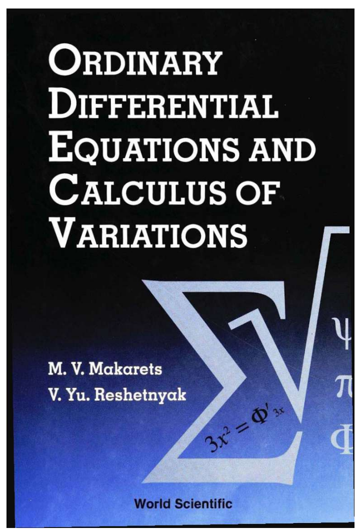
A Solution Manual For

Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995



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Contents

1 Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7

1.1 problem 1

Internal problem ID [4960]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x^2}{y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)=x^2/y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{6x^3 + 9c_1}}{3}$$
$$y(x) = \frac{\sqrt{6x^3 + 9c_1}}{3}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 50

```
DSolve[y'[x]==x^2/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{2}{3}}\sqrt{x^3 + 3c_1}$$
$$y(x) \to \sqrt{\frac{2}{3}}\sqrt{x^3 + 3c_1}$$

1.2 problem 2

Internal problem ID [4961]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x^2}{y(x^3+1)} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(y(x),x)=x^2/(y(x)*(1+x^3)),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{6\ln(x^3+1)+9c_1}}{3}$$
$$y(x) = \frac{\sqrt{6\ln(x^3+1)+9c_1}}{3}$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 56

```
DSolve[y'[x]==x^2/(y[x]*(1+x^3)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{2}{3}}\sqrt{\log(x^3+1)+3c_1}$$
$$y(x) \to \sqrt{\frac{2}{3}}\sqrt{\log(x^3+1)+3c_1}$$

1.3 problem 3

Internal problem ID [4962]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y\sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=y(x)*sin(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 19

```
DSolve[y'[x]==y[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\cos(x)}$$
$$y(x) \to 0$$

1.4 problem 4

Internal problem ID [4963]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - \sqrt{1 - y^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*diff(y(x),x)=sqrt(1-y(x)^2),y(x), singsol=all)
```

$$y(x) = \sin(\ln(x) + c_1)$$

✓ Solution by Mathematica

Time used: 0.266 (sec). Leaf size: 29

```
DSolve[x*y'[x]==Sqrt[1-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(\log(x) + c_1)$$
$$y(x) \to -1$$
$$y(x) \to 1$$
$$y(x) \to \text{Interval}[\{-1, 1\}]$$

1.5 problem 5

Internal problem ID [4964]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x^2}{1 + y^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 353

```
dsolve(diff(y(x),x)=x^2/(1+y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{1}{3}}}{2} - \frac{2}{\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{1}{3}}}{4} + \frac{1}{\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{1}{3}}}{2} + \frac{2}{\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{1}{3}}}{4} + \frac{1}{\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{1}{3}}}{2} + \frac{2}{\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 2.092 (sec). Leaf size: 307

```
DSolve[y'[x]==x^2/(1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2 + \sqrt[3]{2}\left(x^3 + \sqrt{x^6 + 6c_1x^3 + 4 + 9{c_1}^2} + 3c_1\right)^{2/3}}{2^{2/3}\sqrt[3]{x^3 + \sqrt{x^6 + 6c_1x^3 + 4 + 9{c_1}^2} + 3c_1}}$$

$$y(x) \to \frac{i\left(\sqrt{3} + i\right)\sqrt[3]{x^3 + \sqrt{x^6 + 6c_1x^3 + 4 + 9{c_1}^2} + 3c_1}}{2\sqrt[3]{2}} + \frac{1 + i\sqrt{3}}{2^{2/3}\sqrt[3]{x^3 + \sqrt{x^6 + 6c_1x^3 + 4 + 9{c_1}^2} + 3c_1}}$$

$$y(x) \to \frac{1 - i\sqrt{3}}{2^{2/3}\sqrt[3]{x^3 + \sqrt{x^6 + 6c_1x^3 + 4 + 9{c_1}^2} + 3c_1}} - \frac{\left(1 + i\sqrt{3}\right)\sqrt[3]{x^3 + \sqrt{x^6 + 6c_1x^3 + 4 + 9{c_1}^2} + 3c_1}}{2\sqrt[3]{2}}$$

1.6 problem 6

Internal problem ID [4965]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xyy' - \sqrt{1+y^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*y(x)*diff(y(x),x)=sqrt(1+y(x)^2),y(x), singsol=all)
```

$$\ln(x) - \sqrt{1 + y(x)^2} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.221 (sec). Leaf size: 59

```
DSolve[x*y[x]*y'[x]==Sqrt[1+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{(\log(x) - 1 + c_1)(\log(x) + 1 + c_1)}$$
$$y(x) \to \sqrt{(\log(x) - 1 + c_1)(\log(x) + 1 + c_1)}$$
$$y(x) \to -i$$
$$y(x) \to i$$

1.7 problem 7

Internal problem ID [4966]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2-1)\,y'+2xy^2=0}$$

With initial conditions

$$[y(0)=1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 27

```
dsolve([(x^2-1)*diff(y(x),x)+2*x*y(x)^2=0,y(0) = 1],y(x), singsol=all)
```

$$y(x)=\frac{i}{\pi+i\ln(x-1)+i\ln(x+1)+i}$$

✓ Solution by Mathematica

Time used: 0.152 (sec). Leaf size: 20

```
DSolve[{(x^2-1)*y'[x]+2*x*y[x]^2==0,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{\log(x^2-1)-i\pi+1}$$

1.8 problem 8

Internal problem ID [4967]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 3y^{\frac{2}{3}} = 0}$$

With initial conditions

$$[y(2) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=3*y(x)^(2/3),y(2) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 6

```
DSolve[{y'[x]==3*y[x]^(2/3),{y[2]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

1.9 problem 9

Internal problem ID [4968]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x + y - y^2 = 0}$$

With initial conditions

$$\left[y(1) = \frac{1}{2}\right]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 9

```
dsolve([x*diff(y(x),x)+y(x)=y(x)^2,y(1) = 1/2],y(x), singsol=all)
```

$$y(x) = \frac{1}{x+1}$$

✓ Solution by Mathematica

Time used: 0.23 (sec). Leaf size: 10

```
DSolve[{x*y'[x]+y[x]==y[x]^2,{y[1]==1/2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x+1}$$

1.10 problem 10

Internal problem ID [4969]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2y'x^2y + y^2 - 2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(2*x^2*y(x)*diff(y(x),x)+y(x)^2=2,y(x), singsol=all)
```

$$y(x) = \sqrt{\mathrm{e}^{\frac{1}{x}} c_1 + 2}$$
$$y(x) = -\sqrt{\mathrm{e}^{\frac{1}{x}} c_1 + 2}$$

✓ Solution by Mathematica

Time used: 0.27 (sec). Leaf size: 70

```
DSolve[2*x*y[x]*y'[x]+y[x]^2==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2x + e^{2c_1}}}{\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{2x + e^{2c_1}}}{\sqrt{x}}$$
$$y(x) \to -\sqrt{2}$$
$$y(x) \to \sqrt{2}$$

1.11 problem 11

Internal problem ID [4970]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - xy^2 - 2xy = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)-x*y(x)^2=2*x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{2}{-1 + 2c_1 e^{-x^2}}$$

✓ Solution by Mathematica

Time used: 0.266 (sec). Leaf size: 31

```
DSolve[y'[x]-2*x*y[x]^2==2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + \frac{1}{1 - e^{x^2 + c_1}}$$
$$y(x) \to -1$$
$$y(x) \to 0$$

1.12 problem 12

Internal problem ID [4971]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(1 + z')\, e^{-z} - 1 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve((1+diff(z(t),t))*exp(-z(t))=1,z(t), singsol=all)
```

$$z(t) = \ln\left(-\frac{1}{c_1 e^t - 1}\right)$$

✓ Solution by Mathematica

Time used: 0.705 (sec). Leaf size: 21

```
DSolve[(1+z'[t])*Exp[-z[t]]==1,z[t],t,IncludeSingularSolutions -> True]
```

$$z(t) \to \log\left(\frac{1}{1 + e^{t+c_1}}\right)$$
$$z(t) \to 0$$

1.13 problem 13

Internal problem ID [4972]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{3x^2 + 4x + 2}{2y - 2} = 0}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 19

```
dsolve([diff(y(x),x)=(3*x^2+4*x+2)/(2*(y(x)-1)),y(0) = -1],y(x), singsol=all)
```

$$y(x) = 1 - \sqrt{(x+2)(x^2+2)}$$

✓ Solution by Mathematica

Time used: 0.123 (sec). Leaf size: 22

```
DSolve[{y'[x]==(3*x^2+4*x+2)/(2*(y[x]-1)),{y[0]==-1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to 1 - \sqrt{(x+2)(x^2+2)}$$

1.14 problem 14

Internal problem ID [4973]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{e^x - (e^x + 1)\, yy' = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 19

```
dsolve([exp(x)-(1+exp(x))*y(x)*diff(y(x),x)=0,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \sqrt{1 - 2\ln(2) + 2\ln(e^x + 1)}$$

✓ Solution by Mathematica

Time used: 0.16 (sec). Leaf size: 23

```
DSolve[{Exp[x]-(1+Exp[x])*y[x]*y'[x]==0,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{2\log(e^x + 1) + 1 - \log(4)}$$

1.15 problem 15

Internal problem ID [4974]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{y}{x-1}+\frac{xy'}{1+y}=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(y(x)/(x-1)+x/(y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{x}{c_1x-c_1-1}$$

✓ Solution by Mathematica

Time used: 0.384 (sec). Leaf size: 33

```
DSolve[y[x]/(x-1)+x/(y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{e^{c_1}x}{x+e^{c_1}x-1}$$
$$y(x)\to -1$$
$$y(x)\to 0$$

1.16 problem 16

Internal problem ID [4975]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x + 2x^3 + (2y^3 + y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 113

```
dsolve((x+2*x^3)+(y(x)+2*y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2 - 2\sqrt{-4x^4 - 4x^2 - 8c_1 - 1}}}{2}$$
$$y(x) = \frac{\sqrt{-2 - 2\sqrt{-4x^4 - 4x^2 - 8c_1 - 1}}}{2}$$
$$y(x) = -\frac{\sqrt{-2 + 2\sqrt{-4x^4 - 4x^2 - 8c_1 - 1}}}{2}$$
$$y(x) = \frac{\sqrt{-2 + 2\sqrt{-4x^4 - 4x^2 - 8c_1 - 1}}}{2}$$

✓ Solution by Mathematica

Time used: 2.063 (sec). Leaf size: 147

```
DSolve[(x+2*x^3)+(y[x]+2*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-1-\sqrt{-4(x^4+x^2)+1+8c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-1-\sqrt{-4(x^4+x^2)+1+8c_1}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{-1+\sqrt{-4(x^4+x^2)+1+8c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-1+\sqrt{-4(x^4+x^2)+1+8c_1}}}{\sqrt{2}}$$

1.17 problem 17

Internal problem ID [4976]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{1}{\sqrt{x}}+\frac{y'}{\sqrt{y}}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(1/sqrt(x)+diff(y(x),x)/sqrt(y(x))=0,y(x), singsol=all)
```

$$\sqrt{y(x)}+\sqrt{x}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.118 (sec). Leaf size: 21

```
DSolve[1/Sqrt[x]+y'[x]/Sqrt[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{4}\left(-2\sqrt{x}+c_1\right)^2$$

