3}} = 0}$$

With initial conditions

$$[y(3) = -7]$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 19

```
dsolve([diff(y(x),x)=3*x*(y(x)-1)^(1/3),y(3) = -7],y(x), singsol=all)
```

$$y(x) = 1 + \left(-11 + 2i\sqrt{3} + x^2\right)^{\frac{3}{2}}$$

✓ Solution by Mathematica

Time used: 0.131 (sec). Leaf size: 49

```
DSolve[{y'[x]==3*x*(y[x]-1)^(1/3),y[3]==-7},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + \left(x^2 - 2i\sqrt{3} - 11\right)^{3/2}$$
$$y(x) \to 1 + \left(x^2 + 2i\sqrt{3} - 11\right)^{3/2}$$

5 Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

5.1 problem Example 1

Internal problem ID [975]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: Example 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - y - xy^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)-y(x)=x*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{1 + e^{-x}c_1 - x}$$

✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 25

```
DSolve[y'[x]-y[x]==x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{-x + c_1 e^{-x} + 1}$$
$$y(x) \to 0$$

5.2 problem Example 2

Internal problem ID [976]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: Example 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$y' - \frac{y + x\,\mathrm{e}^{-\frac{y}{x}}}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=(y(x)+x*exp(-y(x)/x))/x,y(x), singsol=all)
```

$$y(x) = \ln\left(\ln\left(x\right) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.37 (sec). Leaf size: 13

```
DSolve[y'[x]==(y[x]+x*Exp[-y[x]/x])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \log(\log(x) + c_1)$$

5.3 problem Example 3(a) (As Riccati)

Internal problem ID [977]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: Example 3(a) (As Riccati).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y'x^2 - y^2 - yx + x^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x^2*diff(y(x),x)=y(x)^2+x*y(x)-x^2,y(x), singsol=all)
```

$$y(x) = -\tanh(\ln(x) + c_1)\,x$$

✓ Solution by Mathematica

Time used: 0.265 (sec). Leaf size: 244

```
DSolve[y'[x]==y[x]^2+x*y[x]-x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{10}\left(5\left(\sqrt{5}-1\right)x + \frac{-5\left(\sqrt{5}-1\right)x\,\text{Hypergeometric1F1}\left(\frac{5}{4}-\frac{1}{4\sqrt{5}},\frac{3}{2},\frac{\sqrt{5}x^2}{2}\right)+2\sqrt{5\left(3\sqrt{5}-5\right)}c_1\,\text{HermiteH}\left(\frac{1}{10}\left(-15+\sqrt{5}\right)\right.}{\text{Hypergeometric1F1}\left(\frac{1}{20}\left(5-\sqrt{5}\right),\frac{1}{2},\frac{\sqrt{5}x^2}{2}\right)+c_1\,\text{HermiteH}\left(\frac{1}{10}\left(-5+\sqrt{5}\right),\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)}\right.$$

$$y(x) \to \frac{\sqrt{\frac{3}{\sqrt{5}}-1}\,\text{HermiteH}\left(\frac{1}{10}\left(-15+\sqrt{5}\right),\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)}{\text{HermiteH}\left(\frac{1}{10}\left(-5+\sqrt{5}\right),\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)} + \frac{1}{2}\left(\sqrt{5}-1\right)x$$

5.4 problem Example 3(b)

Internal problem ID [978]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: Example 3(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y'x^2 - y^2 - yx + x^2 = 0}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 19

```
dsolve([x^2*diff(y(x),x)=y(x)^2+x*y(x)-x^2,y(1) = 2],y(x), singsol=all)
```

$$y(x) = -\frac{x(x^2+3)}{x^2-3}$$

✓ Solution by Mathematica

Time used: 0.49 (sec). Leaf size: 20

```
DSolve[{x^2*y'[x]==y[x]^2+x*y[x]-x^2,y[1]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x(x^2+3)}{x^2-3}$$

5.5 problem 1

Internal problem ID [979]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+y(x)=y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{1 + c_1 e^x}$$

✓ Solution by Mathematica

Time used: 0.721 (sec). Leaf size: 54

```
DSolve[y'[x]+y[x]==y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{1 + e^{2(x+c_1)}}}$$
$$y(x) \to \frac{1}{\sqrt{1 + e^{2(x+c_1)}}}$$
$$y(x) \to -1$$
$$y(x) \to 0$$
$$y(x) \to 1$$

5.6 problem 2

Internal problem ID [980]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{7y'x - 2y + \frac{x^2}{y^6} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 213

```
dsolve(7*x*diff(y(x),x)-2*y(x)=-x^2/y(x)^6,y(x), singsol=all)
```

$$y(x) = \left(-\ln(x)\,x^2 + c_1x^2\right)^{\frac{1}{7}}$$

$$y(x) = \left(-\cos\left(\frac{\pi}{7}\right) - i\cos\left(\frac{5\pi}{14}\right)\right)\left(-\ln(x)\,x^2 + c_1x^2\right)^{\frac{1}{7}}$$

$$y(x) = \left(-\cos\left(\frac{\pi}{7}\right) + i\cos\left(\frac{5\pi}{14}\right)\right)\left(-\ln(x)\,x^2 + c_1x^2\right)^{\frac{1}{7}}$$

$$y(x) = \left(\cos\left(\frac{2\pi}{7}\right) - i\cos\left(\frac{3\pi}{14}\right)\right)\left(-\ln(x)\,x^2 + c_1x^2\right)^{\frac{1}{7}}$$

$$y(x) = \left(\cos\left(\frac{2\pi}{7}\right) + i\cos\left(\frac{3\pi}{14}\right)\right)\left(-\ln(x)\,x^2 + c_1x^2\right)^{\frac{1}{7}}$$

$$y(x) = \left(-\cos\left(\frac{3\pi}{7}\right) - i\cos\left(\frac{\pi}{14}\right)\right)\left(-\ln(x)\,x^2 + c_1x^2\right)^{\frac{1}{7}}$$

$$y(x) = \left(-\cos\left(\frac{3\pi}{7}\right) + i\cos\left(\frac{\pi}{14}\right)\right)\left(-\ln(x)\,x^2 + c_1x^2\right)^{\frac{1}{7}}$$

✓ Solution by Mathematica

Time used: 0.225 (sec). Leaf size: 181

```
DSolve[7*x*y'[x]-2*y[x]==-x^2/y[x]^6,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{2/7} \sqrt[7]{-\log(x) + c_1}$$
$$y(x) \to -\sqrt[7]{-1} x^{2/7} \sqrt[7]{-\log(x) + c_1}$$
$$y(x) \to (-1)^{2/7} x^{2/7} \sqrt[7]{-\log(x) + c_1}$$
$$y(x) \to -(-1)^{3/7} x^{2/7} \sqrt[7]{-\log(x) + c_1}$$
$$y(x) \to (-1)^{4/7} x^{2/7} \sqrt[7]{-\log(x) + c_1}$$
$$y(x) \to -(-1)^{5/7} x^{2/7} \sqrt[7]{-\log(x) + c_1}$$
$$y(x) \to (-1)^{6/7} x^{2/7} \sqrt[7]{-\log(x) + c_1}$$

5.7 problem 3

Internal problem ID [981]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y'x^2 + 2y - 2\,\mathrm{e}^{\frac{1}{x}}\sqrt{y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x)+2*y(x)=2*exp(1/x)*y(x)^(1/2),y(x), singsol=all)
```

$$\sqrt{y\left(x\right)} - \left(-\frac{1}{x} + c_1\right)\mathrm{e}^{\frac{1}{x}} = 0$$

✓ Solution by Mathematica

Time used: 0.27 (sec). Leaf size: 39

```
DSolve[y'[x]+2*y[x]==2*Exp[1/x]*y[x]^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}\left(\int_1^x 2e^{K[1]+\frac{1}{K[1]}}dK[1] + 2c_1\right)^2$$

5.8 problem 4

Internal problem ID [982]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(x^2+1)\,y'+2yx-\frac{1}{(x^2+1)\,y}=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 38

```
dsolve((1+x^2)*diff(y(x),x)+2*x*y(x)=1/((1+x^2)*y(x)),y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{2x+c_1}}{x^2+1}$$
$$y(x)=-\frac{\sqrt{2x+c_1}}{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.26 (sec). Leaf size: 46

```
DSolve[(1+x^2)*y'[x]+2*x*y[x]==1/((1+x^2)*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{\sqrt{2x+c_1}}{x^2+1}$$
$$y(x)\to\frac{\sqrt{2x+c_1}}{x^2+1}$$

5.9 problem 5

Internal problem ID [983]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{-yx + y' - x^3y^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x)-x*y(x)=x^3*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{\mathrm{e}^{-x^2}c_1 - x^2 + 1}}$$
$$y(x) = -\frac{1}{\sqrt{\mathrm{e}^{-x^2}c_1 - x^2 + 1}}$$

✓ Solution by Mathematica

Time used: 1.861 (sec). Leaf size: 80

```
DSolve[y'[x]-x*y[x]==x^3*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ie^{\frac{x^2}{2}}}{\sqrt{e^{x^2}(x^2-1) - c_1}}$$
$$y(x) \to \frac{ie^{\frac{x^2}{2}}}{\sqrt{e^{x^2}(x^2-1) - c_1}}$$
$$y(x) \to 0$$

5.10 problem 6

Internal problem ID [984]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{y' - \frac{(x+1)\,y}{3x} - y^4 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 179

```
dsolve(diff(y(x),x)-(1+x)/(3*x)*y(x)=y(x)^4,y(x), singsol=all)
```

$$y(x) = \frac{\left(x(\mathrm{e}^{-x}c_1 - 3x + 3)^2\right)^{\frac{1}{3}}}{\mathrm{e}^{-x}c_1 - 3x + 3}$$

$$y(x) = -\frac{\left(x(\mathrm{e}^{-x}c_1 - 3x + 3)^2\right)^{\frac{1}{3}}}{2\left(\mathrm{e}^{-x}c_1 - 3x + 3\right)} - \frac{i\sqrt{3}\left(x(\mathrm{e}^{-x}c_1 - 3x + 3)^2\right)^{\frac{1}{3}}}{2\left(\mathrm{e}^{-x}c_1 - 3x + 3\right)}$$

$$y(x) = -\frac{\left(x(\mathrm{e}^{-x}c_1 - 3x + 3)^2\right)^{\frac{1}{3}}}{2\left(\mathrm{e}^{-x}c_1 - 3x + 3\right)} + \frac{i\sqrt{3}\left(x(\mathrm{e}^{-x}c_1 - 3x + 3)^2\right)^{\frac{1}{3}}}{2\,\mathrm{e}^{-x}c_1 - 6x + 6}$$

✓ Solution by Mathematica

Time used: 1.858 (sec). Leaf size: 120

```
DSolve[y'[x]-(1+x)/(3*x)*y[x]==y[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{x/3}\sqrt[3]{x}}{\sqrt[3]{3e^x(x-1)-c_1}}$$
$$y(x) \to \frac{\sqrt[3]{-1}e^{x/3}\sqrt[3]{x}}{\sqrt[3]{3e^x(x-1)-c_1}}$$
$$y(x) \to -\frac{(-1)^{2/3}e^{x/3}\sqrt[3]{x}}{\sqrt[3]{3e^x(x-1)-c_1}}$$
$$y(x) \to 0$$

5.11 problem 7

Internal problem ID [985]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - 2y - x y^3 = 0}$$

With initial conditions

$$\left[y(0) = 2\sqrt{2}\right]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)-2*y(x)=x*y(x)^3,y(0) = 2*2^(1/2)],y(x), singsol=all)
```

$$y(x) = \frac{4}{\sqrt{-8x+2}}$$

✓ Solution by Mathematica

Time used: 1.842 (sec). Leaf size: 34

```
DSolve[{y'[x]-2*y[x]==x*y[x]^3,y[0]==2*Sqrt[2]},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2i\sqrt{2}e^{2x}}{\sqrt{e^{4x}(4x-1)}}$$

5.12 problem 8

Internal problem ID [986]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-yx + y' - xy^{\frac{3}{2}} = 0}$$

With initial conditions

$$[y(1) = 4]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 21

```
dsolve([diff(y(x),x)-x*y(x)=x*y(x)^(3/2),y(1) = 4],y(x), singsol=all)
```

$$y(x) = \frac{4}{\left(-2 + 3\,\mathrm{e}^{-\frac{(x-1)(x+1)}{4}}\right)^2}$$

✓ Solution by Mathematica

Time used: 0.33 (sec). Leaf size: 55

```
DSolve[{y'[x]-x*y[x]==x*y[x]^(3/2),y[1]==4},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(\tanh\left(\frac{1}{8}(-8\operatorname{arctanh}(3) - x^2 + 1)\right) - 1\right)^2$$
$$y(x) \to \frac{1}{4}\left(\tanh\left(\operatorname{arctanh}(5) - \frac{x^2}{8} + \frac{1}{8}\right) - 1\right)^2$$

5.13 problem 9

Internal problem ID [987]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y + y'x - x^4y^4 = 0}$$

With initial conditions

$$\left[y(1) = \frac{1}{2}\right]$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 35

```
dsolve([x*diff(y(x),x)+y(x)=x^4*y(x)^4,y(1) = 1/2],y(x), singsol=all)
```

$$y(x) = \frac{\left(-(3x-11)^2\right)^{\frac{1}{3}} \left(i\sqrt{3}-1\right)}{6x^2-22x}$$

✓ Solution by Mathematica

Time used: 0.397 (sec). Leaf size: 18

```
DSolve[{x*y'[x]+y[x]==x^4*y[x]^4,y[1]==1/2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\sqrt[3]{(11-3x)x^3}}$$

5.14 problem 10

Internal problem ID [988]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 2y - 2\sqrt{y} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 16

```
dsolve([diff(y(x),x)-2*y(x)=2*y(x)^(1/2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = 4\,e^{2x} - 4\,e^{x} + 1$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 14

```
DSolve[{y'[x]-2*y[x]==2*y[x]^(1/2),y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (1 - 2e^{x})^2$$

5.15 problem 11

Internal problem ID [989]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{-4y + y' - \frac{48x}{y^2} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)-4*y(x)=48*x/y(x)^2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \left(2\,\mathrm{e}^{12x} - 12x - 1\right)^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 3.916 (sec). Leaf size: 21

```
DSolve[{y'[x]-4*y[x]==48*x/y[x]^2,y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{-12x + 2e^{12x} - 1}$$

5.16 problem 12

Internal problem ID [990]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y'x^2 + 2yx - y^3 = 0}$$

With initial conditions

$$\left[y(1) = \frac{\sqrt{2}}{2}\right]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 26

```
dsolve([x^2*diff(y(x),x)+2*x*y(x)=y(x)^3,y(1) = 1/2*2^(1/2)],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{10}\,\sqrt{4x^6 + x}}{8x^5 + 2}$$

✓ Solution by Mathematica

Time used: 0.427 (sec). Leaf size: 29

```
DSolve[{x^2*y'[x]+2*x*y[x]==y[x]^3,y[1]==1/Sqrt[2]},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\frac{5}{2}}\sqrt{x}}{\sqrt{4x^5 + 1}}$$

5.17 problem 13

Internal problem ID [991]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - y - \sqrt{y}\, x = 0}$$

With initial conditions

$$[y(0) = 4]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 26

```
dsolve([diff(y(x),x)-y(x)=x*y(x)^(1/2),y(0) = 4],y(x), singsol=all)
```

$$y(x) = (-8x - 16)\, e^{\frac{x}{2}} + x^2 + 4x + 16\, e^{x} + 4$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 28

```
DSolve[{y'[x]-y[x]==x*y[x]^(1/2),y[0]==4},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(x - 4e^{x/2} + 2\right)^2$$
$$y(x) \to (x + 2)^2$$

5.18 problem 15

Internal problem ID [992]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{x+y}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=(y(x)+x)/x,y(x), singsol=all)
```

$$y(x) = (\ln(x) + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 12

```
DSolve[y'[x]==(y[x]+x)/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(\log(x) + c_1)$$

5.19 problem 16

Internal problem ID [993]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y' - \frac{y^2 + 2yx}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=(y(x)^2+2*x*y(x))/x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{c_1 - x}$$

✓ Solution by Mathematica

Time used: 0.128 (sec). Leaf size: 22

```
DSolve[y'[x]==(y[x]^2+2*x*y[x])/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{-x + c_1}$$
$$y(x) \to 0$$

5.20 problem 17

Internal problem ID [994]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{xy^3y' - y^4 - x^4 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
dsolve(x*y(x)^3*diff(y(x),x)=y(x)^4+x^4,y(x), singsol=all)
```

$$y(x) = (4\ln(x) + c_1)^{\frac{1}{4}} x$$
$$y(x) = -(4\ln(x) + c_1)^{\frac{1}{4}} x$$
$$y(x) = -i(4\ln(x) + c_1)^{\frac{1}{4}} x$$
$$y(x) = i(4\ln(x) + c_1)^{\frac{1}{4}} x$$

✓ Solution by Mathematica

Time used: 0.181 (sec). Leaf size: 76

```
DSolve[x*y[x]^3*y'[x]==y[x]^4+x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt[4]{4\log(x) + c_1}$$
$$y(x) \to -ix\sqrt[4]{4\log(x) + c_1}$$
$$y(x) \to ix\sqrt[4]{4\log(x) + c_1}$$
$$y(x) \to x\sqrt[4]{4\log(x) + c_1}$$

5.21 problem 18

Internal problem ID [995]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y' - \frac{y}{x} - \sec\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=y(x)/x+sec(y(x)/x),y(x), singsol=all)
```

$$y(x) = \arcsin\left(\ln\left(x\right) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.357 (sec). Leaf size: 13

```
DSolve[y'[x]==y[x]/x+Sec[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin(\log(x) + c_1)$$

5.22 problem 19

Internal problem ID [996]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y'x^2 - x^2 - yx - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve(x^2*diff(y(x),x)=x*y(x)+x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \tan(\ln(x) + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.168 (sec). Leaf size: 13

```
DSolve[x^2*y'[x]==x*y[x]+x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \tan(\log(x) + c_1)$$

5.23 problem 20

Internal problem ID [997]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{xyy' - x^2 - 2y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(x*y(x)*diff(y(x),x)=x^2+2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x^2 - 1}\, x$$
$$y(x) = -\sqrt{c_1 x^2 - 1}\, x$$

✓ Solution by Mathematica

Time used: 0.385 (sec). Leaf size: 38

```
DSolve[x*y[x]*y'[x]==x^2+2*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{-1 + c_1 x^2}$$
$$y(x) \to x\sqrt{-1 + c_1 x^2}$$

5.24 problem 21

Internal problem ID [998]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘]]`

$$\boxed{y' - \frac{2y^2 + x^2 \mathrm{e}^{-\frac{y^2}{x^2}}}{2yx} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)=(2*y(x)^2+x^2*exp(- (y(x)/x)^2 ))/(2*x*y(x)),y(x), singsol=all)
```

$$y(x) = \sqrt{\ln(\ln(x) + c_1)}\, x$$
$$y(x) = -\sqrt{\ln(\ln(x) + c_1)}\, x$$

✓ Solution by Mathematica

Time used: 2.12 (sec). Leaf size: 38

```
DSolve[y'[x]==(2*y[x]^2+x^2*Exp[- (y[x]/x)^2 ])/(2*x*y[x]),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -x\sqrt{\log(\log(x) + 2c_1)}$$
$$y(x) \to x\sqrt{\log(\log(x) + 2c_1)}$$

5.25 problem 22

Internal problem ID [999]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{yx + y^2}{x^2} = 0}$$

With initial conditions

$$[y(-1) = 2]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 21

```
dsolve([diff(y(x),x)=(x*y(x)+y(x)^2)/x^2,y(-1) = 2],y(x), singsol=all)
```

$$y(x) = -\frac{2ix}{2i\ln(x) + i + 2\pi}$$

✓ Solution by Mathematica

Time used: 0.128 (sec). Leaf size: 21

```
DSolve[{y'[x]==(x*y[x]+y[x]^2)/x^2,y[-1]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x}{2\log(x) - 2i\pi + 1}$$

5.26 problem 23

Internal problem ID [1000]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y' - \frac{x^3 + y^3}{y^2 x} = 0}$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=(x^3+y(x)^3)/(x*y(x)^2),y(1) = 3],y(x), singsol=all)
```

$$y(x) = (3\ln(x) + 27)^{\frac{1}{3}} x$$

✓ Solution by Mathematica

Time used: 0.185 (sec). Leaf size: 20

```
DSolve[{y'[x]==(x^3+y[x]^3)/(x*y[x]^2),y[1]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{3} x \sqrt[3]{\log(x) + 9}$$

5.27 problem 24

Internal problem ID [1001]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{xyy' + x^2 + y^2 = 0}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 18

```
dsolve([x*y(x)*diff(y(x),x)+x^2+y(x)^2=0,y(1) = 2],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-2x^4 + 18}}{2x}$$

✓ Solution by Mathematica

Time used: 0.202 (sec). Leaf size: 25

```
DSolve[{x*y[x]*y'[x]+x^2+y[x]^2==0,y[1]==2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{9 - x^4}}{\sqrt{2}x}$$

5.28 problem 25

Internal problem ID [1002]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y' - \frac{y^2 - 3yx - 5x^2}{x^2} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.39 (sec). Leaf size: 23

```
dsolve([diff(y(x),x)=(y(x)^2-3*x*y(x)-5*x^2)/x^2,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{-2x^7 + 5x}{2x^6 + 1}$$

✓ Solution by Mathematica

Time used: 1.891 (sec). Leaf size: 20

```
DSolve[{y'[x]==(y[x]^2-3*x*y[x]-5*x^2)/x^2,y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(\frac{6}{2x^6 + 1} - 1\right)$$

5.29 problem 26

Internal problem ID [1003]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y'x^2 - 2x^2 - y^2 - 4yx = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 21

```
dsolve([x^2*diff(y(x),x)=2*x^2+y(x)^2+4*x*y(x),y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{-4x^2 + 3x}{2x - 3}$$

✓ Solution by Mathematica

Time used: 0.428 (sec). Leaf size: 19

```
DSolve[{x^2*y'[x]==2*x^2+y[x]^2+4*x*y[x],y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(3 - 4x)x}{2x - 3}$$

5.30 problem 27

Internal problem ID [1004]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{xyy' - 3x^2 - 4y^2 = 0}$$

With initial conditions

$$\left[y(1) = \sqrt{3}\right]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 15

```
dsolve([x*y(x)*diff(y(x),x)=3*x^2+4*y(x)^2,y(1) = 3^(1/2)],y(x), singsol=all)
```

$$y(x) = \sqrt{4x^6 - 1}\, x$$

✓ Solution by Mathematica

Time used: 0.566 (sec). Leaf size: 18

```
DSolve[{x*y[x]*y'[x]==3*x^2+4*y[x]^2,y[1]==Sqrt[3]},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\sqrt{4x^6 - 1}$$

5.31 problem 28

Internal problem ID [1005]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{x+y}{x-y} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=(x+y(x))/(x-y(x)),y(x), singsol=all)
```

$$y(x) = \tan\left(\operatorname{RootOf}\left(-2\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(x) + 2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 36

```
DSolve[y'[x]==(x+y[x])/(x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right) - \arctan\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

5.32 problem 29

Internal problem ID [1006]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘]]`

$$\boxed{(y'x - y)(\ln(y) - \ln(x)) - x = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 19

```
dsolve((diff(y(x),x)*x-y(x))*(ln(y(x))-ln(x))=x,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{LambertW}\left(\ln(c_1 x)\mathrm{e}^{-1}\right)+1} x$$

✓ Solution by Mathematica

Time used: 60.156 (sec). Leaf size: 21

```
DSolve[(y'[x]*x-y[x])*(Log[y[x]]-Log[x])==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x e^{1+W\left(\frac{\log(x)+c_1}{e}\right)}$$

5.33 problem 30

Internal problem ID [1007]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$y' - \frac{y^3 + 2xy^2 + x^2y + x^3}{x(x+y)^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 96

```
dsolve(diff(y(x),x)=(y(x)^3+2*x*y(x)^2+x^2*y(x)+x^3)/(x*(y(x)+x)^2),y(x), singsol=all)
```

$$y(x) = (3\ln(x) + 3c_1)^{\frac{1}{3}} x - x$$
$$y(x) = \left(-\frac{(3\ln(x) + 3c_1)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\,(3\ln(x) + 3c_1)^{\frac{1}{3}}}{2}\right) x - x$$
$$y(x) = \left(-\frac{(3\ln(x) + 3c_1)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\,(3\ln(x) + 3c_1)^{\frac{1}{3}}}{2}\right) x - x$$

✓ Solution by Mathematica

Time used: 1.285 (sec). Leaf size: 109

```
DSolve[y'[x]==(y[x]^3+2*x*y[x]^2+x^2*y[x]+x^3)/(x*(y[x]+x)^2),y[x],x,IncludeSingularSolutions
```

$$y(x) \to -x + \sqrt[3]{x^3(3\log(x) + 1 + 3c_1)}$$
$$y(x) \to -x + \frac{1}{2}i\left(\sqrt{3} + i\right)\sqrt[3]{x^3(3\log(x) + 1 + 3c_1)}$$
$$y(x) \to -x - \frac{1}{2}\left(1 + i\sqrt{3}\right)\sqrt[3]{x^3(3\log(x) + 1 + 3c_1)}$$

5.34 problem 31

Internal problem ID [1008]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001

Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68

Problem number: 31.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{x + 2y}{2x + y} = 0}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 397

```
dsolve(diff(y(x),x)=(x+2*y(x))/(2*x+y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{x\left(-c_1^2 - c_1^2\left(\frac{\left(3\sqrt{3}\,\sqrt{27c_1^2x^2-1}+27c_1x\right)^{\frac{1}{3}}}{3xc_1} + \frac{1}{xc_1\left(3\sqrt{3}\,\sqrt{27c_1^2x^2-1}+27c_1x\right)^{\frac{1}{3}}}\right)\right)}{c_1^2}$$

$$y(x) = -\frac{x\left(-c_1^2 - c_1^2\left(-\frac{\left(3\sqrt{3}\,\sqrt{27c_1^2x^2-1}+27c_1x\right)^{\frac{1}{3}}}{6xc_1} - \frac{1}{2xc_1\left(3\sqrt{3}\,\sqrt{27c_1^2x^2-1}+27c_1x\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(3\sqrt{3}\,\sqrt{27c_1^2x^2-1}+27c_1x\right)^{\frac{1}{3}}}{3xc_1} - \frac{}{xc_1\left(3\sqrt{3}\right.}\right.}{2}\right.\right.}{c_1^2}$$

$$y(x) = -\frac{x\left(-c_1^2 - c_1^2\left(-\frac{\left(3\sqrt{3}\,\sqrt{27c_1^2x^2-1}+27c_1x\right)^{\frac{1}{3}}}{6xc_1} - \frac{1}{2xc_1\left(3\sqrt{3}\,\sqrt{27c_1^2x^2-1}+27c_1x\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(3\sqrt{3}\,\sqrt{27c_1^2x^2-1}+27c_1x\right)^{\frac{1}{3}}}{3xc_1} - \frac{}{xc_1\left(3\sqrt{3}\right.}\right.}{2}\right.\right.}{c_1^2}$$

✓ Solution by Mathematica

Time used: 30.908 (sec). Leaf size: 338

```
DSolve[y'[x]==(x+2*y[x])/(2*x+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{\sqrt{3}\sqrt{27e^{4c_1}x^2+e^{6c_1}}-9e^{2c_1}x}}{3^{2/3}} - \frac{e^{2c_1}}{\sqrt[3]{3}\sqrt[3]{\sqrt{3}\sqrt{27e^{4c_1}x^2+e^{6c_1}}-9e^{2c_1}x}} + x$$

$$y(x) \to \left(-\frac{1}{3}\right)^{2/3}\sqrt[3]{\sqrt{3}\sqrt{27e^{4c_1}x^2+e^{6c_1}}-9e^{2c_1}x} + \frac{\sqrt[3]{-\frac{1}{3}}e^{2c_1}}{\sqrt[3]{\sqrt{3}\sqrt{27e^{4c_1}x^2+e^{6c_1}}-9e^{2c_1}x}} + x$$

$$y(x) \to \frac{1}{3}\left(-\sqrt[3]{-3}\sqrt[3]{\sqrt{3}\sqrt{27e^{4c_1}x^2+e^{6c_1}}-9e^{2c_1}x} - \frac{(-3)^{2/3}e^{2c_1}}{\sqrt[3]{\sqrt{3}\sqrt{27e^{4c_1}x^2+e^{6c_1}}-9e^{2c_1}x}} + 3x\right)$$

5.35 problem 32

Internal problem ID [1009]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{y}{y - 2x} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 335

```
dsolve(diff(y(x),x)=y(x)/(y(x)-2*x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-12c_1 + 8x^3 + 4\sqrt{-12c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{2x^2}{\left(-12c_1 + 8x^3 + 4\sqrt{-12c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} + x$$

$$y(x) = -\frac{\left(-12c_1 + 8x^3 + 4\sqrt{-12c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^2}{\left(-12c_1 + 8x^3 + 4\sqrt{-12c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} + x - \frac{i\sqrt{3}\left(\frac{\left(-12c_1+8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{2x^2}{\left(-12c_1+8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(-12c_1 + 8x^3 + 4\sqrt{-12c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^2}{\left(-12c_1 + 8x^3 + 4\sqrt{-12c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} + x + \frac{i\sqrt{3}\left(\frac{\left(-12c_1+8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{2x^2}{\left(-12c_1+8x^3+4\sqrt{-12c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 35.624 (sec). Leaf size: 479

```
DSolve[y'[x]==y[x]/(y[x]-2*x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}x^2}{\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}} + x$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}}{2\sqrt[3]{2}} - \frac{(1+i\sqrt{3})\,x^2}{2^{2/3}\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}} + x$$

$$y(x) \to -\frac{(1+i\sqrt{3})\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}}{2\sqrt[3]{2}} + \frac{i(\sqrt{3}+i)\,x^2}{2^{2/3}\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}} + x$$

$$y(x) \to 0$$

$$y(x) \to -\frac{i\left(\sqrt[3]{x^3} - x\right)\left(\left(\sqrt{3} - i\right)\sqrt[3]{x^3} - 2ix\right)}{2x}$$

$$y(x) \to \frac{i\left(\sqrt[3]{x^3} - x\right)\left(\left(\sqrt{3} + i\right)\sqrt[3]{x^3} + 2ix\right)}{2x}$$

$$y(x) \to \sqrt[3]{x^3} + \frac{(x^3)^{2/3}}{x} + x$$

5.36 problem 33

Internal problem ID [1010]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y' - \frac{x y^2 + 2 y^3}{x^3 + x^2 y + x y^2} = 0}$$

✓ Solution by Maple

Time used: 1.109 (sec). Leaf size: 129

```
dsolve(diff(y(x),x)=(x*y(x)^2+2*y(x)^3)/(x^3+x^2*y(x)+x*y(x)^2),y(x), singsol=all)
```

$$\begin{aligned} y(x) = {} & c_1 \operatorname{RootOf}\left(\_Z^8 c_1 x^2 + 2\_Z^6 c_1 x^2 + \_Z^4 c_1 x^2 - 2\_Z^2 - 1\right)^6 x^3 \\ & + 2c_1 \operatorname{RootOf}\left(\_Z^8 c_1 x^2 + 2\_Z^6 c_1 x^2 + \_Z^4 c_1 x^2 - 2\_Z^2 - 1\right)^4 x^3 \\ & + c_1 \operatorname{RootOf}\left(\_Z^8 c_1 x^2 + 2\_Z^6 c_1 x^2 + \_Z^4 c_1 x^2 - 2\_Z^2 - 1\right)^2 x^3 - x \end{aligned}$$

✓ Solution by Mathematica

Time used: 60.15 (sec). Leaf size: 1989

```
DSolve[y'[x]==(x*y[x]^2+2*y[x]^3)/(x^3+x^2*y[x]+x*y[x]^2),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{6}\left(-\sqrt{3}\sqrt{-2e^{2c_1}x^4+3x^2+\frac{e^{4c_1}x^8}{\sqrt[3]{e^{6c_1}x^{12}+54e^{2c_1}x^8+6\sqrt{3}\sqrt{e^{4c_1}x^{16}\left(27+e^{4c_1}x^4\right)}}}+\sqrt[3]{e^{6c_1}x^{12}+54e^{2c_1}x^8}\cdots} - \sqrt{3}\sqrt{-4e^{2c_1}x^4+6x^2-\frac{e^{4c_1}x^8}{\sqrt[3]{e^{6c_1}x^{12}+54e^{2c_1}x^8+6\sqrt{3}\sqrt{e^{4c_1}x^{16}\left(27+e^{4c_1}x^4\right)}}}-\sqrt[3]{e^{6c_1}x^{12}+54e^{2c_1}x^8+6\cdots}\cdots} + 3x\right)$$

$$y(x) \to \frac{1}{6}\left(-\sqrt{3}\sqrt{-2e^{2c_1}x^4+3x^2+\frac{e^{4c_1}x^8}{\sqrt[3]{e^{6c_1}x^{12}+54e^{2c_1}x^8+6\sqrt{3}\sqrt{e^{4c_1}x^{16}\left(27+e^{4c_1}x^4\right)}}}+\sqrt[3]{e^{6c_1}x^{12}+54e^{2c_1}x^8}\cdots} + \sqrt{3}\sqrt{-4e^{2c_1}x^4+6x^2-\frac{e^{4c_1}x^8}{\sqrt[3]{e^{6c_1}x^{12}+54e^{2c_1}x^8+6\sqrt{3}\sqrt{e^{4c_1}x^{16}\left(27+e^{4c_1}x^4\right)}}}-\sqrt[3]{e^{6c_1}x^{12}+54e^{2c_1}x^8+6\cdots}\cdots} + 3x\right)$$

$$y(x)$$

5.37 problem 34

Internal problem ID [1011]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y' - \frac{x^3 + x^2y + 3y^3}{x^3 + 3xy^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 369

```
dsolve(diff(y(x),x)=(x^3+x^2*y(x)+3*y(x)^3)/(x^3+3*x*y(x)^2),y(x), singsol=all)
```

$$y(x) = \left(\frac{\left(108\ln(x)+108c_1+12\sqrt{12+81\ln(x)^2+162\ln(x)c_1+81c_1^2}\right)^{\frac{1}{3}}}{6} - \frac{2}{\left(108\ln(x)+108c_1+12\sqrt{12+81\ln(x)^2+162\ln(x)c_1+81c_1^2}\right)^{\frac{1}{3}}}\right)x$$

$$y(x) = \left(-\frac{\left(108\ln(x)+108c_1+12\sqrt{12+81\ln(x)^2+162\ln(x)c_1+81c_1^2}\right)^{\frac{1}{3}}}{12} + \frac{1}{\left(108\ln(x)+108c_1+12\sqrt{12+81\ln(x)^2+162\ln(x)c_1+81c_1^2}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(108\ln(x)+108c_1+12\sqrt{12+81\ln(x)^2+162\ln(x)c_1+81c_1^2}\right)^{\frac{1}{3}}}{6} + \frac{2}{\left(108\ln(x)+108c_1+12\sqrt{12+81\ln(x)^2+162\ln(x)c_1+81c_1^2}\right)^{\frac{1}{3}}}\right)}{2}\right)$$

$$y(x) = \left(-\frac{\left(108\ln(x)+108c_1+12\sqrt{12+81\ln(x)^2+162\ln(x)c_1+81c_1^2}\right)^{\frac{1}{3}}}{12} + \frac{1}{\left(108\ln(x)+108c_1+12\sqrt{12+81\ln(x)^2+162\ln(x)c_1+81c_1^2}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(108\ln(x)+108c_1+12\sqrt{12+81\ln(x)^2+162\ln(x)c_1+81c_1^2}\right)^{\frac{1}{3}}}{6} + \frac{2}{\left(108\ln(x)+108c_1+12\sqrt{12+81\ln(x)^2+162\ln(x)c_1+81c_1^2}\right)^{\frac{1}{3}}}\right)}{2}\right)$$

✓ Solution by Mathematica

Time used: 27.048 (sec). Leaf size: 398

```
DSolve[y'[x]==(x^3+x^2*y[x]+3*y[x]^3)/(x^3+3*x*y[x]^2),y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{-2\sqrt[3]{3}x^2 + \sqrt[3]{2}\left(\sqrt{3}\sqrt{x^6\left(4+27(\log(x)+c_1)^2\right)} + 9x^3\log(x) + 9c_1x^3\right)^{2/3}}{6^{2/3}\sqrt[3]{\sqrt{3}\sqrt{x^6\left(4+27(\log(x)+c_1)^2\right)} + 9x^3\log(x) + 9c_1x^3}}$$

$$y(x) \to \frac{2\sqrt[3]{3}(1+i\sqrt{3})\,x^2 + i\sqrt[3]{2}(\sqrt{3}+i)\left(\sqrt{3}\sqrt{x^6\left(4+27(\log(x)+c_1)^2\right)} + 9x^3\log(x) + 9c_1x^3\right)^{2/3}}{2\ 6^{2/3}\sqrt[3]{\sqrt{3}\sqrt{x^6\left(4+27(\log(x)+c_1)^2\right)} + 9x^3\log(x) + 9c_1x^3}}$$

$$y(x) \to \frac{2\sqrt[3]{3}(1-i\sqrt{3})\,x^2 - \sqrt[3]{2}(1+i\sqrt{3})\left(\sqrt{3}\sqrt{x^6\left(4+27(\log(x)+c_1)^2\right)} + 9x^3\log(x) + 9c_1x^3\right)^{2/3}}{2\ 6^{2/3}\sqrt[3]{\sqrt{3}\sqrt{x^6\left(4+27(\log(x)+c_1)^2\right)} + 9x^3\log(x) + 9c_1x^3}}$$

5.38 problem 35(a)

Internal problem ID [1012]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 35(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Riccati]`

$$\boxed{y'x^2 - y^2 - yx + 4x^2 = 0}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.406 (sec). Leaf size: 19

```
dsolve([x^2*diff(y(x),x)=y(x)^2+x*y(x)-4*x^2,y(-1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{-2x^5 + 2x}{x^4 + 1}$$

✓ Solution by Mathematica

Time used: 2.085 (sec). Leaf size: 18

```
DSolve[{x^2*y'[x]==y[x]^2+x*y[x]-4*x^2,y[-1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(\frac{4}{x^4 + 1} - 2\right)$$

5.39 problem 36(a)

Internal problem ID [1013]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 36(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{xyy' - x^2 + yx - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve(x*y(x)*diff(y(x),x)=x^2-x*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(\frac{\mathrm{e}^{-1}\mathrm{e}^{-c_1}}{x}\right)-c_1-1} + x$$

✓ Solution by Mathematica

Time used: 3.588 (sec). Leaf size: 25

```
DSolve[x*y[x]*y'[x]==x^2-x*y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(1 + W\left(\frac{e^{-1+c_1}}{x}\right)\right)$$
$$y(x) \to x$$

5.40 problem 37(a)

Internal problem ID [1014]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 37(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$y' - \frac{2y^2 - yx + 2x^2}{yx + 2x^2} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 43

```
dsolve(diff(y(x),x)=(2*y(x)^2-x*y(x)+2*x^2)/(x*y(x)+2*x^2),y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(\_Z^4 + c_1x + 16 + (-3c_1x - 32)\_Z + (3c_1x + 24)\_Z^2 + (-c_1x - 8)\_Z^3\right)x$$

✓ Solution by Mathematica

Time used: 60.158 (sec). Leaf size: 1913

```
DSolve[y'[x]==(2*y[x]^2-x*y[x]+2*x^2)/(x*y[x]+2*x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(-\sqrt{9e^{2c_1}x^4+6\,2^{2/3}\sqrt[3]{3\sqrt{3}\sqrt{e^{3c_1}x^{15}\left(-256+27e^{c_1}x\right)}+27e^{2c_1}x^8}+24e^{c_1}x^3\left(-3+\frac{2}{\sqrt[3]{\sqrt{3}\sqrt{e^{3c_1}x^{15}\left(\right.}}}\right.}\right.$$

$$-6\sqrt{\frac{1}{2}x^2\left(-8+e^{c_1}x\right)^2+4x^2\left(-8+e^{c_1}x\right)-\frac{\sqrt[3]{27e^{2c_1}x^8+\sqrt{729e^{4c_1}x^{16}-6912e^{3c_1}x^{15}}}}{3\sqrt[3]{2}}-\frac{}{\sqrt[3]{\sqrt{3}\sqrt{e^{3c_1}x^{15}\left(\right.}}}}$$

$$\left.-3x\left(-8+e^{c_1}x\right)\right)$$

$$y(x) \to \frac{1}{12}\left(-\sqrt{9e^{2c_1}x^4+6\,2^{2/3}\sqrt[3]{3\sqrt{3}\sqrt{e^{3c_1}x^{15}\left(-256+27e^{c_1}x\right)}+27e^{2c_1}x^8}+24e^{c_1}x^3\left(-3+\frac{2}{\sqrt[3]{\sqrt{3}\sqrt{e^{3c_1}x^{15}\left(\right.}}}\right.}\right.$$

$$+6\sqrt{\frac{1}{2}x^2\left(-8+e^{c_1}x\right)^2+4x^2\left(-8+e^{c_1}x\right)-\frac{\sqrt[3]{27e^{2c_1}x^8+\sqrt{729e^{4c_1}x^{16}-6912e^{3c_1}x^{15}}}}{3\sqrt[3]{2}}-\frac{}{\sqrt[3]{\sqrt{3}\sqrt{e^{3c_1}x^{15}\left(\right.}}}}$$

5.41 problem 38

Internal problem ID [1015]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{x^2 + yx + y^2}{yx} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)=(x*y(x)+x^2+y(x)^2)/(x*y(x)),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-1}\mathrm{e}^{-c_1}}{x}\right)-c_1-1} - x$$

✓ Solution by Mathematica

Time used: 4.386 (sec). Leaf size: 31

```
DSolve[y'[x]==(x*y[x]+x^2+y[x]^2)/(x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\left(1 + W\left(-\frac{e^{-1-c_1}}{x}\right)\right)$$

$$y(x) \to -x$$

5.42 problem 41

Internal problem ID [1016]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{-6x + y - 3}{2x - y - 1} = 0}$$

✓ Solution by Maple

Time used: 0.421 (sec). Leaf size: 51

```
dsolve(diff(y(x),x)=(-6*x+y(x)-3)/(2*x-y(x)-1),y(x), singsol=all)
```

$$y(x) = -3 - \frac{\frac{\operatorname{RootOf}\left(\_Z^{25} - 5(x+1)^5 c_1 \_Z^5 - (x+1)^5 c_1\right)^{20}}{c_1} - 3(x+1)^5}{(x+1)^4}$$

✓ Solution by Mathematica

Time used: 60.098 (sec). Leaf size: 3011

```
DSolve[y'[x]==(-6*x+y[x]-3)/(2*x-y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

5.43 problem 42

Internal problem ID [1017]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{2x + y + 1}{x + 2y - 4} = 0}$$

✓ Solution by Maple

Time used: 1.594 (sec). Leaf size: 64

```
dsolve(diff(y(x),x)=(2*x+y(x)+1)/(x+2*y(x)-4),y(x), singsol=all)
```

$$y(x) = 3 + \frac{(2+x)\left(\operatorname{RootOf}\left(\_Z^{16} + 2(2+x)^4 c_1\_Z^4 - (2+x)^4 c_1\right)^4 - 1\right)}{\operatorname{RootOf}\left(\_Z^{16} + 2(2+x)^4 c_1\_Z^4 - (2+x)^4 c_1\right)^4}$$

✓ Solution by Mathematica

Time used: 60.306 (sec). Leaf size: 8077

```
DSolve[y'[x]==(2*x+y[x]+1)/(x+2*y[x]-4),y[x],x,IncludeSingularSolutions -> True]
```

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5.44 problem 43

Internal problem ID [1018]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{-x+3y-14}{x+y-2} = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)=(-x+3*y(x)-14)/(x+y(x)-2),y(x), singsol=all)
```

$$y(x) = 4 + \frac{(2+x)\left(\operatorname{LambertW}\left(-2c_1(2+x)\right)+2\right)}{\operatorname{LambertW}\left(-2c_1\left(2+x\right)\right)}$$

✓ Solution by Mathematica

Time used: 1.003 (sec). Leaf size: 144

```
DSolve[y'[x]==(-x+3*y[x]-14)/(x+y[x]-2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{2^{2/3}\left(x\log\left(\frac{y(x)-x-6}{y(x)+x-2}\right)-(x+6)\log\left(\frac{x+2}{y(x)+x-2}\right)+6\log\left(\frac{y(x)-x-6}{y(x)+x-2}\right)+y(x)\left(\log\left(\frac{x+2}{y(x)+x-2}\right)-\log\left(\frac{y}{y}\right.\right.\right.}{9(-y(x)+x+6)}\right.$$

5.45 problem 44

Internal problem ID [1019]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{3y^2y'x - y^3 - x = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 78

```
dsolve(3*x*y(x)^2*diff(y(x),x)=y(x)^3+x,y(x), singsol=all)
```

$$y(x) = (x\ln(x) + c_1x)^{\frac{1}{3}}$$
$$y(x) = -\frac{(x\ln(x) + c_1x)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\,(x\ln(x) + c_1x)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{(x\ln(x) + c_1x)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\,(x\ln(x) + c_1x)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 0.183 (sec). Leaf size: 69

```
DSolve[3*x*y[x]^2*y'[x]==y[x]^3+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{x}\sqrt[3]{\log(x) + c_1}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{x}\sqrt[3]{\log(x) + c_1}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{x}\sqrt[3]{\log(x) + c_1}$$

5.46 problem 45

Internal problem ID [1020]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$xyy' - 3x^6 - 6y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(x*y(x)*diff(y(x),x)=3*x^6+6*y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1x^6 - 1}\, x^3$$
$$y(x) = -\sqrt{c_1x^6 - 1}\, x^3$$

✓ Solution by Mathematica

Time used: 0.415 (sec). Leaf size: 42

```
DSolve[x*y[x]*y'[x]==3*x^6+6*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^3\sqrt{-1 + c_1x^6}$$
$$y(x) \to x^3\sqrt{-1 + c_1x^6}$$

5.47 problem 46

Internal problem ID [1021]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{x^3y' - 2y^2 - 2x^2y + 2x^4 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x^3*diff(y(x),x)=2*(y(x)^2+x^2*y(x)-x^4),y(x), singsol=all)
```

$$y(x) = \tanh\left(-2\ln\left(x\right) + 2c_1\right) x^2$$

✓ Solution by Mathematica

Time used: 0.885 (sec). Leaf size: 62

```
DSolve[x^3*y'[x]==2*(y[x]^2+x^2*y[x]-x^4),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to ix^2 \tan(2i \log(x) + c_1)$$

$$y(x) \to \frac{x^2\left(-x^4 + e^{2i\text{Interval}[\{0,\pi\}]}\right)}{x^4 + e^{2i\text{Interval}[\{0,\pi\}]}}$$

5.48 problem 47

Internal problem ID [1022]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$\boxed{y' - y^2 \mathrm{e}^{-x} - 4y - 2\,\mathrm{e}^{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)=y(x)^2*exp(-x)+4*y(x)+2*exp(x),y(x), singsol=all)
```

$$y(x) = -\frac{2\,\mathrm{e}^{x}(c_1 \mathrm{e}^{x} - 1)}{-2 + c_1 \mathrm{e}^{x}}$$

✓ Solution by Mathematica

Time used: 0.27 (sec). Leaf size: 30

```
DSolve[y'[x]==y[x]^2*Exp[-x]+4*y[x]+2*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2e^{x} + \frac{1}{e^{-x} + c_1}$$
$$y(x) \to -2e^{x}$$

5.49 problem 48

Internal problem ID [1023]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \frac{y^2 + y\tan(x) + \tan(x)^2}{\sin(x)^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 115

```
dsolve(diff(y(x),x)=(y(x)^2+y(x)*tan(x)+tan(x)^2)/sin(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)\left(-c_1 \sin\left(-\ln(\sin(x)) + \frac{\ln(\sin(x)+1)}{2} + \frac{\ln(\sin(x)-1)}{2}\right) + \cos\left(-\ln(\sin(x)) + \frac{\ln(\sin(x)+1)}{2} + \frac{\ln(\sin}{}\right.\right.}{(\sin(x)+1)(\sin(x)-1)\left(c_1 \cos\left(-\ln(\sin(x)) + \frac{\ln(\sin(x)+1)}{2} + \frac{\ln(\sin(x)-1)}{2}\right) + \sin\left(-\ln(\sin(x)) + \frac{\ln(\sin}{}\right.\right.}$$

✓ Solution by Mathematica

Time used: 0.651 (sec). Leaf size: 20

```
DSolve[y'[x]==(y[x]^2+y[x]*Tan[x]+Tan[x]^2)/Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(x)\tan(\log(\sin(x)) - \log(\cos(x)) + c_1)$$

5.50 problem 49

Internal problem ID [1024]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]'], _Riccati]`

$$x \ln(x)^2 y' + 4\ln(x)^2 - \ln(x) y - y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*(ln(x))^2*diff(y(x),x)=-4*(ln(x))^2+y(x)*ln(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = 2i \tan(2i \ln(\ln(x)) + c_1) \ln(x)$$

✓ Solution by Mathematica

Time used: 1.04 (sec). Leaf size: 64

```
DSolve[x*(Log[x])^2*y'[x]==-4*(Log[x])^2+y[x]*Log[x]+y[x]^2,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to 2i \log(x) \tan(2i \log(\log(x)) + c_1)$$

$$y(x) \to \frac{2\log(x) \left(-\log^4(x) + e^{2i\text{Interval}[\{0,\pi\}]}\right)}{\log^4(x) + e^{2i\text{Interval}[\{0,\pi\}]}}$$

5.51 problem 50

Internal problem ID [1025]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$2x\left(y+2\sqrt{x}\right)y'-\left(y+\sqrt{x}\right)^2=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 64

```
dsolve(2*x*(y(x)+2*sqrt(x))*diff(y(x),x)=(y(x)+sqrt(x))^2,y(x), singsol=all)
```

$$y(x)=\frac{-2x+\sqrt{\ln\left(x\right)x^2-c_1x^2+4x^2}}{\sqrt{x}}$$

$$y(x)=-\frac{2x+\sqrt{\ln\left(x\right)x^2-c_1x^2+4x^2}}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.527 (sec). Leaf size: 68

```
DSolve[2*x*(y[x]+2*Sqrt[x])*y'[x]==(y[x]+Sqrt[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -2\sqrt{x}-\sqrt{\frac{1}{x^2}}x\sqrt{x(\log(x)+4+c_1)}$$

$$y(x)\to -2\sqrt{x}+\sqrt{\frac{1}{x^2}}x\sqrt{x(\log(x)+4+c_1)}$$

5.52 problem 51

Internal problem ID [1026]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(y)]‘], [_Abel, ‘2nd type‘`

$$\left(y+\mathrm{e}^{x^2}\right) y'-2x\left(y^2+y\,\mathrm{e}^{x^2}+\mathrm{e}^{2x^2}\right)=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve((y(x)+exp(x^2))*diff(y(x),x)=2*x*(y(x)^2+y(x)*exp(x^2)+exp(2*x^2)),y(x), singsol=all)
```

$$y(x)=\left(-1-\sqrt{2x^2-2c_1+1}\right)\mathrm{e}^{x^2}$$
$$y(x)=\left(-1+\sqrt{2x^2-2c_1+1}\right)\mathrm{e}^{x^2}$$

✓ Solution by Mathematica

Time used: 0.701 (sec). Leaf size: 76

```
DSolve[(y[x]+Exp[x^2])*y'[x]==2*x*(y[x]^2+y[x]*Exp[x^2]+Exp[2*x^2]),y[x],x,IncludeSingularSol
```

$$y(x)\to -e^{x^2}-\frac{\sqrt{2x^2+1+c_1}}{\sqrt{e^{-2x^2}}}$$
$$y(x)\to -e^{x^2}+\frac{\sqrt{2x^2+1+c_1}}{\sqrt{e^{-2x^2}}}$$

5.53 problem 52

Internal problem ID [1027]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' + \frac{2y}{x} - \frac{3x^2y^2 + 6yx + 2}{x^2\,(2yx + 3)} = 0}$$

With initial conditions

$$[y(2) = 2]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 18

```
dsolve([diff(y(x),x)+2/x*y(x)=(3*x^2*y(x)^2+6*x*y(x)+2)/(x^2*(2*x*y(x)+3)),y(2) = 2],y(x), si
```

$$y(x) = \frac{-3 + \sqrt{60x + 1}}{2x}$$

✓ Solution by Mathematica

Time used: 0.637 (sec). Leaf size: 35

```
DSolve[{y'[x]+2/x*y[x]==(3*x^2*y[x]^2+6*x*y[x]+2)/(x^2*(2*x*y[x]+3)),y[2]==2},y[x],x,IncludeS
```

$$y(x) \to \frac{\sqrt{\frac{1}{x^2}}\sqrt{x^2(60x + 1)} - 3}{2x}$$

5.54 problem 53

Internal problem ID [1028]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$y' + \frac{3y}{x} - \frac{3y^2x^4 + 10x^2y + 6}{x^3(2x^2y + 5)} = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 18

```
dsolve([diff(y(x),x)+3/x*y(x)=(3*x^4*y(x)^2+10*x^2*y(x)+6)/(x^3*(2*x^2*y(x)+5)),y(1) = 1],y(x
```

$$y(x) = \frac{-5 + \sqrt{48x + 1}}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.764 (sec). Leaf size: 37

```
DSolve[{y'[x]+3/x*y[x]==(3*x^4*y[x]^2+10*x^2*y[x]+6)/(x^3*(2*x^2*y[x]+5)),y[1]==1},y[x],x,Inc
```

$$y(x) \to \frac{\sqrt{\frac{1}{x^2}}\sqrt{x^4(48x + 1)} - 5x}{2x^3}$$

5.55 problem 56

Internal problem ID [1029]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Transformation of Nonlinear Equations into Separable Equations. Section 2.4 Page 68
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - 1 - x + (1 + 2x)\, y - x y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=1+x-(1+2*x)*y(x)+x*y(x)^2,y(x), singsol=all)
```

$$y(x) = 1 - \frac{2\,\mathrm{e}^{-x}}{c_1 - 2\,(x+1)\,\mathrm{e}^{-x}}$$

✓ Solution by Mathematica

Time used: 0.148 (sec). Leaf size: 23

```
DSolve[y'[x]==1+x-(1+2*x)*y[x]+x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + \frac{1}{x + c_1 e^x + 1}$$
$$y(x) \to 1$$

6 Chapter 2, First order equations. Exact equations. Section 2.5 Page 79

6.1 problem 1

Internal problem ID [1030]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{6x^2y^2 + 4x^3yy' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(6*x^2*y(x)^2+4*x^3*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -\frac{\sqrt{-2c_1x}}{2x^2}$$
$$y(x) = \frac{\sqrt{-2c_1x}}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 23

```
DSolve[6*x^2*y[x]^2+4*x^3*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to \frac{c_1}{x^{3/2}}$$
$$y(x) \to 0$$

6.2 problem 2

Internal problem ID [1031]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{3\cos(x)\,y + 4\,\mathrm{e}^{x}x + 2x^{3}y + (3\sin(x) + 3)\,y' = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 488

```
dsolve((3*y(x)*cos(x)+4*x*exp(x)+2*x^3*y(x))+(3*sin(x)+3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(1-i\mathrm{e}^{ix})^{-4x^2}\left(\int -\frac{4x(1-i\mathrm{e}^{ix})^{4x^2}\left(\mathrm{e}^{-\frac{4ix^3\mathrm{e}^{ix}+24ix\,\mathrm{polylog}(2,i\mathrm{e}^{ix})\mathrm{e}^{ix}+3x-3ix\,\mathrm{e}^{ix}-24i\,\mathrm{polylog}(3,i\mathrm{e}^{ix})-3ix-24\,\mathrm{polylog}(3,i\mathrm{e}^{ix})\mathrm{e}^{ix}-3x\,\mathrm{e}^{ix}-24\cdots}{3(\mathrm{e}^{ix}+i)}}\right.}{\cdots}\right.}{\cdots}$$

✓ Solution by Mathematica

Time used: 26.035 (sec). Leaf size: 182

```
DSolve[(3*y[x]*Cos[x]+4*x*Exp[x]+2*x^3*y[x])+(3*Sin[x]+3)*y'[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{\exp\left(-8ix\,\mathrm{PolyLog}(2,-ie^{-ix}) - 8\,\mathrm{PolyLog}(3,-ie^{-ix}) + \frac{4x^3}{3e^{ix}+3i}\right)(\sin(x)+i\cos(x)+1)^{-4x^2}\left(\int_1^x -\frac{4}{3}\exp\cdots\right.}{\cdots}$$

6.3 problem 3

Internal problem ID [1032]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{14x^2y^3 + 21x^2y^2y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((14*x^2*y(x)^3)+(21*x^2*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1 e^{-\frac{2x}{3}}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 25

```
DSolve[(14*x^2*y[x]^3)+(21*x^2*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to c_1 e^{-2x/3}$$
$$y(x) \to 0$$

6.4 problem 4

Internal problem ID [1033]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{2x - 2y^2 + \left(12y^2 - 4yx\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 531

```
dsolve((2*x-2*y(x)^2)+(12*y(x)^2-4*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-27x^2 - 27c_1 + x^3 + 3\sqrt{-6x^5 - 6c_1x^3 + 81x^4 + 162c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}{6} + \frac{x^2}{6\left(-27x^2 - 27c_1 + x^3 + 3\sqrt{-6x^5 - 6c_1x^3 + 81x^4 + 162c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}} + \frac{x}{6}$$

$$y(x) = -\frac{\left(-27x^2 - 27c_1 + x^3 + 3\sqrt{-6x^5 - 6c_1x^3 + 81x^4 + 162c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}{12} - \frac{x^2}{12\left(-27x^2 - 27c_1 + x^3 + 3\sqrt{-6x^5 - 6c_1x^3 + 81x^4 + 162c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}} + \frac{x}{6} - \frac{i\sqrt{3}\left(\frac{\left(-27x^2-27c_1+x^3+3\sqrt{-6x^5-6c_1x^3+81x^4+162c_1x^2+81c_1^2}\right)^{\frac{1}{3}}}{6} - \frac{x^2}{6\left(-27x^2-27c_1+x^3+3\sqrt{-6x^5-6c_1x^3+81x^4+162c_1x^2+81c_1^2}\right)}\right.}{2}$$

$$y(x) = -\frac{\left(-27x^2 - 27c_1 + x^3 + 3\sqrt{-6x^5 - 6c_1x^3 + 81x^4 + 162c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}}{12} - \frac{x^2}{12\left(-27x^2 - 27c_1 + x^3 + 3\sqrt{-6x^5 - 6c_1x^3 + 81x^4 + 162c_1x^2 + 81c_1^2}\right)^{\frac{1}{3}}} + \frac{x}{6} + \frac{i\sqrt{3}\left(\frac{\left(-27x^2-27c_1+x^3+3\sqrt{-6x^5-6c_1x^3+81x^4+162c_1x^2+81c_1^2}\right)^{\frac{1}{3}}}{6} - \frac{x^2}{6\left(-27x^2-27c_1+x^3+3\sqrt{-6x^5-6c_1x^3+81x^4+162c_1x^2+81c_1^2}\right)}\right.}{2}$$

✓ Solution by Mathematica

Time used: 4.722 (sec). Leaf size: 408

```
DSolve[(2*x-2*y[x]^2)+(12*y[x]^2-4*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{3\,2^{2/3}\sqrt[3]{-2x^3+54x^2+\sqrt{-4x^6+4\left((x-27)x^2-54c_1\right)^2}+108c_1}} - \frac{\sqrt[3]{-2x^3+54x^2+\sqrt{-4x^6+4\left((x-27)x^2-54c_1\right)^2}+108c_1}}{6\sqrt[3]{2}} + \frac{x}{6}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)x^2}{6\,2^{2/3}\sqrt[3]{-2x^3+54x^2+\sqrt{-4x^6+4\left((x-27)x^2-54c_1\right)^2}+108c_1}} + \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-2x^3+54x^2+\sqrt{-4x^6+4\left((x-27)x^2-54c_1\right)^2}+108c_1}}{12\sqrt[3]{2}} + \frac{x}{6}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)x^2}{6\,2^{2/3}\sqrt[3]{-2x^3+54x^2+\sqrt{-4x^6+4\left((x-27)x^2-54c_1\right)^2}+108c_1}} + \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-2x^3+54x^2+\sqrt{-4x^6+4\left((x-27)x^2-54c_1\right)^2}+108c_1}}{12\sqrt[3]{2}} + \frac{x}{6}$$

6.5 problem 5

Internal problem ID [1034]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(x+y)^2 + (x+y)^2 y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
dsolve((x+y(x))^2+(x+y(x))^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -x$$
$$y(x) = c_1 - x$$
$$y(x) = -\frac{c_1}{2} - \frac{i\sqrt{3}\,c_1}{2} - x$$
$$y(x) = -\frac{c_1}{2} + \frac{i\sqrt{3}\,c_1}{2} - x$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[(x+y[x])^2+(x+y[x])^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x$$
$$y(x) \to -x + c_1$$

6.6 problem 6

Internal problem ID [1035]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cla`

$$4x + 7y + (3x + 4y)\, y' = 0$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 55

```
dsolve((4*x+7*y(x))+(3*x+4*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x\left(2\,\mathrm{RootOf}\left(\_Z^{36} + 3\_Z^{6} c_1 x^6 - 2c_1 x^6\right)^6 - 1\right)}{\mathrm{RootOf}\left(\_Z^{36} + 3\_Z^{6} c_1 x^6 - 2c_1 x^6\right)^6}$$

✓ Solution by Mathematica

Time used: 2.981 (sec). Leaf size: 409

```
DSolve[(4*x+7*y[x])+(3*x+4*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \mathrm{Root}\left[2\#1^6 + 21\#1^5 x + 90\#1^4 x^2 + 200\#1^3 x^3 + 240\#1^2 x^4 + 144\#1 x^5 + 32x^6 - e^{3c_1}\&, 1\right]$$
$$y(x) \to \mathrm{Root}\left[2\#1^6 + 21\#1^5 x + 90\#1^4 x^2 + 200\#1^3 x^3 + 240\#1^2 x^4 + 144\#1 x^5 + 32x^6 - e^{3c_1}\&, 2\right]$$
$$y(x) \to \mathrm{Root}\left[2\#1^6 + 21\#1^5 x + 90\#1^4 x^2 + 200\#1^3 x^3 + 240\#1^2 x^4 + 144\#1 x^5 + 32x^6 - e^{3c_1}\&, 3\right]$$
$$y(x) \to \mathrm{Root}\left[2\#1^6 + 21\#1^5 x + 90\#1^4 x^2 + 200\#1^3 x^3 + 240\#1^2 x^4 + 144\#1 x^5 + 32x^6 - e^{3c_1}\&, 4\right]$$
$$y(x) \to \mathrm{Root}\left[2\#1^6 + 21\#1^5 x + 90\#1^4 x^2 + 200\#1^3 x^3 + 240\#1^2 x^4 + 144\#1 x^5 + 32x^6 - e^{3c_1}\&, 5\right]$$
$$y(x) \to \mathrm{Root}\left[2\#1^6 + 21\#1^5 x + 90\#1^4 x^2 + 200\#1^3 x^3 + 240\#1^2 x^4 + 144\#1 x^5 + 32x^6 - e^{3c_1}\&, 6\right]$$

6.7 problem 7

Internal problem ID [1036]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{-2\sin\left(x\right)y^{2}+3y^{3}-2x+\left(4\cos\left(x\right)y+9xy^{2}\right)y'=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 758

```
dsolve((-2*y(x)^2*sin(x)+3*y(x)^3-2*x)+(4*y(x)*cos(x)+9*x*y(x)^2)*diff(y(x),x)=0,y(x), singso
```

$$y(x) = \frac{\left(972x^4 + 36\sqrt{3}\,\sqrt{243x^6 - 32\cos(x)^3 x^2 - 486x^4 c_1 + 32\cos(x)^3 c_1 + 243c_1^2 x^2}\,x - 64\cos(x)^3 - 972c_1 x^2\right.}{18x}$$
$$+ \frac{8\cos(x)^2}{9x\left(972x^4 + 36\sqrt{3}\,\sqrt{243x^6 - 32\cos(x)^3 x^2 - 486x^4 c_1 + 32\cos(x)^3 c_1 + 243c_1^2 x^2}\,x - 64\cos(x)^3 - 972\right.}$$
$$- \frac{2\cos(x)}{9x}$$

$$y(x) =$$
$$- \frac{\left(972x^4 + 36\sqrt{3}\,\sqrt{243x^6 - 32\cos(x)^3 x^2 - 486x^4 c_1 + 32\cos(x)^3 c_1 + 243c_1^2 x^2}\,x - 64\cos(x)^3 - 972c_1 x\right.}{36x}$$
$$- \frac{4\cos(x)^2}{9x\left(972x^4 + 36\sqrt{3}\,\sqrt{243x^6 - 32\cos(x)^3 x^2 - 486x^4 c_1 + 32\cos(x)^3 c_1 + 243c_1^2 x^2}\,x - 64\cos(x)^3 - 972\right.}$$
$$- \frac{2\cos(x)}{9x}$$
$$- \frac{i\sqrt{3}\left(\frac{\left(972x^4+36\sqrt{3}\,\sqrt{243x^6-32\cos(x)^3x^2-486x^4c_1+32\cos(x)^3c_1+243c_1^2x^2}\,x-64\cos(x)^3-972c_1x^2\right)^{\frac{1}{3}}}{18x} - \frac{}{9x\left(972x^4+36\sqrt{3}\,\sqrt{243x^6-\right.}\right.}{2}$$

$$y(x) =$$
$$- \frac{\left(972x^4 + 36\sqrt{3}\,\sqrt{243x^6 - 32\cos(x)^3 x^2 - 486x^4 c_1 + 32\cos(x)^3 c_1 + 243c_1^2 x^2}\,x - 64\cos(x)^3 - 972c_1 x\right.}{36x}$$
$$- \frac{4\cos(x)^2}{9x\left(972x^4 + 36\sqrt{3}\,\sqrt{243x^6 - 32\cos(x)^3 x^2 - 486x^4 c_1 + 32\cos(x)^3 c_1 + 243c_1^2 x^2}\,x - 64\cos(x)^3 - 972\right.}$$
$$- \frac{2\cos(x)}{9x}$$
$$+ \frac{i\sqrt{3}\left(\frac{\left(972x^4+36\sqrt{3}\,\sqrt{243x^6-32\cos(x)^3x^2-486x^4c_1+32\cos(x)^3c_1+243c_1^2x^2}\,x-64\cos(x)^3-972c_1x^2\right)^{\frac{1}{3}}}{18x} - \frac{}{9x\left(972x^4+36\sqrt{3}\,\sqrt{243x^6-\right.}\right.}{2}$$

✓ Solution by Mathematica

Time used: 33.84 (sec). Leaf size: 465

```
DSolve[(-2*y[x]^2*Sin[x]+3*y[x]^3-2*x)+(4*y[x]*Cos[x]+9*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSing
```

$$y(x) \to \frac{2^{2/3}\sqrt[3]{243x^2\left(x^2+c_1\right)+9\sqrt{729x^4\left(x^2+c_1\right)^2-96x^2\left(x^2+c_1\right)\cos^3(x)}-16\cos^3(x)}+\frac{\cdots}{\sqrt[3]{-8\cos^3(x)+\frac{9}{2}\left(\cdots\right.}}}{18x}$$

$$y(x) \to \frac{i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{243x^2\left(x^2+c_1\right)+9\sqrt{729x^4\left(x^2+c_1\right)^2-96x^2\left(x^2+c_1\right)\cos^3(x)}-16\cos^3(x)}-\frac{\cdots}{\sqrt[3]{243x^2\cdots}}}{36x}$$

$$y(x) \to -\frac{\cos(x)\left(8-\frac{16(-1)^{2/3}\cos(x)}{\sqrt[3]{-8\cos^3(x)+\frac{9}{2}\left(27x^2\left(x^2+c_1\right)+\sqrt{729x^4\left(x^2+c_1\right)^2-96x^2\left(x^2+c_1\right)\cos^3(x)}\right)}}\right)+2^{2\cdots}}{36x}$$

6.8 problem 8

Internal problem ID [1037]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{2x + y + (2y + 2x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 46

```
dsolve((2*x+y(x))+(2*y(x)+2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{3x}{4} + \frac{\sqrt{7}\, x \tan\left(\operatorname{RootOf}\left(\sqrt{7}\, \ln\left(\frac{7x^2}{8} + \frac{7x^2 \tan(\_Z)^2}{8}\right) + 2\sqrt{7}\, c_1 + 2\_Z\right)\right)}{4}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 62

```
DSolve[(2*x+y[x])+(2*y[x]+2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\arctan\left(\frac{\frac{4y(x)}{x}+3}{\sqrt{7}}\right)}{2\sqrt{7}} + \frac{1}{4}\log\left(\frac{2y(x)^2}{x^2} + \frac{3y(x)}{x} + 2\right) = -\frac{\log(x)}{2} + c_1, y(x)\right]$$

6.9 problem 9

Internal problem ID [1038]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{3x^2 + 2yx + 4y^2 + (x^2 + 8yx + 18y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 75

```
dsolve((3*x^2+2*x*y(x)+4*y(x)^2)+(x^2+8*x*y(x)+18*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x^2 + \sqrt{-15x^4 - 36x^3 - 16c_1x - 36c_1}}{8x + 18}$$
$$y(x) = -\frac{x^2 + \sqrt{-15x^4 - 36x^3 - 16c_1x - 36c_1}}{2(4x + 9)}$$

✓ Solution by Mathematica

Time used: 0.54 (sec). Leaf size: 84

```
DSolve[(3*x^2+2*x*y[x]+4*y[x]^2)+(x^2+8*x*y[x]+18*y[x])*y'[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to -\frac{x^2 + \sqrt{-3x^3(5x + 12) + 4c_1(4x + 9)}}{8x + 18}$$
$$y(x) \to \frac{-x^2 + \sqrt{-3x^3(5x + 12) + 4c_1(4x + 9)}}{8x + 18}$$

6.10 problem 10

Internal problem ID [1039]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$2x^2 + 8yx + y^2 + \left(2x^2 + \frac{xy^3}{3}\right) y' = 0$$

✗ Solution by Maple

```
dsolve((2*x^2+8*x*y(x)+y(x)^2)+(2*x^2+x*y(x)^3/3)*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(2*x^2+8*x*y[x]+y[x]^2)+(2*x^2+x*y[x]^3/3)*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

Not solved

6.11 problem 11

Internal problem ID [1040]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{1}{x}+2x+\left(\frac{1}{y}+2y\right)y'=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve((1/x+2*x)+(1/y(x)+2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\mathrm{e}^{-\frac{\operatorname{LambertW}\left(\frac{2\,\mathrm{e}^{-2x^2-2c_1}}{x^2}\right)}{2}-x^2-c_1}}{x}$$

✓ Solution by Mathematica

Time used: 7.957 (sec). Leaf size: 71

```
DSolve[(1/x+2*x)+(1/y[x]+2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{\sqrt{W\left(\frac{2e^{-2x^2+2c_1}}{x^2}\right)}}{\sqrt{2}}$$
$$y(x)\to\frac{\sqrt{W\left(\frac{2e^{-2x^2+2c_1}}{x^2}\right)}}{\sqrt{2}}$$
$$y(x)\to 0$$

6.12 problem 12

Internal problem ID [1041]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{\sin(x)\, y^2 + x y^3 \cos(x) + \left(\sin(x)\, y x + x y^3 \cos(x)\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 5

```
dsolve((y(x)*sin(x)*y(x)+x*y(x)^2*cos(x)*y(x))+(x*sin(x)*y(x)+x*y(x)^2*cos(x)*y(x))*diff(y(x)
```

$$y(x) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y[x]*Sin[x]*y[x]+x*y[x]^2*Cos[x]*y[x])+(x*Sin[x]*y[x]+x*y[x]^2*Cos[x]*y[x])*y'[x]==0,
```

Not solved

6.13 problem 13

Internal problem ID [1042]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\frac{x}{(x^2+y^2)^{\frac{3}{2}}} + \frac{yy'}{(x^2+y^2)^{\frac{3}{2}}} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve((x/(x^2+y(x)^2)^(3/2))+(y(x)/(x^2+y(x)^2)^(3/2))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2+c_1}$$
$$y(x) = -\sqrt{-x^2+c_1}$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 39

```
DSolve[(x/(x^2+y[x]^2)^(3/2))+(y[x]/(x^2+y[x]^2)^(3/2))*y'[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to -\sqrt{-x^2+2c_1}$$
$$y(x) \to \sqrt{-x^2+2c_1}$$

6.14 problem 14

Internal problem ID [1043]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_Abel, '2nd type', 'class B']]`

$$\boxed{e^{x}\left(x^{2}y^{2}+2x y^{2}\right)+6x+\left(2x^{2}y\,e^{x}+2\right)y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 66

```
dsolve((exp(x)*(x^2*y(x)^2+2*x*y(x)^2)+6*x)+(2*x^2*y(x)*exp(x)+2)*diff(y(x),x)=0,y(x), singso
```

$$y(x)=\frac{\left(-1+\sqrt{-3\,e^{x}x^{4}-e^{x}c_{1}x^{2}+1}\right)e^{-x}}{x^{2}}$$
$$y(x)=-\frac{\left(1+\sqrt{-3\,e^{x}x^{4}-e^{x}c_{1}x^{2}+1}\right)e^{-x}}{x^{2}}$$

✓ Solution by Mathematica

Time used: 35.852 (sec). Leaf size: 74

```
DSolve[(Exp[x]*(x^2*y[x]^2+2*x*y[x]^2)+6*x)+(2*x^2*y[x]*Exp[x]+2)*y'[x]==0,y[x],x,IncludeSing
```

$$y(x)\to -\frac{e^{-x}\left(1+\sqrt{1+e^{x}x^{2}\left(-3x^{2}+c_{1}\right)}\right)}{x^{2}}$$
$$y(x)\to \frac{e^{-x}\left(-1+\sqrt{1+e^{x}x^{2}\left(-3x^{2}+c_{1}\right)}\right)}{x^{2}}$$

6.15 problem 15

Internal problem ID [1044]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$x^2 e^{y+x^2}\left(2x^2+3\right)+4x+\left(x^3 e^{y+x^2}-12y^2\right)y'=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve((x^2*exp(x^2+y(x))*(2*x^2+3)+4*x)+(x^3*exp(x^2+y(x))-12*y(x)^2)*diff(y(x),x)=0,y(x), s
```

$$x^3 e^{x^2+y(x)}-4y(x)^3+2x^2+c_1=0$$

✓ Solution by Mathematica

Time used: 0.422 (sec). Leaf size: 30

```
DSolve[(x^2*Exp[x^2+y[x]]*(2*x^2+3)+4*x)+(x^3*Exp[x^2+y[x]]-12*y[x]^2)*y'[x]==0,y[x],x,Includ
```

$$\text{Solve}\left[2x^2+x^3e^{x^2+y(x)}-4y(x)^3=c_1,y(x)\right]$$

6.16 problem 16

Internal problem ID [1045]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{e^{yx}\left(y x^4+4 x^3\right)+3 y+\left(x^5 e^{yx}+3 x\right) y'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve((exp(x*y(x))*(x^4*y(x)+4*x^3)+3*y(x))+( x^5*exp(x*y(x))+3*x)*diff(y(x),x)=0,y(x), sing
```

$$y(x)=-\frac{3\,\mathrm{LambertW}\left(\frac{x^4 e^{-\frac{c_1}{3}}}{3}\right)+c_1}{3x}$$

✓ Solution by Mathematica

Time used: 4.266 (sec). Leaf size: 33

```
DSolve[(Exp[x*y[x]]*(x^4*y[x]+4*x^3)+3*y[x])+(x^5*Exp[x*y[x]]+3*x)*y'[x]==0,y[x],x,IncludeSin
```

$$y(x)\to\frac{c_1-3W\left(\frac{1}{3}e^{\frac{c_1}{3}}x^4\right)}{3x}$$

6.17 problem 17

Internal problem ID [1046]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, ‘2nd type‘, ‘class B‘]]`

$$3y\cos(x)\,x^2 - x^3y^2\sin(x) + 4x + \left(8y - x^4\sin(x)\,y\right)y' = 0$$

✗ Solution by Maple

```
dsolve((3*x^2*cos(x)*y(x)-x^3*y(x)*sin(x)*y(x)+4*x)+(8*y(x)-x^4*sin(x)*y(x))*diff(y(x),x)=0,y
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(3*x^2*Cos[x]*y[x]-x^3*y[x]*Sin[x]*y[x]+4*x)+(8*y[x]-x^4*Sin[x]*y[x])*y'[x]==0,y[x],x,
```

Not solved

6.18 problem 18

Internal problem ID [1047]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$4x^3y^2 - 6x^2y - 2x - 3 + (2yx^4 - 2x^3)\, y' = 0$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 22

```
dsolve([(4*x^3*y(x)^2-6*x^2*y(x)-2*x-3)+(2*x^4*y(x)-2*x^3)*diff(y(x),x)=0,y(1) = 3],y(x), sin
```

$$y(x) = \frac{x + \sqrt{2x^2 + 3x - 1}}{x^2}$$

✓ Solution by Mathematica

Time used: 0.648 (sec). Leaf size: 30

```
DSolve[{(4*x^3*y[x]^2-6*x^2*y[x]-2*x-3)+(2*x^4*y[x]-2*x^3)*y'[x]==0,y[1]==3},y[x],x,IncludeSi
```

$$y(x) \to \frac{\sqrt{x^4(x(2x+3)-1)} + x^3}{x^4}$$

6.19 problem 19

Internal problem ID [1048]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel, '2`

$$-4\cos(x)\,y + 4\cos(x)\sin(x) + \sec(x)^2 + (4y - 4\sin(x))\,y' = 0$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = 0\right]$$

✓ Solution by Maple

Time used: 0.797 (sec). Leaf size: 32

```
dsolve([(-4*y(x)*cos(x)+4*sin(x)*cos(x)+sec(x)^2)+(4*y(x)-4*sin(x))*diff(y(x),x)=0,y(1/4*Pi)
```

$$y(x) = \sin(x) - \frac{\sec(x)^2 \sqrt{2}\,\sqrt{\cos(x)^3 (2\cos(x) - \sin(x))}}{2}$$

✓ Solution by Mathematica

Time used: 13.006 (sec). Leaf size: 38

```
DSolve[{(-4*y[x]*Cos[x]+4*Sin[x]*Cos[x]+Sec[x]^2)+(4*y[x]-4*Sin[x])*y'[x]==0,y[Pi/4]==0},y[x]
```

$$y(x) \to \sin(x) + \frac{1}{2}\sqrt{-\sec^2(x)}\sqrt{\sin(2x) - 2\cos(2x) - 2}$$

6.20 problem 20

Internal problem ID [1049]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(y^3-1)\,e^x+3y^2(e^x+1)\,y'=0}$$

With initial conditions

$$[y(0)=0]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 91

```
dsolve([((y(x)^3-1)*exp(x))+(3*y(x)^2*(exp(x)+1))*diff(y(x),x)=0,y(0) = 0],y(x), singsol=all)
```

$$y(x)=\frac{\left((e^x-1)\,(1+e^x)^2\right)^{\frac{1}{3}}}{1+e^x}$$

$$y(x)=\frac{\left(i\sqrt{3}-1\right)\left((e^x-1)\,(1+e^x)^2\right)^{\frac{1}{3}}}{2+2\,e^x}$$

$$y(x)=-\frac{\left(1+i\sqrt{3}\right)\left((e^x-1)\,(1+e^x)^2\right)^{\frac{1}{3}}}{2+2\,e^x}$$

