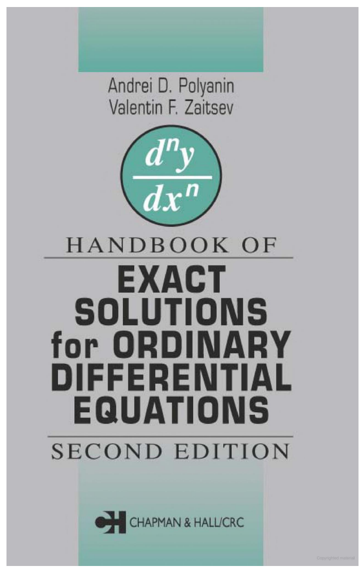
A Solution Manual For

Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition



Nasser M. Abbasi

October 12, 2023

Contents

1 Chapter 1, First-Order differential equations

1.1 problem 1.1.1

Internal problem ID [9578]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - f(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=f(x),y(x), singsol=all)
```

$$y(x) = \int f(x)\,dx + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[y'[x]==f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \int_1^x f(K[1])dK[1] + c_1$$

1.2 problem 1.1.2

Internal problem ID [9579]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - f(y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=f(y(x)),y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{1}{f(\_a)} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.296 (sec). Leaf size: 33

```
DSolve[y'[x]==f[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{f(K[1])} dK[1]\&\right][x + c_1]$$

$$y(x) \to f^{(-1)}(0)$$

1.3 problem 1.1.3

Internal problem ID [9580]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - f(x)\,g(y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)=f(x)*g(y(x)),y(x), singsol=all)
```

$$\int f(x)\,dx - \left(\int^{y(x)} \frac{1}{g\left(\_a\right)} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.303 (sec). Leaf size: 42

```
DSolve[y'[x]==f[x]*g[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{g(K[1])} dK[1]\&\right]\left[\int_1^x f(K[2])dK[2] + c_1\right]$$

$$y(x) \to g^{(-1)}(0)$$

1.4 problem 1.1.4

Internal problem ID [9581]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{g(x)\, y' - f_1(x)\, y - f_0(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(g(x)*diff(y(x),x)=f__1(x)*y(x)+f__0(x),y(x), singsol=all)
```

$$y(x) = \left(\int \frac{f_0(x)\, \mathrm{e}^{-\left(\int \frac{f_1(x)}{g(x)}dx\right)}}{g\left(x\right)}dx + c_1\right) \mathrm{e}^{\int \frac{f_1(x)}{g(x)}dx}$$

✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 64

```
DSolve[g[x]*y'[x]==f1[x]*y[x]+f0[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(\int_1^x \frac{\mathrm{f1}(K[1])}{g(K[1])}dK[1]\right) \left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} \frac{\mathrm{f1}(K[1])}{g(K[1])}dK[1]\right) \mathrm{f0}(K[2])}{g(K[2])}dK[2] + c_1\right)$$

1.5 problem 1.1.5

Internal problem ID [9582]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{g(x)\,y' - f_1(x)\,y - f_n(x)\,y^n = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 119

```
dsolve(g(x)*diff(y(x),x)=f__1(x)*y(x)+f__n(x)*y(x)^n,y(x), singsol=all)
```

$$y(x) = \left(\int \left(-\frac{n\,e^{\int \left(\frac{f_1(x)n}{g(x)}-\frac{f_1(x)}{g(x)}\right)dx} f_n(x)}{g(x)} + \frac{e^{\int \left(\frac{f_1(x)n}{g(x)}-\frac{f_1(x)}{g(x)}\right)dx} f_n(x)}{g(x)}\right) dx + c_1\right)^{-\frac{1}{n-1}} e^{\frac{\left(\int \frac{f_1(x)}{g(x)}dx\right)n}{n-1}} e^{\int -\frac{f_1(x)}{(n-1)g(x)}dx}$$

✓ Solution by Mathematica

Time used: 10.848 (sec). Leaf size: 84

```
DSolve[g[x]*y'[x]==f1[x]*y[x]+fn[x]*y[x]^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\exp\left(-\left((n-1)\int_1^x \frac{\text{f1}(K[1])}{g(K[1])}dK[1]\right)\right)\left(-(n-1)\int_1^x \frac{\exp\left((n-1)\int_1^{K[2]} \frac{\text{f1}(K[1])}{g(K[1])}dK[1]\right)\text{fn}(K[2])}{g(K[2])}dK[2] + c_1\right)\right)^{\frac{1}{1-n}}$$

1.6 problem 1.1.6

Internal problem ID [9583]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y' - f\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)=f(y(x)/x),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\int^{\_Z} \frac{1}{-f(\_a) + \_a} d\_a + \ln(x) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 33

```
DSolve[y'[x]==f[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$\operatorname{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{1}{K[1] - f(K[1])} dK[1] = -\log(x) + c_1, y(x)\right]$$

2 Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions

2.1 problem 1

Internal problem ID [9584]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 a - bx - c = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=a*y(x)^2+b*x+c,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{b}{\sqrt{a}}\right)^{\frac{1}{3}}\left(\operatorname{AiryAi}\left(1, -\frac{xb+c}{\left(\frac{b}{\sqrt{a}}\right)^{\frac{2}{3}}}\right)c_1 + \operatorname{AiryBi}\left(1, -\frac{xb+c}{\left(\frac{b}{\sqrt{a}}\right)^{\frac{2}{3}}}\right)\right)}{\sqrt{a}\left(c_1 \operatorname{AiryAi}\left(-\frac{xb+c}{\left(\frac{b}{\sqrt{a}}\right)^{\frac{2}{3}}}\right) + \operatorname{AiryBi}\left(-\frac{xb+c}{\left(\frac{b}{\sqrt{a}}\right)^{\frac{2}{3}}}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.188 (sec). Leaf size: 143

```
DSolve[y'[x]==a*y[x]^2+b*x+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b\left(\operatorname{AiryBiPrime}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right) + c_1 \operatorname{AiryAiPrime}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right)\right)}{(-ab)^{2/3}\left(\operatorname{AiryBi}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right) + c_1 \operatorname{AiryAi}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right)\right)}$$

$$y(x) \to \frac{b \operatorname{AiryAiPrime}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right)}{(-ab)^{2/3} \operatorname{AiryAi}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right)}$$

2.2 problem 2

Internal problem ID [9585]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 + a^2x^2 - 3a = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 108

```
dsolve(diff(y(x),x)=y(x)^2-a^2*x^2+3*a,y(x), singsol=all)
```

$$y(x) = \frac{e^{\frac{a\,x^2}{2}} c_1 a x + \left(\left(-c_1 x^2 \sqrt{\pi}\,(-a)^{\frac{3}{2}} - \sqrt{\pi}\,\sqrt{-a}\,c_1\right) \operatorname{erf}\left(x\sqrt{-a}\right) + a\,x^2 - 1\right) e^{-\frac{a\,x^2}{2}}}{e^{\frac{a\,x^2}{2}} c_1 + \left(\operatorname{erf}\left(x\sqrt{-a}\right)\sqrt{\pi}\,\sqrt{-a}\,c_1 x + x\right) e^{-\frac{a\,x^2}{2}}}$$

✓ Solution by Mathematica

Time used: 0.491 (sec). Leaf size: 94

```
DSolve[y'[x]==y[x]^2-a^2*x^2+3*a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to ax + \frac{\sqrt{a}\left(\sqrt{\pi}\left(\operatorname{erfi}\left(\sqrt{a}x\right) + i\right) - \sqrt{2}c_1\right)}{2e^{ax^2}\,\text{HermiteH}\left(-2, i\sqrt{a}x\right) + \sqrt{2}\sqrt{a}c_1 x}$$

$$y(x) \to ax - \frac{1}{x}$$

2.3 problem 3

Internal problem ID [9586]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - a^2x^2 - bx - c = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 688

```
dsolve(diff(y(x),x)=y(x)^2+a^2*x^2+b*x+c,y(x), singsol=all)
```

$$y(x) = -\frac{(48a^7c_1x^2i - 16a^6cc_1x^2 + 48a^5bc_1xi + 4a^4b^2c_1x^2 - 16a^4bcc_1x + 12a^3b^2c_1i + 4a^2b^3c_1x - 4a^2b^2cc_1 + b^4c_1}{24a^4\left((2a^2c_1x + c_1b)\,\text{hypergeom}\left(\left[\frac{4ia^2c+12a^3-ib^2}{16a^3}\right], \left[\frac{3}{2}\right], \frac{i(2a^2x+b)^2}{4a^3}\right) + \text{hypergeom}\left(\left[\frac{4ia}{}\right.\right.\right.}$$

$$-\frac{(24ia^5x - 24a^4cx + 12ia^3b + 6a^2b^2x - 12a^2bc + 3b^3)\,\text{hypergeom}\left(\left[\frac{4ia^2c+20a^3-ib^2}{16a^3}\right], \left[\frac{3}{2}\right], \frac{i(2a^2x+b}{4a^3}\right.}{24a^4\left((2a^2c_1x + c_1b)\,\text{hypergeom}\left(\left[\frac{4ia^2c+12a^3-ib^2}{16a^3}\right], \left[\frac{3}{2}\right], \frac{i(2a^2x+b)^2}{4a^3}\right) + \text{hypergeom}\left(\left[\frac{4ia^2c+4a^3-ib^2}{16a^3}\right], \left[\frac{1}{2}\right], \right.\right.}$$

$$-\frac{(-48a^7c_1x^2i - 48a^5bc_1xi + 48a^6c_1 - 12a^3b^2c_1i)\,\text{hypergeom}\left(\left[\frac{4ia^2c+12a^3-ib^2}{16a^3}\right], \left[\frac{3}{2}\right], \frac{i(2a^2x+b)^2}{4a^3}\right) + (-24ia}{24a^4\left((2a^2c_1x + c_1b)\,\text{hypergeom}\left(\left[\frac{4ia^2c+12a^3-ib^2}{16a^3}\right], \left[\frac{3}{2}\right], \frac{i(2a^2x+b)^2}{4a^3}\right) + \text{hypergeom}\right.}$$

✓ Solution by Mathematica

Time used: 0.946 (sec). Leaf size: 602

```
DSolve[y'[x]==y[x]^2+a^2*x^2+b*x+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4(-1)^{3/4}a^{3/2}\,\text{ParabolicCylinderD}\left(\frac{1}{8}\left(4-\frac{i(b^2-4a^2c)}{a^3}\right),-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)(2xa^2+b)}{a^{3/2}}\right)+i\sqrt{2}(2a^2x+b)\,\text{ParabolicCylind}}{2\sqrt{2}a\left(\text{ParabolicCyli}\right.}$$

$$y(x) \to \frac{(1+i)\sqrt{a}\,\text{ParabolicCylinderD}\left(\frac{1}{8}\left(\frac{i(b^2-4a^2c)}{a^3}+4\right),\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(2xa^2+b)}{a^{3/2}}\right)}{\text{ParabolicCylinderD}\left(\frac{1}{8}\left(\frac{i(b^2-4a^2c)}{a^3}-4\right),\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(2xa^2+b)}{a^{3/2}}\right)}-\frac{i(2a^2x+b)}{2a}$$

$$y(x) \to \frac{(1+i)\sqrt{a}\,\text{ParabolicCylinderD}\left(\frac{1}{8}\left(\frac{i(b^2-4a^2c)}{a^3}+4\right),\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(2xa^2+b)}{a^{3/2}}\right)}{\text{ParabolicCylinderD}\left(\frac{1}{8}\left(\frac{i(b^2-4a^2c)}{a^3}-4\right),\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(2xa^2+b)}{a^{3/2}}\right)}-\frac{i(2a^2x+b)}{2a}$$

2.4 problem 4

Internal problem ID [9587]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - y^2 a - b\,x^n = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 207

```
dsolve(diff(y(x),x)=a*y(x)^2+b*x^n,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{ba}\,x^{\frac{n}{2}+1}\,\mathrm{BesselJ}\left(\frac{n+3}{n+2}, \frac{2\sqrt{ba}\,x^{\frac{n}{2}+1}}{n+2}\right) c_1 + \mathrm{BesselY}\left(\frac{n+3}{n+2}, \frac{2\sqrt{ba}\,x^{\frac{n}{2}+1}}{n+2}\right)\sqrt{ba}\,x^{\frac{n}{2}+1} - c_1\,\mathrm{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{ba}\,x^{\frac{n}{2}+1}}{n+2}\right)}{xa\left(c_1\,\mathrm{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{ba}\,x^{\frac{n}{2}+1}}{n+2}\right) + \mathrm{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{ba}\,x^{\frac{n}{2}+1}}{n+2}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.416 (sec). Leaf size: 433

```
DSolve[y'[x]==a*y[x]^2+b*x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}\left(-2\,\text{Gamma}\left(1+\frac{1}{n+2}\right)\text{BesselJ}\left(\frac{1}{n+2}-1,\frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right)+c_1\,\text{Gamma}\left(\frac{n+1}{n+2}\right)\left(\text{BesselJ}\left(\frac{n+1}{n+2},\frac{2\sqrt{a}\cdots\right.\right.\right.}{2ax\left(\text{Gamma}\left(1+\frac{1}{n+2}\right)\text{BesselJ}\left(\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right)+c_1\,\text{G}\cdots\right.}$$

$$y(x) \to \frac{\frac{{}_0F_1\left(;-\frac{1}{n+2};-\frac{abx^{n+2}}{(n+2)^2}\right)}{{}_0F_1\left(;\frac{n+1}{n+2};-\frac{abx^{n+2}}{(n+2)^2}\right)}-1}{ax}$$

$$y(x) \to \frac{\frac{{}_0F_1\left(;-\frac{1}{n+2};-\frac{abx^{n+2}}{(n+2)^2}\right)}{{}_0F_1\left(;\frac{n+1}{n+2};-\frac{abx^{n+2}}{(n+2)^2}\right)}-1}{ax}$$

2.5 problem 5

Internal problem ID [9588]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - an\,x^{n-1} + a^2x^{2n} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 555

```
dsolve(diff(y(x),x)=y(x)^2+a*n*x^(n-1)-a^2*x^(2*n),y(x), singsol=all)
```

$$y(x) = \frac{2a\,x^{1+n}\mathrm{e}^{-\frac{a\,x^{1+n}}{1+n}} + \left(2x^{-\frac{3n}{2}-1}c_1n^3 + 11x^{-\frac{3n}{2}-1}c_1n^2 + 20x^{-\frac{3n}{2}-1}c_1n + 12x^{-\frac{3n}{2}-1}c_1\right)\mathrm{WhittakerM}\left(\frac{3n+4}{2+2n}, \frac{3+2n}{2+2n}\right.}{}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a*n*x^(n-1)-a^2*x^(2*n),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.6 problem 6

Internal problem ID [9589]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 a - b\,x^{2n} - c\,x^{n-1} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 499

```
dsolve(diff(y(x),x)=a*y(x)^2+b*x^(2*n)+c*x^(n-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-2b^{\frac{3}{2}}c_1 n - 2b^{\frac{3}{2}}c_1\right)\operatorname{WhittakerW}\left(-\frac{i\sqrt{a}\,c-2\sqrt{b}\,n-2\sqrt{b}}{2\sqrt{b}\,(1+n)}, \frac{1}{2+2n}, \frac{2i\sqrt{b}\,\sqrt{a}\,x^{1+n}}{1+n}\right)}{2b^{\frac{3}{2}}\left(\operatorname{WhittakerW}\left(-\frac{i\sqrt{a}\,c}{2\sqrt{b}\,(1+n)}, \frac{1}{2+2n}, \frac{2i\sqrt{b}\,\sqrt{a}\,x^{1+n}}{1+n}\right)c_1 + \operatorname{WhittakerM}\left(-\frac{i\sqrt{a}\,c}{2\sqrt{b}\,(1+n)}, \frac{1}{2+2n}, \frac{2i\sqrt{b}\,\sqrt{a}\,x^{1+n}}{1+n}\right)\right)ax}$$
$$-\frac{\left(2i\sqrt{a}\,x^{1+n}c_1 b^2 + i\sqrt{a}\,c_1 bc - b^{\frac{3}{2}}c_1 n\right)\operatorname{WhittakerW}\left(-\frac{i\sqrt{a}\,c}{2\sqrt{b}\,(1+n)}, \frac{1}{2+2n}, \frac{2i\sqrt{b}\,\sqrt{a}\,x^{1+n}}{1+n}\right) + \left(-i\sqrt{a}\,bc + b^{\frac{3}{2}}n +\right.}{2b^{\frac{3}{2}}\left(\operatorname{WhittakerW}\left(-\frac{i\sqrt{a}\,c}{2\sqrt{b}\,(1+n)}, \frac{1}{2+2n}\right.\right.}$$

✓ Solution by Mathematica

Time used: 0.981 (sec). Leaf size: 982

```
DSolve[y'[x]==a*y[x]^2+b*x^(2*n)+c*x^(n-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^n\left(\sqrt{b}c_1(n+1)\sqrt{-(n+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{b}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+c_1\left(\sqrt{a}c(n\right.\right.}{\sqrt{a}(n+1)^2\left(\right.}$$

$$y(x) \to \frac{x^n\left(-\frac{\left(\sqrt{a}c(n+1)+\sqrt{b}\sqrt{-(n+1)^2}n\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}+2\right),\frac{n}{n+1}+1,\frac{2\sqrt{a}\sqrt{b}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{b}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}-\sqrt{b}\sqrt{-(n+1)^2}(n+\right.}{\sqrt{a}(n+1)^2}$$

$$y(x) \to \frac{x^n\left(-\frac{\left(\sqrt{a}c(n+1)+\sqrt{b}\sqrt{-(n+1)^2}n\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}+2\right),\frac{n}{n+1}+1,\frac{2\sqrt{a}\sqrt{b}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{b}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}-\sqrt{b}\sqrt{-(n+1)^2}(n+\right.}{\sqrt{a}(n+1)^2}$$

2.7 problem 7

Internal problem ID [9590]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Riccati]`

$$\boxed{y' - a\,x^n y^2 - b\,x^{-n-2} = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 61

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*x^(-n-2),y(x), singsol=all)
```

$$y(x) = -\frac{x^{-1-n}\left(n+1+\tan\left(\frac{\sqrt{4ba-n^2-2n-1}\,(-\ln(x)+c_1)}{2}\right)\sqrt{4ba-n^2-2n-1}\right)}{2a}$$

✓ Solution by Mathematica

Time used: 0.481 (sec). Leaf size: 135

```
DSolve[y'[x]==a*x^n*y[x]^2+b*x^(-n-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-n-1}\left(-\left(\sqrt{(n+1)^2-4ab}+n+1\right)x^{\sqrt{(n+1)^2-4ab}}+c_1\left(\sqrt{(n+1)^2-4ab}-n-1\right)\right)}{2a\left(x^{\sqrt{(n+1)^2-4ab}}+c_1\right)}$$

$$y(x) \to \frac{x^{-n-1}\left(\sqrt{(n+1)^2-4ab}-n-1\right)}{2a}$$

2.8 problem 8

Internal problem ID [9591]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 - b\,x^m = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 177

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*x^m,y(x), singsol=all)
```

$$y(x) = \frac{\left(\operatorname{BesselY}\left(\frac{m+1}{m+n+2}, \frac{2\sqrt{ba}\,x^{\frac{m}{2}+\frac{n}{2}+1}}{m+n+2}\right) c_1 + \operatorname{BesselJ}\left(\frac{m+1}{m+n+2}, \frac{2\sqrt{ba}\,x^{\frac{m}{2}+\frac{n}{2}+1}}{m+n+2}\right)\right) x^{\frac{m}{2}+\frac{n}{2}+1}\sqrt{ba}\,x^{-n}}{\left(\operatorname{BesselY}\left(-\frac{1+n}{m+n+2}, \frac{2\sqrt{ba}\,x^{\frac{m}{2}+\frac{n}{2}+1}}{m+n+2}\right) c_1 + \operatorname{BesselJ}\left(-\frac{1+n}{m+n+2}, \frac{2\sqrt{ba}\,x^{\frac{m}{2}+\frac{n}{2}+1}}{m+n+2}\right)\right) ax}$$

✓ Solution by Mathematica

Time used: 1.482 (sec). Leaf size: 992

```
DSolve[y'[x]==a*x^n*y[x]^2+b*x^m,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(n+1)x^{-n-1}\left((m+n+1)^{\frac{2(n+1)}{m+n+2}}\,\text{Gamma}\left(\frac{n+1}{m+n+2}\right)\left(-\sqrt{a}\sqrt{b}(m+n+1)\left(x^{m+n+1}\right)^{\frac{m+n+2}{2(m+n+1)}}\,\text{BesselJ}\left(-\frac{\cdots}{m\cdots}\right.\right.\right.}{\cdots}$$

$$y(x) \to \frac{x^{-n-1}\left(\frac{\frac{ab\left(x^{m+n+1}\right)^{\frac{1}{m+n+1}+1}{}_0F_1\left(;\frac{m+1}{m+n+2}+1;-\frac{ab\left(x^{m+n+1}\right)^{1+\frac{1}{m+n+1}}}{(m+n+2)^2}\right)}{m+1}+(n+1)\,{}_0F_1\left(;-\frac{n+1}{m+n+2};-\frac{ab\left(x^{m+n+1}\right)^{1+\frac{1}{m+n+1}}}{(m+n+2)^2}\right)}{{}_0F_1\left(;\frac{m+1}{m+n+2};-\frac{ab\left(x^{m+n+1}\right)^{1+\frac{1}{m+n+1}}}{(m+n+2)^2}\right)}-n-\cdots\right.}{2a}$$

$$y(x) \to \frac{x^{-n-1}\left(\frac{\frac{ab\left(x^{m+n+1}\right)^{\frac{1}{m+n+1}+1}{}_0F_1\left(;\frac{m+1}{m+n+2}+1;-\frac{ab\left(x^{m+n+1}\right)^{1+\frac{1}{m+n+1}}}{(m+n+2)^2}\right)}{m+1}+(n+1)\,{}_0F_1\left(;-\frac{n+1}{m+n+2};-\frac{ab\left(x^{m+n+1}\right)^{1+\frac{1}{m+n+1}}}{(m+n+2)^2}\right)}{{}_0F_1\left(;\frac{m+1}{m+n+2};-\frac{ab\left(x^{m+n+1}\right)^{1+\frac{1}{m+n+1}}}{(m+n+2)^2}\right)}-n-\cdots\right.}{2a}$$

2.9 problem 9

Internal problem ID [9592]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - k(ax+b)^n (cx+d)^{-n-4} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+k*(a*x+b)^n*(c*x+d)^(-n-4),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+k*(a*x+b)^n*(c*x+d)^(-n-4),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.10 problem 10

Internal problem ID [9593]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 - bm\,x^{m-1} + a\,b^2 x^{n+2m} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 1166

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*m*x^(m-1)-a*b^2*x^(n+2*m),y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*x^n*y[x]^2+b*m*x^(m-1)-a*b^2*x^(n+2*m),y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

2.11 problem 11

Internal problem ID [9594]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \left(x^{2n} a + b\, x^{n-1}\right) y^2 - c = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8014

```
dsolve(diff(y(x),x)=(a*x^(2*n)+b*x^(n-1))*y(x)^2+c,y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(a*x^(2*n)+b*x^(n-1))*y[x]^2+c,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.12 problem 12

Internal problem ID [9595]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(a_2x+b_2)\left(y'+\lambda y^2\right)+a_0x+b_0=0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 1030

```
dsolve((a__2*x+b__2)*(diff(y(x),x)+lambda*y(x)^2)+a__0*x+b__0=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.974 (sec). Leaf size: 471

```
DSolve[(a2*x+b2)*(y'[x]+\[Lambda]*y[x]^2)+a0*x+b0==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\frac{i\text{a2}^{3/2}c_1\sqrt{\lambda}(\text{a2b0}-\text{a0b2})\,\text{HypergeometricU}\left(\frac{i\sqrt{\lambda}(\text{a2b0}-\text{a0b2})}{2\sqrt{\text{a0}}\text{a2}^{3/2}}+1,1,\frac{2i\sqrt{\text{a0}}(\text{b2}+\text{a2}x)\sqrt{\lambda}}{\text{a2}^{3/2}}\right)+2\sqrt{\text{a0}}\text{a2}^3\,\text{LaguerreL}\left(\frac{i(\text{a0b2}-\text{a2b0})}{2\sqrt{\text{a0}}\text{a2}^{3/2}}\cdots\right.}{2\sqrt{\text{a0}}\sqrt{\lambda}(\text{a2}x+\text{b2})\,\text{Hypergeometric1F1}\left(\frac{i\sqrt{\lambda}(\text{a2b0}-\text{a0b2})}{2\sqrt{\text{a0}}\text{a2}^{3/2}}+1,2,\frac{2i\sqrt{\text{a0}}(\text{b2}+\text{a2}x)\sqrt{\lambda}}{\text{a2}^{3/2}}\right)+i\text{a2}^{3/2}c_1\,\text{HypergeometricU}\left(\frac{i(\text{a2b0}-\text{a0b2})}{2\sqrt{\text{a0}}\text{a2}^{3/2}}\cdots\right.}}{\text{a2}^2\sqrt{\lambda}}$$

$$y(x) \to \frac{(\text{a2b0}-\text{a0b2})\,\text{HypergeometricU}\left(\frac{i\sqrt{\lambda}(\text{a2b0}-\text{a0b2})}{2\sqrt{\text{a0}}\text{a2}^{3/2}}+1,1,\frac{2i\sqrt{\text{a0}}(\text{b2}+\text{a2}x)\sqrt{\lambda}}{\text{a2}^{3/2}}\right)}{\text{a2}^2\,\text{HypergeometricU}\left(\frac{i(\text{a2b0}-\text{a0b2})\sqrt{\lambda}}{2\sqrt{\text{a0}}\text{a2}^{3/2}},0,\frac{2i\sqrt{\text{a0}}(\text{b2}+\text{a2}x)\sqrt{\lambda}}{\text{a2}^{3/2}}\right)} - \frac{i\sqrt{\text{a0}}}{\sqrt{\text{a2}}\sqrt{\lambda}}$$

2.13 problem 13

Internal problem ID [9596]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Riccati, _special]]`

$$\boxed{x^2y' - a\,x^2y^2 - b = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 40

```
dsolve(x^2*diff(y(x),x)=a*x^2*y(x)^2+b,y(x), singsol=all)
```

$$y(x) = -\frac{1 + \tan\left(\frac{\sqrt{4ba-1}\,(-\ln(x)+c_1)}{2}\right)\sqrt{4ba-1}}{2ax}$$

✓ Solution by Mathematica

Time used: 0.212 (sec). Leaf size: 77

```
DSolve[x^2*y'[x]==a*x^2*y[x]^2+b,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-1 + \sqrt{1-4ab}\left(-1 + \frac{2c_1}{x^{\sqrt{1-4ab}}+c_1}\right)}{2ax}$$

$$y(x) \to \frac{\sqrt{1-4ab}-1}{2ax}$$

2.14 problem 14

Internal problem ID [9597]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^2y' - y^2x^2 + a^2x^4 - a(1-2b)\,x^2 + b(1+b) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 121

```
dsolve(x^2*diff(y(x),x)=x^2*y(x)^2-a^2*x^4+a*(1-2*b)*x^2-b*(b+1),y(x), singsol=all)
```

$$y(x) = \frac{2(-a\,x^2)^{b-\frac{1}{2}}\,c_1 a x\,\mathrm{e}^{a\,x^2}}{c_1\Gamma\left(b+\frac{1}{2}\right) - c_1\Gamma\left(b+\frac{1}{2}, -a\,x^2\right) + 1} + \frac{(-ac_1x^2 - c_1b)\,\Gamma\left(b+\frac{1}{2}, -a\,x^2\right) + (ac_1x^2 + c_1b)\,\Gamma\left(b+\frac{1}{2}\right) + a\,x^2 + b}{x\left(c_1\Gamma\left(b+\frac{1}{2}\right) - c_1\Gamma\left(b+\frac{1}{2}, -a\,x^2\right) + 1\right)}$$

✓ Solution by Mathematica

Time used: 0.639 (sec). Leaf size: 106

```
DSolve[x^2*y'[x]==x^2*y[x]^2-a^2*x^4+a*(1-2*b)*x^2-b*(b+1),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{x^{2b+1}(ax^2+b)\,\text{ExpIntegralE}\left(\frac{1}{2}-b, -ax^2\right) - 2c_1(ax^2+b) + 2e^{ax^2}x^{2b+1}}{x^{2b+2}\,\text{ExpIntegralE}\left(\frac{1}{2}-b, -ax^2\right) - 2c_1x}$$

$$y(x) \to ax + \frac{b}{x}$$

2.15 problem 15

Internal problem ID [9598]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^2y' - a\,x^2y^2 - b\,x^n - c = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 239

```
dsolve(x^2*diff(y(x),x)=a*x^2*y(x)^2+b*x^n+c,y(x), singsol=all)
```

$$y(x) = -\frac{\left(\sqrt{-4ac+1}\,c_1 + c_1\right)\text{BesselY}\left(\frac{\sqrt{-4ac+1}}{n}, \frac{2\sqrt{ba}\,x^{\frac{n}{2}}}{n}\right) - 2\,\text{BesselY}\left(\frac{\sqrt{-4ac+1}+n}{n}, \frac{2\sqrt{ba}\,x^{\frac{n}{2}}}{n}\right)\sqrt{ba}\,x^{\frac{n}{2}}c_1 + \left(\sqrt{-4a\right.}{2xa\left(\text{BesselY}\left(\frac{\sqrt{-4ac+1}}{n}, \frac{2\sqrt{ba}\,x^{\frac{n}{2}}}{n}\right)c_1 + \text{Bessel}\right.}$$

✓ Solution by Mathematica

Time used: 0.93 (sec). Leaf size: 1779

```
DSolve[x^2*y'[x]==a*x^2*y[x]^2+b*x^n+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-a^{\frac{i\sqrt{4ac-1}}{n}+\frac{1}{2}}n^{\frac{2\sqrt{(1-4ac)n^2}}{n^2}+1}(x^n)^{\frac{i\sqrt{4ac-1}}{n}+1}\text{BesselJ}\left(\frac{\sqrt{(1-4ac)n^2}}{n^2}-1,\frac{2\sqrt{a}\sqrt{b}\sqrt{x^n}}{n}\right)\text{Gamma}\left(\frac{n+\sqrt{1-4ac}}{n}\right)b^{\frac{i\sqrt{4ac-1}}{n}+\cdots}}{\cdots}$$

$$y(x) \to \frac{\frac{\sqrt{a}\sqrt{b}\sqrt{x^n}\left(\text{BesselJ}\left(1-\frac{\sqrt{(1-4ac)n^2}}{n^2},\frac{2\sqrt{a}\sqrt{b}\sqrt{x^n}}{n}\right)-\text{BesselJ}\left(-\frac{\sqrt{(1-4ac)n^2}}{n^2}-1,\frac{2\sqrt{a}\sqrt{b}\sqrt{x^n}}{n}\right)\right)}{\text{BesselJ}\left(-\frac{\sqrt{(1-4ac)n^2}}{n^2},\frac{2\sqrt{a}\sqrt{b}\sqrt{x^n}}{n}\right)}-\frac{\sqrt{n^2(1-4ac)}}{n}+i\sqrt{4ac-1}-1}{2ax}$$

2.16 problem 16

Internal problem ID [9599]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$x^2y' - y^2x^2 - a\,x^{2m}(b\,x^m + c)^n + \frac{n^2}{4} - \frac{1}{4} = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)=x^2*y(x)^2+a*x^(2*m)*(b*x^m+c)^n+1/4*(1-n^2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==x^2*y[x]^2+a*x^(2*m)*(b*x^m+c)^n+1/4*(1-n^2),y[x],x,IncludeSingularSolution
```

Not solved

2.17 problem 17

Internal problem ID [9600]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(c_2x^2 + b_2x + a_2)(y' + \lambda y^2) + a_0 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 2906

```
dsolve((c__2*x^2+b__2*x+a__2)*(diff(y(x),x)+lambda*y(x)^2)+a__0=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 1.869 (sec). Leaf size: 846

```
DSolve[(c2*x^2+b2*x+a2)*(y'[x]+\[Lambda]*y[x]^2)+a0==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{(\mathrm{b2}+2\mathrm{c2}x)\left(8\mathrm{c2}\left(\mathrm{b2}^2-4\mathrm{a2c2}\right) G^{2,0}_{2,2}\left(-\frac{4\mathrm{c2}(\mathrm{a2}+x(\mathrm{b2}+\mathrm{c2}x))}{\mathrm{b2}^2-4\mathrm{a2c2}}\middle| \begin{matrix} \frac{1}{4}-\frac{\sqrt{\mathrm{c2}-4\mathrm{a0}\lambda}}{4\sqrt{\mathrm{c2}}}, \frac{1}{4}\left(\frac{\sqrt{\mathrm{c2}-4\mathrm{a0}\lambda}}{\sqrt{\mathrm{c2}}}+1\right) \\ 0,0 \end{matrix}\right)+4\mathrm{c2}c \ldots\right.}{2\lambda\left(\mathrm{b2}^2-4\mathrm{a2c2}\right)^2\left(G^{2,0}_{2,2}\left(- \ldots\right.\right.}$$

$$y(x) \to \frac{(\mathrm{b2}+2\mathrm{c2}x)\left(\frac{2}{\mathrm{a2}+x(\mathrm{b2}+\mathrm{c2}x)}-\frac{(\mathrm{a0}\lambda+2\mathrm{c2})\,\mathrm{Hypergeometric2F1}\left(\frac{7\mathrm{c2}+\sqrt{\mathrm{c2}(\mathrm{c2}-4\mathrm{a0}\lambda)}}{4\mathrm{c2}},\frac{1}{4}\left(7-\frac{\sqrt{\mathrm{c2}(\mathrm{c2}-4\mathrm{a0}\lambda)}}{\mathrm{c2}}\right),3,-\frac{4\mathrm{c2}(\mathrm{a2}+ \ldots}{\mathrm{b2}^2 \ldots}\right.}{\left(\mathrm{b2}^2-4\mathrm{a2c2}\right)\mathrm{Hypergeometric2F1}\left(\frac{3\mathrm{c2}+\sqrt{\mathrm{c2}(\mathrm{c2}-4\mathrm{a0}\lambda)}}{4\mathrm{c2}},\frac{1}{4}\left(3-\frac{\sqrt{\mathrm{c2}(\mathrm{c2}-4\mathrm{a0}\lambda)}}{\mathrm{c2}}\right),2,-\frac{4\mathrm{c2}(\mathrm{a2} \ldots}{\mathrm{b2} \ldots}\right.}\right.}{2\lambda}$$

$$y(x) \to \frac{(\mathrm{b2}+2\mathrm{c2}x)\left(\frac{2}{\mathrm{a2}+x(\mathrm{b2}+\mathrm{c2}x)}-\frac{(\mathrm{a0}\lambda+2\mathrm{c2})\,\mathrm{Hypergeometric2F1}\left(\frac{7}{4}-\frac{\sqrt{\mathrm{c2}(\mathrm{c2}-4\mathrm{a0}\lambda)}}{4\mathrm{c2}},\frac{1}{4}\left(\frac{\sqrt{\mathrm{c2}(\mathrm{c2}-4\mathrm{a0}\lambda)}}{\mathrm{c2}}+7\right),3,-\frac{4\mathrm{c2}(\mathrm{a2}+x \ldots}{\mathrm{b2}^2- \ldots}\right.}{\left(\mathrm{b2}^2-4\mathrm{a2c2}\right)\mathrm{Hypergeometric2F1}\left(\frac{3\mathrm{c2}+\sqrt{\mathrm{c2}(\mathrm{c2}-4\mathrm{a0}\lambda)}}{4\mathrm{c2}},\frac{1}{4}\left(3-\frac{\sqrt{\mathrm{c2}(\mathrm{c2}-4\mathrm{a0}\lambda)}}{\mathrm{c2}}\right),2,-\frac{4\mathrm{c2}(\mathrm{a2} \ldots}{\mathrm{b2} \ldots}\right.}\right.}{2\lambda}$$

2.18 problem 18

Internal problem ID [9601]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Riccati, _special]]`

$$\boxed{x^4y' + x^4y^2 + a^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(x^4*diff(y(x),x)=-x^4*y(x)^2-a^2,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{a^2}\,\tan\left(\frac{\sqrt{a^2}\,(xc_1-1)}{x}\right) - x}{x^2}$$

✓ Solution by Mathematica

Time used: 0.526 (sec). Leaf size: 54

```
DSolve[x^4*y'[x]==-x^4*y[x]^2-a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x + a\left(\frac{1}{ac_1e^{\frac{2ia}{x}} - \frac{i}{2}} - i\right)}{x^2}$$

$$y(x) \to \frac{x - ia}{x^2}$$

2.19 problem 19

Internal problem ID [9602]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{a\,x^2(x-1)^2\left(y'+\lambda y^2\right)+b\,x^2+cx+s=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 2636

```
dsolve(a*x^2*(x-1)^2*(diff(y(x),x)+lambda*y(x)^2)+b*x^2+c*x+s=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 61.157 (sec). Leaf size: 266473

```
DSolve[a*x^2*(x-1)^2*(y'[x]+\[Lambda]*y[x]^2)+b*x^2+c*x+s==0,y[x],x,IncludeSingularSolutions
```

Too large to display

2.20 problem 20

Internal problem ID [9603]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{\left(a\,x^2+bx+c\right)^2\left(y'+y^2\right)+A=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 846

```
dsolve((a*x^2+b*x+c)^2*(diff(y(x),x)+y(x)^2)+A=0,y(x), singsol=all)
```

$$y(x) = -\frac{2\left(-i\sqrt{-\frac{4ac-b^2+4A}{a^2}}\,\sqrt{4ac-b^2}\left(\frac{i\sqrt{4ac-b^2}-2ax-b}{2ax+b+i\sqrt{4ac-b^2}}\right)^{-\frac{a\sqrt{-\frac{4ac-b^2+4A}{a^2}}}{2\sqrt{-4ac+b^2}}} c_1 a + i\sqrt{-\frac{4ac-b^2+4A}{a^2}}\,\sqrt{4ac-b^2}\left(\frac{i\sqrt{4ac-b^2}-}{2ax+b+i\sqrt{}}\right.\right.}{\sqrt{-4ac}}$$

✓ Solution by Mathematica

Time used: 2.707 (sec). Leaf size: 312

```
DSolve[(a*x^2+b*x+c)^2*(y'[x]+y[x]^2)+A==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\frac{2(-4ac-4A+b^2)}{1+c_1\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}\exp\left(\frac{2\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right)}+2ax\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}+b\sqrt{b^2-4ac}\sqrt{1-\cdots}}{2\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}(x(ax+b)+c)}$$

$$y(x) \to \frac{-\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}+2ax+b}{2(x(ax+b)+c)}$$

2.21 problem 21

Internal problem ID [9604]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^{1+n}y' - x^{2n}y^2a - c\,x^m - d = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 270

```
dsolve(x^(n+1)*diff(y(x),x)=a*x^(2*n)*y(x)^2+c*x^m+d,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(-\sqrt{-4ad+n^2}\,c_1 - c_1 n\right)\operatorname{BesselY}\left(\frac{\sqrt{-4ad+n^2}}{m}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}}}{m}\right) + 2x^{\frac{m}{2}}\sqrt{ac}\,\operatorname{BesselY}\left(\frac{\sqrt{-4ad+n^2}+m}{m}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}}}{m}\right)c_1 +}{2xa\left(\operatorname{BesselY}\left(\frac{\sqrt{-4ad+n^2}}{m}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}}}{m}\right)c\right.}$$

✓ Solution by Mathematica

Time used: 1.097 (sec). Leaf size: 948

```
DSolve[x^(n+1)*y'[x]==a*x^(2*n)*y[x]^2+c*x^m+d,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-n}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{x^m}}{m}\right)^{-\frac{\sqrt{m^2(n^2-4ad)}}{m^2}}\left(a^{\frac{\sqrt{n^2-4ad}}{m}}c^{\frac{\sqrt{n^2-4ad}}{m}}m^{\frac{2\sqrt{m^2(n^2-4ad)}}{m^2}}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{x^m}}{m}\right)^{\frac{2\sqrt{m^2(n^2-4ad)}}{m^2}}\text{Gamma}\left(\frac{\sqrt{n^2-4ad}}{m}\right)\right.}{2a}$$

$$y(x) \to \frac{x^{-n}\left(\frac{2acx^m\,{}_0\tilde{F}_1\left(;2-\frac{\sqrt{m^2(n^2-4ad)}}{m^2};-\frac{acx^m}{m^2}\right)}{m\,{}_0\tilde{F}_1\left(;1-\frac{\sqrt{m^2(n^2-4ad)}}{m^2};-\frac{acx^m}{m^2}\right)}+\sqrt{n^2-4ad}-n\right)}{2a}$$

2.22 problem 22

Internal problem ID [9605]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(x^n a + b)\, y' - b y^2 - a\, x^{n-2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 333

```
dsolve((a*x^n+b)*diff(y(x),x)=b*y(x)^2+a*x^(n-2),y(x), singsol=all)
```

$$y(x) = -\frac{(-x^{2n}c_1a^2n - x^n c_1 abn)\operatorname{hypergeom}\left(\left[2, \frac{1+n}{n}\right], \left[\frac{2n-1}{n}\right], -\frac{a\,x^n}{b}\right)}{\left(\operatorname{hypergeom}\left(\left[\frac{2}{n}\right], [], -\frac{a\,x^n}{b}\right)x + \operatorname{hypergeom}\left(\left[1, \frac{1}{n}\right], \left[\frac{n-1}{n}\right], -\frac{a\,x^n}{b}\right)c_1\right)(n-1)\,b^2x} - \frac{(x^n c_1 abn - x^n c_1 ab)\operatorname{hypergeom}\left(\left[1, \frac{1}{n}\right], \left[\frac{n-1}{n}\right], -\frac{a\,x^n}{b}\right) + (2abn\,x^{1+n} - 2ab\,x^{1+n} + b^2nx - b^2x)\operatorname{hyperge}}{\left(\operatorname{hypergeom}\left(\left[\frac{2}{n}\right], [], -\frac{a\,x^n}{b}\right)x + \operatorname{hyperge}\right.}$$

✓ Solution by Mathematica

Time used: 1.088 (sec). Leaf size: 250

```
DSolve[(a*x^n+b)*y'[x]==b*y[x]^2+a*x^(n-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1(a(n-1)x^n + bn)\,\text{Hypergeometric2F1}\left(-\frac{1}{n}, \frac{n-2}{n}, \frac{n-1}{n}, -\frac{ax^n}{b}\right) - b\left((-1)^{\frac{1}{n}}\left(-\frac{ax^n}{b}\right)^{\frac{1}{n}} + c_1 n\,\text{Hypergeomet}\right.}{bx\left((-1)^{\frac{1}{n}}\left(-\frac{ax^n}{b}\right)^{\frac{1}{n}} + c_1\,\text{Hypergeometric2F1}\left(-\frac{1}{n}, \frac{n-2}{n}, \frac{n-1}{n}, -\frac{ax^n}{b}\right)\right)}$$

$$y(x) \to \frac{ax^{n-1}\left(\frac{n(ax^n+b)\,\text{Hypergeometric2F1}\left(2,1+\frac{1}{n},2-\frac{1}{n},-\frac{ax^n}{b}\right)}{\text{Hypergeometric2F1}\left(1,\frac{1}{n},\frac{n-1}{n},-\frac{ax^n}{b}\right)} + b(-n) + b\right)}{b^2(n-1)}$$

2.23 problem 23

Internal problem ID [9606]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(x^n a + b\,x^m + c)\left(y' - y^2\right) + an(n-1)\,x^{n-2} + bm(m-1)\,x^{m-2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 173

```
dsolve((a*x^n+b*x^m+c)*(diff(y(x),x)-y(x)^2)+a*n*(n-1)*x^(n-2)+b*m*(m-1)*x^(m-2)=0,y(x), sing
```

$$y(x) = -\frac{(a^2x^{2n}n + abm\,x^{m+n} + abn\,x^{m+n} + x^{2m}b^2m + can\,x^n + x^m bcm)\left(\int \frac{1}{(a\,x^n+b\,x^m+c)^2}dx\right) + x^{2n}c_1a^2n + abn}{(a\,x^n + b\,x^m + c)^2\,x\left(c_1 + \int \frac{1}{(a\,x^n+b\,x^m+c)^2}dx\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b*x^m+c)*(y'[x]-y[x]^2)+a*n*(n-1)*x^(n-2)+b*m*(m-1)*x^(m-2)==0,y[x],x,IncludeSi
```

Not solved

2.24 problem 24

Internal problem ID [9607]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 a - by - cx - k = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 194

```
dsolve(diff(y(x),x)=a*y(x)^2+b*y(x)+c*x+k,y(x), singsol=all)
```

$$y(x) = \frac{2\sqrt{a}\left(\frac{c}{\sqrt{a}}\right)^{\frac{1}{3}}\left(\operatorname{AiryAi}\left(1, -\frac{a(cx+k)-\frac{b^2}{4}}{\left(\frac{c}{\sqrt{a}}\right)^{\frac{2}{3}} a}\right) c_1 + \operatorname{AiryBi}\left(1, -\frac{a(cx+k)-\frac{b^2}{4}}{\left(\frac{c}{\sqrt{a}}\right)^{\frac{2}{3}} a}\right)\right) - \left(c_1 \operatorname{AiryAi}\left(-\frac{a(cx+k)-\frac{b^2}{4}}{\left(\frac{c}{\sqrt{a}}\right)^{\frac{2}{3}} a}\right) + \right.}{2a\left(c_1 \operatorname{AiryAi}\left(-\frac{a(cx+k)-\frac{b^2}{4}}{\left(\frac{c}{\sqrt{a}}\right)^{\frac{2}{3}} a}\right) + \operatorname{AiryBi}\left(-\frac{a(cx+k)-\frac{b^2}{4}}{\left(\frac{c}{\sqrt{a}}\right)^{\frac{2}{3}} a}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.303 (sec). Leaf size: 264

```
DSolve[y'[x]==a*y[x]^2+b*y[x]+c*x+k,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{b \operatorname{AiryBi}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right) + bc_1 \operatorname{AiryAi}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right) + 2\sqrt[3]{-ac}\left(\operatorname{AiryBiPrime}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right) + c_1 \operatorname{AiryAiPr}\right.}{2a\left(\operatorname{AiryBi}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right) + c_1 \operatorname{AiryAi}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)\right)}$$

$$y(x) \to -\frac{\frac{2\sqrt[3]{-ac}\operatorname{AiryAiPrime}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)}{\operatorname{AiryAi}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)} + b}{2a}$$

2.25 problem 25

Internal problem ID [9608]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\,x^n y - a\,x^{n-1} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 385

```
dsolve(diff(y(x),x)=y(x)^2+a*x^n*y(x)+a*x^(n-1),y(x), singsol=all)
```

$$y(x) = \frac{\cdots}{x^2\left(c_1 - \frac{\left(\frac{a}{-1-n}\right)^{\frac{1}{1+n}}\left(\frac{(-1-n)^2 x^{-1-\frac{1}{1+n}-\frac{n}{1+n}-n}\left(\frac{a}{-1-n}\right)^{-\frac{1}{1+n}}\left(\frac{x^{1+n}a\,n^2}{-1-n}+\frac{2x^{1+n}an}{-1-n}+n^2+\frac{x^{1+n}a}{-1-n}+n\right)\left(\frac{x^{1+n}a}{-1-n}\right)^{-\frac{n}{2(1+n)}}\mathrm{e}^{-\frac{x^{1+n}a}{2(-1-n)}}\,\mathrm{Whi}\cdots}{n(2n+1)a}\cdots\right.}{\cdots}\right.\cdots} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 1.868 (sec). Leaf size: 72

```
DSolve[y'[x]==y[x]^2+a*x^n*y[x]+a*x^(n-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1 + \frac{(n+1)e^{\frac{ax^{n+1}}{n+1}}}{-\mathrm{ExpIntegralE}\left(1+\frac{1}{n+1},-\frac{ax^{n+1}}{n+1}\right)+c_1(n+1)x}}{x}$$

$$y(x) \to -\frac{1}{x}$$

2.26 problem 26

Internal problem ID [9609]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\,x^n y - b\,x^{n-1} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 376

```
dsolve(diff(y(x),x)=y(x)^2+a*x^n*y(x)+b*x^(n-1),y(x), singsol=all)
```

$$y(x) = \frac{(c_1 an + c_1 a)\operatorname{KummerU}\left(-\frac{an+b}{a(1+n)}, \frac{n+2}{1+n}, \frac{a\,x^{1+n}}{1+n}\right)}{\left(\operatorname{KummerU}\left(\frac{a-b}{a(1+n)}, \frac{n+2}{1+n}, \frac{a\,x^{1+n}}{1+n}\right) c_1 + \operatorname{KummerM}\left(\frac{a-b}{a(1+n)}, \frac{n+2}{1+n}, \frac{a\,x^{1+n}}{1+n}\right)\right) ax}$$
$$+ \frac{(-x^{1+n} c_1 a^2 + c_1 an + c_1 b)\operatorname{KummerU}\left(\frac{a-b}{a(1+n)}, \frac{n+2}{1+n}, \frac{a\,x^{1+n}}{1+n}\right) + (-an - a - b)\operatorname{KummerM}\left(-\frac{an+b}{a(1+n)}, \frac{n+2}{1+n}, \right.}{\left(\operatorname{KummerU}\left(\frac{a-b}{a(1+n)}, \frac{n+2}{1+n}, \frac{a\,x^{1+n}}{1+n}\right) c_1 + \operatorname{KummerM}\left(\frac{a-b}{a(1+n)}, \right.\right.}$$

✓ Solution by Mathematica

Time used: 0.532 (sec). Leaf size: 427

```
DSolve[y'[x]==y[x]^2+a*x^n*y[x]+b*x^(n-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x^n)^{\frac{1}{n}}\left(e^{\frac{i\pi}{n+1}}na^{\frac{1}{n+1}}\left(\frac{(x^n)^{\frac{1}{n}+1}(an+a+b)\text{Hypergeometric1F1}\left(\frac{a-b}{na+a},2+\frac{1}{n+1},\frac{a(x^n)^{1+\frac{1}{n}}}{n+1}\right)}{n+2}-\left(a(x^n)^{\frac{1}{n}+1}+1\right)\text{Hypergeomet}\right.\right.}{nx\left((-1)^{\frac{1}{n+1}}a^{\frac{1}{n+1}}(x^n)^{\frac{1}{n}}\text{Hypergeometric1F1}\left(\frac{a-b}{na+a},1+\frac{1}{n+1},\frac{a(x^n}{n}\right.\right.}$$

$$y(x) \to \frac{bx^{n-1}(x^n)^{\frac{1}{n}}\text{Hypergeometric1F1}\left(1-\frac{b}{na+a},2-\frac{1}{n+1},\frac{a(x^n)^{1+\frac{1}{n}}}{n+1}\right)}{n\,\text{Hypergeometric1F1}\left(-\frac{b}{na+a},\frac{n}{n+1},\frac{a(x^n)^{1+\frac{1}{n}}}{n+1}\right)}$$

2.27 problem 27

Internal problem ID [9610]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - (\alpha x + \beta)\, y - a\, x^2 - bx - c = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 4562

```
dsolve(diff(y(x),x)=y(x)^2+(alpha*x+beta)*y(x)+a*x^2+b*x+c,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 2.452 (sec). Leaf size: 1291

```
DSolve[y'[x]==y[x]^2+(\[Alpha]*x+\[Beta])*y[x]+a*x^2+b*x+c,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{2\left(2b + 4ax + \left(\sqrt{\alpha^2-4a}-\alpha\right)(x\alpha+\beta)\right) \text{Hypergeometric1F1}\left(-\frac{2b^2-2\alpha\beta b+\alpha^2\left(2c+\alpha-\sqrt{\alpha^2-4a}\right)+2a\left(\beta^2-4c-2a \cdots\right.}{4(\alpha^2-4a)^{3/2}} \cdots\right.}{\cdots}$$

$$y(x) \to \frac{(4a-\alpha^2)\left(\left(\sqrt{\alpha^2-4a}-\alpha\right)(\beta+\alpha x)+4ax+2b\right) - \frac{\sqrt{2}\sqrt[4]{\alpha^2-4a}\left(2a\left(2\sqrt{\alpha^2-4a}-2\alpha+\beta^2-4c\right)+\alpha^2\left(-\sqrt{\alpha^2-4a}+\alpha+2c \cdots\right.\right.}{\text{HermiteH}\left(-\frac{-2b^2+2\alpha\beta b+\alpha^2 \cdots}{\cdots}\right.}}{2(\alpha^2-4a)^{3/2} \cdots}$$

2.28 problem 28

Internal problem ID [9611]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - a\,x^n y + ab\,x^n + b^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 89

```
dsolve(diff(y(x),x)=y(x)^2+a*x^n*y(x)-a*b*x^n-b^2,y(x), singsol=all)
```

$$c_1 + \int^{x} -\frac{(-\_ab + \_ay(x))\,\mathrm{e}^{\frac{2\_abn+a\_a^{1+n}+2\_ab}{1+n}}}{(b-y(x))\_a} d\_a - \frac{\mathrm{e}^{\frac{2bnx+a\,x^{1+n}+2xb}{1+n}}}{b-y(x)} = 0$$

✓ Solution by Mathematica

Time used: 1.254 (sec). Leaf size: 195

```
DSolve[y'[x]==y[x]^2+a*x^n*y[x]-a*b*x^n-b^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{e^{\frac{ax^{n+1}}{n+1}+2bx}}{an(K[2]-b)^2} - \int_1^x\left(\frac{e^{\frac{aK[1]^{n+1}}{n+1}+2bK[1]}(aK[1]^n+b+K[2])}{an(b-K[2])^2} + \frac{e^{\frac{aK[1]^{n+1}}{n+1}+2bK[1]}}{an(b-K[2])}\right)dK[1]\right)dK[2] + \int_1^x \frac{e^{\frac{aK[1]^{n+1}}{n+1}+2bK[1]}(aK[1]^n+b+y(x))}{an(b-y(x))}dK[1] = c_1, y(x)\right]$$

2.29 problem 29

Internal problem ID [9612]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (1+n)\, x^n y^2 - a\, x^{1+m+n} + a\, x^m = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=-(n+1)*x^n*y(x)^2+a*x^(n+m+1)-a*x^m,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(n+1)*x^n*y[x]^2+a*x^(n+m+1)-a*x^m,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.30 problem 30

Internal problem ID [9613]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 - b\,x^m y - x^m bc + a\,c^2 x^n = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 149

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*x^m*y(x)+b*c*x^m-a*c^2*x^n,y(x), singsol=all)
```

$$c_1 + \int^x -\frac{\left(-ac\_a^{1+n} - ay(x)\_a^{1+n}\right) e^{-\frac{2ac\_a^{1+n}m - \_a^{m+1}bn + 2ac\_a^{1+n} - \_a^{m+1}b}{(1+n)(m+1)}}}{(c+y(x))\_a} d\_a + \frac{e^{-\frac{2ac\,x^{1+n}m - x^{m+1}bn + 2ac\,x^{1+n} - x^{m+1}b}{(1+n)(m+1)}}}{c+y(x)} = 0$$

✓ Solution by Mathematica

Time used: 2.15 (sec). Leaf size: 286

```
DSolve[y'[x]==a*x^n*y[x]^2+b*x^m*y[x]+b*c*x^m-a*c^2*x^n,y[x],x,IncludeSingularSolutions -> Tr
```

$$\text{Solve}\Bigg[\int_1^{y(x)} \Bigg(\frac{e^{\frac{bx^{m+1}}{m+1} - \frac{2acx^{n+1}}{n+1}}}{ab(m-n)(c+K[2])^2} - \int_1^x \Bigg(-\frac{\exp\left(\frac{bK[1]^{m+1}}{m+1} - \frac{2acK[1]^{n+1}}{n+1}\right) K[1]^n}{b(m-n)(c+K[2])} - \frac{\exp\left(\frac{bK[1]^{m+1}}{m+1} - \frac{2acK[1]^{n+1}}{n+1}\right) \left(-bK[1]^m + acK[1]^n - aK[2]K[1\right.}{ab(m-n)(c+K[2])^2}$$

$$+ \int_1^x \frac{\exp\left(\frac{bK[1]^{m+1}}{m+1} - \frac{2acK[1]^{n+1}}{n+1}\right) \left(-bK[1]^m + acK[1]^n - ay(x)K[1]^n\right)}{ab(m-n)(c+y(x))} dK[1] = c_1, y(x)\Bigg]$$

2.31 problem 31

Internal problem ID [9614]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,x^n y^2 + a\,x^n (b\,x^m + c)\,y - bm\,x^{m-1} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2-a*x^n*(b*x^m+c)*y(x)+b*m*x^(m-1),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*x^n*y[x]^2-a*x^n*(b*x^m+c)*y[x]+b*m*x^(m-1),y[x],x,IncludeSingularSolutions -
```

Not solved

2.32 problem 32

Internal problem ID [9615]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + an\,x^{n-1}y^2 - c\,x^m(x^n a + b)\,y + c\,x^m = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 400

```
dsolve(diff(y(x),x)=-a*n*x^(n-1)*y(x)^2+c*x^m*(a*x^n+b)*y(x)-c*x^m,y(x), singsol=all)
```

$$y(x) = -\frac{-\left(\int -\frac{e^{\frac{cx\,x^m(x^n am + a\,x^n + mb + bn + b)}{(m+1)(m+n+1)}} an\,x^n}{(a\,x^n+b)^2 x}dx\right) x^n a - c_1 a\,x^n - \left(\int -\frac{e^{\frac{cx\,x^m(x^n am + a\,x^n + mb + bn + b)}{(m+1)(m+n+1)}} an}{(a\,x^n+b)^2 x}\right.}{\left(\int -\frac{e^{\frac{cx\,x^m(x^n am + a\,x^n + mb + bn + b)}{(m+1)(m+n+1)}} an\,x^n}{(a\,x^n+b)^2 x}dx\right) x^{2n}a^2 + x^{2n}c_1 a^2 + 2\left(\int -\frac{e^{\frac{cx\,x^m(x^n am + a\,x^n + mb + bn + b)}{(m+1)(m+n+1)}} an\,x^n}{(a\,x^n+b)^2 x}dx\right) x^n ab +}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-a*n*x^(n-1)*y[x]^2+c*x^m*(a*x^n+b)*y[x]-c*x^m,y[x],x,IncludeSingularSolutions
```

Not solved

2.33 problem 33

Internal problem ID [9616]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 - b\,x^m y - ck\,x^{k-1} + bc\,x^{m+k} + a\,c^2 x^{n+2k} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*x^m*y(x)+c*k*x^(k-1)-b*c*x^(m+k)-a*c^2*x^(n+2*k),y(x), sin
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*x^n*y[x]^2+b*x^m*y[x]+c*k*x^(k-1)-b*c*x^(m+k)-a*c^2*x^(n+2*k),y[x],x,IncludeS
```

Not solved

2.34 problem 34

Internal problem ID [9617]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - y^2a - by - c\,x^{2b} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 38

```
dsolve(x*diff(y(x),x)=a*y(x)^2+b*y(x)+c*x^(2*b),y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\frac{\sqrt{a}\,\sqrt{c}\,x^b - c_1 b}{b}\right)\sqrt{c}\,x^b}{\sqrt{a}}$$

✓ Solution by Mathematica

Time used: 0.345 (sec). Leaf size: 139

```
DSolve[x*y'[x]==a*y[x]^2+b*y[x]+c*x^(2*b),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{c}x^b\left(-\cos\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right) + c_1\sin\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)\right)}{\sqrt{a}\left(\sin\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right) + c_1\cos\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)\right)}$$

$$y(x) \to \frac{\sqrt{c}x^b\tan\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)}{\sqrt{a}}$$

2.35 problem 35

Internal problem ID [9618]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - y^2a - by - c\,x^n = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 225

```
dsolve(x*diff(y(x),x)=a*y(x)^2+b*y(x)+c*x^n,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{ac}\,x^{\frac{n}{2}} c_1 \operatorname{BesselY}\left(\frac{b+n}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)}{a\left(\operatorname{BesselY}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right) c_1 + \operatorname{BesselJ}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\right)} + \frac{\operatorname{BesselJ}\left(\frac{b+n}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\sqrt{ac}\,x^{\frac{n}{2}} - \operatorname{BesselY}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right) c_1 b - b\operatorname{BesselJ}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)}{a\left(\operatorname{BesselY}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right) c_1 + \operatorname{BesselJ}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.322 (sec). Leaf size: 205

```
DSolve[x*y'[x]==a*y[x]^2+b*y[x]+c*x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{c}x^{n/2}\left(-\operatorname{BesselJ}\left(\frac{b}{n}-1, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right) + c_1 \operatorname{BesselJ}\left(1-\frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)\right)}{\sqrt{a}\left(\operatorname{BesselJ}\left(\frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right) + c_1 \operatorname{BesselJ}\left(-\frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)\right)}$$

$$y(x) \to \frac{cx^n\,{}_0\tilde{F}_1\left(;2-\frac{b}{n};-\frac{acx^n}{n^2}\right)}{n\,{}_0\tilde{F}_1\left(;1-\frac{b}{n};-\frac{acx^n}{n^2}\right)}$$

2.36 problem 36

Internal problem ID [9619]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - y^2a - (n + b\,x^n)\,y - c\,x^{2n} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
dsolve(x*diff(y(x),x)=a*y(x)^2+(n+b*x^n)*y(x)+c*x^(2*n),y(x), singsol=all)
```

$$y(x) = \frac{x^{2n-1}\left(\sqrt{4b^2ac-b^4}\,\tan\left(\frac{\sqrt{4b^2ac-b^4}\,(b\,x^n+c_1n)}{2b^2n}\right)-b^2\right)x^{-n+1}}{2ab}$$

✓ Solution by Mathematica

Time used: 0.653 (sec). Leaf size: 94

```
DSolve[x*y'[x]==a*y[x]^2+(n+b*x^n)*y[x]+c*x^(2*n),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^n\left(-b+\sqrt{b^2-4ac}\left(-1+\frac{2c_1}{e^{\frac{x^n\sqrt{b^2-4ac}}{n}}+c_1}\right)\right)}{2a}$$

$$y(x) \to \frac{x^n\left(\sqrt{b^2-4ac}-b\right)}{2a}$$

2.37 problem 37

Internal problem ID [9620]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - y^2x - ya - b\,x^n = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 216

```
dsolve(x*diff(y(x),x)=x*y(x)^2+a*y(x)+b*x^n,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^{\frac{n}{2}+\frac{1}{2}}\sqrt{b}\,\mathrm{BesselY}\left(-\frac{a-n}{1+n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{1+n}\right)}{\left(\mathrm{BesselY}\left(-\frac{a+1}{1+n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{1+n}\right)c_1 + \mathrm{BesselJ}\left(-\frac{a+1}{1+n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{1+n}\right)\right)x} + \frac{\mathrm{BesselJ}\left(-\frac{a-n}{1+n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{1+n}\right)\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{\left(\mathrm{BesselY}\left(-\frac{a+1}{1+n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{1+n}\right)c_1 + \mathrm{BesselJ}\left(-\frac{a+1}{1+n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{1+n}\right)\right)x}$$

✓ Solution by Mathematica

Time used: 0.736 (sec). Leaf size: 855

```
DSolve[x*y'[x]==x*y[x]^2+a*y[x]+b*x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to$$

$$-\frac{\sqrt{b}(x^n)^{\frac{n+1}{2n}}\,\text{Gamma}\left(\frac{a+n+2}{n+1}\right)\text{BesselJ}\left(\frac{a-n}{n+1},\frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)-\sqrt{b}(x^n)^{\frac{n+1}{2n}}\,\text{Gamma}\left(\frac{a+n+2}{n+1}\right)\text{BesselJ}\left(\frac{a+n+2}{n+1},\right.}{}$$

$$y(x) \to -\frac{\frac{\sqrt{b}(x^n)^{\frac{n+1}{2n}}\left(\text{BesselJ}\left(-\frac{a+n+2}{n+1},\frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)-\text{BesselJ}\left(\frac{n-a}{n+1},\frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)\right)}{\text{BesselJ}\left(-\frac{a+1}{n+1},\frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)}+a+1}{2x}$$

$$y(x) \to -\frac{\frac{\sqrt{b}(x^n)^{\frac{n+1}{2n}}\left(\text{BesselJ}\left(-\frac{a+n+2}{n+1},\frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)-\text{BesselJ}\left(\frac{n-a}{n+1},\frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)\right)}{\text{BesselJ}\left(-\frac{a+1}{n+1},\frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)}+a+1}{2x}$$

2.38 problem 38

Internal problem ID [9621]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'x + a_3xy^2 + a_2y + a_1x + a_0 = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 848

```
dsolve(x*diff(y(x),x)+a__3*x*y(x)^2+a__2*y(x)+a__1*x+a__0=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.478 (sec). Leaf size: 421

```
DSolve[x*y'[x]+a3*x*y[x]^2+a2*y[x]+a1*x+a0==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\left(\sqrt{\text{a1}}c_1 \text{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\text{a3}}\text{a0}}{\sqrt{\text{a1}}} + \text{a2}\right), \text{a2}, 2i\sqrt{\text{a1}}\sqrt{\text{a3}}x\right) + c_1\left(\sqrt{\text{a1}}\text{a2} + i\text{a0}\sqrt{\text{a3}}\right) \text{Hypergeomet}}{\sqrt{\text{a3}}\left(c_1 \text{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\text{a3}}\text{a0}}{\sqrt{\text{a1}}}\right.\right.\right.}$$

$$y(x) \to \frac{\frac{\left(\text{a0}\sqrt{\text{a3}} - i\sqrt{\text{a1}}\text{a2}\right) \text{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\text{a3}}\text{a0}}{\sqrt{\text{a1}}} + \text{a2} + 2\right), \text{a2} + 1, 2i\sqrt{\text{a1}}\sqrt{\text{a3}}x\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\text{a3}}\text{a0}}{\sqrt{\text{a1}}} + \text{a2}\right), \text{a2}, 2i\sqrt{\text{a1}}\sqrt{\text{a3}}x\right)} - i\sqrt{\text{a1}}}{\sqrt{\text{a3}}}$$

2.39 problem 39

Internal problem ID [9622]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Riccati]`

$$\boxed{y'x - a\,x^n y^2 - by - c\,x^{-n} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 69

```
dsolve(x*diff(y(x),x)=a*x^n*y(x)^2+b*y(x)+c*x^(-n),y(x), singsol=all)
```

$$y(x) = -\frac{x^{-n}\left(b+n+\tan\left(\frac{\sqrt{4ac-b^2-2bn-n^2}\,(-\ln(x)+c_1)}{2}\right)\sqrt{4ac-b^2-2bn-n^2}\right)}{2a}$$

✓ Solution by Mathematica

Time used: 0.632 (sec). Leaf size: 103

```
DSolve[x*y'[x]==a*x^n*y[x]^2+b*y[x]+c*x^(-n),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-n}\left(\sqrt{(b+n)^2-4ac}\left(-1+\frac{2c_1}{x^{\sqrt{(b+n)^2-4ac}}+c_1}\right)-b-n\right)}{2a}$$

$$y(x) \to \frac{x^{-n}\left(\sqrt{(b+n)^2-4ac}-b-n\right)}{2a}$$

2.40 problem 40

Internal problem ID [9623]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - a\,x^n y^2 - my + a\,b^2 x^{n+2m} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x)=a*x^n*y(x)^2+m*y(x)-a*b^2*x^(n+2*m),y(x), singsol=all)
```

$$y(x) = i\tan\left(\frac{iab\,x^{m+n} + c_1 m + c_1 n}{m+n}\right) b\,x^m$$

✓ Solution by Mathematica

Time used: 1.05 (sec). Leaf size: 43

```
DSolve[x*y'[x]==a*x^n*y[x]^2+m*y[x]-a*b^2*x^(n+2*m),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{-b^2}x^m \tan\left(\frac{a\sqrt{-b^2}x^{m+n}}{m+n} + c_1\right)$$

2.41 problem 41

Internal problem ID [9624]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - x^{2n}y^2 - (m-n)\,y - x^{2m} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x*diff(y(x),x)=x^(2*n)*y(x)^2+(m-n)*y(x)+x^(2*m),y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{-c_1 m - c_1 n + x^{m+n}}{m+n}\right) x^{m-n}$$

✓ Solution by Mathematica

Time used: 0.48 (sec). Leaf size: 28

```
DSolve[x*y'[x]==x^(2*n)*y[x]^2+(m-n)*y[x]+x^(2*m),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{m-n} \tan\left(\frac{x^{m+n}}{m+n} + c_1\right)$$

2.42 problem 42

Internal problem ID [9625]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - a\,x^n y^2 - by - c\,x^m = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 172

```
dsolve(x*diff(y(x),x)=a*x^(n)*y(x)^2+b*y(x)+c*x^(m),y(x), singsol=all)
```

$$y(x) = \frac{\left(\text{BesselY}\left(-\frac{b-m}{m+n}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}+\frac{n}{2}}}{m+n}\right)c_1 + \text{BesselJ}\left(-\frac{b-m}{m+n}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}+\frac{n}{2}}}{m+n}\right)\right)x^{\frac{m}{2}+\frac{n}{2}}\sqrt{ac}\,x^{-n+1}}{\left(\text{BesselY}\left(-\frac{b+n}{m+n}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}+\frac{n}{2}}}{m+n}\right)c_1 + \text{BesselJ}\left(-\frac{b+n}{m+n}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}+\frac{n}{2}}}{m+n}\right)\right)ax}$$

✓ Solution by Mathematica

Time used: 0.802 (sec). Leaf size: 698

```
DSolve[x*y'[x]==a*x^(n)*y[x]^2+b*y[x]+c*x^(m),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-n}\left(\frac{\sqrt{a}\sqrt{c}x^{m+n}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{x^{m+n}}}{\sqrt{(m+n)^2}}\right)^{-\frac{b+m+2n}{m+n}}\left(-2(m+n)^{\frac{2b+3m+5n}{m+n}}\text{Gamma}\left(\frac{b+m+2n}{m+n}\right)\left(\frac{\sqrt{a}\sqrt{c}\sqrt{x^{m+n}}}{\sqrt{(m+n)^2}}\right)^{\frac{2(b+n)}{m+n}}{}_0\tilde{F}_1\left(;\frac{b+n}{m+n};-\frac{acx^{m+n}}{(m+n)^2}\right)+c\right.}{(m+n)^2}\right.}{2a\sqrt{(m+n)^2}\sqrt{x^{m+n}}\left((m+n)^{\frac{2(b+n)}{m+n}}\text{Gamm}\right.}$$

$$y(x) \to \frac{x^{-n}\left(acx^{m+n}\,{}_0\tilde{F}_1\left(;\frac{m-b}{m+n}+1;-\frac{acx^{m+n}}{(m+n)^2}\right)-(b+n)(m+n)\,{}_0\tilde{F}_1\left(;\frac{m-b}{m+n};-\frac{acx^{m+n}}{(m+n)^2}\right)-(m+n)^2\,{}_0\tilde{F}_1\left(;-\frac{b+n}{m+n};\right.\right.}{2a(m+n)\,{}_0\tilde{F}_1\left(;\frac{m-b}{m+n};-\frac{acx^{m+n}}{(m+n)^2}\right)}$$

2.43 problem 43

Internal problem ID [9626]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'x - x^{2n}y^2a - (b\,x^n - n)\,y - c = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 72

```
dsolve(x*diff(y(x),x)=a*x^(2*n)*y(x)^2+(b*x^n-n)*y(x)+c,y(x), singsol=all)
```

$$y(x) = \frac{\left(\sqrt{4b^2ac - b^4}\,\tan\left(\frac{\sqrt{4b^2ac-b^4}\,(b\,x^n + c_1 n)}{2b^2 n}\right) - b^2\right) x^{-n}}{2ab}$$

✓ Solution by Mathematica

Time used: 0.693 (sec). Leaf size: 98

```
DSolve[x*y'[x]==a*x^(2*n)*y[x]^2+(b*x^n-n)*y[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-n}\left(-b + \sqrt{b^2 - 4ac}\left(-1 + \frac{2c_1}{e^{\frac{x^n\sqrt{b^2-4ac}}{n}} + c_1}\right)\right)}{2a}$$

$$y(x) \to \frac{x^{-n}\left(\sqrt{b^2 - 4ac} - b\right)}{2a}$$

2.44 problem 44

Internal problem ID [9627]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - a\,x^{m+2n}y^2 - \left(b\,x^{m+n} - n\right)y - c\,x^m = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 87

```
dsolve(x*diff(y(x),x)=a*x^(2*n+m)*y(x)^2+(b*x^(n+m)-n)*y(x)+c*x^m,y(x), singsol=all)
```

$$y(x) = \frac{x^{m-1}\left(\sqrt{4b^2ac - b^4}\,\tan\left(\frac{\sqrt{4b^2ac-b^4}\,(x^{m+n}b + c_1m + c_1n)}{2b^2(m+n)}\right) - b^2\right)x^{-m-n+1}}{2ab}$$

✓ Solution by Mathematica

Time used: 0.987 (sec). Leaf size: 102

```
DSolve[x*y'[x]==a*x^(2*n+m)*y[x]^2+(b*x^(n+m)-n)*y[x]+c*x^m,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{x^{-n}\left(-b + \sqrt{b^2 - 4ac}\left(-1 + \frac{2c_1}{e^{\frac{\sqrt{b^2-4ac}x^{m+n}}{m+n}} + c_1}\right)\right)}{2a}$$

$$y(x) \to \frac{x^{-n}\left(\sqrt{b^2 - 4ac} - b\right)}{2a}$$

2.45 problem 45

Internal problem ID [9628]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(a_2x+b_2)\left(y'+\lambda y^2\right)+(a_1x+b_1)\,y+a_0x+b_0=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 964

```
dsolve((a__2*x+b__2)*(diff(y(x),x)+lambda*y(x)^2)+(a__1*x+b__1)*y(x)+a__0*x+b__0=0,y(x), sing
```

Expression too large to display

✓ Solution by Mathematica

Time used: 1.904 (sec). Leaf size: 1418

```
DSolve[(a2*x+b2)*(y'[x]+\[Lambda]*y[x]^2)+(a1*x+b1)*y[x]+a0*x+b0==0,y[x],x,IncludeSingularSol
```