1.18 problem 18

Internal problem ID [4977]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{1}{\sqrt{-x^2+1}}+\frac{y'}{\sqrt{1-y^2}}=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve(1/sqrt(1-x^2)+diff(y(x),x)/sqrt(1-y(x)^2)=0,y(x), singsol=all)
```

$$y(x)=-\sin\left(\arcsin\left(x\right)+c_1\right)$$

✓ Solution by Mathematica

Time used: 0.325 (sec). Leaf size: 35

```
DSolve[1/Sqrt[1-x^2]+y'[x]/Sqrt[1-y[x]^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\cos\left(2\cot^{-1}\left(\frac{x+1}{\sqrt{1-x^2}}\right)+c_1\right)$$
$$y(x)\to\text{Interval}[\{-1,1\}]$$

1.19 problem 19

Internal problem ID [4978]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2x\sqrt{1-y^2}+yy'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(2*x*sqrt(1-y(x)^2)+y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$c_1+x^2+\frac{(y(x)-1)\,(y(x)+1)}{\sqrt{1-y\,(x)^2}}=0$$

✓ Solution by Mathematica

Time used: 0.283 (sec). Leaf size: 67

```
DSolve[2*x*Sqrt[1-y[x]^2]+y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
y(x) &\to -\sqrt{(x^2+1-c_1)\,(-x^2+1+c_1)} \\
y(x) &\to \sqrt{(x^2+1-c_1)\,(-x^2+1+c_1)} \\
y(x) &\to -1 \\
y(x) &\to 1
\end{aligned}$$

1.20 problem 20

Internal problem ID [4979]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (y - 1)(1 + x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=(y(x)-1)*(x+1),y(x), singsol=all)
```

$$y(x) = 1 + \mathrm{e}^{\frac{x(x+2)}{2}} c_1$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 25

```
DSolve[y'[x]==(y[x]-1)*(x+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + c_1 e^{\frac{1}{2}x(x+2)}$$
$$y(x) \to 1$$

1.21 problem 21

Internal problem ID [4980]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{-y+x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=exp(x-y(x)),y(x), singsol=all)
```

$$y(x) = \ln\left(e^x + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.689 (sec). Leaf size: 12

```
DSolve[y'[x]==Exp[x-y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(e^x + c_1\right)$$

1.22 problem 22

Internal problem ID [4981]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\sqrt{y}}{\sqrt{x}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=sqrt(y(x))/sqrt(x),y(x), singsol=all)
```

$$\sqrt{y(x)} - \sqrt{x} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.14 (sec). Leaf size: 26

```
DSolve[y'[x]==Sqrt[y[x]]/Sqrt[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(2\sqrt{x} + c_1\right)^2$$
$$y(x) \to 0$$

1.23 problem 23

Internal problem ID [4982]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{\sqrt{y}}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=sqrt(y(x))/x,y(x), singsol=all)
```

$$\sqrt{y(x)} - \frac{\ln(x)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 21

```
DSolve[y'[x]==Sqrt[y[x]]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(\log(x) + c_1)^2$$
$$y(x) \to 0$$

1.24 problem 24

Internal problem ID [4983]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{z' - 10^{x+z} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(z(x),x)=10^(x+z(x)),z(x), singsol=all)
```

$$z(x) = \frac{\ln\left(-\frac{1}{c_1 \ln(10)+10^x}\right)}{\ln(10)}$$

✓ Solution by Mathematica

Time used: 0.871 (sec). Leaf size: 24

```
DSolve[z'[x]==10^(x+z[x]),z[x],x,IncludeSingularSolutions -> True]
```

$$z(x) \to -\frac{\log\left(-10^x + c_1(-\log(10))\right)}{\log(10)}$$

1.25 problem 25

Internal problem ID [4984]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' + t - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(x(t),t)+t=1,x(t), singsol=all)
```

$$x(t) = -\frac{1}{2}t^2 + t + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 16

```
DSolve[x'[t]+t==1,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{t^2}{2} + t + c_1$$

1.26 problem 26

Internal problem ID [4985]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \cos(-y + x) = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=cos(y(x)-x),y(x), singsol=all)
```

$$y(x) = x - 2\arctan\left(\frac{1}{-x + c_1}\right)$$

✓ Solution by Mathematica

Time used: 0.425 (sec). Leaf size: 40

```
DSolve[y'[x]==Cos[y[x]-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + 2\cot^{-1}\left(x - \frac{c_1}{2}\right)$$
$$y(x) \to x + 2\cot^{-1}\left(x - \frac{c_1}{2}\right)$$
$$y(x) \to x$$

1.27 problem 27

Internal problem ID [4986]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y - 2x + 3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)-y(x)=2*x-3,y(x), singsol=all)
```

$$y(x) = -2x + 1 + e^x c_1$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 16

```
DSolve[y'[x]-y[x]==2*x-3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x + c_1 e^x + 1$$

1.28 problem 28

Internal problem ID [4987]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], [_Abel, ‘2nd type‘, ‘class C‘], _dA`

$$\boxed{(x + 2y)\, y' - 1 = 0}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 9

```
dsolve([(x+2*y(x))*diff(y(x),x)=1,y(0) = -1],y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} - 1$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 12

```
DSolve[{(x+2*y[x])*y'[x]==1,{y[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{2} - 1$$

1.29 problem 29

Internal problem ID [4988]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y - 1 - 2x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)=2*x+1,y(x), singsol=all)
```

$$y(x) = 2x - 1 + \mathrm{e}^{-x}c_1$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]==2*x+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x + c_1e^{-x} - 1$$

1.30 problem 30

Internal problem ID [4989]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \cos(x - y - 1) = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=cos(x-y(x)-1),y(x), singsol=all)
```

$$y(x) = x - 1 - 2\arctan\left(\frac{1}{-x + c_1}\right)$$

✓ Solution by Mathematica

Time used: 0.546 (sec). Leaf size: 50

```
DSolve[y'[x]==Cos[x-y[x]-1],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - 2\cot^{-1}\left(-x + 1 + \frac{c_1}{2}\right) - 1$$
$$y(x) \to x - 2\cot^{-1}\left(-x + 1 + \frac{c_1}{2}\right) - 1$$
$$y(x) \to x - 1$$

1.31 problem 31

Internal problem ID [4990]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' + \sin(y+x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+sin(x+y(x))^2=0,y(x), singsol=all)
```

$$y(x) = -x - \arctan(-x + c_1)$$

✓ Solution by Mathematica

Time used: 0.203 (sec). Leaf size: 27

```
DSolve[y'[x]+Sin[x+y[x]]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[2(\tan(y(x)+x) - \arctan(\tan(y(x)+x))) + 2y(x) = c_1, y(x)]$$

1.32 problem 32

Internal problem ID [4991]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - 2\sqrt{2x + y + 1} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 56

```
dsolve(diff(y(x),x)=2*sqrt(2*x+y(x)+1),y(x), singsol=all)
```

$$x - \sqrt{2x + y(x) + 1} - \frac{\ln\left(-1 + \sqrt{2x + y(x) + 1}\right)}{2} + \frac{\ln\left(\sqrt{2x + y(x) + 1} + 1\right)}{2} + \frac{\ln(y(x) + 2x)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 8.962 (sec). Leaf size: 47

```
DSolve[y'[x]==2*Sqrt[2*x+y[x]+1],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x + W\left(-e^{-x-\frac{3}{2}+c_1}\right)\left(2 + W\left(-e^{-x-\frac{3}{2}+c_1}\right)\right)$$
$$y(x) \to -2x$$

1.33 problem 33

Internal problem ID [4992]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (y + x + 1)^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=(x+y(x)+1)^2,y(x), singsol=all)
```

$$y(x) = -x - 1 - \tan(-x + c_1)$$

✓ Solution by Mathematica

Time used: 0.483 (sec). Leaf size: 15

```
DSolve[y'[x]==(x+y[x]+1)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + \tan(x + c_1) - 1$$

1.34 problem 34

Internal problem ID [4993]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 + xy^2 + (x^2 - x^2 y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve((y(x)^2+x*y(x)^2)+(x^2-x^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{\ln(x)x + \operatorname{LambertW}\left(-\frac{e^{-c_1+\frac{1}{x}}}{x}\right)x + c_1 x - 1}{x}}$$

✓ Solution by Mathematica

Time used: 5.024 (sec). Leaf size: 30

```
DSolve[(y[x]^2+x*y[x]^2)+(x^2-x^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{W\left(-\frac{e^{\frac{1}{x}-c_1}}{x}\right)}$$

$$y(x) \to 0$$

1.35 problem 35

Internal problem ID [4994]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\left(1+y^2\right)\left(\mathrm{e}^{2x}-\mathrm{e}^{y} y'\right)-(1+y)\,y'=0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 30

```
dsolve((1+y(x)^2)*(exp(2*x)-exp(y(x))*diff(y(x),x))-(1+y(x))*diff(y(x),x)=0,y(x), singsol=all
```

$$\frac{\mathrm{e}^{2x}}{2}-\arctan\left(y(x)\right)-\frac{\ln\left(1+y(x)^2\right)}{2}-\mathrm{e}^{y(x)}+c_1=0$$

✓ Solution by Mathematica

Time used: 0.687 (sec). Leaf size: 70

```
DSolve[(1+y[x]^2)*(Exp[2*x]-Exp[y[x]]*y'[x])-(1+y[x])*y'[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to \text{InverseFunction}\left[e^{\#1}+\left(\frac{1}{2}-\frac{i}{2}\right)\log(-\#1+i)+\left(\frac{1}{2}+\frac{i}{2}\right)\log(\#1+i)\&\right]\left[\frac{e^{2x}}{2}+c_1\right]$$

$y(x)\to -i$

$y(x)\to i$

2 Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12

2.1 problem 1

Internal problem ID [4995]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{x - y + (y + x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x-y(x))+(x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \tan\left(\operatorname{RootOf}\left(2\_Z + \ln\left(\frac{1}{\cos\left(\_Z\right)^2}\right) + 2\ln\left(x\right) + 2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 34

```
DSolve[(x-y[x])+(x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{y(x)}{x}\right) + \frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + 1\right) = -\log(x) + c_1, y(x)\right]$$

2.2 problem 2

Internal problem ID [4996]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - 2xy + x^2y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((y(x)-2*x*y(x))+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{1}{x}} x^2$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 21

```
DSolve[(y[x]-2*x*y[x])+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{1}{x}} x^2$$
$$y(x) \to 0$$

2.3 problem 3

Internal problem ID [4997]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{2y'x - y(2x^2 - y^2) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 74

```
dsolve(2*x*diff(y(x),x)=y(x)*(2*x^2-y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-2\left(-2c_1 + \text{Ei}_1\left(-x^2\right)\right)\text{e}^{x^2}}}{-2c_1 + \text{Ei}_1\left(-x^2\right)}$$
$$y(x) = -\frac{\sqrt{-2\left(-2c_1 + \text{Ei}_1\left(-x^2\right)\right)\text{e}^{x^2}}}{-2c_1 + \text{Ei}_1\left(-x^2\right)}$$

✓ Solution by Mathematica

Time used: 0.261 (sec). Leaf size: 65

```
DSolve[2*x*y'[x]==y[x]*(2*x^2-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{\frac{x^2}{2}}}{\sqrt{\frac{\text{ExpIntegralEi}(x^2)}{2} + c_1}}$$
$$y(x) \to \frac{e^{\frac{x^2}{2}}}{\sqrt{\frac{\text{ExpIntegralEi}(x^2)}{2} + c_1}}$$
$$y(x) \to 0$$