✓ Solution by Mathematica

Time used: 0.916 (sec). Leaf size: 73

```
DSolve[{((y[x]^3-1)*Exp[x])+(3*y[x]^2*(Exp[x]+1))*y'[x]==0,y[0]==0},y[x],x,IncludeSingularSol
```

$$y(x) \to \sqrt[3]{\tanh\left(\frac{x}{2}\right)}$$

$$y(x) \to -\frac{1}{2}i\left(\sqrt{3}-i\right)\sqrt[3]{\tanh\left(\frac{x}{2}\right)}$$

$$y(x) \to \frac{1}{2}i\left(\sqrt{3}+i\right)\sqrt[3]{\tanh\left(\frac{x}{2}\right)}$$

6.21 problem 21

Internal problem ID [1050]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\sin(x) - \sin(x)\, y - 2\cos(x) + y' \cos(x) = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve([(sin(x)-y(x)*sin(x)-2*cos(x))+(cos(x))*diff(y(x),x)=0,y(0) = 1],y(x), singsol=all)
```

$$y(x) = 2\tan(x) + 1$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 11

```
DSolve[{(Sin[x]-y[x]*Sin[x]-2*Cos[x])+(Cos[x])*y'[x]==0,y[0]==1},y[x],x,IncludeSingularSoluti
```

$$y(x) \to 2\tan(x) + 1$$

6.22 problem 22

Internal problem ID [1051]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(2x-1)(y-1)+(2+x)(x-3)y'=0}$$

With initial conditions

$$[y(1)=-1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve([((2*x-1)*(y(x)-1))+((x+2)*(x-3))*diff(y(x),x)=0,y(1) = -1],y(x), singsol=all)
```

$$y(x)=\frac{x^2-x+6}{(2+x)(x-3)}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 23

```
DSolve[{((2*x-1)*(y[x]-1))+((x+2)*(x-3))*y'[x]==0,y[1]==-1},y[x],x,IncludeSingularSolutions -
```

$$y(x)\to \frac{(x-1)x+6}{(x-3)(x+2)}$$

6.23 problem 23

Internal problem ID [1052]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd typ`

$$\boxed{7x + 4y + (4x + 3y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 53

```
dsolve((7*x+4*y(x))+(4*x+3*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-\frac{4c_1x}{3} - \frac{\sqrt{-5c_1^2x^2+3}}{3}}{c_1}$$
$$y(x) = \frac{-\frac{4c_1x}{3} + \frac{\sqrt{-5c_1^2x^2+3}}{3}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.448 (sec). Leaf size: 118

```
DSolve[(7*x+4*y[x])+(4*x+3*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(-4x - \sqrt{-5x^2 + 3e^{2c_1}}\right)$$
$$y(x) \to \frac{1}{3}\left(-4x + \sqrt{-5x^2 + 3e^{2c_1}}\right)$$
$$y(x) \to \frac{1}{3}\left(-\sqrt{5}\sqrt{-x^2} - 4x\right)$$
$$y(x) \to \frac{1}{3}\left(\sqrt{5}\sqrt{-x^2} - 4x\right)$$

6.24 problem 24

Internal problem ID [1053]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _Bernoulli]`

$$\boxed{e^{x}\left(y^{2} x^{4}+4 x^{3} y^{2}+1\right)+\left(2 x^{4} y\, e^{x}+2 y\right) y^{\prime}=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 66

```
dsolve((exp(x)*(x^4*y(x)^2+4*x^3*y(x)^2+1))+(2*x^4*y(x)*exp(x)+2*y(x))*diff(y(x),x)=0,y(x), s
```

$$y(x)=\frac{\sqrt{-\left(e^{x} x^{4}+1\right)\left(-c_{1}+e^{x}\right)}}{e^{x} x^{4}+1}$$
$$y(x)=-\frac{\sqrt{-\left(e^{x} x^{4}+1\right)\left(-c_{1}+e^{x}\right)}}{e^{x} x^{4}+1}$$

✓ Solution by Mathematica

Time used: 1.021 (sec). Leaf size: 64

```
DSolve[(Exp[x]*(x^4*y[x]^2+4*x^3*y[x]^2+1))+(2*x^4*y[x]*Exp[x]+2*y[x])*y'[x]==0,y[x],x,Includ
```

$$y(x)\to -\frac{\sqrt{-2e^{x}+c_{1}}}{\sqrt{2e^{x}x^{4}+2}}$$
$$y(x)\to \frac{\sqrt{-2e^{x}+c_{1}}}{\sqrt{2e^{x}x^{4}+2}}$$

6.25 problem 25

Internal problem ID [1054]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{x^3y^4 + x + (x^4y^3 + y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 111

```
dsolve((x^3*y(x)^4+x)+(x^4*y(x)^3+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-1 - \sqrt{-2x^6 - 4x^4c_1 + 1}}}{x^2}$$
$$y(x) = \frac{\sqrt{-1 + \sqrt{-2x^6 - 4x^4c_1 + 1}}}{x^2}$$
$$y(x) = -\frac{\sqrt{-1 - \sqrt{-2x^6 - 4x^4c_1 + 1}}}{x^2}$$
$$y(x) = -\frac{\sqrt{-1 + \sqrt{-2x^6 - 4x^4c_1 + 1}}}{x^2}$$

✓ Solution by Mathematica

Time used: 11.574 (sec). Leaf size: 135

```
DSolve[(x^3*y[x]^4+x)+(x^4*y[x]^3+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-\frac{1+\sqrt{-2x^6+4c_1x^4+1}}{x^4}}$$
$$y(x) \to \sqrt{-\frac{1+\sqrt{-2x^6+4c_1x^4+1}}{x^4}}$$
$$y(x) \to -\sqrt{\frac{-1+\sqrt{-2x^6+4c_1x^4+1}}{x^4}}$$
$$y(x) \to \sqrt{\frac{-1+\sqrt{-2x^6+4c_1x^4+1}}{x^4}}$$

6.26 problem 26

Internal problem ID [1055]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'],`

$$\boxed{3x^2 + 2y + (2y + 2x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve((3*x^2+2*y(x))+(2*y(x)+2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -x - \sqrt{-x^3 + x^2 - c_1}$$
$$y(x) = -x + \sqrt{-x^3 + x^2 - c_1}$$

✓ Solution by Mathematica

Time used: 0.12 (sec). Leaf size: 49

```
DSolve[(3*x^2+2*y[x])+(2*y[x]+2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \sqrt{-x^3 + x^2 + c_1}$$
$$y(x) \to -x + \sqrt{-x^3 + x^2 + c_1}$$

6.27 problem 27(a)

Internal problem ID [1056]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 27(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{x^3y^4 + 2x + \left(x^4y^3 + 3y\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 111

```
dsolve((x^3*y(x)^4+2*x)+(x^4*y(x)^3+3*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-3 - \sqrt{-4x^6 - 4x^4c_1 + 9}}}{x^2}$$
$$y(x) = \frac{\sqrt{-3 + \sqrt{-4x^6 - 4x^4c_1 + 9}}}{x^2}$$
$$y(x) = -\frac{\sqrt{-3 - \sqrt{-4x^6 - 4x^4c_1 + 9}}}{x^2}$$
$$y(x) = -\frac{\sqrt{-3 + \sqrt{-4x^6 - 4x^4c_1 + 9}}}{x^2}$$

✓ Solution by Mathematica

Time used: 11.275 (sec). Leaf size: 135

```
DSolve[(x^3*y[x]^4+2*x)+(x^4*y[x]^3+3*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-\frac{3+\sqrt{-4x^6+4c_1x^4+9}}{x^4}}$$

$$y(x) \to \sqrt{-\frac{3+\sqrt{-4x^6+4c_1x^4+9}}{x^4}}$$

$$y(x) \to -\sqrt{\frac{-3+\sqrt{-4x^6+4c_1x^4+9}}{x^4}}$$

$$y(x) \to \sqrt{\frac{-3+\sqrt{-4x^6+4c_1x^4+9}}{x^4}}$$

6.28 problem 28(a)

Internal problem ID [1057]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 28(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _Bernoulli]`

$$\boxed{x^2 + y^2 + 2xyy' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve((x^2+y(x)^2)+(2*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{3}\,\sqrt{x\,(-x^3 + 3c_1)}}{3x}$$
$$y(x) = \frac{\sqrt{3}\,\sqrt{x\,(-x^3 + 3c_1)}}{3x}$$

✓ Solution by Mathematica

Time used: 0.205 (sec). Leaf size: 60

```
DSolve[(x^2+y[x]^2)+(2*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt{-x^3 + 3c_1}}{\sqrt{3}\sqrt{x}}$$
$$y(x) \rightarrow \frac{\sqrt{-x^3 + 3c_1}}{\sqrt{3}\sqrt{x}}$$

6.29 problem 38

Internal problem ID [1058]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{y' + \frac{2y}{x} + \frac{2xy}{x^2 + 2x^2y + 1} = 0}$$

With initial conditions

$$[y(1) = -2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)+2/x*y(x)= -(2*x*y(x))/(x^2+2*x^2*y(x)+1),y(1) = -2],y(x), singsol=all)
```

$$y(x) = \frac{-x^2 - 1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.599 (sec). Leaf size: 39

```
DSolve[{y'[x]+2/x*y[x]== -(2*x*y[x])/(x^2+2*x^2*y[x]+1),y[1]==-2},y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{1}{2}\left(-\frac{x^{3/2} + \sqrt{x^3 (x^2+1)^2}}{x^{7/2}} - 1\right)$$

6.30 problem 39

Internal problem ID [1059]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y' - \frac{3y}{x} - \frac{2x^4(4x^3 - 3y)}{3x^5 + 3x^3 + 2y} = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 30

```
dsolve([diff(y(x),x)-3/x*y(x)= (2*x^4*(4*x^3-3*y(x)))/(3*x^5+3*x^3+2*y(x)),y(1) = 1],y(x), si
```

$$y(x) = \frac{\left(-3x^2 + \sqrt{9x^4 + 34x^2 + 21} - 3\right) x^3}{2}$$

✓ Solution by Mathematica

Time used: 0.669 (sec). Leaf size: 47

```
DSolve[{y'[x]-3/x*y[x]== (2*x^4*(4*x^3-3*y[x]))/(3*x^5+3*x^3+2*y[x]),y[1]==1},y[x],x,IncludeS
```

$$y(x) \to \frac{1}{2} x^3 \left(x^2 \left(\sqrt{\frac{1}{x^7}} x \sqrt{x \left(x^2 + 3\right) \left(9x^2 + 7\right)} - 3 \right) - 3 \right)$$

6.31 problem 40

Internal problem ID [1060]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Section 2.5 Page 79
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$y' + 2yx + \frac{e^{-x^2}\left(3x + 2y\,e^{x^2}\right)}{2x + 3y\,e^{x^2}} = 0$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 36

```
dsolve([diff(y(x),x)+2*x*y(x)= -exp(-x^2)*(3*x+2*y(x)*exp(x^2))/(2*x+3*y(x)*exp(x^2)),y(0) =
```

$$y(x) = -\frac{\left(2x\,e^{x^2} + \sqrt{e^{2x^2}\left(-5x^2+9\right)}\right)e^{-2x^2}}{3}$$

✓ Solution by Mathematica

Time used: 32.678 (sec). Leaf size: 44

```
DSolve[{y'[x]+2*x*y[x]== -Exp[-x^2]*(3*x+2*y[x]*Exp[x^2])/(2*x+3*y[x]*Exp[x^2]),y[0]==-1},y[x
```

$$y(x) \to -\frac{1}{3}e^{-2x^2}\left(2e^{x^2}x + \sqrt{e^{2x^2}\left(9-5x^2\right)}\right)$$

7 Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91

7.1 problem 1(a)

Internal problem ID [1061]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y + \left(2x + \frac{1}{y}\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(y(x)+(2*x+1/y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-1 + \sqrt{4c_1x + 1}}{2x}$$
$$y(x) = -\frac{1 + \sqrt{4c_1x + 1}}{2x}$$

✓ Solution by Mathematica

Time used: 0.3 (sec). Leaf size: 54

```
DSolve[y[x]+(2*x+1/y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1 + \sqrt{1 + 4c_1x}}{2x}$$
$$y(x) \to \frac{-1 + \sqrt{1 + 4c_1x}}{2x}$$
$$y(x) \to 0$$

7.2 problem 2(a)

Internal problem ID [1062]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-y^2 + y'x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(-y(x)^2+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{c_1 x + 1}$$

✓ Solution by Mathematica

Time used: 0.105 (sec). Leaf size: 21

```
DSolve[-y[x]^2+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{1 - c_1 x}$$
$$y(x) \to 0$$

7.3 problem 3

Internal problem ID [1063]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - y'x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve(y(x)-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 14

```
DSolve[y[x]-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x$$
$$y(x) \to 0$$

7.4 problem 4

Internal problem ID [1064]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{3x^2y + 2x^3y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(3*x^2*y(x)+2*x^3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 18

```
DSolve[3*x^2*y[x]+2*x^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^{3/2}}$$
$$y(x) \to 0$$

7.5 problem 5

Internal problem ID [1065]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{2y^3 + 3y^2y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(2*y(x)^3+3*y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1 \mathrm{e}^{-\frac{2x}{3}}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 25

```
DSolve[2*y[x]^3+3*y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to c_1 e^{-2x/3}$$
$$y(x) \to 0$$

7.6 problem 6

Internal problem ID [1066]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{5yx + 2y + 5 + 2y'x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((5*x*y(x)+2*y(x)+5)+(2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{x} + \frac{\mathrm{e}^{-\frac{5x}{2}} c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 21

```
DSolve[(5*x*y[x]+2*y[x]+5)+(2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-1 + c_1 e^{-5x/2}}{x}$$

7.7 problem 7

Internal problem ID [1067]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{yx + x + 2y + 1 + (x + 1)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((x*y(x)+x+2*y(x)+1)+(x+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(-x\,\mathrm{e}^{x} + c_1)\,\mathrm{e}^{-x}}{x + 1}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 23

```
DSolve[(x*y[x]+x+2*y[x]+1)+(x+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x - c_1 e^{-x}}{x + 1}$$

7.8 problem 8

Internal problem ID [1068]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{27xy^2 + 8y^3 + \left(18x^2y + 12xy^2\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.313 (sec). Leaf size: 33

```
dsolve((27*x*y(x)^2+8*y(x)^3)+(18*x^2*y(x)+12*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \mathrm{RootOf}\left(4\_Z^{15}c_1x^5 + 9\_Z^{10}c_1x^5 - 1\right)^5 x$$

✓ Solution by Mathematica

Time used: 53.668 (sec). Leaf size: 532

```
DSolve[(27*x*y[x]^2+8*y[x]^3)+(18*x^2*y[x]+12*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to 0$$

$$y(x) \to \frac{1}{4}\left(\frac{9x^2}{\sqrt[3]{\frac{-27x^5+4\sqrt{e^{6c_1}\left(-27x^5+4e^{6c_1}\right)}+8e^{6c_1}}{x^2}}} + \sqrt[3]{\frac{-27x^5+4\sqrt{e^{6c_1}\left(-27x^5+4e^{6c_1}\right)}+8e^{6c_1}}{x^2}} - 3x\right)$$

$$y(x) \to \frac{1}{8}\left(\frac{\left(-9-9i\sqrt{3}\right)x^2}{\sqrt[3]{\frac{-27x^5+4\sqrt{e^{6c_1}\left(-27x^5+4e^{6c_1}\right)}+8e^{6c_1}}{x^2}}} + i\left(\sqrt{3}+i\right)\sqrt[3]{\frac{-27x^5+4\sqrt{e^{6c_1}\left(-27x^5+4e^{6c_1}\right)}+8e^{6c_1}}{x^2}} - 6x\right)$$

$$y(x) \to \frac{1}{8}\left(\frac{9i\left(\sqrt{3}+i\right)x^2}{\sqrt[3]{\frac{-27x^5+4\sqrt{e^{6c_1}\left(-27x^5+4e^{6c_1}\right)}+8e^{6c_1}}{x^2}}} + \left(-1-i\sqrt{3}\right)\sqrt[3]{\frac{-27x^5+4\sqrt{e^{6c_1}\left(-27x^5+4e^{6c_1}\right)}+8e^{6c_1}}{x^2}} - 6x\right)$$

$$y(x) \to 0$$

$$y(x) \to \frac{3\left(\sqrt[3]{-x^3}+x\right)\left(-2x+\left(1-i\sqrt{3}\right)\sqrt[3]{-x^3}\right)}{8x}$$

$$y(x) \to \frac{3\left(\sqrt[3]{-x^3}+x\right)\left(-2x+\left(1+i\sqrt{3}\right)\sqrt[3]{-x^3}\right)}{8x}$$

$$3\left(-x\sqrt[3]{-x^3}+\left(-x^3\right)^{2/3}+x^2\right)$$

7.9 problem 9

Internal problem ID [1069]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{6xy^2 + 2y + (12x^2y + 12xy^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 5

```
dsolve((6*x*y(x)^2+2*y(x))+(12*x^2*y(x)+12*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(6*x*y[x]^2+2*y[x])+(12*x^2*y[x]+12*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions
```

Not solved

7.10 problem 10

Internal problem ID [1070]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y^2 + \left(xy^2 + 6yx + \frac{1}{y}\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve((y(x)^2)+(x*y(x)^2+3*x*y(x)+3*x*y(x)+1/y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$x - \frac{3}{y(x)^4} + \frac{1}{y(x)^3} + \frac{6}{y(x)^5} - \frac{6}{y(x)^6} - \frac{e^{-y(x)} c_1}{y(x)^6} = 0$$

✓ Solution by Mathematica

Time used: 0.192 (sec). Leaf size: 41

```
DSolve[(y[x]^2)+(x*y[x]^2+3*x*y[x]+3*x*y[x]+1/y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[x = -\frac{y(x)^3 - 3y(x)^2 + 6y(x) - 6}{y(x)^6} + \frac{c_1 e^{-y(x)}}{y(x)^6}, y(x)\right]$$

7.11 problem 11

Internal problem ID [1071]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{12x^3y + 24x^2y^2 + (9x^4 + 32x^3y + 4y)\,y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve((12*x^3*y(x)+24*x^2*y(x)^2)+(9*x^4+32*x^3*y(x)+4*y(x))*diff(y(x),x)=0,y(x), singsol=al
```

$$3x^4y(x)^3 + 8x^3y(x)^4 + y(x)^4 + c_1 = 0$$

✓ Solution by Mathematica

Time used: 61.7 (sec). Leaf size: 1733

```
DSolve[(12*x^3*y[x]+24*x^2*y[x]^2)+(9*x^4+32*x^3*y[x]+4*y[x])*y'[x]==0,y[x],x,IncludeSingular
```

$$y(x) \to -\frac{3x^4}{32x^3+4} + \frac{1}{2}\sqrt{\frac{9x^8}{4(8x^3+1)^2} + \frac{\sqrt[3]{\sqrt{59049c_1{}^2x^{16}+6912c_1{}^3(8x^3+1)^3}-243c_1x^8}}{3\sqrt[3]{2}(8x^3+1)} - \frac{4\sqrt[3]{2}c_1}{\sqrt[3]{\sqrt{59049c_1{}^2x^{16}+6912c_1{}^3(8x\cdots}}}} - \frac{1}{2}\sqrt{\frac{9x^8}{2(8x^3+1)^2} - \frac{\sqrt[3]{\sqrt{59049c_1{}^2x^{16}+6912c_1{}^3(8x^3+1)^3}-243c_1x^8}}{3\sqrt[3]{2}(8x^3+1)} + \frac{4\sqrt[3]{2}c_1}{\sqrt[3]{\sqrt{59049c_1{}^2x^{16}+6912c_1{}^3(8x\cdots}}}\cdots}$$

$$y(x) \to -\frac{3x^4}{32x^3+4} + \frac{1}{2}\sqrt{\frac{9x^8}{4(8x^3+1)^2} + \frac{\sqrt[3]{\sqrt{59049c_1{}^2x^{16}+6912c_1{}^3(8x^3+1)^3}-243c_1x^8}}{3\sqrt[3]{2}(8x^3+1)} - \frac{4\sqrt[3]{2}c_1}{\sqrt[3]{\sqrt{59049c_1{}^2x^{16}+6912c_1{}^3(8x\cdots}}}} + \frac{1}{2}\sqrt{\frac{9x^8}{2(8x^3+1)^2} - \frac{\sqrt[3]{\sqrt{59049c_1{}^2x^{16}+6912c_1{}^3(8x^3+1)^3}-243c_1x^8}}{3\sqrt[3]{2}(8x^3+1)} + \frac{4\sqrt[3]{2}c_1}{\sqrt[3]{\sqrt{59049c_1{}^2x^{16}+6912c_1{}^3(8x\cdots}}}\cdots}$$

$$y(x) \to -\frac{3x^4}{32x^3+4} - \frac{1}{2}\sqrt{\frac{9x^8}{4(8x^3+1)^2} + \frac{\sqrt[3]{\sqrt{59049c_1{}^2x^{16}+6912c_1{}^3(8x^3+1)^3}-243c_1x^8}}{3\sqrt[3]{2}(8x^3+1)} - \frac{4\sqrt[3]{2}c_1}{\sqrt[3]{\sqrt{59049c_1{}^2x^{16}+6912c_1{}^3(8x\cdots}}}} - \frac{1}{2}\sqrt{\frac{9x^8}{2(8x^3+1)^2} - \frac{\sqrt[3]{\sqrt{59049c_1{}^2x^{16}+6912c_1{}^3(8x^3+1)^3}-243c_1x^8}}{3\sqrt[3]{2}(8x^3+1)} + \frac{4\sqrt[3]{2}c_1}{\sqrt[3]{\sqrt{59049c_1{}^2x^{16}+6912c_1{}^3(8x\cdots}}}\cdots}$$

7.12 problem 12

Internal problem ID [1072]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2y + 4yx + 2y + (x^2 + x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((x^2*y(x)+4*x*y(x)+2*y(x))+(x^2+x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{-x}}{x^2\,(x+1)}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 26

```
DSolve[(x^2*y[x]+4*x*y[x]+2*y[x])+(x^2+x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 e^{-x}}{x^2(x+1)}$$

$$y(x) \to 0$$

7.13 problem 13

Internal problem ID [1073]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-y + (x^4 - x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(-y(x)+(x^4-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2 + x + 1)^{\frac{1}{3}} (x - 1)^{\frac{1}{3}}}{x}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 27

```
DSolve[-y[x]+(x^4-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 \sqrt[3]{1 - x^3}}{x}$$
$$y(x) \to 0$$

7.14 problem 14

Internal problem ID [1074]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{\cos(x)\cos(y) + (\sin(x)\cos(y) - \sin(x)\sin(y) + y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve((cos(x)*cos(y(x)))+(sin(x)*cos(y(x))-sin(x)*sin(y(x))+y(x))*diff(y(x),x)=0,y(x), sings
```

$$\cos(y(x))\, e^{y(x)} \sin(x) + (y(x) - 1)\, e^{y(x)} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.23 (sec). Leaf size: 28

```
DSolve[(Cos[x]*Cos[y[x]])+(Sin[x]*Cos[y[x]]-Sin[x]*Sin[y[x]]+y[x])*y'[x]==0,y[x],x,IncludeSin
```

$$\text{Solve}\left[-2e^{y(x)}(y(x) - 1) - 2e^{y(x)} \sin(x)\cos(y(x)) = c_1, y(x)\right]$$

7.15 problem 15

Internal problem ID [1075]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{2yx + y^2 + (2yx + x^2 - 2xy^2 - 2xy^3)\, y' = 0}$$

✗ Solution by Maple

```
dsolve((2*x*y(x)+y(x)^2)+(2*x*y(x)+x^2-2*x*y(x)^2-2*x*y(x)^3)*diff(y(x),x)=0,y(x), singsol=al
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(2*x*y[x]+y[x]^2)+(2*x*y[x]+x^2-2*x*y[x]^2-2*x*y[x]^3)*y'[x]==0,y[x],x,IncludeSingular
```

Not solved

7.16 problem 16

Internal problem ID [1076]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y\sin(y) + x(\sin(y) - y\cos(y))\, y' = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 16

```
dsolve((y(x)*sin(y(x)))+(x*(sin(y(x))-y(x)*cos(y(x))))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\ln(x) + \ln(y(x)) - \ln(\sin(y(x))) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.564 (sec). Leaf size: 31

```
DSolve[(y[x]*Sin[y[x]])+(x*(Sin[y[x]]-y[x]*Cos[y[x]]))*y'[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to \text{InverseFunction}[-\log(\#1) + \log(\tan(\#1)) + \log(\cos(\#1))\&][\log(x) + c_1]$$
$$y(x) \to 0$$

7.17 problem 18

Internal problem ID [1077]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$ay + bxy + (cx + dxy)\, y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve((a*y(x)+b*x*y(x))+(c*x+d*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{a\ln(x)+bx+c\,\mathrm{LambertW}\left(\frac{d\,x^{-\frac{a}{c}}\,\mathrm{e}^{-\frac{bx}{c}-\frac{c_1}{c}}}{c}\right)+c_1}{c}}$$

✓ Solution by Mathematica

Time used: 1.001 (sec). Leaf size: 42

```
DSolve[(a*y[x]+b*x*y[x])+(c*x+d*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{cW\left(\frac{dx^{-\frac{a}{c}}e^{\frac{-bx+c_1}{c}}}{c}\right)}{d}$$

$$y(x) \to 0$$

7.18 problem 19

Internal problem ID [1078]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class C‘]]`

$$3x^2y^3 - y^2 + y + (-yx + 2x)\, y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve((3*x^2*y(x)^3-y(x)^2+y(x))+(-x*y(x)+2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{4}{\sqrt{x}\,\sqrt{\frac{c_1x+48x^2+4}{x}} + 2}$$
$$y(x) = -\frac{4}{\sqrt{x}\,\sqrt{\frac{c_1x+48x^2+4}{x}} - 2}$$

✓ Solution by Mathematica

Time used: 0.744 (sec). Leaf size: 78

```
DSolve[(3*x^2*y[x]^3-y[x]^2+y[x])+(-x*y[x]+2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{2}{1 + \sqrt{-\frac{1}{x^2}}x\sqrt{-1 - 4x(3x + c_1)}}$$
$$y(x) \to \frac{2x}{x + \frac{\sqrt{-1-4x(3x+c_1)}}{\sqrt{-\frac{1}{x^2}}}}$$
$$y(x) \to 0$$

7.19 problem 20

Internal problem ID [1079]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2y + 3(x^2 + x^2y^3)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve((2*y(x))+3*(x^2+x^2*y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x \,\mathrm{LambertW}\left(\mathrm{e}^{\frac{2}{x}-2c_1}\right)+2c_1x-2}{3x}}$$

✓ Solution by Mathematica

Time used: 4.329 (sec). Leaf size: 82

```
DSolve[(2*y[x])+3*(x^2+x^2*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{W\left(e^{\frac{2}{x}+3c_1}\right)}$$

$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{W\left(e^{\frac{2}{x}+3c_1}\right)}$$

$$y(x) \to (-1)^{2/3}\sqrt[3]{W\left(e^{\frac{2}{x}+3c_1}\right)}$$

$$y(x) \to 0$$

7.20 problem 21

Internal problem ID [1080]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$a\cos(x)\,y-\sin(x)\,y^2+(b\cos(x)\,y-\sin(x)\,yx)\,y'=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve((a*cos(x)*y(x)-y(x)*sin(x)*y(x))+(b*cos(x)*y(x)-x*sin(x)*y(x))*diff(y(x),x)=0,y(x), si
```

$$y(x)=0$$
$$y(x)=\left(\int -\frac{a\cos(x)\,\mathrm{e}^{\int -\frac{\sin(x)}{-\sin(x)x+\cos(x)b}dx}}{-\sin(x)\,x+\cos(x)\,b}dx+c_1\right)\mathrm{e}^{\int \frac{\sin(x)}{-\sin(x)x+\cos(x)b}dx}$$

✓ Solution by Mathematica

Time used: 3.479 (sec). Leaf size: 85

```
DSolve[(a*Cos[x]*y[x]-y[x]*Sin[x]*y[x])+(b*Cos[x]*y[x]-x*Sin[x]*y[x])*y'[x]==0,y[x],x,Include
```

$$y(x)\to 0$$
$$y(x)\to \exp\left(\int_1^x \frac{1}{b\cot(K[1])-K[1]}dK[1]\right)\left(\int_1^x -\frac{a\exp\left(-\int_1^{K[2]}\frac{1}{b\cot(K[1])-K[1]}dK[1]\right)}{b-K[2]\tan(K[2])}dK[2]+c_1\right)$$
$$y(x)\to 0$$

7.21 problem 22

Internal problem ID [1081]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^4y^4 + x^5y^3y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((x^4*y(x)^4)+(x^5*y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 21

```
DSolve[(x^4*y[x]^4)+(x^5*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to 0$$

7.22 problem 23

Internal problem ID [1082]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y(x\cos(x) + 2\sin(x)) + x(1 + y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 33

```
dsolve((y(x)*(x*cos(x)+2*sin(x)))+(x*(y(x)+1))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(\mathrm{e}^{-\sin(x)-2\operatorname{Si}(x)-c_1}\right)-\sin(x)-2\operatorname{Si}(x)-c_1}$$

✓ Solution by Mathematica

Time used: 2.59 (sec). Leaf size: 24

```
DSolve[(y[x]*(x*Cos[x]+2*Sin[x]))+(x*(y[x]+1))*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to W\left(e^{-2\operatorname{Si}(x)-\sin(x)+c_1}\right)$$
$$y(x) \to 0$$

7.23 problem 24

Internal problem ID [1083]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x^4y^3 + y + (y^2x^5 - x)\, y' = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 304

```
dsolve((x^4*y(x)^3+y(x))+(x^5*y(x)^2-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\frac{\left(4x^3+4\sqrt{x^6-4c_1^3}\right)^{\frac{1}{3}}}{2x^2} + \frac{2c_1}{x^2\left(4x^3+4\sqrt{x^6-4c_1^3}\right)^{\frac{1}{3}}}}{\sqrt{c_1}}$$

$$y(x) = \frac{-\frac{\left(4x^3+4\sqrt{x^6-4c_1^3}\right)^{\frac{1}{3}}}{4x^2} - \frac{c_1}{x^2\left(4x^3+4\sqrt{x^6-4c_1^3}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(4x^3+4\sqrt{x^6-4c_1^3}\right)^{\frac{1}{3}}}{2x^2} - \frac{2c_1}{x^2\left(4x^3+4\sqrt{x^6-4c_1^3}\right)^{\frac{1}{3}}}\right)}{2}}{\sqrt{c_1}}$$

$$y(x) = \frac{-\frac{\left(4x^3+4\sqrt{x^6-4c_1^3}\right)^{\frac{1}{3}}}{4x^2} - \frac{c_1}{x^2\left(4x^3+4\sqrt{x^6-4c_1^3}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(4x^3+4\sqrt{x^6-4c_1^3}\right)^{\frac{1}{3}}}{2x^2} - \frac{2c_1}{x^2\left(4x^3+4\sqrt{x^6-4c_1^3}\right)^{\frac{1}{3}}}\right)}{2}}{\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 31.623 (sec). Leaf size: 267

```
DSolve[(x^4*y[x]^3+y[x])+(x^5*y[x]^2-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2 + \frac{\sqrt[3]{2}\left(3c_1x^9 + \sqrt{x^{12}(-4+9c_1{}^2x^6)}\right)^{2/3}}{x^4}}{2^{2/3}\sqrt[3]{3c_1x^9 + \sqrt{x^{12}\left(-4+9c_1{}^2x^6\right)}}}$$

$$y(x) \to \frac{-2\sqrt[3]{-2}x^4 + (-2)^{2/3}\left(3c_1x^9 + \sqrt{x^{12}\left(-4+9c_1{}^2x^6\right)}\right)^{2/3}}{2x^4\sqrt[3]{3c_1x^9 + \sqrt{x^{12}\left(-4+9c_1{}^2x^6\right)}}}$$

$$y(x) \to \frac{2(-1)^{2/3}x^4 - \sqrt[3]{-2}\left(3c_1x^9 + \sqrt{x^{12}\left(-4+9c_1{}^2x^6\right)}\right)^{2/3}}{2^{2/3}x^4\sqrt[3]{3c_1x^9 + \sqrt{x^{12}\left(-4+9c_1{}^2x^6\right)}}}$$

7.24 problem 25

Internal problem ID [1084]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$3yx + 2y^2 + y + (x^2 + 2yx + x + 2y)\, y' = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 103

```
dsolve((3*x*y(x)+2*y(x)^2+y(x))+(x^2+2*x*y(x)+x+2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-\frac{c_1 x^2}{2} - \frac{c_1 x}{2} - \frac{\sqrt{c_1^2 x^4 + 2c_1^2 x^3 + c_1^2 x^2 + 4}}{2}}{c_1 (x+1)}$$

$$y(x) = \frac{-\frac{c_1 x^2}{2} - \frac{c_1 x}{2} + \frac{\sqrt{c_1^2 x^4 + 2c_1^2 x^3 + c_1^2 x^2 + 4}}{2}}{c_1 (x+1)}$$

✓ Solution by Mathematica

Time used: 14.38 (sec). Leaf size: 105

```
DSolve[(3*x*y[x]+2*y[x]^2+y[x])+(x^2+2*x*y[x]+x+2*y[x])*y'[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{1}{2}\left(-x - \sqrt{x^2 + \frac{4e^{c_1}}{(x+1)^2}}\right)$$

$$y(x) \to \frac{1}{2}\left(-x + \sqrt{x^2 + \frac{4e^{c_1}}{(x+1)^2}}\right)$$

$$y(x) \to \frac{1}{2}\left(-\sqrt{x^2} - x\right)$$

$$y(x) \to \frac{1}{2}\left(\sqrt{x^2} - x\right)$$

7.25 problem 26

Internal problem ID [1085]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{12yx + 6y^3 + (9x^2 + 10xy^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 35

```
dsolve((12*x*y(x)+6*y(x)^3)+(9*x^2+10*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$\ln(x) - c_1 + \frac{6\ln\left(\frac{y(x)}{\sqrt{x}}\right)}{11} + \frac{2\ln\left(\frac{2y(x)^2+3x}{x}\right)}{11} = 0$$

✓ Solution by Mathematica

Time used: 7.541 (sec). Leaf size: 151

```
DSolve[(12*x*y[x]+6*y[x]^3)+(9*x^2+10*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \text{Root}\left[2\#1^5x^3 + 3\#1^3x^4 - c_1\&, 1\right]$$
$$y(x) \to \text{Root}\left[2\#1^5x^3 + 3\#1^3x^4 - c_1\&, 2\right]$$
$$y(x) \to \text{Root}\left[2\#1^5x^3 + 3\#1^3x^4 - c_1\&, 3\right]$$
$$y(x) \to \text{Root}\left[2\#1^5x^3 + 3\#1^3x^4 - c_1\&, 4\right]$$
$$y(x) \to \text{Root}\left[2\#1^5x^3 + 3\#1^3x^4 - c_1\&, 5\right]$$

7.26 problem 27

Internal problem ID [1086]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 2, First order equations. Exact equations. Integrating factors. Section 2.6 Page 91
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{3x^2y^2 + 2y + 2y'x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((3*x^2*y(x)^2+2*y(x))+(2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2}{(3x + 2c_1)\, x}$$

✓ Solution by Mathematica

Time used: 0.144 (sec). Leaf size: 25

```
DSolve[(3*x^2*y[x]^2+2*y[x])+(2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3x^2 + 2c_1x}$$

$$y(x) \to 0$$

8 Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203

8.1 problem 1

Internal problem ID [1087]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 7y' + 10y = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)-7*diff(y(x),x)+10*y(x)=0,y(0) = -1, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = e^{5x} - 2e^{2x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 18

```
DSolve[{y''[x]-7*y'[x]+10*y[x]==0,{y[0]==-1,y'[0]==1}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{2x}(e^{3x} - 2)$$

8.2 problem 2c

Internal problem ID [1088]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 2c.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 2y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = -2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=0,y(0) = 3, D(y)(0) = -2],y(x), singsol=all)
```

$$y(x) = e^x(-5\sin(x) + 3\cos(x))$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{y''[x]-2*y'[x]+2*y[x]==0,{y[0]==3,y'[0]==-2}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^x(3\cos(x) - 5\sin(x))$$

8.3 problem 2d

Internal problem ID [1089]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 2d.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 2y = 0}$$

With initial conditions

$$[y(0) = k_0, y'(0) = k_1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=0,y(0) = k__0, D(y)(0) = k__1],y(x), singsol=all
```

$$y(x) = e^x((k_1 - k_0)\sin(x) + k_0\cos(x))$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[{y''[x]-2*y'[x]+2*y[x]==0,{y[0]==k0,y'[0]==k1}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^x((\text{k1} - \text{k0})\sin(x) + \text{k0}\cos(x))$$

8.4 problem 3c

Internal problem ID [1090]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 3c.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + y = 0}$$

With initial conditions

$$[y(0) = 7, y'(0) = 4]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(0) = 7, D(y)(0) = 4],y(x), singsol=all)
```

$$y(x) = e^x(7 - 3x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 14

```
DSolve[{y''[x]-2*y'[x]+y[x]==0,{y[0]==7,y'[0]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(7 - 3x)$$

8.5 problem 3d

Internal problem ID [1091]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 3d.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + y = 0}$$

With initial conditions

$$[y(0) = k_0, y'(0) = k_1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(0) = k__0, D(y)(0) = k__1],y(x), singsol=all)
```

$$y(x) = -((x-1)\,k_0 - xk_1)\,\mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{y''[x]-2*y'[x]+y[x]==0,{y[0]==k0,y'[0]==k1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(\mathrm{k0}(-x) + \mathrm{k0} + \mathrm{k1}x)$$

8.6 problem 4

Internal problem ID [1092]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2-1)\, y'' + 4y'x + 2y = 0}$$

With initial conditions

$$[y(0) = -5, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([(x^2-1)*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=0,y(0) = -5, D(y)(0) = 1],y(x), singso
```

$$y(x) = \frac{-x+5}{x^2-1}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 18

```
DSolve[{(x^2-1)*y''[x]+4*x*y'[x]+2*y[x]==0,{y[0]==-5,y'[0]==1}},y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{5-x}{x^2-1}$$

8.7 problem 10

Internal problem ID [1093]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' - 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 \mathrm{e}^{3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[y''[x]-2*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2 e^{4x} + c_1)$$

8.8 problem 11

Internal problem ID [1094]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} + c_2 e^{3x} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[y''[x]-6*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}(c_2 x + c_1)$$

8.9 problem 12

Internal problem ID [1095]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2ay' + ya^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-2*a*diff(y(x),x)+a^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{ax} + c_2 e^{ax} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[y''[x]-2*a*y'[x]+a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{ax}(c_2 x + c_1)$$

8.10 problem 13

Internal problem ID [1096]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2x$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x} + c_2x$$

8.11 problem 14

Internal problem ID [1097]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x + c_2x\ln(x)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 15

```
DSolve[x^2*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2\log(x) + c_1)$$

8.12 problem 15

Internal problem ID [1098]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2y'' - (2a - 1)\,xy' + ya^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-(2*a-1)*x*diff(y(x),x)+a^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^a + c_2x^a \ln(x)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-(2*a-1)*x*y'[x]+a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^a(ac_2\log(x) + c_1)$$

8.13 problem 16

Internal problem ID [1099]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4y'x + (-16x^2 + 3)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(3-16*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\sinh(2x) + c_2\sqrt{x}\,\cosh(2x)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 32

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(3-16*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}\sqrt{x}\left(c_2e^{4x} + 4c_1\right)$$

8.14 problem 17

Internal problem ID [1100]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\, y''-y'x+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+\mathrm{e}^{x}c_2$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^x-c_2x$$

8.15 problem 18

Internal problem ID [1101]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 2y'x + (x^2 + 2)\, y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x)\, x + c_2 \cos(x)\, x$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

8.16 problem 19

Internal problem ID [1102]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2 \sin(x)\, y'' - 4x(x\cos(x) + \sin(x))\, y' + (2x\cos(x) + 3\sin(x))\, y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 17

```
dsolve(4*x^2*sin(x)*diff(y(x),x$2)-4*x*(x*cos(x)+sin(x))*diff(y(x),x)+(2*x*cos(x)+3*sin(x))*y
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x}\,\cos(x)$$

✓ Solution by Mathematica

Time used: 0.172 (sec). Leaf size: 21

```
DSolve[4*x^2*Sin[x]*y''[x]-4*x*(x*Cos[x]+Sin[x])*y'[x]+(2*x*Cos[x]+3*Sin[x])*y[x]==0,y[x],x,I
```

$$y(x) \to \sqrt{\arccos(\cos(x))}(c_2\cos(x) + c_1)$$

8.17 problem 20

Internal problem ID [1103]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(3x-1)\,y''-(3x+2)\,y'+(6x-8)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 89

```
dsolve((3*x-1)*diff(y(x),x$2)-(3*x+2)*diff(y(x),x)+(6*x-8)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{-\frac{x(i\sqrt{7}-1)}{2}}(3x-1)^2\,\mathrm{KummerM}\left(\frac{3}{2}-\frac{5i\sqrt{7}}{14},3,\frac{i\sqrt{7}\,(3x-1)}{3}\right)+c_2\mathrm{e}^{-\frac{x(i\sqrt{7}-1)}{2}}(3x-1)^2\,\mathrm{KummerU}\left(\frac{3}{2}-\frac{5i\sqrt{7}}{14},3,\frac{i\sqrt{7}\,(3x-1)}{3}\right)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 109

```
DSolve[(3*x-1)*y''[x]-(3*x+2)*y'[x]+(6*x-8)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 4e^{\frac{1}{6}\left(1-i\sqrt{7}\right)(3x-1)}(1-3x)^2\left(c_1\,\mathrm{HypergeometricU}\left(\frac{3}{2}-\frac{5i}{2\sqrt{7}},3,\frac{1}{3}i\sqrt{7}(3x-1)\right)+c_2L^2_{-\frac{3}{2}+\frac{5i}{2\sqrt{7}}}\left(\frac{1}{3}i\sqrt{7}(3x-1)\right)\right)$$

8.18 problem 21

Internal problem ID [1104]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2-4)\, y''+4y'x+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((x^2-4)*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x+c_2}{x^2-4}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 20

```
DSolve[(x^2-4)*y''[x]+4*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_2x+c_1}{x^2-4}$$

8.19 problem 22

Internal problem ID [1105]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(1+2x)\,y'' - 2\left(2x^2-1\right)y' - 4(x+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 32

```
dsolve((2*x+1)*diff(y(x),x$2)-2*(2*x^2-1)*diff(y(x),x)-4*(x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{HeunB}\left(-\frac{1}{2}, -2, -\frac{1}{2}, 3, \frac{1}{2}+x\right) + c_2 \operatorname{HeunB}\left(\frac{1}{2}, -2, -\frac{1}{2}, 3, \frac{1}{2}+x\right)\sqrt{2+4x}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(2*x+1)*y''[x]-2*(2*x^2-1)*y'[x]-4*(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

8.20 problem 23

Internal problem ID [1106]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.1 Homogeneous linear equations. Page 203
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 - 2x)\, y'' + (-x^2 + 2)\, y' + (2x - 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((x^2-2*x)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)+(2*x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^2 + e^x c_2$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 18

```
DSolve[(x^2-2*x)*y''[x]+(2-x^2)*y'[x]+(2*x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_2 x^2 + c_1 e^x$$

9 Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253

9.1 problem 1

Internal problem ID [1107]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1+2x)\,y''-2y'-(2x+3)\,y-(1+2x)^2=0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{-x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve([(2*x+1)*diff(y(x),x$2)-2*diff(y(x),x)-(2*x+3)*y(x)=(2*x+1)^2,exp(-x)],y(x), singsol=a
```

$$y(x) = \mathrm{e}^{-x}c_2 + x\,\mathrm{e}^{x}c_1 + 1 - 2x$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 33

```
DSolve[(2*x+1)*y''[x]-2*y'[x]-(2*x+3)*y[x]==(2*x+1)^2,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_1 e^{-x-\frac{1}{2}} + x\left(-2 + c_2 e^{x+\frac{1}{2}}\right) + 1$$

9.2 problem 2

Internal problem ID [1108]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + y'x - y - \frac{4}{x^2} = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=4/x^2,x],y(x), singsol=all)
```

$$y(x) = c_2x + \frac{c_1}{x} + \frac{4}{3x^2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==4/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4}{3x^2} + \frac{c_1}{x} + c_2x$$

9.3 problem 3

Internal problem ID [1109]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x + y - x = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=x,x],y(x), singsol=all)
```

$$y(x) = c_2x + \ln(x)\, c_1x + \frac{\ln(x)^2 x}{2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]-x*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x\left(\log^2(x) + 2c_2\log(x) + 2c_1\right)$$

9.4 problem 4

Internal problem ID [1110]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y - \frac{1}{1 + e^{-x}} = 0}$$

Given that one solution of the ode is

$$y_1 = e^{2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve([diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=1/(1+exp(-x)),exp(2*x)],y(x), singsol=all)
```

$$y(x) = (c_1 e^x - \ln(e^x) - \ln(e^x) e^x + \ln(1 + e^x)(1 + e^x) - 1 + c_2) e^x$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 34

```
DSolve[y''[x]-3*y'[x]+2*y[x]==1/(1+Exp[-x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(2(e^x + 1)\operatorname{arctanh}(2e^x + 1) + c_2 e^x - 1 + c_1)$$

9.5 problem 5

Internal problem ID [1111]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y - 7x^{\frac{3}{2}} \mathrm{e}^{x} = 0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+y(x)=7*x^(3/2)*exp(x),exp(x)],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} c_2 + x\,\mathrm{e}^{x} c_1 + \frac{4x^{\frac{7}{2}} \mathrm{e}^{x}}{5}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 29

```
DSolve[y''[x]-2*y'[x]+y[x]==7*x^(3/2)*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{5} e^{x} \left(4x^{7/2} + 5c_2 x + 5c_1\right)$$

9.6 problem 6

Internal problem ID [1112]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$4x^2y'' + (-8x^2 + 4x)\,y' + (4x^2 - 4x - 1)\,y - 4\sqrt{x}\,e^x(4x + 1) = 0$$

Given that one solution of the ode is

$$y_1 = \sqrt{x}\,e^x$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve([4*x^2*diff(y(x),x$2)+(4*x-8*x^2)*diff(y(x),x)+(4*x^2-4*x-1)*y(x)=4*x^(1/2)*exp(x)*(1+
```

$$y(x) = \frac{e^x c_2}{\sqrt{x}} + \sqrt{x}\,e^x c_1 + e^x\sqrt{x}\,(\ln(x) + 2x - 1)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 30

```
DSolve[4*x^2*y''[x]+(4*x-8*x^2)*y'[x]+(4*x^2-4*x-1)*y[x]==4*x^(1/2)*Exp[x]*(1+4*x),y[x],x,Inc
```

$$y(x) \to \frac{e^x(x\log(x) + x(2x - 1 + c_2) + c_1)}{\sqrt{x}}$$

9.7 problem 7

Internal problem ID [1113]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + 2y - e^{x} \sec(x) = 0}$$

Given that one solution of the ode is

$$y_1 = e^{x} \cos(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=exp(x)*sec(x),exp(x)*cos(x)],y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) e^{x} + c_1 \cos(x) e^{x} + e^{x}(\sin(x) x - \cos(x) \ln(\sec(x)))$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 26

```
DSolve[y''[x]-2*y'[x]+2*y[x]==Exp[x]*Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}((x + c_1)\sin(x) + \cos(x)(\log(\cos(x)) + c_2))$$

9.8 problem 8

Internal problem ID [1114]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y'x + (4x^2 + 2)\, y - 8\,\mathrm{e}^{-x(2+x)} = 0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{-x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve([diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+2)*y(x)=8*exp(-x*(x+2)),exp(-x^2)],y(x), sings
```

$$y(x) = \mathrm{e}^{-x^2} c_2 + \mathrm{e}^{-x^2} x c_1 + 2\,\mathrm{e}^{-x(2+x)}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 29

```
DSolve[y''[x]+4*x*y'[x]+(4*x^2+2)*y[x]==8*Exp[-x*(x+2)],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to e^{-x(x+2)}\left(2 + e^{2x}(c_2 x + c_1)\right)$$

9.9 problem 9

Internal problem ID [1115]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x - 4y + 6x + 4 = 0}$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=-6*x-4,x^2],y(x), singsol=all)
```

$$y(x) = x^2c_2 + \frac{c_1}{x^2} + 1 + 2x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+x*y'[x]-4*y[x]==-6*x-4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^2 + \frac{c_1}{x^2} + 2x + 1$$

9.10 problem 10

Internal problem ID [1116]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 2x(x-1)\,y' + \left(x^2 - 2x + 2\right)y - \mathrm{e}^{2x}x^3 = 0}$$

Given that one solution of the ode is

$$y_1 = x\,\mathrm{e}^{-x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve([x^2*diff(y(x),x$2)+2*x*(x-1)*diff(y(x),x)+(x^2-2*x+2)*y(x)=x^3*exp(2*x),x*exp(-x)],y(
```

$$y(x) = c_2\mathrm{e}^{-x}x + x^2\mathrm{e}^{-x}c_1 + \frac{\mathrm{e}^{2x}x}{9}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 30

```
DSolve[x^2*y''[x]+2*x*(x-1)*y'[x]+(x^2-2*x+2)*y[x]==x^3*Exp[2*x],y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{1}{9}e^{-x}x\left(e^{3x} + 9(c_2x + c_1)\right)$$

9.11 problem 11

Internal problem ID [1117]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2y'' - x(2x-1)\,y' + \left(x^2-x-1\right)y - x^2\mathrm{e}^x = 0$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^x x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve([x^2*diff(y(x),x$2)-x*(2*x-1)*diff(y(x),x)+(x^2-x-1)*y(x)=x^2*exp(x),x*exp(x)],y(x), s
```

$$y(x) = \frac{\mathrm{e}^x c_2}{x} + x\,\mathrm{e}^x c_1 + \frac{x^2\mathrm{e}^x}{3}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]-x*(2*x-1)*y'[x]+(x^2-x-1)*y[x]==x^2*Exp[x],y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^x(2x^3 + 3c_2x^2 + 6c_1)}{6x}$$

9.12 problem 12

Internal problem ID [1118]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{(1-2x)\,y''+2y'+(-3+2x)\,y-\left(4x^2-4x+1\right)\mathrm{e}^x=0}$$

Given that one solution of the ode is

$$y_1=\mathrm{e}^x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve([(1-2*x)*diff(y(x),x$2)+2*diff(y(x),x)+(2*x-3)*y(x)=(1-4*x+4*x^2)*exp(x),exp(x)],y(x),
```

$$y(x)=\mathrm{e}^x c_2+c_1 x\,\mathrm{e}^{-x}-\frac{x(x-1)\,\mathrm{e}^x}{2}$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 75

```
DSolve[(1-2*x)*y''[x]+2*y'[x]+(2*x-3)*y[x]==(1-4*x+4*x^2)*Exp[x],y[x],x,IncludeSingularSoluti
```

$$y(x)\to -\frac{1}{2}e^x(x-1)x+\frac{c_1e^{x-\frac{1}{2}}\sqrt{1-2x}}{\sqrt{2x-1}}+\frac{c_2\sqrt{e-2ex}x(\sinh(x)-\cosh(x))}{\sqrt{2x-1}}$$

9.13 problem 13

Internal problem ID [1119]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 3y'x + 4y - 4x^4 = 0}$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=4*x^4,x^2],y(x), singsol=all)
```

$$y(x) = x^2c_2 + \ln(x)\, c_1x^2 + x^4$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==4*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2\left(x^2 + 2c_2\log(x) + c_1\right)$$

9.14 problem 14

Internal problem ID [1120]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2xy'' + (4x+1)\,y' + (1+2x)\,y - 3\sqrt{x}\,e^{-x} = 0}$$

Given that one solution of the ode is

$$y_1 = e^{-x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve([2*x*diff(y(x),x$2)+(4*x+1)*diff(y(x),x)+(2*x+1)*y(x)=3*x^(1/2)*exp(-x),exp(-x)],y(x),
```

$$y(x) = e^{-x}c_2 + \sqrt{x}\,e^{-x}c_1 + x^{\frac{3}{2}}e^{-x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 26

```
DSolve[2*x*y''[x]+(4*x+1)*y'[x]+(2*x+1)*y[x]==3*x^(1/2)*Exp[-x],y[x],x,IncludeSingularSolutio
```

$$y(x) \to e^{-x}\left(\sqrt{x}(x+2c_2)+c_1\right)$$

9.15 problem 15

Internal problem ID [1121]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' - (1+2x)\,y' + (x+1)\,y + \mathrm{e}^{-x} = 0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve([x*diff(y(x),x$2)-(2*x+1)*diff(y(x),x)+(x+1)*y(x)=-exp(-x),exp(x)],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} c_2 + \mathrm{e}^{x} c_1 x^2 - \mathrm{Ei}_1\,(2x)\, x^2 \mathrm{e}^{x} + \frac{\mathrm{e}^{-x}(2x-1)}{4}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 46

```
DSolve[x*y''[x]-(2*x+1)*y'[x]+(x+1)*y[x]==-Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^{-x}\left(2e^{2x}\left(2x^2\,\text{ExpIntegralEi}(-2x) + c_2 x^2 + 2c_1\right) + 2x - 1\right)$$

9.16 problem 16

Internal problem ID [1122]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{4x^2y'' - 4x(x+1)\,y' + (2x+3)\,y - 4\,\mathrm{e}^{2x}x^{\frac{5}{2}} = 0}$$

Given that one solution of the ode is

$$y_1 = \sqrt{x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve([4*x^2*diff(y(x),x$2)-4*x*(x+1)*diff(y(x),x)+(2*x+3)*y(x)=4*x^(5/2)*exp(2*x),x^(1/2)],
```

$$y(x) = \sqrt{x}\,c_2 + \sqrt{x}\,\mathrm{e}^{x}c_1 + \frac{\sqrt{x}\,\mathrm{e}^{2x}}{2}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 31

```
DSolve[4*x^2*y''[x]-4*x*(x+1)*y'[x]+(2*x+3)*y[x]==4*x^(5/2)*Exp[2*x],y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{1}{2}\sqrt{x}\left(e^{2x} + 2c_2e^{x} + 2c_1\right)$$

9.17 problem 17

Internal problem ID [1123]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 5y'x + 8y - 4x^2 = 0}$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve([x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+8*y(x)=4*x^2,x^2],y(x), singsol=all)
```

$$y(x) = c_2x^4 + c_1x^2 + x^2(-1 - 2\ln(x))$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]-5*x*y'[x]+8*y[x]==4*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2\left(c_2x^2 - 2\log(x) - 1 + c_1\right)$$

9.18 problem 18

Internal problem ID [1124]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (-2x + 2)\, y' + (-2 + x)\, y = 0}$$

Given that one solution of the ode is

$$y_1 = e^x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([x*diff(y(x),x$2)+(2-2*x)*diff(y(x),x)+(x-2)*y(x)=0,exp(x)],y(x), singsol=all)
```

$$y(x) = c_1 e^x + \frac{e^x c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 19

```
DSolve[x*y''[x]+(2-2*x)*y'[x]+(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x (c_2 x + c_1)}{x}$$

9.19 problem 19

Internal problem ID [1125]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$\boxed{x^2y'' - 4y'x + 6y = 0}$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,x^2],y(x), singsol=all)
```

$$y(x) = c_1x^3 + x^2c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(c_2x + c_1)$$

9.20 problem 20

Internal problem ID [1126]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 \ln(x)^2 y'' - 2x \ln(x) y' + (2 + \ln(x)) y = 0}$$

Given that one solution of the ode is

$$y_1 = \ln(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([x^2*(ln(x))^2*diff(y(x),x$2)-2*x*ln(x)*diff(y(x),x)+(2+ln(x))*y(x)=0,ln(x)],y(x), sin
```

$$y(x) = \ln(x) c_1 + c_2 x \ln(x)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 15

```
DSolve[x^2*Log[x]^2*y''[x]-2*x*Log[x]*y'[x]+(2+Log[x])*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to (c_2 x + c_1) \log(x)$$

9.21 problem 21

Internal problem ID [1127]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$\boxed{4xy'' + 2y' + y = 0}$$

Given that one solution of the ode is

$$y_1 = \sin\left(\sqrt{x}\right)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([4*x*diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,sin(sqrt(x))],y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\sqrt{x}\right) + c_2 \cos\left(\sqrt{x}\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 24

```
DSolve[4*x*y''[x]+2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\sqrt{x}\right) + c_2 \sin\left(\sqrt{x}\right)$$

9.22 problem 22

Internal problem ID [1128]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2+2x)\,y' + (2+x)\,y = 0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([x*diff(y(x),x$2)-(2*x+2)*diff(y(x),x)+(x+2)*y(x)=0,exp(x)],y(x), singsol=all)
```

$$y(x) = c_1\mathrm{e}^x + c_2\mathrm{e}^x x^3$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 23

```
DSolve[x*y''[x]-(2*x+2)*y'[x]+(x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}e^x\left(c_2x^3 + 3c_1\right)$$

9.23 problem 23

Internal problem ID [1129]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - (2a-1)\,xy' + ya^2 = 0}$$

Given that one solution of the ode is

$$y_1 = x^a$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([x^2*diff(y(x),x$2)-(2*a-1)*x*diff(y(x),x)+a^2*y(x)=0,x^a],y(x), singsol=all)
```

$$y(x) = c_1x^a + c_2x^a \ln(x)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-(2*a-1)*x*y'[x]+a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^a(ac_2\log(x) + c_1)$$

9.24 problem 24

Internal problem ID [1130]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + (x^2 + 2)\, y = 0}$$

Given that one solution of the ode is

$$y_1 = \sin(x)\, x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,x*sin(x)],y(x), singsol=all)
```

$$y(x) = c_1 \sin(x)\, x + c_2 \cos(x)\, x$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x$$

9.25 problem 25

Internal problem ID [1131]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (4x+1)\,y' + (4x+2)\,y = 0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve([x*diff(y(x),x$2)-(4*x+1)*diff(y(x),x)+(4*x+2)*y(x)=0,exp(2*x)],y(x), singsol=all)
```

$$y(x) = c_1\mathrm{e}^{2x} + c_2\mathrm{e}^{2x}x^2$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 25

```
DSolve[x*y''[x]-(4*x+1)*y'[x]+(4*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{2x}\left(c_2x^2 + 2c_1\right)$$

9.26 problem 26

Internal problem ID [1132]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2 \sin(x)\, y'' - 4x(x\cos(x) + \sin(x))\, y' + (2x\cos(x) + 3\sin(x))\, y = 0}$$

Given that one solution of the ode is

$$y_1 = \sqrt{x}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 17

```
dsolve([4*x^2*sin(x)*diff(y(x),x$2)-4*x*(x*cos(x)+sin(x))*diff(y(x),x)+(2*x*cos(x)+3*sin(x))*
```

$$y(x) = c_1\sqrt{x} + c_2\sqrt{x}\,\cos(x)$$

✓ Solution by Mathematica

Time used: 0.146 (sec). Leaf size: 21

```
DSolve[4*x^2*Sin[x]*y''[x]-4*x*(x*Cos[x]+Sin[x])*y'[x]+(2*x*Cos[x]+3*Sin[x])*y[x]==0,y[x],x,I
```

$$y(x) \to \sqrt{\arccos(\cos(x))}(c_2\cos(x) + c_1)$$

9.27 problem 27

Internal problem ID [1133]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4y'x + (-16x^2 + 3)\, y = 0}$$

Given that one solution of the ode is

$$y_1 = e^{2x}\sqrt{x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve([4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(3-16*x^2)*y(x)=0,sqrt(x)*exp(2*x)],y(x), sings
```

$$y(x) = c_1\sqrt{x}\, \sinh(2x) + c_2\sqrt{x}\, \cosh(2x)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 32

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(3-16*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}\sqrt{x}\left(c_2e^{4x} + 4c_1\right)$$

9.28 problem 28

Internal problem ID [1134]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1+2x)\,xy''-2(2x^2-1)\,y'-4(x+1)\,y=0}$$

Given that one solution of the ode is

$$y_1=\frac{1}{x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([(2*x+1)*x*diff(y(x),x$2)-2*(2*x^2-1)*diff(y(x),x)-4*(x+1)*y(x)=0,1/x],y(x), singsol=a
```

$$y(x)=\frac{c_1}{x}+c_2\mathrm{e}^{2x}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 28

```
DSolve[(2*x+1)*x*y''[x]-2*(2*x^2-1)*y'[x]-4*(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{c_2e^{2x+1}x+c_1}{\sqrt{e}x}$$

9.29 problem 29

Internal problem ID [1135]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-2x\right)y''+\left(-x^2+2\right)y'+\left(2x-2\right)y=0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([(x^2-2*x)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)+(2*x-2)*y(x)=0,exp(x)],y(x), singsol=al
```

$$y(x) = c_1x^2 + \mathrm{e}^x c_2$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 18

```
DSolve[(x^2-2*x)*y''[x]+(2-x^2)*y'[x]+(2*x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_2x^2 + c_1e^x$$

9.30 problem 30

Internal problem ID [1136]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (4x+1)\,y' + (4x+2)\,y = 0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve([x*diff(y(x),x$2)-(4*x+1)*diff(y(x),x)+(4*x+2)*y(x)=0,exp(2*x)],y(x), singsol=all)
```

$$y(x) = c_1\mathrm{e}^{2x} + c_2\mathrm{e}^{2x}x^2$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 25

```
DSolve[x*y''[x]-(4*x+1)*y'[x]+(4*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{2x}\left(c_2x^2 + 2c_1\right)$$

9.31 problem 31

Internal problem ID [1137]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 3y'x + 4y - 4x^4 = 0}$$

Given that one solution of the ode is

$$y_1 = x^2$$

With initial conditions

$$[y(-1) = 7, y'(-1) = -8]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve([x^2*diff(diff(y(x),x),x)-3*x*diff(y(x),x)+4*y(x) = 4*x^4, x^2, y(-1) = 7, D(y)(-1) =
```

$$y(x) = x^2(8i\pi + x^2 - 8\ln(x) + 6)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==4*x^2,{y[-1]==7,y'[-1]==8},y[x],x,IncludeSingularSolution
```

$$y(x) \to x^2(2\log(x)(\log(x) - 2i\pi - 11) - 2\pi(\pi - 11i) + 7)$$

9.32 problem 32

Internal problem ID [1138]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x-1)\,y''-(3x+2)\,y'-(6x-8)\,y=0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{2x}$$

With initial conditions

$$[y(0)=2, y'(0)=3]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([(3*x-1)*diff(diff(y(x),x),x)-(3*x+2)*diff(y(x),x)-(6*x-8)*y(x) = 0, exp(2*x), y(0) =
```

$$y(x) = 2\,\mathrm{e}^{2x} - x\,\mathrm{e}^{-x}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(3*x-1)*y''[x]-(3*x+2)*x*y'[x]-(6*x-8)*y[x]==0,{y[0]==2,y'[0]==3},y[x],x,IncludeSingul
```

Not solved

9.33 problem 33

Internal problem ID [1139]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{(x+1)^2 y'' - 2(x+1) y' - \left(x^2+2x-1\right) y - (x+1)^3 \mathrm{e}^x = 0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^x (x+1)$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([(1+x)^2*diff(diff(y(x),x),x)-2*(1+x)*diff(y(x),x)-(x^2+2*x-1)*y(x) = (1+x)^3*exp(x),
```

$$y(x) = \frac{(x+1)\left(x\,\mathrm{e}^x - 5\sinh(x) + 2\cosh(x)\right)}{2}$$

✓ Solution by Mathematica

Time used: 10.662 (sec). Leaf size: 5749

```
DSolve[(x+1)^2*y''[x]-2*(x+1)*x*y'[x]-(x^2+2*x-1)*y[x]==(x+1)^3*Exp[x],{y[0]==1,y'[0]==-1},y[
```

Too large to display

9.34 problem 34

Internal problem ID [1140]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2y'x - 2y - x^2 = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

With initial conditions

$$\left[y(1) = \frac{5}{4}, y'(1) = \frac{3}{2}\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve([x^2*diff(diff(y(x),x),x)+2*x*diff(y(x),x)-2*y(x) = x^2, x, y(1) = 5/4, D(y)(1) = 3/2]
```

$$y(x) = x + \frac{1}{4}x^2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 13

```
DSolve[x^2*y''[x]+2*x*y'[x]-2*y[x]==x^2,{y[1]==5/4,y'[1]==3/2},y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{1}{4}x(x + 4)$$

9.35 problem 35

Internal problem ID [1141]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{(x^2-4)\, y''+4y'x+2y-2-x=0}$$

Given that one solution of the ode is

$$y_1=\frac{1}{-2+x}$$

With initial conditions

$$\left[y(0)=-\frac{1}{3}, y'(0)=-1\right]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve([(x^2-4)*diff(diff(y(x),x),x)+4*x*diff(y(x),x)+2*y(x) = 2+x, 1/(-2+x), y(0) = -1/3, D(
```

$$y(x)=\frac{x^3+6x^2+24x+8}{6x^2-24}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 27

```
DSolve[(x^2-4)*y''[x]+4*x*y'[x]+2*y[x]==x+2,{y[1]==5/4,y'[1]==3/2},y[x],x,IncludeSingularSolu
```

$$y(x)\to\frac{2(x-3)x(x+9)-5}{12\,(x^2-4)}$$

9.36 problem 38 part (a)

Internal problem ID [1142]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 38 part (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y^2 + k^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)+y(x)^2+k^2=0,y(x), singsol=all)
```

$$y(x) = -\tan\left(k(c_1 + x)\right)k$$

✓ Solution by Mathematica

Time used: 3.44 (sec). Leaf size: 35

```
DSolve[y'[x]+y[x]^2+k^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -k\tan(k(x - c_1))$$
$$y(x) \to -ik$$
$$y(x) \to ik$$

9.37 problem 38 part (b)

Internal problem ID [1143]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 38 part (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y^2 - 3y + 2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)+y(x)^2-3*y(x)+2=0,y(x), singsol=all)
```

$$y(x) = \frac{2c_1 e^x - 1}{c_1 e^x - 1}$$

✓ Solution by Mathematica

Time used: 0.744 (sec). Leaf size: 31

```
DSolve[y'[x]+y[x]^2-3*y[x]+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + \frac{1}{1 - e^{-x+c_1}}$$
$$y(x) \to 1$$
$$y(x) \to 2$$

9.38 problem 38 part (c)

Internal problem ID [1144]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 38 part (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y^2 + 5y - 6 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)+y(x)^2+5*y(x)-6=0,y(x), singsol=all)
```

$$y(x) = \frac{6 + e^{7x} c_1}{e^{7x} c_1 - 1}$$

✓ Solution by Mathematica

Time used: 0.546 (sec). Leaf size: 34

```
DSolve[y'[x]+y[x]^2+5*y[x]-6==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-5 + 7\coth\left(\frac{7(x - c_1)}{2}\right)\right)$$
$$y(x) \to -6$$
$$y(x) \to 1$$

9.39 problem 38 part (d)

Internal problem ID [1145]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 38 part (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y^2 + 8y + 7 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)+y(x)^2+8*y(x)+7=0,y(x), singsol=all)
```

$$y(x) = -\frac{-7 + e^{6x} c_1}{e^{6x} c_1 - 1}$$

✓ Solution by Mathematica

Time used: 0.548 (sec). Leaf size: 28

```
DSolve[y'[x]+y[x]^2+8*y[x]+7==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4 + 3\coth(3(x - c_1))$$
$$y(x) \to -7$$
$$y(x) \to -1$$

9.40 problem 38 part (e)

Internal problem ID [1146]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 38 part (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y^2 + 14y + 50 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+y(x)^2+14*y(x)+50=0,y(x), singsol=all)
```

$$y(x) = -7 - \tan(c_1 + x)$$

✓ Solution by Mathematica

Time used: 0.449 (sec). Leaf size: 30

```
DSolve[y'[x]+y[x]^2+14*y[x]+50==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -7 - \tan(x - c_1)$$
$$y(x) \to -7 - i$$
$$y(x) \to -7 + i$$

9.41 problem 38 part (f)

Internal problem ID [1147]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 38 part (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{6y' + 6y^2 - y - 1 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(6*diff(y(x),x)+6*y(x)^2-y(x)-1=0,y(x), singsol=all)
```

$$y(x) = \frac{1 + e^{\frac{5x}{6}} c_1}{2\, e^{\frac{5x}{6}} c_1 - 3}$$

✓ Solution by Mathematica

Time used: 0.223 (sec). Leaf size: 45

```
DSolve[6*y'[x]+6*y[x]^2-y[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{3} + \frac{1}{\frac{6}{5} + \frac{9}{5}e^{-\frac{5x}{6}+5c_1}}$$
$$y(x) \to -\frac{1}{3}$$
$$y(x) \to \frac{1}{2}$$

9.42 problem 38 part (g)

Internal problem ID [1148]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 38 part (g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{36y' + 36y^2 - 12y + 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(36*diff(y(x),x)+36*y(x)^2-12*y(x)+1=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 + x + 6}{6x + 6c_1}$$

✓ Solution by Mathematica

Time used: 0.118 (sec). Leaf size: 24

```
DSolve[36*y'[x]+36*y[x]^2-12*y[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6} + \frac{1}{x - 36c_1}$$
$$y(x) \to \frac{1}{6}$$

9.43 problem 39 part(a)

Internal problem ID [1149]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 39 part(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccat`

$$\boxed{x^2(y'+y^2)-x(2+x)\,y+x+2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*(diff(y(x),x)+y(x)^2)-x*(x+2)*y(x)+x+2=0,y(x), singsol=all)
```

$$y(x)=-\frac{\mathrm{e}^x}{-\mathrm{e}^x+c_1}+\frac{1}{x}$$

✓ Solution by Mathematica

Time used: 0.168 (sec). Leaf size: 49

```
DSolve[x^2*(y'[x]+y[x])-x*(x+2)+y[x]+x+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{\frac{1}{x}-x}\left(\int_1^x\frac{e^{K[1]-\frac{1}{K[1]}}(K[1]^2+K[1]-2)}{K[1]^2}dK[1]+c_1\right)$$

9.44 problem 39 part(b)

Internal problem ID [1150]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 39 part(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' + y^2 + 4yx + 4x^2 + 2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)+y(x)^2+4*x*y(x)+4*x^2+2=0,y(x), singsol=all)
```

$$y(x) = -\frac{2c_1x - 2x^2 + 1}{c_1 - x}$$

✓ Solution by Mathematica

Time used: 0.122 (sec). Leaf size: 22

```
DSolve[y'[x]+y[x]^2+4*x*y[x]+4*x^2+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x + \frac{1}{x + c_1}$$
$$y(x) \to -2x$$

9.45 problem 39 part(c)

Internal problem ID [1151]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 39 part(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(1+2x)\left(y'+y^2\right)-2y-2x-3=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve((2*x+1)*(diff(y(x),x)+y(x)^2)-2*y(x)-(2*x+3)=0,y(x), singsol=all)
```

$$y(x)=-1-\frac{e^{2x}+2\,e^{2x}x}{-\frac{x\left(e^{2x}+2\,e^{2x}x\right)}{1+2x}-c_1}$$

✓ Solution by Mathematica

Time used: 0.377 (sec). Leaf size: 41

```
DSolve[(2*x+1)*(y'[x]+y[x]^2)-2*y[x]-(2*x+3)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{e^{2x+1}(x+1)-c_1}{e^{2x+1}x+c_1}$$
$$y(x)\to-1$$

9.46 problem 39 part(d)

Internal problem ID [1152]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 39 part(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(3x-1)\left(y'+y^2\right)-(3x+2)\,y-6x+8=0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 31

```
dsolve((3*x-1)*(diff(y(x),x)+y(x)^2)-(3*x+2)*y(x)-6*x+8=0,y(x), singsol=all)
```

$$y(x)=\frac{-c_1x+2\,\mathrm{e}^{3x-1}+c_1}{c_1x+\mathrm{e}^{3x-1}}$$

✓ Solution by Mathematica

Time used: 0.494 (sec). Leaf size: 34

```
DSolve[(3*x-1)*(y'[x]+y[x]^2)-(3*x+2)*y[x]-6*x+8==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 2+\frac{e(2-6x)}{2ex+c_1e^{3x}}$$
$$y(x)\to 2$$

9.47 problem 39 part(e)

Internal problem ID [1153]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 39 part(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, ‘_with_symmetry_[F(x),G(x)]‘], _Riccat`

$$\boxed{x^2(y'+y^2)+yx+x^2-\frac{1}{4}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x^2*(diff(y(x),x)+y(x)^2)+x*y(x)+x^2-1/4=0,y(x), singsol=all)
```

$$y(x)=\frac{2ix-1}{2x}-\frac{e^{-2ix}}{c_1-\frac{ie^{-2ix}}{2}}$$

✓ Solution by Mathematica

Time used: 0.357 (sec). Leaf size: 22

```
DSolve[x^2*(y'[x]+y[x]^2)+x*y[x]+x^2-1/4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{2x}-\tan(x-c_1)$$

9.48 problem 39 part(f)

Internal problem ID [1154]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.6 Reduction or order. Page 253
Problem number: 39 part(f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{x^2(y' + y^2) - 7yx + 7 = 0}$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 24

```
dsolve(x^2*(diff(y(x),x)+y(x)^2)-7*x*y(x)+7=0,y(x), singsol=all)
```

$$y(x) = \frac{-7x^6 + c_1}{x(-x^6 + c_1)}$$

✓ Solution by Mathematica

Time used: 0.186 (sec). Leaf size: 34

```
DSolve[x^2*(y'[x]+y[x]^2)-7*x*y[x]+7==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{7x^6 - 6c_1}{x^7 - 6c_1 x}$$

$$y(x) \to \frac{1}{x}$$

10 Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262

10.1 problem 1

Internal problem ID [1155]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y - \tan(3x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+9*y(x)=tan(3*x),y(x), singsol=all)
```

$$y(x) = \sin(3x)\, c_2 + \cos(3x)\, c_1 - \frac{\cos(3x)\ln(\sec(3x) + \tan(3x))}{9}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 31

```
DSolve[y''[x]+9*y[x]==Tan[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sin(3x) - \frac{1}{9}\cos(3x)(\text{arctanh}(\sin(3x)) - 9c_1)$$

10.2 problem 2

Internal problem ID [1156]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y - \sin(2x)\sec(2x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$2)+4*y(x)=sin(2*x)*sec(2*x)^2,y(x), singsol=all)
```

$$y(x) = \sin(2x)\,c_2 + \cos(2x)\,c_1 + \frac{\ln(\sec(2x))\sin(2x)}{4} - \frac{\sin(2x)}{4} + \frac{x\cos(2x)}{2}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 33

```
DSolve[y''[x]+4*y[x]==Sin[2*x]*Sec[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (-x + c_1)\cos(2x) + \sin(x)\cos(x)(2\log(\cos(x)) - 1 + 2c_2)$$

10.3 problem 3

Internal problem ID [1157]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y - \frac{4}{1 + e^{-x}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=4/(1+exp(-x)),y(x), singsol=all)
```

$$y(x) = \left(c_1 e^x - 4\ln\left(e^x\right) - 4\ln\left(e^x\right)e^x + 4\ln\left(1 + e^x\right)\left(1 + e^x\right) - 4 + c_2\right)e^x$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 34

```
DSolve[y''[x]-3*y'[x]+2*y[x]==4/(1+Exp[-x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(8(e^x + 1)\operatorname{arctanh}(2e^x + 1) + c_2 e^x - 4 + c_1)$$

10.4 problem 4

Internal problem ID [1158]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + 2y - 3\,\mathrm{e}^{x} \sec(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=3*exp(x)*sec(x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(x)\,\mathrm{e}^{x} + c_1 \cos(x)\,\mathrm{e}^{x} - 3\,\mathrm{e}^{x}(\cos(x)\ln(\sec(x)) - \sin(x)\,x)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 30

```
DSolve[y''[x]-2*y'[x]+2*y[x]==3*Exp[x]*Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}((3x + c_1)\sin(x) + \cos(x)(3\log(\cos(x)) + c_2))$$

10.5 problem 5

Internal problem ID [1159]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y - 14x^{\frac{3}{2}} \mathrm{e}^{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=14*x^(3/2)*exp(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} c_2 + x\,\mathrm{e}^{x} c_1 + \frac{8x^{\frac{7}{2}} \mathrm{e}^{x}}{5}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 29

```
DSolve[y''[x]-2*y'[x]+y[x]==14*x^(3/2)*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{5} e^{x} \left(8x^{7/2} + 5c_2 x + 5c_1\right)$$

10.6 problem 6

Internal problem ID [1160]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y - \frac{4\,\mathrm{e}^{-x}}{1 - \mathrm{e}^{-2x}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)-y(x)=4*exp(-x)/(1-exp(-2*x)),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_2 + c_1 \mathrm{e}^{x} + \ln\left(1 - \mathrm{e}^{-2x}\right) \mathrm{e}^{x} + \mathrm{e}^{-x}\left(\ln\left(\mathrm{e}^{-2x}\right) - \ln\left(-1 + \mathrm{e}^{-2x}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 49

```
DSolve[y''[x]-y[x]==4*Exp[-x]/(1-Exp[-2*x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2e^x \log\left(e^x\right) + (c_1 + c_2)\cosh(x) + \sinh(x)\left(2\log\left(e^x - 1\right) + 2\log\left(e^x + 1\right) + c_1 - c_2\right)$$

10.7 problem 7

Internal problem ID [1161]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + y'x - y - 2x^2 - 2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=2*x^2+2,y(x), singsol=all)
```

$$y(x) = c_2x + \frac{2x^2}{3} + \frac{c_1}{x} - 2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 24

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==2*x^2+2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^2}{3} + c_2x + \frac{c_1}{x} - 2$$

10.8 problem 8

Internal problem ID [1162]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2y'' + (-2x+2)\,y' + (-2+x)\,y - \mathrm{e}^{2x} = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x$2)+(2-2*x)*diff(y(x),x)+(x-2)*y(x)=exp(2*x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]+(2-2*x)*y'[x]+(x-2)*y[x]==exp(2*x),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

10.9 problem 9

Internal problem ID [1163]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{4x^2y'' + \left(-8x^2 + 4x\right) y' + \left(4x^2 - 4x - 1\right) y - 4\sqrt{x}\, \mathrm{e}^{x} = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 29

```
dsolve(4*x^2*diff(y(x),x$2)+(4*x-8*x^2)*diff(y(x),x)+(4*x^2-4*x-1)*y(x)=4*x^(1/2)*exp(x),y(x)
```

$$y(x) = \frac{\mathrm{e}^{x} c_2}{\sqrt{x}} + \sqrt{x}\, \mathrm{e}^{x} c_1 + \sqrt{x}\, \mathrm{e}^{x}(-1 + \ln (x))$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 27

```
DSolve[4*x^2*y''[x]+(4*x-8*x^2)*y'[x]+(4*x^2-4*x-1)*y[x]==4*x^(1/2)*Exp[x],y[x],x,IncludeSing
```

$$y(x) \to \frac{e^x (x \log(x) + (-1 + c_2)x + c_1)}{\sqrt{x}}$$

10.10 problem 10

Internal problem ID [1164]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y'x + (4x^2 + 2)\, y - 4\,\mathrm{e}^{-x(2+x)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+2)*y(x)=4*exp(-x*(x+2)),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x^2} c_2 + \mathrm{e}^{-x^2} x c_1 + \mathrm{e}^{-x(2+x)}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 27

