A Solution Manual For

# Own collection of miscellaneous problems

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# Contents

# 1 section 1.0

## 1.1 problem 1

Internal problem ID [6291]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 1.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_separable]

$$
y' - \frac{\cos(y)\sec(x)}{x} = 0
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 73

```
dsolve(diff(y(x),x) = cos(y(x))*sec(x)/x,y(x), singsol=all)
```

$$
y(x) = \arctan\left(\frac{e^{2\left(\int \frac{\sec(x)}{x}dx\right)}c_1^2 - 1}{e^{2\left(\int \frac{\sec(x)}{x}dx\right)}c_1^2 + 1}, \frac{2e^{\int \frac{\sec(x)}{x}dx}c_1}{e^{2\left(\int \frac{\sec(x)}{x}dx\right)}c_1^2 + 1}\right)
$$

✓ Solution by Mathematica

Time used: 4.625 (sec). Leaf size: 49

```
DSolve[y'[x]== Cos[y[x]]*Sec[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to 2\arctan\left(\tanh\left(\frac{1}{2}\left(\int_1^x \frac{\sec(K[1])}{K[1]}dK[1] + c_1\right)\right)\right)
$$

$$
y(x) \to -\frac{\pi}{2}
$$

$$
y(x) \to \frac{\pi}{2}
$$

## 1.2 problem 2

Internal problem ID [6292]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 2.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_separable]

$$y' - x(\cos(y) + y) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(diff(y(x),x) = x*(cos(y(x))+y(x)),y(x), singsol=all)
```

$$\frac{x^2}{2} - \left(\int^{y(x)} \frac{1}{\cos(\_a) + \_a} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.619 (sec). Leaf size: 33

```
DSolve[y'[x] == x*(Cos[y[x]]+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{\cos(K[1]) + K[1]} dK[1] \&\right]\left[\frac{x^2}{2} + c_1\right]$$

## 1.3 problem 3

Internal problem ID [6293]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 3.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_separable]

$$y' - \frac{\sec(x)(\sin(y) + y)}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x) = sec(x)*(sin(y(x))+y(x))/x,y(x), singsol=all)
```

$$\int \frac{\sec(x)}{x} dx - \left( \int^{y(x)} \frac{1}{\sin(\_a) + \_a} d\_a \right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.178 (sec). Leaf size: 41

```
DSolve[y'[x]== Sec[x]*(Sin[y[x]]+y[x])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction} \left[ \int_1^{\#1} \frac{1}{K[1] + \sin(K[1])} dK[1] \& \right] \left[ \int_1^x \frac{\sec(K[2])}{K[2]} dK[2] + c_1 \right]$$

## 1.4 problem 4

Internal problem ID [6294]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 4.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_separable]

$$y' - \left(5 + \frac{\sec(x)}{x}\right)(\sin(y) + y) = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) = (5+sec(x)/x)*(sin(y(x))+y(x)),y(x), singsol=all)
```

$$\int \frac{5x + \sec(x)}{x} dx - \left(\int^{y(x)} \frac{1}{\sin(\_a) + \_a} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 20.728 (sec). Leaf size: 168

```
DSolve[y'[x] == (5+Sec[x]/x)*(Sin[y[x]]+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x \left(-\frac{2\sec(K[1])}{K[1]}\right.\right.$$
$$\left. - \frac{5(-\sec(K[1])\sin(K[1] - y(x)) + \sec(K[1])\sin(K[1] + y(x)) + 2y(x))}{\sin(y(x)) + y(x)}\right) dK[1]$$
$$+ \int_1^{y(x)} \left(\frac{2}{K[2] + \sin(K[2])}\right.$$
$$\left. - \int_1^x \left(\frac{5(\cos(K[2]) + 1)(2K[2] - \sec(K[1])\sin(K[1] - K[2]) + \sec(K[1])\sin(K[1] + K[2]))}{(K[2] + \sin(K[2]))^2} - \frac{5(\cos(K[1] - \right.$$

## 1.5 problem 5

Internal problem ID [6295]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 5.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$y' - y - 1 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x) = y(x)+1,y(x), singsol=all)
```

$$y(x) = -1 + e^x c_1$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 18

```
DSolve[y'[x] == y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -1 + c_1 e^x$$
$$y(x) \rightarrow -1$$

## 1.6 problem 6

Internal problem ID [6296]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 6.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$y' - 1 - x = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) = 1+x,y(x), singsol=all)
```

$$y(x) = \frac{1}{2}x^2 + x + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 16

```
DSolve[y'[x]== 1+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^2}{2} + x + c_1$$

## 1.7 problem 7

Internal problem ID [6297]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 7.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$y' - x = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = x,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 15

```
DSolve[y'[x] == x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + c_1$$

## 1.8 problem 8

Internal problem ID [6298]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 8.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$y' - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x) = e^x c_1$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 16

```
DSolve[y'[x] == y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to 0$$

## 1.9 problem 9

Internal problem ID [6299]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 9.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$\boxed{y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

## 1.10 problem 10

Internal problem ID [6300]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 10.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$y' - 1 - \frac{\sec(x)}{x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) = 1+sec(x)/x,y(x), singsol=all)
```

$$y(x) = \int \frac{\sec(x)}{x} dx + x + c_1$$

✓ Solution by Mathematica

Time used: 0.747 (sec). Leaf size: 25

```
DSolve[y'[x] == 1+Sec[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \int_1^x \left( \frac{\sec(K[1])}{K[1]} + 1 \right) dK[1] + c_1$$

## 1.11 problem 11

Internal problem ID [6301]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 11.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_linear]

$$y' - x - \frac{\sec(x)\,y}{x} = 0$$

✔ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x) = x+sec(x)*y(x)/x,y(x), singsol=all)
```

$$y(x) = \left( \int x\,\mathrm{e}^{-\left(\int \frac{\sec(x)}{x} dx\right)} dx + c_1 \right) \mathrm{e}^{\int \frac{\sec(x)}{x} dx}$$

✔ Solution by Mathematica

Time used: 0.438 (sec). Leaf size: 56

```
DSolve[y'[x] == x+Sec[x]*y[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left( \int_1^x \frac{\sec(K[1])}{K[1]} dK[1] \right) \left( \int_1^x \exp\left( - \int_1^{K[2]} \frac{\sec(K[1])}{K[1]} dK[1] \right) K[2] dK[2] + c_1 \right)$$

## 1.12 problem 12

Internal problem ID [6302]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 12.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_separable]

$$\boxed{y' - \frac{2y}{x} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 9

```
dsolve([diff(y(x),x) = 2*y(x)/x,y(0) = 0],y(x), singsol=all)
```

$$y(x) = c_1 x^2$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x] == 2*y[x]/x,y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

## 1.13 problem 13

Internal problem ID [6303]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 13.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_separable]

$$y' - \frac{2y}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x) = 2*y(x)/x,y(x), singsol=all)
```

$$y(x) = c_1 x^2$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 16

```
DSolve[y'[x] == 2*y[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 x^2$$
$$y(x) \rightarrow 0$$

## 1.14 problem 14

Internal problem ID [6304]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 14.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_separable]

$$y' - \frac{\ln\left(y^2 + 1\right)}{\ln\left(x^2 + 1\right)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)=ln(y(x)^2+1)/ln(x^2+1),y(x), singsol=all)
```

$$\int \frac{1}{\ln\left(x^2 + 1\right)}dx - \left(\int^{y(x)} \frac{1}{\ln\left(\_a^2 + 1\right)}d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.584 (sec). Leaf size: 48

```
DSolve[y'[x] == Log[1+y[x]^2]/Log[1+x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{\log\left(K[1]^2 + 1\right)}dK[1]\&\right]\left[\int_1^x \frac{1}{\log\left(K[2]^2 + 1\right)}dK[2] + c_1\right]$$

$$y(x) \to 0$$

## 1.15   problem 15

Internal problem ID [6305]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 15.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$y' - \frac{1}{x} = 0$$

✓  Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=1/x,y(x), singsol=all)
```

$$y(x) = \ln(x) + c_1$$

✓  Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 10

```
DSolve[y'[x] == 1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \log(x) + c_1$$

## 1.16 problem 16

Internal problem ID [6306]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 16.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [[_homogeneous, 'class G'], _rational, [_Abel, '2nd type', 'cla

$$y' - \frac{-yx - 1}{4yx^3 - 2x^2} = 0$$

✓ Solution by Maple

Time used: 0.344 (sec). Leaf size: 37

```
dsolve(diff(y(x),x)=(-x*y(x)-1)/(4*x^3*y(x)-2*x^2),y(x), singsol=all)
```

$$y(x) = \frac{\text{RootOf}\left(\_Z^{25}c_1 - 10\_Z^{20}c_1 + 25\_Z^{15}c_1 - 16x^5\right)^5 - 1}{4x}$$

✓ Solution by Mathematica

Time used: 13.595 (sec). Leaf size: 391

```
DSolve[y'[x] == (-x*y[x]-1)/(4*x^3*y[x]-2*x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[64\#1^5 c_1{}^5 x^5 - 80\#1^4 c_1{}^5 x^4 - 20\#1^3 c_1{}^5 x^3 + 25\#1^2 c_1{}^5 x^2 + 10\#1 c_1{}^5 x - x^5 \right. $$
$$\left. + c_1{}^5\&, 1\right]$$

$$y(x) \to \text{Root}\left[64\#1^5 c_1{}^5 x^5 - 80\#1^4 c_1{}^5 x^4 - 20\#1^3 c_1{}^5 x^3 + 25\#1^2 c_1{}^5 x^2 + 10\#1 c_1{}^5 x - x^5 \right. $$
$$\left. + c_1{}^5\&, 2\right]$$

$$y(x) \to \text{Root}\left[64\#1^5 c_1{}^5 x^5 - 80\#1^4 c_1{}^5 x^4 - 20\#1^3 c_1{}^5 x^3 + 25\#1^2 c_1{}^5 x^2 + 10\#1 c_1{}^5 x - x^5 \right. $$
$$\left. + c_1{}^5\&, 3\right]$$

$$y(x) \to \text{Root}\left[64\#1^5 c_1{}^5 x^5 - 80\#1^4 c_1{}^5 x^4 - 20\#1^3 c_1{}^5 x^3 + 25\#1^2 c_1{}^5 x^2 + 10\#1 c_1{}^5 x - x^5 \right. $$
$$\left. + c_1{}^5\&, 4\right]$$

$$y(x) \to \text{Root}\left[64\#1^5 c_1{}^5 x^5 - 80\#1^4 c_1{}^5 x^4 - 20\#1^3 c_1{}^5 x^3 + 25\#1^2 c_1{}^5 x^2 + 10\#1 c_1{}^5 x - x^5 \right. $$
$$\left. + c_1{}^5\&, 5\right]$$

## 1.17 problem 17

Internal problem ID [6307]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 17.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\frac{y'^2}{4} - y'x + y = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 19

```
dsolve((1/4)*diff(y(x),x)^2-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2$$

$$y(x) = -\frac{1}{4}c_1^2 + c_1 x$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 25

```
DSolve[(1/4)*(y'[x])^2-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{c_1^2}{4}$$

$$y(x) \to x^2$$

## 1.18   problem 18

Internal problem ID [6308]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 18.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$
y' - \sqrt{\frac{y+1}{y^2}} = 0
$$

With initial conditions

$$
[y(0) = 1]
$$

✓ Solution by Maple

Time used: 0.329 (sec). Leaf size: 148

```
dsolve([diff(y(x),x)=sqrt( (1+y(x))/y(x)^2),y(0) = 1],y(x), singsol=all)
```

$$
y(x) =
$$

$$
-\frac{\left(1 + i\sqrt{3}\right)\left(-12\sqrt{2}\,x + 9x^2 + \sqrt{\left(-12\sqrt{2}\,x + 9x^2 - 8\right)\left(3x - 2\sqrt{2}\right)^2}\right)^{\frac{2}{3}} - 4i\sqrt{3} - 4\left(-12\sqrt{2}\,x + 9x^2 - \right.}{4\left(-12\sqrt{2}\,x + 9x^2 + \sqrt{\left(-12\sqrt{2}\,x + 9x^2 - 8\right)\left(3x - 2\sqrt{2}\right)^2}\right)}
$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 122

```
DSolve[{y'[x]==Sqrt[ (1+y[x])/y[x]^2],y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to \frac{1}{4}\left(-1 - i\sqrt{3}\right)\sqrt[3]{9x^2 + \sqrt{9x^2\left(9x^2 - 24\sqrt{2}\,x + 32\right) - 64} - 12\sqrt{2}\,x}
$$

$$
+ \frac{-1 + i\sqrt{3}}{\sqrt[3]{9x^2 + \sqrt{9x^2\left(9x^2 - 24\sqrt{2}\,x + 32\right) - 64} - 12\sqrt{2}\,x}} + 1
$$

## 1.19 problem 19

Internal problem ID [6309]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 19.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [`y=_G(x,y')`]

$$y' - \sqrt{1 - x^2 - y^2} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=sqrt( 1-x^2-y(x)^2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==Sqrt[ 1-x^2-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 1.20 problem 20

Internal problem ID [6310]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 20.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_Bernoulli]

$$y' + \frac{y}{3} - \frac{(1 - 2x) y^4}{3} = 0$$

✓  Solution by Maple

Time used: 0.016 (sec). Leaf size: 83

```
dsolve(diff(y(x),x)+y(x)/3= (1-2*x)/3*y(x)^4,y(x), singsol=all)
```

$$y(x) = \frac{1}{(e^x c_1 - 2x - 1)^{\frac{1}{3}}}$$

$$y(x) = -\frac{1}{2 (e^x c_1 - 2x - 1)^{\frac{1}{3}}} - \frac{i\sqrt{3}}{2 (e^x c_1 - 2x - 1)^{\frac{1}{3}}}$$

$$y(x) = -\frac{1}{2 (e^x c_1 - 2x - 1)^{\frac{1}{3}}} + \frac{i\sqrt{3}}{2 (e^x c_1 - 2x - 1)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 3.864 (sec). Leaf size: 76

```
DSolve[y'[x]+y[x]/3== (1-2*x)/3*y[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{\sqrt[3]{-2x + c_1 e^x - 1}}$$

$$y(x) \rightarrow -\frac{\sqrt[3]{-1}}{\sqrt[3]{-2x + c_1 e^x - 1}}$$

$$y(x) \rightarrow \frac{(-1)^{2/3}}{\sqrt[3]{-2x + c_1 e^x - 1}}$$

$$y(x) \rightarrow 0$$

## 1.21 problem 21

Internal problem ID [6311]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 21.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Chini]`

$$y' - \sqrt{y} - x = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 65

```
dsolve(diff(y(x),x)=sqrt(y(x))+x,y(x), singsol=all)
```

$$-\frac{2 \operatorname{arctanh}\left(2\sqrt{\frac{y(x)}{x^2}}\right)}{3} + \frac{4 \operatorname{arctanh}\left(\sqrt{\frac{y(x)}{x^2}}\right)}{3}$$
$$-\frac{\ln\left(-\frac{x^2-4y(x)}{x^2}\right)}{3} - \frac{2\ln\left(-\frac{2(x^2-y(x))}{x^2}\right)}{3} - 2\ln(x) + c_1 = 0$$

✓  Solution by Mathematica

Time used: 46.25 (sec). Leaf size: 716

```
DSolve[y'[x]==Sqrt[y[x]]+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(3x^2 + \frac{e^{3c_1}x(8 + e^{3c_1}x^3)}{\sqrt[3]{8\sqrt{-e^{24c_1}(-1+e^{3c_1}x^3)^3} + e^{12c_1}(-e^{6c_1}x^6 + 20e^{3c_1}x^3 + 8)}}\right.$$
$$\left. + e^{-6c_1}\sqrt[3]{8\sqrt{-e^{24c_1}(-1+e^{3c_1}x^3)^3} + e^{12c_1}(-e^{6c_1}x^6 + 20e^{3c_1}x^3 + 8)}\right)$$

$$y(x) \to \frac{1}{72}\left(54x^2 + \frac{(-9 - 9i\sqrt{3})e^{3c_1}x(8 + e^{3c_1}x^3)}{\sqrt[3]{8\sqrt{-e^{24c_1}(-1+e^{3c_1}x^3)^3} + e^{12c_1}(-e^{6c_1}x^6 + 20e^{3c_1}x^3 + 8)}}\right.$$
$$\left. + 9i\left(\sqrt{3}+i\right)e^{-6c_1}\sqrt[3]{8\sqrt{-e^{24c_1}(-1+e^{3c_1}x^3)^3} + e^{12c_1}(-e^{6c_1}x^6 + 20e^{3c_1}x^3 + 8)}\right)$$

$$y(x) \to \frac{1}{72}\left(54x^2 + \frac{9i\left(\sqrt{3}+i\right)e^{3c_1}x(8 + e^{3c_1}x^3)}{\sqrt[3]{8\sqrt{-e^{24c_1}(-1+e^{3c_1}x^3)^3} + e^{12c_1}(-e^{6c_1}x^6 + 20e^{3c_1}x^3 + 8)}}\right.$$
$$\left. - 9\left(1 + i\sqrt{3}\right)e^{-6c_1}\sqrt[3]{8\sqrt{-e^{24c_1}(-1+e^{3c_1}x^3)^3} + e^{12c_1}(-e^{6c_1}x^6 + 20e^{3c_1}x^3 + 8)}\right)$$

$$y(x) \to \frac{1}{4}\left(\sqrt[3]{-x^6} + 3x^2 + \frac{x^4}{\sqrt[3]{-x^6}}\right)$$

$$y(x) \to \frac{(1+i\sqrt{3})(-x^6)^{2/3} + 6x^4 + i(\sqrt{3}+i)\sqrt[3]{-x^6}x^2}{8x^2}$$

$$y(x) \to \frac{1}{8}x^2\left(\frac{(1+i\sqrt{3})x^4}{(-x^6)^{2/3}} + \frac{i(\sqrt{3}+i)x^2}{\sqrt[3]{-x^6}} + 6\right)$$

## 1.22 problem 23

Internal problem ID [6312]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 23.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cla

$$y'x^2 + y^2 - yxy' = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x)+y(x)^2=x*y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = e^{-\operatorname{LambertW}\left(-\frac{e^{-c_1}}{x}\right) - c_1}$$

✓ Solution by Mathematica

Time used: 2.25 (sec). Leaf size: 25

```
DSolve[x^2*y'[x]+y[x]^2==x*y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -xW\left(-\frac{e^{-c_1}}{x}\right)$$

$$y(x) \to 0$$

## 1.23 problem 24

Internal problem ID [6313]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 24.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type [_separable]

$$y - y'x - x^2 y'^2 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 97

```
dsolve(y(x)=x*diff(y(x),x)+x^2*diff(y(x),x)^2,y(x), singsol=all)
```

$$\ln(x) - \sqrt{4y(x)+1} - \frac{\ln\left(\sqrt{4y(x)+1}-1\right)}{2} + \frac{\ln\left(\sqrt{4y(x)+1}+1\right)}{2} - \frac{\ln(y(x))}{2} - c_1 = 0$$

$$\ln(x) + \sqrt{4y(x)+1} + \frac{\ln\left(\sqrt{4y(x)+1}-1\right)}{2} - \frac{\ln\left(\sqrt{4y(x)+1}+1\right)}{2} - \frac{\ln(y(x))}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 21.391 (sec). Leaf size: 72

```
DSolve[y[x]==x*y'[x]+x^2*(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}W\left(-e^{-1-2c_1}x\right)\left(2 + W\left(-e^{-1-2c_1}x\right)\right)$$

$$y(x) \to \frac{1}{4}W\left(e^{-1+2c_1}x\right)\left(2 + W\left(e^{-1+2c_1}x\right)\right)$$

$$y(x) \to 0$$

## 1.24 problem 25

Internal problem ID [6314]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 25.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$(x+y)\,y' = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve((x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -x$$
$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 14

```
DSolve[(x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x$$
$$y(x) \to c_1$$

## 1.25 problem 26

Internal problem ID [6315]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 26.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$y'x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1$$

## 1.26 problem 27

Internal problem ID [6316]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 27.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$\boxed{\frac{y'}{x+y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(1/(x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[1/(x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

## 1.27 problem 28

Internal problem ID [6317]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 28.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$\boxed{\frac{y'}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(1/x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[1/x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

## 1.28 problem 29

Internal problem ID [6318]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 29.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1$$

## 1.29 problem 30

Internal problem ID [6319]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 30.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, _dAlembert]`

$$y - xy'^2 - y'^2 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 99

```
dsolve(y(x)=x*diff(y(x),x)^2+diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{x\left(x + 1 + \sqrt{c_1 x + c_1 + x + 1}\right)^2}{(x+1)^2} + \frac{\left(x + 1 + \sqrt{c_1 x + c_1 + x + 1}\right)^2}{(x+1)^2}$$

$$y(x) = \frac{x\left(-x - 1 + \sqrt{c_1 x + c_1 + x + 1}\right)^2}{(x+1)^2} + \frac{\left(-x - 1 + \sqrt{c_1 x + c_1 + x + 1}\right)^2}{(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 57

```
DSolve[y[x]==x*(y'[x])^2+(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - c_1\sqrt{x+1} + 1 + \frac{c_1^2}{4}$$

$$y(x) \to x + c_1\sqrt{x+1} + 1 + \frac{c_1^2}{4}$$

$$y(x) \to 0$$

## 1.30   problem 31

Internal problem ID [6320]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 31.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Riccati]`

$$y' - \frac{5x^2 - yx + y^2}{x^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)=(5*x^2-x*y(x)+y(x)^2)/x^2,y(x), singsol=all)
```

$$y(x) = 2\tan\left(2\ln\left(x\right) + 2c_1\right)x + x$$

✓ Solution by Mathematica

Time used: 0.737 (sec). Leaf size: 18

```
DSolve[y'[x]==(5*x^2-x*y[x]+y[x]^2)/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + 2x\tan(2(\log(x) + c_1))$$

## 1.31 problem 32

Internal problem ID [6321]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 32.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cla

$$2t + 3x + (x + 2)\, x' = 0$$

✓ Solution by Maple

Time used: 2.219 (sec). Leaf size: 29

```
dsolve(2*t+3*x(t)+(x(t)+2)*diff(x(t),t)=0,x(t), singsol=all)
```

$$x(t) = -2 - \frac{4(t-3)\, c_1 + 1 + \sqrt{4(t-3)\, c_1 + 1}}{2c_1}$$

✓  Solution by Mathematica

Time used: 60.111 (sec). Leaf size: 365

```
DSolve[2*t+3*x[t]+(x[t]+2)*x'[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow -2 + \cfrac{1}{\cfrac{1}{6-2t} - \cfrac{1}{2}\sqrt{\cfrac{1}{(t-3)^2 + e^{\frac{4c_1}{9}}(t-3)^4}} - \sqrt{-\cfrac{1}{2(t-3)^4 + e^{\frac{4c_1}{9}}(t-3)^6 + e^{-\frac{4c_1}{9}}(t-3)^2}}}$$

$$x(t) \rightarrow -2 + \cfrac{2}{\cfrac{1}{3-t} + \sqrt{\cfrac{1}{(t-3)^2 + e^{\frac{4c_1}{9}}(t-3)^4}} - \sqrt{-\cfrac{1}{2(t-3)^4 + e^{\frac{4c_1}{9}}(t-3)^6 + e^{-\frac{4c_1}{9}}(t-3)^2}}}$$

$$x(t) \rightarrow -2 + \cfrac{1}{\cfrac{1}{6-2t} - \cfrac{1}{2}\sqrt{\sqrt{-\cfrac{1}{2(t-3)^4 + e^{\frac{4c_1}{9}}(t-3)^6 + e^{-\frac{4c_1}{9}}(t-3)^2}} + \cfrac{1}{(t-3)^2 + e^{\frac{4c_1}{9}}(t-3)^4}}}$$

$$x(t) \rightarrow -2 + \cfrac{2}{\cfrac{1}{3-t} + \sqrt{\sqrt{-\cfrac{1}{2(t-3)^4 + e^{\frac{4c_1}{9}}(t-3)^6 + e^{-\frac{4c_1}{9}}(t-3)^2}} + \cfrac{1}{(t-3)^2 + e^{\frac{4c_1}{9}}(t-3)^4}}}$$

## 1.32 problem 33

Internal problem ID [6322]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 33.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$y' - \frac{1}{1-y} = 0$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(t),t)=1/(1-y(t)),y(0) = 2],y(t), singsol=all)
```

$$y(t) = 1 + \sqrt{1 - 2t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{y'[t]==1/(1-y[t]),y[0]==2},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sqrt{1 - 2t} + 1$$

## 1.33 problem 34

Internal problem ID [6323]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 34.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$p' - ap + bp^2 = 0$$

With initial conditions

$$[p(\text{t0}) = \text{p0}]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve([diff(p(t),t)=a*p(t)-b*p(t)^2,p(t0) = p0],p(t), singsol=all)
```

$$p(t) = \frac{a\,\text{p0}}{(-\,\text{p0}\,b + a)\,\mathrm{e}^{-a(t-\text{t0})} + \text{p0}\,b}$$

✓ Solution by Mathematica

Time used: 0.76 (sec). Leaf size: 36

```
DSolve[{p'[t]==a*p[t]-b*p[t]^2,p[t0]==p0},p[t],t,IncludeSingularSolutions -> True]
```

$$p(t) \to \frac{a\text{p0}e^{at}}{b\text{p0}e^{at} + e^{at0}(a - b\text{p0})}$$

## 1.34 problem 35

Internal problem ID [6324]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 35.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [[_homogeneous, 'class G'], _exact, _rational, _Bernoulli]

$$y^2 + \frac{2}{x} + 2yxy' = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 42

```
dsolve((y(x)^2+2/x)+2*y(x)*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-x\left(2\ln\left(x\right) - c_1\right)}}{x}$$

$$y(x) = -\frac{\sqrt{-x\left(2\ln\left(x\right) - c_1\right)}}{x}$$

✓ Solution by Mathematica
Time used: 0.198 (sec). Leaf size: 44

```
DSolve[(y[x]^2+2/x)+2*y[x]*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-2\log(x) + c_1}}{\sqrt{x}}$$

$$y(x) \to \frac{\sqrt{-2\log(x) + c_1}}{\sqrt{x}}$$

## 1.35   problem 36

Internal problem ID [6325]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 36.
**ODE order**: 1.
**ODE degree**: 0.

CAS Maple gives this as type [_Clairaut]

$$ xf' - f - \frac{f'^2\left(1 - f'^\lambda\right)^2}{\lambda^2} = 0 $$

✓  <u>Solution by Maple</u>

Time used: 0.453 (sec). Leaf size: 648

```
dsolve(x*diff(f(x),x)-f(x)=diff(f(x),x)^2/lambda^2*(1-diff(f(x),x)^lambda)^2,f(x), singsol=al
```

$$ f(x) = 0 $$

Expression too large to display

$$ f(x) = c_1 x - \frac{c_1^2\left(1 - c_1^\lambda\right)^2}{\lambda^2} $$

✓  <u>Solution by Mathematica</u>

Time used: 15.316 (sec). Leaf size: 30

```
DSolve[x*f'[x]-f[x]==f'[x]^2/\[Lambda]^2*(1-f'[x]^\[Lambda])^2,f[x],x,IncludeSingularSolution
```

$$ f(x) \to c_1\left(x - \frac{c_1\left(-1 + c_1^\lambda\right)^2}{\lambda^2}\right) $$

$$ f(x) \to 0 $$

## 1.36 problem 37

Internal problem ID [6326]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 37.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_rational, _Riccati]

$$y'x - 2y + by^2 - cx^4 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x)-2*y(x)+b*y(x)^2=c*x^4,y(x), singsol=all)
```

$$y(x) = \frac{i \tan\left(-\frac{ix^2\sqrt{b}\sqrt{c}}{2} + c_1\right) x^2 \sqrt{c}}{\sqrt{b}}$$

✓ Solution by Mathematica

Time used: 0.241 (sec). Leaf size: 145

```
DSolve[x*y'[x]-2*y[x]+b*y[x]^2==c*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{c}x^2\left(-\cosh\left(\frac{1}{2}\sqrt{b}\sqrt{c}x^2\right) + c_1\sin\left(\frac{1}{2}\sqrt{-b}\sqrt{c}x^2\right)\right)}{\sqrt{-b}\left(\sin\left(\frac{1}{2}\sqrt{-b}\sqrt{c}x^2\right) + c_1\cosh\left(\frac{1}{2}\sqrt{b}\sqrt{c}x^2\right)\right)}$$

$$y(x) \to \frac{\sqrt{c}x^2\tanh\left(\frac{1}{2}\sqrt{b}\sqrt{c}x^2\right)}{\sqrt{b}}$$

## 1.37 problem 38

Internal problem ID [6327]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 38.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_rational, _Riccati]

$$y'x - y + y^2 - x^{\frac{2}{3}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(x*diff(y(x),x)-y(x)+y(x)^2=x^(2/3),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\left(-|3x^{\frac{1}{3}} - 1|c_1 - \text{abs}\left(1, 3x^{\frac{1}{3}} - 1\right)c_1\right)e^{3x^{\frac{1}{3}}} + 3\,e^{-3x^{\frac{1}{3}}}x^{\frac{1}{3}}\right)x^{\frac{1}{3}}}{c_1 e^{3x^{\frac{1}{3}}}|3x^{\frac{1}{3}} - 1| + \left(3x^{\frac{1}{3}} + 1\right)e^{-3x^{\frac{1}{3}}}}$$

✓ Solution by Mathematica

Time used: 0.211 (sec). Leaf size: 103

```
DSolve[x*y'[x]-y[x]+y[x]^2==x^(2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x^{2/3}}{3ic_1\sqrt[3]{x} + \frac{3(1+c_1^2)\sqrt[3]{x}\cosh\left(3\sqrt[3]{x}\right)}{\sinh\left(3\sqrt[3]{x}\right)+ic_1\cosh\left(3\sqrt[3]{x}\right)} - 1}$$

$$y(x) \to \frac{3x^{2/3}}{3\sqrt[3]{x}\tanh\left(3\sqrt[3]{x}\right) - 1}$$

## 1.38 problem 39

Internal problem ID [6328]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 39.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$u' + u^2 - \frac{1}{x^{\frac{4}{5}}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve(diff(u(x),x)+u(x)^2=x^(-4/5),u(x), singsol=all)
```

$$u(x) = \frac{\text{BesselI}\left(-\frac{1}{6}, \frac{5x^{\frac{3}{5}}}{3}\right) c_1 - \text{BesselK}\left(\frac{1}{6}, \frac{5x^{\frac{3}{5}}}{3}\right)}{x^{\frac{2}{5}}\left(c_1 \text{BesselI}\left(\frac{5}{6}, \frac{5x^{\frac{3}{5}}}{3}\right) + \text{BesselK}\left(\frac{5}{6}, \frac{5x^{\frac{3}{5}}}{3}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.233 (sec). Leaf size: 139

```
DSolve[u'[x]+u[x]^2==x^(-4/5),u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{(-1)^{5/6}\,\text{Gamma}\left(\frac{11}{6}\right)\text{BesselI}\left(-\frac{1}{6}, \frac{5x^{3/5}}{3}\right) + c_1\,\text{Gamma}\left(\frac{1}{6}\right)\text{BesselI}\left(\frac{1}{6}, \frac{5x^{3/5}}{3}\right)}{x^{2/5}\left((-1)^{5/6}\,\text{Gamma}\left(\frac{11}{6}\right)\text{BesselI}\left(\frac{5}{6}, \frac{5x^{3/5}}{3}\right) + c_1\,\text{Gamma}\left(\frac{1}{6}\right)\text{BesselI}\left(-\frac{5}{6}, \frac{5x^{3/5}}{3}\right)\right)}$$

$$u(x) \to \frac{\text{BesselI}\left(\frac{1}{6}, \frac{5x^{3/5}}{3}\right)}{x^{2/5}\,\text{BesselI}\left(-\frac{5}{6}, \frac{5x^{3/5}}{3}\right)}$$

## 1.39 problem 40

Internal problem ID [6329]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 40.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cla

$$yy' - y - x = 0$$

✓ Solution by Maple

Time used: 0.406 (sec). Leaf size: 53

```
dsolve(y(x)*diff(y(x),x)-y(x)=x,y(x), singsol=all)
```

$$-\frac{\ln\left(-\frac{x^2 + y(x)x - y(x)^2}{x^2}\right)}{2} - \frac{\sqrt{5}\,\text{arctanh}\left(\frac{(x - 2y(x))\sqrt{5}}{5x}\right)}{5} - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 63

```
DSolve[y[x]*y'[x] - y[x] == x,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{10}\left(\left(5 + \sqrt{5}\right)\log\left(-\frac{2y(x)}{x} + \sqrt{5} + 1\right) - \left(\sqrt{5} - 5\right)\log\left(\frac{2y(x)}{x} + \sqrt{5} - 1\right)\right) = \right.$$
$$\left. -\log(x) + c_1, y(x)\right]$$

## 1.40   problem 41

Internal problem ID [6330]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 41.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x]]

$$y'' + 2y' + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + c_2e^{-x}x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[y''[x]+2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2x + c_1)$$

## 1.41 problem 41

Internal problem ID [6331]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 41.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x]]

$$5y'' + 2y' + 4y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 5]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 20

```
dsolve([5*diff(y(x),x$2)+2*diff(y(x),x)+4*y(x)=0,y(0) = 0, D(y)(0) = 5],y(x), singsol=all)
```

$$y(x) = \frac{25\sqrt{19}\,e^{-\frac{x}{5}}\sin\left(\frac{\sqrt{19}\,x}{5}\right)}{19}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 6