Too large to display

2.46 problem 46

Internal problem ID [9629]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Riccati]`

$$\boxed{(xa+c)\,y'-\alpha(ya+bx)^2-\beta(ya+bx)+bx-\gamma=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 94

```
dsolve((a*x+c)*diff(y(x),x)=alpha*(a*y(x)+b*x)^2+beta*(a*y(x)+b*x)-b*x+gamma,y(x), singsol=al
```

$$y(x)=\frac{-2a^2\alpha bx-a^2\beta+\tan\left(\frac{-2c_1a^2+\ln(ax+c)\sqrt{a^3(4\alpha\gamma a-a\,\beta^2+4\alpha bc)}}{2a^2}\right)\sqrt{a^3\,(4\alpha\gamma a-a\,\beta^2+4\alpha bc)}}{2a^3\alpha}$$

✓ Solution by Mathematica

Time used: 60.358 (sec). Leaf size: 98

```
DSolve[(a*x+c)*y'[x]==\[Alpha]*(a*y[x]+b*x)^2+\[Beta]*(a*y[x]+b*x)-b*x+\[Gamma],y[x],x,Includ
```

$$y(x)\to-\frac{-a\alpha\sqrt{\frac{4a\alpha\gamma-a\beta^2+4\alpha bc}{a^3\alpha^2}}\tan\left(\frac{1}{2}a\alpha\log(ax+c)\sqrt{\frac{4a\alpha\gamma-a\beta^2+4\alpha bc}{a^3\alpha^2}}+c_1\right)+2\alpha bx+\beta}{2a\alpha}$$

2.47 problem 47

Internal problem ID [9630]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{2x^2y' - 2y^2 - xy + 2a^2x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(2*x^2*diff(y(x),x)=2*y(x)^2+x*y(x)-2*a^2*x,y(x), singsol=all)
```

$$y(x) = -i \tan\left(\frac{2ia - c_1\sqrt{x}}{\sqrt{x}}\right) a\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.413 (sec). Leaf size: 43

```
DSolve[2*x^2*y'[x]==2*y[x]^2+x*y[x]-2*a^2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-a^2}\sqrt{x} \tan\left(\frac{2\sqrt{-a^2}}{\sqrt{x}} - c_1\right)$$

2.48 problem 48

Internal problem ID [9631]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{2x^2y' - 2y^2 - 3xy + 2a^2x = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 102

```
dsolve(2*x^2*diff(y(x),x)=2*y(x)^2+3*x*y(x)-2*a^2*x,y(x), singsol=all)
```

$$y(x) = \frac{\left(-2c_1x\sqrt{-\frac{a^2}{x}} - x\right)\sin\left(2\sqrt{-\frac{a^2}{x}}\right) - x\left(c_1 - 2\sqrt{-\frac{a^2}{x}}\right)\cos\left(2\sqrt{-\frac{a^2}{x}}\right)}{2\cos\left(2\sqrt{-\frac{a^2}{x}}\right)c_1 + 2\sin\left(2\sqrt{-\frac{a^2}{x}}\right)}$$

✓ Solution by Mathematica

Time used: 0.26 (sec). Leaf size: 66

```
DSolve[2*x^2*y'[x]==2*y[x]^2+3*x*y[x]-2*a^2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4a^2c_1\sqrt{x}}{e^{\frac{4a}{\sqrt{x}}} - 2ac_1} + a\sqrt{x} - \frac{x}{2}$$

$$y(x) \to a(-\sqrt{x}) - \frac{x}{2}$$

2.49 problem 49

Internal problem ID [9632]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{x^2y' - a\,x^2y^2 - bxy - c = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 57

```
dsolve(x^2*diff(y(x),x)=a*x^2*y(x)^2+b*x*y(x)+c,y(x), singsol=all)
```

$$y(x) = -\frac{b + 1 + \tan\left(\frac{\sqrt{4ac-b^2-2b-1}\,(-\ln(x)+c_1)}{2}\right)\sqrt{4ac-b^2-2b-1}}{2ax}$$

✓ Solution by Mathematica

Time used: 0.29 (sec). Leaf size: 93

```
DSolve[x^2*y'[x]==a*x^2*y[x]^2+b*x*y[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{(b+1)^2-4ac}\left(1 - \frac{2c_1}{x^{\sqrt{(b+1)^2-4ac}}+c_1}\right) + b + 1}{2ax}$$

$$y(x) \to \frac{\sqrt{(b+1)^2-4ac} - b - 1}{2ax}$$

2.50 problem 50

Internal problem ID [9633]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x^2y' - y^2c\,x^2 - \left(a\,x^2 + bx\right)y - \alpha\,x^2 - \beta x - \gamma = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 724

```
dsolve(x^2*diff(y(x),x)=c*x^2*y(x)^2+(a*x^2+b*x)*y(x)+alpha*x^2+beta*x+gamma,y(x), singsol=al
```

$$y(x) = -\frac{\left(\sqrt{a^2 - 4\alpha c}\,c_1a^3x - 4\sqrt{a^2 - 4\alpha c}\,c_1a\alpha cx + c_1a^4x - 8c_1a^2\alpha cx + 16c_1\alpha^2c^2x + \sqrt{a^2 - 4\alpha c}\,c_1a^2b - 4\sqrt{a^2}\right.}{}$$

✓ Solution by Mathematica

Time used: 0.938 (sec). Leaf size: 1312

```
DSolve[x^2*y'[x]==c*x^2*y[x]^2+(a*x^2+b*x)*y[x]+\[Alpha]*x^2+\[Beta]*x+\[Gamma],y[x],x,Includ
```

$$y(x) \to$$

$$-\frac{\left(b+ax-x\sqrt{a^2-4c\alpha}+\sqrt{b^2+2b-4c\gamma+1}+1\right)c_1 \text{HypergeometricU}\left(\frac{ab-2c\beta+\sqrt{a^2-4c\alpha}\left(\sqrt{b^2+2b-4c\gamma+1}+\right.}{2\sqrt{a^2-4c\alpha}}\right.}{}$$

$$y(x)$$

$$\to \frac{\frac{\left(\sqrt{a^2-4\alpha c}\left(\sqrt{b^2+2b-4c\gamma+1}+1\right)+ab-2\beta c\right)\text{HypergeometricU}\left(\frac{ab-2c\beta+\sqrt{a^2-4c\alpha}\left(\sqrt{b^2+2b-4c\gamma+1}+3\right)}{2\sqrt{a^2-4c\alpha}},\sqrt{b^2+2b-4c\gamma+1}+2,x\sqrt{a^2-4c\alpha}\right)}{\text{HypergeometricU}\left(\frac{ab-2c\beta+\sqrt{a^2-4c\alpha}\left(\sqrt{b^2+2b-4c\gamma+1}+1\right)}{2\sqrt{a^2-4c\alpha}},\sqrt{b^2+2b-4c\gamma+1}+1,x\sqrt{a^2-4c\alpha}\right)} - }{2c}$$

2.51 problem 51

Internal problem ID [9634]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x^2y' - a\,x^2y^2 - bxy - c\,x^n - s = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 299

```
dsolve(x^2*diff(y(x),x)=a*x^2*y(x)^2+b*x*y(x)+c*x^n+s,y(x), singsol=all)
```

$$y(x) = \frac{\left(-\sqrt{-4as+b^2+2b+1}\,c_1 - c_1 b - c_1\right)\text{BesselY}\left(\frac{\sqrt{-4as+b^2+2b+1}}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right) + 2\sqrt{ac}\,\text{BesselY}\left(\frac{\sqrt{-4as+b^2+2b+1}}{n}\right.}{2xa\left(\text{BesselY}\left(\frac{\sqrt{-4as+b^2}}{n}\right.\right.}$$

✓ Solution by Mathematica

Time used: 1.41 (sec). Leaf size: 2281

```
DSolve[x^2*y'[x]==a*x^2*y[x]^2+b*x*y[x]+c*x^n+s,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

2.52 problem 52

Internal problem ID [9635]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^2y' - a\,x^2y^2 - bxy - c\,x^{2n} - s\,x^n = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 442

```
dsolve(x^2*diff(y(x),x)=a*x^2*y(x)^2+b*x*y(x)+c*x^(2*n)+s*x^n,y(x), singsol=all)
```

$$y(x) = -\frac{\left(2ix^n\sqrt{a}\,c_1c + i\sqrt{a}\,c_1s + \sqrt{c}\,c_1b - \sqrt{c}\,c_1n + \sqrt{c}\,c_1\right)\mathrm{KummerU}\left(\frac{i\sqrt{a}\,s+\sqrt{c}\,b+\sqrt{c}\,n+\sqrt{c}}{2\sqrt{c}\,n}, \frac{b+n+1}{n}, \frac{2i\sqrt{a}\,\sqrt{c}\,x^n}{n}\right) -}{}$$

✓ Solution by Mathematica

Time used: 1.062 (sec). Leaf size: 638

```
DSolve[x^2*y'[x]==a*x^2*y[x]^2+b*x*y[x]+c*x^(2*n)+s*x^n,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{i\sqrt{a}c_1x^n\left(\sqrt{c}(b+n+1)-i\sqrt{a}s\right)\text{HypergeometricU}\left(\frac{b+3n-\frac{i\sqrt{a}s}{\sqrt{c}}+1}{2n},\frac{b+2n+1}{n},-\frac{2i\sqrt{a}\sqrt{c}x^n}{n}\right)+c_1n\left(i\sqrt{a}\sqrt{c}x^n\right.}{anx\left(c_1\,\text{Hypergeon}\right.}$$

$$y(x) \to -\frac{\frac{\sqrt{a}x^n\left(\sqrt{a}s+i\sqrt{c}(b+n+1)\right)\text{HypergeometricU}\left(\frac{b+3n-\frac{i\sqrt{a}s}{\sqrt{c}}+1}{2n},\frac{b+2n+1}{n},-\frac{2i\sqrt{a}\sqrt{c}x^n}{n}\right)}{n\,\text{HypergeometricU}\left(\frac{b+n-\frac{i\sqrt{a}s}{\sqrt{c}}+1}{2n},\frac{b+n+1}{n},-\frac{2i\sqrt{a}\sqrt{c}x^n}{n}\right)}+i\sqrt{a}\sqrt{c}x^n+b+1}{ax}$$

2.53 problem 53

Internal problem ID [9636]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^2y' - y^2c\,x^2 - (x^n a + b)\,xy - \alpha\,x^{2n} - \beta\,x^n - \gamma = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 1036

```
dsolve(x^2*diff(y(x),x)=c*x^2*y(x)^2+(a*x^n+b)*x*y(x)+alpha*x^(2*n)+beta*x^n+gamma,y(x), sing
```

Expression too large to display

✓ Solution by Mathematica

Time used: 2.16 (sec). Leaf size: 2380

```
DSolve[x^2*y'[x]==c*x^2*y[x]^2+(a*x^n+b)*x*y[x]+\[Alpha]*x^(2*n)+\[Beta]*x^n+\[Gamma],y[x],x,
```

Too large to display

2.54 problem 54

Internal problem ID [9637]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x^2 - \left(\alpha\, x^{2n} + \beta\, x^n + \gamma\right) y^2 - (a\, x^n + b)\, xy - c\, x^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 215224

```
dsolve(x^2*diff(y(x),x)=(alpha*x^(2*n)+beta*x^n+gamma)*y(x)^2+(a*x^n+b)*x*y(x)+c*x^2,y(x), si
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==(\[Alpha]*x^(2*n)+\[Beta]*x^n+\[Gamma])*y[x]^2+(a*x^n+b)*x*y[x]+c*x^2,y[x],
```

Not solved

2.55 problem 55

Internal problem ID [9638]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(x^2-1)\,y'+\lambda(y^2-2xy+1)=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 231

```
dsolve((x^2-1)*diff(y(x),x)+lambda*(y(x)^2-2*x*y(x)+1)=0,y(x), singsol=all)
```

$$y(x) = \frac{8c_1\left(\left(\lambda-\frac{1}{2}\right)x-\frac{\lambda}{2}+\frac{1}{2}\right)(x+1)\operatorname{HeunC}\left(0,-2\lambda+1,0,0,\lambda^2-\lambda+\frac{1}{2},\frac{2}{x+1}\right)-\lambda\left(-\frac{x}{2}-\frac{1}{2}\right)^{-2\lambda+1}(x+1)\operatorname{Heu}}{4\lambda\left(\operatorname{HeunC}\left(0,-2\lambda+1,0,0\right.\right.}$$

✓ Solution by Mathematica

Time used: 0.387 (sec). Leaf size: 47

```
DSolve[(x^2-1)*y'[x]+\[Lambda]*(y[x]^2-2*x*y[x]+1)==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to\frac{\operatorname{LegendreQ}(\lambda,x)+c_1\operatorname{LegendreP}(\lambda,x)}{\operatorname{LegendreQ}(\lambda-1,x)+c_1\operatorname{LegendreP}(\lambda-1,x)}$$

$$y(x)\to\frac{\operatorname{LegendreP}(\lambda,x)}{\operatorname{LegendreP}(\lambda-1,x)}$$

2.56 problem 56

Internal problem ID [9639]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(a\,x^2+b)\,y'+\alpha y^2+\beta xy+\frac{b(a+\beta)}{\alpha}=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 563

```
dsolve((a*x^2+b)*diff(y(x),x)+alpha*y(x)^2+beta*x*y(x)+b/alpha*(a+beta)=0,y(x), singsol=all)
```

$$y(x) = -\frac{b\left(-\frac{\left(-\frac{-ax+\sqrt{-ba}}{2\sqrt{-ba}}\right)^{\frac{a+\beta}{\alpha}}(ax-\sqrt{-ba})\,\mathrm{HeunC}\left(0,\frac{-a-\beta}{\alpha},\frac{2a+\beta}{2a},0,\frac{2a^2+2a\beta+\beta^2}{4a^2},\frac{2\sqrt{-ba}}{-ax+\sqrt{-ba}}\right)}{2}+\left(-\frac{-ax+\sqrt{-ba}}{2\sqrt{-ba}}\right)^{\frac{a+\beta}{\alpha}}(ax+\sqrt{-ba})}{\alpha\left(\left(-\right.\right.}$$

✓ Solution by Mathematica

Time used: 0.671 (sec). Leaf size: 27

```
DSolve[(a*x^2+b)*y'[x]+\[Alpha]*y[x]^2+\[Beta]*x*y[x]+b/\[Alpha]*(a+\[Beta])==0,y[x],x,Includ
```

$$y(x)\to -\frac{x(a+\beta)}{\alpha}$$
$$y(x)\to -\frac{x(a+\beta)}{\alpha}$$

2.57 problem 57

Internal problem ID [9640]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(a\,x^2+b)\,y'+\alpha y^2+\beta xy+\gamma=0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 682

```
dsolve((a*x^2+b)*diff(y(x),x)+alpha*y(x)^2+beta*x*y(x)+gamma=0,y(x), singsol=all)
```

$y(x) =$

$$-\frac{}{2c_1\left(\frac{\left(-\sqrt{-ba}\,x-b\right)\sqrt{4\gamma\alpha ba+b^2\beta^2}}{2}+b\left(\frac{\sqrt{-ba}\,\beta x}{2}+\left(a-\frac{\beta}{2}\right)b+a\,x^2\,(a-\beta)\right)\right)\operatorname{HeunC}\left(0,\frac{a-\beta}{a},-\frac{\sqrt{4\gamma\alpha ba+b^2\beta^2}}{2ab},0,\right.}$$

✓ Solution by Mathematica

Time used: 0.667 (sec). Leaf size: 598

```
DSolve[(a*x^2+b)*y'[x]+\[Alpha]*y[x]^2+\[Beta]*x*y[x]+\[Gamma]==0,y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{i\left(c_1\left(\sqrt{4a\alpha\gamma+b\beta^2}-2a\sqrt{b}-\sqrt{b}\beta\right)P_{\frac{\beta}{2a}+1}^{\frac{\sqrt{b\beta^2+4a\alpha\gamma}}{2a\sqrt{b}}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)+2i\sqrt{a}x(a+\beta)Q_{\frac{\beta}{2a}}^{\frac{\sqrt{b\beta^2+4a\alpha\gamma}}{2a\sqrt{b}}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)+\left(\sqrt{4a\alpha\gamma+\cdots\right.\right.}{2\sqrt{a}\alpha\left(c_1P_{\frac{\beta}{2a}}^{\frac{\sqrt{b\beta^2+4a\alpha\gamma}}{2a\sqrt{b}}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)+Q_{\frac{\beta}{2a}}^{\frac{\sqrt{b\beta^2+4\cdots}}{2a\sqrt{b}}}\cdots\right.}$$

$$y(x) \to \frac{-2x(a+\beta)+\frac{i\left(\sqrt{4a\alpha\gamma+b\beta^2}-2a\sqrt{b}-\sqrt{b}\beta\right)P_{\frac{\beta}{2a}+1}^{\frac{\sqrt{b\beta^2+4a\alpha\gamma}}{2a\sqrt{b}}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}P_{\frac{\beta}{2a}}^{\frac{\sqrt{b\beta^2+4a\alpha\gamma}}{2a\sqrt{b}}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)}}{2\alpha}$$

2.58 problem 58

Internal problem ID [9641]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccat`

$$\boxed{(a\,x^2+b)\,y'+y^2-2xy+(-a+1)\,x^2-b=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve((a*x^2+b)*diff(y(x),x)+y(x)^2-2*x*y(x)+(1-a)*x^2-b=0,y(x), singsol=all)
```

$$y(x)=x+\frac{1}{c_1+\frac{\arctan\left(\frac{ax}{\sqrt{ba}}\right)}{\sqrt{ba}}}$$

✓ Solution by Mathematica

Time used: 0.398 (sec). Leaf size: 41

```
DSolve[(a*x^2+b)*y'[x]+y[x]^2-2*x*y[x]+(1-a)*x^2-b==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to x+\frac{1}{\frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}+c_1}$$

$$y(x)\to x$$

2.59 problem 59

Internal problem ID [9642]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccat`

$$\boxed{(a\,x^2+bx+c)\,y'-y^2-(2\lambda x+b)\,y-\lambda(\lambda-a)\,x^2-\mu=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5761

```
dsolve((a*x^2+b*x+c)*diff(y(x),x)=y(x)^2+(2*lambda*x+b)*y(x)+lambda*(lambda-a)*x^2+mu,y(x), s
```

Expression too large to display

✓ Solution by Mathematica

Time used: 11.43 (sec). Leaf size: 93

```
DSolve[(a*x^2+b*x+c)*y'[x]==y[x]^2+(2*\[Lambda]*x+b)*y[x]+\[Lambda]*(\[Lambda]-a)*x^2+\[Mu],y
```

$$y(x)\to \frac{1}{2}\left(\sqrt{4(c\lambda+\mu)-b^2}\tan\left(\frac{\sqrt{-b^2+4c\lambda+4\mu}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+c_1\right)-b-2\lambda x\right)$$

2.60 problem 60

Internal problem ID [9643]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(a\,x^2+bx+c)\,y'-y^2-(ax+\mu)\,y+\lambda^2x^2-\lambda(b-\mu)\,x-c\lambda=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 476004

```
dsolve((a*x^2+b*x+c)*diff(y(x),x)=y(x)^2+(a*x+mu)*y(x)-lambda^2*x^2+lambda*(b-mu)*x+lambda*c,
```

Expression too large to display

✓ Solution by Mathematica

Time used: 12.45 (sec). Leaf size: 183

```
DSolve[(a*x^2+b*x+c)*y'[x]==y[x]^2+(a*x+\[Mu])*y[x]-\[Lambda]^2*x^2+\[Lambda]*(b-\[Mu])*x+\[L
```

$$y(x)\to \lambda x-\frac{(x(ax+b)+c)^{\frac{\lambda}{a}+\frac{1}{2}}\exp\left(-\frac{(a(b-2\mu)+2b\lambda)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a\sqrt{4ac-b^2}}\right)}{\int_1^x\exp\left(-\frac{\frac{2(ab+2\lambda b-2a\mu)\arctan\left(\frac{b+2aK[1]}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+(a-2\lambda)\log(c+K[1](b+aK[1]))}{2a}\right)dK[1]+c_1}$$

$$y(x)\to \lambda x$$

2.61 problem 61

Internal problem ID [9644]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\left(a_2x^2+b_2x+c_2\right)y'-y^2-\left(a_1x+b_1\right)y+\lambda(\lambda+a_1-a_2)\,x^2-\lambda(b_2-b_1)\,x-\lambda c_2=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 545907

```
dsolve((a__2*x^2+b__2*x+c__2)*diff(y(x),x)=y(x)^2+(a__1*x+b__1)*y(x)-lambda*(lambda+a__1-a__2
```

Expression too large to display

✓ Solution by Mathematica

Time used: 22.144 (sec). Leaf size: 189

```
DSolve[(a2*x^2+b2*x+c2)*y'[x]==y[x]^2+(a1*x+b1)*y[x]-\[Lambda]*(\[Lambda]+a1-a2)*x^2+\[Lambda
```

$$y(x)\to \lambda x$$

$$-\frac{(x(\text{a2}x+\text{b2})+\text{c2})^{\frac{\text{a1}+2\lambda}{2\text{a2}}}\exp\left(\frac{(2\text{a2}\text{b1}-\text{b2}(\text{a1}+2\lambda))\arctan\left(\frac{2\text{a2}x+\text{b2}}{\sqrt{4\text{a2}\text{c2}-\text{b2}^2}}\right)}{\text{a2}\sqrt{4\text{a2}\text{c2}-\text{b2}^2}}\right)}{\int_1^x\exp\left(\frac{(\text{a1}-2\text{a2}+2\lambda)\log(\text{c2}+K[1](\text{b2}+\text{a2}K[1]))-\frac{2(-2\text{a2}\text{b1}+\text{a1}\text{b2}+2\text{b2}\lambda)\arctan\left(\frac{\text{b2}+2\text{a2}K[1]}{\sqrt{4\text{a2}\text{c2}-\text{b2}^2}}\right)}{\sqrt{4\text{a2}\text{c2}-\text{b2}^2}}}{2\text{a2}}\right)dK[1]+c_1}$$

$$y(x)\to \lambda x$$

2.62 problem 62

Internal problem ID [9645]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(a_2x^2 + b_2x + c_2)\,y' - y^2 - (a_1x + b_1)\,y - a_0x^2 - b_0x - c_0 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 404913

```
dsolve((a__2*x^2+b__2*x+c__2)*diff(y(x),x)=y(x)^2+(a__1*x+b__1)*y(x)+a__0*x^2+b__0*x+c__0,y(x
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a2*x^2+b2*x+c2)*y'[x]==y[x]^2+(a1*x+b1)*y[x]+a0*x^2+b0*x+c0,y[x],x,IncludeSingularSol
```

Not solved

2.63 problem 63

Internal problem ID [9646]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccat`

$$(-a+x)(x-b)y'+y^2+k(y+x-a)(y+x-b)=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 128

```
dsolve((x-a)*(x-b)*diff(y(x),x)+y(x)^2+k*(y(x)+x-a)*(y(x)+x-b)=0,y(x), singsol=all)
```

$$y(x)=\frac{k\left(\frac{bc_1(-x+b)^k}{c_1(-x+b)^k+(-x+a)^k}-\frac{xc_1(-x+b)^k}{c_1(-x+b)^k+(-x+a)^k}+\frac{a(-x+a)^k}{c_1(-x+b)^k+(-x+a)^k}-\frac{x(-x+a)^k}{c_1(-x+b)^k+(-x+a)^k}\right)}{1+k}$$

✓ Solution by Mathematica

Time used: 60.359 (sec). Leaf size: 99

```
DSolve[(x-a)*(x-b)*y'[x]+y[x]^2+k*(y[x]+x-a)*(y[x]+x-b)==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{1}{2}\left(\frac{k(a+b-2x)}{k+1}+\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}\tan\left(\frac{(k+1)\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}(\log(x-b)-\log(x-a))}{2(a-b)}+c_1\right)\right)$$

2.64 problem 64

Internal problem ID [9647]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\left(c_2x^2+b_2x+a_2\right)\left(y'+\lambda y^2\right)+\left(b_1x+a_1\right)y+a_0=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18039

```
dsolve((c__2*x^2+b__2*x+a__2)*(diff(y(x),x)+lambda*y(x)^2)+(b__1*x+a__1)*y(x)+a__0=0,y(x), si
```

Expression too large to display

✓ Solution by Mathematica

Time used: 7.783 (sec). Leaf size: 1986

```
DSolve[(c2*x^2+b2*x+a2)*(y'[x]+\[Lambda]*y[x]^2)+(b1*x+a1)*y[x]+a0==0,y[x],x,IncludeSingularS
```

Too large to display

2.65 problem 65

Internal problem ID [9648]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^3y' - a\,x^3y^2 - \left(b\,x^2 + c\right)y - sx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 327

```
dsolve(x^3*diff(y(x),x)=a*x^3*y(x)^2+(b*x^2+c)*y(x)+s*x,y(x), singsol=all)
```

$$y(x) = -\frac{2c_1 \operatorname{KummerU}\left(\frac{\sqrt{-4as+b^2+2b+1}}{4} + \frac{b}{4} + \frac{1}{4}, 1 + \frac{\sqrt{-4as+b^2+2b+1}}{2}, \frac{c}{2x^2}\right)}{xa\left(\operatorname{KummerU}\left(\frac{5}{4} + \frac{\sqrt{-4as+b^2+2b+1}}{4} + \frac{b}{4}, 1 + \frac{\sqrt{-4as+b^2+2b+1}}{2}, \frac{c}{2x^2}\right)c_1 + \operatorname{KummerM}\left(\frac{5}{4} + \frac{\sqrt{-4as+b^2+2b+1}}{4} + \right.\right.}$$
$$+\frac{\left(-1 + \sqrt{-4as + b^2 + 2b + 1} - b\right)\operatorname{KummerM}\left(\frac{\sqrt{-4as+b^2+2b+1}}{4} + \frac{b}{4} + \frac{1}{4}, 1 + \frac{\sqrt{-4as+b^2+}}{2}\right.}{2xa\left(\operatorname{KummerU}\left(\frac{5}{4} + \frac{\sqrt{-4as+b^2+2b+1}}{4} + \frac{b}{4}, 1 + \frac{\sqrt{-4as+b^2+2b+1}}{2}, \frac{c}{2x^2}\right)c_1 + \operatorname{KummerM}\left(\frac{5}{4} + \frac{\sqrt{-4as+b^2+2b+1}}{4} - \right.\right.}$$

✓ Solution by Mathematica

Time used: 1.828 (sec). Leaf size: 756

```
DSolve[x^3*y'[x]==a*x^3*y[x]^2+(b*x^2+c)*y[x]+s*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{-\left(\sqrt{(b+1)^2-4as}-b-1\right)c^{\frac{1}{2}\sqrt{(b+1)^2-4as}}\left(\frac{1}{x}\right)^{\sqrt{(b+1)^2-4as}}\left(2x^2\,\text{Hypergeometric1F1}\left(\frac{1}{4}\left(-b+\sqrt{(b+1\right.\right.\right.}{4ax^3\left(c^{\frac{1}{2}\sqrt{}}\right.}$$

$$y(x) \to -\frac{\left(\sqrt{(b+1)^2-4as}+b+1\right)\left(2x^2\,{}_1\tilde{F}_1\left(\frac{1}{4}\left(-b-\sqrt{(b+1)^2-4as}-1\right);1-\frac{1}{2}\sqrt{(b+1)^2-4as};-\frac{c}{2x^2}\right)-\right.}{4ax^3\,{}_1\tilde{F}_1\left(\frac{1}{4}\left(-b-\sqrt{(b+1)^2-4as}-1\right);1-\frac{1}{2}\sqrt{(b\right.}$$

2.66 problem 66

Internal problem ID [9649]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^3y' - a\,x^3y^2 - x(bx+c)\,y - \alpha x - \beta = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 615

```
dsolve(x^3*diff(y(x),x)=a*x^3*y(x)^2+x*(b*x+c)*y(x)+alpha*x+beta,y(x), singsol=all)
```

$$y(x) = \frac{(2a^2\beta^2c_1x + 2a\alpha\, c^2c_1x - 2ab\beta cc_1x - 6a\beta cc_1x + 2b\,c^2c_1x + 4c^2c_1x)\,\mathrm{KummerU}\left(\frac{\sqrt{-4a\alpha+b^2+2b+1}\,c-2}{2c}\right.}{2x^2c^2a\left(\mathrm{KummerU}\left(\frac{\sqrt{-4a\alpha+b^2+2b+1}\,c-2a\beta+bc+3c}{2c}, 1+\sqrt{-4a\alpha+b^2+2b+1}, \frac{c}{x}\right)c_1 + \mathrm{KummerM}\left(\frac{\sqrt{-4a\alpha+b^2+}}{}\right.\right.}$$
$$+ \frac{(2a\beta cc_1x - 2b\,c^2c_1x - 2c^3c_1 - 4c^2c_1x)\,\mathrm{KummerU}\left(\frac{\sqrt{-4a\alpha+b^2+2b+1}\,c-2a\beta+bc+3c}{2c}, 1+\sqrt{-4a\alpha+b^2+2b+1}\right.}{2x^2c^2a\left(\mathrm{Ku}\right.}$$

✓ Solution by Mathematica

Time used: 1.338 (sec). Leaf size: 1044

```
DSolve[x^3*y'[x]==a*x^3*y[x]^2+x*(b*x+c)*y[x]+\[Alpha]*x+\[Beta],y[x],x,IncludeSingularSoluti
```

$y(x) \rightarrow$

$$-\frac{\text{Gamma}\left(\sqrt{(b+1)^2-4a\alpha}+1\right)c^{\sqrt{(b+1)^2-4a\alpha}}\left(\frac{1}{x}\right)^{\sqrt{(b+1)^2-4a\alpha}}\left(\left(-\sqrt{(b+1)^2-4a\alpha}+b+1\right){}_1\tilde{F}_1\left(\frac{1}{2}\left(-\right.\right.\right.}{2ax}$$

$y(x)$

$$\rightarrow \frac{\frac{\left(-2a\beta\left(\sqrt{(b+1)^2-4a\alpha}+1\right)+bc\left(\sqrt{(b+1)^2-4a\alpha}+b+3\right)+2c\left(-2a\alpha+\sqrt{(b+1)^2-4a\alpha}+1\right)\right)\text{Hypergeometric1F1}\left(\frac{2a\beta-c\left(b+\sqrt{(b+1)^2-4a\alpha}-1\right)}{2c},2-\right.}{\text{Hypergeometric1F1}\left(\frac{2a\beta-c\left(b+\sqrt{(b+1)^2-4a\alpha}+1\right)}{2c},1-\sqrt{(b+1)^2-4a\alpha},-\frac{c}{x}\right)}}{2ax^2(4a\alpha-b(b+2))}$$

$y(x)$

$$\rightarrow \frac{\frac{\left(-2a\beta\left(\sqrt{(b+1)^2-4a\alpha}+1\right)+bc\left(\sqrt{(b+1)^2-4a\alpha}+b+3\right)+2c\left(-2a\alpha+\sqrt{(b+1)^2-4a\alpha}+1\right)\right)\text{Hypergeometric1F1}\left(\frac{2a\beta-c\left(b+\sqrt{(b+1)^2-4a\alpha}-1\right)}{2c},2-\right.}{\text{Hypergeometric1F1}\left(\frac{2a\beta-c\left(b+\sqrt{(b+1)^2-4a\alpha}+1\right)}{2c},1-\sqrt{(b+1)^2-4a\alpha},-\frac{c}{x}\right)}}{2ax^2(4a\alpha-b(b+2))}$$

2.67 problem 67

Internal problem ID [9650]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x\left(x^{2}+a\right)\left(y'+\lambda y^{2}\right)+\left(b\,x^{2}+c\right)y+sx=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 1348

```
dsolve(x*(x^2+a)*(diff(y(x),x)+lambda*y(x)^2)+(b*x^2+c)*y(x)+s*x=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 1.592 (sec). Leaf size: 783

```
DSolve[x*(x^2+a)*(y'[x]+\[Lambda]*y[x]^2)+(b*x^2+c)*y[x]+s*x==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{\frac{a^{\frac{1}{2}\left(\frac{c}{a}-5\right)}x^{-\frac{c}{a}}\left(a(3a^2-4ac+c^2)\,\text{Hypergeometric2F1}\left(\frac{a\left(b-\sqrt{(b-1)^2-4s\lambda}+1\right)-2c}{4a},\frac{a\left(b+\sqrt{(b-1)^2-4s\lambda}+1\right)-2c}{4a},\frac{3}{2}-\frac{c}{2a},-\frac{x^2}{a}\right)-x^2\left(a^2(b+\lambda s)-a(b-\right.\right.}{3a-c}}{\lambda a^{\frac{1}{2}\left(\frac{c}{a}-1\right)}x^{1-\frac{c}{a}}\,\text{Hypergeometric2F1}\left(\frac{a\left(b-\sqrt{(b-1)^2-4s\lambda}+1\right)-2c}{4a},\frac{a\left(b+\sqrt{(b-1)^2-4s\lambda}+1\right)\right.}{4a}\right.}$$

$$y(x) \to -\frac{sx\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{(b-1)^2-4s\lambda}+3\right),\frac{1}{4}\left(b+\sqrt{(b-1)^2-4s\lambda}+3\right),\frac{1}{2}\left(\frac{c}{a}+3\right),-\frac{x^2}{a}\right)}{(a+c)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{(b-1)^2-4s\lambda}-1\right),\frac{1}{4}\left(b+\sqrt{(b-1)^2-4s\lambda}-1\right),\frac{a+c}{2a},-\frac{x^2}{a}\right)}$$

$$y(x) \to -\frac{sx\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{(b-1)^2-4s\lambda}+3\right),\frac{1}{4}\left(b+\sqrt{(b-1)^2-4s\lambda}+3\right),\frac{1}{2}\left(\frac{c}{a}+3\right),-\frac{x^2}{a}\right)}{(a+c)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{(b-1)^2-4s\lambda}-1\right),\frac{1}{4}\left(b+\sqrt{(b-1)^2-4s\lambda}-1\right),\frac{a+c}{2a},-\frac{x^2}{a}\right)}$$

2.68 problem 68

Internal problem ID [9651]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 68.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^2(x+a)\left(y'+\lambda y^2\right)+x(bx+c)\,y+\alpha x+\beta=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5852

```
dsolve(x^2*(x+a)*(diff(y(x),x)+lambda*y(x)^2)+x*(b*x+c)*y(x)+alpha*x+beta=0,y(x), singsol=all
```

Expression too large to display

✓ Solution by Mathematica

Time used: 3.056 (sec). Leaf size: 1545

```
DSolve[x^2*(x+a)*(y'[x]+\[Lambda]*y[x]^2)+x*(b*x+c)*y[x]+\[Alpha]*x+\[Beta]==0,y[x],x,Include
```

$$y(x) \to \frac{c_1\left(\frac{2x\left((b+2\alpha\lambda)a^2-\left(cb+\sqrt{(a-c)^2-4a\beta\lambda}b+c+2\beta\lambda\right)a+c\left(c+\sqrt{(a-c)^2-4a\beta\lambda}\right)\right)\text{Hypergeometric2F1}\left(-\frac{c+a\left(-b+\sqrt{(b-1)^2-4\alpha\lambda}-2\right)+\sqrt{(a-c)^2}}{2a}\right.}{\sqrt{(a-c)^2-4a\beta\lambda}-a}\right.}{}$$

$$y(x) \to \frac{\left(-2\alpha a^3\lambda-a^2\left(2\alpha\lambda\sqrt{(a-c)^2-4a\beta\lambda}+4b\beta\lambda+bc-2\beta\lambda\right)+a\left(bc\sqrt{(a-c)^2-4a\beta\lambda}+2\beta\lambda\sqrt{(a-c)^2-4a\beta\lambda}+c(bc+4\beta\lambda+c)\right)-c^2\left(\sqrt{(a-c)^2-4a\beta\lambda}+c\right)\right)}{\text{Hypergeometric2F1}\left(-\frac{c+a\left(\sqrt{(b-1)^2-4\alpha\lambda}-b\right)+\sqrt{(a-c)^2-4a\beta\lambda}}{2a},-\frac{c-}{}\right.}$$

2.69 problem 69

Internal problem ID [9652]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Riccati]`

$$(a\,x^2+bx+e)\,(y'x-y)-y^2+x^2=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 58

```
dsolve((a*x^2+b*x+e)*(x*diff(y(x),x)-y(x))-y(x)^2+x^2=0,y(x), singsol=all)
```

$$y(x)=-\tanh\left(\frac{c_1\sqrt{4ea-b^2}+2\arctan\left(\frac{2ax+b}{\sqrt{4ea-b^2}}\right)}{\sqrt{4ea-b^2}}\right)x$$

✓ Solution by Mathematica

Time used: 1.255 (sec). Leaf size: 62

```
DSolve[(a*x^2+b*x+e)*(x*y'[x]-y[x])-y[x]^2+x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x\tanh\left(\frac{2\arctan\left(\frac{2ax+b}{\sqrt{4ae-b^2}}\right)}{\sqrt{4ae-b^2}}+c_1\right)$$

$$y(x)\to -x$$

$$y(x)\to x$$

2.70 problem 70

Internal problem ID [9653]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 70.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x^2(x^2+a)\left(y'+\lambda y^2\right)+x\left(b\,x^2+c\right)y+s=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5730

```
dsolve(x^2*(x^2+a)*(diff(y(x),x)+lambda*y(x)^2)+x*(b*x^2+c)*y(x)+s=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 3.815 (sec). Leaf size: 1687

```
DSolve[x^2*(x^2+a)*(y'[x]+\[Lambda]*y[x]^2)+x*(b*x^2+c)*y[x]+s==0,y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{\left(a-c-\sqrt{(a-c)^2-4as\lambda}\right)c_1\,\mathrm{Gamma}\left(1-\frac{\sqrt{(a-c)^2-4as\lambda}}{2a}\right)\left(4\,{}_2\tilde{F}_1\left(-\frac{-a+c+\sqrt{(a-c)^2-4as\lambda}}{4a},-\frac{-2ba+a+c+\sqrt{(a-}}{4a}\right.\right.}{}$$

$$y(x) \to \frac{x\left(a^3(-b)+a^2\left(b\sqrt{(a-c)^2-4a\lambda s}-4(b-1)\lambda s+c\right)+a\left(bc\left(\sqrt{(a-c)^2-4a\lambda s}+c\right)-c\sqrt{(a-c)^2-}\right.\right.}{2a^2\lambda\left(3a^2+2a(c+2\lambda s)-c^2\right)\mathrm{I}}$$
$$-\frac{\sqrt{(a-c)^2-4a\lambda s}-a+c}{2a\lambda x}$$

$$y(x) \to \frac{x\left(a^3(-b)+a^2\left(b\sqrt{(a-c)^2-4a\lambda s}-4(b-1)\lambda s+c\right)+a\left(bc\left(\sqrt{(a-c)^2-4a\lambda s}+c\right)-c\sqrt{(a-c)^2-}\right.\right.}{2a^2\lambda\left(3a^2+2a(c+2\lambda s)-c^2\right)\mathrm{I}}$$
$$-\frac{\sqrt{(a-c)^2-4a\lambda s}-a+c}{2a\lambda x}$$

2.71 problem 71

Internal problem ID [9654]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 71.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$a(x^2-1)\left(y'+\lambda y^2\right)+bx(x^2-1)\,y+c\,x^2+dx+s=0$$

✗ Solution by Maple

```
dsolve(a*(x^2-1)*(diff(y(x),x)+lambda*y(x)^2)+b*x*(x^2-1)*y(x)+c*x^2+d*x+s=0,y(x), singsol=al
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a*(x^2-1)*(y'[x]+\[Lambda]*y[x]^2)+b*x*(x^2-1)*y[x]+c*x^2+d*x+s==0,y[x],x,IncludeSingu
```

Not solved

2.72 problem 72

Internal problem ID [9655]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 72.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^{1+n}y' - x^{2n}y^2a - x^n y b - c\,x^m - d = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 333

```
dsolve(x^(n+1)*diff(y(x),x)=a*x^(2*n)*y(x)^2+b*x^n*y(x)+c*x^m+d,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(-\sqrt{-4ad+b^2+2bn+n^2}\,c_1 - c_1 b - c_1 n\right)\operatorname{BesselY}\left(\frac{\sqrt{-4ad+b^2+2bn+n^2}}{m}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}}}{m}\right) + 2\sqrt{ac}\operatorname{BesselY}\left(\frac{\sqrt{-4ad}}{}\right.\right.}{2xa\left(\operatorname{BesselY}\right.}$$

✓ Solution by Mathematica

Time used: 1.708 (sec). Leaf size: 2576

```
DSolve[x^(n+1)*y'[x]==a*x^(2*n)*y[x]^2+b*x^n*y[x]+c*x^m+d,y[x],x,IncludeSingularSolutions ->
```

Too large to display

2.73 problem 73

Internal problem ID [9656]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 73.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x\left(a\,x^{k}+b\right)y'-\alpha\,x^{n}y^{2}-\left(\beta-an\,x^{k}\right)y-\gamma\,x^{-n}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 257

```
dsolve(x*(a*x^k+b)*diff(y(x),x)=alpha*x^n*y(x)^2+(beta-a*n*x^k)*y(x)+gamma*x^(-n),y(x), sings
```

$$y(x) = -\frac{\left(x^{n}x^{-n}b^{2}n^{2}+2x^{n}x^{-n}b\beta n+x^{n}x^{-n}\beta^{2}+\tanh\left(\frac{\sqrt{b^{4}n^{4}+4b^{3}\beta\,n^{3}-4\alpha\,b^{2}\gamma\,n^{2}+6b^{2}\beta^{2}n^{2}-8\alpha b\beta\gamma n+4b\,\beta^{3}n-4\alpha\,\beta^{2}\gamma+\beta^{4}}\,(\ln(x}{2bk(b^{2}n^{2}+2b\beta n+\beta^{2})}\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 2.921 (sec). Leaf size: 604

```
DSolve[x*(a*x^k+b)*y'[x]==\[Alpha]*x^n*y[x]^2+(\[Beta]-a*n*x^k)*y[x]+\[Gamma]*x^(-n),y[x],x,I
```

$$y(x) \rightarrow \frac{x^{-n}\exp\left(-\frac{(bn+\beta)\left(\log\left(ax^{k}+b\right)+\log(b)-k\log(x)+\log(k)\right)}{bk}\right)\left((bn+\beta)\left(-\exp\left(\frac{\left(\log\left(ax^{k}+b\right)+\log(b)-k\log(x)+\log(k)\right)\left(-\sqrt{\alpha}\sqrt{\right.}}{2bk}\right.\right.\right.}{}$$

$$y(x) \rightarrow \frac{x^{-n}\left(\sqrt{\alpha}\sqrt{\gamma}\sqrt{\frac{(bn+\beta)^{2}}{\alpha\gamma}-4}-bn-\beta\right)}{2\alpha}$$

2.74 problem 74

Internal problem ID [9657]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 74.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^2(a\,x^n-1)\left(y'+\lambda y^2\right)+(p\,x^n+q)\,xy+r\,x^n+s=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 3716

```
dsolve(x^2*(a*x^n-1)*(diff(y(x),x)+lambda*y(x)^2)+(p*x^n+q)*x*y(x)+r*x^n+s=0,y(x), singsol=al
```

Expression too large to display

✓ Solution by Mathematica

Time used: 5.06 (sec). Leaf size: 1619

```
DSolve[x^2*(a*x^n-1)*(y'[x]+\[Lambda]*y[x]^2)+(p*x^n+q)*x*y[x]+r*x^n+s==0,y[x],x,IncludeSingu
```

$$y(x) \to \frac{a^{\frac{n+\sqrt{(q+1)^2+4s\lambda}}{n}}e^{\frac{i\pi\sqrt{(q+1)^2+4s\lambda}}{n}}\left(\frac{2n\left(2r\lambda+2as\lambda+p\left(q+\sqrt{(q+1)^2+4s\lambda}+1\right)+aq\left(q+\sqrt{(q+1)^2+4s\lambda}+1\right)\right)\text{Hypergeometric2F1}\left(\frac{p+a\left(2n+q+\right.}{}\right.}{n+\sqrt{(q+1)}}\right.}{}$$

$$y(x) \to \frac{\frac{x^n\left(a\left(q^2\left(n+\sqrt{(q+1)^2+4\lambda s}-2\right)+q\left(n\left(-\sqrt{(q+1)^2+4\lambda s}\right)+n+\sqrt{(q+1)^2+4\lambda s}-4\lambda s-1\right)+2\lambda s\left(n+\sqrt{(q+1)^2+4\lambda s}\right)-q^3\right)+p\left(n\left(-\sqrt{(q+1)^2+4\lambda\right.\right.}{(n^2-(q+1)^2-4\lambda s)\text{ Hypergeomet}}}{}$$

2.75 problem 75

Internal problem ID [9658]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 75.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(x^n a + b\,x^m + c)\,y' - cy^2 + b\,x^{m-1}y - a\,x^{n-2} = 0$$

✗ Solution by Maple

```
dsolve((a*x^n+b*x^m+c)*diff(y(x),x)=c*y(x)^2-b*x^(m-1)*y(x)+a*x^(n-2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b*x^m+c)*y'[x]==c*y[x]^2-b*x^(m-1)*y[x]+a*x^(n-2),y[x],x,IncludeSingularSolutio
```

Not solved

2.76 problem 76

Internal problem ID [9659]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 76.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(x^n a + b\,x^m + c)\,y' - a\,x^{n-2}y^2 - b\,x^{m-1}y - c = 0$$

✗ Solution by Maple

```
dsolve((a*x^n+b*x^m+c)*diff(y(x),x)=a*x^(n-2)*y(x)^2+b*x^(m-1)*y(x)+c,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b*x^m+c)*y'[x]==a*x^(n-2)*y[x]^2+b*x^(m-1)*y[x]+c,y[x],x,IncludeSingularSolutio
```

Not solved

2.77 problem 77

Internal problem ID [9660]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 77.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(x^n a + b\,x^m + c)\,y' - \alpha\,x^k y^2 - \beta\,x^s y + \alpha\,\lambda^2 x^k - \beta\lambda\,x^s = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 225

```
dsolve((a*x^n+b*x^m+c)*diff(y(x),x)=alpha*x^k*y(x)^2+beta*x^s*y(x)-alpha*lambda^2*x^k+beta*la
```

$$y(x) = -\frac{\left(\left(\int \frac{\alpha\,x^k \mathrm{e}^{\int -\frac{2x^k\alpha\lambda-\beta\,x^s}{a\,x^n+b\,x^m+c}dx}}{a\,x^n+b\,x^m+c}dx\right)\mathrm{e}^{\int \frac{2x^k\alpha\lambda-\beta\,x^s}{a\,x^n+b\,x^m+c}dx}\lambda + c_1\mathrm{e}^{\int \frac{2x^k\alpha\lambda-\beta\,x^s}{a\,x^n+b\,x^m+c}dx}\lambda + 1\right)\mathrm{e}^{\int -\frac{2x^k\alpha\lambda-\beta\,x^s}{a\,x^n+b\,x^m+c}dx}}{c_1 + \int \frac{\alpha\,x^k \mathrm{e}^{\int -\frac{2x^k\alpha\lambda-\beta\,x^s}{a\,x^n+b\,x^m+c}dx}}{a\,x^n+b\,x^m+c}dx}$$

✓ Solution by Mathematica

Time used: 8.557 (sec). Leaf size: 389

```
DSolve[(a*x^n+b*x^m+c)*y'[x]==\[Alpha]*x^k*y[x]^2+\[Beta]*x^s*y[x]-\[Alpha]*\[Lambda]^2*x^k+\
```

$$\mathrm{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[6]} -\frac{\beta K[5]^s-2\alpha\lambda K[5]^k}{bK[5]^m+aK[5]^n+c}dK[5]\right)\left(-\alpha\lambda K[6]^k+\alpha y(x)K[6]^k+\beta K[6]^s\right)}{(k-s)\alpha\beta\left(bK[6]^m+aK[6]^n+c\right)(\lambda+y(x))}dK[6]\right.$$
$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{\exp\left(-\int_1^{K[6]} -\frac{\beta K[5]^s-2\alpha\lambda K[5]^k}{bK[5]^m+aK[5]^n+c}dK[5]\right)K[6]^k}{(k-s)\beta\left(bK[6]^m+aK[6]^n+c\right)(\lambda+K[7])}-\frac{\exp\left(-\int_1^{K[6]} -\frac{\beta K[5]^s-2\alpha\lambda K[5]^k}{bK[5]^m+aK[5]^n+c}dK[5]\right)(-\alpha\lambda}{(k-s)\alpha\beta\left(bK[6]^m+aK[6]^n\right.}\right.\right.$$
$$\left.\left.-\frac{\exp\left(-\int_1^x -\frac{\beta K[5]^s-2\alpha\lambda K[5]^k}{bK[5]^m+aK[5]^n+c}dK[5]\right)}{(k-s)\alpha\beta(\lambda+K[7])^2}\right)dK[7]=c_1,y(x)\right]$$

2.78 problem 78

Internal problem ID [9661]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 78.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$(x^n a + b\,x^m + c)\,(y'x - y) + s\,x^k\,(y^2 - \lambda\,x^2) = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 42

```
dsolve((a*x^n+b*x^m+c)*(x*diff(y(x),x)-y(x))+s*x^k*(y(x)^2-lambda*x^2)=0,y(x), singsol=all)
```

$$y(x) = \tanh\left(\left(\left(\int \frac{x^k}{a\,x^n + b\,x^m + c}dx\right) s\sqrt{\lambda} + c_1 s\sqrt{\lambda}\right) x\sqrt{\lambda}$$

✓ Solution by Mathematica

Time used: 14.26 (sec). Leaf size: 53

```
DSolve[(a*x^n+b*x^m+c)*(x*y'[x]-y[x])+s*x^k*(y[x]^2-\[Lambda]*x^2)==0,y[x],x,IncludeSingularS
```

$$y(x) \to \sqrt{\lambda}(-x)\tanh\left(\sqrt{\lambda}\left(\int_1^x -\frac{sK[1]^k}{bK[1]^m + aK[1]^n + c}dK[1] + c_1\right)\right)$$

3 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions

3.1 problem 1

Internal problem ID [9662]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 a - b\,\mathrm{e}^{\lambda x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 144

```
dsolve(diff(y(x),x)=a*y(x)^2+b*exp(lambda*x),y(x), singsol=all)
```

$$y(x) = \left(\frac{\sqrt{b}\,c_1 \operatorname{BesselY}\left(1, \frac{2\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{\frac{\lambda x}{2}}}{\lambda}\right)}{\sqrt{a}\left(c_1 \operatorname{BesselY}\left(0, \frac{2\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{\frac{\lambda x}{2}}}{\lambda}\right) + \operatorname{BesselJ}\left(0, \frac{2\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{\frac{\lambda x}{2}}}{\lambda}\right)\right)} + \frac{\sqrt{b}\operatorname{BesselJ}\left(1, \frac{2\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{\frac{\lambda x}{2}}}{\lambda}\right)}{\sqrt{a}\left(c_1 \operatorname{BesselY}\left(0, \frac{2\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{\frac{\lambda x}{2}}}{\lambda}\right) + \operatorname{BesselJ}\left(0, \frac{2\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{\frac{\lambda x}{2}}}{\lambda}\right)\right)}\right)\mathrm{e}^{\frac{\lambda x}{2}}$$

✓ Solution by Mathematica

Time used: 0.323 (sec). Leaf size: 163

```
DSolve[y'[x]==a*y[x]^2+b*Exp[\[Lambda]*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{be^{\lambda x}}\left(2Y_1\left(\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right) + c_1 \operatorname{BesselJ}\left(1, \frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)\right)}{\sqrt{a}\left(2Y_0\left(\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right) + c_1\, {}_0\tilde{F}_1\left(;1;-\frac{abe^{x\lambda}}{\lambda^2}\right)\right)}$$

$$y(x) \to \frac{be^{\lambda x}\, {}_0\tilde{F}_1\left(;2;-\frac{abe^{x\lambda}}{\lambda^2}\right)}{\lambda\, {}_0\tilde{F}_1\left(;1;-\frac{abe^{x\lambda}}{\lambda^2}\right)}$$

3.2 problem 2

Internal problem ID [9663]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\lambda\,\mathrm{e}^{\lambda x} + a^2\mathrm{e}^{2\lambda x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda*exp(lambda*x)-a^2*exp(2*lambda*x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{Ei}_1\left(-\frac{2\,\mathrm{e}^{\lambda x}a}{\lambda}\right)\mathrm{e}^{\lambda x}c_1 a + \mathrm{e}^{\frac{2\,\mathrm{e}^{\lambda x}a}{\lambda}}c_1\lambda + \mathrm{e}^{\lambda x}a}{\mathrm{Ei}_1\left(-\frac{2\,\mathrm{e}^{\lambda x}a}{\lambda}\right)c_1 + 1}$$

✓ Solution by Mathematica

Time used: 1.169 (sec). Leaf size: 57

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]*Exp[\[Lambda]*x]-a^2*Exp[2*\[Lambda]*x],y[x],x,IncludeSingul
```

$$y(x) \to ae^{\lambda x} - \frac{\lambda e^{\frac{2ae^{\lambda x}}{\lambda}}}{\text{ExpIntegralEi}\left(\frac{2ae^{x\lambda}}{\lambda}\right) + c_1}$$

$$y(x) \to ae^{\lambda x}$$

3.3 problem 3

Internal problem ID [9664]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \sigma y^2 - a - b\,e^{\lambda x} - c\,e^{2\lambda x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 650

```
dsolve(diff(y(x),x)=sigma*y(x)^2+a+b*exp(lambda*x)+c*exp(2*lambda*x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(2i\,\mathrm{WhittakerW}\left(-\frac{i\sqrt{\sigma}\,b}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right)\sqrt{\sigma}\,c_1 c^2 + 2i\,\mathrm{WhittakerM}\left(-\frac{i\sqrt{\sigma}\,b}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right)\sqrt{\sigma}}{2c^{\frac{3}{2}}\sigma\left(\mathrm{WhittakerW}\left(-\frac{i\sqrt{\sigma}\,b}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right)c_1 + \mathrm{WhittakerM}\left(-\frac{i\sqrt{\sigma}\,b}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right)\right)}$$
$$+ \frac{c_1\lambda\,\mathrm{WhittakerW}\left(-\frac{i\sqrt{\sigma}\,b-2\lambda\sqrt{c}}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right)}{\sigma\left(\mathrm{WhittakerW}\left(-\frac{i\sqrt{\sigma}\,b}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right)c_1 + \mathrm{WhittakerM}\left(-\frac{i\sqrt{\sigma}\,b}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right)\right)}$$
$$- \frac{\left(i\sqrt{\sigma}\,c_1 bc - c^{\frac{3}{2}}c_1\lambda\right)\mathrm{WhittakerW}\left(-\frac{i\sqrt{\sigma}\,b}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right) + \left(2i\sqrt{\sigma}\,\sqrt{a}\,c^{\frac{3}{2}} - i\sqrt{\sigma}\,bc + c^{\frac{3}{2}}\lambda\right)\mathrm{Whitta}}{2c^{\frac{3}{2}}\sigma\left(\mathrm{WhittakerW}\left(-\frac{i\sqrt{\sigma}\,b}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\,\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{c}\,\sqrt{\sigma}\,e^{\lambda x}}{\lambda}\right)c_1}$$

✓ Solution by Mathematica

Time used: 1.47 (sec). Leaf size: 842

```
DSolve[y'[x]==sigma*y[x]^2+a+b*Exp[\[Lambda]*x]+c*Exp[2*\[Lambda]*x],y[x],x,IncludeSingularSo
```

$$y(x) \to -\frac{i\left(c_1\lambda\left(\sqrt{a}-\sqrt{c}e^{\lambda x}\right)\text{HypergeometricU}\left(\frac{\frac{i\sqrt{\sigma}b}{\sqrt{c}}+\lambda+2i\sqrt{a}\sqrt{\sigma}}{2\lambda},\frac{2i\sqrt{a}\sqrt{\sigma}}{\lambda}+1,\frac{2i\sqrt{c}e^{x\lambda}\sqrt{\sigma}}{\lambda}\right)-ic_1e^{\lambda x}\left(b\sqrt{\sigma}+\sqrt{c}(2\cdots\right.}{\lambda\sqrt{\sigma}\left(c_1\,\text{Hypergeo}\cdots\right.}$$

$$y(x) \to \frac{-\frac{e^{\lambda x}\left(b\sqrt{\sigma}+\sqrt{c}\left(2\sqrt{a}\sqrt{\sigma}-i\lambda\right)\right)\text{HypergeometricU}\left(\frac{\frac{i\sqrt{\sigma}b}{\sqrt{c}}+3\lambda+2i\sqrt{a}\sqrt{\sigma}}{2\lambda},\frac{2i\sqrt{a}\sqrt{\sigma}}{\lambda}+2,\frac{2i\sqrt{c}e^{x\lambda}\sqrt{\sigma}}{\lambda}\right)}{\lambda\,\text{HypergeometricU}\left(\frac{\frac{i\sqrt{\sigma}b}{\sqrt{c}}+\lambda+2i\sqrt{a}\sqrt{\sigma}}{2\lambda},\frac{2i\sqrt{a}\sqrt{\sigma}}{\lambda}+1,\frac{2i\sqrt{c}e^{x\lambda}\sqrt{\sigma}}{\lambda}\right)}-i\left(\sqrt{a}-\sqrt{c}e^{\lambda x}\right)}{\sqrt{\sigma}}$$

3.4 problem 4

Internal problem ID [9665]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \sigma y^2 - ya - b\,\mathrm{e}^x - c = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 317

```
dsolve(diff(y(x),x)=sigma*y(x)^2+a*y(x)+b*exp(x)+c,y(x), singsol=all)
```

$$y(x) = \left(\frac{\sqrt{b}\,c_1 \operatorname{BesselY}\left(\sqrt{a^2-4c\sigma}+1, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right)}{\sqrt{\sigma}\left(\operatorname{BesselY}\left(\sqrt{a^2-4c\sigma}, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right)c_1 + \operatorname{BesselJ}\left(\sqrt{a^2-4c\sigma}, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right)\right)} + \frac{\operatorname{BesselJ}\left(\sqrt{a^2-4c\sigma}+1, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right)\sqrt{b}}{\sqrt{\sigma}\left(\operatorname{BesselY}\left(\sqrt{a^2-4c\sigma}, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right)c_1 + \operatorname{BesselJ}\left(\sqrt{a^2-4c\sigma}, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right)\right)}\right)\mathrm{e}^{\frac{x}{2}} + \frac{\left(-\sqrt{a^2-4c\sigma}\,c_1 - c_1 a\right)\operatorname{BesselY}\left(\sqrt{a^2-4c\sigma}, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right) + \left(-\sqrt{a^2-4c\sigma} - a\right)\operatorname{BesselJ}\left(\sqrt{a^2-4c\sigma}, 2\right.}{2\sigma\left(\operatorname{BesselY}\left(\sqrt{a^2-4c\sigma}, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right)c_1 + \operatorname{BesselJ}\left(\sqrt{a^2-4c\sigma}, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.596 (sec). Leaf size: 482

```
DSolve[y'[x]==sigma*y[x]^2+a*y[x]+b*Exp[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{b\sigma e^x}\left(\text{Gamma}\left(\sqrt{a^2-4c\sigma}+1\right)\left(2\,\text{BesselJ}\left(\sqrt{a^2-4c\sigma}+1,2\sqrt{be^x\sigma}\right)-\frac{\left(\sqrt{a^2-4c\sigma}+a\right)\text{BesselJ}\left(\sqrt{a^2-4c\sigma},2\sqrt{b\cdots}\right.}{\sqrt{b\sigma e^x}}\right.\right.}{2\left(\sigma\,\text{Gamma}\left(\sqrt{a^2-4c\sigma}+1\right)\text{BesselJ}\left(\sqrt{a^2-4c\cdots}\right.\right.}$$

$$y(x) \to -\frac{\frac{2\,{}_0\tilde{F}_1\left(;-\sqrt{a^2-4c\sigma};-be^x\sigma\right)}{{}_0\tilde{F}_1\left(;1-\sqrt{a^2-4c\sigma};-be^x\sigma\right)}+\sqrt{a^2-4c\sigma}+a}{2\sigma}$$

$$y(x) \to -\frac{\frac{2\,{}_0\tilde{F}_1\left(;-\sqrt{a^2-4c\sigma};-be^x\sigma\right)}{{}_0\tilde{F}_1\left(;1-\sqrt{a^2-4c\sigma};-be^x\sigma\right)}+\sqrt{a^2-4c\sigma}+a}{2\sigma}$$

3.5 problem 5

Internal problem ID [9666]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - by - a(\lambda - b)\,\mathrm{e}^{\lambda x} + a^2\mathrm{e}^{2\lambda x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 117

```
dsolve(diff(y(x),x)=y(x)^2+b*y(x)+a*(lambda-b)*exp(lambda*x)-a^2*exp(2*lambda*x),y(x), singso
```

$$y(x) = -\frac{\left(-\left(\int \mathrm{e}^{xb+\frac{2\,\mathrm{e}^{\lambda x}a}{\lambda}}dx\right)a - c_1 a\right)\mathrm{e}^{\frac{2\,\mathrm{e}^{\lambda x}a}{\lambda}}\mathrm{e}^{\lambda x-\frac{2\,\mathrm{e}^{\lambda x}a}{\lambda}}}{\int \mathrm{e}^{xb+\frac{2\,\mathrm{e}^{\lambda x}a}{\lambda}}dx + c_1} - \frac{\mathrm{e}^{xb}\mathrm{e}^{\frac{2\,\mathrm{e}^{\lambda x}a}{\lambda}}}{\int \mathrm{e}^{xb+\frac{2\,\mathrm{e}^{\lambda x}a}{\lambda}}dx + c_1}$$

✓ Solution by Mathematica

Time used: 1.565 (sec). Leaf size: 174

```
DSolve[y'[x]==y[x]^2+b*y[x]+a*(\[Lambda]-b)*Exp[\[Lambda]*x]-a^2*Exp[2*\[Lambda]*x],y[x],x,In
```

$$y(x) \to \frac{2^{b/\lambda}\left(\frac{ae^{\lambda x}}{\lambda}\right)^{b/\lambda}\left(\left(ae^{\lambda x}-b\right)L_{-\frac{b}{\lambda}}^{\frac{b}{\lambda}}\left(\frac{2ae^{x\lambda}}{\lambda}\right)+2ae^{\lambda x}L_{-\frac{b+\lambda}{\lambda}}^{\frac{b+\lambda}{\lambda}}\left(\frac{2ae^{x\lambda}}{\lambda}\right)\right)+ac_1e^{\lambda x}}{2^{b/\lambda}\left(\frac{ae^{\lambda x}}{\lambda}\right)^{b/\lambda}L_{-\frac{b}{\lambda}}^{\frac{b}{\lambda}}\left(\frac{2ae^{x\lambda}}{\lambda}\right)+c_1}$$

$$y(x) \to ae^{\lambda x}$$

3.6 problem 6

Internal problem ID [9667]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\,\mathrm{e}^{\lambda x} y + ab\,\mathrm{e}^{\lambda x} + b^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve(diff(y(x),x)=y(x)^2+a*exp(lambda*x)*y(x)-a*b*exp(lambda*x)-b^2,y(x), singsol=all)
```

$$y(x) = b - \frac{\mathrm{e}^{\frac{\mathrm{e}^{\lambda x} a}{\lambda}+2xb}}{\int \mathrm{e}^{\frac{\mathrm{e}^{\lambda x} a}{\lambda}+2xb} dx - c_1}$$

✓ Solution by Mathematica

Time used: 0.558 (sec). Leaf size: 82

```
DSolve[y'[x]==y[x]^2+a*Exp[\[Lambda]*x]*y[x]-a*b*Exp[\[Lambda]*x]-b^2,y[x],x,IncludeSingularS
```

$$y(x) \to b - \frac{2b\lambda e^{\frac{ae^{\lambda x}}{\lambda}} \left(-\frac{ae^{\lambda x}}{\lambda}\right)^{\frac{2b}{\lambda}}}{2b\Gamma\left(\frac{2b}{\lambda}, 0, -\frac{ae^{x\lambda}}{\lambda}\right) + c_1\lambda(-1)^{b/\lambda}}$$

$$y(x) \to b$$

3.7 problem 7

Internal problem ID [9668]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - a\,e^{2\lambda x}\left(e^{\lambda x} + b\right)^n + \frac{\lambda^2}{4} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 739

```
dsolve(diff(y(x),x)=y(x)^2+a*exp(2*lambda*x)*(exp(lambda*x)+b)^n-1/4*lambda^2,y(x), singsol=a
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a*Exp[2*\[Lambda]*x]*(Exp[\[Lambda]*x]+b)^n-1/4*\[Lambda]^2,y[x],x,Inclu
```

Not solved

3.8 problem 8

Internal problem ID [9669]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - a\,e^{8x\lambda} - b\,e^{6x\lambda} - c\,e^{4x\lambda} + \lambda^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 2495

```
dsolve(diff(y(x),x)=y(x)^2+a*exp(8*lambda*x)+b*exp(6*lambda*x)+c*exp(4*lambda*x)-lambda^2,y(x
```

Expression too large to display

✓ Solution by Mathematica

Time used: 2.408 (sec). Leaf size: 1239

```
DSolve[y'[x]==y[x]^2+a*Exp[8*\[Lambda]*x]+b*Exp[6*\[Lambda]*x]+c*Exp[4*\[Lambda]*x]-\[Lambda]
```

$y(x)$

$$\rightarrow \frac{8\lambda\left(-2ie^{4x\lambda}a+2\lambda\sqrt{a}-ibe^{2x\lambda}\right)\text{Hypergeometric1F1}\left(\frac{1}{4}-\frac{i(b^2-4ac)}{32a^{3/2}\lambda},\frac{1}{2},\frac{i\left(2e^{2x\lambda}a+b\right)^2}{8a^{3/2}\lambda}\right)a^{3/2}+\frac{(1+i)2^{\frac{i\left(b^2-4ac\right)}{16a^{3/2}\lambda}+4}\cdots}{\cdots}}{\cdots}$$

$$y(x)\rightarrow\frac{\left(\frac{1}{8}-\frac{i}{8}\right)e^{2\lambda x}\left(8ia^{3/2}\lambda-4ac+b^2\right)\text{HermiteH}\left(\frac{i(b^2-4ac)}{16a^{3/2}\lambda}-\frac{3}{2},\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(2e^{2x\lambda}a+b\right)}{a^{3/4}\sqrt{\lambda}}\right)}{a^{5/4}\sqrt{\lambda}\,\text{HermiteH}\left(\frac{i(b^2-4ac)}{16a^{3/2}\lambda}-\frac{1}{2},\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(2e^{2x\lambda}a+b\right)}{a^{3/4}\sqrt{\lambda}}\right)}+\frac{ie^{2\lambda x}\left(2ae^{2\lambda x}+b\right)}{2\sqrt{a}}+\lambda$$

$$y(x)\rightarrow\frac{\left(\frac{1}{8}-\frac{i}{8}\right)e^{2\lambda x}\left(8ia^{3/2}\lambda-4ac+b^2\right)\text{HermiteH}\left(\frac{i(b^2-4ac)}{16a^{3/2}\lambda}-\frac{3}{2},\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(2e^{2x\lambda}a+b\right)}{a^{3/4}\sqrt{\lambda}}\right)}{a^{5/4}\sqrt{\lambda}\,\text{HermiteH}\left(\frac{i(b^2-4ac)}{16a^{3/2}\lambda}-\frac{1}{2},\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(2e^{2x\lambda}a+b\right)}{a^{3/4}\sqrt{\lambda}}\right)}+\frac{ie^{2\lambda x}\left(2ae^{2\lambda x}+b\right)}{2\sqrt{a}}+\lambda$$

3.9 problem 9

Internal problem ID [9670]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{kx} y^2 - b\,\mathrm{e}^{sx} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 145

```
dsolve(diff(y(x),x)=a*exp(k*x)*y(x)^2+b*exp(s*x),y(x), singsol=all)
```

$$y(x) = \frac{\left(\mathrm{BesselY}\left(\frac{s}{s+k}, \frac{2\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right) c_1 + \mathrm{BesselJ}\left(\frac{s}{s+k}, \frac{2\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right)\right)\sqrt{b}\,\mathrm{e}^{-xk+\frac{x(s+k)}{2}}}{\sqrt{a}\left(\mathrm{BesselY}\left(-\frac{k}{s+k}, \frac{2\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right) c_1 + \mathrm{BesselJ}\left(-\frac{k}{s+k}, \frac{2\sqrt{b}\,\sqrt{a}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right)\right)}$$

✓ Solution by Mathematica

Time used: 3.587 (sec). Leaf size: 1097

```
DSolve[y'[x]==a*Exp[k*x]*y[x]^2+b*Exp[s*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-kx}\left(-kK_{\frac{k\log\left(e^{k+s}\right)}{(k+s)^2}}\left(2\sqrt{-\frac{ab\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right) - c_1k(-1)^{\frac{k\log\left(e^{k+s}\right)}{(k+s)^2}}\text{BesselI}\left(\frac{k\log\left(e^{k+s}\right)}{(k+s)^2}, 2\sqrt{-\frac{ab(\cdots}{}}\right.\right.}{}$$

$$y(x) \to \frac{e^{-kx}\left(-\frac{(k+s)\sqrt{-\frac{ab\log^2\left(e^{k+s}\right)\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}}{(k+s)^4}}\left(\text{BesselI}\left(\frac{k\log\left(e^{k+s}\right)}{(k+s)^2}-1, 2\sqrt{-\frac{ab\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right)+\text{BesselI}\left(\frac{k\log(e\cdots}{(k+\cdots}\right.\right.}{\text{BesselI}\left(\frac{k\log\left(e^{k+s}\right)}{(k+s)^2}, 2\sqrt{-\frac{ab\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right)}\right.}{2a}$$

$$y(x) \to \frac{e^{-kx}\left(-\frac{(k+s)\sqrt{-\frac{ab\log^2\left(e^{k+s}\right)\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}}{(k+s)^4}}\left(\text{BesselI}\left(\frac{k\log\left(e^{k+s}\right)}{(k+s)^2}-1, 2\sqrt{-\frac{ab\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right)+\text{BesselI}\left(\frac{k\log(e\cdots}{(k+\cdots}\right.\right.}{\text{BesselI}\left(\frac{k\log\left(e^{k+s}\right)}{(k+s)^2}, 2\sqrt{-\frac{ab\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right)}\right.}{2a}$$

3.10 problem 10

Internal problem ID [9671]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - b\,e^{\mu x}y^2 - a\lambda\,e^{\lambda x} + a^2 b\,e^{(\mu+2\lambda)x} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=b*exp(mu*x)*y(x)^2+a*lambda*exp(lambda*x)-a^2*b*exp((mu+2*lambda)*x),y(x)
```

No solution found

✓ Solution by Mathematica

Time used: 4.311 (sec). Leaf size: 844

```
DSolve[y'[x]==b*Exp[\[Mu]*x]*y[x]^2+a*\[Lambda]*Exp[\[Lambda]*x]-a^2*b*Exp[(\[Mu]+2*\[Lambda]
```

$$y(x) \to \frac{e^{\mu(-x)}\left(-2ab\log\left(e^{\lambda+\mu}\right)\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\left(2(\lambda+\mu)L_{-\frac{\log\left(e^{\lambda+\mu}\right)}{2(\lambda+\mu)}-\frac{3}{2}}^{\frac{\mu\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}+1}\left(-\frac{2ab\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}\right)+c_1\right.}{}$$

$$y(x) \to -\frac{ae^{\mu(-x)}\log\left(e^{\lambda+\mu}\right)\left(\log\left(e^{\lambda+\mu}\right)+\lambda+\mu\right)\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\log\left(e^{\lambda+\mu}\right)}{\lambda+\mu}+3\right),\frac{2(\lambda+\mu)^2}{(\right.}{(\lambda+\mu)^2\,\text{HypergeometricU}\left(\frac{\lambda+\mu+\log\left(e^{\lambda+\mu}\right)}{2(\lambda+\mu)},\frac{\mu\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}+1,-\frac{2ab\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+}{\log(e}}}{(\lambda+\mu}\right.}$$

$$-\frac{e^{\mu(-x)}\left(\log\left(e^{\lambda+\mu}\right)\left(2ab\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}+\mu\right)+\mu(\lambda+\mu)\right)}{2b(\lambda+\mu)}$$

3.11 problem 11

Internal problem ID [9672]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{\lambda x} y^2 - by - c\,\mathrm{e}^{-\lambda x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 165

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*y(x)^2+b*y(x)+c*exp(-lambda*x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-\lambda x}\left(\sqrt{4b^2ac + 8abc\lambda + 4\lambda^2 ac - b^4 - 4b^3\lambda - 6b^2\lambda^2 - 4b\,\lambda^3 - \lambda^4}\,\tan\left(\frac{\sqrt{4b^2ac+8abc\lambda+4\lambda^2ac-b^4-4b^3\lambda-6b^2\lambda^2-4b\,\lambda}}{2b^2+4b\lambda+2\lambda^2}\right.\right.}{2a\,(b+\lambda)}$$

✓ Solution by Mathematica

Time used: 0.588 (sec). Leaf size: 115

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*y[x]^2+b*y[x]+c*Exp[-\[Lambda]*x],y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{e^{\lambda(-x)}\left(-\sqrt{(b+\lambda)^2 - 4ac} + \frac{2}{\frac{1}{\sqrt{(b+\lambda)^2-4ac}} + c_1 e^{x\sqrt{(b+\lambda)^2-4ac}}} - b - \lambda\right)}{2a}$$

$$y(x) \to -\frac{e^{\lambda(-x)}\left(\sqrt{(b+\lambda)^2 - 4ac} + b + \lambda\right)}{2a}$$

3.12 problem 12

Internal problem ID [9673]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{\mu x} y^2 - y\lambda + a\,b^2 \mathrm{e}^{(\mu+2\lambda)x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 86

```
dsolve(diff(y(x),x)=a*exp(mu*x)*y(x)^2+lambda*y(x)-a*b^2*exp((mu+2*lambda)*x),y(x), singsol=a
```

$$y(x) = -\frac{b\left(c_1 \sinh\left(\frac{ba\,\mathrm{e}^{x(\lambda+\mu)}}{\lambda+\mu}\right) + \cosh\left(\frac{ba\,\mathrm{e}^{x(\lambda+\mu)}}{\lambda+\mu}\right)\right)\mathrm{e}^{x(\lambda+\mu)-\mu x}}{c_1 \cosh\left(\frac{ba\,\mathrm{e}^{x(\lambda+\mu)}}{\lambda+\mu}\right) + \sinh\left(\frac{ba\,\mathrm{e}^{x(\lambda+\mu)}}{\lambda+\mu}\right)}$$

✓ Solution by Mathematica

Time used: 0.727 (sec). Leaf size: 282

