A Solution Manual For

Second order enumerated odes

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Contents

1 section 1

1.1 problem 1

Internal problem ID [6633]

Book: Second order enumerated odes
Section: section 1
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x + c_1$$

1.2 problem 2

Internal problem ID [6634]

Book: Second order enumerated odes
Section: section 1
Problem number: 2.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y''^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x$2)^2=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 12

```
DSolve[(y''[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x + c_1$$

1.3 problem 3

Internal problem ID [6635]

Book: Second order enumerated odes
Section: section 1
Problem number: 3.
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y''^n = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x$2)^n=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[(y''[x])^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}0^{\frac{1}{n}}x^2 + c_2x + c_1$$

1.4 problem 4

Internal problem ID [6636]

Book: Second order enumerated odes
Section: section 1
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{ay'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(a*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[a*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x + c_1$$

1.5 problem 5

Internal problem ID [6637]

Book: Second order enumerated odes
Section: section 1
Problem number: 5.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{ay''^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 9

```
dsolve(a*diff(y(x),x$2)^2=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[a*(y''[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x + c_1$$

1.6 problem 6

Internal problem ID [6638]

Book: Second order enumerated odes
Section: section 1
Problem number: 6.
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$a{y''}^{n} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(a*diff(y(x),x$2)^n=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[a*(y''[x])^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} 0^{\frac{1}{n}} x^2 + c_2 x + c_1$$

1.7 problem 7

Internal problem ID [6639]

Book: Second order enumerated odes
Section: section 1
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)=1,y(x), singsol=all)
```

$$y(x) = \frac{1}{2}x^2 + xc_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 19

```
DSolve[y''[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + c_2x + c_1$$

1.8 problem 8

Internal problem ID [6640]

Book: Second order enumerated odes
Section: section 1
Problem number: 8.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y''^2 - 1 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)^2=1,y(x), singsol=all)
```

$$y(x) = \frac{1}{2}x^2 + xc_1 + c_2$$
$$y(x) = -\frac{1}{2}x^2 + xc_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 37

```
DSolve[(y''[x])^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{2} + c_2x + c_1$$
$$y(x) \to \frac{x^2}{2} + c_2x + c_1$$

1.9 problem 9

Internal problem ID [6641]

Book: Second order enumerated odes
Section: section 1
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)=x,y(x), singsol=all)
```

$$y(x) = \frac{1}{6}x^3 + xc_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 19

```
DSolve[y''[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{6} + c_2x + c_1$$

1.10 problem 10

Internal problem ID [6642]

Book: Second order enumerated odes
Section: section 1
Problem number: 10.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y''^2 - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)^2=x,y(x), singsol=all)
```

$$y(x) = \frac{4x^{\frac{5}{2}}}{15} + xc_1 + c_2$$
$$y(x) = -\frac{4x^{\frac{5}{2}}}{15} + xc_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 41

```
DSolve[(y''[x])^2==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{4x^{5/2}}{15} + c_2x + c_1$$
$$y(x) \to \frac{4x^{5/2}}{15} + c_2x + c_1$$

1.11 problem 11

Internal problem ID [6643]

Book: Second order enumerated odes
Section: section 1
Problem number: 11.
ODE order: 2.
ODE degree: 3.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y''^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x$2)^3=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[(y''[x])^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x + c_1$$

1.12 problem 12

Internal problem ID [6644]

Book: Second order enumerated odes
Section: section 1
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 17

```
DSolve[y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - c_1 e^{-x}$$

1.13 problem 13

Internal problem ID [6645]

Book: Second order enumerated odes
Section: section 1
Problem number: 13.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y''^2 + y' = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)^2+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$
$$y(x) = -\frac{1}{12}x^3 + \frac{1}{2}x^2c_1 - c_1^2x + c_2$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 59

```
DSolve[(y''[x])^2+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{12}x\left(x^2 + 3ic_1x - 3c_1{}^2\right)$$
$$y(x) \to c_2 - \frac{1}{12}x\left(x^2 - 3ic_1x - 3c_1{}^2\right)$$

1.14 problem 14

Internal problem ID [6646]

Book: Second order enumerated odes
Section: section 1
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible,`

$$\boxed{y'' + y'^2 = 0}$$

✓ Solution by Maple

Time used: 1.516 (sec). Leaf size: 10

```
dsolve(diff(y(x),x$2)+diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = \ln(xc_1 + c_2)$$

✓ Solution by Mathematica

Time used: 0.191 (sec). Leaf size: 15

```
DSolve[y''[x]+(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x - c_1) + c_2$$

1.15 problem 15

Internal problem ID [6647]

Book: Second order enumerated odes
Section: section 1
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-x}c_1 + x + c_2$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 18

```
DSolve[y''[x]+y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - c_1e^{-x} + c_2$$

1.16 problem 16

Internal problem ID [6648]

Book: Second order enumerated odes
Section: section 1
Problem number: 16.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y''^2 + y' - 1 = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)^2+diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = c_1 + x$$
$$y(x) = -\frac{1}{12}x^3 + \frac{1}{2}x^2c_1 - c_1^2x + x + c_2$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 59

```
DSolve[(y''[x])^2+y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{12}x\left(x^2 + 3c_1x + 3\left(-4 + c_1{}^2\right)\right)$$
$$y(x) \to c_2 - \frac{1}{12}x\left(x^2 - 3c_1x + 3\left(-4 + c_1{}^2\right)\right)$$

1.17 problem 17

Internal problem ID [6649]

Book: Second order enumerated odes
Section: section 1
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' + y'^2 - 1 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+diff(y(x),x)^2=1,y(x), singsol=all)
```

$$y(x) = x + \ln\left(\frac{e^{-2x}c_1}{2} - \frac{c_2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.308 (sec). Leaf size: 46

```
DSolve[y''[x]+(y'[x])^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log(e^x) + \log\left(e^{2x} + e^{2c_1}\right) + c_2$$
$$y(x) \to -\log(e^x) + \log\left(e^{2x}\right) + c_2$$

1.18 problem 18

Internal problem ID [6650]

Book: Second order enumerated odes
Section: section 1
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} - \mathrm{e}^{-x}c_1 - x + c_2$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 25

```
DSolve[y''[x]+y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(x-2)x - c_1e^{-x} + c_2$$

1.19 problem 19

Internal problem ID [6651]

Book: Second order enumerated odes
Section: section 1
Problem number: 19.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y''^2 + y' - x = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 122

```
dsolve(diff(y(x),x$2)^2+diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = \int \left(-e^{2\,\mathrm{RootOf}\left(\_Z - x - 2\,e^{-\_Z} + 2 + c_1 - \ln\left(e^{-\_Z}\left(e^{-\_Z} - 2\right)^2\right)\right)} + 2\,e^{\mathrm{RootOf}\left(\_Z - x - 2\,e^{-\_Z} + 2 + c_1 - \ln\left(e^{-\_Z}\left(e^{-\_Z} - 2\right)^2\right)\right)} + x\right) dx - x + c_2$$

$$y(x) = \frac{2\,\mathrm{LambertW}\left(-c_1 e^{-\frac{x}{2}-1}\right)^3}{3} + 3\,\mathrm{LambertW}\left(-c_1 e^{-\frac{x}{2}-1}\right)^2 + 4\,\mathrm{LambertW}\left(-c_1 e^{-\frac{x}{2}-1}\right) + \frac{x^2}{2} - x + c_2$$

✓ Solution by Mathematica

Time used: 23.232 (sec). Leaf size: 166

```
DSolve[(y''[x])^2+y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3} W\left(e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right)^3 + 3W\left(e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right)^2 + 4W\left(e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right) + \frac{1}{2}(x-2)x + c_2$$

$$y(x) \to \frac{2}{3} W\left(-e^{\frac{1}{2}(-x-2+c_1)}\right)^3 + 3W\left(-e^{\frac{1}{2}(-x-2+c_1)}\right)^2 + 4W\left(-e^{\frac{1}{2}(-x-2+c_1)}\right) + \frac{1}{2}(x-2)x + c_2$$

$$y(x) \to \frac{1}{2}(x-2)x + c_2$$

1.20 problem 20

Internal problem ID [6652]

Book: Second order enumerated odes
Section: section 1
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' + {y'}^2 - x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+diff(y(x),x)^2=x,y(x), singsol=all)
```

$$y(x) = \ln\left(\operatorname{AiryAi}(x) c_1\pi - c_2 \operatorname{AiryBi}(x)\pi\right)$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 15

```
DSolve[y''[x]+(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x - c_1) + c_2$$

1.21 problem 21

Internal problem ID [6653]

Book: Second order enumerated odes
Section: section 1
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y' + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) + c_2 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[y''[x]+y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

1.22 problem 22

Internal problem ID [6654]

Book: Second order enumerated odes
Section: section 1
Problem number: 22.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y''^2 + y' + y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)^2+diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y''[x])^2+y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.23 problem 23

Internal problem ID [6655]

Book: Second order enumerated odes
Section: section 1
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y'^2 + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$2)+diff(y(x),x)^2+y(x)=0,y(x), singsol=all)
```

$$\int^{y(x)} -\frac{2}{\sqrt{2 + 4\,\mathrm{e}^{-2\_a} c_1 - 4\_a}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} \frac{2}{\sqrt{2 + 4\,\mathrm{e}^{-2\_a} c_1 - 4\_a}} d\_a - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.327 (sec). Leaf size: 90

```
DSolve[y''[x]+(y'[x])^2+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{2}}{\sqrt{2e^{-2K[1]} c_1 - 2K[1] + 1}} dK[1]\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt{2}}{\sqrt{2e^{-2K[2]} c_1 - 2K[2] + 1}} dK[2]\&\right][x + c_2]$$

1.24 problem 24

Internal problem ID [6656]

Book: Second order enumerated odes
Section: section 1
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' + y - 1 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=1,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) c_1 + 1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 44

```
DSolve[y''[x]+y'[x]+y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + e^{-x/2}\left(c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

1.25 problem 25

Internal problem ID [6657]

Book: Second order enumerated odes
Section: section 1
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' + y - x = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right) c_2 + e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) c_1 + x - 1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 45

```
DSolve[y''[x]+y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + e^{-x/2}\left(c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right) - 1$$

1.26 problem 26

Internal problem ID [6658]

Book: Second order enumerated odes
Section: section 1
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y' + y - 1 - x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=1+x,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) c_1 + x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 44