2.4 problem 4

Internal problem ID [4998]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y^2 + x^2 y' - x y y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(y(x)^2+x^2*diff(y(x),x)=x*y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-c_1}}{x}\right)-c_1}$$

✓ Solution by Mathematica

Time used: 2.045 (sec). Leaf size: 25

```
DSolve[y[x]^2+x^2*y'[x]==x*y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -xW\left(-\frac{e^{-c_1}}{x}\right)$$

$$y(x) \to 0$$

2.5 problem 5

Internal problem ID [4999]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(y^2 + x^2)\, y' - 2xy = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 45

```
dsolve((x^2+y(x)^2)*diff(y(x),x)=2*x*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{-1 + \sqrt{4c_1^2x^2 + 1}}{2c_1}$$
$$y(x) = \frac{1 + \sqrt{4c_1^2x^2 + 1}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.907 (sec). Leaf size: 70

```
DSolve[(x^2+y[x]^2)*y'[x]==2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-\sqrt{4x^2 + e^{2c_1}} - e^{c_1}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{4x^2 + e^{2c_1}} - e^{c_1}\right)$$
$$y(x) \to 0$$

2.6 problem 6

Internal problem ID [5000]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{-y + y'x - \tan\left(\frac{y}{x}\right)x = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x)-y(x)=x*tan(y(x)/x),y(x), singsol=all)
```

$$y(x) = \arcsin(c_1 x)\, x$$

✓ Solution by Mathematica

Time used: 3.759 (sec). Leaf size: 19

```
DSolve[x*y'[x]-y[x]==x*Tan[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin\left(e^{c_1} x\right)$$
$$y(x) \to 0$$

2.7 problem 7

Internal problem ID [5001]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'x - y + x\,\mathrm{e}^{\frac{y}{x}} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)=y(x)-x*exp(y(x)/x),y(x), singsol=all)
```

$$y(x) = -\ln\left(\ln\left(x\right) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.348 (sec). Leaf size: 16

```
DSolve[x*y'[x]==y[x]-x*Exp[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\log(\log(x) - c_1)$$

2.8 problem 8

Internal problem ID [5002]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{-y + y'x - (y + x)\ln\left(\frac{y + x}{x}\right) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x)-y(x)=(x+y(x))*ln((x+y(x))/x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{c_1 x} x - x$$

✓ Solution by Mathematica

Time used: 0.383 (sec). Leaf size: 24

```
DSolve[x*y'[x]-y[x]==(x+y[x])*Log[ (x+y[x])/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(-1 + e^{e^{-c_1 x}}\right)$$
$$y(x) \to 0$$

2.9 problem 9

Internal problem ID [5003]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'x - y\cos\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)=y(x)*cos(y(x)/x),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\ln(x) + c_1 - \left(\int^{\_Z} \frac{1}{\_a(-1+\cos(\_a))}d\_a\right)\right)x$$

✓ Solution by Mathematica

Time used: 2.015 (sec). Leaf size: 33

```
DSolve[x*y'[x]==y[x]*Cos[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{1}{(\cos(K[1])-1)K[1]}dK[1] = \log(x) + c_1, y(x)\right]$$

2.10 problem 10

Internal problem ID [5004]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y + \sqrt{xy} - y'x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((y(x)+sqrt(x*y(x)))-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$-\frac{y(x)}{\sqrt{y(x)\,x}} + \frac{\ln(x)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.157 (sec). Leaf size: 17

```
DSolve[(y[x]+Sqrt[x*y[x]])-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}x(\log(x) + c_1)^2$$

2.11 problem 11

Internal problem ID [5005]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y'x - \sqrt{x^2 - y^2} - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)-sqrt(x^2-y(x)^2)-y(x)=0,y(x), singsol=all)
```

$$-\arctan\left(\frac{y(x)}{\sqrt{x^2 - y(x)^2}}\right) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.258 (sec). Leaf size: 18

```
DSolve[x*y'[x]-Sqrt[x^2-y[x]^2]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\cosh(i\log(x) + c_1)$$

2.12 problem 12

Internal problem ID [5006]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{x + y - (-y + x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x+y(x))-(x-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \tan\left(\operatorname{RootOf}\left(-2\_Z + \ln\left(\frac{1}{\cos\left(\_Z\right)^2}\right) + 2\ln\left(x\right) + 2c_1\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 36

```
DSolve[(x+y[x])-(x-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + 1\right) - \arctan\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

2.13 problem 13

Internal problem ID [5007]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$x^2 + 2xy - y^2 + (y^2 + 2xy - x^2)\, y' = 0$$

With initial conditions

$$[y(1) = -1]$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 7

```
dsolve([(x^2+2*x*y(x)-y(x)^2)+(y(x)^2+2*x*y(x)-x^2)*diff(y(x),x)=0,y(1) = -1],y(x), singsol=a
```

$$y(x) = -x$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(x^2+2*x*y[x]-y[x]^2)+(y[x]^2+2*x*y[x]-x^2)*y'[x]==0,{y[1]==-1}},y[x],x,IncludeSingul
```

{}

2.14 problem 14

Internal problem ID [5008]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{-y + y'x - yy' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)-y(x)=y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = e^{\operatorname{LambertW}(-x\,e^{-c_1})+c_1}$$

✓ Solution by Mathematica

Time used: 3.871 (sec). Leaf size: 25

```
DSolve[x*y'[x]-y[x]==y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{W(-e^{-c_1}x)+c_1}$$
$$y(x) \to 0$$

2.15 problem 15

Internal problem ID [5009]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y^2 + \left(-xy + x^2\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(y(x)^2+(x^2-x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-c_1}}{x}\right)-c_1}$$

✓ Solution by Mathematica

Time used: 1.95 (sec). Leaf size: 25

```
DSolve[y[x]^2+(x^2-x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -xW\left(-\frac{e^{-c_1}}{x}\right)$$
$$y(x) \to 0$$

2.16 problem 16

Internal problem ID [5010]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Riccati]`

$$\boxed{x^2 + xy + y^2 - x^2y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve((x^2+x*y(x)+y(x)^2)=x^2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \tan(\ln(x) + c_1)\,x$$

✓ Solution by Mathematica

Time used: 0.179 (sec). Leaf size: 13

```
DSolve[(x^2+x*y[x]+y[x]^2)==x^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\tan(\log(x) + c_1)$$

2.17 problem 17

Internal problem ID [5011]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{\frac{1}{x^2 - xy + y^2} - \frac{y'}{2y^2 - xy} = 0}$$

✓ Solution by Maple

Time used: 0.968 (sec). Leaf size: 40

```
dsolve(1/(x^2-x*y(x)+y(x)^2)=1/(2*y(x)^2-x*y(x))*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \left(\operatorname{RootOf}\left(\_Z^8 c_1 x^2 + 2\_Z^6 c_1 x^2 - \_Z^4 - 2\_Z^2 - 1\right)^2 + 2\right) x$$

✓ Solution by Mathematica

Time used: 60.191 (sec). Leaf size: 1805

```
DSolve[1/(x^2-x*y[x]+y[x]^2)==1/(2*y[x]^2-x*y[x])*y'[x],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{6}\left(-\sqrt{3}\sqrt{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{6c_1}}+\frac{e^{4c_1}}{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e}}} \right.$$
$$-\sqrt{3}\sqrt{-\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{6c_1}}-\frac{e^{4c_1}}{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{6c}}}}$$
$$\left. +9x \right)$$

$$y(x) \to \frac{1}{6}\left(-\sqrt{3}\sqrt{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{6c_1}}+\frac{e^{4c_1}}{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e}}} \right.$$
$$+\sqrt{3}\sqrt{-\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{6c_1}}-\frac{e^{4c_1}}{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{6c}}}}$$
$$\left. +9x \right)$$

$$y(x)$$

2.18 problem 18

Internal problem ID [5012]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y' - \frac{2xy}{3x^2 - y^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 402

```
dsolve(diff(y(x),x)=2*x*y(x)/(3*x^2-y(x)^2),y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= \frac{\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{6c_1} \\
&\quad + \frac{2}{3c_1\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}+\frac{1}{3c_1}
\end{aligned}
$$

$$
\begin{aligned}
y(x) &= -\frac{\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{12c_1} \\
&\quad - \frac{1}{3c_1\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}+\frac{1}{3c_1} \\
&\quad - \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{6c_1}-\frac{2}{3c_1\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}\right)}{2}
\end{aligned}
$$

$$
\begin{aligned}
y(x) &= -\frac{\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{12c_1} \\
&\quad - \frac{1}{3c_1\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}+\frac{1}{3c_1} \\
&\quad + \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{6c_1}-\frac{2}{3c_1\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}\right)}{2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 60.184 (sec). Leaf size: 458

```
DSolve[y'[x]==2*x*y[x]/(3*x^2-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(\frac{\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}e^{2c_1}}{\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - e^{c_1}\right)$$

$$y(x) \to \frac{i\left(\sqrt{3}+i\right)\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{6\sqrt[3]{2}} - \frac{i\left(\sqrt{3}-i\right)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

$$y(x) \to -\frac{i\left(\sqrt{3}-i\right)\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{6\sqrt[3]{2}} + \frac{i\left(\sqrt{3}+i\right)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

2.19 problem 19

Internal problem ID [5013]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y' - \frac{x}{y} - \frac{y}{x} = 0}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 34

```
dsolve([diff(y(x),x)=x/y(x)+y(x)/x,y(-1) = 0],y(x), singsol=all)
```

$$y(x) = \sqrt{2\ln(x) - 2i\pi}\, x$$
$$y(x) = -\sqrt{2\ln(x) - 2i\pi}\, x$$

✓ Solution by Mathematica

Time used: 0.179 (sec). Leaf size: 48

```
DSolve[{y'[x]==x/y[x]+y[x]/x,{y[-1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}x\sqrt{\log(x) - i\pi}$$
$$y(x) \to \sqrt{2}x\sqrt{\log(x) - i\pi}$$

2.20 problem 20

Internal problem ID [5014]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y'x - y - \sqrt{y^2 - x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x)=y(x)+sqrt(y(x)^2-x^2),y(x), singsol=all)
```

$$\frac{y(x)}{x^2} + \frac{\sqrt{y(x)^2 - x^2}}{x^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.363 (sec). Leaf size: 14

```
DSolve[x*y'[x]==y[x]+Sqrt[y[x]^2-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \cosh(\log(x) + c_1)$$

2.21 problem 21

Internal problem ID [5015]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y + (2\sqrt{xy} - x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(y(x)+(2*sqrt(x*y(x))-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$\ln\left(y(x)\right) + \frac{x}{\sqrt{y\left(x\right)x}} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.222 (sec). Leaf size: 33

```
DSolve[y[x]+(2*Sqrt[x*y[x]]-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2}{\sqrt{\frac{y(x)}{x}}} + 2\log\left(\frac{y(x)}{x}\right) = -2\log(x) + c_1, y(x)\right]$$

2.22 problem 22

Internal problem ID [5016]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'x - y\ln\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)=y(x)*ln(y(x)/x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{c_1x+1}x$$

✓ Solution by Mathematica

Time used: 0.205 (sec). Leaf size: 24

```
DSolve[x*y'[x]==y[x]*Log[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to xe^{1+e^{c_1}x}$$
$$y(x) \to ex$$