```
DSolve[y'[x]==a*Exp[\[Mu]*x]*y[x]^2+\[Lambda]*y[x]-a*b^2*Exp[(\[Mu]+2*\[Lambda])*x],y[x],x,In
```

$$y(x) \to \boxed{-\frac{\tan\left(\frac{ab^2 e^{x(2\lambda+\mu)}\sqrt{-\frac{e^{-2x\lambda}}{b^2}}}{\lambda+\mu} - c_1\right)}{\sqrt{-\frac{e^{-2x\lambda}}{b^2}}} \text{ if condition}}$$

3.13 problem 13

Internal problem ID [9674]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - e^{\lambda x} y^2 - a\,e^{\mu x} y - a\lambda\,e^{(-\lambda+\mu)x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 140

```
dsolve(diff(y(x),x)=exp(lambda*x)*y(x)^2+a*exp(mu*x)*y(x)+a*lambda*exp((mu-lambda)*x),y(x), s
```

$$y(x) = \left(-\frac{\lambda\,\text{hypergeom}\left(\left[-\frac{\lambda-\mu}{\mu}\right], \left[-\frac{\lambda-2\mu}{\mu}\right], \frac{a\,e^{\mu x}}{\mu}\right) c_1 a\,e^{\mu x}}{(\lambda-\mu)\left(c_1\,\text{hypergeom}\left(\left[-\frac{\lambda}{\mu}\right], \left[-\frac{\lambda-\mu}{\mu}\right], \frac{a\,e^{\mu x}}{\mu}\right) + e^{\lambda x}\right)} - \frac{\lambda\,e^{\lambda x}}{c_1\,\text{hypergeom}\left(\left[-\frac{\lambda}{\mu}\right], \left[-\frac{\lambda-\mu}{\mu}\right], \frac{a\,e^{\mu x}}{\mu}\right) + e^{\lambda x}}\right) e^{-\lambda x}$$

✓ Solution by Mathematica

Time used: 2.058 (sec). Leaf size: 82

```
DSolve[y'[x]==Exp[\[Lambda]*x]*y[x]^2+a*Exp[\[Mu]*x]*y[x]+a*\[Lambda]*Exp[(\[Mu]-\[Lambda])*x
```

$$y(x) \to e^{\lambda(-x)}\left(-\lambda - \frac{\mu e^{\frac{a e^{\mu x}}{\mu}}}{-\text{ExpIntegralE}\left(\frac{\lambda+\mu}{\mu}, -\frac{a e^{x\mu}}{\mu}\right) + c_1 \left(e^{\mu x}\right)^{\lambda/\mu}}\right)$$

$$y(x) \to \lambda\left(-e^{\lambda(-x)}\right)$$

3.14 problem 14

Internal problem ID [9675]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + \lambda\,\mathrm{e}^{\lambda x} y^2 - a\,\mathrm{e}^{\mu x} y + a\,\mathrm{e}^{(-\lambda+\mu)x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 136

```
dsolve(diff(y(x),x)=-lambda*exp(lambda*x)*y(x)^2+a*exp(mu*x)*y(x)-a*exp((mu-lambda)*x),y(x),
```

$$y(x) = \left(\frac{\operatorname{hypergeom}\left(\left[-\frac{\lambda-\mu}{\mu}\right], \left[-\frac{\lambda-2\mu}{\mu}\right], \frac{a\,\mathrm{e}^{\mu x}}{\mu}\right) c_1 a\,\mathrm{e}^{\mu x}}{(\lambda-\mu)\left(c_1 \operatorname{hypergeom}\left(\left[-\frac{\lambda}{\mu}\right], \left[-\frac{\lambda-\mu}{\mu}\right], \frac{a\,\mathrm{e}^{\mu x}}{\mu}\right) + \mathrm{e}^{\lambda x}\right)} + \frac{\mathrm{e}^{\lambda x}}{c_1 \operatorname{hypergeom}\left(\left[-\frac{\lambda}{\mu}\right], \left[-\frac{\lambda-\mu}{\mu}\right], \frac{a\,\mathrm{e}^{\mu x}}{\mu}\right) + \mathrm{e}^{\lambda x}}\right) \mathrm{e}^{-\lambda x}$$

✓ Solution by Mathematica

Time used: 2.015 (sec). Leaf size: 78

```
DSolve[y'[x]==-\[Lambda]*Exp[\[Lambda]*x]*y[x]^2+a*Exp[\[Mu]*x]*y[x]-a*Exp[(\[Mu]-\[Lambda])*
```

$$y(x) \to e^{\lambda(-x)}\left(1 + \frac{\mu e^{\frac{ae^{\mu x}}{\mu}}}{-\lambda \operatorname{ExpIntegralE}\left(\frac{\lambda+\mu}{\mu}, -\frac{ae^{x\mu}}{\mu}\right) + c_1\lambda\left(e^{\mu x}\right)^{\lambda/\mu}}\right)$$

$$y(x) \to e^{\lambda(-x)}$$

3.15 problem 15

Internal problem ID [9676]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,e^{\mu x} y^2 - ab\,e^{x(\lambda+\mu)} y + b\lambda\,e^{\lambda x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 1648

```
dsolve(diff(y(x),x)=a*exp(mu*x)*y(x)^2+a*b*exp((lambda+mu)*x)*y(x)-b*lambda*exp(lambda*x),y(x
```

Expression too large to display

✓ Solution by Mathematica

Time used: 6.254 (sec). Leaf size: 902

```
DSolve[y'[x]==a*Exp[\[Mu]*x]*y[x]^2+a*b*Exp[(\[Lambda]+\[Mu])*x]*y[x]-b*\[Lambda]*Exp[\[Lambd
```

$$y(x) \rightarrow \frac{e^{\mu(-x)}\left(ab\log\left(e^{\lambda+\mu}\right)\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\left(2(\lambda+\mu)L_{-\frac{\log\left(e^{\lambda+\mu}\right)}{2(\lambda+\mu)}-\frac{3}{2}}^{\frac{\mu\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}+1}\left(\frac{ab\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}\right)+c_1\left(\log\left(e\right.\right.\right.\right.}{}$$

$$y(x) \rightarrow \frac{be^{\mu(-x)}\log\left(e^{\lambda+\mu}\right)\left(\log\left(e^{\lambda+\mu}\right)+\lambda+\mu\right)\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\log\left(e^{\lambda+\mu}\right)}{\lambda+\mu}+3\right),\frac{\mu\log\left(e^{\lambda+\mu}\right.}{(\lambda+\mu)^2}\right.}{2(\lambda+\mu)^2\,\text{HypergeometricU}\left(\frac{\lambda+\mu+\log\left(e^{\lambda+\mu}\right)}{2(\lambda+\mu)},\frac{\mu\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}+1,\frac{ab\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\log}{(\lambda+\mu)^2}\right.}$$

$$-\frac{e^{\mu(-x)}\left((\lambda+\mu)\left(ab\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}+\mu\right)+\log\left(e^{\lambda+\mu}\right)\left(\mu-ab\left(\left(e^{\lambda+\mu}\right)^x\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\right)\right)}{2a(\lambda+\mu)}$$

3.16 problem 16

Internal problem ID [9677]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition

Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions

Problem number: 16.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,\mathrm{e}^{kx}y^2 - by - c\,\mathrm{e}^{sx} - d\,\mathrm{e}^{-kx} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 563

```
dsolve(diff(y(x),x)=a*exp(k*x)*y(x)^2+b*y(x)+c*exp(s*x)+d*exp(-k*x),y(x), singsol=all)
```

$$y(x) = \left(\frac{\sqrt{c}\,c_1\,\mathrm{BesselY}\left(\frac{\sqrt{-4ad+b^2+2bk+k^2}+s+k}{s+k}, \frac{2\sqrt{a}\,\sqrt{c}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right)}{\sqrt{a}\left(\mathrm{BesselY}\left(\frac{\sqrt{-4ad+b^2+2bk+k^2}}{s+k}, \frac{2\sqrt{a}\,\sqrt{c}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right)c_1 + \mathrm{BesselJ}\left(\frac{\sqrt{-4ad+b^2+2bk+k^2}}{s+k}, \frac{2\sqrt{a}\,\sqrt{c}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right)\right)} + \frac{\mathrm{BesselJ}\left(\frac{\sqrt{-4ad+b^2+2bk+k^2}+s+k}{s+k}, \frac{2\sqrt{a}\,\sqrt{c}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right)\sqrt{c}}{\sqrt{a}\left(\mathrm{BesselY}\left(\frac{\sqrt{-4ad+b^2+2bk+k^2}}{s+k}, \frac{2\sqrt{a}\,\sqrt{c}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right)c_1 + \mathrm{BesselJ}\left(\frac{\sqrt{-4ad+b^2+2bk+k^2}}{s+k}, \frac{2\sqrt{a}\,\sqrt{c}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right)\right)}\right)\mathrm{e}^{-xk}\mathrm{e} \ldots + \frac{\left(\left(-\sqrt{-4ad+b^2+2bk+k^2}\,c_1 - c_1 b - c_1 k\right)\mathrm{BesselY}\left(\frac{\sqrt{-4ad+b^2+2bk+k^2}}{s+k}, \frac{2\sqrt{a}\,\sqrt{c}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right) + \left(-\sqrt{-4ad+b \ldots\right.\right.}{2\left(\mathrm{BesselY}\left(\frac{\sqrt{-4ad+b^2+2bk+k^2}}{s+k}, \frac{2\sqrt{a}\,\sqrt{c}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right)c_1 + \mathrm{BesselJ}\left(\frac{\sqrt{-4 \ldots}}{}\right.\right.}$$

✓ Solution by Mathematica

Time used: 10.383 (sec). Leaf size: 1636

```
DSolve[y'[x]==a*Exp[k*x]*y[x]^2+b*y[x]+c*Exp[s*x]+d*Exp[-k*x],y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^{-kx}\left(-\left((b+k)K_{\frac{\sqrt{\left(b^2+2kb+k^2-4ad\right)(k+s)^4\log^2\left(e^{k+s}\right)}}{(k+s)^4}}\left(2\sqrt{-\frac{ac\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right)\right)+(-1)^{\frac{k^4+4sk^3+6s^2k^2+}{}}\right.}{}$$

$$y(x) \to \frac{e^{-kx}\left(-(b+k)(k+s)^3\sqrt{-\frac{ac\log^2\left(e^{k+s}\right)\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}}{(k+s)^4}}\,\text{BesselI}\left(\frac{\sqrt{(b^2+2kb+k^2-4ad)(k+s)^4\log^2\left(e^{k+s}\right)}}{(k+s)^4},2\sqrt{-\frac{ac((e^{}}{}}\right.\right.}{}$$

3.17 problem 17

Internal problem ID [9678]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,\mathrm{e}^{(\mu+2\lambda)x}y^2 - \left(b\,\mathrm{e}^{x(\lambda+\mu)} - \lambda\right)y - c\,\mathrm{e}^{\mu x} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 87

```
dsolve(diff(y(x),x)=a*exp((2*lambda+mu)*x)*y(x)^2+(b*exp((lambda+mu)*x)-lambda)*y(x)+c*exp(mu
```

$$y(x) = \frac{\mathrm{e}^{\mu x}\left(\sqrt{4b^2ac-b^4}\,\tan\left(\frac{\sqrt{4b^2ac-b^4}\,(\mathrm{e}^{x(\lambda+\mu)}b+c_1\lambda+c_1\mu)}{2b^2(\lambda+\mu)}\right)-b^2\right)\mathrm{e}^{-x(\lambda+\mu)}}{2ab}$$

✓ Solution by Mathematica

Time used: 3.175 (sec). Leaf size: 348

```
DSolve[y'[x]==a*Exp[(2*\[Lambda]+\[Mu])*x]*y[x]^2+(b*Exp[(\[Lambda]+\[Mu])*x]-\[Lambda])*y[x]
```

$$y(x) \to \frac{e^{\lambda(-x)}\left(b^2e^{x(\lambda+\mu)}\left(\pi+ic_1\left(e^{\sqrt{\frac{(b^2-4ac)e^{2x(\lambda+\mu)}}{(\lambda+\mu)^2}}}-1\right)\right)-b(\lambda+\mu)\sqrt{\frac{(b^2-4ac)e^{2x(\lambda+\mu)}}{(\lambda+\mu)^2}}\left(\pi-ic_1\left(e^{\sqrt{\frac{(b^2-4ac)e^{2x(}}{(\lambda+\mu)^2}}}\right.\right.\right.}{2a(\lambda+\mu)\sqrt{\frac{(b^2-4ac)e^{2x(\lambda+\mu)}}{(\lambda+\mu)^2}}\left(\pi-ic_1\left(e^{\sqrt{\frac{(b^2-4ac)e^{2x(\lambda}}{(\lambda+\mu)^2}}}\right.\right.}$$

$$y(x) \to -\frac{e^{-x(2\lambda+\mu)}\left(\sqrt{b^2-4ac}\sqrt{e^{2x(\lambda+\mu)}}\tanh\left(\frac{\sqrt{b^2-4ac}\sqrt{e^{2x(\lambda+\mu)}}}{2(\lambda+\mu)}\right)+be^{x(\lambda+\mu)}\right)}{2a}$$

3.18 problem 18

Internal problem ID [9679]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,e^{kx}y^2 - by - c\,e^{knx} - d\,e^{k(1+2n)x} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*exp(k*x)*y(x)^2+b*y(x)+c*exp(k*n*x)+d*exp(k*(2*n+1)*x),y(x), singsol=al
```

No solution found

✓ Solution by Mathematica

Time used: 21.812 (sec). Leaf size: 2503

```
DSolve[y'[x]==a*Exp[k*x]*y[x]^2+b*y[x]+c*Exp[k*n*x]+d*Exp[k*(2*n+1)*x],y[x],x,IncludeSingular
```

Too large to display

3.19 problem 19

Internal problem ID [9680]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - e^{\mu x}\left(y - b\, e^{\lambda x}\right)^2 - b\lambda\, e^{\lambda x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 74

```
dsolve(diff(y(x),x)=exp(mu*x)*(y(x)-b*exp(lambda*x))^2+b*lambda*exp(lambda*x),y(x), singsol=a
```

$$y(x) = \left(e^{x(\lambda+\mu)}b - \mu + \frac{e^{-\mu x - \frac{2b\, e^{\lambda x+\mu x}}{\lambda+\mu} + \frac{2\, e^{x(\lambda+\mu)}b}{\lambda+\mu}}}{c_1 + \frac{e^{-\mu x}}{\mu}}\right) e^{-\mu x}$$

✓ Solution by Mathematica

Time used: 0.975 (sec). Leaf size: 40

```
DSolve[y'[x]==Exp[\[Mu]*x]*(y[x]-b*Exp[\[Lambda]*x])^2+b*\[Lambda]*Exp[\[Lambda]*x],y[x],x,In
```

$$y(x) \to be^{\lambda x} + \frac{\mu}{-e^{\mu x} + c_1\mu}$$

$$y(x) \to be^{\lambda x}$$

3.20 problem 20

Internal problem ID [9681]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{\left(\mathrm{e}^{\lambda x} a+\mathrm{e}^{\mu x} b+c\right) y'-y^{2}-k\,\mathrm{e}^{\nu x} y+m^{2}-k m\,\mathrm{e}^{\nu x}=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 129

```
dsolve((a*exp(lambda*x)+b*exp(mu*x)+c)*diff(y(x),x)=y(x)^2+k*exp(nu*x)*y(x)-m^2+k*m*exp(nu*x)
```

$$y(x)=-m-\frac{\mathrm{e}^{\int \frac{k\,\mathrm{e}^{\nu x}}{\mathrm{e}^{\lambda x} a+b\,\mathrm{e}^{\mu x}+c}d x-2 m\left(\int \frac{1}{\mathrm{e}^{\lambda x} a+b\,\mathrm{e}^{\mu x}+c}d x\right)}}{\int \frac{\mathrm{e}^{\int \frac{k\,\mathrm{e}^{\nu x}}{\mathrm{e}^{\lambda x} a+b\,\mathrm{e}^{\mu x}+c}d x-2 m\left(\int \frac{1}{\mathrm{e}^{\lambda x} a+b\,\mathrm{e}^{\mu x}+c}d x\right)}}{\mathrm{e}^{\lambda x} a+b\,\mathrm{e}^{\mu x}+c}d x-c_{1}}$$

✓ Solution by Mathematica

Time used: 9.77 (sec). Leaf size: 358

```
DSolve[(a*Exp[\[Lambda]*x]+b*Exp[\[Mu]*x]+c)*y'[x]==y[x]^2+k*Exp[\[Nu]*x]*y[x]-m^2+k*m*Exp[\[
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[6]} -\frac{e^{\nu K[5]}k-2m}{e^{\lambda K[5]}a+be^{\mu K[5]}+c}dK[5]\right)\left(e^{\nu K[6]}k-m+y(x)\right)}{\left(e^{\lambda K[6]}a+be^{\mu K[6]}+c\right)k\nu(m+y(x))}dK[6]\right.$$
$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^{x} -\frac{e^{\nu K[5]}k-2m}{e^{\lambda K[5]}a+be^{\mu K[5]}+c}dK[5]\right)}{k\nu(m+K[7])^2}\right.$$
$$-\int_1^x\left(\frac{\exp\left(-\int_1^{K[6]} -\frac{e^{\nu K[5]}k-2m}{e^{\lambda K[5]}a+be^{\mu K[5]}+c}dK[5]\right)\left(e^{\nu K[6]}k-m+K[7]\right)}{\left(e^{\lambda K[6]}a+be^{\mu K[6]}+c\right)k\nu(m+K[7])^2}-\frac{\exp\left(-\int_1^{K[6]} -\frac{e^{\nu K[5]}k-2m}{e^{\lambda K[5]}a+be^{\mu K[5]}+c}dK[5]\right)}{\left(e^{\lambda K[6]}a+be^{\mu K[6]}+c\right)k\nu(m+K[7])}\right.$$

3.21 problem 21

Internal problem ID [9682]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(e^{\lambda x}a + e^{\mu x}b + c)\left(y' - y^2\right) + a\,\lambda^2 e^{\lambda x} + \mu^2 e^{\mu x}b = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 350

```
dsolve((a*exp(lambda*x)+b*exp(mu*x)+c)*(diff(y(x),x)-y(x)^2)+a*lambda^2*exp(lambda*x)+b*mu^2*
```

$$y(x) = -\frac{\left((ab\lambda + \mu ba)\left(\int \frac{1}{(e^{\lambda x}a+b\,e^{\mu x}+c)^2}dx\right) + c_1ab\lambda + c_1ab\mu\right)e^{\lambda x+\mu x}}{(e^{\lambda x}a + b\,e^{\mu x} + c)^2\left(c_1 + \int \frac{1}{(e^{\lambda x}a+b\,e^{\mu x}+c)^2}dx\right)}$$
$$- \frac{\left(\left(\int \frac{1}{(e^{\lambda x}a+b\,e^{\mu x}+c)^2}dx\right)b^2\mu + c_1b^2\mu\right)e^{2\mu x}}{(e^{\lambda x}a + b\,e^{\mu x} + c)^2\left(c_1 + \int \frac{1}{(e^{\lambda x}a+b\,e^{\mu x}+c)^2}dx\right)}$$
$$- \frac{\left(\left(\int \frac{1}{(e^{\lambda x}a+b\,e^{\mu x}+c)^2}dx\right)a^2\lambda + c_1a^2\lambda\right)e^{2\lambda x}}{(e^{\lambda x}a + b\,e^{\mu x} + c)^2\left(c_1 + \int \frac{1}{(e^{\lambda x}a+b\,e^{\mu x}+c)^2}dx\right)}$$
$$- \frac{\left(\left(\int \frac{1}{(e^{\lambda x}a+b\,e^{\mu x}+c)^2}dx\right)bc\mu + c_1bc\mu\right)e^{\mu x} + 1 + \left(\left(\int \frac{1}{(e^{\lambda x}a+b\,e^{\mu x}+c)^2}dx\right)ac\lambda + c_1ac\lambda\right)e^{\lambda x}}{(e^{\lambda x}a + b\,e^{\mu x} + c)^2\left(c_1 + \int \frac{1}{(e^{\lambda x}a+b\,e^{\mu x}+c)^2}dx\right)}$$

✓ Solution by Mathematica

Time used: 21.067 (sec). Leaf size: 393

```
DSolve[(a*Exp[\[Lambda]*x]+b*Exp[\[Mu]*x]+c)*(y'[x]-y[x]^2)+a*\[Lambda]^2*Exp[\[Lambda]*x]+b*
```

$$\text{Solve}\left[\int_1^x -\frac{-ae^{\lambda K[1]}\lambda^2 - be^{\mu K[1]}\mu^2 + ae^{\lambda K[1]}y(x)^2 + be^{\mu K[1]}y(x)^2 + cy(x)^2}{\left(e^{\lambda K[1]}a + be^{\mu K[1]} + c\right)\left(ae^{\lambda K[1]}\lambda + be^{\mu K[1]}\mu + ae^{\lambda K[1]}y(x) + be^{\mu K[1]}y(x) + cy(x)\right)^2}dK[1]\right.$$
$$+\int_1^{y(x)}\left(\frac{1}{\left(ae^{x\lambda}\lambda + be^{x\mu}\mu + ae^{x\lambda}K[2] + be^{x\mu}K[2] + cK[2]\right)^2}\right.$$
$$-\int_1^x\left(\frac{2\left(-ae^{\lambda K[1]}\lambda^2 - be^{\mu K[1]}\mu^2 + ae^{\lambda K[1]}K[2]^2 + be^{\mu K[1]}K[2]^2 + cK[2]^2\right)}{\left(ae^{\lambda K[1]}\lambda + be^{\mu K[1]}\mu + ae^{\lambda K[1]}K[2] + be^{\mu K[1]}K[2] + cK[2]\right)^3} - \frac{2ae^{\lambda K[1]}}{\left(e^{\lambda K[1]}a + be^{\mu K[1]} + c\right)\left(ae^{\lambda K[1]}\right.}\right.$$

4 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions

4.1 problem 22

Internal problem ID [9683]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax\,e^{\lambda x} y - e^{\lambda x} a = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 76

```
dsolve(diff(y(x),x)=y(x)^2+a*x*exp(lambda*x)*y(x)+a*exp(lambda*x),y(x), singsol=all)
```

$$y(x) = \frac{e^{\frac{ax\,e^{\lambda x}}{\lambda} - \frac{a\,e^{\lambda x}}{\lambda^2}}}{x^2\lambda^2\left(c_1 - \left(\int \frac{e^{\frac{ax\,e^{\lambda x}}{\lambda} - \frac{a\,e^{\lambda x}}{\lambda^2}}}{x^2\lambda^2}dx\right)\right)} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 1.107 (sec). Leaf size: 75

```
DSolve[y'[x]==y[x]^2+a*x*Exp[\[Lambda]*x]*y[x]+a*Exp[\[Lambda]*x],y[x],x,IncludeSingularSolut
```

$$y(x) \to -\frac{x + \frac{e^{\frac{ae^{\lambda x}(\lambda x-1)}{\lambda^2}}}{\int_1^x \frac{e^{\frac{ae^{\lambda K[1]}(\lambda K[1]-1)}{\lambda^2}}}{K[1]^2}dK[1]+c_1}}{x^2}$$

$$y(x) \to -\frac{1}{x}$$

4.2 problem 23

Internal problem ID [9684]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{\lambda x} y^2 - \mathrm{e}^{-\lambda x} b = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 75

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*y(x)^2+b*exp(-lambda*x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-\mathrm{e}^{\lambda x}\mathrm{e}^{-\lambda x}\lambda^2 + \tan\left(\frac{\sqrt{4ab\,\lambda^2-\lambda^4}\,(\lambda x+c_1)}{2\lambda^2}\right)\sqrt{4ab\,\lambda^2-\lambda^4}\right)\mathrm{e}^{-\lambda x}}{2a\lambda}$$

✓ Solution by Mathematica

Time used: 0.396 (sec). Leaf size: 103

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*y[x]^2+b*Exp[-\[Lambda]*x],y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{e^{\lambda(-x)}\left(-\sqrt{\lambda^2-4ab} + \frac{2}{\frac{1}{\sqrt{\lambda^2-4ab}} + c_1 e^{x\sqrt{\lambda^2-4ab}}} - \lambda\right)}{2a}$$

$$y(x) \to -\frac{e^{\lambda(-x)}\left(\sqrt{\lambda^2-4ab}+\lambda\right)}{2a}$$

4.3 problem 24

Internal problem ID [9685]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,e^{\lambda x} y^2 - bn\,x^{n-1} + a\,b^2 e^{\lambda x} x^{2n} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*y(x)^2+b*n*x^(n-1)-a*b^2*exp(lambda*x)*x^(2*n),y(x), sing
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*y[x]^2+b*n*x^(n-1)-a*b^2*Exp[\[Lambda]*x]*x^(2*n),y[x],x,Inc
```

Not solved

4.4 problem 25

Internal problem ID [9686]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - e^{\lambda x} y^2 - a\,x^n y - a\lambda\,x^n e^{-\lambda x} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 89

```
dsolve(diff(y(x),x)=exp(lambda*x)*y(x)^2+a*x^(n)*y(x)+a*lambda*x^n*exp(-lambda*x),y(x), sings
```

$$y(x) = -\frac{\left(\left(\int e^{\frac{x(a\,x^n - \lambda n - \lambda)}{1+n}} dx\right)\lambda + c_1\lambda + e^{\frac{x(a\,x^n - \lambda n - \lambda)}{1+n}}\right) e^{-\lambda x}}{c_1 + \int e^{\frac{x(a\,x^n - \lambda n - \lambda)}{1+n}} dx}$$

✓ Solution by Mathematica

Time used: 1.231 (sec). Leaf size: 254

```
DSolve[y'[x]==Exp[\[Lambda]*x]*y[x]^2+a*x^(n)*y[x]+a*\[Lambda]*x^n*Exp[-\[Lambda]*x],y[x],x,I
```

$$\text{Solve}\left[\int_1^{y(x)} \left(\frac{e^{\frac{ax^{n+1}}{n+1}}}{(\lambda + e^{x\lambda}K[2])^2} - \int_1^x \left(\frac{2e^{\frac{aK[1]^{n+1}}{n+1}}\left(a\lambda K[1]^n + ae^{\lambda K[1]}K[2]K[1]^n + e^{2\lambda K[1]}K[2]^2\right)}{(\lambda + e^{\lambda K[1]}K[2])^3} - \frac{e^{\frac{aK[1]^{n+1}}{n+1} - \lambda K[1]}\left(ae^{\lambda K[1]}K[1]^n + 2e^{2\lambda K[1]}K[\right.}{(\lambda + e^{\lambda K[1]}K[2])^2}\right.\right.\right.$$

$$\left. + \int_1^x -\frac{e^{\frac{aK[1]^{n+1}}{n+1} - \lambda K[1]}\left(a\lambda K[1]^n + ae^{\lambda K[1]}y(x)K[1]^n + e^{2\lambda K[1]}y(x)^2\right)}{(\lambda + e^{\lambda K[1]}y(x))^2} dK[1] = c_1, y(x)\right]$$

4.5 problem 26

Internal problem ID [9687]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + \lambda\,\mathrm{e}^{\lambda x} y^2 - a\,x^n \mathrm{e}^{\lambda x} y + x^n a = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 135

```
dsolve(diff(y(x),x)=-lambda*exp(lambda*x)*y(x)^2+a*x^(n)*exp(lambda*x)*y(x)-a*x^n,y(x), sings
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\lambda x} \mathrm{e}^{-\lambda x + a\left(\int \mathrm{e}^{\lambda x} x^n dx\right)}}{\lambda^2 \left(\left(\int \frac{\mathrm{e}^{-\lambda x + a\left(\int \mathrm{e}^{\lambda x} x^n dx\right)}}{\lambda} dx\right) c_1 + 1\right)} + \frac{\mathrm{e}^{-\lambda x} \left(\left(\int \frac{\mathrm{e}^{-\lambda x + a\left(\int \mathrm{e}^{\lambda x} x^n dx\right)}}{\lambda} dx\right) c_1 \lambda^2 + \lambda^2\right)}{\lambda^2 \left(\left(\int \frac{\mathrm{e}^{-\lambda x + a\left(\int \mathrm{e}^{\lambda x} x^n dx\right)}}{\lambda} dx\right) c_1 + 1\right)}$$

✓ Solution by Mathematica

Time used: 3.295 (sec). Leaf size: 110

```
DSolve[y'[x]==-\[Lambda]*Exp[\[Lambda]*x]*y[x]^2+a*x^(n)*Exp[\[Lambda]*x]*y[x]-a*x^n,y[x],x,I
```

$$y(x) \to e^{-2\lambda x} \left(e^{\lambda x} + \frac{\left(e^{\lambda x}\right)^{-\frac{a\left(\frac{\log\left(e^{\lambda x}\right)}{\lambda}\right)^n \text{ExpIntegralE}\left(-n, -\log\left(e^{x\lambda}\right)\right)}{\lambda}}}{\int_1^{e^{x\lambda}} K[1]^{-\frac{a \text{ExpIntegralE}(-n, -\log(K[1]))\left(\frac{\log(K[1])}{\lambda}\right)^n}{\lambda} - 2} dK[1] + c_1} \right)$$

$$y(x) \to e^{\lambda(-x)}$$

4.6 problem 27

Internal problem ID [9688]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{\lambda x}y^2 + ab\,x^n\mathrm{e}^{\lambda x}y - bn\,x^{n-1} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 143

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*y(x)^2-a*b*x^(n)*exp(lambda*x)*y(x)+b*n*x^(n-1),y(x), sin
```

$$y(x) = -\frac{c_1\lambda\,\mathrm{e}^{-\lambda x}\mathrm{e}^{\lambda x+ab\left(\int \mathrm{e}^{\lambda x}x^n dx\right)}}{a\left(\left(\int \lambda\,\mathrm{e}^{\lambda x+ab\left(\int \mathrm{e}^{\lambda x}x^n dx\right)}dx\right)c_1+1\right)} - \frac{\left(-x^n\left(\int \lambda\,\mathrm{e}^{\lambda x+ab\left(\int \mathrm{e}^{\lambda x}x^n dx\right)}dx\right)c_1ab - x^n ab\right)\mathrm{e}^{\lambda x}\mathrm{e}^{-\lambda x}}{a\left(\left(\int \lambda\,\mathrm{e}^{\lambda x+ab\left(\int \mathrm{e}^{\lambda x}x^n dx\right)}dx\right)c_1+1\right)}$$

✓ Solution by Mathematica

Time used: 61.176 (sec). Leaf size: 109

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*y[x]^2-a*b*x^(n)*Exp[\[Lambda]*x]*y[x]+b*n*x^(n-1),y[x],x,In
```

$$y(x) \to b\left(\frac{\log\left(e^{\lambda x}\right)}{\lambda}\right)^n - \frac{c_1\lambda\left(e^{\lambda x}\right)^{-\frac{ab\left(\frac{\log\left(e^{\lambda x}\right)}{\lambda}\right)^n\text{ExpIntegralE}\left(-n,-\log\left(e^{x\lambda}\right)\right)}{\lambda}}}{a + ac_1\int_1^{e^{x\lambda}} K[1]^{-\frac{ab\,\text{ExpIntegralE}(-n,-\log(K[1]))\left(\frac{\log(K[1])}{\lambda}\right)^n}{\lambda}}dK[1]}$$

4.7 problem 28

Internal problem ID [9689]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 - b\lambda\,e^{\lambda x} + a\,b^2 x^n e^{2\lambda x} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*lambda*exp(lambda*x)-a*b^2*x^n*exp(2*lambda*x),y(x), sings
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*x^n*y[x]^2+b*\[Lambda]*Exp[\[Lambda]*x]-a*b^2*x^n*Exp[2*\[Lambda]*x],y[x],x,I
```

Not solved

4.8 problem 29

Internal problem ID [9690]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 - y\lambda + a\,b^2 x^n \mathrm{e}^{2\lambda x} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 79

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+lambda*y(x)-a*b^2*x^n*exp(2*lambda*x),y(x), singsol=all)
```

$$y(x) = -i \tan\left(\frac{i\Gamma(n)\,an\,x^n(-\lambda x)^{-n}\,b - ian\,x^n\Gamma(n,-\lambda x)\,(-\lambda x)^{-n}\,b - iba\,\mathrm{e}^{\lambda x}x^n - c_1\lambda}{\lambda}\right) b\,\mathrm{e}^{\lambda x}$$

✓ Solution by Mathematica

Time used: 1.061 (sec). Leaf size: 49

```
DSolve[y'[x]==a*x^n*y[x]^2+\[Lambda]*y[x]-a*b^2*x^n*Exp[2*\[Lambda]*x],y[x],x,IncludeSingular
```

$$y(x) \to \sqrt{-b^2}e^{\lambda x} \tan\left(-a\sqrt{-b^2}x^{n+1}\,\mathrm{ExpIntegralE}(-n,-x\lambda) + c_1\right)$$

4.9 problem 30

Internal problem ID [9691]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,x^n y^2 + ab\,x^n e^{\lambda x} y - b\lambda\,e^{\lambda x} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2-a*b*x^n*exp(lambda*x)*y(x)+b*lambda*exp(lambda*x),y(x), sing
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*x^n*y[x]^2-a*b*x^n*Exp[\[Lambda]*x]*y[x]+b*\[Lambda]*Exp[\[Lambda]*x],y[x],x,
```

Not solved

4.10 problem 31

Internal problem ID [9692]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (1+k)\,x^k y^2 - a\,x^{1+k} e^{\lambda x} y + e^{\lambda x} a = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 196

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+a*x^(k+1)*exp(lambda*x)*y(x)-a*exp(lambda*x),y(x), sing
```

$$y(x) = \frac{\left(e^{\int \frac{x^2 x^k e^{\lambda x} a - 2k - 2}{x} dx} x\, x^k + \int \left(x^k k\, e^{a\left(\int x^{1+k} e^{\lambda x} dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} + x^k e^{a\left(\int x^{1+k} e^{\lambda x} dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)}\right) dx +}{x\left(\int \left(x^k k\, e^{a\left(\int x^{1+k} e^{\lambda x} dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} + x^k e^{a\left(\int x^{1+k} e^{\lambda x} dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)}\right) dx + c_1\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+a*x^(k+1)*Exp[\[Lambda]*x]*y[x]-a*Exp[\[Lambda]*x],y[x],x,Inc
```

Not solved

4.11 problem 32

Internal problem ID [9693]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 + a\,x^n \left(b\,e^{\lambda x} + c\right) y - c\,x^n = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2-a*x^n*(b*exp(lambda*x)+c)*y(x)+c*x^n,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*x^n*y[x]^2-a*x^n*(b*Exp[\[Lambda]*x]+c)*y[x]+c*x^n,y[x],x,IncludeSingularSolu
```

Not solved

4.12 problem 33

Internal problem ID [9694]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n e^{2\lambda x} y^2 - \left(e^{\lambda x} x^n b - \lambda\right) y - c\,x^n = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 114

```
dsolve(diff(y(x),x)=a*x^n*exp(2*lambda*x)*y(x)^2+(b*x^n*exp(lambda*x)-lambda)*y(x)+c*x^n,y(x)
```

$$y(x) = \frac{\left(\tan\left(\frac{\sqrt{4b^2ac-b^4}\left(x^n(-\lambda x)^{-n}\Gamma(n,-\lambda x)bn-x^n\Gamma(n)(-\lambda x)^{-n}bn+x^n e^{\lambda x}b+c_1\lambda\right)}{2b^2\lambda}\right)\sqrt{4b^2ac-b^4}-b^2\right)e^{-\lambda x}}{2ab}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*x^n*Exp[2*\[Lambda]*x]*y[x]^2+(b*x^n*Exp[\[Lambda]*x]-\[Lambda])*y[x]+c*x^n,y
```

Timed out

4.13 problem 34

Internal problem ID [9695]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - a\,e^{\lambda x}(y - b\,x^n - c)^2 - bn\,x^{n-1} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = \frac{\left(ba\,e^{\lambda x}x^n + e^{\lambda x}ac - \lambda + \frac{e^{-\lambda x}}{c_1 + \frac{e^{-\lambda x}}{\lambda}}\right)e^{-\lambda x}}{a}$$

✓ Solution by Mathematica

Time used: 1.029 (sec). Leaf size: 40

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{\lambda}{-ae^{\lambda x} + c_1\lambda} + bx^n + c$$
$$y(x) \to bx^n + c$$

4.14 problem 35

Internal problem ID [9696]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - a\,\mathrm{e}^{\lambda x} y^2 - ky - a\,b^2 x^{2k} \mathrm{e}^{\lambda x} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 54

```
dsolve(x*diff(y(x),x)=a*exp(lambda*x)*y(x)^2+k*y(x)+a*b^2*x^(2*k)*exp(lambda*x),y(x), singsol
```

$$y(x) = -\tan\left(-a\,x^k(-\lambda x)^{-k}\,\Gamma(k)\,b + a\,x^k(-\lambda x)^{-k}\,\Gamma(k,-\lambda x)\,b + c_1\right) b\,x^k$$

✓ Solution by Mathematica

Time used: 1.024 (sec). Leaf size: 43

```
DSolve[x*y'[x]==a*Exp[\[Lambda]*x]*y[x]^2+k*y[x]+a*b^2*x^(2*k)*Exp[\[Lambda]*x],y[x],x,Includ
```

$$y(x) \to \sqrt{b^2}x^k \tan\left(-a\sqrt{b^2}x^k\,\mathrm{ExpIntegralE}(1-k,-x\lambda) + c_1\right)$$

4.15 problem 36

Internal problem ID [9697]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - a\,x^{2n}\mathrm{e}^{\lambda x}y^2 - \left(\mathrm{e}^{\lambda x}x^n b - n\right)y - \mathrm{e}^{\lambda x}c = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 97

```
dsolve(x*diff(y(x),x)=a*x^(2*n)*exp(lambda*x)*y(x)^2+(b*x^n*exp(lambda*x)-n)*y(x)+c*exp(lambd
```

$$y(x) = -\frac{\left(\tan\left(\frac{\sqrt{4b^2ac-b^4}\left(x^n(-\lambda x)^{-n}\Gamma(n,-\lambda x)b-x^n\Gamma(n)(-\lambda x)^{-n}b-c_1\right)}{2b^2}\right)\sqrt{4b^2ac-b^4}+b^2\right)x^{-n}}{2ab}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==a*x^(2*n)*Exp[\[Lambda]*x]*y[x]^2+(b*x^n*Exp[\[Lambda]*x]-n)*y[x]+c*Exp[\[Lam
```

Timed out

4.16 problem 37

Internal problem ID [9698]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - 2a\lambda x\, e^{\lambda\, x^2} + a^2 e^{2\lambda\, x^2} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+2*a*lambda*x*exp(lambda*x^2)-a^2*exp(2*lambda*x^2),y(x), singsol=a
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+2*a*\[Lambda]*x*Exp[\[Lambda]*x^2]-a^2*Exp[2*\[Lambda]*x^2],y[x],x,Inclu
```

Not solved

4.17 problem 38

Internal problem ID [9699]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{-\lambda\,x^2} y^2 - y\lambda x - a\,b^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 49

```
dsolve(diff(y(x),x)=a*exp(-lambda*x^2)*y(x)^2+lambda*x*y(x)+a*b^2,y(x), singsol=all)
```

$$y(x) = -\tan\left(\frac{-ba\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\frac{\sqrt{2}\,\sqrt{\lambda}\,x}{2}\right) + 2c_1\sqrt{\lambda}}{2\sqrt{\lambda}}\right) b\,\mathrm{e}^{\frac{x^2\lambda}{2}}$$

✓ Solution by Mathematica

Time used: 1.46 (sec). Leaf size: 63

```
DSolve[y'[x]==a*Exp[-\[Lambda]*x^2]*y[x]^2+\[Lambda]*x*y[x]+a*b^2,y[x],x,IncludeSingularSolut
```

$$y(x) \to \sqrt{b^2} e^{\frac{\lambda x^2}{2}} \tan\left(\frac{\sqrt{\frac{\pi}{2}} a\sqrt{b^2}\mathrm{erf}\left(\frac{\sqrt{\lambda}x}{\sqrt{2}}\right)}{\sqrt{\lambda}} + c_1\right)$$

4.18 problem 39

Internal problem ID [9700]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 - y\lambda x - a\,b^2 x^n \mathrm{e}^{\lambda\,x^2} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 141

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+lambda*x*y(x)+a*b^2*x^n*exp(lambda*x^2),y(x), singsol=all)
```

$$y(x) = -\tan\left(-\frac{ba2^{\frac{n}{2}+\frac{1}{2}}\lambda^{-\frac{n}{2}-\frac{1}{2}}(-1)^{-\frac{n}{2}}\,x^{1+n}\lambda^{\frac{n}{2}+\frac{1}{2}}(-1)^{\frac{n}{2}}\,(-x^2\lambda)^{-\frac{n}{2}-\frac{1}{2}}\,\Gamma\left(\frac{n}{2}+\frac{1}{2}\right)}{2} + \frac{ba2^{\frac{n}{2}+\frac{1}{2}}\lambda^{-\frac{n}{2}-\frac{1}{2}}(-1)^{-\frac{n}{2}}\,x^{1+n}\lambda^{\frac{n}{2}+\frac{1}{2}}(-1)^{\frac{n}{2}}\,(-x^2\lambda)^{-\frac{n}{2}-\frac{1}{2}}\,\Gamma\left(\frac{n}{2}+\frac{1}{2},-\frac{x^2\lambda}{2}\right)}{2} + c_1\right) b\,\mathrm{e}^{\frac{x^2\lambda}{2}}$$

✓ Solution by Mathematica

Time used: 1.535 (sec). Leaf size: 62

```
DSolve[y'[x]==a*x^n*y[x]^2+\[Lambda]*x*y[x]+a*b^2*x^n*Exp[\[Lambda]*x^2],y[x],x,IncludeSingul
```

$$y(x) \to \sqrt{b^2}e^{\frac{\lambda x^2}{2}}\tan\left(-\frac{1}{2}a\sqrt{b^2}x^{n+1}\,\text{ExpIntegralE}\left(\frac{1}{2}-\frac{n}{2},-\frac{x^2\lambda}{2}\right)+c_1\right)$$

4.19 problem 40

Internal problem ID [9701]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$x^4(y'-y^2)-a-b\,e^{\frac{k}{x}}-c\,e^{\frac{2k}{x}}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 564

```
dsolve(x^4*(diff(y(x),x)-y(x)^2)=a+b*exp(k/x)+c*exp(2*k/x),y(x), singsol=all)
```

$$y(x) = \frac{\left(2i\,\mathrm{WhittakerW}\left(-\frac{ib}{2k\sqrt{c}},\frac{i\sqrt{a}}{k},\frac{2i\sqrt{c}\,e^{\frac{k}{x}}}{k}\right)c_1c^2+2i\,\mathrm{WhittakerM}\left(-\frac{ib}{2k\sqrt{c}},\frac{i\sqrt{a}}{k},\frac{2i\sqrt{c}\,e^{\frac{k}{x}}}{k}\right)c^2\right)e^{\frac{k}{x}}}{2c^{\frac{3}{2}}x^2\left(\mathrm{WhittakerW}\left(-\frac{ib}{2k\sqrt{c}},\frac{i\sqrt{a}}{k},\frac{2i\sqrt{c}\,e^{\frac{k}{x}}}{k}\right)c_1+\mathrm{WhittakerM}\left(-\frac{ib}{2k\sqrt{c}},\frac{i\sqrt{a}}{k},\frac{2i\sqrt{c}\,e^{\frac{k}{x}}}{k}\right)\right)}$$
$$-\frac{c_1k\,\mathrm{WhittakerW}\left(-\frac{ib-2k\sqrt{c}}{2k\sqrt{c}},\frac{i\sqrt{a}}{k},\frac{2i\sqrt{c}\,e^{\frac{k}{x}}}{k}\right)}{x^2\left(\mathrm{WhittakerW}\left(-\frac{ib}{2k\sqrt{c}},\frac{i\sqrt{a}}{k},\frac{2i\sqrt{c}\,e^{\frac{k}{x}}}{k}\right)c_1+\mathrm{WhittakerM}\left(-\frac{ib}{2k\sqrt{c}},\frac{i\sqrt{a}}{k},\frac{2i\sqrt{c}\,e^{\frac{k}{x}}}{k}\right)\right)}$$
$$+\frac{\left(-c^{\frac{3}{2}}c_1k-2c^{\frac{3}{2}}c_1x+ic_1bc\right)\mathrm{WhittakerW}\left(-\frac{ib}{2k\sqrt{c}},\frac{i\sqrt{a}}{k},\frac{2i\sqrt{c}\,e^{\frac{k}{x}}}{k}\right)+\left(2i\sqrt{a}\,c^{\frac{3}{2}}+c^{\frac{3}{2}}k-ibc\right)\mathrm{WhittakerM}}{2c^{\frac{3}{2}}x^2\left(\mathrm{WhittakerW}\left(-\frac{ib}{2k\sqrt{c}},\frac{i\sqrt{a}}{k},\frac{2i\sqrt{c}\,e^{\frac{k}{x}}}{k}\right)c_1+\mathrm{Wl}\right.}$$

✓ Solution by Mathematica

Time used: 1.945 (sec). Leaf size: 940

```
DSolve[x^4*(y'[x]-y[x]^2)==a+b*Exp[k/x]+c*Exp[2*k/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{k/x}\log\left(e^{k/x}\right)\left(c_1\left(b+\sqrt{c}(2\sqrt{a}-ik)\right)\text{HypergeometricU}\left(\frac{\frac{ib}{\sqrt{c}}+3k+2i\sqrt{a}}{2k},\frac{2i\sqrt{a}}{k}+2,\frac{2i\sqrt{c}e^{k/x}}{k}\right)-2i\sqrt{c}kL^{\frac{2i\sqrt{a}}{k}}_{-\frac{ib}{\sqrt{c}}\cdots}\cdots\right.}{kx^2\log\left(e^{k/x}\right)\left(\cdots\right.}$$

$$y(x) \to \frac{\frac{e^{k/x}\left(b+\sqrt{c}(2\sqrt{a}-ik)\right)\text{HypergeometricU}\left(\frac{\frac{ib}{\sqrt{c}}+3k+2i\sqrt{a}}{2k},\frac{2i\sqrt{a}}{k}+2,\frac{2i\sqrt{c}e^{k/x}}{k}\right)}{k\,\text{HypergeometricU}\left(\frac{\frac{ib}{\sqrt{c}}+k+2i\sqrt{a}}{2k},\frac{2i\sqrt{a}}{k}+1,\frac{2i\sqrt{c}e^{k/x}}{k}\right)}+i\left(\sqrt{a}-\sqrt{c}e^{k/x}\right)-\frac{k}{\log\left(e^{k/x}\right)}}{x^2}$$

$$y(x) \to \frac{\frac{e^{k/x}\left(b+\sqrt{c}(2\sqrt{a}-ik)\right)\text{HypergeometricU}\left(\frac{\frac{ib}{\sqrt{c}}+3k+2i\sqrt{a}}{2k},\frac{2i\sqrt{a}}{k}+2,\frac{2i\sqrt{c}e^{k/x}}{k}\right)}{k\,\text{HypergeometricU}\left(\frac{\frac{ib}{\sqrt{c}}+k+2i\sqrt{a}}{2k},\frac{2i\sqrt{a}}{k}+1,\frac{2i\sqrt{c}e^{k/x}}{k}\right)}+i\left(\sqrt{a}-\sqrt{c}e^{k/x}\right)-\frac{k}{\log\left(e^{k/x}\right)}}{x^2}$$

5 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine

5.1 problem 1

Internal problem ID [9702]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 + a^2 - a\lambda \sinh(\lambda x) + a^2 \sinh(\lambda x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 508

```
dsolve(diff(y(x),x)=y(x)^2-a^2+a*lambda*sinh(lambda*x)-a^2*sinh(lambda*x)^2,y(x), singsol=all
```

$$y(x) = \frac{\cosh(\lambda x)\left(i \operatorname{HeunCPrime}\left(\frac{4ia}{\lambda}, \frac{1}{2}, -\frac{1}{2}, \frac{2ia}{\lambda}, -\frac{8ia-3\lambda}{8\lambda}, \frac{1}{2} - \frac{i\sinh(\lambda x)}{2}\right) c_1\lambda - 2\operatorname{HeunC}\left(\frac{4ia}{\lambda}, \frac{1}{2}, -\frac{1}{2}, \frac{2ia}{\lambda}, -\frac{8ia-3\lambda}{8\lambda},\right.\right.}{2\sqrt{\sinh(\lambda x)+i}\left(\operatorname{HeunC}\left(\frac{4ia}{\lambda}, \frac{1}{2}, -\frac{1}{2}, \frac{2ia}{\lambda}, -\frac{8ia-3\lambda}{8\lambda}, \frac{1}{2} - \frac{i\sinh(\lambda x)}{2}\right)\sqrt{\sinh(\lambda x)+i}\, c_1 + \operatorname{HeunC}\left(\frac{4ia}{\lambda}, -\frac{1}{2}, -\right.\right.}$$
$$+ \frac{\left((-2ac_1 i - c_1\lambda)\operatorname{HeunC}\left(\frac{4ia}{\lambda}, \frac{1}{2}, -\frac{1}{2}, \frac{2ia}{\lambda}, -\frac{8ia-3\lambda}{8\lambda}, \frac{1}{2} - \frac{i\sinh(\lambda x)}{2}\right) + i\lambda \operatorname{HeunCPrime}\left(\frac{4ia}{\lambda}, -\frac{1}{2}, -\frac{1}{2}, \frac{2ia}{\lambda}, -\right.\right.}{2\sqrt{\sinh(\lambda x)+i}\left(\operatorname{HeunC}\left(\frac{4ia}{\lambda},\right.\right.}$$

✓ Solution by Mathematica

Time used: 5.255 (sec). Leaf size: 75

```
DSolve[y'[x]==y[x]^2-a^2+a*\[Lambda]*Sinh[\[Lambda]*x]-a^2*Sinh[\[Lambda]*x]^2,y[x],x,Include
```

$$y(x) \to a\cosh(\lambda x) - \frac{\lambda e^{\frac{2a\sinh(\lambda x)}{\lambda}}}{\int_1^{e^{x\lambda}} \frac{e^{\frac{a\left(K[1]^2-1\right)}{\lambda K[1]}}}{K[1]} dK[1] + c_1}$$

$$y(x) \to a\cosh(\lambda x)$$

5.2 problem 2

Internal problem ID [9703]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - a\sinh(\beta x)\,y - ab\sinh(\beta x) + b^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve(diff(y(x),x)=y(x)^2+a*sinh(beta*x)*y(x)+a*b*sinh(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = -b - \frac{e^{\frac{a\cosh(\beta x)}{\beta}-2xb}}{\int e^{\frac{a\cosh(\beta x)}{\beta}-2xb}dx - c_1}$$

✓ Solution by Mathematica

Time used: 5.903 (sec). Leaf size: 183

```
DSolve[y'[x]==y[x]^2+a*Sinh[\[Beta]*x]*y[x]+a*b*Sinh[\[Beta]*x]-b^2,y[x],x,IncludeSingularSol
```

$$\text{Solve}\left[\int_1^x -\frac{e^{\frac{a\cosh(\beta K[1])}{\beta}-2bK[1]}(-b+a\sinh(\beta K[1])+y(x))}{a\beta(b+y(x))}dK[1] + \int_1^{y(x)}\left(\frac{e^{\frac{a\cosh(x\beta)}{\beta}-2bx}}{a\beta(b+K[2])^2} - \int_1^x\left(\frac{e^{\frac{a\cosh(\beta K[1])}{\beta}-2bK[1]}(-b+K[2]+a\sinh(\beta K[1]))}{a\beta(b+K[2])^2} - \frac{e^{\frac{a\cosh(\beta K[1])}{\beta}-2bK[1]}}{a\beta(b+K[2])}\right)dK[1]\right)dK[2] = c_1, y(x)\right]$$

5.3 problem 3

Internal problem ID [9704]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax\sinh(bx)^m y - a\sinh(bx)^m = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*sinh(b*x)^m*y(x)+a*sinh(b*x)^m,y(x), singsol=all)
```

$$y(x) = -\frac{e^{\int \frac{a\sinh(xb)^m x^2-2}{x}dx} x + \int e^{\int \frac{a\sinh(xb)^m x^2-2}{x}dx}dx - c_1}{\left(-c_1 + \int e^{\int \frac{a\sinh(xb)^m x^2-2}{x}dx}dx\right)x}$$

✓ Solution by Mathematica

Time used: 4.12 (sec). Leaf size: 232

```
DSolve[y'[x]==y[x]^2+a*x*Sinh[b*x]^m*y[x]+a*Sinh[b*x]^m,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to$$

$$-\frac{x + \frac{\exp\left(-\frac{a\left(1-e^{2bx}\right)^{-m}\sinh^m(bx)\left({}_3F_2\left(-m,-\frac{m}{2},-\frac{m}{2};1-\frac{m}{2},1-\frac{m}{2};e^{2bx}\right)+bmx\,\text{Hypergeometric2F1}\left(-m,-\frac{m}{2},1-\frac{m}{2},e^{2bx}\right)\right)}{b^2m^2}\right)}{\int_1^x \frac{\exp\left(-\frac{a\left(1-e^{2bK[1]}\right)^{-m}\left({}_3F_2\left(-m,-\frac{m}{2},-\frac{m}{2};1-\frac{m}{2},1-\frac{m}{2};e^{2bK[1]}\right)+bm\,\text{Hypergeometric2F1}\left(-m,-\frac{m}{2},1-\frac{m}{2},e^{2bK[1]}\right)K[1]\right)\sinh^m(bK[1])}{b^2m^2}\right)}{K[1]^2}dK}}{x^2}$$

$$y(x) \to -\frac{1}{x}$$

5.4 problem 4

Internal problem ID [9705]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \sinh(\lambda x)\, y^2 + \lambda \sinh(\lambda x)^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve(diff(y(x),x)=lambda*sinh(lambda*x)*y(x)^2-lambda*sinh(lambda*x)^3,y(x), singsol=all)
```

$$y(x) = -\frac{2c_1 e^{\cosh(\lambda x)^2}}{\sqrt{\pi}\,(\operatorname{erfi}(\cosh(\lambda x))\, c_1 + 1)} + \frac{\cosh(\lambda x)\sqrt{\pi}\, \operatorname{erfi}(\cosh(\lambda x))\, c_1 + \cosh(\lambda x)\sqrt{\pi}}{\sqrt{\pi}\,(\operatorname{erfi}(\cosh(\lambda x))\, c_1 + 1)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Sinh[\[Lambda]*x]*y[x]^2-\[Lambda]*Sinh[\[Lambda]*x]^3,y[x],x,Include
```

Not solved

5.5 problem 5

Internal problem ID [9706]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \left(a \sinh\left(\lambda x\right)^2 - \lambda\right) y^2 + a \sinh\left(\lambda x\right)^2 - \lambda + a = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 470

```
dsolve(diff(y(x),x)=(a*sinh(lambda*x)^2-lambda)*y(x)^2-a*sinh(lambda*x)^2+lambda-a,y(x), sing
```

$$y(x) = \frac{\sinh\left(2\lambda x\right)\left(-4\cosh\left(2\lambda x\right)\sqrt{-1+\cosh\left(2\lambda x\right)}\, c_1 a\lambda + 4\sqrt{-1+\cosh\left(2\lambda x\right)}\, c_1 a\lambda + 8\sqrt{-1+\cosh\left(2\lambda x\right)}\, c_1\lambda}{2\left(-1+\cosh\left(2\lambda x\right)\right)^2\sqrt{1+\cosh\left(2\lambda x\right)}\,\left(\sinh\left(\lambda x\right)^2 a - \lambda\right)\left(\left(\int \frac{2(a\cosh(2\lambda x)-a-2\lambda)e^{\frac{a\cosh(2\lambda x)}{2\lambda}}\lambda\sinh(2\lambda x)}{(-1+\cosh(2\lambda x))^{\frac{3}{2}}\sqrt{1+\cosh(2\lambda x)}}dx\right)\right.} + \frac{\sinh\left(2\lambda x\right)\left(\left(\cosh\left(2\lambda x\right)^2\sqrt{1+\cosh\left(2\lambda x\right)}\, c_1 a + \left(-2\sqrt{1+\cosh\left(2\lambda x\right)}\, c_1 a - 2\sqrt{1+\cosh\left(2\lambda x\right)}\, c_1\lambda\right) c\right.\right.}{}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(a*Sinh[\[Lambda]*x]^2-\[Lambda])*y[x]^2-a*Sinh[\[Lambda]*x]^2+\[Lambda]-a,y[x]
```

Not solved

5.6 problem 6

Internal problem ID [9707]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(\sinh(\lambda x)\, a + b)\, y' - y^2 - c\sinh(\mu x)\, y + d^2 - cd\sinh(\mu x) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 147

```
dsolve((a*sinh(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*sinh(mu*x)*y(x)-d^2+c*d*sinh(mu*x),y(x), si
```

$$y(x) = -d - \frac{e^{\int \frac{c\sinh(\mu x)}{\sinh(\lambda x)a+b}dx - \frac{4d \operatorname{arctanh}\left(\frac{2b\tanh\left(\frac{\lambda x}{2}\right)-2a}{2\sqrt{a^2+b^2}}\right)}{\lambda\sqrt{a^2+b^2}}}}{\int \frac{e^{\int \frac{c\sinh(\mu x)}{\sinh(\lambda x)a+b}dx - \frac{4d \operatorname{arctanh}\left(\frac{2b\tanh\left(\frac{\lambda x}{2}\right)-2a}{2\sqrt{a^2+b^2}}\right)}{\lambda\sqrt{a^2+b^2}}}}{\sinh(\lambda x)a+b}dx - c_1}$$

✓ Solution by Mathematica

Time used: 17.459 (sec). Leaf size: 289

```
DSolve[(a*Sinh[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Sinh[\[Mu]*x]*y[x]-d^2+c*d*Sinh[\[Mu]*x],y[x],
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[6]}\frac{2d-c\sinh(\mu K[5])}{b+a\sinh(\lambda K[5])}dK[5]\right)(-d+c\sinh(\mu K[6])+y(x))}{c\mu(b+a\sinh(\lambda K[6]))(d+y(x))}dK[6]\right.$$
$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x\frac{2d-c\sinh(\mu K[5])}{b+a\sinh(\lambda K[5])}dK[5]\right)}{c\mu(d+K[7])^2}\right.$$
$$-\int_1^x\left(\frac{\exp\left(-\int_1^{K[6]}\frac{2d-c\sinh(\mu K[5])}{b+a\sinh(\lambda K[5])}dK[5]\right)(-d+K[7]+c\sinh(\mu K[6]))}{c\mu(d+K[7])^2(b+a\sinh(\lambda K[6]))}-\frac{\exp\left(-\int_1^{K[6]}\frac{2d-c\sinh(\mu K[5])}{b+a\sinh(\lambda K[5])}dK[5]\right)}{c\mu(d+K[7])(b+a\sinh(\lambda K[6]))}\right.$$

5.7 problem 7

Internal problem ID [9708]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(\sinh(\lambda x)\, a + b)\left(y' - y^2\right) + a\,\lambda^2 \sinh(\lambda x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 938

```
dsolve((a*sinh(lambda*x)+b)*(diff(y(x),x)-y(x)^2)+a*lambda^2*sinh(lambda*x)=0,y(x), singsol=a
```

Expression too large to display

✓ Solution by Mathematica

Time used: 23.625 (sec). Leaf size: 202

```
DSolve[(a*Sinh[\[Lambda]*x]+b)*(y'[x]-y[x]^2)+a*\[Lambda]^2*Sinh[\[Lambda]*x]==0,y[x],x,Inclu
```

$$y(x) \to -\frac{\lambda\left(\sqrt{-a^2-b^2}(b-a\sinh(\lambda x))+a\cosh(\lambda x)\left(2b\arctan\left(\frac{a-b\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{-a^2-b^2}}\right)-c_1\lambda(-a^2-b^2)^{3/2}\right)\right)}{-a\sqrt{-a^2-b^2}\cosh(\lambda x)+(a\sinh(\lambda x)+b)\left(2b\arctan\left(\frac{a-b\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{-a^2-b^2}}\right)-c_1\lambda\left(-a^2-b^2\right)^{3/2}\right)}$$

$$y(x) \to -\frac{a\lambda\cosh(\lambda x)}{a\sinh(\lambda x)+b}$$

5.8 problem 8

Internal problem ID [9709]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \alpha y^2 - \beta - \gamma \cosh(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 70

```
dsolve(diff(y(x),x)=alpha*y(x)^2+beta+gamma*cosh(x),y(x), singsol=all)
```

$$y(x) = -\frac{i\left(c_1 \operatorname{MathieuSPrime}\left(-4\alpha\beta, 2\gamma\alpha, \frac{ix}{2}\right) + \operatorname{MathieuCPrime}\left(-4\alpha\beta, 2\gamma\alpha, \frac{ix}{2}\right)\right)}{2\alpha\left(c_1 \operatorname{MathieuS}\left(-4\alpha\beta, 2\gamma\alpha, \frac{ix}{2}\right) + \operatorname{MathieuC}\left(-4\alpha\beta, 2\gamma\alpha, \frac{ix}{2}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.308 (sec). Leaf size: 140

```
DSolve[y'[x]==\[Alpha]*y[x]^2+\[Beta]+\[Gamma]*Cosh[x],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\frac{ic_1 \text{MathieuCPrime}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right] - i\text{MathieuSPrime}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right]}{2\alpha c_1 \text{MathieuC}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right] - 2\alpha\text{MathieuS}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right]}$$

$$y(x) \to -\frac{i\text{MathieuCPrime}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right]}{2\alpha\text{MathieuC}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right]}$$

5.9 problem 9

Internal problem ID [9710]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\cosh(\beta x)\, y - ab\cosh(\beta x) + b^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x)=y(x)^2+a*cosh(beta*x)*y(x)+a*b*cosh(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = -b - \frac{\mathrm{e}^{\frac{a\sinh(\beta x)}{\beta}-2xb}}{\int \mathrm{e}^{\frac{a\sinh(\beta x)}{\beta}-2xb}dx - c_1}$$

✓ Solution by Mathematica

Time used: 4.594 (sec). Leaf size: 189

```
DSolve[y'[x]==y[x]^2+a*Cosh[\[Beta]*x]*y[x]+a*b*Cosh[\[Beta]*x]-b^2,y[x],x,IncludeSingularSol
```

$$y(x) \to -b - \frac{\beta\left(e^{\beta x}\right)^{-\frac{2b}{\beta}}\left(\sinh\left(\frac{a\sinh(\beta x)}{\beta}\right) + \cosh\left(\frac{a\sinh(\beta x)}{\beta}\right)\right)}{\int_1^{e^{x\beta}} e^{\frac{a\left(K[1]^2-1\right)}{2\beta K[1]}} K[1]^{-\frac{2b}{\beta}-1}dK[1] + c_1}$$

$$y(x) \to -b$$

$$y(x) \to -\frac{\beta\left(e^{\beta x}\right)^{-\frac{2b}{\beta}}\left(\sinh\left(\frac{a\sinh(\beta x)}{\beta}\right) + \cosh\left(\frac{a\sinh(\beta x)}{\beta}\right)\right)}{\int_1^{e^{x\beta}} e^{\frac{a\left(K[1]^2-1\right)}{2\beta K[1]}} K[1]^{-\frac{2b}{\beta}-1}dK[1]} - b$$

5.10 problem 10

Internal problem ID [9711]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax\cosh(bx)^m y - a\cosh(bx)^m = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*cosh(b*x)^m*y(x)+a*cosh(b*x)^m,y(x), singsol=all)
```

$$y(x) = -\frac{e^{\int \frac{a\cosh(xb)^m x^2-2}{x}dx} x + \int e^{\int \frac{a\cosh(xb)^m x^2-2}{x}dx}dx - c_1}{\left(-c_1 + \int e^{\int \frac{a\cosh(xb)^m x^2-2}{x}dx}dx\right) x}$$

✓ Solution by Mathematica

Time used: 4.172 (sec). Leaf size: 236

```
DSolve[y'[x]==y[x]^2+a*x*Cosh[b*x]^m*y[x]+a*Cosh[b*x]^m,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to$$
$$-\frac{x + \frac{\exp\left(-\frac{a\left(e^{2bx}+1\right)^{-m}\cosh^m(bx)\left({}_3F_2\left(-m,-\frac{m}{2},-\frac{m}{2};1-\frac{m}{2},1-\frac{m}{2};-e^{2bx}\right)+bmx\,\text{Hypergeometric2F1}\left(-m,-\frac{m}{2},1-\frac{m}{2},-e^{2bx}\right)\right)}{b^2m^2}\right)}{\int_1^x \frac{\exp\left(-\frac{a\left(1+e^{2bK[1]}\right)^{-m}\cosh^m(bK[1])\left({}_3F_2\left(-m,-\frac{m}{2},-\frac{m}{2};1-\frac{m}{2},1-\frac{m}{2};-e^{2bK[1]}\right)+bm\,\text{Hypergeometric2F1}\left(-m,-\frac{m}{2},1-\frac{m}{2},-e^{2bK[1]}\right)K[1]\right)}{b^2m^2}\right)}{K[1]^2}}}{x^2}$$

$$y(x) \to -\frac{1}{x}$$

5.11 problem 11

Internal problem ID [9712]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \left(a\cosh\left(\lambda x\right)^2 - \lambda\right)y^2 - a - \lambda + a\cosh\left(\lambda x\right)^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 464

```
dsolve(diff(y(x),x)=(a*cosh(lambda*x)^2-lambda)*y(x)^2+a+lambda-a*cosh(lambda*x)^2,y(x), sing
```

$$y(x) = \frac{\sinh\left(2\lambda x\right)\left(-4\cosh\left(2\lambda x\right)\sqrt{1+\cosh\left(2\lambda x\right)}\,c_1 a\lambda - 4\sqrt{1+\cosh\left(2\lambda x\right)}\,c_1 a\lambda + 8\sqrt{1+\cosh\left(2\lambda x\right)}\,c_1\lambda^2\right)e^{\frac{a}{}}}{2\left(1+\cosh\left(2\lambda x\right)\right)^2\sqrt{-1+\cosh\left(2\lambda x\right)}\,\left(a\cosh\left(\lambda x\right)^2-\lambda\right)\left(\left(\int\frac{2\left(a\cosh(2\lambda x)+a-2\lambda\right)e^{\frac{a\cosh(2\lambda x)}{2\lambda}}\lambda\sinh(2\lambda x)}{\sqrt{-1+\cosh(2\lambda x)}\,\left(1+\cosh(2\lambda x)\right)^{\frac{3}{2}}}dx\right)c_1\right.} + \frac{\sinh\left(2\lambda x\right)\left(\left(\cosh\left(2\lambda x\right)^2\sqrt{-1+\cosh\left(2\lambda x\right)}\,c_1 a + \left(2\sqrt{-1+\cosh\left(2\lambda x\right)}\,c_1 a - 2\sqrt{-1+\cosh\left(2\lambda x\right)}\,c_1\right.\right.\right.}{}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(a*Cosh[\[Lambda]*x]^2-\[Lambda])*y[x]^2+a+\[Lambda]-a*Cosh[\[Lambda]*x]^2,y[x]
```

Not solved

5.12 problem 12

Internal problem ID [9713]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$2y' - (a - \lambda + a\cosh(\lambda x))\, y^2 - a - \lambda + a\cosh(\lambda x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 255

```
dsolve(2*diff(y(x),x)=(a-lambda+a*cosh(lambda*x))*y(x)^2+a+lambda-a*cosh(lambda*x),y(x), sing
```

$$y(x) = -\frac{2c_1\lambda\sinh(\lambda x)\,\mathrm{e}^{\frac{a\cosh(\lambda x)}{\lambda}}}{(\cosh(\lambda x)+1)^{\frac{3}{2}}\left(\left(\int\frac{(a-\lambda+\cosh(\lambda x)a)\mathrm{e}^{\frac{a\cosh(\lambda x)}{\lambda}}\lambda\sinh(\lambda x)}{\sqrt{\cosh(\lambda x)-1}\,(\cosh(\lambda x)+1)^{\frac{3}{2}}}dx\right)c_1+1\right)\sqrt{\cosh(\lambda x)-1}}$$
$$+\frac{\left(\left(\cosh(\lambda x)\sqrt{\cosh(\lambda x)-1}\,c_1+\sqrt{\cosh(\lambda x)-1}\,c_1\right)\left(\int\frac{(a-\lambda+\cosh(\lambda x)a)\mathrm{e}^{\frac{a\cosh(\lambda x)}{\lambda}}\lambda\sinh(\lambda x)}{\sqrt{\cosh(\lambda x)-1}\,(\cosh(\lambda x)+1)^{\frac{3}{2}}}dx\right)+\cosh(\lambda}{\left(\left(\int\frac{(a-\lambda+\cosh(\lambda x)a)\mathrm{e}^{\frac{a\cosh(\lambda x)}{\lambda}}\lambda\sinh(\lambda x)}{\sqrt{\cosh(\lambda x)-1}\,(\cosh(\lambda x)+1)^{\frac{3}{2}}}dx\right)c_1+1\right)\sqrt{\cosh(\lambda x)-1}\,(\mathrm{co}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*y'[x]==(a-\[Lambda]+a*Cosh[\[Lambda]*x])*y[x]^2+a+\[Lambda]-a*Cosh[\[Lambda]*x],y[x]
```

Not solved

5.13 problem 13

Internal problem ID [9714]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 + \lambda^2 - a\cosh(\lambda x)^n \sinh(\lambda x)^{-n-4} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2-lambda^2+a*cosh(lambda*x)^n*sinh(lambda*x)^(-n-4),y(x), singsol=al
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-\[Lambda]^2+a*Cosh[\[Lambda]*x]^n*Sinh[\[Lambda]*x]^(-n-4),y[x],x,Includ
```

Not solved

5.14 problem 14

Internal problem ID [9715]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \sinh(\lambda x)\, y^2 a - b \sinh(\lambda x) \cosh(\lambda x)^n = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 333

```
dsolve(diff(y(x),x)=a*sinh(lambda*x)*y(x)^2+b*sinh(lambda*x)*cosh(lambda*x)^n,y(x), singsol=a
```

$$y(x) = \frac{\frac{\sqrt{b}\cosh(\lambda x)^{\frac{n}{2}+1} c_1 \operatorname{BesselY}\left(\frac{n+3}{n+2}, \frac{2\sqrt{a}\sqrt{b}\cosh(\lambda x)^{\frac{n}{2}+1}}{\lambda(n+2)}\right)}{\sqrt{a}\left(\operatorname{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{a}\sqrt{b}\cosh(\lambda x)^{\frac{n}{2}+1}}{\lambda(n+2)}\right)c_1 + \operatorname{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{a}\sqrt{b}\cosh(\lambda x)^{\frac{n}{2}+1}}{\lambda(n+2)}\right)\right)} + \frac{\operatorname{BesselJ}\left(\frac{n+3}{n+2}, \frac{2\sqrt{a}\sqrt{b}\cosh(\lambda x)^{\frac{n}{2}+1}}{\lambda(n+2)}\right)\sqrt{a}\sqrt{b}\cosh(\lambda x)^{\frac{n}{2}}}{\left(\operatorname{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{a}\sqrt{b}\,\mathrm{c}}{\lambda(}\right.\right.}}{\cosh(\lambda x)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*Sinh[\[Lambda]*x]*y[x]^2+b*Sinh[\[Lambda]*x]*Cosh[\[Lambda]*x]^n,y[x],x,Inclu
```

Not solved

5.15 problem 15

Internal problem ID [9716]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 \cosh(\lambda x)\, a - b \cosh(\lambda x) \sinh(\lambda x)^n = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 333

```
dsolve(diff(y(x),x)=a*cosh(lambda*x)*y(x)^2+b*cosh(lambda*x)*sinh(lambda*x)^n,y(x), singsol=a
```

$$y(x) = \frac{\frac{\sqrt{b}\,\sinh(\lambda x)^{\frac{n}{2}+1} c_1 \operatorname{BesselY}\left(\frac{n+3}{n+2}, \frac{2\sqrt{a}\,\sqrt{b}\,\sinh(\lambda x)^{\frac{n}{2}+1}}{\lambda(n+2)}\right)}{\sqrt{a}\left(\operatorname{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{a}\,\sqrt{b}\,\sinh(\lambda x)^{\frac{n}{2}+1}}{\lambda(n+2)}\right) c_1 + \operatorname{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{a}\,\sqrt{b}\,\sinh(\lambda x)^{\frac{n}{2}+1}}{\lambda(n+2)}\right)\right)} + \frac{\operatorname{BesselJ}\left(\frac{n+3}{n+2}, \frac{2\sqrt{a}\,\sqrt{b}\,\sinh(\lambda x)^{\frac{n}{2}+1}}{\lambda(n+2)}\right)\sqrt{a}\,\sqrt{b}\,\sinh(\lambda x)^{\frac{n}{2}}}{\left(\operatorname{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{a}\,\sqrt{b}\,\sin}{\lambda(n}\right.\right.}}{\sinh(\lambda x)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*Cosh[\[Lambda]*x]*y[x]^2+b*Cosh[\[Lambda]*x]*Sinh[\[Lambda]*x]^n,y[x],x,Inclu
```

Not solved

5.16 problem 16

Internal problem ID [9717]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(a\cosh(\lambda x)+b)\,y'-y^2-c\cosh(\mu x)\,y+d^2-cd\cosh(\mu x)=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 149

```
dsolve((a*cosh(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*cosh(mu*x)*y(x)-d^2+c*d*cosh(mu*x),y(x), si
```

$$y(x)=-d-\frac{e^{\int \frac{c\cosh(\mu x)}{\cosh(\lambda x)a+b}dx-\frac{4d\arctan\left(\frac{(a-b)\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\lambda\sqrt{(a-b)(a+b)}}}}{\int \frac{e^{\int \frac{c\cosh(\mu x)}{\cosh(\lambda x)a+b}dx-\frac{4d\arctan\left(\frac{(a-b)\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\lambda\sqrt{(a-b)(a+b)}}}}{\cosh(\lambda x)a+b}dx-c_1}$$

✓ Solution by Mathematica

Time used: 15.017 (sec). Leaf size: 289

```
DSolve[(a*Cosh[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Cosh[\[Mu]*x]*y[x]-d^2+c*d*Cosh[\[Mu]*x],y[x],
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[6]}\frac{2d-c\cosh(\mu K[5])}{b+a\cosh(\lambda K[5])}dK[5]\right)(-d+c\cosh(\mu K[6])+y(x))}{c\mu(b+a\cosh(\lambda K[6]))(d+y(x))}dK[6]\right.$$
$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^{x}\frac{2d-c\cosh(\mu K[5])}{b+a\cosh(\lambda K[5])}dK[5]\right)}{c\mu(d+K[7])^2}\right.$$
$$-\int_1^x\left(\frac{\exp\left(-\int_1^{K[6]}\frac{2d-c\cosh(\mu K[5])}{b+a\cosh(\lambda K[5])}dK[5]\right)(-d+c\cosh(\mu K[6])+K[7])}{c\mu(b+a\cosh(\lambda K[6]))(d+K[7])^2}-\frac{\exp\left(-\int_1^{K[6]}\frac{2d-c\cosh(\mu K[5])}{b+a\cosh(\lambda K[5])}dK[5]\right.}{c\mu(b+a\cosh(\lambda K[6]))(d+K[7]}\right.$$

5.17 problem 17

Internal problem ID [9718]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(a\cosh(\lambda x)+b)\left(y'-y^2\right)+a\,\lambda^2\cosh(\lambda x)=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 758

```
dsolve((a*cosh(lambda*x)+b)*(diff(y(x),x)-y(x)^2)+a*lambda^2*cosh(lambda*x)=0,y(x), singsol=a
```

$$y(x) = \frac{\lambda\left(\left(\left(\left(2\sqrt{a^2-b^2}\,a^3b-4\sqrt{a^2-b^2}\,a^2b^2+2\sqrt{a^2-b^2}\,a\,b^3\right)\arctan\left(\frac{(a-b)\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)-2\sqrt{a^2-b^2}\,c_1a^3+4\sqrt{\right.\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 5.573 (sec). Leaf size: 242

```
DSolve[(a*Cosh[\[Lambda]*x]+b)*(y'[x]-y[x]^2)+a*\[Lambda]^2*Cosh[\[Lambda]*x]==0,y[x],x,Inclu
```

$$y(x) \to \frac{\lambda\left(2ab\sinh(\lambda x)\arctan\left(\frac{(b-a)\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)+a\sqrt{(a-b)(a+b)}(\cosh(\lambda x)+c_1\lambda(a-b)(a+b)\sinh\right.}{-a\sqrt{a^2-b^2}\sinh(\lambda x)+2b^2\cot^{-1}\left(\frac{(a+b)\coth\left(\frac{\lambda x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)-bc_1\lambda((a-b)(a+b))^{3/2}+a\cosh(\lambda x)\left(2b\cot^{-1}\left(\frac{(a+}{\sqrt{}}\right.\right.}$$

$$y(x) \to -\frac{a\lambda\sinh(\lambda x)}{a\cosh(\lambda x)+b}$$

6 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.

