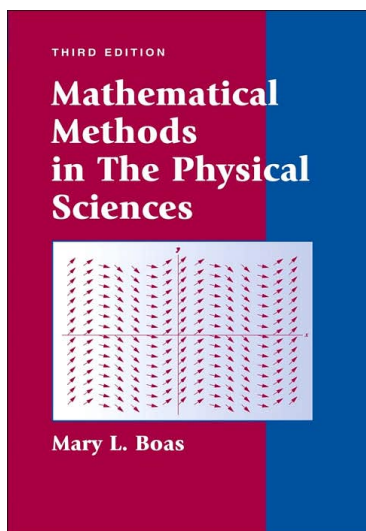


A Solution Manual For

**Mathematical Methods in the
Physical Sciences. third edition.
Mary L. Boas. John Wiley. 2006**



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October 12, 2023

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1 Chapter 8, Ordinary differential equations. Section 1. Introduction. page 394

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1.1 problem 1

Internal problem ID [4239]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 1. Introduction. page 394

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^x$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^x + c_2 e^{-x}$$

2 Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

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2.1 problem 1

Internal problem ID [4240]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$-y + y'x = 0$$

With initial conditions

$$[y(2) = 3]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve([x*diff(y(x),x)=y(x),y(2) = 3],y(x), singsol=all)
```

$$y(x) = \frac{3x}{2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 10

```
DSolve[{x*y'[x]==y[x],{y[2]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{3x}{2}$$

2.2 problem 2

Internal problem ID [4241]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$x\sqrt{1-y^2} + y\sqrt{-x^2+1}y' = 0$$

With initial conditions

$$\left[y\left(\frac{1}{2}\right) = \frac{1}{2} \right]$$

✓ Solution by Maple

Time used: 0.375 (sec). Leaf size: 26

```
dsolve([x*sqrt(1-y(x)^2)+y(x)*sqrt(1-x^2)*diff(y(x),x)=0,y(1/2) = 1/2],y(x), singsol=all)
```

$$y(x) = \sqrt{2\sqrt{3}\sqrt{-x^2+1} + x^2 - 3}$$

✓ Solution by Mathematica

Time used: 3.589 (sec). Leaf size: 38

```
DSolve[{x*Sqrt[1-y[x]^2]+y[x]*Sqrt[1-x^2]*y'[x]==0,{y[1/2]==1/2}},y[x],x,IncludeSingularSolut
```

$$y(x) \rightarrow \sqrt{x^2}$$

$$y(x) \rightarrow \sqrt{x^2 + 2\sqrt{3} - 3x^2 - 3}$$

2.3 problem 3

Internal problem ID [4242]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' \sin(x) - y \ln(y) = 0$$

With initial conditions

$$\left[y\left(\frac{\pi}{3}\right) = e \right]$$

✓ Solution by Maple

Time used: 0.438 (sec). Leaf size: 16

```
dsolve([diff(y(x),x)*sin(x)=y(x)*ln(y(x)),y(1/3*Pi) = exp(1)],y(x), singsol=all)
```

$$y(x) = e^{(\csc(x) - \cot(x))\sqrt{3}}$$

✓ Solution by Mathematica

Time used: 0.209 (sec). Leaf size: 19

```
DSolve[{y'[x]*Sin[x]==y[x]*Log[y[x]],{y[Pi/3]==Exp[1]}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \rightarrow e^{\sqrt{3} \tan\left(\frac{x}{2}\right)}$$

2.4 problem 4

Internal problem ID [4243]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$1 + y^2 + xyy' = 0$$

With initial conditions

$$[y(5) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 34

```
dsolve([(1+y(x)^2)+x*y(x)*diff(y(x),x)=0,y(5) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-x^2 + 25}}{x}$$

$$y(x) = -\frac{\sqrt{-x^2 + 25}}{x}$$

✓ Solution by Mathematica

Time used: 0.327 (sec). Leaf size: 40

```
DSolve[{(1+y[x]^2)+x*y[x]*y'[x]==0,{y[5]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt{25 - x^2}}{x}$$

$$y(x) \rightarrow \frac{\sqrt{25 - x^2}}{x}$$

2.5 problem 5

Internal problem ID [4244]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$xyy' - xy - y = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 8

```
dsolve([x*y(x)*diff(y(x),x)-x*y(x)=y(x),y(1) = 1],y(x), singsol=all)
```

$$y(x) = x + \ln(x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 9

```
DSolve[{x*y[x]*y'[x]-x*y[x]==y[x],{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x + \log(x)$$

2.6 problem 6

Internal problem ID [4245]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - \frac{2xy^2 + x}{x^2y - y} = 0$$

With initial conditions

$$[y(\sqrt{2}) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve([diff(y(x),x)=(2*x*y(x)^2+x)/(x^2*y(x)-y(x)),y(2^(1/2)) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2x^2 - 4x}}{2}$$

$$y(x) = \frac{\sqrt{2x^2 - 4x}}{2}$$

✓ Solution by Mathematica

Time used: 3.887 (sec). Leaf size: 48

```
DSolve[{y'[x]==(2*x*y[x]^2+x)/(x^2*y[x]-y[x]),{y[Sqrt[2]]==0}},y[x],x,IncludeSingularSolution
```

$$y(x) \rightarrow -\frac{\sqrt{x^2(x^2 - 2)}}{\sqrt{2}}$$

$$y(x) \rightarrow \frac{\sqrt{x^2(x^2 - 2)}}{\sqrt{2}}$$

2.7 problem 7

Internal problem ID [4246]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$yy' + xy^2 - 8x = 0$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 17

```
dsolve([y(x)*diff(y(x),x)+(x*y(x)^2-8*x)=0,y(1) = 3],y(x), singsol=all)
```

$$y(x) = \sqrt{e^{-(x-1)(x+1)} + 8}$$

✓ Solution by Mathematica

Time used: 1.925 (sec). Leaf size: 39

```
DSolve[{y[x]*y'[x]+(x*y[x]^2-8*x)==0,{y[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt{e^{1-x^2} + 8}$$

$$y(x) \rightarrow \sqrt{e^{1-x^2} + 8}$$

2.8 problem 8

Internal problem ID [4247]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' + 2xy^2 = 0$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.359 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)+2*x*y(x)^2=0,y(2) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{x^2 - 3}$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 12

```
DSolve[{y'[x]+2*x*y[x]^2==0,{y[2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{x^2 - 3}$$

2.9 problem 9

Internal problem ID [4248]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$(1 + y)y' - y = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 7

```
dsolve([(1+y(x))*diff(y(x),x)=y(x),y(1) = 1],y(x), singsol=all)
```

$$y(x) = \text{LambertW}(e^x)$$

✓ Solution by Mathematica

Time used: 2.168 (sec). Leaf size: 9

```
DSolve[{(1+y[x])*y'[x]==y[x],{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow W(e^x)$$

2.10 problem 10

Internal problem ID [4249]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - xy - x = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)-x*y(x)=x,y(0) = 1],y(x), singsol=all)
```

$$y(x) = -1 + 2e^{\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 20

```
DSolve[{y'[x]-x*y[x]==x,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2e^{\frac{1}{2}(x^2-1)} - 1$$

2.11 problem 11

Internal problem ID [4250]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$2y' - 3(y - 2)^{\frac{1}{3}} = 0$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve([2*diff(y(x),x)=3*(y(x)-2)^(1/3),y(1) = 3],y(x), singsol=all)
```

$$y(x) = 2 + x^{\frac{3}{2}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 12

```
DSolve[{2*y'[x]==3*(y[x]-2)^(1/3),{y[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^{3/2} + 2$$

2.12 problem 12

Internal problem ID [4251]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$(xy + x)y' + y = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 11

```
dsolve([(x+x*y(x))*diff(y(x),x)+y(x)=0,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \text{LambertW}\left(\frac{e}{x}\right)$$

✓ Solution by Mathematica

Time used: 2.104 (sec). Leaf size: 11

```
DSolve[{(x+x*y[x])*y'[x]+y[x]==0,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow W\left(\frac{e}{x}\right)$$

3 Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

3.1	problem 1	18
3.2	problem 2	19
3.3	problem 3	20
3.4	problem 4	21
3.5	problem 5	22
3.6	problem 6	23
3.7	problem 7	24
3.8	problem 8	25
3.9	problem 9	26
3.10	problem 10	27
3.11	problem 11	28
3.12	problem 12	29
3.13	problem 13	30
3.14	problem 14	31

3.1 problem 1

Internal problem ID [4252]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y' + y - e^x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \frac{e^x}{2} + c_1 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 21

```
DSolve[y'[x]+y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^x}{2} + c_1 e^{-x}$$

3.2 problem 2

Internal problem ID [4253]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$x^2y' + 3xy - 1 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x)+3*x*y(x)=1,y(x), singsol=all)
```

$$y(x) = \frac{\frac{x^2}{2} + c_1}{x^3}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 20

```
DSolve[x^2*y'[x]+3*x*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^2 + 2c_1}{2x^3}$$

3.3 problem 3

Internal problem ID [4254]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' + 2xy - x e^{-x^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)+2*x*y(x)-x*exp(-x^2)=0,y(x), singsol=all)
```

$$y(x) = \left(\frac{x^2}{2} + c_1 \right) e^{-x^2}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 24

```
DSolve[y'[x]+2*x*y[x]-x*Exp[-x^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} e^{-x^2} (x^2 + 2c_1)$$

3.4 problem 4

Internal problem ID [4255]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$2y'x + y - 2x^{\frac{5}{2}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*x*diff(y(x),x)+y(x)=2*x^(5/2),y(x), singsol=all)
```

$$y(x) = \frac{\frac{x^3}{3} + c_1}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 22

```
DSolve[2*x*y'[x]+y[x]==2*x^(5/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^3 + 3c_1}{3\sqrt{x}}$$

