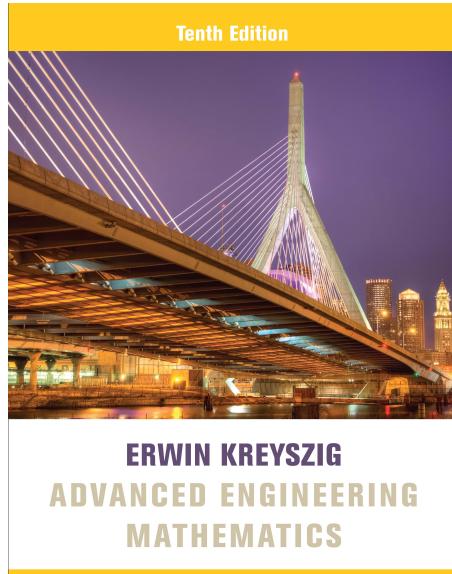


A Solution Manual For

**ADVANCED ENGINEERING  
MATHEMATICS. ERWIN  
KREYSZIG, HERBERT  
KREYSZIG, EDWARD J.  
NORMINTON. 10th edition.  
John Wiley USA. 2011**



# **Nasser M. Abbasi**

March 3, 2024

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# **1 Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.1. page 174**

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## 1.1 problem 6

Internal problem ID [5623]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.1. page 174

**Problem number:** 6.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y'(1+x) - y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
Order:=6;
dsolve((1+x)*diff(y(x),x)=y(x),y(x),type='series',x=0);
```

$$y(x) = y(0)(x + 1)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 9

```
AsymptoticDSolveValue[(1+x)*y'[x]==y[x],y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1(x + 1)$$

## 1.2 problem 7

Internal problem ID [5624]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.1. page 174

**Problem number:** 7.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$2yx + y' = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
Order:=6;
dsolve(diff(y(x),x)=-2*x*y(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 + \frac{1}{2}x^4\right)y(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 20

```
AsymptoticDSolveValue[y'[x]==-2*x*y[x],y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left( \frac{x^4}{2} - x^2 + 1 \right)$$

### 1.3 problem 8

Internal problem ID [5625]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.1. page 174

**Problem number:** 8.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$-3y + y'x = k$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
Order:=6;
dsolve(x*diff(y(x),x)-3*y(x)=k,y(x),type='series',x=0);
```

$$y(x) = c_1 x^3 \left(1 + O(x^6)\right) + \left(-\frac{k}{3} + O(x^6)\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 15

```
AsymptoticDSolveValue[x*y'[x]-3*y[x]==k,y[x],{x,0,5}]
```

$$y(x) \rightarrow -\frac{k}{3} + c_1 x^3$$

## 1.4 problem 9

Internal problem ID [5626]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.1. page 174

**Problem number:** 9.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x]`

$$y'' + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4\right)y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2\left(\frac{x^5}{120} - \frac{x^3}{6} + x\right) + c_1\left(\frac{x^4}{24} - \frac{x^2}{2} + 1\right)$$

## 1.5 problem 10

Internal problem ID [5627]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.1. page 174

**Problem number:** 10.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' - y' + yx = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)-diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) &= \left(1 - \frac{1}{6}x^3 - \frac{1}{24}x^4 - \frac{1}{120}x^5\right)y(0) \\ &\quad + \left(x + \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{24}x^4 - \frac{1}{30}x^5\right)D(y)(0) + O(x^6) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x] - y'[x] + x*y[x] == 0, y[x], {x, 0, 5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{x^5}{120} - \frac{x^4}{24} - \frac{x^3}{6} + 1\right) + c_2 \left(-\frac{x^5}{30} - \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x\right)$$

## 1.6 problem 11

Internal problem ID [5628]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.1. page 174

**Problem number:** 11.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' - y' + yx^2 = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=6;
dsolve(diff(y(x),x$2)-diff(y(x),x)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{12}x^4 - \frac{1}{60}x^5\right)y(0) + \left(x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{24}x^5\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[y''[x]-y'[x]+x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1\left(-\frac{x^5}{60} - \frac{x^4}{12} + 1\right) + c_2\left(-\frac{x^5}{24} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x\right)$$

## 1.7 problem 12

Internal problem ID [5629]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.1. page 174

**Problem number:** 12.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Gegenbauer]

$$(-x^2 + 1) y'' - 2y'x + 2y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 - \frac{1}{3}x^4\right) y(0) + D(y)(0)x + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 25

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left( -\frac{x^4}{3} - x^2 + 1 \right) + c_2 x$$

## 1.8 problem 13

Internal problem ID [5630]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.1. page 174

**Problem number:** 13.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' + y(x^2 + 1) = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+(1+x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{24}x^4\right)y(0) + \left(x - \frac{1}{6}x^3 - \frac{1}{24}x^5\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+(1+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2\left(-\frac{x^5}{24} - \frac{x^3}{6} + x\right) + c_1\left(-\frac{x^4}{24} - \frac{x^2}{2} + 1\right)$$

## 1.9 problem 14

Internal problem ID [5631]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.1. page 174

**Problem number:** 14.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' - 4y'x + (4x^2 - 2)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
Order:=6;
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x^2 + \frac{1}{2}x^4\right)y(0) + \left(x + x^3 + \frac{1}{2}x^5\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2\left(\frac{x^5}{2} + x^3 + x\right) + c_1\left(\frac{x^4}{2} + x^2 + 1\right)$$

## 1.10 problem 16

Internal problem ID [5632]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.1. page 174

**Problem number:** 16.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$4y + y' = 1$$

With initial conditions

$$\left[ y(0) = \frac{5}{4} \right]$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x)+4*y(x)=1,y(0) = 5/4],y(x),type='series',x=0);
```

$$y(x) = \frac{5}{4} - 4x + 8x^2 - \frac{32}{3}x^3 + \frac{32}{3}x^4 - \frac{128}{15}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{y'[x]+4*y[x]==1,{y[0]==125/100}],y[x],{x,0,5}]
```

$$y(x) \rightarrow -\frac{128x^5}{15} + \frac{32x^4}{3} - \frac{32x^3}{3} + 8x^2 - 4x + \frac{5}{4}$$

## 1.11 problem 17

Internal problem ID [5633]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.1. page 174

**Problem number:** 17.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' + 3y'x + 2y = 0$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)+3*x*diff(y(x),x)+2*y(x)=0,y(0) = 1, D(y)(0) = 1],y(x),type='series',x)
```

$$y(x) = 1 + x - x^2 - \frac{5}{6}x^3 + \frac{2}{3}x^4 + \frac{11}{24}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[{y''[x]+3*x*y'[x]+2*y[x]==0,{y[0]==1,y'[0]==1}],y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{11x^5}{24} + \frac{2x^4}{3} - \frac{5x^3}{6} - x^2 + x + 1$$

## 1.12 problem 18

Internal problem ID [5634]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.1. page 174

**Problem number:** 18.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Gegenbauer]

$$(-x^2 + 1) y'' - 2y'x + 30y = 0$$

With initial conditions

$$\left[ y(0) = 0, y'(0) = \frac{15}{8} \right]$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([(1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+30*y(x)=0,y(0) = 0, D(y)(0) = 15/8],y(x),typ
```

$$y(x) = \frac{15}{8}x - \frac{35}{4}x^3 + \frac{63}{8}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 23

```
AsymptoticDSolveValue[{{(1-x^2)*y''[x]-2*x*y'[x]+30*y[x]==0,{y[0]==0,y'[0]==1875/1000}},y[x],
```

$$y(x) \rightarrow \frac{63x^5}{8} - \frac{35x^3}{4} + \frac{15x}{8}$$

## 1.13 problem 19

Internal problem ID [5635]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.1. page 174

**Problem number:** 19.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$(x - 2) y' - yx = 0$$

With initial conditions

$$[y(0) = 4]$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
Order:=6;
dsolve([(x-2)*diff(y(x),x)=x*y(x),y(0) = 4],y(x),type='series',x=0);
```

$$y(x) = 4 - x^2 - \frac{1}{3}x^3 + \frac{1}{30}x^5 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 24

```
AsymptoticDSolveValue[{{(x-2)*y'[x]==x*y[x],{y[0]==4}},y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{x^5}{30} - \frac{x^3}{3} - x^2 + 4$$

## **2 Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186**

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## 2.1 problem 2

Internal problem ID [5636]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 2.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _exact, _linear, _homogeneous]`

$$(x - 2)^2 y'' + (x + 2) y' - y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve((x-2)^2*diff(y(x),x$2)+(x+2)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{8}x^2 + \frac{1}{48}x^3 - \frac{1}{480}x^5\right)y(0) + \left(x - \frac{1}{4}x^2 - \frac{1}{24}x^3 + \frac{1}{240}x^5\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 56

```
AsymptoticDSolveValue[(x-2)^2*y''[x]+(x+2)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{x^5}{480} + \frac{x^3}{48} + \frac{x^2}{8} + 1\right) + c_2 \left(\frac{x^5}{240} - \frac{x^3}{24} - \frac{x^2}{4} + x\right)$$

## 2.2 problem 3

Internal problem ID [5637]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 3.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Lienard]

$$xy'' + 2y' + yx = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
Order:=6;
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 \left( 1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + O(x^6) \right) + \frac{c_2(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + O(x^6))}{x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 42

```
AsymptoticDSolveValue[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left( \frac{x^3}{24} - \frac{x}{2} + \frac{1}{x} \right) + c_2 \left( \frac{x^4}{120} - \frac{x^2}{6} + 1 \right)$$