6.1 problem 18

Internal problem ID [9719]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \lambda a + a(a+\lambda)\tanh(\lambda x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 198

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda-a*(a+lambda)*tanh(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\left((c_1 a + c_1\lambda)\operatorname{LegendreQ}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right) + (a+\lambda)\operatorname{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right)\right)\tanh(\lambda x)}{c_1\operatorname{LegendreQ}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right) + \operatorname{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right)} + \frac{c_1\lambda\operatorname{LegendreQ}\left(\frac{a+\lambda}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right)}{c_1\operatorname{LegendreQ}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right) + \operatorname{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right)} + \frac{\operatorname{LegendreP}\left(\frac{a+\lambda}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right)\lambda}{c_1\operatorname{LegendreQ}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right) + \operatorname{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh(\lambda x)\right)}$$

✓ Solution by Mathematica

Time used: 4.355 (sec). Leaf size: 162

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]-a*(a+\[Lambda])*Tanh[\[Lambda]*x]^2,y[x],x,IncludeSingularSo
```

$$y(x) \to -\frac{2ae^{\lambda x}\left(2\lambda\left(e^{2\lambda x}+1\right)^{\frac{2a}{\lambda}}\cosh(\lambda x) + \sinh(\lambda x)\left(\lambda\,\text{Hypergeometric2F1}\left(-\frac{2a}{\lambda},-\frac{a}{\lambda},1-\frac{a}{\lambda},-e^{2x\lambda}\right) - 2ac_1\left(e^{\lambda\ldots\right.\right.\right.}{\left(e^{2\lambda x}+1\right)\left(-\lambda\,\text{Hypergeometric2F1}\left(-\frac{2a}{\lambda},-\frac{a}{\lambda},1-\frac{a}{\lambda},-e^{2x\lambda}\right) + 2ac_1\left(e^{\lambda x}\right)^{\frac{2a}{\lambda}}\right)}$$

$$y(x) \to a\tanh(\lambda x)$$

6.2 problem 19

Internal problem ID [9720]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - 3\lambda a + \lambda^2 + a(a+\lambda)\tanh(\lambda x)^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 245

```
dsolve(diff(y(x),x)=y(x)^2+3*a*lambda-lambda^2-a*(a+lambda)*tanh(lambda*x)^2,y(x), singsol=al
```

$$y(x) = \frac{\left((-c_1 a - c_1\lambda)\,\mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right) + (-a-\lambda)\,\mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right)\right)\tanh(\lambda x)}{c_1\,\mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right)} + \frac{2c_1\lambda\,\mathrm{LegendreQ}\left(\frac{a+\lambda}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right)}{c_1\,\mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right)} + \frac{2\,\mathrm{LegendreP}\left(\frac{a+\lambda}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right)\lambda}{c_1\,\mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(\lambda x)\right)}$$

✓ Solution by Mathematica

Time used: 12.997 (sec). Leaf size: 341

```
DSolve[y'[x]==y[x]^2+3*a*\[Lambda]-\[Lambda]^2-a*(a+\[Lambda])*Tanh[\[Lambda]*x]^2,y[x],x,Inc
```

$$y(x)$$

$$\to \frac{-2e^{2\lambda x}\left(e^{2\lambda x}-1\right)^2\left((\lambda-a)\cosh(2\lambda x)+a+\lambda\right)\int_1^{e^{x\lambda}}\frac{K[1]^{1-\frac{2a}{\lambda}}\left(K[1]^2+1\right)^{\frac{2a}{\lambda}}}{\left(K[1]^2-1\right)^2}dK[1]+4e^{4\lambda x}\sinh(\lambda x)\left(\lambda\left(-\left(e^{2\lambda x}\right.\right.\right.}{\left(e^{2\lambda x}-1\right)^3\left(e^{2\lambda x}+1\right)\left(\int_1^{e^{x\lambda}}\frac{K[1]^{1-\frac{2a}{\lambda}}\left(K[1]^2+1\right)^{\frac{2a}{\lambda}}}{\left(K[1]^2-1\right)^2}\right.}$$

$$y(x)\to a\tanh(\lambda x)-\lambda\coth(\lambda x)$$

$$y(x)\to -\frac{\lambda\left(e^{2\lambda x}+1\right)^{\frac{2a}{\lambda}}\left(e^{\lambda x}\right)^{-\frac{2a}{\lambda}}\operatorname{csch}^2(\lambda x)}{4\int_1^{e^{x\lambda}}\frac{K[1]^{1-\frac{2a}{\lambda}}\left(K[1]^2+1\right)^{\frac{2a}{\lambda}}}{\left(K[1]^2-1\right)^2}dK[1]}+a\tanh(\lambda x)-\lambda\coth(\lambda x)$$

6.3 problem 20

Internal problem ID [9721]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - ax\tanh(bx)^m y - a\tanh(bx)^m = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*tanh(b*x)^m*y(x)+a*tanh(b*x)^m,y(x), singsol=all)
```

$$y(x) = -\frac{e^{\int \frac{a\tanh(xb)^m x^2 - 2}{x}dx} x + \int e^{\int \frac{a\tanh(xb)^m x^2 - 2}{x}dx}dx - c_1}{\left(-c_1 + \int e^{\int \frac{a\tanh(xb)^m x^2 - 2}{x}dx}dx\right) x}$$

✓ Solution by Mathematica

Time used: 6.923 (sec). Leaf size: 86

```
DSolve[y'[x]==y[x]^2+a*x*Tanh[b*x]^m*y[x]+a*Tanh[b*x]^m,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{x + \frac{\exp\left(-\int_1^x -aK[5]\tanh^m(bK[5])dK[5]\right)}{\int_1^x \frac{\exp\left(-\int_1^{K[6]} -aK[5]\tanh^m(bK[5])dK[5]\right)}{K[6]^2}dK[6] + c_1}}{x^2}$$

$$y(x) \to -\frac{1}{x}$$

6.4 problem 21

Internal problem ID [9722]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(a\tanh(\lambda x)+b)\,y'-y^2-c\tanh(\mu x)\,y+d^2-cd\tanh(\mu x)=0$$

✗ Solution by Maple

```
dsolve((a*tanh(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*tanh(mu*x)*y(x)-d^2+c*d*tanh(mu*x),y(x), si
```

No solution found

✓ Solution by Mathematica

Time used: 102.677 (sec). Leaf size: 800

```
DSolve[(a*Tanh[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Tanh[\[Mu]*x]*y[x]-d^2+c*d*Tanh[\[Mu]*x],y[x],
```

$$\text{Solve}\left[\int_1^x \frac{e^{-\int_1^{K[6]}\frac{\operatorname{sech}(\mu K[5])(2d\cosh(\lambda K[5]-\mu K[5])+2d\cosh(\lambda K[5]+\mu K[5])+c\sinh(\lambda K[5]-\mu K[5])-c\sinh(\lambda K[5]+\mu K[5]))}{2(b\cosh(\lambda K[5])+a\sinh(\lambda K[5]))}dK[5]}(d\cosh(\lambda K[6]-\right.}{c\mu(b\cosh(\lambda K[6]-\mu K[6])+b\cos}\right.$$

$$+\int_1^{y(x)}\left(\frac{e^{-\int_1^x\frac{\operatorname{sech}(\mu K[5])(2d\cosh(\lambda K[5]-\mu K[5])+2d\cosh(\lambda K[5]+\mu K[5])+c\sinh(\lambda K[5]-\mu K[5])-c\sinh(\lambda K[5]+\mu K[5]))}{2(b\cosh(\lambda K[5])+a\sinh(\lambda K[5]))}dK[5]}}{c\mu(d+K[7])^2}\right.$$

$$-\int_1^x\left(\frac{e^{-\int_1^{K[6]}\frac{\operatorname{sech}(\mu K[5])(2d\cosh(\lambda K[5]-\mu K[5])+2d\cosh(\lambda K[5]+\mu K[5])+c\sinh(\lambda K[5]-\mu K[5])-c\sinh(\lambda K[5]+\mu K[5]))}{2(b\cosh(\lambda K[5])+a\sinh(\lambda K[5]))}dK[5]}(-\cosh(\lambda K[6]-\mu}{c\mu(d+K[7])(b\cosh(\lambda K[6]-\mu K[6])+b\cosh(\lambda K[6]+\mu K[6])+a\sinh(\lambda K[6]-\mu K[6])+}\right.$$

6.5 problem 22

Internal problem ID [9723]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \lambda a + a(a+\lambda)\coth(\lambda x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 198

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda-a*(a+lambda)*coth(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\left((c_1 a + c_1\lambda)\operatorname{LegendreQ}\left(\frac{a}{\lambda},\frac{a}{\lambda},\coth(\lambda x)\right) + (a+\lambda)\operatorname{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\coth(\lambda x)\right)\right)\coth(\lambda x)}{c_1\operatorname{LegendreQ}\left(\frac{a}{\lambda},\frac{a}{\lambda},\coth(\lambda x)\right) + \operatorname{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\coth(\lambda x)\right)} + \frac{c_1\lambda\operatorname{LegendreQ}\left(\frac{a+\lambda}{\lambda},\frac{a}{\lambda},\coth(\lambda x)\right)}{c_1\operatorname{LegendreQ}\left(\frac{a}{\lambda},\frac{a}{\lambda},\coth(\lambda x)\right) + \operatorname{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\coth(\lambda x)\right)} + \frac{\operatorname{LegendreP}\left(\frac{a+\lambda}{\lambda},\frac{a}{\lambda},\coth(\lambda x)\right)\lambda}{c_1\operatorname{LegendreQ}\left(\frac{a}{\lambda},\frac{a}{\lambda},\coth(\lambda x)\right) + \operatorname{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\coth(\lambda x)\right)}$$

✓ Solution by Mathematica

Time used: 4.231 (sec). Leaf size: 156

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]-a*(a+\[Lambda])*Coth[\[Lambda]*x]^2,y[x],x,IncludeSingularSo
```

$$y(x) \to -\frac{ae^{\lambda x}(\coth(\lambda x)-1)\left(2\lambda\left(1-e^{2\lambda x}\right)^{\frac{2a}{\lambda}}\sinh(\lambda x) + \cosh(\lambda x)\left(\lambda\operatorname{Hypergeometric2F1}\left(-\frac{2a}{\lambda},-\frac{a}{\lambda},1-\frac{a}{\lambda},e^{2x\lambda}\right.\right.\right.}{-\lambda\operatorname{Hypergeometric2F1}\left(-\frac{2a}{\lambda},-\frac{a}{\lambda},1-\frac{a}{\lambda},e^{2x\lambda}\right) + 2ac_1\left(e^{\lambda x}\right)^{\frac{2a}{\lambda}}}$$

$$y(x) \to a\coth(\lambda x)$$

6.6 problem 23

Internal problem ID [9724]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 + \lambda^2 - 3\lambda a + a(a+\lambda)\coth(\lambda x)^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 240

```
dsolve(diff(y(x),x)=y(x)^2-lambda^2+3*a*lambda-a*(a+lambda)*coth(lambda*x)^2,y(x), singsol=al
```

$$y(x) = -\frac{\left((c_1 a + c_1\lambda)\,\mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right) + (a+\lambda)\,\mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right)\right)\coth(\lambda x)}{c_1\,\mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right)} + \frac{2c_1\lambda\,\mathrm{LegendreQ}\left(\frac{a+\lambda}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right)}{c_1\,\mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right)} + \frac{2\,\mathrm{LegendreP}\left(\frac{a+\lambda}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right)\lambda}{c_1\,\mathrm{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right) + \mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(\lambda x)\right)}$$

✓ Solution by Mathematica

Time used: 5.717 (sec). Leaf size: 267

```
DSolve[y'[x]==y[x]^2-\[Lambda]^2+3*a*\[Lambda]-a*(a+\[Lambda])*Coth[\[Lambda]*x]^2,y[x],x,Inc
```

$$y(x) \to \frac{(a-2\lambda)e^{3\lambda x}(\coth(\lambda x)-1)\left(-\lambda e^{\lambda x}\operatorname{AppellF1}\left(1-\frac{a}{\lambda},-\frac{2a}{\lambda},2,2-\frac{a}{\lambda},e^{2x\lambda},-e^{2x\lambda}\right)((a-\lambda)\cosh(2\lambda x)+a}{(2\lambda-a)\left(e^{2\lambda x}+1\right)\left(\lambda e^{2\lambda x}\operatorname{AppellF1}\left(1-\frac{a}{\lambda},-\frac{2a}{\lambda},2,2-\frac{a}{\lambda},e^{2x\lambda},\right.\right.}$$

$$y(x) \to a\coth(\lambda x) - \lambda\tanh(\lambda x)$$

6.7 problem 24

Internal problem ID [9725]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax\coth(bx)^m y - a\coth(bx)^m = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*coth(b*x)^m*y(x)+a*coth(b*x)^m,y(x), singsol=all)
```

$$y(x) = -\frac{e^{\int \frac{a\coth(xb)^m x^2-2}{x}dx}x + \int e^{\int \frac{a\coth(xb)^m x^2-2}{x}dx}dx - c_1}{\left(-c_1 + \int e^{\int \frac{a\coth(xb)^m x^2-2}{x}dx}dx\right)x}$$

✓ Solution by Mathematica

Time used: 6.74 (sec). Leaf size: 86

```
DSolve[y'[x]==y[x]^2+a*x*Coth[b*x]^m*y[x]+a*Coth[b*x]^m,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{x + \frac{\exp\left(-\int_1^x -a\coth^m(bK[5])K[5]dK[5]\right)}{\int_1^x \frac{\exp\left(-\int_1^{K[6]} -a\coth^m(bK[5])K[5]dK[5]\right)}{K[6]^2}dK[6]+c_1}}{x^2}$$
$$y(x) \to -\frac{1}{x}$$

6.8 problem 25

Internal problem ID [9726]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(\coth(\lambda x)\, a + b)\, y' - y^2 - c\coth(\mu x)\, y + d^2 - cd\coth(\mu x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 217

```
dsolve((a*coth(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*coth(mu*x)*y(x)-d^2+c*d*coth(mu*x),y(x), si
```

$$y(x) = -d - \frac{e^{\int \frac{c\coth(\mu x)}{a\coth(\lambda x)+b}dx}(a\coth(\lambda x)+b)^{-\frac{2ad}{\lambda(a-b)(a+b)}}(\coth(\lambda x)-1)^{\frac{d}{\lambda(a+b)}}(\coth(\lambda x)+1)^{\frac{d}{\lambda(a-b)}}}{\int \frac{e^{\int \frac{c\coth(\mu x)}{a\coth(\lambda x)+b}dx}(a\coth(\lambda x)+b)^{-\frac{2ad}{\lambda(a-b)(a+b)}}(\coth(\lambda x)-1)^{\frac{d}{\lambda(a+b)}}(\coth(\lambda x)+1)^{\frac{d}{\lambda(a-b)}}}{a\coth(\lambda x)+b}dx - c_1}$$

✓ Solution by Mathematica

Time used: 87.742 (sec). Leaf size: 808

```
DSolve[(a*Coth[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Coth[\[Mu]*x]*y[x]-d^2+c*d*Coth[\[Mu]*x],y[x],
```

$$\text{Solve}\Bigg[\int_1^x -\frac{e^{-\int_1^{K[6]} \frac{\text{csch}(\mu K[5])(-2d\cosh(\lambda K[5]-\mu K[5])+2d\cosh(\lambda K[5]+\mu K[5])-c\sinh(\lambda K[5]-\mu K[5])-c\sinh(\lambda K[5]+\mu K[5]))}{2(a\cosh(\lambda K[5])+b\sinh(\lambda K[5]))}dK[5]}(d\cosh(\lambda K[6]-\mu K[6])}{c\mu(b\cosh(\lambda K[6]-\mu K[6])-b\cosh(\lambda K[6]}$$
$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{e^{-\int_1^{K[6]} \frac{\text{csch}(\mu K[5])(-2d\cosh(\lambda K[5]-\mu K[5])+2d\cosh(\lambda K[5]+\mu K[5])-c\sinh(\lambda K[5]-\mu K[5])-c\sinh(\lambda K[5]+\mu K[5]))}{2(a\cosh(\lambda K[5])+b\sinh(\lambda K[5]))}dK[5]}(d\cosh(}{c\mu(d+K[7])^2(b\cosh(\lambda K[}\right.\right.$$
$$\left.-\frac{e^{-\int_1^x \frac{\text{csch}(\mu K[5])(-2d\cosh(\lambda K[5]-\mu K[5])+2d\cosh(\lambda K[5]+\mu K[5])-c\sinh(\lambda K[5]-\mu K[5])-c\sinh(\lambda K[5]+\mu K[5]))}{2(a\cosh(\lambda K[5])+b\sinh(\lambda K[5]))}dK[5]}}{c\mu(d+K[7])^2}\right)dK[7]=c_1,y(x)\Bigg]$$

6.9 problem 26

Internal problem ID [9727]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 + 2\lambda^2 \tanh(\lambda x)^2 + 2\lambda^2 \coth(\lambda x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 330

```
dsolve(diff(y(x),x)=y(x)^2-2*lambda^2*tanh(lambda*x)^2-2*lambda^2*coth(lambda*x)^2,y(x), sing
```

$$y(x) = \frac{\lambda\left(\left(\left(-3\coth(\lambda x)^2 c_1 - c_1\right)\operatorname{csch}(\lambda x)^2 \sinh(\lambda x)^2 + 2\coth(\lambda x)\cosh(\lambda x)\sinh(\lambda x)\operatorname{csch}(\lambda x)^2 c_1\right)\ln(\coth(\right.}{}$$

✓ Solution by Mathematica

Time used: 3.746 (sec). Leaf size: 65

```
DSolve[y'[x]==y[x]^2-2*\[Lambda]^2*Tanh[\[Lambda]*x]^2-2*\[Lambda]^2*Coth[\[Lambda]*x]^2,y[x]
```

$$y(x) \to -\frac{2\lambda\left(\cosh(4\lambda x) - \coth(2\lambda x)\left(-2\log\left(e^{2\lambda x}\right) + c_1\right) - 3\right)}{-2\log\left(e^{2\lambda x}\right) + \sinh(4\lambda x) + c_1}$$

$$y(x) \to 2\lambda\coth(2\lambda x)$$

6.10 problem 27

Internal problem ID [9728]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \lambda a - b\lambda + 2ab + a(a+\lambda)\tanh(\lambda x)^2 + b(b+\lambda)\coth(\lambda x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 1111

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda+b*lambda-2*a*b-a*(a+lambda)*tanh(lambda*x)^2-b*(b+lambda)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 17.263 (sec). Leaf size: 162

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]+b*\[Lambda]-2*a*b-a*(a+\[Lambda])*Tanh[\[Lambda]*x]^2-b*(b+\
```

$$y(x) \to \frac{1}{2}\left(\frac{4\lambda(a+b)\left(e^{2\lambda x}+1\right)^{\frac{2a}{\lambda}}\left(1-e^{2\lambda x}\right)^{\frac{2b}{\lambda}}}{\lambda \operatorname{AppellF1}\left(-\frac{a+b}{\lambda},-\frac{2b}{\lambda},-\frac{2a}{\lambda},-\frac{a+b-\lambda}{\lambda},e^{2x\lambda},-e^{2x\lambda}\right)-c_1(a+b)\left(e^{2\lambda x}\right)^{\frac{a+b}{\lambda}}} + 2a\tanh(\lambda x) + b\tanh\left(\frac{\lambda x}{2}\right) + b\coth\left(\frac{\lambda x}{2}\right)\right)$$

$$y(x) \to a\tanh(\lambda x) + b\coth(\lambda x)$$

7 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions

7.1 problem 1

Internal problem ID [9729]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\ln(x)^n y^2 - bm\,x^{m-1} + a\,b^2 x^{2m}\ln(x)^n = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*(ln(x))^n*y(x)^2+b*m*x^(m-1)-a*b^2*x^(2*m)*(ln(x))^n,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*(Log[x])^n*y[x]^2+b*m*x^(m-1)-a*b^2*x^(2*m)*(Log[x])^n,y[x],x,IncludeSingular
```

Not solved

7.2 problem 2

Internal problem ID [9730]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `unknown`

$$\boxed{y'x - y^2a - b\ln(x) - c = 0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=a*y(x)^2+b*ln(x)+c,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==a*y[x]^2+b*Log[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.3 problem 3

Internal problem ID [9731]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - y^2a - b\ln(x)^k - c\ln(x)^{2k+2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 660

```
dsolve(x*diff(y(x),x)=a*y(x)^2+b*(ln(x))^k+c*(ln(x))^(2*k+2),y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(-i\sqrt{a}\,\ln(x)^{k+2}\sqrt{c}\,c_1k^2 - 4i\sqrt{a}\,\ln(x)^{k+2}\sqrt{c}\,c_1k - 3i\sqrt{a}\,\ln(x)^{k+2}\sqrt{c}\,c_1 + \ln(x)^{k+2}c_1abk + \ln(x)^{k+2}c\right.\right.}{}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==a*y[x]^2+b*(Log[x])^k+c*(Log[x])^(2*k+2),y[x],x,IncludeSingularSolutions -> T
```

Not solved

7.4 problem 4

Internal problem ID [9732]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - y^2x + a^2x\ln(\beta x)^2 - a = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=x*y(x)^2-a^2*x*(ln(beta*x))^2+a,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==x*y[x]^2-a^2*x*(Log[\[Beta]*x])^2+a,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.5 problem 5

Internal problem ID [9733]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - y^2x + a^2x\ln(\beta x)^{2k} - ak\ln(\beta x)^{k-1} = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=x*y(x)^2-a^2*x*(ln(beta*x))^(2*k)+a*k*(ln(beta*x))^(k-1),y(x), singsol=
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==x*y[x]^2-a^2*x*(Log[\[Beta]*x])^(2*k)+a*k*(Log[\[Beta]*x])^(k-1),y[x],x,Inclu
```

Not solved

7.6 problem 6

Internal problem ID [9734]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - a\,x^n y^2 - b + a\,b^2 x^n \ln\left(x\right)^2 = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=a*x^n*y(x)^2+b-a*b^2*x^n*(ln(x))^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==a*x^n*y[x]^2+b-a*b^2*x^n*(Log[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.7 problem 7

Internal problem ID [9735]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$x^2y' - y^2x^2 - a\ln(x)^2 - b\ln(x) - c = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 850

```
dsolve(x^2*diff(y(x),x)=x^2*y(x)^2+a*(ln(x))^2+b*ln(x)+c,y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==x^2*y[x]^2+a*(Log[x])^2+b*Log[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.8 problem 8

Internal problem ID [9736]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$x^2y' - y^2x^2 - a(b\ln(x) + c)^n - \frac{1}{4} = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)=x^2*y(x)^2+a*(b*ln(x)+c)^n+1/4,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==x^2*y[x]^2+a*(b*Log[x]+c)^n+1/4,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.9 problem 9

Internal problem ID [9737]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^2 \ln (ax) (y' - y^2) - 1 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve(x^2*ln(a*x)*(diff(y(x),x)-y(x)^2)=1,y(x), singsol=all)
```

$$y(x) = -\frac{c_1 \operatorname{Ei}_1(-\ln(ax)) - 1}{x((c_1 \operatorname{Ei}_1(-\ln(ax)) - 1)\ln(ax) + axc_1)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*Log[a*x]*(y'[x]-y[x]^2)==1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

8 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2

8.1 problem 10

Internal problem ID [9738]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\ln(\beta x)\, y + ab\ln(\beta x) + b^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x)=y(x)^2+a*ln(beta*x)*y(x)-a*b*ln(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = b - \frac{(\beta x)^{ax}\, e^{-ax} e^{2xb}}{\int (\beta x)^{ax}\, e^{-ax} e^{2xb} dx - c_1}$$

✓ Solution by Mathematica

Time used: 0.985 (sec). Leaf size: 187

```
DSolve[y'[x]==y[x]^2+a*Log[\[Beta]*x]*y[x]-a*b*Log[\[Beta]*x]-b^2,y[x],x,IncludeSingularSolut
```

$$\text{Solve}\left[\int_1^x \frac{e^{2bK[1]-aK[1]}(\beta K[1])^{aK[1]}(b + a\log(\beta K[1]) + y(x))}{a(b - y(x))} dK[1] + \int_1^{y(x)} \left(\frac{e^{2bx-ax}(x\beta)^{ax}}{a(K[2]-b)^2} - \int_1^x \left(\frac{e^{2bK[1]-aK[1]}(b + K[2] + a\log(\beta K[1]))(\beta K[1])^{aK[1]}}{a(b-K[2])^2} + \frac{e^{2bK[1]-aK[1]}(\beta K[1])^{aK[1]}}{a(b-K[2])}\right) dK[1]\right) dK[2] = c_1, y\right.$$

8.2 problem 11

Internal problem ID [9739]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax\ln(bx)^m y - a\ln(bx)^m = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x)=y(x)^2+a*x*(ln(b*x))^m*y(x)+a*(ln(b*x))^m,y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{a\ln(xb)^m x^2-2}{x}dx}}{c_1 - \left(\int e^{\int \frac{a\ln(xb)^m x^2-2}{x}dx}dx\right)} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 2.007 (sec). Leaf size: 93

```
DSolve[y'[x]==y[x]^2+a*x*(Log[b*x])^m*y[x]+a*(Log[b*x])^m,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{x + \frac{\exp\left(-\frac{a\log^{m+1}(bx)\,\text{ExpIntegralE}(-m,-2\log(bx))}{b^2}\right)}{\int_1^x \frac{\exp\left(-\frac{a\,\text{ExpIntegralE}(-m,-2\log(bK[1]))\log^{m+1}(bK[1])}{b^2}\right)}{K[1]^2}dK[1]+c_1}}{x^2}$$

$$y(x) \to -\frac{1}{x}$$

8.3 problem 12

Internal problem ID [9740]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,x^n y^2 + ab\,x^{1+n} \ln(x)\,y - b\ln(x) - b = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2-a*b*x^(n+1)*ln(x)*y(x)+b*ln(x)+b,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*x^n*y[x]^2-a*b*x^(n+1)*Log[x]*y[x]+b*Log[x]+b,y[x],x,IncludeSingularSolutions
```

Not solved

8.4 problem 13

Internal problem ID [9741]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + (1+n)\,x^{n}y^{2} - a\,x^{1+n}\ln(x)^{m}\,y + a\ln(x)^{m} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 201

```
dsolve(diff(y(x),x)=-(n+1)*x^n*y(x)^2+a*x^(n+1)*(ln(x))^m*y(x)-a*(ln(x))^m,y(x), singsol=all)
```

$$y(x) = \frac{\left(-\mathrm{e}^{\int \frac{\ln(x)^{m}x^{n}a\,x^{2}-2n-2}{x}dx}x^{n}x + \int \left(-x^{n}n\,\mathrm{e}^{a\left(\int x^{1+n}\ln(x)^{m}dx\right)-2n\left(\int \frac{1}{x}dx\right)-2\left(\int \frac{1}{x}dx\right)} - x^{n}\mathrm{e}^{a\left(\int x^{1+n}\ln(x)^{m}dx\right)-2n\left(\int \frac{1}{x}dx\right)-2}\right.}{x\left(\int \left(-x^{n}n\,\mathrm{e}^{a\left(\int x^{1+n}\ln(x)^{m}dx\right)-2n\left(\int \frac{1}{x}dx\right)-2\left(\int \frac{1}{x}dx\right)} - x^{n}\mathrm{e}^{a\left(\int x^{1+n}\ln(x)^{m}dx\right)-2n\left(\int \frac{1}{x}dx\right)-2\left(\int \frac{1}{x}dx\right)}\right)dx}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(n+1)*x^n*y[x]^2+a*x^(n+1)*(Log[x])^m*y[x]-a*(Log[x])^m,y[x],x,IncludeSingular
```

Not solved

8.5 problem 14

Internal problem ID [9742]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - a\ln(x)^n y + abx\ln(x)^{1+n} y - b\ln(x) - b = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(diff(y(x),x)=a*(ln(x))^n*y(x)-a*b*x*(ln(x))^(n+1)*y(x)+b*ln(x)+b,y(x), singsol=all)
```

$$y(x) = \left(\int e^{a\left(\int \ln(x)^n(-1+\ln(x)bx)dx\right)} b(\ln(x)+1)\,dx + c_1\right) e^{\int\left(-\ln(x)^{1+n}abx + a\ln(x)^n\right)dx}$$

✓ Solution by Mathematica

Time used: 0.535 (sec). Leaf size: 96

```
DSolve[y'[x]==a*(Log[x])^n*y[x]-a*b*x*(Log[x])^(n+1)*y[x]+b*Log[x]+b,y[x],x,IncludeSingularSo
```

$$y(x) \to \exp\left(a\log^{n+1}(x)(b\log(x)\,\text{ExpIntegralE}(-n-1,-2\log(x)) - \text{ExpIntegralE}(-n,-\log(x)))\right)\left(\int_1^x b\exp\left(a\log^{n+1}(K[1])(\text{ExpIntegralE}(-n,-\log(K[1]))-b\,\text{ExpInteg}\right. \right.$$
$$\left. + 1)dK[1] + c_1\right)$$

8.6 problem 15

Internal problem ID [9743]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(x)]‘], _Riccati]`

$$y' - a\ln(x)^k (y - b\,x^n - c)^2 - bn\,x^{n-1} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)=a*(ln(x))^k*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-2a\ln(x)^k x^n b - 2a\ln(x)^k c\right)\ln(x)^{-k}}{2a} + \frac{1}{c_1 - \left(\int a\ln(x)^k\,dx\right)}$$

✓ Solution by Mathematica

Time used: 0.999 (sec). Leaf size: 44

```
DSolve[y'[x]==a*(Log[x])^k*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{a\log^{k+1}(x)\,\text{ExpIntegralE}(-k, -\log(x)) + c_1} + bx^n + c$$

$$y(x) \to bx^n + c$$

8.7 problem 16

Internal problem ID [9744]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\ln(x)^n y^2 - b\ln(x)^m y - bc\ln(x)^m + a c^2 \ln(x)^n = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 134

```
dsolve(diff(y(x),x)=a*(ln(x))^n*y(x)^2+b*(ln(x))^m*y(x)+b*c*(ln(x))^m-a*c^2*(ln(x))^n,y(x), s
```

$$y(x) = -\frac{\left(\left(\int a\ln(x)^n \mathrm{e}^{\int(-2\ln(x)^n ac+\ln(x)^m b)dx}dx\right)\mathrm{e}^{\int(2\ln(x)^n ac-\ln(x)^m b)dx}c + c_1\mathrm{e}^{\int(2\ln(x)^n ac-\ln(x)^m b)dx}c + 1\right)\mathrm{e}^{\int(-2\ln(x)^n a}}{c_1 + \int a\ln(x)^n \mathrm{e}^{\int(-2\ln(x)^n ac+\ln(x)^m b)dx}dx}$$

✓ Solution by Mathematica

Time used: 2.485 (sec). Leaf size: 385

```
DSolve[y'[x]==a*(Log[x])^n*y[x]^2+b*(Log[x])^m*y[x]+b*c*(Log[x])^m-a*c^2*(Log[x])^n,y[x],x,In
```

$$\text{Solve}\Bigg[\int_1^x \frac{\exp\left(b\Gamma(m+1,-\log(K[1]))(-\log(K[1]))^{-m}\log^m(K[1]) - 2ac\Gamma(n+1,-\log(K[1]))(-\log(K[1]\right.}{ab(m-n)(c+y(x))}$$
$$+\int_1^{y(x)}\left(\frac{\exp\left(b\Gamma(m+1,-\log(x))(-\log(x))^{-m}\log^m(x) - 2ac\Gamma(n+1,-\log(x))(-\log(x))^{-n}\log^n(x)\right)}{ab(m-n)(c+K[2])^2}\right.$$
$$-\int_1^x\left(-\frac{\exp\left(b\Gamma(m+1,-\log(K[1]))(-\log(K[1]))^{-m}\log^m(K[1]) - 2ac\Gamma(n+1,-\log(K[1]))(-\log(K[1])\right.}{b(m-n)(c+K[2])}\right.$$

8.8 problem 17

Internal problem ID [9745]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$\boxed{y'x - (ya + b\ln(x))^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(x*diff(y(x),x)=(a*y(x)+b*ln(x))^2,y(x), singsol=all)
```

$$y(x) = -\frac{\ln(x)\, ab - \tan\left(c_1 a\sqrt{ba} + \ln(x)\sqrt{ba}\right)\sqrt{ba}}{a^2}$$

✓ Solution by Mathematica

Time used: 4.084 (sec). Leaf size: 43

```
DSolve[x*y'[x]==(a*y[x]+b*Log[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{b\log(x)}{a} + \sqrt{\frac{b}{a^3}}\tan\left(a^2\sqrt{\frac{b}{a^3}}\log(x) + c_1\right)$$

8.9 problem 18

Internal problem ID [9746]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - a\ln(\lambda x)^m y^2 - ky - a\,b^2x^{2k}\ln(\lambda x)^m = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 36

```
dsolve(x*diff(y(x),x)=a*(ln(lambda*x))^m*y(x)^2+k*y(x)+a*b^2*x^(2*k)*(ln(lambda*x))^m,y(x), s
```

$$y(x) = -\tan\left(-ba\left(\int \frac{x^k \ln(\lambda x)^m}{x}dx\right) + c_1\right) b\,x^k$$

✓ Solution by Mathematica

Time used: 1.362 (sec). Leaf size: 59

```
DSolve[x*y'[x]==a*(Log[\[Lambda]*x])^m*y[x]^2+k*y[x]+a*b^2*x^(2*k)*(Log[\[Lambda]*x])^m,y[x],
```

$$y(x) \to \sqrt{b^2}x^k \tan\left(-a\sqrt{b^2}x^k(\lambda x)^{-k}\log^{m+1}(\lambda x)\,\text{ExpIntegralE}(-m, -k\log(x\lambda)) + c_1\right)$$

8.10 problem 19

Internal problem ID [9747]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(x)]‘], _Riccati]`

$$\boxed{y'x - a\,x^n(y + b\ln(x))^2 + b = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x)=a*x^n*(y(x)+b*ln(x))^2-b,y(x), singsol=all)
```

$$y(x) = -\ln(x)\,b + \frac{1}{c_1 - \frac{a\,x^n}{n}}$$

✓ Solution by Mathematica

Time used: 0.425 (sec). Leaf size: 35

```
DSolve[x*y'[x]==a*x^n*(y[x]+b*Log[x])^2-b,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -b\log(x) + \frac{n}{-ax^n + c_1 n}$$
$$y(x) \to -b\log(x)$$

8.11 problem 20

Internal problem ID [9748]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - a\,x^{2n}\ln(x)\,y^2 - (x^n b\ln(x) - n)\,y - c\ln(x) = 0$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 83

```
dsolve(x*diff(y(x),x)=a*x^(2*n)*ln(x)*y(x)^2+(b*x^n*ln(x)-n)*y(x)+c*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(\tan\left(\frac{\sqrt{4b^2ac-b^4}\,(\ln(x)x^n bn + c_1n^2 - b\,x^n)}{2b^2n^2}\right)\sqrt{4b^2ac-b^4} - b^2\right)x^{-n}}{2ab}$$

✓ Solution by Mathematica

Time used: 1.112 (sec). Leaf size: 104

```
DSolve[x*y'[x]==a*x^(2*n)*Log[x]*y[x]^2+(b*x^n*Log[x]-n)*y[x]+c*Log[x],y[x],x,IncludeSingular
```

$$y(x) \to \frac{x^{-n}\left(-b + \sqrt{b^2-4ac}\left(-1 + \frac{2c_1}{e^{\frac{x^n\sqrt{b^2-4ac}(n\log(x)-1)}{n^2}} + c_1}\right)\right)}{2a}$$

$$y(x) \to \frac{x^{-n}\left(\sqrt{b^2-4ac} - b\right)}{2a}$$

8.12 problem 21

Internal problem ID [9749]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^2y' - a^2x^2y^2 + xy - b^2\ln(x)^n = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 324

```
dsolve(x^2*diff(y(x),x)=a^2*x^2*y(x)^2-x*y(x)+b^2*(ln(x))^n,y(x), singsol=all)
```

$$y(x) = \frac{\frac{\ln(x)^{\frac{n}{2}+1}\sqrt{b^2a^2}\, c_1 \operatorname{BesselY}\left(\frac{n+3}{n+2}, \frac{2\sqrt{b^2a^2}\ \ln(x)^{\frac{n}{2}+1}}{n+2}\right)}{\left(\operatorname{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{b^2a^2}\ \ln(x)^{\frac{n}{2}+1}}{n+2}\right)c_1 + \operatorname{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{b^2a^2}\ \ln(x)^{\frac{n}{2}+1}}{n+2}\right)\right)a^2x} + \frac{\operatorname{BesselJ}\left(\frac{n+3}{n+2}, \frac{2\sqrt{b^2a^2}\ \ln(x)^{\frac{n}{2}+1}}{n+2}\right)\sqrt{b^2a^2}\ \ln(x)^{\frac{n}{2}+1} - \operatorname{BesselY}}{\left(\operatorname{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{b^2a^2}\ \ln(x)^{\frac{n}{2}+1}}{n+2}\right)c}}{\ln(x)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==a^2*x^2*y[x]^2-x*y[x]+b^2*(Log[x])^n,y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

8.13 problem 22

Internal problem ID [9750]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(a\ln(x)+b)\,y'-y^2-c\ln(x)^n\,y+\lambda^2-\lambda c\ln(x)^n=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 160

```
dsolve((a*ln(x)+b)*diff(y(x),x)=y(x)^2+c*(ln(x))^n*y(x)-lambda^2+lambda*c*(ln(x))^n,y(x), sin
```

$$y(x)=-\frac{\left(\left(\int\frac{e^{\int\frac{\ln(x)^n c-2\lambda}{a\ln(x)+b}dx}}{a\ln(x)+b}dx\right)e^{\int-\frac{\ln(x)^n c-2\lambda}{a\ln(x)+b}dx}\lambda+c_1e^{\int-\frac{\ln(x)^n c-2\lambda}{a\ln(x)+b}dx}\lambda+1\right)e^{\int\frac{\ln(x)^n c-2\lambda}{a\ln(x)+b}dx}}{c_1+\int\frac{e^{\int\frac{\ln(x)^n c-2\lambda}{a\ln(x)+b}dx}}{a\ln(x)+b}dx}$$

✓ Solution by Mathematica

Time used: 3.348 (sec). Leaf size: 275

```
DSolve[(a*Log[x]+b)*y'[x]==y[x]^2+c*(Log[x])^n*y[x]-\[Lambda]^2+\[Lambda]*c*(Log[x])^n,y[x],x
```

$$\text{Solve}\left[\int_1^x-\frac{\exp\left(-\int_1^{K[6]}\frac{2\lambda-c\log^n(K[5])}{b+a\log(K[5])}dK[5]\right)(c\log^n(K[6])-\lambda+y(x))}{cn(b+a\log(K[6]))(\lambda+y(x))}dK[6]\right.$$
$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x\frac{2\lambda-c\log^n(K[5])}{b+a\log(K[5])}dK[5]\right)}{cn(\lambda+K[7])^2}\right.$$
$$-\int_1^x\left(\frac{\exp\left(-\int_1^{K[6]}\frac{2\lambda-c\log^n(K[5])}{b+a\log(K[5])}dK[5]\right)(c\log^n(K[6])-\lambda+K[7])}{cn(\lambda+K[7])^2(b+a\log(K[6]))}-\frac{\exp\left(-\int_1^{K[6]}\frac{2\lambda-c\log^n(K[5])}{b+a\log(K[5])}dK[5]\right)}{cn(\lambda+K[7])(b+a\log(K[6]))}\right)d$$

8.14 problem 23

Internal problem ID [9751]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(a\ln(x)+b)\,y'-\ln(x)^n\,y^2-cy+\lambda^2\ln(x)^n-c\lambda=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 168

```
dsolve((a*ln(x)+b)*diff(y(x),x)=(ln(x))^n*y(x)^2+c*y(x)-lambda^2*(ln(x))^n+c*lambda,y(x), sin
```

$$y(x)=-\frac{\left(\left(\int\frac{\ln(x)^n\mathrm{e}^{\int-\frac{2\ln(x)^n\lambda-c}{a\ln(x)+b}dx}}{a\ln(x)+b}dx\right)\mathrm{e}^{\int\frac{2\ln(x)^n\lambda-c}{a\ln(x)+b}dx}\lambda+c_1\mathrm{e}^{\int\frac{2\ln(x)^n\lambda-c}{a\ln(x)+b}dx}\lambda+1\right)\mathrm{e}^{\int-\frac{2\ln(x)^n\lambda-c}{a\ln(x)+b}dx}}{c_1+\int\frac{\ln(x)^n\mathrm{e}^{\int-\frac{2\ln(x)^n\lambda-c}{a\ln(x)+b}dx}}{a\ln(x)+b}dx}$$

✓ Solution by Mathematica

Time used: 3.235 (sec). Leaf size: 286

```
DSolve[(a*Log[x]+b)*y'[x]==(Log[x])^n*y[x]^2+c*y[x]-\[Lambda]^2*(Log[x])^n+c*\[Lambda],y[x],x
```

$$\text{Solve}\left[\int_1^x\frac{\exp\left(-\int_1^{K[2]}-\frac{c-2\lambda\log^n(K[1])}{b+a\log(K[1])}dK[1]\right)(-\lambda\log^n(K[2])+y(x)\log^n(K[2])+c)}{cn(b+a\log(K[2]))(\lambda+y(x))}dK[2]\right.$$
$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]}-\frac{c-2\lambda\log^n(K[1])}{b+a\log(K[1])}dK[1]\right)\log^n(K[2])}{cn(\lambda+K[3])(b+a\log(K[2]))}-\frac{\exp\left(-\int_1^{K[2]}-\frac{c-2\lambda\log^n(K[1])}{b+a\log(K[1])}dK[1]\right)(-\lambda\mathrm{l}}{cn(\lambda+K[3])^2(b+}\right.\right.$$
$$\left.\left.-\frac{\exp\left(-\int_1^x-\frac{c-2\lambda\log^n(K[1])}{b+a\log(K[1])}dK[1]\right)}{cn(\lambda+K[3])^2}\right)dK[3]=c_1,y(x)\right]$$

9 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine

9.1 problem 1

Internal problem ID [9752]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \alpha y^2 - \beta - \gamma \sin(\lambda x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 110

```
dsolve(diff(y(x),x)=alpha*y(x)^2+beta+gamma*sin(lambda*x),y(x), singsol=all)
```

$$y(x) = -\frac{\lambda\left(c_1 \,\mathrm{MathieuSPrime}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, -\frac{\pi}{4}+\frac{\lambda x}{2}\right) + \mathrm{MathieuCPrime}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, -\frac{\pi}{4}+\frac{\lambda x}{2}\right)\right)}{2\alpha\left(c_1 \,\mathrm{MathieuS}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, -\frac{\pi}{4}+\frac{\lambda x}{2}\right) + \mathrm{MathieuC}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, -\frac{\pi}{4}+\frac{\lambda x}{2}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.342 (sec). Leaf size: 191

```
DSolve[y'[x]==\[Alpha]*y[x]^2+\[Beta]+\[Gamma]*Sin[\[Lambda]*x],y[x],x,IncludeSingularSolutio
```

$$y(x) \to -\frac{\lambda\left(\mathrm{MathieuSPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(\pi - 2\lambda x)\right] + c_1 \mathrm{MathieuCPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(2\lambda x - \pi)\right]\right)}{2\alpha\left(\mathrm{MathieuS}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(2\lambda x - \pi)\right] + c_1 \mathrm{MathieuC}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(\pi - 2\lambda x)\right]\right)}$$

$$y(x) \to \frac{\lambda \mathrm{MathieuCPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(\pi - 2\lambda x)\right]}{2\alpha \mathrm{MathieuC}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(\pi - 2\lambda x)\right]}$$

9.2 problem 2

Internal problem ID [9753]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 + a^2 - a\lambda \sin(\lambda x) - a^2 \sin(\lambda x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 415

```
dsolve(diff(y(x),x)=y(x)^2-a^2+a*lambda*sin(lambda*x)+a^2*sin(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\left(\left(4\sqrt{-\cos(\lambda x)^2+1}\,c_1 a + 4c_1 a + 2c_1\lambda\right)\operatorname{HeunC}\left(\frac{4a}{\lambda}, \frac{1}{2}, -\frac{1}{2}, -\frac{2a}{\lambda}, \frac{8a+3\lambda}{8\lambda}, \frac{\sqrt{-\cos(\lambda x)^2+1}}{2} + \frac{1}{2}\right) + 2a\operatorname{Heu}\right.}{2\sqrt{2\sqrt{}}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-a^2+a*\[Lambda]*Sin[\[Lambda]*x]+a^2*Sin[\[Lambda]*x]^2,y[x],x,IncludeSi
```

Not solved

9.3 problem 3

Internal problem ID [9754]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \lambda^2 - c\sin(\lambda x + a)^n \sin(\lambda x + b)^{-n-4} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+c*sin(lambda*x+a)^n*sin(lambda*x+b)^(-n-4),y(x), singsol=
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+c*Sin[\[Lambda]*x+a]^n*Sin[\[Lambda]*x+b]^(-n-4),y[x],x,Incl
```

Not solved

9.4 problem 4

Internal problem ID [9755]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\sin(\beta x)\, y - ab\sin(\beta x) + b^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)=y(x)^2+a*sin(beta*x)*y(x)+a*b*sin(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = -b - \frac{\mathrm{e}^{-\frac{a\cos(\beta x)}{\beta} - 2xb}}{\int \mathrm{e}^{-\frac{a\cos(\beta x)}{\beta} - 2xb} dx - c_1}$$

✓ Solution by Mathematica

Time used: 5.665 (sec). Leaf size: 187

```
DSolve[y'[x]==y[x]^2+a*Sin[\[Beta]*x]*y[x]+a*b*Sin[\[Beta]*x]-b^2,y[x],x,IncludeSingularSolut
```

$$\text{Solve}\left[\int_1^x -\frac{e^{-\frac{a\cos(\beta K[1])}{\beta}-2bK[1]}(-b + a\sin(\beta K[1]) + y(x))}{a\beta(b+y(x))} dK[1] + \int_1^{y(x)} \left(\frac{e^{-2bx-\frac{a\cos(x\beta)}{\beta}}}{a\beta(b+K[2])^2} - \int_1^x \left(\frac{e^{-\frac{a\cos(\beta K[1])}{\beta}-2bK[1]}(-b+K[2]+a\sin(\beta K[1]))}{a\beta(b+K[2])^2} - \frac{e^{-\frac{a\cos(\beta K[1])}{\beta}-2bK[1]}}{a\beta(b+K[2])}\right) dK[1]\right) dK[2] = c_1, y(x)\right]$$

9.5 problem 5

Internal problem ID [9756]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - a\sin(bx)^m y - a\sin(bx)^m = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+a*sin(b*x)^m*y(x)+a*sin(b*x)^m,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a*Sin[b*x]^m*y[x]+a*Sin[b*x]^m,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

9.6 problem 6

Internal problem ID [9757]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \sin(\lambda x) y^2 - \lambda \sin(\lambda x)^3 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
dsolve(diff(y(x),x)=lambda*sin(lambda*x)*y(x)^2+lambda*sin(lambda*x)^3,y(x), singsol=all)
```

$$y(x) = \frac{2c_1 e^{\cos(\lambda x)^2}}{\sqrt{\pi}\,(\operatorname{erfi}(\cos(\lambda x))\,c_1 + 1)} - \frac{\left(\sqrt{\pi}\,\operatorname{erfi}(\cos(\lambda x))\,c_1 + \sqrt{\pi}\right)\cos(\lambda x)}{\sqrt{\pi}\,(\operatorname{erfi}(\cos(\lambda x))\,c_1 + 1)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Sin[\[Lambda]*x]*y[x]^2+\[Lambda]*Sin[\[Lambda]*x]^3,y[x],x,IncludeSi
```

Not solved

9.7 problem 7

Internal problem ID [9758]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$2y' - (\lambda + a - \sin(\lambda x)\, a)\, y^2 - \lambda + a + \sin(\lambda x)\, a = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 3136

```
dsolve(2*diff(y(x),x)=(lambda+a-a*sin(lambda*x))*y(x)^2+lambda-a-a*sin(lambda*x),y(x), singso
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*y'[x]==(\[Lambda]+a-a*Sin[\[Lambda]*x])*y[x]^2+\[Lambda]-a-a*Sin[\[Lambda]*x],y[x],x
```

Not solved

9.8 problem 8

Internal problem ID [9759]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \left(\lambda + \sin(\lambda x)^2 a\right) y^2 - \lambda + a - \sin(\lambda x)^2 a = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 467

```
dsolve(diff(y(x),x)=(lambda+a*sin(lambda*x)^2)*y(x)^2+lambda-a+a*sin(lambda*x)^2,y(x), singso
```

$y(x) =$

$$-\frac{\left(\left(-4\cos(2\lambda x)\sqrt{-1+\cos(2\lambda x)}\,c_1 a\lambda + 4\sqrt{-1+\cos(2\lambda x)}\,c_1 a\lambda + 8\sqrt{-1+\cos(2\lambda x)}\,c_1\lambda^2\right) e^{\frac{a\cos(2\lambda x)}{2\lambda}}}{}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(\[Lambda]+a*Sin[\[Lambda]*x]^2)*y[x]^2+\[Lambda]-a+a*Sin[\[Lambda]*x]^2,y[x],x
```

Not solved

9.9 problem 9

Internal problem ID [9760]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + (1+k)\,x^k y^2 - a\,x^{1+k}\sin(x)^m\,y + a\sin(x)^m = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 172

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+a*x^(k+1)*sin(x)^m*y(x)-a*sin(x)^m,y(x), singsol=all)
```

$$y(x) = \frac{\left(\mathrm{e}^{\int \frac{\sin(x)^m x^k a\,x^2-2k-2}{x}dx} x\,x^k + \left(\int x^k \mathrm{e}^{\int \frac{a\,x^{k+2}\sin(x)^m-2k-2}{x}dx}dx\right)k + \int x^k \mathrm{e}^{\int \frac{a\,x^{k+2}\sin(x)^m-2k-2}{x}dx}dx - c_1\right)x^{-k}}{x\left(\left(\int x^k \mathrm{e}^{\int \frac{a\,x^{k+2}\sin(x)^m-2k-2}{x}dx}dx\right)k + \int x^k \mathrm{e}^{\int \frac{a\,x^{k+2}\sin(x)^m-2k-2}{x}dx}dx - c_1\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+a*x^(k+1)*Sin[x]^m*y[x]-a*Sin[x]^m,y[x],x,IncludeSingularSolu
```

Not solved

9.10 problem 10

Internal problem ID [9761]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - a\sin(\lambda x + \mu)^k (y - b\,x^n - c)^2 - bn\,x^{n-1} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 102

```
dsolve(diff(y(x),x)=a*sin(lambda*x+mu)^k*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-2a\,x^n(\sin(\lambda x)\cos(\mu)+\cos(\lambda x)\sin(\mu))^k b - 2ac(\sin(\lambda x)\cos(\mu)+\cos(\lambda x)\sin(\mu))^k\right)\sin(\lambda x+\mu)}{2a} + \frac{1}{c_1 - \left(\int a(\sin(\lambda x)\cos(\mu)+\cos(\lambda x)\sin(\mu))^k\,dx\right)}$$

✓ Solution by Mathematica

Time used: 3.654 (sec). Leaf size: 93

```
DSolve[y'[x]==a*Sin[\[Lambda]*x+\[Mu]]^k*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingularS
```

$$y(x) \to \frac{1}{-\frac{a\sqrt{\cos^2(\mu+\lambda x)}\sec(\mu+\lambda x)\sin^{k+1}(\mu+\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\sin^2(x\lambda+\mu)\right)}{(k+1)\lambda} + c_1} + bx^n + c$$

$$y(x) \to bx^n + c$$

9.11 problem 11

Internal problem ID [9762]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - a\sin(\lambda x)^m y^2 - ky - a\,b^2x^{2k}\sin(\lambda x)^m = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve(x*diff(y(x),x)=a*sin(lambda*x)^m*y(x)^2+k*y(x)+a*b^2*x^(2*k)*sin(lambda*x)^m,y(x), sin
```

$$y(x) = -\tan\left(-ba\left(\int \frac{x^k \sin(\lambda x)^m}{x}dx\right) + c_1\right) b\,x^k$$

✓ Solution by Mathematica

Time used: 1.116 (sec). Leaf size: 50

```
DSolve[x*y'[x]==a*Sin[\[Lambda]*x]^m*y[x]^2+k*y[x]+a*b^2*x^(2*k)*Sin[\[Lambda]*x]^m,y[x],x,In
```

$$y(x) \to \sqrt{b^2}x^k \tan\left(\sqrt{b^2}\int_1^x aK[1]^{k-1}\sin^m(\lambda K[1])dK[1] + c_1\right)$$

9.12 problem 12

Internal problem ID [9763]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(\sin(\lambda x)\,a+b)\,y'-y^2-c\sin(\mu x)\,y+d^2-cd\sin(\mu x)=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 155

```
dsolve((a*sin(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*sin(mu*x)*y(x)-d^2+c*d*sin(mu*x),y(x), sings
```

$$y(x)=-d-\frac{e^{\int\frac{c\sin(\mu x)}{a\sin(\lambda x)+b}dx-\frac{4d\arctan\left(\frac{2b\tan\left(\frac{\lambda x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{\lambda\sqrt{-a^2+b^2}}}}{\int\frac{e^{\int\frac{c\sin(\mu x)}{a\sin(\lambda x)+b}dx-\frac{4d\arctan\left(\frac{2b\tan\left(\frac{\lambda x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{\lambda\sqrt{-a^2+b^2}}}}{a\sin(\lambda x)+b}dx-c_1}$$

✓ Solution by Mathematica

Time used: 9.668 (sec). Leaf size: 289

```
DSolve[(a*Sin[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Sin[\[Mu]*x]*y[x]-d^2+c*d*Sin[\[Mu]*x],y[x],x,I
```

$$\text{Solve}\left[\int_1^x-\frac{\exp\left(-\int_1^{K[6]}\frac{2d-c\sin(\mu K[5])}{b+a\sin(\lambda K[5])}dK[5]\right)(-d+c\sin(\mu K[6])+y(x))}{c\mu(b+a\sin(\lambda K[6]))(d+y(x))}dK[6]\right.$$
$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x\frac{2d-c\sin(\mu K[5])}{b+a\sin(\lambda K[5])}dK[5]\right)}{c\mu(d+K[7])^2}\right.$$
$$-\int_1^x\left(\frac{\exp\left(-\int_1^{K[6]}\frac{2d-c\sin(\mu K[5])}{b+a\sin(\lambda K[5])}dK[5]\right)(-d+K[7]+c\sin(\mu K[6]))}{c\mu(d+K[7])^2(b+a\sin(\lambda K[6]))}-\frac{\exp\left(-\int_1^{K[6]}\frac{2d-c\sin(\mu K[5])}{b+a\sin(\lambda K[5])}dK[5]\right)}{c\mu(d+K[7])(b+a\sin(\lambda K[6]))}\right)$$

9.13 problem 13

Internal problem ID [9764]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(\sin(\lambda x)\, a + b)\left(y' - y^2\right) - \lambda^2 \sin(\lambda x)\, a = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 650

```
dsolve((a*sin(lambda*x)+b)*(diff(y(x),x)-y(x)^2)-a*lambda^2*sin(lambda*x)=0,y(x), singsol=all
```

$y(x) =$

$$-\frac{\lambda\left(\left(\left(-2\arctan\left(\frac{b\tan\left(\frac{\lambda x}{2}\right)+a}{\sqrt{-a^2+b^2}}\right)\sqrt{-a^2+b^2}\, a\, b^3 + a^4 b - b^3 a^2\right)\sin\left(\frac{\lambda x}{2}\right)^2 + \left(2\arctan\left(\frac{b\tan\left(\frac{\lambda x}{2}\right)+a}{\sqrt{-a^2+b^2}}\right)\sqrt{-}\right.\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 23.754 (sec). Leaf size: 186

```
DSolve[(a*Sin[\[Lambda]*x]+b)*(y'[x]-y[x]^2)-a*\[Lambda]^2*Sin[\[Lambda]*x]==0,y[x],x,Include
```

$$y(x) \to \frac{\lambda\left(2ab\cos(\lambda x)\arctan\left(\frac{a+b\tan\left(\frac{\lambda x}{2}\right)}{\sqrt{b^2-a^2}}\right) + \sqrt{b^2-a^2}\left(ac_1\lambda(b-a)(a+b)\cos(\lambda x) - a\sin(\lambda x) + b\right)\right)}{-2b(a\sin(\lambda x)+b)\arctan\left(\frac{a+b\tan\left(\frac{\lambda x}{2}\right)}{\sqrt{b^2-a^2}}\right) + \sqrt{b^2-a^2}\left(-a\cos(\lambda x) + c_1\lambda(a-b)(a+b)(a\sin(\lambda x)+b)\right)}$$

$$y(x) \to -\frac{a\lambda\cos(\lambda x)}{a\sin(\lambda x)+b}$$

10 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.

10.1 problem 14

Internal problem ID [9765]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \alpha y^2 - \beta - \gamma \cos(\lambda x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 94

```
dsolve(diff(y(x),x)=alpha*y(x)^2+beta+gamma*cos(lambda*x),y(x), singsol=all)
```

$$y(x) = -\frac{\lambda\left(c_1 \operatorname{MathieuSPrime}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, \frac{\lambda x}{2}\right) + \operatorname{MathieuCPrime}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, \frac{\lambda x}{2}\right)\right)}{2\alpha\left(c_1 \operatorname{MathieuS}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, \frac{\lambda x}{2}\right) + \operatorname{MathieuC}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, \frac{\lambda x}{2}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.314 (sec). Leaf size: 163

```
DSolve[y'[x]==\[Alpha]*y[x]^2+\[Beta]+\[Gamma]*Cos[\[Lambda]*x],y[x],x,IncludeSingularSolutio
```

$$y(x) \to -\frac{\lambda\left(\text{MathieuSPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right] + c_1\text{MathieuCPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right]\right)}{2\alpha\left(\text{MathieuS}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right] + c_1\text{MathieuC}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right]\right)}$$

$$y(x) \to -\frac{\lambda\text{MathieuCPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right]}{2\alpha\text{MathieuC}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right]}$$

10.2 problem 15

Internal problem ID [9766]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 + a^2 - \lambda \cos(\lambda x) a - a^2 \cos(\lambda x)^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 467

```
dsolve(diff(y(x),x)=y(x)^2-a^2+a*lambda*cos(lambda*x)+a^2*cos(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = \left(\frac{\left(4\,\mathrm{HeunC}\left(\frac{4a}{\lambda}, \frac{1}{2}, -\frac{1}{2}, -\frac{2a}{\lambda}, \frac{8a+3\lambda}{8\lambda}, \frac{\cos(\lambda x)}{2} + \frac{1}{2}\right) c_1 a + 2\,\mathrm{HeunCPrime}\left(\frac{4a}{\lambda}, \frac{1}{2}, -\frac{1}{2}, -\frac{2a}{\lambda}, \frac{8a+3\lambda}{8\lambda}, \frac{\cos(\lambda x)}{2}\right.\right.}{2\sqrt{2\cos(\lambda x) + 2}\,\left(\sqrt{2\cos(\lambda x) + 2}\,\mathrm{HeunC}\left(\frac{4a}{\lambda}, \frac{1}{2}, -\frac{1}{2}, -\frac{2a}{\lambda}, \frac{8a+3\lambda}{8\lambda}, \frac{\cos(\lambda x)}{2} + \frac{1}{2}\right) c_1 + \mathrm{HeunC}\left(\frac{4a}{\lambda}, -\frac{1}{2}, -\right.\right.} + \frac{(4c_1 a + 2c_1\lambda)\,\mathrm{HeunC}\left(\frac{4a}{\lambda}, \frac{1}{2}, -\frac{1}{2}, -\frac{2a}{\lambda}, \frac{8a+3\lambda}{8\lambda}, \frac{\cos(\lambda x)}{2} + \frac{1}{2}\right) + 2a\,\mathrm{HeunC}\left(\frac{4a}{\lambda}, -\frac{1}{2}, -\frac{1}{2}, -\frac{2a}{\lambda}, \frac{8a+3\lambda}{8\lambda}, \frac{\cos(\lambda x)}{2} +\right.}{2\sqrt{2\cos(\lambda x) + 2}\,\left(\sqrt{2\cos(\lambda x) + 2}\,\mathrm{Heun}\right.}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-a^2+a*\[Lambda]*Cos[\[Lambda]*x]+a^2*Cos[\[Lambda]*x]^2,y[x],x,IncludeSi
```

Not solved

10.3 problem 16

Internal problem ID [9767]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda^2 - c\cos(\lambda x + a)^n \cos(\lambda x + b)^{-n-4} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+c*cos(lambda*x+a)^n*cos(lambda*x+b)^(-n-4),y(x), singsol=
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+c*Cos[\[Lambda]*x+a]^n*Cos[\[Lambda]*x+b]^(-n-4),y[x],x,Incl
```

Not solved

10.4 problem 17

Internal problem ID [9768]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\cos(\beta x)\, y - ab\cos(\beta x) + b^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x)=y(x)^2+a*cos(beta*x)*y(x)+a*b*cos(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = -b - \frac{\mathrm{e}^{\frac{a\sin(\beta x)}{\beta} - 2xb}}{\int \mathrm{e}^{\frac{a\sin(\beta x)}{\beta} - 2xb} dx - c_1}$$

✓ Solution by Mathematica

Time used: 5.682 (sec). Leaf size: 183

```
DSolve[y'[x]==y[x]^2+a*Cos[\[Beta]*x]*y[x]+a*b*Cos[\[Beta]*x]-b^2,y[x],x,IncludeSingularSolut
```

$$\text{Solve}\left[\int_1^x \frac{e^{\frac{a\sin(\beta K[1])}{\beta} - 2bK[1]}(-b + a\cos(\beta K[1]) + y(x))}{a\beta(b + y(x))} dK[1] + \int_1^{y(x)} \left(-\int_1^x \left(\frac{e^{\frac{a\sin(\beta K[1])}{\beta} - 2bK[1]}}{a\beta(b + K[2])} - \frac{e^{\frac{a\sin(\beta K[1])}{\beta} - 2bK[1]}(-b + a\cos(\beta K[1]) + K[2])}{a\beta(b + K[2])^2}\right) dK[1] - \frac{e^{\frac{a\sin(x\beta)}{\beta} - 2bx}}{a\beta(b + K[2])^2}\right) dK[2] = c_1, y(x)\right]$$

10.5 problem 18

Internal problem ID [9769]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\cos(bx)^m y - a\cos(bx)^m = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+a*cos(b*x)^m*y(x)+a*cos(b*x)^m,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a*Cos[b*x]^m*y[x]+a*Cos[b*x]^m,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

10.6 problem 19

Internal problem ID [9770]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \cos(\lambda x)\, y^2 - \lambda \cos(\lambda x)^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x)=lambda*cos(lambda*x)*y(x)^2+lambda*cos(lambda*x)^3,y(x), singsol=all)
```

$$y(x) = \sin(\lambda x) + \frac{2c_1 - 1}{\left(\mathrm{KummerU}\left(1, \frac{3}{2}, -\sin(\lambda x)^2\right) c_1 + \mathrm{KummerM}\left(1, \frac{3}{2}, -\sin(\lambda x)^2\right)\right) \sin(\lambda x)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Cos[\[Lambda]*x]*y[x]^2+\[Lambda]*Cos[\[Lambda]*x]^3,y[x],x,IncludeSi
```

Not solved

10.7 problem 20

Internal problem ID [9771]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{2y' - (\lambda + a - \cos(\lambda x)\, a)\, y^2 - \lambda + a + \cos(\lambda x)\, a = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 307

```
dsolve(2*diff(y(x),x)=(lambda+a-a*cos(lambda*x))*y(x)^2+lambda-a-a*cos(lambda*x),y(x), singso
```

$$y(x) = \left(-\frac{2c_1\lambda\, e^{\frac{\cos(\lambda x)a}{\lambda}}}{(\cos(\lambda x)-1)^{\frac{3}{2}}\left(\left(\int -\frac{(-\lambda-a+\cos(\lambda x)a)e^{\frac{\cos(\lambda x)a}{\lambda}}\lambda\sin(\lambda x)}{(\cos(\lambda x)-1)^{\frac{3}{2}}\sqrt{\cos(\lambda x)+1}}dx\right)c_1+1\right)\sqrt{\cos(\lambda x)+1}} \right.$$
$$+ \frac{\left(\left(\int -\frac{(-\lambda-a+\cos(\lambda x)a)e^{\frac{\cos(\lambda x)a}{\lambda}}\lambda\sin(\lambda x)}{(\cos(\lambda x)-1)^{\frac{3}{2}}\sqrt{\cos(\lambda x)+1}}dx\right)\sqrt{\cos(\lambda x)+1}\,c_1 + \sqrt{\cos(\lambda x)+1}\right)\cos(\lambda x) - \left(\int -\frac{(-\lambda-a+\cos}{(\cos(\lambda}\right.}{\left(\left(\int -\frac{(-\lambda-a+\cos(\lambda x)a)e^{\frac{\cos(\lambda x)a}{\lambda}}\lambda\sin(\lambda x)}{(\cos(\lambda x)-1)^{\frac{3}{2}}\sqrt{\cos(\lambda x)+1}}dx\right)c_1+1\right)\sqrt{\cos(\lambda x)+}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*y'[x]==(\[Lambda]+a-a*Cos[\[Lambda]*x])*y[x]^2+\[Lambda]-a-a*Cos[\[Lambda]*x],y[x],x
```

Not solved

10.8 problem 21

Internal problem ID [9772]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \left(\lambda + \cos(\lambda x)^2 a\right) y^2 - \lambda + a - \cos(\lambda x)^2 a = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 550

```
dsolve(diff(y(x),x)=(lambda+a*cos(lambda*x)^2)*y(x)^2+lambda-a+a*cos(lambda*x)^2,y(x), singso
```

$$y(x) = \left(\frac{\left(4\cos(2\lambda x)\sqrt{1+\cos(2\lambda x)}\, c_1 a\lambda + 4\sqrt{1+\cos(2\lambda x)}\, c_1 a\lambda + 8\sqrt{1+\cos(2\lambda x)}\, c_1 \lambda^2\right) e^{-\frac{a\cos(2\lambda x)}{2\lambda}}}{2\left(1+\cos(2\lambda x)\right)^2 \sqrt{-1+\cos(2\lambda x)}\, \left(\lambda + a\cos(\lambda x)^2\right) \left(\left(\int -\frac{2(a\cos(2\lambda x)+a+2\lambda)e^{-\frac{a\cos(2\lambda x)}{2\lambda}}\sin(2\lambda x)\lambda}{\sqrt{-1+\cos(2\lambda x)}\,(1+\cos(2\lambda x))^{\frac{3}{2}}}dx\right) c_1 + \right.} + \frac{\left(\left(\int -\frac{2(a\cos(2\lambda x)+a+2\lambda)e^{-\frac{a\cos(2\lambda x)}{2\lambda}}\sin(2\lambda x)\lambda}{\sqrt{-1+\cos(2\lambda x)}\,(1+\cos(2\lambda x))^{\frac{3}{2}}}dx\right)\sqrt{-1+\cos(2\lambda x)}\, c_1 a + a\sqrt{-1+\cos(2\lambda x)}\right)\cos(2\lambda x)^2 +}{} \right.$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(\[Lambda]+a*Cos[\[Lambda]*x]^2)*y[x]^2+\[Lambda]-a+a*Cos[\[Lambda]*x]^2,y[x],x
```

Not solved

10.9 problem 22

Internal problem ID [9773]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (1+k)\,x^k y^2 - a\,x^{1+k}\cos(x)^m\, y + a\cos(x)^m = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 168

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+a*x^(k+1)*cos(x)^m*y(x)-a*cos(x)^m,y(x), singsol=all)
```

$$y(x) = \frac{\left(\mathrm{e}^{\int \frac{x^k \cos(x)^m a\,x^2-2k-2}{x}dx} x^k x + \left(\int x^k \mathrm{e}^{\int \frac{a\,x^{k+2}\cos(x)^m-2k-2}{x}dx}dx\right)k + \int x^k \mathrm{e}^{\int \frac{a\,x^{k+2}\cos(x)^m-2k-2}{x}dx}dx + c_1\right)x^{-k}}{x\left(\left(\int x^k \mathrm{e}^{\int \frac{a\,x^{k+2}\cos(x)^m-2k-2}{x}dx}dx\right)k + \int x^k \mathrm{e}^{\int \frac{a\,x^{k+2}\cos(x)^m-2k-2}{x}dx}dx + c_1\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+a*x^(k+1)*Cos[x]^m*y[x]-a*Cos[x]^m,y[x],x,IncludeSingularSolu
```

Not solved

10.10 problem 23

Internal problem ID [9774]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, `_with_symmetry_[F(x),G(x)]`], _Riccati]`

$$y' - a\cos(\lambda x+\mu)^k (y - b\,x^n - c)^2 - bn\,x^{n-1} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 106

```
dsolve(diff(y(x),x)=a*cos(lambda*x+mu)^k*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-2a\,x^n(\cos(\lambda x)\cos(\mu)-\sin(\lambda x)\sin(\mu))^k b - 2ac(\cos(\lambda x)\cos(\mu)-\sin(\lambda x)\sin(\mu))^k\right)\cos(\lambda x+\mu)}{2a} + \frac{1}{c_1 - \left(\int a(\cos(\lambda x)\cos(\mu)-\sin(\lambda x)\sin(\mu))^k dx\right)}$$

✓ Solution by Mathematica

Time used: 3.805 (sec). Leaf size: 92

```
DSolve[y'[x]==a*Cos[\[Lambda]*x+\[Mu]]^k*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingularS
```

$$y(x) \to \frac{1}{\frac{a\sqrt{\sin^2(\mu+\lambda x)}\csc(\mu+\lambda x)\cos^{k+1}(\mu+\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\cos^2(x\lambda+\mu)\right)}{(k+1)\lambda} + c_1} + bx^n + c$$

$$y(x) \to bx^n + c$$

10.11 problem 24

Internal problem ID [9775]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - a\cos(\lambda x)^m y^2 - ky - a\,b^2x^{2k}\cos(\lambda x)^m = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 36

```
dsolve(x*diff(y(x),x)=a*cos(lambda*x)^m*y(x)^2+k*y(x)+a*b^2*x^(2*k)*cos(lambda*x)^m,y(x), sin
```

$$y(x) = -\tan\left(-ba\left(\int \frac{\cos(\lambda x)^m x^k}{x}dx\right) + c_1\right) b\,x^k$$

✓ Solution by Mathematica

Time used: 1.031 (sec). Leaf size: 50

```
DSolve[x*y'[x]==a*Cos[\[Lambda]*x]^m*y[x]^2+k*y[x]+a*b^2*x^(2*k)*Cos[\[Lambda]*x]^m,y[x],x,In
```

$$y(x) \to \sqrt{b^2}x^k \tan\left(\sqrt{b^2}\int_1^x a\cos^m(\lambda K[1])K[1]^{k-1}dK[1] + c_1\right)$$

10.12 problem 25

Internal problem ID [9776]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(\cos(\lambda x) a + b) y' - y^2 - c\cos(\mu x) y + d^2 - cd\cos(\mu x) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 149

```
dsolve((a*cos(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*cos(mu*x)*y(x)-d^2+c*d*cos(mu*x),y(x), sings
```

$$y(x) = -d - \frac{e^{\int \frac{c\cos(\mu x)}{\cos(\lambda x)a+b}dx - \frac{4d\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{\lambda x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\lambda\sqrt{(a-b)(a+b)}}}}{\int \frac{e^{\int \frac{c\cos(\mu x)}{\cos(\lambda x)a+b}dx - \frac{4d\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{\lambda x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\lambda\sqrt{(a-b)(a+b)}}}}{\cos(\lambda x)a+b}dx - c_1}$$

✓ Solution by Mathematica

Time used: 7.795 (sec). Leaf size: 289

```
DSolve[(a*Cos[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Cos[\[Mu]*x]*y[x]-d^2+c*d*Cos[\[Mu]*x],y[x],x,I
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[6]} \frac{2d-c\cos(\mu K[5])}{b+a\cos(\lambda K[5])}dK[5]\right)(-d+c\cos(\mu K[6])+y(x))}{c\mu(b+a\cos(\lambda K[6]))(d+y(x))}dK[6]\right.$$
$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{\exp\left(-\int_1^{K[6]} \frac{2d-c\cos(\mu K[5])}{b+a\cos(\lambda K[5])}dK[5]\right)}{c\mu(b+a\cos(\lambda K[6]))(d+K[7])} - \frac{\exp\left(-\int_1^{K[6]} \frac{2d-c\cos(\mu K[5])}{b+a\cos(\lambda K[5])}dK[5]\right)(-d+c\cos(\mu K[6])}{c\mu(b+a\cos(\lambda K[6]))(d+K[7])^2}\right.\right.$$
$$\left.\left.-\frac{\exp\left(-\int_1^x \frac{2d-c\cos(\mu K[5])}{b+a\cos(\lambda K[5])}dK[5]\right)}{c\mu(d+K[7])^2}\right)dK[7] = c_1, y(x)\right]$$

10.13 problem 26

Internal problem ID [9777]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(\cos(\lambda x) a + b)(y' - y^2) - a\lambda^2 \cos(\lambda x) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 539

```
dsolve((a*cos(lambda*x)+b)*(diff(y(x),x)-y(x)^2)-a*lambda^2*cos(lambda*x)=0,y(x), singsol=all
```

$$y(x) = \frac{\lambda\left(\left(\left(a^4 - b a^3 - b^2 a^2 + a b^3\right)\sin\left(\frac{\lambda x}{2}\right)^2 + \left(-4\sqrt{a^2-b^2}\, a^2 b + 4\sqrt{a^2-b^2}\, a b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{\lambda x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)\cos\left(\right.\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 5.913 (sec). Leaf size: 184

```
DSolve[(a*Cos[\[Lambda]*x]+b)*(y'[x]-y[x]^2)-a*\[Lambda]^2*Cos[\[Lambda]*x]==0,y[x],x,Include
```

$$y(x) \to \frac{\lambda\left(2ab\sin(\lambda x)\coth^{-1}\left(\frac{(a+b)\cot\left(\frac{\lambda x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right) + \sqrt{a^2-b^2}\left(ac_1\lambda(b-a)(a+b)\sin(\lambda x) + a\cos(\lambda x) - b\right)\right)}{2b(a\cos(\lambda x)+b)\coth^{-1}\left(\frac{(a+b)\cot\left(\frac{\lambda x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right) - \sqrt{a^2-b^2}\left(a\sin(\lambda x) + c_1\lambda(a-b)(a+b)(a\cos(\lambda x)+b)\right)}$$

$$y(x) \to \frac{a\lambda\sin(\lambda x)}{a\cos(\lambda x)+b}$$

11 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.