3.5 problem 5

Internal problem ID [4256]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' \cos(x) + y - \cos(x)^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)*cos(x)+y(x)=cos(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x - \cos(x) + c_1}{\sec(x) + \tan(x)}$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 25

```
DSolve[y'[x]*Cos[x]+y[x]==Cos[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)}(x - \cos(x) + c_1)$$

3.6 problem 6

Internal problem ID [4257]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$y' + \frac{y}{\sqrt{x^2 + 1}} - \frac{1}{x + \sqrt{x^2 + 1}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)+y(x)/sqrt(x^2+1)=1/(x+sqrt(x^2+1)),y(x), singsol=all)
```

$$y(x) = \frac{x + c_1}{x + \sqrt{x^2 + 1}}$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 23

```
DSolve[y'[x]+y[x]/Sqrt[x^2+1]==1/(x+Sqrt[x^2+1]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \left(\sqrt{x^2 + 1} - x \right) (x + c_1)$$

3.7 problem 7

Internal problem ID [4258]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$(e^x + 1)y' + 2e^xy - (e^x + 1)e^x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve((1+exp(x))*diff(y(x),x)+2*exp(x)*y(x)=(1+exp(x))*exp(x),y(x), singsol=all)
```

$$y(x) = \frac{e^{2x} + e^x + \frac{e^{3x}}{3} + c_1}{(e^x + 1)^2}$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 25

```
DSolve[(1+Exp[x])*y'[x]+2*Exp[x]*y[x]==(1+Exp[x])*Exp[x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \rightarrow \frac{1}{3}(e^x + 1) + \frac{c_1}{(e^x + 1)^2}$$

3.8 problem 8

Internal problem ID [4259]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$x \ln(x) y' + y - \ln(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x*ln(x))*diff(y(x),x)+y(x)=ln(x),y(x), singsol=all)
```

$$y(x) = \frac{\ln(x)}{2} + \frac{c_1}{\ln(x)}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 19

```
DSolve[(x*Log[x])*y'[x]+y[x]==Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\log(x)}{2} + \frac{c_1}{\log(x)}$$

3.9 problem 9

Internal problem ID [4260]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$(-x^2 + 1) y' - xy - 2\sqrt{-x^2 + 1} x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve((1-x^2)*diff(y(x),x)=x*y(x)+2*x*sqrt(1-x^2),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{\sqrt{-x^2 + 1}} + \frac{c_1}{\sqrt{x - 1} \sqrt{x + 1}}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 33

```
DSolve[(1-x^2)*y'[x]==x*y[x]+2*x*Sqrt[1-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^2}{\sqrt{1 - x^2}} + \frac{c_1}{\sqrt{x^2 - 1}}$$

3.10 problem 10

Internal problem ID [4261]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$y' + y \tanh(x) - 2e^x = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)+y(x)*tanh(x)=2*exp(x),y(x), singsol=all)
```

$$y(x) = \frac{\cosh(2x) + 1 + \sinh(2x) + 2x + 2c_1}{2 \cosh(x)}$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 29

```
DSolve[y'[x]+y[x]*Tanh[x]==2*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^x(2x + e^{2x} + c_1)}{e^{2x} + 1}$$

3.11 problem 11

Internal problem ID [4262]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$y' + y \cos(x) - \sin(2x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+y(x)*cos(x)=sin(2*x),y(x), singsol=all)
```

$$y(x) = 2 \sin(x) - 2 + e^{-\sin(x)} c_1$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 20

```
DSolve[y'[x]+y[x]*Cos[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2 \sin(x) + c_1 e^{-\sin(x)} - 2$$

3.12 problem 12

Internal problem ID [4263]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x' - \cos(y) + x \tan(y) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(x(y),y)=cos(y)-x(y)*tan(y),x(y), singsol=all)
```

$$x(y) = (y + c_1) \cos(y)$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 12

```
DSolve[x'[y]==Cos[y]-x[y]*Tan[y],x[y],y,IncludeSingularSolutions -> True]
```

$$x(y) \rightarrow (y + c_1) \cos(y)$$

3.13 problem 13

Internal problem ID [4264]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$x' + x - e^y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(x(y),y)+(x(y)-exp(y))=0,x(y), singsol=all)
```

$$x(y) = \frac{e^y}{2} + e^{-y}c_1$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 21

```
DSolve[x'[y]+(x[y]-Exp[y])==0,x[y],y,IncludeSingularSolutions -> True]
```

$$x(y) \rightarrow \frac{e^y}{2} + c_1 e^{-y}$$

3.14 problem 14

Internal problem ID [4265]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 14.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$x' - \frac{3y^{\frac{2}{3}} - x}{3y} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve(diff(x(y),y)=(3*y^(2/3)-x(y))/(3*y),x(y), singsol=all)
```

$$x(y) = \frac{y + c_1}{y^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 15

```
DSolve[x'[y]==(3*y^(2/3)-x[y])/(3*y),x[y],y,IncludeSingularSolutions -> True]
```

$$x(y) \rightarrow \frac{y + c_1}{\sqrt[3]{y}}$$

**4 Chapter 8, Ordinary differential equations. Section
4. OTHER METHODS FOR FIRST-ORDER
EQUATIONS. page 406**

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4.1 problem 1

Internal problem ID [4266]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_Bernoulli]

$$y' + y - xy^{\frac{2}{3}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)+y(x)=x*y(x)^(2/3),y(x), singsol=all)
```

$$-x + 3 - e^{-\frac{x}{3}}c_1 + y(x)^{\frac{1}{3}} = 0$$

✓ Solution by Mathematica

Time used: 0.175 (sec). Leaf size: 27

```
DSolve[y'[x]+y[x]==x*y[x]^(2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x} (e^{x/3} (x - 3) + c_1)^3$$

4.2 problem 2

Internal problem ID [4267]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$y' + \frac{y}{x} - 2x^{\frac{3}{2}}\sqrt{y} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)+1/x*y(x)=2*x^(3/2)*y(x)^(1/2),y(x), singsol=all)
```

$$\sqrt{y(x)} - \frac{\frac{x^3}{3} + c_1}{\sqrt{x}} = 0$$

✓ Solution by Mathematica

Time used: 0.169 (sec). Leaf size: 22

```
DSolve[y'[x]+1/x*y[x]==2*x^(3/2)*y[x]^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{(x^3 + 3c_1)^2}{9x}$$

4.3 problem 3

Internal problem ID [4268]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$3xy^2y' + 3y^3 - 1 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 96

```
dsolve(3*x*y(x)^2*diff(y(x),x)+3*y(x)^3=1,y(x), singsol=all)
```

$$y(x) = \frac{(9x^3 + 27c_1)^{\frac{1}{3}}}{3x}$$

$$y(x) = \frac{-\frac{(9x^3+27c_1)^{\frac{1}{3}}}{6} - \frac{i\sqrt{3}(9x^3+27c_1)^{\frac{1}{3}}}{6}}{x}$$

$$y(x) = \frac{-\frac{(9x^3+27c_1)^{\frac{1}{3}}}{6} + \frac{i\sqrt{3}(9x^3+27c_1)^{\frac{1}{3}}}{6}}{x}$$

✓ Solution by Mathematica

Time used: 0.284 (sec). Leaf size: 195

```
DSolve[3*x*y[x]^2*y'[x]+3*y[x]^3==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt[3]{-\frac{1}{3}\sqrt[3]{x^3 + e^{9c_1}}}}{x}$$

$$y(x) \rightarrow \frac{\sqrt[3]{x^3 + e^{9c_1}}}{\sqrt[3]{3}x}$$

$$y(x) \rightarrow \frac{(-1)^{2/3}\sqrt[3]{x^3 + e^{9c_1}}}{\sqrt[3]{3}x}$$

$$y(x) \rightarrow -\sqrt[3]{-\frac{1}{3}}$$

$$y(x) \rightarrow \frac{1}{\sqrt[3]{3}}$$

$$y(x) \rightarrow \frac{(-1)^{2/3}}{\sqrt[3]{3}}$$

$$y(x) \rightarrow -\frac{\sqrt[3]{-\frac{1}{3}\sqrt[3]{x^3}}}{x}$$

$$y(x) \rightarrow \frac{\sqrt[3]{x^3}}{\sqrt[3]{3}x}$$

$$y(x) \rightarrow \frac{(-1)^{2/3}\sqrt[3]{x^3}}{\sqrt[3]{3}x}$$

4.4 problem 4

Internal problem ID [4269]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_exact]

$$2x e^{3y} + e^x + (3x^2 e^{3y} - y^2) y' = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 22

```
dsolve((2*x*exp(3*y(x))+exp(x))+(3*x^2*exp(3*y(x))-y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$x^2 e^{3y(x)} + e^x - \frac{y(x)^3}{3} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.273 (sec). Leaf size: 28

```
DSolve[(2*x*Exp[3*y[x]]+Exp[x])+(3*x^2*Exp[3*y[x]]-y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSol
```

$$\text{Solve} \left[x^2 e^{3y(x)} - \frac{1}{3} y(x)^3 + e^x = c_1, y(x) \right]$$

4.5 problem 5

Internal problem ID [4270]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _exact, _rational, [_Abel, '2nd typ`

$$(-y + x)y' + y + x + 1 = 0$$

✓ Solution by Maple

Time used: 0.344 (sec). Leaf size: 36

```
dsolve((x-y(x))*diff(y(x),x)+(y(x)+x+1)=0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2} - \frac{-(1+2x)c_1 + \sqrt{2(1+2x)^2 c_1^2 + 1}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 51

```
DSolve[(x-y[x])*y'[x]+(y[x]+x+1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x - i\sqrt{-2x(x+1)} - c_1$$

$$y(x) \rightarrow x + i\sqrt{-2x(x+1)} - c_1$$

4.6 problem 6

Internal problem ID [4271]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_exact]

$$\cos(x) \cos(y) + \sin(x)^2 - (\sin(x) \sin(y) + \cos(y)^2) y' = 0$$

✓ Solution by Maple

Time used: 0.438 (sec). Leaf size: 35

```
dsolve((cos(x)*cos(y(x))+sin(x)^2)-(sin(x)*sin(y(x))+cos(y(x))^2)*diff(y(x),x)=0,y(x), singso
```

$$c_1 + x - y(x) - \frac{\sin(2x)}{2} + \sin(x + y(x)) + \sin(-y(x) + x) - \frac{\sin(2y(x))}{2} = 0$$

✓ Solution by Mathematica

Time used: 0.38 (sec). Leaf size: 43

```
DSolve[(Cos[x]*Cos[y[x]]+Sin[x]^2)-(Sin[x]*Sin[y[x]]+Cos[y[x]]^2)*y'[x]==0,y[x],x,IncludeSing
```

$$\text{Solve} \left[2 \left(\frac{y(x)}{2} + \frac{1}{4} \sin(2y(x)) \right) - 2 \sin(x) \cos(y(x)) - x + \frac{1}{2} \sin(2x) = c_1, y(x) \right]$$

4.7 problem 7

Internal problem ID [4272]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _Bernoulli]`

$$x^2y' + y^2 - xy = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x^2*diff(y(x),x)+(y(x)^2-x*y(x))=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\ln(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 19