2.23 problem 23

Internal problem ID [5017]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 23.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'(y'+y)-x(y+x)=0}$$

With initial conditions

$$[y(0)=0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 9

```
dsolve([diff(y(x),x)*(diff(y(x),x)+y(x))=x*(x+y(x)),y(0) = 0],y(x), singsol=all)
```

$$y(x)=\frac{x^2}{2}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 27

```
DSolve[{y'[x]*(y'[x]+y[x])==x*(x+y[x]),{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{x^2}{2}$$
$$y(x)\to -x+\sinh(x)-\cosh(x)+1$$

2.24 problem 24

Internal problem ID [5018]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 24.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$(y'x + y)^2 - y^2 y' = 0$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 125

```
dsolve((x*diff(y(x),x)+y(x))^2=y(x)^2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = 4x$$
$$y(x) = 0$$
$$y(x) = -\frac{2c_1^2(\sqrt{2}\, c_1 - x)}{2c_1^2 - x^2}$$
$$y(x) = \frac{2c_1^2(\sqrt{2}\, c_1 + x)}{2c_1^2 - x^2}$$
$$y(x) = -\frac{c_1^2(\sqrt{2}\, c_1 - 2x)}{2\,(c_1^2 - 2x^2)}$$
$$y(x) = \frac{c_1^2(\sqrt{2}\, c_1 + 2x)}{2c_1^2 - 4x^2}$$

✓ Solution by Mathematica

Time used: 0.641 (sec). Leaf size: 61

```
DSolve[(x*y'[x]+y[x])^2==y[x]^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{4e^{-2c_1}}{2 + e^{2c_1}x}$$
$$y(x) \to \frac{1}{-4e^{4c_1}x - 2e^{2c_1}}$$
$$y(x) \to 0$$
$$y(x) \to 4x$$

2.25 problem 25

Internal problem ID [5019]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 25.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$x^2y'^2 - 3xyy' + 2y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x)^2-3*x*y(x)*diff(y(x),x)+2*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1x^2$$
$$y(x) = c_1x$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 24

```
DSolve[x^2*(y'[x])^2-3*x*y[x]*y'[x]+2*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1x$$
$$y(x) \to c_1x^2$$
$$y(x) \to 0$$

2.26 problem 26

Internal problem ID [5020]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$-y + y'x - \sqrt{y^2 + x^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{y(x)}{x^2} + \frac{\sqrt{x^2 + y(x)^2}}{x^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.346 (sec). Leaf size: 27

```
DSolve[x*y'[x]-y[x]==Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-c_1} \left(-1 + e^{2c_1} x^2\right)$$

2.27 problem 27

Internal problem ID [5021]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 27.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$yy'^2 + 2y'x - y = 0$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 75

```
dsolve(y(x)*diff(y(x),x)^2+2*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = 0$$
$$y(x) = \sqrt{c_1^2 - 2c_1x}$$
$$y(x) = \sqrt{c_1^2 + 2c_1x}$$
$$y(x) = -\sqrt{c_1^2 - 2c_1x}$$
$$y(x) = -\sqrt{c_1^2 + 2c_1x}$$

✓ Solution by Mathematica

Time used: 0.473 (sec). Leaf size: 126

```
DSolve[y[x]*(y'[x])^2+2*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{\frac{c_1}{2}}\sqrt{-2x+e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{2}}\sqrt{-2x+e^{c_1}}$$
$$y(x) \to -e^{\frac{c_1}{2}}\sqrt{2x+e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{2}}\sqrt{2x+e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -ix$$
$$y(x) \to ix$$

2.28 problem 28

Internal problem ID [5022]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{x + 2y}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)+(x+2*y(x))/x=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{3} + \frac{c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 17

```
DSolve[y'[x]+(x+2*y[x])/x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{3} + \frac{c_1}{x^2}$$

2.29 problem 29

Internal problem ID [5023]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{y}{y + x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=y(x)/(x+y(x)),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{LambertW}(x\,\mathrm{e}^{c_1})-c_1}$$

✓ Solution by Mathematica

Time used: 3.413 (sec). Leaf size: 23

```
DSolve[y'[x]==y[x]/(x+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{W\left(e^{-c_1}x\right)}$$
$$y(x) \to 0$$

2.30 problem 30

Internal problem ID [5024]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - x - \frac{y}{2} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 13

```
dsolve([x*diff(y(x),x)=x+1/2*y(x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 2x + c_1\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 17

```
DSolve[{x*y'[x]==x+1/2*y[x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x + c_1\sqrt{x}$$

2.31 problem Example 3

Internal problem ID [5025]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: Example 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{x + y - 2}{y - x - 4} = 0}$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)=(x+y(x)-2)/(y(x)-x-4),y(x), singsol=all)
```

$$y(x) = 3 - \frac{-c_1(x+1) + \sqrt{2\left(x+1\right)^2 c_1^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.12 (sec). Leaf size: 59

```
DSolve[y'[x]==(x+y[x]-2)/(y[x]-x-4),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - i\sqrt{-2(x(x+2)+8) - c_1} + 4$$
$$y(x) \to x + i\sqrt{-2(x(x+2)+8) - c_1} + 4$$

2.32 problem Example 4

Internal problem ID [5026]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: Example 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{2x - 4y + 6 + (x + y - 2) y' = 0}$$

✓ Solution by Maple

Time used: 0.171 (sec). Leaf size: 300

```
dsolve((2*x-4*y(x)+6)+(x+y(x)-2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{5}{3} + \frac{\left(12\sqrt{3}\,(3x-1)\sqrt{\frac{(3x-1)(27(3x-1)c_1-4)}{c_1}}\,c_1^2 + 108(3x-1)^2 c_1^2 - 72(3x-1)\,c_1 + 8\right)^{\frac{1}{3}}}{36c_1}$$
$$- \frac{6(3x-1)\,c_1 - 1}{9c_1\left(12\sqrt{3}\,(3x-1)\sqrt{\frac{(3x-1)(27(3x-1)c_1-4)}{c_1}}\,c_1^2 + 108\,(3x-1)^2 c_1^2 - 72\,(3x-1)\,c_1 + 8\right)^{\frac{1}{3}}}$$
$$+ \frac{6(3x-1)\,c_1 - 1}{9c_1}$$
$$- \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,(3x-1)\sqrt{\frac{(3x-1)(27(3x-1)c_1-4)}{c_1}}\,c_1^2+108(3x-1)^2c_1^2-72(3x-1)c_1+8\right)^{\frac{1}{3}}}{6c_1} + \frac{4(3x-1)c_1-\frac{2}{3}}{c_1\left(12\sqrt{3}\,(3x-1)\sqrt{\frac{(3x-1)(27(3x-1)c_1-4)}{c_1}}\,c_1^2+108\right.}\right.}{6}$$

✓ Solution by Mathematica

Time used: 60.092 (sec). Leaf size: 2563

```
DSolve[(2*x-4*y[x]+6)+(x+y[x]-2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

2.33 problem 31

Internal problem ID [5027]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$y' - \frac{2y - x + 5}{2x - y - 4} = 0$$

✓ Solution by Maple

Time used: 0.313 (sec). Leaf size: 182

```
dsolve(diff(y(x),x)=(2*y(x)-x+5)/(2*x-y(x)-4),y(x), singsol=all)
```

$$y(x) = -2 - \frac{(x-1)\left(c_1^2\left(-\frac{\left(3\sqrt{3}\,\sqrt{27c_1^2(x-1)^2-1}+27c_1(x-1)\right)^{\frac{1}{3}}}{6c_1(x-1)} - \frac{1}{2c_1(x-1)\left(3\sqrt{3}\,\sqrt{27c_1^2(x-1)^2-1}+27c_1(x-1)\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(3\sqrt{3}\,\sqrt{27c_1^2(x-\cdots}{3\cdots}\right.}{\cdots}\right.\right.}{c_1^2}$$

✓ Solution by Mathematica

Time used: 60.161 (sec). Leaf size: 629

```
DSolve[y'[x]==(2*y[x]-x+5)/(2*x-y[x]-4),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2(x-2) + \frac{3(x-1)}{\frac{1}{\sqrt[3]{-e^{\frac{3c_1}{4}}(x-1)^4 + 2e^{\frac{3c_1}{8}}(x-1)^2 + \sqrt{e^{\frac{3c_1}{8}}(x-1)^2\left(-1+e^{\frac{3c_1}{8}}(x-1)^2\right)^3} - 1}} - \frac{\sqrt[3]{-e^{\frac{3c_1}{4}}(x-1)^4 + 2e}}{}}$$

$$y(x) \to 2\left(x + \frac{3(x-1)}{\frac{-1-i\sqrt{3}}{\sqrt[3]{-e^{\frac{3c_1}{4}}(x-1)^4 + 2e^{\frac{3c_1}{8}}(x-1)^2 + \sqrt{e^{\frac{3c_1}{8}}(x-1)^2\left(-1+e^{\frac{3c_1}{8}}(x-1)^2\right)^3} - 1}} + \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-e^{\frac{3c_1}{4}}(x-1}}{}} - 2\right)$$

$$y(x) \to 2\left(x\right.$$

2.34 problem 32

Internal problem ID [5028]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$y' + \frac{4x + 3y + 15}{2x + y + 7} = 0$$

✓ Solution by Maple

Time used: 0.718 (sec). Leaf size: 204

```
dsolve(diff(y(x),x)=-(4*x+3*y(x)+15)/(2*x+y(x)+7),y(x), singsol=all)
```

$$y(x) = -1 - \frac{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{6} \Big(-2\left(4(x+3)^{3}c_{1}+4\sqrt{-4(x+3)^{9}c_{1}^{3}+(x+3)^{6}c_{1}^{2}}\right)^{\frac{1}{3}} - \frac{8(x+3)^{3}c_{1}}{\left(4(x+3)^{3}c_{1}+4\sqrt{-4(x+3)^{9}c_{1}^{3}+(x+3)^{6}c_{1}^{2}}\right)^{\frac{1}{3}}} + 4i\sqrt{3}\Big(\frac{\big(4(x+3)^{3}c_{1}+4\sqrt{-4(x+3)^{9}c \cdots}{2} \cdots}{64c_{1}}}{(x+3)^{2}}$$