11.1 problem 27

Internal problem ID [9778]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda a - a(\lambda - a)\tan(\lambda x)^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 380

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda+a*(lambda-a)*tan(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{c_1\lambda\,\mathrm{LegendreQ}\left(\frac{2a+\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\sqrt{-\cos(\lambda x)^2+1}\right)}{\sqrt{-\cos(\lambda x)^2+1}\left(\mathrm{LegendreQ}\left(\frac{2a-\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\sqrt{-\cos(\lambda x)^2+1}\right)c_1+\mathrm{LegendreP}\left(\frac{2a-\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\sqrt{-\cos(\lambda x)^2+1}\right)\right)} + \frac{-\mathrm{LegendreQ}\left(\frac{2a-\lambda}{2\lambda},\frac{2a-}{2\lambda}\right.}{}\right.}{}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]+a*(\[Lambda]-a)*Tan[\[Lambda]*x]^2,y[x],x,IncludeSingularSol
```

Not solved

11.2 problem 28

Internal problem ID [9779]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda^2 - 3\lambda a - a(\lambda - a)\tan(\lambda x)^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 320

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+3*a*lambda+a*(lambda-a)*tan(lambda*x)^2,y(x), singsol=all
```

$$y(x) = \frac{\left(\left(-\sqrt{-\cos(\lambda x)^2+1}\,c_1 a - \sqrt{-\cos(\lambda x)^2+1}\,c_1\lambda\right)\operatorname{LegendreQ}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \sqrt{-\cos(\lambda x)^2+1}\right) + 2\,\mathrm{Le}}{\sqrt{-\cos(\lambda x)^2}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+3*a*\[Lambda]+a*(\[Lambda]-a)*Tan[\[Lambda]*x]^2,y[x],x,Incl
```

Not solved

11.3 problem 29

Internal problem ID [9780]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 a - b\tan(x)\, y - c = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 359

```
dsolve(diff(y(x),x)=a*y(x)^2+b*tan(x)*y(x)+c,y(x), singsol=all)
```

$$y(x) = \frac{\left((-c_1 b + c_1)\operatorname{LegendreQ}\left(\frac{\sqrt{4ac+b^2}}{2} - \frac{1}{2}, \frac{b}{2} - \frac{1}{2}, \sin(x)\right) + (1-b)\operatorname{LegendreP}\left(\frac{\sqrt{4ac+b^2}}{2} - \frac{1}{2}, \frac{b}{2} - \frac{1}{2}, \sin(x)\right)\right)}{}$$

✓ Solution by Mathematica

Time used: 1.153 (sec). Leaf size: 599

```
DSolve[y'[x]==a*y[x]^2+b*Tan[x]*y[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x)\left(-(b-3)(b-1)(b+1)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(-b-\sqrt{b^2+4ac}+2\right), \frac{1}{4}\left(-b+\sqrt{b^2+4ac}+2\right), \frac{3-}{}\right.\right.}{a(b-3)(b+1)\left(\cos(x)\,\text{Hyperg}\right.}$$

$$y(x) \to -\frac{c\sin(2x)\,{}_2\tilde{F}_1\left(\frac{1}{4}\left(b-\sqrt{b^2+4ac}+4\right), \frac{1}{4}\left(b+\sqrt{b^2+4ac}+4\right); \frac{b+3}{2}; \cos^2(x)\right)}{4\,{}_2\tilde{F}_1\left(\frac{1}{4}\left(b-\sqrt{b^2+4ac}\right), \frac{1}{4}\left(b+\sqrt{b^2+4ac}\right); \frac{b+1}{2}; \cos^2(x)\right)}$$

$$y(x) \to -\frac{c\sin(2x)\,{}_2\tilde{F}_1\left(\frac{1}{4}\left(b-\sqrt{b^2+4ac}+4\right), \frac{1}{4}\left(b+\sqrt{b^2+4ac}+4\right); \frac{b+3}{2}; \cos^2(x)\right)}{4\,{}_2\tilde{F}_1\left(\frac{1}{4}\left(b-\sqrt{b^2+4ac}\right), \frac{1}{4}\left(b+\sqrt{b^2+4ac}\right); \frac{b+1}{2}; \cos^2(x)\right)}$$

11.4 problem 30

Internal problem ID [9781]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - y^2 a - 2ab\tan(x)\, y - b(ab-1)\tan(x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 60

```
dsolve(diff(y(x),x)=a*y(x)^2+2*a*b*tan(x)*y(x)+b*(a*b-1)*tan(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{-b\tan(x)\, a - i\sqrt{b}\,\sqrt{a} + \frac{e^{-2i\sqrt{b}\,\sqrt{a}\,x}}{c_1 - \frac{i e^{-2i\sqrt{b}\,\sqrt{a}\,x}}{2\sqrt{b}\,\sqrt{a}}}}{a}$$

✓ Solution by Mathematica

Time used: 8.415 (sec). Leaf size: 37

```
DSolve[y'[x]==a*y[x]^2+2*a*b*Tan[x]*y[x]+b*(a*b-1)*Tan[x]^2,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -b\tan(x) + \sqrt{\frac{b}{a}}\tan\left(ax\sqrt{\frac{b}{a}} + c_1\right)$$

11.5 problem 31

Internal problem ID [9782]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - a\tan(\beta x)\, y - ab\tan(\beta x) + b^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(diff(y(x),x)=y(x)^2+a*tan(beta*x)*y(x)+a*b*tan(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = -b - \frac{\left(1+\tan(\beta x)^2\right)^{\frac{a}{2\beta}} \mathrm{e}^{-2xb}}{\int \left(1+\tan(\beta x)^2\right)^{\frac{a}{2\beta}} \mathrm{e}^{-2xb} dx - c_1}$$

✓ Solution by Mathematica

Time used: 16.405 (sec). Leaf size: 329

```
DSolve[y'[x]==y[x]^2+a*Tan[\[Beta]*x]*y[x]+a*b*Tan[\[Beta]*x]-b^2,y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{2^{-\frac{a}{\beta}}(a+2ib+2\beta)\cos^{-\frac{a}{\beta}}(\beta x)\left(b\left(2ia\,\text{Hypergeometric2F1}\left(1,-\frac{a-2ib}{2\beta},\frac{a+2ib}{2\beta}+1,-e^{2ix\beta}\right)-ia+2b\right)(\text{si}\right.}{(a-2ib)\left((2b-ia)e^{2i\beta x}\,\text{Hypergeometric2F1}\left(1,1-\frac{a-2ib}{2\beta},\frac{a+2ib}{2\beta}+2,-e^{2ix\beta}\right)+(a+2ib+2\beta)\left(a\beta c_1(a\right.\right.}$$

$$y(x) \to -b$$

11.6 problem 32

Internal problem ID [9783]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - ax\tan(bx)^m y - a\tan(bx)^m = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*tan(b*x)^m*y(x)+a*tan(b*x)^m,y(x), singsol=all)
```

$$y(x) = -\frac{e^{\int \frac{a\tan(xb)^m x^2-2}{x}dx} x + \int e^{\int \frac{a\tan(xb)^m x^2-2}{x}dx} dx - c_1}{x\left(-c_1 + \int e^{\int \frac{a\tan(xb)^m x^2-2}{x}dx} dx\right)}$$

✓ Solution by Mathematica

Time used: 4.737 (sec). Leaf size: 86

```
DSolve[y'[x]==y[x]^2+a*x*Tan[b*x]^m*y[x]+a*Tan[b*x]^m,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{x + \frac{\exp\left(-\int_1^x -aK[5]\tan^m(bK[5])dK[5]\right)}{\int_1^x \frac{\exp\left(-\int_1^{K[6]} -aK[5]\tan^m(bK[5])dK[5]\right)}{K[6]^2}dK[6]+c_1}}{x^2}$$

$$y(x) \to -\frac{1}{x}$$

11.7 problem 33

Internal problem ID [9784]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (1+k)\, x^k y^2 - a\, x^{1+k} \tan(x)^m y + a \tan(x)^m = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 172

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+a*x^(k+1)*tan(x)^m*y(x)-a*tan(x)^m,y(x), singsol=all)
```

$$y(x) = \frac{\left(\mathrm{e}^{\int \frac{x^k \tan(x)^m a\, x^2-2k-2}{x}dx} x\, x^k + \left(\int x^k \mathrm{e}^{\int \frac{a\, x^{k+2} \tan(x)^m-2k-2}{x}dx}dx\right) k + \int x^k \mathrm{e}^{\int \frac{a\, x^{k+2} \tan(x)^m-2k-2}{x}dx}dx - c_1\right) x^{-k}}{x\left(\left(\int x^k \mathrm{e}^{\int \frac{a\, x^{k+2} \tan(x)^m-2k-2}{x}dx}dx\right) k + \int x^k \mathrm{e}^{\int \frac{a\, x^{k+2} \tan(x)^m-2k-2}{x}dx}dx - c_1\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+a*x^(k+1)*Tan[x]^m*y[x]-a*Tan[x]^m,y[x],x,IncludeSingularSolu
```

Not solved

11.8 problem 34

Internal problem ID [9785]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\tan(\lambda x)^n y^2 + a\,b^2\tan(\lambda x)^{n+2} - b\lambda\tan(\lambda x)^2 - b\lambda = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*tan(lambda*x)^n*y(x)^2-a*b^2*tan(lambda*x)^(n+2)+b*lambda*tan(lambda*x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*Tan[\[Lambda]*x]^n*y[x]^2-a*b^2*Tan[\[Lambda]*x]^(n+2)+b*\[Lambda]*Tan[\[Lamb
```

Not solved

11.9 problem 35

Internal problem ID [9786]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - a\tan(\lambda x+\mu)^k (y - b\,x^n - c)^2 - bn\,x^{n-1} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 134

```
dsolve(diff(y(x),x)=a*tan(lambda*x+mu)^k*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-2a\,x^n\left(\frac{-\tan(\mu)-\tan(\lambda x)}{\tan(\mu)\tan(\lambda x)-1}\right)^k b - 2ac\left(\frac{-\tan(\mu)-\tan(\lambda x)}{\tan(\mu)\tan(\lambda x)-1}\right)^k\right)\left(-\frac{\tan(\mu)+\tan(\lambda x)}{\tan(\mu)\tan(\lambda x)-1}\right)^{-k}}{2a} + \frac{1}{c_1 - \left(\int a\left(\frac{-\tan(\mu)-\tan(\lambda x)}{\tan(\mu)\tan(\lambda x)-1}\right)^k dx\right)}$$

✓ Solution by Mathematica

Time used: 3.804 (sec). Leaf size: 75

```
DSolve[y'[x]==a*Tan[\[Lambda]*x+mu]^k*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{1}{-\frac{a\tan^{k+1}(\mu+\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},-\tan^2(\mu+x\lambda)\right)}{(k+1)\lambda} + c_1} + bx^n + c$$

$$y(x) \to bx^n + c$$

11.10 problem 36

Internal problem ID [9787]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - a\tan(\lambda x)^m y^2 - ky - a\,b^2 x^{2k}\tan(\lambda x)^m = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 36

```
dsolve(x*diff(y(x),x)=a*tan(lambda*x)^m*y(x)^2+k*y(x)+a*b^2*x^(2*k)*tan(lambda*x)^m,y(x), sin
```

$$y(x) = -\tan\left(-ba\left(\int \frac{x^k \tan(\lambda x)^m}{x} dx\right) + c_1\right) b\,x^k$$

✓ Solution by Mathematica

Time used: 1.156 (sec). Leaf size: 50

```
DSolve[x*y'[x]==a*Tan[\[Lambda]*x]^m*y[x]^2+k*y[x]+a*b^2*x^(2*k)*Tan[\[Lambda]*x]^m,y[x],x,In
```

$$y(x) \to \sqrt{b^2} x^k \tan\left(\sqrt{b^2}\int_1^x aK[1]^{k-1}\tan^m(\lambda K[1])dK[1] + c_1\right)$$

11.11 problem 37

Internal problem ID [9788]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(a\tan(\lambda x)+b)\,y'-y^2-k\tan(\mu x)\,y+d^2-kd\tan(\mu x)=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 213

```
dsolve((a*tan(lambda*x)+b)*diff(y(x),x)=y(x)^2+k*tan(mu*x)*y(x)-d^2+k*d*tan(mu*x),y(x), sings
```

$$y(x)=-d-\frac{e^{\int \frac{\tan(\mu x)k}{a\tan(\lambda x)+b}dx}(a\tan(\lambda x)+b)^{-\frac{2ad}{\lambda(a^2+b^2)}}\left(1+\tan(\lambda x)^2\right)^{\frac{ad}{\lambda(a^2+b^2)}}e^{-\frac{2db\arctan(\tan(\lambda x))}{\lambda(a^2+b^2)}}}{\int \frac{e^{\int \frac{\tan(\mu x)k}{a\tan(\lambda x)+b}dx}(a\tan(\lambda x)+b)^{-\frac{2ad}{\lambda(a^2+b^2)}}\left(1+\tan(\lambda x)^2\right)^{\frac{ad}{\lambda(a^2+b^2)}}e^{-\frac{2db\arctan(\tan(\lambda x))}{\lambda(a^2+b^2)}}}{a\tan(\lambda x)+b}dx-c_1}$$

✓ Solution by Mathematica

Time used: 85.063 (sec). Leaf size: 800

```
DSolve[(a*Tan[\[Lambda]*x]+b)*y'[x]==y[x]^2+k*Tan[\[Mu]*x]*y[x]-d^2+k*d*Tan[\[Mu]*x],y[x],x,I
```

$$\text{Solve}\left[\int_1^x \frac{e^{-\int_1^{K[6]}\frac{\sec(\mu K[5])(2d\cos(\lambda K[5]-\mu K[5])+2d\cos(\lambda K[5]+\mu K[5])+k\sin(\lambda K[5]-\mu K[5])-k\sin(\lambda K[5]+\mu K[5]))}{2(b\cos(\lambda K[5])+a\sin(\lambda K[5]))}dK[5]}(d\cos(\lambda K[6]-\mu K[6]}{k\mu(b\cos(\lambda K[6]-\mu K[6])+b\cos(\lambda K}\right.$$

$$+\int_1^{y(x)}\left(\frac{e^{-\int_1^{x}\frac{\sec(\mu K[5])(2d\cos(\lambda K[5]-\mu K[5])+2d\cos(\lambda K[5]+\mu K[5])+k\sin(\lambda K[5]-\mu K[5])-k\sin(\lambda K[5]+\mu K[5]))}{2(b\cos(\lambda K[5])+a\sin(\lambda K[5]))}dK[5]}}{k\mu(d+K[7])^2}\right.$$

$$-\int_1^x\left(\frac{e^{-\int_1^{K[6]}\frac{\sec(\mu K[5])(2d\cos(\lambda K[5]-\mu K[5])+2d\cos(\lambda K[5]+\mu K[5])+k\sin(\lambda K[5]-\mu K[5])-k\sin(\lambda K[5]+\mu K[5]))}{2(b\cos(\lambda K[5])+a\sin(\lambda K[5]))}dK[5]}(-\cos(\lambda K[6]-\mu K[6])}{k\mu(d+K[7])(b\cos(\lambda K[6]-\mu K[6])+b\cos(\lambda K[6]+\mu K[6])+a\sin(\lambda K[6]-\mu K[6])+a\sin(}\right.$$

12 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.

12.1 problem 38

Internal problem ID [9789]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \lambda a - a(\lambda - a)\cot(\lambda x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 353

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda+a*(lambda-a)*cot(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\mathrm{LegendreQ}\left(\frac{2a-\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(\lambda x)\right) c_1 a + \mathrm{LegendreP}\left(\frac{2a-\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(\lambda x)\right) a\right)\cos(3\lambda x) + \left(-2\,\mathrm{LegendreQ}\right.}{}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]+a*(\[Lambda]-a)*Cot[\[Lambda]*x]^2,y[x],x,IncludeSingularSol
```

Not solved

12.2 problem 39

Internal problem ID [9790]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda^2 - 3\lambda a - a(\lambda - a)\cot(\lambda x)^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 436

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+3*a*lambda+a*(lambda-a)*cot(lambda*x)^2,y(x), singsol=all
```

$$y(x) = -\frac{\left(\left((2c_1a + 3c_1\lambda)\,\mathrm{LegendreQ}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(\lambda x)\right) + (2a + 3\lambda)\,\mathrm{LegendreP}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(\lambda x)\right)\right)\cos(\lambda\right.}{2\left(\cos(\lambda x)^2 - 1\right)\left(\mathrm{LegendreQ}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(\lambda x\right.\right.} - \frac{\left(\mathrm{LegendreQ}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(\lambda x)\right)c_1\lambda + \mathrm{LegendreP}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(\lambda x)\right)\lambda\right)\cos(\lambda x)^3 + \left(-\mathrm{LegendreQ}\right.}{2\left(\cos(\lambda x)^2 - 1\right)\left(\mathrm{LegendreQ}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(\lambda x)\right)c_1 + \mathrm{Legend}\right.}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+3*a*\[Lambda]+a*(\[Lambda]-a)*Cot[\[Lambda]*x]^2,y[x],x,Incl
```

Not solved

12.3 problem 40

Internal problem ID [9791]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 + 2ab\cot(ax)\,y - b^2 + a^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 565

```
dsolve(diff(y(x),x)=y(x)^2-2*a*b*cot(a*x)*y(x)+b^2-a^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{\left(bac_1-\sqrt{b^2a^2-a^2+b^2}\,c_1-c_1a\right)\mathrm{LegendreQ}\left(\frac{a+2\sqrt{b^2a^2-a^2+b^2}}{2a},b-\frac{1}{2},\sqrt{-\sin(ax)^2+1}\right)}{\sqrt{-\sin(ax)^2+1}\left(\mathrm{LegendreQ}\left(\frac{-a+2\sqrt{b^2a^2-a^2+b^2}}{2a},b-\frac{1}{2},\sqrt{-\sin(ax)^2+1}\right)c_1+\mathrm{LegendreP}\left(\frac{-a+2\sqrt{b^2a^2-a^2+b^2}}{2a},b-\frac{1}{2},\sqrt{-\sin(ax)^2+1}\right)\right)} + \cdots\right.}{\cdots}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-2*a*b*Cot[a*x]*y[x]+b^2-a^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

12.4 problem 41

Internal problem ID [9792]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - a\cot(\beta x)\, y - ab\cot(\beta x) + b^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 61

```
dsolve(diff(y(x),x)=y(x)^2+a*cot(beta*x)*y(x)+a*b*cot(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = -b - \frac{\left(\cot(\beta x)^2 + 1\right)^{-\frac{a}{2\beta}} e^{-2xb}}{\int \left(\cot(\beta x)^2 + 1\right)^{-\frac{a}{2\beta}} e^{-2xb} dx - c_1}$$

✓ Solution by Mathematica

Time used: 82.06 (sec). Leaf size: 19223

```
DSolve[y'[x]==y[x]^2+a*Cot[\[Beta]*x]*y[x]+a*b*Cot[\[Beta]*x]-b^2,y[x],x,IncludeSingularSolut
```

Too large to display

12.5 problem 42

Internal problem ID [9793]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax\cot(bx)^m y - a\cot(bx)^m = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*cot(b*x)^m*y(x)+a*cot(b*x)^m,y(x), singsol=all)
```

$$y(x) = -\frac{e^{\int \frac{a\cot(xb)^m x^2-2}{x}dx} x + \int e^{\int \frac{a\cot(xb)^m x^2-2}{x}dx}dx - c_1}{\left(-c_1 + \int e^{\int \frac{a\cot(xb)^m x^2-2}{x}dx}dx\right)x}$$

✓ Solution by Mathematica

Time used: 4.797 (sec). Leaf size: 86

```
DSolve[y'[x]==y[x]^2+a*x*Cot[b*x]^m*y[x]+a*Cot[b*x]^m,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{x + \frac{\exp\left(-\int_1^x -a\cot^m(bK[5])K[5]dK[5]\right)}{\int_1^x \frac{\exp\left(-\int_1^{K[6]} -a\cot^m(bK[5])K[5]dK[5]\right)}{K[6]^2}dK[6]+c_1}}{x^2}$$

$$y(x) \to -\frac{1}{x}$$

12.6 problem 43

Internal problem ID [9794]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (1+k)\, x^k y^2 - a\, x^{1+k} \cot(x)^m\, y + a \cot(x)^m = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 168

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+a*x^(k+1)*cot(x)^m*y(x)-a*cot(x)^m,y(x), singsol=all)
```

$$y(x) = \frac{\left(\mathrm{e}^{\int \frac{x^k \cot(x)^m a\, x^2 - 2k - 2}{x} dx} x^k x + \left(\int x^k \mathrm{e}^{\int \frac{a\, x^{k+2} \cot(x)^m - 2k - 2}{x} dx} dx\right) k + \int x^k \mathrm{e}^{\int \frac{a\, x^{k+2} \cot(x)^m - 2k - 2}{x} dx} dx + c_1\right) x^{-k}}{x \left(\left(\int x^k \mathrm{e}^{\int \frac{a\, x^{k+2} \cot(x)^m - 2k - 2}{x} dx} dx\right) k + \int x^k \mathrm{e}^{\int \frac{a\, x^{k+2} \cot(x)^m - 2k - 2}{x} dx} dx + c_1\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+a*x^(k+1)*Cot[x]^m*y[x]-a*Cot[x]^m,y[x],x,IncludeSingularSolu
```

Not solved

12.7 problem 44

Internal problem ID [9795]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - a\cot(\lambda x+\mu)^k (y - b\,x^n - c)^2 - bn\,x^{n-1} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 118

```
dsolve(diff(y(x),x)=a*cot(lambda*x+mu)^k*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-2x^n ab\left(\frac{-1+\cot(\mu)\cot(\lambda x)}{\cot(\mu)+\cot(\lambda x)}\right)^k - 2ac\left(\frac{-1+\cot(\mu)\cot(\lambda x)}{\cot(\mu)+\cot(\lambda x)}\right)^k\right)\left(\frac{-1+\cot(\mu)\cot(\lambda x)}{\cot(\mu)+\cot(\lambda x)}\right)^{-k}}{2a} + \frac{1}{c_1 - \left(\int a\left(\frac{-1+\cot(\mu)\cot(\lambda x)}{\cot(\mu)+\cot(\lambda x)}\right)^k dx\right)}$$

✓ Solution by Mathematica

Time used: 3.665 (sec). Leaf size: 74

```
DSolve[y'[x]==a*Cot[\[Lambda]*x+mu]^k*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{1}{\frac{a\cot^{k+1}(\mu+\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},-\cot^2(\mu+x\lambda)\right)}{(k+1)\lambda} + c_1} + bx^n + c$$

$$y(x) \to bx^n + c$$

12.8 problem 45

Internal problem ID [9796]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - a\cot(\lambda x)^m y^2 - ky - a\,b^2x^{2k}\cot(\lambda x)^m = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve(x*diff(y(x),x)=a*cot(lambda*x)^m*y(x)^2+k*y(x)+a*b^2*x^(2*k)*cot(lambda*x)^m,y(x), sin
```

$$y(x) = -\tan\left(-ba\left(\int \frac{x^k\cot(\lambda x)^m}{x}dx\right) + c_1\right) b\,x^k$$

✓ Solution by Mathematica

Time used: 1.165 (sec). Leaf size: 50

```
DSolve[x*y'[x]==a*Cot[\[Lambda]*x]^m*y[x]^2+k*y[x]+a*b^2*x^(2*k)*Cot[\[Lambda]*x]^m,y[x],x,In
```

$$y(x) \to \sqrt{b^2}x^k\tan\left(\sqrt{b^2}\int_1^x a\cot^m(\lambda K[1])K[1]^{k-1}dK[1] + c_1\right)$$

12.9 problem 46

Internal problem ID [9797]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(a\cot(\lambda x)+b)\,y'-y^2-c\cot(\mu x)\,y+d^2-cd\cot(\mu x)=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 251

```
dsolve((a*cot(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*cot(mu*x)*y(x)-d^2+c*d*cot(mu*x),y(x), sings
```

$$y(x) = -d - \frac{e^{\int \frac{c\cot(\mu x)}{a\cot(\lambda x)+b}dx}(a\cot(\lambda x)+b)^{\frac{2ad}{\lambda(a^2+b^2)}}\left(\cot(\lambda x)^2+1\right)^{-\frac{ad}{\lambda(a^2+b^2)}} e^{\frac{\pi bd}{\lambda(a^2+b^2)}} e^{-\frac{2db\,\operatorname{arccot}(\cot(\lambda x))}{\lambda(a^2+b^2)}}}{\int \frac{e^{\int \frac{c\cot(\mu x)}{a\cot(\lambda x)+b}dx}(a\cot(\lambda x)+b)^{\frac{2ad}{\lambda(a^2+b^2)}}\left(\cot(\lambda x)^2+1\right)^{-\frac{ad}{\lambda(a^2+b^2)}} e^{\frac{\pi bd}{\lambda(a^2+b^2)}} e^{-\frac{2db\,\operatorname{arccot}(\cot(\lambda x))}{\lambda(a^2+b^2)}}}{a\cot(\lambda x)+b}dx - c_1}$$

✓ Solution by Mathematica

Time used: 52.682 (sec). Leaf size: 799

```
DSolve[(a*Cot[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Cot[\[Mu]*x]*y[x]-d^2+c*d*Cot[\[Mu]*x],y[x],x,I
```

$$\text{Solve}\Bigg[\int_1^x \frac{e^{-\int_1^{K[6]} -\frac{\csc(\mu K[5])(-2d\cos(\lambda K[5]-\mu K[5])+2d\cos(\lambda K[5]+\mu K[5])+c\sin(\lambda K[5]-\mu K[5])+c\sin(\lambda K[5]+\mu K[5]))}{2(a\cos(\lambda K[5])+b\sin(\lambda K[5]))}dK[5]}(-d\cos(\lambda K[6]-\mu}{c\mu(b\cos(\lambda K[6]-\mu K[6])-b\cos(\lambda}$$
$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{e^{-\int_1^{K[6]} -\frac{\csc(\mu K[5])(-2d\cos(\lambda K[5]-\mu K[5])+2d\cos(\lambda K[5]+\mu K[5])+c\sin(\lambda K[5]-\mu K[5])+c\sin(\lambda K[5]+\mu K[5]))}{2(a\cos(\lambda K[5])+b\sin(\lambda K[5]))}dK[5]}(\cos(\lambda K[6]}{c\mu(d+K[7])(b\cos(\lambda K[6]-\mu K[6])-b\cos(\lambda K[6]+\mu K[6])-a\sin(\lambda K[6]-\mu K[6}\right.\right.$$
$$\left.-\frac{e^{-\int_1^x -\frac{\csc(\mu K[5])(-2d\cos(\lambda K[5]-\mu K[5])+2d\cos(\lambda K[5]+\mu K[5])+c\sin(\lambda K[5]-\mu K[5])+c\sin(\lambda K[5]+\mu K[5]))}{2(a\cos(\lambda K[5])+b\sin(\lambda K[5]))}dK[5]}}{c\mu(d+K[7])^2}\right)dK[7]=c_1,y(x)\Bigg]$$

13 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.

13.1 problem 47

Internal problem ID [9798]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda^2 - c\sin(\lambda x)^n \cos(\lambda x)^{-n-4} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+c*sin(lambda*x)^n*cos(lambda*x)^(-n-4),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+c*Sin[\[Lambda]*x]^n*Cos[\[Lambda]*x]^(-n-4),y[x],x,IncludeS
```

Not solved

13.2 problem 48

Internal problem ID [9799]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \sin(\lambda x)\, y^2 a - b \sin(\lambda x) \cos(\lambda x)^n = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 334

```
dsolve(diff(y(x),x)=a*sin(lambda*x)*y(x)^2+b*sin(lambda*x)*cos(lambda*x)^n,y(x), singsol=all)
```

$$y(x) = \frac{-\frac{\sqrt{\frac{ab}{\lambda^2}}\cos(\lambda x)^{\frac{n}{2}+1} c_1 \lambda \operatorname{BesselY}\left(\frac{n+3}{n+2}, \frac{2\sqrt{\frac{ab}{\lambda^2}}\cos(\lambda x)^{\frac{n}{2}+1}}{n+2}\right)}{\left(\operatorname{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{\frac{ab}{\lambda^2}}\cos(\lambda x)^{\frac{n}{2}+1}}{n+2}\right)c_1 + \operatorname{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{\frac{ab}{\lambda^2}}\cos(\lambda x)^{\frac{n}{2}+1}}{n+2}\right)\right)a} - \frac{\left(\operatorname{BesselJ}\left(\frac{n+3}{n+2}, \frac{2\sqrt{\frac{ab}{\lambda^2}}\cos(\lambda x)^{\frac{n}{2}+1}}{n+2}\right)\sqrt{\frac{ab}{\lambda^2}}\cos(\lambda x)^{\frac{n}{2}+1}}{\left(\operatorname{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{\frac{ab}{\lambda^2}}\cos(}{n+2}\right.\right.}}{\cos(\lambda x)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*Sin[\[Lambda]*x]*y[x]^2+b*Sin[\[Lambda]*x]*Cos[\[Lambda]*x]^n,y[x],x,IncludeS
```

Not solved

13.3 problem 49

Internal problem ID [9800]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \sin(\lambda x)\, y^2 - a\cos(\lambda x)^n\, y + a\cos(\lambda x)^{n-1} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*sin(lambda*x)*y(x)^2+a*cos(lambda*x)^n*y(x)-a*cos(lambda*x)^(n-1),
```

No solution found

✓ Solution by Mathematica

Time used: 103.065 (sec). Leaf size: 467

```
DSolve[y'[x]==\[Lambda]*Sin[\[Lambda]*x]*y[x]^2+a*Cos[\[Lambda]*x]^n*y[x]-a*Cos[\[Lambda]*x]^
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\frac{a\cos^{n+1}(\lambda K[1])\csc(\lambda K[1])\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(\lambda K[1])\right)\sqrt{\sin^2(\lambda K[1])}}{(n+1)\lambda}\right)\tan(\lambda K[1])\left(-a\csc(\lambda K[1])\cos\right.}{(\cos(\lambda K[1])y(x)-1)^2}\right.$$

$$+\int_1^{y(x)}\left(\frac{\exp\left(-\frac{a\cos^{n+1}(x\lambda)\csc(x\lambda)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(x\lambda)\right)\sqrt{\sin^2(x\lambda)}}{(n+1)\lambda}\right)}{(\cos(x\lambda)K[2]-1)^2}\right.$$

$$-\int_1^x\left(\frac{2\exp\left(-\frac{a\cos^{n+1}(\lambda K[1])\csc(\lambda K[1])\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(\lambda K[1])\right)\sqrt{\sin^2(\lambda K[1])}}{(n+1)\lambda}\right)\left(-a\csc(\lambda K[1])\cos^n(\lambda\right.}{(\cos(\lambda K[1])K[2]-1)^3}\right.$$

13.4 problem 50

Internal problem ID [9801]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \cos(\lambda x)\, y^2 a - b\cos(\lambda x)\sin(\lambda x)^n = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 284

```
dsolve(diff(y(x),x)=a*cos(lambda*x)*y(x)^2+b*cos(lambda*x)*sin(lambda*x)^n,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(\sin(\lambda x)^{n+2} c_1 abn + \sin(\lambda x)^{n+2} c_1 ab\right) \operatorname{hypergeom}\left([\,], \left[\frac{2n+5}{n+2}\right], -\frac{\sin(\lambda x)^{n+2} ab}{\lambda^2 (n+2)^2}\right) + \left(-c_1\lambda^2 n^2 - 4c_1\lambda^2 n - 3c\right.}{(1+n)\,\lambda \sin(\lambda x)\,(n+3)\, a\left(c_1 \sin(\lambda x) \operatorname{hyperge}\right.}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*Cos[\[Lambda]*x]*y[x]^2+b*Cos[\[Lambda]*x]*Sin[\[Lambda]*x]^n,y[x],x,IncludeS
```

Not solved

13.5 problem 51

Internal problem ID [9802]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \sin(\lambda x)\, y^2 - a\, x^n \cos(\lambda x)\, y + x^n a = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 122

```
dsolve(diff(y(x),x)=lambda*sin(lambda*x)*y(x)^2+a*x^n*cos(lambda*x)*y(x)-a*x^n,y(x), singsol=
```

$$y(x) = \frac{c_1 e^{\int \frac{\sqrt{\frac{1}{2}-\frac{\cos(2\lambda x)}{2}}\left(\cos(\lambda x)\sin(\lambda x)x^n a+2\sqrt{\frac{1}{2}-\frac{\cos(2\lambda x)}{2}}\,\lambda\tan(\lambda x)\right)}{\sin(\lambda x)^2}dx}}{\left(\int -e^{\int \frac{\sqrt{\frac{1}{2}-\frac{\cos(2\lambda x)}{2}}\left(\cos(\lambda x)\sin(\lambda x)x^n a+2\sqrt{\frac{1}{2}-\frac{\cos(2\lambda x)}{2}}\,\lambda\tan(\lambda x)\right)}{\sin(\lambda x)^2}dx}\sin(\lambda x)\,\lambda dx\right)c_1+1} + \frac{1}{\cos(\lambda x)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Sin[\[Lambda]*x]*y[x]^2+a*x^n*Cos[\[Lambda]*x]*y[x]-a*x^n,y[x],x,Incl
```

Not solved

13.6 problem 52

Internal problem ID [9803]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\sin(2x)^{1+n} y' - a y^2 \sin(x)^{2n} - b\cos(x)^{2n} = 0$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 325

```
dsolve(sin(2*x)^(n+1)*diff(y(x),x)=a*y(x)^2*sin(x)^(2*n)+b*cos(x)^(2*n),y(x), singsol=all)
```

$$y(x) = -\frac{\sin(2x)^n \left(-\sin(x)^{-2n+1-\frac{\sqrt{n^2-ab4^{-n}}}{2}} \cos(x)^{\frac{\sqrt{n^2-ab4^{-n}}}{2}} \sqrt{n^2-ab4^{-n}}\, c_1 + \sin(x)^{-2n+1-\frac{\sqrt{n^2-ab4^{-n}}}{2}} \cos(x)^{\sqrt{\cdots}} \cdots\right.}{a\left(\sin(x)^{-\frac{\sqrt{n^2-ab4^{-n}}}{2}} \cos(x)^{\frac{\sqrt{n^2-ab4^{-n}}}{2}} \cdots\right.}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[Sin[2*x]^(n+1)*y'[x]==a*y[x]^2*Sin[x]^(2*n)+b*Cos[x]^(2*n),y[x],x,IncludeSingularSolut
```

Timed out

13.7 problem 53

Internal problem ID [9804]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 + \tan(x)\, y - a(-a+1)\cot(x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 70

```
dsolve(diff(y(x),x)=y(x)^2-y(x)*tan(x)+a*(1-a)*cot(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{a c_1 \sin(x)}{c_1 \sin(x)+\sin(x)^{2a}} - \frac{c_1 \sin(x)}{c_1 \sin(x)+\sin(x)^{2a}} - \frac{\sin(x)^{2a} a}{c_1 \sin(x)+\sin(x)^{2a}}\right)\cos(x)}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 4.518 (sec). Leaf size: 141

```
DSolve[y'[x]==y[x]^2-y[x]*Tan[x]+a*(1-a)*Cot[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\cot(x)\left(-1 - i\sqrt{\frac{1}{a}+\frac{1}{1-a}-4}\sqrt{a-1}\sqrt{a}\left(1-\frac{2c_1}{\left(-\sin^2(x)\right)^{\frac{1}{2}i\sqrt{\frac{1}{a}+\frac{1}{1-a}-4}\sqrt{a-1}\sqrt{a}}+c_1}\right)\right)$$

$$y(x) \to \frac{1}{2}i\left(\sqrt{a-1}\sqrt{a}\sqrt{\frac{1}{a-a^2}-4}+i\right)\cot(x)$$

13.8 problem 54

Internal problem ID [9805]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 + my\tan(x) - b^2\cos(x)^{2m} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 346

```
dsolve(diff(y(x),x)=y(x)^2-m*y(x)*tan(x)+b^2*cos(x)^(2*m),y(x), singsol=all)
```

$$y(x) = -\frac{\left((m-1)\,\mathrm{hypergeom}\left(\left[\frac{3}{2}, -\frac{m}{2}+\frac{3}{2}\right], \left[\frac{5}{2}\right], \sin(x)^2\right)\sin(x)^2 - 3\,\mathrm{hypergeom}\left(\left[\frac{1}{2}, -\frac{m}{2}+\frac{1}{2}\right], \left[\frac{3}{2}\right], \sin(x)^2\right)\right)}{3\left(c_1\cos\left(b\sqrt{\cos(x)^{-2+2m}}\,\cos(x)^{-m+1}\sin(x)\,\mathrm{hypergeom}\left(\left[\frac{1}{2}, -\frac{m}{2}+\frac{1}{2}\right], \left[\right.\right.\right.\right.}$$
$$+\frac{b\sqrt{\cos(x)^{2m}}\,\cos(x)^{-m}\cos(x)\left((m-1)\,\mathrm{hypergeom}\left(\left[\frac{3}{2}, -\frac{m}{2}+\frac{3}{2}\right], \left[\frac{5}{2}\right], \sin(x)^2\right)\sin(x)^2 - 3\,\mathrm{hypergeom}\right.}{3c_1\cos\left(b\sqrt{\cos(x)^{-2+2m}}\,\cos(x)^{-m+1}\sin(x)\,\mathrm{hypergeom}\left(\left[\frac{1}{2}, -\frac{m}{2}+\frac{1}{2}\right], \left[\frac{3}{2}\right], \sin(x\right.\right.}$$

✓ Solution by Mathematica

Time used: 2.733 (sec). Leaf size: 73

```
DSolve[y'[x]==y[x]^2-m*y[x]*Tan[x]+b^2*Cos[x]^(2*m),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{b^2}\cos^m(x)\tan\left(-\frac{\sqrt{b^2}\sqrt{\sin^2(x)}\csc(x)\cos^{m+1}(x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(x)\right)}{m+1}+c_1\right)$$

13.9 problem 55

Internal problem ID [9806]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - my\cot(x) - b^2\sin(x)^{2m} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 301

```
dsolve(diff(y(x),x)=y(x)^2+m*y(x)*cot(x)+b^2*sin(x)^(2*m),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\left(\csc(x)^2\right)^{-m-2}}\left(\csc(x)^2\right)^{\frac{m}{2}+1} b\left((-m-2)\operatorname{hypergeom}\left(\left[\frac{3}{2}, 2+\frac{m}{2}\right], \left[\frac{5}{2}\right], -\cot(x)^2\right)\cot(x)^4 + ((-m-}{}$$

✓ Solution by Mathematica

Time used: 1.5 (sec). Leaf size: 233

```
DSolve[y'[x]==y[x]^2+m*y[x]*Cot[x]+b^2*Sin[x]^(2*m),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b\sqrt{\cos^2(x)}\sec(x)\sin^m(x)\left(-\cos\left(\frac{b\sin^{m+1}(x)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(x)\right)}{m+1}\right) + c_1\sin\left(\frac{b\sin^{m+1}(x)\operatorname{Hypergeom}}{}\right.\right.}{\sin\left(\frac{b\sin^{m+1}(x)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(x)\right)}{m+1}\right) + c_1\cos\left(\frac{b\sin^{m+1}(x)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m}{}\right.}{m+1}\right.}$$

$$y(x) \to b\sqrt{\cos^2(x)}\sec(x)\sin^m(x)\tan\left(\frac{b\sin^{m+1}(x)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(x)\right)}{m+1}\right)$$

13.10 problem 56

Internal problem ID [9807]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 + 2\lambda^2 \tan(x)^2 + 2\lambda^2 \cot(\lambda x)^2 = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2-2*lambda^2*tan(x)^2-2*lambda^2*cot(lambda*x)^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-2*\[Lambda]^2*Tan[x]^2-2*\[Lambda]^2*Cot[\[Lambda]*x]^2,y[x],x,IncludeSi
```

Not solved

13.11 problem 57

Internal problem ID [9808]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda a - b\lambda - 2ab - a(\lambda - a)\tan(\lambda x)^2 - b(\lambda - b)\cot(\lambda x)^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 885

```
dsolve(diff(y(x),x)=y(x)^2+lambda*a+lambda*b+2*a*b+a*(lambda-a)*tan(lambda*x)^2+b*(lambda-b)*
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]*a+\[Lambda]*b+2*a*b+a*(\[Lambda]-a)*Tan[\[Lambda]*x]^2+b*(\[La
```

Not solved

13.12 problem 58

Internal problem ID [9809]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 + \frac{\lambda^2}{2} + \frac{3\tan(\lambda x)^2 \lambda^2}{4} - a\cos(\lambda x)^2 \sin(\lambda x)^n = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 411

```
dsolve(diff(y(x),x)=y(x)^2-1/2*lambda^2-3/4*lambda^2*tan(lambda*x)^2+a*cos(lambda*x)^2*sin(la
```

$$y(x) = -\frac{(c_1\lambda^2 n^2 + 4c_1\lambda^2 n + 3c_1\lambda^2)\operatorname{hypergeom}\left([], \left[\frac{n+3}{n+2}\right], -\frac{\sin(\lambda x)^{n+2} a}{\lambda^2 (n+2)^2}\right)\sin(\lambda x)^3 + (\lambda^2 n^2 + 4\lambda^2 n + 3\lambda^2)\operatorname{hyperg}}{}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-1/2*\[Lambda]^2-3/4*\[Lambda]^2*Tan[\[Lambda]*x]^2+a*Cos[\[Lambda]*x]^2*
```

Not solved

13.13 problem 59

Internal problem ID [9810]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \sin(\lambda x) y^2 - a \sin(\lambda x) y + a \tan(\lambda x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
dsolve(diff(y(x),x)=lambda*sin(lambda*x)*y(x)^2+a*sin(lambda*x)*y(x)-a*tan(lambda*x),y(x), si
```

$$y(x) = \frac{\mathrm{Ei}_1\left(\frac{\cos(\lambda x) a}{\lambda}\right) c_1 a - 1}{\mathrm{Ei}_1\left(\frac{\cos(\lambda x) a}{\lambda}\right) \cos(\lambda x) c_1 a - \mathrm{e}^{-\frac{\cos(\lambda x) a}{\lambda}} c_1 \lambda - \cos(\lambda x)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Sin[\[Lambda]*x]*y[x]^2+a*Sin[\[Lambda]*x]*y[x]-a*Tan[\[Lambda]*x],y[
```

Not solved

14 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.

14.1 problem 1

Internal problem ID [9811]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda \arcsin(x)^n y + a^2 - a\lambda \arcsin(x)^n = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 97

```
dsolve(diff(y(x),x)=y(x)^2+lambda*arcsin(x)^n*y(x)-a^2+a*lambda*arcsin(x)^n,y(x), singsol=all
```

$$y(x) = -\frac{\left(\left(\int e^{\int (\arcsin(x)^n \lambda - 2a)dx} dx\right) e^{\int (-\arcsin(x)^n \lambda + 2a)dx} a + c_1 e^{\int (-\arcsin(x)^n \lambda + 2a)dx} a + 1\right) e^{\int (\arcsin(x)^n \lambda - 2a)dx}}{c_1 + \int e^{\int (\arcsin(x)^n \lambda - 2a)dx} dx}$$

✓ Solution by Mathematica

Time used: 4.245 (sec). Leaf size: 398

```
DSolve[y'[x]==y[x]^2+\[Lambda]*ArcSin[x]^n*y[x]-a^2+a*\[Lambda]*ArcSin[x]^n,y[x],x,IncludeSin
```

$$\text{Solve}\Bigg[\int_1^x$$

$$-\frac{\exp\left(\frac{1}{2}i\lambda\arcsin(K[1])^n\left(\arcsin(K[1])^2\right)^{-n}\left((-i\arcsin(K[1]))^n\Gamma(n+1,i\arcsin(K[1]))-(i\arcsin(K[1]))^n\Gamma\right.\right.}{n\lambda(a+y(x))}$$

$$+\int_1^{y(x)}\left(\frac{\exp\left(\frac{1}{2}i\lambda\arcsin(x)^n\left(\arcsin(x)^2\right)^{-n}\left((-i\arcsin(x))^n\Gamma(n+1,i\arcsin(x))-(i\arcsin(x))^n\Gamma(n+1,-\right.\right.}{n\lambda(a+K[2])^2}\right.$$

$$-\int_1^x\left(\frac{\exp\left(\frac{1}{2}i\lambda\arcsin(K[1])^n\left(\arcsin(K[1])^2\right)^{-n}\left((-i\arcsin(K[1]))^n\Gamma(n+1,i\arcsin(K[1]))-(i\arcsin(K\right.\right.}{n\lambda(a+K[2])^2}\right.$$

14.2 problem 2

Internal problem ID [9812]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \lambda x \arcsin(x)^n y - \arcsin(x)^n \lambda = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve(diff(y(x),x)=y(x)^2+lambda*x*arcsin(x)^n*y(x)+lambda*arcsin(x)^n,y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{\arcsin(x)^n \lambda x^2 - 2}{x} dx}}{c_1 - \left(\int e^{\int \frac{\arcsin(x)^n \lambda x^2 - 2}{x} dx} dx\right)} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 2.151 (sec). Leaf size: 171

```
DSolve[y'[x]==y[x]^2+\[Lambda]*x*ArcSin[x]^n*y[x]+\[Lambda]*ArcSin[x]^n,y[x],x,IncludeSingula
```

$$y(x) \to -\frac{x + \frac{\exp\left(\lambda(-2^{-n-3}) \arcsin(x)^n \left(\arcsin(x)^2\right)^{-n} ((-i \arcsin(x))^n \Gamma(n+1, 2i \arcsin(x)) + (i \arcsin(x))^n \Gamma(n+1, -2i \arcsin(x)))\right)}{\int_1^x \frac{\exp\left(-2^{-n-3} \lambda \arcsin(K[1])^n \left(\arcsin(K[1])^2\right)^{-n} (\Gamma(n+1, 2i \arcsin(K[1]))(-i \arcsin(K[1]))^n + (i \arcsin(K[1]))^n \Gamma(n+1, -2i \arcsin(K[1])))\right)}{K[1]^2} dK[1] + c}}{x^2}$$

$$y(x) \to -\frac{1}{x}$$

14.3 problem 3

Internal problem ID [9813]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (1+k)\,x^k y^2 - \lambda \arcsin(x)^n \left(x^{1+k} y - 1\right) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 196

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+lambda*arcsin(x)^n*(x^(k+1)*y(x)-1),y(x), singsol=all)
```

$$y(x) = \frac{\left(e^{\int \frac{x^k \arcsin(x)^n \lambda x^2 - 2k - 2}{x} dx} x^k x + \int \left(x^k k\, e^{\lambda\left(\int x^{1+k} \arcsin(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} + x^k e^{\lambda\left(\int x^{1+k} \arcsin(x)^n dx\right) - 2k\left(\int \frac{1}{x} d\right.}\right.\right.}{x \left(\int \left(x^k k\, e^{\lambda\left(\int x^{1+k} \arcsin(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} + x^k e^{\lambda\left(\int x^{1+k} \arcsin(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)}\right)\right.}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+\[Lambda]*ArcSin[x]^n*(x^(k+1)*y[x]-1),y[x],x,IncludeSingular
```

Not solved

14.4 problem 4

Internal problem ID [9814]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arcsin(x)^n y^2 - ya - ab + b^2 \lambda \arcsin(x)^n = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 114

```
dsolve(diff(y(x),x)=lambda*arcsin(x)^n*y(x)^2+a*y(x)+a*b-b^2*lambda*arcsin(x)^n,y(x), singsol
```

$$y(x) = -\frac{\left(\left(\int \arcsin(x)^n \lambda \, e^{\int (-2\arcsin(x)^n \lambda b + a)dx} dx\right) e^{\int (2\arcsin(x)^n \lambda b - a)dx} b + c_1 e^{\int (2\arcsin(x)^n \lambda b - a)dx} b + 1\right) e^{\int (-2\arcsin(x)^n}}{c_1 + \int \arcsin(x)^n \lambda \, e^{\int (-2\arcsin(x)^n \lambda b + a)dx} dx}$$

✓ Solution by Mathematica

Time used: 4.504 (sec). Leaf size: 428

```
DSolve[y'[x]==\[Lambda]*ArcSin[x]^n*y[x]^2+a*y[x]+a*b-b^2*\[Lambda]*ArcSin[x]^n,y[x],x,Includ
```

$$\text{Solve}\left[\int_1^x \frac{i \exp\left(aK[1] - ib\lambda \arcsin(K[1])^n \left(\arcsin(K[1])^2\right)^{-n} \left((-i\arcsin(K[1]))^n \Gamma(n+1, i\arcsin(K[1])) - \right.\right.}{an\lambda(b+}\right.$$
$$+\int_1^{y(x)} \left(-\int_1^x \left(\frac{i \exp\left(aK[1] - ib\lambda \arcsin(K[1])^n \left(\arcsin(K[1])^2\right)^{-n} \left((-i\arcsin(K[1]))^n \Gamma(n+1, i\arcsin(K\right.\right.}{an(b+K[2])}\right.\right.$$
$$-\frac{i \exp\left(ax - ib\lambda \arcsin(x)^n \left(\arcsin(x)^2\right)^{-n} \left((-i\arcsin(x))^n \Gamma(n+1, i\arcsin(x)) - (i\arcsin(x))^n \Gamma(n+1, -\right.\right.}{an\lambda(b+K[2])^2}$$

14.5 problem 5

Internal problem ID [9815]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \arcsin(x)^n y^2 + b\lambda x^m \arcsin(x)^n y - bm x^{m-1} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arcsin(x)^n*y(x)^2-b*lambda*x^m*arcsin(x)^n*y(x)+b*m*x^(m-1),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcSin[x]^n*y[x]^2-b*\[Lambda]*x^m*ArcSin[x]^n*y[x]+b*m*x^(m-1),y[x],
```

Not solved

14.6 problem 6

Internal problem ID [9816]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arcsin(x)^n y^2 - \beta m\, x^{m-1} + \lambda \beta^2 x^{2m} \arcsin(x)^n = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arcsin(x)^n*y(x)^2+beta*m*x^(m-1)-lambda*beta^2*x^(2*m)*arcsin(x)^
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcSin[x]^n*y[x]^2+\[Beta]*m*x^(m-1)-\[Lambda]*\[Beta]^2*x^(2*m)*ArcS
```

Not solved

14.7 problem 7

Internal problem ID [9817]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arc-sine.
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \lambda \arcsin(x)^n (y - a\,x^m - b)^2 - am\,x^{m-1} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)=lambda*arcsin(x)^n*(y(x)-a*x^m-b)^2+a*m*x^(m-1),y(x), singsol=all)
```

$$y(x) = -\frac{(-2a\,x^m \arcsin(x)^n \lambda - 2\arcsin(x)^n \lambda b)\arcsin(x)^{-n}}{2\lambda} + \frac{1}{c_1 - \left(\int \arcsin(x)^n \lambda dx\right)}$$

✓ Solution by Mathematica

Time used: 2.634 (sec). Leaf size: 87

```
DSolve[y'[x]==\[Lambda]*ArcSin[x]^n*(y[x]-a*x^m-b)^2+a*m*x^(m-1),y[x],x,IncludeSingularSoluti
```

$$y(x) \to ax^m + \frac{1}{\frac{1}{2}i\lambda \arcsin(x)^n \left(\arcsin(x)^2\right)^{-n} \left((i\arcsin(x))^n\Gamma(n+1, -i\arcsin(x)) - (-i\arcsin(x))^n\Gamma(n+1, i\arcsin(x$$
$$+ b$$

$$y(x) \to ax^m + b$$

14.8 problem 8

Internal problem ID [9818]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - \lambda \arcsin (x)^n y^2 - ky - \lambda b^2 x^{2k} \arcsin (x)^n = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x)=lambda*arcsin(x)^n*y(x)^2+k*y(x)+lambda*b^2*x^(2*k)*arcsin(x)^n,y(x), s
```

$$y(x) = -\tan\left(-b\lambda\left(\int \frac{\arcsin (x)^n x^k}{x} dx\right) + c_1\right) b\, x^k$$

✓ Solution by Mathematica

Time used: 1.143 (sec). Leaf size: 48

```
DSolve[x*y'[x]==\[Lambda]*ArcSin[x]^n*y[x]^2+k*y[x]+\[Lambda]*b^2*x^(2*k)*ArcSin[x]^n,y[x],x,
```

$$y(x) \to \sqrt{b^2} x^k \tan\left(\sqrt{b^2} \int_1^x \lambda \arcsin(K[1])^n K[1]^{k-1} dK[1] + c_1\right)$$

14.9 problem 9

Internal problem ID [9819]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - \left(a\,x^{2m}y^2 + x^n yb + c\right)\arcsin\left(x\right)^m + yn = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=(a*x^(2*m)*y(x)^2+b*x^n*y(x)+c)*arcsin(x)^m-n*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==(a*x^(2*m)*y[x]^2+b*x^n*y[x]+c)*ArcSin[x]^m-n*y[x],y[x],x,IncludeSingularSolu
```

Not solved

15 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.

15.1 problem 10

Internal problem ID [9820]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arc-cosine.
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \lambda \arccos(x)^n y + a^2 - a\lambda \arccos(x)^n = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 595

```
dsolve(diff(y(x),x)=y(x)^2+lambda*arccos(x)^n*y(x)-a^2+a*lambda*arccos(x)^n,y(x), singsol=all
```

Expression too large to display

✓ Solution by Mathematica

Time used: 5.271 (sec). Leaf size: 404

```
DSolve[y'[x]==y[x]^2+\[Lambda]*ArcCos[x]^n*y[x]-a^2+a*\[Lambda]*ArcCos[x]^n,y[x],x,IncludeSin
```

$$\text{Solve}\left[\int_1^x \frac{i\exp\left(\frac{1}{2}\lambda\arccos(K[1])^n\Gamma(n+1,-i\arccos(K[1]))(-i\arccos(K[1]))^{-n}+\frac{1}{2}\lambda(i\arccos(K[1]))^{-n}\text{arc}}{n\lambda(a+y(x))}\right.$$
$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{i\exp\left(\frac{1}{2}\lambda\arccos(K[1])^n\Gamma(n+1,-i\arccos(K[1]))(-i\arccos(K[1]))^{-n}+\frac{1}{2}\lambda(i\arccos(K[1])\right.}{n\lambda(a+K[2])}\right.\right.$$
$$-\frac{i\exp\left(\frac{1}{2}\lambda\arccos(x)^n\Gamma(n+1,-i\arccos(x))(-i\arccos(x))^{-n}-2ax+\frac{1}{2}\lambda(i\arccos(x))^{-n}\arccos(x)^n\Gamma(n+1\right.}{n\lambda(a+K[2])^2}$$

15.2 problem 11

Internal problem ID [9821]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda x \arccos(x)^n y - \arccos(x)^n \lambda = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x)=y(x)^2+lambda*x*arccos(x)^n*y(x)+lambda*arccos(x)^n,y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{\arccos(x)^n \lambda x^2 - 2}{x} dx}}{c_1 - \left(\int e^{\int \frac{\arccos(x)^n \lambda x^2 - 2}{x} dx} dx\right)} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 3.067 (sec). Leaf size: 169

```
DSolve[y'[x]==y[x]^2+\[Lambda]*x*ArcCos[x]^n*y[x]+\[Lambda]*ArcCos[x]^n,y[x],x,IncludeSingula
```

$$y(x) \to -\frac{x + \frac{\exp\left(\lambda 2^{-n-3} \arccos(x)^n \left(\arccos(x)^2\right)^{-n} \left((-i \arccos(x))^n \Gamma(n+1, 2i \arccos(x)) + (i \arccos(x))^n \Gamma(n+1, -2i \arccos(x))\right)\right)}{\int_1^x \frac{\exp\left(2^{-n-3} \lambda \arccos(K[1])^n \left(\arccos(K[1])^2\right)^{-n} \left(\Gamma(n+1, 2i \arccos(K[1]))(-i \arccos(K[1]))^n + (i \arccos(K[1]))^n \Gamma(n+1, -2i \arccos(K[1]))\right)\right)}{K[1]^2} dK[1] + c_1}}{x^2}$$

$$y(x) \to -\frac{1}{x}$$

15.3 problem 12

Internal problem ID [9822]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (1+k)\, x^k y^2 - \lambda \arccos(x)^n \left(x^{1+k} y - 1\right) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 205

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+lambda*arccos(x)^n*(x^(k+1)*y(x)-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\mathrm{e}^{\int \frac{x^k \arccos(x)^n \lambda x^2 - 2k - 2}{x} dx} x^k x - \left(\int \left(-x^k k\, \mathrm{e}^{\lambda\left(\int x^{1+k} \arccos(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} - x^k \mathrm{e}^{\lambda\left(\int x^{1+k} \arccos(x)^n dx\right) -}\right.\right.\right.}{x \left(\int \left(-x^k k\, \mathrm{e}^{\lambda\left(\int x^{1+k} \arccos(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} - x^k \mathrm{e}^{\lambda\left(\int x^{1+k} \arccos(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2(}\right.\right.}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+\[Lambda]*ArcCos[x]^n*(x^(k+1)*y[x]-1),y[x],x,IncludeSingular
```

Not solved

15.4 problem 13

Internal problem ID [9823]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \arccos(x)^n y^2 - ya - ab + b^2\lambda \arccos(x)^n = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 587

```
dsolve(diff(y(x),x)=lambda*arccos(x)^n*y(x)^2+a*y(x)+a*b-b^2*lambda*arccos(x)^n,y(x), singsol
```

Expression too large to display

✓ Solution by Mathematica

Time used: 5.843 (sec). Leaf size: 420

```
DSolve[y'[x]==\[Lambda]*ArcCos[x]^n*y[x]^2+a*y[x]+a*b-b^2*\[Lambda]*ArcCos[x]^n,y[x],x,Includ
```

$$\text{Solve}\left[\int_1^x -\frac{i\exp\left(-b\lambda\arccos(K[1])^n\Gamma(n+1,-i\arccos(K[1]))(-i\arccos(K[1]))^{-n} - b\lambda(i\arccos(K[1]))^{-n}\arccos(K[\right.}{an\lambda(b+y(x))}\right.$$

$$+\int_1^{y(x)}\left(\frac{i\exp\left(-b\lambda\arccos(x)^n\Gamma(n+1,-i\arccos(x))(-i\arccos(x))^{-n} + ax - b\lambda(i\arccos(x))^{-n}\arccos(x)^n\right.}{an\lambda(b+K[2])^2}\right.$$

$$-\int_1^x\left(\frac{i\exp\left(-b\lambda\arccos(K[1])^n\Gamma(n+1,-i\arccos(K[1]))(-i\arccos(K[1]))^{-n} - b\lambda(i\arccos(K[1]))^{-n}\arccos\right.}{an\lambda(b+K[\right.}\right.$$

15.5 problem 14

Internal problem ID [9824]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arccos(x)^n y^2 + b\lambda x^m \arccos(x)^n y - bm\, x^{m-1} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arccos(x)^n*y(x)^2-b*lambda*x^m*arccos(x)^n*y(x)+b*m*x^(m-1),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcCos[x]^n*y[x]^2-b*\[Lambda]*x^m*ArcCos[x]^n*y[x]+b*m*x^(m-1),y[x],
```

Not solved

15.6 problem 15

Internal problem ID [9825]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \arccos(x)^n y^2 - \beta m\, x^{m-1} + \lambda \beta^2 x^{2m} \arccos(x)^n = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arccos(x)^n*y(x)^2+beta*m*x^(m-1)-lambda*beta^2*x^(2*m)*arccos(x)^
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcCos[x]^n*y[x]^2+\[Beta]*m*x^(m-1)-\[Lambda]*\[Beta]^2*x^(2*m)*ArcC
```

Not solved

15.7 problem 16

Internal problem ID [9826]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \lambda \arccos(x)^n (y - a x^m - b)^2 - am x^{m-1} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 162

```
dsolve(diff(y(x),x)=lambda*arccos(x)^n*(y(x)-a*x^m-b)^2+a*m*x^(m-1),y(x), singsol=all)
```

$$y(x) = -\frac{(-2a x^m \arccos(x)^n \lambda - 2\arccos(x)^n \lambda b)\arccos(x)^{-n}}{2\lambda} + \frac{1}{c_1 + \lambda\sqrt{\pi}\, 2^n \left(\frac{\arccos(x)^{1+n} 2^{-n}\sqrt{-x^2+1}}{\sqrt{\pi}\,(n+2)} - \frac{2^{-n}\sqrt{\arccos(x)}\ \mathrm{LommelS1}\left(\frac{3}{2}+n, \frac{3}{2}, \arccos(x)\right)\sqrt{-x^2+1}}{\sqrt{\pi}\,(n+2)} - \frac{3\, 2^{-1-n}\left(\frac{2n}{3}+\frac{4}{3}\right)\left(\arccos(x)\right.}{\sqrt{\pi}}\right.}$$

✓ Solution by Mathematica

Time used: 3.028 (sec). Leaf size: 84

```
DSolve[y'[x]==\[Lambda]*ArcCos[x]^n*(y[x]-a*x^m-b)^2+a*m*x^(m-1),y[x],x,IncludeSingularSoluti
```

$$y(x) \to ax^m + \frac{1}{c_1 - \frac{1}{2}\lambda \arccos(x)^n (\arccos(x)^2)^{-n} ((-i\arccos(x))^n \Gamma(n+1, i\arccos(x)) + (i\arccos(x))^n \Gamma(n+1, -i \arc} + b$$

$$y(x) \to ax^m + b$$

15.8 problem 17

Internal problem ID [9827]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arc-cosine.
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - \lambda \arccos(x)^n y^2 - ky - \lambda b^2 x^{2k} \arccos(x)^n = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x)=lambda*arccos(x)^n*y(x)^2+k*y(x)+lambda*b^2*x^(2*k)*arccos(x)^n,y(x), s
```

$$y(x) = -\tan\left(-b\lambda\left(\int \frac{x^k \arccos(x)^n}{x}dx\right) + c_1\right) b\, x^k$$

✓ Solution by Mathematica

Time used: 1.349 (sec). Leaf size: 48

```
DSolve[x*y'[x]==\[Lambda]*ArcCos[x]^n*y[x]^2+k*y[x]+\[Lambda]*b^2*x^(2*k)*ArcCos[x]^n,y[x],x,
```

$$y(x) \to \sqrt{b^2} x^k \tan\left(\sqrt{b^2} \int_1^x \lambda \arccos(K[1])^n K[1]^{k-1} dK[1] + c_1\right)$$

15.9 problem 18

Internal problem ID [9828]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arc-cosine.
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - \left(a\,x^{2m}y^2 + x^n yb + c\right)\arccos\left(x\right)^m + yn = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=(a*x^(2*m)*y(x)^2+b*x^n*y(x)+c)*arccos(x)^m-n*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==(a*x^(2*m)*y[x]^2+b*x^n*y[x]+c)*ArcCos[x]^m-n*y[x],y[x],x,IncludeSingularSolu
```

Not solved

16 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.

16.1 problem 19

Internal problem ID [9829]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda \arctan(x)^n y + a^2 - a\lambda \arctan(x)^n = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 97

```
dsolve(diff(y(x),x)=y(x)^2+lambda*arctan(x)^n*y(x)-a^2+a*lambda*arctan(x)^n,y(x), singsol=all
```

$$y(x) = -\frac{\left(\left(\int e^{\int (\arctan(x)^n\lambda - 2a)dx}dx\right) e^{\int (-\arctan(x)^n\lambda + 2a)dx} a + c_1 e^{\int (-\arctan(x)^n\lambda + 2a)dx} a + 1\right) e^{\int (\arctan(x)^n\lambda - 2a)dx}}{c_1 + \int e^{\int (\arctan(x)^n\lambda - 2a)dx}dx}$$

✓ Solution by Mathematica

Time used: 5.043 (sec). Leaf size: 210

```
DSolve[y'[x]==y[x]^2+\[Lambda]*ArcTan[x]^n*y[x]-a^2+a*\[Lambda]*ArcTan[x]^n,y[x],x,IncludeSin
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[6]} (2a - \lambda \arctan(K[5])^n)\, dK[5]\right) (-\lambda \arctan(K[6])^n + a - y(x))}{n\lambda(a + y(x))} dK[6]\right.$$
$$+ \int_1^{y(x)} \left(\frac{\exp\left(-\int_1^x (2a - \lambda \arctan(K[5])^n)\, dK[5]\right)}{n\lambda(a + K[7])^2}\right.$$
$$- \int_1^x \left(-\frac{\exp\left(-\int_1^{K[6]} (2a - \lambda \arctan(K[5])^n)\, dK[5]\right) (-\lambda \arctan(K[6])^n + a - K[7])}{n\lambda(a + K[7])^2} - \frac{\exp\left(-\int_1^{K[6]} (2a - \right.}{n\lambda(}\right.$$

16.2 problem 20

Internal problem ID [9830]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda x \arctan(x)^n y - \arctan(x)^n \lambda = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x)=y(x)^2+lambda*x*arctan(x)^n*y(x)+lambda*arctan(x)^n,y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{\arctan(x)^n \lambda x^2 - 2}{x} dx}}{c_1 - \left(\int e^{\int \frac{\arctan(x)^n \lambda x^2 - 2}{x} dx} dx\right)} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 4.342 (sec). Leaf size: 82

```
DSolve[y'[x]==y[x]^2+\[Lambda]*x*ArcTan[x]^n*y[x]+\[Lambda]*ArcTan[x]^n,y[x],x,IncludeSingula
```

$$y(x) \to -\frac{x + \frac{\exp\left(-\int_1^x -\lambda \arctan(K[5])^n K[5] dK[5]\right)}{\int_1^x \frac{\exp\left(-\int_1^{K[6]} -\lambda \arctan(K[5])^n K[5] dK[5]\right)}{K[6]^2} dK[6] + c_1}}{x^2}$$

$$y(x) \to -\frac{1}{x}$$

16.3 problem 21

Internal problem ID [9831]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (1+k)\, x^k y^2 - \lambda \arctan(x)^n \left(x^{1+k} y - 1\right) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 205

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+lambda*arctan(x)^n*(x^(k+1)*y(x)-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(e^{\int \frac{x^k \arctan(x)^n \lambda\, x^2 - 2k - 2}{x} dx} x^k x - \left(\int \left(-x^k k\, e^{\lambda\left(\int x^{1+k} \arctan(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} - x^k e^{\lambda\left(\int x^{1+k} \arctan(x)^n dx\right) - \cdots}\right.\right.\right.}{x\left(\int \left(-x^k k\, e^{\lambda\left(\int x^{1+k} \arctan(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} - x^k e^{\lambda\left(\int x^{1+k} \arctan(x)^n dx\right) - 2k\left(\int \frac{1}{x} dx\right) - 2\cdots}\right.\right.}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+\[Lambda]*ArcTan[x]^n*(x^(k+1)*y[x]-1),y[x],x,IncludeSingular
```

Not solved

16.4 problem 22

Internal problem ID [9832]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arctan(x)^n y^2 - ya - ab + b^2\lambda \arctan(x)^n = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 114

```
dsolve(diff(y(x),x)=lambda*arctan(x)^n*y(x)^2+a*y(x)+a*b-b^2*lambda*arctan(x)^n,y(x), singsol
```

$$y(x) = -\frac{\left(\left(\int \arctan(x)^n \lambda \, e^{\int (-2\arctan(x)^n\lambda b+a)dx}dx\right) e^{\int (2\arctan(x)^n\lambda b-a)dx} b + c_1 e^{\int (2\arctan(x)^n\lambda b-a)dx} b + 1\right) e^{\int (-2\arctan(}}{c_1 + \int \arctan(x)^n \lambda \, e^{\int (-2\arctan(x)^n\lambda b+a)dx}dx}$$

✓ Solution by Mathematica

Time used: 6.922 (sec). Leaf size: 240

```
DSolve[y'[x]==\[Lambda]*ArcTan[x]^n*y[x]^2+a*y[x]+a*b-b^2*\[Lambda]*ArcTan[x]^n,y[x],x,Includ
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[2]} (2b\lambda \arctan(K[1])^n - a)\, dK[1]\right)(-b\lambda \arctan(K[2])^n + \lambda y(x) \arctan(K[2])^n + a)}{an\lambda(b+y(x))} dK\right.$$
$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]} (2b\lambda \arctan(K[1])^n - a)\, dK[1]\right)\arctan(K[2])^n}{an(b+K[3])} - \frac{\exp\left(-\int_1^{K[2]} (2b\lambda \arctan(K\right.}{}\right.\right.$$
$$\left.\left. - \frac{\exp\left(-\int_1^x (2b\lambda \arctan(K[1])^n - a)\, dK[1]\right)}{an\lambda(b+K[3])^2}\right) dK[3] = c_1, y(x)\right]$$

16.5 problem 23

Internal problem ID [9833]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arctan(x)^{n} y^{2} + b\lambda x^{m} \arctan(x)^{n} y - bm x^{m-1} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arctan(x)^n*y(x)^2-b*lambda*x^m*arctan(x)^n*y(x)+b*m*x^(m-1),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcTan[x]^n*y[x]^2-b*\[Lambda]*x^m*ArcTan[x]^n*y[x]+b*m*x^(m-1),y[x],
```

Not solved

16.6 problem 24

Internal problem ID [9834]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arctan(x)^n y^2 - \beta m x^{m-1} + \lambda \beta^2 x^{2m} \arctan(x)^n = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arctan(x)^n*y(x)^2+beta*m*x^(m-1)-lambda*beta^2*x^(2*m)*arctan(x)^
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcTan[x]^n*y[x]^2+\[Beta]*m*x^(m-1)-\[Lambda]*\[Beta]^2*x^(2*m)*ArcT
```

Not solved

16.7 problem 25

Internal problem ID [9835]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \lambda \arctan(x)^n (y - a\,x^m - b)^2 - am\,x^{m-1} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)=lambda*arctan(x)^n*(y(x)-a*x^m-b)^2+a*m*x^(m-1),y(x), singsol=all)
```

$$y(x) = -\frac{(-2a\,x^m \arctan(x)^n \lambda - 2\arctan(x)^n \lambda b) \arctan(x)^{-n}}{2\lambda} + \frac{1}{c_1 - \left(\int \arctan(x)^n \lambda dx\right)}$$

✓ Solution by Mathematica

Time used: 1.36 (sec). Leaf size: 44

```
DSolve[y'[x]==\[Lambda]*ArcTan[x]^n*(y[x]-a*x^m-b)^2+a*m*x^(m-1),y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{1}{-\int_1^x \lambda \arctan(K[2])^n dK[2] + c_1} + ax^m + b$$

$$y(x) \to ax^m + b$$

16.8 problem 26

Internal problem ID [9836]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - \lambda \arctan(x)^n y^2 - ky - \lambda b^2 x^{2k} \arctan(x)^n = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x)=lambda*arctan(x)^n*y(x)^2+k*y(x)+lambda*b^2*x^(2*k)*arctan(x)^n,y(x), s
```

$$y(x) = -\tan\left(-b\lambda\left(\int \frac{\arctan(x)^n x^k}{x} dx\right) + c_1\right) b\, x^k$$

✓ Solution by Mathematica

Time used: 1.283 (sec). Leaf size: 48

```
DSolve[x*y'[x]==\[Lambda]*ArcTan[x]^n*y[x]^2+k*y[x]+\[Lambda]*b^2*x^(2*k)*ArcTan[x]^n,y[x],x,
```

$$y(x) \to \sqrt{b^2} x^k \tan\left(\sqrt{b^2} \int_1^x \lambda \arctan(K[1])^n K[1]^{k-1} dK[1] + c_1\right)$$

16.9 problem 27

Internal problem ID [9837]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - \left(a\,x^{2m}y^2 + x^n y b + c\right)\arctan\left(x\right)^m + yn = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=(a*x^(2*m)*y(x)^2+b*x^n*y(x)+c)*arctan(x)^m-n*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==(a*x^(2*m)*y[x]^2+b*x^n*y[x]+c)*ArcTan[x]^m-n*y[x],y[x],x,IncludeSingularSolu
```

Not solved

17 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-4. Equations containing arccotangent.