## 2.3 problem 4

Internal problem ID [5638]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 4.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$xy'' + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 58

```
Order:=6;
dsolve(x*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) &= c_1 x \left( 1 - \frac{1}{2}x + \frac{1}{12}x^2 - \frac{1}{144}x^3 + \frac{1}{2880}x^4 - \frac{1}{86400}x^5 + O(x^6) \right) \\ &\quad + c_2 \left( \ln(x) \left( -x + \frac{1}{2}x^2 - \frac{1}{12}x^3 + \frac{1}{144}x^4 - \frac{1}{2880}x^5 + O(x^6) \right) \right. \\ &\quad \left. + \left( 1 - \frac{3}{4}x^2 + \frac{7}{36}x^3 - \frac{35}{1728}x^4 + \frac{101}{86400}x^5 + O(x^6) \right) \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x*y''[x] + y[x] == 0, y[x], {x, 0, 5}]
```

$$y(x) \rightarrow c_1 \left( \frac{1}{144} x (x^3 - 12x^2 + 72x - 144) \log(x) + \frac{-47x^4 + 480x^3 - 2160x^2 + 1728x + 1728}{1728} \right) + c_2 \left( \frac{x^5}{2880} - \frac{x^4}{144} + \frac{x^3}{12} - \frac{x^2}{2} + x \right)$$

## 2.4 problem 5

Internal problem ID [5639]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 5.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$xy'' + (1 + 2x)y' + (1 + x)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(2*x+1)*diff(y(x),x)+(x+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5\right)(\ln(x)c_2 + c_1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 78

```
AsymptoticDSolveValue[x*y''[x]+(2*x+1)*y'[x]+(x+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(-\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1\right) + c_2 \left(-\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1\right) \log(x)$$

## 2.5 problem 6

Internal problem ID [5640]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 6.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' + 2y'x^3 + (x^2 - 2)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
Order:=6;
dsolve(x*diff(y(x),x$2)+2*x^3*diff(y(x),x)+(x^2-2)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = & c_1 x \left( 1 + x + \frac{1}{3}x^2 - \frac{7}{36}x^3 - \frac{97}{360}x^4 - \frac{517}{5400}x^5 + O(x^6) \right) \\ & + c_2 \left( \ln(x) \left( 2x + 2x^2 + \frac{2}{3}x^3 - \frac{7}{18}x^4 - \frac{97}{180}x^5 + O(x^6) \right) \right. \\ & \quad \left. + \left( 1 - 3x^2 - \frac{31}{18}x^3 - \frac{85}{216}x^4 + \frac{4067}{5400}x^5 + O(x^6) \right) \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 83

```
AsymptoticDSolveValue[x*y''[x] + 2*x^3*y'[x] + (x^2 - 2)*y[x] == 0, y[x], {x, 0, 5}]
```

$$y(x) \rightarrow c_1 \left( \frac{1}{216} (-x^4 - 516x^3 - 1080x^2 - 432x + 216) \right. \\ \left. - \frac{1}{18} x (7x^3 - 12x^2 - 36x - 36) \log(x) \right) + c_2 \left( -\frac{97x^5}{360} - \frac{7x^4}{36} + \frac{x^3}{3} + x^2 + x \right)$$

## 2.6 problem 7

Internal problem ID [5641]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 7.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (x - 1)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)+(x-1)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) &= \left(1 + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{30}x^5\right) y(0) \\ &\quad + \left(x + \frac{1}{6}x^3 - \frac{1}{12}x^4 + \frac{1}{120}x^5\right) D(y)(0) + O(x^6) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]+(x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left( \frac{x^5}{120} - \frac{x^4}{12} + \frac{x^3}{6} + x \right) + c_1 \left( -\frac{x^5}{30} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} + 1 \right)$$

## 2.7 problem 8

Internal problem ID [5642]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 8.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Lienard]

$$xy'' + y' + yx = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)c_2 + c_1) \left(1 - \frac{1}{4}x^2 + \frac{1}{64}x^4 + O(x^6)\right) + \left(\frac{1}{4}x^2 - \frac{3}{128}x^4 + O(x^6)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left( \frac{x^4}{64} - \frac{x^2}{4} + 1 \right) + c_2 \left( -\frac{3x^4}{128} + \frac{x^2}{4} + \left( \frac{x^4}{64} - \frac{x^2}{4} + 1 \right) \log(x) \right)$$

## 2.8 problem 9

Internal problem ID [5643]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 9.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Jacobi]

$$2x(x - 1)y'' - y'(1 + x) + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
Order:=6;
dsolve(2*x*(x-1)*diff(y(x),x$2)-(x+1)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x} (1 + O(x^6)) + c_2(1 + x + O(x^6))$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 18

```
AsymptoticDSolveValue[2*x*(x-1)*y''[x]-(x+1)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1\sqrt{x} + c_2(x + 1)$$

## 2.9 problem 10

Internal problem ID [5644]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 10.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$xy'' + 2y' + 4yx = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
Order:=6;
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+4*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 \left( 1 - \frac{2}{3}x^2 + \frac{2}{15}x^4 + O(x^6) \right) + \frac{c_2(1 - 2x^2 + \frac{2}{3}x^4 + O(x^6))}{x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 40

```
AsymptoticDSolveValue[x*y''[x]+2*y'[x]+4*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left( \frac{2x^3}{3} - 2x + \frac{1}{x} \right) + c_2 \left( \frac{2x^4}{15} - \frac{2x^2}{3} + 1 \right)$$

## 2.10 problem 11

Internal problem ID [5645]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 11.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$xy'' + (2 - 2x)y' + (x - 2)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(2-2*x)*diff(y(x),x)+(x-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 \left( 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + O(x^6) \right) \\ + \frac{c_2(1 + 2x + \frac{3}{2}x^2 + \frac{2}{3}x^3 + \frac{5}{24}x^4 + \frac{1}{20}x^5 + O(x^6))}{x}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x*y''[x]+(2-2*x)*y'[x]+(x-2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left( \frac{5x^3}{24} + \frac{2x^2}{3} + \frac{3x}{2} + \frac{1}{x} + 2 \right) + c_2 \left( \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right)$$

## 2.11 problem 12

Internal problem ID [5646]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 12.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$x^2y'' + 6y'x + (4x^2 + 6)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+6*x*diff(y(x),x)+(4*x^2+6)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1(1 - \frac{2}{3}x^2 + \frac{2}{15}x^4 + O(x^6))x + c_2(1 - 2x^2 + \frac{2}{3}x^4 + O(x^6))}{x^3}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 38

```
AsymptoticDSolveValue[x^2*y''[x]+6*x*y'[x]+(4*x^2+6)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1\left(\frac{1}{x^3} + \frac{2x}{3} - \frac{2}{x}\right) + c_2\left(\frac{2x^2}{15} + \frac{1}{x^2} - \frac{2}{3}\right)$$

## 2.12 problem 13

Internal problem ID [5647]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 13.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$xy'' + (-2x + 1)y' + (x - 1)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)+(x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)c_2 + c_1) \left( 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 \right) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 74

```
AsymptoticDSolveValue[x*y''[x]+(1-2*x)*y'[x]+(x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left( \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right) + c_2 \left( \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right) \log(x)$$

## 2.13 problem 15

Internal problem ID [5648]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 15.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _exact, _linear, _homogeneous]`

$$2x(1-x)y'' - (1+6x)y' - 2y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(2*x*(1-x)*diff(y(x),x$2)-(1+6*x)*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) &= c_1 x^{\frac{3}{2}} \left( 1 + \frac{5}{2}x + \frac{35}{8}x^2 + \frac{105}{16}x^3 + \frac{1155}{128}x^4 + \frac{3003}{256}x^5 + O(x^6) \right) \\ &\quad + c_2 \left( 1 - 2x - 8x^2 - 16x^3 - \frac{128}{5}x^4 - \frac{256}{7}x^5 + O(x^6) \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 79

```
AsymptoticDSolveValue[2*x*(1-x)*y''[x]-(1+6*x)*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) &\rightarrow c_2 \left( -\frac{256x^5}{7} - \frac{128x^4}{5} - 16x^3 - 8x^2 - 2x + 1 \right) \\ &\quad + c_1 \left( \frac{3003x^5}{256} + \frac{1155x^4}{128} + \frac{105x^3}{16} + \frac{35x^2}{8} + \frac{5x}{2} + 1 \right) x^{3/2} \end{aligned}$$

## 2.14 problem 16

Internal problem ID [5649]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 16.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Jacobi]

$$x(1-x)y'' + \left(\frac{1}{2} + 2x\right)y' - 2y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
Order:=6;
dsolve(x*(1-x)*diff(y(x),x$2)+(1/2+2*x)*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 \sqrt{x} \left( 1 + \frac{1}{2}x - \frac{1}{40}x^2 - \frac{1}{560}x^3 - \frac{1}{2688}x^4 - \frac{1}{8448}x^5 + O(x^6) \right) + c_2(1 + 4x + O(x^6))$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 55

```
AsymptoticDSolveValue[x*(1-x)*y''[x]+(1/2+2*x)*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt{x} \left( -\frac{x^5}{8448} - \frac{x^4}{2688} - \frac{x^3}{560} - \frac{x^2}{40} + \frac{x}{2} + 1 \right) + c_2(4x + 1)$$

## 2.15 problem 17

Internal problem ID [5650]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 17.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$4xy'' + y' + 8y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(4*x*diff(y(x),x$2)+diff(y(x),x)+8*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) &= c_1 x^{\frac{3}{4}} \left( 1 - \frac{8}{7}x + \frac{32}{77}x^2 - \frac{256}{3465}x^3 + \frac{512}{65835}x^4 - \frac{4096}{7571025}x^5 + O(x^6) \right) \\ &\quad + c_2 \left( 1 - 8x + \frac{32}{5}x^2 - \frac{256}{135}x^3 + \frac{512}{1755}x^4 - \frac{4096}{149175}x^5 + O(x^6) \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 83

```
AsymptoticDSolveValue[4*x*y''[x]+y'[x]+8*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) &\rightarrow c_2 \left( -\frac{4096x^5}{149175} + \frac{512x^4}{1755} - \frac{256x^3}{135} + \frac{32x^2}{5} - 8x + 1 \right) \\ &\quad + c_1 x^{3/4} \left( -\frac{4096x^5}{7571025} + \frac{512x^4}{65835} - \frac{256x^3}{3465} + \frac{32x^2}{77} - \frac{8x}{7} + 1 \right) \end{aligned}$$

## 2.16 problem 18

Internal problem ID [5651]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 18.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4(t^2 - 3t + 2) y'' - 2y' + y = 0$$