```
DSolve[y''[x]+y'[x]+y[x]==1+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + e^{-x/2}\left(c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

1.27 problem 27

Internal problem ID [6659]

Book: Second order enumerated odes
Section: section 1
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' + y - x^2 - x - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=1+x+x^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) c_1 + x^2 - x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 48

```
DSolve[y''[x]+y'[x]+y[x]==1+x+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x-1)x + e^{-x/2}\left(c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

1.28 problem 28

Internal problem ID [6660]

Book: Second order enumerated odes
Section: section 1
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y - x^3 - x^2 - x - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=1+x+x^2+x^3,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) c_1 + x^3 - 2x^2 - x + 6$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 53

```
DSolve[y''[x]+y'[x]+y[x]==1+x+x^2+x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x((x-2)x-1) + e^{-x/2}\left(c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right) + 6$$

1.29 problem 29

Internal problem ID [6661]

Book: Second order enumerated odes
Section: section 1
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y - \sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) c_1 - \cos(x)$$

✓ Solution by Mathematica

Time used: 0.166 (sec). Leaf size: 47

```
DSolve[y''[x]+y'[x]+y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\cos(x) + e^{-x/2}\left(c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

1.30 problem 30

Internal problem ID [6662]

Book: Second order enumerated odes
Section: section 1
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y' + y - \cos(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) c_1 + \sin(x)$$

✓ Solution by Mathematica

Time used: 0.313 (sec). Leaf size: 45

```
DSolve[y''[x]+y'[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + e^{-x/2}\left(c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

1.31 problem 31

Internal problem ID [6663]

Book: Second order enumerated odes
Section: section 1
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-x} c_1 + x + c_2$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 18

```
DSolve[y''[x]+y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - c_1 e^{-x} + c_2$$

1.32 problem 32

Internal problem ID [6664]

Book: Second order enumerated odes
Section: section 1
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' + y' - x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} - \mathrm{e}^{-x} c_1 - x + c_2$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 25

```
DSolve[y''[x]+y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(x-2)x - c_1 e^{-x} + c_2$$

1.33 problem 33

Internal problem ID [6665]

Book: Second order enumerated odes
Section: section 1
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' + y' - 1 - x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=1+x,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} - e^{-x}c_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 24

```
DSolve[y''[x]+y'[x]==1+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} - c_1e^{-x} + c_2$$

1.34 problem 34

Internal problem ID [6666]

Book: Second order enumerated odes
Section: section 1
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' - x^2 - x - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=1+x+x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{3} - e^{-x}c_1 - \frac{x^2}{2} + 2x + c_2$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 31

```
DSolve[y''[x]+y'[x]==1+x+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}x(x(2x-3)+12) - c_1e^{-x} + c_2$$

1.35 problem 35

Internal problem ID [6667]

Book: Second order enumerated odes
Section: section 1
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' - x^3 - x^2 - x - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=1+x+x^2+x^3,y(x), singsol=all)
```

$$y(x) = \frac{x^4}{4} - e^{-x}c_1 + \frac{5x^2}{2} - \frac{2x^3}{3} - 4x + c_2$$

✓ Solution by Mathematica

Time used: 0.129 (sec). Leaf size: 35

```
DSolve[y''[x]+y'[x]==1+x+x^2+x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}x(x(x(3x - 8) + 30) - 48) - c_1e^{-x} + c_2$$

1.36 problem 36

Internal problem ID [6668]

Book: Second order enumerated odes
Section: section 1
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' - \sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=sin(x),y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-x} c_1 - \frac{\sin(x)}{2} - \frac{\cos(x)}{2} + c_2$$

✓ Solution by Mathematica

Time used: 0.115 (sec). Leaf size: 29

```
DSolve[y''[x]+y'[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sin(x)}{2} - \frac{\cos(x)}{2} + c_1(-e^{-x}) + c_2$$

1.37 problem 37

Internal problem ID [6669]

Book: Second order enumerated odes
Section: section 1
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' - \cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=cos(x),y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-x}c_1 + \frac{\sin(x)}{2} - \frac{\cos(x)}{2} + c_2$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 28

```
DSolve[y''[x]+y'[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\sin(x) - \cos(x) - 2c_1e^{-x}\right) + c_2$$

1.38 problem 38

Internal problem ID [6670]

Book: Second order enumerated odes
Section: section 1
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+y(x)=1,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + 1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 17

```
DSolve[y''[x]+y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + c_2 \sin(x) + 1$$

1.39 problem 39

Internal problem ID [6671]

Book: Second order enumerated odes
Section: section 1
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+y(x)=x,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 17

```
DSolve[y''[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1 \cos(x) + c_2 \sin(x)$$

1.40 problem 40

Internal problem ID [6672]

Book: Second order enumerated odes
Section: section 1
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y - 1 - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+y(x)=1+x,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + x + 1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[y''[x]+y[x]==1+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1 \cos(x) + c_2 \sin(x) + 1$$

1.41 problem 41

Internal problem ID [6673]

Book: Second order enumerated odes
Section: section 1
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y - x^2 - x - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+y(x)=1+x+x^2,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + x^2 + x - 1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 21

```
DSolve[y''[x]+y[x]==1+x+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + x + c_1 \cos(x) + c_2 \sin(x) - 1$$

1.42 problem 42

Internal problem ID [6674]

Book: Second order enumerated odes
Section: section 1
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - x^3 - x^2 - x - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=1+x+x^2+x^3,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + x^3 + x^2 - 5x - 1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[y''[x]+y[x]==1+x+x^2+x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x^2 + x - 5) + c_1 \cos(x) + c_2 \sin(x) - 1$$

1.43 problem 43

Internal problem ID [6675]

Book: Second order enumerated odes
Section: section 1
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + \cos(x)\, c_1 + \frac{\sin(x)}{2} - \frac{\cos(x)\, x}{2}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{x}{2} + c_1\right)\cos(x) + c_2 \sin(x)$$

1.44 problem 44

Internal problem ID [6676]

Book: Second order enumerated odes
Section: section 1
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y - \cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + \frac{x \sin(x)}{2}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 28

```
DSolve[y''[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(x \sin(x) + \cos(x) + 2c_1 \cos(x) + 2c_2 \sin(x))$$

1.45 problem 45

Internal problem ID [6677]

Book: Second order enumerated odes
Section: section 1
Problem number: 45.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y{y''}^2 + y' = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 229

```
dsolve(y(x)*diff(y(x),x$2)^2+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

$$y(x) = 0$$

$$\int^{y(x)} -\frac{\_a}{\left(c_1\_a^{\frac{3}{2}} - 3\_a^2\right)^{\frac{2}{3}}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} -\frac{\_a}{\left(c_1\_a^{\frac{3}{2}} + 3\_a^2\right)^{\frac{2}{3}}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} -\frac{4\_a}{\left(c_1\_a^{\frac{3}{2}} - 3\_a^2\right)^{\frac{2}{3}} \left(1 + i\sqrt{3}\right)^2} d\_a - x - c_2 = 0$$

$$\int^{y(x)} -\frac{4\_a}{\left(c_1\_a^{\frac{3}{2}} - 3\_a^2\right)^{\frac{2}{3}} \left(i\sqrt{3} - 1\right)^2} d\_a - x - c_2 = 0$$

$$\int^{y(x)} -\frac{4\_a}{\left(c_1\_a^{\frac{3}{2}} + 3\_a^2\right)^{\frac{2}{3}} \left(1 + i\sqrt{3}\right)^2} d\_a - x - c_2 = 0$$

$$\int^{y(x)} -\frac{4\_a}{\left(c_1\_a^{\frac{3}{2}} + 3\_a^2\right)^{\frac{2}{3}} \left(i\sqrt{3} - 1\right)^2} d\_a - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 1.026 (sec). Leaf size: 23861

```
DSolve[y[x]*y''[x]^2+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

1.46 problem 46

Internal problem ID [6678]

Book: Second order enumerated odes
Section: section 1
Problem number: 46.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y {y''}^{2}+{y'}^{3}=0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 166

```
dsolve(y(x)*diff(y(x),x$2)^2+diff(y(x),x)^3=0,y(x), singsol=all)
```

$$y(x) = c_1$$

$$y(x) = 0$$

$$y(x) = \frac{c_2 \left(\operatorname{LambertW}\left(c_1 \mathrm{e}^{-1+\frac{x}{2}}\right)+1\right)^2}{\operatorname{LambertW}\left(c_1 \mathrm{e}^{-1+\frac{x}{2}}\right)^2}$$

$$y(x) = \frac{c_2 \left(\operatorname{LambertW}\left(-c_1 \mathrm{e}^{-1+\frac{x}{2}}\right)+1\right)^2}{\operatorname{LambertW}\left(-c_1 \mathrm{e}^{-1+\frac{x}{2}}\right)^2}$$

$$y(x) = \mathrm{e}^{-\left(\int \mathrm{e}^{2 \operatorname{RootOf}\left(\mathrm{e}^{\_Z} \ln\left(\left(\mathrm{e}^{\_Z}+1\right)^2\right)+c_1 \mathrm{e}^{\_Z}-2\_Z \mathrm{e}^{\_Z}+x \,\mathrm{e}^{\_Z}+\ln\left(\left(\mathrm{e}^{\_Z}+1\right)^2\right)+c_1-2\_Z+x-2\right)} dx\right)-2\left(\int \mathrm{e}^{\operatorname{RootOf}\left(\mathrm{e}^{\_Z} \ln\left(\left(\mathrm{e}^{\_Z}+1\right)^2\right)+c_1 \mathrm{e}^{\_Z}-2\_Z \mathrm{e}^{\_Z}\right.}}$$

✓ Solution by Mathematica

Time used: 0.571 (sec). Leaf size: 117

```
DSolve[y[x]*y''[x]^2+y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-4\left(\frac{1}{2}\log\left(2\sqrt{\#1}-ic_1\right)-\frac{ic_1}{2\left(2\sqrt{\#1}-ic_1\right)}\right)\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[-4\left(\frac{ic_1}{2\left(2\sqrt{\#1}+ic_1\right)}+\frac{1}{2}\log\left(2\sqrt{\#1}+ic_1\right)\right)\&\right][x+c_2]$$

1.47 problem 47

Internal problem ID [6679]

Book: Second order enumerated odes
Section: section 1
Problem number: 47.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y^2 y''^2 + y' = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 205