```
DSolve[{5*y''[x]+2*y'[x]+4*y[x]==0,{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to 0$$

## 1.42   problem 42

Internal problem ID [6332]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 42.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x]]

$$y'' + y' + 4y - 1 = 0$$

✓   Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+4*y(x)=1,y(x), singsol=all)
```

$$y(x) = e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{15}\,x}{2}\right) c_2 + e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{15}\,x}{2}\right) c_1 + \frac{1}{4}$$

✓   Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 46

```
DSolve[y''[x]+y'[x]+4*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} + e^{-x/2}\left(c_2 \cos\left(\frac{\sqrt{15}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{15}x}{2}\right)\right)$$

## 1.43 problem 43

Internal problem ID [6333]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 43.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + y' + 4y - \sin(x) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+4*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{15}\,x}{2}\right) c_2 + e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{15}\,x}{2}\right) c_1 + \frac{3\sin(x)}{10} - \frac{\cos(x)}{10}$$

✓ Solution by Mathematica

Time used: 0.732 (sec). Leaf size: 55

```
DSolve[y''[x]+y'[x]+4*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3\sin(x)}{10} - \frac{\cos(x)}{10} + e^{-x/2}\left(c_2 \cos\left(\frac{\sqrt{15}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{15}x}{2}\right)\right)$$

## 1.44 problem 44

Internal problem ID [6334]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 44.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type [[_homogeneous, 'class A'], _rational, _dAlembert]

$$\boxed{y - xy'^2 = 0}$$

✓  Solution by Maple

Time used: 0.031 (sec). Leaf size: 39

```
dsolve(y(x)=x*(diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{\left(x + \sqrt{c_1 x}\right)^2}{x}$$

$$y(x) = \frac{\left(-x + \sqrt{c_1 x}\right)^2}{x}$$

✓  Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 46

```
DSolve[y[x]==x*(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-2\sqrt{x} + c_1\right)^2$$

$$y(x) \to \frac{1}{4}\left(2\sqrt{x} + c_1\right)^2$$

$$y(x) \to 0$$

## 1.45   problem 45

Internal problem ID [6335]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 45.
**ODE order**: 1.
**ODE degree**: 3.

CAS Maple gives this as type [_dAlembert]

$$yy' - 1 + xy'^3 = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 2255

```
dsolve(diff(y(x),x)*y(x)=1-x*(diff(y(x),x))^3,y(x), singsol=all)
```

$$c_1 x^2 \left( \left( \sqrt{3} \sqrt{\frac{4y(x)^3+27x}{x}} + 9 \right) x^2 \right)^{\frac{2}{3}} \left( 2 \left( \sqrt{3} \sqrt{\frac{4y(x)^3+27x}{x}} + 9 \right) x^2 18^{\frac{1}{3}} \left( \left( \sqrt{3} \sqrt{\frac{4y(x)^3+27x}{x}} + 9 \right) x^2 \right)^{\frac{1}{3}} + 12\,12^{\frac{1}{3}} \right)$$

$$\div \left( y(x)\,12^{\frac{2}{3}} x - 12^{\frac{1}{3}} \left( \left( \sqrt{3} \sqrt{\frac{4y(x)^3+27x}{x}} + 9 \right)^2 x^4 \right)^{\frac{1}{3}} - 6x \left( \left( \sqrt{3} \sqrt{\frac{4y(x)^3+27x}{x}} + 9 \right) x^2 \right)^{\frac{1}{3}} \right)^2 \left( y(x) \right.$$

$$+ x$$

$$- \frac{3x^2 \left( 6\sqrt{3}\,12^{\frac{1}{3}} \sqrt{\frac{4y(x)^3+27x}{x}}\,x^3 - 12x^2 y(x)\,18^{\frac{1}{3}} \left( \left( \sqrt{3} \sqrt{\frac{4y(x)^3+27x}{x}} + 9 \right) x^2 \right)^{\frac{1}{3}} + 54\,12^{\frac{1}{3}} x^3 + 18x^2 \left( \left( \sqrt{3} \sqrt{\frac{4y}{}} \right. \right. \right)}{\left( y(x)\,12^{\frac{2}{3}} x - 12^{\frac{1}{3}} \left( \left( \sqrt{3} \sqrt{\frac{4y(x)^3+27x}{x}} + 9 \right)^2 x^4 \right)^{\frac{1}{3}} - 6x \left( \left( \sqrt{3} \sqrt{\frac{4y(x)^3+27x}{x}} + 9 \right) x^2 \right)^{\frac{1}{3}} \right)^2 \left( y \right.}$$

$$= 0$$

Expression too large to display

Expression too large to display

✓ Solution by Mathematica

Time used: 87.737 (sec). Leaf size: 20717

```
DSolve[y'[x]*y[x]==1-x*(y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

## 1.46 problem 46

Internal problem ID [6336]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 46.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$f' - \frac{1}{f} = 0$$

✓  Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(f(x),x)=f(x)^(-1),f(x), singsol=all)
```

$$f(x) = \sqrt{2x + c_1}$$
$$f(x) = -\sqrt{2x + c_1}$$

✓  Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 38

```
DSolve[f'[x]==f[x]^(-1),f[x],x,IncludeSingularSolutions -> True]
```

$$f(x) \to -\sqrt{2}\sqrt{x + c_1}$$
$$f(x) \to \sqrt{2}\sqrt{x + c_1}$$

## 1.47 problem 47

Internal problem ID [6337]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 47.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_y]]

$$ty'' + 4y' - t^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(t*diff(y(t),t$2)+4*diff(y(t),t)=t^2,y(t), singsol=all)
```

$$y(t) = \frac{t^3}{18} - \frac{c_1}{3t^3} + c_2$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 23

```
DSolve[t*y''[t]+4*y'[t]==t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t^6 - 6c_1}{18t^3} + c_2$$

## 1.48  problem 48

Internal problem ID [6338]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 48.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_y]]

$$(t^2 + 9)\, y'' + 2ty' = 0$$

With initial conditions

$$\left[ y(3) = 2\pi, y'(3) = \frac{2}{3} \right]$$

✓  Solution by Maple

Time used: 0.063 (sec). Leaf size: 12

```
dsolve([(t^2+9)*diff(y(t),t$2)+2*t*diff(y(t),t)=0,y(3) = 2*Pi, D(y)(3) = 2/3],y(t), singsol=a
```

$$y(t) = \pi + 4\arctan\left(\frac{t}{3}\right)$$

✓  Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 15

```
DSolve[{(t^2+9)*y''[t]+2*t*y'[t]==0,{y[3]==2*Pi,y'[3]==2/3}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to 4\arctan\left(\frac{t}{3}\right) + \pi$$

## 1.49   problem 49

Internal problem ID [6339]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 49.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$t^2 y'' - 3ty' + 5y = 0$$

✓  Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(t^2*diff(y(t),t$2)-3*t*diff(y(t),t)+5*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t^2 \sin\left(\ln\left(t\right)\right) + c_2 t^2 \cos\left(\ln\left(t\right)\right)$$

✓  Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 22

```
DSolve[t^2*y''[t]-3*t*y'[t]+5*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t^2(c_2 \cos(\log(t)) + c_1 \sin(\log(t)))$$

## 1.50   problem 50

Internal problem ID [6340]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 50.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_y]]

$$ty'' + y' = 0$$

✓   Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(t*diff(y(t),t$2)+diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = c_2 \ln(t) + c_1$$

✓   Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 13

```
DSolve[t*y''[t]+y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \log(t) + c_2$$

## 1.51 problem 51

Internal problem ID [6341]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 51.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_y]]

$$t^2 y'' - 2y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(t^2*diff(y(t),t$2)-2*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = c_1 + \left( t\,e^{-\frac{2}{t}} - 2\,\text{Ei}_1\left(\frac{2}{t}\right) \right) c_2$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 29

```
DSolve[t^2*y''[t]-2*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2c_1\,\text{ExpIntegralEi}\left(-\frac{2}{t}\right) + c_1 e^{-2/t}t + c_2$$

## 1.52 problem 52

Internal problem ID [6342]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 52.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' + \frac{(t^2 - 1)\, y'}{t} + \frac{t^2 y}{\left(1 + e^{\frac{t^2}{2}}\right)^2} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 84

```
dsolve(diff(y(t),t$2)+(t^2-1)/t*diff(y(t),t)+t^2/(1 + exp(t^2/2))^2*y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{\left(c_1\left(1 + e^{\frac{t^2}{2}}\right)^{-\frac{i\sqrt{3}}{2}}\left(e^{\frac{t^2}{2}}\right)^{\frac{i\sqrt{3}}{2}} + c_2\left(1 + e^{\frac{t^2}{2}}\right)^{\frac{i\sqrt{3}}{2}}\left(e^{\frac{t^2}{2}}\right)^{-\frac{i\sqrt{3}}{2}}\right)\sqrt{1 + e^{\frac{t^2}{2}}}}{\sqrt{e^{\frac{t^2}{2}}}}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[t]+(t^2-1)/t*t'[t]+t^2/(1 + Exp[t^2/2])^2*y[t]==0,y[t],t,IncludeSingularSolutions
```

Not solved

## 1.53 problem 53

Internal problem ID [6343]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 53.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(

$$ty'' - y' + 4t^3 y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(t*diff(y(t),t$2)-diff(y(t),t)+4*t^3*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \sin\left(t^2\right) + c_2 \cos\left(t^2\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 20

```
DSolve[t*y''[t]-y'[t]+4*t^3*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \cos\left(t^2\right) + c_2 \sin\left(t^2\right)$$

## 1.54 problem 54

Internal problem ID [6344]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 54.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$y'' = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(t),t$2)=0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 12

```
DSolve[y''[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_2 t + c_1$$

## 1.55   problem 55

Internal problem ID [6345]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 55.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$y'' - 1 = 0$$

✓  Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(t),t$2)=1,y(t), singsol=all)
```

$$y(t) = \frac{1}{2}t^2 + c_1 t + c_2$$

✓  Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 19

```
DSolve[y''[t]==1,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t^2}{2} + c_2 t + c_1$$

## 1.56 problem 56

Internal problem ID [6346]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 56.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$y'' - f(t) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(t),t$2)=f(t),y(t), singsol=all)
```

$$y(t) = \int \left( \int f(t) \, dt \right) dt + c_1 t + c_2$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 30

```
DSolve[y''[t]==f[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \int_1^t \int_1^{K[2]} f(K[1]) dK[1] dK[2] + c_2 t + c_1$$

## 1.57   problem 57

Internal problem ID [6347]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 57.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _quadrature]]

$$y'' - k = 0$$

✓   Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(t),t$2)=k,y(t), singsol=all)
```

$$y(t) = \frac{1}{2}k\,t^2 + c_1 t + c_2$$

✓   Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y''[t]==k,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{kt^2}{2} + c_2 t + c_1$$

## 1.58   problem 58

Internal problem ID [6348]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 58.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [[_homogeneous, 'class C'], _dAlembert]

$$y' + 4\sin\left(-y + x\right) + 4 = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=4*sin(y(x)-x)-4,y(x), singsol=all)
```

$$y(x) = x + 2\arctan\left(\frac{3\tan\left(-\frac{3x}{2} + \frac{3c_1}{2}\right)}{5} + \frac{4}{5}\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==4*Sin[y[x]-x]-4,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

## 1.59 problem 59

Internal problem ID [6349]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 59.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$y' + \sin(-y + x) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)-sin(y(x)-x)=0,y(x), singsol=all)
```

$$y(x) = x + 2\arctan\left(\frac{c_1 - x - 2}{-x + c_1}\right)$$

✓ Solution by Mathematica

Time used: 37.14 (sec). Leaf size: 529

```
DSolve[y'[x]-Sin[y[x]-x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\arccos\left(\frac{(-x+2+c_1)\cos\left(\frac{x}{2}\right)+(x-c_1)\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2(1+c_1)x+2+c_1(2+c_1)}}\right)$$

$$y(x) \to 2\arccos\left(\frac{(-x+2+c_1)\cos\left(\frac{x}{2}\right)+(x-c_1)\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2(1+c_1)x+2+c_1(2+c_1)}}\right)$$

$$y(x) \to -2\arccos\left(\frac{(x-2-c_1)\cos\left(\frac{x}{2}\right)+(-x+c_1)\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2(1+c_1)x+2+c_1(2+c_1)}}\right)$$

$$y(x) \to 2\arccos\left(\frac{(x-2-c_1)\cos\left(\frac{x}{2}\right)+(-x+c_1)\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2(1+c_1)x+2+c_1(2+c_1)}}\right)$$

$$y(x) \to -2\arccos\left(\frac{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$

$$y(x) \to 2\arccos\left(\frac{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$

$$y(x) \to -2\arccos\left(\frac{\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$

$$y(x) \to 2\arccos\left(\frac{\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$

$$y(x) \to -2\arccos\left(\frac{(x-2)\cos\left(\frac{x}{2}\right)-x\sin\left(\frac{x}{2}\right)}{\sqrt{2(x-2)x+4}}\right)$$

$$y(x) \to 2\arccos\left(\frac{(x-2)\cos\left(\frac{x}{2}\right)-x\sin\left(\frac{x}{2}\right)}{\sqrt{2(x-2)x+4}}\right)$$

$$y(x) \to -2\arccos\left(\frac{x\sin\left(\frac{x}{2}\right)-(x-2)\cos\left(\frac{x}{2}\right)}{\sqrt{2(x-2)x+4}}\right)$$

$$y(x) \to 2\arccos\left(\frac{x\sin\left(\frac{x}{2}\right)-(x-2)\cos\left(\frac{x}{2}\right)}{\sqrt{2(x-2)x+4}}\right)$$

## 1.60 problem 60

Internal problem ID [6350]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 60.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _quadrature]]

$$y'' - 4\sin(x) + 4 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)=4*sin(x)-4,y(x), singsol=all)
```

$$y(x) = -2x^2 - 4\sin(x) + c_1 x + c_2$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 20

```
DSolve[y''[x]==4*Sin[x]-4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -4\sin(x) + x(-2x + c_2) + c_1$$

## 1.61 problem 61

Internal problem ID [6351]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 61.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _quadrature]]

$$yy'' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(y(x)*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1 x + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 17

```
DSolve[y[x]*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to c_2 x + c_1$$

## 1.62 problem 62

Internal problem ID [6352]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 62.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]

$$yy'' - 1 = 0$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 51

```
dsolve(y(x)*diff(y(x),x$2)=1,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{2\ln(\_a) - c_1}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} -\frac{1}{\sqrt{2\ln(\_a) - c_1}} d\_a - x - c_2 = 0$$

✓ Solution by Mathematica
Time used: 0.066 (sec). Leaf size: 93

```
DSolve[y[x]*y''[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(-\text{erf}^{-1}\left(-i\sqrt{\frac{2}{\pi}}\sqrt{e^{c_1}(x + c_2)^2}\right)^2 - \frac{c_1}{2}\right)$$

$$y(x) \to \exp\left(-\text{erf}^{-1}\left(i\sqrt{\frac{2}{\pi}}\sqrt{e^{c_1}(x + c_2)^2}\right)^2 - \frac{c_1}{2}\right)$$

## 1.63    problem 63

Internal problem ID [6353]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 63.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_Emden, _Fowler], [_2nd_order, _with_linear_symmetries]]

$$yy'' - x = 0$$

**✗  Solution by Maple**

```
dsolve(y(x)*diff(y(x),x$2)=x,y(x), singsol=all)
```

No solution found

**✗  Solution by Mathematica**
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y''[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 1.64 problem 64

Internal problem ID [6354]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 64.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_Emden, _Fowler], [_2nd_order, _with_linear_symmetries]]

$$y^2 y'' - x = 0$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 106

```
dsolve(y(x)^2*diff(y(x),x$2)=x,y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(\ln(x) + 2^{\frac{1}{3}}\left(\int^{-Z}\frac{1}{2^{\frac{1}{3}}\_f + 2\,\text{RootOf}\left(\text{AiryBi}\left(\frac{2\_f\_Z^2+2^{\frac{2}{3}}}{2\_f}\right)c_1\_Z + \_Z\,\text{AiryAi}\left(\frac{2\_f\_Z^2+2^{\frac{2}{3}}}{2\_f}\right) + \text{AiryBi}\left(1, \frac{2}{}\right)} \right. \right.$$
$$\left. \left. - c_2\right)x\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]^2*y''[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 1.65 problem 65

Internal problem ID [6355]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 65.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _quadrature]]

$$y^2 y'' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(y(x)^2*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1 x + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 17

```
DSolve[y[x]^2*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to c_2 x + c_1$$

## 1.66  problem 66

Internal problem ID [6356]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 66.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[NONE]`

$$3yy'' - \sin(x) = 0$$

✗  Solution by Maple

```
dsolve(3*y(x)*diff(y(x),x$2)=sin(x),y(x), singsol=all)
```

No solution found

✗  Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[3*y[x]*y''[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 1.67 problem 67

Internal problem ID [6357]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 67.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]

$$3yy'' + y - 5 = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 59

```
dsolve(3*y(x)*diff(y(x),x$2)+y(x)=5,y(x), singsol=all)
```

$$\int^{y(x)} -\frac{3}{\sqrt{30\ln(\_a) + 9c_1 - 6\_a}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} \frac{3}{\sqrt{30\ln(\_a) + 9c_1 - 6\_a}} d\_a - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.312 (sec). Leaf size: 41

```
DSolve[3*y[x]*y''[x]+y[x]==5,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\sqrt{c_1 + \frac{2}{3}(5\log(K[1]) - K[1])}} dK[1]^2 = (x + c_2)^2, y(x)\right]$$

## 1.68 problem 68

Internal problem ID [6358]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 68.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]

$$ayy'' + by - c = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 68

```
dsolve(a*y(x)*diff(y(x),x$2)+b*y(x)=c,y(x), singsol=all)
```

$$\int^{y(x)} \frac{a}{\sqrt{a\left(2c\ln\left(\_a\right)+c_1 a - 2\_ab\right)}}\,d\_a - x - c_2 = 0$$

$$\int^{y(x)} -\frac{a}{\sqrt{a\left(2c\ln\left(\_a\right)+c_1 a - 2\_ab\right)}}\,d\_a - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.395 (sec). Leaf size: 43

```
DSolve[a*y[x]*y''[x]+b*y[x]==c,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\sqrt{c_1 + \frac{2(c\log(K[1]) - bK[1])}{a}}}\,dK[1]^2 = (x + c_2)^2, y(x)\right]$$

## 1.69   problem 69

Internal problem ID [6359]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 69.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]

$$ay^2 y'' + by^2 - c = 0$$

✓  Solution by Maple

Time used: 0.109 (sec). Leaf size: 74

```
dsolve(a*y(x)^2*diff(y(x),x$2)+b*y(x)^2=c,y(x), singsol=all)
```

$$\int^{y(x)} \frac{\_aa}{\sqrt{\_aa \left(c_1 \_aa - 2b\_a^2 - 2c\right)}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} -\frac{\_aa}{\sqrt{\_aa \left(c_1 \_aa - 2b\_a^2 - 2c\right)}} d\_a - x - c_2 = 0$$

✓  Solution by Mathematica

Time used: 0.732 (sec). Leaf size: 346

```
DSolve[a*y[x]^2*y''[x]+b*y[x]^2==c,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{\left(\sqrt{-16bc + a^2 c_1^2} - ac_1\right)\left(\sqrt{-16bc + a^2 c_1^2} + ac_1\right)^2 \left(1 + \frac{4by(x)}{\sqrt{-16bc+a^2 c_1^2} - ac_1}\right)\left(1 - \frac{4by(x)}{\sqrt{-16bc+a^2 c_1^2} + ac_1}\right)}{}\right.$$

## 1.70 problem 70

Internal problem ID [6360]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 70.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _quadrature]]

$$ayy'' + by = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(a*y(x)*diff(y(x),x$2)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = -\frac{b\,x^2}{2a} + c_1 x + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 28

```
DSolve[a*y[x]*y''[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

$$y(x) \to -\frac{bx^2}{2a} + c_2 x + c_1$$

## 1.71  problem 71

Internal problem ID [6361]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 71.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
x'(t) = 9x(t) + 4y(t)
$$
$$
y'(t) = -6x(t) - y(t)
$$
$$
z'(t) = 6x(t) + 4y(t) + 3z(t)
$$

✓  Solution by Maple

Time used: 0.125 (sec). Leaf size: 66

```
dsolve([diff(x(t),t)=9*x(t)+4*y(t),diff(y(t),t)=-6*x(t)-y(t),diff(z(t),t)=6*x(t)+4*y(t)+3*z(t
```

$$
x(t) = c_2 e^{5t} + \frac{2c_3 e^{3t}}{3} - \frac{2c_1 e^{3t}}{3}
$$

$$
y(t) = -c_2 e^{5t} - c_3 e^{3t} + c_1 e^{3t}
$$

$$
z(t) = c_2 e^{5t} + c_3 e^{3t}
$$

✓  Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 95

```
DSolve[{x'[t]==9*x[t]+4*y[t],y'[t]==-6*x[t]-y[t],z'[t]==6*x[t]+4*y[t]+3*z[t]},{x[t],y[t],z[t]
```

$$
x(t) \to e^{4t}(c_1 \cosh(t) + (5c_1 + 4c_2)\sinh(t))
$$
$$
y(t) \to 3(c_1 + c_2)e^{3t} - (3c_1 + 2c_2)e^{5t}
$$
$$
z(t) \to e^{3t}\left(3c_1\left(e^{2t} - 1\right) + 2c_2\left(e^{2t} - 1\right) + c_3\right)
$$

## 1.72 problem 72

Internal problem ID [6362]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 72.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t) - 3y(t)$$
$$y'(t) = 3x(t) + 7y(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=x(t)-3*y(t),diff(y(t),t)=3*x(t)+7*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^{4t}(3c_2 t + 3c_1 - c_2)}{3}$$

$$y(t) = e^{4t}(c_2 t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 44

```
DSolve[{x'[t]==x[t]-3*y[t],y'[t]==3*x[t]+7*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$x(t) \to e^{4t}(-3c_1 t - 3c_2 t + c_1)$$
$$y(t) \to e^{4t}(3(c_1 + c_2)t + c_2)$$

## 1.73 problem 73

Internal problem ID [6363]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 73.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t) - 2y(t)$$
$$y'(t) = 2x(t) + 5y(t)$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 35

```
dsolve([diff(x(t),t) = x(t)-2*y(t), diff(y(t),t) = 2*x(t)+5*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^{3t}(2c_2 t + 2c_1 - c_2)}{2}$$

$$y(t) = e^{3t}(c_2 t + c_1)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 44

```
DSolve[{x'[t]== x[t]-2*y[t],y'[t] == 2*x[t]+5*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to e^{3t}(-2c_1 t - 2c_2 t + c_1)$$
$$y(t) \to e^{3t}(2(c_1 + c_2)t + c_2)$$

## 1.74 problem 74

Internal problem ID [6364]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 74.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 7x(t) + y(t)$$
$$y'(t) = -4x(t) + 3y(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 33

```
dsolve([diff(x(t),t) = 7*x(t)+y(t), diff(y(t),t) = -4*x(t)+3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^{5t}(2c_2t + 2c_1 + c_2)}{4}$$

$$y(t) = e^{5t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 45

```
DSolve[{x'[t]== 7*x[t]+y[t],y'[t] == -4*x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -
```

$$x(t) \rightarrow e^{5t}(2c_1t + c_2t + c_1)$$
$$y(t) \rightarrow e^{5t}(c_2 - 2(2c_1 + c_2)t)$$

## 1.75 problem 75

Internal problem ID [6365]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 75.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t) + y(t)$$
$$y'(t) = y(t)$$
$$z'(t) = z(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 27

```
dsolve([diff(x(t),t)=x(t)+y(t),diff(y(t),t)=y(t),diff(z(t),t)=z(t)],[x(t), y(t), z(t)], sings
```

$$x(t) = (c_2 t + c_1) \, e^t$$

$$y(t) = c_2 e^t$$

$$z(t) = c_3 e^t$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 62

```
DSolve[{x'[t]== x[t]+y[t],y'[t] == y[t],z'[t]==z[t]},{x[t],y[t],z[t]},t,IncludeSingularSoluti
```

$$x(t) \rightarrow e^t(c_2 t + c_1)$$

$$y(t) \rightarrow c_2 e^t$$

$$z(t) \rightarrow c_3 e^t$$

$$x(t) \rightarrow e^t(c_2 t + c_1)$$

$$y(t) \rightarrow c_2 e^t$$

$$z(t) \rightarrow 0$$

## 1.76 problem 76

Internal problem ID [6366]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 76.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) &= 2x(t) + y(t) - z(t) \\
y'(t) &= -x(t) + 2z(t) \\
z'(t) &= -x(t) - 2y(t) + 4z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 59

```
dsolve([diff(x(t),t)=2*x(t)+y(t)-z(t),diff(y(t),t)=-x(t)+2*z(t),diff(z(t),t)=-x(t)-2*y(t)+4*z
```

$$
x(t) = -e^{2t}(2c_3 t + c_2 - 4c_3)
$$

$$
y(t) = e^{2t}\left(c_3 t^2 + c_2 t + c_1 - 2c_3\right)
$$

$$
z(t) = e^{2t}\left(c_3 t^2 + c_2 t + c_1\right)
$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 94

```
DSolve[{x'[t]== 2*x[t]+y[t]-z[t],y'[t] == -x[t]+2*z[t],z'[t]==-x[t]-2*y[t]+4*z[t]},{x[t],y[t]
```

$$
x(t) \rightarrow e^{2t}((c_2 - c_3)t + c_1)
$$

$$
y(t) \rightarrow \frac{1}{2}e^{2t}(-2c_1 t - (c_2 - c_3)t(t+4) + 2c_2)
$$

$$
z(t) \rightarrow \frac{1}{2}e^{2t}(-2c_1 t - c_2 t(t+4) + c_3(t(t+4) + 2))
$$

## 1.77 problem 77

Internal problem ID [6367]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 77.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$
x' - 4Ak\left(\frac{x}{A}\right)^{\frac{3}{4}} + 3kx = 0
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 83

```
dsolve(diff(x(t),t)=4*A*k*(x(t)/A)^(3/4)-3*k*x(t),x(t), singsol=all)
```

$$
t - \frac{-\frac{\ln(256A-81x(t))}{3} - \frac{\ln\left(9\sqrt{\frac{x(t)}{A}}-16\right)}{3} + \frac{\ln\left(9\sqrt{\frac{x(t)}{A}}+16\right)}{3} - \frac{2\ln\left(3\left(\frac{x(t)}{A}\right)^{\frac{1}{4}}-4\right)}{3} + \frac{2\ln\left(3\left(\frac{x(t)}{A}\right)^{\frac{1}{4}}+4\right)}{3}}{k}
$$
$$
+ c_1 = 0
$$

✓ Solution by Mathematica

Time used: 0.37 (sec). Leaf size: 51

```
DSolve[x'[t]==4*A*k*(x[t]/A)^(3/4)-3*k*x[t],x[t],t,IncludeSingularSolutions -> True]
```

$$
x(t) \to \frac{1}{81}Ae^{-3kt}\left(4e^{\frac{3kt}{4}} + e^{\frac{3c_1}{4}}\right)^4
$$
$$
x(t) \to 0
$$
$$
x(t) \to \frac{256A}{81}
$$

## 1.78   problem 78

Internal problem ID [6368]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 78.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{\dfrac{y'y}{1+\dfrac{\sqrt{1+y'^2}}{2}} + x = 0}$$

✓   Solution by Maple

Time used: 1.265 (sec). Leaf size: 198

```
dsolve(diff(y(x),x)*y(x)/(1+1/2*sqrt(1+diff(y(x),x)^2))=-x,y(x), singsol=all)
```

$$y(x) = -\sqrt{-x^2 + c_1}\left(1 + \frac{\sqrt{\frac{x^2}{-x^2+c_1}+1}}{2}\right)$$

$$y(x) = \sqrt{-x^2 + c_1}\left(1 + \frac{\sqrt{\frac{x^2}{-x^2+c_1}+1}}{2}\right)$$

$$y(x) = -\frac{\sqrt{-9x^2 + 15c_1 - 6\sqrt{-3c_1x^2 + 4c_1^2}}}{3}$$

$$y(x) = \frac{\sqrt{-9x^2 + 15c_1 - 6\sqrt{-3c_1x^2 + 4c_1^2}}}{3}$$

$$y(x) = -\frac{\sqrt{-9x^2 + 15c_1 + 6\sqrt{-3c_1x^2 + 4c_1^2}}}{3}$$

$$y(x) = \frac{\sqrt{-9x^2 + 15c_1 + 6\sqrt{-3c_1x^2 + 4c_1^2}}}{3}$$

✓  Solution by Mathematica

Time used: 2.035 (sec). Leaf size: 153

```
DSolve[y'[x]*y[x]/(1+1/2*Sqrt[1+(y'[x])^2])==-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(e^{c_1} - \sqrt{-9x^2 + 4e^{2c_1}}\right)$$

$$y(x) \to \frac{1}{3}\left(\sqrt{-9x^2 + 4e^{2c_1}} + e^{c_1}\right)$$

$$y(x) \to -\sqrt{-x^2 + 4e^{2c_1}} - e^{c_1}$$

$$y(x) \to \sqrt{-x^2 + 4e^{2c_1}} - e^{c_1}$$

$$y(x) \to -\sqrt{-x^2}$$

$$y(x) \to \sqrt{-x^2}$$

## 1.79 problem 78

Internal problem ID [6369]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 78.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type [[_homogeneous, 'class A'], _dAlembert]

$$\frac{y'y}{1 + \frac{\sqrt{1+y'^2}}{2}} + x = 0$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 2.625 (sec). Leaf size: 29

```
dsolve([diff(y(x),x)*y(x)/(1+1/2*sqrt(1+diff(y(x),x)^2))=-x,y(0) = 3],y(x), singsol=all)
```

$$y(x) = -3 + \sqrt{-x^2 + 36}$$
$$y(x) = 1 + \sqrt{-x^2 + 4}$$

✓ Solution by Mathematica

Time used: 0.526 (sec). Leaf size: 35

```
DSolve[{y'[x]*y[x]/(1+1/2*Sqrt[1+(y'[x])^2])==-x,y[0]==3},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \sqrt{4 - x^2} + 1$$
$$y(x) \to \sqrt{36 - x^2} - 3$$

## 1.80 problem 79

Internal problem ID [6370]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 79.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_separable]

$$
y' - \frac{y\left(1 + \frac{a^2 x}{\sqrt{a^2(x^2+1)}}\right)}{\sqrt{a^2(x^2+1)}} = 0
$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x) = y(x)*(1+ a^2*x/sqrt(a^2*(x^2+1)))/sqrt(a^2*(x^2+1)),y(x), singsol=all)
```

$$
y(x) = c_1 \left(\frac{a^2 x}{\sqrt{a^2}} + \sqrt{a^2 x^2 + a^2}\right)^{\frac{1}{\sqrt{a^2}}} \sqrt{x^2 + 1}
$$

✓ Solution by Mathematica
Time used: 0.339 (sec). Leaf size: 105

```
DSolve[y'[x]== y[x]*(1+ a^2*x/Sqrt[a^2*(x^2+1)])/Sqrt[a^2*(x^2+1)],y[x],x,IncludeSingularSolu
```

$$
y(x) \to
$$
$$
-\frac{\sqrt{a^2} c_1 \sqrt{a^2(x^2+1)}\left(a\left(-\sqrt{a^2(x^2+1)} + \sqrt{a^2} + ax\right)\right)^{-1/a}\left(a\left(\sqrt{a^2(x^2+1)} - \sqrt{a^2} + ax\right)\right)^{\frac{1}{a}}}{2a^4}
$$
$$
y(x) \to 0
$$

## 1.81 problem 80

Internal problem ID [6371]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 80.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$y' - y^2 - x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x)=x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(-\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right)c_1 - \operatorname{BesselY}\left(-\frac{3}{4}, \frac{x^2}{2}\right)\right)x}{c_1 \operatorname{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \operatorname{BesselY}\left(\frac{1}{4}, \frac{x^2}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.127 (sec). Leaf size: 93

```
DSolve[y'[x]==x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\left(-\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right) + c_1 \operatorname{BesselJ}\left(\frac{3}{4}, \frac{x^2}{2}\right)\right)}{\operatorname{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{x^2}{2}\right)}$$

$$y(x) \to \frac{x \operatorname{BesselJ}\left(\frac{3}{4}, \frac{x^2}{2}\right)}{\operatorname{BesselJ}\left(-\frac{1}{4}, \frac{x^2}{2}\right)}$$

## 1.82 problem 81

Internal problem ID [6372]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 81.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$y' - 2\sqrt{y} = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 5

```
dsolve([diff(y(x),x) = 2*sqrt(y(x)),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 8

```
DSolve[{y'[x]==2*Sqrt[y[x]],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2$$

## 1.83 problem 82

Internal problem ID [6373]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 82.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$z'' + 3z' + 2z - 24\,\mathrm{e}^{-3t} + 24\,\mathrm{e}^{-4t} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(z(t),t$2)+3*diff(z(t),t)+2*z(t)=24*(exp(-3*t)-exp(-4*t)),z(t), singsol=all)
```

$$z(t) = \left(-\mathrm{e}^{-t}c_1 - 4\,\mathrm{e}^{-3t} + 12\,\mathrm{e}^{-2t} + c_2\right)\mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 32