✓ Solution by Mathematica

Time used: 60.065 (sec). Leaf size: 763

```
DSolve[y'[x]==-(4*x+3*y[x]+15)/(2*x+y[x]+7),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{}{\text{Root}\left[\#1^6\left(16x^6+288x^5+2160x^4+8640x^3+19440x^2+23328x+11664+16e^{12c_1}\right)+\#1^4\left(-24x^4-2\right.\right.} - 2x - 7$$

$$y(x) \to \frac{}{\text{Root}\left[\#1^6\left(16x^6+288x^5+2160x^4+8640x^3+19440x^2+23328x+11664+16e^{12c_1}\right)+\#1^4\left(-24x^4-2\right.\right.} - 2x - 7$$

$$y(x) \to \frac{}{\text{Root}\left[\#1^6\left(16x^6+288x^5+2160x^4+8640x^3+19440x^2+23328x+11664+16e^{12c_1}\right)+\#1^4\left(-24x^4-2\right.\right.} - 2x - 7$$

$$y(x) \to \frac{}{\text{Root}\left[\#1^6\left(16x^6+288x^5+2160x^4+8640x^3+19440x^2+23328x+11664+16e^{12c_1}\right)+\#1^4\left(-24x^4-2\right.\right.} - 2x - 7$$

$$y(x) \to \frac{}{\text{Root}\left[\#1^6\left(16x^6+288x^5+2160x^4+8640x^3+19440x^2+23328x+11664+16e^{12c_1}\right)+\#1^4\left(-24x^4-2\right.\right.} - 2x - 7$$

$$y(x) \to \frac{}{\text{Root}\left[\#1^6\left(16x^6+288x^5+2160x^4+8640x^3+19440x^2+23328x+11664+16e^{12c_1}\right)+\#1^4\left(-24x^4-2\right.\right.} - 2x - 7$$

2.35 problem 33

Internal problem ID [5029]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y' - \frac{x + 3y - 5}{x - y - 1} = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)=(x+3*y(x)-5)/(x-y(x)-1),y(x), singsol=all)
```

$$y(x) = 1 - \frac{(x-2)\left(\text{LambertW}\left(2c_1(x-2)\right)+2\right)}{\text{LambertW}\left(2c_1\left(x-2\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.986 (sec). Leaf size: 148

```
DSolve[y'[x]==(x+3*y[x]-5)/(x-y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{2^{2/3}\left(x\log\left(-\frac{y(x)+x-3}{-y(x)+x-1}\right)-(x-3)\log\left(\frac{x-2}{-y(x)+x-1}\right)-3\log\left(-\frac{y(x)+x-3}{-y(x)+x-1}\right)-y(x)\left(\log\left(\frac{x-2}{-y(x)+x-1}\right)\right.\right.}{9(y(x)+x-3)}\right.$$

2.36 problem 34

Internal problem ID [5030]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational]`

$$\boxed{y' - \frac{2(y+2)^2}{(y+x+1)^2} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=2*((y(x)+2)/(x+y(x)+1))^2,y(x), singsol=all)
```

$$y(x) = -2 - \tan\left(\operatorname{RootOf}\left(-2\_Z + \ln\left(\tan\left(\_Z\right)\right) + \ln\left(x-1\right) + c_1\right)\right)(x-1)$$

✓ Solution by Mathematica

Time used: 0.129 (sec). Leaf size: 27

```
DSolve[y'[x]==2*((y[x]+2)/(x+y[x]+1))^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{1-x}{y(x)+2}\right) + \log(y(x)+2) = c_1, y(x)\right]$$

2.37 problem 35

Internal problem ID [5031]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{2x + y + 1 - (4x + 2y - 3)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve((2*x+y(x)+1)-(4*x+2*y(x)-3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(-2\,\mathrm{e}^{-5x}\mathrm{e}^{2}\mathrm{e}^{5c_1}\right)-5x+2+5c_1} + 1 - 2x$$

✓ Solution by Mathematica

Time used: 3.211 (sec). Leaf size: 35

```
DSolve[(2*x+y[x]+1)-(4*x+2*y[x]-3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2} W\left(-e^{-5x-1+c_1}\right) - 2x + 1$$
$$y(x) \to 1 - 2x$$

2.38 problem 36

Internal problem ID [5032]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd typ`

$$x - y - 1 + (y - x + 2)\, y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve((x-y(x)-1)+(y(x)-x+2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x - 2 - \sqrt{2c_1 - 2x + 4}$$
$$y(x) = x - 2 + \sqrt{2c_1 - 2x + 4}$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 49

```
DSolve[(x-y[x]-1)+(y[x]-x+2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - i\sqrt{2x - 4 - c_1} - 2$$
$$y(x) \to x + i\sqrt{2x - 4 - c_1} - 2$$

2.39 problem 37

Internal problem ID [5033]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(x+4y)\,y'-2x-3y+5=0}$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 64

```
dsolve((x+4*y(x))*diff(y(x),x)=2*x+3*y(x)-5,y(x), singsol=all)
```

$$y(x)=-1+\frac{(x-4)\left(\operatorname{RootOf}\left(\_Z^{36}+3(x-4)^{6}c_1\_Z^{6}-2(x-4)^{6}c_1\right)^{6}-1\right)}{\operatorname{RootOf}\left(\_Z^{36}+3\,(x-4)^{6}c_1\_Z^{6}-2\,(x-4)^{6}c_1\right)^{6}}$$

✓ Solution by Mathematica

Time used: 60.042 (sec). Leaf size: 805

```
DSolve[(x+4*y[x])*y'[x]==2*x+3*y[x]-5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{4}$$
$$+ \frac{}{4\text{Root}\left[\#1^6\left(-3125x^6 + 75000x^5 - 750000x^4 + 4000000x^3 - 12000000x^2 + 19200000x - 12800000 + \right.\right.}$$

$$y(x) \to -\frac{x}{4}$$
$$+ \frac{}{4\text{Root}\left[\#1^6\left(-3125x^6 + 75000x^5 - 750000x^4 + 4000000x^3 - 12000000x^2 + 19200000x - 12800000 + \right.\right.}$$

$$y(x) \to -\frac{x}{4}$$
$$+ \frac{}{4\text{Root}\left[\#1^6\left(-3125x^6 + 75000x^5 - 750000x^4 + 4000000x^3 - 12000000x^2 + 19200000x - 12800000 + \right.\right.}$$

$$y(x) \to -\frac{x}{4}$$
$$+ \frac{}{4\text{Root}\left[\#1^6\left(-3125x^6 + 75000x^5 - 750000x^4 + 4000000x^3 - 12000000x^2 + 19200000x - 12800000 + \right.\right.}$$

$$y(x) \to -\frac{x}{4}$$
$$+ \frac{}{4\text{Root}\left[\#1^6\left(-3125x^6 + 75000x^5 - 750000x^4 + 4000000x^3 - 12000000x^2 + 19200000x - 12800000 + \right.\right.}$$

$$y(x) \to -\frac{x}{4}$$
$$+ \frac{}{4\text{Root}\left[\#1^6\left(-3125x^6 + 75000x^5 - 750000x^4 + 4000000x^3 - 12000000x^2 + 19200000x - 12800000 + \right.\right.}$$

2.40 problem 38

Internal problem ID [5034]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y + 2 - (2x + y - 4)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(y(x)+2=(2*x+y(x)-4)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{1 - 4c_1 + \sqrt{4c_1x - 12c_1 + 1}}{2c_1}$$
$$y(x) = -\frac{-1 + 4c_1 + \sqrt{4c_1x - 12c_1 + 1}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.219 (sec). Leaf size: 82

```
DSolve[y[x]+2==(2*x+y[x]-4)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{1 + 4c_1(x - 3)} - 1 + 4c_1}{2c_1}$$
$$y(x) \to \frac{\sqrt{1 + 4c_1(x - 3)} + 1 - 4c_1}{2c_1}$$
$$y(x) \to -2$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to 1 - x$$

2.41 problem 39

Internal problem ID [5035]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _dAlembert]`

$$(1+y')\ln\left(\frac{y+x}{x+3}\right)-\frac{y+x}{x+3}=0$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 27

```
dsolve((diff(y(x),x)+1)*ln((y(x)+x)/(x+3))=(y(x)+x)/(x+3),y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\mathrm{LambertW}\left(\frac{\mathrm{e}^{-1}}{(x+3)c_1}\right)+1}(x+3)-x$$

✓ Solution by Mathematica

Time used: 0.216 (sec). Leaf size: 30

```
DSolve[(y'[x]+1)*Log[(y[x]+x)/(x+3)]==(y[x]+x)/(x+3),y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[-y(x)+(y(x)+x)\log\left(\frac{y(x)+x}{x+3}\right)-x=c_1,y(x)\right]$$

2.42 problem 40

Internal problem ID [5036]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$y' - \frac{x - 2y + 5}{y - 2x - 4} = 0$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 184

```
dsolve(diff(y(x),x)=(x-2*y(x)+5)/(y(x)-2*x-4),y(x), singsol=all)
```

$$y(x) = 2 + \frac{(x+1)\left(-c_1^2 - c_1^2\left(-\frac{\left(27c_1(x+1)+3\sqrt{3}\sqrt{27(x+1)^2c_1^2-1}\right)^{\frac{1}{3}}}{6c_1(x+1)} - \frac{1}{2c_1(x+1)\left(27c_1(x+1)+3\sqrt{3}\sqrt{27(x+1)^2c_1^2-1}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(27c_1 \cdots\right.}{\cdots}\right.}{\cdots}\right.\right.}{c_1^2}$$

✓ Solution by Mathematica

Time used: 60.172 (sec). Leaf size: 629

```
DSolve[y'[x]==(x-2*y[x]+5)/(y[x]-2*x-4),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2(x+2) + \frac{3(x+1)}{\frac{1}{\sqrt[3]{-e^{\frac{3c_1}{4}}(x+1)^4 + 2e^{\frac{3c_1}{8}}(x+1)^2 + \sqrt{e^{\frac{3c_1}{8}}(x+1)^2\left(-1+e^{\frac{3c_1}{8}}(x+1)^2\right)^3} - 1}} - \frac{\sqrt[3]{-e^{\frac{3c_1}{4}}(x+1)^4 + 2e}}{}}$$

$$y(x) \to 2\left(x + \frac{3(x+1)}{\frac{-1-i\sqrt{3}}{\sqrt[3]{-e^{\frac{3c_1}{4}}(x+1)^4 + 2e^{\frac{3c_1}{8}}(x+1)^2 + \sqrt{e^{\frac{3c_1}{8}}(x+1)^2\left(-1+e^{\frac{3c_1}{8}}(x+1)^2\right)^3} - 1}} + \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-e^{\frac{3c_1}{4}}(x+1}}{}} + 2\right)$$

$$y(x) \to 2\left(x\right.$$

2.43 problem 41

Internal problem ID [5037]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$y' - \frac{3x - y + 1}{2x + y + 4} = 0$$

✓ Solution by Maple

Time used: 0.562 (sec). Leaf size: 77

```
dsolve(diff(y(x),x)=(3*x-y(x)+1)/(2*x+y(x)+4),y(x), singsol=all)
```

$$-\frac{\ln\left(-\frac{3(x+1)^2+3(x+1)(-y(x)-2)-(-y(x)-2)^2}{(x+1)^2}\right)}{2} + \frac{\sqrt{21}\,\operatorname{arctanh}\left(\frac{(2y(x)+7+3x)\sqrt{21}}{21x+21}\right)}{21} - \ln(x+1) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.132 (sec). Leaf size: 79

```
DSolve[y'[x]==(3*x-y[x]+1)/(2*x+y[x]+4),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\sqrt{21}\text{arctanh}\left(\frac{-\frac{10(x+1)}{y(x)+2(x+2)} - 1}{\sqrt{21}}\right) + 21\left(\log\left(-\frac{-3x^2 + y(x)^2 + (3x+7)y(x) + 7}{5(x+1)^2}\right) + 2\log(x+1) - 10c_1\right) = 0, y(x)\right]$$

2.44 problem Example 5

Internal problem ID [5038]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: Example 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{2y'x + (x^2y^4 + 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 67

```
dsolve(2*x*diff(y(x),x)+(x^2*y(x)^4+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{\sqrt{2\ln(x) + c_1}\, x}}$$

$$y(x) = \frac{1}{\sqrt{-\sqrt{2\ln(x) + c_1}\, x}}$$

$$y(x) = -\frac{1}{\sqrt{\sqrt{2\ln(x) + c_1}\, x}}$$

$$y(x) = -\frac{1}{\sqrt{-\sqrt{2\ln(x) + c_1}\, x}}$$

✓ Solution by Mathematica

Time used: 0.369 (sec). Leaf size: 92

```
DSolve[2*x*y'[x]+(x^2*y[x]^4+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt[4]{x^2(2\log(x)+c_1)}}$$

$$y(x) \to -\frac{i}{\sqrt[4]{x^2(2\log(x)+c_1)}}$$

$$y(x) \to \frac{i}{\sqrt[4]{x^2(2\log(x)+c_1)}}$$

$$y(x) \to \frac{1}{\sqrt[4]{x^2(2\log(x)+c_1)}}$$

$$y(x) \to 0$$

2.45 problem Example 6

Internal problem ID [5039]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: Example 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{2xy'(x-y^2)+y^3=0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 19