```
DSolve[4*x^2*y''[x]+(4*x-8*x^2)*y'[x]+(4*x^2-4*x-1)*y[x]==4*x^(1/2)*Exp[x],y[x],x,IncludeSing
```

$$y(x) \rightarrow \frac{e^x(x \log(x) + (-1 + c_2)x + c_1)}{\sqrt{x}}$$

10.11 problem 11

Internal problem ID [1165]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 4y'x + 6y - x^{\frac{5}{2}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=x^(5/2),y(x), singsol=all)
```

$$y(x) = x^3c_2 + c_1x^2 - 4x^{\frac{5}{2}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==x^(5/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2\left(-4\sqrt{x} + c_2x + c_1\right)$$

10.12 problem 12

Internal problem ID [1166]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 3y'x + 3y - 2\sin(x)\, x^4 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+3*y(x)=2*x^4*sin(x),y(x), singsol=all)
```

$$y(x) = \left(\frac{c_1 x^2}{2} - 2\cos(x) - 2\sin(x)\, x + c_2\right) x$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]-3*x*y'[x]+3*y[x]==2*x^4*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(c_2 x^2 - 2x\sin(x) - 2\cos(x) + c_1\right)$$

10.13 problem 13

Internal problem ID [1167]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{(1+2x)\,y''-2y'-(2x+3)\,y-(1+2x)^2\,\mathrm{e}^{-x}=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve((2*x+1)*diff(y(x),x$2)-2*diff(y(x),x)-(2*x+3)*y(x)=(2*x+1)^2*exp(-x),y(x), singsol=all
```

$$y(x)=\mathrm{e}^{-x}c_2+x\,\mathrm{e}^{x}c_1-\frac{(x+1)\,x\,\mathrm{e}^{-x}}{2}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 42

```
DSolve[(2*x+1)*y''[x]-2*y'[x]-(2*x+3)*y[x]==(2*x+1)^2*Exp[-x],y[x],x,IncludeSingularSolutions
```

$$y(x)\to -\frac{1}{2}e^{-x}x(x+1)+c_1e^{-x-\frac{1}{2}}+c_2e^{x+\frac{1}{2}}x$$

10.14 problem 14

Internal problem ID [1168]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2xy'' + 2y' + 2y - \sin\left(\sqrt{x}\right) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 72

```
dsolve(2*x*diff(y(x),x$2)+2*diff(y(x),x)+2*y(x)=sin(sqrt(x)),y(x), singsol=all)
```

$$y(x) = \text{BesselJ}\left(0, 2\sqrt{x}\right) c_2 + \text{BesselY}\left(0, 2\sqrt{x}\right) c_1 + \frac{\pi\left(\left(\int \text{BesselJ}\left(0, 2\sqrt{x}\right) \sin\left(\sqrt{x}\right) dx\right) \text{BesselY}\left(0, 2\sqrt{x}\right) - \left(\int \text{BesselY}\left(0, 2\sqrt{x}\right) \sin\left(\sqrt{x}\right) dx\right) \text{BesselJ}\left(0, 2\right.}{2}$$

✓ Solution by Mathematica

Time used: 5.178 (sec). Leaf size: 88

```
DSolve[2*x*y''[x]+2*y'[x]+2*y[x]==Sin[Sqrt[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to {}_0\tilde{F}_1(;1;-x) \int_1^x -\frac{1}{2}\pi Y_0\left(2\sqrt{K[1]}\right) \sin\left(\sqrt{K[1]}\right) dK[1] + 2Y_0\left(2\sqrt{x}\right)\left(\int_1^x \frac{1}{4}\pi\, {}_0\tilde{F}_1(;1;-K[2]) \sin\left(\sqrt{K[2]}\right) dK[2] + c_2\right) + c_1\, {}_0\tilde{F}_1(;1;-x)$$

10.15 problem 15

Internal problem ID [1169]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' - (2+2x)\,y' + (2+x)\,y - 6\,\mathrm{e}^{x} x^{3} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x$2)-(2*x+2)*diff(y(x),x)+(x+2)*y(x)=6*x^3*exp(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} c_2 + \mathrm{e}^{x} x^{3} c_1 + \frac{3\,\mathrm{e}^{x} x^{4}}{2}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 29

```
DSolve[x*y''[x]-(2*x+2)*y'[x]+(x+2)*y[x]==6*x^3*Exp[x],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{6} e^{x} \left(9x^4 + 2c_2 x^3 + 6c_1\right)$$

10.16 problem 16

Internal problem ID [1170]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - (2a-1)\,xy' + ya^2 - x^{1+a} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)-(2*a-1)*x*diff(y(x),x)+a^2*y(x)=x^(a+1),y(x), singsol=all)
```

$$y(x) = x^a c_2 + x^a \ln(x)\, c_1 + x^{a+1}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 19

```
DSolve[x^2*y''[x]-(2*a-1)*x*y'[x]+a^2*y[x]==x^(a+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^a(ac_2 \log(x) + x + c_1)$$

10.17 problem 17

Internal problem ID [1171]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 2y'x + (x^2+2)\,y - x^3\cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=x^3*cos(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\,xc_2 + \cos(x)\,xc_1 + \frac{\sin(x)\,x^2}{2}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 40

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==x^3*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}x((1+4c_1-2ic_2)\cos(x)+2(x-2ic_1+c_2)\sin(x))$$

10.18 problem 18

Internal problem ID [1172]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' - y' - 4x^3y - 8x^5 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)-4*x^3*y(x)=8*x^5,y(x), singsol=all)
```

$$y(x) = \sinh\left(x^2\right)c_2 + \cosh\left(x^2\right)c_1 - 2x^2$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 28

```
DSolve[x*y''[x]-y'[x]-4*x^3*y[x]==8*x^5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x^2 + c_1\cosh\left(x^2\right) + ic_2\sinh\left(x^2\right)$$

10.19 problem 19

Internal problem ID [1173]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{\sin(x)\, y'' + (2\sin(x) - \cos(x))\, y' + (\sin(x) - \cos(x))\, y - e^{-x} = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve(sin(x)*diff(y(x),x$2)+(2*sin(x)-cos(x))*diff(y(x),x)+(sin(x)-cos(x))*y(x)=exp(-x),y(x)
```

$$y(x) = e^{\frac{\pi}{2}-x}c_2 + \cos(x)\, e^{-x}c_1 - \sin(x)\, e^{-x}$$

✓ Solution by Mathematica

Time used: 0.343 (sec). Leaf size: 52

```
DSolve[Sin[x]*y''[x]+(2*Sin[x]-Cos[x])*y'[x]+(Sin[x]-Cos[x])*y[x]==Exp[-x],y[x],x,IncludeSing
```

$$y(x) \to -\sqrt{\sin^2(x)}e^{-\arccos(\cos(x))} + (c_2\cos(x) + c_1)e^{-2\sqrt{\sin^2(x)}\csc(x)\cot^{-1}(\cot(x)+\csc(x))}$$

10.20 problem 20

Internal problem ID [1174]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{4x^2y'' - 4y'x + (-16x^2 + 3)\,y - 8x^{\frac{5}{2}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(3-16*x^2)*y(x)=8*x^(5/2),y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,\sinh(2x)\,c_2 + \sqrt{x}\,\cosh(2x)\,c_1 - \frac{\sqrt{x}}{2}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 39

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(3-16*x^2)*y[x]==8*x^(5/2),y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{4}e^{-2x}\sqrt{x}\left(-2e^{2x} + c_2e^{4x} + 4c_1\right)$$

10.21 problem 21

Internal problem ID [1175]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{4x^2y'' - 4y'x + (4x^2 + 3)\, y - x^{\frac{7}{2}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2+3)*y(x)=x^(7/2),y(x), singsol=all)
```

$$y(x) = \sqrt{x}\, \sin(x)\, c_2 + \sqrt{x}\, \cos(x)\, c_1 + \frac{x^{\frac{3}{2}}}{4}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 40

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+(4*x^2+3)*y[x]==x^(7/2),y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{4}\sqrt{x}\left(x + 4c_1e^{-ix} - 2ic_2e^{ix}\right)$$

10.22 problem 22

Internal problem ID [1176]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 2y'x - (x^2 - 2)\, y - 3x^4 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)-(x^2-2)*y(x)=3*x^4,y(x), singsol=all)
```

$$y(x) = \sinh(x)\, xc_2 + \cosh(x)\, xc_1 - 3x^2$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 29

```
DSolve[x^2*y''[x]-2*x*y'[x]-(x^2-2)*y[x]==3*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x\left(-6x + 2c_1e^{-x} + c_2e^x\right)$$

10.23 problem 23

Internal problem ID [1177]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2y'' - 2x(x+1)\,y' + \left(x^2+2x+2\right)y - e^x x^3 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x$2)-2*x*(x+1)*diff(y(x),x)+(x^2+2*x+2)*y(x)=x^3*exp(x),y(x), singsol=al
```

$$y(x) = c_2 e^x x + e^x c_1 x^2 + \frac{e^x x^3}{2}$$

✓ Solution by Mathematica

Time used: 0.412 (sec). Leaf size: 210

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2*x+2)*y[x]==x^3*Exp[x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{ix}x\Bigg(\text{HypergeometricU}(-i,0,$$
$$-2ix)\left(\int_1^x \frac{e^{(1-i)K[1]}\,\text{Hypergeometric1F1}(1-i,2,-2iK}{2\,\text{Hypergeometric1F1}(1-i,2,-2iK[1])\,\text{HypergeometricU}(1-i,1,-2iK[1])K[1]-\text{Hyper}}\right.$$
$$\left.+c_1\right) + 2ix\,\text{Hypergeometric1F1}(1-i,2,$$
$$-2ix)\left(\int_1^x \frac{ie^{(1-i)K[2]}\,\text{HypergeometricU}(-i,0,-2i}{4\,\text{Hypergeometric1F1}(1-i,2,-2iK[2])\,\text{HypergeometricU}(1-i,1,-2iK[2])K[2]-2\,\text{Hype}}\right.$$
$$\left.\left.+c_2\right)\right)$$

10.24 problem 24

Internal problem ID [1178]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - y'x - 3y - x^{\frac{3}{2}} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)-3*y(x)=x^(3/2),y(x), singsol=all)
```

$$y(x) = x^3c_2 - \frac{4x^{\frac{3}{2}}}{15} + \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]-x*y'[x]-3*y[x]==x^(3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{4x^{3/2}}{15} + c_2x^3 + \frac{c_1}{x}$$

10.25 problem 25

Internal problem ID [1179]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - x(x+4)\,y' + 2(x+3)\,y - x^4\mathrm{e}^x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-x*(x+4)*diff(y(x),x)+2*(x+3)*y(x)=x^4*exp(x),y(x), singsol=all)
```

$$y(x) = c_2x^2\mathrm{e}^x + c_1x^2 + \mathrm{e}^x x^3$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]-x*(x+4)*y'[x]+2*(x+3)*y[x]==x^4*Exp[x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to x^2(e^x(x-1+c_2)+c_1)$$

10.26 problem 26

Internal problem ID [1180]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 2x(2+x)\,y' + \left(x^2+4x+6\right)y - 2\,\mathrm{e}^x x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-2*x*(x+2)*diff(y(x),x)+(x^2+4*x+6)*y(x)=2*x*exp(x),y(x), singsol=al
```

$$y(x) = c_2x^2\mathrm{e}^x + \mathrm{e}^x x^3c_1 + x\,\mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]-2*x*(x+2)*y'[x]+(x^2+4*x+6)*y[x]==2*x*Exp[x],y[x],x,IncludeSingularSolution
```

$$y(x) \to e^x x(1 + x(c_2x + c_1))$$

10.27 problem 27

Internal problem ID [1181]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 4y'x + (x^2 + 6)\, y - x^4 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(x^2+6)*y(x)=x^4,y(x), singsol=all)
```

$$y(x) = \sin(x)\, x^2 c_2 + \cos(x)\, x^2 c_1 + x^2$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]-4*x*y'[x]+(x^2+6)*y[x]==x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x^2 \left(2c_1 e^{-ix} + c_2(\sin(x) - i\cos(x)) + 2\right)$$

10.28 problem 28

Internal problem ID [1182]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-y'x+y-2(x-1)^2\,\mathrm{e}^x=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve((x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=2*(x-1)^2*exp(x),y(x), singsol=all)
```

$$y(x)=c_2x+c_1\mathrm{e}^x+x(-2+x)\,\mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 23

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==2*(x-1)^2*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^x((x-2)x+c_1)-c_2x$$

10.29 problem 29

Internal problem ID [1183]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{4x^2y'' - 4x(x+1)\,y' + (2x+3)\,y - x^{\frac{5}{2}}\mathrm{e}^x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*(x+1)*diff(y(x),x)+(2*x+3)*y(x)=x^(5/2)*exp(x),y(x), singsol=
```

$$y(x) = \sqrt{x}\,c_2 + \sqrt{x}\,\mathrm{e}^x c_1 + \frac{x^{\frac{3}{2}}\mathrm{e}^x}{4}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 30

```
DSolve[4*x^2*y''[x]-4*x*(x+1)*y'[x]+(2*x+3)*y[x]==x^(5/2)*Exp[x],y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{1}{4}\sqrt{x}(e^x(x-1+4c_2)+4c_1)$$

10.30 problem 30

Internal problem ID [1184]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(3x-1)\,y''-(3x+2)\,y'-(6x-8)\,y-(3x-1)^2\,e^{2x}=0$$

With initial conditions

$$[y(0)=1, y'(0)=2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve([(3*x-1)*diff(y(x),x$2)-(3*x+2)*diff(y(x),x)-(6*x-8)*y(x)=(3*x-1)^2*exp(2*x),y(0) = 1,
```

$$y(x)=\frac{(3x^2-2x+6)\,e^{2x}}{6}+\frac{x\,e^{-x}}{3}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 33

```
DSolve[{(3*x-1)*y''[x]-(3*x+2)*y'[x]-(6*x-8)*y[x]==(3*x-1)^2*Exp[2*x],{y[0]==1,y'[0]==2}},y[x
```

$$y(x)\to\frac{1}{6}e^{-x}\left(2x+e^{3x}(x(3x-2)+6)\right)$$

10.31 problem 31

Internal problem ID [1185]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x-1)^2 y'' - 2(x-1) y' + 2y - (x-1)^2 = 0$$

With initial conditions

$$[y(0) = 3, y'(0) = -6]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve([(x-1)^2*diff(y(x),x$2)-2*(x-1)*diff(y(x),x)+2*y(x)=(x-1)^2,y(0) = 3, D(y)(0) = -6],y(
```

$$y(x) = -(x-1)\left(i\pi x - i\pi - \ln(x-1)x + \ln(x-1) - 2x + 3\right)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 30

```
DSolve[{(x-1)^2*y''[x]-2*(x-1)*y'[x]+2*y[x]==(x-1)^2,{y[0]==3,y'[0]==-6}},y[x],x,IncludeSingu
```

$$y(x) \to (x-1)(-i\pi(x-1) + 2x + (x-1)\log(x-1) - 3)$$

10.32 problem 32

Internal problem ID [1186]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(x-1)^2 y'' - (x^2-1) y' + (x-1)^3 y - (x-1)^3 e^x = 0$$

With initial conditions

$$[y(0) = 4, y'(0) = -6]$$

✗ Solution by Maple

```
dsolve([(x-1)^2*diff(y(x),x$2)-(x^2-1)*diff(y(x),x)+(x-1)^3*y(x)=(x-1)^3*exp(x),y(0) = 4, D(y
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(x-1)^2*y''[x]-(x^2-1)*y'[x]+(x-1)^3*y[x]==(x-1)^3*Exp[x],{y[0]==4,y'[0]==-6}},y[x],x
```

Not solved

10.33 problem 33

Internal problem ID [1187]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{(x-1)^2 y'' + 4y'x + 2y - 2x = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = -2]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 67

```
dsolve([(x-1)^2*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=2*x,y(0) = 0, D(y)(0) = -2],y(x), sing
```

$$y(x) = \frac{x^3 + 28\,e^{\frac{4}{x-1}}\,\mathrm{Ei}_1\left(\frac{4}{x-1}\right) - 28\,e^{\frac{4}{x-1}}\,\mathrm{Ei}_1\left(-4\right) - 7\,e^{\frac{4x}{x-1}} - 2x^2 - 6x + 7}{3\left(x-1\right)^2}$$

✓ Solution by Mathematica

Time used: 0.115 (sec). Leaf size: 70

```
DSolve[{(x-1)^2*y''[x]+4*x*y'[x]+2*y[x]==2*x,{y[0]==0,y'[0]==-2}},y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{-28e^{\frac{4}{x-1}}\,\mathrm{ExpIntegralEi}\left(-\frac{4}{x-1}\right) + 28\,\mathrm{ExpIntegralEi}(4)e^{\frac{4}{x-1}} + (x-1)((x-1)x-7) - 7e^{\frac{4x}{x-1}}}{3(x-1)^2}$$

10.34 problem 34

Internal problem ID [1188]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2y'x - 2y + 2x^2 = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=-2*x^2,y(1) = 1, D(y)(1) = -1],y(x), sings
```

$$y(x) = \frac{1}{2x^2} + x - \frac{x^2}{2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 21

```
DSolve[{x^2*y''[x]+2*x*y'[x]-2*y[x]==-2*x^2,{y[1]==1,y'[1]==-1}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to -\frac{x^2}{2} + \frac{1}{2x^2} + x$$

10.35 problem 35

Internal problem ID [1189]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 5 linear second order equations. Section 5.7 Variation of Parameters. Page 262
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$(x+1)(2x+3)y''+2(2+x)y'-2y-(2x+3)^2=0$$

With initial conditions

$$[y(0)=0, y'(0)=0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve([(x+1)*(2*x+3)*diff(y(x),x$2)+2*(x+2)*diff(y(x),x)-2*y(x)=(2*x+3)^2,y(0) = 0, D(y)(0)
```

$$y(x)=\frac{x^2(4x+9)}{6x+6}$$

✓ Solution by Mathematica

Time used: 0.344 (sec). Leaf size: 22

```
DSolve[{(x+1)*(2*x+3)*y''[x]+2*(x+2)*y'[x]-2*y[x]==(2*x+3)^2,{y[0]==0,y'[0]==0}},y[x],x,Inclu
```

$$y(x)\to\frac{x^2(4x+9)}{6(x+1)}$$

11 Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318

11.1 problem 11

Internal problem ID [1190]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2+x)\,y''+y'x+3y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve((2+x)*diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{3}{4}x^2+\frac{1}{8}x^3+\frac{1}{8}x^4-\frac{9}{160}x^5\right)y(0)+\left(x-\frac{1}{3}x^3+\frac{1}{12}x^4+\frac{1}{40}x^5\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(2+x)*y''[x]+x*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{x^5}{40}+\frac{x^4}{12}-\frac{x^3}{3}+x\right)+c_1\left(-\frac{9x^5}{160}+\frac{x^4}{8}+\frac{x^3}{8}-\frac{3x^2}{4}+1\right)$$

11.2 problem 12

Internal problem ID [1191]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x^2+1)\,y''+3y'x^2-2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
Order:=6;
dsolve((1+3*x^2)*diff(y(x),x$2)+3*x^2*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+x^2-\frac{1}{3}x^4-\frac{3}{10}x^5\right)y(0)+\left(x+\frac{1}{3}x^3-\frac{1}{4}x^4-\frac{4}{15}x^5\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 52

```
AsymptoticDSolveValue[(1+3*x^2)*y''[x]+3*x^2*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(-\frac{4x^5}{15}-\frac{x^4}{4}+\frac{x^3}{3}+x\right)+c_1\left(-\frac{3x^5}{10}-\frac{x^4}{3}+x^2+1\right)$$

11.3 problem 13

Internal problem ID [1192]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2+1)\,y''+(-3x+2)\,y'+4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((1+2*x^2)*diff(y(x),x$2)+(2-3*x)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-2x^2+\frac{4}{3}x^3-\frac{1}{3}x^4-\frac{1}{3}x^5\right)y(0)+\left(x-x^2+\frac{1}{2}x^3-\frac{1}{12}x^4-\frac{17}{120}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 66

```
AsymptoticDSolveValue[(1+2*x^2)*y''[x]+(2-3*x)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{x^5}{3}-\frac{x^4}{3}+\frac{4x^3}{3}-2x^2+1\right)+c_2\left(-\frac{17x^5}{120}-\frac{x^4}{12}+\frac{x^3}{2}-x^2+x\right)$$

11.4 problem 14

Internal problem ID [1193]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''+(2-x)\,y'+3y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
Order:=6;
dsolve((1+x^2)*diff(y(x),x$2)+(2-x)*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{3}{2}x^2+x^3-\frac{1}{8}x^4-\frac{1}{4}x^5\right)y(0)+\left(x-x^2+\frac{1}{3}x^3+\frac{1}{12}x^4-\frac{2}{15}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 64

```
AsymptoticDSolveValue[(1+x^2)*y''[x]+(2-x)*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{x^5}{4}-\frac{x^4}{8}+x^3-\frac{3x^2}{2}+1\right)+c_2\left(-\frac{2x^5}{15}+\frac{x^4}{12}+\frac{x^3}{3}-x^2+x\right)$$

11.5 problem 15

Internal problem ID [1194]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x^2+1)\, y'' - 2y'x + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
Order:=6;
dsolve((1+3*x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (x^4 - 2x^2 + 1)\, y(0) + \left(x - \frac{1}{3}x^3 + \frac{4}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[(1+3*x^2)*y''[x]-2*x*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2\left(\frac{4x^5}{15} - \frac{x^3}{3} + x\right) + c_1(x^4 - 2x^2 + 1)$$

11.6 problem 16

Internal problem ID [1195]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (2x+4)\,y' + (2+x)\,y = 0}$$

With the expansion point for the power series method at $x = -1$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(4+2*x)*diff(y(x),x)+(2+x)*y(x)=0,y(x),type='series',x=-1);
```

$$y(x) = \left(1 + \frac{(x+1)^2}{2} + \frac{2(x+1)^3}{3} + \frac{7(x+1)^4}{8} + \frac{17(x+1)^5}{15}\right) y(-1) + \left(x + 1 + (x+1)^2 + \frac{3(x+1)^3}{2} + 2(x+1)^4 + \frac{103(x+1)^5}{40}\right) D(y)(-1) + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 81

```
AsymptoticDSolveValue[(x)*y''[x]+(4+2*x)*y'[x]+(2+x)*y[x]==0,y[x],{x,-1,5}]
```

$$y(x) \to c_1\left(\frac{17}{15}(x+1)^5 + \frac{7}{8}(x+1)^4 + \frac{2}{3}(x+1)^3 + \frac{1}{2}(x+1)^2 + 1\right) + c_2\left(\frac{103}{40}(x+1)^5 + 2(x+1)^4 + \frac{3}{2}(x+1)^3 + (x+1)^2 + x + 1\right)$$

11.7 problem 17

Internal problem ID [1196]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 2y'x - 3yx = 0}$$

With the expansion point for the power series method at $x = 2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-3*x*y(x)=0,y(x),type='series',x=2);
```

$$y(x) = \left(1 + \frac{3(-2+x)^2}{4} - \frac{3(-2+x)^3}{8} + \frac{9(-2+x)^4}{32} - \frac{27(-2+x)^5}{160}\right) y(2) + \left(-2+x - \frac{(-2+x)^2}{2} + \frac{(-2+x)^3}{2} - \frac{5(-2+x)^4}{16} + \frac{31(-2+x)^5}{160}\right) D(y)(2) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x^2*y''[x]+2*x*y'[x]-3*x*y[x]==0,y[x],{x,2,5}]
```

$$y(x) \to c_1\left(-\frac{27}{160}(x-2)^5 + \frac{9}{32}(x-2)^4 - \frac{3}{8}(x-2)^3 + \frac{3}{4}(x-2)^2 + 1\right) + c_2\left(\frac{31}{160}(x-2)^5 - \frac{5}{16}(x-2)^4 + \frac{1}{2}(x-2)^3 - \frac{1}{2}(x-2)^2 + x - 2\right)$$

11.8 problem 18

Internal problem ID [1197]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(2-x)\,y''+2y=0}$$

With initial conditions

$$[y(0)=a_0, y'(0)=a_1]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
Order:=6;
dsolve([(2-x)*diff(y(x),x$2)+2*y(x)=0,y(0) = a__0, D(y)(0) = a__1],y(x),type='series',x=0);
```

$$y(x)=a_0+a_1x-\frac{1}{2}a_0x^2+\left(-\frac{a_1}{6}-\frac{a_0}{12}\right)x^3+\left(\frac{a_0}{48}-\frac{a_1}{24}\right)x^4+\left(-\frac{a_1}{240}+\frac{a_0}{96}\right)x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 79

```
AsymptoticDSolveValue[{(2-x)*y''[x]+2*y[x]==0,{y[0]==a0,y'[0]==a1}},y[x],{x,0,5}]
```

$$y(x)\to\frac{1}{20}x^5\left(\frac{1}{6}\left(\frac{\mathrm{a0}}{2}+\mathrm{a1}\right)+\frac{\mathrm{a0}}{8}-\frac{\mathrm{a1}}{4}\right)+\frac{1}{12}x^4\left(\frac{\mathrm{a0}}{4}-\frac{\mathrm{a1}}{2}\right)+\frac{1}{6}x^3\left(-\frac{\mathrm{a0}}{2}-\mathrm{a1}\right)-\frac{\mathrm{a0}x^2}{2}+\mathrm{a0}+\mathrm{a1}x$$

11.9 problem 19

Internal problem ID [1198]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+1)\,y''+2(x-1)^2\,y'+3y=0}$$

With initial conditions

$$[y(1)=a_0, y'(1)=a_1]$$

With the expansion point for the power series method at $x=1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
Order:=6;
dsolve([(1+x)*diff(y(x),x$2)+2*(x-1)^2*diff(y(x),x)+3*y(x)=0,y(1) = a__0, D(y)(1) = a__1],y(x
```

$$y(x)=a_0+a_1(x-1)-\frac{3}{4}a_0(x-1)^2+\left(\frac{a_0}{8}-\frac{a_1}{4}\right)(x-1)^3+\left(\frac{a_0}{16}-\frac{a_1}{48}\right)(x-1)^4+\left(\frac{3a_0}{64}+\frac{a_1}{40}\right)(x-1)^5+\mathrm{O}\left((x-1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 95

```
AsymptoticDSolveValue[{(1+x)*y''[x]+2*(x-1)^2*y'[x]+3*y[x]==0,{y[1]==a0,y'[1]==a1}},y[x],{x,1
```

$$y(x)\to\frac{1}{20}(x-1)^5\left(\frac{1}{4}\left(\frac{3\text{a1}}{2}-\frac{3\text{a0}}{4}\right)+\frac{9\text{a0}}{8}+\frac{\text{a1}}{8}\right)+\frac{1}{12}(x-1)^4\left(\frac{3\text{a0}}{4}-\frac{\text{a1}}{4}\right)+\frac{1}{6}(x-1)^3\left(\frac{3\text{a0}}{4}-\frac{3\text{a1}}{2}\right)-\frac{3}{4}\text{a0}(x-1)^2+\text{a0}+\text{a1}(x-1)$$

11.10 problem 21

Internal problem ID [1199]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x)\,y'' + x(x+4)\,y' + (2-x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
Order:=6;
dsolve(x^2*(1-x)*diff(y(x),x$2)+x*(4+x)*diff(y(x),x)+(2-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{\ln(x)\left(9x + 18x^2 + 3x^3 + \mathrm{O}(x^6)\right)c_2 + c_1\left(1 + 2x + \frac{1}{3}x^2 + \mathrm{O}(x^6)\right)x + \left(1 - 5x - 55x^2 - \frac{53}{3}x^3 + \mathrm{O}(x^6)\right)c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 56

```
AsymptoticDSolveValue[x^2*(1-x)*y''[x]+x*(4+x)*y'[x]+(2-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{3(x^2+6x+3)\log(x)}{x} - \frac{21x^3+75x^2+15x-1}{x^2}\right) + c_2\left(\frac{x}{3} + \frac{1}{x} + 2\right)$$

11.11 problem 22

Internal problem ID [1200]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+1)\,y''+x(1+2x)\,y'-(6x+4)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 51

```
Order:=6;
dsolve(x^2*(1+x)*diff(y(x),x$2)+x*(1+2*x)*diff(y(x),x)-(4+6*x)*y(x)=0,y(x),type='series',x=0)
```

$$y(x)=c_1x^2\left(1+\mathrm{O}\left(x^6\right)\right)+\frac{c_2(\ln(x)\left(576x^4+\mathrm{O}\left(x^6\right)\right)+\left(-144+192x-288x^2+576x^3-576x^4-576x^5+\mathrm{O}\left(x^6\right)\right))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 48

```
AsymptoticDSolveValue[x^2*(1+x)*y''[x]+x*(1+2*x)*y'[x]-(4+6*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2x^2+c_1\left(\frac{3x^4-12x^3+6x^2-4x+3}{3x^2}-4x^2\log(x)\right)$$

11.12 problem 23

Internal problem ID [1201]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+1)\,y'' - x\left(-x^2-6x+1\right)y' + \left(x^2+6x+1\right)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
Order:=6;
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(1-6*x-x^2)*diff(y(x),x)+(1+6*x+x^2)*y(x)=0,y(x),type='seri
```

$$y(x) = x\left((c_2\ln(x)+c_1)\left(1-12x+\frac{119}{2}x^2-\frac{583}{3}x^3+\frac{1981}{4}x^4-\frac{80287}{75}x^5+\mathrm{O}\left(x^6\right)\right)\right.$$
$$\left.+\left(17x-\frac{471}{4}x^2+445x^3-\frac{118285}{96}x^4+\frac{702451}{250}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 114

```
AsymptoticDSolveValue[x^2*(1+x)*y''[x]-x*(1-6*x-x^2)*y'[x]+(1+6*x+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x\left(-\frac{80287x^5}{75}+\frac{1981x^4}{4}-\frac{583x^3}{3}+\frac{119x^2}{2}-12x+1\right)$$
$$+c_2\left(x\left(\frac{702451x^5}{250}-\frac{118285x^4}{96}+445x^3-\frac{471x^2}{4}+17x\right)\right.$$
$$\left.+x\left(-\frac{80287x^5}{75}+\frac{1981x^4}{4}-\frac{583x^3}{3}+\frac{119x^2}{2}-12x+1\right)\log(x)\right)$$

11.13 problem 24

Internal problem ID [1202]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$x^2(3x+1)\,y''+x\left(x^2+12x+2\right)y'+2x(x+3)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
Order:=6;
dsolve(x^2*(1+3*x)*diff(y(x),x$2)+x*(2+12*x+x^2)*diff(y(x),x)+2*x*(3+x)*y(x)=0,y(x),type='ser
```

$$y(x)=c_1\left(1-3x+\frac{26}{3}x^2-\frac{101}{4}x^3+\frac{4441}{60}x^4-\frac{26141}{120}x^5+\mathrm{O}\left(x^6\right)\right)+\frac{c_2\left(1-6x+\frac{35}{2}x^2-\frac{101}{2}x^3+\frac{1177}{8}x^4-\frac{17251}{40}x^5+\mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x^2*(1+3*x)*y''[x]+x*(2+12*x+x^2)*y'[x]+2*x*(3+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{571x^3}{8}-\frac{49x^2}{2}+\frac{17x}{2}+\frac{1}{x}-3\right)+c_2\left(\frac{4441x^4}{60}-\frac{101x^3}{4}+\frac{26x^2}{3}-3x+1\right)$$

11.14 problem 25

Internal problem ID [1203]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x^2+1)\,y''+x(2x^2+4)\,y'+2(-x^2+1)\,y=0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
Order:=6;
dsolve(x^2*(1+2*x^2)*diff(y(x),x$2)+x*(4+2*x^2)*diff(y(x),x)+2*(1-x^2)*y(x)=0,y(x),type='seri
```

$$y(x)=\frac{c_1(1+\mathrm{O}(x^6))\,x+c_2\left(1-3x^2-\frac{1}{2}x^4+\mathrm{O}(x^6)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 25

```
AsymptoticDSolveValue[x^2*(1+2*x^2)*y''[x]+x*(4+2*x^2)*y'[x]+2*(1-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{x^2}{2}+\frac{1}{x^2}-3\right)+\frac{c_2}{x}$$

11.15 problem 26

Internal problem ID [1204]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.1 Exercises. Page 318
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x^2+2)\,y''+2x(x^2+5)\,y'+2(-x^2+3)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(x^2*(2+x^2)*diff(y(x),x$2)+2*x*(x^2+5)*diff(y(x),x)+2*(3-x^2)*y(x)=0,y(x),type='series
```

$$y(x) = \frac{c_1\left(1+\frac{1}{8}x^2+\mathrm{O}\left(x^6\right)\right)}{x}+\frac{c_2\left(\ln\left(x\right)\left(2x^2+\frac{1}{4}x^4+\mathrm{O}\left(x^6\right)\right)+\left(-2-\frac{3}{2}x^2-\frac{1}{4}x^4+\mathrm{O}\left(x^6\right)\right)\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 51

```
AsymptoticDSolveValue[x^2*(2+x^2)*y''[x]+2*x*(x^2+5)*y'[x]+2*(3-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{x^4+7x^2+4}{4x^3}-\frac{(x^2+8)\log(x)}{8x}\right)+c_2\left(\frac{x}{8}+\frac{1}{x}\right)$$

12 Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329

12.1 problem 1

Internal problem ID [1205]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''+6y'x+6y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((1+x^2)*diff(y(x),x$2)+6*x*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(5x^4-3x^2+1\right)y(0)+\left(3x^5-2x^3+x\right)D(y)(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[(1+x^2)*y''[x]+6*x*y'[x]+6*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(3x^5-2x^3+x\right)+c_1\left(5x^4-3x^2+1\right)$$

12.2 problem 2

Internal problem ID [1206]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''+2y'x-2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
Order:=6;
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+x^2-\frac{1}{3}x^4\right)y(0)+D(y)\,(0)\,x+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 23

```
AsymptoticDSolveValue[(1+x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{x^4}{3}+x^2+1\right)+c_2x$$

12.3 problem 3

Internal problem ID [1207]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-8y'x+20y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((1+x^2)*diff(y(x),x$2)-8*x*diff(y(x),x)+20*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(5x^4-10x^2+1)\,y(0)+\left(x-2x^3+\frac{1}{5}x^5\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[(1+x^2)*y''[x]-8*x*y'[x]+20*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{x^5}{5}-2x^3+x\right)+c_1(5x^4-10x^2+1)$$

12.4 problem 4

Internal problem ID [1208]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\,y''-8y'x-12y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-8*x*diff(y(x),x)-12*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(15x^4+6x^2+1)\,y(0)+\left(x+\frac{10}{3}x^3+7x^5\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-8*x*y'[x]-12*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(7x^5+\frac{10x^3}{3}+x\right)+c_1(15x^4+6x^2+1)$$

12.5 problem 5

Internal problem ID [1209]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2+1)\,y''+7y'x+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((1+2*x^2)*diff(y(x),x$2)+7*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-x^2+\frac{5}{3}x^4\right)y(0)+\left(x-\frac{3}{2}x^3+\frac{21}{8}x^5\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[(1+2*x^2)*y''[x]+7*x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{21x^5}{8}-\frac{3x^3}{2}+x\right)+c_1\left(\frac{5x^4}{3}-x^2+1\right)$$

12.6 problem 6

Internal problem ID [1210]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''+2y'x+\frac{y}{4}=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)+1/4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{8}x^2+\frac{25}{384}x^4\right)y(0)+\left(x-\frac{3}{8}x^3+\frac{147}{640}x^5\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(1+x^2)*y''[x]+2*x*y'[x]+1/4*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{147x^5}{640}-\frac{3x^3}{8}+x\right)+c_1\left(\frac{25x^4}{384}-\frac{x^2}{8}+1\right)$$

12.7 problem 7

Internal problem ID [1211]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\, y'' - 5y'x - 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-5*x*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + 2x^2 + \frac{8}{3}x^4\right) y(0) + \left(x + \frac{3}{2}x^3 + \frac{15}{8}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-5*x*y'[x]-4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{15x^5}{8} + \frac{3x^3}{2} + x\right) + c_1\left(\frac{8x^4}{3} + 2x^2 + 1\right)$$

12.8 problem 8

Internal problem ID [1212]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-10y'x+28y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
Order:=6;
dsolve((1+x^2)*diff(y(x),x$2)-10*x*diff(y(x),x)+28*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{35}{3}x^4-14x^2\right)y(0)+\left(x-3x^3+\frac{3}{5}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 38

```
AsymptoticDSolveValue[(1+x^2)*y''[x]-10*x*y'[x]+28*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{3x^5}{5}-3x^3+x\right)+c_1\left(\frac{35x^4}{3}-14x^2+1\right)$$

12.9 problem 9(a)

Internal problem ID [1213]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 9(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 + \frac{1}{3}x^4\right) y(0) + \left(x - \frac{1}{2}x^3 + \frac{1}{8}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{8} - \frac{x^3}{2} + x\right) + c_1\left(\frac{x^4}{3} - x^2 + 1\right)$$

12.10 problem 10

Internal problem ID [1214]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y'x + 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{3}{2}x^2 + \frac{7}{8}x^4\right) y(0) + \left(x - \frac{5}{6}x^3 + \frac{3}{8}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+2*x*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{3x^5}{8} - \frac{5x^3}{6} + x\right) + c_1\left(\frac{7x^4}{8} - \frac{3x^2}{2} + 1\right)$$

12.11 problem 11

Internal problem ID [1215]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$\boxed{(x^2+1)\, y''+y'x+y=0}$$

With initial conditions

$$[y(0)=2, y'(0)=-1]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(1+x^2)*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(0) = 2, D(y)(0) = -1],y(x),type='serie
```

$$y(x)=2-x-x^2+\frac{1}{3}x^3+\frac{5}{12}x^4-\frac{1}{6}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{(1+x^2)*y''[x]+x*y'[x]+y[x]==0,{y[0]==2,y'[0]==-1}},y[x],{x,0,5}]
```

$$y(x)\to -\frac{x^5}{6}+\frac{5x^4}{12}+\frac{x^3}{3}-x^2-x+2$$

12.12 problem 12

Internal problem ID [1216]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2+1)\, y'' - 9y'x - 6y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(1+2*x^2)*diff(y(x),x$2)-9*x*diff(y(x),x)-6*y(x)=0,y(0) = 1, D(y)(0) = -1],y(x),type=
```

$$y(x) = 1 - x + 3x^2 - \frac{5}{2}x^3 + 5x^4 - \frac{21}{8}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[{(1+2*x^2)*y''[x]-9*x*y'[x]-6*y[x]==0,{y[0]==1,y'[0]==-1}},y[x],{x,0,5}
```

$$y(x) \to -\frac{21x^5}{8} + 5x^4 - \frac{5x^3}{2} + 3x^2 - x + 1$$

12.13 problem 13

Internal problem ID [1217]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(8x^2+1)\,y''+2y=0}$$

With initial conditions

$$[y(0)=2, y'(0)=-1]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(1+8*x^2)*diff(y(x),x$2)+2*y(x)=0,y(0) = 2, D(y)(0) = -1],y(x),type='series',x=0);
```

$$y(x)=2-x-2x^2+\frac{1}{3}x^3+3x^4-\frac{5}{6}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[{(1+8*x^2)*y''[x]+2*y[x]==0,{y[0]==2,y'[0]==-1}},y[x],{x,0,5}]
```

$$y(x)\to -\frac{5x^5}{6}+3x^4+\frac{x^3}{3}-2x^2-x+2$$

12.14 problem 16

Internal problem ID [1218]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

With the expansion point for the power series method at $x = 3$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)-y(x)=0,y(x),type='series',x=3);
```

$$y(x) = \left(1 + \frac{(x-3)^2}{2} + \frac{(x-3)^4}{24}\right) y(3) + \left(x - 3 + \frac{(x-3)^3}{6} + \frac{(x-3)^5}{120}\right) D(y)(3) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 51

```
AsymptoticDSolveValue[y''[x]-y[x]==0,y[x],{x,3,5}]
```

$$y(x) \to c_1\left(\frac{1}{24}(x-3)^4 + \frac{1}{2}(x-3)^2 + 1\right) + c_2\left(\frac{1}{120}(x-3)^5 + \frac{1}{6}(x-3)^3 + x - 3\right)$$

12.15 problem 17

Internal problem ID [1219]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' - (x-3)\,y' - y = 0}$$

With the expansion point for the power series method at $x = 3$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)-(x-3)*diff(y(x),x)-y(x)=0,y(x),type='series',x=3);
```

$$y(x) = \left(1 + \frac{(x-3)^2}{2} + \frac{(x-3)^4}{8}\right) y(3) + \left(x - 3 + \frac{(x-3)^3}{3} + \frac{(x-3)^5}{15}\right) D(y)\,(3) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 51

```
AsymptoticDSolveValue[y''[x]-(x-3)*y'[x]-y[x]==0,y[x],{x,3,5}]
```

$$y(x) \to c_1\left(\frac{1}{8}(x-3)^4 + \frac{1}{2}(x-3)^2 + 1\right) + c_2\left(\frac{1}{15}(x-3)^5 + \frac{1}{3}(x-3)^3 + x - 3\right)$$

12.16 problem 18

Internal problem ID [1220]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(2x^2 - 4x + 1)\, y'' + 10(x-1)\, y' + 6y = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((1-4*x+2*x^2)*diff(y(x),x$2)+10*(x-1)*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 + 3(x-1)^2 + \frac{15(x-1)^4}{2}\right) y(1) + \left(x - 1 + \frac{8(x-1)^3}{3} + \frac{32(x-1)^5}{5}\right) D(y)\,(1) + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 49

```
AsymptoticDSolveValue[(1-4*x+2*x^2)*y''[x]+10*(x-1)*y'[x]+6*y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1\left(\frac{15}{2}(x-1)^4 + 3(x-1)^2 + 1\right) + c_2\left(\frac{32}{5}(x-1)^5 + \frac{8}{3}(x-1)^3 + x - 1\right)$$

12.17 problem 19

Internal problem ID [1221]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2-8x+11)\,y''-16(-2+x)\,y'+36y=0}$$

With the expansion point for the power series method at $x=2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve((11-8*x+2*x^2)*diff(y(x),x$2)-16*(x-2)*diff(y(x),x)+36*y(x)=0,y(x),type='series',x=2);
```

$$y(x)=\left(1-6(-2+x)^2+\frac{4(-2+x)^4}{3}\right)y(2)+\left(-2+x-\frac{10(-2+x)^3}{9}\right)D(y)\,(2)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[(11-8*x+2*x^2)*y''[x]-16*(x-2)*y'[x]+36*y[x]==0,y[x],{x,2,5}]
```

$$y(x)\to c_1\left(\frac{4}{3}(x-2)^4-6(x-2)^2+1\right)+c_2\left(-\frac{10}{9}(x-2)^3+x-2\right)$$

12.18 problem 20

Internal problem ID [1222]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(3x^2+6x+5)\,y''+9(x+1)\,y'+3y=0}$$

With the expansion point for the power series method at $x=-1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((5+6*x+3*x^2)*diff(y(x),x$2)+9*(x+1)*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=-1);
```

$$y(x)=\left(1-\frac{3(x+1)^2}{4}+\frac{27(x+1)^4}{32}\right)y(-1)+\left(x+1-(x+1)^3+\frac{6(x+1)^5}{5}\right)D(y)(-1)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 70

```
AsymptoticDSolveValue[(5+6*x+2*x^2)*y''[x]+9*(x+1)*y'[x]+3*y[x]==0,y[x],{x,-1,5}]
```

$$y(x)\to c_1\left(-\frac{93}{20}(x+1)^5+\frac{17}{8}(x+1)^4+(x+1)^3-\frac{3}{2}(x+1)^2+1\right)+c_2\left(\frac{9}{5}(x+1)^5+2(x+1)^4-2(x+1)^3+x+1\right)$$

12.19 problem 21

Internal problem ID [1223]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(x^2-4)\,y''-y'x-3y=0$$

With initial conditions

$$[y(0)=-1, y'(0)=2]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve([(x^2-4)*diff(y(x),x$2)-x*diff(y(x),x)-3*y(x)=0,y(0) = -1, D(y)(0) = 2],y(x),type='ser
```

$$y(x)=-1+2x+\frac{3}{8}x^2-\frac{1}{3}x^3-\frac{3}{128}x^4+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 29

```
AsymptoticDSolveValue[{(x^2-4)*y''[x]-x*y'[x]-3*y[x]==0,{y[0]==-1,y'[0]==2}},y[x],{x,0,5}]
```

$$y(x)\to-\frac{3x^4}{128}-\frac{x^3}{3}+\frac{3x^2}{8}+2x-1$$

12.20 problem 22

Internal problem ID [1224]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (x-3)\,y' + 3y = 0}$$

With initial conditions

$$[y(3) = -2, y'(3) = 3]$$

With the expansion point for the power series method at $x = 3$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)+(x-3)*diff(y(x),x)+3*y(x)=0,y(3) = -2, D(y)(3) = 3],y(x),type='series'
```

$$y(x) = -2 + 3(x-3) + 3(x-3)^2 - 2(x-3)^3 - \frac{5}{4}(x-3)^4 + \frac{3}{5}(x-3)^5 + \mathrm{O}\left((x-3)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[{y''[x]+(x-3)*y'[x]+3*y[x]==0,{y[3]==-2,y'[3]==3}},y[x],{x,3,5}]
```

$$y(x) \to \frac{3}{5}(x-3)^5 - \frac{5}{4}(x-3)^4 - 2(x-3)^3 + 3(x-3)^2 + 3(x-3) - 2$$

12.21 problem 23

Internal problem ID [1225]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x^2 - 6x + 5)\, y'' + (x - 1)\, y' + 12y = 0}$$

With initial conditions

$$[y(1) = -1, y'(1) = 1]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
Order:=6;
dsolve([(5-6*x+3*x^2)*diff(y(x),x$2)+(x-1)*diff(y(x),x)+12*y(x)=0,y(1) = -1, D(y)(1) = 1],y(x
```

$$y(x) = -1 + (x - 1) + 3(x - 1)^2 - \frac{13}{12}(x - 1)^3 - \frac{5}{2}(x - 1)^4 + \frac{143}{160}(x - 1)^5 + \mathrm{O}\left((x - 1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[{(5-6*x+3*x^2)*y''[x]+(x-1)*y'[x]+12*y[x]==0,{y[1]==-1,y'[1]==1}},y[x],
```

$$y(x) \to \frac{143}{160}(x - 1)^5 - \frac{5}{2}(x - 1)^4 - \frac{13}{12}(x - 1)^3 + 3(x - 1)^2 + x - 2$$

12.22 problem 24

Internal problem ID [1226]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(4x^2 - 24x + 37)\, y'' + y = 0}$$

With initial conditions

$$[y(3) = 4, y'(3) = -6]$$

With the expansion point for the power series method at $x = 3$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(4*x^2-24*x+37)*diff(y(x),x$2)+y(x)=0,y(3) = 4, D(y)(3) = -6],y(x),type='series',x=3)
```

$$y(x) = 4 - 6(x-3) - 2(x-3)^2 + (x-3)^3 + \frac{3}{2}(x-3)^4 - \frac{5}{4}(x-3)^5 + \mathrm{O}\left((x-3)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[{(4*x^2-24*x+37)*y''[x]+y[x]==0,{y[3]==4,y'[3]==-6}},y[x],{x,3,5}]
```

$$y(x) \to -\frac{5}{4}(x-3)^5 + \frac{3}{2}(x-3)^4 + (x-3)^3 - 2(x-3)^2 - 6(x-3) + 4$$

12.23 problem 25

Internal problem ID [1227]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 - 8x + 14)\, y'' - 8(x - 4)\, y' + 20y = 0}$$

With initial conditions

$$[y(4) = 3, y'(4) = -4]$$

With the expansion point for the power series method at $x = 4$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(x^2-8*x+14)*diff(y(x),x$2)-8*(x-4)*diff(y(x),x)+20*y(x)=0,y(4) = 3, D(y)(4) = -4],y(
```

$$y(x) = 3 - 4(x - 4) + 15(x - 4)^2 - 4(x - 4)^3 + \frac{15}{4}(x - 4)^4 - \frac{1}{5}(x - 4)^5 + \mathrm{O}\left((x - 4)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 44

```
AsymptoticDSolveValue[{(x^2-8*x+14)*y''[x]+8*(x-4)*y'[x]+20*y[x]==0,{y[4]==3,y'[4]==-4}},y[x]
```

$$y(x) \to -\frac{35}{3}(x - 4)^5 + \frac{95}{4}(x - 4)^4 - \frac{28}{3}(x - 4)^3 + 15(x - 4)^2 - 4(x - 4) + 3$$

12.24 problem 26

Internal problem ID [1228]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2+4x+5)\,y''-20(x+1)\,y'+60y=0}$$

With initial conditions

$$[y(-1)=3, y'(-1)=-3]$$

With the expansion point for the power series method at $x=-1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(2*x^2+4*x+5)*diff(y(x),x$2)-20*(x+1)*diff(y(x),x)+60*y(x)=0,y(-1) = 3, D(y)(-1) = -3
```

$$y(x)=3-3(x+1)-30(x+1)^2+\frac{20}{3}(x+1)^3+20(x+1)^4-\frac{4}{3}(x+1)^5+\mathrm{O}\left((x+1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[{(2*x^2+4*x+5)*y''[x]-20*(x+1)*y'[x]+60*y[x]==0,{y[-1]==3,y'[-1]==-3}},
```

$$y(x)\to -\frac{4}{3}(x+1)^5+20(x+1)^4+\frac{20}{3}(x+1)^3-30(x+1)^2-3(x+1)+3$$

12.25 problem 27

Internal problem ID [1229]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2+1)\,y''+4y'x+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
Order:=6;
dsolve((1+x^2)*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(x^4-x^2+1)\,y(0)+(x^5-x^3+x)\,D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 30

```
AsymptoticDSolveValue[(1+x^2)*y''[x]+4*x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2(x^5-x^3+x)+c_1(x^4-x^2+1)$$

12.26 problem 31

Internal problem ID [1230]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 2\alpha y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
Order:=6;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+2*alpha*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \alpha\, x^2 + \frac{\alpha(\alpha-2)\, x^4}{6}\right) y(0) + \left(x - \frac{(\alpha-1)\, x^3}{3} + \frac{(\alpha^2-4\alpha+3)\, x^5}{30}\right) D(y)\,(0) + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 78

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]+2*\[Alpha]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{\alpha^2 x^5}{30} - \frac{2\alpha x^5}{15} + \frac{x^5}{10} - \frac{\alpha x^3}{3} + \frac{x^3}{3} + x\right) + c_1\left(\frac{\alpha^2 x^4}{6} - \frac{\alpha x^4}{3} - \alpha x^2 + 1\right)$$

12.27 problem 33

Internal problem ID [1231]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^3}{6}\right) y(0) + \left(x + \frac{1}{12}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^4}{12} + x\right) + c_1\left(\frac{x^3}{6} + 1\right)$$

12.28 problem 34

Internal problem ID [1232]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(-2x^3+1)\, y''-10y'x^2-8yx=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve((1-2*x^3)*diff(y(x),x$2)-10*x^2*diff(y(x),x)-8*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{4x^3}{3}\right)y(0)+\left(x+\frac{3}{2}x^4\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[(1-2*x^3)*y''[x]-10*x^2*y'[x]-8*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{3x^4}{2}+x\right)+c_1\left(\frac{4x^3}{3}+1\right)$$

12.29 problem 35

Internal problem ID [1233]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^3+1)\,y''+7y'x^2+9yx=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve((1+x^3)*diff(y(x),x$2)+7*x^2*diff(y(x),x)+9*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{3x^3}{2}\right)y(0)+\left(x-\frac{4}{3}x^4\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[(1+x^3)*y''[x]+7*x^2*y'[x]+9*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(x-\frac{4x^4}{3}\right)+c_1\left(1-\frac{3x^3}{2}\right)$$

12.30 problem 36

Internal problem ID [1234]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(-2x^3+1)\,y''+6y'x^2+24yx=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve((1-2*x^3)*diff(y(x),x$2)+6*x^2*diff(y(x),x)+24*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(-4x^3+1)\,y(0)+\left(x-\frac{5}{2}x^4\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 26

```
AsymptoticDSolveValue[(1-2*x^3)*y''[x]+6*x^2*y'[x]+24*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(x-\frac{5x^4}{2}\right)+c_1(1-4x^3)$$

12.31 problem 37

Internal problem ID [1235]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-x^3+1)\,y''+15y'x^2-36yx=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
Order:=6;
dsolve((1-x^3)*diff(y(x),x$2)+15*x^2*diff(y(x),x)-36*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(6x^3+1)\,y(0)+\left(x+\frac{7}{4}x^4\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[(1-2*x^3)*y''[x]-10*x^2*y'[x]-8*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{3x^4}{2}+x\right)+c_1\left(\frac{4x^3}{3}+1\right)$$

12.32 problem 39

Internal problem ID [1236]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^5+1)\,y''+14y'x^4+10x^3y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
Order:=6;
dsolve((1+2*x^5)*diff(y(x),x$2)+14*x^4*diff(y(x),x)+10*x^3*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{x^5}{2}\right)y(0)+D(y)(0)\,x+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 20

```
AsymptoticDSolveValue[(1+2*x^5)*y''[x]+14*x^4*y'[x]+10*x^3*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(1-\frac{x^5}{2}\right)+c_2x$$

12.33 problem 40

Internal problem ID [1237]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + x^2 y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^4}{12}\right) y(0) + \left(x - \frac{1}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^5}{20}\right) + c_1\left(1 - \frac{x^4}{12}\right)$$

12.34 problem 41

Internal problem ID [1238]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x^6 + 7yx^5 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
Order:=6;
dsolve(diff(y(x),x$2)+x^6*diff(y(x),x)+7*x^5*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + D(y)(0)\,x$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 10

```
AsymptoticDSolveValue[y''[x]+x^6*y'[x]+7*x^5*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 x + c_1$$

12.35 problem 42

Internal problem ID [1239]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^8+1)\,y''-16y'x^7+72yx^6=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
Order:=6;
dsolve((1+x^8)*diff(y(x),x$2)-16*x^7*diff(y(x),x)+72*x^6*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=y(0)+D(y)\,(0)\,x$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 10

```
AsymptoticDSolveValue[(1+x^8)*y''[x]-16*x^7*y'[x]+72*x^6*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2x+c_1$$

12.36 problem 43

Internal problem ID [1240]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{\left(-x^{6}+1\right) y''-12y'x^{5}-30y x^{4}=0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve((1-x^6)*diff(y(x),x$2)-12*x^5*diff(y(x),x)-30*x^4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + D(y)\,(0)\,x + O\left(x^{6}\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 10

```
AsymptoticDSolveValue[(1-x^6)*y''[x]-12*x^5*y'[x]-30*x^4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 x + c_1$$

12.37 problem 44

Internal problem ID [1241]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.2 SERIES SOLUTIONS NEAR AN ORDINARY POINT I. Exercises 7.2. Page 329
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x^5 + 6yx^4 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve(diff(y(x),x$2)+x^5*diff(y(x),x)+6*x^4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + D(y)(0)\,x + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 10

```
AsymptoticDSolveValue[y''[x]+x^5*y'[x]+6*x^4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2x + c_1$$

13 Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338

13.1 problem 1

Internal problem ID [1242]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x+1)\,y''+y'x+2y=0}$$

With initial conditions

$$[y(0)=2, y'(0)=-3]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(1+3*x)*diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(0) = 2, D(y)(0) = -3],y(x),type='ser
```

$$y(x)=2-3x-2x^2+\frac{7}{2}x^3-\frac{55}{12}x^4+\frac{59}{8}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{(1+3*x)*y''[x]+x*y'[x]+2*y[x]==0,{y[0]==2,y'[0]==-3}},y[x],{x,0,5}]
```

$$y(x)\to\frac{59x^5}{8}-\frac{55x^4}{12}+\frac{7x^3}{2}-2x^2-3x+2$$

13.2 problem 2

Internal problem ID [1243]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(2x^2 + x + 1)\, y'' + (2 + 8x)\, y' + 4y = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve([(1+x+2*x^2)*diff(y(x),x$2)+(2+8*x)*diff(y(x),x)+4*y(x)=0,y(0) = -1, D(y)(0) = 2],y(x)
```

$$y(x) = -1 + 2x - 4x^3 + 4x^4 + 4x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 23

```
AsymptoticDSolveValue[{(1+x+2*x^2)*y''[x]+(2+8*x)*y'[x]+4*y[x]==0,{y[0]==-1,y'[0]==2}},y[x],{
```

$$y(x) \to 4x^5 + 4x^4 - 4x^3 + 2x - 1$$

13.3 problem 3

Internal problem ID [1244]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(-2x^2+1)\,y''+(2-6x)\,y'-2y=0}$$

With initial conditions

$$[y(0)=1, y'(0)=0]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
Order:=6;
dsolve([(1-2*x^2)*diff(y(x),x$2)+(2-6*x)*diff(y(x),x)-2*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),ty
```

$$y(x)=1+x^2-\frac{2}{3}x^3+\frac{11}{6}x^4-\frac{9}{5}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 29

```
AsymptoticDSolveValue[{(1-2*x^2)*y''[x]+(2-6*x)*y'[x]-2*y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0
```

$$y(x)\to-\frac{9x^5}{5}+\frac{11x^4}{6}-\frac{2x^3}{3}+x^2+1$$

13.4 problem 4

Internal problem ID [1245]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x^2 + x + 1)\, y'' + (2 + 15x)\, y' + 12y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve([(1+x+3*x^2)*diff(y(x),x$2)+(2+15*x)*diff(y(x),x)+12*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x
```

$$y(x) = x - x^2 - \frac{7}{2}x^3 + \frac{15}{2}x^4 + \frac{45}{8}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 31

```
AsymptoticDSolveValue[{(1+x+3*x^2)*y''[x]+(2+15*x)*y'[x]+12*y[x]==0,{y[0]==0,y'[0]==1}},y[x],
```

$$y(x) \to \frac{45x^5}{8} + \frac{15x^4}{2} - \frac{7x^3}{2} - x^2 + x$$

13.5 problem 5

Internal problem ID [1246]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2+x)\,y''+(x+1)\,y'+3y=0}$$

With initial conditions

$$[y(0)=4, y'(0)=3]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(2+x)*diff(y(x),x$2)+(1+x)*diff(y(x),x)+3*y(x)=0,y(0) = 4, D(y)(0) = 3],y(x),type='se
```

$$y(x)=4+3x-\frac{15}{4}x^2+\frac{1}{4}x^3+\frac{11}{16}x^4-\frac{5}{16}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{(2+x)*y''[x]+(1+x)*y'[x]+3*y[x]==0,{y[0]==4,y'[0]==3}},y[x],{x,0,5}]
```

$$y(x)\to -\frac{5x^5}{16}+\frac{11x^4}{16}+\frac{x^3}{4}-\frac{15x^2}{4}+3x+4$$

13.6 problem 6

Internal problem ID [1247]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(x^2 + 3x + 3)\, y'' + (6 + 4x)\, y' + 2y = 0$$

With initial conditions

$$[y(0) = 7, y'(0) = 3]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(3+3*x+x^2)*diff(y(x),x$2)+(6+4*x)*diff(y(x),x)+2*y(x)=0,y(0) = 7, D(y)(0) = 3],y(x),
```

$$y(x) = 7 + 3x - \frac{16}{3}x^2 + \frac{13}{3}x^3 - \frac{23}{9}x^4 + \frac{10}{9}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{(3+3*x+x^2)*y''[x]+(6+4*x)*y'[x]+2*y[x]==0,{y[0]==7,y'[0]==3}},y[x],{x
```

$$y(x) \to \frac{10x^5}{9} - \frac{23x^4}{9} + \frac{13x^3}{3} - \frac{16x^2}{3} + 3x + 7$$

13.7 problem 7

Internal problem ID [1248]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+4)\,y''+(2+x)\,y'+2y=0}$$

With initial conditions

$$[y(0)=2, y'(0)=5]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
Order:=6;
dsolve([(4+x)*diff(y(x),x$2)+(2+x)*diff(y(x),x)+2*y(x)=0,y(0) = 2, D(y)(0) = 5],y(x),type='se
```

$$y(x)=2+5x-\frac{7}{4}x^2-\frac{3}{16}x^3+\frac{37}{192}x^4-\frac{7}{192}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{(4+x)*y''[x]+(2+x)*y'[x]+2*y[x]==0,{y[0]==4,y'[0]==3}},y[x],{x,0,5}]
```

$$y(x)\to -\frac{7x^5}{192}+\frac{25x^4}{192}+\frac{x^3}{16}-\frac{7x^2}{4}+3x+4$$

13.8 problem 8

Internal problem ID [1249]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(2x^2 - 3x + 2)\, y'' - (4 - 6x)\, y' + 2y = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = -1]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve([(2-3*x+2*x^2)*diff(y(x),x$2)-(4-6*x)*diff(y(x),x)+2*y(x)=0,y(1) = 1, D(y)(1) = -1],y(
```

$$y(x) = 1 - (x-1) + \frac{4}{3}(x-1)^3 - \frac{4}{3}(x-1)^4 - \frac{4}{5}(x-1)^5 + \mathrm{O}\left((x-1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 35

```
AsymptoticDSolveValue[{(2-3*x+2*x^2)*y''[x]-(4-6*x)*y'[x]+2*y[x]==0,{y[1]==1,y'[1]==-1}},y[x]
```

$$y(x) \to -\frac{4}{5}(x-1)^5 - \frac{4}{3}(x-1)^4 + \frac{4}{3}(x-1)^3 - x + 2$$

13.9 problem 9

Internal problem ID [1250]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2 + 3x)\, y'' + 10(x + 1)\, y' + 8y = 0}$$

With initial conditions

$$[y(-1) = 1, y'(-1) = -1]$$

With the expansion point for the power series method at $x = -1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(3*x+2*x^2)*diff(y(x),x$2)+10*(1+x)*diff(y(x),x)+8*y(x)=0,y(-1) = 1, D(y)(-1) = -1],y
```

$$y(x) = 1 - (x + 1) + 4(x + 1)^2 - \frac{13}{3}(x + 1)^3 + \frac{77}{6}(x + 1)^4 - \frac{278}{15}(x + 1)^5 + \mathrm{O}\left((x + 1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 41

```
AsymptoticDSolveValue[{(3*x+2*x^2)*y''[x]+10*(1+x)*y'[x]+8*y[x]==0,{y[-1]==1,y'[-1]==-1}},y[x
```

$$y(x) \to -\frac{278}{15}(x + 1)^5 + \frac{77}{6}(x + 1)^4 - \frac{13}{3}(x + 1)^3 + 4(x + 1)^2 - x$$

13.10 problem 10

Internal problem ID [1251]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2 - x + 1)\, y'' - (-4x + 1)\, y' + 2y = 0}$$

With initial conditions

$$[y(1) = 2, y'(1) = -1]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(1-x+x^2)*diff(y(x),x$2)-(1-4*x)*diff(y(x),x)+2*y(x)=0,y(1) = 2, D(y)(1) = -1],y(x),t
```

$$y(x) = 2 - (x-1) - \frac{1}{2}(x-1)^2 + \frac{5}{3}(x-1)^3 - \frac{19}{12}(x-1)^4 + \frac{7}{30}(x-1)^5 + \mathrm{O}\left((x-1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 44

```
AsymptoticDSolveValue[{(1-x+x^2)*y''[x]-(1-4*x)*y'[x]+2*y[x]==0,{y[1]==2,y'[1]==-1}},y[x],{x,
```

$$y(x) \to \frac{7}{30}(x-1)^5 - \frac{19}{12}(x-1)^4 + \frac{5}{3}(x-1)^3 - \frac{1}{2}(x-1)^2 - x + 3$$

13.11 problem 11

Internal problem ID [1252]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(2+x)\,y''+(2+x)\,y'+y=0}$$

With initial conditions

$$[y(-1)=-2, y'(-1)=3]$$

With the expansion point for the power series method at $x=-1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(2+x)*diff(y(x),x$2)+(2+x)*diff(y(x),x)+y(x)=0,y(-1) = -2, D(y)(-1) = 3],y(x),type='s
```

$$y(x)=-2+3(x+1)-\frac{1}{2}(x+1)^2-\frac{2}{3}(x+1)^3+\frac{5}{8}(x+1)^4-\frac{11}{30}(x+1)^5+\mathrm{O}\left((x+1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{(2+x)*y''[x]+(2+x)*y'[x]+y[x]==0,{y[-1]==-2,y'[-1]==3}},y[x],{x,-1,5}]
```

$$y(x)\to -\frac{11}{30}(x+1)^5+\frac{5}{8}(x+1)^4-\frac{2}{3}(x+1)^3-\frac{1}{2}(x+1)^2+3(x+1)-2$$

13.12 problem 12

Internal problem ID [1253]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - (6-7x)\,y' + 8y = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = -2]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([x^2*diff(y(x),x$2)-(6-7*x)*diff(y(x),x)+8*y(x)=0,y(1) = 1, D(y)(1) = -2],y(x),type='s
```

$$y(x) = 1 - 2(x-1) - 3(x-1)^2 + 8(x-1)^3 - 4(x-1)^4 - \frac{42}{5}(x-1)^5 + \mathrm{O}\left((x-1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[{x^2*y''[x]-(6-7*x)*y'[x]+8*y[x]==0,{y[1]==1,y'[1]==-2}},y[x],{x,1,5}]
```

$$y(x) \to -\frac{42}{5}(x-1)^5 - 4(x-1)^4 + 8(x-1)^3 - 3(x-1)^2 - 2(x-1) + 1$$

13.13 problem 13

Internal problem ID [1254]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2 + x + 1)\, y'' + (1 + 7x)\, y' + 2y = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve([(1+x+2*x^2)*diff(y(x),x$2)+(1+7*x)*diff(y(x),x)+2*y(x)=0,y(1) = 1, D(y)(1) = 0],y(x),
```

$$y(x) = 1 - \frac{1}{4}(x-1)^2 + \frac{13}{48}(x-1)^3 - \frac{77}{384}(x-1)^4 + \frac{287}{2560}(x-1)^5 + \mathrm{O}\left((x-1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 41

```
AsymptoticDSolveValue[{(1+x+2*x^2)*y''[x]+(1+7*x)*y'[x]+2*y[x]==0,{y[1]==1,y'[1]==0}},y[x],{x
```

$$y(x) \to \frac{287(x-1)^5}{2560} - \frac{77}{384}(x-1)^4 + \frac{13}{48}(x-1)^3 - \frac{1}{4}(x-1)^2 + 1$$

13.14 problem 14

Internal problem ID [1255]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+3)\,y''+(1+2x)\,y'-(2-x)\,y=0}$$

With initial conditions

$$[y(-1)=1, y'(-1)=0]$$

With the expansion point for the power series method at $x=-1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve([(3+x)*diff(y(x),x$2)+(1+2*x)*diff(y(x),x)-(2-x)*y(x)=0,y(-1) = 1, D(y)(-1) = 0],y(x),
```

$$y(x)=1+\frac{3}{4}(x+1)^2-\frac{1}{12}(x+1)^3-\frac{1}{48}(x+1)^4-\frac{1}{120}(x+1)^5+\mathrm{O}\left((x+1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 41

```
AsymptoticDSolveValue[{(3+x)*y''[x]+(1+2*x)*y'[x]-(2-x)*y[x]==0,{y[-1]==1,y'[-1]==0}},y[x],{x
```

$$y(x)\to -\frac{1}{120}(x+1)^5-\frac{1}{48}(x+1)^4-\frac{1}{12}(x+1)^3+\frac{3}{4}(x+1)^2+1$$

13.15 problem 15

Internal problem ID [1256]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y'x + (2x^2 + 4)\, y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)+3*x*diff(y(x),x)+(4+2*x^2)*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='se
```

$$y(x) = 1 - 2x^2 + \frac{3}{2}x^4 + \mathrm{O}(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 17

```
AsymptoticDSolveValue[{y''[x]+3*x*y'[x]+(4+2*x^2)*y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{3x^4}{2} - 2x^2 + 1$$

13.16 problem 19

Internal problem ID [1257]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(4x+2)\,y''-4y'-(6+4x)\,y=0}$$

With initial conditions

$$[y(0)=2, y'(0)=-7]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(2+4*x)*diff(y(x),x$2)-4*diff(y(x),x)-(6+4*x)*y(x)=0,y(0) = 2, D(y)(0) = -7],y(x),typ
```

$$y(x)=2-7x-4x^2-\frac{17}{6}x^3-\frac{3}{4}x^4-\frac{9}{40}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{(2+4*x)*y''[x]-4*y'[x]-(6+4*x)*y[x]==0,{y[0]==2,y'[0]==-7}},y[x],{x,0,
```

$$y(x)\to-\frac{9x^5}{40}-\frac{3x^4}{4}-\frac{17x^3}{6}-4x^2-7x+2$$

13.17 problem 20

Internal problem ID [1258]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1+2x)\,y''-(1-2x)\,y'-(-2x+3)\,y=0}$$

With initial conditions

$$[y(1)=1, y'(1)=-2]$$

With the expansion point for the power series method at $x=1$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
Order:=6;
dsolve([(1+2*x)*diff(y(x),x$2)-(1-2*x)*diff(y(x),x)-(3-2*x)*y(x)=0,y(1) = 1, D(y)(1) = -2],y(
```

$$y(x)=1-2(x-1)+\frac{1}{2}(x-1)^2-\frac{1}{6}(x-1)^3+\frac{5}{36}(x-1)^4-\frac{73}{1080}(x-1)^5+\mathrm{O}\left((x-1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{(1+2*x)*y''[x]-(1-2*x)*y'[x]-(3-2*x)*y[x]==0,{y[1]==1,y'[1]==-2}},y[x]
```

$$y(x)\to-\frac{73(x-1)^5}{1080}+\frac{5}{36}(x-1)^4-\frac{1}{6}(x-1)^3+\frac{1}{2}(x-1)^2-2(x-1)+1$$

13.18 problem 21

Internal problem ID [1259]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x+5)\,y''-y'+(x+5)\,y=0}$$

With initial conditions

$$[y(-2)=2, y'(-2)=-1]$$

With the expansion point for the power series method at $x=-2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(5+2*x)*diff(y(x),x$2)-diff(y(x),x)+(5+x)*y(x)=0,y(-2) = 2, D(y)(-2) = -1],y(x),type=
```

$$y(x)=2-(2+x)-\frac{7}{2}(2+x)^2+\frac{4}{3}(2+x)^3-\frac{1}{24}(2+x)^4+\frac{1}{60}(2+x)^5+\mathrm{O}\left((2+x)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 43

```
AsymptoticDSolveValue[{(5+2*x)*y''[x]-y'[x]+(5+x)*y[x]==0,{y[-2]==2,y'[-2]==-1}},y[x],{x,-2,5
```

$$y(x)\to\frac{1}{60}(x+2)^5-\frac{1}{24}(x+2)^4+\frac{4}{3}(x+2)^3-\frac{7}{2}(x+2)^2-x$$

13.19 problem 22

Internal problem ID [1260]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+4)\,y''-(2x+4)\,y'+y(x+6)=0}$$

With initial conditions

$$[y(-3)=2, y'(-3)=-2]$$

With the expansion point for the power series method at $x=-3$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(4+x)*diff(y(x),x$2)-(4+2*x)*diff(y(x),x)+(6+x)*y(x)=0,y(-3) = 2, D(y)(-3) = -2],y(x)
```

$$y(x)=2-2(x+3)-(x+3)^2+(x+3)^3-\frac{11}{12}(x+3)^4+\frac{67}{60}(x+3)^5+\mathrm{O}\left((x+3)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[{(4+x)*y''[x]-(4+2*x)*y'[x]+(6+x)*y[x]==0,{y[-3]==2,y'[-3]==-2}},y[x],{
```

$$y(x)\to\frac{67}{60}(x+3)^5-\frac{11}{12}(x+3)^4+(x+3)^3-(x+3)^2-2(x+3)+2$$

13.20 problem 23

Internal problem ID [1261]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x+2)\,y''-y'x+2yx=0}$$

With initial conditions

$$[y(0)=-1,y'(0)=2]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve([(2+3*x)*diff(y(x),x$2)-x*diff(y(x),x)+2*x*y(x)=0,y(0) = -1, D(y)(0) = 2],y(x),type='s
```

$$y(x)=-1+2x+\frac{1}{3}x^3-\frac{5}{12}x^4+\frac{2}{5}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 29

```
AsymptoticDSolveValue[{(2+3*x)*y''[x]-x*y'[x]+2*x*y[x]==0,{y[0]==-1,y'[0]==2}},y[x],{x,0,5}]
```

$$y(x)\to\frac{2x^5}{5}-\frac{5x^4}{12}+\frac{x^3}{3}+2x-1$$

13.21 problem 24

Internal problem ID [1262]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x+3)\,y''+3y'-yx=0}$$

With initial conditions

$$[y(-1)=2, y'(-1)=-3]$$

With the expansion point for the power series method at $x=-1$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
Order:=6;
dsolve([(3+2*x)*diff(y(x),x$2)+3*diff(y(x),x)-x*y(x)=0,y(-1) = 2, D(y)(-1) = -3],y(x),type='s
```

$$y(x)=2-3(x+1)+\frac{7}{2}(x+1)^2-5(x+1)^3+\frac{197}{24}(x+1)^4-\frac{287}{20}(x+1)^5+\mathrm{O}\left((x+1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 44

```
AsymptoticDSolveValue[{(3+2*x)*y''[x]+3*y'[x]-x*y[x]==0,{y[-1]==2,y'[-1]==-3}},y[x],{x,-1,5}]
```

$$y(x)\to -\frac{287}{20}(x+1)^5+\frac{197}{24}(x+1)^4-5(x+1)^3+\frac{7}{2}(x+1)^2-3(x+1)+2$$

13.22 problem 25

Internal problem ID [1263]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x+3)\,y''-3y'-(2+x)\,y=0}$$

With initial conditions

$$[y(-2)=-2, y'(-2)=3]$$

With the expansion point for the power series method at $x=-2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(3+2*x)*diff(y(x),x$2)-3*diff(y(x),x)-(2+x)*y(x)=0,y(-2) = -2, D(y)(-2) = 3],y(x),typ
```

$$y(x)=-2+3(2+x)-\frac{9}{2}(2+x)^2+\frac{11}{6}(2+x)^3+\frac{5}{24}(2+x)^4+\frac{7}{20}(2+x)^5+\mathrm{O}\left((2+x)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{(3+2*x)*y''[x]-3*y'[x]-(2+x)*y[x]==0,{y[-2]==-2,y'[-2]==3}},y[x],{x,-2
```

$$y(x)\to\frac{7}{20}(x+2)^5+\frac{5}{24}(x+2)^4+\frac{11}{6}(x+2)^3-\frac{9}{2}(x+2)^2+3(x+2)-2$$

13.23 problem 26

Internal problem ID [1264]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(10-2x)\,y''+(x+1)\,y=0}$$

With initial conditions

$$[y(2)=2,y'(2)=-4]$$

With the expansion point for the power series method at $x=2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(10-2*x)*diff(y(x),x$2)+(1+x)*y(x)=0,y(2) = 2, D(y)(2) = -4],y(x),type='series',x=2);
```

$$y(x)=2-4(-2+x)-\frac{1}{2}(-2+x)^2+\frac{2}{9}(-2+x)^3+\frac{49}{432}(-2+x)^4+\frac{23}{1080}(-2+x)^5+\mathrm{O}\left((-2+x)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{(10-2*x)*y''[x]+(1+x)*y[x]==0,{y[2]==2,y'[2]==-4}},y[x],{x,2,5}]
```

$$y(x)\to\frac{23(x-2)^5}{1080}+\frac{49}{432}(x-2)^4+\frac{2}{9}(x-2)^3-\frac{1}{2}(x-2)^2-4(x-2)+2$$

13.24 problem 27

Internal problem ID [1265]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(7+x)\,y''+(2x+8)\,y'+(x+5)\,y=0}$$

With initial conditions

$$[y(-4)=1, y'(-4)=2]$$

With the expansion point for the power series method at $x=-4$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(7+x)*diff(y(x),x$2)+(8+2*x)*diff(y(x),x)+(5+x)*y(x)=0,y(-4) = 1, D(y)(-4) = 2],y(x),
```

$$y(x)=1+2(x+4)-\frac{1}{6}(x+4)^2-\frac{10}{27}(x+4)^3+\frac{19}{648}(x+4)^4+\frac{13}{324}(x+4)^5+\mathrm{O}\left((x+4)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{(7+x)*y''[x]+(8+2*x)*y'[x]+(5+x)*y[x]==0,{y[-4]==1,y'[-4]==2}},y[x],{x
```

$$y(x)\to\frac{13}{324}(x+4)^5+\frac{19}{648}(x+4)^4-\frac{10}{27}(x+4)^3-\frac{1}{6}(x+4)^2+2(x+4)+1$$

13.25 problem 28

Internal problem ID [1266]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(6+4x)\,y''+(1+2x)\,y=0}$$

With initial conditions

$$[y(-1)=-1, y'(-1)=2]$$

With the expansion point for the power series method at $x=-1$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
Order:=6;
dsolve([(6+4*x)*diff(y(x),x$2)+(1+2*x)*y(x)=0,y(-1) = -1, D(y)(-1) = 2],y(x),type='series',x=
```

$$y(x)=-1+2(x+1)-\frac{1}{4}(x+1)^2+\frac{1}{2}(x+1)^3-\frac{65}{96}(x+1)^4+\frac{67}{80}(x+1)^5+\mathrm{O}\left((x+1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{(6+4*x)*y''[x]+(1+2*x)*y[x]==0,{y[-1]==-1,y'[-1]==2}},y[x],{x,-1,5}]
```

$$y(x)\to\frac{67}{80}(x+1)^5-\frac{65}{96}(x+1)^4+\frac{1}{2}(x+1)^3-\frac{1}{4}(x+1)^2+2(x+1)-1$$