```
DSolve[z''[t]+3*z'[t]+2*z[t]==24*(Exp[-3*t]-Exp[-4*t]),z[t],t,IncludeSingularSolutions -> Tru
```

$$z(t) \to e^{-4t}\left(-4 + e^t\left(12 + e^t\left(c_2 e^t + c_1\right)\right)\right)$$

## 1.84 problem 83

Internal problem ID [6374]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 83.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$y' - \sqrt{1 - y^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=sqrt(1-y(x)^2),y(x), singsol=all)
```

$$y(x) = \sin(x + c_1)$$

✓ Solution by Mathematica

Time used: 0.174 (sec). Leaf size: 28

```
DSolve[y'[x]==Sqrt[1-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x + c_1)$$
$$y(x) \to -1$$
$$y(x) \to 1$$
$$y(x) \to \text{Interval}[\{-1, 1\}]$$

## 1.85 problem 84

Internal problem ID [6375]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 84.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_Riccati]

$$y' - x^2 - y^2 + 1 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 140

```
dsolve(diff(y(x),x)=x^2+y(x)^2-1,y(x), singsol=all)
```

$$y(x) = \frac{2c_1 \operatorname{WhittakerW}\left(1 + \frac{i}{4}, \frac{1}{4}, ix^2\right)}{x\left(c_1 \operatorname{WhittakerW}\left(\frac{i}{4}, \frac{1}{4}, ix^2\right) + \operatorname{WhittakerM}\left(\frac{i}{4}, \frac{1}{4}, ix^2\right)\right)}$$
$$- \frac{\left(2c_1 x^2 i - c_1 i - c_1\right)\operatorname{WhittakerW}\left(\frac{i}{4}, \frac{1}{4}, ix^2\right) + (3+i)\operatorname{WhittakerM}\left(1 + \frac{i}{4}, \frac{1}{4}, ix^2\right) + (2ix^2 - i - 1)\operatorname{Whit}}{2x\left(c_1 \operatorname{WhittakerW}\left(\frac{i}{4}, \frac{1}{4}, ix^2\right) + \operatorname{WhittakerM}\left(\frac{i}{4}, \frac{1}{4}, ix^2\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.195 (sec). Leaf size: 153

```
DSolve[y'[x]==x^2+y[x]^2-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to \frac{i\left(x \operatorname{ParabolicCylinderD}\left(-\frac{1}{2} - \frac{i}{2}, (-1+i)x\right) + (1+i)\operatorname{ParabolicCylinderD}\left(\frac{1}{2} - \frac{i}{2}, (-1+i)x\right) - c_1 x \operatorname{Pa}\right.}{\operatorname{ParabolicCylinderD}\left(-\frac{1}{2} - \frac{i}{2}, (-1+i)x\right) + c_1 \operatorname{Para}}$$

$$y(x) \to \frac{(1+i)\operatorname{ParabolicCylinderD}\left(\frac{1}{2} + \frac{i}{2}, (1+i)x\right)}{\operatorname{ParabolicCylinderD}\left(-\frac{1}{2} + \frac{i}{2}, (1+i)x\right)} - ix$$

## 1.86 problem 85

Internal problem ID [6376]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 85.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - 2y(x\sqrt{y} - 1) = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 9

```
dsolve([diff(y(x),x)= 2*y(x)*(x*sqrt(y(x)) - 1),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{(x+1)^2}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]==2*y[x]*(x*sqrt[y[x]-1]),{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

{}

## 1.87 problem 86

Internal problem ID [6377]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 86.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _exact, _nonlinear], [_2nd_order, _with_linear_sy

$$y'' - \frac{1}{y} + \frac{xy'}{y^2} = 0$$

✔ Solution by Maple
Time used: 0.016 (sec). Leaf size: 88

```
dsolve(diff(y(x),x$2)=1/y(x)-x/y(x)^2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(\_Z^2 - e^{\text{RootOf}\left(x^2\left(\tanh\left(\frac{\sqrt{c_1^2+4}\,(2c_2+\_Z+2\ln(x))}{2c_1}\right)\right)^2 c_1^2 + 4\tanh\left(\frac{\sqrt{c_1^2+4}\,(2c_2+\_Z+2\ln(x))}{2c_1}\right)^2 - c_1^2 - 4e^{-Z} - 4\right)} - 1 + \_Zc_1\right)x$$

✔ Solution by Mathematica
Time used: 0.187 (sec). Leaf size: 77

```
DSolve[y''[x]==1/y[x]-x/y[x]^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(-\frac{y(x)^2}{x^2} - \frac{c_1 y(x)}{x} + 1\right) - \frac{c_1 \arctan\left(\frac{\frac{2y(x)}{x}+c_1}{\sqrt{-4-c_1^2}}\right)}{\sqrt{-4-c_1^2}} = -\log(x)+c_2, y(x)\right]$$

## 1.88 problem 87

Internal problem ID [6378]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 87.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x]]

$$y'' + y' + y = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(0) = 0],y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[{y''[x]+y'[x]+y[x]==0,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x/2} \sin\left(\frac{\sqrt{3}x}{2}\right)$$

## 1.89 problem 88

Internal problem ID [6379]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 88.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y' + y = 0$$

With initial conditions

$$[y'(0) = 0]$$

✔ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve([diff(y(x),x$2)+diff(y(x),x)+y(x)=0,D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = e^{-\frac{x}{2}} c_1 \left( \sqrt{3} \cos\left( \frac{\sqrt{3}\,x}{2} \right) + \sin\left( \frac{\sqrt{3}\,x}{2} \right) \right)$$

✔ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 44

```
DSolve[{y''[x]+y'[x]+y[x]==0,{y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x/2} \left( \sin\left( \frac{\sqrt{3}x}{2} \right) + \sqrt{3} \cos\left( \frac{\sqrt{3}x}{2} \right) \right)$$

## 1.90 problem 88

Internal problem ID [6380]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 88.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y' + y = 0$$

With initial conditions

$$[y'(0) = 0, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve([diff(y(x),x$2)+diff(y(x),x)+y(x)=0,D(y)(0) = 0, y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{e^{-\frac{x}{2}}\left(\sqrt{3}\sin\left(\frac{\sqrt{3}\,x}{2}\right) + 3\cos\left(\frac{\sqrt{3}\,x}{2}\right)\right)}{3}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 47

```
DSolve[{y''[x]+y'[x]+y[x]==0,{y'[0]==0,y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}e^{-x/2}\left(\sqrt{3}\sin\left(\frac{\sqrt{3}x}{2}\right) + 3\cos\left(\frac{\sqrt{3}x}{2}\right)\right)$$

## 1.91   problem 89

Internal problem ID [6381]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 89.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _exact, _nonlinear], [_2nd_order, _reducible, _mu

$$y'' - yy' - 2x = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 227

```
dsolve(diff(y(x),x$2)-diff(y(x),x)*y(x)=2*x,y(x), singsol=all)
```

$$y(x) = \frac{4c_2 \operatorname{WhittakerW}\left(\frac{ic_1\sqrt{2}}{8} + 1, \frac{1}{4}, \frac{i\sqrt{2}\,x^2}{2}\right)}{x\left(c_2 \operatorname{WhittakerW}\left(\frac{ic_1\sqrt{2}}{8}, \frac{1}{4}, \frac{i\sqrt{2}\,x^2}{2}\right) + \operatorname{WhittakerM}\left(\frac{ic_1\sqrt{2}}{8}, \frac{1}{4}, \frac{i\sqrt{2}\,x^2}{2}\right)\right)}$$

$$+ \frac{\left(-2i\sqrt{2}\,c_2x^2 + i\sqrt{2}\,c_1c_2 + 2c_2\right)\operatorname{WhittakerW}\left(\frac{ic_1\sqrt{2}}{8}, \frac{1}{4}, \frac{i\sqrt{2}\,x^2}{2}\right) + \left(-ic_1\sqrt{2} - 6\right)\operatorname{WhittakerM}\left(\frac{ic_1\sqrt{2}}{8} + 1\right.}{2x\left(c_2 \operatorname{WhittakerW}\left(\frac{ic_1\sqrt{2}}{8}, \frac{1}{4}, \frac{i\sqrt{2}\,x^2}{2}\right) + \operatorname{WhittakerM}\right.}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 163

```
DSolve[y''[x]+y'[x]*y[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{2}\,x$$
$$-\frac{2\sqrt[4]{2}\left(\sqrt[4]{2}\,x\operatorname{ParabolicCylinderD}\left(\frac{1}{4}\left(-\sqrt{2}c_1 - 2\right), i\sqrt[4]{2}\,x\right) + i\operatorname{ParabolicCylinderD}\left(\frac{1}{4}\left(2 - \sqrt{2}c_1\right), i\sqrt[4]{2}\,x\right.\right.}{\operatorname{ParabolicCylinderD}\left(\frac{1}{4}\left(-\sqrt{2}c_1 - 2\right), i\sqrt[4]{2}\,x\right) + c_2\operatorname{ParabolicCylinderD}\left(\right.}$$

## 1.92 problem 90

Internal problem ID [6382]

**Book**: Own collection of miscellaneous problems
**Section**: section 1.0
**Problem number**: 90.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_Riccati]

$$y' - y^2 - x - x^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 280

```
dsolve(diff(y(x),x)-y(x)^2-x-x^2=0,y(x), singsol=all)
```

$y(x) =$

$$-\frac{(48c_1x^2i + 4c_1x^2 + 48c_1xi + 4c_1x + 12c_1i + c_1)\,\text{hypergeom}\left(\left[\frac{7}{4} - \frac{i}{16}\right], \left[\frac{5}{2}\right], \frac{i(2x+1)^2}{4}\right)}{24\left((2c_1x + c_1)\,\text{hypergeom}\left(\left[\frac{3}{4} - \frac{i}{16}\right], \left[\frac{3}{2}\right], \frac{i(2x+1)^2}{4}\right) + \text{hypergeom}\left(\left[\frac{1}{4} - \frac{i}{16}\right], \left[\frac{1}{2}\right], \frac{i(2x+1)^2}{4}\right)\right)}$$

$$-\frac{(24ix + 6x + 12i + 3)\,\text{hypergeom}\left(\left[\frac{5}{4} - \frac{i}{16}\right], \left[\frac{3}{2}\right], \frac{i(2x+1)^2}{4}\right)}{24\left((2c_1x + c_1)\,\text{hypergeom}\left(\left[\frac{3}{4} - \frac{i}{16}\right], \left[\frac{3}{2}\right], \frac{i(2x+1)^2}{4}\right) + \text{hypergeom}\left(\left[\frac{1}{4} - \frac{i}{16}\right], \left[\frac{1}{2}\right], \frac{i(2x+1)^2}{4}\right)\right)}$$

$$-\frac{(-48c_1x^2i - 48c_1xi - 12c_1i + 48c_1)\,\text{hypergeom}\left(\left[\frac{3}{4} - \frac{i}{16}\right], \left[\frac{3}{2}\right], \frac{i(2x+1)^2}{4}\right) + (-24ix - 12i)\,\text{hypergeom}\left(}{24\left((2c_1x + c_1)\,\text{hypergeom}\left(\left[\frac{3}{4} - \frac{i}{16}\right], \left[\frac{3}{2}\right], \frac{i(2x+1)^2}{4}\right) + \text{hypergeom}\left(\left[\frac{1}{4} - \frac{i}{16}\right], \left[\frac{1}{2}\right], \frac{i(2x}\right.}$$

✓ Solution by Mathematica

Time used: 0.202 (sec). Leaf size: 233

```
DSolve[y'[x]-y[x]^2-x-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\to \frac{i\left((2x+1)\operatorname{ParabolicCylinderD}\left(-\frac{1}{2}-\frac{i}{8},\left(-\frac{1}{2}+\frac{i}{2}\right)(2x+1)\right)+(2+2i)\operatorname{ParabolicCylinderD}\left(\frac{1}{2}-\frac{i}{8},\left(-\frac{1}{2}\right.\right.}{2\left(\operatorname{ParabolicCylinderD}\left(-\frac{1}{2}-\frac{i}{8},\left(-\right.\right.}$$

$$y(x) \to \frac{(1+i)\operatorname{ParabolicCylinderD}\left(\frac{1}{2}+\frac{i}{8},(1+i)x+\left(\frac{1}{2}+\frac{i}{2}\right)\right)}{\operatorname{ParabolicCylinderD}\left(-\frac{1}{2}+\frac{i}{8},(1+i)x+\left(\frac{1}{2}+\frac{i}{2}\right)\right)} - \frac{1}{2}i(2x+1)$$

# 2 section 2.0

## 2.1 problem 1

Internal problem ID [6383]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y'x - yx - x = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x=0,y(x), singsol=all)
```

$$y(x) = e^{-x}(x+2)c_2 + \left(i\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\sqrt{\pi}\,\sqrt{2}\,(x+2)\,e^{-x-2} + 2\,e^{\frac{x(x+2)}{2}}\right)c_1 - 1$$

✓ Solution by Mathematica

Time used: 1.95 (sec). Leaf size: 104

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{2}e^{-x}(x+2)\left(\int_1^x \frac{1}{2}e^{K[1]}K[1]\left(\sqrt{2} - 2\,\mathrm{DawsonF}\left(\frac{K[1]+2}{\sqrt{2}}\right)(K[1]+2)\right)dK[1] \right.$$
$$\left. + c_1\right) - \left(c_2 e^{\frac{x^2}{2}+x+2} + x + 1\right)\left(\sqrt{2}(x+2)\,\mathrm{DawsonF}\left(\frac{x+2}{\sqrt{2}}\right) - 1\right)$$

## 2.2   problem 2

Internal problem ID [6384]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - y'x - yx - 2x = 0$$

✓   Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-2*x=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}(x+2)\,c_2 + \left(i\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\sqrt{\pi}\,\sqrt{2}\,(x+2)\,\mathrm{e}^{-x-2} + 2\,\mathrm{e}^{\frac{x(x+2)}{2}}\right)c_1 - 2$$

✓   Solution by Mathematica

Time used: 0.426 (sec). Leaf size: 122

```
DSolve[y''[x]-x*y'[x]-x*y[x]-2*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{2}e^{-x}(x+2)\left(\int_1^x e^{K[1]}K[1]\left(\sqrt{2} - 2\,\mathrm{DawsonF}\left(\frac{K[1]+2}{\sqrt{2}}\right)(K[1]+2)\right)dK[1]\right.$$
$$\left. - \frac{1}{2}\sqrt{\pi}c_2\mathrm{erfi}\left(\frac{x+2}{\sqrt{2}}\right) + c_1\right)$$
$$- 2(x+1)\left(\sqrt{2}(x+2)\,\mathrm{DawsonF}\left(\frac{x+2}{\sqrt{2}}\right) - 1\right) + c_2 e^{\frac{x^2}{2}+x+2}$$

## 2.3 problem 3

Internal problem ID [6385]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - y'x - yx - 3x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-3*x=0,y(x), singsol=all)
```

$$y(x) = e^{-x}(x+2)c_2 + \left(i\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\sqrt{\pi}\,\sqrt{2}\,(x+2)\,e^{-x-2} + 2\,e^{\frac{x(x+2)}{2}}\right)c_1 - 3$$

✓ Solution by Mathematica

Time used: 0.638 (sec). Leaf size: 125

```
DSolve[y''[x]-x*y'[x]-x*y[x]-3*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{2}e^{-x}(x+2)\left(\int_1^x \frac{3}{2}e^{K[1]}K[1]\left(\sqrt{2} - 2\operatorname{DawsonF}\left(\frac{K[1]+2}{\sqrt{2}}\right)(K[1]+2)\right)dK[1]\right.$$
$$\left. - \frac{1}{2}\sqrt{\pi}c_2\operatorname{erfi}\left(\frac{x+2}{\sqrt{2}}\right) + c_1\right)$$
$$- 3(x+1)\left(\sqrt{2}(x+2)\operatorname{DawsonF}\left(\frac{x+2}{\sqrt{2}}\right) - 1\right) + c_2 e^{\frac{x^2}{2}+x+2}$$

## 2.4 problem 4

Internal problem ID [6386]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y'x - yx - x^2 - x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 57

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x^2-x=0,y(x), singsol=all)
```

$$y(x) = e^{-x}(x + 2) c_2 + \left( i \operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right) \sqrt{\pi}\,\sqrt{2}\,(x+2)\,e^{-x-2} + 2\,e^{\frac{x(x+2)}{2}} \right) c_1 - x$$

✓ Solution by Mathematica

Time used: 0.921 (sec). Leaf size: 72

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x^2-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x} \left( -\sqrt{2\pi} c_2 (x+2)\operatorname{erfi}\left(\frac{x+2}{\sqrt{2}}\right) - 2e^x x + 2\sqrt{2} c_1 (x+2) + 2c_2 e^{\frac{1}{2}(x+2)^2} \right)$$

## 2.5   problem 5

Internal problem ID [6387]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - y'x - yx - x^3 + 2 = 0$$

✓  Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x^3+2=0,y(x), singsol=all)
```

$$y(x) = e^{-x}(x+2)\, c_2 + \left( i \operatorname{erf}\left( \frac{i\sqrt{2}\,(x+2)}{2} \right) \sqrt{\pi}\,\sqrt{2}\,(x+2)\, e^{-x-2} + 2\, e^{\frac{x(x+2)}{2}} \right) c_1 - x^2 + 2x - 2$$

✓  Solution by Mathematica

Time used: 2.335 (sec). Leaf size: 78

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x^3+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x} \left( -\sqrt{2\pi}\, c_2 (x+2) \operatorname{erfi}\left( \frac{x+2}{\sqrt{2}} \right) - 2e^x((x-2)x+2) + 2\sqrt{2}\, c_1 (x+2) + 2c_2 e^{\frac{1}{2}(x+2)^2} \right)$$

## 2.6 problem 6

Internal problem ID [6388]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$ y'' - y'x - yx - x^4 - 6 = 0 $$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x^4-6=0,y(x), singsol=all)
```

$$ y(x) = e^{-x}(x+2) c_2 + \left( i\,\mathrm{erf}\left( \frac{i\sqrt{2}\,(x+2)}{2} \right) \sqrt{\pi}\,\sqrt{2}\,(x+2)\,e^{-x-2} \right. $$
$$ \left. + 2\,e^{\frac{x(x+2)}{2}} \right) c_1 - x^3 + 3x^2 - 6x $$

✓ Solution by Mathematica

Time used: 3.34 (sec). Leaf size: 79

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x^4-6==0,y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to \frac{1}{2}e^{-x}\left( -\sqrt{2\pi}c_2(x+2)\mathrm{erfi}\left( \frac{x+2}{\sqrt{2}} \right) - 2e^x x((x-3)x+6) + 2\sqrt{2}c_1(x+2) + 2c_2 e^{\frac{1}{2}(x+2)^2} \right) $$

## 2.7 problem 7

Internal problem ID [6389]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - y'x - yx - x^5 + 24 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 73

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x^5+24=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}(x+2)\, c_2 + \left( i\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\sqrt{\pi}\,\sqrt{2}\,(x+2)\,\mathrm{e}^{-x-2} + 2\,\mathrm{e}^{\frac{x(x+2)}{2}}\right) c_1$$
$$- x^4 + 4x^3 - 12x^2 + 12x + 12$$

✓ Solution by Mathematica

Time used: 1.378 (sec). Leaf size: 86

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x^5+24==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2(x+2)\operatorname{erfi}\left(\frac{x+2}{\sqrt{2}}\right) + e^x(24 - 2x(x((x-4)x+12) - 12))\right.$$
$$\left. + 2\sqrt{2}c_1(x+2) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

## 2.8 problem 8

Internal problem ID [6390]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - y'x - yx - x = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x=0,y(x), singsol=all)
```

$$y(x) = e^{-x}(x+2)\, c_2 + \left( i \operatorname{erf}\left( \frac{i\sqrt{2}\,(x+2)}{2} \right) \sqrt{\pi}\,\sqrt{2}\,(x+2)\, e^{-x-2} + 2\, e^{\frac{x(x+2)}{2}} \right) c_1 - 1$$

✓ Solution by Mathematica

Time used: 0.197 (sec). Leaf size: 104

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{2}e^{-x}(x+2) \left( \int_1^x \frac{1}{2} e^{K[1]} K[1] \left( \sqrt{2} - 2\operatorname{DawsonF}\left( \frac{K[1]+2}{\sqrt{2}} \right) (K[1]+2) \right) dK[1] \right.$$
$$\left. + c_1 \right) - \left( c_2 e^{\frac{x^2}{2}+x+2} + x + 1 \right) \left( \sqrt{2}(x+2)\operatorname{DawsonF}\left( \frac{x+2}{\sqrt{2}} \right) - 1 \right)$$

## 2.9 problem 9

Internal problem ID [6391]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - y'x - yx - x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x^2=0,y(x), singsol=all)
```

$$y(x) = e^{-x}(x+2) c_2 + \left( i \operatorname{erf}\left( \frac{i\sqrt{2}\,(x+2)}{2} \right) \sqrt{\pi}\,\sqrt{2}\,(x+2)\, e^{-x-2} + 2\, e^{\frac{x(x+2)}{2}} \right) c_1 + 1 - x$$

✓ Solution by Mathematica

Time used: 1.778 (sec). Leaf size: 140

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \left( \sqrt{2}(x+2) \int_1^x \frac{1}{2} e^{K[1]} K[1]^2 \left( \sqrt{2} - 2\, \text{DawsonF}\left( \frac{K[1]+2}{\sqrt{2}} \right) (K[1]+2) \right) dK[1] \right.$$
$$\left. + e^x \left( x^2 + x + 1 \right) + \sqrt{2}c_1(x+2) + c_2 e^{\frac{1}{2}(x+2)^2} \right)$$
$$- \sqrt{2}(x+2) \left( x^2 + c_2 e^{\frac{x^2}{2}+x+2} + x + 1 \right) \text{DawsonF}\left( \frac{x+2}{\sqrt{2}} \right)$$

## 2.10 problem 10

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - y'x - yx - x^3 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 243

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x^3=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}(x+2)\,c_2 + \left(i\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\sqrt{\pi}\,\sqrt{2}\,(x+2)\,\mathrm{e}^{-x-2} + 2\,\mathrm{e}^{\frac{x(x+2)}{2}}\right)c_1$$

$$+\frac{\left(\mathrm{e}^{\frac{(x+2)^2}{2}}(x+2)\left(\int\left(i\sqrt{\pi}\,\sqrt{2}\,(x+2)\,\mathrm{e}^{-2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right) + 2\,\mathrm{e}^{\frac{x(x+4)}{2}}\right)\mathrm{e}^{-\frac{x(x+2)}{2}}x^3 dx\right) - 2i\,\mathrm{erf}\left(\frac{\sqrt{2}\,(x+1)}{2}\right)\pi(}{}$$

✓  Solution by Mathematica

Time used: 2.878 (sec). Leaf size: 237

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \rightarrow \frac{1}{2} e^{-\frac{1}{2}(x+2)^2} \Bigg( \sqrt{2}(x
$$
$$
+ 2) \left( 2 e^{\frac{x^2}{2}+x+2} \int_1^x \frac{1}{2} e^{K[1]} K[1]^3 \left( \sqrt{2} - 2\,\mathrm{DawsonF}\left( \frac{K[1]+2}{\sqrt{2}} \right) (K[1]+2) \right) dK[1]
$$
$$
- \sqrt{\pi} x \left(x^2 + x + 2\right) \mathrm{erfi}\left( \frac{x+2}{\sqrt{2}} \right) \right)
$$
$$
- 2 e^{(x+1)(x+2)} \left( \sqrt{e\pi}\left( \sqrt{2} - 2(x+2)\,\mathrm{DawsonF}\left( \frac{x+2}{\sqrt{2}} \right) \right) \mathrm{erf}\left( \frac{x+1}{\sqrt{2}} \right) \right.
$$
$$
\left. + \sqrt{2} e^2 c_2 (x+2)\,\mathrm{DawsonF}\left( \frac{x+2}{\sqrt{2}} \right) \right)
$$
$$
+ 2 e^{\frac{x^2}{2}+x+2} \left( e^x x \left(x^2 + x + 2\right) + \sqrt{2} c_1 (x+2) + c_2 e^{\frac{1}{2}(x+2)^2} \right) \Bigg)
$$

## 2.11 problem 11

Internal problem ID [6393]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - axy' - bxy - cx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 83

```
dsolve(diff(y(x),x$2)-a*x*diff(y(x),x)-b*x*y(x)-c*x=0,y(x), singsol=all)
```

$$y(x) = e^{-\frac{bx}{a}} \operatorname{KummerM}\left(-\frac{b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x + 2b)^2}{2a^3}\right) c_2$$

$$+ e^{-\frac{bx}{a}} \operatorname{KummerU}\left(-\frac{b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x + 2b)^2}{2a^3}\right) c_1 - \frac{c}{b}$$

✓ Solution by Mathematica

Time used: 2.512 (sec). Leaf size: 565

```
DSolve[y''[x]-a*x*y'[x]-b*x*y[x]-c*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\rightarrow e^{-\frac{bx}{a}} \left( \operatorname{HermiteH}\left(\frac{b^2}{a^3}, \frac{xa^2 + 2b}{\sqrt{2}a^{3/2}}\right) \int_1^x \frac{a^4 c e^{\frac{bK[1]}{a}}}{b^2 \left(\sqrt{2}\operatorname{HermiteH}\left(\frac{b^2}{a^3} - 1, \frac{K[1]a^2 + 2b}{\sqrt{2}a^{3/2}}\right)\operatorname{Hypergeometric1F1}\left(-\frac{b^2}{2a^3}, \frac{1}{2}\right)} \right.$$

## 2.12 problem 12

Internal problem ID [6394]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - axy' - bxy - c\,x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 88

```
dsolve(diff(y(x),x$2)-a*x*diff(y(x),x)-b*x*y(x)-c*x^2=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{bx}{a}} \operatorname{KummerM}\left(-\frac{b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x+2b)^2}{2a^3}\right) c_2$$
$$+ \mathrm{e}^{-\frac{bx}{a}} \operatorname{KummerU}\left(-\frac{b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x+2b)^2}{2a^3}\right) c_1 + \frac{c(-bx+a)}{b^2}$$

✓ Solution by Mathematica

Time used: 1.963 (sec). Leaf size: 569

```
DSolve[y''[x]-a*x*y'[x]-b*x*y[x]-c*x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$\to e^{-\frac{bx}{a}}\left( \operatorname{HermiteH}\left(\frac{b^2}{a^3}, \frac{xa^2+2b}{\sqrt{2}a^{3/2}}\right) \int_1^x \frac{a^4 c e^{\frac{bK[1]}{a}}}{b^2\left(\sqrt{2}\operatorname{HermiteH}\left(\frac{b^2}{a^3}-1, \frac{K[1]a^2+2b}{\sqrt{2}a^{3/2}}\right) \operatorname{Hypergeometric1F1}\left(-\frac{b^2}{2a^3}, \frac{1}{2}\right.}\right.$

## 2.13 problem 13

Internal problem ID [6395]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - axy' - bxy - x^3 c = 0$$

✔ Solution by Maple

Time used: 0.015 (sec). Leaf size: 522

```
dsolve(diff(y(x),x$2)-a*x*diff(y(x),x)-b*x*y(x)-c*x^3=0,y(x), singsol=all)
```

$y(x)$

$$= \mathrm{e}^{-\frac{bx}{a}} \mathrm{KummerM}\left(-\frac{b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x + 2b)^2}{2a^3}\right) c_2 + \mathrm{e}^{-\frac{bx}{a}} \mathrm{KummerU}\left(-\frac{b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x + 2b)^2}{2a^3}\right) c_1$$

$$+ \frac{2a^4 c}{} \left(\left(\int \frac{(a^2x+2b)x^3 \mathrm{KummerU}\left(-\frac{b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x+2b)^2}{2a^3}\right) \mathrm{e}^{\frac{bx}{a}}}{\mathrm{KummerM}\left(-\frac{b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x+2b)^2}{2a^3}\right)(a^3-b^2)\mathrm{KummerU}\left(\frac{2a^3-b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x+2b)^2}{2a^3}\right) - 2\mathrm{KummerU}\left(-\frac{b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x+2b)^2}{2a^3}\right)\mathrm{Kummer}}$$

✔ Solution by Mathematica

Time used: 1.053 (sec). Leaf size: 569

```
DSolve[y''[x]-a*x*y'[x]-b*x*y[x]-c*x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\rightarrow e^{-\frac{bx}{a}}\left(\mathrm{HermiteH}\left(\frac{b^2}{a^3}, \frac{xa^2 + 2b}{\sqrt{2}a^{3/2}}\right)\int_1^x \frac{1}{b^2\left(\sqrt{2}\,\mathrm{HermiteH}\left(\frac{b^2}{a^3} - 1, \frac{K[1]a^2+2b}{\sqrt{2}a^{3/2}}\right)\mathrm{Hypergeometric1F1}\left(-\frac{b^2}{2a^3}, \frac{1}{2}\right)} \quad \frac{a^4 c e^{\frac{bK[1]}{a}}}{}$$

## 2.14 problem 14

Internal problem ID [6396]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y' - yx - x = 0$$

✔ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-x*y(x)-x=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{2}} \, \mathrm{AiryAi}\left(\frac{1}{4} + x\right) c_2 + e^{\frac{x}{2}} \, \mathrm{AiryBi}\left(\frac{1}{4} + x\right) c_1 - 1$$

✔ Solution by Mathematica

Time used: 6.656 (sec). Leaf size: 99

```
DSolve[y''[x]-y'[x]-x*y[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/2}\left(\mathrm{AiryAi}\left(x + \frac{1}{4}\right)\int_1^x -e^{-\frac{K[1]}{2}}\pi\,\mathrm{AiryBi}\left(K[1] + \frac{1}{4}\right)K[1]dK[1]\right.$$
$$\left. + \mathrm{AiryBi}\left(x + \frac{1}{4}\right)\int_1^x e^{-\frac{K[2]}{2}}\pi\,\mathrm{AiryAi}\left(K[2] + \frac{1}{4}\right)K[2]dK[2] + c_1\,\mathrm{AiryAi}\left(x + \frac{1}{4}\right)\right.$$
$$\left. + c_2\,\mathrm{AiryBi}\left(x + \frac{1}{4}\right)\right)$$

## 2.15 problem 15

Internal problem ID [6397]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 15.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - y' - yx - x^2 = 0$$

✔ Solution by Maple

Time used: 0.0 (sec). Leaf size: 71

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-x*y(x)-x^2=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{2}} \operatorname{AiryAi}\left(\frac{1}{4} + x\right) c_2 + e^{\frac{x}{2}} \operatorname{AiryBi}\left(\frac{1}{4} + x\right) c_1$$
$$+ \pi e^{\frac{x}{2}} \left( -\left( \int x^2 \operatorname{AiryBi}\left(\frac{1}{4} + x\right) e^{-\frac{x}{2}} dx \right) \operatorname{AiryAi}\left(\frac{1}{4} + x\right) \right.$$
$$\left. + \left( \int x^2 \operatorname{AiryAi}\left(\frac{1}{4} + x\right) e^{-\frac{x}{2}} dx \right) \operatorname{AiryBi}\left(\frac{1}{4} + x\right) \right)$$

✔ Solution by Mathematica

Time used: 4.768 (sec). Leaf size: 103

```
DSolve[y''[x]-y'[x]-x*y[x]-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/2} \left( \operatorname{AiryAi}\left(x + \frac{1}{4}\right) \int_1^x -e^{-\frac{K[1]}{2}} \pi \operatorname{AiryBi}\left(K[1] + \frac{1}{4}\right) K[1]^2 dK[1] \right.$$
$$+ \operatorname{AiryBi}\left(x + \frac{1}{4}\right) \int_1^x e^{-\frac{K[2]}{2}} \pi \operatorname{AiryAi}\left(K[2] + \frac{1}{4}\right) K[2]^2 dK[2]$$
$$\left. + c_1 \operatorname{AiryAi}\left(x + \frac{1}{4}\right) + c_2 \operatorname{AiryBi}\left(x + \frac{1}{4}\right) \right)$$

## 2.16 problem 16

Internal problem ID [6398]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 16.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - y' - yx - x^2 - 1 = 0$$

✔ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-x*y(x)-x^2-1=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{2}} \operatorname{AiryAi}\left(\frac{1}{4} + x\right) c_2 + e^{\frac{x}{2}} \operatorname{AiryBi}\left(\frac{1}{4} + x\right) c_1 - x$$

✔ Solution by Mathematica

Time used: 2.036 (sec). Leaf size: 107

```
DSolve[y''[x]-y'[x]-x*y[x]-x^2-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to e^{x/2}\Bigg( &\operatorname{AiryAi}\left(x + \frac{1}{4}\right)\int_1^x -e^{-\frac{K[1]}{2}} \pi \operatorname{AiryBi}\left(K[1] + \frac{1}{4}\right)\left(K[1]^2 + 1\right) dK[1] \\
&+ \operatorname{AiryBi}\left(x + \frac{1}{4}\right)\int_1^x e^{-\frac{K[2]}{2}} \pi \operatorname{AiryAi}\left(K[2] + \frac{1}{4}\right)\left(K[2]^2 + 1\right) dK[2] \\
&+ c_1 \operatorname{AiryAi}\left(x + \frac{1}{4}\right) + c_2 \operatorname{AiryBi}\left(x + \frac{1}{4}\right)\Bigg)
\end{aligned}
$$

## 2.17 problem 16

Internal problem ID [6399]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 16.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - y' - yx - x^2 - 1 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-x*y(x)-x^2-1=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{2}} \operatorname{AiryAi}\left(\frac{1}{4} + x\right) c_2 + e^{\frac{x}{2}} \operatorname{AiryBi}\left(\frac{1}{4} + x\right) c_1 - x$$

✓ Solution by Mathematica

Time used: 0.498 (sec). Leaf size: 107

