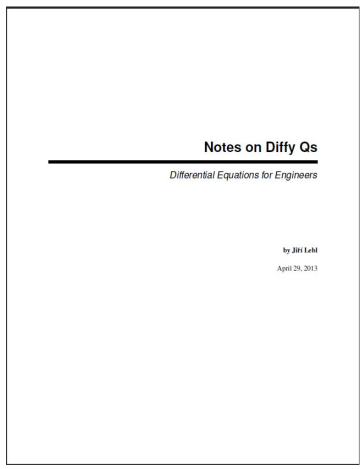
A Solution Manual For

Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.



Nasser M. Abbasi

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1 Chapter 7. POWER SERIES METHODS. 7.2.1 Exercises. page 290

1.1 problem 7.2.1

Internal problem ID [4749]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.2.1 Exercises. page 290
Problem number: 7.2.1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 - \frac{(x-1)^2}{2} + \frac{(x-1)^4}{24}\right) y(1) + \left(x - 1 - \frac{(x-1)^3}{6} + \frac{(x-1)^5}{120}\right) D(y)(1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 51

```
AsymptoticDSolveValue[y''[x]+y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1 \left(\frac{1}{24}(x-1)^4 - \frac{1}{2}(x-1)^2 + 1\right) + c_2 \left(\frac{1}{120}(x-1)^5 - \frac{1}{6}(x-1)^3 + x - 1\right)$$

1.2 problem 7.2.2

Internal problem ID [4750]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.2.1 Exercises. page 290
Problem number: 7.2.2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + 4xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+4*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{2x^3}{3}\right) y(0) + \left(x - \frac{1}{3}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+4*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^4}{3}\right) + c_1\left(1 - \frac{2x^3}{3}\right)$$

1.3 problem 7.2.3

Internal problem ID [4751]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.2.1 Exercises. page 290
Problem number: 7.2.3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' - xy = 0$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)-x*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{6} + \frac{(x-1)^4}{24} + \frac{(x-1)^5}{30}\right) y(1) + \left(x - 1 + \frac{(x-1)^3}{6} + \frac{(x-1)^4}{12} + \frac{(x-1)^5}{120}\right) D(y)(1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 78

```
AsymptoticDSolveValue[y''[x]-x*y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1\left(\frac{1}{30}(x-1)^5 + \frac{1}{24}(x-1)^4 + \frac{1}{6}(x-1)^3 + \frac{1}{2}(x-1)^2 + 1\right) + c_2\left(\frac{1}{120}(x-1)^5 + \frac{1}{12}(x-1)^4 + \frac{1}{6}(x-1)^3 + x - 1\right)$$

1.4 problem 7.2.4

Internal problem ID [4752]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.2.1 Exercises. page 290
Problem number: 7.2.4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + x^2 y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^4}{12}\right) y(0) + \left(x - \frac{1}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(x - \frac{x^5}{20}\right) + c_1 \left(1 - \frac{x^4}{12}\right)$$

1.5 problem 7.2.5

Internal problem ID [4753]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.2.1 Exercises. page 290
Problem number: 7.2.5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
Order:=6;
dsolve(diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{8}x^4\right)y(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 22

```
AsymptoticDSolveValue[y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{8} + \frac{x^2}{2} + 1\right)$$

1.6 problem 7.2.6

Internal problem ID [4754]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.2.1 Exercises. page 290
Problem number: 7.2.6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{(-x^2+1)\,y''-y'x+p^2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 71

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+p^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{p^2x^2}{2}+\frac{p^2(p^2-4)\,x^4}{24}\right)y(0)+\left(x-\frac{(p^2-1)\,x^3}{6}+\frac{(p^4-10p^2+9)\,x^5}{120}\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 88

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-x*y'[x]+p^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{p^4x^5}{120}-\frac{p^2x^5}{12}-\frac{p^2x^3}{6}+\frac{3x^5}{40}+\frac{x^3}{6}+x\right)+c_1\left(\frac{p^4x^4}{24}-\frac{p^2x^4}{6}-\frac{p^2x^2}{2}+1\right)$$

1.7 problem 7.2.7

Internal problem ID [4755]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.2.1 Exercises. page 290
Problem number: 7.2.7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-2y'x+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve((1+x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=y(0)+D(y)\,(0)\,x-x^2y(0)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 18

```
AsymptoticDSolveValue[(1+x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(1-x^2\right)+c_2x$$