13.26 problem 29

Internal problem ID [1267]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(\beta x^2 + x\alpha + 1\right) y'' + (\delta x + \gamma) y' + \epsilon y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 366

```
Order:=6;
dsolve((1+alpha*x+beta*x^2)*diff(y(x),x$2)+(gamma+delta*x)*diff(y(x),x)+epsilon*y(x)=0,y(x),t
```

$$y(x) = \left(1 - \frac{\epsilon x^2}{2} + \frac{\epsilon(\alpha+\gamma) x^3}{6} + \frac{\epsilon\left(-\alpha^2 - \frac{3}{2}\alpha\gamma - \frac{1}{2}\gamma^2 + \beta + \delta + \frac{1}{2}\epsilon\right) x^4}{12} - \frac{\left(\frac{\left(\alpha+\frac{\gamma}{2}\right)\epsilon}{3} - \frac{\gamma^3}{12} - \frac{\alpha\gamma^2}{2} + \frac{\left(-\frac{11\alpha^2}{4}+2\beta+\frac{5\delta}{4}\right)\gamma}{3} + \alpha\left(-\frac{\alpha^2}{2} + \beta + \frac{3\delta}{4}\right)\right)\epsilon x^5}{10}\right) y(0) + \left(x - \frac{\gamma x^2}{2} + \frac{(\alpha\gamma + \gamma^2 - \delta - \epsilon) x^3}{6} + \frac{((2\alpha + 2\gamma)\epsilon - \gamma^3 - 3\alpha\gamma^2 + (-2\alpha^2 + 2\beta + 3\delta)\gamma + 2\delta\alpha) x^4}{24} + \frac{\left(\epsilon^2 + (-6\alpha^2 - 9\alpha\gamma - 3\gamma^2 + 6\beta + 4\delta)\epsilon + \gamma^4 + 6\alpha\gamma^3 + (11\alpha^2 - 8\beta - 6\delta)\gamma^2 - 12\left(-\frac{\alpha^2}{2} + \beta + \frac{7\delta}{6}\right)\alpha\gamma - \right.}{120}$$

$$+ O(x^6)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 561

```
AsymptoticDSolveValue[(1+\[Alpha]*x+\[Beta]*x^2)*y''[x]+(\[Gamma]+\[Delta]*x)*y'[x]+\[Epsilon
```

$$
\begin{aligned}
y(x) \to\ & c_1\left(-\frac{1}{20}x^5\epsilon\left(2\alpha\beta-\alpha^3\right)-\frac{1}{20}x^5\epsilon\left(\alpha\delta-\gamma\left(\alpha^2-\beta\right)\right)+\frac{1}{60}\gamma x^5\epsilon\left(\alpha^2-\beta\right)+\frac{1}{120}\alpha\gamma^2x^5\epsilon\right.\\
&+\frac{1}{40}\alpha x^5\epsilon(\alpha\gamma-\delta)+\frac{1}{24}\gamma x^5\epsilon(\alpha\gamma-\delta)-\frac{1}{30}\alpha x^5\epsilon^2+\frac{1}{120}\gamma^3x^5\epsilon-\frac{1}{60}\gamma x^5\epsilon^2-\frac{1}{12}x^4\epsilon\left(\alpha^2-\beta\right)\\
&\left.-\frac{1}{12}x^4\epsilon(\alpha\gamma-\delta)-\frac{1}{24}\alpha\gamma x^4\epsilon-\frac{1}{24}\gamma^2x^4\epsilon+\frac{x^4\epsilon^2}{24}+\frac{1}{6}\alpha x^3\epsilon+\frac{1}{6}\gamma x^3\epsilon-\frac{x^2\epsilon}{2}+1\right)\\
&+c_2\left(\frac{1}{60}\gamma x^5\left(\gamma\left(\alpha^2-\beta\right)-\alpha\delta\right)-\frac{1}{20}\gamma x^5\left(\alpha\delta-\gamma\left(\alpha^2-\beta\right)\right)-\frac{1}{20}x^5\epsilon\left(\alpha^2-\beta\right)\right.\\
&-\frac{1}{20}x^5\left(\alpha^3(-\gamma)+\alpha^2\delta+2\alpha\beta\gamma-\beta\delta\right)+\frac{1}{24}\gamma^2x^5(\alpha\gamma-\delta)-\frac{1}{120}\gamma^2x^5(\delta-\alpha\gamma)\\
&-\frac{1}{40}x^5\epsilon(\alpha\gamma-\delta)+\frac{1}{120}x^5\epsilon(\delta-\alpha\gamma)-\frac{1}{40}x^5(\alpha\gamma-\delta)(\delta-\alpha\gamma)-\frac{1}{24}\alpha\gamma x^5\epsilon+\frac{\gamma^4x^5}{120}\\
&-\frac{1}{40}\gamma^2x^5\epsilon+\frac{x^5\epsilon^2}{120}-\frac{1}{12}x^4\left(\gamma\left(\alpha^2-\beta\right)-\alpha\delta\right)-\frac{1}{12}\gamma x^4(\alpha\gamma-\delta)+\frac{1}{24}\gamma x^4(\delta-\alpha\gamma)\\
&\left.+\frac{1}{12}\alpha x^4\epsilon-\frac{\gamma^3x^4}{24}+\frac{1}{12}\gamma x^4\epsilon-\frac{1}{6}x^3(\delta-\alpha\gamma)+\frac{\gamma^2x^3}{6}-\frac{x^3\epsilon}{6}-\frac{\gamma x^2}{2}+x\right)
\end{aligned}
$$

13.27 problem 31(a)

Internal problem ID [1268]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 31(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(2x^2 + 3x + 1)\, y'' + (6 + 8x)\, y' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
Order:=6;
dsolve((1+3*x+2*x^2)*diff(y(x),x$2)+(6+8*x)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (30x^5 - 14x^4 + 6x^3 - 2x^2 + 1)\, y(0) + (31x^5 - 15x^4 + 7x^3 - 3x^2 + x)\, D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 54

```
AsymptoticDSolveValue[(1+3*x+2*x^2)*y''[x]+(6+8*x)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1(30x^5 - 14x^4 + 6x^3 - 2x^2 + 1) + c_2(31x^5 - 15x^4 + 7x^3 - 3x^2 + x)$$

13.28 problem 31(b)

Internal problem ID [1269]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 31(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(6x^2 - 5x + 1)\, y'' - (10 - 24x)\, y' + 12y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((1-5*x+6*x^2)*diff(y(x),x$2)-(10-24*x)*diff(y(x),x)+12*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = &\left(-390x^5 - 114x^4 - 30x^3 - 6x^2 + 1\right) y(0) \\ &+ \left(211x^5 + 65x^4 + 19x^3 + 5x^2 + x\right) D(y)(0) + O(x^6) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 54

```
AsymptoticDSolveValue[(1-5*x+6*x^2)*y''[x]-(10-24*x)*y'[x]+12*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-390x^5 - 114x^4 - 30x^3 - 6x^2 + 1\right) + c_2\left(211x^5 + 65x^4 + 19x^3 + 5x^2 + x\right)$$

13.29 problem 31(c)

Internal problem ID [1270]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 31(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(4x^2 - 4x + 1)\, y'' - (8 - 16x)\, y' + 8y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
Order:=6;
dsolve((1-4*x+4*x^2)*diff(y(x),x$2)-(8-16*x)*diff(y(x),x)+8*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (-128x^5 - 48x^4 - 16x^3 - 4x^2 + 1)\, y(0) + (80x^5 + 32x^4 + 12x^3 + 4x^2 + x)\, D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 54

```
AsymptoticDSolveValue[(1-4*x+4*x^2)*y''[x]-(8-16*x)*y'[x]+8*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 (-128x^5 - 48x^4 - 16x^3 - 4x^2 + 1) + c_2 (80x^5 + 32x^4 + 12x^3 + 4x^2 + x)$$

13.30 problem 31(d)

Internal problem ID [1271]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 31(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(x^2+4x+4)\, y''+(8+4x)\, y'+2y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((4+4*x+x^2)*diff(y(x),x$2)+(8+4*x)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{4}x^2+\frac{1}{4}x^3-\frac{3}{16}x^4+\frac{1}{8}x^5\right)y(0)+\left(x-x^2+\frac{3}{4}x^3-\frac{1}{2}x^4+\frac{5}{16}x^5\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 68

```
AsymptoticDSolveValue[(4+4*x+x^2)*y''[x]+(8+4*x)*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{x^5}{8}-\frac{3x^4}{16}+\frac{x^3}{4}-\frac{x^2}{4}+1\right)+c_2\left(\frac{5x^5}{16}-\frac{x^4}{2}+\frac{3x^3}{4}-x^2+x\right)$$

13.31 problem 31(e)

Internal problem ID [1272]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 31(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(3x^2 + 8x + 4)\, y'' + (16 + 12x)\, y' + 6y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((4+8*x+3*x^2)*diff(y(x),x$2)+(16+12*x)*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & \left(1 - \frac{3}{4}x^2 + \frac{3}{2}x^3 - \frac{39}{16}x^4 + \frac{15}{4}x^5\right) y(0) \\ & + \left(x - 2x^2 + \frac{13}{4}x^3 - 5x^4 + \frac{121}{16}x^5\right) D(y)\,(0) + O(x^6) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 66

```
AsymptoticDSolveValue[(4+8*x+3*x^2)*y''[x]+(16+12*x)*y'[x]+6*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{15x^5}{4} - \frac{39x^4}{16} + \frac{3x^3}{2} - \frac{3x^2}{4} + 1\right) + c_2\left(\frac{121x^5}{16} - 5x^4 + \frac{13x^3}{4} - 2x^2 + x\right)$$

13.32 problem 32

Internal problem ID [1273]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y'x + (2x^2 + 3)\, y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)+2*x*diff(y(x),x)+(3+2*x^2)*y(x)=0,y(0) = 1, D(y)(0) = -2],y(x),type='s
```

$$y(x) = 1 - 2x - \frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{17}{24}x^4 - \frac{11}{20}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{y''[x]+2*x*y'[x]+(3+2*x^2)*y[x]==0,{y[0]==1,y'[0]==-2}},y[x],{x,0,5}]
```

$$y(x) \to -\frac{11x^5}{20} + \frac{17x^4}{24} + \frac{5x^3}{3} - \frac{3x^2}{2} - 2x + 1$$

13.33 problem 33

Internal problem ID [1274]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y'x + (2x^2 + 5)\, y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)-3*x*diff(y(x),x)+(5+2*x^2)*y(x)=0,y(0) = 1, D(y)(0) = -2],y(x),type='s
```

$$y(x) = 1 - 2x - \frac{5}{2}x^2 + \frac{2}{3}x^3 - \frac{3}{8}x^4 + \frac{1}{3}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{y''[x]-3*x*y'[x]+(5+2*x^2)*y[x]==0,{y[0]==1,y'[0]==-2}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^5}{3} - \frac{3x^4}{8} + \frac{2x^3}{3} - \frac{5x^2}{2} - 2x + 1$$

13.34 problem 34

Internal problem ID [1275]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 5y'x - (-x^2 + 3)\, y = 0}$$

With initial conditions

$$[y(0) = 6, y'(0) = -2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)+5*x*diff(y(x),x)-(3-x^2)*y(x)=0,y(0) = 6, D(y)(0) = -2],y(x),type='ser
```

$$y(x) = 6 - 2x + 9x^2 + \frac{2}{3}x^3 - \frac{23}{4}x^4 - \frac{3}{10}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{y''[x]+5*x*y'[x]-(3-x^2)*y[x]==0,{y[0]==6,y'[0]==-2}},y[x],{x,0,5}]
```

$$y(x) \to -\frac{3x^5}{10} - \frac{23x^4}{4} + \frac{2x^3}{3} + 9x^2 - 2x + 6$$

13.35 problem 35

Internal problem ID [1276]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x - (3x^2 + 2)\, y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -5]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)-2*x*diff(y(x),x)-(2+3*x^2)*y(x)=0,y(0) = 2, D(y)(0) = -5],y(x),type='s
```

$$y(x) = 2 - 5x + 2x^2 - \frac{10}{3}x^3 + \frac{3}{2}x^4 - \frac{25}{12}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{y''[x]-2*x*y'[x]-(2+3*x^2)*y[x]==0,{y[0]==2,y'[0]==-5}},y[x],{x,0,5}]
```

$$y(x) \to -\frac{25x^5}{12} + \frac{3x^4}{2} - \frac{10x^3}{3} + 2x^2 - 5x + 2$$

13.36 problem 36

Internal problem ID [1277]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y'x + (4x^2 + 2)\, y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 6]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)+3*x*diff(y(x),x)+(2+4*x^2)*y(x)=0,y(0) = 3, D(y)(0) = 6],y(x),type='se
```

$$y(x) = 3 + 6x - 3x^2 - 5x^3 + x^4 + \frac{31}{20}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[{y''[x]+3*x*y'[x]+(2+4*x^2)*y[x]==0,{y[0]==3,y'[0]==6}},y[x],{x,0,5}]
```

$$y(x) \to \frac{31x^5}{20} + x^4 - 5x^3 - 3x^2 + 6x + 3$$

13.37 problem 37

Internal problem ID [1278]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y'' + 5y'x + (2x^2 + 4)\, y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = -2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
Order:=6;
dsolve([2*diff(y(x),x$2)+5*x*diff(y(x),x)+(4+2*x^2)*y(x)=0,y(0) = 3, D(y)(0) = -2],y(x),type=
```

$$y(x) = 3 - 2x - 3x^2 + \frac{3}{2}x^3 + \frac{3}{2}x^4 - \frac{49}{80}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{2*y''[x]+5*x*y'[x]+(4+2*x^2)*y[x]==0,{y[0]==3,y'[0]==-2}},y[x],{x,0,5}
```

$$y(x) \to -\frac{49x^5}{80} + \frac{3x^4}{2} + \frac{3x^3}{2} - 3x^2 - 2x + 3$$

13.38 problem 38

Internal problem ID [1279]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3y'' + 2y'x + \left(-x^2 + 4\right) y = 0}$$

With initial conditions

$$[y(0) = -2, y'(0) = 3]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([3*diff(y(x),x$2)+2*x*diff(y(x),x)+(4-x^2)*y(x)=0,y(0) = -2, D(y)(0) = 3],y(x),type='s
```

$$y(x) = -2 + 3x + \frac{4}{3}x^2 - x^3 - \frac{19}{54}x^4 + \frac{13}{60}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{3*y''[x]+2*x*y'[x]+(4-x^2)*y[x]==0,{y[0]==-2,y'[0]==3}},y[x],{x,0,5}]
```

$$y(x) \to \frac{13x^5}{60} - \frac{19x^4}{54} - x^3 + \frac{4x^2}{3} + 3x - 2$$

13.39 problem 39 (a)

Internal problem ID [1280]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 39 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y'x + (4x^2 + 2)\, y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)+4*x*diff(y(x),x)+(2+4*x^2)*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='se
```

$$y(x) = 1 - x^2 + \frac{1}{2}x^4 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 17

```
AsymptoticDSolveValue[{y''[x]+4*x*y'[x]+(2+4*x^2)*y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^4}{2} - x^2 + 1$$

13.40 problem 39 (b)

Internal problem ID [1281]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 39 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y'x + (4x^2 + 2)\, y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)+4*x*diff(y(x),x)+(2+4*x^2)*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='se
```

$$y(x) = x - x^3 + \frac{1}{2}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 17

```
AsymptoticDSolveValue[{y''[x]+4*x*y'[x]+(2+4*x^2)*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^5}{2} - x^3 + x$$

13.41 problem 40

Internal problem ID [1282]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+1)\,y''+y'x^2+(1+2x)\,y=0}$$

With initial conditions

$$[y(0)=-2, y'(0)=3]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(1+x)*diff(y(x),x$2)+x^2*diff(y(x),x)+(1+2*x)*y(x)=0,y(0) = -2, D(y)(0) = 3],y(x),typ
```

$$y(x)=-2+3x+x^2-\frac{1}{6}x^3-\frac{3}{4}x^4+\frac{31}{120}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[{(1+x)*y''[x]+x^2*y'[x]+(1+2*x)*y[x]==0,{y[0]==-2,y'[0]==3}},y[x],{x,0,
```

$$y(x)\to\frac{31x^5}{120}-\frac{3x^4}{4}-\frac{x^3}{6}+x^2+3x-2$$

13.42 problem 41

Internal problem ID [1283]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (x^2 + 2x + 1)\, y' + 2y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 3]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)+(1+2*x+x^2)*diff(y(x),x)+2*y(x)=0,y(0) = 2, D(y)(0) = 3],y(x),type='se
```

$$y(x) = 2 + 3x - \frac{7}{2}x^2 - \frac{5}{6}x^3 + \frac{41}{24}x^4 + \frac{41}{120}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{y''[x]+(1+2*x+x^2)*y'[x]+2*y[x]==0,{y[0]==2,y'[0]==3}},y[x],{x,0,5}]
```

$$y(x) \to \frac{41x^5}{120} + \frac{41x^4}{24} - \frac{5x^3}{6} - \frac{7x^2}{2} + 3x + 2$$

13.43 problem 42

Internal problem ID [1284]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''+(x^2+2)\,y'+yx=0}$$

With initial conditions

$$[y(0)=-3, y'(0)=5]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(1+x^2)*diff(y(x),x$2)+(2+x^2)*diff(y(x),x)+x*y(x)=0,y(0) = -3, D(y)(0) = 5],y(x),typ
```

$$y(x)=-3+5x-5x^2+\frac{23}{6}x^3-\frac{23}{12}x^4+\frac{11}{30}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{(1+x^2)*y''[x]+(2+x^2)*y'[x]+x*y[x]==0,{y[0]==-3,y'[0]==5}},y[x],{x,0,
```

$$y(x)\to\frac{11x^5}{30}-\frac{23x^4}{12}+\frac{23x^3}{6}-5x^2+5x-3$$

13.44 problem 43

Internal problem ID [1285]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x+1)\,y'' + \left(2x^2 - 3x + 1\right) y' - y(x-4) = 0$$

With initial conditions

$$[y(1) = -2, y'(1) = 3]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(1+x)*diff(y(x),x$2)+(1-3*x+2*x^2)*diff(y(x),x)-(x-4)*y(x)=0,y(1) = -2, D(y)(1) = 3],
```

$$y(x) = -2 + 3(x-1) + \frac{3}{2}(x-1)^2 - \frac{17}{12}(x-1)^3 - \frac{1}{12}(x-1)^4 + \frac{1}{8}(x-1)^5 + \mathrm{O}\left((x-1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{(1+x)*y''[x]+(1-3*x+x^2)*y'[x]-(x-4)*y[x]==0,{y[1]==-2,y'[1]==3}},y[x]
```

$$y(x) \to -\frac{13}{240}(x-1)^5 - \frac{1}{96}(x-1)^4 - \frac{2}{3}(x-1)^3 + \frac{9}{4}(x-1)^2 + 3(x-1) - 2$$

13.45 problem 44

Internal problem ID [1286]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (3x^2 + 12x + 13)\, y' + (2x + 5)\, y = 0}$$

With initial conditions

$$[y(-2) = 2, y'(-2) = -3]$$

With the expansion point for the power series method at $x = -2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)+(13+12*x+3*x^2)*diff(y(x),x)+(5+2*x)*y(x)=0,y(-2) = 2, D(y)(-2) = -3],
```

$$y(x) = 2 - 3(2 + x) + \frac{1}{2}(2 + x)^2 - \frac{1}{3}(2 + x)^3 + \frac{31}{24}(2 + x)^4 - \frac{53}{120}(2 + x)^5 + \mathrm{O}\left((2 + x)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{y''[x]+(13+12*x+3*x^2)*y'[x]+(5+2*x)*y[x]==0,{y[-2]==2,y'[-2]==-3}},y[
```

$$y(x) \to -\frac{53}{120}(x + 2)^5 + \frac{31}{24}(x + 2)^4 - \frac{1}{3}(x + 2)^3 + \frac{1}{2}(x + 2)^2 - 3(x + 2) + 2$$

13.46 problem 45

Internal problem ID [1287]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x^2+2x+1)\,y''+(-x^2+2)\,y'+(x+1)\,y=0}$$

With initial conditions

$$[y(0)=1, y'(0)=-2]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(1+2*x+3*x^2)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)+(1+x)*y(x)=0,y(0) = 1, D(y)(0) = -2
```

$$y(x)=1-2x+\frac{3}{2}x^2-\frac{11}{6}x^3+\frac{15}{8}x^4-\frac{71}{60}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{(1+2*x+3*x^2)*y''[x]+(2-x^2)*y'[x]+(1+x)*y[x]==0,{y[0]==1,y'[0]==-2}},
```

$$y(x)\to -\frac{71x^5}{60}+\frac{15x^4}{8}-\frac{11x^3}{6}+\frac{3x^2}{2}-2x+1$$

13.47 problem 46

Internal problem ID [1288]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+4x+3)\,y''-(-x^2+4x+5)\,y'-(2+x)\,y=0}$$

With initial conditions

$$[y(-2)=2, y'(-2)=-1]$$

With the expansion point for the power series method at $x=-2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(3+4*x+x^2)*diff(y(x),x$2)-(5+4*x-x^2)*diff(y(x),x)-(2+x)*y(x)=0,y(-2) = 2, D(y)(-2)
```

$$y(x)=2-(2+x)-\frac{7}{2}(2+x)^2-\frac{43}{6}(2+x)^3-\frac{203}{24}(2+x)^4-\frac{167}{30}(2+x)^5+\mathrm{O}\left((2+x)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 43

```
AsymptoticDSolveValue[{(3+4*x+x^2)*y''[x]-(5+4*x-x^2)*y'[x]-(2+x)*y[x]==0,{y[-2]==2,y'[-2]==-
```

$$y(x)\to -\frac{167}{30}(x+2)^5-\frac{203}{24}(x+2)^4-\frac{43}{6}(x+2)^3-\frac{7}{2}(x+2)^2-x$$

13.48 problem 47

Internal problem ID [1289]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 + 2x + 1)\, y'' + (1 - x)\, y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(1+2*x+x^2)*diff(y(x),x$2)+(1-x)*y(x)=0,y(0) = 2, D(y)(0) = -1],y(x),type='series',x=
```

$$y(x) = 2 - x - x^2 + \frac{7}{6}x^3 - x^4 + \frac{89}{120}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[{(1+2*x+x^2)*y''[x]+(1-x)*y[x]==0,{y[0]==2,y'[0]==-1}},y[x],{x,0,5}]
```

$$y(x) \to \frac{89x^5}{120} - x^4 + \frac{7x^3}{6} - x^2 - x + 2$$

13.49 problem 48

Internal problem ID [1290]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 48.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(-2x^2+x\right)y''+\left(-x^2+3x+1\right)y'+(2+x)\,y=0}$$

With initial conditions

$$[y(1)=1, y'(1)=0]$$

With the expansion point for the power series method at $x=1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
Order:=6;
dsolve([(x-2*x^2)*diff(y(x),x$2)+(1+3*x-x^2)*diff(y(x),x)+(2+x)*y(x)=0,y(1) = 1, D(y)(1) = 0]
```

$$y(x)=1+\frac{3}{2}(x-1)^2+\frac{1}{6}(x-1)^3-\frac{1}{8}(x-1)^5+\mathrm{O}\left((x-1)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[{(x-2*x^2)*y''[x]+(1+3*x-x^2)*y'[x]+(2+x)*y[x]==0,{y[1]==1,y'[1]==0}},y
```

$$y(x)\to -\frac{1}{8}(x-1)^5+\frac{1}{6}(x-1)^3+\frac{3}{2}(x-1)^2+1$$

13.50 problem 49

Internal problem ID [1291]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.3 SERIES SOLUTIONS NEAR AN ORDINARY POINT II. Exercises 7.3. Page 338
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2 - 11x + 16)\, y'' + (x^2 - 6x + 10)\, y' - (2 - x)\, y = 0}$$

With initial conditions

$$[y(3) = 1, y'(3) = -2]$$

With the expansion point for the power series method at $x = 3$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(16-11*x+2*x^2)*diff(y(x),x$2)+(10-6*x+x^2)*diff(y(x),x)-(2-x)*y(x)=0,y(3) = 1, D(y)(
```

$$y(x) = 1 - 2(x-3) + \frac{1}{2}(x-3)^2 - \frac{1}{6}(x-3)^3 + \frac{1}{4}(x-3)^4 - \frac{1}{6}(x-3)^5 + O\left((x-3)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 46

```
AsymptoticDSolveValue[{(16-11*x+2*x^2)*y''[x]+(10-6*x+x^2)*y'[x]-(2-x)*y[x]==0,{y[3]==1,y'[3]
```

$$y(x) \to -\frac{1}{6}(x-3)^5 + \frac{1}{4}(x-3)^4 - \frac{1}{6}(x-3)^3 + \frac{1}{2}(x-3)^2 - 2(x-3) + 1$$

14 Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358

14.1 problem Example 7.5.1 page 353

Internal problem ID [1292]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: Example 7.5.1 page 353.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x^2+x+1)\,y''+x(11x^2+11x+9)\,y'+(7x^2+10x+6)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*x^2*(1+x+x^2)*diff(y(x),x$2)+x*(9+11*x+11*x^2)*diff(y(x),x)+(6+10*x+7*x^2)*y(x)=0,y(
```

$$y(x) = \frac{\left(1+\frac{1}{2}x^2-\frac{1}{3}x^3+\frac{1}{8}x^4+\frac{1}{30}x^5+\mathrm{O}(x^6)\right)c_1\sqrt{x}+\left(1-\frac{1}{3}x+\frac{2}{5}x^2-\frac{5}{21}x^3+\frac{7}{135}x^4+\frac{76}{1155}x^5+\mathrm{O}(x^6)\right)c_2x}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 83

```
AsymptoticDSolveValue[2*x^2*(1+x+x^2)*y''[x]+x*(9+11*x+11*x^2)*y'[x]+(6+10*x+7*x^2)*y[x]==0,y
```

$$y(x)\to\frac{c_2\left(\frac{x^5}{30}+\frac{x^4}{8}-\frac{x^3}{3}+\frac{x^2}{2}+1\right)}{x^2}+\frac{c_1\left(\frac{76x^5}{1155}+\frac{7x^4}{135}-\frac{5x^3}{21}+\frac{2x^2}{5}-\frac{x}{3}+1\right)}{x^{3/2}}$$

14.2 problem Example 7.5.2 page 354

Internal problem ID [1293]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: Example 7.5.2 page 354.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2(x+3)\,y''+5x(x+1)\,y'-(-4x+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*(3+x)*diff(y(x),x$2)+5*x*(1+x)*diff(y(x),x)-(1-4*x)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}}\left(1-\frac{7}{9}x+\frac{35}{81}x^2-\frac{455}{2187}x^3+\frac{1820}{19683}x^4-\frac{6916}{177147}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1+x-x^2+\frac{3}{5}x^3-\frac{3}{10}x^4+\frac{3}{22}x^5+\mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 82

```
AsymptoticDSolveValue[x^2*(3+x)*y''[x]+5*x*(1+x)*y'[x]-(1-4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt[3]{x}\left(-\frac{6916x^5}{177147}+\frac{1820x^4}{19683}-\frac{455x^3}{2187}+\frac{35x^2}{81}-\frac{7x}{9}+1\right)+\frac{c_2\left(\frac{3x^5}{22}-\frac{3x^4}{10}+\frac{3x^3}{5}-x^2+x+1\right)}{x}$$

14.3 problem Example 7.5.3 page 356

Internal problem ID [1294]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: Example 7.5.3 page 356.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(-x^2+2)\,y''-x(4x^2+3)\,y'+(2+2x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*(2-x^2)*diff(y(x),x$2)-x*(3+4*x^2)*diff(y(x),x)+(2+2*x)*y(x)=0,y(x),type='series',
```

$$y(x)=c_1\sqrt{x}\left(1+2x-\frac{9}{8}x^2+\frac{7}{4}x^3-\frac{607}{640}x^4+\frac{13347}{11200}x^5+\mathrm{O}\left(x^6\right)\right)+c_2x^2\left(1-\frac{2}{5}x+\frac{27}{35}x^2-\frac{34}{105}x^3+\frac{584}{1155}x^4-\frac{768}{3575}x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 86

```
AsymptoticDSolveValue[x^2*(2-x^2)*y''[x]-x*(3+4*x^2)*y'[x]+(2+2*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{768x^5}{3575}+\frac{584x^4}{1155}-\frac{34x^3}{105}+\frac{27x^2}{35}-\frac{2x}{5}+1\right)x^2+c_2\left(\frac{13347x^5}{11200}-\frac{607x^4}{640}+\frac{7x^3}{4}-\frac{9x^2}{8}+2x+1\right)\sqrt{x}$$

14.4 problem 1

Internal problem ID [1295]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2+x+1)\,y''+x(5x^2+3x+3)\,y'+y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 1243

```
Order:=6;
dsolve(2*x^2*(1+x+x^2)*diff(y(x),x$2)+x*(3+3*x+5*x^2)*diff(y(x),x)+y(x)=0,y(x),type='series',
```

$$y(x) = \frac{c_1 x^{-\frac{i\sqrt{7}}{4}}\left(1-\frac{1}{-2+i\sqrt{7}}x+\frac{1}{4}\frac{11+i\sqrt{7}}{\left(-2+i\sqrt{7}\right)\left(i\sqrt{7}-4\right)}x^2-\frac{1}{12}\frac{49i\sqrt{7}-89}{\left(-2+i\sqrt{7}\right)\left(i\sqrt{7}-4\right)\left(i\sqrt{7}-6\right)}x^3-\frac{1}{48}\frac{395i\sqrt{7}+1553}{\left(-2+i\sqrt{7}\right)\left(i\sqrt{7}-4\right)\left(i\sqrt{7}-6\right)\left(i\right.}\right.}{}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 4838

```
AsymptoticDSolveValue[2*x^2*(1+x+x^2)*y''[x]+x*(3+3*x+5*x^2)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

Too large to display

14.5 problem 2

Internal problem ID [1296]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, ‘`

$$3x^2y'' + 2x(-2x^2 + x + 1)\, y' + (-8x^2 + 2x)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(3*x^2*diff(y(x),x$2)+2*x*(1+x-2*x^2)*diff(y(x),x)+(2*x-8*x^2)*y(x)=0,y(x),type='series
```

$$y(x) = c_1 x^{\frac{1}{3}} \left(1 - \frac{2}{3}x + \frac{8}{9}x^2 - \frac{40}{81}x^3 + \frac{92}{243}x^4 - \frac{664}{3645}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 - x + \frac{6}{5}x^2 - \frac{4}{5}x^3 + \frac{32}{55}x^4 - \frac{24}{77}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 83

```
AsymptoticDSolveValue[3*x^2*y''[x]+2*x*(1+x-2*x^2)*y'[x]+(2*x-8*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \sqrt[3]{x}\left(-\frac{664x^5}{3645} + \frac{92x^4}{243} - \frac{40x^3}{81} + \frac{8x^2}{9} - \frac{2x}{3} + 1\right) + c_2\left(-\frac{24x^5}{77} + \frac{32x^4}{55} - \frac{4x^3}{5} + \frac{6x^2}{5} - x + 1\right)$$

14.6 problem 3

Internal problem ID [1297]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+3x+3)\,y''+x(7x^2+8x+5)\,y'-(-9x^2-2x+1)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 43

```
Order:=6;
dsolve(x^2*(3+3*x+x^2)*diff(y(x),x$2)+x*(5+8*x+7*x^2)*diff(y(x),x)-(1-2*x-9*x^2)*y(x)=0,y(x),
```

$$y(x) = \frac{c_2x^{\frac{4}{3}}\left(1-\frac{4}{7}x-\frac{7}{45}x^2+\frac{970}{2457}x^3-\frac{5707}{22680}x^4+\frac{13568}{300105}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1-x^2+\frac{2}{3}x^3-\frac{10}{33}x^5+\mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 74

```
AsymptoticDSolveValue[x^2*(3+3*x+x^2)*y''[x]+x*(5+8*x+7*x^2)*y'[x]-(1-2*x-9*x^2)*y[x]==0,y[x]
```

$$y(x)\to\frac{c_2\left(-\frac{10x^5}{33}+\frac{2x^3}{3}-x^2+1\right)}{x}+c_1\sqrt[3]{x}\left(\frac{13568x^5}{300105}-\frac{5707x^4}{22680}+\frac{970x^3}{2457}-\frac{7x^2}{45}-\frac{4x}{7}+1\right)$$

14.7 problem 4

Internal problem ID [1298]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + x(4x^2 + 2x + 7)\, y' - (-7x^2 - 4x + 1)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+x*(7+2*x+4*x^2)*diff(y(x),x)-(1-4*x-7*x^2)*y(x)=0,y(x),type='seri
```

$$y(x) = \frac{c_2 x^{\frac{5}{4}}\left(1 - \frac{1}{2}x - \frac{19}{104}x^2 + \frac{1571}{10608}x^3 + \frac{3225}{198016}x^4 - \frac{752183}{29702400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_1\left(1 + 2x - \frac{11}{6}x^2 - \frac{1}{7}x^3 + \frac{895}{1848}x^4 - \frac{4}{13}\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 86

```
AsymptoticDSolveValue[4*x^2*y''[x]+x*(7+2*x+4*x^2)*y'[x]-(1-4*x-7*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \sqrt[4]{x}\left(-\frac{752183x^5}{29702400} + \frac{3225x^4}{198016} + \frac{1571x^3}{10608} - \frac{19x^2}{104} - \frac{x}{2} + 1\right) + \frac{c_2\left(-\frac{499x^5}{13860} + \frac{895x^4}{1848} - \frac{x^3}{7} - \frac{11x^2}{6} + 2x + 1\right)}{x}$$

14.8 problem 5

Internal problem ID [1299]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$12x^2(x+1)\,y''+x\left(3x^2+35x+11\right)y'-\left(-5x^2-10x+1\right)y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
Order:=6;
dsolve(12*x^2*(1+x)*diff(y(x),x$2)+x*(11+35*x+3*x^2)*diff(y(x),x)-(1-10*x-5*x^2)*y(x)=0,y(x),
```

$$y(x)=\frac{c_2x^{\frac{7}{12}}\left(1-x+\frac{28}{31}x^2-\frac{1111}{1333}x^3+\frac{57493}{73315}x^4-\frac{3668716}{4912105}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1-x+\frac{7}{8}x^2-\frac{19}{24}x^3+\frac{283}{384}x^4-\frac{1339}{1920}x^5+\right.}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 86

```
AsymptoticDSolveValue[12*x^2*(1+x)*y''[x]+x*(11+35*x+3*x^2)*y'[x]-(1-10*x-5*x^2)*y[x]==0,y[x]
```

$$y(x)\to c_1\sqrt[3]{x}\left(-\frac{3668716x^5}{4912105}+\frac{57493x^4}{73315}-\frac{1111x^3}{1333}+\frac{28x^2}{31}-x+1\right)+\frac{c_2\left(-\frac{1339x^5}{1920}+\frac{283x^4}{384}-\frac{19x^3}{24}+\frac{7x^2}{8}-x+1\right)}{\sqrt[4]{x}}$$

14.9 problem 6

Internal problem ID [1300]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$x^2(10x^2+x+5)\,y''+x(48x^2+3x+4)\,y'+(36x^2+x)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 44

```
Order:=6;
dsolve(x^2*(5+x+10*x^2)*diff(y(x),x$2)+x*(4+3*x+48*x^2)*diff(y(x),x)+(x+36*x^2)*y(x)=0,y(x),t
```

$$y(x)=c_1x^{\frac{1}{5}}\left(1-\frac{6}{25}x-\frac{1217}{625}x^2+\frac{41972}{46875}x^3+\frac{1447799}{390625}x^4-\frac{375253322}{146484375}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(1-\frac{1}{4}x-\frac{35}{18}x^2+\frac{11}{12}x^3+\frac{632}{171}x^4-\frac{2671}{1026}x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x^2*(5+x+10*x^2)*y''[x]+x*(4+3*x+48*x^2)*y'[x]+(x+36*x^2)*y[x]==0,y[x],
```

$$y(x)\to c_1\sqrt[5]{x}\left(-\frac{375253322x^5}{146484375}+\frac{1447799x^4}{390625}+\frac{41972x^3}{46875}-\frac{1217x^2}{625}-\frac{6x}{25}+1\right)+c_2\left(-\frac{2671x^5}{1026}+\frac{632x^4}{171}+\frac{11x^3}{12}-\frac{35x^2}{18}-\frac{x}{4}+1\right)$$

14.10 problem 7

Internal problem ID [1301]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$8x^2y'' - 2x(-x^2 - 4x + 3)\,y' + (x^2 + 6x + 3)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(8*x^2*diff(y(x),x$2)-2*x*(3-4*x-x^2)*diff(y(x),x)+(3+6*x+x^2)*y(x)=0,y(x),type='series
```

$$y(x) = c_1 x^{\frac{1}{4}}\left(1 + 4x - \frac{131}{24}x^2 + \frac{39}{14}x^3 - \frac{19865}{29568}x^4 + \frac{4421}{110880}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 x^{\frac{3}{2}}\left(1 - x + \frac{11}{26}x^2 - \frac{109}{1326}x^3 + \frac{5}{12376}x^4 + \frac{229}{71400}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 86

```
AsymptoticDSolveValue[8*x^2*y''[x]-2*x*(3-4*x-x^2)*y'[x]+(3+6*x+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{4421x^5}{110880} - \frac{19865x^4}{29568} + \frac{39x^3}{14} - \frac{131x^2}{24} + 4x + 1\right)\sqrt[4]{x} + c_1\left(\frac{229x^5}{71400} + \frac{5x^4}{12376} - \frac{109x^3}{1326} + \frac{11x^2}{26} - x + 1\right)x^{3/2}$$

14.11 problem 8

Internal problem ID [1302]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{18x^2(x+1)\,y''+3x\left(x^2+11x+5\right)y'-\left(-5x^2-2x+1\right)y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
Order:=6;
dsolve(18*x^2*(1+x)*diff(y(x),x$2)+3*x*(5+11*x+x^2)*diff(y(x),x)-(1-2*x-5*x^2)*y(x)=0,y(x),ty
```

$$y(x) = \frac{c_2\sqrt{x}\left(1-\frac{1}{3}x+\frac{2}{15}x^2-\frac{5}{63}x^3+\frac{23}{405}x^4-\frac{458}{10395}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1-\frac{1}{12}x^2+\frac{1}{18}x^3-\frac{11}{288}x^4+\frac{31}{1080}x^5+\mathrm{O}\left(x^6\right.\right.}{x^{\frac{1}{6}}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 85

```
AsymptoticDSolveValue[18*x^2*(1+x)*y''[x]+3*x*(5+11*x+x^2)*y'[x]-(1-2*x-5*x^2)*y[x]==0,y[x],{
```

$$y(x)\to c_1\sqrt[3]{x}\left(-\frac{458x^5}{10395}+\frac{23x^4}{405}-\frac{5x^3}{63}+\frac{2x^2}{15}-\frac{x}{3}+1\right)+\frac{c_2\left(\frac{31x^5}{1080}-\frac{11x^4}{288}+\frac{x^3}{18}-\frac{x^2}{12}+1\right)}{\sqrt[6]{x}}$$

14.12 problem 9

Internal problem ID [1303]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x^2+x+3)\,y''+(-x^2+x+4)\,y'+yx=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
Order:=6;
dsolve(x*(3+x+x^2)*diff(y(x),x$2)+(4+x-x^2)*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2\left(1-\frac{1}{14}x^2+\frac{1}{105}x^3-\frac{1}{3640}x^4-\frac{23}{54600}x^5+\mathrm{O}\left(x^6\right)\right)x^{\frac{1}{3}}+c_1\left(1-\frac{1}{18}x-\frac{71}{405}x^2+\frac{719}{34992}x^3-\frac{1678}{1082565}x^4-\frac{513547}{9920232}\right.}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 80

```
AsymptoticDSolveValue[x*(3+x+x^2)*y''[x]+(4+x-x^2)*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{23x^5}{54600}-\frac{x^4}{3640}+\frac{x^3}{105}-\frac{x^2}{14}+1\right)+\frac{c_2\left(-\frac{513547x^5}{992023200}-\frac{1678x^4}{1082565}+\frac{719x^3}{34992}-\frac{71x^2}{405}-\frac{x}{18}+1\right)}{\sqrt[3]{x}}$$

14.13 problem 10

Internal problem ID [1304]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$10x^2(2x^2+x+1)\,y''+x(66x^2+13x+13)\,y'-(10x^2+4x+1)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 47

```
Order:=6;
dsolve(10*x^2*(1+x+2*x^2)*diff(y(x),x$2)+x*(13+13*x+66*x^2)*diff(y(x),x)-(1+4*x+10*x^2)*y(x)=
```

$$y(x) = \frac{c_2 x^{\frac{7}{10}}\left(1+\frac{3}{17}x-\frac{7}{153}x^2-\frac{547}{5661}x^3+\frac{26942}{266067}x^4+\frac{200432}{3991005}x^5+\mathrm{O}(x^6)\right)+c_1\left(1+x+\frac{14}{13}x^2-\frac{556}{897}x^3-\frac{5314}{9867}x^4+\frac{20}{21}\right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 86

```
AsymptoticDSolveValue[10*x^2*(1+x+2*x^2)*y''[x]+x*(13+13*x+66*x^2)*y'[x]-(1+4*x+10*x^2)*y[x]=
```

$$y(x)\to c_1\sqrt[5]{x}\left(\frac{200432x^5}{3991005}+\frac{26942x^4}{266067}-\frac{547x^3}{5661}-\frac{7x^2}{153}+\frac{3x}{17}+1\right)+\frac{c_2\left(\frac{2092186x^5}{2121405}-\frac{5314x^4}{9867}-\frac{556x^3}{897}+\frac{14x^2}{13}+x+1\right)}{\sqrt{x}}$$

14.14 problem 14

Internal problem ID [1305]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' + x(2x+3)\,y' - (1-x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+x*(3+2*x)*diff(y(x),x)-(1-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2x^{\frac{3}{2}}\left(1 - \frac{2}{5}x + \frac{4}{35}x^2 - \frac{8}{315}x^3 + \frac{16}{3465}x^4 - \frac{32}{45045}x^5 + \mathrm{O}\left(x^6\right)\right) + c_1\left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \mathrm{O}\left(x\right.\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 86

```
AsymptoticDSolveValue[2*x^2*y''[x]+x*(3+2*x)*y'[x]-(1-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{32x^5}{45045} + \frac{16x^4}{3465} - \frac{8x^3}{315} + \frac{4x^2}{35} - \frac{2x}{5} + 1\right) + \frac{c_2\left(-\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1\right)}{x}$$

14.15 problem 15

Internal problem ID [1306]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2(x+3)\,y''+x(4x+5)\,y'-(1-2x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
Order:=6;
dsolve(x^2*(3+x)*diff(y(x),x$2)+x*(5+4*x)*diff(y(x),x)-(1-2*x)*y(x)=0,y(x),type='series',x=0)
```

$$y(x)=\frac{c_2x^{\frac{4}{3}}\left(1-\frac{4}{9}x+\frac{14}{81}x^2-\frac{140}{2187}x^3+\frac{455}{19683}x^4-\frac{1456}{177147}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1+\mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 53

```
AsymptoticDSolveValue[x^2*(3+x)*y''[x]+x*(5+4*x)*y'[x]-(1-2*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt[3]{x}\left(-\frac{1456x^5}{177147}+\frac{455x^4}{19683}-\frac{140x^3}{2187}+\frac{14x^2}{81}-\frac{4x}{9}+1\right)+\frac{c_2}{x}$$

14.16 problem 16

Internal problem ID [1307]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + x(x+5)\,y' - (-3x+2)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+x*(5+x)*diff(y(x),x)-(2-3*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{5}{2}}\left(1-\frac{1}{2}x+\frac{1}{8}x^2-\frac{1}{48}x^3+\frac{1}{384}x^4-\frac{1}{3840}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1+\frac{1}{3}x+\frac{1}{3}x^2-\frac{1}{3}x^3+\frac{1}{9}x^4-\frac{1}{45}x^5+\mathrm{O}\left(x^6\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 88

```
AsymptoticDSolveValue[2*x^2*y''[x]+x*(5+x)*y'[x]-(2-3*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{x^5}{3840}+\frac{x^4}{384}-\frac{x^3}{48}+\frac{x^2}{8}-\frac{x}{2}+1\right)+\frac{c_2\left(-\frac{x^5}{45}+\frac{x^4}{9}-\frac{x^3}{3}+\frac{x^2}{3}+\frac{x}{3}+1\right)}{x^2}$$

14.17 problem 17

Internal problem ID [1308]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2y'' + x(x+1)\,y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
dsolve(3*x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}}\left(1 - \frac{1}{7}x + \frac{1}{70}x^2 - \frac{1}{910}x^3 + \frac{1}{14560}x^4 - \frac{1}{276640}x^5 + \mathrm{O}\left(x^6\right)\right) + c_1\left(1 - \frac{1}{3}x + \frac{1}{18}x^2 - \frac{1}{162}x^3 + \frac{1}{1944}x^4 - \frac{1}{29160}\right.}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 86

```
AsymptoticDSolveValue[3*x^2*y''[x]+x*(1+x)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x\left(-\frac{x^5}{276640} + \frac{x^4}{14560} - \frac{x^3}{910} + \frac{x^2}{70} - \frac{x}{7} + 1\right) + \frac{c_2\left(-\frac{x^5}{29160} + \frac{x^4}{1944} - \frac{x^3}{162} + \frac{x^2}{18} - \frac{x}{3} + 1\right)}{\sqrt[3]{x}}$$

14.18 problem 18

Internal problem ID [1309]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - y'x + (1-2x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+(1-2*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 + 2x + \frac{2}{3}x^2 + \frac{4}{45}x^3 + \frac{2}{315}x^4 + \frac{4}{14175}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2x\left(1 + \frac{2}{3}x + \frac{2}{15}x^2 + \frac{4}{315}x^3 + \frac{2}{2835}x^4 + \frac{4}{155925}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 84

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-2*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x\left(\frac{4x^5}{155925} + \frac{2x^4}{2835} + \frac{4x^3}{315} + \frac{2x^2}{15} + \frac{2x}{3} + 1\right) + c_2\sqrt{x}\left(\frac{4x^5}{14175} + \frac{2x^4}{315} + \frac{4x^3}{45} + \frac{2x^2}{3} + 2x + 1\right)$$

14.19 problem 19

Internal problem ID [1310]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' + 9y'x - (3x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(9*x^2*diff(y(x),x$2)+9*x*diff(y(x),x)-(1+3*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{2}{3}}\left(1+\frac{1}{5}x+\frac{1}{80}x^2+\frac{1}{2640}x^3+\frac{1}{147840}x^4+\frac{1}{12566400}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1+x+\frac{1}{8}x^2+\frac{1}{168}x^3+\frac{1}{6720}x^4+\frac{1}{43680}\right.}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 86

```
AsymptoticDSolveValue[9*x^2*y''[x]+9*x*y'[x]-(1+3*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt[3]{x}\left(\frac{x^5}{12566400}+\frac{x^4}{147840}+\frac{x^3}{2640}+\frac{x^2}{80}+\frac{x}{5}+1\right) + \frac{c_2\left(\frac{x^5}{436800}+\frac{x^4}{6720}+\frac{x^3}{168}+\frac{x^2}{8}+x+1\right)}{\sqrt[3]{x}}$$

14.20 problem 20

Internal problem ID [1311]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2y'' + x(x+1)\,y' - (3x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
Order:=6;
dsolve(3*x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-(1+3*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}}\left(1+\frac{2}{7}x+\frac{1}{70}x^2+\mathrm{O}\left(x^6\right)\right)+c_1\left(1-\frac{10}{3}x-\frac{35}{18}x^2-\frac{14}{81}x^3-\frac{7}{3888}x^4+\frac{7}{320760}x^5+\mathrm{O}\left(x^6\right)\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 65

```
AsymptoticDSolveValue[3*x^2*y''[x]+x*(1+x)*y'[x]-(1+3*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x\left(\frac{x^2}{70}+\frac{2x}{7}+1\right)+\frac{c_2\left(\frac{7x^5}{320760}-\frac{7x^4}{3888}-\frac{14x^3}{81}-\frac{35x^2}{18}-\frac{10x}{3}+1\right)}{\sqrt[3]{x}}$$

14.21 problem 21

Internal problem ID [1312]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x+3)\,y''+x(5x+1)\,y'+(x+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(2*x^2*(3+x)*diff(y(x),x$2)+x*(1+5*x)*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0)
```

$$y(x)=c_1x^{\frac{1}{3}}\left(1-\frac{4}{9}x+\frac{14}{81}x^2-\frac{140}{2187}x^3+\frac{455}{19683}x^4-\frac{1456}{177147}x^5+\mathrm{O}\left(x^6\right)\right)\\+c_2\sqrt{x}\left(1-\frac{3}{7}x+\frac{15}{91}x^2-\frac{15}{247}x^3+\frac{27}{1235}x^4-\frac{297}{38285}x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 90

```
AsymptoticDSolveValue[2*x^2*(3+x)*y''[x]+x*(1+5*x)*y'[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt{x}\left(-\frac{297x^5}{38285}+\frac{27x^4}{1235}-\frac{15x^3}{247}+\frac{15x^2}{91}-\frac{3x}{7}+1\right)\\+c_2\sqrt[3]{x}\left(-\frac{1456x^5}{177147}+\frac{455x^4}{19683}-\frac{140x^3}{2187}+\frac{14x^2}{81}-\frac{4x}{9}+1\right)$$

14.22 problem 22

Internal problem ID [1313]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x+4)\,y''-x(-3x+1)\,y'+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*(4+x)*diff(y(x),x$2)-x*(1-3*x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^{\frac{1}{4}}\left(1-\frac{9}{16}x+\frac{117}{512}x^2-\frac{663}{8192}x^3+\frac{13923}{524288}x^4-\frac{69615}{8388608}x^5+\mathrm{O}\left(x^6\right)\right)+c_2x\left(1-\frac{3}{7}x+\frac{12}{77}x^2-\frac{4}{77}x^3+\frac{24}{1463}x^4-\frac{24}{4807}x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 86

```
AsymptoticDSolveValue[x^2*(4+x)*y''[x]-x*(1-3*x)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1x\left(-\frac{24x^5}{4807}+\frac{24x^4}{1463}-\frac{4x^3}{77}+\frac{12x^2}{77}-\frac{3x}{7}+1\right)+c_2\sqrt[4]{x}\left(-\frac{69615x^5}{8388608}+\frac{13923x^4}{524288}-\frac{663x^3}{8192}+\frac{117x^2}{512}-\frac{9x}{16}+1\right)$$

14.23 problem 23

Internal problem ID [1314]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + 5y'x + (x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - x + \frac{1}{6}x^2 - \frac{1}{90}x^3 + \frac{1}{2520}x^4 - \frac{1}{113400}x^5 + \mathrm{O}\left(x^6\right)\right)\sqrt{x} + c_2\left(1 - \frac{1}{3}x + \frac{1}{30}x^2 - \frac{1}{630}x^3 + \frac{1}{22680}x^4 - \frac{1}{1247400}x\right.}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 86

```
AsymptoticDSolveValue[2*x^2*y''[x]+5*x*y'[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1\left(-\frac{x^5}{1247400} + \frac{x^4}{22680} - \frac{x^3}{630} + \frac{x^2}{30} - \frac{x}{3} + 1\right)}{\sqrt{x}} + \frac{c_2\left(-\frac{x^5}{113400} + \frac{x^4}{2520} - \frac{x^3}{90} + \frac{x^2}{6} - x + 1\right)}{x}$$

14.24 problem 24

Internal problem ID [1315]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2(4x+3)\,y''+x(5+18x)\,y'-(1-12x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*(3+4*x)*diff(y(x),x$2)+x*(5+18*x)*diff(y(x),x)-(1-12*x)*y(x)=0,y(x),type='series',
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}}\left(1-\frac{22}{9}x+\frac{374}{81}x^2-\frac{17204}{2187}x^3+\frac{249458}{19683}x^4-\frac{3492412}{177147}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1+2x-6x^2+12x^3-21x^4+\frac{378}{11}\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 80

```
AsymptoticDSolveValue[x^2*(3+4*x)*y''[x]+x*(5+18*x)*y'[x]-(1-12*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt[3]{x}\left(-\frac{3492412x^5}{177147}+\frac{249458x^4}{19683}-\frac{17204x^3}{2187}+\frac{374x^2}{81}-\frac{22x}{9}+1\right)+\frac{c_2\left(\frac{378x^5}{11}-21x^4+12x^3-6x^2+2x+1\right)}{x}$$

14.25 problem 25

Internal problem ID [1316]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{6x^2y'' + x(10-x)\,y' - (2+x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
Order:=6;
dsolve(6*x^2*diff(y(x),x$2)+x*(10-x)*diff(y(x),x)-(2+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}} \left(1 + \frac{2}{21}x + \frac{1}{180}x^2 + \frac{1}{4212}x^3 + \frac{1}{124416}x^4 + \frac{1}{4432320}x^5 + \mathrm{O}\left(x^6\right)\right) + c_1\left(1 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 53

```
AsymptoticDSolveValue[6*x^2*y''[x]+x*(10-x)*y'[x]-(2+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \sqrt[3]{x}\left(\frac{x^5}{4432320} + \frac{x^4}{124416} + \frac{x^3}{4212} + \frac{x^2}{180} + \frac{2x}{21} + 1\right) + \frac{c_2}{x}$$

14.26 problem 28

Internal problem ID [1317]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x+8)\,y''+x(3x+2)\,y'+(x+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*(8+x)*diff(y(x),x$2)+x*(2+3*x)*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^{\frac{1}{4}}\left(1-\frac{25}{96}x+\frac{675}{14336}x^2-\frac{38025}{5046272}x^3+\frac{732615}{645922816}x^4-\frac{9230949}{56103010304}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\sqrt{x}\left(1-\frac{9}{40}x+\frac{5}{128}x^2-\frac{245}{39936}x^3+\frac{6615}{7241728}x^4-\frac{7623}{57933824}x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 90

```
AsymptoticDSolveValue[x^2*(8+x)*y''[x]+x*(2+3*x)*y'[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt{x}\left(-\frac{7623x^5}{57933824}+\frac{6615x^4}{7241728}-\frac{245x^3}{39936}+\frac{5x^2}{128}-\frac{9x}{40}+1\right)+c_2\sqrt[4]{x}\left(-\frac{9230949x^5}{56103010304}+\frac{732615x^4}{645922816}-\frac{38025x^3}{5046272}+\frac{675x^2}{14336}-\frac{25x}{96}+1\right)$$

14.27 problem 29

Internal problem ID [1318]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(4x+3)\,y''+x(11+4x)\,y'-(4x+3)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
Order:=6;
dsolve(x^2*(3+4*x)*diff(y(x),x$2)+x*(11+4*x)*diff(y(x),x)-(3+4*x)*y(x)=0,y(x),type='series',x
```

$$y(x) = \frac{c_2 x^{\frac{10}{3}}\left(1+\frac{32}{117}x-\frac{28}{1053}x^2+\frac{4480}{540189}x^3-\frac{15680}{4113747}x^4+\frac{401408}{185118615}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1+\frac{32}{7}x+\frac{48}{7}x^2+\mathrm{O}\left(x^6\right)\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 67

```
AsymptoticDSolveValue[x^2*(3+4*x)*y''[x]+x*(11+4*x)*y'[x]-(3+4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to \frac{c_2\left(\frac{48x^2}{7}+\frac{32x}{7}+1\right)}{x^3}+c_1\sqrt[3]{x}\left(\frac{401408x^5}{185118615}-\frac{15680x^4}{4113747}+\frac{4480x^3}{540189}-\frac{28x^2}{1053}+\frac{32x}{117}+1\right)$$

14.28 problem 30

Internal problem ID [1319]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(3x+2)\,y''+x(4+11x)\,y'-(1-x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(2*x^2*(2+3*x)*diff(y(x),x$2)+x*(4+11*x)*diff(y(x),x)-(1-x)*y(x)=0,y(x),type='series',x
```

$$y(x) = \frac{c_1 x\left(1-\frac{5}{8}x+\frac{55}{96}x^2-\frac{935}{1536}x^3+\frac{4301}{6144}x^4-\frac{124729}{147456}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(1-\frac{5}{4}x+\frac{25}{32}x^2-\frac{275}{384}x^3+\frac{4675}{6144}x^4-\frac{21505}{24576}x^5\right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 94

```
AsymptoticDSolveValue[2*x^2*(2+3*x)*y''[x]+x*(4+11*x)*y'[x]-(1-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{935x^{7/2}}{6144}+\frac{55x^{5/2}}{384}-\frac{5x^{3/2}}{32}+\frac{\sqrt{x}}{4}+\frac{1}{\sqrt{x}}\right)+c_2\left(\frac{4301x^{9/2}}{6144}-\frac{935x^{7/2}}{1536}+\frac{55x^{5/2}}{96}-\frac{5x^{3/2}}{8}+\sqrt{x}\right)$$

14.29 problem 31

Internal problem ID [1320]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(2+x)\,y'' + 5x(1-x)\,y' - (-8x+2)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*(2+x)*diff(y(x),x$2)+5*x*(1-x)*diff(y(x),x)-(2-8*x)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = \frac{c_2 x^{\frac{5}{2}}\left(1 - \frac{3}{4}x + \frac{5}{96}x^2 + \frac{5}{4224}x^3 + \frac{5}{292864}x^4 - \frac{1}{3514368}x^5 + \mathrm{O}\left(x^6\right)\right) + c_1\left(1 + 8x + 60x^2 - 160x^3 + 40x^4 + \mathrm{O}\left(x\right.\right.}{x^2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 73

```
AsymptoticDSolveValue[x^2*(2+x)*y''[x]+5*x*(1-x)*y'[x]-(2-8*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_2(40x^4 - 160x^3 + 60x^2 + 8x + 1)}{x^2} + c_1\sqrt{x}\left(-\frac{x^5}{3514368} + \frac{5x^4}{292864} + \frac{5x^3}{4224} + \frac{5x^2}{96} - \frac{3x}{4} + 1\right)$$

14.30 problem 32

Internal problem ID [1321]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x+6)\,y''+x(11+4x)\,y'+(1+2x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*(6+x)*diff(y(x),x$2)+x*(11+4*x)*diff(y(x),x)+(1+2*x)*y(x)=0,y(x),type='series',x=0
```

$$y(x) = \frac{\left(1-\frac{10}{63}x+\frac{200}{7371}x^2-\frac{17600}{3781323}x^3+\frac{3872}{4861701}x^4-\frac{921536}{6782072895}x^5+\mathrm{O}\left(x^6\right)\right)c_2x^{\frac{1}{6}}+\left(1-\frac{3}{20}x+\frac{9}{352}x^2-\frac{105}{23936}x^3+\cdots\right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 90

```
AsymptoticDSolveValue[x^2*(6+x)*y''[x]+x*(11+4*x)*y'[x]+(1+2*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to \frac{c_1\left(-\frac{921536x^5}{6782072895}+\frac{3872x^4}{4861701}-\frac{17600x^3}{3781323}+\frac{200x^2}{7371}-\frac{10x}{63}+1\right)}{\sqrt[3]{x}}+\frac{c_2\left(-\frac{11907x^5}{92889088}+\frac{6615x^4}{8808448}-\frac{105x^3}{23936}+\frac{9x^2}{352}-\frac{3x}{20}+1\right)}{\sqrt{x}}$$

14.31 problem 33

Internal problem ID [1322]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{8x^2y'' + x(x^2+2)\,y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
dsolve(8*x^2*diff(y(x),x$2)+x*(2+x^2)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{4}}\left(1 - \frac{1}{112}x^2 + \frac{3}{17920}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\sqrt{x}\left(1 - \frac{1}{72}x^2 + \frac{5}{19584}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 52

```
AsymptoticDSolveValue[8*x^2*y''[x]+x*(2+x^2)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{5x^4}{19584} - \frac{x^2}{72} + 1\right) + c_2\sqrt[4]{x}\left(\frac{3x^4}{17920} - \frac{x^2}{112} + 1\right)$$

14.32 problem 34

Internal problem ID [1323]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$8x^2(-x^2+1)\,y''+2x(-13x^2+1)\,y'+(-9x^2+1)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(8*x^2*(1-x^2)*diff(y(x),x$2)+2*x*(1-13*x^2)*diff(y(x),x)+(1-9*x^2)*y(x)=0,y(x),type='s
```

$$y(x)=c_1x^{\frac{1}{4}}\left(1+\frac{1}{2}x^2+\frac{3}{8}x^4+\mathrm{O}\left(x^6\right)\right)+c_2\sqrt{x}\left(1+\frac{5}{9}x^2+\frac{65}{153}x^4+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 52

```
AsymptoticDSolveValue[8*x^2*(1-x^2)*y''[x]+2*x*(1-13*x^2)*y'[x]+(1-9*x^2)*y[x]==0,y[x],{x,0,5
```

$$y(x)\to c_1\sqrt{x}\left(\frac{65x^4}{153}+\frac{5x^2}{9}+1\right)+c_2\sqrt[4]{x}\left(\frac{3x^4}{8}+\frac{x^2}{2}+1\right)$$

14.33 problem 35

Internal problem ID [1324]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x^2+1)\,y''-2x(-x^2+2)\,y'+4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
Order:=6;
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-2*x*(2-x^2)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^4\left(1-2x^2+3x^4+\mathrm{O}\left(x^6\right)\right)+c_2x\left(12+12x^2-36x^4+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 34

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]-2*x*(2-x^2)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-3x^5+x^3+x\right)+c_2\left(3x^8-2x^6+x^4\right)$$

14.34 problem 36

Internal problem ID [1325]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(x^2+3)\,y''+(-x^2+2)\,y'-8yx=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
Order:=6;
dsolve(x*(3+x^2)*diff(y(x),x$2)+(2-x^2)*diff(y(x),x)-8*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^{\frac{1}{3}}\left(1+\frac{11}{18}x^2+\frac{55}{648}x^4+\mathrm{O}\left(x^6\right)\right)+c_2\left(1+\frac{4}{5}x^2+\frac{8}{55}x^4+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 47

```
AsymptoticDSolveValue[x*(3+x^2)*y''[x]+(2-x^2)*y'[x]-8*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt[3]{x}\left(\frac{55x^4}{648}+\frac{11x^2}{18}+1\right)+c_2\left(\frac{8x^4}{55}+\frac{4x^2}{5}+1\right)$$

14.35 problem 37

Internal problem ID [1326]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2\left(-x^2+1\right)y''+x\left(-19x^2+7\right)y'-\left(14x^2+1\right)y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(4*x^2*(1-x^2)*diff(y(x),x$2)+x*(7-19*x^2)*diff(y(x),x)-(1+14*x^2)*y(x)=0,y(x),type='se
```

$$y(x)=\frac{c_2x^{\frac{5}{4}}\left(1+\frac{9}{13}x^2+\frac{51}{91}x^4+\mathrm{O}\left(x^6\right)\right)+c_1\left(1+\frac{1}{2}x^2+\frac{3}{8}x^4+\mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 50

```
AsymptoticDSolveValue[4*x^2*(1-x^2)*y''[x]+x*(7-19*x^2)*y'[x]-(1+14*x^2)*y[x]==0,y[x],{x,0,5}
```

$$y(x)\to c_1\sqrt[4]{x}\left(\frac{51x^4}{91}+\frac{9x^2}{13}+1\right)+\frac{c_2\left(\frac{3x^4}{8}+\frac{x^2}{2}+1\right)}{x}$$

14.36 problem 38

Internal problem ID [1327]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2(-x^2+2)\,y''+x(-11x^2+1)\,y'+(-5x^2+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(3*x^2*(2-x^2)*diff(y(x),x$2)+x*(1-11*x^2)*diff(y(x),x)+(1-5*x^2)*y(x)=0,y(x),type='ser
```

$$y(x)=c_1x^{\frac{1}{3}}\left(1+\frac{4}{11}x^2+\frac{40}{253}x^4+\mathrm{O}\left(x^6\right)\right)+c_2\sqrt{x}\left(1+\frac{3}{8}x^2+\frac{21}{128}x^4+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 52

```
AsymptoticDSolveValue[3*x^2*(2-x^2)*y''[x]+x*(1-11*x^2)*y'[x]+(1-5*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt{x}\left(\frac{21x^4}{128}+\frac{3x^2}{8}+1\right)+c_2\sqrt[3]{x}\left(\frac{40x^4}{253}+\frac{4x^2}{11}+1\right)$$

14.37 problem 39

Internal problem ID [1328]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2+2)\,y'' - x(-7x^2+12)\,y' + (3x^2+7)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)-x*(12-7*x^2)*diff(y(x),x)+(7+3*x^2)*y(x)=0,y(x),type='ser
```

$$y(x) = \sqrt{x}\left(x^3\left(1-\frac{9}{8}x^2+\frac{117}{128}x^4+\mathrm{O}\left(x^6\right)\right)c_1+\left(12+9x^2-\frac{63}{4}x^4+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 58

```
AsymptoticDSolveValue[2*x^2*(2+x^2)*y''[x]-x*(12-7*x^2)*y'[x]+(7+3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{21x^{9/2}}{16}+\frac{3x^{5/2}}{4}+\sqrt{x}\right)+c_2\left(\frac{117x^{15/2}}{128}-\frac{9x^{11/2}}{8}+x^{7/2}\right)$$

14.38 problem 40

Internal problem ID [1329]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x^2+2)\,y''+x(7x^2+4)\,y'-(-3x^2+1)\,y=0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)+x*(4+7*x^2)*diff(y(x),x)-(1-3*x^2)*y(x)=0,y(x),type='seri
```

$$y(x)=\frac{c_1x\left(1-\frac{1}{4}x^2+\frac{7}{80}x^4+\mathrm{O}\left(x^6\right)\right)+c_2\left(1-\frac{1}{8}x^2+\frac{5}{128}x^4+\mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 58

```
AsymptoticDSolveValue[2*x^2*(2+x^2)*y''[x]+x*(4+7*x^2)*y'[x]-(1-3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{5x^{7/2}}{128}-\frac{x^{3/2}}{8}+\frac{1}{\sqrt{x}}\right)+c_2\left(\frac{7x^{9/2}}{80}-\frac{x^{5/2}}{4}+\sqrt{x}\right)$$

14.39 problem 41

Internal problem ID [1330]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(2x^2+1)\,y''+5x(6x^2+1)\,y'-(-40x^2+2)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
dsolve(2*x^2*(1+2*x^2)*diff(y(x),x$2)+5*x*(1+6*x^2)*diff(y(x),x)-(2-40*x^2)*y(x)=0,y(x),type=
```

$$y(x)=\frac{c_2x^{\frac{5}{2}}\left(1-3x^2+\frac{15}{2}x^4+\mathrm{O}\left(x^6\right)\right)+c_1\left(1+2x^2-\frac{20}{3}x^4+\mathrm{O}\left(x^6\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 46

```
AsymptoticDSolveValue[2*x^2*(1+2*x^2)*y''[x]+5*x*(1+6*x^2)*y'[x]-(2-40*x^2)*y[x]==0,y[x],{x,0
```

$$y(x)\to c_1\sqrt{x}\left(\frac{15x^4}{2}-3x^2+1\right)+\frac{c_2\left(-\frac{20x^4}{3}+2x^2+1\right)}{x^2}$$

14.40 problem 42

Internal problem ID [1331]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2(x^2+1)\,y''+5x(x^2+1)\,y'-(-5x^2+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
dsolve(3*x^2*(1+x^2)*diff(y(x),x$2)+5*x*(1+x^2)*diff(y(x),x)-(1-5*x^2)*y(x)=0,y(x),type='seri
```

$$y(x)=\frac{c_2x^{\frac{4}{3}}\left(1-\frac{3}{10}x^2+\frac{39}{320}x^4+\mathrm{O}\left(x^6\right)\right)+c_1\left(1-\frac{3}{2}x^2+\frac{15}{32}x^4+\mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 50

```
AsymptoticDSolveValue[3*x^2*(1+x^2)*y''[x]+5*x*(1+x^2)*y'[x]-(1-5*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt[3]{x}\left(\frac{39x^4}{320}-\frac{3x^2}{10}+1\right)+\frac{c_2\left(\frac{15x^4}{32}-\frac{3x^2}{2}+1\right)}{x}$$

14.41 problem 43

Internal problem ID [1332]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(x^2+1)\,y''+(7x^2+4)\,y'+8yx=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
Order:=6;
dsolve(x*(1+x^2)*diff(y(x),x$2)+(4+7*x^2)*diff(y(x),x)+8*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1\left(1-\frac{4}{5}x^2+\frac{24}{35}x^4+\mathrm{O}\left(x^6\right)\right)+\frac{c_2\left(12-6x^2+\frac{9}{2}x^4+\mathrm{O}\left(x^6\right)\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 42

```
AsymptoticDSolveValue[x*(1+x^2)*y''[x]+(4+7*x^2)*y'[x]+8*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{1}{x^3}+\frac{3x}{8}-\frac{1}{2x}\right)+c_2\left(\frac{24x^4}{35}-\frac{4x^2}{5}+1\right)$$

14.42 problem 44

Internal problem ID [1333]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+2)\,y''+x(x^2+3)\,y'-y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*(2+x^2)*diff(y(x),x$2)+x*(3+x^2)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\frac{c_2x^{\frac{3}{2}}\left(1-\frac{1}{56}x^2+\frac{25}{9856}x^4+\mathrm{O}\left(x^6\right)\right)+c_1\left(1-\frac{1}{2}x^2+\frac{1}{40}x^4+\mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 50

```
AsymptoticDSolveValue[x^2*(2+x^2)*y''[x]+x*(3+x^2)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt{x}\left(\frac{25x^4}{9856}-\frac{x^2}{56}+1\right)+\frac{c_2\left(\frac{x^4}{40}-\frac{x^2}{2}+1\right)}{x}$$

14.43 problem 45

Internal problem ID [1334]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x^2+1)\,y''+x(8x^2+3)\,y'-(-4x^2+3)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
Order:=6;
dsolve(2*x^2*(1+x^2)*diff(y(x),x$2)+x*(3+8*x^2)*diff(y(x),x)-(3-4*x^2)*y(x)=0,y(x),type='seri
```

$$y(x)=\frac{c_2x^{\frac{5}{2}}\left(1-\frac{2}{3}x^2+\frac{20}{39}x^4+\mathrm{O}\left(x^6\right)\right)+c_1\left(1-\frac{1}{4}x^2+\frac{5}{32}x^4+\mathrm{O}\left(x^6\right)\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 48

```
AsymptoticDSolveValue[2*x^2*(1+x^2)*y''[x]+x*(3+8*x^2)*y'[x]-(3-4*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1x\left(\frac{20x^4}{39}-\frac{2x^2}{3}+1\right)+\frac{c_2\left(\frac{5x^4}{32}-\frac{x^2}{4}+1\right)}{x^{3/2}}$$

14.44 problem 46

Internal problem ID [1335]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' + 3x(x^2+3)\,y' - (-5x^2+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(9*x^2*diff(y(x),x$2)+3*x*(3+x^2)*diff(y(x),x)-(1-5*x^2)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = \frac{c_2x^{\frac{2}{3}}\left(1-\frac{1}{8}x^2+\frac{1}{112}x^4+\mathrm{O}\left(x^6\right)\right)+c_1\left(1-\frac{1}{6}x^2+\frac{1}{72}x^4+\mathrm{O}\left(x^6\right)\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 52

```
AsymptoticDSolveValue[9*x^2*y''[x]+3*x*(3+x^2)*y'[x]-(1-5*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt[3]{x}\left(\frac{x^4}{112}-\frac{x^2}{8}+1\right)+\frac{c_2\left(\frac{x^4}{72}-\frac{x^2}{6}+1\right)}{\sqrt[3]{x}}$$

14.45 problem 47

Internal problem ID [1336]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{6x^2y'' + x(6x^2+1)\,y' + (9x^2+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(6*x^2*diff(y(x),x$2)+x*(1+6*x^2)*diff(y(x),x)+(1+9*x^2)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = c_1 x^{\frac{1}{3}}\left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\sqrt{x}\left(1 - \frac{6}{13}x^2 + \frac{36}{325}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 52

```
AsymptoticDSolveValue[6*x^2*y''[x]+x*(1+6*x^2)*y'[x]+(1+9*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{36x^4}{325} - \frac{6x^2}{13} + 1\right) + c_2\sqrt[3]{x}\left(\frac{x^4}{8} - \frac{x^2}{2} + 1\right)$$

14.46 problem 48

Internal problem ID [1337]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 48.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2(x^2+8)\,y''+7x(x^2+2)\,y'-(-9x^2+2)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*(8+x^2)*diff(y(x),x$2)+7*x*(2+x^2)*diff(y(x),x)-(2-9*x^2)*y(x)=0,y(x),type='series
```

$$y(x)=\frac{c_2x^{\frac{5}{4}}\left(1-\frac{13}{64}x^2+\frac{273}{8192}x^4+\mathrm{O}\left(x^6\right)\right)+c_1\left(1-\frac{1}{3}x^2+\frac{2}{33}x^4+\mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 50

```
AsymptoticDSolveValue[x^2*(8+x^2)*y''[x]+7*x*(2+x^2)*y'[x]-(2-9*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt[4]{x}\left(\frac{273x^4}{8192}-\frac{13x^2}{64}+1\right)+\frac{c_2\left(\frac{2x^4}{33}-\frac{x^2}{3}+1\right)}{x}$$

14.47 problem 49

Internal problem ID [1338]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2(x^2+1)\,y''+3x(13x^2+3)\,y'-(-25x^2+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(9*x^2*(1+x^2)*diff(y(x),x$2)+3*x*(3+13*x^2)*diff(y(x),x)-(1-25*x^2)*y(x)=0,y(x),type='
```

$$y(x)=\frac{c_2x^{\frac{2}{3}}\left(1-\frac{3}{4}x^2+\frac{9}{14}x^4+\mathrm{O}\left(x^6\right)\right)+c_1\left(1-\frac{2}{3}x^2+\frac{5}{9}x^4+\mathrm{O}\left(x^6\right)\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 52

```
AsymptoticDSolveValue[9*x^2*(1+x^2)*y''[x]+3*x*(3+13*x^2)*y'[x]-(1-25*x^2)*y[x]==0,y[x],{x,0,
```

$$y(x)\to c_1\sqrt[3]{x}\left(\frac{9x^4}{14}-\frac{3x^2}{4}+1\right)+\frac{c_2\left(\frac{5x^4}{9}-\frac{2x^2}{3}+1\right)}{\sqrt[3]{x}}$$

14.48 problem 50

Internal problem ID [1339]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2+1)\,y''+4x(6x^2+1)\,y'-(-25x^2+1)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+4*x*(1+6*x^2)*diff(y(x),x)-(1-25*x^2)*y(x)=0,y(x),type='s
```

$$y(x)=\frac{c_1x\left(1-\frac{3}{2}x^2+\frac{15}{8}x^4+\mathrm{O}\left(x^6\right)\right)+c_2\left(1-2x^2+\frac{8}{3}x^4+\mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 56

```
AsymptoticDSolveValue[4*x^2*(1+x^2)*y''[x]+4*x*(1+6*x^2)*y'[x]-(1-25*x^2)*y[x]==0,y[x],{x,0,5
```

$$y(x)\to c_1\left(\frac{8x^{7/2}}{3}-2x^{3/2}+\frac{1}{\sqrt{x}}\right)+c_2\left(\frac{15x^{9/2}}{8}-\frac{3x^{5/2}}{2}+\sqrt{x}\right)$$

14.49 problem 51

Internal problem ID [1340]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 51.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{8x^2(2x^2+1)\,y''+2x(34x^2+5)\,y'-(-30x^2+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(8*x^2*(1+2*x^2)*diff(y(x),x$2)+2*x*(5+34*x^2)*diff(y(x),x)-(1-30*x^2)*y(x)=0,y(x),type
```

$$y(x)=\frac{c_2x^{\frac{3}{4}}\left(1-x^2+\frac{3}{2}x^4+\mathrm{O}\left(x^6\right)\right)+c_1\left(1-\frac{2}{5}x^2+\frac{36}{65}x^4+\mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 50

```
AsymptoticDSolveValue[8*x^2*(1+2*x^2)*y''[x]+2*x*(5+34*x^2)*y'[x]-(1-30*x^2)*y[x]==0,y[x],{x,
```

$$y(x)\to c_1\sqrt[4]{x}\left(\frac{3x^4}{2}-x^2+1\right)+\frac{c_2\left(\frac{36x^4}{65}-\frac{2x^2}{5}+1\right)}{\sqrt{x}}$$

14.50 problem 61

Internal problem ID [1341]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 61.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x+1)\,y'' - x(-3x+1)\,y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*x^2*(1+x)*diff(y(x),x$2)-x*(1-3*x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(-x^5 + x^4 - x^3 + x^2 - x + 1\right)\left(c_1\sqrt{x} + c_2 x\right) + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 58

```
AsymptoticDSolveValue[2*x^2*(1+x)*y''[x]-x*(1-3*x)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x\left(-x^5 + x^4 - x^3 + x^2 - x + 1\right) + c_2\sqrt{x}\left(-x^5 + x^4 - x^3 + x^2 - x + 1\right)$$

14.51 problem 62

Internal problem ID [1342]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 62.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$6x^2(2x^2+1)\,y''+x(50x^2+1)\,y'+(30x^2+1)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(6*x^2*(1+2*x^2)*diff(y(x),x$2)+x*(1+50*x^2)*diff(y(x),x)+(1+30*x^2)*y(x)=0,y(x),type='
```

$$y(x)=(4x^4-2x^2+1)\,x^{\frac{1}{3}}\left(c_2x^{\frac{1}{6}}+c_1\right)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 44

```
AsymptoticDSolveValue[6*x^2*(1+2*x^2)*y''[x]+x*(1+50*x^2)*y'[x]+(1+30*x^2)*y[x]==0,y[x],{x,0,
```

$$y(x)\to c_1\sqrt{x}(4x^4-2x^2+1)+c_2\sqrt[3]{x}(4x^4-2x^2+1)$$

14.52 problem 63

Internal problem ID [1343]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 63.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$28x^2(-3x+1)\,y''-7x(5+9x)\,y'+7(2+9x)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
Order:=6;
dsolve(28*x^2*(1-3*x)*diff(y(x),x$2)-7*x*(5+9*x)*diff(y(x),x)+7*(2+9*x)*y(x)=0,y(x),type='ser
```

$$y(x)=\left(243x^5+81x^4+27x^3+9x^2+3x+1\right)\left(c_1x^{\frac{1}{4}}+x^2c_2\right)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 68

```
AsymptoticDSolveValue[28*x^2*(1-3*x)*y''[x]-7*x*(5+9*x)*y'[x]+7*(2+9*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(243x^5+81x^4+27x^3+9x^2+3x+1\right)x^2+c_2\left(243x^5+81x^4+27x^3+9x^2+3x+1\right)\sqrt[4]{x}$$

14.53 problem 64

Internal problem ID [1344]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 64.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(x+5)\,y''+9x(5+9x)\,y'-(5-8x)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(9*x^2*(5+x)*diff(y(x),x$2)+9*x*(5+9*x)*diff(y(x),x)-(5-8*x)*y(x)=0,y(x),type='series',
```

$$y(x) = \frac{c_2x^{\frac{2}{3}}\left(1-\frac{11}{25}x+\frac{11}{50}x^2-\frac{1}{10}x^3+\frac{29}{700}x^4-\frac{4727}{297500}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1+x-\frac{1}{2}x^2+\frac{17}{70}x^3-\frac{187}{1750}x^4+\frac{24497}{568750}x^5+\right.}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 86

```
AsymptoticDSolveValue[9*x^2*(5+x)*y''[x]+9*x*(5+9*x)*y'[x]-(5-8*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt[3]{x}\left(-\frac{4727x^5}{297500}+\frac{29x^4}{700}-\frac{x^3}{10}+\frac{11x^2}{50}-\frac{11x}{25}+1\right)+\frac{c_2\left(\frac{24497x^5}{568750}-\frac{187x^4}{1750}+\frac{17x^3}{70}-\frac{x^2}{2}+x+1\right)}{\sqrt[3]{x}}$$

14.54 problem 65

Internal problem ID [1345]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 65.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{8x^2(-x^2+2)\,y''+2x(-21x^2+10)\,y'-(35x^2+2)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(8*x^2*(2-x^2)*diff(y(x),x$2)+2*x*(10-21*x^2)*diff(y(x),x)-(2+35*x^2)*y(x)=0,y(x),type=
```

$$y(x)=\frac{\left(1+\frac{1}{2}x^2+\frac{1}{4}x^4\right)\left(x^{\frac{3}{4}}c_2+c_1\right)}{\sqrt{x}}+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 52