```
DSolve[x^2*y'[x]+(y[x]^2-x*y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x}{\log(x) + c_1}$$

$$y(x) \rightarrow 0$$

4.8 problem 8

Internal problem ID [4273]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _dAlembert]`

$$yy' + x - \sqrt{y^2 + x^2} = 0$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 28

```
dsolve(y(x)*diff(y(x),x)=-x+sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$-c_1 + \frac{x}{y(x)^2} + \frac{\sqrt{x^2 + y(x)^2}}{y(x)^2} = 0$$

✓ Solution by Mathematica

Time used: 0.388 (sec). Leaf size: 57

```
DSolve[y[x]*y'[x]==-x+Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -e^{\frac{c_1}{2}} \sqrt{2x + e^{c_1}}$$

$$y(x) \rightarrow e^{\frac{c_1}{2}} \sqrt{2x + e^{c_1}}$$

$$y(x) \rightarrow 0$$

4.9 problem 9

Internal problem ID [4274]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _dAlembert]`

$$xy + (y^2 - x^2)y' = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(x*y(x)+(y(x)^2-x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-\frac{1}{\text{LambertW}(-c_1 x^2)}} x$$

✓ Solution by Mathematica

Time used: 8.512 (sec). Leaf size: 56

```
DSolve[x*y[x]+(y[x]^2-x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{ix}{\sqrt{W(-e^{-2c_1 x^2})}}$$

$$y(x) \rightarrow \frac{ix}{\sqrt{W(-e^{-2c_1 x^2})}}$$

$$y(x) \rightarrow 0$$

4.10 problem 10

Internal problem ID [4275]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cla`

$$y^2 - xy + (xy + x^2) y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve((y(x)^2-x*y(x))+(x^2+x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\text{LambertW}(c_1 x^2)}$$

✓ Solution by Mathematica

Time used: 4.294 (sec). Leaf size: 25

```
DSolve[(y[x]^2-x*y[x])+(x^2+x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x}{W(e^{-c_1 x^2})}$$

$$y(x) \rightarrow 0$$

4.11 problem 11

Internal problem ID [4276]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class C'], _dAlembert]`

$$y' - \cos(y + x) = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=cos(x+y(x)),y(x), singsol=all)
```

$$y(x) = -x - 2 \arctan(c_1 - x)$$

✓ Solution by Mathematica

Time used: 0.946 (sec). Leaf size: 59

```
DSolve[y'[x]==Cos[x+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x + 2 \arctan\left(x + \frac{c_1}{2}\right)$$

$$y(x) \rightarrow -x + 2 \arctan\left(x + \frac{c_1}{2}\right)$$

$$y(x) \rightarrow -x - \pi$$

$$y(x) \rightarrow \pi - x$$

4.12 problem 12

Internal problem ID [4277]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A'], _dAlembert]`

$$y' - \frac{y}{x} + \tan\left(\frac{y}{x}\right) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=y(x)/x- tan(y(x)/x),y(x), singsol=all)
```

$$y(x) = x \arcsin\left(\frac{1}{c_1 x}\right)$$

✓ Solution by Mathematica

Time used: 14.539 (sec). Leaf size: 21

```
DSolve[y'[x]==y[x]/x- Tan[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x \arcsin\left(\frac{e^{c_1}}{x}\right)$$

$$y(x) \rightarrow 0$$

4.13 problem 13

Internal problem ID [4278]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$(x - 1)y' + y - \frac{1}{x^2} + \frac{2}{x^3} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((x-1)*diff(y(x),x)+y(x)-1/x^2+2/x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x - 1} - \frac{1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 20

```
DSolve[(x-1)*y'[x]+y[x]-1/x^2+2/x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{x^2} - \frac{c_1}{x - 1}$$

4.14 problem 25 part (a)

Internal problem ID [4279]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 25 part (a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Riccati]`

$$y' - xy^2 + \frac{2y}{x} + \frac{1}{x^3} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)= x*y(x)^2-2/x*y(x)-1/x^3,y(x), singsol=all)
```

$$y(x) = \frac{\tanh(-\ln(x) + c_1)}{x^2}$$

✓ Solution by Mathematica

Time used: 1.207 (sec). Leaf size: 63

```
DSolve[y'[x]== x*y[x]^2-2/x*y[x]-1/x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{i \tan(i \log(x) + c_1)}{x^2}$$

$$y(x) \rightarrow \frac{-x^2 + e^{2i \text{Interval}\{0,\pi\}}}{x^4 + x^2 e^{2i \text{Interval}\{0,\pi\}}}$$

4.15 problem 25 part (b)

Internal problem ID [4280]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 25 part (b).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class D'], _rational, _Riccati]`

$$y' - \frac{2y^2}{x} - \frac{y}{x} + 2x = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)= 2/x*y(x)^2+1/x*y(x)-2*x,y(x), singsol=all)
```

$$y(x) = -\tanh(2x + 2c_1)x$$

✓ Solution by Mathematica

Time used: 0.722 (sec). Leaf size: 27

```
DSolve[y'[x]== 2/x*y[x]^2+1/x*y[x]-2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x \tanh(2x + c_1)$$

$$y(x) \rightarrow -x$$

$$y(x) \rightarrow x$$

4.16 problem 25 part (c)

Internal problem ID [4281]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 25 part (c).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_1st_order, _with_linear_symmetries], _Riccati]`

$$y' - y^2 e^{-x} - y + e^x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)= exp(-x)*y(x)^2+y(x)-exp(x),y(x), singsol=all)
```

$$y(x) = i \tan(ix + c_1) e^x$$

✓ Solution by Mathematica

Time used: 0.3 (sec). Leaf size: 19

```
DSolve[y'[x]== Exp[-x]*y[x]^2+y[x]-Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -e^x \tanh(x - ic_1)$$

**5 Chapter 8, Ordinary differential equations. Section
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5.1 problem 1

Internal problem ID [4282]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y' - 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[y''[x]+y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-2x} + c_2 e^x$$

5.2 problem 2

Internal problem ID [4283]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _missing_x]`

$$y'' - 4y' + 4y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{2x} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[y''[x]-4*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{2x}(c_2 x + c_1)$$

5.3 problem 3

Internal problem ID [4284]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 9y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)+9*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{-9x}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 19

```
DSolve[y''[x]+9*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 - \frac{1}{9}c_1 e^{-9x}$$

5.4 problem 4

Internal problem ID [4285]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 2y' + 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} \sin(x) + c_2 e^{-x} \cos(x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[y''[x]+2*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(c_2 \cos(x) + c_1 \sin(x))$$

5.5 problem 5

Internal problem ID [4286]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 2y' + 6y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x \sin(\sqrt{5}x) + c_2 e^x \cos(\sqrt{5}x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 32

```
DSolve[y''[x]-2*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x \left(c_2 \cos(\sqrt{5}x) + c_1 \sin(\sqrt{5}x) \right)$$

5.6 problem 6

Internal problem ID [4287]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 16y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(4x) + c_2 \cos(4x)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y''[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos(4x) + c_2 \sin(4x)$$

5.7 problem 7

Internal problem ID [4288]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _missing_x]`

$$y'' - 5y' + 6y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[y''[x]-5*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{2x}(c_2 e^x + c_1)$$

5.8 problem 8

Internal problem ID [4289]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 5y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)+5*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{-5x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 19

```
DSolve[y''[x]+5*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 - \frac{1}{5}c_1 e^{-5x}$$

5.9 problem 9

Internal problem ID [4290]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _missing_x]`

$$y'' - 4y' + 13y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+13*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} \sin(3x) + c_2 e^{2x} \cos(3x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y''[x]-4*y'[x]+13*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{2x}(c_2 \cos(3x) + c_1 \sin(3x))$$

5.10 problem 12

Internal problem ID [4291]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$2y'' + y' - y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(2*diff(y(x),x$2)+diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} + c_2 e^{\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[2*y'[x]+y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x} (c_1 e^{3x/2} + c_2)$$

5.11 problem 19

Internal problem ID [4292]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + (1 + 2i)y' + (-1 + i)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+(1+2*I)*diff(y(x),x)+(I-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{(-1-i)x} + c_2 e^{-ix}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[y''[x]+(1+2*I)*y'[x]+(I-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{(-1-i)x}(c_2 e^x + c_1)$$

5.12 problem 20

Internal problem ID [4293]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + (1 + 2i)y' + (-1 + i)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+(1+2*I)*diff(y(x),x)+(I-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{(-1-i)x} + c_2 e^{-ix}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[y''[x]+(1+2*I)*y'[x]+(I-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{(-1-i)x}(c_2 e^x + c_1)$$

5.13 problem 24

Internal problem ID [4294]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 24.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' + y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$3)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} + c_2 e^{\frac{x}{2}} \sin\left(\frac{\sqrt{3}x}{2}\right) + c_3 e^{\frac{x}{2}} \cos\left(\frac{\sqrt{3}x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 51

```
DSolve[y'''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-x} + e^{x/2} \left(c_3 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_2 \sin\left(\frac{\sqrt{3}x}{2}\right) \right)$$

5.14 problem 25

Internal problem ID [4295]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 25.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' + y'' - 6y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-6*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + e^{-3x}c_2 + c_3e^{2x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 30

```
DSolve[y'''[x]+y''[x]-6*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{3}c_1e^{-3x} + \frac{1}{2}c_2e^{2x} + c_3$$

5.15 problem 26

Internal problem ID [4296]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 26.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' + 3y'' - 9y' - 5y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 170

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)-9*diff(y(x),x)-5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{\left(i(-3+i\sqrt{55})^{\frac{2}{3}}\sqrt{3}-4i\sqrt{3}+(-3+i\sqrt{55})^{\frac{2}{3}}+2(-3+i\sqrt{55})^{\frac{1}{3}}+4\right)x}{2(-3+i\sqrt{55})^{\frac{1}{3}}}} + c_2 e^{\frac{\left(i(-3+i\sqrt{55})^{\frac{2}{3}}\sqrt{3}-4i\sqrt{3}-(-3+i\sqrt{55})^{\frac{2}{3}}-2(-3+i\sqrt{55})^{\frac{1}{3}}-4\right)x}{2(-3+i\sqrt{55})^{\frac{1}{3}}}} + c_3 e^{\frac{\left((-3+i\sqrt{55})^{\frac{2}{3}}-(-3+i\sqrt{55})^{\frac{1}{3}}+4\right)x}{(-3+i\sqrt{55})^{\frac{1}{3}}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 87

```
DSolve[y'''[x]+3*y''[x]-9*y'[x]-5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 \exp\left(x\text{Root}\left[\#1^3 + 3\#1^2 - 9\#1 - 5\&, 2\right]\right) + c_3 \exp\left(x\text{Root}\left[\#1^3 + 3\#1^2 - 9\#1 - 5\&, 3\right]\right) + c_1 \exp\left(x\text{Root}\left[\#1^3 + 3\#1^2 - 9\#1 - 5\&, 1\right]\right)$$

5.16 problem 28

Internal problem ID [4297]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 28.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[_high_order, _missing_x]`

$$y'''' + 4y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$4)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} \sin(x) + c_2 e^{-x} \cos(x) + c_3 e^x \sin(x) + c_4 e^x \cos(x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 38