1.8 problem 7.2.8 part(a)

Internal problem ID [4756]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.2.1 Exercises. page 290
Problem number: 7.2.8 part(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(x^2+1)\,y''+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((x^2+1)*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{2}x^2+\frac{1}{8}x^4\right)y(0)+\left(x-\frac{1}{6}x^3+\frac{7}{120}x^5\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(x^2+1)*y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{7x^5}{120}-\frac{x^3}{6}+x\right)+c_1\left(\frac{x^4}{8}-\frac{x^2}{2}+1\right)$$

1.9 problem 7.2.8 part(b)

Internal problem ID [4757]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.2.1 Exercises. page 290
Problem number: 7.2.8 part(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(x*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{6} - \frac{(x-1)^4}{24} + \frac{(x-1)^5}{60}\right) y(1) + \left(x - 1 - \frac{(x-1)^3}{6} + \frac{(x-1)^4}{12} - \frac{(x-1)^5}{24}\right) D(y)(1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 78

```
AsymptoticDSolveValue[x*y''[x]+y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1\left(\frac{1}{60}(x-1)^5 - \frac{1}{24}(x-1)^4 + \frac{1}{6}(x-1)^3 - \frac{1}{2}(x-1)^2 + 1\right) + c_2\left(-\frac{1}{24}(x-1)^5 + \frac{1}{12}(x-1)^4 - \frac{1}{6}(x-1)^3 + x - 1\right)$$

1.10 problem 7.2.101

Internal problem ID [4758]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.2.1 Exercises. page 290
Problem number: 7.2.101.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + 2yx^3 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
Order:=6;
dsolve(diff(y(x),x$2)+2*x^3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^5}{10}\right) y(0) + D(y)(0)\, x + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 20

```
AsymptoticDSolveValue[y''[x]+2*x^3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(1 - \frac{x^5}{10}\right) + c_2 x$$

1.11 problem 7.2.102

Internal problem ID [4759]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.2.1 Exercises. page 290
Problem number: 7.2.102.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - xy - \frac{1}{1-x} = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
Order:=6;
dsolve([diff(y(x),x$2)-x*y(x)=1/(1-x),y(0) = 0, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y(x) = \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{12}x^4 + \frac{3}{40}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 56

```
AsymptoticDSolveValue[{y''[x]-x*y[x]==1/(1-x),{}},y[x],{x,0,5}]
```

$$y(x) \to \frac{3x^5}{40} + \frac{x^4}{12} + c_2\left(\frac{x^4}{12} + x\right) + \frac{x^3}{6} + c_1\left(\frac{x^3}{6} + 1\right) + \frac{x^2}{2}$$

1.12 problem 7.2.103

Internal problem ID [4760]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.2.1 Exercises. page 290
Problem number: 7.2.103.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left(x^{-\frac{\sqrt{5}}{2}}c_1 + x^{\frac{\sqrt{5}}{2}}c_2\right) + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 38

```
AsymptoticDSolveValue[x^2*y''[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x^{\frac{1}{2}\left(1+\sqrt{5}\right)} + c_2 x^{\frac{1}{2}\left(1-\sqrt{5}\right)}$$

2 Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300

2.1 problem 7.3.3

Internal problem ID [4761]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + (x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^{-i}\left(1+\left(-\frac{1}{5}-\frac{2i}{5}\right)x+\left(-\frac{1}{40}+\frac{3i}{40}\right)x^2+\left(\frac{3}{520}-\frac{7i}{1560}\right)x^3+\left(-\frac{1}{2496}+\frac{i}{12480}\right)x^4 + \left(\frac{9}{603200}+\frac{i}{361920}\right)x^5 + \mathrm{O}\left(x^6\right)\right) + c_2x^i\left(1+\left(-\frac{1}{5}+\frac{2i}{5}\right)x+\left(-\frac{1}{40}-\frac{3i}{40}\right)x^2 + \left(\frac{3}{520}+\frac{7i}{1560}\right)x^3+\left(-\frac{1}{2496}-\frac{i}{12480}\right)x^4+\left(\frac{9}{603200}-\frac{i}{361920}\right)x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 90

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \left(\frac{1}{12480}+\frac{i}{2496}\right)c_2x^{-i}\left(ix^4-(8+16i)x^3+(168+96i)x^2-(1056-288i)x + (480-2400i)\right) - \left(\frac{1}{2496}+\frac{i}{12480}\right)c_1x^i\left(x^4-(16+8i)x^3+(96+168i)x^2 + (288-1056i)x-(2400-480i)\right)$$

2.2 problem 7.3.4

Internal problem ID [4762]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left(x^{-\frac{\sqrt{5}}{2}}c_1 + x^{\frac{\sqrt{5}}{2}}c_2\right) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 38