```
AsymptoticDSolveValue[8*x^2*(2-x^2)*y''[x]+2*x*(10-21*x^2)*y'[x]-(2+35*x^2)*y[x]==0,y[x],{x,0
```

$$y(x)\to c_1\sqrt[4]{x}\left(\frac{x^4}{4}+\frac{x^2}{2}+1\right)+\frac{c_2\left(\frac{x^4}{4}+\frac{x^2}{2}+1\right)}{\sqrt{x}}$$

14.55 problem 66

Internal problem ID [1346]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 66.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(x^2+3x+1)\,y''-4x(-3x^2-3x+1)\,y'+3(x^2-x+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(4*x^2*(1+3*x+x^2)*diff(y(x),x$2)-4*x*(1-3*x-3*x^2)*diff(y(x),x)+3*(1-x+x^2)*y(x)=0,y(x
```

$$y(x)=\sqrt{x}\left(x\left(1-3x+8x^2-21x^3+55x^4-144x^5+\mathrm{O}\left(x^6\right)\right)c_1 + \left(1-6x+17x^2-45x^3+118x^4-309x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 78

```
AsymptoticDSolveValue[4*x^2*(1+3*x+x^2)*y''[x]-4*x*(1-3*x-3*x^2)*y'[x]+3*(1-x+x^2)*y[x]==0,y[
```

$$y(x)\to c_1\left(55x^{9/2}-21x^{7/2}+8x^{5/2}-3x^{3/2}+\sqrt{x}\right)+c_2\left(55x^{11/2}-21x^{9/2}+8x^{7/2}-3x^{5/2}+x^{3/2}\right)$$

14.56 problem 67

Internal problem ID [1347]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 67.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2(x+1)^2 y'' - x(-11x^2 - 10x + 1) y' + (5x^2 + 1) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;
dsolve(3*x^2*(1+x)^2*diff(y(x),x$2)-x*(1-10*x-11*x^2)*diff(y(x),x)+(1+5*x^2)*y(x)=0,y(x),type
```

$$y(x) = (-6x^5 + 5x^4 - 4x^3 + 3x^2 - 2x + 1)\left(c_1 x^{\frac{1}{3}} + c_2 x\right) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 66

```
AsymptoticDSolveValue[3*x^2*(1+x)^2*y''[x]-x*(1-10*x-11*x^2)*y'[x]+(1+5*x^2)*y[x]==0,y[x],{x,
```

$$y(x) \to c_1 x(-6x^5 + 5x^4 - 4x^3 + 3x^2 - 2x + 1) + c_2 \sqrt[3]{x}(-6x^5 + 5x^4 - 4x^3 + 3x^2 - 2x + 1)$$

14.57 problem 68

Internal problem ID [1348]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.5 THE METHOD OF FROBENIUS I. Exercises 7.5. Page 358
Problem number: 68.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2+2x+3)\,y''-x(-15x^2-14x+3)\,y'+(7x^2+3)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(4*x^2*(3+2*x+x^2)*diff(y(x),x$2)-x*(3-14*x-15*x^2)*diff(y(x),x)+(3+7*x^2)*y(x)=0,y(x),
```

$$y(x)=\left(1-\frac{2}{3}x+\frac{1}{9}x^2+\frac{4}{27}x^3-\frac{11}{81}x^4+\frac{10}{243}x^5\right)\left(c_1x^{\frac{1}{4}}+c_2x\right)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 86

```
AsymptoticDSolveValue[4*x^2*(3+2*x+x^2)*y''[x]-x*(3-14*x-15*x^2)*y'[x]+(3+7*x^2)*y[x]==0,y[x]
```

$$y(x)\to c_1x\left(\frac{10x^5}{243}-\frac{11x^4}{81}+\frac{4x^3}{27}+\frac{x^2}{9}-\frac{2x}{3}+1\right)+c_2\sqrt[4]{x}\left(\frac{10x^5}{243}-\frac{11x^4}{81}+\frac{4x^3}{27}+\frac{x^2}{9}-\frac{2x}{3}+1\right)$$

15 Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374

15.1 problem Example 7.6.1 page 367

Internal problem ID [1349]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: Example 7.6.1 page 367.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2-2x+1)\,y''-x(x+3)\,y'+(x+4)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*(1-2*x+x^2)*diff(y(x),x$2)-x*(3+x)*diff(y(x),x)+(4+x)*y(x)=0,y(x),type='series',x=
```

$$y(x)=x^2\left((c_2\ln(x)+c_1)\left(1+5x+17x^2+\frac{143}{3}x^3+\frac{355}{3}x^4+\frac{4043}{15}x^5+\mathrm{O}(x^6)\right)+\left((-3)\,x-\frac{29}{2}x^2-\frac{859}{18}x^3-\frac{4693}{36}x^4-\frac{285181}{900}x^5+\mathrm{O}(x^6)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 118

```
AsymptoticDSolveValue[x^2*(1-2*x+x^2)*y''[x]-x*(3+x)*y'[x]+(4+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{4043x^5}{15}+\frac{355x^4}{3}+\frac{143x^3}{3}+17x^2+5x+1\right)x^2+c_2\left(\left(-\frac{285181x^5}{900}-\frac{4693x^4}{36}-\frac{859x^3}{18}-\frac{29x^2}{2}-3x\right)x^2+\left(\frac{4043x^5}{15}+\frac{355x^4}{3}+\frac{143x^3}{3}+17x^2+5x+1\right)x^2\log(x)\right)$$

15.2 problem Example 7.6.2 page 369

Internal problem ID [1350]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: Example 7.6.2 page 369.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(2+x)\,y''+5y'x^2+(x+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(2*x^2*(2+x)*diff(y(x),x$2)+5*x^2*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\sqrt{x}\left((c_2\ln(x)+c_1)\left(1-\frac{3}{4}x+\frac{15}{32}x^2-\frac{35}{128}x^3+\frac{315}{2048}x^4-\frac{693}{8192}x^5+\mathrm{O}\left(x^6\right)\right)+\left(\frac{1}{4}x-\frac{13}{64}x^2+\frac{101}{768}x^3-\frac{641}{8192}x^4+\frac{7303}{163840}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 134

```
AsymptoticDSolveValue[2*x^2*(2+x)*y''[x]+5*x^2*y'[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt{x}\left(-\frac{693x^5}{8192}+\frac{315x^4}{2048}-\frac{35x^3}{128}+\frac{15x^2}{32}-\frac{3x}{4}+1\right)+c_2\left(\sqrt{x}\left(\frac{7303x^5}{163840}-\frac{641x^4}{8192}+\frac{101x^3}{768}-\frac{13x^2}{64}+\frac{x}{4}\right)+\sqrt{x}\left(-\frac{693x^5}{8192}+\frac{315x^4}{2048}-\frac{35x^3}{128}+\frac{15x^2}{32}-\frac{3x}{4}+1\right)\log(x)\right)$$

15.3 problem Example 7.6.3 page 370

Internal problem ID [1351]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: Example 7.6.3 page 370.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(-x^2+2)\,y''-2x(2x^2+1)\,y'+(-2x^2+2)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*(2-x^2)*diff(y(x),x$2)-2*x*(1+2*x^2)*diff(y(x),x)+(2-2*x^2)*y(x)=0,y(x),type='seri
```

$$y(x)=x\left((c_2\ln(x)+c_1)\left(1+\frac{3}{4}x^2+\frac{15}{32}x^4+\mathrm{O}\left(x^6\right)\right)+\left(-\frac{1}{8}x^2-\frac{13}{128}x^4+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 65

```
AsymptoticDSolveValue[x^2*(2-x^2)*y''[x]-2*x*(1+2*x^2)*y'[x]+(2-2*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1x\left(\frac{15x^4}{32}+\frac{3x^2}{4}+1\right)+c_2\left(x\left(-\frac{13x^4}{128}-\frac{x^2}{8}\right)+x\left(\frac{15x^4}{32}+\frac{3x^2}{4}+1\right)\log(x)\right)$$

15.4 problem Example 7.6.4 page 372

Internal problem ID [1352]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: Example 7.6.4 page 372.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(-x+5)\,y' + (-4x+9)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(5-x)*diff(y(x),x)+(9-4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^3\left((c_2 \ln(x) + c_1)\left(1 + x + \mathrm{O}\left(x^6\right)\right) + \left((-3)\,x - \frac{1}{4}x^2 + \frac{1}{36}x^3 - \frac{1}{288}x^4 + \frac{1}{2400}x^5 + \mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 62

```
AsymptoticDSolveValue[x^2*y''[x]-x*(5-x)*y'[x]+(9-4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1(x+1)x^3 + c_2\left((x+1)x^3\log(x) + \left(\frac{x^5}{2400} - \frac{x^4}{288} + \frac{x^3}{36} - \frac{x^2}{4} - 3x\right)x^3\right)$$

15.5 problem 1

Internal problem ID [1353]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(1-x)\,y' + \left(-x^2+1\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 75

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)-x*(1-x)*diff(y(x),x)+(1-x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x\Bigg((c_2\ln(x)+c_1)\left(1 - x + \frac{3}{4}x^2 - \frac{13}{36}x^3 + \frac{79}{576}x^4 - \frac{67}{1600}x^5 + \frac{5593}{518400}x^6 - \frac{60859}{25401600}x^7 + \mathrm{O}\left(x^8\right)\right) + \left(x - x^2 + \frac{65}{108}x^3 - \frac{895}{3456}x^4 + \frac{12547}{144000}x^5 - \frac{41729}{1728000}x^6 + \frac{10121677}{1778112000}x^7 + \mathrm{O}\left(x^8\right)\right)c_2\Bigg)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 154

```
AsymptoticDSolveValue[x^2*y''[x]-x*(1-x)*y'[x]+(1-x^2)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 x \left(-\frac{60859x^7}{25401600} + \frac{5593x^6}{518400} - \frac{67x^5}{1600} + \frac{79x^4}{576} - \frac{13x^3}{36} + \frac{3x^2}{4} - x + 1\right) + c_2 \left(x\left(\frac{10121677x^7}{1778112000} - \frac{41729x^6}{1728000} + \frac{12547x^5}{144000} - \frac{895x^4}{3456} + \frac{65x^3}{108} - x^2 + x\right) + x\left(-\frac{60859x^7}{25401600} + \frac{5593x^6}{518400} - \frac{67x^5}{1600} + \frac{79x^4}{576} - \frac{13x^3}{36} + \frac{3x^2}{4} - x + 1\right)\log(x)\right)$$

15.6 problem 2

Internal problem ID [1354]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2(2x^2+x+1)\,y''+x(7x^2+6x+3)\,y'+(-3x^2+6x+1)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 81

```
Order:=8;
dsolve(x^2*(1+x+2*x^2)*diff(y(x),x$2)+x*(3+6*x+7*x^2)*diff(y(x),x)+(1+6*x-3*x^2)*y(x)=0,y(x),
```

$$y(x) = \frac{(c_2\ln(x)+c_1)\left(1-2x+\frac{9}{2}x^2-\frac{20}{3}x^3+\frac{173}{24}x^4-\frac{93}{20}x^5-\frac{419}{720}x^6+\frac{6697}{1260}x^7+\mathrm{O}(x^8)\right)+\left(x-\frac{15}{4}x^2+\frac{133}{18}x^3-\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 162

```
AsymptoticDSolveValue[x^2*(1+x+2*x^2)*y''[x]+x*(3+6*x+7*x^2)*y'[x]+(1+6*x-3*x^2)*y[x]==0,y[x]
```

$$y(x)\to\frac{c_1\left(\frac{6697x^7}{1260}-\frac{419x^6}{720}-\frac{93x^5}{20}+\frac{173x^4}{24}-\frac{20x^3}{3}+\frac{9x^2}{2}-2x+1\right)}{x}+c_2\left(\frac{-\frac{125221x^7}{29400}-\frac{70949x^6}{14400}+\frac{4217x^5}{400}-\frac{3077x^4}{288}+\frac{133x^3}{18}-\frac{15x^2}{4}+x}{x}+\frac{\left(\frac{6697x^7}{1260}-\frac{419x^6}{720}-\frac{93x^5}{20}+\frac{173x^4}{24}-\frac{20x^3}{3}+\frac{9x^2}{2}-2x+1\right)\log(x)}{x}\right)$$

15.7 problem 3

Internal problem ID [1355]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$x^2(x^2+2x+1)\,y''+x(4x^2+3x+1)\,y'-x(1-2x)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
Order:=8;
dsolve(x^2*(1+2*x+x^2)*diff(y(x),x$2)+x*(1+3*x+4*x^2)*diff(y(x),x)-x*(1-2*x)*y(x)=0,y(x),type
```

$$y(x)=(c_2\ln(x)+c_1)\left(1+x-x^2+\frac{1}{3}x^3+\frac{1}{3}x^4-\frac{11}{15}x^5+\frac{37}{45}x^6-\frac{209}{315}x^7+\mathrm{O}(x^8)\right)+\left((-3)\,x+\frac{1}{2}x^2+\frac{31}{18}x^3-\frac{91}{36}x^4+\frac{1897}{900}x^5-\frac{301}{300}x^6-\frac{3901}{14700}x^7+\mathrm{O}(x^8)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 145

```
AsymptoticDSolveValue[x^2*(1+2*x+x^2)*y''[x]+x*(1+3*x+4*x^2)*y'[x]-x*(1-2*x)*y[x]==0,y[x],{x,
```

$$y(x)\to c_1\left(-\frac{209x^7}{315}+\frac{37x^6}{45}-\frac{11x^5}{15}+\frac{x^4}{3}+\frac{x^3}{3}-x^2+x+1\right)+c_2\left(-\frac{3901x^7}{14700}-\frac{301x^6}{300}+\frac{1897x^5}{900}-\frac{91x^4}{36}+\frac{31x^3}{18}+\frac{x^2}{2}+\left(-\frac{209x^7}{315}+\frac{37x^6}{45}-\frac{11x^5}{15}+\frac{x^4}{3}+\frac{x^3}{3}-x^2+x+1\right)\log(x)-3x\right)$$

15.8 problem 4

Internal problem ID [1356]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2+x+1)\,y''+12x^2(x+1)\,y'+(3x^2+3x+1)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 81

```
Order:=8;
dsolve(4*x^2*(1+x+x^2)*diff(y(x),x$2)+12*x^2*(1+x)*diff(y(x),x)+(1+3*x+3*x^2)*y(x)=0,y(x),typ
```

$$y(x)=\sqrt{x}\left((c_2\ln(x)+c_1)\left(1-2x+\frac{5}{2}x^2-2x^3+\frac{5}{8}x^4+\frac{17}{20}x^5-\frac{121}{80}x^6+x^7+\mathrm{O}(x^8)\right)\right.$$
$$\left.+\left(x-\frac{9}{4}x^2+\frac{17}{6}x^3-\frac{205}{96}x^4+\frac{481}{1200}x^5+\frac{2109}{1600}x^6-\frac{1063}{560}x^7+\mathrm{O}(x^8)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 156

```
AsymptoticDSolveValue[4*x^2*(1+x+x^2)*y''[x]+12*x^2*(1+x)*y'[x]+(1+3*x+3*x^2)*y[x]==0,y[x],{x
```

$$y(x)\to c_1\sqrt{x}\left(x^7-\frac{121x^6}{80}+\frac{17x^5}{20}+\frac{5x^4}{8}-2x^3+\frac{5x^2}{2}-2x+1\right)$$
$$+c_2\left(\sqrt{x}\left(-\frac{1063x^7}{560}+\frac{2109x^6}{1600}+\frac{481x^5}{1200}-\frac{205x^4}{96}+\frac{17x^3}{6}-\frac{9x^2}{4}+x\right)\right.$$
$$\left.+\sqrt{x}\left(x^7-\frac{121x^6}{80}+\frac{17x^5}{20}+\frac{5x^4}{8}-2x^3+\frac{5x^2}{2}-2x+1\right)\log(x)\right)$$

15.9 problem 5

Internal problem ID [1357]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+x+1)\,y''-x(-2x^2-4x+1)\,y'+y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 75

```
Order:=8;
dsolve(x^2*(1+x+x^2)*diff(y(x),x$2)-x*(1-4*x-2*x^2)*diff(y(x),x)+y(x)=0,y(x),type='series',x=
```

$$y(x)=x\left((c_2\ln(x)+c_1)\left(1-4x+\frac{19}{2}x^2-\frac{49}{3}x^3+\frac{515}{24}x^4-\frac{319}{15}x^5+\frac{10093}{720}x^6-\frac{647}{360}x^7+\mathrm{O}\left(x^8\right)\right)+\left(3x-\frac{43}{4}x^2+\frac{208}{9}x^3-\frac{10379}{288}x^4+\frac{76321}{1800}x^5-\frac{172499}{4800}x^6+\frac{39091}{2400}x^7+\mathrm{O}\left(x^8\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 158

```
AsymptoticDSolveValue[x^2*(1+x+x^2)*y''[x]-x*(1-4*x-2*x^2)*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 x \left(-\frac{647x^7}{360} + \frac{10093x^6}{720} - \frac{319x^5}{15} + \frac{515x^4}{24} - \frac{49x^3}{3} + \frac{19x^2}{2} - 4x + 1\right) + c_2 \left(x \left(\frac{39091x^7}{2400} - \frac{172499x^6}{4800} + \frac{76321x^5}{1800} - \frac{10379x^4}{288} + \frac{208x^3}{9} - \frac{43x^2}{4} + 3x\right) + x \left(-\frac{647x^7}{360} + \frac{10093x^6}{720} - \frac{319x^5}{15} + \frac{515x^4}{24} - \frac{49x^3}{3} + \frac{19x^2}{2} - 4x + 1\right) \log(x)\right)$$

15.10 problem 6

Internal problem ID [1358]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' + 3x(-2x^2 + 3x + 5)\,y' + (-14x^2 + 12x + 1)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 81

```
Order:=8;
dsolve(9*x^2*diff(y(x),x$2)+3*x*(5+3*x-2*x^2)*diff(y(x),x)+(1+12*x-14*x^2)*y(x)=0,y(x),type='
```

$$y(x) = \frac{(c_2\ln(x) + c_1)\left(1 - x + \frac{5}{6}x^2 - \frac{1}{2}x^3 + \frac{19}{72}x^4 - \frac{43}{360}x^5 + \frac{319}{6480}x^6 - \frac{167}{9072}x^7 + \mathrm{O}(x^8)\right) + \left(x - \frac{11}{12}x^2 + \frac{25}{36}x^3 - \frac{113}{288}\right.}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 168

```
AsymptoticDSolveValue[9*x^2*y''[x]+3*x*(5+3*x-2*x^2)*y'[x]+(1+12*x-14*x^2)*y[x]==0,y[x],{x,0,
```

$$y(x) \to \frac{c_1\left(-\frac{167x^7}{9072} + \frac{319x^6}{6480} - \frac{43x^5}{360} + \frac{19x^4}{72} - \frac{x^3}{2} + \frac{5x^2}{6} - x + 1\right)}{\sqrt[3]{x}} + c_2\left(\frac{\frac{126647x^7}{3810240} - \frac{32773x^6}{388800} + \frac{4211x^5}{21600} - \frac{113x^4}{288} + \frac{25x^3}{36} - \frac{11x^2}{12} + x}{\sqrt[3]{x}} + \frac{\left(-\frac{167x^7}{9072} + \frac{319x^6}{6480} - \frac{43x^5}{360} + \frac{19x^4}{72} - \frac{x^3}{2} + \frac{5x^2}{6} - x + 1\right)\log(x)}{\sqrt[3]{x}}\right)$$

15.11 problem 7

Internal problem ID [1359]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(x^2+x+1)\,y' + x(2-x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 71

```
Order:=8;
dsolve(x^2*diff(y(x),x$2)+x*(1+x+x^2)*diff(y(x),x)+x*(2-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 - 2x + \frac{7}{4}x^2 - \frac{7}{9}x^3 + \frac{77}{576}x^4 + \frac{217}{7200}x^5 - \frac{8813}{518400}x^6 + \frac{143}{453600}x^7 + \mathrm{O}(x^8)\right) + \left(3x - \frac{15}{4}x^2 + \frac{239}{108}x^3 - \frac{2021}{3456}x^4 - \frac{1241}{54000}x^5 + \frac{93859}{1728000}x^6 - \frac{311177}{42336000}x^7 + \mathrm{O}(x^8)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 153

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1+x+x^2)*y'[x]+x*(2-x)*y[x]==0,y[x],{x,0,7}]
```

$$
\begin{aligned}
y(x) \to\; & c_1\left(\frac{143x^7}{453600}-\frac{8813x^6}{518400}+\frac{217x^5}{7200}+\frac{77x^4}{576}-\frac{7x^3}{9}+\frac{7x^2}{4}-2x+1\right)\\
& + c_2\left(-\frac{311177x^7}{42336000}+\frac{93859x^6}{1728000}-\frac{1241x^5}{54000}-\frac{2021x^4}{3456}+\frac{239x^3}{108}-\frac{15x^2}{4}\right.\\
& \left.+\left(\frac{143x^7}{453600}-\frac{8813x^6}{518400}+\frac{217x^5}{7200}+\frac{77x^4}{576}-\frac{7x^3}{9}+\frac{7x^2}{4}-2x+1\right)\log(x)+3x\right)
\end{aligned}
$$

15.12 problem 8

Internal problem ID [1360]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1+2x)\,y''+x\left(3x^2+14x+5\right)y'+\left(12x^2+18x+4\right)y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 79

```
Order:=8;
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(5+14*x+3*x^2)*diff(y(x),x)+(4+18*x+12*x^2)*y(x)=0,y(x),t
```

$$y(x) = \frac{(c_2\ln(x)+c_1)\left(1-2x+\frac{5}{2}x^2-3x^3+\frac{33}{8}x^4-\frac{129}{20}x^5+\frac{867}{80}x^6-\frac{1059}{56}x^7+\mathrm{O}\left(x^8\right)\right)+\left(\frac{3}{4}x^2-\frac{13}{6}x^3+\frac{407}{96}x^4-\right.}{x^2}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 157

```
AsymptoticDSolveValue[x^2*(1+2*x)*y''[x]+x*(5+14*x+3*x^2)*y'[x]+(4+18*x+12*x^2)*y[x]==0,y[x],
```

$$y(x)\to\frac{c_1\left(-\frac{1059x^7}{56}+\frac{867x^6}{80}-\frac{129x^5}{20}+\frac{33x^4}{8}-3x^3+\frac{5x^2}{2}-2x+1\right)}{x^2}+c_2\left(\frac{-\frac{559033x^7}{23520}+\frac{63851x^6}{4800}-\frac{9047x^5}{1200}+\frac{407x^4}{96}-\frac{13x^3}{6}+\frac{3x^2}{4}}{x^2}+\frac{\left(-\frac{1059x^7}{56}+\frac{867x^6}{80}-\frac{129x^5}{20}+\frac{33x^4}{8}-3x^3+\frac{5x^2}{2}-2x+1\right)\log(x)}{x^2}\right)$$

15.13 problem 9

Internal problem ID [1361]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 2x(x^2 + x + 4) y' + (3x^2 + 5x + 1) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 81

```
Order:=8;
dsolve(4*x^2*diff(y(x),x$2)+2*x*(4+x+x^2)*diff(y(x),x)+(1+5*x+3*x^2)*y(x)=0,y(x),type='series
```

$$y(x) = \frac{(c_2 \ln(x) + c_1)\left(1 - x + \frac{1}{4}x^2 + \frac{1}{18}x^3 - \frac{37}{1152}x^4 - \frac{17}{28800}x^5 + \frac{593}{259200}x^6 - \frac{1913}{12700800}x^7 + \mathrm{O}(x^8)\right) + \left(\frac{3}{2}x - \frac{13}{16}x^2 + \right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 172

```
AsymptoticDSolveValue[4*x^2*y''[x]+2*x*(4+x+x^2)*y'[x]+(1+5*x+3*x^2)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to \frac{c_1\left(-\frac{1913x^7}{12700800} + \frac{593x^6}{259200} - \frac{17x^5}{28800} - \frac{37x^4}{1152} + \frac{x^3}{18} + \frac{x^2}{4} - x + 1\right)}{\sqrt{x}} + c_2\left(\frac{\frac{982189x^7}{889056000} - \frac{98531x^6}{20736000} - \frac{19507x^5}{1728000} + \frac{1103x^4}{13824} + \frac{x^3}{54} - \frac{13x^2}{16} + \frac{3x}{2}}{\sqrt{x}} + \frac{\left(-\frac{1913x^7}{12700800} + \frac{593x^6}{259200} - \frac{17x^5}{28800} - \frac{37x^4}{1152} + \frac{x^3}{18} + \frac{x^2}{4} - x + 1\right)\log(x)}{\sqrt{x}}\right)$$

15.14 problem 10

Internal problem ID [1362]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$16x^2y'' + 4x(2x^2 + x + 6)\,y' + (18x^2 + 5x + 1)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 81

```
Order:=8;
dsolve(16*x^2*diff(y(x),x$2)+4*x*(6+x+2*x^2)*diff(y(x),x)+(1+5*x+18*x^2)*y(x)=0,y(x),type='se
```

$$y(x) = \frac{(c_2\ln(x) + c_1)\left(1 - \frac{1}{4}x - \frac{7}{32}x^2 + \frac{23}{384}x^3 + \frac{145}{6144}x^4 - \frac{881}{122880}x^5 - \frac{4919}{2949120}x^6 + \frac{47207}{82575360}x^7 + \mathrm{O}(x^8)\right) + \left(\frac{1}{4}x + \frac{5}{64}\right.}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 176

```
AsymptoticDSolveValue[16*x^2*y''[x]+4*x*(6+x+2*x^2)*y'[x]+(1+5*x+18*x^2)*y[x]==0,y[x],{x,0,7}
```

$$y(x) \to \frac{c_1\left(\frac{47207x^7}{82575360} - \frac{4919x^6}{2949120} - \frac{881x^5}{122880} + \frac{145x^4}{6144} + \frac{23x^3}{384} - \frac{7x^2}{32} - \frac{x}{4} + 1\right)}{\sqrt[4]{x}} + c_2\left(\frac{-\frac{953509x^7}{1284505600} + \frac{50791x^6}{58982400} + \frac{65017x^5}{7372800} - \frac{841x^4}{73728} - \frac{157x^3}{2304} + \frac{5x^2}{64} + \frac{x}{4}}{\sqrt[4]{x}} + \frac{\left(\frac{47207x^7}{82575360} - \frac{4919x^6}{2949120} - \frac{881x^5}{122880} + \frac{145x^4}{6144} + \frac{23x^3}{384} - \frac{7x^2}{32} - \frac{x}{4} + 1\right)\log(x)}{\sqrt[4]{x}}\right)$$

15.15 problem 11

Internal problem ID [1363]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(x+1)\,y''+3x\left(-x^2+11x+5\right)y'+\left(-7x^2+16x+1\right)y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 79

```
Order:=8;
dsolve(9*x^2*(1+x)*diff(y(x),x$2)+3*x*(5+11*x-x^2)*diff(y(x),x)+(1+16*x-7*x^2)*y(x)=0,y(x),ty
```

$$y(x)=\frac{(c_2\ln(x)+c_1)\left(1-x+\frac{7}{6}x^2-\frac{23}{18}x^3+\frac{11}{8}x^4-\frac{1577}{1080}x^5+\frac{3319}{2160}x^6-\frac{72853}{45360}x^7+\mathrm{O}\left(x^8\right)\right)+\left(-\frac{1}{12}x^2+\frac{13}{108}x^3-\frac{13}{86}\right.}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 167

```
AsymptoticDSolveValue[9*x^2*(1+x)*y''[x]+3*x*(5+11*x-x^2)*y'[x]+(1+16*x-7*x^2)*y[x]==0,y[x],{
```

$$y(x)\to\frac{c_1\left(-\frac{72853x^7}{45360}+\frac{3319x^6}{2160}-\frac{1577x^5}{1080}+\frac{11x^4}{8}-\frac{23x^3}{18}+\frac{7x^2}{6}-x+1\right)}{\sqrt[3]{x}}+c_2\left(\frac{\frac{4118557x^7}{19051200}-\frac{76919x^6}{388800}+\frac{11449x^5}{64800}-\frac{131x^4}{864}+\frac{13x^3}{108}-\frac{x^2}{12}}{\sqrt[3]{x}}+\frac{\left(-\frac{72853x^7}{45360}+\frac{3319x^6}{2160}-\frac{1577x^5}{1080}+\frac{11x^4}{8}-\frac{23x^3}{18}+\frac{7x^2}{6}-x+1\right)\log(x)}{\sqrt[3]{x}}\right)$$

15.16 problem 12

Internal problem ID [1364]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + (4x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+(1+4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left((c_2\ln(x)+c_1)\left(1-x+\frac{1}{4}x^2-\frac{1}{36}x^3+\frac{1}{576}x^4-\frac{1}{14400}x^5+\mathrm{O}\left(x^6\right)\right)\right.$$
$$\left.+\left(2x-\frac{3}{4}x^2+\frac{11}{108}x^3-\frac{25}{3456}x^4+\frac{137}{432000}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 128

```
AsymptoticDSolveValue[4*x^2*y''[x]+(1+4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{x^5}{14400}+\frac{x^4}{576}-\frac{x^3}{36}+\frac{x^2}{4}-x+1\right)$$
$$+c_2\left(\sqrt{x}\left(\frac{137x^5}{432000}-\frac{25x^4}{3456}+\frac{11x^3}{108}-\frac{3x^2}{4}+2x\right)\right.$$
$$\left.+\sqrt{x}\left(-\frac{x^5}{14400}+\frac{x^4}{576}-\frac{x^3}{36}+\frac{x^2}{4}-x+1\right)\log(x)\right)$$

15.17 problem 13

Internal problem ID [1365]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$36x^2(1-2x)\,y''+24x(1-9x)\,y'+(1-70x)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 69

```
Order:=6;
dsolve(36*x^2*(1-2*x)*diff(y(x),x$2)+24*x*(1-9*x)*diff(y(x),x)+(1-70*x)*y(x)=0,y(x),type='ser
```

$$y(x)=x^{\frac{1}{6}}\left((c_2\ln(x)+c_1)\left(1+\frac{8}{3}x+\frac{56}{9}x^2+\frac{1120}{81}x^3+\frac{7280}{243}x^4+\frac{46592}{729}x^5+\mathrm{O}\left(x^6\right)\right)+\left(-\frac{2}{3}x-2x^2-\frac{1192}{243}x^3-\frac{8168}{729}x^4-\frac{270112}{10935}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 132

```
AsymptoticDSolveValue[36*x^2*(1-2*x)*y''[x]+24*x*(1-9*x)*y'[x]+(1-70*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt[6]{x}\left(\frac{46592x^5}{729}+\frac{7280x^4}{243}+\frac{1120x^3}{81}+\frac{56x^2}{9}+\frac{8x}{3}+1\right)+c_2\left(\sqrt[6]{x}\left(-\frac{270112x^5}{10935}-\frac{8168x^4}{729}-\frac{1192x^3}{243}-2x^2-\frac{2x}{3}\right)+\sqrt[6]{x}\left(\frac{46592x^5}{729}+\frac{7280x^4}{243}+\frac{1120x^3}{81}+\frac{56x^2}{9}+\frac{8x}{3}+1\right)\log(x)\right)$$

15.18 problem 14

Internal problem ID [1366]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x+1)\,y'' - x(-x+3)\,y' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(3-x)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^2\left((c_2 \ln(x) + c_1)\left(1 - 4x + 9x^2 - 16x^3 + 25x^4 - 36x^5 + \mathrm{O}\left(x^6\right)\right) + \left(4x - 12x^2 + 24x^3 - 40x^4 + 60x^5 + \mathrm{O}\left(x^6\right)\right) c_2\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 98

```
AsymptoticDSolveValue[x^2*(1+x)*y''[x]-x*(3-x)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-36x^5 + 25x^4 - 16x^3 + 9x^2 - 4x + 1\right) x^2 + c_2\left(\left(60x^5 - 40x^4 + 24x^3 - 12x^2 + 4x\right) x^2 + \left(-36x^5 + 25x^4 - 16x^3 + 9x^2 - 4x + 1\right) x^2 \log(x)\right)$$

15.19 problem 15

Internal problem ID [1367]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-2x)\,y''-x(5-4x)\,y'+(-4x+9)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*(1-2*x)*diff(y(x),x$2)-x*(5-4*x)*diff(y(x),x)+(9-4*x)*y(x)=0,y(x),type='series',x=
```

$$y(x)=x^3\left((c_2\ln(x)+c_1)\left(1+4x+12x^2+32x^3+80x^4+192x^5+\mathrm{O}\left(x^6\right)\right)+\left((-2)\,x-8x^2-24x^3-64x^4-160x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 98

```
AsymptoticDSolveValue[x^2*(1-2*x)*y''[x]-x*(5-4*x)*y'[x]+(9-4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(192x^5+80x^4+32x^3+12x^2+4x+1\right)x^3+c_2\left(\left(-160x^5-64x^4-24x^3-8x^2-2x\right)x^3+\left(192x^5+80x^4+32x^3+12x^2+4x+1\right)x^3\log(x)\right)$$

15.20 problem 16

Internal problem ID [1368]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{25x^2y'' + x(15+x)\,y' + (x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(25*x^2*diff(y(x),x$2)+x*(15+x)*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^{\frac{1}{5}}\left((c_2\ln(x)+c_1)\left(1-\frac{6}{125}x+\frac{33}{31250}x^2-\frac{88}{5859375}x^3+\frac{77}{488281250}x^4-\frac{1001}{762939453125}x^5+\mathrm{O}\left(x^6\right)\right)+\left(\frac{7}{125}x-\frac{113}{62500}x^2+\frac{1091}{35156250}x^3-\frac{1721}{4687500000}x^4+\frac{609221}{183105468750000}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 134

```
AsymptoticDSolveValue[25*x^2*y''[x]+x*(15+x)*y'[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to\ & c_1 \sqrt[5]{x}\left(-\frac{1001x^5}{762939453125}+\frac{77x^4}{488281250}-\frac{88x^3}{5859375}+\frac{33x^2}{31250}-\frac{6x}{125}+1\right) \\
& + c_2\left(\sqrt[5]{x}\left(\frac{609221x^5}{183105468750000}-\frac{1721x^4}{4687500000}+\frac{1091x^3}{35156250}-\frac{113x^2}{62500}+\frac{7x}{125}\right)\right. \\
& \left. + \sqrt[5]{x}\left(-\frac{1001x^5}{762939453125}+\frac{77x^4}{488281250}-\frac{88x^3}{5859375}+\frac{33x^2}{31250}-\frac{6x}{125}+1\right)\log(x)\right)
\end{aligned}
$$

15.21 problem 17

Internal problem ID [1369]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(2+x)\,y''+y'x^2+(1-x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(2*x^2*(2+x)*diff(y(x),x$2)+x^2*diff(y(x),x)+(1-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\sqrt{x}\left((c_2\ln(x)+c_1)\left(1+\frac{1}{4}x-\frac{1}{32}x^2+\frac{1}{128}x^3-\frac{5}{2048}x^4+\frac{7}{8192}x^5+\mathrm{O}\left(x^6\right)\right)+\left(-\frac{3}{4}x+\frac{3}{64}x^2-\frac{7}{768}x^3+\frac{61}{24576}x^4-\frac{391}{491520}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 134

```
AsymptoticDSolveValue[2*x^2*(2+x)*y''[x]+x^2*y'[x]+(1-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt{x}\left(\frac{7x^5}{8192}-\frac{5x^4}{2048}+\frac{x^3}{128}-\frac{x^2}{32}+\frac{x}{4}+1\right)+c_2\left(\sqrt{x}\left(-\frac{391x^5}{491520}+\frac{61x^4}{24576}-\frac{7x^3}{768}+\frac{3x^2}{64}-\frac{3x}{4}\right)+\sqrt{x}\left(\frac{7x^5}{8192}-\frac{5x^4}{2048}+\frac{x^3}{128}-\frac{x^2}{32}+\frac{x}{4}+1\right)\log(x)\right)$$

15.22 problem 18

Internal problem ID [1370]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(4x+9)\,y''+3y'x+(x+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*(9+4*x)*diff(y(x),x$2)+3*x*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=x^{\frac{1}{3}}\left((c_2\ln(x)+c_1)\left(1-\frac{1}{81}x+\frac{25}{26244}x^2-\frac{3025}{19131876}x^3+\frac{874225}{24794911296}x^4-\frac{18498601}{2008387814976}x^5+\mathrm{O}\left(x^6\right)\right)+\left(\frac{14}{81}x-\frac{35}{2916}x^2+\frac{110495}{57395628}x^3-\frac{62786185}{148769467776}x^4+\frac{1315043653}{12050326889856}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 134

```
AsymptoticDSolveValue[x^2*(9+4*x)*y''[x]+3*x*y'[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \sqrt[3]{x}\left(-\frac{18498601x^5}{2008387814976}+\frac{874225x^4}{24794911296}-\frac{3025x^3}{19131876}+\frac{25x^2}{26244}-\frac{x}{81}+1\right) + c_2\left(\sqrt[3]{x}\left(\frac{1315043653x^5}{12050326889856}-\frac{62786185x^4}{148769467776}+\frac{110495x^3}{57395628}-\frac{35x^2}{2916}+\frac{14x}{81}\right) + \sqrt[3]{x}\left(-\frac{18498601x^5}{2008387814976}+\frac{874225x^4}{24794911296}-\frac{3025x^3}{19131876}+\frac{25x^2}{26244}-\frac{x}{81}+1\right)\log(x)\right)$$

15.23 problem 19

Internal problem ID [1371]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(-2x+3)\,y' + (3x+4)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(3-2*x)*diff(y(x),x)+(4+3*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_2\ln(x)+c_1)\left(1-7x+\frac{63}{4}x^2-\frac{77}{4}x^3+\frac{1001}{64}x^4-\frac{3003}{320}x^5+\mathrm{O}\left(x^6\right)\right) + \left(12x-\frac{157}{4}x^2+\frac{2063}{36}x^3-\frac{59875}{1152}x^4+\frac{323399}{9600}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)x^2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 122

```
AsymptoticDSolveValue[x^2*y''[x]-x*(3-2*x)*y'[x]+(4+3*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{3003x^5}{320}+\frac{1001x^4}{64}-\frac{77x^3}{4}+\frac{63x^2}{4}-7x+1\right)x^2 + c_2\left(\left(\frac{323399x^5}{9600}-\frac{59875x^4}{1152}+\frac{2063x^3}{36}-\frac{157x^2}{4}+12x\right)x^2 + \left(-\frac{3003x^5}{320}+\frac{1001x^4}{64}-\frac{77x^3}{4}+\frac{63x^2}{4}-7x+1\right)x^2\log(x)\right)$$

15.24 problem 20

Internal problem ID [1372]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2(-4x+1)\,y''+3x(1-6x)\,y'+(1-12x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*(1-4*x)*diff(y(x),x$2)+3*x*(1-6*x)*diff(y(x),x)+(1-12*x)*y(x)=0,y(x),type='series'
```

$$y(x) = \frac{(c_2\ln(x)+c_1)\left(1+2x+6x^2+20x^3+70x^4+252x^5+\mathrm{O}(x^6)\right)+\left(2x+7x^2+\frac{74}{3}x^3+\frac{533}{6}x^4+\frac{1627}{5}x^5+\mathrm{O}\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 104

```
AsymptoticDSolveValue[x^2*(1-4*x)*y''[x]+3*x*(1-6*x)*y'[x]+(1-12*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to\frac{c_1(252x^5+70x^4+20x^3+6x^2+2x+1)}{x}+c_2\left(\frac{\frac{1627x^5}{5}+\frac{533x^4}{6}+\frac{74x^3}{3}+7x^2+2x}{x}+\frac{(252x^5+70x^4+20x^3+6x^2+2x+1)\log(x)}{x}\right)$$

15.25 problem 21

Internal problem ID [1373]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2(1+2x)\,y''+x(3+5x)\,y'+(1-2x)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(3+5*x)*diff(y(x),x)+(1-2*x)*y(x)=0,y(x),type='series',x=
```

$$y(x) = \frac{(c_2\ln(x)+c_1)\left(1+3x+\frac{3}{2}x^2-\frac{1}{2}x^3+\frac{3}{8}x^4-\frac{3}{8}x^5+\mathrm{O}(x^6)\right)+\left((-5)\,x-\frac{25}{4}x^2+\frac{5}{4}x^3-\frac{25}{32}x^4+\frac{113}{160}x^5+\mathrm{O}\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 122

```
AsymptoticDSolveValue[x^2*(1+2*x)*y''[x]+x*(3+5*x)*y'[x]+(1-2*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to \frac{c_1\left(-\frac{3x^5}{8}+\frac{3x^4}{8}-\frac{x^3}{2}+\frac{3x^2}{2}+3x+1\right)}{x}+c_2\left(\frac{\frac{113x^5}{160}-\frac{25x^4}{32}+\frac{5x^3}{4}-\frac{25x^2}{4}-5x}{x}+\frac{\left(-\frac{3x^5}{8}+\frac{3x^4}{8}-\frac{x^3}{2}+\frac{3x^2}{2}+3x+1\right)\log(x)}{x}\right)$$

15.26 problem 22

Internal problem ID [1374]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x+1)\,y'' - x(-x+6)\,y' + (8-x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
Order:=6;
dsolve(2*x^2*(1+x)*diff(y(x),x$2)-x*(6-x)*diff(y(x),x)+(8-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_2 \ln(x) + c_1)\left(1 - \frac{5}{2}x + \frac{35}{8}x^2 - \frac{105}{16}x^3 + \frac{1155}{128}x^4 - \frac{3003}{256}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(\frac{3}{2}x - \frac{57}{16}x^2 + \frac{583}{96}x^3 - \frac{13771}{1536}x^4 + \frac{187339}{15360}x^5 + \mathrm{O}\left(x^6\right)\right)c_2\right)x^2$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 128

```
AsymptoticDSolveValue[2*x^2*(1+x)*y''[x]-x*(6-x)*y'[x]+(8-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{3003x^5}{256} + \frac{1155x^4}{128} - \frac{105x^3}{16} + \frac{35x^2}{8} - \frac{5x}{2} + 1\right)x^2 + c_2\left(\left(\frac{187339x^5}{15360} - \frac{13771x^4}{1536} + \frac{583x^3}{96} - \frac{57x^2}{16} + \frac{3x}{2}\right)x^2 + \left(-\frac{3003x^5}{256} + \frac{1155x^4}{128} - \frac{105x^3}{16} + \frac{35x^2}{8} - \frac{5x}{2} + 1\right)x^2\log(x)\right)$$

15.27 problem 23

Internal problem ID [1375]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1+2x)\,y''+x(5+9x)\,y'+(3x+4)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 81

```
Order:=8;
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(5+9*x)*diff(y(x),x)+(4+3*x)*y(x)=0,y(x),type='series',x=
```

$$y(x) = \frac{(c_2\ln(x)+c_1)\left(1+3x+\frac{3}{2}x^2-\frac{1}{2}x^3+\frac{3}{8}x^4-\frac{3}{8}x^5+\frac{7}{16}x^6-\frac{9}{16}x^7+\mathrm{O}\left(x^8\right)\right)+\left((-5)\,x-\frac{25}{4}x^2+\frac{5}{4}x^3-\frac{25}{32}x\right.}{x^2}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 164

```
AsymptoticDSolveValue[x^2*(1+2*x)*y''[x]+x*(5+9*x)*y'[x]+(4+3*x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to\frac{c_1\left(-\frac{9x^7}{16}+\frac{7x^6}{16}-\frac{3x^5}{8}+\frac{3x^4}{8}-\frac{x^3}{2}+\frac{3x^2}{2}+3x+1\right)}{x^2}+c_2\left(\frac{\frac{2123x^7}{2240}-\frac{247x^6}{320}+\frac{113x^5}{160}-\frac{25x^4}{32}+\frac{5x^3}{4}-\frac{25x^2}{4}-5x}{x^2}+\frac{\left(-\frac{9x^7}{16}+\frac{7x^6}{16}-\frac{3x^5}{8}+\frac{3x^4}{8}-\frac{x^3}{2}+\frac{3x^2}{2}+3x+1\right)\log(x)}{x^2}\right)$$

15.28 problem 24

Internal problem ID [1376]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-2x)\,y''-x(4x+5)\,y'+(4x+9)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 81

```
Order:=8;
dsolve(x^2*(1-2*x)*diff(y(x),x$2)-x*(5+4*x)*diff(y(x),x)+(9+4*x)*y(x)=0,y(x),type='series',x=
```

$$y(x)=\left((c_2\ln(x)+c_1)\left(1+20x+180x^2+1120x^3+5600x^4+24192x^5+94080x^6+337920x^7+\mathrm{O}\left(x^8\right)\right)+\left((-26)\,x-324x^2-\frac{6968}{3}x^3-\frac{37780}{3}x^4-57360x^5-\frac{694736}{3}x^6-\frac{2566144}{3}x^7+\mathrm{O}\left(x^8\right)\right)c_2\right)x^3$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 136

```
AsymptoticDSolveValue[x^2*(1-2*x)*y''[x]-x*(5+4*x)*y'[x]+(9+4*x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(337920x^7+94080x^6+24192x^5+5600x^4+1120x^3+180x^2+20x+1\right)x^3+c_2\left(\left(-\frac{2566144x^7}{3}-\frac{694736x^6}{3}-57360x^5-\frac{37780x^4}{3}-\frac{6968x^3}{3}-324x^2-26x\right)x^3+\left(337920x^7+94080x^6+24192x^5+5600x^4+1120x^3+180x^2+20x+1\right)x^3\log(x)\right)$$

15.29 problem 25

Internal problem ID [1377]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(4x+1)\,y''-x(-4x+1)\,y'+(x+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 75

```
Order:=8;
dsolve(x^2*(1+4*x)*diff(y(x),x$2)-x*(1-4*x)*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0)
```

$$y(x)=\left((c_2\ln(x)+c_1)\left(1-5x+\frac{85}{4}x^2-\frac{3145}{36}x^3+\frac{204425}{576}x^4-\frac{825877}{576}x^5+\frac{119752165}{20736}x^6-\frac{23591176505}{1016064}x^7+\mathrm{O}\left(x^8\right)\right)+\left(2x-\frac{39}{4}x^2+\frac{4499}{108}x^3-\frac{594305}{3456}x^4+\frac{2420617}{3456}x^5-\frac{117547073}{41472}x^6+\frac{162576422327}{14224896}x^7+\mathrm{O}\left(x^8\right)\right)c_2\right)x$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 158

```
AsymptoticDSolveValue[x^2*(1+4*x)*y''[x]-x*(1-4*x)*y'[x]+(1+x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 x \left(-\frac{23591176505x^7}{1016064} + \frac{119752165x^6}{20736} - \frac{825877x^5}{576} + \frac{204425x^4}{576} - \frac{3145x^3}{36} + \frac{85x^2}{4} - 5x + 1\right) + c_2 \left(x \left(\frac{162576422327x^7}{14224896} - \frac{117547073x^6}{41472} + \frac{2420617x^5}{3456} - \frac{594305x^4}{3456} + \frac{4499x^3}{108} - \frac{39x^2}{4} + 2x\right) + x \left(-\frac{23591176505x^7}{1016064} + \frac{119752165x^6}{20736} - \frac{825877x^5}{576} + \frac{204425x^4}{576} - \frac{3145x^3}{36} + \frac{85x^2}{4} - 5x + 1\right) \log(x)\right)$$

15.30 problem 26

Internal problem ID [1378]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+1)\,y''+x(1+2x)\,y'+yx=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 71

```
Order:=8;
dsolve(x^2*(1+x)*diff(y(x),x$2)+x*(1+2*x)*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(c_2\ln(x)+c_1)\left(1-x+\frac{3}{4}x^2-\frac{7}{12}x^3+\frac{91}{192}x^4-\frac{637}{1600}x^5+\frac{19747}{57600}x^6-\frac{17329}{57600}x^7+\mathrm{O}\left(x^8\right)\right)+\left(x-\frac{3}{4}x^2+\frac{5}{9}x^3-\frac{499}{1152}x^4+\frac{16919}{48000}x^5-\frac{56861}{192000}x^6+\frac{1027717}{4032000}x^7+\mathrm{O}\left(x^8\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 151

```
AsymptoticDSolveValue[x^2*(1+x)*y''[x]+x*(1+2*x)*y'[x]+x*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to c_1\left(-\frac{17329x^7}{57600}+\frac{19747x^6}{57600}-\frac{637x^5}{1600}+\frac{91x^4}{192}-\frac{7x^3}{12}+\frac{3x^2}{4}-x+1\right)+c_2\left(\frac{1027717x^7}{4032000}-\frac{56861x^6}{192000}+\frac{16919x^5}{48000}-\frac{499x^4}{1152}+\frac{5x^3}{9}-\frac{3x^2}{4}+\left(-\frac{17329x^7}{57600}+\frac{19747x^6}{57600}-\frac{637x^5}{1600}+\frac{91x^4}{192}-\frac{7x^3}{12}+\frac{3x^2}{4}-x+1\right)\log(x)+x\right)$$

15.31 problem 27

Internal problem ID [1379]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x)\,y''+x(7+x)\,y'+(-x+9)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
Order:=8;
dsolve(x^2*(1-x)*diff(y(x),x$2)+x*(7+x)*diff(y(x),x)+(9-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(c_2\ln(x)+c_1)\left(1+16x+36x^2+16x^3+x^4+\mathrm{O}\left(x^8\right)\right)+\left((-40)\,x-150x^2-\frac{280}{3}x^3-\frac{25}{3}x^4+\mathrm{O}\left(x^8\right)\right)c_2}{x^3}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 83

```
AsymptoticDSolveValue[x^2*(1-x)*y''[x]+x*(7+x)*y'[x]+(9-x)*y[x]==0,y[x],{x,0,7}]
```

$$y(x)\to\frac{c_1(x^4+16x^3+36x^2+16x+1)}{x^3}+c_2\left(\frac{-\frac{25x^4}{3}-\frac{280x^3}{3}-150x^2-40x}{x^3}+\frac{(x^4+16x^3+36x^2+16x+1)\log(x)}{x^3}\right)$$

15.32 problem 28

Internal problem ID [1380]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(-x^2+1)\,y' + y(x^2+1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(1-x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x\left((c_2\ln(x)+c_1)\left(1-\frac{1}{2}x^2+\frac{1}{8}x^4+\mathrm{O}\left(x^6\right)\right)+\left(\frac{1}{4}x^2-\frac{3}{32}x^4+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 65

```
AsymptoticDSolveValue[x^2*y''[x]-x*(1-x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x\left(\frac{x^4}{8}-\frac{x^2}{2}+1\right)+c_2\left(x\left(\frac{x^2}{4}-\frac{3x^4}{32}\right)+x\left(\frac{x^4}{8}-\frac{x^2}{2}+1\right)\log(x)\right)$$

15.33 problem 29

Internal problem ID [1381]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''-3x(-x^2+1)\,y'+4y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 51

```
Order:=6;
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-3*x*(1-x^2)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left((c_2\ln(x)+c_1)\left(1-2x^2+3x^4+\mathrm{O}\left(x^6\right)\right)+\left(\frac{1}{2}x^2-x^4+\mathrm{O}\left(x^6\right)\right)c_2\right)x^2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 61

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]-3*x*(1-x^2)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1(3x^4-2x^2+1)\,x^2+c_2\left(\left(\frac{x^2}{2}-x^4\right)x^2+(3x^4-2x^2+1)\,x^2\log(x)\right)$$

15.34 problem 30

Internal problem ID [1382]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 2x^3y' + (3x^2 + 1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+2*x^3*diff(y(x),x)+(1+3*x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left((c_2 \ln(x) + c_1)\left(1 - \frac{1}{4}x^2 + \frac{1}{32}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(\frac{1}{8}x^2 - \frac{3}{128}x^4 + \mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 77

```
AsymptoticDSolveValue[4*x^2*y''[x]+2*x^3*y'[x]+(1+3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^4}{32} - \frac{x^2}{4} + 1\right) + c_2\left(\sqrt{x}\left(\frac{x^2}{8} - \frac{3x^4}{128}\right) + \sqrt{x}\left(\frac{x^4}{32} - \frac{x^2}{4} + 1\right)\log(x)\right)$$

15.35 problem 31

Internal problem ID [1383]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x^2+1)\,y''-x(-2x^2+1)\,y'+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1-2*x^2)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=x\left((c_2\ln(x)+c_1)\left(1-\frac{1}{2}x^2+\frac{3}{8}x^4+\mathrm{O}(x^6)\right)+\left(-\frac{1}{4}x^2+\frac{7}{32}x^4+\mathrm{O}(x^6)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 65

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]-x*(1-2*x^2)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1x\left(\frac{3x^4}{8}-\frac{x^2}{2}+1\right)+c_2\left(x\left(\frac{7x^4}{32}-\frac{x^2}{4}\right)+x\left(\frac{3x^4}{8}-\frac{x^2}{2}+1\right)\log(x)\right)$$

15.36 problem 32

Internal problem ID [1384]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x^2+2)\,y''+7x^3y'+(3x^2+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
dsolve(2*x^2*(2+x^2)*diff(y(x),x$2)+7*x^3*diff(y(x),x)+(1+3*x^2)*y(x)=0,y(x),type='series',x=
```

$$y(x)=\sqrt{x}\left((c_2\ln(x)+c_1)\left(1-\frac{3}{8}x^2+\frac{21}{128}x^4+\mathrm{O}(x^6)\right)+\left(-\frac{1}{16}x^2+\frac{17}{512}x^4+\mathrm{O}(x^6)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 77

```
AsymptoticDSolveValue[2*x^2*(2+x^2)*y''[x]+7*x^3*y'[x]+(1+3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt{x}\left(\frac{21x^4}{128}-\frac{3x^2}{8}+1\right)+c_2\left(\sqrt{x}\left(\frac{17x^4}{512}-\frac{x^2}{16}\right)+\sqrt{x}\left(\frac{21x^4}{128}-\frac{3x^2}{8}+1\right)\log(x)\right)$$

15.37 problem 33

Internal problem ID [1385]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x^2+1)\,y''-x(-4x^2+1)\,y'+y(2x^2+1)=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1-4*x^2)*diff(y(x),x)+(1+2*x^2)*y(x)=0,y(x),type='series
```

$$y(x)=x\left((c_2\ln(x)+c_1)\left(1-\frac{3}{2}x^2+\frac{15}{8}x^4+\mathrm{O}(x^6)\right)+\left(\frac{1}{4}x^2-\frac{13}{32}x^4+\mathrm{O}(x^6)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 65

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]-x*(1-4*x^2)*y'[x]+(1+2*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1x\left(\frac{15x^4}{8}-\frac{3x^2}{2}+1\right)+c_2\left(x\left(\frac{x^2}{4}-\frac{13x^4}{32}\right)+x\left(\frac{15x^4}{8}-\frac{3x^2}{2}+1\right)\log(x)\right)$$

15.38 problem 34

Internal problem ID [1386]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(x^2+4)\,y''+3x(3x^2+8)\,y'+(-9x^2+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
dsolve(4*x^2*(4+x^2)*diff(y(x),x$2)+3*x*(8+3*x^2)*diff(y(x),x)+(1-9*x^2)*y(x)=0,y(x),type='se
```

$$y(x)=\frac{(c_2\ln(x)+c_1)\left(1+\frac{5}{32}x^2-\frac{15}{2048}x^4+\mathrm{O}(x^6)\right)+\left(-\frac{13}{64}x^2+\frac{13}{8192}x^4+\mathrm{O}(x^6)\right)c_2}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 77

```
AsymptoticDSolveValue[4*x^2*(4+x^2)*y''[x]+3*x*(8+3*x^2)*y'[x]+(1-9*x^2)*y[x]==0,y[x],{x,0,5}
```

$$y(x)\to\frac{c_1\left(-\frac{15x^4}{2048}+\frac{5x^2}{32}+1\right)}{\sqrt[4]{x}}+c_2\left(\frac{\frac{13x^4}{8192}-\frac{13x^2}{64}}{\sqrt[4]{x}}+\frac{\left(-\frac{15x^4}{2048}+\frac{5x^2}{32}+1\right)\log(x)}{\sqrt[4]{x}}\right)$$

15.39 problem 35

Internal problem ID [1387]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2(x^2+3)y''+x(11x^2+3)y'+(5x^2+1)y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
dsolve(3*x^2*(3+x^2)*diff(y(x),x$2)+x*(3+11*x^2)*diff(y(x),x)+(1+5*x^2)*y(x)=0,y(x),type='ser
```

$$y(x)=x^{\frac{1}{3}}\left((c_2\ln(x)+c_1)\left(1-\frac{2}{9}x^2+\frac{5}{81}x^4+\mathrm{O}\left(x^6\right)\right)+\left(-\frac{1}{18}x^2+\frac{1}{54}x^4+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
AsymptoticDSolveValue[3*x^2*(3+x^2)*y''[x]+x*(3+11*x^2)*y'[x]+(1+5*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to \frac{c_1\left(-\frac{1139x^5}{405}+\frac{53x^4}{81}+\frac{7x^3}{9}-\frac{11x^2}{9}+x+1\right)}{\sqrt[3]{x}}$$

15.40 problem 36

Internal problem ID [1388]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(4x^2+1)\,y''+32x^3y'+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
dsolve(4*x^2*(1+4*x^2)*diff(y(x),x$2)+32*x^3*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\sqrt{x}\left((c_2\ln(x)+c_1)\left(1-\frac{3}{4}x^2+\frac{105}{64}x^4+\mathrm{O}\left(x^6\right)\right)+\left(-\frac{5}{4}x^2+\frac{389}{128}x^4+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 77

```
AsymptoticDSolveValue[4*x^2*(1+4*x^2)*y''[x]+32*x^3*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt{x}\left(\frac{105x^4}{64}-\frac{3x^2}{4}+1\right)+c_2\left(\sqrt{x}\left(\frac{389x^4}{128}-\frac{5x^2}{4}\right)+\sqrt{x}\left(\frac{105x^4}{64}-\frac{3x^2}{4}+1\right)\log(x)\right)$$

15.41 problem 37

Internal problem ID [1389]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' - 3x(-2x^2+7)\, y' + (2x^2+25)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
dsolve(9*x^2*diff(y(x),x$2)-3*x*(7-2*x^2)*diff(y(x),x)+(25+2*x^2)*y(x)=0,y(x),type='series',x
```

$$y(x) = x^{\frac{5}{3}}\left((c_2\ln(x) + c_1)\left(1 - \frac{1}{3}x^2 + \frac{1}{18}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(\frac{1}{6}x^2 - \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 77

```
AsymptoticDSolveValue[9*x^2*y''[x]-3*x*(7-2*x^2)*y'[x]+(25+2*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1\left(\frac{x^4}{18} - \frac{x^2}{3} + 1\right)x^{5/3} + c_2\left(\left(\frac{x^2}{6} - \frac{x^4}{24}\right)x^{5/3} + \left(\frac{x^4}{18} - \frac{x^2}{3} + 1\right)x^{5/3}\log(x)\right)$$

15.42 problem 38

Internal problem ID [1390]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2(2x^2+1)\, y''+x(7x^2+3)\, y'+(-3x^2+1)\, y=0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
dsolve(x^2*(1+2*x^2)*diff(y(x),x$2)+x*(3+7*x^2)*diff(y(x),x)+(1-3*x^2)*y(x)=0,y(x),type='seri
```

$$y(x)=\frac{(c_2\ln(x)+c_1)\left(1+\frac{3}{2}x^2-\frac{3}{8}x^4+\mathrm{O}(x^6)\right)+\left(-\frac{7}{4}x^2-\frac{7}{32}x^4+\mathrm{O}(x^6)\right)c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 71

```
AsymptoticDSolveValue[x^2*(1+2*x^2)*y''[x]+x*(3+7*x^2)*y'[x]+(1-3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to\frac{c_1\left(-\frac{3x^4}{8}+\frac{3x^2}{2}+1\right)}{x}+c_2\left(\frac{-\frac{7x^4}{32}-\frac{7x^2}{4}}{x}+\frac{\left(-\frac{3x^4}{8}+\frac{3x^2}{2}+1\right)\log(x)}{x}\right)$$

15.43 problem 39

Internal problem ID [1391]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2(x^2+1)\,y''+x(8x^2+3)\,y'+(12x^2+1)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(3+8*x^2)*diff(y(x),x)+(1+12*x^2)*y(x)=0,y(x),type='serie
```

$$y(x)=\frac{(c_2\ln(x)+c_1)\left(1-\frac{3}{2}x^2+\frac{15}{8}x^4+\mathrm{O}(x^6)\right)+\left(\frac{1}{4}x^2-\frac{13}{32}x^4+\mathrm{O}(x^6)\right)c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 71

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]+x*(3+8*x^2)*y'[x]+(1+12*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to\frac{c_1\left(\frac{15x^4}{8}-\frac{3x^2}{2}+1\right)}{x}+c_2\left(\frac{\frac{x^2}{4}-\frac{13x^4}{32}}{x}+\frac{\left(\frac{15x^4}{8}-\frac{3x^2}{2}+1\right)\log(x)}{x}\right)$$

15.44 problem 40

Internal problem ID [1392]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(-x^2+1)\, y' + y(x^2+1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(1-x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x\left((c_2 \ln(x) + c_1)\left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 + O(x^6)\right) + \left(\frac{1}{4}x^2 - \frac{3}{32}x^4 + O(x^6)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 65

```
AsymptoticDSolveValue[x^2*y''[x]-x*(1-x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x\left(\frac{x^4}{8} - \frac{x^2}{2} + 1\right) + c_2\left(x\left(\frac{x^2}{4} - \frac{3x^4}{32}\right) + x\left(\frac{x^4}{8} - \frac{x^2}{2} + 1\right)\log(x)\right)$$

15.45 problem 41

Internal problem ID [1393]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-2x^2+1)\,y''+x(-9x^2+5)\,y'+(-3x^2+4)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
Order:=6;
dsolve(x^2*(1-2*x^2)*diff(y(x),x$2)+x*(5-9*x^2)*diff(y(x),x)+(4-3*x^2)*y(x)=0,y(x),type='seri
```

$$y(x)=\frac{(c_2\ln(x)+c_1)\left(1-\frac{3}{4}x^2-\frac{9}{64}x^4+\mathrm{O}(x^6)\right)+\left(\frac{1}{2}x^2-\frac{21}{128}x^4+\mathrm{O}(x^6)\right)c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 71

```
AsymptoticDSolveValue[x^2*(1-2*x^2)*y''[x]+x*(5-9*x^2)*y'[x]+(4-3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to\frac{c_1\left(-\frac{9x^4}{64}-\frac{3x^2}{4}+1\right)}{x^2}+c_2\left(\frac{\frac{x^2}{2}-\frac{21x^4}{128}}{x^2}+\frac{\left(-\frac{9x^4}{64}-\frac{3x^2}{4}+1\right)\log(x)}{x^2}\right)$$

15.46 problem 42

Internal problem ID [1394]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x^2+2)\,y''+x(-x^2+14)\,y'+2(x^2+9)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
dsolve(x^2*(2+x^2)*diff(y(x),x$2)+x*(14-x^2)*diff(y(x),x)+2*(9+x^2)*y(x)=0,y(x),type='series'
```

$$y(x)=\frac{(c_2\ln(x)+c_1)\left(1-\frac{17}{8}x^2+\frac{85}{256}x^4+\mathrm{O}(x^6)\right)+\left(\frac{25}{8}x^2-\frac{471}{512}x^4+\mathrm{O}(x^6)\right)c_2}{x^3}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 71

```
AsymptoticDSolveValue[x^2*(2+x^2)*y''[x]+x*(14-x^2)*y'[x]+2*(9+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to\frac{c_1\left(\frac{85x^4}{256}-\frac{17x^2}{8}+1\right)}{x^3}+c_2\left(\frac{\frac{25x^2}{8}-\frac{471x^4}{512}}{x^3}+\frac{\left(\frac{85x^4}{256}-\frac{17x^2}{8}+1\right)\log(x)}{x^3}\right)$$

15.47 problem 43

Internal problem ID [1395]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x^2+1)\,y''+x(7x^2+3)\,y'+(8x^2+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
Order:=6;
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(3+7*x^2)*diff(y(x),x)+(1+8*x^2)*y(x)=0,y(x),type='series
```

$$y(x)=\frac{(c_2\ln(x)+c_1)\left(1-\frac{3}{4}x^2+\frac{45}{64}x^4+\mathrm{O}(x^6)\right)+\left(-\frac{1}{4}x^2+\frac{33}{128}x^4+\mathrm{O}(x^6)\right)c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 71

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]+x*(3+7*x^2)*y'[x]+(1+8*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to\frac{c_1\left(\frac{45x^4}{64}-\frac{3x^2}{4}+1\right)}{x}+c_2\left(\frac{\frac{33x^4}{128}-\frac{x^2}{4}}{x}+\frac{\left(\frac{45x^4}{64}-\frac{3x^2}{4}+1\right)\log(x)}{x}\right)$$

15.48 problem 44

Internal problem ID [1396]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-2x)\,y''+3y'x+(4x+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*(1-2*x)*diff(y(x),x$2)+3*x*diff(y(x),x)+(1+4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\frac{(c_2\ln(x)+c_1)\,(1+\mathrm{O}(x^6))+\left((-6)\,x+6x^2-\frac{8}{3}x^3+\mathrm{O}(x^6)\right)c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 40

```
AsymptoticDSolveValue[x^2*(1-2*x)*y''[x]+3*x*y'[x]+(1+4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{-\frac{8x^3}{3}+6x^2-6x}{x}+\frac{\log(x)}{x}\right)+\frac{c_1}{x}$$

15.49 problem 45

Internal problem ID [1397]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x+1)\,y''+(1-x)\,y'+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(x*(1+x)*diff(y(x),x$2)+(1-x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(c_2\ln(x)+c_1)\left(1-x+\mathrm{O}\left(x^6\right)\right)+\left(4x+\mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 27

```
AsymptoticDSolveValue[x*(1+x)*y''[x]+(1-x)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1(1-x)+c_2(4x+(1-x)\log(x))$$

15.50 problem 46

Internal problem ID [1398]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2(1-x)\,y''+x(-2x+3)\,y'+(1+2x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 57

```
Order:=6;
dsolve(x^2*(1-x)*diff(y(x),x$2)+x*(3-2*x)*diff(y(x),x)+(1+2*x)*y(x)=0,y(x),type='series',x=0)
```

$$y(x)=\frac{(c_2\ln(x)+c_1)\left(1-2x+x^2+\mathrm{O}(x^6)\right)+\left(3x-3x^2+\frac{1}{3}x^3+\frac{1}{12}x^4+\frac{1}{30}x^5+\mathrm{O}(x^6)\right)c_2}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 70

```
AsymptoticDSolveValue[x^2*(1-x)*y''[x]+x*(3-2*x)*y'[x]+(1+2*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to\frac{c_1(x^2-2x+1)}{x}+c_2\left(\frac{(x^2-2x+1)\log(x)}{x}+\frac{\frac{x^5}{30}+\frac{x^4}{12}+\frac{x^3}{3}-3x^2+3x}{x}\right)$$

15.51 problem 47

Internal problem ID [1399]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x+1)\,y'' + 4y'x^2 + (1-5x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
Order:=6;
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x^2*diff(y(x),x)+(1-5*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left((c_2\ln(x)+c_1)\left(1+x-\frac{1}{4}x^2+\frac{5}{36}x^3-\frac{55}{576}x^4+\frac{209}{2880}x^5+\mathrm{O}\left(x^6\right)\right) + \left((-3)\,x+\frac{1}{4}x^2-\frac{5}{54}x^3+\frac{175}{3456}x^4-\frac{2863}{86400}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 124

```
AsymptoticDSolveValue[4*x^2*(1+x)*y''[x]+4*x^2*y'[x]+(1-5*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{209x^5}{2880}-\frac{55x^4}{576}+\frac{5x^3}{36}-\frac{x^2}{4}+x+1\right) + c_2\left(\sqrt{x}\left(-\frac{2863x^5}{86400}+\frac{175x^4}{3456}-\frac{5x^3}{54}+\frac{x^2}{4}-3x\right) + \sqrt{x}\left(\frac{209x^5}{2880}-\frac{55x^4}{576}+\frac{5x^3}{36}-\frac{x^2}{4}+x+1\right)\log(x)\right)$$

15.52 problem 48

Internal problem ID [1400]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 48.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-x)\,y''-x(3-5x)\,y'+(4-5x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
Order:=6;
dsolve(x^2*(1-x)*diff(y(x),x$2)-x*(3-5*x)*diff(y(x),x)+(4-5*x)*y(x)=0,y(x),type='series',x=0)
```

$$y(x)=x^2\left((c_2\ln(x)+c_1)\left(1-3x+3x^2-x^3+\mathrm{O}\left(x^6\right)\right)+\left(4x-7x^2+\frac{11}{3}x^3-\frac{1}{4}x^4-\frac{1}{20}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 84

```
AsymptoticDSolveValue[x^2*(1-x)*y''[x]-x*(3-5*x)*y'[x]+(4-5*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-x^3+3x^2-3x+1\right)x^2+c_2\left(\left(-x^3+3x^2-3x+1\right)x^2\log(x)+\left(-\frac{x^5}{20}-\frac{x^4}{4}+\frac{11x^3}{3}-7x^2+4x\right)x^2\right)$$

15.53 problem 49

Internal problem ID [1401]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''-x(9x^2+1)\,y'+(25x^2+1)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(1+9*x^2)*diff(y(x),x)+(1+25*x^2)*y(x)=0,y(x),type='serie
```

$$y(x)=x\left((c_2\ln(x)+c_1)\left(1-4x^2+x^4+\mathrm{O}\left(x^6\right)\right)+\left(6x^2-3x^4+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 49

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]-x*(1+9*x^2)*y'[x]+(1+25*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1x\left(x^4-4x^2+1\right)+c_2\left(x\left(6x^2-3x^4\right)+x\left(x^4-4x^2+1\right)\log(x)\right)$$

15.54 problem 50

Internal problem ID [1402]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' + 3x(-x^2+1)\,y' + (7x^2+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
dsolve(9*x^2*diff(y(x),x$2)+3*x*(1-x^2)*diff(y(x),x)+(1+7*x^2)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = x^{\frac{1}{3}}\left((c_2\ln(x)+c_1)\left(1-\frac{1}{6}x^2+\mathrm{O}\left(x^6\right)\right)+\left(\frac{1}{4}x^2-\frac{1}{288}x^4+\mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 63

```
AsymptoticDSolveValue[9*x^2*y''[x]+3*x*(1-x^2)*y'[x]+(1+7*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt[3]{x}\left(1-\frac{x^2}{6}\right)+c_2\left(\sqrt[3]{x}\left(1-\frac{x^2}{6}\right)\log(x)+\sqrt[3]{x}\left(\frac{x^2}{4}-\frac{x^4}{288}\right)\right)$$

15.55 problem 51

Internal problem ID [1403]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 51.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(x^2+1)\,y''+(-x^2+1)\,y'-8yx=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
Order:=6;
dsolve(x*(1+x^2)*diff(y(x),x$2)+(1-x^2)*diff(y(x),x)-8*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(c_2\ln(x)+c_1)\left(1+2x^2+x^4+\mathrm{O}\left(x^6\right)\right)+\left(-\frac{3}{2}x^2-\frac{3}{2}x^4+\mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 48

```
AsymptoticDSolveValue[x*(1+x^2)*y''[x]+(1-x^2)*y'[x]-8*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(x^4+2x^2+1\right)+c_2\left(-\frac{3x^4}{2}-\frac{3x^2}{2}+\left(x^4+2x^2+1\right)\log(x)\right)$$

15.56 problem 52

Internal problem ID [1404]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 52.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 2x(-x^2+4)\,y' + (7x^2+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+2*x*(4-x^2)*diff(y(x),x)+(1+7*x^2)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = \frac{(c_2\ln(x)+c_1)\left(1-\frac{1}{2}x^2+\frac{1}{32}x^4+\mathrm{O}(x^6)\right)+\left(\frac{5}{8}x^2-\frac{9}{128}x^4+\mathrm{O}(x^6)\right)c_2}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 77

```
AsymptoticDSolveValue[4*x^2*y''[x]+2*x*(4-x^2)*y'[x]+(1+7*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1\left(\frac{x^4}{32}-\frac{x^2}{2}+1\right)}{\sqrt{x}} + c_2\left(\frac{\frac{5x^2}{8}-\frac{9x^4}{128}}{\sqrt{x}} + \frac{\left(\frac{x^4}{32}-\frac{x^2}{2}+1\right)\log(x)}{\sqrt{x}}\right)$$

15.57 problem 58

Internal problem ID [1405]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 58.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(x+1)\,y''+8y'x^2+(x+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
Order:=6;
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+8*x^2*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\sqrt{x}\left(-x^5+x^4-x^3+x^2-x+1\right)\left(c_2\ln(x)+c_1\right)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 64

```
AsymptoticDSolveValue[4*x^2*(1+x)*y''[x]+8*x^2*y'[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt{x}\left(-x^5+x^4-x^3+x^2-x+1\right)+c_2\sqrt{x}\left(-x^5+x^4-x^3+x^2-x+1\right)\log(x)$$

15.58 problem 59

Internal problem ID [1406]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 59.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(x+3)\,y''+3x(3+7x)\,y'+(4x+3)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
Order:=6;
dsolve(9*x^2*(3+x)*diff(y(x),x$2)+3*x*(3+7*x)*diff(y(x),x)+(3+4*x)*y(x)=0,y(x),type='series',
```

$$y(x)=x^{\frac{1}{3}}\left(1-\frac{1}{3}x+\frac{1}{9}x^2-\frac{1}{27}x^3+\frac{1}{81}x^4-\frac{1}{243}x^5\right)(c_2\ln(x)+c_1)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 92

```
AsymptoticDSolveValue[9*x^2*(3+x)*y''[x]+3*x*(3+7*x)*y'[x]+(3+4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt[3]{x}\left(-\frac{x^5}{243}+\frac{x^4}{81}-\frac{x^3}{27}+\frac{x^2}{9}-\frac{x}{3}+1\right)+c_2\sqrt[3]{x}\left(-\frac{x^5}{243}+\frac{x^4}{81}-\frac{x^3}{27}+\frac{x^2}{9}-\frac{x}{3}+1\right)\log(x)$$

15.59 problem 60

Internal problem ID [1407]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 60.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-x^2+2)\,y''-x(3x^2+2)\,y'+(-x^2+2)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
Order:=6;
dsolve(x^2*(2-x^2)*diff(y(x),x$2)-x*(2+3*x^2)*diff(y(x),x)+(2-x^2)*y(x)=0,y(x),type='series',
```

$$y(x)=\left(1+\frac{1}{2}x^2+\frac{1}{4}x^4\right)x(c_2\ln(x)+c_1)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 46

```
AsymptoticDSolveValue[x^2*(2-x^2)*y''[x]-x*(2+3*x^2)*y'[x]+(2-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1x\left(\frac{x^4}{4}+\frac{x^2}{2}+1\right)+c_2x\left(\frac{x^4}{4}+\frac{x^2}{2}+1\right)\log(x)$$

15.60 problem 61

Internal problem ID [1408]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 61.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16x^2(x^2+1)\,y''+8x(9x^2+1)\,y'+(49x^2+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
Order:=6;
dsolve(16*x^2*(1+x^2)*diff(y(x),x$2)+8*x*(1+9*x^2)*diff(y(x),x)+(1+49*x^2)*y(x)=0,y(x),type='
```

$$y(x)=x^{\frac{1}{4}}(x^4-x^2+1)\,(c_2\ln(x)+c_1)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 42

```
AsymptoticDSolveValue[16*x^2*(1+x^2)*y''[x]+8*x*(1+9*x^2)*y'[x]+(1+49*x^2)*y[x]==0,y[x],{x,0,
```

$$y(x)\to c_1\sqrt[4]{x}(x^4-x^2+1)+c_2\sqrt[4]{x}(x^4-x^2+1)\log(x)$$

15.61 problem 62

Internal problem ID [1409]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 62.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(3x+4)\,y''-x(4-3x)\,y'+4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*(4+3*x)*diff(y(x),x$2)-x*(4-3*x)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{3}{4}x+\frac{9}{16}x^2-\frac{27}{64}x^3+\frac{81}{256}x^4-\frac{243}{1024}x^5\right)x(c_2\ln(x)+c_1)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 84

```
AsymptoticDSolveValue[x^2*(4+3*x)*y''[x]-x*(4-3*x)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1x\left(-\frac{243x^5}{1024}+\frac{81x^4}{256}-\frac{27x^3}{64}+\frac{9x^2}{16}-\frac{3x}{4}+1\right)+c_2x\left(-\frac{243x^5}{1024}+\frac{81x^4}{256}-\frac{27x^3}{64}+\frac{9x^2}{16}-\frac{3x}{4}+1\right)\log(x)$$

15.62 problem 63

Internal problem ID [1410]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 63.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x^2+3x+1)\,y''+8x^2(2x+3)\,y'+(9x^2+3x+1)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
Order:=6;
dsolve(4*x^2*(1+3*x+x^2)*diff(y(x),x$2)+8*x^2*(3+2*x)*diff(y(x),x)+(1+3*x+9*x^2)*y(x)=0,y(x),
```

$$y(x)=\sqrt{x}\,(-144x^5+55x^4-21x^3+8x^2-3x+1)\,(c_2\ln(x)+c_1)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 72

```
AsymptoticDSolveValue[4*x^2*(1+3*x+x^2)*y''[x]+8*x^2*(3+2*x)*y'[x]+(1+3*x+9*x^2)*y[x]==0,y[x]
```

$$\begin{aligned}y(x)\to\ & c_1\sqrt{x}(-144x^5+55x^4-21x^3+8x^2-3x+1)\\ &+c_2\sqrt{x}(-144x^5+55x^4-21x^3+8x^2-3x+1)\log(x)\end{aligned}$$

15.63 problem 64

Internal problem ID [1411]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 64.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(1-x)^2 y'' - x(-3x^2+2x+1) y' + y(x^2+1) = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*(1-x)^2*diff(y(x),x$2)-x*(1+2*x-3*x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x),type='seri
```

$$y(x) = (6x^5+5x^4+4x^3+3x^2+2x+1)\, x(c_2 \ln(x) + c_1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 64

```
AsymptoticDSolveValue[x^2*(1-x)^2*y''[x]-x*(1+2*x-3*x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x(6x^5+5x^4+4x^3+3x^2+2x+1) + c_2 x(6x^5+5x^4+4x^3+3x^2+2x+1)\log(x)$$

15.64 problem 65

Internal problem ID [1412]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS II. Exercises 7.6. Page 374
Problem number: 65.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2(x^2+x+1)\,y''+3x(13x^2+7x+1)\,y'+(25x^2+4x+1)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
Order:=6;
dsolve(9*x^2*(1+x+x^2)*diff(y(x),x$2)+3*x*(1+7*x+13*x^2)*diff(y(x),x)+(1+4*x+25*x^2)*y(x)=0,y
```

$$y(x)=x^{\frac{1}{3}}\left(-x^4+x^3-x+1\right)\left(c_2\ln\left(x\right)+c_1\right)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 48

```
AsymptoticDSolveValue[9*x^2*(1+x+x^2)*y''[x]+3*x*(1+7*x+13*x^2)*y'[x]+(1+4*x+25*x^2)*y[x]==0,
```

$$y(x)\to c_1\sqrt[3]{x}\left(-x^4+x^3-x+1\right)+c_2\sqrt[3]{x}\left(-x^4+x^3-x+1\right)\log(x)$$