With the expansion point for the power series method at  $t = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
Order:=6;
dsolve(4*(t^2-3*t+2)*diff(y(t),t$2)-2*diff(y(t),t)+y(t)=0,y(t),type='series',t=0);
```

$$\begin{aligned} y(t) &= \left(1 - \frac{1}{16}t^2 - \frac{7}{192}t^3 - \frac{73}{3072}t^4 - \frac{1037}{61440}t^5\right)y(0) \\ &\quad + \left(t + \frac{1}{8}t^2 + \frac{5}{96}t^3 + \frac{47}{1536}t^4 + \frac{643}{30720}t^5\right)D(y)(0) + O(t^6) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 70

```
AsymptoticDSolveValue[4*(t^2-3*t+2)*y''[t]-2*y'[t]+y[t]==0,y[t],{t,0,5}]
```

$$y(t) \rightarrow c_1 \left( -\frac{1037t^5}{61440} - \frac{73t^4}{3072} - \frac{7t^3}{192} - \frac{t^2}{16} + 1 \right) + c_2 \left( \frac{643t^5}{30720} + \frac{47t^4}{1536} + \frac{5t^3}{96} + \frac{t^2}{8} + t \right)$$

## 2.17 problem 19

Internal problem ID [5652]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 19.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$2(t^2 - 5t + 6) y'' + (2t - 3) y' - 8y = 0$$

With the expansion point for the power series method at  $t = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
Order:=6;
dsolve(2*(t^2-5*t+6)*diff(y(t),t$2)+(2*t-3)*diff(y(t),t)-8*y(t)=0,y(t),type='series',t=0);
```

$$\begin{aligned} y(t) &= \left( 1 + \frac{1}{3}t^2 + \frac{13}{108}t^3 + \frac{299}{5184}t^4 + \frac{923}{34560}t^5 \right) y(0) \\ &\quad + \left( t + \frac{1}{8}t^2 + \frac{37}{288}t^3 + \frac{851}{13824}t^4 + \frac{2627}{92160}t^5 \right) D(y)(0) + O(t^6) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 70

```
AsymptoticDSolveValue[2*(t^2-5*t+6)*y''[t]+(2*t-3)*y'[t]-8*y[t]==0,y[t],{t,0,5}]
```

$$y(t) \rightarrow c_1 \left( \frac{923t^5}{34560} + \frac{299t^4}{5184} + \frac{13t^3}{108} + \frac{t^2}{3} + 1 \right) + c_2 \left( \frac{2627t^5}{92160} + \frac{851t^4}{13824} + \frac{37t^3}{288} + \frac{t^2}{8} + t \right)$$

## 2.18 problem 20

Internal problem ID [5653]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.3. Extended Power Series Method: Frobenius Method page 186

**Problem number:** 20.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$3t(t+1)y'' + ty' - y = 0$$

With the expansion point for the power series method at  $t = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
Order:=6;
dsolve(3*t*(1+t)*diff(y(t),t$2)+t*diff(y(t),t)-y(t)=0,y(t),type='series',t=0);
```

$$\begin{aligned} y(t) &= c_1 t \left(1 + O(t^6)\right) + \left(\frac{1}{3}t + O(t^6)\right) \ln(t) c_2 \\ &\quad + \left(1 - \frac{1}{3}t - \frac{2}{9}t^2 + \frac{7}{81}t^3 - \frac{35}{729}t^4 + \frac{91}{2916}t^5 + O(t^6)\right) c_2 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 43

```
AsymptoticDSolveValue[3*t*(1+t)*y''[t]+t*y'[t]-y[t]==0,y[t],{t,0,5}]
```

$$y(t) \rightarrow c_1 \left( \frac{1}{729} (-35t^4 + 63t^3 - 162t^2 + 243t + 729) + \frac{1}{3}t \log(t) \right) + c_2 t$$

### **3 Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.4. Bessels Equation page 195**

3.1	problem 2 . . . . .	38
3.2	problem 3 . . . . .	39
3.3	problem 4 . . . . .	40
3.4	problem 6 . . . . .	41
3.5	problem 7 . . . . .	42
3.6	problem 8 . . . . .	43

### 3.1 problem 2

Internal problem ID [5654]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.4. Bessels Equation page 195

**Problem number:** 2.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$x^2y'' + y'x + \left(x^2 - \frac{4}{49}\right)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-4/49)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{4}{7}} \left(1 - \frac{7}{36}x^2 + \frac{49}{4608}x^4 + O(x^6)\right) + c_1 \left(1 - \frac{7}{20}x^2 + \frac{49}{1920}x^4 + O(x^6)\right)}{x^{\frac{2}{7}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 52

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-4/49)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x^{2/7} \left(\frac{49x^4}{4608} - \frac{7x^2}{36} + 1\right) + \frac{c_2 \left(\frac{49x^4}{1920} - \frac{7x^2}{20} + 1\right)}{x^{2/7}}$$

## 3.2 problem 3

Internal problem ID [5655]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.4. Bessel's Equation page 195

**Problem number:** 3.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$xy'' + y' + \frac{y}{4} = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+1/4*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = & (\ln(x)c_2 + c_1) \left( 1 - \frac{1}{4}x + \frac{1}{64}x^2 - \frac{1}{2304}x^3 + \frac{1}{147456}x^4 - \frac{1}{14745600}x^5 + O(x^6) \right) \\ & + \left( \frac{1}{2}x - \frac{3}{64}x^2 + \frac{11}{6912}x^3 - \frac{25}{884736}x^4 + \frac{137}{442368000}x^5 + O(x^6) \right) c_2 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 117

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+1/4*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \rightarrow & c_1 \left( -\frac{x^5}{14745600} + \frac{x^4}{147456} - \frac{x^3}{2304} + \frac{x^2}{64} - \frac{x}{4} + 1 \right) + c_2 \left( \frac{137x^5}{442368000} - \frac{25x^4}{884736} \right. \\ & \left. + \frac{11x^3}{6912} - \frac{3x^2}{64} + \left( -\frac{x^5}{14745600} + \frac{x^4}{147456} - \frac{x^3}{2304} + \frac{x^2}{64} - \frac{x}{4} + 1 \right) \log(x) + \frac{x}{2} \right) \end{aligned}$$

### 3.3 problem 4

Internal problem ID [5656]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.4. Bessels Equation page 195

**Problem number:** 4.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' + \left(e^{-2x} - \frac{1}{9}\right) y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)+(exp(-2*x)-1/9)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) &= \left(1 - \frac{4}{9}x^2 + \frac{1}{3}x^3 - \frac{65}{486}x^4 + \frac{1}{135}x^5\right) y(0) \\ &\quad + \left(x - \frac{4}{27}x^3 + \frac{1}{6}x^4 - \frac{227}{2430}x^5\right) D(y)(0) + O(x^6) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]+(Exp[-2*x]-1/9)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(-\frac{227x^5}{2430} + \frac{x^4}{6} - \frac{4x^3}{27} + x\right) + c_1 \left(\frac{x^5}{135} - \frac{65x^4}{486} + \frac{x^3}{3} - \frac{4x^2}{9} + 1\right)$$

### 3.4 problem 6

Internal problem ID [5657]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.4. Bessel's Equation page 195

**Problem number:** 6.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + \frac{(x + \frac{3}{4}) y}{4} = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+1/4*(x+3/4)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) &= c_1 x^{\frac{1}{4}} \left( 1 - \frac{1}{2}x + \frac{1}{24}x^2 - \frac{1}{720}x^3 + \frac{1}{40320}x^4 - \frac{1}{3628800}x^5 + O(x^6) \right) \\ &\quad + c_2 x^{\frac{3}{4}} \left( 1 - \frac{1}{6}x + \frac{1}{120}x^2 - \frac{1}{5040}x^3 + \frac{1}{362880}x^4 - \frac{1}{39916800}x^5 + O(x^6) \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 90

```
AsymptoticDSolveValue[x^2*y'[x]+1/4*(x+3/4)*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) &\rightarrow c_2 \sqrt[4]{x} \left( -\frac{x^5}{3628800} + \frac{x^4}{40320} - \frac{x^3}{720} + \frac{x^2}{24} - \frac{x}{2} + 1 \right) \\ &\quad + c_1 x^{3/4} \left( -\frac{x^5}{39916800} + \frac{x^4}{362880} - \frac{x^3}{5040} + \frac{x^2}{120} - \frac{x}{6} + 1 \right) \end{aligned}$$

### 3.5 problem 7

Internal problem ID [5658]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.4. Bessels Equation page 195

**Problem number:** 7.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$x^2y'' + y'x + \frac{y(x^2 - 1)}{4} = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+1/4*(x^2-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x \left(1 - \frac{1}{24} x^2 + \frac{1}{1920} x^4 + O(x^6)\right) + c_2 \left(1 - \frac{1}{8} x^2 + \frac{1}{384} x^4 + O(x^6)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+1/4*(x^2-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left( \frac{x^{7/2}}{384} - \frac{x^{3/2}}{8} + \frac{1}{\sqrt{x}} \right) + c_2 \left( \frac{x^{9/2}}{1920} - \frac{x^{5/2}}{24} + \sqrt{x} \right)$$

### 3.6 problem 8

Internal problem ID [5659]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.4. Bessels Equation page 195

**Problem number:** 8.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$(1+2x)^2 y'' + 2(1+2x) y' + 16x(1+x) y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
Order:=6;
dsolve((2*x+1)^2*diff(y(x),x$2)+2*(2*x+1)*diff(y(x),x)+16*x*(x+1)*y(x)=0,y(x),type='series',
```

$$\begin{aligned} y(x) &= \left(1 - \frac{8}{3}x^3 + \frac{16}{3}x^4 - \frac{152}{15}x^5\right) y(0) \\ &\quad + \left(x - x^2 + \frac{4}{3}x^3 - \frac{10}{3}x^4 + \frac{104}{15}x^5\right) D(y)(0) + O(x^6) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 61