17.1 problem 28

Internal problem ID [9838]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-4. Equations containing arc-cotangent.
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda \operatorname{arccot}(x)^n y + a^2 - a\lambda \operatorname{arccot}(x)^n = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 97

```
dsolve(diff(y(x),x)=y(x)^2+lambda*arccot(x)^n*y(x)-a^2+a*lambda*arccot(x)^n,y(x), singsol=all
```

$$y(x) = -\frac{\left(\left(\int e^{\int (\operatorname{arccot}(x)^n\lambda - 2a)dx}dx\right) e^{\int (-\operatorname{arccot}(x)^n\lambda + 2a)dx} a + c_1 e^{\int (-\operatorname{arccot}(x)^n\lambda + 2a)dx} a + 1\right) e^{\int (\operatorname{arccot}(x)^n\lambda - 2a)dx}}{c_1 + \int e^{\int (\operatorname{arccot}(x)^n\lambda - 2a)dx}dx}$$

✓ Solution by Mathematica

Time used: 5.451 (sec). Leaf size: 210

```
DSolve[y'[x]==y[x]^2+\[Lambda]*ArcCot[x]^n*y[x]-a^2+a*\[Lambda]*ArcCot[x]^n,y[x],x,IncludeSin
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[6]} \left(2a - \lambda \cot^{-1}(K[5])^n\right) dK[5]\right) \left(-\lambda \cot^{-1}(K[6])^n + a - y(x)\right)}{n\lambda(a + y(x))} dK[6]\right.$$
$$+ \int_1^{y(x)} \left(-\int_1^x \left(\frac{\exp\left(-\int_1^{K[6]} \left(2a - \lambda \cot^{-1}(K[5])^n\right) dK[5]\right) \left(-\lambda \cot^{-1}(K[6])^n + a - K[7]\right)}{n\lambda(a + K[7])^2} + \frac{\exp\left(-\int_1^{K[6]}\right.}{}\right.\right.$$
$$\left.\left. - \frac{\exp\left(-\int_1^x \left(2a - \lambda \cot^{-1}(K[5])^n\right) dK[5]\right)}{n\lambda(a + K[7])^2}\right) dK[7] = c_1, y(x)\right]$$

18 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.

18.1 problem 29

Internal problem ID [9839]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \lambda x \operatorname{arccot}(x)^n y - \operatorname{arccot}(x)^n \lambda = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 55

```
dsolve(diff(y(x),x)=y(x)^2+lambda*x*arccot(x)^n*y(x)+lambda*arccot(x)^n,y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{\operatorname{arccot}(x)^n \lambda x^2 - 2}{x} dx}}{c_1 - \left(\int e^{\int \frac{\operatorname{arccot}(x)^n \lambda x^2 - 2}{x} dx} dx\right)} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 4.496 (sec). Leaf size: 82

```
DSolve[y'[x]==y[x]^2+\[Lambda]*x*ArcCot[x]^n*y[x]+\[Lambda]*ArcCot[x]^n,y[x],x,IncludeSingula
```

$$y(x) \to -\frac{x + \frac{\exp\left(-\int_1^x -\lambda \cot^{-1}(K[5])^n K[5] dK[5]\right)}{\int_1^x \frac{\exp\left(-\int_1^{K[6]} -\lambda \cot^{-1}(K[5])^n K[5] dK[5]\right)}{K[6]^2} dK[6] + c_1}}{x^2}$$

$$y(x) \to -\frac{1}{x}$$

18.2 problem 30

Internal problem ID [9840]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (1+k)\,x^k y^2 - \lambda \operatorname{arccot}(x)^n \left(x^{1+k} y - 1\right) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 205

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+lambda*arccot(x)^n*(x^(k+1)*y(x)-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\mathrm{e}^{\int \frac{x^k \operatorname{arccot}(x)^n \lambda\, x^2-2k-2}{x} dx} x^k x - \left(\int \left(-x^k k\, \mathrm{e}^{\lambda\left(\int x^{1+k} \operatorname{arccot}(x)^n dx\right)-2k\left(\int \frac{1}{x} dx\right)-2\left(\int \frac{1}{x} dx\right)} - x^k \mathrm{e}^{\lambda\left(\int x^{1+k} \operatorname{arccot}(x)^n dx\right)-}\right.\right.\right.}{x \left(\int \left(-x^k k\, \mathrm{e}^{\lambda\left(\int x^{1+k} \operatorname{arccot}(x)^n dx\right)-2k\left(\int \frac{1}{x} dx\right)-2\left(\int \frac{1}{x} dx\right)} - x^k \mathrm{e}^{\lambda\left(\int x^{1+k} \operatorname{arccot}(x)^n dx\right)-2k\left(\int \frac{1}{x} dx\right)-2(}\right.\right.}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+\[Lambda]*ArcCot[x]^n*(x^(k+1)*y[x]-1),y[x],x,IncludeSingular
```

Not solved

18.3 problem 31

Internal problem ID [9841]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \operatorname{arccot}(x)^n y^2 - ya - ab + b^2\lambda \operatorname{arccot}(x)^n = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 114

```
dsolve(diff(y(x),x)=lambda*arccot(x)^n*y(x)^2+a*y(x)+a*b-b^2*lambda*arccot(x)^n,y(x), singsol
```

$$y(x) = -\frac{\left(\left(\int \operatorname{arccot}(x)^n \lambda\, e^{\int(-2\operatorname{arccot}(x)^n\lambda b+a)dx}dx\right) e^{\int(2\operatorname{arccot}(x)^n\lambda b-a)dx} b + c_1 e^{\int(2\operatorname{arccot}(x)^n\lambda b-a)dx} b + 1\right) e^{\int(-2\operatorname{arccot}(x)}}{c_1 + \int \operatorname{arccot}(x)^n \lambda\, e^{\int(-2\operatorname{arccot}(x)^n\lambda b+a)dx}dx}$$

✓ Solution by Mathematica

Time used: 7.443 (sec). Leaf size: 240

```
DSolve[y'[x]==\[Lambda]*ArcCot[x]^n*y[x]^2+a*y[x]+a*b-b^2*\[Lambda]*ArcCot[x]^n,y[x],x,Includ
```

$$\text{Solve}\Bigg[\int_1^x -\frac{\exp\left(-\int_1^{K[2]}\left(2b\lambda\cot^{-1}(K[1])^n-a\right)dK[1]\right)\left(-b\lambda\cot^{-1}(K[2])^n+\lambda y(x)\cot^{-1}(K[2])^n+a\right)}{an\lambda(b+y(x))}dK[2]$$
$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x\left(2b\lambda\cot^{-1}(K[1])^n-a\right)dK[1]\right)}{an\lambda(b+K[3])^2}\right.$$
$$-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]}\left(2b\lambda\cot^{-1}(K[1])^n-a\right)dK[1]\right)\left(-b\lambda\cot^{-1}(K[2])^n+\lambda K[3]\cot^{-1}(K[2])^n+a\right)}{an\lambda(b+K[3])^2}-\frac{\exp\left(-\right.}{}\right.$$

18.4 problem 32

Internal problem ID [9842]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \operatorname{arccot}(x)^n y^2 + b\lambda x^m \operatorname{arccot}(x)^n y - bm x^{m-1} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arccot(x)^n*y(x)^2-b*lambda*x^m*arccot(x)^n*y(x)+b*m*x^(m-1),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcCot[x]^n*y[x]^2-b*\[Lambda]*x^m*ArcCot[x]^n*y[x]+b*m*x^(m-1),y[x],
```

Not solved

18.5 problem 33

Internal problem ID [9843]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \operatorname{arccot}(x)^n y^2 - \beta m x^{m-1} + \lambda \beta^2 x^{2m} \operatorname{arccot}(x)^n = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arccot(x)^n*y(x)^2+beta*m*x^(m-1)-lambda*beta^2*x^(2*m)*arccot(x)^
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcCot[x]^n*y[x]^2+\[Beta]*m*x^(m-1)-\[Lambda]*\[Beta]^2*x^(2*m)*ArcC
```

Not solved

18.6 problem 34

Internal problem ID [9844]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \lambda \operatorname{arccot}(x)^n (y - a x^m - b)^2 - am x^{m-1} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)=lambda*arccot(x)^n*(y(x)-a*x^m-b)^2+a*m*x^(m-1),y(x), singsol=all)
```

$$y(x) = -\frac{(-2a x^m \operatorname{arccot}(x)^n \lambda - 2 \operatorname{arccot}(x)^n \lambda b) \operatorname{arccot}(x)^{-n}}{2\lambda} + \frac{1}{c_1 - \left(\int \operatorname{arccot}(x)^n \lambda dx\right)}$$

✓ Solution by Mathematica

Time used: 1.45 (sec). Leaf size: 44

```
DSolve[y'[x]==\[Lambda]*ArcCot[x]^n*(y[x]-a*x^m-b)^2+a*m*x^(m-1),y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{1}{-\int_1^x \lambda \cot^{-1}(K[2])^n dK[2] + c_1} + ax^m + b$$

$$y(x) \to ax^m + b$$

18.7 problem 35

Internal problem ID [9845]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - \lambda \operatorname{arccot}(x)^n y^2 - ky - \lambda b^2 x^{2k} \operatorname{arccot}(x)^n = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x)=lambda*arccot(x)^n*y(x)^2+k*y(x)+lambda*b^2*x^(2*k)*arccot(x)^n,y(x), s
```

$$y(x) = -\tan\left(-b\lambda\left(\int \frac{x^k \operatorname{arccot}(x)^n}{x}dx\right) + c_1\right) b\, x^k$$

✓ Solution by Mathematica

Time used: 1.646 (sec). Leaf size: 48

```
DSolve[x*y'[x]==\[Lambda]*ArcCot[x]^n*y[x]^2+k*y[x]+\[Lambda]*b^2*x^(2*k)*ArcCot[x]^n,y[x],x,
```

$$y(x) \to \sqrt{b^2} x^k \tan\left(\sqrt{b^2} \int_1^x \lambda \cot^{-1}(K[1])^n K[1]^{k-1} dK[1] + c_1\right)$$

18.8 problem 36

Internal problem ID [9846]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - \left(a\,x^{2m}y^2 + x^n yb + c\right) \operatorname{arccot}\left(x\right)^m + yn = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=(a*x^(2*m)*y(x)^2+b*x^n*y(x)+c)*arccot(x)^m-n*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==(a*x^(2*m)*y[x]^2+b*x^n*y[x]+c)*ArcCot[x]^m-n*y[x],y[x],x,IncludeSingularSolu
```

Not solved

19 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).

19.1 problem 1

Internal problem ID [9847]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - f(x)\,y + a^2 + af(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x)=y(x)^2+f(x)*y(x)-a^2-a*f(x),y(x), singsol=all)
```

$$y(x) = a - \frac{e^{\int f(x)dx+2ax}}{\int e^{\int f(x)dx+2ax}dx - c_1}$$

✓ Solution by Mathematica

Time used: 0.45 (sec). Leaf size: 166

```
DSolve[y'[x]==y[x]^2+f[x]*y[x]-a^2-a*f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[6]}(-2a-f(K[5]))dK[5]\right)(a+f(K[6])+y(x))}{a-y(x)}dK[6] + \int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x(-2a-f(K[5]))dK[5]\right)}{(K[7]-a)^2} - \int_1^x\left(\frac{\exp\left(-\int_1^{K[6]}(-2a-f(K[5]))dK[5]\right)(a+f(K[6])+K[7])}{(a-K[7])^2} + \frac{\exp\left(-\int_1^{K[6]}(-2a-f(K[5]))dK[5]\right)}{a-K[7]}\right.\right.\right.$$

19.2 problem 2

Internal problem ID [9848]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - f(x)\, y^2 + ya + ab + b^2 f(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 83

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*y(x)-a*b-b^2*f(x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\left(\int f(x)\, \mathrm{e}^{\int (-2bf(x)-a)dx} dx\right) \mathrm{e}^{\int (2bf(x)+a)dx} b + c_1 \mathrm{e}^{\int (2bf(x)+a)dx} b + 1\right) \mathrm{e}^{\int (-2bf(x)-a)dx}}{c_1 + \int f(x)\, \mathrm{e}^{\int (-2bf(x)-a)dx} dx}$$

✓ Solution by Mathematica

Time used: 0.607 (sec). Leaf size: 185

```
DSolve[y'[x]==f[x]*y[x]^2-a*y[x]-a*b-b^2*f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[2]} (a + 2bf(K[1]))dK[1]\right) (a + bf(K[2]) - f(K[2])y(x))}{a(b + y(x))} dK[2]\right.$$
$$+ \int_1^{y(x)} \left(\frac{\exp\left(-\int_1^x (a + 2bf(K[1]))dK[1]\right)}{a(b + K[3])^2}\right.$$
$$- \int_1^x \left(-\frac{\exp\left(-\int_1^{K[2]} (a + 2bf(K[1]))dK[1]\right) f(K[2])}{a(b + K[3])} - \frac{\exp\left(-\int_1^{K[2]} (a + 2bf(K[1]))dK[1]\right) (a + bf(K[2]}{a(b + K[3])^2}\right.$$

19.3 problem 3

Internal problem ID [9849]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - xf(x)\, y - f(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
dsolve(diff(y(x),x)=y(x)^2+x*f(x)*y(x)+f(x),y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{f(x)x^2-2}{x}dx}}{c_1 - \left(\int e^{\int \frac{f(x)x^2-2}{x}dx}dx\right)} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 0.508 (sec). Leaf size: 76

```
DSolve[y'[x]==y[x]^2+x*f[x]*y[x]+f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x + \frac{\exp\left(-\int_1^x -f(K[5])K[5]dK[5]\right)}{\int_1^x \frac{\exp\left(-\int_1^{K[6]} -f(K[5])K[5]dK[5]\right)}{K[6]^2}dK[6]+c_1}}{x^2}$$

$$y(x) \to -\frac{1}{x}$$

19.4 problem 4

Internal problem ID [9850]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - f(x)\, y^2 + a\, x^n f(x)\, y - an\, x^{n-1} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*x^n*f(x)*y(x)+a*n*x^(n-1),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*x^n*f[x]*y[x]+a*n*x^(n-1),y[x],x,IncludeSingularSolutions -> True
```

Not solved

19.5 problem 5

Internal problem ID [9851]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\, y^2 - an\, x^{n-1} + a^2 x^{2n} f(x) = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+a*n*x^(n-1)-a^2*x^(2*n)*f(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+a*n*x^(n-1)-a^2*x^(2*n)*f[x],y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

19.6 problem 6

Internal problem ID [9852]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + (1+n)\, x^n y^2 - x^{1+n} f(x)\, y + f(x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 182

```
dsolve(diff(y(x),x)=-(n+1)*x^n*y(x)^2+x^(n+1)*f(x)*y(x)-f(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-\mathrm{e}^{\int \frac{x^n f(x) x^2 - 2n - 2}{x} dx} x^n x + \int \left(-x^n n\, \mathrm{e}^{\int x^{1+n} f(x) dx - 2n\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} - x^n \mathrm{e}^{\int x^{1+n} f(x) dx - 2n\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)}\right) dx \cdots}{x \left(\int \left(-x^n n\, \mathrm{e}^{\int x^{1+n} f(x) dx - 2n\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)} - x^n \mathrm{e}^{\int x^{1+n} f(x) dx - 2n\left(\int \frac{1}{x} dx\right) - 2\left(\int \frac{1}{x} dx\right)}\right) dx + c_1\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(n+1)*x^n*y[x]^2+x^(n+1)*f[x]*y[x]-f[x],y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

19.7 problem 7

Internal problem ID [9853]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - f(x)\, y^2 - yn - f(x)\, x^{2n}a = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 35

```
dsolve(x*diff(y(x),x)=f(x)*y(x)^2+n*y(x)+a*x^(2*n)*f(x),y(x), singsol=all)
```

$$y(x) = -\tan\left(-\sqrt{a}\left(\int \frac{x^n f(x)}{x} dx\right) + c_1\right)\sqrt{a}\, x^n$$

✓ Solution by Mathematica

Time used: 0.369 (sec). Leaf size: 41

```
DSolve[x*y'[x]==f[x]*y[x]^2+n*y[x]+a*x^(2*n)*f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{a}x^n \tan\left(\sqrt{a}\int_1^x f(K[1])K[1]^{n-1}dK[1] + c_1\right)$$

19.8 problem 8

Internal problem ID [9854]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - x^{2n} f(x)\, y^2 - (f(x)\, x^n a - n)\, y - f(x)\, b = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 64

```
dsolve(x*diff(y(x),x)=x^(2*n)*f(x)*y(x)^2+(a*x^n*f(x)-n)*y(x)+b*f(x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\tanh\left(\frac{\sqrt{a^4-4a^2b}\left(a\left(\int \frac{x^n f(x)}{x} dx\right)+c_1\right)}{2a^2}\right)\sqrt{a^4-4a^2b}+a^2\right) x^{-n}}{2a}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==x^(2*n)*f[x]*y[x]^2+(a*x^n*f[x]-n)*y[x]+b*f[x],y[x],x,IncludeSingularSolution
```

Timed out

19.9 problem 9

Internal problem ID [9855]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - f(x)\, y^2 - g(x)\, y + f(x)\, a^2 + ag(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+g(x)*y(x)-a^2*f(x)-a*g(x),y(x), singsol=all)
```

$$y(x) = a - \frac{e^{\int g(x)dx + 2a\left(\int f(x)dx\right)}}{\int e^{\int g(x)dx + 2a\left(\int f(x)dx\right)} f(x)\, dx - c_1}$$

✓ Solution by Mathematica

Time used: 0.716 (sec). Leaf size: 201

```
DSolve[y'[x]==f[x]*y[x]^2+g[x]*y[x]-a^2*f[x]-a*g[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\Bigg[\int_1^x -\frac{\exp\left(-\int_1^{K[6]}(-2af(K[5]) - g(K[5]))dK[5]\right)(af(K[6]) + y(x)f(K[6]) + g(K[6]))}{a - y(x)} dK[6] + \int_1^{y(x)} \left(-\int_1^x \left(-\frac{\exp\left(-\int_1^{K[6]}(-2af(K[5]) - g(K[5]))dK[5]\right) f(K[6])}{a - K[7]} - \frac{\exp\left(-\int_1^{K[6]}(-2af(K[5]) - g(\right.}{}\right.\right.$$
$$\left. - \frac{\exp\left(-\int_1^x(-2af(K[5]) - g(K[5]))dK[5]\right)}{(K[7] - a)^2}\right) dK[7] = c_1, y(x)\Bigg]$$

19.10 problem 10

Internal problem ID [9856]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\, y^2 - g(x)\, y - an\, x^{n-1} + a\, x^n g(x) + a^2 x^{2n} f(x) = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+g(x)*y(x)+a*n*x^(n-1)-a*x^n*g(x)-a^2*f(x)*x^(2*n),y(x), sings
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+g[x]*y[x]+a*n*x^(n-1)-a*x^n*g[x]-a^2*f[x]*x^(2*n),y[x],x,IncludeSin
```

Not solved

19.11 problem 11

Internal problem ID [9857]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\,y^2 + a\,x^n g(x)\,y - an\,x^{n-1} - a^2x^{2n}(g(x) - f(x)) = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*x^n*g(x)*y(x)+a*n*x^(n-1)+a^2*x^(2*n)*(g(x)-f(x)),y(x), sin
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*x^n*g[x]*y[x]+a*n*x^(n-1)+a^2*x^(2*n)*(g[x]-f[x]),y[x],x,IncludeS
```

Not solved

19.12 problem 12

Internal problem ID [9858]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{\lambda x} y^2 - a\,\mathrm{e}^{\lambda x} f(x)\,y - \lambda f(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 139

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*y(x)^2+a*exp(lambda*x)*f(x)*y(x)+lambda*f(x),y(x), singso
```

$$y(x) = -\frac{\mathrm{e}^{-\lambda x} c_1 \mathrm{e}^{-\lambda x + a\left(\int f(x)\mathrm{e}^{\lambda x}dx\right)}}{\lambda a\left(\left(\int \frac{\mathrm{e}^{-\lambda x + a\left(\int f(x)\mathrm{e}^{\lambda x}dx\right)}}{\lambda}dx\right)c_1 + 1\right)} - \frac{\mathrm{e}^{-\lambda x}\left(\left(\int \frac{\mathrm{e}^{-\lambda x + a\left(\int f(x)\mathrm{e}^{\lambda x}dx\right)}}{\lambda}dx\right)c_1\lambda^2 + \lambda^2\right)}{\lambda a\left(\left(\int \frac{\mathrm{e}^{-\lambda x + a\left(\int f(x)\mathrm{e}^{\lambda x}dx\right)}}{\lambda}dx\right)c_1 + 1\right)}$$

✓ Solution by Mathematica

Time used: 2.065 (sec). Leaf size: 118

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*y[x]^2+a*Exp[\[Lambda]*x]*f[x]*y[x]+\[Lambda]*f[x],y[x],x,In
```

$$y(x) \to \frac{\lambda e^{-2\lambda x}\left(-e^{\lambda x} - \frac{\exp\left(-\int_1^{e^{x\lambda}} -\frac{af\left(\frac{\log(K[5])}{\lambda}\right)}{\lambda}dK[5]\right)}{\int_1^{e^{x\lambda}} \frac{\exp\left(-\int_1^{K[6]} -\frac{af\left(\frac{\log(K[5])}{\lambda}\right)}{\lambda}dK[5]\right)}{K[6]^2}dK[6] + c_1}\right)}{a}$$

$$y(x) \to -\frac{\lambda e^{\lambda(-x)}}{a}$$

19.13 problem 13

Internal problem ID [9859]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\,y^2 + a\,\mathrm{e}^{\lambda x} f(x)\,y - a\lambda\,\mathrm{e}^{\lambda x} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*exp(lambda*x)*f(x)*y(x)+a*lambda*exp(lambda*x),y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*Exp[\[Lambda]*x]*f[x]*y[x]+a*\[Lambda]*Exp[\[Lambda]*x],y[x],x,In
```

Not solved

19.14 problem 14

Internal problem ID [9860]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\, y^2 - a\lambda\, e^{\lambda x} + f(x)\, e^{2\lambda x} a^2 = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+a*lambda*exp(lambda*x)-a^2*exp(2*lambda*x)*f(x),y(x), singsol
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+a*\[Lambda]*Exp[\[Lambda]*x]-a^2*Exp[2*\[Lambda]*x]*f[x],y[x],x,Inc
```

Not solved

19.15 problem 15

Internal problem ID [9861]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - f(x)\, y^2 - y\lambda - f(x)\, \mathrm{e}^{2\lambda x} a^2 = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+lambda*y(x)+a^2*exp(2*lambda*x)*f(x),y(x), singsol=all)
```

$$y(x) = -\tan\left(-a\left(\int f(x)\, \mathrm{e}^{\lambda x} dx\right) + c_1\right) a\, \mathrm{e}^{\lambda x}$$

✓ Solution by Mathematica

Time used: 0.399 (sec). Leaf size: 47

```
DSolve[y'[x]==f[x]*y[x]^2+\[Lambda]*y[x]+a^2*Exp[2*\[Lambda]*x]*f[x],y[x],x,IncludeSingularSo
```

$$y(x) \to \sqrt{a^2} e^{\lambda x} \tan\left(\sqrt{a^2} \int_1^x e^{\lambda K[1]} f(K[1]) dK[1] + c_1\right)$$

19.16 problem 16

Internal problem ID [9862]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\, y^2 + f(x) \left(\mathrm{e}^{\lambda x} a + b\right) y - a\lambda\, \mathrm{e}^{\lambda x} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-f(x)*(a*exp(lambda*x)+b)*y(x)+a*lambda*exp(lambda*x),y(x), si
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-f[x]*(a*Exp[\[Lambda]*x]+b)*y[x]+a*\[Lambda]*Exp[\[Lambda]*x],y[x],
```

Not solved

19.17 problem 17

Internal problem ID [9863]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - e^{\lambda x} f(x)\, y^2 - (a f(x) - \lambda)\, y - b\, e^{-\lambda x} f(x) = 0$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 68

```
dsolve(diff(y(x),x)=exp(lambda*x)*f(x)*y(x)^2+(a*f(x)-lambda)*y(x)+b*exp(-lambda*x)*f(x),y(x)
```

$$y(x) = -\frac{\left(e^{\lambda x} e^{-\lambda x} a^2 + \tanh\left(\frac{\sqrt{a^4-4a^2b}\,\left(a\left(\int f(x)dx\right)+c_1\right)}{2a^2}\right)\sqrt{a^4-4a^2b}\right) e^{-\lambda x}}{2a}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==Exp[\[Lambda]*x]*f[x]*y[x]^2+(a*f[x]-\[Lambda])*y[x]+b*Exp[-\[Lambda]*x]*f[x],y
```

Timed out

19.18 problem 18

Internal problem ID [9864]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\, y^2 - g(x)\, y - a\lambda\, e^{\lambda x} + a\, e^{\lambda x} g(x) + f(x)\, e^{2\lambda x} a^2 = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+g(x)*y(x)+a*lambda*exp(lambda*x)-a*exp(lambda*x)*g(x)-a^2*exp
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+g[x]*y[x]+a*\[Lambda]*Exp[\[Lambda]*x]-a*Exp[\[Lambda]*x]*g[x]-a^2*
```

Not solved

19.19 problem 19

Internal problem ID [9865]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\,y^2 + a\,\mathrm{e}^{\lambda x} g(x)\,y - a\lambda\,\mathrm{e}^{\lambda x} - a^2 \mathrm{e}^{2\lambda x}(g(x) - f(x)) = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*exp(lambda*x)*g(x)*y(x)+a*lambda*exp(lambda*x)+a^2*exp(2*la
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*Exp[\[Lambda]*x]*g[x]*y[x]+a*\[Lambda]*Exp[\[Lambda]*x]+a^2*Exp[2
```

Not solved

19.20 problem 20

Internal problem ID [9866]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\, y^2 - 2a\lambda x\, e^{\lambda x^2} + a^2 f(x)\, e^{2\lambda x^2} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+2*a*lambda*x*exp(lambda*x^2)-a^2*f(x)*exp(2*lambda*x^2),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+2*a*\[Lambda]*x*Exp[\[Lambda]*x^2]-a^2*f[x]*Exp[2*\[Lambda]*x^2],y[
```

Not solved

19.21 problem 21

Internal problem ID [9867]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\, y^2 - y\lambda x - f(x)\, e^{\lambda x} a = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+lambda*x*y(x)+a*f(x)*exp(lambda*x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+\[Lambda]*x*y[x]+a*f[x]*Exp[\[Lambda]*x],y[x],x,IncludeSingularSolu
```

Not solved

19.22 problem 22

Internal problem ID [9868]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\, y^2 + a \tanh(\lambda x)^2 (a f(x) + \lambda) - \lambda a = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*tanh(lambda*x)^2*(a*f(x)+lambda)+a*lambda,y(x), singsol=all
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*Tanh[\[Lambda]*x]^2*(a*f[x]+\[Lambda])+a*\[Lambda],y[x],x,Include
```

Not solved

19.23 problem 23

Internal problem ID [9869]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\, y^2 + a \coth(\lambda x)^2 \,(a f(x) + \lambda) - \lambda a = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*coth(lambda*x)^2*(a*f(x)+lambda)+a*lambda,y(x), singsol=all
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*Coth[\[Lambda]*x]^2*(a*f[x]+\[Lambda])+a*\[Lambda],y[x],x,Include
```

Not solved

19.24 problem 24

Internal problem ID [9870]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x) y^2 + f(x) a^2 - a\lambda \sinh(\lambda x) + f(x) \sinh(\lambda x)^2 a^2 = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a^2*f(x)+a*lambda*sinh(lambda*x)-a^2*f(x)*sinh(lambda*x)^2,y(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a^2*f[x]+a*\[Lambda]*Sinh[\[Lambda]*x]-a^2*f[x]*Sinh[\[Lambda]*x]^2
```

Not solved

19.25 problem 25

Internal problem ID [9871]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - f(x)\,y^2 - a + a^2 f(x)\ln(x)^2 = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=f(x)*y(x)^2+a-a^2*f(x)*(ln(x))^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==f[x]*y[x]^2+a-a^2*f[x]*(Log[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

19.26 problem 26

Internal problem ID [9872]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y'x - f(x)\left(y + a\ln\left(x\right)\right)^2 + a = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x*diff(y(x),x)=f(x)*(y(x)+a*ln(x))^2-a,y(x), singsol=all)
```

$$y(x) = -a\ln\left(x\right) + \frac{1}{c_1 - \left(\int \frac{f(x)}{x}dx\right)}$$

✓ Solution by Mathematica

Time used: 0.307 (sec). Leaf size: 42

```
DSolve[x*y'[x]==f[x]*(y[x]+a*Log[x])^2-a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -a\log(x) + \frac{1}{-\int_1^x \frac{f(K[2])}{K[2]}dK[2] + c_1}$$

$$y(x) \to -a\log(x)$$

19.27 problem 27

Internal problem ID [9873]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\, y^2 + ax \ln(x)\, f(x)\, y - a \ln(x) - a = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*x*ln(x)*f(x)*y(x)+a*ln(x)+a,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*x*Log[x]*f[x]*y[x]+a*Log[x]+a,y[x],x,IncludeSingularSolutions ->
```

Not solved

19.28 problem 28

Internal problem ID [9874]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + a\ln(x)\, y^2 - af(x)\left(\ln(x)\, x - x\right) y + f(x) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 348

```
dsolve(diff(y(x),x)=-a*ln(x)*y(x)^2+a*f(x)*(x*ln(x)-x)*y(x)-f(x),y(x), singsol=all)
```

$$y(x) = \frac{x\,e^{\int \frac{f(x)\ln(x)^2 a\,x^2-2f(x)\ln(x)a\,x^2+a\,x^2 f(x)-2\ln(x)}{x(-1+\ln(x))}dx}\ln(x) - x\,e^{\int \frac{f(x)\ln(x)^2 a\,x^2-2f(x)\ln(x)a\,x^2+a\,x^2 f(x)-2\ln(x)}{x(-1+\ln(x))}dx} - c_1 a +}{ax\left(-\ln(x)\,c_1 a + \ln(x)\left(\int \ln(x)\,e^{a\left(\int \frac{x f(x)\ln(x)^2}{-1+\ln(x)}dx\right)-2a\left(\int \frac{x f(x)\ln(x)}{-1+\ln(x)}dx\right)+a\left(\int \frac{x f(x)}{-1+\ln(x)}dx\right)-2\left(\int \frac{\ln(x)}{x(-1+\ln(x))}dx\right)}dx\right) + \right.}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-a*Log[x]*y[x]^2+a*f[x]*(x*Log[x]-x)*y[x]-f[x],y[x],x,IncludeSingularSolutions
```

Not solved

19.29 problem 29

Internal problem ID [9875]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \sin(\lambda x)\, y^2 - f(x) \cos(\lambda x)\, y + f(x) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 118

```
dsolve(diff(y(x),x)=lambda*sin(lambda*x)*y(x)^2+f(x)*cos(lambda*x)*y(x)-f(x),y(x), singsol=al
```

$$y(x) = \frac{c_1 e^{\int \frac{\sqrt{\frac{1}{2}-\frac{\cos(2\lambda x)}{2}}\left(f(x)\sin(\lambda x)\cos(\lambda x)+2\sqrt{\frac{1}{2}-\frac{\cos(2\lambda x)}{2}}\,\lambda\tan(\lambda x)\right)}{\sin(\lambda x)^2}dx}}{\left(\int -e^{\int \frac{\sqrt{\frac{1}{2}-\frac{\cos(2\lambda x)}{2}}\left(f(x)\sin(\lambda x)\cos(\lambda x)+2\sqrt{\frac{1}{2}-\frac{\cos(2\lambda x)}{2}}\,\lambda\tan(\lambda x)\right)}{\sin(\lambda x)^2}dx}\sin(\lambda x)\lambda dx\right)c_1 + 1} + \frac{1}{\cos(\lambda x)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Sin[\[Lambda]*x]*y[x]^2+f[x]*Cos[\[Lambda]*x]*y[x]-f[x],y[x],x,Includ
```

Not solved

19.30 problem 30

Internal problem ID [9876]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\,y^2 + f(x)\,a^2 - a\lambda \sin(\lambda x) - a^2 f(x) \sin(\lambda x)^2 = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a^2*f(x)+a*lambda*sin(lambda*x)+a^2*f(x)*sin(lambda*x)^2,y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a^2*f[x]+a*\[Lambda]*Sin[\[Lambda]*x]+a^2*f[x]*Sin[\[Lambda]*x]^2,y
```

Not solved

19.31 problem 31

Internal problem ID [9877]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\, y^2 + f(x)\, a^2 - \lambda \cos(\lambda x)\, a - f(x)\, a^2 \cos(\lambda x)^2 = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a^2*f(x)+a*lambda*cos(lambda*x)+a^2*f(x)*cos(lambda*x)^2,y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a^2*f[x]+a*\[Lambda]*cos[\[Lambda]*x]+a^2*f[x]*Cos[\[Lambda]*x]^2,y
```

Not solved

19.32 problem 32

Internal problem ID [9878]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\, y^2 + a \tan(\lambda x)^2 (a f(x) - \lambda) - \lambda a = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*tan(lambda*x)^2*(a*f(x)-lambda)+a*lambda,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*Tan[\[Lambda]*x]^2*(a*f[x]-\[Lambda])+a*\[Lambda],y[x],x,IncludeS
```

Not solved

19.33 problem 33

Internal problem ID [9879]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\,y^2 + a\cot\left(\lambda x\right)^2\left(af(x) - \lambda\right) - \lambda a = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*cot(lambda*x)^2*(a*f(x)-lambda)+a*lambda,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*Cot[\[Lambda]*x]^2*(a*f[x]-\[Lambda])+a*\[Lambda],y[x],x,IncludeS
```

Not solved

20 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.

20.1 problem 34

Internal problem ID [9880]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 + f(x)^2 - f'(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)=y(x)^2-f(x)^2+diff(f(x),x),y(x), singsol=all)
```

$$y(x) = f(x) + \frac{e^{\int 2f(x)dx}}{c_1 - \left(\int e^{\int 2f(x)dx}dx\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-f[x]^2+f'[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

20.2 problem 35

Internal problem ID [9881]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\,y^2 + f(x)\,g(x)\,y - g'(x) = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-f(x)*g(x)*y(x)+diff(g(x),x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-f[x]*g[x]*y[x]+g'[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

20.3 problem 36

Internal problem ID [9882]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + f'(x)\, y^2 - f(x)\, g(x)\, y + g(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 102

```
dsolve(diff(y(x),x)=-diff(f(x),x)*y(x)^2+f(x)*g(x)*y(x)-g(x),y(x), singsol=all)
```

$$y(x) = \frac{f(x)\, e^{\int \frac{g(x)f(x)^2 - 2\frac{d}{dx}f(x)}{f(x)}dx} + \int \left(\frac{d}{dx}f(x)\right) e^{\int g(x)f(x)dx - 2\left(\int \frac{\frac{d}{dx}f(x)}{f(x)}dx\right)} dx - c_1}{f\,(x)\left(\int \left(\frac{d}{dx}f\,(x)\right) e^{\int g(x)f(x)dx - 2\left(\int \frac{\frac{d}{dx}f(x)}{f(x)}dx\right)} dx - c_1\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-f'[x]*y[x]^2+f[x]*g[x]*y[x]-g[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

20.4 problem 37

Internal problem ID [9883]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - g(x)\left(y - f(x)\right)^2 - f'(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=g(x)*(y(x)-f(x))^2+diff(f(x),x),y(x), singsol=all)
```

$$y(x) = f(x) + \frac{1}{c_1 - \left(\int g(x)\, dx\right)}$$

✓ Solution by Mathematica

Time used: 0.23 (sec). Leaf size: 31

```
DSolve[y'[x]==g[x]*(y[x]-f[x])^2+f'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to f(x) + \frac{1}{-\int_1^x g(K[2])dK[2] + c_1}$$

$$y(x) \to f(x)$$

20.5 problem 38

Internal problem ID [9884]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \frac{f'(x)\, y^2}{g(x)} + \frac{g'(x)}{f(x)} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 57

```
dsolve(diff(y(x),x)=diff(f(x),x)/g(x)*y(x)^2-diff(g(x),x)/f(x),y(x), singsol=all)
```

$$y(x) = -\frac{g(x)\left(\int \frac{\frac{d}{dx}f(x)}{g(x)f(x)^2}dx\right) f(x) + c_1 f(x)\, g(x) + 1}{f(x)^2 \left(\int \frac{\frac{d}{dx}f(x)}{g(x)f(x)^2}dx + c_1\right)}$$

✓ Solution by Mathematica

Time used: 0.224 (sec). Leaf size: 160

```
DSolve[y'[x]==f'[x]/g[x]*y[x]^2-g'[x]/f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\Bigg[\int_1^{y(x)} \Bigg(\frac{1}{(g(x)+f(x)K[2])^2} - \int_1^x \left(\frac{2(f(K[1])K[2]^2 f'(K[1]) - g(K[1])g'(K[1]))}{g(K[1])(g(K[1])+f(K[1])K[2])^3} - \frac{2K[2]f'(K[1])}{g(K[1])(g(K[1])+f(K[1])K[2])^2}\right) dK[1]\Bigg) dK[2] + \int_1^x -\frac{f(K[1])y(x)^2 f'(K[1]) - g(K[1])g'(K[1])}{f(K[1])g(K[1])(g(K[1])+f(K[1])y(x))^2} dK[1] = c_1, y(x)\Bigg]$$

20.6 problem 39

Internal problem ID [9885]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$f(x)^2 y' - f'(x) y^2 + g(x) (y - f(x)) = 0$$

✗ Solution by Maple

```
dsolve(f(x)^2*diff(y(x),x)-diff(f(x),x)*y(x)^2+g(x)*(y(x)-f(x))=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x]^2*y'[x]-f'[x]*y[x]^2+g[x]*(y[x]-f[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

20.7 problem 40

Internal problem ID [9886]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f'(x)\, y^2 - a\,\mathrm{e}^{\lambda x} f(x)\, y - \mathrm{e}^{\lambda x} a = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 110

```
dsolve(diff(y(x),x)=diff(f(x),x)*y(x)^2+a*exp(lambda*x)*f(x)*y(x)+a*exp(lambda*x),y(x), sings
```

$$y(x) = -\frac{f(x)\,\mathrm{e}^{\int \frac{f(x)^2 \mathrm{e}^{\lambda x} a - 2\frac{d}{dx}f(x)}{f(x)}dx} + \int \left(\frac{d}{dx}f(x)\right) \mathrm{e}^{a\left(\int f(x)\mathrm{e}^{\lambda x}dx\right) - 2\left(\int \frac{\frac{d}{dx}f(x)}{f(x)}dx\right)} dx + c_1}{f(x)\left(\int \left(\frac{d}{dx}f(x)\right) \mathrm{e}^{a\left(\int f(x)\mathrm{e}^{\lambda x}dx\right) - 2\left(\int \frac{\frac{d}{dx}f(x)}{f(x)}dx\right)} dx + c_1\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f'[x]*y[x]^2+a*Exp[\[Lambda]*x]*f[x]*y[x]+a*Exp[\[Lambda]*x],y[x],x,IncludeSing
```

Not solved

20.8 problem 41

Internal problem ID [9887]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - f(x)\, y^2 - y g'(x) - a f(x)\, \mathrm{e}^{2g(x)} = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+diff(g(x),x)*y(x)+a*f(x)*exp(2*g(x)),y(x), singsol=all)
```

$$y(x) = -\tan\left(-\sqrt{a}\left(\int f(x)\, \mathrm{e}^{g(x)} dx\right) + c_1\right)\sqrt{a}\, \mathrm{e}^{g(x)}$$

✓ Solution by Mathematica

Time used: 0.421 (sec). Leaf size: 41

```
DSolve[y'[x]==f[x]*y[x]^2+g'[x]*y[x]+a*f[x]*Exp[2*g[x]],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \sqrt{a} e^{g(x)} \tan\left(\sqrt{a} \int_1^x e^{g(K[1])} f(K[1]) dK[1] + c_1\right)$$

20.9 problem 42

Internal problem ID [9888]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 + \frac{f''(x)}{f(x)} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve(diff(y(x),x)=y(x)^2-diff(f(x),x$2)/f(x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\int \frac{1}{f(x)^2}dx\right)\left(\frac{d}{dx}f(x)\right) + c_1\left(\frac{d}{dx}f(x)\right)}{\left(\int \frac{1}{f(x)^2}dx + c_1\right)f(x)} - \frac{1}{\left(\int \frac{1}{f(x)^2}dx + c_1\right)f(x)^2}$$

✓ Solution by Mathematica

Time used: 0.236 (sec). Leaf size: 132

```
DSolve[y'[x]==y[x]^2-f''[x]/f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{1}{(f(x)K[2]+f'(x))^2} - \int_1^x\left(\frac{2(f(K[1])K[2]^2 - f''(K[1]))}{(f(K[1])K[2]+f'(K[1]))^3} - \frac{2K[2]}{(f(K[1])K[2]+f'(K[1]))^2}\right)dK[1]\right)dK[2] + \int_1^x -\frac{f(K[1])y(x)^2 - f''(K[1])}{f(K[1])\left(f(K[1])y(x)+f'(K[1])\right)^2}dK[1] = c_1, y(x)\right]$$

21 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations

21.1 problem 1

Internal problem ID [9889]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a^2 f(ax + b) = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+a^2*f(a*x+b),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a^2*f[a*x+b],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.2 problem 2

Internal problem ID [9890]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \frac{f\left(\frac{1}{x}\right)}{x^4} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+1/x^4*f(1/x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+1/x^4*f[1/x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.3 problem 3

Internal problem ID [9891]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \frac{f\left(\frac{ax+b}{cx+d}\right)}{(cx+d)^4} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+1/(c*x+d)^4*f((a*x+b)/(c*x+d)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+1/(c*x+d)^4*f[(a*x+b)/(c*x+d)],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.4 problem 4

Internal problem ID [9892]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^2y' - x^4 f(x)\, y^2 - 1 = 0}$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)=x^4*f(x)*y(x)^2+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==x^4*f[x]*y[x]^2+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.5 problem 5

Internal problem ID [9893]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$x^2y' - x^4y^2 - x^{2n}f(x^n a + b) + \frac{n^2}{4} - \frac{1}{4} = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)=x^4*y(x)^2+x^(2*n)*f(a*x^n+b)+1/4*(1-n^2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==x^4*y[x]^2+x^(2*n)*f[a*x^n+b]+1/4*(1-n^2),y[x],x,IncludeSingularSolutions -
```

Not solved

21.6 problem 6

Internal problem ID [9894]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - f(x)\, y^2 - g(x)\, y - h(x) = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+g(x)*y(x)+h(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+g[x]*y[x]+h[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.7 problem 7

Internal problem ID [9895]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - e^{2\lambda x} f\left(e^{\lambda x}\right) + \frac{\lambda^2}{4} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+exp(2*lambda*x)*f(exp(lambda*x))-1/4*lambda^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+Exp[2*\[Lambda]*x]*f[Exp[\[Lambda]*x]]-1/4*\[Lambda]^2,y[x],x,IncludeSin
```

Not solved

21.8 problem 8

Internal problem ID [9896]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 + \frac{\lambda^2}{4} - \frac{e^{2\lambda x} f\left(\frac{e^{\lambda x} a+b}{e^{\lambda x} c+d}\right)}{\left(e^{\lambda x} c + d\right)^4} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2-lambda^2/4+exp(2*lambda*x)/(c*exp(lambda*x)+d)^4*f((a*exp(lambda*x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-\[Lambda]^2/4+Exp[2*\[Lambda]*x]/(c*Exp[\[Lambda]*x]+d)^4*f[(a*Exp[\[Lam
```

Not solved

21.9 problem 9

Internal problem ID [9897]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 + \lambda^2 - \frac{f(\coth(\lambda x))}{\sinh(\lambda x)^4} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2-lambda^2+sinh(lambda*x)^(-4)*f(coth(lambda*x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-\[Lambda]^2+Sinh[\[Lambda]*x]^(-4)*f[Coth[\[Lambda]*x]],y[x],x,IncludeSi
```

Not solved

21.10 problem 10

Internal problem ID [9898]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 + \lambda^2 - \frac{f(\tanh(\lambda x))}{\cosh(\lambda x)^4} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2-lambda^2+cosh(lambda*x)^(-4)*f(tanh(lambda*x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-\[Lambda]^2+Cosh[\[Lambda]*x]^(-4)*f[Tanh[\[Lambda]*x]],y[x],x,IncludeSi
```

Not solved

21.11 problem 11

Internal problem ID [9899]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$x^2y' - y^2x^2 - f(a\ln(x) + b) - \frac{1}{4} = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)=x^2*y(x)^2+f(a*ln(x)+b)+1/4,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==x^2*y[x]^2+f[a*Log[x]+b]+1/4,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.12 problem 12

Internal problem ID [9900]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda^2 - \frac{f(\cot(\lambda x))}{\sin(\lambda x)^4} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+sin(lambda*x)^(-4)*f(cot(lambda*x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+Sin[\[Lambda]*x]^(-4)*f[Cot[\[Lambda]*x]],y[x],x,IncludeSing
```

Not solved

21.13 problem 13

Internal problem ID [9901]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda^2 - \frac{f(\tan(\lambda x))}{\cos(\lambda x)^4} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+cos(lambda*x)^(-4)*f(tan(lambda*x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+Cos[\[Lambda]*x]^(-4)*f[Tan[\[Lambda]*x]],y[x],x,IncludeSing
```

Not solved

21.14 problem 14

Internal problem ID [9902]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda^2 - \frac{f\left(\frac{\sin(\lambda x+a)}{\sin(\lambda x+b)}\right)}{\sin(\lambda x+b)^4} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+sin(lambda*x+b)^(-4)*f(sin(lambda*x+a)/sin(lambda*x+b)),y
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+Sin[\[Lambda]*x+b]^(-4)*f[Sin[\[Lambda]*x+a]/Sin[\[Lambda]*x
```

Not solved

22 Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions

22.1 problem 1

Internal problem ID [9903]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'y - y - A = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(y(x)*diff(y(x),x)-y(x)=A,y(x), singsol=all)
```

$$y(x) = -A\left(\mathrm{LambertW}\left(-\frac{\mathrm{e}^{-1-\frac{c_1}{A}-\frac{x}{A}}}{A}\right)+1\right)$$

✓ Solution by Mathematica

Time used: 60.021 (sec). Leaf size: 28

```
DSolve[y[x]*y'[x]-y[x]==A,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -A\left(1+W\left(-\frac{e^{-\frac{A+x+c_1}{A}}}{A}\right)\right)$$

22.2 problem 2

Internal problem ID [9904]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{y'y - y - xA - B = 0}$$

✓ Solution by Maple

Time used: 1.0 (sec). Leaf size: 119

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x+B,y(x), singsol=all)
```

$$y(x) = -\frac{\operatorname{RootOf}\left(\_Z^2 - A + e^{\operatorname{RootOf}\left((xA+B)^2\left(4\tanh\left(-\frac{\_Z\sqrt{4A+1}}{2}+\ln(xA+B)\sqrt{4A+1}+c_1\sqrt{4A+1}\right)^2 A+\tanh\left(-\frac{\_Z\sqrt{4A+1}}{2}+\ln(xA+B)\sqrt{4 \right.\right.\right.}\right.}{A}$$

✓ Solution by Mathematica

Time used: 0.115 (sec). Leaf size: 88

```
DSolve[y[x]*y'[x]-y[x]==A*x+B,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{\frac{2\arctan\left(\frac{\frac{2Ay(x)}{Ax+B}-1}{\sqrt{-4A-1}}\right)}{\sqrt{-4A-1}}+\log\left(-\frac{Ay(x)^2}{(Ax+B)^2}+\frac{y(x)}{Ax+B}+1\right)}{2A}=\frac{\log(Ax+B)}{A}+c_1,y(x)\right]$$

22.3 problem 3

Internal problem ID [9905]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{2x}{9} - A - \frac{B}{\sqrt{x}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 89

```
dsolve(y(x)*diff(y(x),x)-y(x)=-2/9*x+A+B*x^(-1/2),y(x), singsol=all)
```

$$y(x) = -\frac{A\left(9A\sqrt{x} - 2x^{\frac{3}{2}} + 9B\right)}{3\left(A\sqrt{x} + \mathrm{RootOf}\left(9A^3\left(\int^{-Z} \frac{1}{-2\_a^3B^2 + 9\_aA^3 - 9A^3} d\_a\right) + \int -\frac{9A}{2(9xA - 2x^2 + 9B\sqrt{x})} dx + c_1\right) B\right)}$$

✓ Solution by Mathematica

Time used: 6.249 (sec). Leaf size: 415

```
DSolve[y[x]*y'[x]-y[x]==-2/9*x+A+B*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\Bigg[6\text{RootSum}\Big[8\#1^6 - 72\#1^4A - 36\#1^4y(x) - 72\#1^3B + 162\#1^2A^2 + 162\#1^2Ay(x) + 54\#1^2y(x)^2 + 324\#1AB + 162\#1By(x) - 81Ay(x)^2 + 162B^2 - 27y(x)^3\&, \frac{-2\#1^3\log\left(\sqrt{x}-\#1\right) + 9\#1A\log\left(\sqrt{x}-\#1\right) + 9B\log\left(\sqrt{x}-\#1\right) + 9\#1y(x)\log\left(\sqrt{x}-\right.}{8\#1^5 - 48\#1^3A - 24\#1^3y(x) - 36\#1^2B + 54\#1A^2 + 54\#1Ay(x) + 18\#1y(x)^2 + 54AB + 27} + \int_1^{y(x)}\left(\frac{162K[1]}{8x^3 - 72Ax^2 - 36K[1]x^2 - 72Bx^{3/2} + 162A^2x + 54K[1]^2x + 162AK[1]x + 324AB\sqrt{x} + 162BK[} + \frac{162K[1]}{-8x^3 + 72Ax^2 + 36K[1]x^2 + 72Bx^{3/2} - 162A^2x - 54K[1]^2x - 162AK[1]x - 324AB\sqrt{x} - 162BK[1]\sqrt{x}}\right.$$

22.4 problem 4

Internal problem ID [9906]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y - 2A\left(\sqrt{x} + 4A + \frac{3A^2}{\sqrt{x}}\right) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 109

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*A*(x^(1/2)+4*A+3*A^2*x^(-1/2)),y(x), singsol=all)
```

$$c_1 + \frac{4\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{A^2}{y(x)}}\,(3A+\sqrt{x})}{\sqrt{\frac{-3A^2-4A\sqrt{x}-x+y(x)}{y(x)}}\,A}\right)\sqrt{-\frac{A^2}{y(x)}} - \sqrt{2}\,\sqrt{\frac{-6A^2-8A\sqrt{x}-2x+2y(x)}{y(x)}}}{\sqrt{-\frac{A^2}{y(x)}}} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*A*(x^(1/2)+4*A+3*A^2*x^(-1/2)),y[x],x,IncludeSingularSolutions -> T
```

Not solved

22.5 problem 5

Internal problem ID [9907]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$
y'y - y - xA - \frac{B}{x} + \frac{B^2}{x^3} = 0
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 171

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x+B/x-B^2*x^(-3),y(x), singsol=all)
```

$$
c_1 + \frac{\left(-y(x)\,B\,x^2 - B^2 x\right)\left(\int^{-\frac{x^2}{2xy(x)+2B}} \frac{e^{\frac{2\,\operatorname{arctanh}\left(\frac{4A\_a-1}{\sqrt{4A+1}}\right)}{\sqrt{4A+1}}}\left(4A\_a^2-2\_a-1\right)}{\_a^2}d\_a\right) + 2y(x)\,e^{-\frac{2\,\operatorname{arctanh}\left(\frac{2A\,x^2+xy(x)+B}{\sqrt{4A+1}\,(xy(x)+B)}\right)}{\sqrt{4A+1}}}\left(\right.}{x\left(xy\left(x\right)+B\right)} = 0
$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*x+B/x-B^2*x^(-3),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.6 problem 6

Internal problem ID [9908]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - y - A\,x^{k-1} + kB\,x^k - k\,B^2x^{2k-1} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^(k-1)-k*B*x^k+k*B^2*x^(2*k-1),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*x^(k-1)-k*B*x^k+k*B^2*x^(2*k-1),y[x],x,IncludeSingularSolutions ->
```

Not solved

22.7 problem 7

Internal problem ID [9909]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - y - \frac{A}{x} + \frac{A^2}{x^3} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 117

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^(-1)-A^2*x^(-3),y(x), singsol=all)
```

$$y(x) = \frac{\left(x^2 c_1 - A\,\mathrm{e}^{\mathrm{RootOf}\left(2\_ZA\,\mathrm{e}^{2\_Z} - \mathrm{e}^{2\_Z}x^2 + 2c_1\mathrm{e}^{\_Z}x^2 - c_1^2x^2 - 2A\,\mathrm{e}^{2\_Z} + 2Ac_1\mathrm{e}^{\_Z}\right)}\right)\mathrm{e}^{-\mathrm{RootOf}\left(2\_ZA\,\mathrm{e}^{2\_Z} - \mathrm{e}^{2\_Z}x^2 + 2c_1\mathrm{e}^{\_Z}x^2 - c_1^2x^2 - 2A\right.}}{x}$$

✓ Solution by Mathematica

Time used: 0.356 (sec). Leaf size: 63

```
DSolve[y[x]*y'[x]-y[x]==A*x^(-1)-A^2*x^(-3),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x^2\left(-\frac{1}{A} + \frac{2x^2\log\left(\frac{x^2}{A+xy(x)}\right) + 2A - c_1x^2 + 2xy(x)}{(A - x^2 + xy(x))^2}\right) = 0, y(x)\right]$$

22.8 problem 8

Internal problem ID [9910]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - y - A - B\,\mathrm{e}^{-\frac{2x}{A}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 76

```
dsolve(y(x)*diff(y(x),x)-y(x)=A+B*exp(-2*x/A),y(x), singsol=all)
```

$$c_1 - 2A\arctan\left(\frac{y(x)+A}{y\left(x\right)\sqrt{\frac{-AB\,\mathrm{e}^{-\frac{2x}{A}}-(y(x)+A)^2}{y(x)^2}}}\right) - 2\sqrt{\frac{-AB\,\mathrm{e}^{-\frac{2x}{A}}-\left(y\left(x\right)+A\right)^2}{y\left(x\right)^2}}\,y(x) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A+B*Exp[-2*x/A],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.9 problem 9

Internal problem ID [9911]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - y - A\left(\mathrm{e}^{\frac{2x}{A}} - 1\right) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 82

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*(exp(2*x/A)-1),y(x), singsol=all)
```

$$c_1 + 2\arctan\left(\frac{A - y(x)}{y(x)\sqrt{\frac{\mathrm{e}^{\frac{2x}{A}}A^2 - (A - y(x))^2}{y(x)^2}}}\right)A + 2\sqrt{\frac{\mathrm{e}^{\frac{2x}{A}}A^2 - (A - y(x))^2}{y(x)^2}}\, y(x) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*(Exp[2*x/A]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$\{\}$$

22.10 problem 10

Internal problem ID [9912]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - y + \frac{2+2m}{(m+3)^2} - A x^m = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-(2*(m+1))/(m+3)^2+A*x^m,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-(2*(m+1))/(m+3)^2+A*x^m,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.11 problem 11

Internal problem ID [9913]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - y + \frac{2x}{9} - 6A^2\left(\frac{2A}{\sqrt{x}} + 1\right) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 331

```
dsolve(y(x)*diff(y(x),x)-y(x)=-2/9*x+6*A^2*(1+2*A*x^(-1/2)),y(x), singsol=all)
```

$$y(x) = \frac{-}{3\,\mathrm{e}^{\mathrm{RootOf}\left(18A^2\ln\left(\frac{4(3A-\sqrt{x})(6A-\sqrt{x})(36A^2-x)}{(9A^2-x)(6A+\sqrt{x})(3A+\sqrt{x})(\mathrm{e}^{\_Z}+9)^2}\right)\mathrm{e}^{\_Z}+108A^2c_1\mathrm{e}^{\_Z}+36A^2\mathrm{e}^{\_Z}\_Z+3A\sqrt{x}\ln\left(\frac{4(3A-\sqrt{x})(6A-\sqrt{x})(36A^2-x)}{(9A^2-x)(6A+\sqrt{x})(3A+\sqrt{x})(\mathrm{e}^{\_Z}+9)^2}\right)}}$$

✓ Solution by Mathematica

Time used: 9.417 (sec). Leaf size: 488

```
DSolve[y[x]*y'[x]-y[x]==-2/9*x+6*A^2*(1+2*A*x^(-1/2)),y[x],x,IncludeSingularSolutions -> True
```

$$\text{Solve}\left[\frac{2^{2/3}\left(\frac{-\frac{6(6A-\sqrt{x})(3A+\sqrt{x})^2}{y(x)}-9\sqrt{x}}{\sqrt[3]{A^3}}+54\right)\left(\frac{6(6A-\sqrt{x})(3A+\sqrt{x})^2+9\sqrt{x}y(x)}{\sqrt[3]{A^3}y(x)}+27\right)\left(-\frac{\left(3\left(3\sqrt[3]{A^3}+\sqrt{x}\right)y(x)+2(6A-\sqrt{x})(\cdots\right.}{\cdots}\right.}{6561\left(\frac{\left(2(6A-\sqrt{\cdots}\right.}{\cdots}\right.}\right.$$

$$\left.+c_1, y(x)\right]$$

22.12 problem 12

Internal problem ID [9914]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y - \frac{2(m-1)}{(-3+m)^2} - \frac{2A\left(m(m+3)\sqrt{x} + (4m^2+3m+9)A + \frac{3m(m+3)A^2}{\sqrt{x}}\right)}{(-3+m)^2} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*(m-1)/(m-3)^2+2*A/(m-3)^2*(m*(m+3)*x^(1/2)+(4*m^2+3*m+9)*A+3*
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*(m-1)/(m-3)^2+2*A/(m-3)^2*(m*(m+3)*x^(1/2)+(4*m^2+3*m+9)*A+3*m*(m+3
```

Not solved

22.13 problem 13

Internal problem ID [9915]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y - \frac{(1+2m)\,x}{4m^2} - \frac{A}{x} + \frac{A^2}{x^3} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 165

```
dsolve(y(x)*diff(y(x),x)-y(x)=(2*m+1)/(4*m^2)*x+A*1/x-A^2*1/(x^3),y(x), singsol=all)
```

$$c_1 + \frac{y(x)\,2^{-\frac{m}{m+1}}\left(\frac{-2y(x)mx-2Am-x^2}{2xy(x)+2A}\right)^{\frac{1}{m+1}}(xy(x)+A)\left(\frac{(-2x^2+2xy(x)+2A)m-x^2}{xy(x)+A}\right)^{\frac{1+2m}{m+1}} - \left(\int^{-\frac{x^2}{2xy(x)+2A}} \frac{(-m+\_a)^{\frac{1}{m+1}}((2}{-}\right.}{x} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==(2*m+1)/(4*m^2)*x+A*1/x-A^2*1/(x^3),y[x],x,IncludeSingularSolutions -
```

Not solved

22.14 problem 14

Internal problem ID [9916]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - y - \frac{4x}{9} - 2A\,x^2 - 2A^2x^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 177

```
dsolve(y(x)*diff(y(x),x)-y(x)=4/9*x+2*A*x^2+2*A^2*x^3,y(x), singsol=all)
```

$$c_1 - \frac{9\left(-\frac{\sqrt{\frac{(3xA+1)^2}{1+(3x-9y(x))A}}\left(\frac{1}{3}+(-3y(x)+x)A\right)\left(\int^{\frac{(3xA+1)^2}{1+(3x-9y(x))A}} \frac{(\_a^2-1)^{\frac{1}{4}}}{\sqrt{\_a}}d\_a\right)}{3} + \sqrt{3}\,Ay(x)\,(3xA+1)\left(\frac{(A\,x^2+\frac{x}{3}+y(x))A(\frac{2}{9}+A}{(\frac{1}{3}+(-3y(x)+}\right.\right.}{\sqrt{\frac{(3xA+1)^2}{1+(3x-9y(x))A}}\,(1+3\,(-3y\,(x)+x)\,A)} = 0$$

✓ Solution by Mathematica

Time used: 2.044 (sec). Leaf size: 170

```
DSolve[y[x]*y'[x]-y[x]==4/9*x+2*A*x^2+2*A^2*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\sqrt[4]{\frac{(-9Ay(x)+3Ax+1)^2}{(3Ax+1)^4}-1}\left(\frac{(-9Ay(x)+3Ax+1)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{(3Ax-}{(3}\right.}{2\sqrt[4]{3}(3Ax+1)\sqrt{(3Ax+1)^2}\sqrt[4]{\frac{A\left(6(3Ax+1)y(x)-27Ay(x)^2+x(3\right.}{(3Ax+1)^4}}}\right.\right.$$

$$\left.\left.+\sqrt{(3Ax+1)^2}\right)+c_1=0,y(x)\right]$$

22.15 problem 15

Internal problem ID [9917]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{3x}{16} - \frac{5A}{x^{\frac{1}{3}}} + \frac{12A^2}{x^{\frac{5}{3}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-3/16*x+5*A*x^(-1/3)-12*A^2*x^(-5/3),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-3/16*x+5*A*x^(-1/3)-12*A^2*x^(-5/3),y[x],x,IncludeSingularSolutions
```

Not solved

22.16 problem 16

Internal problem ID [9918]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{y'y - y - \frac{A}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 57

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*1/x,y(x), singsol=all)
```

$$c_1 + \frac{-\operatorname{erf}\left(\frac{(x-y(x))\sqrt{2}}{2\sqrt{-A}}\right)\sqrt{2}\,\sqrt{\pi}\,x - 2\,\mathrm{e}^{\frac{(x-y(x))^2}{2A}}\sqrt{-A}}{x} = 0$$

✓ Solution by Mathematica

Time used: 0.518 (sec). Leaf size: 64

```
DSolve[y[x]*y'[x]-y[x]==A*1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x}{\sqrt{A}} = \frac{2e^{\frac{(x-y(x))^2}{2A}}}{\sqrt{2\pi}\text{erfi}\left(\frac{y(x)-x}{\sqrt{2}\sqrt{A}}\right) + 2c_1}, y(x)\right]$$

22.17 problem 17

Internal problem ID [9919]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{x}{4} - \frac{A\left(\sqrt{x} + 5A + \frac{3A^2}{\sqrt{x}}\right)}{4} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 252

```
dsolve(y(x)*diff(y(x),x)-y(x)=-1/4*x+1/4*A*(x^(1/2)+5*A+3*A^2*x^(-1/2)),y(x), singsol=all)
```

$$c_1 - \frac{-2A\left(\int^{\frac{6A\sqrt{x}-2x+3y(x)}{12A^2-4A\sqrt{x}+2y(x)}} \frac{e^{-\frac{2}{2\_a+1}}\sqrt{2\_a+1}}{\sqrt{2\_a-3}}d\_a\right)\left(6A^2-2A\sqrt{x}+y(x)\right)\sqrt{-\frac{(3A-\sqrt{x})^2}{6A^2-2A\sqrt{x}+y(x)}} + y(x)\,e^{\frac{-6A^2+2A\sqrt{x}-}{3A^2+2A\sqrt{x}-x+}}}{\sqrt{-\frac{(3A-\sqrt{x})^2}{6A^2-2A\sqrt{x}+y(x)}}\left(6A^2-2A\sqrt{x}+y(x)\right)} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-1/4*x+1/4*A*(x^(1/2)+5*A+3*A^2*x^(-1/2)),y[x],x,IncludeSingularSolut
```

Not solved

22.18 problem 18

Internal problem ID [9920]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$y'y - y - \frac{2a^2}{\sqrt{8a^2 + x^2}} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 720

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*a^2/sqrt(x^2+8*a^2),y(x), singsol=all)
```

$$c_1 + \frac{128\sqrt{-\sqrt{8a^2+x^2}\,x+4a^2+x^2}\,\sqrt{2}\,\sqrt{\pi}\left(\frac{\left(-\frac{33a^4x}{16}+a^4y(x)-\frac{23a^2x^3}{32}+\frac{21a^2x^2y(x)}{32}-\frac{x^5}{32}+\frac{x^4y(x)}{32}\right)\sqrt{8a^2+x^2}}{4}+a^6+\frac{75a^4x^2}{64}-\right.}{\sqrt{-\sqrt{}}}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*a^2/Sqrt[x^2+8*a^2],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.19 problem 19

Internal problem ID [9921]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{y'y - y - 2x - \frac{A}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 169

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*x+A*x^(-2),y(x), singsol=all)
```

$$c_1 + \frac{-6\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{x\left(A^2\right)^{\frac{1}{3}}}{A}}\,(-y(x)+2x)}{\sqrt{\frac{\left(4x^3-4y(x)x^2+xy(x)^2+2A\right)\left(A^2\right)^{\frac{1}{3}}}{y(x)^2A}}\,y(x)}\right)Ax\sqrt{\frac{x(A^2)^{\frac{1}{3}}}{A}} + \sqrt{\frac{\left(4x^3-4y(x)x^2+xy(x)^2+2A\right)(A^2)^{\frac{1}{3}}}{y(x)^2A}}\,y(x)\left(-2x^3\right.}{x\sqrt{\frac{x(A^2)^{\frac{1}{3}}}{A}}} = 0$$

✓ Solution by Mathematica

Time used: 1.287 (sec). Leaf size: 233

```
DSolve[y[x]*y'[x]-y[x]==2*x+A*x^(-2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\Bigg[c_1 = -\frac{i\sqrt{-\frac{2A+4x^3-4x^2y(x)+xy(x)^2}{A}}\left(-6\sqrt{A}x^{3/2}\text{arcsinh}\left(\frac{\sqrt{x}(2x-y(x))}{\sqrt{2}\sqrt{A}}\right)+x^2(-y(x))\sqrt{\frac{2A+4x^3-4x^2y(x)+xy(x)^2}{A}}+xy(x)^2\sqrt{\right.}{4\sqrt{A}x^{3/2}\sqrt{\frac{2A+4x^3-4x^2y(x)+xy(x)^2}{A}}}$$

22.20 problem 20

Internal problem ID [9922]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{6X}{25} - \frac{2A\left(2\sqrt{x} + 19A + \frac{6A^2}{\sqrt{x}}\right)}{25} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-6/25*X+2/25*A*(2*x^(1/2)+19*A+6*A^2*x^(-1/2)),y(x), singsol=al
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-6/25*X+2/25*A*(2*x^(1/2)+19*A+6*A^2*x^(-1/2)),y[x],x,IncludeSingular
```

Not solved

22.21 problem 21

Internal problem ID [9923]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$y'y - y - \frac{3x}{8} - \frac{3\sqrt{a^2 + x^2}}{8} + \frac{a^2}{16\sqrt{a^2 + x^2}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=3/8*x+3/8*sqrt(x^2+a^2)-a^2/(16*sqrt(x^2+a^2)),y(x), singsol=al
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==3/8*x+3/8*Sqrt[x^2+a^2]-a^2/(16*Sqrt[x^2+a^2]),y[x],x,IncludeSingular
```

Not solved

22.22 problem 22

Internal problem ID [9924]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - y + \frac{4x}{25} - \frac{A}{\sqrt{x}} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 165

```
dsolve(y(x)*diff(y(x),x)-y(x)=-4/25*x+A*x^(-1/2),y(x), singsol=all)
```

$$c_1 + \frac{\frac{4\left(10xA-4\sqrt{A\,x^{\frac{3}{2}}}\,x+5\sqrt{A\,x^{\frac{3}{2}}}\,y(x)\right)\left(10xA+4\sqrt{A\,x^{\frac{3}{2}}}\,x-5\sqrt{A\,x^{\frac{3}{2}}}\,y(x)\right)(4x-5y(x))\left(150A\sqrt{x}-16x^2+40xy(x)-25y(x)^2\right)}{A^2x^{\frac{3}{2}}\left(100A\sqrt{x}-16x^2+40xy(x)-25y(x)^2\right)} - 5000A}{\left(\frac{100A\sqrt{x}-16x^2+40xy(x)-25y(x)^2}{A\sqrt{x}}\right)^{\frac{3}{2}}\sqrt{A\,x^{\frac{3}{2}}}} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-4/25*x+A*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.23 problem 23

Internal problem ID [9925]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - y + \frac{9x}{100} - \frac{A}{x^{\frac{5}{3}}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 551

```
dsolve(y(x)*diff(y(x),x)-y(x)=-9/100*x+A*x^(-5/3),y(x), singsol=all)
```

$$c_1 + \frac{10460353203000A\,x^{\frac{44}{3}} - 895371796800000A\,x^{\frac{35}{3}}y(x)^3 + 1205308188000000A\,x^{\frac{32}{3}}y(x)^4 - 89282088000000}{}$$

$= 0$

✓ Solution by Mathematica

Time used: 60.339 (sec). Leaf size: 7909

```
DSolve[y[x]*y'[x]-y[x]==-9/100*x+A*x^(-5/3),y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

22.24 problem 24

Internal problem ID [9926]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{12x}{49} - \frac{2A\left(5\sqrt{x} + 34A + \frac{15A^2}{\sqrt{x}}\right)}{49} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 274

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+2/49*A*(5*x^(1/2)+34*A+15*A^2*x^(-1/2)),y(x), singsol=
```

$$\frac{\left(3A - \sqrt{x}\right)\left(36A^4 + 120A^3\sqrt{x} - 80A\,x^{\frac{3}{2}} + 52A^2x + 84A^2y(x) + 140A\sqrt{x}\,y(x) + 16x^2 - 56xy(x) + 49y(x)\right.}{8\left(\frac{15A^2+4A\sqrt{x}-3x+7y(x)}{6A^2-2A\sqrt{x}+y(x)}\right)^{\frac{3}{2}}\sqrt{-\frac{\left(3A-\sqrt{x}\right)^2}{6A^2-2A\sqrt{x}+y(x)}}\left(6A^2 - 2A\sqrt{x} + y(x)\right)^3 A}$$
$$- \frac{\left(54A^2 + 6A\sqrt{x} - 8x + 21y(x)\right)\sqrt{-\frac{2\left(9A^2-6A\sqrt{x}+x\right)}{6A^2-2A\sqrt{x}+y(x)}}\sqrt{2}}{12\left(6A^2 - 2A\sqrt{x} + y(x)\right)\sqrt{\frac{15A^2+4A\sqrt{x}-3x+7y(x)}{6A^2-2A\sqrt{x}+y(x)}}} + c_1 = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+2/49*A*(5*x^(1/2)+34*A+15*A^2*x^(-1/2)),y[x],x,IncludeSingul
```

Not solved

22.25 problem 25

Internal problem ID [9927]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{12x}{49} - \frac{A\left(25\sqrt{x} + 41A + \frac{10A^2}{\sqrt{x}}\right)}{98} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 1093

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+1/98*A*(25*x^(1/2)+41*A+10*A^2*x^(-1/2)),y(x), singsol
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+1/98*A*(25*x^(1/2)+41*A+10*A^2*x^(-1/2)),y[x],x,IncludeSingu
```

Not solved

22.26 problem 26

Internal problem ID [9928]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - y + \frac{2x}{9} - \frac{A}{\sqrt{x}} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 148

```
dsolve(y(x)*diff(y(x),x)-y(x)=-2/9*x+A*x^(-1/2),y(x), singsol=all)
```

$$y(x) = \frac{2\,6^{\frac{1}{3}}\sqrt{3}\left(-2x^{\frac{3}{2}} + 9A\right)}{3\sqrt{x}\left(9\tan\left(\mathrm{RootOf}\left(18\sqrt{3}\,6^{\frac{1}{3}}\left(\int \frac{\left(\frac{A}{x^{\frac{3}{2}}}\right)^{\frac{2}{3}}\sqrt{x}}{-2x^{\frac{3}{2}}+9A}dx\right) + \ln\left(\frac{\tan(\_Z)^{4}-4\sqrt{3}\,\tan(\_Z)^{3}+18\tan(\_Z)^{2}-12\sqrt{3}\,\tan(\_}{\tan(\_Z)^{4}+2\tan(\_Z)^{2}+1}\right.\right.\right.\right.}$$

✓ Solution by Mathematica

Time used: 0.784 (sec). Leaf size: 282

```
DSolve[y[x]*y'[x]-y[x]==-2/9*x+A*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\log\left(9A^{2/3}+3\sqrt[3]{6}\sqrt[3]{A}\sqrt{x}\right.\right.$$

$$\left.+6^{2/3}x\right)+2\sqrt{3}\arctan\left(\frac{-\frac{6\sqrt[3]{6}(9A-2x^{3/2}+3\sqrt{x}y(x))}{\sqrt[3]{A}y(x)}-27}{27\sqrt{3}}\right)+2\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{6}\sqrt{x}}{\sqrt[3]{A}}+3}{3\sqrt{3}}\right)+2\log\left(\frac{1}{27}\left(27-\frac{3\sqrt[3]{6}\cdots}{\cdots}\right.\right.$$

22.27 problem 27

Internal problem ID [9929]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - y + \frac{5x}{36} - \frac{A}{x^{\frac{7}{5}}} = 0}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-5/36*x+A*x^(-7/5),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-5/36*x+A*x^(-7/5),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.28 problem 28

Internal problem ID [9930]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{12x}{49} - \frac{6A\left(-3\sqrt{x} + 23A + \frac{12A^2}{\sqrt{x}}\right)}{49} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+6/49*A*(-3*x^(1/2)+23*A+12*A^2*x^(-1/2)),y(x), singsol
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+6/49*A*(-3*x^(1/2)+23*A+12*A^2*x^(-1/2)),y[x],x,IncludeSingu
```