16 Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389

16.1 problem Example 7.7.1 page 381

Internal problem ID [1413]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: Example 7.7.1 page 381.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(2+x)\,y'' - x(-7x+4)\,y' - (5-3x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 65

```
Order:=6;
dsolve(2*x^2*(2+x)*diff(y(x),x$2)-x*(4-7*x)*diff(y(x),x)-(5-3*x)*y(x)=0,y(x),type='series',x=
```

$$y(x) = \frac{c_1 x^3\left(1 - \frac{7}{4}x + \frac{63}{32}x^2 - \frac{231}{128}x^3 + \frac{3003}{2048}x^4 - \frac{9009}{8192}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(-\frac{45}{32}x^3 + \frac{315}{128}x^4 - \frac{2835}{1024}x^5 + \mathrm{O}\left(x^6\right)\right)\right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 98

```
AsymptoticDSolveValue[2*x^2*(2+x)*y''[x]-x*(4-7*x)*y'[x]-(5-3*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{3003x^{13/2}}{2048} - \frac{231x^{11/2}}{128} + \frac{63x^{9/2}}{32} - \frac{7x^{7/2}}{4} + x^{5/2}\right) + c_1\left(\frac{15}{512}(7x-4)x^{5/2}\log(x) + \frac{809x^4 - 548x^3 + 96x^2 + 128x + 1024}{1024\sqrt{x}}\right)$$

16.2 problem Example 7.7.2 page 383

Internal problem ID [1414]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: Example 7.7.2 page 383.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1-2x)\,y''+x(8-9x)\,y'+(6-3x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*(1-2*x)*diff(y(x),x$2)+x*(8-9*x)*diff(y(x),x)+(6-3*x)*y(x)=0,y(x),type='series',x=
```

$$y(x)=\frac{c_1\left(1-\frac{1}{3}x-\frac{1}{14}x^2-\frac{1}{28}x^3-\frac{25}{1008}x^4-\frac{1}{48}x^5+\mathrm{O}\left(x^6\right)\right)}{x}+\frac{c_2\left(2880-23760x+71280x^2-83160x^3+62370x^5+\mathrm{O}\left(x^6\right)\right)}{x^6}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 61

```
AsymptoticDSolveValue[x^2*(1-2*x)*y''[x]+x*(8-9*x)*y'[x]+(6-3*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(-\frac{25x^3}{1008}-\frac{x^2}{28}-\frac{x}{14}+\frac{1}{x}-\frac{1}{3}\right)+c_1\left(\frac{1}{x^6}-\frac{33}{4x^5}+\frac{99}{4x^4}-\frac{231}{8x^3}\right)$$

16.3 problem Example 7.7.3 page 385

Internal problem ID [1415]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: Example 7.7.3 page 385.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''+x(10x^2+3)\,y'-(-14x^2+15)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(3+10*x^2)*diff(y(x),x)-(15-14*x^2)*y(x)=0,y(x),type='ser
```

$$y(x)=c_1x^3\left(1-\frac{5}{2}x^2+\frac{35}{8}x^4+\mathrm{O}\left(x^6\right)\right)+\frac{c_2(-203212800+101606400x^2-25401600x^4+\mathrm{O}\left(x^6\right))}{x^5}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 46

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]+x*(3+10*x^2)*y'[x]-(15-14*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{1}{x^5}-\frac{1}{2x^3}+\frac{1}{8x}\right)+c_2\left(\frac{35x^7}{8}-\frac{5x^5}{2}+x^3\right)$$

16.4 problem Example 7.7.4 page 387

Internal problem ID [1416]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: Example 7.7.4 page 387.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-2x^2+1)\,y''+x(-13x^2+7)\,y'-14x^2y=0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
Order:=6;
dsolve(x^2*(1-2*x^2)*diff(y(x),x$2)+x*(7-13*x^2)*diff(y(x),x)-14*x^2*y(x)=0,y(x),type='series
```

$$y(x)=c_1\left(1+\frac{7}{8}x^2+\frac{77}{80}x^4+\mathrm{O}\left(x^6\right)\right)+\frac{c_2(-86400+216000x^2-54000x^4+\mathrm{O}\left(x^6\right))}{x^6}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 44

```
AsymptoticDSolveValue[x^2*(1-2*x^2)*y''[x]+x*(7-13*x^2)*y'[x]-14*x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{77x^4}{80}+\frac{7x^2}{8}+1\right)+c_1\left(\frac{1}{x^6}-\frac{5}{2x^4}+\frac{5}{8x^2}\right)$$

16.5 problem 1

Internal problem ID [1417]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 3y'x + (4x+3)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+(3+4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x\left(c_1x^2\left(1-\frac{4}{3}x+\frac{2}{3}x^2-\frac{8}{45}x^3+\frac{4}{135}x^4-\frac{16}{4725}x^5+\mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(16x^2-\frac{64}{3}x^3+\frac{32}{3}x^4-\frac{128}{45}x^5+\mathrm{O}\left(x^6\right)\right) + \left(-2-8x+\frac{256}{9}x^3-\frac{200}{9}x^4+\frac{5024}{675}x^5+\mathrm{O}\left(x^6\right)\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x^2*y''[x]-3*x*y'[x]+(3+4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{9}x\left(124x^4-176x^3+36x^2+36x+9\right)-\frac{8}{3}x^3\left(2x^2-4x+3\right)\log(x)\right) + c_2\left(\frac{4x^7}{135}-\frac{8x^6}{45}+\frac{2x^5}{3}-\frac{4x^4}{3}+x^3\right)$$

16.6 problem 2

Internal problem ID [1418]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 58

```
Order:=6;
dsolve(x*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x\left(1 - \frac{1}{2}x + \frac{1}{12}x^2 - \frac{1}{144}x^3 + \frac{1}{2880}x^4 - \frac{1}{86400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(-x + \frac{1}{2}x^2 - \frac{1}{12}x^3 + \frac{1}{144}x^4 - \frac{1}{2880}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(1 - \frac{3}{4}x^2 + \frac{7}{36}x^3 - \frac{35}{1728}x^4 + \frac{101}{86400}x^5 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x*y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{144}x\left(x^3 - 12x^2 + 72x - 144\right)\log(x) + \frac{-47x^4 + 480x^3 - 2160x^2 + 1728x + 1728}{1728}\right) + c_2\left(\frac{x^5}{2880} - \frac{x^4}{144} + \frac{x^3}{12} - \frac{x^2}{2} + x\right)$$

16.7 problem 3

Internal problem ID [1419]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(x+1)\,y''+4x(1+2x)\,y'-(3x+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(1+2*x)*diff(y(x),x)-(1+3*x)*y(x)=0,y(x),type='series',
```

$$y(x) = \frac{c_1(1+\mathrm{O}(x^6))\,x+(x+\mathrm{O}(x^6))\ln(x)\,c_2+\left(1-x-x^2+\frac{1}{2}x^3-\frac{1}{3}x^4+\frac{1}{4}x^5+\mathrm{O}(x^6)\right)c_2}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 53

```
AsymptoticDSolveValue[4*x^2*(1+x)*y''[x]+4*x*(1+2*x)*y'[x]-(1+3*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\sqrt{x}\log(x)-\frac{2x^4-3x^3+6x^2+6x-6}{6\sqrt{x}}\right)+c_2\sqrt{x}$$

16.8 problem 4

Internal problem ID [1420]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x+1)\,y''+y'x+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 60

```
Order:=6;
dsolve(x*(1+x)*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x\left(1-x+\frac{5}{6}x^2-\frac{25}{36}x^3+\frac{85}{144}x^4-\frac{221}{432}x^5+\mathrm{O}\left(x^6\right)\right) \\ & +\left(-x+x^2-\frac{5}{6}x^3+\frac{25}{36}x^4-\frac{85}{144}x^5+\mathrm{O}\left(x^6\right)\right)\ln\left(x\right)c_2 \\ & +\left(1-x+\frac{1}{2}x^2-\frac{7}{18}x^3+\frac{145}{432}x^4-\frac{257}{864}x^5+\mathrm{O}\left(x^6\right)\right)c_2 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x*(1+x)*y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \to {} & c_1\left(\frac{1}{36}x\left(25x^3-30x^2+36x-36\right)\log(x)+\frac{1}{432}\left(-455x^4+552x^3-648x^2+432x+432\right)\right) \\ & +c_2\left(\frac{85x^5}{144}-\frac{25x^4}{36}+\frac{5x^3}{6}-x^2+x\right) \end{aligned}$$

16.9 problem 5

Internal problem ID [1421]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(3x+2)\,y''+x(4+21x)\,y'-(1-9x)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
Order:=6;
dsolve(2*x^2*(2+3*x)*diff(y(x),x$2)+x*(4+21*x)*diff(y(x),x)-(1-9*x)*y(x)=0,y(x),type='series'
```

$$y(x) = \frac{c_1x\left(1-\frac{9}{4}x+\frac{135}{32}x^2-\frac{945}{128}x^3+\frac{25515}{2048}x^4-\frac{168399}{8192}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(\ln\left(x\right)\left(-\frac{3}{4}x+\frac{27}{16}x^2-\frac{405}{128}x^3+\frac{2835}{512}x^4-\right.\right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 108

```
AsymptoticDSolveValue[2*x^2*(2+3*x)*y''[x]+x*(4+21*x)*y'[x]-(1-9*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{25515x^{9/2}}{2048}-\frac{945x^{7/2}}{128}+\frac{135x^{5/2}}{32}-\frac{9x^{3/2}}{4}+\sqrt{x}\right)+c_1\left(\frac{3}{512}\sqrt{x}\left(945x^3-540x^2+288x-128\right)\log(x)+\frac{8613x^4-5076x^3+2880x^2-1536x+1024}{1024\sqrt{x}}\right)$$

16.10 problem 6

Internal problem ID [1422]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(2+x)\,y' - (-3x+2)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(2+x)*diff(y(x),x)-(2-3*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^3\left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2(\ln\left(x\right)\left(\left(-6\right)x^3 + 6x^4 - 3x^5 + \mathrm{O}\left(x^6\right)\right) + (12 + 6x}{x^2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 74

```
AsymptoticDSolveValue[x^2*y''[x]+x*(2+x)*y'[x]-(2-3*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4 - 3x^3 + 2x^2 + 2x + 4}{4x^2} + \frac{1}{2}(x-1)x\log(x)\right) + c_2\left(\frac{x^5}{24} - \frac{x^4}{6} + \frac{x^3}{2} - x^2 + x\right)$$

16.11 problem 7

Internal problem ID [1423]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4y'x - (-x+9)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)-(9-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^3\left(1 - \frac{1}{16}x + \frac{1}{640}x^2 - \frac{1}{46080}x^3 + \frac{1}{5160960}x^4 - \frac{1}{825753600}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(-\frac{1}{64}x^3 + \frac{1}{1024}x^4 - \frac{1}{40960}x^5\right.\right.}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 96

```
AsymptoticDSolveValue[4*x^2*y''[x]+4*x*y'[x]-(9-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^{11/2}}{5160960} - \frac{x^{9/2}}{46080} + \frac{x^{7/2}}{640} - \frac{x^{5/2}}{16} + x^{3/2}\right) + c_1\left(\frac{(x-16)x^{3/2}\log(x)}{12288} - \frac{19x^4 - 64x^3 - 2304x^2 - 18432x - 147456}{147456x^{3/2}}\right)$$

16.12 problem 8

Internal problem ID [1424]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 10y'x + (14 + x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 59

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+10*x*diff(y(x),x)+(14+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{6}x + \frac{1}{84}x^2 - \frac{1}{2016}x^3 + \frac{1}{72576}x^4 - \frac{1}{3628800}x^5 + \mathrm{O}\left(x^6\right)\right)x^5 + c_2(\ln(x)\left(-x^5 + \mathrm{O}\left(x^6\right)\right) + (2880 + 720x}{x^7}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 68

```
AsymptoticDSolveValue[x^2*y''[x]+10*x*y'[x]+(14+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^2}{72576} + \frac{1}{x^2} - \frac{x}{2016} - \frac{1}{6x} + \frac{1}{84}\right) + c_1\left(\frac{1}{x^7} + \frac{1}{4x^6} + \frac{1}{24x^5} + \frac{1}{144x^4} + \frac{1}{576x^3}\right)$$

16.13 problem 9

Internal problem ID [1425]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x+1)\,y'' + 4x(3+8x)\,y' - (5-49x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 65

```
Order:=6;
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(3+8*x)*diff(y(x),x)-(5-49*x)*y(x)=0,y(x),type='series'
```

$$y(x) = \frac{c_1 x^3(1 - 4x + 10x^2 - 20x^3 + 35x^4 - 56x^5 + \mathrm{O}(x^6)) + c_2(\ln(x)\,((-36)\,x^3 + 144x^4 - 360x^5 + \mathrm{O}(x^6)) + (}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 86

```
AsymptoticDSolveValue[4*x^2*(1+x)*y''[x]+4*x*(3+8*x)*y'[x]-(5-49*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(35x^{9/2} - 20x^{7/2} + 10x^{5/2} - 4x^{3/2} + \sqrt{x}\right) + c_1\left(\frac{62x^4 - 20x^3 + 2x^2 + x + 2}{2x^{5/2}} + 3\sqrt{x}(4x-1)\log(x)\right)$$

16.14 problem 10

Internal problem ID [1426]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x+1)\,y''-x(3+10x)\,y'+30yx=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 48

```
Order:=6;
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(3+10*x)*diff(y(x),x)+30*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^4\left(1-\frac{2}{5}x+\mathrm{O}\left(x^6\right)\right)+\left(43200x^4-17280x^5+\mathrm{O}\left(x^6\right)\right)\ln\left(x\right)c_2 \\ +\left(-144-1440x-7200x^2-28800x^3-90720x^4+82944x^5+\mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 48

```
AsymptoticDSolveValue[x^2*(1+x)*y''[x]-x*(3+10*x)*y'[x]+30*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(x^4-\frac{2x^5}{5}\right)+c_1\left(745x^4-300x^4\log(x)+200x^3+50x^2+10x+1\right)$$

16.15 problem 11

Internal problem ID [1427]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(x+1)\,y' - 3y(x+3) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(1+x)*diff(y(x),x)-3*(3+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^3\left(1 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2(-86400 + 103680x - 64800x^2 + 28800x^3 - 10800x^4 + 4320x^5 + \mathrm{O}\left(x^6\right))}{x^3}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 39

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1+x)*y'[x]-3*(3+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2x^3 + c_1\left(\frac{1}{x^3} - \frac{6}{5x^2} + \frac{x}{8} + \frac{3}{4x} - \frac{1}{3}\right)$$

16.16 problem 12

Internal problem ID [1428]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(1-2x)\,y' - (x+4)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 61

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(1-2*x)*diff(y(x),x)-(4+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^4\left(1 + x + \frac{7}{12}x^2 + \frac{1}{4}x^3 + \frac{11}{128}x^4 + \frac{143}{5760}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(9x^4 + 9x^5 + \mathrm{O}\left(x^6\right)\right) + \left(-144 - 144x -\right.\right.}{x^2}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 75

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1-2*x)*y'[x]-(4+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{3x^4 - 16x^3 + 48x^2 + 192x + 192}{192x^2} - \frac{1}{16}x^2\log(x)\right) + c_2\left(\frac{11x^6}{128} + \frac{x^5}{4} + \frac{7x^4}{12} + x^3 + x^2\right)$$

16.17 problem 13

Internal problem ID [1429]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(x+1)\,y''-4y'-2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 38.375 (sec). Leaf size: 40

```
Order:=6;
dsolve(x*(1+x)*diff(y(x),x$2)-4*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^5\left(1-3x+6x^2-10x^3+15x^4-21x^5+\mathrm{O}\left(x^6\right)\right)\\+c_2\left(2880-1440x+480x^2-480x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 48

```
AsymptoticDSolveValue[x*(1+x)*y''[x]-4*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{x^2}{6}-\frac{x}{2}+1\right)+c_2\left(15x^9-10x^8+6x^7-3x^6+x^5\right)$$

16.18 problem 14

Internal problem ID [1430]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+2x)\,y''+x(9+13x)\,y'+(7+5x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 41

```
Order:=6;
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*(9+13*x)*diff(y(x),x)+(7+5*x)*y(x)=0,y(x),type='series',x
```

$$\begin{aligned} y(x) = {} & \frac{c_1\left(1+\frac{4}{7}x-\frac{5}{28}x^2+\frac{5}{42}x^3-\frac{5}{48}x^4+\frac{7}{66}x^5+\mathrm{O}\left(x^6\right)\right)}{x} \\ & + \frac{c_2(-86400-449280x-617760x^2+\mathrm{O}\left(x^6\right))}{x^7} \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 54

```
AsymptoticDSolveValue[x^2*(1+2*x)*y''[x]+x*(9+13*x)*y'[x]+(7+5*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(-\frac{5x^3}{48}+\frac{5x^2}{42}-\frac{5x}{28}+\frac{1}{x}+\frac{4}{7}\right)+c_1\left(\frac{1}{x^7}+\frac{26}{5x^6}+\frac{143}{20x^5}\right)$$

16.19 problem 15

Internal problem ID [1431]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(1+2x)\,y'' - 2x(-x+4)\,y' - (7+5x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
Order:=6;
dsolve(4*x^2*(1+2*x)*diff(y(x),x$2)-2*x*(4-x)*diff(y(x),x)-(7+5*x)*y(x)=0,y(x),type='series',
```

$$y(x) = \frac{c_1x^4\left(1-\frac{18}{5}x+\frac{39}{4}x^2-\frac{663}{28}x^3+\frac{13923}{256}x^4-\frac{7735}{64}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(-144-\frac{405}{8}x^4+\frac{729}{4}x^5+\mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 67

```
AsymptoticDSolveValue[4*x^2*(1+2*x)*y''[x]-2*x*(4-x)*y'[x]-(7+5*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{\sqrt{x}}-\frac{35x^{7/2}}{128}\right)+c_2\left(\frac{13923x^{15/2}}{256}-\frac{663x^{13/2}}{28}+\frac{39x^{11/2}}{4}-\frac{18x^{9/2}}{5}+x^{7/2}\right)$$

16.20 problem 16

Internal problem ID [1432]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3x^2(x+3)\,y''-x(15+x)\,y'-20y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(3*x^2*(3+x)*diff(y(x),x$2)-x*(15+x)*diff(y(x),x)-20*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^4\left(1-\frac{4}{9}x+\frac{13}{81}x^2-\frac{832}{15309}x^3+\frac{2470}{137781}x^4-\frac{21736}{3720087}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(-144-\frac{64}{3}x+\frac{16}{27}x^2-\frac{112}{6561}x^4+\frac{448}{59049}\right.}{x^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 85

```
AsymptoticDSolveValue[3*x^2*(3+x)*y''[x]-x*(15+x)*y'[x]-20*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{7x^{10/3}}{59049}-\frac{x^{4/3}}{243}+\frac{1}{x^{2/3}}+\frac{4\sqrt[3]{x}}{27}\right)+c_2\left(\frac{2470x^{22/3}}{137781}-\frac{832x^{19/3}}{15309}+\frac{13x^{16/3}}{81}-\frac{4x^{13/3}}{9}+x^{10/3}\right)$$

16.21 problem 17

Internal problem ID [1433]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x+1)\,y''+x(1-10x)\,y'-(9-10x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*(1+x)*diff(y(x),x$2)+x*(1-10*x)*diff(y(x),x)-(9-10*x)*y(x)=0,y(x),type='series',x=
```

$$y(x)=c_1x^3\left(1+2x+\frac{9}{4}x^2+\frac{5}{3}x^3+\frac{5}{6}x^4+\frac{3}{11}x^5+\mathrm{O}\left(x^6\right)\right)$$
$$+\frac{c_2(-86400-898560x-4043520x^2-9884160x^3-12355200x^4+\mathrm{O}\left(x^6\right))}{x^3}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 64

```
AsymptoticDSolveValue[x^2*(1+x)*y''[x]+x*(1-10*x)*y'[x]-(9-10*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{1}{x^3}+\frac{52}{5x^2}+143x+\frac{234}{5x}+\frac{572}{5}\right)+c_2\left(\frac{5x^7}{6}+\frac{5x^6}{3}+\frac{9x^5}{4}+2x^4+x^3\right)$$

16.22 problem 18

Internal problem ID [1434]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x+1)\,y''+3y'x^2-(-x+6)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
Order:=6;
dsolve(x^2*(1+x)*diff(y(x),x$2)+3*x^2*diff(y(x),x)-(6-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^3\left(1-\frac{8}{3}x+\frac{100}{21}x^2-\frac{50}{7}x^3+\frac{175}{18}x^4-\frac{112}{9}x^5+\mathrm{O}\left(x^6\right)\right)+\frac{c_2(2880+720x+\mathrm{O}\left(x^6\right))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 53

```
AsymptoticDSolveValue[x^2*(1+x)*y''[x]+3*x^2*y'[x]-(6-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{1}{x^2}+\frac{1}{4x}\right)+c_2\left(\frac{175x^7}{18}-\frac{50x^6}{7}+\frac{100x^5}{21}-\frac{8x^4}{3}+x^3\right)$$

16.23 problem 19

Internal problem ID [1435]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+2x)\,y''-2x(3+14x)\,y'+(6+100x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
Order:=6;
dsolve(x^2*(1+2*x)*diff(y(x),x$2)-2*x*(3+14*x)*diff(y(x),x)+(6+100*x)*y(x)=0,y(x),type='serie
```

$$y(x)=c_1x^6\left(1+\frac{4}{3}x+\frac{8}{7}x^2+\frac{4}{7}x^3+\frac{8}{63}x^4+\mathrm{O}\left(x^6\right)\right)\\
+c_2x\left(2880+51840x+414720x^2+1935360x^3+5806080x^4+11612160x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 64

```
AsymptoticDSolveValue[x^2*(1+2*x)*y''[x]-2*x*(3+14*x)*y'[x]+(6+100*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(2016x^5+672x^4+144x^3+18x^2+x\right)+c_2\left(\frac{8x^{10}}{63}+\frac{4x^9}{7}+\frac{8x^8}{7}+\frac{4x^7}{3}+x^6\right)$$

16.24 problem 20

Internal problem ID [1436]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+1)\,y'' - x(6+11x)\,y' + (6+32x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
Order:=6;
dsolve(x^2*(1+x)*diff(y(x),x$2)-x*(6+11*x)*diff(y(x),x)+(6+32*x)*y(x)=0,y(x),type='series',x=
```

$$y(x) = c_1x^6\left(1+\frac{2}{3}x+\frac{1}{7}x^2+\mathrm{O}\left(x^6\right)\right) + c_2x\left(2880+15120x+30240x^2+25200x^3-15120x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 51

```
AsymptoticDSolveValue[x^2*(1+x)*y''[x]-x*(6+11*x)*y'[x]+(6+32*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^8}{7}+\frac{2x^7}{3}+x^6\right)+c_1\left(\frac{35x^4}{4}+\frac{21x^3}{2}+\frac{21x^2}{4}+x\right)$$

16.25 problem 21

Internal problem ID [1437]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2(x+1)\,y''+4x(4x+1)\,y'-(49+27x)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(4*x^2*(1+x)*diff(y(x),x$2)+4*x*(1+4*x)*diff(y(x),x)-(49+27*x)*y(x)=0,y(x),type='series
```

$$y(x) = \frac{c_1x^7(1-2x+3x^2-4x^3+5x^4-6x^5+\mathrm{O}(x^6))+c_2(3628800-3024000x+2419200x^2-1814400x^3+1}{x^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 86

```
AsymptoticDSolveValue[4*x^2*(1+x)*y''[x]+4*x*(1+4*x)*y'[x]-(49+27*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{2}{3x^{3/2}}-\frac{5}{6x^{5/2}}+\frac{1}{x^{7/2}}+\frac{\sqrt{x}}{3}-\frac{1}{2\sqrt{x}}\right)+c_2\left(5x^{15/2}-4x^{13/2}+3x^{11/2}-2x^{9/2}+x^{7/2}\right)$$

16.26 problem 22

Internal problem ID [1438]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$x^2(1+2x)\,y''-x(9+8x)\,y'-12yx=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(x^2*(1+2*x)*diff(y(x),x$2)-x*(9+8*x)*diff(y(x),x)-12*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^{10}\left(1-8x+40x^2-160x^3+560x^4-1792x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(-1316818944000+1755758592000x-2194698240000x^2+2508226560000x^3-2508226560000x^4+2006581248000x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 62

```
AsymptoticDSolveValue[x^2*(1+2*x)*y''[x]-x*(9+8*x)*y'[x]-12*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{40x^4}{21}-\frac{40x^3}{21}+\frac{5x^2}{3}-\frac{4x}{3}+1\right)+c_2\left(560x^{14}-160x^{13}+40x^{12}-8x^{11}+x^{10}\right)$$

16.27 problem 23

Internal problem ID [1439]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''-x(-2x^2+7)\,y'+12y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
Order:=6;
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(7-2*x^2)*diff(y(x),x)+12*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(c_1x^4\left(1-\frac{7}{2}x^2+\frac{63}{8}x^4+\mathrm{O}\left(x^6\right)\right)+c_2\left(\ln(x)\left(1080x^4+\mathrm{O}\left(x^6\right)\right)+\left(-144-216x^2+2106x^4+\mathrm{O}\left(x^6\right)\right)\right)\right)x^2$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 57

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]-x*(7-2*x^2)*y'[x]+12*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{63x^{10}}{8}-\frac{7x^8}{2}+x^6\right)+c_1\left(-\frac{15}{2}x^6\log(x)-\frac{1}{4}\left(31x^4-6x^2-4\right)x^2\right)$$

16.28 problem 24

Internal problem ID [1440]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - x(-x^2+7)\,y' + 12y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(7-x^2)*diff(y(x),x)+12*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(c_1x^4\left(1-\frac{1}{2}x^2+\frac{1}{8}x^4+\mathrm{O}\left(x^6\right)\right)\right. \\ \left.+c_2\left(\ln(x)\left(72x^4+\mathrm{O}\left(x^6\right)\right)+\left(-144-72x^2+54x^4+\mathrm{O}\left(x^6\right)\right)\right)\right)x^2$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 55

```
AsymptoticDSolveValue[x^2*y''[x]-x*(7-x^2)*y'[x]+12*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^{10}}{8}-\frac{x^8}{2}+x^6\right)+c_1\left(-\frac{1}{2}x^6\log(x)-\frac{1}{4}\left(x^4-2x^2-4\right)x^2\right)$$

16.29 problem 25

Internal problem ID [1441]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' - 5y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
Order:=6;
dsolve(x*diff(y(x),x$2)-5*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^6 \left(1 - \frac{1}{16}x^2 + \frac{1}{640}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(-86400 - 10800x^2 - 1350x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 44

```
AsymptoticDSolveValue[x*y''[x]-5*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{64} + \frac{x^2}{8} + 1\right) + c_2\left(\frac{x^{10}}{640} - \frac{x^8}{16} + x^6\right)$$

16.30 problem 26

Internal problem ID [1442]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(2x^2+1)\, y' - (-10x^2+1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(1+2*x^2)*diff(y(x),x)-(1-10*x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1 - \frac{3}{2}x^2 + x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(8x^2 - 12x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-2 + 2x^2 + 4x^4 + \mathrm{O}\left(x^6\right)\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 52

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1+2*x^2)*y'[x]-(1-10*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x^5 - \frac{3x^3}{2} + x\right) + c_1\left(2x\left(3x^2-2\right)\log(x) - \frac{5x^4 - x^2 - 1}{x}\right)$$

16.31 problem 27

Internal problem ID [1443]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - y'x - (-x^2 + 3)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)-(3-x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^4\left(1 - \frac{1}{12}x^2 + \frac{1}{384}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(9x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-144 - 36x^2 + \mathrm{O}\left(x^6\right)\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 52

```
AsymptoticDSolveValue[x^2*y''[x]-x*y'[x]-(3-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{(x^2+8)^2}{64x} - \frac{1}{16}x^3\log(x)\right) + c_2\left(\frac{x^7}{384} - \frac{x^5}{12} + x^3\right)$$

16.32 problem 28

Internal problem ID [1444]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 2x(x^2+8)\,y' + (3x^2+5)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+2*x*(8+x^2)*diff(y(x),x)+(5+3*x^2)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = \frac{c_1\left(1-\frac{1}{16}x^2+\frac{1}{256}x^4+\mathrm{O}\left(x^6\right)\right)x^2+c_2\left(\ln(x)\left(-\frac{1}{2}x^2+\frac{1}{32}x^4+\mathrm{O}\left(x^6\right)\right)+\left(-2+\frac{1}{2}x^2-\frac{3}{128}x^4+\mathrm{O}\left(x^6\right)\right)\right)}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 72

```
AsymptoticDSolveValue[4*x^2*y''[x]+2*x*(8+x^2)*y'[x]+(5+3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^{7/2}}{256}-\frac{x^{3/2}}{16}+\frac{1}{\sqrt{x}}\right)+c_1\left(\frac{5x^4-96x^2+256}{256x^{5/2}}-\frac{(x^2-16)\log(x)}{64\sqrt{x}}\right)$$

16.33 problem 29

Internal problem ID [1445]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + x(x^2+1)\,y' - (-3x^2+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(1+x^2)*diff(y(x),x)-(1-3*x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1-\frac{1}{2}x^2+\frac{1}{8}x^4+\mathrm{O}\left(x^6\right)\right)+c_2(\ln\left(x\right)\left(2x^2-x^4+\mathrm{O}\left(x^6\right)\right)+\left(-2+x^2+\mathrm{O}\left(x^6\right)\right))}{x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 51

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1+x^2)*y'[x]-(1-3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{8}-\frac{x^3}{2}+x\right)+c_1\left(\frac{1}{2}x\left(x^2-2\right)\log(x)-\frac{x^4-4}{4x}\right)$$

16.34 problem 30

Internal problem ID [1446]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(-2x^2+1)\,y' - 4y(2x^2+1) = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(1-2*x^2)*diff(y(x),x)-4*(1+2*x^2)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = \frac{c_1x^4\left(1+x^2+\frac{1}{2}x^4+\mathrm{O}\left(x^6\right)\right)+c_2\left(\ln\left(x\right)\left(288x^4+\mathrm{O}\left(x^6\right)\right)+\left(-144+144x^2+216x^4+\mathrm{O}\left(x^6\right)\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 45

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1-2*x^2)*y'[x]-4*(1+2*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-2x^2\log(x) - \frac{x^4+x^2-1}{x^2}\right) + c_2\left(\frac{x^6}{2}+x^4+x^2\right)$$

16.35 problem 31

Internal problem ID [1447]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 8y'x - \left(-x^2 + 35\right) y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+8*x*diff(y(x),x)-(35-x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^6 \left(1 - \frac{1}{64}x^2 + \frac{1}{10240}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(-86400 - 2700x^2 - \frac{675}{8}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 58

```
AsymptoticDSolveValue[4*x^2*y''[x]+8*x*y'[x]-(35-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{32x^{3/2}} + \frac{1}{x^{7/2}} + \frac{\sqrt{x}}{1024}\right) + c_2\left(\frac{x^{13/2}}{10240} - \frac{x^{9/2}}{64} + x^{5/2}\right)$$

16.36 problem 32

Internal problem ID [1448]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$9x^2y'' - 3x(2x^2+11)\,y' + (10x^2+13)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
Order:=6;
dsolve(9*x^2*diff(y(x),x$2)-3*x*(11+2*x^2)*diff(y(x),x)+(13+10*x^2)*y(x)=0,y(x),type='series'
```

$$y(x) = \left(x^4\left(1+\frac{4}{27}x^2+\frac{7}{486}x^4+\mathrm{O}\left(x^6\right)\right)c_1 + c_2\left(\ln(x)\left(-\frac{32}{9}x^4+\mathrm{O}\left(x^6\right)\right)+\left(-144-32x^2-\frac{8}{3}x^4+\mathrm{O}\left(x^6\right)\right)\right)\right)x^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 62

```
AsymptoticDSolveValue[9*x^2*y''[x]-3*x*(11+2*x^2)*y'[x]+(13+10*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{7x^{25/3}}{486}+\frac{4x^{19/3}}{27}+x^{13/3}\right)+c_1\left(\frac{2}{81}x^{13/3}\log(x)+\frac{1}{81}\left(x^2+9\right)^2\sqrt[3]{x}\right)$$

16.37 problem 33

Internal problem ID [1449]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$\boxed{x^2y'' + x(-2x^2+1)\,y' - 4(1-x^2)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(1-2*x^2)*diff(y(x),x)-4*(1-x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2(\ln(x)\,(288x^4 + \mathrm{O}(x^6)) + (-144 - 288x^2 - 216x^4 + \mathrm{O}(x^6)))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 37

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1-2*x^2)*y'[x]-4*(1-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2x^2 + c_1\left(\frac{2x^4 + 2x^2 + 1}{x^2} - 2x^2\log(x)\right)$$

16.38 problem 34

Internal problem ID [1450]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(-3x^2+1)\,y' - 4(-3x^2+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*(1-3*x^2)*diff(y(x),x)-4*(1-3*x^2)*y(x)=0,y(x),type='series',x=0)
```

$$y(x) = c_1x^2\left(1 - \frac{1}{2}x^2 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2(\ln(x)\left(1944x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-144 - 648x^2 - 810x^4 + \mathrm{O}\left(x^6\right)\right))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 50

```
AsymptoticDSolveValue[x^2*y''[x]+x*(1-3*x^2)*y'[x]-4*(1-3*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x^2 - \frac{x^4}{2}\right) + c_1\left(\frac{18x^4 + 9x^2 + 2}{2x^2} - \frac{27}{2}x^2\log(x)\right)$$

16.39 problem 35

Internal problem ID [1451]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x^2+1)\,y''+x(11x^2+5)\,y'+24x^2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
Order:=6;
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(5+11*x^2)*diff(y(x),x)+24*x^2*y(x)=0,y(x),type='series',
```

$$y(x)=c_1\left(1-2x^2+3x^4+\mathrm{O}\left(x^6\right)\right)+\frac{c_2(-144+432x^4+\mathrm{O}\left(x^6\right))}{x^4}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 27

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]+x*(5+11*x^2)*y'[x]+24*x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{1}{x^4}-1\right)+c_2\left(3x^4-2x^2+1\right)$$

16.40 problem 36

Internal problem ID [1452]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(x^2+1)\,y''+8y'x-(-x^2+35)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+8*x*diff(y(x),x)-(35-x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\frac{c_1x^6\left(1-\frac{1}{4}x^2+\frac{1}{10}x^4+\mathrm{O}\left(x^6\right)\right)+c_2\left(-86400-172800x^2-86400x^4+\mathrm{O}\left(x^6\right)\right)}{x^{\frac{7}{2}}}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 52

```
AsymptoticDSolveValue[4*x^2*(1+x^2)*y''[x]+8*x*y'[x]-(35-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{2}{x^{3/2}}+\frac{1}{x^{7/2}}+\sqrt{x}\right)+c_2\left(\frac{x^{13/2}}{10}-\frac{x^{9/2}}{4}+x^{5/2}\right)$$

16.41 problem 37

Internal problem ID [1453]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x^2+1)\,y''-x(-x^2+5)\,y'-(25x^2+7)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*(1+x^2)*diff(y(x),x$2)-x*(5-x^2)*diff(y(x),x)-(7+25*x^2)*y(x)=0,y(x),type='series'
```

$$y(x)=c_1x^7\left(1-\frac{6}{5}x^2+\frac{7}{5}x^4+\mathrm{O}\left(x^6\right)\right)+\frac{c_2(-203212800+406425600x^2-609638400x^4+\mathrm{O}\left(x^6\right))}{x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 40

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]-x*(5-x^2)*y'[x]-(7+25*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(3x^3-2x+\frac{1}{x}\right)+c_2\left(\frac{7x^{11}}{5}-\frac{6x^9}{5}+x^7\right)$$

16.42 problem 38

Internal problem ID [1454]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2+1)\,y''+x(2x^2+5)\,y'-21y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*(5+2*x^2)*diff(y(x),x)-21*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^3\left(1-\frac{1}{2}x^2+\frac{15}{56}x^4+\mathrm{O}\left(x^6\right)\right)+\frac{c_2(-1316818944000-3456649728000x^2-2880541440000x^4+\mathrm{O}\left(x^6\right))}{x^7}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 46

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]+x*(5+2*x^2)*y'[x]-21*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{1}{x^7}+\frac{21}{8x^5}+\frac{35}{16x^3}\right)+c_2\left(\frac{15x^7}{56}-\frac{x^5}{2}+x^3\right)$$

16.43 problem 39

Internal problem ID [1455]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(1+2x^2)\,y'' - x(x^2+3)\,y' - 2yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
Order:=6;
dsolve(x^2*(1+2*x^2)*diff(y(x),x$2)-x*(3+x^2)*diff(y(x),x)-2*x*y(x)=0,y(x),type='series',x=0)
```

$$\begin{aligned} y(x) = {} & c_1 x^4\left(1+\frac{2}{5}x-\frac{8}{5}x^2-\frac{86}{105}x^3+\frac{445}{168}x^4+\frac{9571}{6300}x^5+\mathrm{O}\left(x^6\right)\right) \\ & + c_2\left(\ln(x)\left(24x^4+\frac{48}{5}x^5+\mathrm{O}\left(x^6\right)\right)\right. \\ & \left. + \left(-144+96x-48x^2+210x^4+\frac{1812}{25}x^5+\mathrm{O}\left(x^6\right)\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 71

```
AsymptoticDSolveValue[x^2*(1+2*x^2)*y''[x]-x*(3+x^2)*y'[x]-2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{12}(3x^4+4x^2-8x+12)-\frac{1}{6}x^4\log(x)\right)+c_2\left(\frac{445x^8}{168}-\frac{86x^7}{105}-\frac{8x^6}{5}+\frac{2x^5}{5}+x^4\right)$$

16.44 problem 40

Internal problem ID [1456]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 7 Series Solutions of Linear Second Equations. 7.6 THE METHOD OF FROBENIUS III. Exercises 7.7. Page 389
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2(x^2+1)\,y''+4x(x^2+2)\,y'-(x^2+15)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(4*x^2*(1+x^2)*diff(y(x),x$2)+4*x*(2+x^2)*diff(y(x),x)-(15+x^2)*y(x)=0,y(x),type='serie
```

$$y(x)=\frac{c_1x^4\left(1-\frac{1}{6}x^2+\frac{1}{16}x^4+\mathrm{O}\left(x^6\right)\right)+c_2\left(-144-216x^2-54x^4+\mathrm{O}\left(x^6\right)\right)}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 58

```
AsymptoticDSolveValue[4*x^2*(1+x^2)*y''[x]+4*x*(2+x^2)*y'[x]-(15+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{3x^{3/2}}{8}+\frac{1}{x^{5/2}}+\frac{3}{2\sqrt{x}}\right)+c_2\left(\frac{x^{11/2}}{16}-\frac{x^{7/2}}{6}+x^{3/2}\right)$$

17 Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471

17.1 problem section 9.1, problem 2

Internal problem ID [1457]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471
Problem number: section 9.1, problem 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^3y''' - x^2y'' - 2y'x + 6y = 0}$$

With initial conditions

$$[y(-1) = -4, y'(-1) = -14, y''(-1) = -20]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 20

```
dsolve([x^3*diff(y(x),x$3)-x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+6*y(x)=0,y(-1) = -4, D(y)(-1)
```

$$y(x) = \frac{22x^2}{3} + \frac{25}{3x} + 3x^3$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 23

```
DSolve[{x^3*y'''[x]-x^2*y''[x]-2*x*y'[x]+6*y[x]==0,{y[-1]==-4,y'[-1]==-14,y''[-1]==-20}},y[x]
```

$$y(x) \to \frac{(9x + 22)x^3 + 25}{3x}$$

17.2 problem section 9.1, problem 3

Internal problem ID [1458]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471
Problem number: section 9.1, problem 3.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + y''' - 7y'' - y' + 6y = 0}$$

With initial conditions

$$[y(0) = 5, y'(0) = -6, y''(0) = 10, y'''(0) = -36]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$4)+diff(y(x),x$3)-7*diff(y(x),x$2)-diff(y(x),x)+6*y(x)=0,y(0) = 5, D(y)(0
```

$$y(x) = \left(-e^{5x} + 2\,e^{4x} + 3\,e^{2x} + 1\right) e^{-3x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 30

```
DSolve[{y''''[x]+y'''[x]-7*y''[x]-y'[x]+6*y[x]==0,{y[0]==5,y'[0]==-6,y''[0]==10,y'''[0]==-36}
```

$$y(x) \to e^{-3x} + 3e^{-x} + 2e^{x} - e^{2x}$$

17.3 problem section 9.1, problem 5(b) 1

Internal problem ID [1459]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471
Problem number: section 9.1, problem 5(b) 1.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^3y''' - x^2y'' - 2y'x + 6y = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = 0, y''(1) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve([x^3*diff(y(x),x$3)-x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+6*y(x)=0,y(1) = 1, D(y)(1) = 0
```

$$y(x) = x^2 + \frac{1}{2x} - \frac{x^3}{2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 23

```
DSolve[{x^3*y'''[x]-x^2*y''[x]-2*x*y'[x]+6*y[x]==0,{y[1]==1,y'[1]==0,y''[1]==0}},y[x],x,Inclu
```

$$y(x) \to -\frac{x^3}{2} + x^2 + \frac{1}{2x}$$

17.4 problem section 9.1, problem 5(b) 2

Internal problem ID [1460]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471
Problem number: section 9.1, problem 5(b) 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^3y''' - x^2y'' - 2y'x + 6y = 0}$$

With initial conditions

$$[y(1) = 0, y'(1) = 1, y''(1) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([x^3*diff(y(x),x$3)-x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+6*y(x)=0,y(1) = 0, D(y)(1) = 1
```

$$y(x) = \frac{x^3 - 1}{3x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 17

```
DSolve[{x^3*y'''[x]-x^2*y''[x]-2*x*y'[x]+6*y[x]==0,{y[1]==0,y'[1]==1,y''[1]==0}},y[x],x,Inclu
```

$$y(x) \to \frac{x^3 - 1}{3x}$$

17.5 problem section 9.1, problem 5(b) 3

Internal problem ID [1461]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471
Problem number: section 9.1, problem 5(b) 3.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^3 y''' - x^2 y'' - 2y'x + 6y = 0}$$

With initial conditions

$$[y(1) = 0, y'(1) = 0, y''(1) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve([x^3*diff(y(x),x$3)-x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+6*y(x)=0,y(1) = 0, D(y)(1) = 0
```

$$y(x) = \frac{3x^4 - 4x^3 + 1}{12x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 23

```
DSolve[{x^3*y'''[x]-x^2*y''[x]-2*x*y'[x]+6*y[x]==0,{y[1]==0,y'[1]==0,y''[1]==1}},y[x],x,Inclu
```

$$y(x) \to \frac{(3x - 4)x^3 + 1}{12x}$$

17.6 problem section 9.1, problem 5(b)

Internal problem ID [1462]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471
Problem number: section 9.1, problem 5(b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^3y''' - x^2y'' - 2y'x + 6y = 0}$$

With initial conditions

$$[y(1) = k_0, y'(1) = k_1, y''(1) = k_2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 40

```
dsolve([x^3*diff(y(x),x$3)-x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+6*y(x)=0,y(1) = k__0, D(y)(1)
```

$$y(x) = \frac{3(k_2 - 2k_0)\,x^4 + 4(3k_0 + k_1 - k_2)\,x^3 + 6k_0 - 4k_1 + k_2}{12x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 43

```
DSolve[{x^3*y'''[x]-x^2*y''[x]-2*x*y'[x]+6*y[x]==0,{y[1]==k0,y'[1]==k1,y''[1]==k2}},y[x],x,In
```

$$y(x) \to \frac{4x^3(3\text{k0} + \text{k1} - \text{k2}) + 3x^4(\text{k2} - 2\text{k0}) + 6\text{k0} - 4\text{k1} + \text{k2}}{12x}$$

17.7 problem section 9.1, problem 6(a)

Internal problem ID [1463]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471
Problem number: section 9.1, problem 6(a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y'' - y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + \mathrm{e}^{-x} c_2 + c_3 \mathrm{e}^{-x} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[y'''[x]+y''[x]-y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2 x + c_1) + c_3 e^{x}$$

17.8 problem section 9.1, problem 6(b)

Internal problem ID [1464]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.1. Page 471
Problem number: section 9.1, problem 6(b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y'' + 7y' - 5y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+7*diff(y(x),x)-5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^x \sin(2x) + c_3 e^x \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y'''[x]-3*y''[x]+7*y'[x]-5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(c_2 \cos(2x) + c_1 \sin(2x) + c_3)$$

18 Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483

18.1 problem section 9.2, problem 1

Internal problem ID [1465]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 1.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y'' + 3y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+3*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^x x + c_3 e^x x^2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 21

```
DSolve[y'''[x]-3*y''[x]+3*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x (x(c_3 x + c_2) + c_1)$$

18.2 problem section 9.2, problem 2

Internal problem ID [1466]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 2.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 8y'' - 9y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)+8*diff(y(x),x$2)-9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{x} + e^{-x} c_2 + c_3 \sin(3x) + c_4 \cos(3x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 34

```
DSolve[y''''[x]+8*y''[x]-9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 e^{-x} + c_4 e^{x} + c_1 \cos(3x) + c_2 \sin(3x)$$

18.3 problem section 9.2, problem 3

Internal problem ID [1467]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 3.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y'' + 16y' - 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+16*diff(y(x),x)-16*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 \sin(4x) + c_3 \cos(4x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y'''[x]-y''[x]+16*y'[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 e^x + c_1 \cos(4x) + c_2 \sin(4x)$$

18.4 problem section 9.2, problem 4

Internal problem ID [1468]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 4.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{2y''' + 3y'' - 2y' - 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(2*diff(y(x),x$3)+3*diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + e^{-x} c_2 + c_3 e^{-\frac{3x}{2}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[2*y'''[x]+3*y''[x]-2*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-3x/2} + c_2 e^{-x} + c_3 e^x$$

18.5 problem section 9.2, problem 5

Internal problem ID [1469]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 5.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 5y'' + 9y' + 5y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$3)+5*diff(y(x),x$2)+9*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}c_1 + c_2\mathrm{e}^{-2x}\sin(x) + c_3\mathrm{e}^{-2x}\cos(x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 28

```
DSolve[y'''[x]+5*y''[x]+9*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(c_3e^x + c_2\cos(x) + c_1\sin(x))$$

18.6 problem section 9.2, problem 6

Internal problem ID [1470]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 6.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{4y''' - 8y'' + 5y' - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(4*diff(y(x),x$3)-8*diff(y(x),x$2)+5*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^{\frac{x}{2}} + c_3 e^{\frac{x}{2}} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 27

```
DSolve[4*y'''[x]-8*y''[x]+5*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/2}(c_2 x + c_1) + c_3 e^x$$

18.7 problem section 9.2, problem 7

Internal problem ID [1471]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 7.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{27y''' + 27y'' + 9y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(27*diff(y(x),x$3)+27*diff(y(x),x$2)+9*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x}{3}} + c_2 e^{-\frac{x}{3}} x + c_3 e^{-\frac{x}{3}} x^2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[27*y'''[x]+27*y''[x]+9*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/3}(x(c_3 x + c_2) + c_1)$$

18.8 problem section 9.2, problem 8

Internal problem ID [1472]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 8.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + y'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$4)+diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 x + c_3 \sin(x) + c_4 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 24

```
DSolve[y''''[x]+y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_4 x - c_1 \cos(x) - c_2 \sin(x) + c_3$$

18.9 problem section 9.2, problem 9

Internal problem ID [1473]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 9.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)-16*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{-2x} + c_3 \sin(2x) + c_4 \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 36

```
DSolve[y''''[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2x} + c_3 e^{-2x} + c_2 \cos(2x) + c_4 \sin(2x)$$

18.10 problem section 9.2, problem 10

Internal problem ID [1474]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 10.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 12y'' + 36y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$4)+12*diff(y(x),x$2)+36*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\sqrt{6}\, x\right) + c_2 \cos\left(\sqrt{6}\, x\right) + c_3 \sin\left(\sqrt{6}\, x\right) x + c_4 \cos\left(\sqrt{6}\, x\right) x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 38

```
DSolve[y''''[x]+12*y''[x]+36*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (c_2 x + c_1) \cos\left(\sqrt{6} x\right) + (c_4 x + c_3) \sin\left(\sqrt{6} x\right)$$

18.11 problem section 9.2, problem 11

Internal problem ID [1475]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 11.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{16y'''' - 72y'' + 81y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(16*diff(y(x),x$4)-72*diff(y(x),x$2)+81*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{3x}{2}} + c_2 \mathrm{e}^{\frac{3x}{2}} x + c_3 \mathrm{e}^{-\frac{3x}{2}} + c_4 \mathrm{e}^{-\frac{3x}{2}} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 33

```
DSolve[16*y''''[x]-72*y''[x]+81*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x/2} \left(c_2 x + e^{3x} (c_4 x + c_3) + c_1\right)$$

18.12 problem section 9.2, problem 12

Internal problem ID [1476]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 12.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$6y'''' + 5y''' + 7y'' + 5y' + y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(6*diff(y(x),x$4)+5*diff(y(x),x$3)+7*diff(y(x),x$2)+5*diff(y(x),x)+y(x)=0,y(x), singsol
```

$$y(x) = c_1 e^{-\frac{x}{2}} + c_2 e^{-\frac{x}{3}} + c_3 \sin(x) + c_4 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 37

```
DSolve[6*y''''[x]+5*y'''[x]+7*y''[x]+5*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(c_3 e^{x/6} + c_4\right) + c_1 \cos(x) + c_2 \sin(x)$$

18.13 problem section 9.2, problem 13

Internal problem ID [1477]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 13.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{4y'''' + 12y''' + 3y'' - 13y' - 6y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(4*diff(y(x),x$4)+12*diff(y(x),x$3)+3*diff(y(x),x$2)-13*diff(y(x),x)-6*y(x)=0,y(x), sin
```

$$y(x) = c_1 e^{-\frac{x}{2}} + e^x c_2 + c_3 e^{-2x} + c_4 e^{-\frac{3x}{2}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 41

```
DSolve[4*y''''[x]+12*y'''[x]+3*y''[x]-13*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{-2x}\left(e^{x/2}\left(c_2 e^x + c_4 e^{5x/2} + c_1\right) + c_3\right)$$

18.14 problem section 9.2, problem 14

Internal problem ID [1478]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 14.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 4y''' + 7y'' - 6y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+7*diff(y(x),x$2)-6*diff(y(x),x)+2*y(x)=0,y(x), singsol
```

$$y(x) = c_1 e^x + c_2 e^x x + c_3 \sin(x) e^x + c_4 \cos(x) e^x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y''''[x]-4*y'''[x]+7*y''[x]-6*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(c_4 x + c_2 \cos(x) + c_1 \sin(x) + c_3)$$

18.15 problem section 9.2, problem 15

Internal problem ID [1479]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 15.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 2y'' + 4y' - 8y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -2, y''(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$3)-2*diff(y(x),x$2)+4*diff(y(x),x)-8*y(x)=0,y(0) = 2, D(y)(0) = -2, (D@@2
```

$$y(x) = \frac{5\,\mathrm{e}^{2x}}{4} - \frac{9\sin(2x)}{4} + \frac{3\cos(2x)}{4}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 21

```
DSolve[{y'''[x]-2*y''[x]+4*y'[x]-8*y[x]==0,{y[0]==2,y'[0]==-2,y''[0]==0}},y[x],x,IncludeSingu
```

$$y(x) \to e^{2x} - 2\sin(2x) + \cos(2x)$$

18.16 problem section 9.2, problem 16

Internal problem ID [1480]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 16.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 3y'' - y' - 3y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 14, y''(0) = -40]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$3)+3*diff(y(x),x$2)-diff(y(x),x)-3*y(x)=0,y(0) = 0, D(y)(0) = 14, (D@@2)(
```

$$y(x) = (2\,e^{4x} + 3\,e^{2x} - 5)\,e^{-3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[{y'''[x]+3*y''[x]-y'[x]-3*y[x]==0,{y[0]==0,y'[0]==14,y''[0]==-40}},y[x],x,IncludeSingu
```

$$y(x) \to -5e^{-3x} + 3e^{-x} + 2e^{x}$$

18.17 problem section 9.2, problem 17

Internal problem ID [1481]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 17.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y'' - y' + y = 0}$$

With initial conditions

$$[y(0) = -2, y'(0) = 9, y''(0) = 4]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 20

```
dsolve([diff(y(x),x$3)-diff(y(x),x$2)-diff(y(x),x)+y(x)=0,y(0) = -2, D(y)(0) = 9, (D@@2)(y)(0
```

$$y(x) = -4\,\mathrm{e}^{-x} + (3x + 2)\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[{y'''[x]-y''[x]-y'[x]+y[x]==0,{y[0]==-2,y'[0]==9,y''[0]==4}},y[x],x,IncludeSingularSol
```

$$y(x) \to e^{x}(3x + 2) - 4e^{-x}$$

18.18 problem section 9.2, problem 18

Internal problem ID [1482]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 18.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 2y' - 4y = 0}$$

With initial conditions

$$[y(0) = 6, y'(0) = 3, y''(0) = 22]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve([diff(y(x),x$3)-2*diff(y(x),x)-4*y(x)=0,y(0) = 6, D(y)(0) = 3, (D@@2)(y)(0) = 22],y(x)
```

$$y(x) = (2\cos(x) - 3\sin(x))\,\mathrm{e}^{-x} + 4\,\mathrm{e}^{2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 27

```
DSolve[{y'''[x]-2*y'[x]-4*y[x]==0,{y[0]==6,y'[0]==3,y''[0]==22}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to e^{-x}\left(4e^{3x} - 3\sin(x) + 2\cos(x)\right)$$

18.19 problem section 9.2, problem 19

Internal problem ID [1483]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 19.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{3y''' - y'' - 7y' + 5y = 0}$$

With initial conditions

$$\left[y(0) = \frac{14}{5}, y'(0) = 0, y''(0) = 10\right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve([3*diff(y(x),x$3)-diff(y(x),x$2)-7*diff(y(x),x)+5*y(x)=0,y(0) = 14/5, D(y)(0) = 0, (D@
```

$$y(x) = \mathrm{e}^{-\frac{5x}{3}}\left(\frac{9}{5} + (1 + 2x)\,\mathrm{e}^{\frac{8x}{3}}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[{3*y'''[x]-y''[x]-7*y'[x]+5*y[x]==0,{y[0]==14/5,y'[0]==0,y''[0]==10}},y[x],x,IncludeSi
```

$$y(x) \to e^x(2x + 1) + \frac{9}{5}e^{-5x/3}$$

18.20 problem section 9.2, problem 20

Internal problem ID [1484]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 20.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 6y'' + 12y' - 8y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1, y''(0) = -4]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$3)-6*diff(y(x),x$2)+12*diff(y(x),x)-8*y(x)=0,y(0) = 1, D(y)(0) = -1, (D@@
```

$$y(x) = e^{2x}(2x^2 - 3x + 1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 19

```
DSolve[{y'''[x]-6*y''[x]+12*y'[x]-8*y[x]==0,{y[0]==1,y'[0]==-1,y''[0]==-4}},y[x],x,IncludeSin
```

$$y(x) \to e^{2x}(x - 1)(2x - 1)$$

18.21 problem section 9.2, problem 21

Internal problem ID [1485]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 21.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{2y''' - 11y'' + 12y' + 9y = 0}$$

With initial conditions

$$[y(0) = 6, y'(0) = 3, y''(0) = 13]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 21

```
dsolve([2*diff(y(x),x$3)-11*diff(y(x),x$2)+12*diff(y(x),x)+9*y(x)=0,y(0) = 6, D(y)(0) = 3, (D
```

$$y(x) = 4\,\mathrm{e}^{-\frac{x}{2}} + (2-x)\,\mathrm{e}^{3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[{2*y'''[x]-11*y''[x]+12*y'[x]+9*y[x]==0,{y[0]==6,y'[0]==3,y''[0]==13}},y[x],x,IncludeS
```

$$y(x) \to 4e^{-x/2} - e^{3x}(x-2)$$

18.22 problem section 9.2, problem 22

Internal problem ID [1486]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 22.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{8y''' - 4y'' - 2y' + y = 0}$$

With initial conditions

$$[y(0) = 4, y'(0) = -3, y''(0) = -1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve([8*diff(y(x),x$3)-4*diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(0) = 4, D(y)(0) = -3, (D@@2
```

$$y(x) = 3\,\mathrm{e}^{-\frac{x}{2}} + \mathrm{e}^{\frac{x}{2}} - 2\,\mathrm{e}^{\frac{x}{2}} x$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 56

```
DSolve[{8*y'''[x]-4*y''[x]-2*y'[x]-2*y[x]==0,{y[0]==4,y'[0]==-3,y''[0]==-1}},y[x],x,IncludeSi
```

$$y(x) \to \frac{2}{21}e^{-x/4}\left(51\cos\left(\frac{\sqrt{3}x}{4}\right) - 13\sqrt{3}\sin\left(\frac{\sqrt{3}x}{4}\right)\right) - \frac{6e^x}{7}$$

18.23 problem section 9.2, problem 23

Internal problem ID [1487]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 23.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 16y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 2, y''(0) = -2, y'''(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve([diff(y(x),x$4)-16*y(x)=0,y(0) = 2, D(y)(0) = 2, (D@@2)(y)(0) = -2, (D@@3)(y)(0) = 0],
```

$$y(x) = \frac{5\,e^{2x}}{8} + \frac{e^{-2x}}{8} + \frac{\sin(2x)}{2} + \frac{5\cos(2x)}{4}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 34

```
DSolve[{y''''[x]-16*y[x]==0,{y[0]==2,y'[0]==2,y''[0]==-2,y'''[0]==0}},y[x],x,IncludeSingularS
```

$$y(x) \to \frac{1}{4}(2\sin(2x) + 5\cos(2x) + 2\sinh(2x) + 3\cosh(2x))$$

18.24 problem section 9.2, problem 24

Internal problem ID [1488]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 24.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 6y''' + 7y'' + 6y' - 8y = 0}$$

With initial conditions

$$[y(0) = -2, y'(0) = -8, y''(0) = -14, y'''(0) = -62]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$4)-6*diff(y(x),x$3)+7*diff(y(x),x$2)+6*diff(y(x),x)-8*y(x)=0,y(0) = -2, D
```

$$y(x) = -e^{4x} - 4e^{x} + e^{2x} + 2e^{-x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[{y''''[x]-6*y'''[x]+7*y''[x]+6*y'[x]-8*y[x]==0,{y[0]==-2,y'[0]==-8,y''[0]==-14,y'''[0]
```

$$y(x) \to 2e^{-x} - 4e^{x} + e^{2x} - e^{4x}$$

18.25 problem section 9.2, problem 25

Internal problem ID [1489]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 25.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{4y'''' - 13y'' + 9y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 3, y''(0) = 1, y'''(0) = 3]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 15

```
dsolve([4*diff(y(x),x$4)-13*diff(y(x),x$2)+9*y(x)=0,y(0) = 1, D(y)(0) = 3, (D@@2)(y)(0) = 1,
```

$$y(x) = 2\,e^{x} - e^{-x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 12

```
DSolve[{4*y''''[x]-13*y''[x]+9*y[x]==0,{y[0]==1,y'[0]==3,y''[0]==1,y'''[0]==3}},y[x],x,Includ
```

$$y(x) \to 3\sinh(x) + \cosh(x)$$

18.26 problem section 9.2, problem 26

Internal problem ID [1490]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 26.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2y''' - 2y'' - 8y' - 8y = 0}$$

With initial conditions

$$[y(0) = 5, y'(0) = -2, y''(0) = 6, y'''(0) = 8]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 28

```
dsolve([diff(y(x),x$4)+2*diff(y(x),x$3)-2*diff(y(x),x$2)-8*diff(y(x),x)-8*y(x)=0,y(0) = 5, D(
```

$$y(x) = \left(\mathrm{e}^{4x} + 1 + (3\cos(x) + \sin(x))\,\mathrm{e}^{x}\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 29

```
DSolve[{y''''[x]+2*y'''[x]-2*y''[x]-8*y'[x]-8*y[x]==0,{y[0]==5,y'[0]==-2,y''[0]==6,y'''[0]==8
```

$$y(x) \to e^{-2x}\left(e^{4x} + e^{x}(\sin(x) + 3\cos(x)) + 1\right)$$

18.27 problem section 9.2, problem 27

Internal problem ID [1491]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 27.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{4y'''' + 8y''' + 19y'' + 32y' + 12y = 0}$$

With initial conditions

$$\left[y(0) = 3, y'(0) = -3, y''(0) = -\frac{7}{2}, y'''(0) = \frac{31}{4}\right]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 21

```
dsolve([4*diff(y(x),x$4)+8*diff(y(x),x$3)+19*diff(y(x),x$2)+32*diff(y(x),x)+12*y(x)=0,y(0) =
```

$$y(x) = 2\,\mathrm{e}^{-\frac{x}{2}} - \sin(2x) + \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 29

```
DSolve[{y''''[x]+2*y'''[x]-2*y''[x]-8*y'[x]-8*y[x]==0,{y[0]==5,y'[0]==-2,y''[0]==6,y'''[0]==8
```

$$y(x) \to e^{-2x}\left(e^{4x} + e^x(\sin(x) + 3\cos(x)) + 1\right)$$

18.28 problem section 9.2, problem 43(a)

Internal problem ID [1492]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 43(a).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$4)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{x} + e^{-x} c_2 + c_3 \sin(x) + c_4 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 30

```
DSolve[y''''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_3 e^{-x} + c_2 \cos(x) + c_4 \sin(x)$$

18.29 problem section 9.2, problem 43(b)

Internal problem ID [1493]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 43(b).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
dsolve(diff(y(x),x$4)+y(x)=0,y(x), singsol=all)
```

$$y(x) = -c_1 e^{-\frac{\sqrt{2}\,x}{2}} \sin\left(\frac{\sqrt{2}\,x}{2}\right) - c_2 e^{\frac{\sqrt{2}\,x}{2}} \sin\left(\frac{\sqrt{2}\,x}{2}\right) + c_3 e^{-\frac{\sqrt{2}\,x}{2}} \cos\left(\frac{\sqrt{2}\,x}{2}\right) + c_4 e^{\frac{\sqrt{2}\,x}{2}} \cos\left(\frac{\sqrt{2}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 65

```
DSolve[y''''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x}{\sqrt{2}}} \left(\left(c_1 e^{\sqrt{2}x} + c_2\right) \cos\left(\frac{x}{\sqrt{2}}\right) + \left(c_4 e^{\sqrt{2}x} + c_3\right) \sin\left(\frac{x}{\sqrt{2}}\right) \right)$$

18.30 problem section 9.2, problem 43(c)

Internal problem ID [1494]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 43(c).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 64y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$4)+64*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-2x} \sin(2x) + c_2 \mathrm{e}^{-2x} \cos(2x) + c_3 \mathrm{e}^{2x} \sin(2x) + c_4 \mathrm{e}^{2x} \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 44

```
DSolve[y''''[x]+64*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(\left(c_4 e^{4x} + c_1\right)\cos(2x) + \left(c_3 e^{4x} + c_2\right)\sin(2x)\right)$$

18.31 problem section 9.2, problem 43(d)

Internal problem ID [1495]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 43(d).
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(6)} - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 67

```
dsolve(diff(y(x),x$6)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + e^{-x} c_2 + c_3 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) + c_4 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) + c_5 e^{\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) + c_6 e^{\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 71

```
DSolve[y''''''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + c_4 e^{-x} + e^{-x/2}\left((c_2 e^x + c_3)\cos\left(\frac{\sqrt{3}x}{2}\right) + (c_6 e^x + c_5)\sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

18.32 problem section 9.2, problem 43(e)

Internal problem ID [1496]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 43(e).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 64y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$4)+64*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-2x} \sin(2x) + c_2 e^{-2x} \cos(2x) + c_3 e^{2x} \sin(2x) + c_4 e^{2x} \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 44

```
DSolve[y''''[x]+64*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(\left(c_4 e^{4x} + c_1\right)\cos(2x) + \left(c_3 e^{4x} + c_2\right)\sin(2x)\right)$$

18.33 problem section 9.2, problem 43(g)

Internal problem ID [1497]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.2. constant coefficient. Page 483
Problem number: section 9.2, problem 43(g).
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)}+y''''+y'''+y''+y'+y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
dsolve(diff(y(x),x$5)+diff(y(x),x$4)+diff(y(x),x$3)+diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x),
```

$$y(x)=\mathrm{e}^{-x}c_1+c_2\mathrm{e}^{-\frac{x}{2}}\sin\left(\frac{\sqrt{3}\,x}{2}\right)+c_3\mathrm{e}^{-\frac{x}{2}}\cos\left(\frac{\sqrt{3}\,x}{2}\right)+c_4\mathrm{e}^{\frac{x}{2}}\sin\left(\frac{\sqrt{3}\,x}{2}\right)+c_5\mathrm{e}^{\frac{x}{2}}\cos\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 65

```
DSolve[y'''''[x]+y''''[x]+y'''[x]+y''[x]+y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to e^{-x}\left(e^{x/2}\left((c_3e^x+c_2)\cos\left(\frac{\sqrt{3}x}{2}\right)+(c_4e^x+c_1)\sin\left(\frac{\sqrt{3}x}{2}\right)\right)+c_5\right)$$

19 Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495

19.1 problem section 9.3, problem 1

Internal problem ID [1498]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 1.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 6y'' + 11y' - 6y + e^{x}\left(-24x^{2} + 76x + 4\right) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+11*diff(y(x),x)-6*y(x)=-exp(x)*(4+76*x-24*x^2),y(x), s
```

$$y(x) = \frac{x(4x^{2} - x - 17)\left(24x^{2}e^{x} - 76x\,e^{x} - 4\,e^{x}\right)}{24x^{2} - 76x - 4} + c_{1}e^{x} + c_{2}e^{2x} + c_{3}e^{3x}$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 44

```
DSolve[y'''[x]-6*y''[x]+11*y'[x]-6*y[x]==-Exp[x]*(4+76*x-24*x^2),y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{1}{2}e^{x}(2x(x(4x - 1) - 17) + 2e^{x}(c_{3}e^{x} + c_{2}) - 49 + 2c_{1})$$

19.2 problem section 9.3, problem 2

Internal problem ID [1499]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 2y'' - 5y' + 6y - e^{-3x}(6x^2 - 23x + 32) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=exp(-3*x)*(32-23*x+6*x^2),y(x),
```

$$y(x) = -\frac{(x^2 - x + 3)\, e^{-3x}}{4} + c_1 e^x + c_2 e^{-2x} + c_3 e^{3x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 44

```
DSolve[y'''[x]-2*y''[x]-5*y'[x]+6*y[x]==Exp[-3*x]*(32-23*x+6*x^2),y[x],x,IncludeSingularSolut
```

$$y(x) \to -\frac{1}{4} e^{-3x}((x-1)x+3) + c_1 e^{-2x} + c_2 e^x + c_3 e^{3x}$$

19.3 problem section 9.3, problem 3

Internal problem ID [1500]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 3.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{4y''' + 8y'' - y' - 2y + e^{x}\left(6x^{2} + 45x + 4\right) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 64

```
dsolve(4*diff(y(x),x$3)+8*diff(y(x),x$2)-diff(y(x),x)-2*y(x)=-exp(x)*(4+45*x+6*x^2),y(x), sin
```

$$y(x) = \frac{(18x^{2} + 27x - 149)\left(-6x^{2}e^{x} - 45x\, e^{x} - 4\, e^{x}\right)}{162x^{2} + 1215x + 108} + e^{-2x}c_{1} + c_{2}e^{-\frac{x}{2}} + c_{3}e^{\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 51

```
DSolve[4*y'''[x]+8*y''[x]-y'[x]-2*y[x]==-Exp[x]*(4+45*x+6*x^2),y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{1}{27}e^{x}(149 - 9x(2x+3)) + c_{1}e^{-x/2} + c_{2}e^{x/2} + c_{3}e^{-2x}$$

19.4 problem section 9.3, problem 4

Internal problem ID [1501]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 4.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + 3y'' - y' - 3y - e^{-2x}(3x^2 - 17x + 2) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)-diff(y(x),x)-3*y(x)=exp(-2*x)*(2-17*x+3*x^2),y(x), sin
```

$$y(x) = (x^2 - 5x + 1)\,e^{-2x} + c_1 e^x + c_2 e^{-3x} + c_3 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 39

```
DSolve[y'''[x]+3*y''[x]-y'[x]-3*y[x]==Exp[-2*x]*(2-17*x+3*x^2),y[x],x,IncludeSingularSolution
```

$$y(x) \to e^{-3x}\left(e^x\left((x-5)x + c_2 e^x + c_3 e^{3x} + 1\right) + c_1\right)$$

19.5 problem section 9.3, problem 5

Internal problem ID [1502]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 5.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + 3y'' - y' - 3y - e^x(16x^3 + 24x^2 + 2x - 1) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 76

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)-diff(y(x),x)-3*y(x)=exp(x)*(-1+2*x+24*x^2+16*x^3),y(x)
```

$$y(x) = \frac{x(x^3 - x^2 + x - 1)(16\,e^x x^3 + 24x^2 e^x + 2x\,e^x - e^x)}{32x^3 + 48x^2 + 4x - 2} + c_1 e^x + c_2 e^{-3x} + c_3 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 46

```
DSolve[y'''[x]+3*y''[x]-y'[x]-3*y[x]==Exp[x]*(-1+2*x+24*x^2+16*x^3),y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{1}{4}e^x(2(x-1)x(x^2+1) + 1 + 4c_3) + c_1 e^{-3x} + c_2 e^{-x}$$

19.6 problem section 9.3, problem 6

Internal problem ID [1503]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 6.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + y'' - 2y - e^x(15x^2 + 34x + 14) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-2*y(x)=exp(x)*(14+34*x+15*x^2),y(x), singsol=all)
```

$$y(x) = \frac{x^2(x+1)(15x^2 e^x + 34x e^x + 14 e^x)}{15x^2 + 34x + 14} + c_1 e^x + c_2 \cos(x) e^{-x} + c_3 e^{-x} \sin(x)$$

✓ Solution by Mathematica

Time used: 0.156 (sec). Leaf size: 44

```
DSolve[y'''[x]+y''[x]-2*y[x]==Exp[x]*(14+34*x+15*x^2),y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{5} e^x (5(x+1)x^2 - 2 + 5c_3) + e^{-x}(c_2 \cos(x) + c_1 \sin(x))$$

19.7 problem section 9.3, problem 7

Internal problem ID [1504]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 7.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{4y''' + 8y'' - y' - 2y + e^{-2x}(1 - 15x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(4*diff(y(x),x$3)+8*diff(y(x),x$2)-diff(y(x),x)-2*y(x)=-exp(-2*x)*(1-15*x),y(x), singso
```

$$y(x) = \frac{x(2+x)\,e^{-2x}}{2} + e^{-2x}c_1 + c_2e^{-\frac{x}{2}} + c_3e^{\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 44

```
DSolve[4*y'''[x]+8*y''[x]-y'[x]-2*y[x]==-Exp[-2*x]*(1-15*x),y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{10}e^{-2x}\left(5x(x+2) + 10e^{3x/2}(c_2e^x + c_1) + 8 + 10c_3\right)$$

19.8 problem section 9.3, problem 8

Internal problem ID [1505]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 8.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' - y' + y + e^x(7 + 6x) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)-diff(y(x),x)+y(x)=-exp(x)*(7+6*x),y(x), singsol=all)
```

$$y(x) = \frac{(2+x)\,x^2(-6x\,e^x - 7\,e^x)}{12x + 14} + c_1e^x + e^{-x}c_2 + c_3e^xx$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 40

```
DSolve[y'''[x]-y''[x]-y'[x]+y[x]==-Exp[x]*(7+6*x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x\left(-\frac{1}{2}(x+2)x^2 + (1+c_3)x - \frac{1}{2} + c_2\right) + c_1e^{-x}$$

19.9 problem section 9.3, problem 9

Internal problem ID [1506]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 9.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{2y''' - 7y'' + 4y' + 4y - e^{2x}(17 + 30x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve(2*diff(y(x),x$3)-7*diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=exp(2*x)*(17+30*x),y(x), sings
```

$$y(x) = \frac{x^2(1 + 2x)\left(30\,e^{2x}x + 17\,e^{2x}\right)}{34 + 60x} + c_1e^{2x} + c_2e^{-\frac{x}{2}} + c_3e^{2x}x$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 46

```
DSolve[2*y'''[x]-7*y''[x]+4*y'[x]+4*y[x]==Exp[2*x]*(17+30*x),y[x],x,IncludeSingularSolutions
```

$$y(x) \to e^{2x}\left(x^3 + \frac{x^2}{2} + \left(-\frac{2}{5} + c_3\right)x + \frac{4}{25} + c_2\right) + c_1e^{-x/2}$$

19.10 problem section 9.3, problem 10

Internal problem ID [1507]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 10.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 5y'' + 3y' + 9y - 2\,\mathrm{e}^{3x}(11 - 24x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$3)-5*diff(y(x),x$2)+3*diff(y(x),x)+9*y(x)=2*exp(3*x)*(11-24*x),y(x), sings
```

$$y(x) = \frac{x^2(8x-17)\left(-48x\,\mathrm{e}^{3x} + 22\,\mathrm{e}^{3x}\right)}{192x - 88} + \mathrm{e}^{-x}c_1 + c_2\mathrm{e}^{3x} + c_3\mathrm{e}^{3x}x$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 45

```
DSolve[y'''[x]-5*y''[x]+3*y'[x]+9*y[x]==2*Exp[3*x]*(11-24*x),y[x],x,IncludeSingularSolutions
```

$$y(x) \to c_1e^{-x} + e^{3x}\left(\frac{1}{8}x(2(17-8x)x - 17 + 8c_3) + \frac{17}{32} + c_2\right)$$

19.11 problem section 9.3, problem 11

Internal problem ID [1508]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 11.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 7y'' + 8y' + 16y - 2\,\mathrm{e}^{4x}(13 + 15x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$3)-7*diff(y(x),x$2)+8*diff(y(x),x)+16*y(x)=2*exp(4*x)*(13+15*x),y(x), sing
```

$$y(x) = \frac{(2 + x)\,x^2(30\,\mathrm{e}^{4x}x + 26\,\mathrm{e}^{4x})}{30x + 26} + \mathrm{e}^{-x}c_1 + c_2\mathrm{e}^{4x} + c_3\mathrm{e}^{4x}x$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 39

```
DSolve[y'''[x]-7*y''[x]+8*y'[x]+16*y[x]==2*Exp[4*x]*(13+15*x),y[x],x,IncludeSingularSolutions
```

$$y(x) \to c_1e^{-x} + e^{4x}\left(x\left(x(x+2) - \frac{4}{5} + c_3\right) + \frac{4}{25} + c_2\right)$$

19.12 problem section 9.3, problem 12

Internal problem ID [1509]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 12.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{8y''' - 12y'' + 6y' - y - e^{\frac{x}{2}}(4x+1) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 51

```
dsolve(8*diff(y(x),x$3)-12*diff(y(x),x$2)+6*diff(y(x),x)-y(x)=exp(x/2)*(1+4*x),y(x), singsol=
```

$$y(x) = \left(\frac{1}{192}x^3 + \frac{1}{256}x^2\right)\left(4\,e^{\frac{x}{2}}x + e^{\frac{x}{2}}\right) + c_1 e^{\frac{x}{2}} + c_2 x^2 e^{\frac{x}{2}} + c_3 e^{\frac{x}{2}} x$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 39

```
DSolve[8*y'''[x]-12*y''[x]+6*y'[x]-y[x]==Exp[x/2]*(1+4*x),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{48}e^{x/2}\left(x^4 + x^3 + 48c_3x^2 + 48c_2x + 48c_1\right)$$

19.13 problem section 9.3, problem 13

Internal problem ID [1510]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 13.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 3y''' - 3y'' - 7y' + 6y + 3\,e^{-x}(-8x^2 + 8x + 12) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 62

```
dsolve(diff(y(x),x$4)+3*diff(y(x),x$3)-3*diff(y(x),x$2)-7*diff(y(x),x)+6*y(x)=-3*exp(-x)*(12+
```

$$y(x) = \frac{(x^2 - 2x + 1)(24x^2 - 24x - 36)\,e^{-x}}{8x^2 - 8x - 12} + c_1 e^x + c_2 e^{-3x} + c_3 e^{-2x} + c_4 x\,e^x$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 44

```
DSolve[y''''[x]+3*y'''[x]-3*y''[x]-7*y'[x]+6*y[x]==-3*Exp[-x]*(12+8*x-8*x^2),y[x],x,IncludeSi
```

$$y(x) \to e^{-3x}\left(e^x\left(3e^x(x-1)^2 + e^{3x}(c_4 x + c_3) + c_2\right) + c_1\right)$$

19.14 problem section 9.3, problem 14

Internal problem ID [1511]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 14.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 3y''' + y'' - 3y' - 2y + 3\,\mathrm{e}^{2x}(11 + 12x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$4)+3*diff(y(x),x$3)+diff(y(x),x$2)-3*diff(y(x),x)-2*y(x)=-3*exp(2*x)*(11+1
```

$$y(x) = \frac{(x-1)\left(-36\,\mathrm{e}^{2x}x - 33\,\mathrm{e}^{2x}\right)}{36x + 33} + c_1\mathrm{e}^{x} + c_2\mathrm{e}^{-2x} + c_3\mathrm{e}^{-x} + c_4x\,\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 43

```
DSolve[y''''[x]+3*y'''[x]+y''[x]-3*y'[x]-2*y[x]==-3*Exp[2*x]*(11+12*x),y[x],x,IncludeSingular
```

$$y(x) \to e^{-2x}\left(-e^{4x}(x-1) + e^{x}(c_3x + c_2) + c_4e^{3x} + c_1\right)$$

19.15 problem section 9.3, problem 15

Internal problem ID [1512]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 15.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$y'''' + 8y''' + 24y'' + 32y' + 16\,\mathrm{e}^{-2x}\left(-x^3 + x^2 + x + 1\right) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 94

```
dsolve(diff(y(x),x$4)+8*diff(y(x),x$3)+24*diff(y(x),x$2)+32*diff(y(x),x)=-16*exp(-2*x)*(1+x+x
```

$$y(x) = \frac{c_3\mathrm{e}^{-2x}(-2\sin(2x) - 2\cos(2x))}{8} + 2c_2\left(\frac{(-2\cos(x) + 2\sin(x))\,\mathrm{e}^{-2x}\cos(x)}{8} - \frac{\mathrm{e}^{-2x}}{8}\right) + \frac{c_2\mathrm{e}^{-2x}}{2} - \frac{c_1\mathrm{e}^{-4x}}{4} - \mathrm{e}^{-2x}x^3 + \mathrm{e}^{-2x}x^2 + \mathrm{e}^{-2x}x + \mathrm{e}^{-2x} + c_4$$

✓ Solution by Mathematica

Time used: 0.702 (sec). Leaf size: 62

```
DSolve[y''''[x]+8*y'''[x]+24*y''[x]+32*y'[x]==-16*Exp[-2*x]*(1+x+x^2-x^3),y[x],x,IncludeSingu
```

$$y(x) \to \frac{1}{4}e^{-2x}\left(4x\left(-x^2 + x + 1\right) - c_3e^{-2x} - (c_1 + c_2)\cos(2x) + (c_2 - c_1)\sin(2x) + 4\right) + c_4$$

19.16 problem section 9.3, problem 16

Internal problem ID [1513]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 16.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{4y'''' - 11y'' - 9y' - 2y + \mathrm{e}^{x}(1 - 6x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
dsolve(4*diff(y(x),x$4)-11*diff(y(x),x$2)-9*diff(y(x),x)-2*y(x)=-exp(x)*(1-6*x),y(x), singsol
```

$$y(x) = -\frac{(x-1)\,(6x\,\mathrm{e}^{x} - \mathrm{e}^{x})}{3\,(-1+6x)} + \mathrm{e}^{-x}c_1 + c_2\mathrm{e}^{2x} + c_3\mathrm{e}^{-\frac{x}{2}} + c_4\mathrm{e}^{-\frac{x}{2}}x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 47