```
AsymptoticDSolveValue[x^2*y''[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x^{\frac{1}{2}\left(1+\sqrt{5}\right)} + c_2x^{\frac{1}{2}\left(1-\sqrt{5}\right)}$$

2.3 problem 7.3.5

Internal problem ID [4763]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + \frac{y'}{x} - xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x) c_2 + c_1)\left(1 + \frac{1}{9}x^3 + \mathrm{O}(x^6)\right) + \left(-\frac{2}{27}x^3 + \mathrm{O}(x^6)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 39

```
AsymptoticDSolveValue[y''[x]+1/x*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^3}{9} + 1\right) + c_2\left(\left(\frac{x^3}{9} + 1\right)\log(x) - \frac{2x^3}{27}\right)$$

2.4 problem 7.3.6

Internal problem ID [4764]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2xy'' + y' - x^2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)+diff(y(x),x)-x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{21}x^3 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 + \frac{1}{15}x^3 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 33

```
AsymptoticDSolveValue[2*x*y''[x]+y'[x]-x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^3}{21} + 1\right) + c_2\left(\frac{x^3}{15} + 1\right)$$

2.5 problem 7.3.7

Internal problem ID [4765]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x\left(x^{-\sqrt{2}}c_1 + x^{\sqrt{2}}c_2\right) + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 30

```
AsymptoticDSolveValue[x^2*y''[x]-x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x^{1+\sqrt{2}} + c_2x^{1-\sqrt{2}}$$

2.6 problem 7.3.8 (a)

Internal problem ID [4766]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.8 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x^2+1)\,y''+xy=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
Order:=6;
dsolve(x^2*(1+x^2)*diff(y(x),x$2)+x*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x) = c_1x\left(1-\frac{1}{2}x+\frac{1}{12}x^2+\frac{11}{144}x^3-\frac{83}{2880}x^4-\frac{2557}{86400}x^5+\mathrm{O}\left(x^6\right)\right)\\
+c_2\left(\ln\left(x\right)\left(-x+\frac{1}{2}x^2-\frac{1}{12}x^3-\frac{11}{144}x^4+\frac{83}{2880}x^5+\mathrm{O}\left(x^6\right)\right)\right.\\
\left.+\left(1-\frac{3}{4}x^2+\frac{13}{36}x^3+\frac{25}{1728}x^4-\frac{8743}{86400}x^5+\mathrm{O}\left(x^6\right)\right)\right)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x^2*(1+x^2)*y''[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned}y(x)\to c_1\left(\frac{157x^4+768x^3-2160x^2+1728x+1728}{1728}\right.\\
\left.-\frac{1}{144}x\left(11x^3+12x^2-72x+144\right)\log(x)\right)+c_2\left(-\frac{83x^5}{2880}+\frac{11x^4}{144}+\frac{x^3}{12}-\frac{x^2}{2}+x\right)\end{aligned}$$

2.7 problem 7.3.8 (b)

Internal problem ID [4767]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.8 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 84

```
AsymptoticDSolveValue[x^2*y''[x]+y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 e^{\frac{1}{x}} \left(\frac{59241x^5}{40} + \frac{1911x^4}{8} + \frac{91x^3}{2} + \frac{21x^2}{2} + 3x + 1\right) x^2 + c_1 \left(-\frac{91x^5}{40} + \frac{7x^4}{8} - \frac{x^3}{2} + \frac{x^2}{2} - x + 1\right)$$

2.8 problem 7.3.8 (c)