```
DSolve[y''[x]-y'[x]-x*y[x]-x^2-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to e^{x/2} \bigg( &\operatorname{AiryAi}\left(x + \frac{1}{4}\right) \int_1^x -e^{-\frac{K[1]}{2}} \pi \operatorname{AiryBi}\left(K[1] + \frac{1}{4}\right) \left(K[1]^2 + 1\right) dK[1] \\
&+ \operatorname{AiryBi}\left(x + \frac{1}{4}\right) \int_1^x e^{-\frac{K[2]}{2}} \pi \operatorname{AiryAi}\left(K[2] + \frac{1}{4}\right) \left(K[2]^2 + 1\right) dK[2] \\
&+ c_1 \operatorname{AiryAi}\left(x + \frac{1}{4}\right) + c_2 \operatorname{AiryBi}\left(x + \frac{1}{4}\right) \bigg)
\end{aligned}
$$

## 2.18 problem 17

Internal problem ID [6400]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 17.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y' - yx - x^2 - 2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-x*y(x)-x^2-2=0,y(x), singsol=all)
```

$$y(x) = e^x \operatorname{AiryAi}(x + 1) c_2 + e^x \operatorname{AiryBi}(x + 1) c_1 - x$$

✓ Solution by Mathematica

Time used: 2.801 (sec). Leaf size: 87

```
DSolve[y''[x]-2*y'[x]-x*y[x]-x^2-2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x \Bigg( \operatorname{AiryAi}(x+1) \int_1^x -e^{-K[1]} \pi \operatorname{AiryBi}(K[1]+1) \left( K[1]^2 + 2 \right) dK[1]$$

$$+ \operatorname{AiryBi}(x+1) \int_1^x e^{-K[2]} \pi \operatorname{AiryAi}(K[2]+1) \left( K[2]^2 + 2 \right) dK[2] + c_1 \operatorname{AiryAi}(x+1)$$

$$+ c_2 \operatorname{AiryBi}(x+1) \Bigg)$$

## 2.19 problem 18

Internal problem ID [6401]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 18.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - 4y' - yx - x^2 - 4 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)-x*y(x)-x^2-4=0,y(x), singsol=all)
```

$$y(x) = e^{2x} \operatorname{AiryAi}(x + 4) c_2 + e^{2x} \operatorname{AiryBi}(x + 4) c_1 - x$$

✓ Solution by Mathematica

Time used: 2.977 (sec). Leaf size: 89

```
DSolve[y''[x]-4*y'[x]-x*y[x]-x^2-4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to e^{2x} \Bigg( & \operatorname{AiryAi}(x + 4) \int_1^x -e^{-2K[1]} \pi \operatorname{AiryBi}(K[1] + 4) \left( K[1]^2 + 4 \right) dK[1] \\
& + \operatorname{AiryBi}(x + 4) \int_1^x e^{-2K[2]} \pi \operatorname{AiryAi}(K[2] + 4) \left( K[2]^2 + 4 \right) dK[2] + c_1 \operatorname{AiryAi}(x + 4) \\
& + c_2 \operatorname{AiryBi}(x + 4) \Bigg)
\end{aligned}
$$

## 2.20 problem 19

Internal problem ID [6402]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 19.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - y' - yx - x^3 + 1 = 0$$

✔ Solution by Maple

Time used: 0.0 (sec). Leaf size: 75

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-x*y(x)-x^3+1=0,y(x), singsol=all)
```

$$
y(x) = e^{\frac{x}{2}} \operatorname{AiryAi}\left(\frac{1}{4} + x\right) c_2 + e^{\frac{x}{2}} \operatorname{AiryBi}\left(\frac{1}{4} + x\right) c_1
$$
$$
+ \pi e^{\frac{x}{2}} \left(-\left(\int (x^3 - 1) \operatorname{AiryBi}\left(\frac{1}{4} + x\right) e^{-\frac{x}{2}} dx\right) \operatorname{AiryAi}\left(\frac{1}{4} + x\right)\right.
$$
$$
\left. + \left(\int (x^3 - 1) \operatorname{AiryAi}\left(\frac{1}{4} + x\right) e^{-\frac{x}{2}} dx\right) \operatorname{AiryBi}\left(\frac{1}{4} + x\right)\right)
$$

✔ Solution by Mathematica

Time used: 1.781 (sec). Leaf size: 107

```
DSolve[y''[x]-y'[x]-x*y[x]-x^3+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to e^{x/2} \left( \operatorname{AiryAi}\left(x + \frac{1}{4}\right) \int_1^x -e^{-\frac{K[1]}{2}} \pi \operatorname{AiryBi}\left(K[1] + \frac{1}{4}\right) (K[1]^3 - 1) \, dK[1] \right.
$$
$$
+ \operatorname{AiryBi}\left(x + \frac{1}{4}\right) \int_1^x e^{-\frac{K[2]}{2}} \pi \operatorname{AiryAi}\left(K[2] + \frac{1}{4}\right) (K[2]^3 - 1) \, dK[2]
$$
$$
\left. + c_1 \operatorname{AiryAi}\left(x + \frac{1}{4}\right) + c_2 \operatorname{AiryBi}\left(x + \frac{1}{4}\right) \right)
$$

## 2.21 problem 20

Internal problem ID [6403]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 20.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - 2y' - yx - x^3 - x^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-x*y(x)-x^3-x^2=0,y(x), singsol=all)
```

$$y(x) = e^x \operatorname{AiryAi}(x+1) c_2 + e^x \operatorname{AiryBi}(x+1) c_1 - x^2 - x + 4$$

✓ Solution by Mathematica

Time used: 4.164 (sec). Leaf size: 91

```
DSolve[y''[x]-2*y'[x]-x*y[x]-x^3-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
y(x) \to e^x \Bigg( &\operatorname{AiryAi}(x+1) \int_1^x -e^{-K[1]} \pi \operatorname{AiryBi}(K[1]+1) K[1]^2 (K[1]+1) dK[1] \\
&+ \operatorname{AiryBi}(x+1) \int_1^x e^{-K[2]} \pi \operatorname{AiryAi}(K[2]+1) K[2]^2 (K[2]+1) dK[2] \\
&+ c_1 \operatorname{AiryAi}(x+1) + c_2 \operatorname{AiryBi}(x+1) \Bigg)
\end{aligned}$$

## 2.22 problem 21

Internal problem ID [6404]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 21.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - y' - yx - x^3 + 2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-x*y(x)-x^3+2=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{2}} \text{AiryAi}\left(\frac{1}{4} + x\right) c_2 + e^{\frac{x}{2}} \text{AiryBi}\left(\frac{1}{4} + x\right) c_1 - x^2 + 2$$

✓ Solution by Mathematica

Time used: 1.757 (sec). Leaf size: 107

```
DSolve[y''[x]-y'[x]-x*y[x]-x^3+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \rightarrow e^{x/2} \Bigg( &\text{AiryAi}\left(x + \frac{1}{4}\right) \int_1^x -e^{-\frac{K[1]}{2}} \pi \, \text{AiryBi}\left(K[1] + \frac{1}{4}\right) \left(K[1]^3 - 2\right) dK[1] \\
&+ \text{AiryBi}\left(x + \frac{1}{4}\right) \int_1^x e^{-\frac{K[2]}{2}} \pi \, \text{AiryAi}\left(K[2] + \frac{1}{4}\right) \left(K[2]^3 - 2\right) dK[2] \\
&+ c_1 \text{AiryAi}\left(x + \frac{1}{4}\right) + c_2 \text{AiryBi}\left(x + \frac{1}{4}\right) \Bigg)
\end{aligned}
$$

## 2.23   problem 22

Internal problem ID [6405]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 22.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' - yx - x^3 + 2 = 0$$

✓   Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-x*y(x)-x^3+2=0,y(x), singsol=all)
```

$$y(x) = e^x \operatorname{AiryAi}(x+1) c_2 + e^x \operatorname{AiryBi}(x+1) c_1 - x^2 + 4$$

✓   Solution by Mathematica

Time used: 1.147 (sec). Leaf size: 87

```
DSolve[y''[x]-2*y'[x]-x*y[x]-x^3+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to e^x \bigg( & \operatorname{AiryAi}(x+1) \int_1^x -e^{-K[1]} \pi \operatorname{AiryBi}(K[1]+1) \left(K[1]^3 - 2\right) dK[1] \\
& + \operatorname{AiryBi}(x+1) \int_1^x e^{-K[2]} \pi \operatorname{AiryAi}(K[2]+1) \left(K[2]^3 - 2\right) dK[2] + c_1 \operatorname{AiryAi}(x+1) \\
& \hspace{8cm} + c_2 \operatorname{AiryBi}(x+1) \bigg)
\end{aligned}
$$

## 2.24 problem 23

Internal problem ID [6406]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 23.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y' - yx - x^3 + 2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)-x*y(x)-x^3+2=0,y(x), singsol=all)
```

$$y(x) = e^{2x} \operatorname{AiryAi}(x + 4) c_2 + e^{2x} \operatorname{AiryBi}(x + 4) c_1 - x^2 + 8$$

✓ Solution by Mathematica

Time used: 1.209 (sec). Leaf size: 89

```
DSolve[y''[x]-4*y'[x]-x*y[x]-x^3+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to e^{2x} \bigg( &\operatorname{AiryAi}(x+4) \int_1^x -e^{-2K[1]} \pi \operatorname{AiryBi}(K[1]+4) \left(K[1]^3 - 2\right) dK[1] \\
&+ \operatorname{AiryBi}(x+4) \int_1^x e^{-2K[2]} \pi \operatorname{AiryAi}(K[2]+4) \left(K[2]^3 - 2\right) dK[2] + c_1 \operatorname{AiryAi}(x+4) \\
&+ c_2 \operatorname{AiryBi}(x+4) \bigg)
\end{aligned}
$$

## 2.25 problem 24

Internal problem ID [6407]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 24.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - 6y' - yx - x^3 + 2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)-x*y(x)-x^3+2=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{3x} \operatorname{AiryAi}(9 + x) c_2 + \mathrm{e}^{3x} \operatorname{AiryBi}(9 + x) c_1 - x^2 + 12$$

✓ Solution by Mathematica

Time used: 3.215 (sec). Leaf size: 89

```
DSolve[y''[x]-6*y'[x]-x*y[x]-x^3+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to e^{3x} \bigg( &\operatorname{AiryAi}(x + 9) \int_1^x -e^{-3K[1]} \pi \operatorname{AiryBi}(K[1] + 9) \left(K[1]^3 - 2\right) dK[1] \\
&+ \operatorname{AiryBi}(x + 9) \int_1^x e^{-3K[2]} \pi \operatorname{AiryAi}(K[2] + 9) \left(K[2]^3 - 2\right) dK[2] + c_1 \operatorname{AiryAi}(x + 9) \\
&+ c_2 \operatorname{AiryBi}(x + 9) \bigg)
\end{aligned}
$$

## 2.26 problem 25

Internal problem ID [6408]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 25.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 8y' - yx - x^3 + 2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-8*diff(y(x),x)-x*y(x)-x^3+2=0,y(x), singsol=all)
```

$$y(x) = e^{4x} \operatorname{AiryAi}(16 + x) c_2 + e^{4x} \operatorname{AiryBi}(16 + x) c_1 - x^2 + 16$$

✓ Solution by Mathematica

Time used: 3.178 (sec). Leaf size: 89

```
DSolve[y''[x]-8*y'[x]-x*y[x]-x^3+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to e^{4x} \bigg( & \operatorname{AiryAi}(x + 16) \int_1^x -e^{-4K[1]} \pi \operatorname{AiryBi}(K[1] + 16) \left( K[1]^3 - 2 \right) dK[1] \\
& + \operatorname{AiryBi}(x + 16) \int_1^x e^{-4K[2]} \pi \operatorname{AiryAi}(K[2] + 16) \left( K[2]^3 - 2 \right) dK[2] \\
& + c_1 \operatorname{AiryAi}(x + 16) + c_2 \operatorname{AiryBi}(x + 16) \bigg)
\end{aligned}
$$

## 2.27 problem 26

Internal problem ID [6409]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 26.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - y' - yx - x^4 + 3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-x*y(x)-x^4+3=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{2}} \operatorname{AiryAi}\left(\frac{1}{4} + x\right) c_2 + e^{\frac{x}{2}} \operatorname{AiryBi}\left(\frac{1}{4} + x\right) c_1 - x^3 + 3x - 6$$

✓ Solution by Mathematica

Time used: 1.792 (sec). Leaf size: 107

```
DSolve[y''[x]-y'[x]-x*y[x]-x^4+3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to e^{x/2} \bigg( & \operatorname{AiryAi}\left(x + \frac{1}{4}\right) \int_1^x -e^{-\frac{K[1]}{2}} \pi \operatorname{AiryBi}\left(K[1] + \frac{1}{4}\right) \left(K[1]^4 - 3\right) dK[1] \\
& + \operatorname{AiryBi}\left(x + \frac{1}{4}\right) \int_1^x e^{-\frac{K[2]}{2}} \pi \operatorname{AiryAi}\left(K[2] + \frac{1}{4}\right) \left(K[2]^4 - 3\right) dK[2] \\
& + c_1 \operatorname{AiryAi}\left(x + \frac{1}{4}\right) + c_2 \operatorname{AiryBi}\left(x + \frac{1}{4}\right) \bigg)
\end{aligned}
$$

## 2.28 problem 27

Internal problem ID [6410]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 27.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - y' - yx - x^3 = 0$$

✔ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-x*y(x)-x^3=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{2}} \operatorname{AiryAi}\left(\frac{1}{4} + x\right) c_2 + e^{\frac{x}{2}} \operatorname{AiryBi}\left(\frac{1}{4} + x\right) c_1$$
$$+ \pi e^{\frac{x}{2}} \left( -\left( \int x^3 \operatorname{AiryBi}\left(\frac{1}{4} + x\right) e^{-\frac{x}{2}} dx \right) \operatorname{AiryAi}\left(\frac{1}{4} + x\right) \right.$$
$$\left. + \left( \int x^3 \operatorname{AiryAi}\left(\frac{1}{4} + x\right) e^{-\frac{x}{2}} dx \right) \operatorname{AiryBi}\left(\frac{1}{4} + x\right) \right)$$

✔ Solution by Mathematica

Time used: 5.055 (sec). Leaf size: 103

```
DSolve[y''[x]-y'[x]-x*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/2} \left( \operatorname{AiryAi}\left(x + \frac{1}{4}\right) \int_1^x -e^{-\frac{K[1]}{2}} \pi \operatorname{AiryBi}\left(K[1] + \frac{1}{4}\right) K[1]^3 dK[1] \right.$$
$$+ \operatorname{AiryBi}\left(x + \frac{1}{4}\right) \int_1^x e^{-\frac{K[2]}{2}} \pi \operatorname{AiryAi}\left(K[2] + \frac{1}{4}\right) K[2]^3 dK[2]$$
$$\left. + c_1 \operatorname{AiryAi}\left(x + \frac{1}{4}\right) + c_2 \operatorname{AiryBi}\left(x + \frac{1}{4}\right) \right)$$

## 2.29 problem 28

Internal problem ID [6411]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 28.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - yx - x^3 + 2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-x*y(x)-x^3+2=0,y(x), singsol=all)
```

$$y(x) = \mathrm{AiryAi}\,(x)\,c_2 + \mathrm{AiryBi}\,(x)\,c_1 - x^2$$

✓ Solution by Mathematica

Time used: 0.207 (sec). Leaf size: 141

```
DSolve[y''[x]-x*y[x]-x^3+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to {} & \frac{1}{4}\, _0F_1\left(;\frac{4}{3};\frac{x^3}{9}\right)\left(x^5\, _1F_2\left(\frac{4}{3};\frac{2}{3},\frac{7}{3};\frac{x^3}{9}\right) - 8x^2\, _1F_2\left(\frac{1}{3};\frac{2}{3},\frac{4}{3};\frac{x^3}{9}\right)\right) \\
& + {} _0F_1\left(;\frac{2}{3};\frac{x^3}{9}\right)\left(x^2\, _1F_2\left(\frac{2}{3};\frac{4}{3},\frac{5}{3};\frac{x^3}{9}\right) - \frac{1}{5}x^5\, _1F_2\left(\frac{5}{3};\frac{4}{3},\frac{8}{3};\frac{x^3}{9}\right)\right) \\
& + c_1\,\mathrm{AiryAi}(x) + c_2\,\mathrm{AiryBi}(x)
\end{aligned}
$$

## 2.30 problem 29

Internal problem ID [6412]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 29.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - yx - x^6 + 64 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 143

```
dsolve(diff(y(x),x$2)-x*y(x)-x^6+64=0,y(x), singsol=all)
```

$$y(x) = \text{AiryAi}(x)\, c_2 + \text{AiryBi}(x)\, c_1$$
$$- \frac{\left(-\frac{16x^6\pi\left(\text{AiryBi}(x)3^{\frac{1}{3}} - 3^{\frac{5}{6}}\,\text{AiryAi}(x)\right)\text{hypergeom}\left(\left[\frac{7}{3}\right],\left[\frac{2}{3},\frac{10}{3}\right],\frac{x^3}{9}\right)}{21} + x^7\Gamma\left(\frac{2}{3}\right)^2\left(3^{\frac{1}{6}}\,\text{AiryBi}(x) + 3^{\frac{2}{3}}\,\text{AiryAi}(x)\right)\text{hyper}}{}$$

✓ Solution by Mathematica

Time used: 0.216 (sec). Leaf size: 142

```
DSolve[y''[x]-x*y[x]-x^6+64==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to {}_0F_1\left(;\frac{4}{3};\frac{x^3}{9}\right)\left(\frac{1}{7}x^8\,{}_1F_2\left(\frac{7}{3};\frac{2}{3},\frac{10}{3};\frac{x^3}{9}\right) - 64x^2\,{}_1F_2\left(\frac{1}{3};\frac{2}{3},\frac{4}{3};\frac{x^3}{9}\right)\right)$$
$$+ {}_0F_1\left(;\frac{2}{3};\frac{x^3}{9}\right)\left(32x^2\,{}_1F_2\left(\frac{2}{3};\frac{4}{3},\frac{5}{3};\frac{x^3}{9}\right) - \frac{1}{8}x^8\,{}_1F_2\left(\frac{8}{3};\frac{4}{3},\frac{11}{3};\frac{x^3}{9}\right)\right)$$
$$+ c_1\,\text{AiryAi}(x) + c_2\,\text{AiryBi}(x)$$

## 2.31 problem 30

Internal problem ID [6413]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 30.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - yx - x = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)-x*y(x)-x=0,y(x), singsol=all)
```

$$y(x) = \operatorname{AiryAi}(x) c_2 + \operatorname{AiryBi}(x) c_1 - 1$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 28

```
DSolve[y''[x]-x*y[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \pi \operatorname{AiryAiPrime}(x) \operatorname{AiryBi}(x) + c_2 \operatorname{AiryBi}(x) + \operatorname{AiryAi}(x)(-\pi \operatorname{AiryBiPrime}(x) + c_1)$$

## 2.32 problem 31

Internal problem ID [6414]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 31.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - yx - x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)-x*y(x)-x^2=0,y(x), singsol=all)
```

$$y(x) = \text{AiryAi}(x) c_2 + \text{AiryBi}(x) c_1 - x$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 30

```
DSolve[y''[x]-x*y[x]-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \pi x \, \text{AiryAiPrime}(x) \, \text{AiryBi}(x) + c_2 \, \text{AiryBi}(x) + \text{AiryAi}(x)(-\pi x \, \text{AiryBiPrime}(x) + c_1)$$

## 2.33   problem 32

Internal problem ID [6415]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 32.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - yx - x^3 = 0$$

✓   Solution by Maple

Time used: 0.015 (sec). Leaf size: 81

```
dsolve(diff(y(x),x$2)-x*y(x)-x^3=0,y(x), singsol=all)
```

$$y(x) = \text{AiryAi}(x)\,c_2 + \text{AiryBi}(x)\,c_1$$
$$- \frac{\left(-\frac{5\pi\left(\text{AiryBi}(x)3^{\frac{1}{3}} - 3^{\frac{5}{6}}\,\text{AiryAi}(x)\right)\text{hypergeom}\left(\left[\frac{4}{3}\right],\left[\frac{2}{3},\frac{7}{3}\right],\frac{x^3}{9}\right)}{6} + x\,\text{hypergeom}\left(\left[\frac{5}{3}\right],\left[\frac{4}{3},\frac{8}{3}\right],\frac{x^3}{9}\right)\Gamma\left(\frac{2}{3}\right)^2\left(3^{\frac{1}{6}}\,\text{AiryBi}(x)\right)}{10\Gamma\left(\frac{2}{3}\right)}$$

✓   Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 88

```
DSolve[y''[x]-x*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{20}x^5\left(5\,_0F_1\left(;\frac{4}{3};\frac{x^3}{9}\right)\,_1F_2\left(\frac{4}{3};\frac{2}{3},\frac{7}{3};\frac{x^3}{9}\right) - 4\,_0F_1\left(;\frac{2}{3};\frac{x^3}{9}\right)\,_1F_2\left(\frac{5}{3};\frac{4}{3},\frac{8}{3};\frac{x^3}{9}\right)\right)$$
$$+ c_1\,\text{AiryAi}(x) + c_2\,\text{AiryBi}(x)$$

## 2.34 problem 33

Internal problem ID [6416]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 33.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - yx - x^6 - x^3 + 42 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-x*y(x)-x^6-x^3+42=0,y(x), singsol=all)
```

$$y(x) = \text{AiryAi}(x)\, c_2 + \text{AiryBi}(x)\, c_1 - x^5 - 21x^2$$

✓ Solution by Mathematica

Time used: 0.538 (sec). Leaf size: 190

```
DSolve[y''[x]-x*y[x]-x^6-x^3+42==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to {}& \frac{1}{280} x^2 \left( 10\, {}_0F_1\left(; \frac{4}{3}; \frac{x^3}{9}\right) \left( 7x^3\, {}_1F_2\left(\frac{4}{3}; \frac{2}{3}, \frac{7}{3}; \frac{x^3}{9}\right) - 1176\, {}_1F_2\left(\frac{1}{3}; \frac{2}{3}, \frac{4}{3}; \frac{x^3}{9}\right) \right.\right. \\
& \left. + 4x^6\, {}_1F_2\left(\frac{7}{3}; \frac{2}{3}, \frac{10}{3}; \frac{x^3}{9}\right) \right) + 7\, {}_0F_1\left(; \frac{2}{3}; \frac{x^3}{9}\right) \left( -8x^3\, {}_1F_2\left(\frac{5}{3}; \frac{4}{3}, \frac{8}{3}; \frac{x^3}{9}\right) \right. \\
& \left.\left. + 840\, {}_1F_2\left(\frac{2}{3}; \frac{4}{3}, \frac{5}{3}; \frac{x^3}{9}\right) - 5x^6\, {}_1F_2\left(\frac{8}{3}; \frac{4}{3}, \frac{11}{3}; \frac{x^3}{9}\right) \right) \right) + c_1 \text{AiryAi}(x) + c_2 \text{AiryBi}(x)
\end{aligned}
$$

## 2.35   problem 34

Internal problem ID [6417]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 34.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - yx^2 - x^2 = 0$$

✓  Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-x^2*y(x)-x^2=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\, \text{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_2 + \sqrt{x}\, \text{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_1 - 1$$

✓  Solution by Mathematica

Time used: 2.957 (sec). Leaf size: 213

```
DSolve[y''[x]-x^2*y[x]-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to \text{ParabolicCylinderD}\left(-\frac{1}{2}, \sqrt{2}x\right)\left(\int_1^x \frac{K[1]^2\, \text{ParabolicCylin}}{\sqrt{2}\left(\text{HermiteH}\left(-\frac{1}{2}, K[1]\right)\left(i\,\text{HermiteH}\left(\frac{1}{2}, iK[1]\right) + 2\,\text{HermiteH}\right.} \right.$$
$$\left. + c_1 \right)$$
$$+ \text{ParabolicCylinderD}\left(-\frac{1}{2}, i\sqrt{2}x\right)\left(\int_1^x \frac{K[2]^2\, \text{ParabolicC}}{\sqrt{2}\left(\text{HermiteH}\left(-\frac{1}{2}, iK[2]\right)\text{HermiteH}\left(\frac{1}{2}, K[2]\right) + \text{HermiteH}\left(- \right.\right.}\right.$$
$$\left. + c_2 \right)$$

## 2.36 problem 35

Internal problem ID [6418]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 35.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - yx^2 - x^3 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)-x^2*y(x)-x^3=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\, \text{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_2 + \sqrt{x}\, \text{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_1 - x$$

✓ Solution by Mathematica

Time used: 2.729 (sec). Leaf size: 213

```
DSolve[y''[x]-x^2*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\to \text{ParabolicCylinderD}\left(-\frac{1}{2}, \sqrt{2}x\right)\left(\int_1^x \frac{K[1]^3 \text{ParabolicCylin}}{\sqrt{2}\left(\text{HermiteH}\left(-\frac{1}{2}, K[1]\right)\left(i\,\text{HermiteH}\left(\frac{1}{2}, iK[1]\right) + 2\,\text{HermiteH}\right.\right.} + c_1\right)$$

$$+ \text{ParabolicCylinderD}\left(-\frac{1}{2}, i\sqrt{2}x\right)\left(\int_1^x \frac{K[2]^3 \text{ParabolicC}}{\sqrt{2}\left(\text{HermiteH}\left(-\frac{1}{2}, iK[2]\right)\text{HermiteH}\left(\frac{1}{2}, K[2]\right) + \text{HermiteH}\left(\right.\right.} + c_2\right)$$

## 2.37 problem 36

Internal problem ID [6419]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 36.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - yx^2 - x^4 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 120

```
dsolve(diff(y(x),x$2)-x^2*y(x)-x^4=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\, \text{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_2 + \sqrt{x}\, \text{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_1$$

$$- \frac{x^{\frac{11}{2}}\left(\pi\Gamma\left(\frac{3}{4}\right)^2 \text{hypergeom}\left(\left[\frac{3}{2}\right], \left[\frac{19}{8}, \frac{5}{2}\right], \frac{x^4}{16}\right)\sqrt{2}\, x\, \text{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) + 2\Gamma\left(\frac{3}{4}\right)^2 \text{hypergeom}\left(\left[\frac{3}{2}\right], \left[\frac{5}{4}, \frac{5}{2}\right], \frac{x^4}{16}\right)\right)}{12\Gamma\left(\frac{3}{4}\right)\pi}$$

✓ Solution by Mathematica

Time used: 2.783 (sec). Leaf size: 213

```
DSolve[y''[x]-x^2*y[x]-x^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\to \text{ParabolicCylinderD}\left(-\frac{1}{2}, \sqrt{2}x\right)\left(\int_1^x \frac{K[1]^4\,\text{ParabolicCylin}}{\sqrt{2}\left(\text{HermiteH}\left(-\frac{1}{2}, K[1]\right)\left(i\,\text{HermiteH}\left(\frac{1}{2}, iK[1]\right) + 2\,\text{HermiteH}\right.}\right.$$
$$\left.\left. + c_1\right)$$

$$+ \text{ParabolicCylinderD}\left(-\frac{1}{2}, i\sqrt{2}x\right)\left(\int_1^x \frac{K[2]^4\,\text{ParabolicC}}{\sqrt{2}\left(\text{HermiteH}\left(-\frac{1}{2}, iK[2]\right)\text{HermiteH}\left(\frac{1}{2}, K[2]\right) + \text{HermiteH}\left(-\right.}\right.$$
$$\left.\left. + c_2\right)$$

## 2.38 problem 37

Internal problem ID [6420]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 37.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - yx^2 - x^4 + 2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-x^2*y(x)-x^4+2=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\, \text{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_2 + \sqrt{x}\, \text{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_1 - x^2$$

✓ Solution by Mathematica

Time used: 3.427 (sec). Leaf size: 217

```
DSolve[y''[x]-x^2*y[x]-x^4+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{ParabolicCylinderD}\left(-\frac{1}{2}, \sqrt{2}x\right)\left(\int_1^x \right.$$

$$-\frac{(K[1]^4 - 2)\,\text{ParabolicCylinderD}\left(-\frac{1}{2}, i\sqrt{2}K[1]\right)}{\sqrt{2}\left(\text{HermiteH}\left(-\frac{1}{2}, iK[1]\right)\text{HermiteH}\left(\frac{1}{2}, K[1]\right) + \text{HermiteH}\left(-\frac{1}{2}, K[1]\right)\left(-i\,\text{HermiteH}\left(\frac{1}{2}, iK[1]\right) - 2\right.}$$

$$\left. + c_1 \right)$$

$$+\text{ParabolicCylinderD}\left(-\frac{1}{2}, i\sqrt{2}x\right)\left(\int_1^x \frac{(K[2]^4 - 2)\,\text{Parabol}}{\sqrt{2}\left(\text{HermiteH}\left(-\frac{1}{2}, iK[2]\right)\text{HermiteH}\left(\frac{1}{2}, K[2]\right) + \text{HermiteH}\left(-\right.\right.}$$

$$\left. + c_2 \right)$$

## 2.39 problem 38

Internal problem ID [6421]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 38.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2yx^2 - x^4 + 1 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)-2*x^2*y(x)-x^4+1=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\, \text{BesselI}\left(\frac{1}{4}, \frac{\sqrt{2}\,x^2}{2}\right) c_2 + \sqrt{x}\, \text{BesselK}\left(\frac{1}{4}, \frac{\sqrt{2}\,x^2}{2}\right) c_1 - \frac{x^2}{2}$$

✓ Solution by Mathematica

Time used: 1.79 (sec). Leaf size: 288

```
DSolve[y''[x]-2*x^2*y[x]-x^4+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\to \text{ParabolicCylinderD}\left(-\frac{1}{2}, 2^{3/4}x\right)\left(\int_1^x \frac{1}{i2^{3/4}\,\text{HermiteH}\left(-\frac{1}{2}, \sqrt[4]{2}K[1]\right)\text{HermiteH}\left(\frac{1}{2}, i\sqrt[4]{2}K[1]\right) + \text{Parab}}\right.$$

## 2.40 problem 39

Internal problem ID [6422]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 39.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - yx^3 - x^3 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-x^3*y(x)-x^3=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,\text{BesselI}\left(\frac{1}{5}, \frac{2x^{\frac{5}{2}}}{5}\right)c_2 + \sqrt{x}\,\text{BesselK}\left(\frac{1}{5}, \frac{2x^{\frac{5}{2}}}{5}\right)c_1 - 1$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 217

```
DSolve[y''[x]-x^3*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to \frac{\sqrt[5]{-1}\,\text{Gamma}\left(\frac{4}{5}\right)\left(5^{4/5}x^5\,\text{Gamma}\left(\frac{6}{5}\right)\,_0\tilde{F}_1\left(;\frac{9}{5};\frac{x^5}{25}\right)\text{BesselI}\left(\frac{1}{5}, \frac{2x^{5/2}}{5}\right) + 5\,5^{2/5}\left(x^{5/2}\right)^{2/5}\text{BesselI}\left(-\frac{1}{5}, \frac{2x^{5/2}}{5}\right)\right)}{25\sqrt[5]{x^{5/2}}\,\text{Root}\left[25\#1^5 + 1\&, 5\right]}$$
$$+ \frac{c_1\sqrt{x}\,\text{Gamma}\left(\frac{4}{5}\right)\text{BesselI}\left(-\frac{1}{5}, \frac{2x^{5/2}}{5}\right)}{\sqrt[5]{5}} + \sqrt[5]{-\frac{1}{5}}c_2\sqrt{x}\,\text{Gamma}\left(\frac{6}{5}\right)\text{BesselI}\left(\frac{1}{5}, \frac{2x^{5/2}}{5}\right)$$

## 2.41 problem 40

Internal problem ID [6423]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 40.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - yx^3 - x^4 = 0$$

✔ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)-x^3*y(x)-x^4=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,\text{BesselI}\left(\frac{1}{5}, \frac{2x^{\frac{5}{2}}}{5}\right) c_2 + \sqrt{x}\,\text{BesselK}\left(\frac{1}{5}, \frac{2x^{\frac{5}{2}}}{5}\right) c_1 - x$$

✔ Solution by Mathematica

Time used: 0.103 (sec). Leaf size: 109

```
DSolve[y''[x]-x^3*y[x]-x^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\sqrt{2\left(1 + \sqrt{5}\right)}\pi c_1 \sqrt{x}\,\text{BesselI}\left(-\frac{1}{5}, \frac{2x^{5/2}}{5}\right)}{5^{9/20}\,\text{Gamma}\left(\frac{1}{5}\right)}$$
$$+ \frac{\text{Gamma}\left(\frac{1}{5}\right)\left(\left(x^{5/2}\right)^{4/5} + \frac{\sqrt[5]{-1}c_2 x^2}{5^{2/5}}\right)\text{BesselI}\left(\frac{1}{5}, \frac{2x^{5/2}}{5}\right)}{5^{4/5}x^{3/2}} - x$$

## 2.42   problem 41

Internal problem ID [6424]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 41.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - y'x^2 - yx^2 - x^2 = 0$$

✓   Solution by Maple

Time used: 0.031 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-x^2*y(x)-x^2=0,y(x), singsol=all)
```

$$y(x) = \text{HeunT}\left(3^{\frac{2}{3}}, 3, 2\,3^{\frac{1}{3}}, \frac{3^{\frac{2}{3}}x}{3}\right) e^{-x} c_2 + \text{HeunT}\left(3^{\frac{2}{3}}, -3, 2\,3^{\frac{1}{3}}, -\frac{3^{\frac{2}{3}}x}{3}\right) e^{\frac{x\left(x^2+3\right)}{3}} c_1 - 1$$

✗   Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-x^2*y'[x]-x^2*y[x]-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 2.43   problem 42

Internal problem ID [6425]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 42.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - y'x^3 - yx^3 - x^3 = 0$$

**✗ Solution by Maple**