```
DSolve[y''''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(c_1 \cos(x) + c_2 \sin(x)) + e^x(c_4 \cos(x) + c_3 \sin(x))$$

**6 Chapter 8, Ordinary differential equations. Section
6. SECOND-ORDER LINEAR
EQUATIONSWITH CONSTANT
COEFFICIENTS AND RIGHT-HAND SIDE NOT
ZERO. page 422**

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6.1 problem 1

Internal problem ID [4298]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 4y' - 10 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)=10,y(x), singsol=all)
```

$$y(x) = \frac{e^{4x}c_1}{4} - \frac{5x}{2} + c_2$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 24

```
DSolve[y''[x]-4*y'[x]==10,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{5x}{2} + \frac{1}{4}c_1e^{4x} + c_2$$

6.2 problem 2

Internal problem ID [4299]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 4y' + 4y - 16 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=16,y(x), singsol=all)
```

$$y(x) = c_2 e^{2x} + c_1 e^{2x} x + 4$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[y''[x]-4*y'[x]+4*y[x]==16,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 4 + e^{2x}(c_2 x + c_1)$$

6.3 problem 3

Internal problem ID [4300]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y' - 2y - e^{2x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = e^x c_2 + c_1 e^{-2x} + \frac{e^{2x}}{4}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 29

```
DSolve[y''[x]+y'[x]-2*y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{2x}}{4} + c_1 e^{-2x} + c_2 e^x$$

6.4 problem 4

Internal problem ID [4301]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y' - 3y - 24e^{-3x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=24*exp(-3*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + e^{3x} c_1 + 2 e^{-3x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 29

```
DSolve[y''[x]-2*y'[x]-3*y[x]==24*Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-3x} (c_1 e^{2x} + c_2 e^{6x} + 2)$$

6.5 problem 5

Internal problem ID [4302]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 5.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y - 2e^x = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+y(x)=2*exp(x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + c_1 \cos(x) + e^x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 19

```
DSolve[y''[x]+y[x]==2*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x + c_1 \cos(x) + c_2 \sin(x)$$

6.6 problem 6

Internal problem ID [4303]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 6y' + 9y - 12e^{-x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=12*exp(-x),y(x), singsol=all)
```

$$y(x) = e^{-3x}c_2 + xe^{-3x}c_1 + 3e^{-x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 25

```
DSolve[y''[x]+6*y'[x]+9*y[x]==12*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-3x}(3e^{2x} + c_2x + c_1)$$

6.7 problem 7

Internal problem ID [4304]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y' - 2y - 3e^{2x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=3*exp(2*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + c_1 e^{2x} + e^{2x} x$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 27

```
DSolve[y''[x]-y'[x]-2*y[x]==3*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-x} + e^{2x} \left(x - \frac{1}{3} + c_2 \right)$$

6.8 problem 8

Internal problem ID [4305]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 16y - 40e^{4x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-16*y(x)=40*exp(4*x),y(x), singsol=all)
```

$$y(x) = e^{4x}c_2 + e^{-4x}c_1 + 5e^{4x}x$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 29

```
DSolve[y''[x]-16*y[x]==40*Exp[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{4x} \left(5x - \frac{5}{8} + c_1 \right) + c_2 e^{-4x}$$

6.9 problem 9

Internal problem ID [4306]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2y' + y - 2e^{-x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=2*exp(-x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + e^{-x} c_1 x + e^{-x} x^2$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 20

```
DSolve[y''[x]+2*y'[x]+y[x]==2*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(x(x + c_2) + c_1)$$

6.10 problem 10

Internal problem ID [4307]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 6y' + 9y - 6e^{3x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=6*exp(3*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{3x} + e^{3x} c_1 x + 3x^2 e^{3x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 22

```
DSolve[y''[x]-6*y'[x]+9*y[x]==6*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{3x}(x(3x + c_2) + c_1)$$

6.11 problem 11

Internal problem ID [4308]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 11.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 10y - 100 \cos(4x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+10*y(x)=100*cos(4*x),y(x), singsol=all)
```

$$y(x) = e^{-x} \sin(3x) c_2 + e^{-x} \cos(3x) c_1 - 6 \cos(4x) + 8 \sin(4x)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 39

```
DSolve[y''[x]+2*y'[x]+10*y[x]==100*Cos[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 8 \sin(4x) - 6 \cos(4x) + e^{-x}(c_2 \cos(3x) + c_1 \sin(3x))$$

6.12 problem 12

Internal problem ID [4309]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y' + 12y - 80 \sin(2x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+12*y(x)=80*sin(2*x),y(x), singsol=all)
```

$$y(x) = e^{-2x} \sin\left(2\sqrt{2}x\right) c_2 + e^{-2x} \cos\left(2\sqrt{2}x\right) c_1 + 5 \sin(2x) - 5 \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 49

```
DSolve[y''[x]+4*y'[x]+12*y[x]==80*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 5 \sin(2x) - 5 \cos(2x) + e^{-2x} \left(c_2 \cos\left(2\sqrt{2}x\right) + c_1 \sin\left(2\sqrt{2}x\right) \right)$$

6.13 problem 13

Internal problem ID [4310]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + y - 2 \cos(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=2*cos(x),y(x), singsol=all)
```

$$y(x) = e^x c_2 + e^x c_1 x - \sin(x)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 21

```
DSolve[y''[x]-2*y'[x]+y[x]==2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sin(x) + e^x(c_2 x + c_1)$$

6.14 problem 14

Internal problem ID [4311]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 8y' + 25y - 120 \sin(5x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+8*diff(y(x),x)+25*y(x)=120*sin(5*x),y(x), singsol=all)
```

$$y(x) = e^{-4x} \sin(3x) c_2 + e^{-4x} \cos(3x) c_1 - 3 \cos(5x)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 33

```
DSolve[y''[x]+8*y'[x]+25*y[x]==120*Sin[5*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -3 \cos(5x) + e^{-4x} (c_2 \cos(3x) + c_1 \sin(3x))$$

6.15 problem 15

Internal problem ID [4312]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$5y'' + 12y' + 20y - 120 \sin(2x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(5*diff(y(x),x$2)+12*diff(y(x),x)+20*y(x)=120*sin(2*x),y(x), singsol=all)
```

$$y(x) = e^{-\frac{6x}{5}} \sin\left(\frac{8x}{5}\right) c_2 + e^{-\frac{6x}{5}} \cos\left(\frac{8x}{5}\right) c_1 - 5 \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 39

```
DSolve[5*y'[x]+12*y'[x]+20*y[x]==120*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -5 \cos(2x) + e^{-6x/5} \left(c_2 \cos\left(\frac{8x}{5}\right) + c_1 \sin\left(\frac{8x}{5}\right) \right)$$

6.16 problem 16

Internal problem ID [4313]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y - 30 \sin(3x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+9*y(x)=30*sin(3*x),y(x), singsol=all)
```

$$y(x) = \sin(3x) c_2 + \cos(3x) c_1 - 5 \cos(3x) x$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 31

```
DSolve[y''[x]+9*y[x]==30*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (-5x + c_1) \cos(3x) + \frac{1}{6}(5 + 6c_2) \sin(3x)$$

6.17 problem 17

Internal problem ID [4314]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 16y - 16 \cos(4x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+16*y(x)=16*cos(4*x),y(x), singsol=all)
```

$$y(x) = \sin(4x) c_2 + \cos(4x) c_1 + \frac{\cos(4x)}{2} + 2 \sin(4x) x$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 28

```
DSolve[y''[x]+16*y[x]==16*Cos[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \left(\frac{1}{4} + c_1\right) \cos(4x) + (2x + c_2) \sin(4x)$$

6.18 problem 18

Internal problem ID [4315]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 17y - 60e^{-4x} \sin(5x) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+17*y(x)=60*exp(-4*x)*sin(5*x),y(x), singsol=all)
```

$$y(x) = e^{-x} \sin(4x) c_2 + e^{-x} \cos(4x) c_1 + 2e^{-4x} \cos(5x)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 38

```
DSolve[y''[x]+2*y'[x]+17*y[x]==60*Exp[-4*x]*Sin[5*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2e^{-4x} \cos(5x) + e^{-x}(c_2 \cos(4x) + c_1 \sin(4x))$$

6.19 problem 19

Internal problem ID [4316]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$4y'' + 4y' + 5y - 40e^{-\frac{3x}{2}} \sin(2x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(4*dif(y(x),x$2)+4*dif(y(x),x)+5*y(x)=40*exp(-3*x/2)*sin(2*x),y(x), singsol=all)
```

$$y(x) = e^{-\frac{x}{2}} \sin(x) c_2 + e^{-\frac{x}{2}} \cos(x) c_1 + e^{-\frac{3x}{2}} (-\sin(2x) + 2 \cos(2x))$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 41

```
DSolve[4*y''[x]+4*y'[x]+5*y[x]==40*Exp[-3*x/2]*Sin[2*x],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \rightarrow e^{-3x/2}(-\sin(2x) + 2 \cos(2x) + e^x(c_2 \cos(x) + c_1 \sin(x)))$$

6.20 problem 20

Internal problem ID [4317]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y' + 8y - 30e^{-\frac{x}{2}} \cos\left(\frac{5x}{2}\right) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+8*y(x)=30*exp(-x/2)*cos(5/2*x),y(x), singsol=all)
```