```
dsolve(2*x*diff(y(x),x)*(x-y(x)^2)+y(x)^3=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\frac{\mathrm{LambertW}\left(-\frac{\mathrm{e}^{c_1}}{x}\right)}{2}+\frac{c_1}{2}}$$

✓ Solution by Mathematica

Time used: 2.348 (sec). Leaf size: 60

```
DSolve[2*x*y'[x]*(x-y[x]^2)+y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\rightarrow -i\sqrt{x}\sqrt{W\left(-\frac{e^{c_1}}{x}\right)}$$
$$y(x)\rightarrow i\sqrt{x}\sqrt{W\left(-\frac{e^{c_1}}{x}\right)}$$
$$y(x)\rightarrow 0$$

2.46 problem 42

Internal problem ID [5040]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{x^3(y'-x)-y^2=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^3*(diff(y(x),x)-x)=y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{x^2(\ln(x)-c_1-1)}{\ln(x)-c_1}$$

✓ Solution by Mathematica

Time used: 0.134 (sec). Leaf size: 27

```
DSolve[x^3*(y'[x]-x)==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x^2\left(1-\frac{1}{\log(x)+c_1}\right)$$
$$y(x)\to x^2$$

2.47 problem 43

Internal problem ID [5041]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{2x^2y' - y^3 - xy = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve(2*x^2*diff(y(x),x)=y(x)^3+x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-(\ln(x) - c_1)\,x}}{\ln(x) - c_1}$$
$$y(x) = -\frac{\sqrt{-(\ln(x) - c_1)\,x}}{\ln(x) - c_1}$$

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 49

```
DSolve[2*x^2*y'[x]==y[x]^3+x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x}}{\sqrt{-\log(x) + c_1}}$$
$$y(x) \to \frac{\sqrt{x}}{\sqrt{-\log(x) + c_1}}$$
$$y(x) \to 0$$

2.48 problem 44

Internal problem ID [5042]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y + x(2xy + 1)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 18

```
dsolve(y(x)+x*(2*x*y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{2\,\text{LambertW}\left(\frac{c_1}{2x}\right) x}$$

✓ Solution by Mathematica

Time used: 60.49 (sec). Leaf size: 36

```
DSolve[y[x]+x*(2*x*y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2xW\left(\frac{e^{\frac{1}{2}\left(-2-9\sqrt[3]{-2}c_1\right)}}{x}\right)}$$

2.49 problem 45

Internal problem ID [5043]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Chini]`

$$\boxed{2y' + x - 4\sqrt{y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 111

```
dsolve(2*diff(y(x),x)+x=4*sqrt(y(x)),y(x), singsol=all)
```

$$-\frac{4i\sqrt{-\frac{y(x)}{x^2}}\,x^2 - 2i\arctan\left(2\sqrt{-\frac{y(x)}{x^2}}\right)x^2 + 8i\arctan\left(2\sqrt{-\frac{y(x)}{x^2}}\right)y(x) + \ln\left(\frac{x^2-4y(x)}{x^2}\right)x^2 - 4\ln\left(\frac{x^2-4y(x)}{x^2}\right.}{x^2 - 4y\left(x\right)}$$
$$-2\ln\left(x\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 49

```
DSolve[2*y'[x]+x==4*Sqrt[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[4\left(\frac{4}{4\sqrt{\frac{y(x)}{x^2}}+2} + 2\log\left(4\sqrt{\frac{y(x)}{x^2}}+2\right)\right) = -8\log(x) + c_1, y(x)\right]$$

2.50 problem 46

Internal problem ID [5044]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Riccati, _special]]`

$$\boxed{y' - y^2 + \frac{2}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=y(x)^2-2/x^2,y(x), singsol=all)
```

$$y(x) = \frac{2x^3 + c_1}{(-x^3 + c_1)\, x}$$

✓ Solution by Mathematica

Time used: 0.139 (sec). Leaf size: 29

```
DSolve[y'[x]==y[x]^2-2/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x} - \frac{3x^2}{x^3 + c_1}$$
$$y(x) \to \frac{1}{x}$$

2.51 problem 47

Internal problem ID [5045]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{2y'x + y - y^2\sqrt{x - x^2y^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve(2*x*diff(y(x),x)+y(x)=y(x)^2*sqrt(x-x^2*y(x)^2),y(x), singsol=all)
```

$$-\frac{-1 + xy(x)^2}{y(x)\sqrt{x - y(x)^2 x^2}} + \frac{\ln(x)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.751 (sec). Leaf size: 62

```
DSolve[2*x*y'[x]+y[x]==y[x]^2*Sqrt[x-x^2*y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{\sqrt{x(\log^2(x) - 2c_1\log(x) + 4 + c_1^2)}}$$

$$y(x) \to \frac{2}{\sqrt{x(\log^2(x) - 2c_1\log(x) + 4 + c_1^2)}}$$

$$y(x) \to 0$$

2.52 problem 48

Internal problem ID [5046]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{\frac{2xyy'}{3}-\sqrt{x^6-y^4}-y^2=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 100

```
dsolve(2/3*x*y(x)*diff(y(x),x)=sqrt(x^6-y(x)^4)+y(x)^2,y(x), singsol=all)
```

$$\int_{\_b}^{x}-\frac{\sqrt{\_a^6-y\left(x\right)^4}+y(x)^2}{\sqrt{\_a^6-y\left(x\right)^4}\,\_a}d\_a+\int^{y(x)}\frac{2\_f\left(3\sqrt{x^6-\_f^4}\left(\int_{\_b}^{x}\frac{\_a^5}{\left(\_a^6-\_f^4\right)^{\frac{3}{2}}}d\_a\right)+1\right)}{3\sqrt{x^6-\_f^4}}d\_f+c_1=0$$

✓ Solution by Mathematica

Time used: 5.398 (sec). Leaf size: 128

```
DSolve[2/3*x*y[x]*y'[x]==Sqrt[x^6-y[x]^4]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^{3/2}}{\sqrt[4]{\sec^2\left(-\frac{\log(x^6)}{2}+3c_1\right)}}$$

$$y(x) \to -\frac{ix^{3/2}}{\sqrt[4]{\sec^2\left(-\frac{\log(x^6)}{2}+3c_1\right)}}$$

$$y(x) \to \frac{ix^{3/2}}{\sqrt[4]{\sec^2\left(-\frac{\log(x^6)}{2}+3c_1\right)}}$$

$$y(x) \to \frac{x^{3/2}}{\sqrt[4]{\sec^2\left(-\frac{\log(x^6)}{2}+3c_1\right)}}$$

2.53 problem 49

Internal problem ID [5047]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{2y + (x^2y + 1)\, xy' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 16

```
dsolve(2*y(x)+(x^2*y(x)+1)*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\text{LambertW}\left(\frac{c_1}{x^2}\right) x^2}$$

✓ Solution by Mathematica

Time used: 60.366 (sec). Leaf size: 33

```
DSolve[2*y[x]+(x^2*y[x]+1)*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x^2 W\left(\frac{e^{\frac{1}{2}\left(-2-9\sqrt[3]{-2}c_1\right)}}{x^2}\right)}$$

2.54 problem 50

Internal problem ID [5048]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y(xy+1)+(1-xy)\,xy'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(y(x)*(1+x*y(x))+(1-x*y(x))*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{1}{\text{LambertW}\left(-\frac{c_1}{x^2}\right)x}$$

✓ Solution by Mathematica

Time used: 5.495 (sec). Leaf size: 35

```
DSolve[y[x]*(1+x*y[x])+(1-x*y[x])*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{xW\left(\frac{e^{-1+\frac{9c_1}{2^{2/3}}}}{x^2}\right)}$$

$$y(x)\to 0$$

2.55 problem 51

Internal problem ID [5049]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$y(x^2y^2+1)+(x^2y^2-1)\,xy'=0$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 23

```
dsolve(y(x)*(x^2*y(x)^2+1)+(x^2*y(x)^2-1)*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\frac{\mathrm{LambertW}\left(-x^4\mathrm{e}^{-4c_1}\right)}{2}-2c_1}x$$

✓ Solution by Mathematica

Time used: 4.883 (sec). Leaf size: 60

```
DSolve[y[x]*(x^2*y[x]^2+1)+(x^2*y[x]^2-1)*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{i\sqrt{W\left(-e^{-2c_1}x^4\right)}}{x}$$
$$y(x)\to\frac{i\sqrt{W\left(-e^{-2c_1}x^4\right)}}{x}$$
$$y(x)\to 0$$

2.56 problem 52

Internal problem ID [5050]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{(x^2 - y^4)\, y' - xy = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 97

```
dsolve((x^2-y(x)^4)*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2c_1 - 2\sqrt{c_1^2 - 4x^2}}}{2}$$
$$y(x) = \frac{\sqrt{2c_1 - 2\sqrt{c_1^2 - 4x^2}}}{2}$$
$$y(x) = -\frac{\sqrt{2c_1 + 2\sqrt{c_1^2 - 4x^2}}}{2}$$
$$y(x) = \frac{\sqrt{2c_1 + 2\sqrt{c_1^2 - 4x^2}}}{2}$$

✓ Solution by Mathematica

Time used: 2.14 (sec). Leaf size: 122

```
DSolve[(x^2-y[x]^4)*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-\sqrt{-x^2 + c_1{}^2} - c_1}$$
$$y(x) \to \sqrt{-\sqrt{-x^2 + c_1{}^2} - c_1}$$
$$y(x) \to -\sqrt{\sqrt{-x^2 + c_1{}^2} - c_1}$$
$$y(x) \to \sqrt{\sqrt{-x^2 + c_1{}^2} - c_1}$$
$$y(x) \to 0$$

2.57 problem 53

Internal problem ID [5051]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$y\left(1+\sqrt{x^2y^4-1}\right)+2y'x=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(y(x)*(1+sqrt(x^2*y(x)^4-1))+2*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\operatorname{RootOf}\left(-\ln(x)+c_1-2\left(\int^{\_Z}\frac{1}{\_a\sqrt{\_a^4-1}}d\_a\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.152 (sec). Leaf size: 40

```
DSolve[y[x]*(1+Sqrt[x^2*y[x]^4-1])+2*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\sqrt{x^2y(x)^4-1}\right)+\frac{1}{2}\log\left(x^2y(x)^4\right)-2\log(y(x))=c_1,y(x)\right]$$

3 Chapter 1. First order differential equations. Section 1.3. Exact equations problems. page 24

3.1 problem 1

Internal problem ID [5052]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.3. Exact equations problems. page 24
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$x(2-9xy^2)+y(4y^2-6x^3)\,y'=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 125

```
dsolve(x*(2-9*x*y(x)^2)+y(x)*(4*y(x)^2-6*x^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{\sqrt{6x^3-2\sqrt{9x^6-4x^2-4c_1}}}{2}$$
$$y(x)=\frac{\sqrt{6x^3-2\sqrt{9x^6-4x^2-4c_1}}}{2}$$
$$y(x)=-\frac{\sqrt{6x^3+2\sqrt{9x^6-4x^2-4c_1}}}{2}$$
$$y(x)=\frac{\sqrt{6x^3+2\sqrt{9x^6-4x^2-4c_1}}}{2}$$