```
dsolve(diff(y(x),x$2)-x^3*diff(y(x),x)-x^3*y(x)-x^3=0,y(x), singsol=all)
```

No solution found

**✗ Solution by Mathematica**

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-x^3*y'[x]-x^3*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 2.44 problem 43

Internal problem ID [6426]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 43.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y'x - yx - x = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}(x+2)\,c_2 + \left(\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)\pi(x+2)\,\mathrm{e}^{-x-2} - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}}\right)c_1 - 1$$

✓ Solution by Mathematica

Time used: 0.231 (sec). Leaf size: 104

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{2}e^{-x}(x+2)\left(\int_1^x \frac{1}{2}e^{K[1]}K[1]\left(\sqrt{2} - 2\,\mathrm{DawsonF}\left(\frac{K[1]+2}{\sqrt{2}}\right)(K[1]+2)\right)dK[1]\right.$$
$$\left. + c_1\right) - \left(c_2 e^{\frac{x^2}{2}+x+2} + x + 1\right)\left(\sqrt{2}(x+2)\,\mathrm{DawsonF}\left(\frac{x+2}{\sqrt{2}}\right) - 1\right)$$

## 2.45 problem 44

Internal problem ID [6427]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 44.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - y'x^2 - yx - x^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-x*y(x)-x^2=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x^3}{6}} \sqrt{x}\, \text{BesselI}\left(\frac{1}{6}, \frac{x^3}{6}\right) c_2 + e^{\frac{x^3}{6}} \sqrt{x}\, \text{BesselK}\left(\frac{1}{6}, \frac{x^3}{6}\right) c_1 - \frac{x}{2}$$

✓ Solution by Mathematica

Time used: 0.144 (sec). Leaf size: 224

```
DSolve[y''[x]-x^2*y'[x]-x*y[x]-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow$$
$$-\frac{e^{\frac{x^3}{6}}\left(12(x^3)^{5/6} \text{Gamma}\left(\frac{1}{6}\right) \text{Gamma}\left(\frac{7}{6}\right) \text{BesselI}\left(\frac{1}{6}, \frac{x^3}{6}\right) {}_1F_1\left(-\frac{2}{3}; -\frac{1}{3}; -\frac{x^3}{3}\right) + \sqrt[3]{2}3^{2/3}\sqrt[6]{x^3}x^6 \text{Gamma}\left(\frac{1}{6}\right)}$$

## 2.46 problem 45

Internal problem ID [6428]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 45.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - y'x^2 - yx^2 - x^3 - x^2 = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 57

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-x^2*y(x)-x^3-x^2=0,y(x), singsol=all)
```

$$y(x) = \mathrm{HeunT}\left(3^{\frac{2}{3}}, 3, 2\,3^{\frac{1}{3}}, \frac{3^{\frac{2}{3}}x}{3}\right)\mathrm{e}^{-x}c_2 + \mathrm{HeunT}\left(3^{\frac{2}{3}}, -3, 2\,3^{\frac{1}{3}}, -\frac{3^{\frac{2}{3}}x}{3}\right)\mathrm{e}^{\frac{x\left(x^2+3\right)}{3}}c_1 - x$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-x^2*y'[x]-x^2*y[x]-x^3-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 2.47 problem 46

Internal problem ID [6429]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 46.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - y'x^2 - yx^3 - x^4 - x^2 = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 75

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-x^3*y(x)-x^4-x^2=0,y(x), singsol=all)
```

$$y(x) = e^{-\frac{1}{2}x^2+x} \operatorname{HeunT}\left(2\,3^{\frac{2}{3}}, -3, -3\,3^{\frac{1}{3}}, \frac{3^{\frac{2}{3}}(x+1)}{3}\right) c_2$$

$$+ e^{\frac{1}{3}x^3+\frac{1}{2}x^2-x} \operatorname{HeunT}\left(2\,3^{\frac{2}{3}}, 3, -3\,3^{\frac{1}{3}}, -\frac{3^{\frac{2}{3}}(x+1)}{3}\right) c_1 - x$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-x^2*y'[x]-x^3*y[x]-x^4-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 2.48 problem 47

Internal problem ID [6430]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 47.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - \frac{y'}{x} - yx - x^2 - \frac{1}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-1/x*diff(y(x),x)-x*y(x)-x^2-1/x=0,y(x), singsol=all)
```

$$y(x) = x \operatorname{BesselI}\left(\frac{2}{3}, \frac{2x^{\frac{3}{2}}}{3}\right) c_2 + x \operatorname{BesselK}\left(\frac{2}{3}, \frac{2x^{\frac{3}{2}}}{3}\right) c_1 - x$$

✓ Solution by Mathematica

Time used: 0.216 (sec). Leaf size: 138

```
DSolve[y''[x]-1/x*y'[x]-x*y[x]-x^2-1/x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to {} & \frac{1}{4} x \, {}_0F_1\left(; \frac{5}{3}; \frac{x^3}{9}\right) \left(x^3 \, {}_1F_2\left(\frac{2}{3}; \frac{1}{3}, \frac{5}{3}; \frac{x^3}{9}\right) - 2 \, {}_1F_2\left(-\frac{1}{3}; \frac{1}{3}, \frac{2}{3}; \frac{x^3}{9}\right)\right) \\
& - \frac{1}{8} x \, {}_0F_1\left(; \frac{1}{3}; \frac{x^3}{9}\right) \left(x^3 \, {}_1F_2\left(\frac{4}{3}; \frac{5}{3}, \frac{7}{3}; \frac{x^3}{9}\right) + 4 \, {}_1F_2\left(\frac{1}{3}; \frac{4}{3}, \frac{5}{3}; \frac{x^3}{9}\right)\right) \\
& + c_1 \operatorname{AiryAiPrime}(x) + c_2 \operatorname{AiryBiPrime}(x)
\end{aligned}
$$

## 2.49   problem 48

Internal problem ID [6431]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 48.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{y'}{x} - yx^2 - x^3 - \frac{1}{x} = 0$$

✓   Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-1/x*diff(y(x),x)-x^2*y(x)-x^3-1/x=0,y(x), singsol=all)
```

$$y(x) = \sinh\left(\frac{x^2}{2}\right) c_2 + \cosh\left(\frac{x^2}{2}\right) c_1 - x$$

✓   Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 34

```
DSolve[y''[x]-1/x*y'[x]-x^2*y[x]-x^3-1/x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cosh\left(\frac{x^2}{2}\right) + i c_2 \sinh\left(\frac{x^2}{2}\right) - x$$

## 2.50 problem 49

Internal problem ID [6432]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 49.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{y'}{x} - yx^3 - x^4 - \frac{1}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-1/x*diff(y(x),x)-x^3*y(x)-x^4-1/x=0,y(x), singsol=all)
```

$$y(x) = x \, \mathrm{BesselI}\left(\frac{2}{5}, \frac{2x^{\frac{5}{2}}}{5}\right) c_2 + x \, \mathrm{BesselK}\left(\frac{2}{5}, \frac{2x^{\frac{5}{2}}}{5}\right) c_1 - x$$

✓ Solution by Mathematica

Time used: 0.161 (sec). Leaf size: 316

```
DSolve[y''[x]-1/x*y'[x]-x^3*y[x]-x^4-1/x==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\rightarrow \frac{\frac{5(x^{5/2})^{13/5} \, \mathrm{Gamma}\left(\frac{4}{5}\right) \mathrm{Gamma}\left(\frac{7}{5}\right) \mathrm{BesselI}\left(\frac{2}{5}, \frac{2x^{5/2}}{5}\right) {}_1F_2\left(\frac{4}{5}; \frac{3}{5}, \frac{9}{5}; \frac{x^5}{25}\right)}{\mathrm{Gamma}\left(\frac{9}{5}\right)} - \frac{\sqrt[5]{5}(x^{5/2})^{7/5} \, \mathrm{Gamma}\left(\frac{1}{5}\right) \mathrm{Gamma}\left(\frac{3}{5}\right) \mathrm{BesselI}\left(-\frac{2}{5}, \frac{2x^{5/2}}{5}\right) {}_1F_2\left(\right.}{\mathrm{Gamma}\left(\frac{6}{5}\right)}$$

## 2.51 problem 50

Internal problem ID [6433]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 50.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y'x^3 - yx - x^3 - x^2 = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)-x^3*diff(y(x),x)-x*y(x)-x^3-x^2=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-x^3*y'[x]-x*y[x]-x^3-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 2.52 problem 51

Internal problem ID [6434]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 51.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - y'x^3 - yx^2 - x^3 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-x^3*diff(y(x),x)-x^2*y(x)-x^3=0,y(x), singsol=all)
```

$$y(x) = x \operatorname{KummerM}\left(\frac{1}{2}, \frac{5}{4}, \frac{x^4}{4}\right) c_2 + x \operatorname{KummerU}\left(\frac{1}{2}, \frac{5}{4}, \frac{x^4}{4}\right) c_1 - \frac{x}{2}$$

✓ Solution by Mathematica

Time used: 0.49 (sec). Leaf size: 274

```
DSolve[y''[x]-x^3*y'[x]-x^2*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\rightarrow \text{Hypergeometric1F1}\left(\frac{1}{4}, \frac{3}{4}, \frac{x^4}{4}\right) \left( \int_1^x \frac{1}{\text{Hypergeometric1F1}\left(\frac{1}{4}, \frac{3}{4}, \frac{K[1]^4}{4}\right) \left( \frac{3\,\text{Hypergeometric1F1}\left(\frac{1}{2}, \frac{9}{4}, \frac{K[1]^4}{4}\right)}{5\,\text{Hypergeometric1F1}\left(\frac{1}{2}, \frac{5}{4}, \frac{K[1]^4}{4}\right)} - \overline{K} \right.} \right.$$

$$\left. + c_1 \right)$$

$$+ \frac{1}{2} x \,\text{Hypergeometric1F1}\left(\frac{1}{2}, \frac{5}{4}, \frac{x^4}{4}\right) \left( \sqrt[4]{-1}\sqrt{2} \int_1^x \frac{1}{3\,\text{Hypergeometric1F1}\left(\frac{1}{4}, \frac{3}{4}, \frac{K[2]^4}{4}\right) \left( 2\,\text{Hypergeomet} \right.} \right.$$

$$\left. + (1+i)c_2 \right)$$

## 2.53   problem 52

Internal problem ID [6435]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 52.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y'x^3 - yx^3 - x^4 - x^3 = 0$$

✗  Solution by Maple

```
dsolve(diff(y(x),x$2)-x^3*diff(y(x),x)-x^3*y(x)-x^4-x^3=0,y(x), singsol=all)
```

No solution found

✗  Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-x^3*y'[x]-x^3*y[x]-x^4-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 2.54 problem 50

Internal problem ID [6436]

**Book**: Own collection of miscellaneous problems
**Section**: section 2.0
**Problem number**: 50.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' - y'x^3 - yx^2 - x^3 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$3)-x^3*diff(y(x),x)-x^2*y(x)-x^3=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + c_1 \, \text{hypergeom}\left(\left[\frac{1}{5}\right], \left[\frac{3}{5}, \frac{4}{5}\right], \frac{x^5}{25}\right)$$
$$+ c_2 x \, \text{hypergeom}\left(\left[\frac{2}{5}\right], \left[\frac{4}{5}, \frac{6}{5}\right], \frac{x^5}{25}\right) + c_3 x^2 \, \text{hypergeom}\left(\left[\frac{3}{5}\right], \left[\frac{6}{5}, \frac{7}{5}\right], \frac{x^5}{25}\right)$$

✓ Solution by Mathematica

Time used: 9.591 (sec). Leaf size: 2542

```
DSolve[y'''[x]-x^3*y'[x]-x^2*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

# 3 section 3.0

## 3.1 problem 1

Internal problem ID [6437]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x]]

$$y'' + y'c + ky = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$2)+c*diff(y(x),x)+k*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\left(-\frac{c}{2} + \frac{\sqrt{c^2-4k}}{2}\right)x} + c_2 e^{\left(-\frac{c}{2} - \frac{\sqrt{c^2-4k}}{2}\right)x}$$

✓ Solution by Mathematica

Time used: 8.448 (sec). Leaf size: 2542

```
DSolve[y'''[x]-x^3*y'[x]-x^2*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

## 3.2 problem 2

Internal problem ID [6438]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 2.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$w' + \frac{1}{2} + \frac{\sqrt{1 - 12w}}{2} = 0$$

With initial conditions

$$[w(1) = -1]$$

✓ Solution by Maple

Time used: 0.687 (sec). Leaf size: 66

```
dsolve([diff(w(z),z) = -1/2 - sqrt(1/4 - 3*w(z)),w(1) = -1],w(z), singsol=all)
```

$$w(z) = \text{RootOf}\left(-i\pi - \ln\left(1 + \sqrt{13}\right) + \ln\left(-1 + \sqrt{13}\right) + 2\sqrt{13} - 2\sqrt{1 - 12\_Z} + \ln\left(\_Z\right)\right.$$
$$\left. - \ln\left(-1 + \sqrt{1 - 12\_Z}\right) + \ln\left(1 + \sqrt{1 - 12\_Z}\right) + 6z - 6\right)$$

✓ Solution by Mathematica

Time used: 12.334 (sec). Leaf size: 105

```
DSolve[{w'[z] == -1/2 - Sqrt[1/4 - 3*w[z]],{w[1] == -1}},w[z],z,IncludeSingularSolutions -> T
```

$$w(z) \to -\frac{1}{12}W\left(\left(\sqrt{13} - 1\right)e^{-3z + \sqrt{13} + 2}\right)\left(W\left(\left(\sqrt{13} - 1\right)e^{-3z + \sqrt{13} + 2}\right) + 2\right)$$

$$w(z) \to -\frac{1}{12}W\left(\left(\sqrt{13} - 1\right)e^{-3z + \sqrt{13} + 2}\right)\left(W\left(\left(\sqrt{13} - 1\right)e^{-3z + \sqrt{13} + 2}\right) + 2\right)$$

## 3.3 problem 3

Internal problem ID [6439]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + y - \sin(x) = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve([diff(y(x),x$2)+y(x)=sin(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)(1 + 2c_2)}{2} - \frac{\cos(x)(x - 2)}{2}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 20

```
DSolve[{y''[x]+y[x]==Sin[x],{y[0] == 1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}x\cos(x) + \cos(x) + c_2\sin(x)$$

## 3.4 problem 4

Internal problem ID [6440]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + y - \sin(x) = 0$$

With initial conditions

$$[y'(0) = 1]$$

✔ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)+y(x)=sin(x),D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{(2c_1 - x)\cos(x)}{2} + \frac{3\sin(x)}{2}$$

✔ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 23

```
DSolve[{y''[x]+y[x]==Sin[x],{y'[0] == 1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3\sin(x)}{2} + \left(-\frac{x}{2} + c_1\right)\cos(x)$$

## 3.5   problem 5

Internal problem ID [6441]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + y - \sin(x) = 0$$

With initial conditions

$$[y'(0) = 1, y(0) = 0]$$

✓  Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)+y(x)=sin(x),D(y)(0) = 1, y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{3\sin(x)}{2} - \frac{\cos(x)\,x}{2}$$

✓  Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 19

```
DSolve[{y''[x]+y[x]==Sin[x],{y'[0] == 1,y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(3\sin(x) - x\cos(x))$$

## 3.6 problem 6

Internal problem ID [6442]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + y - \sin(x) = 0$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 32

```
dsolve([diff(y(x),x$2)+y(x)=sin(x),y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{((-2c_2 - 1)\tan(1) - x + 1)\cos(x)}{2} + \frac{\sin(x)(1 + 2c_2)}{2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 18

```
DSolve[{y''[x]+y[x]==Sin[x],{y[0] == 0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}x\cos(x) + c_2\sin(x)$$

## 3.7 problem 7

Internal problem ID [6443]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + y - \sin(x) = 0$$

With initial conditions

$$[y'(1) = 0]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 27

```
dsolve([diff(y(x),x$2)+y(x)=sin(x),D(y)(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(2c_2 \cot(1) - x + 1) \cos(x)}{2} + \frac{\sin(x)(1 + 2c_2)}{2}$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 33

```
DSolve[{y''[x]+y[x]==Sin[x],{y'[1] == 0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\sin(x) - ((x - 2c_1)\cos(x)) + (-1 + 2c_1)\tan(1)\sin(x))$$

## 3.8 problem 8

Internal problem ID [6444]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + y - \sin(x) = 0$$

With initial conditions

$$[y'(1) = 0, y(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve([diff(y(x),x$2)+y(x)=sin(x),D(y)(1) = 0, y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(1 - \tan(1))\sin(x)}{2} - \frac{\cos(x)\,x}{2}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 23

```
DSolve[{y''[x]+y[x]==Sin[x],{y'[1] == 0,y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\sin(x) - x\cos(x) - \tan(1)\sin(x))$$

## 3.9 problem 9

Internal problem ID [6445]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - \sin(x) = 0$$

With initial conditions

$$[y'(1) = 0, y(2) = 0]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 77

```
dsolve([diff(y(x),x$2)+y(x)=sin(x),D(y)(1) = 0, y(2) = 0],y(x), singsol=all)
```

$$y(x)$$
$$= \frac{(((-x+2)\cos(x)+\sin(x))\cos(2)-\cos(x)\sin(2))\cos(1)-\sin(1)(-\sin(x)\cos(2)+\cos(x)\sin(2)(x}{2\cos(2)\cos(1)+2\sin(2)\sin(1)}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 35

```
DSolve[{y''[x]+y[x]==Sin[x],{y'[1] == 0,y[2]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(-2(1+\sin(2))(\tan(1)-1)\sin(x)-2\cos(x)(x-1+\sin(2)-\cos(2)))$$

## 3.10 problem 10

Internal problem ID [6446]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - \sin(x) = 0$$

With initial conditions

$$[y'(1) = 0, y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve([diff(y(x),x$2)+y(x)=sin(x),D(y)(1) = 0, y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(1 - \tan(1))\sin(x)}{2} - \frac{\cos(x)x}{2}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 23

```
DSolve[{y''[x]+y[x]==Sin[x],{y'[1] == 0,y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\sin(x) - x\cos(x) - \tan(1)\sin(x))$$

## 3.11 problem 11

Internal problem ID [6447]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y' + y - \sin(x) = 0$$

With initial conditions

$$[y'(1) = 0, y(2) = 0]$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 156

```
dsolve([diff(y(x),x$2)+diff(y(x),x)+y(x)=sin(x),D(y)(1) = 0, y(2) = 0],y(x), singsol=all)
```

$y(x)$
$$= \frac{4\sin(1)\left(\left(-\cos\left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2}\right)\sin\left(\frac{\sqrt{3}\,x}{2}\right) + \cos\left(\frac{\sqrt{3}\,x}{2}\right)\cos\left(\frac{\sqrt{3}}{2}\right)\sin\left(\frac{\sqrt{3}}{2}\right)\right)e^{-\frac{x}{2}+\frac{1}{2}} + \cos(2)\left(\left(\sqrt{3}\,\sin\left(\frac{\sqrt{?}}{?}\right.\right.\right.}{\sqrt{3}\cos\left(\right.}$$

✓ Solution by Mathematica

Time used: 0.989 (sec). Leaf size: 12765

```
DSolve[{y'''[x]+y'[x]+y[x]==Sin[x],{y'[1] == 0,y[2]==0}},y[x],x,IncludeSingularSolutions -> T
```

Too large to display

## 3.12 problem 12

Internal problem ID [6448]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + y' + y - \sin(x) = 0$$

With initial conditions

$$[y'(1) = 0]$$

✔ Solution by Maple

Time used: 0.079 (sec). Leaf size: 80

```
dsolve([diff(y(x),x$2)+diff(y(x),x)+y(x)=sin(x),D(y)(1) = 0],y(x), singsol=all)
```

$$y(x)$$
$$= \frac{2\cos\left(\frac{\sqrt{3}\,x}{2}\right)\sin(1)\,e^{-\frac{x}{2}+\frac{1}{2}} + c_2 e^{-\frac{x}{2}}\left(\sqrt{3}\cos\left(\frac{\sqrt{3}}{2}\right) - \sin\left(\frac{\sqrt{3}}{2}\right)\right)\cos\left(\frac{\sqrt{3}\,x}{2}\right) + \left(\sqrt{3}\sin\left(\frac{\sqrt{3}}{2}\right) + \cos\left(\frac{\sqrt{3}}{2}\right)\right)}{\sqrt{3}\sin\left(\frac{\sqrt{3}}{2}\right) + \cos\left(\frac{\sqrt{3}}{2}\right)}$$

✔ Solution by Mathematica

Time used: 0.478 (sec). Leaf size: 4176

```
DSolve[{y'''[x]+y'[x]+y[x]==Sin[x],{y'[1] == 0}},y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

## 3.13    problem 13

Internal problem ID [6449]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y' + y - \sin(x) = 0$$

With initial conditions

$$[y'(1) = 0, y(2) = 0]$$

✓  Solution by Maple

Time used: 0.0 (sec). Leaf size: 156

```
dsolve([diff(y(x),x$2)+diff(y(x),x)+y(x)=sin(x),D(y)(1) = 0, y(2) = 0],y(x), singsol=all)
```

$y(x)$
$$= \frac{4\sin(1)\left(\left(-\cos\left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2}\right)\sin\left(\frac{\sqrt{3}\,x}{2}\right) + \cos\left(\frac{\sqrt{3}\,x}{2}\right)\cos\left(\frac{\sqrt{3}}{2}\right)\sin\left(\frac{\sqrt{3}}{2}\right)\right)e^{-\frac{x}{2}+\frac{1}{2}} + \cos(2)\left(\left(\sqrt{3}\sin\left(\frac{\sqrt{2}}{2}\right)\right.\right.}{\sqrt{3}\cos\left(\right.}$$

✓  Solution by Mathematica

Time used: 0.841 (sec). Leaf size: 12765

```
DSolve[{y'''[x]+y'[x]+y[x]==Sin[x],{y'[1] == 0,y[2]==0}},y[x],x,IncludeSingularSolutions -> T
```

Too large to display

## 3.14 problem 14

Internal problem ID [6450]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 14.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type [[_3rd_order, _with_linear_symmetries]]

$$y''' + y' + y - x = 0$$

With initial conditions

$$[y'(0) = 0, y(0) = 0, y''(0) = 1]$$

✓ Solution by Maple

Time used: 0.625 (sec). Leaf size: 359

```
dsolve([diff(y(x),x$3)+diff(y(x),x)+y(x)=x,D(y)(0) = 0, y(0) = 0, (D@@2)(y)(0) = 1],y(x), sin
```

$y(x)$

$$= \frac{10\,e^{-\frac{\left(-12+\left(-9+\sqrt{93}\right)\left(108+12\sqrt{93}\right)^{\frac{1}{3}}\right)\left(108+12\sqrt{93}\right)^{\frac{1}{3}}x}{144}}\left(\left(108+12\sqrt{3}\sqrt{31}\right)^{\frac{1}{3}}\sqrt{3}\sqrt{31}+\frac{3\sqrt{3}\left(108+12\sqrt{3}\sqrt{31}\right)^{\frac{2}{3}}\sqrt{31}}{5}-\frac{6\sqrt{3}\sqrt{31}}{5}-\frac{39\left(108+12\sqrt{3}\sqrt{31}\right)^{\frac{1}{3}}}{5}\right)}{3}}{3}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 1546

```
DSolve[{y'''[x]+y'[x]+y[x]==x,{y'[1] == 0,y[0]==0,y''[0]==1}},y[x],x,IncludeSingularSolutions
```

Too large to display

## 3.15 problem 15

Internal problem ID [6451]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 15.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$x^4 y'' + y' x^3 - 4yx^2 - 1 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(x^4*diff(y(x),x$2)+x^3*diff(y(x),x)-4*x^2*y(x)=1,y(x), singsol=all)
```

$$y(x) = \frac{c_2}{x^2} + x^2 c_1 + \frac{-4\ln(x) - 1}{16x^2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 29

```
DSolve[x^4*y''[x]+x^3*y'[x]-4*x^2*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{16c_2 x^4 - 4\log(x) - 1 + 16c_1}{16x^2}$$

## 3.16   problem 16

Internal problem ID [6452]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 16.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$x^4 y'' + y' x^3 - 4yx^2 - x = 0$$

✓  Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x^4*diff(y(x),x$2)+x^3*diff(y(x),x)-4*x^2*y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{c_2}{x^2} + x^2 c_1 - \frac{1}{3x}$$

✓  Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 25

```
DSolve[x^4*y''[x]+x^3*y'[x]-4*x^2*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x^2 + \frac{c_1}{x^2} - \frac{1}{3x}$$

## 3.17   problem 17

Internal problem ID [6453]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 17.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y''x^2 + y'x - 4y - x = 0$$

✓   Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(x^2*diff(diff(y(x),x),x)+x*diff(y(x),x)-4*y(x) = x,y(x), singsol=all)
```

$$y(x) = \frac{c_2}{x^2} + x^2 c_1 - \frac{x}{3}$$

✓   Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]+x*y'[x]-4*y[x] == x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x^2 + \frac{c_1}{x^2} - \frac{x}{3}$$

## 3.18 problem 18

Internal problem ID [6454]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 18.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type [[_3rd_order, _with_linear_symmetries]]

$$x^4 y''' + x^3 y'' + y' x^2 + yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 182

```
dsolve(x^4*diff(y(x),x$3)+x^3*diff(y(x),x$2)+x^2*diff(y(x),x)+x*y(x)= 0,y(x), singsol=all)
```

$$y(x) = c_1 x^{-\frac{\left(188+12\sqrt{249}\right)^{\frac{2}{3}} - 4\left(188+12\sqrt{249}\right)^{\frac{1}{3}} - 8}{6\left(188+12\sqrt{249}\right)^{\frac{1}{3}}}}$$

$$+ c_2 x^{\frac{-8+\left(188+12\sqrt{249}\right)^{\frac{2}{3}} + 8\left(188+12\sqrt{249}\right)^{\frac{1}{3}}}{12\left(188+12\sqrt{249}\right)^{\frac{1}{3}}}} \sin\left(\frac{\left(\left(188+12\sqrt{249}\right)^{\frac{2}{3}}\sqrt{3} + 8\sqrt{3}\right)\ln(x)}{12\left(188+12\sqrt{249}\right)^{\frac{1}{3}}}\right)$$

$$+ c_3 x^{\frac{-8+\left(188+12\sqrt{249}\right)^{\frac{2}{3}} + 8\left(188+12\sqrt{249}\right)^{\frac{1}{3}}}{12\left(188+12\sqrt{249}\right)^{\frac{1}{3}}}} \cos\left(\frac{\left(\left(188+12\sqrt{249}\right)^{\frac{2}{3}}\sqrt{3} + 8\sqrt{3}\right)\ln(x)}{12\left(188+12\sqrt{249}\right)^{\frac{1}{3}}}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 81

```
DSolve[x^4*y'''[x]+x^3*y''[x]+x^2*y'[x]+x*y[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^{\text{Root}\left[\#1^3 - 2\#1^2 + 2\#1 + 1\&,1\right]} + c_3 x^{\text{Root}\left[\#1^3 - 2\#1^2 + 2\#1 + 1\&,3\right]} + c_2 x^{\text{Root}\left[\#1^3 - 2\#1^2 + 2\#1 + 1\&,2\right]}$$

## 3.19   problem 19

Internal problem ID [6455]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 19.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type [[_3rd_order, _with_linear_symmetries]]

$$x^4 y''' + x^3 y'' + y' x^2 + yx - x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 220

```
dsolve(x^4*diff(y(x),x$3)+x^3*diff(y(x),x$2)+x^2*diff(y(x),x)+x*y(x)= x,y(x), singsol=all)
```

$$y(x) = 1 + c_1 x^{\frac{\left(188+12\sqrt{249}\right)^{\frac{2}{3}}\sqrt{249}}{32} - \frac{47\left(188+12\sqrt{249}\right)^{\frac{2}{3}}}{96} - \frac{\left(188+12\sqrt{249}\right)^{\frac{1}{3}}}{6} + \frac{2}{3}}$$

$$+ c_2 x^{-\frac{\left(188+12\sqrt{249}\right)^{\frac{2}{3}}\sqrt{249}}{64} + \frac{47\left(188+12\sqrt{249}\right)^{\frac{2}{3}}}{192} + \frac{\left(188+12\sqrt{249}\right)^{\frac{1}{3}}}{12} + \frac{2}{3}} \cos\left(\frac{\left(188+12\sqrt{249}\right)^{\frac{1}{3}}\sqrt{3}\left(3\left(188+12\sqrt{249}\right)^{\frac{1}{3}}\sqrt{24}\right)}{192}\right)$$

$$+ c_3 x^{-\frac{\left(188+12\sqrt{249}\right)^{\frac{2}{3}}\sqrt{249}}{64} + \frac{47\left(188+12\sqrt{249}\right)^{\frac{2}{3}}}{192} + \frac{\left(188+12\sqrt{249}\right)^{\frac{1}{3}}}{12} + \frac{2}{3}} \sin\left(\frac{\left(188+12\sqrt{249}\right)^{\frac{1}{3}}\sqrt{3}\left(3\left(188+12\sqrt{249}\right)^{\frac{1}{3}}\sqrt{24}\right)}{192}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 82

```
DSolve[x^4*y'''[x]+x^3*y''[x]+x^2*y'[x]+x*y[x]== x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^{\text{Root}\left[\#1^3 - 2\#1^2 + 2\#1 + 1\&, 1\right]} + c_3 x^{\text{Root}\left[\#1^3 - 2\#1^2 + 2\#1 + 1\&, 3\right]}$$
$$+ c_2 x^{\text{Root}\left[\#1^3 - 2\#1^2 + 2\#1 + 1\&, 2\right]} + 1$$

## 3.20 problem 20

Internal problem ID [6456]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 20.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type [[_high_order, _with_linear_symmetries]]

$$5x^5 y'''' + 4x^4 y''' + y'x^2 + yx = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 38

```
dsolve(5*x^5*diff(y(x),x$4)+4*x^4*diff(y(x),x$3)+x^2*diff(y(x),x)+x*y(x)= 0,y(x), singsol=all
```

$$y(x) = \sum_{\_a=1}^{4} x^{\mathrm{RootOf}\left(5\_Z^4 - 26\_Z^3 + 43\_Z^2 - 21\_Z + 1, \mathrm{index}=\_a\right)} \_C\_a$$

✓ Solution by Mathematica

Time used: 0.997 (sec). Leaf size: 1931

```
DSolve[5*x^5*y''''[x]+4*x^4*y'''[x]+x^2*y'[x]+x*y[x]== Sin[x],y[x],x,IncludeSingularSolutions
```

Too large to display

## 3.21 problem 21

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 21.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]

$$(x^2 + 1) y'' + 1 + {y'}^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve((1+x^2)*diff(y(x),x$2)+1+diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{c_1} - \frac{(-c_1^2 - 1)\ln(c_1 x - 1)}{c_1^2} + c_2$$

✓ Solution by Mathematica

Time used: 7.608 (sec). Leaf size: 33

```
DSolve[(1+x^2)*y''[x]+1+(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \cot(c_1) + \csc^2(c_1) \log(-x \sin(c_1) - \cos(c_1)) + c_2$$

## 3.22 problem 22

Internal problem ID [6458]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 22.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\left(x^2 + 1\right) y'' + 1 + y'^2 - x = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 982

```
dsolve((1+x^2)*diff(y(x),x$2)+1+diff(y(x),x)^2=x,y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1+x^2)*y''[x]+1+(y'[x])^2==x,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 3.23 problem 23

Internal problem ID [6459]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 23.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]

$$\left(x^2 + 1\right) y'' + xy'^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve((1+x^2)*diff(y(x),x$2)+1+x*diff(y(x),x)^2=1,y(x), singsol=all)
```

$$y(x) = \int \frac{2}{\ln\left(x^2 + 1\right) + 2c_1} dx + c_2$$

✓ Solution by Mathematica

Time used: 60.256 (sec). Leaf size: 31

```
DSolve[(1+x^2)*y''[x]+1+x*(y'[x])^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \int_1^x \frac{2}{\log\left(K[1]^2 + 1\right) - 2c_1} dK[1] + c_2$$

## 3.24 problem 24

Internal problem ID [6460]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 24.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [NONE]

$$\left(x^2 + 1\right) y'' + yy'^2 = 0$$

✗ Solution by Maple

```
dsolve((1+x^2)*diff(y(x),x$2)+y(x)*diff(y(x),x)^2=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1+x^2)*y''[x]+y[x]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 3.25 problem 25

Internal problem ID [6461]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 25.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]

$$\left(x^2 + 1\right) y'' + y'^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((1+x^2)*diff(y(x),x$2)+diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = \int \frac{1}{\arctan(x) + c_1} \, dx + c_2$$

✓ Solution by Mathematica

Time used: 60.272 (sec). Leaf size: 25

```
DSolve[(1+x^2)*y''[x]+(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \int_1^x \frac{1}{\arctan(K[1]) - c_1} \, dK[1] + c_2$$

## 3.26 problem 26

Internal problem ID [6462]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 26.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible,

$$y'' + \sin{(y)}\, y'^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+sin(y(x))*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$\int^{y(x)} \mathrm{e}^{-\cos(\_a)} d\_a - xc_1 - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.697 (sec). Leaf size: 37