```
dsolve(y(x)^2*diff(y(x),x$2)^2+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$
$$y(x) = 0$$
$$\int^{y(x)} -\frac{4}{(-12\ln(\_a) + 8c_1)^{\frac{2}{3}}} d\_a - x - c_2 = 0$$
$$\int^{y(x)} -\frac{4}{(12\ln(\_a) - 8c_1)^{\frac{2}{3}}} d\_a - x - c_2 = 0$$
$$\int^{y(x)} -\frac{16}{(-12\ln(\_a) + 8c_1)^{\frac{2}{3}} \left(1 + i\sqrt{3}\right)^2} d\_a - x - c_2 = 0$$
$$\int^{y(x)} -\frac{16}{(-12\ln(\_a) + 8c_1)^{\frac{2}{3}} \left(i\sqrt{3} - 1\right)^2} d\_a - x - c_2 = 0$$
$$\int^{y(x)} -\frac{16}{(12\ln(\_a) - 8c_1)^{\frac{2}{3}} \left(1 + i\sqrt{3}\right)^2} d\_a - x - c_2 = 0$$
$$\int^{y(x)} -\frac{16}{(12\ln(\_a) - 8c_1)^{\frac{2}{3}} \left(i\sqrt{3} - 1\right)^2} d\_a - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.435 (sec). Leaf size: 145

```
DSolve[y[x]^2*y''[x]^2+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{\left(\frac{2}{3}\right)^{2/3} e^{-ic_1}(-\log(\#1) - ic_1)^{2/3}\Gamma\left(\frac{1}{3}, -ic_1 - \log(\#1)\right)}{(c_1 - i\log(\#1))^{2/3}}\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\left(\frac{2}{3}\right)^{2/3} e^{ic_1}(-\log(\#1) + ic_1)^{2/3}\Gamma\left(\frac{1}{3}, ic_1 - \log(\#1)\right)}{(i\log(\#1) + c_1)^{2/3}}\&\right][x + c_2]$$

1.48 problem 48

Internal problem ID [6680]

Book: Second order enumerated odes
Section: section 1
Problem number: 48.
ODE order: 2.
ODE degree: 4.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{yy''^4 + y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 2698

```
dsolve(y(x)*diff(y(x),x$2)^4+diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1$$

$$y(x) = 0$$

$$\int^{y(x)} \frac{\_a^2}{\sqrt{-\_a\left(\left(-2\_a+(c_1\_a)^{\frac{1}{4}}\right)\_a^2\right)^{\frac{4}{3}}}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} \frac{\_a^2}{\sqrt{-\_a\left(-\left(2\_a+(c_1\_a)^{\frac{1}{4}}\right)\_a^2\right)^{\frac{4}{3}}}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} -\frac{\_a^2}{\sqrt{-\_a\left(\left(-2\_a+(c_1\_a)^{\frac{1}{4}}\right)\_a^2\right)^{\frac{4}{3}}}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} \frac{\_a^2}{\sqrt{-\_a\left(\left(i\,(c_1\_a)^{\frac{1}{4}}-2\_a\right)\_a^2\right)^{\frac{4}{3}}}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} -\frac{\_a^2}{\sqrt{-\_a\left(-\left(2\_a+(c_1\_a)^{\frac{1}{4}}\right)\_a^2\right)^{\frac{4}{3}}}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} \frac{\_a^2}{\sqrt{-\_a\left(-\left(i\,(c_1\_a)^{\frac{1}{4}}+2\_a\right)\_a^2\right)^{\frac{4}{3}}}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} -\frac{\_a^2}{\sqrt{-\_a\left(\left(i\,(c_1\_a)^{\frac{1}{4}}-2\_a\right)\_a^2\right)^{\frac{4}{3}}}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} -\frac{\_a^2}{\sqrt{-\_a\left(-\left(i\,(c_1\_a)^{\frac{1}{4}}+2\_a\right)\_a^2\right)^{\frac{4}{3}}}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} \frac{\_a^2\sqrt{2}}{\sqrt{\_a\left(\left(-2\_a+(c_1\_a)^{\frac{1}{4}}\right)\_a^2\right)^{\frac{4}{3}}\left(1+i\sqrt{3}\right)}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} -\frac{2\_a^2}{\sqrt{-2\_a\left(\left(-2\_a+(c_1\_a)^{\frac{1}{4}}\right)\_a^2\right)^{\frac{4}{3}}\left(i\sqrt{3}-1\right)}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} \frac{2\_a^2}{\sqrt{\cdots}} d\_a - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 1.058 (sec). Leaf size: 405

```
DSolve[y[x]*y''[x]^4+y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{\#1\left(1-\frac{\left(\frac{2}{3}+\frac{2i}{3}\right)\sqrt{2}\#1^{3/4}}{c_1}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},\frac{\left(\frac{2}{3}+\frac{2i}{3}\right)\sqrt{2}\#1^{3/4}}{c_1}\right)}{\left(\frac{3c_1}{2}-(1+i)\sqrt{2}\#1^{3/4}\right)^{2/3}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\#1\left(1-\frac{\left(\frac{2}{3}-\frac{2i}{3}\right)\sqrt{2}\#1^{3/4}}{c_1}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},\frac{\left(\frac{2}{3}-\frac{2i}{3}\right)\sqrt{2}\#1^{3/4}}{c_1}\right)}{\left(\frac{3c_1}{2}-(1-i)\sqrt{2}\#1^{3/4}\right)^{2/3}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\#1\left(1+\frac{\left(\frac{2}{3}-\frac{2i}{3}\right)\sqrt{2}\#1^{3/4}}{c_1}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{\left(\frac{2}{3}-\frac{2i}{3}\right)\sqrt{2}\#1^{3/4}}{c_1}\right)}{\left((1-i)\sqrt{2}\#1^{3/4}+\frac{3c_1}{2}\right)^{2/3}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\#1\left(1+\frac{\left(\frac{2}{3}+\frac{2i}{3}\right)\sqrt{2}\#1^{3/4}}{c_1}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{\left(\frac{2}{3}+\frac{2i}{3}\right)\sqrt{2}\#1^{3/4}}{c_1}\right)}{\left((1+i)\sqrt{2}\#1^{3/4}+\frac{3c_1}{2}\right)^{2/3}}\&\right][x+c_2]$$

1.49 problem 49

Internal problem ID [6681]

Book: Second order enumerated odes
Section: section 1
Problem number: 49.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y^3 {y''}^2 + y y' = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 205

```
dsolve(y(x)^3*diff(y(x),x$2)^2+y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$
$$y(x) = 0$$
$$\int^{y(x)} -\frac{4}{\left(-12\ln\left(\_a\right)+8c_1\right)^{\frac{2}{3}}} d\_a - x - c_2 = 0$$
$$\int^{y(x)} -\frac{4}{\left(12\ln\left(\_a\right)-8c_1\right)^{\frac{2}{3}}} d\_a - x - c_2 = 0$$
$$\int^{y(x)} -\frac{16}{\left(-12\ln\left(\_a\right)+8c_1\right)^{\frac{2}{3}}\left(1+i\sqrt{3}\right)^2} d\_a - x - c_2 = 0$$
$$\int^{y(x)} -\frac{16}{\left(-12\ln\left(\_a\right)+8c_1\right)^{\frac{2}{3}}\left(i\sqrt{3}-1\right)^2} d\_a - x - c_2 = 0$$
$$\int^{y(x)} -\frac{16}{\left(12\ln\left(\_a\right)-8c_1\right)^{\frac{2}{3}}\left(1+i\sqrt{3}\right)^2} d\_a - x - c_2 = 0$$
$$\int^{y(x)} -\frac{16}{\left(12\ln\left(\_a\right)-8c_1\right)^{\frac{2}{3}}\left(i\sqrt{3}-1\right)^2} d\_a - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.378 (sec). Leaf size: 150

```
DSolve[y[x]^3*y''[x]^2+y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

$$y(x) \to \text{InverseFunction}\left[\frac{\left(\frac{2}{3}\right)^{2/3} e^{-ic_1}(-\log(\#1) - ic_1)^{2/3}\Gamma\left(\frac{1}{3}, -ic_1 - \log(\#1)\right)}{(c_1 - i\log(\#1))^{2/3}}\&\right][x + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\left(\frac{2}{3}\right)^{2/3} e^{ic_1}(-\log(\#1) + ic_1)^{2/3}\Gamma\left(\frac{1}{3}, ic_1 - \log(\#1)\right)}{(i\log(\#1) + c_1)^{2/3}}\&\right][x + c_2]$$

1.50 problem 50

Internal problem ID [6682]

Book: Second order enumerated odes
Section: section 1
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$yy'' + y'^3 = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x$2)+diff(y(x),x)^3=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1$$
$$y(x) = e^{\mathrm{LambertW}\left((x+c_2)e^{c_1}e^{-1}\right)-c_1+1}$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 25

```
DSolve[y[x]*y''[x]+y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{W\left(e^{-1-c_1}(x+c_2)\right)+1+c_1}$$

1.51 problem 51

Internal problem ID [6683]

Book: Second order enumerated odes
Section: section 1
Problem number: 51.
ODE order: 2.
ODE degree: 3.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y y''^3 + y^3 y' = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 124

```
dsolve(y(x)*diff(y(x),x$2)^3+y(x)^3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1$$
$$y(x) = \mathrm{e}^{\int \mathrm{RootOf}\left(x+\int^{\_Z} \frac{1}{\_f^2-(-\_f)^{\frac{1}{3}}} d\_f+c_1\right) dx+c_2}$$
$$y(x) = \mathrm{e}^{\int \mathrm{RootOf}\left(x+2\left(\int^{\_Z} \frac{1}{i\sqrt{3}\,(-\_f)^{\frac{1}{3}}+2\_f^2+(-\_f)^{\frac{1}{3}}} d\_f\right)+c_1\right) dx+c_2}$$
$$y(x) = \mathrm{e}^{\int \mathrm{RootOf}\left(x-2\left(\int^{\_Z} \frac{1}{i\sqrt{3}\,(-\_f)^{\frac{1}{3}}-2\_f^2-(-\_f)^{\frac{1}{3}}} d\_f\right)+c_1\right) dx+c_2}$$

✓ Solution by Mathematica

Time used: 0.762 (sec). Leaf size: 263