✓ Solution by Mathematica

Time used: 5.766 (sec). Leaf size: 163

```
DSolve[x*(2-9*x*y[x]^2)+y[x]*(4*y[x]^2-6*x^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\frac{\sqrt{3x^3 - \sqrt{9x^6 - 4x^2 + 4c_1}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{3x^3 - \sqrt{9x^6 - 4x^2 + 4c_1}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{3x^3 + \sqrt{9x^6 - 4x^2 + 4c_1}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{3x^3 + \sqrt{9x^6 - 4x^2 + 4c_1}}}{\sqrt{2}}$$

3.2 problem 2

Internal problem ID [5053]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.3. Exact equations problems. page 24
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{\frac{y}{x}+\left(y^{3}+\ln \left(x \right)\right) y^{\prime}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(y(x)/x+(y(x)^3+ln(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) \ln \left(x \right)+\frac{y(x)^{4}}{4}+c_{1} = 0$$

✓ Solution by Mathematica

Time used: 60.193 (sec). Leaf size: 1012

```
DSolve[y[x]/x+(y[x]^3+Log[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\frac{\sqrt[3]{3}\left(9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}\right)^{2/3}-4\ 3^{2/3}c_1}{\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}}}}{\sqrt{6}} - \frac{1}{2}\sqrt{\frac{8c_1}{\sqrt[3]{3}\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}} - \frac{2\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}}{3^{2/3}} - \frac{\cdots}{\sqrt{\frac{\sqrt[3]{3}\left(9\log^2(x)+\cdots\right.}{\sqrt[3]{9\log^2(x)\cdots}}}}\cdots}$$

$$y(x) \to \frac{1}{2}\left(\sqrt{\frac{2\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}}{3^{2/3}} - \frac{8c_1}{\sqrt[3]{3}\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}}}\right.$$

$$+\sqrt{\frac{8c_1}{\sqrt[3]{3}\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}} - \frac{2\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}}{3^{2/3}} - \frac{4\cdots}{\sqrt{\frac{\sqrt[3]{3}\left(9\log^2(x)+\sqrt{\cdots}\right.}{\sqrt[3]{9\log^2(x)\cdots}}}}\cdots}$$

$$y(x) \to -\frac{\sqrt{\frac{\sqrt[3]{3}\left(9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}\right)^{2/3}-4\ 3^{2/3}c_1}{\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}}}}{\sqrt{6}} - \frac{1}{2}\sqrt{\frac{8c_1}{\sqrt[3]{3}\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}} - \frac{2\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}}{3^{2/3}} + \frac{\cdots}{\sqrt{\frac{\sqrt[3]{3}\left(9\log^2(x)+\cdots\right.}{\sqrt[3]{9\log^2(x)\cdots}}}}\cdots}$$

3.3 problem 3

Internal problem ID [5054]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.3. Exact equations problems. page 24
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2x + 3 + (2y - 2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve((2*x+3)+(2*y(x)-2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 1 - \sqrt{-x^2 - c_1 - 3x + 1}$$
$$y(x) = 1 + \sqrt{-x^2 - c_1 - 3x + 1}$$

✓ Solution by Mathematica

Time used: 0.118 (sec). Leaf size: 47

```
DSolve[(2*x+3)+(2*y[x]-2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 - \sqrt{-x(x+3) + 1 + 2c_1}$$
$$y(x) \to 1 + \sqrt{-x(x+3) + 1 + 2c_1}$$

3.4 problem 4

Internal problem ID [5055]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.3. Exact equations problems. page 24
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{2x + 4y + (2x - 2y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 56

```
dsolve((2*x+4*y(x))+(2*x-2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$-\frac{\ln\left(-\frac{x^2+3y(x)x-y(x)^2}{x^2}\right)}{2} - \frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{(3x-2y(x))\sqrt{13}}{13x}\right)}{13} - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 47

```
DSolve[(2*x+3)+(2*y[x]-2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 - \sqrt{-x(x+3)+1+2c_1}$$
$$y(x) \to 1 + \sqrt{-x(x+3)+1+2c_1}$$

4 Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95

4.1 problem 49

Internal problem ID [5056]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\left(\sqrt{2}-1\right)x} + c_2 \mathrm{e}^{-\left(1+\sqrt{2}\right)x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 34

```
DSolve[y''[x]+2*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\left(\left(1+\sqrt{2}\right)x\right)}\left(c_2 e^{2\sqrt{2}x} + c_1\right)$$

4.2 problem 50

Internal problem ID [5057]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + \frac{y'}{x} - \frac{y}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)-1/x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + xc_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 16

```
DSolve[y''[x]+1/x*y'[x]-1/x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x} + c_2x$$

4.3 problem 51

Internal problem ID [5058]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 51.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$\boxed{(x^2+1)\, y''+y'x+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sin(\operatorname{arcsinh}(x))+c_2\cos(\operatorname{arcsinh}(x))$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 18

```
DSolve[(x^2+1)*y''[x]+x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cos(\operatorname{arcsinh}(x))+c_2\sin(\operatorname{arcsinh}(x))$$

4.4 problem 52

Internal problem ID [5059]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 52.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' \cot(x) + y \cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.625 (sec). Leaf size: 65

```
dsolve(diff(y(x),x$2)-cot(x)*diff(y(x),x)+cos(x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1(\cos(x)+1)\,\mathrm{HeunC}\left(0,1,-1,-2,\frac{3}{2},\frac{\cos(x)}{2}+\frac{1}{2}\right) + c_2(\cos(x)+1)\,\mathrm{HeunC}\left(0,1,-1,-2,\frac{3}{2},\frac{\cos(x)}{2}+\frac{1}{2}\right)\left(\int^{\cos(x)} \frac{1}{(\_a+1)^2\,\mathrm{HeunC}\left(0,1,-1,-2,\frac{3}{2},\frac{\_a}{2}+\frac{1}{2}\right)^2} d\_a\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-Cot[x]*y'[x]+Cos[x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.5 problem 53

Internal problem ID [5060]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 53.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + \frac{y'}{x} + x^2 y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)+x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(0, \frac{x^2}{2}\right) + c_2 \operatorname{BesselY}\left(0, \frac{x^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 31

```
DSolve[y''[x]+1/x*y'[x]+x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(0, \frac{x^2}{2}\right) + 2c_2 Y_0\left(\frac{x^2}{2}\right)$$

4.6 problem 54

Internal problem ID [5061]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 54.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-x^2+1)\,y''+2x(-x^2+1)\,y'-2y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(x^2*(1-x^2)*diff(y(x),x$2)+2*x*(1-x^2)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1(x^2-1)}{x^2}+\frac{c_2(\ln(x-1)\,x^2-\ln(x+1)\,x^2-\ln(x-1)+\ln(x+1)-2x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 35

```
DSolve[x^2*(1-x^2)*y''[x]+2*x*(1-x^2)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to\frac{c_2((x^2-1)\operatorname{arctanh}(x)+x)-2c_1(x^2-1)}{2x^2}$$

4.7 problem 55

Internal problem ID [5062]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 55.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(-x^2+1)\,y''-y'x+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1x+c_2\sqrt{x-1}\,\sqrt{x+1}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 93

```
DSolve[(1-x^2)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cosh\left(\frac{2\sqrt{1-x^2}\cot^{-1}\left(\frac{x+1}{\sqrt{1-x^2}}\right)}{\sqrt{x^2-1}}\right)-ic_2\sinh\left(\frac{2\sqrt{1-x^2}\cot^{-1}\left(\frac{x+1}{\sqrt{1-x^2}}\right)}{\sqrt{x^2-1}}\right)$$

4.8 problem 56

Internal problem ID [5063]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 56.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - 2xy'' + 4x^2y' + 8yx^3 = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$3)-2*x*diff(y(x),x$2)+4*x^2*diff(y(x),x)+8*x^3*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'''[x]-2*x*y''[x]+4*x^2*y'[x]+8*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.9 problem 57

Internal problem ID [5064]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 57.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + x(1-x)\, y' + e^x y = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+x*(1-x)*diff(y(x),x)+exp(x)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+x*(1-x)*y'[x]+Exp[x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.10 problem 58

Internal problem ID [5065]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 58.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 2y'x + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{\sqrt{15}\,\ln(x)}{2}\right)}{\sqrt{x}} + \frac{c_2 \cos\left(\frac{\sqrt{15}\,\ln(x)}{2}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]+2*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \cos\left(\frac{1}{2}\sqrt{15}\log(x)\right) + c_1 \sin\left(\frac{1}{2}\sqrt{15}\log(x)\right)}{\sqrt{x}}$$

4.11 problem 59

Internal problem ID [5066]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 59.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{x^4 y'''' - x^2 y'' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(x^4*diff(y(x),x$4)-x^2*diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \sum_{\_a=1}^{4} x^{\mathrm{RootOf}\left(\_Z^4-6\_Z^3+10\_Z^2-5\_Z+1,\mathrm{index}=\_a\right)} \_C_{\_a}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 130

```
DSolve[x^4*y''''[x]-x^2*y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_4 x^{\mathrm{Root}\left[\#1^4-6\#1^3+10\#1^2-5\#1+1\&,4\right]} + c_3 x^{\mathrm{Root}\left[\#1^4-6\#1^3+10\#1^2-5\#1+1\&,3\right]} + c_1 x^{\mathrm{Root}\left[\#1^4-6\#1^3+10\#1^2-5\#1+1\&,1\right]} + c_2 x^{\mathrm{Root}\left[\#1^4-6\#1^3+10\#1^2-5\#1+1\&,2\right]}$$

4.12 problem 60

Internal problem ID [5067]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 60.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$\boxed{(x^2+1)\, y''+y'x+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+x^2)*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sin(\operatorname{arcsinh}(x))+c_2\cos(\operatorname{arcsinh}(x))$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 18

```
DSolve[(1+x^2)*y''[x]+x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cos(\operatorname{arcsinh}(x))+c_2\sin(\operatorname{arcsinh}(x))$$