```
DSolve[y''[x]+y[x]*Sin[y[x]](y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{e^{\sin(K[1])-\cos(K[1])K[1]}}{c_1} dK[1]\&\right][x + c_2]$$

## 3.27 problem 27

Internal problem ID [6463]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 27.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]

$$\left(x^2 + 1\right) y'' + y'^3 = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 33

```
dsolve((1+x^2)*diff(y(x),x$2)+diff(y(x),x)^3=0,y(x), singsol=all)
```

$$y(x) = \int \frac{1}{\sqrt{c_1 + 2\arctan(x)}} dx + c_2$$

$$y(x) = \int -\frac{1}{\sqrt{c_1 + 2\arctan(x)}} dx + c_2$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1+x^2)*y''[x]+y[x]*(y'[x])^3==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 3.28 problem 28

Internal problem ID [6464]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 28.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$y' - e^{-\frac{y}{x}} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=exp(-y(x)/x),y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(\int^{-Z} \frac{1}{-e^{-\_a} + \_a} d\_a + \ln(x) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.157 (sec). Leaf size: 39

```
DSolve[y'[x]==Exp[-y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{e^{K[1]}}{e^{K[1]}K[1] - 1} dK[1] = -\log(x) + c_1, y(x)\right]$$

## 3.29 problem 29

Internal problem ID [6465]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 29.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [[_homogeneous, 'class D']]

$$y' - 2x^2 \sin\left(\frac{y}{x}\right)^2 - \frac{y}{x} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)= 2*x^2 * sin(y(x)/x)^2 + y(x)/x,y(x), singsol=all)
```

$$y(x) = -\left(-\pi + \operatorname{arccot}\left(x^2 + 2c_1\right)\right)x$$

✓ Solution by Mathematica
Time used: 0.313 (sec). Leaf size: 22

```
DSolve[y'[x]== 2*x^2 * Sin[y[x]/x]^2 + y[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \cot^{-1}\left(x^2 - 2c_1\right)$$
$$y(x) \to 0$$

## 3.30 problem 30

Internal problem ID [6466]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 30.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$4y''x^2 + y - 8\sqrt{x}\left(1 + \ln(x)\right) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(4*x^2*diff(y(x),x$2)+ y(x) = 8*sqrt(x)*(1+ln(x)),y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,c_2 + \sqrt{x}\,\ln(x)\,c_1 + \frac{\sqrt{x}\,\ln(x)^2\left(3 + \ln(x)\right)}{3}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 35

```
DSolve[4*x^2*y''[x]+y[x] == 8*Sqrt[x]*(1+Log[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\sqrt{x}(\log(x)(2\log(x)(\log(x) + 3) + 3c_2) + 6c_1)$$

## 3.31 problem 31

Internal problem ID [6467]

**Book**: Own collection of miscellaneous problems
**Section**: section 3.0
**Problem number**: 31.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_rational, _Bernoulli]

$$
vv' - \frac{2v^2}{r^3} - \frac{\lambda r}{3} = 0
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 101

```
dsolve(v(r)*diff(v(r),r)=2*v(r)^2/r^3+1/3*lambda*r,v(r), singsol=all)
```

$$
v(r) = -\frac{e^{-\frac{2}{r^2}}\sqrt{3}\sqrt{e^{\frac{2}{r^2}}\left(\lambda e^{\frac{2}{r^2}}r^2 + 2\lambda\,\mathrm{Ei}_1\left(-\frac{2}{r^2}\right) + 3c_1\right)}}{3}
$$

$$
v(r) = \frac{e^{-\frac{2}{r^2}}\sqrt{3}\sqrt{e^{\frac{2}{r^2}}\left(\lambda e^{\frac{2}{r^2}}r^2 + 2\lambda\,\mathrm{Ei}_1\left(-\frac{2}{r^2}\right) + 3c_1\right)}}{3}
$$

✓ Solution by Mathematica

Time used: 9.418 (sec). Leaf size: 86

```
DSolve[v[r]*v'[r]==2*v[r]^2/r^3+1/3*\[Lambda]*r,v[r],r,IncludeSingularSolutions -> True]
```

$$
v(r) \to -\frac{\sqrt{\lambda r^2 + e^{-\frac{2}{r^2}}\left(-2\lambda\,\mathrm{ExpIntegralEi}\left(\frac{2}{r^2}\right) + 3c_1\right)}}{\sqrt{3}}
$$

$$
v(r) \to \frac{\sqrt{\lambda r^2 + e^{-\frac{2}{r^2}}\left(-2\lambda\,\mathrm{ExpIntegralEi}\left(\frac{2}{r^2}\right) + 3c_1\right)}}{\sqrt{3}}
$$

# 4 section 4.0

## 4.1 problem 1

Internal problem ID [6468]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$2y''x^2 - y'x + (1 - x^2)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1 \sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + O\left(x^6\right)\right) + c_2 x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + O\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 48

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x\left(\frac{x^4}{360} + \frac{x^2}{10} + 1\right) + c_2 \sqrt{x}\left(\frac{x^4}{168} + \frac{x^2}{6} + 1\right)$$

## 4.2 problem 2

Internal problem ID [6469]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 - y'x + (1 - x^2)\,y - 1 = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = 1,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + O\left(x^6\right)\right)$$
$$+ c_2 x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + O\left(x^6\right)\right) + \left(1 + \frac{1}{3}x^2 + \frac{1}{63}x^4 + O\left(x^6\right)\right)$$

✔ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 176

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==1,y[x],{x,0,5}]
```

$$y(x) \to c_2 x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$
$$+ c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x}\left(-\frac{x^{11/2}}{154440} - \frac{x^{7/2}}{1260} - \frac{x^{3/2}}{15}\right)$$
$$+ \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{x^5}{55440} + \frac{x^3}{504} + \frac{x}{6} - \frac{1}{x}\right)\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

## 4.3 problem 3

Internal problem ID [6470]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 - y'x + \left(1 - x^2\right)y - 1 - x = 0$$

With the expansion point for the power series method at $x = 0$.

✗  Solution by Maple

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = 1+x,y(x),type='series',x=0);
```

No solution found

✓  Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 224

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==1+x,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left( \frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1 \right) + c_2 x \left( \frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1 \right)$$
$$+ \sqrt{x} \left( -\frac{x^{11/2}}{154440} - \frac{x^{9/2}}{1620} - \frac{x^{7/2}}{1260} - \frac{x^{5/2}}{25} - \frac{x^{3/2}}{15} - 2\sqrt{x} \right.$$
$$\left. + \frac{2}{\sqrt{x}} \right) \left( \frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1 \right) + x \left( \frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1 \right) \left( \frac{x^6}{66528} + \frac{x^5}{55440} + \frac{x^4}{672} + \frac{x^3}{504} + \frac{x^2}{12} + \frac{x}{6} - \frac{1}{x} + 1 \right)$$

## 4.4 problem 4

Internal problem ID [6471]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 - y'x + \left(1 - x^2\right)y - x = 0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = x,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 166

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==x,y[x],{x,0,5}]
```

$$y(x) \to x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)\left(\frac{x^6}{66528} + \frac{x^4}{672} + \frac{x^2}{12} + \log(x)\right)$$
$$+ c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + c_2 x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$
$$+ \sqrt{x}\left(-\frac{x^{9/2}}{1620} - \frac{x^{5/2}}{25} - 2\sqrt{x}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right)$$

## 4.5 problem 5

Internal problem ID [6472]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 - y'x + (1 - x^2) y - x^2 - x - 1 = 0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = 1+x+x^2,y(x),type='series',x
```

No solution found

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 224

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==1+x+x^2,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left( \frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1 \right) + c_2 x \left( \frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1 \right)$$
$$+ \sqrt{x} \left( -\frac{79x^{11/2}}{154440} - \frac{x^{9/2}}{1620} - \frac{37x^{7/2}}{1260} - \frac{x^{5/2}}{25} - \frac{11x^{3/2}}{15} - 2\sqrt{x} \right.$$
$$\left. + \frac{2}{\sqrt{x}} \right) \left( \frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1 \right) + x \left( \frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1 \right) \left( \frac{x^6}{66528} + \frac{67x^5}{55440} + \frac{x^4}{672} + \frac{29x^3}{504} + \frac{x^2}{12} + \frac{7x}{6} - \frac{1}{x} \right)$$

## 4.6 problem 6

Internal problem ID [6473]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 - y'x + (1 - x^2) y - x^2 = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = x^2,y(x),type='series',x=0);
```

$$y(x) = c_1 \sqrt{x} \left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + O(x^6)\right)$$
$$+ c_2 x \left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + O(x^6)\right) + x^2 \left(\frac{1}{3} + \frac{1}{63}x^2 + O(x^4)\right)$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 160

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==x^2,y[x],{x,0,5}]
```

$$y(x) \to c_2 x \left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + c_1 \sqrt{x} \left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x} \left(-\frac{x^{11/2}}{1980} - \frac{x^{7/2}}{35}\right)$$
$$- \frac{2x^{3/2}}{3}\right) \left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x \left(\frac{x^5}{840} + \frac{x^3}{18} + x\right) \left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

## 4.7 problem 7

Internal problem ID [6474]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 - y'x + \left(1 - x^2\right)y - x^2 - 1 = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = 1+x^2,y(x),type='series',x=0
```

$$y(x) = c_1 \sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + O\left(x^6\right)\right)$$
$$+ c_2 x \left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + O\left(x^6\right)\right) + \left(1 + \frac{2}{3}x^2 + \frac{2}{63}x^4 + O\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 176

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==1+x^2,y[x],{x,0,5}]
```

$$y(x)$$
$$\to c_2 x \left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$
$$+ c_1 \sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x}\left(-\frac{79x^{11/2}}{154440} - \frac{37x^{7/2}}{1260} - \frac{11x^{3/2}}{15}\right)$$
$$+ \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{67x^5}{55440} + \frac{29x^3}{504} + \frac{7x}{6} - \frac{1}{x}\right)\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

## 4.8 problem 8

Internal problem ID [6475]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 - y'x + \left(1 - x^2\right) y - x^4 = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = x^4,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right)$$
$$+ c_2 x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right) + x^4\left(\frac{1}{21} + \mathrm{O}\left(x^2\right)\right)$$

✔ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 150

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==x^4,y[x],{x,0,5}]
```

$$y(x) \to c_2 x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x}\left(-\frac{x^{11/2}}{55}\right.$$
$$\left. - \frac{2x^{7/2}}{7}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{x^5}{30} + \frac{x^3}{3}\right)\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

## 4.9 problem 9

Internal problem ID [6476]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2y''x^2 - y'x + (1 - x^2)\, y - \sin(x) = 0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = sin(x),y(x),type='series',x=
```

No solution found

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 159

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==Sin[x],y[x],{x,0,5}]
```

$$y(x) \to x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)\left(\frac{x^6}{20790} - \frac{17x^4}{5040} + \log(x)\right) + c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right)$$
$$+ c_2 x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + \sqrt{x}\left(\frac{x^{9/2}}{810} + \frac{2x^{5/2}}{75} - 2\sqrt{x}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right)$$

## 4.10 problem 10

Internal problem ID [6477]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 - y'x + (1 - x^2)\,y - 1 - \sin(x) = 0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = 1+sin(x),y(x),type='series',
```

No solution found

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 232

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==1+sin(x),y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to {} & c_1 \sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + c_2 x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) \\
& + \sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right)\left(-\frac{x^{11/2}}{154440} - \frac{x^{7/2}}{1260} - \frac{x^{3/2}}{15} - \frac{x^{9/2}\sin}{1620} - \frac{1}{25}x^{5/2}\sin + \frac{2}{\sqrt{x}}\right. \\
& \left. -2\sqrt{x}\sin\right) + x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)\left(\frac{x^6\sin}{66528} + \frac{x^5}{55440} + \frac{x^4\sin}{672} + \frac{x^3}{504} + \frac{x^2\sin}{12} + \frac{x}{6} - \frac{1}{x} + \sin\log(x)\right)
\end{aligned}
$$

## 4.11 problem 11

Internal problem ID [6478]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 - y'x + (1 - x^2)y - x\sin(x) = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = x*sin(x),y(x),type='series',
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + O\left(x^6\right)\right)$$
$$+ c_2 x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + O\left(x^6\right)\right) + x^2\left(\frac{1}{3} + \frac{1}{126}x^2 + O\left(x^4\right)\right)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 167

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==x*sin(x),y[x],{x,0,5}]
```

$$y(x) \to c_2 x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right)$$
$$+ \sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right)\left(-\frac{x^{11/2}\sin}{1980} - \frac{1}{35}x^{7/2}\sin\right.$$
$$\left. - \frac{2}{3}x^{3/2}\sin\right) + x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)\left(\frac{x^5\sin}{840} + \frac{x^3\sin}{18} + x\sin\right)$$

## 4.12 problem 12

Internal problem ID [6479]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2y''x^2 - y'x + \left(1 - x^2\right)y - \cos(x) - \sin(x) = 0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = sin(x)+cos(x),y(x),type='ser
```

No solution found

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 217

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==Sin[x]+Cos[x],y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + c_2 x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

$$+ \sqrt{x}\left(-\frac{x^{11/2}}{3861} + \frac{x^{9/2}}{810} + \frac{x^{7/2}}{630} + \frac{2x^{5/2}}{75} + \frac{4x^{3/2}}{15} - 2\sqrt{x}\right)$$

$$+ \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)\left(\frac{x^6}{20790} + \frac{37x^5}{69300} - \frac{17x^4}{5040} - \frac{x^3}{84} - \frac{x}{3} - \frac{1}{x} + \log(x$$

## 4.13 problem 13

Internal problem ID [6480]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$x^2 y'' + (\cos(x) - 1) y' + e^x y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 1171

```
Order:=6;
dsolve(x^2*diff(y(x), x$2) + (cos(x)-1)*diff(y(x), x) + exp(x)*y(x) = 0,y(x),type='series',x=
```

$$
y(x) = \sqrt{x} \left( c_2 x^{\frac{i\sqrt{3}}{2}} \left( 1 + \frac{1}{4} i\sqrt{3} x + \frac{-i\sqrt{3} - 11}{32 i\sqrt{3} + 64} x^2 - \frac{55}{288} \frac{i\sqrt{3} - 3}{(1 + i\sqrt{3})(i\sqrt{3} + 2)(i\sqrt{3} + 3)} x^3 \right. \right.
$$
$$
+ \frac{1}{384} \frac{112 i\sqrt{3} + 199}{(1 + i\sqrt{3})(i\sqrt{3} + 2)(i\sqrt{3} + 3)(i\sqrt{3} + 4)} x^4
$$
$$
\left. + \frac{\frac{18491 i\sqrt{3}}{38400} - \frac{4387}{12800}}{(1 + i\sqrt{3})(i\sqrt{3} + 2)(i\sqrt{3} + 3)(i\sqrt{3} + 4)(i\sqrt{3} + 5)} x^5 + O\left(x^6\right) \right)
$$
$$
+ c_1 x^{-\frac{i\sqrt{3}}{2}} \left( 1 - \frac{1}{4} i\sqrt{3} x + \frac{-i\sqrt{3} + 11}{32 i\sqrt{3} - 64} x^2 - \frac{55}{288} \frac{i\sqrt{3} + 3}{(i\sqrt{3} - 1)(i\sqrt{3} - 2)(i\sqrt{3} - 3)} x^3 \right.
$$
$$
- \frac{1}{384} \frac{112 i\sqrt{3} - 199}{(i\sqrt{3} - 1)(i\sqrt{3} - 2)(i\sqrt{3} - 3)(i\sqrt{3} - 4)} x^4
$$
$$
\left. \left. + \frac{\frac{18491 i\sqrt{3}}{38400} + \frac{4387}{12800}}{(i\sqrt{3} - 1)(i\sqrt{3} - 2)(i\sqrt{3} - 3)(i\sqrt{3} - 4)(i\sqrt{3} - 5)} x^5 + O\left(x^6\right) \right) \right)
$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 2502

```
AsymptoticDSolveValue[x^2*y''[x] + (Cos[x]-1)*y'[x] + Exp[x]*y[x] ==0,y[x],{x,0,5}]
```

Too large to display

## 4.14 problem 14

Internal problem ID [6481]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$(x - 2) y'' + \frac{y'}{x} + (1 + x) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
Order:=6;
dsolve((x-2)*diff(y(x), x$2) + 1/x*diff(y(x), x) + (x+1)*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{3}{2}} \left( 1 + \frac{3}{20}x + \frac{25}{224}x^2 + \frac{1361}{17280}x^3 + \frac{80753}{2365440}x^4 + \frac{616517}{38707200}x^5 + O\left(x^6\right) \right)$$
$$+ c_2 \left( 1 + \frac{1}{2}x^2 + \frac{2}{9}x^3 + \frac{11}{120}x^4 + \frac{82}{1575}x^5 + O\left(x^6\right) \right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 80

```
AsymptoticDSolveValue[(x-2)*y''[x] + 1/x*y'[x] + (x+1)*y[x] ==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left( \frac{82x^5}{1575} + \frac{11x^4}{120} + \frac{2x^3}{9} + \frac{x^2}{2} + 1 \right)$$
$$+ c_1 \left( \frac{616517x^5}{38707200} + \frac{80753x^4}{2365440} + \frac{1361x^3}{17280} + \frac{25x^2}{224} + \frac{3x}{20} + 1 \right) x^{3/2}$$

## 4.15   problem 15

Internal problem ID [6482]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 15.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$(x - 2) y'' + \frac{y'}{x} + (1 + x) y = 0$$

With the expansion point for the power series method at $x = 2$.

✓   Solution by Maple

Time used: 0.031 (sec). Leaf size: 46

```
Order:=6;
dsolve((x-2)*diff(y(x), x$2) + 1/x*diff(y(x), x) + (x+1)*y(x) = 0,y(x),type='series',x=2);
```

$$
\begin{aligned}
y(x) = c_1 \sqrt{x - 2} \, \bigg( &1 - \frac{23}{12}(x - 2) + \frac{127}{160}(x - 2)^2 + \frac{1621}{40320}(x - 2)^3 - \frac{426599}{5806080}(x - 2)^4 \\
&+ \frac{4670443}{425779200}(x - 2)^5 + \mathrm{O}\left((x - 2)^6\right) \bigg) + c_2 \bigg( 1 - 6(x - 2) + \frac{31}{6}(x - 2)^2 - \frac{37}{45}(x - 2)^3 \\
&- \frac{299}{840}(x - 2)^4 + \frac{6743}{56700}(x - 2)^5 + \mathrm{O}\left((x - 2)^6\right) \bigg)
\end{aligned}
$$

✓   Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 105

```
AsymptoticDSolveValue[(x-2)*y''[x] + 1/x*y'[x] + (x+1)*y[x] ==0,y[x],{x,2,5}]
```

$$
\begin{aligned}
y(x) \rightarrow c_1 \bigg( &\frac{4670443(x - 2)^5}{425779200} - \frac{426599(x - 2)^4}{5806080} + \frac{1621(x - 2)^3}{40320} + \frac{127}{160}(x - 2)^2 - \frac{23(x - 2)}{12} \\
&+ 1 \bigg) \sqrt{x - 2} \\
+ c_2 \bigg( &\frac{6743(x - 2)^5}{56700} - \frac{299}{840}(x - 2)^4 - \frac{37}{45}(x - 2)^3 + \frac{31}{6}(x - 2)^2 - 6(x - 2) + 1 \bigg)
\end{aligned}
$$

## 4.16 problem 16

Internal problem ID [6483]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 16.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$(1+x)(3x-1)y'' + y'\cos(x) - 3yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
Order:=6;
dsolve((x+1)*(3*x-1)*diff(y(x),x$2)+cos(x)*diff(y(x),x)-3*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^3 - \frac{5}{8}x^4 - \frac{53}{40}x^5\right)y(0) + \left(x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{7}{12}x^4 + \frac{7}{6}x^5\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(x+1)*(3*x-1)*y''[x]+Cos[x]*y'[x]-3*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{53x^5}{40} - \frac{5x^4}{8} - \frac{x^3}{2} + 1\right) + c_2\left(\frac{7x^5}{6} + \frac{7x^4}{12} + \frac{x^3}{2} + \frac{x^2}{2} + x\right)$$

## 4.17 problem 17

Internal problem ID [6484]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 17.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [_Lienard]

$$xy'' + 2y' + yx = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='series',x=
```

$$y(x) = 1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{x*y''[x]+2*y'[x]+x*y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^4}{120} - \frac{x^2}{6} + 1$$

## 4.18 problem 18

Internal problem ID [6485]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 18.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$2y''x^2 + 3y'x - yx - x^2 - 2x = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.015 (sec). Leaf size: 60

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-x*y(x)=x^2+2*x,y(x),type='series',x=0);
```

$$y(x)$$
$$= \frac{c_2\left(1 + \frac{1}{3}x + \frac{1}{30}x^2 + \frac{1}{630}x^3 + \frac{1}{22680}x^4 + \frac{1}{1247400}x^5 + O\left(x^6\right)\right)\sqrt{x} + x^{\frac{3}{2}}\left(\frac{2}{3} + \frac{1}{6}x + \frac{1}{126}x^2 + \frac{1}{4536}x^3 + \frac{1}{249480}x^4 + \right.}{\sqrt{x}}$$

✔ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 239

```
AsymptoticDSolveValue[2*x^2*y''[x]+3*x*y'[x]-x*y[x]==x^2+2*x,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{1247400} + \frac{x^4}{22680} + \frac{x^3}{630} + \frac{x^2}{30} + \frac{x}{3} + 1\right) + \frac{c_2\left(\frac{x^5}{113400} + \frac{x^4}{2520} + \frac{x^3}{90} + \frac{x^2}{6} + x + 1\right)}{\sqrt{x}}$$
$$+ \frac{\left(\frac{x^5}{113400} + \frac{x^4}{2520} + \frac{x^3}{90} + \frac{x^2}{6} + x + 1\right)\left(-\frac{19x^{11/2}}{62370} - \frac{23x^{9/2}}{2835} - \frac{4x^{7/2}}{35} - \frac{2x^{5/2}}{3} - \frac{4x^{3/2}}{3}\right)}{\sqrt{x}}$$
$$+ \left(\frac{x^5}{1247400} + \frac{x^4}{22680} + \frac{x^3}{630} + \frac{x^2}{30} + \frac{x}{3} + 1\right)\left(\frac{47x^6}{680400} + \frac{x^5}{420} + \frac{17x^4}{360} + \frac{4x^3}{9} + \frac{3x^2}{2} + 2x\right)$$

## 4.19 problem 19

Internal problem ID [6486]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 19.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2y''x^2 - y'x + \left(1 - x^2\right)y - 1 = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (-x^2 + 1)*y(x) = 1,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + O\left(x^6\right)\right)$$
$$+ c_2 x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + O\left(x^6\right)\right) + \left(1 + \frac{1}{3}x^2 + \frac{1}{63}x^4 + O\left(x^6\right)\right)$$

✔ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 176

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-x^2)*y[x]==1,y[x],{x,0,5}]
```

$$y(x) \to c_2 x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$
$$+ c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x}\left(-\frac{x^{11/2}}{154440} - \frac{x^{7/2}}{1260} - \frac{x^{3/2}}{15}\right)$$
$$+ \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{x^5}{55440} + \frac{x^3}{504} + \frac{x}{6} - \frac{1}{x}\right)\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

## 4.20 problem 20

Internal problem ID [6487]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 20.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2y''x^2 + 2y'x - yx - 1 = 0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) + 2*x*diff(y(x), x) - x*y(x) = 1,y(x),type='series',x=0);
```

No solution found

✓  Solution by Mathematica

Time used: 0.142 (sec). Leaf size: 360

```
AsymptoticDSolveValue[2*x^2*y''[x]+2*x*y'[x]-x*y[x]==1,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to \; & c_2\left(\frac{x^5}{460800} + \frac{x^4}{9216} + \frac{x^3}{288} + \frac{x^2}{16} + \frac{x}{2} + 1\right) \\
& + c_1\left(x^5\left(\frac{\log(x)}{460800} - \frac{107}{13824000}\right) + x^4\left(\frac{\log(x)}{9216} - \frac{19}{55296}\right) + x^3\left(\frac{\log(x)}{288} - \frac{1}{108}\right)\right. \\
& \qquad\qquad \left. + x^2\left(\frac{\log(x)}{16} - \frac{1}{8}\right) + x\left(\frac{\log(x)}{2} - \frac{1}{2}\right) + \log(x) + 1\right) \\
& + \left(-\frac{137x^6}{1990656000} + \frac{x^5}{4608000} + \frac{x^4}{73728} + \frac{x^3}{1728} + \frac{x^2}{64} + \frac{x}{4}\right. \\
& \quad \left. + \frac{\log(x)}{2}\right)\left(x^5\left(\frac{\log(x)}{460800} - \frac{107}{13824000}\right) + x^4\left(\frac{\log(x)}{9216} - \frac{19}{55296}\right) + x^3\left(\frac{\log(x)}{288} - \frac{1}{108}\right)\right. \\
& \quad \left. + x^2\left(\frac{\log(x)}{16} - \frac{1}{8}\right) + x\left(\frac{\log(x)}{2} - \frac{1}{2}\right) + \log(x) + 1\right) + \left(\frac{x^5}{460800} + \frac{x^4}{9216} + \frac{x^3}{288}\right. \\
& \quad \left. + \frac{x^2}{16} + \frac{x}{2} + 1\right)\left(\frac{137x^6(6\log(x) + 5)}{11943936000} + \frac{x^5(113 - 30\log(x))}{138240000} + \frac{x^4(41 - 12\log(x))}{884736}\right. \\
& \quad \left.\left. + \frac{x^3(3 - \log(x))}{1728} + \frac{1}{128}x^2(5 - 2\log(x)) + \frac{1}{4}x(2 - \log(x)) - \frac{1}{4}\log(x)(\log(x) + 2)\right)\right)
\end{aligned}
$$

## 4.21 problem 21

Internal problem ID [6488]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 21.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' + (x - 6)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(diff(y(x), x, x) + (x-6)*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + 3x^2 - \frac{1}{6}x^3 + \frac{3}{2}x^4 - \frac{1}{5}x^5\right) y(0) + \left(x + x^3 - \frac{1}{12}x^4 + \frac{3}{10}x^5\right) D(y)\,(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 57

```
AsymptoticDSolveValue[y''[x]+(x-6)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(\frac{3x^5}{10} - \frac{x^4}{12} + x^3 + x\right) + c_1 \left(-\frac{x^5}{5} + \frac{3x^4}{2} - \frac{x^3}{6} + 3x^2 + 1\right)$$

## 4.22 problem 22

Internal problem ID [6489]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 22.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y''x^2 + (3x^2 + 2x) \, y' - 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x), x, x) + (2*x+3*x^2)*diff(y(x),x)-2*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1 x \left( 1 - \frac{3}{4}x + \frac{9}{20}x^2 - \frac{9}{40}x^3 + \frac{27}{280}x^4 - \frac{81}{2240}x^5 + \mathrm{O}\left(x^6\right) \right)$$
$$+ \frac{c_2 \left( 12 - 36x + 54x^2 - 54x^3 + \frac{81}{2}x^4 - \frac{243}{10}x^5 + \mathrm{O}\left(x^6\right) \right)}{x^2}$$

✔ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 64

```
AsymptoticDSolveValue[x^2*y''[x]+(2*x+3*x^2)*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left( \frac{27x^2}{8} + \frac{1}{x^2} - \frac{9x}{2} - \frac{3}{x} + \frac{9}{2} \right) + c_2 \left( \frac{27x^5}{280} - \frac{9x^4}{40} + \frac{9x^3}{20} - \frac{3x^2}{4} + x \right)$$

## 4.23 problem 23

Internal problem ID [6490]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 23.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 - y'x + (1 - x^2)\, y - x^2 - \cos(x) = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) - x*diff(y(x), x) + (-x^2 + 1)*y(x) = x^2+cos(x),y(x),type='ser
```

$$y(x) = c_1 \sqrt{x} \left( 1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + O\left(x^6\right) \right)$$
$$+ c_2 x \left( 1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + O\left(x^6\right) \right) + \left( 1 + \frac{1}{2}x^2 + \frac{13}{504}x^4 + O\left(x^6\right) \right)$$

✓ Solution by Mathematica

Time used: 0.102 (sec). Leaf size: 176

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-x^2)*y[x]==x^2+Cos[x],y[x],{x,0,5}]
```

$$y(x)$$
$$\to c_2 x \left( \frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1 \right)$$
$$+ c_1 \sqrt{x} \left( \frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1 \right) + \sqrt{x} \left( -\frac{59x^{11/2}}{77220} - \frac{17x^{7/2}}{630} - \frac{2x^{3/2}}{5} \right)$$
$$+ \frac{2}{\sqrt{x}} \left( \frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1 \right) + x \left( \frac{239x^5}{138600} + \frac{11x^3}{252} + \frac{2x}{3} - \frac{1}{x} \right) \left( \frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1 \right)$$

## 4.24 problem 24

Internal problem ID [6491]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 24.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 - y'x + (1 - x^2)\, y - \cos(x) = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) - x*diff(y(x), x) + (-x^2 + 1)*y(x) = cos(x),y(x),type='series'
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right)$$
$$+ c_2 x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(1 + \frac{1}{6}x^2 + \frac{5}{504}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✔ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 176

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-x^2)*y[x]==Cos[x],y[x],{x,0,5}]
```

$$y(x) \to c_2 x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$
$$+ c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x}\left(-\frac{x^{11/2}}{3861} + \frac{x^{7/2}}{630} + \frac{4x^{3/2}}{15}\right)$$
$$+ \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{37x^5}{69300} - \frac{x^3}{84} - \frac{x}{3} - \frac{1}{x}\right)\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

## 4.25   problem 24

Internal problem ID [6492]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 24.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 - y'x + (1 - x^2) y - x^3 - \cos(x) = 0$$

With the expansion point for the power series method at $x = 0$.