Not solved

22.29 problem 29

Internal problem ID [9931]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{30x}{121} - \frac{3A\left(21\sqrt{x} + 35A + \frac{6A^2}{\sqrt{x}}\right)}{242} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-30/121*x+3/242*A*(21*x^(1/2)+35*A+6*A^2*x^(-1/2)),y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-30/121*x+3/242*A*(21*x^(1/2)+35*A+6*A^2*x^(-1/2)),y[x],x,IncludeSing
```

Not solved

22.30 problem 30

Internal problem ID [9932]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - y + \frac{3x}{16} - \frac{A}{x^{\frac{5}{3}}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8477

```
dsolve(y(x)*diff(y(x),x)-y(x)=-3/16*x+A*x^(-5/3),y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-3/16*x+A*x^(-5/3),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.31 problem 31

Internal problem ID [9933]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{12x}{49} - \frac{4A\left(-10\sqrt{x} + 27A + \frac{10A^2}{\sqrt{x}}\right)}{49} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+4/49*A*(-10*x^(1/2)+27*A+10*A^2*x^(-1/2)),y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+4/49*A*(-10*x^(1/2)+27*A+10*A^2*x^(-1/2)),y[x],x,IncludeSing
```

Not solved

22.32 problem 32

Internal problem ID [9934]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - y - \frac{A}{\sqrt{x}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 159

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^(-1/2),y(x), singsol=all)
```

$$c_1 + \frac{-2^{\frac{2}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{1}{3}} \operatorname{AiryAi}\left(\frac{2^{\frac{1}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{2}{3}}(x-y(x))}{2A^2x}\right) - 2\operatorname{AiryAi}\left(1, \frac{2^{\frac{1}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{2}{3}}(x-y(x))}{2A^2x}\right)A}{2^{\frac{2}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{1}{3}} \operatorname{AiryBi}\left(\frac{2^{\frac{1}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{2}{3}}(x-y(x))}{2A^2x}\right) + 2\operatorname{AiryBi}\left(1, \frac{2^{\frac{1}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{2}{3}}(x-y(x))}{2A^2x}\right)A}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 0.321 (sec). Leaf size: 139

```
DSolve[y[x]*y'[x]-y[x]==A*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt[3]{-1}2^{2/3}\sqrt{x}\,\text{AiryAi}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}(x-y(x))}{A^{2/3}}\right)+2\sqrt[3]{A}\,\text{AiryAiPrime}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}(x-y(x))}{A^{2/3}}\right)}{\sqrt[3]{-1}2^{2/3}\sqrt{x}\,\text{AiryBi}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}(x-y(x))}{A^{2/3}}\right)+2\sqrt[3]{A}\,\text{AiryBiPrime}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}(x-y(x))}{A^{2/3}}\right)}+c_1=0,y(x)\right]$$

22.33 problem 33

Internal problem ID [9935]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{y'y - y - \frac{A}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 197

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^(-2),y(x), singsol=all)
```

$$c_1 + \frac{-2^{\frac{1}{3}} A(x-y(x)) \operatorname{AiryAi}\left(-\frac{\left(x^3-2y(x)x^2+xy(x)^2+2A\right)2^{\frac{2}{3}}}{4(-A^2)^{\frac{1}{3}}x}\right) + 2\operatorname{AiryAi}\left(1, -\frac{\left(x^3-2y(x)x^2+xy(x)^2+2A\right)2^{\frac{2}{3}}}{4(-A^2)^{\frac{1}{3}}x}\right)(-A^2)^{\frac{2}{3}}}{2^{\frac{1}{3}} A(x-y(x)) \operatorname{AiryBi}\left(-\frac{\left(x^3-2y(x)x^2+xy(x)^2+2A\right)2^{\frac{2}{3}}}{4(-A^2)^{\frac{1}{3}}x}\right) - 2\operatorname{AiryBi}\left(1, -\frac{\left(x^3-2y(x)x^2+xy(x)^2+2A\right)2^{\frac{2}{3}}}{4(-A^2)^{\frac{1}{3}}x}\right)(-A^2)^{\frac{2}{3}}} = 0$$

✓ Solution by Mathematica

Time used: 0.595 (sec). Leaf size: 201

```
DSolve[y[x]*y'[x]-y[x]==A*x^(-2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\text{AiryAiPrime}\left(\frac{x^3-2y(x)x^2+y(x)^2x+2A}{2\sqrt[3]{2}A^{2/3}x}\right)-\frac{(x-y(x))\,\text{AiryAi}\left(\frac{x^3-2y(x)x^2+y(x)^2x+2A}{2\sqrt[3]{2}A^{2/3}x}\right)}{2^{2/3}\sqrt[3]{A}}}{\text{AiryBiPrime}\left(\frac{x^3-2y(x)x^2+y(x)^2x+2A}{2\sqrt[3]{2}A^{2/3}x}\right)-\frac{(x-y(x))\,\text{AiryBi}\left(\frac{x^3-2y(x)x^2+y(x)^2x+2A}{2\sqrt[3]{2}A^{2/3}x}\right)}{2^{2/3}\sqrt[3]{A}}}+c_1=0,y(x)\right]$$

22.34 problem 34

Internal problem ID [9936]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{y'y - y - A(n+2)\left(\sqrt{x} + 2(n+2)\,A + \frac{(1+n)\,(n+3)\,A^2}{\sqrt{x}}\right) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 309

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*(n+2)*(x^(1/2)+2*(n+2)*A+(n+1)*(n+3)*A^2*x^(-1/2)),y(x), sing
```

$$c_1 + \frac{A\sqrt{\frac{2(n+2)A\sqrt{x}+A^2(n^2+4n+3)+x-y(x)}{(n+2)^2A^2}}\,(n+2)\,\mathrm{BesselK}\left(\frac{n+3}{n+2}, -\sqrt{\frac{2(n+2)A\sqrt{x}+A^2(n^2+4n+3)+x-y(x)}{(n+2)^2A^2}}\right) - \mathrm{BesselK}\left(\frac{1}{n+2}\right.}{A\sqrt{\frac{2(n+2)A\sqrt{x}+A^2(n^2+4n+3)+x-y(x)}{(n+2)^2A^2}}\,(n+2)\,\mathrm{BesselI}\left(\frac{n+3}{n+2}, -\sqrt{\frac{2(n+2)A\sqrt{x}+A^2(n^2+4n+3)+x-y(x)}{(n+2)^2A^2}}\right) + \mathrm{BesselI}\left(\frac{1}{n+2}\right.}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*(n+2)*(x^(1/2)+2*(n+2)*A+(n+1)*(n+3)*A^2*x^(-1/2)),y[x],x,IncludeSi
```

Not solved

22.35 problem 35

Internal problem ID [9937]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y - A(n+2)\left(\sqrt{x} + 2(n+2)A + \frac{(3+2n)A^2}{\sqrt{x}}\right) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 359

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*(n+2)*(x^(1/2)+2*(n+2)*A+(2*n+3)*A^2*x^(-1/2)),y(x), singsol=
```

$$c_1 + \frac{\left(A\sqrt{\frac{(1+n)^2}{(n+2)^2}}\,(n+2) - \sqrt{x} + (-n-2)A\right)\operatorname{BesselK}\left(\sqrt{\frac{(1+n)^2}{(n+2)^2}}, -\sqrt{\frac{2(n+2)A\sqrt{x}+(3+2n)A^2+x-y(x)}{(n+2)^2A^2}}\right) + \operatorname{BesselK}\left(1\right.}{\left(-A\sqrt{\frac{(1+n)^2}{(n+2)^2}}\,(n+2) + \sqrt{x} + (n+2)A\right)\operatorname{BesselI}\left(\sqrt{\frac{(1+n)^2}{(n+2)^2}}, -\sqrt{\frac{2(n+2)A\sqrt{x}+(3+2n)A^2+x-y(x)}{(n+2)^2A^2}}\right) + A\operatorname{BesselI}\left(\right.}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*(n+2)*(x^(1/2)+2*(n+2)*A+(2*n+3)*A^2*x^(-1/2)),y[x],x,IncludeSingul
```

Not solved

22.36 problem 36

Internal problem ID [9938]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{y'y - y - A\sqrt{x} - 2A^2 - \frac{B}{\sqrt{x}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 273

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^(1/2)+2*A^2+B*x^(-1/2),y(x), singsol=all)
```

$$c_1 + \frac{\left(\sqrt{\frac{A^3-B}{A^3}}\,A - A - \sqrt{x}\right)\operatorname{BesselK}\left(\sqrt{\frac{A^3-B}{A^3}}, -\sqrt{\frac{2A^2\sqrt{x}+(x-y(x))A+B}{A^3}}\right) + \operatorname{BesselK}\left(1+\sqrt{\frac{A^3-B}{A^3}}, -\sqrt{\frac{2A^2\sqrt{x}+(\cdots}{}}\right.}{\left(-\sqrt{\frac{A^3-B}{A^3}}\,A + A + \sqrt{x}\right)\operatorname{BesselI}\left(\sqrt{\frac{A^3-B}{A^3}}, -\sqrt{\frac{2A^2\sqrt{x}+(x-y(x))A+B}{A^3}}\right) + A\operatorname{BesselI}\left(1+\sqrt{\frac{A^3-B}{A^3}}, -\sqrt{\frac{2A^2\sqrt{x}\cdots}{}}\right.}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*x^(1/2)+2*A^2+B*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.37 problem 37

Internal problem ID [9939]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - y - 2A^2 + A\sqrt{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 156

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*A^2-A*x^(1/2),y(x), singsol=all)
```

$$c_1 + \frac{\left(-2A+\sqrt{x}\right)\operatorname{BesselK}\left(1, -\sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}}\right) + \operatorname{BesselK}\left(0, -\sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}}\right)\sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}}\,A}{A\operatorname{BesselI}\left(0, \sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}}\right)\sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}} + \left(-2A+\sqrt{x}\right)\operatorname{BesselI}\left(1, \sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}}\right)} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*A^2-A*x^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.38 problem 38

Internal problem ID [9940]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{x}{4} - \frac{6A\left(\sqrt{x} + 8A + \frac{5A^2}{\sqrt{x}}\right)}{49} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/48*x+6/49*A*(x^(1/2)+8*A+5*A^2*x^(-1/2)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/48*x+6/49*A*(x^(1/2)+8*A+5*A^2*x^(-1/2)),y[x],x,IncludeSingularSo
```

Not solved

22.39 problem 39

Internal problem ID [9941]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{6x}{25} - \frac{6A\left(2\sqrt{x} + 7A + \frac{4A^2}{\sqrt{x}}\right)}{25} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-6/25*x+6/25*A*(2*x^(1/2)+7*A+4*A^2*x^(-1/2)),y(x), singsol=all
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-6/25*x+6/25*A*(2*x^(1/2)+7*A+4*A^2*x^(-1/2)),y[x],x,IncludeSingularS
```

Not solved

22.40 problem 40

Internal problem ID [9942]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - y + \frac{3x}{16} - \frac{3A}{x^{\frac{1}{3}}} + \frac{12A^2}{x^{\frac{5}{3}}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9138

```
dsolve(y(x)*diff(y(x),x)-y(x)=-3/16*x+3*A*x^(-1/3)-12*A^2*x^(-5/3),y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-3/16*x+3*A*x^(-1/3)-12*A^2*x^(-5/3),y[x],x,IncludeSingularSolutions
```

Not solved

22.41 problem 41

Internal problem ID [9943]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$y'y - y - \frac{3x}{8} - \frac{3\sqrt{b^2 + x^2}}{8} - \frac{3b^2}{16\sqrt{b^2 + x^2}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=3/8*x+3/8*sqrt(x^2+b^2)+3*b^2/(16*sqrt(x^2+b^2)),y(x), singsol=
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==3/8*x+3/8*Sqrt[x^2+b^2]+3*b^2/(16*Sqrt[x^2+b^2]),y[x],x,IncludeSingul
```

Not solved

22.42 problem 42

Internal problem ID [9944]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$y'y - y - \frac{9x}{32} - \frac{15\sqrt{b^2 + x^2}}{32} - \frac{3b^2}{64\sqrt{b^2 + x^2}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=9/32*x+15/32*sqrt(x^2+b^2)+3*b^2/(64*sqrt(x^2+b^2)),y(x), sings
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==9/32*x+15/32*Sqrt[x^2+b^2]+3*b^2/(64*Sqrt[x^2+b^2]),y[x],x,IncludeSin
```

Not solved

22.43 problem 43

Internal problem ID [9945]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{3x}{32} + \frac{3\sqrt{a^2+x^2}}{32} - \frac{15a^2}{64\sqrt{a^2+x^2}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-3/32*x-3/32*sqrt(x^2+a^2)+15*a^2/(64*sqrt(x^2+a^2)),y(x), sing
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-3/32*x-3/32*Sqrt[x^2+a^2]+15*a^2/(64*Sqrt[x^2+a^2]),y[x],x,IncludeSi
```

Not solved

22.44 problem 44

Internal problem ID [9946]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - y - A\,x^2 + \frac{9}{625A} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 196

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^2-9/625*A^(-1),y(x), singsol=all)
```

$$c_1 + \frac{2\left(\frac{(25xA+3)^{\frac{3}{2}}}{50xA-125Ay(x)+6}\right)^{\frac{1}{3}}(6+(50x-125y(x))A)\left(\int^{\frac{2(25xA+3)^{\frac{3}{2}}}{6+(50x-125y(x))A}}\frac{(\_a^2-6)^{\frac{1}{6}}}{\_a^{\frac{1}{3}}}d\_a\right) - 125\,2^{\frac{5}{6}}\left(\frac{-54+31250A^3x^3+(3\cdots}{\cdots}\right.}{\left(\frac{(25xA+3)^{\frac{3}{2}}}{6+(50x-125y(x))A}\right)^{\frac{1}{3}}(12+(100x-250y(x)\cdots}$$

$$= 0$$

✓ Solution by Mathematica

Time used: 1.436 (sec). Leaf size: 198

```
DSolve[y[x]*y'[x]-y[x]==A*x^2-9/625*A^(-1),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt[6]{\frac{46875A^2y(x)^2-1500A(25Ax+3)y(x)-2(25Ax-3)(25Ax+3)^2}{(25Ax+3)^3}}\left(\frac{(-125Ay(x)+50\cdots}{\sqrt[3]{2}\sqrt{3}(25Ax+3)^{3/2}\sqrt[6]{\frac{-46875A\cdots}{\cdots}}}\cdots\right.}{\sqrt[6]{2}\cdots}\right.$$

$$\left.+c_1=0,y(x)\right]$$

22.45 problem 45

Internal problem ID [9947]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - y + \frac{6x}{25} + A\,x^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 160

```
dsolve(y(x)*diff(y(x),x)-y(x)=-6/25*x-A*x^2,y(x), singsol=all)
```

$$c_1 + \frac{(2x - 5y(x))\left(\int^{\frac{10\sqrt{-xA}\,x}{2x-5y(x)}} \frac{(\_a^2-6)^{\frac{1}{6}}}{\_a^{\frac{1}{3}}} d\_a\right) - \frac{5\,2^{\frac{1}{6}}\left(-\frac{50x^3A}{(2x-5y(x))^2} - \frac{12x^2}{(2x-5y(x))^2} + \frac{60y(x)x}{(2x-5y(x))^2} - \frac{75y(x)^2}{(2x-5y(x))^2}\right)^{\frac{1}{6}} 10^{\frac{2}{3}}\sqrt{-xA}\,y(x)}{2\left(\frac{\sqrt{-xA}\,x}{2x-5y(x)}\right)^{\frac{1}{3}}}}{2x - 5y(x)} = 0$$

✓ Solution by Mathematica

Time used: 1.313 (sec). Leaf size: 162

```
DSolve[y[x]*y'[x]-y[x]==-6/25*x-A*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[c_1 = \frac{i\sqrt[6]{\frac{-2x^2(25Ax+6)+60xy(x)-75y(x)^2}{Ax^3}}\left(25Ax^2 - \frac{\sqrt[6]{2}\sqrt[3]{5}(2x-5y(x))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},-\frac{3(2x-\cdots}{50\cdots}\right.}{\sqrt[6]{\frac{2x^2(25Ax+6)-60xy(x)+75y(x\cdots}{Ax^3}}}\right.}{5\,2^{2/3}\sqrt{3}\sqrt[3]{5}\sqrt{A}x^{3/2}}\right.$$

22.46 problem 46

Internal problem ID [9948]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - y - \frac{6x}{25} + A\,x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 196

```
dsolve(y(x)*diff(y(x),x)-y(x)=6/25*x-A*x^2,y(x), singsol=all)
```

$$c_1 + \frac{-125\,5^{\frac{1}{3}}2^{\frac{5}{6}}\left(\frac{-1250A^3x^3+\left(600x^2+1500xy(x)-1875y(x)^2\right)A^2+(-72x-360y(x))A}{(50xA-125Ay(x)-12)^2}\right)^{\frac{1}{6}} Ay(x)\sqrt{-25xA+6}+100\left(-\frac{6}{25}+\left(x\right.\right.}{\left(-\frac{(-25xA+6)^{\frac{3}{2}}}{-12+(50x-125y(x))A}\right)^{\frac{1}{3}}(-24+(100x-250}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 2.015 (sec). Leaf size: 189

```
DSolve[y[x]*y'[x]-y[x]==6/25*x-A*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt[3]{5}\sqrt[6]{-\frac{A(1875Ay(x)^2-60(25Ax-6)y(x)+2x(6-25Ax)^2)}{(25Ax-6)^3}}\left(\frac{(-125Ay(x)+50Ax-12)\,\text{Hypergeo}}{\sqrt[3]{10}\sqrt{18-75Ax}(25Ax-6)\sqrt[6]{\frac{A(1875Ay(}{}}}\right.}{\sqrt[6]{2}}\right.$$

$$\left.+c_1=0,y(x)\right]$$

22.47 problem 47

Internal problem ID [9949]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - y - 12x - \frac{A}{x^{\frac{5}{2}}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 119

```
dsolve(y(x)*diff(y(x),x)-y(x)=12*x+A*x^(-5/2),y(x), singsol=all)
```

$$c_1 + \frac{-168x^{\frac{5}{2}}\sqrt{3}\,(-y(x)+4x)\operatorname{hypergeom}\left(\left[-\frac{1}{6},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^{\frac{3}{2}}(-y(x)+4x)^2}{4A}\right) + 3\,2^{\frac{2}{3}}\left(\frac{48x^{\frac{7}{2}}-24y(x)x^{\frac{5}{2}}+3y(x)^2x^{\frac{3}{2}}+4A}{A}\right)}{\sqrt{-A\,x^{\frac{7}{2}}}} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==12*x+A*x^(-5/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.48 problem 48

Internal problem ID [9950]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - y - \frac{63x}{4} - \frac{A}{x^{\frac{5}{3}}} = 0}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=63/4*x+A*x^(-5/3),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==63/4*x+A*x^(-5/3),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.49 problem 49

Internal problem ID [9951]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - y - 2x - 2A\left(10\sqrt{x} + 31A + \frac{30A^2}{\sqrt{x}}\right) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 196

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*x+2*A*(10*x^(1/2)+31*A+30*A^2*x^(-1/2)),y(x), singsol=all)
```

$$c_1 - \frac{\left(3A+\sqrt{x}\right) 2^{\frac{1}{3}} \left(\frac{12A^2+10A\sqrt{x}+2x-y(x)}{6A^2+2A\sqrt{x}+y(x)}\right)^{\frac{1}{3}} \left(\frac{15A^2+8A\sqrt{x}+x+y(x)}{6A^2+2A\sqrt{x}+y(x)}\right)^{\frac{1}{6}} y(x)}{4\sqrt{\frac{\left(3A+\sqrt{x}\right)^2}{6A^2+2A\sqrt{x}+y(x)}}\left(6A^2+2A\sqrt{x}+y\left(x\right)\right) A} - \left(\int^{\frac{6A\sqrt{x}+2x-3y(x)}{12A^2+4A\sqrt{x}+2y(x)}} \frac{\left(\_a+1\right)^{\frac{1}{3}} \left(2\_a+5\right)^{\frac{1}{6}}}{\sqrt{2\_a+3}} d\_a\right) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*x+2*A*(10*x^(1/2)+31*A+30*A^2*x^(-1/2)),y[x],x,IncludeSingularSolut
```

Not solved

22.50 problem 50

Internal problem ID [9952]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - y - 2x - 2A\left(-10\sqrt{x} + 19A + \frac{30A^2}{\sqrt{x}}\right) = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*x+2*A*(-10*x^(1/2)+19*A+30*A^2*x^(-1/2)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*x+2*A*(-10*x^(1/2)+19*A+30*A^2*x^(-1/2)),y[x],x,IncludeSingularSolu
```

Not solved

22.51 problem 51

Internal problem ID [9953]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - y + \frac{28x}{121} - \frac{2A\left(5\sqrt{x} + 106A + \frac{65A^2}{\sqrt{x}}\right)}{121} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-28/121*x+2/121*A*(5*x^(1/2)+106*A+65*A^2*x^(-1/2)),y(x), sings
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-28/121*x+2/121*A*(5*x^(1/2)+106*A+65*A^2*x^(-1/2)),y[x],x,IncludeSin
```

Not solved

22.52 problem 52

Internal problem ID [9954]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{12x}{49} - \frac{A\left(5\sqrt{x} + 262A + \frac{65A^2}{\sqrt{x}}\right)}{49} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 697

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+1/49*A*(5*x^(1/2)+262*A+65*A^2*x^(-1/2)),y(x), singsol
```

$$c_1 + \frac{2i\sqrt{3}\,4^{\frac{2}{3}}\left(\left(\left(A\left(3+\frac{5i\sqrt{3}}{3}\right)\sqrt{x} + \frac{i(-25A^2-x)\sqrt{3}}{6} - x + \frac{7y(x)}{4} + 10A^2\right)\sqrt{-35A^2+7A\sqrt{x}} + \frac{7iAx\sqrt{3}}{6} + \left(-\frac{35iA^2}{3}\right.\right.\right.}{\left(\frac{i(5A-\sqrt{x})\sqrt{3}\sqrt{-35A^2+7A\sqrt{x}}}{10i\sqrt{3}\left(A-\frac{\sqrt{x}}{5}\right)\sqrt{-35A^2+7A\sqrt{x}}-120A^2-36A\sqrt{x}+12x-21y(x)}\right)^{\frac{1}{3}}\left(2\left(\left(2A\left(18-7i\sqrt{3}\right)\sqrt{x}+70iA^2\sqrt{3}+120A^2-1\right.\right.\right.}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-28/121*x+2/121*A*(5*x^(1/2)+262*A+65*A^2*x^(-1/2)),y[x],x,IncludeSin
```

Not solved

22.53 problem 53

Internal problem ID [9955]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - y + \frac{12x}{49} - A\sqrt{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 127

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+A*x^(1/2),y(x), singsol=all)
```

$$c_1 + \frac{-7\,14^{\frac{1}{3}}A\sqrt{3} + \frac{\sqrt{3}\,(4x-7y(x))\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{7}{6}\right],\left[\frac{3}{2}\right],\frac{3(4x-7y(x))^2}{196x^{\frac{3}{2}}A}\right)\left(\frac{196A\,x^{\frac{3}{2}}-48x^2+168xy(x)-147y(x)^2}{A\,x^{\frac{3}{2}}}\right)^{\frac{1}{6}}}{7\sqrt{x}}}{\left(\frac{196A\,x^{\frac{3}{2}}-48x^2+168xy(x)-147y(x)^2}{A\,x^{\frac{3}{2}}}\right)^{\frac{1}{6}}\sqrt{A\sqrt{x}}} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+A*x^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.54 problem 54

Internal problem ID [9956]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - y - 6x - \frac{A}{x^4} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 308

```
dsolve(y(x)*diff(y(x),x)-y(x)=6*x+A*x^(-4),y(x), singsol=all)
```

$$c_1 + \frac{2\left(-\frac{x^{\frac{11}{2}} 625^{\frac{5}{6}} 2^{\frac{2}{3}} 16^{\frac{1}{6}} A \operatorname{hypergeom}\left(\left[\frac{1}{6}, \frac{2}{3}\right], \left[\frac{5}{3}\right], -\frac{2A}{3x^3(y(x)+2x)^2}\right) 243^{\frac{1}{6}} \left(-\frac{1}{x^{\frac{3}{2}}(y(x)+2x)}\right)^{\frac{5}{3}} \left(\frac{y(x)}{2}+x\right)^3 \left(\frac{12x^5+12x^4y(x)+3x^3y(x)^2+2A}{x^9(y(x)+2x)^6}\right)^{\frac{1}{6}} \left(-\frac{3}{x^{\frac{3}{2}}} \cdots\right.}{10} \cdots}{\left(-\frac{2}{x^{\frac{3}{2}}(10x+5y(x))}\right) \cdots} = 0$$

✓ Solution by Mathematica

Time used: 1.189 (sec). Leaf size: 213

```
DSolve[y[x]*y'[x]-y[x]==6*x+A*x^(-4),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[c_1 = \frac{i\left(-\frac{2A+12x^5+12x^4y(x)+3x^3y(x)^2}{A}\right)^{5/6}\left(-10\, 2^{5/6}x^5 \text{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, -\frac{3x^3(2x+y(x))^2}{2A}\right) - 5\, 2 \cdots\right.}{2\sqrt[3]{2}\sqrt{3}\sqrt{A}x^{5/2}\left(\frac{2A+12x^5+\cdots}{}\right.\cdots}\right.$$

22.55 problem 55

Internal problem ID [9957]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y - 20x - \frac{A}{\sqrt{x}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=20*x+A*x^(-1/2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==20*x+A*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.56 problem 56

Internal problem ID [9958]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y - \frac{15x}{4} - \frac{A}{x^7} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=15/4*x+A*x^(-7),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==15/4*x+A*x^(-7),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.57 problem 57

Internal problem ID [9959]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{10x}{49} - \frac{2A\left(4\sqrt{x} + 61A + \frac{12A^2}{\sqrt{x}}\right)}{49} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 200

```
dsolve(y(x)*diff(y(x),x)-y(x)=-10/49*x+2/49*A*(4*x^(1/2)+61*A+12*A^2*x^(-1/2)),y(x), singsol=
```

$$c_1 - \frac{(3A+\sqrt{x})\, 2^{\frac{2}{3}} \left(\frac{3A^2+16A\sqrt{x}+5x-7y(x)}{6A^2+2A\sqrt{x}+y(x)}\right)^{\frac{5}{6}} y(x)}{2\sqrt{\frac{(3A+\sqrt{x})^2}{6A^2+2A\sqrt{x}+y(x)}}\left(\frac{-24A^2-2A\sqrt{x}+2x-7y(x)}{6A^2+2A\sqrt{x}+y(x)}\right)^{\frac{1}{3}} \left(6A^2+2A\sqrt{x}+y(x)\right) A}$$
$$- \left(\int^{\frac{6A\sqrt{x}+2x-3y(x)}{12A^2+4A\sqrt{x}+2y(x)}} \frac{(10\_a+1)^{\frac{5}{6}}}{\sqrt{2\_a+3}\,(\_a-2)^{\frac{1}{3}}} d\_a\right) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-10/49*x+2/49*A*(4*x^(1/2)+61*A+12*A^2*x^(-1/2)),y[x],x,IncludeSingul
```

Not solved

22.58 problem 58

Internal problem ID [9960]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{12x}{49} - \frac{2A\left(\sqrt{x} + 166A + \frac{55A^2}{\sqrt{x}}\right)}{49} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 683

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+2/49*A*(x^(1/2)+166*A+55*A^2*x^(-1/2)),y(x), singsol=a
```

$$c_1 + \frac{3i\sqrt{6}\,4^{\frac{2}{3}}\left(\left(\left(A\left(3 + \frac{5i\sqrt{6}}{3}\right)\sqrt{x} + \frac{i(25A^2+x)\sqrt{6}}{6} - \right.\right.\right.}{4\left(\frac{i(5A+\sqrt{x})\sqrt{6}\,\sqrt{-35A^2-7A\sqrt{x}}}{10i\sqrt{6}\left(A+\frac{\sqrt{x}}{5}\right)\sqrt{-35A^2-7A\sqrt{x}-120A^2+36A\sqrt{x}+12x-21y(x)}}\right)^{\frac{1}{3}}\left(\left(\left(2A\left(18-7i\sqrt{6}\right)\sqrt{x} - 70iA^2\sqrt{6} - 120A^2 + 12x\right.\right.\right.}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+2/49*A*(x^(1/2)+166*A+55*A^2*x^(-1/2)),y[x],x,IncludeSingula
```

Not solved

22.59 problem 59

Internal problem ID [9961]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{4x}{25} - \frac{A\left(7\sqrt{x} + 49A + \frac{6A^2}{\sqrt{x}}\right)}{50} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-4/25*x+1/50*A*(7*x^(1/2)+49*A+6*A^2*x^(-1/2)),y(x), singsol=al
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-4/25*x+1/50*A*(7*x^(1/2)+49*A+6*A^2*x^(-1/2)),y[x],x,IncludeSingular
```

Not solved

22.60 problem 60

Internal problem ID [9962]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - y - \frac{15x}{4} - \frac{6A}{x^{\frac{1}{3}}} + \frac{3A^2}{x^{\frac{5}{3}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=15/4*x+6*A*x^(-1/3)-3*A^2*x^(-5/3),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==15/4*x+6*A*x^(-1/3)-3*A^2*x^(-5/3),y[x],x,IncludeSingularSolutions ->
```

Not solved

22.61 problem 61

Internal problem ID [9963]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{3x}{16} - \frac{A}{x^{\frac{1}{3}}} - \frac{B}{x^{\frac{5}{3}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-3/16*x+A*x^(-1/3)+B*x^(-5/3),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-3/16*x+A*x^(-1/3)+B*x^(-5/3),y[x],x,IncludeSingularSolutions -> True
```

Not solved

22.62 problem 62

Internal problem ID [9964]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{5x}{36} - \frac{A}{x^{\frac{3}{5}}} + \frac{B}{x^{\frac{7}{5}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-5/36*x+A*x^(-3/5)-B*x^(-7/5),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-5/36*x+A*x^(-3/5)-B*x^(-7/5),y[x],x,IncludeSingularSolutions -> True
```

Timed out

22.63 problem 63

Internal problem ID [9965]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$y'y - y - \frac{k}{\sqrt{A\,x^2 + Bx + c}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=k*(A*x^2+B*x+c)^(-1/2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==k*(A*x^2+B*x+c)^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.64 problem 64

Internal problem ID [9966]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$
\boxed{y'y - y + \frac{12x}{49} - 3A\left(\frac{1}{49} + B\right)\sqrt{x} - 3A^2\left(\frac{4}{49} - \frac{5B}{2}\right) - \frac{15A^3\left(\frac{1}{49} - \frac{5B}{4}\right)}{4\sqrt{x}} = 0}
$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+3*A*(1/49+B)*x^(1/2)+3*A^2*(4/49-5/2*B)+15/4*A^3*(1/49
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+3*A*(1/49+B)*x^(1/2)+3*A^2*(4/49-5/2*B)+15/4*A^3*(1/49-5/4*B
```

Not solved

22.65 problem 65

Internal problem ID [9967]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - y + \frac{6x}{25} - \frac{4B^2\left((2-A)\,x^{\frac{1}{3}} - \frac{3B(2A+1)}{2} + \frac{B^2(1-3A)}{x^{\frac{1}{3}}} - \frac{A\,B^3}{x^{\frac{2}{3}}}\right)}{75} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 2369

```
dsolve(y(x)*diff(y(x),x)-y(x)=-6/25*x+4/75*B^2*((2-A)*x^(1/3)-3/2*B*(2*A+1)+B^2*(1-3*A)*x^(-1
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-6/25*x+4/75*B^2*((2-A)*x^(1/3)-3/2*B*(2*A+1)+B^2*(1-3*A)*x^(-1/3)-A*
```

Not solved

22.66 problem 66

Internal problem ID [9968]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y - \frac{3x}{4} + \frac{3A\,x^{\frac{1}{3}}}{2} - \frac{3A^2}{4x^{\frac{1}{3}}} + \frac{27A^4}{625x^{\frac{5}{3}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=3/4*x-3/2*A*x^(1/3)+3/4*A^2*x^(-1/3)-27/625*A^4*x^(-5/3),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==3/4*x-3/2*A*x^(1/3)+3/4*A^2*x^(-1/3)-27/625*A^4*x^(-5/3),y[x],x,Inclu
```

Not solved

22.67 problem 67

Internal problem ID [9969]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{6x}{25} - \frac{7A\,x^{\frac{1}{3}}}{5} - \frac{31A^2}{3x^{\frac{1}{3}}} + \frac{100A^4}{3x^{\frac{5}{3}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-6/25*x+7/5*A*x^(1/3)+31/3*A^2*x^(-1/3)-100/3*A^4*x^(-5/3),y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-6/25*x+7/5*A*x^(1/3)+31/3*A^2*x^(-1/3)-100/3*A^4*x^(-5/3),y[x],x,Inc
```

Not solved

22.68 problem 68

Internal problem ID [9970]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 68.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{10x}{49} - \frac{13A^2}{5x^{\frac{1}{5}}} + \frac{7A^3}{20x^{\frac{4}{5}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-10/49*x+13/5*A^2*x^(-1/5)-7/20*A^3*x^(-4/5),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-10/49*x+13/5*A^2*x^(-1/5)-7/20*A^3*x^(-4/5),y[x],x,IncludeSingularSo
```

Not solved

22.69 problem 69

Internal problem ID [9971]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{33x}{169} - \frac{286A^2}{3x^{\frac{5}{11}}} + \frac{770A^3}{9x^{\frac{13}{11}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-33/169*x+286/3*A^2*x^(-5/11)-770/9*A^3*x^(-13/11),y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-33/169*x+286/3*A^2*x^(-5/11)-770/9*A^3*x^(-13/11),y[x],x,IncludeSing
```

Not solved

22.70 problem 70

Internal problem ID [9972]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 70.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - y + \frac{21x}{100} - \frac{7A^2\left(\frac{123}{x^{\frac{1}{7}}} + \frac{280A}{x^{\frac{5}{7}}} - \frac{400A^2}{x^{\frac{9}{7}}}\right)}{9} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-21/100*x+7/9*A^2*(123*x^(-1/7)+280*A*x^(-5/7)-400*A^2*x^(-9/7)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-21/100*x+7/9*A^2*(123*x^(-1/7)+280*A*x^(-5/7)-400*A^2*x^(-9/7)),y[x]
```

Not solved

22.71 problem 71

Internal problem ID [9973]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 71.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class A‘]]`

$$\boxed{y'y - y - ax - b\,x^m = 0}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=a*x+b*x^m,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==a*x+b*x^m,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.72 problem 72

Internal problem ID [9974]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 72.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - y + \frac{(m+1)\,x}{(m+2)^2} - A\,x^{1+2m} - B\,x^{3m+1} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-(m+1)/(m+2)^2*x+A*x^(2*m+1)+B*x^(3*m+1),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-(m+1)/(m+2)^2*x+A*x^(2*m+1)+B*x^(3*m+1),y[x],x,IncludeSingularSoluti
```

Not solved

22.73 problem 73

Internal problem ID [9975]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 73.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - y - a^2\lambda\, e^{2\lambda x} + a(b\lambda + 1)\, e^{\lambda x} - b = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=a^2*lambda*exp(2*lambda*x)-a*(b*lambda+1)*exp(lambda*x)+b,y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==a^2*\[Lambda]*Exp[2*\[Lambda]*x]-a*(b*\[Lambda]+1)*Exp[\[Lambda]*x]+b
```

Not solved

22.74 problem 74

Internal problem ID [9976]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 74.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - y - a^2\lambda\,e^{2\lambda x} - e^{\lambda x}a\lambda x - b\,e^{\lambda x} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=a^2*lambda*exp(2*lambda*x)+a*lambda*x*exp(lambda*x)+b*exp(lambd
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==a^2*\[Lambda]*Exp[2*\[Lambda]*x]+a*\[Lambda]*x*Exp[\[Lambda]*x]+b*Exp
```

Not solved

22.75 problem 75

Internal problem ID [9977]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 75.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - y - 2a^2\lambda \sin(2\lambda x) - 2\sin(\lambda x)\, a = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*a^2*lambda*sin(2*lambda*x)+2*a*sin(lambda*x),y(x), singsol=al
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*a^2*\[Lambda]*Sin[2*\[Lambda]*x]+2*a*Sin[\[Lambda]*x],y[x],x,Includ
```

Not solved

22.76 problem 76

Internal problem ID [9978]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 76.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$y'y - y - a^2 f'(x)\, f''(x) + \frac{(f(x)+b)^2\, f''(x)}{f'(x)^3} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=a^2*diff(f(x),x)*diff(f(x),x$2)-(f(x)+b)^2/( diff(f(x),x)^3)*di
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==a^2*f'[x]*f''[x]-(f[x]+b)^2/( (f'[x])^3)*f''[x],y[x],x,IncludeSingula
```

Timed out

23 Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$

23.1 problem 1

Internal problem ID [9979]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - (ax + b)\, y - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 191

```
dsolve(y(x)*diff(y(x),x)=(a*x+b)*y(x)+1,y(x), singsol=all)
```

$$c_1 + \frac{-2^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}(ax+b)\operatorname{AiryAi}\left(-\frac{(a^2x^2+(2xb-2y(x))a+b^2)2^{\frac{2}{3}}}{4(-a^2)^{\frac{1}{3}}}\right) - 2\operatorname{AiryAi}\left(1, -\frac{(a^2x^2+(2xb-2y(x))a+b^2)2^{\frac{2}{3}}}{4(-a^2)^{\frac{1}{3}}}\right)a}{2^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}(ax+b)\operatorname{AiryBi}\left(-\frac{(a^2x^2+(2xb-2y(x))a+b^2)2^{\frac{2}{3}}}{4(-a^2)^{\frac{1}{3}}}\right) + 2\operatorname{AiryBi}\left(1, -\frac{(a^2x^2+(2xb-2y(x))a+b^2)2^{\frac{2}{3}}}{4(-a^2)^{\frac{1}{3}}}\right)a} = 0$$

✓ Solution by Mathematica

Time used: 0.532 (sec). Leaf size: 161

```
DSolve[y[x]*y'[x]==(a*x+b)*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt[3]{2}(ax+b)\,\text{AiryAi}\left(\frac{(b+ax)^2-2ay(x)}{2\sqrt[3]{2}a^{2/3}}\right) - 2\sqrt[3]{a}\,\text{AiryAiPrime}\left(\frac{(b+ax)^2-2ay(x)}{2\sqrt[3]{2}a^{2/3}}\right)}{\sqrt[3]{2}(ax+b)\,\text{AiryBi}\left(\frac{(b+ax)^2-2ay(x)}{2\sqrt[3]{2}a^{2/3}}\right) - 2\sqrt[3]{a}\,\text{AiryBiPrime}\left(\frac{(b+ax)^2-2ay(x)}{2\sqrt[3]{2}a^{2/3}}\right)} + c_1 = 0, y(x)\right]$$

23.2 problem 2

Internal problem ID [9980]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - \frac{y}{(ax+b)^2} - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 415

```
dsolve(y(x)*diff(y(x),x)=(a*x+b)^(-2)*y(x)+1,y(x), singsol=all)
```

$$c_1 + \frac{-a2^{\frac{1}{3}}(y(x)\,a^2x + y(x)\,ab + 1)\operatorname{AiryAi}\left(-\frac{2^{\frac{2}{3}}\left(-\frac{1}{2}+x^2\left(-\frac{y(x)^2}{2}+x\right)a^4+3xb\left(-\frac{y(x)^2}{3}+x\right)a^3+\left(\left(-\frac{y(x)^2}{2}+3x\right)b^2-xy(x)\right)a^2+b(b}{2(a^2)^{\frac{1}{3}}(ax+b)^2}\right.}{a2^{\frac{1}{3}}\,(y\,(x)\,a^2x + y\,(x)\,ab + 1)\operatorname{AiryBi}\left(-\frac{2^{\frac{2}{3}}\left(-\frac{1}{2}+x^2\left(-\frac{y(x)^2}{2}+x\right)a^4+3xb\left(-\frac{y(x)^2}{3}+x\right)a^3+\left(\left(-\frac{y(x)^2}{2}+3x\right)b^2-xy(x)\right)a^2+b(b}{2(a^2)^{\frac{1}{3}}(ax+b)^2}\right.} = 0$$

✓ Solution by Mathematica

Time used: 1.367 (sec). Leaf size: 561

```
DSolve[y[x]*y'[x]==(a*x+b)^(-2)*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{ay(x)(ax+b)\,\text{AiryAi}\left(\frac{-2x^3a^4-6bx^2a^3+(b+ax)^2y(x)^2a^2-6b^2xa^2-2b^3a+2(b+ax)y(x)a+1}{2\sqrt[3]{2}(a(b+ax)^3)^{2/3}}\right)+\text{AiryAi}\left(\frac{-2x^3a^4-6bx^2a^3}{}\right.}{ay(x)(ax+b)\,\text{AiryBi}\left(\frac{-2x^3a^4-6bx^2a^3+(b+ax)^2y(x)^2a^2-6b^2xa^2-2b^3a+2(b+ax)y(x)a+1}{2\sqrt[3]{2}(a(b+ax)^3)^{2/3}}\right)+\text{AiryBi}\left(\frac{-2x^3a^4-6bx^2a^3}{}\right.}\right.$$

$$\left.+c_1=0,y(x)\right]$$

23.3 problem 3

Internal problem ID [9981]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - \left(a - \frac{1}{ax}\right)y - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(y(x)*diff(y(x),x)=(a-1/(a*x))*y(x)+1,y(x), singsol=all)
```

$$y(x) = \frac{a^2x - \operatorname{RootOf}\left(-\mathrm{e}^{\_Z} - \operatorname{Ei}_1\left(-\_Z\right)a^2x + c_1a^2x\right)}{a}$$

✓ Solution by Mathematica

Time used: 0.168 (sec). Leaf size: 37

```
DSolve[y[x]*y'[x]==(a-1/(a*x))*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{ExpIntegralEi}(a(ax - y(x))) + c_1 = \frac{e^{a(ax-y(x))}}{a^2x}, y(x)\right]$$

23.4 problem 4

Internal problem ID [9982]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], [_Abel, '2nd type', 'cl`

$$yy' - \frac{y}{\sqrt{xa+b}} - 1 = 0$$

✓ Solution by Maple

Time used: 0.344 (sec). Leaf size: 171

```
dsolve(y(x)*diff(y(x),x)=(a*x+b)^(-1/2)*y(x)+1,y(x), singsol=all)
```

$$-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{ax+b}\,y(x)a-ax-b}{\sqrt{(2a+1)(ax+b)^2}}\right)ax}{\sqrt{(2a+1)(ax+b)^2}} + \ln\left(ay(x)^2\sqrt{ax+b} - 2\sqrt{ax+b}\,ax - 2axy(x) - 2\sqrt{ax+b}\,b - 2by(x)\right) - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{ax+b}\,y(x)a-ax-b}{\sqrt{(2a+1)(ax+b)^2}}\right)b}{\sqrt{(2a+1)(ax+b)^2}} - \frac{\ln(ax+b)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.16 (sec). Leaf size: 90

```
DSolve[y[x]*y'[x]==(a*x+b)^(-1/2)*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{\frac{2\arctan\left(\frac{\frac{ay(x)}{\sqrt{ax+b}}-1}{\sqrt{-2a-1}}\right)}{\sqrt{-2a-1}}+\log\left(-\frac{ay(x)^2}{ax+b}+\frac{2y(x)}{\sqrt{ax+b}}+2\right)}{a}=\frac{\log(ax+b)}{a}+c_1,y(x)\right]$$

23.5 problem 5

Internal problem ID [9983]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - \frac{3y}{\sqrt{a\,x^{\frac{3}{2}} + 8x}} - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 292

```
dsolve(y(x)*diff(y(x),x)=3*(a*x^(3/2)+8*x)^(-1/2)*y(x)+1,y(x), singsol=all)
```

$$c_1 + \frac{\left(-\frac{a\sqrt{x}\left(-2a\,x^{\frac{3}{2}}+\sqrt{x}\,ay(x)^2-8\sqrt{x(8+a\sqrt{x})}\,y(x)-16x\right)}{\left(\sqrt{x}\,ay(x)-4\sqrt{x(8+a\sqrt{x})}\right)^2}\right)^{\frac{1}{4}}\sqrt{2a\sqrt{x}+16}\,a\sqrt{x}\,y(x)+4\left(\int^{-\frac{\sqrt{2a\sqrt{x}+16}\,\sqrt{x(8+a\sqrt{x})}}{\sqrt{x}\,ay(x)-4\sqrt{x(8+a\sqrt{x})}}}\frac{(\_a^2-1)^{\frac{1}{4}}}{\sqrt{\_a}}\right.}{\sqrt{-\frac{\sqrt{2a\sqrt{x}+16}\,\sqrt{x(8+a\sqrt{x})}}{\sqrt{x}\,ay(x)-4\sqrt{x(8+a\sqrt{x})}}}\left(\sqrt{x}\,ay\,(x)-4\sqrt{x\,(8+a}\right.}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==3*(a*x^(3/2)+8*x)^(-1/2)*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

23.6 problem 6

Internal problem ID [9984]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - \left(\frac{a}{x^{\frac{2}{3}}} - \frac{2}{3a\,x^{\frac{1}{3}}}\right)y - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 128

```
dsolve(y(x)*diff(y(x),x)=(a*x^(-2/3)-2/3*a^(-1)*x^(-1/3))*y(x)+1,y(x), singsol=all)
```

$$c_1 + \frac{\operatorname{BesselK}\left(1, -\frac{2\sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}}{3}\right)\sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}\,a^2 - x^{\frac{1}{3}}\operatorname{BesselK}\left(0, -\frac{2\sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}}{3}\right)}{-\operatorname{BesselI}\left(1, \frac{2\sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}}{3}\right)\sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}\,a^2 + x^{\frac{1}{3}}\operatorname{BesselI}\left(0, \frac{2\sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}}{3}\right)} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*x^(-2/3)-2/3*a^(-1)*x^(-1/3))*y[x]+1,y[x],x,IncludeSingularSolutions ->
```

Not solved

23.7 problem 7

Internal problem ID [9985]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - a\,e^{\lambda x}y - 1 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 84

```
dsolve(y(x)*diff(y(x),x)=a*exp(lambda*x)*y(x)+1,y(x), singsol=all)
```

$$c_1 - a\,\mathrm{erf}\left(\frac{\left(-\lambda y(x) + e^{\lambda x} a\right)\sqrt{2}}{2\sqrt{-\lambda}}\right)\sqrt{2}\,\sqrt{\pi} - 2\sqrt{-\lambda}\,e^{\frac{a^2 e^{2\lambda x} - 2\,e^{\lambda x} a \lambda y(x) + \lambda^2 y(x)^2 - 2\lambda^2 x}{2\lambda}} = 0$$

✓ Solution by Mathematica

Time used: 1.077 (sec). Leaf size: 83

```
DSolve[y[x]*y'[x]==a*Exp[\[Lambda]*x]*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[-\frac{ae^{\lambda x}}{\sqrt{\lambda}} = \frac{2e^{\frac{\left(ae^{\lambda x} - \lambda y(x)\right)^2}{2\lambda}}}{\sqrt{2\pi}\mathrm{erfi}\left(\frac{\lambda y(x) - ae^{\lambda x}}{\sqrt{2}\sqrt{\lambda}}\right) + 2c_1}, y(x)\right]$$

23.8 problem 8

Internal problem ID [9986]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, ‘2nd type‘, ‘class A‘]]`

$$y'y - \left(e^{\lambda x}a + e^{-\lambda x}b\right) y - 1 = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*exp(lambda*x)+b*exp(-lambda*x))*y(x)+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*Exp[\[Lambda]*x]+b*Exp[-\[Lambda]*x])*y[x]+1,y[x],x,IncludeSingularSolu
```

Not solved

23.9 problem 9

Internal problem ID [9987]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - ay\cosh(x) - 1 = 0}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*y(x)*cosh(x)+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*y[x]*Cosh[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

23.10 problem 10

Internal problem ID [9988]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - ay\sinh(x) - 1 = 0}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*y(x)*sinh(x)+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*y[x]*Sinh[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

23.11 problem 11

Internal problem ID [9989]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - a\cos(\lambda x)\, y - 1 = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*cos(lambda*x)*y(x)+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*Cos[\[Lambda]*x]*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

23.12 problem 12

Internal problem ID [9990]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - a\sin(\lambda x)\, y - 1 = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*sin(lambda*x)*y(x)+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*Sin[\[Lambda]*x]*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

24 Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$

24.1 problem 1

Internal problem ID [9991]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - (ax + 3b)\, y - c\, x^3 + a\, x^2 b + 2b^2 x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 233

```
dsolve(y(x)*diff(y(x),x)=(a*x+3*b)*y(x)+c*x^3-a*b*x^2-2*b^2*x,y(x), singsol=all)
```

$$c_1 + \frac{x\left(\frac{-ab\,x^3+c\,x^4+a\,x^2y(x)-2b^2x^2+4bxy(x)-2y(x)^2}{(xb-y(x))^2}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{a\,\operatorname{arctanh}\left(\frac{-2c\,x^2+a(xb-y(x))}{(xb-y(x))\sqrt{a^2+8c}}\right)}{2\sqrt{a^2+8c}}} y(x) + \sqrt{-\frac{x^2}{xb-y(x)}}\left(\int^{-\frac{x^2}{xb-y(x)}} \frac{(\_a^2c+\_\!\_}{}\right.}{\sqrt{-\frac{x^2}{xb-y(x)}}\,(xb-y(x))}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*x+3*b)*y[x]+c*x^3-a*b*x^2-2*b^2*x,y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

24.2 problem 2

Internal problem ID [9992]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - (3ax + b)\,y + a^2x^3 + a\,x^2b - cx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 943

```
dsolve(y(x)*diff(y(x),x)=(3*a*x+b)*y(x)-a^2*x^3-a*b*x^2+c*x,y(x), singsol=all)
```

$$y(x) = -\frac{}{-9a\,b^2x - 27acx + b\,\mathrm{e}^{\mathrm{RootOf}\left(2a\,b^8\operatorname{arctanh}\left(\frac{b^2\left(9b^2+27c+2\,\mathrm{e}^{\_Z}\right)}{9\sqrt{b^8+10b^6c+33b^4c^2+36b^2c^3}}\right)+20a\,b^6c\operatorname{arctanh}\left(\frac{b^2\left(9b^2+27c+2\,\mathrm{e}^{\_Z}\right)}{9\sqrt{b^8+10b^6c+33b^4c^2+36b^2c^3}}\right)+66a\,b^4}}$$

✓ Solution by Mathematica

Time used: 4.22 (sec). Leaf size: 194

```
DSolve[y[x]*y'[x]==(3*a*x+b)*y[x]-a^2*x^3-a*b*x^2+c*x,y[x],x,IncludeSingularSolutions -> True
```

$$\text{Solve}\left[\frac{2ab\left(\text{RootSum}\left[\#1^4a^2 + \#1^3ab - 2\#1^2ay(x) - \#1^2c - \#1by(x) + y(x)^2\&, \frac{-2\#1^3a^2\log(x-\#1)-\#1^2a}{-}\right.\right.}{c(3a+b+}\right.$$

24.3 problem 3

Internal problem ID [9993]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$2y'y - (7ax + 5b)\,y + 3a^2x^3 + 2c\,x^2 + 3b^2x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 2909

```
dsolve(2*y(x)*diff(y(x),x)=(7*a*x+5*b)*y(x)-3*a^2*x^3-2*c*x^2-3*b^2*x,y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*y[x]*y'[x]==(7*a*x+5*b)*y[x]-3*a^2*x^3-2*c*x^2-3*b^2*x,y[x],x,IncludeSingularSolutio
```

Not solved

24.4 problem 4

Internal problem ID [9994]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - ((3-m)x-1)y + (m-1)ax = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=((3-m)*x-1)*y(x)+(m-1)*(x^2-x^2-a*x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==((3-m)*x-1)*y[x]+(m-1)*(x^2-x^2-a*x),y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

24.5 problem 5

Internal problem ID [9995]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class A‘]]`

$$\boxed{y'y + x(a\,x^2 + b)\,y + x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 157

```
dsolve(y(x)*diff(y(x),x)+x*(a*x^2+b)*y(x)+x=0,y(x), singsol=all)
```

$$c_1 + \frac{-2\,\mathrm{AiryAi}\left(1, \frac{a^2x^4+2ab\,x^2+4ay(x)+b^2}{4a^{\frac{2}{3}}}\right)a^{\frac{1}{3}} + (-a\,x^2 - b)\,\mathrm{AiryAi}\left(\frac{a^2x^4+2ab\,x^2+4ay(x)+b^2}{4a^{\frac{2}{3}}}\right)}{(a\,x^2+b)\,\mathrm{AiryBi}\left(\frac{a^2x^4+2ab\,x^2+4ay(x)+b^2}{4a^{\frac{2}{3}}}\right) + 2\,\mathrm{AiryBi}\left(1, \frac{a^2x^4+2ab\,x^2+4ay(x)+b^2}{4a^{\frac{2}{3}}}\right)a^{\frac{1}{3}}} = 0$$

✓ Solution by Mathematica

Time used: 0.304 (sec). Leaf size: 143

```
DSolve[y[x]*y'[x]+x*(a*x^2+b)*y[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{(ax^2+b)\,\mathrm{AiryAi}\left(\frac{(ax^2+b)^2+4ay(x)}{4a^{2/3}}\right) + 2\sqrt[3]{a}\,\mathrm{AiryAiPrime}\left(\frac{(ax^2+b)^2+4ay(x)}{4a^{2/3}}\right)}{(ax^2+b)\,\mathrm{AiryBi}\left(\frac{(ax^2+b)^2+4ay(x)}{4a^{2/3}}\right) + 2\sqrt[3]{a}\,\mathrm{AiryBiPrime}\left(\frac{(ax^2+b)^2+4ay(x)}{4a^{2/3}}\right)} + c_1 = 0, y(x)\right]$$

24.6 problem 6

Internal problem ID [9996]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'y + a\left(1 - \frac{1}{x}\right) y - a^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve(y(x)*diff(y(x),x)+a*(1-x^(-1))*y(x)=a^2,y(x), singsol=all)
```

$$y(x) = -ax + \text{RootOf}\left(-e^{-\_Z} - \text{Ei}_1\left(-\_Z\right) x + x c_1\right) a$$

✓ Solution by Mathematica

Time used: 0.132 (sec). Leaf size: 30

```
DSolve[y[x]*y'[x]+a*(1-x^(-1))*y[x]==a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{ExpIntegralEi}\left(x + \frac{y(x)}{a}\right) + c_1 = \frac{e^{\frac{y(x)}{a}+x}}{x}, y(x)\right]$$

24.7 problem 7

Internal problem ID [9997]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - a\left(1 - \frac{b}{x}\right)y - b\,a^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(y(x)*diff(y(x),x)-a*(1-b*x^(-1))*y(x)=a^2*b,y(x), singsol=all)
```

$$y(x) = -\operatorname{RootOf}\left(-b\,\mathrm{e}^{-Z} - \operatorname{Ei}_1\left(-\_Z\right)x + xc_1\right)ba + ax$$

✓ Solution by Mathematica

Time used: 0.187 (sec). Leaf size: 45

```
DSolve[y[x]*y'[x]-a*(1-b*x^(-1))*y[x]==a^2*b,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{ExpIntegralEi}\left(\frac{ax - y(x)}{ab}\right) + c_1 = \frac{be^{\frac{ax-y(x)}{ab}}}{x}, y(x)\right]$$

24.8 problem 8

Internal problem ID [9998]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class A‘]]`

$$y'y - x^{n-1}((1+2n)\,x + an)\,y + n\,x^{2n}(a+x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 153

```
dsolve(y(x)*diff(y(x),x)=x^(n-1)*((1+2*n)*x+a*n)*y(x)-n*x^(2*n)*(x+a),y(x), singsol=all)
```

$$y(x) = -\frac{2\left(-\frac{\sqrt{-n^2}\,\tan\left(\frac{\operatorname{RootOf}\left(-\sqrt{-n^2}\,\tan\left(\frac{\_a\sqrt{-n^2}}{2}\right)\_Z x-2an\,e^{\_a+\_Z}-nx\,e^{\_a+\_Z}+2xc_1e^{\_a}\right)\sqrt{-n^2}}{2}\right)x}{2}+n\left(a+\frac{x}{2}\right)\right)x^n}{\tan\left(\frac{\operatorname{RootOf}\left(-\sqrt{-n^2}\,\tan\left(\frac{\_a\sqrt{-n^2}}{2}\right)\_Z x-2an\,e^{\_a+\_Z}-nx\,e^{\_a+\_Z}+2xc_1e^{\_a}\right)\sqrt{-n^2}}{2}\right)\sqrt{-n^2}-n}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==x^(n-1)*((1+2*n)*x+a*n)*y[x]-n*x^(2*n)*(x+a),y[x],x,IncludeSingularSolutio
```

Not solved

24.9 problem 9

Internal problem ID [9999]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - a(-bn + x)\, x^{n-1}y - c\left(x^2 - (1+2n)\, bx + n(1+n)\, b^2\right) x^{-1+2n} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10211

```
dsolve(y(x)*diff(y(x),x)=a*(x-n*b)*x^(n-1)*y(x)+c*(x^2-(2*n+1)*b*x+n*(n+1)*b^2)*x^(2*n-1),y(x
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.463 (sec). Leaf size: 200

```
DSolve[y[x]*y'[x]==a*(x-n*b)*x^(n-1)*y[x]+c*(x^2-(2*n+1)*b*x+n*(n+1)*b^2)*x^(2*n-1),y[x],x,In
```

$$\text{Solve}\left[\frac{a^2\left(-\frac{2a\,\text{arctanh}\left(\frac{a^2-\frac{2ac(n+1)y(x)}{-bcx^n-bcnx^n+cx^{n+1}}}{a\sqrt{a^2+4c(n+1)}}\right)}{\sqrt{a^2+4c(n+1)}}-\log\left(a^2\left(\frac{ay(x)}{-bcx^n-bcnx^n+cx^{n+1}}+1\right)-\frac{a^2c(n+1)y(x)^2}{(-bcx^n-bcnx^n+cx^{n+1})^2}\right)\right)}{2c(n+1)}=\right.$$

$$\left.+c_1,y(x)\right]$$

24.10 problem 10

Internal problem ID [10000]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - (a(k+2n)\,x^k + b)\,x^{n-1}y - (-a^2x^{2k}n - ab\,x^k + c)\,x^{-1+2n} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*(2*n+k)*x^k+b)*x^(n-1)*y(x)+(-a^2*n*x^(2*k)-a*b*x^k+c)*x^(2*n-1),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*(2*n+k)*x^k+b)*x^(n-1)*y[x]+(-a^2*n*x^(2*k)-a*b*x^k+c)*x^(2*n-1),y[x],x
```

Not solved

24.11 problem 11

Internal problem ID [10001]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \left(a(k+2n)\,x^{2k} + b(2m-k)\right)x^{m-k-1}y + \frac{a^2m\,x^{4k} + c\,x^{2k} + m\,b^2}{x} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*(2*n+k)*x^(2*k)+b*(2*m-k))*x^(m-k-1)*y(x)-(a^2*m*x^(4*k)+c*x^(2*k
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*(2*n+k)*x^(2*k)+b*(2*m-k))*x^(m-k-1)*y[x]-(a^2*m*x^(4*k)+c*x^(2*k)+b^2*
```

Timed out

24.12 problem 12

Internal problem ID [10002]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{((m+2L-3)x+n-2L+3)y}{x} - \left((m-L-1)x^2+(n-m-2L+3)x-n+L-2\right)x^{1-2L} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=((m+2*L-3)*x+n-2*L+3)*1/x*y(x)+((m-L-1)*x^2+(n-m-2*L+3)*x-n+L-2)*x^(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==((m+2*L-3)*x+n-2*L+3)*1/x*y[x]+((m-L-1)*x^2+(n-m-2*L+3)*x-n+L-2)*x^(1-2*L)
```

Timed out

24.13 problem 13

Internal problem ID [10003]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - \left(a(1+2n)\,x^2 + cx + b(-1+2n)\right)x^{n-2}y + \left(n\,a^2x^4 + ac\,x^3 + b^2n + bcx + d\,x^2\right)x^{-3+2n} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*(2*n+1)*x^2+c*x+b*(2*n-1))*x^(n-2)*y(x)-(n*a^2*x^4+a*c*x^3+d*x^2+
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*(2*n+1)*x^2+c*x+b*(2*n-1))*x^(n-2)*y[x]-(n*a^2*x^4+a*c*x^3+d*x^2+b*c*x+
```

Timed out

24.14 problem 14

Internal problem ID [10004]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - (a(n-1)\,x + b(2\lambda+n))\,x^{\lambda-1}(ax+b)^{-\lambda-2}\,y + (anx + b(\lambda+n))\,x^{2\lambda-1}(ax+b)^{-2\lambda-3} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*(n-1)*x+b*(2*lambda+n))*x^(lambda-1)*(a*x+b)^(-lambda-2)*y(x)-(a*
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*(n-1)*x+b*(2*\[Lambda]+n))*x^(\[Lambda]-1)*(a*x+b)^(-\[Lambda]-2)*y[x]-
```

Not solved

24.15 problem 15

Internal problem ID [10005]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{a((m-1)x+1)y}{x} - \frac{a^2(mx+1)(x-1)}{x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 521

```
dsolve(y(x)*diff(y(x),x)-a*((m-1)*x+1)*1/x*y(x)=a^2*1/x*(m*x+1)*(x-1),y(x), singsol=all)
```

$$c_1 - \frac{9(3mx-m+1)\,m\left(\frac{(amx+a-y(x))m}{am-y(x)m+2a-2y(x)}\right)^{\frac{m}{m+1}}\left(\frac{a\,m^2x}{am-y(x)m-a+y(x)}\right)^{-\frac{m}{m+1}}\left(\frac{a\,m^2x}{am-y(x)m-a+y(x)}\right)^{-\frac{1}{m+1}}\left(\frac{(ax-a+y(x)}{2am-2y(x)m+}\right.}{2m^3+3m^2-3m-2}$$

$$-\left(\int^{-\frac{9m(3amx+y(x)m-am-y(x)+a)}{2y(x)m^3-2a\,m^3+3y(x)m^2-3a\,m^2-3y(x)m+3am-2y(x)+2a}} \frac{\_a(2\_a\,m^2-\_am-\_a+9m)^{\frac{2m^3}{(m+1)(2m^2-m-1)}}(2\_a\,m^2-}{}\right.$$

$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*((m-1)*x+1)*1/x*y[x]==a^2*1/x*(m*x+1)*(x-1),y[x],x,IncludeSingularSolutio
```

Not solved

24.16 problem 16

Internal problem ID [10006]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y'y - a\left(1 - \frac{b}{\sqrt{x}}\right)y - \frac{a^2 b}{\sqrt{x}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 183

```
dsolve(y(x)*diff(y(x),x)-a*(1-b*x^(-1/2))*y(x)=a^2*b*x^(-1/2),y(x), singsol=all)
```

$$c_1 + \frac{-(b^2)^{\frac{1}{3}} 2^{\frac{2}{3}} \left(-\sqrt{x}+b\right) \operatorname{AiryAi}\left(\frac{2^{\frac{1}{3}}\left(-2\sqrt{x}\,ab+(b^2+x)a-y(x)\right)}{2(b^2)^{\frac{1}{3}}a}\right) - 2\operatorname{AiryAi}\left(1, \frac{2^{\frac{1}{3}}\left(-2\sqrt{x}\,ab+(b^2+x)a-y(x)\right)}{2(b^2)^{\frac{1}{3}}a}\right)b}{(b^2)^{\frac{1}{3}} 2^{\frac{2}{3}} \left(-\sqrt{x}+b\right) \operatorname{AiryBi}\left(\frac{2^{\frac{1}{3}}\left(-2\sqrt{x}\,ab+(b^2+x)a-y(x)\right)}{2(b^2)^{\frac{1}{3}}a}\right) + 2\operatorname{AiryBi}\left(1, \frac{2^{\frac{1}{3}}\left(-2\sqrt{x}\,ab+(b^2+x)a-y(x)\right)}{2(b^2)^{\frac{1}{3}}a}\right)b} = 0$$

✓ Solution by Mathematica

Time used: 1.841 (sec). Leaf size: 323

```
DSolve[y[x]*y'[x]-a*(1-b*x^(-1/2))*y[x]==a^2*b*x^(-1/2),y[x],x,IncludeSingularSolutions -> Tr
```

$$\text{Solve}\left[\frac{\sqrt[3]{-1}2^{2/3}\sqrt[3]{\left(b-\sqrt{x}\right)^3}\,\text{AiryAi}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}\left(\left(b-\sqrt{x}\right)^3\right)^{2/3}\left(a\left(b-\sqrt{x}\right)^2-y(x)\right)}{ab^{2/3}\left(b-\sqrt{x}\right)^2}\right)-2\sqrt[3]{b}\,\text{AiryAiPrime}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}\left(\left(b-\right.\right.}{}\right.}{\sqrt[3]{-1}2^{2/3}\sqrt[3]{\left(b-\sqrt{x}\right)^3}\,\text{AiryBi}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}\left(\left(b-\sqrt{x}\right)^3\right)^{2/3}\left(a\left(b-\sqrt{x}\right)^2-y(x)\right)}{ab^{2/3}\left(b-\sqrt{x}\right)^2}\right)-2\sqrt[3]{b}\,\text{AiryBiPrime}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}\left(\left(b-\right.\right.}{}\right.}\right.$$

$$\left.+c_1=0,y(x)\right]$$

24.17 problem 17

Internal problem ID [10007]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$y'y - \frac{3y}{(ax+b)^{\frac{1}{3}} x^{\frac{5}{3}}} - \frac{3}{(ax+b)^{\frac{2}{3}} x^{\frac{7}{3}}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 147

```
dsolve(y(x)*diff(y(x),x)=3*(a*x+b)^(-1/3)*x^(-5/3)*y(x)+3*(a*x+b)^(-2/3)*x^(-7/3),y(x), sings
```

$$y(x) = -\frac{6\sqrt{3}}{(ax+b)^{\frac{1}{3}} x^{\frac{2}{3}} \left(\left(\frac{a}{(ax+b)^2 x^4}\right)^{\frac{1}{3}} \sqrt{3}\,(ax+b)^{\frac{1}{3}} x^{\frac{5}{3}} - 3x^{\frac{5}{3}} \tan\left(\operatorname{RootOf}\left(\sqrt{3}\ln\left(\frac{\tan(\_Z)^2+1}{\left(\sqrt{3}-\tan(\_Z)\right)^2}\right) + 6\sqrt{3}\,c_1\right.\right.\right.}$$

✓ Solution by Mathematica

Time used: 1.667 (sec). Leaf size: 312

```
DSolve[y[x]*y'[x]==3*(a*x+b)^(-1/3)*x^(-5/3)*y[x]+3*(a*x+b)^(-2/3)*x^(-7/3),y[x],x,IncludeSin
```

$$\text{Solve}\left[\frac{1}{6}\left(2\sqrt{3}\arctan\left(\frac{-\frac{2\left(x^{2/3}y(x)\sqrt[3]{ax+b}+3\right)}{\sqrt[3]{ax^3}y(x)}-1}{\sqrt{3}}\right)+2\log\left(\frac{-x^{2/3}y(x)\sqrt[3]{ax+b}-3}{\sqrt[3]{ax^3}y(x)}+1\right)-\log\left(\frac{\left(x^{2/3}y(\right.}{(a}\right.\right.\right.$$

24.18 problem 18

Internal problem ID [10008]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$3y'y - \frac{(-7\lambda s(3s+4\lambda)\,x + 6s - 2\lambda)\,y}{x^{\frac{1}{3}}} - \frac{6(\lambda s x - 1)}{x^{\frac{2}{3}}} - 2(\lambda s(3s+4\lambda)\,x + 5\lambda)\,(-\lambda s(3s+4\lambda)\,x + 3s + 4\lambda$$

✗ Solution by Maple

```
dsolve(3*y(x)*diff(y(x),x)=(-7*lambda*s*(3*s+4*lambda)*x+6*s-2*lambda)*x^(-1/3)*y(x)+6*(lambd
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[3*y[x]*y'[x]==(-7*\[Lambda]*s*(3*s+4*\[Lambda])*x+6*s-2*\[Lambda])*x^(-1/3)*y[x]+6*(\[
```

Timed out

24.19 problem 19

Internal problem ID [10009]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a(6x-1)\,y}{2x} + \frac{a^2(x-1)\,(4x-1)}{2x} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 364

```
dsolve(y(x)*diff(y(x),x)+1/2*a*(6*x-1)*1/x*y(x)=-1/2*a^2*(x-1)*(4*x-1)*1/x,y(x), singsol=all)
```

$$c_1 + \frac{\sqrt{2}\left(\frac{i(i\sqrt{-x}\,a+2ax+y(x)-a)\sqrt{-x}}{xa}\right)^{\frac{3}{2}}\left(-\frac{i(i\sqrt{-x}\,a+2ax+y(x)-a)\sqrt{-x}\ \mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{2}\right],\left[\frac{7}{2}\right],\frac{i(i\sqrt{-x}\,a+2ax+y(x)-a)\sqrt{-x}}{2xa}\right)}{8xa} + \frac{5\left(-4i\sqrt{2}\right.}{}\right.}{2\left(\frac{3}{2}-\frac{-4i\sqrt{2}\,x+6i\sqrt{-x}+i\sqrt{2}-4x-2}{2\left(-4i\sqrt{2}\,x+2i\sqrt{-x}+4\sqrt{2}\,\sqrt{-x}-i\sqrt{2}-4x+2\right)}\right)\mathrm{hypergeom}\left(\left[-2,-1\right],\left[-\frac{1}{2}\right],\frac{i(i\sqrt{-x}\,a+2ax}{2x}\right.}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/2*a*(6*x-1)*1/x*y[x]==-1/2*a^2*(x-1)*(4*x-1)*1/x,y[x],x,IncludeSingularSo
```

Not solved

24.20 problem 20

Internal problem ID [10010]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{a\left(1 + \frac{2b}{x^2}\right) y}{2} - \frac{a^2\left(3x + \frac{4b}{x}\right)}{16} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/2*a*(1+2*b*x^(-2))*y(x)=1/16*a^2*(3*x+4*b/x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/2*a*(1+2*b*x^(-2))*y[x]==1/16*a^2*(3*x+4*b/x),y[x],x,IncludeSingularSolut
```

Not solved

24.21 problem 21

Internal problem ID [10011]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a(13x-20)\,y}{14x^{\frac{9}{7}}} + \frac{3a^2(x-1)\,(x-8)}{14x^{\frac{11}{17}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/14*a*(13*x-20)*x^(-9/7)*y(x)=-3/14*a^2*(x-1)*(x-8)*x^(-11/17),y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/14*a*(13*x-20)*x^(-9/7)*y[x]==-3/14*a^2*(x-1)*(x-8)*x^(-11/17),y[x],x,Inc
```

Timed out

24.22 problem 22

Internal problem ID [10012]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{5a(23x-16)\,y}{56x^{\frac{9}{7}}} + \frac{3a^2(x-1)\,(25x-32)}{56x^{\frac{11}{17}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+5/56*a*(23*x-16)*x^(-9/7)*y(x)=-3/56*a^2*(x-1)*(25*x-32)*x^(-11/17),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+5/56*a*(23*x-16)*x^(-9/7)*y[x]==-3/56*a^2*(x-1)*(25*x-32)*x^(-11/17),y[x],x
```

Timed out

24.23 problem 23

Internal problem ID [10013]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a(19x+85)\,y}{26x^{\frac{18}{13}}} + \frac{3a^2(x-1)\,(x+25)}{26x^{\frac{23}{13}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/26*a*(19*x+85)*x^(-18/13)*y(x)=-3/26*a^2*(x-1)*(x+25)*x^(-23/13),y
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/26*a*(19*x+85)*x^(-18/13)*y[x]==-3/26*a^2*(x-1)*(x+25)*x^(-23/13),y[x],x,
```

Timed out

24.24 problem 24

Internal problem ID [10014]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a(13x-18)\,y}{15x^{\frac{7}{5}}} + \frac{4a^2(x-1)\,(x-6)}{15x^{\frac{9}{5}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/15*a*(13*x-18)*x^(-7/5)*y(x)=-4/15*a^2*(x-1)*(x-6)*x^(-9/5),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/15*a*(13*x-18)*x^(-7/5)*y[x]==-4/15*a^2*(x-1)*(x-6)*x^(-9/5),y[x],x,Inclu
```

Timed out

24.25 problem 25

Internal problem ID [10015]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, `2nd type`, `class B`]]`

$$y'y + \frac{a(5x+1)\,y}{2\sqrt{x}} - a^2(-x^2+1) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13536

```
dsolve(y(x)*diff(y(x),x)+1/2*a*(5*x+1)*x^(-1/2)*y(x)=a^2*(1-x^2),y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/2*a*(5*x+1)*x^(-1/2)*y[x]==a^2*(1-x^2),y[x],x,IncludeSingularSolutions ->
```

Not solved

24.26 problem 26

Internal problem ID [10016]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y + \frac{3a(19x-14)\,x^{\frac{7}{5}}y}{35} + \frac{4a^2(x-1)\,(9x-14)\,x^{\frac{9}{5}}}{35} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+3/35*a*(19*x-14)*x^(7/5)*y(x)=-4/35*a^2*(x-1)*(9*x-14)*x^(9/5),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+3/35*a*(19*x-14)*x^(7/5)*y[x]==-4/35*a^2*(x-1)*(9*x-14)*x^(9/5),y[x],x,Incl
```