```
DSolve[y[x]*y''[x]^3+y[x]^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

$$y(x) \to \text{InverseFunction}\left[\frac{\#1\left(1-\frac{3\#1^{5/3}}{5c_1}\right)^{3/5}\text{Hypergeometric2F1}\left(\frac{3}{5},\frac{3}{5},\frac{8}{5},\frac{3\#1^{5/3}}{5c_1}\right)}{\left(-\#1^{5/3}+\frac{5c_1}{3}\right)^{3/5}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\#1\left(1+\frac{3\sqrt[3]{-1}\#1^{5/3}}{5c_1}\right)^{3/5}\text{Hypergeometric2F1}\left(\frac{3}{5},\frac{3}{5},\frac{8}{5},-\frac{3\sqrt[3]{-1}\#1^{5/3}}{5c_1}\right)}{\left(\sqrt[3]{-1}\#1^{5/3}+\frac{5c_1}{3}\right)^{3/5}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\#1\left(1-\frac{3(-1)^{2/3}\#1^{5/3}}{5c_1}\right)^{3/5}\text{Hypergeometric2F1}\left(\frac{3}{5},\frac{3}{5},\frac{8}{5},\frac{3(-1)^{2/3}\#1^{5/3}}{5c_1}\right)}{\left(-(-1)^{2/3}\#1^{5/3}+\frac{5c_1}{3}\right)^{3/5}}\&\right][x+c_2]$$

1.52 problem 52

Internal problem ID [6684]

Book: Second order enumerated odes
Section: section 1
Problem number: 52.
ODE order: 2.
ODE degree: 3.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$yy''^3 + y^3 y'^5 = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 248

```
dsolve(y(x)*diff(y(x),x$2)^3+y(x)^3*diff(y(x),x)^5=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = c_1$$

$$\int^{y(x)} \frac{1}{\mathrm{RootOf}\left(-5\left(\int_{\_g}^{\_Z} \frac{1}{-\_a\left(-\_a^2\_f^2\right)^{\frac{1}{3}}+5\_f} d\_f\right) - \ln\left(\_a^5+125\right)+5c_1\right)} d\_a - x - c_2 = 0$$

$$\int^{y(x)} \frac{1}{\mathrm{RootOf}\left(-i\ln\left(\_a^5+125\right)+\sqrt{3}\,\ln\left(\_a^5+125\right)+20\left(\int_{\_g}^{\_Z} \frac{i\sqrt{3}-1}{\left(5i\sqrt{3}\_f-2\_a\left(-\_a^2\_f^2\right)^{\frac{1}{3}}-5\_f\right)\left(\sqrt{3}+i\right)} d\_\right.\right.}$$

$$- x - c_2 = 0$$

$$\int^{y(x)} \frac{1}{\mathrm{RootOf}\left(i\ln\left(\_a^5+125\right)+\sqrt{3}\,\ln\left(\_a^5+125\right)-20\left(\int_{\_g}^{\_Z} -\frac{1+i\sqrt{3}}{\left(5i\sqrt{3}\_f+2\_a\left(-\_a^2\_f^2\right)^{\frac{1}{3}}+5\_f\right)\left(-i+\sqrt{3}\right)} d\right.\right.}$$

$$- x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 11.799 (sec). Leaf size: 148

```
DSolve[y[x]*y''[x]^3+y[x]^3*y'[x]^5==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

$$y(x) \to \text{InverseFunction}\left[\frac{27\#1\,\text{Hypergeometric2F1}\left(\frac{3}{5},3,\frac{8}{5},\frac{3\#1^{5/3}}{5c_1}\right)}{c_1{}^3}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{27\#1\,\text{Hypergeometric2F1}\left(\frac{3}{5},3,\frac{8}{5},-\frac{3i\left(-i+\sqrt{3}\right)\#1^{5/3}}{10c_1}\right)}{c_1{}^3}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{27\#1\,\text{Hypergeometric2F1}\left(\frac{3}{5},3,\frac{8}{5},\frac{3i\left(i+\sqrt{3}\right)\#1^{5/3}}{10c_1}\right)}{c_1{}^3}\&\right][x+c_2]$$

2 section 2

2.1 problem 1

Internal problem ID [6685]

Book: Second order enumerated odes
Section: section 2
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' + y'x + yy'^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = -i \operatorname{RootOf}\left(i\sqrt{\pi}\, \operatorname{erf}\left(\frac{\sqrt{2}\, x}{2}\right) c_1 + i\sqrt{2}\, c_2 - \operatorname{erf}\left(\_Z\right)\sqrt{\pi}\right)\sqrt{2}$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 44

```
DSolve[y''[x]+x*y'[x]+y[x]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i\sqrt{2}\operatorname{erf}^{-1}\left(i\left(\sqrt{\frac{2}{\pi}}c_2 - c_1\operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right)\right)$$

2.2 problem 2

Internal problem ID [6686]

Book: Second order enumerated odes
Section: section 2
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_xy]]`

$$y'' + y' \sin(x) + y{y'}^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)+sin(x)*diff(y(x),x)+y(x)*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = -i \operatorname{RootOf}\left(i\sqrt{2}\, c_1 \left(\int \mathrm{e}^{\cos(x)} dx\right) + i\sqrt{2}\, c_2 - \operatorname{erf}(\_Z)\sqrt{\pi}\right)\sqrt{2}$$

✓ Solution by Mathematica

Time used: 0.266 (sec). Leaf size: 47

```
DSolve[y''[x]+Sin[x]*y'[x]+y[x]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i\sqrt{2}\operatorname{erf}^{-1}\left(i\sqrt{\frac{2}{\pi}}\left(\int_1^x -e^{\cos(K[2])}c_1 dK[2] + c_2\right)\right)$$

2.3 problem 3

Internal problem ID [6687]

Book: Second order enumerated odes
Section: section 2
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_x_y1], [_2nd_order, _`

$$y'' + (1-x)\,y' + y^2 {y'}^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 64

```
dsolve(diff(y(x),x$2)+(1-x)*diff(y(x),x)+y(x)^2*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$c_1 \operatorname{erf}\left(\frac{i\sqrt{2}\,x}{2} - \frac{i\sqrt{2}}{2}\right) - c_2 + \frac{2\,3^{\frac{5}{6}} y(x)\,\pi}{9\Gamma\left(\frac{2}{3}\right)\left(-y\left(x\right)^3\right)^{\frac{1}{3}}} - \frac{y(x)\,\Gamma\left(\frac{1}{3}, -\frac{y(x)^3}{3}\right)}{\left(-9y\left(x\right)^3\right)^{\frac{1}{3}}} = 0$$

✓ Solution by Mathematica

Time used: 0.192 (sec). Leaf size: 67

```
DSolve[y''[x]+(1-x)*y'[x]+y[x]^2*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{\#1\Gamma\left(\frac{1}{3}, -\frac{\#1^3}{3}\right)}{3^{2/3}\sqrt[3]{-\#1^3}}\&\right]\left[c_2 - \sqrt{\frac{\pi}{2e}}c_1\text{erfi}\left(\frac{x-1}{\sqrt{2}}\right)\right]$$

2.4 problem 4

Internal problem ID [6688]

Book: Second order enumerated odes
Section: section 2
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_x_y1], [_2nd_order, _`

$$y'' + (\sin(x) + 2x)\,y' + \cos(y)\,y{y'}^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)+(sin(x)+2*x)*diff(y(x),x)+cos(y(x))*y(x)*diff(y(x),x)^2=0,y(x), singsol
```

$$\int^{y(x)} \mathrm{e}^{\cos(\_a)+\sin(\_a)\_a} d\_a - c_1 \left(\int \mathrm{e}^{-x^2+\cos(x)} dx\right) - c_2 = 0$$

✓ Solution by Mathematica

Time used: 1.013 (sec). Leaf size: 53

```
DSolve[y''[x]+(Sin[x]+2*x)*y'[x]+Cos[y[x]]*y[x]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} e^{\cos(K[2])+K[2]\sin(K[2])} dK[2]\&\right]\left[\int_1^x -e^{\cos(K[3])-K[3]^2} c_1 dK[3] + c_2\right]$$

2.5 problem 5

Internal problem ID [6689]

Book: Second order enumerated odes
Section: section 2
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$y''y' + y^2 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$2)*diff(y(x),x)+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \mathrm{e}^{\frac{\sqrt{3}\left(\int \tan\left(\mathrm{RootOf}\left(-\sqrt{3}\,\ln\left(\cos\left(\_Z\right)^2\right)-2\sqrt{3}\,\ln\left(\tan\left(\_Z\right)+\sqrt{3}\right)+6\sqrt{3}\,c_1+6\sqrt{3}\,x+6\_Z\right)\right)dx\right)}{2}+c_2+\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.117 (sec). Leaf size: 179

```
DSolve[y''[x]*y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{}{\sqrt[3]{1+\text{InverseFunction}\left[\frac{1}{6}\log\left(\#1^2-\#1+1\right)+\frac{\arctan\left(\frac{2\#1-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(\#1+1)\&\right][-x+c_1]}\sqrt[3]{1+}}$$

2.6 problem 6

Internal problem ID [6690]

Book: Second order enumerated odes
Section: section 2
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y''y' + y^n = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 174