✔  Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) - x*diff(y(x), x) + (-x^2 + 1)*y(x) = x^3+cos(x),y(x),type='ser
```

$$y(x) = c_1 \sqrt{x} \left( 1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + O\left(x^6\right) \right) + c_2 x \left( 1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + O\left(x^6\right) \right)$$
$$+ \left( 1 + \frac{1}{6}x^2 + \frac{1}{10}x^3 + \frac{5}{504}x^4 + \frac{1}{360}x^5 + O\left(x^6\right) \right)$$

✔  Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 176

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-x^2)*y[x]==Cos[x],y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 x \left( \frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1 \right)$$
$$+ c_1 \sqrt{x} \left( \frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1 \right) + \sqrt{x} \left( -\frac{x^{11/2}}{3861} + \frac{x^{7/2}}{630} + \frac{4x^{3/2}}{15} \right.$$
$$\left. + \frac{2}{\sqrt{x}} \right) \left( \frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1 \right) + x \left( \frac{37x^5}{69300} - \frac{x^3}{84} - \frac{x}{3} - \frac{1}{x} \right) \left( \frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1 \right)$$

## 4.26 problem 24

Internal problem ID [6493]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 24.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 - y'x + \left(1 - x^2\right)y - x^3\cos\left(x\right) = 0$$

With the expansion point for the power series method at $x = 0$.

✓  Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) - x*diff(y(x), x) + (-x^2 + 1)*y(x) = x^3*cos(x),y(x),type='ser
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right)$$
$$+ c_2x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right) + x^3\left(\frac{1}{10} - \frac{1}{90}x^2 + \mathrm{O}\left(x^4\right)\right)$$

✓  Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 215

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-x^2)*y[x]==x^3+Cos[x],y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + c_2x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$
$$+ \sqrt{x}\left(-\frac{x^{11/2}}{3861} - \frac{x^{9/2}}{45} + \frac{x^{7/2}}{630} - \frac{2x^{5/2}}{5} + \frac{4x^{3/2}}{15}\right.$$
$$\left. + \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)\left(\frac{x^6}{1008} + \frac{37x^5}{69300} + \frac{x^4}{24} - \frac{x^3}{84} + \frac{x^2}{2} - \frac{x}{3} - \frac{1}{x}\right)$$

## 4.27 problem 24

Internal problem ID [6494]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 24.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 - y'x + \left(1 - x^2\right)y - x^3 \cos\left(x\right) - \sin\left(x\right)^2 = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) - x*diff(y(x), x) + (-x^2 + 1)*y(x) = x^3*cos(x)+sin(x)^2,y(x),
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + O\left(x^6\right)\right)$$
$$+ c_2 x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + O\left(x^6\right)\right) + x^2\left(\frac{1}{3} + \frac{1}{10}x - \frac{1}{90}x^3 + O\left(x^4\right)\right)$$

✓ Solution by Mathematica

Time used: 0.471 (sec). Leaf size: 199

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-x^2)*y[x]==x^3*Cos[x]+Sin[x]^2,y[x],{x,0,5}]
```

$$y(x) \to c_2 x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$
$$+ c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x}\left(-\frac{x^{11/2}}{396} + \frac{4x^{9/2}}{45} + \frac{x^{7/2}}{15} - \frac{2x^{5/2}}{5}\right.$$
$$\left. - \frac{2x^{3/2}}{3}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(-\frac{x^6}{168} - \frac{13x^5}{12600} - \frac{x^4}{12} - \frac{x^3}{18} + \frac{x^2}{2} + x\right)\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

## 4.28 problem 24

Internal problem ID [6495]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 24.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 - y'x + (1 - x^2)\, y - \ln(x) = 0$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) - x*diff(y(x), x) + (-x^2 + 1)*y(x) = ln(x),y(x),type='series',
```

$$
\begin{aligned}
y(x) = {} & \left(1 + \frac{(x-1)^3}{6} - \frac{5(x-1)^4}{48} + \frac{37(x-1)^5}{480}\right) y(1) \\
& + \left(x - 1 + \frac{(x-1)^2}{4} - \frac{(x-1)^3}{24} + \frac{19(x-1)^4}{192} - \frac{119(x-1)^5}{1920}\right) D(y)(1) \\
& + \frac{(x-1)^3}{12} - \frac{3(x-1)^4}{32} + \frac{89(x-1)^5}{960} + O(x^6)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 105

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-x^2)*y[x]==Log[x],y[x],{x,1,5}]
```

$$
\begin{aligned}
y(x) \to {} & \frac{89}{960}(x-1)^5 - \frac{3}{32}(x-1)^4 + \frac{1}{12}(x-1)^3 + c_1 \left(\frac{37}{480}(x-1)^5 - \frac{5}{48}(x-1)^4 + \frac{1}{6}(x-1)^3 + 1\right) \\
& + c_2 \left(-\frac{119(x-1)^5}{1920} + \frac{19}{192}(x-1)^4 - \frac{1}{24}(x-1)^3 + \frac{1}{4}(x-1)^2 + x - 1\right)
\end{aligned}
$$

## 4.29 problem 25

Internal problem ID [6496]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 25.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$2x^2\left(x^2 + x + 1\right) y'' + x\left(11x^2 + 11x + 9\right) y' + \left(7x^2 + 10x + 6\right) y = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*x^2*(1+x+x^2)*diff(y(x), x$2) + x*(9+11*x+11*x^2)*diff(y(x), x) + (6+10*x+7*x^2)*y(x
```

$$y(x)$$
$$= \frac{\left(1 + \frac{1}{2}x^2 - \frac{1}{3}x^3 + \frac{1}{8}x^4 + \frac{1}{30}x^5 + O\left(x^6\right)\right) c_1 \sqrt{x} + \left(1 - \frac{1}{3}x + \frac{2}{5}x^2 - \frac{5}{21}x^3 + \frac{7}{135}x^4 + \frac{76}{1155}x^5 + O\left(x^6\right)\right) c_2 x}{x^{\frac{5}{2}}}$$

✔ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 83

```
AsymptoticDSolveValue[2*x^2*(1+x+x^2)*y''[x] + x*(9+11*x+11*x^2)*y'[x] + (6+10*x+7*x^2)*y[x]
```

$$y(x) \to \frac{c_2\left(\frac{x^5}{30} + \frac{x^4}{8} - \frac{x^3}{3} + \frac{x^2}{2} + 1\right)}{x^2} + \frac{c_1\left(\frac{76x^5}{1155} + \frac{7x^4}{135} - \frac{5x^3}{21} + \frac{2x^2}{5} - \frac{x}{3} + 1\right)}{x^{3/2}}$$

## 4.30 problem 26

Internal problem ID [6497]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 26.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2(x+3)\,y'' + 5x(1+x)\,y' - (1-4x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓  Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*(3+x)*diff(y(x), x$2) + 5*x*(1+x)*diff(y(x), x) - (1-4*x)*y(x) = 0,y(x),type='seri
```

$$y(x)$$
$$= \frac{c_2 x^{\frac{4}{3}}\left(1 - \frac{7}{9}x + \frac{35}{81}x^2 - \frac{455}{2187}x^3 + \frac{1820}{19683}x^4 - \frac{6916}{177147}x^5 + O\left(x^6\right)\right) + c_1\left(1 + x - x^2 + \frac{3}{5}x^3 - \frac{3}{10}x^4 + \frac{3}{22}x^5 + O\left(x\right.\right.}{x}$$

✓  Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 82

```
AsymptoticDSolveValue[x^2*(3+x)*y''[x] + 5*x*(1+x)*y'[x] - (1-4*x)*y[x] == 0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \sqrt[3]{x}\left(-\frac{6916x^5}{177147} + \frac{1820x^4}{19683} - \frac{455x^3}{2187} + \frac{35x^2}{81} - \frac{7x}{9} + 1\right)$$
$$+ \frac{c_2\left(\frac{3x^5}{22} - \frac{3x^4}{10} + \frac{3x^3}{5} - x^2 + x + 1\right)}{x}$$

## 4.31 problem 27

Internal problem ID [6498]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 27.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2\left(-x^2+2\right)y'' - x\left(4x^2+3\right)y' + \left(-2x^2+2\right)y = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*(2-x^2)*diff(y(x), x$2) - x*(3+4*x^2)*diff(y(x), x) + (2-2*x^2)*y(x) = 0,y(x),type
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{15}{8}x^2 + \frac{189}{128}x^4 + O\left(x^6\right)\right) + c_2 x^2\left(1 + \frac{6}{7}x^2 + \frac{45}{77}x^4 + O\left(x^6\right)\right)$$

✔ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 50

```
AsymptoticDSolveValue[x^2*(2-x^2)*y''[x] - x*(3+4*x^2)*y'[x] + (2-2*x^2)*y[x] == 0,y[x],{x,0,
```

$$y(x) \to c_1\left(\frac{45x^4}{77} + \frac{6x^2}{7} + 1\right)x^2 + c_2\left(\frac{189x^4}{128} + \frac{15x^2}{8} + 1\right)\sqrt{x}$$

## 4.32 problem 28

Internal problem ID [6499]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 28.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type [`y=_G(x,y')`]

$$y'^2 + y^2 - \sec(x)^4 = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)^2+y(x)^2=sec(x)^4,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]^2+y[x]^2==Sec[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 4.33 problem 29

Internal problem ID [6500]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 29.
**ODE order**: 1.
**ODE degree**: 3.

CAS Maple gives this as type [[_1st_order, _with_linear_symmetries], _dAlembert]

$$\boxed{\left(y - 2y'x\right)^2 - y'^3 = 0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 75

```
dsolve((y(x)-2*x*diff(y(x),x))^2= diff(y(x),x)^3,y(x), singsol=all)
```

$$y(x) = 0$$

$$\left[ x(\_T) = \frac{\frac{3}{5}\_T^{\frac{5}{2}} + c_1}{\_T^2}, y(\_T) = \frac{\frac{6}{5}\_T^{\frac{5}{2}} + 2c_1}{\_T} - \_T^{\frac{3}{2}} \right]$$

$$\left[ x(\_T) = \frac{-\frac{3}{5}\_T^{\frac{5}{2}} + c_1}{\_T^2}, y(\_T) = \frac{-\frac{6}{5}\_T^{\frac{5}{2}} + 2c_1}{\_T} + \_T^{\frac{3}{2}} \right]$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y[x]-2*x*y'[x])^2== y'[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

## 4.34 problem 31

Internal problem ID [6501]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 31.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y''x^2 + y = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
Order:=6;
dsolve(x^2*diff(y(x), x$2) +y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left(c_1 x^{-\frac{i\sqrt{3}}{2}} + c_2 x^{\frac{i\sqrt{3}}{2}}\right) + O\left(x^6\right)$$

✔ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
AsymptoticDSolveValue[x^2*y''[x] +y[x] == 0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x^{-(-1)^{2/3}} + c_2 x^{\sqrt[3]{-1}}$$

## 4.35 problem 32

Internal problem ID [6502]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 32.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_Emden, _Fowler]]

$$xy'' + y' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x), x$2) +diff(y(x),x)-y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1) \left(1 + x + \frac{1}{4}x^2 + \frac{1}{36}x^3 + \frac{1}{576}x^4 + \frac{1}{14400}x^5 + O\left(x^6\right)\right)$$
$$+ \left((-2)x - \frac{3}{4}x^2 - \frac{11}{108}x^3 - \frac{25}{3456}x^4 - \frac{137}{432000}x^5 + O\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 107

```
AsymptoticDSolveValue[x*y''[x] +y'[x]-y[x] == 0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right)$$
$$+ c_2 \left(-\frac{137x^5}{432000} - \frac{25x^4}{3456} - \frac{11x^3}{108} - \frac{3x^2}{4} + \left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right) \log(x) - 2x\right)$$

## 4.36 problem 33

Internal problem ID [6503]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 33.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$4xy'' + 2y' + y = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(4*x*diff(y(x), x$2) +2*diff(y(x),x)+y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 - \frac{1}{6}x + \frac{1}{120}x^2 - \frac{1}{5040}x^3 + \frac{1}{362880}x^4 - \frac{1}{39916800}x^5 + \mathrm{O}\left(x^6\right)\right)$$
$$+ c_2\left(1 - \frac{1}{2}x + \frac{1}{24}x^2 - \frac{1}{720}x^3 + \frac{1}{40320}x^4 - \frac{1}{3628800}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✔ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 85

```
AsymptoticDSolveValue[4*x*y''[x] +2*y'[x]+y[x] == 0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{x^5}{39916800} + \frac{x^4}{362880} - \frac{x^3}{5040} + \frac{x^2}{120} - \frac{x}{6} + 1\right)$$
$$+ c_2\left(-\frac{x^5}{3628800} + \frac{x^4}{40320} - \frac{x^3}{720} + \frac{x^2}{24} - \frac{x}{2} + 1\right)$$

## 4.37 problem 34

Internal problem ID [6504]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 34.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_Emden, _Fowler]]

$$xy'' + y' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.015 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x), x$2) +diff(y(x),x)-y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 + x + \frac{1}{4}x^2 + \frac{1}{36}x^3 + \frac{1}{576}x^4 + \frac{1}{14400}x^5 + O\left(x^6\right)\right)$$
$$+ \left((-2)x - \frac{3}{4}x^2 - \frac{11}{108}x^3 - \frac{25}{3456}x^4 - \frac{137}{432000}x^5 + O\left(x^6\right)\right)c_2$$

✔ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 107

```
AsymptoticDSolveValue[x*y''[x] +y'[x]-y[x] == 0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right)$$
$$+ c_2\left(-\frac{137x^5}{432000} - \frac{25x^4}{3456} - \frac{11x^3}{108} - \frac{3x^2}{4} + \left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right)\log(x) - 2x\right)$$

## 4.38 problem 35

Internal problem ID [6505]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 35.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$xy'' + (1 + x) y' + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.016 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x), x$2) +(1+x)*diff(y(x),x)+2*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1) \left( 1 - 2x + \frac{3}{2}x^2 - \frac{2}{3}x^3 + \frac{5}{24}x^4 - \frac{1}{20}x^5 + O\left(x^6\right) \right)$$
$$+ \left( 3x - \frac{13}{4}x^2 + \frac{31}{18}x^3 - \frac{173}{288}x^4 + \frac{187}{1200}x^5 + O\left(x^6\right) \right) c_2$$

✔ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 111

```
AsymptoticDSolveValue[x*y''[x] +(1+x)*y'[x]+2*y[x] == 0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left( -\frac{x^5}{20} + \frac{5x^4}{24} - \frac{2x^3}{3} + \frac{3x^2}{2} - 2x + 1 \right)$$
$$+ c_2 \left( \frac{187x^5}{1200} - \frac{173x^4}{288} + \frac{31x^3}{18} - \frac{13x^2}{4} + \left( -\frac{x^5}{20} + \frac{5x^4}{24} - \frac{2x^3}{3} + \frac{3x^2}{2} - 2x + 1 \right) \log(x) + 3x \right)$$

## 4.39 problem 36

Internal problem ID [6506]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 36.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x(-1+x)\,y'' + 3y'x + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
Order:=6;
dsolve(x*(x-1)*diff(y(x), x$2) +3*x*diff(y(x),x)+y(x) = 0,y(x),type='series',x=0);
```

$$
\begin{aligned}
y(x) = {}& \ln(x)\left(x + 2x^2 + 3x^3 + 4x^4 + 5x^5 + O\left(x^6\right)\right)c_2 \\
& + c_1 x\left(1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + O\left(x^6\right)\right) \\
& + \left(1 + 3x + 5x^2 + 7x^3 + 9x^4 + 11x^5 + O\left(x^6\right)\right)c_2
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 63

```
AsymptoticDSolveValue[x*(x-1)*y''[x] +3*x*y'[x]+y[x] == 0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1\left(x^4 + x^3 + x^2 + \left(4x^3 + 3x^2 + 2x + 1\right)x\log(x) + x + 1\right) + c_2\left(5x^5 + 4x^4 + 3x^3 + 2x^2 + x\right)$$

## 4.40 problem 37

Internal problem ID [6507]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 37.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$x^2 (x^2 - 2x + 1) y'' - x(x + 3) y' + (x + 4) y = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*(1-2*x+x^2)*diff(y(x), x$2) -x*(3+x)*diff(y(x),x)+(4+x)*y(x) = 0,y(x),type='series
```

$$y(x) = x^2 \left( (c_2 \ln(x) + c_1) \left( 1 + 5x + 17x^2 + \frac{143}{3}x^3 + \frac{355}{3}x^4 + \frac{4043}{15}x^5 + O(x^6) \right) \right.$$
$$\left. + \left( (-3)x - \frac{29}{2}x^2 - \frac{859}{18}x^3 - \frac{4693}{36}x^4 - \frac{285181}{900}x^5 + O(x^6) \right) c_2 \right)$$

✔ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 118

```
AsymptoticDSolveValue[x^2*(1-2*x+x^2)*y''[x] -x*(3+x)*y'[x]+(4+x)*y[x] == 0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left( \frac{4043x^5}{15} + \frac{355x^4}{3} + \frac{143x^3}{3} + 17x^2 + 5x + 1 \right) x^2$$
$$+ c_2 \left( \left( -\frac{285181x^5}{900} - \frac{4693x^4}{36} - \frac{859x^3}{18} - \frac{29x^2}{2} - 3x \right) x^2 \right.$$
$$\left. + \left( \frac{4043x^5}{15} + \frac{355x^4}{3} + \frac{143x^3}{3} + 17x^2 + 5x + 1 \right) x^2 \log(x) \right)$$

## 4.41    problem 38

Internal problem ID [6508]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 38.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$2x^2(x+2)\,y'' + 5y'x^2 + (1+x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓  Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(2*x^2*(2+x)*diff(y(x), x$2) +5*x^2*diff(y(x),x)+(1+x)*y(x) = 0,y(x),type='series',x=0)
```

$$y(x) = \sqrt{x}\left( (c_2 \ln(x) + c_1)\left(1 - \frac{3}{4}x + \frac{15}{32}x^2 - \frac{35}{128}x^3 + \frac{315}{2048}x^4 - \frac{693}{8192}x^5 + O(x^6)\right) \right.$$
$$\left. + \left(\frac{1}{4}x - \frac{13}{64}x^2 + \frac{101}{768}x^3 - \frac{641}{8192}x^4 + \frac{7303}{163840}x^5 + O(x^6)\right)c_2 \right)$$

✓  Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 134

```
AsymptoticDSolveValue[2*x^2*(2+x)*y''[x] +5*x^2*y'[x]+(1+x)*y[x] == 0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \sqrt{x}\left(-\frac{693x^5}{8192} + \frac{315x^4}{2048} - \frac{35x^3}{128} + \frac{15x^2}{32} - \frac{3x}{4} + 1\right)$$
$$+ c_2\left(\sqrt{x}\left(\frac{7303x^5}{163840} - \frac{641x^4}{8192} + \frac{101x^3}{768} - \frac{13x^2}{64} + \frac{x}{4}\right)\right.$$
$$\left. + \sqrt{x}\left(-\frac{693x^5}{8192} + \frac{315x^4}{2048} - \frac{35x^3}{128} + \frac{15x^2}{32} - \frac{3x}{4} + 1\right)\log(x)\right)$$

## 4.42 problem 39

Internal problem ID [6509]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 39.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$2y''x^2 + y'x + (x - 5)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 665

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) +x*diff(y(x), x) +(x-5)*y(x) = 0,y(x),type='series',x=0);
```

$$
y(x) = x^{\frac{1}{4}}\left( c_1 x^{-\frac{\sqrt{41}}{4}} \left( 1 + \frac{1}{-2+\sqrt{41}} x + \frac{1}{2} \frac{1}{(-2+\sqrt{41})(-4+\sqrt{41})} x^2 \right.\right.
$$
$$
+ \frac{1}{6} \frac{1}{(-2+\sqrt{41})(-4+\sqrt{41})(-6+\sqrt{41})} x^3
$$
$$
+ \frac{1}{24} \frac{1}{(-2+\sqrt{41})(-4+\sqrt{41})(-6+\sqrt{41})(-8+\sqrt{41})} x^4
$$
$$
+ \frac{1}{120} \frac{1}{(-2+\sqrt{41})(-4+\sqrt{41})(-6+\sqrt{41})(-8+\sqrt{41})(-10+\sqrt{41})} x^5
$$
$$
\left. + O\left(x^6\right) \right) + c_2 x^{\frac{\sqrt{41}}{4}} \left( 1 + \frac{1}{-2-\sqrt{41}} x + \frac{1}{2} \frac{1}{(2+\sqrt{41})(4+\sqrt{41})} x^2 \right.
$$
$$
- \frac{1}{6} \frac{1}{(2+\sqrt{41})(4+\sqrt{41})(6+\sqrt{41})} x^3
$$
$$
+ \frac{1}{24} \frac{1}{(2+\sqrt{41})(4+\sqrt{41})(6+\sqrt{41})(8+\sqrt{41})} x^4
$$
$$
\left.\left. - \frac{1}{120} \frac{1}{(2+\sqrt{41})(4+\sqrt{41})(6+\sqrt{41})(8+\sqrt{41})(10+\sqrt{41})} x^5 + O\left(x^6\right) \right)\right)
$$

✓ <u>Solution by Mathematica</u>

Time used: 0.003 (sec). Leaf size: 1668

```
AsymptoticDSolveValue[2*x^2*y''[x]+x*y'[x]+(x-5)*y[x]==0,y[x],{x,0,5}]
```

Too large to display

## 4.43 problem 40

Internal problem ID [6510]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 40.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2y''x^2 + 2y'x - yx - \sin(x) = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 77

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) + 2*x*diff(y(x), x) - x*y(x) = sin(x),y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 + \frac{1}{2}x + \frac{1}{16}x^2 + \frac{1}{288}x^3 + \frac{1}{9216}x^4 + \frac{1}{460800}x^5 + O\left(x^6\right)\right)$$
$$+ x\left(\frac{1}{2} + \frac{1}{16}x - \frac{5}{864}x^2 - \frac{5}{27648}x^3 + \frac{1127}{6912000}x^4 + \frac{1127}{497664000}x^5 + O\left(x^6\right)\right)$$
$$+ \left(-x - \frac{3}{16}x^2 - \frac{11}{864}x^3 - \frac{25}{55296}x^4 - \frac{137}{13824000}x^5 + O\left(x^6\right)\right)c_2$$

✓  Solution by Mathematica

Time used: 0.143 (sec). Leaf size: 340

```
AsymptoticDSolveValue[2*x^2*y''[x]+2*x*y'[x]-x*y[x]==Sin[x],y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to\ & c_2\left(\frac{x^5}{460800} + \frac{x^4}{9216} + \frac{x^3}{288} + \frac{x^2}{16} + \frac{x}{2} + 1\right) \\
& + c_1\left(x^5\left(\frac{\log(x)}{460800} - \frac{107}{13824000}\right) + x^4\left(\frac{\log(x)}{9216} - \frac{19}{55296}\right) + x^3\left(\frac{\log(x)}{288} - \frac{1}{108}\right)\right. \\
& \quad + x^2\left(\frac{\log(x)}{16} - \frac{1}{8}\right) + x\left(\frac{\log(x)}{2} - \frac{1}{2}\right) + \log(x) + 1\right) + \left(\frac{4963 x^6}{16588800} - \frac{91 x^5}{460800}\right. \\
& \quad \left. - \frac{23 x^4}{2304} - \frac{5 x^3}{288} + \frac{x^2}{8} + \frac{x}{2}\right)\left(x^5\left(\frac{\log(x)}{460800} - \frac{107}{13824000}\right) + x^4\left(\frac{\log(x)}{9216} - \frac{19}{55296}\right)\right. \\
& \quad \left. + x^3\left(\frac{\log(x)}{288} - \frac{1}{108}\right) + x^2\left(\frac{\log(x)}{16} - \frac{1}{8}\right) + x\left(\frac{\log(x)}{2} - \frac{1}{2}\right) + \log(x) + 1\right) + \left(\frac{x^5}{460800}\right. \\
& \quad + \frac{x^4}{9216} + \frac{x^3}{288} + \frac{x^2}{16} + \frac{x}{2} + 1\right)\left(\frac{x^6(66968 - 74445\log(x))}{248832000} + \frac{13 x^5(210\log(x) - 3107)}{13824000}\right. \\
& \quad \left.\left. + \frac{x^4(276\log(x) - 325)}{27648} + \frac{1}{864}x^3(15\log(x) + 37) + \frac{1}{16}x^2(3 - 2\log(x)) - \frac{1}{2}x\log(x)\right)\right)
\end{aligned}
$$

## 4.44 problem 41

Internal problem ID [6511]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 41.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 + 2y'x - yx - x\sin(x) = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 75

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) + 2*x*diff(y(x), x) - x*y(x) = x*sin(x),y(x),type='series',x=0)
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 + \frac{1}{2}x + \frac{1}{16}x^2 + \frac{1}{288}x^3 + \frac{1}{9216}x^4 + \frac{1}{460800}x^5 + O(x^6)\right)$$
$$+ x^2\left(\frac{1}{8} + \frac{1}{144}x - \frac{23}{4608}x^2 - \frac{23}{230400}x^3 + O(x^4)\right)$$
$$+ \left(-x - \frac{3}{16}x^2 - \frac{11}{864}x^3 - \frac{25}{55296}x^4 - \frac{137}{13824000}x^5 + O(x^6)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.163 (sec). Leaf size: 328

```
AsymptoticDSolveValue[2*x^2*y''[x]+2*x*y'[x]-x*y[x]==x*Sin[x],y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to\ & c_2\left(\frac{x^5}{460800} + \frac{x^4}{9216} + \frac{x^3}{288} + \frac{x^2}{16} + \frac{x}{2} + 1\right) \\
& + c_1\left(x^5\left(\frac{\log(x)}{460800} - \frac{107}{13824000}\right) + x^4\left(\frac{\log(x)}{9216} - \frac{19}{55296}\right) + x^3\left(\frac{\log(x)}{288} - \frac{1}{108}\right)\right. \\
& + x^2\left(\frac{\log(x)}{16} - \frac{1}{8}\right) + x\left(\frac{\log(x)}{2} - \frac{1}{2}\right) + \log(x) + 1\right) + \left(-\frac{91x^6}{552960} - \frac{23x^5}{2880} - \frac{5x^4}{384} + \frac{x^3}{12}\right. \\
& + \frac{x^2}{4}\right)\left(x^5\left(\frac{\log(x)}{460800} - \frac{107}{13824000}\right) + x^4\left(\frac{\log(x)}{9216} - \frac{19}{55296}\right) + x^3\left(\frac{\log(x)}{288} - \frac{1}{108}\right)\right. \\
& + x^2\left(\frac{\log(x)}{16} - \frac{1}{8}\right) + x\left(\frac{\log(x)}{2} - \frac{1}{2}\right) + \log(x) + 1\right) + \left(\frac{x^5}{460800} + \frac{x^4}{9216} + \frac{x^3}{288}\right. \\
& + \frac{x^2}{16} + \frac{x}{2} + 1\right)\left(\frac{13x^6(21\log(x) - 310)}{1658880} + \frac{x^5(345\log(x) - 389)}{43200}\right. \\
& + \frac{x^4(20\log(x) + 51)}{1536} + \frac{1}{36}x^3(4 - 3\log(x)) + \frac{1}{8}x^2(-2\log(x) - 1)\right)
\end{aligned}
$$

## 4.45 problem 42

Internal problem ID [6512]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 42.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 + 2y'x - yx - \cos(x)\sin(x) = 0$$

With the expansion point for the power series method at $x = 0$.

✓  Solution by Maple

Time used: 0.015 (sec). Leaf size: 77

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) + 2*x*diff(y(x), x) - x*y(x) = cos(x)*sin(x),y(x),type='series'
```

$$
\begin{aligned}
y(x) = {}& (c_2 \ln(x) + c_1) \left(1 + \frac{1}{2}x + \frac{1}{16}x^2 + \frac{1}{288}x^3 + \frac{1}{9216}x^4 + \frac{1}{460800}x^5 + O(x^6)\right) \\
& + x\left(\frac{1}{2} + \frac{1}{16}x - \frac{29}{864}x^2 - \frac{29}{27648}x^3 + \frac{18287}{6912000}x^4 + \frac{18287}{497664000}x^5 + O(x^6)\right) \\
& + \left(-x - \frac{3}{16}x^2 - \frac{11}{864}x^3 - \frac{25}{55296}x^4 - \frac{137}{13824000}x^5 + O(x^6)\right)c_2
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.141 (sec). Leaf size: 340

```
AsymptoticDSolveValue[2*x^2*y''[x]+2*x*y'[x]-x*y[x]==Cos[x]*Sin[x],y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to\ & c_2 \left( \frac{x^5}{460800} + \frac{x^4}{9216} + \frac{x^3}{288} + \frac{x^2}{16} + \frac{x}{2} + 1 \right) \\
& + c_1 \left( x^5 \left( \frac{\log(x)}{460800} - \frac{107}{13824000} \right) + x^4 \left( \frac{\log(x)}{9216} - \frac{19}{55296} \right) + x^3 \left( \frac{\log(x)}{288} - \frac{1}{108} \right) \right. \\
& \left. + x^2 \left( \frac{\log(x)}{16} - \frac{1}{8} \right) + x \left( \frac{\log(x)}{2} - \frac{1}{2} \right) + \log(x) + 1 \right) + \left( \frac{88963 x^6}{16588800} + \frac{4229 x^5}{460800} \right. \\
& \left. - \frac{95 x^4}{2304} - \frac{29 x^3}{288} + \frac{x^2}{8} + \frac{x}{2} \right) \left( x^5 \left( \frac{\log(x)}{460800} - \frac{107}{13824000} \right) + x^4 \left( \frac{\log(x)}{9216} - \frac{19}{55296} \right) \right. \\
& \left. + x^3 \left( \frac{\log(x)}{288} - \frac{1}{108} \right) + x^2 \left( \frac{\log(x)}{16} - \frac{1}{8} \right) + x \left( \frac{\log(x)}{2} - \frac{1}{2} \right) + \log(x) + 1 \right) \\
& + \left( \frac{x^5}{460800} + \frac{x^4}{9216} + \frac{x^3}{288} + \frac{x^2}{16} + \frac{x}{2} + 1 \right) \left( \frac{x^6 (1476968 - 1334445 \log(x))}{248832000} \right. \\
& + \frac{x^5 (-126870 \log(x) - 273671)}{13824000} + \frac{5 x^4 (228 \log(x) - 281)}{27648} + \frac{1}{864} x^3 (87 \log(x) + 85) \\
& \left. + \frac{1}{16} x^2 (3 - 2 \log(x)) - \frac{1}{2} x \log(x) \right)
\end{aligned}
$$

## 4.46 problem 43

Internal problem ID [6513]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 43.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$2y''x^2 + 2y'x - yx - x^3 - x\sin(x) = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 75

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) + 2*x*diff(y(x), x) - x*y(x) = x^3+x*sin(x),y(x),type='series',
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 + \frac{1}{2}x + \frac{1}{16}x^2 + \frac{1}{288}x^3 + \frac{1}{9216}x^4 + \frac{1}{460800}x^5 + O\left(x^6\right)\right)$$

$$+ x^2\left(\frac{1}{8} + \frac{1}{16}x - \frac{5}{1536}x^2 - \frac{1}{15360}x^3 + O\left(x^4\right)\right)$$

$$+ \left(-x - \frac{3}{16}x^2 - \frac{11}{864}x^3 - \frac{25}{55296}x^4 - \frac{137}{13824000}x^5 + O\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.274 (sec). Leaf size: 268

```
AsymptoticDSolveValue[2*x^2*y''[x]+2*x*y'[x]-x*y[x]==x^3*x*Sin[x],y[x],{x,0,5}]
```

$$
y(x) \rightarrow c_2\left(\frac{x^5}{460800} + \frac{x^4}{9216} + \frac{x^3}{288} + \frac{x^2}{16} + \frac{x}{2} + 1\right)
$$
$$
+ c_1\left(x^5\left(\frac{\log(x)}{460800} - \frac{107}{13824000}\right) + x^4\left(\frac{\log(x)}{9216} - \frac{19}{55296}\right) + x^3\left(\frac{\log(x)}{288} - \frac{1}{108}\right)\right.
$$
$$
\left. + x^2\left(\frac{\log(x)}{16} - \frac{1}{8}\right) + x\left(\frac{\log(x)}{2} - \frac{1}{2}\right) + \log(x) + 1\right)
$$
$$
+ \left(\frac{x^5}{460800} + \frac{x^4}{9216} + \frac{x^3}{288} + \frac{x^2}{16} + \frac{x}{2} + 1\right)\left(\frac{1}{144}x^6(7 - 6\log(x)) + \frac{1}{50}x^5(-5\log(x) - 4)\right)
$$
$$
+ \left(\frac{x^6}{24} + \frac{x^5}{10}\right)\left(x^5\left(\frac{\log(x)}{460800} - \frac{107}{13824000}\right) + x^4\left(\frac{\log(x)}{9216} - \frac{19}{55296}\right)\right.
$$
$$
\left. + x^3\left(\frac{\log(x)}{288} - \frac{1}{108}\right) + x^2\left(\frac{\log(x)}{16} - \frac{1}{8}\right) + x\left(\frac{\log(x)}{2} - \frac{1}{2}\right) + \log(x) + 1\right)
$$

## 4.47 problem 44

Internal problem ID [6514]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 44.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' \cos(x) + 2y'x - yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
Order:=6;
dsolve(cos(x)*diff(y(x), x, x) + 2*x*diff(y(x), x) - x*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{6}x^3 - \frac{1}{40}x^5\right) y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{12}x^4 + \frac{1}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 49

```
AsymptoticDSolveValue[Cos[x]*y''[x]+2*x*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^5}{40} + \frac{x^3}{6} + 1\right) + c_2\left(\frac{x^5}{20} + \frac{x^4}{12} - \frac{x^3}{3} + x\right)$$

## 4.48 problem 45

Internal problem ID [6515]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 45.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y''x^2 + 4y'x + (x^2 + 2) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x), x, x) + 4*x*diff(y(x), x) + (x^2+2)*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + O\left(x^6\right)\right)x + c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + O\left(x^6\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 40

```
AsymptoticDSolveValue[x^2*y''[x]+4*x*y'[x]+(x^2+2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^3}{120} - \frac{x}{6} + \frac{1}{x}\right) + c_1\left(\frac{x^2}{24} + \frac{1}{x^2} - \frac{1}{2}\right)$$

## 4.49 problem 46

Internal problem ID [6516]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 46.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_Emden, _Fowler]]

$$y''x^2 + y'x - yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 59

```
Order:=6;
dsolve(x^2*diff(y(x), x, x) + x*diff(y(x), x) - x*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1) \left( 1 + x + \frac{1}{4}x^2 + \frac{1}{36}x^3 + \frac{1}{576}x^4 + \frac{1}{14400}x^5 + O\left(x^6\right) \right)$$
$$+ \left( (-2)x - \frac{3}{4}x^2 - \frac{11}{108}x^3 - \frac{25}{3456}x^4 - \frac{137}{432000}x^5 + O\left(x^6\right) \right) c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 107

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left( \frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1 \right)$$
$$+ c_2 \left( -\frac{137x^5}{432000} - \frac{25x^4}{3456} - \frac{11x^3}{108} - \frac{3x^2}{4} + \left( \frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1 \right) \log(x) - 2x \right)$$

## 4.50 problem 47

Internal problem ID [6517]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 47.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y''x^2 + y'x + \left(x^2 - \frac{1}{4}\right)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + O\left(x^6\right)\right)x + c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + O\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^{7/2}}{24} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}}\right) + c_2\left(\frac{x^{9/2}}{120} - \frac{x^{5/2}}{6} + \sqrt{x}\right)$$

## 4.51 problem 48

Internal problem ID [6518]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 48.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 - x)\, y'' - y'x + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
Order:=6;
dsolve((x^2-x)*diff(y(x), x$2)-x*diff(y(x), x)+y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1 x \left(1 + O\left(x^6\right)\right) + \left(x + O\left(x^6\right)\right) \ln(x)\, c_2 + \left(1 - x + O\left(x^6\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 20

```
AsymptoticDSolveValue[(x^2-x)*y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 x + c_1 (-3x + x \log(x) + 1)$$

## 4.52 problem 49

Internal problem ID [6519]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 49.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '

$$y''x^2 + (x^2 + 6x) y' + yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(x^2*diff(y(x), x$2)+(6*x+x^2)*diff(y(x), x)+x*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1 \left( 1 - \frac{1}{6}x + \frac{1}{42}x^2 - \frac{1}{336}x^3 + \frac{1}{3024}x^4 - \frac{1}{30240}x^5 + O\left(x^6\right) \right)$$
$$+ \frac{c_2(2880 - 2880x + 1440x^2 - 480x^3 + 120x^4 - 24x^5 + O\left(x^6\right))}{x^5}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 68

```
AsymptoticDSolveValue[x^2*y''[x]+(6*x+x^2)*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left( \frac{x^4}{3024} - \frac{x^3}{336} + \frac{x^2}{42} - \frac{x}{6} + 1 \right) + c_1 \left( \frac{1}{x^5} - \frac{1}{x^4} + \frac{1}{2x^3} - \frac{1}{6x^2} + \frac{1}{24x} \right)$$

## 4.53 problem 50

Internal problem ID [6520]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 50.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y''x^2 - y'x + (x^2 - 8)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x), x$2)-x*diff(y(x), x)+(x^2-8)*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^4 \left(1 - \frac{1}{16}x^2 + \frac{1}{640}x^4 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2(-86400 - 10800x^2 - 1350x^4 + \mathrm{O}\left(x^6\right))}{x^2}$$

✔ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 42

```
AsymptoticDSolveValue[x^2*y''[x]-x*y'[x]+(x^2-8)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^2}{64} + \frac{1}{x^2} + \frac{1}{8}\right) + c_2\left(\frac{x^8}{640} - \frac{x^6}{16} + x^4\right)$$

## 4.54 problem 51

Internal problem ID [6521]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 51.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y''x^2 - 9y'x + 25y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve(x^2*diff(y(x), x$2)-9*x*diff(y(x), x)+25*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = x^5(c_2 \ln(x) + c_1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
AsymptoticDSolveValue[x^2*y''[x]-9*x*y'[x]+25*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x^5 + c_2 x^5 \log(x)$$

## 4.55   problem 52

Internal problem ID [6522]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 52.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y''x^2 - y'x - \left(x^2 + \frac{5}{4}\right)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓  Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)-(x^2+5/4)*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^3 \left(1 + \frac{1}{10}x^2 + \frac{1}{280}x^4 + O\left(x^6\right)\right) + c_2 \left(12 - 6x^2 - \frac{3}{2}x^4 + O\left(x^6\right)\right)}{\sqrt{x}}$$

✓  Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]-x*y'[x]-(x^2+5/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(-\frac{x^{7/2}}{8} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}}\right) + c_2 \left(\frac{x^{13/2}}{280} + \frac{x^{9/2}}{10} + x^{5/2}\right)$$

## 4.56   problem 53

Internal problem ID [6523]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 53.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$ y''x^2 + y'x + \left( x^2 - \frac{1}{4} \right) y = 0 $$

With the expansion point for the power series method at $x = 0$.