Internal problem ID [4768]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.8 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + x^3y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
Order:=6;
dsolve(x*diff(y(x),x$2)+x^3*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x \left(1 - \frac{1}{2}x + \frac{1}{12}x^2 - \frac{13}{144}x^3 + \frac{157}{2880}x^4 - \frac{877}{86400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln\left(x\right)\left(-x + \frac{1}{2}x^2 - \frac{1}{12}x^3 + \frac{13}{144}x^4 - \frac{157}{2880}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(1 - \frac{3}{4}x^2 + \frac{7}{36}x^3 + \frac{25}{1728}x^4 + \frac{6377}{86400}x^5 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x*y''[x]+x^3*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{1}{144}x\left(13x^3 - 12x^2 + 72x - 144\right)\log(x) + \frac{-131x^4 + 480x^3 - 2160x^2 + 1728x + 1728}{1728}\right) + c_2\left(\frac{157x^5}{2880} - \frac{13x^4}{144} + \frac{x^3}{12} - \frac{x^2}{2} + x\right)$$

2.9 problem 7.3.8 (d)

Internal problem ID [4769]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.8 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + y'x - e^x y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
Order:=6;
dsolve(x*diff(y(x),x$2)+x*diff(y(x),x)-exp(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x\left(1 + \frac{1}{6}x^2 + \frac{1}{72}x^3 + \frac{7}{480}x^4 + \frac{29}{10800}x^5 + O\left(x^6\right)\right) + c_2\left(\ln(x)\left(x + \frac{1}{6}x^3 + \frac{1}{72}x^4 + \frac{7}{480}x^5 + O\left(x^6\right)\right) + \left(1 - x - \frac{2}{9}x^3 - \frac{11}{864}x^4 - \frac{109}{4800}x^5 + O\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 70

```
AsymptoticDSolveValue[x*y''[x]+x*y'[x]-Exp[x]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{7x^5}{480} + \frac{x^4}{72} + \frac{x^3}{6} + x\right) + c_1\left(\frac{1}{864}\left(-23x^4 - 336x^3 - 1728x + 864\right) + \frac{1}{72}x\left(x^3 + 12x^2 + 72\right)\log(x)\right)$$

2.10 problem 7.3.8 (e)

Internal problem ID [4770]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.8 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x^2y'' + x^2y' + x^2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{120}x^5\right) y(0) + \left(x - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{120}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[x^2*y''[x]+x^2*y'[x]+x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(-\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^2}{2} + x\right) + c_1\left(-\frac{x^5}{120} + \frac{x^3}{6} - \frac{x^2}{2} + 1\right)$$

2.11 problem 7.3.101 (a)

Internal problem ID [4771]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.101 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(\frac{1}{24}x^4 - \frac{1}{2}x^2 + 1\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{120} - \frac{x^3}{6} + x\right) + c_1\left(\frac{x^4}{24} - \frac{x^2}{2} + 1\right)$$

2.12 problem 7.3.101 (b)

Internal problem ID [4772]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.101 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y'' + (x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^3*diff(y(x),x$2)+(1+x)*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 222

```
AsymptoticDSolveValue[x^3*y''[x]+(1+x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 e^{-\frac{2i}{\sqrt{x}}} x^{3/4} \left(\frac{520667425699057 i x^{9/2}}{131941395333120} - \frac{21896102683 i x^{7/2}}{21474836480} + \frac{19100991 i x^{5/2}}{41943040} - \frac{3367 i x^{3/2}}{8192} - \frac{194208949785748261 x^5}{21110623253299200} + \frac{5189376335871 x^4}{2748779069440} - \frac{846810601 x^3}{1342177280} + \frac{205387 x^2}{524288} - \frac{273 x}{512} + \frac{13 i \sqrt{x}}{16} + 1 \right) + c_2 e^{\frac{2i}{\sqrt{x}}} x^{3/4} \left(-\frac{520667425699057 i x^{9/2}}{131941395333120} + \frac{21896102683 i x^{7/2}}{21474836480} - \frac{19100991 i x^{5/2}}{41943040} + \frac{3367 i x^{3/2}}{8192} - \frac{19420894\cdots}{211106\cdots}\right.$$

2.13 problem 7.3.101 (c)