```
dsolve(diff(y(x),x$2)*diff(y(x),x)+y(x)^n=0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{-\frac{\left((-3\_a^{1+n}+c_1)(1+n)^2\right)^{\frac{1}{3}}}{2(1+n)} - \frac{i\sqrt{3}\left((-3\_a^{1+n}+c_1)(1+n)^2\right)^{\frac{1}{3}}}{2(1+n)}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} \frac{1}{-\frac{\left((-3\_a^{1+n}+c_1)(1+n)^2\right)^{\frac{1}{3}}}{2(1+n)} + \frac{i\sqrt{3}\left((-3\_a^{1+n}+c_1)(1+n)^2\right)^{\frac{1}{3}}}{2+2n}} d\_a - x - c_2 = 0$$

$$\int^{y(x)} \frac{1+n}{\left((-3\_a^{1+n}+c_1)(1+n)^2\right)^{\frac{1}{3}}} d\_a - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.492 (sec). Leaf size: 298

```
DSolve[y''[x]*y'[x]+y[x]^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{\#1\sqrt[3]{n+1}\sqrt[3]{1-\frac{\#1^{n+1}}{c_1(n+1)}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n+1},1+\frac{1}{n+1},\frac{\#1^{n+1}}{(n+1)c_1}\right)}{\sqrt[3]{-3\#1^{n+1}+3c_1(n+1)}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{(-1)^{2/3}\#1\sqrt[3]{n+1}\sqrt[3]{1-\frac{\#1^{n+1}}{c_1(n+1)}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n+1},1+\frac{1}{n+1},\frac{\#1^{n+1}}{(n+1)c_1}\right)}{\sqrt[3]{-3\#1^{n+1}+3c_1(n+1)}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt[3]{-\frac{1}{3}}\#1\sqrt[3]{n+1}\sqrt[3]{1-\frac{\#1^{n+1}}{c_1(n+1)}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n+1},1+\frac{1}{n+1},\frac{\#1^{n+1}}{(n+1)c_1}\right)}{\sqrt[3]{-\#1^{n+1}+c_1(n+1)}}\&\right][x+c_2]$$

2.7 problem 8

Internal problem ID [6691]

Book: Second order enumerated odes
Section: section 2
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{y' - (x + y)^4 = 0}$$

✓ Solution by Maple

Time used: 0.532 (sec). Leaf size: 880

```
dsolve(diff(y(x), x) = (x + y(x))^4,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 88

```
DSolve[y'[x] == (x + y[x])^4,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{4}\text{RootSum}\left[\#1^4 + 4\#1^3 y(x) + 6\#1^2 y(x)^2 + 4\#1 y(x)^3 + y(x)^4 + 1\&, \frac{\log(x - \#1)}{\#1^3 + 3\#1^2 y(x) + 3\#1 y(x)^2 + y(x)^3}\&\right] - x = c_1, y(x)\right]$$

2.8 problem 9

Internal problem ID [6692]

Book: Second order enumerated odes
Section: section 2
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_x_y1], [_2nd_order, _`

$$y'' + (x+3)\, y' + \left(3 + y^2\right) y'^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+(3+x)*diff(y(x),x)+(3+y(x)^2)*(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$c_1 \operatorname{erf}\left(\frac{\sqrt{2}\, x}{2} + \frac{3\sqrt{2}}{2}\right) - c_2 + \int^{y(x)} \mathrm{e}^{\frac{1}{3}\_a^3 + 3\_a} d\_a = 0$$

✓ Solution by Mathematica

Time used: 0.19 (sec). Leaf size: 61

```
DSolve[y''[x]+(3+x)*y'[x]+(3+y[x]^2)*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} e^{\frac{K[1]^3}{3} + 3K[1]} dK[1]\&\right]\left[c_2 - e^{9/2}\sqrt{\frac{\pi}{2}} c_1 \operatorname{erf}\left(\frac{x+3}{\sqrt{2}}\right)\right]$$

2.9 problem 10

Internal problem ID [6693]

Book: Second order enumerated odes
Section: section 2
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_xy]]`

$$y'' + y'x + yy'^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)*(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$y(x) = -i \operatorname{RootOf}\left(i\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{2}\,x}{2}\right)c_1 + i\sqrt{2}\,c_2 - \operatorname{erf}(\_Z)\sqrt{\pi}\right)\sqrt{2}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 44

```
DSolve[y''[x]+x*y'[x]+y[x]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i\sqrt{2}\operatorname{erf}^{-1}\left(i\left(\sqrt{\frac{2}{\pi}}c_2 - c_1\operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right)\right)$$

2.10 problem 11

Internal problem ID [6694]

Book: Second order enumerated odes
Section: section 2
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], _Liouville, [_2nd_order, _reducible,`

$$y'' + y' \sin(x) + y'^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+sin(x)*diff(y(x),x)+(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$y(x) = \ln\left(c_1\left(\int e^{\cos(x)}dx\right) + c_2\right)$$

✓ Solution by Mathematica

Time used: 60.088 (sec). Leaf size: 43

```
DSolve[y''[x]+Sin[x]*y'[x]+(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \int_1^x \frac{e^{\cos(K[2])}}{c_1 - \int_1^{K[2]} -e^{\cos(K[1])}dK[1]}dK[2] + c_2$$

2.11 problem 12

Internal problem ID [6695]

Book: Second order enumerated odes
Section: section 2
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_x_y1], [_2nd_order, _`

$$3y'' + y' \cos(x) + \sin(y) y'^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(3*diff(y(x),x$2)+cos(x)*diff(y(x),x)+sin(y(x))*(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$\int^{y(x)} \mathrm{e}^{-\frac{\cos(\_a)}{3}} d\_a - c_1 \left(\int \mathrm{e}^{-\frac{\sin(x)}{3}} dx \right) - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.543 (sec). Leaf size: 47

```
DSolve[3*y''[x]+Cos[x]*y'[x]+Sin[y[x]]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} e^{-\frac{1}{3}\cos(K[2])} dK[2]\&\right] \left[\int_1^x -e^{-\frac{1}{3}\sin(K[3])} c_1 dK[3] + c_2\right]$$

2.12 problem 13

Internal problem ID [6696]

Book: Second order enumerated odes
Section: section 2
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _with_linear_symmetries], [_2nd_order`

$$10y'' + y'x^2 + \frac{3y'^2}{y} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 70

```
dsolve(10*diff(y(x),x$2)+x^2*diff(y(x),x)+3/y(x)*(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$\frac{10y(x)^{\frac{13}{10}}}{13} - \frac{xc_1 \operatorname{WhittakerM}\left(\frac{1}{6}, \frac{2}{3}, \frac{x^3}{30}\right) \mathrm{e}^{-\frac{x^3}{60}} 3^{\frac{1}{3}} 300000^{\frac{5}{6}}}{40000 \left(x^3\right)^{\frac{1}{6}}} - \frac{30c_1 \mathrm{e}^{-\frac{x^3}{60}} \operatorname{WhittakerM}\left(\frac{7}{6}, \frac{2}{3}, \frac{x^3}{30}\right) 30^{\frac{1}{6}}}{x^2 \left(x^3\right)^{\frac{1}{6}}} - c_2 = 0$$

✓ Solution by Mathematica

Time used: 6.562 (sec). Leaf size: 52

```
DSolve[10*y''[x]+x^2*y'[x]+3/y[x]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \exp\left(\int_1^x \frac{30e^{-\frac{1}{30}K[1]^3}}{30c_1 - 13\,\text{ExpIntegralE}\left(\frac{2}{3}, \frac{K[1]^3}{30}\right) K[1]} dK[1]\right)$$

2.13 problem 14

Internal problem ID [6697]

Book: Second order enumerated odes
Section: section 2
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_x_y1], [_2nd_order, _`

$$10y'' + (e^x + 3x)\, y' + \frac{3\, e^y y'^2}{\sin(y)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(10*diff(y(x),x$2)+(exp(x)+3*x)*diff(y(x),x)+3/sin(y(x))*exp(y(x))*(diff(y(x),x))^2=0,y
```

$$\int^{y(x)} e^{\int \frac{3\, e^{\_b}}{10 \sin(\_b)} d\_b} d\_b - c_1 \left(\int e^{-\frac{3x^2}{20} - \frac{e^x}{10}} dx \right) - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.142 (sec). Leaf size: 90

```
DSolve[10*y''[x]+(Exp[x]+3*x)*y'[x]+3/Sin[y[x]]*Exp[y[x]]*(y'[x])^2==0,y[x],x,IncludeSingular
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \exp\left(\left(-\frac{3}{10} - \frac{3i}{10} \right) e^{(1+i)K[2]} \text{Hypergeometric2F1}\left(\frac{1}{2} - \frac{i}{2}, 1, \frac{3}{2} - \frac{i}{2}, e^{2iK[2]} \right) \right) dK[2] - e^{\frac{1}{20}\left(-3K[3]^2 - 2e^{K[3]}\right)} c_1 dK[3] + c_2 \right]$$

2.14 problem 15

Internal problem ID [6698]

Book: Second order enumerated odes
Section: section 2
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - \frac{2y}{x^2} - x\,\mathrm{e}^{-\sqrt{x}} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(diff(y(x),x),x)-2/x^2*y(x) = x*exp(-x^(1/2)),y(x), singsol=all)
```

$$y(x) = \frac{c_2}{x} + x^2 c_1 + \frac{4\,\mathrm{e}^{-\sqrt{x}}\left(7x^{\frac{5}{2}} + 140x^{\frac{3}{2}} + x^3 + 35x^2 + 840\sqrt{x} + 420x + 840\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 53

```
DSolve[y''[x]-2/x^2*y[x] == x*Exp[-x^(1/2)],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3(c_2 x^3 + c_1) - 2e^{-\sqrt{x}}\left(x^{7/2} + x^3\right) + 2\Gamma\left(8, \sqrt{x}\right)}{3x}$$

2.15 problem 16

Internal problem ID [6699]

Book: Second order enumerated odes
Section: section 2
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - \frac{y'}{\sqrt{x}} + \frac{\left(x + \sqrt{x} - 8\right) y}{4x^2} - x = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 56

```
dsolve(diff(y(x),x$2)-1/sqrt(x)*diff(y(x),x)+1/(4*x^2)*(x+sqrt(x)-8)*y(x)=x,y(x), singsol=all
```

$$y(x) = \frac{e^{\sqrt{x}} c_2}{x} + e^{\sqrt{x}} x^2 c_1 + \frac{28x^{\frac{5}{2}} + 560x^{\frac{3}{2}} + 4x^3 + 140x^2 + 3360\sqrt{x} + 1680x + 3360}{x}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 52

```
DSolve[y''[x]-1/Sqrt[x]*y'[x]+1/(4*x^2)*(x+Sqrt[x]-8)*y[x]==x,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{-2\left(x^{7/2} + x^3\right) + e^{\sqrt{x}}\left(c_2 x^3 + 2\Gamma\left(8, \sqrt{x}\right) + 3c_1\right)}{3x}$$