```
AsymptoticDSolveValue[(2*x+1)^2*y''[x]+2*(2*x+1)*y'[x]+16*x*(x+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left( -\frac{152x^5}{15} + \frac{16x^4}{3} - \frac{8x^3}{3} + 1 \right) + c_2 \left( \frac{104x^5}{15} - \frac{10x^4}{3} + \frac{4x^3}{3} - x^2 + x \right)$$

## **4 Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.5. Bessel Functions $Y(x)$ . General Solution page 200**

4.1	problem 1 . . . . .	45
4.2	problem 2 . . . . .	47
4.3	problem 3 . . . . .	48
4.4	problem 4 . . . . .	49
4.5	problem 5 . . . . .	50
4.6	problem 6 . . . . .	51
4.7	problem 7 . . . . .	53
4.8	problem 8 . . . . .	54
4.9	problem 9 . . . . .	55

## 4.1 problem 1

Internal problem ID [5660]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.5. Bessel Functions  $Y(x)$ . General Solution page 200

**Problem number:** 1.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Bessel]

$$x^2y'' + y'x + (x^2 - 6)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 97

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-6)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) &= c_1 x^{-\sqrt{6}} \left( 1 + \frac{1}{-4 + 4\sqrt{6}} x^2 + \frac{1}{32} \frac{1}{(-2 + \sqrt{6})(-1 + \sqrt{6})} x^4 + O(x^6) \right) \\ &\quad + c_2 x^{\sqrt{6}} \left( 1 - \frac{1}{4 + 4\sqrt{6}} x^2 + \frac{1}{32} \frac{1}{(2 + \sqrt{6})(1 + \sqrt{6})} x^4 + O(x^6) \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 210

```
AsymptoticDSolveValue[x^2*y''[x] + x*y'[x] + (x^2 - 6)*y[x] == 0, y[x], {x, 0, 5}]
```

$$y(x) \rightarrow c_2 \left( \frac{x^4}{(-4 - \sqrt{6} + (1 - \sqrt{6})(2 - \sqrt{6})) (-2 - \sqrt{6} + (3 - \sqrt{6})(4 - \sqrt{6}))} \right. \\ \left. - \frac{x^2}{-4 - \sqrt{6} + (1 - \sqrt{6})(2 - \sqrt{6})} + 1 \right) x^{-\sqrt{6}} \\ + c_1 \left( \frac{x^4}{(-4 + \sqrt{6} + (1 + \sqrt{6})(2 + \sqrt{6})) (-2 + \sqrt{6} + (3 + \sqrt{6})(4 + \sqrt{6}))} \right. \\ \left. - \frac{x^2}{-4 + \sqrt{6} + (1 + \sqrt{6})(2 + \sqrt{6})} + 1 \right) x^{\sqrt{6}}$$

## 4.2 problem 2

Internal problem ID [5661]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.5. Bessel Functions Y(x). General Solution page 200

**Problem number:** 2.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Lienard]

$$xy'' + 5y' + yx = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=6;
dsolve(x*diff(y(x),x$2)+5*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^4 \left(1 - \frac{1}{12} x^2 + \frac{1}{384} x^4 + O(x^6)\right) + c_2 (\ln(x) (9x^4 + O(x^6)) + (-144 - 36x^2 + O(x^6)))}{x^4}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 47

```
AsymptoticDSolveValue[x*y''[x]+5*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left( \frac{x^4}{384} - \frac{x^2}{12} + 1 \right) + c_1 \left( \frac{(x^2 + 8)^2}{64x^4} - \frac{\log(x)}{16} \right)$$

## 4.3 problem 3

Internal problem ID [5662]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.5. Bessel Functions  $Y(x)$ . General Solution page 200

**Problem number:** 3.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$9x^2y'' + 9y'x + (36x^4 - 16)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
Order:=6;
dsolve(9*x^2*diff(y(x),x$2)+9*x*diff(y(x),x)+(36*x^4-16)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{8}{3}} (1 - \frac{3}{20} x^4 + O(x^6)) + c_1 (1 - \frac{3}{4} x^4 + O(x^6))}{x^{\frac{4}{3}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 38

```
AsymptoticDSolveValue[9*x^2*y''[x]+9*x*y'[x]+(36*x^4-16)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left(1 - \frac{3x^4}{20}\right) x^{4/3} + \frac{c_2 \left(1 - \frac{3x^4}{4}\right)}{x^{4/3}}$$

## 4.4 problem 4

Internal problem ID [5663]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.5. Bessel Functions Y(x). General Solution page 200

**Problem number:** 4.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' + yx = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^3}{6}\right)y(0) + \left(x - \frac{1}{12}x^4\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2\left(x - \frac{x^4}{12}\right) + c_1\left(1 - \frac{x^3}{6}\right)$$

## 4.5 problem 5

Internal problem ID [5664]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.5. Bessel Functions Y(x). General Solution page 200

**Problem number:** 5.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$4xy'' + 4y' + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 59

```
Order:=6;
dsolve(4*x*diff(y(x),x$2)+4*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = & (\ln(x)c_2 + c_1) \left( 1 - \frac{1}{4}x + \frac{1}{64}x^2 - \frac{1}{2304}x^3 + \frac{1}{147456}x^4 - \frac{1}{14745600}x^5 + O(x^6) \right) \\ & + \left( \frac{1}{2}x - \frac{3}{64}x^2 + \frac{11}{6912}x^3 - \frac{25}{884736}x^4 + \frac{137}{442368000}x^5 + O(x^6) \right) c_2 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 117

```
AsymptoticDSolveValue[4*x*y''[x]+4*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \rightarrow & c_1 \left( -\frac{x^5}{14745600} + \frac{x^4}{147456} - \frac{x^3}{2304} + \frac{x^2}{64} - \frac{x}{4} + 1 \right) + c_2 \left( \frac{137x^5}{442368000} - \frac{25x^4}{884736} \right. \\ & \left. + \frac{11x^3}{6912} - \frac{3x^2}{64} + \left( -\frac{x^5}{14745600} + \frac{x^4}{147456} - \frac{x^3}{2304} + \frac{x^2}{64} - \frac{x}{4} + 1 \right) \log(x) + \frac{x}{2} \right) \end{aligned}$$

## 4.6 problem 6

Internal problem ID [5665]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.5. Bessel Functions Y(x). General Solution page 200

**Problem number:** 6.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$xy'' + y' + 36y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+36*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) &= (\ln(x)c_2 + c_1) \left( 1 - 36x + 324x^2 - 1296x^3 + 2916x^4 - \frac{104976}{25}x^5 + O(x^6) \right) \\ &\quad + \left( 72x - 972x^2 + 4752x^3 - 12150x^4 + \frac{2396952}{125}x^5 + O(x^6) \right) c_2 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 93

```
AsymptoticDSolveValue[x*y''[x] + y'[x] + 36*y[x] == 0, y[x], {x, 0, 5}]
```

$$\begin{aligned}y(x) \rightarrow & c_1 \left( -\frac{104976x^5}{25} + 2916x^4 - 1296x^3 + 324x^2 - 36x + 1 \right) \\& + c_2 \left( \frac{2396952x^5}{125} - 12150x^4 + 4752x^3 - 972x^2 \right. \\& \left. + \left( -\frac{104976x^5}{25} + 2916x^4 - 1296x^3 + 324x^2 - 36x + 1 \right) \log(x) + 72x \right)\end{aligned}$$

## 4.7 problem 7

Internal problem ID [5666]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.5. Bessel Functions Y(x). General Solution page 200

**Problem number:** 7.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' + k^2x^2y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
Order:=6;
dsolve(diff(y(x),x$2)+k^2*x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{k^2 x^4}{12}\right) y(0) + \left(x - \frac{1}{20} k^2 x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[y''[x]+k^2*x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(x - \frac{k^2 x^5}{20}\right) + c_1 \left(1 - \frac{k^2 x^4}{12}\right)$$

## 4.8 problem 8

Internal problem ID [5667]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.5. Bessel Functions Y(x). General Solution page 200

**Problem number:** 8.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' + x^4 k^2 y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
Order:=6;
dsolve(diff(y(x),x$2)+k^2*x^4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + D(y)(0)x + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 10

```
AsymptoticDSolveValue[y''[x]+k^2*x^4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 x + c_1$$

## 4.9 problem 9

Internal problem ID [5668]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. Special Functions. Problem set 5.5. Bessel Functions Y(x). General Solution page 200

**Problem number:** 9.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Lienard]

$$xy'' - 5y' + yx = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
Order:=6;
dsolve(x*diff(y(x),x$2)-5*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^6 \left( 1 - \frac{1}{16} x^2 + \frac{1}{640} x^4 + O(x^6) \right) + c_2 (-86400 - 10800 x^2 - 1350 x^4 + O(x^6))$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 44

```
AsymptoticDSolveValue[x*y''[x]-5*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left( \frac{x^4}{64} + \frac{x^2}{8} + 1 \right) + c_2 \left( \frac{x^{10}}{640} - \frac{x^8}{16} + x^6 \right)$$

## **5 Chapter 5. Series Solutions of ODEs. REVIEW QUESTIONS. page 201**

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## 5.1 problem 11

Internal problem ID [5669]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. REVIEW QUESTIONS. page 201

**Problem number:** 11.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x]`

$$y'' + 4y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - 2x^2 + \frac{2}{3}x^4\right)y(0) + \left(x - \frac{2}{3}x^3 + \frac{2}{15}x^5\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[y''[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left( \frac{2x^5}{15} - \frac{2x^3}{3} + x \right) + c_1 \left( \frac{2x^4}{3} - 2x^2 + 1 \right)$$

## 5.2 problem 12

Internal problem ID [5670]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. REVIEW QUESTIONS. page 201