$$y(x) = e^{-2x} \sin(2x) c_2 + e^{-2x} \cos(2x) c_1 + 4e^{-\frac{x}{2}} \sin\left(\frac{5x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 41

```
DSolve[y''[x]+4*y'[x]+8*y[x]==30*Exp[-x/2]*Cos[5/2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x} \left(4e^{3x/2} \sin\left(\frac{5x}{2}\right) + c_2 \cos(2x) + c_1 \sin(2x) \right)$$

6.21 problem 21

Internal problem ID [4318]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$5y'' + 6y' + 2y - x^2 - 6x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(5*diff(y(x),x$2)+6*diff(y(x),x)+2*y(x)=x^2+6*x,y(x), singsol=all)
```

$$y(x) = e^{-\frac{3x}{5}} \sin\left(\frac{x}{5}\right) c_2 + e^{-\frac{3x}{5}} \cos\left(\frac{x}{5}\right) c_1 + \frac{x^2}{2} - \frac{5}{2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 42

```
DSolve[5*y''[x]+6*y'[x]+2*y[x]==x^2+6*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}(x^2 - 5) + e^{-3x/5} \left(c_2 \cos\left(\frac{x}{5}\right) + c_1 \sin\left(\frac{x}{5}\right) \right)$$

6.22 problem 22

Internal problem ID [4319]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$2y'' + y' - 2x = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(2*diff(y(x),x$2)+diff(y(x),x)=2*x,y(x), singsol=all)
```

$$y(x) = -2e^{-\frac{x}{2}}c_1 + x^2 - 4x + c_2$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 22

```
DSolve[y''[x]+y'[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (x - 2)x - c_1e^{-x} + c_2$$

6.23 problem 23

Internal problem ID [4320]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 23.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - 2e^x x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+y(x)=2*x*exp(x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + c_1 \cos(x) + (x - 1) e^x$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==2*x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x(x - 1) + c_1 \cos(x) + c_2 \sin(x)$$

6.24 problem 24

Internal problem ID [4321]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 6y' + 9y - 12e^{3x}x = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=12*x*exp(3*x),y(x), singsol=all)
```

$$y(x) = c_2e^{3x} + e^{3x}c_1x + 2x^3e^{3x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 23

```
DSolve[y''[x]-6y'[x]+9*y[x]==12*x*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{3x}(2x^3 + c_2x + c_1)$$

6.25 problem 25

Internal problem ID [4322]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 25.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - 2y' - 3y - 16x^2e^{-x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=16*x^2*exp(-x),y(x), singsol=all)
```

$$y(x) = c_2e^{-x} + e^{3x}c_1 - \frac{x(8x^2 + 6x + 3)e^{-x}}{6}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 37

```
DSolve[y''[x]-2*y'[x]-3*y[x]==16*x*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}e^{-x}(-4x(2x + 1) + 4c_2e^{4x} - 1 + 4c_1)$$

6.26 problem 26

Internal problem ID [4323]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 26.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - 8 \sin(x) x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+y(x)=8*x*sin(x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + c_1 \cos(x) + 2x(-x \cos(x) + \sin(x))$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 27

```
DSolve[y''[x]+y[x]==8*x*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (-2x^2 + 1 + c_1) \cos(x) + (2x + c_2) \sin(x)$$

6.27 problem 33

Internal problem ID [4324]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 33.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - x^3 + 1 - 2 \cos(x) - (2 - 4x)e^x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)+y(x)=x^3-1+2*cos(x)+(2-4*x)*exp(x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + c_1 \cos(x) + (-2x + 3)e^x + x^3 + x \sin(x) - 6x + \cos(x) - 1$$

✓ Solution by Mathematica

Time used: 0.286 (sec). Leaf size: 38

```
DSolve[y''[x]+y[x]==x^3-1+2*Cos[x]+(2-4*x)*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^3 - 6x + e^x(3 - 2x) + \left(\frac{1}{2} + c_1\right) \cos(x) + (x + c_2) \sin(x) - 1$$

6.28 problem 34

Internal problem ID [4325]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 34.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 5y' + 6y - 2e^x - 6x + 5 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=2*exp(x)+6*x-5,y(x), singsol=all)
```

$$y(x) = c_2 e^{2x} + e^{3x} c_1 + x + e^x$$

✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 26

```
DSolve[y''[x]-5*y'[x]+6*y[x]==2*Exp[x]+6*x-5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x + e^x + c_1 e^{2x} + c_2 e^{3x}$$

6.29 problem 35

Internal problem ID [4326]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 35.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - y - \sinh(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-y(x)=sinh(x),y(x), singsol=all)
```

$$y(x) = e^x c_2 + c_1 e^{-x} + \frac{x e^{-x}}{4} + \frac{e^x(-1 + 2x)}{8}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 38

```
DSolve[y''[x]-y[x]==Sinh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{8} e^{-x} (2x + e^{2x} (2x - 1 + 8c_1) + 1 + 8c_2)$$

6.30 problem 36

Internal problem ID [4327]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 36.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - 2 \sin(x) - 4x \cos(x) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+y(x)=2*sin(x)+4*x*cos(x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + c_1 \cos(x) + \sin(x)(x^2 - 1)$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 28

```
DSolve[y''[x]+y[x]==2*Sin[x]+4*x*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}(2x^2 - 1 + 2c_2) \sin(x) + c_1 \cos(x)$$

6.31 problem 37

Internal problem ID [4328]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 37.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + y - 4e^x - (1 - x)(e^{2x} - 1) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=4*exp(x)+(1-x)*(exp(2*x)-1),y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + e^{-x} c_1 x + \frac{(-3x + 5)e^{2x}}{27} + x + e^x - 3$$

✓ Solution by Mathematica

Time used: 0.263 (sec). Leaf size: 38

```
DSolve[y''[x]+2*y'[x]+y[x]==4*Exp[x]+(1-x)*(Exp[2*x]-1),y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \rightarrow \frac{1}{27} e^{2x} (5 - 3x) + e^x + x + e^{-x} (c_2 x + c_1) - 3$$

6.32 problem 38

Internal problem ID [4329]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 38.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' - 2y' - 9e^{-x}x + 6x^2 - 4e^{2x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)=9*x*exp(-x)-6*x^2+4*exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{2x}}{2} + \frac{3x^2}{2} + x^3 + 2e^{2x}x - e^{2x} + 3xe^{-x} + 4e^{-x} + \frac{3x}{2} + c_2$$

✓ Solution by Mathematica

Time used: 0.493 (sec). Leaf size: 48

```
DSolve[y''[x]-2*y'[x]==9*x*Exp[-x]-6*x^2+4*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}(e^{-x}(6x + 8) + x(x(2x + 3) + 3) + e^{2x}(4x - 2 + c_1)) + c_2$$

7 Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

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7.1 problem 1 (a)

Internal problem ID [4330]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 1 (a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _L`

$$y'' + yy' = 0$$

With initial conditions

$$[y(0) = 5, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$2)+y(x)*diff(y(x),x)=0,y(0) = 5, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = 5$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]+y[x]*y'[x]==0,{y[0]==5,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

{}

7.2 problem 1 (b)

Internal problem ID [4331]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 1 (b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _L`

$$y'' + yy' = 0$$

With initial conditions

$$[y(0) = 2, y'(0) = -2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)+y(x)*diff(y(x),x)=0,y(0) = 2, D(y)(0) = -2],y(x), singsol=all)
```

$$y(x) = \frac{2}{x+1}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]+y[x]*y'[x]==0,{y[0]==2,y'[0]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

```
{}
```

7.3 problem 1 (c)

Internal problem ID [4332]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 1 (c).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _L`

$$y'' + yy' = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)+y(x)*diff(y(x),x)=0,y(0) = 1, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = \cot\left(\frac{x}{2} + \frac{\pi}{4}\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]+y[x]*y'[x]==0,{y[0]==1,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

{}

7.4 problem 1 (d)

Internal problem ID [4333]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 1 (d).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _L`

$$y'' + yy' = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 8

```
dsolve([diff(y(x),x$2)+y(x)*diff(y(x),x)=0,y(0) = 0, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = 2 \tanh(x)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 9

```
DSolve[{y'[x]+y[x]*y'[x]==0,{y[0]==0,y'[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2 \tanh(x)$$

7.5 problem 2

Internal problem ID [4334]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' + 2y'x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + \operatorname{erf}(x) c_2$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 21

```
DSolve[y''[x]+2*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}\sqrt{\pi}c_1\operatorname{erf}(x) + c_2$$

7.6 problem 3

Internal problem ID [4335]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 3.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible,`

$$2yy'' - y'^2 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 27

```
dsolve(2*y(x)*diff(y(x),x$2)=(diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{1}{4}c_1^2x^2 + \frac{1}{2}c_1c_2x + \frac{1}{4}c_2^2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[2*y[x]*y'[x]==(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{(c_1x + 2c_2)^2}{4c_2}$$

7.7 problem 4

Internal problem ID [4336]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]`

$$xy'' - y' - y'^3 = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x$2)=diff(y(x),x)+(diff(y(x),x))^3,y(x), singsol=all)
```

$$y(x) = -\sqrt{-x^2 + c_1} + c_2$$

$$y(x) = \sqrt{-x^2 + c_1} + c_2$$

✓ Solution by Mathematica

Time used: 1.506 (sec). Leaf size: 103

```
DSolve[x*y''[x]==y'[x]+(y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 - ie^{-c_1} \sqrt{-1 + e^{2c_1} x^2}$$

$$y(x) \rightarrow ie^{-c_1} \sqrt{-1 + e^{2c_1} x^2} + c_2$$

$$y(x) \rightarrow c_2 - i\sqrt{x^2}$$

$$y(x) \rightarrow i\sqrt{x^2} + c_2$$

7.8 problem 5

Internal problem ID [4337]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 5.

ODE order: 2.

ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y''^2 - k^2(1 + y'^2) = 0$$

✓ Solution by Maple

Time used: 0.75 (sec). Leaf size: 67

```
dsolve((diff(y(x),x$2))^2=k^2*(1+(diff(y(x),x))^2),y(x), singsol=all)
```

$$y(x) = -ix + c_1$$

$$y(x) = ix + c_1$$

$$y(x) = c_1 + c_2 e^{kx} + \frac{e^{-kx}}{4k^2 c_2}$$

$$y(x) = c_1 + \frac{e^{kx}}{4c_2 k^2} + c_2 e^{-kx}$$

✓ Solution by Mathematica

Time used: 0.451 (sec). Leaf size: 40

```
DSolve[(y'[x])^2==k^2*(1+(y'[x])^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2 - \frac{\cosh(kx - c_1)}{k}$$

$$y(x) \rightarrow \frac{\cosh(kx + c_1)}{k} + c_2$$

7.9 problem 6

Internal problem ID [4338]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 6.