5 Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104

5.1 problem 1

Internal problem ID [5068]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y'x + y - 2x\,\mathrm{e}^{x} + 1 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)=2*x*exp(x)-1,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x^2}{2}} \operatorname{erf}\left(\frac{i\sqrt{2}\,x}{2}\right) c_1 + \mathrm{e}^{-\frac{x^2}{2}} c_2 + \left(2i\sqrt{\pi}\,\mathrm{e}^{-\frac{1}{2}}\sqrt{2}\operatorname{erf}\left(\frac{i\sqrt{2}\,x}{2} + \frac{i\sqrt{2}}{2}\right) + 2\,\mathrm{e}^{x+\frac{1}{2}x^2} - \mathrm{e}^{\frac{x^2}{2}}\right)\mathrm{e}^{-\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.12 (sec). Leaf size: 53

```
DSolve[y''[x]+x*y'[x]+y[x]==2*x*Exp[x]-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}}\left(\int_1^x e^{\frac{K[1]^2}{2}}\left(c_1 + 2e^{K[1]}(K[1]-1) - K[1]\right)dK[1] + c_2\right)$$

5.2 problem 2

Internal problem ID [5069]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + y'x - y - x^2 - 2x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=x^2+2*x,y(x), singsol=all)
```

$$y(x) = \left(-\frac{e^{-x}}{x} + \mathrm{Ei}_1(x)\right) x c_2 + c_1 x + x^2$$

✓ Solution by Mathematica

Time used: 0.134 (sec). Leaf size: 29

```
DSolve[x*y''[x]+x*y'[x]-y[x]==x^2+2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(-c_2 \,\mathrm{ExpIntegralEi}(-x) + x + c_1) - c_2 e^{-x}$$

5.3 problem 3

Internal problem ID [5070]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$x^2y'' + y'x - y - x^2 - 2x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=x^2+2*x,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + xc_2 + \frac{(x + 3\ln(x))\,x}{3}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 31

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==x^2+2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{3} + x\log(x) + \left(-\frac{1}{2} + c_2\right)x + \frac{c_1}{x}$$

5.4 problem 4

Internal problem ID [5071]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3y'' + y'x - y - \cos\left(\frac{1}{x}\right) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^3*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=cos(1/x),y(x), singsol=all)
```

$$y(x) = e^{\frac{1}{x}}xc_2 + c_1x - \frac{x\left(\cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right)\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 32

```
DSolve[x^3*y''[x]+x*y'[x]-y[x]==Cos[1/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}x\left(\sin\left(\frac{1}{x}\right) + \cos\left(\frac{1}{x}\right) - 2\left(c_1e^{\frac{1}{x}} + c_2\right)\right)$$

5.5 problem 5

Internal problem ID [5072]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x(1+x)\,y'' + (2+x)\,y' - y - x - \frac{1}{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve(x*(1+x)*diff(y(x),x$2)+(x+2)*diff(y(x),x)-y(x)=x+1/x,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + \frac{(x+1)^2 c_2}{x} + \frac{2\ln(x)\,x^2 + 4\ln(x)\,x + 6x + 5}{4x}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 37

```
DSolve[x*(1+x)*y''[x]+(x+2)*y'[x]-y[x]==x+1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(x+2)\log(x) + \frac{1+c_1}{x} + \frac{1}{4}(-1+2c_2)x + 1 + c_2$$

5.6 problem 6

Internal problem ID [5073]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + (x-2)\,y' - y - x^2 + 1 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(2*x*diff(y(x),x$2)+(x-2)*diff(y(x),x)-y(x)=x^2-1,y(x), singsol=all)
```

$$y(x) = (x-2)\,c_2 + c_1 \mathrm{e}^{-\frac{x}{2}} + x^2 + 1$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 30

```
DSolve[2*x*y''[x]+(x-2)*y'[x]-y[x]==x^2-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 - 4x + c_1 e^{-x/2} + 2c_2(x-2) + 9$$

5.7 problem 7

Internal problem ID [5074]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2(1+x)\,y''+x(4x+3)\,y'-y-x-\frac{1}{x}=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 644

```
dsolve(x^2*(x+1)*diff(y(x),x$2)+x*(4*x+3)*diff(y(x),x)-y(x)=x+1/x,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= x^{-1-\sqrt{2}}\operatorname{hypergeom}\left(\left[2-\sqrt{2},-1-\sqrt{2}\right],\left[1-2\sqrt{2}\right],-x\right)c_2 \\ &+ x^{\sqrt{2}-1}\operatorname{hypergeom}\left(\left[\sqrt{2}-1,2+\sqrt{2}\right],\left[1+2\sqrt{2}\right],-x\right)c_1 \\ &+ \frac{3\sqrt{2}\left(x^{-\sqrt{2}}\operatorname{hypergeom}\left(\left[2-\sqrt{2},-1-\sqrt{2}\right],\left[1-2\sqrt{2}\right],-x\right)\left(\sqrt{2}-\frac{5}{3}\right)\left(\int \frac{}{\left(-7\sqrt{2}\operatorname{hypergeom}\left(\left[\sqrt{2}-1,\sqrt{2}-1\right]\right.\right.}\right.\right.\right.}{} \end{aligned}$$

✓ Solution by Mathematica

Time used: 4.099 (sec). Leaf size: 568

```
DSolve[x^2*(x+1)*y''[x]+x*(4*x+3)*y'[x]-y[x]==x+1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{\sqrt{2}-1}\,\text{Hypergeometric2F1}\left(-1+\sqrt{2},2+\sqrt{2},1+2\sqrt{2},\right.$$

$$\left.-x\right)\left(\int_1^x \frac{}{(K[2]+1)\left(\left(4+\sqrt{2}\right)\text{Hypergeometric2F1}\left(-\sqrt{2},3-\sqrt{2},2-2\sqrt{2},-K[2]\right)\text{Hypergeometric2}\right.}\right.$$

$$\left.+c_2\right)+x^{-1-\sqrt{2}}\,\text{Hypergeometric2F1}\left(-1-\sqrt{2},2-\sqrt{2},1-2\sqrt{2},-x\right)\left(\int_1^x\right.$$

$$-\frac{}{(K[1]+1)\left(\left(4+\sqrt{2}\right)\text{Hypergeometric2F1}\left(-\sqrt{2},3-\sqrt{2},2-2\sqrt{2},-K[1]\right)\text{Hypergeometric2F1}\left(-1+\right.\right.}$$

$$\left.+c_1\right)$$

5.8 problem 8

Internal problem ID [5075]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(\ln(x)-1)\,y''-y'x+y-x(-\ln(x)+1)^2=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 26

```
dsolve(x^2*(ln(x)-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=x*(1-ln(x))^2,y(x), singsol=all)
```

$$y(x)=\left(\frac{\ln(x)^2}{2}-\frac{c_1\ln(x)}{x}-\ln(x)+c_2\right)x$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 27

```
DSolve[x^2*(Log[x]-1)*y''[x]-x*y'[x]+y[x]==x*(1-Log[x])^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{1}{2}x\log^2(x)+c_1x-(x+c_2)\log(x)$$

5.9 problem 9

Internal problem ID [5076]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' + 2y' + xy - \sec(x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=sec(x),y(x), singsol=all)
```

$$y(x) = \frac{\sin(x) c_2}{x} + \frac{\cos(x) c_1}{x} + \frac{-\ln(\sec(x))\cos(x) + \sin(x) x}{x}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 59

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(\log(1 + e^{2ix}) + e^{2ix}(\log(1 + e^{-2ix}) - ic_2) + 2c_1)}{2x}$$

5.10 problem 10

Internal problem ID [5077]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(-x^2+1)\,y''-y'x+\frac{y}{4}+\frac{x^2}{2}-\frac{1}{2}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+1/4*y(x)=1/2*(1-x^2),y(x), singsol=all)
```

$$y(x)=\frac{c_2}{\sqrt{x+\sqrt{x^2-1}}}+\sqrt{x+\sqrt{x^2-1}}\,c_1+\frac{2x^2}{15}+\frac{14}{15}$$

✓ Solution by Mathematica

Time used: 9.129 (sec). Leaf size: 216

```
DSolve[(1-x^2)*y''[x]-x*y'[x]+1/4*y[x]==1/2*(1-x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}y(x)&\\ \to\cosh\left(\frac{\sqrt{1-x^2}\cot^{-1}\left(\frac{x+1}{\sqrt{1-x^2}}\right)}{\sqrt{x^2-1}}\right)&\left(\int_1^x\sqrt{K[1]^2-1}\sinh\left(\frac{\cot^{-1}\left(\frac{K[1]+1}{\sqrt{1-K[1]^2}}\right)\sqrt{1-K[1]^2}}{\sqrt{K[1]^2-1}}\right)dK[1]\right.\\ &\left.+c_1\right)-i\sinh\left(\frac{\sqrt{1-x^2}\cot^{-1}\left(\frac{x+1}{\sqrt{1-x^2}}\right)}{\sqrt{x^2-1}}\right)\left(\int_1^x\right.\\ &\left.-i\cosh\left(\frac{\cot^{-1}\left(\frac{K[2]+1}{\sqrt{1-K[2]^2}}\right)\sqrt{1-K[2]^2}}{\sqrt{K[2]^2-1}}\right)\sqrt{K[2]^2-1}dK[2]+c_2\right)\end{aligned}$$

5.11 problem 11

Internal problem ID [5078]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(\cos(x)+\sin(x))\,y''-2y'\cos(x)+(-\sin(x)+\cos(x))\,y-(\cos(x)+\sin(x))^2\,e^{2x}=0$$

✓ Solution by Maple

Time used: 0.359 (sec). Leaf size: 323

```
dsolve((cos(x)+sin(x))*diff(y(x),x$2)-2*cos(x)*diff(y(x),x)+(cos(x)-sin(x))*y(x)=(cos(x)+sin(
```

$$y(x)=c_2\cos(x)+\cos(x)\left(\int -e^{\int \frac{(-3\cot(x)-1)\cos(x)+2\csc(x)+2\sec(x)}{\cos(x)+\sin(x)}dx}\sin(x)\,dx\right)c_1$$
$$+\cos(x)\left(\left(\left(\int e^{2x+3\left(\int \frac{\cos(x)\cot(x)}{\cos(x)+\sin(x)}dx\right)-2\left(\int \frac{\sec(x)}{\cos(x)+\sin(x)}dx\right)-2\left(\int \frac{\csc(x)}{\cos(x)+\sin(x)}dx\right)+\int \frac{\cos(x)}{\cos(x)+\sin(x)}dx}(\csc(x)+\sec(x))\,dx\right)\right.\right.$$
$$-\left(\int e^{2x+3\left(\int \frac{\cos(x)\cot(x)}{\cos(x)+\sin(x)}dx\right)-2\left(\int \frac{\sec(x)}{\cos(x)+\sin(x)}dx\right)-2\left(\int \frac{\csc(x)}{\cos(x)+\sin(x)}dx\right)+\int \frac{\cos(x)}{\cos(x)+\sin(x)}dx}(\csc(x)+\sec(x))\left(\int e^{-3\left(\int \frac{\cos}{\cos}\right.}\right.\right.$$

✓ Solution by Mathematica

Time used: 2.216 (sec). Leaf size: 337

```
DSolve[(Cos[x]+Sin[x])*y''[x]-2*Cos[x]*y'[x]+(Cos[x]-Sin[x])*y[x]==(Cos[x]+Sin[x])^2*Exp[2*x]
```

$$y(x) \to \frac{(e^{ix})^{-1-2i}\left(e^{6ix}\sqrt{1+e^{-4ix}}+e^{8ix}\sqrt{1+e^{-4ix}}+e^{10ix}\sqrt{1+e^{-4ix}}+(1+i)\sqrt{-e^{4ix}(1+e^{4ix})}\sqrt{-(1+e^{4ix})^2}\right.}{}$$

5.12 problem 12

Internal problem ID [5079]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(-\sin(x)+\cos(x))\,y''-2y'\sin(x)+(\cos(x)+\sin(x))\,y-(-\sin(x)+\cos(x))^2=0$$

✓ Solution by Maple

Time used: 62.813 (sec). Leaf size: 7363

```
dsolve((cos(x)-sin(x))*diff(y(x),x$2)-2*sin(x)*diff(y(x),x)+(cos(x)+sin(x))*y(x)=(cos(x)-sin(
```

Expression too large to display

✓ Solution by Mathematica

Time used: 8.61 (sec). Leaf size: 7186

```
DSolve[(Cos[x]-Sin[x])*y''[x]-2*Sin[x]*y'[x]+(Cos[x]+Sin[x])*y[x]==(Cos[x]-Sin[x])^2,y[x],x,I
```

Too large to display