Timed out

24.27 problem 27

Internal problem ID [10017]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{3a(3x+7)\,y}{10x^{\frac{13}{10}}} + \frac{a^2(x-1)\,(x+9)}{5x^{\frac{8}{5}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+3/10*a*(3*x+7)*x^(-13/10)*y(x)=-1/5*a^2*(x-1)*(x+9)*x^(-8/5),y(x), s
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+3/10*a*(3*x+7)*x^(-13/10)*y[x]==-1/5*a^2*(x-1)*(x+9)*x^(-8/5),y[x],x,Includ
```

Timed out

24.28 problem 28

Internal problem ID [10018]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$
\boxed{y'y + \frac{a(7x-12)\,y}{10x^{\frac{7}{5}}} + \frac{a^2(x-1)\,(x-16)}{10x^{\frac{9}{5}}} = 0}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 290

```
dsolve(y(x)*diff(y(x),x)+1/10*a*(7*x-12)*x^(-7/5)*y(x)=-1/10*a^2*(x-1)*(x-16)*x^(-9/5),y(x),
```

$$
c_1 + \frac{\left(-\frac{x^{\frac{8}{5}}a+4x^{\frac{3}{5}}a-5x^{\frac{11}{10}}a+xy(x)}{10x^{\frac{11}{10}}a}\right)^{\frac{3}{2}}\left(\frac{5\left(x^{\frac{8}{5}}a+4x^{\frac{3}{5}}a-5x^{\frac{11}{10}}a+xy(x)\right)\operatorname{hypergeom}\left(\left[-\frac{3}{2}\right],[],-\frac{x^{\frac{8}{5}}a+4x^{\frac{3}{5}}a-5x^{\frac{11}{10}}a+xy(x)}{10x^{\frac{11}{10}}a}\right)}{8x^{\frac{11}{10}}a} + \frac{25(-x-2+3\sqrt{x})}{\cdots}\right.}{\frac{\left(\frac{3}{2}-\frac{5(-x-2+3\sqrt{x})}{2(-x+2+\sqrt{x})}\right)\operatorname{hypergeom}\left([-4,1],\left[-\frac{1}{2}\right],-\frac{x^{\frac{8}{5}}a+4x^{\frac{3}{5}}a-5x^{\frac{11}{10}}a+xy(x)}{10x^{\frac{11}{10}}a}\right)}{2} + \frac{2\left(x^{\frac{8}{5}}a+4x^{\frac{3}{5}}a-5x^{\frac{11}{10}}a+xy(x)\right)\operatorname{hypergeom}\left([-\cdots\right.}{5x^{\frac{11}{10}}a}}
= 0
$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/10*a*(7*x-12)*x^(-7/5)*y[x]==-1/10*a^2*(x-1)*(x-16)*x^(-9/5),y[x],x,Inclu
```

Timed out

24.29 problem 29

Internal problem ID [10019]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{3a(13x-8)\,y}{20x^{\frac{7}{5}}} + \frac{a^2(x-1)\,(27x-32)}{20x^{\frac{9}{5}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+3/20*a*(13*x-8)*x^(-7/5)*y(x)=-1/20*a^2*(x-1)*(27*x-32)*x^(-9/5),y(x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+3/20*a*(13*x-8)*x^(-7/5)*y[x]==-1/20*a^2*(x-1)*(27*x-32)*x^(-9/5),y[x],x,In
```

Timed out

24.30 problem 30

Internal problem ID [10020]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{3a(3x+11)\,y}{14x^{\frac{10}{7}}} + \frac{a^2(x-1)\,(x-27)}{14x^{\frac{13}{7}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+3/14*a*(3*x+11)*x^(-10/7)*y(x)=-1/14*a^2*(x-1)*(x-27)*x^(-13/7),y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+3/14*a*(3*x+11)*x^(-10/7)*y[x]==-1/14*a^2*(x-1)*(x-27)*x^(-13/7),y[x],x,Inc
```

Timed out

24.31 problem 31

Internal problem ID [10021]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{a(x+1)\,y}{2x^{\frac{7}{4}}} - \frac{a^2(x-1)\,(3x+5)}{4x^{\frac{5}{2}}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 191

```
dsolve(y(x)*diff(y(x),x)-1/2*a*(x+1)*x^(-7/4)*y(x)=1/4*a^2*(x-1)*(3*x+5)*x^(-5/2),y(x), sings
```

$$c_1 - \frac{\left(\int^{-\frac{90\left(2x^{\frac{3}{4}}y(x)+2ax-15a\right)}{143\left(x^{\frac{3}{4}}y(x)+ax\right)}} \frac{\_a\sqrt{11\_a-90}\,(13\_a+90)^{\frac{5}{6}}}{(143\_a+180)^{\frac{4}{3}}\,(-\frac{20449}{1458000}\_a^3+\frac{49}{60}\_a+1)}d\_a\right)x\left(\frac{a}{x^{\frac{3}{4}}y(x)+ax}\right)^{\frac{4}{3}} - \frac{\sqrt{78}\,11^{\frac{1}{6}}\left(\frac{(3x+5)a+3x^{\frac{3}{4}}y(x)}{x^{\frac{3}{4}}y(x)+ax}\right)^{\frac{5}{6}}3\ldots}{4\ldots}}{\left(\frac{a}{x^{\frac{3}{4}}y(x)+ax}\right)^{\frac{4}{3}}x}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/2*a*(x+1)*x^(-7/4)*y[x]==1/4*a^2*(x-1)*(3*x+5)*x^(-5/2),y[x],x,IncludeSin
```

Not solved

24.32 problem 32

Internal problem ID [10022]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{a(x+1)\,y}{2x^{\frac{7}{4}}} - \frac{a^2(x-1)\,(x+5)}{4x^{\frac{5}{2}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/2*a*(x+1)*x^(-7/4)*y(x)=1/4*a^2*(x-1)*(x+5)*x^(-5/2),y(x), singsol
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/2*a*(x+1)*x^(-7/4)*y[x]==1/4*a^2*(x-1)*(x+5)*x^(-5/2),y[x],x,IncludeSingu
```

Not solved

24.33 problem 33

Internal problem ID [10023]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - \frac{a(4x+3)\,y}{14x^{\frac{8}{7}}} + \frac{a^2(x-1)\,(16x+5)}{14x^{\frac{9}{7}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/14*a*(4*x+3)*x^(-8/7)*y(x)=-1/14*a^2*(x-1)*(16*x+5)*x^(-9/7),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/14*a*(4*x+3)*x^(-8/7)*y[x]==-1/14*a^2*(x-1)*(16*x+5)*x^(-9/7),y[x],x,Incl
```

Timed out

24.34 problem 34

Internal problem ID [10024]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a(13x-3)\,y}{6x^{\frac{2}{3}}} + \frac{a^2(x-1)\,(5x-1)}{6x^{\frac{1}{3}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/6*a*(13*x-3)*x^(-2/3)*y(x)=-1/6*a^2*(x-1)*(5*x-1)*x^(-1/3),y(x), s
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/6*a*(13*x-3)*x^(-2/3)*y[x]==-1/6*a^2*(x-1)*(5*x-1)*x^(-1/3),y[x],x,Includ
```

Not solved

24.35 problem 35

Internal problem ID [10025]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{a(8x-1)\,y}{28x^{\frac{8}{7}}} - \frac{a^2(x-1)\,(32x+3)}{28x^{\frac{9}{7}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/28*a*(8*x-1)*x^(-8/7)*y(x)=1/28*a^2*(x-1)*(32*x+3)*x^(-9/7),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/28*a*(8*x-1)*x^(-8/7)*y[x]==1/28*a^2*(x-1)*(32*x+3)*x^(-9/7),y[x],x,Inclu
```

Timed out

24.36 problem 36

Internal problem ID [10026]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{y'y - \frac{a(5x-4)\,y}{x^4} - \frac{a^2(x-1)\,(3x-1)}{x^7} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 169

```
dsolve(y(x)*diff(y(x),x)-a*(5*x-4)*x^(-4)*y(x)=a^2*(x-1)*(3*x-1)*x^(-7),y(x), singsol=all)
```

$$c_1 - \frac{4^{\frac{1}{3}}27^{\frac{2}{3}}5^{\frac{1}{6}}\left(x-\frac{3}{4}\right)\sqrt{\frac{y(x)x^2+a-\frac{a}{x}}{y(x)x^2+a}}}{5x\left(\frac{3y(x)x^2+3a-\frac{a}{x}}{y(x)x^2+a}\right)^{\frac{1}{6}}\left(\frac{a}{x(-y(x)x^2-a)}\right)^{\frac{1}{3}}} - \left(\int^{\frac{\frac{9y(x)x^3}{5}+\frac{9ax}{5}-\frac{27a}{20}}{x\left(y(x)x^2+a\right)}} \frac{\_a\sqrt{20\_a-9}}{(9+4\_a)^{\frac{1}{6}}\,(5\_a-9)^{\frac{1}{3}}\left(\frac{400}{729}\_a^3-\frac{7}{3}\_a+1\right)}d\_a\right) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*(5*x-4)*x^(-4)*y[x]==a^2*(x-1)*(3*x-1)*x^(-7),y[x],x,IncludeSingularSolut
```

Not solved

24.37 problem 37

Internal problem ID [10027]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - \frac{2a(3x-10)\,y}{5x^4} - \frac{a^2(x-1)\,(8x-5)}{5x^7} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-2/5*a*(3*x-10)*x^(-4)*y(x)=1/5*a^2*(x-1)*(8*x-5)*x^(-7),y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-2/5*a*(3*x-10)*x^(-4)*y[x]==1/5*a^2*(x-1)*(8*x-5)*x^(-7),y[x],x,IncludeSing
```

Not solved

24.38 problem 38

Internal problem ID [10028]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a(39x-4)\,y}{42x^{\frac{9}{7}}} + \frac{a^2(x-1)\,(9x-1)}{42x^{\frac{11}{7}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/42*a*(39*x-4)*x^(-9/7)*y(x)=-1/42*a^2*(x-1)*(9*x-1)*x^(-11/7),y(x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/42*a*(39*x-4)*x^(-9/7)*y[x]==-1/42*a^2*(x-1)*(9*x-1)*x^(-11/7),y[x],x,Inc
```

Timed out

24.39 problem 39

Internal problem ID [10029]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$
\boxed{y'y + \frac{a(x-2)\,y}{x} - \frac{2a^2(x-1)}{x} = 0}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 116

```
dsolve(y(x)*diff(y(x),x)+a*(x-2)*x^(-1)*y(x)=2*a^2*(x-1)*x^(-1),y(x), singsol=all)
```

$$
c_1 + \frac{x\sqrt{\frac{a}{ax+y(x)}}\,(ax+y(x))\left(\int^{\frac{a}{ax+y(x)}} \frac{\sqrt{\_a-1}\,\mathrm{e}^{\frac{1}{2\_a}}}{\sqrt{\_a}} d\_a\right) + \sqrt{\frac{(1-x)a-y(x)}{ax+y(x)}}\,\mathrm{e}^{\frac{ax+y(x)}{2a}}\,y(x)}{\sqrt{\frac{a}{ax+y(x)}}\,(ax+y(x))\,x} = 0
$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a*(x-2)*x^(-1)*y[x]==2*a^2*(x-1)*x^(-1),y[x],x,IncludeSingularSolutions ->
```

Not solved

24.40 problem 40

Internal problem ID [10030]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a(3x-2)\,y}{x} + \frac{2a^2(x-1)^2}{x} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+a*(3*x-2)*x^(-1)*y(x)=-2*a^2*(x-1)^2*x^(-1),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a*(3*x-2)*x^(-1)*y[x]==-2*a^2*(x-1)^2*x^(-1),y[x],x,IncludeSingularSolution
```

Not solved

24.41 problem 41

Internal problem ID [10031]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a\left(1 - \frac{b}{x^2}\right)y}{x} - \frac{a^2 b}{x} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+a*(1-b*x^(-2))*x^(-1)*y(x)=a^2*b*x^(-1),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a*(1-b*x^(-2))*x^(-1)*y[x]==a^2*b*x^(-1),y[x],x,IncludeSingularSolutions ->
```

Not solved

24.42 problem 42

Internal problem ID [10032]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{a(3x-4)\,y}{4x^{\frac{5}{2}}} - \frac{a^2(x-1)\,(x+2)}{4x^4} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/4*a*(3*x-4)*x^(-5/2)*y(x)=1/4*a^2*(x-1)*(x+2)*x^(-4),y(x), singsol
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/4*a*(3*x-4)*x^(-5/2)*y[x]==1/4*a^2*(x-1)*(x+2)*x^(-4),y[x],x,IncludeSingu
```

Not solved

24.43 problem 43

Internal problem ID [10033]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a(33x+2)\,y}{30x^{\frac{6}{5}}} + \frac{a^2(x-1)\,(9x-4)}{30x^{\frac{7}{5}}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 4330

```
dsolve(y(x)*diff(y(x),x)+1/30*a*(33*x+2)*x^(-6/5)*y(x)=-1/30*a^2*(x-1)*(9*x-4)*x^(-7/5),y(x),
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/30*a*(33*x+2)*x^(-6/5)*y[x]==-1/30*a^2*(x-1)*(9*x-4)*x^(-7/5),y[x],x,Incl
```

Timed out

24.44 problem 44

Internal problem ID [10034]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - \frac{a(x-8)\,y}{8x^{\frac{5}{2}}} + \frac{a^2(x-1)\,(3x-4)}{8x^4} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/8*a*(x-8)*x^(-5/2)*y(x)=-1/8*a^2*(x-1)*(3*x-4)*x^(-4),y(x), singso
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/8*a*(x-8)*x^(-5/2)*y[x]==-1/8*a^2*(x-1)*(3*x-4)*x^(-4),y[x],x,IncludeSing
```

Not solved

24.45 problem 45

Internal problem ID [10035]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y + \frac{a(17x+18)\,y}{30x^{\frac{22}{15}}} + \frac{a^2(x-1)\,(x+4)}{30x^{\frac{29}{15}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/30*a*(17*x+18)*x^(-22/15)*y(x)=-1/30*a^2*(x-1)*(x+4)*x^(-29/15),y(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/30*a*(17*x+18)*x^(-22/15)*y[x]==-1/30*a^2*(x-1)*(x+4)*x^(-29/15),y[x],x,I
```

Timed out

24.46 problem 46

Internal problem ID [10036]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - \frac{a(6x-13)\,y}{13x^{\frac{5}{2}}} + \frac{a^2(x-1)\,(x-13)}{26x^4} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/13*a*(6*x-13)*x^(-5/2)*y(x)=-1/26*a^2*(x-1)*(x-13)*x^(-4),y(x), si
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/13*a*(6*x-13)*x^(-5/2)*y[x]==-1/26*a^2*(x-1)*(x-13)*x^(-4),y[x],x,Include
```

Not solved

24.47 problem 47

Internal problem ID [10037]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a(24x+11)\,x^{\frac{27}{20}}y}{30} + \frac{a^2(x-1)\,(9x+1)}{60x^{\frac{17}{10}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/30*a*(24*x+11)*x^(27/20)*y(x)=-1/60*a^2*(x-1)*(9*x+1)*x^(-17/10),y
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/30*a*(24*x+11)*x^(27/20)*y[x]==-1/60*a^2*(x-1)*(9*x+1)*x^(-17/10),y[x],x,
```

Timed out

24.48 problem 48

Internal problem ID [10038]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - \frac{2a(3x+2)\,y}{5x^{\frac{8}{5}}} - \frac{a^2(x-1)\,(8x+1)}{5x^{\frac{11}{5}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-2/5*a*(3*x+2)*x^(-8/5)*y(x)=1/5*a^2*(x-1)*(8*x+1)*x^(-11/5),y(x), si
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-2/5*a*(3*x+2)*x^(-8/5)*y[x]==1/5*a^2*(x-1)*(8*x+1)*x^(-11/5),y[x],x,Include
```

Timed out

24.49 problem 49

Internal problem ID [10039]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{6a(4x+1)\,y}{5x^{\frac{7}{5}}} - \frac{a^2(x-1)\,(27x+8)}{5x^{\frac{9}{5}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-6/5*a*(4*x+1)*x^(-7/5)*y(x)=1/5*a^2*(x-1)*(27*x+8)*x^(-9/5),y(x), si
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-6/5*a*(4*x+1)*x^(-7/5)*y[x]==1/5*a^2*(x-1)*(27*x+8)*x^(-9/5),y[x],x,Include
```

Timed out

24.50 problem 50

Internal problem ID [10040]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{a(x+4)\,y}{5x^{\frac{8}{5}}} - \frac{a^2(x-1)\,(3x+7)}{5x^{\frac{3}{5}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/5*a*(x+4)*x^(-8/5)*y(x)=1/5*a^2*(x-1)*(3*x+7)*x^(-3/5),y(x), sings
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/5*a*(x+4)*x^(-8/5)*y[x]==1/5*a^2*(x-1)*(3*x+7)*x^(-3/5),y[x],x,IncludeSin
```

Not solved

24.51 problem 51

Internal problem ID [10041]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - \frac{a(x+4)\,y}{5x^{\frac{8}{5}}} - \frac{a^2(x-1)\,(3x+7)}{5x^{\frac{11}{5}}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 194

```
dsolve(y(x)*diff(y(x),x)-1/5*a*(x+4)*x^(-8/5)*y(x)=1/5*a^2*(x-1)*(3*x+7)*x^(-11/5),y(x), sing
```

$$c_1 - \frac{\left(\int^{-\frac{315\left(4y(x)x^{\frac{3}{5}}+4ax-21a\right)}{884\left(y(x)x^{\frac{3}{5}}+ax\right)}} \frac{\_a\sqrt{52\_a-315}\,(68\_a+315)^{\frac{7}{6}}}{(221\_a+315)^{\frac{5}{3}}\,(-\frac{781456}{31255875}\_a^3+\frac{79}{105}\_a+1)} d\_a\right) x \left(\frac{a}{y(x)x^{\frac{3}{5}}+ax}\right)^{\frac{5}{3}} - \frac{8\,105^{\frac{1}{6}}\left(\frac{(3x+7)a+3y(x)x^{\frac{3}{5}}}{y(x)x^{\frac{3}{5}}+ax}\right)^{\frac{7}{6}}}{}}{\left(\frac{a}{y(x)x^{\frac{3}{5}}+ax}\right)^{\frac{5}{3}} x}$$

$= 0$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/5*a*(x+4)*x^(-8/5)*y[x]==1/5*a^2*(x-1)*(3*x+7)*x^(-11/5),y[x],x,IncludeSi
```

Timed out

24.52 problem 52

Internal problem ID [10042]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - \frac{a(2x-1)\,y}{x^{\frac{5}{2}}} - \frac{a^2(x-1)\,(3x+1)}{2x^4} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 195

```
dsolve(y(x)*diff(y(x),x)-a*(2*x-1)*x^(-5/2)*y(x)=1/2*a^2*(x-1)*(3*x+1)*x^(-4),y(x), singsol=a
```

$$c_1 - \frac{\left(\int^{\frac{-\frac{18x^{\frac{3}{2}}y(x)}{35}+\frac{9(-2x-3)a}{35}}{x(\sqrt{x}\,y(x)+a)}} \frac{\_a(5\_a-9)^{\frac{1}{6}}\sqrt{7\_a+9}}{(35\_a+18)^{\frac{2}{3}}\left(-\frac{1225}{1458}\_a^3+\frac{13}{6}\_a+1\right)}d\_a\right) x\left(\frac{a}{x(-\sqrt{x}\,y(x)-a)}\right)^{\frac{2}{3}} - \frac{6\,7^{\frac{2}{3}}\left(x+\frac{3}{2}\right)\sqrt{\frac{(x-1)a+x^{\frac{3}{2}}y(x)}{x(\sqrt{x}\,y(x)+a)}}\,21^{\frac{1}{6}}\,3^{\frac{5}{6}}\cdots}{1225\cdots}}{\left(-\frac{a}{x(\sqrt{x}\,y(x)+a)}\right)^{\frac{2}{3}}x}$$
$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*(2*x-1)*x^(-5/2)*y[x]==1/2*a^2*(x-1)*(3*x+1)*x^(-4),y[x],x,IncludeSingula
```

Not solved

24.53 problem 53

Internal problem ID [10043]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a(x-6)\,y}{5x^{\frac{7}{5}}} - \frac{2a^2(x-1)\,(x+4)}{5x^{\frac{9}{5}}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 157

```
dsolve(y(x)*diff(y(x),x)+1/5*a*(x-6)*x^(-7/5)*y(x)=2/5*a^2*(x-1)*(x+4)*x^(-9/5),y(x), singsol
```

$$c_1 + \frac{\left(-12ay(x)\,x^{\frac{2}{5}} - \frac{3x^{\frac{4}{5}}y(x)^2}{2} + \left(-\frac{y(x)x^{\frac{7}{5}}}{2} + a(x+24)\,(x-1)\right)a\right)\left(8ay(x)\,x^{\frac{2}{5}} + x^{\frac{4}{5}}y(x)^2 + a\left(2y(x)\,x^{\frac{7}{5}} + a(\cdots\right.\right.}{54\left(\frac{a}{y(x)x^{\frac{2}{5}}+ax}\right)^{\frac{5}{2}}\left(a\,(x+4) + y\,(x)\,x^{\frac{2}{5}}\right)^2 x\left(y\,(x)\,x^{\frac{2}{5}} + ax\right)^2} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/5*a*(x-6)*x^(-7/5)*y[x]==2/5*a^2*(x-1)*(x+4)*x^(-9/5),y[x],x,IncludeSingu
```

Timed out

24.54 problem 54

Internal problem ID [10044]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y + \frac{a(21x+19)\, y}{5x^{\frac{7}{5}}} + \frac{2a^2(x-1)\,(9x-4)}{5x^{\frac{9}{5}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/5*a*(21*x+19)*x^(-7/5)*y(x)=-2/5*a^2*(x-1)*(9*x-4)*x^(-9/5),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/5*a*(21*x+19)*x^(-7/5)*y[x]==-2/5*a^2*(x-1)*(9*x-4)*x^(-9/5),y[x],x,Inclu
```

Timed out

24.55 problem 55

Internal problem ID [10045]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y'y - \frac{3ay}{x^{\frac{7}{4}}} - \frac{a^2(x-1)(x-9)}{4x^{\frac{5}{2}}} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-3*a*x^(-7/4)*y(x)=1/4*a^2*(x-1)*(x-9)*x^(-5/2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-3*a*x^(-7/4)*y[x]==1/4*a^2*(x-1)*(x-9)*x^(-5/2),y[x],x,IncludeSingularSolut
```

Not solved

24.56 problem 56

Internal problem ID [10046]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$y'y - \frac{a((1+k)\,x-1)\,y}{x^2} - \frac{a^2(1+k)\,(x-1)}{x^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 141

```
dsolve(y(x)*diff(y(x),x)-a*((k+1)*x-1)*x^(-2)*y(x)=a^2*(k+1)*(x-1)*x^(-2),y(x), singsol=all)
```

$$c_1 + \frac{(xy(x)-a)\left(\int^{\frac{ax}{-xy(x)+a}} (\_a-1)^{\frac{1}{1+k}} \mathrm{e}^{\frac{1}{(1+k)\_a}} \_a^{-\frac{1}{1+k}} d\_a\right) + \left(\frac{ax}{-xy(x)+a}\right)^{-\frac{1}{1+k}} x^2 \left(\frac{(x-1)a+xy(x)}{-xy(x)+a}\right)^{\frac{1}{1+k}} \mathrm{e}^{\frac{-xy(x)+a}{a(1+k)x}} y(}{-xy\left(x\right)+a} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*((k+1)*x-1)*x^(-2)*y[x]==a^2*(k+1)*(x-1)*x^(-2),y[x],x,IncludeSingularSol
```

Not solved

24.57 problem 57

Internal problem ID [10047]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - a((k-2)x + 2k - 3)x^{-k}y - a^2(k-2)(x-1)^2x^{1-2k} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-a*( (k-2)*x + 2*k - 3)*x^(-k)*y(x)=a^2*(k-2)*(x-1)^2*x^(1-2*k),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*( (k-2)*x + 2*k - 3)*x^(-k)*y[x]==a^2*(k-2)*(x-1)^2*x^(1-2*k),y[x],x,Incl
```

Not solved

24.58 problem 58

Internal problem ID [10048]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - \frac{a((4k-7)\,x - 4k + 5)\,x^{-k}y}{2} - \frac{a^2(-3+2k)\,(x-1)^2\,x^{1-2k}}{2} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/2*a*( (4*k-7)*x - 4*k + 5)*x^(-k)*y(x)=1/2*a^2*(2*k-3)*(x-1)^2*x^(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/2*a*( (4*k-7)*x - 4*k + 5)*x^(-k)*y[x]==1/2*a^2*(2*k-3)*(x-1)^2*x^(1-2*k)
```

Not solved

24.59 problem 59

Internal problem ID [10049]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class A‘]]`

$$
\boxed{y'y - ((-1+2n)x - an)\,x^{-n-1}y - n(-a+x)\,x^{-2n} = 0}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 150

```
dsolve(y(x)*diff(y(x),x)-((2*n-1)*x-a*n)*x^(-n-1)*y(x)=n*(x-a)*x^(-2*n),y(x), singsol=all)
```

$$
y(x) = -\frac{\left(\sqrt{-n^2}\,x\tan\left(\frac{\mathrm{RootOf}\left(-\sqrt{-n^2}\,\tan\left(\frac{\_a\sqrt{-n^2}}{2}\right)\_Zx - 2\,\mathrm{e}^{\_Z}na\,\mathrm{e}^{\_a} + nx\,\mathrm{e}^{\_Z}\mathrm{e}^{\_a} + 2xc_1\mathrm{e}^{\_a}\right)\sqrt{-n^2}}{2}\right) - 2an + nx\right)x^{-n}}{\sqrt{-n^2}\,\tan\left(\frac{\mathrm{RootOf}\left(-\sqrt{-n^2}\,\tan\left(\frac{\_a\sqrt{-n^2}}{2}\right)\_Zx - 2\,\mathrm{e}^{\_Z}na\,\mathrm{e}^{\_a} + nx\,\mathrm{e}^{\_Z}\mathrm{e}^{\_a} + 2xc_1\mathrm{e}^{\_a}\right)\sqrt{-n^2}}{2}\right) + n}
$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-((2*n-1)*x-a*n)*x^(-n-1)*y[x]==n*(x-a)*x^(-2*n),y[x],x,IncludeSingularSolut
```

Not solved

24.60 problem 60

Internal problem ID [10050]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, ‘2nd type‘, ‘class A‘]]`

$$y'y - ((1+n)\,x - an)\,x^{n-1}(-a+x)^{-n-2}\,y - n\,x^{2n}(-a+x)^{-2n-3} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-((n+1)*x-a*n)*x^(n-1)*(x-a)^(-n-2)*y(x)=n*x^(2*n)*(x-a)^(-2*n-3),y(x
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-((n+1)*x-a*n)*x^(n-1)*(x-a)^(-n-2)*y[x]==n*x^(2*n)*(x-a)^(-2*n-3),y[x],x,In
```

Not solved

24.61 problem 61

Internal problem ID [10051]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - a((-3+2k)\,x+1)\,x^{-k}y - a^2(k-2)\,((k-1)\,x+1)\,x^{2-2k} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-a*((2*k-3)*x+1)*x^(-k)*y(x)=a^2*(k-2)*((k-1)*x+1)*x^(2*(1-k)),y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*((2*k-3)*x+1)*x^(-k)*y[x]==a^2*(k-2)*((k-1)*x+1)*x^(2*(1-k)),y[x],x,Inclu
```

Not solved

24.62 problem 62

Internal problem ID [10052]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class A‘]]`

$$y'y - a((n+2k-3)\,x+3-2k)\,x^{-k}y - a^2\left((k-1+n)\,x^2-(n+2k-3)\,x+k-2\right)x^{1-2k} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-a*((n+2*k-3)*x+3-2*k)*x^(-k)*y(x)=a^2*((n+k-1)*x^2-(n+2*k-3)*x+k-2)*
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*((n+2*k-3)*x+3-2*k)*x^(-k)*y[x]==a^2*((n+k-1)*x^2-(n+2*k-3)*x+k-2)*x^(1-2
```

Timed out

24.63 problem 63

Internal problem ID [10053]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - \frac{a((n+2)\,x-2)\,x^{-\frac{1+2n}{n}}y}{n} - \frac{a^2((1+n)\,x^2-2x-n+1)\,x^{-\frac{2+3n}{n}}}{n} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-a/n*((n+2)*x-2)*x^(-(2*n+1)/n)*y(x)=a^2/n*((n+1)*x^2-2*x-n+1)*x^(-(3
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a/n*((n+2)*x-2)*x^(-(2*n+1)/n)*y[x]==a^2/n*((n+1)*x^2-2*x-n+1)*x^(-(3*n+2)/
```

Not solved

24.64 problem 64

Internal problem ID [10054]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - \frac{a\left(\frac{(n+4)x}{n+2} - 2\right)x^{-\frac{1+2n}{n}}y}{n} - \frac{a^2(2x^2 + (n^2+n-4)\,x - (n-1)\,(n+2))\,x^{-\frac{2+3n}{n}}}{n\,(n+2)} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-a/n*((n+4)/(n+2)*x-2)*x^(-(2*n+1)/n)*y(x)=a^2/(n*(n+2))*(2*x^2+(n^2+
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a/n*((n+4)/(n+2)*x-2)*x^(-(2*n+1)/n)*y[x]==a^2/(n*(n+2))*(2*x^2+(n^2+n-4)*x
```

Not solved

24.65 problem 65

Internal problem ID [10055]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y + \frac{a\left(\frac{(5+3n)x}{2} + \frac{n-1}{1+n}\right)x^{-\frac{n+4}{n+3}}y}{n+3} + \frac{a^2\left((1+n)\,x^2 - \frac{(n^2+2n+5)x}{1+n} + \frac{4}{1+n}\right)x^{-\frac{5+n}{n+3}}}{2n+6} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+a/(n+3)*((3*n+5)/(2)*x+(n-1)/(n+1))*x^(-(n+4)/(n+3))*y(x)=-a^2/(2*(n
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a/(n+3)*((3*n+5)/(2)*x+(n-1)/(n+1))*x^(-(n+4)/(n+3))*y[x]==-a^2/(2*(n+3))*(
```

Timed out

24.66 problem 66

Internal problem ID [10056]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y'y - a\left(\frac{n+2}{n} + b\,x^n\right)y + \frac{a^2 x\left(\frac{1+n}{n} + b\,x^n\right)}{n} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 202

```
dsolve(y(x)*diff(y(x),x)-a*((n+2)/n+b*x^n)*y(x)=-a^2/n*x*((n+1)/n+b*x^n),y(x), singsol=all)
```

$$c_1 - n\sqrt{-\frac{(1+n)^2}{n^2}}\left(\int^{\frac{2\arctan\left(\frac{2abn\,x^{1+n}+anx-n^2y(x)+ax-y(x)n}{n\sqrt{-\frac{(1+n)^2}{n^2}}\,(ax-y(x)n)}\right)}{\sqrt{-\frac{(1+n)^2}{n^2}}}} \tan\left(\frac{\_a\sqrt{-\frac{(1+n)^2}{n^2}}}{2}\right)\mathrm{e}^{-\_a}d\_a\right) + (-2x^n nb - n - 1)\,\mathrm{e}^{-\frac{2\arctan\left(\frac{2abn\,x^{1+n}+anx-n^2y(x)+ax-y(x)n}{n\sqrt{-\frac{(1+n)^2}{n^2}}\,(ax-y(x)n)}\right)}{\sqrt{-\frac{(1+n)^2}{n^2}}}} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*((n+2)/n+b*x^n)*y[x]==-a^2/n*x*((n+1)/n+b*x^n),y[x],x,IncludeSingularSolu
```

Not solved

24.67 problem 67

Internal problem ID [10057]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - (a\,e^{x} + b)\,y - c\,e^{2x} + ab\,e^{x} + b^{2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 154

```
dsolve(y(x)*diff(y(x),x)=(a*exp(x)+b)*y(x)+c*exp(2*x)-a*b*exp(x)-b^2,y(x), singsol=all)
```

$$c_1 + \sqrt{\frac{c\,e^{2x} - (b - y(x))\,(e^{x}a + b - y(x))}{(b - y(x))^{2}}}\; y(x)\, e^{-\frac{a \operatorname{arctanh}\left(\frac{(b-y(x))a - 2\,e^{x}c}{\sqrt{a^{2}+4c}\,(b-y(x))}\right)}{\sqrt{a^{2}+4c}}} - b\left(\int^{-\frac{e^{x}}{b-y(x)}} \frac{\sqrt{\_a^{2}c + \_aa - 1}\, e^{-\frac{a \operatorname{arctanh}\left(\frac{2\_ac + a}{\sqrt{a^{2}+4c}}\right)}{\sqrt{a^{2}+4c}}}}{\_a} d\_a\right) = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*Exp[x]+b)*y[x]+c*Exp[2*x]-a*b*Exp[x]-b^2,y[x],x,IncludeSingularSolution
```

Not solved

24.68 problem 68

Internal problem ID [10058]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 68.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - \left(a(\lambda + 2\mu)\,\mathrm{e}^{\lambda x} + b\right)\mathrm{e}^{\mu x} y - \left(-a^2\mu\,\mathrm{e}^{2\lambda x} - ab\,\mathrm{e}^{\lambda x} + c\right)\mathrm{e}^{2\mu x} = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*(2*mu+lambda)*exp(lambda*x)+b)*exp(mu*x)*y(x)+(-a^2*mu*exp(2*lamb
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*(2*\[Mu]+\[Lambda])*Exp[\[Lambda]*x]+b)*Exp[\[Mu]*x]*y[x]+(-a^2*\[Mu]*E
```

Not solved

24.69 problem 69

Internal problem ID [10059]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - \left(\mathrm{e}^{\lambda x}a + b\right) y - c\left(a^2\mathrm{e}^{2\lambda x} + ab(\lambda x + 1)\,\mathrm{e}^{\lambda x} + b^2 x\lambda\right) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 545

```
dsolve(y(x)*diff(y(x),x)=(a*exp(lambda*x)+b)*y(x)+c*(a^2*exp(2*lambda*x)+a*b*(lambda*x+1)*exp
```

$$-\frac{(3c\lambda+1)\left(6\,\mathrm{arctanh}\left(\frac{6b\,c^2\lambda^2x+6\,\mathrm{e}^{\lambda x}a\,c^2\lambda+2bc\lambda x+2\,\mathrm{e}^{\lambda x}ac+3c\lambda y(x)+y(x)}{y(x)\sqrt{36c^3\lambda^3+33\lambda^2c^2+10c\lambda+1}}\right)c\lambda+2\ln\left(\frac{9\left(3bc\,\lambda^2x+3\,\mathrm{e}^{\lambda x}ac\lambda+b\lambda x+\mathrm{e}^{\lambda x}a\right)c}{y(x)}\right)\sqrt{3}\cdots\right.}{\cdots}$$

$$+\frac{(3c\lambda+1)\ln\left(b\lambda x+\mathrm{e}^{\lambda x}a\right)}{3c\lambda}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.312 (sec). Leaf size: 134

```
DSolve[y[x]*y'[x]==(a*Exp[\[Lambda]*x]+b)*y[x]+c*(a^2*Exp[2*\[Lambda]*x]+a*b*(\[Lambda]*x+1)*
```

$$\text{Solve}\left[-\frac{\frac{2\arctan\left(\frac{\frac{2c\lambda y(x)}{ace^{\lambda x}+bc\lambda x}-1}{\sqrt{-4c\lambda-1}}\right)}{\sqrt{-4c\lambda-1}}+\log\left(-\frac{c\lambda y(x)^2}{(ace^{\lambda x}+bc\lambda x)^2}+\frac{y(x)}{ace^{\lambda x}+bc\lambda x}+1\right)}{2c\lambda}=\frac{\log\left(ace^{\lambda x}+bc\lambda x\right)}{c\lambda}\right.$$

$$\left.+c_1,y(x)\right]$$

24.70 problem 70

Internal problem ID [10060]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 70.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - e^{\lambda x}(2a\lambda x + a + b)\, y + e^{2\lambda x}\left(a^2 \lambda\, x^2 + abx + c\right) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 119

```
dsolve(y(x)*diff(y(x),x)=exp(lambda*x)*(2*a*lambda*x+a+b)*y(x)-exp(2*lambda*x)*(a^2*lambda*x^
```

$$y(x) = \frac{\left(\tan\left(\frac{\operatorname{RootOf}\left(2\,e^{-\_Z} a\lambda x\, e^{-a\sqrt{-\frac{b^2-4c\lambda}{a^2}}\tan\left(\frac{a\sqrt{-\frac{b^2-4c\lambda}{a^2}}\,\_Z}{2}\right)}\_Za + e^{-\_Z} b\, e^{-a} + 2c_1 a\, e^{-a}\right)\sqrt{-\frac{b^2-4c\lambda}{a^2}}}{2}\right) a\sqrt{-\frac{b^2-4c\lambda}{a^2}} + 2a\lambda\right.}{2\lambda}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==Exp[\[Lambda]*x]*(2*a*\[Lambda]*x+a+b)*y[x]-Exp[2*\[Lambda]*x]*(a^2*\[Lamb
```

Not solved

24.71 problem 71

Internal problem ID [10061]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 71.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, ‘2nd type‘, ‘class A‘]]`

$$y'y - e^{ax}(2a\,x^2 + b + 2x)\,y - e^{2ax}(-x^4 a - b\,x^2 + c) = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=exp(a*x)*(2*a*x^2+2*x+b)*y(x)+exp(2*a*x)*(-a*x^4-b*x^2+c),y(x), sing
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==Exp[a*x]*(2*a*x^2+2*x+b)*y[x]+Exp[2*a*x]*(-a*x^4-b*x^2+c),y[x],x,IncludeSi
```

Not solved

24.72 problem 72

Internal problem ID [10062]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 72.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{y'y + a(2bx+1)\,\mathrm{e}^{bx}y + a^2b\,x^2\mathrm{e}^{2bx} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve(y(x)*diff(y(x),x)+a*(1+2*b*x)*exp(b*x)*y(x)=-a^2*b*x^2*exp(2*b*x),y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{e}^{xb}a\left(bx\,\mathrm{RootOf}\left(-\mathrm{e}^{\_Z}xb - \mathrm{Ei}_1\left(-\_Z\right) + c_1\right) - 1\right)}{\mathrm{RootOf}\left(-\mathrm{e}^{\_Z}xb - \mathrm{Ei}_1\left(-\_Z\right) + c_1\right)b}$$

✓ Solution by Mathematica

Time used: 0.466 (sec). Leaf size: 59

```
DSolve[y[x]*y'[x]+a*(1+2*b*x)*Exp[b*x]*y[x]==-a^2*b*x^2*Exp[2*b*x],y[x],x,IncludeSingularSolu
```

$$\mathrm{Solve}\left[bxe^{\frac{ae^{bx}}{abxe^{bx}+by(x)}} = \mathrm{ExpIntegralEi}\left(\frac{ae^{bx}}{abe^{bx}x + by(x)}\right) + c_1, y(x)\right]$$

24.73 problem 73

Internal problem ID [10063]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 73.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - a(1 + 2n + 2n(1 + n)\,x)\,e^{(1+n)x}y + a^2n(1 + n)\,(nx + 1)\,x\,e^{2(1+n)x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 130

```
dsolve(y(x)*diff(y(x),x)-a*(1+2*n+2*n*(n+1)*x)*exp((n+1)*x)*y(x)=-a^2*n*(n+1)*(1+n*x)*x*exp(2
```

$$y(x) = \frac{\left(1 + 2n^2x + \left(\sqrt{-\frac{(1+n)^2}{n^2}}\ \tan\left(\frac{\mathrm{RootOf}\left(2x\,n^2\mathrm{e}^{-\_a+\_Z} - \tan\left(\frac{\_a\sqrt{-\frac{(1+n)^2}{n^2}}}{2}\right)\_Z\sqrt{-\frac{(1+n)^2}{n^2}}\,n + 2nx\,\mathrm{e}^{-\_a+\_Z} + n\,\mathrm{e}^{-\_a+\_Z} + 2c_1}{2}\right.\right.\right.}{2 + 2n}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*(1+2*n+2*n*(n+1)*x)*Exp[(n+1)*x]*y[x]==-a^2*n*(n+1)*(1+n*x)*x*Exp[2*(n+1)
```

Not solved

24.74 problem 74

Internal problem ID [10064]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 74.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, ‘2nd type‘, ‘class A‘]]`

$$y'y + a\left(1 + 2b\sqrt{x}\right) e^{2b\sqrt{x}} y + a^2 b\, x^{\frac{3}{2}} e^{4b\sqrt{x}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 209

```
dsolve(y(x)*diff(y(x),x)+a*(1+2*b*x^(1/2))*exp(2*b*x^(1/2))*y(x)=-a^2*b*x^(3/2)*exp(4*b*x^(1/
```

$$c_1 + \frac{-\operatorname{BesselK}\left(1, -\sqrt{\frac{a\, e^{2b\sqrt{x}}}{b^2\left(e^{2b\sqrt{x}}ax+y(x)\right)}}\right)\sqrt{\frac{a\, e^{2b\sqrt{x}}}{b^2\left(e^{2b\sqrt{x}}ax+y(x)\right)}}\, b\sqrt{x} + \operatorname{BesselK}\left(0, -\sqrt{\frac{a\, e^{2b\sqrt{x}}}{b^2\left(e^{2b\sqrt{x}}ax+y(x)\right)}}\right)}{\operatorname{BesselI}\left(1, \sqrt{\frac{a\, e^{2b\sqrt{x}}}{b^2\left(e^{2b\sqrt{x}}ax+y(x)\right)}}\right)\sqrt{\frac{a\, e^{2b\sqrt{x}}}{b^2\left(e^{2b\sqrt{x}}ax+y(x)\right)}}\, b\sqrt{x} - \operatorname{BesselI}\left(0, \sqrt{\frac{a\, e^{2b\sqrt{x}}}{b^2\left(e^{2b\sqrt{x}}ax+y(x)\right)}}\right)} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a*(1+2*b*x^(1/2))*Exp[2*b*x^(1/2)]*y[x]==-a^2*b*x^(3/2)*exp(4*b*x^(1/2)),y[
```

Not solved

24.75 problem 75

Internal problem ID [10065]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 75.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - (a\cosh(x) + b)\,y + ab\sinh(x) - c = 0}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*cosh(x)+b)*y(x)-a*b*sinh(x)+c,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*Cosh[x]+b)*y[x]-a*b*Sinh[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

24.76 problem 76

Internal problem ID [10066]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 76.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - (a\sinh(x) + b)\,y + ab\cosh(x) - c = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*sinh(x)+b)*y(x)-a*b*cosh(x)+c,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*Sinh[x]+b)*y[x]-a*b*Cosh[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

24.77 problem 77

Internal problem ID [10067]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 77.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - (2\ln(x) + a + 1)\, y - x\left(-\ln(x)^2 - a\ln(x) + b\right) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 163

```
dsolve(y(x)*diff(y(x),x)=(2*ln(x)+a+1)*y(x)+x*( -(ln(x))^2-a*ln(x)+b),y(x), singsol=all)
```

$$y(x) = \frac{x\left(-\tanh\left(\frac{\mathrm{RootOf}\left(\mathrm{e}^{-\frac{2\,\mathrm{arctanh}\left(\frac{2\_a-a}{\sqrt{a^2+4b}}\right)}{\sqrt{a^2+4b}}}\tanh\left(\frac{\_Z\sqrt{a^2+4b}}{2}\right)\sqrt{a^2+4b}-\sqrt{a^2+4b}\,\tanh\left(\frac{\_Z\sqrt{a^2+4b}}{2}\right)\mathrm{e}^{\_Z}+2\,\mathrm{e}^{\_Z}\ln(x)-\mathrm{e}^{-\frac{2\,\mathrm{arctanh}(\cdots}{\sqrt{a^2\cdots}}}\cdots\right.}{2}\cdots\right.\right.}{2}\cdots$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(2*Log[x]+a+1)*y[x]+x*( -(Log[x])^2-a*Log[x]+b),y[x],x,IncludeSingularSolu
```

Not solved

24.78 problem 78

Internal problem ID [10068]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 78.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$y'y - \left(2\ln(x)^2 + 2\ln(x) + a\right)y - x\left(-\ln(x)^4 - a\ln(x)^2 + b\right) = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(2*(ln(x))^2+2*ln(x)+a)*y(x)+x*(- (ln(x))^4-a*(ln(x))^2+b),y(x), sin
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(2*(Log[x])^2+2*Log[x]+a)*y[x]+x*(- (Log[x])^4-a*(Log[x])^2+b),y[x],x,Incl
```

Not solved

24.79 problem 79

Internal problem ID [10069]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, ‘2nd type‘, ‘class A‘]]`

$$y'y - ax\cos\left(\lambda\,x^2\right)y - x = 0$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*x*cos(lambda*x^2)*y(x)+x,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*x*Cos[\[Lambda]*x^2]*y[x]+x,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

24.80 problem 80

Internal problem ID [10070]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 80.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{y'y - ax\sin(\lambda x^2)\, y - x = 0}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*x*sin(lambda*x^2)*y(x)+x,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*x*Sin[\[Lambda]*x^2]*y[x]+x,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

25 Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x) + g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$

25.1 problem 1

Internal problem ID [10071]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x) + g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd typ`

$$(yA + Bx + a)\, y' + By + kx + b = 0$$

✓ Solution by Maple

Time used: 0.703 (sec). Leaf size: 113

```
dsolve((A*y(x)+B*x+a)*diff(y(x),x)+B*y(x)+k*x+b=0,y(x), singsol=all)
```

$$y(x) = \frac{-Bb + ak + \frac{B(x(Ak-B^2)+Ab-aB)c_1+\sqrt{-A\,c_1^2k(x(Ak-B^2)+Ab-aB)^2+B^2(x(Ak-B^2)+Ab-aB)^2c_1^2+A}}{Ac_1}}{-Ak + B^2}$$

✓ Solution by Mathematica

Time used: 16.738 (sec). Leaf size: 106

```
DSolve[(A*y[x]+B*x+a)*y'[x]+B*y[x]+k*x+b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\frac{\sqrt{\frac{(a+Bx)^2}{A}+Ac_1-x(2b+kx)}}{\sqrt{\frac{1}{A}}} + a + Bx}{A}$$

$$y(x) \to -\frac{a + Bx}{A} + \sqrt{\frac{1}{A}}\sqrt{\frac{(a + Bx)^2}{A} + Ac_1 - x(2b + kx)}$$

25.2 problem 2

Internal problem ID [10072]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x) + g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(y + ax + b)\, y' - \alpha y - \beta x - \gamma = 0}$$

✓ Solution by Maple

Time used: 0.672 (sec). Leaf size: 186

```
dsolve((y(x)+a*x+b)*diff(y(x),x)=alpha*y(x)+beta*x+gamma,y(x), singsol=all)
```

$$y(x) = \frac{a\gamma - b\beta - \frac{(x(a\alpha-\beta)+b\alpha-\gamma)\left(\tan\left(\operatorname{RootOf}\left(\sqrt{-a^2+2a\alpha-\alpha^2-4\beta}\,\ln\left(-\frac{(x(a\alpha-\beta)+b\alpha-\gamma)^2\left(\tan\left(\_Z\right)^2 a^2-2\tan\left(\_Z\right)^2 a\alpha+\alpha^2\tan\left(\_Z\right)^2\right.}{4}\right.\right.\right.\right.}{2}}{-a\alpha+\beta}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y[x]*a*x+b)*y'[x]==\[Alpha]*y[x]+\[Beta]*x+\[Gamma],y[x],x,IncludeSingularSolutions -
```

Not solved

25.3 problem 3

Internal problem ID [10073]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x) + g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{(y + x^2ak + bx + c)\, y' + y^2a - 2akxy - my - k(k + b - m)\, x - s = 0}$$

✗ Solution by Maple

```
dsolve((y(x)+a*k*x^2+b*x+c)*diff(y(x),x)=-a*y(x)^2+2*a*k*x*y(x)+m*y(x)+k*(k+b-m)*x+s,y(x), si
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y[x]+a*k*x^2+b*x+c)*y'[x]==-a*y[x]^2+2*a*k*x*y[x]+m*y[x]+k*(k+b-m)*x+s,y[x],x,Include
```

Timed out

25.4 problem 4

Internal problem ID [10074]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x)+g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'],`

$$(y + A\,x^n + a)\,y' + nA\,x^{n-1}y + k\,x^m + b = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 299

```
dsolve((y(x)+A*x^n+a)*diff(y(x),x)+n*A*x^(n-1)*y(x)+k*x^m+b=0,y(x), singsol=all)
```

$$y(x) = -\frac{Am\,x^n + A\,x^n + am - \sqrt{A^2x^{2n}m^2 + 2A^2x^{2n}m + 2A\,x^n a\,m^2 + x^{2n}A^2 + 4A\,x^n am + m^2a^2 - 2b\,m^2x + 2}}{m+1}$$

$$y(x) = -\frac{Am\,x^n + A\,x^n + am + \sqrt{A^2x^{2n}m^2 + 2A^2x^{2n}m + 2A\,x^n a\,m^2 + x^{2n}A^2 + 4A\,x^n am + m^2a^2 - 2b\,m^2x + 2}}{m+1}$$

✓ Solution by Mathematica

Time used: 14.082 (sec). Leaf size: 118

```
DSolve[(y[x]+A*x^n+a)*y'[x]+n*A*x^(n-1)*y[x]+k*x^m+b==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\sqrt{\frac{1}{x}}\sqrt{x\left((a+Ax^n)^2 - \frac{2x\,(bm+b+kx^m)}{m+1} + c_1\right)} - a - Ax^n$$

$$y(x) \to \sqrt{\frac{1}{x}}\sqrt{x\left((a+Ax^n)^2 - \frac{2x\,(bm+b+kx^m)}{m+1} + c_1\right)} - a - Ax^n$$

25.5 problem 5

Internal problem ID [10075]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x) + g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{\left(y+a\,x^{1+n}+b\,x^{n}\right)y'-\left(an\,x^{n}+c\,x^{n-1}\right)y=0}$$

✗ Solution by Maple

```
dsolve((y(x)+a*x^(n+1)+b*x^n)*diff(y(x),x)=(a*n*x^n+c*x^(n-1))*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y[x]+a*x^(n+1)+b*x^n)*y'[x]==(a*n*x^n+c*x^(n-1))*y[x],y[x],x,IncludeSingularSolutions
```

Not solved

25.6 problem 6

Internal problem ID [10076]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x) + g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$yxy' - y^2a - by - c\,x^n - s = 0$$

✗ Solution by Maple

```
dsolve(x*y(x)*diff(y(x),x)=a*y(x)^2+b*y(x)+c*x^n+s,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y[x]*y'[x]==a*y[x]^2+b*y[x]+c*x^n+s,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

25.7 problem 7

Internal problem ID [10077]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x) + g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yxy' + y^2n - a(1+2n)\,xy - by + a^2n\,x^2 + abx - c = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 223

```
dsolve(x*y(x)*diff(y(x),x)=-n*y(x)^2+a*(2*n+1)*x*y(x)+b*y(x)-a^2*n*x^2-a*b*x+c,y(x), singsol=
```

$$c_1 + \frac{\left(\frac{1}{ax-y(x)}\right)^{\frac{1}{n}}\left(\frac{-n\,a^2x^2-x(-2y(x)n+b)a-ny(x)^2+by(x)+c}{(ax-y(x))^2}\right)^{-\frac{1}{2n}} y(x)\,\mathrm{e}^{\frac{b\,\operatorname{arctanh}\left(\frac{b(ax-y(x))-2c}{\sqrt{b^2+4cn}\,(ax-y(x))}\right)}{\sqrt{b^2+4cn}\,n}} - \left(\int^{\frac{1}{ax-y(x)}} \_a^{\frac{1}{n}}\left(\_a^2c -\right.\right.}{(ax-y(x))\,x} = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y[x]*y'[x]==-n*y[x]^2+a*(2*n+1)*x*y[x]+b*y[x]-a^2*n*x^2-a*b*x+c,y[x],x,IncludeSingul
```

Not solved

26 Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213

26.1 problem 1

Internal problem ID [10078]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + ya = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\sqrt{a}\, x\right) + c_2 \cos\left(\sqrt{a}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y''[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\sqrt{a}x\right) + c_2 \sin\left(\sqrt{a}x\right)$$

26.2 problem 2

Internal problem ID [10079]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (ax + b)\, y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)-(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{AiryAi}\left(\frac{ax + b}{(-a)^{\frac{2}{3}}}\right) + c_2 \operatorname{AiryBi}\left(\frac{ax + b}{(-a)^{\frac{2}{3}}}\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 36

```
DSolve[y''[x]-(a*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{AiryAi}\left(\frac{b + ax}{a^{2/3}}\right) + c_2 \operatorname{AiryBi}\left(\frac{b + ax}{a^{2/3}}\right)$$

26.3 problem 3

Internal problem ID [10080]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (a^2x^2 + a)\, y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-(a^2*x^2+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{a\,x^2}{2}} + c_2 \mathrm{e}^{\frac{a\,x^2}{2}} \operatorname{erf}\left(\sqrt{a}\,x\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 36

```
DSolve[y''[x]-(a^2*x^2+a)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \,\text{ParabolicCylinderD}\left(-1, \sqrt{2}\sqrt{a}x\right) + c_2 e^{\frac{ax^2}{2}}$$

26.4 problem 4

Internal problem ID [10081]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(a\,x^2 + b\right)y = 0}$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$2)-(a*x^2+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{WhittakerM}\left(-\frac{b}{4\sqrt{a}}, \frac{1}{4}, x^2\sqrt{a}\right)}{\sqrt{x}} + \frac{c_2 \operatorname{WhittakerW}\left(-\frac{b}{4\sqrt{a}}, \frac{1}{4}, x^2\sqrt{a}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 68

```
DSolve[y''[x]-(a*x^2+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \rightarrow\ & c_1 \operatorname{ParabolicCylinderD}\left(-\frac{b}{2\sqrt{a}} - \frac{1}{2}, \sqrt{2}\sqrt[4]{a}x\right) \\ & + c_2 \operatorname{ParabolicCylinderD}\left(\frac{1}{2}\left(\frac{b}{\sqrt{a}} - 1\right), i\sqrt{2}\sqrt[4]{a}x\right) \end{aligned}$$

26.5 problem 5

Internal problem ID [10082]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + a^3 x(-ax+2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$2)+a^3*x*(2-a*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{ax(ax-2)}{2}} + c_2 \mathrm{e}^{-\frac{1}{2}a^2x^2+ax} \operatorname{erf}(iax - i)$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 50

```
DSolve[y''[x]+a^3*x*(2-a*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{1}{2}ax(ax-2)-1}\left(2eac_1 - \sqrt{\pi}c_2 \operatorname{erfi}(1-ax)\right)}{2a}$$

26.6 problem 6

Internal problem ID [10083]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \left(a\,x^2 + bcx\right) y = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 119

```
dsolve(diff(y(x),x$2)-(a*x^2+b*x*c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{hypergeom}\left(\left[-\frac{b^2c^2 - 4a^{\frac{3}{2}}}{16a^{\frac{3}{2}}}\right], \left[\frac{1}{2}\right], \frac{(2ax+bc)^2}{4a^{\frac{3}{2}}}\right) \mathrm{e}^{-\frac{x(ax+bc)}{2\sqrt{a}}} + c_2(2ax+bc) \operatorname{hypergeom}\left(\left[-\frac{b^2c^2 - 12a^{\frac{3}{2}}}{16a^{\frac{3}{2}}}\right], \left[\frac{3}{2}\right], \frac{(2ax+bc)^2}{4a^{\frac{3}{2}}}\right) \mathrm{e}^{-\frac{x(ax+bc)}{2\sqrt{a}}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 92

```
DSolve[y''[x]-(a*x^2+b*x*c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \operatorname{ParabolicCylinderD}\left(-\frac{b^2c^2}{8a^{3/2}} - \frac{1}{2}, \frac{i(bc+2ax)}{\sqrt{2}a^{3/4}}\right) + c_1 \operatorname{ParabolicCylinderD}\left(\frac{1}{8}\left(\frac{b^2c^2}{a^{3/2}} - 4\right), \frac{bc+2ax}{\sqrt{2}a^{3/4}}\right)$$

26.7 problem 7

Internal problem ID [10084]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - a\,x^n y = 0}$$

✓ Solution by Maple

Time used: 0.875 (sec). Leaf size: 65

```
dsolve(diff(y(x),x$2)-a*x^n*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x}\,\mathrm{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{-a}\,x^{\frac{n}{2}+1}}{n+2}\right) + c_2\sqrt{x}\,\mathrm{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{-a}\,x^{\frac{n}{2}+1}}{n+2}\right)$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 119

```
DSolve[y''[x]-a*x^n*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (n+2)^{-\frac{1}{n+2}}\sqrt{x}a^{\frac{1}{2n+4}}\left(c_1\,\mathrm{Gamma}\left(\frac{n+1}{n+2}\right)\mathrm{BesselI}\left(-\frac{1}{n+2}, \frac{2\sqrt{a}x^{\frac{n}{2}+1}}{n+2}\right) + c_2(-1)^{\frac{1}{n+2}}\,\mathrm{Gamma}\left(1+\frac{1}{n+2}\right)\mathrm{BesselI}\left(\frac{1}{n+2}, \frac{2\sqrt{a}x^{\frac{n}{2}+1}}{n+2}\right)\right)$$

26.8 problem 8

Internal problem ID [10085]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - a\left(x^{2n}a + n\,x^{n-1}\right)y = 0}$$

✓ Solution by Maple

Time used: 0.407 (sec). Leaf size: 137

```
dsolve(diff(y(x),x$2)-a*(a*x^(2*n)+n*x^(n-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{a\,x^{1+n}}{1+n}} + c_2 \left(\frac{x^{-\frac{3n}{2}-1}(n+2)^2 \operatorname{WhittakerM}\left(\frac{n+2}{2+2n}, \frac{3+2n}{2+2n}, \frac{2a\,x^{1+n}}{1+n}\right)}{2} + \operatorname{WhittakerM}\left(-\frac{n}{2+2n}, \frac{3+2n}{2+2n}, \frac{2a\,x^{1+n}}{1+n}\right)\left(\left(\frac{n}{2}+1\right)x^{-\frac{3n}{2}-1} + a\,x^{-\frac{n}{2}}\right)(1+n)\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-a*(a*x^(2*n)+n*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

26.9 problem 9

Internal problem ID [10086]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - a\,x^{n-2}(x^n a + n + 1)\,y = 0$$

✓ Solution by Maple

Time used: 0.562 (sec). Leaf size: 109

```
dsolve(diff(y(x),x$2)-a*x^(n-2)*(a*x^n+n+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\,\mathrm{e}^{\frac{a\,x^n}{n}} + c_2\left(\left(\left(n^2-n\right)x^{-\frac{3n}{2}+\frac{1}{2}} + 2an\,x^{-\frac{n}{2}+\frac{1}{2}}\right)\mathrm{WhittakerM}\left(-\frac{1}{2}-\frac{1}{2n}, -\frac{1}{2n}+1, \frac{2a\,x^n}{n}\right) + x^{-\frac{3n}{2}+\frac{1}{2}}\,\mathrm{WhittakerM}\left(\frac{1}{2}-\frac{1}{2n}, -\frac{1}{2n}+1, \frac{2a\,x^n}{n}\right)(n-1)^2\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-a*x^(n-2)*(a*x^n+n+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

26.10 problem 10

Internal problem ID [10087]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(x^{2n}a + b\,x^{n-1}\right) y = 0}$$

✓ Solution by Maple

Time used: 0.86 (sec). Leaf size: 95

```
dsolve(diff(y(x),x$2)+(a*x^(2*n)+b*x^(n-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\mathrm{WhittakerM}\left(-\frac{ib}{\sqrt{a}\,(2+2n)}, \frac{1}{2+2n}, \frac{2i\sqrt{a}\,x^{1+n}}{1+n}\right) x^{-\frac{n}{2}} + c_2 \,\mathrm{WhittakerW}\left(-\frac{ib}{\sqrt{a}\,(2+2n)}, \frac{1}{2+2n}, \frac{2i\sqrt{a}\,x^{1+n}}{1+n}\right) x^{-\frac{n}{2}}$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 199

```
DSolve[y''[x]+(a*x^(2*n)+b*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2^{\frac{n}{2n+2}} x^{-n/2} \left(x^{n+1}\right)^{\frac{n}{2n+2}} e^{-\frac{\sqrt{a}x^{n+1}}{\sqrt{-(n+1)^2}}} \left(c_1 \,\mathrm{HypergeometricU}\left(\frac{n^2+n-\frac{b\sqrt{-(n+1)^2}}{\sqrt{a}}}{2(n+1)^2}, \frac{n}{n+1}, \frac{2\sqrt{a}x^{n+1}}{\sqrt{-(n+1)^2}}\right) + c_2 L^{-\frac{1}{n+1}}_{-\frac{n^2+n-\frac{b\sqrt{-(n+1)^2}}{\sqrt{a}}}{2(n+1)^2}}\left(\frac{2\sqrt{a}x^{n+1}}{\sqrt{-(n+1)^2}}\right)\right)$$

27 Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$

27.1 problem 11

Internal problem ID [10088]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + ay' + by = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\left(-\frac{a}{2}+\frac{\sqrt{a^2-4b}}{2}\right)x} + c_2 \mathrm{e}^{\left(-\frac{a}{2}-\frac{\sqrt{a^2-4b}}{2}\right)x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 47

```
DSolve[y''[x]+a*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}x\left(\sqrt{a^2-4b}+a\right)}\left(c_2 e^{x\sqrt{a^2-4b}} + c_1\right)$$

27.2 problem 12

Internal problem ID [10089]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + ay' + (bx + c)\,y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+(b*x+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{ax}{2}} \operatorname{AiryAi}\left(\frac{a^2 - 4xb - 4c}{4b^{\frac{2}{3}}}\right) + c_2 \mathrm{e}^{-\frac{ax}{2}} \operatorname{AiryBi}\left(\frac{a^2 - 4xb - 4c}{4b^{\frac{2}{3}}}\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 67

```
DSolve[y''[x]+a*y'[x]+(b*x+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{ax}{2}} \left(c_1 \operatorname{AiryAi}\left(\frac{a^2 - 4(c + bx)}{4(-b)^{2/3}}\right) + c_2 \operatorname{AiryBi}\left(\frac{a^2 - 4(c + bx)}{4(-b)^{2/3}}\right)\right)$$

27.3 problem 13

Internal problem ID [10090]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + ay' - \left(b\,x^2 + c\right) y = 0}$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 85

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)-(b*x^2+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \,\mathrm{KummerM}\left(\frac{a^2 + 12\sqrt{b} + 4c}{16\sqrt{b}}, \frac{3}{2}, \sqrt{b}\,x^2\right) \mathrm{e}^{-\frac{x\left(\sqrt{b}\,x+a\right)}{2}} + c_2 x \,\mathrm{KummerU}\left(\frac{a^2 + 12\sqrt{b} + 4c}{16\sqrt{b}}, \frac{3}{2}, \sqrt{b}\,x^2\right) \mathrm{e}^{-\frac{x\left(\sqrt{b}\,x+a\right)}{2}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 94

```
DSolve[y''[x]+a*y'[x]-(b*x^2+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}x\left(a+\sqrt{b}x\right)} \left(c_1 \,\mathrm{HermiteH}\left(-\frac{a^2 + 4\left(c + \sqrt{b}\right)}{8\sqrt{b}}, \sqrt[4]{b}x\right) + c_2 \,\mathrm{Hypergeometric1F1}\left(\frac{a^2 + 4\left(c + \sqrt{b}\right)}{16\sqrt{b}}, \frac{1}{2}, \sqrt{b}x^2\right)\right)$$

27.4 problem 14

Internal problem ID [10091]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$\boxed{y'' + ay' + b(-b\,x^2 + ax + 1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*(-b*x^2+a*x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \operatorname{erf}\left(x\sqrt{-b} + \frac{a}{2\sqrt{-b}}\right) \mathrm{e}^{-\frac{b\,x^2}{2}} c_1 + c_2 \mathrm{e}^{-\frac{b\,x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 56

```
DSolve[y''[x]+a*y'[x]+b*(-b*x^2+a*x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{bx^2}{2}} \left(\frac{c_2 e^{x(bx-a)} \operatorname{DawsonF}\left(\frac{2bx-a}{2\sqrt{b}}\right)}{\sqrt{b}} + c_1 \right)$$

27.5 problem 15

Internal problem ID [10092]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$y'' + ay' + bx\left(-b\,x^3 + ax + 2\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*x*(-b*x^3+a*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\left(\int \mathrm{e}^{-ax}\mathrm{e}^{\frac{2x^3b}{3}}dx\right)c_1 + c_2\right)\mathrm{e}^{-\frac{x^3b}{3}}$$

✓ Solution by Mathematica

Time used: 0.123 (sec). Leaf size: 46

```
DSolve[y''[x]+a*y'[x]+b*x*(-b*x^3+a*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{bx^3}{3}}\left(c_2\int_1^x e^{\frac{2}{3}bK[1]^3-aK[1]}dK[1] + c_1\right)$$

27.6 problem 16

Internal problem ID [10093]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear, '`

$$y'' + ay' + b\left(-b\,x^{2n} + x^n a + n\,x^{n-1}\right) y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*(-b*x^(2*n)+a*x^n+n*x^(n-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\left(\int e^{-ax} e^{\frac{2b\,x^{1+n}}{1+n}} dx\right) c_1 + c_2\right) e^{-\frac{b\,x^{1+n}}{1+n}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+a*y'[x]+b*(-b*x^(2*n)+a*x^n+n*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolutions
```

Not solved

27.7 problem 17

Internal problem ID [10094]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + ay' + b\left(-b\,x^{2n} - x^n a + n\,x^{n-1}\right) y = 0$$

✓ Solution by Maple

Time used: 0.406 (sec). Leaf size: 80

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*(-b*x^(2*n)-a*x^n+n*x^(n-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{anx+b\,x^{1+n}+ax}{1+n}} + c_2 \left(\int \mathrm{e}^{\frac{anx+2b\,x^{1+n}+ax}{1+n}} dx\right) \mathrm{e}^{-\frac{anx+b\,x^{1+n}+ax}{1+n}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+a*y'[x]+b*(-b*x^(2*n)-a*x^n+n*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolutions
```

Not solved

27.8 problem 18

Internal problem ID [10095]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + (n-1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+(n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x^2}{2}} \operatorname{KummerM}\left(\frac{3}{2} - \frac{n}{2}, \frac{3}{2}, \frac{x^2}{2}\right) x + c_2 \mathrm{e}^{-\frac{x^2}{2}} \operatorname{KummerU}\left(\frac{3}{2} - \frac{n}{2}, \frac{3}{2}, \frac{x^2}{2}\right) x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 51

```
DSolve[y''[x]+x*y'[x]+(n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} \left(c_1 \operatorname{HermiteH}\left(n-2, \frac{x}{\sqrt{2}}\right) + c_2 \operatorname{Hypergeometric1F1}\left(1 - \frac{n}{2}, \frac{1}{2}, \frac{x^2}{2}\right)\right)$$

27.9 problem 19

Internal problem ID [10096]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 2yn = 0}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+2*n*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x \operatorname{KummerM}\left(-\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, x^2\right) + c_2 x \operatorname{KummerU}\left(-\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, x^2\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 27

```
DSolve[y''[x]-2*x*y'[x]+2*n*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{HermiteH}(n, x) + c_2 \operatorname{Hypergeometric1F1}\left(-\frac{n}{2}, \frac{1}{2}, x^2\right)$$

27.10 problem 20

Internal problem ID [10097]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'ax + by = 0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 65

```
dsolve(diff(y(x),x$2)+a*x*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{a\,x^2}{2}} \operatorname{KummerM}\left(\frac{2a-b}{2a}, \frac{3}{2}, \frac{a\,x^2}{2}\right) x + c_2 \mathrm{e}^{-\frac{a\,x^2}{2}} \operatorname{KummerU}\left(\frac{2a-b}{2a}, \frac{3}{2}, \frac{a\,x^2}{2}\right) x$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 67

```
DSolve[y''[x]+a*x*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{ax^2}{2}} \left(c_1 \operatorname{HermiteH}\left(\frac{b}{a} - 1, \frac{\sqrt{a}x}{\sqrt{2}}\right) + c_2 \operatorname{Hypergeometric1F1}\left(\frac{a-b}{2a}, \frac{1}{2}, \frac{ax^2}{2}\right)\right)$$

27.11 problem 21

Internal problem ID [10098]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'ax + bxy = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)+a*x*diff(y(x),x)+b*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{bx}{a}} \operatorname{KummerM}\left(\frac{b^2}{2a^3}, \frac{1}{2}, -\frac{(a^2x-2b)^2}{2a^3}\right) + c_2 e^{-\frac{bx}{a}} \operatorname{KummerU}\left(\frac{b^2}{2a^3}, \frac{1}{2}, -\frac{(a^2x-2b)^2}{2a^3}\right)$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 96

```
DSolve[y''[x]+a*x*y'[x]+b*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{bx}{a}-\frac{ax^2}{2}} \left(c_2 \operatorname{Hypergeometric1F1}\left(\frac{1}{2}-\frac{b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x-2b)^2}{2a^3}\right) + c_1 \operatorname{HermiteH}\left(\frac{b^2}{a^3}-1, \frac{a^2x-2b}{\sqrt{2}a^{3/2}}\right)\right)$$

27.12 problem 22

Internal problem ID [10099]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'ax + (bx + c)\, y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 89

```
dsolve(diff(y(x),x$2)+a*x*diff(y(x),x)+(b*x+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{bx}{a}} \operatorname{KummerM}\left(\frac{c\,a^2 + b^2}{2a^3}, \frac{1}{2}, -\frac{(a^2x - 2b)^2}{2a^3}\right) + c_2 e^{-\frac{bx}{a}} \operatorname{KummerU}\left(\frac{c\,a^2 + b^2}{2a^3}, \frac{1}{2}, -\frac{(a^2x - 2b)^2}{2a^3}\right)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 107

```
DSolve[y''[x]+a*x*y'[x]+(b*x+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{bx}{a} - \frac{ax^2}{2}} \left(c_2 \operatorname{Hypergeometric1F1}\left(-\frac{(c-a)a^2 + b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x - 2b)^2}{2a^3}\right) + c_1 \operatorname{HermiteH}\left(\frac{b^2}{a^3} + \frac{c}{a} - 1, \frac{a^2x - 2b}{\sqrt{2}a^{3/2}}\right)\right)$$

27.13 problem 23

Internal problem ID [10100]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y'ax + \left(b\,x^4 + a^2x^2 + cx + a\right)y = 0}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 97

```
dsolve(diff(y(x),x$2)+2*a*x*diff(y(x),x)+(b*x^4+a^2*x^2+c*x+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x\, \mathrm{e}^{-\frac{x^2\left(i\sqrt{b}\,x+\frac{3a}{2}\right)}{3}}\,\mathrm{KummerM}\left(\frac{ic+4\sqrt{b}}{6\sqrt{b}}, \frac{4}{3}, \frac{2i\sqrt{b}\,x^3}{3}\right) + c_2 x\, \mathrm{e}^{-\frac{x^2\left(i\sqrt{b}\,x+\frac{3a}{2}\right)}{3}}\,\mathrm{KummerU}\left(\frac{ic+4\sqrt{b}}{6\sqrt{b}}, \frac{4}{3}, \frac{2i\sqrt{b}\,x^3}{3}\right)$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 119

```
DSolve[y''[x]+2*a*x*y'[x]+(b*x^4+a^2*x^2+c*x+a)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{\sqrt[3]{2}\sqrt[3]{x^3}e^{\frac{1}{6}x^2\left(-3a+2i\sqrt{b}x\right)}\left(c_1\,\mathrm{HypergeometricU}\left(\frac{1}{3}-\frac{ic}{6\sqrt{b}}, \frac{2}{3}, -\frac{2}{3}i\sqrt{b}x^3\right) + c_2 L_{\frac{ic}{6\sqrt{b}}-\frac{1}{3}}^{-\frac{1}{3}}\left(-\frac{2}{3}i\sqrt{b}x^3\right)\right)}{x}$$

27.14 problem 24

Internal problem ID [10101]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y'(ax + b) - ya = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 66

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)-a*y(x)=0,y(x), singsol=all)
```

$$y(x) = (ax + b)\, c_1 + c_2 \left(\pi(ax + b)\, \mathrm{e}^{\frac{b^2}{2a}} \operatorname{erf}\left(\frac{\sqrt{2}\,(ax + b)}{2\sqrt{a}}\right) + \sqrt{2}\,\sqrt{\pi}\,\sqrt{a}\,\mathrm{e}^{-\frac{x(ax+2b)}{2}}\right)$$

✓ Solution by Mathematica

Time used: 0.274 (sec). Leaf size: 82

```
DSolve[y''[x]+(a*x+b)*y'[x]-a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(ax + b)\left(-\frac{\sqrt{\frac{\pi}{2}}\, c_2 \operatorname{erf}\left(\frac{ax+b}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}} - \frac{c_2 e^{-\frac{(ax+b)^2}{2a}}}{a(ax+b)} + c_1\right)}{b}$$

27.15 problem 25

Internal problem ID [10102]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y'(ax + b) + ya = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = \operatorname{erf}\left(-\frac{\sqrt{-2a}\,x}{2} + \frac{b}{\sqrt{-2a}}\right) e^{-\frac{1}{2}a\,x^2 - xb} c_1 + c_2 e^{-\frac{1}{2}a\,x^2 - xb}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 53

```
DSolve[y''[x]+(a*x+b)*y'[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{2}c_1 \operatorname{DawsonF}\left(\frac{b+ax}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}} + c_2 e^{-\frac{1}{2}x(ax+2b)}$$

27.16 problem 26

Internal problem ID [10103]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-1 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'(ax + b) + c(ax + b - c)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+c*(a*x+b-c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-cx} + c_2 \mathrm{e}^{-cx} \operatorname{erf}\left(\frac{\sqrt{2}\,(ax + b - 2c)}{2\sqrt{a}}\right)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 63

```
DSolve[y''[x]+(a*x+b)*y'[x]+c*(a*x+b-c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{\frac{(b-2c)^2}{2a} - cx} \left(\sqrt{\pi} c_1 \operatorname{erfc}\left(\frac{ax + b - 2c}{\sqrt{2}\sqrt{a}}\right) + 2c_2\right)$$