ODE order: 2.

ODE degree: 2.

CAS Maple gives this as type `[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear]`

$$k - \frac{y''}{(1 + y')^{\frac{3}{2}}} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 19

```
dsolve(k=diff(y(x),x$2)*(1+ (diff(y(x),x)))^(-3/2),y(x), singsol=all)
```

$$y(x) = -x - \frac{4}{k^2(x + c_1)} + c_2$$

✓ Solution by Mathematica

Time used: 0.518 (sec). Leaf size: 73

```
DSolve[k==y'[x]*(1+ (y'[x])^2)^(-3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2 k - i \sqrt{(kx - 1 + c_1)(kx + 1 + c_1)}}{k}$$

$$y(x) \rightarrow \frac{i \sqrt{(kx - 1 + c_1)(kx + 1 + c_1)} + c_2 k}{k}$$

7.10 problem 16 (a)

Internal problem ID [4339]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 16 (a).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_Emden, _Fowler]`

$$x^2 y'' + 3y'x - 3y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x + \frac{c_2}{x^3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[x^2*y'[x]+3*x*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1}{x^3} + c_2 x$$

7.11 problem 16 (b)

Internal problem ID [4340]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 16 (b).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$x^2 y'' + y'x - 4y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^2} + c_2 x^2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_2 x^4 + c_1}{x^2}$$

7.12 problem 16 (c)

Internal problem ID [4341]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 16 (c).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_Emden, _Fowler]`

$$x^2 y'' + 7y'x + 9y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)+7*x*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x^3} + \frac{c_2 \ln(x)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[x^2*y'[x]+7*x*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{3c_2 \log(x) + c_1}{x^3}$$

7.13 problem 16 (d)

Internal problem ID [4342]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 16 (d).

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_Emden, _Fowler]`

$$x^2 y'' - y' x + 6y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin(\sqrt{5} \ln(x)) + c_2 x \cos(\sqrt{5} \ln(x))$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]-x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x \left(c_2 \cos(\sqrt{5} \log(x)) + c_1 \sin(\sqrt{5} \log(x)) \right)$$

7.14 problem 17

Internal problem ID [4343]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 17.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$x^2 y'' + y' x - 16y - 8x^4 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-16*y(x)=8*x^4,y(x), singsol=all)
```

$$y(x) = x^4 c_2 + \frac{c_1}{x^4} + x^4 \left(-\frac{1}{8} + \ln(x) \right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 28

```
DSolve[x^2*y''[x]+x*y'[x]-16*y[x]==8*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^4 \log(x) + \left(-\frac{1}{8} + c_2 \right) x^4 + \frac{c_1}{x^4}$$

7.15 problem 18

Internal problem ID [4344]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$x^2 y'' + y' x - y - x + \frac{1}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=x-1/x,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2 x + \frac{2x^2 \ln(x) + 2 \ln(x) + 1}{4x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 37

```
DSolve[x^2*y'[x]+x*y'[x]-y[x]==x-1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2(x^2 + 1) \log(x) + (-1 + 4c_2)x^2 + 1 + 4c_1}{4x}$$

7.16 problem 19

Internal problem ID [4345]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$x^2 y'' - 5y'x + 9y - 2x^3 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+9*y(x)=2*x^3,y(x), singsol=all)
```

$$y(x) = c_2 x^3 + x^3 \ln(x) c_1 + \ln(x)^2 x^3$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 22

```
DSolve[x^2*y'[x]-5*x*y'[x]+9*y[x]==2*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^3(\log^2(x) + 3c_2 \log(x) + c_1)$$

7.17 problem 20

Internal problem ID [4346]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2 y'' - 3y'x + 4y - 6 \ln(x) x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=6*x^2*ln(x),y(x), singsol=all)
```

$$y(x) = c_2 x^2 + x^2 \ln(x) c_1 + \ln(x)^3 x^2$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==6*x^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2(\log^3(x) + 2c_2 \log(x) + c_1)$$

7.18 problem 21

Internal problem ID [4347]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + y - 3x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(x^2*diff(y(x),x$2)+y(x)=3*x^2,y(x), singsol=all)
```

$$y(x) = \sqrt{x} \sin\left(\frac{\sqrt{3} \ln(x)}{2}\right) c_2 + \sqrt{x} \cos\left(\frac{\sqrt{3} \ln(x)}{2}\right) c_1 + x^2$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 47

```
DSolve[x^2*y''[x]+y[x]==3*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt{x} \left(x^{3/2} + c_1 \cos\left(\frac{1}{2}\sqrt{3} \log(x)\right) + c_2 \sin\left(\frac{1}{2}\sqrt{3} \log(x)\right) \right)$$

7.19 problem 22

Internal problem ID [4348]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$x^2 y'' + y' x + y - 2x = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=2*x,y(x), singsol=all)
```

$$y(x) = \sin(\ln(x)) c_2 + \cos(\ln(x)) c_1 + x$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 19

```
DSolve[x^2*y''[x]+x*y'[x]+y[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x + c_1 \cos(\log(x)) + c_2 \sin(\log(x))$$

7.20 problem 25

Internal problem ID [4349]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 25.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$x^2(-x + 2)y'' + 2y'x - 2y = 0$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([x^2*(2-x)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,x],y(x), singsol=all)
```

$$y(x) = c_1x + \frac{c_2(x - 1)}{x}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 24

```
DSolve[x^2*(2-x)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1(x - 2)^2 + c_2(x - 1)}{x}$$

7.21 problem 26

Internal problem ID [4350]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 26.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$(x^2 + 1)y'' - 2y'x + 2y = 0$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([(x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,x],y(x), singsol=all)
```

$$y(x) = c_1x + c_2(x^2 - 1)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2x - c_1(x - i)^2$$

7.22 problem 27

Internal problem ID [4351]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 27.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' - 2y'(x+1) + (2+x)y = 0$$

Given that one solution of the ode is

$$y_1 = e^x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([x*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+(x+2)*y(x)=0,exp(x)],y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^x x^3$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 23

```
DSolve[x*y''[x]-2*(x+1)*y'[x]+(x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{3} e^x (c_2 x^3 + 3c_1)$$

7.23 problem 28

Internal problem ID [4352]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 28.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3xy'' - 2(3x - 1)y' + (3x - 2)y = 0$$

Given that one solution of the ode is

$$y_1 = e^x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([3*x*diff(y(x),x$2)-2*(3*x-1)*diff(y(x),x)+(3*x-2)*y(x)=0,exp(x)],y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 x^{\frac{1}{3}} e^x$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 21

```
DSolve[3*x*y'[x]-2*(3*x-1)*y'[x]+(3*x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x (3c_2 \sqrt[3]{x} + c_1)$$

7.24 problem 29

Internal problem ID [4353]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 29.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2 y'' + y'(x+1) - y = 0$$

Given that one solution of the ode is

$$y_1 = x + 1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve([x^2*diff(y(x),x$2)+(x+1)*diff(y(x),x)-y(x)=0,x+1],y(x), singsol=all)
```

$$y(x) = \left(\frac{c_1(x+1)e^{-\frac{1}{x}}}{x} + c_2 \right) e^{\frac{1}{x}x}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]+(x+1)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{\frac{1}{x}x} + c_2(x+1)$$

7.25 problem 30

Internal problem ID [4354]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 30.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$x(x+1)y'' - (x-1)y' + y = 0$$

Given that one solution of the ode is

$$y_1 = x - 1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve([x*(x+1)*diff(y(x),x$2)-(x-1)*diff(y(x),x)+y(x)=0,x-1],y(x), singsol=all)
```

$$y(x) = c_1(x - 1) + c_2(-4 + (x - 1) \ln(x))$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 23

```
DSolve[x*(x+1)*y''[x]-(x-1)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1(x - 1) + c_2((x - 1) \log(x) - 4)$$

8 Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

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8.1 problem 1

Internal problem ID [4355]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$x^2 y' - xy - \frac{1}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x)-x*y(x)=1/x,y(x), singsol=all)
```

$$y(x) = \left(-\frac{1}{3x^3} + c_1 \right) x$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 17

```
DSolve[x^2*y'[x]-x*y[x]==1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{3x^2} + c_1 x$$

8.2 problem 2

Internal problem ID [4356]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$x \ln(y) y' - \ln(x) y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve(x*ln(y(x))*diff(y(x),x)-y(x)*ln(x)=0,y(x), singsol=all)
```

$$y(x) = e^{\sqrt{\ln(x)^2 + 2c_1}}$$

$$y(x) = e^{-\sqrt{\ln(x)^2 + 2c_1}}$$

✓ Solution by Mathematica

Time used: 0.81 (sec). Leaf size: 60

```
DSolve[x*Log[y[x]]*y'[x]-y[x]*Log[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-\sqrt{\log^2(x) + 2c_1}}$$

$$y(x) \rightarrow e^{\sqrt{\log^2(x) + 2c_1}}$$

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow e^{2i\text{Interval}\{0,\pi\}}$$

8.3 problem 3

Internal problem ID [4357]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 3.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' + 2y'' + 2y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)+2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{-x} \sin(x) + c_3 e^{-x} \cos(x)$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 37

```
DSolve[y'''[x]+2*y''[x]+2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2} e^{-x} ((c_2 - c_1) \sin(x) - (c_1 + c_2) \cos(x)) + c_3$$

8.4 problem 4

Internal problem ID [4358]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$r'' - 6r' + 9r = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(r(t),t$2)-6*diff(r(t),t)+9*r(t)=0,r(t), singsol=all)
```

$$r(t) = c_1 e^{3t} + c_2 e^{3t} t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[r''[t]-6*r'[t]+9*r[t]==0,r[t],t,IncludeSingularSolutions -> True]
```

$$r(t) \rightarrow e^{3t}(c_2 t + c_1)$$

8.5 problem 5

Internal problem ID [4359]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, ' _with_symmetry_[F(x),G(x)] '], [_Abel, '2`

$$2x - y \sin(2x) - (\sin(x)^2 - 2y) y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 75

```
dsolve(2*x-y(x)*sin(2*x)=(sin(x)^2-2*y(x))*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{1}{4} - \frac{\cos(2x)}{4} - \frac{\sqrt{\cos(2x)^2 - 16x^2 - 2\cos(2x) - 16c_1 + 1}}{4}$$

$$y(x) = \frac{1}{4} - \frac{\cos(2x)}{4} + \frac{\sqrt{\cos(2x)^2 - 16x^2 - 2\cos(2x) - 16c_1 + 1}}{4}$$

✓ Solution by Mathematica

Time used: 0.255 (sec). Leaf size: 87