**Problem number:** 12.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$xy'' + (-2x + 1)y' + (x - 1)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)+(x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)c_2 + c_1) \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5\right) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 74

```
AsymptoticDSolveValue[x*y''[x]+(1-2*x)*y'[x]+(x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left( \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right) + c_2 \left( \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right) \log(x)$$

### 5.3 problem 13

Internal problem ID [5671]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. REVIEW QUESTIONS. page 201

**Problem number:** 13.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$(x - 1)^2 y'' - (x - 1) y' - 35y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((x-1)^2*diff(y(x),x$2)-(x-1)*diff(y(x),x)-35*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) &= \left( 1 + \frac{35}{2}x^2 + \frac{35}{6}x^3 + \frac{665}{12}x^4 + \frac{259}{4}x^5 \right) y(0) \\ &\quad + \left( x - \frac{1}{2}x^2 + \frac{35}{6}x^3 + \frac{35}{12}x^4 + \frac{49}{4}x^5 \right) D(y)(0) + O(x^6) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 70

```
AsymptoticDSolveValue[(x-1)^2*y''[x]-(x-1)*y'[x]-35*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left( \frac{259x^5}{4} + \frac{665x^4}{12} + \frac{35x^3}{6} + \frac{35x^2}{2} + 1 \right) + c_2 \left( \frac{49x^5}{4} + \frac{35x^4}{12} + \frac{35x^3}{6} - \frac{x^2}{2} + x \right)$$

## 5.4 problem 14

Internal problem ID [5672]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. REVIEW QUESTIONS. page 201

**Problem number:** 14.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_Emden, _Fowler]`

$$16(1+x)^2 y'' + 3y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(16*(x+1)^2*diff(y(x),x$2)+3*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) &= \left(1 - \frac{3}{32}x^2 + \frac{1}{16}x^3 - \frac{93}{2048}x^4 + \frac{9}{256}x^5\right) y(0) \\ &\quad + \left(x - \frac{1}{32}x^3 + \frac{1}{32}x^4 - \frac{57}{2048}x^5\right) D(y)(0) + O(x^6) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[16*(x+1)^2*y''[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left(-\frac{57x^5}{2048} + \frac{x^4}{32} - \frac{x^3}{32} + x\right) + c_1 \left(\frac{9x^5}{256} - \frac{93x^4}{2048} + \frac{x^3}{16} - \frac{3x^2}{32} + 1\right)$$

## 5.5 problem 15

Internal problem ID [5673]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. REVIEW QUESTIONS. page 201

**Problem number:** 15.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Bessel]

$$x^2y'' + y'x + (x^2 - 5)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 97

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-5)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) &= c_1 x^{-\sqrt{5}} \left( 1 + \frac{1}{-4 + 4\sqrt{5}} x^2 + \frac{1}{32} \frac{1}{(-2 + \sqrt{5})(\sqrt{5} - 1)} x^4 + O(x^6) \right) \\ &\quad + c_2 x^{\sqrt{5}} \left( 1 - \frac{1}{4 + 4\sqrt{5}} x^2 + \frac{1}{32} \frac{1}{(2 + \sqrt{5})(\sqrt{5} + 1)} x^4 + O(x^6) \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 210

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-5)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left( \frac{x^4}{(-3 - \sqrt{5} + (1 - \sqrt{5}) (2 - \sqrt{5})) (-1 - \sqrt{5} + (3 - \sqrt{5}) (4 - \sqrt{5}))} \right. \\ \left. - \frac{x^2}{-3 - \sqrt{5} + (1 - \sqrt{5}) (2 - \sqrt{5})} + 1 \right) x^{-\sqrt{5}} \\ + c_1 \left( \frac{x^4}{(-3 + \sqrt{5} + (1 + \sqrt{5}) (2 + \sqrt{5})) (-1 + \sqrt{5} + (3 + \sqrt{5}) (4 + \sqrt{5}))} \right. \\ \left. - \frac{x^2}{-3 + \sqrt{5} + (1 + \sqrt{5}) (2 + \sqrt{5})} + 1 \right) x^{\sqrt{5}}$$

## 5.6 problem 16

Internal problem ID [5674]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. REVIEW QUESTIONS. page 201

**Problem number:** 16.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$x^2y'' + 2y'x^3 + (x^2 - 2)y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+2*x^3*diff(y(x),x)+(x^2-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^2 \left( 1 - \frac{1}{2} x^2 + \frac{9}{56} x^4 + O(x^6) \right) + \frac{c_2 (12 - 6x^2 + \frac{9}{2} x^4 + O(x^6))}{x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 44

```
AsymptoticDSolveValue[x^2*y''[x]+2*x^3*y'[x]+(x^2-2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left( \frac{3x^3}{8} - \frac{x}{2} + \frac{1}{x} \right) + c_2 \left( \frac{9x^6}{56} - \frac{x^4}{2} + x^2 \right)$$

## 5.7 problem 17

Internal problem ID [5675]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. REVIEW QUESTIONS. page 201

**Problem number:** 17.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type [\_Laguerre]

$$xy'' - y'(1+x) + y = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=6;
dsolve(x*diff(y(x),x$2)-(x+1)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) &= c_1 x^2 \left( 1 + \frac{1}{3}x + \frac{1}{12}x^2 + \frac{1}{60}x^3 + \frac{1}{360}x^4 + \frac{1}{2520}x^5 + O(x^6) \right) \\ &\quad + c_2 \left( -2 - 2x - x^2 - \frac{1}{3}x^3 - \frac{1}{12}x^4 - \frac{1}{60}x^5 + O(x^6) \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 66

```
AsymptoticDSolveValue[x*y''[x]-(x+1)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left( \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right) + c_2 \left( \frac{x^6}{360} + \frac{x^5}{60} + \frac{x^4}{12} + \frac{x^3}{3} + x^2 \right)$$

## 5.8 problem 18

Internal problem ID [5676]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. REVIEW QUESTIONS. page 201

**Problem number:** 18.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_Emden, _Fowler]`

$$xy'' + 3y' + 4yx^3 = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
Order:=6;
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+4*x^3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 \left( 1 - \frac{1}{6}x^4 + O(x^6) \right) + \frac{c_2(-2 + x^4 + O(x^6))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 30

```
AsymptoticDSolveValue[x*y''[x]+3*y'[x]+4*x^3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2 \left( 1 - \frac{x^4}{6} \right) + c_1 \left( \frac{1}{x^2} - \frac{x^2}{2} \right)$$

## 5.9 problem 19

Internal problem ID [5677]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. REVIEW QUESTIONS. page 201

**Problem number:** 19.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' + \frac{y}{4x} = 0$$

With the expansion point for the power series method at  $x = 0$ .

 Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
Order:=6;
dsolve(diff(y(x),x$2)+1/(4*x)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = & c_1 x \left( 1 - \frac{1}{8}x + \frac{1}{192}x^2 - \frac{1}{9216}x^3 + \frac{1}{737280}x^4 - \frac{1}{88473600}x^5 + O(x^6) \right) \\ & + c_2 \left( \ln(x) \left( -\frac{1}{4}x + \frac{1}{32}x^2 - \frac{1}{768}x^3 + \frac{1}{36864}x^4 - \frac{1}{2949120}x^5 + O(x^6) \right) \right. \\ & \left. + \left( 1 - \frac{3}{64}x^2 + \frac{7}{2304}x^3 - \frac{35}{442368}x^4 + \frac{101}{88473600}x^5 + O(x^6) \right) \right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 85

```
AsymptoticDSolveValue[y''[x]+1/(4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left( \frac{x(x^3 - 48x^2 + 1152x - 9216) \log(x)}{36864} + \frac{-47x^4 + 1920x^3 - 34560x^2 + 110592x + 442368}{442368} \right) + c_2 \left( \frac{x^5}{737280} - \frac{x^4}{9216} + \frac{x^3}{192} - \frac{x^2}{8} + x \right)$$

## 5.10 problem 20

Internal problem ID [5678]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 5. Series Solutions of ODEs. REVIEW QUESTIONS. page 201

**Problem number:** 20.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$xy'' + y' - yx = 0$$

With the expansion point for the power series method at  $x = 0$ .

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)c_2 + c_1) \left( 1 + \frac{1}{4}x^2 + \frac{1}{64}x^4 + O(x^6) \right) + \left( -\frac{1}{4}x^2 - \frac{3}{128}x^4 + O(x^6) \right) c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x*y''[x]+y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \left( \frac{x^4}{64} + \frac{x^2}{4} + 1 \right) + c_2 \left( -\frac{3x^4}{128} - \frac{x^2}{4} + \left( \frac{x^4}{64} + \frac{x^2}{4} + 1 \right) \log(x) \right)$$

## **6 Chapter 6. Laplace Transforms. Problem set 6.2, page 216**

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## 6.1 problem 1

Internal problem ID [5679]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.2, page 216

**Problem number:** 1.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y' + \frac{26y}{5} = \frac{97 \sin(2t)}{5}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(t),t)+52/10*y(t)=194/10*sin(2*t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{5 \cos(2t)}{4} + \frac{13 \sin(2t)}{4} + \frac{5 e^{-\frac{26t}{5}}}{4}$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 31

```
DSolve[{y'[t]+52/10*y[t]==194/10*Sin[2*t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{4}(5e^{-26t/5} + 13 \sin(2t) - 5 \cos(2t))$$

## 6.2 problem 2

Internal problem ID [5680]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.2, page 216

**Problem number:** 2.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$2y + y' = 0$$

With initial conditions

$$\left[ y(0) = \frac{3}{2} \right]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(t),t)+2*y(t)=0,y(0) = 3/2],y(t), singsol=all)
```

$$y(t) = \frac{3 e^{-2t}}{2}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 31

```
DSolve[{y'[t]+52/10*y[t]==194/10*Sin[2*t],{y[0]==15/10}],y[t],t,IncludeSingularSolutions ->
```

$$y(t) \rightarrow \frac{1}{4} (11e^{-26t/5} + 13 \sin(2t) - 5 \cos(2t))$$

### 6.3 problem 3

Internal problem ID [5681]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.2, page 216

**Problem number:** 3.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x]`

$$y'' - y' - 6y = 0$$

With initial conditions

$$[y(0) = 11, y'(0) = 28]$$

#### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)-diff(y(t),t)-6*y(t)=0,y(0) = 11, D(y)(0) = 28],y(t), singsol=all)
```

$$y(t) = (10 e^{5t} + 1) e^{-2t}$$

#### ✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 18