```
DSolve[4*y''''[x]-11*y''[x]-9*y'[x]-2*y[x]==-Exp[x]*(1-6*x),y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -\frac{1}{3}e^{x}(x-1) + e^{-x/2}(c_2x + c_1) + c_3e^{-x} + c_4e^{2x}$$

19.17 problem section 9.3, problem 17

Internal problem ID [1514]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 17.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 2y''' + 3y' - y - e^x(x^2 + 4x + 3) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 118

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$3)+0*diff(y(x),x$2)+3*diff(y(x),x)-y(x)=exp(x)*(3+4*x+x^2
```

$$y(x) = \frac{(x+1)(x^2 e^x + 4x e^x + 3 e^x)}{x+3} + c_1 e^{\mathrm{RootOf}(\_Z^4-2\_Z^3+3\_Z-1, \mathrm{index}=1)x} + c_2 e^{\mathrm{RootOf}(\_Z^4-2\_Z^3+3\_Z-1, \mathrm{index}=2)x} + c_3 e^{\mathrm{RootOf}(\_Z^4-2\_Z^3+3\_Z-1, \mathrm{index}=3)x} + c_4 e^{\mathrm{RootOf}(\_Z^4-2\_Z^3+3\_Z-1, \mathrm{index}=4)x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 123

```
DSolve[y''''[x]-2*y'''[x]+0*y''[x]+3*y'[x]-y[x]==Exp[x]*(3+4*x+x^2),y[x],x,IncludeSingularSol
```

$$y(x) \to c_2 \exp\left(x\mathrm{Root}\left[\#1^4 - 2\#1^3 + 3\#1 - 1\&, 2\right]\right) + c_3 \exp\left(x\mathrm{Root}\left[\#1^4 - 2\#1^3 + 3\#1 - 1\&, 3\right]\right) + c_4 \exp\left(x\mathrm{Root}\left[\#1^4 - 2\#1^3 + 3\#1 - 1\&, 4\right]\right) + c_1 \exp\left(x\mathrm{Root}\left[\#1^4 - 2\#1^3 + 3\#1 - 1\&, 1\right]\right) + e^x(x+1)^2$$

19.18 problem section 9.3, problem 18

Internal problem ID [1515]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 18.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 4y''' + 6y'' - 4y' + 2y - e^{2x}(x^4 + x + 24) = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 126

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+6*diff(y(x),x$2)-4*diff(y(x),x)+2*y(x)=exp(2*x)*(24+x+
```

$$y(x) = \frac{(x^4 - 8x^3 + 12x^2 + 49x - 62)(e^{2x}x^4 + e^{2x}x + 24\,e^{2x})}{2x^4 + 2x + 48} + c_1 e^{\frac{(2+\sqrt{2})x}{2}} \cos\left(\frac{\sqrt{2}\,x}{2}\right) + c_2 e^{\frac{(2+\sqrt{2})x}{2}} \sin\left(\frac{\sqrt{2}\,x}{2}\right) + c_3 e^{-\frac{(\sqrt{2}-2)x}{2}} \cos\left(\frac{\sqrt{2}\,x}{2}\right) + c_4 e^{-\frac{(\sqrt{2}-2)x}{2}} \sin\left(\frac{\sqrt{2}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 98

```
DSolve[y''''[x]-4*y'''[x]+6*y''[x]-4*y'[x]+2*y[x]==Exp[2*x]*(24+x+x^4),y[x],x,IncludeSingular
```

$$y(x) \to \frac{1}{2}e^{x-\frac{x}{\sqrt{2}}}\left(e^{\frac{x}{\sqrt{2}}+x}(x((x-6)(x-2)x+49)-62) + 2\left(c_4 e^{\sqrt{2}x} + c_2\right)\cos\left(\frac{x}{\sqrt{2}}\right) + 2\left(c_1 e^{\sqrt{2}x} + c_3\right)\sin\left(\frac{x}{\sqrt{2}}\right)\right)$$

19.19 problem section 9.3, problem 19

Internal problem ID [1516]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 19.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{2y'''' + 5y''' - 5y' - 2y - 18\,e^{x}(2x+5) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(2*diff(y(x),x$4)+5*diff(y(x),x$3)+0*diff(y(x),x$2)-5*diff(y(x),x)-2*y(x)=18*exp(x)*(5+
```

$$y(x) = \frac{x(2+x)\left(36x\,e^{x} + 90\,e^{x}\right)}{36x+90} + c_1 e^{x} + c_2 e^{-2x} + c_3 e^{-x} + c_4 e^{-\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 47

```
DSolve[2*y''''[x]+5*y'''[x]+0*y''[x]-5*y'[x]-2*y[x]==18*Exp[x]*(5+2*x),y[x],x,IncludeSingular
```

$$y(x) \to e^{-2x}\left(c_1 e^{3x/2} + c_3 e^{x} + e^{3x}\left(x(x+2) - \frac{40}{9} + c_4\right) + c_2\right)$$

19.20 problem section 9.3, problem 20

Internal problem ID [1517]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 20.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + y''' - 2y'' - 6y' - 4y + e^{2x}(15x^2 + 28x + 4) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 76

```
dsolve(1*diff(y(x),x$4)+1*diff(y(x),x$3)-2*diff(y(x),x$2)-6*diff(y(x),x)-4*y(x)=-exp(2*x)*(4+
```

$$y(x) = \frac{x(x^2-1)\left(-15\,e^{2x}x^2 - 28\,e^{2x}x - 4\,e^{2x}\right)}{90x^2 + 168x + 24} + e^{-x}c_1 + c_2e^{2x} + c_3e^{-x}\cos(x) + c_4e^{-x}\sin(x)$$

✓ Solution by Mathematica

Time used: 0.187 (sec). Leaf size: 48

```
DSolve[1*y''''[x]+1*y'''[x]-2*y''[x]-6*y'[x]-4*y[x]==-Exp[2*x]*(4+28*x+15*x^2),y[x],x,Include
```

$$y(x) \to \frac{1}{90}e^{2x}\left(-15x^3 + 15x - 11 + 90c_4\right) + e^{-x}(c_2\cos(x) + c_1\sin(x) + c_3)$$

19.21 problem section 9.3, problem 21

Internal problem ID [1518]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 21.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{2y'''' + y''' - 2y' - y - 3\,\mathrm{e}^{-\frac{x}{2}}(1 - 6x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 68

```
dsolve(2*diff(y(x),x$4)+1*diff(y(x),x$3)-0*diff(y(x),x$2)-2*diff(y(x),x)-1*y(x)=3*exp(-x/2)*(
```

$$y(x) = -\frac{4(x+1)\,x\left(-18\,\mathrm{e}^{-\frac{x}{2}}x + 3\,\mathrm{e}^{-\frac{x}{2}}\right)}{3\,(-1+6x)} + c_1\mathrm{e}^{x} + c_2\mathrm{e}^{-\frac{x}{2}} + c_3\mathrm{e}^{-\frac{x}{2}}\cos\left(\frac{\sqrt{3}\,x}{2}\right) + c_4\mathrm{e}^{-\frac{x}{2}}\sin\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.626 (sec). Leaf size: 58

```
DSolve[2*y''''[x]+1*y'''[x]-0*y''[x]-2*y'[x]-1*y[x]==3*Exp[-x/2]*(1-6*x),y[x],x,IncludeSingul
```

$$y(x) \to c_4e^{x} + e^{-x/2}\left(4x(x+1) + c_2\cos\left(\frac{\sqrt{3}x}{2}\right) + c_1\sin\left(\frac{\sqrt{3}x}{2}\right) - 8 + c_3\right)$$

19.22 problem section 9.3, problem 22

Internal problem ID [1519]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 22.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 5y'' + 4y - e^x(-3x^2 + x + 3) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
dsolve(1*diff(y(x),x$4)+0*diff(y(x),x$3)-5*diff(y(x),x$2)-0*diff(y(x),x)+4*y(x)=exp(x)*(3+x-3
```

$$y(x) = -\frac{x(x^2+1)(-3x^2 e^x + x e^x + 3 e^x)}{6(3x^2 - x - 3)} + c_1 e^x + c_2 e^{-2x} + c_3 e^{-x} + c_4 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 50

```
DSolve[1*y''''[x]+0*y'''[x]-5*y''[x]-0*y'[x]+4*y[x]==Exp[x]*(3+x-3*x^2),y[x],x,IncludeSingula
```

$$y(x) \to \frac{1}{36} e^x (6(x^3 + x) + 7 + 36c_3) + c_1 e^{-2x} + c_2 e^{-x} + c_4 e^{2x}$$

19.23 problem section 9.3, problem 23

Internal problem ID [1520]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 23.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 2y''' - 3y'' + 4y' + 4y - e^{2x}(18x^2 + 33x + 13) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 77

```
dsolve(1*diff(y(x),x$4)-2*diff(y(x),x$3)-3*diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=exp(2*x)*(13+
```

$$y(x) = \frac{x^2(x^2 + x + 1)(18e^{2x}x^2 + 33e^{2x}x + 13e^{2x})}{108x^2 + 198x + 78} + e^{-x}c_1 + c_2e^{2x} + c_3e^{-x}x + c_4e^{2x}x$$

✓ Solution by Mathematica

Time used: 0.1 (sec). Leaf size: 50

```
DSolve[1*y''''[x]-2*y'''[x]-3*y''[x]+4*y'[x]+4*y[x]==Exp[2*x]*(13+33*x+18*x^2),y[x],x,Include
```

$$y(x) \to \frac{1}{54}e^{2x}\left(9x\left(x^3 + x^2 + x - 2 + 6c_4\right) + 10 + 54c_3\right) + e^{-x}(c_2x + c_1)$$

19.24 problem section 9.3, problem 24

Internal problem ID [1521]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 24.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' - 3y''' + 4y' - e^{2x}(12x^2 + 26x + 15) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 76

```
dsolve(1*diff(y(x),x$4)-3*diff(y(x),x$3)-0*diff(y(x),x$2)+4*diff(y(x),x)+0*y(x)=exp(2*x)*(15+
```

$$y(x) = \frac{c_2 e^{2x}}{2} - e^{-x} c_1 + \frac{e^{2x} x^3}{6} + \frac{e^{2x} x^4}{6} + \frac{e^{2x} x^2}{2} - \frac{e^{2x} x}{2} + \frac{e^{2x}}{4} + c_3 \left(\frac{e^{2x} x}{2} - \frac{e^{2x}}{4}\right) + c_4$$

✓ Solution by Mathematica

Time used: 0.186 (sec). Leaf size: 54

```
DSolve[1*y''''[x]-3*y'''[x]-0*y''[x]+4*y'[x]+0*y[x]==Exp[2*x]*(15+26*x+12*x^2),y[x],x,Include
```

$$y(x) \to \frac{1}{12} e^{2x} \left(2x\left(x\left(x^2 + x + 3\right) - 6\right) + 3c_3(2x - 1) + 8 + 6c_2\right) + c_1\left(-e^{-x}\right) + c_4$$

19.25 problem section 9.3, problem 25

Internal problem ID [1522]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 25.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 2y''' + 2y' - y - e^x(x+1) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve(1*diff(y(x),x$4)-2*diff(y(x),x$3)-0*diff(y(x),x$2)+2*diff(y(x),x)-1*y(x)=exp(x)*(1+x),
```

$$y(x) = \frac{x^3(2+x)\left(e^x + x\,e^x\right)}{48x+48} + c_1e^x + e^{-x}c_2 + c_3e^xx + c_4e^xx^2$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 49

```
DSolve[1*y''''[x]-2*y'''[x]-0*y''[x]+2*y'[x]-1*y[x]==Exp[x]*(1+x),y[x],x,IncludeSingularSolut
```

$$y(x) \to c_1e^{-x} + \frac{1}{96}e^x(2x(x(x(x+2)-3+48c_4)+3+48c_3)-3+96c_2)$$

19.26 problem section 9.3, problem 26

Internal problem ID [1523]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 26.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{2y'''' - 5y''' + 3y'' + y' - y - e^{x}(11 + 12x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
dsolve(2*diff(y(x),x$4)-5*diff(y(x),x$3)+3*diff(y(x),x$2)+1*diff(y(x),x)-1*y(x)=exp(x)*(11+12
```

$$y(x) = \frac{x^{3}(x+1)\left(12x\,e^{x} + 11\,e^{x}\right)}{66 + 72x} + c_{1}e^{x} + c_{2}e^{x}x + c_{3}e^{x}x^{2} + c_{4}e^{-\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 51

```
DSolve[2*y''''[x]-5*y'''[x]+3*y''[x]+1*y'[x]-1*y[x]==Exp[x]*(11+12*x),y[x],x,IncludeSingularS
```

$$y(x) \to \frac{1}{54}e^{x}\left(3x\left(3x\left(x^{2} + x - 2 + 6c_{4}\right) + 8 + 18c_{3}\right) - 16 + 54c_{2}\right) + c_{1}e^{-x/2}$$

19.27 problem section 9.3, problem 27

Internal problem ID [1524]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 27.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$y'''' + 3y''' + 3y'' + y' - e^{-x}(10x^2 - 24x + 5) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 56

```
dsolve(1*diff(y(x),x$4)+3*diff(y(x),x$3)+3*diff(y(x),x$2)+1*diff(y(x),x)-0*y(x)=exp(-x)*(5-24
```

$$y(x) = -\frac{(x^5 - x^4 + 6c_3x^2 + x^3 + 6c_2x + 12c_3x + 3x^2 + 6c_1 + 6c_2 + 12c_3 + 6x + 6)\,e^{-x}}{6} + c_4$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 64

```
DSolve[1*y''''[x]+3*y'''[x]+3*y''[x]+1*y'[x]-0*y[x]==Exp[-x]*(5-24*x+10*x^2),y[x],x,IncludeSi
```

$$y(x) \to \frac{1}{6}e^{-x}\left(-x\left(x^4 - x^3 + x^2 + 3x + 6c_3(x+2) + 6 + 6c_2\right) - 6(-c_4e^x + 1 + c_1 + c_2 + 2c_3)\right)$$

19.28 problem section 9.3, problem 28

Internal problem ID [1525]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 28.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 7y''' + 18y'' - 20y' + 8y - e^{2x}(-5x^2 - 8x + 3) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 79

```
dsolve(1*diff(y(x),x$4)-7*diff(y(x),x$3)+18*diff(y(x),x$2)-20*diff(y(x),x)+8*y(x)=exp(2*x)*(3
```

$$y(x) = \frac{x^3(x^2 - x - 2)\left(-5\,e^{2x}x^2 - 8\,e^{2x}x + 3\,e^{2x}\right)}{60x^2 + 96x - 36} + c_1e^x + c_2e^{2x} + c_3e^{2x}x + c_4e^{2x}x^2$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 55

```
DSolve[1*y''''[x]-7*y'''[x]+18*y''[x]-20*y'[x]+8*y[x]==Exp[2*x]*(3-8*x-5*x^2),y[x],x,IncludeS
```

$$y(x) \to \frac{1}{12}e^{2x}\left(x\left(x\left(x\left(-x^2 + x + 2\right) - 6 + 12c_4\right) + 12(1 + c_3)\right) + 12(-1 + c_2)\right) + c_1e^x$$

19.29 problem section 9.3, problem 29

Internal problem ID [1526]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 29.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' - 4y' + 4y - e^{-x}((16 + 10x)\cos(x) + (30 - 10x)\sin(x)) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(0*diff(y(x),x$4)+1*diff(y(x),x$3)-1*diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=exp(-x)*((16+
```

$$y(x) = \cos(x) e^{-x} x + e^{-x} \cos(x) - \sin(x) e^{-x} x + 2\sin(x) e^{-x} + c_1 e^{x} + c_2 e^{-2x} + c_3 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 47

```
DSolve[0*y''''[x]+1*y'''[x]-1*y''[x]-4*y'[x]+4*y[x]==Exp[-x]*((16+10*x)*Cos[x]+(30-10*x)*Sin[
```

$$y(x) \to e^{-2x}\left(e^{3x}(c_3 e^{x} + c_2) + e^{x}((x+1)\cos(x) - (x-2)\sin(x)) + c_1\right)$$

19.30 problem section 9.3, problem 30

Internal problem ID [1527]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 30.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + y'' - 4y' - 4y - e^{-x}((1 - 22x)\cos(2x) - (6x + 1)\sin(2x)) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(1*diff(y(x),x$3)+1*diff(y(x),x$2)-4*diff(y(x),x)-4*y(x)=exp(-x)*((1-22*x)*cos(2*x)-(1+
```

$$y(x) = -(x - 1)e^{-x}\cos(2x) + (x + 1)e^{-x}\sin(2x) - \frac{5e^{-x}}{3} + e^{-2x}c_1 + e^{-x}c_2 + c_3e^{2x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 46

```
DSolve[1*y'''[x]+1*y''[x]-4*y'[x]-4*y[x]==Exp[-x]*((1-22*x)*Cos[2*x]-(1+6*x)*Sin[2*x]),y[x],x
```

$$y(x) \to e^{-2x}\left(c_3e^{4x} + e^x((x + 1)\sin(2x) - (x - 1)\cos(2x) + c_2) + c_1\right)$$

19.31 problem section 9.3, problem 31

Internal problem ID [1528]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 31.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' + 2y' - 2y - e^{2x}\left(\left(-x^2+5x+27\right)\cos(x) + \left(9x^2+13x+2\right)\sin(x)\right) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 217

```
dsolve(1*diff(y(x),x$3)-1*diff(y(x),x$2)+2*diff(y(x),x)-2*y(x)=exp(2*x)*((27+5*x-x^2)*cos(1*x
```

$$y(x) = -\frac{5\,e^{2x}\cos(x)\,x^2}{3} + 5\,e^{2x}\cos(x) + \frac{10\,e^{2x}\sin(x)\,x}{3} + \frac{4\,e^{2x}\sin(x)\,x^2}{3} + \frac{7\,e^{2x}\sin(x)}{3} + \frac{5\,e^{2x}\cos(x)\,x}{3} + \left(\int \frac{\sqrt{2}\left(\cos(x)\,x^2 - 9\sin(x)\,x^2 - 5\cos(x)\,x - 13\sin(x)\,x - 27\cos(x) - 2\sin(x)\right)\left(\sqrt{2}\cos\left(\sqrt{2}\,x\right) - \right.}{6}\right. + \left(\int \frac{\sqrt{2}\left(\cos(x)\,x^2 - 9\sin(x)\,x^2 - 5\cos(x)\,x - 13\sin(x)\,x - 27\cos(x) - 2\sin(x)\right)\left(\sqrt{2}\sin\left(\sqrt{2}\,x\right) + \right.}{6}\right. + c_1 e^{x} + c_2\cos\left(\sqrt{2}\,x\right) + c_3\sin\left(\sqrt{2}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 59

```
DSolve[1*y'''[x]-1*y''[x]+2*y'[x]-2*y[x]==Exp[2*x]*((27+5*x-x^2)*Cos[1*x]+(2+13*x+9*x^2)*Sin[
```

$$y(x) \to e^{2x}\left(\left(-x^2+x+1\right)\cos(x) + 2x\sin(x) + \sin(x)\right) + c_3 e^{x} + c_1\cos\left(\sqrt{2}x\right) + c_2\sin\left(\sqrt{2}x\right)$$

19.32 problem section 9.3, problem 32

Internal problem ID [1529]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 32.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 2y'' + y' - 2y + e^x\left(\left(4x^2 + 5x + 9\right)\cos(2x) - \left(-3x^2 - 5x + 6\right)\sin(2x)\right) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(1*diff(y(x),x$3)-2*diff(y(x),x$2)+1*diff(y(x),x)-2*y(x)=-exp(x)*((9+5*x+4*x^2)*cos(2*x
```

$$y(x) = \frac{(55x + 61)e^x\cos(2x)}{50} + \frac{(25x^2 + 15x - 27)e^x\sin(2x)}{50} + c_1\cos(x) + c_2\sin(x) + c_3e^{2x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 65

```
DSolve[1*y'''[x]-2*y''[x]+1*y'[x]-2*y[x]==Exp[2*x]*((9+5*x+4*x^2)*Cos[2*x]-(6-5*x-3*x^2)*Sin[
```

$$y(x) \to c_3e^{2x} + \frac{e^{2x}((520(34 - 13x)x + 29907)\sin(2x) - 2(65x(91x + 113) + 3928)\cos(2x))}{43940} + c_1\cos(x) + c_2\sin(x)$$

19.33 problem section 9.3, problem 33

Internal problem ID [1530]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 33.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + 3y'' + 4y' + 12y - 8\cos(2x) + 16\sin(2x) = 0}$$

✓ Solution by Maple

Time used: 0.36 (sec). Leaf size: 43

```
dsolve(1*diff(y(x),x$3)+3*diff(y(x),x$2)+4*diff(y(x),x)+12*y(x)=8*cos(2*x)-16*sin(2*x),y(x),
```

$$y(x) = \left(\frac{56}{169} + \frac{8x}{13}\right)\cos(2x) + \left(-\frac{136}{169} + \frac{14x}{13}\right)\sin(2x) + \cos(2x)\,c_1 + c_2\mathrm{e}^{-3x} + c_3\sin(2x)$$

✓ Solution by Mathematica

Time used: 0.196 (sec). Leaf size: 47

```
DSolve[1*y'''[x]+3*y''[x]+4*y'[x]+12*y[x]==8*Cos[2*x]-16*Sin[2*x],y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{1}{169}\left(169c_3e^{-3x} + (104x + 43 + 169c_1)\cos(2x) + (182x - 32 + 169c_2)\sin(2x)\right)$$

19.34 problem section 9.3, problem 34

Internal problem ID [1531]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 34.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' + 2y - e^x((20 + 4x)\cos(x) - (12x + 12)\sin(x)) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 55

```
dsolve(1*diff(y(x),x$3)-1*diff(y(x),x$2)+0*diff(y(x),x)+2*y(x)=exp(x)*((20+4*x)*cos(x)-(12+12
```

$$y(x) = \frac{e^x(5x^2 + 5x + 22)\cos(x)}{5} + \frac{e^x(5x^2 + 15x + 1)\sin(x)}{5} + e^{-x}c_1 + c_2\cos(x)e^x + c_3\sin(x)e^x$$

✓ Solution by Mathematica

Time used: 0.299 (sec). Leaf size: 52

```
DSolve[1*y'''[x]-1*y''[x]+0*y'[x]+2*y[x]==Exp[x]*((20+4*x)*Cos[x]-(12+12*x)*Sin[x]),y[x],x,In
```

$$y(x) \to c_3e^{-x} + \frac{1}{10}e^x((10x(x + 1) + 23 + 10c_2)\cos(x) + (10x(x + 3) - 21 + 10c_1)\sin(x))$$

19.35 problem section 9.3, problem 35

Internal problem ID [1532]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 35.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 7y'' + 20y' - 24y + e^{2x}((13 - 8x)\cos(2x) - (8 - 4x)\sin(2x)) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 66

```
dsolve(1*diff(y(x),x$3)-7*diff(y(x),x$2)+20*diff(y(x),x)-24*y(x)=-exp(2*x)*((13-8*x)*cos(2*x)
```

$$y(x) = -\frac{(20x^2 - 60x + 83)e^{2x}\cos(2x)}{40} + \frac{(10x - 47)e^{2x}\sin(2x)}{20} + c_1e^{3x} + c_2\cos(2x)e^{2x} + c_3e^{2x}\sin(2x)$$

✓ Solution by Mathematica

Time used: 0.945 (sec). Leaf size: 53

```
DSolve[1*y'''[x]-7*y''[x]+20*y'[x]-24*y[x]==-Exp[2*x]*((13-8*x)*Cos[2*x]-(8-4*x)*Sin[2*x]),y[
```

$$y(x) \to \frac{1}{40}e^{2x}(40c_3e^x + (-20(x-3)x + 21 + 40c_2)\cos(2x) + (20x - 37 + 40c_1)\sin(2x))$$

19.36 problem section 9.3, problem 36

Internal problem ID [1533]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 36.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$y''' - 6y'' + 18y' + e^{3x}((-3x + 2)\cos(3x) - (3x + 3)\sin(3x)) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 94

```
dsolve(1*diff(y(x),x$3)-6*diff(y(x),x$2)+18*diff(y(x),x)-0*y(x)=-exp(3*x)*((2-3*x)*cos(3*x)-(
```

$$y(x) = -\frac{e^{3x}\cos(3x)\,x^2}{12} + \frac{11\cos(3x)\,e^{3x}}{36} - \frac{e^{3x}\sin(3x)\,x}{12} - \frac{37\,e^{3x}\sin(3x)}{24} + \frac{\cos(3x)\,e^{3x}c_1}{6} + \frac{c_1e^{3x}\sin(3x)}{6} - \frac{c_2\cos(3x)\,e^{3x}}{6} + \frac{e^{3x}\sin(3x)\,c_2}{6} + c_3$$

✓ Solution by Mathematica

Time used: 1.816 (sec). Leaf size: 58

```
DSolve[1*y'''[x]-6*y''[x]+18*y'[x]-0*y[x]==-Exp[3*x]*((2-3*x)*Cos[3*x]-(3+3*x)*Sin[3*x]),y[x]
```

$$y(x) \to c_3 - \frac{1}{216}e^{3x}\left(6\left(3x^2 + 1 + 6c_1 - 6c_2\right)\cos(3x) + \sin(3x) + 18(x - 2(c_1 + c_2))\sin(3x)\right)$$

19.37 problem section 9.3, problem 37

Internal problem ID [1534]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 37.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 2y''' - 2y'' - 8y' - 8y - e^x(8\cos(x) + 16\sin(x)) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(1*diff(y(x),x$4)+2*diff(y(x),x$3)-2*diff(y(x),x$2)-8*diff(y(x),x)-8*y(x)=exp(x)*(8*cos
```

$$y(x) = -\frac{\cos(x)\,e^x}{10} - \frac{7\sin(x)\,e^x}{10} + e^{-2x}c_1 + c_2e^{2x} + c_3e^{-x}\cos(x) + c_4e^{-x}\sin(x)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 53

```
DSolve[1*y''''[x]+2*y'''[x]-2*y''[x]-8*y'[x]-8*y[x]==Exp[x]*(8*Cos[x]+16*Sin[x]),y[x],x,Inclu
```

$$y(x) \to c_3e^{-2x} + c_4e^{2x} - \frac{1}{10}e^x(7\sin(x) + \cos(x)) + e^{-x}(c_2\cos(x) + c_1\sin(x))$$

19.38 problem section 9.3, problem 38

Internal problem ID [1535]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 38.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 3y''' + 2y'' + 2y' - 4y - e^x(2\cos(2x) - \sin(2x)) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(1*diff(y(x),x$4)-3*diff(y(x),x$3)+2*diff(y(x),x$2)+2*diff(y(x),x)-4*y(x)=exp(x)*(2*cos
```

$$y(x) = \frac{e^x \cos(2x)}{12} - \frac{e^x \sin(2x)}{12} + e^{-x}c_1 + c_2 e^{2x} + c_3 \cos(x) e^x + c_4 \sin(x) e^x$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 51

```
DSolve[1*y''''[x]-3*y'''[x]+2*y''[x]+2*y'[x]-4*y[x]==Exp[x]*(2*Cos[2*x]-Sin[2*x]),y[x],x,Incl
```

$$y(x) \to c_3 e^{-x} + c_4 e^{2x} + \frac{1}{12} e^x (\cos(2x) + 12c_1 \sin(x) - 2\cos(x)(\sin(x) - 6c_2))$$

19.39 problem section 9.3, problem 39

Internal problem ID [1536]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 39.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 8y''' + 24y'' - 32y' + 15y - e^{2x}(15x\cos(2x) + 32\sin(2x)) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
dsolve(1*diff(y(x),x$4)-8*diff(y(x),x$3)+24*diff(y(x),x$2)-32*diff(y(x),x)+15*y(x)=exp(2*x)*(
```

$$y(x) = \cos(2x)\,e^{2x}x + c_1e^{x} + c_2e^{3x} + c_3e^{2x}\cos(x) + c_4e^{2x}\sin(x)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 41

```
DSolve[1*y''''[x]-8*y'''[x]+24*y''[x]-32*y'[x]+15*y[x]==Exp[2*x]*(15*x*Cos[2*x]+32*Sin[2*x]),
```

$$y(x) \to e^{x}\left(c_4e^{2x} + e^{x}(x\cos(2x) + c_2\cos(x) + c_1\sin(x)) + c_3\right)$$

19.40 problem section 9.3, problem 40

Internal problem ID [1537]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 40.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 6y''' + 13y'' + 12y' + 4y - e^{-x}((-x + 4)\cos(x) - (x + 5)\sin(x)) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 65

```
dsolve(1*diff(y(x),x$4)+6*diff(y(x),x$3)+13*diff(y(x),x$2)+12*diff(y(x),x)+4*y(x)=exp(-1*x)*(
```

$$y(x) = -\frac{e^{-x}\cos(x)}{2} + \frac{\sin(x)e^{-x}x}{2} - \sin(x)e^{-x} - \frac{\cos(x)e^{-x}x}{2} + e^{-2x}c_1 + e^{-x}c_2 + c_3e^{-x}x + c_4e^{-2x}x$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 51

```
DSolve[1*y''''[x]+6*y'''[x]+13*y''[x]+12*y'[x]+4*y[x]==Exp[-1*x]*((4-x)*Cos[x]-(5+x)*Sin[x]),
```

$$y(x) \to \frac{1}{2}e^{-2x}(2(c_2x + c_1) + e^x((x - 2)\sin(x) - (x + 1)\cos(x) + 2(c_4x + c_3)))$$

19.41 problem section 9.3, problem 41

Internal problem ID [1538]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 41.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 3y''' + 2y'' - 2y' - 4y + e^{-x}(\cos(x) - \sin(x)) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 57

```
dsolve(1*diff(y(x),x$4)+3*diff(y(x),x$3)+2*diff(y(x),x$2)-2*diff(y(x),x)-4*y(x)=-exp(-1*x)*(c
```

$$y(x) = \frac{e^{-x}(5x+14)\cos(x)}{50} + \frac{e^{-x}(5x-1)\sin(x)}{25} + c_1 e^x + c_2 e^{-2x} + c_3 e^{-x}\cos(x) + c_4 e^{-x}\sin(x)$$

✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 57

```
DSolve[1*y''''[x]+3*y'''[x]+2*y''[x]-2*y'[x]-4*y[x]==-Exp[-1*x]*(Cos[x]-Sin[x]),y[x],x,Includ
```

$$y(x) \to \frac{1}{50} e^{-2x} \left(50\left(c_4 e^{3x} + c_3\right) + e^x\left((5x + 14 + 50c_2)\cos(x) + (10x - 7 + 50c_1)\sin(x)\right)\right)$$

19.42 problem section 9.3, problem 42

Internal problem ID [1539]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 42.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - 5y''' + 13y'' - 19y' + 10y - e^x(\cos(2x) + \sin(2x)) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 61

```
dsolve(1*diff(y(x),x$4)-5*diff(y(x),x$3)+13*diff(y(x),x$2)-19*diff(y(x),x)+10*y(x)=exp(x)*(co
```

$$y(x) = \frac{(15x+1)\,e^x\cos(2x)}{200} - \frac{(10x+39)\,e^x\sin(2x)}{400} + \frac{e^x}{8} + c_1e^x + c_2e^{2x} + c_3e^x\cos(2x) + c_4e^x\sin(2x)$$

✓ Solution by Mathematica

Time used: 0.192 (sec). Leaf size: 53

```
DSolve[1*y''''[x]-5*y'''[x]+13*y''[x]-19*y'[x]+10*y[x]==Exp[x]*(Cos[2*x]+Sin[2*x]),y[x],x,Inc
```

$$y(x) \to \frac{1}{400}e^x(400(c_4e^x + c_3) + (30x - 13 + 400c_2)\cos(2x) - 2(5x + 17 - 200c_1)\sin(2x))$$

19.43 problem section 9.3, problem 43

Internal problem ID [1540]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 43.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 8y''' + 32y'' + 64y' + 39y - e^{-2x}((4-15x)\cos(3x) - (4+15x)\sin(3x)) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 77

```
dsolve(1*diff(y(x),x$4)+8*diff(y(x),x$3)+32*diff(y(x),x$2)+64*diff(y(x),x)+39*y(x)=exp(-2*x)*
```

$$y(x) = -\frac{e^{-2x}(30x^2 - 30x - 11)\cos(3x)}{240} + \frac{e^{-2x}(30x^2 + 30x - 11)\sin(3x)}{240} + c_1e^{-3x} + e^{-x}c_2 + c_3e^{-2x}\cos(3x) + c_4e^{-2x}\sin(3x)$$

✓ Solution by Mathematica

Time used: 0.815 (sec). Leaf size: 67

```
DSolve[1*y''''[x]+8*y'''[x]+32*y''[x]+64*y'[x]+39*y[x]==Exp[-2*x]*((4-15*x)*Cos[3*x]-(4+15*x)
```

$$y(x) \to e^{-3x}(c_4e^{2x} + c_3) + \frac{1}{720}e^{-2x}((-90(x-1)x + 25 + 720c_2)\cos(3x) + (90x(x+1) - 41 + 720c_1)\sin(3x))$$

19.44 problem section 9.3, problem 44

Internal problem ID [1541]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 44.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 5y''' + 13y'' - 19y' + 10y - e^x((8x+7)\cos(2x) + (8-4x)\sin(2x)) = 0$$

✓ Solution by Maple

Time used: 0.765 (sec). Leaf size: 61

```
dsolve(1*diff(y(x),x$4)-5*diff(y(x),x$3)+13*diff(y(x),x$2)-19*diff(y(x),x)+10*y(x)=exp(x)*((7
```

$$y(x) = \frac{3\,e^x\cos(2x)}{4} - \frac{(4x^2+4x+23)\,e^x\sin(2x)}{16} + \frac{7\,e^x}{2} + c_1e^x + c_2e^{2x} + c_3e^x\cos(2x) + c_4e^x\sin(2x)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 172

```
DSolve[1*y''''[x]-5*y'''[x]+13*y''[x]-19*y'[x]-10*y[x]==Exp[x]*((7+8*x)*Cos[2*x]+(8-4*x)*Sin[
```

$$\begin{aligned} y(x) \to\ & c_1\exp\left(x\mathrm{Root}\left[\#1^4 - 5\#1^3 + 13\#1^2 - 19\#1 - 10\&, 1\right]\right) \\ & + c_3\exp\left(x\mathrm{Root}\left[\#1^4 - 5\#1^3 + 13\#1^2 - 19\#1 - 10\&, 3\right]\right) \\ & + c_4\exp\left(x\mathrm{Root}\left[\#1^4 - 5\#1^3 + 13\#1^2 - 19\#1 - 10\&, 4\right]\right) \\ & + c_2\exp\left(x\mathrm{Root}\left[\#1^4 - 5\#1^3 + 13\#1^2 - 19\#1 - 10\&, 2\right]\right) \\ & - \frac{1}{100}e^x(-20x\sin(2x) + 64\sin(2x) + 40x\cos(2x) + 67\cos(2x)) \end{aligned}$$

19.45 problem section 9.3, problem 45

Internal problem ID [1542]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 45.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 4y''' + 8y'' + 8y' + 4y + 2(\cos(x) - \sin(x))\,e^{x} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 51

```
dsolve(1*diff(y(x),x$4)+4*diff(y(x),x$3)+8*diff(y(x),x$2)+8*diff(y(x),x)+4*y(x)=-2*exp(x)*(co
```

$$y(x) = -\frac{\cos(x)\,e^{x}}{16} - \frac{\sin(x)\,e^{x}}{16} + c_1 e^{-x}\cos(x) + c_2 \sin(x)\,e^{-x} + c_3 \cos(x)\,e^{-x} x + c_4 \sin(x)\,e^{-x} x$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 45

```
DSolve[1*y''''[x]+4*y'''[x]+8*y''[x]+8*y'[x]+4*y[x]==-2*Exp[x]*(Cos[1*x]-Sin[1*x]),y[x],x,Inc
```

$$y(x) \to -\frac{1}{16}e^{x}(\sin(x) + \cos(x)) + e^{-x}((c_4 x + c_3)\cos(x) + (c_2 x + c_1)\sin(x))$$

19.46 problem section 9.3, problem 46

Internal problem ID [1543]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 46.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 8y''' + 32y'' - 64y' + 64y - e^{2x}(\cos(2x) - \sin(2x)) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 87

```
dsolve(1*diff(y(x),x$4)-8*diff(y(x),x$3)+32*diff(y(x),x$2)-64*diff(y(x),x)+64*y(x)=exp(2*x)*(
```

$$y(x) = -\frac{e^{2x}(4x^2 - 6x - 41)\cos(2x)}{128} + \frac{e^{2x}(18x^2 + 9x - 517)\sin(2x)}{576} + c_1\cos(2x)e^{2x} + c_2\sin(2x)e^{2x} + c_3\cos(2x)e^{2x}x + c_4\sin(2x)x e^{2x}$$

✓ Solution by Mathematica

Time used: 0.196 (sec). Leaf size: 59

```
DSolve[1*y''''[x]-8*y'''[x]+32*y''[x]-64*y'[x]+64*y[x]==Exp[2*x]*(Cos[2*x]-Sin[2*x]),y[x],x,I
```

$$y(x) \to \frac{1}{256}e^{2x}((4x(-2x + 1 + 64c_4) + 5 + 256c_3)\cos(2x) + (8x(x + 1 + 32c_2) - 1 + 256c_1)\sin(2x))$$

19.47 problem section 9.3, problem 47

Internal problem ID [1544]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 47.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 8y''' + 26y'' - 40y' + 25y - e^{2x}(3\cos(x) - (3x+1)\sin(x)) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
dsolve(1*diff(y(x),x$4)-8*diff(y(x),x$3)+26*diff(y(x),x$2)-40*diff(y(x),x)+25*y(x)=exp(2*x)*(
```

$$y(x) = \frac{e^{2x}(4x+9)\cos(x)}{16} + \frac{e^{2x}(x^3+x^2+3x-2)\sin(x)}{8} + c_1e^{2x}\cos(x) + c_2e^{2x}\sin(x) + c_3e^{2x}\cos(x)\,x + c_4e^{2x}\sin(x)\,x$$

✓ Solution by Mathematica

Time used: 0.231 (sec). Leaf size: 56

```
DSolve[1*y''''[x]-8*y'''[x]+26*y''[x]-40*y'[x]+25*y[x]==Exp[2*x]*(3*Cos[1*x]-(1+3*x)*Sin[1*x]
```

$$y(x) \to \frac{1}{16}e^{2x}((2(1+8c_4)x+3+16c_3)\cos(x) + (x(2x(x+1)+9+16c_2)-1+16c_1)\sin(x))$$

19.48 problem section 9.3, problem 48

Internal problem ID [1545]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 48.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 4y'' + 5y' - 2y - e^{2x} + 4\,e^{x} + 2\cos(x) - 4\sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve(1*diff(y(x),x$3)-4*diff(y(x),x$2)+5*diff(y(x),x)-2*y(x)=exp(2*x)-4*exp(x)-2*cos(x)+4*s
```

$$y(x) = -\left(\cos(x)\,e^{x} - 2\,e^{2x}x^{2} + 2\,e^{3x} - 4\,e^{2x}x - x\,e^{3x} - 4\,e^{2x}\right)e^{-x} + c_1 e^{x} + c_2 e^{2x} + c_3 e^{x} x$$

✓ Solution by Mathematica

Time used: 0.362 (sec). Leaf size: 36

```
DSolve[1*y'''[x]-4*y''[x]+5*y'[x]-2*y[x]==Exp[2*x]-4*Exp[x]-2*Cos[x]+4*Sin[x],y[x],x,IncludeS
```

$$y(x) \to -\cos(x) + e^{x}(x(2x + 4 + c_2) + e^{x}(x - 2 + c_3) + 4 + c_1)$$

19.49 problem section 9.3, problem 49

Internal problem ID [1546]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 49.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' + y' - y - 5\,\mathrm{e}^{2x} - 2\,\mathrm{e}^{x} + 4\cos(x) - 4\sin(x) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
dsolve(1*diff(y(x),x$3)-1*diff(y(x),x$2)+1*diff(y(x),x)-1*y(x)=5*exp(2*x)+2*exp(x)-4*cos(x)+4
```

$$y(x) = (2x+2)\cos(x) + x\,\mathrm{e}^{x} - 2\sin(x) - \mathrm{e}^{x} + \mathrm{e}^{2x} + c_1\cos(x) + \mathrm{e}^{x}c_2 + c_3\sin(x)$$

✓ Solution by Mathematica

Time used: 0.287 (sec). Leaf size: 35

```
DSolve[1*y'''[x]-1*y''[x]+1*y'[x]-1*y[x]==5*Exp[2*x]+2*Exp[x]-4*Cos[x]+4*Sin[x],y[x],x,Includ
```

$$y(x) \to e^x(x + e^x - 1 + c_3) + (2x + 1 + c_1)\cos(x) + (-2 + c_2)\sin(x)$$

19.50 problem section 9.3, problem 50

Internal problem ID [1547]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 50.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - y' + 2 + 2x - 4\,\mathrm{e}^{x} + 6\,\mathrm{e}^{-x} - 96\,\mathrm{e}^{3x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(1*diff(y(x),x$3)-0*diff(y(x),x$2)-1*diff(y(x),x)-0*y(x)=-2*(1+x)+4*exp(x)-6*exp(-x)+96
```

$$y(x) = x^2 + 2x\,\mathrm{e}^{x} - 3\,\mathrm{e}^{x} - 3x\,\mathrm{e}^{-x} - \frac{9\,\mathrm{e}^{-x}}{2} + \mathrm{e}^{x} c_2 - \mathrm{e}^{-x} c_1 + 4\,\mathrm{e}^{3x} + 2x + c_3$$

✓ Solution by Mathematica

Time used: 0.649 (sec). Leaf size: 49

```
DSolve[1*y'''[x]-0*y''[x]-1*y'[x]-0*y[x]==-2*(1+x)+4*Exp[x]-6*Exp[-x]+96*Exp[3*x],y[x],x,Incl
```

$$y(x) \to x(x+2) + 4e^{3x} + e^{x}(2x - 3 + c_1) - \frac{1}{2}e^{-x}(6x + 9 + 2c_2) + c_3$$

19.51 problem section 9.3, problem 51

Internal problem ID [1548]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 51.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 4y'' + 9y' - 10y - 10\,\mathrm{e}^{2x} - 20\,\mathrm{e}^{x}\sin(2x) + 10 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 91

```
dsolve(1*diff(y(x),x$3)-4*diff(y(x),x$2)+9*diff(y(x),x)-10*y(x)=10*exp(2*x)+20*exp(x)*sin(2*x
```

$$y(x) = \frac{\mathrm{e}^{-2x}\mathrm{e}^{3x}(5x-8)\cos(2x)}{5} - \frac{\mathrm{e}^{-2x}\mathrm{e}^{3x}(20x+13)\sin(2x)}{10} + \frac{\mathrm{e}^{-2x}(10\,\mathrm{e}^{4x}x + 5\,\mathrm{e}^{2x} - 4\,\mathrm{e}^{4x})}{5} + c_1\mathrm{e}^{2x} + c_2\mathrm{e}^{x}\cos(2x) + c_3\mathrm{e}^{x}\sin(2x)$$

✓ Solution by Mathematica

Time used: 0.975 (sec). Leaf size: 58

```
DSolve[1*y'''[x]-4*y''[x]+9*y'[x]-10*y[x]==10*Exp[2*x]+20*Exp[x]*Sin[2*x]-10,y[x],x,IncludeSi
```

$$y(x) \to e^{2x}\left(2x - \frac{4}{5} + c_3\right) + \frac{1}{20}e^{x}((20x - 22 + 20c_2)\cos(2x) + (-40x - 21 + 20c_1)\sin(2x)) + 1$$

19.52 problem section 9.3, problem 52

Internal problem ID [1549]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 52.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + 3y'' + 3y' + y - 12\,\mathrm{e}^{-x} - 9\cos(2x) + 13\sin(2x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve(1*diff(y(x),x$3)+3*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=12*exp(-x)+9*cos(2*x)-13*sin(2*x
```

$$y(x) = 2\,\mathrm{e}^{-x}x^3 - \cos(2x) + \sin(2x) + \mathrm{e}^{-x}c_1 + c_2x\,\mathrm{e}^{-x} + c_3x^2\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.292 (sec). Leaf size: 38

```
DSolve[1*y'''[x]+3*y''[x]+3*y'[x]+1*y[x]==12*Exp[-x]+9*Cos[2*x]-13*Sin[2*x],y[x],x,IncludeSin
```

$$y(x) \to \sin(2x) - \cos(2x) + e^{-x}(x(x(2x + c_3) + c_2) + c_1)$$

19.53 problem section 9.3, problem 53

Internal problem ID [1550]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 53.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + y'' - y' - y - 4\,e^{-x}(1-6x) + 2x\cos(x) - 2\sin(x)(x+1) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(1*diff(y(x),x$3)+1*diff(y(x),x$2)-1*diff(y(x),x)-1*y(x)=4*exp(-x)*(1-6*x)-2*x*cos(x)+2
```

$$y(x) = e^{-x}\left(e^{x}\cos(x)\,x + 2x^3 - 2\sin(x)\,e^{x} + 2x^2 + 2x + 1\right) + c_1 e^{x} + e^{-x}c_2 + c_3 e^{-x}x$$

✓ Solution by Mathematica

Time used: 0.619 (sec). Leaf size: 42

```
DSolve[1*y'''[x]+1*y''[x]-1*y'[x]-1*y[x]==4*Exp[-x]*(1-6*x)-2*x*Cos[x]+2*(1+x)*Sin[x],y[x],x,
```

$$y(x) \to -2\sin(x) + x\cos(x) + e^{-x}(x(2x(x+1) + 2 + c_2) + 1 + c_1) + c_3 e^{x}$$

19.54 problem section 9.3, problem 54

Internal problem ID [1551]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 54.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 5y'' + 4y + 12\,e^{x} - 6\,e^{-x} - 10\cos(x) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 68

```
dsolve(diff(y(x),x$4)-0*diff(y(x),x$3)-5*diff(y(x),x$2)-0*diff(y(x),x)+4*y(x)=-12*exp(x)+6*ex
```

$$y(x) = \frac{\left(2\,e^{4x} + 6\,e^{3x}\cos(x) - e^{2x} + 12\,e^{4x}x + 6\,e^{2x}x\right)e^{-3x}}{6} + c_1 e^{x} + c_2 e^{-2x} + c_3 e^{-x} + c_4 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.187 (sec). Leaf size: 58

```
DSolve[y''''[x]-0*y'''[x]-5*y''[x]-0*y'[x]+4*y[x]==-12*Exp[x]+6*Exp[-x]+10*Cos[x],y[x],x,Incl
```

$$y(x) \to \cos(x) + \frac{1}{6}e^{-2x}\left(e^{x}\left(6x + 2e^{2x}(6x + 3c_4e^{x} + 1 + 3c_3) - 1 + 6c_2\right) + 6c_1\right)$$

19.55 problem section 9.3, problem 55

Internal problem ID [1552]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 55.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 4y''' + 11y'' - 14y' + 10y + e^x(\sin(x) + 2\cos(2x)) = 0$$

✓ Solution by Maple

Time used: 0.609 (sec). Leaf size: 63

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+11*diff(y(x),x$2)-14*diff(y(x),x)+10*y(x)=-exp(x)*(sin
```

$$y(x) = \frac{7\,e^x \cos(2x)}{18} + \frac{e^x \sin(2x)\,x}{6} + \frac{e^x \cos(x)\,x}{6} + \frac{\sin(x)\,e^x}{9} + c_1 \cos(x)\,e^x + c_2 \sin(x)\,e^x + c_3 e^x \cos(2x) + c_4 e^x \sin(2x)$$

✓ Solution by Mathematica

Time used: 0.141 (sec). Leaf size: 53

```
DSolve[y''''[x]-4*y'''[x]+11*y''[x]-14*y'[x]+10*y[x]==-Exp[x]*(Sin[x]+2*Cos[2*x]),y[x],x,Incl
```

$$y(x) \to \frac{1}{36} e^x((11 + 36c_2)\cos(2x) + (1 + 36c_3)\sin(x) + 6\cos(x)(x + 2(x + 6c_1)\sin(x) + 6c_4))$$

19.56 problem section 9.3, problem 56

Internal problem ID [1553]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 56.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 2y''' - 3y'' - 4y' + 4y - 2\,\mathrm{e}^{x}(x+1) - \mathrm{e}^{-2x} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 74

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)-3*diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=2*exp(x)*(1+x)+
```

$$y(x) = \frac{\mathrm{e}^{-2x}(18x^3\mathrm{e}^{3x} + 18x^2\mathrm{e}^{3x} - 36x\,\mathrm{e}^{3x} + 27x^2 + 20\,\mathrm{e}^{3x} + 36x + 18)}{486} + c_1\mathrm{e}^{x} + c_2\mathrm{e}^{-2x} + c_3\mathrm{e}^{x}x + c_4\mathrm{e}^{-2x}x$$

✓ Solution by Mathematica

Time used: 0.235 (sec). Leaf size: 58

```
DSolve[y''''[x]+2*y'''[x]-3*y''[x]-4*y'[x]+4*y[x]==2*Exp[x]*(1+x)+Exp[-2*x],y[x],x,IncludeSin
```

$$y(x) \to \frac{1}{243}e^{x}\left(9x\left(x^2 + x - 2 + 27c_4\right) + 10 + 243c_3\right) + \frac{1}{54}e^{-2x}(x(3x + 4 + 54c_2) + 2 + 54c_1)$$

19.57 problem section 9.3, problem 57

Internal problem ID [1554]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 57.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 4y - \sinh(x)\cos(x) + \cosh(x)\sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 73

```
dsolve(diff(y(x),x$4)+0*diff(y(x),x$3)-0*diff(y(x),x$2)-0*diff(y(x),x)+4*y(x)=sinh(x)*cos(x)-
```

$$y(x) = \frac{3\,\mathrm{e}^{-x}(\mathrm{e}^{2x} - 1)\cos(x)}{64} + \frac{\mathrm{e}^{-x}(4\,\mathrm{e}^{2x}x - 3\,\mathrm{e}^{2x} - 4x - 3)\sin(x)}{64} + c_1\cos(x)\,\mathrm{e}^{x} + c_2\sin(x)\,\mathrm{e}^{x} + c_3\mathrm{e}^{-x}\cos(x) + c_4\mathrm{e}^{-x}\sin(x)$$

✓ Solution by Mathematica

Time used: 0.65 (sec). Leaf size: 63

```
DSolve[y''''[x]+0*y'''[x]-0*y''[x]-0*y'[x]+4*y[x]==Sinh[x]*Cos[x]-Cosh[x]*Sin[x],y[x],x,Inclu
```

$$y(x) \to \frac{1}{64}e^{-x}\left(\left((3+64c_4)e^{2x} - 3 + 64c_1\right)\cos(x) + \left(-4x + e^{2x}(4x - 3 + 64c_3) - 3 + 64c_2\right)\sin(x)\right)$$

19.58 problem section 9.3, problem 58

Internal problem ID [1555]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 58.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 5y''' + 9y'' + 7y' + 2y - e^{-x}(30 + 24x) + e^{-2x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 72

```
dsolve(diff(y(x),x$4)+5*diff(y(x),x$3)+9*diff(y(x),x$2)+7*diff(y(x),x)+2*y(x)=exp(-x)*(30+24*
```

$$y(x) = -\left(-x^4 - x^3 - 6x + 3x^2 + 6 - x\,e^{-x} - 3\,e^{-x}\right)e^{-x} + e^{-2x}c_1 + e^{-x}c_2 + c_3e^{-x}x + c_4x^2e^{-x}$$

✓ Solution by Mathematica

Time used: 0.269 (sec). Leaf size: 40

```
DSolve[y''''[x]+5*y'''[x]+9*y''[x]+7*y'[x]+2*y[x]==Exp[-x]*(30+24*x)-Exp[-2*x],y[x],x,Include
```

$$y(x) \rightarrow e^{-2x}\left(e^x\left(x\left(x\left(x^2 + x - 3 + c_4\right) + 6 + c_3\right) - 6 + c_2\right) + x + 3 + c_1\right)$$

19.59 problem section 9.3, problem 59

Internal problem ID [1556]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 59.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 4y''' + 7y'' - 6y' + 2y - e^x(12x - 2\cos(x) + 2\sin(x)) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+7*diff(y(x),x$2)-6*diff(y(x),x)+2*y(x)=exp(x)*(12*x-2*
```

$$y(x) = e^x(x+3)\cos(x) + e^x(-2+x)\sin(x) + 2e^x x(x^2-6) + c_1 e^x + c_2 x e^x + c_3 \cos(x) e^x + c_4 \sin(x) e^x$$

✓ Solution by Mathematica

Time used: 0.139 (sec). Leaf size: 39

```
DSolve[y''''[x]-4*y'''[x]+7*y''[x]-6*y'[x]+2*y[x]==Exp[x]*(12*x-2*Cos[x]+2*Sin[x]),y[x],x,Inc
```

$$y(x) \to e^x(x(2x^2 - 12 + c_4) + (x + 3 + c_2)\cos(x) + (x - 2 + c_1)\sin(x) + c_3)$$

19.60 problem section 9.3, problem 60

Internal problem ID [1557]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 60.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' - y' + y - e^{2x}(10 + 3x) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$3)-1*diff(y(x),x$2)-1*diff(y(x),x)+1*y(x)=exp(2*x)*(10+3*x),y(x), singsol=
```

$$y(x) = \frac{(x+1)\left(3\,e^{2x}x + 10\,e^{2x}\right)}{10+3x} + c_1 e^{x} + e^{-x}c_2 + c_3 e^{x} x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 34

```
DSolve[y'''[x]-1*y''[x]-1*y'[x]+1*y[x]==Exp[2*x]*(10+3*x),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{2x}(x+1) + c_1 e^{-x} + e^{x}(c_3 x + c_2)$$

19.61 problem section 9.3, problem 61

Internal problem ID [1558]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 61.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + y'' - 2y + e^{3x}(17x^2 + 67x + 9) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 70

```
dsolve(diff(y(x),x$3)+1*diff(y(x),x$2)-0*diff(y(x),x)-2*y(x)=-exp(3*x)*(9+67*x+17*x^2),y(x),
```

$$y(x) = \frac{(x^2 + 2x - 2)(-17x^2 e^{3x} - 67x\, e^{3x} - 9\, e^{3x})}{34x^2 + 134x + 18} + c_1 e^x + c_2 \cos(x)\, e^{-x} + c_3 e^{-x} \sin(x)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 45

```
DSolve[y'''[x]+1*y''[x]-0*y'[x]-2*y[x]==-Exp[3*x]*(9+67*x+17*x^2),y[x],x,IncludeSingularSolut
```

$$y(x) \to -\frac{1}{2} e^{3x}(x(x+2) - 2) + c_3 e^x + e^{-x}(c_2 \cos(x) + c_1 \sin(x))$$

19.62 problem section 9.3, problem 62

Internal problem ID [1559]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 62.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 6y'' + 11y' - 6y - e^{2x}(-3x^2 - 4x + 5) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 67

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+11*diff(y(x),x)-6*y(x)=exp(2*x)*(5-4*x-3*x^2),y(x), si
```

$$y(x) = -\frac{x(x^2 + 2x + 1)(-3e^{2x}x^2 - 4e^{2x}x + 5e^{2x})}{3x^2 + 4x - 5} + c_1e^x + c_2e^{2x} + c_3e^{3x}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 33

```
DSolve[y'''[x]-6*y''[x]+11*y'[x]-6*y[x]==Exp[2*x]*(5-4*x-3*x^2),y[x],x,IncludeSingularSolutio
```

$$y(x) \to e^x(e^x(x(x+1)^2 + c_3e^x + 4 + c_2) + c_1)$$

19.63 problem section 9.3, problem 63

Internal problem ID [1560]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 63.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 2y'' + y' + 2\,\mathrm{e}^{-x}\left(6x^2 - 18x + 7\right) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)+1*diff(y(x),x)-0*y(x)=-2*exp(-x)*(7-18*x+6*x^2),y(x),
```

$$y(x) = \mathrm{e}^{-x}\left(x^4 - 2x^3 - c_1x + x^2 - c_1 - c_2 + 2x + 2\right) + c_3$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 42

```
DSolve[y'''[x]+2*y''[x]+1*y'[x]-0*y[x]==-2*Exp[-x]*(7-18*x+6*x^2),y[x],x,IncludeSingularSolut
```

$$y(x) \to e^{-x}\left(x\left(x(x-1)^2 + 2 - c_2\right) + c_3e^x + 2 - c_1 - c_2\right)$$

19.64 problem section 9.3, problem 64

Internal problem ID [1561]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 64.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 3y'' + 3y' - y - e^{x}(x+1) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+3*diff(y(x),x)-y(x)=exp(x)*(1+x),y(x), singsol=all)
```

$$y(x) = \left(\frac{1}{24}x^{3} + \frac{1}{8}x^{2}\right)\left(e^{x} + x\,e^{x}\right) + c_1 e^{x} + c_2 x\,e^{x} + c_3 e^{x} x^{2}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 36

```
DSolve[y'''[x]-3*y''[x]+3*y'[x]-1*y[x]==Exp[x]*(1+x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{24}e^{x}(x(x(x(x+4) + 24c_3) + 24c_2) + 24c_1)$$

19.65 problem section 9.3, problem 65

Internal problem ID [1562]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 65.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 2y'' + y + e^{-x}(3x^2 - 9x + 4) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 66

```
dsolve(diff(y(x),x$4)-0*diff(y(x),x$3)-2*diff(y(x),x$2)+0*diff(y(x),x)+y(x)=-exp(-x)*(4-9*x+3
```

$$y(x) = \frac{x^2(x^2 - 2x - 1)(-3x^2 + 9x - 4)e^{-x}}{48x^2 - 144x + 64} + c_1 e^x + e^{-x} c_2 + c_3 e^x x + c_4 x e^{-x}$$

✓ Solution by Mathematica

Time used: 0.112 (sec). Leaf size: 52

```
DSolve[y''''[x]-0*y'''[x]-2*y''[x]+0*y'[x]+1*y[x]==-Exp[-x]*(4-9*x+3*x^2),y[x],x,IncludeSingu
```

$$y(x) \to \frac{1}{32} e^{-x}\left(2x\left(-x^3 + 2x^2 + x - 1 + 16c_2\right) - 3 + 32c_1\right) + e^x(c_4 x + c_3)$$

19.66 problem section 9.3, problem 66

Internal problem ID [1563]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 66.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + 2y'' - y' - 2y - e^{-2x}((23 - 2x)\cos(x) + (8 - 9x)\sin(x)) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)-diff(y(x),x)-2*y(x)=exp(-2*x)*((23-2*x)*cos(x)+(8-9*x)
```

$$y(x) = \frac{\cos(x)\,e^{-2x}x}{2} + e^{-2x}\cos(x) - 2\sin(x)\,e^{-2x}x + \frac{3\,e^{-2x}\sin(x)}{2} + c_1e^{x} + c_2e^{-2x} + c_3e^{-x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 48

```
DSolve[y'''[x]+2*y''[x]-y'[x]-2*y[x]==Exp[-2*x]*((23-2*x)*Cos[x]+(8-9*x)*Sin[x]),y[x],x,Inclu
```

$$y(x) \to \frac{1}{2}e^{-2x}\left((3 - 4x)\sin(x) + (x + 2)\cos(x) + 2\left(c_2e^{x} + c_3e^{3x} + c_1\right)\right)$$

19.67 problem section 9.3, problem 67

Internal problem ID [1564]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 67.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$y'''' - 3y''' + 4y'' - 2y' - e^x((28 + 6x)\cos(2x) + (11 - 12x)\sin(2x)) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$4)-3*diff(y(x),x$3)+4*diff(y(x),x$2)-2*diff(y(x),x)-0*y(x)=exp(x)*((28+6*x
```

$$y(x) = -e^x \sin(2x)x + \frac{3e^x}{2} - \frac{c_3 \cos(x) e^x}{2} + \frac{c_3 \sin(x) e^x}{2} + \frac{c_2 e^x \cos(x)}{2} + \frac{c_2 \sin(x) e^x}{2} + c_1 e^x + c_4$$

✓ Solution by Mathematica

Time used: 0.276 (sec). Leaf size: 78

```
DSolve[y''''[x]-3*y'''[x]+4*y''[x]-2*y'[x]-0*y[x]==Exp(x)*((28+6*x)*Cos[2*x]+(11-12*x)*Sin[2*
```

$$y(x) \to \frac{\operatorname{Exp}(20x(15x-8)-819)\sin(2x) + 2\operatorname{Exp}(5x(60x+143)+676)\cos(2x)}{1000} + c_3 e^x + \frac{1}{2}e^x((c_2 - c_1)\cos(x) + (c_1 + c_2)\sin(x)) + c_4$$

19.68 problem section 9.3, problem 68

Internal problem ID [1565]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 68.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - 4y''' + 14y'' - 20y' + 25y - e^x((6x + 2)\cos(2x) + 3\sin(2x)) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 75

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+14*diff(y(x),x$2)-20*diff(y(x),x)+25*y(x)=exp(x)*((2+6
```

$$y(x) = -\frac{e^x(4x^3 + 4x^2 + 126x - 41)\cos(2x)}{64} + \frac{e^x(6x + 1111)\sin(2x)}{192} + c_1e^x\cos(2x) + c_2e^x\sin(2x) + c_3e^x\cos(2x)x + c_4e^x\sin(2x)x$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 74

```
DSolve[y''''[x]-4*y'''[x]+14*y''[x]-20*y'[x]+25*y[x]==Exp(x)*((2+6*x)*Cos[2*x]+3*Sin[2*x]),y[
```

$$y(x) \to -\frac{\mathrm{Exp}(17x(816x + 3053) + 68676)\sin(2x)}{83521} - \frac{2\mathrm{Exp}(255x(51x + 245) + 100292)\cos(2x)}{83521} + e^x(c_4x + c_3)\cos(2x) + e^x(c_2x + c_1)\sin(2x)$$

19.69 problem section 9.3, problem 69

Internal problem ID [1566]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 69.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 2y'' - 5y' + 6y - 2\,\mathrm{e}^{x}(1 - 6x) = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 7, y''(0) = 9]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve([diff(y(x),x$3)-2*diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=2*exp(x)*(1-6*x),y(0) = 2, D(y)
```

$$y(x) = \left(\mathrm{e}^{5x} + x^2\mathrm{e}^{3x} + 2\,\mathrm{e}^{3x} - 1\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 27

```
DSolve[{y'''[x]-2*y''[x]-5*y'[x]+6*y[x]==2*Exp[x]*(1-6*x),{y[0]==2,y'[0]==7,y''[0]==9}},y[x],
```

$$y(x) \to e^x\left(x^2 + 2\right) - e^{-2x} + e^{3x}$$

19.70 problem section 9.3, problem 70

Internal problem ID [1567]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 70.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - y'' - y' + y + \mathrm{e}^{-x}(4 - 8x) = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 0, y''(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$3)-1*diff(y(x),x$2)-1*diff(y(x),x)+1*y(x)=-exp(-x)*(4-8*x),y(0) = 2, D(y)
```

$$y(x) = (x^2 + x + 1)\,\mathrm{e}^{-x} - (x - 1)\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 26

```
DSolve[{y'''[x]-1*y''[x]-1*y'[x]+1*y[x]==-Exp[-x]*(4-8*x),{y[0]==2,y'[0]==0,y''[0]==0}},y[x],
```

$$y(x) \to e^{-x}(x^2 + x + 1) - e^{x}(x - 1)$$

19.71 problem section 9.3, problem 71

Internal problem ID [1568]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 71.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{4y''' - 3y' - y - \mathrm{e}^{-\frac{x}{2}}(-3x + 2) = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 15, y''(0) = -17]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 22

```
dsolve([4*diff(y(x),x$3)-0*diff(y(x),x$2)-3*diff(y(x),x)-1*y(x)=exp(-x/2)*(2-3*x),y(0) = -1,
```

$$y(x) = \frac{(x^3 + 192x)\,\mathrm{e}^{-\frac{x}{2}}}{12} - \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 35

```
DSolve[{4*y'''[x]-0*y''[x]-3*y'[x]-1*y[x]==Exp[-x/2]*(2-3*x),{y[0]==2,y'[0]==0,y''[0]==0}},y[
```

$$y(x) \to \frac{1}{36}e^{-x/2}\left(3x\left(x^2 + 8\right) + 8e^{3x/2} + 64\right)$$

19.72 problem section 9.3, problem 72

Internal problem ID [1569]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 72.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 2y''' + 2y'' + 2y' + y - e^{-x}(20 - 12x) = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = -4, y''(0) = 7, y'''(0) = -22]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve([diff(y(x),x$4)+2*diff(y(x),x$3)+2*diff(y(x),x$2)+2*diff(y(x),x)+1*y(x)=exp(-x)*(20-12
```

$$y(x) = \left(-x^3 + 2x^2 - x + 2\right) e^{-x} + \cos(x) - \sin(x)$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 27

```
DSolve[{y''''[x]+2*y'''[x]+2*y''[x]+2*y'[x]+1*y[x]==Exp[-x]*(20-12*x),{y[0]==3,y'[0]==-4,y''[
```

$$y(x) \to -e^{-x}(x-2)\left(x^2+1\right) - \sin(x) + \cos(x)$$

19.73 problem section 9.3, problem 73

Internal problem ID [1570]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 73.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + 2y'' + y' + 2y - 30\cos(x) + 10\sin(x) = 0$$

With initial conditions

$$[y(0) = 3, y'(0) = -4, y''(0) = 16]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve([0*diff(y(x),x$4)+1*diff(y(x),x$3)+2*diff(y(x),x$2)+1*diff(y(x),x)+2*y(x)=30*cos(x)-10
```

$$y(x) = e^{-2x} + (2 - x)\cos(x) + (7x - 1)\sin(x)$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 26

```
DSolve[{0*y''''[x]+1*y'''[x]+2*y''[x]+1*y'[x]+2*y[x]==30*Cos[x]-10*Sin[x],{y[0]==3,y'[0]==-4,
```

$$y(x) \to e^{-2x} + (7x - 1)\sin(x) - ((x - 2)\cos(x))$$

19.74 problem section 9.3, problem 74

Internal problem ID [1571]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.3. Undetermined Coefficients for Higher Order Equations. Page 495
Problem number: section 9.3, problem 74.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' - 3y''' + 5y'' - 2y' + 2(\cos(x) - \sin(x))\,e^{x} = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 0, y''(0) = -1, y'''(0) = -5]$$

✓ Solution by Maple

Time used: 42.938 (sec). Leaf size: 1299

```
dsolve([1*diff(y(x),x$4)-3*diff(y(x),x$3)+5*diff(y(x),x$2)-2*diff(y(x),x)+0*y(x)=-2*exp(x)*(c
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 3484

```
DSolve[{1*y''''[x]-3*y'''[x]+5*y''[x]-2*y'[x]+0*y[x]==-2*Exp[x]*(Cos[x]-Sin[x]),{y[0]==2,y'[0
```

Too large to display

20 Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503

20.1 problem section 9.4, problem 3

Internal problem ID [1572]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 3.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' - 3x^2y'' + 6y'x - 6y - 2x = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(x^3*diff(y(x),x$3)-3*x^2*diff(y(x),x$2)+6*x*diff(y(x),x)-6*y(x)=2*x,y(x), singsol=all)
```

$$y(x) = x\ln(x) + \frac{3x}{2} + c_3x^3 + x^2c_2 + c_1x$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 24

```
DSolve[x^3*y'''[x]-3*x^2*y''[x]+6*x*y'[x]-6*y[x]==2*x,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to x\left(\log(x) + x(c_3x + c_2) + \frac{3}{2} + c_1\right)$$

20.2 problem section 9.4, problem 8

Internal problem ID [1573]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 8.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{4x^3y''' + 4x^2y'' - 5y'x + 2y - 30x^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(4*x^3*diff(y(x),x$3)+4*x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+2*y(x)=30*x^2,y(x), singsol
```

$$y(x) = 2\ln(x)\,x^2 - \frac{32x^2}{15} + c_1x^2 + \frac{c_2}{\sqrt{x}} + c_3\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 38

```
DSolve[4*x^3*y'''[x]+4*x^2*y''[x]-5*x*y'[x]+2*y[x]==30*x^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to 2x^2\log(x) + \frac{\left(-\frac{32}{15} + c_3\right)x^{5/2} + c_2x + c_1}{\sqrt{x}}$$

20.3 problem section 9.4, problem 11

Internal problem ID [1574]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 11.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^3y''' + x^2y'' - 2y'x + 2y - x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^3*diff(y(x),x$3)+x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = c_2x + c_3x^2 + \frac{2x^3\ln(x) + c_1}{6x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 35

```
DSolve[x^3*y'''[x]+x^2*y''[x]-2*x*y'[x]+2*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}x^2\log(x) + \left(-\frac{4}{9} + c_3\right)x^2 + c_2x + \frac{c_1}{x}$$

20.4 problem section 9.4, problem 14

Internal problem ID [1575]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 14.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$16x^4y'''' + 96x^3y''' + 72x^2y'' - 24y'x + 9y - 96x^{\frac{5}{2}} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(16*x^4*diff(y(x),x$4)+96*x^3*diff(y(x),x$3)+72*x^2*diff(y(x),x$2)-24*x*diff(y(x),x)+9*
```

$$y(x) = \frac{x^{\frac{5}{2}}}{4} + \frac{c_1}{x^{\frac{3}{2}}} + \frac{c_2}{\sqrt{x}} + c_3\sqrt{x} + c_4x^{\frac{3}{2}}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 41

```
DSolve[16*x^4*y''''[x]+96*x^3*y'''[x]+72*x^2*y''[x]-24*x*y'[x]+9*y[x]==96*x^(5/2),y[x],x,Incl
```

$$y(x) \to \frac{x^4 + 4c_4x^3 + 4c_3x^2 + 4c_2x + 4c_1}{4x^{3/2}}$$

20.5 problem section 9.4, problem 16

Internal problem ID [1576]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 16.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{x^4y'''' - 4x^3y''' + 12x^2y'' - 24y'x + 24y - x^4 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve(x^4*diff(y(x),x$4)-4*x^3*diff(y(x),x$3)+12*x^2*diff(y(x),x$2)-24*x*diff(y(x),x)+24*y(x
```

$$y(x) = -\frac{11x^4}{36} + \frac{\ln(x)\,x^4}{6} + c_4x^4 + c_3x^3 + x^2c_2 + c_1x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 38

```
DSolve[x^4*y''''[x]-4*x^3*y'''[x]+12*x^2*y''[x]-24*x*y'[x]+24*y[x]==x^4,y[x],x,IncludeSingula
```

$$y(x) \to \frac{1}{6}x^4\log(x) + x\left(x\left(x\left(\left(-\frac{11}{36} + c_4\right)x + c_3\right) + c_2\right) + c_1\right)$$

20.6 problem section 9.4, problem 18

Internal problem ID [1577]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 18.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _exact, _linear, _nonhomogeneous]]`

$$x^4y'''' + 6x^3y''' + 2x^2y'' - 4y'x + 4y - 12x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x^4*diff(y(x),x$4)+6*x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+4*y(x)=1
```

$$y(x) = x^2c_2 + c_3x + \frac{c_4}{x^2} + \frac{12x^3 \ln(x) - 15x^3 + 2c_1}{12x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 38

```
DSolve[x^4*y''''[x]+6*x^3*y'''[x]+2*x^2*y''[x]-4*x*y'[x]+4*y[x]==12*x^2,y[x],x,IncludeSingula
```

$$y(x) \to \frac{x^4 \log(x) + \left(-\frac{19}{12} + c_4\right)x^4 + c_3x^3 + c_2x + c_1}{x^2}$$

20.7 problem section 9.4, problem 22

Internal problem ID [1578]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 22.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' - 2x^2y'' + 3y'x - 3y - 4x = 0}$$

With initial conditions

$$[y(1) = 4, y'(1) = 4, y''(1) = 2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve([x^3*diff(y(x),x$3)-2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-3*y(x)=4*x,y(1) = 4, D(y)(1)
```

$$y(x) = x\left(x^2 - \ln\left(x\right)^2 - 2\ln\left(x\right) + 3\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 20

```
DSolve[{x^3*y'''[x]-2*x^2*y''[x]+3*x*y'[x]-3*y[x]==4*x,{y[1]==4,y'[1]==4,y''[1]==2}},y[x],x,I
```

$$y(x) \to x\left(x^2 - \log(x)(\log(x) + 2) + 3\right)$$

20.8 problem section 9.4, problem 23

Internal problem ID [1579]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 23.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3y''' - 5x^2y'' + 14y'x - 18y - x^3 = 0$$

With initial conditions

$$[y(1) = 0, y'(1) = 1, y''(1) = 7]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve([x^3*diff(y(x),x$3)-5*x^2*diff(y(x),x$2)+14*x*diff(y(x),x)-18*y(x)=x^3,y(1) = 0, D(y)(
```

$$y(x) = \frac{x^2 \left(\ln(x)^2 x + 4x \ln(x) - 2x + 2\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 25

```
DSolve[{x^3*y'''[x]-5*x^2*y''[x]+14*x*y'[x]-18*y[x]==x^3,{y[1]==0,y'[1]==1,y''[1]==7}},y[x],x
```

$$y(x) \to \frac{1}{2}x^2(-2x + x\log(x)(\log(x) + 4) + 2)$$

20.9 problem section 9.4, problem 25

Internal problem ID [1580]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 25.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' - 6x^2y'' + 16y'x - 16y - 9x^4 = 0}$$

With initial conditions

$$[y(1) = 2, y'(1) = 1, y''(1) = 5]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 29

```
dsolve([x^3*diff(y(x),x$3)-6*x^2*diff(y(x),x$2)+16*x*diff(y(x),x)-16*y(x)=9*x^4,y(1) = 2, D(y
```

$$y(x) = -x^4 + \frac{3\ln(x)^2 x^4}{2} + 2\ln(x)x^4 + 3x$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 29

```
DSolve[{x^3*y'''[x]-6*x^2*y''[x]+16*x*y'[x]-16*y[x]==9*x^4,{y[1]==2,y'[1]==1,y''[1]==5}},y[x]
```

$$y(x) \to \frac{1}{2}x^4\log(x)(3\log(x) + 4) - x(x^3 - 3)$$

20.10 problem section 9.4, problem 27

Internal problem ID [1581]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 27.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^3y''' + x^2y'' - 2y'x + 2y - x(x+1) = 0}$$

With initial conditions

$$\left[y(-1) = -6, y'(-1) = \frac{43}{6}, y''(-1) = -\frac{5}{2}\right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 30

```
dsolve([x^3*diff(y(x),x$3)+x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=x*(x+1),y(-1) = -6, D(y
```

$$y(x) = -\frac{x(2i\pi x - 3i\pi - 2x\ln(x) + 3\ln(x) + 12x - 24)}{6}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 34

```
DSolve[{x^3*y'''[x]+x^2*y''[x]-2*x*y'[x]+2*y[x]==x*(x+1),{y[-1]==-6,y'[-1]==43/6,y''[-1]==-5/
```

$$y(x) \to \frac{1}{6}x(i\pi(3-2x) - 12(x-2) + (2x-3)\log(x))$$

20.11 problem section 9.4, problem 30

Internal problem ID [1582]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 30.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^4y'''' + 3x^3y''' - x^2y'' + 2y'x - 2y - 9x^2 = 0}$$

With initial conditions

$$[y(1) = -7, y'(1) = -11, y''(1) = -5, y'''(1) = 6]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 14

```
dsolve([x^4*diff(y(x),x$4)+3*x^3*diff(y(x),x$3)-x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=9*
```

$$y(x) = x^2(-7 + 3\ln(x))$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 15

```
DSolve[{x^4*y''''[x]+3*x^3*y'''[x]-x^2*y''[x]+2*x*y'[x]-2*y[x]==9*x^2,{y[1]==-7,y'[1]==-11,y'
```

$$y(x) \to x^2(3\log(x) - 7)$$

20.12 problem section 9.4, problem 32

Internal problem ID [1583]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 32.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{4x^4y'''' + 24x^3y''' + 23x^2y'' - y'x + y - 6x = 0}$$

With initial conditions

$$\left[y(1) = 2, y'(1) = 0, y''(1) = 4, y'''(1) = -\frac{37}{4}\right]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 27

```
dsolve([4*x^4*diff(y(x),x$4)+24*x^3*diff(y(x),x$3)+23*x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=
```

$$y(x) = \frac{\ln(x)\, x^{\frac{5}{2}} - x^2 + x^{\frac{5}{2}} + \sqrt{x} + x}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 26

```
DSolve[{4*x^4*y''''[x]+24*x^3*y'''[x]+23*x^2*y''[x]-x*y'[x]+y[x]==6*x,{y[1]==2,y'[1]==0,y''[1
```

$$y(x) \to x - \sqrt{x} + \frac{1}{\sqrt{x}} + \frac{1}{x} + x\log(x)$$

20.13 problem section 9.4, problem 33

Internal problem ID [1584]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 33.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _exact, _linear, _nonhomogeneous]]`

$$x^4y'''' + 5x^3y''' - 3x^2y'' - 6y'x + 6y - 40x^3 = 0$$

With initial conditions

$$[y(-1) = -1, y'(-1) = -7, y''(-1) = -1, y'''(-1) = -31]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 30

```
dsolve([x^4*diff(y(x),x$4)+5*x^3*diff(y(x),x$3)-3*x^2*diff(y(x),x$2)-6*x*diff(y(x),x)+6*y(x)=
```

$$y(x) = \frac{\ln(x)\, x^5 - 1 + (-i\pi - 2)\, x^5 + x^3 + x}{x^2}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 32

```
DSolve[{x^4*y''''[x]+5*x^3*y'''[x]-3*x^2*y''[x]-6*x*y'[x]+6*y[x]==40*x^3,{y[-1]==-1,y'[-1]==-
```

$$y(x) \to \frac{(-2 - i\pi)x^5 + x^5\log(x) + x^3 + x - 1}{x^2}$$

20.14 problem section 9.4, problem 35

Internal problem ID [1585]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 35.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + 2y'' - y' - 2y - F(x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)-diff(y(x),x)-2*y(x)=F(x),y(x), singsol=all)
```

$$y(x) = \left(\int \frac{e^{-x} F(x)}{6} dx\right) e^{x} + \left(\int \frac{F(x) e^{2x}}{3} dx\right) e^{-2x} - \left(\int \frac{e^{x} F(x)}{2} dx\right) e^{-x} + c_1 e^{x} + c_2 e^{-2x} + c_3 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 88

```
DSolve[y'''[x]+2*y''[x]-y'[x]-2*y[x]==f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x} \left(\int_1^x \frac{1}{3} e^{2K[1]} f(K[1]) dK[1] + e^{x} \left(\int_1^x -\frac{1}{2} e^{K[2]} f(K[2]) dK[2] + c_2\right) + e^{3x} \left(\int_1^x \frac{1}{6} e^{-K[3]} f(K[3]) dK[3] + c_3\right) + c_1\right)$$

20.15 problem section 9.4, problem 36

Internal problem ID [1586]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 36.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^3y''' + x^2y'' - 2y'x + 2y - F(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(x^3*diff(y(x),x$3)+x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=F(x),y(x), singsol=all)
```

$$y(x) = \left(c_3 + \int \frac{c_2 - \left(\int -\frac{c_1 - \left(\int -F(x)dx\right)}{x^3}dx\right)}{x^2}dx\right)x^2$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 80