```
DSolve[2*x-y[x]*Sin[2*x]==(Sin[x]^2-2*y[x])*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4} \left(-\sqrt{-16x^2 + (\cos(2x) - 2) \cos(2x) + 1 + 16c_1} - \cos(2x) + 1 \right)$$

$$y(x) \rightarrow \frac{1}{4} \left(\sqrt{-16x^2 + (\cos(2x) - 2) \cos(2x) + 1 + 16c_1} - \cos(2x) + 1 \right)$$

8.6 problem 6

Internal problem ID [4360]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + 2y - 10e^x - 6e^{-x} \cos(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+2*y(x)=10*exp(x)+6*exp(-x)*cos(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} \sin(x) + e^{-x} \cos(x) c_1 + (3x \sin(x) + 3 \cos(x)) e^{-x} + 2 e^x$$

✓ Solution by Mathematica

Time used: 0.101 (sec). Leaf size: 41

```
DSolve[y''[x]+2*y'[x]+2*y[x]==10*Exp[x]+6*Exp[-x]*Cos[x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \rightarrow \frac{1}{2} e^{-x} (4e^{2x} + (3 + 2c_2) \cos(x) + 2(3x + c_1) \sin(x))$$

8.7 problem 7

Internal problem ID [4361]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$3x^3y^2y' - y^3x^2 - 1 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 105

```
dsolve(3*x^3*y(x)^2*diff(y(x),x)-x^2*y(x)^3=1,y(x), singsol=all)
```

$$y(x) = \frac{((27c_1x^3 - 9)x)^{\frac{1}{3}}}{3x}$$

$$y(x) = -\frac{((27c_1x^3 - 9)x)^{\frac{1}{3}}}{6x} - \frac{i\sqrt{3}((27c_1x^3 - 9)x)^{\frac{1}{3}}}{6x}$$

$$y(x) = -\frac{((27c_1x^3 - 9)x)^{\frac{1}{3}}}{6x} + \frac{i\sqrt{3}((27c_1x^3 - 9)x)^{\frac{1}{3}}}{6x}$$

✓ Solution by Mathematica

Time used: 0.514 (sec). Leaf size: 85

```
DSolve[3*x^3*y[x]^2*y'[x]-x^2*y[x]^3==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt[3]{-\frac{1}{3}} \sqrt[3]{-1 + 3c_1x^3}}{x^{2/3}}$$

$$y(x) \rightarrow \frac{\sqrt[3]{-\frac{1}{3} + c_1x^3}}{x^{2/3}}$$

$$y(x) \rightarrow \frac{(-1)^{2/3} \sqrt[3]{-\frac{1}{3} + c_1x^3}}{x^{2/3}}$$

8.8 problem 8

Internal problem ID [4362]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - y'x + y - x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = c_2x + x \ln(x) c_1 + \frac{\ln(x)^2 x}{2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]-x*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{2}x(\log^2(x) + 2c_2 \log(x) + 2c_1)$$

8.9 problem 9

Internal problem ID [4363]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_1st_order, _with_linear_symmetries], _Bernoulli]`

$$y' - 2y - y^2 e^{3x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)-(2*y(x)+y(x)^2*exp(3*x))=0,y(x), singsol=all)
```

$$y(x) = -\frac{5}{-5c_1 e^{-2x} + e^{3x}}$$

✓ Solution by Mathematica

Time used: 0.221 (sec). Leaf size: 29

```
DSolve[y'[x]-(2*y[x]+y[x]^2*Exp[3*x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{5e^{2x}}{e^{5x} - 5c_1}$$

$$y(x) \rightarrow 0$$

8.10 problem 10

Internal problem ID [4364]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$u(1 - v) + v^2(1 - u)u' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(u(v)*(1-v)+v^2*(1-u(v))*diff(u(v),v)=0,u(v), singsol=all)
```

$$u(v) = e^{\frac{\ln(v)v - \text{LambertW}\left(-v e^{c_1 + \frac{1}{v}}\right)v + c_1 v + 1}{v}}$$

✓ Solution by Mathematica

Time used: 3.83 (sec). Leaf size: 26

```
DSolve[u[v]*(1-v)+v^2*(1-u[v])*u'[v]==0,u[v],v,IncludeSingularSolutions -> True]
```

$$u(v) \rightarrow -W\left(v\left(-e^{\frac{1}{v}-c_1}\right)\right)$$

$$u(v) \rightarrow 0$$

8.11 problem 11

Internal problem ID [4365]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y + 2x - y'x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((y(x)+2*x)-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (2 \ln(x) + c_1) x$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 14

```
DSolve[(y[x]+2*x)-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(2 \log(x) + c_1)$$

8.12 problem 12

Internal problem ID [4366]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$xy'' + y' - 4x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x$2)+diff(y(x),x)=4*x,y(x), singsol=all)
```

$$y(x) = x^2 + c_1 \ln(x) + c_2$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 16

```
DSolve[x*y''[x]+y'[x]==4*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2 + c_1 \log(x) + c_2$$

8.13 problem 13

Internal problem ID [4367]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 4y' + 5y - 26e^{3x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=26*exp(3*x),y(x), singsol=all)
```

$$y(x) = \sin(x) e^{-2x} c_2 + \cos(x) e^{-2x} c_1 + e^{3x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 27

```
DSolve[y''[x]+4*y'[x]+5*y[x]==26*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x}(e^{5x} + c_2 \cos(x) + c_1 \sin(x))$$

8.14 problem 14

Internal problem ID [4368]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y' + 5y - 2e^{-2x} \cos(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=2*exp(-2*x)*cos(x),y(x), singsol=all)
```

$$y(x) = \sin(x) e^{-2x} c_2 + \cos(x) e^{-2x} c_1 + e^{-2x} \sin(x) x$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 26

```
DSolve[y''[x]+4*y'[x]+5*y[x]==2*Exp[-2*x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x}((1 + c_2) \cos(x) + (x + c_1) \sin(x))$$

8.15 problem 15

Internal problem ID [4369]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' - 4y' + 4y - 6e^{2x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=6*exp(2*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{2x} + c_1 e^{2x} x + 3x^2 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 22

```
DSolve[y''[x]-4*y'[x]+4*y[x]==6*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{2x}(x(3x + c_2) + c_1)$$

8.16 problem 16

Internal problem ID [4370]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' - 5y' + 6y - e^{2x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{2x} + e^{3x} c_1 - e^{2x} x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 24

```
DSolve[y''[x]-5*y'[x]+6*y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{2x}(-x + c_2 e^x - 1 + c_1)$$

8.17 problem 17

Internal problem ID [4371]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 17.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _exact, _rational, [_Abel, '2nd typ`

$$(y + 2x)y' - x + 2y = 0$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 51

```
dsolve((2*x+y(x))*diff(y(x),x)-(x-2*y(x))=0,y(x), singsol=all)
```

$$y(x) = \frac{-2c_1x - \sqrt{5c_1^2x^2 + 1}}{c_1}$$

$$y(x) = \frac{-2c_1x + \sqrt{5c_1^2x^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.452 (sec). Leaf size: 94

```
DSolve[(2*x+y[x])*y'[x]-(x-2*y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -2x - \sqrt{5x^2 + e^{2c_1}}$$

$$y(x) \rightarrow -2x + \sqrt{5x^2 + e^{2c_1}}$$

$$y(x) \rightarrow -\sqrt{5}\sqrt{x^2} - 2x$$

$$y(x) \rightarrow \sqrt{5}\sqrt{x^2} - 2x$$

8.18 problem 18

Internal problem ID [4372]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 18.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$(x \cos(y) - e^{-\sin(y)}) y' + 1 = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 17

```
dsolve((x*cos(y(x)) - exp(-sin(y(x))))*diff(y(x),x)+1=0,y(x), singsol=all)
```

$$x - (y(x) + c_1) e^{-\sin(y(x))} = 0$$

✓ Solution by Mathematica

Time used: 0.742 (sec). Leaf size: 26

```
DSolve[(x*Cos[y[x]] - Exp[-Sin[y[x]]])*y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[x = y(x)e^{-\sin(y(x))} + c_1e^{-\sin(y(x))}, y(x)]$$

8.19 problem 19

Internal problem ID [4373]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 19.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' \sin(x)^2 + \sin(x)^2 + (y + x) \sin(2x) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(sin(x)^2*diff(y(x),x)+(sin(x)^2+(x+y(x))*sin(2*x))=0,y(x), singsol=all)
```

$$y(x) = -\frac{2c_1}{-1 + \cos(2x)} - x$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 16

```
DSolve[Sin[x]^2*y'[x]+(Sin[x]^2+(x+y[x])*Sin[2*x])==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \rightarrow -x + c_1 \csc^2(x)$$

8.20 problem 20

Internal problem ID [4374]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + 5y - 5x - 4e^x(1 + \sin(2x)) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+5*y(x)=5*x+4*exp(x)*(1+sin(2*x)),y(x), singsol=all)
```

$$y(x) = e^x \sin(2x) c_2 + e^x \cos(2x) c_1 - (x - 1) e^x \cos(2x) + \frac{e^x \sin(2x)}{2} + x + e^x + \frac{2}{5}$$

✓ Solution by Mathematica

Time used: 0.642 (sec). Leaf size: 44

```
DSolve[y''[x]-2*y'[x]+5*y[x]==5*x+4*Exp[x]*(1+Sin[2*x]),y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \rightarrow x + e^x + \frac{1}{4} e^x (4(-x + c_2) \cos(2x) + (1 + 4c_1) \sin(2x)) + \frac{2}{5}$$