```
DSolve[{y''[t] - y'[t] - 6*y[t] == 0, {y[0] == 11, y'[0] == 28}}, y[t], t, IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow e^{-2t} + 10e^{3t}$$

## 6.4 problem 4

Internal problem ID [5682]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.2, page 216

**Problem number:** 4.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' + 9y = 10e^{-t}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+9*y(t)=10*exp(-t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\sin(3t)}{3} - \cos(3t) + e^{-t}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 25

```
DSolve[{y''[t]+9*y[t]==10*Exp[-t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow e^{-t} + \frac{1}{3} \sin(3t) - \cos(3t)$$

## 6.5 problem 5

Internal problem ID [5683]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.2, page 216

**Problem number:** 5.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x]`

$$y'' - \frac{y}{4} = 0$$

With initial conditions

$$[y(0) = 12, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)-1/4*y(t)=0,y(0) = 12, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = 6e^{-\frac{t}{2}} + 6e^{\frac{t}{2}}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 19

```
DSolve[{y''[t]-1/4*y[t]==0,{y[0]==12,y'[0]==0}],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow 6e^{-t/2}(e^t + 1)$$

## 6.6 problem 6

Internal problem ID [5684]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.2, page 216

**Problem number:** 6.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' - 6y' + 5y = 29 \cos(2t)$$

With initial conditions

$$\left[ y(0) = \frac{16}{5}, y'(0) = \frac{31}{5} \right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve([diff(y(t),t$2)-6*diff(y(t),t)+5*y(t)=29*cos(2*t),y(0) = 16/5, D(y)(0) = 31/5],y(t),
```

$$y(t) = 2e^{5t} + e^t + \frac{\cos(2t)}{5} - \frac{12\sin(2t)}{5}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 32

```
DSolve[{y''[t]-6*y'[t]+5*y[t]==29*Cos[2*t],{y[0]==32/10,y'[0]==62/10}],y[t],t,IncludeSingular
```

$$y(t) \rightarrow e^t + 2e^{5t} - \frac{12}{5}\sin(2t) + \frac{1}{5}\cos(2t)$$

## 6.7 problem 7

Internal problem ID [5685]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.2, page 216

**Problem number:** 7.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' + 7y' + 12y = 21e^{3t}$$

With initial conditions

$$\left[ y(0) = \frac{7}{2}, y'(0) = -10 \right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(t),t$2)+7*diff(y(t),t)+12*y(t)=21*exp(3*t),y(0) = 7/2, D(y)(0) = -10],y(t), s)
```

$$y(t) = \frac{(e^{7t} + e^t + 5)e^{-4t}}{2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 28

```
DSolve[{y''[t]+7*y'[t]+12*y[t]==21*Exp[3*t],{y[0]==32/10,y'[0]==62/10}],y[t],t,IncludeSingul
```

$$y(t) \rightarrow \frac{1}{10} e^{-4t} (155e^t + 5e^{7t} - 128)$$

## 6.8 problem 8

Internal problem ID [5686]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.2, page 216

**Problem number:** 8.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x]`

$$y'' - 4y' + 4y = 0$$

With initial conditions

$$\left[ y(0) = \frac{81}{10}, y'(0) = \frac{39}{10} \right]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)+4*y(t)=0,y(0) = 81/10, D(y)(0) = 39/10],y(t), singsol=
```

$$y(t) = -\frac{3e^{2t}(-27 + 41t)}{10}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 19

```
DSolve[{y''[t]-4*y'[t]+4*y[t]==0,{y[0]==81/10,y'[0]==39/10}],y[t],t,IncludeSingularSolutions
```

$$y(t) \rightarrow -\frac{3}{10}e^{2t}(41t - 27)$$

## 6.9 problem 9

Internal problem ID [5687]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.2, page 216

**Problem number:** 9.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' - 4y' + 3y = 6t - 8$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)+3*y(t)=6*t-8,y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = e^t - e^{3t} + 2t$$

### ✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 19

```
DSolve[{y''[t]-4*y'[t]+3*y[t]==6*t-8,{y[0]==0,y'[0]==0}],y[t],t,IncludeSingularSolutions ->
```

$$y(t) \rightarrow 2t + e^t - e^{3t}$$

## 6.10 problem 10

Internal problem ID [5688]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.2, page 216

**Problem number:** 10.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' + \frac{y}{25} = \frac{t^2}{50}$$

With initial conditions

$$[y(0) = -25, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve([diff(y(t),t$2)+4/100*y(t)=2/100*t^2,y(0) = -25, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{t^2}{2} - 25$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 14

```
DSolve[{y''[t]+4/100*y[t]==2/100*t^2,{y[0]==-25,y'[0]==0}],y[t],t,IncludeSingularSolutions -
```

$$y(t) \rightarrow \frac{1}{2}(t^2 - 50)$$

## 6.11 problem 11

Internal problem ID [5689]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.2, page 216

**Problem number:** 11.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + 3y' + \frac{9y}{4} = 9t^3 + 64$$

With initial conditions

$$\left[ y(0) = 1, y'(0) = \frac{63}{2} \right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)+225/100*y(t)=9*t^3+64,y(0) = 1, D(y)(0) = 63/2],y(t),
```

$$y(t) = e^{-\frac{3t}{2}} + e^{-\frac{3t}{2}} t + 4t^3 - 16t^2 + 32t$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 28

```
DSolve[{y''[t]+3*y'[t]+225/100*y[t]==9*t^3+64,{y[0]==1,y'[0]==315/10}],y[t],t,IncludeSingular
```

$$y(t) \rightarrow 4t(t^2 - 4t + 8) + e^{-3t/2}(t + 1)$$

## 6.12 problem 12

Internal problem ID [5690]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.2, page 216

**Problem number:** 12.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x]`

$$y'' - 2y' - 3y = 0$$

With initial conditions

$$[y(4) = -3, y'(4) = -17]$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)-3*y(t)=0,y(4) = -3, D(y)(4) = -17],y(t), singsol=all)
```

$$y(t) = 2e^{4-t} - 5e^{-12+3t}$$

### ✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 24

```
DSolve[{y''[t]-2*y'[t]-3*y[t]==0,{y[4]==-3,y'[4]==-17}],y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \rightarrow 2e^{4-t} - 5e^{3(t-4)}$$

## 6.13 problem 13

Internal problem ID [5691]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.2, page 216

**Problem number:** 13.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_quadrature]

$$y' - 6y = 0$$

With initial conditions

$$[y(-1) = 4]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(t),t)-6*y(t)=0,y(-1) = 4],y(t), singsol=all)
```

$$y(t) = 4 e^{6t+6}$$

### ✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 14

```
DSolve[{y'[t]-6*y[t]==0,{y[-1]==4}],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow 4e^{6t+6}$$

## 6.14 problem 14

Internal problem ID [5692]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.2, page 216

**Problem number:** 14.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' + 2y' + 5y = 50t - 100$$

With initial conditions

$$[y(2) = -4, y'(2) = 14]$$

### ✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 37

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+5*y(t)=50*t-100,y(2) = -4, D(y)(2) = 14],y(t), singsol)
```

$$y(t) = 2 \sin(2t) \cos(4) e^{-t+2} - 2 \cos(2t) \sin(4) e^{-t+2} + 10t - 24$$

### ✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 25

```
DSolve[{y''[t]+2*y'[t]+5*y[t]==50*t-100,{y[2]==-4,y'[2]==14}],y[t],t,IncludeSingularSolution]
```

$$y(t) \rightarrow 10t - 2e^{2-t} \sin(4 - 2t) - 24$$

## 6.15 problem 15

Internal problem ID [5693]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.2, page 216

**Problem number:** 15.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' + 3y' - 4y = 6e^{2t-3}$$

With initial conditions

$$\left[ y\left(\frac{3}{2}\right) = 4, y'\left(\frac{3}{2}\right) = 5 \right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)-4*y(t)=6*exp(2*t-3),y(3/2) = 4, D(y)(3/2) = 5],y(t), s)
```

$$y(t) = 3e^{t-\frac{3}{2}} + e^{2t-3}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 22

```
DSolve[{y''[t]+3*y'[t]-4*y[t]==6*Exp[2*t-3],{y[15/10]==4,y'[15/10]==5}],y[t],t,IncludeSingular
```

$$y(t) \rightarrow 3e^{t-\frac{3}{2}} + e^{2t-3}$$

## **7 Chapter 6. Laplace Transforms. Problem set 6.3, page 224**

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## 7.1 problem 18

Internal problem ID [5694]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.3, page 224

**Problem number:** 18.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _missing_x]`

$$9y'' - 6y' + y = 0$$

With initial conditions

$$[y(0) = 3, y'(0) = 1]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve([9*diff(y(t),t$2)-6*diff(y(t),t)+y(t)=0,y(0) = 3, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = 3e^{\frac{t}{3}}$$

### ✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 14

```
DSolve[{9*y''[t]-6*y'[t]+y[t]==0,{y[0]==3,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow 3e^{t/3}$$

## 7.2 problem 19

Internal problem ID [5695]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.3, page 224

**Problem number:** 19.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + 6y' + 8y = e^{-3t} - e^{-5t}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

### ✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve([diff(y(t),t$2)+6*diff(y(t),t)+8*y(t)=exp(-3*t)-exp(-5*t),y(0) = 0, D(y)(0) = 0],y(t))
```

$$y(t) = -\frac{(e^{-3t} - 3e^{-2t} + 3e^{-t} - 1)e^{-2t}}{3}$$

### ✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 21

```
DSolve[{y''[t]+6*y'[t]+8*y[t]==Exp[-3*t]-Exp[-5*t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingular
```

$$y(t) \rightarrow \frac{1}{3}e^{-5t}(e^t - 1)^3$$

### 7.3 problem 20

Internal problem ID [5696]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.3, page 224

**Problem number:** 20.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _with_linear_symmetries]`

$$y'' + 10y' + 24y = 144t^2$$

With initial conditions

$$\left[ y(0) = \frac{19}{12}, y'(0) = -5 \right]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(y(t),t$2)+10*diff(y(t),t)+24*y(t)=144*t^2,y(0) = 19/12, D(y)(0) = -5],y(t), sin
```

$$y(t) = 6t^2 - 5t + \frac{19}{12}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 17