Internal problem ID [4773]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.101 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + y'x^5 + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
Order:=6;
dsolve(x*diff(y(x),x$2)+x^5*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x) = {} & c_1 x\left(1 - \frac{1}{2}x + \frac{1}{12}x^2 - \frac{1}{144}x^3 + \frac{1}{2880}x^4 - \frac{2881}{86400}x^5 + \mathrm{O}\left(x^6\right)\right) \\ & + c_2\left(\ln\left(x\right)\left(-x + \frac{1}{2}x^2 - \frac{1}{12}x^3 + \frac{1}{144}x^4 - \frac{1}{2880}x^5 + \mathrm{O}\left(x^6\right)\right)\right. \\ & \left. + \left(1 - \frac{3}{4}x^2 + \frac{7}{36}x^3 - \frac{35}{1728}x^4 + \frac{101}{86400}x^5 + \mathrm{O}\left(x^6\right)\right)\right)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x*y''[x]+x^5*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned}y(x) \to {} & c_1\left(\frac{1}{144}x\left(x^3 - 12x^2 + 72x - 144\right)\log(x) + \frac{-47x^4 + 480x^3 - 2160x^2 + 1728x + 1728}{1728}\right) \\ & + c_2\left(\frac{x^5}{2880} - \frac{x^4}{144} + \frac{x^3}{12} - \frac{x^2}{2} + x\right)\end{aligned}$$

2.14 problem 7.3.101 (d)

Internal problem ID [4774]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.101 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\sin(x)\, y'' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 58

```
Order:=6;
dsolve(sin(x)*diff(y(x),x$2)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x \left(1 + \frac{1}{2}x + \frac{1}{12}x^2 + \frac{1}{48}x^3 + \frac{1}{192}x^4 + \frac{37}{28800}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln(x)\left(x + \frac{1}{2}x^2 + \frac{1}{12}x^3 + \frac{1}{48}x^4 + \frac{1}{192}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(1 - \frac{3}{4}x^2 - \frac{1}{6}x^3 - \frac{5}{192}x^4 - \frac{257}{28800}x^5 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 85

```
AsymptoticDSolveValue[Sin[x]*y''[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{1}{48}x\left(x^3 + 4x^2 + 24x + 48\right)\log(x) + \frac{1}{64}\left(-3x^4 - 16x^3 - 80x^2 - 64x + 64\right)\right) + c_2 \left(\frac{x^5}{192} + \frac{x^4}{48} + \frac{x^3}{12} + \frac{x^2}{2} + x\right)$$

2.15 problem 7.3.101 (e)

Internal problem ID [4775]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.101 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\cos(x)\, y'' - y\sin(x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
Order:=6;
dsolve(cos(x)*diff(y(x),x$2)-sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{6}x^3 + \frac{1}{60}x^5\right) y(0) + \left(x + \frac{1}{12}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 35

```
AsymptoticDSolveValue[Cos[x]*y''[x]-Sin[x]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^4}{12} + x\right) + c_1\left(\frac{x^5}{60} + \frac{x^3}{6} + 1\right)$$

2.16 problem 7.3.102

Internal problem ID [4776]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.102.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left(x^{-\frac{\sqrt{5}}{2}}c_1 + x^{\frac{\sqrt{5}}{2}}c_2\right) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 38

```
AsymptoticDSolveValue[x^2*y''[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x^{\frac{1}{2}\left(1+\sqrt{5}\right)} + c_2x^{\frac{1}{2}\left(1-\sqrt{5}\right)}$$

2.17 problem 7.3.103

Internal problem ID [4777]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.103.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + \left(x - \frac{3}{4}\right)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 65

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+(x-3/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1 - \frac{1}{3}x + \frac{1}{24}x^2 - \frac{1}{360}x^3 + \frac{1}{8640}x^4 - \frac{1}{302400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(x^2 - \frac{1}{3}x^3 + \frac{1}{24}x^4 - \frac{1}{360}x^5 + \mathrm{O}\left(x^6\right)\right.\right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 101

```
AsymptoticDSolveValue[x^2*y''[x]+(x-3/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^{11/2}}{8640} - \frac{x^{9/2}}{360} + \frac{x^{7/2}}{24} - \frac{x^{5/2}}{3} + x^{3/2}\right) + c_1\left(\frac{31x^4 - 176x^3 + 144x^2 + 576x + 576}{576\sqrt{x}} - \frac{1}{48}x^{3/2}\left(x^2 - 8x + 24\right)\log(x)\right)$$

2.18 problem 7.3.104 (d)

Internal problem ID [4778]

Book: Notes on Diffy Qs. Differential Equations for Engineers. By by Jiri Lebl, 2013.
Section: Chapter 7. POWER SERIES METHODS. 7.3.2 The method of Frobenius. Exercises. page 300
Problem number: 7.3.104 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x(\ln(x) c_2 + c_1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 14

```
AsymptoticDSolveValue[x^2*y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x + c_2x\log(x)$$