```
DSolve[x^3*y'''[x]+x^2*y''[x]-2*x*y'[x]+2*y[x]==f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2\left(\int_1^x -\frac{f(K[2])}{2K[2]^2}dK[2] + x\int_1^x \frac{f(K[3])}{3K[3]^3}dK[3]\right) + \int_1^x \frac{1}{6}f(K[1])dK[1] + x^2(c_3x + c_2) + c_1}{x}$$

20.16 problem section 9.4, problem 39

Internal problem ID [1587]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 39.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - 5y'' + 4y - F(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 85

```
dsolve(diff(y(x),x$4)-5*diff(y(x),x$2)+4*y(x)=F(x),y(x), singsol=all)
```

$$y(x) = -\left(\int \frac{e^{-x} F(x)}{6} dx\right) e^{x} - \left(\int \frac{F(x) e^{2x}}{12} dx\right) e^{-2x} + \left(\int \frac{e^{x} F(x)}{6} dx\right) e^{-x} + \left(\int \frac{F(x) e^{-2x}}{12} dx\right) e^{2x} + c_1 e^{x} + c_2 e^{-2x} + c_3 e^{-x} + c_4 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 117

```
DSolve[y''''[x]-5*y''[x]+4*y[x]==f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(\int_1^x -\frac{1}{12}e^{2K[1]}f(K[1])dK[1] + e^{x}\left(\int_1^x \frac{1}{6}e^{K[2]}f(K[2])dK[2] + c_2\right) + e^{3x}\left(\int_1^x -\frac{1}{6}e^{-K[3]}f(K[3])dK[3] + c_3\right) + e^{4x}\left(\int_1^x \frac{1}{12}e^{-2K[4]}f(K[4])dK[4] + c_4\right) + c_1\right)$$

20.17 problem section 9.4, problem 41

Internal problem ID [1588]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 9 Introduction to Linear Higher Order Equations. Section 9.4. Variation of Parameters for Higher Order Equations. Page 503
Problem number: section 9.4, problem 41.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _exact, _linear, _nonhomogeneous]]`

$$x^4y'''' + 6x^3y''' + 2x^2y'' - 4y'x + 4y - F(x) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
dsolve(x^4*diff(y(x),x$4)+6*x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+4*y(x)=F
```

$$y(x) = \frac{c_4 + \int \left(2c_2x + c_3 - \left(\int \left(\int -\frac{c_1 - (\int -F(x)dx)}{x^4} dx\right) dx\right)\right) x^2 dx}{x^2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 102

```
DSolve[x^4*y''''[x]+6*x^3*y'''[x]+2*x^2*y''[x]-4*x*y'[x]+4*y[x]==f[x],y[x],x,IncludeSingularS
```

$$y(x) \to \frac{x^3\left(\int_1^x -\frac{f(K[3])}{6K[3]^2}dK[3] + x\int_1^x \frac{f(K[4])}{12K[4]^3}dK[4]\right) + x\int_1^x \frac{1}{6}f(K[2])dK[2] + \int_1^x -\frac{1}{12}f(K[1])K[1]dK[1] + x^3(c_4x -}{x^2}$$

21 Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540

21.1 problem section 10.4, problem 1

Internal problem ID [1589]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540
Problem number: section 10.4, problem 1.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = y_1(t) + 2y_2(t)$$
$$y_2'(t) = 2y_1(t) + y_2(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve([diff(y__1(t),t)=y__1(t)+2*y__2(t),diff(y__2(t),t)=2*y__1(t)+1*y__2(t)],[y__1(t), y__2
```

$$y_1(t) = -e^{-t}c_1 + c_2e^{3t}$$

$$y_2(t) = e^{-t}c_1 + c_2e^{3t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 46

```
DSolve[{y1'[t]==y1[t]+2*y2[t],y2'[t]==2*y1[t]+y2[t]},{y1[t],y2[t]},t,IncludeSingularSolutions
```

$$\text{y1}(t) \to e^t(c_1 \cosh(2t) + c_2 \sinh(2t))$$
$$\text{y2}(t) \to e^t(c_2 \cosh(2t) + c_1 \sinh(2t))$$

21.2 problem section 10.4, problem 2

Internal problem ID [1590]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540
Problem number: section 10.4, problem 2.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -\frac{5y_1(t)}{4} + \frac{3y_2(t)}{4}$$
$$y_2'(t) = \frac{3y_1(t)}{4} - \frac{5y_2(t)}{4}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 35

```
dsolve([diff(y__1(t),t)=-5/4*y__1(t)+3/4*y__2(t),diff(y__2(t),t)=3/4*y__1(t)-5/4*y__2(t)],[y_
```

$$y_1(t) = c_1 e^{-\frac{t}{2}} - e^{-2t} c_2$$

$$y_2(t) = c_1 e^{-\frac{t}{2}} + e^{-2t} c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 66

```
DSolve[{y1'[t]==-5/4*y1[t]+3/4*y2[t],y2'[t]==3/4*y1[t]-5/4*y2[t]},{y1[t],y2[t]},t,IncludeSing
```

$$\text{y1}(t) \to \frac{1}{2} e^{-2t} \left((c_1 + c_2) e^{3t/2} + c_1 - c_2 \right)$$
$$\text{y2}(t) \to \frac{1}{2} e^{-2t} \left((c_1 + c_2) e^{3t/2} - c_1 + c_2 \right)$$

21.3 problem section 10.4, problem 3

Internal problem ID [1591]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540
Problem number: section 10.4, problem 3.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -\frac{4y_1(t)}{5} + \frac{3y_2(t)}{5}$$
$$y_2'(t) = -\frac{2y_1(t)}{5} - \frac{11y_2(t)}{5}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve([diff(y__1(t),t)=-4/5*y__1(t)+3/5*y__2(t),diff(y__2(t),t)=-2/5*y__1(t)-11/5*y__2(t)],[
```

$$y_1(t) = -3\,\mathrm{e}^{-t}c_1 - \frac{\mathrm{e}^{-2t}c_2}{2}$$

$$y_2(t) = \mathrm{e}^{-t}c_1 + \mathrm{e}^{-2t}c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 67

```
DSolve[{y1'[t]==-4/5*y1[t]+3/5*y2[t],y2'[t]==-2/5*y1[t]-11/5*y2[t]},{y1[t],y2[t]},t,IncludeSi
```

$$\mathrm{y1}(t) \to \frac{1}{5}e^{-2t}\left(c_1\left(6e^t - 1\right) + 3c_2\left(e^t - 1\right)\right)$$
$$\mathrm{y2}(t) \to \frac{1}{5}e^{-2t}\left(-(2c_1 + c_2)e^t + 2c_1 + 6c_2\right)$$

21.4 problem section 10.4, problem 4

Internal problem ID [1592]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540
Problem number: section 10.4, problem 4.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -y_1(t) - 4y_2(t)$$
$$y_2'(t) = -y_1(t) - y_2(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 32

```
dsolve([diff(y__1(t),t)=-1*y__1(t)-4*y__2(t),diff(y__2(t),t)=-1*y__1(t)-1*y__2(t)],[y__1(t),
```

$$y_1(t) = 2c_1 e^{-3t} - 2c_2 e^t$$

$$y_2(t) = c_1 e^{-3t} + c_2 e^t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 56

```
DSolve[{y1'[t]==-1*y1[t]-4*y2[t],y2'[t]==-1*y1[t]-1*y2[t]},{y1[t],y2[t]},t,IncludeSingularSol
```

$$\text{y1}(t) \to e^{-t}(c_1 \cosh(2t) - 2c_2 \sinh(2t))$$
$$\text{y2}(t) \to \frac{1}{2}e^{-t}(2c_2 \cosh(2t) - c_1 \sinh(2t))$$

21.5 problem section 10.4, problem 5

Internal problem ID [1593]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540
Problem number: section 10.4, problem 5.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = 2y_1(t) - 4y_2(t)$$
$$y_2'(t) = -y_1(t) - y_2(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve([diff(y__1(t),t)=2*y__1(t)-4*y__2(t),diff(y__2(t),t)=-1*y__1(t)-1*y__2(t)],[y__1(t), y
```

$$y_1(t) = -4c_1 e^{3t} + e^{-2t} c_2$$

$$y_2(t) = c_1 e^{3t} + e^{-2t} c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 67

```
DSolve[{y1'[t]==2*y1[t]-4*y2[t],y2'[t]==-1*y1[t]-1*y2[t]},{y1[t],y2[t]},t,IncludeSingularSolu
```

$$\text{y1}(t) \to \frac{1}{5} e^{-2t} \left(4(c_1 - c_2) e^{5t} + c_1 + 4c_2\right)$$
$$\text{y2}(t) \to \frac{1}{5} e^{-2t} \left((c_2 - c_1) e^{5t} + c_1 + 4c_2\right)$$

21.6 problem section 10.4, problem 6

Internal problem ID [1594]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540
Problem number: section 10.4, problem 6.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = 4y_1(t) - 3y_2(t)$$
$$y_2'(t) = 2y_1(t) - y_2(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve([diff(y__1(t),t)=4*y__1(t)-3*y__2(t),diff(y__2(t),t)=2*y__1(t)-1*y__2(t)],[y__1(t), y_
```

$$y_1(t) = \frac{3c_1\mathrm{e}^{2t}}{2} + c_2\mathrm{e}^t$$

$$y_2(t) = c_1\mathrm{e}^{2t} + c_2\mathrm{e}^t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 56

```
DSolve[{y1'[t]==4*y1[t]-3*y2[t],y2'[t]==2*y1[t]-1*y2[t]},{y1[t],y2[t]},t,IncludeSingularSolut
```

$$\text{y1}(t) \to e^t\left(c_1\left(3e^t - 2\right) - 3c_2\left(e^t - 1\right)\right)$$
$$\text{y2}(t) \to e^t\left(2c_1\left(e^t - 1\right) + c_2\left(3 - 2e^t\right)\right)$$

21.7 problem section 10.4, problem 7

Internal problem ID [1595]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540
Problem number: section 10.4, problem 7.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -6y_1(t) - 3y_2(t)$$
$$y_2'(t) = y_1(t) - 2y_2(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve([diff(y__1(t),t)=-6*y__1(t)-3*y__2(t),diff(y__2(t),t)=1*y__1(t)-2*y__2(t)],[y__1(t), y
```

$$y_1(t) = -c_1 e^{-3t} - 3c_2 e^{-5t}$$

$$y_2(t) = c_1 e^{-3t} + c_2 e^{-5t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 61

```
DSolve[{y1'[t]==-6*y1[t]-3*y2[t],y2'[t]==1*y1[t]-2*y2[t]},{y1[t],y2[t]},t,IncludeSingularSolu
```

$$\mathrm{y1}(t) \to \frac{1}{2}e^{-5t}\left(3(c_1 + c_2) - (c_1 + 3c_2)e^{2t}\right)$$
$$\mathrm{y2}(t) \to e^{-4t}(c_2 \cosh(t) + (c_1 + 2c_2)\sinh(t))$$

21.8 problem section 10.4, problem 8

Internal problem ID [1596]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540
Problem number: section 10.4, problem 8.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = y_1(t) - y_2(t) - 2y_3(t)$$
$$y_2'(t) = y_1(t) - 2y_2(t) - 3y_3(t)$$
$$y_3'(t) = -4y_1(t) + y_2(t) - y_3(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 73

```
dsolve([diff(y__1(t),t)=1*y__1(t)-1*y__2(t)-2*y__3(t),diff(y__2(t),t)=1*y__1(t)-2*y__2(t)-3*y
```

$$y_1(t) = c_1 e^{-3t} - c_2 e^{-t} - c_3 e^{2t}$$

$$y_2(t) = 2c_1 e^{-3t} - 4c_2 e^{-t} - c_3 e^{2t}$$

$$y_3(t) = c_1 e^{-3t} + c_2 e^{-t} + c_3 e^{2t}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 102

```
DSolve[{y1'[t]==1*y1[t]-1*y2[t]-2*y3[t],y2'[t]==1*y1[t]-2*y2[t]-3*y3[t],y1'[t]==-4*y1[t]+1*y2
```

$$y1(t) \to \frac{1}{576}e^{-3t}\left(c_1\left(63-128e^t\right)+c_2\left(64e^t-27\right)\right)$$

$$y2(t) \to \frac{1}{864}e^{-3t}\left(c_2\left(224e^t-81\right)-7c_1\left(64e^t-27\right)\right)$$

$$y3(t) \to \frac{e^{-3t}(c_1(189-128e^t)+c_2(64e^t-81))}{1728}$$

21.9 problem section 10.4, problem 9

Internal problem ID [1597]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540
Problem number: section 10.4, problem 9.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= -6y_1(t) - 4y_2(t) - 8y_3(t)\\
y_2'(t) &= -4y_1(t) - 4y_3(t)\\
y_3'(t) &= -8y_1(t) - 4y_2(t) - 6y_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 66

```
dsolve([diff(y__1(t),t)=-6*y__1(t)-4*y__2(t)-8*y__3(t),diff(y__2(t),t)=-4*y__1(t)-0*y__2(t)-4
```

$$y_1(t) = c_2 e^{-16t} - \frac{5c_3 e^{2t}}{4} - \frac{c_1 e^{2t}}{2}$$

$$y_2(t) = \frac{c_2 e^{-16t}}{2} + \frac{c_3 e^{2t}}{2} + c_1 e^{2t}$$

$$y_3(t) = c_2 e^{-16t} + c_3 e^{2t}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 110

```
DSolve[{y1'[t]==-6*y1[t]-4*y2[t]-8*y3[t],y2'[t]==-4*y1[t]-0*y2[t]-4*y3[t],y1'[t]==-8*y1[t]-4*
```

$$
\begin{aligned}
\text{y1}(t) &\to \frac{2(4c_1+c_2)e^{-16t}}{44217} - \frac{1}{9}(c_1-2c_2)e^{2t} \\
\text{y2}(t) &\to \frac{4}{9}(c_1-2c_2)e^{2t} + \frac{(4c_1+c_2)e^{-16t}}{44217} \\
\text{y3}(t) &\to \frac{2(4c_1+c_2)e^{-16t}}{44217} - \frac{1}{9}(c_1-2c_2)e^{2t}
\end{aligned}
$$

21.10 problem section 10.4, problem 10

Internal problem ID [1598]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540
Problem number: section 10.4, problem 10.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = 3y_1(t) + 5y_2(t) + 8y_3(t)$$
$$y_2'(t) = y_1(t) - y_2(t) - 2y_3(t)$$
$$y_3'(t) = -y_1(t) - y_2(t) - y_3(t)$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 61

```
dsolve([diff(y__1(t),t)=3*y__1(t)+5*y__2(t)+8*y__3(t),diff(y__2(t),t)=1*y__1(t)-1*y__2(t)-2*y
```

$$y_1(t) = -\frac{7c_2e^{2t}}{4} - \frac{2c_3e^{t}}{3} - c_1e^{-2t}$$

$$y_2(t) = -\frac{4c_3e^{t}}{3} - \frac{5c_2e^{2t}}{4} + c_1e^{-2t}$$

$$y_3(t) = c_2e^{2t} + c_3e^{t}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 193

```
DSolve[{y1'[t]==3*y1[t]+5*y2[t]+8*y3[t],y2'[t]==1*y1[t]-1*y2[t]-2*y3[t],y1'[t]==-1*y1[t]-1*y2
```

$$y1(t) \to \frac{e^{-t/9}\left(\sqrt{35}(2c_2 - 121c_1)\sin\left(\frac{\sqrt{35}t}{9}\right) - 7(74c_1 + 53c_2)\cos\left(\frac{\sqrt{35}t}{9}\right)\right)}{1575}$$

$$y2(t) \to \frac{e^{-t/9}\left(7(901c_1 + 202c_2)\cos\left(\frac{\sqrt{35}t}{9}\right) - \sqrt{35}(34c_1 + 379c_2)\sin\left(\frac{\sqrt{35}t}{9}\right)\right)}{4725}$$

$$y3(t) \to \frac{e^{-t/9}\left(2\sqrt{35}(92c_1 + 125c_2)\sin\left(\frac{\sqrt{35}t}{9}\right) - 14(251c_1 + 32c_2)\cos\left(\frac{\sqrt{35}t}{9}\right)\right)}{4725}$$

21.11 problem section 10.4, problem 11

Internal problem ID [1599]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540
Problem number: section 10.4, problem 11.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= y_1(t) - y_2(t) + 2y_3(t) \\
y_2'(t) &= 12y_1(t) - 4y_2(t) + 10y_3(t) \\
y_3'(t) &= -6y_1(t) + y_2(t) - 7y_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 74

```
dsolve([diff(y__1(t),t)=1*y__1(t)-1*y__2(t)+2*y__3(t),diff(y__2(t),t)=12*y__1(t)-4*y__2(t)+10
```

$$y_1(t) = -c_1 e^{-3t} - \frac{2c_2 e^{-5t}}{3} - c_3 e^{-2t}$$

$$y_2(t) = -2c_1 e^{-3t} - 2c_2 e^{-5t} - c_3 e^{-2t}$$

$$y_3(t) = c_1 e^{-3t} + c_2 e^{-5t} + c_3 e^{-2t}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 193

```
DSolve[{y1'[t]==1*y1[t]-1*y2[t]+2*y3[t],y2'[t]==12*y1[t]-4*y2[t]+10*y3[t],y1'[t]==-6*y1[t]+1*
```

$$\text{y1}(t) \to \frac{e^{-7t/6}\left(71(77c_1 - 109c_2)\cos\left(\frac{\sqrt{71}t}{6}\right) + \sqrt{71}(143c_2 - 2479c_1)\sin\left(\frac{\sqrt{71}t}{6}\right)\right)}{340800}$$

$$\text{y2}(t) \to \frac{e^{-7t/6}\left(71(2071c_1 - 407c_2)\cos\left(\frac{\sqrt{71}t}{6}\right) - \sqrt{71}(2717c_1 + 5411c_2)\sin\left(\frac{\sqrt{71}t}{6}\right)\right)}{852000}$$

$$\text{y3}(t) \to \frac{e^{-7t/6}\left(639(23c_1 + 9c_2)\cos\left(\frac{\sqrt{71}t}{6}\right) + 3\sqrt{71}(937c_1 - 329c_2)\sin\left(\frac{\sqrt{71}t}{6}\right)\right)}{568000}$$

21.12 problem section 10.4, problem 12

Internal problem ID [1600]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540
Problem number: section 10.4, problem 12.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= 4y_1(t) - y_2(t) - 4y_3(t) \\
y_2'(t) &= 4y_1(t) - 3y_2(t) - 2y_3(t) \\
y_3'(t) &= y_1(t) - y_2(t) - y_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 71

```
dsolve([diff(y__1(t),t)=4*y__1(t)-1*y__2(t)-4*y__3(t),diff(y__2(t),t)=4*y__1(t)-3*y__2(t)-2*y
```

$$y_1(t) = e^{-t}c_1 + 11c_2e^{3t} + c_3e^{-2t}$$

$$y_2(t) = e^{-t}c_1 + 7c_2e^{3t} + 2c_3e^{-2t}$$

$$y_3(t) = e^{-t}c_1 + c_2e^{3t} + c_3e^{-2t}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 102

```
DSolve[{y1'[t]==4*y1[t]-1*y2[t]-4*y3[t],y2'[t]==4*y1[t]-3*y2[t]-2*y3[t],y1'[t]==1*y1[t]-1*y2[
```

$$y1(t) \to \frac{1}{216}e^{-2t}\left(c_1\left(54e^t-8\right)+c_2\left(8-27e^t\right)\right)$$
$$y2(t) \to \frac{1}{216}e^{-2t}\left(2c_1\left(27e^t-8\right)+c_2\left(16-27e^t\right)\right)$$
$$y3(t) \to \frac{1}{216}e^{-2t}\left(c_1\left(54e^t-8\right)+c_2\left(8-27e^t\right)\right)$$

21.13 problem section 10.4, problem 13

Internal problem ID [1601]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540
Problem number: section 10.4, problem 13.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -2y_1(t) + 2y_2(t) - 6y_3(t)$$
$$y_2'(t) = 2y_1(t) + 6y_2(t) + 2y_3(t)$$
$$y_3'(t) = -2y_1(t) - 2y_2(t) + 2y_3(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 67

```
dsolve([diff(y__1(t),t)=-2*y__1(t)+2*y__2(t)-6*y__3(t),diff(y__2(t),t)=2*y__1(t)+6*y__2(t)+2*
```

$$y_1(t) = 4\,e^{-4t}c_1 - c_2e^{4t} - c_3e^{6t}$$

$$y_2(t) = -e^{-4t}c_1 - c_3e^{6t}$$

$$y_3(t) = e^{-4t}c_1 + c_2e^{4t} + c_3e^{6t}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 192

```
DSolve[{y1'[t]==-2*y1[t]+2*y2[t]-6*y3[t],y2'[t]==2*y1[t]+6*y2[t]+2*y3[t],y1'[t]==-2*y1[t]-2*y
```

$$y1(t) \to \frac{e^{5t/2}\left(73(198c_1+25c_2)\cosh\left(\frac{\sqrt{73}t}{2}\right)-\sqrt{73}(1682c_1+171c_2)\sinh\left(\frac{\sqrt{73}t}{2}\right)\right)}{299008}$$

$$y2(t) \to -\frac{e^{5t/2}\left(73(50c_1+27c_2)\cosh\left(\frac{\sqrt{73}t}{2}\right)+\sqrt{73}(143c_2-342c_1)\sinh\left(\frac{\sqrt{73}t}{2}\right)\right)}{299008}$$

$$y3(t) \to -\frac{e^{5t/2}\left(73(50c_1+27c_2)\cosh\left(\frac{\sqrt{73}t}{2}\right)+\sqrt{73}(143c_2-342c_1)\sinh\left(\frac{\sqrt{73}t}{2}\right)\right)}{598016}$$

21.14 problem section 10.4, problem 14

Internal problem ID [1602]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540
Problem number: section 10.4, problem 14.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= 3y_1(t) + 2y_2(t) - 2y_3(t) \\
y_2'(t) &= -2y_1(t) + 7y_2(t) - 2y_3(t) \\
y_3'(t) &= -10y_1(t) + 10y_2(t) - 5y_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 66

```
dsolve([diff(y__1(t),t)=3*y__1(t)+2*y__2(t)-2*y__3(t),diff(y__2(t),t)=-2*y__1(t)+7*y__2(t)-2*
```

$$y_1(t) = -\frac{4c_2 e^{5t}}{5} + \frac{c_3 e^{-5t}}{5} + c_1 e^{5t}$$

$$y_2(t) = \frac{c_2 e^{5t}}{5} + \frac{c_3 e^{-5t}}{5} + c_1 e^{5t}$$

$$y_3(t) = c_2 e^{5t} + c_3 e^{-5t}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 122

```
DSolve[{y1'[t]==3*y1[t]+2*y2[t]-2*y3[t],y2'[t]==-2*y1[t]+7*y2[t]-2*y3[t],y1'[t]==-10*y1[t]+10
```

$$
\begin{aligned}
\mathrm{y1}(t) &\to \frac{1}{64}(c_1 - c_2)e^{5t} - \frac{27(2c_1 - c_2)e^{25t/3}}{10648} \\
\mathrm{y2}(t) &\to \frac{1}{32}(c_1 - c_2)e^{5t} - \frac{27(2c_1 - c_2)e^{25t/3}}{10648} \\
\mathrm{y3}(t) &\to \frac{1}{64}(c_1 - c_2)e^{5t} + \frac{45(2c_1 - c_2)e^{25t/3}}{10648}
\end{aligned}
$$

21.15 problem section 10.4, problem 15

Internal problem ID [1603]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.4, constant coefficient homogeneous system. Page 540
Problem number: section 10.4, problem 15.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = 3y_1(t) + y_2(t) - y_3(t)$$
$$y_2'(t) = 3y_1(t) + 5y_2(t) + y_3(t)$$
$$y_3'(t) = -6y_1(t) + 2y_2(t) + 4y_3(t)$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 54

```
dsolve([diff(y__1(t),t)=3*y__1(t)+1*y__2(t)-1*y__3(t),diff(y__2(t),t)=3*y__1(t)+5*y__2(t)+1*y
```

$$y_1(t) = -\frac{c_3 e^{6t}}{2} + \frac{c_2}{2} + \frac{e^{6t} c_1}{3}$$

$$y_2(t) = -\frac{c_2}{2} - \frac{c_3 e^{6t}}{2} + e^{6t} c_1$$

$$y_3(t) = c_2 + c_3 e^{6t}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 91

```
DSolve[{y1'[t]==3*y1[t]+1*y2[t]-1*y3[t],y2'[t]==3*y1[t]+5*y2[t]+1*y3[t],y1'[t]==-6*y1[t]+2*y2
```

$$
\begin{aligned}
\text{y1}(t) &\to -\frac{1}{625}(c_1+c_2)e^{6t}+\frac{4c_1}{5}-\frac{c_2}{5} \\
\text{y2}(t) &\to \frac{1}{5}(c_2-4c_1)-\frac{4}{625}(c_1+c_2)e^{6t} \\
\text{y3}(t) &\to -\frac{1}{625}(c_1+c_2)e^{6t}+\frac{8c_1}{5}-\frac{2c_2}{5}
\end{aligned}
$$

22 Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555

22.1 problem section 10.5, problem 1

Internal problem ID [1604]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 1.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = 3y_1(t) + 4y_2(t)$$
$$y_2'(t) = -y_1(t) + 7y_2(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 34

```
dsolve([diff(y__1(t),t)=3*y__1(t)+4*y__2(t),diff(y__2(t),t)=-1*y__1(t)+7*y__2(t)],[y__1(t), y
```

$$y_1(t) = e^{5t}(2c_2t + 2c_1 - c_2)$$

$$y_2(t) = e^{5t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{y1'[t]==3*y1[t]+4*y2[t],y2'[t]==-1*y1[t]+7*y2[t]},{y1[t],y2[t]},t,IncludeSingularSolu
```

$$\text{y1}(t) \to e^{5t}(-2c_1t + 4c_2t + c_1)$$
$$\text{y2}(t) \to e^{5t}(c_1(-t) + 2c_2t + c_2)$$

22.2 problem section 10.5, problem 2

Internal problem ID [1605]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 2.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -y_2(t)$$
$$y_2'(t) = y_1(t) - 2y_2(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve([diff(y__1(t),t)=0*y__1(t)-1*y__2(t),diff(y__2(t),t)=1*y__1(t)-2*y__2(t)],[y__1(t), y_
```

$$y_1(t) = \mathrm{e}^{-t}(c_2 t + c_1 + c_2)$$

$$y_2(t) = \mathrm{e}^{-t}(c_2 t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 44

```
DSolve[{y1'[t]==0*y1[t]-1*y2[t],y2'[t]==1*y1[t]-2*y2[t]},{y1[t],y2[t]},t,IncludeSingularSolut
```

$$\text{y1}(t) \to e^{-t}(c_1(t+1) - c_2 t)$$
$$\text{y2}(t) \to e^{-t}((c_1 - c_2)t + c_2)$$

22.3 problem section 10.5, problem 3

Internal problem ID [1606]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 3.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= -7y_1(t) + 4y_2(t) \\
y_2'(t) &= -y_1(t) - 11y_2(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 33

```
dsolve([diff(y__1(t),t)=-7*y__1(t)+4*y__2(t),diff(y__2(t),t)=-1*y__1(t)-11*y__2(t)],[y__1(t),
```

$$y_1(t) = -e^{-9t}(2c_2t + 2c_1 + c_2)$$

$$y_2(t) = e^{-9t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{y1'[t]==-7*y1[t]+4*y2[t],y2'[t]==-1*y1[t]-11*y2[t]},{y1[t],y2[t]},t,IncludeSingularSo
```

$$
\begin{aligned}
\text{y1}(t) &\to e^{-9t}(2c_1t + 4c_2t + c_1) \\
\text{y2}(t) &\to e^{-9t}(c_2 - (c_1 + 2c_2)t)
\end{aligned}
$$

22.4 problem section 10.5, problem 4

Internal problem ID [1607]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 4.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = 3y_1(t) + y_2(t)$$
$$y_2'(t) = -y_1(t) + y_2(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 30

```
dsolve([diff(y__1(t),t)=3*y__1(t)+1*y__2(t),diff(y__2(t),t)=-1*y__1(t)+1*y__2(t)],[y__1(t), y
```

$$y_1(t) = -e^{2t}(c_2t + c_1 + c_2)$$

$$y_2(t) = e^{2t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 42

```
DSolve[{y1'[t]==3*y1[t]+1*y2[t],y2'[t]==-1*y1[t]+1*y2[t]},{y1[t],y2[t]},t,IncludeSingularSolu
```

$$\text{y1}(t) \to e^{2t}(c_1(t+1) + c_2t)$$
$$\text{y2}(t) \to e^{2t}(c_2 - (c_1 + c_2)t)$$

22.5 problem section 10.5, problem 5

Internal problem ID [1608]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 5.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = 4y_1(t) + 12y_2(t)$$
$$y_2'(t) = -3y_1(t) - 8y_2(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 33

```
dsolve([diff(y__1(t),t)=4*y__1(t)+12*y__2(t),diff(y__2(t),t)=-3*y__1(t)-8*y__2(t)],[y__1(t),
```

$$y_1(t) = -\frac{e^{-2t}(6c_2t + 6c_1 + c_2)}{3}$$

$$y_2(t) = e^{-2t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{y1'[t]==4*y1[t]+12*y2[t],y2'[t]==-3*y1[t]-8*y2[t]},{y1[t],y2[t]},t,IncludeSingularSol
```

$$\text{y1}(t) \to e^{-2t}(6c_1t + 12c_2t + c_1)$$
$$\text{y2}(t) \to e^{-2t}(c_2 - 3(c_1 + 2c_2)t)$$

22.6 problem section 10.5, problem 6

Internal problem ID [1609]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 6.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -10y_1(t) + 9y_2(t)$$
$$y_2'(t) = -4y_1(t) + 2y_2(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve([diff(y__1(t),t)=-10*y__1(t)+9*y__2(t),diff(y__2(t),t)=-4*y__1(t)+2*y__2(t)],[y__1(t),
```

$$y_1(t) = \frac{e^{-4t}(6c_2t + 6c_1 - c_2)}{4}$$

$$y_2(t) = e^{-4t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{y1'[t]==-10*y1[t]+9*y2[t],y2'[t]==-4*y1[t]+2*y2[t]},{y1[t],y2[t]},t,IncludeSingularSo
```

$$\text{y1}(t) \to e^{-4t}(-6c_1t + 9c_2t + c_1)$$
$$\text{y2}(t) \to e^{-4t}(-4c_1t + 6c_2t + c_2)$$

22.7 problem section 10.5, problem 7

Internal problem ID [1610]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 7.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -13y_1(t) + 16y_2(t)$$
$$y_2'(t) = -9y_1(t) + 11y_2(t)$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 35

```
dsolve([diff(y__1(t),t)=-13*y__1(t)+16*y__2(t),diff(y__2(t),t)=-9*y__1(t)+11*y__2(t)],[y__1(t
```

$$y_1(t) = \frac{e^{-t}(12c_2t + 12c_1 - c_2)}{9}$$

$$y_2(t) = e^{-t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{y1'[t]==-13*y1[t]+16*y2[t],y2'[t]==-9*y1[t]+11*y2[t]},{y1[t],y2[t]},t,IncludeSingular
```

$$\text{y1}(t) \to e^{-t}(-12c_1t + 16c_2t + c_1)$$
$$\text{y2}(t) \to e^{-t}(-9c_1t + 12c_2t + c_2)$$

22.8 problem section 10.5, problem 8

Internal problem ID [1611]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 8.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = 2y_2(t) + y_3(t)$$
$$y_2'(t) = -4y_1(t) + 6y_2(t) + y_3(t)$$
$$y_3'(t) = 4y_2(t) + 2y_3(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 71

```
dsolve([diff(y__1(t),t)=0*y__1(t)+2*y__2(t)+1*y__3(t),diff(y__2(t),t)=-4*y__1(t)+6*y__2(t)+1*
```

$$y_1(t) = \frac{c_2 e^{4t}}{2} + \frac{c_3 e^{4t} t}{2} - \frac{c_1}{2}$$

$$y_2(t) = \frac{c_2 e^{4t}}{2} + \frac{c_3 e^{4t} t}{2} + \frac{c_3 e^{4t}}{4} - \frac{c_1}{2}$$

$$y_3(t) = c_1 + c_2 e^{4t} + c_3 e^{4t} t$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 118

```
DSolve[{y1'[t]==0*y1[t]+2*y2[t]+1*y3[t],y2'[t]==-4*y1[t]+6*y2[t]+1*y3[t],y3'[t]==0*y1[t]+4*y2
```

$$\text{y1}(t) \to \frac{1}{4}\left(e^{4t}(c_1(2-8t)+8c_2t+c_3)+2c_1-c_3\right)$$

$$\text{y2}(t) \to \frac{1}{4}\left(e^{4t}(-2c_1(4t+1)+c_2(8t+4)+c_3)+2c_1-c_3\right)$$

$$\text{y3}(t) \to \frac{1}{2}\left(e^{4t}(c_1(2-8t)+8c_2t+c_3)-2c_1+c_3\right)$$

22.9 problem section 10.5, problem 9

Internal problem ID [1612]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 9.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = \frac{y_1(t)}{3} + \frac{y_2(t)}{3} - y_3(t)$$
$$y_2'(t) = -\frac{4y_1(t)}{3} - \frac{4y_2(t)}{3} + y_3(t)$$
$$y_3'(t) = -\frac{2y_1(t)}{3} + \frac{y_2(t)}{3}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 75

```
dsolve([diff(y__1(t),t)=1/3*y__1(t)+1/3*y__2(t)-1*y__3(t),diff(y__2(t),t)=-4/3*y__1(t)-4/3*y_
```

$$y_1(t) = -c_1 e^t + c_2 e^{-t} + c_3 e^{-t} t$$

$$y_2(t) = c_1 e^t - c_2 e^{-t} - c_3 e^{-t} t + 3c_3 e^{-t}$$

$$y_3(t) = c_1 e^t + c_2 e^{-t} + c_3 e^{-t} t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 136

```
DSolve[{y1'[t]==1/3*y1[t]+1/3*y2[t]-1*y3[t],y2'[t]==-4/3*y1[t]-4/3*y2[t]+1*y3[t],y3'[t]==-2/3
```

$$y1(t) \to \frac{1}{6}e^{-t}\left(c_1\left(2t+3e^{2t}+3\right)+2c_2t-3c_3\left(e^{2t}-1\right)\right)$$
$$y2(t) \to \frac{1}{6}e^{-t}\left(c_1\left(-2t-3e^{2t}+3\right)-2c_2(t-3)+3c_3\left(e^{2t}-1\right)\right)$$
$$y3(t) \to \frac{1}{6}e^{-t}\left(c_1\left(2t-3e^{2t}+3\right)+2c_2t+3c_3\left(e^{2t}+1\right)\right)$$

22.10 problem section 10.5, problem 10

Internal problem ID [1613]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 10.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= -y_1(t) + y_2(t) - y_3(t) \\
y_2'(t) &= -2y_1(t) + 2y_3(t) \\
y_3'(t) &= -y_1(t) + 3y_2(t) - y_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 60

```
dsolve([diff(y__1(t),t)=-1*y__1(t)+1*y__2(t)-1*y__3(t),diff(y__2(t),t)=-2*y__1(t)+0*y__2(t)+2
```

$$
y_1(t) = \frac{e^{-2t}(2c_3t + 2c_2 + c_3)}{2}
$$

$$
y_2(t) = c_1 e^{2t} + \frac{c_3 e^{-2t}}{2}
$$

$$
y_3(t) = c_1 e^{2t} + e^{-2t} c_2 + c_3 e^{-2t} t
$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 101

```
DSolve[{y1'[t]==-1*y1[t]+1*y2[t]-1*y3[t],y2'[t]==-2*y1[t]+0*y2[t]+2*y3[t],y3'[t]==-1*y1[t]+3*
```

$$y1(t) \to e^{-2t}(c_1(t+1)+(c_2-c_3)t)$$
$$y2(t) \to c_2\cosh(2t)+(c_3-c_1)\sinh(2t)$$
$$y3(t) \to \frac{1}{2}e^{-2t}\left(2c_1t+2c_2t-2c_3t+(-c_1+c_2+c_3)e^{4t}+c_1-c_2+c_3\right)$$

22.11 problem section 10.5, problem 11

Internal problem ID [1614]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 11.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= 4y_1(t) - 2y_2(t) - 2y_3(t) \\
y_2'(t) &= -2y_1(t) + 3y_2(t) - y_3(t) \\
y_3'(t) &= 2y_1(t) - y_2(t) + 3y_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 69

```
dsolve([diff(y__1(t),t)=4*y__1(t)-2*y__2(t)-2*y__3(t),diff(y__2(t),t)=-2*y__1(t)+3*y__2(t)-1*
```

$$y_1(t) = -2c_1 e^{2t} + \frac{c_3 e^{4t}}{2}$$

$$y_2(t) = -3c_1 e^{2t} - c_2 e^{4t} - c_3 e^{4t} t$$

$$y_3(t) = c_1 e^{2t} + c_2 e^{4t} + c_3 e^{4t} t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 125

```
DSolve[{y1'[t]==4*y1[t]-2*y2[t]-2*y3[t],y2'[t]==-2*y1[t]+3*y2[t]-1*y3[t],y3'[t]==2*y1[t]-1*y2
```

$$y1(t) \to e^{2t}\left(-(-c_1+c_2+c_3)e^{2t}+c_2+c_3\right)$$
$$y2(t) \to \frac{1}{2}\left(3(c_2+c_3)e^{2t}-e^{4t}(4(c_1-c_2-c_3)t+c_2+3c_3)\right)$$
$$y3(t) \to -\frac{1}{2}e^{2t}\left(-e^{2t}(4(c_1-c_2-c_3)t+c_2+3c_3)+c_2+c_3\right)$$

22.12 problem section 10.5, problem 12

Internal problem ID [1615]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 12.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= 6y_1(t) - 5y_2(t) + 3y_3(t) \\
y_2'(t) &= 2y_1(t) - y_2(t) + 3y_3(t) \\
y_3'(t) &= 2y_1(t) + y_2(t) + y_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 80

```
dsolve([diff(y__1(t),t)=6*y__1(t)-5*y__2(t)+3*y__3(t),diff(y__2(t),t)=2*y__1(t)-1*y__2(t)+3*y
```

$$y_1(t) = -c_1 e^{-2t} + c_2 e^{4t} + c_3 e^{4t} t + \frac{c_3 e^{4t}}{2}$$

$$y_2(t) = -c_1 e^{-2t} + c_2 e^{4t} + c_3 e^{4t} t$$

$$y_3(t) = c_1 e^{-2t} + c_2 e^{4t} + c_3 e^{4t} t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 127

```
DSolve[{y1'[t]==6*y1[t]-5*y2[t]+3*y3[t],y2'[t]==2*y1[t]-1*y2[t]+3*y3[t],y3'[t]==2*y1[t]+1*y2[
```

$$y1(t) \to \frac{1}{2}e^{-2t}\left(e^{6t}(c_1(4t+2)-c_2(4t+1)+c_3)+c_2-c_3\right)$$
$$y2(t) \to \frac{1}{2}e^{-2t}\left(e^{6t}(4(c_1-c_2)t+c_2+c_3)+c_2-c_3\right)$$
$$y3(t) \to \frac{1}{2}e^{-2t}\left(e^{6t}(4(c_1-c_2)t+c_2+c_3)-c_2+c_3\right)$$

22.13 problem section 10.5, problem 13

Internal problem ID [1616]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 13.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -11y_1(t) + 8y_2(t)$$
$$y_2'(t) = -2y_1(t) - 3y_2(t)$$

With initial conditions

$$[y_1(0) = 6, y_2(0) = 2]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 29

```
dsolve([diff(y__1(t),t) = -11*y__1(t)+8*y__2(t), diff(y__2(t),t) = -2*y__1(t)-3*y__2(t), y__1
```

$$y_1(t) = \frac{e^{-7t}(-16t + 12)}{2}$$

$$y_2(t) = e^{-7t}(-4t + 2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[{y1'[t]==-11*y1[t]+8*y2[t],y2'[t]==-2*y1[t]-3*y2[t]},{y1[0]==6,y2[0]==2},{y1[t],y2[t]}
```

$$\text{y1}(t) \to e^{-7t}(6 - 8t)$$
$$\text{y2}(t) \to e^{-7t}(2 - 4t)$$

22.14 problem section 10.5, problem 14

Internal problem ID [1617]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 14.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = 15y_1(t) - 9y_2(t)$$
$$y_2'(t) = 16y_1(t) - 9y_2(t)$$

With initial conditions

$$[y_1(0) = 5, y_2(0) = 8]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 29

```
dsolve([diff(y__1(t),t) = 15*y__1(t)-9*y__2(t), diff(y__2(t),t) = 16*y__1(t)-9*y__2(t), y__1(
```

$$y_1(t) = \frac{e^{3t}(-192t + 80)}{16}$$

$$y_2(t) = e^{3t}(-16t + 8)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 31

```
DSolve[{y1'[t]==15*y1[t]-9*y2[t],y2'[t]==16*y1[t]-9*y2[t]},{y1[0]==5,y2[0]==8},{y1[t],y2[t]},
```

$$\text{y1}(t) \to e^{3t}(5 - 12t)$$
$$\text{y2}(t) \to -8e^{3t}(2t - 1)$$

22.15 problem section 10.5, problem 15

Internal problem ID [1618]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 15.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -3y_1(t) - 4y_2(t)$$
$$y_2'(t) = y_1(t) - 7y_2(t)$$

With initial conditions

$$[y_1(0) = 2, y_2(0) = 3]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 28

```
dsolve([diff(y__1(t),t) = -3*y__1(t)-4*y__2(t), diff(y__2(t),t) = y__1(t)-7*y__2(t), y__1(0)
```

$$y_1(t) = e^{-5t}(-8t + 2)$$

$$y_2(t) = e^{-5t}(-4t + 3)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[{y1'[t]==-3*y1[t]-4*y2[t],y2'[t]==1*y1[t]-7*y2[t]},{y1[0]==2,y2[0]==3},{y1[t],y2[t]},t
```

$$\text{y1}(t) \to e^{-5t}(2 - 8t)$$
$$\text{y2}(t) \to e^{-5t}(3 - 4t)$$

22.16 problem section 10.5, problem 16

Internal problem ID [1619]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 16.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -7y_1(t) + 24y_2(t)$$
$$y_2'(t) = -6y_1(t) + 17y_2(t)$$

With initial conditions

$$[y_1(0) = 3, y_2(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve([diff(y__1(t),t) = -7*y__1(t)+24*y__2(t), diff(y__2(t),t) = -6*y__1(t)+17*y__2(t), y__
```

$$y_1(t) = \frac{e^{5t}(-72t + 18)}{6}$$

$$y_2(t) = e^{5t}(-6t + 1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 31

```
DSolve[{y1'[t]==-7*y1[t]+24*y2[t],y2'[t]==-6*y1[t]+17*y2[t]},{y1[0]==3,y2[0]==1},{y1[t],y2[t]
```

$$\text{y1}(t) \to -3e^{5t}(4t - 1)$$
$$\text{y2}(t) \to e^{5t}(1 - 6t)$$

22.17 problem section 10.5, problem 17

Internal problem ID [1620]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 17.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -7y_1(t) + 3y_2(t)$$
$$y_2'(t) = -3y_1(t) - y_2(t)$$

With initial conditions

$$[y_1(0) = 0, y_2(0) = 2]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 25

```
dsolve([diff(y__1(t),t) = -7*y__1(t)+3*y__2(t), diff(y__2(t),t) = -3*y__1(t)-y__2(t), y__1(0)
```

$$y_1(t) = 6\,\mathrm{e}^{-4t}t$$

$$y_2(t) = \mathrm{e}^{-4t}(6t + 2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 27

```
DSolve[{y1'[t]==-7*y1[t]+3*y2[t],y2'[t]==-3*y1[t]-1*y2[t]},{y1[0]==0,y2[0]==2},{y1[t],y2[t]},
```

$$\mathrm{y1}(t) \to 6e^{-4t}t$$
$$\mathrm{y2}(t) \to e^{-4t}(6t + 2)$$

22.18 problem section 10.5, problem 18

Internal problem ID [1621]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 18.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= -y_1(t) + y_2(t)\\
y_2'(t) &= y_1(t) - y_2(t) - 2y_3(t)\\
y_3'(t) &= -y_1(t) - y_2(t) - y_3(t)
\end{aligned}
$$

With initial conditions

$$[y_1(0) = 6, y_2(0) = 5, y_3(0) = -7]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 57

```
dsolve([diff(y__1(t),t) = -y__1(t)+y__2(t), diff(y__2(t),t) = y__1(t)-y__2(t)-2*y__3(t), diff
```

$$y_1(t) = 4\,\mathrm{e}^{t} + 2\,\mathrm{e}^{-2t} - t\,\mathrm{e}^{-2t}$$

$$y_2(t) = t\,\mathrm{e}^{-2t} + 8\,\mathrm{e}^{t} - 3\,\mathrm{e}^{-2t}$$

$$y_3(t) = -6\,\mathrm{e}^{t} - \mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 55

```
DSolve[{y1'[t]==-1*y1[t]+1*y2[t]+0*y3[t],y2'[t]==1*y1[t]-1*y2[t]-2*y3[t],y3'[t]==-1*y1[t]-1*y
```

$$
\begin{aligned}
\text{y1}(t) &\to 4e^t - e^{-2t}(t-2) \\
\text{y2}(t) &\to e^{-2t}(t-3) + 8e^t \\
\text{y3}(t) &\to -e^{-2t} - 6e^t
\end{aligned}
$$

22.19 problem section 10.5, problem 19

Internal problem ID [1622]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 19.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= -2y_1(t) + 2y_2(t) + y_3(t) \\
y_2'(t) &= -2y_1(t) + 2y_2(t) + y_3(t) \\
y_3'(t) &= -3y_1(t) + 3y_2(t) + 2y_3(t)
\end{aligned}
$$

With initial conditions

$$[y_1(0) = -6, y_2(0) = -2, y_3(0) = 0]$$

✓ Solution by Maple

Time used: 38.844 (sec). Leaf size: 41

```
dsolve([diff(y__1(t),t) = -2*y__1(t)+2*y__2(t)+y__3(t), diff(y__2(t),t) = -2*y__1(t)+2*y__2(t
```

$$y_1(t) = 2t + 3\,\mathrm{e}^{2t} - 9$$

$$y_2(t) = 2t + 3\,\mathrm{e}^{2t} - 5$$

$$y_3(t) = -6 + 6\,\mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 44

```
DSolve[{y1'[t]==-2*y1[t]+2*y2[t]+1*y3[t],y2'[t]==-2*y1[t]+2*y2[t]+1*y3[t],y3'[t]==-3*y1[t]+3*
```

$$
\begin{aligned}
\text{y1}(t) &\to 2t + 3e^{2t} - 9 \\
\text{y2}(t) &\to 2t + 3e^{2t} - 5 \\
\text{y3}(t) &\to 6\left(e^{2t} - 1\right)
\end{aligned}
$$

22.20 problem section 10.5, problem 20

Internal problem ID [1623]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 20.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= -7y_1(t) - 4y_2(t) + 4y_3(t) \\
y_2'(t) &= y_1(t) + y_3(t) \\
y_3'(t) &= -9y_1(t) - 5y_2(t) + 6y_3(t)
\end{aligned}
$$

With initial conditions

$$
[y_1(0) = -6, y_2(0) = 9, y_3(0) = -1]
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 80

```
dsolve([diff(y__1(t),t) = -7*y__1(t)-4*y__2(t)+4*y__3(t), diff(y__2(t),t) = y__1(t)+y__3(t),
```

$$
y_1(t) = \frac{4\,\mathrm{e}^{-t}\left(-\frac{17\sin(2t)}{2} - \frac{51\cos(2t)}{2}\right)}{17}
$$

$$
y_2(t) = \frac{9\,\mathrm{e}^{t}}{2} - \frac{7\,\mathrm{e}^{-t}\sin(2t)}{2} + \frac{9\,\mathrm{e}^{-t}\cos(2t)}{2}
$$

$$
y_3(t) = \frac{9\,\mathrm{e}^{t}}{2} - \frac{7\,\mathrm{e}^{-t}\sin(2t)}{2} - \frac{11\,\mathrm{e}^{-t}\cos(2t)}{2}
$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 50

```
DSolve[{y1'[t]==-7*y1[t]-4*y2[t]+4*y3[t],y2'[t]==-1*y1[t]-0*y2[t]+1*y3[t],y3'[t]==-9*y1[t]-5*
```

$$y1(t) \to -2e^{-3t} - 4e^t$$
$$y2(t) \to e^t(9 - 4t)$$
$$y3(t) \to e^t(1 - 4t) - 2e^{-3t}$$

22.21 problem section 10.5, problem 21

Internal problem ID [1624]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 21.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= -y_1(t) - 4y_2(t) - y_3(t) \\
y_2'(t) &= 3y_1(t) + 6y_2(t) + y_3(t) \\
y_3'(t) &= -3y_1(t) - 2y_2(t) + 3y_3(t)
\end{aligned}
$$

With initial conditions

$$[y_1(0) = -2, y_2(0) = 1, y_3(0) = 3]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 63

```
dsolve([diff(y__1(t),t) = -y__1(t)-4*y__2(t)-y__3(t), diff(y__2(t),t) = 3*y__1(t)+6*y__2(t)+y
```

$$y_1(t) = -2\,e^{4t} + 3\,e^{2t}t$$

$$y_2(t) = 2\,e^{4t} - e^{2t} - 3\,e^{2t}t$$

$$y_3(t) = 2\,e^{4t} + e^{2t} + 3\,e^{2t}t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 63

```
DSolve[{y1'[t]==-1*y1[t]-4*y2[t]-1*y3[t],y2'[t]==3*y1[t]+6*y2[t]+1*y3[t],y3'[t]==-3*y1[t]-2*y
```

$$y1(t) \to 3e^{2t}t - 2e^{4t}$$
$$y2(t) \to e^{2t}\left(-3t + 2e^{2t} - 1\right)$$
$$y3(t) \to e^{2t}\left(3t + 2e^{2t} + 1\right)$$

22.22 problem section 10.5, problem 22

Internal problem ID [1625]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 22.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = 4y_1(t) - 8y_2(t) - 4y_3(t)$$
$$y_2'(t) = -3y_1(t) - y_2(t) - 4y_3(t)$$
$$y_3'(t) = y_1(t) - y_2(t) + 9y_3(t)$$

With initial conditions

$$[y_1(0) = -4, y_2(0) = 1, y_3(0) = -3]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 62

```
dsolve([diff(y__1(t),t) = 4*y__1(t)-8*y__2(t)-4*y__3(t), diff(y__2(t),t) = -3*y__1(t)-y__2(t)
```

$$y_1(t) = -\frac{50\,e^{7t}}{11} + \frac{22\,e^{9t}}{13} - \frac{164\,e^{-4t}}{143}$$

$$y_2(t) = \frac{5\,e^{7t}}{11} + \frac{22\,e^{9t}}{13} - \frac{164\,e^{-4t}}{143}$$

$$y_3(t) = \frac{5\,e^{7t}}{2} - \frac{11\,e^{9t}}{2}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 57

```
DSolve[{y1'[t]==4*y1[t]-8*y2[t]-4*y3[t],y2'[t]==-3*y1[t]-1*y2[t]-3*y3[t],y3'[t]==1*y1[t]-1*y2
```

$$y1(t) \to e^{8t}(8t-3) - e^{-4t}$$
$$y2(t) \to 2e^{8t} - e^{-4t}$$
$$y3(t) \to -e^{8t}(8t+3)$$

22.23 problem section 10.5, problem 23

Internal problem ID [1626]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 23.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -5y_1(t) - y_2(t) + 11y_3(t)$$
$$y_2'(t) = -7y_1(t) + y_2(t) + 13y_3(t)$$
$$y_3'(t) = -4y_1(t) + 8y_3(t)$$

With initial conditions

$$[y_1(0) = 0, y_2(0) = 2, y_3(0) = 2]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 44

```
dsolve([diff(y__1(t),t) = -5*y__1(t)-y__2(t)+11*y__3(t), diff(y__2(t),t) = -7*y__1(t)+y__2(t)
```

$$y_1(t) = -3 + 3\,e^{4t} + 8t$$

$$y_2(t) = 6\,e^{4t} - 4 + 4t$$

$$y_3(t) = -1 + 4t + 3\,e^{4t}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 47

```
DSolve[{y1'[t]==-5*y1[t]-1*y2[t]+11*y3[t],y2'[t]==-7*y1[t]+1*y2[t]+13*y3[t],y3'[t]==-4*y1[t]-
```

$$\text{y1}(t)\to 8t+3e^{4t}-3$$
$$\text{y2}(t)\to 4t+6e^{4t}-4$$
$$\text{y3}(t)\to 4t+3e^{4t}-1$$

22.24 problem section 10.5, problem 24

Internal problem ID [1627]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 24.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= 5y_1(t) - y_2(t) + y_3(t) \\
y_2'(t) &= -y_1(t) + 9y_2(t) - 3y_3(t) \\
y_3'(t) &= -2y_1(t) + 2y_2(t) + 4y_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 67

```
dsolve([diff(y__1(t),t)=5*y__1(t)-1*y__2(t)+1*y__3(t),diff(y__2(t),t)=-1*y__1(t)+9*y__2(t)-3*
```

$$y_1(t) = -\frac{e^{6t}(2c_3t + c_2 - c_3)}{4}$$

$$y_2(t) = \frac{e^{6t}(4c_3t^2 + 4c_2t + 2c_3t + 4c_1 + c_2 + c_3)}{4}$$

$$y_3(t) = e^{6t}\left(c_3t^2 + c_2t + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 96

```
DSolve[{y1'[t]==5*y1[t]-1*y2[t]+1*y3[t],y2'[t]==-1*y1[t]+9*y2[t]-3*y3[t],y3'[t]==-2*y1[t]+2*y
```

$$
\begin{aligned}
\mathrm{y1}(t) &\to e^{6t}(c_1(-t) - c_2t + c_3t + c_1) \\
\mathrm{y2}(t) &\to e^{6t}(c_1t(2t-1) + (c_2 - c_3)t(2t+3) + c_2) \\
\mathrm{y3}(t) &\to e^{6t}(2t(c_1(t-1) + (c_2 - c_3)(t+1)) + c_3)
\end{aligned}
$$

22.25 problem section 10.5, problem 25

Internal problem ID [1628]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 25.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= y_1(t) + 10y_2(t) - 12y_3(t) \\
y_2'(t) &= 2y_1(t) + 2y_2(t) + 3y_3(t) \\
y_3'(t) &= 2y_1(t) - y_2(t) + 6y_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 77

```
dsolve([diff(y__1(t),t)=1*y__1(t)+10*y__2(t)-12*y__3(t),diff(y__2(t),t)=2*y__1(t)+2*y__2(t)+3
```

$$y_1(t) = -\frac{e^{3t}(18c_3t^2 + 18c_2t - 18c_3t + 18c_1 - 9c_2 - c_3)}{18}$$

$$y_2(t) = \frac{e^{3t}(9c_3t^2 + 9c_2t + 9c_1 + c_3)}{9}$$

$$y_3(t) = e^{3t}\left(c_3t^2 + c_2t + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 112

```
DSolve[{y1'[t]==1*y1[t]+10*y2[t]-12*y3[t],y2'[t]==2*y1[t]+2*y2[t]+3*y3[t],y3'[t]==2*y1[t]-1*y
```

$$\text{y1}(t) \to -e^{3t}(c_1(2t-1)+c_2t(9t-10)+3c_3(4-3t)t)$$
$$\text{y2}(t) \to e^{3t}(t(c_2(9t-1)+3c_3(1-3t)+2c_1)+c_2)$$
$$\text{y3}(t) \to e^{3t}(t(c_2(9t-1)+3c_3(1-3t)+2c_1)+c_3)$$

22.26 problem section 10.5, problem 26

Internal problem ID [1629]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 26.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= -6y_1(t) - 4y_2(t) - 4y_3(t) \\
y_2'(t) &= 2y_1(t) - y_2(t) + y_3(t) \\
y_3'(t) &= 2y_1(t) + 3y_2(t) + y_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 67

```
dsolve([diff(y__1(t),t)=-6*y__1(t)-4*y__2(t)-4*y__3(t),diff(y__2(t),t)=2*y__1(t)-1*y__2(t)+1*
```

$$y_1(t) = -\frac{e^{-2t}(4c_3t + 2c_2 - 3c_3)}{2}$$

$$y_2(t) = -e^{-2t}(c_3t^2 + c_2t - 2c_3t + c_1 - c_2 + c_3)$$

$$y_3(t) = e^{-2t}(c_3t^2 + c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 95

```
DSolve[{y1'[t]==-6*y1[t]-4*y2[t]-4*y3[t],y2'[t]==2*y1[t]-1*y2[t]+1*y3[t],y3'[t]==2*y1[t]+3*y2
```

$$
\begin{aligned}
\text{y1}(t) &\to e^{-2t}(c_1(1-4t) - 4(c_2+c_3)t) \\
\text{y2}(t) &\to e^{-2t}\left(-2(c_1+c_2+c_3)t^2 + (2c_1+c_2+c_3)t + c_2\right) \\
\text{y3}(t) &\to e^{-2t}(2c_1t(t+1) + (c_2+c_3)t(2t+3) + c_3)
\end{aligned}
$$

22.27 problem section 10.5, problem 27

Internal problem ID [1630]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 27.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= 2y_2(t) - 2y_3(t) \\
y_2'(t) &= -y_1(t) + 5y_2(t) - 3y_3(t) \\
y_3'(t) &= y_1(t) + y_2(t) + y_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 71

```
dsolve([diff(y__1(t),t)=0*y__1(t)+2*y__2(t)-2*y__3(t),diff(y__2(t),t)=-1*y__1(t)+5*y__2(t)-3*
```

$$y_1(t) = \frac{e^{2t}(4c_3t + 2c_2 - c_3)}{4}$$

$$y_2(t) = \frac{e^{2t}(4c_3t^2 + 4c_2t + 4c_3t + 4c_1 + 2c_2 + c_3)}{4}$$

$$y_3(t) = e^{2t}\left(c_3t^2 + c_2t + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 100

```
DSolve[{y1'[t]==0*y1[t]+2*y2[t]-2*y3[t],y2'[t]==-1*y1[t]+5*y2[t]-3*y3[t],y3'[t]==1*y1[t]+1*y2
```

$$\mathrm{y1}(t) \to e^{2t}(-2c_1t + 2c_2t - 2c_3t + c_1)$$
$$\mathrm{y2}(t) \to e^{2t}(c_1(-t)(2t+1) + (c_2 - c_3)t(2t+3) + c_2)$$
$$\mathrm{y3}(t) \to e^{2t}(c_1(1-2t)t + (c_2 - c_3)t(2t+1) + c_3)$$

22.28 problem section 10.5, problem 28

Internal problem ID [1631]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 28.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= -2y_1(t) - 12y_2(t) + 10y_3(t) \\
y_2'(t) &= 2y_1(t) - 24y_2(t) + 11y_3(t) \\
y_3'(t) &= 2y_1(t) - 24y_2(t) + 8y_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 86

```
dsolve([diff(y__1(t),t)=-2*y__1(t)-12*y__2(t)+10*y__3(t),diff(y__2(t),t)=2*y__1(t)-24*y__2(t)
```

$$y_1(t) = -\frac{e^{-6t}(6c_3t^2 + 6c_2t + 6c_3t + 6c_1 + 3c_2 + 2c_3)}{6}$$

$$y_2(t) = \frac{e^{-6t}(18c_3t^2 + 18c_2t - 6c_3t + 18c_1 - 3c_2 - c_3)}{36}$$

$$y_3(t) = e^{-6t}\left(c_3t^2 + c_2t + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 125

```
DSolve[{y1'[t]==-2*y1[t]-12*y2[t]+10*y3[t],y2'[t]==2*y1[t]-24*y2[t]+11*y3[t],y3'[t]==2*y1[t]-
```

$$y1(t) \to e^{-6t}\left(c_1\left(6t^2+4t+1\right)+2t(c_3(12t+5)-6c_2(3t+1))\right)$$
$$y2(t) \to e^{-6t}(t(c_1(2-3t)+18c_2(t-1)+c_3(11-12t))+c_2)$$
$$y3(t) \to e^{-6t}(2t(c_1(1-3t)+6c_2(3t-2)+c_3(7-12t))+c_3)$$

22.29 problem section 10.5, problem 29

Internal problem ID [1632]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 29.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= -y_1(t) - 12y_2(t) + 8y_3(t) \\
y_2'(t) &= y_1(t) - 9y_2(t) + 4y_3(t) \\
y_3'(t) &= y_1(t) - 6y_2(t) + y_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 49

```
dsolve([diff(y__1(t),t)=-1*y__1(t)-12*y__2(t)+8*y__3(t),diff(y__2(t),t)=1*y__1(t)-9*y__2(t)+4
```

$$y_1(t) = e^{-3t}(2c_3t + 6c_1 + 2c_2 + c_3)$$

$$y_2(t) = e^{-3t}(c_3t + c_1 + c_2)$$

$$y_3(t) = e^{-3t}(c_3t + c_2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 79

```
DSolve[{y1'[t]==-1*y1[t]-12*y2[t]+8*y3[t],y2'[t]==1*y1[t]-9*y2[t]+4*y3[t],y3'[t]==1*y1[t]-6*y
```

$$
\begin{aligned}
\text{y1}(t) &\to e^{-3t}(2c_1t - 12c_2t + 8c_3t + c_1) \\
\text{y2}(t) &\to e^{-3t}((c_1 - 6c_2 + 4c_3)t + c_2) \\
\text{y3}(t) &\to e^{-3t}((c_1 - 6c_2 + 4c_3)t + c_3)
\end{aligned}
$$

22.30 problem section 10.5, problem 30

Internal problem ID [1633]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 30.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= -4y_1(t) - y_3(t) \\
y_2'(t) &= -y_1(t) - 3y_2(t) - y_3(t) \\
y_3'(t) &= y_1(t) - 2y_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 49

```
dsolve([diff(y__1(t),t)=-4*y__1(t)-0*y__2(t)-1*y__3(t),diff(y__2(t),t)=-1*y__1(t)-3*y__2(t)-1
```

$$y_1(t) = -\mathrm{e}^{-3t}(c_3t + c_2 - c_3)$$

$$y_2(t) = \mathrm{e}^{-3t}(-c_3t + c_1 - c_2)$$

$$y_3(t) = \mathrm{e}^{-3t}(c_3t + c_2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 63

```
DSolve[{y1'[t]==-4*y1[t]-0*y2[t]-1*y3[t],y2'[t]==-1*y1[t]-3*y2[t]-1*y3[t],y3'[t]==1*y1[t]-0*y
```

$$
\begin{aligned}
\mathrm{y1}(t) &\to e^{-3t}(c_1(-t) - c_3t + c_1) \\
\mathrm{y2}(t) &\to e^{-3t}(c_2 - (c_1 + c_3)t) \\
\mathrm{y3}(t) &\to e^{-3t}((c_1 + c_3)t + c_3)
\end{aligned}
$$

22.31 problem section 10.5, problem 31

Internal problem ID [1634]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 31.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -3y_1(t) - 3y_2(t) + 4y_3(t)$$
$$y_2'(t) = 4y_1(t) + 5y_2(t) - 8y_3(t)$$
$$y_3'(t) = 2y_1(t) + 3y_2(t) - 5y_3(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 55

```
dsolve([diff(y__1(t),t)=-3*y__1(t)-3*y__2(t)+4*y__3(t),diff(y__2(t),t)=4*y__1(t)+5*y__2(t)-8*
```

$$y_1(t) = -\frac{e^{-t}(2c_3t + 3c_1 + 2c_2 - c_3)}{2}$$

$$y_2(t) = e^{-t}(2c_3t + c_1 + 2c_2)$$

$$y_3(t) = e^{-t}(c_3t + c_2)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 83

```
DSolve[{y1'[t]==-3*y1[t]-3*y2[t]+4*y3[t],y2'[t]==4*y1[t]+5*y2[t]-8*y3[t],y3'[t]==2*y1[t]+3*y2
```

$$\text{y1}(t) \to e^{-t}(-2c_1t - 3c_2t + 4c_3t + c_1)$$
$$\text{y2}(t) \to e^{-t}((4c_1 + 6c_2 - 8c_3)t + c_2)$$
$$\text{y3}(t) \to e^{-t}((2c_1 + 3c_2 - 4c_3)t + c_3)$$

22.32 problem section 10.5, problem 32

Internal problem ID [1635]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.5, constant coefficient homogeneous system II. Page 555
Problem number: section 10.5, problem 32.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -3y_1(t) - y_2(t)$$
$$y_2'(t) = y_1(t) - y_2(t)$$
$$y_3'(t) = -y_1(t) - y_2(t) - 2y_3(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 51

```
dsolve([diff(y__1(t),t)=-3*y__1(t)-1*y__2(t)+0*y__3(t),diff(y__2(t),t)=1*y__1(t)-1*y__2(t)+0*
```

$$y_1(t) = -\mathrm{e}^{-2t}(-c_3 t + c_1 - c_2 + c_3)$$

$$y_2(t) = \mathrm{e}^{-2t}(-c_3 t + c_1 - c_2)$$

$$y_3(t) = \mathrm{e}^{-2t}(c_3 t + c_2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 63

```
DSolve[{y1'[t]==-3*y1[t]-1*y2[t]+0*y3[t],y2'[t]==1*y1[t]-1*y2[t]+0*y3[t],y3'[t]==-1*y1[t]-1*y
```

$$\text{y1}(t) \to e^{-2t}(c_1(-t) - c_2 t + c_1)$$
$$\text{y2}(t) \to e^{-2t}((c_1 + c_2)t + c_2)$$
$$\text{y3}(t) \to e^{-2t}(c_3 - (c_1 + c_2)t)$$

23 Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566

23.1 problem section 10.6, problem 1

Internal problem ID [1636]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566
Problem number: section 10.6, problem 1.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -y_1(t) + 2y_2(t)$$
$$y_2'(t) = -5y_1(t) + 5y_2(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 48

```
dsolve([diff(y__1(t),t)=-1*y__1(t)+2*y__2(t),diff(y__2(t),t)=-5*y__1(t)+5*y__2(t)],[y__1(t),
```

$$y_1(t) = \frac{e^{2t}(3c_1 \sin(t) + c_2 \sin(t) - \cos(t) c_1 + 3c_2 \cos(t))}{5}$$

$$y_2(t) = e^{2t}(c_1 \sin(t) + c_2 \cos(t))$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 55

```
DSolve[{y1'[t]==-1*y1[t]+2*y2[t],y2'[t]==-5*y1[t]+5*y2[t]},{y1[t],y2[t]},t,IncludeSingularSol
```

$$\text{y1}(t) \to e^{2t}(c_1 \cos(t) + (2c_2 - 3c_1)\sin(t))$$
$$\text{y2}(t) \to e^{2t}(c_2(3\sin(t) + \cos(t)) - 5c_1 \sin(t))$$

23.2 problem section 10.6, problem 2

Internal problem ID [1637]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566
Problem number: section 10.6, problem 2.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -11y_1(t) + 4y_2(t)$$
$$y_2'(t) = -26y_1(t) + 9y_2(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 60

```
dsolve([diff(y__1(t),t)=-11*y__1(t)+4*y__2(t),diff(y__2(t),t)=-26*y__1(t)+9*y__2(t)],[y__1(t)
```

$$y_1(t) = \frac{e^{-t}(5c_1 \sin(2t) + c_2 \sin(2t) - c_1 \cos(2t) + 5c_2 \cos(2t))}{13}$$

$$y_2(t) = e^{-t}(c_1 \sin(2t) + c_2 \cos(2t))$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 64

```
DSolve[{y1'[t]==-11*y1[t]+4*y2[t],y2'[t]==-26*y1[t]+9*y2[t]},{y1[t],y2[t]},t,IncludeSingularS
```

$$\text{y1}(t) \to e^{-t}(c_1 \cos(2t) + (2c_2 - 5c_1)\sin(2t))$$
$$\text{y2}(t) \to e^{-t}(c_2 \cos(2t) + (5c_2 - 13c_1)\sin(2t))$$

23.3 problem section 10.6, problem 3

Internal problem ID [1638]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566
Problem number: section 10.6, problem 3.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = y_1(t) + 2y_2(t)$$
$$y_2'(t) = -4y_1(t) + 5y_2(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 58

```
dsolve([diff(y__1(t),t)=1*y__1(t)+2*y__2(t),diff(y__2(t),t)=-4*y__1(t)+5*y__2(t)],[y__1(t), y
```

$$y_1(t) = \frac{e^{3t}(c_1 \sin(2t) + c_2 \sin(2t) - c_1 \cos(2t) + c_2 \cos(2t))}{2}$$

$$y_2(t) = e^{3t}(c_1 \sin(2t) + c_2 \cos(2t))$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 60

```
DSolve[{y1'[t]==1*y1[t]+2*y2[t],y2'[t]==-4*y1[t]+5*y2[t]},{y1[t],y2[t]},t,IncludeSingularSolu
```

$$\text{y1}(t) \to e^{3t}(c_1 \cos(2t) + (c_2 - c_1)\sin(2t))$$
$$\text{y2}(t) \to e^{3t}(c_2 \cos(2t) + (c_2 - 2c_1)\sin(2t))$$

23.4 problem section 10.6, problem 4

Internal problem ID [1639]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566
Problem number: section 10.6, problem 4.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = 5y_1(t) - 6y_2(t)$$
$$y_2'(t) = 3y_1(t) - y_2(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 57

```
dsolve([diff(y__1(t),t)=5*y__1(t)-6*y__2(t),diff(y__2(t),t)=3*y__1(t)-1*y__2(t)],[y__1(t), y_
```

$$y_1(t) = \mathrm{e}^{2t}(c_1 \cos(3t) + c_2 \cos(3t) + c_1 \sin(3t) - c_2 \sin(3t))$$

$$y_2(t) = \mathrm{e}^{2t}(c_1 \sin(3t) + c_2 \cos(3t))$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 60

```
DSolve[{y1'[t]==5*y1[t]-6*y2[t],y2'[t]==3*y1[t]-1*y2[t]},{y1[t],y2[t]},t,IncludeSingularSolut
```

$$\mathrm{y1}(t) \to e^{2t}(c_1 \cos(3t) + (c_1 - 2c_2)\sin(3t))$$
$$\mathrm{y2}(t) \to e^{2t}(c_2 \cos(3t) + (c_1 - c_2)\sin(3t))$$

23.5 problem section 10.6, problem 5

Internal problem ID [1640]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566
Problem number: section 10.6, problem 5.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = -3y_1(t) - 3y_2(t) + y_3(t)$$
$$y_2'(t) = 2y_2(t) + 2y_3(t)$$
$$y_3'(t) = 5y_1(t) + y_2(t) + y_3(t)$$

✓ Solution by Maple

Time used: 0.563 (sec). Leaf size: 2820

```
dsolve([diff(y__1(t),t)=-3*y__1(t)-3*y__2(t)+1*y__3(t),diff(y__2(t),t)=0*y__1(t)+2*y__2(t)+2*
```

Expression too large to display

Expression too large to display

$$y_3(t) = c_2 e^{\frac{\left(\frac{(540+6\sqrt{6042})^{\frac{2}{3}}}{6}+7\right)t}{(540+6\sqrt{6042})^{\frac{1}{3}}}} \sin\left(\frac{\left((540+6\sqrt{6042})^{\frac{2}{3}}-42\right)t\sqrt{3}\,36^{\frac{1}{3}}}{36\,(90+\sqrt{6042})^{\frac{1}{3}}}\right) + c_3 e^{\frac{\left(\frac{(540+6\sqrt{6042})^{\frac{2}{3}}}{6}+7\right)t}{(540+6\sqrt{6042})^{\frac{1}{3}}}} \cos\left(\frac{\left((540+6\sqrt{6042})^{\frac{2}{3}}-42\right)t\sqrt{3}\,36^{\frac{1}{3}}}{36\,(90+\sqrt{6042})^{\frac{1}{3}}}\right) + c_1 e^{-\frac{\left((540+6\sqrt{6042})^{\frac{2}{3}}+42\right)t}{3(540+6\sqrt{6042})^{\frac{1}{3}}}}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 177

```
DSolve[{y1'[t]==3*y1[t]-3*y2[t]+1*y3[t],y2'[t]==0*y1[t]+2*y2[t]+2*y3[t],y3'[t]==5*y1[t]+1*y2[
```

$$\text{y1}(t) \to \frac{1}{4}e^{-2t}\left(e^{6t}((3c_1 - c_2 + c_3)\cos(2t) + (c_1 - 3c_2 - c_3)\sin(2t)) + c_1 + c_2 - c_3\right)$$
$$\text{y2}(t) \to \frac{1}{4}e^{-2t}\left(e^{6t}((-c_1 + 3c_2 + c_3)\cos(2t) + (3c_1 - c_2 + c_3)\sin(2t)) + c_1 + c_2 - c_3\right)$$
$$\text{y3}(t) \to \frac{1}{2}e^{-2t}\left(e^{6t}((c_1 + c_2 + c_3)\cos(2t) + 2(c_1 - c_2)\sin(2t)) - c_1 - c_2 + c_3\right)$$

23.6 problem section 10.6, problem 6

Internal problem ID [1641]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566
Problem number: section 10.6, problem 6.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= -3y_1(t) + 3y_2(t) + y_3(t) \\
y_2'(t) &= y_1(t) - 5y_2(t) - 3y_3(t) \\
y_3'(t) &= -3y_1(t) + 7y_2(t) + 3y_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 142

```
dsolve([diff(y__1(t),t)=-3*y__1(t)+3*y__2(t)+1*y__3(t),diff(y__2(t),t)=1*y__1(t)-5*y__2(t)-3*
```

$$y_1(t) = -e^{-t}c_1 + \frac{c_2 e^{-2t}\sin(2t)}{2} + \frac{c_2 e^{-2t}\cos(2t)}{2} + \frac{c_3 e^{-2t}\cos(2t)}{2} - \frac{c_3 e^{-2t}\sin(2t)}{2}$$

$$y_2(t) = -e^{-t}c_1 + \frac{c_2 e^{-2t}\cos(2t)}{2} - \frac{c_3 e^{-2t}\sin(2t)}{2} - \frac{c_2 e^{-2t}\sin(2t)}{2} - \frac{c_3 e^{-2t}\cos(2t)}{2}$$

$$y_3(t) = e^{-t}c_1 + c_2 e^{-2t}\sin(2t) + c_3 e^{-2t}\cos(2t)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 158

```
DSolve[{y1'[t]==-3*y1[t]+3*y2[t]+1*y3[t],y2'[t]==1*y1[t]-5*y2[t]-3*y3[t],y3'[t]==-3*y1[t]+7*y
```

$$y1(t) \to e^{-2t}\left((c_1 - c_2 - c_3)e^t + (c_2 + c_3)\cos(2t) + (-c_1 + 2c_2 + c_3)\sin(2t)\right)$$
$$y2(t) \to e^{-2t}\left((c_1 - c_2 - c_3)e^t + (-c_1 + 2c_2 + c_3)\cos(2t) - (c_2 + c_3)\sin(2t)\right)$$
$$y3(t) \to e^{-2t}\left((-c_1 + c_2 + c_3)e^t + (c_1 - c_2)\cos(2t) + (-c_1 + 3c_2 + 2c_3)\sin(2t)\right)$$

23.7 problem section 10.6, problem 7

Internal problem ID [1642]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566
Problem number: section 10.6, problem 7.
ODE order: 1.
ODE degree: 1.

Solve

$$y_1'(t) = 2y_1(t) + y_2(t) - y_3(t)$$
$$y_2'(t) = y_2(t) + y_3(t)$$
$$y_3'(t) = y_1(t) + y_3(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 70

```
dsolve([diff(y__1(t),t)=2*y__1(t)+1*y__2(t)-1*y__3(t),diff(y__2(t),t)=0*y__1(t)+1*y__2(t)+1*y
```

$$y_1(t) = c_1 e^{2t} + c_2 e^t \cos(t) - c_3 e^t \sin(t)$$

$$y_2(t) = c_1 e^{2t} - c_2 e^t \cos(t) + c_3 e^t \sin(t)$$

$$y_3(t) = c_1 e^{2t} + c_2 e^t \sin(t) + c_3 e^t \cos(t)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 129

```
DSolve[{y1'[t]==2*y1[t]+1*y2[t]-1*y3[t],y2'[t]==0*y1[t]+1*y2[t]+1*y3[t],y3'[t]==1*y1[t]+0*y2[
```

$$\text{y1}(t) \to \frac{1}{2}e^t\left(-2c_3\sin(t) + c_2\left(e^t + \sin(t) - \cos(t)\right) + c_1\left(e^t + \sin(t) + \cos(t)\right)\right)$$

$$\text{y2}(t) \to \frac{1}{2}e^t\left((c_1 + c_2)e^t + (c_2 - c_1)\cos(t) - (c_1 + c_2 - 2c_3)\sin(t)\right)$$

$$\text{y3}(t) \to \frac{1}{2}e^t\left((c_1 + c_2)e^t - (c_1 + c_2 - 2c_3)\cos(t) + (c_1 - c_2)\sin(t)\right)$$

23.8 problem section 10.6, problem 8

Internal problem ID [1643]

Book: Elementary differential equations with boundary value problems. William F. Trench. Brooks/Cole 2001
Section: Chapter 10 Linear system of Differential equations. Section 10.6, constant coefficient homogeneous system III. Page 566
Problem number: section 10.6, problem 8.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
y_1'(t) &= -3y_1(t) + y_2(t) - 3y_3(t) \\
y_2'(t) &= 4y_1(t) - y_2(t) + 2y_3(t) \\
y_3'(t) &= 4y_1(t) - 2y_2(t) + 3y_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 112

```
dsolve([diff(y__1(t),t)=-3*y__1(t)+1*y__2(t)-3*y__3(t),diff(y__2(t),t)=4*y__1(t)-1*y__2(t)+2*
```

$$y_1(t) = -c_1 e^t - \frac{c_2 e^{-t} \sin(2t)}{2} - \frac{c_3 e^{-t} \cos(2t)}{2} + \frac{c_2 e^{-t} \cos(2t)}{2} - \frac{c_3 e^{-t} \sin(2t)}{2}$$

$$y_2(t) = -c_1 e^t + c_2 e^{-t} \sin(2t) + c_3 e^{-t} \cos(2t)$$

$$y_3(t) = c_1 e^t + c_2 e^{-t} \sin(2t) + c_3 e^{-t} \cos(2t)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 163

```
DSolve[{y1'[t]==-3*y1[t]+1*y2[t]-3*y3[t],y2'[t]==4*y1[t]-1*y2[t]+2*y3[t],y3'[t]==4*y1[t]-2*y2
```

$$\text{y1}(t) \to \frac{1}{2}e^{-t}\left((c_2 - c_3)e^{2t} + (2c_1 - c_2 + c_3)\cos(2t) - 2(c_1 + c_3)\sin(2t)\right)$$
$$\text{y2}(t) \to \frac{1}{2}e^{-t}\left((c_2 - c_3)e^{2t} + (c_2 + c_3)\cos(2t) + (4c_1 - c_2 + 3c_3)\sin(2t)\right)$$
$$\text{y3}(t) \to \frac{1}{2}e^{-t}\left((c_3 - c_2)e^{2t} + (c_2 + c_3)\cos(2t) + (4c_1 - c_2 + 3c_3)\sin(2t)\right)$$