✓  Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x),type='series',x=0);
```

$$ y(x) = \frac{c_1 \left( 1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + O\left(x^6\right) \right) x + c_2 \left( 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + O\left(x^6\right) \right)}{\sqrt{x}} $$

✓  Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],{x,0,5}]
```

$$ y(x) \to c_1 \left( \frac{x^{7/2}}{24} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}} \right) + c_2 \left( \frac{x^{9/2}}{120} - \frac{x^{5/2}}{6} + \sqrt{x} \right) $$

## 4.57 problem 54

Internal problem ID [6524]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 54.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _exact, _linear, _homogeneous]]

$$xy'' + (2 - x)\,y' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(2-x)*diff(y(x),x)-y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 + \frac{1}{2}x + \frac{1}{6}x^2 + \frac{1}{24}x^3 + \frac{1}{120}x^4 + \frac{1}{720}x^5 + O\left(x^6\right)\right)$$
$$+ \frac{c_2\left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + O\left(x^6\right)\right)}{x}$$

✔ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 62

```
AsymptoticDSolveValue[x*y''[x]+(2-x)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^3}{24} + \frac{x^2}{6} + \frac{x}{2} + \frac{1}{x} + 1\right) + c_2\left(\frac{x^4}{120} + \frac{x^3}{24} + \frac{x^2}{6} + \frac{x}{2} + 1\right)$$

## 4.58   problem 55

Internal problem ID [6525]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 55.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$2y''x^2 + 3y'x - y = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \frac{x^{\frac{3}{2}} c_2 + c_1}{x} + O\left(x^6\right)$$

✔ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
AsymptoticDSolveValue[2*x^2*y''[x]+3*x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \sqrt{x} + \frac{c_2}{x}$$

## 4.59  problem 56

Internal problem ID [6526]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 56.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$2y''x^2 + 5y'x + 4y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+4*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \frac{x^{-\frac{i\sqrt{23}}{4}}c_1 + x^{\frac{i\sqrt{23}}{4}}c_2}{x^{\frac{3}{4}}} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 44

```
AsymptoticDSolveValue[2*x^2*y''[x]+5*x*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x^{\frac{1}{4}\left(-3+i\sqrt{23}\right)} + c_2 x^{\frac{1}{4}\left(-3-i\sqrt{23}\right)}$$

## 4.60 problem 57

Internal problem ID [6527]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 57.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_Emden, _Fowler]]

$$y''x^2 + 3y'x + 4x^4y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+4*x^4*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 - \frac{1}{6}x^4 + O\left(x^6\right)\right) + \frac{c_2(-2 + x^4 + O\left(x^6\right))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 30

```
AsymptoticDSolveValue[x^2*y''[x]+3*x*y'[x]+4*x^4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(1 - \frac{x^4}{6}\right) + c_1\left(\frac{1}{x^2} - \frac{x^2}{2}\right)$$

## 4.61  problem 58

Internal problem ID [6528]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 58.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_Emden, _Fowler]]

$$y''x^2 - yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 58

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1 x \left(1 + \frac{1}{2}x + \frac{1}{12}x^2 + \frac{1}{144}x^3 + \frac{1}{2880}x^4 + \frac{1}{86400}x^5 + O\left(x^6\right)\right)$$
$$+ c_2 \left(\ln(x)\left(x + \frac{1}{2}x^2 + \frac{1}{12}x^3 + \frac{1}{144}x^4 + \frac{1}{2880}x^5 + O\left(x^6\right)\right)\right.$$
$$\left. + \left(1 - \frac{3}{4}x^2 - \frac{7}{36}x^3 - \frac{35}{1728}x^4 - \frac{101}{86400}x^5 + O\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x^2*y''[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{1}{144}x\left(x^3 + 12x^2 + 72x + 144\right)\log(x) + \frac{-47x^4 - 480x^3 - 2160x^2 - 1728x + 1728}{1728}\right)$$
$$+ c_2 \left(\frac{x^5}{2880} + \frac{x^4}{144} + \frac{x^3}{12} + \frac{x^2}{2} + x\right)$$

## 4.62 problem 59

Internal problem ID [6529]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 59.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$\left(1 - x^2\right) y'' + y' + y - x\, e^x = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)+diff(y(x),x)+y(x)=x*exp(x),y(x),type='series',x=0);
```

$$
\begin{aligned}
y(x) = {}& \left(1 - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4 + \frac{7}{120}x^5\right) y(0) \\
& + \left(x - \frac{1}{2}x^2 - \frac{1}{24}x^4 + \frac{1}{120}x^5\right) D(y)\,(0) + \frac{x^3}{6} + \frac{x^4}{24} + \frac{7x^5}{120} + O\!\left(x^6\right)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(1-x^2)*y''[x]+y'[x]+y[x]==x*Exp[x],y[x],{x,0,5}]
```

$$
y(x) \to c_2\left(\frac{x^5}{120} - \frac{x^4}{24} - \frac{x^2}{2} + x\right) + c_1\left(\frac{7x^5}{120} - \frac{x^4}{12} + \frac{x^3}{6} - \frac{x^2}{2} + 1\right)
$$

## 4.63   problem 60

Internal problem ID [6530]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 60.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$y' - y(1 - y^2) = 0$$

✓   Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)=y(x)*(1-y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{e^{-2x}c_1 + 1}}$$

$$y(x) = -\frac{1}{\sqrt{e^{-2x}c_1 + 1}}$$

✓  Solution by Mathematica

Time used: 0.715 (sec). Leaf size: 100

```
DSolve[y'[x]==y[x]*(1-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^x}{\sqrt{e^{2x} + e^{2c_1}}}$$

$$y(x) \to \frac{e^x}{\sqrt{e^{2x} + e^{2c_1}}}$$

$$y(x) \to -1$$

$$y(x) \to 0$$

$$y(x) \to 1$$

$$y(x) \to -\frac{e^x}{\sqrt{e^{2x}}}$$

$$y(x) \to \frac{e^x}{\sqrt{e^{2x}}}$$

## 4.64 problem 61

Internal problem ID [6531]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 61.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$\boxed{\frac{xy''}{1-x} + y - \frac{1}{1-x} = 0}$$

✓  Solution by Maple

Time used: 0.047 (sec). Leaf size: 155

```
dsolve(x/(1-x)*diff(y(x),x$2)+y(x)=1/(1-x),y(x), singsol=all)
```

$$
\begin{aligned}
y(x) = {}& \left(\text{BesselI}\left(0, -x\right) + \text{BesselI}\left(1, -x\right)\right) x c_2 \\
& + x\left(-\text{BesselK}\left(0, -x\right) + \text{BesselK}\left(1, -x\right)\right) c_1 - x\bigg((\text{BesselI}\left(0, -x\right) \\
& + \text{BesselI}\left(1, -x\right))\left(\int \frac{-\text{BesselK}\left(0, -x\right) + \text{BesselK}\left(1, -x\right)}{\text{BesselK}\left(1, -x\right)\text{BesselI}\left(0, x\right) x^2 - \text{BesselK}\left(0, -x\right)\text{BesselI}\left(1, x\right) x^2 + x + 1} dx\right) \\
& + \left(\int \frac{\text{BesselI}\left(0, -x\right) + \text{BesselI}\left(1, -x\right)}{\text{BesselK}\left(1, -x\right)\text{BesselI}\left(0, x\right) x^2 - \text{BesselK}\left(0, -x\right)\text{BesselI}\left(1, x\right) x^2 + x + 1} dx\right)\left(\text{BesselK}\left(0, -x\right)\right. \\
& \left.- \text{BesselK}\left(1, -x\right)\right)\bigg)
\end{aligned}
$$

✓  Solution by Mathematica

Time used: 0.116 (sec). Leaf size: 89

```
DSolve[x/(1-x)*y''[x]+y[x]==1/(1-x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(\text{BesselI}(0, x) - \text{BesselI}(1, x)) \left( \int_1^x (K_0(K[1]) + K_1(K[1]))dK[1] + c_2 \right)$$
$$+ e^{-x}x \, \text{HypergeometricU} \left( \frac{1}{2}, 2, 2x \right) \left( -2\sqrt{\pi}x \, _1F_2 \left( \frac{1}{2}; 1, \frac{3}{2}; \frac{x^2}{4} \right) + 2\sqrt{\pi} \, \text{BesselI}(0, x) \right.$$
$$\left. + c_1 \right)$$

## 4.65 problem 62

Internal problem ID [6532]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 62.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$\frac{xy''}{1-x} + yx = 0$$

✓  Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x/(1-x)*diff(y(x),x$2)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{AiryAi}(x-1) + c_2 \operatorname{AiryBi}(x-1)$$

✓  Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[x/(1-x)*y''[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \operatorname{AiryAi}(x-1) + c_2 \operatorname{AiryBi}(x-1)$$

## 4.66 problem 63

Internal problem ID [6533]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 63.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\frac{xy''}{1-x} + y - \cos(x) = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 162

```
dsolve(x/(1-x)*diff(y(x),x$2)+y(x)=cos(x),y(x), singsol=all)
```

$$
\begin{aligned}
y(x) = {}& (\text{BesselI}(0,-x) + \text{BesselI}(1,-x))\, x c_2 \\
& + x(-\text{BesselK}(0,-x) + \text{BesselK}(1,-x))\, c_1 + x \Big( (\text{BesselI}(0,-x) \\
& + \text{BesselI}(1,-x)) \left( \int \frac{(-\text{BesselK}(0,-x) + \text{BesselK}(1,-x)) \cos(x)(x-1)}{\text{BesselK}(1,-x)\,\text{BesselI}(0,x)\,x^2 - \text{BesselK}(0,-x)\,\text{BesselI}(1,x)\,x^2 + x + 1} dx \right) \\
& + \left( \int \frac{(\text{BesselI}(0,x) - \text{BesselI}(1,x)) \cos(x)(x-1)}{\text{BesselK}(1,-x)\,\text{BesselI}(0,x)\,x^2 - \text{BesselK}(0,-x)\,\text{BesselI}(1,x)\,x^2 + x + 1} dx \right) (\text{BesselK}(0,-x) \\
& - \text{BesselK}(1,-x)) \Big)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 3.942 (sec). Leaf size: 95

```
DSolve[x/(1-x)*y''[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} x \, \text{HypergeometricU}\left(\frac{1}{2}, 2, 2x\right) \left(\int_1^x 2\sqrt{\pi}(\text{BesselI}(0, K[1]) \right.$$
$$\left. - \text{BesselI}(1, K[1])) \cos(K[1])(K[1] - 1)dK[1] + c_1\right) + x(\text{BesselI}(0, x)$$
$$- \text{BesselI}(1, x)) \left(\int_1^x -((K_0(K[2]) + K_1(K[2])) \cos(K[2])(K[2] - 1))dK[2] + c_2\right)$$

## 4.67 problem 64

Internal problem ID [6534]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 64.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\frac{xy''}{1-x^2} + y = 0$$

✗ Solution by Maple

```
dsolve(x/(1-x^2)*diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x/(1-x^2)*y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 4.68 problem 65

Internal problem ID [6535]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 65.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - \left(x^2 + 3\right) y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)=(x^2+3)*y(x),y(x), singsol=all)
```

$$y(x) = c_1 x\, e^{\frac{x^2}{2}} + c_2 \left( e^{\frac{x^2}{2}} \sqrt{\pi}\, \mathrm{erf}\left(x\right) x + e^{-\frac{x^2}{2}} \right)$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 41

```
DSolve[y''[x]==(x^2+3)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} \left( e^{x^2} x \left(c_1 - \sqrt{\pi} c_2 \mathrm{erf}(x)\right) - c_2 \right)$$

## 4.69 problem 66

Internal problem ID [6536]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 66.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' + y(-1 + x) = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)+(x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{30}x^5\right)y(0) + \left(x + \frac{1}{6}x^3 - \frac{1}{12}x^4 + \frac{1}{120}x^5\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]+(x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{120} - \frac{x^4}{12} + \frac{x^3}{6} + x\right) + c_1\left(-\frac{x^5}{30} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} + 1\right)$$

## 4.70 problem 67

Internal problem ID [6537]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 67.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = x(t) + 2y(t) + 2t + 1$$
$$y'(t) = 5x(t) + y(t) + 3t - 1$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 68

```
dsolve([diff(x(t),t)=x(t)+2*y(t)+2*t+1,diff(y(t),t)=5*x(t)+y(t)+3*t-1],[x(t), y(t)], singsol=
```

$$x(t) = \frac{e^{\left(1+\sqrt{10}\right)t} c_2 \sqrt{10}}{5} - \frac{e^{-\left(-1+\sqrt{10}\right)t} c_1 \sqrt{10}}{5} - \frac{4t}{9} + \frac{17}{81}$$

$$y(t) = e^{\left(1+\sqrt{10}\right)t} c_2 + e^{-\left(-1+\sqrt{10}\right)t} c_1 - \frac{7t}{9} - \frac{67}{81}$$

✓ Solution by Mathematica

Time used: 10.731 (sec). Leaf size: 96

```
DSolve[{x'[t]==x[t]+2*y[t]+2*t+1,y'[t]==5*x[t]+y[t]+3*t-1},{x[t],y[t]},t,IncludeSingularSolut
```

$$x(t) \to -\frac{4t}{9} + c_1 e^t \cosh\left(\sqrt{10}t\right) + \sqrt{\frac{2}{5}} c_2 e^t \sinh\left(\sqrt{10}t\right) + \frac{17}{81}$$

$$y(t) \to -\frac{7t}{9} + c_2 e^t \cosh\left(\sqrt{10}t\right) + \sqrt{\frac{5}{2}} c_1 e^t \sinh\left(\sqrt{10}t\right) - \frac{67}{81}$$

## 4.71 problem 68

Internal problem ID [6538]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 68.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + 20y' + 500y - 100000\cos(100x) = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 37

```
dsolve(diff(diff(y(x),x),x)+20*diff(y(x),x)+500*y(x) = 100000*cos(100*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-10x}\sin(20x)\,c_2 + \mathrm{e}^{-10x}\cos(20x)\,c_1 - \frac{3800\cos(100x)}{377} + \frac{800\sin(100x)}{377}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 44

```
DSolve[y''[x]+20*y'[x]+500*y[x] == 100000*Cos[100*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{200}{377}(19\cos(100x) - 4\sin(100x)) + e^{-10x}(c_2\cos(20x) + c_1\sin(20x))$$

## 4.72 problem 69

Internal problem ID [6539]

**Book**: Own collection of miscellaneous problems
**Section**: section 4.0
**Problem number**: 69.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '

$$y'' \sin(2x)^2 + y' \sin(4x) - 4y = 0$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)*sin(2*x)^2+diff(y(x),x)*sin(4*x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{-2\cos(4x) + 2}} + \frac{c_2\sqrt{\cos(4x) + 1}}{\sqrt{-2\cos(4x) + 2}}$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 23

```
DSolve[y''[x]*Sin[2*x]^2+y'[x]*Sin[4*x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \csc(2x)(c_1 - ic_2\cos(2x))$$

# 5 section 5.0

## 5.1 problem 1

Internal problem ID [6540]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]

$$y'' - Ay^{\frac{2}{3}} = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$2)=A*y(x)^(2/3),y(x), singsol=all)
```

$$y(x) = 0$$

$$\int^{y(x)} -\frac{5}{\sqrt{30\_a^{\frac{5}{3}}A - 5c_1}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} \frac{5}{\sqrt{30\_a^{\frac{5}{3}}A - 5c_1}} d\_a - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 75

```
DSolve[y''[x]==A*y[x]^(2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{y(x)^2\left(1 + \frac{6Ay(x)^{5/3}}{5c_1}\right)\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{5}, \frac{8}{5}, -\frac{6Ay(x)^{5/3}}{5c_1}\right)^2}{\frac{6}{5}Ay(x)^{5/3} + c_1} = (x + c_2)^2, y(x)\right]$$

## 5.2 problem 2

Internal problem ID [6541]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2y'x + (x^2 + 1)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+(x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x^2}{2}} + c_2 x\, e^{-\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 22

```
DSolve[y''[x]+2*x*y'[x]+(x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} (c_2 x + c_1)$$

## 5.3 problem 3

Internal problem ID [6542]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' + 2\cot(x)\,y' - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+2*cot(x)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \csc(x) + c_2 x \csc(x)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 15

```
DSolve[y''[x]+2*Cot[x]*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (c_2 x + c_1) \csc(x)$$

## 5.4 problem 4

Internal problem ID [6543]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' x^2 + y' x + \left( x^2 - \frac{1}{4} \right) y = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x^2*diff(diff(y(x),x),x)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{\sqrt{x}} + \frac{c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix} \left( 2c_1 - ic_2 e^{2ix} \right)}{2\sqrt{x}}$$

## 5.5 problem 5

Internal problem ID [6544]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$4y''x^2 + \left(-8x^2 + 4x\right)y' + \left(4x^2 - 4x - 1\right)y - 4\sqrt{x}\,\mathrm{e}^x = 0$$

✓  Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(4*x^2*diff(diff(y(x),x),x)+(-8*x^2+4*x)*diff(y(x),x)+(4*x^2-4*x-1)*y(x) = 4*x^(1/2)*ex
```

$$y(x) = \frac{\mathrm{e}^x c_2}{\sqrt{x}} + \sqrt{x}\,\mathrm{e}^x c_1 + \sqrt{x}\,\mathrm{e}^x(-1 + \ln(x))$$

✓  Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 27

```
DSolve[4*x^2*y''[x]+(-8*x^2+4*x)*y'[x]+(4*x^2-4*x-1)*y[x] == 4*x^(1/2)*Exp[x],y[x],x,IncludeS
```

$$y(x) \to \frac{\mathrm{e}^x(x \log(x) + (-1 + c_2)x + c_1)}{\sqrt{x}}$$

## 5.6 problem 6

Internal problem ID [6545]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$xy'' - (2x + 2)\, y' + (x + 2)\, y - 6x^3 \mathrm{e}^x = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(x*diff(diff(y(x),x),x)-(2*x+2)*diff(y(x),x)+(2+x)*y(x) = 6*x^3*exp(x),y(x), singsol=al
```

$$y(x) = \mathrm{e}^x c_2 + \mathrm{e}^x x^3 c_1 + \frac{3\,\mathrm{e}^x x^4}{2}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 29

```
DSolve[x*y''[x]-(2*x+2)*y'[x]+(2+x)*y[x] == 6*x^3*Exp[x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{6}\mathrm{e}^x \left(9x^4 + 2c_2 x^3 + 6c_1\right)$$

## 5.7 problem 7

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 7.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + y - \frac{1}{x} = 0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x)+y(x)=1/x,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 113

```
AsymptoticDSolveValue[y'[x]+y[x]==1/x,y[x],{x,0,5}]
```

$$y(x) \rightarrow \left( -\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1 \right) \left( \frac{x^6}{2160} + \frac{x^5}{600} + \frac{x^4}{96} + \frac{x^3}{18} + \frac{x^2}{4} + x + \log(x) \right)$$
$$+ c_1 \left( -\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1 \right)$$

## 5.8  problem 8

Internal problem ID [6547]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 8.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [[_linear, 'class A']]

$$y' + y - \frac{1}{x^2} = 0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x)+y(x)=1/x^2,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 122

```
AsymptoticDSolveValue[y'[x]+y[x]==1/x^2,y[x],{x,0,5}]
```

$$y(x) \to \left( -\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1 \right) \left( \frac{x^6}{2160} + \frac{x^5}{1800} + \frac{x^4}{480} + \frac{x^3}{72} + \frac{x^2}{12} + \frac{x}{2} - \frac{1}{x} + \log(x) \right)$$
$$+ c_1 \left( -\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1 \right)$$

## 5.9 problem 9

Internal problem ID [6548]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 9.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_separable]

$$y'x + y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
Order:=6;
dsolve(x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1}{x} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 9

```
AsymptoticDSolveValue[x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1}{x}$$

## 5.10   problem 10

Internal problem ID [6549]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 10.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_quadrature]

$$y' - \frac{1}{x} = 0$$

With the expansion point for the power series method at $x = 0$.

✗  Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x)=1/x,y(x),type='series',x=0);
```

No solution found

✓  Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 8

```
AsymptoticDSolveValue[y'[x]==1/x,y[x],{x,0,5}]
```

$$y(x) \rightarrow \log(x) + c_1$$

## 5.11 problem 11

Internal problem ID [6550]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' - \frac{1}{x} = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x$2)=1/x,y(x),type='series',x=0);
```

No solution found

✔ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 17

```
AsymptoticDSolveValue[y''[x]==1/x,y[x],{x,0,5}]
```

$$y(x) \to -x + x \log(x) + c_2 x + c_1$$

## 5.12   problem 12

Internal problem ID [6551]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' + y' - \frac{1}{x} = 0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x$2)+diff(y(x),x)=1/x,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 159

```
AsymptoticDSolveValue[y''[x]+y'[x]==1/x,y[x],{x,0,5}]
```

$$y(x) \to -\frac{x^6}{4320} - \frac{x^5}{600} - \frac{x^4}{96} - \frac{x^3}{18} - \frac{x^2}{4} + c_2 \left( -\frac{x^5}{720} + \frac{x^4}{120} - \frac{x^3}{24} + \frac{x^2}{6} - \frac{x}{2} + 1 \right) x + \left( -\frac{x^5}{720} \right.$$
$$\left. + \frac{x^4}{120} - \frac{x^3}{24} + \frac{x^2}{6} - \frac{x}{2} + 1 \right) x \left( \frac{x^6}{2160} + \frac{x^5}{600} + \frac{x^4}{96} + \frac{x^3}{18} + \frac{x^2}{4} + x + \log(x) \right) - x + c_1$$

## 5.13   problem 13

Internal problem ID [6552]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y - \frac{1}{x} = 0$$

With the expansion point for the power series method at $x = 0$.

✗  Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x$2)+y(x)=1/x,y(x),type='series',x=0);
```

No solution found

✓  Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 148

```
AsymptoticDSolveValue[y''[x]+y[x]==1/x,y[x],{x,0,5}]
```

$$y(x) \to x\left(-\frac{x^6}{5040} + \frac{x^4}{120} - \frac{x^2}{6} + 1\right)\left(-\frac{x^6}{4320} + \frac{x^4}{96} - \frac{x^2}{4} + \log(x)\right) + c_1\left(-\frac{x^6}{720} + \frac{x^4}{24} - \frac{x^2}{2} + 1\right)$$

$$+ c_2 x\left(-\frac{x^6}{5040} + \frac{x^4}{120} - \frac{x^2}{6} + 1\right) + \left(-\frac{x^5}{600} + \frac{x^3}{18} - x\right)\left(-\frac{x^6}{720} + \frac{x^4}{24} - \frac{x^2}{2} + 1\right)$$

## 5.14    problem 14

Internal problem ID [6553]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$ y'' + y' + y - \frac{1}{x} = 0 $$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=1/x,y(x),type='series',x=0);
```

No solution found

✔ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 152

```
AsymptoticDSolveValue[y''[x]+y'[x]+y[x]==1/x,y[x],{x,0,5}]
```

$$ y(x) \to c_2 x \left( -\frac{x^4}{120} + \frac{x^3}{24} - \frac{x}{2} + 1 \right) + c_1 \left( \frac{x^3}{6} - \frac{x^2}{2} + 1 \right) $$
$$ + x \left( -\frac{x^4}{120} + \frac{x^3}{24} - \frac{x}{2} + 1 \right) \left( \frac{41x^6}{4320} + \frac{x^5}{120} - \frac{x^4}{96} - \frac{x^3}{18} + x + \log(x) \right) $$
$$ + \left( \frac{x^3}{6} - \frac{x^2}{2} + 1 \right) \left( -\frac{x^6}{180} + \frac{x^5}{600} + \frac{x^4}{96} - \frac{x^2}{4} - x \right) $$

## 5.15   problem 15

Internal problem ID [6554]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 15.
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type [_quadrature]

$$
h^2 + \frac{2ah}{\sqrt{1 + h'^2}} - b^2 = 0
$$

✓   Solution by Maple

Time used: 0.719 (sec). Leaf size: 108

```
dsolve(h(u)^2 + 2*a*h(u)/sqrt(1 + diff(h(u), u)^2) = b^2,h(u), singsol=all)
```

$$
u - \left( \int^{h(u)} \frac{(\_a + b)(\_a - b)}{\sqrt{-(\_a^2 + 2\_aa - b^2)(\_a^2 - 2\_aa - b^2)}} d\_a \right) - c_1 = 0
$$

$$
u - \left( \int^{h(u)} -\frac{(\_a + b)(\_a - b)}{\sqrt{-(\_a^2 + 2\_aa - b^2)(\_a^2 - 2\_aa - b^2)}} d\_a \right) - c_1 = 0
$$

✓ Solution by Mathematica

Time used: 24.533 (sec). Leaf size: 913

```
DSolve[h[u]^2 + 2*a*h[u]/Sqrt[1 + (h'[u])^2] == b^2,h[u],u,IncludeSingularSolutions -> True]
```

$h(u)$

$$\to \text{InverseFunction}\left[-\frac{i\sqrt{\left(b^2-\#1^2\right)^2}\sqrt{1-\frac{\#1^2}{-2\sqrt{a^2(a^2+b^2)}+2a^2+b^2}}\sqrt{1-\frac{\#1^2}{2\sqrt{a^2(a^2+b^2)}+2a^2+b^2}}\left(\left(2\sqrt{a^2\left(a^2+b^2\right)}-\right.\right.}{}\right.$$

$$\left. + c_1\right]$$

$h(u)$

$$\to \text{InverseFunction}\left[-\frac{i\sqrt{\left(b^2-\#1^2\right)^2}\sqrt{1-\frac{\#1^2}{-2\sqrt{a^2(a^2+b^2)}+2a^2+b^2}}\sqrt{1-\frac{\#1^2}{2\sqrt{a^2(a^2+b^2)}+2a^2+b^2}}\left(\left(2\sqrt{a^2\left(a^2+b^2\right)}-\right.\right.}{}\right.$$

$$\left. + c_1\right]$$

$$h(u) \to -\sqrt{a^2+b^2}-a$$

$$h(u) \to \sqrt{a^2+b^2}-a$$

## 5.16 problem 16

Internal problem ID [6555]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 16.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + 2y' - 24y - 16 + (x+2)\,\mathrm{e}^{4x} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-24*y(x)=16-(x+2)*exp(4*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{4x} c_2 + \mathrm{e}^{-6x} c_1 - \frac{2}{3} + \frac{(-50x^2 - 190x + 19)\,\mathrm{e}^{4x}}{1000}$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 40

```
DSolve[y''[x]+2*y'[x]-24*y[x]==16-(x+2)*Exp[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-6x} + \frac{e^{4x}(-10x(5x + 19) + 19 + 1000c_2)}{1000} - \frac{2}{3}$$

## 5.17 problem 17

Internal problem ID [6556]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 17.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' + 3y' - 4y - 6\,\mathrm{e}^{2t-2} = 0$$

With initial conditions

$$[y(1) = 4, y'(1) = 5]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)-4*y(t)=6*exp(2*t-2),y(1) = 4, D(y)(1) = 5],y(t), singso
```

$$y(t) = 3\,\mathrm{e}^{t-1} + \mathrm{e}^{2t-2}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 18

```
DSolve[{y''[t]+3*y'[t]-4*y[t]==6*Exp[2*t-2],{y[1]==4,y'[1]==5}},y[t],t,IncludeSingularSolutio
```

$$y(t) \to \mathrm{e}^{t-2}\left(\mathrm{e}^t + 3e\right)$$

## 5.18 problem 18

Internal problem ID [6557]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 18.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + y - e^{a\cos(x)} = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple

Time used: 0.0 (sec). Leaf size: 76

```
Order:=8;
dsolve(diff(y(x),x$2)+y(x)=exp(a*cos(x)),y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6\right)y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7\right)D(y)(0)$$
$$+ \frac{e^a x^2}{2} + \frac{(-a-1)e^a x^4}{24} + \frac{(3a^2 + 2a + 1)e^a x^6}{720} + O(x^8)$$

✔ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 239

```
AsymptoticDSolveValue[y''[x]+y[x]==Exp[a*Cos[x]],y[x],{x,0,7}]
```

$$y(x) \to \left(-\frac{x^7}{5040} + \frac{x^5}{120} - \frac{x^3}{6} + x\right)\left(\frac{1}{120}(3a^2 + 7a + 1)e^a x^5 - \frac{(15a^3 + 60a^2 + 31a + 1)e^a x^7}{5040}\right.$$
$$-\frac{1}{6}(a+1)e^a x^3 + e^a x\right) + \left(-\frac{x^6}{720} + \frac{x^4}{24} - \frac{x^2}{2} + 1\right)\left(-\frac{1}{720}(15a^2 + 15a + 1)e^a x^6\right.$$
$$+ \frac{(105a^3 + 210a^2 + 63a + 1)e^a x^8}{40320} + \frac{1}{24}(3a+1)e^a x^4 - \frac{e^a x^2}{2}\right)$$
$$+ c_2\left(-\frac{x^7}{5040} + \frac{x^5}{120} - \frac{x^3}{6} + x\right) + c_1\left(-\frac{x^6}{720} + \frac{x^4}{24} - \frac{x^2}{2} + 1\right)$$

## 5.19 problem 19

Internal problem ID [6558]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 19.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y' - \frac{y}{2y \ln(y) + y - x} = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=y(x)/(2*y(x)*ln(y(x))+y(x)-x),y(x), singsol=all)
```

$$y(x) = e^{\text{RootOf}\left(\_Z e^{2\_Z} - x e^{\_Z} + c_1\right)}$$

✓ Solution by Mathematica
Time used: 0.195 (sec). Leaf size: 19

```
DSolve[y'[x]==y[x]/(2*y[x]*Log[y[x]]+y[x]-x),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = y(x) \log(y(x)) + \frac{c_1}{y(x)}, y(x)\right]$$

## 5.20 problem 20

Internal problem ID [6559]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 20.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' - (2x + 1) y' + (1 + x) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)-(2*x+1)*diff(y(x),x)+(x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x + c_2 \mathrm{e}^x x^2$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 23

```
DSolve[x*y''[x]-(2*x+1)*y'[x]+(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^x \left( c_2 x^2 + 2c_1 \right)$$

## 5.21 problem 21

Internal problem ID [6560]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 21.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type [_separable]

$$y'x^2 + e^{-y} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x)+exp(-y(x))=0,y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{c_1 x - 1}{x}\right)$$

✓ Solution by Mathematica
Time used: 0.425 (sec). Leaf size: 12

```
DSolve[x^2*y'[x]+Exp[-y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(\frac{1}{x} + c_1\right)$$

## 5.22 problem 22

Internal problem ID [6561]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 22.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]

$$y'' + e^y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+exp(y(x))=0,y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{\tanh\left(\frac{x+c_2}{2c_1}\right)^2 - 1}{2c_1^2}\right)$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 31

```
DSolve[y''[x]+Exp[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log(2) + \log\left(c_1 \operatorname{sech}^2\left(\frac{1}{2}\sqrt{c_1}(x + c_2)\right)\right)$$

## 5.23   problem 23

Internal problem ID [6562]

**Book**: Own collection of miscellaneous problems
**Section**: section 5.0
**Problem number**: 23.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y' - \frac{yx + 3x - 2y + 6}{yx - 3x - 2y + 6} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=(x*y(x)+3*x-2*y(x)+6)/(x*y(x)-3*x-2*y(x)+6),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(x*y[x]+3*x-2*y[x]+6)/(x*y[x]-3*x-2*y[x]+6),y[x],x,IncludeSingularSolutions ->
```

Not solved