2.16 problem 17

Internal problem ID [6700]

Book: Second order enumerated odes
Section: section 2
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$y'' + \frac{2y'}{x} + \frac{a^2 y}{x^4} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+2/x*diff(y(x),x)+a^2/x^4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\frac{a}{x}\right) + c_2 \cos\left(\frac{a}{x}\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 25

```
DSolve[y''[x]+2/x*y'[x]+a^2/x^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\frac{a}{x}\right) - c_2 \sin\left(\frac{a}{x}\right)$$

2.17 problem 18

Internal problem ID [6701]

Book: Second order enumerated odes
Section: section 2
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$(1 - x^2)\, y'' - y'x - c^2 y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)-c^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(x + \sqrt{x^2 - 1}\right)^{ic} + c_2 \left(x + \sqrt{x^2 - 1}\right)^{-ic}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 42

```
DSolve[(1-x^2)*y''[x]-x*y'[x]-c^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(c \operatorname{arctanh}\left(\frac{x}{\sqrt{x^2 - 1}}\right)\right) + c_2 \sin\left(c \operatorname{arctanh}\left(\frac{x}{\sqrt{x^2 - 1}}\right)\right)$$

2.18 problem 19

Internal problem ID [6702]

Book: Second order enumerated odes
Section: section 2
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^6 y'' + 3y' x^5 + a^2 y - \frac{1}{x^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x^6*diff(y(x),x$2)+3*x^5*diff(y(x),x)+a^2*y(x)=1/x^2,y(x), singsol=all)
```

$$y(x) = \sin\left(\frac{a}{2x^2}\right) c_2 + \cos\left(\frac{a}{2x^2}\right) c_1 + \frac{1}{a^2 x^2}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 38

```
DSolve[x^6*y''[x]+3*x^5*y'[x]+a^2*y[x]==1/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{a^2 x^2} + c_1 \cos\left(\frac{a}{2x^2}\right) - c_2 \sin\left(\frac{a}{2x^2}\right)$$

2.19 problem 20

Internal problem ID [6703]

Book: Second order enumerated odes
Section: section 2
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 - 3y'x + 3y - 2x^3 + x^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+3*y(x)=2*x^3-x^2,y(x), singsol=all)
```

$$y(x) = \left(x^2 \ln(x) - \frac{x^2}{2} + x + \frac{x^2 c_1}{2} + c_2\right) x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]-3*x*y'[x]+3*y[x]==2*x^3-x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(x^2 \log(x) + \left(-\frac{1}{2} + c_2\right) x^2 + x + c_1\right)$$

2.20 problem 21

Internal problem ID [6704]

Book: Second order enumerated odes
Section: section 2
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$y'' + \cot(x)\, y' + 4y \csc(x)^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+cot(x)*diff(y(x),x)+4*y(x)*csc(x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1 (\csc(x) + \cot(x))^{-2i} + c_2 (\csc(x) + \cot(x))^{2i}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 53

```
DSolve[y''[x]+Cot[x]*y'[x]+4*y[x]*Csc[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(2\left(\log\left(\cos\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right)\right)\right)\right) - c_2 \sin\left(2\left(\log\left(\cos\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right)\right)\right)\right)$$

2.21 problem 22

Internal problem ID [6705]

Book: Second order enumerated odes
Section: section 2
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(x^2+1)\,y''+(1+x)\,y'+y-4\cos(\ln(1+x))=0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 407

```
dsolve((1+x^2)*diff(y(x),x$2)+(1+x)*diff(y(x),x)+y(x)=4*cos(ln(1+x)),y(x), singsol=all)
```

$$y(x) = \text{hypergeom}\left([i,-i],\left[\frac{1}{2}+\frac{i}{2}\right],\frac{1}{2}-\frac{ix}{2}\right)c_2$$
$$+(x+i)^{\frac{1}{2}-\frac{i}{2}}\,\text{hypergeom}\left(\left[\frac{1}{2}+\frac{i}{2},\frac{1}{2}-\frac{3i}{2}\right],\left[\frac{3}{2}-\frac{i}{2}\right],\frac{1}{2}-\frac{ix}{2}\right)c_1$$
$$+80\left(\int\frac{\cos(\ln(x+1))\,(x}{7\left(\frac{10((1-i+(-1-i)x)\,\text{hypergeom}([1-i,1+i],[\frac{3}{2}+\frac{i}{2}],\frac{1}{2}-\frac{ix}{2})+(-1+i)\,\text{hypergeom}([i,-i],[\frac{1}{2}+\frac{i}{2}],\frac{1}{2}-\frac{ix}{2}))\,\text{hypergeom}([\frac{1}{2}+\frac{i}{2},\frac{1}{2}-\frac{3i}{2}}{7}\right.}\right.$$
$$-i],\left[\frac{1}{2}+\frac{i}{2}\right],\frac{1}{2}-\frac{ix}{2}\right)$$
$$-80\left(\int\frac{\text{hypergeom}([i,}{7\left(\frac{10((1-i+(-1-i)x)\,\text{hypergeom}([1-i,1+i],[\frac{3}{2}+\frac{i}{2}],\frac{1}{2}-\frac{ix}{2})+(-1+i)\,\text{hypergeom}([i,-i],[\frac{1}{2}+\frac{i}{2}],\frac{1}{2}-\frac{ix}{2}))\,\text{hypergeom}([\frac{1}{2}+\frac{i}{2},\frac{1}{2}-\frac{3i}{2}}{7}\right.}\right.$$
$$+i)^{\frac{1}{2}-\frac{i}{2}}\,\text{hypergeom}\left(\left[\frac{1}{2}+\frac{i}{2},\frac{1}{2}-\frac{3i}{2}\right],\left[\frac{3}{2}-\frac{i}{2}\right],\frac{1}{2}-\frac{ix}{2}\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1+x^2)*y''[x]+(1+x)*y'[x]+y[x]==4*Cos[Log[1+x]],y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

2.22 problem 23

Internal problem ID [6706]

Book: Second order enumerated odes
Section: section 2
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \tan(x)\, y' + \cos(x)^2\, y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+tan(x)*diff(y(x),x)+cos(x)^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(\sin(x)) + c_2 \cos(\sin(x))$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 18

```
DSolve[y''[x]+Tan[x]*y'[x]+Cos[x]^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sin(\sin(x)) + c_1 \cos(\sin(x))$$

2.23 problem 24

Internal problem ID [6707]

Book: Second order enumerated odes
Section: section 2
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$xy'' - y' + 4yx^3 - 8x^3 \sin(x)^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 124

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)+4*x^3*y(x)=8*x^3*sin(x)^2,y(x), singsol=all)
```

$$y(x) = \sin(x^2)\, c_2 + \cos(x^2)\, c_1 + 1 - \cos(2x) - \frac{\sqrt{\pi}\,\sqrt{2}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,(x-1)}{\sqrt{\pi}}\right)\sin(x^2+1)}{2} + \frac{\sqrt{\pi}\,\sqrt{2}\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,(x-1)}{\sqrt{\pi}}\right)\cos(x^2+1)}{2} + \frac{\sqrt{\pi}\,\sqrt{2}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,(x+1)}{\sqrt{\pi}}\right)\sin(x^2+1)}{2} - \frac{\sqrt{\pi}\,\sqrt{2}\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,(x+1)}{\sqrt{\pi}}\right)\cos(x^2+1)}{2}$$

✓ Solution by Mathematica

Time used: 0.492 (sec). Leaf size: 113

```
DSolve[x*y''[x]-y'[x]+4*x^3*y[x]==8*x^3*Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{\frac{\pi}{2}}\left(\left(\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}(x+1)\right) - \mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}(x-1)\right)\right)\sin(x^2+1) + \left(\mathrm{FresnelS}\left(\sqrt{\frac{2}{\pi}}(x-1)\right) - \mathrm{FresnelS}\left(\sqrt{\frac{2}{\pi}}(x+1)\right)\right)\cos(x^2+1)\right) + c_1\cos(x^2) + c_2\sin(x^2) + 2\sin^2(x)$$

2.24 problem 25

Internal problem ID [6708]

Book: Second order enumerated odes
Section: section 2
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$xy'' - y' + 4yx^3 - x^5 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)+4*x^3*y(x)=x^5,y(x), singsol=all)
```

$$y(x) = \sin\left(x^2\right) c_2 + \cos\left(x^2\right) c_1 + \frac{x^2}{4}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 27

```
DSolve[x*y''[x]-y'[x]+4*x^3*y[x]==x^5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{4} + c_1 \cos\left(x^2\right) + c_2 \sin\left(x^2\right)$$

2.25 problem 25

Internal problem ID [6709]

Book: Second order enumerated odes
Section: section 2
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\cos(x) y'' + y' \sin(x) - 2y \cos(x)^3 - 2\cos(x)^5 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(cos(x)*diff(y(x),x$2)+sin(x)*diff(y(x),x)-2*y(x)*cos(x)^3=2*cos(x)^5,y(x), singsol=all
```

$$y(x) = \sinh\left(\sin(x)\sqrt{2}\right) c_2 + \cosh\left(\sin(x)\sqrt{2}\right) c_1 + \frac{1}{2} - \frac{\cos(2x)}{2}$$

✓ Solution by Mathematica

Time used: 8.575 (sec). Leaf size: 132

```
DSolve[Cos[x]*y''[x]+Sin[x]*y'[x]-2*y[x]*Cos[x]^3==2*Cos[x]^5,y[x],x,IncludeSingularSolutions
```

$$\begin{aligned} y(x) \\ \to \sin\left(\sqrt{-\cos(2x)-1}\tan(x)\right)\left(\int_1^x \sqrt{2}(-\cos^2(K[2]))^{3/2}\cos\left(\sqrt{-\cos(2K[2])-1}\tan(K[2])\right)dK[2] \right. \\ \left. + c_2\right) \\ +\cos\left(\sqrt{-\cos(2x)-1}\tan(x)\right)\left(\int_1^x \cos^2(K[1])\sqrt{-\cos(2K[1])-1}\sin\left(\sqrt{-\cos(2K[1])-1}\tan(K[1])\right) \right. \\ \left. + c_1\right) \end{aligned}$$

2.26 problem 26

Internal problem ID [6710]