```
DSolve[{y''[t]+10*y'[t]+24*y[t]==144*t^2,{y[0]==19/12,y'[0]==-5}],y[t],t,IncludeSingularSolu
```

$$y(t) \rightarrow 6t^2 - 5t + \frac{19}{12}$$

## 7.4 problem 21

Internal problem ID [5697]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.3, page 224

**Problem number:** 21.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + 9y = \begin{cases} 8 \sin(t) & 0 < t < \pi \\ 0 & \pi < t \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = 4]$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 35

```
dsolve([diff(y(t),t$2)+9*y(t)=piecewise(0<t and t<Pi,8*sin(t),t>Pi,0),y(0) = 0, D(y)(0) = 4])
```

$$y(t) = 4 \left( \begin{cases} \frac{\sin(3t)}{3} & t < 0 \\ \sin(t) \cos(t)^2 & t < \pi \\ \frac{\sin(3t)}{3} & \pi \leq t \end{cases} \right)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 30

```
DSolve[{y''[t] + 9*y[t] == Piecewise[{{{8*Sin[t], 0 < t < Pi}, {0, t > Pi}}}], {y[0] == 0, y'[0] == 4}}, y[t], t, In]
```

$$y(t) \rightarrow \begin{cases} \frac{4}{3} \sin(3t) & t > \pi \vee t \leq 0 \\ \sin(t) + \sin(3t) & \text{True} \end{cases}$$

## 7.5 problem 22

Internal problem ID [5698]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.3, page 224

**Problem number:** 22.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + 3y' + 2y = \begin{cases} 4t & 0 < t < 1 \\ 8 & 1 < t \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 62

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=piecewise(0<t and t<1,4*t,t>1,8),y(0) = 0, D(y)(0) = 0])
```

$$y(t) = \begin{cases} 0 & t < 0 \\ 2t - e^{-2t} - 3 + 4e^{-t} & t < 1 \\ 3e^{-2t+2} - 8e^{-t+1} - e^{-2t} + 4 + 4e^{-t} & 1 \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 70

```
DSolve[{y''[t] + 3*y'[t] + 2*y[t] == Piecewise[{{4*t, 0 < t < 1}, {8, t > 1}}], {y[0] == 0, y'[0] == 0}}, y[t], t, I]
```

$$y(t) \rightarrow \begin{cases} 0 & t \leq 0 \\ 2t - e^{-2t} + 4e^{-t} - 3 & 0 < t \leq 1 \\ e^{-2t}(-1 + 3e^2 + 4e^t + 4e^{2t} - 8e^{t+1}) & \text{True} \end{cases}$$

## 7.6 problem 23

Internal problem ID [5699]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.3, page 224

**Problem number:** 23.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + y' - 2y = \begin{cases} 3\sin(t) - \cos(t) & 0 < t < 2\pi \\ 3\sin(2t) - \cos(2t) & 2\pi < t \end{cases}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 48

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-2*y(t)=piecewise(0<t and t<2*Pi,3*sin(t)-cos(t),t>2*Pi,3*sin(t)+cos(t))],y(t))
```

$$y(t) = \begin{cases} \frac{(2e^{3t}+1)e^{-2t}}{3} & t < 0 \\ e^t - \sin(t) & t < 2\pi \\ e^t - \sin(t)\cos(t) & 2\pi \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 55

```
DSolve[{y''[t] + y'[t] - 2*y[t] == Piecewise[{{3*Sin[t] - Cos[t], 0 < t < 2*Pi}, {3*Sin[2*t] - Cos[2*t], t > 2*Pi}}]}]
```

$$y(t) \rightarrow \begin{cases} \frac{e^{-2t}}{3} + \frac{2e^t}{3} & t \leq 0 \\ e^t - \sin(t) & 0 < t \leq 2\pi \\ e^t - \cos(t) \sin(t) & \text{True} \end{cases}$$

## 7.7 problem 24

Internal problem ID [5700]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.3, page 224

**Problem number:** 24.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + 3y' + 2y = \begin{cases} 1 & 0 < t < 1 \\ 0 & 1 < t \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 56

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=piecewise(0<t and t<1,1,t>1,0),y(0) = 0, D(y)(0)
```

$$y(t) = \frac{\begin{cases} 0 & t < 0 \\ 1 - 2e^{-t} + e^{-2t} & t < 1 \\ 2e^{-t+1} - e^{-2t+2} - 2e^{-t} + e^{-2t} & 1 \leq t \end{cases}}{2}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 57

```
DSolve[{y''[t] + 3*y'[t] + 2*y[t] == Piecewise[{{1, 0 < t < 1}, {0, t > 1}}], {y[0] == 0, y'[0] == 0}}, y[t], t, Inc]
```

$$y(t) \rightarrow \begin{cases} 0 & t \leq 0 \\ \frac{1}{2}e^{-2t}(-1 + e^t)^2 & 0 < t \leq 1 \\ \frac{1}{2}(-1 + e)e^{-2t}(-1 - e + 2e^t) & \text{True} \end{cases}$$

## 7.8 problem 25

Internal problem ID [5701]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.3, page 224

**Problem number:** 25.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \begin{cases} t & 0 < t < 1 \\ 0 & 1 < t \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 34

```
dsolve([diff(y(t),t$2)+y(t)=piecewise(0<t and t<1,t,t>1,0),y(0) = 0, D(y)(0) = 0],y(t), singu
```

$$y(t) = \begin{cases} 0 & t < 0 \\ -\sin(t) + t & t < 1 \\ -\sin(t) + \cos(t-1) + \sin(t-1) & 1 \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 44

```
DSolve[{y''[t]+y[t]==Piecewise[{{t,0<t<1},{0,t>1}}],{y[0]==0,y'[0]==0}],y[t],t,IncludeSingu
```

$$y(t) \rightarrow \begin{cases} t - \sin(t) & 0 < t \leq 1 \\ \cos(1-t) - \sin(1-t) - \sin(t) & t > 1 \end{cases}$$

## 7.9 problem 26

Internal problem ID [5702]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.3, page 224

**Problem number:** 26.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + 2y' + 5y = \begin{cases} 10 \sin(t) & 0 < t < 2\pi \\ 0 & 2\pi < t \end{cases}$$

With initial conditions

$$[y(\pi) = 1, y'(\pi) = 2e^{-\pi} - 2]$$

✓ Solution by Maple

Time used: 0.594 (sec). Leaf size: 102

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+5*y(t)=piecewise(0<t and t<2*Pi,10*sin(t),t>2*Pi,0),y(
```

$$y(t) = \begin{cases} -\frac{e^{-t}(2\cos(2t)-3\sin(2t))}{2} & t < 0 \\ 2\cos(t)e^{-t}\sin(t) - \cos(t) + 2\sin(t) & t < 2\pi \\ 2\cos(t)e^{-t}\sin(t) - 2\cos(t)^2 e^{2\pi-t} + \sin(t)\cos(t)e^{2\pi-t} + e^{2\pi-t} & 2\pi \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 94

```
DSolve[{y''[t] + 2*y'[t] + 5*y[t] == Piecewise[{{10*Sin[t], 0 < t < 2*Pi}, {0, t > 2*Pi}}]}, {y[Pi] == 1, y'[Pi]
```

$$y(t) \rightarrow \begin{cases} \frac{1}{2}e^{-t}(3\sin(2t) - 2\cos(2t)) & t \leq 0 \\ -\cos(t) + 2\sin(t) + e^{-t}\sin(2t) & 0 < t \leq 2\pi \\ \frac{1}{2}e^{-t}((2 + e^{2\pi})\sin(2t) - 2e^{2\pi}\cos(2t)) & \text{True} \end{cases}$$

## 7.10 problem 27

Internal problem ID [5703]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.3, page 224

**Problem number:** 27.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y = \begin{cases} 8t^2 & 0 < t < 5 \\ 0 & 5 < t \end{cases}$$

With initial conditions

$$[y(1) = 1 + \cos(2), y'(1) = 4 - 2\sin(2)]$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 47

```
dsolve([diff(y(t),t$2)+4*y(t)=piecewise(0<t and t<5,8*t^2,t>5,0),y(1) = 1+cos(2), D(y)(1) =
```

$$y(t) = \begin{cases} 0 & t < 0 \\ 2t^2 - 1 + \cos(2t) & t < 5 \\ 49\cos(2t - 10) + 10\sin(2t - 10) + \cos(2t) & 5 \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 51

```
DSolve[{y''[t]+4*y[t]==Piecewise[{{8*t^2,0<t<5},{0,t>5}}],y[1]==1+Cos[2],y'[1]==4-2*Sin[2]}]
```

$$y(t) \rightarrow \begin{cases} 2t^2 + \cos(2t) - 1 & 0 < t \leq 5 \\ 49\cos(2(t - 5)) + \cos(2t) - 10\sin(10 - 2t) & t > 5 \end{cases}$$

## **8 Chapter 6. Laplace Transforms. Problem set 6.4, page 230**

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## 8.1 problem 3

Internal problem ID [5704]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.4, page 230

**Problem number:** 3.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + 4y = \delta(-\pi + t)$$

With initial conditions

$$[y(0) = 8, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)+4*y(t)=Dirac(t-Pi),y(0) = 8, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = 8 \cos(2t) + \frac{\text{Heaviside}(-\pi + t) \sin(2t)}{2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 23

```
DSolve[{y''[t]+4*y[t]==DiracDelta[t-Pi],{y[0]==8,y'[0]==0}],y[t],t,IncludeSingularSolutions
```

$$y(t) \rightarrow \theta(t - \pi) \sin(t) \cos(t) + 8 \cos(2t)$$

## 8.2 problem 4

Internal problem ID [5705]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.4, page 230

**Problem number:** 4.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + 16y = 4(\delta(t - 3\pi))$$

With initial conditions

$$[y(0) = 2, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([diff(y(t),t$2)+16*y(t)=4*Dirac(t-3*Pi),y(0) = 2, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = 2 \cos(4t) + \text{Heaviside}(t - 3\pi) \sin(4t)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 23