8.21 problem 21

Internal problem ID [4375]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 21.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' + xy - \frac{x}{y} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)+x*y(x)=x/y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{e^{-x^2}c_1 + 1}$$

$$y(x) = -\sqrt{e^{-x^2}c_1 + 1}$$

✓ Solution by Mathematica

Time used: 1.922 (sec). Leaf size: 57

```
DSolve[y'[x]+x*y[x]==x/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt{1 + e^{-x^2+2c_1}}$$

$$y(x) \rightarrow \sqrt{1 + e^{-x^2+2c_1}}$$

$$y(x) \rightarrow -1$$

$$y(x) \rightarrow 1$$

8.22 problem 22

Internal problem ID [4376]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 22.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[_high_order, _missing_x]`

$$y'''' - 2y''' + 13y'' - 18y' + 36y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$3)+13*diff(y(x),x$2)-18*diff(y(x),x)+36*y(x)=0,y(x),sing
```

$$y(x) = c_1 \sin(3x) + c_2 \cos(3x) + c_3 e^x \sin(\sqrt{3}x) + c_4 e^x \cos(\sqrt{3}x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 47

```
DSolve[y''''[x]-2*y'''[x]+13*y''[x]-18*y'[x]+36*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \rightarrow c_3 \cos(3x) + c_4 \sin(3x) + e^x \left(c_2 \cos(\sqrt{3}x) + c_1 \sin(\sqrt{3}x) \right)$$

8.23 problem 23

Internal problem ID [4377]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 23.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\sin(\theta) \cos(\theta) r' - \sin(\theta)^2 - r \cos(\theta)^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(sin(theta)*cos(theta)*diff(r(theta),theta)-sin(theta)^2=r(theta)*cos(theta)^2,r(theta))
```

$$r(\theta) = (\ln(\sec(\theta) + \tan(\theta)) + c_1) \sin(\theta)$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 14

```
DSolve[Sin[\[Theta]]*Cos[\[Theta]]*r'[\[Theta]]-Sin[\[Theta]]^2==r[\[Theta]]*Cos[\[Theta]]^2,
```

$$r(\theta) \rightarrow \sin(\theta)(\operatorname{arctanh}(\sin(\theta)) + c_1)$$

8.24 problem 24

Internal problem ID [4378]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], _Liouville, [_2nd_order, _wi`

$$x(yy'' + y'^2) - yy' = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve(x*(y(x)*diff(y(x),x$2) + diff(y(x),x)^2)= y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \sqrt{c_1x^2 + 2c_2}$$

$$y(x) = -\sqrt{c_1x^2 + 2c_2}$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 18

```
DSolve[x*(y[x]*y'[x]+(y'[x])^2)==y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2\sqrt{x^2 + c_1}$$

8.25 problem 25

Internal problem ID [4379]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 25.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$3x^2y + x^3y' = 0$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve([3*x^2*y(x)+x^3*diff(y(x),x)=0,y(1) = 2],y(x), singsol=all)
```

$$y(x) = \frac{2}{x^3}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 10

```
DSolve[{3*x^2*y[x]+x^3*y'[x]==0,{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2}{x^3}$$

8.26 problem 26

Internal problem ID [4380]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 26.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [linear]

$$-y + y'x - x^2 = 0$$

With initial conditions

$$[y(2) = 6]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve([x*diff(y(x),x)-y(x)=x^2,y(2) = 6],y(x), singsol=all)
```

$$y(x) = x(x + 1)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 10

```
DSolve[{x*y'[x]-y[x]==x^2,{y[2]==6}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(x + 1)$$

8.27 problem 27

Internal problem ID [4381]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 27.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y' - 6y - 6 = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 4]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)+diff(y(x),x)-6*y(x)=6,y(0) = 1, D(y)(0) = 4],y(x), singsol=all)
```

$$y(x) = -1 + 2e^{2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 14

```
DSolve[{y'[x]+y'[x]-6*y[x]==6,{y[0]==1,y'[0]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2e^{2x} - 1$$

8.28 problem 28

Internal problem ID [4382]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 28.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$yy'' + y'^2 + 4 = 0$$

With initial conditions

$$[y(1) = 3, y'(1) = 0]$$

✓ Solution by Maple

Time used: 0.547 (sec). Leaf size: 16

```
dsolve([y(x)*diff(y(x),x$2)+diff(y(x),x)^2+4=0,y(1) = 3, D(y)(1) = 0],y(x), singsol=all)
```

$$y(x) = \sqrt{-4x^2 + 8x + 5}$$

✓ Solution by Mathematica

Time used: 0.154 (sec). Leaf size: 17

```
DSolve[{y[x]*y'[x]+y'[x]^2+4==0,{y[1]==3,y'[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sqrt{5 - 4(x - 2)x}$$

9 Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

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9.1 problem 1, using series method

Internal problem ID [4383]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 1, using series method.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y'x - xy - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
Order:=6;
dsolve(x*diff(y(x),x)=x*y(x)+y(x),y(x),type='series',x=0);
```

$$y(x) = c_1 x \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 \right) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 38

```
AsymptoticDSolveValue[x*y'[x]==x*y[x]+y[x],y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x \left(\frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right)$$

9.2 problem 1, using elementary method

Internal problem ID [4384]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 1, using elementary method.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y'x - xy - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*diff(y(x),x)=x*y(x)+y(x),y(x), singsol=all)
```

$$y(x) = e^x c_1 x$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 17

```
DSolve[x*y'[x]==x*y[x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^x x$$

$$y(x) \rightarrow 0$$

9.3 problem 2, using series method

Internal problem ID [4385]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 2, using series method.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - 3x^2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve(diff(y(x),x)=3*x^2*y(x),y(x),type='series',x=0);
```

$$y(x) = (x^3 + 1) y(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 11

```
AsymptoticDSolveValue[y'[x]==3*x^2*y[x],y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1(x^3 + 1)$$

9.4 problem 2, using elementary method

Internal problem ID [4386]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 2, using elementary method.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - 3x^2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=3*x^2*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{x^3}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 18

```
DSolve[y'[x]==3*x^2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{x^3}$$

$$y(x) \rightarrow 0$$

9.5 problem 3, using series method

Internal problem ID [4387]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 3, using series method.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$-y + y'x = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
Order:=6;
dsolve(x*diff(y(x),x)=y(x),y(x),type='series',x=0);
```

$$y(x) = c_1x + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 7

```
AsymptoticDSolveValue[x*y'[x]==y[x],y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1x$$

9.6 problem 3, using elementary method

Internal problem ID [4388]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 3, using elementary method.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$-y + y'x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve(x*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = c_1x$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 14

```
DSolve[x*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1x$$

$$y(x) \rightarrow 0$$

9.7 problem 4, using series method

Internal problem ID [4389]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 4, using series method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 4y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)=-4*y(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 - 2x^2 + \frac{2}{3}x^4\right) y(0) + \left(x - \frac{2}{3}x^3 + \frac{2}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[y''[x]==-4*y[x],y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{2x^5}{15} - \frac{2x^3}{3} + x \right) + c_1 \left(\frac{2x^4}{3} - 2x^2 + 1 \right)$$

9.8 problem 4, using elementary method

Internal problem ID [4390]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 4, using elementary method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 4y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)=-4*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \sin(2x) + c_2 \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[y''[x]==-4*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \cos(2x) + c_2 \sin(2x)$$

9.9 problem 5, using series method

Internal problem ID [4391]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 5, using series method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)=y(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{24}x^4\right) y(0) + \left(x + \frac{1}{6}x^3 + \frac{1}{120}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]==y[x],y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^5}{120} + \frac{x^3}{6} + x \right) + c_1 \left(\frac{x^4}{24} + \frac{x^2}{2} + 1 \right)$$

9.10 problem 5, using elementary method

Internal problem ID [4392]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 5, using elementary method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y''[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^x + c_2 e^{-x}$$

9.11 problem 6, using series method

Internal problem ID [4393]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 6, using series method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 2y' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
Order:=6;
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{8}x^4 - \frac{1}{30}x^5\right) y(0) + \left(x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4 + \frac{1}{24}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 66

```
AsymptoticDSolveValue[y''[x]-2*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{x^5}{30} - \frac{x^4}{8} - \frac{x^3}{3} - \frac{x^2}{2} + 1 \right) + c_2 \left(\frac{x^5}{24} + \frac{x^4}{6} + \frac{x^3}{2} + x^2 + x \right)$$

9.12 problem 6, using elementary method

Internal problem ID [4394]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 6, using elementary method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 2y' + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^x x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[y''[x]-2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x(c_2 x + c_1)$$

9.13 problem 7, using series method

Internal problem ID [4395]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 7, using series method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_Emden, _Fowler]`

$$x^2 y'' - 3y'x + 3y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^3 (1 + O(x^6)) + c_2 x (-2 + O(x^6))$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 14

```
AsymptoticDSolveValue[x^2*y''[x]-3*x*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 x^3 + c_1 x$$

9.14 problem 7, using elementary method

Internal problem ID [4396]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 7, using elementary method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_Emden, _Fowler]`

$$x^2 y'' - 3y'x + 3y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_2 x^3 + c_1 x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[x^2*y'[x]-3*x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(c_2 x^2 + c_1)$$

9.15 problem 8, using series method

Internal problem ID [4397]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 8, using series method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 2x)y'' - 2y'(x + 1) + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
Order:=6;
dsolve((x^2+2*x)*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (1 + O(x^6)) c_1 x^2 + c_2 \left(-2 - 2x - \frac{1}{2}x^2 + O(x^6) \right)$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 23

```
AsymptoticDSolveValue[(x^2+2*x)*y'[x]-2*(x+1)*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 x^2 + c_1 \left(\frac{x^2}{4} + x + 1 \right)$$

9.16 problem 8, using elementary method

Internal problem ID [4398]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 8, using elementary method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$(x^2 + 2x)y'' - 2y'(x + 1) + 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+2*x)*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2 + c_2(x + 1)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 19

```
DSolve[(x^2+2*x)*y'[x]-2*(x+1)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1x^2 - c_2(x + 1)$$

9.17 problem 9, using series method

Internal problem ID [4399]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 9, using series method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 1)y'' - 2y'x + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve((x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + D(y)(0)x - y(0)x^2$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 18

```
AsymptoticDSolveValue[(x^2+1)*y'[x]-2*x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1(1 - x^2) + c_2x$$

9.18 problem 9, using elementary method

Internal problem ID [4400]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 9, using elementary method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$(x^2 + 1)y'' - 2y'x + 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x + c_2(x^2 - 1)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_2x - c_1(x - i)^2$$

9.19 problem 10, using series method

Internal problem ID [4401]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 10, using series method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 4y'x + (4x^2 - 2)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
Order:=6;
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x^2 + \frac{1}{2}x^4\right) y(0) + \left(x + x^3 + \frac{1}{2}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(\frac{x^5}{2} + x^3 + x \right) + c_1 \left(\frac{x^4}{2} + x^2 + 1 \right)$$

9.20 problem 10, using elementary method

Internal problem ID [4402]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 10, using elementary method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' - 4y'x + (4x^2 - 2)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{x^2} + c_2 x e^{x^2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{x^2}(c_2 x + c_1)$$