Book: Second order enumerated odes
Section: section 2
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + \left(1 - \frac{1}{x}\right) y' + 4x^2 y\,\mathrm{e}^{-2x} - 4\left(x^3 + x^2\right)\mathrm{e}^{-3x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+(1-1/x)*diff(y(x),x)+4*x^2*y(x)*exp(-2*x)=4*(x^2+x^3)*exp(-3*x),y(x), s
```

$$y(x) = \sin\left(2(x+1)\,\mathrm{e}^{-x}\right) c_2 + \cos\left(2(x+1)\,\mathrm{e}^{-x}\right) c_1 + (x+1)\,\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.291 (sec). Leaf size: 44

```
DSolve[y''[x]+(1-1/x)*y'[x]+4*x^2*y[x]*Exp[-2*x]==4*(x^2+x^3)*Exp[-3*x],y[x],x,IncludeSingula
```

$$y(x) \to e^{-x}(x+1) + c_1 \cos\left(2e^{-x}(x+1)\right) - c_2 \sin\left(2e^{-x}(x+1)\right)$$

2.27 problem 27

Internal problem ID [6711]

Book: Second order enumerated odes
Section: section 2
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y'x^2 + yx - x^{m+1} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 207

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)+x*y(x)=x^(m+1),y(x), singsol=all)
```

$$y(x) = c_2 x + \frac{\left(-3^{\frac{1}{3}}(-x^3)^{\frac{2}{3}} \mathrm{e}^{\frac{x^3}{3}} + x^3\left(\Gamma\left(\frac{2}{3}\right) - \Gamma\left(\frac{2}{3}, -\frac{x^3}{3}\right)\right)\right) c_1}{x^2} + \frac{x(m+3)\left(\int x^{m+1}\left((-x^3)^{\frac{1}{3}} 3^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right) \mathrm{e}^{-\frac{x^3}{3}} - (-x^3)^{\frac{1}{3}} 3^{\frac{2}{3}}\Gamma\left(\frac{2}{3}, -\frac{x^3}{3}\right) \mathrm{e}^{-\frac{x^3}{3}} + 3\right) dx\right) + \mathrm{WhittakerM}\left(\frac{m}{6}, \frac{m}{6}\right.}{3m+9}$$

✓ Solution by Mathematica

Time used: 0.193 (sec). Leaf size: 90

```
DSolve[y''[x]-x^2*y'[x]+x*y[x]==x^(m+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \int_1^x \frac{1}{3} e^{-\frac{1}{3}K[1]^3} \mathrm{ExpIntegralE}\left(\frac{4}{3}, -\frac{1}{3}K[1]^3\right) K[1]^{m+1} dK[1] + \frac{1}{9} \mathrm{ExpIntegralE}\left(\frac{4}{3}, -\frac{x^3}{3}\right)\left(x^{m+3}\,\mathrm{ExpIntegralE}\left(-\frac{m}{3}, \frac{x^3}{3}\right) - 3c_2\right) + c_1 x$$

2.28 problem 28

Internal problem ID [6712]

Book: Second order enumerated odes
Section: section 2
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{y'}{\sqrt{x}} + \frac{\left(x + \sqrt{x} - 8\right) y}{4x^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-1/x^(1/2)*diff(y(x),x)+y(x)/(4*x^2)*(-8+x^(1/2)+x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{\sqrt{x}}}{x} + c_2 e^{\sqrt{x}} x^2$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 30

```
DSolve[y''[x]-1/x^(1/2)*y'[x]+y[x]/(4*x^2)*(-8+x^(1/2)+x)==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^{\sqrt{x}}(c_2 x^3 + 3c_1)}{3x}$$

2.29 problem 29

Internal problem ID [6713]

Book: Second order enumerated odes
Section: section 2
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{\cos(x)^2 y'' - 2\cos(x)\sin(x)\, y' + \cos(x)^2 y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(cos(x)^2*diff(y(x),x$2)-2*cos(x)*sin(x)*diff(y(x),x)+y(x)*cos(x)^2=0,y(x), singsol=all
```

$$y(x) = c_1 \sec(x)\sin\left(\sqrt{2}\, x\right) + c_2 \sec(x)\cos\left(\sqrt{2}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 51

```
DSolve[Cos[x]^2*y''[x]-2*Cos[x]*Sin[x]*y'[x]+y[x]*Cos[x]^2==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{4} e^{-i\sqrt{2}x}\left(4c_1 - i\sqrt{2}c_2 e^{2i\sqrt{2}x}\right)\sec(x)$$

2.30 problem 30

Internal problem ID [6714]

Book: Second order enumerated odes
Section: section 2
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y'x + (4x^2 - 1)\, y + 3\,\mathrm{e}^{x^2} \sin(x) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-1)*y(x)=-3*exp(x^2)*sin(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x^2} \cos(x)\, c_2 + \mathrm{e}^{x^2} \sin(x)\, c_1 - \frac{3\,\mathrm{e}^{x^2} (-\cos(x)\, x + \sin(x))}{2}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 47

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-1)*y[x]==-3*Exp[x^2]*Sin[x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{4} e^{x^2} ((6x + 4c_1 - 2ic_2) \cos(x) + (-3 - 4ic_1 + 2c_2) \sin(x))$$

2.31 problem 31

Internal problem ID [6715]

Book: Second order enumerated odes
Section: section 2
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2bxy' + b^2x^2y - x = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 116

```
dsolve(diff(y(x),x$2)-2*b*x*diff(y(x),x)+b^2*x^2*y(x)=x,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x(xb+2\sqrt{-b})}{2}} c_2 + \mathrm{e}^{\frac{x(xb-2\sqrt{-b})}{2}} c_1 + \frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-\frac{1}{2}+\frac{b\,x^2}{2}-x\sqrt{-b}}\left(-\operatorname{erf}\left(\frac{\sqrt{2}\,(xb+\sqrt{-b})}{2\sqrt{b}}\right)\mathrm{e}^{2x\sqrt{-b}} + \operatorname{erf}\left(\frac{\sqrt{2}\,(-xb+\sqrt{-b})}{2\sqrt{b}}\right)\right)}{4b^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.186 (sec). Leaf size: 116

```
DSolve[y''[x]-2*b*x*y'[x]+b^2*x^2*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\frac{bx^2}{2}-i\sqrt{b}x}\left(4b^{3/2}c_1 - 2ibc_2e^{2i\sqrt{b}x}\right) - 2i\sqrt{2}\left(\operatorname{DawsonF}\left(\frac{1-i\sqrt{b}x}{\sqrt{2}}\right) - \operatorname{DawsonF}\left(\frac{i\sqrt{b}x+1}{\sqrt{2}}\right)\right)}{4b^{3/2}}$$

2.32 problem 32

Internal problem ID [6716]

Book: Second order enumerated odes
Section: section 2
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y'x + (4x^2 - 3)\, y - e^{x^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-3)*y(x)=exp(x^2),y(x), singsol=all)
```

$$y(x) = e^{x(x+1)} c_2 + e^{x(x-1)} c_1 - e^{x^2}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 33

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-3)*y[x]==Exp[x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{(x-1)x} (e^x(-2 + c_2 e^x) + 2c_1)$$

2.33 problem 33

Internal problem ID [6717]

Book: Second order enumerated odes
Section: section 2
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2\tan(x)\, y' + 5y - e^{x^2}\sec(x) = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 96

```
dsolve(diff(y(x),x$2)-2*tan(x)*diff(y(x),x)+5*y(x)=exp(x^2)*sec(x),y(x), singsol=all)
```

$$y(x) = \sec(x)\sin\left(x\sqrt{6}\right) c_2 + \sec(x)\cos\left(x\sqrt{6}\right) c_1 - \frac{\left(\left(i\sin\left(x\sqrt{6}\right) - \cos\left(x\sqrt{6}\right)\right)\operatorname{erf}\left(ix - \frac{\sqrt{6}}{2}\right) + \left(i\sin\left(x\sqrt{6}\right) + \cos\left(x\sqrt{6}\right)\right)\operatorname{erf}\left(ix + \frac{\sqrt{6}}{2}\right)\right)\sec(x)\sqrt{6}\,\sqrt{\pi}}{24}$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 107

```
DSolve[y''[x]-2*Tan[x]*y'[x]+5*y[x]==Exp[x^2]*Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{24}e^{-i\sqrt{6}x}\sec(x)\left(-e^{3/2}\sqrt{6\pi}\left(\operatorname{erf}\left(\sqrt{\frac{3}{2}} - ix\right) + e^{2i\sqrt{6}x}\operatorname{erf}\left(\sqrt{\frac{3}{2}} + ix\right)\right) - 2i\sqrt{6}c_2e^{2i\sqrt{6}x} + 24c_1\right)$$

2.34 problem 34

Internal problem ID [6718]

Book: Second order enumerated odes
Section: section 2
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 - 2y'x + 2(x^2 + 1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*(1+x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \sin\left(\sqrt{2}\,x\right) + c_2 x \cos\left(\sqrt{2}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 48

```
DSolve[x^2*y''[x]-2*x*y'[x]+2*(1+x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-i\sqrt{2}x} x - \frac{i c_2 e^{i\sqrt{2}x} x}{2\sqrt{2}}$$

2.35 problem 35

Internal problem ID [6719]

Book: Second order enumerated odes
Section: section 2
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4y''x^2 + 4y'x^5 + (x^8 + 6x^4 + 4)\, y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve(4*x^2*diff(y(x),x$2)+4*x^5*diff(y(x),x)+(x^8+6*x^4+4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{1}{2}+\frac{i\sqrt{3}}{2}} \mathrm{e}^{-\frac{x^4}{8}} + c_2 x^{\frac{1}{2}-\frac{i\sqrt{3}}{2}} \mathrm{e}^{-\frac{x^4}{8}}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 62

```
DSolve[4*x^2*y''[x]+4*x^5*y'[x]+(x^8+6*x^4+4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{3} e^{-\frac{x^4}{8}} x^{\frac{1}{2}-\frac{i\sqrt{3}}{2}} \left(3c_1 - i\sqrt{3} c_2 x^{i\sqrt{3}}\right)$$

2.36 problem 36

Internal problem ID [6720]