```
DSolve[{y''[t]+16*y[t]==4*DiracDelta[t-3*Pi],{y[0]==2,y'[0]==0}},y[t],t,IncludeSingularSolut
```

$$y(t) \rightarrow \theta(t - 3\pi) \sin(4t) + 2 \cos(4t)$$

### 8.3 problem 5

Internal problem ID [5706]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.4, page 230

**Problem number:** 5.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + y = \delta(-\pi + t) - (\delta(-2\pi + t))$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve([diff(y(t),t$2)+y(t)=Dirac(t-Pi)-Dirac(t-2*Pi),y(0) = 0, D(y)(0) = 1],y(t), singsol=a)
```

$$y(t) = -\sin(t) (\text{Heaviside}(-\pi + t) + \text{Heaviside}(-2\pi + t) - 1)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 23

```
DSolve[{y''[t] + y[t] == DiracDelta[t - Pi] - DiracDelta[t - 2*Pi], {y[0] == 0, y'[0] == 1}}, y[t], t, IncludesS]
```

$$y(t) \rightarrow -((\theta(t - 2\pi) + \theta(t - \pi) - 1) \sin(t))$$

## 8.4 problem 6

Internal problem ID [5707]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.4, page 230

**Problem number:** 6.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + 4y' + 5y = \delta(t - 1)$$

With initial conditions

$$[y(0) = 0, y'(0) = 3]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 28

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+5*y(t)=Dirac(t-1),y(0) = 0, D(y)(0) = 3],y(t), singsol)
```

$$y(t) = 3e^{-2t} \sin(t) + \text{Heaviside}(t - 1)e^{-2t+2} \sin(t - 1)$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 31

```
DSolve[{y''[t]+4*y'[t]+5*y[t]==DiracDelta[t-1],{y[0]==0,y'[0]==3}},y[t],t,IncludeSingularSol]
```

$$y(t) \rightarrow e^{-2t} (3 \sin(t) - e^2 \theta(t - 1) \sin(1 - t))$$

## 8.5 problem 7

Internal problem ID [5708]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.4, page 230

**Problem number:** 7.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$4y'' + 24y' + 37y = 17e^{-t} + \delta\left(t - \frac{1}{2}\right)$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

### ✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve([4*diff(y(t),t$2)+24*diff(y(t),t)+37*y(t)=17*exp(-t)+Dirac(t-1/2),y(0) = 1, D(y)(0) = 1])
```

$$y(t) = \frac{e^{-3t} \left( \text{Heaviside}\left(t - \frac{1}{2}\right) e^{\frac{3}{2}} \sin\left(-\frac{1}{4} + \frac{t}{2}\right) + 2e^{2t} + 8 \sin\left(\frac{t}{2}\right) \right)}{2}$$

### ✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 63

```
DSolve[{4*y''[t]+24*y'[t]+37*y[t]==17*Exp[-t]+DiracDelta[t-1/2],{y[0]==1,y'[0]==1}},y[t],t]
```

$$y(t) \rightarrow \frac{1}{84} e^{-9t/2} (7e^{3/4} (e^{3t} - e^{3/2}) \theta(2t - 1) + 12(-7e^{3t} + 17e^{7t/2} - 3))$$

## 8.6 problem 8

Internal problem ID [5709]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.4, page 230

**Problem number:** 8.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + 3y' + 2y = 10 \sin(t) + 10(\delta(t - 1))$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=10*(sin(t)+Dirac(t-1)),y(0) = 1, D(y)(0) = -1],
```

$$\begin{aligned} y(t) = & -10 \text{Heaviside}(t - 1) e^{-2t+2} + 10 \text{Heaviside}(t - 1) e^{-t+1} \\ & - 3 \cos(t) + \sin(t) - 2 e^{-2t} + 6 e^{-t} \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.165 (sec). Leaf size: 46

```
DSolve[{y''[t]+3*y'[t]+2*y[t]==10*(Sin[t]+DiracDelta[t-1]),{y[0]==1,y'[0]==-1}},y[t],t,IncludeUnitary]
```

$$y(t) \rightarrow 10e^{1-2t}(e^t - e)\theta(t - 1) - 2e^{-2t} + 6e^{-t} + \sin(t) - 3\cos(t)$$

## 8.7 problem 9

Internal problem ID [5710]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.4, page 230

**Problem number:** 9.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + 4y' + 5y = (1 - \text{Heaviside}(-10 + t)) e^t - e^{10}(\delta(-10 + t))$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

### ✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 49

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+5*y(t)=(1-Heaviside(t-10))*exp(t)-exp(10)*Dirac(t-10),
```

$$y(t) = \frac{-e^{-2t}((-e^{30} \cos(t-10) + 7e^{30} \sin(t-10) + e^{3t}) \text{Heaviside}(t-10) + \cos(t) - 7 \sin(t) - e^{3t})}{10}$$

### ✓ Solution by Mathematica

Time used: 0.571 (sec). Leaf size: 94

```
DSolve[{y''[t] + 4*y'[t] + 5*y[t] == (1 - UnitStep[t-10])*Exp[t] - Exp[10]*DiracDelta[t-10], y[0]==0, y'
```

$$y(t) \rightarrow \frac{1}{10} e^{-2t} (10e^{30} \theta(t-10) \sin(10-t) + \theta(10-t) (e^{3t} + 3e^{30} \sin(10-t) - e^{30} \cos(10-t)) - 3e^{30} \sin(10-t) + 7 \sin(t) + e^{30} \cos(10-t) - \cos(t))$$

## 8.8 problem 10

Internal problem ID [5711]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.4, page 230

**Problem number:** 10.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 5y' + 6y = \delta\left(t - \frac{\pi}{2}\right) + \cos(t) \text{Heaviside}(-\pi + t)$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 79

```
dsolve([diff(y(t),t$2)+5*diff(y(t),t)+6*y(t)=Dirac(t-1/2*Pi)+Heaviside(t-Pi)*cos(t),y(0) = 0,y'(0) = 0])
```

$$\begin{aligned} y(t) = & -\text{Heaviside}\left(t - \frac{\pi}{2}\right) e^{-3t+\frac{3\pi}{2}} - \frac{3 \text{Heaviside}(-\pi + t) e^{-3t+3\pi}}{10} \\ & + \frac{2 \text{Heaviside}(-\pi + t) e^{-2t+2\pi}}{5} + \text{Heaviside}\left(t - \frac{\pi}{2}\right) e^{-2t+\pi} \\ & + \frac{\text{Heaviside}(-\pi + t) (\cos(t) + \sin(t))}{10} \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.511 (sec). Leaf size: 85

```
DSolve[{y''[t]+5*y'[t]+6*y[t]==DiracDelta[t-1/2*Pi]+UnitStep[t-Pi]*Cos[t],{y[0]==0,y'[0]==0}]
```

$$\begin{aligned} y(t) \rightarrow & \frac{1}{10} e^{-3t} ((\theta(\pi - t) - 1) (-4e^{t+2\pi} - e^{3t} \sin(t) - e^{3t} \cos(t) + 3e^{3\pi}) \\ & - 10e^\pi (e^{\pi/2} - e^t) \theta(2t - \pi)) \end{aligned}$$

## 8.9 problem 11

Internal problem ID [5712]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.4, page 230

**Problem number:** 11.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + 5y' + 6y = \text{Heaviside}(t - 1) + \delta(-2 + t)$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 68

```
dsolve([diff(y(t),t$2)+5*diff(y(t),t)+6*y(t)=Heaviside(t-1)+Dirac(t-2),y(0) = 0, D(y)(0) = 1])
```

$$\begin{aligned} y(t) = & e^{-2t} - e^{-3t} + \frac{\text{Heaviside}(t - 1)}{6} - \frac{\text{Heaviside}(t - 1)e^{-2t+2}}{2} \\ & + \text{Heaviside}(t - 2)e^{-2t+4} + \frac{\text{Heaviside}(t - 1)e^{-3t+3}}{3} - \text{Heaviside}(t - 2)e^{-3t+6} \end{aligned}$$

### ✓ Solution by Mathematica

Time used: 0.208 (sec). Leaf size: 80

```
DSolve[{y''[t]+5*y'[t]+6*y[t]==UnitStep[t-1]+DiracDelta[t-2],{y[0]==0,y'[0]==1}},y[t],t,Inclu
```

$$\begin{aligned} y(t) \rightarrow & \frac{1}{6}e^{-3t} \left( 6e^4(e^t - e^2) \theta(t - 2) - \left( (e^t + 2e)(e - e^t)^2 \theta(1 - t) \right) + 6e^t + e^{3t} - 3e^{t+2} \right. \\ & \left. + 2e^3 - 6 \right) \end{aligned}$$

## 8.10 problem 12

Internal problem ID [5713]

**Book:** ADVANCED ENGINEERING MATHEMATICS. ERWIN KREYSZIG, HERBERT KREYSZIG, EDWARD J. NORMINTON. 10th edition. John Wiley USA. 2011

**Section:** Chapter 6. Laplace Transforms. Problem set 6.4, page 230

**Problem number:** 12.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[_2nd_order, _linear, _nonhomogeneous]`

$$y'' + 2y' + 5y = 25t - 100(\delta(-\pi + t))$$

With initial conditions

$$[y(0) = -2, y'(0) = 5]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+5*y(t)=25*t-100*Dirac(t-Pi),y(0) = -2, D(y)(0) = 5],y(t))
```

$$y(t) = -50 \operatorname{Heaviside}(-\pi + t) \sin(2t) e^{\pi - t} + 5t - 2$$

✓ Solution by Mathematica

Time used: 0.271 (sec). Leaf size: 29

```
DSolve[{y''[t] + 2*y'[t] + 5*y[t] == 25*t - 100*DiracDelta[t - Pi], {y[0] == -2, y'[0] == 5}}, y[t], t, Includes
```

$$y(t) \rightarrow -50e^{\pi-t}\theta(t-\pi)\sin(2t) + 5t - 2$$