Book: Second order enumerated odes
Section: section 2
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y''x^2 + (y'x - y)^2 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)+(x*diff(y(x),x)-y(x))^2=0,y(x), singsol=all)
```

$$y(x) = \left(-\mathrm{e}^{c_1} \operatorname{Ei}_1\left(-\ln\left(\frac{1}{x}\right) + c_1\right) + c_2\right) x$$

✓ Solution by Mathematica

Time used: 40.59 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]+(x*y'[x]-y[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(e^{c_1} \operatorname{ExpIntegralEi}(-c_1 - \log(x)) + c_2)$$

2.37 problem 37

Internal problem ID [6721]

Book: Second order enumerated odes
Section: section 2
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 2y' - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh(x)}{x} + \frac{c_2 \cosh(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 28

```
DSolve[x*y''[x]+2*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-x} + c_2 e^x}{2x}$$

2.38 problem 38

Internal problem ID [6722]

Book: Second order enumerated odes
Section: section 2
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x)}{x} + \frac{c_2 \cos(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-ix} - ic_2 e^{ix}}{2x}$$

2.39 problem 39

Internal problem ID [6723]

Book: Second order enumerated odes
Section: section 2
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cot(x) - 2\cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+y(x)*cot(x)=2*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{-\frac{\cos(2x)}{2} + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 17

```
DSolve[y'[x]+y[x]*Cot[x]==2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + \left(-\frac{1}{2} + c_1\right)\csc(x)$$

2.40 problem 40

Internal problem ID [6724]

Book: Second order enumerated odes
Section: section 2
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{2y^2x - y + (y^2 + x + y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 28

```
dsolve((2*x*y(x)^2-y(x))+(y(x)^2+x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^{\mathrm{RootOf}\left(x^2 e^{\_Z} + e^{2\_Z} + c_1 e^{\_Z} + \_Z e^{\_Z} - x\right)}$$

✓ Solution by Mathematica

Time used: 0.178 (sec). Leaf size: 22

```
DSolve[(2*x*y[x]^2-y[x])+(y[x]^2+x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[x^2 - \frac{x}{y(x)} + y(x) + \log(y(x)) = c_1, y(x)\right]$$

2.41 problem 41

Internal problem ID [6725]

Book: Second order enumerated odes
Section: section 2
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - x + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)=x-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{AiryAi}(1, x) + \operatorname{AiryBi}(1, x)}{c_1 \operatorname{AiryAi}(x) + \operatorname{AiryBi}(x)}$$

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 118

```
DSolve[y'[x]==x-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{i\sqrt{x}\left(\operatorname{BesselJ}\left(-\frac{2}{3}, \frac{2}{3}ix^{3/2}\right) - c_1 \operatorname{BesselJ}\left(\frac{2}{3}, \frac{2}{3}ix^{3/2}\right)\right)}{\operatorname{BesselJ}\left(\frac{1}{3}, \frac{2}{3}ix^{3/2}\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{3}, \frac{2}{3}ix^{3/2}\right)}$$

$$y(x) \to \frac{3 \operatorname{AiryAiPrime}(x) + \sqrt{3} \operatorname{AiryBiPrime}(x)}{3 \operatorname{AiryAi}(x) + \sqrt{3} \operatorname{AiryBi}(x)}$$

2.42 problem 42

Internal problem ID [6726]

Book: Second order enumerated odes
Section: section 2
Problem number: 42.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - y''' - 3y'' + 5y' - 2y - x\,\mathrm{e}^{x} - 3\,\mathrm{e}^{-2x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 78

```
dsolve(diff(y(x),x$4)-diff(y(x),x$3)-3*diff(y(x),x$2)+5*diff(y(x),x)-2*y(x)=x*exp(x)+3*exp(-2
```

$$y(x) = -\frac{\mathrm{e}^{-2x}\left(-27x^{4}\mathrm{e}^{3x} + 36x^{3}\mathrm{e}^{3x} + 24\,\mathrm{e}^{3x}x - 36\,\mathrm{e}^{3x}x^{2} - 8\,\mathrm{e}^{3x} + 216x + 216\right)}{1944} + c_{1}\mathrm{e}^{x} + c_{2}\mathrm{e}^{-2x} + c_{3}x\,\mathrm{e}^{x} + c_{4}\mathrm{e}^{x}x^{2}$$

✓ Solution by Mathematica

Time used: 0.216 (sec). Leaf size: 59

```
DSolve[y''''[x]-y'''[x]-3*y''[x]+5*y'[x]-2*y[x]==x*Exp[x]+3*Exp[-2*x],y[x],x,IncludeSingularS
```

$$y(x) \to e^{x}\left(\frac{1}{648}x(3x(x(3x-4)+4+216c_{4})-8+648c_{3})+\frac{1}{243}+c_{2}\right)-\frac{1}{9}e^{-2x}(x+1-9c_{1})$$

2.43 problem 43

Internal problem ID [6727]

Book: Second order enumerated odes
Section: section 2
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 - x(x+6)\,y' + 10y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 65

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(x+6)*diff(y(x),x)+10*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^2\left(c_1x^3\left(1+\frac{5}{4}x+\frac{3}{4}x^2+\frac{7}{24}x^3+\frac{1}{12}x^4+\frac{3}{160}x^5+\mathrm{O}\left(x^6\right)\right)\right.$$
$$+ c_2\left(\ln\left(x\right)\left(24x^3+30x^4+18x^5+\mathrm{O}\left(x^6\right)\right)\right.$$
$$\left.\left.+\left(12-12x+18x^2+26x^3+x^4-9x^5+\mathrm{O}\left(x^6\right)\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 84

```
AsymptoticDSolveValue[x^2*y''[x]-x*(x+6)*y'[x]+10*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{2}x^5(5x+4)\log(x)-\frac{1}{4}x^2\left(3x^4-6x^3-6x^2+4x-4\right)\right)$$
$$+c_2\left(\frac{x^9}{12}+\frac{7x^8}{24}+\frac{3x^7}{4}+\frac{5x^6}{4}+x^5\right)$$

2.44 problem 44

Internal problem ID [6728]

Book: Second order enumerated odes
Section: section 2
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{y''x^2 + y'x + (x^2 - 5)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 97

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-5)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-\sqrt{5}} \left(1 + \frac{1}{-4 + 4\sqrt{5}} x^2 + \frac{1}{32} \frac{1}{(-2 + \sqrt{5})(\sqrt{5} - 1)} x^4 + \mathrm{O}(x^6)\right) + c_2 x^{\sqrt{5}} \left(1 - \frac{1}{4 + 4\sqrt{5}} x^2 + \frac{1}{32} \frac{1}{(\sqrt{5} + 2)(\sqrt{5} + 1)} x^4 + \mathrm{O}(x^6)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 210

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-5)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^4}{\left(-3-\sqrt{5}+\left(1-\sqrt{5}\right)\left(2-\sqrt{5}\right)\right)\left(-1-\sqrt{5}+\left(3-\sqrt{5}\right)\left(4-\sqrt{5}\right)\right)} - \frac{x^2}{-3-\sqrt{5}+\left(1-\sqrt{5}\right)\left(2-\sqrt{5}\right)}+1\right)x^{-\sqrt{5}} + c_1\left(\frac{x^4}{\left(-3+\sqrt{5}+\left(1+\sqrt{5}\right)\left(2+\sqrt{5}\right)\right)\left(-1+\sqrt{5}+\left(3+\sqrt{5}\right)\left(4+\sqrt{5}\right)\right)} - \frac{x^2}{-3+\sqrt{5}+\left(1+\sqrt{5}\right)\left(2+\sqrt{5}\right)}+1\right)x^{\sqrt{5}}$$

2.45 problem 45

Internal problem ID [6729]

Book: Second order enumerated odes
Section: section 2
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{y''x^2 + y'x + (x^2 - 5)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-5)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(\sqrt{5}, x\right) + c_2 \operatorname{BesselY}\left(\sqrt{5}, x\right)$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-5)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(\sqrt{5}, x\right) + c_2 Y_{\sqrt{5}}(x)$$

2.46 problem 46

Internal problem ID [6730]

Book: Second order enumerated odes
Section: section 2
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, ‘_with_symmetry_[0,F(`

$$y''x^2 - 4y'x + 6y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_2x^3 + x^2c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(c_2x + c_1)$$

2.47 problem 47

Internal problem ID [6731]

Book: Second order enumerated odes
Section: section 2
Problem number: 47.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$3)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\mathrm{hypergeom}\left([\,], \left[\frac{1}{2}, \frac{3}{4}\right], \frac{x^4}{64}\right) + c_2 x \,\mathrm{hypergeom}\left([\,], \left[\frac{3}{4}, \frac{5}{4}\right], \frac{x^4}{64}\right) + c_3 x^2 \,\mathrm{hypergeom}\left([\,], \left[\frac{5}{4}, \frac{3}{2}\right], \frac{x^4}{64}\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 76

```
DSolve[y'''[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \, {}_0F_2\left(;\frac{1}{2}, \frac{3}{4}; \frac{x^4}{64}\right) + \frac{1}{8} x \left((2+2i) c_2 \, {}_0F_2\left(;\frac{3}{4}, \frac{5}{4}; \frac{x^4}{64}\right) + i c_3 x \, {}_0F_2\left(;\frac{5}{4}, \frac{3}{2}; \frac{x^4}{64}\right)\right)$$

2.48 problem 48

Internal problem ID [6732]

Book: Second order enumerated odes
Section: section 2
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^{\frac{1}{3}} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=y(x)^(1/3),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 21

```
DSolve[{y'[x]==y[x]^(1/3),{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3}\sqrt{\frac{2}{3}}x^{3/2}$$

2.49 problem 49

Internal problem ID [6733]

Book: Second order enumerated odes
Section: section 2
Problem number: 49.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 3x(t) + y(t)$$
$$y'(t) = -x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 30

```
dsolve([diff(x(t),t)=3*x(t)+y(t),diff(y(t),t)=-x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -e^{2t}(c_2t + c_1 + c_2)$$

$$y(t) = e^{2t}(c_2t + c_1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[{x'[t]==3*x[t]+y[t],y'[t]==-x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{2t}(c_1(t+1) + c_2t)$$
$$y(t) \to e^{2t}(c_2 - (c_1 + c_2)t)$$